TIME REVERSAL VIOLATION IN NUCLEAR EFFECTIVE FIELD THEORY

by

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A Dissertation Submitted to the Faculty of the

DEPARTMENT OF PHYSICS

In Partial Fulfillment of the Requirements
For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

THE UNIVERSITY OF ARIZONA

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THE UNIVERSITY OF ARIZONA
GRADUATE COLLEGE

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entitled Time Reversal Violation in Nuclear Effective Field Theory
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William Hill Hockings

ACKNOWLEDGEMENTS

I would like to sincerely thank my advisor Bira van Kolck, who provided me with both an education in nuclear effective field theory and the opportunity to be a part of the nuclear physics community through conferences, summer schools and programs. Thanks also for a graduate research associateship.

I would like to thank the members of my committee for being willing to take time out of their summers to read my thrilling and provocative work.

Collaborations with Prof. Claudio Maekawa resulted in much of the content of Chapter 3 in this thesis. Discussions with Profs. Judith McGovern, Dick Furnstahl, Rob Timmermans, Dan Phillips, Ed Stephenson and Susan Gardner, as well as Steve Puglia, were helpful in one way or another to my understanding of the physics involved with this thesis.

Thank you to Profs. Jan Rafelski and Mike Scadron for helping me along in my professional development with their encouragement, unique insights and viewpoints.

My mother, Susan Rimbey, deserves acknowledgement for everything she has done for me, not the least of which is teaching me how to be a good person.

Thank you to Jane and Mandy Wang, whose mere presence kept me in a pleasant mood during the last two years of my graduate study. May you both achieve all of your dreams.

Finally, I thank my wife Jennifer for being with me through the beauty and the sadness and everything in between. She helped me during my graduate studies in too many ways to count, and her love will carry me through the rest of my days.

DEDICATION

To Jennifer, the love of my life

and to that little orange fuzzball who alternately elated and exasperated

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ABSTRACT

The lack of invariance with respect to time reversal (T) of the weak interactions has long been known. However, T violation has yet to be observed from flavor-diagonal sources, where the primary quantities of interest are electric dipole moments (EDMs). Weak T violation gives EDMs that are far too small, but strong T violation via flavor-diagonal sources could give EDMs strong enough to be detected in the near future. It is thus important to understand precisely how various quark-level sources of T violation manifest themselves in hadronic physics.

A useful technique for dealing with low-energy phenomena involving nucleons, nuclei, and various mesons, is effective field theory (EFT). The formalism and methodology of EFT are presented, followed by an introduction to the construction of chiral Lagrangians. A motivation for the study of T violation beyond the weak interactions is then given, with brief introductions to the most important sources of T violation.

The QCD $\bar{\theta}$ term is looked at using two different approaches. First, enforcing vacuum stability at quark level, a series of T-violating interactions ensue. Second, enforcing vacuum stability at hadronic level via field redefinitions, spurious interactions are demonstrated to be avoidable. Both approaches involve a constraining relationship between $\bar{\theta}$-term T violation and up-down quark-mass-difference isospin violation. The quark chromo-EDMs are shown to be identical to the $\bar{\theta}$ term in their chiral symmetry properties. The quark EDMs and Weinberg operator, conversely, are shown to generate new interactions in addition to those generated by the $\bar{\theta}$ term, differing nucleon EDM contributions in particular.

The electric dipole form factor (EDFF) of the nucleon, with a $\bar{\theta}$ term source, is calculated in both leading and subleading orders in chiral perturbation theory, with the momentum dependence at both orders given entirely by contributions from the

pion cloud. The leading result is purely isovector, while an isoscalar result appears in subleading order. The isoscalar EDM is used as a lower-bound estimate of the deuteron EDM. The momentum dependence of the EDFF for small momentum transfer is related to the electromagnetic nucleon Schiff moment, which is computed to subleading order.

CHAPTER 1

INTRODUCTION

1.1 What is effective field theory?

Contrary to what one might expect, effective field theory (EFT) has a rather long history. The first example of an EFT is probably due to Euler and Heisenberg in the context of QED (Heisenber and Euler, 1936). The modern notion of EFT originated in the late 1960's when Weinberg found that a chiral Lagrangian used at tree level was a field theoretical realization of current algebra (Weinberg, 1967). The Standard Model itself, which was developed around this same time, can be viewed as a low-energy effective field theoretic approximation of a more fundamental theory (Georgi and Glashow, 1974; Pati and Salam, 1974). EFT was put on a firm theoretical foundation in the 1970's with the aid of Wilson's work on the renormalization group (Wilson and Kogut, 1974), and the concept of effective Lagrangians soon became the accepted paradigm of particle physics (Weinberg, 1979, 1980).

EFT[1] is, first and foremost, a technique for dealing with problems that involve multiple energy scales. It can be applied to situations in which we seek to understand the physics at some low-energy scale as the limiting case of a more general problem whose full features manifest themselves only at some higher energy. It is often the case that the physics of a system appears radically different at different energy scales, whether due to symmetries or to restrictions on available degrees of freedom. EFT takes this apparent complication, a separation of energy regimes, and turns it

[1]There are a number of fine introductions and reviews of EFT, each with a unique perspective and aim, see for example (van Kolck, 1999; Polchinski, 1992; Kaplan, 1995; Phillips, 2002; Georgi, 1994)

into an advantage.

In order to understand what an effective field theory is, consider a field theory given by some Lagrangian $\mathcal{L}(\Psi)$ written in terms of some ("elementary") fields Ψ. We will call this theory the "underlying" theory and assume that it accurately describes the physics over a certain range of energy. In other words, the S-matrix elements can be calculated from the path integral

$$Z = \int \mathcal{D}\Psi e^{i \int \mathcal{L}(\Psi)}. \tag{1.1}$$

The Lagrangian $\mathcal{L}$ may or may not be known. Even if $\mathcal{L}$ is known, it may not be possible to solve for the dynamics of the underlying theory. Therefore, it is often necessary to set an energy cutoff Λ within the range of validity of the theory, where Λ provides a division between energies which require a full understanding of $\mathcal{L}$ and energies at which some of the degrees of freedom of $\mathcal{L}$ can be neglected. By splitting the fields Ψ into a "fast" component Ψ_h and a "slow" component Ψ_l according to whether their momenta are greater or less than Λ, respectively, the advantages of EFT become manifest.

Integrating over the degrees of freedom in Ψ_h,

$$Z = \int \mathcal{D}\Psi_l e^{i \int \mathcal{L}_{eff}(\Psi_l)}, \tag{1.2}$$

where the effective Lagrangian $\mathcal{L}_{eff}$ is given, in D spacetime dimensions, by

$$\int d^D x \mathcal{L}_{eff}(\Psi_l) = -i \ln \int \mathcal{D}\Psi_h e^{i \int \mathcal{L}(\Psi_h, \Psi_l)}. \tag{1.3}$$

Since $\mathcal{L}_{eff}$ is a function only of the slow fields Ψ_l, it can be given the series representation

$$\int d^D x \mathcal{L}_{eff}(\Psi_l) = \int d^D x \sum_i g_i(\Lambda) \mathcal{O}_i(\Psi_l), \tag{1.4}$$

The operators $\mathcal{O}_i$ can in general be quite complicated, but there are two important properties that they must possess. First, they are local operators in the sense that they involve only fields at the same spacetime point. This necessarily entails an arbitrary number of derivatives, due to the uncertainty principle: particles with momentum $\lesssim \Lambda$ can only probe length scales $\gtrsim 1/\Lambda$, so that length scales smaller than $1/\Lambda$ can only be sensed in an average sense, and this averaging requires high-order derivatives. Second, the operators $\mathcal{O}_i$ must possess the same symmetries as the underlying theory in the following sense: If the underlying Lagrangian is symmetric under a (nonanomalous) transformation, then the effective Lagrangian is as well. If a particular symmetry is broken in the underlying theory, it will also be broken in the effective theory via operators that incorporate the breaking.

The coefficients g_i are independent of the soft momentum carried by the fields of the effective theory, but depend explicitly on Λ. Hence the g_i are "running coupling constants." They are also functions of the parameters of the underlying theory, and consequently they retain the effects of the high-momentum fields that have been integrated out. The g_i are usually called low-energy constants, since they contain information from the underlying theory necessary to compute low-energy observables. The dependence of the g_i's on Λ is dictated by the principle of renormalization group invariance, namely that physical observables must be insensitive to changes in Λ.

If the underlying theory contains a characteristic mass scale M, then the effective Lagrangian for $\Lambda < M$ is generally written in terms of a different set of fields rather than the elementary fields Ψ. These "effective" fields are some combinations of the Ψ_l. When the degrees of freedom that are inaccessible at lower energies are integrated out, leaving the degrees of freedom corresponding to the Ψ_l fields, one is then selecting only those aspects of the underlying theory that are important at low energies.

What the mass scale M is depends on the underlying theory. Often it is the mass of a physical particle. In this case, whether or not the particle is stable determines its role in the effective theory. If the particle is not effectively stable in the underlying theory, then it does not appear at low energies and no field Ψ_l need be associated with it. If the particle is stable for timescales relevant to the effective theory, then it can be treated nonrelativistically. Production of such a particle involves large momenta that are beyond the range of validity of the EFT, and effects of virtual exchange are of short range and are thus incorporated into the g_i coefficients (Appelquist and Carazzone, 1975).

If, on the other hand, M is the scale associated with some scalar field which spontaneously breaks a continuous internal symmetry group G down to a subgroup H and has a non-vanishing vacuum expectation value, then a massless spin-zero particle–a Goldstone boson–will appear in the effective theory (Goldstone et al., 1962). If the symmetry is approximate, then there will be small symmetry-breaking terms in the theory that lead to low-mass pseudo-Goldstone bosons (Weinberg, 1972). This new particle will generally have a field associated with it in the effective theory, with the field being a parameterization of the coset space G/H at each spacetime point. The parameterization used is not unique, and one can always be found in which all interactions of the Goldstone boson are derivative (Coleman et al., 1969; Callan et al., 1969). This assures that the particle's interactions are weak at low energy. It is important to note that this symmetry breaking can only be treated effectively if the dimensionful parameters associated with the breaking are small compared to M.

If the underlying theory is known, then the g_i's can be calculated and $\mathcal{L}_{eff}$ can be determined (to the accuracy of the approximation involved). However, when the underlying theory is unknown (as in electroweak theory) or difficult to solve (as in nuclear physics), there is a justification for using an effective theory given by the following realization (not yet formally embodied in a theorem) due to Weinberg (Weinberg, 1979): "if one writes down the most general possible Lagrangian, including all terms consistent with assumed symmetry principles, and then calculates S-matrix elements with this Lagrangian to any order in perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles." Once the most general Lagrangian is constructed, one computes the contributions from all diagrams with momenta $Q < \Lambda$ and relates the coefficients g_i to physical observables via renormalization. The Lagrangian will contain an infinite number of terms. As terms with more and more derivatives are considered, coupling constants of higher and higher inverse mass dimension will be required. The coupling constants are expected to be of $\mathcal{O}(1)$ (expressed in units of M) unless there is a symmetry or dynamical mechanism that suppresses or enhances them. This is called the "naturalness" assumption.

The effective Lagrangian is only useful if the infinite number of contributions to any observable can be ordered according to their expected sizes. This ordering is known as "power counting." It was first formalized in the context of nuclear physics by Weinberg (Weinberg, 1990, 1991, 1992). The effective Lagrangian will generate diagrams in which the particles involved all have three-momenta of order $Q \ll M$, so Q/M serves as a natural expansion parameter. Since physical observables are cutoff independent, the expansion is valid for any cutoff. To any given order in Q/M, only a finite number of g_i's need to be considered. This is because varying the cutoff simply shifts around contributions to observables at each order in Q/M. The g_i's can be determined via experimental data, with an error given by the estimated size of higher-order terms. Thus EFT provides a controlled expansion of the most general dynamics, giving a model-independent method for developing theoretical predictions that can be compared (in most cases) to experiment.

The EFT most appropriate for the problems considered in this thesis is chiral perturbation theory (χPT). This is an EFT which straightforwardly allows one to compute amplitudes for processes involving at most one heavy particle. The next section will provide an introduction to χPT and how to construct chiral Lagrangians by first looking at the properties of the particles which are the building blocks of the theory, then using symmetry considerations to construct the terms that make up the effective theory Lagrangian, and finally ordering their interactions using Weinberg's power counting.

1.2 Construction of chiral Lagrangians

The Standard Model (SM) of particle physics is perhaps the most successful example of an EFT. There is little doubt that the SM is an effective theory of some underlying "theory of everything." In the regime of energies that have been probed experimentally, the SM gives an apparently complete description of elementary particle interactions. This thesis will be concerned primarily with the area of the SM dealing with strong interactions, namely quantum chromodynamics (QCD). At an energy scale of a few GeV, the degrees of freedom are the lightest quarks, gluons, leptons and the photon. The leptons can be neglected if we are only concerned with strong interactions (along with electromagnetic interactions between quarks and photons). Heavy quarks can be integrated out. For simplicity, consider only interactions involving the two lightest quarks, up (u) and down (d), which can be arranged in a flavor doublet $q = (u \quad d)^T$. Most of what follows can be straightforwardly extended to include the strange quark. However, since the strange quark mass is about one to two orders of magnitude heavier than the up/down quark mass ($m_{u,d} = \mathcal{O}(1 - 10 \text{ MeV})$, $m_s = \mathcal{O}(100 \text{ MeV})$), its contributions in the effective theory will generally be less important than the contributions from the up and down quarks. The relevant terms in the effective Lagrangian at this scale are

$$\begin{aligned}\mathcal{L} = &- \bar{q}(\partial\!\!\!/ - ig_s G\!\!\!/ - ieQ A\!\!\!/) - \frac{1}{2}(m_u + m_d)\bar{q}q + \frac{1}{2}(m_d - m_u)\bar{q}\tau_3 q \\ &- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}Tr[G_{\mu\nu}G^{\mu\nu}] + \frac{\bar{\theta}g_s^2}{32\pi^2}\varepsilon_{\mu\nu\alpha\beta}Tr[G^{\mu\nu}G^{\alpha\beta}] + \ldots,\end{aligned} \tag{1.5}$$

where g_s is the strong gauge coupling, e is the negative of the electron charge, G_μ (A_μ) is the gluon (photon) field of strength $G_{\mu\nu}$ ($F_{\mu\nu}$), m_u (m_d) is the up (down) quark mass, τ_3 is the usual Pauli matrix, $Q = 1/6 + \tau_3/2$ is the quark charge matrix, $\bar{\theta}$ is the QCD theta angle, and $''\ldots''$ denotes terms that are of higher dimension. The traces in Eq. (1.5) are over the color indices of the gluons, i.e. $Tr[G_{\mu\nu}G^{\mu\nu}] = G^a_{\mu\nu}G^{\mu\nu,a}$, where $G^a_{\mu\nu} = G^{\mu\nu}t^a$, with t^a the Gell-Mann color $SU(3)$ matrices. The $\bar{\theta}$-term in Eq. (1.5) is a consequence of the vacuum structure of QCD and violates both time-reversal (T) and parity (P) invariance. This term along with higher dimension P- and T-violating terms omitted from Eq. (1.5) will be studied in detail in the following chapter.

In the limit where the up and down quark masses as well as the QCD theta angle are neglected, the Lagrangian (1.5) has a global $SU(2)_L \times SU(2)_R \sim SO(4)$ symmetry, called chiral symmetry. The absence of degenerate parity doublets but presence of (approximate) isospin multiplets in the hadron spectrum makes it a reasonable assumption that chiral symmetry is spontaneously broken down to its diagonal subgroup, $SU(2)_{L+R} \sim SO(3)$ of isospin. As a result, Goldstone's theorem (Goldstone et al., 1962) states that there exist massless Goldstone bosons associated with the three broken generators, whose fields live in the three-sphere $S^3 \sim SO(4)/SO(3)$. The three-sphere is formed from the degenerate minima of the effective potential of QCD, and is often called the "chiral circle." Although this potential cannot presently be calculated, it must have a "Mexican hat" shape when plotted as a function of the quark bilinears $(\bar{q}i\boldsymbol{\tau}\gamma_5 q, \bar{q}q)$. The chiral circle has a radius which turns out to be equal to the pion decay constant $f_\pi \approx 93$ MeV. The pions represent the excitations along the circle, and their interactions in the EFT must reproduce the symmetry properties inherent in Eq. (1.5).

There are several different ways to represent the pions in the EFT, corresponding to different parameterizations of the chiral circle on which the pions live. In this thesis, the pions will be viewed in the stereographic projection (see Appendix A for details). Whatever parameterization is employed, the procedure for constructing the effective Lagrangian is well-established (Coleman et al., 1969; Callan et al., 1969). First of all, the degrees of freedom in the theory must be given. The degrees of freedom in the low-energy EFT are hadrons, since they represent the only asymptotic states (a consequence of confinement). Since most hadron masses are of order 1 GeV or higher, the underlying scale for the effective theory (which sets the range of validity) is $M_{QCD} \sim 1$ GeV. In addition to pions, nucleons should be included, as well as the delta isobar (since $m_\Delta - m_N \approx 300$ MeV$\ll M_{QCD}$). However, the delta isobar will contribute at a higher order in the theory than we are considering in the problems dealt with in this thesis, so it will be neglected. A nucleon N is described by a Pauli spinor in both spin and isospin spaces, with generators $\boldsymbol{\sigma}/2$ and $\boldsymbol{\tau}/2$, respectively. Throughout this thesis, we work in the heavy baryon formalism in which the nucleon field is redefined as $N \to e^{-im_N v\cdot x}N$. This has the effect of eliminating the nucleon mass term, where m_N can refer either to the physical nucleon mass or the nucleon mass in the chiral limit (Jenkins and Manohar, 1991). In this formalism, the nucleon is characterized by its velocity v_μ and spin S_μ, which are given in the rest frame of the nucleon by $v_\mu = (1, \mathbf{0})$ and $S_\mu = (0, \boldsymbol{\sigma}/2)$, respectively.

The derivative nature of Goldstone boson interactions is implemented in the low energy theory via the pion covariant derivative

$$\mathbf{D}_\mu = D^{-1}\partial_\mu \boldsymbol{\pi}, \tag{1.6}$$

where $D \equiv 1 + \boldsymbol{\pi}^2/(2f_\pi)^2$.

1.2.1 Chiral invariant operators

The effective chiral Lagrangian can now be constructed using all of the isoscalar, parity and time-reversal invariant operators that can be built from the fields $\mathbf{D}_\mu$, N and their covariant derivatives

$$\mathcal{D}_\mu \mathbf{D}_\nu = \partial_\mu \mathbf{D}_\nu + i\mathbf{E}_\mu \times \mathbf{D}_\nu \tag{1.7}$$

and

$$\mathcal{D}_\mu N = \partial_\mu N + \frac{1}{2}\boldsymbol{\tau} \cdot \mathbf{E}_\mu N, \tag{1.8}$$

where

$$\mathbf{E}_\mu \equiv \frac{i}{2f_\pi^2}\boldsymbol{\pi} \times \mathbf{D}_\mu. \tag{1.9}$$

Terms in the effective Lagrangian will be chiral invariant so long as they are isospin invariant and built from covariant quantities. Particularly important among the chiral-invariant operators that can be formed are the pion kinetic term $\sim \boldsymbol{D}_\mu \cdot \boldsymbol{D}^\mu$ and the pion-nucleon coupling $\sim \bar{N} S_\mu \boldsymbol{\tau} N \cdot \boldsymbol{D}^\mu$.

1.2.2 Operators due to the sum and difference of quark masses

Chiral symmetry is a good approximate symmetry of the QCD Lagrangian. This is primarily because the light quarks are very far below the chiral symmetry breaking scale $\Lambda_{\chi SB} \sim 4\pi f_\pi$, and because the electromagnetic coupling constant is much less than unity. Although the explicit breaking of chiral symmetry in QCD is small, the pattern of symmetry breaking nevertheless has significant consequences for the hadronic theory.

The quark mass terms in the QCD Lagrangian (1.5) break chiral symmetry, and each of these two terms generates particular interactions, such as the pion mass term or various isospin-breaking terms in the low-energy theory. However, the fact that the up and down quarks have different charges indicates that chiral symmetry is also broken electromagnetically. Both of these mechanisms will be discussed below. The method of constructing chiral- symmetry-breaking operators is given in Appendix B.

In the limit where the quark masses are zero, the QCD Lagrangian (without photons) exhibits a global chiral $SO(4)$ symmetry which is spontaneously broken down to its $SO(3)$ subgroup of isospin. It is this spontaneous symmetry breaking that gives the triplet of Goldstone bosons, the pions $\boldsymbol{\pi}$, which inhabit the three-sphere $S^3 \sim SO(4)/SO(3)$. Tuning the QCD Lagrangian back to reality by including the quark mass terms from Eq. (1.5),

$$\mathcal{L}_{mass} = -\frac{1}{2}(m_u + m_d)\bar{q}q - \frac{1}{2}(m_d - m_u)(-\bar{q}\tau_3 q), \tag{1.10}$$

where

$$q = \begin{pmatrix} u \\ d \end{pmatrix}, \tag{1.11}$$

chiral symmetry is then explicitly broken. This is because both of the terms in Eq. (1.10) transform as vectors under the full $SO(4)$ group. The first term is the fourth component of an $SO(4)$ vector S, with

$$S = (\bar{q}i\gamma_5\boldsymbol{\tau}q, \bar{q}q), \tag{1.12}$$

and the second term is the third component of another $SO(4)$ vector P (with opposite transformation properties from S under parity and time reversal), with

$$P = (-\bar{q}\boldsymbol{\tau}q, \bar{q}i\gamma_5 q). \tag{1.13}$$

Because there is an $SO(3)$ subgroup that does not affect the fourth component of $SO(4)$ vectors, only the second term in Eq. (1.10) breaks isospin.

These symmetry breaking terms will produce S-matrix elements that transform as tensor products of the vectors S and P under chiral symmetry. This requires that there be terms in the effective Lagrangian with the same transformation properties. The procedure to determine what these terms are should then be to construct all tensors $T_{\alpha\beta\ldots}[\boldsymbol{\pi};\boldsymbol{D}_\mu,N]$ from covariant objects only, and then rotate them:

$$T_{\alpha\beta\ldots}[\boldsymbol{\pi};\boldsymbol{D}_\mu,N]=\sum_{\alpha'\beta'\ldots}R_{\alpha\alpha'}[\boldsymbol{\pi}]R_{\beta\beta'}[\boldsymbol{\pi}]\ldots T_{\alpha'\beta'\ldots}[0;\boldsymbol{D}_\mu,N], \tag{1.14}$$

with $R[\boldsymbol{\pi}]$ given by Eq. (A.3), then select third and fourth components.

