# EXPLORING EARLY ALGEBRA:

# MAKING MEANINGFUL CONNECTIONS IN THE ELEMENTARY CLASSROOM

By

LYNETTE DEAUN GUZMAN

A Thesis Submitted to The Honors College

In Partial Fulfillment of the Bachelors degree

With Honors in

Mathematics

THE UNIVERSITY OF ARIZONA

M A Y 2 0 1 2

Approved by:

Mathew D. Felton-Koestler, Ph.D.
Department of Mathematics

## STATEMENT BY AUTHOR

This thesis has been submitted in partial fulfillment of requirements for a degree at The University of Arizona and is deposited in the University Library to be made available to borrowers under rules of the Library.

Signed: ______________________

# Abstract

Algebra is traditionally absent from elementary classrooms. Historically, learning theorists believed that young students are restricted in cognitive competence for abstract thinking; however, research in mathematics education demonstrates that children are capable of algebraic reasoning in the elementary grades. Currently, topics in elementary mathematics focus on particular instances rather than generality, and students often struggle when first introduced to algebraic concepts. A suggestion to ease a shift in thinking about particular numbers to thinking about relations among sets of numbers is the integration of algebra in the elementary grades, described as early algebra.

In this paper, I provide a definition of early algebra and propose a research study to investigate whether elementary students are able to make meaningful connections between their mathematics knowledge and early algebra activities. In particular, I am interested in the following questions: How do elementary students make meaningful connections in early algebra activities and what role does natural language play in facilitating algebraic reasoning in the elementary grades?



*Keywords:* early algebra, elementary mathematics, children's algebraic reasoning

Viewed as an essential strand of mathematics education, algebra has been used as a tool for manipulating symbols by using language to describe relationships as a first introduction to abstract thinking in mathematics. Often considered “the gateway to higher mathematics”, algebra serves as an entry-level skill for professions in science, business, and industry; however, students struggle with this abstract system (Hatfield, Edwards, Bitter, & Morrow, 2000). Topics in elementary mathematics focus on particular instances rather than generality, which contributes to the difficulties students may face when introduced to algebra. For example, many students have trouble with algebraic concepts such as assigning meaning to variables or distinguishing multiple uses of variables, especially when variability concepts are misunderstood (Rakes et al., 2010).

A suggestion to ease this shift in thinking from particular numbers to relations among sets of numbers is the integration of algebra in elementary grades, often described as early algebra. The National Council of Teachers of Mathematics (NCTM) acknowledges the importance of algebraic competence throughout grades K-12 and explicitly advocates the idea of integrating algebra as a continuous strand throughout a student’s mathematics exposure in school. In reviewing early algebra research findings thus far, I consider a variety of perspectives for conducting my own investigation of whether elementary students build meaningful connections between their elementary mathematics knowledge and early algebra activities.

## History

Learning theorists have contributed to mathematics education and the study of how children learn mathematics. Jean Piaget supported the theory that learning occurs through a process of developmental stages in action (Hatfield et al., 2000). In Piagetian theory, people traverse through these defined stages in a particular sequence and cannot advance to the following stage until the previous stage is completed. “Formal operations”, the final developmental stage, involves abstract reasoning such as algebraic skills. Piagetian developmental theory exhibited a strong presence and influence in educational psychology research with the belief that young students have difficulties learning algebra due to a restriction in cognitive competence. Consequently, this belief is one reason why algebra is traditionally absent from elementary classrooms.

In addition to the historical legacy of Piagetian developmental theory, Carraher, Schliemann, Brizuela, and Earnest (2006) identified a second reason why algebra is traditionally absent from elementary classrooms. They state that because there is a distinction between arithmetic and algebra in the organization of mathematics in school, arithmetic and algebra are not integrated in the classroom. Thus, pre-algebra becomes a designated transitional period in a student's mathematics sequence that bridges topics of arithmetic and algebra.

