ACTIVE REFLECTIVE COMPONENTS FOR ADAPTIVE OPTICAL ZOOM SYSTEMS

by
Matthew Edward Lewis Jungwirth

A Dissertation Submitted to the Faculty of the
COLLEGE OF OPTICAL SCIENCES
In Partial Fulfillment of the Requirements
For the Degree of
DOCTOR OF PHILOSOPHY
In the Graduate College
THE UNIVERSITY OF ARIZONA

2012
THE UNIVERSITY OF ARIZONA
GRADUATE COLLEGE

As members of the Dissertation Committee, we certify that we have read the dissertation
prepared by Matthew Edward Lewis Jungwirth
entitled Active Reflective Components for Adaptive Optical Zoom Systems

and recommend that it be accepted as fulfilling the dissertation requirement for the
Degree of Doctor of Philosophy.

________________________________________  Date: 11/09/2012
Eustace L. Dereniak

________________________________________  Date: 11/09/2012
David V. Wick

________________________________________  Date: 11/09/2012
Jose M. Sasián

________________________________________  Date:

________________________________________  Date:

Final approval and acceptance of this dissertation is contingent upon the candidate’s submission
of the final copies of the dissertation to the Graduate College.

I hereby certify that I have read this dissertation prepared under my direction and recommend that
it be accepted as fulfilling the dissertation requirement.

________________________________________  Date: 11/09/2012
Dissertation Director: Eustace L. Dereniak
STATEMENT BY AUTHOR

This dissertation has been submitted in partial fulfillment of requirements for an advanced degree at The University of Arizona and is deposited in the University Library to be made available to borrowers under rules of the Library.

Brief quotations from this dissertation are allowable without special permission, provided that accurate acknowledgement of source is made. Requests for permission for extended quotation from or reproduction of this manuscript in whole or in part may be granted by the author.

Signed: Matthew Edward Lewis Jungwirth
ACKNOWLEDGEMENTS

I have so many people to thank for the success of this work. My gratitude is organized into sections, as any good engineer should do.

- Family: First and foremost, my lovely wife Erika who endured many weeks and months of widowhood in my effort to earn this degree. Second, my parents who supported me both emotionally and financially throughout my nearly decade-long stretch of higher education. Third, to my sister Emily, brother Aaron, and parents-in-law Laurence and Katy for their love and support.

- The Optical Detection Lab at the University of Arizona: I need to especially thank Eustace Dereniak for the wonderful research and career advising, and to Trish Pettijohn for keeping the lab running. I would especially like to thank Mike Kudenov, for the research opportunities and mentoring, and my former office mate Julia Craven-Jones, for many hours of comprehensive quizzing and many friendly discussions over coffee.

- Sandia National Laboratories: I would like to thank David Wick and Robert Spulak for giving me the opportunity to complete an exciting and rewarding project at a world-class research facility. The mentoring in research practice and in being a research engineer I received from David Wick and Brett Bagwell is invaluable and will benefit me throughout my career. Thank you also to Jared Milinazzo and Brian Clark for fabrication of mechanical parts, and especially to Freddie Santiago for being a springboard to many ideas and for patiently discussing research headaches and concepts.

- Research partners: This work would not have succeeded without the research groundwork and software development by Chris Wilcox. I would also like to thank Mike Baker for creating the FEM and diligently simulating actuation modalities, and to Clint Hobart for mechanical engineering support with the opto-mechanical structure of the active CFRP mirror.

- Finally, I would like to thank the patriarch of my maternal family Lewis Larson, whose own dream I believe I am at least partially fulfilling with the completion of this document.

All work was completed at Sandia National Laboratories. Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy’s National Nuclear Security Administration under contract DE-AC04-94AL85000.
DEDICATION

For my wonderful wife,
Erika,
who always listened
to my ‘squishy mirror’
progress with enthusiasm
and vigor.
## TABLE OF CONTENTS

LIST OF FIGURES ......................................................................................................................... 9

LIST OF TABLES ............................................................................................................................ 15

ABSTRACT ................................................................................................................................... 17

CHAPTER 1: INTRODUCTION .................................................................................................. 18
   1.1 Motivation ........................................................................................................................... 18
   1.2 Research Objectives ............................................................................................................ 18
   1.3 Aberration Theory ............................................................................................................... 19
      1.3.1 Individual Aberrations ................................................................................................. 20
      1.3.2 Zernike Polynomials .................................................................................................... 22
   1.4 Adaptive Optics ............................................................................................................ ....... 23
      1.4.1 Wavefront Sensing ....................................................................................................... 25
      1.4.2 Shack-Hartmann ........................................................................................................... 25
      1.4.3 Deformable Mirror ....................................................................................................... 28
      1.4.4 Closed-Loop Feedback Algorithm ............................................................................... 29

CHAPTER 2: DISCUSSION OF ZOOM SYSTEMS ................................................................... 32
   2.1 Basics of Zoom............................................................................................................. ....... 32
      2.1.1 Optical Zoom ............................................................................................................. ...32
   2.2 Mechanical Optical Zoom ................................................................................................... 33
      2.2.1 Mechanical Zoom Paraxial Theory .............................................................................. 34
      2.2.2 Mechanical and Optical Compensation ........................................................................ 37
   2.3 Digital Zoom ....................................................................................................................... 38
   2.4 Adaptive Optical Zoom ....................................................................................................... 39
      2.4.1 AOZ Paraxial Theory ................................................................................................... 39
      2.4.2 Mechanical and Adaptive Zoom Comparison .............................................................. 42
      2.4.3 Other Design Theories of AOZ ....................................................................................43
      2.4.4 Experimental Examples of AOZ .................................................................................. 46

CHAPTER 3: THEORY OF ADAPTIVE OPTICAL ZOOM ...................................................... 50
   3.1 Third-Order Design Theory................................................................................................. 50
      3.1.1 Cassegrain Telescope Design ....................................................................................... 50
      3.1.2 Cassegrain AOZ Design .............................................................................................. 51
   3.2 Aberration Simulation ......................................................................................................... 54
      3.2.1 Bilateral System Theory ............................................................................................... 54
      3.2.2 Ray Tracing .................................................................................................................. 56
      3.2.3 AOZ Aberration Simulation ......................................................................................... 56
   3.3 Simulation Code .................................................................................................................. 57
      3.3.1 Logic Flow ................................................................................................................... 57
      3.3.2 Determination of Obscuration Ratio ............................................................................ 58
      3.3.3 Code Description ......................................................................................................... 59
      3.3.4 Verification ............................................................................................................... 60
## TABLE OF CONTENTS - Continued

3.4 Tradespace Analysis........................................................................................................ 62  
3.4.1 Set Primary and Image Diameters............................................................................. 63  
3.4.2 Set Construction Parameters.................................................................................... 64  
3.5 Optical Design Results.................................................................................................... 64  
3.6 Residual Aberration Correction...................................................................................... 67

CHAPTER 4: CARBON FIBER REINFORCED POLYMER MIRRORS .................................. 69  
4.1 Material Properties ........................................................................................................ 69  
4.2 Fabrication Processes and Concerns ............................................................................. 72  
4.2.1 Mirror Fabrication...................................................................................................... 72  
4.2.2 Fiber Print-through.................................................................................................... 74  
4.3 CFRP Mirrors.................................................................................................................. 76  
4.3.1 Finite Element Modeling............................................................................................ 76  
4.3.2 Recent Static Mirrors.................................................................................................. 78  
4.3.3 Recent Active Mirrors............................................................................................... 79  
4.4 CFRP & Glass Comparison............................................................................................. 82

CHAPTER 5: ACTIVE MIRROR TESTBED .............................................................................. 85  
5.1 Optical Design................................................................................................................ 85  
5.1.1 Design Requirements ................................................................................................. 85  
5.1.2 First-Order Design...................................................................................................... 86  
5.1.3 Final Optical Layout.................................................................................................... 87  
5.1.4 Component Selection................................................................................................. 89  
5.2 Closed-Loop Control...................................................................................................... 89  
5.2.1 CASAO Inputs ............................................................................................................ 90  
5.2.2 CASAO Closed-Loop Execution ............................................................................. 91  
5.2.3 HASO Wavefront Graphs........................................................................................... 91  
5.3 System Verification......................................................................................................... 92  
5.3.1 Mirao Error .............................................................................................................. 92  
5.3.2 Pupil Conjugation....................................................................................................... 93  
5.3.3 Comparison to Zemax............................................................................................... 93  
5.4 COTS Mirror Aberration Correction............................................................................. 96  
5.5 Zygo Testbed................................................................................................................... 98

CHAPTER 6: ACTIVE OPTICAL CFRP MIRROR ................................................................. 100  
6.1 FEM of Actuation Modalities........................................................................................ 100  
6.1.1 Point-Load and Radial Force Actuation................................................................. 100  
6.1.2 Radial/Ring Actuation............................................................................................... 100  
6.1.3 Annular Ring Actuation............................................................................................. 102  
6.1.4 Moment Actuation.................................................................................................... 103  
6.1.5 Actuation Modality Selection................................................................................. 104  
6.1.6 Actuator Selection..................................................................................................... 105  
6.2 Open-Loop ROC Increase............................................................................................ 106  
6.2.1 Open-Loop Mirror and Apparatus.......................................................................... 107  
6.2.2 Open-Loop Results.................................................................................................. 108  
6.3 Annular Ring Opto-Mechanical Apparatus................................................................. 110
TABLE OF CONTENTS - Continued

6.3.1 CFRP Mirror .............................................................................................................. 110
6.3.2 General Description.................................................................................................... 112
6.3.3 Effect of Edge Constrain ............................................................................................ 113
6.3.4 Plunger Mount............................................................................................................ 114
6.3.5 Mirror & Plunger Alignment...................................................................................... 116
6.3.6 Inner Ring Alignment................................................................................................. 123
6.3.7 Outer Ring Alignment................................................................................................ 124
6.3.8 Actuator Influence Function....................................................................................... 124
6.3.9 Final Apparatus .......................................................................................................... 125
6.4 Closed-Loop ROC Increase ............................................................................................... 126
6.4.1 Actuator Engagement................................................................................................. 126
6.4.2 ROC Change Procedure ............................................................................................. 128
6.4.3 Experimental Results.................................................................................................. 129
6.4.4 Repeatability between ROCs ...................................................................................... 135
6.4.5 Annular Ring Sag Change.......................................................................................... 136
6.4.6 FEM Predicted WFE .................................................................................................. 136

CHAPTER 7: POTENTIAL APPLICATIONS........................................................................... 138
7.1 Active Secondary Mirror ............................................................................................... 138
7.2 Phase Diversity ............................................................................................................ 139
7.3 Adaptive Optical Zoom ................................................................................................. 139

CHAPTER 8: CONCLUSIONS .................................................................................................. 141
8.1 Summary of Research ................................................................................................. 141
8.2 Final Remarks .............................................................................................................. 141

APPENDIX A: AOZ THEORY CODE....................................................................................... 143
APPENDIX B: ANNULAR RING SAG CHANGE ................................................................. 145
REFERENCES ........................................................................................................................ 149
LIST OF FIGURES

Figure 1. Schematic of a base-level reflective adaptive optics system. ......................................................... 25
Figure 2. Schematic of a SHWS with a partially aberrated beam incident upon a 1-D lenslet array. The dotted line represents the local optical axis for a single lens within the array. ................................. 26
Figure 3. Schematic of a DM. The ‘flexible face sheet’ is the reflecting surface that is moved by the actuators behind it. ......................................................................................................................... 28
Figure 4. Schematic of electrostatically driven DM. The dotted line represents a varying potential across the face sheet. ..................................................................................................................... 28
Figure 5. Specific pictures of mirao. (a) Layout of the 52 actuators, and WFE of the mirao with (b) convex and (c) concave curvatures. ........................................................................................................... 29
Figure 6. Mechanical zoom system with only one moving element. Notice that only two positions result in zero displacement of the image plane. ................................................................. 33
Figure 7. Layout of a two-element mechanical zoom system for paraxial theory derivation. Positive distances move from left to right, negative distances vice versa. ........................................... 35
Figure 8. An example of (a) a mechanically compensated zoom system and (b) an optically compensated zoom system. ........................................................................................................... 38
Figure 9. Image of an airport zoomed electronically and optically by 8X. a) Unzoomed image, b) digital zoom with no resolution gain, and c) adaptive optical zoom with increased resolution. ................. 38
Figure 10. Layout of two-element AOZ system for paraxial theory – (a) system at unit magnification, and (b) Gaussian reduced system at unit magnification. P and P’ denote the system principal planes. ................................. 41
Figure 11. Correlation between the primary and secondary sag changes for a given zoom ratio and field angle. ........................................................................................................................... 44
Figure 12. “Enhanced first-order design” of an AOZ system. Three states are displayed, with zoom ratios of (a) 1X, (b) 1.5X, and (c) 3X. ................................................................. 45
Figure 13. A 3.9X AOZ system designed in Zemax. The top layout is the unzoomed state, corresponding to the ‘F’ on the left, and the bottom layout is the zoomed state, corresponding to the large ‘F’ on the right. 46
Figure 14. Air Force resolution chart enlarged 3.3X using an SLM-based active optical zoom sensor. ................................. 47
Figure 15. An SLM-based 5X AOZ system. By adding optical tilt, any portion of the Air Force chart can be magnified. 49
Figure 16. Experimental AOZ system. (a) Schematic of layout with three states, with zoom ratios of 1X, 1.5X, and 3X from left to right, respectively. (b) Image taken with system at 3X zoom state. 48
LIST OF FIGURES - Continued

Figure 17. Coordinate system for third-order design, with the chief and marginal rays displayed. The stop is at the primary. Labels: $D_p$ is the diameter of the primary mirror, $XP$ is the exit pupil, $d$ is the axial separation between the primary and secondary mirrors, $l_n$ is the distance between the exit pupil and image plane, $WD$ is the axial distance between the primary apex and the image plane (‘working distance’), $D_I$ is the diameter of the image, $\theta_n$ is the maximum field angle in object space (HFOV) and $\theta_n'$ is the maximum field angle in image space.

Figure 18. Schematic of logic for simulation program. Labels: Y stands for ‘yes’ and N stands for ‘no.’

Figure 19. Paraxial layout of the two-element AOZ system to derive the obscuration ratio. The mirrors are represented by thin-lenses.

Figure 20. Verification of simulation code for tangential and sagittal OPD plots (top and bottom, respectively) for an on-axis field.

Figure 21. Verification of simulation code for tangential and sagittal OPD plots (top and bottom, respectively) at an HFOV of 0.5º.

Figure 22. Number of viable AOZ designs found for a Cassegrain objective as the image diameter $D_I$ and primary diameter $D_p$ vary. The colorbar’s units are number of designs.

Figure 23. Results from a parameter sweep with $D_I = 7.5$mm and $D_p = 375$mm. The obscuration ratio on the y-axis is for the unzoomed state. Designs to the right of the dashed red line are the ‘best’ for this analysis.

Figure 24. AOZ system with 375mm diameter primary. (a) System layout and (b) spot diagrams for the unzoomed state, and (c) system layout and (d) spot diagrams for the zoomed state. Normalized field heights of 0.0, 0.7, and 1.0 (clockwise from upper left) are displayed for each state along with the Airy disc.

Figure 25. Schematic of additional AO system for aberration correction. The marginal ray is displayed as a dotted line. Labels: $M_1$ is the primary mirror, $M_2$ is the secondary mirror, $L_1$ is the collimating lens, $L_2$ and $L_3$ are an image relay system, $L_4$ is the imaging lens, $BS_1$ and $BS_2$ are 50/50 beamsplitters, $DM$ is a deformable mirror, $SHWS$ is a Shack-Hartmann wavefront sensor, and $\hat{y}_0$ is the image plane.

Figure 26. An example of a CFRP mirror. This mirror was fabricated by Composite Mirror Applications.

Figure 27. (a) Diagram of four-point bend test. (b) Results from bend test for a 24-ply CFRP coupon. The y-axis has units of pounds-force (lbf).

Figure 28. Outline of CFRP fabrication by CMA.

Figure 29. Ronchigrams of (left) glass mandrel and (right) CFRP mirror replicated from the mandrel.

Figure 30. Example of two different lay-up schemes.

Figure 31. Examples of FPT causing a (a) periodic structure and (b) a random structure as several layers are visible.
LIST OF FIGURES - Continued

Figure 32. Control of FPT using (a) an extra resin layer and (b) a proprietary method. 75

Figure 33. Gravitationally induced deformation predicted by FEM. The dimples are point-load actuators attached to the back of the mirror. 77

Figure 34. FEM results of CFRP mirror actuation. (a) Predicted wavefront due to apex point-load and (b) measured wavefront. (c) Predicted wavefront for uniform pressure and (d) cross-sectional error. 78

Figure 35. Wavefront of a flat static CFRP mirror after manual polishing. 79

Figure 36. Pictures of static CFRP mirrors produced by CMA, (a) with and (b) without a central aperture. 79

Figure 37. Images of the first active CFRP mirror. (a) Schematic of actuator layout, (b) wavefront after open loop correction, and (c) comparison of actuator influence function between FEM (red) and experiment (blue). 80

Figure 38. (a) 80mm diameter CFRP mirror with seven actuators, (b) interferogram showing influence of glue spots to hold neodymium magnets, and (c) corrected wavefront. 81

Figure 39. Simulated force vs. displacement curves for Zerodur (blue) and CFRP (green). Notice that the same amount of force produces more deflection for CFRP than for Zerodur. 83

Figure 40. First-order layout of AMT. The solid line is the optical axis, the dotted lines are the beam, and the double arrows are beamsplitters. Other labels: $L_i$ are lenses, $f_i$ are focal lengths, and $R$ is the ROC of the CFRP mirror. 87

Figure 41. Final active mirror testbed – (a) schematic and (b) picture with beam paths in red. Note that (b) shows the AMT with a flat mirror at the system reference point labeled in (a). Labels: PH is pinhole, BS$_1$ and BS$_2$ are cube beamsplitters, and ‘AMT Ref’ is where the system reference is measured. 88

Figure 42. Screenshots of CASAO software showing how to (a) calculate the influence matrix, (b) calculate the command matrix, and (c) run closed-loop correction. 92

Figure 43. Mirao at ‘flat’ setting, measured with the Zygo. 92

Figure 44. Images for experimental verification of correct pupil conjugation – (a) a ruler taped in front of the COTS mirror in Section 5.3.4 and (b) the image of the ruler at the SHWS’ entrance pupil. 93

Figure 45. Zemax-predicted error of the AMT. 94

Figure 46. Residual error within the AMT - (a) WFE without the mirao for comparison to Figure 45, (b) with the mirao, and (c) AMT after closed-loop correction. 95

Figure 47. Zernike aberration coefficients for the AMT. Labels: ‘F’ is focus, ‘A0’ is astigmatism at 0º, ‘A45’ is astigmatism at 45º, ‘C0’ is coma at 0º, ‘C90’ is coma at 90º, ‘SA’ is spherical aberration, ‘T0’ is trefoil at 0º, and ‘T90’ is trefoil at 90º. 96

Figure 48. WFE of testbed. (a) Zygo shot of COTS mirror, WFE of mirror with (b) no correction, (c) with correction, and (d) Zernike aberrations coefficients. Focus was removed in all graphs. Labels in (d) are as in Figure 47. 97
LIST OF FIGURES - Continued

Figure 49. Picture of Zygo testbed. The arrows show the folded ‘W’ path the test beam travels from the Zygo to the test mirror. The return beam follows an identical path. ...............................98

Figure 50. FEM results for (a) point-load actuation and (b) radial inward force. The scales on the right are in microns. Note that ‘FEA’ stands for finite element analysis, an alternate acronym for FEM. ............................................................................................................................................ 101

Figure 51. (a) 2D phase map and (b) 1D cut of radial force and annular ring combination, with the ring 120 mm in diameter. The scales are in microns. Again, FEA is another acronym for FEM. (c) Potential apparatus for radial/ring modality. Note that the annular ring is not pictured. ....... 102

Figure 52. Cross-section of a CFRP mirror at 4000mm ROC when pushing on (a) a single annular and (b) two annular rings. Note the difference in y-axis scales. .............................................................................. 103

Figure 53. (a) 2D phase map and (b) 1D cut of moment actuation method. The scales are in microns. .........................................................................................................................................104

Figure 54. Open-loop CFRP mirror – (a) picture of mirror and (b) Zygo shot of WFE. .......... 107

Figure 55. Opto-mechanical apparatus for open-loop ROC increase. (a) Schematic of actuator and ring layout, (b) back of mirror with actuators protruding, and (c) predicted WFE for $\Delta$ROC = 100%. ........................................................................................................................................... 108

