Auctions with Buy Prices

by

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Abstract

Internet auction sites eBay and Yahoo have developed an innovative hybrid auction designs that incorporate buy prices. My dissertation focuses on eBay's Buy It Now (BIN, hereafter) auction. The BIN hybrid auction combines a standard ascending bid auction with a posted-price offer "buy price".

Chapter 1: Risk aversion and impatience of either the bidders or the seller have mainly been used to explain the popularity of BIN auctions in IPV models. Using a pure common value framework, we model auctions with buy prices, when the bidders and the seller are either risk neutral or risk averse. It characterizes equilibrium bidding strategies in a general setup and then analyzes a seller's incentive to post a buy price. There is no incentive to post a buy price for a risk neutral seller. But when the seller is risk averse, a suitably chosen buy price can raise the seller's expected utility.

Chapter 2: The theoretical predictions from Wooders and Reynolds (2003) and Chapter 1 show that the introduction of a buy price causes the seller's revenue to move in opposite directions in private value and common value settings. Our results show that a buy price raises seller revenue in private value auctions. The results for common value auctions are inconsistent with the theoretical predictions. As a result, we develop and estimate a behavioral model of common value BIN auctions based on the winner's curse and overweighting of a bidder's private information which explains all the departures from the rational model.

Chapter 3: Haile and Tamer (2003) first used an incomplete econometric model in an auction context, assuming that bidders bid up to their values and do not allow an opponent to win at a price they are willing to beat. Chapter 3 extends these incomplete models to eBay's BIN auctions. We develop and estimate an equilibrium model for BIN independent private value auctions with a stochastic and unknown number of potential bidders who enter the auction sequentially using a new data set

of 3245 eBay auctions of Pentium-3 laptops that ran between 22 July to 10 August 2005.

Chapter 1

Common Value Auctions with Buy Prices

1.1 Introduction:

Web-based Internet auctions began with the initiatives of Onsale in May 1995, followed by eBay in September 1995 (see Lucking-Reiley (2000)). The other three major firms that entered this market in the following years were uBid, Yahoo! and Amazon. The expansion of internet auctions since then has set off renewed interest in auction theory and has provided research ideas based on new auction formats. One prominent example is the introduction of auctions with "buy prices" by Yahoo! in 1999, known as *Buy Now* auctions. Then, in 2000 eBay introduced *Buy It Now* auctions, its version of such hybrid auctions.

In an auction with a buy price the seller sets a "buy price" at which he commits to sell his item to a bidder who agrees to buy at that price (as long as the buy price is available at that time). A buy price thus allows a bidder to end and win an auction early, sometimes even without going to the auction at all. A buy price can be either *temporary* (eBay, LabX, Mackley and Company) or *permanent* (Yahoo!, uBid, Bid or Buy, MSN, Amazon). (See Mathews (2006).) A temporary buy price is available only at the beginning of an auction when bidding has not yet started and disappears as soon as someone places a bid. But a permanent buy price is available during the entire time an auction takes place.

This paper, using a common value framework, characterizes symmetric equilibrium bidding strategies in an auction with a temporary buy price and then analyzes the effects of such a buy price on the seller's expected utility. An auction without a buy price is modeled as a second price sealed bid auction. We show that, for both risk neutral and risk averse bidders, it is not possible for the seller to raise his expected revenue by introducing a buy price. This is a significant difference with a private value model where an appropriately chosen buy price can always raise seller revenue when bidders are risk averse. But we find that a risk averse seller can increase his expected utility by using an appropriately chosen buy price when bidders are either risk neutral or risk averse.

The implication of the type of information available to the bidders regarding their valuation of the item has never been investigated in the context of buy price auctions. All the theoretical investigations into buy price auctions assume that each bidder has his own private valuation for the item (we discuss some of these studies below). But it might be the case that a bidder only has partial information (a signal) regarding his value for the item and other bidders possess information that, if known, would affect his assessment of valuation. This paper assumes a pure common value structure in which the *ex post* value is common to all bidders but unknown at the time they participate in an auction. Each bidder's *ex ante* information regarding the value consists only of a signal. Within this structure, this paper analyzes second price sealed bid auctions with temporary buy prices. The focus of the paper is on the impact of risk preferences (of either the bidders or the seller) on seller utility in auctions with and without eBay-style temporary buy prices.

The appropriateness of private and common value frameworks in modeling online auctions is an open question. For some goods a private value model seems to be suitable, while for others a common value model appears to be more compelling. Bajari and Hortaçsu (2003) use data from eBay coin auctions to explore the determinants of bidder and seller behavior. They argue that a common value model is the appropriate one to use for coin auctions as they found an inverse relation between the sale price and number of bidders.[1] This type of analysis and results will probably fit into

[1] The presence of the winner's curse in a common value auction requires bidders to shade their expected values and bids in an equilibrium, and this shading of bids will be an increasing function of the number of bidders. Bajari and Hortaçsu (2003) found that, for an average auction, the bidders lowered their bids by 3.2% per additional competitor.

many of the buy price auctions on eBay and other online auction sites. It is also of theoretical interest to see if a seller's incentive to post a buy price can be explained in a common value auction.

eBay's value of sales in the fixed price platform (Half.com and buy price auctions together) totaled approximately \$5.5 billion during the fourth quarter and \$18.9 billion during the whole year of 2006 which is about 36% of gross merchandise sales of \$52.4 billion in 2006.[2] The growing popularity of buy price auctions on eBay and other sites suggests a substantial motivation for sellers to post and for bidders to accept buy prices. For bidders, a buy price (temporary or permanent) reduces the uncertainty of winning and payment and the time (and other transaction costs) involved in obtaining the item in an auction. For a seller, a buy price reduces the time it takes to complete a trade and the variability in trading prices. The literature on auctions with buy prices has utilized these issues in explaining the popularity of auctions with buy prices in private value auctions (see Ockenfels et al. (2006)).

Budish and Takeyama (2001) were the first to analyze buy price auctions. They considered two risk averse bidders with only two possible valuations in an ascending bid auction with a permanent buy price. They showed that the buy price raises expected seller revenue. Reynolds and Wooders (2005) use a symmetric independent private values (IPV) framework with a continuous distribution of values for the bidders and showed that both temporary and permanent buy prices can raise seller revenue compared to a standard ascending bid auction when the bidders have constant absolute risk aversion (CARA). In fact, they show that the more risk averse the bidders are the easier it is for the seller to generate higher revenues with buy prices. The reason for this, as they note, is that it's easier to make the buy price attractive to the bidders when they are more risk averse since they are willing to pay an even higher risk premium to avoid the uncertainty involved in an ascending bid auction.

[2] These data are collected from eBay's Annual and Quarterly Financial Results press releases (visit http://investor.ebay.com/releases.cfm).

Mathews and Katzman (2006) develops an IPV model which for the first time allows a risk averse seller. They find that a temporary buy price added to a second price sealed bid auction with risk neutral bidders gives the seller a higher expected utility. Since a buy price reduces the variance of seller revenue, the seller's expected utility can increase even when his monetary payoffs are lower. Hidvégi, Wang and Whinston (2006) examine an IPV model for a modified English auction in which a bidder does not observe rival dropouts. They show that a properly set permanent buy price increases social welfare and the utility of each agent when either the bidders or the seller is risk averse.

This paper differs from the previous studies as it considers a common value model with risk aversion for both the bidders and the seller. The implications of a buy price are different in private and common values auctions. A buy price helps a bidder to eliminate the uncertainty of winning and payment in both of these auction formats. But there is an additional source of uncertainty in a common value auction arising from the unknown valuation of the item. A buy price cannot eliminate this uncertainty and hence the winning bidder can suffer from a winner's curse. But in an auction without a buy price a bidder can avoid the winner's curse by shading his bid appropriately. This feature of a common value auction makes a buy price relatively less attractive to the bidders. To summarize, we find that a buy price that is accepted by the bidders with a positive probability reduces the expected revenue of the seller but when a seller is risk averse an appropriately chosen buy price can raise his expected utility. Intuitively, a buy price can be attractive to a risk averse seller since it reduces the uncertainty of revenue.

The rest of this paper is organized as follows. Section 2 describes a common value model of auctions with and without a buy price. Section 3 characterizes equilibrium bidding strategies (for both risk neutral and risk averse bidders) in a general setup and applies them in a simple case of two bidders with signals that are identically and independently distributed as $U[0,1]$. In the two bidders case, we also show that the equilibrium is unique. Section 4 analyzes the seller's incentive to post a buy price when he is either risk neutral or risk averse and there are two bidders, both risk neutral or risk averse. The signals are assumed to be distributed identically and independently as $U[0,1]$. Section 6 makes some concluding remarks.

1.2 The Model:

Consider a seller selling a single item through auction. There are $n \geq 2$ bidders in the market who are identical with respect to their risk aversion attitudes.

Each bidder i receives a signal x_i which is identically and independently distributed according to a cumulative distribution function F over the support of $[\underline{x}, \overline{x}]$. Let $f(x) = F'(x)$ be the probability density function. We consider a *pure common value model* in which the *ex post* valuation of the item is the same for each bidder and equals the average of all the signals.[3] So, for bidder i,

$$v_i = v = \frac{x_1 + x_2 + ... + x_n}{n}.$$

In this formulation, each bidder's valuation is a symmetric function of other bidders' signals, the function v is nonnegative, continuous and nondecreasing in all its variables and twice continuously differentiable. Since each bidder observes only his own signal the true value of the item is unknown to a bidder when he participates in the auction.

If bidder i buys the item and pays a price p, he receives a payoff of $u(v-p)$, otherwise he receives a zero payoff. We assume a *Constant Absolute Risk Aversion* (CARA) utility function, $u(v-p) = \frac{1-e^{-\alpha(v-p)}}{\alpha}$, where α is the index of risk aversion. Notice that $\lim\limits_{\alpha \to 0} u(v-p) = v - p$. As a result, $\alpha = 0$ corresponds to risk neutral bidders. If the seller earns a revenue s, he receives a payoff of $\widetilde{u}(s)$. Assume that $\widetilde{u}(s)$

[3] This common value formulation has been used in Bikhchandani and Riley (1991), Albers and Harstad (1991), Klemperer (1998), Bulow, Huang and Klemperer (1999), Goeree and Offerman (2002) and Goeree and Offerman (2003).

is continuous, $\widetilde{u}(0) = 0$, $\widetilde{u}'(s) > 0$ and $\widetilde{u}''(s) \leq 0$. When the seller is risk neutral, $\widetilde{u}(s) = s$ and $\widetilde{u}''(s) = 0$, but when he is risk averse, $\widetilde{u}''(s) < 0$.

We consider two auction formats. The first format is an auction without a buy price and the auction proceeds as a second price sealed bid (SPSB, hereafter) auction. Bidders simultaneously and independently submit their private bids and the bidder with the highest bid wins the item paying a price equal to the second highest bid. The second format introduces a buy price B and consists of two stages. In the first stage, the seller posts B and the bidders simultaneously and independently decide whether to accept B or not.[4] The auction ends immediately if one or more bidders decide to accept B. If only one of the bidders accepts B, he wins the item for sure. When multiple bidders accept B, the winner is chosen randomly from the bidders who have accepted B. When none of the bidders accepts B, the option to buy at B disappears and the auction proceeds to the second stage. In the second stage, the item is sold via a SPSB auction.

1.3 Equilibrium Bidding Strategies

1.3.1 An Auction without a Buy Price:

In a SPSB auction, a strategy for a bidder is a function mapping his signal x into his bid b. Since this paper concentrates on symmetric auctions, throughout this paper I will focus only on the decisions of bidder 1. The unique symmetric equilibrium bidding function $b(x)$ for bidder 1 with signal $x_1 = x$ is known from Milgrom and Weber (1982). It is defined by the following equation:

$$E[u(v - b(x))|x_1 = x, z = x] = 0, \tag{1.1}$$

where $z = \max\{x_2, ..., x_n\}$ is distributed according to the cumulative distribution

[4] A rational seller posts a buy price larger than the minimum possible value of the item to the bidders.

function $H(z) = F(z)^{n-1}$. When bidders are risk averse and have a CARA utility function, $b(x)$ takes the following form.

$$b_\alpha(x) = \frac{1}{\alpha} \ln \left[\frac{1}{E\left[e^{-\alpha v} | x_1 = x, z = x\right]} \right]. \tag{1.2}$$

When bidders are risk neutral (i.e., $\alpha = 0$), then (1.1) reduces to:

$$b_0(x) = E\left[v | x_1 = x, z = x\right]. \tag{1.3}$$

From (1.1) it is easy to find that $b(\underline{x}) = \underline{x}$. Since $b(x)$ is an increasing function of x, the bidder with the highest signal draw wins the auction.[5] Notice that $b(x)$ does not depend on the upper bound of the support (i.e., $\overline{x}$) since the maximum value that any of the random variables may take in (1.1) is x. Another important thing to notice is that the concavity of the utility function in (1.1) implies that $b_\alpha(x) < b_0(x)$.

1.3.2 An Auction with a Buy Price:

A strategy for a bidder in the first stage is a function mapping his signal x into his decision of whether to accept or reject the buy price B. This paper focuses on cutoff strategies. A cutoff strategy for bidder i is defined by a constant $c \in [\underline{x}, \overline{x}]$ such that bidder i accepts B if $x_i > c$ and he rejects B if $x_i < c$.

Suppose all the rivals employ the same cutoff c. Then, given a signal $x_1 = x$, the expected utility to bidder 1 for accepting B is given by

$$U^A(x, c) = \sum_{l=0}^{n-1} \left[\binom{n-1}{l} F(c)^{n-1-l} (1 - F(c))^l \left(\frac{1}{l+1} \right) u_l \right] \tag{1.4}$$

where $u_l = E[u(v - B) | x_1 = x, T_l]$ is the expected utility to a bidder winning the item at price B when his own signal is x, l rival bidders have signals above c and $n - 1 - l$

[5] For an example, if bidder 1 has the highest signal, he wins the item and pays $b(z)$.

rivals have signals below c. Let T_l denote the condition "l rivals have signals above c".[6] Notice that $U^A(x,\bar{x}) = E[u(v-B)|x_1 = x]$.

When a bidder rejects B he wins only when all his rivals also reject B and he has the highest signal draw. In the SPSB auction stage that follows each bidder knows that all his rivals have signals less than c. Since $b(x)$ is independent of the upper bound of the support it remains the equilibrium bidding function in the subgame that follows when everyone rejects B. So, given a signal $x_1 = x$, the expected utility to bidder 1 for rejecting B is given by

$$\begin{aligned} & U^R(x,c) \\ = \; & E[u(v-b(z))|x_1 = x, z < \min\{x,c\}]H(\min\{x,c\}) \\ = \; & \int_{\underline{x}}^{\min\{x,c\}} \left[\int_{\underline{x}}^{z} \ldots \int_{\underline{x}}^{z} u\left(\frac{x+z+\sum_{l>2} x_l}{n} - b(z) \right) \frac{f(x_3)}{F(z)} dx_3 \ldots \frac{f(x_n)}{F(z)} dx_n \right] h(z) dz \\ = \; & (n-1) \int_{\underline{x}}^{\min\{x,c\}} \int_{\underline{x}}^{x_2} \ldots \int_{\underline{x}}^{x_2} u\left(\frac{x+\sum_{l>1} x_l}{n} - b(x_2) \right) dF(x_n) \ldots dF(x_2). \qquad (1.5) \end{aligned}$$

Notice that $U^R(x,\underline{x}) = 0$.

A cutoff c^* is a **symmetric Bayes Nash equilibrium** if a bidder receives a higher expected utility by accepting B (than rejecting it) when $x > c^*$, and he receives a higher expected utility by rejecting B (than accepting it) when $x < c^*$. In other words, $U^A(x,c^*) > U^R(x,c^*)$ when $x > c^*$ and $U^A(x,c^*) < U^R(x,c^*)$ when $x < c^*$. Before we propose an equilibrium we will define two quantities, denoted by $\overline{\gamma}$ and $\underline{\gamma}$, which are useful in characterising the candidate equilibrium.

(1) Let $\overline{\gamma}$ be the price that makes bidder 1 with a signal $x_1 = \overline{x}$ indifferent

[6] We can express u_l as

$$\begin{aligned} u_l \; = \; & \int_c^{\bar{v}} \cdots \int_c^{\bar{v}} \int_{\underline{v}}^{c} \cdots \int_{\underline{v}}^{c} u\left(\frac{x+\sum_{j>1} x_j}{n} - B \right) \frac{f(x_2)}{F(c)} dx_2 \ldots \frac{f(x_{n-l})}{F(c)} dx_{n-l} \\ & \times \frac{f(x_{n-l+1})}{1-F(c)} dx_{n-l+1} \ldots \frac{f(x_n)}{1-F(c)} dx_n. \end{aligned}$$

between buying the item at $\overline{\gamma}$ and participating in the SPSB auction. So, $\overline{\gamma}$ is defined by

$$E[u(v-\overline{\gamma})|x_1=\overline{x}] = E[u(v-b(z))|x_1=\overline{x}]. \tag{1.6}$$

The right side of (1.6) is the bidder's expected utility in the SPSB auction (he is the winning bidder as he has the highest signal), while the left side is his expected utility when he buys the item at a price of $\overline{\gamma}$. For risk averse bidders with a CARA utility function, (1.6) simplifies to:

$$\overline{\gamma}_\alpha = \frac{1}{\alpha}\ln\left[\frac{E[e^{-\alpha(v-b(z))}|x_1=\overline{x}]}{E[e^{-\alpha v}|x_1=\overline{x}]}\right] \tag{1.7}$$

and for risk neutral bidders, (1.6) simplifies to:

$$\overline{\gamma}_0 = E[b(z)|x_1=\overline{x}]. \tag{1.8}$$

(2) Let $\underline{\gamma}$ be the maximum willingness to pay for bidder 1 with a signal $x_1=\underline{x}$ which is defined by

$$E[u(v-\underline{\gamma})|x_1=\underline{x}] = 0. \tag{1.9}$$

For $\alpha>0$ and $\alpha=0$, (1.9) simplifies to the following two equations, respectively:

$$\underline{\gamma}_\alpha = \frac{1}{\alpha}\ln\left[\frac{1}{E[e^{-\alpha v}|x_1=\underline{x}]}\right] \tag{1.10}$$

and

$$\underline{\gamma}_0 = E[v|x_1=\underline{x}]. \tag{1.11}$$

Proposition 1 below characterizes the equilibrium cutoff c^* in an auction with a buy price B.

Proposition 1:

For an auction with a buy price B,

(i) when $B \in (\underline{\gamma}, \overline{\gamma})$ there exists a symmetric equilibrium cutoff $c^* \in (\underline{x}, \overline{x})$ defined by:

$$U^A(c^*, c^*) = U^R(c^*, c^*) \tag{1.12}$$

(ii) when $B \leq \underline{\gamma}$, $c^* = \underline{x}$ is a symmetric equilibrium cutoff, that is, everyone accepts B.

(iii) when $B \geq \overline{\gamma}$, $c^* = \overline{x}$ is a symmetric equilibrium cutoff, that is, everyone rejects B.

Proof: Appendix-A.

Next I consider a case of two bidders. This helps us understand Proposition 1. The uniqueness properties of the proposed equilibrium are also established in this case.

1.3.3 A Case of Two Bidders:

Let $n = 2$ and the signals be identically and independently distributed as an uniform distribution over the support $[0, 1]$. From (1.2) and (1.3) it is easy to find that $b(x) = x$.

Risk Neutral Bidders: Given the above simplifying assumptions, some simple calculations show that $\underline{\gamma}_0 = \frac{1}{4}$ and $\overline{\gamma}_0 = \frac{1}{2}$ (from (1.11) and (1.8) , respectively). Proposition 1 implies the existence of some equilibrium cutoff $c_0^* \in (0, 1)$ when $B \in (\frac{1}{4}, \frac{1}{2})$ along with $c^* = 0$ when $B \leq \frac{1}{4}$ and $c^* = 1$ when $B \geq \frac{1}{2}$. Now let's try intuitively to explain these bounds on B. In order to explain the lower bound of $\underline{\gamma}_0 = \frac{1}{4}$, fix bidder 2's cutoff at $c = 0$. In this situation, bidder 1's expected payoff by rejecting B is zero (as bidder 2 accepts B and wins the item for sure). Suppose bidder 1 has a signal $x_1 = x$. So, by accepting B bidder 1 gets a payoff of $\frac{1}{2}(E[v|x_1 = x] \ - B) = \frac{1}{2}((\frac{x}{2} + \frac{1}{4}) - B)$.[7] If $x = 0$, bidder 1 is indifferent between accepting and rejecting B when $B = \frac{1}{4}$ which is consistent with $c = 0$ for bidder 1. For any $B < \frac{1}{4}$, bidder 1 prefers accepting B, which is also consistent with $c = 0$ for bidder 1. If $x > 0$, $E[v|x_1 = x] > \frac{1}{4}$. So, bidder 1 accepts any $B \leq \frac{1}{4}$ which is consistent with $c = 0$ for bidder 1.

In order to explain the upper bound of $\overline{\gamma}_0 = \frac{1}{2}$, fix bidder 2's cutoff at $c = 1$ and let $x_1 = x$. Bidder 1 wins the item for sure if he accepts B and gets a payoff $(\frac{x}{2} + \frac{1}{4} - B)$. But if he rejects B he gets a payoff $\frac{x^2}{4}$. If $x = 1$, bidder 1 is indifferent between accepting and rejecting B when $B = \frac{1}{2}$ which is consistent with $c = 1$ for bidder 1. So, bidder 1 definitely rejects any $B > \frac{1}{2}$ which is consistent with $c = 1$ for bidder 1. If $x < 1$, $(\frac{x}{2} + \frac{1}{4} - B) < \frac{x^2}{4}$. So, bidder 1 rejects any $B \geq \frac{1}{2}$ which is consistent with $c = 1$ for bidder 1.

For any $B \in (\frac{1}{4}, \frac{1}{2})$ neither $c = 0$ nor $c = 1$ can be an equilibrium cutoff. To see this, fix $B \in (\frac{1}{4}, \frac{1}{2})$, bidder 2's cutoff at $c = 0$ and $x_1 = x$. If $x = 0$, bidder 1 gets a negative expected payoff by accepting $B > \frac{1}{4}$. So he prefers rejecting B which is inconsistent with $c = 0$ for bidder 1. Again, fix bidder 2's cutoff at $c = 1$ and $x_1 = x$. If $x = 1$, bidder 1 gets a higher payoff by accepting $B < \frac{1}{2}$ which is inconsistent with $c = 1$ for bidder 1. So, an equilibrium cutoff needs to be at the interior when $B \in (\frac{1}{4}, \frac{1}{2})$. Proposition 1 characterizes these interior equilibrium cutoffs.

Now, proposition 2 establishes the uniqueness property of an interior equilibrium cutoff c_0^*.

Proposition 2: *For $n = 2$, $x \sim U[0,1]$ and $\alpha = 0$, the equilibrium cutoff $c_0^* \in (0,1)$ corresponding to $B \in (\frac{1}{4}, \frac{1}{2})$ (as defined in Proposition 1) is unique.*

Proof: Appendix-A.

[7] Calculations for $U^A(x,c)$ and $U^R(x,c)$ are shown in the proof of Proposition 2.

Risk Averse Bidders: From (1.10) and (1.7) , $\underline{\gamma}_\alpha = \frac{1}{\alpha}\ln\left[\frac{\alpha e^{\frac{\alpha}{2}}}{2e^{\frac{\alpha}{2}}-2}\right]$ and $\overline{\gamma}_\alpha = \frac{1}{2}$. According to Proposition 1 an equilibrium cutoff $c^*_\alpha \in (0,1)$ exists when $B \in \left(\frac{1}{\alpha}\ln\left[\frac{\alpha e^{\frac{\alpha}{2}}}{2e^{\frac{\alpha}{2}}-2}\right], \frac{1}{2}\right)$ along with $c^*_\alpha = 0$ when $B \le \frac{1}{\alpha}\ln\left[\frac{\alpha e^{\frac{\alpha}{2}}}{2e^{\frac{\alpha}{2}}-2}\right]$ and $c^*_\alpha = 1$ when $B \ge \frac{1}{2}$. It is easy to show that the lower bound $\underline{\gamma}_\alpha$ is positive[8]. Similar to the risk neutral case, $c^*_\alpha = 0$ can be justified when $B \le \frac{1}{\alpha}\ln\left[\frac{\alpha e^{\frac{\alpha}{2}}}{2e^{\frac{\alpha}{2}}-2}\right]$ $\underline{\gamma}_\alpha$ by fixing $c = 0$ for bidder 2 and looking at bidder 1's optimal decisions. It can also be shown that $\underline{\gamma}_\alpha < \underline{\gamma}_0 (= \frac{1}{4})$.[9] This is derived below[10]:

$$e^{\frac{\alpha}{2}} - 1 > \left(\frac{\alpha}{2}\right)e^{\frac{\alpha}{4}} \;\Rightarrow\; \frac{e^{\frac{\alpha}{2}}}{e^{\frac{\alpha}{2}}-1} < \left(\frac{\alpha}{2}\right)\frac{e^{\frac{\alpha}{2}}}{e^{\frac{\alpha}{4}}} \Rightarrow \frac{\frac{\alpha}{2}e^{\frac{\alpha}{2}}}{e^{\frac{\alpha}{2}}-1} < e^{\frac{\alpha}{4}}$$
$$\Rightarrow \ln\left[\frac{\alpha e^{\frac{\alpha}{2}}}{2e^{\frac{\alpha}{2}}-2}\right] < \frac{\alpha}{4} \;\Rightarrow\; \frac{1}{\alpha}\ln\left[\frac{\alpha e^{\frac{\alpha}{2}}}{2e^{\frac{\alpha}{2}}-2}\right] < \frac{1}{4}.$$

The reason for a smaller lower bound is that it's harder to make B attractive to bidder 1 when he is risk averse. Although accepting B might give him a positive payoff (compared to the zero payoff for rejecting) he faces the uncertainty regarding bidder 2's signal which enters his valuation for the item. As a result, B needs to be lower than $\frac{1}{4}$ to elicit indifference between accepting and rejecting B for a risk averse with $x = 0$.[11]

Notice that the upper bound of $\overline{\gamma} = \frac{1}{2}$ is the same under both risk neutrality and risk aversion. In order to understand this, once again, fix $c = 1$ for bidder 2 and let $x = 1$ for bidder 1. Bidder 1's surplus upon winning is $\left(\frac{x}{2} + \frac{x_2}{2} - B\right)$ when he accepts B, while it is $\left(\frac{x}{2} - \frac{x_2}{2}\right)$ when he rejects B. These two expressions will have exactly

[8] Since $-\alpha/2 \neq 0$, we have $e^{-\alpha/2} > 1 - \alpha/2 \Rightarrow \alpha/2 > 1 - e^{-\alpha/2} \Rightarrow \frac{\alpha/2}{1-e^{-\alpha/2}} > 1 \Rightarrow \frac{\alpha e^{\alpha/2}}{2(e^{\alpha/2}-1)} > 1 \Rightarrow \frac{1}{\alpha}\ln\left[\frac{\alpha e^{\alpha/2}}{2e^{\alpha/2}-2}\right] > 0$

[9] For example, when $\alpha = 1$ we get $\underline{\gamma}_\alpha = \frac{1}{\alpha}\ln\left[\frac{\alpha e^{\frac{\alpha}{2}}}{2e^{\frac{\alpha}{2}}-2}\right] = \frac{6}{25}$ which is smaller than $\frac{1}{4}$.

[10] The first line of this derivation uses the fact that $e^m > 1 + me^{\frac{X}{2}}$ when $m > 0$. Since $\alpha > 0$ it is the case that $\frac{\alpha}{2} > 0$.

[11] Notice that this lower bound as given by $\frac{1}{\alpha}\ln\left[\frac{\alpha e^{\alpha/2}}{2e^{\alpha/2}-2}\right]$ is a decreasing function of α, the index for risk aversion. This implies that the higher the level of risk aversion, the lower this lower bound has to be. The more risk averse the bidders are the harder it is to make the buy price attractive.

the same mean and variance when $B = \frac{1}{2}$. Notice that there is no scope for B to reduce uncertainty of winning for bidder 1 (since he wins for sure by either accepting or rejecting B) and the elements of uncertainty regarding bidder 2's signal enter the surplus the same way in cases of both accepting and rejecting B. As a result, the value of B that induces indifference remains the same under both risk neutrality and risk aversion.

For any $B \in \left(\frac{1}{\alpha}\ln\left[\frac{\alpha e^{\frac{\alpha}{2}}}{2e^{\frac{\alpha}{2}}-2}\right], \frac{1}{2}\right)$ Proposition 1 characterizes an interior cutoff. An interior cutoff is defined implicitly by (1.12) [12]:

$$\begin{aligned} & \frac{1}{\alpha}\left(\frac{1}{2} + \frac{c}{2} + \frac{1}{\alpha}e^{-\alpha(c-B)} - \frac{2}{\alpha}e^{-\alpha(\frac{c}{2}-B)} + \frac{1}{\alpha}e^{-\alpha(\frac{c+1}{2}-B)}\right) \\ = \; & \frac{1}{\alpha}\left(c - \frac{2}{\alpha}e^{(-\frac{\alpha c}{2}+\frac{\alpha}{2}c)} + \frac{2}{\alpha}e^{-\frac{\alpha c}{2}}\right). \end{aligned} \tag{1.13}$$

Now, proposition 3 establishes the uniqueness property of an interior equilibrium cutoff c^*_α.

Proposition 3: *For $n = 2$, $x \sim U[0,1]$ and $\alpha > 0$, the equilibrium cutoff $c^*_\alpha \in (0,1)$ corresponding to $B \in \left(\frac{1}{\alpha}\ln\left[\frac{\alpha e^{\frac{\alpha}{2}}}{2e^{\frac{\alpha}{2}}-2}\right], \frac{1}{2}\right)$ (as defined in Proposition 1) is unique.*

Proof: Appendix-A.