The number of algebra education researchers increased in the late 1970s, and they focused on how students make meaning of algebra and on the different ways to make meaning of algebra learning (Kieran, 2007). As a growing community, researchers explored the ideas of Piaget and ultimately discovered that children are capable of algebraic reasoning at younger ages than previously believed.

Bastable and Schifter (2008) claimed that students are more likely to demonstrate algebraic thinking when classroom instruction builds upon their own mathematical ideas. In their research, Bastable and Schifter found that students are able to generalize properties about number operations such as the commutative property of addition. Beginning with particular examples, students began to realize that the order of addends does not matter and eventually reached conceptualization where they did not have to try every combination to find the sum.

An important strand in current early algebra research focuses on elementary classroom practices that promote algebraic reasoning by eliciting concepts through existing curriculum (Blanton & Kaput, 2011). For instance, in Blanton and Kaput's work, a team of researchers and classroom teachers collaborated and conducted lessons together in the classroom. Many working definitions of early algebra have emerged through these investigations. Overall, researchers identify major components of early algebra while exploring tools that express and highlight these components.

## Defining Early Algebra

In my definition, early algebra is focused on *generalization* and *functional thinking* in elementary mathematics. Generalization is demonstrated when students make arguments about whole classes of numbers at once, as opposed to reasoning about individual numbers. For instance, if a child states that "any number plus zero is the same number" she is generalizing by making a claim about all numbers (or perhaps all whole numbers) instead of evaluating a specific case. As another aspect of early algebra, functional thinking involves thinking about how one set of numbers can be transformed through a fixed process to another set of numbers. Related to algebra, functional thinking describes *covariation,* the relationship and behavior of two variables such that when one variable changes then the other variable changes in a corresponding way. Functional thinking may be viewed as a special form of generalization since arguments concern a set of numbers, which is considered a class. However, functional thinking stands out because of the unique role it plays and its prominence in early algebra research. For example, if a student wants to add three to any number and identifies the relationship as "three more than any number", functional thinking would allow the student to acknowledge a transformation by adding three to any number, thus making a statement about all numbers. Additionally, notational systems may be used as tools that support students in engaging in the generalization and functional thinking involved in early algebra reasoning. Notational systems are defined as being any form of representation that is “external to the mind” (Brizuela & Earnest, 2008, p. 279). Notational systems may be used as a means of communication in mathematics classes and include instances of tabular, graphical, symbolic, or verbal representations. In the elementary grades, natural language serves as an important notational system that allows students to articulate mathematical statements and develop algebraic reasoning. Together, notational systems and the role of language mediate mathematical reasoning, in general, and early algebra, in particular. In the following section, I will discuss notational systems and the role of language.

**Notational Systems and the Role of Language**

In algebra, the role of language facilitates making meaning of symbolic representations or variables. Early algebra research places a large emphasis on elementary students’ use of informal language because this use of informal language supports the students in understanding the real world and mathematical meaning in problems.

Carraher, Schliemann, and Schwartz (2008) identified three distinguishing characteristics of early algebra: “it builds upon background contexts of problems, it only introduces formal notation gradually, and it interweaves existing topic of early mathematics” (pp. 236-237). The second characteristic of early algebra highlights a gradual introduction of formal notation, which provides a structure and guide for written notation in algebra. It is unlikely that elementary students will independently invent a symbolic system for variables that can express unknown amounts (Carraher, Schliemann, & Schwartz, 2008). For this reason, elementary teachers may guide their students by working with a concept first through classroom exploration, then later introduce formal notation to identify that concept. This structure of gradually exposing elementary students to formal notation allows teachers to respond to and facilitate classroom discussion that accommodates initial student interpretations of a concept and ultimately provides students with the opportunity to expand and shape their own understanding of notations.