A simple example of a term that can be formed from a scalar operator is the pion mass term. Without nucleons, only numbers are available to construct operators, so that the only operator at our disposal is

$$S[0;0,0]=(\mathbf{0},1). \tag{1.15}$$

Following Eq. (1.14), we rotate with the matrix (A.3) to give

$$S[\boldsymbol{\pi};0,0]=\left(\frac{\boldsymbol{\pi}}{f_\pi D},\frac{1}{D}\Big(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}\Big)\right). \tag{1.16}$$

The third component S_3 is a pseudoscalar that will be shown later to be unphysical, but the fourth component directly gives the Lagrangian

$$\mathcal{L}^{(0)}_{\pi,qm}=-\frac{1}{2D}m_\pi^2\boldsymbol{\pi}^2+\text{constant}. \tag{1.17}$$

This introduces a quantity, m_π, whose square is proportional to the average light quark mass $\hat{m}\equiv(m_u+m_d)/2$, i.e $m_\pi^2=\mathcal{O}(\hat{m}M_{QCD})$, since Eq. (1.17) is generated by the first term in Eq. (1.10).

A very important example of a quantity generated by a pseudoscalar operator is the quark-mass contribution to the nucleon mass difference, denoted δm_N. Consider the operator

$$P[0;0,N] = (\bar{N}\boldsymbol{\tau}N, 0). \tag{1.18}$$

Upon rotation with the matrix (A.3), the fourth component of P gives a T-violating pion-nucleon coupling interaction which will be studied in Chapter 2, while the third component gives the following isospin-breaking terms:

$$\mathcal{L}^{(1)}_{N,qm} = -\frac{\delta m_N}{2}\left(\bar{N}\tau_3 N - \frac{1}{2f_\pi^2 D}\pi_3\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\pi}N\right). \tag{1.19}$$

Here $\delta m_N = \mathcal{O}(\varepsilon\hat{m}) = \mathcal{O}(\varepsilon m_\pi^2/M_{QCD})$, where

$$\varepsilon = \frac{m_d - m_u}{m_u + m_d}, \tag{1.20}$$

since the Lagrangian (1.19) is generated by the second term in Eq. (1.10). The nucleon mass difference contribution (1.19) is thus proportional to m_π^2, with a suppression of $\varepsilon \approx 0.3$ (Weinberg, 1977).

1.2.3 Operators due to electromagnetic interactions

The quark mass terms (1.10) are not the only source of chiral symmetry breaking in the QCD Lagrangian (1.5). Including photons via minimal coupling in the quark kinetic term gives the coupling of the quark doublet (1.11) to the photon field A_μ,

$$\mathcal{L}_{em} = ie\bar{q}Q\gamma_\mu A^\mu q, \tag{1.21}$$

via the quark charge matrix

$$Q = \frac{1}{6} + \frac{1}{2}\tau_3. \tag{1.22}$$

The τ_3 part of Q breaks chiral symmetry generally and isospin in particular. There are two classes of interactions that are generated by Eq. (1.21) (van Kolck, 1993). The first class that we will consider are those that involve hard photon (momenta greater than M) exchange between quarks. These hard photons can be integrated out, producing operators that do not explicitly involve the electromagnetic field. The second class of interactions, which will be examined later in this section, includes explicit soft (momentum less than M) photons, where the photons couple to pions and nucleons in the most general way that respects gauge invariance.

In order to determine what types of indirect electromagnetic interactions are generated by the quark-photon coupling (1.21), it is necessary to first know its chiral symmetry properties. To this end, let us reexpress Eq. (1.21) as

$$\mathcal{L}_{em} = e\left(\frac{1}{6}c^{\mu} + \frac{1}{2}i_3^{\mu}\right)A_{\mu}, \tag{1.23}$$

where

$$c^{\mu} \equiv \bar{q}i\gamma^{\mu}q \tag{1.24}$$

$$\boldsymbol{i}^{\mu} \equiv \bar{q}i\gamma^{\mu}\boldsymbol{\tau}q. \tag{1.25}$$

The quantity c^{μ} is a chiral scalar, and hence does not break chiral symmetry. The quantity $\boldsymbol{i}^{\mu}$ is more complicated, however. It transforms as a vector under isospin, but does not transform as a vector under chiral $SO(4)$ symmetry. Under an axial transformation, $\boldsymbol{i}^{\mu}$ goes into $\boldsymbol{j}^{\mu} \equiv \bar{q}i\gamma^{\mu}\gamma_5\boldsymbol{\tau}q$, which is also an isovector. Thus $\boldsymbol{i}^{\mu}$ is a sum and $\boldsymbol{j}^{\mu}$ is a difference of two isovectors which transform under $SU(2)_L$ and $SU(2)_R$, respectively. These isovectors form an $(1,0)+(0,1)$ representation of $SU(2)_L \times SU(2)_R \sim SO(4)$. It follows that $\boldsymbol{i}^{\mu}$ and $\boldsymbol{j}^{\mu}$ together form a rank-two antisymmetric $SO(4)$ tensor

$$F^\mu = (F^\mu)_{ab} = \begin{pmatrix} \varepsilon_{ijk} j_k^\mu & i_j^\mu \\ -i_i^\mu & 0 \end{pmatrix}. \tag{1.26}$$

The chiral transformation properties of Eq. (1.21) are then made clear if it is expressed as

$$\mathcal{L}_{em} = e\left(\frac{1}{6}c^\mu + \frac{1}{2}F_{34}^\mu\right)A_\mu. \tag{1.27}$$

Thus photons couple to the quark doublet (1.11) with one contribution that is isoscalar and chiral invariant, and another contribution that is isovector and transforms as the 34-component of an antisymmetric $SO(4)$ tensor.

Considering specifically interactions from hard photon exchange, integrating out photons in Eq. (1.21) produces effective four-quark contact interactions. This is done schematically by joining two photon-quark vertices via a photon propagator, which gives the effective interaction

$$\mathcal{L}_{em,eff} = -e^2\left(\frac{1}{36}c^\mu D_{\mu\nu}c^\nu + \frac{1}{6}c^\mu D_{\mu\nu}F_{34}^\nu + \frac{1}{4}F_{34}^\mu D_{\mu\nu}F_{34}^\nu\right), \tag{1.28}$$

where $D_{\mu\nu}$ stands for the photon propagator. From the way Eq. (1.28) is written, it is clear that the first term is chiral invariant, while the second and third terms transform under $SO(4)$ as F_{34} and $F_{34}F_{34}$, respectively.

In order to construct objects that break chiral symmetry in the same way as the terms in Eq. (1.28) do, we construct chiral scalars from chiral invariant quantities like $\bar{N}N$ and $\boldsymbol{D}_\mu \cdot \boldsymbol{D}^\mu$, and antisymmetric rank-two tensors built out of covariant quantities such as $\boldsymbol{D}_\mu$, N and their covariant derivatives, and then select the 34-components of these tensors. Unlike the previous section, where the sources of chiral symmetry breaking are the quark masses, the operators constructed with the chiral structure of the terms in Eq. (1.28) will not vanish in the chiral limit ($m_{u,d} \to 0$).

Ref. (van Kolck, 1993) details the tensors that can be formed, and a number of these tensors will be discussed in Chapter 2 as they are needed. For now, we focus on a particularly useful example that will complement an example from the previous section, the electromagnetic contribution to the nucleon mass difference.

With nucleon fields, there are several rank-two tensors that can be constructed. The tensor

$$F[0;0,N] = \begin{pmatrix} 0 & \bar{N}\tau_i N \\ -\bar{N}\tau_j N & 0 \end{pmatrix} \tag{1.29}$$

generates the isospin-breaking terms

$$\mathcal{L}^{(-1)}_{N,em} = -\frac{\bar{\delta}m_N}{2}\left[\bar{N}\tau_3 N - \frac{1}{2f_\pi^2 D}\bar{N}(\boldsymbol{\pi}^2\tau_3 - \pi_3\boldsymbol{\tau}\cdot\boldsymbol{\pi})N\right], \tag{1.30}$$

where $\bar{\delta}m_N$ is the electromagnetic contribution to the nucleon mass difference. Its value is not known precisely, but it is fair to assume that $\bar{\delta}m_N = \mathcal{O}(\alpha M_{QCD})$, with $\alpha = e^2/4\pi$ the fine structure constant. The actual nucleon mass difference $\Delta m_N = m_p - m_n = \delta m_N + \bar{\delta}m_N = -1.3$ MeV, which is the same order of magnitude as the estimates for δm_N and $\bar{\delta}m_N$ above. Note how the different chiral properties of the up-down quark-mass difference and the electromagnetic quark coupling generate different pion interactions.

Turning to operators involving explicit soft photons, these operators will give interactions with the same chiral symmetry properties as Eq. (1.21). An interesting example that incorporates chiral symmetry breaking both from soft photons and from the quark masses is the magnetic dipole moment of the nucleon.

The nucleon magnetic dipole moment receives both isoscalar and isovector contributions which are of short range. The isoscalar part generated from Eq. (1.21) is given by the chiral invariant operator

$$c_{mag} = \varepsilon_{\mu\nu\alpha\beta}\bar{N}v^{\mu}S^{\nu}NF^{\alpha\beta}, \tag{1.31}$$

while the isovector part comes from the tensor operator

$$(F_{mag})_{ab}[0;0,N] = \varepsilon_{\mu\nu\alpha\beta}\begin{pmatrix} 0 & \bar{N}\tau_j v^{\mu}S^{\nu}N \\ -\bar{N}\tau_i v^{\mu}S^{\nu}N & 0 \end{pmatrix}F^{\alpha\beta}. \tag{1.32}$$

Clearly, the operator (1.31) will not acquire any pion interactions through a transformation, since it is $SO(4)$-invariant. Chiral invariant factors such as $\boldsymbol{D}_{\mu}\cdot\boldsymbol{D}^{\mu}$ can give pionic contributions, however, at the expense of increasing the dimension. Applying the rotation (A.3) to Eq. (1.32) and combining the result with the magnetic moment interaction from Eq. (1.31), we have the lowest-dimension short-range contributions to the nucleon magnetic dipole moment generated by the quark electromagnetic coupling (1.21)

$$\mathcal{L}^{(0)}_{mag} = \varepsilon_{\mu\nu\alpha\beta}\bar{N}\left\{\tilde{\mu}_0^{em} + \tilde{\mu}_1^{em}\left[\frac{1}{D}\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\left(\tau_3 - \frac{1}{2f_\pi^2 D}\pi_3\boldsymbol{\tau}\cdot\boldsymbol{\pi}\right)\right]\right\}v^{\mu}S^{\nu}NF^{\alpha\beta}, \tag{1.33}$$

where $\tilde{\mu}_0^{em}$ ($\tilde{\mu}_1^{em}$) is a short-range contribution to the isoscalar (isovector) nucleon magnetic moment, and $\tilde{\mu}_i^{em} = \mathcal{O}(e/M_{QCD})$. Not carrying any dependence on quark masses, the contributions to the magnetic dipole moment from Eq. (1.21) will be finite in the chiral limit.

The quark-photon coupling (1.21) is not the only source that can generate the magnetic dipole moment, however. One can also generate the magnetic dipole interaction from a combination of the quark mass terms and the electromagnetic quark coupling. Interactions generated by this combination will carry factors of m_π^2 due to the quark masses, and thus will vanish in the chiral limit. Contributions to the isoscalar magnetic moment can be constructed using products of chiral scalars times Lorentz scalar $SO(4)$ vectors. Contributions to the isovector magnetic moment can be constructed both from the product of a chiral scalar times a Lorentz pseudoscalar $SO(4)$ vector and from the product of a Lorentz scalar $SO(4)$ vector times an antisymmetric $SO(4)$ tensor.

The operator that generates the leading contribution to the isoscalar nucleon magnetic moment is simply

$$cS[0;0,0] = \varepsilon_{\mu\nu\alpha\beta}\bar{N}v^{\mu}S^{\nu}NF^{\alpha\beta}S[0;0,0], \tag{1.34}$$

where the 'c' is meant to remind us that there is to be an electromagnetic, chiral scalar contribution to the generated interaction, and $S[0;0,0]$ is given by (1.15). Upon rotating in pions and selecting the fourth component of the resulting vector, Eq. (1.34) gives

$$\mathcal{L}^{(2)}_{mag,isosc.} = \frac{\tilde{\mu}_0^{qm}}{D}\varepsilon_{\mu\nu\alpha\beta}\left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\bar{N}v^{\mu}S^{\nu}NF^{\alpha\beta}, \tag{1.35}$$

where $\tilde{\mu}_0^{qm} = \mathcal{O}(e\hat{m}/M_{QCD}^2) = \mathcal{O}(em_\pi^2/M_{QCD}^3)$ is a short-range contribution to the isoscalar nucleon magnetic dipole moment generated by Eq. (1.10).

The operator

$$cP[0;0,N] = \varepsilon_{\mu\nu\alpha\beta}(\bar{N}\boldsymbol{\tau}v^{\mu}S^{\nu}N, 0)F^{\alpha\beta} \tag{1.36}$$

generates

$$\mathcal{L}^{(2)}_{mag,isovec} = \tilde{\mu}_1^{qmd}\varepsilon_{\mu\nu\alpha\beta}\bar{N}\left[\tau_3 - \frac{1}{2f_\pi^2 D}\pi_3\boldsymbol{\tau}\cdot\boldsymbol{\pi}\right]v^{\mu}S^{\nu}NF^{\alpha\beta}, \tag{1.37}$$

where $\tilde{\mu}_1^{qmd} = \mathcal{O}(e\varepsilon\hat{m}/M_{QCD}^2) = \mathcal{O}(e\varepsilon m_\pi^2/M_{QCD}^3)$ is a short-range contribution to the isovector nucleon magnetic dipole moment generated by the quark-mass difference, and the operator

$$S[0;0,0]F_{mag}[0;0,N] \tag{1.38}$$

generates

$$\mathcal{L}^{(2)}_{mag,isovec.} = \frac{\tilde{\mu}_1^{qm}}{D^2}\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\varepsilon_{\mu\nu\alpha\beta}\bar{N}\left[\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\tau_3+\frac{1}{f_\pi^2 D}\pi_3\boldsymbol{\tau}\cdot\boldsymbol{\pi}\right]v^\mu S^\nu N F^{\alpha\beta}, \tag{1.39}$$

where $\tilde{\mu}_1^{qm} = \mathcal{O}(e\hat{m}/M_{QCD}^2) = \mathcal{O}(em_\pi^2/M_{QCD}^3)$ is a short-range contribution to the isovector nucleon magnetic dipole moment generated by the sum of the light quark masses. The magnetic moment contributions (1.35), (1.37) and (1.39) that are generated by the quark-mass terms (1.10) are expected to be about two orders of magnitude down from the lowest dimension contributions (1.33).

The analysis of the nucleon magnetic dipole moment above allows us to know precisely how the magnetic moment should be affected by the presence of pions. It also serves as a primer for computing short-range contributions to the nucleon electric dipole moment, which will be done in the following chapter.

Electromagnetic chiral symmetry breaking arising from gauge-invariant coupling of hard photons to quarks has been discussed above, but soft photons can also be minimally coupled to nucleons and pions directly. Soft photons couple to nucleons and pions in the most general gauge-invariant way. The photon field A_μ enters either through gauge invariant objects constructed from the field strength $F_{\mu\nu}$ or through minimal substitution in covariant derivatives:

$$D_\mu\pi_a \rightarrow D_\mu\pi_a - eA_\mu\varepsilon_{3ab}\pi_b \tag{1.40}$$

$$\mathcal{D}_\mu N \rightarrow \mathcal{D}_\mu N - ieQA_\mu N \tag{1.41}$$

where $Q = (1+\tau_3)/2$ is the nucleon charge matrix. This is not the whole story, however, because there is an anomaly in the third (isospin) component of the axial current which explicitly breaks chiral symmetry (Bijnens, 1993). There must, then, be terms in the effective Lagrangian that reproduce this anomaly when a chiral transformation is performed. However, this subtlety will not be relevant for this thesis.

1.2.4 Power counting

It remains now to order the terms that appear in the low-energy effective chiral Lagrangian via power counting. The Lagrangian can be written as a sum

$$\mathcal{L} = \sum_{\Delta=0}^{\infty} \mathcal{L}^{(\Delta)}, \tag{1.42}$$

where $\Delta \equiv d + f/2 - 2$ is called the chiral index, with d being the number of derivatives and/or powers of m_π, and f being the number of fermion fields. When we form terms that are chiral invariant or break chiral symmetry as the quark masses, we find that the index $\Delta \geqslant 0$. This is because terms with only pions will contain at least two derivatives or powers of m_π, and terms with a nucleon bilinear will contain at least one derivative. Thus chiral symmetry *guarantees* a natural perturbative low-energy theory. When electromagnetic chiral symmetry breaking is taken into account, powers of e may be included in d. This is because in minimal coupling, e essentially takes the place of the derivative. In this thesis, we will generally keep track of powers of e explicitly, and hence maintain d as being the number of derivatives/powers of m_π. The terms in the leading and next-to-leading order Lagrangians that are relevant to this thesis are given by (Ordóñez and van Kolck, 1992; Ordóñez et al., 1996; van Kolck, 1994; Bernard et al., 1995)

$$\mathcal{L}^{(0)} = \frac{1}{2}\boldsymbol{D}_\mu \cdot \boldsymbol{D}^\mu - \frac{1}{2D} m_\pi^2 \boldsymbol{\pi}^2 + \bar{N} i v \cdot \mathcal{D} N - \frac{g_A}{f_\pi} \bar{N}\big(S_\mu \boldsymbol{\tau} \cdot \boldsymbol{D}^\mu\big) N + \dots \tag{1.43}$$

$$
\begin{aligned}
\mathcal{L}^{(1)} =& \frac{1}{2m_N}\Big\{\bar{N}\big((v\cdot\mathcal{D})^2-\mathcal{D}^2\big)N+\frac{g_A}{f_\pi}\big[i\bar{N}(v_\mu\boldsymbol{\tau}\cdot\boldsymbol{D}^\mu)S\cdot\mathcal{D}N+\text{H.c.}\big]\Big\}\\
&+\frac{e}{2}\varepsilon_{\mu\nu\alpha\beta}\bar{N}\big(1+\kappa_0+(1+\kappa_1)\tau_3\big)v^\mu S^\nu N F^{\alpha\beta}\\
&-\frac{B_1}{4f_\pi^2}\bar{N}N\boldsymbol{D}_\mu\cdot\boldsymbol{D}^\mu-\frac{B_2}{2f_\pi^2}\varepsilon_{\alpha\beta\mu\nu}v^\alpha\bar{N}S^\beta\boldsymbol{\tau}N\cdot(\boldsymbol{D}^\mu\times\boldsymbol{D}^\nu)\\
&-\frac{B_3}{4f_\pi^2 D}m_\pi^2\bar{N}N\boldsymbol{\pi}^2-\frac{B_4}{4f_\pi^2}\bar{N}N(v\cdot\boldsymbol{D})^2+\ldots,
\end{aligned}
\tag{1.44}
$$

together with the minimal couplings (1.40) and (1.41). Here $g_A = 1.267$ is the pion-nucleon coupling, $\kappa_0 = -0.12$ and $\kappa_1 = 3.7$ are the isoscalar and isovector anomalous magnetic photon-nucleon couplings, respectively, B_i are coefficients of order $\mathcal{O}(1/M_{QCD})$, and "..." stands for terms with additional fermion fields that will not be needed in this thesis. The undetermined coefficients in Eq. (1.44) can be determined either by solving QCD or fitting to data.

Processes that involve at most one nucleon ($A \leqslant 1$) can be straightforwardly described using EFT. Assuming all particles have momenta $Q = \mathcal{O}(m_\pi)$, there are only two energy scales in the theory: Q and M_{QCD}. Then a generic contribution to an amplitude can be written as

$$
T \sim \left(\frac{Q}{M_{QCD}}\right)^\nu F_\nu\left(\frac{Q}{m_\pi}\right), \tag{1.45}
$$

where F is a dimensionless non-analytic function, and ν is an exponent determined as follows: The diagrams corresponding to the amplitude (1.45) are irreducible diagrams (diagrams which cannot be separated by cutting the lines of initial or final particles in an intermediate state). An irreducible diagram with $2A$ external nucleon lines, L loops and V_i vertices of type i which have chiral indices $\Delta_i = d_i + f_i/2 - 2$ is of order $\mathcal{O}(Q^\nu)$, where

$$
\nu = 2(1-A+L)+\sum_i V_i\Delta_i. \tag{1.46}
$$

Summing the diagrams contributing to a given process results in an expansion in Q/M_{QCD}. It starts at $\nu = \nu_{min} = 2 - A$ (for strong interactions) with tree diagrams ($L = 0$) built with index-0 vertices, then continues at $\nu = \nu_{min} + 1$ with diagrams built with one vertex of index 1 and the remaining vertices all having index 0. Loop diagrams first contribute at $\nu = \nu_{min}+2$, where they are built exclusively with index-0 vertices. Of course, at this order there are also tree diagrams with one index-2 vertex or two index-1 vertices. Generalizing to higher orders is straightforward. In the context above, the EFT is called Chiral Perturbation theory (χPT). [2] For a discussion of the subtleties involved with systems with $A \geq 2$, see for example Ref. (Bedaque and van Kolck, 2002).

1.3 T violation and its sources

Prior to the 1950's, it was thought that the laws of physics should be invariant under the transformation $t \to -t$. However, Ref. (Purcell and Ramsey, 1950) noted that an elementary particle having a permanent electric dipole moment (EDM) necessarily indicates time reversal (T) violation. From then on, efforts to search for intrinsic EDMs in nature have comprised an intense effort in the study of fundamental symmetry violation.

Assuming the validity of the CPT theorem, nonconservation of T is synonymous with violation of CP. Within the SM, there are two sources of CP violation: the complex phase in the Cabibo-Kobayashi-Maskawa (CKM) quark mixing matrix (Kobayashi and Maskawa, 1973) and the $\bar{\theta}$ term in the QCD Lagrangian (1.5) (Belavin et al., 1975; 'tHooft, 1976; Jackiw and Rebbi, 1976; Callan et al., 1976). CP violation of the CKM type has been seen in both the neutral kaon (Christensen et al., 1964) and B meson (Aubert et al., 2001; Abe et al., 2001) systems, but CP violation from the $\bar{\theta}$ term has thus far not been detected. However, for cosmological

[2] For a review, see, for example (Weinberg, 1996; Bernard et al., 1995).

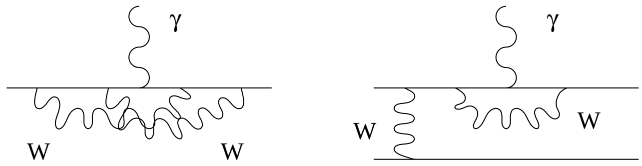

Figure 1.1: Contributions to an electric dipole moment via W boson exchange.

reasons, weak CP violation alone is insufficient to account for all of the CP violation in nature. In particular, electroweak baryogenesis could not have occurred without additional CP violation beyond what can be provided by the CKM matrix (see, for example, Ref. (Cohen et al., 1993)). Direct evidence for non-CKM CP violation would come from a nonzero neutron EDM.

