BUILDING ALLIANCES: A PARTNERSHIP BETWEEN A MIDDLE SCHOOL MATHEMATICS TEACHER AND A UNIVERSITY RESEARCHER

by

Anthony M. A. Fernandes

A Dissertation Submitted to the Faculty of the

DEPARTMENT OF MATHEMATICS

In Partial Fulfillment of the Requirements
For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

THE UNIVERSITY OF ARIZONA

2 0 0 7

THE UNIVERSITY OF ARIZONA
GRADUATE COLLEGE

As members of the Dissertation Committee, we certify that we have read the dissertation

prepared by Anthony M.A. Fernandes

entitled BUILDING ALLIANCES: A PARTNERSHIP BETWEEN A MIDDLE SCHOOL MATHEMATICS TEACHER AND A UNIVERSITY RESEARCHER

and recommend that it be accepted as fulfilling the dissertation requirement for the

Degree of Doctor of Philosophy

Peter Wiles
____ Date: June 29, 2007

Marta Civil
____ Date: June 29, 2007

Rebecca McGraw
____ Date: June 29, 2007

Walter Doyle
____ Date: June 29, 2007

Alberto Arenas
____ Date: June 29, 2007

Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copies of the dissertation to the Graduate College.

I hereby certify that I have read this dissertation prepared under my direction and recommend that it be accepted as fulfilling the dissertation requirement.

Peter Wiles
____ Date: June 29, 2007
Dissertation Director: Peter Wiles

STATEMENT BY AUTHOR

SIGNED: Anthony M. A. Fernandes

ACKNOWLEDGEMENTS

Looking back over the years one realizes that all milestones are arrived at with a lot of support from others. I would like to take this opportunity thank my wonderful family, teachers, and friends for their love and sacrifices through the years. I learnt so much and could not have made it without you.

I would like to thank my committee members, Drs. Peter Wiles, Marta Civil, Rebecca McGraw, Walter Doyle, and Alberto Arenas, who have guided me selflessly through the process of my work on this study.

I am grateful to Dr. Peter Wiles, my advisor, committee chair and mentor. His guidance and direction have broadened my horizons and instilled a love for teacher education. He offered unconditional support as I pursued my degree and always raised the bar knowing that I could achieve the goals he set.

I greatly appreciate Dr. Marta Civil for her continual support and sharing through my graduate experience and especially for introducing me to the teacher. Through her guidance I have learnt about other areas in mathematics education and my thinking has grown considerably through this collaboration.

I wish to thank Dr. Rebecca McGraw for her guidance and interesting discussions on the philosophies of mathematics education. I would like to thank Drs. Walter Doyle, and Alberto Arenas for sharing their expertise and guiding me at crucial stages of this study; Dr. Carol Larson for listening and guiding me; and Dr. Leslie Kahn for her patience, listening, and help.

I would like to thank the Center for the Mathematics Education of Latinos/as (CEMELA[1]) for providing me with resources and financial support for two semesters as I collected my data and completed my analysis. A special thanks to Derek Griffith for helping me with the data and patiently taking care of all video editing needs. I am grateful to all the CEMELA members for their comments and feedback during the course of this study. It was a pleasure being a part of your team.

Finally, I am indebted to the wonderful teacher who welcomed me into her classroom and life. She made this study a reality despite all the time pressures that she faced. Her hard work and dedication through the months were an inspiration for me. Her knowledge of the school, teachers, and students was invaluable and shared generously. I would like to thank her for the patience she displayed throughout the study and as a result I have come out as a better person and researcher.

[1] This study was supported through the Center for the Mathematics Education of Latinos/as (CEMELA). CEMELA is a Center for Learning and Teaching supported by the National Science Foundation, grant number ESI-042983. The viewpoints expressed by the author do not necessarily reflect those of the NSF.

DEDICATION

I dedicate this to my family,

Zeffrey and Stella Fernandes

Zita, Donald, Tisha, and Francesco

Benny, Marisa, and Samuela

For their love and support through the years

TABLE OF CONTENTS

TABLE OF CONTENTS – Continued

TABLE OF CONTENTS – Continued

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

This case study examined the evolution of a partnership between a middle school mathematics teacher and a university researcher around discussions on the content and teaching of mathematics. In particular, the study sought to examine the evolution of the partnership, the constraints present for the teacher and researcher, the impact of the partnership on the mathematical and pedagogical issues that arose in planning, teaching, and assessment, and the impact on the tasks that the teacher chose and implemented in the classroom.

Drawing from the literature on collaborations and the emergent perspective, the evolution of the partnership occurred through three stages, determined by the content-teaching tensions. The first stage focused on the mathematics content, with the agenda being set and run by the researcher. The second stage gave rise to the content-teaching tensions as the teacher shifted the discussions from content to a focus on lesson planning and teaching. Tensions were resolved in the third stage with the teacher taking a proactive role in the discussions of lesson design and teaching. The mathematical issues in planning and teaching reflected the shift in the partnership where in the beginning the discussions focused on the mathematical content, later discussions centered on a combination of content, pedagogy, and student thinking. The assessment discussions addressed differences between the language of the curriculum and the district and state tests.

The shift in the partnership can be attributed to the teacher's choice of high level mathematics tasks, the subsequent adoption of a conceptually based mathematics

curriculum and the effective management of the dialectic tensions by both partners. This study illustrated that generating perturbations and effective management of dialectical tensions has the potential for a fruitful collaboration between teachers and researchers.

CHAPTER 1: INTRODUCTION

My study seeks to understand the evolution of a partnership between a middle school mathematics teacher and myself, a researcher, as we had discussions centered on the content and teaching of mathematics. The successful establishment of a discourse community between teachers and researchers has been cited as a crucial piece in recent initiatives of teacher professional development (Putnam & Borko, 2000; Wilson & Berne, 1999). However, knowledge of building and sustaining these communities is more fragmented. The major goal of this study is to examine a teacher-researcher partnership, and to shed light on the larger problem of establishing learning communities consisting of teachers and researchers.

There is a current push for reform mathematics to break away from the traditional "banking" concept of education (Freire, 1970) where the students are viewed as receptacles that need to be "filled" by the teacher via showing and telling. After making "deposits" the teacher then makes "withdrawals" from the students. Generally, in this model, the knowledge is far from the experiences of the student, but considered important by an outside authority.

Recently (NCTM, 2000) there has been a push to adopt ideas based on constructivist philosophy (von Glasersfeld, 1989), where the students actively engage in the construction of their own knowledge (Goldsmith & Schifter, 1997; NCTM, 2000). This construction is facilitated by engaging the students in challenging problems, inventing procedures, justifying the validity of these procedures and communicating these ideas to peers (NCTM, 2000; Simon, 1997). The teacher also negotiates a classroom culture that encourages students to discuss and build on each others ideas and collectively work to understand the mathematics. Thus teaching based on constructivist principles requires a radical change from the way teachers currently teach (Cohen & Ball, 1990; National Research Council, 2001). This can be facilitated by understanding both the content and the connections between mathematical concepts, and using this to build on students' emerging ideas in the classroom (NCTM, 2000; NCTM, 2006). According to the recent Curriculum Focal Points,

> To achieve the best results with students when teaching for the depth, understanding and proficiency sought by the curriculum focal points, teachers themselves will need a deep understanding of the mathematics and facility with the relationships among mathematical ideas. (NCTM, 2006, p. 7)

This deep and flexible understanding of the mathematics, as described above, is beyond what many teachers experienced in their pre-service education courses (Ball, 2000a; NCTM, 2000) and hence this makes long term ongoing professional development essential. There have been a number of models of ongoing professional development that focused on teacher knowledge (Birchak et al., 1998; Cohen, 2004), instruction (Fernandez, 2002), instruction and student thinking (Cobb, Wood, & Yackel, 1990), and student thinking (Carpenter & Fennema, 1992; Fennema et al., 1996). All these examples of professional development involved prolonged interactions between teachers and researchers. Reports of these professional development projects focused on the content of the professional development and the impact that this had on various aspects of the teachers practice, knowledge, and beliefs. For example, in the professional development project of Cognitively Guided Instruction (Carpenter & Fennema, 1992; Fennema et al., 1996) the researchers helped the teachers to understand the relationship between the research based model of students thinking and the thinking that was displayed by the teacher's current students. The teachers reflected on these relationships together with the researchers and incorporated this in their instruction. In most of the examples of ongoing professional development models given above, the importance of collegiality and collaboration are underlined as factors that promoted the professional development. Further, the formation of learning communities was emphasized as a factor that was important for sustaining the professional development over time. However, reports about these studies lack an in-depth examination of the collaboration between the researcher(s) and teacher(s) and how this shifted over time. In my study I intend to examine the simplest case of collaboration between a middle school mathematics teacher and myself, a university researcher, as I provide ongoing professional development through discussions of the mathematics content and teaching. I will examine how this ongoing professional development partnership evolves over time. In this exploratory qualitative case study (Merriam, 1988), I seek to answer the following main question:

What is the nature of a partnership between a Middle School teacher and a researcher centered on conversations about the mathematics content and teaching?

This main question can be sub-divided into additional questions:

1. How does the nature of the partnership evolve over time? What are the constraints on the teacher and the researcher in such a partnership?
2. How does the partnership contribute to mathematical and pedagogical issues that arise in planning, teaching, and assessment?

3. How does the partnership influence the curriculum with respect to the tasks the teacher chooses to use in the classroom and how the tasks are utilized by the teacher?

There are diverse meanings to the term “partnership”, such as symbolic arrangements, service agreements, information-sharing systems, and collaborations between equal partners. In this study, “partnership” will refer to symbiotic partnerships which, according to Goodland (1988), involved dissimilar partners coming together to help each other achieve their goals.

Symbiotic partnerships were used extensively by Goodlad (1988) in reference to large school university partnerships. Goodlad (1988) envisioned symbiotic partnerships as a way to stop the stagnation and deterioration in the universities and schools. These partnerships facilitated a continuous infusion of new ideas and knowledge for both institutions and contributed to their educational renewal. Even though I am looking at a small scale partnership between a teacher and researcher, this collaboration satisfied the conditions outlined for a symbiotic partnership of being from diverse backgrounds and coming together to mutually satisfy our goals. In the case of the teacher, learning mathematics was a primary goal and my goal was to study the evolution of our partnership as we shared ideas around the content and teaching. The next chapter focuses on the review of the literature and theoretical framework that guided my study.

CHAPTER 2: CONCEPTUAL FRAMEWORK

This chapter is divided into two major sections. The first section reviews the literature and the second examines my theoretical perspective. The literature review covers the major areas that informed my collaboration and interactions with Linda (the teacher). The theoretical framework examines my perspective on the learning and teaching of mathematics and it also informs my data collection and analysis.

Review of Selected Literature

My literature review consists of three major sections that have influenced the partnership. The first section examines collaborations and partnerships and I drew on this literature to inform my partnership with Linda. There was a dearth of literature in the area of one-on-one collaborations in mathematics education that focused on the partnership between a school teacher and a university researcher. Thus my focus expanded to include areas where there were numerous interactions between school and university personnel, namely collaborative action research and teacher study groups. I examine each area and focus on the important collaborative aspects like the roles of the researchers and teachers in the partnership and the possible tensions that could arise in the relationship. The second section examines the literature of tasks and the use of the *Mathematics Task Framework* (MTF) (Stein, Smith, Henningsen, & Silver, 2000) as a lens for the teacher and as a mediating factor in our interactions (McGraw, Lynch, Koc, Kapusuz, & Brown, 2007). Finally, the third section focuses on the mathematics knowledge that teachers need to teach effectively. The second and third sections informed the work that Linda and I did as part of our partnership.

Collaborations & Partnerships

Collaborations between schools and universities have been around from 1892, the first being the Committee of Ten which was a meeting between college and school teachers that discussed the curriculum, teaching, and the preparation of teachers. Thereafter collaborations have evolved into a number of different entities and in general there were two broad categories cited in the literature. The first examined collaborations by their structural aspects and the second category examined collaborations via relationship between the partners. Handler and Ravid (2001) compiled a summary of the various structural partnerships that could be set up, namely, Professional Development Schools (PDS), consultation models, one-on-one collaborations and umbrella models. Whitford, Schlechty, and Shelor (1987) summarized the various relationships that defined collaborations namely, cooperative, symbiotic, and organic. I will briefly touch upon the structural and relationship models that have been reported in collaborations.

Handler and Ravid (2001) outlined the four structural models for collaboration between institutions and individuals. The PDS sought to simultaneously address the needs of the university and those of the schools. In the case of the university this could be student teaching of pre-service teachers with a mentor teacher in the school and for the school it could be professional development of in-service teachers. Both the university and the school benefited through this exchange. The second collaborative structure was the consultation model where one or more university members worked with one or more teachers in training teachers (e.g. in the use of a new technology) or procured their help (e.g. to test a new curriculum in the teachers classroom). The third type of collaborative structure was the one-on-one collaboration between individuals from the university and schools. These collaborations were popular due to their ease in set-up and they benefited both parties. Finally, Umbrella models consisted of an independent organization known as the umbrella organization that facilitated the collaboration between and among individual teams that wanted to collaborate.

Collaborations between partners could also be defined by their relationship, as opposed to the structure described above. Whitford, Schlechty, and Shelor (1987) distinguished three major partnerships as cooperative, symbiotic and organic. Cooperative collaborations were based on personal contacts, operated for a limited time, and were based on giving and receiving a service. For example, a university member could train the school faculty in the use of a new technology. In the interactions it was assumed that the university members delivered a service and the school faculty, cooperated, for a finite amount of time, to receive the service. Further, these collaborations could be either formal or informal, depending on whether they operated in the organization or outside of it.

Symbiotic partnerships (Goodlad, 1988) involved unlike partners who came together for the mutual satisfaction of their goals and each partner displayed enough selflessness to ensure the satisfaction of the other’s goals. Schools and universities, like all institutions, deteriorated when things were left unattended and Goodlad (1988) suggested symbiotic partnerships between schools and universities as a way for renewal for the individuals and institutions involved. According to Goodlad (1988), symbiotic partnerships addressed the two needs of (1) renewal through individuals with varied expertise coming in contact with others who had expertise in similar work and (2) ensured a continuous flow of knowledge and alternative ideas. These conditions were met through symbiotic partnerships.

Organic partnerships involved partners who performed unique functions but had common goals. These partnerships could be thought of as different body parts working jointly for the functioning of the body. The common goals were made explicit and neither partner 'owned' the goals. This differed from symbiotic partnerships, where a goal was owned by one of the partners. Whitford, Schlechty, and Shelor (1987) pointed to professionlization of teaching as an example of an issue that was boundary spanning and could foster organic partnerships. Here the two institutions could be jointly responsible for the development of teachers from the recruitment, selection, preparation to the ongoing professional development. In the next section I will review selected collaborative research literature and examine important aspects of these collaborations.

Collaborative Research

Collaborative research can assume different forms, as seen above, and the focus could either be on the structural aspects or on the relationships. Collaborative research between teacher(s) and researcher(s) that focuses on the issues determined by the teacher is called action research. Miller and Pine (1990) defined action research as "an ongoing process of systematic study in which teachers examine their own teaching and students' learning through descriptive reporting, purposeful conversation, collegial sharing, and critical reflection for the purpose of improving classroom practice" (p. 57). Action research projects became collaborative action research projects when university researcher(s) collaborated with teacher(s) to provide professional development and support for the teacher to address the teachers' concerns (Clift, Veal, Johnson, & Holland, 1990; Lieberman, 1986).

Collaborative action research could be either initiated by the teacher(s) (Raymond, 1995; Raymond & Leinenbach, 2000) or by researcher(s) (Cryns & Johnston, 1993; Edwards & Hensien, 1999; Garrido, Pimenta, Moura, & Fusari, 1999; Hsiung, Chen, & Wang, 2000; Hunsaker & Johnston, 1992) and occurred due to a variety of reasons. A teacher challenged with a new curriculum sought help from a university researcher. This could begin as a consultation model but then evolve into collaboration. For example, Raymond and Leinenbach (2000) examined a collaboration initiated by an 8^{th} grade teacher and a university researcher. The teacher sought guidance from the researcher to implement algebra manipulatives in the classroom. The collaborative study later examined the effect of the manipulatives on the students' confidence, interest, algebraic equation solving ability, and the retention of the algebraic skills (developed with manipulatives) at the next grade level.

Collaborations initiated by the university researchers generally arose out of professional development of teachers or efforts to study the environment of the teachers, and could be part of a larger project. For example, Edwards and Hensien (1999) detailed a study, initiated by a university researcher, to help a middle school teacher to align her teaching to the vision outlined in the Standards (NCTM, 2000). The study by Cardelle-Elawar (1993) was another example of collaboration initiated by researchers in the quest to promote teachers as classroom researchers through action research projects. In this study (Cardelle-Elawar, 1993), the teachers were guided in identifying a problem, define this problem, explore strategies to solve this problem, and finally assessed their success or failure in solving the problem. Collaborations could also be agreed to achieve a common goal. For example, Freedman and Salmon (2001) described in their partnership as they developed, implemented, and evaluated literacy curriculum that addressed individual and classroom social needs of students.

Other action research studies went outside the classroom environment to engage the teachers in collaborative research that focused on school issues. For example, Garrido, Pimenta, Moura, and Fusari (1999) discussed a study where the teachers were engaged in examining the relationship between school and society and the critical aspects of their school. These teachers developed and engaged their students in projects and then reported on this process.

Collaborative action research, though essentially focused on the work of the teacher in a classroom, could simultaneously address one or more of the following goals:

1. close the gap between universities and schools
2. provide opportunities for professional development of the teachers
3. stimulate reform efforts in the classroom
4. increase professionalization of teaching
5. improve teaching and learning and
6. generate theory and knowledge. (Raymond & Leinenbach, 2000)

Communication between teachers and university researchers via sharing of expertise and extended interactions helped to build relationships between members of the two communities and closed the gap that was present between the two (Cardelle-Elawar, 1993; Miller & Pine, 1990). Collaborative research engaged teachers in thinking about classroom issues as part of their ongoing professional development, increasing the likelihood that they would implement reform efforts in the classroom (Clift et al., 1990; Raymond & Leinenbach, 2000). As teachers reflected on their work, made improvements, and shared their work with others, they set an example for the other teachers and thus increased the professional status of teachers (Miller & Pine, 1990). Though, the results generated by the teacher research could also add to the existing bodies of research (Miller & Pine, 1990), there was a debate if action research should be considered a contribution to the research base on education since it was located in a particular context and was primarily seen as a tool for the professional development of teachers (Noffke, 1994).

Collaborations have had various meanings in the literature, and issues of power and control were addressed in various ways in studies. Olson (1997) argued that building collaborative relationships was challenging since traditionally, learning relationships have been hierarchical rather than collaborative. She examined two taken-as-granted cultural structures present in educational institutions of ‘getting an education’ and ‘becoming more experienced’. The metaphor ‘getting an education’ viewed knowledge as objective, certain, received and accumulated. The ‘becoming more experienced’ metaphor was more subjective and was constructed continuously through interactions with others. Each of these cultural structures led to different approaches in research and 'becoming more experienced' was a metaphor that was better suited to developing collaborative relationships.

> Collaboration involves much more than getting an education. Here diverse knowledge is shared which takes into account individuals' experiential knowledge of situations. As new views are shared, each participant in the collaborative endeavor is provided with new ways to reconstruct past knowledge and imagine future possibilities. (Olson, 1997, p. 24)

The next section examines the literature that seeks to address the above mentioned issues of power and control that are present in collaborations between teachers and researchers.

Teacher/Researcher roles

Cole and Knowles (1993) assumed a hermeneutic perspective, based on empirical evidence from their work with teachers. They stated that for the teacher-researcher relationship to be truly collaborative it must move away from the traditional model of research that sought to exclude the teacher's participation from the major phases of research, namely planning, data collection, data analysis, and reporting. Their studies indicated that a lack of conscious effort to negotiate the involvement of the teacher from the beginning of the project and in all phases, led to a breakdown of communication and hence the collaboration. This point of involving the teacher as an equal partner was also emphasized by others in the literature (Blond & Webb, 1997; Hord, 1981; Little, 1981; Oja & Pine, 1987). In collaboration, which was characterized by involving the teacher in all stages of the research, reporting generally took the form of narratives or stories (Connelly & Clandinin, 1990) that the teachers and researcher developed through negotiation and collaboration to preserve the voice of the teacher. This exemplified research ‘with’ rather than ‘on’ the teacher (Blond & Webb, 1997; Day, 1991), presented their multiple perspectives, and challenged the notion of the ‘researcher as expert’ who had the right to interpret the data.

Other researchers (Clark et al., 1996; Kreisberg, 1992) argued that this mutuality between the partners was difficult to achieve in all aspects of the research. This would force the teachers to enact the roles of the researchers in addition to their roles as teachers. For example, the teacher would have to take time out to write a report that is more valuable for the researcher in the context of the institutional demand to publish. Clark et al. (1996) cited dialogue between the partners as the major focus of the collaboration and through this dialogue there should be mutual professional development for *both* partners. Thus parity between the partners was achieved not by “doing the same work” but rather by “understanding the work of one another” (p. 196). According to Clark et al. (1996) collaborative research involved

> opportunities that allow all of us to reflect on our practices, engage in shared critique of those practices, and support one another in making professional choices and changes. Successful collaboration involves increasing our understanding of one another’s worlds and roles through shared dialogue, as opposed to shared work.… Thus, collaboration is not defined by the fact that everyone does everything but rather by the fact that everyone gains from the interaction. (p. 227)

Clark et al. (1996) examined three groups of teachers-researcher collaborations who came together to discuss collaborations at their individual sites and also shared excerpts from their journals in the discussions. All the interactions and writing were captured and weaved into a collaborative script (by two researchers) that reflected the voices of both the teachers and the researchers. This removed the burden of writing from the teachers and both parties benefited through the dialogue.

Tensions

Tensions are a common theme in partnerships and arise when the partners are from different institutions that have different cultures and expectations. Resolving these tensions allowed the partnership to move forward. Freedman and Salomon (2001) described the evolution of their partnership to create a new Relational Literacy Curriculum through the lens of Rawlins' (1992) dialectical tensions. In the first phase the partners resolved the tension of judgment and acceptance by accepting the other person's strengths and weaknesses. In the second phase the tensions centered on negotiating independence-dependence. This tension referred to the partners' freedom to be independent and yet to compromise this independence in times of need for the other partner (dependence). In the study the teacher and the researcher came to the collaborations with their own goals and there was a strong dependence on each other to accomplish their goals. This dependence was reduced after renegotiating roles of the researcher teaching in the classroom and the teacher doing research. Through this renegotiation, the partners' independent goals became less distinct and interdependence grew and fostered shared goals and a realization that the strength of the project was in the interconnectedness. Phase three, during the fifth and sixth year of the partnership, involved the resolution of the tension to be expressive and protective. This referred "to the delicate balance between communicating one's feelings and needs and strategically protecting oneself or the feelings of others" (Freedman & Salomon, 2001, p. 189).

Understanding and respect between the partners contributed to resolving this more personal tension.

Attributes of caring and humility are some of the affective factors pointed out in the literature that could impact collaborations. Day (1991) highlighted the caring nature of partnership between the teacher and researcher as a means of fostering a more lasting connection. The researcher should have

> …affective, human-relating skills and qualities and, by implication, places the researcher into an ethical relationship with the research object which now becomes the research subject. Caring also implies that this relationship will be open-ended that is, the researcher will not compartmentalize, override, or hijack the teachers' concerns, and will focus upon personal, professional, and interpersonal as well as cognitive development. (Day, 1991, p. 537)

Further,

> Anyone seriously considering the building of a collaborative enterprise needs to have a hard-nosed sense of modesty and a real capacity for making mid-course adjustments in the process. (Bikel & Hattrupp, 1995, p. 51)

In summary, negotiating tensions effectively contributed to the development and growth of the partnership. On the other hand ignoring the tensions could cause the partnership to stall. The human relating skills and affective characteristics played an important role to keep the partnership on track. The next section examines another major collaboration that arose among educators, namely the formation of Teacher Study Groups (TSGs). In the following section, I discuss the major ideas of TSGs.

Teacher Study Groups

Teacher Study Groups were developed to facilitate ongoing professional development of teachers and to banish the isolation that is generally associated with teachers in the profession (Little, 1990; Miller & Pine, 1990). Arbaugh (2003) defined a Study Group as “a group of educators who come together on a regular basis to support each other as they work collaboratively to both develop professionally and to change their practice” (p. 141). TSGs were an example of collaborative professional activity and they may or may not have enlisted the guidance of a researcher(s).

Makibbin & Sprague (1991) outlined four models of study groups namely, implementation study group, institutionalization study group, research-sharing study group, and an investigation study group. The implementation study group aimed to support prior learning of new methods or strategies by teachers as they implemented these in their classroom. The institutionalization study groups were set up so that teachers who have learned a new innovation gained mastery through discussion and sharing with the other teachers. As the name suggests, research-sharing promoted the reading and sharing of new research that was applicable to the teachers. The teachers in an investigative study group investigated an agreed upon topic through cycles of reading, discussion, and implementation.

Investigative study groups have been prominent in recent years and some topics that have been examined include, development of an inquiry-based geometry curriculum (Arbaugh, 2003), support for English Language Learners (Clair, 1998), and classroom issues (Birchak et al., 1998). A major theme of investigative study groups has been development of the content knowledge of the teachers by rooting the professional development in the work of the teacher (Putnam & Borko, 2000). For example the teachers could engage with the content through open ended problems (Crespo, 2002;

Crockett, 2002) or examining students' work on mathematics problems (Crespo, 2002; Crockett, 2002; Kazemi & Franke, 2004). Developing lessons was another way of rooting the professional development in the work of the teacher. The Japanese Lesson Study Model (Fernandez, 2002; Puchner & Taylor, 2006) involved 'research lessons' designed by a group of teachers and implemented by one of the teachers. After the group observed the teaching in the classroom, they debriefed and improvements were made in the next iteration based on the feedback. The cycle was then repeated in a different teacher's classroom.

Research on TSGs has focused on the participation of the teachers and the activities that facilitated this. Crockett (2002) examined the teacher participation around the four activities of working on an open-ended problem, a video teaching vignette, lesson planning, and analyzing student work. The goal was to investigate the practical activity that facilitated the most discussion and as a result led to teacher learning. The last item, analyzing student work, was the one that challenged the current beliefs and practices of the teachers and hence had the most potential for teacher growth. Kazemi and Franke (2004) also detailed the 'shifts' in participation among a group of teachers as they analyzed student work. These 'shifts' occurred as the teachers tried to make sense of the strategies that the students were using to solve the problems and again when they tried to develop possible instructional strategies based on this. These discussions helped form a closely knit working group among the teachers. Crespo (2006) examined the conversations of teachers in a TSG and isolated the discourse into two categories namely, exploratory and expository. Exploratory talk occurred as the teachers discussed their own mathematical work and expository talk occurred when they discussed their students work.

All collaborations, as seen in the previous section, developed tensions as people tried to work together and there was potential for the collaborations to dissolve. This made the role of the facilitator crucial in a TSG. Birchak et al. (1998) reported that the facilitator could be another teacher or an outsider and there were advantages and disadvantages to both. If the facilitator was another teacher, the group members would feel comfortable sharing their ideas and the relationships would be less hierarchical. On the flip side, the internal politics of the school could ruin cooperation. An outside researcher would not be part of the internal politics of the school, but the teachers could also look to this person to provide solutions to their problems. Birchak et al. (1998) also reported another option of facilitation being rotated among the members of the group (or a smaller subset) as the topics changed (Birchak et al., 1998). Generally the facilitator's role involved, setting up the logistics of the group, like the time and place of the meetings, the readings that had to be done by the teachers, and gathered the materials that were necessary for the session. During the sessions, the facilitator ensured that general focus was maintained in the discussions, all participants voices were being heard, and they resolved tensions that arose among group members (Birchak et al., 1998). Some other factors that contributed to the formation and maintenance of a TSG were release time for the teachers, the place of meeting, the amount of work that the teachers had outside the TSG, and the number of members in the group (Arbaugh, 2003; Birchak et al., 1998).

Study groups were valued as a route for ongoing professional development as they facilitated the development for teachers in a safe collegial environment that promoted growth, reflection, professionalism, and self-efficacy (Arbaugh, 2003; Birchak et al., 1998; Boggs, 1996; Clair, 1998). The format of the meetings facilitated the development of knowledge by allowing exploration, conjecturing, independent and group problem solving, discussion among members and valuing each members input (Birchak et al., 1998). There was also significant teacher ownership over the products and ideas that were finally developed (Arbaugh, 2003; Birchak et al., 1998).

Summary

In the above section I outlined two major manifestations of collaborations in collaborative action research and teacher study groups. In both cases there were issues of power and control that could give rise to tensions between the teachers and researchers. These tensions could be resolved by involving the teacher in all stages of the research or through effective dialogue. Effective management of the dialectical tensions and a caring attitude along with a degree of humility also contributed in resolving the tensions. In the case of the TSGs, the facilitator assumed an important role in resolving tensions among members of the group.

As we have seen in the literature till this point, partnerships cannot exist in vacuum and there has to be a purpose for the teachers and researchers to come together. A major reason so far has been the professional development of teachers, which has been a focus of a number of collaborations. My collaboration with Linda also had the underlying basis of her professional development as she requested specifically for content development. I decided to focus on the tasks literature to inform my work with Linda. Further, tasks also had ties to my theoretical framework (see Teaching Framework in the Theoretical Framework section). The next section examines the literature on tasks and their use in mediating discussions.

Tasks

The Standards (NCTM, 2000) considered problem solving to be a focus in the development of mathematical knowledge and habits in students. The teacher assumes an important role in choosing and setting-up worthwhile problems for the students. In my work with the teacher, I focused on examining the selection and setting-up of the tasks in the classroom. Below I outline the literature on tasks.

Based on the work of sociologists and historians of education, Doyle (1983) agreed that there was a relationship between schooling and adult work since both emphasized developing characteristics like punctuality, patience, production schedules and obedience. This was regarded as the “hidden” curriculum. Thus using the metaphor of work, the curriculum could be viewed as a collection of academic tasks, which consisted of four basic parts,

1. A goal state or an end product to be achieved.
2. A problem space or a set of conditions and resources available to accomplish the task.
3. The operations involved in assembling and using the resources to reach the goal.

4. The importance of the task in the overall work system of the class. (Doyle, 1988)

Stein et al. (2000) classified tasks based on the level of thinking required by the students or the *Level of Cognitive Demand* (LCD). They developed the *Task Analysis Guide* (TAG) (see Appendix D for details) that classified mathematical tasks into four categories of *memorization*, *procedures without connections* (both were of lower-level cognitive demand) , *procedures with connections*, and *doing mathematics* (both were of higher-level cognitive demand). This classification was similar to that developed previously by Doyle (1988), for more general tasks (not subject specific) based on ambiguity and risk involved in tasks (Figure 1). Ambiguity referred to the extent to which a precise answer could be defined in advance or a precise formula for the generation of an answer was available, while risk referred to the strictness of the evaluative criteria a teacher used and the likelihood that these criteria could be met on a given occasion.

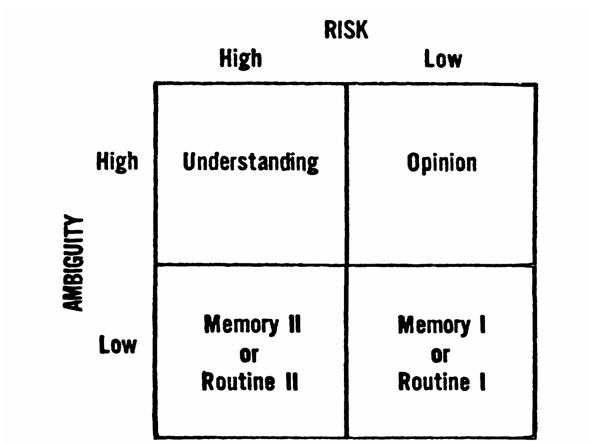


Figure 1: Ambiguity and risk associated with tasks (Doyle, 1983, p. 183)

Stein et al. (2000) used mathematical tasks as the basic unit of analysis in their *Mathematics Task Framework* (MTF) (Figure 2) where the tasks were seen to flow through four stages. The first stage referred to the tasks as they were represented in the curriculum, the second stage discussed the way the tasks were set up in the classroom by the teacher. The third stage was the enactment of the tasks by the students in the classroom. Finally, the last stage pointed to the student learning that took place. The researchers stated that "the tasks used in mathematics classrooms highly influence the kinds of thinking processes in which students engage, which, in turn, influence student learning outcomes." (p. 462)

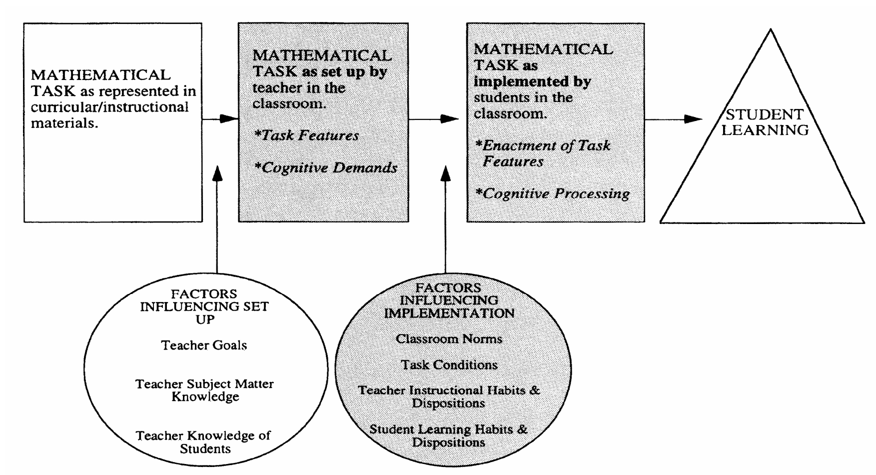


Figure 2: The MTF (Stein, Grover, & Henningsen, 1996, p. 459)

Tasks were embedded in the classroom and hence worked under the evaluative system and elements of ambiguity and risk worked together to make the tasks 'novel' or 'familiar' and as a result the classroom flow could be 'bumpy' or 'smooth' respectively (Doyle, 1983; Doyle, 1988). Zaslavsky (2005) claimed that risk in the task could be achieved by incorporating competing claims, unknown path or questionable conclusions, and non-readily verifiable outcomes into the assigned tasks. Teachers could manipulate ambiguity and risk via the evaluation of tasks (e.g. generous grading) and hence control classroom management and student engagement. This had two important consequences, first answers favorable to the teacher defined the real task and secondly the strictness of the grading encouraged students to focus on maximizing their score (Doyle, 1988). Further, the nature of the classroom caused the students to selectively listen to information sources to accomplish the task at hand and peers served as valuable resources for solving the problems (Doyle, 1988). Similar results were obtained by Stein et al. (1996), in terms of mathematical tasks, where the cognitive demands of tasks either remained the same or declined between the set up and implementation.

Stein et al. (2000) isolated some major factors that contributed to lowering the level of cognitive demand, which included tasks being made less challenging, students' lack of engagement, and lack of motivation or knowledge. Cognitive demand was also lowered with a shift of focus to the correct answer, non-optimal time distribution and a lack of accountability on the part of the students. Lowering the LCD of tasks are reported to have other subtle effects on students acquiring the belief that mathematics problems got solved in a short period of time (Schoenfeld, 1988). There was also a risk that independent thinking and problem solving skills of the students could be curtailed (Corno, 1988; Greeno, 1983).

Factors that contributed to the maintenance of the high level of cognitive demand included tasks that were built on the prior knowledge of the students and tasks that had an appropriate allotment of time. Modeling of high level performance, done by the teacher or another student, also helped maintain a high LCD. Also continuous pressure by the teacher for justifications and explanations through questioning, comments and feedback further pushed the students to think (Stein et al., 2000). Scaffolding by the teacher in a way that did not take away the complexity of the task also contributed to the maintenance of a high LCD. Other factors included the students self-monitoring and the teacher making conceptual connections for the students (Doyle, 1988).