The following example is an instance of the role of language in mathematics. Brizuela and Earnest (2008) examined multiple notational systems in stages, beginning with verbal reactions and instantiations. According to Brizuela and Earnest, natural language in verbal representations elicits “real-life nuances through dialogue” (p.283) and captures articulated thoughts in a student’s problem solving process. In this study a member of the research team served as both an interviewer and teacher while working with a small group of students on algebraic tasks. For example, elementary students were presented with the following problem:

> Let’s Make a Deal!
>
> Raymond has some money. His grandmother offers him two deals:
>
> Deal 1: She will double his money.
>
> Deal 2: She will triple his money and then take away 7.
>
> Raymond wants to choose the best deal. What should he do? How would you figure out and show him what is the best to do? Is one deal always better?
>
> Show this on a piece of paper. (Brizuela & Earnest, 2008, p. 281)

Initially, many students claimed that Deal 1 would be the better deal. When asked to explain their reasoning, one student stated, “She [the grandmother] doesn’t take away $7” (Brizuela & Earnest, 2008, p. 283). A number of students agreed that the context of money being taken away was undesirable. Another student countered this claim by providing the situation of Raymond having $15. In this case, Deal 2 would triple the amount that Raymond had, 15 × 3 = 45, with the grandmother taking away 7, meaning Raymond ended up with 45 – 7 = 38. Deal 1 would be double the amount Raymond had, $15 \times 2 = 30$, which showed that Deal 2 is the better deal if Raymond had $15.

This example highlights how verbal representations supported the students in beginning to attribute meaning to problems by making connections with real-life situations. Students are tasked to choose the best deal, which indicates that students must identify which one of the two scenarios benefits Raymond more. Verbal representations helped students make connections to the amount of money that Raymond would end up with; however, some of the students found it undesirable to take money away regardless of whether Raymond ended up with more money. In mathematics, this type of nuanced information is usually ignored. Although the role of language assisted students with figuring out the amount of money that Raymond would end up with for each deal, it also interfered with some students' selection of the best deal. Instead of comparing amounts and choosing the deal where Raymond ended up with the more money, some students focused on the aspect of taking money away and ultimately did not correctly use their mathematical skills to select the best deal.

As seen above, context serves as an important aspect in verbal representations and allows students to think about applications of mathematics when presented with concepts. In another example, Carraher et al. (2006) introduced number lines to elementary students by creating a line of strung twine across the classroom with numbered markers attached at regular intervals. Students interpreted displacement of different terms by physically standing near the markers and shifting to different markers in accordance with the situations given to them by the researchers. For example, if a group of eight-year-old students were told to stand at the marker of their current age and then move to how old they would be in three years, then the students would be able to physically walk from marker 8 to marker 11.

When considering the situation of earning and spending money, discussion about debt was “crucial to clarifying what negative numbers mean” (Carraher et al., 2006, p. 95). For example, when comparing having $0 to owing $2 (or having -$2), both situations involve not having any money; however, the difference between the two situations captures characteristics of a negative number. Having $0 corresponds to not having money in possession and also not owing money while having -$2 corresponds to not having money in possession and also owing $2 to an outside recipient. The amount of money in -$2 still maintains the amount of $2, but that amount belongs to another receiver. This provides an example of how informal language contributes to children’s understanding of mathematical concepts, and in this case, showing how negative numbers can be associated with the idea of debt. These understanding can be built on to develop more formal mathematical concepts and notational systems.

The role of language is bidirectional: introducing notational systems is sometimes needed to progress with algebra; however, students’ natural language can often be built upon to introduce notational systems. It is important to realize that it is not essential for students to invent a symbolic system to represent unknown amounts. Instead, what researchers have found is that it is important for students to accept symbolic systems as their tool and also have the ability to access and apply the system in situations they encounter (Carraher, et al., 2006).

**Generalization**

Students exhibit understanding of generalization by making arguments about classes of numbers. In the elementary classroom, this is often seen when extending single cases of arithmetic to properties of operations, particularly with addition and multiplication.

