A TRANSLOG COST FUNCTION ANALYSIS OF INPUT SUBSTITUTION IN THE U.S. COPPER SMELTING INDUSTRY 1960 - 1991

by

Morris Michael Pitts Jr.

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As members of the Dissertation Committee, we certify that we have read the dissertation

prepared by Morris Michael Pitts Jr.

entitled A Translog Cost Function Analysis of Input Substitution in the U.S. Copper Smelting Industry 1960 – 1991

and recommended that it be accepted as fulfilling the dissertation requirement for the

Degree of Doctor of Philosophy

_______________________________________ Date: August 21, 1996
DeVerle Harris

_______________________________________ Date: August 21, 1996
Michael Rieber

_______________________________________ Date: August 21, 1996
Richard T. Newcomb

_______________________________________ Date: August 21, 1996
Satheesh V. Aradhyula

_______________________________________ Date: May 10, 2005
Mary M. Poulton

_______________________________________ Date: May 10, 2005
William G. Davenport

Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copies of the dissertation to the Graduate College.

I hereby certify that I have read this dissertation prepared under my direction and recommend that it be accepted as fulfilling the dissertation requirement.

_______________________________________ Date: May 10, 2005
Dissertation Director: DeVerle Harris

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SIGNED: ____
Morris Michael Pitts Jr.

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ABSTRACT

The copper smelting industry has under gone extreme change over the past three decades. These changes have reordered dramatically the demand for inputs and the way in which those inputs have been utilized. The stimulus for change has come from multiple sources, and chief among these stimuli has been the mandate to sharply curtail the atmospheric release of sulfur dioxide. Even though the total emissions were lower than those from steam generation of electricity and from the refinery and petro-chemical industry, the perceived local and regional impact of sulfur dioxide forced extreme changes in the utilization of fundamental inputs of capital, labor, energy and materials.

This study attempts to analyze these input use changes by modeling the industry as a translog cost function and by generating a number of associated elasticities. In addition to the four basic inputs, the model includes as control variables output, and other variables that represent pollution abatement and technical change.

The challenge of estimating a large model on a limited number of observations has delivered information that is more limited in scope than was originally desired. The proxy for technical change did not produce significant parameters and the pollution abatement proxy is limited in its participation in the results. The range of elasticities computed reveal a picture of an industry characterized by inelasticity, in general, labor and energy being part of the exceptions. The industry is found to be sensitive to output level in its degree of elasticity among inputs.

The translog model is found to be an effective tool for industry analysis. The promise of detailed analytical information may be even greater at the firm level where data are more accurate and the number of observations far greater.

CHAPTER 1

BACKGROUND AND INTRODUCTION

1.1 Introduction

Copper demand has grown in conjunction with expansion of the use of electricity, military demands, and later by the expanding of the infrastructure that supports the U.S. industrial base and the subsequent prosperity of the U.S. population. The development of the U.S. copper industry can be viewed as a series of responses to challenges against growth and profitable operation.

Early copper mines were developed on rich veins that yielded direct smelting ores. Depletion of the early deposits and the development of flotation methods led to the increase in production available from open pit porphyritic ores. Mining these ores required new milling and concentration techniques to produce a sustainable ore concentrate for smelter feed. Smelters of the basic reverberatory design increased in size but the base technology was held constant as purchased fuel was used to smelt ore, to concentrate, and to produce matte, and ultimately, refined copper products.

For many years the U.S. was largely self sufficient in copper production. The discovery and exploitation of major copper deposits in Africa and in South America provided metal for growing world consumption, but had relatively little impact on the domestic markets of the U.S. producers.

While the desired domestic ore grade declined and advances in mining and milling methods led to larger scale and more efficient extraction methodologies, operational costs were prevented from rising while production of copper was sustained or expanded.

With population growth and deepening concern about environmental degradation, the non-ferrous mining and smelting industry came under scrutiny for land, water, and air pollution concerns. Demands to meet air quality regulations were most severe, although the relative quantity of pollutants released to the atmosphere by the copper smelting industry was small by comparison to that from other industries. The demand for sulfur emission reduction increased, and the laws subsequently adopted resulted in serious technical problems for all smelting operations.

During the three decades from 1961 to 1991, the U.S. copper industry has been presented with some of the greatest challenges to its economic viability ever encountered. Any one of the combined environmental, energy, or labor difficulties encountered in this period of time presented a formidable problem. All three problems occurring in the same time frame created an era of considerable crisis.

While the U.S. copper industry struggled to survive the various domestic challenges, international lending institutions were placing low interest loans into lesser developed countries expressly for expansion of their copper production. Chile especially benefitted from these low interest loans expanding production while simultaneously inflating the peso. The inflated peso delivered additional competitive advantage in pricing upon international markets by reducing the equivalent cost of locally purchased inputs.

TABLE 1.1

Significant Events Affecting Copper Prices and Output

1959-1960........	Labor strike, 5 months; 17% U.S. consumption increase over 1958.
1961-1963........	Voluntary U.S. production curtailments.
1965-1973........	Vietnam conflict; period of extraordinary consumption growth; releases from stockpile; Coinage Act of 1965; price controls 1971-1973; U.S. producers maintain two-tier pricing relative to LME price; Chilean strike; African supply problems; nationalizations of U.S.-owned foreign properties. Demonetization of gold and silver.
1967-1968........	Labor strike, 9 months; formation of CIPEC.
1969	Congress passes National Environmental Protection Act (NEPA).
1970	Creation of the Environmental Protection Agency and passage of the clean air act.
1973-1974........	End of price controls; OPEC oil embargo; last stockpile release, 229,000 tons; fixed exchange rates abandoned.
1975-1977........	Economic recession; record copper inventories; amendment to clean air act; national smelter order extends deadlines to 1988.
1979-1980........	Record copper consumption; energy price increases; labor strike, 5 months.
1981.................	Change from copper to zinc-based U.S. coins.
1982-1984........	Economic recession; inventory buildup; strong U.S. currency impacts competitiveness.
1984-1986........	Work down of high copper inventories; cutback in capacity at U.S. mines; cost-cutting and efficiency moves.
1987-1989........	Historically low inventories; increased world consumption and U.S. production.

Modified after J. Jolly “Nonferrous Metal Prices in the United States through 1988", U.S. Bureau of Mines, Washington, D.C., 1990.

The U.S. copper industry was required to make changes in production procedures to continue operation. The often quoted productivity factor of tons of copper per man hour is not adequate to describe the realignment of factors of production that took place nor to describe the effect of challenges in technology and scale economies. The need for a national environmental legal framework and a stronger Federal presence became more and more evident as environmental problems grew and public opinion became more sensitive. State legislation was non-uniform and became a potential source of competitive disadvantages among states. As a consequence the states were often reluctant to initiate remediation initiatives. Reacting to a strong environmental movement, Congress passed the National Environmental Protection Act (NEPA) in 1969.

The institutional basis for enforcement of the environmental laws was created in 1970. There were several governmental agencies already in charge of implementing and enforcing the several diverse laws that were intended to protect the environment. The decision was taken to create a new, separate agency called the Environmental Protection Agency (EPA). The rationale behind the creation of the EPA was the need for an independent institution with the necessary expertise to formulate environmental regulations in accordance with the mandate of the Congress and to oversee their implementation and enforcement.

The environmental impacts of mining include land, water and air pollution. Land disturbance is produced by exploration, development, and mining activities. The pollution of surface and ground water results from the release of metals and toxic chemicals.

The pollution of air comes about as a result of the emission of sulfur dioxide, lead, or other components of flu gas during smelting operations.

Much of the original regulatory effort was well intended trial and error. There were no existing proven technologies to accomplish the goals of restricting emissions. The resultant activities directed toward meeting the mandates were very costly in failed retrofit attempts and ultimately in subsequent smelter shutdowns and closures.

The impact on local economies in mining areas and on general employment was large. Also striking was the effect on input utilization in smelting activities such as fuel, especially on labor use and the impact on the relationships with organized labor, and on the capital utilization due to smelter closings and retrofitting, and ultimately to the building of new facilities.

1.2 Objective

The objective of this study is to estimate the cost function of the U.S. copper smelting industry. A translog cost function is specified to facilitate a more complete extraction of economic parameters and information. The translog cost function allows one to examine the relationships between inputs and to quantify the effect of output pollution abatement and technical change on the distribution and substitution of input factors. Successfully applied, this analysis may provide insight into the economic effects of the major stimuli to the industry during the years 1960 to 1991. This information may then be compared with information made available in other studies of U.S. manufacturing.

1.2.1 Setting the Stage for Further Work

A secondary objective is to set the stage for further work by documenting available information in terms of data for computation and revealing modifications in methodology. Hopefully findings of this study will serve to direct activity more effectively toward analysis at the firm level.

CHAPTER 2

TRANSLOG ECONOMETRIC COST MODELS

2.1 Production Theory and Historical Perspective

During the 1970's, the neoclassical cost function gained renewed popularity as a tool for estimating the structure of production. This surge of popularity is attributed to the wide-spread application of duality theory to economic analysis and the concomitant development of flexible functional forms.

Typical duality theorems state that (1) a concave production function yields a cost function homogeneous of degree one in input prices, given specified regularity conditions;[1] (2) a cost function homogeneous of degree one in input prices yields a concave production function, given specified regularity conditions; and (3) the cost function derived from a particular production function will in turn yield that production function.

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[1] The production function is assumed to be a single-valued, continuous, twice differentiable function. The production function is assumed to be increasing its inputs, i.e., the partial derivative with respect to each argument in the production function is assumed positive. Further, it is assumed that the production function is a regular strictly quasi-concave function when cost is minimized.

2.2 Early Work

Empirical research on estimating cost and production relationships has a long history in the economics profession. Work done in agricultural economics by Earl Heady and associates led to the analysis of more general forms of cost and production functions. They experimented with Taylor's series expansions as polynomial approximations to unknown algebraic forms in the late 1940s. In their 1961 book, Agricultural Production Functions, Heady and John Dillon explicitly considered a second-degree polynomial production function expressed in terms of logarithms. It was this same functional form that Christensen, Jorgenson, and Lau employed in their 1971 paper, "Conjugate Duality and the Transcendental Logarithmic Production Function", which they called translog.

The estimation of a production function necessitates that input quantities appear as right-hand variables. At the firm level of analysis, it is plausible that input quantities are endogenous. If this is the case, then the explanatory variables and the disturbance term in the production function are correlated, and ordinary least-squares estimation of the production equation would lead to simultaneous-equation bias. The result is parameter estimates that are biased and inconsistent. In cases where the data used are at the firm level, it is more credible to assume that input prices, rather than input quantities, are exogenous. This suggests that endogeneity is less of a problem in the estimation of a translog cost function than it is in the estimation of a translog production function.

2.3 Production Duality Theory and Flexible Functional Forms

Daniel McFadden developed the theory and applications of duality in production. In turn, it was a student of McFadden's at the University of California, Berkeley, W. Erwin Diewert, who solved the problem of developing flexible functional forms with three or more inputs. Diewert's Ph.D. dissertation at Berkeley and his subsequent 1971 article, "An Application of the Shephard Duality Theorem: A Generalized Linear Production Function", made functional forms widely available that placed no *a priori* restrictions on substitution elasticities, yet were consistent with the constraints typically assumed in economic theory.

CHAPTER 3

MODELING THE COPPER SMELTER COST FUNCTION

3.1 Translog Cost Function

The output of the copper smelting industry is produced using capital, labor, energy, ore or concentrate, and other materials and inputs. Pollution abatement and technology enter the production function in that they can alter total output by increasing or decreasing the marginal productivity of the inputs. If weak function separability is assumed to eliminate the minor inputs, the net output function is:

$$Y = f(k,p,s,e,m,A,T)$$

Where:
Y = Output
k = Capital
p = Production Labor
s = Salaried Labor
e = Energy
m = Materials (Ore)
A = Pollution Abatement
T = Technical Change

As discussed in the introduction, the purpose of this study is to determine the elasticities of substitution among the inputs and, if possible, gauge the effect of pollution abatement and technical change on input substitution. While it is possible to estimate these directly from the production function, it is normally simpler and expressive of economic theory to estimate these using the cost function which is dual to this production function.[2] The production function may be specified as:

$$TC = C(P_k, P_p, P_s, P_e, P_m, Y, A, T)$$

Where: P_k = Price of capital.
P_p = Price of hourly labor.
P_s = Price of salaried labor.
P_e = Price of energy.
P_m = Price of ore (concentrate).
Y = Copper production.
A = Pollution abatement
T = Technical change

3.1

It is assumed that the cost function is a positive continuous function in output and prices which approaches infinity as output approaches infinity and that the cost function is concave and linearly homogeneous in prices.[3]

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[2] Every production function would generate a "dual" production function from which it is possible to ascertain the properties of the production function.

[3] The concavity consumption guarantees the globality and uniqueness of any extremum. The assumption of linear homogeneity that only, if all prices were doubled, total cost would double.

Using the principles of duality[4], if the cost function is minimized with respect to the input prices, then the existence of the dual production function is guaranteed; and, while its parametric form is unknown, its properties may be ascertained from parameters of the cost function.

For purposes of estimation it is necessary to specify the cost function in parametric form. The transcendental logarithmic (translog) form used for empirical estimation of the cost function below was specified in translog form by Christiansen, Jorgenson, and Lau (1973) with five input variables (W_k, W_p, W_s, W_e, and W_m) and three control variables (A, Y, and T):

Where i,j = k,p,s,e,m.

$$\begin{gathered} InC = a_o + a_Y \ln Y + a_A \ln A + \Sigma_i \beta_i \ln W_i + y_T \ln T + 1/2\, a_{YY} (\ln Y)^2 \\ + a_{YA} \ln Y \ln A + \Sigma_i \delta_{Yi} \ln Y \ln W_i + \rho_{YT} \ln Y \ln T \\ + 1/2\, a_{AA} (\ln A)^2 + \Sigma_i \delta_{Ai} \ln A \ln W_i + \rho_{AT} \ln A \ln T \\ + 1/2\, \Sigma_i \Sigma_j \beta_{ij} \ln W_i \ln W_j + \Sigma_i \Phi_{iT} \ln W_i \ln T + 1/2\, y_{TT} (\ln T)^2, \end{gathered} \tag{3.2}$$

---

[4] Every production function would generate a "dual" cost function from which it is possible to ascertain the properties of the production function.

The function is a general, log-quadratic local approximation to an arbitrary cost function and entails no *a priori* restrictions on values of elasticities of substitution nor does it dictate baric constancy. Since the translog cost function approximates a twice differentiable production surface, the cross partial derivatives of the cost function imply the symmetry condition of:

$$\delta_{ij} = \delta_{ji} \tag{3.3}$$

Where i,j = k,p,s,e,m

The function is assumed to be monotonically increasing, linearly homogeneous, and concave in input prices. Linear homogeneity in input prices implies the parametric restrictions:

$$\Sigma\beta_i = 1,\ \Sigma\delta_{ij} = \Sigma\delta_{ji} = \Sigma\delta_{iY} = \Sigma\delta_{iA} = \Sigma\delta_{iT} = 0 \tag{3.4}$$

The relationships expressed in (3.4) imply that the ordinary least squares residuals will sum to zero at each observation, i.e.:

$$\Sigma\ \Phi_n = 0 \textit{ for } n = 1.....5. \tag{3.5}$$

Hence only four of the five share equations will be linearly independent and the residual covariance matrix will be singular. If the residual in the first share is positive, the share is over predicted, so that at least one other share must be under predicted; i.e. at least one other residual must be negative. The presence of singularity among the share equations makes it mechanically impossible to compute ordinary least squares estimates, i.e. the estimating procedure simply breaks down for mathematical reasons. A more complete discussion of these features is included in Chapter 4 concerning estimation of the model.

The translog functional form is a second degree polynomial, meaning that there are squared terms as well as cross-product terms between each of the variables. Inspection of the general form shows precisely these terms. The inclusion of second degree terms in the cost function allows for flexibility in the specification of the cost function. The second order approximation is generally the limiting development of the translog equation to be estimated because of the exponentially increasing complexity of models as they move to third or fourth order form.

Since the true form of the production function underlying the cost model is unknown, one would like a model specification which is general enough to allow the model, through the estimation procedure, to determine which terms should appropriately be included and which should not be included in the estimated cost function. This selection is made by examining the asymptotic standard errors of the estimated parameters to determine the level of significance of the respective terms of the equation.

3.2 Share Equations

The cost function alone could be estimated by ordinary least squares. However, if the cost function is estimated directly, the resulting information implies only an engineering relationship of cost to explanatory variables under some arbitrary condition. On the other hand, if the cost function is to be interpreted as the dual to the overlying production function, then the cost function must be consistent with the cost minimizing behavior for the firm. To obtain the cost minima associated with the data set requires the introduction of cost shares.

Assuming perfect competition in the input markets, input prices are treated as exogenous. Given the level of output, cost minimizing input demand functions are derived by logarithmically differentiating the cost function with respect to each of the input prices. It can be shown that the partial derivative of a cost function with respect to input prices gives the cost minimizing values for the inputs (Henderson and Quandt, 1980):

$$\delta C/\delta W_i = X_i \qquad 3.6$$

for i = k,p,s,e,m

and, given X_i from (3.6), cost share, S_i, of the i^{th} factor is:

$$S_i = \frac{W_i X_i}{C} = \frac{W_i}{C} \frac{\delta C}{\delta W_i}$$

where $\frac{W_i}{C} \frac{\delta C}{\delta W_i}$ = the elasticity of cost with regard to W_i

Applying Shephard's Lemma, the cost share equations for the model are obtained in the form:

$$S_i = \frac{\delta C}{\delta W_i}\left(\frac{W_i}{C}\right) = \frac{d lnC}{d\, lnW_i} = \beta_i + \delta_{Yi}\, lnY + \delta_{Ai}\, lnA + \Sigma_j\, \beta_{ij}\, lnW_j + \gamma_{iT}\, lnT$$

3.7

Clearly $\underset{I}{\Sigma} S_i = 1$.

In addition, a disturbance term, θ_n for n = 1,....,5, is added to each share equation to reflect errors in optimization. Because $\Sigma_I S_i = 1$ and input prices appear in each of the equations, the simple arithmetic of equation by equation ordinary least squares estimation will yield parameter estimates that will always obey the "adding up" conditions as shown below:

$$\sigma_i = 1, \mathit{for}\ i = k, p, s, e, m$$

3.8

$$\sigma\ \Phi_{ij} = 0, \mathit{for}\ j = k,p,s,e,m\ \mathit{for\ all}\ i$$

3.9

One share equation is redundant and may be dropped. Maximum likelihood estimation procedures are invariant as to which equation is dropped. The coefficients of the redundant equation are estimated using the homogeneity relationships in (3.4). These assumptions assure that $\Sigma\Phi_n = 0$; n = 1, ..., 5.

3.3 Cost Elasticities

Differentiating logarithmically the cost function with respect to Y, A, T yields cost elasticities for output, abatement, and technical change.

$$\in_{CY} \equiv \frac{dC}{dY}\left(\frac{Y}{C}\right) = \frac{dlnC}{d\ln Y} = a_Y + a_{YY}\, lnY + a_{YA}\, lnA + \Sigma_i\, \delta_{Yi}\, lnW_i + \rho_{YT}\, lnT,$$

3.10

$$\in_{CT} \equiv \frac{dC}{dT}\left(\frac{T}{C}\right) = \frac{dlnC}{dlnT} = \alpha_T + \alpha_{TT}\, lnT + \alpha_{YT}\, lnY + \Sigma_i\, \delta_{iT}\, lnW_i + \rho_{AT}\, lnA,$$

3.11

$$\in_{CA} \equiv \frac{dC}{dA}\left(\frac{A}{C}\right) = \frac{dlnC}{d\ln Y} = a_A + a_{YA}\, lnY + a_{AA}\, lnA + \Sigma_i\, \delta_{Ai}\, lnW_i + \rho_{AT}\, lnT,$$

3.12

The value of $\in_{CY}$ measures the degree of scale economies. $\in_{CY} \succ 1$ implies the existence of economies of scale; $\in_{CY} = 1$ implies constant returns to scale; and $\in_{CY} \prec 1$ implies diseconomies of scale.

3.4 Allen Elasticities of Substitution

The AES have become the most popular method of expressing relationships between inputs. The AES communicate by sign the complimentarily (-) or substitute (+) nature of the input relationship. Moreover, the absolute value of the AES indicates relative elasticity where the interdependence relationship is seen as: elastic (>1), unitary elastic (=1), or inelastic (<1). The magnitude of the absolute value infers semi-quantitatively an approximation of the marginal rate of substitution.

Allen elasticities are functions of cost shares (S_i) and parameters of the translog cost function: own price elasticities, σ_{ii}, are computed as follows:

$$\sigma_{ii} = \frac{\gamma_{ii} + S_i^2 - S_i}{S_i^2}$$

3.13

For cross price elasticities:

$$\sigma_{ij} = \frac{\gamma_{ij} + S_i S_j}{S_i S_j}$$

3.14

for i = k,p,s,e,m
i ≠j

3.5 Compensated Price Elasticities of Input Demand

The own price and cross price compensated elasticities of input demand with the output held constant are obtained from the share equations as:

$$\eta_{ii} \equiv \frac{dln\, X_i}{dln\, W_i} = \frac{1}{S_i}(\beta_{ii} + S_i(S_i - 1)),$$

3.15

$$\eta_{ij} \equiv \frac{dln\,X_i}{dln\,W_j} = \frac{1}{S_i}(\beta_{ij} + S_i S_j).(i \neq j)$$

3.16

Compensated price elasticities reflect the rate of technical substitution along an isoquant or fixed level of output of the production function.

3.6 Uncompensated Price Elasticities

In contrast to the compensated price elasticities (3.9) and (3.10), the uncompensated price elasticities allow for changes in the output level, which are obtained as:

$$n_{ij}^{u} = n_{ij} + n_{iY} \in_{CY}^{-1}.$$

3.17

Uncompensated Price Elasticities, which reflect substitution between inputs as output varies, are commonly different not only in magnitude of absolute value but also in sign, reflecting the very different considerations of input substitution under varying rather than fixed output conditions.