Assume that the neutron EDM is generated by W boson exchange among quarks, with examples of contributions given in Fig. 1.1. We can then obtain a reasonable estimate of the neutron EDM based on dimensional analysis alone (c.f. (Donoghue et al., 1992)). The EDM must contain a factor of the Jarlskog determinant (Jarlskog, 1985), which is given in terms of the elements of the CKM matrix as

$$J_{CP} = Im(V_{ij}V_{kl}V^*_{kj}V^*_{il}) \tag{1.47}$$

In the Wolfenstein parameterization of the CKM matrix (Wolfenstein, 1983)

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & \lambda^3 A[\rho - i\eta(1-\frac{1}{2}\lambda^2)] \\ -\lambda & 1-\frac{1}{2}\lambda^2 - i\eta A^2\lambda^4 & \lambda^2 A(1+i\eta\lambda^2) \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix}, \tag{1.48}$$

where $\lambda \cong 0.22$ and $A, \rho, \eta = \mathcal{O}(1)$, $J_{CP} = A^2\lambda^6\eta + \mathcal{O}(\lambda^8) \approx 3 \cdot 10^{-5}$. An EDM can only occur at second order (or higher) in the weak interactions, so there will be a factor of $G_F^2/(4\pi)^4 \sim 10^{-15}\text{GeV}^{-4}$, with factors of 4π from loop diagrams included. In addition, the GIM mechanism (Glashow et al., 1970) would cancel contributions of degenerate mass quarks, so there should be a contribution of the form $(m_j^2 - m_k^2)/M_W^2$. Then

$$d_n \sim e\frac{G_F^2}{(4\pi)^4}\left(\frac{m_t}{M_W}\right)^2 J_{CP}(4\pi f_\pi)^3 \approx 10^{-32} e \cdot \mathrm{cm}, \tag{1.49}$$

where $4\pi f_\pi$ is included as a typical hadronic scale for dimensional correctness, and factors of 4π from loop diagrams have been included. This estimate is six orders of magnitude smaller than the current experimental value (Baker et al., 2006)

$$|d_n| = (0.6 \pm 2.3) \cdot 10^{-26} e \text{ cm}, \tag{1.50}$$

and is several orders of magnitude smaller than the limits that experiments can expect to attain in the near future. Thus there are compelling reasons to expect that CP violation exists beyond the weak interactions, with the $\bar{\theta}$ term being a prime candidate.

The $\bar{\theta}$ term, given in Eq. (1.5), is a purely topological boundary term which is required by the instanton solution to the "axial $U(1)$" problem. Naively, one would expect that $\bar{\theta} \sim \mathcal{O}(1)$. This would lead to a prediction for the size of the neutron electric dipole moment large enough that it should have been detected. However, as we shall see in Chapter 3, the current experimental bound on the neutron EDM implies that

$$|\bar{\theta}| \lesssim 10^{-10}. \tag{1.51}$$

This is the so-called *strong CP problem*, and it has yet to be satisfactorily resolved. There have been many attempts to use physical arguments to eliminate the $\bar{\theta}$ term entirely, most notably through the introduction of a $U(1)$ symmetry (called Peccei-Quinn symmetry) that causes the $\bar{\theta}$ term to vanish at the expense of adding a new dynamical field (the axion) to the theory (Peccei and Quinn, 1977a,b). However, none of these attempts have been satisfactory (in particular, the existence of the axion has been almost ruled out for experimental and cosmological reasons).

The best probes for detecting flavor-diagonal CP violation like that caused by the $\bar{\theta}$ term are EDMs, and the precision to which intrinsic EDMs are known to vanish makes them among the most important precision tests of the SM.

Beyond the $\bar{\theta}$ term, which is dimension four, there are additional sources of CP violation that can be classified by their scale dimension. Formally at dimension five are the EDMs and chromoelectric dipole moments of the quarks:

$$\mathcal{L}^{(5)}_{QCD,\not{T}} = \frac{i}{2}\bar{q}(d_s + d_v\tau_3)F^{\mu\nu}\sigma_{\mu\nu}\gamma_5 q + \frac{i}{2}\bar{q}(\breve{d}_s + \breve{d}_v\tau_3)G^{\mu\nu}\sigma_{\mu\nu}\gamma_5 q. \tag{1.52}$$

Here $d_s = (d_u + d_d)/2$ ($d_v = (d_u - d_d)/2$) is the isoscalar (isovector) component of the quark EDM d_q, $\breve{d}_s = (\breve{d}_u + \breve{d}_d)/2$ ($\breve{d}_v = (\breve{d}_u - \breve{d}_d)/2$) is the isoscalar (isovector) component of the quark color EDM $\breve{d}_q$, $\sigma_{\mu\nu} \equiv (i/2)[\gamma_\mu, \gamma_\nu]$, and the superscript in $\mathcal{L}^{(5)}_{QCD,\not{T}}$ indicates the scale dimension of each term. In the minimal SM, the operators in Eq. (1.52) are effectively dimension six operators because the quark EDMs and quark chromo-EDMs scale as $d_q \sim m_q/M^2$, where m_q is a light quark mass and M is some large mass scale far beyond M_{QCD}. The reason for this is that if one goes to a chiral basis, the operators in Eq. (1.52) connect left- and right-handed quarks (fermions), which requires a chirality flip. This is usually supplied by an insertion of the quark (fermion) mass, implying that $d_q \sim m_q/M^2$. To be consistent then, dimension six operators should be included along with the dimension five operators (1.52). At dimension six one encounters the Weinberg three-gluon operator (Weinberg, 1989) and a number of four-quark interactions:

$$\mathcal{L}^{(6)}_{QCD,\not{T}} = \frac{1}{3}wf^{abc}G^a_{\mu\rho}G^{\rho,b}_{\nu}G^c_{\lambda\sigma}\varepsilon^{\mu\nu\lambda\sigma} + C\bar{q}q\bar{q}i\gamma_5 q + \ldots. \tag{1.53}$$

Here f^{abc} are the totally antisymmetric Gell-Mann coefficients, and "…" indicates four-quark interactions with nontrivial Lorentz and/or gauge structures. In the same way that the terms in Eq. (1.52) are effectively dimension six, the four-quark interactions in Eq. (1.53) are effectively of dimension eight (require two chirality flips when looked at in a chiral basis). Nevertheless, in some cases the four-quark interactions may be non-negligible.

As will be discussed in the next chapter, each of the above sources of T violation generates T-violating hadronic interactions in a manner determined by the chiral symmetry properties of the sources. Determining the forms of these interactions and properly understanding their potential consequences for real-world experiments shall be the focus of the remainder of this thesis. Naturally, doing this will require the application of many of the ideas presented in this chapter.

CHAPTER 2

THE T-VIOLATING EFFECTIVE CHIRAL LAGRANGIAN

2.1 Introduction

Relating T violation among quarks and gluons to T violation among baryons and mesons is a topic that has been considered a number of times in various contexts in the literature, but has yet to receive a formal treatment in EFT. The aim of this thesis is to provide such a treatment, utilizing the powerful tool that is chiral symmetry. As discussed in Chapter 1, terms in the low-energy effective Lagrangian must have the same symmetry properties as the terms in the Lagrangian of the underlying theory. This requirement highly constrains the types of T-violating interactions that arise from the sources discussed in Section 1.3.

Knowing the T-violating chiral Lagrangian for a given source can provide useful phenomenological information. Once strong (flavor-diagonal) T violation is seen in nature, it can be understood in the context of being generated by some more fundamental interaction if there exists a signature for T violation from a specific source. For example, if a particular T-violating process is seen to break chiral symmetry or isospin in a particular manner, this symmetry-breaking can be compared to the expected breaking in the interactions generated by the sources of T violation that are understood. Formulating an effective Lagrangian for the interactions generated by a variety of quark-level T-violating sources can thus provide a guidebook for experimentalists, an indication of exactly what kind of T-violating interactions should be seen based upon what is causing the T violation.

In this chapter, the various flavor-diagonal quark-level sources of T violation will be studied in order of their scale dimensions. The lowest dimension term is the $\bar{\theta}$

term. Being the lowest dimension source, it *a priori* generates the most important contributions to hadronic T violation. It also provides a straightforward setting for comparing and contrasting two different approaches to ensuring the stability of the QCD vacuum under perturbations arising from sources of T violation. Once the T-violating effective chiral Lagrangian has been presented for the $\bar{\theta}$ term, the same will be done for the quark EDM, quark color EDM and Weinberg three-gluon operator, which are all higher dimension compared to the $\bar{\theta}$ term.

2.2 The QCD $\bar{\theta}$ term and Baluni's method

The QCD $\bar{\theta}$ term was introduced in Section 1.2:

$$\mathcal{L}^{(4)}_{QCD,\not{T}} = \frac{g_s^2\bar{\theta}}{16\pi^2} Tr(G_{\mu\nu}\tilde{G}^{\mu\nu}) \tag{2.1}$$

Here $\tilde{G}^{\mu\nu} = (1/2)\varepsilon^{\mu\nu\alpha\beta}G_{\alpha\beta}$ is the dual of the gluon field strength. Although not obvious, Eq. (2.1) is a total divergence:

$$G^a_{\mu\nu}\tilde{G}^{\mu\nu,a} = \varepsilon^{\mu\nu\rho\sigma}\partial_\mu(A^a_\nu G^a_{\rho\sigma} - \frac{2}{3}g_s f^{abc}A^a_\nu A^b_\rho A^c_\sigma)$$

Consequently, Eq. (2.1) does not contribute to the physics either classically or in perturbation theory, but only nonperturbatively via QCD vacuum effects. Since T violation due to the $\bar{\theta}$ term has not been observed, it is tempting to set $\bar{\theta}$ to zero. However, the nature of the QCD vacuum dictates that doing this has consequences ('tHooft, 1976; Callan et al., 1976; Jackiw and Rebbi, 1976). The vacuum can be written as a sum of vacua with different phases:

$$|\text{vac}\rangle_\theta = \sum_{n=-\infty}^{\infty} e^{in\theta}|n\rangle\,, \tag{2.2}$$

where $|n\rangle$ is the vacuum state with winding number n, and transition matrix elements between vacua can be written as

$$
\begin{aligned}
{}_{\theta'}\langle \text{vac}|\, e^{-iHt}\, |\text{vac}\rangle_{\theta'} &= \sum_{m,n} e^{im\theta'} e^{-in\theta} \langle m|\, e^{-iHt}\, |n\rangle \\
&= \delta(\theta' - \theta) \sum_{n} e^{-i\nu\theta} \int [DA_\mu]_\nu e^{-iS}, \qquad (2.3)
\end{aligned}
$$

where $\nu \equiv n - m$, the delta function is obtained by summing over m, and the matrix element $\langle m|\, e^{-iHt}\, |n\rangle$ is an integration of the action S over all gauge fields A_μ of the same homotopic class with winding number ν. The phase factor $\exp(-i\nu\theta)$ in Eq. (2.3) can be absorbed into the action, and since the winding number $\nu = (g_s^2/16\pi^2)\int d^4x TrG\tilde{G}$, the effective QCD Lagrangian is

$$
\mathcal{L} = \mathcal{L}_{QCD} + \theta\nu = \mathcal{L}_{QCD} + \theta\frac{g_s^2}{16\pi^2}\text{tr}(G_{\mu\nu}\tilde{G}^{\mu\nu}). \qquad (2.4)
$$

Thus the $\bar{\theta}$ term arises naturally when the effects of tunnelling between degenerate vacua, i.e. instanton effects, are included.

Although the effects of Eq. (2.1) cannot be eliminated, the term itself can be eliminated in favor of T violation from the quark mass terms (1.10) by performing a chiral rotation on the quark fields. This is due to the fact that under a chiral rotation of the quark fields, i.e. $q(x) \to \exp(i\alpha\gamma_5)q(x)$, the measure of integration for the fermionic path integral changes (Fujikawa, 1979):

$$
[dq][d\bar{q}] \to \exp\Big\{-i\frac{\alpha g_s^2}{16\pi^2}\int d^4x TrG_{\mu\nu}\tilde{G}^{\mu\nu}\Big\}[dq][d\bar{q}] \qquad (2.5)
$$

Consider the particular chiral rotation

$$
q \to \exp\left[\frac{i}{4}(1 + \alpha\tau_3)\gamma_5\right]q, \qquad (2.6)
$$

with α an arbitrary real number. Acting on the QCD Lagrangian (1.5), the change in path integral measure has the effect that

$$\bar{\theta} \to \bar{\theta} - \frac{1}{2}(1+\alpha)\bar{\theta} - \frac{1}{2}(1-\alpha)\bar{\theta} = 0.$$

Meanwhile, the quark mass terms (1.10) become

$$\begin{aligned}\mathcal{L}_{mass} \to \hat{m}\cos\frac{\alpha}{2}\bar{q}\Bigg\{ & -1 + \varepsilon\frac{\bar{\theta}}{2}\tan\frac{\alpha}{2} - \left(\varepsilon + \frac{\bar{\theta}}{2}\tan\frac{\alpha}{2}\right)\tau_3 \\ & + i\gamma_5\left[\varepsilon\tan\frac{\alpha}{2} + \frac{\bar{\theta}}{2} + \left(\tan\frac{\alpha}{2} + \varepsilon\frac{\bar{\theta}}{2}\right)\tau_3\right]\Bigg\}q + \mathcal{O}(\bar{\theta}^2),\end{aligned} \tag{2.7}$$

where $\hat{m} \equiv (m_u + m_d)/2$ and $\varepsilon \equiv (m_d - m_u)/(m_d + m_u) \approx 0.3$ (Weinberg, 1977). There is a constraint that can be used to relate α and $\bar{\theta}$ (Baluni, 1979). Looked at as a perturbation of the QCD Lagrangian (1.5) in the chiral limit, the quark mass terms should satisfy a condition. Namely they should not cause instability of the vacuum, or in other words, the perturbation to the vacuum should be a minimum:

$$\frac{d}{d\alpha}\langle\text{vac}|\,\mathcal{L}_{mass}\,|\text{vac}\rangle = 0. \tag{2.8}$$

This gives the condition that $\tan(\alpha/2) = -(\varepsilon/2)\bar{\theta} + \mathcal{O}(\bar{\theta}^2)$, which allows us to reexpress Eq. (2.7) as

$$\mathcal{L}_{mass} \to -\hat{m}\bar{q}q + \varepsilon\hat{m}\bar{q}\tau_3 q + m_*\bar{\theta}\bar{q}i\gamma_5 q + \mathcal{O}(\bar{\theta}^2), \tag{2.9}$$

where $m_* \equiv \hat{m}(1-\varepsilon^2)/2 = m_u m_d/(m_u+m_d)$. Notice from Eq. (2.9) that T violation disappears if either of the light quark masses are zero. The most likely candidate would be the up quark, and $m_u = 0$ would allow us to simply rotate away the $\bar{\theta}$ term with no consequences for the (remaining) quark mass terms. However, analyses of the influence of quark masses on baryon and meson masses effectively rules out this possibility (Gasser and Leutwyler, 1982; Leutwyler, 1996).

The above argument is an application of Dashen's theorem (Dashen, 1971). This theorem essentially states that for a system with Hamiltonian H and vacuum $|0\rangle$ such that $H|0\rangle = 0$, if $|0\rangle$ is the $\delta H \to 0$ limit of $|\text{vac}\rangle$, where $(H + \delta H)|\text{vac}\rangle = 0$, then $\langle 0|\,\delta H\,|0\rangle$ is a minimum. A simple example is sufficient to illustrate this theorem. Consider a ferromagnet with ground state magnetization $\mathbf{M}$. If the ferromagnet is placed in the presence of an external magnetic field $\mathbf{B}$, then the ground state $|\mathbf{M}\rangle$ will only be maintained as $\mathbf{B} \to 0$ if $\mathbf{B}$ is parallel to $\mathbf{M}$, i.e. if the perturbation to the ground state is minimized.

Looking at Eq. (2.9), the second and third terms are the third and fourth components, respectively, of the $SO(4)$ vector $P = (-\bar{q}\boldsymbol{\tau}q, \bar{q}i\gamma_5 q)$ (c.f. Eq. (1.13)). The term in Eq. (2.9) that transforms as P_3 (P_4) generates isospin-violating (T-violating) hadronic interactions in the low-energy EFT (see Section 1.2.2 and Appendix B). Since P_3 and P_4 are components of the same $SO(4)$ vector, the coefficients of the terms that each generates in the low-energy theory will be linked.

To generate the terms in the low-energy theory that stem from the QCD $\bar{\theta}$ term and the quark mass difference, we construct all possible tensors out of π, N and their covariant derivatives (as outlined in Section 1.2.2 and Appendix B), and then select the fourth and third components of those terms which are Lorentz scalars and pseudoscalars, respectively. These terms will appear in the resulting effective Lagrangian with coefficients proportional to powers of $m_*\bar{\theta}$ and $\varepsilon\hat{m}$, respectively.

Since $m_\pi^2 = \mathcal{O}(\hat{m}M_{QCD})$, we have $\varepsilon\hat{m} = \mathcal{O}(\varepsilon m_\pi^2/M_{QCD})$. Likewise, for $\varepsilon^2 \ll 1$, $m_* = \hat{m}(1-\varepsilon^2)/2 \approx \hat{m}/2 = \mathcal{O}(m_\pi^2/M_{QCD})$. As before, Eq. (1.14) can be used to construct terms. Pseudoscalars cannot be formed from numbers alone, and vectors that are constructed will involve nucleon bilinears, excepting the case where photons are explicitly included. The leading such operator that can be formed is

$$P_1[0;0,N] = (\bar{N}\boldsymbol{\tau}N, 0). \tag{2.10}$$

Upon applying the rotation (A.3) to include pions, this operator generates the following terms with index one:

$$\mathcal{L}^{(1)}_{\not{I},P_1} + \mathcal{L}^{(1)}_{\not{T},P_1} = -\frac{\delta m_N}{2}\bar{N}\left[\tau_3 - \frac{1}{2f_\pi^2 D}\pi_3\boldsymbol{\tau}\cdot\boldsymbol{\pi}\right]N - \frac{\bar{g}_0}{D}\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\pi}N \tag{2.11}$$

where $\delta m_N = \mathcal{O}(\varepsilon\hat{m}) = \mathcal{O}(\varepsilon m_\pi^2/M_{QCD})$ is a contribution to the nucleon mass splitting due to isospin violation from the quark masses (see Eq. (1.19)), and $\bar{g}_0 = \mathcal{O}(m_*\bar{\theta}/2f_\pi) = \mathcal{O}(m_\pi^2\bar{\theta}/2f_\pi M_{QCD})$ is an isospin-zero T-violating pion-nucleon coupling. The subscript "P_1" on the left-hand side of Eq. (2.11) indicates which operator was used to generate the interactions, and this notation will be used throughout this chapter. Since the coefficients of the terms in Eq. (2.11) are related by Eq. (2.9),

$$\bar{g}_0 = \frac{\delta m_N}{8f_\pi}\frac{1-\varepsilon^2}{\varepsilon}\bar{\theta}. \tag{2.12}$$

The quantity δm_N is not known precisely[1]. It is one of two contributions to the nucleon mass difference $m_n - m_p = 1.29$ MeV (Eidelman et al., 2004), with the other being an electromagnetic contribution, $\bar{\delta}m_N$ (see Eq. 1.30). Taking the estimate from Ref. (Gasser and Leutwyler, 1982), $\bar{\delta}m_N \approx -0.76$ MeV, which employed a renormalized Cottingham sum rule (Cottingham, 1963), we estimate that $\delta m_N = m_n - m_p - \bar{\delta}m_N \approx 2.1$ MeV. This gives $\bar{g}_0 \approx 8.5 \times 10^{-3}\bar{\theta}$.

The next operators that need to be considered are those that generate terms in the effective Lagrangian with chiral index 2, obtained by including one derivative or

[1] Efforts to determine δm_N model-independently have focused on charge symmetry breaking (which is a subclass of isospin breaking) productions and decays, such as $np \to d\pi^0$ and $dd \to \alpha\pi^0$ (see Ref. (Miller et al., 2006) for a review of these efforts).

by having two pairs of nucleon fields with no derivatives. There is a pair of operators with one derivative:

$$P_{2,3}[0;\boldsymbol{D}_\mu, N] = (\bar{N}S_\mu N\boldsymbol{D}^\mu, 0), (0, \bar{N}S_\mu \mathcal{D}^\mu N + \text{H.c.}) \tag{2.13}$$

These operators, which are related via integration by parts, generate the following index-2 terms:

$$\mathcal{L}^{(2)}_{\not{I},P_{2,3}} + \mathcal{L}^{(2)}_{\not{T},P_{2,3}} = -\frac{\beta_1}{2f_\pi}\left(D_{\mu,3} - \frac{1}{2f_\pi^2 D}\pi_3 \boldsymbol{\pi}\cdot\boldsymbol{D}_\mu\right)\bar{N}S^\mu N + \frac{2\bar{h}_0}{D}\boldsymbol{\pi}\cdot\boldsymbol{D}_\mu \bar{N}S^\mu N, \tag{2.14}$$

where $\beta_1 = \mathcal{O}(\varepsilon\hat{m}/M_{QCD}) = \mathcal{O}(\varepsilon m_\pi^2/M_{QCD}^2)$ and $\bar{h}_0 = \mathcal{O}(m_*\bar{\theta}/4f_\pi^2 M_{QCD}) = \mathcal{O}(m_\pi^2\bar{\theta}/4f_\pi^2 M_{QCD}^2)$. The ratio $2\bar{h}_0/(\beta_1/2f_\pi)$ is equal to the ratio $\bar{g}_0/(\delta m_N/2)$ according to Eq. (2.9), so

$$\bar{h}_0 = \frac{\beta_1}{16f_\pi^2}\frac{1-\varepsilon^2}{\varepsilon}\bar{\theta} \tag{2.15}$$

An upper bound exists for β_1 from the Nijmegen phase-shift analysis of nucleon-nucleon scattering data(van Kolck et al., 1996), $\beta_1 \lesssim 9\times 10^{-3}$, which gives the bound $\bar{h}_0 \lesssim 3\times 10^{-7}\bar{\theta}$ MeV^{-2}.