The following example shows how early algebra captures the way children express ideas and identify generalizations about number. In a classroom story exploring multiplication from

Bastable and Schifter (2008), third-grade students found number patterns in the factors of 36. Students made the statement $3 \times 12 = 36$, but also noticed $12 \times 3 = 36$ and described it as being "backwards" of the first statement. Also realizing that $9 \times 4 = 36$ and $4 \times 9 = 36$, students noticed once again that the second equivalent statement was "backwards" of the first statement. Soon following these new examples, one student asked, "does that always work?" (p. 167). This inquiry demonstrates generalization because it is the beginning stage of making a statement about a set of numbers, in this case, the factors of 36 in multiplication equations.

A few weeks later, further classroom discussion highlighted a variety of ways in which students tried to prove their previous conjecture by unknowingly exploring the commutative property of multiplication. Some instances involved students finding other combinations of multiplied numbers that followed the conjecture while others used manipulatives to model situations. Eventually, one student created an array that modeled the multiplication of two integers, $3 \times 7 = 21$. The student then proceeded to rotate the array 90 degrees to demonstrate the "backwards" model, $7 \times 3 = 21$. The student pointed out that even though the order of the two numbers was reversed, the areas of the two representations (the products) were equal with the same physical amount. These elementary students demonstrated two forms of generalization in their work. First, students were able to make a claim about any two whole numbers being multiplied with the conjecture $a \times b = b \times a$, although not explicitly stated in symbolic notation. Next, students were able to discuss a general argument that would work for any two numbers by using specific numbers (3 and 7).

The third-grade students in this classroom worked with the commutative property of multiplication without having a formal introduction or definition of properties of operations. Through their own inquiry and exploration, students made a conjecture as a class and individually attempted to justify their thinking. Even without symbolic notation, these students were able to use generalization in their understanding that this “backwards” property was applicable and valid for multiple instances as long as it followed the pattern of involving the operation of multiplication and the switching the order of the factors.

Another classroom of second-grade students explored articulating, refining, and editing conjectures about properties of operations. Carpenter, Franke, and Levi (2003) found that number sentences identified by a truth-value gives context to assist “children to articulate basic mathematical principles” (p. 48). Table 1 below shows a sample of the investigated properties of operations in this classroom, along with typical student language used to state these conjectures and number sentence examples.

| **Number Sentence** | **Children’s Conjectures** | **T/F, Open Number Sentences** |
|---|---|---|
| Addition and subtraction involving zero | | |
| 1a. $a + 0 = a$ | When you add zero to a number, you get the number you started with. | 5,467 + 0 = 5,467<br>23 + 7 = 23*<br>40 + 0 = 400*<br>89 + $c$ = 89 |
| 1b. $0 + a = a$ | When you add a number to zero, you get that number (you added). | 0 + 5,467 = 5,467<br>23 + 7 = 7*<br>$c$ + 89 = 89 |
| Multiplication and division involving 1 | | |
| 4a. $a \times 1 = a$ | When you multiply a number times 1, you get the number you started with. | 5,467 × 1 = 5,467<br>48 × 1 = 49*<br>8.4 × .01 = 8.4*<br>$c$ × 1 = 76 |
| 4b. $1 \times a = a$ | When you multiply 1 times a number, you get that number. | 1 × 5,467 = 5,467<br>1 × $c$ = 76 |
| Commutative properties for addition and multiplication | | |
| 9. $a + b = b + a$ | When you add two numbers, you can change the order of the numbers you add, and you will still get the same number. | 34 + 58 = 58 + 34<br>95 + 87 = $c$ + 95 |
| 10. $a \times b = b \times a$ | When you multiply two numbers, you can change the order of the numbers you multiply, and you will get | 34 × 58 = 58 × 34<br>95 × 87 = $c$ × 87 |

| | | |
|---|---|---|
| | the same number. | |
| *False number sentence | | |

Table 1. Selected examples of basic properties of addition and multiplication (Carpenter, Franke, & Levi, 2003, pp. 54-55)

In the work of Carpenter, et al. (2003), students' generalizations are usually stated in natural language. The basis of articulating ideas began with looking at true and false number sentences, determining a pattern or common characteristic, and finally making a claim about a class of numbers.