1.4 A Seller's Motivation for a Buy Price:

In our model the seller has the option to choose an auction with or without a buy price. He chooses the auction format that maximizes his expected utility. Let $x_{(n)}$ and $x_{(n-1)}$ be the highest and the second highest order statistics among n draws and $\widetilde{f}$ be their joint density function. So, $\widetilde{f}(q,r) = n(n-1)f(q)f(r)F(q)^{n-2}$. Now, the expected utility to the seller in an auction without a buy price is

[12] Calculations for $U^A(x,c)$ and $U^R(x,c)$ are shown in the proof of Proposition 3.

$$\widetilde{U}_{WBP} = \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{r} \widetilde{u}\,(b(q))\, \widetilde{f}(q,r)dqdr. \tag{1.14}$$

In an auction with a buy price B, let $\beta(c)$ be the function that maps an equilibrium cuoff c^* to a buy price B . From the indifference equation in Proposition 1 it is easy to show that this function exists. So, the expected utility to the seller in an auction with a buy price as a function of the equilibrium cutoff c^* is

$$\widetilde{U}_{BP}(c^*) = \int_{\underline{x}}^{c^*} \int_{\underline{x}}^{r} \widetilde{u}\,(b(q))\, \widetilde{f}(q,r)dqdr + \int_{c^*}^{\overline{x}} \int_{\underline{x}}^{r} \widetilde{u}\,(\beta(c^*))\, \widetilde{f}(q,r)dqdr. \tag{1.15}$$

Notice that $\widetilde{U}_{BP}(\overline{x}) = \widetilde{U}_{WBP}$. This means that when the seller sets B high enough so that none of the bidders ever accepts B the seller's expected utilities from the two auction formats are the same. But if he sets B so that it is accepted with positive a probability the revenues from the two auction formats will have different variances and may also have different means. So the seller's decision to post a buy price will be affected by his attitude towards risk.

1.4.1 A Risk Neutral Seller:

In a common value auction (with or without a buy price) bidders' bidding behaviors are affected by their risk preferences. As a result, the outcome of an auction will be different depending on these preferences. I will treat risk neutral and risk averse bidders separately. Let's assume that $n = 2$ and $x \sim U[0, 1]$.

When both the bidders are either risk neutral or risk averse, $b(x) = x$. From (1.14) , $\widetilde{U}_{WBP} = \frac{1}{3}$.[13] When bidders are risk neutral, any buy price $B < \overline{\gamma}_0 = \frac{1}{2}$ has a positive probability of being accepted. For such buy price, Proposition 4 compares $\widetilde{U}_{WBP}$ and $\widetilde{U}_{BP}$ when the bidders are risk neutral.

[13] For $n = 2$, $x \sim U[0, 1]$ we get $\widetilde{f}(q, r) = 2$.

Proposition 4: *For $n = 2$, $x \sim U[0,1]$, and $\alpha = 0$, an auction with a buy price $B < \overline{\gamma}_0 = \frac{1}{2}$ generates a smaller expected seller revenue than an auction without any buy price, that is, for a risk neutral seller $\widetilde{U}_{BP} < \widetilde{U}_{WBP} = \frac{1}{3}$.*

Proof: Appendix-A.

In order to understand Proposition 4 let's consider the different possible values of B. The expected seller revenue in an auction with $B < \underline{\gamma}_0 = \frac{1}{4}$ is B (since everyone accepts B) and this is smaller than $\frac{1}{3}$. When $B \in (\frac{1}{4}, \frac{1}{2})$ the expected seller revenue for each of possible B in this range is always less than $\frac{1}{3}$. Figure 1.1 graphically evaluates $\widetilde{U}_{WBP}$ at different values of $B \in [\frac{1}{4}, \frac{1}{2}]$. The seller revenue is maximized at $B = \frac{1}{2}$ and is equal to $\frac{1}{3}$, while for $B \in (\frac{1}{4}, \frac{1}{2})$ the seller revenue is strictly less than $\frac{1}{3}$. As a result, $\widetilde{U}_{BP} < \widetilde{U}_{WBP} = \frac{1}{3}$ when the bidders are risk neutral and $B < \overline{\gamma}_0 = \frac{1}{2}$.

For the revenue implications when the bidders are risk averse, let's assume $\alpha = 1$. So, $\underline{\gamma}_1 = \frac{6}{25}$ and $\overline{\gamma}_1 = \frac{1}{2}$. It can numerically be shown that an auction with $B < \overline{\gamma}_1 = \frac{1}{2}$ generates an expected seller revenue less than $\frac{1}{3}$. The expected seller revenue for $B < \underline{\gamma}_1 = \frac{6}{25}$ is B (since everyone accepts B) which is smaller than $\frac{1}{3}$. When $B \in (\frac{6}{25}, \frac{1}{2})$, B is accepted with positive probabilities and the expected seller revenue for each possible B in this range is always less than $\frac{1}{3}$ (as Figure 1.2 shows). So, $\widetilde{U}_{WBP} > \widetilde{U}_{BP}$ when $B < \overline{\gamma}_1 = \frac{1}{2}$.

In order to understand the intuition behind the above result let's consider the set of buy prices $(\frac{6}{25}, \frac{1}{2})$ and divide this into two sub-sets: (i) $B \in \left(\frac{6}{25}, \frac{1}{3}\right)$ and (ii) $B \in \left(\frac{1}{3}, \frac{1}{2}\right)$. Let c_1^* and c_0^* denote the equilibrium cutoffs corresponding to $\alpha = 1$ and $\alpha = 0$. When $B \in \left(\frac{6}{25}, \frac{1}{3}\right)$ surely $c_1^* < 1$, and since $B < \frac{1}{3}$ it follows from (1.15) that $\widetilde{U}_{BP} = c_1^{*2}[\frac{1}{3}c_1^*] + (1 - c_1^{*2})B < \frac{1}{3} = \widetilde{U}_{WBP}$. Let's fix a $B \in \left(\frac{1}{3}, \frac{1}{2}\right)$, say $B = \frac{2}{5}$. With $B = \frac{2}{5}$, once again, $\widetilde{U}_{BP} = c_1^{*2}[\frac{1}{3}c_1^*] + (1 - c_1^{*2})B$ from (1.15) . A buy price $B = \frac{2}{5}$ is accepted with positive probabilities also under when $\alpha = 0$ and Proposition 4 (together with (1.15)) says that $c_0^{*2}[\frac{1}{3}c_0^*] + (1 - c_0^{*2})B < \frac{1}{3}$. As $\frac{1}{3}c_0^* < \frac{1}{3}$ and $B = \frac{2}{5} > \frac{1}{3}$, the only way $c_0^{*2}[\frac{1}{3}c_0^*] + (1 - c_0^{*2})B$ may be at least as high as $\frac{1}{3}$ is when

c_0^*'s are replaced with smaller values. But $c_1^* = 0.62 > c_0^* = 0.60$ when $B = \frac{2}{5}$. So, $\widetilde{U}_{BP} = c_1^{*2}[\frac{1}{3}c_1^{*3}] + (1 - c_1^{*2})B < \frac{1}{3} = \widetilde{U}_{WBP}$ for $B = \frac{2}{5}$. Similar reasonings can be made for any other $B \in \left(\frac{1}{3}, \frac{1}{2}\right)$.

A buy price helps a risk averse bidder to avoid some of the uncertainties regarding winning and payment upon winning in an auction. This may give the seller a chance to extract some risk premium in the form of a higher B (as it happens in the case of a private value model in Reynolds and Wooders (2005)). But there is another source of uncertainty arising from the unobservability of the rival's signal which enters a bidder's expected payoff when he accepts B. Risk averse bidders dislike this uncertainty. This feature of the common value model, as it turns out, leads to the complete failure of the seller in extracting any risk premium from the bidders. A risk neutral seller prefers an auction witout a buy price as an auction with B fails to generate any revenue advantage[14].

1.4.2 A Risk Averse Seller:

A risk averse decision maker cares for not only the monetary payoffs but also the level of uncertainties involved in different stakes. Although an auction with a buy price has no revenue advantage it might still be attarctive to a risk averse seller as it involves less uncertainties regarding the revenue he obtains. In particular, an auction with a buy price that is accepted with positive probabilities entails lesser variance of revenue than an auction without any buy price. Let's assume that $n = 2$ and $x \sim U[0, 1]$. Now, Proposition 5 states that a risk averse seller actually prefers an auction with a buy price to an auction without any buy price.

Proposition 5: *For $n = 2$ (both risk neutral or risk averse) and $x \sim U[0, 1]$, there exists some buy price $B < \overline{\gamma} = \frac{1}{2}$ for which an auction with a buy price generates*

[14]This result remained the same in different examples involving more than two bidders with different degrees of risk aversion.

a higher expected utility for the seller than an auction without any buy price, that is, $\widetilde{U}_{BP} > \widetilde{U}_{WBP}$.

Proof: Appendix-A.

When $B = \overline{\gamma}_0 = \frac{1}{2}$ none of the bidders ever accepts B (that is, $c^* = 1$) and $\widetilde{U}_{BP}(1) = \widetilde{U}_{WBP} = \frac{1}{3}$. The proof of Proposition 5 relies on the existence of some $B < \overline{\gamma} = \frac{1}{2}$ which generates an equilibrium cutoff $c^* < 1$ for which $\widetilde{U}_{BP}(c^*) > \widetilde{U}_{BP}(1) = \widetilde{U}_{WBP} = \frac{1}{3}$. In order to show this existence it is sufficient to establish that $\left.\frac{d\widetilde{U}_{BP}(c^*)}{dc^*}\right|_{c^*=1} < 0$. Intuitively, the lower variance of revenue in an auction with a buy price (which is high enough but still accepted with positive probabilities) offsets the effect of lower expected revenue compared to an auction without any buy price.

1.5 Conclusion:

In this paper, I have modeled auctions with and without a buy price. An auction without a buy price is a SPSB auction, while an auction with a buy price adds a temporary buy price to a SPSB auction. A pure common value framework is used where bidders' signals are independent and identical draws from some continuous distribution and the common value of the item is the average of all signals. A symmetric equilibrium in the general case of n CARA bidders is then characterized. In order to derive some intuitions and establish uniqueness properties a simplifying case of two bidders is considered where the signals are distributed as $U[0,1]$. I then analyze a seller's motivation to post a buy price in this two-bidders case.

A buy price fails to give a risk neutral seller any revenue advantage with both risk neutral and risk averse bidders. This is a major departure from the results of the private value model. Reynolds and Wooders (2005) identifies a range of buy prices that gives a seller higher expected revenue when bidders are risk averse and values are private. This resutls from the fact that accepting a buy price permits a bidder to avoid the uncertainty about winning and payment in an acution. A bidder who accepts a buy price has a higher chance of winning the auction and earn a certain surplus of his value minus the buy price. Then it becomes easier to make a higher buy price attractive since risk averse bidders view this increase in a buy price as a premium they pay to escape some uncertainties. This, in turn, translates into a higher seller revenue. But under the common value framework a buy price is unable to offer that much reduction in bidder uncertainties. The elements of uncertainties about rivals' information can lead to a winner's curse when a bidder accepts a buy price. These uncertainties enter a bidder's surplus upon winning via the common value structure of the model. This makes it harder for the seller to extract any risk premium from the bidders in the form of a higher buy price, and the risk neutral seller, in fact, ends up with a lower expected revenue by posting a buy price which is accepted with positive probabilities. But when a buy price is accepted with positive probabilities the variance of revenue decreases. This gives a risk averse seller an additional motivation to post buy prices. I find that a suitably chosen buy price raises the expected utility of a risk averse seller when bidders are either risk neutral or risk averse. So, the risk aversion of a seller justifies posting buy prices in common value auctions.

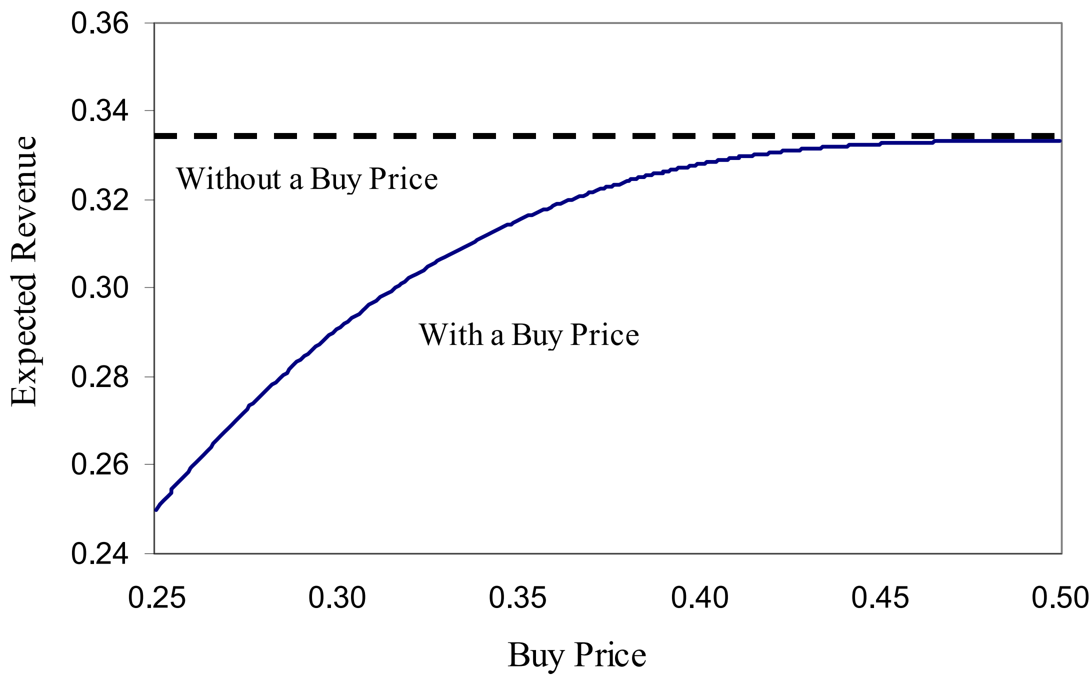

FIGURE 1.1. Expected Seller Revenue (Risk Neutral Bidders)

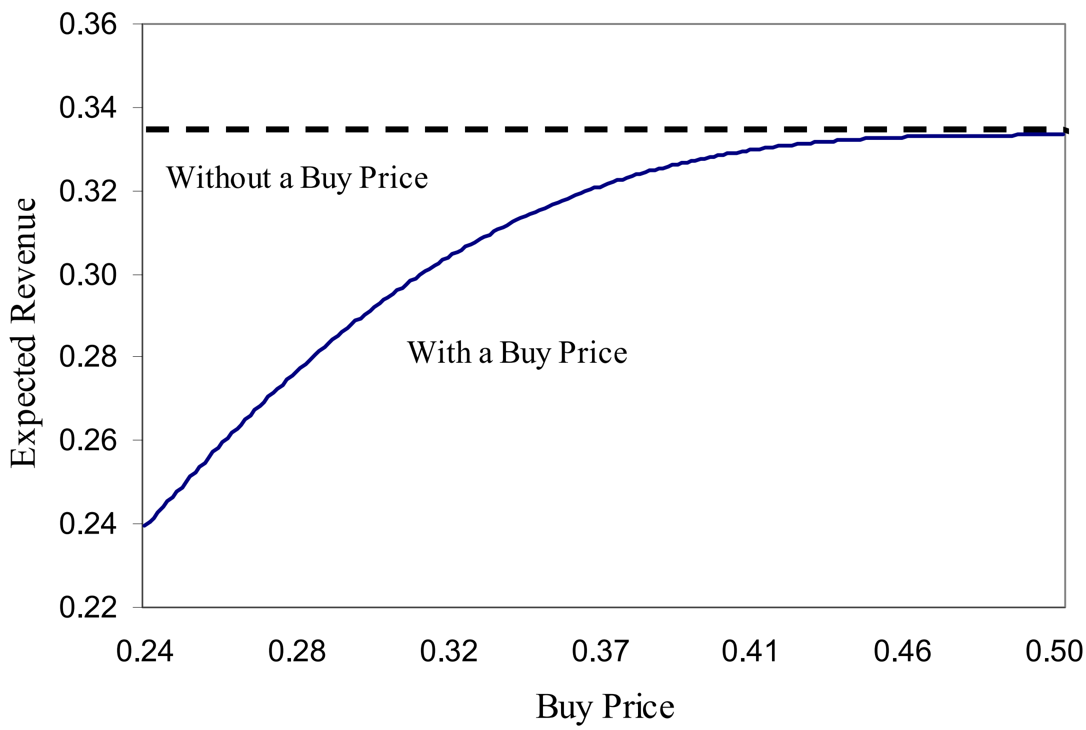

FIGURE 1.2. Expected Seller Revenue (Risk Averse Bidders)

Chapter 2

An Experimental Study of Auctions with a Buy Price Under Private and Common Values

2.1 Introduction

[1]In a "buy it now" auction the seller sets a fixed price, termed a "buy price," at which a bidder may purchase the item, thereby ending the auction immediately. If no bidder accepts the buy price, then in the ascending bid auction that follows, the bidder with the highest bid wins and pays the second highest bid. The buy-now auction format has proven to be extremely popular. eBay introduced its buy-now auction format in 2000 and by the end of 2001 about 40% of all eBay auctions were "buy-now" auctions (see Hof (2001)).

Several theoretical explanations for the popularity of buy prices have been proposed for private-value auctions. Reynolds and Wooders (2003) show for both eBay and Yahoo auctions that when bidders are risk averse then a suitably chosen buy price raises seller revenue; in this case the buy price extracts a risk premium from bidders who wish to avoid uncertainty over whether they win and the price that they pay. Mathews (2003) shows for eBay auctions that when bidders are impatient, then a seller can increase his revenue by setting a buy price; in this case the buy price extracts a premium from bidders who end the auction early. Mathews and Katzman (2006) consider eBay auctions with risk-neutral bidders and risk averse sellers, in which case a buy price may be advantageous for the seller as it reduces the variance of seller revenue.

In the present paper we investigate experimentally the properties of eBay buy-now auctions in both pure private value and pure common value settings. For private

[1]This is a joint work with John Wooders.

value auctions our objective is to determine whether a buy price raises seller revenue and whether it reduces the variance of revenue, as theory suggests it can. We find that in private value auctions the use of a buy price has a positive and statistically significant effect on seller revenue. We also find that a buy price lowers the standard deviation of revenue. In fact, the empirical cdf of revenue in buy price auctions second order stochastically dominates revenue in auctions without a buy price, suggesting that a risk averse seller is better off with a buy price. These experimental results provide support to the main theoretical explanations offered for the use of buy prices in private value auctions.[2]

In private value auctions a bidder who accepts a buy price eliminates uncertainty about his payment (and hence his payoff). In common-value auctions, in constrast, a bidder who accepts a buy price eliminates uncertainty about his payment but not about his payoff, since he is also uncertain about the item's value. Common value buy price auctions have, as a result, quite different theoretical properties than private value auctions: In the common value setting we study, there is no buy price that raises seller revenue, whether bidders are risk neutral or risk averse; unlike the private value case, seller revenue and the probability the buy price is accepted are both decreasing in the degree of bidder risk aversion. These different theoretical properties suggest that buy prices are worth studying experimentally in common value settings. Common value auctions are also of practical interest as well since in some eBay auctions a common value model may be more appropriate than a private value model. Bajari and Hortaçsu (2003), for example, argue that a common value model is correct for eBay coin auctions.[3]

In a common value ascending bid auction, a rational (risk neutral) bidder drops out of the auction when the bid reaches his expected value for the item conditional on his

[2]Since there is no meaningful delay in laboratory experiments, the results do not provide insight into whether a buy price can be used to exploit bidder impatience.

[3]They find the sale price is decreasing in the number of bidders, which is consistent with common values.

own signal and conditional on the highest signal of a rival bidder being equal to his own signal (see Milgrom and Weber (1982)). In common value buy price auctions we focus on equilibria in which bidders follow "cutoff" strategies, with bidders accepting the buy price if their signal exceeds some cutoff "c," and rejecting it otherwise. Consider a rational bidder whose signal is less than c, and who therefore rejects the buy price. If the buy price is also rejected by all the other bidders, then this has no effect on the bidder's dropout price – i.e., his dropout price is the same after the buy price is rejected as it would be in the auction in which no buy price is offered, when his own signal is the same in each case. Observing that all his rivals reject the buy price, and hence have signals below c, is not informative to a rational bidder who conditions on the highest signal of a rival being equal to his own, since his own signal is less than c.

Prior experimental studies of common value auctions have found that bidders are naive in common value auctions, conditioning only on their own signal (and failing to condition on their rival's signals) when forming their bid. When all his rivals reject the buy price, this is informative to a naive bidder who then infers that all his rivals have signals less than the cutoff c. A naive bidder, consequently, drops out earlier when the buy price is rejected than in an ascending bid auction where no buy price is offered. The different behaviors predicted by the rational and by the naive models for buy price auctions provide a new mechanism to distinguish rational from naive bidding in common value auctions.

We find that, in fact, bidders drop out earlier when the buy price is rejected than they do in identical auctions were no buy price has been offered. This finding provides additional support for the naive bidding model. We find that in the ascending bid phase of the auction, bidders tend to overbid relative to equilibrium when they have low signals and underbid when they have high signals. This last finding is consistent with prior studies of ascending bid common-value auctions without a buy price. In contrast to the predictions of the rational model: the use of a buy price in common value auctions had a small positive (although statistically insignificant) effect on seller revenue; the buy price set in the experiment is accepted with high frequency, although theoretically it is always rejected if bidders are risk neutral or risk averse. We also find the variance of seller revenue is statistically significantly lower in the buy-now auctions.

As in prior experimental studies (e.g.,) we find that bidders suffer from the winner's curse, apparently conditioning only on their own signal and failing to condition on having the highest signal. We also find that bidders bid less agressively, choosing lower dropout prices, after the buy price is rejected. Following

In order to better explain the common value data, we develop and estimate a behavioral model of common value buy-now auctions. In the model a bidder fails to condition his value for the item on winning, both when deciding whether to accept the buy price and when deciding whether to drop out in the ascending bid phase of the auction (i.e., a bidder suffers from the "winner's curse" when making either type of decision). The model also allows bidders to overweight their private information, and we find that overweighting of own signal is statistically significant and is important in explaining the high frequency with which the buy price is accepted. The behavioral model explains all the departures from the rational model mentioned above.

Results From Field Data

There are several important empirical studies based on field data from eBay auctions. We focus here on their finding regarding which sellers tend to use a buy price and the revenue effects of a buy price. In an extensive study of Palm Vx auctions, Anderson, Friedman, Milam, and Singh (2004) find that sellers are more likely to use the buy-it-now auction format as they are more experienced. However, since their sample consists entirely of auctions which end with a sale, the effect on seller revenue of employing a buy-it-now price is not clear. Durham, Roelofs, and Standifird (2004), henceforth DR&S, studies a sample of 138 auctions of American silver dollars ending with a sale. They find that the 41 auctions listed with a buy price had an average selling price of \$10.27, while the remaining auctions had an average selling price of \$9.56, a statistically significant difference. They also find that a buy price tended to be offered by the more experienced sellers. DR&S reports in addition the results of a field experiment in which they conducted 84 eBay auctions of 2001 American Eagle Silver dollars, with varying buy prices but the same low reserve price of \$1.00. All of these auctions ended with a sale. The average sale price in auctions without a buy price was \$8.82. Setting a buy price equal to \$8.80 raised average revenue to \$9.83, a difference which was statistically significant. [4]

There are several reasons why laboratory experiments complement the analysis of field data obtained from either naturally occurring or experimental auctions. First, the theory doesn't make unambiguous predictions regarding the revenue effect of buy prices since, as shown in Mathews (2004), a risk averse seller may have an incentive to set a buy price even if it reduces expected revenue. Second, as noted earlier, the theoretical properties of a buy price differ depending on whether the bidders' values are characterized by private or by common values. In the field it may be difficult to determine whether either pure private values or pure common values is appropriate, whereas in the lab the experimenter controls the structure of values. Third, in the field bidders may be both risk averse and impatient. The short duration of experiments in the lab allow the experimenter to focus on the effects of bidder risk aversion in isolation. Fourth, when analyzing field data one must control for differences the seller reputation (which is known to have an effect on price), whereas reputation is not a factor in lab experiments. Finally, an a laboratory experiment one can focus on the effect of a buy price by varying whether or not the auction has a buy price, while holding all the other aspects of the auction – e.g., the number of bidders, the reserve

[4] DR&S remark that this difference is largely driven by several outliers in the buy-it-now auction where the sale price was unusually high. Since the \$8.80 buy-price was accepted in only 1 of the 21 buy-price auctions, the revenue difference must be the result of a difference in bidding behavior when no buy price was present and behavior when a buy price was originally offered but disappeared following a bid.

price – fixed.

Section 2 presents the experimental design. The theoretical background on buy-now auction is presented in Section 3. Section 4 presents our experimental results for both the private value and common value auctions. In Section 5 we estimate a behavioral model of common value buy-now auctions. Section 6 concludes.

2.2 Experimental Design

We conducted experiments for the four treatments shown in Table 2.1. In the private value (henceforth PV) treatments, each bidder's value for the item was an independent draw from the $U[\$0, \$10]$ distribution. In the common value (henceforth CV) treatments each bidder's *signal* was an independent draw from the $U[\$0, \$10]$ distribution; the value of the item was the same for each bidder and equal to the average of the signals.[5] For expositional convenience we used the same distribution for value/signal draws in both the private and common value treatments. This implies, however, no theoretical connection between the two treatments.

We conducted both ascending bid auctions and buy-now auctions. In the ascending bid auctions the price increased by $.25 per second so long as at least one bidder remained active. At any point a bidder could choose to exit the auction by clicking on a "Drop Out" button. Bidders did not observe the number of bidders remaining in the auction, i.e., they did not observe when a rival bidder dropped out. The auction ended when only one bidder remained, the remaining bidder won the auction and paid a price equal to the amount at which the last bidder dropped out.[6] There was no reserve price, and the clock began ascending from a bid of $.05.

The buy-now auctions had two stages. At the first stage the four bidders simultaneously decided whether to accept or reject the buy price. The buy price was $8.10

[5] Experimental papers in which the common value is the average of the bidders' signals include Avery and Kagel (1997), Holt and Sherman (2000), Goeree and Offerman (2002).

[6] This auction format is sometimes referred to as a Japanese, or button, auction.

in the private value auctions and \$5.60 in the common value auctions. If a bidder accepted the buy price, then he won the item at the buy price and the auction ended.[7] If all the bidders rejected the buy price, then at the second stage the item was sold via the ascending auction described above.

The experiments were conducted at the University of Arizona where subjects were recruited in groups of eight. Each group of 8 subjects was split into two groups of four bidders, and each group of four bidders participated in 30 periods of an auction.[8] We refer to a single group of four bidders participating in 30 rounds of a given auction format as a "session." We conducted six sessions for each of the four treatments, and hence a total of 96 subjects participated in the experiments. The bidders values/signals were determined randomly once, i.e., the same set of values/signals was used in all 24 sessions. Table 2.1 summarizes our experimental design.

In common value auctions it has been observed in prior experiments that subjects sometimes go "bankrupt," with their accumulated earnings becoming negative. Bankruptcy affects a bidder's incentives as he is no longer liable for his losses. A low but positive balance also affects a bidder's incentives as he can lose at most his current balance if wins an auction and the price exceeds the item's value. In our experiment a bidder was declared bankrupt if his current balance fell below \$5.[9] A bankrupt bidder would exit the experiment and be replaced by an additional subject who was standing by. In the common value sessions each subject began with an initial balance of \$10 and, in fact, none of our subjects went bankrupt over the course of the experiment. Each session lasted between 30 and 40 minutes. In the private

[7] If more than one bidder accepted the buy price, then the item was randomly allocated to one of the accepting bidders.

[8] The composition of a group was fixed over the course of a session but, to minimize the potential for repeated game effects, the subjects were not informed of this fact.

[9] The \$5 amount was chosen as it is the most a bidder can lose as a result of winning an auction (when his rivals follow the symmetric equilibrium). With risk-neutral bidders, the highest equilibrium drop out price is \$7.50 (for a bidder whose signal is \$10.00) and hence the winning bidder pays at most \$7.50. In this case, the value of the item is at least (\$10.00+\$0+\$0+\$0)/4=\$2.50 and hence a winning bidder loses at most \$5.00.

value auctions subjects earned (excluding the show up fee) an average of \$13.02 and \$16.09, respectively, in auctions with and without a buy price. In the common values subjects earned an average of \$15.32 and \$16.35, respectively.[10]

2.3 Theoretical Background

There are n bidders who are assumed to have constant absolute risk aversion (CARA) with utility function $u(x) = (1 - e^{-\alpha x})/\alpha$, where $\alpha \geq 0$ is the index of risk aversion. Since $\lim_{\alpha \to 0} u(x) = x$, then $\alpha = 0$ corresponds to risk neutrality. Let B denote the buy price in a buy-now auction. Denote by $F(v)$ the cumulative distribution function of values/signals with support $[\underline{v}, \bar{v}]$. Let $G(v) = F(v)^{n-1}$ be the *c.d.f.* of the highest of $n-1$ values/signals. The densities of F and G are denoted by f and g, respectively. For our experimental design we have $n = 4$ and $F(x) = \frac{x}{10}$ for $x \in [0, 10]$.

Private Value Auctions

In an ascending bid private-value auction it is a dominant strategy for a bidder to drop out of the auction when the bid reaches his value. We characterize equilibrium in a private-value buy price auction by a cutoff c such that a bidder accepts the buy price if his values exceeds c and rejects it otherwise. Suppose that a bidder's signal is x and all his rivals employ the same cutoff c. His payoff to accepting the buy price is

$$U_\alpha^A(v, c) = u(v - B)Q(c),$$

where $Q(c)$ is the probability that the bidder wins the auction if he accepts the buy price.[11] If the bidder rejects the buy price, then he wins the auction only if all his

[10] Under risk neutrality, each bidder's equilibrium expected cumulative earnings over 30 periods is \$15.00 in the private value auctions and is \$3.75 in the common value auctions.