3.7 Elasticities of Input Demand

The elasticities of input demand with respect to output (Y), abatement (A), and technical change (T) are given by:

$$\eta_{iY} \equiv \frac{dlnX_i}{dlnY} = \frac{\delta_{Yi}}{S_i} + \in_{CY} ,$$

3.18

$$\eta_{iT} \equiv \frac{dlnX_i}{dlnT} = \frac{\theta\,\delta_{Yi}}{S_i} + \in_{CT} ,$$

3.19

$$\eta_{iA} \equiv \frac{dlnX_i}{dlnA} = \frac{\delta_{Ai}}{S_i} + \in_{CA} ,$$

3.20

The cost elasticity of a variable can be shown to be the weighted sum of the input demand elasticities given in (3.11)-(3.14), with the weight given by the cost shares. For example, $\in_{CY} = \Sigma_i S_i \eta_{IY}$.

CHAPTER 4

TRANSLOG MODEL ESTIMATION

4.1 Econometric Issues

In order to model cost minimizing economic behavior it is necessary to include additional estimating equations which employ economic theory and support the behavioral assumptions of cost minimization. This information is included in the model through the various share equations. The model is estimated using a system of five equations (the cost function and four share equations) which include the restrictions previously described as components of the structural model.

The cost equation and the input share equations are connected, not because they interact, but because the model consists of a series of endogenous variables (cost and input shares) which bear a close conceptional relationship to each other. Consequently, the error terms of the equations are related. Correlation of error terms arises because of the cross-equation restrictions which are imposed; for example input cost shares sum to one. Efficient estimation is achieved using a systems method of estimation by means of nonlinear multi-variate regression (seemingly unrelated regression) developed by Zellner.[5] Importantly, estimates derived through this procedure are invariant to the choice of the omitted equation. The Time Series Processor (TSP version 4.3) software is used in the

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[5] See, A. Zellner {1962}.

empirical estimation portion of this research. Maximum likelihood is the estimation procedure used when programming non-linear system equations with TSP.

4.2 Econometric Problems in Estimating the Translog Cost Model

Following ordinary least squares computation the residuals across equations will sum to zero at each observation. Hence only four of the five share equations will be linearly independent, and the residual covariance matrix will be singular. If the residual in the first share is positive, the share is over predicted so that at least one other share must be under predicted; i.e. at least one other residual must be negative. The presence of singularity among the share equations makes it mechanically impossible to compute ordinary least squares estimates; i.e., the estimating procedures simply break down for mathematical reasons just as if one tried to divide by zero. Another problem that arises with equation by equation ordinary least squares estimation is that the symmetry restriction of $\delta_{ij} = \delta_{ji}$ can not be imposed.

Historically, researchers who wanted to employ ordinary least squares but also wanted to deal with problems of cross-equation symmetry constraints have done so by arbitrarily deleting one of the linearly dependent share equations, and then estimating the modified remaining equations. This procedure allowed for the symmetry conditions to be imposed. However, there are at least two problems with this estimation procedure. First, the parameter estimates and standard errors lack invariance to the equation deleted. This is problematic in that it permits manipulation of the model and allows the researcher to report only those estimation results most agreeable with prior beliefs. Second, the standard error estimates of this procedure are biased.

4.3 Correct Econometric Specification

An appropriate procedure is to employ some form of generalized least squares. This avoids the problems associated with an equation-by-equation ordinary least squares procedure.

Suppose the fifth share equation is omitted and the remaining share equations and the cost equation are modified, based on their relationships detailed in Appendix A. This five equation model (the cost function and four share equations) is then estimated. If the symmetry restriction of $\delta_{ij} = \delta_{ji}$ is also imposed, then the system of equations can be estimated.

4.4 Maximum Likelihood Estimation

The maximum likelihood estimation procedure is based on the notion that the sample data is more likely to have come from a “real world” characterized by one particular set of parameter values than from a “real world” characterized by any other set of parameter values. The maximum likelihood estimate for the coefficients in the model is the set of values which gives the greatest probability of obtaining the observed data.

Maximum likelihood estimation will develop the set of parameters most consistent with the probability density function of the data set.

4.5 Estimation Properties of Maximum Likelihood Estimators

In addition to the intuitive appeal of matching the probability density function of the data, the maximum likelihood estimator has several other desirable estimating properties. First, the maximum likelihood estimator is asymptotically unbiased; it is consistent; and it is distributed asymptotically normally. As the sample size approaches the size of the population, the expected value of the parameter becomes equal to the true population value. Second, the maximum likelihood estimator is consistent if the calculated sample distribution for successively larger samples sizes tends to become concentrated on a particular value. Third, the maximum likelihood estimator is asymptotically efficient if its variance is smaller than the variance of all other consistent estimators as the sample size increases. These properties, along with intuitive appeal, make maximum likelihood a powerful estimation procedure.

Maximum likelihood estimation is very appealing for situations in which it is impossible to find estimators with desirable small sample properties. Two limitations that have historically limited the use of the maximum likelihood estimation are the required assumption that the disturbance term is normally distributed and the fact that the technique can be computationally difficult.

CHAPTER 5

DESCRIPTION OF THE DATA SET

5.1 Introduction

The compilation of time series data to support the estimation of the translog cost function of the Copper Smelting Industry proved a challenging task. Some series of prices and quantity of production, input variables, and control variables were relatively accessible from government and industry publications. Other series, such as fuel, were incomplete for the three decade time span, or the information available was extremely disjointed, broken as a series or reported in arbitrarily chosen units by a succession of compilers.

5.2 Copper Smelter Production and Price of Copper

Quantity and price for copper production were relatively easy to obtain. There are numerous sources such as the Annual Report of the President, Annual Survey of Manufacturers, American Bureau of Metal Statistics, and the U.S. Bureau of Mines Minerals Yearbook.

However, in searching the various publications for a series of years, it becomes evident that the units and/or product definitions change frequently, producing considerable difficulty in obtaining long series.

The best source of information was found in the Annual Data (various years) published by the Copper Development Association in New York City.

The Copper Development Association maintains uniform definition of product classifications, and year to year the data have been reconciled so that one can follow an unbroken flow of copper material and mining activity from 1948 to the present.
The only exception is for production by Solvent Extraction Electro Winning (SXEW). For these data, the work of Janice Jolly in the Mineral Yearbook of the U.S. Bureau of Mines was most valuable.

Price of copper is easier to find, but it too varies by source. Prices used in the study were taken from nonferrous metal prices in the United States through 1988, U.S. Bureau of Mines years through 1991 were supplemented from the Annual Report of the President. All prices are reported in 1982 constant dollars.

TABLE 5.1
Total Smelter Production and Price

Year	Thousands of Short Tons	$/Ton *
1961	1285.69995	1994.73328
1962	1341.50000	2040.00000
1963	1355.90002	2040.00000
1964	385.50000	2088.88892
1965	1460.80005	2244.67944
1966	504.30005	2274.84277
1967	933.00000	2366.93506
1968	1351.30005	2505.80835
1969	1662.80005	2700.79541
1670	1720.19995	3118.91895
1971	1566.30005	2699.89502
1972	1759.40002	2537.19287
1973	1821.80005	2730.95117
1974	1649.40002	2767.11182
1975	1496.50000	2089.96704
1976	1585.69995	2124.19751
1977	1484.59998	1874.87183
1978	1480.00000	1707.37537
1979	1538.69995	2115.32642
1980	1161.00000	2088.89795
1981	1518.59998	1663.23730
1982	1125.19995	1458.18005
1983	1088.00000	1511.86414
1984	1304.50000	1272.77405
1985	1312.69995	1269.42883
1986	1318.30005	1277.70752
1987	1376.50000	1527.23169
1988	1515.19995	2006.85767
1989	1630.90002	2096.01953
1990	1613.00000	2017.63049
1991	1639.40002	1853.05090

* Constant Dollars 1982 Index

5.3 Capital Expenditures and Price of Capital

5.3.1 Capital Expenditures

Annual capital expenditures are reported in the Annual Survey of Manufacturers (ASM) published by the Bureau of the Census. All data are taken from SIC 3331, Copper Smelting Industry. Capital stock or inventory was reported in short series in the Census of Manufacturers of the Bureau of the Census.

For this study a capital stock series was calculated from the annual investment series from the ASM using a perpetual inventory and assumed constant rate of depreciation. The capital stock series was benchmarked near the mid point of the short section of capital stock reported in the Census of Manufacturers, and numerous iterations were performed to find the depreciation rate that produced the best fit to the reported capital stock series. That depreciation rate was found to be 4%. This figure seems unrealistically low, but it is the rate found to match the capital series curve closely with the limited amount of reported dates.

5.3.2 Price of Capital

Price of capital was problematic. At the single firm level, there are numerous ways to arrive at capital charges including direct application of capital service figures from published reports.

In the aggregate, difficulties arise because the industry, although highly concentrated, has a strong variance in horizontal and vertical integration among its participants. As a result, no clear picture would be presented using direct data on the forms or data extracted from a single form to apply to the group. Attempts were made to use earnings and capital expenditures extracted from Value Line reports both individually and calculated as a divisia index. Using Asarco, Cyprus-Amax, and Phelps Dodge individually and as an aggregate was totally unsuccessful due to auto-correlation and other statistical factors that caused failure to converge in the Maximum Likelihood calculation.

In the end, the most logical approach was to use the annual series of interest paid on Moody's Baa corporate bonds as the price of capital for this aggregate of firms. Prices are therefore represented by the surrogate of average annual interest rates reported in the Annual Report of the President. All values indicated are stated in terms of real dollar values indexed to the 1982 Producer Price Index or Consumer Price Index.

$$K_t = (1-\delta)\ K_{t-1} + I_{t-1}$$

Where: K= capital stock
δ= depreciation rate
I= gross investment

TABLE 5.2
Annual Capital Stock and Price

YEAR	CAPITAL STOCK*	PRICE **
1961	1595.08948	5.08000
1962	1629.47192	5.02000
1963	1647.87720	4.86000
1964	1631.43640	4.83000
1965	1613.54907	4.87000
1966	1619.78955	5.67000
1967	1702.29773	6.23000
1968	1784.56189	6.94000
1969	1803.64575	7.81000
1670	1882.52551	9.11000
1971	1995.45703	8.56000
1972	2106.68262	8.16000
1973	2208.49365	8.24000
1974	1937.98206	9.50000
1975	1967.27917	10.61000
1976	2081.39600	9.75000
1977	1976.71973	8.97000
1978	2099.18726	9.49000
1979	1957.02722	10.69000
1980	1835.42737	13.67000
1981	1812.88782	16.04000
1982	1894.32227	16.11000
1983	1930.27087	13.55000
1984	2136.12695	14.19000
1985	2329.22485	12.72000
1986	2395.77905	10.39000
1987	2273.14795	10.58000
1988	2041.73743	10.83000
1989	1972.90198	10.18000
1990	2036.96924	10.36000
1991	2143.67407	9.80000

* Constant Dollars 1982 Index
** Moodys Baa Corporate Bond Rates

5.4 Labor Inputs, Quantity and Price

In 1960, there were 16,122 total workers in copper smelting; 86% or 13,834 were production workers. Production workers are defined as those at or below the first supervisory level. By 1991 total workers had dropped to 4,500; of these 3,500 or 78% were classed as production workers.

Labor figures were taken from the series available in the Annual Survey of Manufacturers SIC 3331, Primary Copper. Data are reported for “All Employees” (number and payroll) and for “Production Workers” (number, hours, wages).

The separate reporting of all workers and production workers provided an opportunity to examine the changes in each category and look for substitution between the two categories. Production labor and non-production labor were calculated from the ASM data, and series were formed for constant 1982 dollar wages per hour. Quantity of labor of both types is expressed as millions of hours per year.

TABLE 5.4 and TABLE 5.3 present quantity and price data for two categories equivalent to blue collar and white collar employment.

5.4.1 Production Labor and Price

Data on the quantity and price of labor were obtained from the Annual Census of Manufacturers. The information on hand for the full time span of the study allowed the extraction of quantity and price information for production and non-production labor.

With the shift in technology and the strong reduction in the power of trade unions, there was considerable interest in determining from the model the changes in the mix of the two labor categories and their relationship to the other factors.

5.4.2 Non-Production Labor and Price

Non-production workers include all others in the ranks of supervisory, engineering, clerical, marketing, etc.

TABLE 5.3
Production Labor and Cost of Labor

Years	Million Hours/Year	$/Hour *
1961	21.76509	11.12518
1962	21.81437	11.49069
1963	21.61589	11.59580
1964	21.47651	11.67066
1965	23.09737	11.69652
1966	23.57721	12.03034
1967	14.35517	12.72257
1968	19.16522	12.36975
1969	24.93099	12.20808
1670	25.92167	12.25938
1971	21.76364	13.19451
1972	24.02791	13.75992
1973	23.63824	14.15862
1974	22.32000	15.19481
1975	18.81355	16.35102
1976	17.85874	16.70993
1977	16.66870	16.58213
1978	17.20462	16.44765
1979	15.97647	16.71708
1980	11.79612	17.07819
1981	13.42523	17.47034
1982	9.31579	18.79922
1983	8.54328	18.05124
1984	7.28333	18.01151
1985	5.73565	18.03433
1986	4.76923	17.06499
1987	4.09697	15.68490
1988	4.35556	15.91423
1989	5.21053	13.79032
1990	6.02609	14.05519
1991	5.98889	13.81786

* Constant Dollars 1982 Index

TABLE 5.4
Non-Production Labor and Price

Year	Thousand Workers	Salary/$1000 Per Year *
1961	4.23491	16.11070
1962	4.18563	15.98025
1963	3.88411	16.32260
1964	3.52349	17.12013
1965	3.70263	17.14783
1966	4.22279	15.93354
1967	3.74483	15.67031
1968	3.63478	17.15545
1969	4.26901	16.40364
1670	4.97833	16.04893
1971	4.83636	16.08187
1972	4.67209	17.92174
1973	4.86176	17.32586
1974	5.58000	17.12142
1975	5.28645	17.89663
1976	4.74126	19.27513
1977	3.93130	20.90359
1978	3.89538	20.43476
1979	3.42353	21.76641
1980	3.20388	24.73485
1981	3.47477	21.46546
1982	2.68421	18.49233
1983	2.25672	22.28956
1984	2.21667	19.45190
1985	2.02435	17.62925
1986	1.23077	20.38663
1987	1.10303	19.79183
1988	1.24444	19.35908
1989	1.38947	17.35398
1990	1.67391	15.90636
1991	1.71111	16.21965

* Constant Dollars 1982 Index

5.5 Energy and Materials

Energy and material data are aggregate or co-mingled in the statistical record. Considerable effort was required to develop useful time series for the respective price and quantities.

5.5.1 Energy

Primary copper was produced historically from reverberatory furnaces fired by purchased fuel. Some production came from electric furnaces powered by electricity generated from firing boilers with natural gas.

By the 1990's, all copper was produced in furnaces either flash or electric arc that use oxygen to accelerate the oxidation of sulfur in the ore. This chemical reaction provides more heat than required for basic smelting purposes, enabling the production of electricity on site to meet most operating needs.

Compilation of suitable energy input data was difficult. Fuel consumption information is very scarce for the years preceding the early 1970's. Energy data keeping began with the onset of the energy problems of the 1970's and several years lapsed before figures were kept in a consistent manner with uniform recording methodology.

The ASM reports “Cost of Materials” as the dollar sum of fuel and materials. The desired information is embedded in this series as the fuel input share added to the materials (in this case, ore). A method was needed to allow the preparation of separate energy and materials data.

5.5.2 Materials (Ore)

As previously mentioned in section 5.5.1 (fuel) the ASM reports materials as a figure combining value of materials which in this case is primarily ore and the value of fuel.

The value of ore was best determined by using the annual copper content of ore calculated by annual ore tonnage and average annual ore grade. This copper content was then multiplied by the annual price of copper. The resultant value of ore per ton was applied to annual ore production tonnage figures to determine the value of ore as a component of the ASM materials value. To do this, value of ore was subtracted from the combined figure for fuel and material reported in the Annual Survey of Manufacturers to extract the value of fuel. A price series was then constructed from the Commerce Department's, "Business Statistics" reporting of the index of fuels and Related Products and Power. The transformation of the price figure for value of fuels to a figure representing fuel in trillion btu units was accomplished by developing a benchmark price from the short price/quantity series of 1974 to 1980 and expanding the index to a price equivalent level.

TABLE 5.5 and TABLE 5.6 display the data produced by the methodology described above.

TABLE 5.5
Annual Energy Consumption

Year	Trillion BTUs	$/Trillion BTU*
1961	144.03812	1.30340
1962	150.28944	1.30340
1963	151.90269	1.29409
1964	155.21881	1.25685
1965	163.65472	1.28478
1966	168.52808	1.31271
1967	104.52483	1.34064
1968	151.38734	1.33133
1969	186.28497	1.35926
1670	192.71555	1.42443
1971	175.40689	1.54546
1972	188.80836	1.59201
1973	176.37431	1.80614
1974	105.98755	2.80231
1975	95.48364	3.29574
1976	81.86430	3.56573
1977	69.44312	4.05916
1978	72.87432	4.32915
1979	59.32193	5.48359
1980	26.36779	7.70868
1981	18.28340	9.32862
1982	18.64662	9.31000
1983	21.48973	8.92829
1984	19.79220	8.82588
1985	14.31044	8.50934
1986	23.03858	6.49838
1987	17.34959	6.53562
1988	20.03903	6.20977
1989	17.96789	6.78699
1990	17.51815	7.65282
1991	16.50242	7.55972

* Constant Dollars 1982 Index

5.6 Material (Ore)

The cost of material for copper smelting as reported in the Annual Census of Manufacturers combines both fuel and materials. Materials in the case of copper smelting consists principally of ore. The value of the fuel component was determined by calculating the annual tonnage of copper value in the materials figure. Thus, simple subtraction reduced the materials figure and revealed the value of the fuel component. Dividing by the fuel price thus expressed the annual quantity of fuel used.

5.6.1 Quantity (Ore)

In considering the inputs to smelting, the cost of ore overshadows the increments of cost for various other maintenance supplies, etc., that are utilized at the smelter site annually. The annual quantity of ore is taken from various issues of the U.S. Bureau of Mines, Minerals Yearbook.

5.6.2 Price (Ore)

The cost of material (ore) in millions of constant dollars is divided by the quantity of ore mined in that year to establish the price per ton of mined ore. Prices are adjusted to 1982 constant dollars using the Producer Price Index for materials from the Annual Report of the President.

TABLE 5.6
Annual Ore Production

Year	Million Tons	$ Value/Ton *
1961	142.72200	14.96050
1962	150.21700	15.30000
1963	146.45000	15.09600
1964	155.20000	15.24889
1965	173.28600	15.71276
1966	186.9600	15.24145
1967	127.06600	14.91169
1968	170.05400	15.03485
1969	223.75200	16.20477
1670	257.72900	18.40162
1971	242.65601	14.84942
1972	266.38101	13.95456
1973	289.99799	14.47404
1974	293.44299	13.55885
1975	263.00000	9.82285
1976	283.73300	10.83341
1977	262.17499	9.74933
1978	263.72198	8.70761
1979	305.92401	9.94203
1980	244.26601	10.02671
1981	306.07999	8.48251
1982	200.55701	8.01999
1983	196.13200	7.71051
1984	189.39101	7.38209
1985	178.80400	7.87046
1986	186.55099	7.92179
1987	223.36099	8.70522
1988	246.44800	12.04115
1989	261.57700	12.78572
1990	275.02399	12.50931
1991	308.52399	10.56239

* Constant Dollars 1982 Index

5.7 Pollution Abatement and Changing Technologies

Although there were many points of challenge to continuing operation of the U.S. smelting industry, in general the most difficult and costly challenge was presented by the legally mandated control of sulfur emissions into the atmosphere.

In the beginning there was no suitable technology to stem the emissions, and efforts to retrofit existing plants proved to be a disastrously expensive experiment. Finding successful retrofit to be impractical, older smelters were shut down in the 1985 - 1987 period precedent to the final emissions deadline in 1988.

The U.S. copper industry was reducing its production capacity in the face of an abnormally strong U.S. dollar made strong by high interest rates. Imported copper, primarily from Chile, easily took a large share of the U.S. market, and exports of U.S. produced copper concentrate increased (Copper Development Association Annual Production Report). Prices fell, but the Chilean peso under strong devaluation allowed profitable delivery of copper from distant production sites.

While suffering low prices and strong import competition, U.S. smelters were forced to replace reverberatory furnaces and upgrade electric furnaces usually with oxygen enrichment and sulfur dioxide capture facility. This meant building on-site oxygen production plants and building sulfuric acid production plants. Sites equipped with reverberatory furnaces were abandoned or used as locations for installations of flash technology.

The abundance of sulfuric acid drove down the price and made leach technologies more cost effective. The early 1970's saw a sharp expansion in the implementation of Solvent Extraction Electro Winning (SXEW) practice. Quality of SXEW production quickly reached the quality of traditional refined copper and SXEW production grew rapidly as a percentage of total U.S. copper production.

The pressure applied to accomplish pollution control is clearly seen to be the driving force behind technical change in the industry. The influence of these two factors is tested by inserting proxies as control variables.

TABLE 5.7 displays the quantity of sulfuric acid produced by copper smelters. The data is from the U.S. Bureau of Mines Minerals Yearbook and represents the growing compliance with air pollution regulations.

Technical change is represented by the percentage of U.S. refined copper produced from SXEW operation. There is clearly a link between air pollution control and growth of SXEW production. Having reviewed and tested other proxies, such as a measure of sulfur reduction in Arizona, these data are considered the best available proxy.

A better figure for technical change would have been the shift in percentage copper output produced from reverberatory vs. oxygen enriched technologies. The change over was not gradual but precipitous, and the data are not known to be available.

5.7.1 Pollution Abatement

Compliance with air pollution abatement regulations was virtually impossible with the operation of the traditional reverberatory furnaces used in copper smelting. The sulfur dioxide content of the gas stream was too low for efficient capture with flue gas scrubbers. Attempts were more successful in curtailing emissions from convertors and roasters (Davenport and Partelpog, 1987) but in the end, the reverberatory smelters were shutdown and eventually dismantled. Those locations sufficiently well situated to serve captive and/or toll concentrate production were rebuilt with smelting units based on flash smelting or oxygen enrichment technology. By the end of the study period the conversion to new technologies was complete. U.S. smelter capacity had completed the cycle; it had declined and rebounded with the startup of newly designed smelters.

The new smelter designs very efficiently capture highly concentrated sulfur dioxide off-gas streams and are able to produce low cost sulfuric acid for the commercial market and for the leach stage of SXEW copper allowing production from previously non-commercial low grade oxide ore in dump leach and heap leach operations.