Doyle (1988) observed that in classrooms there was a big pressure from the students to turn tasks into familiar work, where meaning was sacrificed and the curriculum got conceptualized as a set of discrete skills. Herbst (2003) provided a reason why it was difficult to sustain novel tasks in the classroom. He described the tensions faced by the teacher in negotiating the unfolding curriculum in the classroom and the complexity could prevent the teacher from maintaining a high LCD. The teacher had to manage the tension between the explicit product of the task and new ideas that emerged in the process. In managing the resources there was a tension to maintain the balance between information provided in concrete representations and the more abstract underlying ideas. Further, in negotiating the operations that were used to solve the tasks, the teacher had to balance between maintaining ambiguity and specificity in operations to ensure student thinking was preserved but at the same time frustration was kept at manageable levels. Herbst (2003) conjectured that these tensions could explain the lower cognitive demand that was observed in the enactment of the tasks in the classroom.

The MTF as a Tool for Professional Development

Encouraging teachers to be reflective of their tasks and practice via the MTF has proved to be useful for professional development (Arbaugh, 2005; Stein et al., 2000). Arbaugh (2005) discussed the importance of tasks and the need for teachers to be able to distinguish between the various cognitive levels (Stein et al., 2000) in order to better impact their students learning. After introduction to the MTF, the teachers analyzed tasks that they assigned in their classrooms, based on this framework, and discussed their classification with the others in the group. The MTF also provided an effective framing concept for professional development since it allowed initial examination of teaching in a non-threatening way and also allowed for the development of Pedagogical Content Knowledge (PCK) (outlined further in the section on Teacher Knowledge) of the teachers. The change in the teachers' understanding of the framework and application in the classroom were examined through their responses to a task sorting activity before and after an intervention. This intervention included discussions and reflections on the tasks, justification of the categories into which tasks were sorted, reading literature on the MTF, finding activities of high LCD to use in their classes, discussing the results of using these tasks in their classroom, "tweaking" tasks that were of lower LCD so that they were high LCD, and using the MTF in their reflections of their classroom teaching. The MTF could also be used as a mediating tool for discussions between different groups of pre-service teachers, in-service teachers, and researchers. McGraw, Lynch, Koc, Kapusuz, and Brown (2007) facilitated a discussion based on multimedia cases and tasks between a diverse group of pre-service teachers, in-service teachers, mathematicians and mathematics teacher educators. Here the MTF proved useful “as a lens through which to interpret the participants’ comments” (p. 9).

Summary

The literature showed that the MTF provided a useful lens for both the teacher and researcher to discuss tasks that were assigned and set-up in the classroom. Balancing the LCD in the tasks through the ambiguity and risks had important consequences both for the student learning and the classroom management. Choosing appropriate tasks for the class and handling the tasks in the classroom formed a big part of the teachers work. The teacher’s knowledge about mathematics and teaching played a vital role in this aspect. I outline the literature on the mathematics knowledge required for teaching in the next section.

Teacher Knowledge

The teacher’s knowledge is a crucial component in teaching (Ball, 1990; Elbaz, 1983; Schifter, 1998; L. Shulman, 1986) and this knowledge cannot be acquired by just taking higher mathematics courses. Shulman (1986) identified seven areas of knowledge that would comprise the vast and highly organized bodies of knowledge the teacher needed to teach: knowledge of Subject Matter Knowledge (SMK), Pedagogical Content Knowledge (PCK), knowledge of other content, knowledge of the curriculum, knowledge of learners, knowledge of educational aims, and general pedagogical knowledge. PCK was one that has been predominant from the time it was introduced by Shulman (1986). He defined PCK as,

> The most useful forms of representation of … ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others … [It] also includes an understanding of what makes the learning of specific concepts easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning. (p. 9)

Ball (1990, 2000a) and Ball and Bass (2000) also highlighted that subject matter was important, but was only part of the knowledge that was required for teaching. Ball (1990) agreed with Shulman (1986) that there were specialized bodies of knowledge that teachers required to “unpack” the curriculum for the students; their work intended to isolate this “subject matter knowledge for teaching” (p. 457) or mathematics knowledge for teaching. In a study with pre-service teachers, Ball (1990) highlighted three characteristics of the teachers’ knowledge, namely knowledge of correct concepts and the procedures, understanding of the underlying principles and meanings, and an appreciation and understanding of the connections among mathematical ideas. Another important characteristic of the mathematics knowledge for teaching was the teachers’ knowledge about mathematics, such as, what counted as an answer and what established validity of an answer. These conceptions of mathematics influenced the representations that the teachers used. Ball (1990) found that the prospective teachers who were part of the study, had a procedural and rule based conception of mathematics. Further, the rules also served as a means of justification of procedures and thus they considered remembering rules as an essential part of doing mathematics.

Ball and Bass (2000) claimed that even though the PCK could help the teacher anticipate some of the difficulties that the students might have or the advantages of using a particular representation over another, the PCK did not completely prepare the teacher to handle the ongoing "dynamic interplay of content and pedagogy" (p. 88) during problem solving. In the classroom, teachers required knowledge of the content to flexibly listen to the students, create opportunities for learning that were based on the students' interests, and to work in a diverse classroom (Ball & Bass, 2000). It was this pedagogically useful mathematical understanding (p. 89) that they sought to find through the examination of the teacher's practice rather than by examining the curriculum. For them, teaching was a combination of both regularities and uncertainties, and knowing mathematics for teaching had to take this into account. The researchers isolated three problems that had to be solved in order to bridge the gap between content and pedagogy, the first was "What mathematics is entailed by teaching?" (p. 95). For example, if various solutions were presented in the classroom for an assigned problem, the teacher would need to examine them and justify why they are the same. The second question was "What makes mathematical knowledge usable for teaching?" (p. 96). Here the teachers are required to 'unpack' the usual compression of knowledge in mathematics, since he or she would be working with students' ideas that were still in formation. The teacher would also have to recognize the possible trajectories that could be taken at various stages of a problem. Finally, the last question was, "How might teachers develop usable mathematical understanding?" (p. 99). Some suggestions were to design and explore the "core activities of teaching" either through a simulation in the teacher preparation (professional development) or in the context of the classroom. Another suggestion was to use the students' work as a site for learning the content. Ball and Bass (2000) felt that resolving these three questions could bring content and pedagogy closer.

Ma (1999) picked up from Ball's (1990) work and studied the SMK and the PCK of major topics among a group of elementary mathematics teachers from US and China. She concluded that the US teachers had a fragmented knowledge of the mathematics content compared to the Chinese teachers whose knowledge was deeper and coherent. Ma (1999) recommended that the teachers needed to acquire a Profound Understanding of Fundamental Mathematics (PUFM), which referred to an intertwining of the procedural and conceptual understanding of the topics and the interconnections among the various topics. Teachers with a PUFM were able to teach mathematics with connectedness, multiple perspectives, basic ideas, and longitudinal coherence. In order to acquire such a PUFM, Chinese teachers studied the curriculum materials intensively when they taught it, formed study groups with other teachers, were open to new ideas from the students and developed knowledge by doing mathematics themselves. Ma (1999) claimed that without a thorough understanding of the mathematics concepts, just knowing how to represent the material was not enough to be an effective teacher.

Summary

This short section pointed to the important knowledge that teachers need to teach mathematics effectively. The ideas of PCK consisting of representations and understanding students' difficulties were important points to consider when working with a task. The mathematics knowledge for teaching went beyond the PCK and examined knowledge that was useful during teaching where the teacher 'unpacked' the mathematics for the students. Further, the literature points to the teachers acquiring PCK and mathematics knowledge for teaching through reflection on the day-to-day aspects of their work of the curriculum or their students work. This last point helped guide my work in the study as it helped me to make informed decisions about the tasks that I chose to discuss with Linda.

The next section examines the theoretical framework, which together with the literature reviews, guides my study. The theoretical framework informs my perspective on the learning and teaching of mathematics and serves as a guidepost to the discussions that I have with Linda, data collection, and the analysis of this data.

Theoretical Framework

There have been debates (Steffe & Gale, 1995) about the learning process being primarily cognitive or social. The first position had its origins in the work of Piaget (1970) and assumed that learning was a constructive process that occurred primarily within the individual. Some of these theorists acknowledged that the social aspects played some part in facilitating an individuals' construction, but learning was still primarily an individual process. On the other hand, theorists who assumed that learning was primarily a social process viewed the social aspect as primary and the individual as secondary. These ideas found their genesis in the work of Vygotsky (1978). My overall perspective for this study has been guided by a position that does not assume either extreme outlined above, but is a coordination of the psychological and social perspectives of learning, and referred to as the emergent perspective (Cobb, 1994; Cobb & Bauersfeld, 1995; Cobb, Jaworski, & Presmeg, 1996). This perspective combined the ideas of radical constructivism (von Glasersfeld, 1987, 1991, 1995) with those of Symbolic Interactionism (SI) (Bauersfeld, 1995; Blumer, 1969). I first examine each of these theories separately and then their coordination in the emergent perspective. Further, I examine how the emergent perspective informs a theory of teaching and I also detail the use of this theoretical framework in my study.

Radical Constructivism

Ernst von Glasersfeld (1987, 1991, 1995) built on the ideas of Piaget (1970) but went beyond in his epistemology of radical constructivism. This philosophical position assumed that a person had no access to objective reality, but instead constructed knowledge of the world through perceptions and experiences, which were in turn based on previous experiences. Learning was seen as a process where the individual adapted to the experiential world. Since there was no way of knowing the “truth”, the individual constructed “viable” models. The individual analyzed the experiential world through schemas, which were built up from previous experiences. When faced with a new situation, the individual experienced disequilibrium and she/he sought to resolve it to ‘fit’ the existing schemas. If the new situation ‘fit’ the current schema, then the new situation was assimilated into the schema. On the other hand if the new situation did not ‘fit’ the existing schema, then the current schema was adapted; this was known as accommodation. The individual always worked to achieve equilibrium, and schemas evolved to better fit the individual’s experiential world. Reflection on the successful adaptive processes (termed Reflective Abstraction) led to the development of new concepts in the individual. In this theory, the individual was the central focus and even interactions that the individual had, for example with the teacher and other students, helped to facilitate disequilibrium. The focus on the individual had been problematic with some researchers and there have been debates about developmental knowledge being more social (Steffe & Gale, 1995).

Symbolic Interactionism

Symbolic Interactionism (SI), introduced by (Blumer, 1969) rests on three pillars of meaning, language, and thought. Individuals act towards objects based on the meaning that they assign to them. These meanings are socially negotiated through interactions in language and can be dealt with and modified via thought.

By focusing on the nature of social interactions, SI rejected the view of a passive individual being influenced by the environment or by others; instead there was a constant back and forth between individuals as they interpreted meaning and based their actions on these interpretations. Individuals 'emerged' as they dealt with the current situations based on their definition of the current situation and thus could not be considered to be a product of their past or the past of their social group. The active nature of the individual in group life led to the assumption in SI that groups or societies existed in action. The principles of SI also implied that the existing world was composed of physical, social and abstract objects that were socially created and derived their meaning through interactions (they do not possess any inherent meaning). An important class of social objects was symbols, which were used for communication and representation. In this category of symbols, words had a special place as they allowed thought and imposition of the individual in the world.

The construction of the ‘self’ was an important part of SI. The individual created this definition of ‘self’ through interactions with others. Thus they saw themselves, partly, as others saw them by placing themselves in the role of the other. The individual could use this process of defining ‘self’ to make changes and grow as they learned more about themselves via interactions.

The Emergent Perspective

This perspective coordinated the ideas of radical constructivism (von Glasersfeld, 1987, 1991, 1995) and Symbolic Interactionism (Blumer, 1969) and claimed that learning was both a psychological and a social process and there is a reflexive relationship between the two (Cobb & Bauersfeld, 1995). So individuals contributed to the group activity as they reorganized their thinking and these reorganizations were expanded or constrained by the individuals’ participation in the group activity. The psychological process differed for each member in the group and the social process accounted for the shared knowledge through coordination of the individuals’ activities. Thus the emergent perspective sought to account for the growth of the individual as they participated in the group activities and developed taken-as-shared knowledge (Cobb, Yackel, & Wood, 1992). The individual development could not be viewed as independent to the social aspects of the group and the development of the joint taken-as-shared knowledge could not be accounted for by ignoring the individual participants.

Teaching Framework

The emergent perspective did not prescribe a method for teaching, but Simon (1997) conjectured problem posing and promoting discussions around these problems as a possible route for teaching. Posing problems provided perturbations for the individual to either assimilate or accommodate, and finally make generalizations on the basis of reflective abstraction. The individual's learning depended on the opportunities that they had for reflective abstraction and this could be promoted by journal writing or group and whole class discussions where the students interacted to communicate ideas. It was through participation in the classroom community that individuals learned about the nature of mathematics and the teacher played an important role in aligning this community to the practices of the wider mathematics community. By orchestrating discussions through the ideas put forth by the students, the teacher could create value for certain types of reasoning and communication among the students. The teacher could also promote discourse around mathematical justifications and gradually allow the students to negotiate the validity of a solution through negotiation among them.

How I Use this Framework in this Study

In this study, the group consisted of a dyad and the development of the teacher is examined as she participated in the mathematics content and teaching issues as part of the dyadic relationship with me. The study began with ideas from the teaching framework of posing appropriate mathematical problems and sustaining discourse around these problems. The problems were meant to be a perturbation for the teacher and through our discussions, learning would be furthered.

In the data collection and analysis I focused on capturing and analyzing the teacher's work as she displayed her thinking via interactions and also focused on aspects that we agreed upon and acted upon in discussions. Symbolic Interactionism claimed that meaning could either be changed or modified through thought, which in turn was influenced by interactions. By building an environment where the teacher grappled with challenging problems and discussed these problems, the aim was for the teacher to acquire meaning aligned to the Standards (NCTM, 2000) that would guide her future work. The classroom and the students were a central source of perturbations both for the teacher and me, as we worked on mathematics content, planned lessons for the class, discussed issues in mathematics and teaching that arose in the classroom, and discussed the student thinking.

Summary

This chapter summarized the literature and my theoretical perspective, both of which inform my decisions in the study and interactions with Linda. The literature on collaborative research highlighted that tensions are integral to collaborations between teachers and researchers. Further, these tensions of power and equity can be managed by either sharing the work or sharing an understanding of each other's roles through dialogue. The literature on tasks isolated the MTF as a useful concept to promote growth in (a) the practice of the teacher, (b) reflection on her teaching, and (c) communication between the teacher and researcher. The PCK and the mathematical knowledge for teaching guided the choice and implementation of tasks to facilitate student thinking. These concepts also promoted reflection by the teacher on her practice. The teaching framework by Simon (1997) emphasized the importance in promoting learning by using tasks as a source of perturbations. Further, Ball and Bass (2000) and Ma (1999) pointed to the importance for the professional development to be based in the curriculum and the teacher's everyday practice. The idea of situating the professional development in the everyday practice motivated me to choose problems from the *Connected Mathematics Project* (Lappan, Friel, Fey, & Phillips, 1998) curriculum for my discussions with Linda. These tasks were of a high LCD, or could be raised, so as to challenge Linda's thinking on the material and would provide perturbations that would reorganize her knowledge. Interactions between Linda and me around the tasks would also contribute to Linda's growth.

The theoretical perspective informed my decisions about the data I collected and the analysis as I sought to study the evolution of the partnership. The next chapter outlines the methods used for this study, the data collection, and the analysis that I did.

CHAPTER 3: METHODS

The qualitative case study approach (Merriam, 1988) was suitable to address the research questions in my study as the aim was to gain "an in depth understanding of the situation and its meaning for those involved. The interest is in the process rather than outcomes, in context rather than a specific variable, in discovery rather than confirmation." (p. xii). The qualitative case study is also suitable to examine critical problems of practice and increase the knowledge base of the mathematics education community (Merriam, 1988).

This chapter will describe the context of the study, the data collection and the analysis. The questions that guided my research design are reproduced below:

What is the nature of a partnership between a Middle School teacher and a university researcher centered on conversations about the mathematics content and teaching?

In particular:

1. How does the nature of the partnership evolve over time? What are the constraints on the teacher and the researcher in such a partnership?
2. How does the partnership contribute to mathematical and pedagogical issues that arise in planning, teaching, and assessment?
3. How does the partnership influence the curriculum with respect to both the tasks the teacher chooses to use in the classroom and how the tasks are utilized by the teacher?

Context of the Study

This section will outline the context of the study namely the school, the classes, the teacher's background (whom I will refer to as Linda), my background, the history of the dyad, and my roles in the study. These roles included being a participant-observer in Linda's classroom and an insider (Ball, 2000b) in the one-on-one after class discussions with three different foci of the mathematics content, Mathematics Task Framework, and the pedagogy.

The School

Linda's school was located in the south west of the country, in a predominantly working class Latino neighborhood. The demographics of the school given below in the tables indicate that there were 76% Latino students in the school (Table 1), 74% of the student body was on free or reduced lunch, and 14% were English Language Learners (ELL) (Table 2). Recently there was falling enrollment in the school with a number of 'feeder' elementary schools opting to start their own sixth grade class.

Ethnicity	School	State Average
Hispanic	76%	39%
White	12%	47%
Native American	6%	6%
Black	5%	5%
Asian	1%	2%

Table 1: Ethnicity in the school and the state.

Student subgroups	School	State Average
Free or reduced-price lunch	74%	49%
English Language Learners	14%	15%
Home language of ELL (Spanish)	93%	82%

Table 2: Student subgroups in the school and the state

The Classes

The study spanned two different academic years, and observations were made in the classes that were scheduled just before Linda's planning period. In spring 2006, I observed in Linda's seventh grade classroom and in fall 2006, I observed Linda's sixth grade classroom. Each class had between 19 and 24 students. The varying number of students was due to families of students who moved, either in or out of the area. Each class was of 45 minutes on Monday, Tuesday, Thursday, and Friday and 25 minutes on Wednesday. The students were seated in groups of three or four and Linda expected them to work collaboratively on assigned activities.

Teacher's Background

Linda grew up in the same town in which the school was located. She attended the local community college part time and took general courses that could be transferred to the university, like writing and mathematics. After four years she joined the Elementary Education program at the local university and was certified to teach kindergarten through eighth grade at the end of three years. Linda volunteered at an elementary school on a regular basis and that was a big motivation for choosing to become an Elementary Education major and teach. After her degree, she taught kindergarten for six years before she got the current job as a mathematics teacher in the middle school. In this elementary school she used a reform curriculum *Investigations* (TERC, 1995) and hence was not new to ideas of reform. Linda liked mathematics and preferred teaching mathematics rather than mathematics and science. She had been at this school for eight years and taught sixth and seventh grade classes along with a Yearbook class. She loved being a teacher, but there were also a number of challenges in her work. Some of them, in her view, were the growing lack of respect that the students had for the teachers and the lack of parental involvement. In some instances it was impossible to contact parents whose children were failing throughout the year. Other pressures were the increasing accountability requirements due to Federal policies. For example, detailed plans for improving failing students had to be made and submitted to the school. The parents of the failing students had to be aware of the plans, but generally it was hard to contact them. These challenges took time away from the teaching of mathematics. Another challenge was addressing all the topics in the curriculum in the given time frame. Linda could not be sure if she had spent enough time on a topic so that the students had the opportunity to build the required concepts.

My Background

I grew up in eastern India and attended a Christian Brother school till Grade 12. In India Grades K-12 were all in one school and all the teachers interacted with one another.

Discipline was strict and students had to wear a uniform to school. Teaching here was lecture driven and the teacher was the only person talking at any given moment unless a student was called on to answer a question or speak. Homework was a regular feature and the teachers were strict about collecting and grading the work. There were two or three tests in the year that determined the class score (out of a 100) and anything below 40 was a considered a fail in the respective subject. There were at most eight subjects in K-10 and at most six subjects in Grades 11 and 12. Failure in two or more subjects resulted in repeating the class for another year. In this, the teacher's decision was final. Students competed among themselves, and from my perspective the parents were very involved in their children's academics, always pushing them to do more. All students paid for this education and the parents viewed education as an investment to a better life. The mathematics curriculum featured challenging word problems and geometry, and students were expected to memorize formulas that they needed. Calculators were not allowed in the classroom or in the exams and as a result all calculations had to be done by hand. I learned mathematics in the traditional way, attuning to lectures and solving assigned problems. It was natural for us students to discuss elegant solutions together in school. Thus even though there was no social environment in the classroom, this existed in the informal networks outside the classroom.

Every school in India was part of an all India board of education and my school was associated to one of these boards. Students had to pass public exams in grades ten and twelve respectively. These were gate keeping exams and higher grades opened up better academic avenues. I entered college to do a Mathematics major and then went on to do a Masters in a university in the west of India. Later I went to Hong Kong to do a Masters in Financial Mathematics.

My first experience teaching was in a problem session as a Graduate Teaching Assistant in Hong Kong and it was here that I first started liking to teach. Explaining abstract concepts of topology to students who did not have English as their first language was difficult, and I had to support my explanations with plenty of pictures and written work. At this stage I decided that I did not want to pursue a career in Financial Mathematics and decided to combine my love for mathematics and teaching to do a doctorate in Mathematics Education. I wanted to get a sense of teaching in school first before I made up my mind fully and went back to India to teach in a school. This school was one of the elite schools in the city and attracted some very talented students. I taught mathematics in four classes from Grades 9 through 12 and the whole experience was very enjoyable. There were always students who wanted to learn more and as a result pushed me to understand the subject better. After teaching for two years at this school I moved to the United States to pursue a PhD in Mathematics Education.

In the doctorate program I was required to do three mathematics graduate courses for the first year and pass the mathematics qualifying exams at the end of the year. These exams were the same for the pure Mathematics and Mathematics Education students. My previous background in mathematics helped me but I did not have any exposure to education research during the first year. During this year I taught College Algebra that consisted mostly of entering freshmen students. The first semester was challenging and teaching in Hong Kong and India did not prepare me for the experience. I had to relearn how to explain very basic mathematics as my students struggled with topics I assumed they knew from school. Gradually interacting with the students, I came to understand the system of education and why students had such varied proficiency in mathematics; the major reason being that all the students opted for different mathematics courses in school. The students' written work was not clear, and explanations of their thinking were almost non existent. I had to work with the students on their writing, and explaining the basics that they missed in school. Given this situation I tried to ensure that the students understood the 'why' behind the procedures that they were using and the connection between the topics that they were studying. I would have students in the office hour work out their questions on the board with help from their peers and I would query the students about the steps they took. Once we started with their idea, I would try to push them to think about the next step and proceeded with this line of inquiry as far as possible. In some cases I intervened with hints that may have been linked to work that we had done in the classroom. I felt that the students would learn as they grappled with and talked about the concepts in the public space and use the feedback that they got. I engaged the students in simple problems in the beginning and gradually increased the complexity in order to help them learn the mathematics. This required that the students fit their prior knowledge in innovative ways to solve the current problem. I also liked to encourage students to think about removing or adding conditions to a problem and reflect on the effect that this had on the solution. Having the facility for symbol manipulation freed up the teachers' thinking to focus on the students' thinking and also gave the teacher confidence in the classroom. Effective use of symbols, by the teacher, could promote the same with the students. However, meaning was always at the heart of all endeavors.

Besides the regular mathematics courses, I also got an opportunity to teach the content courses for the pre-service elementary and middle school teachers. These classes were set up with a constructivist philosophy and were supposed to expose and engage the students in the big ideas of mathematics at the K-8 level. Interactions and group work was a central piece of the pedagogy and the teacher was not at the center of the classroom. This experience was difficult for me to adjust to as I was used to being the center of attention. Overall I felt that teaching these courses was very satisfying and I learnt how to question the students in the whole class discussions so that they could talk about the mathematics. The students were expected to present their solutions to the whole class and I had to learn how to build on the students' emerging ideas. By not being at the center of attention, I had to learn a degree of humility when I did not know how to connect the students' ideas to the topic at hand. These experiences in learning and teaching mathematics have shaped the way I think and interact with others about mathematics, and shaped the orientation I took at the beginning of our partnership.

History of the Dyad

I was introduced to Linda by one of the principal investigators of the Center for the Mathematics Education of Latinos/as (CEMELA) project. Linda was part of the first CEMELA cohort of Middle school teachers attending classes at the university. These classes were a combination of a mathematics content area with a focus on working with Latino students. Linda wanted to become a highly qualified mathematics middle school teacher. Linda's school did not have a partnership with CEMELA and she was the only one from her school who was working with CEMELA. She recognized that her mathematics background could be improved (she mentioned not having taken a college algebra course) and she also expressed a desire to learn more mathematics and become a better teacher. The Principle Investigator of the project was impressed with Linda's motivation to learn more mathematics. This factored heavily in her decision to recommend for me to work with Linda. I related the goals of the study with Linda, which were (a) to have discussions based on the mathematics content for her professional growth and (b) to observe our partnership evolve over time. I also outlined the benefits that she and the students could derive from my help in the classroom. At our initial meeting Linda expressed her reservations for video recordings, but later she agreed to video recordings on the condition that they were only viewed by me. She wanted the study to focus on her as a learner in our one-on-one discussions and was reluctant to be videotaped in the classroom.

We began communication with each other around the beginning of January and I visited her classroom at the end of January. I continued to visit Linda's classroom informally until the beginning of the study in mid-March. These initial visits were to observe the classroom setting, Linda's style of teaching, and to interact and build rapport with Linda and the students. At this point I had envisioned a study that would focus on the Mathematics Task Framework (MTF) (Stein et al., 2000) and planned to introduce Linda to the MTF, observe how she incorporated these ideas into her classroom, and her thinking. As a result, I paid particular attention to the tasks that she assigned in the classroom, how she set them up, and what the students at my group did with the assigned task. Linda used our meetings to ask me suggestions about good tasks that she could supplement her regular curriculum *Middle School Mathematics* (Charles et al., 1999) and it was at this time that I showed her the *Connected Mathematics* (CMP) curriculum (Lappan, Fritzgerald, Friel, Fey, & Phillips, 1998) as an example of tasks that she might want to use in her classroom. Though my initial intent in bringing the tasks was for Linda's own mathematical learning, she used these tasks to supplement her regular curriculum when she felt it was necessary. My observations up to this point (mid April) suggested that just focusing on the Mathematics Task Framework did not capture the exact nature of the work that I was doing with Linda and so I broadened my research questions to address the nature of my partnership with Linda that included conversations of the mathematics content, pedagogy, and the Mathematics Task Framework.

My Roles

My roles were split between my time in the classroom as a participant-observer and as a facilitator/insider (Ball, 2000b) in my one-on-one discussions with Linda. I observed Linda teach and immediately after I assumed the role of a facilitator, as Linda and I discussed mathematics content, the Mathematics Task Framework, and pedagogy issues. I outline each of these roles in more detail below.

Participant Observer

My participation observations focused on the instructional moves that Linda made in the classroom in terms of the routines that she had in place, the tasks that she assigned, the way the tasks were set-up for the students and her interactions with them after she set-up the task. I took fieldnotes of my analysis of the tasks using the lens of the Mathematics Task Framework (MTF) (see Appendix D for details of levels). Some of the aspects that I noted were the cognitive demand that was present in the task as it was in the curriculum and the way the task fit into the previous work that the students had been doing. I also examined the teaching moves that Linda made as she set-up the task in the classroom, with particular attention to her efforts that changed the level of cognitive demand. I also noted the questions that the students asked and the overall support that Linda provided them. The class was arranged in groups of three or four and I would sit at a table and observe and interact with these students as they attempted the assigned task. I would engage the students in a discussion if I saw that they were not making progress with the problem or if they were drifting off-task. Further, I also made an effort to keep track of Linda as she moved among the groups and the time that she spent at each. The information that I collected from my classroom informed my role as a facilitator in the one-on-one discussions, outlined in the next section.

Insider

The partnership that I focused on in this study was based in the professional development that Linda had requested at the beginning. Linda had specifically requested that this be rooted in developing her own mathematics content knowledge. She felt that it could impact her teaching and, ultimately, the learning of her students. The classroom observations allowed me to ground this development in the context of her classroom so that it would be more meaningful for Linda. This aspect of my role made me an insider

(Ball, 2000b) to the process that I was going to study. As an insider I was required to separate my roles as a researcher and as a facilitator. This resulted in a methodological challenge; I had to avoid ongoing data analysis to prevent a constant switching of roles between facilitator and insider. I worked around these difficulties by being a facilitator during the course of the study and then analyzing the data at the end. This enabled me to maintain separate roles and prevented constant switching. Despite the difficulties of this form of research in studying the process I was creating, in this case it was the best avenue to take since I could tailor the content to suit the needs of Linda. Observing a teacher who was already a part of ongoing professional development project would not have got to the phenomenon that I was seeking to observe, namely the growth of a partnership between a teacher and researcher. It was vital therefore, that I focus on myself and my role as much as on Linda. In my role as a facilitator I discussed the mathematics content, the MTF, and pedagogy. These issues were not distinct, for example discussions involving the MTF were based in the content and raised issues of pedagogy. I outline my role in each of these areas in more detail below.

Content. The four major topics that Linda and I discussed were probability (seventh grade), functions (seventh grade), statistics (sixth grade) and algebra (sixth grade). Simon's (1997) ideas about teaching with a constructivist perspective guided my work with the teacher as I tried to pose problems and facilitate discussions around these problems. My goal was to challenge Linda's thinking in ways that would enable her to 'unpack' (Ball & Bass, 2000) the mathematics concepts. Linda would inform me in advance about the topic that the class would be working on and I would plan activities or problems for our discussions or some for her to work on independently (see Appendix B for selected problems). Based on our discussions of the problems, I would decide if Linda needed to work on additional material that might further support her understanding of the central mathematical ideas. Later in the study, the discussions were based on the needs of Linda and my role changed to preparing a broad range of activities and problems. I read widely on the topic of focus in the classroom and also watched professional teaching videos (Annenberg Media, 2001) for ideas that could be useful in our discussions. The activities that I chose or assigned for our one-on-one discussions were sometimes modified for use in the classroom, but this was not my explicit intention at the outset of the study. The focus in the beginning was Linda's growth as a mathematics learner.

The mathematics task framework. Our study began with a task sort activity (see Appendix A for details of the tasks used) that Linda did at home and reported on her classification during our first meeting. This was supposed to initiate Linda's thinking about tasks through their cognitive demand. As Linda reported on her task sort, I probed her reasons for classification and asked clarifying questions about her thinking. During the course of the study, I assigned Linda readings from *Implementing standards-based mathematics instruction: A casebook for professional development* (Stein et al., 2000). The time pressures in this study meant that we only discussed the first two chapters in the book. Our second discussion on the MTF involved a discussion on the first chapter that was an introduction to the MTF and the *Levels of Cognitive Demand* (LCD) of tasks. As an assignment to tie the MTF with Linda's work, I requested her to reflect on the tasks that she had been assigning in her classroom and determine the categories they fit based on the *Task Analysis Guide* (TAG) (see Appendix D). Our third discussion on the MTF covered the second chapter in the book that focused on the factors that changed the LCD in the classroom. Again I asked Linda to read the chapter keeping in mind her classroom. During our discussion, I also asked her to think of a task that she considered of a low LCD and how she might raise the LCD for the students.

Pedagogy. I supported Linda with planning of lessons especially if they were based on an activity that we had discussed. After implementation, we would usually debrief about the issues that arose from the classroom and these issues would inform further planning. I generally sought to focus Linda's attention to the mathematics that was present in the activity and how this related to our one-on-one discussions. Another aspect that guided my support for Linda was the Pedagogical Content Knowledge (PCK) and the mathematics knowledge for teaching that was discussed in Chapter 2. PCK emphasized representations and knowledge of students' strengths and weaknesses, so I paid attention to instances where these could be brought to the fore. I pointed out connections that were present between the current activity and other topics or problems that the students had done previously. Linda contributed to our discussions with her knowledge of students' thinking and evaluated the 'fit' of tasks in her classroom. Linda also shared ideas about teaching and sought feedback from me. On one occasion she consulted me on the size of groups in the class that would counter the off-task behavior that she observed sometime. Thus I was a resource for new ideas in teaching and I also helped Linda with thinking about her own pedagogical ideas.

Data Collection

Data collection for this study began on the 13th of March 2006 and ended on September 29th, 2006. Data from 13th of March till May 12th was from Linda's seventh grade class and the data from August 24th till September 29th was from the sixth grade classroom. The entire data set consisted of 35 classroom observations and 32 after class discussions (ACDs). There were a mixture of fieldnotes (see My Role as a Participant Observer for details of the methodology), audio, and video recordings of the classroom observations and the ACDs. The audio and the video recording in the classroom were mainly to fill out my fieldnotes and to record Linda's task set-up. The central idea in the data collection in the ACD was to closely record all the interactions between Linda and me using a combination of audio and/or video. Note that even though my research questions changed (in mid April) from a focus of the Mathematics Task Framework to the nature of my partnership with Linda, the data that I colleted always included our interactions around the problems we discussed. As soon as I got permission from all the students in the seventh grade classroom, I videotaped in the classroom for a short period. Prior to this Linda moved around this seventh grade classroom with the audiotape so that I could record her talk while she was teaching. After August, there was a new sixth grade classroom, and I did not get permission to videotape from a number of students who either did not return the consent forms or refused to be videotaped. I decided not to video or audiotape in this sixth grade classroom and instead relied on my fieldnotes. I used the ideas of the MTF (Stein et al., 2000) to organize my fieldnotes by dividing the class time into the task set-up and task implementation. I focused on Linda and the moves that she made in the classroom, such as taking the attendance, instructing the students on the plan for the day, setting up the problem for the students, asking them questions, and assigning them work to do in their groups. The details of the data collection are laid out in the Table 3 below, where the '*' represents data collected. Besides this data, Linda wrote three reflections and I kept a personal research journal through the duration of the study.

		Classroom			After Class Discussion		
Date	Day	Notes	Audio	Video	Notes	Audio	Video
03/13	M	*			*	*	
03/14	T	*			*	*	
03/16	TH	*			*	*	
03/20	M	*			*	*	
03/21	T	*			*		*
03/22	W	*			*	*	
03/23	TH	*				*	*
03/24	F	*	*		*		
03/27	M	*	*		*		*
03/28	T	*	*		*	*	
03/29	W	*	*		*		*
03/30	TH	*	*		*		
04/03	M	*			*		*
04/04	T				*	*	*
04/18	T		*				*
04/19	W		*			*	*
04/20	TH		*			*	*
04/21	F		*				*
04/24	M		*	*			
04/25	T		*	*			
05/01	M		*			*	*
05/02	T			*		*	*
05/03	W			*		*	
05/05	F		*	*		*	*
05/09	T		*	*		*	
05/10	W		*	*		*	
05/12	F		*	*			
08/24	TH	*				*	
08/28	M	*				*	
08/29	T	*				*	
08/30	W	*				*	
09/01	F	*				*	
09/15	M	*				*	*
09/26	T	*				*	*
09/27	W	*				*	*
09/29	F	*					

Table 3: Details of data collection

Data Analysis

The insider perspective (Ball, 2000b) calls for a separation between the phenomenon and the analysis of the phenomenon,

> …researchers using this approach must be able to treat their experiments, settings, and work as matters for scrutiny. They must be able to view the teaching, the students, and the learning in the context of, but also apart from, their efforts and desires (p. 392)

In this study the bulk of the data analysis was done at the end of the data collection period and interactions with Linda. Thus during data collection and the one-on-one interactions with Linda, I assumed an insider perspective, and later I assumed the role of a researcher as I analyzed the data. This change in role after the study prevented me from doing ongoing data analysis, which would have involved a continuous switching of roles. My data analysis followed the cycles outlined in Creswell's (1998) 'Data analysis spiral'. I outline the general course for my analysis and then the data analysis for each question in more detail. The four stages that I followed (Creswell, 1998) were as follows:

1. Data management: I transcribed all the audio and video of the ACD sessions that I had with the teacher. I also transcribed parts where Linda set up a problem in the classroom, which I used to answer my third research question. Note that the videos were first converted into DVD's with help from the CEMELA video editor. I organized these transcriptions along with my fieldnotes from the classroom observations, and reflections by the teacher into NVivo 7 (QSR International, 2006) files. Each file would have some general comments about observations that I made or things that came up in informal conversations with the teacher, the classroom fieldnotes for the day, and the transcript of the audio or video of the ACD. I maintained a research journal from the beginning of the

study in NVivo 7. To help me manage the large volume of data I made tables to record the type of data that I had for classroom observations and the ACDs (see Table 3). I also made a table that would outline the topic and content of the classroom teaching and our ACDs (Appendix F).