[11] The probability the bidder wins if exactly k other bidders accept the buy price is $\frac{1}{k+1}$, and hence

$$Q(c) = \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{1}{k+1} (1 - F(c))^k F(c)^{n-1-k}.$$

rivals also wait and he has the highest value. His expected payoff is

$$U_\alpha^R(v,c) = \int_0^{\min\{v,c\}} u(v-y)dG(y).$$

A cutoff c^* is a **symmetric Bayes Nash equilibrium** if $U_\alpha^R(v,c^*) > U_\alpha^A(v,c^*)$ for all $v \in [\underline{v}, c^*)$ and $U_\alpha^R(x,c^*) < U_\alpha^A(x,c^*)$ for all $x \in (c^*, \bar{v}]$.[12]

Figure 2.1 shows seller revenue as a function of the buy price when $\alpha = 0$ (i.e., bidders are risk neutral) and when $\alpha = 1$. When bidders are risk neutral, then an ascending auction is an optimal auction and, as Figure 2.1 shows, any buy price below \$7.50 is accepted in equilibrium with positive probability and reduces the seller's expected revenue. A buy price above \$7.50 is not accepted in equilibrium and hence the seller obtains the expected revenue of the ascending bid auction (\$6.00). In contrast, when $\alpha = 1$ then there is a range of buy prices which raise seller revenue. If, for example, the buy price is \$8.10 (as in our experiment), then the equilibrium cutoff is \$9.11, the buy price is accepted (by one or more bidders) with probability 0.31, and the seller's expected revenue is \$6.29.[13] A buy price which exceeds \$8.60 is rejected by all bidders, and again yields the expected revenue of the ascending auction.

Common Value Auctions

In an ascending bid common value auction without a buy price, the symmetric equilibrium bidding function when bidders have index of risk aversion α, denoted by $b_\alpha(x)$, satisfies

$$E\left[u(v - b_\alpha(x)) | x_1 = x, z = x\right] = 0, \tag{2.1}$$

where $v = (x_1 + \cdots + x_n)/n$ is the true value of the item and $z = \max\{x_2, \ldots, x_n\}$ is the highest signal of a rival bidder.[14] (See Milgrom and Weber (1982).) When

[12] Reynolds and Wooders (2005) establish that when bidders are CARA risk averse then there is a unique symmetric equilibrium in cutoff strategies.

[13] Any buy price above \$7.50 will raise seller revenue provided the bidders are sufficiently risk averse it is accepted with positive probability.

[14] Since bidders do not observe the drop out prices of their rivals, the auction is strategically equivalent to a second-price sealed-bid auction.

bidders have CARA then

$$b_\alpha(x) = \frac{1}{\alpha} \ln \left[\frac{1}{E[e^{-\alpha v}|x_1 = x, z = x]} \right],$$

which reduces to $b_0(x) = \frac{n+2}{2n}x$ when bidders are risk neutral. In common value auctions bidders drop out earlier as they become more risk averse; in particular, $b_\alpha(x) < b_0(x)$ for $x > \underline{v}$.

We characterize equilibrium in a common-value buy price auction by a cutoff c such that a bidder accepts the buy price if his signal exceeds c and rejects it otherwise. Suppose that a bidder's signal is x and all his rivals employ the same cutoff c. Then his payoff to accepting the buy price is

$$\hat{U}^A_\alpha(x, c) = \sum_{l=0}^{n-1} \left[\binom{n-1}{l} F(c)^{n-1-l}(1 - F(c))^l \frac{1}{l+1} u_l \right],$$

where u_l is the expected utility to a bidder of winning the item at price B when his own signal is x, l rival bidders have a value above c, and $n - 1 - l$ rivals have signals below c.[15]

If all the bidders reject the buy price, then the equilibrium bidding function in the ascending auction that follows continues to be given by (2.1). In other words, equilibrium dropout prices are the same in an ascending bid auction without a buy price and the ascending bid auction reached following the rejection of a buy price. Intuitively this is because a bidder drops out at the bid where he is indifferent between winning or losing the auction, conditional on the highest signal of a rival (z) being equal to his own signal. Hence, a bidder who rejects the buy price (since his signal

[15] In particular

$$\begin{aligned} u_l \quad = \quad & \int_c^{\bar{v}} \cdots \int_c^{\bar{v}} \int_{\underline{v}}^c \cdots \int_{\underline{v}}^c u \left(\frac{x + \sum_{j>1} x_j}{n} - B \right) \frac{f(x_2)}{F(c)} dx_2 \ldots \frac{f(x_{n-l})}{F(c)} dx_{n-l} \\ & \times \frac{f(x_{n-l+1})}{1 - F(c)} dx_{n-l+1} \cdots \frac{f(x_n)}{1 - F(c)} dx_n. \end{aligned}$$

x is less than c), and who observes that all his rivals reject the buy price, can infer that $z < c$. This additional information has no effect on his dropout price since he forms it by conditioning on $z = x$ (which is less than c).

Consider a bidder with signal x who rejects the buy price. He wins the ascending bid auction (and pays $b_\alpha(z)$ where z is the highest signal of a rival bidder) if he has the highest signal (i.e., $z < x$) and no other bidder accepts the buy price (i.e., $z < c$). Hence, his payoff to rejecting the buy price is

$$\begin{aligned}\hat{U}_\alpha^R(x,c) &= \int_{\underline{v}}^{\min\{x,c\}} \left[\int_{\underline{v}}^{z} \cdots \int_{\underline{v}}^{z} u\left(\frac{x+z+\sum_{l>2} x_l}{n} - b_\alpha(z)\right)\right. \\ &\quad \left. \times \frac{f(x_3)}{F(z)} dx_3 \ldots \frac{f(x_n)}{F(z)} dx_n\right] g(z)dz.\end{aligned}$$

A cutoff c^* is a **symmetric Bayes Nash equilibrium** if $\hat{U}_\alpha^R(x,c^*) > \hat{U}_\alpha^A(x,c^*)$ for all $x \in [\underline{v}, c^*)$ and $\hat{U}_\alpha^R(x,c^*) < \hat{U}_\alpha^A(x,c^*)$ for all $x \in (c^*, \bar{v}]$. Shahriar (2005) establishes that a symmetric BNE exists under general conditions when bidders have constant absolute risk aversion.

Figure 2.2 illustrates the equilibrium cutoff for our experimental design when $\alpha = 0$ and when $\alpha = 1$.[16] When $\alpha = 0$, the equilibrium cutoff is $c = \$9.96$. In particular, a bidder with a signal $x > \$9.96$ obtains a higher payoff accepting the buy price than rejecting it (since $U_0^R(x, 9.96) < U_0^A(x, 9.96)$), when all his rivals employ a cutoff of \$9.96. A bidder with a signal $x < \$9.96$ obtains a higher payoff rejecting the buy price. If $\alpha = 1$, the equilibrium cutoff is $c^* = \$10$ and the buy price is rejected by all bidders (since $U_1^R(x, 10) > U_1^A(x, 10)$ for all $x \leq 10$). As illustrated in Figure 2.2, when values are common then bidders are less likely to accept the buy price as they become more risk averse. In our experiment, no bidder had a signal above \$9.96 and hence, according to the theory, no risk neutral or risk averse bidder will accept the buy price.

[16] In each case numerical calculations establish that there is only one symmetric equilibrium in cutoff strategies.

Figure 2.3 shows seller revenue as a function of the buy price for the cases $\alpha = 0$ and $\alpha = 1$. If $\alpha = 0$ then any buy price above \$5.63 is rejected, whereas if $\alpha = 1$ then any buy price above \$4.98 is rejected. In both cases, the introduction of a buy price reduces seller revenue if the buy price is accepted with positive probability. Furthermore, seller revenue is lower when bidders are risk averse than when they are risk neutral.

Risk aversion has dramatically different effects in private and common value buy-now auctions. In private value auctions, seller revenue and the probability the buy price is accepted both increase as bidders become more risk averse. In common value auctions an increase in risk aversion has the opposite effect.

2.4 Results

Private Value Auctions

The left panel of Table 2.2 shows the mean per-period seller revenue, the standard deviation of revenue, and auction efficiency in each of the six private value buy-now sessions. The buy price was accepted in 81 of the 180 auctions. The right panel of Table 2.2 shows the same data for the six sessions of private value ascending auctions. Averaging across all 6 sessions, seller revenue was \$6.06 in the ascending auctions, which is less than the mean revenue of \$6.47 in the buy-now auctions.[17]

The lowest revenue achieved in a buy-now auction (\$6.25) exceeded the highest revenue achieved in any ascending auction (\$6.17). Applying the Mann-Whitney U test to the two samples, we can reject at the 1% level the hypothesis that the revenues of each auction format are drawn from the same distribution. Table 2.2 also shows that the standard deviation of revenue is lower in the buy-now auctions, a difference that is significant at the 1% level.

[17] Since values are distributed U[\$0,\$10], *ex-ante* expected revenue is \$6.00. Conditional on the values actually used in the experiment, mean theoretical revenue is \$6.19, which is slightly more than the \$6.06 actually obtained.

Further insight into the effect of introducing a buy price is revealed by Figure 2.4, which shows the empirical *c.d.f.* of revenue in the 180 buy-now auctions and the 180 ascending auctions.

A buy price reduces the likelihood that seller revenue is either very low or very high. Introducing a buy price of \$8.10 reduces by about 10% the chance that the seller will obtain less than \$6.00. On the other hand, in an ascending auction the seller has nearly a 25% chance of a revenue of more than \$8.10, while with a buy price the seller almost surely obtains at most \$8.10.[18] In the data, these two effects are roughly a wash for the seller. The revenue advantage to the buy price comes from auctions where revenue would have been between \$6.00 and \$8.10. In almost all of these auctions a bidder took the buy price and the seller obtained \$8.10. The empirical cdf of revenue in a buy price auction (approximately) second-order stochastically dominates the cdf of revenue in the ascending auctions, which suggests that a risk averse seller is better off with a buy price.

An auction is efficient if the bidder with the highest value wins. Theoretically, the ascending bid auction is 100% efficient since the bidder with the highest value wins the auction. The introduction of a buy price, however, reduces auction efficiency since when more than one bidder accepts the buy price then the item is allocated randomly to one of these bidders. Table 2.2 shows that the introduction of a buy price leads only to a modest reduction in auction efficiency (from 93.33% to 89.44%). Applying the Mann-Whitney U test we cannot reject that hypothesis that the efficiency of the two auction formats is the same.

In private value buy-now auctions where the buy price has been rejected, it is a dominant strategy in the ascending bid auction that follows for a bidder to remain active until the bid reaches his value. It's possible, however, that the revenue advantage

[18] In a buy-now auction it is theoretically possible for seller revenue to exceed B. In particular, if both the highest and second highest values are below the equilibrium cutoff but above B, then all bidders reject the buy price and the price in the ascending bid auction that follows is the second highest value.

of the buy price comes from more aggressive bidding when the buy price is offered but rejected. Table 2.3 compares the buy-now auctions in which the buy price was rejected to *identical* ascending auctions, i.e., auctions where the bidders values are same in each case.[19] Consider, for example, the first row of Table 2.3. The buy price was rejected in 18 of the Session 1 auctions and in these auctions mean seller revenue with \$5.16. We compare revenue in these auctions with the 18 identical auctions in Session 7 where the bidders had the same values. The mean revenue in these auctions was \$5.28. Since Sessions 1 and 7 use different subjects, the revenues \$5.16 and \$5.28 are independent draws from the same distribution under the null hypothesis that bidding behavior is unchanged following the rejection of the buy price.

Mean seller revenue when the buy price is rejected (\$5.14) is nearly identical to seller revenue in the identical ascending bid auctions (\$5.09). Using the Mann-Whitney U test, we cannot reject that the two samples are drawn from the same distribution. Thus there is no evidence that bidding behavior differs following the rejection of the buy price. Conditional on the buy price being accepted, it increases revenue by \$.86 (= \$8.10-\$7.24) beyond what the seller would have obtained without a buy price.

The index of bidder risk aversion can be estimated from the bidders' decisions to accept or reject the buy price. (The bidders' dropout prices are uninformative when estimating α since in an ascending bid auction it is a dominant strategy for a bidder to drop out when the bid reaches his value, independently of α.) Let c^*_α denote the equilibrium cutoff when bidders have index of risk aversion α, i.e., c^*_α is the value of c solving $U^A_\alpha(c,c) = U^R_\alpha(c,c)$. For a bidder with value v, the difference in the payoff between accepting or rejecting the buy price is denoted by $\Delta_\alpha(v)$, where

$$\Delta_\alpha(v) = U^A_\alpha(v, c^*_\alpha) - U^R_\alpha(v, c^*_\alpha).$$

[19] This comparison is meaningful since the same set of 120 values/signals (4 bidders per auction and 30 auctions) were used in all sessions.

Let D_i^t be a dummy variable indicating whether bidder i in auction t accepted the buy price, and let v_i^t be the value of bidder i in auction t. Our econometric model is that

$$D_i^t = \begin{cases} 1 & \text{if } \Delta_\alpha(v_i^t) \geq \varepsilon_i \\ 0 & \text{if } \Delta_\alpha(v_i^t) < \varepsilon_i, \end{cases}$$

where ε_i is $N(0, \sigma_1)$. The likelihood function for the t^{th} buy-now auction is

$$L^t = \begin{cases} 1 - \prod_{i=1}^4 \left[1 - \Phi\left(\frac{\Delta_\alpha(v_i^t)}{\sigma_1}\right)\right] & \text{if } D_i^t = 1 \text{ for some } i \\ \prod_{i=1}^4 \left[1 - \Phi\left(\frac{\Delta_\alpha(v_i^t)}{\sigma_1}\right)\right] & \text{otherwise,} \end{cases}$$

where Φ and ϕ are, respectively, the *c.d.f.* and *p.d.f.* of the standard normal. Maximizing $\frac{1}{T}\sum L^t$ with respect to α (and σ_1) yields the estimate $\hat{\alpha} = 1.11$ (0.003) and an equilibrium cutoff of $c^*(\hat{\alpha}) = 9.03$ (0.002), where the standard deviations are given in parentheses.

Common Values

The left panel of Table 2.4 shows the seller's average per-period revenue in the six common value buy-now auction sessions. The buy price was accepted in 142 of the 180 auctions. The right panel shows the seller's average revenue in the six ascending auction sessions.

Introducing the \$5.60 buy price raises revenue an average of \$.21 per auction, but this difference is insignificant according to the Mann-Whitney U test (p-value of 0.197). Introducing the buy price reduces the standard deviation of seller revenue; we can reject at the 5% level that the standard deviation of revenue is the same in the two auction formats. The reduction in the standard deviation of seller revenue is apparent in Figure 2.5, which shows the empirical *c.d.f.* of revenue in the 180 common value buy-now auctions and the 180 common-value ascending auctions.

Theoretically, the price at which a bidder drops out in the ascending bid auction is the same whether there is no buy price or whether there was a buy price but it was rejected by all the bidders. In each case a bidder's drop out price is given by (**??**), and hence in auctions where the buy price is rejected seller revenue should be the same as if no buy price had been offered.

Table 2.5 compares seller revenue in the two auction formats, conditional on whether the buy price is rejected or accepted, and is constructed in the same fashion as Table 2.3. Mean seller revenue in the 38 auctions where the buy price was rejected was \$3.93. Contrary to the theory, seller revenue was on average \$.67 higher in auctions where no buy price was offered, but which were otherwise identical. Hence bidders tend to drop out earlier in the ascending bid auction when the auction had a buy price. Applying the Mann-Whitney U test, we can reject at the 10% level the null hypothesis that the two samples are drawn from the same distribution.

The revenue advantage of the buy price comes from the auctions where the buy price is accepted. The seller obtains \$5.60 when the buy price is accepted, which is \$.45 higher than what he obtains on average in the identical ascending bid auctions.

Figure 2.6 shows observed drop out prices in the ascending bid auctions. Comparing dropout prices to the risk-neutral rational bidding function $b_0(x) = \frac{3}{4}x$, it is apparent that bidders overbid for low signals (i.e., drop out too late) but underbid when they have high signals. The figure does not conclusively show underbidding for high signals since there is a selection bias – we don't observe the highest drop out price in an auction.

To better understand behavior in the ascending bid common value auctions, we estimated linear bid functions using a censored regression. In each auction we observe the dropout prices of three bidders, but not the dropout price of the bidder who wins the auction. We assume that the bid of the i-th bidder, $i \in \{1, 2, 3, 4\}$, in auction t is given by

$$b_i^t = \gamma + \beta x_i^t + \varepsilon_i^t,$$

where x_i^t is the signal of bidder i in auction t, and ε_i is distributed according to

$N(0,\sigma_2)$. For auction t, let $k^t \in \{1,2,3,4\}$ be the winner of the auction t, i.e., bidder k^t has the highest (but unobserved) dropout price, and let $\bar{b}^t$ be the highest observed dropout price. The censored regression likelihood function is given by

$$L = \frac{1}{T}\sum_{t=1}^{T} \ln \left[\left[1 - \Phi\left(\frac{\bar{b}^t - \gamma - \beta x^t_{k^t}}{\sigma_2} \right) \right] \prod_{i \in \{1,2,3,4\} \setminus \{k^t\}} \frac{1}{\sigma_2} \phi \left(\frac{b^t_i - \gamma - \beta x^t_i}{\sigma_2} \right) \right],$$

where Φ and ϕ are, respectively, the c.d.f. and p.d.f. of the standard normal, and where T is the number of auctions.

Table 2.6 reports the results of maximum likelihood estimation on the data from the ascending bid auctions and from the ascending bid phase of auctions with a buy price (standard errors of the estimates are reported within parentheses).

The estimated intercept (γ) and the slope (β) are each significantly different from zero in the both types of auctions. One can reject the null hypothesis that γ is the same for both types of auctions (p-value of 0.08) – overbidding by bidders with low signals is reduced following the rejection of a buy price. One cannot reject that β is the same for both types of auctions (p-value of 0.64) – the rejection of the buy price has no statistically significant effect on the responsiveness of bids to signals. Taken together, these results suggest that bidders employ lower dropout prices following the rejection of the buy price, a finding which is consistent with Table 2.5, but which is inconsistent with rational bidding.

The results for common values auctions depart from the rational bidding theory in four significant ways. (i) The buy price is accepted too frequently. For the signals received by bidders in our experiment, the buy price should never be accepted if bidders are either risk neutral or risk averse. In fact it was accepted in 79% of all auctions. (ii) Subjects bid less in auctions where the buy price has been rejected than in identical auctions where no buy price is offered. (iii) Subjects overbid relative to theoretical predictions when they have low signals, but underbid when they have high signals. (iv) The buy price raises seller revenue rather than reducing it. In the next section we introduce a model which explains all these features of the data.

2.5 A Behavioral Model

In this section we introduce a behavioral model in order to explain the results from the common value buy-now auctions. We first focus on bidding behavior in the ascending auctions, and then consider the decision to accept or reject the buy price.

Bidding in Common Value Auctions

The expected value (EV) bidding model is a simple model of the winner's curse in common-value auctions. According to this model, bidders fail to condition on having the highest signal when they win the auction; instead, they bid (when risk neutral) up to their expected value of the item conditional on only their own signal. The EV bidding function, when bidders have index of risk aversion α, is denoted by $b_\alpha^{EV}(x)$ and satisfies

$$E[u(v - b_\alpha^{EV}(x))|x_1 = x] = 0.$$

We also consider the possibility that bidders may overweight their own signal when calculating the value of the item. We denote the EV bidding function augmented with overweighting of own signal by $b_\alpha^{EV+}(x)$. It satisfies

$$E\left[u\left(\frac{\lambda x_1 + x_2 + \cdots + x_n}{n-1+\lambda} - b_\alpha^{EV+}(x)\right)|x_1 = x\right] = 0,$$

where λ denotes the degree to which a bidder overweights (if $\lambda > 1$) or under weights (if $\lambda < 1$) his own signal. If $\lambda = 1$ this model reduces to the expected value bidding model.

In a buy-now auction, the ascending bid phase of the auction is only reached if all the bidders reject the buy price at the first stage. Hence, in the ascending bid phase of the auction a bidder will condition on all of his rivals having a signal below the equilibrium cutoff c. A bidder who correctly conditions on his rivals' signals being less than the cutoff, but who fails to condition on his signal being highest when he wins, will bid according to $b_\alpha^{EV}(x,c)$ defined by

$$E[u(v - b_\alpha^{EV}(x,c))|x_1 = x, z \leq c] = 0.$$

If in addition the bidder overweights his own signal, then he bids according to $b_\alpha^{EV+}(x,c)$ defined by

$$E\left[u\left(\frac{\lambda x_1 + x_2 + \cdots + x_n}{n-1+\lambda} - b_\alpha^{EV+}(x,c)\right)|x_1 = x, z \leq c\right] = 0.$$

In contrast to the rational model, the EV and EV+ models both predict less aggressive bidding following the rejection of the buy price, so long as $c < \bar{v}$ $(= \$10)$.

Table 2.7 shows the theoretical bidding functions for the three alternative bidding models (rational, EV, and EV+) for the parameterization of our experiment and, for comparison, it shows on the last row the estimated bid functions from Table 2.6. For simplicity, the table provides the risk-neutral bidding functions, in which case all three bidding functions are linear.[20] The rational bidding function has a zero intercept, while the EV and EV+ bidding functions both have positive intercepts and hence predict overbidding by bidders with low signals. However, the intercept is lower in both models if the buy price is rejected when bidders employ a cutoff $c <$ \$10. In the EV+ model bidders overweight their own signals and hence the bidding function is steeper than in the EV model.

For the bidding functions reported on the last row of Table 2.7, the estimated

[20] The EV+ bidding function for the ascending bid phase of a buy-now auction is computed as

$$\begin{aligned} & b_0^{EV+}(x,c) \\ = \; & E\left(\frac{\lambda x_1 + x_2 + x_3 + x_4}{3+\lambda}|x_1 = x, z \leq c\right) \\ = \; & \int_0^c \int_0^c \int_0^c \frac{\lambda x + x_2 + x_3 + x_4}{3+\lambda} \frac{1}{c^3} dx_2 dx_3 dx_4 = \frac{3}{2(3+\lambda)} c + \frac{\lambda}{3+\lambda} x. \end{aligned}$$

intercepts are positive and statistically significant, both in the ascending auctions and the ascending bid phase of the buy-now auctions, which is inconsistent with the rational model.[21] For the ascending bid auction, the estimated intercept of 2.80 is less than the theoretical risk-neutral intercept in the EV and EV+ model of $\frac{3}{8}(10) = 3.75$. The theoretical intercept is decreasing in the index of bidder risk aversion, and the 2.80 estimate implies an index of risk aversion of $\alpha = 1.32$. For both the ascending auction and the buy-now auction, the estimated slopes (0.43 and 0.35, respectively) are greater than $\frac{1}{4}$, which suggests that bidders overweight their own signals. The 0.43 estimate for the ascending auctions implies a value of $\lambda = 1.72$.[22] Since the data suggests that bidders suffer from the winner's curse and they also overweight their own signals, henceforth we focus on the EV+ model.

The Decision to Accept or Reject the Buy Price

Next we investigate whether the winner's curse and overweighting of own signal can explain the high frequency with which the buy price is accepted. Suppose that all of a bidder's rivals follow the cutoff strategy c, with each bidder accepting the buy price if his signal exceeds c and rejecting it otherwise. A bidder who accepts the buy price wins for sure if all his rivals have values below c and wins with probability $\frac{1}{1+l}$ if exactly l of his rivals have value above c. A rational bidder accounts for the fact that if he accepts the buy price, he is more likely to win as more of his rivals have values below c. We suppose instead that bidders are also subject to the winner's

[21] The equilibrium rational bidding function has a zero intercept for any level of risk aversion, hence the rational bidding function does not explain the observed overbidding for low signals even if bidders are risk averse.

[22] When bidders have index of risk aversion α, then the equilibrium bid function in the EV+ model of the ascending auction is

$$b_{\alpha}^{EV+}(x) = -\frac{1}{\alpha}\ln\Big(\int_0^{10}\int_0^{10}\int_0^{10} e^{-\alpha(\frac{x_2+x_3+x_4}{3+\lambda})}\frac{1}{10^3}dx_2dx_3dx_4\Big) + \frac{\lambda}{3+\lambda}x.$$

Hence λ can be inferred from the estimate of β (since $\beta = \lambda/4$) and, similarly, α can be inferred from γ. When there is a buy price, then the intercept depends on both α and c; hence α can no longer be inferred from γ.

curse when accepting the buy price, with a bidder whose signal is x computing the expected utility to accepting the buy price as

$$U^A(x, c; \lambda, \alpha) = E\left[u\left(\frac{\lambda x_1 + x_2 + \cdots + x_n}{n - 1 + \lambda} - B\right) |x_1 = x\right] Q(F(c)),$$

where $Q(F(c))$ is the bidder's probability of winning the item when all his rivals employ the cutoff c. A bidder computes the expected utility of rejecting the buy price as

$$\begin{aligned} U^R(x, c; \lambda, \alpha) &= E\left[u\left(\frac{\lambda x_1 + x_2 + \cdots + x_n}{n - 1 + \lambda} - b_\alpha^{EV+}(z, c)\right) |x_1 = x, z \le c\right] \\ &\quad \times G(\min\{x, c\}). \end{aligned}$$

Such a bidder correctly calculates the probability of ultimately winning the auction if he rejects the buy price. He also conditions on all his rivals' signals being less than c if no bidder accepts the buy price. However, consistent with the winner's curse, he fails to condition on his signal being highest when bidding in the ascending bid auction.

A cutoff c^* is a **symmetric Bayes Nash equilibrium** if $U^R(x, c^*; \lambda, \alpha) > U^A(x, c^*; \lambda, \alpha)$ for all $x \in [\underline{v}, c^*)$ and $U^R(x, c^*; \lambda, \alpha) < U^A(x, c^*; \lambda, \alpha)$ for all $x \in (c^*, \bar{v}]$. We write $c^*(\lambda, \alpha)$ for the equilibrium cutoff as a function of λ and α.

We use maximum likelihood techniques to estimate α and λ from the buy-now auction data. In each auction we observe whether a bidder accepts the buy price. If no bidder accepts the buy price, then we observe the dropout price of each bidder (except the winning bidder) in the ascending bid auction. Define

$$\Delta(x; \lambda, \alpha) = U^A(x, c^*(\lambda, \alpha); \lambda, \alpha) - U^R(x, c^*(\lambda, \alpha); \lambda, \alpha)$$

as the difference between the payoff to accepting and rejecting the buy price when all of a bidder's rivals employ the cutoff $c^*(\lambda, \alpha)$. Let D_i^t be a dummy variable which equals 1 if bidder i in auction t accepts the buy price and which equals zero otherwise.

The econometric model underlying our estimation is

$$D_i^t = \begin{cases} 1 & \text{if } \Delta(x_i^t;\lambda,\alpha) \geq \varepsilon_i^t \\ 0 & \text{if } \Delta(x_i^t;\lambda,\alpha) < \varepsilon_i^t, \end{cases}$$

where ε_i is $N(0,\sigma_1')$. We assume that bidder i's bid in auction t is given by

$$b_i^t = b_\alpha^{EV+}(x_i^t, c^*(\lambda,\alpha)) + \eta_i^t,$$

where η_i^t is distributed $N(0,\sigma_2')$.[23] Let $k^t \in \{1,2,3,4\}$ be the winner of auction t, i.e., bidder k^t has the highest (but unobserved) dropout price, and let $\bar{b}^t$ be the highest observed dropout price.

The likelihood function for auction t is given by

$$L^t = \begin{cases} 1 - \prod_{i=1}^4 \left[1 - \Phi\left(\frac{\Delta(x_i^t;\lambda,\alpha)}{\sigma_1'}\right)\right] & \text{if } D_i^t = 1 \\ \prod_{i=1}^4 \left[1 - \Phi\left(\frac{\Delta(x_i^t;\lambda,\alpha)}{\sigma_1'}\right)\right]\left[1 - \Phi\left(\frac{\bar{b}^t - b_\alpha^{EV+}(x_{k^t}^t, c^*(\lambda,\alpha))}{\sigma_2'}\right)\right] & \\ \quad \times \prod_{i\in\{1,2,3,4\}\backslash\{k^t\}} \frac{1}{\sigma_2'}\phi\left(\frac{b_i^t - b_\alpha^{EV+}(x_i^t, c^*(\lambda,\alpha))}{\sigma_2'}\right) & \text{otherwise} \end{cases}$$

Maximizing $\frac{1}{T}\sum L^t$ with respect to α and λ (and σ_1' and σ_2') yields the following maximum likelihood estimates (standard errors of the estimates are reported within parentheses).

We can reject the null hypothesis that bidders are risk neutral. The estimated value of λ is significantly different from 1, with the results suggesting that bidders do indeed overweight their own signals.

We conclude this section by comparing the predictions of the rational model and

[23] In particular

$$\begin{aligned} & b_\alpha^{EV+}(x_i^t, c^*(\lambda,\alpha)) \\ = \; & -\frac{1}{\alpha}\ln\left(\int_0^{c^*(\lambda,\alpha)}\int_0^{c^*(\lambda,\alpha)}\int_0^{c^*(\lambda,\alpha)} e^{-\alpha(\frac{x_2+x_3+x_4}{3+\lambda})}\frac{1}{c^*(\lambda,\alpha)^3}dx_2dx_3dx_4\right) + \frac{\lambda}{3+\lambda}x_i^t. \end{aligned}$$

the EV+ model. Recall that under the rational model, in equilibrium the \$5.60 buy price is always rejected and seller revenue is \$4.50 if bidders are risk neutral (and less if bidders are risk averse). In contrast, in the data the buy price was taken in 79% of all auctions and mean seller revenue was \$5.25. Based on our parameter estimates, in the equilibrium of the EV+ model the buy price is taken in 77% of the auctions and seller revenue is \$5.21. Figure 2.7 shows the empirical c.d.f. of seller revenue in the common value buy-now auctions, and the predicted c.d.f. of revenue for the estimated model. It shows that that EV+ model explains the seller revenue data well. However, the EV+ model does not fully explain the high rate at which the buy price is accepted and the high revenue sellers obtain in common value buy-now auctions.

2.6 Conclusion

In this study we conducted lab experiments to test several theoretical models of eBay's Buy It Now auctions. We test these predictions using both private and common values frameworks. Consistent with the theoretical predictions for private value auctions, we find that a suitably chosen buy price raises revenue and reduces the variance of revenue. Hence a buy price is advantageous for a risk neutral or risk averse seller. On the other hand, in contrast to the theoretical predictions for common value auctions, a buy price did not lower seller revenue and was accepted by the bidders with high frequency. We develop a behavioral model for common values auctions with a buy price that incorporates the winner's curse and overweighting of a bidder's own signal. We find that this behavioral model explains the data far better than the rational model. We conclude that such a model rationalizes a seller's decision to post a buy price when the bidders have common values.