The capture of sulfur dioxide that was previously released into the atmosphere and its conversion into sulfuric acid is reflected in the data for sulfuric acid production from copper smelters. The information is reported from data in the Minerals Yearbook of the U.S. Bureau of Mines, various years. Data representing copper smelter production, therefore, serves well to illustrate the level of compliance with pollution abatement regulations.

5.7.2 Technical Change

Technological change that occurred between 1960 and 1991 was influenced by a number of factors. Flash smelting along with other enriched oxygen designs such as the electric furnace, Noranda, and Mitsubishi designs all produced easily captured sulfur dioxide rich gas streams. All of the oxygen enriched smelter designs offer sharp reduction or near complete elimination of purchased fuel requirements as a result of the exothermic oxidation of sulfur associated with copper in the ore.

As the older reverberatory furnaces were dismantled, the new units that replaced them on specific sites were much higher in capacity. So, while the number of smelters fell initially, and reduced the U.S. smelter capacity, some recovery in capacity has been made with the installation of more recently designed units.

The switch to enriched oxygen furnaces with near complete capture of sulfur dioxide and conversion of that sulfur dioxide to large quantities of relatively cheap sulfuric acid enhanced the expansion of hydrometallurgy. Hydrometallurgical copper production became a parallel growth component in the supply of refined copper for market. The growth of hydrometallurgical copper output as a percentage of total refined copper output is taken as an indicator of technical change in the copper smelting industry for purposes of this study.

TABLE 5.7
Annual Smelter Production and Index of Technological Change of Sulfuric Acid - Ton/Year

Year	Thousand Tons Sulfuric Acid	Index of Technical Change
1961	362.63000	1.11000
1962	403.68301	1.12000
1963	358.50299	1.13000
1964	330.27301	1.14000
1965	369.32101	1.15000
1966	469.72800	1.16000
1967	348.49701	1.17000
1968	483.10800	1.18000
1969	685.77502	1.19000
1670	747.78400	1.77383
1971	803.28400	1.89962
1972	1010.61401	1.37124
1973	1088.32202	1.73784
1974	1277.43994	1.58306
1975	1784.74402	2.20937
1976	2281.59106	5.20008
1977	2357.36597	6.94186
1978	2738.23608	4.92915
1979	2770.11890	4.69023
1980	2312.28589	6.43878
1981	2859.10400	7.27537
1982	2072.25610	7.14889
1983	2025.83704	6.04886
1984	2251.31201	6.33572
1985	2230.25708	5.95259
1986	2308.80396	7.52792
1987	2542.60205	9.47155
1988	2892.65503	10.95801
1989	3075.85889	13.74705
1990	3380.93994	16.31985
1991	3819.43896	18.11073

CHAPTER 6

INTERPRETATION OF RESULTS

6.1 Introduction

As discussed in previous chapters, a model of a production process, whether examining the production function or the cost function, must be examined relative to economic theory in order to facilitate proper interpretation. How well the model results compare with the theoretical expectation constitutes part of the information derived from the modeling process.

The translog form was chosen to represent the cost function because it is highly flexible in conforming to data and does not impose restrictions on elasticities. With the exception of symmetry and homogeneity of input prices there are no restrictions that would bias the results of statistical estimation.

Production theory describes the shape of the production surface in terms of regularity. Within the concept of regularity are the expectations that certain basic principles of economics will be manifested by the estimated model. Chief among these regularity conditions are (1) the negative slope of input demand curve, (2) monotonicity, indicating that the production process will respond predictably to changes in inputs price, (3) strict quasi-concavity, which requires that the production surface be relatively smooth and that irregularities that do exist remain within prescribed limits.

In addition to the requirements imposed in modeling a production process there is the presence of an accounting relationship implied in the cost shares. Caution must be used to see that prices and input units meet the required accounting relationship without distortions.

All production processes are related each day to the events of the prior day. This linkage often produces strong serial correlation of the error terms. In the translog cost model, where a system of simultaneous equations are employed, not only are the individual equations likely to exhibit auto-correlated error terms but the degree of auto-correlation may vary between equations as well as the direction (positive or negative) of the auto-correlation. Auto-correlation complications may defy the best efforts to find a single co efficient that will correct for the degree of auto-correlation among all equations equally. In developing the coefficient for correction of auto-correlation (Rho) in the copper industry model an iterative search was performed. Various values of Rho were tested until that value was found which most closely satisfied or corrected most of the auto-correlation exhibited by the five equations. While theory dictates that Rho for the cost share equations must be the same for all equations in order to preserve symmetry, it is not required that the Rho for the cost function be the same as for the share equations. However, in this model there was found, in examining the relationship between Rho of the cost function and the cost share equations that there was no gain in estimation efficiency discovered by using different values of Rho for these respective equations.

6.2 Regularity Conditions

Regularity requirements demand an overview of the model and its results. Ideally the process of model estimation will conform in all ways to the dictates of production theory, but conditions and results may not be ideal. Unfortunately, firms do not always conform to cost minimizing or profit maximizing behavior; and markets may be upset by extraneous stimuli; or governmental interference; such as taxation, embargos, price freezes, or other breaches of free market operation.

6.2.1 Monotonicity

Monotonicity implies that inputs and outputs will increase together in some orderly manner according to the structure of the production or cost function. The test for monotonicity is made by examination of the input share values compared to the fitted share values. The primary requirement of the test is that all shares remain positive or that there be no major sign reversals. The calculations of the monotonicity test, which are presented in Appendix D, satisfactorily indicate that the total cost and share equations are monotonic.

6.2.2 Input Demand Curve Slope

The sign of the own price Allen Elasticities of Substitution should all be negative, indicating that the demand curve is sloped negatively according to the dictates of economic theory. These elasticities are computed for each year of the time series. Certain of the Allen own price elasticities of substitution (Appendix F) do not exhibit this expected negative sign for all years. Both capital and materials (ore) exhibit an unexplained positive sign on portions of their time series of elasticities over the study period.

There is not sufficient information on the absolute accuracy of the data to justify an attempt at a detailed explanation other than to note that sign reversals are evident following a declining trend of absolute values for AES and occur around or associated with the period of the 1980s which has been discussed earlier as a period of severe upset and realignment in the industry.

6.2.3 Strict Quasi-Concavity

Data from a period of three decades is used to develop and examine the model. As is often the case in matters related to the minerals industry, there are large movements in business cycles, political events, and technology that impact the decision making process of the firm and influence the shape of the production and cost surfaces.

Strict quasi-concavity requires that variations in the cost surface or production surface remain within certain limits. Breaking out of these limits indicates departure from optimality conditions that are required by static analysis. Analysis by partial and cross partial derivatives implies that the model is being examined at the cost minimum or profit maximum. The translog cost share equations establish the required optimality conditions within the model. To see that the computations in fact are expressing optimality an observation by observation test of the model illustrated in Appendix E seeks to satisfactorily test for optimality conditions at each observation.

The model was tested at each observation for strict quasi con-cavity of the cost surface curvature. The purpose was to indicate consistency of cost optimizing behavior and is one of the key components of the set of regularity conditions to which the cost function must conform.

At each observation the matrices of Allen Elasticities were computed producing 2x2, 3x3, 4x4, and 5x5 matrices. Strict quasi-concavity is indicated if the signs of the determinants are negative for the 2x2 matrix and alternate in signs for the next larger matrices in turn. The final matrix in this case the 5x5 matrix should have determinant with a value very close to zero (Berndt, 1991).

The test of strict quasi-concavity is favorable only in the period of 1982 to 1986. This is also the five year period when the Allen own price elasticities have negative signs for all inputs. Historically this time frame was one when capital inventory was rebuilding in the face of high interest rates and foreign competition, made possible by the high trade weighted value of U.S. currency, had made considerable inroads into domestic U.S. markets. The tonnage of ore mined and processed was reduced not only by foreign competition but by smelter closure and a lower share value of the materials (ore) component of the cost structure. The prolonged strikes of the early 1980s were settled with new work rules in place allowing more optimal employment of labor. It is perhaps in view of these preceding points that this five year span is seen as the only period where all regularity conditions are met over the 31 year period of the study.

In view of the limited number of observations where regularity prevails, elasticities will be examined at the mid-point of the five year span, specifically for the year 1984.

6.3 Parameter Estimates

Table 6.1 presents the parameter estimates and the associated t-statistics. A total of 44 parameter values are present, 36 parameters were estimated directly, the other eight were calculated respecting the necessity of having dropped a share equation and the need for calculation of the parameters utilizing the homogeneity restrictions.

Of the 44 parameter values, 17 are found to be statistically significant at the 95% confidence level. Significantly, all inputs are represented by reasonable R-squared values and small standard errors. Elasticities are calculated for all inputs and presented in table form regardless of the statistical significance of the associated parameters. Such a presentation may be worthwhile if one were at a point in the future to consider exercise of a lower confidence level in considering the significance of the elasticities.

TABLE 6.1
Parameter Estimates of the Cost and Demand Functions

Parameter	Estimate	t-statistic
α_{0}	101.267	2.72176
α_{Y}	-13.0540	-2.36782
α_{A}	.1.02096	1.22368
α_{T}	-.032487	-.113918
α_{k}	1.22010	5.79895
α_{p}	-.059433	-.704350
α_{s}	.076658	1.49058
α_{e}	.040595	.567137
α_{m}	-.277923	-1.12338
γ_{kk}	.084530	6.82258
γ_{kp}	-.430815E-02	-.761529
γ_{ks}	.328413E-02	.948796
γ_{ke}	-.117448E-02	-.270819
γ_{km}	-.082332	-7.45662
γ_{pp}	.021056	4.36321
γ_{ps}	-.364825E-02	-1.25364
γ_{pe}	.893666E-02	3.23060
γ_{pm}	-.022036	-4.40480
γ_{ss}	.418897E-02	1.65369
γ_{se}	.691212E-02	3.30268
γ_{sm}	-.010737	-3.49203
γ_{ee}	-.260891E-02	-.809311

TABLE 6.1 (cont.)

Parameter Estimates of the Cost and Input Demand Functions

Parameter	Estimate	t-statistic
γ_{mm}	-.012065	-3.12645
γ_{em}	.127171	8.43255
δ_{kY}	-.016830	-.929083
δ_{pY}	-.895602E-02	-1.43522
δ_{sY}	-.018040	-4.92246
δ_{eY}	-.010430	-2.11314
δ_{mY}	.054257	2.35772
δ_{kA}	-.036058	-2.12849
δ_{pA}	.654233E-02	1.11505
δ_{sA}	.593764E-02	1.72244
δ_{eA}	.639742E-02	1.38839
δ_{mA}	.017181	.796975
δ_{kT}	.304965E-02	.348993
δ_{pT}	-.404272E-02	-1.43214
δ_{sT}	-.253043E-02	-1.53192
δ_{eT}	-.346456E-02	-1.56035
δ_{mT}	.698806E-02	.630988
ρ_{YY}	.503074	2.54484
ρ_{AA}	-.038653	-1.34744
ρ_{TT}	.479862E-02	.217669
ρ_{YA}	-.254558	-1.43352
ρ_{YT}	-.059777	-.164473
ρ_{AT}	.159451	.587506

Table 6.2 presents a statistical summary of the estimated equations. Auto-correlation correction as previously mentioned was required and a value of the coefficient of auto-correlation (Rho) of 0.95 was utilized. The resultant Durbin-Watson statistics rest in the range of values allowing the acceptance of the null hypothesis of no auto-correlation. Two values remain in the region of uncertainty. Auto-correlation within data related to industrial production is to be expected. In these cases the correction should be sufficient to allow useful interpretation of the parameters and subsequent elasticities.

TABLE 6.2
Summary Statistics

Equation	Std. Error	R^2	D-W
Cost	.056828	.82	1.48
Capital	.010340	.64	1.82
Production Labor	.327743E-02	.57	1.86
Non-Production Labor	.190137E-02	.72	1.79
Energy	.257614E-02	.52	2.50

Log of Likelihood Function = 737.275
Number of Observations = 31

6.4 Analysis of Input Elasticities

Input elasticities calculated by various techniques express the relative change in input utilization under the influence of another input or under the influence of a control variable such as output or technical change.

Several methodologies have been utilized to express relationships where inputs substitute for one another. The elasticities employed in this study for insight and comparison of input substitution are:

Allen Elasticities of Substitution

Compensated Price Elasticities

Uncompensated Price Elasticities

Cost Elasticities

Input Elasticities with Respect to Output, Abatement, and Technical Change

6.4.1 Allen Elasticities of Substitution (AES)

The AES are one of the most commonly used methods of expressing relationships between inputs. The AES communicate by sign the complimentarily (negative) or substitution (plus) nature of the input relationships. The AES communicate by absolute value the relative elasticity: elastic (>1), unitary elastic or constant returns to scale (=1), or inelastic interdependence(<1). The magnitude of the absolute value infers semiquantatively an approximation of the marginal rate of substitution. The AES conform to the restriction of $\delta_{ij} = \delta_{ij}$ and therefore are symmetrical.

The AES are first reviewed for own price sign that would indicate correct slope of the demand curve described by the share equations. The full range of series of AES for the observations (1960-1991) is in Appendix F.

The AES for 1984 are of the correct own price sign. Most are inelastic, those indicating elastic relations among inputs are:

Capital and Non-Production Labor	Substitutes
Production Labor and Non-Production Labor	Compliments
Production Labor and Energy	Substitutes
Non-Production Labor and Energy	Substitutes

Table 6.1 lists the first, middle, and last elasticities, ie 1960-1976, 1991 of this series. The sign reversals and magnitude of the co-efficiencies is seen in a way that suggest statistically the basic changes that were known to have occurred.

TABLE 6.3
Estimated Allen Elasticities of Substitution
for the Year 1984

Input	Elasticity with respect to the price of				
	Capital	Production Labor	Non-Prod' Labor	Energy	Materials (Ore)
Capital	-1.95434	0.23275	1.86163	0.069739	0.33382
Production Labor		-10.25772	-3.56749	11.98786	0.14914
Non-Prod' Labor			-34.22003	13.51991	0.38927
Energy				-45.81697	0.32601
Materials (Ore)					-0.10139

6.4.2 Compensated Price Elasticities (CPE)

Compensated price elasticities are presented in TABLE 6.4. They represent the rate of technical substitution at a given level of production. Each elasticity figure is coupled to the production level for that annual observation. CPE is calculated from the share values at each observation and the parameter estimated for the cross products of the inputs to be compared.

The signs of the own price elasticities are negative as are own price AES. The CPE lack the symmetry restrictions of the AES. The opposing pairs of cross price elasticities are different but remain close in sign and value. The lack of symmetry restrictions in the CPE did not produce changes in sign so the substitute versus compliment information remains in agreement.

Elasticity coefficients for CPE are smaller than AES and can be interpreted as the percentage change in input usage with a 1% change in price of the cross-price input.

All CPE are inelastic with signs in agreement with the AES. The most elastic inputs, as with the AES, are production labor, non-production labor, and energy. The least elastic is materials (ore).

Materials (ore) is inelastic with respect to all other inputs, a reasonable observation considering the run / no-run decision that management must make in the operation of a smelter. The decision is not one of using less ore but of using no ore at all. The smelter range of operation relative to the through put is relatively inflexible, especially for flash and very inflexible for SXEW. Mining and milling can continue while concentrate is stockpiled, but there is considerable expense to the stockpiled material. For the most part, the operating decision is not one of using more or less ore but of using design levels of ore in the smelter or not running the smelter at all.

6.4.3 Uncompensated Price Elasticities (UPE)

Uncompensated price elasticities are calculated by adding the quotient of the input elasticity with respect to output demand divided by the cost elasticity of output to the compensated price elasticity. The resulting UPE values thus reflect the effect of the production level in the period of the observation.

The number of values close to one suggests unitary or constant returns to scale for many input cross price elasticities. This is a strong change from the general picture presented by the CPE wherein elasticities prevailed.

As with the Compensated Price Elasticity the Uncompensated Price Elasticity indicates a (%) change in the use of the input with a one percent change in the price of the comparative input. Thus production labor use would increase by 1.06% with a one percent increase in the price of capital.

The own price signs of the UPE are positive, indicating a possible computational problem. The elasticities with respect to output seem abnormally high; this effect appears to be dominant and may have biased the results.

If the uncompensated elasticities are accurate, the near unitary elasticity of most input elasticities is notable. Perhaps, these values computed with a more compact model or a substantially stronger data set would provide an answer to the suggestion of a shift toward unit elasticity under the influence of changing levels of output.

TABLE 6.4
Estimated Price Elasticities of Input Demand - 1984

Input	Elasticity with respect to the price of				
	Capital	Production Labor	Non-Prod. Labor	Energy	Materials (Ore)
	Compensated Price Elasticities				
Capital	-0.31991	0.00798	0.043348	0.016535	0.25204
Prod' Labor	0.03899	-0.35187	-0.083069	0.28423	0.11261
Non-Prod' Labor	0.30473	-0.12237	-0.79682	0.32056	0.29390
Energy	0.11416	0.41122	0.31481	-1.08632	0.24614
Materials (Ore)	0.054644	0.005716	0.009064	0.007730	-0.076553
	Uncompensated Price Elasticities				
Capital	0.68810	1.01599	1.05135	1.02454	1.26004
Production Labor	1.05843	0.66846	0.93726	1.30456	1.13293
Non-Prod' Labor	1.36505	0.93795	0.26351	1.38088	1.35422
Energy	1.14841	1.44547	1.34906	-0.052074	1.28039
Materials (Ore)	1.04905	0.99952	1.00347	1.00213	0.91785

6.4.4 Input Demand Elasticities

Input demand elasticities are computed for each of the five inputs with respect to output, pollution abatement, and technical change. The results are presented in TABLE 6.5. All elasticities carry a negative sign indicating that a 1% increase in output, abatement, or technical change would produce a decline in use of each input. One cannot accept the results as being correct indicators, since a 1% increase in output will be accomplished by a 12.7% decrease in ore usage. Clearly something is wrong.

Regrettably, the information that was desired by including the control variables in the model is not available. A smaller model in inputs run against fewer control variables could possible produce better results.

TABLE 6.5
Estimated Input Demand Elasticities with Respect to Output, Pollution Abatement, and Technical Change for the Year 1984

	Elasticity with respect to		
Input	Output	Pollution Abatement	Technical Change
Capital	-12.94670	-13.06417	-12.82526
Production Labor	-13.10497	-12.65316	-12.96174
Non-Prod' Labor	-13.61865	-12.58889	-12.95256
Energy	-13.28379	-12.57407	-12.99001
Materials (Ore)	-12.77202	-12.82113	-12.83463
Cost Elasticity	-12.84389	-2.56356	0.044788

6.4.5 Cost Elasticities with Respect to Output, Pollution Abatement and Technical Change

Cost elasticities should indicate the percentage response of total cost to the percentage of change in the respective variable (output, pollution abatement, or technical change). Each elasticity is computed as the logarithmic derivative of the cost function with respect to the given variable. The elasticity of total cost with respect to output carries information on scale economies. If an increase in output would reduce the total cost, there are economies of scale yet unused.

It is now recognized that pollution abatement has reduced the total cost of copper smelting through the introduction of new technologies to contain sulfur dioxide off gases. It is questionable whether a 1% increase in abatement, however, would produce a 2.6% decrease in cost.

Cost elasticity with respect to technical change is a smaller coefficient and would indicate that a 1% increase in copper produced by SXEW methodology would increase copper smelting total cost by 0.05%. Since SXEW production as a percentage of total copper production is used as a technical change proxy there is no direct causal link. Otherwise, it is clear that technical change has produced strong reductions in cost exemplified by the shift from reverbratory to flash furnaces.

6.5 Summary

Estimating the translog cost function for the U.S. Copper Smelting Industry produced some results that were anticipated and other results that were not foreseen. Viewed relative to the heavy demand such a large model placed upon a limited number of observations, the amount of information that can be obtained and the completeness of the information is a priori somewhat limited.

Breach of regularity conditions can be seen as a function of the realignments of the 1980s. Further work with limited (smaller) models would reveal more on the lack of global correctness in cost surface curvature.

The Allen, Compensated and Uncompensated, elasticities appear to deliver the clearest information about the industry and its inputs.

Most cross input coefficients are inelastic. The labor inputs and energy show the most elastic response. The most inelastic input is materials.

Production Labor and Non-Production Labor are established as compliments. With increasing automation, the possibility that non-production labor might be substituting for production labor may have been a logical intuitive conclusion.

Capital substitutes for non-production workers but not for production workers.

Energy use declines sharply over the end of the study period and gives a statistical response indicating energy and labor of both types are substitutes.

Technical change and pollution abatement did not prove to be statistically significant in the model. Pollution abatement has some significant cross products in the parameter estimates, but technical change as defined in this study is not a significant factor in describing the total cost of copper smelting.

6.6 Conclusions

While some observations of input substitution have come to light from this analysis, the total information derived falls short of the original goals. In retrospect there are changes in study format that would possibly have provided a better approach to the study. The effort toward obtaining a wealth of information from a single large complex model may have shortened the yield from this effort.

The data were difficult to develop and even with careful preparation certainly contain some flaws. Labor might have been included as a single production labor variable or combined with non-production labor to form a single variable which would have given a single labor input to calculate into substitution elasticities.

Fuel and materials were combined in the root obtained from the Bureau of the Census Annual Survey of Manufacturers and required careful work to disaggregate. The energy variable thus generated is clearly of interest because of declining energy use accompanying technical change. Material (ore) might have been excluded from the model because of its relative inelasticity, but the overwhelming position it occupies in dominance as a percentage of the value of the total shares of the total cost function calls for its inclusion.

An interesting model may have been produced by examining the substitution between one or both types of labor and capital, and/or energy.

Output is represented correctly as smelter production and refined copper price. Exclusion of output from the model would have set up the condition of constant returns to scale, a condition present in many earlier studies as a precondition or basis for hypothesis testing for the presence of returns to scale.

Pollution abatement is well represented by the proxy of smelter acid production, and exhibits significance as a cross product parameter. A smaller model might have been designed for hypothesis testing of the significance of the presence or absence of pollution control in determining cost structure.

Technical change is often represented by a simple time series rather than as a series of compiled variables. As with the pollution abatement variable, a comparison of the relative usefulness of the differently configured data series for technical change might have been tested as a means of judging the worth of the contrasting methodologies.