There are two more operators with two pairs of nucleon fields:

$$P_{4,5}[0;0,N] = (\bar{N}\boldsymbol{\tau}N\bar{N}N, 0), (\bar{N}\boldsymbol{\tau}S_\mu N\bar{N}S^\mu N, 0). \tag{2.16}$$

These operators generate the additional index-2 terms

$$\mathcal{L}^{(2)}_{\not{I},P_{4,5}} + \mathcal{L}^{(2)}_{\not{T},P_{4,5}} = \gamma_s \bar{N}\left[\tau_3 - \frac{1}{2f_\pi^2 D}\pi_3 \boldsymbol{\tau}\cdot\boldsymbol{\pi}\right] N\bar{N}N + \frac{\bar{j}_0}{D}\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\pi}N\bar{N}N$$
$$+ \gamma_\sigma \bar{N}\left[\tau_3 S_\mu - \frac{1}{2f_\pi^2 D}\pi_3 \boldsymbol{\tau}\cdot\boldsymbol{\pi}S_\mu\right] N\bar{N}S^\mu N + \frac{\bar{k}_0}{D}\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\pi}S_\mu N\bar{N}S^\mu N, \tag{2.17}$$

where we expect γ_s and γ_σ to be of order $\mathcal{O}(\varepsilon\hat{m}/4f_\pi^2 M_{QCD}) = \mathcal{O}(\varepsilon m_\pi^2/4f_\pi^2 M_{QCD}^2)$, and $\bar{j}_0$ and $\bar{k}_0$ are both of order $\mathcal{O}(m_*\bar{\theta}/8f_\pi^3 M_{QCD}) = \mathcal{O}(m_\pi^2\bar{\theta}/8f_\pi^3 M_{QCD}^2)$. The terms on the first (second) line of Eq. (2.17) are both generated by the operator P_4 (P_5), so that the coefficients γ_s and $\bar{j}_0$ (γ_σ and $\bar{k}_0$) satisfy

$$\bar{j}_0 = \frac{\gamma_s}{4f_\pi}\frac{1-\varepsilon^2}{\varepsilon}\bar{\theta} \tag{2.18}$$

and

$$\bar{k}_0 = \frac{\gamma_\sigma}{4f_\pi}\frac{1-\varepsilon^2}{\varepsilon}\bar{\theta} \tag{2.19}$$

The parameter γ_s has been estimated based on its contribution from the mechanism of ρ-ω mixing (van Kolck et al., 1996). This estimate is of the same order of magnitude as above, roughly $\mathcal{O}(10^{-7}\ \text{MeV}^{-2})$, from which we would then expect $\bar{j}_0 = \mathcal{O}(10^{-9}\bar{\theta}\ \text{MeV}^{-3})$. The parameter γ_σ has been estimated under the assumption that the dominant contribution to it comes from mixing of the axial-vector mesonic resonances a_1 and f_1 (Coon et al., 1996), where the authors arrive at a γ_σ value of order $\mathcal{O}(10^{-8}\ \text{MeV}^{-2})$. This implies that the T-violating coupling $\bar{k}_0 = \mathcal{O}(10^{-10}\bar{\theta}$ MeV^{-3}).

When photons are incorporated along with pions and nucleons, additional operators can be formed. There is a single operator with no nucleons:

$$P_6[0;0,0] = F_{\mu\nu}\tilde{F}^{\mu\nu}S_1[0;0,0] \tag{2.20}$$

This operator generates an effective interaction which contributes to the decay rate for $\pi^0 \to \gamma\gamma$, giving a correction to the axial anomaly, as well as a T-violating pion-photon interaction:

$$\mathcal{L}^{(2)}_{\not{I},P_6} + \mathcal{L}^{(2)}_{\not{T},P_6} = \frac{\eta}{D}\pi_3 F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\bar{\imath}_0}{D}\left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right) F_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{2.21}$$

The coefficient η is expected to be $\mathcal{O}(e^2\varepsilon\hat{m}/(8\pi^2 f_\pi M_{QCD})) = \mathcal{O}(e^2\varepsilon m_\pi^2/8\pi^2 f_\pi M_{QCD}^2)$, where factors of e^2 and $4\pi^2$ are included due to the fact that the process $\pi^0 \to \gamma\gamma$ can only occur (minimally) via a quark loop. The coefficient $\bar{\imath}_0 = \mathcal{O}(m_*\bar{\theta}/M_{QCD}) = \mathcal{O}(m_\pi^2\bar{\theta}/M_{QCD}^2)$.

The QCD axial anomaly gives rise to $\pi^0 \to \gamma\gamma$ decay:

$$\mathcal{L}_{anom} = a\pi_3 F_{\mu\nu}\tilde{F}^{\mu\nu} + \ldots, \tag{2.22}$$

where a can be computed exactly in QCD to give (Adler, 1969)

$$a = \frac{\alpha_{em} N_C}{24\pi f_\pi} \approx 3.1 \times 10^{-6}\text{MeV}^{-1}, \tag{2.23}$$

where $\alpha_{em} = e^2/4\pi$ is the fine structure constant, and $N_C = 3$ is the number of color degrees of freedom available in strong interactions. The expression (2.23) is about an order of magnitude larger than the estimate for the correction η above. Eq. (2.9) then implies that $\bar{\imath}_0$ is related to η by

$$\bar{\imath}_0 = \eta f_\pi \frac{1-\varepsilon^2}{\varepsilon}\bar{\theta} = \mathcal{O}(10^{-5}\bar{\theta}). \tag{2.24}$$

Note that, although $F_{\mu\nu}\tilde{F}^{\mu\nu}$ is a total derivative, the T-violating interaction in Eq. (2.21) does not vanish, since the part proportional to $\boldsymbol{\pi}^2$ is not a total derivative.

Before concluding this section, there is one more interaction that can be generated by the $\bar{\theta}$ term which is of central importance in this thesis. Short-range contributions to the EDM of the nucleon are generated by the insertion of the $\bar{\theta}$

term operator in a diagram with a vertex from the electromagnetic quark coupling (1.21). Consider the operators

$$c_{edm} P_7[0; 0, N] = \big(0, \bar{N}(v_\mu S_\nu - S_\mu v_\nu) N F^{\mu\nu}\big), \tag{2.25}$$

where we take the chiral scalar $c_{edm} = 1$, and

$$P_8[0; 0, N] F_{mag}[0; 0, N] = \varepsilon_{\mu\nu\alpha\beta} F^{\alpha\beta} \begin{pmatrix} 0 & \varepsilon^{\mu\nu\rho\sigma} \bar{N} \tau_i v_\rho S_\sigma N \\ -\varepsilon^{\mu\nu\rho\sigma} \bar{N} \tau_i v_\rho S_\sigma N & 0 \end{pmatrix} \tag{2.26}$$

The dual field strength $\varepsilon_{\mu\nu\alpha\beta} F^{\mu\nu}$ can be considered the fourth component of a P vector in Eq. (2.26) in the sense that it is isoscalar and odd under parity. The operators (2.25) and (2.26) generate both isospin-violating, T-conserving terms (which will not be necessary for anything that follows) and terms which give the leading short-range $\bar{\theta}$ term contributions to the nucleon EDM. Upon rotating with the usual rotation (A.3), the operators (2.25) and (2.26) give

$$\begin{aligned} \mathcal{L}^{(2)}_{edm,\bar{\theta}} = \frac{1}{D}\left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right) \bar{N} \left\{ \tilde{d}_0 + \frac{\tilde{d}_1}{D}\left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\left(\tau_3 - \frac{1}{2f_\pi^2 D}\pi_3 \boldsymbol{\tau}\cdot\boldsymbol{\pi}\right)\right\} \\ \times \big(v_\mu S_\nu - S_\mu v_\nu\big) N F^{\mu\nu}. \end{aligned} \tag{2.27}$$

Here $\tilde{d}_0$ ($\tilde{d}_1$) is a short-range contribution to the isoscalar (isovector) nucleon EDM stemming from the $\bar{\theta}$ term, and $\tilde{d}_i = \mathcal{O}(em_*\bar{\theta}/M_{QCD}^2) = \mathcal{O}(em_\pi^2\bar{\theta}/M_{QCD}^3)$. Here we employ the power counting where factors of e are tracked separately from powers of m_π or derivatives. Note that the isoscalar and isovector nucleon EDMs from (2.27) are expected to be roughly the same size. This will not be true for all quark-level T violation sources.

Of course, the process of forming operators and generating hadronic interactions can be continued to whatever order one wishes. At this point, there are several remarks to be made. First, the index 1 and 2 Lagrangians (2.11), (2.14) and (2.17) indicate a strong connection between T violation and isospin violation. While each of the terms in (2.11) and (separately) Eq. (2.14) and Eq. (2.17) occur at the same order in the chiral expansion, they have very different strengths due to the unnaturally small dimensionless parameter $\bar{\theta}$. Were $\bar{\theta}$ of order unity, a correlation between $\bar{\theta}$-term T violation and isospin violation would likely be seen experimentally. However, the disparate magnitudes of these two phenomena preclude any tangible connection.

The results in this section for the $\Delta = 1$ and $\Delta = 2$ pieces of the T-violating effective chiral Lagrangian with a QCD $\bar{\theta}$ term source are based on the form of the $\bar{\theta}$ term derived by Baluni. This application of Dashen's theorem was surely an important step in our understanding of T violation in strong interactions, particularly in obtaining theoretical estimates of meson-loop contributions to baryonic EDMs. However, it is not known whether Baluni's argument can provide information regarding T violation sources of higher scale dimension. For example, are the quark EDM and quark chromo-EDM interactions (1.52) combinations of isoscalar and isovector contributions, or is there a source of vacuum instability that would indicate that one contribution or the other is unphysical?

There is an alternative approach to deriving the T-violating effective chiral Lagrangian that one can take which does not require applying constraints to the vacuum. Rather, one can retain a more general set of operators, making sure that all of the relevant symmetries are satisfied. Upon generating the interactions that arise from these operators, one can then transform the fields in the theory to eliminate any terms which lead to instability of the vacuum. The next section will revisit the QCD $\bar{\theta}$ term, taking this alternative path to what, of course, must be the same resulting Lagrangian (or, more precisely, the Lagrangians must give the same S-matrix elements).

2.3 The QCD $\bar{\theta}$ term and field redefinitions

The QCD $\bar{\theta}$ term, upon applying a chiral rotation and Dashen's theorem, gives a pseudoscalar quark density which is purely isoscalar. However, there is no symmetry argument forbidding an isovector contribution. This is clear from Eq. (2.7).

Suppose that a chiral rotation has been performed on the QCD Lagrangian (1.5) which has the effect of eliminating the $\bar{\theta}$ term. Upon rotating, the quark mass terms (1.10) become (see Eq. (2.7))

$$\mathcal{L}_{mass} \to \hat{m}\left[A(\varepsilon,\bar{\theta})\bar{q}q + \ldots + B(\varepsilon,\bar{\theta})\bar{q}i\gamma_5\tau_3 q\right] + \mathcal{O}(\bar{\theta}^2), \tag{2.28}$$

where $A(\varepsilon,\bar{\theta})$ and $B(\varepsilon,\bar{\theta})$ are functions which depend on the chiral rotation used, and $''\ldots''$ represents terms which transform as P_3 and P_4, and thus generate the Lagrangians (2.11), (2.14) and (2.17).

The terms in Eq. (2.28) with coefficients A and B are the fourth and third components, respectively, of the $SO(4)$ vector $S = (\bar{q}i\gamma_5\boldsymbol{\tau}q, \bar{q}q)$ (c.f. Eq. (1.12)). The $\bar{q}q$ term is isospin-symmetric and generates the pion mass term (see Section 1.2 and Appendix B), while the $\bar{q}i\gamma_5\tau_3 q$ term violates both isospin symmetry and T. Since the two terms in Eq. (2.28) are components of the same $SO(4)$ vector, the coefficients $A(\varepsilon,\bar{\theta})$ and $B(\varepsilon,\bar{\theta})$ are related, so that terms in the EFT must be generated by the combination

$$S_4 + \frac{B}{A}S_3. \tag{2.29}$$

2.3.1 The theory without nucleons

Since S is a Lorentz scalar, numbers are available as well as vectors and tensors of various ranks. The leading operator that can be formed is simply (c.f. Eq. (1.15))

$$S_1[0;0,0] = (\mathbf{0}, 1). \tag{2.30}$$

This operator will generate both chiral symmetry-breaking and T-violating interactions involving only pions.

Upon rotating in pions, the following terms are generated by the operator (2.30):

$$\mathcal{L}^{(0)}_{\not\chi,S_1} + \mathcal{L}^{(0)}_{\not T,S_1} = \frac{1}{2D} m_\pi^2 (4f_\pi^2 - \boldsymbol{\pi}^2) + \frac{\bar{f}_1}{D}\pi_3, \tag{2.31}$$

where $m_\pi^2 = \mathcal{O}(\hat{m} M_{QCD})$ and Eq. (2.29) implies that

$$\bar{f}_1 = \frac{B}{A} 2 m_\pi^2 f_\pi. \tag{2.32}$$

As in Section 2.2, the subscript 'S_1' in Eq. (2.31) indicates that the terms in the Lagrangians are generated from the operator S_1. There is a serious problem with the T-violating term in Eq. (2.31). The fact that it is linear in the pion field indicates that the vacuum can create pions to no end, and thus can lower its energy without bound. This is precisely the type of vacuum instability that was avoided in Section 2.2 by employing Dashen's theorem. However, this unphysicality can also be avoided by an axial transformation of the pion fields. Consider the transformation (A.5), with $\epsilon_{A,i} = (B/A)\delta_{i3}$:

$$\pi_i \to \pi_i + f_\pi \frac{B}{A}\left[\left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\delta_{i3} + \frac{\pi_3 \pi_i}{2f_\pi^2}\right]. \tag{2.33}$$

Acting on the Lagrangian (2.31), the transformation (2.33) eliminates the T-violating interaction, leaving only the pion mass term. One can check that this transformation also leaves the pion kinetic term from Eq. (1.43) invariant. Since the axial transformation (2.33) is contained in the chiral group, we would expect that it leaves chiral-invariant objects unchanged. Additionally, there are terms generated by Eq. (2.33) that are proportional to $(B/A)^2$. Since T-violating interactions (from the $\bar{\theta}$ term) that transform as third components of S carry a dependence on $\bar{\theta}$, it follows that $B/A \sim \bar{\theta}$. Thus terms proportional to $(B/A)^2$ may be safely neglected.

The next order operator that can be formed increases the index by two (required by Lorentz invariance) and is given by

$$S_2[0; \boldsymbol{D}_\mu, 0] = \boldsymbol{D}_\mu \cdot \boldsymbol{D}^\mu S_1[0; 0, 0]. \tag{2.34}$$

The operator (2.34) generates a T-conserving, isospin-symmetric term and a T-violating term:

$$\mathcal{L}^{(2)}_{\not{\chi},S_2} + \mathcal{L}^{(2)}_{\not{T},S_2} = \frac{\alpha}{D}\left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\boldsymbol{D}_\mu \cdot \boldsymbol{D}^\mu + \frac{\bar{h}_1}{D}\pi_3 \boldsymbol{D}_\mu \cdot \boldsymbol{D}^\mu, \tag{2.35}$$

where $\alpha = \mathcal{O}(m_\pi^2/M_{QCD}^2)$, and Eq. (2.29) implies that

$$\bar{h}_1 = \frac{B}{A}\frac{\alpha}{f_\pi}. \tag{2.36}$$

Similar to $\mathcal{L}^{(0)}_{\not{T},S_1}$, $\mathcal{L}^{(2)}_{\not{T},S_2}$ can be eliminated by acting on Eq. (2.35) with the axial transformation (2.33). As we go to higher chiral indices, Eq. (2.33) acts on the chiral symmetry-breaking terms to eliminate the accompanying T-violating terms at each index.

An additional type of operator can be formed without nucleons. Operators with explicit photon fields can be formed using the field strength $F_{\mu\nu}$. The leading such operator that can be formed is

$$S_3[0; 0, 0] = F_{\mu\nu}F^{\mu\nu} S_1[0; 0, 0]. \tag{2.37}$$

This operator generates the $\Delta = 2$ terms

$$\mathcal{L}^{(2)}_{\chi,S_3} + \mathcal{L}^{(2)}_{\not{T},S_3} = \frac{\rho}{D}\left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right) F_{\mu\nu}F^{\mu\nu} + \frac{\bar{i}_1}{D}\pi_3 F_{\mu\nu}F^{\mu\nu}, \tag{2.38}$$

where $\rho = \mathcal{O}(m_\pi^2/M_{QCD}^2)$ and $\bar{i}_1 = \mathcal{O}(m_\pi^2\bar{\theta}/2f_\pi M_{QCD}^2)$. The term in (2.38) proportional to $\boldsymbol{\pi}^2 F_{\mu\nu}F^{\mu\nu}$ is a contribution to the pion magnetic polarizability which vanishes in the chiral limit. The coefficients of the terms in Eq. (2.38) are related according to (2.29) as

$$\bar{i}_1 = \frac{B}{A}\frac{\rho}{f_\pi}. \tag{2.39}$$

The term $\mathcal{L}^{(2)}_{\not{T},S_3}$ can be eliminated in precisely the same way as the T-violating interactions in Eqs. (2.31) and (2.35).

2.3.2 The theory with nucleons

Incorporating nucleons, the leading (Lorentz scalar) operator that can be formed with a pair of nucleon fields is

$$S_4[0;0,N] = \bar{N}NS_1[0;0,0]. \tag{2.40}$$

This operator generates the $\Delta = 1$ terms

$$\mathcal{L}^{(1)}_{\chi,S_4} + \mathcal{L}^{(1)}_{\not{T},S_4} = \frac{m_\pi^2 B_3}{D}\left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\bar{N}N + \frac{\bar{g}_1}{D}\pi_3\bar{N}N, \tag{2.41}$$

where $B_3 = \mathcal{O}(1/M_{QCD})$ (the coefficient is chosen to match that in Eq. (1.44)), and Eq. (2.29) implies that

$$\bar{g}_1 = \frac{B}{A}\frac{m_\pi^2 B_3}{f_\pi}. \tag{2.42}$$

Note that the term in Eq. (2.41) proportional to $\boldsymbol{\pi}^2\bar{N}N$ is a contribution to the nucleon mass which vanishes in the chiral limit. The T-violating term in Eq. (2.41) vanishes when we apply the axial transformation (2.33) to the pion fields, and the axial transformation (A.15) with $\epsilon_{A,i} = (B/A)\delta_{i3}$,

$$N \to N - \frac{i}{2f_\pi}\frac{B}{A}\varepsilon_{3kl}\tau_k\pi_l N, \tag{2.43}$$

to the nucleon fields. The chiral-invariant nucleon kinetic term and pion-nucleon coupling term (see Eq. (1.43)) are also invariant under Eq. (2.43).

The index can be increased by one by including one derivative or an additional pair of nucleon fields. There is a pair of vectors with one derivative:

$$S_{5,6}[0; \boldsymbol{D}_\mu, N] = (0, \bar{N}S^\mu\boldsymbol{\tau}\cdot N\boldsymbol{D}_\mu), (\bar{N}\boldsymbol{\tau}S^\mu\mathcal{D}_\mu N + \text{H.c.}, 0). \tag{2.44}$$

The following terms are generated by these vectors (which are related by an integration by parts):

$$\mathcal{L}^{(2)}_{\not{\chi},S_{5,6}} + \mathcal{L}^{(2)}_{\not{T},S_{5,6}} = \frac{\beta}{D}\left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\bar{N}S^\mu\boldsymbol{\tau}\cdot N\boldsymbol{D}_\mu + \frac{\bar{k}_1}{D}\pi_3\bar{N}S^\mu\boldsymbol{\tau}\cdot N\boldsymbol{D}_\mu. \tag{2.45}$$

The coefficient $\beta = \mathcal{O}(m_\pi^2/f_\pi M_{QCD}^2)$, while $\bar{k}_1 = (B/A)(\beta/f_\pi)$. Again, performing the transformations (2.33) and (2.43) on Eq. (2.45) eliminates the T-violating term.

There are four additional vectors that have two pairs of nucleon fields:

$$S_{7,8}[0; 0, N] = \bar{N}N\bar{N}NS_1[0; 0, 0] \quad , \quad \bar{N}S_\mu N\bar{N}S^\mu NS_1[0; 0, 0] \tag{2.46}$$

$$S_{9,10}[0; 0, N] = \bar{N}\boldsymbol{\tau}N\cdot\bar{N}\boldsymbol{\tau}NS_1[0; 0, 0] \quad , \quad \bar{N}\boldsymbol{\tau}S_\mu N\cdot\bar{N}\boldsymbol{\tau}S^\mu NS_1[0; 0, 0]. \tag{2.47}$$

These operators generate a number of similar chiral symmetry-breaking and T-violating terms, such as

$$\mathcal{L}^{(2)}_{\chi,S_7} + \mathcal{L}^{(2)}_{\not{T},S_7} = \frac{\gamma_1}{D}\left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\bar{N}N\bar{N}N + \frac{\bar{j}_1}{D}\pi_3\bar{N}N\bar{N}N, \tag{2.48}$$

where $\gamma_1 = \mathcal{O}(m_\pi^2/f_\pi^2 M_{QCD}^2)$ and $\bar{j}_1 = (B/A)(\gamma_1/f_\pi)$. There are three additional pairs of interactions which can simply be obtained by inserting spin and isospin vectors into the expression for Eq. (2.48), in the same way that S_8, S_9 and S_{10} can be obtained from S_7 in Eq. (2.46). As with the T-violating term in Eq. (2.45), the T-violating term in Eq. (2.48) (and the three related terms described above) can be eliminated by transforming the pion and nucleon fields according to Eqs. (2.33) and (2.43), respectively.

The procedure described above can be carried out to whatever order in the chiral expansion one wishes. It is wise at this point to think about what is happening at the level of the QCD vacuum when one performs the field redefinitions described above. When Dashen's theorem was applied in the previous section, it had the effect of selecting the "correct" vacuum, i.e. the vacuum about which perturbative calculations could be carried out. Baluni accomplished this by rotating the quark fields appropriately. In this section, the "wrong" vacuum was selected, meaning that a vacuum was selected that would not allow perturbative calculations. Instead, one needed to rotate the hadronic fields in order to align the vacuum of the theory so that it corresponds to the proper vacuum, i.e. the vacuum that satisfies Dashen's theorem.

The goal of the current section has been accomplished. This goal was to confirm that an isovector T-violating term (at quark level), obtained by rotating away the QCD $\bar{\theta}$ term, is unphysical and to show that all interactions in the low-energy theory that were generated by this unphysical term could be eliminated by appropriately rotating the pion and nucleon fields. In some sense, this has to be the case, so long as one accepts the validity of Baluni's application of Dashen's theorem (the author is unaware of any who dispute it). The question now becomes what can this procedure tell us about the T-violating interactions that are generated by quark-level sources of T violation that are formally of higher scale dimension? That is the subject of the following section.

2.4 T violation sources of higher scale dimension

Having determined the T-violating effective chiral Lagrangian in the first few orders for a $\bar{\theta}$-term source, we now turn our attention to the next highest dimension operators available. These are (naively) the electric dipole moment operators and the color electric dipole moment operators for the quarks, both of which are formally of dimension five. However, both of these operators involve a change of chirality. This must be accompanied by the insertion of a Higgs field, so that the coefficients of the EDM and color EDM operators will implicitly carry a factor of the Higgs vacuum expectation value. Hence the coefficients scale as $1/M^2$, where M is some large mass scale that is large compared to the scale of electroweak symmetry breaking.

The Weinberg operator, which is dimension six, has no such suppression. Therefore, it is expected to contribute to T-violating hadronic operators with a strength comparable to the quark EDM and chromo-EDM. The types of interactions that each operator generates are different, however. This is because the chiral symmetry properties of each operator are different. The transformation properties of each of the effective dimension six operators will be examined, and from this the T-violating chiral Lagrangians can be derived.