Although there is abundant field data on buy-now auctions, experimental tests of the theory offer several advantages. In a laboratory test the experimenter controls whether values are private or common, as well as the distribution of values. In field auctions, in contrast, bidder values may have both common and private attributes. As illustrated in this paper, this is important since the theoretical predictions of the effect of buy prices differ substantially in the two settings. Bidder impatience may also be important in field auctions, while in the lab we can eliminate impatience as a factor.

		Buy Price	Number Sessions	Periods/ Session	Bidders/ Session	Dist. of Values/Signals
PV	Ascending	na	6	30	4	$U[\$0, \$10]$
	Buy-now	\$8.10	6	30	4	$U[\$0, \$10]$
CV	Ascending	na	6	30	4	$U[\$0, \$10]$
	Buy-now	\$5.60	6	30	4	$U[\$0, \$10]$

TABLE 2.1. Experimental Design

	Buy-now, $B = \$8.10$				Ascending		
Session	Rev.	s.d. Rev.	Efficiency	Session	Rev.	s.d. Rev.	Efficiency
1	\$6.33	\$1.84	86.67	7	\$6.20	\$1.97	100.00
2	\$6.50	\$1.91	96.67	8	\$6.04	\$2.06	93.33
3	\$6.55	\$1.84	93.33	9	\$5.98	\$2.00	90.00
4	\$6.50	\$1.91	93.33	10	\$6.17	\$2.08	96.67
5	\$6.25	\$1.82	80.00	11	\$6.11	\$2.08	86.67
6	\$6.72	\$1.88	86.67	12	\$5.86	\$2.25	93.33
Mean	\$6.47	\$1.87	89.44	Mean	\$6.06	\$2.07	93.33

TABLE 2.2. Revenue and Efficiency With and Without a Buy Price

	Buy-now, B=\$8.10			Identical Ascending	
Session	B rejected	B accepted	Session	B rejected	B accepted
1	\$5.16 (18 auc)	\$8.10	7	\$5.28 (18 auc)	\$7.57
2	\$5.81 (21 auc)	\$8.10	8	\$5.67 (21 auc)	\$6.89
3	\$5.19 (16 auc)	\$8.10	9	\$5.09 (16 auc)	\$7.00
4	\$4.67 (14 auc)	\$8.10	10	\$4.71 (14 auc)	\$7.43
5	\$5.01 (18 auc)	\$8.10	11	\$5.05 (18 auc)	\$7.70
6	\$4.65 (12 auc)	\$8.10	12	\$4.28 (12 auc)	\$6.91
Mean	\$5.14 (99 auc)	\$8.10	Mean	\$5.09 (99 auc)	\$7.24

TABLE 2.3. Seller Revenue Conditional on Acceptance/Rejection of the Buy Price

	Buy-now, $B = \$5.60$			Ascending	
Session	Rev.	s.d. Rev.	Session	Rev.	s.d. Rev.
1	\$5.05	\$0.85	7	\$4.84	\$0.85
2	\$5.46	\$0.51	8	\$4.64	\$0.97
3	\$5.23	\$0.85	9	\$5.11	\$1.15
4	\$5.43	\$0.47	10	\$5.37	\$1.03
5	\$5.35	\$0.78	11	\$4.81	\$0.79
6	\$5.00	\$1.08	12	\$5.49	\$1.36
Mean	\$5.25	\$0. 76	Mean	\$5.04	\$1. 03

TABLE 2.4. Revenue With and Without a Buy Price

	Buy-now, B=\$5.60			Identical Ascending	
Session	B rejected	B accepted	Session	B rejected	B accepted
13	\$4.33 (13 auc)	\$5.60	19	\$4.29 (13 auc)	\$5.28
14	\$4.15 (3 auc)	\$5.60	20	\$3.90 (3 auc)	\$4.72
15	\$3.39 (5 auc)	\$5.60	21	\$4.21 (5 auc)	\$5.28
16	\$4.31 (4 auc)	\$5.60	22	\$4.66 (4 auc)	\$5.47
17	\$3.10 (3 auc)	\$5.60	23	\$4.05 (3 auc)	\$4.89
18	\$3.81 (10 auc)	\$5.60	24	\$5.68 (10 auc)	\$5.39
Mean	\$3.96 (38 auc)	\$5.60	Mean	\$4.63 (38 auc)	\$5.15

TABLE 2.5. Seller Revenue Conditional on Acceptance/Rejection of Buy Price

	Ascending	Buy-now (B rejected)	z-test (p-value)
$\hat{\gamma}$	2.15 (0.121)	1.77 (0.192)	0.08
$\hat{\beta}$	0.43 (0.020)	0.34 (0.047)	0.64

TABLE 2.6. ML Estimated Bidding Functions

	Ascending	Buy-now (B rejected)
Rational	$b_0(x) = \frac{3}{4}x$	$b_0(x, c) = \frac{3}{4}x$ for $x \in [0, c)$
EV	$b_0^{EV}(x) = \frac{3}{8}(10) + \frac{1}{4}x$	$b_0^{EV}(x, c) = \frac{3}{8}c + \frac{1}{4}x$
EV+	$b_0^{EV+}(x) = \frac{3}{2(3+\lambda)}(10) + \frac{\lambda}{3+\lambda}x$	$b_0^{EV+}(x, c) = \frac{3}{2(3+\lambda)}c + \frac{\lambda}{3+\lambda}x$
Estimated	$b(x) = 2.15 + 0.43x$	$b(x) = 1.77 + 0.34x$

TABLE 2.7. Risk Neutral Bidding Functions (cutoff of c)

	Buy-now	Ascending	Buy-now (B rejected)
$\hat{\alpha}$	0.001 (0.000)	1.617 (0.229)	3.433 (0.541)
$\hat{\lambda}$	5.118 (0.066)	2.320 (0.191)	1.513 (0.321)
$c^*(\hat{\alpha}, \hat{\lambda})$	6.842 (0.000)		

TABLE 2.8. ML Estimates for EV+

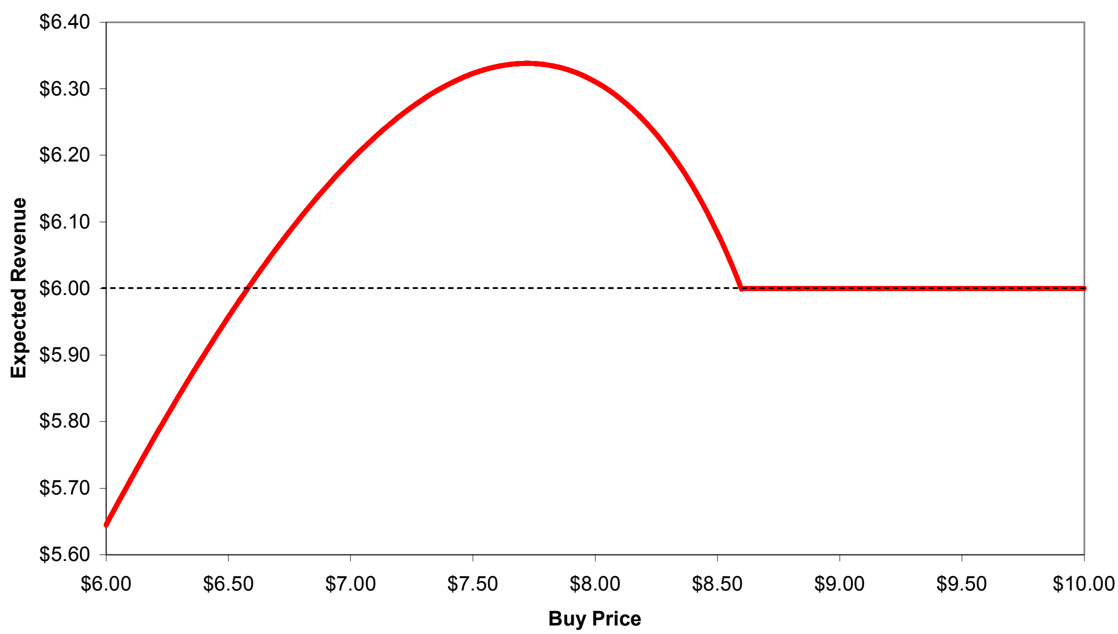

FIGURE 2.1. Expected Seller Revenue as a Function of the Buy Price, $\alpha = 1$

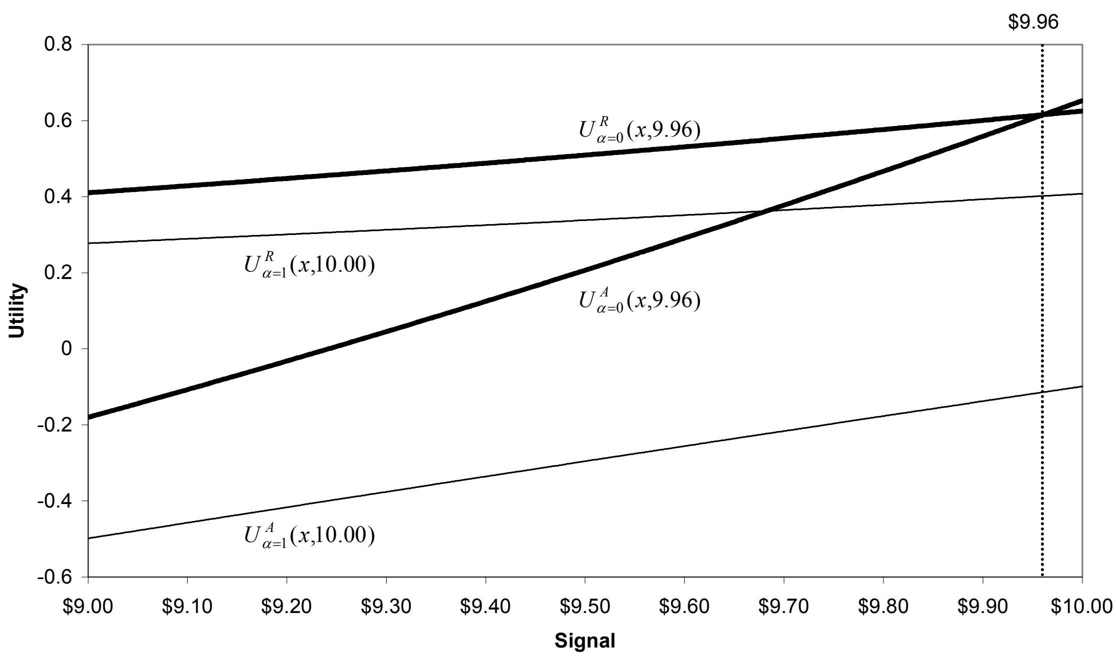

FIGURE 2.2. Expected Utilities as a Function of the Bidder's Signal

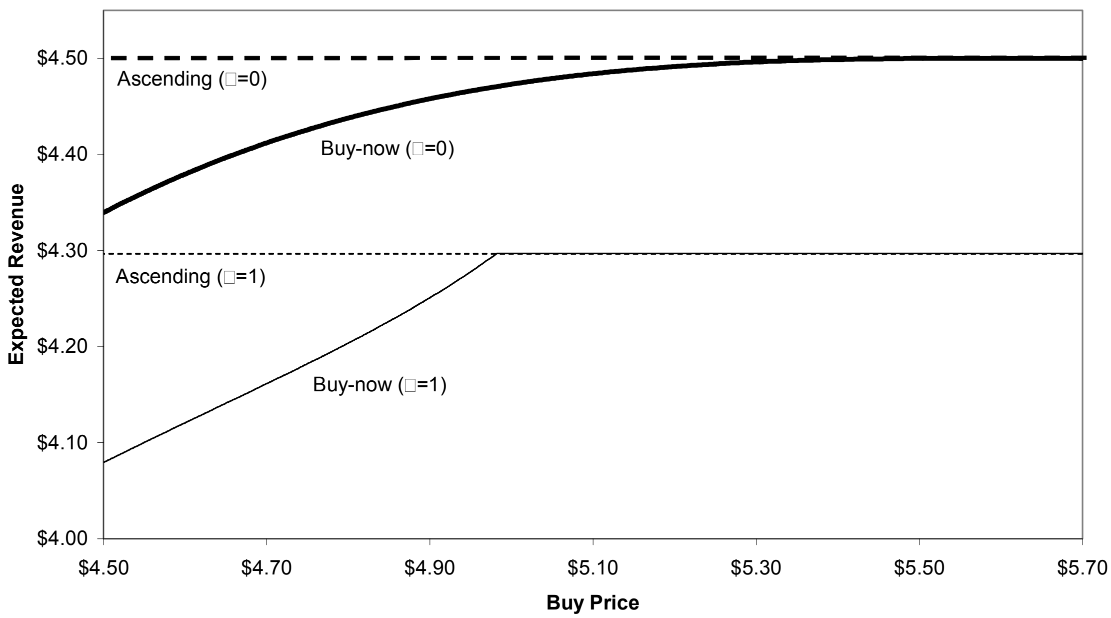

FIGURE 2.3. Expected Seller Revenue as a Function of the Buy Price, $\alpha = 0$ and $\alpha = 1$

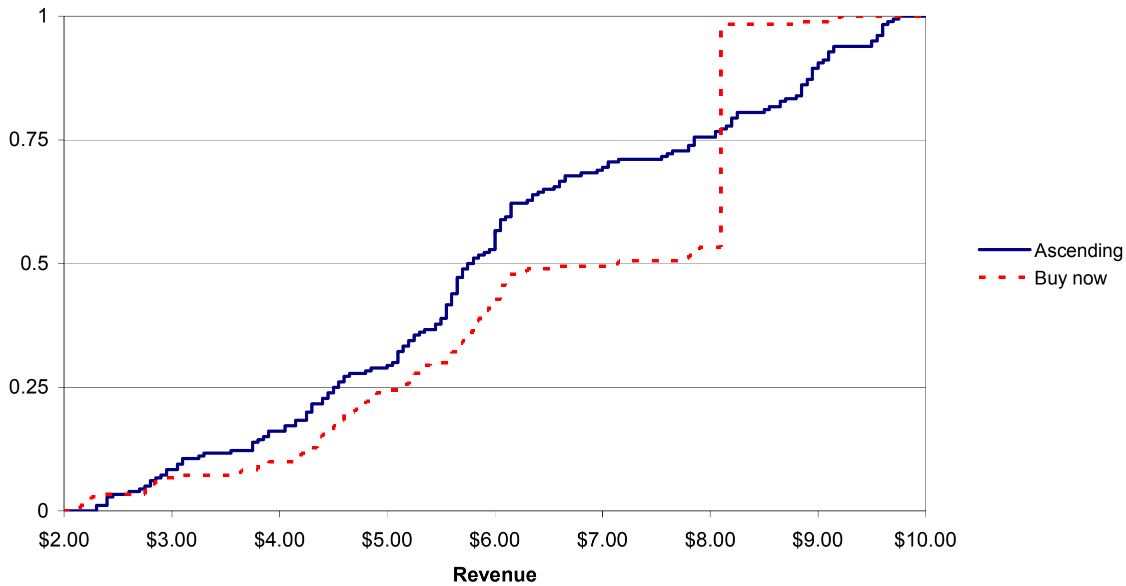

FIGURE 2.4. c.d.f of Seller Revenue in Private Value Auctions (With and Without a Buy Price of $8.10)

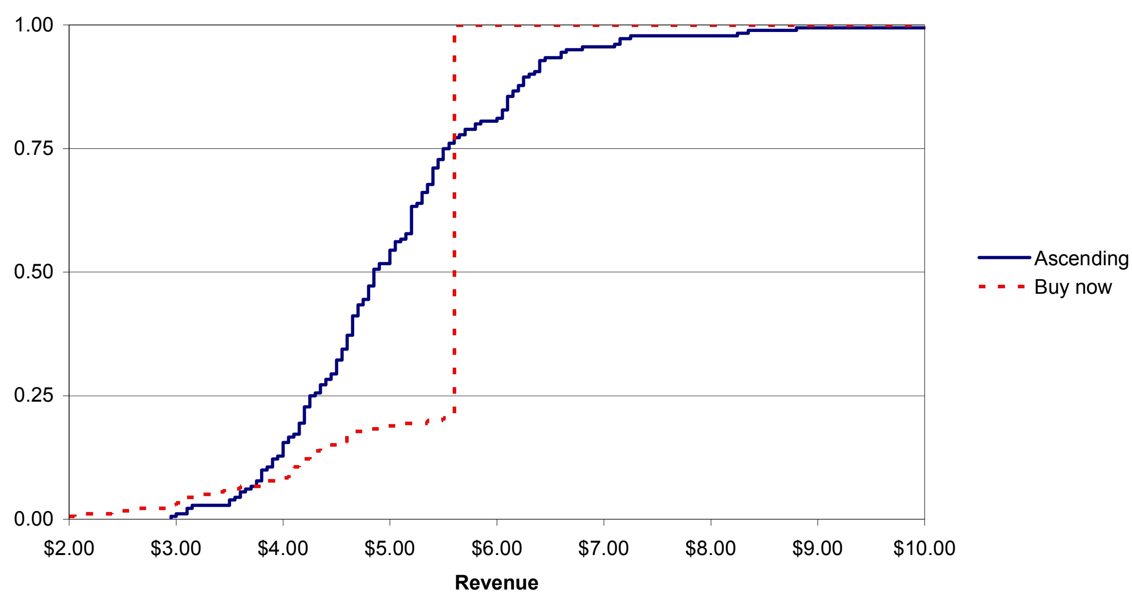

FIGURE 2.5. c.d.f. of Seller Revenue in Common-Value Auctions With and Without a Buy Price of $5.60

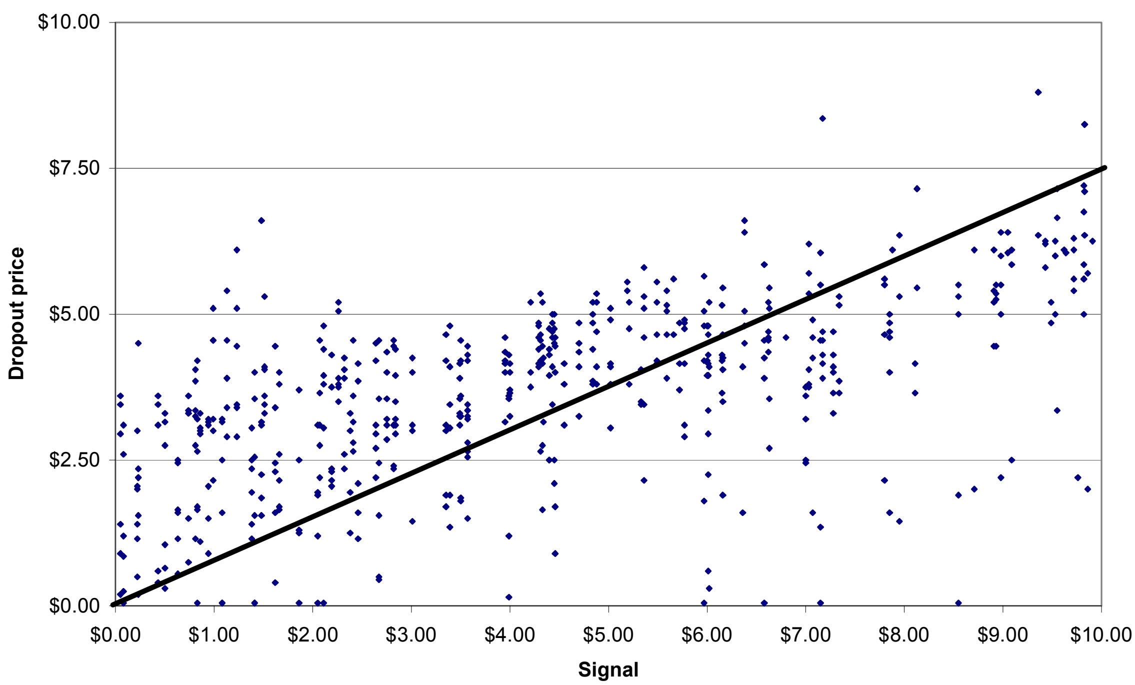

FIGURE 2.6. Dropout Prices in Ascending Common-Value Auctions

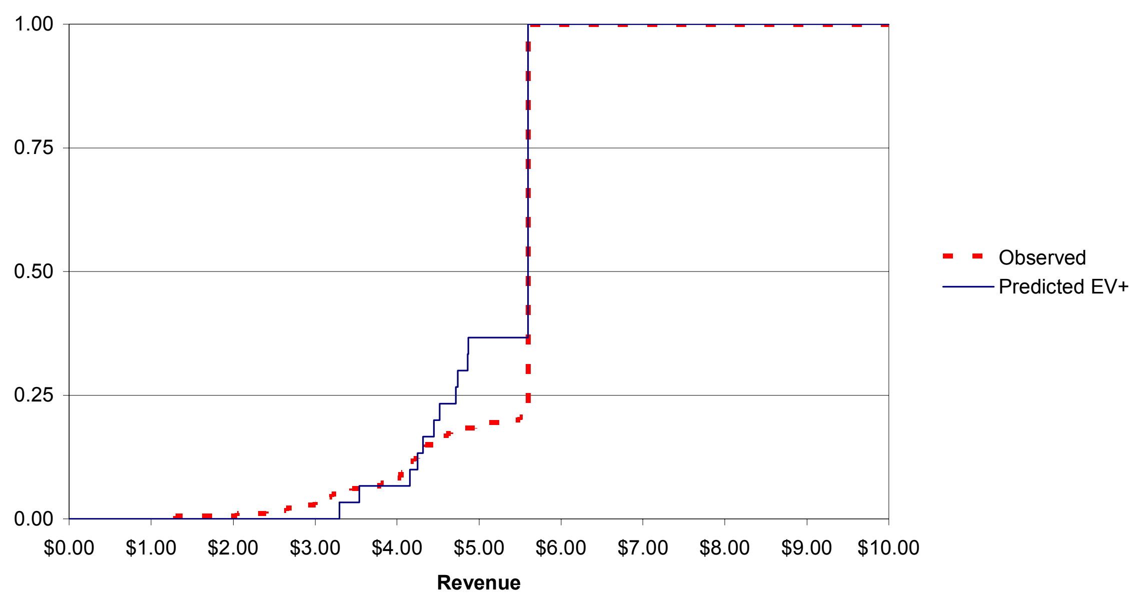

FIGURE 2.7. c.d.f. of Seller Revenue in Common-Value Buy-now Auctions (Observed vs. Predicted EV+)

Chapter 3

The Buy-it-now Option, Risk Aversion, and Impatience in an Empirical Model of eBay Bidding

3.1 Introduction

[1]In 2000, the online auction site eBay introduced its "buy-it-now" (BIN) feature, which allows sellers to use hybrid auctions with a fixed price component. In a BIN auction, the seller lists a "buy price" for the good. A bidder may purchase the item immediately at the buy price and end the auction, or she may place a bid below the buy price, in which case the BIN option disappears and the auction proceeds as a standard (oral ascending bid) eBay auction. The BIN feature has become very popular; in 2005, fixed price sales (in BIN auctions and sales through Half.com) totaled $13.8 billion, comprising 33.1% of eBay's total sales.[2]

Theory suggests that a buy price may increase seller revenue if bidders are risk averse or impatient. Reynolds and Wooders (2006) show that, in an independent private values (IPV) setting, risk averse bidders will pay a premium to obtain the good with certainty. Thus, an appropriately chosen buy price can raise seller revenue. Mathews (2003) considers auctions with impatient bidders and shows that a similar effect holds.[3] These models, however, abstract signficantly from the form of

[1]This is a joint work with Daniel Ackerberg and Keisuke Hirano.

[2]BIN auctions in eBay can take three forms: (a) a BIN auction where bidding is possible; (b) a BIN auction without bidding, which amounts to a posted-price offer; and (c) a BIN auction where bidders can place their best offers. In "best-offer" auctions, the bidder whose offer is accepted by the seller wins the item at the bidder's offered price. In this paper, we only focus on the first type of BIN auction, where bidding can follow rejection of the BIN price, and we refer to such auctions as BIN auctions.

[3]In addition, Mathews and Katzman (2006) find that risk averse *sellers* may find buy prices advantageous, because they reduce the variance of revenue when buyers are risk-neutral. We focus on identifying risk aversion and time preferences in buyers in this application, but note that our

eBay auctions, by assuming that the number of bidders is fixed and known to all participants, and that all bidders choose simultaneously whether to accept the buy price. In actual eBay auctions, bidders arrive over time and the number of bidders is not known in advance. In addition, the buy price disappears as soon as any bidder makes a bid, so bidders who arrive later in the auction may not have the opportunity to accept the good at the buy price. One of our goals in this paper is to construct a detailed empirical model that captures both the effects described by Reynolds and Wooders (2006) and Mathews (2003), and these specific features of eBay auctions.

We construct an empirical model of bidding behavior in auctions for laptops, both with and without a BIN feature. Since buy prices affect bidder behavior through risk aversion and impatience, our structural model of bidder behavior allows us to recover risk aversion and time preference parameters from the data. As Athey and Haile (2005) note, risk aversion is important for substantive questions such as the determination of optimal auction format and reserve prices, but it is difficult to identify risk aversion in standard auction data. For example, in standard ascending auctions with independent private values, bidders will bid their valuations, hence risk aversion cannot be detected. In first-price auctions, risk aversion causes aggressive bidding. The actual identification and estimation of the index of risk aversion is, however, a challenging task. Campo, Guerre, Perrigne and Vuong (2002) start with a general utility function in an IPV setting and show that the model is non-identified even when constant relative risk aversion (CRRA) is assumed, and that the same bid data can be rationalized by risk neutrality and many different specifications of risk aversion. Identification is only achieved using a parametric CRRA utility function and the assumption that the distribution of value conditional on the item characteristics satisfies a parametric quantile restriction. Perrigne (2003) makes similar assumptions for identification and extends the study by Campo, Guerre, Perrigne and Vuong

empirical results could then be used to calculate the value to the seller of using a well chosen buy price.

(2002) to auctions with random reserve prices. Our paper extends this literature by proposing an alternative approach to identifying risk aversion (and impatience), by examining bidder behavior in BIN auctions.[4]

In our model, bidders arrive in continuous time according to a Poisson point process, with a time-varying arrival rate.[5] Bidders' valuations are assumed to be independent draws from a distribution (the distribution being common knowledge among bidders). If the BIN option is available, a bidder who has arrived may choose to buy the good immediate at the buy price, or start bidding. This decision will depend on the distribution of valuations, the arrival rate function, and the amount of time remaining in the auction. Bidders can be risk averse and/or impatient, which may make the BIN option attractive.

Empirical data from eBay exhibits some characteristics that are difficult to capture in standard models of bidding. Bidders enter their "maximum" bid to a proxy (making the auction similar to theoretical models of "button" or "clock" auctions). Thus, bidding one's true valuation is a weakly dominant strategy. However, in practice we observe many bidders raising the "maximum" amount they are willing to pay later in the auction, in some cases multiple times within the same auction. It is not clear why this rebidding happens, and standard theoretical models do not predict such rebidding. Rather than fully specify a particular model for rebidding, we set up an incomplete model of bidding, following Haile and Tamer (2003) and Canals-Cerda and Pearcy (2004). This allows us to estimate our main structural parameters without making any strong assumptions on why the rebidding occurs.

Although we make parametric assumptions about the arrival process and the distribution of valuations, our model is incomplete because we do not specify the

[4] Our continuous-time model of eBay BIN auctions is complicated to analyze using formal semi-parametric methods, as in Campo, Guerre, Perrigne, and Vuong (2002). However, we provide below an intuitive discussion of how bidder reaction to the BIN option should identify risk aversion and impatience.

[5] In our current structural estimates, we assume a constant arrival rate, but we plan to estimate versions of the model with time-varying arrival rates in future work.

rebidding process. As a consequence, the model does not possess a likelihood function in the usual sense. (Alternatively, it can be viewed as a semiparametric model with an infinite-dimensional nuisance component.) We show, however, that a partial likelihood can be constructed based on a portion of the data, by considering a subset of the data vector as a marked point process. Additional information about model parameters can also be obtained using simulated moment equalities and inequalities.[6] Our estimator combines a marked point process partial likelihood component and simulated moment conditions.

The next section briefly describes earlier literature on eBay auctions and the buy-it-now option. Section 3 describes the data set we have collected and provides some preliminary evidence on the role of buy prices on bidder behavior. Section 4 describes our basic theoretical model for bidding in eBay, sets up our structural model and estimation technique, and discusses our preliminary estimates of the structural parameters.

3.2 Previous Literature

Several studies on eBay auctions look at the relationship between seller and auction characteristics, and sale prices. McDonald and Slawson (2002) and Lucking-Reiley et al (2000) find that auctions with high minimum bids have fewer bidders. Evidence is mixed on whether longer auctions attract more bidders. Lucking-Reiley et al (2000) find that the number of observed bidders is higher in longer auctions, while McDonald and Slawson (2002) find no significant difference. Houser and Wooders (2006) and McDonald and Slawson (2002) did not find any evidence of a relationship between auction length and sale price. However, both Lucking-Reiley et al. (2000) and Dewally and Ederington (2006) find that longer auctions are associated with higher sale prices. Studies have found that sellers' reputation measures are strongly related to sale price:

[6]Currently, we only simulate some basic moment equality restrictions and use these in our estimation scheme. We plan to extend this method in future versions of the paper.

positive reputations are associate with higher prices (Houser and Wooders (2006) and Lucking-Reiley et al (2000)). Melnik and Alm (2005) suggest that the reputation measures are more important when the items being auctioned are heterogeneous and the bidders are uncertain about the quality of the items.

A few studies take a structural approach to analyze eBay auctions. Bajari and Hortacsu (2004) analyzes bidder and seller behavior in eBay coin auctions. They find that an additional bidder reduces the average bid by 3.2%, and argue that these auctions should therefore be viewed as common value auctions. Song (2004) proposes a nonparametric identification method for the valuation distribution which does not requires that the number of potential bidders be known or constant across auctions. The identification result requires that we observe two order statistics from the valuation distribution, which may be difficult if we do not assume that bids correspond exactly to underlying values.