2. Reading and Memoing: I read through all the transcripts of the ACDs and immersed myself in the data to get a sense of the whole data. I kept track of important ideas by writing them in a notebook and in my personal journal in NVivo 7.

3. Describing, classifying and interpreting: I went through the transcripts with one research question in mind and coded, with free nodes, themes connected with the given question. I also marked other interesting themes that occurred. In the first research question, I organized the free codes into broader categories using tree nodes. I interpreted these broader categories and described them, and in some cases my analysis took me back to the raw data.

4. Representing and Visualizing: I packaged the data to represent my interpretations like the stages in my evolution with the teacher.

Analysis of the First Question

Research Question 1: How does the nature of the partnership evolve over time? What are the constraints on the teacher and the researcher in such a partnership?

The data analysis for this question was guided by the social aspect of the Emergent perspective and led to a focus on our interactions that were directly observable in the data I had collected. By focusing on Linda's interactions, I free coded the after class discussion data for themes like 'Linda answers my questions' or 'Linda on student thinking'. I examined my interactions and got codes such as 'I teach' or 'I suggest' that referred to the instances when I taught Linda mathematics or made suggestions about her teaching. I went through the entire data and arrived with a set of 28 codes. Examining the codes closely I came up with the categories 'Linda's interactions' and 'My interactions' that categorized various free codes that were associate with each of our interactions. There were codes that did not fit into these categories like 'disciple issues', 'student pressure' and 'Linda's beliefs'. Examining the codes closely, there were two codes that captured our discourse, rather than just one persons talk (these were already in the categories 'Linda's interactions' and 'My interactions'). The 'conceptual discussion' captured our discussions that involved mathematics concepts and 'Disconnect' examined the sudden shifts in the discourse from a more general mathematical topic to something specific about planning of an activity. I wanted to find out the reason for the occurrence of the 'Disconnect' code and how it was distributed in the data. In NVivo 7 this was easily done by generating a report that gave the exact location of the data that was coded. Once I observed that the 'Disconnect' codes were clustered at a certain time, I reexamined the category 'Linda's interactions' before and after the cluster of the 'Disconnect' code and observed a shift. For example, the code 'Linda on student thinking' had an instance (after the 'Disconnect' cluster) when Linda selected the Median problem (Appendix C) to perturb students' thinking based on her observations in the classroom. I also observed shifts in the 'I teach' code in the 'My interactions' category. This led to a conjecture that the partnership could be evolving in three stages with the 'Disconnect' being characteristic of the second stage. At this point I made an association between my ‘Disconnect’ code and the appearance of tensions in collaborations in the literature. Once I theorized the partnership evolving through three stages, I went over the data again and examined how they fit (or didn’t) into the stages. There was enough evidence to convince me that the three stages were reasonable, but there was also evidence of overlap in the stages. I have outlined the stages and the anomalies in Chapter 4.

Analysis of the Second Question

Research Question 2: How does the partnership contribute to mathematical and pedagogical issues that arise in planning, teaching, and assessment?

I went through the entire transcribed data set and isolated all the planning sessions, discussions about teaching, and discussions about assessments. I free coded this data using broad categories of planning, teaching, and assessments. I generated a report in NVivo 7 for each of the categories of planning, teaching, and assessments and read the entire report. I also went back to the raw data and listened to the audio or watched the video segments that related to this part. I made notes about the mathematical issues that arose and the stages in the partnership that they occurred. For example in planning, there were three major tasks that were planned in each of the stages, namely the Tile problem (Stage 1), the Lollipop problem (Stage 2) and the Shoe problem (Stage 3). I examined and reported on the mathematical issues that arose in each of these tasks and how these were typical of that stage in the partnership. I also did this for the reports on teaching and assessments. In the case of assessments, all the discussions occurred during the first stage of the partnership.

Analysis of the Third Question

Research Question 3: How does the partnership influence the curriculum with respect to the tasks the teacher chooses to use in the classroom and how the tasks are utilized by the teacher?

I isolated 13 tasks that were representative of each of the three stages in our partnership. Next, I listed the tasks and the transcript of Linda setting up the task in the classroom. I then analyzed each task as they appeared in the curriculum using the Task Analysis Guide (TAG) (Appendix D) and assigned one of the four categories of *memorization*, *procedures without connections*, *procedures with connections*, and *doing mathematics*. I also justified my classification. The next step involved reading the transcript of Linda setting up the task in the classroom and using the TAG as a lens to examine the cognitive demand of the task at this point. I searched for factors that might have either lowered or maintained the level of cognitive demand and summarized my findings. I constructed the table (Table 4) using the summaries that I just made and this helped me write the report. The summaries of my classification of the tasks using the TAG can be found in Appendix E.

Stages	Date	Classroom task	Level of Cognitive Demand of task in the Curriculum	Level of Cognitive Demand of task in the Set-up
First	3/24/06	Tile problem	*Procedures with connection* (PWC)	PWC
	3/27/06	Dice problem	PWC	*Procedures without connections* (PWOC)
	3/29/06	Crime Scene Investigation problem	PWOC	*Memorization* (M)
	3/30/06	Match/No Match problem	PWC	PWC
	4/19/06	Bike trip Day 1	PWC	PWC
Second	4/20/06	Bike trip Day 2	PWC	PWC
	4/21/06	Bike trip: Problem 2.2 Follow up	PWC	PWC
	4/24/06	Lollipop problem	PWC	PWC
Third	5/3/06	Bike trip: Temperature problem	PWC	PWC
	5/5/06	Bike trip: Day 4	*Doing mathematics* (DM)	PWC
	8/28/06	Median problem	PWC	PWC
	8/29/06	Shoe problem	PWC	PWC
	9/26/06	Patterns problem	PWC	PWC

Table 4: The level of cognitive demand of classroom tasks

Validity

In this study, I tried to adhere to conditions for validity outlined by Creswell (1998). I spent a significant amount of time at the school, in the classroom, and conversing with the teacher. Besides our regular discussions, we would also talk about other aspects in our lives and developed a relationship before I began data collection. I have outlined my perspective and my conceptions of what it means to study mathematics as well as my learning and teaching experiences that give a perspective to the reader. For example, my technique of getting Linda to solve problems at the board is reflective of the way I learnt the subject and also the way I interacted with my teachers at the university. I have used thick descriptions to give the reader a sense of the work we did, the problems I chose, my motivations for choosing them, and Linda's background. I have also outlined how issues of validity were addressed during the study in Chapter 5. The next chapter examines the results of this study.

CHAPTER 4: RESULTS

In this chapter I outline the results, from the study, for each of the three research questions given below.

What is the nature of a partnership between a Middle School teacher and a university researcher centered on conversations about the mathematics content and teaching?

In particular:

1. How does the nature of the partnership evolve over time? What are the constraints on the teacher and the researcher in such a partnership?
2. How does this partnership contribute to mathematical and pedagogical issues that arise in planning, teaching and assessment?
3. How does the partnership influence the curriculum with respect to the tasks the teacher chooses to use in the classroom and how the tasks are utilized by the teacher?

First I begin with a brief outline of Linda's teaching portrait at the start of the study. I then outline the stages of evolution of our partnership and the constraints that were present in the partnership. Next I examine each of the stages of evolution, in detail, with respect to the mathematical issues that arose in planning of lessons, teaching the lessons in class, and the assessments. The last section examines the evolution of the cognitive demand of the tasks that Linda assigned in the classroom. In this section I also outline the discussion that we had based on the task sort and readings about the Mathematical Task Framework that Linda had done.

Initial Teaching Portrait

Linda's typical class began with the students recording the goals for the day's work from a chart that was put up near the door. This chart was based on the Performance Objectives (POs) laid out in the State Standards. Once the students were at their seats they were required to record their name, the date, and the topic for the day on a sheet of paper. The students also recorded the details from where the problems were chosen, either the textbook or the associated workbook, along with the page numbers. While the students were engaged in setting their things up for the class, Linda would take the attendance. If they were starting a new topic then Linda would begin by introducing a related problem or activity, from the textbook, and engage the students in a discussion. Linda spent time on direct instruction on topics if she felt were challenging to the students and she would explain the procedures by breaking them into small parts. If the students were continuing from the previous day's work, Linda would ask the students to recall things that they did and then proceed with the day's work. Linda chose her activities from the textbook based on the Performance Objectives that were outlined in the State Standards. At the end of the class period, Linda collected the sheets that recorded the students' class work for grading. Linda graded the students on completion and the work was kept in the students' folder in the classroom.

Linda would usually spend time at the beginning outlining the question(s) being asked and ensure that the students understood the question(s). In some cases Linda engaged the students in a discussion about initial ideas of the solution and then allowed them to work in their groups. Linda would move around the groups and ask the students about their work. If students were unable to make progress, Linda would explain things to the students in detail by breaking up the problem into smaller steps. The students would wait for Linda's help, instead of trying to resolve the problem through discussion in their groups. Thus Linda was pressured into discussing with all the groups and invariably this left her with little time to have a closing whole class discussion. It seemed that the students were dependent on her help and this was a point of discussion at some of our meetings.

Evolution of the Partnership

The partnership can be categorized into three stages. In the first stage our discussions focused on the mathematics content and I controlled the agenda through introduction of the Mathematics Task Framework (MTF) (Stein et al., 2000) and problems of probability and functions. The second stage was characterized by the appearance of content-teaching tensions between my agenda of discussing the mathematics content and Linda's need to discuss planning. In the third stage the tensions were resolved through a change in focus to planning of lessons and discussions of the content when Linda desired or the situation arose. Even though I have classified these stages, the boundaries between them are blurred and there is evidence of overlap, which I outline below.

The results of the study are extensive and the tables below could serve as an organizer. The three stages of the partnership are the key points with respect to which other issues are discussed. Organizer (a), (b), and (c) relate to the first, second, and third research questions respectively.

	Stages	Key concepts at each stage
Discussions	Stage 1	Focused on mathematical concepts by working on challenging problems.
	Stage 2	Appearance of content-teaching tensions in discussions of the mathematical content. Teaching issues are discussed.
	Stage 3	Focused on teaching, student thinking, and mathematics content. Alignment with mathematics knowledge for teaching.
Roles	Stage 1	I determined the material discussed. Linda was a teacher-as-learner.
	Stage 2	Oscillated between Linda and me as Linda pushes to discuss teaching issues.
	Stage 3	Linda decided the topics for discussion. I supported her by being a resource.

Table 5: Organizer (a)

	Stages	Key concepts at each stage
Mathematical and pedagogical issues in planning	Stage 1	Focused on mathematical concepts in the problem. Little focus in pedagogical issues.
	Stage 2	Linda sought my input on mathematical issues that arose in the problems she selected. Efforts to focus discussions from logistics to the mathematics.
	Stage 3	Linda adapted a problem and evaluated tasks by examining them through the students' eyes. Incorporated student thinking in planning tasks. Linda learned to use technology in problems to share with her students.
Mathematical and pedagogical issues in teaching	Stage 1	Minimal.
	Stage 2	Focused on obtaining the correct solution for the problem. Discussion on students' difficulties that arose in the classroom.
	Stage 3	Linda reflected on her teaching. Attempted to challenge students further.
Mathematical and pedagogical issues in assessments	Stage 1	Language of the exams versus language of the CMP. Time pressures related to 'covering' the curriculum.
	Stage 2	Not discussed.
	Stage 3	Not discussed.

Table 6: Organizer (b)

	Stages	Key concepts at each stage
Task selection	Stage 1	CMP tasks that addressed the State Performance Objectives. Procedures with connections tasks.
	Stage 2	Task from our one-on-one discussions and a CMP tasks. Both addressed the Performance Objectives. Procedures with connections tasks.
	Stage 3	Linda adapted or chose tasks to suit need of students and also incorporated the State Performance Objectives. Procedures with connections.
Task set-up	Stage 1	Same as the tasks appeared in the curriculum. No extra information provided. Procedures with connections tasks.
	Stage 2	Some ambiguity was reduced with a push for comprehension. Linda attempted to set up norms to promote independent and group thinking. Procedures with connections tasks.
	Stage 3	Push to promote independent and group thinking. Procedures with connections tasks.

Table 7: Organizer (c)

Stage 1

The agenda at this stage, near the beginning of the study, was determined by me. I introduced Linda to the MTF through a task sort and readings from *Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development* (Stein et al., 2000) and then discussed probability, which was a topic Linda would be teaching in the coming weeks. We discussed probability through problems in the *Connected Mathematics Project* (CMP) curriculum (Lappan et al., 1998) and *Mathematics for Elementary School Teachers* (Bassarear, 2005). Our sessions involved Linda working on a set of problems that emphasized problem solving and conceptual connections and we had discussions around her solutions or attempts. In the process Linda was introduced to the definitions and procedures. Aspects of my sharing included, answering Linda's questions, guiding her thinking through questioning, explaining a concept or talking about general ideas in mathematics or teaching.

I always sought to finish a given set of problems in a time frame but after the first few interactions, I adjusted to Linda's pace. Linda's discourse at this stage involved answers to questions I posed and clarifications she sought on concepts and steps in the solutions that were not clear to her. Linda's participation in the discussions can be characterized as a teacher-as-learner. Below I provide two vignettes that illustrate a typical discussion around the content.

Interactions around a Problem in Stage One

I had just worked out the Basketball problem 1, where the probability of getting zero, one or two points had to be calculated in a one-and-one free-throw situation, given a success rate of 60% (see Appendix B for more details). I then asked Linda to solve the problem if the condition of the second throw was changed (Basketball problem 2, Appendix B). She could use the representations that were on the board, namely the tree diagram (Figure 3) and the area model (Figure 4), to work out her solution. The following tree diagram was on the board and the probability on each branch was either 0.6 (success) or 0.4 (failure).

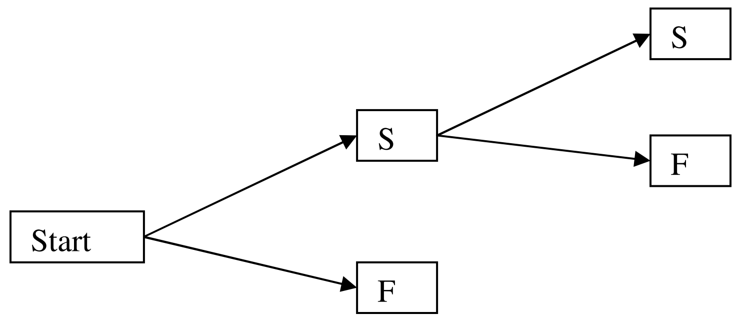


Figure 3: Basketball problem 1 (Tree diagram)

The final probabilities were also worked out using a unit square as follows:

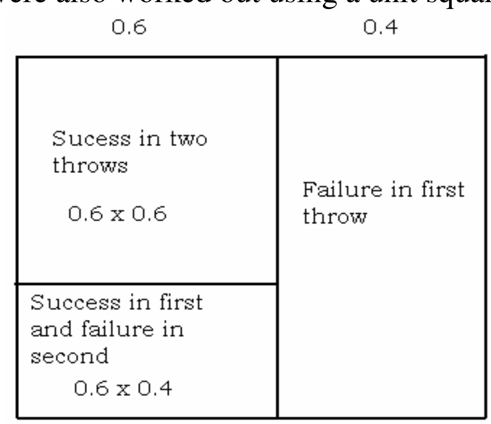


Figure 4: Basketball problem 1 (Area model)

The discourse below represents Linda's attempt to make the modifications to the above representations so that the new condition was reflected and the new probabilities could be worked out for getting zero, one, and two points respectively.

L: So if there is (pause) if the first throw doesn't count, then you would just do 60 percent you don't need … (A interrupts)

A: No first … she is given two throws, regardless of whether the first throw was a success or failure … like here when it was a failure, she was not given a second throw. But now suppose we throw away that rule and say that regardless … you are given two throws regardless … so then how would you change this model. How would the problem change? (pause)

L: So she always gets two throws? (A: Right) regardless … would these like have to change (gets up from her seat and approaches the white board and she points to the sample space and says) would these like have to change to 00, 01 and then and this one would be 10 and 01 [the 0 represents failure and 1- success] (A: Right)

A: So the sample space itself is changing …

L: It gets larger (Video transcription, 3/21/06)

This pattern of interaction, of me providing an example and then allowing Linda to think about extensions of a problem was typical at this stage of our partnership. In other cases Linda would begin a problem and we would discuss areas that she was unable to work through. In these cases, the agenda and the discourse were mostly controlled by me and Linda reacted to questions that I asked or asked questions of her own about the mathematics involved.

My control of the agenda and discourse as described above also filtered into the lesson planning and there was minimal interaction from Linda. The second vignette represents a typical discussion based on teaching in the first stage.

Planning the First Probability Lesson

I suggested to Linda that using the Tile activity would be a good introduction to the concepts of experimental and theoretical probability, which also matched the Performance Objectives (POs) laid out in the State Standards. In this activity the students were required to find the fraction of tiles of each of three colors that were present in a bucket (see Appendix B for more details). I outlined for Linda the important ideas to focus on like the higher the number of trials, the closer one would get to the actual fraction of a color. Linda mentioned that it was hard for her to keep track of all the things that she had to mention in class. The following dialogue below was an example of the detail that I spoke about during the planning session.

> A: This is very similar to the activity that we had done … except now you have a bucket with these cubes and then you can have each one … quite a few of them come up and choose. But … I guess when you present the problem that's … when you present it you don't really tell them that is what we are trying to find out. All you tell them is that there are some blue, yellow and red … generate a discussion from there ... how they are going to figure out the … the fraction of each color that is present in this box. And so one thing that we have to think of is if no one comes up with an idea, of course someone will come up with some sort of an idea, then what sort of ideas do they come up with and how can we lead them from those ideas to where we want them to go. What do you think they will come up with? (Video transcript, 3/13/06)

With minimal interaction from Linda, I controlled the planning session and so Linda did not have ownership over the activity and relied on remembering the important aspects that she would present to her students. On enactment, the students followed the procedures that were outlined for the activity, but did not explore the important ideas of experimental and theoretical probability. After debriefing, Linda attempted to address these aspects in the next lesson by choosing the Dice problem where the students rolled the dice 12 and 24 times and calculated the experimental and theoretical probabilities (see Appendix C for more details). The students later compared their results to the theoretical probability of the same events and decided that the 24 rolls gave a better approximation. Revisiting ideas through similar problems was a theme that reoccurred in Linda's teaching.

Stage 2

This short stage represented the appearance of tensions in the discourse between the content and the immediate planning needs of Linda. Until this point Linda followed along with our discussions about the content and general mathematics concepts and applications. But at this stage the discourse indicated a push by Linda to attend to her planning needs. Linda sought to discuss her planning and the important ideas that she should focus on in her lessons. In these discussions about the planning of lessons, I tried to ensure that the mathematical connections in the problems were brought to the fore. Attention was also given to student thinking that could arise in the lessons.

The following vignettes illustrate the surfacing of a tension in the discourse and the push, by Linda, to shift the discourse to planning of a lesson.

Appearance of Tensions

The discussion below was part of a larger discussion about algebraic word problems. I tried to emphasize the need for procedures in the promotion of algebraic thinking and Linda suddenly changed the topic of discussion to her planning needs.

> A: … just doing the word problems and knowing how to represent them won't get you to the solution because you still have to solve the [equations].
>
> L: Now going back ... to the lollipop problem, if I were to do that on Monday you know, just give them some fun before I leave … I mean it's basically what we have done, but how can I set it so that it is adding something else to it?
>
> (Video transcription, 4/20/06)

The above discussion represented a sudden change in our discourse from a broader discussion of algebra, towards the specific planning of a lesson that Linda wanted to teach. Changes in the discourse usually occurred when I spoke about general ideas in mathematics that were not immediately applicable in Linda's classroom. This stage contrasted with similar interactions in the first stage where Linda would not interrupt our discussions with planning needs.

With the appearance of tensions there was a shift in our discussions towards teaching; however the discussions on teaching were intertwined with discussions of the mathematical content. In the following vignette, Linda shifted the more general discussion we were having about applications of mathematics, to feedback she needed on the Lollipop problem. This was a problem that Linda and I had done as part of our discussions on functions and change. The Lollipop problem involved finding the time it would take to suck a lollipop by recording the circumference after fixed time intervals and doing an 'approximate' regression line. (See Appendix B for more details).

L: Well back to class. This one [Referring to the Lollipop problem].

….

L: So Tuesday ... tomorrow wrap up Lollipop?

A: Yeah

L: And key thing is ... I was thinking of maybe having them define ... what was the line of best fit, see if they remember what that is [italics added].

A: Ok

L: What do you think?

A: Yeah

L: And why it's important?

A: Yeah (Audio transcript, 4/20/06)

Linda wanted to go over the questions that she would ask the students to ensure that the students were working on important aspects of the problem. I then maneuvered the discussion to focus on the content by looking deeper at how the Lollipop problem was different to the previous Bike problem that the students had worked on. The Bike problem was taken from the CMP curriculum (Lappan et al., 1998) and involved graphical and tabular information about the distance and time covered on each day of a five day bike trip. The questions involved working out the rates at which the bikers were moving, converting verbal information into a table or graph, and interpreting graphs (see Appendix C for more details). Both the Bike problem and the Lollipop problem involved graphing change, in the first case the coordinate points were joined by straight lines to form a piecewise defined function, and the second case involved a regression line. I felt that it was important for Linda to understand the differences and the reasons behind the difference. We had the discussion below around this aspect.

A: I was just trying to find the tie up you know between what they did for that Bike problem and what they are doing for this problem (Lollipop problem) … like there they joined the points and here they just drew one line. Trying to get them … to see if they thought that there was anything different …. and why they didn't join the points in this one too.

(pause)

L: And the purpose was …they didn't join 'em because we had to estimate what the…how long it would take for them to finish right?

A: Right

L: Where as in the bike one we knew the beginning and the end right?

A: Because in the bike one you didn't really have to go beyond … you were estimating … all your things were within that interval … in this time period ….

A: They were always working within that interval that was already given … I mean they didn't have to predict things about … like if they…did this journey for 20 hours, what speed they would be moving at in that point in time?

L: Ok

A: So they didn't have to do that sort of thing … outside the range … but this is … slightly different where you are asking them. They [are] only worked within a 6 minute range, but you are asking them to predict on 15 minutes … you have to find out some thing which is constant ... line through the data … it makes things easier to get to that prediction. While if you have these other piecewise lines … then it is little harder …because when you come to the last part, you don't really know how to draw the final one (L interrupts)

L: Ok … so go over the line that best fits …. and then what about the slope? (Audio transcription, 5/01/06)

At the end of this discussion, Linda again focused on the planning or implementation. In general, the mathematical issues were intertwined with discussions of planning and this was a characteristic that was present in the next stage of our partnership. At this stage Linda looked to me to provide guidance in teaching issues that arose in the class. This would be different in the next stage as Linda took a more proactive role in thinking and resolving teaching issues that arose in her class.

Stage 3

This stage was characterized by a move towards resolution of the tensions, from the second stage, by shifting the discourse towards planning lessons and debriefing. The mathematics content was intertwined with the discussions of planning and student thinking. In this stage, Linda took an active role in adapting problems for the class and reflected on the difficulties that arose in the class discussions. I assumed the role of being a resource for Linda by suggesting activities (but letting her decide if she wants to use them or not) and by giving feedback for her ideas and answering her questions. At this stage, Linda's agenda featured prominently and I supported her endeavors. I will highlight Linda's role in this stage with two examples, (1) Linda adapting an activity for her class and (2) Linda reflecting on a teaching issue.

Linda Creates an Activity

This example was part of a planning session in Statistics. Linda expressed a desire to do an activity with the students that would build on their previous lessons on various data representations. We decided to brainstorm and came up with a number of activities before Linda decided to adapt the Hat problem into the Shoe problem, which was appropriate for the students in her class. The Hat problem involved a person starting a hat shop who had to determine the number of hats he should purchase, given that hats came in lots of a thousand (see Appendix B for further details). The Shoe problem was similar, but involved shoe sizes of middle school students (See Appendix B for more details).

> L: I mean we could say that you know that [Name of School] has a new line of tennis shoes. They have partnered up with Nike and

> they need to know what sizes they need to buy for Middle School kids. So you can ask them to interview 25 people or whatever ... between the ages of 13 and 15 … or 11 and 14 … I don't know … I'm gonna give this out, but I don't know how much I'll get back and… whether we will be able to do it. (Audio transcript, 08/24/06)

This was a contrast to the role that Linda played in the planning of a lesson in the first stage where she accepted my suggestions without much discussion. Further, in this stage the activities were not adopted directly from the curriculum but were adapted to suit Linda's and the students' needs.

The activities that Linda assigned the students differed from the first two quarters before the study and as a result there were student difficulties that had to be resolved. In the previous stages Linda would seek my guidance in resolving these issues but in the third stage Linda attempted to resolve these issues on her own. The vignette below illustrates this point.

Linda Reflects on her Teaching

Linda introduced the students to the Shoe problem that she created through our discussion. As the students attempted to work on the problem, they could not recall how to make a frequency chart. This was a surprise to Linda as she had introduced them to the frequency chart in the past week. In fact she expected that they would be challenged by the proportional reasoning required in the problem. But the students were able to use proportional reasoning when required.

> L: Because right away when I said we have 20 pairs of shoes how many need to be 5 you know how many people … had a size 5 … that was easy for them. But getting them to just put the

> frequency chart that was a struggle (laughs). I mean a real struggle. Then I'm thinking to myself, well how much of it do I do as a whole group because then they are just going to emulate what I'm doing and not going to think about it themselves. So I kind of just played with it all day long. Some classes I went as far as making a frequency chart.
>
> (Audio transcript, 08/28/06)

This episode was typical at this stage in the partnership when Linda was actively reflecting on the students' thinking and the amount of guidance that she was providing them. This differed to her reaction in the second stage when she looked to me to provide guidance in resolving teaching issues that arose in the class.

Overlap of Stages

The stages as described above seem to be clear cut, but this was not the case and there was some overlap. In the first stage, there was one instance of the content-teaching tension, but I did not begin the second stage at this point since later there was a cluster of four instances of content-teaching tensions that could better characterize this stage. In the first stage there was also evidence of interactions consistent with the third stage when Linda reflected on the Dart problem that she had assigned her students. In the Dart problem, the students had to find the probability of landing in a certain region (See Figure 5 below) of the dart board shown below (see Appendix C for more details):

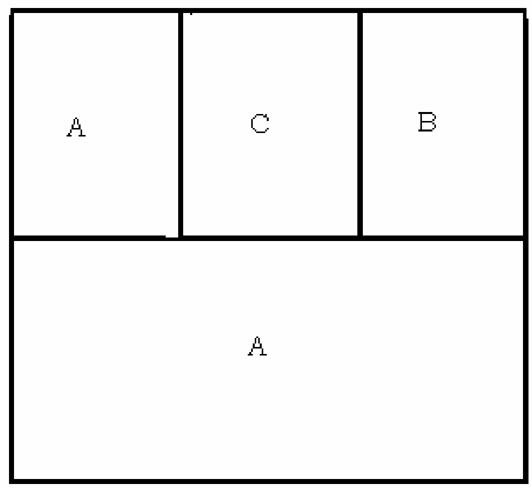


Figure 5: Dart board

This problem arose as part of our larger Mathematical Task Framework discussion on the factors that lowered or maintained the level of cognitive demand (Stein et al., 2000) of high-level tasks that were implemented in the classroom. Linda used the Dart problem to show the complexity of balancing student engagement with the level of cognitive demand of the task:

> L: So I wasn't sure how exactly to hold them accountable, but yet not lower the cognitive demand of the tasks …. And then today I found myself ok that's my task and everybody sat there not knowing what to do …. I'm trying to get the right answer, so lowering the demand again so… I found myself, because they are sitting there and they don't even know how to start. They have their diagrams they have everything correct but "Miss what do we do?" I'm like trying to question them but nothing works, so then I'm like ok let's break it down, so then I had to go draw another one, but I didn't know if I had to but I went [ahead] and drew another one where I only had 2 pieces and I put A and B and then I said what's the probability of getting A …. but I had to go back. So what should I have done [differently] at that point so that (shrugs).

This discourse was characteristic of the third stage where Linda showed evidence of reflecting on her teaching. This was sporadic in the first stage.

The second stage was elusive and I made a decision to begin and end the stage to include a cluster of four tensions that I coded. However, these tensions were interspersed in larger discussions of the mathematics content, which was characteristic of the first stage. The unclear boundary between the second and third stages was exemplified by an episode where we discussed group arrangements in the classroom and algebra word problems. There was no occurrence of content-teaching tensions in these episodes, but I categorized them as part of the second stage as they were distinct from the third stage.

There were two discussions of algebra at the end of the third stage that did not fit with the general characteristics of this stage described previously. These discussions focused on the nature of variables in algebra and a discussion on monomials, polynomials, and operations on polynomials. The focus on the mathematical content mirrored the first stage, however Linda's engagement differed as she participated actively in asking reasons of why things were true and worked independently on the problems at the white board with little help from me.

Summary

The partnership flowed through three stages where our roles shifted over time. In the first stage, I was active in bringing problems and engaging Linda in discussions of the content. At this stage Linda focused on learning the content during our sessions. In the second stage, Linda's need to discuss her planning with me manifested itself in the form of content-teaching tensions. These tensions were resolved in the third stage as I assumed a more supportive role and Linda was proactive during the sessions with topics that she wanted to discuss. In the last stage, Linda actively adapted an activity that built on students' knowledge and became reflective of her practice. The stages were not distinct and there was a degree of overlap.

There were issues that had to be negotiated both by Linda and me during the course of the partnership. The next section examines the constraints that were present for Linda and me in the partnership. The word 'constraints' seems to convey a negative connotation, but this was more a reflection of the dynamic nature of the partnership.

Constraints in the Partnership

This section highlights the constraints that were present both for Linda and me in the partnership. The partnership was evolving over time as we got to know each other and there were constant adjustments that had to be made. Other constraints include the amount of guidance I should provide Linda, the frequent assessments that the school had, Linda's perception of me being the expert, time, and the classroom issues.

Initial Planning

In the first stage, I controlled the agenda and planned the activities and problems that Linda would work on in the after class discussions. Setting up these activities and problems was challenging for me. During the planning of these sessions, I considered problems that would illustrate powerful connections in the mathematics to other parts of the curriculum and some or all of the following factors:

1. The prior knowledge of the teacher: Linda had schooled knowledge, knowledge acquired through professional development activities and also through teaching. I wanted to build on this knowledge.
2. Her teaching needs: I was reluctant to discuss mathematics that was far removed from her practice in the classroom. My observations of her class and interactions with the students also informed my planning.
3. Level of challenge for the teacher: The activities needed to challenge the teacher and allow for the construction of new knowledge and address misconceptions (if present).

If we examine the first point, Linda came to the study with a varied mathematics background. For example she did not study functions as part of any of her courses in college and she relied on her professional development and teaching experiences in this area. On the other hand, Linda had a numerous experiences with Statistics as part of her college courses, professional development activities, and teaching. Thus she expressed a greater level of comfort in Statistics and sought to learn functions through our sessions.

Once I decided on the four criteria listed above, I had to find activities that would address them. I looked at various sources such as professional development books and videos, pre-service teacher education textbooks, college texts, and curriculum materials. I also used my own knowledge and experience in teaching mathematics. For example, to prepare for our sessions in probability I looked at the following:

1. Curriculum: *Connected Mathematics Project* curriculum (Lappan et al., 1998)

2. Professional Development videos: Annenberg videos related to probability (Annenberg Media, 2001).
3. Pre service Teacher Education texts: *Mathematics for Elementary School Teachers* (Bassarear, 2005)
4. Other resources: *Principles and Standards for School Mathematics* (NCTM, 2000)

After reviewing the above resources, I decided to combine the research based activities in the CMP curriculum (Lappan et al., 1998) and supplement this with activities from Bassarear (2005) (see Appendix B for list of activities). These activities allowed for mathematical discussion around important concepts. The activities allowed the teacher to grapple with concepts of experimental and theoretical probability (the Tile problem), independent and dependent events (The Maze problem). Further, I assumed that the activities would challenge her mathematically and illustrate connections to other parts of the curriculum. Below I elaborate on decisions that I made as the study was in progress.

Ongoing Adjustments

Once I had decided the activities of the after class discussions and introduced them to Linda, there were a number of ongoing decisions and adjustments that had to be made. These adjustments changed as the stages of our partnership progressed. Before enacting the activities/problems with Linda, I expected that there would be a number of adjustments that would have to be made as this was my first time working with a teacher and I was not familiar with her prior knowledge. I had to uncover the latter through our interactions with the problems and thus considered the adjustments as constraints. Some adjustments were difficult to expect before hand and had the tendency to take the focus away from the mathematics (stages 2 & 3 below). Below I look at each stage in detail and the adjustments that were made.

Adjustments at Stage 1

Linda wanted to study the mathematics content and so there was a focus on this during the first stage. I wanted our discussions to center on material that would be challenging and allow for some abstraction, so that Linda developed knowledge that could be applied in various contexts. This was a challenge since I was not familiar with Linda's actual mathematical background and had to learn this as she solved problems. For example, I chose the Basketball problem 4, which involved finding the success rate of doing a basketball free-throw if the expected value was one (see Appendix B for more details). Linda had the opportunity of building on our discussion of Basketball problem 3, which involved finding the expected value of points in a free-throw situation given a success rate of 60% (See Appendix B for more details). Linda and I discussed the Basketball problem 3 in detail and then I assumed that she would be able to build on these ideas to solve Basketball problem 4. In the problem, Linda had to assume that the probability was p and rework Basketball problem 3 and finally solve for p using the quadratic formula. However as the discourse below reveals, Linda struggled with a number of aspects of the problem, namely the meaning of the variable 'p', the probability notation (e.g. P(S) for the probability of an event S) and whether the 'p' referred to a percent, fraction or a decimal. Further, she was reluctant to begin the problem with a letter (the 'p') as she assumed that the probability had to have a numerical value. In the end Linda could not start the problem and we spent the remaining time in this session on this problem.

L: Oh I am just seeing that out of a 100 throws she would have to make a 100 of them to get 1.

A: Can you rework … what we have from here? [pointing to the work from the previous problem that is there on the board]. So now you don't know numbers, but you know symbols for each of them so can you use the symbols (L interrupts)

L: And *p* is the probability of *S* a success and *p* [has to be 1- *p*] is a probability of *F* a failure, (A; Right) but what you want to know is what's her percent? Of accuracy? What's her *S*

A: What's *p*, yeah.

L: What's her P(S) probability of success to get a one?

A: Yeah … we have said instead of writing *x* here which is unknown, I just called it little *p*. So you want probability of success is *p*…you don't know what *p* is but you want to find this out, given that we have the expected value of *p*. This is like translating things into algebra, if you have an unknown here (L interrupts)

L: Right, but I'm still trying to figure out the unknown. What you mean. What's your unknown, percent?

….

A: It's this value, 0.6 [referring to the tree diagram from the previous problem]

L: Ok so that's what we are trying to figure out.

A: Once you get this [the probability of success] then you get this [the probability of failure]

L: Right … so you want the decimal portion of that.

A: And we called the decimal part *p*

L: Ok *p*

(pause)

….

A: So can you work out the probability of zero points, probability of 1 point and probability of 2 points just as … you did before?

(pause)

L: But we don't know the percent, we don't know the probability?

A: Oh we have the *p*

L: But *p* is unknown?

A: Yeah, but we can work with the letter all the way till you get to the last step where … something will come up. I think you have to work it step by step and then you will see finally it will unfold.

(pause)

L: I don't even know how to get started.