APPENDIX A

Derivation of the Translog Cost Function

Starting with the general form:

$$\log c = \beta_0 + \Sigma\beta_i \log P_i + 1/2\Sigma\Sigma\varphi_{ij} \log P_i * \log P_j + \alpha_y \log Y +$$

$$1/2\alpha_{yy}(\log Y)^2 + \Sigma\varphi_{iy} \log P_i * \log Y + \alpha_t T + 1/2\alpha_{tt}(T)^2 +$$

$$\Sigma\varphi_i T \log P_i * T + \varphi_{yt} \log Y * T$$

for i,j = k, p, s, e, m

where φij = φφji from the symmetry condition;

and where,

Pk = capital price

Pp = price of production labor

Ps = price of non-production labor

Pe = price of energy

Pm = price of material (ore)

Y = output (KWH)

T = technical change

Assuming a well-behaved cost function with homogeneity of degree one in prices, given Y and T, implies the following restrictions.

$$\beta_k + \beta_p + \beta_s + \beta_e + \beta_m = 1$$

$$>> \beta_m = 1 - (\beta_k + \beta_p + \beta_s + \beta_e)$$

$$\varphi_{kp} + \varphi_{pp} + \varphi_{ps} + \varphi_{pe} + \varphi_{pm} = 1$$

$$>> \varphi_{pm} = 1 - (\varphi_{kp} + \varphi_{pp} + \varphi_{ps} + \varphi_{pe})$$

$$\varphi_{ks} + \varphi_{ps} + \varphi_{ss} + \varphi_{se} + \varphi_{sm} = 1$$

$$>> \varphi_{sm} = 1 - (\varphi_{ks} + \varphi_{ps} + \varphi_{ss} + \varphi_{se})$$

$$\varphi_{ke} + \varphi_{pe} + \varphi_{se} + \varphi_{ee} + \varphi_{em} = 1$$

$$>> \varphi_{em} = 1 - (\varphi_{ke} + \varphi_{pe} + \varphi_{se} + \varphi_{ee})$$

$$\varphi_{km} + \varphi_{pm} + \varphi_{sm} + \varphi_{em} + \varphi_{mm} = 1$$

$$>> \varphi_{mm} = 1 - (\varphi_{km} + \varphi_{pm} + \varphi_{sm} + \varphi_{em})$$

$$\varphi_{ky} + \varphi_{py} + \varphi_{sy} + \varphi_{ey} + \varphi_{my} = 0$$

$$>> \varphi_{my} = - (\varphi_{ky} + \varphi_{py} + \varphi_{sy} + \varphi_{ey})$$

$$\varphi_{kt} + \varphi_{pt} + \varphi_{st} + \varphi_{et} + \varphi_{mt} = 0$$

$$>> \varphi_{mt} = - (\varphi_{kt} + \varphi_{pt} + \varphi_{st} + \varphi_{et})$$

(A-2)

Expanding expression A-1 for the five inputs and specifying the restrictions in expression A-2 into the cost function gives the cost equation in expression A-3.

$$
\begin{aligned}
\text{Log } C = {} & \beta_0 + \alpha_y \log Y + 1/2\alpha_{yy} \log(Y)^2 + \alpha_t + 1/2\alpha_{tt}(T)^2 + \\
& \beta_k \log P_k + \beta_p \log P_p + \beta_s \log P_s + \beta_e \log P_e + \log P_m - \\
& \beta_k \log P_m - \beta_p \log P_m - \beta_s - \beta_e \log P_m \; 1/2\varphi_{kk}(\log P_k)^2 + \\
& \varphi_{kp} \log P_k \log P_p + \varphi_{ks} \log P_k \log P_s + \varphi_{ke} \log P_k \log P_e - \\
& \varphi_{kk} \log P_{kk} \log P_m - \varphi_{kp} \log P_k \log P_m - \varphi_{ks} \log P_k \log P_m - \\
& \varphi_{ke} \log P_k \log P_m + 1/2\varphi_{pp}(\log P_p)^2 + \varphi_{ps} \log P_p \log P_s + \\
& \varphi_{pe} \log P_p \log P_e - \varphi_{kp} \log P_p \log P_m - \varphi_{pp} \log P_p \log P_m - \\
& \varphi_{ps} \log P_p \log P_m - \varphi_{pe} \log P_p \log P_m - 1/2\varphi_{ss}(\log P_s)^2 + \\
& \varphi_{se} \log P_s \log P_e - \varphi_{ks} \log P_s \log P_m - \varphi_{ps} \log P_s \log P_m - \\
& \varphi_{ss} \log P_s \log P_m - \varphi_{se} \log P_s \log P_m + 1/2\varphi_{ee}(\log P_e)^2 - \\
& \varphi_{ke} \log P_e \log P_m - \varphi_{pe} \log P_e \log P_m - \varphi_{se} \log P_e \log P_m - \\
& \varphi_{ee} \log P_e \log P_m + 1/2\varphi_{kk}(\log P_m)^2 + 1/2\varphi_{pp}(\log P_p)^2 + \\
& 1/2\varphi_{ss} \log P_m)^2 + 1/2\varphi_{e}(\log P_m)^2 + \varphi_{ke}(\log P_{pm})^2 + \\
& \varphi_{ks}(\log P_m)^2 + \varphi_{kp}(\log P_m)^2 + \varphi_{ps}(\log P_m)^2 + \\
& \varphi_{pe}(\log P_m)^2 + \varphi_{se}(\log P_m)^2 + \varphi_{ky} \log P_k \log Y + \\
& \varphi_{py} \log P_p \log Y + \varphi_{sy} \log P_s \log Y + \varphi_{ey} \log P_e \log Y - \\
& \varphi_{ky} \log P_m \log Y - \varphi_{py} \log P_m \log Y - \varphi_{sy} \log P_m \log Y - \\
& \varphi_{ey} \log P_m \log Y + \varphi_{kt} \log P_k {*} T + \varphi_{pt} \log P_p {*} T + \\
& \varphi_{st} \log P_s {*} T + \varphi_{et} \log P_e {*} T - \varphi_{kt} \log P_m {*} T - \\
& \varphi_{pt} \log P_m {*} T + \varphi_{st} \log P_m {*} T - \varphi_{et} \log P_m {*} T + \varphi_{yt} \log {*} T
\end{aligned}
\tag{A-3}
$$

Combining terms and simplifying gives equation

$\log C - \log P_m = \beta_0 + \alpha_y \log Y + \frac{1}{2} \alpha_{yy}(\log Y)^2 + \alpha_t +$

$\frac{1}{2} \alpha_{tt}(T)^2 + \beta_k(\log P_k - \log P_m)\ \beta_p \log P_p -$

$\log P_m)+\beta_s(\log P_s-\log P_m)+$

$\beta_e(\log P_e-\log P_m)+\varphi_{kk}[1/2(\log P_k)^2-$

$\log P_{kk}\log P_m+1/2(\log P_m)^2]+$

$\varphi_{kp}[\log P_k \log P_p-\log P_p \log P_m-$

$\log P_k \log P_m+(\log P_m)^2]+$

$\varphi_{ks}[\log P_k \log P_s-\log_p k \log P_m-$

$\log P_s \log P_m+(\log P_m)^2]+$

$\varphi k_e[\log_p k \log P_e-\log P_e \log P_m-$

$\log P_e \log P_m+(\log P_m)^2]+$

$\varphi_p p[1/2(\log Pp)^2-\log P_p \log P_m+$

$1/2 \log P_m)^2]+ \varphi ps[\log P_p \log P_s-$

$\log P_p \log P_m-\log P_s \log P_m+(\log P_m)^2]+$

$\varphi_{pt}[\log P_p \log P_e-\log P_p \log P_m-$

$\log P_e \log P_m+(\log P_m)^2]+$

$\varphi_{ss}[1/2(\log P_s)^2-\log P_s \log P_m+$

$\frac{1}{2}(\log P_m)^2]+\varphi_{se}[\log P_s \log P_e -$

$\log P_s \log P_m - \log P_e \log P_m + (\log P_m)^2]+$

$\varphi_{ee}[1/2(\log P_e)^2 - \log P_e \log P_m +$

$\frac{1}{2}(\log P_m)^2] + \varphi_{ky} \log Y[\log P_k-\log P_m] +$

$\varphi_{py}\log Y[\log P_p - \log P_m] +$

$\varphi_{sy}\log Y[\log P_s - \log P_m] + \varphi_{ey}\log Y[\log P_e -$

$\log P_m] + \varphi_{kt}{*}T[\log P_k - \log P_m] +$

$\varphi_{pt}{*}T[\log P_p - \log P_m] +$

$\varphi_{st}{*}T[\log P_s - \log P_m] + \varphi_{et}{*}T[\log P_e -$

$\log P_m] + \varphi_{yt}\log Y{*}T$

(A-4)

Renaming the cross-product terms gives equation A-5 the form of the cost equation used for estimating model using the TSP software. The coding and data set are shown in appendix B.

$\log C = \beta_0 + \alpha_y\log Y + 1/2\alpha_{yy}(\log)2 + \alpha_t + 1/2\alpha_{tt}(T)^2 +$

$\beta k\log(Pk/Pm)\ \beta p\log(Pp/Pm) + \beta s\log(Ps/Pm) +$

$\beta e\log(Pe/Pm) + \varphi kk[X1] + \varphi kp[X2] + \varphi ks[X3] +$

$\varphi ke[X4] + \varphi pp[X5] + \varphi ps[X6] + \varphi pe[X7] +$

$\varphi ss[X8] + \varphi se[X9] + \varphi ee[X10] + \varphi ky[X18] +$

$\varphi py[X19] + \varphi sy[X20] + \varphi ey[X21] + \varphi kt[X14] +$

$\varphi pt[X15] + \varphi st[X16] + \varphi et[X17] + \varphi yt\log Y{*}T$

(A-5)

Where;

$\log C = \log C - \log P_m$

$X_1 = ½(\log P_k)^2 - \log P_{kk}\log P_m + ½(\log P_m)^2$

$X_2 = \log P_k\log P_p - \log P_p\log P_m - \log P_k\log P_m + (\log P_m)^2$

$X_3 = \log P_k\log P_s - \log P_k\log P_m - \log P_s\log P_m + (\log P_m)^2$

$X_4 = \log P_k \log P_e - \log P_k \log P_m - \log P_e \log P_m + (\log P_m)^2$

$X_5 = ½(\log P_p)^2 - \log P_p \log P_m + ½(\log P_m)^2$

$X_6 = \log P_p \log P_s - \log P_p \log P_m - \log P_s \log P_m + (\log P_m)^2$

$X_7 = \log P_p \log P_e - \log P_p \log P_m - \log P_e \log P_m + (\log P_m)^2$

$X_8 = ½(\log P_s)^2 - \log P_s \log P_m + ½(\log P_m)^2$

$X_9 = \log P_s \log P_e - \log P_s \log P_m - \log P_e \log P_m + (\log P_m)^2$

$X_{10} = ½(\log P_e)^2 - \log P_e \log P_m + ½(\log P_m)^2$

$X_{11} = \log P_k * \log S - \log P_m * \log S$

$X_{12} = \log P_p * \log S - \log P_m * \log S$

$X_{13} = \log P_e * \log S - \log P_m * \log S$

$X_{14} = \log P_k * T - \log P_m * T$

$X_{15} = \log P_p * T - \log P_m * T$

$X_{16} = \log P_s * T - \log P_m * T$

$X_{17} = \log P_e * T - \log P_m * T$

$X_{18} = \log P_k * LNY - \log P_m * LNY$

$X_{18} = \log P_p * LNY - \log P_m * LNY$

$X_{20} = \log P_s * LNY - \log P_m * LNY$

$X_{21} = \log P_e * LNY - \log P_m * LNY$

APPENDIX B

Time Series Processor Coding for Input Substitution Study - U.S. Copper Smelting Industry

```
 PROGRAM LINE
**************************************************************
     1  OPTIONS CRT;
     2  FREQ A;
     3  SMPL 60 91;
     4  LOAD(FILE='C:\QPW\KLEM\CAPITAL.WQ1')NCAP,MOODY,PPI;
     5  LOAD(FILE='C:\QPW\KLEM\LABOR.WQ1')QP,PHW_R,QS,SAW_R;
     6  LOAD(FILE='C:\QPW\KLEM\OUTPUT.WQ1')SMLT_TTL,PYTON,T,A;
     7  LOAD(FILE='C:\QPW\KLEM\ENGY_MAT.WQ1')T_BTU,P_TBTU,ORE_PRD,
     7      DLR_TN_O;
     8  ?
     8  GENR QY=SMLT_TTL;  ?MILLIONS SHORT TONS SMELTER OUTPUT
     9  GENR PY=PYTON;  ? SETS VALUE TO $/SHORT TON OF COPPER
    10  CAPITL(BENCHVAL=1782.2,BENCHOBS=80)NCAP,.01,KINV;
    11  GENR PK=MOODY/100;
    12  GENR QK=((KINV/(PPI/100))*1000000);  ? CAPITAL STOCK, 1982 DOLLARS
    13  GENR QP=QP*1000000; ? MILLION HRS OF PRODUCTION LABOR
    14  GENR PP=PHW_R; ? PRODUCTION WAGE RATE (REAL) $/HOUR
    15  GENR QS=QS; ?THOUSAND SALARIED (NON-PRODUCTION) WORKERS
    16  GENR PS=SAW_R; ? REAL AVERAGE ANNUAL SALARY, NON-PROD'
    17  GENR QE=T_BTU;   ? SETS VALUES TO TRILLION BTUs FUEL QUANTITY
    18  GENR PE=P_TBTU;? PRICE PER MILLION BTUs
    19  GENR QM=ORE_PRD;
    20  GENR PM=DLR_TN_O;
    21  ?
    21  GENR C5 = ((QK*PK)+(QP*PP)+(QS*PS)+(QE*PE)+(QM*PM));
    22  ?
    22  TITLE 'COPPER SMELTING / REFINING - AUTOCORRELATION CORRECTION
    22  MODEL';
    23  SET RHO = 0.95;
    24  ?
    24  ? SHARE EQUATIONS
    24  GENR SK = (QK*PK)/((QK*PK)+(QP*PP)+(QS*PS)+(QE*PE)+(QM*PM));
    25  GENR SP = (QP*PP)/((QK*PK)+(QP*PP)+(QS*PS)+(QE*PE)+(QM*PM));
    26  GENR SS = (QS*PS)/((QK*PK)+(QP*PP)+(QS*PS)+(QE*PE)+(QM*PM));
    27  GENR SE = (QE*PE)/((QK*PK)+(QP*PP)+(QS*PS)+(QE*PE)+(QM*PM));
    28  GENR SM = (QM*PM)/((QK*PK)+(QP*PP)+(QS*PS)+(QE*PE)+(QM*PM));
    29  SMPL 61 91;
    30  ? AUTOCORRELATION CORRECTION FOR SHARES
    30  GENR ZSK = SK-RHO*SK(-1);
    31  GENR ZSP = SP-RHO*SP(-1);
    32  GENR ZSS = SS-RHO*SS(-1);
    33  GENR ZSE = SE-RHO*SE(-1);
    34  GENR ZSM = SM-RHO*SM(-1);
    35  ?
    35  ? LOGARITHMIC TRANSFORMATIONS AND INPUT PRICES AND (P#/PM)
    35  SMPL 60 91;
    36  GENR LNC5 = LOG(C5);
    37  GENR LNC5M= LNC5-LOG(PM);
    38  GENR LNPK = LOG(PK);
    39  GENR LNPP = LOG(PP);
```

```
40  GENR LNPS = LOG(PS);
41  GENR LNPE = LOG(PE);
42  GENR LNPM = LOG(PM);
43  GENR LNPM2 = LOG(PM)**2;
44  GENR LNQY = LOG(QY);
45  GENR LNQY2 = LOG(QY)**2;
46  GENR LNPKM = LOG(PK/PM);
47  GENR LNPPM = LOG(PP/PM);
48  GENR LNPSM = LOG(PS/PM);
49  GENR LNPEM = LOG(PE/PM);
50  GENR LNT = LOG(T);
51  GENR LNT2 = LOG(T)**2;
52  GENR LNA = LOG(A);
53  GENR LNA2 = LOG(A)**2;
54  ?
54  ? AUTOCORRELATION CORRECTION OF LOGARITHMIC
54  ? TRANSFORMATIONS FOR SHARE AND COST EQUATIONS
54  SMPL 61 91;
55  GENR LGC5=LNC5-RHO*LNC5(-1);
56  GENR LGC5M=LNC5M-RHO*LNC5M(-1);
57  GENR LGPK=LNPK-RHO*LNPK(-1);
58  GENR LGPP=LNPP-RHO*LNPP(-1);
59  GENR LGPS=LNPS-RHO*LNPS(-1);
60  GENR LGPE=LNPE-RHO*LNPE(-1);
61  GENR LGPM=LNPM-RHO*LNPM(-1);
62  GENR LGPM2=LNPM2-RHO*LNPM2(-1);
63  GENR LGQY =LNQY-RHO*LNQY(-1);
64  GENR LGQY2 =LNQY2-RHO*LNQY2(-1);
65  GENR LGPKM =LNPKM-RHO*LNPKM(-1);
66  GENR LGPPM =LNPPM-RHO*LNPPM(-1);
67  GENR LGPSM =LNPSM-RHO*LNPSM(-1);
68  GENR LGPEM =LNPEM-RHO*LNPEM(-1);
69  GENR LGT =LNT-RHO*LNT(-1);
70  GENR LGT2 =LNT2-RHO*LNT2(-1);
71  GENR LGA =LNA-RHO*LNA(-1);
72  GENR LGA2=LNA2-RHO*LNA2(-1);
73  ?
73  ? CROSS PRODUCT TERMS FOR SHARE AND COST EQUATIONS
73  SMPL 60 91;
74  GENR X1=.5*LNPK**2-LNPK*LNPM+.5*LNPM2;
75  GENR X2=LNPK*LNPP-LNPK*LNPM-LNPP*LNPM+LNPM2;
76  GENR X3=LNPK*LNPS-LNPK*LNPM-LNPS*LNPM+LNPM2;
77  GENR X4=LNPK*LNPE-LNPK*LNPM-LNPE*LNPM+LNPM2;
78  GENR X5=.5*LNPP**2-LNPP*LNPM+.5*LNPM2;
79  GENR X6=LNPP*LNPS-LNPP*LNPM-LNPS*LNPM+LNPM2;
80  GENR X7=LNPP*LNPE-LNPP*LNPM-LNPE*LNPM+LNPM2;
81  GENR X8=.5*LNPS**2-LNPS*LNPM+.5*LNPM2;
82  GENR X9=LNPS*LNPE-LNPS*LNPM-LNPE*LNPM+LNPM2;
83  GENR X10=.5*LNPE**2-LNPE*LNPM+.5*LNPM2;
84  GENR X11=LNPK*LNQY-LNQY*LNPM;
85  GENR X12=LNPP*LNQY-LNQY*LNPM;
86  GENR X13=LNPS*LNQY-LNQY*LNPM;
87  GENR X14=LNPE*LNQY-LNQY*LNPM;
88  GENR X15=LNPK*LNA-LNA*LNPM;
89  GENR X16=LNPP*LNA-LNA*LNPM;
90  GENR X17=LNPS*LNA-LNA*LNPM;
91  GENR X18=LNPE*LNA-LNA*LNPM;
92  GENR X19=LNPK*LNT-LNT*LNPM;
93  GENR X20=LNPP*LNT-LNT*LNPM;
```

```
94  GENR X21=LNPS*LNT-LNT*LNPM;
95  GENR X22=LNPE*LNT-LNT*LNPM;
96  ?
96  ? AUTOCORRELATION CORRECTION OF CROSS PRODUCT TERMS
96  ? FOR SHARE EQUATIONS
96  SMPL 61 91;
97  GENR y1=X1-rho*x1(-1);
98  GENR y2=X2-rho*x2(-1);
99  GENR y3=X3-rho*x3(-1);
100  GENR y4=X4-rho*x4(-1);
101  GENR y5=X5-rho*x5(-1);
102  GENR y6=X6-rho*x6(-1);
103  GENR y7=X7-rho*x7(-1);
104  GENR y8=X8-rho*x8(-1);
105  GENR y9=X9-rho*x9(-1);
106  GENR y10=x10-rho*x10(-1);
107  GENR y11=X11-rho*x11(-1);
108  GENR y12=X12-rho*x12(-1);
109  GENR y13=X13-rho*x13(-1);
110  GENR y14=X14-rho*x14(-1);
111  GENR y15=X15-rho*x15(-1);
112  GENR y16=X16-rho*x16(-1);
113  GENR y17=X17-rho*x17(-1);
114  GENR y18=X18-rho*x18(-1);
115  GENR y19=X19-rho*x19(-1);
116  GENR y20=X20-rho*x20(-1);
117  GENR y21=X21-rho*x21(-1);
118  GENR y22=X22-rho*x22(-1);
119  ?
119  ?                 72 CHARACTER MEASURE
119  ?-----------------------------------------------------]
119  ?INITIALIZE PARAMETERS
119  PARAM
119  aO 0 aK 0 aP 0 aS 0 aE 0 aM 0
119  aY 0 aA 0 aT 0
119  gKK 0 gKP 0 gKS 0 gKE 0 gKM 0
119  gPP 0 gPS 0 gPE 0 gPM 0
119  gSS 0 gSE 0 gSM 0
119  gEE 0
119  gEM 0 gMM 0
119  dKY 0  dKA 0 dKT 0
119  dPY 0 dPA 0 dPT 0
119  dSY 0 dSA 0 dST 0
119  dEY 0 dEA 0 dET 0
119  dMY 0 dMA 0
119  rYY 0 rYA 0 rYT 0
119  rAA 0 rTT 0 rAT 0;
120  ?
120  ? CONTROL AND CHOICE OF CONSTANTS wrt Y, A, AND T
120  ?CONST aT 0 rTT 0 dKT 0 dPT 0 dST 0 dET 0
120  ?     gKT 0 gPT 0 gST 0 gET 0 gMT 0;
120  ?CONST aA 0 rAA 0 dKA 0 dPA 0 dSA 0 dEA 0
120  ?     gKA 0 gPA 0 gSA 0 gEA 0 gMA 0;
120  ?CONST aY 0 rYY 0 dKY 0 dPY 0 dSY 0 dEY 0
120  ?     gKY 0 gPY 0 gSY 0 gEY 0 dMY 0;
120  ?CONST rYA 0;
120  ?CONST rAT 0;
120  ?CONST rYT 0;
120  ?CONST dKY 0 dPY 0 dSY 0 dEY 0 dMY 0;
```

```
120  ?CONST dKA 0 dPA 0 dSA 0 dEY 0 dMY 0;
120  ?CONST dKT 0 dPT 0 dST 0 dET 0 dMT 0;
120  ?
120  ?EQUATIONS TO BE SOLVED
120  FRML EQN1 LGC5M = aO*(1-RHO)+
120      aY*LGQY+aA*LGA+aT*LGT+
120      aK*LGPKM+aP*LGPPM+aS*LGPSM+aE*LGPEM+
120      gKK*Y1+gKP*Y2+gKS*Y3+gKE*Y4+
120      gPP*Y5+gPS*Y6+gPE*Y7+
120      gSS*Y8+gSE*Y9+
120      gEE*Y10+
120      dKY*Y11+dPY*Y12+dSY*Y13+dEY*Y14+
120      dKA*Y15+dPA*Y16+dSA*Y17+dEA*Y18+
120      dKT*Y19+dPT*Y20+dST*Y21+dET*Y22+
120      rYY*LGQY2+rAA*LGA2+rTT*LGT2+
120      rYA*(LGQY*LGA)+rYT*(LGQY*LGT)+rAT*(LGA*LGT);
121  ?
121  FRML EQN2
121  ZSK=aK*(1-RHO)+gKK*LGPKM+gKP*LGPPM+gKS*LGPSM+gKE*LGPEM+
121      dKY*LGQY+dKA*LGA+dKT*LGT;
122  ?
122  FRML EQN3
122  ZSP=aP*(1-RHO)+gKP*LGPKM+gPP*LGPPM+gPS*LGPSM+gPE*LGPEM+
122      dPY*LGQY+dPA*LGA+dPT*LGT;
123  ?
123  FRML EQN4
123  ZSS=aS*(1-RHO)+gKS*LGPKM+gPS*LGPPM+gSS*LGPSM+gSE*LGPEM+
123      dSY*LGQY+dSA*LGA+dST*LGT;
124  ?
124  FRML EQN5
124  ZSE=aE*(1-RHO)+gKE*LGPKM+gPE*LGPPM+gSE*LGPSM+gEE*LGPEM+
124      dEY*LGQY+dEA*LGA+dET*LGT;
125  ?
125  ? COMMAND TO ESTIMATE BY SUR
125  LSQ (MAXIT=500) EQN1 EQN2 EQN3 EQN4 EQN5;
126  ?
126  ? ESTMATE MISSING PARAMETERS
126  FRML EQN6  aM  = 1-(aK+aP+aS+aE);
127  FRML EQN7  gKM = -(gKK+gKP+gKS+gKE);
128  FRML EQN8  gPM = -(gKP+gPP+gPS+gPE);
129  FRML EQN9  gSM = -(gKS+gPS+gSS+gSE);
130  FRML EQN10 gEM = -(gKE+gPE+gSE+gEE);
131  FRML EQN11 gMM = gKK+gpp+gSS+gEE+
131              2*(gKP+gKS+gKE+gPS+gPE+gSE);
132  FRML EQN12 dMY = -(dKY+dPY+dSY+dEY);
133  FRML EQN13 dMA = -(dKA+dPA+dSA+dEA);
134  FRML EQN14 dMT = -(dKT+dPT+dST+dET);
135  ?
135  ? VALUES, COVARIANCE, MATRIX AND THE WALD TEST
135  ? THAT THE FUNCTIONS ARE JOINTLY ZERO
135  ANALYZ EQN6 EQN7 EQN8 EQN9 EQN10 EQN11 EQN12 EQN13
135      EQN14;
136  ?
136  ? TEST FOR MONOTONICITY ( COMPARE SHARES W/ FITTED SHARES )
136  GENR XZSK=aK*(1-RHO)+gKK*LGPKM+gKP*LGPPM+gKS*LGPSM+gKE*LGPEM+
136      dKY*LGQY+dKA*LGA+dKT*LGT;
137  GENR XZSP=aP*(1-RHO)+gKP*LGPKM+gPP*LGPPM+gPS*LGPSM+gPE*LGPEM+
137      dPY*LGQY+dPA*LGA+dPT*LGT;
138  GENR XZSS=aS*(1-RHO)+gKS*LGPKM+gPS*LGPPM+gSS*LGPSM+gSE*LGPEM+
```