2.4.1 The quark chromoelectric dipole moment

The quark chromoelectric dipole moment operator is given by (see Eq. (1.52))

$$\mathcal{L}_{qced} = \frac{i}{2}\bar{q}(\breve{d}_s + \breve{d}_v\tau_3)G^{\mu\nu}\sigma_{\mu\nu}\gamma_5 q, \tag{2.49}$$

with $\breve{d}_s = (\breve{d}_u + \breve{d}_d)/2$ ($\breve{d}_v = (\breve{d}_u - \breve{d}_d)/2$) the isoscalar (isovector) component of the quark color EDM $\breve{d}_q$. The chiral symmetry properties of Eq. (2.49) can be easily determined. The spaces of color and isospin do not interact, so a chiral transformation has no effect on $G_{\mu\nu} \equiv G^a_{\mu\nu}t^a$, where t^a are the Gell-Mann color $SU(3)$ matrices. Since $\sigma_{\mu\nu} \equiv (i/2)[\gamma_\mu, \gamma_\nu]$ involves a pair of gamma matrices, it will produce a net factor of $(-1)^2$ under a chiral transformation, thus there is no effect. What is left, then, is clearly a pair of operators that transform as P_4 (isoscalar part) and S_3 (isovector part). This is the same chiral structure as the T-violating terms in Eq. (2.28) that arose from the $\bar{\theta}$ term, except that there is no connection between T violation and isospin/chiral symmetry breaking in this case. The terms in Eq. (2.49) do not arise from the quark mass terms, so there are *separate* $SO(4)$ vectors that give the operators generating T violation (from P_4) and isospin violation (from P_3), or T violation (from S_3) and (T-conserving) chiral symmetry breaking (from S_4).

The interactions generated by Lorentz pseudoscalar operators, such as Eqs. (2.10) and (2.13), will be identical in structure to those generated by the $\bar{\theta}$ term. They will have different strengths, however. Thus the leading T-violating term that is formed is the $\Delta = 1$ term (c.f. Eq. (2.11))

$$\mathcal{L}^{(1)}_{\not{T},P_1} = -\frac{\bar{g}_0}{D}\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\pi}N, \tag{2.50}$$

where now $\bar{g}_0 = \mathcal{O}(\breve{d}_s M^2_{QCD}/f_\pi)$. Similar to the case of $\bar{\theta}$-term T violation, Eq. (2.50) is effectively $\Delta = 1$ due to the implicit factor of a light quark mass in $\breve{d}_s$, which gives a dependence on m_π^2. At $\Delta = 2$, there is (c.f. Eq. (2.14))

$$\mathcal{L}^{(2)}_{\not{T},P_{2,3}} = \frac{2\bar{h}_0}{D}\boldsymbol{\pi}\cdot\boldsymbol{D}_\mu\bar{N}S^\mu N, \tag{2.51}$$

where $\bar{h}_0 = \mathcal{O}(\breve{d}_s M_{QCD}/f_\pi^2)$, as well as terms with the same form as the T-violating interactions in Eqs. (2.17) and (2.21).

Of course, the quark color EDM also generates nucleon EDM contributions, just as the $\bar{\theta}$ term does. The form of the resulting interactions is identical to Eq. (2.27), with $\tilde{d}_i = \mathcal{O}(e\breve{d}_s)$. Thus we expect that the nucleon EDM is of the same order as the light quark color EDMs (modulo e), under the assumption that the color EDM is the dominant source which generates the nucleon EDM.

Without accurately knowing the values of $\bar{\theta}$ and $\breve{d}_s$, it is not possible to say for certain which of the $\bar{\theta}$ term or chromo-EDM is stronger in nature. Constraints from the upper bound of the neutron EDM, for example, are not sufficient to determine which source(s) of T violation is(are) dominant. In fact, no low-energy measurement will disentangle the parameters $\bar{\theta}$ and $\breve{d}_s$. One would have to probe the quark-level sources directly in order to observe any distinction.

The interactions generated by Lorentz scalar operators, such as Eqs. (2.30), (2.34), (2.37) and (2.40) create the same vacuum instability problems whether the source is the isovector piece of the $\bar{\theta}$ term or the isovector quark chromo-EDM. The T-violating interactions generated by the isovector quark chromo-EDM have forms identical to those T-violating interactions discussed in Section 2.3, and they can be eliminated in a similar manner to that described in Section 2.3. However, because there is no constraint tying quark chromo-EDM T violation to some other phenomena, a single-parameter axial transformation is not sufficient. Instead, the axial transformation parameter will consist of an infinite series of terms.

Consider the T-violating terms in Eqs. (2.31), (2.35), (2.38), (2.41), (2.45) and (2.48). These terms can all be eliminated by the transformations (c.f. Eqs. (2.33) and (2.43))

$$\pi_i \to \pi_i + f_\pi \left[\left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2} \right) \epsilon_{A,i} + \frac{1}{2f_\pi^2} \boldsymbol{\pi} \cdot \boldsymbol{\epsilon}_A \pi_i \right], \tag{2.52}$$

and

$$N \to N - \frac{i}{2f_\pi}\varepsilon_{jkl}\tau_j\epsilon_{A,k}\pi_l N, \tag{2.53}$$

where $\boldsymbol{\epsilon}_A$ is given by

$$\begin{aligned}
\epsilon_{A,i} = \frac{1}{2f_\pi m_\pi^2}\Bigg[&\bar{f}_1 \\
&+\left(\bar{h}_1 - \frac{\alpha\bar{f}_1}{2f_\pi^2 m_\pi^2}\right)\boldsymbol{D}_\mu\cdot\boldsymbol{D}^\mu + \ldots \\
&+\left(\bar{g}_1 - \frac{B_3\bar{f}_1}{2f_\pi^2}\right)\bar{N}N + \ldots \\
&+\left(\bar{i}_1 - \frac{\rho\bar{f}_1}{2f_\pi^2 m_\pi^2}\right)F_{\mu\nu}F^{\mu\nu} + \ldots \\
&+\left(\bar{k}_1 - \frac{\beta\bar{f}_1}{2f_\pi^2 m_\pi^2}\right)\bar{N}S^\mu\boldsymbol{\tau}N\cdot\boldsymbol{D}_\mu + \ldots \\
&+\left(\bar{j}_1 - \frac{\gamma_1\bar{f}_1}{2f_\pi^2 m_\pi^2}\right)(\bar{N}N)^2 + \ldots\Bigg]\delta_{i3}.
\end{aligned} \tag{2.54}$$

The terms shown eliminate the T-violating interactions in Eqs. (2.31), (2.35), (2.38), (2.41), (2.45) and (2.48), as well as the interactions induced on the T-conserving terms in each of these Lagrangians by the first piece of the axial parameter (2.54). In the second through sixth lines of Eq. (2.54), "..." refers to further terms in the axial parameter that compensate for additional terms that are generated. These further terms will involve higher powers of the chiral-invariant quantities, i.e. $\boldsymbol{D}_\mu\cdot\boldsymbol{D}^\mu$, $\bar{N}N$, etc. Since each term in Eq. (2.54) involves only chiral-invariant quantities, the transformations (2.52) and (2.53) do not alter any chiral-invariant terms in the effective Lagrangian.

With the field transformations above, we can eliminate all terms generated by the isovector quark chromo-EDM, just as was done in Section 2.3. The difference in this case is that the axial transformation parameter will have an infinite number of terms. This indicates that, at least as far as low-energy physics is concerned, the isovector contribution to the quark chromo-EDM can be rotated away, just as the term in Eq. (2.28) of the form $\bar{q}i\gamma_5\tau_3 q$ can be rotated away. Thus, for the purposes of the T-violating effective chiral Lagrangian, the quark chromo-EDM interaction can simply be written as

$$\mathcal{L}_{qced} = \frac{i}{2}\breve{d}_s \bar{q} G^{\mu\nu} \sigma_{\mu\nu}\gamma_5 q. \tag{2.55}$$

This term then generates Eqs. (2.50), (2.51) and terms corresponding to the T-violating interactions in Eqs.(2.17) and (2.21) at the first two orders.

2.4.2 The quark electric dipole moment

The quark electric dipole moment operator is given by (see Eq. (1.52))

$$\mathcal{L}_{qed} = \frac{i}{2}\bar{q}(d_s + d_v\tau_3)\sigma_{\mu\nu}\gamma_5 q F^{\mu\nu}, \tag{2.56}$$

with $d_s = (d_u + d_d)/2$ ($d_v = (d_u - d_d)/2$) the isoscalar (isovector) component of the quark EDM d_q. Unlike the QCD $\bar{\theta}$ term and the quark chromo-EDM, the quark EDM explicitly involves photons. This creates a class of T-violating interactions distinct from those generated by the $\bar{\theta}$ term and the quark color EDMs (van Kolck, 1993). Analogous to the quark minimal coupling to photons considered in Section 1.2.3, exchange of photons between quarks that involves a T-violating (quark EDM) vertex generates two classes of interactions in the low-energy theory (see Section 1.2.3). One class involves soft photons (momenta below M_{QCD}), which couple to nucleons and pions in the most general way that respects gauge invariance. The other class involves hard photons (momenta above M_{QCD}), which can be integrated out to produce operators that represent effective four-quark contact interactions, with no explicit electromagnetic field. We first consider the class involving hard photons.

The class of interactions involving hard photons requires the development of an effective T-violating four-quark interaction. Similar to the construction of the effective four-quark T-even electromagnetic interaction (1.28), the effecive four-quark Lagrangian for interactions between the quark electric dipole current and a T-conserving electromagnetic quark current is

$$\mathcal{L}_{qed}^{eff} = e(d_s K^\mu + d_v K_3^\mu) D_{\mu\nu} \left(\frac{1}{6} c^\nu + \frac{1}{2} i_3^\nu \right) . \tag{2.57}$$

where $K^\mu \equiv k_\nu D_s \bar{q} i \gamma_5 \sigma^{\mu\nu} q$ is isoscalar and transforms as P_4 under chiral symmetry, $K_i^\mu \equiv k_\nu \bar{q} i \tau_i \gamma_5 \sigma^{\mu\nu} q$ is isovector and transforms as S_i under chiral symmetry, while $c^\nu \equiv \bar{q} i \gamma^\nu q$ (c.f. Eq. (1.24)) and $\boldsymbol{i}^\nu \equiv \bar{q} i \boldsymbol{\tau} \gamma^\nu q$ (c.f. (1.25)), with the coefficients of these currents coming from the quark charge matrix $Q = 1/6 + (1/2)\tau_3$. Finally, $D_{\mu\nu}$ stands for the photon propagator. The chiral symmetry properties of c^ν and $\boldsymbol{i}^\nu$ were discussed in Section 1.2.3, where it was shown that c^ν is a chiral scalar and $\boldsymbol{i}_3^\nu$ transforms as the 34-component of the rank-two antisymmetric $SO(4)$ tensor (c.f. Eq. (1.26))

$$(F^\mu)_{ab} = \begin{pmatrix} \varepsilon_{ijk} j_k^\mu & i_j^\mu \\ -i_i^\mu & 0 \end{pmatrix} . \tag{2.58}$$

Just as the $SO(4)$ vectors S and P can be translated from quark-level operators to hadronic operators, so too can F. There are various possibilities, and finding them requires employing the same strategy laid out in Section 1.2.3 and Appendix B. Starting with antisymmetric rank-two tensors built out of covariant quantities, i.e. N, $\boldsymbol{D}_\mu$ and their covariant derivatives, we then apply the rotation (A.3) and select the 34-components of the resulting tensors.

A number of tensors can be formed, with any number of Lorentz indices. With no Lorentz index, there is one possibility:

$$(F_1)_{ab}[0;0,N] = \begin{pmatrix} i\varepsilon_{ijk}\bar{N}\tau_k\gamma_5 N & \bar{N}\tau_j N \\ -\bar{N}\tau_i N & 0 \end{pmatrix}. \tag{2.59}$$

In constructing tensors with one Lorentz index, there must be at least one derivative. Two tensors can be formed with $\boldsymbol{D}_\mu$:

$$(F_2^\mu)_{ab}[0;\boldsymbol{D}_\mu,0] = \begin{pmatrix} \varepsilon_{ijk}D_k^\mu & 0 \\ 0 & 0 \end{pmatrix} \tag{2.60}$$

and

$$(F_3^\mu)_{ab}[0;\boldsymbol{D}_\mu,0] = \begin{pmatrix} 0 & D_i^\mu \\ -D_j^\mu & 0 \end{pmatrix}. \tag{2.61}$$

Because these tensors are four vectors, they cannot be used directly. There are four tensors that have one derivative and a pair of nucleon fields:

$$(F_4)_{ab}[0;\boldsymbol{D}_\mu,N] = \bar{N}N(F_2^0)_{ab}, \tag{2.62}$$

$$(F_5)_{ab}[0;\boldsymbol{D}_\mu,N] = \bar{N}S_\mu N(F_3^\mu)_{ab}, \tag{2.63}$$

$$(F_6)_{ab}[0;\mathcal{D}_\mu,N] = \begin{pmatrix} \varepsilon_{ijk}(\bar{N}\tau_k S_\mu \mathcal{D}^\mu N + H.c.) & 0 \\ 0 & 0 \end{pmatrix} \tag{2.64}$$

and

$$(F_7)_{ab}[0;\boldsymbol{D}_\mu,N] = \begin{pmatrix} 0 & \varepsilon_{ijk}D_\mu\pi_j\bar{N}\tau_k S^\mu N \\ \varepsilon_{ijk}D_\mu\pi_i\bar{N}\tau_k S^\mu N & 0 \end{pmatrix}. \tag{2.65}$$

The tensors (2.59) through (2.65) have the expected transformation to incorporate pions:

$$(F_i)_{ab}[\boldsymbol{\pi};\boldsymbol{D}_\mu,N]=R_{aa'}(\boldsymbol{\pi})R_{bb'}(\boldsymbol{\pi})(F_i)_{a'b'}[0;\boldsymbol{D}_\mu,N], \tag{2.66}$$

The rotation matrix $R_{\alpha\beta}(\boldsymbol{\pi})$ is given by Eq. (A.3).

Looking back at Eq. (2.57), the chiral symmetry properties of each of the four terms are as follows (excluding the factors that are irrelevant to chiral symmetry):

$$K^\mu c^\nu \quad \leftrightarrow \quad P_4 \tag{2.67}$$
$$K_3^\mu c^\nu \quad \leftrightarrow \quad S_3 \tag{2.68}$$
$$K^\mu i_3^\nu \quad \leftrightarrow \quad P_4F_{34} \tag{2.69}$$
$$K_3^\mu i_3^\nu \quad \leftrightarrow \quad S_3F_{34}. \tag{2.70}$$

The hadronic interactions generated by each of these structures will be examined in turn, starting with the two structures that already appeared in the context of the QCD $\bar\theta$ term and the quark chromoelectric dipole moment.

The $K^\mu c^\nu$ piece of Eq. (2.57) has the same chiral symmetry properties as the rotated $\bar\theta$ term in Eq. (2.9) and the "effective" quark chromo-EDM term (2.55). Thus it will generate interactions of exactly the same form as Eqs. (2.50) and (2.51) in the first two orders. Of course, the coefficients will be modified accordingly, so that $\bar g_0=\mathcal{O}(ed_sM_{QCD}^2/f_\pi)$ and $\bar h_0=\mathcal{O}(ed_sM_{QCD}/f_\pi^2)$.

The $K_3^\mu c^\nu$ piece of Eq. (2.57) has the same chiral structure and suffers from the same vacuum instability problems as the S_3-type terms in Eqs. (2.28) and (2.49). The interactions generated by the structure (2.68) can all be eliminated by the axial transformation (2.52) (pions) + (2.53) (nucleons) with axial parameter (2.54). This is precisely the same transformation used in Section 2.4.1. However, the structure (2.68) cannot simply be neglected, because the factor K_3^μ is shared by both Eqs.

(2.68) and (2.70). Thus it appears that the isovector quark EDM leads to vacuum instability and can be rotated out of Eq. (2.56) for the purpose of constructing the T-violating chiral Lagrangian. However, in order to safely omit this term, it must first be shown that any interactions arising from the $K_3^\mu i_3^\nu$ structure of Eq. (2.57) can be safely eliminated.

The leading term that is generated by $K_3^\mu i_3^\nu$ comes from the tensor product $S_1[0;0,0](F_1)_{ab}[0;0,N]$:

$$\mathcal{L}^{(1)}_{\not{T},S_1F_1} = \frac{\bar{g}_2}{D}\pi_3\bar{N}\tau_3 N. \tag{2.71}$$

Here $\bar{g}_2 = \mathcal{O}(ed_v M^2_{QCD}/f_\pi)$. One can eliminate this term in a manner similar to that discussed in section 2.4.1 and earlier in this section. Applying the same axial transformation (2.52) (for pions) + (2.53) (for nucleons), with axial parameter $\epsilon_{A,i} = (\bar{g}_2/2f_\pi m_\pi^2)\delta_{i3}\bar{N}\tau_3 N$, the term (2.71) is eliminated. However, unlike the $\bar{\theta}$ term and quark color EDM cases, $\epsilon_{A,i}$ in this case includes explicit isospin violation. Thus terms in the effective Lagrangian that are chiral invariant will not be invariant under the transformation above. While this makes the procedure of eliminating terms more complicated, the complications are lessened by the fact that the transformation effectively induces terms that have a relative chiral index of one compared to the original terms which appear in the Lagrangian. This is because they include a nucleon bilinear. These induced terms can then be eliminated by adding additional pieces to the axial parameter. The process will continue indefinitely, but the terms induced by the axial transformation will always be of a higher chiral index than the terms from which they are generated.

The above procedure can be carried out for other terms that have the structure S_3F_{34}. Including one derivative or another nucleon bilinear can be done using the operators $S_3[0;0,N](F_1)_{ab}[0;0,N]$ and $S_1[0;0,0](F_7)_{ab}[0;\boldsymbol{D}_\mu,N]$, respectively. These operators generate the T-violating terms

$$\mathcal{L}^{(0)}_{\not{T},S_3F_1,S_1F_7} = \frac{\bar{f}_2}{D}\pi_3\bar{N}\tau_3N\bar{N}N + \frac{\bar{h}_2}{D}\pi_3\bar{N}S_\mu(\boldsymbol{\tau}\times\boldsymbol{D}^\mu)_3N, \tag{2.72}$$

where $\bar{f}_2 = \mathcal{O}(ed_vM_{QCD}/f_\pi^3)$ and $\bar{h}_2 = \mathcal{O}(ed_vM_{QCD}/f_\pi^2)$. Just as with Eq. (2.71), the terms in Eq. (2.72) can be eliminated by the transformation (2.52) (pions) + (2.53) (nucleons), with axial parameter

$$\epsilon_{A,i} = \ldots + \frac{1}{2f_\pi m_\pi^2}\left(\bar{f}_2\bar{N}N\bar{N}\tau_3N + \bar{h}_2\varepsilon_{3ij}\bar{N}S_\mu\tau_iD_j^\mu N\right)\delta_{i3}. \tag{2.73}$$

As was the case with the transformation that eliminated Eq. (2.71), the above transformation brings about a number of new terms when it acts on T-conserving terms in the effective Lagrangian. These unwanted terms will have a chiral index of two relative to the original terms in the Lagrangian from which they originate. This leads to a controlled procedure for eliminating all effects of the interactions (2.72). This procedure can be continued for other interactions generated by Eq. (2.70).

At this point, we may conclude that the isovector piece of the quark EDM interaction (2.56) does not generate any new interactions at low energies. Since the object K_3^μ, through operators like Eq. (2.30), generates terms that directly cause vacuum instability, this must be the case. The term (2.71), while often quoted as giving a long-range contribution to the EDM of the neutron via pion loops, is inconsistent with the chiral symmetry properties of the $\bar{\theta}$ term, the quark EDM and quark chromo-EDM.

Finally, we examine the interactions generated by the $K^\mu i_3^\nu$ piece of Eq. (2.57), which transforms as P_4F_{34}. Since neither the vector P nor the tensor F_{ab} suffers from any vacuum stability problems, the product PF_{ab} does not either. Hence $K^\mu i_3^\nu$ generates real physical interactions. With no external photons involved, interactions will necessarily involve two or more nucleon bilinears, since neither the vector P nor the tensor F_{ab} can form hadronic operators that do not include at least one nucleon bilinear.

The leading terms that can be generated have no derivatives and are formed out of the product $P_1[0;0,N](F_1)_{ab}[0;0,N]$:

$$\mathcal{L}^{(0)}_{\not{T},P_1F_1} = \frac{\bar{j}_2}{D}\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\pi}N\bar{N}\tau_3 N + \frac{\bar{k}_2}{D}\pi_3(\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\pi}N)^2, \tag{2.74}$$

where $\bar{j}_2 = \mathcal{O}(ed_s M_{QCD}/f_\pi^3)$ and $\bar{k}_2 = \mathcal{O}(ed_s/M_{QCD}f_\pi^5)$.

Inclusion of one derivative can be accomplished using the operator products $P_2[0;\boldsymbol{D}_\mu,N](F_1)_{ab}[0;0,N]$ and $P_1[0;0,N](F_7)_{ab}[0;\boldsymbol{D}_\mu,N]$. Other products generate terms that are either identical to those generated by these operators or terms that can be derived from them. These operators yield a variety of interactions:

$$\begin{aligned}\mathcal{L}^{(1)}_{\not{T},P_2F_1,P_1F_7} &= \frac{\bar{c}_2^1}{D}\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\bar{N}S_\mu N\bar{N}\tau_3 N D^\mu\pi_3 + \frac{\bar{c}_2^2}{D^2}\pi_3\bar{N}S_\mu N\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\pi}ND^\mu\pi_3\\ &+ \frac{\bar{c}_2^3}{D^2}\pi_3\bar{N}S_\mu N\bar{N}\tau_3 N\boldsymbol{\pi}\cdot\boldsymbol{D}^\mu + \frac{\bar{c}_2^4}{D}\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\bar{N}S_\mu(\boldsymbol{\tau}\times\boldsymbol{D}^\mu)_3 N\bar{N}\tau_3 N\\ &+ \frac{\bar{c}_2^5}{D^2}\pi_3\bar{N}S_\mu\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\boldsymbol{D}^\mu)N\bar{N}\tau_3 N + \frac{\bar{c}_2^6}{D^2}\pi_3\bar{N}S_\mu(\boldsymbol{\tau}\times\boldsymbol{D}^\mu)_3 N\bar{N}\boldsymbol{\pi}\cdot\boldsymbol{\tau}N.\end{aligned} \tag{2.75}$$

Here $\bar{c}_2^i = \mathcal{O}(ed_s/f_\pi^3)$ for $i = 1,4$ and $\mathcal{O}(ed_s/f_\pi^5)$ for $i = 2,3,5,6$. Of course, this process can be continued to generate terms of higher index. As with the quark chromo-EDM, the quark EDM Lagrangian (2.56) can be written as a purely isoscalar interaction as far as low-energy physics is concerned:

$$\mathcal{L}_{qed} = \frac{i}{2}d_s\bar{q}\sigma_{\mu\nu}\gamma_5 q F^{\mu\nu}. \tag{2.76}$$

This term then generates Eqs. (2.50) and (2.51) (with modified coefficients), (2.74) and (2.75) to the first two orders.