(Audio transcription, 3/22/06)

I used the experience from this session to inform my selection of problems for later sessions. For example, I reduced the amount of symbolic procedures that Linda would have to do in a problem since working through the procedures took up significant amounts of time. The procedures also shifted the focus from the concepts, which was the major goal. Linda wanted to learn or review procedures so we focused on these as the occasion arose. I cut down the number of problems that we discussed and focused in depth on fewer problems. I avoided problems that were very abstract and chose problems that illustrated key concepts and could be modified and implemented in the classroom.

Adjustments at Stage 2

At this stage the tensions between the content and the teaching needs of Linda appeared in the discussions and as a result these factored into my preparation. On one hand I had to address the tensions and on the other hand I was concerned with keeping the mathematics at the fore. Since I could not predict where and when tensions would arise, it seemed that I made adjustments on the spot to bring the mathematics into the discussion. It was important that Linda cooperated in keeping the mathematics in the discussions. An example of such a discussion, around the Lollipop problem (see Appendix B), is illustrated below. Linda was planning to assign the Lollipop problem and prior to this had already assigned the Bike problem. The graphs of these two problems were different as one was a regression line and the other was a piecewise defined function. In the conversation below, I attempted to focus Linda's attention to the difference in the two graphs and the reasons for this.

> L: Now going back to … the lollipop problem. If I were to do that on Monday you know, just give them some fun before I leave. I mean it's basically what we have done, but how can I set it so that it is adding something else to it?
>
> A: Ok. Like what?
>
> L: Like so far … they will do the experiment, they will time it, they'll chart it … and they could put it in a coordinate graph. So what (Interrupting L)
>
> A: But this graph is going to be different to the graph that we know. So far they have been working segmented lines. But this one you are going to ask them to make one straight line that will try and capture as many dots as they see in the graph …. So what you are doing is the line of best fit ….

L: But if it’s … not really a discrete item, why couldn't they just connect ‘em [the dots]?

A: Because if you … just connect them, remember what we are trying to do with this line is try to predict if you have a lollipop that is half the size

L: Oh ok. (Video transcription, 4/20/06).

In this discussion Linda focused on the sequencing of the activity by saying “they will time it, they’ll chart it…” and I chose to bring the mathematics issues of connection between the graphs in the Lollipop (Appendix B) and Bike problems (Appendix C) into the discussion. By bringing the mathematics to the fore, Linda raised the mathematical issue of connecting the dots in the graph, “…why couldn’t they just connect ‘em [the dots]?” These adjustments had to be done on the spot at the second stage.

Adjustments at Stage 3

Linda took charge of the discussions at this stage and they generally focused on lesson planning. My role changed from being a facilitator to a supportive role and I discussed the issues that Linda desired. By relinquishing control of the agenda in the sessions, I reacted to Linda’s suggestions and there was more unpredictability about the discussions. For example in Statistics, she expressed a desire to build on work that she had done with the students using graphs and through our spontaneous discussions, she came up with the Shoe activity (Appendix B). As Linda took a more active role in the discussions, my planning for our sessions expanded. It was hard to predict the direction of our discussions and I prepared for the sessions by taking note of important concepts and activities that could provide an opportunity for Linda make mathematical connections. I also tried to anticipate possible difficulties that could arise. Linda had more ownership over the activities and topics that were discussed and hence it was more beneficial for her.

With discussions focused on teaching, there was evidence that our talk could slide into the logistics of a lesson and anticipating difficulties that the students might or might not have. We entered these discussions with different perspectives about students' knowledge and expectations. I assumed that the students knew the required material from the previous grade, but Linda's experience had been different. Thus there is an adjustment that we had to make in terms of framing a problem or activity so that the students had a fair chance of working on it. This point is illustrated below in the discussion around the Shoe problem (Appendix B). Linda wanted to outline a table that the students would have to fill in and arrive at the number of shoes that had to be ordered of each size. She felt that the students might not recall things that they did before and wanted to guide them. I wanted Linda to give the problem to the students first and then deal with the issues as they arose. But there was uncertainty that Linda would have to deal with.

> L: Now we haven't gone into percentages or anything. So what would be the most efficient way to show them [how to solve the problem without percents]? I haven't formally covered it this year.
>
> A: They know like things from fifth grade?
>
> L: Oh, I hope so (laughs) that's the goal. So formally I haven't shown it to them. So what would be (A interrupts)

A: I was thinking should you just leave it as a “what fraction of” … then you can teach them ok if you have … $\frac{1}{10}$ of 100 or 200?

L: They won’t know ... that.

A: $\frac{1}{10}$ of?

L: I don’t think so.

A: Yeah?

L: I don’t THINK so!

(pause)

A: Cause did they do fractions, I can’t remember? from last year?

L: They should know fractions, but I don’t know …. So still going through a whole cycle, what would be the best way to get to that section if they don’t remember?

A: I think here then you may have to help them a little bit because they need that foundation.

L: (Interrupting A) Now would I say for instance that 3 [referring to a shoe size of 3] is going to be two out of 20? And then do I have them reduce it or do I just (A interrupting)

A: They can do proportions but again they may not have done that, right? If it is two out of 20 people, out of 100 people how many should it be? But that’s … ratio tables and all that.

(pause)

….

L: Ok so just planning out what I m gonna do. Like a little schedule time chart. So tomorrow with their data they are gonna do that, they are gonna make a line plot … for their own table, a line plot and the relative frequency. But how should I get … the relative frequency?

> A: I think this ... maybe you'll have to take one groups thing and illustrate it to them and somehow show them, ok for this thing this is what you got to do .... Think of it as a review for fractions ... because they need to know fractions anyway.
>
> (Audio transcription, 8/28/06)

The major point of our discussion was a debate of the ambiguity (Doyle, 1988) that should be present in the task and this was negotiated between Linda and me in the partnership. This discussion took a route where we tried to anticipate the obstacles for the students and attempted to smooth out the path to their solution. At points like this, I was concerned about tensions that might arise due to different views of presenting the activity. I did not want to come across as imposing my views on Linda and supported her in any direction that she chose to take.

Guidance

In this section I will describe the evolution of the guidance I provided Linda through the course of the study. Constraints arose out of the need for guidance for Linda in the content and due to tensions in opposing views on teaching. I provide the details below.

Guidance in Stage 1

Our mathematics discussions in the first stage required a delicate balance between the amount of information that I provided and the amount of independent thinking done by Linda. When Linda had queries about the mathematics, I guided her through questioning and asked her to relate it back to previous work. In this process and I wanted her to internalize these problem solving methods. This was a delicate issue, as at no point did I want Linda to feel that I was trying to test her knowledge. The following discussion is an example of how I chose to guide her when she encountered a difficulty with the Quiz problem, which involved finding the probability that a person would get a 70% on a multiple-choice quiz with ten questions, given that they guessed on all the questions (see Appendix B for more details). This was one of the problems that Linda had taken home to work on and encountered difficulty. In this case I chose to direct her to the Siblings problem 1 that involved finding the probability that a couple would have two boys and a girl, given they had three children (see Appendix B for more details). I focused Linda's attention on the similarity of the two problems and encouraged her to think of the counting techniques that we had used. In both problems, the counting could be done using Pascal's triangle.

> L: Because you told me not to sit there too long and I sat there already for 2 hours and I couldn't figure it out (laughs) …
>
> A: Let's see what the sample space for this [was].
>
> L: Well I came up with a 1024 possibilities and then I didn't know what to do. Well a 1024 is my sample space, but I don't know about the event.
>
> A: Ok and so how ... would a typical outcome in that sample space look? …
>
> L: Well you could have 10 correct, 10 wrong and then I just didn't go through each single one … [I am at the board]
>
> A: You'd have all right or you may have one right and 9 things right here [I am referring to a string of r's and w's that I had written on the board]
>
> L: Right, but I knew that I had to have seven
>
> A: Right! And then you'd have a huge sample space.
>
> L: Right

A: … if you think back to the first problem that we did, can you make any relationships?

….

A: What is the probability of them having 2 boys and 1 girl and in that we wrote down that the sample space was either it is 3 boys, 2 boys and a girl (inaudible) and so here all you are doing is you are doing the same problem except instead of writing b's [for boys] and g's [for girls] you are writing r [for right] and w [for wrong].

L: And you are going 10 times.

A: Right, and instead of having 3 you have 10 … so when we were doing this, this was one of the representations [a string e.g. bbb or bbg etc.], but we had another representation which was the tree diagram, and then we had another representation we used, which one was that?

….

L: Right, Oh! we used the triangle.

A: Yeah we used Pascal's triangle …So can you use Pascal's triangle to do this? (Audio transcript, 3/20/06)

Guidance at this stage involved similar strategies as outlined above. Linda moved forward with her solution in this case, but there was always a balance for me to decide on the questioning strategy so that I allowed space for Linda's thinking.

Guidance in Stage 2

The second stage of the partnership was characterized by the tensions that arose in the discussion with a shift towards teaching. Linda brought up issues of teaching and in general they were related to a task that she was about to assign her students. I would generally guide Linda in focusing on the mathematical issues that were important in these tasks. For example, when Linda expressed a desire to teach the Lollipop problem

(Appendix B) to her students, I brought up the issue of the difference between the graphs in the Lollipop problem and the Bike problem (Appendix C) and this may have been different to what Linda had in mind.

L: So Tuesday ... tomorrow wrap up Lollipop?

A: Yeah

L: And key thing is ... I was thinking of maybe having them define ... what was the line of best fit, see if they remember what that is.

A: Ok

L: What do you think?

A: Yeah

L: And why it's important?

…

A**:** I was just trying to find the tie up you know between what they did for that Bike problem and what they are doing for this problem (Lollipop problem) … like there they joined the points and here they just drew one line. Trying to get them …to see if they thought that there was anything different ….and why they didn't join the points in this one too.

(pause)

L: And the purpose was … they didn't join 'em because we had to estimate what the…how long It would take for them to finish right?

A: Right

L: Where as in the bike one we knew the beginning and the end right?

A: Because in the bike one you didn't really have to go beyond … you were estimating…all your things were within that interval … in this time period …. (Audio transcript 5/1/06)

Providing this type of guidance was typical at this stage as I sought to bring the mathematics in focus for Linda. Besides issues of teaching, Linda also worked out problems that I brought and we would have discussions based on her work or queries. The guidance that I provided in these issues was similar to that described above in the first stage.

Guidance in Stage 3

The mathematics and the teaching were intertwined at this stage and my guidance was for both these aspects as they arose in the discussions. We approached teaching from two different perspectives based on different experiences. My experience was based in college teaching and I did not experience too much off task behavior and most students were willing to engage in problem solving. There was also regular homework and students were responsible for their work and grades. Thus it was easier for me to assign higher level tasks, keeping in mind the students' previous knowledge. Linda's perspective was developed through her teaching in schools. She expressed the desire for the students to work in their groups and engage in the mathematics. But so far her experience had been that they got confused and asked her questions rather than engage in the mathematics in their groups. If she was not available for answering their questions then there was a chance that they would engage in off task behavior. There was limited time that could be spent on a topic as the students had to be exposed to the various topics before the assessments. In the sixth grade class that she taught, she expressed that students would come in from the previous grade (these students completed fifth grade in various Elementary Schools) not knowing previous concepts (like fractions and percentages) and thus she felt the need to introduce topics to the students in her class before expecting them to use it in other problems. These factors impacted her teaching decisions and hence our discussions about teaching. We had to negotiate a path that balanced challenging activities with their impact on the classroom management.

My primary goal for providing Linda guidance in teaching was to encourage her to think flexibly and independently in the classroom. Another goal was to ensure that Linda kept the mathematics as the focus in her thinking, planning, and teaching. These reasons prevented me from sharing too much with Linda, even in cases where it may have made Linda's teaching more fluid. For example, when Linda assigned the Triangle and Square numbers problem where the students were required to find the number of dots in the 6^{th} and 7^{th} stages of the pattern (see Appendix C for more details), the students used iterative methods. For example, they added the corresponding odd numbers to the square number at the previous stage, to get the next stage. Later Linda wanted the students to work out the closed form formula, which was straight forward for the square numbers (n^2), but more challenging for the triangular numbers ($\frac{n(n+1)}{2}$). If finding the closed form was one of Linda's goals, then it may have been better for her to use closed form formulas that were accessible to the students as they grappled with the ideas at the beginning. The closed form for the triangular numbers could have been left as a challenge towards the end. Later we had a discussion about the closed form formula of the triangular numbers and Linda decided that she did not want to pursue this with the students as it was too complicated to derive from the table (Note that there is a visual way to get the formula for the triangular numbers by completing the triangle to form a rectangle, but we did not think of it at that time). Linda chose instead to pursue other patterns with the students.

Assessments

This section examines constraints imposed on the partnership by the assessments. There were three district mandated tests (One each in the first three quarters) and one state test (In the third quarter). There were no assessments in the fourth quarter. These assessments influenced Linda as they were high stakes and as a result, impacted our partnership. I outline the exact nature of the influence below.

Constraints on Linda

In the current high stakes testing environment the assessments constrained Linda, since she always had one eye on the test and preparing the students for it. Combined with a shortage of time, assessments played a big role in topics that were emphasized with the students. For example, the assessments had more questions on fractions than probability and Linda mentioned that she emphasized fractions, given that she could not ‘cover’ the entire curriculum for the exam. Below, Linda elaborated on these thoughts.

> L: Yeah but probability wasn’t one of those power questions … where there are a lot of them so you want to focus on that to boost your score so like let’s say the regular year, if you don’t have enough time, probability would be the topic to be thrown out … so I am not as experienced in teaching probability as I am fractions…because you know fractions is always on the test … so fractions, fractions, fractions! (Video transcript, 3/23/06)

This passage revealed that the relative importance of topics was driven by the content in the tests. Fractions got the bulk of attention and topics like probability got marginalized. Thus the high stakes test had a major impact on what mathematics got to the floor and the opportunities that the students had for learning. Combined with the shortage of time, Linda was usually faced with the dilemma of either continuing with a new topic or consolidating the students' current knowledge.

Through the partnership we tried to address this issue of time by focusing on activities that addressed multiple Performance Objectives in the State Standards and illustrated connections in the topics. I have dealt with assessments issues in more detail in the next research question. The assessments constrained Linda, but at the same time constrained the partnership as it brought into question the CMP activities that were being used in the classroom. The next section outlines these constraints.

Constraints on our Engagement

Assessments brought up a point directly related to the activities that we were engaged in. Linda expressed concern for the CMP activities that she implemented in the classroom, since they might not prepare the students for the assessments. Even though she expressed an interest in doing 'hands on' activities with the students, she had doubts that these activities were the best preparation for her students. Linda mentioned that the questions in the CMP activities were worded differently to what students might see on the assessments and thus her students would be at a disadvantage. On the other hand, I felt that the CMP activities got to the mathematical understanding that would help the students with the assessments. The discussion below looks at our concerns about the assessment and the attempts to resolve our different approaches.

L: But I think because of the standardized tests and there are so many of them (right) that if they only get this [CMP activities] and they see that standardized test, … they are not going to be able to answer those kinds of questions when they are used to just…I don't know …I think they need to see…I think that they get confused when they just read a question…this is what you are going to do but here they are doing all kinds of investigations and they are not going to go back. I think that they need a little bit of a, you know (A interrupts)

A: Why do you feel if you do this that they will not get the test. Like for example, if you do the theoretical probability and you do all these calculations and you do the different tree diagrams for the outcomes and you calculate all the expected values and they know what's a fair game and what's not a fair game. You still feel that they need something more than that?

L: I don't think that they need more knowledge. I think that if they do this you know, they are going to get it here or they are going … but I think that's it's the way that they word the testing and the test (ok) that they just need. It's a test taking skill, it's practice with how to interpret the questions and I think these questions are very different than what you are going to see on a test. So like … just on a regular handout (ok) some of those seem similar to what they are going to see on … a test (right…right) because they are going to have multiple choice questions where ... there is not that (right) so … I think it's just a matter of language and … and getting used to the language of an exam (so is there any way…) seems like it's different …

A: Is there anyway that you could merge ... things that would show up in the exam and the language and try to use it with these activities here? … like in probability itself try to see all the different things that they could probably use … in the question and try to frame some of your questions based on that so that they are exposed to those …. the most important thing for them to know is how to think about it …because once they know how to think about it then they can figure out the [other] stuff like … for example the circumference in terms of π. They straight away went into ... circumference is πd and so they got 12π, but they were using π equal to 3.14 times 12, multiplying it and trying to get the answer right away, instead of looking at those choices there and seeing that you didn't really have to … all you had to do was just see that 12π is right there … that

> skill I think maybe you want to talk about a little. So that's
> what I think you are referring to as the test taking skill.

(Video transcript, 3/23/06)

The talk by Linda here is very significant, since it occurred in the first stage when I dominated the discourse. Later in the partnership, Linda agreed that activities from the CMP actually addressed the concepts and made students think. She felt that these activities would also be beneficial for the students in preparing them for the assessments. At various points in our collaboration we also discussed how to use the sample exam questions as vehicles for pushing students' thinking further. These issues are discussed in more detail as part of the next research question.

Expertise

In this one-on-one partnership, it was natural that Linda would view me as the expert. She assumed that I would teach her more mathematics, be a resource for her planning of lessons (by providing activities) and help her students in the classroom. Thus this view of me being the 'expert' constrained our communication in some cases and as a result I dominated the discourse in the first stage. As the study progressed, I waited longer for Linda's responses and made a conscious effort to encourage Linda's interaction. Once we developed a relationship, Linda felt more comfortable sharing her ideas and this is reflected in Linda's role in the third stage.

Time

Pressures of time would always be there for teachers, but Linda's participation in the study required extra commitments on her time. She was in school from 8:20 a.m. till

3:50 p.m. and had two planning periods during the day. In one of these planning periods Linda substituted in another class and the second one was allotted to our discussion sessions. We initially met four days a week, but this could not be sustained as Linda would be called to substitute on one or more days. Besides teaching four mathematics classes, Linda was also in-charge of the school Yearbook and our sessions were usually interrupted by students who needed the camera or wanted to discuss aspects of their design with Linda. Linda had other administrative commitments of attending staff meetings, contacting parents of underperforming students and maintaining records. Thus a combination of one or more of the above factors put constraints on our partnership as Linda had less time to work on problems that I assigned for her homework and she also found it difficult to maintain a reflective journal. Thus the emphasis in our sessions revolved around pressing issues that Linda wanted to discuss as we kept pace with the current topic that Linda was doing in the classroom during our discussions.

The use of activities from the CMP took up time and Linda expressed concern that she would fall behind in covering the content for the classes. For example, in the Shoe problem Linda mentioned,

> L: I don't know why they didn't know …. Then they were getting confused, do they go 3 , 4, 5, 6 you know for the shoe size … then they had to go back and like "oh wait a minute there is a half in here" [referring to the sizes like 4.5, 5.5 etc.] (right) … So there were a lot of [trial and error or uncertainty] and I don't know if that good or bad you know.
>
> A: That is good because then they are exploring themselves.

> L: No I know that’s good but then in the end, now a one day activity is going to be a two day activity and it turns into a whole week and it’s like ahhh! my gosh … .
>
> (Audio transcription; 8/28/06)

Linda expressed her concern for the students struggling with the activity and the students taking more time than allotted. The issue of time was challenging to our partnership and a balance between the students’ understanding the mathematics and moving ahead with the curriculum had to be examined carefully.

Classroom Issues

Issues in the classroom both constrained and expanded the partnership. For example if a lesson did not go well it reinforced, in Linda, the belief that it was better to provide more help for the students, but in the process lowered the level of cognitive demand (Stein et al., 2000). The students asked Linda questions if they faced difficulty with the problem and it was a challenge to get them to discuss in their groups. Thus Linda would have to move among the groups and got overwhelmed with the number of students that were asking her questions. As Linda was engaged with a student or a group, there was potential for the other students to engage in off task behaviors as they waited for Linda. In some of our conversations we focused on how to make the students more independent and Linda tried to reinforce norms in the class that made the students discuss in their groups.

Positive feedback from the students encouraged Linda and reinforced her belief in the CMP activities. An example of this is a positive response from a student who never submitted class work before, but started doing so after working on these activities. Linda credited the activities with this turn around.

> L: Well I never know if you've gotten a chance to sit with C?
>
> A: No … I've been trying to rotate …
>
> L: But she is very defiant … she is just who she is … but … providing these activities … I've moved away from the textbook she's doing a lot more work you know … she's collaborating with K and she's turning in her papers. She's never done that before. So I don't know if it was just the intimidation of seeing you know 30 problems on a textbook page versus seeing one or two things that they need to think about and answer. I don't know what it is, but she wouldn't do anything, she wouldn't turn anything in. It was consistently a failing grade you know … every quarter … and now she's … and I don't know if it is the type of activity.
>
> (Audio transcription, 3/27/06

Combined with other positive feedback from the students, Linda's confidence for the curriculum grew over time.

Summary

The partnership was dynamic and evolving over time and there were adjustments that had to be made both by Linda and myself. I had to adapt to the uncertainty in our discussions in the latter stages as we moved away from the fixed content discussions in stage one. By discussing issues of teaching, I had to ensure that the mathematics was in focus and we also had to negotiate our different approaches to teaching. Issues of guidance had to be handled carefully as I wanted Linda to become independent in thinking about the content and teaching but at the same time I did not want her to get the impression that she was being tested. The assessment issues along with the time pressures on Linda also factored into our partnership. Finally, our work had a direct impact on the classroom and in turn the issues of the classroom either facilitated or constrained our partnership.

The next section goes into more detail about the mathematical and the pedagogical issues that arose in the planning, teaching, and assessments and the contribution that the partnership made in each of these areas.

Mathematical and Pedagogical Issues in Planning

This section will highlight the mathematical and pedagogical issues that arose in the planning sessions at each of the three stages discussed in the previous section. The three major activities planned through the study involved the Tile problem (Appendix C), the Lollipop problem (Appendix C) and the Shoe problem (Appendix C) and these occurred in the first, second, and third stages respectively. I will outline the mathematical issues in each problem that were typical at that stage of the partnership.

Stage 1

Recall that the first stage of the partnership was dominated by my agenda with focus on the mathematical content. Linda worked out problems that I brought and we would have discussions based on her solution or attempt of the problem. Our first planning session was supposed to introduce important ideas of probability that built on the previous knowledge of the students and was guided by the Performance Objectives (POs) in the State Standards. In this case I suggested the Tile problem (Appendix B) to Linda and we discussed how this problem addressed some of the POs and then briefly discussed the possible student misconceptions that could arise. This problem addressed multiple POs, especially the main theme of experimental and theoretical probabilities and their relationship. Our discussion was mostly driven by me asking questions and Linda answering them. In the dialogue below, I reminded Linda about discussing the ideas of sample space with the students (in this case it was blue, red and yellow). Linda had a different view of the sample space as S = {B, B, R, R, R, R, Y, Y, Y, Y, Y, Y} and our dialogue below attempted to resolve whether the sample space for the problem is S = {B, R, Y} or S = {B, B, R, R, R, R, Y, Y, Y, Y, Y, Y}.

L: But your sample space is all your outcomes, isn't it?

A: Right ... and so your outcomes (L interrupts)

L: But at the beginning ... they don't know what their outcomes are? Because they don't know every single color because each color is not an outcome. The outcome is just the red blue or yellow …

A: And so (L interrupts)

L: But the sample space, like let's say … there's 2 blue, 4 and 6, so the sample space wouldn't be the two of those [blue tiles] the four of those [red tiles] and the six [yellow tiles], those would go into the theoretical?

A: Right (L: right) that goes in calculating the fractional value right. But that's not part of the sample space. Sample is the color (L interrupts)

L: Just the color, not how many of each color. Like up there we had the combinations of …success-success and success-failure and failure-success (A: right). So then for this one, the sample space wouldn't be blue, blue, blue; red, red, red; white, white, white [Linda meant yellow] however many (L writes on the paper) it would just be blue red white, that's it?

….

A: So let's think back what sample space is. Sample space consists of all the outcomes of the experiment, and here the only outcomes that … we are not looking really at the two reds, we are looking at red as an outcome…

L: That's the space. (Audio transcription, 3/23/06)

Here there are two issues in Linda's thinking (1) to work out the theoretical probability we need the number of tiles of each color, hence this should feature in the sample space, and (2) the difference between the experimental and theoretical probability. In (1) Linda recalled that the theoretical probability involved working out the fraction of each color given all the information and hence S = {B, B, R, R, R, R, Y, Y, Y, Y, Y, Y}. The second issue is more subtle as Linda assumes that the sample space should keep track of the changing number of tiles for a given number of draws, for example for 10 draws S ={3B, 3R, 4Y} and for 20 draws it could be S = {7B, 5R, 8Y}. We attempted to resolve these issues by focusing our attention to the definition of sample space and what it meant in this case.

The above discussion of a mathematical issue was typical of those at the first stage. We focused on the mathematical content and other issues of pedagogy were mentioned briefly as a base for further discussion on the mathematical issues. Further, the mathematical issues arose out of a queries Linda had about the content.

Stage 2

Planning for the Lollipop problem (Appendix B) occurred during the second stage of the partnership. The second stage reflected the tensions that arose in the partnership between discussions of the mathematical content and planning needs of Linda. We had discussed the Lollipop problem (Appendix B) as an introduction to functions in the after class discussion. Later Linda decided to introduce this problem to the students. She was already doing the Bike problem (Appendix C) in the class and she assumed that the students would have a more authentic experience if they worked with their own data to create a table and a graph. The planning at this stage differed from the last, as Linda suggested the Lollipop problem as a possible activity and wanted to discuss aspects that she should emphasize in the class. The focus though still remained on all the things that she needed to know so that she did not forget important mathematical issues to introduce to the students. Linda tried to go over these issues with me in the planning session. One of the mathematical issues was to find the time it would take to suck a lollipop that was six times the circumference of the original lollipop. Our discussion below referenced our previous work in the after class discussion with this problem. We had previously worked out the equation of the regression line with an initial circumference of 6.5 inches to be $C = -0.4t + 6.5$; C – Circumference, t - time. I suggested that finding the time for a lollipop that was six times the circumference of the original lollipop involved putting $C = 6(6.5)$ in the equation and work out the value of t. But Linda pointed out that this was negative and suggested an alternative way of examining this problem by shifting the graph, parallel to itself and upwards so that the y - intercept was six times the value of the initial circumference.

> L: So this lollipop is six times … then they you would multiply this by 6 … wouldn't they?
>
> A: You multiply by 6, which is essentially extend that backwards which in this case (L interrupts)

L: We can't have a negative.

….

L: But I don't know why. I don't understand why you extend it back, why can't you just raise it up higher? Like in this you know like our circumference is 1, six times bigger is 6, why can't my starting point here just be higher (refer to Figure 5)?

A: Yeah right … so you would have to draw a line which was parallel to that (referring to the lower line in Figure 6).

L: And so that would be the same thing as whatever answer they got here … just multiply that by 6 then it should be somewhat close to there right? (Audio transcription, 5/1/06)

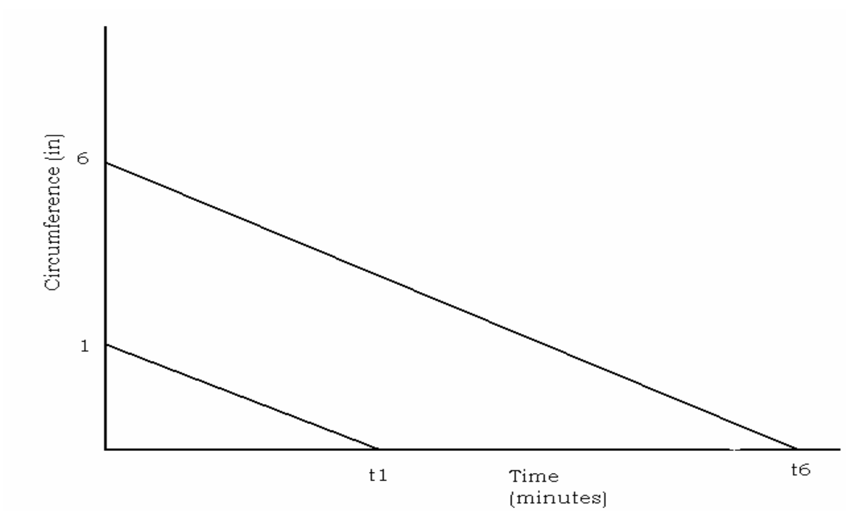


Figure 6: Lollipop problem, time for 1 unit & 6 units

Further mathematical issues that we discussed included working out the time it would take to finish a lollipop that was six times the circumference of the original lollipop, using similar triangles (Refer to Figure 6). The mathematical issues that arose in planning during this stage were unpredictable and had their genesis in the tensions. For example, the above discussion of working out the time it took to suck a lollipop that was six times the circumference of a given lollipop arose as a result of a shift in our conversation given below (see tensions described in Stage 2 in the previous section). This shift occurred when Linda changed the conversation from a discussion of solving word problems to her immediate needs of planning a class that involved the lollipop problem.

> A: … just doing the word problems [I meant just writing the relationships between the variables] and knowing how to represent them won't get you to the solution because you still have to solve the [equations]
>
> L: (Interrupting) Now going back to your, to the Lollipop problem, if I were to do that on Monday you know, just give them some fun before I leave. I mean it's basically what we have done (right) but how can I set it so that it is adding something else to it? (Video transcription, 4/20/06)

Once discussions of teaching were initiated by Linda, I tried to steer the conversation to the mathematical issues that were present in the problem. For example, after our discussion about the Lollipop problem (Appendix B), Linda mentioned that there would be time for another problem and wanted to discuss the Temperature problem that she had chosen, which involved the students making a table out of a graph (Appendix C) and also finding out the rates of change in various time intervals. I suggested that the students could now go beyond visual comparison of the steepness of the line segment in the interval to actually calculating the slope. Further, the students could also connect the negative slopes that they got to the slope of the regression line in the Lollipop problem. Linda liked the idea that the students would be able to use a graph to get a table as they were used to doing it the other way.

> L: Because there were some like in the book … where it gave them the coordinate plane … where it gave 'em the dots ... they had to look at the dots and put it into a table

A: Oh ok

L: Working kind of backwards

….

A: This is an important skill to read graphs (L: right) so you can give them a problem that will be reading the graph

….

A: … they should be able to translate fluidly between…graphs and the table (pause). They should know exactly what these points mean…what are they trying to say (pause) and during which time intervals did the temperature rise the fastest, fall the fastest. The fall will give you the negative slope. You can tie it in to the negative number that they got for slope in the Lollipop.

L: (Interrupting) Now this one in here (points to the graph) are they just going to be looking at the steepness or should I actually have them figure [it] out … is this just gonna be … just a visual?

A: Yeah … I think is visual but you can make … if they say ok "I think this is the most straight", then say ok, prove it. Now ... they can actually find a number for each of those data points … it will be better than just looking.

….

A: Say … find the slope of that point … because then they will use the table to find the slope.

L: So I should have them find he slope of which ever (A: right) one they think is the fastest (A: right) and have them find the slope of whichever one they think is the slowest or the least.

(Audio transcription, 05/01/06)

At this stage it was typical for tensions to shift the focus of our conversations to teaching. The mathematics would arise out of our discussions in teaching in an unpredictable manner and still largely stemmed from my questions and comments.

Stage 3

The planning of the Shoe problem (Appendix B) took place in the third stage when Linda took a proactive role in the agenda of our after class discussions. These sessions focused on planning lessons or debriefing a recently taught lesson. The Shoe problem arose out of a need that Linda had for students to understand the use of various graphical representations in statistics. Unlike other planning sessions, we did not choose an existing activity from the CMP curriculum but adapted an activity for Linda's needs. The discussions did not move in any predictable manner as we evaluated various problems and the mathematical and pedagogical discussions were intermingled. For example, in the following discourse that led Linda to adapt the Hat problem (Appendix B) to the Shoe problem (Appendix B), pedagogical issues like 'whole class data' versus 'group data' were mixed with mathematical issues of the creation of the Shoe problem. The discussion below highlights how the Shoe problem (in italics) arose after we evaluated different ideas one of which was the Hat problem.

> A: You have a business. You want to start a hat shop here you want to know the hat measurements, they come in like ten different sizes it's like the shoe but it's simpler because the shoe has width also … so if you want to keep 1000 hats in the shop how many of each would you keep?
>
> L: So that one you'd have to get into percentages … wouldn't you?
>
> A: Yeah in a way you'd have to get into percentages … but it involves like measuring the circumference of peoples head or something.
>
> ….

L: But then that would be one class graph, one group ... one group analyzing … I mean it will be the class analyzing, the class interpreting , the class doing everything ....

…

L: I don't know … I'm just throwing out ideas.

A: Because then if they measure adults also ... then that like kind of spreads out [unclear] if they measure their mum and dad (L interrupts)

L: So we want to spread out or we don't want to spread out? (I was talking about limiting the age group of kids whose shoe sizes we will be looking at)

A: No it is up to us ... we could...

L: I mean we could say that you know that [Name of School] has a new line of tennis shoes. They have partnered up with Nike and they need to know what sizes they need to buy for Middle School kids. So you can ask them to interview 25 people or whatever ... between the ages of 13 and 15 … or 11 and 14 … I don't know … I'm gonna give this out, but I don't know how much I'll get back and … whether we will be able to do it. (Audio transcription, 8/24/06)

The mathematical issues that arose in the planning at this stage were characterized by either adopting or adapting problems that addressed specific issues Linda observed in the students thinking. Linda also evaluated the tasks carefully to see if they met her goals. For example in the Shoe problem, Linda was interested in getting the students to display their understanding of various representations and pursued this in our discussion. A mathematical issue that arose in this discussion was whether the students would use a bar graph or a line plot as a representation and if one was more efficient than the other. An important point to note is that Linda made the discussion more concrete by working through the ideas using the Raisin problem (Appendix B) which involved the students finding the number of raisins in a standard box by analyzing a sample of boxes from the same company. Later Linda chose not to use the Raisin problem as it was going to be too expensive and some of the students may have dietary restrictions that could prevent them form eating raisins.

> L: So … but I guess I'm still like unclear like as to like ... when would you use certain … graphs …
>
> A: So it depends on what sort of a question you are asking … like for example if you get a line plot say in this case … like would you use the median in the line plot [or] would you use the mode in the line plot? In fact in statistics there is no one correct answer (L interrupts)
>
> L: No. I guess I'm thinking kids might want to use a bar graph because they have seen bar graphs for so long … but …why couldn't they use a bar graph? I mean who's to say that the line plot is better or more efficient or frequency [chart] is more …
>
> A: If they were using a bar graph then how would they represent it? … like on the *x* axis then they would have to write box 1, and then how many, box 2 how many, box 3 how many?
>
> L: So *x* would be the box number and the y axis would be the number in each? I mean I'm just thinking why … what's gonna lead them to which kind of graph …
>
> A: And so when they have this and then in the line plot they'll have the number of … the count of raisins in that box, and then
>
> L: They won't have box 1 …
>
> A: Yeah … how would you represent it if you want to do a line plot? … either you will write (L interrupts)
>
> L: Well first of all, let's say it is the raisin activity (yeah) each table is gonna have 4 boxes (yeah), or one table and then all that in one data … one box here?

> A: No they'll have probably (L: Each person has their own box?)
> yeah … 4 boxes otherwise it will be too small a data set … so
> if each one has a box. (Audio transcription, 8/29/06)

In the above conversation Linda took an active role in bringing up important issues and making 'what if' moves, which characterized her lesson planning at this stage. The 'what if' represented Linda trying to think like her students and visualize the activity being enacted by them. Linda's 'what if' moves and her focus on students' thinking were new aspects to her planning that contrasted this stage from the other two stages. At this stage it was also typical for Linda to evaluate tasks comprehensively before she adopted or adapted them to her classroom. She chose tasks that went beyond satisfying the Performance Objectives to encompass students' thinking and pedagogical issues. Further, this stage could be contrasted with the previous stages as Linda took an active role in bringing up the mathematical issues in planning.

Mathematical issues were brought to the fore in the Shoe problem with the use of technology. We wanted the students see the difference between the students' analysis and an analysis with a larger sample. We used Excel with the 176 data of all Linda's sixth and seventh grade classes and created a table and bar graph representation. Later Linda shared this with her students and they discussed the difference between their group's work and the results of the larger sample.