Both Gonzalez, Hasker, and Sickles (2004) and Canals-Cerda and Pearcy (2004) develop incomplete econometric models of standard eBay auctions (without the BIN option) within the IPV framework based on the assumptions of Haile and Tamer (2003). In both of these studies the sale price is determined by the second highest valuation among the potential bidders, which plays an important role in the identification of underlying valuation distribution. In the former, entry is exogenous and the number of potential bidders is assumed constant across auctions of the same length, while in the later, entry is endogenous and the number of potential bidders is stochastic which is an outcome of a Poisson process. The former study uses a data set of 6543 auctions of computer monitors and utilizes only the winning bid. The later one uses a data set of 4518 auctions of art works (e.g. paitings, collages, sculptures, etc.) and utilizes all the bids.

To our knowledge, there are two empirical studies of eBay's BIN auctions. Anderson et al. (2004) uses a data set of 722 auctions of Palm Vx handheld computers. They use instrumental variables methods to examine the effect of offering the BIN option on seller revenue, conditional on the number of bids. They instrument for the number of bids, but their results are difficult to interpret because the number of bids is an intermediate outcome, which could be directly affected by the BIN option. Thus, even after instrumenting, the coefficient on the BIN option does not have a clear causal interpretation. In addition, many of the auctions in their data had a BIN price equal to the reserve price, making the auctions effectively posted-price offers. In our analysis below, we do not consider posted-price offers. Also, we focus on estimating a detailed model of behavior and recovering underlying behavioral parameters, such as parameters related to risk aversion and underlying valuations.

Wang, Montgomery and Srinivasan (2004) model BIN auctions where bidders incur participation costs by bidding in the standard eBay auctions. Their model show that BIN options can increase the expected profits from an auction when bidder participation costs are high. Their theory predicts that the sellers are more likely to post BIN options when the number of potential bidders are low, and the bidders are more likely to accept BIN options when their participation costs are higher and there are secret reserve prices for the products. The paper tests the predictions in reduced-form settings using some data collected from eBay.

3.3 Data from eBay

Using a set of Perl scripts, we collected data on 3245 auctions of twelve different models of used Pentium-3 Dell Latitude laptops, between 22 July to 10 August 2005. We restricted our data to auctions satisfying the following characteristics: (1) each auction was for a single laptop; (2) the auction either had no buy price (a WBIN auction) or had a buy price with the option to bid (a BIN auction); (3) the auction had a public reserve price; (4) bidder identities were not secret; (5) the laptops being sold were fully functional; and (6) the buy prices were higher than the reserve price.[7]

[7] A buy price equal to the reserve price makes the auction a posted-price offer. We did not include posted-price offers in our data set.

Since the BIN option disappears as soon as any bidder places a bid, we collected pre-auction data as soon as the laptop was listed on eBay. This included characteristics of the laptop (processor speed, hard drive size, RAM, etc.) and auction design features (length of auction, reserve price, buy price). We also collected information on the descriptions of the laptop given by the seller, such as cosmetic appearance and functionality, as well as seller characteristics such as their eBay rating. One month later, we revisited every laptop auction and collected data on the bidding in that auction.

3.3.1 Summary Statistics

Table **??** shows the number of BIN and WBIN auctions in our data, along with buy prices, reserve prices, and sale prices. Of the 3245 auctions, 2243 are WBIN auctions, and the remaining 1002 are BIN auctions. (Tables and figures appear at the end of the paper.) Figure **??** in Appendix B shows the distribution of buy prices in BIN auctions. 95% of the buy prices are between \$225 and \$645, with an average buy price of \$396.

The reserve price ("starting bid" in eBay terminology) is an important auction design parameter. The number of active bidders tends to be lower for auctions with higher reserve prices (Lucking-Reiley, Bryan, Prasa, and Reeves, 2000). The average reserve price in our sample is \$165.9 (including shipping costs), but there are 1306 auctions (1247 WBIN and 59 BIN) with a reserve price (without shipping costs) of \$0.99, which is the minimum allowed by eBay. Table 3.1 shows that BIN auctions have an average reserve price of \$326, which is more than three times the average reserve price in WBIN auctions of \$94.4. Figure **??** in Appendix B shows the distribution of reserve prices among all auctions in our sample.

80% of all auctions ended in a sale of the laptop to some bidder. Of the BIN auctions, about 57% ended in a sale (either through the buy price being accepted, or through the ascending bid phase of the auction). About 22% of BIN auctions ended with a trade at the buy price. The average sale price (including shipping costs) for WBIN and BIN auctions is \$355.9 and \$355.3, respectively. However, in auctions where the buy price was accepted, the average sale price is \$375.4. Below, we explore further the relationship between sale price and various characteristics of the laptop and the auction design. Figure **??** shows a histogram of the sale prices for all auctions ending in a trade (including trades at a BIN price) in our sample.

In addition to auction outcomes, we also collected detailed descriptions of the laptops being auctioned, from the information shown to bidders on eBay. We summarize some of the laptop characteristics in Table **??**. On average, laptops sold in BIN auctions have lower specifications (processor speed, hard drive size and memory) then laptops sold in WBIN auctions. In addition to these basic laptop characteristics, we also collected variables such as: whether or not an operating system was installed; whether a wireless card was included; whether there was additional software or other hardware such as a printer; whether the laptop was under warranty; removable media drives; and other variables.

On eBay, sellers can choose between auctions of 1,3,5,7, or 10 days. Table **??** shows the distribution of auction lengths among different types of auctions. In addition, Table **??** shows average auction length by type of auction. The average length of BIN auctions is higher than in WBIN auctions in our data. Among BIN auctions, the proportion of auctions that traded at the buy price is higher for auctions with higher lengths.

At the end of each auction, we revisited the auction web page and collected the entire available bid history for these auctions. Figure **??** shows a typical bid history page for an eBay auction that ended with a trade. Since each bidder has a unique eBay ID number, it was possible to determine the number of bidders who placed bids and exact time when each bid was placed, including any bid revisions. In addition, by noting the price at which bidders dropped out, we could determine the amount of each bid with the exception of the winning bid. Table **??** indicates that BIN auctions (excluding those where the BIN price was accepted) have fewer bidders and a lower number of total bids than WBIN auctions.

Following each transaction on eBay, the winning bidder and the seller can leave feedback on the other party, and bidders and sellers accumulate feedback scores over time. Previous studies of eBay have shown that these reputation measures are strongly related to auction outcomes (Lucking-Reiley et al (2000), Bajari and Hortacsu (2003), and Houser and Wooders (2006)). We visited each seller's web page, and collected a number of reputation measures: the feedback score (number of positive comments from other parties); the sum of positive and negative feedbacks, and the percent of negative feedback. We also collected information on how long the seller had been an eBay member. Table **??** gives some summaries of these data. We see that sellers in BIN auctions on average have a smaller number of transactions, but longer length of membership, and a higher fraction of positive feedback.

3.3.2 Regression Analysis

Next, we examine the relationship between auction characteristics, including the BIN price, and auction outcomes. From the seller's perspective, the reserve price, the auction length, and the use of the BIN feature are all choices variables, and theory suggests that in some cases, using a positive reserve price and a BIN price could increase expected revenue.

We first consider the choice of whether to offer a BIN price. Table **??** shows ML estimates of a probit model for the existence of a BIN price, given seller and laptop characteristics. We find that "power sellers" are less likely to use the BIN option, as are sellers with an eBay store. The BIN option is used more when the laptop is bundled with a printer, laptop bag, or other item. Table **??** shows least squares estimates in a linear regression of the buy price on seller and laptop characteristics, for auctions with a BIN option. Sellers with an eBay store post a higher buy price, by about \$32. The buy price tends to be higher when the laptop has better features, as one would expect.

Tables **??**-**??** show regressions of seller revenue on auction, seller, and laptop characteristics. The regressions use all the auctions in our data set, with revenue equal to 0 in auctions where the good did not sell, and differ only the auction characteristics included in the specifications. In Table **??**, we use an indicator for the BIN option, but do not control for the specific buy price. We find that having the BIN option is associated with about \$29 higher expected revenue to the seller, which would appear to be consistent with the argument that the BIN option can increase seller revenue. In Table **??**, we include the BIN price as a regressor, in addition to the indicator for the existence of the BIN option. To interpret these results, it is useful to consider the effect of offering the average BIN price of \$420.60. At this price, the expected revenue to the seller would be higher by $\$5.50 + [0.07 \cdot \$420.6] \approx \$35$. The positive coefficient of 0.07 on the BIN price suggests that revenues are roughly increasing in the BIN price, and the coefficient of -.60 on the reserve price suggests that higher reserve price leads to lower revenues.

It seems plausible that both reserve price and BIN price have a nonlinear relationship with seller revenue, with some "optimal" level of both prices maximizing revenue. We tried estimating a regression with quadratic terms in both reserve price and BIN price. Table **??** shows the results. However, the coefficients are difficult to interpret. The relationship between BIN price and revenue appears to be convex, and increasing over most of the observed range for the BIN price. To explore this in slightly more detail, we show in Figure **??** a scatterplot of the residuals from a regression of revenue on all the variables *except* the BIN variables, against the BIN price. (The residuals were generated from a regression using all auctions.) The graph also shows a nonparametric regression curve, generated using the `loess` function in R. There is some weak evidence that the seller revenue is increasing in BIN price over most of the range of the data, and certainly little to indicate a value of BIN price beyond which revenue starts to decline. We may not be conditioning sufficiently finely on the laptop and seller characterisics to be able to make useful policy prescriptions about the auction design. We intend to explore this further in future versions of the paper.

3.4 Structural Empirical Model

In this section, we set up our primary model for bidder behavior in eBay auctions, and take this model to the data we have collected. We will refer to auctions with a buy price B as buy-it-now (BIN) auctions, and auctions without a buy price as without-buy-it-now (WBIN) auctions. A WBIN auction is a standard eBay auction.

The auction takes place from time 0 to a prespecified ending time T. In the time interval $[0, T]$, potential bidders arrive according to a nonhomogeneous Poisson process with intensity function (arrival rate) $\lambda(t)$, for $t \in [0, T]$. We assume that this function is bounded away from 0 and ∞. Thus, the number of bidders is stochastic. We allow the arrival rate to change with time, because bidders appear to arrive more frequently near the end of auctions. At any time $t \in [0, T]$, the probability that exactly n potential bidders will arrive in the remaining time $[t, T]$ is

$$\frac{(\gamma_t)^n e^{-\gamma_t}}{n!}, \quad n = 0, 1, 2, \ldots,$$

where

$$\gamma_t := \int_t^T \lambda(t) dt.$$

A bidder who arrives at time t, and who wins the object at time $\tilde{t} \in [t, T]$ at price p, obtains a utility of $u(v - p, \tilde{t} - t)$.[8] The utility has a constant absolute risk aversion

[8] In practice, $\tilde{t}$ will either be equal to t (in the case that the BIN option is exercised and the auction ends immediately) or T (when there is no BIN option or the option is not exercised, and a standard eBay auction ensues).

form:

$$u(v - p, \tilde{t} - t) = \delta^{\tilde{t}-t}\frac{1 - e^{-\alpha(v-p)}}{\alpha}.$$

Here $\delta \in (0, 1]$ is a discounting factor, v is a "valuation" of the ojbect to the bidder, and α is the bidder's level of risk aversion. When $\alpha = 0$, we set $u(v - p, \tilde{t} - t) = \delta^{\tilde{t}-t}(v - p)$. Currently, we set $\delta = 1$ in our empirical analysis. We plan to estimate δ in future versions of the paper. The risk-aversion parameter α is assumed to be the same across individuals.

The value v is distributed independently and identically across bidders, according to a continuous distribution F with support $[\underline{v}, \bar{v}]$. Let f denote the density of F with respect to Lebesgue measure. Below, we will specify a parametric model for the valuation distribution conditional on observed characteristics of the object being auctioned, and seller characteristics.

3.4.1 WBIN Auctions

We first describe bidder behavior in our model of standard eBay auctions without a buy-it-now price. Our model is essentially the same as that of Canals-Cerda and Pearcy (2004). In the next subsection, we then extend our model to BIN auctions.

In a WBIN auction, the seller sets a starting or reserve price r, assumed to be in $[\underline{v}, \bar{v})$, and the length of the auction T. At any point in time t, there is a current standing bid $s(t)$, which is the second highest of the bids placed up to that time. If fewer than two bids have been received, then $s(t) = r$, the reserve price.

When a bidder arrives at some time t, (according to the Poisson process introduced above), we call her a *potential bidder*. Upon arrival, she decides whether or not to enter the auction. If she exits the auction, she does not enter again at a later time. If she enters the auction, she becomes an *active bidder*. She must place a bid immediately, and the bid must be greater than the standing bid $s(t)$. Active bidders can later raise their bids as many times as they like. At the end of the auction at time T, the highest bidder wins and pays the amount of the second highest bid. Since the auction ends at T, we have $\tilde{t} = T$ and the winner's payoff is discounted by δ^{T-t}.

We assume that any potential bidder with valuation greater than or equal to the current standing bid enters the auction (i.e. places a bid):

In BIN auctions, a potential bidder with valuation v enters the auction if $v \geq s(t)$; otherwise, she exits the auction.

In eBay auctions, bidders enter their "maximum" bid amount, and a computer proxy raises bids as long as the standing bid is below the bidder's maximum bid. Thus, it would seem natural to consider the auction as a standard English or oral ascending bid auction, and assume that bidders bid up to their valuation. However, we observe many bidders raising their "maximum" bids later in the auction. This suggests that bids may not necessarily correspond to valuations. We follow Haile and Tamer (2003) and make a weaker assumption on bidder behavior.

a bidder's bids are less than or equal to her valuation v.

a bidder does not allow an opponent to win at a price they are willing to beat.

Assumption 3.4.1 ensures that the bidder with the highest valuation wins the auction. In general, bids are not necessarily equal to valuations (although this is not ruled out), and the relationship between bids and valuations is not fully specified. In addition, bidders are allowed to raise their bids as many or as few times as they like.

Because eBay uses proxy bidding, Assumption 3.4.1 implies that the bidder with the second highest vauation will (at some point) keep bidding against the bidder with the highest valuation, until eventually she has bid her true valuation. At that point, $s(t)$ will be equal to the second highest valuation and no other bidder will be able to place any more bids. As a result, the sale price at the end of the auction will be equal to the second highest valuation, whenever there are at least two active bidders.[9] This

[9] This argument ignores the fact that eBay has minimum bid increments. Because of these increments, it is possible that the sale price could be higher than the second highest valuation by an amount up to the increment. Since these increments are generally very small, we ignore them in the empirical analysis, but note that it could cause a very small upward bias in our estimated valuation

argument, due to Canals-Cerda and Pearcy (2004), means that in these auctions, the second highest bidder's valuation is observed, providing an additional source of information beyond the inequalities which follow directly from Haile and Tamer's assumptions.

3.4.2 BIN Auctions

In a BIN auction, the seller sets a reserve price $r \in [\underline{v}, \bar{v})$, auction length T, and a buy price $P_B > r$. The BIN feature is available at the start of the auction. It remains available until a bidder arrives and either accepts the buy price (thus ending the auction and winning the item at price P_B), or places a bid b where $r < b < P_B$. If a bidder places a bid below the buy price, the buy-it-now option disappears, and the auction becomes a standard ascending bid auction, with the same rules as in the WBIN auction. Thus, the buy-it-now feature allows a bidder to obtain the item with certainty, and obtain the item earlier than time T.

We assume that when a potential bidder arrives at time t, she decides to enter the auction as long as $v \geq s(t)$. If she does not enter, she disappears from the auction and cannot enter at a later date. If she does enter the auction, she must either buy the item at the buy price P_B (if the buy option is still available), or place a bid immediately.[10] The bid must be greater than the standing bid $s(t)$, and if a buy price is available, must be less than the buy price. In the ascending bid phase that follows, active bidders can revise their bids. Once again, Assumptions 3.4.1 and 3.4.1 restrict bidding behavior in the ascending auction.

There are two benefits for a bidder taking the BIN option First, the bidder gets the item with certainty, rather than potentially being outbid (or paying more) later in the auction. Second, the bidder gets the item earlier, at $\tilde{t} = t$ rathen than $\tilde{t} = T$.

distributions.

[10] While the assumption that a bidder must make a decision immediately upon arrival might appear strong, we could interpret the arrival as the time of the bidder's first decision to take an action.

On the other hand, the cost of taking the BIN option is that it is possible that the bidder could win the item at a lower price by waiting. For bidders who face a choice between the buy price and placing a bid below the buy price, we can characterize the decision whether or not to accept the buy price. Consider a bidder who arrives at time t with value $v > r$ and finds the BIN option available. A strategy for this bidder maps her value into her decision of accepting or rejecting P_B, and conditional on rejecting P_B, the value of her bid b. We will focus on "cutoff strategies" for the bidder. A cutoff strategy at time t is defined by a constant $c_t \in [P_B, \bar{v}]$, such that the bidder accepts the BIN price P_B if $v > c_t$, and she rejects P_B and places a bid if $v < c_t$.

When the bidder accepts P_B she gets a certain payoff. However, if she rejects P_B and places a bid, her payoff is uncertain. In this case, she will win only if she has the highest value among the bidders who arrive in the remaining time. The expected payoffs from accepting and rejecting P_B, denote $U^A(v,t)$ and $U^R(v,t)$, are

$$U^A(v,t) = u(v - P_B, 0),$$

$$U^R(v,t) = \sum_{n=0}^{\infty} E[u(v - \max\{r, y\}, T - t)|y \leq v]G_n(v)\frac{\gamma_t^n e^{-\gamma_t}}{n!},$$

where $G_n(\cdot) = F(\cdot)^n$ is the distribution function of y, and $\gamma_t = \int_t^T \lambda(t)dt$. In the last expression, n is interpreted as the number of potential bidders who arrive after t, and y is the highest among n draws from the distribution F.

A cutoff c_t^* is an equilibrium cutoff if $U^A(v,t) > U^R(v,t)$ for all $v > c_t^*$, and $U^A(v,t) < U^R(v,t)$ for all $v < c_t^*$. It is useful to define $\bar{P}_{Bt}$ as the buy price that makes a bidder with $v = \bar{v}$ indifferent beween accepting and rejecting at time $t \in [0, T)$. This solves:

$$u(\bar{v} - \bar{P}_{Bt}, 0) = \sum_{n=0}^{\infty} E[u(\bar{v} - \max\{r, y\}, T - t)]\frac{\gamma_t^n e^{-\gamma_t}}{n!}.$$

Now, the following proposition characterizes the equilibrium cutoff c_t^*:

Proposition 1. *Consider a BIN auction with a reserve price $r \in [\underline{v}, \bar{v}]$, a buy price P_B, and time length T. Suppose that a bidder with risk aversion index α and discount factor δ arrives at time $t \in [0, T)$, and that P_B is available at time t. Then*

1. *If $P_B \geq \bar{P}_{Bt}$, she does not accept P_B in equilbrium; that is, $c_t^* = \bar{v}$.*

2. *If $P_B < \bar{P}_{Bt}$, there exists a unique equilibrium cutoff $c_t^* \in (P_B, \bar{v})$. This cutoff is implicitly defined by*

$$U^A(c_t^*, t) = U^R(c_t^*, t).$$

The cutoff is c_t^* is decreasing in α and r, and increasing in P_B, t, and δ.

Note that $\lambda(t) < \infty$ ensures that it is not always better for a bidder to accept P_B, while $\lambda(t) > 0$ ensures that it is not always better to reject the buy price.

An important aspect of this result is that we can compute the equilibrium cutoff c_t^* using only assumptions 1-3, without specifying the rebidding process. The reason this is possible is that the expected utility of rejecting the BIN option depends only on the distribution of the winning price. Assumptions 1-3 are sufficient to define this distribution, without fully specifying the complete distribution of bids and rebids.

3.4.3 Parametric Specification

For auction i, let X_i be a vector of laptop characteristics, and let W_i be a vector of seller reputation measures. Then we assume that the valuation distribution in auction i is lognormal, where the log valuation has mean

$$\mu_i = \beta_0 + X_i'\beta_x + W_i'\beta_w.$$

and variance σ^2.

In our initial results reported below, the arrival rate is assumed to be constant over time, and with respect to auction characteristics: $\lambda(t; X_i, W_i) = \lambda$. We currently assume that the discount factor $\delta = 1$, so that there is no impatience. The risk aversion parameter α is constant across auctions, and is a parameter to be estimated within our setup. We will denote the entire parameter vector by θ:

$$\theta := (\beta_0, \beta_x, \beta_w, \sigma, \lambda, \alpha).$$

We will write the distribution of valuations in auction i as $F_\theta(v|X_i, W_i)$, and the associated density as $f_\theta(v|X_i, W_i)$. The cutoff function $c(t)$ also depends on the parameter θ, so we will use $c_\theta(t)$ to make this explicit.

In our analysis, we treat the auction design variables (reserve price, auction length BIN option, BIN price) as exogenous. Implicitly, we are assuming that the auction design variables are not related to the valuation distribution or the other behavioral parameters, after conditioning on the laptop and seller characteristics.

3.4.4 Partial Likelihood based on Arrivals of Bidders and the BIN option

Consider a single auction i with K observed bidders. For now, we drop the "i" subscript and the conditioning on (X_i, W_i) for notational simplicity. We first consider the statistical information contained in the observations on initial arrival times $t_1, \ldots, t_K$, along with the time of acceptance of a buy price (if the BIN option is available). The arrival-time observations can be viewed as a counting process (see Andersen, Borgan, Gill, and Keiding, 1993), and, augmented with the information about acceptance of a buy price, can be viewed as a marked point process (a point process with additional random variables observed at each point). However, the appropriate conditioning set for this process at any point in time t should include the standing bid process $s(t)$ and other observed variables up to time t. Thus, the appropriate filtration for this process is larger than the filtration generated by the marked point process itself (the process is "non-self-exciting"). Cox (1975) noted that one can often construct pseudo-likelihoods by conditioning arguments, and pointed out that such "partial" likelihoods behave like ordinary likelihoods from the point of view of statistical inference. Here, we use this insight to construct a partial likelihood for this component of our data.

To give a heuristic explanation of the partial likelihood, it is useful to (a) normalize $T = 1$ (this can always be done by redefining the arrival rate $\lambda(t)$ appropriately), and (b) discretize the time interval into n periods $1/n, 2/n, \ldots, n/n$. For $j = 1, \ldots, n$, let $\delta_j = 1$ if there is a new arrival (of an active bidder) at time j/n, and 0 otherwise. In addition, let $\tau_j = 1$ if the buy price P_B is accepted at time j/n, and 0 otherwise. (If the BIN option is not available, or has already been rejected at an earlier time period, $\tau_j = 0$.) Collect these two observations at time j/n into

$$\psi_j = (\delta_j, \tau_j),$$

and for any j, also define the sequence of observations up to time j/n as:

$$\Psi_j = (\psi_1, \psi_2, \ldots, \psi_j).$$

There is other information available at any time j/n. Let B_j denote the history of bids up to time j. Then, for an auction with reserve r and buy price P_B, the complete history of the auction before time j/n is

$$\Omega_{j-1} = (\Psi_{j-1}, B_{j-1}, r, P_B),$$

with obvious modification for WBIN auctions. Let $s_j = s(j/n)$ denote the standing bid at time j/n, and let $c_j = c_\theta(j/n)$ denote the cutoff value at time j/n. In addition, let BIN_j be an indicator for whether the BIN option is available at time j. Clearly, s_j, c_j, and BIN_j are all functions of Ω_{j-1}.

In order the derive the distribution of ψ_j, it is useful to define latent variables w_j and v_j, where w_j as an indicator for whether a potential bidder arrives in period j, distributed Bernoulli with probability $\lambda(j/n)/n$, and v_j is an independent draw from the valuation distribution $F(v)$. Then we can define

$$\delta_j := w_j \cdot 1(v_j > s_j),$$

and

$$\tau_j = w_j \cdot 1(v_j > c_j) \cdot \mathrm{BIN}_j.$$

We now focus on obtaining the conditional distribution of ψ_j given Ω_{j-1}. The expressions will be slightly different depending on whether the BIN option is available or not available at time j/n:

BIN Option Not Available at j/n:

In this case, $P_\theta(\tau_j = 0|\Omega_{j-1}, \theta) = 1$, and simple calculations lead to:

$$\begin{aligned} P_\theta(\psi_j = (0,0)|\Omega_{j-1}) &= 1 - \frac{\lambda(j/n)}{n}\left(1 - F_\theta\left(s_j\right)\right); \\ P_\theta(\psi_j = (1,0)|\Omega_{j-1}) &= \frac{\lambda(j/n)}{n}\left(1 - F_\theta\left(s_j\right)\right). \end{aligned}$$

Intuitively, this says that the probability of observing an active bidder enter at period j is equal to the probability of a potential bidder arriving, times the probability that the bidder's valuation is higher than the current standing bid.

BIN Option Available at j/n:

Again, using the latent variable representation of the model, we can derive:

$$\begin{aligned} P_\theta(\psi_j = (0,0)|\Omega_{j-1}) &= = 1 - \frac{\lambda(j/n)}{n}\left(1 - F_\theta(s_j)\right); \\ P_\theta(\psi_j = (1,0)|\Omega_{j-1}) &= \frac{\lambda(j/n)}{n}\left[F_\theta(c_j) - F_\theta(s_j)\right]; \\ P_\theta(\psi_j = (1,1)|\Omega_{j-1}) &= \frac{\lambda(j/n)}{n}\left(1 - F_\theta(c_j)\right). \end{aligned}$$

Here, there are three possibilities. The probability of the BIN option being taken in period j is equal to the probability that a potential bidder arrives, times the probability that the bidder's valuation is above the BIN cutoff c_j. The probability of a standard bid in period j is equal to the probability of a bidder arriving, times the probability that the bidder's valuation is between the current standing bid and the BIN cutoff.

Having calculated these conditional probabilities, we can then define, for a given auction, a partial likelihood composed of the product of these conditional densities:

$$PL_n(\theta) = \prod_{j=1}^{n} P_\theta(\psi_j|\Omega_{j-1}).$$

Notice that $PL_n(\theta)$ is not a sequential decomposition of the joint density of $\psi_1, \ldots, \psi_n$, because the conditioning set Ω_{j-1} at each period includes additional variables beyond the lagged ψ_j. In the terminology of stochastic processes, the process $\{\psi_j\}$ is not self-exciting, because the sequence of conditioning sets (the filtration) is larger than the conditioning sets implied by the lagged values of the process. This is therefore not a standard likelihood, because it is not a joint density for the observed data.

We can take $n \to \infty$ and use standard results on product integration to get the continuous-time version of the partial likelihood. (See Andersen et al, 1993, for a formal derivation of continuous time partial likelihoods of counting processes.) In the case of a WBIN auction with K arrivals at time $t_1, t_2, \ldots, t_K$, that ends in a trade, we have

$$\begin{aligned}
PL_n(\theta) &= \prod_{j=1}^{n} \left[\frac{\lambda(j/n)}{n}(1 - F_\theta(s_j))\right]^{\delta_j} \left[1 - \frac{\lambda(j/n)}{n}\left(1 - F_\theta\left(s_j\right)\right)\right]^{1-\delta_j} \\
&= \prod_{j:\delta_j=1} \frac{\lambda(j/n)}{n}(1 - F_\theta(s_j)) \times \prod_{j:\delta_j=0} \left[1 - \frac{\lambda(j/n)}{n}\left(1 - F_\theta\left(s_j\right)\right)\right] \\
&\propto \prod_{j:\delta_j=1} \lambda(j/n)(1 - F_\theta(s_j)) \times \prod_{j:\delta_j=0} \left[1 - \frac{\lambda(j/n)}{n}\left(1 - F_\theta\left(s_j\right)\right)\right] \\
&\longrightarrow \left[\prod_{k=1}^{K} \lambda(t_k)(1 - F_\theta(s(t_k)))\right] \cdot \exp\left(-\int_0^1 \lambda(t)[1 - F(s(t))]dt\right) \\
=: \; & PL(\theta).
\end{aligned}$$

Hence the continuous-time partial log-likelihood (up to an additive constant C) is:

$$\log PL(\theta) = C + \sum_{k=1}^{K} \log[\lambda(t_k)(1 - F_\theta(s(t_k)))] - \int_0^1 \lambda(t)(1 - F_\theta(s(t)))dt.$$

Similarly, for a BIN auction that ends with the buy-it-now price P_B accepted at time t_1, we have

$$PL(\theta) \propto \lambda(t_1)(1 - F_\theta(c_\theta(t_1))) \exp\left(-\int_0^{t_1} \lambda(t)(1 - F_\theta(s(t)))dt\right).$$

For a BIN auction that ends with a sale through bidding, the partial likelihood reflects the fact that the buy-it-now price was rejected at time t_1:

$$\begin{aligned} PL(\theta) \quad \propto \quad & \lambda(t_1)\left[F_\theta(c_\theta(t_1)) - F_\theta(s(t_1))\right]\left[\prod_{k=2}^{K} \lambda(t_k)(1 - F_\theta(s(t_k)))\right] \\ & \times \exp\left(-\int_0^1 \lambda(t)(1 - F_\theta(s(t)))dt\right). \end{aligned}$$

When an auction (either BIN or WBIN) ends without a trade, the likelihood is:

$$PL(\theta) \propto \exp\left(-\int_0^1 \lambda(t)(1 - F_\theta(s(t)))dt\right).$$

Having constructed the partial likelihood, we can use it like a standard likelihood. In particular, by the argument of Cox (1975), a version of the information equality holds: the expected log partial likelihood

$$E[\log PL(\theta)]$$

is maximized at the true value of θ, and therefore we have the moment equation

$$E\left[\frac{\partial \log PL(\theta)}{\partial \theta}\right] = 0.$$

3.4.5 Moment Conditions Based on Bids

The previous partial likelihood function uses information in the arrival times of bidders and in their decisions of whether to take the BIN option. This is particularly informative about the Poisson arrival process parameter and the risk aversion parameter (which enters the partial likelihood through the cutoff function). However, the partial likelihood does not use any information in the actual bid amounts, which should be quite informative about the distribution of valuations F. To utilize this information, we augment the moment conditions from the partial likelihood approach above witha set of simulated moments related to bid amounts.

The Haile-Tamer assumptions imply various moment inequalities relating observed bids to valuations. In our eBay context, they also imply some moment equalities, because of the fact that they imply that we observe the second highest valuation in every auction with 2 or more bidders. For now we focus on these simpler moment equalities, but we plan to incorporate additional information in moment inequalities in future work.