Blanton and Kaput (2005) also emphasized the importance of patterns in children's algebraic reasoning by claiming that the focus of early algebra is a "shift from arithmetic to pattern building, conjecturing, generalizing, and justifying mathematical facts and relationships" (p. 415). Elementary school students exposed to early algebra participate in a classroom environment that encourages and cultivates deeper understanding. In particular, Blanton and Kaput (2005) saw students generalize about sums and products of even and odd numbers. Using models, students were able to define even numbers as any number that could be broken up into a number of groups of pairs. The students could conclude that an odd number added to an odd number results in an even number because, as one student stated, "if you have leftovers the two leftovers go together [to form a pair]" (Blanton & Kaput, 2005, p. 420). This student's statement shows a general understanding of even and odd numbers because even numbers were generalized as a set of numbers with the characteristic of having the ability to be divided into groups of pairs. Since students only had exposure to the set of natural numbers up to this point in their mathematics careers, the set of odd numbers could be identified as "not even" or a set of numbers with the characteristic of not having the ability to be divided into groups of pairs.

According to Bastable and Schifter (2008), language is used as a tool for "expressing generalizations about number systems" (p. 165). In the previous examples, students are able to make conjectures about properties of number and operations by using natural language. Recognizing patterns and relationships allow elementary students to predict and form generalizations while simultaneously developing observation skills and language skills (Hopkins, Gifford, & Pepperell, 1996).

**Functional Thinking**

Functional thinking involves any case where a student is able to transform an entire set of numbers using operations. This idea involves mapping one set of numbers to another set of numbers. As stated in a previous example, if a student wants to add three to any number and identifies the relationship as "three more than any number", functional thinking would allow the student to acknowledge a transformation by adding three to any number, thus making a statement about all numbers. In elementary mathematics, the integration of early algebra into curriculum involves a shift from thinking about particular numbers and measures toward thinking about relations among sets of numbers and measures. Therefore, functional thinking is a form of generalization by making arguments about classes of numbers but is considered as a distinct aspect of early algebra because of the connection to performing transformations on classes of numbers. During this shift in thinking, functions serve as a major role in exposing algebraic reasoning.

In research done by Carraher et al. (2006), elementary students were introduced to a "variable number line" in a lesson focused on using operations on unknown values (p. 96). An example of the variable number line is shown in Figure 1. For instance, talking about "*N* minus

2” is associated with the displacement of two intervals left from *N*, regardless of what value *N* represents.

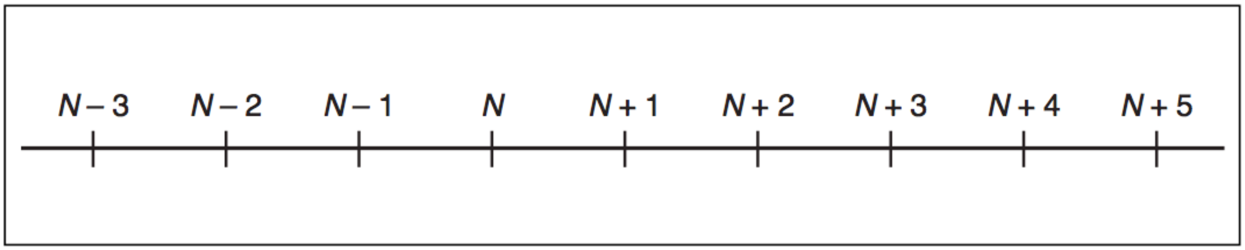



Figure 1. A portion of the variable number line, using *N* as a variable representing an unknown value. (Carraher et al., 2006, p. 97)

Students were presented with two number lines sharing the same metric on an overhead projector: a standard number line with zero at the origin and this new tool, a variable number line. Carraher et al. (2006) overlapped two number lines on the projector to align a specific integer to *N* for students to determine a single instance by assigning a numerical value to *N*. From there, students worked with exploring shifts in sets with regards to a set value for the variable *N*. Additionally, students explored comparisons of $N + 3$ to $N + 7$, inferring that $N + 7$ is positioned four markers to the right of $N + 3$.