Summary

The planning in the first stage was dominated by my ideas as I chose the Tile problem, pointed out the mathematical aspects in the activity, and reminded Linda to emphasize them with the students. In the second stage Linda took the lead on planning for the Lollipop activity and she sought my guidance on the mathematical issues. At this stage there was a tendency for the discussions to slip into the logistics of the implementation in the classroom and I made an effort to ensure that the mathematics remained in focus. In the last stage, Linda and I jointly planned the Shoe problem and the mathematical issues were intertwined with our discussions of teaching. Linda took a proactive role in bringing up the mathematical issues and learning how to use technology in her classroom. I now examine the mathematical and pedagogical issues that arose in teaching and the impact of the partnership on these issues.

Mathematical and Pedagogical Issues in Teaching

Mathematical issues arose in teaching in one or more of the following ways in the stages of the partnership and were discussed in our one-on-one discussions:

1. Linda sought my feedback on mathematical issues that arose in her teaching.
2. Students sought Linda's feedback in the classroom and Linda brought this up in our discussion.
3. We discussed our classroom observations on teaching and student thinking.

In this section I will highlight these ideas, through examples, as they arose in each stage.

Stage 1

Few mathematical issues in teaching arose at this stage as most of our time was spent discussing the problems/activities that I brought. Linda chose activities from the CMP curriculum and from her text, to implement in her classroom. Linda provided the students with scaffolding (for details of level of cognitive demand of tasks see Appendix E) as the students grappled with the new ideas. The mathematical issues that arose were tied to our perceptions of the students' difficulties. One observation that I made in the first probability lesson that involved the Tile problem (Appendix B) was that the ideas of experimental and theoretical probability were not brought to the fore. The students were involved in drawing the tiles and making the table. It was not clear from this if they understood that an increasing number of draws resulted in a better approximation of the theoretical probability. After we briefly spoke about this, Linda implemented the Dice problem (Appendix B) that got to the concepts of theoretical and experimental probabilities from another avenue.

Another issue of the students' thinking arose in our third discussion about the Mathematics Task Framework (Stein et al., 2000). Linda brought up an issue of student thinking with respect to the Dart problem which involved finding the probability that a dart would land in a certain region of a partitioned rectangular dart board (see Appendix C for more details). Linda mentioned that the students had difficulty in even starting the problem and she was not sure if the guidance that she provided them was optimal as she was doing most of the thinking for them. Although this example concerned teaching, Linda brought this up as an example of how the cognitive demand of the task was lowered. We discussed the students' thinking as they tried to solve the problem and conjectured on what they found challenging. The students were unable to solve this problem and in most cases they could not even begin the problem, despite the fact that they had been working on probability for about one week. Linda spoke about her dilemma in providing hints as it lowered the level of cognitive demand of the task (Stein et al., 2000). In trying to conjecture why the students faced this obstacle, Linda mentioned that the students did not think that they could subdivide the given rectangles as that would change the problem.

> L: [student name] attempted it but she did it incorrectly first … so ok I'm trying to get the right answer right…so lowering the demand again … so I found myself … because they are sitting there and they don't even know how to start … they have their diagrams they have everything correct but "Miss what do we do?" I'm like trying to question them but nothing works … so them I'm like ok let's break it down, so then I had to go draw another one, but I didn't know if I had to but I went and drew another one where I only had two pieces and I put A and B and then I said, "What's the probability of getting A?". One out of two … so then I make it a little more difficult, so I drew three pieces. One out of three. But I didn't give 'em a model for equivalent pieces … so what should I have done at that point so that (shrugs)
>
> (I ask Linda why the students were having difficulty)
>
> L: Why couldn't they extend it? I don't think they knew what to do with A (A: ok). They weren't able to break 'A' down … they didn't see that because some of them still said 2 out of 4 … so then I had to go back and ask them, "well are these A's? Or is the bottom A the same size as your C and B?" and then some [students] at that point said … "can we break it up?"… they didn't even feel like they could . "Can we break up A" and I go "well if you break up A is it still the same amount of space?" and they said "yes" … but they felt like they couldn't manipulate the problem. (Audio transcription, 4/3/06)

Discussions of mathematical issues in teaching were rare at this stage as the focus was on the mathematical content that I brought for our discussions. Student thinking was not a major focus as it would become in the latter stages.

Stage 2

This stage was associated with the tensions that arose as a result of Linda's need to plan lessons and Linda brought up issues that arose in her implementation of the activities that she chose. For example, some students came up with the graph below (Figure 7), for the Lollipop problem (Appendix B) as they approximated the regression line using their data. The students could not use this graph to work out the time it would take to complete the lollipop, since the *x*- intercept was outside their graph paper. Linda wanted to know what the students could do in this case and as a result we discussed issues of scale and proportionality.

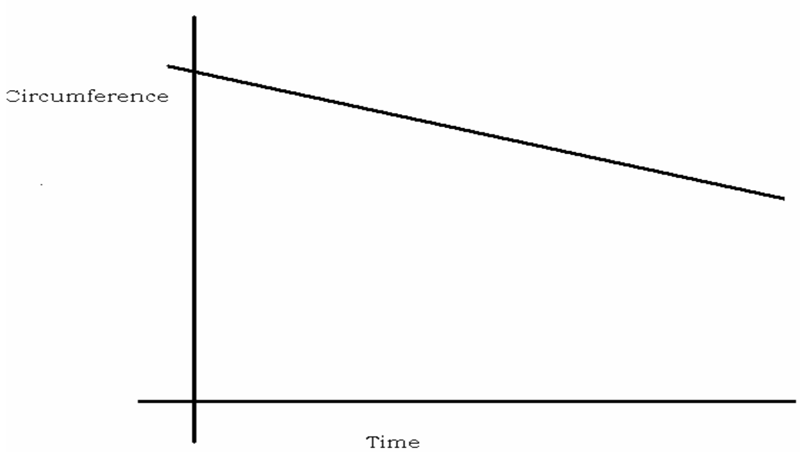


Figure 7: Lollipop problem, Circumference vs Time

Another query that Linda brought up was a mathematical issue that arose in her discussions with the students. In this problem (Bike Trip Day 4; Appendix C), the students had to make a table and a graph using notes that they were given about the trip. This was an open ended activity that required the students to make assumptions about the gaps in the trip. Linda wanted to confirm if their assumptions about one of the hints was correct (At around 3:30 p.m., we reached the north end of the Chesapeake Bay Bridge and Tunnel. We stopped for a few minutes to watch the ships passing by. Since bikes are prohibited on the bridge, the riders put their bikes in the van, and we drove across the bridge). There were two ways to interpret the information; either the table represented the distance actually traveled on their bicycles (for seven hours) or distance of the entire trip which would include the van and the bicycles (for a total of seven and a half hours).

> L: And then at seven and a half … we did this [they put 80 miles] ... and this is right here at 4 ‘o clock because it says that the whole day was 80 miles. But it said that at 3:30 they stopped and picked up ... their bikes. So should it have been that one [at 3:30 p.m. or 7 hours, distance covered is 80 miles] we didn’t do that.
>
> ….
>
> A: So you could look at it in two ways you could be looking at the entire trip which is (L interrupts)
>
> L: They did this ... three and a half hours ... at 3:30 or 7 hours, 4 ‘o clock or seven and a half hours ... that now is wrong?
>
> A: Not wrong
>
> ….
>
> L: But I didn’t make that clear that the bike is in the van so … I think that the kids are still assuming that they are still riding the bikes.
>
> ….
>
> L: I don’t know what they are thinking ... no one [among the students in the class] ever brought that up "But wait a minute Miss, they put the bikes in at 3: 30?"
>
> (Audio transcription, 5/9/06)

At this stage, Linda sought my guidance on mathematical issues that arose during her teaching. The two mathematical issues that she brought up were typical at this stage and either involved mathematical uncertainties the students encountered on an implemented task or issues about the correctness of a solution. Linda paid more attention to the students' difficulties in the last stage.

Stage 3

Linda was proactive at this stage and the agenda focused on planning and implementation of lessons. She took an active role in bringing mathematical issues (to our one-on-one discussions) that arose in connection with the planning and implementation of an activity or problem. For example, in the Triangular and Square numbers problem (Appendix C), the students came up with a recursive formula ($n + a_{n-1}$ or 'n' plus the number of dots in the previous stage) instead of a closed form formula ($\frac{n(n+1)}{2}$). Linda wanted to discuss how she could get the students to discover the closed form formula. First we made a table (see below) with the stage and the number of dots at that stage and then Linda observed that at any stage we could multiply that stage with the previous stage (e.g. In the fourth stage: $\frac{4.3}{2}$) and divide by 2 to get the number of dots.

Stage	Triangular Number
1	1
2	3
3	6
4	10
5	15
6	21
…	…
n	$\frac{n(n+1)}{2}$

Table 8: Triangular numbers

At this stage, Linda paid attention to the students' work as can be seen from the above example. But unlike the previous example, Linda also started reflecting on the mathematical issues independently and reported her thinking to me. For example, Linda reported her handling of the implementation of the Shoe problem (Appendix B) and her reflection on the path that she negotiated.

> L: I said ok they have two out of 20 shoes must be a size 5, 4 out of 20 must be size 6 … I'm like ok, this is 20 groups of shoes but now we want to extend that we are not going to buy 20 pairs of shoes we want to buy 200 pairs of shoes so what can we do if we are going to buy 200 shoes … so a lot of them knew that …
>
> ….
>
> L: Because right away when I said we have 20 pairs of shoes how many need to be 5 … that was easy for them but getting them to just put the frequency chart that was a struggle (laughs). I mean a real struggle … then I'm thinking to myself ... well how much of it do I do as a whole group because then they are just going to emulate what I'm doing and not going to think about it themselves. So I kind of just played with it all day long some classes I went as far as making a frequency chart.
>
> (Audio transcription, 8/29/06)

Linda's independent thinking about the mathematics and the students' thinking distinguished this stage from the previous stages. The Triangular numbers problem still shared similarities with the discussions at the second stage where Linda tried to get at the correct answer. But in this case there was an added goal of Linda attempting to 'smooth' the task out for the students by anticipating and removing the obstacles beforehand. In the reflection of the Shoe problem, Linda grappled with maintaining the level of cognitive demand in the task and at the same time ensuring that the students were not overwhelmed with the task.

Summary

In this section I examined the contribution of the partnership to the mathematical issues as they arose in Linda's teaching. There were three manifestations of this (1) Linda's seeking my feedback on her teaching, (2) the students' thinking in the classroom, and (3) our observations of the classroom interactions. The bulk of the teaching issues arose in the second and the third stages respectively when there was a focus on planning and implementation. In the last stage there was evidence of Linda reflecting independently on the mathematical issues that arose in her teaching. The next section examines the contribution of the partnership in the mathematical and pedagogical issues that arose in assessments.

Mathematical and Pedagogical Issues in Assessments

The partnership brought up issues about the assessment as Linda started using activities from the CMP curriculum. These activities were different from her previous drill and practice problems from the original textbook. The latter problems were more tailored to the tests that the students had to take each quarter. Commenting on this, Linda felt that the difference in the wording and language used in the CMP and the test could put her students at a disadvantage. I consider this as a mathematical issue since it is known that a difference in the language can impact the performance in the content for English Language Learners (Abedi, 2002), who made up a majority of this class.

Linda used the following example from the 7th grade sample of Arizona's Instrument to Measure Standards (AIMS) to make her point that the language the students encountered in the exam was different from their experience in the classroom. In this question her students encountered the term 'common multiple', but they were familiar with the term *least common denominator* in the context of adding fractions. Thus Linda felt that the students needed to be coached in test taking skills so that they could answer these questions.

Byron will add 3/7 to 4/9. Which of the following should be the first step he would use to correctly solve the problem?

A. Find the sum of 3 and 4.
B. Find the sum of 7 and 9.
C. Find a common multiple of 3 and 4.
D. Find a common multiple of 7 and 9.

(Grade 7, Sample AIMS test)

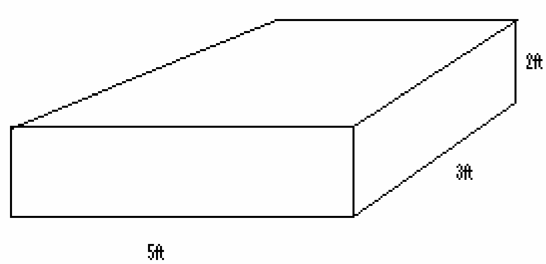
Other assessment issues that were brought to the fore by Linda included the shortage of time to complete the required content for the exams. Thus she was faced with the dilemma of using the limited time before the exam to review or teach a new topic that would be on the exam.

> L: But there is just too much to review, especially since … we haven't even covered every topic yet so then do you go back

> and review what they have covered or we continue covering
> stuff that wasn't covered and still on the test?
>
> (Audio transcription, 3/28/06)

We reflected on this issue and attempted to find possible solution strategies that Linda could incorporate. One of these strategies, that I suggested, was to choose activities that would cover a wide range of Performance Objectives (PO) in a given topic as opposed to doing an activity that addressed a single PO. By emphasizing connections through the activities, students could build understanding. This could be useful in a problem such as the one below where the students could use their understanding of volume to get the correct choice.

A rectangular prism has the dimensions as shown below:



What is the volume of the prism?
A. 10 *ft.*^{2}
B. 10 *cm*^{2}
C. 30 *ft*^{3}.
D. 30 *cm*^{3}.

(Grade 7, Sample AIMS test)

In this question the students did not have to multiply to get the correct answer. Below we discuss that it would be enough for the students to know that the units of volume in this case would be *ft*^{3} and only one of the choices would be possible.

A: You can just show them how to eliminate answers.

L: Well they do have a study skills class and they are supposed to be going over test taking skills.

A: Like the one with the volume, that was an obvious one … there was a volume of some cube … in this one you can immediately tell them that if they look here and here [pointing to the choices in the question] this is out because … (cm square) so it is between this and this …. (I show L how to use elimination in the AIMS for the volume problem)

L: But if the numbers are that small it is probably easier to multiply out.

A: Right. But even if the numbers [are] bigger (L interrupts)

L: If it's bigger then you want to probably do the process of elimination where you [unclear] that don't even match (right) square is area.

A: So they are using their implicit thinking of the unit. (Audio transcription, 3/28/06)

We discussed how Linda could also use the review for the test to challenge students to think about the problems in ways other than recalling facts. Thus the review sessions could become sites for learning and making connections instead of memorization.

Linda mentioned that the time pressures forced her to focus on topics that carried more weight in the assessments. For example, fractions got more attention than probability. Our discussions tried to redress this by finding connections between these topics. For example, the representations for the probabilities of the Maze problem (see Appendix B for a figure of the maze and details of the problem) could be connected to the representation for the addition of fractions as shown below.

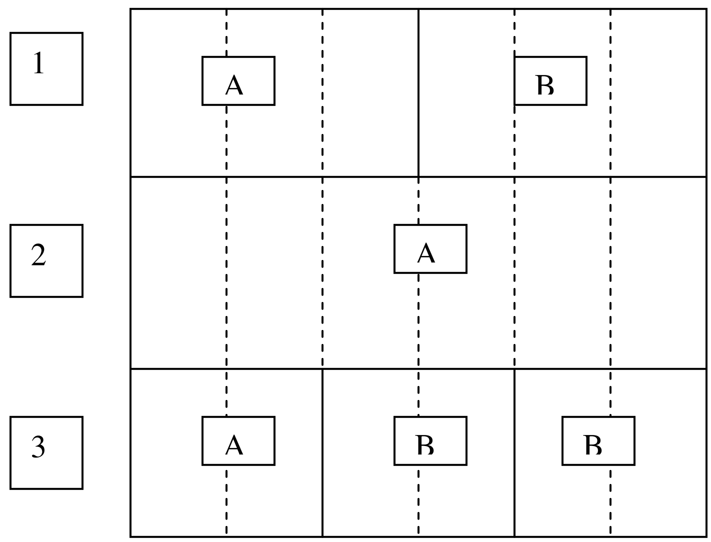


Figure 8: Area model for probabilities in the Maze problem

In the above representation (Figure 8; this is not the maze [see Appendix B]) of the probability, the entire space is represented by a unit square and the paths one, two, and three are represented by an area of one-third the square. Each one-third portion is then divided into further parts depending on the doors that open into A or B. Smaller subdivisions (18 in total) form the unit for calculating the probability of going into A or B. Thus the probability of going into A is: $P(A) = \frac{3}{18} + \frac{6}{18} + \frac{2}{18} = \frac{11}{18}$ and the probability of going into B is: $P(B) = \frac{3}{18} + \frac{2}{18} + \frac{2}{18} = \frac{7}{18}$. This representation could be connected to the addition of fractions and the meaning of the *least common denominator*.

In conclusion, our discussions about the assessments brought out the issues that constrained Linda and allowed us to reflect on them and think about strategies around the constraints. Through our discussion Linda understood that the activities went beyond the assessments to foster understanding of the content. She says:

> L: At first I was like well … maybe they just need to read a straightforward question so that they know what it is like to (pause). So they know what's expected … (pause) but then … I started thinking well maybe if they had more of these tasks and they could start developing general concepts and applying them and start seeing them everywhere.
>
> (Audio transcription, 3/27/06)

All the above discussions about the assessments occurred in the first stage of the partnership. There was no discussion of assessments in the brief second stage or in the third stage. The mathematical issues contributed to resolving the pressures of time that were tied to the exam reviews. The mathematical issues discussed in relation to the assessments also highlighted the importance of connections between topics.

Summary

The results for the second research question show the details of the contribution of the partnership to the mathematical and the pedagogical issues that arose in planning, teaching, and assessments respectively. The mathematical ideas in planning and teaching stemmed from me in the first stage as I chose the activities and discussed possible changes after implementation. The second stage was characterized by Linda raising issues of planning and teaching with me and there was a tendency to for the discussions to drift towards the logistics. At this stage I tried to ensure that the mathematics was brought to the fore in the discussions. In the last stage, Linda and I jointly planned an activity and the mathematical issues were intertwined with our discussions of teaching.

Linda took a proactive role in bringing up the mathematical issues in planning and teaching and students' thinking played a bigger role. Assessment issues were predominant in the first stage with language and time issues being the major concerns.

The next section examines the third research question and outlines the evolution of the curriculum together with the evolution of the cognitive demand of the tasks that Linda assigned in the classroom during the study.

Evolution of the Curriculum

I introduced Linda to the CMP, prior to the study, when she asked for tasks that could supplement her regular curriculum. Linda managed to find old student books that were in the school library but there were only about five copies for each topic and the students at one table had to share a book. Linda would also write the questions on the overhead projector for students' reference. Once the study began, I used the CMP activities as a way for Linda to examine mathematical ideas as a part of our discussions during the first stage. Linda implemented the Tile problem (Appendix C), which was also the first problem that we had discussed in our after class discussions on probability. At this point she also implemented activities such as the Match/No Match problem and the Dart problem independent of me. Linda also worked with the Dice problem and the Crime Scene Investigation problem, which were part of her regular curriculum. In her discussions on functions, Linda independently chose to work with the Bike problem from the CMP and she also did the Lollipop problem, which was a problem I had introduced during the after class discussions. The summer interrupted our study at this point and during this time, Linda placed an order for the CMP curriculum with the school and a district grant was used to purchase an entire set of CMP curriculum for the sixth and seventh grades. The following academic year, the CMP became the official curriculum in Linda's sixth grade classroom and she was the only teacher in the school to use the curriculum.

Evolution of the Cognitive Demand of Tasks

I assumed Doyle's (1983, 1988) framework that the curriculum could be thought of as a sequence of tasks and will also examine the tasks that Linda chose and set-up in her classroom. Stein et al. (2000) stated that the cognitive demand of the task impacted the students' learning and they discussed the *Mathematics Task Framework* (MTF) (Stein et al., 2000) as a means of analyzing tasks from the point they are chosen in the curriculum, set-up in the classroom, worked on by the students, and impact student learning. In this section I will also focus on the discussions that Linda and I had based on the MTF and I will analyze the tasks that she chose and implemented in her classroom. The next section examines Linda's categories in the task sort and her justification on selected tasks.

The MTF Discussions

I assigned the list of 15 tasks (Appendix A) for Linda to sort into any categories that she felt were appropriate. Our first discussion in the study examined her results of the sorting that she had done at home. Linda solved or at least attempted all the problems that were given and isolated two categories, namely tasks that were "straight forward to solve" with "possibly one way to solve" and tasks that were more "open ended" that allowed for a "variety of interpretations and strategies". Linda spent about four hours solving all the 15 tasks before she categorized them and mentioned that there could have been finer categories if she had more time. I was surprised when she classified a word problem (see Task N in Appendix A) as open-ended even though it involved a straight forward procedure. Her reasoning was that it was not clear from the problem whether one should multiply, divide, add or subtract the 30% and thus there was a potential for confusion by the students.

Linda conjectured that if she did the task sort from the students' perspective then she would do it differently since their thinking might cause them to trivialize challenging tasks. In conclusion, Linda's task sort was similar to the Low and High *Levels of Cognitive Demand* (LCD) outlined in the MTF (Stein et al., 2000) and the task sort proved to be a good base for introduction to this framework.

The second discussion on the *Mathematics Task Framework* took place during the first stage in the evolution of the partnership. I assigned Linda to read the first chapter from *Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development* (Stein et al., 2000). Our second session of the MTF involved a discussion about this first chapter and examples of tasks from her class that could be classified in one of the four categories discussed in the reading. Linda commented that she liked the chapter and the things that were mentioned matched her prior knowledge from gifted education.

Linda reflected on her classroom tasks through the lens of the TAG and she concluded that all her tasks were either *procedures without connections* (PWOC) or *procedures with connections* (PWC) and there were no *memorization* or DM tasks. She shared an example from each category and pointed out the complexity in classifying tasks in isolation as there was a connection between tasks from one day to the next. She illustrated her point where a PWC task was followed by a PWOC task. The first task involved approximating the value of π by measuring the circumference and diameter of various circular objects and examining their ratio. This task was followed by tasks that involved using the formulas to work out the circumference of a circle given the radius or diameter.

Linda shared her thinking about the DM tasks and whether it is possible to assign these to her class. She mentioned that most of the tasks assigned in the class involved some sort of direct instruction and hence would not qualify as DM tasks. But she conjectured that it was possible to achieve DM tasks in her classroom if she worked with the same group of students from sixth grade till eighth grade. Linda mentioned that the DM tasks got the students to think and build the concepts, which prepared the students for the assessments. This opinion differed from her previous claim that the higher level tasks from the CMP curriculum did not prepare the students for the assessments as the language used in the test and tasks differed (refer to the previous section on Mathematical and pedagogical issues in assessments). Besides building the conceptual knowledge of the students, Linda pointed out that the students found the CMP tasks interesting and there was one student in particular who started submitting her class work. This student had not done this prior to the assignment of these tasks. Linda conjectured that the tasks she assigned made the difference in the transformation of the student. These instances reinforced the CMP curriculum as a valuable source of high level tasks that could engage the students.

The third MTF discussion took place towards the end of the second stage in the evolution of the partnership. Linda read the second chapter from *Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development* (Stein et al., 2000), which focused on the stages of evolution of tasks from the curriculum to set up and finally to enactment. Factors that maintained or lowered the Level of Cognitive Demand (LCD) were also discussed in the chapter. Our discussion focused on factors that could contribute to maintaining a high LCD in her classroom, the meaning of self monitoring, and a discussion of unsystematic procedures.

Linda wanted to know what could be done to maintain a high LCD by making the students accountable. She used the example of the Dart problem (Appendix C), which the students were unable to start and thus forced her to provide them with hints. Linda wanted to know how she could hold the students accountable in this case, given that they had all the prior knowledge required to solve the problem. We discussed one possible way by having the students keep a record of all the work that they did so that they could refer back to the notes when they encountered a challenging problem.

Unsystematic procedures featured in our discussion and I mentioned the Lollipop problem (Appendix B) as a possible example of such a task. I felt that in the task such as the Lollipop problem it was easy for the students to get distracted by the procedures and lose focus of the mathematics. However, Linda felt that such a problem could provide a marker for certain concepts that could be put to use later. Linda mentioned that these tasks would be remembered by the students and the teacher could ask them to recall this task and the associated concepts in the future.

> L: But then again, they might focus more on the lollipop or time may be an issue. But you can always go back, remember the lollipop and you know they are not going to forget it (right) so it's still going to be an impressive activity you know a fun activity where they will remember and maybe at that time they are focusing on the lollipop.
>
> (Video transcription, 4/3/06)

Here Linda recognizes that some tasks have an affective aspect that could be as important as the mathematics connections. This affective aspect can be used by the teacher as a tool to jog the students' thinking and facilitate connections and building further mathematics. The next section examines the impact of the partnership on the curriculum with respect to the tasks.

Impact on the Curriculum

In this section I will examine the evolution of tasks that Linda selected and her set up of these tasks in the classroom through the course of the partnership. This selection of the tasks and the set up are the first two stages of the Mathematics Task Framework (Stein et al., 2000). Linda and our partnership was the focus of this study and so I did not analyze the third and the fourth stages of the MTF that focus on the students.

Evolution of the Task Selection

I will outline the evolution of the task selection that Linda made in each of the three stages of our partnership.

Stage 1. This stage refers to the first stage in the evolution of the partnership that was described in the first research question. The tasks at this stage were mostly selected by Linda and were at the PWC level based on the TAG. The content centered on probability and the tasks were selected to address the Performance Objectives (POs) laid out in the State Standards. For example the Dice problem, which involved rolling a dice 12 and 24 times respectively and working out the experimental probabilities, was selected with PO7 (Probability) in mind. In this problem, the students carried out the procedure of rolling a dice 12 and 24 times, but it was not mindless as it led the students to construct the concepts of experimental and theoretical probabilities and their connection. There was a broad path that the students would have to take to get the solution, without the use of a standard algorithm and there was some cognitive effort involved. This task was at the PWC level as were most of the other tasks selected from the CMP. At this stage, I influenced her to choose two of the activities, namely the Tile problem (Appendix C) and the use of the tree diagrams to work out the outcomes of a spinner, coin, and dice. But the other activities were selected by Linda independently.

PO 1	Determine the probability that a specific event will occur in a single stage probability experiment (e.g., Find the probability of drawing a red marble from a bag with 3 red, 5 blue, and 9 black marbles.)
PO2	Compare probabilities to determine the fairness of a contextual situation (e.g. If John wins when two or greater shows after a six-sided number cube is rolled and Joaquin wins otherwise, is this a fair game?)
PO3	Predict the outcome of a grade-level appropriate probability experiment
PO4	Record the data from performing a grade-level appropriate probability experiment
PO 5	Compare the outcome of an experiment to predictions made prior to performing the experiment
PO 6	Make predictions from the results of student-generated experiments using objects (e.g., coins, spinners, number cubes, cards)
PO 7	Compare the results of two repetitions of the same grade-level appropriate probability experiment

Table 9: Probability POs

PO 1	Determine all possible outcomes involving the combination of up to three sets of objects (e.g., How many outfits can be made with 3 pants, 2 tee shirts and 2 pairs of shoes?)
PO 2	Determine all possible arrangements of a given set, using a systematic list, table, tree diagram, or other representation

Table 10: Discrete Mathematics POs

Stage 2. At this stage most of the tasks were based on functions and change. The tasks were chosen, by Linda, from the CMP and addressed one PO, namely “Analyze change in various linear contextual situations”. The tasks that Linda chose revolved around a CMP problem based on bike trip that involved analysis of tables, graphs, and verbal descriptions of segments of the trip. She also chose to assign the Lollipop problem as it reinforced the making of a table and graph based on data that they collected. In this case Linda went further and introduced the students to the slope formula. The tasks that were at this stage were interconnected and they all addressed a single PO. All the tasks at this stage were selected by Linda with some feedback from me. The tasks were generally at the PWC level in the TAG.

Stage 3. Linda set the agenda for the third stage and one of the differences between this stage and the others was that Linda and I discussed and adapted a task (Shoe problem). In the other stages Linda adopted existing activities. Even though we designed the Shoe problem, the ambiguity in the task had to be negotiated between Linda and me and it was reduced compared to the original design. As the task was initially designed it could have been a DM task, where the students would have to come up with their own strategies. But asking the students to collect shoe sizes and asking them to make a frequency chart reduced the inherent ambiguity in the task. It was hard to negotiate how much ambiguity could be removed before the task would be suitable in Linda's class. Finally, in the Shoe problem the students filled out a table and in the process lowered the cognitive demand of the task. The cognitive demand could have been preserved if the students were required to come up with their own way of organizing the data and solving the problem.

In this stage, student thinking influenced the choice of tasks along with the POs. For example, Linda decided to expand on her original task for PO5, by assigning the Median problem, which involved the students making a line plot so that the median was the lowest value in the set (see Appendix E for more details). She noticed that the students assumed the median should 'look' to be in the middle of a line plot rather than the midpoint in the data set. Overall the tasks at this stage could be classified as PWC.

PO 1	Formulate questions to collect data in contextual situations.
PO 2	Construct a histogram, line graph, scatter plot, or stem-and-leaf plot with appropriate labels and title from organized data.
PO 3	Interpret simple displays of data including double bar graphs, tally charts, frequency tables, circle graphs, and line graphs.
PO 4	Answer questions based on simple displays of data including double bar graphs, tally charts, frequency tables, circle graphs, and line graphs.
PO 5	Find the mean, median (odd number of data points), mode, range, and extreme values of a given numerical data set.
PO 6	Identify a trend (variable increasing, decreasing, remaining constant) from displayed data
PO 7	Compare trends in data related to the same investigation.
PO 8	Solve contextual problems using bar graphs, tally charts, and frequency tables

Table 11: Data Analysis POs

PO 1	Communicate a grade-level appropriate recursive pattern, using symbols or numbers.
PO 2	Extend a grade-level appropriate iterative pattern.
PO 3	Solve grade-level appropriate iterative pattern problems.

Table 12: Patterns POs

Evolution of the Set-up

I will again refer to the three stages discussed before as a means for framing the cognitive demands of the tasks at the set up by Linda in the classroom.

Stage 1. At this stage all the activities except the Crime Scene Investigation problem (CSI) (Appendix C) were set up as they were in the curriculum. Linda would assign the problem on the over head projector (since each table had a single CMP textbook), read it to the students from the text, and then let them start solving the problem. She would move around the classroom and spend time at each of the groups. In the case of the CSI problem, Linda worked out a very similar problem that involved the outcomes of a spinner with two colors after to spins. She used the tree to represent the outcomes in the spinner problem and suggested that the students do the same for the CSI problem. Since the tasks were set up as they were present in the curriculum, there was no lowering of the level of cognitive demand of the tasks, which were mostly at the PWC level.

Stage 2. At the set up in this stage, most of the ambiguity in the tasks was reduced through scaffolding. Linda questioned them about important aspects of the task and pointed out things that they should pay attention to while they solved the problem. For example, in the Bike problem Day 1, Linda had a student read the problem to the class and after this she asked them pointed questions, which focused the students' attention on the key aspects of the problem. By doing this some of the opportunities to think were reduced for the students, but there was still enough work left for the students to do and I would not categorize this as a task of lower cognitive demand.

[The student had just finished reading the background of this task and the questions. Linda then takes over]. 'S' represents *any* student that answer the question.

L: So at 0 hours what was the dist that they traveled?

S: 0 miles

L: 0 miles ... what does 0.5 mean?

S: 30 minutes

L: 30 minutes ok ... make sure that you are aware of that, that 0.5 means 30 minutes or half an hour …. So what would 1.5 mean?

S: An hour and a half.

L: … so where are you going to be getting this information from?

S: From the chart.

L: From the chart ok. The second question, during which time intervals did the riders make the most progress? And the least progress? How does that relate to yesterday with jumping jacks and rate?

S: You need to know how different variables increase or decrease.

L: Explain that over again. For instance … how are you going to figure out which time interval did the riders make the most progress. Any ideas? That is very similar to what you did yesterday on figuring out your rate. Did your rate increase or did your rate decrease throughout the jumping jack experiment. And then the last bullet says did the riders go further during the first or the second half of the days ride?

S: Miss what does half an hour mean

L: Half an hour means 0.5

(unclear)

L: So this is the problem that you are working on … you are doing problem 2.1 there's a few of the books around the table, so please share them and out them in the center so everybody has the data in front of them. So this is what you are working on today. Use that chart to answer those questions.

(Audio transcription, 4/19/06)

Linda focused the students attention to a particular aspects of the problem like "So at 0 hours what was the distance traveled?" or indicated a relationship to the previous days work by stating, "How does that relate to yesterday with jumping jacks and rate?" I conjectured that these instances were to ensure that the students were clear about the problem and did not lower the cognitive demand of the task since the students still had to think about these hints and follow through with them. The students had to do some thinking and make connections between the problems.

Linda and I both noticed that the students were trying to get help from her and not thinking independently. Usually students would seek out her help if they did not understand a certain aspect of the problem instead of discussing with their partners. Students would wait to ask Linda questions and in some cases this led to off-task behavior. In a bid to get the students to discuss in their groups, Linda and I explored some possibilities of instructing them explicitly or merging groups together. Excerpts of this discussion are below.

A: I mean at least start talking about it in the table. Maybe they don't figure it out but at least you say "I want to hear a conversation where I can see you discussing the problem … and then I'll help you".

L: Going back to the basics again on management

….

A: They should have pleasure of working with the other students. If it is forced then again you would have to be there for them to actually do it [the work]. If you are working with another group then the previous group will just go off task.

(pause)

Yeah that's hard, I think it takes quite some time to build that, and to sustain it….

L: But then even then, they are constantly changing. Attitudes are changing.

(pause)

L: That's what I feel like, everyone is following me. But by the time I get [to them] the bell rings.

A: Right. I know it's very hard once you go from group to group, then it's hard to get stuff done because you work with one group and then there are so many others who are waiting (pause)

L: Wonder how big a group can be when it is not too big?

….

L: I was thinking of maybe combining 2 groups

A: Oh ok. Like 4 people in a group or 6 people?

L: I would just combine these two tables work together and those two tables work. So I don't know if it would promote more discussion or will it promote more off task discussion? Because if there is less groups for me to get to, I can probably get to each group. Is 6 too many?

A: Yeah I think 6 is too many because they will break off into their individual groups in that table. If those three were together before they will probably stick together and the other three will stick together and there won't be any cross discussions.

(Audio transcription, 5/2/06)

The next day, Linda addressed the students about working in their own groups as she set up the task in the classroom. Linda expected that after the students had worked on similar problems for a few days they had enough of experience to work out the current problem. The discourse below represents an instance at the end of the second stage where Linda tries to make her point with the students about working in groups.

> L: None of this is new. You might be doing it differently but it's not new. You still have a graph a table, you still have to figure out the rate, which is fastest which is slowest. It's all review of what you have already done. So what you need to do is you first attempt these problems on your own. You are at a group for a purpose, what's the purpose of the group?
>
> S: To work together
>
> L: To work together. There are a lot of you that just right away you want to … stop, read it freak out and you want to see one of the adults in the class. But you need to be in your seat because we can go back to having desks in here and rows where you don't collaborate with anybody or we can stay in groups where you can collaborate with others but you need to make sure that you are using it the way that it is set up to be …you are a team. A team should work together …that doesn't mean that [Student name] does 1a, [student name] does 1b, they both do 1a and they both do 1b and then they discuss their results. Maybe they got it the same then maybe they are on the right track. Maybe they got different results then that's where they need to dialogue and prove it. Well I know this is right because and then you might say maybe there is another way of thinking about it and you don't need immediate feedback from me. You are going to turn this stuff in and usually there are comments. So you work within your table first. If no one at your table can answer, and don't just give up right away, you need to have actually put some thought into it because I bet you can do it because as soon as we walk to a table and read the question, then you are like 'Oh now I know it'. So we didn't even have to go to the table and read the question to you. So you guys really need to read the question … "oh yeah I know how to do this" and that might help you with the previous one or with the latter one so you need to use your groups the way they are set up to be. You got people in there, if you are working alone I can have you join a group since there is a couple of people absent and you can collaborate, but you need to try. Don't just say "Miss I don't get it" "Anthony I don't get it" you really need to try because a lot of these I know you can answer

> without help, it's just a review now. Now you are applying it to a different situation, that's it. So you are doing problems 1 a through g. (Video transcription, 5/3/07)

By attempting to lay out the norms for engagement in the classroom, Linda maintained the level of cognitive demand for the students. Linda went even further on certain occasions and got the students to share their solutions and thinking with the entire class. In this stage we see a push by Linda to focus the students' attention on the mathematics needed for them to solve the problem and attempt to set up the norms of doing mathematics in the classroom.