The key to our current moment equalities is that Assumptions 1-3 imply that the sale price will equal the second highest valuation if there are two or more bidders, and will equal the reserve price if there is only one bidder. Conceptually, this means that given parameters and a particular auction (i.e. item and seller characteristics), we can very easily simulate possible sale prices. We can then form moments by matching these simulated sale prices to actual sale prices.

More specifically, we simulate sale prices as follows. Consider a particular auction in the data with a given set of item and seller characteristics and auction length. First, given the arrival rate, reserve price, and auction length, we simulate the number of bidder arrivals for the auction by drawing from a Poisson process. Next, given the mean and variance of the valuation distribution for the auction, we take a draw from the valuation distribution for each of these arrivals. Now find the first simulated valuation that is higher than the observed reserve price. This is the first observed bidder. If none of the valuations are higher than the reserve price, the auction ends with no sale. If the first observed bidder has a valuation higher than the BIN threshold, the auction ends with a sale at the BIN price.[11] If the valuation is less than the BIN threshold, the first bidder bids and the auction reverts to standard format. In this case, the auction ends with a sale at either the second highest of the simulated valuations (if at least two are higher than the reserve price) or a sale at the reserve price (if not).

At any given set of parameters, we can simulate these sale prices for each auction NS times. We then construct

$$\begin{aligned} EP_i &= \frac{1}{NS}\sum_{ns} P_{ns} \\ EP_i^2 &= \frac{1}{NS}\sum_{ns} P_{ns}P_{ns} \\ ED_i &= \frac{1}{NS}\sum_{ns} D_{ns} \end{aligned}$$

where P_{ns} is the simulated price in the nsth simulation, and D_{ns} is a dummy variable indicating that the item was sold in the nsth simulation. Thus, EP_i is the simulated price for the ith auction, and ED_i is the simulated probability of sale. EP_i^2 is the simulated second moment of sale price, which should be useful for identifying the variance of bidder valuations.

Given the above simulated values, our additional moment conditions are:

[11] Right now, we are using the BIN cutoff from the observed first bidder's arrival time rather than the simulated first bidder's arrival time. This is because of the computational burden involved in recomputing the cutoffs multiple time. We are working on an alternative conditional moment approach that avoids this problem.

$$\frac{1}{N}\sum_i [P_i - EP_i]$$
$$\frac{1}{N}\sum_i [P_i - EP_i]x_{1i}$$
$$\frac{1}{N}\sum_i [P_i - EP_i]x_{2i}$$
$$\vdots$$
$$\frac{1}{N}\sum_i [P_i - EP_i]x_{ji}$$
$$\frac{1}{N}\sum_i [P_i^2 - EP_i^2]$$
$$\frac{1}{N}\sum_i [D_i - ED_i]$$

where P_i is the observed sale price in auction i (0 if no sale), and D_i is an indicator that auction i ended in a sale. By construction, all these moments should equal zero when the simulated values are computed at the true parameter values. We combine these moments with the score-type moments implied by the partial likelihood function to form one large set of moment conditions. We use a two step GMM procedure for estimation based on these moments. In the second step we weight our moments optimally given consistent estimates from the first stage. Standard errors of our estimates are computed in the usual way. We ignore variance in our estimates due to simulation error, but this should be extremely small. This is because simulation error will only increase the variance of the above subset of the moments by $1/NS$ (McFadden (1989), Pakes and Pollard (1989)) and we are setting NS=300.

Lastly, we briefly discuss identification. As noted above, the patterns of observed arrivals should well identify the parameters of the Poisson process through the partial likelihood function. The parameters of the distribution of valuations should similarly be well identified by the observed sale prices. This leaves us with the critical risk aversion and discounting parameters. If there were no discounting, then the propensity of consumers to take the BIN option (as captured by the partial likelihood function) should identify the risk aversion parameter. The intution here comes from Proposition 1, which says that as the risk aversion parameter increases, the BIN cutoff decreases. Intuitively, the more risk averse buyers are, the more likely they are to favor the certainty of the BIN option. Of course, BIN behavior will also depend on the arrival rates and distribution of valuations, since these impact the amount of risk one faces in not taking the BIN option. But as mentioned above, these should be pinned down by the observed rate of arrivals and the observed sale prices.

When we add discounting, things become a little more complicated. The reason is that the discount factor can also very clearly impact BIN behavior. Again, as pointed out in Proposition 1, as consumers discount the future more, they will be more likely to take the BIN option, as it gets them the item sooner. Fortunately, as noted above, we have data on auctions of various lengths, from 1 day to 10 days. This is the key to separately identifying the discount factor and the level of risk aversion, as the length of the auction should directly impact the effect of discounting (the longer the auction, the more important is discounting), but not directly impact the effect of risk aversion. Of course, there is an indirect impact on the effect of risk aversion because longer auctions will likely have more potential bidders, but this effect can be measured by the Poisson parameters (which are identified from the rate of arrivals), and will be implicitly controlled for in the partial likelihood function. Intuitively, we will identify the discount factor by looking at BIN behavior across auctions of different lengths. If there is no discounting, changes in the number of potential arrivals across these different length auctions should affect BIN behavior in a very specific way (as pinned down by the Poisson parameters). Increased propensity of BIN behavior as auction length increases over and above this specific effect will suggest discounting, and the extent will suggest the level of discounting.

3.4.6 Results and Discussion

Our preliminary GMM structural estimation results are given in Table **??**. The estimated Poisson arrival rate (λ) is 2.96. This implies an estimated value of 2.96 for the expected number of potential arrivals in an 1-day auction. Excluding the BIN auctions where BIN options were accepted, the average number of active bidders per day in our data set is 2.27 which is less than our estimated expected potential arrivals. This makes sense - not all arriving bidders will actually place bids because their valuations may be lower than the standing bid.

The estimates for β_0 and σ, the mean-intercept and standard deviation of $\log(v)$, along with the estimates of the characteristic coefficients, imply that for an auction with the average computer and seller attributes, the median of the valuation distribution is \$352.52. This includes shipping costs (the average shipping cost in our data is \$28.70). The estimated median is slightly lower, as we would expect, than the average sale price of \$355.90 in WBIN auctions. Given our estimate of σ, the standard deviation of $\log(v)$, the 9 decile range of the valuation distribution is from \$262.96 to \$472.57.

Examining the estimates of the effects of item and seller covariates, most appear sensible. Processor speed and more memory significantly increase valuations, as does inclusion of an operating system, wireless, and extra software. Our processor speed and memory variables are normalized to have mean 0 and unit variance, so the estimates imply that a one standard deviation increase in processing power increases the mean valuation by 15% while one standard deviation increase in memory in laptops increases the mean valuation by 1%. The three other item covariates mentioned above are dummy variables, so the coefficients imply that an operating system increases the mean valuation by 25%, a wireless card adds 3% and additional software adds 20%. These laptop characteristics also had positive effects on the sale prices (Tables 3.10-3.12) in our OLS regressions. Hard drive size comes up the wrong sign, but it is insignificant. Examining the three seller covariates, the results are more difficult to interpret. While the proportion of negative feedback significantly decreases valuations, the total number of negative feedback entries significantly increases valuations. Total feedback score is positive but insignificant. It could be that negative feedback is actually picking up a non-linear positive effect of seller experience. Another is that our model is not yet flexible enough to get accurate estimates of these parameters. Right now, both observed arrival rates and observed bids are identifying these parameters. One can easily tell stories how this could generate these results. For example, if sellers with more negative feedback advertise their listings more and hence get more bidder arrivals, it will suggest a positive coefficient in our current model. As noted above, in future work we will allow the arrival rate to vary across both time and item/seller characteristics. In that model, arrival rates and observed bids will be identifying different parameters.

Our estimates suggest that bidders are risk averse. The estimated CARA index (α) is 0.29 which is significantly different from 0. This allows us to reject the null hypothesis of risk neutrality against an alternative hypothesis of risk aversion. The implications of this risk aversion parameter on BIN decisions by bidders is easiest to explain in an example. Consider an item with mean seller and item characteristics. Consider a 5-day auction with the sample average reserve price of \$326.00 and the sample average buy price of \$420.60. The estimates in Table 13 imply that a buyer arriving at the halfway point of the auction will have BIN cutoff of \$421.77. This means that if the buy price is available at this time, the bidder accepts it if his valuation is higher than \$421.77, otherwise she rejects the buy price and places a bid if his valuation is greater than \$326.00. If his valuation is less than \$326.00, she exits the auction. The small interval between the BIN price and the cutoff suggests that bidders are fairly risk averse—virtually any buyer with a valuation higher than the BIN price takes it. This difference increases though, as the BIN price increases. In addition, note that we are assuming a discount factor of 1 currently, which if anything will bias our estimate of the risk aversion parameter upwards.

3.5 Conclusion

Online auctions are playing a big role these days in matching the buyers and the sellers without requiring them to be physically present at the same place. They are also introducing new types of auction formats which never existed in the past and thus providing us with new research areas. Buy price auctions are among these new auction formats.

A buy price auction allows a bidder to buy the item early and with certainty. As a result, a risk averse and impatient bidder may prefer buying the item at the buy price. A bidder's decision thus allows us to identify his risk aversion and time preferences. These information are usually impossible or very difficult to recover from auction data. We used an incomplete model specification which allowed us to obtain these estimates from a dataset of eBay's laptop auctions without fully specifying the bidding and re-bidding process. In our current version we don't have an estimate for time preferences. We also assumed a time-invariant arrival rate for the bidders. We plan to overcome these issues in our future estimates.

	Total Listings	Average Buy Price (SD)	Average Reserve Price (SD)	Successful Trade (%)	Average Sale Price (SD)
All	3245	–	165.9	2610	355.7
			(164.1)	(80.4%)	(92.2)
WBIN	2243	–	94.4	2036	355.9
			(114.6)	(90.8%)	(90.2)
BIN	1002	420.6	326.0	574	355.3
		(102.1)	(143.6)	(57.3%)	(99.1)
Buy Price	219	377.8	311.1	219	375.4
Accepted		(106.2)	(129.7)	–	(104.7)

Average sale price computed using only auctions ending in a trade. All prices are in dollars and include shipping costs. Average shipping cost in our data was $28.7.

Table 3.1. Number of Listings, Trades, and Prices

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	Hard Drive (GB)	Processor (MHZ)	Memory (MB)
All Total	19.2 [2-100]	908.1 [400-2000]	268.5 [64-1024]
Traded	19.4 [2-100]	919.6 [400-2000]	272.6 [64-1024]
WBIN Total	19.6 [2-60]	937.0 [400-2000]	272.0 [64-1024]
Traded	19.6 [2-60]	942.4 [400-2000]	273.7 [64-1024]
BIN Total	18.4 [2-100]	843.0 [400-1300]	260.6 [64-1024]
Traded	18.7 [2-100]	838.5 [400-1300]	268.8 [64-1024]
BIN Accepted	18.1 [2-60]	812.6 [400-1300]	266.2 [64-512]

Minimum and maximum values given in square brackets.

Table 3.2. Laptop Characteristics

		All	WBIN	BIN	Buy Price Accepted
1 Day	Total	1300	1182	118	-
	Traded	1140	1057	83	14
3 Days	Total	972	576	396	-
	Traded	753	543	210	96
5 Days	Total	346	215	131	-
	Traded	288	202	86	35
7 Days	Total	602	251	351	-
	Traded	409	216	193	72
10 Days	Total	25	19	6	-
	Traded	20	18	2	2

Table 3.3. Length of Auctions

		All	WBIN	BIN	Buy Price Accepted
Average Length	Total	3.2	2.7	4.5	-
	Traded	3.0	2.7	4.4	4.6*
Average No. Bidders		7.1	8.8	2.2	-
Average No. Bids**		15.1	18.8	4.5	-

** Average time until buy price accepted: 1.9 days.
*** Excludes auctions where buy price was accepted.

TABLE 3.4. Average Length of Auctions and Number of Bidders and Bids

	Feedback Score		Sum of Positive and Negative Feedback		Negative Feedback as % of Total		Months of Membership to June 2005	
	Total	Traded	Total	Traded	Total	Traded	Total	Traded
All	4019.9	3909.3	4130.5	4024.4	1.3	1.3	44.4	43.1
WBIN	4412.5	4268.3	4547.5	4405.1	1.4	1.4	41.8	41.2
BIN	3141.1	2636.1	3197.1	2674.0	0.8	0.7	50.3	49.8
Buy Price Accepted	-	2519.1	-	2563.3	-	0.9	-	47.4

TABLE 3.5. Seller Characteristics

	Mar. Prob.	SE	$P > \|z\|$
Seller Characteristics			
Power Seller	-0.05	0.02	0.02
eBay Store	-0.06	0.02	0.00
Feedback Score	0.00	0.00	0.01
% Negative Feedback	-0.01	0.00	0.08
Months of Membership	0.00	0.00	0.00
Laptop Characteristics			
Hard Drive (GB)	0.00	0.00	0.16
Processor (MHZ)	0.00	0.00	0.01
Memory (MB)	0.00	0.00	0.72
Operating System	-0.04	0.03	0.12
Wireless Card	0.00	0.02	0.85
Additional Software	-0.13	0.02	0.00
Printer/Laptop Bag/Gifts	0.30	0.03	0.00
Warranty	-0.07	0.02	0.00
Battery_missing	-0.27	0.01	0.00
Powercord_missing	0.27	0.06	0.00
Minor Problem	-0.11	0.02	0.00
CD-Rom	0.04	0.03	0.19
CD-RW	0.38	0.05	0.00
DVD-Rom	0.27	0.04	0.00
CD-DVD Combo	0.38	0.05	0.00
No. of Observations	3211		
Pseudo R^2	0.32		

Estimated coefficients are marginal probabilities at covariate sample means. Regressions also included indicator variables for laptop models. (The same things hold for Table 3.9.)

TABLE 3.6. Probit Regression of Sellers' Decision to Offer BIN Price

	Coefficient	SE	$P > \|z\|$
Seller Characteristics			
Power Seller	7.04	7.48	0.35
eBay Store	32.29	6.92	0.00
Feedback Score	0.00	0.00	0.00
% Negative Feedback	3.63	2.89	0.21
Months of Membership	0.14	0.18	0.43
Laptop Characteristics			
Hard Drive (GB)	0.65	0.34	0.06
Processor (MHZ)	0.22	0.03	0.00
Memory (MB)	0.03	0.02	0.17
Operating System	-5.91	7.01	0.40
Wireless Card	9.78	5.98	0.10
Additional Software	33.50	7.18	0.00
Printer/Laptop Bag/Gifts	17.71	4.87	0.00
Warranty	25.06	5.31	0.00
Battery_missing	-62.06	19.36	0.00
Powercord_missing	-7.74	12.92	0.55
Minor Problem	-31.21	7.73	0.00
CD Rom	3.60	10.50	0.73
CD RW	25.91	10.94	0.02
DVD Rom	19.23	10.64	0.07
CD-DVD Combo	59.45	11.09	0.00
Intercept	132.23	30.02	0.00
No. of Observations	998		
R^2	0.62		

Regressions also included indicator variables for laptop models. Heteroskedasticity-consistent standard errors reported under "SE." (The same things hold for Tables 3.8, 3.10, 3.11 and 3.12.)

TABLE 3.7. OLS Regression of BIN Price

	Coefficient	SE	$P > \|t\|$
Auction Characteristics			
Reserve Price	-0.03	0.00	0.00
3 Day	0.13	0.16	0.40
5 Day	0.37	0.19	0.06
7 Day	0.59	0.20	0.00
10 Day	1.98	0.65	0.00
BIN	-0.73	0.16	0.00
Seller Characteristics			
Power Seller	0.32	0.14	0.02
eBay Store	0.40	0.16	0.01
Feedback Score	-0.00	0.00	0.03
% Negative Feedback	-0.11	0.03	0.00
Months of Membership	0.01	0.00	0.04
Laptop Characteristics			
Hard Drive (GB)	0.02	0.01	0.04
Processor (MHZ)	0.00	0.00	0.00
Memory (MB)	0.00	0.00	0.02
Operating System	1.23	0.15	0.00
Wireless Card	0.24	0.11	0.03
Additional Software	0.59	0.15	0.00
Printer/Laptop Bag/Gifts	-0.21	0.14	0.14
Warranty	0.91	0.12	0.00
Battery_missing	-1.92	0.22	0.00
Powercord_missing	-0.44	0.19	0.02
Minor Problem	-0.55	0.14	0.00
CD-Rom	0.17	0.20	0.39
CD-RW	0.87	0.26	0.00
DVD-Rom	1.05	0.23	0.00
CD-DVD Combo	0.89	0.30	0.00
Intercept	6.12	0.52	0.00
No. of Observations	2992		
R^2	0.77		

TABLE 3.8. Regression of Number of Bidders (excluding BIN auctions with trades at buy price)

	Mar. Prob.	SE	$P > \|z\|$
Auction Characteristics			
Reserve Price	0.00	0.00	0.00
3 Day	0.04	0.05	0.41
5 Day	0.10	0.07	0.12
7 Day	0.05	0.06	0.34
10 Day	0.16	0.22	0.39
BIN Price	0.00	0.00	0.00
Seller Characteristics			
Power Seller	-0.09	0.05	0.06
eBay Store	0.03	0.04	0.48
Feedback Score	0.00	0.00	0.30
% Negative Feedback	0.00	0.01	0.58
Months of Membership	0.00	0.00	1.00
Laptop Characteristics			
Hard Drive (GB)	0.00	0.00	0.97
Processor (MHZ)	0.00	0.00	0.15
Memory (MB)	0.00	0.00	0.07
Operating System	0.05	0.03	0.15
Wireless Card	0.00	0.03	0.98
Additional Software	0.11	0.05	0.01
Printer/Laptop Bag/Gifts	0.01	0.03	0.84
Warranty	-0.11	0.03	0.00
Battery_missing	-0.11	0.06	0.26
Powercord_missing	-0.14	0.04	0.02
Minor Problem	-0.05	0.04	0.20
CD-Rom	-0.03	0.05	0.60
CD-RW	0.00	0.06	1.00
DVD-Rom	0.01	0.06	0.85
CD-DVD Combo	0.18	0.09	0.02
No. of Observations	998		
Pseudo R^2	0.18		

TABLE 3.9. Probit Regression of Bidder Decision to Accept BIN Price

	Coefficient	SE	$P > \|z\|$
Auction Characteristics			
Reserve Price	-0.59	0.02	0.00
3 Day	-8.54	5.47	0.12
5 Day	-14.58	7.25	0.04
7 Day	-30.81	7.44	0.00
10 Day	-4.65	23.36	0.84
BIN	29.48	7.20	0.00
Seller Characteristics			
Power Seller	-4.21	4.75	0.38
eBay Store	15.95	5.19	0.00
Feedback Score	0.00	0.00	0.25
% Negative Feedback	-4.58	1.92	0.02
Months of Membership	0.12	0.11	0.25
Laptop Characteristics			
Hard Drive (GB)	1.17	0.32	0.00
Processor (MHZ)	0.16	0.02	0.00
Memory (MB)	0.12	0.02	0.00
Operating System	27.56	6.20	0.00
Wireless Card	1.44	4.33	0.74
Additional Software	39.45	5.91	0.00
Printer/Laptop Bag/Gifts	-0.67	7.21	0.93
Warranty	-3.86	4.85	0.43
Battery_missing	-53.73	10.36	0.00
Powercord_missing	-25.42	9.13	0.01
Minor Problem	-8.52	4.50	0.06
CD-Rom	-3.28	6.75	0.63
CD-RW	4.74	13.76	0.73
DVD Rom	28.98	9.39	0.00
CD-DVD Combo	49.23	15.93	0.00
Intercept	151.27	20.11	0.00
No. of Observations	3211		
R^2	0.52		

TABLE 3.10. OLS Regression of Revenue to Seller

	Coefficient	SE	$P > \|z\|$
Auction Characteristics			
Reserve Price	-0.60	0.02	0.00
3 Day	-8.33	5.48	0.13
5 Day	-15.09	7.28	0.04
7 Day	-30.45	7.40	0.00
10 Day	-6.23	23.70	0.79
BIN	5.50	31.34	0.86
BIN*BIN Price	0.07	0.09	0.44
Seller Characteristics			
Power Seller	-3.95	4.83	0.41
eBay Store	15.14	5.25	0.00
Feedback Score	0.00	0.00	0.22
% Negative Feedback	-4.64	1.84	0.01
Months of Membership	0.12	0.11	0.27
Laptop Characteristics			
Hard Drive (GB)	1.17	0.31	0.00
Processor (MHZ)	0.16	0.02	0.00
Memory (MB)	0.12	0.02	0.00
Operating System	27.57	6.22	0.00
Wireless Card	1.30	4.37	0.77
Additional Software	38.21	6.00	0.00
Printer/Laptop Bag/Gifts	-0.67	7.21	0.93
Warranty	-4.20	4.92	0.39
Battery_missing	-52.64	10.42	0.00
Powercord_missing	-25.98	9.06	0.00
Minor Problem	-8.06	4.59	0.08
CD-Rom	-3.24	6.75	0.63
CD-RW	3.98	13.90	0.77
DVD Rom	28.48	9.46	0.00
CD-DVD Combo	47.42	15.49	0.00
Intercept	155.64	18.87	0.00
No. of Observations	3211		
R^2	0.52		

TABLE 3.11. OLS Regression of Revenue to Seller

	Coefficient	SE	$P > \|z\|$
Auction Characteristics			
Reserve Price	-0.03	0.09	0.74
Reserve Price^2	0.00	0.00	0.00
3 Day	-1.20	5.70	0.83
5 Day	-13.21	7.26	0.07
7 Day	-23.47	7.41	0.00
10 Day	7.94	22.29	0.72
BIN	182.71	80.46	0.02
BIN*BIN Price	-1.06	0.46	0.02
(BIN*BIN Price)^2	0.00	0.00	0.01
Seller Characteristics			
Power Seller	-3.62	4.68	0.44
eBay Store	19.83	5.18	0.00
Feedback Score	0.00	0.00	0.18
% Negative Feedback	-6.65	1.27	0.00
Months of Membership	0.05	0.11	0.62
Laptop Characteristics			
Hard Drive (GB)	1.35	0.30	0.00
Processor (MHZ)	0.17	0.02	0.00
Memory (MB)	0.12	0.02	0.00
Operating System	34.75	6.15	0.00
Wireless Card	4.14	4.19	0.32
Additional Software	38.16	5.85	0.00
Printer/Laptop Bag/Gifts	-1.84	7.12	0.80
Warranty	1.24	4.83	0.80
Battery_missing	-73.09	10.71	0.00
Powercord_missing	-25.10	9.00	0.01
Minor Problem	-15.14	4.46	0.00
CD-Rom	0.93	6.85	0.89
CD-RW	14.59	14.02	0.30
DVD-Rom	36.47	9.46	0.00
CD-DVD Combo	55.36	15.20	0.00
Intercept	109.98	19.32	0.00
No. of Observations	3211		
R^2	0.54		

TABLE 3.12. OLS Regression of Revenue to Seller

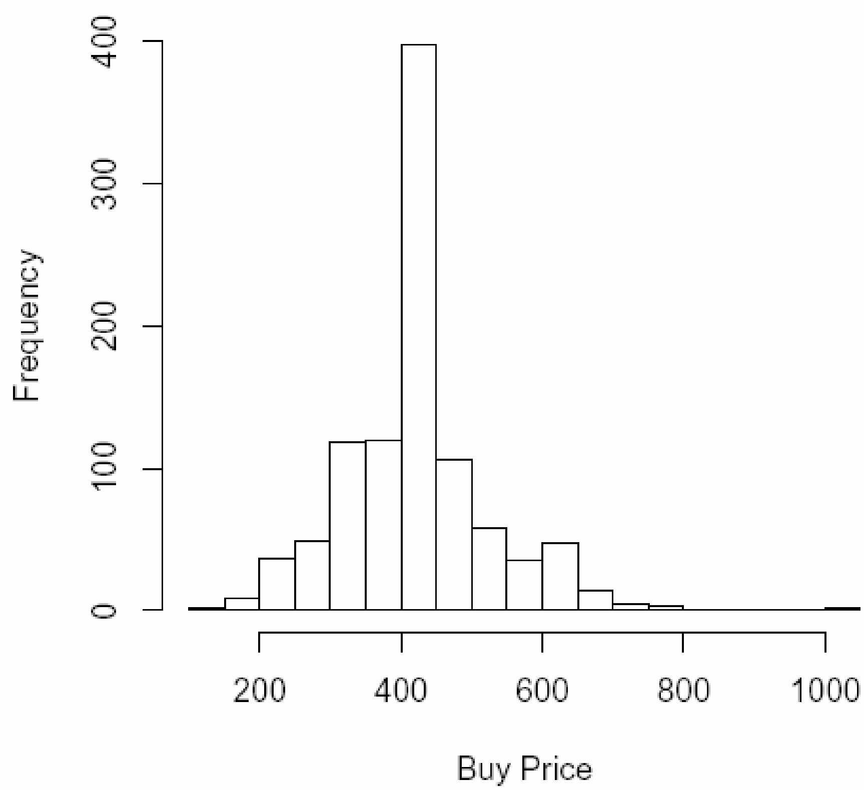

FIGURE 3.1. Histogram of Buy Prices (including shipping costs) in BIN Auctions.

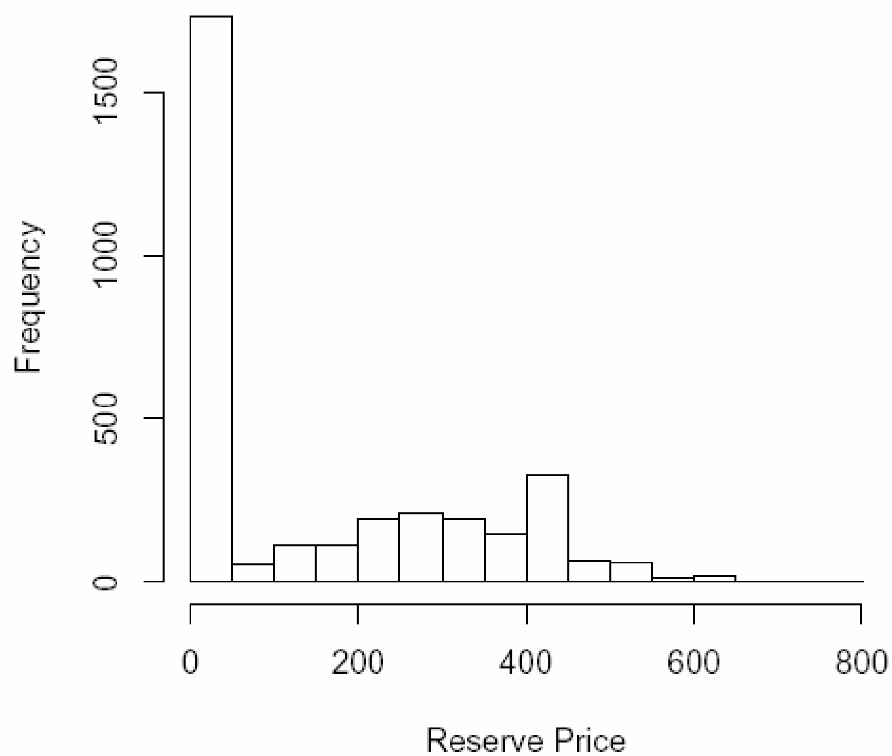

FIGURE 3.2. Histogram of Reserve Prices (including shipping costs) in All Auctions.

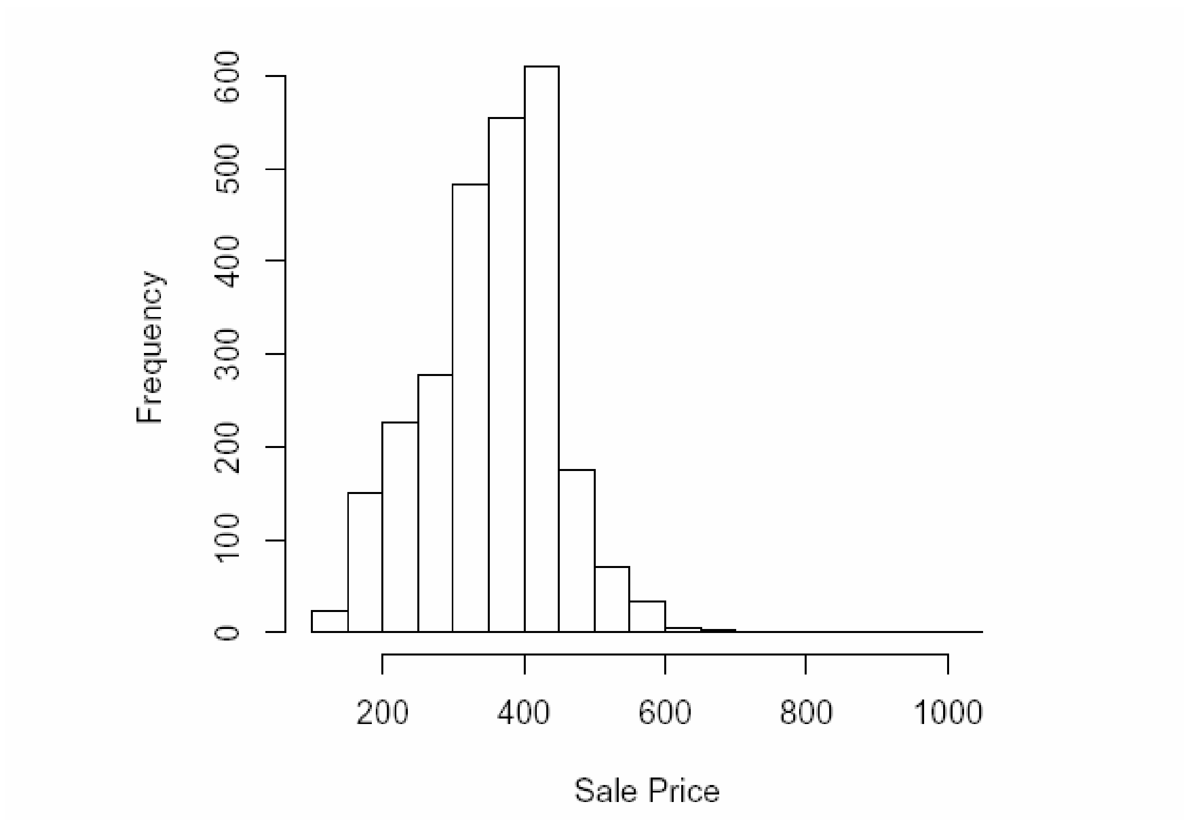

Figure 3.3. Histogram of Sale Prices (including shipping costs) in All Auctions Ending in a Trade.