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138      dSY*LGQY+dSA*LGA+dST*LGT;
139  GENR XZSE=aE*(1-RHO)+gKE*LGPKM+gPE*LGPPM+gSE*LGPSM+gEE*LGPEM+
139      dEY*LGQY+dEA*LGA+dET*LGT;
140  PRINT ZSK XZSK ZSP XZSP;
141  PRINT ZSS XZSS ZSE XZSE;
142  ?
142  GENR SUM_S= SK+SP+SS+SE+SM;
143  PRINT SK SP SS SE SM SUM_S;
144  ? ALLEN ELASTICITIES (OWN)
144  GENR AEKK = (GKK+(SK**2)-SK)/SK**2;
145  GENR AEPP = (GPP+(SP**2)-SP)/SP**2;
146  GENR AESS = (GSS+(SS**2)-SS)/SS**2;
147  GENR AEEE = (GEE+(SE**2)-SE)/SE**2;
148  GENR AEMM = (GMM+(SM**2)-SM)/SM**2;
149  PRINT AEKK AEPP AESS AEEE AEMM;
150  ?
150  ? ALLEN ELASTICITIES (CROSS)
150  GENR AEKP = (GKP+(SK*SP))/(SK*SP);
151  GENR AEKS = (GKS+(SK*SS))/(SK*SS);
152  GENR AEKE = (GKE+(SK*SE))/(SK*SE);
153  GENR AEKM = (GKM+(SK*SM))/(SK*SM);
154  GENR AEPS = (GPS+(SP*SS))/(SP*SS);
155  GENR AEPE = (GPE+(SP*SE))/(SP*SE);
156  GENR AEPM = (GPM+(SP*SM))/(SP*SM);
157  GENR AEES = (GSE+(SS*SE))/(SS*SE);
158  GENR AEEM = (GEM+(SE*SM))/(SE*SM);
159  GENR AEMS = (GSM+(SM*SS))/(SM*SS);
160  PRINT AEKP AEKS AEKE AEKM AEPS AEPE AEPM
160       AEES AEEM AEMS;
161  ?COMPUTE COST ELASTICITIES
161  ?   ELASTICITY OF COST wrt OUTPUT
161  GENR eCY = aY+(rYY*LGQY)+(rYA*LGA)+(rYT*LGT)+(dKY*LGPK)+
161      (dPY*LGPP)+(dSY*LGPS)+(dEY*LGPE)+(dMY*LGPM);
162  ?   ELASTICITY OF COST wrt ABATEMENT
162  GENR eCA = aA+(rYA*LNQY)+(rAA*LGA)+(rAT*LGT)+(dKA*LGPK)+
162     (dPA*LGPP)+(dSA*LGPS)+(dEA*LGPE)+(dMA*LGPM);
163  ?   ELASTICITY OF COST wrt TECHNICAL CHANGE
163  GENR eCT =aT+(rTT*LGT)+(rYT*LGQY)+(rAT*LGA)+(dKT*LGPK)+
163     (dPT*LGPP)+(dST*LGPS)+(dET*LGPE)+(dMT*LGPM);
164  ? MUST COMPUTE THETA(x) OLSQ LOG  C LOG T???
164  ?
164  PRINT eCY eCA eCT;
165  ?
165  ?PRINT QY PY QK PK QP PP QS PS QE PE QM PM A T;
165
165
165  ?
165  SMPL 61 91;
166  ? COMPENSATED PRICE ELASTCITIES OF INPUT DEMAND
166  GENR NKK = (GKK+SK*(SK-1))/SK;
167  GENR NPP = (GPP+SP*(SP-1))/SP;
168  GENR NSS = (GSS+SS*(SS-1))/SS;
169  GENR NEE = (GEE+SE*(SE-1))/SE;
170  GENR NMM = (GMM+SM*(SM-1))/SM;
171  PRINT NKK NPP NSS NEE NMM;
172  ?
172  GENR NKP = (GKP+(SK*SP))/SK;
173  GENR NKS = (GKS+(SK*SS))/SK;
174  GENR NKE = (GKE+(SK*SE))/SK;
```

```
175  GENR NKM = (GKM+(SK*SM))/SK;
176  GENR NPK = (GKP+(SP*SK))/SP;
177  GENR NPS = (GPS+(SP*SS))/SP;
178  GENR NPE = (GPE+(SP*SE))/SP;
179  GENR NPM = (GPM+(SP*SM))/SP;
180  GENR NSK = (GKS+(SS*SK))/SS;
181  GENR NSP = (GPS+(SS*SP))/SS;
182  GENR NSE = (GSE+(SS*SE))/SS;
183  GENR NSM = (GSM+(SS*SM))/SS;
184  GENR NEK = (GKE+(SE*SK))/SE;
185  GENR NEP = (GPE+(SE*SP))/SE;
186  GENR NES = (GSE+(SE*SS))/SE;
187  GENR NEM = (GEM+(SE*SM))/SE;
188  GENR NMK = (GKM+(SM*SK))/SM;
189  GENR NMP = (GPM+(SM*SP))/SM;
190  GENR NMS = (GSM+(SM*SS))/SM;
191  GENR NME = (GEM+(SM*SE))/SM;
192  PRINT NKP NKS NKE NKM NPK NPS NPE NPM
192    NSK NSP NSE NSM NEK NEP NES NEM NMK
192    NMP NMS NME;
193  ?
193  ? ELASTICITIES OF INPUT DEMAND wrt Y, A, T
193  GENR NKY = (dKY/SK)+eCY;
194  GENR NPY = (dPY/SP)+eCY;
195  GENR NSY = (dSY/SS)+eCY;
196  GENR NEY = (dEY/SE)+eCY;
197  GENR NMY = (dMY/SM)+eCY;
198  ?
198  GENR NKA = (dKA/SK)+eCY;
199  GENR NPA = (dPA/SP)+eCY;
200  GENR NSA = (dSA/SS)+eCY;
201  GENR NEA = (dEA/SE)+eCY;
202  GENR NMA = (dMA/SM)+eCY;
203  ?
203  GENR NKT = (dKT/SK)+eCY;
204  GENR NPT = (dPT/SP)+eCY;
205  GENR NST = (dST/SS)+eCY;
206  GENR NET = (dET/SE)+eCY;
207  GENR NMT = (dMT/SM)+eCY;
208  PRINT NKY NPY NSY NEY NMY NKA NPA
208  NSA NEA NMA NKT NPT NST NET NMT;
209  ?
209  ? UNCOMPENSATED PRICE ELASTICITIES
209  GENR NUKK = NKK + (NKY/eCY);
210  GENR NUPP = NPP + (NPY/eCY);
211  GENR NUSS = NSS + (NSY/eCY);
212  GENR NUEE = NEE + (NEY/eCY);
213  GENR NUMM = NMM + (NMY/eCY);
214  ?
214  GENR NUKP = NKP + (NKY/eCY);
215  GENR NUKS = NKS + (NKY/eCY);
216  GENR NUKE = NKE + (NKY/eCY);
217  GENR NUKM = NKM + (NKY/eCY);
218  GENR NUPK = NPK + (NPY/eCY);
219  GENR NUPS = NPS + (NPY/eCY);
220  GENR NUPE = NPE + (NPY/eCY);
221  GENR NUPM = NPM + (NPY/eCY);
222  GENR NUSK = NSK + (NSY/eCY);
223  GENR NUSP = NSP + (NSY/eCY);
```

```
224  GENR NUSE = NSE + (NSY/eCY);
225  GENR NUSM = NSM + (NSY/eCY);
226  GENR NUEK = NEK + (NEY/eCY);
227  GENR NUEP = NEP + (NEY/eCY);
228  GENR NUES = NES + (NEY/eCY);
229  GENR NUEM = NEM + (NEY/eCY);
230  GENR NUMK = NMK + (NMY/eCY);
231  GENR NUMP = NMP + (NMY/eCY);
232  GENR NUMS = NMS + (NMY/eCY);
233  GENR NUME = NME + (NMY/eCY);
234  PRINT NUKK NUPP NUSS NUEE NUMM NUKP
234  NUKS NUKE NUKM NUPK NUPS NUPE NUPM
234  NUSK NUSP NUSE NUSM NUEK NUEP NUES
234  NUEM NUMK NUMP NUMS NUME;
235  ?
235  PRINT QY PY QK PK QP PP QS PS QE PE
235      QM PM A T;
236  ? TO DETERMINE STRICT QUASICONCAVITY
236  TREND T;
237  SET NOB = @NOB;
238  DO I=1,NOB;
239    SELECT T = I;
240  ?
240  MMAKE M21 AEKK AEKP AEKP AEPP;
241  MMAKE M22 AEPP AEPS AEPS AESS;
242  MMAKE M23 AESS AEES AEES AEEE;
243  MMAKE M24 AEEE AEEM AEEM AEMM;
244  MMAKE M25 AEMM AEKM AEKM AEKK;
245  MMAKE M26 AEKK AEKS AEKS AESS;
246  ?
246  MMAKE M31 AEKK AEKP AEKS AEKP AEPP AEPS AEKS AEPS AESS;
247  MMAKE M32 AEPP AEPS AEPE AEPS AESS AEPM AEPE AEPM AEEE;
248  MMAKE M33 AESS AEES AEMS AEES AEEE AEEM AEMS AEEM AEMM;
249  MMAKE M34 AEEE AEEM AEKE AEEM AEMM AEKM AEKE AEKM AEKK;
250  MMAKE M35 AEMM AEKM AEPM AEKM AEKK AEKP AEPM AEKP AEPP;
251  ?
251  MMAKE M41 AEKK AEKP AEKS AEKE AEKP AEPP AEPS AEPE AEKS AEPS
251      AESS AEES AEKE AEPE AEES AEEE;
252  MMAKE M42 AEPP AEPS AEPE AEPM AEPS AESS AEES AEMS AEPE AEES
252      AEEE AEEM AEPM AEMS AEEM AEMM;
253  MMAKE M43 AESS AEES AEMS AEKS AEES AEEE AEEM AEKE AEMS AEEM
253      AEMM AEKM AEKS AEKE AEKM AEKK;
254  MMAKE M44 AEEE AEEM AEKE AEPE AEEM AEMM AEKM AEPM AEKE AEKM
254      AEKK AEKP AEPE AEPM AEKP AEPP;
255  MMAKE M45 AEMM AEKM AEPM AEMS AEKM AEKK AEKP AEKS AEPM AEKP
255      AEPP AEPS AEMS AEKS AEPS AESS;
256  ?
256  MMAKE M51 AEKK AEKP AEKS AEKE AEKM AEKP AEPP AEPS AEPE AEPM
256      AEKS AEPS AESS AEES AEMS AEKE AEPE AEES AEEE AEEM
256      AEKM AEPM AEMS AEEM AEMM;
257  ?
257  MFORM(TYPE=GEN,NROW=2,NCOL=2)M21;
258  MFORM(TYPE=GEN,NROW=2,NCOL=2)M22;
259  MFORM(TYPE=GEN,NROW=2,NCOL=2)M23;
260  MFORM(TYPE=GEN,NROW=2,NCOL=2)M24;
261  MFORM(TYPE=GEN,NROW=2,NCOL=2)M25;
262  MFORM(TYPE=GEN,NROW=2,NCOL=2)M26;
263  ?
263  MFORM(TYPE=GEN,NROW=3,NCOL=3)M31;
```