Figure 56. Open-loop wavefront maps for (a) $\Delta$ROC = 0.0 mm, (b) $\Delta$ROC = 48.7 mm, and (c) $\Delta$ROC = 97.4 mm. The scales on the right are in waves. ............................................................ 109

Figure 57. Side-by-side comparison of glass mirror and CFRP mirror. ...............................111

Figure 58. WFE of CFRP mirror measured using Zygo. ............................................................. 111

Figure 59. A second CFRP mirror rotated by 45º from (a) to (b). Aberrations except for astigmatism were removed from the WFE. ..........................................................112

Figure 60. Annular ring opto-mechanical apparatus - (a) schematic of actuator and ring layout, (b) zoomed picture showing mirror edge and plunger, and (c) mirror mount. .......................113

Figure 61. Hartmanngrams displaying effect of edge constrain. (a) Mirror overconstrained by rubber gasket and (b) mirror underconstrained by plunger. ..............................................................114

Figure 62. CFRP mirror mount (“plunger”). (a) CAD picture of mount and (b) picture displaying magnets ability to hold the CFRP mirror to the plunger. .................................................................115

Figure 63. Neodymium magnet alignment guide. The outside ring is 200mm in diameter, so that it is outside the CFRP mirror’s edge, and the black dots represent magnet placement. ...............116

Figure 64. WFE of CFRP mirror (taken on the Zygo tested) as the mirror is rotated – (a) 0º, (b) 90º, (c) 180º, and (d) 270º. The dark striations are caused by vibrational noise within the testbed. ............................................................................................................................................. 117

Figure 65. Data from the mirror placement test. (a) Schematic of test, (b) WFE as CFRP angle changes with plunger fixed, and (c) WFE as plunger angle changes with CFRP angle fixed. ....... 119

Figure 66. Zygo wavefront maps while CFRP angle is held fixed with respect to the plunger. Displayed are plunger angles of (a) 90º, (b) 135º, (c) 180º, and (d) 315º. .................................120
Figure 67. 1D cuts of the WFE from Figure 66. Displayed are plunger angles of (a) 90°, (b) 135°, (c) 180°, and (d) 315°, i.e., correlated to Figure 66. Note that the wavefront diameter was masked to the proper 160mm clear aperture. ................................................................. 121

Figure 68. CFRP WFE following mirror and plunger alignment – (a) WFE measured with the Zgyo, (b) WFE measured with the AMT, and (c) Zernike coefficients of CFRP mirror. Focus was removed throughout this figure. Labels: ‘F’ is focus, ‘A0’ is astigmatism at 0°, ‘A45’ is astigmatism at 45°, ‘C0’ is coma at 0°, ‘C90’ is coma at 90°, ‘SA’ is spherical aberration, ‘T0’ is trefoil at 0°, and ‘T90’ is trefoil at 90°. ........................................................... 122

Figure 69. Increased WFE caused by weight of rings – (a) with rings attached and (b) without rings. ................................................................................................................................. 123

Figure 70. Inner ring alignment. (a) CAD picture of ring with magnet holes and (b) ring in place on mount and actuators. ................................................................................................................................. 123

Figure 71. Outer ring alignment. (a) Picture of ring alignment tool, and (b) picture displaying how to use tool for alignment. ................................................................................................................................. 124

Figure 72. Close-up of rubber gasket attached to outer ring. ................................................................................................................................. 125

Figure 73. Final opto-mechanical apparatus for the active CFRP mirror – (a) from the front showing the mirror surface and (b) from the back with the actuators protruding. ................................................................................................................................. 125

Figure 74. Screenshots from HASO of wavefronts during actuator engagement – (a) actuator not engaged, and (b) actuator moved forward by 0.05mm. The circles highlights the change in the wavefronts and PV error due to actuator displacement. ................................................................................................................................. 127

Figure 75. Effect of PI actuator engagement – (a) WFE with no actuators engaged, (b) WFE with engaged actuators, and (c) change in low-order Zernike coefficients. Labels: ‘F’ is focus, ‘A0’ is astigmatism at 0°, ‘A45’ is astigmatism at 45°, ‘C0’ is coma at 0°, ‘C90’ is coma at 90°, ‘SA’ is spherical aberration, ‘T0’ is trefoil at 0°, and ‘T90’ is trefoil at 90°. ................................................................................................................................. 128

Figure 76. WFE results for ΔROC = 0mm - (a) no correction, (b) no wavefront graph, (c) correction of (a) with mirao, and (d) aberration coefficients for (a) and (c). ................................................................................................................................. 130

Figure 77. WFE results for ΔROC = 6mm - (a) no correction, (b) correction of (a) with CFRP mirror, (c) correction of (b) with mirao, and (d) aberration coefficients for (a) and (c). ................................................................................................................................. 131

Figure 78. WFE results for ΔROC = 10mm - (a) no correction, (b) correction of (a) with CFRP mirror, (c) correction of (b) with mirao, and (d) aberration coefficients for (a) and (c). ................................................................................................................................. 132

Figure 79. Plot of corrective ability of active CFRP mirror at all tested ROC increases. Only the largest aberration coefficients are included. ................................................................................................................................. 133

Figure 80. Error correction during closed-loop control – (a) PV and (b) RMS WFE during correction. ................................................................................................................................. 134

Figure 81. FEM-predicted and actual WFE of the active CFRP mirror – (a) FEM-predicted for ΔROC = 100.0mm and (b) experimental WFE for ΔROC = 10.0mm. ................................................................................................................................. 137

Figure 82. Active secondary mirror and assembly in place on the MMT. ................................................................................................................................. 138

Figure 83. Schematic of phase diversity. Notice how the two ray bundles vary with defocus. 90 139
LIST OF FIGURES - Continued

Figure 84. Possible AOZ design using an active CFRP mirror. Note that this design was created by David Wick, PhD, of Sandia National Laboratories............................................................... 140

Figure 85. Re-definition of two different spherical surfaces being held at the edge. .................. 146

Figure 86. Sag changes during the ROC increase in Section 6.4.3 – (a) layout of actuator names, (b) inner ring actuators, (c) outer ring actuators, (d) average of inner ring actuators versus predicted, and (e) average of outer ring actuators versus predicted......................................................... 148
LIST OF TABLES

Table 1. Functional forms of the seven Seidel aberrations .............................................................. 20
Table 2. Third order aberrations in Zernike polynomials ............................................................... 23
Table 3. System parameters for a 2.75X mechanical zoom system ................................................ 36
Table 4. System parameters for a 2.75X AOZ system ................................................................. 42
Table 5. Listing of Wetherell and Rimmer equations\textsuperscript{51} utilized in the AOZ derivation along with a brief description of the equation. The equation number listed in the first column is the equation number as listed in Ref. 51. ........................................................................................................... 51
Table 6. Equations from Ref. 52 used during aberration simulation to determine the Seidel coefficients. The right-hand column contains the equation number as listed in the original publication. \( N \) is the total number of optical surfaces ............................................................................. 55
Table 7. Initial criteria used to down-select potential AOZ designs. Labels: \( T_L \) is the threshold on system length, \( T_z \) is the threshold on zoom ratio, and \( T_\varepsilon \) is the threshold on obscuration ratio. .. 58
Table 8. Optical prescription for the non-zooming Cassegrain\textsuperscript{55} used to verify the simulation code ................................................................................................................................................ 60
Table 9. Comparison of the aberration coefficients as determined by Zemax and the simulation code. The agreement between the values is exemplar. ................................................................................................................ 61
Table 10. Construction values and system attributes for the AOZ system in Figure 24 with \( D_s = 7.5 \) mm and \( D_p = 375 \) mm. As before, ‘wv’ are in reference to the wavelength of a HeNe laser. Other labels: \( D_{\text{stop}} \) is the stop diameter and \( D_{\text{secondary}} \) is the secondary diameter. ........................... 66
Table 11. Comparison between simulated and final construction parameters ............................... 67
Table 12. Material properties for CFRP and other materials ‘SiC’ stands for silicon carbide.\textsuperscript{61}... 71
Table 13. Closed-loop parameter settings for correction with the COTS mirror. The parameters are described in Section 5.2.1 ................................................................................................................................. 98
Table 14. Comparison of the five actuation modalities discussed in this dissertation ................. 104
Table 15. Comparison of COTS actuators for the annular ring actuation modality. ................. 106
Table 16. Aberration values for the three tested states with the open-loop mirror. All coefficients are listed in waves. Labels: \( \Delta \text{ROC} = \) change in radius of curvature, \( \text{SA} = \) spherical aberration, and \( \text{PV} = \) peak-to-valley ................................................................................................................................. 110
Table 17. Slope and WFE data for the four examined wavefronts in Figure 65 and Figure 66. The slope values have arbitrary units of waves/pixels. The given PV and RMS data is for the correct 160mm clear aperture. Labels: STD = standard deviation and CV = coefficient of variation..... 121
Table 18. Closed-loop parameter settings for correction with the CFRP mirror and mirao. The parameters are described in Section 5.2.1 ................................................................................................................ 134
Table 19. Repeatability of WFE between ROC states of ΔROC = 6, 10mm. The left section shows results when only the CFRP mirror moves between states and the right section shows results when both the CFRP and mirao move between states. Note that CV = coefficient of variation, and other labels are as previously defined.
ABSTRACT

This dissertation presents the theoretical and experimental exploration of active reflective components specifically for large-aperture adaptive optical zoom systems. An active reflective component can change its focal length by physically deforming its reflecting surface. Adaptive optical zoom (AOZ) utilizes active components in order to change magnification and achieve optical zoom, as opposed to traditional zooming systems that move elements along the optical axis.

AOZ systems are theoretically examined using a novel optical design theory that enables a full-scale tradespace analysis, where optical design begins from a broad perspective and optimizes to a particular system. The theory applies existing strategies for telescope design and aberration simulation to AOZ, culminating in the design of a Cassegrain objective with a 3.3X zoom ratio and a 375mm entrance aperture.

AOZ systems are experimentally examined with the development of a large-aperture active mirror constructed of a composite material called carbon fiber reinforced polymer (CFRP). The active CFRP mirror uses a novel actuation method to change radius of curvature, where actuators press against two annular rings placed on the mirror’s back. This method enables the radius of curvature to increase from 2000mm to 2010mm. Closed-loop control maintains good optical performance of 1.05 waves peak-to-valley (with respect to a HeNe laser) when the active CFRP mirror is used in conjunction with a commercial deformable mirror.
CHAPTER 1: INTRODUCTION

1.1 Motivation
A “zoom” lens system is an image-forming optical system that can vary the object magnification while maintaining the image plane’s absolute position. This capability has far-reaching applications, affecting disciplines as diverse as defense, astronomy, and consumer electronics. Mechanical zoom, a type of zoom system that uses axial movement of optical components, has been fully realized at small apertures (diameters of a few centimeters). Moving to large apertures (greater than ~10cm) has proven difficult due to constraints in size, weight, and power. For example, mechanical zoom requires submillimeter element displacement accuracies, a monumental task for a meter-class glass mirror that weighs a few tons and thus requires a bulky and power-hungry opto-mechanical apparatus to achieve zoom.

Another approach to zoom, called adaptive optical zoom, circumvents this issue by removing the need for element movement. Instead, zoom is achieved with active components, which are optical elements that can alter their focal length through a physical change in shape or phase profile. Reflective adaptive optical zoom, in particular, holds great promise for large aperture systems since the only motions are sagitta (commonly called sag) changes of a few millimeters. The main hurdles to the realization of large aperture adaptive optical zoom systems are the lack of available optical design theories and the dearth of large-aperture active mirrors.

1.2 Research Objectives
The objective of this dissertation is to clear these two hurdles with the development of a novel design theory for adaptive optical zoom and the experimental demonstration of a large-aperture active mirror. The theory applies existing strategies for telescope design and aberration simulation to adaptive optical zoom, culminating in a system with a 3.3X zoom ratio and a 375mm entrance aperture. It also enables a full-scale tradespace analysis, where design begins from a broad perspective and optimizes to a particular design, allowing a designer to choose designs based on a variety of criteria. The active mirror has a 160mm clear aperture and is constructed of a composite called carbon fiber reinforced polymer (CFRP), a material that allows sag changes far beyond the elastic limits of glass or silicon carbide. In fact, a novel actuation method using two annular rings allows the radius of curvature to increase by 10mm while maintaining good optical quality.
The underlying goal of this dissertation is the realization of an experimental adaptive optical zoom system, to be designed using the derived optical theory and utilizing the active CFRP mirror as the primary. To that end, the radius of curvature change would likely need to be greater than ~10%, an increase not realized in this dissertation. However, finite element modeling (performed in Chapter 6) suggests that such increases are possible with the annular ring actuation method.

This dissertation is organized into eight chapters. The current chapter continues with overviews of aberration theory and adaptive optics in order to orient the reader with sufficient background to the presented research. Chapter 2 provides a discussion of zoom systems, including general layouts and designs, with particular emphasis upon recent advances in adaptive optical zoom. Chapter 3 derives and presents the novel optical design theory and tradespace analysis for adaptive zoom systems. Chapter 4 presents the mechanical properties of CFRP, current research into CFRP mirrors, and a quantitative comparison to an active glass mirror. Chapter 5 details the design and verification of the adaptive mirror testbed (AMT), an apparatus constructed specifically for the testing of active reflective components. Chapter 6 presents the opto-mechanical design and experimental testing of a large-aperture active CFRP mirror. Chapter 7 presents several possible applications for the presented research, and Chapter 8 provides a brief conclusion and closing remarks.

1.3 Aberration Theory
A wavefront is a surface (curved, flat, or otherwise) of equal phase located at continuous intervals from a light source. All real wavefronts contain aberrations, which are deviations from a perfect wavefront (i.e., errors). Aberrations are caused by a variety of factors, such as fabrication errors, misalignment of elements, or diffractive effects. Aberrations are measured in the exit pupil of an optical system, the image of the stop in image space.

The level of aberration in a wavefront is relative to a reference wavefront, typically either a perfect spherical or plane wave, although the reference surface can take any shape. Subtracting the phases of the two wavefronts as a function of pupil position results in the wavefront error (WFE) equation $W$, given as

$$ W(\rho, \theta) = W_A(\rho, \theta) - W_R(\rho, \theta), $$

where $W_A$ is the aberrated wavefront, $W_R$ is the reference wavefront, and $\rho$ and $\theta$ are polar coordinates within the exit pupil. Eq. 1 relates optical phase and is generally measured in ‘units’
of optical path difference (OPD). OPD is essential to all wavefront measurement and is commonly specified in terms of the wavelength of the incident light, a unit call ‘waves’ since the OPD is normalized by wavelength. For this dissertation, all OPDs and aberrations are referenced to the main wavelength mode of a helium-neon laser (632.8nm).

1.3.1 Individual Aberrations

For ease in analysis, Eq. 1 is divided into individual aberrations with different functional forms. Each aberration is essentially a separate term in a mathematical expansion. A common wavefront expansion uses a rotationally symmetric power series, given as

\[ W = \sum_{k,l,m} W_{klm} H^k \rho^l \cos^m \theta, \tag{2} \]

where \( W_{klm} \) is an aberration specific coefficient, \( H^k \) is the normalized field height, \( \rho^l \) is the normalized pupil radius, and \( \theta \) is the angle within the pupil plane. Aberrations are delineated by distinct combinations of the indices \( k, l, \) and \( m \).

The first theoretical analysis of optical aberrations was performed by Seidel,\(^7\) describing aberrations to the 3\(^{rd}\) order in functional form, as given by the Maclaurin series expansion of the sine function.\(^8\) Seidel separated these low-order aberrations into seven aberrations, now known as the Seidel aberrations; Table 1 gives the name and functional form of these. The remainder of this section will briefly describe the Seidel aberrations as they form the basis of analysis throughout this dissertation.

Table 1. Functional forms of the seven Seidel aberrations.

<table>
<thead>
<tr>
<th>Aberration</th>
<th>WFE Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tilt</td>
<td>( W_{111} H \rho \cos \theta )</td>
</tr>
<tr>
<td>Focus</td>
<td>( W_{020} \rho^2 )</td>
</tr>
<tr>
<td>Spherical Aberration</td>
<td>( W_{040} \rho^4 )</td>
</tr>
<tr>
<td>Astigmatism</td>
<td>( W_{222} H^2 \rho^2 \cos^2 \theta )</td>
</tr>
<tr>
<td>Coma</td>
<td>( W_{131} H \rho^3 \cos \theta )</td>
</tr>
<tr>
<td>Distortion</td>
<td>( W_{311} H^3 \rho \cos \theta )</td>
</tr>
<tr>
<td>Field Curvature</td>
<td>( W_{220} H^2 \rho^2 )</td>
</tr>
</tbody>
</table>
Tilt
Wavefront tilt or magnification describes the difference between paraxial and actual magnifications within an optical system. It is not actually an optical aberration since it does not describe wavefront curvature. Tilt is mitigated in the Adaptive Mirror Testbed (AMT) with proper element alignment. It is ignored during closed-loop correction in Chapters 5 and 6.

Focus
Focus describes the optical power within a wavefront. It is zero at the paraxial image plane. Changing the radius of curvature (ROC) of an optical element will increase focus unless the image plane is shifted to the paraxial image plane, a vital consequence that affected the design of the AMT in Chapter 5.

Spherical Aberration
Spherical aberration occurs when an element’s focal length varies with pupil radius. This causes a separation between marginal focus, where the marginal rays cross the optical axis, and paraxial focus, where the paraxial rays cross the axis. All spherical surfaces will exhibit spherical aberration unless a spherical beam of the same ROC is incident upon the surface, another design concern of the AMT design.

Astigmatism
Astigmatism occurs when optical power is unequal in orthogonal directions. This causes the image of point source to be a line at tangential and sagittal foci and circular blur in between these two points. Gravity can induce astigmatism in a thin-shelled mirror, such as the CFRP mirror, by increasing ROC in the vertical axis of the mirror, inducing an optical power difference between the sagittal and tangential components.

Coma
Coma causes the magnification to vary with pupil radius, resulting in an asymmetric blur in the tangential plane due to marginal rays being imaged to a different point than paraxial rays. A non-uniform ROC change forces the image point off-axis, inducing coma at the image plane in the AMT.

Distortion
Distortion occurs when the magnification varies with field height. Points image to points in the image plane, so there is not blur, but straight lines map to curved lines, causing the image to be curved at the edge of the field. It is almost completely ignored in this dissertation since it is not directly described by the Zernike polynomial wavefront expansion in Section 1.3.2.
Field Curvature

Field curvature describes the natural tendency for light to focus upon a curved image surface. The majority of optical element surfaces are spherical and thus cause an incoming plane wave to focus as a spherical wave. Controlling field curvature involves forcing the Petzval sum to zero, as a previous adaptive optical zoom (AOZ) design theory utilizes in Section 2.4.3.

1.3.2 Zernike Polynomials

The power series expansion of the WFE function given in Eq. 2 is functional but not orthonormal over a circle. Since the active CFRP mirror is circular, a more appropriate expansion is one that is orthonormal over the unit circle. One such set, Zernike polynomials, have steadily gained in popularity to become the *de facto* description of aberrations. Normalization and numbering schemes for Zernike polynomials abound. For this dissertation, the Zernike formalism is given as

\[
Z^m_n(\rho, \theta) = \begin{cases} 
    N^m_n R^m_n(\rho) \cos(m\theta); & \text{for } m \geq 0 \\
    -N^m_n R^m_n(\rho) \sin(m\theta); & \text{for } m < 0 
\end{cases},
\]

for \( n = 0, 1, 2, \ldots \)

where \( R^m_n(\rho) \) is given as

\[
R^m_n = \sum_{s=0}^{n+|m|} (-1)^s (n-s)! \left\{ \binom{n+|m|}{2} - s \right\} \left\{ \binom{n-|m|}{2} - s \right\} \rho^{n-2s},
\]

and the normalization factor \( N^m_n \) is given by

\[
N^m_n = \sqrt{2(n+1) + \delta_{n0}},
\]

where \( \delta_{n0} \) is the Kronecker delta function. Finally, the wavefront error (WFE) function of Eqs.1 and 2 has become a summation of Zernike polynomials weighted by coefficients, given as

\[
W(\rho, \theta) = \sum_N a_{nm} Z^m_n(\rho, \theta).
\]
where $N$ is number of expansion modes and $a_{nm}$ are aberration coefficients. This method of WFE measurement is called modal estimation,$^{11}$ which is described further in Section 1.4.1.