	Estimates	SE	$P > \|z\|$
λ	2.96	0.03	0.00
α	0.29	0.10	0.00
β_0	5.56	0.01	0.00
σ	0.18	0.01	0.00
Negative Feedback	0.08	0.01	0.00
Negative Feedback/(Total Feedback+1)	-0.01	0.01	0.07
ln(Feedback Score+1)	0.00	0.00	0.86
Processor	0.15	0.01	0.00
Memory	0.01	0.01	0.06
Hard Drive	-0.01	0.01	0.18
Operating System	0.25	0.01	0.00
Wireless Card	0.03	0.01	0.01
Additional Software	0.20	0.01	0.00

Seller characteristics and Processor, Memory, and Hard Drive are standardized to have mean 0 and standard deviation 1.

TABLE 3.13. Structural Estimates.

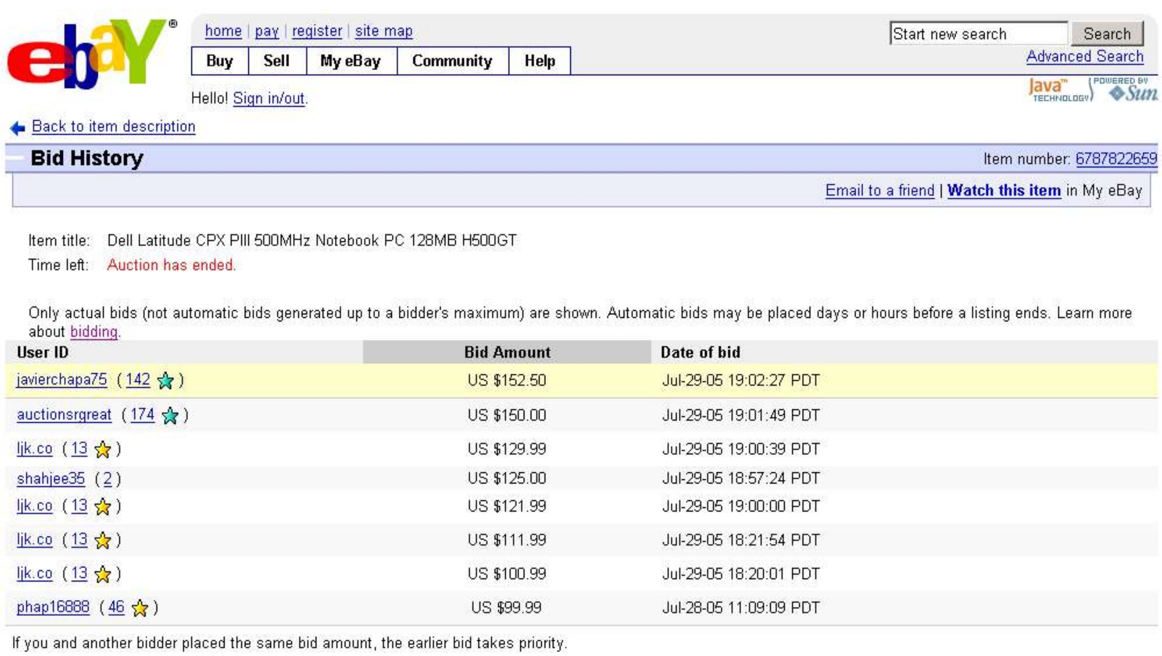

FIGURE 3.4. Typical Bid History

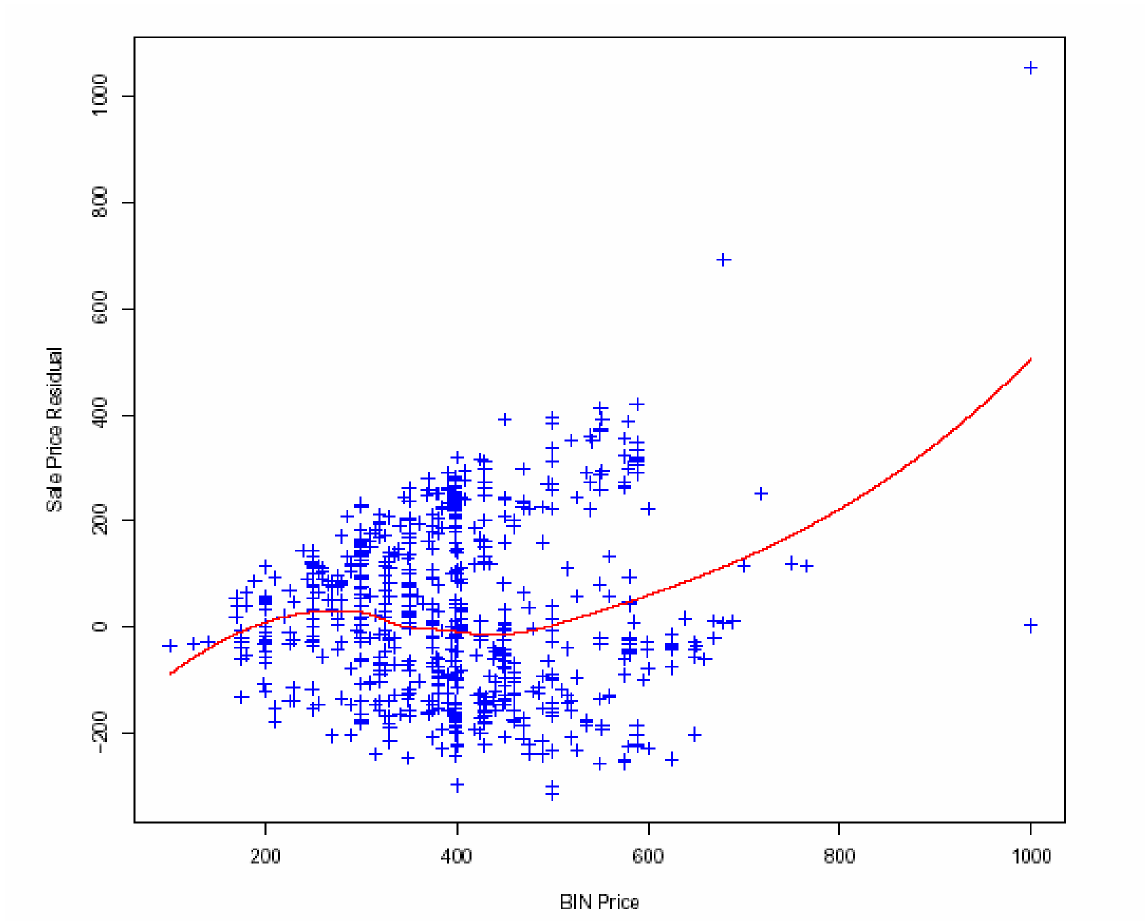

Figure 3.5. Seller Revenue Residuals and BIN Price

Appendix A: Proofs for Chapter 1

Proposition 1:

For an auction with a buy price B,

(i) when $B \in (\underline{\gamma}, \overline{\gamma})$ there exists a symmetric equilibrium cutoff $c^* \in (\underline{x}, \overline{x})$ defined by:

$$U^A(c^*, c^*) \; = U^R(c^*, c^*)$$

(ii) when $B \leq \underline{\gamma}$, $c^* = \underline{x}$ is a symmetric equilibrium cutoff, that is, everyone accepts B.

(iii) when $B \geq \overline{\gamma}$, $c^* = \overline{x}$ is a symmetric equilibrium cutoff, that is, everyone rejects B.

Proof:

(**i**) First I show the existence of some $c \in (\underline{x}, \overline{x})$ that satisfies $U^A(c^*, c^*) = U^R(c^*, c^*)$ when $B \in (\underline{\gamma}, \overline{\gamma})$, and then I show that this c actually is a symmetric equilibrium cutoff c^*. Now, let's define $\hat{U}^A(c)$, $\hat{U}^R(c)$ and $Q(F(a))$ as follows:

$$\begin{aligned}
\hat{U}^A(c) &= U^A(c,c) = \sum_{l=0}^{n-1} \binom{n-1}{l} F(c)^{n-1-l}(1-F(c))^l \left(\frac{1}{l+1}\right) \\
&\qquad E[u(v-B)|x_1 = c, T_l], \\
\hat{U}^R(c) &= U^R(c,c) = E\left[u\left(v - b(z)\right)|x_1 = c, z < c\right].H(c), \text{ and} \\
Q(F(a)) &= \sum_{l=0}^{n-1} \binom{n-1}{l} F(a)^{n-1-l}(1-F(a))^l \left(\frac{1}{l+1}\right) \\
&= \frac{1-F(a)^n}{n(1-F(a))}.
\end{aligned}$$

where T_l denotes the condition "l rivals have signals above c" and $Q(m) = \frac{1-m^n}{n(1-m)}$ is defined for $m \in [0,1)$. Notice that $Q(1) = \lim_{m \to 1} Q(m) = 1$, $Q(0) = \frac{1}{n}$ and, for any

$c < \bar{x}$, $Q(F(c)) > F(c)^{n-1}$.

$\underline{(i)-1}$: For the existence of a c it is sufficient to show that the two curves $\hat{U}^A(c)$ and $\hat{U}^R(c)$ intersect at some $c \in (\underline{x}, \bar{x})$.

Clearly,

$$\begin{aligned}
\hat{U}^A(\underline{x}) &= (\frac{1}{n}).E\left[u\left(v-B\right)|x_1=\underline{x}\right] \text{ since } Q(F(\underline{x})) = \frac{1}{n}, \text{ and} \\
\hat{U}^R(\underline{x}) &= 0 \text{ since } H(\underline{x}) = 0.
\end{aligned}$$

From (1.9) it is easy to see that, by definition of $\underline{\gamma}$, $\hat{U}^A(\underline{x}) = \hat{U}^R(\underline{x})$ when $B = \underline{\gamma}$. Since $\hat{U}^A(\underline{x})$ is a decreasing function of B, for any $B > \underline{\gamma}$ it must be that $\hat{U}^A(\underline{x}) < \hat{U}^R(\underline{x})$.

Again,

$$\begin{aligned}
\hat{U}^A(\bar{x}) &= E\left[u\left(v-B\right)|x_1=\bar{x}\right], \text{ and} \\
\hat{U}^R(\bar{x}) &= E\left[u\left(v-b(z)\right)|x_1=\bar{x}\right].
\end{aligned}$$

Now, from (1.6) , by definition of $\overline{\gamma}$, $\hat{U}^A(\bar{x}) = \hat{U}^R(\bar{x})$ when $B = \overline{\gamma}$. Since $\hat{U}^A(\bar{x})$ is a decreasing function of B, for any $B < \overline{\gamma}$ it must be that $\hat{U}^A(\bar{x}) > \hat{U}^R(\bar{x})$.

Since $\hat{U}^A(c)$ and $\hat{U}^R(c)$ are continuous in c, the above arguments verify that, when $B \in (\underline{\gamma}, \overline{\gamma})$ there exists a $c \in (\underline{x}, \bar{x})$ for which the two curves $\hat{U}^A(c)$ and $\hat{U}^R(c)$ would intersect, that is, $\hat{U}^A(c) = \hat{U}^R(c)$.

Now, I need to show that this c is in the equilibrium.

$\underline{(i)-2}$: Suppose that $B \in (\underline{\gamma}, \overline{\gamma})$ so that there exists a c that satisfies $U^A(c,c) = U^R(c,c)$. Now I show that this c is a symmetric equilibrium cutoff. In order to accomplish this, it is sufficient to show that $U^A(x,c)$ lies above $U^R(x,c)$ for any $x \in (c, \overline{x}]$, and $U^A(x,c)$ lies below $U^R(x,c)$ for any $x \in [\underline{x}, c)$.

For bidder 1,

$$\begin{aligned}
\frac{\partial U^A(x,c)}{\partial x} &= \sum_{l=0}^{n-1} \left(\begin{array}{c} n-1 \\ l \end{array} \right) F(c)^{n-1-l}(1-F(c))^l \left(\frac{1}{l+1}\right) \\
& \quad E\left[u'\left(v-B\right).v_{11} \mid x_1 = x, T_l\right] \\
&= v_{11} \sum_{l=0}^{n-1} \left(\begin{array}{c} n-1 \\ l \end{array} \right) F(c)^{n-1-l}(1-F(c))^l \left(\frac{1}{l+1}\right) \\
& \quad E\left[1-\alpha u\left(v-B\right) \mid x_1 = x, T_l\right] \\
&= v_{11}Q(F(c)) - \alpha v_{11} U^A(x,c), \qquad (3.1)
\end{aligned}$$

where the second line utilizes that $u'(.) = 1 - \alpha u(.)$ for CARA utility function.

Now, for $x \geq c$:

$$\begin{aligned}
\frac{\partial U^R(x,c)}{\partial x} &= v_{11}(n-1) \int_{\underline{x}}^{c} \int_{\underline{x}}^{x_2} .. \int_{\underline{x}}^{x_2} u'\left(v - b(x_2)\right) dF(x_n)..dF(x_2) \\
&= v_{11}(n-1) \int_{\underline{x}}^{c} \int_{\underline{x}}^{x_2} .. \int_{\underline{x}}^{x_2} \left(1 - \alpha u\left(v - b(x_2)\right)\right) dF(x_n)..dF(x_2) \\
&= \left[v_{11}(n-1) \int_{\underline{x}}^{c} \int_{\underline{x}}^{x_2} .. \int_{\underline{x}}^{x_2} dF(x_n)..dF(x_2)\right] \\
& \quad - \left[\alpha v_{11}(n-1) \int_{\underline{x}}^{c} \int_{\underline{x}}^{x_2} .. \int_{\underline{x}}^{x_2} u\left(v - b(x_2)\right) dF(x_n)..dF(x_2)\right] \\
&= v_{11}F(c)^{n-1} - \alpha v_{11} U^R(x,c). \qquad (3.2)
\end{aligned}$$

So,

$$\begin{aligned}
\frac{\partial U^A(x,c)}{\partial x}|_{x=c} &= v_{11}Q(F(c)) - \alpha v_{11} U^A(c,c), \text{ and} \\
\frac{\partial U^R(x,c)}{\partial x}|_{x=c} &= v_{11}F(c)^{n-1} - \alpha v_{11} U^R(c,c),
\end{aligned}$$

Since, by definition of c, $U^A(c,c) = U^R(c,c)$, and $Q(F(c)) > F(c)^{n-1}$ it must be that $\frac{\partial U^A(x,c)}{\partial x}|_{x=c} > \frac{\partial U^R(x,c)}{\partial x}|_{x=c}$.

Now, for $x < c$, applying Leibniz rule:

$$
\begin{aligned}
\frac{\partial U^R(x,c)}{\partial x} &= v_{11}(n-1)\int_{\underline{x}}^{x}\int_{\underline{x}}^{x_2}..\int_{\underline{x}}^{x_2} u'\left(v-b(x_2)\right)dF(x_n)..dF(x_2) \\
&\quad +(n-1)\int_{\underline{x}}^{x}..\int_{\underline{x}}^{x} u\left(v(x,x,..,x_n)-b(x)\right)dF(x_n)..dF(x_3) \\
&= v_{11}(n-1)\int_{\underline{x}}^{x}\int_{\underline{x}}^{x_2}..\int_{\underline{x}}^{x_2}\left(1-\alpha u\left(v-b(x_2)\right)\right)dF(x_n)..dF(x_2) \\
&\quad +F(x)^{n-2}.E[u(v-b(x))|x_1=x,z=x] \\
&= v_{11}F(x)^{n-1}-\alpha v_{11}U^R(x,c)+F(x)^{n-2}.0 \\
&= v_{11}F(x)^{n-1}-\alpha v_{11}U^R(x,c), \qquad (3.3)
\end{aligned}
$$

where the third line utilizes that $E[u(v-b(x))|x_1=x,z=x]=0$ from (1.1) .

Now, if $U^A(x,c)$ and $U^R(x,c)$ intersect at some $x' \in (c,\overline{x}]$, it implies from (3.1) and (3.2) that

$$\frac{\partial U^A(x,c)}{\partial x}|_{x=x'} > \frac{\partial U^R(x,c)}{\partial x}|_{x=x'}$$

which contradicts with

$$\frac{\partial U^A(x,c)}{\partial x}|_{x=c} > \frac{\partial U^R(x,c)}{\partial x}|_{x=c}.$$

Similarly, if $U^A(x,c)$ and $U^R(x,c)$ intersect at some $x'' \in [\underline{x},c)$, it implies from (3.1) and (3.3) that

$$
\begin{aligned}
\frac{\partial U^A(x,c)}{\partial x}|_{x=x''} &> \frac{\partial U^R(x,c)}{\partial x}|_{x=x''} \\
(\text{since } Q(F(c)) > F(c)^{n-1} &> F(x)^{n-1} \text{ for any } x<c)
\end{aligned}
$$

which also contradicts with

$$\frac{\partial U^A(x,c)}{\partial x}|_{x=c} > \frac{\partial U^R(x,c)}{\partial x}|_{x=c}.$$

As a result, $U^A(x,c)$ and $U^R(x,c)$ intersect only at $x=c$, and $U^A(x,c)$ is steeper than $U^R(x,c)$ at this intersection. So, $U^A(x,c)$ lies above $U^R(x,c)$ for any $x \in (c,\overline{x}]$, and $U^A(x,c)$ lies below $U^R(x,c)$ for any $x \in [\underline{x},c)$.

(ii) First I show that when $B=\underline{\gamma}$ and $c=\underline{x}$, $U^A(x,c)$ lies above $U^R(x,c)$ for $x \in (\underline{x},\overline{x}]$. Then I show that when $B<\underline{\gamma}$ and $c=\underline{x}$, once again, $U^A(x,c)$ lies above $U^R(x,c)$ for $x \in (\underline{x},\overline{x}]$. This completes the proof that when $B \leq \underline{\gamma}$, $c=\underline{x}$ is a symmetric equilibrium cutoff.

Suppose $B=\underline{\gamma}$. Fix $c=\underline{x}$. So, $U^A(x,\underline{x})$ and $U^R(x,\underline{x})$ intersect at $x=\underline{x}$. Now, from (3.1) and (3.2) ,

$$\frac{\partial U^A(x,\underline{x})}{\partial x}|_{x=\underline{x}}\left(=\frac{v_{11}}{n}\right) > \frac{\partial U^R(x,\underline{x})}{\partial x}|_{x=\underline{x}}\,(=0)\,. \tag{3.4}$$

If $U^A(x,\underline{x})$ and $U^R(x,\underline{x})$ intersect at some $x' \in (\underline{x},\overline{x}]$, it implies from (3.1) and (3.2) that

$$\frac{\partial U^A(x,\underline{x})}{\partial x}|_{x=x'} > \frac{\partial U^R(x,\underline{x})}{\partial x}|_{x=x'} \tag{3.5}$$

which contradicts (3.4) .

Now, suppose that $B<\underline{\gamma}$. Fix $c=\underline{x}$. So, $U^A(\underline{x},\underline{x}) > U^R(\underline{x},\underline{x})$. If $U^A(x,\underline{x})$ and $U^R(x,\underline{x})$ intersect at some $x' \in (\underline{x},\overline{x}]$, it implies (3.5) which contradicts $U^A(\underline{x},\underline{x})$ $> U^R(\underline{x},\underline{x})$.

So, when $B \leq \underline{\gamma}$ and $c=\underline{x}$, $U^A(x,c)$ and $U^R(x,c)$ do not intersect at any $x' \in (\underline{x},\overline{x}]$, and $U^A(x,c)$ lies above $U^R(x,c)$.

(iii) First I show that when $B=\overline{\gamma}$ and $c=\overline{x}$, $U^A(x,c)$ lies below $U^R(x,c)$ for $x \in [\underline{x},\overline{x})$. Then I show that when $B>\overline{\gamma}$ and $c=\overline{x}$, once again, $U^A(x,c)$ lies below $U^R(x,c)$ for $x \in [\underline{x},\overline{x})$. This completes the proof that when $B \geq \overline{\gamma}$, $c=\overline{x}$ is a symmetric equilibrium cutoff.

Suppose $B=\overline{\gamma}$. Fix $c=\overline{x}$. So, $U^A(x,\overline{x})=U^R(x,\overline{x})$ at $x=\overline{x}$. Now, from (3.1) and (3.3) ,

$$\frac{\partial U^A(x,\overline{x})}{\partial x}|_{x=\overline{x}}\,(=v_{11}) = \frac{\partial U^R(x,\overline{x})}{\partial x}|_{x=\overline{x}}\,(=v_{11})\,. \tag{3.6}$$

For a sufficiently small $\varepsilon > 0$, since $U^A(\overline{x},\overline{x}) = U^R(\overline{x},\overline{x})$, (3.6) implies that $U^A(\overline{x}-\varepsilon,\overline{x}) = U^R(\overline{x}-\varepsilon,\overline{x})$. Then from (3.1) and (3.3) ,

$$\frac{\partial U^A(x,\overline{x})}{\partial x}|_{x=\overline{x}-\varepsilon} > \frac{\partial U^R(x,\overline{x})}{\partial x}|_{x=\overline{x}-\varepsilon}$$

as $v_{11}Q(F(\overline{x})) > v_{11}F(\overline{x}-\varepsilon)^{n-1}$. So, there exist some $x' < \overline{x}-\varepsilon$ for which

$$U^A(x',\overline{x}) < U^R(x',\overline{x}). \tag{3.7}$$

If $U^A(x,\overline{x})$ and $U^R(x,\overline{x})$ intersect at some $x'' \in [\underline{x},\overline{x})$, it implies from (3.1) and (3.3) that

$$\frac{\partial U^A(x,\overline{x})}{\partial x}|_{x=x''} > \frac{\partial U^R(x,\overline{x})}{\partial x}|_{x=x''} \tag{3.8}$$

which contradicts (3.7) .

Now, suppose that $B > \overline{\gamma}$. Fix $c = \overline{x}$. So, $U^A(\overline{x},\overline{x}) < U^R(\overline{x},\overline{x})$. If $U^A(x,\overline{x})$ and $U^R(x,\overline{x})$ intersect at some $x'' \in [\underline{x},\overline{x})$, it implies (3.8) which contradicts $U^A(\overline{x},\overline{x}) < U^R(\overline{x},\overline{x})$.

So, when $B \geq \overline{\gamma}$ and $c = \overline{x}$, $U^A(x,c)$ and $U^R(x,c)$ do not intersect at any $x'' \in [\underline{x},\overline{x})$, and $U^A(x,c)$ lies below $U^R(x,c)$. ■

Proposition 2: *For $n = 2$, $x \sim U[0,1]$ and $\alpha = 0$, the equilibrium cutoff $c_0^* \in (0,1)$ corresponding to $B \in (\frac{1}{4}, \frac{1}{2})$ (as defined in Proposition 1) is unique.*

Proof: Suppose that $B \in (\underline{\gamma}_0, \overline{\gamma}_0)$ where $\underline{\gamma}_0 = \frac{1}{4}$ and $\overline{\gamma}_0 = \frac{1}{2}$. According to Proposition 1 there exist some $c \in (0,1)$ that satisfies $U^A(c,c) = U^R(c,c)$. I show that this c is unique.

With $n = 2$, $x \sim U[0,1]$ and $\alpha = 0$, from (1.4) and (1.5)

$$
\begin{aligned}
U^A(x,c) &= \int_0^c u\left(\frac{x + x_2}{2} - B\right) dF(x_2) + \left(\frac{1}{2}\right) \int_c^1 u\left(\frac{x + x_2}{2} - B\right) dF(x_2) \\
&= \int_0^c \left(\frac{x + x_2}{2} - B\right) dx_2 + \left(\frac{1}{2}\right) \int_c^1 \left(\frac{x + x_2}{2} - B\right) dx_2 \\
&= \frac{xc}{4} + \frac{c^2}{8} - \frac{Bc}{2} + \frac{x}{4} + \frac{1}{8} - \frac{B}{2}, \\
U^R(x,c) &= \int_0^{\min\{x,c\}} u\left(\frac{x + x_2}{2} - x_2\right) dF(x_2) \\
&= \int_0^{\min\{x,c\}} \left(\frac{x + x_2}{2} - x_2\right) dx_2 \\
&= \frac{x}{2} \min\{x,c\} - \frac{1}{4} \left(\min\{x,c\}\right)^2.
\end{aligned}
$$

Now, setting $U^A(c,c) = U^R(c,c)$ gives the equilibrium cutoff as $c_0^* = (4B - 1) \in (0,1)$ for any $B \in (\frac{1}{4}, \frac{1}{2})$. Clearly, for any $B \in (\frac{1}{4}, \frac{1}{2})$ there is one and only one c_0^*. ■

Proposition 3: *For $n = 2$, $x \sim U[0,1]$ and $\alpha > 0$, the equilibrium cutoff $c^*_\alpha \in (0,1)$ corresponding to $B \in \left(\frac{1}{\alpha}\ln\left[\frac{\alpha e^{\frac{\alpha}{2}}}{2e^{\frac{\alpha}{2}}-2}\right], \frac{1}{2}\right)$ (as defined in Proposition 1) is unique.*

Proof: Suppose that $B \in (\underline{\gamma}_\alpha, \overline{\gamma}_\alpha)$ where $\underline{\gamma}_\alpha = \frac{1}{\alpha}\ln\left[\frac{\alpha e^{\frac{\alpha}{2}}}{2e^{\frac{\alpha}{2}}-2}\right]$ and $\overline{\gamma}_\alpha = \frac{1}{2}$. According to Proposition 1 there exist some $c \in (0,1)$ that satisfies $U^A(c,c) = U^R(c,c)$. I want to show that the solution to this equation is unique.

With the simplifying assumptions of $n = 2$ and $x \sim U[0,1]$, from (1.4) and (1.5)

$$
\begin{aligned}
U^A(x,c) &= \int_0^c u\left(\frac{x+x_2}{2} - B\right) dF(x_2) + \left(\frac{1}{2}\right)\int_c^1 u\left(\frac{x+x_2}{2} - B\right) dF(x_2) \\
&= \int_0^c \frac{1}{\alpha}\left(1 - e^{-\alpha(\frac{x+x_2}{2}-B)}\right) dx_2 + \left(\frac{1}{2}\right)\int_c^1 \frac{1}{\alpha}\left(1 - e^{-\alpha(\frac{x+x_2}{2}-B)}\right) dx_2 \\
&= \frac{1}{\alpha}\left(\frac{1}{2} + \frac{c}{2} + \frac{1}{\alpha}e^{-\alpha(\frac{x+c}{2}-B)} - \frac{2}{\alpha}e^{-\alpha(\frac{x}{2}-B)} + \frac{1}{\alpha}e^{-\alpha(\frac{x+1}{2}-B)}\right), \\
U^R(x,c) &= \int_0^{\min\{x,c\}} u\left(\frac{x+x_2}{2} - x_2\right) dF(x_2) \\
&= \int_0^{\min\{x,c\}} \frac{1}{\alpha}\left(1 - e^{-\alpha(\frac{x+x_2}{2}-x_2)}\right) dx_2 \\
&= \frac{1}{\alpha}\left(\min\{x,c\} - \frac{2}{\alpha}e^{(-\frac{\alpha x}{2}+\frac{\alpha}{2}\min\{x,c\})} + \frac{2}{\alpha}e^{-\frac{\alpha x}{2}}\right).
\end{aligned}
$$

Now, setting $U^A(c,c) = U^R(c,c)$ does not give c explicitly as a function of B. Instead, it is possible to express B as a function of c as follows:

$$
B = \frac{1}{\alpha}\ln\left[\frac{[2+\frac{\alpha}{2}-\frac{\alpha c}{2}]e^{\alpha c} - 2e^{\frac{\alpha c}{2}}}{2e^{\frac{\alpha c}{2}} - e^{(\frac{\alpha c}{2}-\frac{\alpha}{2})} - 1}\right].
$$

Let's denote the RHS of the above equation by $\beta(c)$. Now, to show that for any $B \in (\underline{\gamma}_\alpha, \overline{\gamma}_\alpha)$ there is a unique $c \in (0,1)$ that satisfies the equation $B = \beta(c)$, I prove that there exists an inverse to the function $\beta(c)$ such that $c = \beta^{-1}(B) \in (0,1)$ for any $B \in (\underline{\gamma}_\alpha, \overline{\gamma}_\alpha)$. For this, I need to show that $\beta(c)$ is well-defined and monotone increasing or decreasing in $c \in (0,1)$. I proceed in three steps as follows.