A few lessons later, Carraher et al. (2006) presented the following problem to 9 and 10 year old students:

Tom is 4 inches taller than Maria.

Maria is 6 inches shorter than Leslie.

Draw Tom’s height, Maria’s height, and Leslie’s height.

Show what the number 4 and 6 refer to. (p. 104)

This problem demonstrates functional thinking by exploring relations among sets and the potential of shifting a set of numbers to map to another set of numbers. One student responded to the problem by drawing a variable number line, introduced in a previous lesson, with the assumption that Maria's height could be indicated by an unknown value *N*. Many students felt comfortable with using the variable *N* to represent the value of Maria's height.

From there, the same student expressed Tom's height and Leslie's height in relation to Maria's height, $N + 4$, $N + 6$, and *N*, respectively. The facilitators in the classroom then explored this initial reasoning by asking the students to instead assume that Leslie's height was *N*. Students used displacements on their number lines and responded that Tom's height was $N + 2$ and Maria's height was $N - 6$. When asked to assume that Tom's height was *N*, students in the class were able to identify Leslie's height as $N + 2$ and Maria's height as $N - 4$.

This relational thinking using variables to represent unknown values demonstrates algebraic thinking, and in particular, functional thinking. Students used a variable, *N*, to represent Maria's height and then identified how any possible height for Maria could be transformed to find Tom's and created expressions, such as $N + 4$, to represent this. Even when prompted to change what the variable *N* referred to, students adapted their expressions to associate with the correct transformations among the sets. From this classroom episode, it is important to realize that students did not invent the notational system labeled with symbolic expressions on their own; rather, students accepted the presented variable number line as their own tool and were able to access and apply the system in a problem presented in a later lesson (Carraher, et al., 2006).

One lesson given to third grade students by Carraher, Schliemann, and Schwartz (2008), called the "Candy Boxes Problem", follows.

Two boxes are shown to the class. Students are told that:

- The box in the left hand is John's, and all of John's candies are in that box.
- The box in the right hand is Mary's, and Mary's candies include those in the box as well as three additional candies resting atop the box.

- Each box has exactly the same number of candies inside. (p. 238)

Students are asked to say what they know about the number of candies John and Mary each have. At first, a majority of students (63.4%) focused on single instances by attempting to determine particular values to the number of candies each person had. In the majority of the responses, students represented amounts of candy with drawings or integers for particular cases. The facilitator actively creates a table of all suggested possibilities on the overhead projector. After discussing the validity of each proposed pair of numbers, students realize and acknowledge the existence of multiple possibilities. At this point, the focus of discussion moves toward making generalizations. Students agree that the difference between the number of candies that John and Mary have is 3, and they also make the argument that once an amount is assigned to one of the individuals, then the amount that the other individual holds is determinable. This concept, covariation, describes the relationship and behavior of two variables; when one variable changes, the other changes in a corresponding way.

When students work with the idea of indeterminate amounts, formal notation may also be introduced. In another classroom, Carraher, Schliemann, and Schwartz (2008) guided discussion in the "Candy Boxes Problem" by introducing and proposing that students use symbolic notation. When prompted to make a prediction about how many pieces of candy that both John and Mary had, one student was hesitant. The facilitator offered an alternative by suggesting that John has *N* pieces of candy, with *N* representing any amount. When asked how to say how many candies Mary has if John has *N* candies, some students responded, "*N*", because "she could have *any* amount like John" (p. 245). In this case, these students did not yet make a connection between the symbolic notation and its intended meaning (from the facilitator's perspective).