```
MFORM(TYPE=GEN,NROW=3,NCOL=3)M32;
MFORM(TYPE=GEN,NROW=3,NCOL=3)M33;
MFORM(TYPE=GEN,NROW=3,NCOL=3)M34;
MFORM(TYPE=GEN,NROW=3,NCOL=3)M35;
?
MFORM(TYPE=GEN,NROW=4,NCOL=4)M41;
MFORM(TYPE=GEN,NROW=4,NCOL=4)M42;
MFORM(TYPE=GEN,NROW=4,NCOL=4)M43;
MFORM(TYPE=GEN,NROW=4,NCOL=4)M44;
MFORM(TYPE=GEN,NROW=4,NCOL=4)M45;
?
?
?MSD(TERSE,NOPRINT)AEKK AEKP AEKS AEKE AEKM AEKP AEPP AEPS AEPE
?AEPM
    ?AEKS AEPS AESS AEES AEMS AEKE AEPE AEES AEEE AEEM
    ?AEKM AEPM AEMS AEEM AEMM;
MFORM(TYPE=GEN,NROW=5,NCOL=5)M51;
?
MAT D21=DET(M21);
MAT D22=DET(M22);
MAT D23=DET(M23);
MAT D24=DET(M24);
MAT D25=DET(M25);
MAT D26=DET(M26);
?
MAT D31=DET(M31);
MAT D32=DET(M32);
MAT D33=DET(M33);
MAT D34=DET(M34);
MAT D35=DET(M35);
?
MAT D41=DET(M41);
MAT D42=DET(M42);
MAT D43=DET(M43);
MAT D44=DET(M44);
MAT D45=DET(M45);
?
MAT D51=DET(M51);
?
PRINT D21 D22 D23 D24 D25 D26;
PRINT D31 D32 D33 D34 D35;
PRINT D41 D42 D43 D44 D45;
PRINT D51;
?
ENDDO;
 EXECUTION
```

APPENDIX C

Data Set

	QY	PY	QK	PK	QP
1961	1285700.00000	1994.73328	1.59509D+09	0.050800	1.29000D+07
1962	1341500.00000	2040.00000	1.62947D+09	0.050200	1.28000D+07
1963	1355900.00000	2040.00000	1.64788D+09	0.048600	1.28000D+07
1964	1385500.00000	2088.88892	1.63144D+09	0.048300	1.28000D+07
1965	1460800.00000	2244.67944	1.61355D+09	0.048700	1.31000D+07
1966	1504300.00000	2274.84277	1.61979D+09	0.056700	1.34000D+07
1967	933000.00000	2366.93506	1.70230D+09	0.062300	9200000.00000
1968	1351300.00000	2505.80835	1.78456D+09	0.069400	1.16000D+07
1969	1662800.00000	2700.79541	1.80365D+09	0.078100	1.46000D+07
1970	1720200.00000	3118.91895	1.88253D+09	0.091100	1.51000D+07
1971	1566300.00000	2699.89502	1.99546D+09	0.085600	1.35000D+07
1972	1759400.00000	2537.19287	2.10668D+09	0.081600	1.44000D+07
1973	1821800.00000	2730.95117	2.20849D+09	0.082400	1.41000D+07
1974	1649400.00000	2767.11182	1.93798D+09	0.095000	1.40000D+07
1975	1496500.00000	2089.96704	1.96728D+09	0.10610	1.21000D+07
1976	1585700.00000	2124.19751	2.08140D+09	0.097500	1.13000D+07
1977	1484600.00000	1874.87183	1.97672D+09	0.089700	1.06000D+07
1978	1480000.00000	1707.37537	2.09919D+09	0.094900	1.06000D+07
1979	1538700.00000	2115.32642	1.95703D+09	0.106909	800000.00000
1980	1161000.00000	2088.89795	1.83543D+09	0.136708	100000.50000
1981	1518600.00000	1663.23730	1.81289D+09	0.160408	500000.00000
1982	1125200.00000	1458.18005	1.89432D+09	0.16110	5900000.00000
1983	1088000.00000	1511.86414	1.93027D+09	0.13550	5300000.00000
1984	1304500.00000	1272.77405	2.13613D+09	0.14190	4600000.00000
1985	1312700.00000	1269.42883	2.32922D+09	0.12720	3400000.00000
1986	1318300.00000	1277.70752	2.39578D+09	0.10390	3100000.00000
1987	1376500.00000	1527.23169	2.27315D+09	0.10580	2600000.00000
1988	1515200.00000	2006.85767	2.04174D+09	0.10830	2800000.00000
1989	1630900.00000	2096.01953	1.97290D+09	0.10180	3000000.00000
1990	1613000.00000	2017.63049	2.03697D+09	0.10360	3600000.00000
1991	1639400.00000	1853.05090	2.14367D+09	0.098000	3500000.00000

ABBREVIATIONS:

QY = Quantity With Respect To Input
PY = Price With Respect To Input
QK = Quantity With Respect To Capital
PK = Price With Respect To Capital
QP = Quantity With Respect To Production

	PP	QS	PS	QE	PE
1961	9.31310	2510.00024	27182.24023	144.03812	327600.00000
1962	9.64085	2455.99927	27234.30273	150.28944	327600.00000
1963	9.82955	2300.00024	27564.64844	151.90269	325260.00000
1964	10.02581	2099.99951	28725.05273	155.21881	315900.00000
1965	10.08055	2099.99951	30234.32422	163.65472	322920.00000
1966	10.20295	2400.00049	28034.96484	168.52808	329940.00000
1967	10.09032	2400.00049	24451.08984	104.52483	336960.00000
1968	10.39776	2199.99976	28343.78125	151.38734	334620.00000
1969	10.42328	2500.00000	28010.89453	186.28497	341640.00000
1970	10.28426	2899.99951	27550.66211	192.71555	358020.00000
1971	10.79551	3000.00000	25925.92578	175.40689	388440.00000
1972	11.51993	2800.00122	29904.29492	188.80836	400140.00000
1973	11.74332	2899.99951	29046.28516	176.37431	453960.00000
1974	12.15584	3500.00000	27296.43945	105.98755	704340.00000
1975	12.76435	3399.99951	27826.37305	95.48364	828360.00000
1976	13.20435	3000.00000	30462.79883	81.86430	896220.00000
1977	13.41760	2500.00000	32871.28906	69.44312	1020240.00000
1978	13.41116	2399.99951	33167.17969	72.87432	1088100.00000
1979	13.76700	2099.99951	35484.73047	59.32193	1378260.00000
1980	13.43042	2199.99976	36021.62891	26.36779	1937520.00000
1981	13.87831	2199.99976	33903.39453	18.28340	2344680.00000
1982	14.59413	1699.99976	29198.41406	18.64662	2340000.00000
1983	14.27934	1399.99963	35929.43750	21.48973	2244060.00000
1984	13.80882	1400.00012	30798.84375	19.79220	2218320.00000
1985	13.32972	1199.99976	29739.77734	14.31044	2138760.00000
1986	13.56448	800.00018	31364.04492	23.03858	1633320.00000
1987	12.35780	700.00006	31187.12500	17.34959	1642680.00000
1988	12.37773	799.99994	30114.11719	20.03903	1560780.00000
1989	10.88710	799.99994	30141.13281	17.96789	1705860.00000
1990	10.99971	1000.00000	26625.86328	17.51815	1923480.00000
1991	10.74722	1000.00000	27753.62500	16.50242	1900080.00000

ABBREVIATIONS:

PP = Price With Respect to Production Labor
QS = Quantity With Respect To Non-Production Labor
PS = Price With Respect To Non-Production Labor
QE = Quantity With Respect To Energy
PE = Price With Respect To Energy

	QM	PM	A	T
1961	1.42722D+08	14.96050	362630.00000	1.11000
1962	1.50217D+08	15.30000	403680.00000	1.12000
1963	1.46450D+08	15.09600	358500.00000	1.13000
1964	1.55200D+08	15.24889	330270.00000	1.14000
1965	1.73286D+08	15.71276	369320.00000	1.15000
1966	1.86966D+08	15.24145	469730.00000	1.16000
1967	1.27066D+08	14.91169	348500.00000	1.17000
1968	1.70054D+08	15.03485	483110.00000	1.18000
1969	2.23752D+08	16.20477	685780.00000	1.19000
1970	2.57729D+08	18.40162	747784.00000	1.77383
1971	2.42656D+08	14.84942	803284.00000	1.89962
1972	2.66381D+08	13.95456	1010614.00000	1.37124
1973	2.89998D+08	14.47404	1088322.00000	1.73784
1974	2.93443D+08	13.55885	1277440.00000	1.58306
1975	2.63000D+08	9.82285	1784744.00000	2.20937
1976	2.83733D+08	10.83341	2281591.00000	5.20008
1977	2.62175D+08	9.74933	2357366.00000	6.94186
1978	2.63722D+08	8.70761	2738236.00000	4.92915
1979	3.05924D+08	9.94203	2770119.00000	4.69023
1980	2.44266D+08	10.02671	2312286.00000	6.43878
1981	3.06080D+08	8.48251	2859104.00000	7.27537
1982	2.00557D+08	8.01999	2072256.00000	7.14889
1983	1.96132D+08	7.71051	2025837.00000	6.04886
1984	1.89391D+08	7.38209	2251312.00000	6.33572
1985	1.78804D+08	7.87046	2230257.00000	5.95259
1986	1.86551D+08	7.92179	2308804.00000	7.52792
1987	2.23361D+08	8.70522	2542602.00000	9.47155
1988	2.46448D+08	12.04115	2892655.00000	10.95801
1989	2.61577D+08	12.78572	3075859.00000	13.74705
1990	2.75024D+08	12.50931	3380940.00000	16.31985
1991	3.08524D+08	10.56239	3819439.00000	18.11073

ABBREVIATIONS:

QM = Quantity With Respect To Materials (Ore)
PM = Price With Respect To Materials (Ore)
A = Pollution Abatement
T = Technical Change

APPENDIX D

Monotonicity

	ZSK	XZSK	ZSP	XZSP
1961	- 0.00034926	0.010974	-0.002991	0.0003271
1962	- 0.00017183	-0.004726	0.000556	-0.0007191
1963	0.0019967	0.004904	0.004988	-0.0014266
1964	- 0.00070956	0.003921	0.000579	-0.0022187
1965	- 0.0021341	-0.003908	-0.002137	-0.0016601
1966	0.0042135	0.008208	0.001575	0.0006854
1967	0.021224	0.030770	0.001861	0.0015928
1968	- 0.0035581	-0.005163	0.001074	-0.0029167
1969	- 0.0056138	-0.008803	-0.001767	-0.0026576
1970	- 0.0001567	0.002957	-0.005914	-0.0058718
1971	0.010791	0.014031	0.007574	0.0065148
1972	0.00087946	-0.006512	0.005309	0.0033836
1973	- 0.00010875	-0.002055	-0.002234	-0.0008539
1974	0.0041731	0.014887	0.0043887	0.0071619
1975	0.028054	0.029483	0.013597	0.010288
1976	- 0.0070320	-0.017715	-0.005468	-0.0029998
1977	0.0048545	0.006007	0.0074846	0.0034617
1978	0.015614	0.011289	0.0062184	0.0052739
1979	-0.0086049	0.001797	-0.010198	-0.0007280
1980	0.029251	0.035992	0.0008252	0.0015873
1981	0.012029	0.020451	0.0026173	0.0044735
1982	0.057337	0.027155	0.0052495	0.0040287
1983	-0.0042518	-0.002369	-0.0002327	0.0015991
1984	0.036140	0.007254	-0.0026044	0.0001822
1985	0.0077174	-0.007538	-0.0076196	-0.0009879
1986	-0.019152	-0.011688	-0.0007605	-0.0015955
1987	-0.023045	-0.004115	-0.0076404	-0.0041712
1988	-0.033324	-0.025841	-0.0028915	-0.0078271
1989	-0.0087832	-0.009234	-0.0010513	-0.0042250
1990	0.0037325	0.002610	0.0020147	0.0018805
1991	0.0054771	0.007751	0.0005214	0.0032661

ABBREVIATIONS:

ZSK	Auto Correlation Corrected Share of Capital
XZSK	Fitted Auto Correlation Corrected Share of Capital
ZSP	Auto Correlation Corrected Share of Production
XZSP	Fitted Auto Correlation Corrected Share of Production

	ZSS	XZSS	ZSE	XZSE
1961	-0.0000345	-0.0009756	-0.00050847	0.00013839
1962	-0.0009033	-0.0014402	0.00051077	0.00045995
1963	0.0008071	-0.0018691	0.0016778	-0.00022981
1964	-0.0013280	-0.0020934	-0.00028097	-0.00018181
1965	-0.0005396	-0.0012826	-0.000032870	0.00026398
1966	0.0012414	0.00045718	0.00090855	0.0010836
1967	0.0068575	0.0060208	-0.00040143	0.0023051
1968	-0.0040871	-0.0046481	0.0020874	-0.00040854
1969	-0.0030477	-0.0028277	-0.00080945	-0.00083111
1970	-0.0011049	-0.0025778	-0.0016971	-0.0030327
1971	0.0045846	0.0031764	0.0041974	0.0037190
1972	0.0016951	0.00037925	0.0020067	0.0039236
1973	-0.0009883	-0.0011981	-0.00002879	-0.0010954
1974	0.0042229	0.0060464	0.00042552	0.0020893
1975	0.010166	0.0073313	0.0096001	0.0063419
1976	-0.0033533	-0.0023072	-0.0036482	-0.0016249
1977	0.0029552	0.0025260	0.0039820	0.0022108
1978	0.0026856	0.0036879	0.0061298	0.0040580
1979	-0.0060009	0.0004598	-0.0038518	-0.00095221
1980	0.0069752	0.0066229	-0.0045481	0.000058925
1981	-0.0017273	-0.00011325	-0.0027837	0.000077092
1982	0.0010232	0.0038179	0.0078046	0.0019254
1983	0.0032916	0.0020574	0.0049507	0.0035752
1984	-0.0012454	-0.0020290	0.00019252	-0.0013368
1985	-0.0024599	-0.0007460	-0.0056629	-0.00022321
1986	-0.0049781	-0.0023884	0.0045270	0.0014917
1987	-0.0033864	-0.0010146	-0.0069487	-0.0018955
1988	-0.0017990	-0.0049461	-0.0024017	-0.0040712
1989	-0.0003428	-0.0012589	-0.00062504	-0.0027741
1990	0.0007919	0.0008656	0.00096687	-0.00045892
1991	0.0010410	0.0019942	0.00026083	0.0026828

ABBREVIATIONS:

ZSS	Auto Correlation Corrected Share of Non-Production Labor
XZSS	Fitted Auto Correlation Corrected Share of Non-Production Labor
ZSE	Auto Correlation Corrected Share of Energy
XZSE	Fitted Auto Correlation Corrected Share of Energy

APPENDIX E

Test for Strict Quasi-Concavity

```
Current sample:  1961 to 1961


                D21          D22           D23           D24         D25

     Value     -514.79545    311.30188    1519.18997     -1.20417     -2.52739


                D26

     Value    -1442.00820


                D31          D32           D33           D34          D35

     Value    15312.38823  -14830.21879     54.13298     143.68494     18.56478


                D41           D42          D43           D44           D45

     Value   -651645.39372   -938.50088   -2063.02835   -665.35868   -318.25895


     D51 =   0.021459


     Current sample:  1962 to 1962


              D21        D22        D23        D24        D25

     Value     -601.59628    336.73595    1654.17407     -1.58604     -2.60693


              D26

     Value    -1793.15237


                D31          D32           D33          D34          D35
```

```
Value     19185.84427  -16179.43921     64.77242     152.39955      19.38318


                D41            D42            D43            D44           D45

Value    -815323.60466   -1032.78620   -2350.49687    -690.55408    -374.15652


D51 =   -0.015517



Current sample:  1963 to 1963


                D21            D22            D23            D24           D25

Value       -572.35031     337.73066    1609.65603      -1.35894      -2.70238


               D26

Value     -1758.53037


                D31            D32            D33            D34           D35

Value     18561.61710  -15927.08074     58.04409     150.53258      18.87514


                D41            D42            D43            D44           D45

Value    -774012.79989   -1015.70536   -2506.88056    -636.51265    -386.57937


D51 =   0.031528



Current sample:  1964 to 1964


                D21            D22            D23            D24           D25

Value       -699.52990     370.94900    1847.73642      -1.89253      -2.81401
```

```
            D26

Value    -2321.62293


            D31           D32           D33        D34        D35

Value    24686.94148  -18414.73806    74.20712   170.30201   19.47398


             D41           D42          D43          D44         D45

Value   -1.06149D+06  -1176.77076  -3121.15419   -689.63061  -455.66364


D51 =   -0.036199


Current sample:  1965 to 1965


             D21          D22          D23          D24        D25

Value     -980.71836    396.69608   2037.39834    -2.62904    -3.11950


             D26

Value    -3395.89771


             D31           D32          D33        D34        D35

Value    36492.54734  -19746.08799    97.61853   201.41799   23.36516


             D41           D42          D43          D44         D45

Value   -1.55408D+06  -1294.38743  -3655.55528   -845.04766  -585.44927


D51 =   0.018198
```

```
Current sample:  1966 to 1966


              D21           D22          D23           D24        D25

Value      -754.61157    393.85446   2021.85237     -2.47049     -2.38744


             D26

Value     -2594.90999


              D31           D32            D33          D34         D35

Value     27896.57686  -19404.09080     92.46406     153.63169     18.25486


             D41              D42           D43           D44          D45

Value    -1.17965D+06   -1225.34169   -2715.40985    -657.65360    -449.14746


D51 =   0.025252



Current sample:  1967 to 1967


              D21           D22           D23           D24        D25

Value      -178.83369    326.27498   1878.59926     -1.08471     -0.67808


             D26

Value      -510.03802


              D31           D32            D33          D34          D35

Value      5519.37094  -17207.70084     51.76366      48.09157      5.57589


                D41             D42          D43           D44         D45

Value   -255389.53357    -800.08775   -613.02225    -244.96836    -88.24430
```

```
D51 =   0.0049595


Current sample:  1968 to 1968


                  D21            D22            D23            D24          D25

Value      -265.92873     387.93269    2000.47194      -1.69877      -0.84315


                 D26

Value      -909.28691


                  D31            D32            D33            D34            D35

Value      9765.78302  -18719.27760      67.84981       54.17167        6.73669


                  D41              D42            D43            D44            D45

Value   -406802.74334    -954.53996    -905.33188    -241.97636    -159.69634


D51 =   0.0020753


Current sample:  1969 to 1969


                  D21            D22            D23            D24          D25

Value      -456.67918     436.95562    2437.67208      -3.03647      -0.97640


                  D26

Value     -1849.49804


                  D31            D32            D33            D34            D35
```

```
Value     19248.61965  -20879.69382    110.35604     71.16615      7.83966


               D41            D42            D43           D44          D45

Value   -761984.84946  -1150.08846  -1342.26949   -284.22310   -234.75699


D51 =   -0.025501



Current sample:  1970 to 1970


               D21           D22           D23           D24          D25

Value      -442.67090    360.87388   2924.18153     -4.80563     -0.46460


              D26

Value     -2324.72420


               D31           D32           D33           D34          D35

Value     19020.89065   -6265.37228    161.52093     41.42660      4.01898


               D41            D42          D43           D44          D45

Value   -384411.24122   -502.66190   -412.14973   -109.09796    -81.35874


D51 =   -0.0059867



Current sample:  1971 to 1971


              D21           D22           D23           D24          D25

Value      -267.15820    399.92895   2197.74379     -2.31629     -0.62499
```

```
              D26

Value     -1025.12038


               D31          D32           D33         D34        D35

Value     10525.60539  -17181.43825    87.41088    41.88082    5.47889


                D41           D42          D43         D44          D45

Value   -392184.26860   -881.29535   -641.56621   -182.72549   -140.22618


D51 =   -0.012264



Current sample:  1972 to 1972


               D21          D22          D23          D24        D25

Value      -297.94960    405.32518   2019.02817     -1.92640     -0.84833


              D26

Value     -1082.07215


                D31           D32           D33         D34        D35

Value     11508.30920  -18300.57495    75.39039    52.18683    7.03328


                D41           D42          D43         D44          D45

Value   -448820.51355   -959.86935   -893.83914   -227.72896   -185.39633


D51 =   0.014148
```

```
Current sample:  1973 to 1973


               D21          D22          D23          D24        D25

Value      -334.98894    412.19160   2231.88892    -2.59034    -0.72334


              D26

Value     -1359.84367


               D31          D32           D33          D34         D35

Value     13740.11300  -16959.57050    97.12692     47.45465     6.34565


               D41          D42          D43          D44         D45

Value    -490335.62874   -921.65921   -763.03394   -197.46982   -178.41425


D51 =   -0.00074642



Current sample:  1974 to 1974


              D21          D22          D23          D24         D25

Value      -293.78880    382.12005   2107.86775    -2.17041    -0.79366


               D26

Value     -1021.43779


               D31          D32           D33          D34         D35

Value     10784.43446  -17750.16219    82.34062     54.08940     6.82807


               D41          D42          D43          D44          D45

Value    -434426.87452   -930.18747   -794.78385   -250.47875   -152.93156
```

```
D51 =   -0.013140



Current sample:  1975 to 1975


              D21          D22          D23            D24         D25

Value      -52.51136   288.88819   1065.94456     0.76723     -0.34693


             D26

Value     -142.37111


              D31           D32              D33          D34          D35

Value     1505.00044  -10479.86708     -0.028399     13.73010      2.76197


              D41            D42            D43            D44           D45

Value    -50501.32385    -329.67004    -180.50303     -67.73098     -47.33830


D51 =   -0.00091770



Current sample:  1976 to 1976


              D21            D22           D23            D24         D25

Value     -106.90555    339.95462   1520.27277     -0.38743     -0.43993


             D26

Value     -321.69826


              D31            D32            D33            D34          D35
```

```
Value       3474.11822  -14214.74460    29.66932    22.90658    3.80961


                D41           D42          D43          D44          D45

Value   -130671.51931   -569.57375   -307.90825   -115.50817    -74.44730


D51 =   -0.0039875



Current sample:  1977 to 1977


                D21          D22          D23            D24          D25

Value       -92.53613    320.69095   1261.73487      0.14376     -0.49834


               D26

Value      -268.24737


                 D31          D32           D33          D34          D35

Value      2869.10708  -12412.81765     14.59383     21.80894      4.00130


                  D41          D42           D43          D44          D45

Value   -101148.58470   -486.74591   -313.17677   -104.55416    -77.71254



D51 =   0.0033325
```

```
Current sample:  1978 to 1978


                D21          D22          D23          D24        D25

   Value     -38.13965   302.47976   995.03081     0.86466    -0.27200


                D26

   Value     -111.72962


                D31          D32          D33          D34        D35

   Value     1169.98497  -9853.67172    -6.42269     9.20443     2.12978


                D41          D42          D43          D44        D45

   Value    -35518.12912  -267.29661  -136.11429   -42.67748   -42.34928


  D51 =   0.0011302



  Current sample:  1979 to 1979


                D21          D22          D23          D24        D25

   Value     -89.97393   385.51311  1518.15052    -0.52774   -0.32257


                D26

   Value     -320.88816


                D31          D32          D33          D34        D35

   Value     3396.56798 -13607.86108   30.32913    13.89188     2.94644
```

```
              D41            D42           D43           D44          D45

Value   -108441.47031   -513.05823   -213.37809    -65.09386    -78.36305


D51 =   0.0043394



Current sample:  1980 to 1980


              D21           D22           D23           D24        D25

Value      -9.56157    315.85401   1728.27639      1.07527   -0.039781


              D26

Value     -32.74660


              D31            D32           D33           D34        D35

Value     351.12268  -13880.21147     -7.90068      2.55752     0.38631


              D41            D42           D43           D44          D45

Value    -14019.24035   -147.12683    -27.65870    -14.67756     -6.09989


D51 =   0.00049866



Current sample:  1981 to 1981


              D21           D22           D23           D24        D25

Value      -0.14598    349.99765   2384.83276      1.60172  -0.0041715


              D26

Value      -6.43130
```

	D31	D32	D33	D34	D35
Value	73.91501	-18879.36946	-27.63533	0.35810	0.053338

	D41	D42	D43	D44	D45
Value	-3754.23154	-47.09217	-7.75023	-3.09838	-1.02349

D51 = 0.00013683

Current sample: 1982 to 1982

	D21	D22	D23	D24	D25
Value	20.37440	358.89348	1561.68564	4.49030	0.091377

	D26
Value	59.66604

	D31	D32	D33	D34	D35
Value	-634.56567	-14631.59466	-120.14083	-4.61297	-0.76804

	D41	D42	D43	D44	D45
Value	22267.32719	801.57362	63.81230	21.20608	16.38611

D51 = 0.0010507

```
Current sample:  1983 to 1983


              D21         D22          D23          D24       D25

    Value    18.94683   332.31561   1233.08608    3.22840   0.091520


             D26

    Value    51.51548


              D31          D32           D33          D34        D35

    Value   -544.42462  -11189.32102   -77.25567    -3.61807    -0.83781


              D41          D42         D43          D44         D45

    Value   16252.24421   486.14442   40.70216    17.98070    16.52649


       D51 =   0.00092322



    Current sample:  1984 to 1984


              D21         D22          D23          D24       D25

    Value    19.99285   338.29236   1385.06998    4.53927   0.086720


              D26

    Value    63.41176


              D31          D32           D33          D34        D35

    Value   -626.82477  -10594.34349   -126.42647    -3.56444    -0.81741
```

```
               D41          D42         D43        D44        D45

Value     17540.38418   824.48078   36.20677   16.68346   17.29807


D51 =   0.00067356



Current sample:  1985 to 1985


               D21          D22          D23          D24         D25

Value       10.30316    164.50067   2156.11106     5.27609    0.030853


               D26

Value       72.15163


               D31          D32           D33          D34         D35

Value     -298.17985   8219.15353   -161.64099    -2.01386    -0.11956


               D41          D42          D43          D44         D45

Value    -8067.36375   -357.58073    -8.36726    -5.18817    -3.81600


D51 =   -0.00025683



Current sample:  1986 to 1986


               D21          D22          D23          D24         D25

Value        4.50563     17.33240   2021.94277     2.29579    0.016282


               D26

Value       80.85996
```

```
                 D31          D32            D33          D34        D35

Value        18.55907   18908.14824    -84.85473    -0.65143    0.063825


                 D41            D42          D43          D44          D45

Value    -15011.38393   -425.89606    -12.15380    -4.32735    -9.73270


D51 =   -0.00047490



Current sample:  1987 to 1987


                 D21           D22          D23            D24         D25

Value       -35.75252   -2697.65382   2105.87033    -0.66330    -0.015244


                 D26

Value        34.96945


                 D31             D32           D33         D34          D35

Value      2445.66437   409609.05559    27.94121     1.45944     0.33127


                  D41            D42          D43          D44         D45

Value    -12486.59706   -191.02649     -3.40957    -1.57398    -2.68256


D51 =   -0.00015543
```

```
Current sample:  1988 to 1988


               D21          D22            D23            D24         D25

 Value      425.70416  -7567.13400  -2302.77007     -6.83393     0.11804


              D26

 Value    -331.73239


               D31          D32               D33          D34         D35

 Value   -36514.31691 1475131.33804     -8.27480    -16.65406    -2.75093


               D41            D42           D43          D44         D45

 Value  -109474.25255   -607.82920   -14.93219     -7.21427   -12.16043


 D51 =   0.00093118



 Current sample:  1989 to 1989


              D21           D22             D23            D24          D25

 Value     1528.99770  -11816.46073  -7066.09003    -10.14889     0.29332


              D26

 Value     -675.72566


               D31            D32              D33           D34          D35

 Value  -127358.27638 2615399.07158   -241.87057    -49.63877     -4.61693
```

```
                D41             D42             D43          D44          D45
Value    -1.17572D+07  -42399.64070   -1121.30033    -611.14867    -987.49543


D51 =   0.34027


Current sample:  1990 to 1990


                D21             D22             D23          D24          D25
Value      906.96666   -7677.81742   -3877.52373      -8.89189       0.24917


                D26
Value     -648.79234


                D31             D32             D33          D34          D35
Value    -76658.21380  1621554.75000     -97.35089     -38.53205      -4.34510


                D41             D42             D43          D44          D45
Value    -3.06444D+06  -11530.05702    -405.98523    -183.53985    -293.96182


   D51 =   0.11996


Current sample:  1991 to 1991


                D21             D22             D23          D24          D25
Value      764.05566   -7451.02146   -1856.85490      -8.75905       0.20694