As with the Seidel formalism, different values of the indices $n$ and $m$ equate to different aberrations. For completeness and comparison to the Seidel equations, the 3rd order aberrations as Zernike polynomial are listed in Table 2. Note that polar coordinate expressions of Eqs.3-6 have been transformed to Cartesian coordinates with the transform

$$x_p = \rho \sin \theta,$$  \hspace{1cm} (7)

$$y_p = \rho \cos \theta,$$  \hspace{1cm} (8)

where $x_p$ and $y_p$ are normalized pupil coordinates in the $x$ and $y$ axes, respectively. These eight terms will describe the WFE throughout this dissertation.

Table 2. Third order aberrations in Zernike polynomials.$^{12}$

<table>
<thead>
<tr>
<th>Name</th>
<th>n</th>
<th>m</th>
<th>Functional Form, $Z_n^m(x_p,y_p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Astigmatism, 45º</td>
<td>2</td>
<td>-2</td>
<td>$\sqrt{6}(2x_p y_p)$</td>
</tr>
<tr>
<td>Focus</td>
<td>2</td>
<td>0</td>
<td>$\sqrt{3}(2x_p^2 + 2y_p^2 - 1)$</td>
</tr>
<tr>
<td>Astigmatism, 0º</td>
<td>2</td>
<td>2</td>
<td>$\sqrt{6}(x_p^2 - y_p^2)$</td>
</tr>
<tr>
<td>Trefoil, 45º</td>
<td>3</td>
<td>-3</td>
<td>$\sqrt{8}(3x_p^2 y_p - y_p^3)$</td>
</tr>
<tr>
<td>Coma, 90º</td>
<td>3</td>
<td>-1</td>
<td>$\sqrt{8}(3x_p^2 y_p + 3y_p^3 - 2y_p)$</td>
</tr>
<tr>
<td>Coma, 0º</td>
<td>3</td>
<td>1</td>
<td>$\sqrt{8}(3x_p^3 + 3x_p y_p^2 - 2x_p)$</td>
</tr>
<tr>
<td>Trefoil, 0º</td>
<td>3</td>
<td>3</td>
<td>$\sqrt{8}(x_p^3 - 3x_p y_p^2)$</td>
</tr>
<tr>
<td>Spherical Aberration</td>
<td>4</td>
<td>0</td>
<td>$\sqrt{5}(6x_p^4 + 12x_p^2 y_p^2 + 6y_p^4 - 6x_p^2 - 6y_p^2 + 1)$</td>
</tr>
</tbody>
</table>

1.4 Adaptive Optics

Dr. Robert K. Tyson, one of the foremost scientists in adaptive optics, defines adaptive optics as “a scientific and engineering discipline whereby the performance of an optical signal is improved by using information about the environment in which it passes.”$^{13}$ In other words, adaptive optics
attempts to correct for aberrations, either static or continuous, in real-time using a blend of computational and experimental techniques. It is a multi-disciplinary field, covering a vast array of scientific and engineering fields – computer science for computational code, mechanical and electrical engineering for deformable mirrors, controls engineering for closed-loop feedback, and optical engineering for wavefront sensing to name a few.

Adaptive optics systems range in complexity from simple to extreme depending on the temporal stability of the aberrations and the level of correction required. All reflective adaptive systems, (i.e., systems that utilize reflective active optical elements) can be simplified to the system found in Figure 1. An aberrated beam is incident upon a deformable mirror (DM), an optical element that can change its physical shape in order to alter the phase of the reflected beam. The reflected beam is then split by a beamsplitter, sending a portion of the beam towards a wavefront sensor and the remaining portion as an output beam. The wavefront sensor determines the aberrations present within the incident beam. A computer then calculates a control signal that is sent to the DM. This feedback signal is intended to reduce the aberrations in the incident beam by causing the DM to form a shape that is complementary to the aberrated beam, thereby producing a perfect plane wave at the output. This process, called closed-loop feedback, can be repeated *ad nauseum* until the output beam’s aberrations are below a certain threshold or continuously if the aberrations are temporally dependent.

Thus, a reflective adaptive optics system can be broken into three main sub-systems – a wavefront sensor, a DM, and a closed-loop feedback algorithm. The following sections discuss each sub-system in more detail.

One final note concerns a common point of confusion – the difference between ‘active’ and ‘adaptive’ optics. Adaptive optics utilizes real-time control with closed-loop feedback and is a subset of the much broader field of active optics. Active systems do use feedback, but the feedback loop is not fast enough to be considered real-time and is referred to as open-loop feedback.\(^\text{14}\) With these definitions, the AMT is an adaptive optical system, since it uses real-time feedback, while the active CFRP mirror is an active system since it can execute an ROC change open-loop, as in Section 6.4.5.
1.4.1 Wavefront Sensing

Sensing of the wavefront is essential to an adaptive optics system as it provides the necessary feedback for WFE correction. Wavefront sensors separate into two categories, either modal or zonal, depending on how the wavefront is estimated. Modal sensing is when the WFE is described over the entire aperture in terms of the expansion modes of a polynomial, as in Eq. 6. Zonal sensing occurs when the wavefront is divided into small areas, or zones, and the WFE is calculated for each area. Use of either method is largely driven by the sensing method and application of the adaptive optics system. In general, though, modal analysis provides better correction for low-order aberrations, such as the Seidel aberrations, and zonal analysis provides better correction for high-order aberrations. The AMT in Chapter 5 and the active CFRP mirror in Chapter 6 are both optimized to correct for low-order aberrations, and, as such, modal sensing is employed for aberration correction.

1.4.2 Shack-Hartmann

This dissertation utilizes a Shack-Hartmann wavefront sensor (SHWS) for wavefront sensing within the AMT. A SHWS uses the slope of the incident wavefront to measure WFE. A schematic of a SHWS is displayed in Figure 2, where a collimated beam is incident upon a set of
thin lenses. The basic idea is to break the wavefront into a set of sub-apertures, measure each sub-aperture separately, and sum the results to garner the total WFE. Sub-apertures are established with a lenslet array, a grid of small lenses that focuses the incident beam onto the focal plane array. The resulting image, called a Hartmanngram, is an array of focus spots whose positions can be used to determine WFE. One main assumption is that the local wavefront slope $\alpha_{ij}$ is roughly linear over the lenslet sub-aperture, ensuring that the spot position is directly proportional to the slope. This assumption limits the dynamic range of SHWS to beams with slowly varying wavefronts, an important factor in determining the correction orientation of the CFRP mirror in Section 6.3.5.

![Figure 2. Schematic of a SHWS with a partially aberrated beam incident upon a 1-D lenslet array. The dotted line represents the local optical axis for a single lens within the array.](image)

The reference beam of a SHWS is a plane wave since an incident plane wave will focus exactly on the local optical axis of a single lens within the array. Portions of the incident wavefront that are not perfect (i.e., not planar) will focus off-axis at some distance $\Delta x_{ij}$ from the local optical axis, as seen in Figure 2. This shift is directly proportional to wavefront slope, such that

$$\frac{\partial W_p(x_p, y_p)}{\partial x_p} = \frac{\Delta x_{ij}}{f},$$

(9)
where \( f \) is the focal length of a lens in the lenslet array. The total WFE is determined through numerical analysis of all \( x \) and \( y \) slope data, provided that the number of slope measurements exceeds the number of terms used to estimate the wavefront.

Finally, the slopes can be directly related to Zernike polynomials by taking the derivative of Eq. 6 as was done in Eq. 9. Then,

\[
\frac{\partial W(x_p, y_p)}{\partial x_p} = \sum_{n} a_{m} \frac{\partial Z_{m}^u(x_p, y_p)}{\partial x_p} = \sum_{i,j} \frac{\Delta x_{ij}}{f},
\]

where the summation of all the \( x \)-axis slopes is equal to the partial derivatives of the \( x \)-component of the Zernike polynomials. Eq. 10 can be simplified by converting the summation terms into matrices,

\[
a[dZ] = \frac{S}{f}.
\]

Now, \( a \) is an \( Nx1 \) vector containing the Zernike coefficients, \( dZ \) is an \( NxM \) rectangular matrix of Zernike partial derivatives evaluated for each lenslet position, and \( S \) is a \( Mx1 \) vector containing the \( x \)-axis spot shifts of all the lenslets. \( N \) is the number of modes in the expansion and \( M \) is the total number of lenslets within the array. As we will see in Section 1.4.4, the closed-loop algorithm uses the Zernike coefficients to provide the necessary feedback for aberration correction. These can be directly accessed using matrix inversion on Eq. 11. Generalizing to two-dimensions, we have

\[
a = \frac{1}{f} [dZ]^T S,
\]

where \([dZ]^T\) is the adjoint of matrix \( dZ \), and both \([dZ]^T\) and \( S \) have doubled in size to include the \( y \)-axis slope data. If \( dZ \) is not square, the adjoint is found via the pseudoinverse.

Thus, a SHWS’ main benefit is in allowing direct access to the Zernike coefficients. It is also a snapshot sensor, since Eq. 12 can be calculated with a single Hartmanngram, giving it great speed and low sensitivity to temporal noise such as vibration and air currents, attributes that lead to its choice as the wavefront sensor for the AMT.
1.4.3 Deformable Mirror

Deformable mirrors (DMs) can vary focal length by changing the physical shape of their reflecting surface. The DM’s function within Figure 1 is to induce a spatially varying phase profile on the incident wavefront to correct aberrations. Actuators behind the reflecting surface push or pull the surface to change its shape, as in Figure 3. These actuators have a finite amount of stroke, which is the amount that any individual actuator can move up and/or down, placing a limit on the corrective ability of the DM. The other main limitation is the clear aperture of the DM, which here is defined as the width of the actuators, since beam portions outside the width of the actuators cannot be corrected by the DM.

![Figure 3. Schematic of a DM. The ‘flexible face sheet’ is the reflecting surface that is moved by the actuators behind it.](image1)

The DM employed for this dissertation is the Imagine Optic mirao, a continuous face sheet micro-electro-mechanical system (MEMS) device. The mirao is electrostatically driven, as pictured in Figure 4, meaning that the face sheet is displaced via electric attraction and repulsion. A series of voltage sources some distance \( d \) from the face sheet alter their potentials to either pull the mirror towards the electrodes (in attraction) or push the mirror away (in repulsion).

![Figure 4. Schematic of electrostatically driven DM. The dotted line represents a varying potential across the face sheet.](image2)

The mirao has 52 actuators in a square layout, as seen in Figure 5(a), with a 15mm clear aperture (measured edge to edge, not as described above). The mirao’s maximum stroke is
±50 wv of tilt, an enormous displacement, although this stroke decreases as the surface shape becomes more complex. The mirao also has the unique ability to form both convex and concave surfaces (Figure 5(b) and (c), respectively), moving through flat between these curvatures. This gives the mirao the capability to correct Zernike modes up to the 6th order.

1.4.4 Closed-Loop Feedback Algorithm

The closed-loop feedback algorithm’s role is to interpret the data from the wavefront sensor, calculate the control signal, and adjust the DM’s actuators to decrease WFE. The ‘control signal’ is a set of voltages that creates a DM phase profile equivalent to the measured wavefront. Two distinct classes of algorithms exist – direct, where the algorithm calculates a phase map from the

![Figure 5. Specific pictures of mirao. (a) Layout of the 52 actuators, and WFE of the mirao with (b) convex and (c) concave curvatures.](image)
sensor data, and indirect, where the algorithm avoids calculating phase and moves immediately to the control signal calculation.22

Both direct and indirect methods rely on the calculation of the transfer function of the system, a mathematical way to relate the input and output of a system. The transfer function encompasses the response of all components within the loop to an input signal. The responses can be measured individually or en masse with the impulse response, which is the system’s response to a Dirac delta function input. Then, using a delta function, the transfer function can be determined by,

\[ H(s_L) = \frac{G(s_L)}{\delta(s_L)}, \tag{13} \]

where \( H(s_L) \) is the transfer function, \( G(s_L) \) is the system response, \( \delta(s_L) \) is the input Dirac delta function, and \( s_L \) is the Laplace transform variable. The beauty of the impulse response lies in the fact that system output can be calculated for any input with the transfer function in Eq. 13 since any function can be deconvolved into a series of weighted delta functions using the ‘comb’ function.23 Mathematically,

\[ G_o(s_L) = H(s_L)F(s_L), \tag{14} \]

where \( G_o(s_L) \) is system output and \( F(s_L) \) is an input signal. In other words, the impulse response fully quantifies the system.

In adaptive optics, the impulse response is determined through a summation of individual responses. To do this, a single actuator is moved, while the other actuators are unmoved, creating a ‘dimple’ in the face sheet that is optically equivalent to the Dirac delta function. Repeating this process for each actuator and compiling the responses results in the influence matrix \( B \), the transfer function of the entire adaptive optics system. For the AMT, the influence matrix relates the wavefront slopes measured by the SHWS to the control signal. Mathematically,24

\[ S = Bc, \tag{15} \]

where \( S \) is a vector of slope data and \( c \) is a vector containing the control signal. In other words, there is a one-to-one correlation between the shape of the DM (vector \( c \)), and the measured wavefront (vector \( S \)), using the impulse response (matrix \( B \)).

The feedback algorithm in the AMT is buried within the proprietary software package CASAO21 and thus cannot be described here. Instead, let’s work through an example of a simple
feedback algorithm to illustrate the general mathematics. First, we invert the influence matrix in Eq. 15,

\[ c = B^T S_A, \]

where \( S_A \) are the slopes of an aberrated signal. Thus, Eq. 16 now represents the measured WFE. The WFE can be corrected by causing the DM to take on the complementary shape, meaning the new control signal must be the negative of Eq. 16. Accounting for the previous state of the mirror, our final control equation is

\[ c_{i+1} = c_i - g B^T S_A, \]

where \( c_{i+1} \) is the new control signal, \( c_i \) is the previous control signal, and \( g \) is a gain constant between 0 and 1. Performing the calculations in Eq. 17 repeatedly will eventually decrease the numerical value of Eq. 16 to near zero. At this juncture, the system is in steady-state, where \( c_{i+1} \) and \( c_i \) equal each other, and no further wavefront correction can be made. This feedback method is called indirect iterative feedback control.
CHAPTER 2: DISCUSSION OF ZOOM SYSTEMS

A “zoom” lens system can vary the image magnification while maintaining the image plane’s absolute position. This capability has far-reaching applications across optics. Three primary techniques exist for zooming – mechanical, digital, and adaptive. This chapter provides an overview of the theory and capability of each type of zooming method, with particular emphasis placed upon adaptive optical zoom (AOZ) since AOZ is a central theme of this dissertation.

2.1 Basics of Zoom

The word “zoom” refers to the appearance of an image ‘zooming’ towards an observer as the magnification is increased. Zooming systems have a range of focal lengths, providing different object magnifications at each focal length value. The focal length range can be either continuous, where good imaging is found across the entire focal length range, or discrete, where good imaging is only at particular values. The ratio of the maximum to the minimum focal length is called the zoom ratio, a common specification of zoom systems.

A zoom system can be broken into three main parts – a focusing unit at the front, a zoom unit, and an imaging unit near the image plane. The front portion is aptly named as it focuses the incoming light. The zoom unit is the heart of the zoom system as this set of elements provides the necessary system focal length change to increase or decrease magnification. The design of this unit classifies the system as a mechanical, digital, or adaptive zooming system. The final unit images the light onto the focal plane and is designed to reduce the aberrations induced throughout the rest of the system, particularly in the zoom unit.

2.1.1 Optical Zoom

A critical aspect of zoom systems is to maintain the focal ratio (‘F-number’) throughout the zoom range so as to preserve the optical resolution, an effect known as optical zoom. The Rayleigh resolution is given as

\[
(\Delta \lambda)_{\text{min}} = 1.22 \frac{\lambda}{D_{\text{EP}}} = 1.22 \lambda F_
\]

(18)

where \((\Delta \lambda)_{\text{min}}\) is the minimum separation of two point sources (maximum resolution), \(\lambda\) is the wavelength of light, \(f\) is the system focal length, \(D_{\text{EP}}\) is the diameter of the entrance pupil, and \(F_\#\) is the system focal ratio. Thus, if the focal ratio increased with the magnification, the resolution
of the system would decrease, which is generally not desirable. Maintaining a constant focal ratio as the focal length increases thus requires an equivalent increase in the diameter of the entrance pupil.

The main benefit of optical zoom is an increase in object resolution. The cutoff frequency in a system’s modulation transfer function (MTF) is given as

$$\xi_0 = \frac{1}{\lambda F_b},$$

where $\xi_0$ is the spatial frequency cutoff. For optical zoom, the cutoff frequency does not change since the focal ratio is constant. However, the object is magnified during zoom due to the reduction in FOV, meaning that unresolved spatial frequencies at a larger FOV can now be resolved by the zoom system. Thus, an optical zoom system not only increases object magnification but also object resolution.

### 2.2 Mechanical Optical Zoom

Mechanical zoom translates one or more optical elements along the optical axis, using a set of cams or gears to control the motion. Mechanical zoom is optical zoom since the focal ratio remains constant. The most basic mechanical zoom system is pictured in Figure 6, where a negative element moves between two positive elements. Figure 6 clearly demonstrates the three portions of a zoom system – the primary lens focuses the light, the secondary lens is the zoom unit, and the tertiary lens images the beam to the image plane.

![Figure 6. Mechanical zoom system with only one moving element. Notice that only two positions result in zero displacement of the image plane.](image-url)
This particular zoom system has zero image plane displacement at only two axial positions of
the zoom unit. These positions are the reciprocal magnification points, where the magnification of
the zooming element moves from $m$ to $1/m$. All other displacements of the zooming element will
result in defocused images. The axial movement of the zoom unit is generally non-linear, as we
shall see in Section 2.2.1.

2.2.1 Mechanical Zoom Paraxial Theory

Let’s extend the qualitative description above to a derivation of the two element mechanical
zoom system found in Figure 7. Proper sign convention is followed here, where positive distances
move from left to right and negative vice versa. With the thin lens equation, we find that the back focal distance is

$$z_2' = \frac{z_2}{1 + \phi_2 z_2}, \tag{20}$$

where $\phi_2$ is the optical power of the second lens, and the other quantities are as defined in Figure
7. An analogous equation is found for $z_1'$. We relate the two single lens systems with the inter-
element distance $t$.

$$z_2 = z_1' - t. \tag{21}$$

Now, we find an expression for the back focal distance $z_2'$ by substituting Eq. 21 into Eq. 20 along with the analogous equation for $z_1'$. After some work, we arrive at

$$z_2' = \frac{1 - t_1 - t_2 \phi_2}{\phi + \frac{1 - t_1 \phi_1}{z_1}}, \tag{22}$$

where $\phi$ is the system optical power, given as

$$\phi = \phi_1 + \phi_2 - \phi \phi_2 t_1, \tag{23}$$

with Gaussian reduction.
Figure 7. Layout of a two-element mechanical zoom system for paraxial theory derivation. Positive distances move from left to right, negative distances vice versa.

One state is now completely designed, after a few quantitative choices by the designer, leaving the zoom state to be determined. Note that capital letter variables designate analogous distances in the zoom state as in the unzoom state, e.g., $t_1$ and $T_1$ are the inter-element distances for the unzoom and zoom states, respectively. First, we must relate the two states using the total system length,

$$L_4 = -z_1 + t_1 + z_2' = -Z_1 + T_1 + Z_2', \quad (24)$$

where $Z_1$ and $Z_2'$ are the new front and back focal distances, respectively, given as

$$Z_1 = z_1 - L_2, \quad (25)$$

$$Z_2' = z_2' - L_3, \quad (26)$$

where $L_2$ and $L_3$ are the axial displacements for the first and second element, respectively. Then, we can apply Gaussian reduction again to find the new inter-elemental distance $T_i$

$$T_i = \frac{\phi_1 + \phi_2 - \phi'}{\phi_1 \phi_2}, \quad (27)$$

where $\phi'$ is the system power in the second state, given with the zoom ratio $Z_R$ as

$$Z_R = \frac{f}{f'} = \frac{\phi'}{\phi}, \quad (28)$$
where \( f \) and \( f' \) are the system focal lengths in the first and second states, respectively. Next, we find \( Z_i \) by combining Eqs. 24 and 22

\[
Z_i^2a + Z_i b + c = 0 ,
\]

(29)

where

\[
a = -\phi' \\
b = T_i(\phi' - \phi_1 + \phi_2) \\
c = L_i(T_i\phi_2 - 1) - \phi_2 T_i^2
\]

(30)

Note that the terms of Eq. 22 were replaced by their analogous values in the second state to form Eq. 29. Eq. 29 can be solved using the quadratic formula, where typically only one of the two solutions is sensible. Eq. 29 also shows that the axial movement of mechanical zoom units is nonlinear (quadratic in this case) since \( Z_i \) is directly proportional to \( L_2 \) with Eq. 25. Finally, \( Z_2' \) can be found with Eq. 26; thus, all values have been determined.