Stage 3. The set up of the problem at this stage was almost as they appeared in the curriculum or as we had discussed (in the case of the Shoe problem) and Linda refrained from giving the students too many directions. Thus the level of cognitive demand remained unchanged, usually at the *procedures with connections* level. Further, Linda made a sustained push to get the students to think in their groups and report their work to the class. In the Shoe problem, the level of cognitive demand was lowered at the design stage and that level was the same in the set up stage. At this point in time, Linda was teaching a new sixth grade class and she emphasized that they should discuss the problems thoroughly, in their groups, before asking her for help. The push for independent thinking and sharing the work in the class were major factors that contributed to the maintenance of the high level of cognitive demand at this stage.

Summary

This section touched on the evolution of the CMP from a peripheral role to eventually becoming the curriculum in Linda's classroom. The introduction of the tasks from the new CMP curriculum was more challenging for the students than their prior experiences. This caused ‘bumpiness’ (Doyle, 1988) in the flow of the lesson and students tried to get help from Linda or me. Linda judged the students’ understanding through her interactions with the groups as they worked on the problem and also on the written class work that they submitted at the end of each class period. She tried to intervene so that the class flow would be ‘smooth’ (Doyle, 1988). Linda also attempted to maintain a high LCD by getting the students to turn in their written work, share their solutions with the whole class, and work together as a group on a task before getting guidance from her. Through the entire study, the students submitted their class work for a grade. Linda usually assigned similar tasks to reinforce ideas when she observed the students struggling with in the previous task. In her discussions of the MTF, Linda displayed an understanding of this framework through her task sort and points that she raised during the discussions. She also reinforced some of these ideas in her class like the maintenance of accountability by the students. In the selection of the tasks, Linda moved from a focus on the content and the POs, in the first stage, to incorporating ideas of student thinking and adapting tasks, in the last stage. In the set-up Linda moved from setting up tasks the way they appeared in the curriculum to supporting the students’ thinking and pushing for students’ independent thinking and sharing their work with the class. Finally, within a topic there was a gradual progression to more challenging problems that built on prior ideas.

Conclusion of the Results

The partnership evolved in three stages determined by the content-teaching tensions in the second stage. There was a shift in the nature of discussions from the content to the teaching aspects and this induced more active participation from Linda. However, uncertainty was generated with the movement towards teaching and adjustments were required to ensure that the mathematics was part of the discussions. In the last stage the mathematical and teaching issues were intertwined as Linda reflected on her interactions with the students in the class and she selected problems that addressed these issues through planning. In the teaching issues Linda moved from seeking my help with mathematical issues that arose in the class to showing evidence of independently reflecting and resolving these issues in the last stage of the partnership. Differences between the language in the tests and the CMP, dominated the discussion on assessment issues. I illustrated the potential of using the assessments tasks as sites of learning and illustrated the power of connections in tying the ideas together to save time from regular reviewing.

The tasks that were selected though the study, were overwhelmingly at the *procedures with connections* level and they generally stayed at this same level at the set-up phase. Linda made an effort to ensure that the students understood the problems, but her guidance still left room for student thinking and the level of cognitive demand was maintained. Linda made further efforts at maintaining a high LCD by asking the students to discuss in their groups, report to the whole class, and submit all the written class work for grading. In the next chapter I discuss the interpretation of these results.

CHAPTER 5: DISCUSSION

This chapter examines the results for my research question holistically. I begin with a description of the evolution of the partnership and examine key elements that facilitated this evolution. The next section examines the lessons I learned from the study and these include lessons about the mathematical and pedagogical issues in planning and teaching, the negotiation of tasks, and implications for professional development. The final sections outline the limitations of the study and future research questions generated by the study.

The Movement in the Partnership

Whitford, Schlechty, and Shelor (1987) identified partnerships based on relationships and classified them as cooperative, symbiotic and organic. The partnership in this study through the three stages, outlined in the first research question, reflected an evolution from a symbiotic partnership to an organic one. I began with the idea of symbiotic partnership where we had distinct individual goals and helped each other to achieve these goals. Linda’s goals were to further her understanding in the mathematics that she taught and my goal was to expose Linda to the important mathematical ideas and examine the evolution of our partnership as we had these content discussions. These goals were reflected in our interactions at the first stage where the mathematics content was at the core of our discussions. Linda followed as I set the agenda for the discussions. She ensured that I reached my research goals by working with me and even spending her lunch time in some cases. We both made efforts to ensure that the other person achieved his or her goals. At this stage, the classroom and student thinking were not an explicit focus of our discussion.

Gradually, as Linda chose to adopt the CMP curriculum, she had more pressing questions about the activities and implementing them in the classroom. This can be seen in the tensions that arose in the second stage of the partnership. Our content discussions were interrupted by Linda who wanted to discuss a specific activity she had in mind. Other things, such as the homework problems that I assigned her and recording her reflections, took a backseat to her immediate task of implementing the CMP curriculum. The appearance of tensions directed attention away from the content and the original research agenda to the immediate needs of Linda. At this stage the mathematical issues got attention as part of the teaching issues and in general I had to steer Linda's attention to the mathematics that was present in the activity. Linda's major focus was the curriculum with little attention to student thinking, but this changed in the third stage.

In the third stage, Linda and I both focused on the CMP curriculum and the issues that arose in the classroom during implementation. Thus we moved to an organic partnership with the goal being the implementation of the CMP curriculum and reflection on the related classroom issues that arose as a result. Linda focused on the student thinking she observed in the classroom, such as when she assigned the Median problem (Appendix B) to address a misconception that she observed in her students. Our interactions around the design of the Shoe problem (Appendix B) also revealed how the student thinking (or limitations) filtered into our discussions and impacted the activity that was assigned in the classroom. My role at this stage was more supportive as Linda took the lead in setting the agenda and as we acquired a common focus.

Whitford, Schlechty, and Shelor (1987) discussed the common goals that characterized an organic partnership was usually a boundary spanning issue, which is not 'owned' by any one of the partners. In this study the issue of implementing the CMP curriculum and the focus of classroom issues around the implementation could be considered as the boundary spanning issue and was at the heart of the shift from a symbiotic partnership to an organic one. Both of us decided to reconceptualize our original goals with a focus on the new common goal of implementing the CMP curriculum. My original goal of discussing the content still arose, although with significant unpredictability, in our discussions around the activities that Linda wanted to implement in the classroom. In Linda's case, she got to learn mathematics via mathematical issues that arose in the planning and implementation of this new curriculum. Further, the students were exposed to a new curriculum in the middle of the year and a number of pedagogical issues were raised in the classroom. The mathematical and teaching issues acted as perturbations in the reorganization of Linda's mathematics knowledge for teaching (Ball & Bass, 2000) through her efforts to resolve the perturbations by reflection and discussion.

Key Elements for the Movement

In this section I discuss the key factors that caused the partnership to move from a symbiotic partnership to a more organic one. The first factor is the decision of Linda to use the CMP in her classroom and I conjecture that the discussions at the first stage were an important part of this decision. Besides this structural decision of using the CMP, another important aspect contributing to the development of our relationship was the management of the dialectic tensions (Rawlins, 1992) that promoted collegiality.

Stage 1

The first stage of this study was important in setting the pace for the rest of the study. During the first stage Linda grappled with the mathematical concepts in the CMP as part of our after class discussions and she realized the potential of implementing the entire curriculum in her classroom. Linda saw how multiple Performance Objectives were addressed, how the conceptual understanding of the students could be developed, and how connections were made between and within topics through the problems. Thus the students had the opportunity of understanding by building and strengthening their internal networks (Hiebert & Carpenter, 1992). Linda also saw the value of these activities in preparing the students for the assessments and also to address the time pressures that she faced in completing the syllabus on time. She also knew that the activities would be more engaging to the students than their regular textbook exercises. The first stage went a long way in establishing the credibility of the CMP as a valid curriculum in her class. I was also there to help her with the initial start up problems if they arose. The first stage was crucial in determining the path that our partnership took over time.

Managing the Dialectical Tensions

Besides the implementation of the new CMP curriculum and our willingness to reconceptualize our original goals, effectively managing the dialectic tensions (Rawlins, 1992) in the relationship was also a factor that contributed to the shift in the partnership. Rawlins (1992) outlined four major dialectic tensions that arose in relationships that either fostered or hindered the relationship, namely, dialectics of acceptance/judgment, dependence/independence, affection/instrumentality and expressiveness/protectiveness. After outlining each of these dialectic tensions with respect to our partnership, I discuss some of the interconnections among them. The important point of this study is my contribution to the understanding of how these tensions can be managed in a teacher-researcher partnership.

The dialectic of judgment/acceptance referred to the tension between evaluating and holding the other partner to standards or accepting them with their strengths and weaknesses. Discussions of the mathematics content may have been perceived by Linda as an evaluation of her mathematical knowledge and there was potential for our partnership to stall. Later in the study Linda mentioned that she was reluctant to talk too much in the classroom as she was afraid of making a mathematical error in front of me as she perceived me as the ‘expert’. In our interactions around the content, I had to be careful not to appear to be judging Linda’s mathematics knowledge. As a result of this, I would tend to answer the questions that Linda had about the mathematics in the problem, rather than let her solve it independently. Later in the study, with my acceptance of Linda’s need to discuss lesson plans and mathematical issues that arose in teaching, this wait time increased as we better understood each other. As our discussions shifted to teaching and student thinking, Linda gained more confidence as she could participate more in the discussions through her contributions that she had on teaching and student thinking. Two factors that could have contributed to building trust and acceptance in Linda was acknowledging errors that I made in the mathematics content and recognizing that she knew more about her classroom and students. I encouraged her to share her knowledge of the classroom and students as part of the discussions and explicitly recognized her contribution.

The dialectic of dependence/independence referred to the freedom for partners to be independent but also to be there in times of need of the other partner. It is difficult to maintain a relationship when there is too much of dependence or independence. Thus a balance is needed and this had to be managed in our partnership. For example, Linda and I entered into the partnership by choice and there was freedom of independence but also a commitment to some level of dependence on each other. Linda felt free to cancel our session if she had other work that needed attention and I accepted these instances as part of the study. This flexibility went a long way in establishing a trusting relationship. Linda had the option of doing the homework problems and the reflections as time permitted. I was there to support Linda when she wanted suggestions for her class or asked me to comment on activities/problems that she chose for the class. I could also depend on Linda for allowing me to work with her and she was also open to implementing our discussed ideas in the classroom. Thus we could both express our dependence and independence in this partnership and this was one of the reasons that our partnership prospered. If one partner felt restricted by the other then it would have been difficult to make progress.

The dialectical tension of affection/instrumentality referred to one partner helping the other for the sake of helping rather than with the motive of getting the favor returned. In work between teachers and researchers, there has been a tendency to view the teacher as being useful in getting the research done. In our partnership Linda asked me to outline the benefits that she would get in working with me and I gave two, namely learning more mathematics and helping out in her classroom. These were the conditions on which the study started and trust was built by adhering to these conditions during the course of the study. In thinking about this dialectic, I questioned myself after the appearance of the tensions in the second stage and decided to deviate from my research agenda in an attempt to resolve these tensions. Thus I decided to follow Linda's lead in the final stage of our partnership and decided to support her, even though there was a great deal of uncertainty involved in the outcomes. Further there was a risk that it could move away from the focus of my research. Linda also made efforts of spending her lunch hours with me when the need arose, so that I could collect more data for my research.

The dialectic of expressiveness/protectiveness referred to efforts that the partners make to balance restraint and candor so that it furthers trust. In the first stage it was better to exercise restraint with respect to Linda's mathematical knowledge so that there was a degree of trust established before I could be expressive about errors that she made. I could also push for Linda to solve problems independently and could be more candid in my suggestions about the mathematical and teaching issues that arose. The Mathematics

Task Framework contributed to our conversations as it took focus away from Linda's mathematical abilities and allowed us instead to focus on the task. Thus we could be expressive about the tasks and this facilitated protectiveness in aspects of Linda's knowledge.

The above discussion outlined each of the dialectic tensions separately, but there were also interconnections among them. Our partnership began with some level of dependence on each other for our own goals. With this there were moves by both of us to help the other person achieve the other partner's goals and the affection/instrumentality dialectic was in focus. If Linda perceived I was using her to get my research done, there might have been a change in the path of our relationship. Once she perceived that I was assisting her towards her goal, there was more trust and acceptance on her part. She did things for me to further my research and so there was also more trust and acceptance on my part. Further, as discussions of teaching began, Linda gained confidence and developed a degree of independence in the discussions with her ideas. With more acceptance and trust from both sides, there was expressiveness. For example, Linda felt comfortable enough to tell me that she used to perceive me as the 'expert' and this caused her to speak less in the class. With our growth in the partnership, I could be more expressive with Linda and point out mistakes or push her to think independently without fear of our relationship stalling.

A point to note about the report on the management of the dialectic tension is that they were from my point of view. Linda's view was incorporated through my interpretation of her observable behavior in the public domain. Further, my role as an

Insider (Ball, 2000b) limited my access to Linda's feelings that could have been shared in an interview with a third party.

In addition to the dialectic tensions described above, there were other tensions or dilemmas, not dialectic in nature, which were integral to the study. These were the content-teaching tensions described in the second stage and the dilemma related to the amount of guidance to provide Linda (refer to the section on Constraints in Chapter 4). The content-teaching tension was based on my interpretation of the data and was not something that I managed during the course of the study, but something I would be aware of in future studies. The tension of guidance/independent thinking was a dilemma that was exclusive to me during my interactions with Linda and was essential in promoting teacher understanding within acceptable levels of frustration.

The move of the partnership from symbiotic to an organic one via the resolution of the tensions and maintaining the balance of the above four dialectical tensions, addressed the hierarchical nature of the traditional learning relationship (Olson, 1997). The partnership allowed Linda and me to work in a safe zone that brought together the research and practical knowledge and applied it to the classroom. Thus we were "provided with a way to reconstruct [our] past knowledge and imagine future possibilities" (Olson, 1997, p. 24). Our collaboration was achieved by understanding the work of the other partner through joint reflection and discussion and hence mirrored the ideas of Clarke et al. (1996). The adaptation of the Shoe problem (Appendix C) exemplified our shared understanding of each other's roles, namely my role as a resource for mathematical activities and Linda's role in evaluating these activities for her students in the classroom.

This study confirmed claims by Cardelle-Elawar (1993) and Miller and Pine (1990) that the extended interactions between teachers and researchers led to a better relationship. However, as I indicated above (managing the tensions) there was significant work involved by both partners. Clift et al. (1990) and Raymond and Leinenbach (2000) claimed that engaging teachers in research encouraged the teachers to think about classroom issues and increased the chance that they would implement the reform efforts in their classroom. This study illustrated that there are other ways (besides involving them in action research) to get the teachers to think about the classroom issues and implement reform efforts. Linda's implementation of the CMP curriculum was a big step towards shifting to a reform oriented classroom. Additionally, our discussions around the Mathematics Task Framework also facilitated Linda to reflect on classroom issues. Positive feedback from the students was an important factor in sustaining the CMP curriculum in the classroom.

Lessons Learned

The results of the study shed light on the nature of the mathematical and pedagogical issues that arose in planning and teaching and the negotiations that happen around tasks. Further, implications about teacher professional development can also be gleaned from the results. I examine these issues in the following sections.

The Curriculum

In this study neither Linda nor I envisioned the central role that the CMP curriculum would play and the eventual adoption of the curriculum. It is possible that Linda's experience with the *Investigations* curriculum (TERC, 1995) during her teaching in elementary school gave her experience in the reform curriculum and oriented her to take a favorable stance with the CMP curriculum. Our work in the first stage, the students' positive feedback, and my support factored in to make her decision. New curriculum is usually handed down to teachers and they do not get the time and support needed to test it out in their classroom and this study points to one way that teachers could be initiated into a new curriculum. The teacher's willingness to take risks and try out activities in the classroom is also an important factor that I discuss in more detail in the section on Teachers' Risk Behavior.

The Issues in Planning and Teaching

Examining the issues that arose in planning and teaching, there was a shift from a focus on specific mathematics content in the first stage, to issues that were close to the PCK (Shulman, 1986), in the third stage. In the first stage, the issues that were discussed solely involved the mathematical content such as the definition of sample space or the connection between experimental and theoretical probabilities. There was no focus on the students' thinking or on the associated pedagogy of these topics. In the second stage, the issues that arose were normally connected to the planning and implementation of an activity that Linda chose. The issues were based on questions that the students had to answer and Linda wanted to solve these questions beforehand in our discussions. There was a notion of 'correctness' involved with the questions. For example, Linda asked if her interpretation of the Bike trip on Day 4 was 'correct' or she wanted to work out the time to suck a lollipop that was six times the size of the original lollipop to know the 'correct' answer. At the third stage, the student thinking and pedagogical issues entered into the mix along with the content issues described in the first stage and the issues resembled the PCK described by Shulman (1986) or the mathematics knowledge for teaching outlined by Ball and Bass (2000). For example, Linda's 'what if' moves in the Raisin problem is an illustration of the combination of the mathematics knowledge of statistics, with student thinking, and pedagogical issues that contributed to her decision to reject the activity. However, in reflecting about the students' thinking, there was a risk for the discussion to focus on what the students did not know and attempt to frame the activity in a way that prevented the students from grappling with problematic issues. For example, in the planning of the Shoe problem, Linda assumed that the students would have difficulty with fractions and proportions and made the students fill in a table instead of allowing them to grapple with the problem and arrive at their own representations.

Tasks

In our partnership, the ambiguity in the task had to be negotiated between Linda and me as we assumed different views. Linda focused on finishing the content in a reasonable time frame for the tests and wanted to prevent students frustrations on a task and I wanted to preserve the open nature of the tasks to facilitate exploration and grappling with the concepts. Over time, reducing the ambiguity in the tasks to promote a smoother work flow (Doyle, 1983) in the classroom could result in lower expectations for the students. The balance was delicate to maintain in this study and an important goal for a researcher should be to encourage the teacher take calculated risks in relinquishing control over time, even if it means a disruption of the classroom flow. Since the only way that the teacher can learn how to handle the situation is to experience it. At this point trust and the dialectic tension of expressiveness/protectiveness could play a part in the teacher taking a risk with the support of the researcher.

There was evidence that the MTF is a factor that could facilitate a focus on student thinking. For example in one of the MTF discussions, Linda referred to the Dart problem (Appendix C) and the inability for the students to start the problem based on the knowledge that they had. The discussion that followed focused on the reasons why they were having this problem and what could be done. The MTF proved to be a useful framework in focusing attention on the student thinking.

Teachers' Risk Behavior

In the introduction to this study, I cited ongoing professional development as crucial for teachers to align their vision to that created in the Standards (NCTM, 2000). One aspect of this alignment is to use innovative materials that would foster problem solving, connections, representations, communication, and reasoning, and proof as outlined in the Standards (NCTM, 2000). However, Doyle and Ponder (1977) identified the 'practicality ethic' as a crucial piece that influenced the teacher's decision of adopting a new innovation. The practicality ethic referred to "an expression of teacher perceptions of the potential consequences of attempting to implement a change proposal in the classroom." (p. 6). In evaluating a new innovation the teachers sought details of how this innovation was to be implemented in the classroom, if it fit into their view of the classroom conditions, and finally the cost that one had to pay in terms of time and resources for training and addressing issues that could arise on classroom implementation. These factors highlight the inherent risk the teacher has to take in incorporating a new innovation in the classroom.

The nature of risk behavior in people was described by Kahneman and Taversky (1979) in their Prospect Theory and two important ideas being the *framing effects* and *reference point*. According to Prospect Theory, when the alterative to a risky choice involves a gain then the person becomes risk-averse; however, when the alternative to a risky choice is a loss, then the person becomes risk-seeking. The framing of the choices influenced the decisions that people made and this was referred to as the *framing effects*. The other important aspect of Prospect Theory is the *reference point*, which refers to the subjective view that forms the basis of choices that people make. For example, if the overall wealth of a person was $200,000 and this person was used to paying $15 for lunch. A lunch of $200 would be viewed as being $185 more that the expected value, and not as a decrease in the overall wealth to $199,800, which is not too much.

The constructs of *framing effects* and *reference point* could be modified in a way to be useful for teacher professional development. I use my study to illustrate this point. In the current study, Linda had a choice between the traditional curriculum and the reform curriculum. The reform curriculum was more risky to implement due to considerations of the ‘practicality ethic’ (Doyle & Ponder, 1977) and the high stakes testing environment. But despite the risks involved, Linda decided to implement the CMP in her classroom. I conjecture that Linda's *reference point* was that she was not completely satisfied with the traditional curriculum, which she referred to it as being boring. Linda also felt that this feeling may have been conveyed by her subconsciously to her students. Further, Linda was open to alternative curriculum choices, as long as these choices were framed in a way that would address her perceived risks in her current context. Note that my use of *reference point* shares the subjective nature with the *reference point* in Prospect Theory. The first stage in the partnership contributed to the *framing effects* as it addressed the risks by framing the outcomes in a way that tilted the balance in favor of the CMP. We worked together with the CMP and open ended activities to reduce the risk that Linda perceived in them to the level that she was willing to risk these activities in the classroom. This idea of effectively framing a new innovation could promote the use of curriculum aligned with the Standards (NCTM, 2000) or professional development with teachers. Further, the constructs add to the language of researcher as they discuss the teachers' *reference point* and the corresponding *framing effects* that would work towards reducing the perceived risk in the eyes of the teachers. However, more work is needed in defining these constructs in term of teacher professional development and this is something I would pursue in the future.

Validity

During the course of the study I had frequent discussions with my supervisor about the reasonableness of my classifications. He also saw my written work backed up with direct quotes from the data to check my interpretations. Further, this teacher was his student for a semester long course at the university. My supervisor also had one of his student teachers observe in Linda's classroom and so he was also provided with another perspective of her that was different to mine. I looked back through the data once I had conjectured the evolution through the stages and looked for disconfirming evidence that I have listed as anomalies in the general direction of the partnership. In this study I examined the interactions with Linda and thus the main basis for the study were the capturing of the interactions on audio and video. I also met with Linda for lunch during which we discussed some aspects of the study and she was positive about the whole experience in the growth of her mathematical knowledge. She felt that the study had generated a curiosity in her and we even discussed the Triangular numbers problem (Appendix C) during the lunch and I pointed out an elegant visual solution that we had missed when we discussed it during the study. Now that I have a close relationship with the teacher, I feel that she may be reluctant to critique aspects of my study. Even though I had other data such as my research journal, fieldnotes, and reflections by Linda, the tensions were not easy to corroborate as I had to look at the data in its entirety to understand that there was a shift in the interactions. Examining the data locally did not provide me with this perspective. After I arrived with the categories, I sent an email to Linda asking her to verify my outline of the three stages, their characteristics, and my interpretations.

Limitations of the Study

The circumstances in the current study did not allow me to work with a single grade level, which would have been ideal. Working with a single grade level would have eliminated some of the variation present in the student population and the corresponding classroom issues that arose. By spanning the study over the second part of a seventh grade class and the first part of a sixth grade class, the classroom norms were established in the first class and were being set in the second class. This could have affected the way Linda handled the classes and the activities and problems that she chose to assign them. Further, the sixth graders joined this Middle school from various Elementary schools and Linda mentioned that they usually came in with varying mathematical backgrounds and experiences.

My focus of the study was Linda and I did not focus on the students, which would have given me a better overall picture of the context. This would have involved collecting student work, focusing on the interaction of students in the classroom, interviewing certain students, and collecting their test scores. There were English Language Learners in both the classes and I did not examine how this might have affected the way that Linda interacted with the students in the classroom, especially in setting up the problems from the CMP and subsequent interactions. There were instances when Linda spent a significant time in the setup to ensure that the students understood the problem and I interpreted this as a lowering in the cognitive demand of the task. But it just may have been that the cognitive load (Sweller, 1988) on the students was already high and she moved to reduce the load.

The evolution of the stages of our partnership was my interpretation to the data and it is possible that the stages were linked to the content that was discussed in that time frame. During the first stage we discussed probability and Linda expressed that she found this topic challenging. This could explain my dominance in our interactions at that time.

Functions were touched upon during the second stage and Linda was familiar with graphs, tables and symbolic representations of linear functions. This could explain the higher degree of Linda's interactions in the discussions. The tensions appeared as she sought to clarify her ideas with me before she implemented the activity in the classroom. Her knowledge of the topic gave her confidence to shift the direction of the discussions and ask me questions. Linda's interactions increased further in the third stage as she was confident of her knowledge in Statistics and she also tried to adapt the materials.

The three month time frame for the study was short for the development of a partnership, which takes more time. The number of observations and the time spent interacting with Linda compensate for this in some way. In the shorter time span with the number of tests that they had, Linda could not dwell for long on a particular activity as we had to keep moving on.

My background in mathematics, mathematics education, and experiences in teaching mathematics in India, Hong Kong, and the US inform my mathematical perspective in a unique way. Many decisions that I made 'on the fly' would be informed through my perspective and this would change if another researcher replicated the study with Linda. Further, Linda was a teacher who was at a point where she was open to new ideas and motivated to learn more mathematics. Change in these factors could cause the partnership to evolve differently.

Future Research Questions

This study generates a number of directions for future research. The major direction is to work with a group of teachers in a school or school district over a longer period of time and examine the evolution of this collaboration. A similar study could begin with discussions of problems from the CMP, examine the tensions that evolve and how these tensions get resolved in the group. The interactions will extend beyond the teacher-researcher to teacher-teacher; will increase the complexity of the study and the corresponding evolution of the collaboration.

In the current study, Linda was the only teacher in the school to adopt the CMP curriculum. It would be of interest to extend this study for a longer period of time and examine the interactions she has with the other mathematics teachers in the school and if these interactions generate interest in the other teachers for using the CMP curriculum. Recently, the school district has adopted the CMP curriculum for all the schools and a study could look at the leadership role that Linda assumes with the other teachers.

Another aspect of this study would be to examine the influence that the students had on the study by examining student work, interviewing selected students, tracking the interactions of a group of students in the classroom, and examining the scores on the standardized tests. The study could also be extended to study the effects of the language of the CMP curriculum on the English Language Learners and the support that the teacher could render.

Examining the risk construct could be a fruitful avenue of research. In my discussion in a prior section, I conjectured that the *reference point* of the teacher along with the *framing effects* as useful constructs for discussing mathematics teacher education. By understanding the risk behavior, researchers will be able to frame a new innovation in a way that teachers would see the benefits to their everyday work. A survey with questions that would involve choices that teachers normally make as part of their everyday work can be a first step in understanding the risk taking behavior of mathematics teachers. Focusing on the nature of the tasks that teachers assign over a period of time could also be a useful avenue to explore the risk taking behavior of teachers.

APPENDIX A: TASK SORT

Adapted from the *Professional Development Guidebook for Perspectives in the Teaching of Mathematics* (Bright & Rubenstein, 2004)

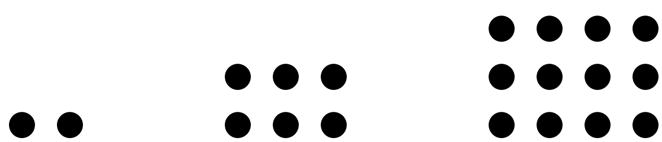
Task sort Activity

Directions for the task-sort:
This packet contains 16 middle school level mathematics tasks. Please read them through, and then sort them into categories of your own making. Write down names of your categories and then list the tasks that fall into each category.

Task A
Manipulatives/Tools available: Calculator
Treena won a 7-day scholarship worth $1,000 to the Pro Shot Basketball Camp. Round-trip travel expenses to the camp are $335 by air or $125 by train. At the camp she must choose between a week of individual instruction at $60 a day or a week of group instruction at $40 a day. Treena's food and other expenses are fixed at $45 a day. If she does not plan to spend any money other than the scholarship, what are all choices of travel and instruction plans she could afford to make? Explain which option you think Treena should select and why.

Task B
Manipulatives/Tools available: Counters

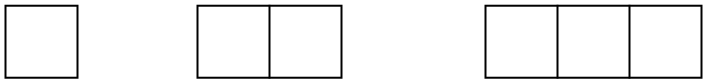
1st Step 2nd Step 3rd Step



2 dots 6 dots 12 dots

Marcy has to determine the number of dots in the 20th step, but she does not want to draw all 20 pictures and then count the dots.
Explain how she could do this and give the answer that Marcy should get for the number of dots.

Task C
Manipulatives/Tools Available: Square Pattern Tiles
Using the side of a square pattern tile as a measure, find the perimeter (i.e. distance around) of each train in the pattern block figure shown below.



Train 1 Train 2 Train 3

Task D
Manipulatives/Tools available: None
Part A: The place kicker on the North High School football team has made 13 out of 20 field goals this season. The place kicker on the South High School football team has made 15 out of 25 field goals so far this season. Which player had made the greater percent of field goals?
Part B: The "better" player had to sit out for the rest of the season, how many field goals would the other player have to make in the next 10 attempts to have the greatest percentage of field goals?

Task E
Manipulatives/Tools available: Calculator
Divide using paper and pencil. Check your answer with a calculator and round the decimal to the nearest thousandth.
525 / 1.3
52.75 / 7.25
30.459 / 0.12

Task F
Manipulatives/Tools available: None
Match the property name with the appropriate equation.

	1. Commutative property of addition.	a. $r(s+t)=rs+rt$
	2. Commutative property of multiplication.	b. $x(\frac{1}{x})=1$
	3. Associative property of addition	c. $-y+x=x+(-y)$
	4. Associative property of multiplication.	d. $\frac{a}{b}+(\frac{-a}{b})=0$
	5. Identity property of addition.	e. $y(zx)=(yz)x$ y
	6. Identity property of multiplication.	f. $1(xy)=xy$
	7. Inverse property of addition.	g. $d\times 0=0$ and $0\times d=0$
	8. Inverse property of multiplication.	h. $x+(b+c)=(x+$
	9. Distributive property.	i. $y+0=y$
	10. Property of zero for multiplication.	j. $p\times q=q\times p$

Task G
Manipulatives/Tools Available: Base 10 blocks, grid paper.

.08 .8 .080 .008000

Make three observations about the relative size of the above four numbers. Be sure to explain your observations as clearly as possible. Feel free to illustrate your observations if you feel it would help others understand them.

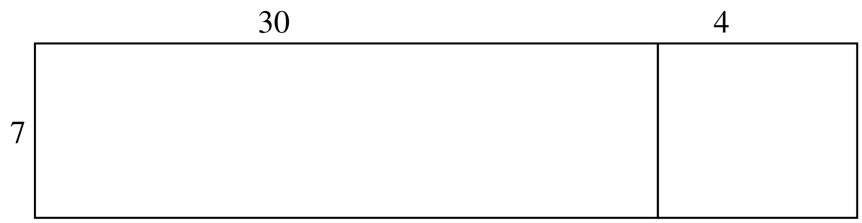
Task H
Manipulatives/Tools Available: Grid paper.
The pairs of numbers in a – d below represent the heights of stacks of cues to be leveled off. On grid paper, sketch the front views of columns of cubes with these heights before and after they are leveled off. Write a statement under the sketches that explain how your method of leveling off is related to finding the average of the two numbers.

a. 14 and 8
b. 16 and 7
c. 7 and 12
d. 13 and 15

By taking 2 blocks off the first stack and giving them to the second stack, I've made the two stacks the same. So the total number of cubes is now distributed into two columns of equal height. And that is what average means.

Task I
Manipulatives/Tools Available: None
Write and solve the proportion for each of the following:
17 is what percent of 68?
What is 15 % of 60?
8 is 10% of what number?
24 is 25% of what number?
28 is what percentage of 140?
What is 60% of 45?
36 is what percent of 90.
What is 80% of 120?
21 is 30% of what number?

Task J
Manipulatives/Tools Available: None
Level: Middle School
One method of mentally computing 7×34 is illustrated in the diagram below:



$7 \times 30 = 210$ and $7 \times 4 = 28$

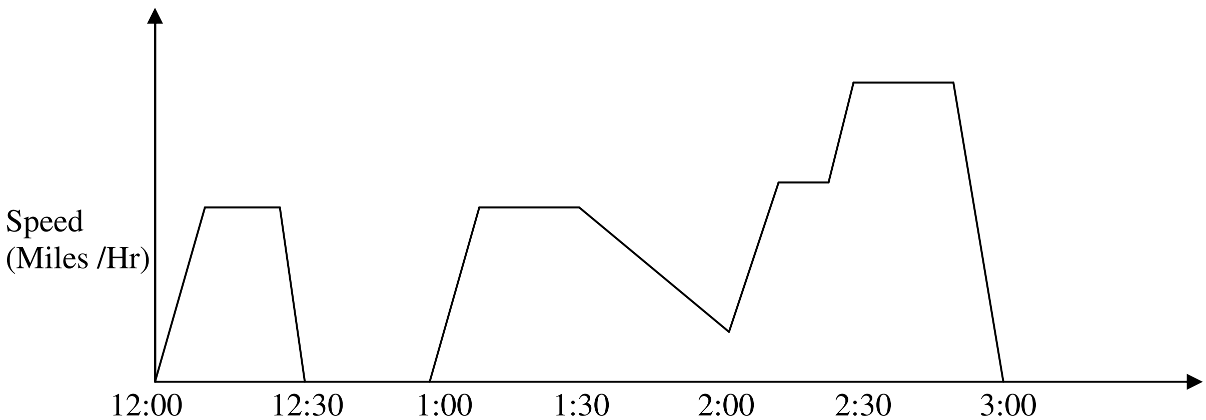
Mentally compute these products. Then sketch a diagram that describes your methods for each.

a. 27×3
b. 325×4

Task K
Manipulatives/Tools available: Calculator with scientific functions
Penny's mother told her that several of her great-great-great-grandparents fought in the Civil War. Penny thought this was interesting and she wondered how many great-great-great grandparents that she actually had. When she found that number, she wondered how many generations back she'd have to go until she could count over 100 ancestral grandparents or 1000, or 10,000, or even 100,000. When she found out she was amazed and she was also pretty glad that she had a calculator. How do you think Penny might have figured out all of this information? Explain and justify you method as clearly and completely as possible.

Task L
Manipulatives/Tools Available: Base 10 blocks
Level: Middle School
Using Base 10 blocks, how could you show that 0.292 is less than 0.3?

Task M
Manipulatives/Tools Available: None
Use the following information and the graph to write a story about Tony's walk :
At noon, Tony started walking to his grandmother's house. He arrived at her house at 3:00. The graph below shows Tony's speed in miles per hour throughout his walk.

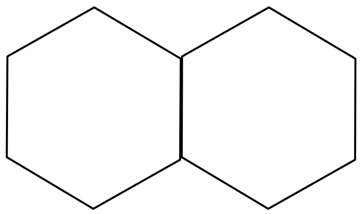


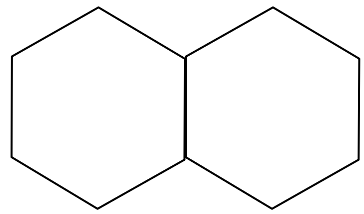
Task N
Manipulatives/Tools Available: None
The cost of a sweater at J. C. Penny's was $45.00. At the "Day and Night Sale" it was marked 30% off of the original price. What was the price of the sweater during the sale? Explain the process you used to find the sale price.

Task O
Manipulatives/Tools: none
Give the fraction and percent for each decimal.