                D26
```

Value -555.28625

	D31	D32	D33	D34	D35
Value	-62519.10526	1654442.15715	-3.18906	-33.17381	-4.21048

	D41	D42	D43	D44	D45
Value	-86585.66484	-359.84433	-11.53649	-6.27946	-8.01611

D51 = 0.00029901

APPENDIX F

Estimated Allen Elasticities of Substitutions
(Own Price)

	AEKK	AEPP	AESS	AEEE	AEMM
1961	48.13111	-10.63842	-29.52592	-58.00219	0.019408
1962	55.67022	-10.73960	-31.73953	-59.59299	0.025407
1963	53.74231	-10.59385	-32.23019	-57.03496	0.022319
1964	64.95636	-10.70345	-35.19186	-61.53163	0.029729
1965	89.51744	-10.86414	-37.36890	-65.39768	0.039460
1966	68.83454	-10.87075	-37.09419	-65.21559	0.037142
1967	16.33834	-10.87238	-30.45662	-71.17427	0.014985
1968	24.27247	-10.86944	-36.58551	-65.16222	0.025400
1969	42.19374	-10.69795	-42.84204	-73.25568	0.041178
1970	48.83483	-8.82725	-46.44377	-89.55566	0.053661
1971	25.08950	-10.51020	-39.82740	-67.96188	0.033564
1972	27.28732	-10.81516	-38.71935	-62.89974	0.029734
1973	31.73007	-10.40894	-41.81412	-66.84257	0.038131
1974	27.11275	-10.71785	-36.82874	-68.81718	0.031088
1975	4.94129	-10.60871	-27.42422	-42.51024	-0.022299
1976	9.77003	-10.87094	-31.82381	-54.18947	0.005373
1977	8.58092	-10.74648	-30.16447	-46.51479	-0.006341
1978	3.62074	-10.52372	-28.96267	-37.51449	-0.029236
1979	8.29892	-10.74080	-37.06727	-47.21626	0.007933
1980	0.88558	-10.64765	-30.32664	-65.16752	-0.016928
1981	0.00891	-10.71687	-33.52248	-85.68133	-0.018732
1982	-1.88229	-10.86644	-33.71825	-52.97163	-0.085919
1983	-1.75888	-10.78935	-31.44473	-43.65253	-0.077130
1984	-1.95434	-10.25772	-34.22003	-45.81697	-0.101390
1985	-1.95374	-5.27524	-39.02557	-67.48238	-0.078273
1986	-1.78157	-2.61035	-49.67461	-53.85283	-0.044007
1987	-0.91448	35.28759	-57.67395	-95.06997	0.006827
1988	4.75692	94.83804	-57.50769	-132.49747	0.050386
1989	10.56721	150.25531	-54.57829	-154.16713	0.063845
1990	9.93481	95.22977	-56.73893	-142.66379	0.060814
1991	8.37754	95.40179	-58.33277	-146.44691	0.058095

ABBREVIATIONS:

AEKK = Allen Elasticity of Capital With Respect To Capital
AEPP = Allen Elasticity of Production Labor With Respect To Production Labor
AESS = Allen Elasticity of Non-Production Labor With Respect To Non-Production Labor
AEEE = Allen Elasticity of Energy With Respect To Energy
AEMM = Allen Elasticity of Material (Ore) With Respect To Material (Ore)

	AEKP	AEKS	AEKE	AEKM	AEPS
1961	-1.66024	4.57087	-0.84645	-1.86051	-1.67549
1962	-1.92887	5.11918	-1.00128	-2.00533	-2.03319
1963	-1.73566	5.13862	-0.89917	-1.97531	-1.92642
1964	-2.06706	5.97395	-1.18832	-2.17833	-2.39276
1965	-2.86154	7.12246	-1.63055	-2.57912	-3.04711
1966	-2.51571	6.44579	-1.35662	-2.22352	-3.06383
1967	-1.09408	3.52526	-0.50467	-0.96068	-2.20480
1968	-1.44931	4.61153	-0.58985	-1.20817	-3.11954
1969	-2.30059	6.46778	-1.15157	-1.64738	-4.62238
1970	-3.40497	7.52667	-1.70285	-1.75645	-7.00691
1971	-1.86077	5.08633	-0.66818	-1.21124	-4.32030
1972	-1.68313	5.05220	-0.60611	-1.28829	-3.66479
1973	-2.17082	5.75141	-0.78765	-1.39042	-4.80095
1974	-1.78838	4.78635	-0.73294	-1.27927	-3.55035
1975	-0.30111	2.61917	0.30736	-0.48656	-1.43083
1976	-0.83440	3.28312	-0.01703	-0.70173	-2.44949
1977	-0.56695	3.06733	0.14245	-0.66628	-1.86305
1978	-0.18982	2.61980	0.41848	-0.40761	-1.52157
1979	-0.91481	3.64277	0.13861	-0.62323	-3.55232
1980	-0.36355	2.42689	0.20845	-0.15745	-2.65582
1981	-0.22455	2.47637	0.08136	-0.06328	-3.04276
1982	0.28178	1.94975	0.61361	0.26523	-2.73931
1983	0.17407	1.94728	0.64664	0.21010	-2.63676
1984	0.23275	1.86163	0.69739	0.33382	-3.56749
1985	-0.05711	2.02337	0.57326	0.34939	-6.43184
1986	-0.38065	2.76382	0.57940	0.24924	-10.59886
1987	-1.86620	4.21571	0.11909	0.09487	-25.73866
1988	-5.04306	7.62713	-0.82554	-0.34878	-45.96974
1989	-7.66697	9.94913	-1.51776	-0.61753	-60.13139
1990	-6.25486	9.22506	-1.32429	-0.59583	-47.69257
1991	-5.93101	8.16095	-1.26671	-0.52893	-43.42776

ABBREVIATIONS:

AEKP = Allen Elasticity of Capital With Respect To Production Labor
AEKS = Allen Elasticity of Capital With Respect To Non-Production Labor
AEKE = Allen Elasticity of Capital With Respect To Energy
AEKM = Allen Elasticity of Capital With Respect To Material (Ore)
AEPS = Allen Elasticity of Production Labor With Respect To Non-Production Labor

	AEPE	AEPM	AEES	AEEM	AEMS
1961	10.47615	0.48361	13.90604	0.28015	0.55695
1962	11.09397	0.46680	15.40388	0.26828	0.52069
1963	10.19839	0.49310	15.11925	0.29324	0.50985
1964	11.22421	0.47766	17.82346	0.25149	0.45856
1965	12.91055	0.42996	20.16039	0.22010	0.42233
1966	13.04568	0.42042	19.93157	0.21974	0.42618
1967	14.07987	0.40047	17.00378	0.13482	0.53790
1968	13.42163	0.39313	19.58370	0.20885	0.42805
1969	16.15415	0.34411	26.47170	0.14129	0.30551
1970	23.71234	0.18524	35.14429	-0.00405	0.22840
1971	15.87690	0.30634	22.56105	0.18763	0.36670
1972	13.66431	0.36531	20.40610	0.23697	0.38734
1973	15.94948	0.29683	23.72918	0.20381	0.32652
1974	15.26503	0.34003	20.65388	0.17623	0.42719
1975	8.12251	0.46229	9.99327	0.42510	0.57230
1976	11.52515	0.38052	14.29134	0.31021	0.50720
1977	9.13478	0.44400	11.88946	0.38888	0.53114
1978	7.20066	0.47204	9.56499	0.48176	0.54060
1979	11.16338	0.32631	15.23244	0.39133	0.40570
1980	14.89126	0.28550	15.74914	0.16697	0.52210
1981	18.23035	0.29848	22.07755	-0.05695	0.45940
1982	11.42016	0.30298	14.98082	0.24689	0.41087
1983	10.29222	0.26934	11.81337	0.37218	0.46437
1984	11.98786	0.14914	13.51991	0.32601	0.38927
1985	22.22712	-0.13839	21.85012	0.07704	0.29560
1986	19.94531	-0.18952	25.55730	0.27220	0.028690
1987	51.17209	-0.81335	58.11360	-0.11932	-0.30037
1988	89.62393	-1.30324	99.61121	-0.39741	-0.61444
1989	118.80521	-1.66222	124.41972	-0.55324	-0.75700
1990	95.25006	-1.27624	109.41716	-0.46463	-0.64946
1991	97.32650	-1.28548	101.97798	-0.50115	-0.50927

ABBREVIATIONS:

AEPE = Allen Elasticity of Production Labor With Respect To Energy
AEPM = Allen Elasticity of Production Labor With Respect To Material (Ore)
AEES = Allen Elasticity of Energy With Respect To Non-Production Labor
AEEM = Allen Elasticity of Energy With Respect To Material (Ore)
AEMS = Allen Elasticity of Material (Ore) With Respect To Non-Production Labor

APPENDIX G

Compensated Price Elasticities

	NKK	NPP	NSS	NEE	NMM
1961	1.59072	-0.52129	-0.82164	-1.11631	0.016902
1962	1.73833	-0.50591	-0.81041	-1.12002	0.022290
1963	1.70153	-0.52694	-0.80780	-1.11404	0.019507
1964	1.90766	-0.51193	-0.79119	-1.12448	0.026222
1965	2.30649	-0.47042	-0.77797	-1.13323	0.035229
1966	1.97493	-0.46429	-0.77969	-1.13282	0.033064
1967	0.79209	-0.46138	-0.81702	-1.14594	0.012979
1968	1.03153	-0.44987	-0.78283	-1.13270	0.022285
1969	1.46663	-0.40172	-0.74030	-1.15042	0.036842
1970	1.60494	-0.26269	-0.71109	-1.18410	0.048769
1971	1.05406	-0.37674	-0.76190	-1.13892	0.029746
1972	1.11307	-0.42572	-0.76930	-1.12760	0.026227
1973	1.22613	-0.36598	-0.74792	-1.13645	0.033987
1974	1.10847	-0.40504	-0.78133	-1.14080	0.027467
1975	0.33054	-0.52511	-0.83152	-1.07757	-0.018462
1976	0.55217	-0.45173	-0.80996	-1.10725	0.004600
1977	0.50238	-0.50467	-0.81849	-1.08813	-0.005352
1978	0.25791	-0.53494	-0.82436	-1.06366	-0.024006
1979	0.49018	-0.40913	-0.77986	-1.08993	0.006812
1980	0.075597	-0.39409	-0.81767	-1.13271	-0.014106
1981	0.000830	-0.40487	-0.80075	-1.17630	-0.015575
1982	-0.27445	-0.44704	-0.79965	-1.10430	-0.066028
1983	-0.23615	-0.41916	-0.81195	-1.08063	-0.059877
1984	-0.31991	-0.35187	-0.79682	-1.08632	-0.076553
1985	-0.31890	-0.13171	-0.76728	-1.13786	-0.060684
1986	-0.24213	-0.059931	-0.68053	-1.10644	-0.035508
1987	-0.096999	0.50005	-0.55530	-1.19499	0.005854
1988	0.32082	1.00250	-0.42256	-1.26395	0.045604
1989	0.58423	1.35093	-0.36227	-1.30077	0.058779
1990	0.55888	1.00525	-0.40272	-1.28146	0.055773
1991	0.49360	1.00645	-0.45405	-1.28787	0.053096

ABBREVIATIONS:

NKK = Compensated Price Elasticity of Capital With Respect To Capital
NPP = Compensated Price Elasticity of Production Labor With Respect To Production Labor
NSS = Compensated Price Elasticity of Non-Production With Respect To Non-Production Labor
NEE = Compensated Price Elasticity of Energy With Respect To Energy
NMM = Compensated Price Elasticity of Material (Ore) With Respect To Material (Ore)

	NKP	NKS	NKE	NKM	NPK
1961	-0.081353	0.12720	-0.016291	-1.62027	-0.054870
1962	-0.090863	0.13071	-0.018818	-1.75935	-0.060230
1963	-0.086332	0.12879	-0.017563	-1.72643	-0.054953
1964	-0.098865	0.13431	-0.021717	-1.92138	-0.060706
1965	-0.12390	0.14828	-0.028255	-2.30261	-0.073730
1966	-0.10745	0.13548	-0.023565	-1.97940	-0.072178
1967	-0.046428	0.094567	-0.008125	-0.83210	-0.053041
1968	-0.059985	0.098674	-0.010253	-1.05997	-0.061593
1969	-0.086391	0.11176	-0.018085	-1.47392	-0.079967
1970	-0.10133	0.11524	-0.022515	-1.59634	-0.11190
1971	-0.066700	0.097301	-0.011198	-1.07347	-0.078175
1972	-0.066253	0.10038	-0.010866	-1.13634	-0.068656
1973	-0.076327	0.10287	-0.013392	-1.23929	-0.083886
1974	-0.067585	0.10154	-0.012150	-1.13028	-0.073116
1975	-0.014905	0.079415	0.007791	-0.40284	-0.020143
1976	-0.034673	0.083560	-0.000348	-0.60071	-0.047158
1977	-0.026625	0.083229	0.003332	-0.56231	-0.033193
1978	-0.009648	0.074567	0.011865	-0.33470	-0.013521
1979	-0.034846	0.076640	0.003199	-0.53518	-0.054034
1980	-0.013456	0.065434	0.003623	-0.13120	-0.031034
1981	-0.008483	0.059153	0.001117	-0.05261	-0.020911
1982	0.011592	0.046240	0.012792	0.20383	0.041086
1983	0.006762	0.050282	0.016008	0.16310	0.023371
1984	0.007983	0.043348	0.016535	0.25204	0.038099
1985	-0.001426	0.039781	0.009666	0.27087	-0.009323
1986	-0.008739	0.037864	0.011904	0.20111	-0.051734
1987	-0.026445	0.040590	0.001496	0.08135	-0.19795
1988	-0.053309	0.056043	-0.007875	-0.31568	-0.34012
1989	-0.068933	0.066039	-0.012806	-0.56853	-0.42388
1990	-0.066027	0.065477	-0.011895	-0.54644	-0.35187
1991	-0.062570	0.063523	-0.011140	-0.48341	-0.34945

ABBREVIATIONS:

NKP = Compensated Price Elasticity of Capital With Respect To Production Labor
NKS = Compensated Price Elasticity of Capital With Respect To Non-Production Labor
NKE = Compensated Price Elasticity of Capital With Respect To Energy
NKM = Compensated Price Elasticity of Capital With Respect To Material (Ore)
NPK = Compensated Price Elasticity of Production With Respect To Capital

	NPS	NPE	NPM	NSK	NSP
1961	-0.046625	0.20162	0.42116	0.15107	-0.082100
1962	-0.051913	0.20851	0.40955	0.15985	-0.095777
1963	-0.048283	0.19920	0.43097	0.16269	-0.095820
1964	-0.053795	0.20512	0.42131	0.17544	-0.11444
1965	-0.063437	0.22372	0.38387	0.18352	-0.13194
1966	-0.064399	0.22661	0.37426	0.18494	-0.13086
1967	-0.059145	0.22669	0.34687	0.17091	-0.093563
1968	-0.066750	0.23331	0.34490	0.19598	-0.12911
1969	-0.079873	0.25369	0.30788	0.22482	-0.17358
1970	-0.10728	0.31352	0.16835	0.24736	-0.20852
1971	-0.082647	0.26607	0.27150	0.21369	-0.15486
1972	-0.072814	0.24496	0.32223	0.20608	-0.14426
1973	-0.085873	0.27117	0.26457	0.22225	-0.16880
1974	-0.075322	0.25305	0.30042	0.19568	-0.13417
1975	-0.043384	0.20589	0.38274	0.17521	-0.070823
1976	-0.062343	0.23549	0.32574	0.18555	-0.10179
1977	-0.050552	0.21369	0.37472	0.17958	-0.087491
1978	-0.043308	0.20416	0.38760	0.18662	-0.077344
1979	-0.074737	0.25769	0.28021	0.21516	-0.13531
1980	-0.071607	0.25883	0.23790	0.20717	-0.098297
1981	-0.072682	0.25028	0.24818	0.23061	-0.11495
1982	-0.064964	0.23808	0.23284	0.28429	-0.11269
1983	-0.068085	0.25479	0.20909	0.26145	-0.10244
1984	-0.083069	0.28423	0.11261	0.30473	-0.12237
1985	-0.12646	0.37479	-0.10729	0.33026	-0.16059
1986	-0.14520	0.40979	-0.15292	0.37563	-0.24334
1987	-0.24782	0.64321	-0.69749	0.44716	-0.36474
1988	-0.33778	0.85496	-1.17957	0.51439	-0.48593
1989	-0.39913	1.00241	-1.53032	0.55006	-0.54063
1990	-0.33851	0.85558	-1.17045	0.51896	-0.50344
1991	-0.33803	0.85590	-1.17487	0.48084	-0.45815

ABBREVIATIONS:

NPS = Compensated Price Elasticity of Production With Respect To Non-Production Labor
NPE = Compensated Price Elasticity of Production With Respect To Energy
NPM = Compensated Price Elasticity of Production With Respect To Material (Ore)
NSK = Compensated Price Elasticity of Non-Production With Respect To Capital
NSP = Compensated Price Elasticity of Non-Production With Respect To Production

	NSE	NSM	NEK	NEP	NES
1961	0.26764	0.48504	-0.027975	0.51334	0.38697
1962	0.28951	0.45683	-0.031265	0.52260	0.39331
1963	0.29532	0.44561	-0.028469	0.50727	0.37894
1964	0.32572	0.40447	-0.034899	0.53684	0.40071
1965	0.34934	0.37705	-0.042012	0.55903	0.41971
1966	0.34622	0.37939	-0.038923	0.55719	0.41894
1967	0.27377	0.46591	-0.024467	0.59749	0.45614
1968	0.34042	0.37554	-0.025067	0.55550	0.41904
1969	0.41572	0.27334	-0.040028	0.60661	0.45742
1970	0.46467	0.20758	-0.055964	0.70566	0.53809
1971	0.37808	0.32499	-0.028072	0.56912	0.43159
1972	0.36582	0.34165	-0.024724	0.53787	0.40544
1973	0.40344	0.29103	-0.030437	0.56079	0.42444
1974	0.34239	0.37744	-0.029965	0.57688	0.43818
1975	0.25332	0.47383	0.020561	0.40205	0.30300
1976	0.29201	0.43418	-0.000962	0.47892	0.36374
1977	0.27813	0.44826	0.008339	0.42898	0.32261
1978	0.27120	0.44389	0.029810	0.36602	0.27225
1979	0.35162	0.34838	0.008187	0.42523	0.32047
1980	0.27375	0.43506	0.017794	0.55116	0.42463
1981	0.30310	0.38199	0.007576	0.68872	0.52736
1982	0.31231	0.31575	0.089468	0.46982	0.35528
1983	0.29244	0.36049	0.086821	0.39985	0.30504
1984	0.32056	0.29390	0.11416	0.41122	0.31481
1985	0.36843	0.22918	0.093570	0.55497	0.42959
1986	0.52509	0.02315	0.078746	0.45793	0.35013
1987	0.73046	-0.25758	0.012631	0.72515	0.55954
1988	0.95023	-0.55613	-0.055676	0.94738	0.73193
1989	1.04978	-0.69693	-0.083912	1.06816	0.82586
1990	0.98283	-0.59562	-0.074498	1.00546	0.77662
1991	0.89681	-0.46545	-0.074633	1.02676	0.79378

ABBREVIATIONS:

NSE = Compensated Price Elasticity of Non-Production With Respect Energy
NSM = Compensated Price Elasticity of Non-Production With Respect To Material (Ore)
NEK = Compensated Price Elasticity of Energy With Respect To Capital
NEP = Compensated Price Elasticity of Energy With Respect To Production
NES = Compensated Price Elasticity of Energy With Respect To Non-Production

	NEM	NMK	NMP	NMS	NME
1961	0.24397	-0.061489	0.023697	0.015499	0.0053917
1962	0.23538	-0.062617	0.021990	0.013295	0.0050422
1963	0.25629	-0.062540	0.024527	0.012779	0.0057278
1964	0.22183	-0.063974	0.022846	0.010309	0.0045960
1965	0.19650	-0.066453	0.018617	0.008792	0.0038140
1966	0.19562	-0.063795	0.017956	0.008957	0.0038170
1967	0.11678	-0.046574	0.016994	0.014430	0.0021707
1968	0.18323	-0.051345	0.016271	0.009159	0.0036305
1969	0.12642	-0.057262	0.012922	0.005279	0.0022189
1970	-0.00369	-0.057725	0.005512	0.003497	-0.0000536
1971	0.16629	-0.050887	0.010981	0.007014	0.0031443
1972	0.20902	-0.052550	0.014380	0.007695	0.0042482
1973	0.18166	-0.053729	0.010437	0.005840	0.0034651
1974	0.15571	-0.052301	0.012850	0.009063	0.0029214
1975	0.35196	-0.032548	0.022882	0.017352	0.010776
1976	0.26556	-0.039660	0.015812	0.012909	0.0063386
1977	0.32820	-0.039008	0.020851	0.014412	0.0090972
1978	0.39558	-0.029035	0.023995	0.015387	0.013660
1979	0.33604	-0.036811	0.012430	0.008535	0.0090334
1980	0.13913	-0.013440	0.010567	0.014077	0.0029022
1981	-0.04735	-0.005893	0.011276	0.010974	-0.0007819
1982	0.18973	0.038672	0.012465	0.009744	0.0051469
1983	0.28892	0.028209	0.010464	0.011991	0.0092133
1984	0.24614	0.054644	0.005116	0.009064	0.0077297
1985	0.05973	0.057028	-0.003455	0.005811	0.0012991
1986	0.21963	0.033874	-0.004351	0.000393	0.0055925
1987	-0.10233	0.010063	-0.011526	-0.002892	-0.0014998
1988	-0.35969	-0.023522	-0.013776	-0.004514	-0.0037910
1989	-0.50934	-0.034141	-0.014945	-0.005024	-0.0046679
1990	-0.42612	-0.033518	-0.013472	-0.004609	-0.0041735
1991	-0.45803	-0.031164	-0.013561	-0.003964	-0.0044072

ABBREVIATIONS:

NEM = Compensated Price Elasticity of Energy With Respect To Material (Ore)
NMK = Compensated Price Elasticity of Material (Ore) With Respect To Capital
NMP = Compensated Price Elasticity of Material (Ore) With Respect To Production
NMS = Compensated Price Elasticity of Material (Ore) With Respect To Non-Production Labor
NME = Compensated Price Elasticity of Material (Ore) With Respect To Energy

APPENDIX H

Uncompensated Price Elasticities