While the above equations determine the mechanical zoom system in Figure 7, it is an underdetermined system as there are 9 equations [22, 23, 24 (both states), 25, 26, 27, 28, 29] and 14 unknowns \([\phi, \phi', \phi_1, \phi_2, z_1, t_1, T_1, z_2', Z_2', L_1, L_2, L_3, Z_R]\). One possible approach to design the system is to assume that the first state is 1:1 imaging and set \( \phi_1, \phi_2, t_1, z_1 \) accordingly, as in Table 3. Also, let’s assume a 2.75X zoom ratio. These two assumptions set five parameters and thus the other nine can be determined; the values are listed in Table 3. We see that both lenses have moved significantly closer to the object and image planes. Note that this is a first-order design that requires further design and optimization in optical design software.

Table 3. System parameters for a 2.75X mechanical zoom system.

<table>
<thead>
<tr>
<th>Independent Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_1 )</td>
<td>50mm</td>
</tr>
<tr>
<td>( \phi_2 )</td>
<td>50mm</td>
</tr>
<tr>
<td>( t_1 )</td>
<td>200mm</td>
</tr>
<tr>
<td>( z_1 )</td>
<td>-100mm</td>
</tr>
<tr>
<td>( Z_R )</td>
<td>2.75</td>
</tr>
</tbody>
</table>
### Dependent Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>-0.04 mm(^{-1} )</td>
</tr>
<tr>
<td>( \phi' )</td>
<td>-0.11 mm(^{-1} )</td>
</tr>
<tr>
<td>( Z_1 )</td>
<td>-5.08 mm</td>
</tr>
<tr>
<td>( T_1 )</td>
<td>375 mm</td>
</tr>
<tr>
<td>( z_2' )</td>
<td>100 mm</td>
</tr>
<tr>
<td>( Z_2' )</td>
<td>19.91 mm</td>
</tr>
<tr>
<td>( L_1 )</td>
<td>400 mm</td>
</tr>
<tr>
<td>( L_2 )</td>
<td>-94.92 mm</td>
</tr>
<tr>
<td>( L_3 )</td>
<td>80.09 mm</td>
</tr>
</tbody>
</table>

#### 2.2.2 Mechanical and Optical Compensation

Increasing the number of zoom states beyond two requires more elements, either static or variable. One scheme, called mechanically compensated zoom, moves two or more elements independently, as in Figure 8(a) and the derivation above. This technique produces continuous zoom with zero defocus at the image plane over the entire zoom range. However, the second moving element must move quadratically (in Figure 8(a)), thereby placing great stress upon the cam system since slight variations in element placement produces large effects in the image plane. The other scheme, called optically compensated zoom, moves two or more elements as a unit around fixed elements, as on the left in Figure 8(b). Optical compensation reduces the stress on the cam system, since both elements move linearly to produce continuous zoom, but induce image shift at the focal plane (defocus), as is graphed on the right in Figure 8(b). Thus, the choice with mechanical zoom is between a difficult cam system and mildly defocused images.
2.3 Digital Zoom

Digital or electronic zoom removes the need for any element motion by performing zoom entirely in code. Digital zoom remaps a section of pixels onto a larger image, making the image appear larger. It is extremely fast and can zoom into any portion of the FOV, as opposed to mechanical zoom which can only perform axial zoom.

However, digital zoom is not optical zoom as the focal ratio is altered between zoom states. With digital zoom, the focal length and entrance pupil are unaltered, meaning that the resolution is also unaltered per Eq. 18. In other words, unresolved object spatial frequencies in the unzoomed image of an airport in Figure 9(a) remain so in the digitally zoomed image in (b). Thus, the usefulness of digital zoom is limited, although certain applications, such as consumer cameras, have utilized digital zoom to artificially extend the zooming ratio beyond the optical zoom limit.

Figure 8. An example of (a) a mechanically compensated zoom system and (b) an optically compensated zoom system.

Figure 9. Image of an airport zoomed electronically and optically by 8X. a) Unzoomed image, b) digital zoom with no resolution gain, and c) adaptive optical zoom with increased resolution.
2.4 Adaptive Optical Zoom

Mechanical zoom works extremely well and is frequently employed across disciplines.\(^{35}\) Element displacement with mechanical zoom is typically on the order of the system focal length.\(^{36}\) Large movement of optical elements, as is inherent with a longer system focal length, can be taxing on weight and power budgets when the elements are more than a few centimeters in diameter. Thus, issues such as size, weight, and power requirements limit mechanical zoom’s use in many military and some industrial applications.\(^{37}\) The opto-mechanical structure is typically large compared to the optics, thus requiring a sizeable amount of power to move and control. Furthermore, displacement tolerances in mechanical zoom system are \(\sim 0.1\) mm,\(^{2}\) an achievable tolerance for small optics, but one that becomes extremely taxing on weight and power budgets for meter-class elements.\(^{35}\)

Circumventing these issues requires another zoom technique called adaptive optical zoom (AOZ), where system magnification is modified via variable focal length elements. Such elements, called active elements, induce a spatially varying phase shift in incoming wavefronts to produce the necessary focal length change for zoom.\(^{38}\) One common active element for reflective AOZ systems is a deformable mirror (DM), where the physical shape of the surface is altered to change the element’s optical power, as described in Section 1.4.3. Two or more DMs are required to alter the system magnification with AOZ.\(^{39}\)

AOZ is true optical zoom as seen Figure 9, where again a picture of an airport is displayed in (a) and the same portion optically zoomed in (c), obtained using an experimental AOZ system.\(^{5}\) As discussed in Section 2.1.1, optical zoom induces an increase in object resolution, which is plainly evident by the ‘C’-shaped white building being unresolved in (a) but visible in (c). Figure 9 also highlights the stark difference in resolution between digitally and optically zoomed images.

2.4.1 AOZ Paraxial Theory

Let’s derive a paraxial AOZ theory\(^{40}\) for comparison to the mechanical zoom theory found in Section 2.2.1. The first-order layout in Figure 10(a) is similar to Figure 7 but has been simplified since the inter-element distance \(t_2\) does not change. First, we Gaussian reduce the system to two principal planes, as in Figure 10(b), using

\[
f = \frac{f_1 f_2}{f_1 + f_2 - t_2},
\]

(31)
where $f$ is the system focal length, $f_1$ is the focal length of the first lens, $f_2$ is the focal length of the second lens. Now, Gaussian imaging equations give us

$$z_3 = \left(\frac{1}{m} - 1\right)f.$$ (32)

Then, $z_3$ can be found by inserting the expressions for the principal plane shift $l$,\(^4^2\)

$$z_3 = z_s + l = z_s + \frac{t_2 f'}{f_2}.$$ (33)

Equating Eqs. 32 and 33, we arrive at

$$z_3 = f \left(\frac{1}{m} - 1 + \frac{t_2}{f_2}\right).$$ (34)

Following a similar procedure for the image space distance $z'_3$, we find

$$z'_3 = f \left(1 - m - \frac{t_2}{f_1}\right).$$ (35)

Next, we know that the total system length $L_4$ is

$$L_4 = -z_3 + t_2 + z'_3.$$ (36)

Substituting Eqs. 34 and 35 into Eq. 36, we arrive at a quadratic equation in terms of $t_2$

$$t_2^2 - L_4t_2 + \left[L_4(f_1 + f_2) + \frac{f_1 f_2}{m} (m - 1)^2\right] = 0.$$ (37)

Eqs. 31, 34, 35, and 37 have analogous expressions in the second (zoom) state, with the system and element focal lengths and magnification changing values. Note that similar AOZ theories (using Gaussian reduction) are found in Refs. 43 and 44.
Figure 10. Layout of two-element AOZ system for paraxial theory – (a) system at unit magnification, and (b) Gaussian reduced system at unit magnification. \( P \) and \( P' \) denote the system principal planes.

The example AOZ system is also underdetermined, as there are 10 equations [28, 31 (both states), 34 (both states), 35 (both states), 36, 37 (both states)] and 13 unknowns \( \{f,f',f_1,f_1',f_2,f_2',z_5,z_6,l_2,L_4,m,m',Z\} \), where primed focal lengths indicate the zoom state. For a true comparison to the mechanical zoom example, we assume that the first state is 1:1 magnification and the zoom ratio is 2.75X. Determining the other unknowns is non-trivial with AOZ since the equations are coupled, requiring a program such as Mathematica to find a solution. The independent and dependent parameters are listed in Table 4. Due to the squared term in Eq. 37, two solutions are found for the dependent parameters, although they are inverses of each other. Note that, as with the mechanical zoom example in Section 2.2.1, this is a first-order design that needs further optimization in optical design software to be viable.
Table 4. System parameters for a 2.75X AOZ system.

<table>
<thead>
<tr>
<th>Independent Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1'$</td>
</tr>
<tr>
<td>$f_2'$</td>
</tr>
<tr>
<td>$m$</td>
</tr>
<tr>
<td>$z_5$</td>
</tr>
<tr>
<td>$Z_R$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
</tr>
<tr>
<td>$f'$</td>
</tr>
<tr>
<td>$f_i'$</td>
</tr>
<tr>
<td>$f_2'$</td>
</tr>
<tr>
<td>$t_2$</td>
</tr>
<tr>
<td>$z_6'$</td>
</tr>
<tr>
<td>$L_4$</td>
</tr>
<tr>
<td>$m'$</td>
</tr>
</tbody>
</table>

2.4.2 Mechanical and Adaptive Zoom Comparison

The derivation for the AOZ system requires fewer steps and can be easier to visualize than the mechanical zoom system derivation since all lengths remain constant. Finding solutions, however, is vastly more complicated with AOZ. Once the independent parameters for a mechanical zoom system are chosen, the dependent parameters are determined in a domino-like fashion requiring only a calculator. AOZ, instead, requires substantial computing power.

The most important comparison is the experimental feasibility of the two zoom systems. Here, both examples exhibit aspects that make them difficult to build. The lenses for the mechanical system move nearly 100mm along the optical axis in different directions, placing immense stress upon the cam system. The ROC change for one of the two lenses with AOZ is nearly a factor of three, a substantial change for some active elements. Of course, a clever optical designer could (and has) found mechanical and adaptive zoom solutions that surpass these
examples, but the overall conclusion is that zoom systems are complicated and present large design challenges both mechanically and optically.

2.4.3 Other Design Theories of AOZ

Another way to describe AOZ is with the Lagrange invariant, an equation that utilizes the linearity of paraxial optics to relate the chief and marginal ray heights and angles. A general expression of the Lagrange invariant is

\[ H = n(\bar{u}y - u\bar{y}) \], \hspace{1cm} (38)

where \( H \) is the invariant (sometimes given as the Russian letter \( \mathcal{Ж} \)), \( n \) is the index of refraction, \( u \) and \( \bar{u} \) are the tangent of the marginal and chief ray angles, respectively, and \( y \) and \( \bar{y} \) are the marginal and chief ray heights, respectively. The Lagrange invariant is constant at any surface within the optical system including upon reflection/refraction and transfer.

One theory\(^{49}\) determined the sag changes of an AOZ system by combining the invariant with first-order imaging equations. The sag is the deviation of a spherical surface from a tangent line at the surface’s apex, given as

\[ Sag = R - \sqrt{R^2 - y_s^2} \], \hspace{1cm} (39)

where \( R \) is the surface’s ROC and \( y_s \) is the height along the tangent line. The Ref. 49 system is similar to Figure 10, a two-element AOZ system, except that the variable lenses are replaced with deformable mirrors (DMs) and the system is derived for infinite conjugates. The theory arrives at the sag changes for the primary and secondary mirrors\(^{49}\)

\[ \Delta Sag_p = \frac{1}{16f} \left( D_p + w \frac{s_1}{q} \right)^2 \frac{q}{s_1} \left( \frac{Z_R - 1}{Z_R} \right), \hspace{1cm} (40) \]

\[ \Delta Sag_s = -\frac{D^2}{16f} \left( \frac{Z_R - 1}{qs_1} \right), \hspace{1cm} (41) \]

where \( \Delta Sag_p \) is the sag change of the primary, \( \Delta Sag_s \) is the sag change of the secondary, \( w \) is the detector width, \( f \) is the system focal length in the unzoomed state, \( D_p \) is the diameter of the primary mirror, \( D_s \) is the diameter of the secondary, and \( q \) and \( s_1 \) are ratios given as
\[ q = \frac{D_t}{D_p}, \quad (42) \]

\[ s_i = \frac{f_i}{f}. \quad (43) \]

Note that \( Z_k \) no longer equals 2.75. The optical layout for a two-element, two DM AOZ system can now be found. It is important to note that this theory provides a first-order theory, since it uses paraxial equations, as opposed to the third-order theory derived in Chapter 3.

The first-order theory above leads to an important conclusion with AOZ systems. The dependence of the primary and secondary sag changes can be found by substituting Eq. 40 into Eq. 41, arriving at

\[
\Delta \text{Sag}_s = \frac{H(Z_R - 1)}{2Z_R} + \frac{(4\Delta \text{Sag}_p)^2 + H^2(Z_R - 1)^2}{16Z_R \Delta \text{Sag}_p}. \quad (44)
\]

Eq. 44 demonstrates that there is a relationship between the deformations of the primary and secondary mirrors, meaning that there is a one-to-one correlation between the two sag changes for a given zoom ratio, as illustrated in Figure 11. Notice that a small sag change on the primary results in a large change for the secondary, an important design consideration for the potential AOZ system given in Section 7.3.

![Figure 11. Correlation between the primary and secondary sag changes for a given zoom ratio and field angle.](image)
Another theory of AOZ minimizes the Petzval sum in the first-order optical layout. The Petzval sum determines the amount of field curvature within a system. For this theory, the Petzval sum is given as

\[
\frac{1}{\hat{r}_p} = \sum_{i=1}^{N} \hat{\phi}_i,
\]

(45)

where \( \hat{r}_p \) is the ROC of the Petzval surface, \( \hat{\phi}_i \) is an element’s optical power, and the ‘hats’ denote a normalized value. Field curvature is zero when the Petzval sum is zero, meaning that the Petzval surface ROC is infinity and the image plane is truly a plane.

The derivation works in normalized distances and optical powers to make the theory as polymorphic as possible, an approach that has the unfortunate side effect of lengthy calculations and equations. As such, the equations will not be repeated here. Suffice it to say that the derivation resulted in the “enhanced first-order design” found in Figure 12 (“enhanced first-order design” implies the design’s generality since normalized quantities are employed). The DMs are the first and third elements. The design is a three-state system with zoom ratios of 1X, 1.5X, and 3X with an acceptably flat Petzval surface in all states. However, as we shall in Section 2.4.4, the three-element system becomes a four-element system, thereby questioning the applicability of the presented theory.

Figure 12. “Enhanced first-order design” of an AOZ system. Three states are displayed, with zoom ratios of (a) 1X, (b) 1.5X, and (c) 3X.
2.4.4 Experimental Examples of AOZ

The first AOZ system was proposed by Tam. Using paraxial theory (similar to Section 2.4.1), he was able to show that the combination of two static lenses and two spatial light modulators (SLMs) could change the system focal length and produce a zoom ratio of 1.5X. The term ‘adaptive optical zoom’ was coined by Wick and Martinez, who extended the concept to a full Zemax design with a 3.9X zoom ratio, found in Figure 13. They showed that the zoom ratio could be greatly increased if the DM and static elements were separated, introducing additional system power since the element separation in Eq. 23 is now non-zero. Finally, the design demonstrated that AOZ could work for reflective components as well as transmissive ones.

Figure 13. A 3.9X AOZ system designed in Zemax. The top layout is the unzoomed state, corresponding to the ‘F’ on the left, and the bottom layout is the zoomed state, corresponding to the large ‘F’ on the right.

Researchers have applied AOZ to tabletop systems, where element diameters are a few centimeters, using a variety of transmissive and reflective active elements. Transmissive elements, such as SLMs or liquid crystal (LC) cells, change focal length by introducing focus. Computers allow precise control of SLMs and LCs, thereby enabling quadratic phase (focus) to be added or subtracted to the incident wave in order to alter the system magnification. Figure 14 displays a standard Air Force resolution chart enlarged by 3.3X using a SLM-based AOZ system.
Mechanical zoom systems are limited to on-axis object magnification, unless the system is on a gimbal to slew the apparatus. AOZ, on the hand, has the potential to zoom in on any portion of the FOV with optical zoom (unlike digital zoom). SLMs change focal length by adding quadratic phase (focus); they can just as easily add linear phase (tilt) to shift the FOV off-axis with respect to the optical axis of the system. Figure 15 demonstrates this concept by magnifying different portions of a standard Air Force chart without physically slewing the system. However, as with lenses, active transmissive elements suffer from chromatic aberration.
Reflective active elements, like all mirrors, avoid chromatic aberration (a major reason this dissertation developed an active mirror). Despite this advantage, only one experimental system can be found in the literature. This system, pictured in Figure 16(a), is a four-element system designed with the Petzval sum, as described in Section 2.4.3. The primary and quaternary components are DMs. All elements are glass, including the DMs, which are actuated by point loads. The system is also unobscured, meaning that no portion of the entrance pupil is blocked, with a zoom ratio of 3X. The difficulty of experimentally realizing an AOZ system, particularly an unobscured version, is highlighted by the authors resorting to aspheric surfaces for all elements, including biconic surfaces for the secondary (which is also off-axis) and tertiary mirrors. The resulting 3X zoomed image, found in Figure 16(b), are acceptable but not diffraction limited. It should also be pointed out that the maximum zoom state, on the right in Figure 16(b), does not preserve the focal ratio.

Figure 16. Experimental AOZ system. (a) Schematic of layout with three states, with zoom ratios of 1X, 1.5X, and 3X from left to right, respectively. (b) Image taken with system at 3X zoom state.
Therefore, the only experimental reflective AOZ system utilizes DMs that are standard glass mirrors, a delicate process as Section 4.3 demonstrates, highlighting the complete dearth of available components for AOZ systems. The active CFRP mirror in Chapter 6 fills this need to prepare for future applications that could benefit from large aperture AOZ systems.
CHAPTER 3: THEORY OF ADAPTIVE OPTICAL ZOOM

This chapter presents a novel optical design theory for reflective adaptive optical zoom (AOZ) systems. The derived theory and subsequent aberration simulation enable a large-scale tradespace analysis, where AOZ systems are examined from a broad perspective and optimized based on given parameter thresholds. Chapter 3 covers the theory derivation, simulation code, and tradespace analysis, culminating with the design of a 3.3X AOZ system featuring a 375mm diameter primary mirror.

3.1 Third-Order Design Theory

Three published AOZ design theories were detailed in Sections 2.4.1 and 2.4.3; however, all theories were determined to be insufficient for large aperture AOZ systems. The first two theories employed Gaussian reduction \(^{40,43,44}\) and the Lagrange invariant, \(^{49}\) both first-order expressions of optical systems. Numerical simulation with these first-order theories showed that the theories designed AOZ systems with significant differences between marginal and chief rays, differences that prevented low wavefront error (WFE) in the final designs. Then, the Petzval sum theory \(^{50}\) did not accurately design the AOZ system, as seen in Section 2.4.4 where the three-element layout became a four-element experimental system. This effect could be exacerbated as the entrance pupil size grows from \(\sim 2\)mm \(^{50}\) to tens or hundreds of millimeters.

Thus, to accurately simulate a large aperture system, terms beyond the first-order must included in the theory. One such example is the third-order theory available for the design of a two-element aplanatic Cassegrain and Gregorian objectives. \(^{51}\) These equations can be re-solved for AOZ systems by letting system focal ratios and element focal lengths possess multiple values, as in Section 2.4.1. Then, residual aberrations can be simulated with a vector aberration theory, originally applied to bilateral systems, \(^{52}\) to increase fidelity between the optical layout and potential experimental system. This aberration theory applies well to AOZ since it can easily handle both axial and non-axial (unobstructed) optical designs and is not computationally intensive as it only requires a paraxial ray trace with the marginal and chief rays.