.20 = __________ = ____________
.25 = __________ = ____________
.33 (recurring) = __________ = ____________
.50 = __________ = ____________
.66(recurring) = __________ = ____________
.75 = __________ = ____________

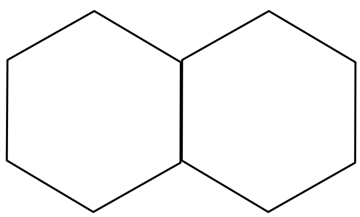
Task P
Manipulatives/Tools Available: Pattern Blocks
Find ½ of 1/3. Use pattern blocks. Draw your answer.

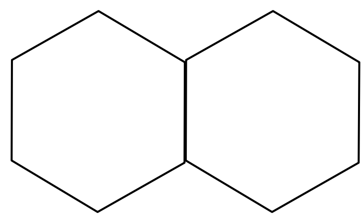




½ of 1/3 or ½ x 1/3 = ?

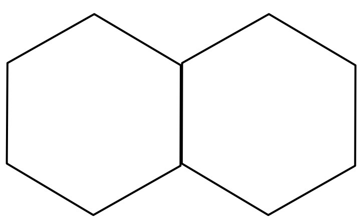
Find 1/3 of ¼. Use pattern blocks. Draw your answer.

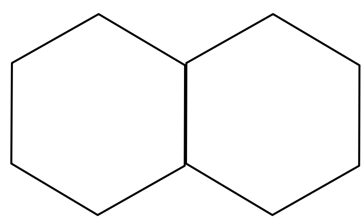




1/3 of ¼ or 1/3 x ¼ = ?

Find ¼ of 1/3. Use pattern blocks. Draw your answer.





¼ of 1/3 or ¼ x 1/3 = ?

APPENDIX B: DISCUSSION TASKS

Independent events

Tile problem
In a bag there are a certain number of tiles of two different colors. Can you predict the fraction of tiles that are blue or yellow? How will you go about doing this?
Hint: Take one tile, note its color, then replace it and repeat the process a certain number of times till you feel that you have a good number to make a prediction. This will be based on the number of trials that you have made till this point.

Siblings problem 1
A couple is planning to begin their family and would like to have 3 children. What is the probability that they will have 2 boys and 1 girl?

First work out all the outcomes and then represent as a tree, Pascal's triangle etc.

Siblings problem 2
A couple is planning to begin their family and would like to have 4 children. What is the probability that they will have 2 boys and 2 girls?

Siblings problem 3
A couple is planning to begin their family and would like to have 4 children. What is the probability that they will have at least 1 girl?

Dice problem
Suppose you roll two dice and add the numbers, what is the probability that the sum will be 7?

Coin problem
Toss a coin 10 times and record the number of heads and tails. Now toss the coin 20 times and do the same. What would happen if you tossed the coin a 100 times, 1000 times and 10,000 times?
(Aside: Instead of tossing a coin 10 times, will this be the same as tossing 10 coins once?)

Outcomes problem
Work out the total number of outcomes of the following events

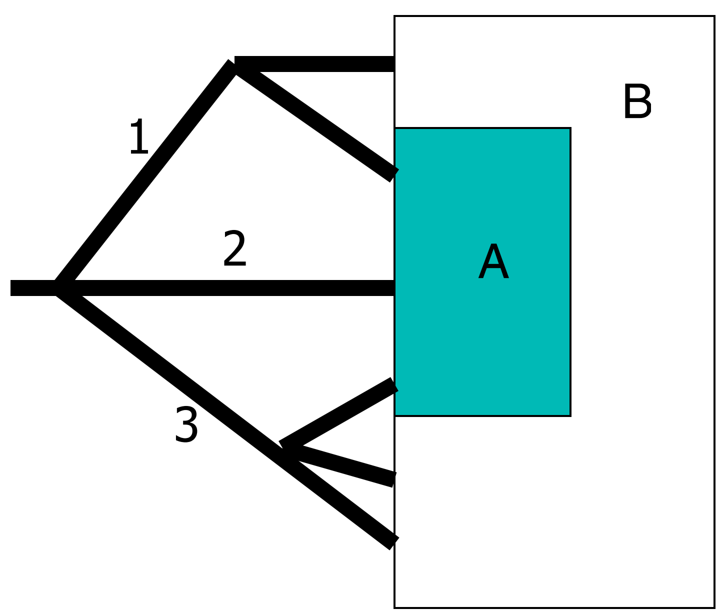
a. A die is tossed.
b. Two dice are tossed.
c. A die is tossed and a coin is flipped.

Siblings problem 4
A couple is planning to begin their family and would like to have 5 children. What is the probability that they will have 3 boys and 2 girls?

Quiz problem
In a true / false quiz with 10 questions, what is the probability of getting a 70%?

Dependent events

Maze problem
The king and queen had arranged for their daughter to be married to a prince, but she fell in love with a peasant. When the king discovered this affair, he ordered that the peasant be thrown into a room full of tigers. However, in response to his daughter's pleas, he agreed to have the peasant walk through a maze to one of two rooms. The princess would wait in one room and the tigers in the other. The princess asked if she could choose the room in which she could wait; the king agreed, for he believed that the chances were the same. If you were the princess, which room would you choose?



Gumball problem
There are 4 white and 2 red gumballs left in a machine. Josie would like 2 gumballs, one for her and another for her brother. What is the probability of getting at least one red gumball?

Expected Value

Basketball problem 1
Nicky is playing basketball on her school team this year. In the district finals, the team is 1 point behind with 2 seconds left in the game. Nicky has just been fouled, and she is in a one-and-one free-throw situation. This means that Nicky will try one shot. If she makes the first shot, she gets to try a second shot. If she misses the first shot, she is done and does not get to try a second shot. Nicky's free throw average is 60%. What is the probability that Nicky will score (a) 0 points (b) 1 point (c) 2 points? How can you show this using a picture?

Basketball problem 2
How does the probabilities and the picture in question 1 change if we assume that Nicky is allowed to try a second shot, regardless of the result of the first one.

Basketball problem 3
Suppose Nicky makes 100 trips to the free-throw line, what total number of points would you expect Nicky to make? What would Nicky's average number of points per trip be? [This is the expected value for this situation.]

Basketball problem 4
What is the shooting average of Nicky if she has an expected value of exactly 1 point per trip.

Carnival problem
The school is having a carnival to raise money for new computer equipment. The students are planning to have games. Two of the games are proposed below. Analyze the games and mathematically justify why you would choose the game for the carnival.

A – A bucket contains 4 blue marbles and one orange marble. Without looking, a player draws one marble from the bucket, replaces it, and then draws a second marble. If the marble is orange on either draw, the player wins. People are charged $2 to play the game and the winners are awarded $5 each.

B - A bucket contains 4 blue marbles and one orange marble. Without looking, a player draws two marbles, one at a time, from the bucket. The player does not replace the first marble before drawing a second marble. If either marble is orange, the player wins. People are charged $2 to play the game and the winners are awarded $5 each.

Permutations and Combinations

1. In how many ways can 9 students line up to take a picture for the yearbook?

2. The student council would like to select a president and a treasurer from among 9 committee members. In how many ways can this be done? Suppose they also want to select a vice president, in how many ways can they select the three officers?

3. Permutation of '*n*' objects taken '*r*' at a time.

4. The school council decides to send two committee members (from among 9) to attend a meeting at another school. In how many ways can these two members be selected? Suppose they decide to send three members, in how many ways can these members be selected?

5. Combination of '*n*' objects taken '*r*' at a time.

The Handshake problem
There are 'n' people at a party and everyone shakes everyone else's hand. What is the total number of handshakes?

The Rectangles problem
In a $n \times n$ grid, how many rectangles of different dimensions can you count. Assume that 1×2 and 2×1 are the same etc.

The Staircase problem
What is the total number of squares present in an '*n*' dimensional staircase.

Connections
How can you relate the Handshake, Rectangle and Staircase problems to Pascal's Triangle?

Functions

The Lollipop problem
Given a lollipop and graph paper, follow the following procedures and carry out the investigation below.

Procedures
1. Draw the x and y axes on the graph paper. Label the *y* axis "circumference" and the *x* axis "time".
2. Get an initial measure of the circumference.
3. Suck the lollipop for 30 seconds and then get another measure. Do this for a total of 3 minutes in 30 second intervals.
4. Suck the lollipop for 1 minute and then repeat for 5 more minutes to get a total of 6 measurements.

Investigation:
1. Draw a line that approximates your data as best you can.
2. Find the slope of the line that you drew. What does the slope mean in this case?
3. What is the sign of the slope? What does that tell you?
4. Find a linear function to model this data. Explain how you got this?
5. What are the domain and the range of the function that you found?
6. How long will it take you to finish a lollipop with a circumference 6 times the size of your lollipop? How long would it take you to finish a lollipop with circumference 1/2 the size of your lollipop?

Domain and Range
Find the domain and range for the following functions (Sketch graphs):
1. $f(x) = 3x - 2$
2. $f(x) = x^2$
3. $f(x) = \dfrac{1}{x}$
4. $f(x) = \sqrt{x}$
5. $f(x) = \dfrac{1}{x^2 + 2}$

Zeros of a function
Find the zeros of the following functions:
1. $z(t) = t^3 + 12t$
2. $f(n) = 45 - 5n^2$
3. $y(x) = \sqrt{2 - 5x}$

4. $g(x) = \dfrac{5}{x^2 - 4}$
5. $z(x) = 3x + 5 - \sqrt{2 - 2x}$

Expressing functions symbolically

1. A box has length and width that are equal, and the height is 4 units less than the width. Express the volume of the box in terms of the width only.
2. A cylindrical can has height twice its radius. Write the surface area of the can as a function of the radius.
3. The length of a rectangle is 2 units longer than its width. Express the length of the rectangle's diagonal in terms of the width only.
1. The area of a circle is A square inches. Find the circumference of the circle as a function of the area.

Algebra word problems

1. A pharmacist has 8 liters of a 15 percent solution of acid. How much distilled water must she add to reduce the concentration of acid to 10 percent? [ACD]
2. By installing a $120 thermostat that reduces the temperature setting at night, a family hopes to cut its annual bill for heating oil by 8 percent, and thereby recover the cost of the thermostat in fuel savings after 2 years. What was the family’s annual fuel bill before installing the thermostat? [HW, ACD]
3. Frank is eight years older than his sister. In three years he will be twice as old as she is. How old are they now? [ACD]
4. Karen is twice as old as Lori. Three years from now the sum of their ages will be 42. How old is Karen? [HW]
5. Dave has six times as much money as Fred, and bill has three times as much money as Fred. Together they have 550.00. How much does each have? [HW]
6. To find the length of a certain rectangle you must triple the width and add 5m. If the perimeter of the rectangle is 74m, find the dimensions. [ACD]
7. Fred gave one half of his baseball cards to Sally. Sally gave Jeff half of the cards that she got from Fred. Jeff gave Allen half of the cards that he got from Sally. Allen got 6 cards. How many cards did Fred originally have? [HW]
8. A lawn-and-garden dealer wants to make a new blend of grass seed by using 200 pounds of $0.45 per pound seed and some $0.65 per pound seed. How much of the $0.65 seed does the dealer need to make a $0.55 per pound blend?
9. A man is 13 times as old as his son is. In 10 years he will be 3 times as old as his son will be. How old are they now?

10. A man bought 20 chickens and ducks altogether, with a $2 discount per chicken and 50 cent discount per duck. He saved $22 in all. How many chickens and how many ducks did he buy?
11. Five years ago Ben was 2/3 as old as Kris. Ten years from now he will be 5/6 as old as Kris. How old are they now?
12. A petroleum distributor has two gasohol storage tanks, the first containing 9 percent alcohol and the second containing 12 percent alcohol. They receive an order for 300,000 gallons of 10 percent alcohol. How can they mix alcohol from the two tanks to fill this order?

Statistics

Median problem
Make a line plot or a bar graph representing 22 names which range from 10 to 21 letters and have a median of 10.

Raisin problem
How many raisins are there in a standard box of raisins?

Hat problem
You would like to open a hat shop and hats come in ten different sizes. You must order the hats in lots of 1000. How many of each size would you order for your middle school?

Shoe problem
[Name of Middle School] has a new line of tennis shoes. They have partnered up with Nike and need to know what sizes to purchase and how many of each assuming that shoes come in fixed lots (later the lots were fixed at 150 or 200 shoes).

Procedures that were given to the students:
1. Collect the shoe sizes from 4 kids between the ages of 11 and 14 from home or school.
2. At each table (with 3 or 4 students), assemble your data into a frequency chart.
[In this frequency chart, the teacher and I, had discussions about adding extra columns with the relative frequency, percentage of shoes of a given size, the number of shoes (given that the total order was either 150 or 200) etc.]
3. Work out the number of shoes of each size that should be purchased, based on your data.

APPENDIX C: CLASSROOM TASKS

Probability

Tile problem
What's in the bucket?
"One day Ms. MacAfee brought a mysterious bucket to class. She did not show her students what was in the bucket, but she told them that it contained blue, yellow, and red blocks. She asked if the could predict, without emptying the bucket, the fraction of tiles that were blue, the fraction that were yellow, and the fraction that were red. [2B, 4R, 6Y]

The class conducted an experiment to help them make their predictions. Each student randomly selected a block from the bucket, and the result was recorded on the board. After each draw, the block was returned to the bucket before the next student selected a block. In this problem, your class will conduct a similar experiment.

Your teacher has prepared a bucket identical to Ms MacAfee's. One at a time, you and each of your classmates will select a block from the bucket, record the result, and return the block to the bucket.

a. How many blocks drawn by your class were blue? How many were yellow? How many were red?
b. Which color block – blue, yellow or red- do you think there are the greatest number of in the bucket? Which color block do you think there are the least number of?
c. Based on your experimental data, predict the fraction of blocks in the bucket that are blue, that are yellow and that are red.
d. After your teacher shows you the blocks in the bucket, find the fraction of blue blocks, the fraction of yellow blocks, and the fraction of red blocks.
e. How do the fraction of the blocks that are blue, yellow and red compare to the fraction of blue, yellow and red blocks drawn during the experiment?

Dice problem
Roll a number cube 12 times. Record your results. What percent of rolls came up 6? What percent came up even? Roll the number cube 12 more times. Compute the percent of sixes and of even rolls for all 24 rolls. What percent were 6? What percent were even? Find the theoretical probability of getting an even?

Crime Scene Investigation problem
What if you worked for CSI and you were investigating some criminal offence and you found a witness and we wanted to come up with some of the characteristics that this witness said this person had and we needed to know all the different possibilities. Go ahead and give me three major hair colors. [student input Brown, Blonde, and Black] and there are three hair lengths short , Medium, and long.

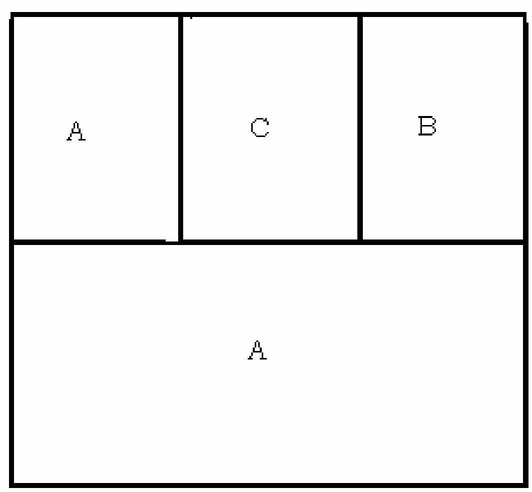
Match/No Match problem

April and Tioko decided to play the Match/No Match game on the spinner below. As in the original game, a turn consists of two spins. Player A scores a point if the spin match, and Player B scores a point if they do not match.

a. Use the counting tree to find all the possible outcomes for this game.
b. What is the theoretical probability of getting a match on a turn?
c. What is the theoretical probability of getting a no-match on a turn?
d. Do you think this is a fair game? If you think the game is fair, explain why. If you think it is not fair, explain how the rules could be changed to make it fair.

Dart problem

A dart is thrown at random at each of the dart boards below.



For each dart board, what is the probability that a dart will land in a region marked A? A region marked B? A region marked C?

Variables and Coordinate graphs

The popularity of bicycle tours gave five college students – Sidney, Celia, Liz, Malcolm, and Theo – an idea for a summer business. They would operate bicycle tours for school and family groups. They chose a route from Philadelphia, Pennsylvania, to Williamsburg, Virginia, including a long stretch along the ocean beaches of New Jersey, Delaware, and Maryland. They decided to name their business Ocean and History Bike Tours.

Jumping Jack problem

While planning their bike tour, the five friends had to determine how far the touring group would be able to ride each day. To figure this out, they took test rides around their hometowns. Although you can't ride your bike around the classroom, you can perform a simple experiment involving jumping jacks. This experiment should give you some idea of the patterns commonly seen in tests of endurance.

Problem 1.1

This experiment requires four people:

- A jumper (to do the jumping jacks)
- A timer (to keep track of the time)
- A counter (to count jumping jacks)
- A recorder (to write down the number of jumping jacks)

As a group, decide who will do each task.

Prepare a table for recording the total number of jumping jacks after every 10 seconds, up to a total of 2 minutes (120 seconds).

Time (sec)	0	10	20	30	40	50 …
Tot. no. of jumping jacks						

Here's how to do the experiment: When the timer says "go," the jumper begins doing jumping jacks. The counter counts the jumping jacks out loud. Every 10 seconds the timer says "time" and the recorder records the total number of jumping jacks the jumper has done so far. Repeat the experiment four times so that everyone has a turn at each of the four tasks.

Problem 1.1 Follow up

Use your table of jumping jack data to answer these questions:

1. How did you jumping jack rate (the number of jumping jacks per second) change as time passed? How is this shown in your table?
2. What might this pattern suggest about how bike-riding speed would change over a day's time on the bicycle tour?

Problem 1.2

a. Make a graph of your jumping jack data.
b. What does your graph show about jumping jack rate as time passes? (Another ay to say this is, What does your graph show about the *relationship* between the number of jumping jacks and time?)

Problem 1.2 Follow up

Is the relationship you found between the number of jumping jacks and time easier to see in the table or the graph? Explain your answer.

Bike trip: Day 1
Philadelphia to Atlantic City. The students began their bike tour near the Liberty Bell and Independence Hall in historic Philadelphia, Pennsylvania. Their goal for the first day was to reach Atlantic City, New Jersey. Sidney, Liz, Sarah, Celia, and Malcolm rode their bicycles. Theo and Tony followed in the van with the camping gear and repair equipment. Tony recorded the distance reading on the van's trip odometer every half hour from 8:00 am to 4:00 pm. Tony's recordings for the first day are given in the table below.

Time (Hours)	Distance (miles)
0.0	0
0.5	9
1.0	19
1.5	26
2.0	28
2.5	38
3.0	47
3.5	47
4.0	47
4.5	54
5.0	59
5.5	67
6.0	73
6.5	78
7.0	80
7.5	86
8.0	89

Problem 2.1
Write a report summarizing the data Tony collected on day 1 of the tour. Describe the distance traveled compared to the time. Look for patterns of change in the data. Be sure to consider the following questions;

- How far did the riders travel in the day? How much time did it take them?
- During which time interval(s) did the riders make the most progress? The least progress?
- Did the riders go further in the first half or the second half of the day's ride?

Problem 2.1 Follow up
Describe any similarities between the jumping jack data that you recorded in Problem 1.1 and the data Tony collected.

Bike trip: Day 2
Atlantic City to Lewes. On the second day of their bicycle trip, the group left Atlantic City and rode five hours south to Cape May, New Jersey. This time,Sidney and Sarah rode in the van. From Cape May, they took a ferry across the Delaware Bay to Lewes, Delaware. They camped out that night in a state park along the ocean. Srah recorded the following data about the distance traveled until they reached the ferry;

Time (Hours)	Distance (miles)
0.0	0
0.5	8
1.0	15
1.5	19
2.0	25
2.5	27
3.0	34
3.5	40
4.0	40
4.5	40
5.0	47

Problem 2.2

a. Make a co ordinate graph of the (time, distance) data given in the table.
b. Sidney wants to write a report describing the day 2 of the tour. Using information from the table or the graph, what could she write about the day's travel? Be sure to consider the following questions:
    - How far did the group travel in the day? How much time did it take them?

- During which time interval(s) did the riders make the most progress? The least progress?
- Did the riders go further in the first half or the second half of the day's ride?

c. By analyzing the table, how can you find the time intervals when the riders made the most progress? The least progress? How can you find these intervals by analyzing the graph?

d. Sidney wants to include either the table or the graph in her report. Which do you think she should include? Why?

Bike trip: Problem 2.2 Follow-up

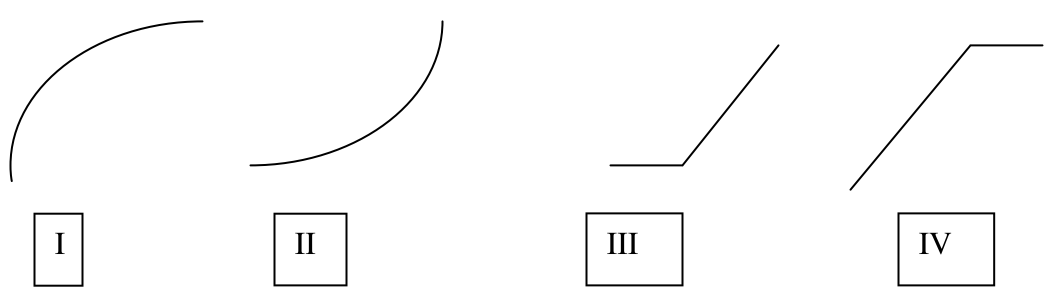
1. Look at the second point on your graph as you count from the left. We can describe this point with the coordinate pair (0.5, 8). The first number in a coordinate pair is the value for the x-coordinate, and the second number is the value for the y-coordinate. Give the coordinate pair for the third point on your graph. What information does this point give?
2. Connecting the points on a graph sometimes helps you see a pattern more clearly. You can connect the points in situations in which it makes sesnse to consider what is happening in the intervals between the points. The points on the graph of the data for day 2 can be connected because the riders were moving during each half-hour interval, so the distance was changing.

a. Connect the points on your graph with straight line segments.

b. How would you use the line segments to help you estimate the distance traveled after ¾ of an hour (0.75 hours)?

3. The straight line segment you drew from 94.5, 40) to (5.0, 45) gives you some idea of how the ride might have gone between the points. It shows you how the ride would have progressed if the riders had traveled at a steady rate for the entire half hour. The actual pace of the group and, of the individual riders, may have varied throughout the half hour. These paths show some possible ways the ride may have progressed:

Match each of these connecting paths with the following travel notes.



a. Celia rode slowly at first and gradually increased her speed.
b. Tony and Liz rode very quickly and reached the campsite early.
c. Malcolm had to fix a flat tire, so he started late.
d. Theo started off fast. After a while, he felt tired and slowed down.

Bike trip: Day 4
Chincoteague Island to Norfolk. On day 4, the group traveled from Chincoteague Island to Norfolk, Virginia. Norfolk is a major base for the United States Navy Atlantic Fleet. Malcolm and Sarah rode in the van. They forgot to record the distance traveled each half hour, but they did write some notes about the trip.

Malcolm and Sarah's Notes

- We started at 8:30 a.m. and rode into a strong wind until our midmorning break.
- About midmorning, the wind shifted to our backs.
- We stopped for lunch at a barbecue stand and rested for about an hour. By this time, we had traveled about halfway to Norfolk.
- At around 2:00 p.m., we stopped for a brief swim in the ocean.
- At around 3:30 p.m., we reached the north end of the Chesapeake Bay Bridge and Tunnel. We stopped for a few minutes to watch the ships passing by. Since bikes are prohibited on the bridge, the riders put their bikes in the van, and we drove across the bridge.
- We took seven and a half hours to complete the day's 80-mile trip.

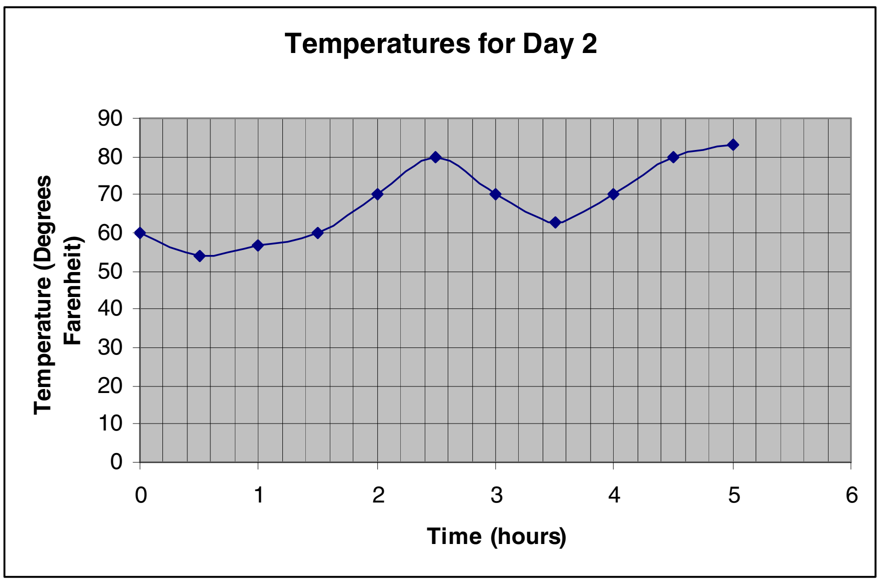
Problem 2.4
a. Make a table of (time, distance) data that reasonably fits the information in Malcolm and Sarah's notes.
b. Sketch a coordinate graph that shows the same information.

Lollipop problem
Procedures
1. Draw the x and y axes on the graph paper. Label the y axis "circumference" and the x axis "time".
2. Get an initial measure of the circumference.
3. Suck the lollipop for 30 seconds and then get another measure. Do this for a total of 3 minutes in 30 second intervals.
4. Suck the lollipop for 1 minute and then repeat for 5 more minutes to get a total of 6 measurements.

Investigation:
1. Draw a line that approximates your data as best you can.

2. Find the slope of the line that you drew. What does the slope mean in this case?
3. What is the sign of the slope? What does that tell you?
4. Find a linear function to model this data. Explain how you got this?
5. What is the domain and the range of the function that you found?
6. How long will it take you to finish a lollipop with a circumference 6 times the size of your lollipop? How long would it take you to finish a lollipop with circumference 1/2 the size of your lollipop?

Bike trip: Temperature problem
Here is the graph of temperature data collected on the students' trip from Atlantic City to Lewes.



[Note that the points are supposed to be connected by straight lines, even though it does not look like that in the above figure]

a. this graph shows the relationship between two variables. What are they?
b. Make a table of data from this graph.
c. What is the difference between the day's lowest and highest temperature?
d. During which time interval(s) did the temperature rise the fastest? Fall the fastest?
e. Is it easier to use the table of the graph to answer part c? why?
f. Is it easier to use the table or the graph to answer part d? Why?
g. On this graph, what information is given by the lines connecting the points? Is it necessarily accurate information? Explain your reasoning.

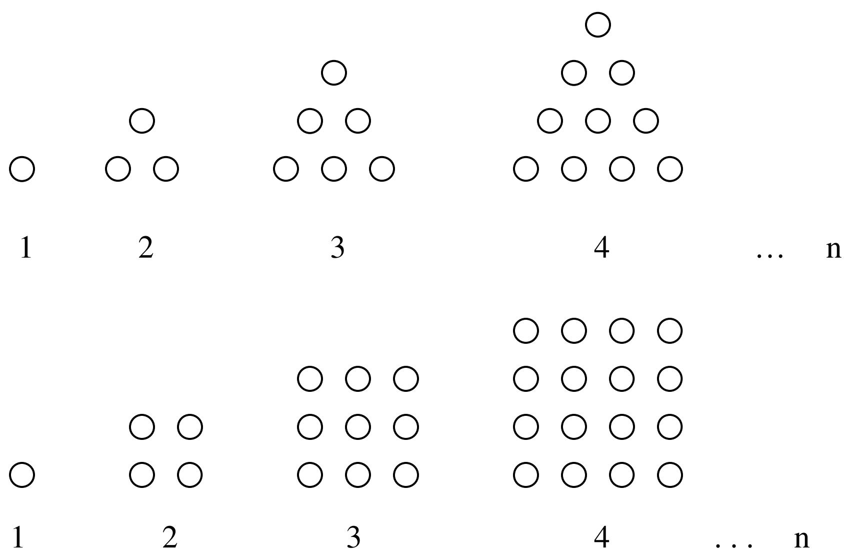
Median problem
Make a line plot or a bar graph representing 22 names which range from 10 to 21 letters and have a median of 10.

Shoe problem
[Name of Middle School] has a new line of tennis shoes. They have partnered up with Nike and need to know what sizes to purchase and how many of each assuming that shoes come in fixed lots (later the lots were fixed at 150 or 200 shoes).

Procedures that were given to the students:
1. Collect the shoe sizes from 4 kids between the ages of 11 and 14 from home or school.
2. At each table (with 3 or 4 students), assemble your data into a frequency chart.
[In this frequency chart, the teacher and I, had discussions about adding extra columns with the relative frequency, percentage of shoes of a given size, the number of shoes (given that the total order was either 150 or 200) etc.]
3. Work out the number of shoes of each size that should be purchased, based on your data.

Patterns problem
Fill in three patterns into a 100's chart and determine if 44 is in the pattern?

Triangular and Square numbers problem



1. What are the next two triangular and square numbers?
2. Other than 1, what is the next number that is both a triangular number and a square number?
3. Write a formula that will give the total number of dots at the n^{th} stage in both cases.

APPENDIX D: TASK ANALYSIS GUIDE

(Adapted from Stein et al., 2000, p. 16)

Lower-level demands

Memorization Tasks

- Involve either reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.
- Have no connection to concepts or meaning that underlie the facts, rules, or definitions being learned or reproduced.

Procedures without connections tasks

- Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.
- Have no connection to the concepts or meanings that underlie the procedure being used.
- Are focused on producing correct answers rather than developing mathematical understanding.
- Requires no explanations, or explanations that focus solely on describing the procedure that was used.

Higher-Level Demands

Procedures with connections tasks

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways (e.g. visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.
- Requires some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.

Doing Mathematics Tasks

- Require complex and non-algorithmic thinking (i.e. there is not a predictable, well rehearsed approach or pathway explicitly suggested by the task, tasks instructions, or a worked-out example).
- Require students to explore and understand the nature of mathematical concepts, processes or relationships.
- Demand self-monitoring or self-regulation of one's own cognitive processes.
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

APPENDIX E: COGNITIVE DEMAND FOR SELECTED CLASSROOM TASKS

Stages	Date	Classroom task	Level of Cognitive Demand of task in the Curriculum	Level of Cognitive Demand of task in the Set-up
First	3/24/06	Tile problem	*Procedures with connection* (PWC)	PWC
	3/27/06	Dice problem	PWC	*Procedures without connections* (PWOC)
	3/29/06	Crime Scene Investigation problem	PWOC	*Memorization* (M)
	3/30/06	Match/No Match problem	PWC	PWC
	4/19/06	Bike trip Day 1	PWC	PWC
Second	4/20/06	Bike trip Day 2	PWC	PWC
	4/21/06	Bike trip: Problem 2.2 Follow up	PWC	PWC
	4/24/06	Lollipop problem	PWC	PWC
Third	5/3/06	Bike trip: Temperature problem	PWC	PWC
	5/5/06	Bike trip: Day 4	*Doing mathematics* (DM)	PWC
	8/28/06	Median problem	PWC	PWC
	8/29/06	Shoe problem	PWC	PWC
	9/26/06	Patterns problem	PWC	PWC

24th March

Task: Tile problem

What’s in the bucket?
One day Ms. MacAfee brought a mysterious bucket to class. She did not show her students what was in the bucket, but she told them that it contained blue, yellow, and red blocks. She asked if the could predict, without emptying the bucket, the fraction of tiles that were blue, the fraction that were yellow, and the fraction that were red.

The class conducted an experiment to help them make their predictions. Each student randomly selected a block from the bucket, and the result was recorded on the board. After each draw, the block was returned to the bucket before the next student selected a block. In this problem, your class will conduct a similar experiment.

Your teacher has prepared a bucket identical to Ms MacAfee's. One at a time, you and each of your classmates will select a block from the bucket, record the result, and return the block to the bucket.

a. How many blocks drawn by your class were blue? How many were yellow? How many were red?
b. Which color block – blue, yellow or red- do you think there are the greatest number of in the bucket? Which color block do you think there are the least number of?
c. Based on your experimental data, predict the fraction of blocks in the bucket that are blue, that are yellow and that are red.
d. After your teacher shows you the blocks in the bucket, find the fraction of blue blocks, the fraction of yellow blocks, and the fraction of red blocks.
e. How do the fraction of the blocks that are blue, yellow and red compare to the fraction of blue, yellow and red blocks drawn during the experiment?

Level of cognitive demand of task in the curriculum
The question (c) and (e) would make this a PWC since the procedure is already stated in the problem and the students have to choose tiles and record their trials. It is not a narrow algorithm as there is still some thinking and observations to be done by the students.

Level of cognitive demand of task as set up by Linda in the class
Linda does the set up very similar to that outlined in the text. In the case of new activities like this some time is spent in the set up. Linda carried out the experiment as the students, picked out the tiles and noted the color on the board. They then answered the questions that were asked. In this process an important idea about the number of trials that should be carried out and why. Linda handled some of the questions that the students had about the draw very well. Manny guess that there were 25 of each color and Ludiz said that there were only two colors. Linda did not choose to pursue why these students mentioned these possibilities.

27th March

Task: Dice problem
In continuation to the ideas above, Linda chose the dice problem from the curriculum to reinforce some of the ideas that they were exposed to previously.
Roll a number cube 12 times. Record your results. What percent of rolls came up 6? What percent came up even? Roll the number cube 12 more times. Compute the percent of sixes and of even rolls for all 24 rolls. What percent were 6? What percent were even? Find the theoretical probability of getting an even?

Level of cognitive demand of task in the curriculum
There are again PWC since the rolling of the dice is tied to the experimental probability, the students work out the theoretical probability too. Students also work out percentages and fractions and see those relationships. However as the task is written, there is no ambiguity and the students could focus on getting the answers, but not think about the purpose of the activity. The connection between the experimental and the theoretical probabilities is left for the teacher to do in a discussion rather than there. This seems to be the draw back of some of the tasks in the SF.

Level of cognitive demand of task as set up by Linda in the class
In the setup phase for this problem, it is important to note that Linda first reviewed what they did for the tile problem and asked students questions about this. Thus Linda laid out some guidelines for them to follow. She mentions that they will be going a comparison between the experimental and theoretical probabilities. This problem spanned over two days. On the first day the students simply did the calculations that were asked in the problem. On the next day, Linda asked a group to share their work on the board and then there is a discussion about the calculations and she tried to make sure that they were all on the same page. In the discussion that followed, Linda questioned them about finding the theoretical probability and one student gave the answer. In this case the other students did not have to think through the problem and all they had to do was follow the procedures that were being talked about. So overall I would consider this a PWOC at the set up.

29th March

Task: Crime Scene Investigation (CSI) problem
What if you worked for CSI and while investigating some criminal offence you found a witness with whom we wanted to come up with some of the characteristics of the suspect based on hair color and length. Suppose there were three hair colors and three different lengths of hair, how many different possibilities of hair color and length could the suspect have?

Level of cognitive demand of task in the curriculum
The task as it was in the curriculum was a PWOC as there was little thinking required in working out the possibilities. There was a slight challenge involved in being systematic and using trees. Further, there was a focus on correct answers rather than development of mathematical understanding. The placement of the activity just after Linda illustrated making trees caused to reduce the LCD.

Level of cognitive demand of task as set up by Linda in the class
Linda reduced the thinking that would have been required in this problem by constantly referring in detail to the previous activity with a spinner that had two colors. In that

problem, the spinner was spun twice and the possible outcomes were listed in a tree. Linda was trying to make explicit the extension that the students had to do to complete the CSI problem based on the tree for the spinners. There was no ambiguity in the task and no connections to concepts. This was a *memorization* task (M).