Although these students had an initial alternative interpretation of using symbols to represent a numerical amount, when asked if Mary has more than, less than, or the same amount of candies as John, they respond with "more" and identify the difference as being three more. From there, the facilitator asks the class how to write "three more than *N*", with *N* representing the amount of candies that John possesses. When a student replies with "*N* plus three", he reasons that Mary "could have any amount plus three" because she has three more candies than John does, identified previously as being the corresponding relationship in this situation. (p. 245)

This dialogue between the students and researchers displays the beginning of meaningful interpretation regarding functional thinking. These instances of dialogue in the classroom demonstrate thinking about the shifting of sets of numbers with a corresponding relationship ("plus three"). Following this dialogue, the facilitator led the class in making a table and students realized that no matter what value John has, Mary has that value shifted up by three. This statement is true for all particular instances and students move from thinking about individual pairs of numbers to the relationship between the two sets of numbers.

In this example, language facilitates algebraic reasoning by connecting algebraic symbols with intended meaning. The number of candies that John has is unknown; however, with symbolic notation, this piece of information can be identified by the variable, $N$. Essentially, students transform natural language, "Mary has three more candies than John has", into the algebraic expression, $N + 3$.

## Proposed Research Study

In this section of the paper, I describe a proposed research study that would deepen my understanding of this content and build on previous research in early algebra. While this study will not be conducted, the following is written in the future tense as would be appropriate for a research proposal. Data collection will involve collecting student work from classroom activities and videotaping small group interviews. This research will involve three mathematics classes of homogeneously grouped students in Grade 4 at a public school in the Tucson area. Each investigation activity will take place once a week for about 30-45 minutes per activity. Some activities may be done for a select group of students in Grade 5 participating in an after-school algebra club. The purpose of this research is to investigate whether elementary students are able to make meaningful connections between their elementary mathematics knowledge and early algebra activities. In particular, I am interested in the following research questions:

1. How do elementary students make meaningful connections in early algebra activities?
2. What role does natural language play in facilitating algebraic reasoning in the elementary grades?

**Classroom Activities**

For implementing classroom activities, I will divide the class into groups of 4-5 students each so the students may work with nearby peers. All students will be provided with paper to record their reasoning, which will be collected as data along with my own recording of open discussion comments facilitated by probing questions and classroom collaboration. All demonstrations of notational systems will be categorized after data collection. The following activities appear in work done by Carraher, Schliemann, and Schwartz (2008):

**Activity 1: Candy boxes problem.** The purpose of reproducing this problem in the classroom is to examine one aspect of the bidirectional role of language in early algebra by exploring the gradual introduction of symbolic notation. *How do students approach the problem? Do the students attempt to form generalizations? When presented with the concept of a* *variable, do students accept symbolic notation as a meaningful representation? In future activities, do students utilize symbolic notation for representing unknown amounts?*

Using props, I have two boxes filled with the same amount of candy in each box. One box has three candies sitting on top. I pick two volunteers, student A and student B, and use them to explain to the class that one box belongs to student A and the other box with three candies on top belongs to student B. The first task is for all students to individually express what they know about the number of candies student A and student B both have. Students are not allowed to open the boxes but may hold and examine each box. During the discussion of various examples of student representations, the focus will be on the relationship between the number of candies student A has compared to the number of candies student B has: *Can the number of candies be expressed for each student even though the exact number of candies in the box is unknown?*

**Activity 2: Wallet problem.** This problem will be used to explore the functional thinking aspect of early algebra. Students are given the following situation:

> Mike has $8 in his hand and the rest of his money is in his wallet; Robin has exactly 3 times as much money as Mike has in his wallet. What can you say about the amounts of money Mike and Robin have? (Carraher, Schliemann, & Schwartz, 2008, p. 248)

Students will need to express the problem in at least two representational forms of their choosing and record their interpretation of the situation on a given sheet of paper. To highlight functional thinking and relationships, we will create a table of possibilities together as a class and introduce other forms of representation such as graphing.