	NUKK	NUPP	NUSS	NUEE	NUMM
1961	2.63035	0.49293	0.22881	-0.074139	1.01205
1962	2.78019	0.50886	0.24447	-0.076917	1.01749
1963	2.74294	0.48709	0.24827	-0.072435	1.01467
1964	2.95230	0.50265	0.27131	-0.080026	1.02143
1965	3.35724	0.54565	0.28937	-0.086459	1.03051
1966	3.02034	0.55194	0.28676	-0.086339	1.02835
1967	1.81872	0.55481	0.23457	-0.096242	1.00817
1968	2.06252	0.56707	0.28314	-0.085752	1.01745
1969	2.50429	0.61683	0.34091	-0.098765	1.03213
1970	2.64465	0.76064	0.38027	-0.12293	1.04414
1971	2.08499	0.64254	0.31090	-0.090875	1.02502
1972	2.14515	0.59197	0.30129	-0.082376	1.02145
1973	2.25992	0.65378	0.33032	-0.088860	1.02927
1974	2.14020	0.61323	0.28421	-0.092301	1.02273
1975	1.34981	0.48874	0.21404	-0.046063	0.97652
1976	1.57513	0.56488	0.24468	-0.067898	0.99971
1977	1.52456	0.51005	0.23281	-0.053732	0.98969
1978	1.27619	0.47869	0.22467	-0.035201	0.97088
1979	1.51231	0.60912	0.28672	-0.054852	1.00191
1980	1.09074	0.62449	0.23371	-0.086630	0.98089
1981	1.01490	0.61359	0.25805	-0.11715	0.97934
1982	0.73445	0.56975	0.25900	-0.065726	0.92853
1983	0.77356	0.59870	0.24217	-0.047994	0.93471
1984	0.68810	0.66846	0.26351	-0.052074	0.91785
1985	0.68911	0.89612	0.30393	-0.089862	0.93389
1986	0.76745	0.97027	0.42142	-0.067135	0.95929
1987	0.91529	1.54899	0.58979	-0.13073	1.00096
1988	1.34019	2.06829	0.76809	-0.17905	1.04095
1989	1.60784	2.42821	0.84859	-0.20486	1.05421
1990	1.58200	2.07081	0.79368	-0.19174	1.05120
1991	1.51567	2.07205	0.72505	-0.19622	1.04851

ABBREVIATIONS:

NUKK = Uncompensated Price Elasticity of Capital With Respect To Capital
NUPP = Uncompensated Price Elasticity of Production With Respect To Production
NUSS = Uncompensated Price Elasticity of Non-Production Labor With Respect To Non-Production Labor
NUEE = Uncompensated Price Elasticity of Energy With Respect To Energy
NUMM = Uncompensated Price Elasticity of Material (Ore) With Respect To Material (Ore)

	NUKP	NUKS	NUKE	NUKM	NUPK
1961	0.95827	1.16682	1.02333	-0.58065	0.95935
1962	0.95100	1.17257	1.02304	-0.71749	0.95454
1963	0.95508	1.17020	1.02385	-0.68501	0.95907
1964	0.94577	1.17895	1.02292	-0.87674	0.95388
1965	0.92685	1.19903	1.02250	-1.25185	0.94234
1966	0.93796	1.18090	1.02184	-0.93399	0.94406
1967	0.98020	1.12120	1.01851	0.19453	0.96315
1968	0.97100	1.12966	1.02073	-0.02898	0.95534
1969	0.95127	1.14942	1.01958	-0.43626	0.93858
1970	0.93838	1.15495	1.01719	-0.55663	0.91143
1971	0.96422	1.12823	1.01973	-0.04254	0.94111
1972	0.96582	1.13245	1.02121	-0.10426	0.94903
1973	0.95746	1.13666	1.02039	-0.20551	0.93587
1974	0.96415	1.13328	1.01958	-0.09854	0.94515
1975	1.00436	1.09868	1.02706	0.61642	0.99371
1976	0.98828	1.10652	1.02261	0.42224	0.96946
1977	0.99555	1.10541	1.02551	0.45986	0.98152
1978	1.00863	1.09285	1.03014	0.68358	1.00011
1979	0.98728	1.09876	1.02532	0.48695	0.96422
1980	1.00169	1.08058	1.01876	0.88394	0.98755
1981	1.00559	1.07322	1.01519	0.96145	0.99755
1982	1.02049	1.05514	1.02169	1.21273	1.05787
1983	1.01647	1.05999	1.02572	1.17281	1.04123
1984	1.01599	1.05135	1.02454	1.26004	1.05843
1985	1.00658	1.04778	1.01767	1.27888	1.01851
1986	1.00085	1.04745	1.02149	1.21069	0.97847
1987	0.98584	1.05288	1.01378	1.09364	0.85099
1988	0.96607	1.07542	1.01150	0.70370	0.72568
1989	0.95468	1.08966	1.01081	0.45509	0.65340
1990	0.95709	1.08859	1.01122	0.47668	0.71369
1991	0.95950	1.08560	1.01093	0.53866	0.71615

ABBREVIATIONS:

NUKP = Uncompensated Price Elasticity of Capital With Respect To Production
NUKS = Uncompensated Price Elasticity of Capital With Respect To Non-Production Labor
NUKE = Uncompensated Price Elasticity of Capital With Respect To Energy
NUKM = ncompensated Price Elasticity of Capital With Respect To Material (Ore)
NUPK = Uncompensated Price Elasticity of Production With Respect To Capital

	NUPS	NUPE	NUPM	NUSK	NUSP
1961	0.96760	1.21585	1.43538	1.20151	0.96835
1962	0.96285	1.22327	1.42431	1.21472	0.95910
1963	0.96574	1.21323	1.44500	1.21877	0.96026
1964	0.96079	1.21971	1.43590	1.23795	0.94806
1965	0.95264	1.23979	1.39994	1.25085	0.93539
1966	0.95183	1.24284	1.39050	1.25138	0.93559
1967	0.95705	1.24288	1.36306	1.22250	0.95803
1968	0.95018	1.25024	1.36184	1.26195	0.93686
1969	0.93868	1.27224	1.32643	1.30602	0.90763
1970	0.91605	1.33686	1.19169	1.33872	0.88284
1971	0.93664	1.28536	1.29078	1.28649	0.91794
1972	0.94487	1.26265	1.33991	1.27667	0.92633
1973	0.93389	1.29093	1.28433	1.30049	0.90943
1974	0.94295	1.27132	1.31869	1.26123	0.93138
1975	0.97047	1.21975	1.39660	1.22077	0.97474
1976	0.95427	1.25211	1.34236	1.24020	0.95286
1977	0.96416	1.22841	1.38943	1.23088	0.96380
1978	0.97032	1.21779	1.40123	1.23565	0.97169
1979	0.94352	1.27595	1.29847	1.28174	0.93127
1980	0.94698	1.27742	1.25648	1.25856	0.95309
1981	0.94578	1.26874	1.26664	1.28941	0.94385
1982	0.95182	1.25486	1.24963	1.34293	0.94595
1983	0.94977	1.27265	1.22695	1.31557	0.95168
1984	0.93726	1.30456	1.13293	1.36505	0.93795
1985	0.90138	1.40262	0.92054	1.40147	0.91061
1986	0.88500	1.43999	0.87728	1.47758	0.85861
1987	0.80112	1.69215	0.35145	1.59226	0.78036
1988	0.72801	1.92075	-0.11377	1.70504	0.70472
1989	0.67815	2.07969	-0.45303	1.76092	0.67023
1990	0.72705	1.92113	-0.10489	1.71535	0.69295
1991	0.72757	1.92150	-0.10927	1.65993	0.72095

ABBREVIATIONS:

NUPS = Uncompensated Price Elasticity of Production With Respect To Non-Production Labor
NUPE = Uncompensated Price Elasticity of Production With Respect To Energy
NUPM = Uncompensated Price Elasticity of Production With Respect To Material (Ore)
NUSK = Uncompensated Price Elasticity of Non-Production Labor With Respect To Capital
NUSP = Uncompensated Price Elasticity of Non-Production Labor With Respect To Production Labor

	NUSE	NUSM	NUEK	NUEP	NUES
1961	1.31808	1.53549	1.01420	1.55551	1.42914
1962	1.34438	1.51170	1.01184	1.56570	1.43641
1963	1.35139	1.50169	1.01313	1.54887	1.42054
1964	1.38823	1.46698	1.00956	1.58130	1.44517
1965	1.41668	1.44438	1.00476	1.60580	1.46648
1966	1.41266	1.44583	1.00756	1.60367	1.46543
1967	1.32536	1.51750	1.02523	1.64719	1.50584
1968	1.40639	1.44152	1.02188	1.60245	1.46599
1969	1.49692	1.35455	1.01163	1.65827	1.50908
1970	1.55603	1.29894	1.00520	1.76682	1.59925
1971	1.45088	1.39779	1.01997	1.61716	1.47964
1972	1.43640	1.41223	1.02050	1.58309	1.45067
1973	1.48168	1.36927	1.01715	1.60838	1.47202
1974	1.40793	1.44299	1.01854	1.62538	1.48668
1975	1.29888	1.51939	1.05207	1.43356	1.33451
1976	1.34666	1.48883	1.03839	1.51827	1.40309
1977	1.32943	1.49956	1.04274	1.46338	1.35701
1978	1.32023	1.49293	1.05827	1.39448	1.30071
1979	1.41820	1.41496	1.04327	1.46031	1.35556
1980	1.32513	1.48644	1.06388	1.59724	1.47072
1981	1.36190	1.44079	1.06673	1.74787	1.58651
1982	1.37095	1.37440	1.12804	1.50839	1.39385
1983	1.34657	1.41462	1.11946	1.43249	1.33768
1984	1.38088	1.35422	1.14841	1.44547	1.34906
1985	1.43963	1.30038	1.14157	1.60297	1.47759
1986	1.62704	1.12510	1.11805	1.49723	1.38943
1987	1.87556	0.88751	1.07689	1.78941	1.62380
1988	2.14088	0.63452	1.02923	2.03229	1.81684
1989	2.26064	0.51393	1.01199	2.16407	1.92177
1990	2.17923	0.60078	1.01523	2.09519	1.86634
1991	2.07590	0.71365	1.01702	2.11841	1.88543

ABBREVIATIONS:

NUSE = Uncompensated Price Elasticity of Non-Production Labor With Respect To Energy
NUSM = Uncompensated Price Elasticity of Non-Production Labor With Respect To Material (Ore)
NUEK = Uncompensated Price Elasticity of Energy With Respect To Capital
NUEP = Uncompensated Price Elasticity of Energy With Respect To Production Labor
NUES = Uncompensated Price Elasticity of Energy With Respect To Non-Production Labor

	NUEM	NUMK	NUMP	NUMS	NUME
1961	1.28614	0.93366	1.01885	1.01065	1.00054
1962	1.27848	0.93258	1.01719	1.00849	1.00024
1963	1.29790	0.93262	1.01969	1.00794	1.00089
1964	1.26629	0.93123	1.01805	1.00552	0.99980
1965	1.24328	0.92882	1.01390	1.00407	0.99909
1966	1.24210	0.93149	1.01324	1.00424	0.99910
1967	1.16647	0.94862	1.01219	1.00962	0.99737
1968	1.23019	0.94382	1.01143	1.00432	0.99879
1969	1.17808	0.93802	1.00821	1.00056	0.99750
1970	1.05748	0.93765	1.00088	0.99887	0.99532
1971	1.21433	0.94439	1.00625	1.00229	0.99842
1972	1.25425	0.94267	1.00960	1.00291	0.99947
1973	1.22924	0.94155	1.00571	1.00112	0.99874
1974	1.20421	0.94297	1.00812	1.00433	0.99819
1975	1.38347	0.96243	1.01786	1.01233	1.00576
1976	1.30491	0.95545	1.01093	1.00802	1.00145
1977	1.36260	0.95603	1.01589	1.00945	1.00414
1978	1.42404	0.96585	1.01888	1.01028	1.00855
1979	1.37113	0.95828	1.00752	1.00363	1.00413
1980	1.18522	0.98156	1.00557	1.00908	0.99790
1981	1.01179	0.98903	1.00620	1.00589	0.99414
1982	1.22831	1.03323	1.00702	1.00430	0.99970
1983	1.32156	1.02279	1.00505	1.00658	1.00380
1984	1.28039	1.04905	0.99952	1.00347	1.00213
1985	1.10773	1.05160	0.99111	1.00038	0.99587
1986	1.25894	1.02867	0.99044	0.99519	1.00039
1987	0.96193	1.00516	0.98357	0.99221	0.99360
1988	0.72521	0.97182	0.98157	0.99083	0.99155
1989	0.58657	0.96129	0.98048	0.99040	0.99076
1990	0.66361	0.96191	0.98196	0.99082	0.99126
1991	0.63362	0.96425	0.98185	0.99145	0.99101

ABBREVIATIONS:

NUEM = Uncompensated Price Elasticity of Energy With Respect To Material (Ore)
NUMK = Uncompensated Price Elasticity of Material (Ore) With Respect To Capital
NUMP = Uncompensated Price Elasticity of Material (Ore) With Respect To Production Labor
NUMS = Uncompensated Price Elasticity of Material (Ore) With Respect To Non-Production Labor
NUME = Uncompensated Price Elasticity of Material (Ore) With Respect To Energy

APPENDIX I

Estimated Cost Elasticities with Respect to Output (ECY), Pollution Abatement (ECA), and Technical Change (ECT)

	ECY	ECA	ECT
1961	-12.85098	-2.56284	0.0054789
1962	-12.87577	-2.58072	0.038679
1963	-12.83609	-2.57476	0.0050314
1964	-12.83767	-2.58184	0.0094813
1965	-12.86942	-2.60244	0.037826
1966	-12.91762	-2.62195	0.060810
1967	-13.03522	-2.47852	0.0071527
1968	-12.77976	-2.59477	0.054669
1969	-12.85658	-2.64937	0.070602
1970	-12.89706	-2.58663	0.043468
1971	-12.95395	-2.60977	0.044570
1972	-12.86422	-2.70493	0.056794
1973	-12.89167	-2.62206	0.042053
1974	-12.97270	-2.65556	0.061395
1975	-13.05834	-2.57520	0.091369
1976	-12.97189	-2.48752	0.075850
1977	-12.96125	-2.54970	0.047072
1978	-12.92576	-2.65782	0.060238
1979	-12.87913	-2.61532	0.039231
1980	-13.02087	-2.48485	0.028972
1981	-12.84426	-2.59784	0.056252
1982	-12.97071	-2.51708	0.0063629
1983	-12.90893	-2.53546	0.034861
1984	-12.84389	-2.56356	0.044788
1985	-12.88661	-2.57007	0.036815
1986	-12.91698	-2.52399	0.044979
1987	-12.91335	-2.54183	0.053743
1988	-12.87793	-2.57540	0.057632
1989	-12.88945	-2.57869	0.047333
1990	-12.94163	-2.58803	0.057078
1991	-12.94094	-2.60281	0.059229

ABBREVIATIONS:

ECY = Elasticity Of Cost With Respect To Output
ECA = Elasticity Of Cost With Respect To Pollution Abatement
ECT = Elasticity Of Cost With Respect To Technical Change

APPENDIX J

Input Demand Elasticities with Respect to Output (Y), Pollution Abatement (A), and Technical Change (T)

	NKY	NPY	NSY	NEY	NMY
1961	-13.36021	-13.03375	-13.49927	-13.39292	-12.78868
1962	-13.41475	-13.06589	-13.58232	-13.43073	-12.81392
1963	-13.36766	-13.01615	-13.55588	-13.37008	-12.77401
1964	-13.41073	-13.02492	-13.64010	-13.40841	-12.77616
1965	-13.52260	-13.07625	-13.73597	-13.47133	-12.80865
1966	-13.50421	-13.12731	-13.77591	-13.51807	-12.85667
1967	-13.38237	-13.24627	-13.70773	-13.68304	-12.97258
1968	-13.17578	-12.99615	-13.62288	-13.37979	-12.71792
1969	-13.34076	-13.09508	-13.90060	-13.52074	-12.79594
1970	-13.40915	-13.19801	-14.07533	-13.68591	-12.83736
1971	-13.35454	-13.20380	-13.89700	-13.57634	-12.89273
1972	-13.27681	-13.09175	-13.77221	-13.44604	-12.80271
1973	-13.32720	-13.14639	-13.90026	-13.50514	-12.83080
1974	-13.38435	-13.20969	-13.82305	-13.60188	-12.91129
1975	-13.30993	-13.23927	-13.65332	-13.46981	-12.99280
1976	-13.26967	-13.18741	-13.68071	-13.48235	-12.90851
1977	-13.24871	-13.15196	-13.62611	-13.40711	-12.89696
1978	-13.16202	-13.10195	-13.55958	-13.29362	-12.85968
1979	-13.16406	-13.11425	-13.73661	-13.33097	-12.81594
1980	-13.21803	-13.26285	-13.68997	-13.62094	-12.95576
1981	-13.02498	-13.08132	-13.59950	-13.60398	-12.77900
1982	-13.08613	-13.18841	-13.73140	-13.47103	-12.90010
1983	-13.03428	-13.13946	-13.60759	-13.33026	-12.83904
1984	-12.94670	-13.10497	-13.61865	-13.28379	-12.77202
1985	-12.98972	-13.24531	-13.80419	-13.50518	-12.81663
1986	-13.04081	-13.30707	-14.23382	-13.42464	-12.84974
1987	-13.07202	-13.54536	-14.78703	-13.74315	-12.85009
1988	-13.12747	-13.72518	-15.33311	-13.97130	-12.81798
1989	-13.19386	-13.88557	-15.60732	-14.12563	-12.83051
1990	-13.24080	-13.79006	-15.48335	-14.10281	-12.88247
1991	-13.22659	-13.78989	-15.25863	-14.12698	-12.88158

	NKA	NPA	NSA	NEA	NMA
1961	-13.94202	-12.71747	-12.63761	-12.51858	-12.83125
1962	-14.03055	-12.73688	-12.64322	-12.53538	-12.85618
1963	-13.97499	-12.70456	-12.59919	-12.50857	-12.81643
1964	-14.06547	-12.70088	-12.57357	-12.48760	-12.81819
1965	-14.26889	-12.71832	-12.58421	-12.50023	-12.85017
1966	-14.17441	-12.76444	-12.63513	-12.54932	-12.89832
1967	-13.77900	-12.88105	-12.81388	-12.63788	-13.01539
1968	-13.62823	-12.62169	-12.50227	-12.41173	-12.76018
1969	-13.89395	-12.68236	-12.51296	-12.44921	-12.83738
1970	-13.99424	-12.67721	-12.50925	-12.41321	-12.87815
1971	-13.81224	-12.77143	-12.64356	-12.57220	-12.93456
1972	-13.74821	-12.69802	-12.56538	-12.50736	-12.84475
1973	-13.82480	-12.70560	-12.55971	-12.51539	-12.87239
1974	-13.85468	-12.79958	-12.69283	-12.58679	-12.95325
1975	-13.59737	-12.92616	-12.86251	-12.80596	-13.03758
1976	-13.60990	-12.81445	-12.73860	-12.65879	-12.95182
1977	-13.57715	-12.82194	-12.74242	-12.68778	-12.94089
1978	-13.43197	-12.79705	-12.71715	-12.70013	-12.90483
1979	-13.48961	-12.70737	-12.59691	-12.60199	-12.85912
1980	-13.44328	-12.84411	-12.80065	-12.65282	-13.00025
1981	-13.23146	-12.67108	-12.59568	-12.37827	-12.82359
1982	-13.21801	-12.81168	-12.72034	-12.66383	-12.94835
1983	-13.17749	-12.74053	-12.67898	-12.65050	-12.88680
1984	-13.06417	-12.65316	-12.58889	-12.57407	-12.82113
1985	-13.10752	-12.62458	-12.58461	-12.50720	-12.86445
1986	-13.18229	-12.63203	-12.48357	-12.60560	-12.89569
1987	-13.25330	-12.45168	-12.29667	-12.40439	-12.89332
1988	-13.41259	-12.25902	-12.06986	-12.20730	-12.85895
1989	-13.54165	-12.16178	-11.99491	-12.13122	-12.87078
1990	-13.58261	-12.32186	-12.10508	-12.22942	-12.92290
1991	-13.55294	-12.32080	-12.17813	-12.21348	-12.92214

	NKT	NPT	NST	NET	NMT
1961	-12.75871	-12.93348	-12.94191	-13.03100	-12.84296
1962	-12.77810	-12.96159	-12.97487	-13.06011	-12.86780
1963	-12.73977	-12.91737	-12.93705	-13.01347	-12.82810
1964	-12.73383	-12.92219	-12.95022	-13.02725	-12.82975
1965	-12.75106	-12.96278	-12.99096	-13.06935	-12.86159
1966	-12.81133	-13.01227	-13.03800	-13.11707	-12.90977
1967	-12.97232	-13.13049	-13.12955	-13.25041	-13.02715
1968	-12.70800	-12.87744	-12.89802	-12.97907	-12.77180
1969	-12.76884	-12.96424	-13.00302	-13.07719	-12.84877
1970	-12.80426	-13.03290	-13.06233	-13.15909	-12.88937
1971	-12.88136	-13.06673	-13.08622	-13.16069	-12.94606
1972	-12.78946	-12.96693	-12.99158	-13.05748	-12.85630
1973	-12.81275	-13.00665	-13.03314	-13.09544	-12.88383
1974	-12.89811	-13.07968	-13.09197	-13.18169	-12.96479
1975	-13.01275	-13.14001	-13.14179	-13.19501	-13.04990
1976	-12.91793	-13.06918	-13.07131	-13.14145	-12.96373
1977	-12.90916	-13.04734	-13.05451	-13.10935	-12.95297
1978	-12.88295	-13.00529	-13.01466	-13.04795	-12.91725
1979	-12.82750	-12.98526	-12.99940	-13.02921	-12.87099
1980	-12.98515	-13.13010	-13.11472	-13.22020	-13.01249
1981	-12.81151	-12.95127	-12.95019	-13.09661	-12.83585
1982	-12.94979	-13.06897	-13.07740	-13.13690	-12.96161
1983	-12.88622	-13.01299	-13.00693	-13.04888	-12.89993
1984	-12.82526	-12.96174	-12.95256	-12.99001	-12.83463
1985	-12.86792	-13.04852	-13.01531	-13.09208	-12.87759
1986	-12.89454	-13.09306	-13.10169	-13.08561	-12.90832
1987	-12.88460	-13.19864	-13.17616	-13.18898	-12.90520
1988	-12.83271	-13.26038	-13.22230	-13.24111	-12.87021
1989	-12.83428	-13.33909	-13.27067	-13.30007	-12.88186
1990	-12.88742	-13.32461	-13.29814	-13.32734	-12.93401
1991	-12.88918	-13.32415	-13.26603	-13.33491	-12.93330

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