We now turn to interactions that include external photons. These interactions directly stem from the quark EDMs. It was shown in Sections 2.2 and 2.4.1 how short-range contributions to the EDM of the nucleon could be obtained from the $\bar{\theta}$ term and the quark color EDMs. Not surprisingly, the quark EDMs can also generate a nucleon EDM. The resulting contributions, however, will differ somewhat from those stemming from the sources of T violation already considered. A contribution to the isoscalar nucleon EDM can be generated directly by the isoscalar quark EDM (the nucleon EDM involves a factor of e, and in any model where the photon is a $U(1)$ gauge boson, the quark EDM should carry a factor of e as well). Since interactions stemming from the isovector quark EDM can be eliminated in the case where there are no external photons, we can safely assume that this can also be done when there are external photons. An isovector nucleon EDM can then be generated by the isoscalar quark EDM with an insertion of the quark mass difference operator in (1.10) . The operators $P_7[0; 0, N]$ from Eq. (2.25) and

$$\varepsilon^{\rho\sigma\alpha\beta} F_{\rho\sigma} \big(\varepsilon_{\mu\nu\alpha\beta} \bar{N} \boldsymbol{\tau} v^{\mu} S^{\nu} N, 0\big) \tag{2.77}$$

generate the following nucleon electric dipole interactions:

$$\mathcal{L}^{(2)}_{edm,qed} = \frac{\tilde{d}_0}{D} \left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right) \bar{N} \big(v_\mu S_\nu - S_\mu v_\nu\big) N F^{\mu\nu}, \tag{2.78}$$

where $\tilde{d}_0 = \mathcal{O}(d_s)$ is the leading short-range contribution to the isoscalar nucleon EDM stemming from the light quark EDMs, and

$$\mathcal{L}^{(4)}_{edm,qed} = \frac{\tilde{d}_1}{D} \left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right) \bar{N} \left[\tau_3 - \frac{1}{2f_\pi^2 D} \pi_3 \boldsymbol{\tau} \cdot \boldsymbol{\pi}\right] (v_\mu S_\nu - S_\mu v_\nu) N F^{\mu\nu}, \tag{2.79}$$

where $\tilde{d}_0 = \mathcal{O}(\varepsilon d_s m_\pi^2 / M_{QCD}^2)$ is the leading short-range contribution to the isovector nucleon EDM stemming from the light quark EDMs. Thus if the nucleon EDM is dominated by the light quark EDMs, the short-range isoscalar component is expected to be significantly larger than the isovector component. This is in distinction to the $\bar{\theta}$ term and quark color EDM. More will be said about this in Section 2.5.

2.4.3 The Weinberg three-gluon operator

The Weinberg three-gluon operator is given by (Weinberg, 1989)

$$\mathcal{L}_{3gluon} = \frac{1}{3} w f^{abc} G^a_{\mu\rho} G^{\rho,b}_{\nu} G^c_{\lambda\sigma} \varepsilon^{\mu\nu\lambda\sigma}. \tag{2.80}$$

The coefficient w was estimated in Ref. (Weinberg, 1989) and improved upon in Ref. (Dicus, 1990), employing the Weinberg (two-Higgs) model of CP violation, by computing the diagrams that give the dominant contributions to the operator (2.80). These diagrams involve the three gluons being attached to a heavy quark loop with a neutral Higgs boson exchanged between the quarks in the loop. Assuming that the top quark mass squared is much larger than the Higgs boson mass squared, Weinberg obtains the upper bound $w \lesssim 10^{-16}$ MeV^{-2}.

The Weinberg operator is unlike any of the sources of T violation considered thus far in that it is purely gluonic with no vacuum peculiarities that relate it to quark densities. Also, unlike the $\bar{\theta}$ term, there is no extra symmetry (such as a Peccei-Quinn symmetry) that can be used to rotate away the Weinberg operator. This operator is a chiral scalar, so any hadronic interactions generated by it must be constructed solely from chiral-invariant quantities, such as $\bar{N}N$, $\boldsymbol{D}_\mu$ and $\mathcal{D}_\mu$ (see Section 1.2.1).

There are many T-violating chiral invariant operators that can be formed. Due to the requirements of hermiticity and Lorentz invariance, there will be at least one derivative in each term. Of great importance at the one derivative level are operators that generate the nucleon EDM. In order to give the nucleon an EDM, the Weinberg operator must be inserted in diagrams with an external photon. Thus the quark electromagnetic coupling term (1.21) will be involved. The operator

$$\varepsilon_{\mu\nu\alpha\beta} \varepsilon^{\rho\sigma\alpha\beta} \bar{N} v^\mu S^\nu N F_{\rho\sigma}, \tag{2.81}$$

which can be viewed as the product of a T-violating chiral scalar $\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{\rho\sigma\alpha\beta}\bar{N}v^{\mu}S^{\nu}NF_{\rho\sigma}$ times 1, generates an isoscalar nucleon EDM. Meanwhile, the operator

$$\varepsilon^{\rho\sigma\alpha\beta}F_{\rho\sigma}\begin{pmatrix} 0 & \varepsilon_{\mu\nu\alpha\beta}\bar{N}\tau_i v^{\mu}S^{\nu}N \\ -\varepsilon_{\mu\nu\alpha\beta}\bar{N}\tau_i v^{\mu}S^{\nu}N & 0 \end{pmatrix} \tag{2.82}$$

can be viewed as a T-violating chiral scalar times a rank-two antisymmetric $SO(4)$ tensor, and generates the isovector nucleon EDM. Applying the rotation (A.3) to Eq. (2.82), the operators (2.81) and (2.82) generate

$$\mathcal{L}^{(2)}_{edm,Wein} = \bar{N}\left\{\tilde{d}_0 + \frac{\tilde{d}_1}{D}\left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\left(\tau_3 - \frac{1}{2f_\pi^2 D}\pi_3\boldsymbol{\tau}\cdot\boldsymbol{\pi}\right)\right\}(v_\mu S_\nu - S_\mu v_\nu)NF^{\mu\nu}, \tag{2.83}$$

where $\tilde{d}_0$ ($\tilde{d}_1$) is the leading short-range contribution to the isoscalar (isovector) nucleon EDM stemming from the Weinberg operator (2.80), with $\tilde{d}_i = \mathcal{O}(ewM_{QCD})$.

Notice that the short-range EDM contributions $\tilde{d}_0$ and $\tilde{d}_1$ are of the same order. The leading order Weinberg operator contributions to the nucleon EDM are thus similar to those generated by the $\bar{\theta}$ term and quark color EDM in the sense that there is no expected separation of the sizes of the isoscalar and isovector nucleon EDM contributions. At subleading order, however, the Weinberg operator can generate isoscalar and isovector contributions of differing sizes. This is because the quark mass terms (1.10) can additionally contribute with the Weinberg operator. Specifically, the operator

$$\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{\rho\sigma\alpha\beta}\bar{N}v^{\mu}S^{\nu}NF_{\rho\sigma}S_1[0;0,0], \tag{2.84}$$

with $S_1[0;0,0]$ given by Eq. (2.30) generates an isoscalar contribution, while

$$\varepsilon^{\rho\sigma\alpha\beta} F_{\rho\sigma}\big(\varepsilon_{\mu\nu\alpha\beta}\bar{N}\boldsymbol{\tau} v^{\mu} S^{\nu} N, 0\big), \tag{2.85}$$

generates an isovector contribution. Performing rotations with Eq. (A.3) on both Eqs. (2.84) and (2.85), we obtain the following terms:

$$\mathcal{L}^{(4)}_{edm,Wein} = \bar{N}\left\{\frac{\tilde{d}'_0}{D}\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)+\tilde{d}'_1\left(\tau_3-\frac{1}{2f_\pi^2 D}\right)\right\}\big(v_\mu S_\nu - S_\mu v_\nu\big) N F^{\mu\nu}, \tag{2.86}$$

where $\tilde{d}'_0$ ($\tilde{d}'_1$) is a sub-leading short-range contribution to the isoscalar (isovector) nucleon EDM stemming from the Weinberg operator, quark-photon coupling and quark mass terms. Now the isoscalar and isovector contributions differ in order of magnitude, with $\tilde{d}'_0 = \mathcal{O}(ewm_\pi^2/M_{QCD})$ and $\tilde{d}'_1 = \mathcal{O}(e\varepsilon wm_\pi^2/M_{QCD})$. While this separation in orders of magnitude is only sub-leading, it could nonetheless be detectable once the neutron and proton EDMs are measured precisely by experiments.

Also at the one-derivative level, one may form operators which give pion-nucleon interactions. For example, the interaction

$$\mathcal{L}^{(2)}_{Wein,\pi N} = \bar{w}_0 \bar{N} i v_\mu \boldsymbol{\tau}\cdot \boldsymbol{D}^\mu N, \tag{2.87}$$

where $\bar{w}_0 = \mathcal{O}(wM_{QCD})$, is T-violating and is distinct from the lowest-index pion-nucleon interactions generated by the lower-dimension sources, i.e. $\mathcal{L}^{(1)}_{\not{T},P_1}$ in Eq. (2.11) and Eq. (2.50). Thus if one were to perform, for example, πN scattering, looking specifically for that part of the pion-nucleon interaction which violates T, then the signature which indicates whether the source of T violation is the Weinberg operator or one of the lower-dimension operators is the momentum dependence of the interaction. The $\bar{\theta}$ term, quark color EDM and quark EDM all give a pion-nucleon couping which is momentum-independent, while the Weinberg operator gives a coupling that becomes stronger as the pion becomes more energetic.

2.5 Conclusion

The preceding sections have accomplished several goals. A method of constructing the T-violating effective chiral Lagrangian for a given quark-level source has been demonstrated. By constructing all possible operators, order by order, that have the desired symmetry properties, it is assured that the set of resulting effective interactions gives a complete description of the phenomena stemming from the underlying theory.

Apart from having the necessary symmetry properties of the underlying theory, the terms in the effective theory must also not make the vacuum unstable. There are two approaches to ensuring this, each approach having its own merits. Enforcing vacuum stability at quark level, via Dashen's theorem, allows one to avoid any spurious effective hadronic interactions. Enforcing vacuum stability at the level of hadrons, via redefinition of the hadronic fields, is readily applicable to any quark-level source that generates hadronic interactions. Also, the field redefinition method allows one to study T violation on the hadronic level without ever having to know anything about the complexities of the QCD vacuum.

The violation of isospin and time reversal symmetries by the light quark mass difference and the QCD $\bar{\theta}$ term, respectively, are closely connected in the effective theory. However, the unnaturally small $\bar{\theta}$ effectively uncouples the two classes of symmetry breaking. This connection of symmetries is nonexistent for other sources of T violation.

The chiral symmetry properties of the $\bar{\theta}$ term and the quark color EDM are identical, hence they give rise to the same interactions in the effective theory. To gain some understanding of the difference in strengths of interactions generated by these sources, consider the isospin zero T-violating pion nucleon coupling $\bar{g}_0$ (c.f. (2.11) and (2.50)). As estimated in Section 2.2, $\bar{g}_0 \approx 1.7 \times 10^{-2}\bar{\theta}$ for a $\bar{\theta}$ term source. Taking 10^{-10} as an order-of-magnitude estimate for $\bar{\theta}$ based on the current bound on the neutron EDM (1.50), this gives $\bar{g}_0 = \mathcal{O}(10^{-12})$. An estimate of the light quark chromoelectric dipole moments can be obtained from Ref. (Khriplovich and Zyablyuk, 1996), which includes a calculation of the down quark color EDM within the supersymmetric $SO(10)$ model, from which we estimate $\breve{d}_q = \mathcal{O}(10^{-25}\text{ cm})$. Given that $\bar{g}_0 = \mathcal{O}(\breve{d}_s M_{QCD}^2/f_\pi)$ for a quark chromo-EDM source, we estimate that $\bar{g}_0 = \mathcal{O}(10^{-10})$. It seems, then, that there is a reasonable distinction between the strengths of interactions generated by the $\bar{\theta}$ term and quark color EDM. However, without an accurate value for $\bar{\theta}$, we cannot reasonably tell which source provides a more significant contribution to hadronic T violation.

The quark EDM generates a number of interactions that differ from those generated by the QCD $\bar{\theta}$ term or the quark color EDM. In particular, the presence of an explicit photon in the quark EDM interaction (2.76) generates both differing nucleon EDM contributions and differing hard-photon interactions. Nevertheless, the quark EDM also gives a set of interactions which are indistinguishable in form from those arising from both the $\bar{\theta}$ term and the color EDM. To have some quantitative measure of the comparative strengths of the light quark EDMs and the quark color EDMs, we can take a result for the light quark EDMs computed in the SM from three-loop diagrams (which are necessary for creating a SM quark EDM) involving W-boson exchange and T violation from the CKM phase (Czarnecki and Krause, 1997). The authors arrive at quark EDM values of the order $d_{u,d} = \mathcal{O}(10^{-34}e\text{ cm})$, which gives a pion-nucleon coupling $\bar{g}_0$ value of order 10^{-19}. This is far smaller than what we would expect from a $\bar{\theta}$ term or quark color EDM source. Thus, assuming a SM quark EDM, it is very unlikely that any light quark EDM contributions to processes involving a $\bar{g}_0$ pion-nucleon vertex will be measured.

Distinguishing between the $\bar{\theta}$ term, quark EDM and quark color EDM in experiments is not easy. In fact, no low-energy experiments can distinguish between effects generated by the $\bar{\theta}$ term and the quark color EDM. The quark EDM and the Weinberg three-gluon operator, on the other hand, do give interactions which could be distinguished by experiment. Of chief importance among these interactions is the nucleon EDM. If the nucleon EDM is dominated by the EDMs of light quarks, then the isoscalar short-range nucleon EDM should be roughly two orders of magnitude larger than the isovector short-range nucleon EDM. Similarly, if the Weinberg operator were the dominant source of the nucleon EDM, there would be a small (sub-leading) order-of-magnitude difference in the sizes of the isoscalar and isovector short-range EDMs. Because the deuteron EDM is given by the isocalar nucleon EDM in the impulse approximation, discrepancies between the sizes of the isoscalar and isovector nucleon EDMs could then be searched for by measuring the neutron EDM (given by $d_0 - d_1$) and the deuteron EDM. Of course, there are both short-range and long-range contributions to EDMs that need to be disentangled, but the long-range contributions are calculable in principle, since they arise chiefly from clouds of mesons.

CHAPTER 3

THE NUCLEON ELECTRIC DIPOLE FORM FACTOR

3.1 Introduction

A natural mechanism for CP violation is provided by the complex phase in the CKM matrix, and this phase provides a consistent explanation for the observed CP violation in a certain class of hadron decays (involving neutral kaons (Christensen et al., 1964) and B mesons (Aubert et al., 2001; Abe et al., 2001)). However, because they involve flavor-diagonal CP violation, electric dipole moments (EDMs) are relatively insensitive to the CKM phase. Indeed, estimates for the CKM contribution to the neutron EDM, d_n, range from 10^{-31} to 10^{-33} e cm (Shabalin, 1983), which is significantly smaller than the current value (Baker et al., 2006)

$$d_n = (0.6 \pm 2.3) \times 10^{-26} e \text{ cm} \tag{3.1}$$

EDMs thus provide a window on sources of CP violation beyond the CKM phase.

Precision experiments to measure the neutron EDM using ultracold neutrons, which are in various stages of preparation at a number of laboratories (LANL, PSI, SNS, ILL, München), promise to provide even higher precision for the neutron EDM than the current value (3.1). A less strict bound on the proton EDM, $|d_p| < 3.8 \times 10^{-24}$ e cm (Dmitriev and Sen'kov, 2003), can be extracted from a calculation of the contribution of the nuclear Schiff moment to the EDM of the ^{199} Hg atom (Romalis et al., 2001). In addition, there are plans to use a storage ring to probe the deuteron EDM at the level of $10^{-27}e$ cm (Semertzidis et al., 2004). While insensitive to weak CP violation, hadronic and nuclear EDMs are sensitive to T violation in strong interactions, in particular the QCD $\bar{\theta}$ angle. (For a review of both experimental and theoretical results, see, for example, Refs. (Khriplovich and Lamoreaux, 1997; Pospelov and Ritz, 2005)).

While hadronic and atomic EDMs can provide insight regarding sources of T violation, the neutron is the most advantageous nuclear system to probe. There are no additional atomic or nuclear effects to consider. Additionally, experimental results for the neutron EDM can be compared with lattice calculations. While neutron experiments yield bounds on the neutron EDM directly, the associated nucleon electric dipole form factor (EDFF) has become an object of recent interest (Hockings and van Kolck, 2005; Dib et al., 2006).

Needless to say, the momentum dependence of the EDFF is not easily accessible experimentally. However, the EDFF can make contact with both experiment and lattice calculations. For atoms or molecules with no unpaired electrons and a nuclear spin of 1/2, the nuclear Schiff moment gives the most important contribution to the EDM (Thomas, 1995). The leading electromagnetic contribution to the Schiff moment of the nucleon is determined by the radius of the nucleon EDFF. In lattice QCD, the nucleon EDM can be obtained (Shintani et al., 2005a,b; Berruto et al., 2005) from the three-point function involving two nucleon fields and one photon field, but it requires (Wilcox, 2002) a calculation of the EDFF followed by its extrapolation to zero momentum (in addition to the required extrapolations in quark masses and volume (O'Connell and Savage, 2005)).

An experimental determination of the nucleon EDFF would contribute to a better understanding of T violation in nuclear systems. First, it would allow an independent determination of the T-violating pion-nucleon coupling. Second, it could reveal information about the deuteron EDM. The deuteron EDM receives contributions from nucleon EDMs as well as pion-exchange currents (Khriplovich and Korkin, 2000; Lebedev et al., 2004) and the T-violating nucleon-nucleon interaction. In particular, the proposed deuteron experiment mentioned above will probe the isoscalar combination of neutron and proton EDMs. Barring cancellations, the isoscalar average of the largest non-analytic contribution to the nucleon EDM provides a lower-bound estimate for the deuteron EDM.

As will be shown below, the pion cloud contributions to the nucleon EDFF for a $\bar{\theta}$ term source are purely isovector to leading order (LO), but do generate an isoscalar contribution at next-to-leading order (NLO). With the bevy of experiments and lattice efforts that are aimed at gaining a greater understanding of T violation from strong interactions, a thorough understanding of the nucleon EDFF is crucial.

3.2 The nucleon EDFF to leading order

The nucleon EDFF is one of four electromagnetic form factors that comprise the nucleon current. The other nucleon form factors have all been calculated previously in χPT at the first few orders: electric and magnetic (Bernard et al., 1992, 1998a), and anapole (Maekawa and van Kolck, 2000; Maekawa et al., 2000). The full nucleon EDFF has been calculated in leading order (LO) in χPT (Hockings and van Kolck, 2005).

At $Q \sim m_\pi$, the nucleon is non-relativistic to a good approximation, since $m_N \sim M_{QCD}$. Pions must be accounted for explicitly in the theory, since they are the light pseudo-Goldstone bosons corresponding to the spontaneous breaking of $SU(2)_l \times SU(2)_R$ down to $SU(2)_{L+R}$. The delta isobar should be included as well, since the mass difference $m_\Delta - m_N \sim 2m_\pi$. The following T-conserving terms, all of which either obey chiral symmetry or break it in the same way as the quark mass terms (1.10) do, will be needed:

$$\mathcal{L}^{(0)}_{str/em} = \frac{1}{2}\boldsymbol{D}_\mu \cdot \boldsymbol{D}^\mu - \frac{1}{2D}m_\pi^2 \boldsymbol{\pi}^2 + \bar{N}iv \cdot \mathcal{D}N - \frac{g_A}{f_\pi}\bar{N}\big(S_\mu \boldsymbol{\tau} \cdot \boldsymbol{D}\big)N + \dots \qquad (3.2)$$

Here $\boldsymbol{\pi}$ denotes the pion field in a stereographic projection of $SO(4)/SO(3)$, with

$D = 1 + \boldsymbol{\pi}^2/4f_\pi^2$ and $f_\pi = 93$ MeV the pion decay constant, $N = (p\ n)^T$ is a heavy-nucleon field of velocity v^μ and spin S^μ ($S^\mu = (0, \vec{\sigma}/2)$ in the nucleon rest frame where $v^\mu = (1, \vec{0})$), $\boldsymbol{D}_\mu$ and $\mathcal{D}_\mu$ are the (both gauge and chiral) covariant derivatives for the pion and nucleon, respectively, with

$$(\boldsymbol{D}_\mu)_{ab} = D^{-1}(\delta_{ab}\partial_\mu + ie\epsilon_{3ab}A_\mu) \tag{3.3}$$

and

$$\mathcal{D}_\mu = \partial_\mu + \frac{i}{4f_\pi^2}\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\boldsymbol{D}_\mu) - \frac{ie}{2}A_\mu(1+\tau_3), \tag{3.4}$$

and "…" stands for other interactions with more pions, nucleons and/or delta fields that are not explicitly needed in the following. Note that the pion mass term comes from the explicit breaking of chiral symmetry by the average quark mass $\hat{m} = (m_u+m_d)/2$, so $m_\pi^2 = \mathcal{O}(\hat{m}M_{QCD})$. Isospin breaking from the quark mass difference, which is proportional to $\hat{m}\varepsilon = \mathcal{O}(\varepsilon m_\pi^2/M_{QCD})$ with $\varepsilon = (m_d - m_u)/(m_u+m_d) \simeq 1/3$, does not appear at this order. Note also that at this order the nucleon is static and couples only to longitudinal photons. Kinetic corrections and magnetic couplings have relative size $\mathcal{O}(Q/M_{QCD})$ and appear in $\mathcal{L}^{(1)}_{str/em}$. The same is true for the delta isobar, including the nucleon-delta transition through coupling to a transverse photon. The pion-nucleon coupling in Eq. (3.2) is not determined from symmetry, but is expected to be $\mathcal{O}(1)$, and indeed $g_A = 1.267$. The Goldberger-Treiman relation $g_A m_N = f_\pi g_{\pi NN}$ holds in lowest order. A term in $\mathcal{L}^{(2)}_{str/em}$ provides an $\mathcal{O}((m_\pi/M_{QCD})^2)$ correction that removes the so-called Goldberger-Treiman discrepancy (Goldberger and Treiman, 1958a; Nambu, 1960).

In addition to T-conserving terms, P- and T-violating interactions are needed. The exact form of these interactions depends on the mechanism for CP violation. Just above M_{QCD}, T-violating interactions involving quarks and gluons can be classified via their scale dimensions, starting with the $\bar{\theta}$ term. In a basis where the quark fields $q = (u \quad d)^T$ have been appropriately rotated (Baluni, 1979),

$$\mathcal{L}_{\not{T}}^{QCD} = m_* \bar{\theta} \bar{q} i \gamma_5 q + \ldots, \tag{3.5}$$

where

$$m_* = \frac{m_u m_d}{m_u + m_d} = \frac{1}{2}\hat{m}(1 - \varepsilon^2) \approx \frac{\hat{m}}{2}. \tag{3.6}$$

Here "..." represents higher-dimension operators–such as the quark EDM and color EDM, the Weinberg three-gluon operator, and four-quark interactions (see Section (1.3))– which will be neglected for this calculation.