3.1.1 Cassegrain Telescope Design

Two basic telescope topologies are the Cassegrain and Gregorian objectives. Both are two-element, axially symmetric aplanats with a parabolic primary mirror. The secondary mirror is a negative hyperbola for the Cassegrain and a positive ellipse for the Gregorian. A thorough study
of Cassegrain and Gregorian telescopes was performed by Wetherell and Rimmer; their intent was to provide the optical designer with all the necessary tools for designing a two-element aplanat. Equations for element curvatures and separations were derived using third-order aberration theory, so chosen since third-order theory adequately describes most two-mirror telescopes. All system attributes are based on the optical designer choosing the system focal ratio ("F-number"), the primary focal ratio, the field-of-view (FOV), and the diameter of the entrance pupil/primary mirror. The design equations from Ref. 51 used for the derivation in Section 3.1.2 are found in Table 5 and are defined in Figure 17. Note $F$ in Table 5 is the system focal ratio, given as

$$F = \frac{f}{D_{EP}},$$  \hspace{1cm} (46)$$

where $f$ is the system focal length and $D_{EP}$ is the diameter of the entrance pupil.

Table 5. Listing of Wetherell and Rimmer equations$^{51}$ utilized in the AOZ derivation along with a brief description of the equation. The equation number listed in the first column is the equation number as listed in Ref. 51.

<table>
<thead>
<tr>
<th>Eq. #</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$m = \frac{F}{F_p}$</td>
<td>Magnification of the secondary mirror</td>
</tr>
<tr>
<td>2</td>
<td>$\eta = \frac{WD}{D_p}$</td>
<td>Normalized working distance</td>
</tr>
<tr>
<td>4</td>
<td>$f_1 = D_p F_p$</td>
<td>Primary focal ratio</td>
</tr>
<tr>
<td>6</td>
<td>$d = D_p \frac{F - \eta}{m + 1}$</td>
<td>Axial separation between primary and secondary</td>
</tr>
<tr>
<td>10</td>
<td>$R_s = 2mD_p \frac{F_p + \eta}{1 - m^2}$</td>
<td>ROC of the secondary mirror</td>
</tr>
</tbody>
</table>

3.1.2 Cassegrain AOZ Design
To make either a Cassegrain or Gregorian objective an AOZ system, both the primary and secondary mirrors must be active elements per Section 2.4. Both elements are assumed to be perfectly spherical. For ease in analysis, the AOZ system has only two states. The object is at
infinity and is one-dimensional on the y-axis, extending from the optical axis to the unvignetted half field-of-view (HFOV). The aperture stop is the primary mirror in both states.

Figure 17. Coordinate system for third-order design, with the chief and marginal rays displayed. The stop is at the primary. Labels: $D_p$ is the diameter of the primary mirror, $XP$ is the exit pupil, $d$ is the axial separation between the primary and secondary mirrors, $l_n$ is the distance between the exit pupil and image plane, $WD$ is the axial distance between the primary apex and the image plane ('working distance'), $D_i$ is the diameter of the image, $\theta_n$ is the maximum field angle in object space (HFOV) and $\theta_n'$ is the maximum field angle in image space.

The five equations listed in Table 5 become nine equations when the focal ratios and lengths are allowed to change between states [Table 5 Eqs. 1(both states), 2, 4 (both states), 6 (both states), and 10 (both states)], resulting in a total of 14 unknowns \[ F_1, F_2, f_{11}, f_{12}, R_{21}, R_{22}, D_p, d, WD, m_1, m_2, \eta \]. The first of two subscripts is the element number (1 is the primary mirror and 2 is the secondary mirror) and the second is the State number (State 1 is the unzoomed state and State 2 is the zoomed state). If only one number is listed, it is the state number. The variables, referring to Figure 17, are given as: $F_1$ and $F_2$ are the system focal ratios, $f_{11}$ and $f_{12}$ are the primary focal lengths, $R_{21}$ and $R_{22}$ are the secondary radii of curvature (ROCs), $D_p$ is the diameter of the primary, $d$ is the axial separation between the primary and secondary mirrors, $WD$ is the ‘working distance,’ $m_1$ and $m_2$ are the secondary magnifications, and $\eta$ is a constant used later in this section.

As in Section 2.4.1, the AOZ equations are coupled, thus requiring substantial computing power to produce an answer. A Mathematica script was written to solve the nine equations and is
found in Appendix A. Independent variables were chosen to be \([ F_1, F_2, f_{11}, f_{12}, D_p ]\). These differ from the original independent variables to more closely match current computer-based design tools such as Zemax and CodeV. Solving for the dependent variables, we have

\[
R_{21} = \frac{2D_p f_{11} F_1 F_2 (f_{11} - f_{12})}{(f_{11} - D_p F_1)(f_{11} F_2 - f_{12} F_1)},
\]

(47)

\[
R_{22} = \frac{2D_p f_{12} F_2 (f_{12} - f_{11})}{(f_{12} - D_p F_2)(f_{12} F_1 - f_{11} F_2)},
\]

(48)

\[
d = \frac{f_{11} f_{12} (F_2 - F_1)}{f_{11} F_2 - f_{12} F_1},
\]

(49)

\[
WD = \frac{D_p f_{12} F_1 F_2 - f_{11} (f_{12} F_1 - f_{12} F_2 + D_p F_1 F_2)}{f_{12} F_1 - f_{11} F_2},
\]

(50)

\[
\eta = -\frac{f_{11} f_{12} F_1 - f_{11} f_{12} F_2 + D_p F_1 F_2 (f_{11} - f_{12})}{D_p (F_1 f_{12} - F_2 f_{11})},
\]

(51)

where all variables are as defined above. Lengths \(d\) and \(WD\) are defined to be positive in the above equations.

The HFOV \(\theta_n\) can be determined using the location of the exit pupil and some trigonometry. Referring to Figure 17, the distance from the exit pupil to the image plane is given by\(^{51}\)

\[
l_n = -\frac{m_n F_n (f_{1n} + D_p \eta)}{m_n F_n + \eta},
\]

(52)

where \(m_n\) is the secondary magnification, as listed in Table 5, and \(n = 1,2\) for the unzoomed and zoomed states, respectively. Next, the image space chief ray angle \(\theta_n'\) is\(^{51}\)

\[
\theta_n' = \theta_n - \frac{m_n F_n + \eta}{m_n (f_{1n} / D_p + \eta)},
\]

(53)
where $\theta_n'$ is as defined in Figure 17. Finally, we can eliminate $\theta_n'$ with trigonometry and solve for $\theta_n$,

$$\tan \theta_n' = \frac{D_i}{2l_n},$$  \hspace{1cm} (54) $$

$$\theta_n' = \tan^{-1}\left(\frac{D_i}{2l_n}\right) = \theta_n \frac{m_n F_n + \eta}{m_n (f_{1n} / D_p + \eta)},$$  \hspace{1cm} (55) $$

$$\theta_n = \frac{m_n (f_{1n} / D_p + \eta)}{m_n F_n + \eta} \tan^{-1}\left(\frac{D_i}{2l_n}\right).$$  \hspace{1cm} (56) $$

Eq. 56 describes the HFOV (chief ray angle) for both states using previously determined values from Eqs. 47-51. The dependent and independent variables in Eqs. 47-50 and Eq. 56 define a set of construction parameters, i.e., a complete set of physical values that define an optical system.

3.2 Aberration Simulation

3.2.1 Bilateral System Theory

Residual aberrations were simulated using Jose Sasian’s theory for bilateral symmetric optical systems found in Ref. 52. A bilateral symmetric system has only one plane of symmetry, meaning that half of the system is a mirror image of the other. Elements can be tilted in one axis, (e.g., the $y$-axis) but not in the orthogonal axis (e.g., the $x$-axis), giving the system planar but not rotational symmetry. Equations were developed to calculate the third-order aberrations of such a system based on a paraxial ray trace of marginal and chief rays. As usual, individual aberrations are summed to garner the wavefront aberration function $W$, given as$^52$

$$W(\tilde{H}, \tilde{\rho}) = \sum_{k,m,n} W_{2k+n,2m+n,n} (\tilde{H} \cdot \tilde{H})^k (\tilde{\rho} \cdot \tilde{\rho})^m (\tilde{H} \cdot \tilde{\rho})^n,$$  \hspace{1cm} (57) $$

where $W_{2k+n,2m+n,n}$ is the coefficient of a particular aberration, $\tilde{H}$ is the normalized field height, $\tilde{\rho}$ is the normalized pupil coordinate, and $k$, $m$, and $n$ are integers. Eq. 57 is the axial aberration function, so given since Figure 17 is a rotationally symmetric system. A rotationally symmetric system is a simplified version of a bilateral system, meaning that the Ref. 52 theory can be readily
applied. The individual aberration equations to determine the Seidel coefficients were applied verbatim but are displayed in Table 6 for completeness. Variables within Table 6 are: \( u \) is the marginal ray angle, \( \bar{u} \) is the chief ray angle, \( x \) is the marginal ray height at a surface, \( \bar{x} \) is the chief ray height at a surface, \( I \) is the angle of the OAR (see Section 3.2.2), \( R \) is the surface ROC, \( n \) is index of refraction, \( H \) is the Lagrange invariant, and primed and unprimed quantities represent the variable after and before refraction/reflection at a surface, respectively.

Table 6. Equations from Ref. 52 used during aberration simulation to determine the Seidel coefficients. The right-hand column contains the equation number as listed in the original publication. \( N \) is the total number of optical surfaces.

<table>
<thead>
<tr>
<th>Eq. #</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>( A = n \left( u + \frac{x \cos(I)}{R} \right) )</td>
<td>Marginal ray angle in medium preceding surface</td>
</tr>
<tr>
<td>9</td>
<td>( \bar{A} = n \left( \bar{u} + \frac{\bar{x} \cos(I)}{R} \right) )</td>
<td>Chief ray angle in medium preceding surface</td>
</tr>
<tr>
<td>N/A</td>
<td>( \Delta \left( \frac{u}{n} \right) = \frac{u'}{n'} - \frac{u}{n} )</td>
<td>Abbe difference operator upon refraction</td>
</tr>
<tr>
<td>10</td>
<td>( S_i = -\frac{1}{8} A^2 \Delta \left( \frac{u}{n} \right) x )</td>
<td>Quantity for aberration calculation</td>
</tr>
<tr>
<td>11</td>
<td>( S_{ii} = -\frac{1}{4R} H^2 \Delta \left( \frac{\cos(I)}{n} \right) )</td>
<td>Quantity for aberration calculation</td>
</tr>
<tr>
<td>28</td>
<td>( \sum_{i=1}^{N} { S_i }_i )</td>
<td>Spherical aberration</td>
</tr>
<tr>
<td>29</td>
<td>( \sum_{i=1}^{N} \left{ 4 \left( \frac{A}{A} \right) S_i \right}_i )</td>
<td>Linear coma</td>
</tr>
<tr>
<td>30</td>
<td>( \sum_{i=1}^{N} \left{ 4 \left( \frac{A}{A} \right)^2 S_i \right}_i )</td>
<td>Quadratic astigmatism</td>
</tr>
<tr>
<td>31</td>
<td>( \sum_{i=1}^{N} \left{ 2 \left( \frac{A}{A} \right)^2 S_i + S_{ii} \right}_i )</td>
<td>Field curvature</td>
</tr>
<tr>
<td>32</td>
<td>( \sum_{i=1}^{N} \left{ 4 \left( \frac{A}{A} \right)^3 S_i + 2 \left( \frac{A}{A} \right) S_{ii} \right}_i )</td>
<td>Cubic distortion</td>
</tr>
</tbody>
</table>
3.2.2 Ray Tracing
An integral part of the bilateral systems theory is the optical axis ray (OAR), which serves as the reference ray for the system and lies in the single plane of symmetry. The traditional optical axis cannot provide the reference for bilateral systems since these systems can be off-axis. The OAR serves the same functions as the traditional optical axis, such as establishing the coordinate axes for the field and pupil vectors and defining the center of the pupils and FOV. However, unlike the traditional optical axis, the OAR changes direction at a reflecting or refracting surface.\(^{52}\)

For example, the OAR in Figure 17 moves from left to right in front of the primary mirror, from right to left between the primary and secondary mirrors, and again from left to right after the secondary due to the mirror reflections. Numerically, the directional change introduces a negative sign into the aberration calculations (when the OAR is moving from right to left), requiring a slight adjustment to the ray trace equations. The reflection and transfer equations used in the simulation code are, respectively,

\[
\begin{align*}
    u' &= u - y \left( \frac{n' - n}{R} \right), \\
    y' &= y + u' \frac{t}{n'},
\end{align*}
\]

where \( t \) is the distance between optical surfaces. Published ray trace equations are\(^{53}\)

\[
\begin{align*}
    n'u' &= nu - y\phi, \\
    y' &= y + u't'.
\end{align*}
\]

The main difference between Eqs. 58 and 59 and the published equations in Eqs. 60 and 61 is the placement of the index of refraction \( n \). Eqs. 58 and 59 were determined via iteration so that the OAR formulism could be accurately employed during the ray trace and subsequent aberration calculation. Although they differ from published ray trace equations, the accuracy of Eqs. 58 and 59 is demonstrated in Section 3.3.4.

3.2.3 AOZ Aberration Simulation
The bilateral theory is intended as an initial design tool and, as such, is not meant to compete with commercially available optical design software in terms of absolute accuracy; using this theory
will give an optical designer a peak-to-valley (PV) error that is about 5-10% away from the error found with software. It does, however, surpass commercial software packages in terms of simulation speed, as only two rays need to be traced, particularly when a large-scale parameter sweep is performed as in Section 3.4.

Also, since the theory is paraxial, the image is assumed to be at the Gaussian image plane, thereby removing a designer’s ability to balance aberrations with defocus. Thus, the total PV error found with the bilateral theory can be larger than is usually permissible since it is assumed that a designer will perform further optimization in commercial software.

### 3.3 Simulation Code

#### 3.3.1 Logic Flow

Matlab scripts were written to simulate AOZ systems using the theory developed in Sections 3.1 and 3.2. First, the five free variables of \([F_1, F_2, f_{i1}, f_{i2}, D_p]\) produced a set of construction parameters per Eqs. 47-50 and 56. Next, this set is down-selected with the criteria in Table 7. Then, the construction parameters are used to paraxially trace marginal and chief rays. Seidel aberration coefficients are calculated from the ray trace using the equations in Table 6. Finally, the construction parameters are recorded if the maximum PV error is below a given threshold. The PV error is determined at standard normalized field heights of \(H = [0.0, 0.7, 1.0]\). A schematic of the logic is found in Figure 18. Repeating the logic of Figure 18 enables a parameter sweep, where many values of the free parameters can be simulated to produce several viable designs, enabling the tradespace analysis found in Section 3.4.

![Schematic of logic for simulation program](image)

Performing the ray trace and calculating the aberration coefficients are the most time-intensive portions of the program. Therefore, more down-selects are placed before the ray trace to make the code as efficient as possible. Note that the threshold parameters in Table 7 are not absolute; i.e., individual designers are free to down-select potential designs using different
parameters. Other possible down-selects could be entrance pupil diameter, focal ratio in either state, and maximum allowable change in ROC for either the primary or secondary mirror.

Table 7. Initial criteria used to down-select potential AOZ designs. Labels: $T_L$ is the threshold on system length, $T_{ZT}$ is the threshold on zoom ratio, and $T_\varepsilon$ is the threshold on obscuration ratio.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d &gt; 0$</td>
<td>The distance $d$ is defined to be positive.</td>
</tr>
<tr>
<td>$WD &gt; 0$</td>
<td>The distance $WD$ is defined to be positive.</td>
</tr>
<tr>
<td>$R_{21} \neq R_{22}$</td>
<td>The secondary ROC must change between states to achieve zoom.</td>
</tr>
<tr>
<td>$(d + WD) &lt; T_L$</td>
<td>The system length must be less than the threshold.</td>
</tr>
<tr>
<td>$Z_T &gt; T_{ZT}$</td>
<td>The zoom ratio must be greater than the threshold.</td>
</tr>
<tr>
<td>$\varepsilon &lt; T_\varepsilon$</td>
<td>The obscuration ratio must be less than the threshold.</td>
</tr>
</tbody>
</table>

### 3.3.2 Determination of Obscuration Ratio

The obscuration ratio $\varepsilon$ is the ratio of the secondary diameter to the primary diameter, given as

$$\varepsilon = \frac{D_s}{D_p}, \quad (62)$$

where $D_s$ is the diameter of the secondary. The obscuration ratio defines the amount of light blocked by the secondary mirror.51 For this dissertation, the obscuration ratio is one of two figures of merit, along with the zoom ratio, used to determine the final AOZ design in Section 3.4.2.

![Figure 19. Paraxial layout of the two-element AOZ system to derive the obscuration ratio. The mirrors are represented by thin-lenses.](image-url)
The obscuration ratio can be calculated before the ray trace using the marginal ray height at the primary and secondary mirrors. For this derivation, the coordinate system of Figure 17 has been greatly simplified to the system in Figure 19, where the mirrors are represented by thin lenses. Then, the tangent of the marginal ray angle between the ‘mirrors’ is given by

\[ u = \frac{h}{d}, \]  

where

\[ h = \frac{D_p - D_s}{2}. \]  

Next, we can relate \( u \) to the primary focal length with Eq. 60,

\[ u = u_0 - y_{in} = -\frac{D_p}{2f_{in}}, \]  

where \( u_0 \) is zero since the initial ray is from infinity. Substituting Eqs. 63 and 64 into Eq. 65 and solving for \( d \), we arrive at an expression containing the obscuration ratio,

\[ d = f_{in} \left( \frac{D_s}{D_p} - 1 \right) = f_{in} (\varepsilon - 1). \]  

Finally, we solve for \( \varepsilon \),

\[ \varepsilon = 1 + \frac{d}{f_{in}}. \]  

Thus, we can calculate the obscuration ratio using the independent parameter \( f_{in} \) and the dependent parameter \( d \) before the simulation ray trace.

### 3.3.3 Code Description

The simulation code to enact the logic in Figure 18, perform the tradespace analysis in Section 3.4, and garner the AOZ design in Section 3.5 encompasses five separate Matlab scripts. The five programs are briefly described below.
1. “tradespaceSimV6.m” – the main program for reflective AOZ design. The inputs are the independent parameters described in Section 3.3.1, and the output is an array containing the construction parameters for AOZ systems that meet the given thresholds.

2. “detSystemPVdia.m” – a sub-program that calculates the PV error. This program uses the construction parameters to trace marginal and chief rays and then determines the PV error using the sub-programs “SeidelCoeffsAxial.m” and “onePoint.m”.

3. “SeidelCoeffsAxial.m” – a sub-program that determines the Seidel aberration coefficients for an axial system. This program uses the heights and angles from the ray trace to calculate the Seidel aberration coefficients with the equations in Table 6.

4. “onePoint.m” – a sub-program that finds the PV error at one point and field angle within the exit pupil. The PV error is determined on a point-by-point basis using the aberration coefficients and Eq. 57.

5. “findGoodSettings.m” – the main program for the AOZ tradespace analysis. This program basically runs “tradespaceSimV6.m” in a giant for loop with different values of the primary and image diameters at each iteration. The output is the graph in Figure 22.

### 3.3.4 Verification

To verify the accuracy of the simulation code, the aberrations of a stock non-zooming Cassegrain objective were simulated. The optical prescription is found in Table 8. Note that the stop is at the primary mirror and the values follow the definitions in Figure 17.

Table 8. Optical prescription for the non-zooming Cassegrain used to verify the simulation code.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_p$</td>
<td>100 mm</td>
</tr>
<tr>
<td>$f_1$</td>
<td>-100 mm</td>
</tr>
<tr>
<td>$f_2$</td>
<td>-25 mm</td>
</tr>
<tr>
<td>$d$</td>
<td>80 mm</td>
</tr>
<tr>
<td>WD</td>
<td>20 mm</td>
</tr>
<tr>
<td>Field Angles</td>
<td>0.0°, 0.5°</td>
</tr>
</tbody>
</table>
The first functionality to check is the accuracy of the simulated aberration coefficient values. This test also verifies the accuracy of the ray trace since the coefficients are calculated using values from the trace. In order to provide what are considered the ‘actual’ aberration values, the system in Table 8 was modeled in Zemax. These values were then compared to the simulated coefficients with the results tabulated in Table 9. Overall, the agreement is exemplary as four of the five aberrations match to the hundredth of a wave. Thus, the ray trace and aberration calculation are functioning correctly.

Table 9. Comparison of the aberration coefficients as determined by Zemax and the simulation code. The agreement between the values is exemplar.