## Small Group Interviews

In smaller group interviews of 3-4 students, I will be collecting data of student written work along with recording dialogue driven by probing questions. The following problems appear in work done by Brizuela and Earnest (2008) and Lannin (2003):

**Activity 3: Let's make a deal problem.** This problem will investigate how context plays a role in semantics. The role of language in early algebra will also be explored. Students are given the following problem:

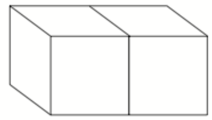
> Raymond has some money. His grandmother offers him two deals:
>
> *Deal 1:* She will double his money.
>
> *Deal 2:* She will triple his money and then take away 7.
>
> Raymond wants to choose the best deal. What should he do? How would you figure out and *show him* what is the best to do? Is one deal *always* better?
>
> Show this on a piece of paper. (Brizuela & Earnest, 2008, p. 281)

The interview will attempt to move through multiple types of representations, including verbal, symbolic, tabular, and graphical.

**Activity 4: Cube sticker problem [adapted from Lannin (2003)].**

Students are given the following problem:

> A company makes colored rods by joining cubes in a row and using a sticker machine to place "smiley" stickers on the rods. The machine places exactly 1 sticker on each exposed face of each cube. Every exposed face of each cube has to have a sticker; this rod of length two (2 cubes) would need 10 stickers.



How many stickers would you need for a rod of:

3 cubes?

4 cubes?

10 cubes?

22 cubes?

What is the rule? (p. 343)

Students will be provided with sheets of paper to record their work and have access to manipulatives, such as blocks. The main interest of this problem is to see what students do in their methods of solving for each prompt and what types of representations they produce.

**References**

Baroody, A. J. (1987). *Children's mathematical thinking.* New York: Teachers College, Columbia University.

Bastable, V., & Schifter, D. (2008). Classroom stories: Examples of elementary students engaged in early algebra. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 165-184). Mahwah, NJ: Lawrence Erlbaum Associates.

Blanton, M. L. , & Kaput, J. J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for Research in Mathematics Education*, *36*(5), 412- 446.

Brizuela, B. M., & Earnest, D. (2008). Multiple notational systems and algebraic understandings: The case of the "best deal" problem. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 273-301). Mahwah, NJ: Lawrence Erlbaum Associates.

Carpenter, T., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Portsmouth, NH: Heinemann.

Carraher, D. W., Schliemann, A. D., Brizuela, B., & Earnest, D. (2006). Arithmetic and algebra in early mathematics education. *Journal for Research in Mathematics Education*, *37*(2), 87-115.

Carraher, D. W., Schliemann, A. D., & Schwartz, J. L. (2008). Early algebra is not the same as algebra early. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 235-272). Mahwah, NJ: Lawrence Erlbaum Associates.

Dougherty, B. (2003). Measure up: A quantitative view of early algebra. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 389-412). Mahwah, NJ: Lawrence Erlbaum Associates.

Hatfield, M. M., Edwards, N. T., Bitter, G. G., & Morrow, J. (2000). *Mathematics methods for elementary and middle school teachers* (4th ed.). New York: John Wiley & Sons, Inc.

Hopkins, C., Gifford, S., & Pepperell, S. (1996). *Mathematics in the primary school: A sense of progression.* London: David Fulton.

Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels: Building meaning for symbols and their manipulation. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learnin*g (pp. 707-762). Charlotte, NC:  Information Age Publishing.

Knuth, E. J., Stephens, A. C., McNeil, N. M., & Alibali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education*, *37*(4), 297-312.

Mihalicek, V., & Wilson, C. (Eds.). (2011). *Language files* (11th ed). Columbus: The Ohio State University Press.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Rakes, C. R., Valentine, J. C., McGatha, M. B., & Ronau, R. N. (2010). Methods of instructional improvement in algebra: A systematic review and meta-analysis. *Review of Educational Research*, *80*(3), 372-400.