(i) Let's define $\lambda(c)$ as follows:

$$\lambda(c) = \frac{[2 + \frac{\alpha}{2} - \frac{\alpha c}{2}]e^{\alpha c} - 2e^{\frac{\alpha c}{2}}}{2e^{\frac{\alpha c}{2}} - e^{(\frac{\alpha c}{2} - \frac{\alpha}{2})} - 1}.$$

Differentiating $\lambda(c)$ with respect to c gives:

$$\lambda'(c) = \frac{\alpha[(4 + 2\alpha - 2\alpha c)e^{\frac{3\alpha c}{2}} + (\alpha c - \alpha - 2)e^{\frac{3\alpha c}{2} - \frac{\alpha}{2}} + (2\alpha c - 2\alpha - 6)e^{\alpha c} + 4e^{\frac{\alpha c}{2}}]}{4[2e^{\frac{\alpha c}{2}} - e^{(\frac{\alpha c}{2} - \frac{\alpha}{2})} - 1]^2}.$$

Now, to show that $\lambda'(c) > 0$, I need to show that both the numerator and the denominator of the above expression have the same sign. Given $\alpha > 0$, the denominator is positive when $c \geq 0$. Let's denote the numerator by $\alpha\eta(c)$, so that

$$\eta(c) = (4 + 2\alpha - 2\alpha c)e^{\frac{3\alpha c}{2}} + (\alpha c - \alpha - 2)e^{\frac{3\alpha c}{2} - \frac{\alpha}{2}} + (2\alpha c - 2\alpha - 6)e^{\alpha c} + 4e^{\frac{\alpha c}{2}}$$

and, differentiating $\eta(c)$ with respect to c gives:

$$\begin{aligned} \eta'(c) \;&=\; 4\alpha e^{\frac{3\alpha c}{2}} + 3\alpha^2 e^{\frac{3\alpha c}{2}} - 3\alpha^2 c e^{\frac{3\alpha c}{2}} - 2\alpha e^{\frac{3\alpha c}{2} - \frac{\alpha}{2}} + \frac{3}{2}\alpha^2 c e^{\frac{3\alpha c}{2} - \frac{\alpha}{2}} - \frac{3}{2}\alpha^2 e^{\frac{3\alpha c}{2} - \frac{\alpha}{2}} \\ &\quad -4\alpha e^{\alpha c} + 2\alpha^2 c e^{\alpha c} - 2\alpha^2 e^{\alpha c} + 2\alpha e^{\frac{\alpha c}{2}} \end{aligned}$$

and, differentiating $\eta'(c)$ with respect to c gives:

$$\begin{aligned} \eta''(c) \;&=\; 3\alpha^2 e^{\frac{3\alpha c}{2}} + \frac{9}{2}\alpha^3 e^{\frac{3\alpha c}{2}} - \frac{9}{2}\alpha^3 c e^{\frac{3\alpha c}{2}} - \frac{3}{2}\alpha^2 e^{\frac{3\alpha c}{2} - \frac{\alpha}{2}} + \frac{9}{4}\alpha^3 c e^{\frac{3\alpha c}{2} - \frac{\alpha}{2}} - \frac{9}{4}\alpha^3 e^{\frac{3\alpha c}{2} - \frac{\alpha}{2}} \\ &\quad -2\alpha^2 e^{\alpha c} + 2\alpha^3 c e^{\alpha c} - 2\alpha^3 e^{\alpha c} + \alpha^2 e^{\frac{\alpha c}{2}} \end{aligned}$$

and, differentiating $\eta''(c)$ with respect to c gives:

$$\begin{aligned}
\eta'''(c) &= \frac{27}{4}\alpha^4 e^{\frac{3\alpha c}{2}}(1-c) + \frac{27}{8}\alpha^4 e^{\frac{3\alpha c}{2}-\frac{\alpha}{2}}(c-1) + 2\alpha^4 e^{\alpha c}(c-1) + \frac{1}{2}\alpha^3 e^{\frac{\alpha c}{2}} \\
&= \frac{27}{4}\alpha^4(1-c)(e^{\frac{3\alpha c}{2}} - \frac{1}{2}e^{\frac{3\alpha c}{2}-\frac{\alpha}{2}} - \frac{8}{27}e^{\alpha c}) + \frac{1}{2}\alpha^3 e^{\frac{\alpha c}{2}} \\
&= \frac{27}{4}\alpha^4(1-c)(\frac{1}{2}e^{\frac{3\alpha c}{2}} - \frac{1}{2}e^{\frac{3\alpha c}{2}-\frac{\alpha}{2}} + \frac{1}{2}e^{\frac{3\alpha c}{2}} - \frac{8}{27}e^{\alpha c}) + \frac{1}{2}\alpha^3 e^{\frac{\alpha c}{2}} \\
&= \frac{27}{4}\alpha^4(1-c)\left[\frac{1}{2}(e^{\frac{3\alpha c}{2}} - e^{\frac{3\alpha c}{2}-\frac{\alpha}{2}}) + \frac{1}{54}(27e^{\frac{3\alpha c}{2}} - 16e^{\alpha c})\right] + \frac{1}{2}\alpha^3 e^{\frac{\alpha c}{2}}.
\end{aligned}$$

Now, given $\alpha > 0$ and $c \geq 0$, it has to be the case that $e^{\frac{3\alpha c}{2}} > e^{\frac{3\alpha c}{2}-\frac{\alpha}{2}}$ and $27e^{\frac{3\alpha c}{2}} > 16e^{\alpha c}$. So, $\eta'''(c) > 0$, which implies that $\eta''(c)$ is a strictly increasing function of $c \geq 0$. Now evaluating $\eta''(c)$ at $c = 0$ gives:

$$\eta''(0) = \frac{5}{2}\alpha^3 - \frac{9}{4}\alpha^3 e^{-\frac{\alpha}{2}} + 2\alpha^2 - \frac{3}{2}\alpha^2 e^{-\frac{\alpha}{2}} = \alpha^3\left(\frac{10}{4} - \frac{9}{4}e^{-\frac{\alpha}{2}}\right) + \alpha^2\left(\frac{4}{2} - \frac{3}{2}e^{-\frac{\alpha}{2}}\right).$$

Since $\alpha > 0$ implies $0 < e^{-\frac{\alpha}{2}} < 1$, it must be that $\eta''(0) > 0$. Now, as $\eta''(c)$ is an increasing function of $c \geq 0$, I have $\eta''(c) > 0$ for $c \in (0,1)$. Then, this implies that $\eta'(c)$ is a strictly increasing function of $c \geq 0$. Now, evaluating $\eta'(c)$ at $c = 0$ gives: $\eta'(0) = 2\alpha(1-e^{-\frac{\alpha}{2}}) + \alpha^2(1-\frac{3}{2}e^{-\frac{\alpha}{2}})$. But, $\frac{d\eta'(0)}{d\alpha} = 2(1-e^{-\frac{\alpha}{2}}) + 2\alpha(1-e^{-\frac{\alpha}{2}}) + \frac{3}{4}\alpha^2 e^{-\frac{\alpha}{2}} \geq 0$ for $\alpha \geq 0$, and $\eta'(0)|_{\alpha=0} = 0$, which imply that $\eta'(0) > 0$ for $\alpha > 0$. Since I have already shown that $\eta'(c)$ is an increasing function of $c \geq 0$, I can say that $\eta'(c) > 0$ for $c \geq 0$ and $\alpha > 0$. Then, this implies that $\eta(c)$ is an increasing function of $c \geq 0$. Now, evaluating $\eta(c)$ at $c = 0$ gives: $\eta(0) = 2 - (2+\alpha)e^{-\frac{\alpha}{2}}$. But, $\frac{d\eta(0)}{d\alpha} = \alpha e^{-\frac{\alpha}{2}} \geq 0$ for $\alpha \geq 0$, and $\eta(0)|_{\alpha=0} = 0$, which imply that $\eta(0) > 0$ for $\alpha > 0$. Since $\eta(c)$ is an increasing function of c, I can say that $\eta(c) > 0$ or $\alpha\eta(c) > 0$ for $c \geq 0$ and $\alpha > 0$. So both the numerator and denominator of $\lambda'(c)$ are positive. So, $\lambda'(c) > 0$ for $c \geq 0$ and $\alpha > 0$, which implies that, when $\alpha > 0$, $\lambda(c)$ is a strictly increasing function of $c \geq 0$.

(ii) In this step I show that $\beta(c)$ is a strictly increasing function of $c \in (0,1)$.

Note that $\beta(c) = \frac{1}{\alpha}\ln[\lambda(c)]$ and $\beta'(c) = \frac{1}{\alpha\lambda(c)}\lambda'(c)$. Now, $\lambda(0) = (\alpha/2)/(1 - e^{-\frac{\alpha}{2}})$. Clearly, for $\alpha > 0$, $\lambda(0) > 0$. Since, given $\alpha > 0$, $\lambda(c)$ is an increasing function of $c \geq 0$, I can say that $\lambda(c) > 0$ for $c \in (0, 1)$ and $\alpha > 0$. As a result, $\beta'(c) > 0$ for $c \in (0, 1)$ and $\alpha > 0$.

(iii) Now I show that $\beta(c) = \frac{1}{\alpha}\ln[\lambda(c)]$ is well-defined, which boils down to showing that $\lambda(c)$ is positive for $c \in (0, 1)$. I have already shown this in (ii).

So, based on the above three steps I can say that $\beta(c)$ is well-defined and monotone increasing in $c \in (0, 1)$. Moreover, $\beta(0) = \frac{1}{\alpha}\ln\left[\frac{\alpha e^{\frac{\alpha}{2}}}{2e^{\frac{\alpha}{2}}-2}\right]$ and $\beta(1) = \frac{1}{2}$. So, there exists an inverse to the function $\beta(c)$ such that $c = \beta^{-1}(B) \in (0, 1)$ for $B \in \left(\frac{1}{\alpha}\ln\left[\frac{\alpha e^{\frac{\alpha}{2}}}{2e^{\frac{\alpha}{2}}-2}\right], \frac{1}{2}\right)$, and this inverse function will be monotone increasing in B. So, there is only one $c \in (0, 1)$ that satisfies $U^A(c, c) = U^R(c, c)$ for a $B \in \left(\frac{1}{\alpha}\ln\left[\frac{\alpha e^{\frac{\alpha}{2}}}{2e^{\frac{\alpha}{2}}-2}\right], \frac{1}{2}\right)$. ■

Proposition 4: *For $n = 2$, $x \sim U[0,1]$, and $\alpha = 0$, an auction with a buy price $B < \overline{\gamma}_0 = \frac{1}{2}$ generates a smaller expected seller revenue than an auction without any buy price, that is, for a risk neutral seller $\widetilde{U}_{BP} < \widetilde{U}_{WBP} = \frac{1}{3}$.*

Proof: With the assumption of $n = 2$ and $x \sim U[0,1]$, (1.3) gives $b_o(x) = x$. So, from (1.14) I find $\widetilde{U}_{WBP} = E[b_0(x_{(1)})] = E[x_{(1)}] = \frac{1}{3}$. Now, I want to show that $\widetilde{U}_{BP} \leq \frac{1}{3}$ for any B when $\alpha = 0$. There are three cases to consider:

(1) When $B < \frac{1}{4}$, both the bidders accept it and the seller revenue is B. But $B < \frac{1}{4} < \frac{1}{3}$. So the expected seller revenue is smaller with B.

(2) If $B > \frac{1}{2}$, none of the bidders accepts it and the auction proceeds to the second stage. So, in this case the expected seller revenue is the same with or without B.

(3) But, when $B \in (\frac{1}{4}, \frac{1}{2})$, as I have shown, there exists an equilibrium cutoff $c_0^* \in (0,1)$. But, if both the bidders' signals are less than or equal to c_0^*, both the auction formats generate the same revenue which is $x_{(1)}$. The difference occurs when B is accepted. This happens when at least one of the bidders' signal is higher than c_0^*, that is, $x_{(2)} > c_0^*$. So, the difference between the expected seller revenue in auctions with and without B is given by:

$$\begin{aligned}
\triangle\Pi &= \int_{c_0^*}^{1} \left[\int_0^r (B - q).\widetilde{f}(q,r) dq \right] dr \\
&= B - Bc_0^{*2} - \frac{1}{3} + \frac{1}{3} c_0^{*3} \\
&= B - B(4B-1)^2 - \frac{1}{3} + \frac{1}{3}(4B-1)^3
\end{aligned}$$

where the second line utilizes that $\widetilde{f}(q,r) = 2$ and the last line utilizes that $c_0^* = (4B-1)$.[12]

It is easy to check that setting $\triangle\Pi = 0$ implies $B = \frac{1}{2}$, and $\frac{d(\triangle\Pi)}{dB} = 4(2B-1)^2 > 0$

[12] This is derived in the proof of Proposition 2.

for $B \neq \frac{1}{2}$ and $\frac{d(\triangle\Pi)}{dB} = 4(2B-1)^2 = 0$ for $B = \frac{1}{2}$. Then, clearly, $\triangle\Pi < 0$ for $B \in (\frac{1}{4}, \frac{1}{2})$. That is, for any $B \in (\frac{1}{4}, \frac{1}{2})$, the buy price B lowers seller revenue. $\blacksquare$

Proposition 5: *For $n = 2$ (both risk neutral or risk averse) and $x \sim U[0,1]$, there exists some buy price $B < \overline{\gamma} = \frac{1}{2}$ for which an auction with a buy price generates a higher expected utility for the seller than an auction without any buy price, that is, $\widetilde{U}_{BP} > \widetilde{U}_{WBP}$.*

Proof: For $n = 2$ (both risk neutral or risk averse) and $x \sim U[0,1]$, Proposition 1 states that $c^* = 1$ when $B \geq \frac{1}{2}$ and $c^* < 1$ when $B < \frac{1}{2}$. Since $\widetilde{U}_{BP}(1) = \widetilde{U}_{WBP}$ it will be sufficient to show that $\left.\frac{d\widetilde{U}_{BP}(c^*)}{dc^*}\right|_{c^*=1} < 0$ to complete this proof.

$n = 2$ and $v = v(x_1, x_2) = \frac{x_1+x_2}{2}$ implies that $b(x) = x$ (from (1.2) and (1.3)), and $n = 2$ and $x \sim U[0,1]$ implies that the joint density function of the highest and second highest signal draws is $\widetilde{f}(q,r) = 2$. Now, (1.15) can be simplified to

$$\begin{aligned}
\widetilde{U}_{BP}(c^*) &= \int_0^{c^*}\int_0^r \widetilde{u}\,(q)\,2dqdr + \int_{c^*}^1\int_0^r \widetilde{u}\,(\beta(c^*))\,2dqdr \\
&= \int_q^{c^*}\int_0^{c^*} \widetilde{u}\,(q)\,2dqdr + \int_{c^*}^1\int_0^r \widetilde{u}\,(\beta(c^*))\,2dqdr \\
&= 2\int_0^{c^*} \widetilde{u}\,(q)\,(c-q)dq + 2\widetilde{u}\,(\beta(c^*))\,(1-c^{*2}).
\end{aligned}$$

Now, $\frac{d\widetilde{U}_{BP}(c^*)}{dc^*} = 2\int_0^{c^*}\widetilde{u}\,(q)\,dq + \widetilde{u}'\,(\beta(c^*))\,\beta'(c^*)(1-c^{*2}) - 2\widetilde{u}\,(\beta(c^*))\,c^*$.

So,

$$\begin{aligned}
\left.\frac{d\widetilde{U}_{BP}(c^*)}{dc^*}\right|_{c^*=1} &= 2\int_0^1 \widetilde{u}\,(q)\,dq - 2\widetilde{u}\,(\beta(1)) \\
&= 2\left[E[\widetilde{u}\,(x)] - \widetilde{u}\left(\frac{1}{2}\right)\right] \\
&= 2\left[E[\widetilde{u}\,(x)] - \widetilde{u}\,(E[x])\right].
\end{aligned}$$

The second line utilizes that $\beta(1) = \frac{1}{2}$ and the last line utilizes that $E[x] = \frac{1}{2}$. Since $\widetilde{u}(s)$ is a strictly concave function, following Jensen's inequality, $E[\widetilde{u}\,(x)] < \widetilde{u}\,(E[x])$. So, $\left.\frac{d\widetilde{U}_{BP}(c^*)}{dc^*}\right|_{c^*=\overline{x}} < 0$. ■

Appendix B: Proof for Chapter 3

Proof of Proposition 1:

We consider a buy price $P_B \in (r, \overline{P}_{Bt})$, and a bidder arriving at time t with a value $v \geq P_B$. Let n denote the number of bidders who arrive after time t. Now, define $U^A(v,t)$ and $U^R(v,t)$ as

$$U^A(v,t) = u(v - P_B, 0), \tag{3.9}$$

and

$$U^R(v,t) = \sum_{n=0}^{\infty} E\left[u\left(v - \max\{r,y\}, T-t\right)|y \leq v\right] G(v) \frac{(\gamma_t)^n e^{-\gamma_t}}{n!}, \tag{3.10}$$

where y is the highest among n value draws, and $G(.) = F(.)^n$ is the distribution function of y. For $n = 0$, $\max\{r,y\} = r$. Whenever (a) and/or (b) do not hold we have $U^R(v,t) = 0$. Notice that $\lambda(t) = 0$ implies $P_0(t) = 1$ and $P_n(t) = 0$ for $n > 0$, which in turn implies that $U^R(v,t) = u(v-r, T-t)$. On the other hand, $U^R(v,t) \to 0$ as $\lambda(t) \to \infty$.

$\overline{P}_{Bt}$, the buy price that makes a bidder with $v = \overline{v}$ indifferent between accepting and rejecting at time $t \in [0,T)$, solves the following equation:

$$u(\overline{v} - \overline{P}_{Bt}, 0) = \sum_{n=0}^{\infty} E\left[u\left(\overline{v} - \max\{r,y\}, T-t\right)\right] \frac{(\gamma_t)^n e^{-\gamma_t}}{n!}. \tag{3.11}$$

Now, we first prove part (2) and then part (1).

Part (2): We start by showing the existence of some value $c_t^* \in (P_B, \overline{v})$ such that the indifference condition in Proposition 1 holds for a bidder with $v = c_t^*$. This is a necessary condition for c_t^* to be in equilibrium. To show this, it is sufficient to show that $U^A(v,t)$ and $U^R(v,t)$ intersect at some $v \in (P_B, \overline{v})$. Then we can set c_t^* equal to this v.

Clearly, $U^A(P_B, t) = 0$, and $v > r$ implies that $U^R(P_B, t) > 0$. So, $U^R(P_B, t) > U^A(P_B, t)$. On the other hand, $P_B < \overline{P}_{Bt}$ implies that

$$\begin{aligned} u(\overline{v} - P_B, T - t) &> u(\overline{v} - \overline{P}_{Bt}, T - t) = U^R(\overline{v}, t); \\ U^A(\overline{v}, t) &> U^R(\overline{v}, t). \end{aligned}$$

Then, since $U^A(v, t)$ and $U^R(v, t)$ are continuous, it must be the case that there is some value $v \in (P_B, \overline{v})$ such that $U^A(v, t) = U^R(v, t)$. We set c_t^* equal to this value.

Now, we show that the value c_t^* satisfying $U^A(c_t^*, t) = U^R(c_t^*, t)$ is unique and actually an equilibrium cutoff. We accomplish both by showing that $U^A(v, t)$ is steeper than $U^R(v, t)$ at any v where $U^A(v, t) = U^R(v, t)$. Lets start by deriving the slope equations for $U^A(v, t)$ and $U^R(v, t)$ as follows:

$$\begin{aligned} \frac{\partial U^A(v, t)}{\partial v} &= u_1(v - P_B, 0) \\ &= \delta^0 - \alpha u(v - P_B, 0) \\ &= 1 - \alpha U^A(v, t) \end{aligned}$$

where the second line utilizes the relationship $u_1(., x) = \delta^x - \alpha u(., x)$ which holds for CARA utility function, and using Leibniz's rule,

$$
\begin{aligned}
&\frac{\partial U^R(v,t)}{\partial v} \\
= \; & \frac{\partial}{\partial v}\left[u(v-r,T-t)e^{-\gamma_t} + \sum_{n=1}^{\infty}\left[u(v-r,T-t)G(r)+\right.\right. \\
& \left.\left.\int_r^v u(v-y,T-t)dG(y)\right]\frac{(\gamma_t)^n e^{-\gamma_t}}{n!}\right] \\
= \; & u_1(v-r,T-t)e^{-\gamma_t} + \sum_{n=1}^{\infty}\left[u_1(v-r,T-t)G(r)\frac{(\gamma_t)^n e^{-\gamma_t}}{n!}\right. \\
& \left. + \frac{(\gamma_t)^n e^{-\gamma_t}}{n!}\left[\int_r^v u_1(v-y,T-t)dG(y) + u(v-v,T-t)\right]\right] \\
= \; & \left[\delta^{T-t} - \alpha u(v-r,T-t)\right]e^{-\gamma_t} + \sum_{n=1}^{\infty}\left[\left(\delta^{T-t} - \alpha u(v-r,T-t)\right)G(r)\frac{(\gamma_t)^n e^{-\gamma_t}}{n!}\right. \\
& \left. + \frac{(\gamma_t)^n e^{-\gamma_t}}{n!}\left[\int_r^v \left(\delta^{T-t} - \alpha u(v-y,T-t)\right)dG(y)\right]\right] \\
= \; & \delta^{T-t}e^{-\gamma_t} + \delta^{T-t}\sum_{n=1}^{\infty}\left[G(r)+G(v)-G(r)\right]\frac{(\gamma_t)^n e^{-\gamma_t}}{n!} \\
& -\alpha\left[u(v-r,T-t)e^{-\gamma_t} + \sum_{n=1}^{\infty}\left[u(v-r,T-t)G(r)+\right.\right. \\
& \left.\left.\int_r^v u(v-y,T-t)dG(y)\right]\frac{(\gamma_t)^n e^{-\gamma_t}}{n!}\right] \\
= \; & \delta^{T-t}\left[e^{-\gamma_t} + \sum_{n=1}^{\infty} G(v)\frac{(\gamma_t)^n e^{-\gamma_t}}{n!}\right] - \alpha U^R(v,t)
\end{aligned}
$$

where the second line utilizes the relationship $u_1(.,x) = \delta^x - \alpha u(.,x)$. Notice that for $v < \overline{v}$,

$$
\begin{aligned}
e^{-\gamma_t} + \sum_{n=1}^{\infty} G(v)\frac{(\gamma_t)^n e^{-\gamma_t}}{n!} &< e^{-\gamma_t} + \sum_{n=1}^{\infty}\frac{(\gamma_t)^n e^{-\gamma_t}}{n!} \\
&= \sum_{n=0}^{\infty}\frac{(\gamma_t)^n e^{-\gamma_t}}{n!} \\
&= 1.
\end{aligned}
$$

The above inequality implies that,

$$\frac{\partial U^A(v,t)}{\partial v} > \frac{\partial U^R(v,t)}{\partial v} \text{ when } U^A(v,t) = U^R(v,t). \tag{3.12}$$

As a result, $U^A(v,t)$ is steeper than $U^R(v,t)$ at $v = c_t^*$. So, $U^A(v,t)$ lies below $U^R(v,t)$ for $v \in [\underline{v}, c_t^*)$ and $U^A(v,t)$ lies above $U^R(v,t)$ for $v \in (c_t^*, \overline{v}]$. This completes the proof that c_t^* is an equilibrium cutoff and is unique.

Now we prove part 1 of Proposition 2.

Part (1): We have two cases to consider.

(1) When $P_B = \overline{P}_{Bt}$: By definition of $\overline{P}_{Bt}$ in equation (maxb), when $P_B = \overline{P}_{Bt}$, $U^A(\overline{v},t) = U^R(\overline{v},t)$. Since, from (slope$_reject B$)

$$\left.\frac{\partial U^R(v,t)}{\partial v}\right|_{v=\overline{v}} = \delta^{T-t} \sum_{n=0}^{\infty} \frac{(\gamma_t)^n e^{-\gamma_t}}{n!} - \alpha U^R(\overline{v},t) = \delta^{T-t} - \alpha U^R(\overline{v},t)$$

it is the case that, when $\delta = 1$,

$$\left.\frac{\partial U^A(v,t)}{\partial v}\right|_{v=\overline{v}} = \left.\frac{\partial U^R(v,t)}{\partial v}\right|_{v=\overline{v}}.$$

Then it has to be that $U^A(v,t)$ lies completely below $U^R(v,t)$ over $[\underline{v}, \overline{v})$, and $U^A(v,t) = U^R(v,t)$ at $v = \overline{v}$. Otherwise, for $P_B < \overline{P}_{Bt}$ or $P_B > \overline{P}_{Bt}$, $U^A(\overline{v},t)$ and $U^R(\overline{v},t)$ would intersect twice over $[\underline{v}, \overline{v})$ which we know cannot happen (from part 2 above). This is because $U^A(\overline{v},t)$ is steeper than $U^R(\overline{v},t)$ at any such intersection. So, $c_t^* = \overline{v}$ when $P_B = \overline{P}_{Bt}$.

Now, when $\delta \in (0,1)$

$$\left.\frac{\partial U^A(v,t)}{\partial v}\right|_{v=\overline{v}} > \left.\frac{\partial U^R(v,t)}{\partial v}\right|_{v=\overline{v}}.$$

As a result, $U^A(v,t)$ will lie completely below $U^R(v,t)$ over $[\underline{v}, \overline{v})$, and $U^A(v,t) = U^R(v,t)$ at $v = \overline{v}$. Otherwise, if they intersect at some $v \in (\underline{v}, \overline{v})$, it will imply that $U^R(v,t)$ is steeper than $U^A(v,t)$ which we know cannot happen (from part 2 above).

(2) When $P_B > \overline{P}_{Bt}$: Given what we found above, for $P_B > \overline{P}_{Bt}$, it will be the case that $U^A(v,t)$ lies completely below $U^R(v,t)$ over $[\underline{v}, \overline{v}]$ which is consistent with $c_t^* = \overline{v}$.

Now we show that c_t^*, the solution to $U^A(c_t^*, t) = U^R(c_t^*, t)$, would be decreasing in α and r, and increasing in P_B, t and δ. We start with the relation between c_t^* and α. To explicitly show the dependence of $U^A(v,t)$ and $U^R(v,t)$ on α we write them as $U_\alpha^A(v,t)$ and $U_\alpha^R(v,t)$. Now, define κ_α which solves

$$U_\alpha^R(v,t) = u(v - \kappa_\alpha, T - t) \sum_{n=0}^{\infty} G(v) \frac{(\gamma_t)^n e^{-\gamma_t}}{n!}.$$

Since κ_α is a certainty equivalent payment, κ_α will be increasing in α. A more risk averse bidder prefers accepting the certain payoff on the RHS of the above equation than rejecting P_B and participate in the auction where the payoff is uncertain.

Lets consider two risk aversion levels, α' and α'' where $\alpha' < \alpha''$. Let $c_t^{*\prime}$ and $c_t^{*\prime\prime}$ be the cutoffs corresponding to α' and α'' which solve the indifference condition in Proposition 1. We want to show that $c_t^{*\prime} > c_t^{*\prime\prime}$. From the indifference condition we get

$$\begin{aligned}
& U_{\alpha'}^A(c_t^{*\prime}, t) = U_{\alpha'}^R(c_t^{*\prime}, t) \\
\Rightarrow \quad & \frac{1 - e^{-\alpha'(c_t^{*\prime} - P_B)}}{\alpha'} = \delta^{T-t} \frac{1 - e^{-\alpha'(c_t^{*\prime} - \kappa_{\alpha'}(c_t^{*\prime}))}}{\alpha'} \sum_{n=0}^{\infty} G(v) \frac{(\gamma_t)^n e^{-\gamma_t}}{n!} \\
\Rightarrow \quad & \frac{1 - e^{-\alpha'(c_t^{*\prime} - P_B)}}{1 - e^{-\alpha'(c_t^{*\prime} - \kappa_{\alpha'}(c_t^{*\prime}))}} = \delta^{T-t} \sum_{n=0}^{\infty} G(v) \frac{(\gamma_t)^n e^{-\gamma_t}}{n!}.
\end{aligned}$$

Since $\sum_{n=0}^{\infty} G(v) \frac{(\gamma_t)^n e^{-\gamma_t}}{n!} < 1$, we have $c_t^{*\prime} - P_B < c_t^{*\prime} - \kappa_{\alpha'}(c_t^{*\prime})$. For some fixed x and y and $x < y$, we can show that $\frac{1 - e^{-\alpha x}}{1 - e^{-\alpha y}}$ is increasing in α. So, if we let $x = c_t^{*\prime} - P_B$

and $y = c_t^{*\prime} - \kappa_{\alpha'}(c_t^{*\prime})$, we have

$$\begin{aligned}
&\frac{1-e^{-\alpha''(c_t^{*\prime}-P_B)}}{1-e^{-\alpha''(c_t^{*\prime}-\kappa_{\alpha'}(c_t^{*\prime}))}} > \frac{1-e^{-\alpha'(c_t^{*\prime}-P_B)}}{1-e^{-\alpha'(c_t^{*\prime}-\kappa_{\alpha'}(c_t^{*\prime}))}} = \delta^{T-t}\sum_{n=0}^{\infty} G(v)\frac{(\gamma_t)^n e^{-\gamma_t}}{n!} \\
\Rightarrow\ &\frac{1-e^{-\alpha''(c_t^{*\prime}-P_B)}}{1-e^{-\alpha''(c_t^{*\prime}-\kappa_{\alpha'}(c_t^{*\prime}))}} > \delta^{T-t}\sum_{n=0}^{\infty} G(v)\frac{(\gamma_t)^n e^{-\gamma_t}}{n!} \\
\Rightarrow\ &\frac{1-e^{-\alpha''(c_t^{*\prime}-P_B)}}{\alpha''} > \delta^{T-t}\frac{1-e^{-\alpha''(c_t^{*\prime}-\kappa_{\alpha'}(c_t^{*\prime}))}}{\alpha''}\sum_{n=0}^{\infty} G(v)\frac{(\gamma_t)^n e^{-\gamma_t}}{n!}.
\end{aligned}$$

As κ_α is increasing in α, we get

$$\begin{aligned}
&\frac{1-e^{-\alpha''(c_t^{*\prime}-P_B)}}{\alpha''} > \delta^{T-t}\frac{1-e^{-\alpha''(c_t^{*\prime}-\kappa_{\alpha''}(c_t^{*\prime}))}}{\alpha''}\sum_{n=0}^{\infty} G(v)\frac{(\gamma_t)^n e^{-\gamma_t}}{n!} \\
\Rightarrow\ &U^A_{\alpha''}(c_t^{*\prime}, t) > U^R_{\alpha''}(c_t^{*\prime}, t).
\end{aligned}$$

Since $U^A_{\alpha''}(c_t^{*\prime\prime}, t) = U^R_{\alpha''}(c_t^{*\prime\prime}, t)$, $U^A_{\alpha''}(v,t)$ lies above $U^R_{\alpha''}(v,t)$ for $v \in (c_t^{*\prime\prime}, \overline{v}]$, and $U^A_{\alpha''}(v,t)$ lies below $U^R_{\alpha''}(v,t)$ for $v \in [\underline{v}, c_t^{*\prime\prime})$, the above inequality implies that $c_t^{*\prime} \in (c_t^{*\prime\prime}, \overline{v})$. So, c_t^* is decreasing in α.

Since $U^A(v,t)$ is steeper than $U^R(v,t)$ at $v = c_t^*$, and $U^R(v,t)$ is decreasing in r as $\max\{r, y\}$ is increasing in r, the solution to the indifference condition in Proposition 1 is decreasing in r. Similarly, since $U^A(v,t)$ is decreasing in P_B, the solution to the indifference condition in Proposition 1 is increasing in P_B. Again, since $U^R(v,t)$ is increasing in t, the solution to the indifference condition is increasing in t. To see this, notice that $U^R(v,t)$ in (u$_r eject) is decreasing in \gamma_t$, while γ_t is decreasing t. Finally, since $U^R(v,t)$ is increasing in δ, the solution to the indifference condition is increasing in δ.

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