The $\bar{\theta}$ term is the fourth component of the $SO(4)$ vector P (1.13), so it will generate interactions in the low-energy EFT that transform as T-violating, fourth components of $SO(4)$ vectors made out of hadronic fields. The lowest-dimensional operator of this type is (c.f. Eq. (2.11))

$$\mathcal{L}_{\not{T}}^{(1)} = -\frac{\bar{g}_0}{D} \bar{N} \boldsymbol{\tau} \cdot \boldsymbol{\pi} N, \tag{3.7}$$

where $\bar{g}_0$ is an $I = 0$ T-violating pion-nucleon coupling. From dimensional analysis,

$$\bar{g}_0 = \mathcal{O}(m_* \bar{\theta}/f_\pi) = \mathcal{O}(m_\pi^2 \bar{\theta}/f_\pi M_{QCD}).$$

Because of the implicit factor of m_π^2 in $\bar{g}_0$, Eq. (3.7) has an effective chiral index $\Delta = 1$.

Interactions of higher index can be similarly constructed (see Section 2.2). T-violating nucleon-delta transitions through pion emission, for example, involve at least one derivative, and are thus suppressed by at least $\mathcal{O}(Q/M_{QCD})$ relative to interactions stemming from Eq. (3.7). Among the higher-order operators, particularly relevant here are short-range contributions to the nucleon EDM (see Eq. (2.27)),

$$\mathcal{L}_{\not{T}}^{(3)} = \bar{N}(\tilde{d}_0 + \tilde{d}_1\tau_3)(S_\mu v_\nu - S_\nu v_\mu)NF^{\mu\nu} + \ldots, \tag{3.8}$$

where $\tilde{d}_0$ ($\tilde{d}_1$) is a short-range contribution to the isoscalar (isovector) EDM of the nucleon. From dimensional analysis,

$$\tilde{d}_i = \mathcal{O}(em_*\bar{\theta}/M_{QCD}^2) = \mathcal{O}(em_\pi^2\bar{\theta}/M_{QCD}^3).$$

Direct short-range contributions to the momentum-dependence of the EDFF first appear in $\mathcal{L}_{\not{T}}^{(5)}$, being further suppressed by $\mathcal{O}((Q/M_{QCD})^2)$.

Denoting by iJ_{ed}^μ the T-violating nucleon current that interacts with the electron current $-ie\bar{e}\gamma^\mu e$ via the photon propagator

$$iD_{\mu\nu} = i\left(\frac{\eta_{\mu\nu}}{q^2} + \ldots\right),$$

a contribution

$$iT = -ie\bar{e}(k')\gamma^\mu e(k)D_{\mu\nu}(q)\bar{N}(p')J_{ed}^\nu(q)N(p), \tag{3.9}$$

to the electron-nucleon S matrix is produced. With $q^2 = (p-p')^2 \equiv Q^2 < 0$,

$$J_{ed}^\mu(q) = 2\left[F_D^{(0)}(-q^2) + F_D^{(1)}(-q^2)\tau_3\right](v\cdot qS^\mu - S\cdot qv^\mu), \tag{3.10}$$

where $F_D^{(0)}(Q^2)$ ($F_D^{(1)}(Q^2)$) is the isoscalar (isovector) EDFF of the nucleon, with $F_D^{(i)}(0) = d_i$ the corresponding EDM.

Considering the sizes of specific contributions to the nucleon EDFF, the first tree-level contribution comes from the vertex generated by the Lagrangian (3.8). The lowest-order one-loop graphs are built out of one vertex from the T-violating Lagrangian (3.7) and all other vertices from the T-conserving Lagrangian (3.2). The one-loop diagrams that *a priori* could contribute to the EDFF are shown in Fig. 3.1.

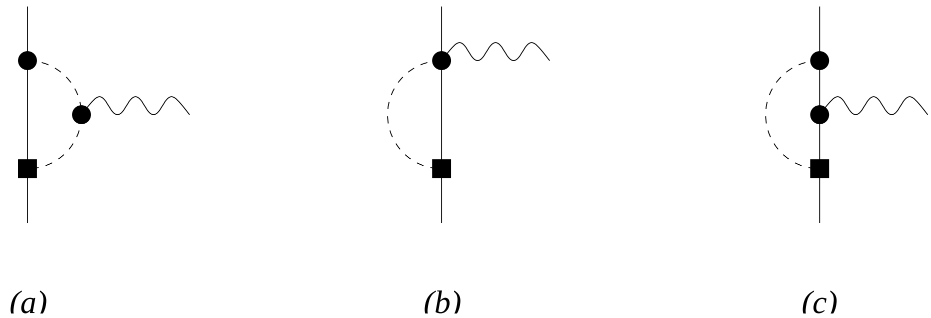

Figure 3.1: One-loop diagrams contributing to the nucleon electric dipole form factor in leading order. Solid, dashed and wavy lines represent nucleon, pion and photon, respectively; circles and squares stand for interactions from $\mathcal{L}^{(0)}_{str/em}$ and $\mathcal{L}^{(1)}_{\not T}$, respectively. For simplicity only one of two possible orderings is shown here.

These one-loop diagrams contribute $\mathcal{O}(e\bar{g}_0 f_\pi Q/(4\pi f_\pi)^2)$ to $J^\mu_{ed}(q)$, thus they give a contribution to the EDFF that is of the same order as the short-range contribution to the EDM from (3.8), which is $\mathcal{O}(em_*\bar{\theta}/M^2_{QCD})$.

The contribution from Fig. 3.1(c) vanishes due to its isospin structure. Due to the static nature of the nucleon in leading order, diagrams in Fig. 3.1 are evaluated at $v \cdot q = 0$. Consequently, diagram 3.1(b) also vanishes. Diagram 3.1(a) gives a non-zero contribution only to the isovector component of the nucleon EDFF. This contribution contains divergent pieces that can be absorbed by redefining the tree-level contribution to the EDFF. Including both the tree-level and one-loop contributions, the isoscalar and isovector EDMs are given, respectively, by

$$d_0 = \tilde{d}_0 \tag{3.11}$$

$$d_1 = \tilde{d}_1 + \frac{eg_A\bar{g}_0}{8\pi^2 f_\pi}\left[\bar{\Delta} + 2\ln\frac{\mu}{m_\pi}\right], \tag{3.12}$$

where μ is the renormalization scale introduced by dimensional regularization, and $\bar{\Delta} \equiv 2/\epsilon - \gamma_E + \ln 4\pi$ ($\epsilon = 4 - d$, with d the spacetime dimension). The piece in (3.12) that is nonanalytic in m^2_π is in agreement with the result of Crewther *et al*

(Crewther et al., 1979) when μ is set to m_N and the Goldberger-Treiman relation $g_{\pi NN} f_\pi = g_A m_N$ (Goldberger and Treiman, 1958b) is assumed. This result has been rederived a number of times since (Cheng, 1991; Pich and de Rafael, 1991; Cho, 1993; Borasoy, 2000). The short- and long-range contributions are in general of the same size, but they are not likely to cancel due to the $\ln m_\pi$ dependence of the long-range pion contribution. Thus the leading non-analytic contribution serves as an estimate of the EDM. Numerically, we find

$$|d_n| \gtrsim \frac{e g_A \bar{g}_0}{4\pi^2 f_\pi} \ln \frac{m_N}{m_\pi} \approx 3.6 \times 10^{-16} \bar{\theta} e \text{ cm} \tag{3.13}$$

using Eq. (3.12). Together with the current experimental value for the neutron EDM (3.1), this implies that $\bar{\theta} \lesssim 10^{-10}$. Additionally, a measurement of the neutron EDM $d_n = d_0 - d_1$ alone is insufficient to separate the short- and long-range physics.

Because the EDFF is given by lowest-order loop graphs, it depends on the combination Q^2/m_π^2 only. Because only Fig. 3.1(a) gives a non-vanishing contribution to this order, the dependence is actually on $Q^2/(2m_\pi)^2$. The EDFF defined in Eq. (3.10) is found to be

$$F_D^{(0)}(Q^2) = d_0, \tag{3.14}$$

$$F_D^{(1)}(Q^2) = d_1 - \frac{e g_A \bar{g}_0}{12\pi^2 f_\pi} F\left(\frac{Q^2}{(2m_\pi)^2}\right), \tag{3.15}$$

where

$$F(x) = 3\left\{\frac{1}{2}\sqrt{1+\frac{1}{x}} \ln\left(\frac{\sqrt{1+1/x}+1}{\sqrt{1+1/x}-1}\right) - 1\right\}. \tag{3.16}$$

One can check that $F(0) = 0$, so indeed Eq. (3.15) does meet the standard of being a form factor. The function F is a testable prediction of χPT, and is plotted as a function of Q^2 in Fig. 3.2.

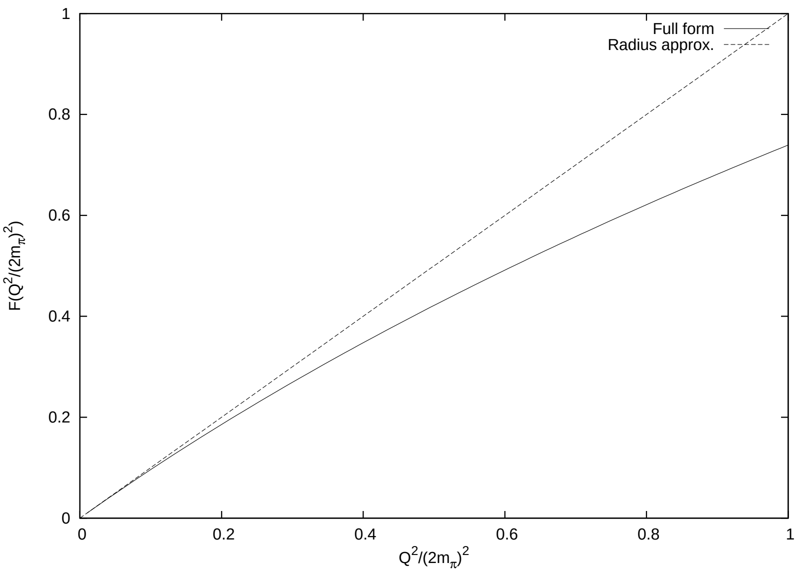

Figure 3.2: The function $F(Q^2/(2m_\pi)^2)$ that enters the isovector electric dipole form factor $F_D^{(1)}$: χPT in leading order, Eq. (3.16) (solid line); and quadratic approximation, Eq. (3.17) (dotted line).

For $x \ll 1$, $F(x)$ can be expanded in powers of x,

$$F(x) = x + \mathcal{O}(x^2) \tag{3.17}$$

This approximation is compared to the full functional form (3.16) in Fig. 3.2. For very small momenta, the variation of the form factor with Q can be characterized by the isovector electromagnetic Schiff moment S_1' or equivalently the EDM square radius (defined in analogy to the charge square radius) (Thomas, 1995), with the leading contribution given by

$$S_1' = -\frac{1}{6} d_1 \langle r_{ed}^2 \rangle^{(1)} = \left(\frac{dF_D^{(1)}}{dQ^2} \right)_{Q^2=0} = -\frac{e g_A \bar{g}_0}{48\pi^2 f_\pi} \frac{1}{m_\pi^2} \tag{3.18}$$

Whereas the EDM vanishes in the chiral limit, the electromagnetic Schiff moment stemming from the EDFF is finite (due to the implicit scaling of $\bar{g}_0$ with m_π^2).

Since it is generated by the pion cloud, the mass scale governing the variation of $F(Q^2/(2m_\pi)^2)$ is in fact $2m_\pi$. The relative importance of the Q^2 variation of $F_D^{(1)}(Q^2)$ depends additionally on the ratio $eg_A\bar{g}_0/12\pi^2 f_\pi d_1$. This ratio is a constant in the chiral limit. Looking at Eq. (3.15), the ratio is expected to be $\mathcal{O}(1)$, and thus it is likely that m_π sets the scale of the variation of $F_D^{(1)}(Q^2)$ as well–an example of a low-energy theorem. Unlike the other nucleon electromagnetic form factors, a dipole approximation with a mass scale close to the rho meson mass, m_ρ, should not be good even as a numerical approximation for the EDFF. Under the assumption that higher-order results are not afflicted by anomalously large dimensionless factors, the error of the above results at momentum Q should be $\sim Q/m_\rho$.

3.3 The nucleon EDFF to sub-leading order

In going from leading order to next-to-leading order (NLO), new T-conserving and T-violating interactions are needed. The additional T-conserving terms that are needed are nucleon recoil corrections to the static limit and photon coupling to the nucleon magnetic moment:

$$\mathcal{L}^{(1)}_{str/em} = \frac{1}{2m_N}\Big\{\bar{N}((v\cdot\mathcal{D})^2 - \mathcal{D}^2)N + \frac{g_A}{f_\pi}\left[i\bar{N}(v_\mu \boldsymbol{\tau}\cdot\boldsymbol{D}^\mu)S\cdot\mathcal{D}N + \text{H.c.}\right] \\ + \frac{e}{2}\varepsilon_{\mu\nu\rho\sigma}\bar{N}\left(1+\kappa_0+(1+\kappa_1)\tau_3\right)v^\mu S^\nu N F^{\rho\sigma} + \dots\Big\} + \dots. \quad (3.19)$$

Here κ_0 and κ_1 are the isoscalar and isovector anomalous magnetic photon-nucleon couplings, respectively. These couplings are not determined from symmetry, but are expected to be $\mathcal{O}(1)$, and indeed $\kappa_0 = -0.12$ and $\kappa_1 = 3.7$. Isospin violation stemming from the down-up quark mass difference, $m_d - m_u = 2\varepsilon\hat{m}$, is neglected here. To the order at which we consider here, isospin violation contributes only through the nucleon mass difference, which is small due to a partial cancellation with electromagnetic effects.

There are two additional T-violating terms needed, with the first being a one-derivative term that, like Eq. (3.7), transforms as the fourth component of Eq. (1.13) (c.f. Eq. (2.14)):

$$\mathcal{L}_{\not{T}}^{(2)} = \frac{2\bar{h}_0}{D}\boldsymbol{\pi}\cdot D_\mu \boldsymbol{\pi} \bar{N} S^\mu N. \tag{3.20}$$

Here $\bar{h}_0$ is an undetermined coefficient of order $\mathcal{O}(m_* \bar{\theta}/f_\pi^2 M_{QCD}) = \mathcal{O}(m_\pi^2 \bar{\theta}/f_\pi^2 M_{QCD})$. As with Eq. (3.7), the contributions of Eq. (3.20) to the nucleon EDFF are calculable, as they are given by long-range physics associated with the pion cloud. While the strong- interaction corrections (3.19) depend on known parameters, Eq. (3.20) introduces a new T-violating parameter.

The other T-violating term that gives contributions at this order is

$$\mathcal{L}_{\not{T}}^{(4)} = \frac{1}{2m_N}\left\{ i\bar{N}(\tilde{d}_0 + \tilde{d}_1\tau_3)\left[S_\mu D_\nu - S_\nu D_\mu\right]N + \text{H.c.}\right\} F^{\mu\nu}, \tag{3.21}$$

which, like Eq. (3.8), gives short-range contributions to the nucleon EDM. Direct short-range contributions to the momentum dependence of the EDFF first appear in $\mathcal{L}_{\not{T}}^{(5)}$, being suppressed by $\mathcal{O}(Q/M_{QCD})$ compared to Eq. (3.21).

The nucleon electric dipole current in Eq. (3.9), given in leading order by Eq. (3.10), lends itself to an expansion in powers of Q/m_N that reads

$$\begin{aligned} J^\mu_{ed}(q,k) = 2\left(F_D^{(0)}(-q^2) + F_D^{(1)}(-q^2)\tau_3\right) \\ \times\left[S^\mu v\cdot q - S\cdot q v^\mu + \frac{1}{m_N}\left(S^\mu q\cdot k - S\cdot q k^\mu\right) + \ldots\right], \end{aligned} \tag{3.22}$$

where $q = p - p'$ is the momentum transfer, and $k = (p+p')/2$. As in Section 3.2, $F_D^{(0)}(Q^2)$ and $F_D^{(1)}(Q^2)$ are the isoscalar and isovector nucleon EDFFs, respectively, with $F_D^{(i)}(0) = d_i$ being the corresponding EDM. The form factors themselves can be expanded in powers of Q/M_{QCD}.

The LO contributions to the current, which were calculated in Section 3.2, are of order $\mathcal{O}(e\bar{g}_0 Q/(4\pi)^2 f_\pi)$. Here we focus on those contributions that are NLO, that is, terms of relative order $\mathcal{O}(Q/M_{QCD})$. The one-loop diagrams contributing to the nucleon EDFF in NLO are shown in Figs. 3.3 and 3.4. They are classified according to the combination of couplings that appear.

The NLO diagrams of Fig. 3.3 are built from the leading interactions in Eqs. (3.2) and (3.7), plus one insertion of an operator from Eq. (3.19). This insertion can be (i) a kinetic correction to the LO diagrams in Fig. 3.1, in which case the correction is to either the nucleon propagator or the external energy; (ii) a recoil correction in pion emission/ absorption; or (iii) a magnetic photon-nucleon interaction. These one-loop diagrams contribute to the current at order $\mathcal{O}(e\bar{g}_0 Q^2/(4\pi)^2 f_\pi m_N)$.

The NLO diagrams in Fig. 3.4 are built from the leading interactions in Eq. (3.2) with an insertion of the operator from Eq. (3.20). These one-loop diagrams *a priori* contribute to the current at order $\mathcal{O}(e\bar{h}_0 Q^2/(4\pi)^2)$, which is precisely the same order as the diagrams in Fig. 3.3.

Note that, as in the LO EDFF calculation, the delta isobar does not contribute to this order. As is the case for the nucleon anapole form factor, the structure of the delta interactions that would contribute at NLO vanish in χPT (Zhu et al., 2000; Maekawa and van Kolck, 2000; Maekawa et al., 2000). The first nonvanishing delta contribution occurs at a higher order than we are considering here.

The diagrams in Figs. 3.3 and 3.4 can be evaluated in a straightforward way. Both diagrams in Fig. 3.4 vanish due to isospin, so the EDFF does not depend on any new T-violating parameters to this order. Diagrams 3.3(c), 3.3(d) and 3.3(k) also vanish due to isospin, while diagram 3.3(j) vanishes due to its spin structure. Since both diagrams 3.3(j) and 3.3(k) vanish, the EDFF does not depend on the anomalous magnetic moments either. Diagrams 3.3(h) and 3.3(i) give both isoscalar

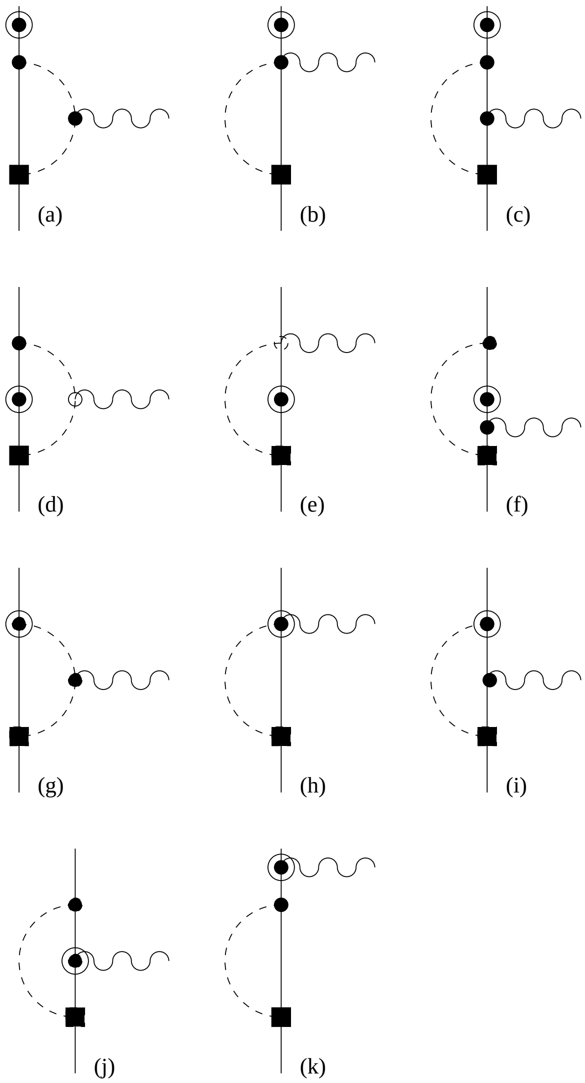

Figure 3.3: One-loop diagrams contributing to the nucleon electric dipole form factor in sub-leading order coming from one insertion of an $\mathcal{L}^{(1)}_{str/em}$ operator. Solid, dashed and wavy lines represent nucleons, pions and (virtual) photons, respectively; squares represent the T-violating vertex from $\mathcal{L}^{(1)}_{\not{T}}$; single filled circles stand for interactions from $\mathcal{L}^{(0)}_{str/em}$ and double circles represent interactions from $\mathcal{L}^{(1)}_{str/em}$. For simplicity only one possible ordering is shown here.

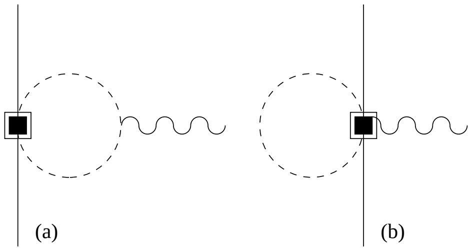

Figure 3.4: Diagrams contributing to the nucleon electric dipole form factor in sub-leading order coming from one insertion of the T-violating vertex from $\mathcal{L}_{\not{T}}^{(2)}$, represented by a double square. Other symbols are as in Fig. 3.1.

and isovector contributions, while all remaining nonvanishing diagrams give purely isovector results. These results are all proportional to $eg_A\bar{g}_0/(4\pi)^2 f_\pi$, as in LO, times the suppression factor m_π/m_N.

Here we concentrate on the isoscalar terms that first appear at NLO. The NLO correction to the LO isoscalar contribution (3.11) is

$$d_0^{NLO} = -\frac{3eg_a\bar{g}_0}{32\pi f_\pi}\frac{m_\pi}{m_N} \tag{3.23}$$

Note that the NLO contribution is enhanced by π over naive dimensional analysis, as sometimes happens in baryon χPT. However, the other dimensionless factors are not large enough to overcome the m_π/m_N suppression. The isoscalar NLO term (3.23) is about 10% of the leading nonanalytic contribution to the isovector nucleon EDM from Eq. (3.12). Eq. (3.23) serves as a lower-bound estimate of the size of the nucleon isoscalar EDM, since the short-range contribution $\tilde{d}_0$ is nominally of lower order.