<table>
<thead>
<tr>
<th>Aberration</th>
<th>Zemax (wv)</th>
<th>Matlab (wv)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical Aberration</td>
<td>237.53</td>
<td>237.53</td>
<td>0.0%</td>
</tr>
<tr>
<td>Coma</td>
<td>-21.58</td>
<td>-21.58</td>
<td>0.0%</td>
</tr>
<tr>
<td>Astigmatism</td>
<td>-0.12</td>
<td>-0.12</td>
<td>0.0%</td>
</tr>
<tr>
<td>Field Curvature</td>
<td>2.25</td>
<td>2.19</td>
<td>-2.6%</td>
</tr>
<tr>
<td>Distortion</td>
<td>-0.33</td>
<td>-0.33</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

A second verification of the simulation code is to ensure that aberrations are correctly calculated across the exit pupil. To do this, optical path difference (OPD) plots were produced by the simulation code and compared to the analogous Zemax OPD plots. Figure 20 and Figure 21 display OPD plots for sagittal and tangential components at the two tested field angles, respectively. The simulation and Zemax plots are superimposed on each other. The residual errors of spherical aberration and defocus are due to the Ref. 55 system not being well-corrected, as opposed to errors within the simulation code. Visually the agreement is extremely good. Numerically, the root-mean-squared (RMS) error between Zemax and the simulated results is below 1% for all four plots. Thus, the entire simulation code performs in a highly accurate manner.
3.4 Tradespace Analysis

Combining the theory in Sections 3.1 and 3.2 and the simulation code in Section 3.3 enables a full-scale analysis of the AOZ tradespace, which allows optical design to begin from a broad perspective and optimize to a particular design. The analysis can be used on any of three basic objective topologies – Cassegrain, Gregorian, or a system that is Cassegrain in one state and
Gregorian in another. This section outlines the steps to perform an AOZ tradespace analysis, focusing only on designing a Cassegrain AOZ objective for simplicity.

Before beginning the tradespace analysis, we must set thresholds on the parameters used to down-select designs during simulation, as detailed in Section 3.3.1 - zoom ratio $Z_R > 3$, $\epsilon < 0.35$ and PV $< 150$wv. A PV error of 150wv may sound large but is easily mitigated when defocus can be introduced (see Section 3.2.3) and the proper aspheric surfaces are employed (parabolic primary and hyperbolic secondary, in this case).

### 3.4.1 Set Primary and Image Diameters

Next, the image diameter $D_I$ and the entrance pupil diameter $D_p$ need to be established as these values are constant during a single parameter sweep. Figure 22 shows simulation results as $D_I$ varies from 7.5-35mm and $D_p$ varies from 250-750mm. Note that each parameter sweep consists of 1,048,576 possible AOZ designs, making the total number of simulated designs in Figure 22 264,241,152. Each box displays the number of AOZ designs that passed all three thresholds, as described in Figure 18; thus, Figure 22 shows ‘hot spots’ where designers should focus further analysis as many AOZ designs result from these values. From the graph, we see that the highest number of designs are found for $D_I = [7.5,15]$mm and $D_p = [300-400]$mm. Let’s chose $D_I = 7.5$mm and $D_p = 375$mm for further analysis since the maximum number of designs were found at these two values.

![Figure 22. Number of viable AOZ designs found for a Cassegrain objective as the image diameter $D_I$ and primary diameter $D_p$ vary. The colorbar’s units are number of designs.](image-url)
3.4.2 Set Construction Parameters
With the primary and image diameters set, we perform a single parameter sweep to set the remaining construction parameters, thus pinpointing a particular AOZ design. As stated in Section 3.3.2, the two figures of merit chosen to determine the final design were the zoom and obscuration ratios. Specifically, the merit function is to maximize the zoom ratio (for maximum image enlargement) and minimize the obscuration ratio (for maximum throughput). Figure 23 shows the obscuration and zoom ratios resulting from a parameter sweep with a 375mm primary diameter and 7.5mm image diameter. The obscuration ratio for the unzoomed state is displayed because $\varepsilon$ is larger in the unzoomed state than the zoomed state for this particular parameter sweep. The focal ratios in both states were allowed to vary between 1-20 and the primary focal lengths between 100-1000mm. The merit function implies that the ‘best’ designs are found to the right of the red line. Putting more emphasis on zoom ratio leads to the design at $\varepsilon = 0.34$ and $Z_R = 3.78$.

![Figure 23. Results from a parameter sweep with $D_i = 7.5$mm and $D_p = 375$mm. The obscuration ratio on the y-axis is for the unzoomed state. Designs to the right of the dashed red line are the ‘best’ for this analysis.](image)

3.5 Optical Design Results
Optimization was performed on the chosen design from Figure 23 using Zemax. The primary’s conic constants $\kappa_{11}$ and $\kappa_{12}$ were both set to -1.0 per the classical Cassegrain topology, while the secondary’s conic constants $\kappa_{21}$ and $\kappa_{22}$ were allowed to vary individually during optimization.
The lengths and element ROCs were also allowed to vary to minimize residual error. The Zemax default merit function was utilized for optimization.

Figure 24. AOZ system with 375mm diameter primary. (a) System layout and (b) spot diagrams for the unzoomed state, and (c) system layout and (d) spot diagrams for the zoomed state. Normalized field heights of 0.0, 0.7, and 1.0 (clockwise from upper left) are displayed for each state along with the Airy disc.

Figure 24 displays the final AOZ system, with (a) and (b) showing the layout and spot diagrams for the unzoomed state, respectively, and (c) and (d) showing the same for the zoomed state. The full prescription is in Table 10. Both states are well-corrected and beyond the diffraction limit, as evidenced by the superimposed Airy disc in Figure 24(b) and (d). Notice that the secondary’s ROC increases by a substantial amount, far more than the primary’s ROC, possibly affecting the practicality of this particular system. The stop diameter is decreased in the unzoomed state to maintain a constant focal ratio and achieve optical zoom, as described in
Section 2.1.1. A focal ratio of 19.2, while slow, is commensurate with other large aperture telescopes such as the F/24 Hubble space telescope.\textsuperscript{56}

Table 10. Construction values and system attributes for the AOZ system in Figure 24 with $D_t = 7.5\text{mm}$ and $D_r = 375\text{ mm}$. As before, ‘wv’ are in reference to the wavelength of a HeNe laser. Other labels: $D_{\text{stop}}$ is the stop diameter and $D_{\text{secondary}}$ is the secondary diameter.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_R$</td>
<td>3.3</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.34</td>
</tr>
<tr>
<td>$F_n$</td>
<td>19.7, 19.7</td>
</tr>
<tr>
<td>$f_{1n}$</td>
<td>1088.9mm, 803.4mm</td>
</tr>
<tr>
<td>$\kappa_{1n}$</td>
<td>-1.0, -1.0</td>
</tr>
<tr>
<td>$D_{\text{stop}}$</td>
<td>113.6mm, 375.0mm</td>
</tr>
<tr>
<td>$R_{2n}$</td>
<td>-1429.6mm, -183.7mm</td>
</tr>
<tr>
<td>$\kappa_{2n}$</td>
<td>-8.37, -1.54</td>
</tr>
<tr>
<td>$D_{\text{secondary}}$</td>
<td>39.2mm, 39.2mm</td>
</tr>
<tr>
<td>$d$</td>
<td>721.5mm</td>
</tr>
<tr>
<td>$WD$</td>
<td>34.4mm</td>
</tr>
<tr>
<td>$HFOV$</td>
<td>0.13°, 0.04°</td>
</tr>
<tr>
<td>$PV\ Error$</td>
<td>0.1wv, 0.07wv</td>
</tr>
</tbody>
</table>

An overall intent of this theory is to accurately simulate AOZ systems in order to shrink the disparity between theory and experiment, a disparity found in Ref. 50 and described in the introduction to Section 3.1. The simulated and final parameter values are listed in Table 11 along with the percent differences between the values. Notice that most differences are ~10\%, as detailed in Section 3.2.3. The largest changes are the working distance and unzoomed focal ratio. The decrease in $WD$ between final and simulated values is expected since the simulation program does not include defocus, as pointed out in Section 3.2.3. The sharp difference in unzoomed focal ratios is due to the lack of a down-select criterion for a constant focal ratio between the two states. This is an obvious point of improvement in further research since a constant focal ratio is tantamount for zoom systems.
Table 11. Comparison between simulated and final construction parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simulated Value</th>
<th>Final Value</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_R$</td>
<td>3.8</td>
<td>3.3</td>
<td>15.1%</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.34</td>
<td>0.34</td>
<td>0.0%</td>
</tr>
<tr>
<td>$F_o$</td>
<td>5.26, 20</td>
<td>19.7, 19.7</td>
<td>-73.3%, 1.5%</td>
</tr>
<tr>
<td>$f_{1o}$</td>
<td>991.8mm, 720.4mm</td>
<td>1088.9mm, 803.4mm</td>
<td>-8.9%, -10.3%</td>
</tr>
<tr>
<td>$R_{2n}$</td>
<td>-1348.6mm, -141.9mm</td>
<td>-1429.6mm, -183.7mm</td>
<td>-5.6%, -22.7%</td>
</tr>
<tr>
<td>$d$</td>
<td>656.2mm</td>
<td>721.5mm</td>
<td>-9.1%</td>
</tr>
<tr>
<td>$WD$</td>
<td>11.86mm</td>
<td>34.4mm</td>
<td>-65.5%</td>
</tr>
<tr>
<td>$HFOV$</td>
<td>0.10°, 0.02°</td>
<td>0.13°, 0.04°</td>
<td>-23.1%, -50.0%</td>
</tr>
</tbody>
</table>

3.6 Residual Aberration Correction

Additional adaptive optics (AO) can be employed to correct residual aberrations. Although not needed for the results in Figure 24, since that system is already diffraction limited, many systems require AO to achieve the diffraction limit. A possible AO layout for the previously-designed Cassegrain AOZ system is found in Figure 25. The Cassegrain objective produces an image behind the primary on the optical axis. The image is then re-collimated by lens $L_1$ and reflects off a deformable mirror (DM). $L_2$ and $L_3$ conjugate the DM and SHWS. A 50/50 beamsplitter sends the beam into a Shack-Hartmann wavefront sensor (SHWS) and through the imaging lens $L_4$ to the focal plane at $f_0^*$. Aberration correction is performed via a closed-loop feedback system between the DM and SHWS, as described in Section 1.4.

The combination of the AOZ layout in Figure 17 and the aberration correction provided by the additional optics in Figure 25 has the potential to create a large-aperture, diffraction limited AOZ Cassegrain objective. One general observation during this analysis was that larger zoom ratios meant larger PV error. Thus, the AO correction could push the zoom ratio of a Cassegrain objective well beyond the 3.3X displayed in Figure 24 while maintaining excellent optical performance.
Figure 25. Schematic of additional AO system for aberration correction. The marginal ray is displayed as a dotted line. Labels: $M_1$ is the primary mirror, $M_2$ is the secondary mirror, $L_1$ is the collimating lens, $L_2$ and $L_3$ are an image relay system, $L_4$ is the imaging lens, $BS_1$ and $BS_2$ are 50/50 beamsplitters, $DM$ is a deformable mirror, $SHWS$ is a Shack-Hartmann wavefront sensor, and $\hat{y}_0$ is the image plane.
CHAPTER 4: CARBON FIBER REINFORCED POLYMER MIRRORS

With the ability to design an adaptive optical zoom (AOZ) system in Chapter 3, we can now explore the experimental execution of AOZ. The second half of this dissertation describes the development of a deformable primary mirror for an AOZ system constructed of carbon fiber reinforced polymer (CFRP). This chapter describes the material properties of CFRP, past results with CFRP mirrors, and compares the deformation ability of CFRP to glass. An example of a CFRP mirror is pictured below in Figure 26 where a standard Air Force resolution chart is clearly visible.

![Figure 26. An example of a CFRP mirror. This mirror was fabricated by Composite Mirror Applications.](image)

4.1 Material Properties

Most testing of material properties involves the application of force and the measurement of resulting displacement. This technique, derived from linear elasticity theory, defines material properties as a relation between the stresses and strains, where the response to a force or other ‘excitation’ produces a linear function of the excitation tensor

\[
\sigma_{ij} = c_{ijkl} \epsilon_{kl}, \quad (68)
\]
where $\sigma_{ij}$ is the stress tensor, $c_{ijkl}$ is the stiffness coefficient, and $\varepsilon_{ij}$ is the strain tensor. Eq. 68 results in 21 elements, an enormous amount of information. Significant reduction occurs when symmetry of the material is employed.\textsuperscript{57}

For the purposes of this dissertation, several elements of Eq. 68 can be combined into three common measurements of materials – Young’s modulus, coefficient of thermal expansion (CTE), and coefficient of moisture expansion (CME). Young’s modulus, or the modulus of elasticity, describes a material’s reaction to stress when the deformation is elastic.\textsuperscript{58} CTE characterizes the amount a material expands when heated and contracts when cooled. CME is related to CTE, defining the amount a material expands in the presence of moisture.\textsuperscript{59} Both CTE and CME are important aspects for zoom since small element deviations can cause large aberrations in the image plane (see Section 2.2.2).

Figure 27. (a) Diagram of four-point bend test. (b) Results from bend test for a 24-ply CFRP coupon.\textsuperscript{70} The $y$-axis has units of pounds-force (lbf).
Young's modulus can be calculated by measuring the individual values of the tensors that comprise it, or \textit{en masse} with the four-point bend test. The four-point bend test, diagrammed in Figure 27(a), involves placing a flat sample of the material (called a ‘coupon’) between four supports, each with a point contact to the coupon. Then, a force is slowly applied from the top and displacement is tracked as a function of force. The test ends when the coupon catastrophically fails and breaks into pieces. Data from the test, shown in Figure 27(b), can be used to determine Young’s modulus and ultimate strength as in Section 4.4. The CFRP coupon tested in Figure 27(b) is mechanically identical to the active mirror in Chapter 6.

Figure 27(b) also displays three distinct regions of a material’s reaction to applied force – a linear region, a non-linear region, and failure. The CFRP’s linear region, extending from 0-100 pounds-force (lbf), is also called the elastic region, since induced displacement will vanish once the force is alleviated. In other words, the coupon would return to its original shape and no residual signs of stress will remain. The non-linear region, extending from 100-110 lbf, occurs when some fibers within the material start breaking, causing permanent damage. The failure region, above 110 lbf, means that the coupon has literally broken in half. From this graph, it is obvious that an active CFRP mirror needs to operate within the linear region, since even a small number of broken fibers would result in performance degradation.

Table 12. Material properties for CFRP and other materials. ‘SiC’ stands for silicon carbide.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg/m$^3$)</th>
<th>Young's Modulus (GPa)</th>
<th>CTE (ppm/K)</th>
<th>CME at 35% RH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>2710</td>
<td>71</td>
<td>23.9</td>
<td>0</td>
</tr>
<tr>
<td>SiC</td>
<td>3140</td>
<td>420</td>
<td>2.2</td>
<td>0</td>
</tr>
<tr>
<td>Zerodur</td>
<td>2520</td>
<td>92.9</td>
<td>0.05</td>
<td>0</td>
</tr>
<tr>
<td>CFRP</td>
<td>1170</td>
<td>101</td>
<td>0.2</td>
<td>9.0-25</td>
</tr>
</tbody>
</table>

A comparison of CFRP to aluminum, silicon carbide (SiC) and Zerodur is found in Table 12, which was originally compiled by Ref. 61. Several important conclusions can be drawn from Table 12:

1. CFRP has a high stiffness in comparison to Zerodur and aluminum.
2. CFRP has a low areal density, resulting in less overall weight and creating an extremely high stiffness-to-weight ratio that is \textasciitilde5X that of steel.
3. Its CTE is very low, but not as low as Zerodur.
4. Its CME is non-zero, meaning that performance will vary as a function of humidity. Thus, mechanically at least, CFRP appears to be a promising material for an active mirror.

4.2 Fabrication Processes and Concerns

4.2.1 Mirror Fabrication

CFRP mirrors are fabricated via replication, where CFRP material is placed upon a glass mandrel, thereby forming the eventual reflecting surface. An outline of the process performed by Composite Mirror Applications (CMA), a major CFRP mirror manufacturer, is found in Figure 28. A glass mandrel, polished to optical quality, defines the mirror shape and ROC by forming the complementary shape; i.e., if a concave CFRP mirror of ROC = 2000mm is desired, the mandrel must be convex with ROC = -2000mm. Then, several layers of prepreg are placed upon the mandrel. ‘Prepreg’ is short for unidirectional pre-impregnated composite fiber material, a type of carbon fiber available where all fibers of a single layer are collinear and held together with resin. Next, the prepreg/mandrel combination is cured at high-temperature and under vacuum causing the resin in the individual layers to fuse into a single mirror. Finally, the mirror is released from the mandrel, typically with the use of a release agent that eases the separation and helps maintain quality.63,64,62,65

Since the mirror’s reflecting surface is formed by the mandrel, the quality of the mandrel has a large effect on the resultant optical quality. Any imperfection on the mandrel will be found in the CFRP mirror. Figure 29 clearly demonstrates this where Ronchigrams of a glass mandrel (left) and CFRP mirror fabricated from that mandrel (right) are nearly identical, particularly in the center.62 A ‘Ronchigram’ is the result of a Ronchi test, which is a non-interferometric, pseudo-quantitative method to measure aberrations. A perfect optic will display straight-line fringes in the Ronchigram,66 as opposed to the aberrated optics in Figure 29. CMA has found that, with their fabrication process at least, the correlation between mandrel and mirror is nearly one-to-one, meaning that a mandrel with 1wv PV of error will produce a mirror with ~1wv PV of error. Thus,
the quality of the mandrel is a main limiting factor in optical performance, provided fiber print-through is mitigated (see Section 4.2.2).

Figure 29. Ronchigrams of (left) glass mandrel and (right) CFRP mirror replicated from the mandrel.

The direction of the individual fiber layers determines the mechanical and thermal properties, meaning that the properties can be tailored with different lay-up schemes. For instance, if the lay-up scheme places all fibers in the same direction, as on the left in Figure 30, the mirror will exhibit substantially different properties in orthogonal directions, such as high strength in the plane parallel to the layers but low in the perpendicular direction. A balanced lay-up, as on the right in Figure 30, will result in properties that are quasi-isotropic. Even with a balanced lay-up, however, the number of layers has a large effect on the overall stiffness of the mirror and its resulting quality – too many layers and the mirror will be very stiff and hard to actuate, too few and the mirror will not be able to maintain good static quality.

Then, the mirror’s ROC can be altered during the cure process or the release from the mandrel. A CFRP mirror with a nickel coating placed between the prepreg and mandrel, added to enable post-polishing, has been observed to decrease the ROC by ~1%. This is almost certainly caused by a CTE mismatch between the prepreg and nickel since a mirror without the coating resulted in no ROC change. Even the release from the mandrel can change the mirror surface shape if force is not applied uniformly.
Finally, although the final resin surface can be mildly reflective, only deposition of a metallic coating realizes a true reflecting surface. Reflective coatings have been successfully applied for wavelengths from the UV (200nm) to the microwave (1cm) regimes. The CFRP fibers and resin cannot be polished directly, but depositing a thick metal layer enables polishing of the mirror’s surface to increase optical quality, as in Section 4.3.2. Care must be given in CTE match, however, to avoid the ROC change described above.

4.2.2 Fiber Print-through
A major concern with CFRP mirrors has been fiber print-through (FPT), which is where the fiber structure can be observed within reflected beams. FPT induces a periodic structure into the reflection, as in Figure 31(a), or a random structure if several layers print-through, as in Figure 31(b). FPT is thought to be caused by chemical and thermal shrinkage of the prepreg during the cure and cooling process, and has been found to be exaggerated by increases in cure pressure and temperature. Interestingly, it does not appear to be caused by the individual fiber diameters, as the fiber spatial frequency does not correlate well with the observed FPT spatial frequency.
Controlling FPT has been attempted in several different ways. The most basic is to add an additional layer of resin to the final resin surface, which serves to ‘fill-in the gaps’ between fibers observed in Figure 31. In fact, a resin layer 0.25mm thick has been found to completely eliminate FPT, as in Figure 32(a), although this can increase RMS surface roughness. A related idea involved placing a 0.1mm thick layer of metal to fill-in the gaps, although no FPT mitigation was observed. A highly successful method, found in Figure 32(b), has reduced FPT to 3nm PV virtually eliminating the problem (the lower frequency oscillation is due to the mirror’s curvature). This process was conceived by CMA and is considered proprietary. CMA mirrors, however, have been observed to exhibit some FPT months to years after fabrication, so FPT is still not a completely solved problem.
Thus, the fabrication of CFRP mirrors is a process wrought with potential errors. Increased effort in process development should further decrease the errors, thereby allowing development into the tantalizing and potentially revolutionary realm of material property tailoring. For now, though, CFRP fabrication only results in highly-stiff static mirrors that are not diffraction limited (see Section 4.3.2). Both are major concerns for the active CFRP mirror in Chapter 6 since high stiffness requires a large force to deform the mirror, and initial mirror aberrations reduce the available DM stroke to correct deformation-induced aberrations.