30th March

Task: Match/No Match problem
April and Tioko decided to play the Match/No Match game on the spinner below [Spinner had three colors, blue, orange, and yellow, each occupying one-third of the spinner]. As in the original game, a turn consists of two spins. Player A scores a point if the spin match, and Player B scores a point if they do not match.

a. Use the counting tree to find all the possible outcomes for this game.
b. What is the theoretical probability of getting a match on a turn?
c. What is the theoretical probability of getting a no-match on a turn?
d. Do you think this is a fair game? If you think the game is fair, explain why. If you think it is not fair, explain how the rules could be changed to make it fair.

Level of cognitive demand of task in the curriculum
This is a PWC since there is a path suggested using the tree to get the outcomes and then find the theoretical probability. There were some deep ideas of fairness that the students had to think about in the game. There was no fixed way of achieving fairness in the game and there were multiple routes that the students could take. There was a cognitive effort on the part of the students.

Level of cognitive demand of task as set up by Linda in the class
At the set up stage, Linda recalls the outcomes that they had worked out for the dice, coin, and spinner before. She then asks them to think of the systematic way in which the data was organized using trees. Thus the students were led into part (a).However that was the only part that Linda provided for the students and they had to the other parts independently.

19th April

Task: Bike trip, Day 1
The popularity of bicycle tours gave five college students – Sidney, Celia, Liz, Malcolm, and Theo – an idea for a summer business. They would operate bicycle tours for school and family groups. They chose a route from Philadelphia, Pennsylvania, to Williamsburg, Virginia, including a long stretch along the ocean beaches of New Jersey, Delaware, and Maryland. They decided to name their business Ocean and History Bike Tours.

Day 1 of tour: Philadelphia to Atlantic City

The students began their bike tour near the Liberty Bell and Independence Hall in historic Philadelphia, Pennsylvania. Their goal for the first day was to reach Atlantic City, New Jersey. Sidney, Liz, Sarah, Celia, and Malcolm rode their bicycles. Theo and Tony followed in the van with the camping gear and repair equipment. Tony recorded the distance reading on the van's trip odometer every half hour from 8:00 am to 4:00 pm. Tony's recordings for the first day are given in the table below.

Time (Hours)	Distance (miles)
0.0	0
0.5	9
1.0	19
1.5	26
2.0	28
2.5	38
3.0	47
3.5	47
4.0	47
4.5	54
5.0	59
5.5	67
6.0	73
6.5	78
7.0	80
7.5	86
8.0	89

Write a report summarizing the data Tony collected on day 1 of the tour. Describe the distance traveled compared to the time. Look for patterns of change in the data. Be sure to consider the following questions;

- How far did the riders travel in the day? How much time did it take them?
- During which time interval(s) did the riders make the most progress? The least progress?
- Did the riders go further in the first half or the second half of the day's ride?

Level of cognitive demand of task in the curriculum
In this task the students are required to derive meaning from the information given in the table. The task involves not just reading the tale, but making meaning, and I would classify it as a PWC.

Level of cognitive demand of task as set up by Linda in the class
In the set up, Linda asks questions to make sure that they have understood the problem as she read it to them. One of the reasons for this could be because most of the students were English Language Learners. Linda also related this problem to a problem that they

had done the previous day and may have lowered the cognitive demand on the task. On the other hand it could also mean that she has pointed out connections that the students could pursue, not just blindly following a procedure. Overall I would classify this task as a PWC.

20th April

Task: Bike trip, Day 2
Atlantic City to Lewes. On the second day of their bicycle trip, the group left Atlantic City and rode five hours south to Cape May, New Jersey. This time, Sidney and Sarah rode in the van. From Cape May, they took a ferry across the Delaware Bay to Lewes, Delaware. They camped out that night in a state park along the ocean. Sarah recorded the following data about the distance traveled until they reached the ferry;

Time (Hours)	Distance (miles)
0.0	0
0.5	8
1.0	15
1.5	19
2.0	25
2.5	27
3.0	34
3.5	40
4.0	40
4.5	40
5.0	47

Problem 2.2

1. Make a coordinate graph of the (time, distance) data given in the table.
2. Sidney wants to write a report describing the day 2 of the tour. Using information from the table or the graph, what could she write about the day's travel? Be sure to consider the following questions:
    a. How far did the group travel in the day? How much time did it take them?
    b. During which time interval(s) did the riders make the most progress? The least progress?
    c. Did the riders go further in the first half or the second half of the day's ride?
3. By analyzing the table, how can you find the time intervals when the riders made the most progress? The least progress? How can you find these intervals by analyzing the graph?
4. Sidney wants to include either the table or the graph in her report. Which do you think she should include? Why?

Level of cognitive demand of task in the curriculum

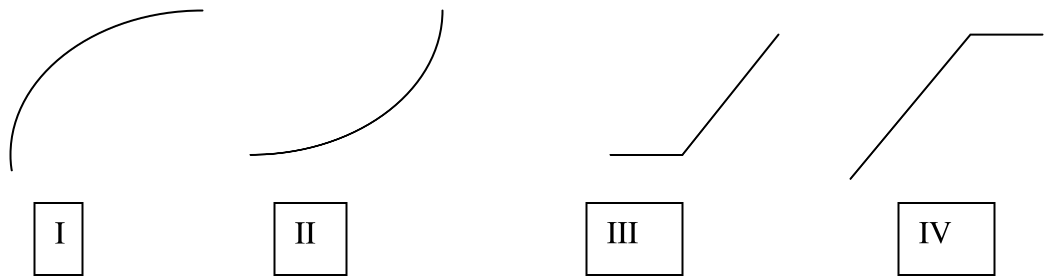
This is a PWC task. In this activity the students connect the table to the graph and are required to move between the two representations. There are no prescribed algorithms and the students follow some general procedures. There are multiple representations of graph and table involved and students need to think about including the graph or the table in the report (Q 4). This will get them to think about connecting the ideas of rates with those of representations.

Level of cognitive demand of task as set up by Linda in the class
In the setup phase, Linda tries to focus the attention of the students on key things in the problem and is eager to connect it to the activity that was done previously (Bike trip, Day 1). However on providing too much support and guidance in publicly talking about the axes, scales, intervals and doing direct comparisons to the table from Day 1, the LCD of the task is lowered. I would not categorize this as a PWOC since the task is not focused on the correct answers or does not require explanations or is algorithmic, but it would be a lower PWC task. Linda's question about the effect on the graph after a change in scales, pushed the students to think and make connections.

21st April

Task: Problem 2.2 Follow-up to Day 2 of bike trip

1. Look at the second point on your graph (that was done in Bike trip Day 2) as you count from the left. We can describe this point with the coordinate pair (0.5, 8). The first number in a coordinate pair is the value for the x-coordinate, and the second number is the value for the y-coordinate. Give the coordinate pair for the third point on your graph. What information does this point give?
2. Connecting the points on a graph sometimes helps you see a pattern more clearly. You can connect the points in situations in which it makes sense to consider what is happening in the intervals between the points. The points on the graph of the data for day 2 can be connected because the riders were moving during each half-hour interval, so the distance was changing.
    c. Connect the points on your graph with straight line segments.
    d. How would you use the line segments to help you estimate the distance traveled after ¾ of an hour (0.75 hours)?
3. The straight line segment you drew from (4.5, 40) to (5.0, 45) gives you some idea of how the ride might have gone between the points. It shows you how the ride would have progressed if the riders had traveled at a steady rate for the entire half hour. The actual pace of the group and, of the individual riders, may have varied throughout the half hour. These paths show some possible ways the ride may have progressed:
    Match each of these connecting paths with the following travel notes.



a. Celia rode slowly at first and gradually increased her speed.
b. Tony and Liz rode very quickly and reached the campsite early.
c. Malcolm had to fix a flat tire, so he started late.
d. Theo started off fast. After a while, he felt tired and slowed down.

Level of cognitive demand of task in the curriculum
This task was at the PWC level. The students had to interpret the coordinate pair in terms of the given problem, using the graph to make estimates. They would understand the way the points were connected in the graph gave rise to different interpretations for the rate of progress in the trip. Thus the students had to think about the concepts and move back and forth between the abstract and the concrete. There was no algorithmic thinking involved and no clear cut ways to think about the problem. For some of the students the elaborate explanations in English could challenge them in addition to the mathematics.

Level of cognitive demand of task as set up by Linda in the class
Linda explained the question to the students and the things that were being asked of them. Essentially her set up did not lower the LCD and it was still at the PWC level.

24th April

Task: Lollipop problem
Procedures
1. Draw the *x* and *y* axes on the graph paper. Label the *y* axis "circumference" and the *x* axis "time".
2. Get an initial measure of the circumference.
3. Suck the lollipop for 30 seconds and then get another measure. Do this for a total of 3 minutes in 30 second intervals.
4. Suck the lollipop for 1 minute and then repeat for 5 more minutes to get a total of 6 measurements.

Investigation
1. Draw a line that approximates your data as best you can. How long will it take to finish sucking the lollipop?

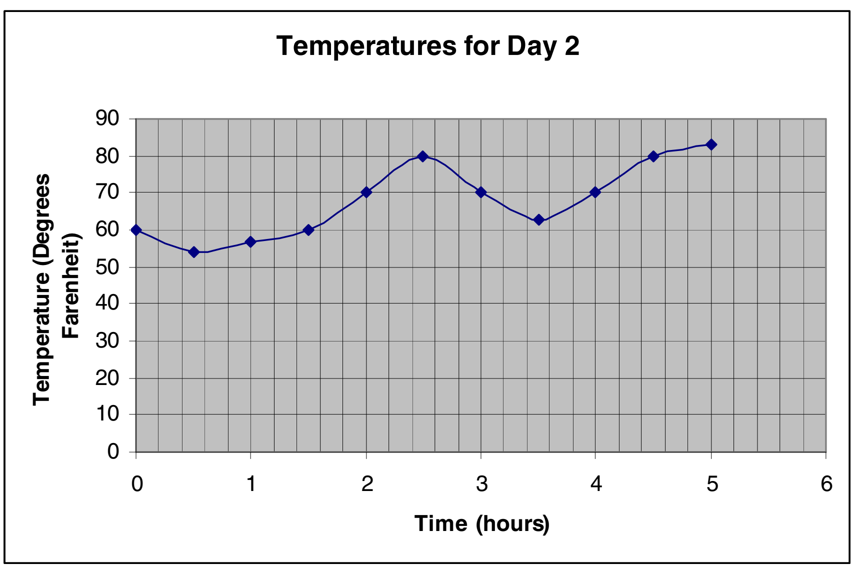
2. Find the slope of the line that you drew. What does the slope mean in this case?
3. How long will it take you to finish a lollipop with a circumference 6 times the size of your lollipop? How long would it take you to finish a lollipop with circumference 1/2 the size of your lollipop?

Level of cognitive demand of task in the curriculum
This is task is at the PWC level. The task relates the students' real life experience to the concepts of change. The students use graphical and tabular representations and grapple with the meaning of slopes and intercepts. There is no algorithm that can be applied and the students need to do some thinking about how they will approximate the regression line.

Level of cognitive demand of task as set up by Linda in the class
Linda laid out the outline that the students were supposed to follow. This lessened the cognitive demand that was required by the students, but I would still consider it at the PWC level, as there was a significant amount of thinking left for the students to do. The thinking that the students did in this problem would depend on the degree to which Linda pushed them to think about the interpretations of the slope of the line, the intercepts, and using these to draw inferences about the questions asked.

3rd May

Task: Bike trip, Temperature problem
Here is the graph of temperature data collected on the students' trip from Atlantic City to Lewes.



[Note that the points are supposed to be connected by straight lines, even though it does not look like that in the above figure]

a. This graph shows the relationship between two variables. What are they?
b. Make a table of data from this graph.
c. What is the difference between the day's lowest and highest temperature?
d. During which time interval(s) did the temperature rise the fastest? Fall the fastest?
e. Is it easier to use the table of the graph to answer part c? why?
f. Is it easier to use the table or the graph to answer part d? Why?
g. On this graph, what information is given by the lines connecting the points? Is it necessarily accurate information? Explain your reasoning.

Level of cognitive demand of task in the curriculum
This was an interesting task that Linda chose because it asked the students to form a table from the graph. This was different to the questions that the students had normally encountered which involved making a graph from a table. Linda also wanted the students to revisit the ideas of rate of change. The task made use of representations, there was a level of thinking involved by the students and procedures could not be followed blindly. This task was at the PWC level.

Level of cognitive demand of task as set up by Linda in the class
Linda reminds the students that they were not doing something new and she expected them to use their experience in the previous problem to answer this question. At the outset, Linda maintains a high expectation of thinking and this task was set up at the PWC level.

5th May

Task: Bike trip, Day 4
Chincoteague Island to Norfolk. On day 4, the group traveled from Chincoteague Island to Norfolk, Virginia. Norfolk is a major base for the United States Navy Atlantic Fleet. Malcolm and Sarah rode in the van. They forgot to record the distance traveled each half hour, but they did write some notes about the trip.

Malcolm and Sarah's Notes

- We started at 8:30 a.m. and rode into a strong wind until our midmorning break.
- About midmorning, the wind shifted to our backs.
- We stopped for lunch at a barbecue stand and rested for about an hour. By this time, we had traveled about halfway to Norfolk.
- At around 2:00 p.m., we stopped for a brief swim in the ocean.
- At around 3:30 p.m., we reached the north end of the Chesapeake Bay Bridge and Tunnel. We stopped for a few minutes to watch the ships passing by. Since bikes are prohibited on the bridge, the riders put their bikes in the van, and we drove across the bridge.

- We took seven and a half hours to complete the day's 80-mile trip.

Problem 2.4

a. Make a table of (time, distance) data that reasonably fits the information in Malcolm and Sarah's notes.
b. Sketch a coordinate graph that shows the same information.

Level of cognitive demand of task in the curriculum
This is a DM task. There is no one answer and the approaches to the solution could vary among the students. However, the things that they come up with have to match the conditions that are given in the notes. The students would have to make conjectures and test them out and repeat the process till the conditions given in the problem are satisfied. Given the students expectation that there should only be one solution to a math problem, this activity will cause some anxiety and frustration. The student will have to reflect on their real life experiences to interpret the problem like riding into a strong wind. There is a demand for the students to keep track of their thinking with the problem and make adjustments to suit the constraints that are given. There is a significant amount of cognitive effort required by the student as they make sense of their previous knowledge and apply it to the situation at hand.
Level of cognitive demand of task as set up by Linda in the class

In the set up Linda points their attention to some things that are given in the table and by doing so reduces the LCD of the task in the set up to PWC. Linda provided the students with hints in the course of her set up like starting, ending times, and the variables that were involved. But the students had to make decisions about the other things like mid morning break and represent this information in the table. There was also a debate of whether traveling in the van should be counted as part of the journey or was the table restricted to the trip made by the bikes. Broad pathways were suggested to the students and there were multiple connections between the table, graph, and the verbal descriptions.

28th August

Task: Median problem
Make a line plot or a bar graph representing 22 names which range from 10 to 21 letters and have a median of 10.

Level of cognitive demand of task in the curriculum
This is a PWC problem as the students had to piece together the information that they knew about median and line plots. There was no algorithm and some trial and error was involved as the students tried different things and monitored their own thinking. They had to examine the constraints closely to make sure that their solution worked. There is quite a bit of thinking for these students especially since the problem was counter intuitive to them. Linda mentioned that she assigned this problem to specifically get at the idea that

the median is not something that 'looks' like it is in the center, which was a misconceptions that she found the students had. Thus we see the tasks are getting tailored to the students thinking.

Level of cognitive demand of task as set up by Linda in the class
At the set up Linda gave them a few pointers by asking them the meaning of median, and range. Linda mentioned that the students were expected to discuss their work in the groups and then ask her for help. Thus there was a focus on pushing the students to think independently about the problem. Overall, there was some direction from Linda but otherwise the task demand was preserved.

29th August

Task: Shoe problem
[Name of middle school] has a new line of tennis shoes. They have partnered up with Nike and need to know what sizes to purchase and how many of each assuming that shoes come in fixed lots (later the lots were fixed at 150 or 200 shoes).

Procedures that were given to the students:
1. Collect the shoe sizes from 4 kids between the ages of 11 and 14 from home or school.
2. At each table (with 3 or 4 students), assemble your data into a frequency chart.
[In this frequency chart, Linda and I, had discussions about adding extra columns with the relative frequency, percentage of shoes of a given size, the number of shoes (given that the total order was either 150 or 200) etc.]
3. Work out the number of shoes of each size that should be purchased, based on your data.

Level of cognitive demand of task in the curriculum
This task was another example of a problem that was created for a specific purpose at that point in time. Linda had completed graphical representations and she wanted a problem that would tie in their knowledge about the different representations. This task would have been a DM task as it was set up in the beginning (without the procedures). However, once the students have a procedure o follow the task was reduced to a PWC. Students needed to make interpretations of their procedures and write a report which justified the number of shoes of each size. There was some cognitive effort required.

Level of cognitive demand of task as set up by Linda in the class
In the setup Linda told the students to organize their data into a chart with the columns for, shoe size, tallies, frequency, relative frequency and number of shoes. The students just had to fill out the chart. But they had to interpret their data and write the report and hence had to engage in the conceptual ideas of this problem. There was some level of thinking that they had to do for this problem after filling out the table. The students would also see that there were different solutions at the various groups and realize that

there was no unique solution to the problem and a degree of frustration was involved. I would consider this a PWC task at the set up.

26th September

Task: Patterns problem
Fill in three patterns into a 100's chart and determine if 44 is in the pattern?

Level of cognitive demand of task in the curriculum
The last part involved the students coming up with their own activity. In this they had to write an explanation for what their pattern was and this served as an introduction to algebraic thinking and symbols. There is thinking involved especially when the students need to create their pattern and exchange it with a partner. The 100's chart is made use of in an effective way to allow the students to see the patterns. There is a general method that the students could follow but there are no fixed algorithms, atleast as emphasized in the problem. This would be a PWC task.

Level of cognitive demand of task as set up by Linda in the class
The problem was set up as it was in the curriculum, but Linda worked out a similar example to ensure that the students understood the question. She insisted that the students justify their work and pushed them to come up with a pattern that would challenge their partner. Thus the elements of thinking were preserved for this task and it would be at the PWC level.

APPENDIX F: SCHEDULE OF TASKS

Date	**Classroom**	**After Class discussion**
3/13/06	Surface area and volume problems	Mathematics Task Framework: Task sort Tile problem Siblings Problem 1 Siblings Problem 2
3/14/06	Surface area and volume	Maze problem Gumball problem
3/16/06	Surface area and volume	Lunch and informal discussion.
3/20/06	Surface Area and volume	Definitions and ideas in probability Quiz problem Siblings Problem 4
3/21/06	Surface area and volume	Basketball problems 1 & 2
3/22/06	Quiz	Basketball problems 3 & 4 Carnival problem
3/23/06	AIMS Review	Plan for probability lesson
3/24/06	Tile problem	Short debrief
3/27/06	Dice problem	Mathematics Task Framework: Chapter 1 Debrief Dice problem Permutations and Combinations
3/28/06	Dice problem continued	outcome of spinners, tree diagrams
3/29/06	Spinner and Crime Scene Investigation problems	Handshake problem Staircase problem
3/30/06	Match/No Match problem	Other things
4/03/06	Dart problem	Mathematics Task Framework: Chapter 2 Focused on the Cognitive Demand as we discussed the student work in the problem just before.
4/04/06	Test	Lollipop problem
4/18/06	Jumping Jack problem continued	Evaluating games for a carnival
4/19/06	Bike trip: Day 1	Evaluating games for the carnival (correcting my mistake) Functions: Domain, range, *x* and *y* intercepts, examples of functions
4/20/06	Bike trip: Day 2	Functions: Finding zeros, factorization Plan Lollipop problem
4/21/06	Bike trip: Problem 2.2 Follow up	Algebra: Word problems Function: zeros of functions Functions: Expressing functions symbolically

4/24/06	Bike trip questions that the kids did before. Lollipop activity	Short talk and Graph and the need to do an approximately.
4/25/06	Lollipop problem	Debrief Lollipop problem
5/1/06	Worksheets	NCTM conference Plan Lollipop problem extensions Plan Bike trip: Temperature problem
5/2/06	Lollipop problem: Slope of line	Algebra: Word problems
5/3/06	Bike trip: Temperature problem	Algebra: Word problems
5/5/06	Bike trip: Day 4	Linear regression using TI73
5/9/06	Bike trip: Day 4 continued	Debrief Bike trip: Day 4 Further planning for Bike trip Calculator: Regression line connected with Lollipop problem
5/10/06	Bike problem: Day 4 continued (Students present their work)	Background about Linda
5/12/06	Calculator	No meeting
8/24/06	Stem and leaf plots	Create Shoe problem
8/28/06	Median problem	Debrief Median problem Further planning for Shoe problem
8/29/06	Shoe problem	Debrief Shoe problem
8/30/06	Shoe problem continued	Further planning and debriefing Shoe problem Excel spreadsheet on Shoe problem data
9/1/06	Quiz	Mean Annenberg video of the 6th grade teacher teaching Statistics lesson
9/15/06	Order of operations	Algebra: Variable and polynomials
9/26/06	Patterns problem	Algebra: Operations with polynomials
9/27/06	Triangular and Square numbers problem	Debrief Triangular and Square numbers problem

REFERENCES

Abedi, J. (2002). Standardized achievement tests and english language learners: Psychometric issues. *Educational Assessment, 8*(3), 231-257.

Annenberg Media. (2001). *Learning math: Data analysis, statistics, and probability.* Retrieved 09/01, 2006, from http://www.learner.org/resources/series158.html

Arbaugh, F. (2003). Study groups as a form of professional development for secondary mathematics teachers. *Journal of Mathematics Teacher Education, 6*, 139-163.

Arbaugh, F. (2005). Analyzing mathematical tasks: A catalyst for change? *Journal of Teacher Education, 8*, 499-536.

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal, 90*(4), 449-466.

Ball, D. L. (2000a). Bridging practices: Intertwining content and pedagogy in teaching and learning to teach. *Journal of Teacher Education, 51*(3), 241-247.

Ball, D. L. (2000b). Working on the inside: Using one's own practice as a site for studying teaching and learning. In A. E. Kelly, & R. A. Lesh (Eds.), *Research design in mathematics and science education* (pp. 365-402). Mahwah, NJ: Lawrence Erlbaum Associates.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83-104). Westport, CT: Ablex.

Bassarear, T. (2005). *Mathematics for elementary school teachers* (3rd ed.). Boston, NY: Houghton Mifflin.

Bauersfeld, H. (1995). Development and function of mathematizing as a social practice. In L. Steffe, & J. Gale (Eds.), *Constructivism in education* (pp. 137-158). Hillsdale, NJ: Lawrence Erlbaum.

Bickel, W. E., & Hattrup, R. A. (1995). Teachers and researchers in collaboration: Reflections on the process. *American Educational Research Journal, 32*(1), 35-62.

Birchak, B., Connor, C., Crawford, K. M., Kahn, L. H., Kaser, S., Turner, S., et al. (1998). *Teacher study groups: Building community through dialogue and reflection*. Urbana, IL: National Council of Teachers of English.

Blond, J., & Webb, K. (1997). Practitioner and researcher perspectives in teacher research and the construction of knowledge. In H. Christiansen, L. Goulet, C. Krentz & M. Maeers (Eds.), *Recreating relationships: Collaboration and educational reform* (pp. 83-103). Albany, NY: State Univesity of New York Press.

Blumer, H. (1969). *Symbolic interactionism: Perspective and method*. Englewood Cliffs, NJ: Prentice-Hall.

Boggs, H. (1996). Launching school change through teacher study groups: An action research project. *Paper Presented at the Annual Conference of the Mid-Western Educational Research Association,* Chicago, IL.

Cardelle-Elawar, M. (1993). The teacher as research in the classroom. *Action in Teacher Education, 15*(1), 49-57.

Carpenter, T. P., & Fennema, E. (1992). Cognitively guided instruction: Building on the knowledge of students and teachers. *International Journal of Educational Research, 17*, 457-470.

Charles, R. I., Dossey, J. A., Leinwand, S. J., Seeley, C. L., Vonder Embse, C. B., & Bolster, L. C. et al. (1999). *Middle school math*. Menlo Park, CA: Scott Forseman - Addison Wesley.

Clair, N. (1998). Teacher study groups: Persistent questions in a promising approach. *Teachers of English to Speakers of Other Languages Quarterly, 32*(3), 465-495.

Clark, C., Moss, P. A., Goering, S., Herter, R. J., Lamar, B., Leonard, D., et al. (1996). Collaboration as dialogue: Teachers and researchers engaged in conversation and professional development. *American Educational Research Journal, 33*(1), 193-231.

Clift, R., Veal, M. L., Johnson, M., & Holland, P. (1990). Restructuring teacher education through collaborative action research. *Journal of Teacher Education, 41*(2), 52-62.

Cobb, P. (1994). Where is the mind? constructivist and sociocultural perspectives on mathematical development. *Educational Researcher, 23*(7), 13-20.

Cobb, P., & Bauersfeld, H. (1995). Introduction: The coordination of the psychological and sociological perspectives in mathematics education. In P. Cobb, & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interactions in classroom cultures* (pp. 1-16). Hillsdale, NJ: Erlbaum.

Cobb, P., Jaworski, B., & Presmeg, N. (1996). Emergent and sociocultural views on mathematical activity. In P. Nesher, L. P. Steffe, P. Cobb, G. Goldin & B. Greer

(Eds.), *Theories of mathematical learning* (pp. 3-19). Mahwah, NJ: Lawrence Erlbaum Associates.

Cobb, P., Wood, T., & Yackel, E. (1990). Classrooms as learning environments for teachers and researchers. In R. Davis, C. Maher & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 125-146). Reston, VA: National Council of Teachers of Mathematics.

Cobb, P., Yackel, E., & Wood, T. (1992). Interaction and learning in mathematics classroom situations. *Educational Studies in Mathematics, 23*(1), 99-122.

Cohen, D. K., & Ball, D. L. (1990). Policy and practice: An overview. *Educational Evaluation and Policy Analysis, 12*(3), 347-353.

Cohen, S. (2004). *Teachers' professional development and the elementary mathematics classroom: Bringing understandings to light*. Mahwah, NJ: Lawrence Erlbaum Associates.

Cole, A. L., & Knowles, J. G. (1993). Teacher development partnership research: A focus on methods and issues. *American Educational Research Journal, 30*(3), 473-495.

Connelly, F. M., & Clandinin, D. J. (1990). Stories of experience and narrative inquiry. *Educational Researcher, 19*(5), 2-14.

Corno, L. (1988). The study of teaching for mathematics learning: Views through two lenses. *Educational Psychologist, 23*(2), 181-202.

Crespo, S. (2002, Oct). Teacher learning in mathematics teacher study groups. *Proceedings of the 24th PME-NA Annual Conference,* Columbus, OH. *, 3* 1439-1450.

Crespo, S. (2006). Elementary teacher talk in mathematics study groups. *Educational Studies in Mathematics, 63*, 29-56.

Creswell, J. W. (1998). *Qualitative inquiry and research design: Choosing among five traditions*. Thousand Oaks, CA: SAGE Publications.

Crockett, M. D. (2002). Inquiry as professional development: Creating dilemmas through teachers' work. *Teaching and Teacher Education, 18*, 609-624.

Cryns, T., & Johnston, M. (1993). A collaborative case study of teacher change: From a personal to a professional perspective. *Teaching and Teacher Education, 9*(2), 147-158.

Day, C. (1991). Roles and relationships in qualitative research on teachers' thinking: A reconsideration
. *Teaching and Teacher Education, 7*(5/6), 537-547.

Doyle, W. (1983). Academic work. *Review of Educational Research, 53*(2), 159-199.

Doyle, W. (1988). Work in the mathematics classes: The context of students' thinking during instruction. *Educational Psychologist, 23*(2), 167-180.

Doyle, W., & Ponder, G. (1977). The practicality ethic in teacher decision making. *Interchange, 8*(3), 1-12.

Edwards, T. G., & Hensien, S. M. (1999). Changing instructional practice through action research. *Journal of Mathematics Teacher Education, 2*, 187-206.

Elbaz, F. (1983). *Teacher thinking: A study of practical knowledge*. London: Croom Helm.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thiking in mathematics instruction. *Journal for Research in Mathematics Education, 27*(4), 403-434.

Fernandez, C. (2002). Learning from japanese approaches to professional development: The case of lesson study. *Journal of Teacher Education, 53*(5), 393-405.

Freedman, R., & Salmon, D. (2001). The dialectic nature of research collaborations: The relational literacy curriculum. In R. Ravid, & M. G. Handler (Eds.), *The many faces of school-university collaboration* (pp. 179-192). Englewood, CO: Teachers Ideas Press.

Freire, P. (1970). *Pedagogy of the opressed*. New York: The Seabury Press.

Garrido, E., Pimenta, S. G., Moura, M. O., & Fusari, M. F. R. (1999). Collaboratibe research as an approach to foster teacher development, teachers' production of knowledge and changes in school practices. *Educational Action Research, 7*(3), 385-397.

Goldsmith, L. T., & Schifter, D. (1997). Understanding teachers in transition: Characteristics of a model for the development of mathematics teachers. In E. Fennema, & B. S. Nelson (Eds.), *Mathematics teachers in transition* (pp. 19-54). Mahwah, NJ: Lawrence Erlbaum Associates.

Goodlad, J. I. (1988). School-university partnerships for educational renewal: Rational and concepts. In K. A. Sirotnik, & J. I. Goodlad (Eds.), *School university* *partnerships in action: Concepts, cases, and concerns* (pp. 3-31). New York, NY: Teacher College Press.

Greeno, J. G. (1983, April). Skills for representing problems. *Paper Presented at the Annual Meeting of the American Educational Research Association,* Montreal.

Handler, M. G., & Ravid, R. (2001). Models of school-university collaboration. In R. Ravid, & M. G. Handler (Eds.), *The many faces of school-university collaboration* (pp. 3-10). Englewood, CO: Teacher Ideas Press.

Herbst, P. G. (2003). Using novel tasks in teaching mathematics: Three tensions affecting the work of the teacher. *American Educational Research Journal, 40*(1), 197-238.

Hiebert, J. (1986). In Hiebert J. (Ed.), *Conceptual and procedural knowledge: The case of mathematics*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 65-97). New York: MacMillan.

Hord, S. M. (1981). *Working together: Cooperation or collaboration*. Austin, Texas: Research and Development Center for Teacher Education.

Hsiung, T. H., Chen, S. L., & Wang, Z. S. (2000, April). Utilizing the science as core course of an integrated curriculum development: A collaborative action research. *Paper Presented at the Annual Meeting of the National Association for Research in Science Teaching,* New Orleans, LA.

Hunsaker, L., & Johnston, M. (1992). Teacher under construction: A collaborative case study of teacher change. *American Educational Research Journal, 29*(2), 350-372.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica, 47*(2), 263-292.

Kazemi, E., & Franke, M. L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of Mathematics Teacher Education, 7*, 203-235.

Kreisberg, S. (1992). *Transforming power: Domination, empowerment, and education*. Albany, NY: SUNY Press.

Lappan, G., Fritzgerald, W., Friel, S., Fey, J., & Phillips, E. (1998). *Connected mathematics*. White Plains, NY: Cuisenaire Dale Seymour Publications.

Lieberman, A. (1986). Collaborative work. *Educational Leadership, 43*(5), 4-8.

Little, J. W. (1981). School success and staff development in urban desegregated schools: A summary of recently completed research. *American Educational Research Association,*

Little, J. W. (1990). The persistence of privacy: Autonomy and initiative in teachers' professional relations. *Teachers College Record, 91*(4), 509-536.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in china and the united states*. Mahwah, NJ: Lawrence Erlbaum.

Makibbin, S. S., & Sprague, M. M. (1991, December). Study groups: Conduit for reform. *Paper Presented at the Meeting of the National Staff Development Council,* St. Louis, MO.

McGraw, R., Lynch, R. K., Koc, Y., Kapusuz, A., & Brown, C. A. (2007). The multimedia case as a tool for professional development: An analysis of online and face-to-face interactions among mathematics pre-service teachers, in-service teachers, mathematicians, and mathematics teacher educators. *Journal of Mathematics Teacher Education, 10*(2), XXX-XXX.

Merriam, S. (1988). *Case study research in education: A qualitative approach*. San Francisco, CA: Jossey-Bass.

Miller, D. M., & Pine, G. J. (1990). Advancing professional inquiry for educational improvement through action research. *Journal for Staff Development, 11*(3), 56-61.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2006). *Curriculum focal points for pre-kindergarten through grade 8 mathematics: A quest for coherence*. Reston, VA: NCTM.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Noffke, S. (1994). Action research: Towards the next generation. *Educational Action Research, 2*(1), 9-21.

Oja, S. N., & Pine, G. J. (1987). Collaborative action research: Teachers' stages of development and school contexts. *Peabody Journal of Education, 64*(2), 96-115.

Olson, M. R. (1997). Collaboration: An epistemological shift. In H. Christiansen, L. Goulet, C. Krentz & M. Maeers (Eds.), *Recreating relationships: Collaboration and educational reform* (pp. 13-25). Albany, NY: State University of New York Press.

Piaget, J. (1970). *Science of education and psychology of the child*. London, UK: Longman.

Puchner, L. D., & Taylor, A. R. (2006). Lesson study, collaboration and teacher efficacy: Stories from two school-based math lesson study groups. *Teaching and Teacher Education, 22*, 922-934.

Putnam, R. T., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher, 29*(1), 4-15.

QSR International. (2006). *NVivo 7*. Doncaster, Australia:

Rawlins, W. K. (1992). *Friendship matters: Communication, dialectics and the life course*. New York, NY: Walter de Gruyter, Inc.

Raymond, A. M. (1995, October). Two collaborative action research partnerships. *Paper Presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education,* Columbus, OH.

Raymond, A. M., & Leinenbach, M. (2000). Collaborative action research on the learning and teaching of algebra: A story of one mathematics teacher's development. *Educational Studies in Mathematics, 41*, 283-307.

Schifter, D. (1998). Learning mathematics for teaching: From a teachers' seminar to the classroom. *Journal of Mathematics Teacher Education, 1*, 55-87.

Schoenfeld, A. (1988). When good teaching leads to bad results: The disasters of "well-taught" mathematics classes. *Educational Psychologist, 23*(2), 145-166.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*, 114-145.

Simon, M. (1997). Developing new models of mathematics teaching: An imperative for research on mathematics teacher development. In E. Fennema, & B. S. Nelson (Eds.), *Mathematics teachers in transition* (pp. 55-86). Mahwah, NJ: Erlbaum.

Steffe, L., & Gale, J. (Eds.). (1995). *Constructivism in education*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455-488.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. New York: Teachers College Press.

Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science, 12*, 257-285.

TERC. (1995). *Investigations*. Palo Alto, CA: Dale Seymour Publications.

von Glasersfeld, E. (1987). Learning as a constructive activity. In C. Janvier (Ed.), *Problems of representation in the learning of mathematics* (pp. 3-17). Hllsdale, NJ: Lawrence Erlbaum Associates.

von Glasersfeld, E. (1989). Constructivism in education. In T. Husen, & N. Postlethwaite (Eds.), *International encyclopedia of education* (pp. 162-163). Oxford, England: Pergamon Press.

von Glasersfeld, E. (1991). *Radical constructivism in mathematics education*. Dordrecht, The Netherlands: Kluwer.

von Glasersfeld, E. (1995). A constructivist approach to teaching. In L. Steffe, & J. Gale (Eds.), *Constructivism in education* (pp. 3-15). Hillsdale, NJ: Lawrence Erlbaum Associates.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Whitford, B. L., Schlechty, P. C., & Shelor, L. G. (1987). Sustaining action research through collaboration: Inquiries for invention. *Peabody Journal of Education, 64*(3), 151-169.

Wilson, S. M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: An examination of research on contemporary professional development. *Review of Research in Education, 24*, 173-209.

Zaslavsky, O. (2005). Seizing the opportunity to create uncertainty in learning mathematics. *Educational Studies in Mathematics, 60*, 297-321.