To NLO, the momentum dependence of the EDFF is completely determined by long-range contributions, i.e. by the pion cloud. While the isoscalar form factor receives no momentum dependence, there is a nonvanishing correction to the isovector momentum dependence:

$$F_D^{(0)}(Q^2) = d_0, \tag{3.24}$$

$$F_D^{(1)}(Q^2) = d_1 - \frac{eg_A\bar{g}_0}{12\pi^2 f_\pi}\left[F\left(\frac{Q^2}{(2m_\pi)^2}\right) + c\frac{m_\pi}{m_N}G\left(\frac{Q^2}{(2m_\pi)^2}\right)\right]. \tag{3.25}$$

Here c is a constant of order $\mathcal{O}(1)$, and $G(Q^2/(2m_\pi)^2)$ is a nonanalytic function which vanishes for $Q^2 = 0$. The fuction $F(Q^2/(2m_\pi)^2))$ is given by Eq. (3.16). The NLO contribution to the isovector EDFF gives a correction to the LO result from Eq. (3.15).

3.4 Discussion

The neutron EDM represents a window of opportunity. Its measurement provides one of the most sensitive probes of physics beyond the Standard Model. There has been much effort to improve the upper bound on the neutron EDM, particularly with a major experiment in the works using ultra-cold neutrons at the Spallation Neutron Source (SNS) at ORNL. This experiment aims to improve the current upper bound by two orders of magnitude. Along with current efforts quoted in Section 3.1 to compute the neutron EDM on the lattice, as well as other experimental efforts internationally to measure the neutron EDM, a concerted theoretical effort is required to have an accurate interpretation of the experimental and lattice results that should emerge in the coming years. In particular, if a better understanding of the sensitivity of the neutron EDM to the quark EDMs and color-EDMs is attained, our current picture of T violation in nature could be significantly improved in the coming years.

In addition to the implications for new physics, the neutron EDM can give insight on the EDM of the deuteron. As with the neutron EDM, the deuteron EDM is a probe of new physics. It also serves as a kind of intermediate step between nucleons and heavier nuclei. In most models (Khriplovich and Korkin, 2000; Lebedev et al., 2004), the neutron and deuteron EDMs are of comparable size, both well above the range where weak T violation can create an EDM. The deuteron EDM is given by the sum of the isoscalar nucleon EDM d_0 and a nuclear bound-state contribution. The leading isoscalar contribution to the nucleon EDM that is nonanalytic in m_π, given by (3.23), thus serves as a lower-bound estimate of the deuteron EDM in the impulse approximation. Numerically, we find

$$|d_d| \gtrsim \frac{3eg_A\bar{g}_0}{32\pi f_\pi}\frac{m_\pi}{m_N} \approx 3.5 \times 10^{-17}\bar{\theta}e \text{ cm} \tag{3.26}$$

This is about 10% of the lower-bound estimate of the neutron EDM (3.13). It is important to keep in mind that this is only a rough bound. It can be greatly improved upon with a model-independent calculation of the two-body contributions to the deuteron EDM in χPT with perturbative pions.

While an equally concerted effort to study the nucleon EDFF does not exist as of yet, measurement of this form factor can still prove valuable. As with any form factor, the EDFF describes the behavior of the T-violating nucleon current without any on-shell constraint. However, it is currently inaccessible experimentally for nonvanishing values of Q^2. Nevertheless, for momenta $Q \ll m_\pi$, information about the EDFF can be obtained by expanded it in powers of Q^2:

$$F_D^{(i)}(Q^2) = d_i - S'_i Q^2 + \ldots \tag{3.27}$$

where S'_0 (S'_1) is the electromagnetic contribution to the isoscalar (isovector) nucleon Schiff moment. The Schiff moment thus arises in the amplitude for low-energy electron-nucleon scattering through the T-violating nucleon current, i.e. Eq. (3.10) or Eq. (3.22). The nucleon Schiff moment is also important for the study of EDMs of particular atoms and molecules. The Schiff moments of individual nucleons contribute incoherently to nuclear Schiff moments, and give the dominant contribution to nuclear Schiff moments for atoms and molecules with zero net intrinsic electronic spin and nuclear spin $\frac{1}{2}$.

Apart from low-energy experiments, the nucleon EDFF has recently become important in lattice QCD attempts to compute the neutron EDM. The CP-PACS (Shintani et al., 2005a,b) and RBC (Berruto et al., 2005) lattice groups have used the method of computing the three-point function $\langle p', s'| J^{\mu}_{ed} |p, s\rangle$, which is then proportional to the EDFF (Wilcox, 2002)

$$\lim_{p,p' \to m_N} \langle p', s'| J^{\mu}_{ed} |p, s\rangle \propto \lim_{p,p' \to m_N} q_\nu F_D(-q^2), \tag{3.28}$$

to extract the EDM in the $q^2 \to 0$ limit. This extraction is necessary if one uses the nucleon-nucleon-photon three-point function since the electric dipole interaction itself includes the momentum q, hence it cannot simply be set to zero on the lattice to obtain the EDM.

The CP-PACS and RBC groups both computed the EDFF at particular values of Q^2. Using quenched QCD with domain-wall quarks and quark masses set to correspond with $m_\pi/m_\rho \cong 0.63$, Shintani *et al* obtained

$$F_D(-q^2 = 0.58\mathrm{GeV}^2) = \begin{cases} +2.1(6) \times 10^{-15}\bar{\theta} e \text{ cm} & \text{proton} \\ -2.4(5) \times 10^{-15}\bar{\theta} e \text{ cm} & \text{neutron} \end{cases} \tag{3.29}$$

Within the error, this is consistent with an isovector nucleon EDFF. Using unquenched QCD with domain-wall quarks and setting $m_u, m_d \approx m_s$, Berruto *et al* obtained

$$\frac{F_D(-q^2 = 0.399\mathrm{GeV}^2)}{G_E^{(p)}(-q^2 = 0.399\mathrm{GeV}^2)} \lesssim 2 \times 10^{-15}\bar{\theta} e \text{ cm} \tag{3.30}$$

for the neutron, where G_E is the electric Sachs form factor, and is $\mathcal{O}(1)$. Taking the isovector EDM (3.12) to be given by the nonanalytic long-range contribution, the isovector EDFF from Eq. (3.15) at $q^2 = -0.58\text{GeV}^2$ has a value

$$F_D^{(1)}(-q^2 = 0.58\text{GeV}^2) = -2.9 \times 10^{-16}\bar{\theta} e \text{ cm} \tag{3.31}$$

which is about 10 times smaller than the lattice result (3.29) and the upper bound (3.30). Since the q^2 values used in Eqs. (3.29) and (3.30) are $\mathcal{O}(m_\rho)$, which is well outside the range of validity of the chiral expansion, these lattice calculations cannot reasonably be compared with the results of this chapter. Obtaining lattice results for significantly lower energies is complicated by finite size effects on the lattice, so a direct comparison between χPT and lattice calculations may not be feasible for the near future. Furthermore, the results (3.29) were computed in quenched QCD, whereas the results from this chapter were computed using unquenched χPT.

The result (3.30) from full QCD does not necessarily disagree with the results of this chapter, however. Since Eq. (3.30) only gives an upper bound, the neutron EDFF could be significantly smaller. When lattice calculations are able to handle momenta in the range of validity for χPT, the EDFF result (3.15) would be appropriate to use.

3.5 Conclusion

The nucleon EDFF is an important quantity in nuclear physics, both as a generalization of the nucleon EDM and in its own right. Its importance should only increase once the EDM of the neutron is measured in experiments. The nucleon EDFF determines the electromagnetic contribution to the nucleon Schiff moment, which in turn contributes to the EDMs of a number of atoms and molecules. The EDFF has also proved valuable in lattice QCD, where it is employed in order to extract the neutron EDM.

In leading order, the nucleon EDFF consists of both short-range contributions and long-range contributions from the pion cloud. The origin of the short-range contributions can only be fully understood using the underlying theory of QCD. The momentum dependence of the LO EDFF is purely isovector and is proportional to a non-derivative T-violating pion-nucleon coupling. The scale for momentum variation, appearing in the nucleon electromagnetic Schiff moment and the radius of the EDFF, is the pion mass. The leading long-range contribution which is nonanalytic in the pion mass and momentum-independent serves as an estimate of the nucleon EDM.

In subleading order, a long-range contribution to the isoscalar nucleon EDM first appears. This isoscalar contribution serves as a lower-bound estimate of the deuteron EDM, since this EDM is given by a sum of the isoscalar nucleon EDM and two-nucleon contributions. The isovector nucleon EDFF receives further momentum dependence, and there is an associated correction to the LO result for the nucleon EDM. No new T-violating parameters contribute at this order.

CHAPTER 4

CONCLUSION

The central goal of this thesis was to present a systematic approach to deriving the T-violating hadronic interactions generated by a variety of quark-level sources of T violation. This was done first for the QCD $\bar{\theta}$ term (2.1). Due to the chiral rotation properties of the $\bar{\theta}$ term, we showed that T violation from the $\bar{\theta}$ term and isospin violation from the up-down quark-mass difference are connected and that there is a constraining relationship between their interaction coefficients at any given chiral index. If Dashen's theorem is used to avoid vacuum instability, then the only chiral symmetry-breaking interactions generated from the quark mass terms and the $\bar{\theta}$ term are those that transform as third and fourth components of P, respectively (see Eq. (1.13)).

On the other hand, if Dashen's theorem is not applied and one instead maintains all of the quark-level terms that result from performing a chiral rotation to eliminate the $\bar{\theta}$ term (2.1), then new interactions will be generated. The T-violating interactions thus generated transform as third components of S (see Eq. (1.12)) and are problematic due to the fact that they cause vacuum instability. We showed that this problem can be resolved by simply applying a specific axial transformation to the pion and nucleon fields in the theory. One is then left with only the T-violating interactions that transform as third components of P. An axial transformation of the hadronic fields in the low-energy theory thus has the same effect (at least at low energies) as a chiral rotation of the quark fields. This fact was previously unknown and allows for a similar procedure to be used to derive the set of interactions generated by other sources of T violation.

The color EDMs of the light quarks (2.49) were found to have the same properties as the QCD $\bar{\theta}$ term under chiral symmetry. This indicates that both sources generate T-violating interactions which differ only in their strengths. It was also shown that a transformation of the hadronic fields could be used to eliminate any T violation generated by the isovector quark color EDM, similar to what was done for the $\bar{\theta}$ term.

The quark EDMs of the light quarks differ from the $\bar{\theta}$ term and the quark color EDMs in that they involve an explicit photon field. This changes the types of T-violating interactions available. One can directly generate hadronic interactions that have external photons, or indirectly generate interactions without external photons by attaching a quark EDM vertex to a quark T-even electromagnetic vertex. The interactions with no external photons are identical in form to those generated by the $\bar{\theta}$ term and quark color EDMs. This includes vacuum-unstable terms generated by the isovector quark EDM, and these terms can be eliminated in a similar manner to the quark isovector color EDM. The interactions with external photons can also arise from the $\bar{\theta}$ term and quark color EDMs, but only indirectly.

The Weinberg three-gluon operator is unlike any of the T-violation sources discussed above in that it does not break chiral symmetry. Thus the interactions that it generates are either chiral invariant or are the result of attaching a Weinberg operator vertex to a quark line in some diagram.

An important quantity that can arise from any of the sources of T violation mentioned above is the EDM of the nucleon. More precisely, short-range contributions to the nucleon EDM can be generated by tree-level diagrams involving these sources. In order to better understand how a nucleon EDM can be generated at quark level, we first showed how one can generate short-range contributions to the nucleon magnetic moment. Generating a nucleon EDM then follows in a similar manner.

We obtained order-of-magnitude estimates of the short-range nucleon EDM contributions stemming from the $\bar{\theta}$ term, quark color EDMs, quark EDMs and the

Weinberg operator. At leading order, the only source which gives differing orders of magnitude for the isoscalar and isovector EDMs is the quark EDM. At sub-leading orders, each source gives isoscalar and isovector EDMs that are of differing orders of magnitude.

In addition to short-range contributions, the nucleon EDM also receives long-range contributions from the pion cloud. More general than the EDM is the electric dipole form factor. We have shown that at leading order, for a $\bar{\theta}$-term source, the pion cloud generates an isovector EDM as well as a momentum-dependent contribution to the EDFF. We have also computed the leading long-range isoscalar nucleon EDM, which appears at next-to-leading order. This isoscalar contribution then serves as a lower-bound estimate for the deuteron EDM.

In the future, it would be advantageous to compute the long-range contributions to the deuteron EDM in χPT. These contributions would then combine with the isoscalar nucleon EDM (as well as any calculable short-range contributions) to give a much better estimate for the deuteron EDM. In order to calculate short-range deuteron EDM contributions, it would additionally be necessary to determine the T-violating component of the nucleon-nucleon interaction.

APPENDIX A

TRANSFORMATION PROPERTIES OF THE HADRONIC FIELDS

Stereographic coordinates are a convenient way to parameterize pion fields in Chiral Perturbation Theory. Being the (pseudo)Goldstone bosons of the spontaneous breaking of $SU(2)_L \times SU(2)_R \sim SO(4)$ down to $SU(2)_V \sim SO(3)$ (local isomorphisms), pions inhabit the three-sphere $SO(4)/SO(3) \sim S^3$ of radius f_π. The sphere can be parameterized in a variety of ways, such as embedding the sphere in the Euclidean space E^4 of cartesian coordinates $\phi = \{\boldsymbol{\phi}, \phi_4 \equiv \sigma\}$ subject to the constraint

$$\sigma^2 + \boldsymbol{\phi}^2 = f_\pi^2. \tag{A.1}$$

The three pion fields $\boldsymbol{\pi}$ can be obtained by applying an $SO(4)$ transformation (four-rotation) $R(\boldsymbol{\pi})$ to the north pole $(\mathbf{0}, f_\pi)$:

$$\phi_\alpha(\boldsymbol{\pi}) = R_{\alpha 4}(\boldsymbol{\pi}) f_\pi. \tag{A.2}$$

In stereographic coordinates, the rotation matrix $R(\boldsymbol{\pi})$ is given by

$$R_{\alpha\beta}[\boldsymbol{\pi}] = \begin{pmatrix} \delta_{ij} - \frac{1}{D}\frac{\pi_i \pi_j}{2f_\pi^2} & \frac{1}{D}\frac{\pi_i}{f_\pi} \\ -\frac{1}{D}\frac{\pi_j}{f_\pi} & \frac{1}{D}\left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right) \end{pmatrix}, \tag{A.3}$$

where $D \equiv 1 + \boldsymbol{\pi}^2/(4f_\pi^2)$. The pion fields simply rotate under infinitesimal $SU(2)_V$ isospin transformations with parameter $\boldsymbol{\epsilon}_V$

$$\delta_V \boldsymbol{\pi} = \boldsymbol{\epsilon}_V \times \boldsymbol{\pi}, \tag{A.4}$$

but transform nonlinearly under axial transformations with parameter $\boldsymbol{\epsilon}_A$:

$$\delta_A \boldsymbol{\pi} = f_\pi \left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right) \boldsymbol{\epsilon}_A + \frac{1}{2f_\pi} \boldsymbol{\pi} \cdot \boldsymbol{\epsilon}_A \boldsymbol{\pi}. \tag{A.5}$$

The pion fields have a covariant derivative

$$\mathbf{D}_\mu = D^{-1} \partial_\mu \boldsymbol{\pi}. \tag{A.6}$$

The covariant derivative (A.6) transforms linearly under isospin

$$\delta_V \boldsymbol{D}_\mu = \boldsymbol{\epsilon}_V \times \boldsymbol{D}_\mu, \tag{A.7}$$

and under $SU(2)_A$ as if under isospin with a field-dependent parameter:

$$\delta_A \boldsymbol{D}_\mu = \left(\boldsymbol{\epsilon}_A \times \frac{\boldsymbol{\pi}}{f_\pi}\right) \times \boldsymbol{D}_\mu. \tag{A.8}$$

The covariant derivative of $\boldsymbol{D}_\mu$, meanwhile, is

$$\mathcal{D}_\mu \boldsymbol{D}_\nu = \partial_\mu \boldsymbol{D}_\nu + i \boldsymbol{E}_\mu \times \boldsymbol{D}_\nu, \tag{A.9}$$

where

$$\boldsymbol{E}_\mu = \frac{i}{2f_\pi^2} \boldsymbol{\pi} \times \boldsymbol{D}_\mu. \tag{A.10}$$

Of course, stereographic coordinates are not the only possible parameterization that can be used for the pion fields. Other choices include the sigma model parameterization

$$u = \sqrt{1 - \frac{\boldsymbol{\pi}^2}{f_\pi^2}} + \frac{i}{f_\pi} \boldsymbol{\tau} \cdot \boldsymbol{\pi} \tag{A.11}$$

and the exponential parameterization

$$u = \exp\left(\frac{i}{2f_\pi}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\right) \tag{A.12}$$

For nucleons, it is most convenient to use a non-linear realization

$$N = \begin{pmatrix} p \\ n \end{pmatrix} = \frac{1}{\sqrt{D}}\left(1 + \frac{i}{2f_\pi}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\right)\Psi_N, \tag{A.13}$$

where Ψ_N transforms linearly under the chiral group. As with $\boldsymbol{D}_\mu$, N transforms linearly under infinitesimal isospin transformations

$$\delta_V N = i\boldsymbol{\epsilon}_V\cdot\boldsymbol{\tau}N, \tag{A.14}$$

and under an infinitesimal axial transformation as if under isospin with a field-dependent parameter:

$$\delta_A N = \frac{i}{2}\left(\boldsymbol{\epsilon}_A \times \frac{\boldsymbol{\pi}}{f_\pi}\right)\cdot\boldsymbol{\tau}N. \tag{A.15}$$

The nucleon covariant derivative is given by

$$\mathcal{D}_\mu N = \left(\partial_\mu + \frac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{E}_\mu\right)N \tag{A.16}$$

Because of the properties of these quantities under axial transformations, a combination of them that is isospin invariant is also chiral invariant.

APPENDIX B

GENERATION OF SYMMETRY-BREAKING OPERATORS

In QCD with two light quarks, chiral $SU(2)_L \times SU(2)_R \sim SO(4)$ symmetry is spontaneously broken down to $SU(2)_V \sim SO(3)$. There are also terms that explicitly break chiral symmetry, call them collectively $\Delta\mathcal{L}$. Suppose that

$$\Delta\mathcal{L} = c_a \mathcal{O}_a, \tag{B.1}$$

where c_a are some coefficients quantifying the strength of the symmetry breaking, and $\mathcal{O}_a$ are operators that transform as

$$\mathcal{O}_a = D[g]_{ab} \mathcal{O}_b. \tag{B.2}$$

with $D[g]$ some representation of $SO(4)$ and g some element of $SO(4)$. The $\mathcal{O}_a$ are constructed out of the pion fields $\boldsymbol{\pi}$ and the nucleon field N as

$$\mathcal{O}_a[\boldsymbol{\pi}, N] = D[\boldsymbol{\pi}]_{ab} \mathcal{O}_b[0, N]. \tag{B.3}$$

This is a more general version of (A.2), and shows that an $SO(4)$ vector involving $\boldsymbol{\pi}$ can be constructed by applying an $SO(4)$ transformation, i.e. rotation, on another that does not. The representation $D[\boldsymbol{\pi}]$ is not unique. A convenient choice is the stereographic representation (A.3).

More generally, we search for tensors $T_{ab\ldots}[0; \mathbf{D}_\mu, N]$ constructed from $\mathbf{D}_\mu$, N and their covariant derivatives, as well as isoscalar products like $\boldsymbol{\tau} \cdot \boldsymbol{D}_\mu$, and then rotate them:

$$T_{ab\ldots}[\boldsymbol{\pi};\mathbf{D}_\mu,N]=D_{aa'}[\boldsymbol{\pi}]D_{bb'}[\boldsymbol{\pi}]\ldots T_{a'b'\ldots}[0;\mathbf{D}_\mu,N]. \tag{B.4}$$

The simplest case is when the tensor does not include N or $\mathbf{D}_\mu$. Then only numbers are available:

$$S[0;0,0]=(\mathbf{0},1). \tag{B.5}$$

Rotating with the matrix (A.3) gives

$$S[\boldsymbol{\pi};0,0]=\left(\frac{\boldsymbol{\pi}}{f_\pi D},\frac{1}{D}\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\right). \tag{B.6}$$

The component $S_3[\boldsymbol{\pi};0,0]$ cannot appear in the Lagrangian because it gives rise to vacuum instability (see Section 2.3), but $S_4[\boldsymbol{\pi};0,0]$ gives

$$\mathcal{L}^{(0)}_{\pi,qm}=\frac{1}{2D}m_\pi^2\boldsymbol{\pi}^2+\text{constant}, \tag{B.7}$$

where m_π^2 is proportional to m_u+m_d, since the pion mass term is generated by the quark mass term in Eq. (1.10) that transforms as S_4. Including $\mathbf{D}_\mu$, we obtain a term related to Eq. (B.5):

$$S[0;\mathbf{D}_\mu,0]=\mathbf{D}_\mu\cdot\mathbf{D}^\mu S[0;0,0] \tag{B.8}$$

where the form is required by Lorentz invariance. Higher chiral-index operators containing pion fields can be constructed similarly.

When nucleon fields are incorporated, Lorentz scalar operators such as

$$S[0;\boldsymbol{D}_\mu,N]=(\bar{N}S_\mu\boldsymbol{\tau}\mathcal{D}^\mu N+H.c.,0),(0,\bar{N}S_\mu\boldsymbol{\tau}N\cdot\boldsymbol{D}^\mu) \tag{B.9}$$

and

$$S[0;0,N] = \bar{N}NS[0;0,0], \tag{B.10}$$

and Lorentz pseudoscalar operators such as

$$P[0;0,N] = (\bar{N}\boldsymbol{\tau}N, 0) \tag{B.11}$$

can be formed, with the specific forms dictated by parity and isospin conservation. Applying the rotation (A.3) generates a variety of terms, including (from $S[0;0,N]$) the pion-nucleon "sigma term"

$$\mathcal{L}^{(1)}_{N,qm} = \frac{\sigma}{2f_\pi^2}\boldsymbol{\pi}^2\bar{N}N, \tag{B.12}$$

where $\sigma = \mathcal{O}(m_\pi^2/M_{QCD})$, and (from $P[0;0,N]$) the isospin zero T-violating term

$$\mathcal{L}^{(1)}_{\not{T},\pi N} = -\frac{\bar{g}_0}{D}\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\pi}N, \tag{B.13}$$

where $\bar{g}_0 = \mathcal{O}(m_*\bar{\theta}/f_\pi)$, assuming that Eq. (B.13) is generated by the QCD $\bar{\theta}$ term.

There are a variety of other vectors that can be formed, as well as tensors of various rank. As long as these objects are constructed with the same symmetry properties as the quark-level interactions from which they are derived, then a consistent connection between the underlying theory (i.e. QCD) and the low-energy EFT will be maintained.

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