4.3 CFRP Mirrors

4.3.1 Finite Element Modeling

Finite element modeling (FEM) is a theoretical tool used to predict and visualize the effect of stress upon a material or structure. FEM is a computational extension of the finite element method, where a material is divided into many small sections and simulated individually. The inputs are the material properties and stresses, and the output is the resulting shape due to the stress.

FEM is a powerful tool for CFRP mirrors as it can be used to analyze the gamut of CFRP mirrors, from fabrication schemes to gravity-induced sag to actuation modalities. As described in Section 4.2.1, the fiber lay-up scheme has a great effect on the resulting mechanical properties. The variety of available prepreg, resins, lay-up angles, mirror thickness, etc., lead to an immense tradespace for CFRP mirror production. Time and money can be saved by modeling different combinations in code using FEM. This analysis can not only reveal a good fabrication method but also give insight into the ability to tailor material properties for specific applications.

Since CFRP mirrors are thin-shelled components, gravity can induce substantial aberrations despite the high stiffness-to-weight ratio. If the mirror is mounted vertically, gravity will induce astigmatism since the curvature in the vertical direction has decreased and has remained constant in the horizontal direction. When the mirror is mounted horizontally, gravity will cause a uniform deformation, as in Figure 33, where most of the mirror has sagged by the same amount (colored in green). The ‘dimples’ observed in Figure 33 are point-load actuators attached to the mirror’s back and do not sag with gravity, an effect also observed experimentally in Figure 38(b).
Most importantly for this dissertation is FEM’s ability to simulate different actuation modalities, where an ‘actuation modality’ is a pattern of applied forces used to change ROC. This model, developed by Sandia National Laboratories and the Naval Research Laboratory, specifically used material properties determined from the four-point bend test in Section 4.1, which, again, is mechanically identical to the active mirror in Chapter 6. To ensure model accuracy, FEM was first used to predict the deformation from a point-load placed at the mirror’s apex. The FEM wavefront in Figure 34(a) and the experimentally measured wavefront in Figure 34(b) agree well with 8.6% RMS error across the entire surface. Note that the experimental CFRP mirror in Figure 34(b) is the same mirror used in Chapter 6.
Figure 34. FEM results of CFRP mirror actuation. (a) Predicted wavefront due to apex point-load and (b) measured wavefront. (c) Predicted wavefront for uniform pressure and (d) cross-sectional error.

Figure 34(a) and (b) establish a reasonable amount of confidence in the FEM’s predictive ability, allowing extension of the model to simulating more complex actuation methods. Several methods are simulated in Ref. 70, with one such example in Figure 34(c) and (d). Here, a constant pressure is applied uniformly to the back of the mirror (analogous to hydraulic machinery) to change the ROC from 2000mm to 1250mm, a substantial change. The resulting wavefront and cross-section in Figure 34(c) and (d), respectively, show that this modality is beyond usefulness for any imaging application with a 105μm PV error.70 A less substantial ROC change would result in lower error, thereby potentially allowing this modality to be utilized in an experimental active mirror system. FEM was used to design the actuation methods described in Section 6.1.

4.3.2 Recent Static Mirrors

The first step to achieving an active CFRP is the successful production of a static mirror. One group71 fabricated a flat CFRP mirror with a thick nickel coating for post-production polishing, as described earlier in Section 4.2.1. The coating induced 65μm of astigmatism due to CTE mismatch during curing. Manual polishing and grinding reduced to overall error to 15.4μm PV, as seen in Figure 35. While a significant improvement, this level of error is still far too large for good imaging.
High quality static CFRP mirrors have been previously produced by CMA. CMA uses cyanate ester as its resin matrix\textsuperscript{57} in order to reduce the large CME commonly observed in CFRP mirrors.\textsuperscript{71} With this and other proprietary processes, CMA has successfully fabricated and delivered a 400mm diameter F/1 parabolic CFRP mirror, pictured in Figure 36, both without a central aperture for a Newtonian telescope (Figure 36(a)) and with an aperture for a Cassegrain telescope (Figure 36(b)). Both mirrors were measured to 0.6\(\mu\text{m}\) of error across the entire aperture. However, CMA has yet to solve the extreme temperature sensitivity in CFRP mirrors, which has been measured to cause a 135% PV error increase when temperature was decreased from 60°F to 20°F.\textsuperscript{62} All experimental CFRP mirrors in Chapter 6 were produced by CMA.

4.3.3 Recent Active Mirrors

The first active CFRP mirror combined the flat mirror from Figure 35 and the actuation modality pictured in Figure 37(a). Seven point-load actuators, arranged hexagonally, have the ability to both push and pull the mirror surface.\textsuperscript{61} Then, the flat mirror was corrected to 11.4\(\mu\text{m}\) PV using
open-loop feedback in Figure 37(b). Open-loop feedback is manual feedback, where the user observes the wavefront change as actuators are displaced. The actuator influence function, required for closed-loop feedback (as described in Section 1.4.4), can be predicted by FEM. Figure 37(c) displays reasonable agreement the predicted and measured influence function.69

![Figure 37](image)

Figure 37. Images of the first active CFRP mirror. (a) Schematic of actuator layout,61 (b) wavefront after open loop correction, and (c) comparison of actuator influence function between FEM (red) and experiment (blue). 69

A similar actuation modality was employed to an 80mm diameter concave mirror. Seven point-load actuators were again utilized to deform the mirror, as in Figure 38(a), although two significant improvements have been made over Figure 37. First, the CFRP mirror was fabricated by CMA and has an initial error of 92nm RMS, immediately improving the results from Figure
Another improvement is a reduction of the actuator influence function on the mirror. The pads connecting the actuators to the mirror create a zone in which the ROC cannot be changed, especially the large pads observed in Figure 37(a). This effect can be directly observed via the circular deflections in Figure 38(b), caused by adhesive holding neodymium magnets to the mirror. (Neodymium magnets coupled the actuators to the mirror through magnetic attraction, a method applied to the opto-mechanical apparatus described in Section 6.3.) The combination of these two improvements results in fantastic error correction to 17.4nm RMS, as in Figure 38(c), again via open loop feedback. The still significant pad influence was removed in Figure 38(c), but has since been mitigated by reducing the amount of adhesive used to affix the magnets.

![Figure 38](image)

Figure 38. (a) 80mm diameter CFRP mirror with seven actuators, (b) interferogram showing influence of glue spots to hold neodymium magnets, and (c) corrected wavefront.
All the aforementioned active mirrors were designed to correct high-order aberrations, in order to garner the best-possible reflecting surface, rather than low-order aberrations, in order to change ROC. In fact, the author and colleagues at Sandia National Laboratories are the first group to successfully change the ROC of a large diameter CFRP mirror, as is done using both open and closed-loop control in Chapter 6.

4.4 CFRP & Glass Comparison

The vast majority of optical elements, both refractive and reflective, are made of glass, begging the question as to why a thin-shelled CFRP mirror was chosen over its glass counterpart. One attraction of CFRP over glass is the weight reduction. Referring to Table 12, CFRP has a 53% lower density than Zerodur, leading to at least an equivalent reduction in weight for mirrors of the same thickness.

Another advantage is reduction of fabrication costs. Glass mirrors need to be individually cast and polished, involving great costs in time and labor. CFRP mirrors, on the other hand, are replicated, thus only limited by the cost of more prepreg and resin since the mandrel and other fixtures need only be purchased once. Also, the overall time is cut since no polishing is required (if CMA mirrors are used, at least).

By far the greatest advantage of CFRP over glass is robustness during actuation. The force required to deform a mirror is given by

$$ C \propto \frac{F}{Et^3}, $$

(69)

where $C$ is element curvature, $F$ is applied force, $E$ is the Young’s modulus, and $t$ is the thickness of the mirror. Curvature is given here since curvature is proportional to deformation. The thickness of a CFRP mirror can be much lower than glass since CFRP is resistant to shattering, unlike glass which is brittle. Thus, let’s assume a 0.8mm thick CFRP mirror and a 2mm thick Zerodur mirror. Then, using values from Table 12, the force required to cause an equivalent curvature change is ~10X higher for Zerodur than for CFRP.

Furthermore, CFRP can sustain a much higher amount of deformation than Zerodur. Figure 39 displays force vs. displacement curves for Zerodur (blue) and CFRP (green) using Eq. 69. (Note that ‘force vs. displacement curves’ are also referred to as ‘stress-strain curves.’) This calculation assumes elastic deformation, i.e., the linear deformation region described in Section 4.1. Notice that the slope of the line is much larger for Zerodur than for CFRP, meaning that
equal amounts of force will produce a much greater ROC change for CFRP than Zerodur. Since one major benefit of AOZ is power budget reduction (see Section 2.4), the lower required force for an active CFRP mirror begs its usage in experimental AOZ systems.

![Figure 39. Simulated force vs. displacement curves for Zerodur (blue) and CFRP (green). Notice that the same amount of force produces more deflection for CFRP than for Zerodur.](image)

Finally, the brittleness of Zerodur leads to a much lower ultimate strength than CFRP. Ultimate strength is a measure of a material’s resistance to breaking and delineates the linear and non-linear regions of the stress-strain curve.\footnote{The ultimate strength for Zerodur is \(~10\) MPa.} We can calculate the ultimate strength for the CFRP mirror using values garnered from the four-point bend test in Section 4.1, given as\footnote{We can calculate the ultimate strength for the CFRP mirror using values garnered from the four-point bend test in Section 4.1, given as}

\[
S = 3F \frac{b-a}{2ct^2},
\]

where \(S\) is ultimate strength, \(F\) is applied force, \(b\) is the span between bottom supports, \(a\) is the span between top supports, \(c\) is the sample width, and \(t\) is the sample thickness (referring to Figure 27(a)). With values from the test, the ultimate strength of the CFRP mirror is \(417\) MPa, a factor of 40 higher stress tolerance than Zerodur. So thin-shelled CFRP mirrors not only require less force to deform than thin-shelled Zerodur mirrors, they can also operate under a much higher force without breaking, leading to a greater ROC change.

Thus, CFRP has many advantages over glass (specifically Zerodur) as an active mirror. Other considerations for thin-shelled active mirrors have been aluminum or silicon carbide. However,
Ref. 67 summarized CFRP’s advantages best, stating that CFRP avoids the “brittleness of Zerodur, the weight of aluminum, or the cost of [silicon carbide].”
CHAPTER 5: ACTIVE MIRROR TESTBED

Testing an active mirror requires a specialized testbed that must include a wavefront sensor and various optics for collimating, beam sizing, and image relaying. This chapter describes the design, construction, and verification of the Active Mirror Testbed (AMT), created specifically to test the active CFRP mirror detailed in Chapter 6.

5.1 Optical Design

5.1.1 Design Requirements

The basic adaptive optics (AO) system is pictured in Figure 1. As opposed to Figure 1, the AMT uses two DMs – the mirao, a commercial off-the-shelf (COTS) DM, and the active CFRP mirror – thereby increasing the overall complexity and necessitating the need for additional optics and specifications. The specific AMT design goals and accompanying explanations are listed below:

1. Minimize total number of optics
   a. Fewer optics will lower system aberrations due to alignment or manufacturing errors.

2. Test CFRP with expanding spherical beam
   a. Since the active CFRP mirror is spherical, an expanding spherical beam will eliminate the naturally-occurring spherical aberration.

3. Restrict system to COTS components
   a. This is a budgetary concern since COTS elements are less expensive than custom elements.

4. Beam diameter on CFRP is 160mm
   a. The clear aperture of the active CFRP mirror, as dictated by the outer ring diameter (see Section 6.3.2) is 160mm.

5. Beam diameter on mirao is 14mm
   a. The clear aperture of the mirao is 14mm as dictated by the actuator width in Figure 5.

6. Beam diameter on SHWS less than 3.6mm
   a. The entrance pupil of the SHWS is rectangular with dimensions 3.6mm x 4.6mm. To avoid vignetting of the circular test beam, the beam diameter must be less than 3.6mm.
7. Correct conjugation of CFRP, mirao, and SHWS planes
   a. Aberrations of the CFRP mirror must be corrected at an image plane of the mirror; thus, the CFRP mirror and mirao need to be conjugated. Then, a SHWS measures the wavefront at its entrance pupil; thus, the plane of the mirao (the location of the corrected beam) must be conjugate to the SHWS’ entrance pupil.

5.1.2 First-Order Design
Using the design goals in Section 5.1.1, we can use paraxial theory to construct the first-order design in Figure 40. The beam incident upon $L_1$ is assumed to be collimated and the stop is at the CFRP mirror. $L_1$ produces an expanding spherical beam incident upon the CFRP mirror to meet Objective 2. Then, using trigonometry, we find that

$$f_1 = \frac{y_1 R}{y_2} \quad (71)$$

where all variables follow the definitions in Figure 40. In order to avoid beam sizing optics between the beamsplitters (Objective 1), we set $y_1 = y_3 = 7\text{mm}$ (per Objective 5). The initial ROC of the CFRP mirror is 2000mm with a clear aperture of 160mm (Objective 4), making $f_1 = 175\text{mm}$ with Eq. 71. Consequently, for proper conjugation of the CFRP and mirao planes (Objective 7), the axial distance between $L_1$ and the mirao is 190.4mm using the thin-lens equation.

Next, Objective 6 can be met by placing a beam reducer between the mirao and the SHWS. Following paraxial afocal theory, we have

$$\left|\frac{f_2}{f_3}\right| = m = \frac{y_2}{y_3}, \quad (72)$$

where $m$ is the magnification of the afocal system. Let’s let $f_2 = 200\text{mm}$, a nice round number to meet Objective 3, making $f_3 = 50\text{mm}$, another nice round number, making the beam reduction 4:1. Now, the beam diameter at the SHWS is 3.5mm, thereby meeting Objective 6. Finally, for proper plane conjugation, the axial distance between the mirao and the SHWS is 500mm, i.e., the distance between the entrance and exit pupils for the beam reducer. Thus, all objective have been met in the first-order design.
5.1.3 Final Optical Layout

The first-order design in Figure 40 is extremely compact. An initial attempt to construct the system showed that it was, in fact, too compact since the image of the CFRP mirror was inside the second beamsplitter. A second relay system, with 1:1 magnification, was thus added between the two beamsplitters to increase the optical path.

The final experimental AMT is shown schematically in Figure 41(a) and pictured in Figure 41(b). A HeNe laser is spatially filtered and re-collimated before passing through the first beamsplitter ($BS_1$). Then, a telephoto lens (Edmund Optics P/N 59817) produces an expanding spherical beam to test the CFRP mirror’s spherical surface. After being reflected by $BS_1$, a 1:1 afocal system relays the intermediate CFRP mirror image to the mirao. The lenses are aspherized achromatic doublets (Edmund Optics P/N 49663) to help reduce spherical aberration. Next, after passing through $BS_2$, the beam reflects off the mirao. Finally, a second lens sub-system reduces the beam by 4:1 and relays the mirao plane to the entrance pupil of the SHWS (Imagine Optic HASO First).
An important note is that the absolute position of the image point from the CFRP mirror must remain constant in order to re-collimate the beam. Thus, the CFRP mirror needs to be displaced along the optical axis when the ROC is altered.

Figure 41. Final active mirror testbed – (a) schematic and (b) picture with beam paths in red. Note that (b) shows the AMT with a flat mirror at the system reference point labeled in (a). Labels: \(PH\) is pinhole, \(BS_1\) and \(BS_2\) are cube beamsplitters, and ‘AMT Ref’ is where the system reference is measured.
5.1.4 Component Selection

The main two components of the AMT are the mirao and HASO First SHWS, both manufactured by Imagine Optic. Two of the higher quality DMs, as measured by their large strokes and clear apertures, were available for use within the AMT – the OKO Mirror from Flexible Optical and the mirao from Imagine Optic. The OKO Mirror is an electrostatically-driven DM, as is the mirao, but with only 37 actuators. In terms of stroke, the OKO Mirror’s ROC range is \([-1500\text{mm}, \infty]\), so it cannot produce a convex wavefront as the mirao can. Thus, the mirao exceeds the OKO Mirror in terms of stroke and actuator density making it the correct choice for the AMT.

SHWS’ are widely available as the majority of optics suppliers sell a version of the popular wavefront sensor. SHWS’ can be optimized for large dynamic range, high measurement speed, high measurement accuracy, or a balanced combination of the three. Speed is not a concern for the AMT, since it is a testbed, nor is extreme measurement accuracy, since the residual error of the AMT is \(\sim 1\text{wv PV}\) (see Figure 46(b)), leaving large dynamic range as a necessity. Another, more subtle attribute with SHWS’ is the agreement between the DM’s actuator density and SHWS’ lenslet array pitch. A high-density DM, like the mirao, needs a low-pitch lenslet array in order to accurately measure the induced spatial frequencies during DM deformation. A natural choice to ensure this accuracy is to employ a SHWS specifically designed for use with the mirao – the HASO First from Imagine Optic. The HASO First also has a very large dynamic range of \(400\lambda\) with an accuracy of \(\lambda/100\) (with respect to \(\lambda = 650\text{nm}\)), making it an excellent choice for the AMT.

5.2 Closed-Loop Control

Two closed-loop methods are available for aberration correction within the AMT – one custom and the other COTS. The custom package is a development of code from Ref. 78. The COTS software is called CASAO and is also from Imagine Optic. Both packages use modal control to correct the wavefront since the AMT and active CFRP mirror in Chapter 6 are optimized for low-order correction (see Section 1.5.1). Also, both packages have the ability to run closed-loop control with the CFRP mirror and the mirao, although testing showed that the error correction is much more pronounced with the CASAO software. As such, only the CASAO software was used to produce the closed-loop results in this chapter and the next, so only CASAO will be described here.
5.2.1 CASAO Inputs

Imagine Optics CASAO closed-loop software\(^{21}\) is a fully-integrated combination of wavefront control and diagnostics. While specifically formatted to control the mirao, any DM can be controlled with a DLL (dynamic-link library) file. A DLL file was used to control the CFRP mirror’s actuators in Ref. 79; a similar DLL allows CASAO to control the CFRP mirror in closed loop. Since CASAO is COTS, the exact algorithm employed is proprietary and cannot be described here. Instead, a brief tutorial of usage is given in this section and the next.

The main inputs to a closed-loop algorithm, referring to Section 1.4.2, are the influence matrix \(B\), the current wavefront error, and the gain \(g\) of the algorithm. The specific inputs to CASAO are more numerous and discussed below:

1. Gain – controls how fast the algorithm will converge. For CASAO, this number ranges from 0-1. A larger value will result in the loop converging faster, but too large may make the loop unstable and cause it to oscillate.
2. Averaging – how many sensor readings are averaged before a new feedback signal is calculated. Increasing averaging will decrease the sensor noise due to vibration or air currents, therefore making the signal more stable, but too much averaging will eliminate the ability to accurately measure high-order aberrations.
3. Actuator displacement – how far the individual actuators displace during the influence matrix calculation. In CASAO, the influence function for each actuator is calculated as both a positive motion, increasing ROC, and a negative motion, decreasing ROC. Iteration has shown that the displacement needs to cover ~75% of the usable range of the actuator.
4. Background removal – removing the effect of extraneous light in code. A SHWS will measure whatever light is in its entrance pupil, including room lights. CASAO allows the measurement and removal of a ‘dark image,’ where the test laser beam is blocked, in order to eliminate extraneous light.
5. Target function – the final wavefront shape after closed-loop correction. Normally the target function is a plane wave, although the feedback loop can push the wavefront to include small amounts of the Seidel aberrations.
6. Number of modes – how many terms are in the Zernike polynomial expansion to estimate the wavefront. Since the reconstruction is modal, \(N\) terms in the Zernike expansion are used to estimate the WFE. Iteration has shown that varying this value has a great impact.
on the quality of the correction, as too few terms (~20) will result in a corrected wavefront with a higher PV error than more terms (~40) are employed.

5.2.2 CASAO Closed-Loop Execution
After choosing the settings for the above six parameters, we can execute closed-loop correction with CASAO. First, we need to calculate the influence matrix of the mirror using the menu found in Figure 42(a). This menu also sets the number of measurements to average (which should be set to stabilize the signal) and the exposure duration (which should be set to keep the pixel saturation at ~90%). Next, an intermediate matrix called the ‘command matrix’ must be calculated using Figure 42(b). The number of modes in the modal expansion is also established in this screen. Finally, Figure 42(c) runs the closed-loop correction, which can be run both continuously or for a finite number of iterations.

5.2.3 HASO Wavefront Graphs
The HASO First SHWS comes with a wavefront reconstruction program called HASOv3. CASAO also includes a wavefront reconstructor, but HASO is used to determine and plot WFE since only HASO determines the Zernike coefficients. Therefore, all quantitative WFE results throughout this dissertation were garnered with HASO. Also, the circular beam incident upon the SHWS may not appear circular in the displayed wavefront maps, a visual inconvenience caused by the SHWS’ large dynamic range. Finally, unless noted, all wavefronts are plotted without tip and tilt.
Figure 42. Screenshots of CASAO software showing how to (a) calculate the influence matrix, (b) calculate the command matrix, and (c) run closed-loop correction.

5.3 System Verification

5.3.1 Mirao Error

The mirao has 52 electrostatically-driven actuators, as briefly described in Section 1.4.3. The actuator positions are tracked by voltages over the range of -1V to +1V, where -1V corresponds to the convex surface in Figure 5(b) and +1V to the concave surface in Figure 5(c). The mirao is calibrated by Imagine Optic for a flat setting (i.e. ROC = ∞), which is not a perfect planar surface. A Zygo shot (see Section 5.5) of the mirao at the ‘flat’ setting is in Figure 43. The residual error is 0.68wv PV and is almost entirely high-order error. The ‘flat’ setting is the initial mirao state within the AMT, during system alignment and prior to closed-loop control, despite its small amount of error.

Figure 43. Mirao at ‘flat’ setting, measured with the Zygo.
5.3.2 Pupil Conjugation

Objective 7 of the first-order design was to properly conjugate the pupil planes, meaning that an image of the CFRP was at the mirao and the SHWS for correct aberration correction and wavefront sensing, respectively. Pupil conjugation was confirmed using a ruler placed in front of a COTS mirror, as in Figure 44(a). A ruler was chosen since the ruler’s edge is similar to a knife-edge, thereby creating an object with a high spatial frequency content. The ruler is ~2mm in front of the reflecting surface, which is considered a negligible amount of defocus for this test. Then, a CMOS camera was placed at the location of the SHWS’ entrance pupil to observe the image of the ruler. As can be seen in Figure 44(b), the ruler’s edge is quite sharp, meaning that all planes have been properly conjugated.

5.3.3 Comparison to Zemax

The AMT was simulated in Zemax in order to predict the residual aberrations of the system. All components were real components using Zemax-included lens prescriptions or prescriptions provided by the individual suppliers. A flat mirror was used to represent the mirao since the mirao is not perfectly flat. The final WFE, found in Figure 45, is 0.73wv PV. The WFE is completely spherical aberration since most of the refracting surfaces are spherical and the only simulated field angle was axial.
The experimental error of the AMT was measured with a flat mirror placed at the back focal plane of $L_1$, as indicated in Figure 41(a). The WFE is displayed in Figure 46(a); note that all aberrations are present in Figure 46(a), including tip and tilt. A flat mirror is placed in lieu of the mirao to make an accurate comparison to the Zemax model. The dominant aberration is coma which is due to misalignment. Spherical aberration, the dominant aberration in Zemax, was 0.84wv (after conversion from Zernike to Seidel coefficients\textsuperscript{81}) 15% larger than predicted by Zemax. The PV error was 0.56wv (0.103wv RMS), which is 23.2% lower than predicted. Thus, coma has balanced the spherical and other aberrations to produce a lower PV than predicted.

With the mirao in place, the error remained constant at 0.55wv PV (0.090wv RMS) as in Figure 46(b). Thus, the non-planar nature of the mirao in Figure 43 was complementary in WFE since the WFE did not increase. The dominant aberrations within the full AMT are astigmatism and coma, again due to alignment errors. Following closed-loop correction with CASAO the error is diffraction limited with 0.08wv PV (0.015wv RMS), as shown in Figure 46(c). The Zernike coefficients both before and after correction are found in Figure 47. Notice the sharp decrease in the low-order aberrations.
Figure 46. Residual error within the AMT - (a) WFE without the mirao for comparison to Figure 45, (b) with the mirao, and (c) AMT after closed-loop correction.

CASAO will correct for all errors within the system, including the residual error of the AMT. Ideally, the AMT error would be zero so that only errors in the CFRP mirror would be corrected. The AMT, however, is optimized for low-order correction, meaning the error in Figure 46(c) is considered the ‘zero point’ of the testbed; in other words, Figure 46(c) is the lowest WFE that the AMT can muster. Then, the wavefront in Figure 46(b) serves as an internal reference for WFE
measurement in order to accurately measure the active CFRP mirror. This functionality is demonstrated below in Figure 48(a) and (b).

5.4 COTS Mirror Aberration Correction

To test the AMT’s measurement and corrective abilities, a COTS mirror from Edmund Optics was used as a test mirror. The mirror has a 146mm clear aperture with ROC = 1828.8mm, and has a WFE of 0.38wv PV (0.048wv RMS) as shown by the Zygo interferogram in Figure 48(a). The high frequency fluctuations in Figure 48(a) are due to vibrational noise within the Zygo testbed in Figure 49. Figure 48(b) shows the WFE as measured with the AMT to be 0.42wv PV error (0.073wv RMS). Thus, the AMT can accurately measure mirror WFE since the AMT and Zygo differ by 0.04wv PV (10.5%) and the same functional form is observed. Then, using the CASAO software and the mirao, the testbed error was driven in closed-loop to the WFE found in Figure 48(c) using the closed-loop settings in Table 13; WFE was 0.10wv PV (0.021wv RMS). Aberration coefficients for both Figure 48(b) and (c) are found in (d). Note that focus was removed throughout Figure 48.
Figure 48. WFE of testbed. (a) Zygo shot of COTS mirror, WFE of mirror with (b) no correction, (c) with correction, and (d) Zernike aberrations coefficients. Focus was removed in all graphs. Labels in (d) are as in Figure 47.
Table 13. Closed-loop parameter settings for correction with the COTS mirror. The parameters are described in Section 5.2.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain</td>
<td>1</td>
</tr>
<tr>
<td>Averaging</td>
<td>5</td>
</tr>
<tr>
<td>Actuator Displacement</td>
<td>±0.2</td>
</tr>
<tr>
<td>Target Function</td>
<td>Plane wave</td>
</tr>
<tr>
<td># of Modes</td>
<td>44</td>
</tr>
</tbody>
</table>

5.5 Zygo Testbed

While not part of the AMT, a second testbed is used in Chapters 5 and 6 to measure WFE. This testbed utilized a Zygo VeriFire interferometer,82 a high-accuracy phase-shifting interferometer that tests optics using an expanding spherical beam, like the AMT. The Zygo performs phase-shifting interferometry by shifting a phase plate within the interferometer, meaning that the beam is susceptible to vibration and air currents, as observed in Figure 48(a).

Figure 49. Picture of Zygo testbed. The arrows show the folded ‘W’ path the test beam travels from the Zygo to the test mirror. The return beam follows an identical path.
The entire ~2500mm beam path between the interferometer and test mirror must fit on a single, floating optical table to reduce noise, a difficult task since the optical table in Figure 49 is ~1800mm in length. This was solved by creating a folded, ‘W’-shaped beam path as displayed by the arrows in Figure 49. (The return beam follows an identical path.)
CHAPTER 6: ACTIVE OPTICAL CFRP MIRROR

As stated earlier, a main hurdle to the experimental realization of a large-aperture adaptive optical zoom (AOZ) system is the lack of a large-aperture deformable mirror. This chapter details the design and testing of an active CFRP mirror that can increase ROC by 10mm while maintaining WFE to ~1wv PV when used in conjunction with the mirao. Two active mirrors are discussed – a preliminary attempt using open-loop feedback and the final apparatus using closed-loop feedback.

6.1 FEM of Actuation Modalities

6.1.1 Point-Load and Radial Force Actuation

The finite element model (FEM) from Ref. 70, constructed in the commercial software package ANSYS, simulates the change in a CFRP mirror’s surface shape under different applied forces. The model assumed that the initial state of the CFRP mirror was a perfect sphere and that the circumferential edge can freely expand in the x-y plane. Two possible methods to actuate a CFRP mirror are the point-load and radial force actuation modalities. An actuation modality is a pattern of applied forces used to change ROC. With the point-load, a single force is placed at the mirror’s apex and pushes or pulls the mirror to increase or decrease ROC, respectively. The radial force method places a force along the circumferential edge and radially pushes or pulls the mirror to decrease or increase ROC, respectively. 70

Figure 50 shows the resulting WFE when a CFRP mirror is actuated from 2000mm to 1250mm ROC using (a) the point-load and (b) radial force modalities. Figure 50 shows that point-load and radial force actuation methods can successfully change a CFRP mirror’s ROC. However, the induced wavefront errors of 493.2wv PV and 116.6wv PV for the point-load and radial forces, respectively, are not acceptable for imaging applications as they would severely degrade the image.

6.1.2 Radial/Ring Actuation

Notice that the phase maps from Figure 50 are somewhat complementary in error. This implies that a combination of radial and point-load forces could reduce the overall aberrations observed in the reflected wavefront.
Figure 50. FEM results for (a) point-load actuation and (b) radial inward force. The scales on the right are in microns. Note that ‘FEA’ stands for finite element analysis, an alternate acronym for FEM.

This combination of forces can be readily examined using FEM. After a few iterations, it became clear that performance could be improved by allowing the point-load to become a ring-load, where force is applied to an annular ring placed on the backside of the mirror. In this scheme, the ROC change is caused by the radial inward force and the pushing force primarily acts a WFE corrector. The resulting 2D phase map, of the same 2000mm to 1250mm ROC change, is found in Figure 51(a) with a 1D cut in (b). Now the PV error is 24.6wv and only 3.6wv over a diameter of 105.6mm.

A potential opto-mechanical apparatus to enact the radial/ring actuation modality is found in Figure 51(c). Here, a force from the top pushes the blue ring down onto the green portion which holds the mirror in a set of 100 ‘teeth,’ an action similar to collets found in lathes. However, the force required to deform the mirror is extreme – a radial force of 20100 N and a ring force of 477 N to produce the wavefront in Figure 51. This immense radial force is caused by the stiffness gradient of the CFRP mirror being parallel to the radius of the mirror, i.e., parallel to the applied force. So the high stiffness of CFRP, one of the benefits of this material outlined in Chapter 4, is actually a hindrance in this situation.
Figure 51. (a) 2D phase map and (b) 1D cut of radial force and annular ring combination, with the ring 120 mm in diameter. The scales are in microns. Again, FEA is another acronym for FEM. (c) Potential apparatus for radial/ring modality. Note that the annular ring is not pictured.

### 6.1.3 Annular Ring Actuation

A derivative of the radial/annular ring method above removes the radial force and only uses annular rings pushing on the back of the CFRP mirror. This method is attractive because the applied force is now perpendicular to the stiffness gradient.

Figure 52(a) shows the simulated cross-section of the WFE as a CFRP is actuated from 2000mm to 4000mm ROC with a single 125mm diameter annular ring. WFE is 50.5wv PV. While this is an improvement over other examined modalities, the WFE is still prohibitive for most applications. However if a second ring is added, as in Figure 52(b), the WFE has decreased to 22wv PV and only 9.5wv PV over a 160mm clear aperture using ring diameters of 35mm and
Adding a third ring only lowers WFE if the ring can be pulled, as described in Section 6.2.1, and it substantially increases the actuation complexity. Since modality complexity is a concern in Section 6.1.4, the annular ring method was limited to two rings. The force required to achieve the wavefront in Figure 52(b) is 25 N and 333 N on the inner and outer rings, respectively.

![Figure 52. Cross-section of a CFRP mirror at 4000mm ROC when pushing on (a) a single annular and (b) two annular rings. Note the difference in y-axis scales.](image)

6.1.4 Moment Actuation

Another simulated actuation modality is edge moment actuation, where a mechanical moment is applied to the CFRP mirror. A tangential or torsional force is applied equally to the circumferential edge of the mirror, thereby causing the mirror’s edge to rotate and inducing the desired ROC change.

The WFE from moment actuation is found in Figure 53 after applying sufficient force to increase the ROC to 4000mm. The WFE is 138.4wv PV with an edge rotation of 1.76º, limiting the usefulness in this application. In addition, this method is severely hampered by experimental feasibility. Applying a moment force to the mirror is possible in principle, but extremely difficult in reality due to the CFRP mirror’s size and high stiffness. Furthermore, designing a mechanical apparatus that can impart a repeatable tangential force with optical tolerances was not found despite a concerted effort. Thus, the moment actuation method does not appear to be experimentally feasible.
6.1.5 Actuation Modality Selection

Five actuation modalities have been discussed in this dissertation – point-load (Section 6.1.1), radial force (Section 6.1.1), radial/ring (Section 6.1.2), constant pressure (Section 4.3.1), and annular ring (Section 6.1.3). Note that the sixth discussed modality – the moment actuation from Section 6.1.4 – is immediately removed from consideration due to its very low experimental feasibility. These five modalities were evaluated based on WFE, total required force, ease of control, and fine control with the results tabulated in Table 14. ‘Ease of control’ assesses the complexity of the actuation scheme and ‘fine control’ assesses the potential for minute adaptive optics (AO) aberration correction.

Table 14. Comparison of the five actuation modalities discussed in this dissertation.

<table>
<thead>
<tr>
<th>Modality</th>
<th>WFE (wv)</th>
<th>Total Force (N)</th>
<th>Ease of Control</th>
<th>Fine Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point-Load</td>
<td>493.2</td>
<td>462</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Radial Force</td>
<td>116.6</td>
<td>12891</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Radial/Ring</td>
<td>24.6</td>
<td>20577</td>
<td>Medium</td>
<td>Medium</td>
</tr>
<tr>
<td>Constant Pressure</td>
<td>166.1</td>
<td>1011</td>
<td>High</td>
<td>Medium</td>
</tr>
<tr>
<td>Annular Ring</td>
<td>22.0</td>
<td>358</td>
<td>Medium</td>
<td>High</td>
</tr>
</tbody>
</table>

The ideal modality would have low WFE, low total force, high ease of control, and high fine control. Since the eventual goal of the active CFRP mirror is an experimental AOZ system, low
WFE is the primary criterion. As such, point-load, radial force, and constant pressure are eliminated from contention since their WFE’s are far too high. The secondary criterion is total force in order to minimize the power budget, an overall goal of AOZ. An initial assessment with the radial/ring modality showed that applying a large force with high resolution, as required to maintain optical tolerances, was improbable even if gears are employed to lessen the force. Thus, the radial/ring actuation is eliminated. The remaining modality is annular ring which has good ratings at both ease of control (medium, since the modality is not single actuator control) and fine control (high, since the two annular rings can move independently). Thus, the annular ring actuation modality was chosen for the active CFRP mirror detailed in Sections 6.3-6.4.

Unfortunately, the 22.0wv of error found with the annular ring modality is still too large for good image resolution. However, if several separate actuators are placed on the annular rings, they could be dithered independently during closed-loop control to further decrease WFE, possibly decreasing the error to within the correction range of COTS DMs. So, the combination of the annular ring CFRP actuation method and a DM could produce a diffraction limited image.

6.1.6 Actuator Selection

The next step is to select an actuator to enact the annular ring actuation modality. Actuators were assessed on the following criteria:

1. Stroke > 1mm
   a. The sag change for an ROC increase from 2000mm to 4000mm is 0.8mm at a 160mm clear aperture. Thus, the linear actuator stroke must be larger than 1mm.

2. Resolution > 150nm
   a. The applied force for the annular ring method is parallel to the sag change, making the resolution of the actuator also the resolution of the sag change. A resolution of 150nm is ~0.25wv, which was deemed to be the ceiling for fine aberration control. Note that some suppliers specify both a resolution (the accuracy limit on position) and minimum incremental motion (the smallest repeatable step size); for this comparison, ‘resolution’ is defined as the smallest repeatable step size.

3. Number of actuators
   a. The total force for a 100% ROC change on the outer ring is 333 N which will be spread out over several actuators as envisioned above. The number of actuators can be found by dividing 333 N by the maximum actuator force. Linear actuators
specify both a holding force, the maximum constantly applied force, and a ‘push-pull’ force, the maximum impulse force; the smaller of these two numbers was employed.

4. Positional feedback

a. Aberrations will be controlled using closed-loop feedback. The eventual goal of the active CFRP mirror is to operate open-loop to further reduce size, weight, and power. Both open and closed-loop control require positional feedback, meaning that the computer knows the exact position of each actuator.

5. Non-rotating tip

a. The actuators will push directly upon the annular rings. A rotating tip could potentially introduce actuation errors by shifting the rings or causing precession of the rings during actuation.

Unlike the actuation modality selection, the chosen actuator must meet all five criteria. ~25 COTS actuators were evaluated using the above criteria; a table comparing the three actuators that met all criteria is found in Table 15. The actuator chosen was the Physik Instrumente M230.10 (which will herein be referred to as the ‘PI actuators’) since it has the lowest resolution and requires the fewest number of actuators.

Table 15. Comparison of COTS actuators for the annular ring actuation modality.

<table>
<thead>
<tr>
<th>Actuator</th>
<th>Stroke (mm)</th>
<th>Resolution (nm)</th>
<th>Number of Actuators</th>
<th>Positional Feedback?</th>
<th>Rotating Tip?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newport TRA25CC</td>
<td>25</td>
<td>200</td>
<td>6</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>PI M-227.10</td>
<td>10</td>
<td>50</td>
<td>9</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>PI M-230.10</td>
<td>10</td>
<td>50</td>
<td>5</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

6.2 Open-Loop ROC Increase

The ROC of a CFRP mirror was first increased with a previously-designed mirror and mount. The apparatus used three annular rings, but was constructed before creation of the FEM leading to detrimental WFE results. However, this research demonstrated the fundamental concept that ROC can be increased using annular ring actuation.
6.2.1 Open-Loop Mirror and Apparatus

The open-loop CFRP mirror was a parabola 406.4mm in diameter with a 3251.2mm ROC. The mirror, pictured in Figure 54(a), was also manufactured by Composite Mirror Applications (CMA) and is mechanically identical to the closed-loop CFRP mirror. A Zygo shot of the mirror is found in Figure 54(b); the WFE is 8.8wv PV over a clear aperture of 127mm. Decreasing the aperture diameter by almost 4X was necessary as the deformation-induced aberrations quickly becomes large, as found in Section 6.2.2. Most of the error is astigmatism (8.4wv) with the remaining a mix of coma and spherical aberration. The large astigmatism could be caused by gravitational sag (as described in Section 4.3.1) or by the mirror itself (as described in Section 6.3.1).

![Figure 54. Open-loop CFRP mirror – (a) picture of mirror and (b) Zygo shot of WFE.](image)

A schematic of the annular ring and actuator layout is found in Figure 55(a), where the dotted lines represent three annular rings and the black dots represent six PI actuators. The three rings, constructed of CFRP, have diameters of 162.5, 243.8, and 320.0 mm. The circumferential edge of the mirror is held fixed inside of the mount, which is also made of CFRP. Six PI actuators are used to increase the ROC, which are affixed to three of the six ‘spokes’ that cross the back of the mirror as seen in Figure 55(b).

FEM can be utilized to predict WFE for the open-loop mirror since it is mechanically identical to the closed-loop mirror. The predicted FEM for a 100% ROC change from 3251.1mm to 6052.2mm is found in Figure 55(c); the PV error is 684wv. Nearly 80% of the error is found on-axis, due to the inner diameter ring being too large to affect the axial portion of the wavefront. Also, FEM showed that the middle ring needs to be pulled, not pushed, to aid in wavefront
correction. A pulling force is impossible with the previously-constructed apparatus in Figure 55(b) since there is no method of tying the PI actuators to the rings; thus, actuators were only placed on the inner and outer rings as shown in Figure 55(a).

![Opto-mechanical apparatus for open-loop ROC increase](image)

Figure 55. Opto-mechanical apparatus for open-loop ROC increase. (a) Schematic of actuator and ring layout, (b) back of mirror with actuators protruding, and (c) predicted WFE for $\Delta$ROC = 100%.

### 6.2.2 Open-Loop Results

The open-loop test was performed on the Zygo testbed pictured in Figure 49. The mirror was translated along the optical axis as the ROC changed to maintain correct positioning within the testbed. A null corrector was not used during testing since aspheric-induced spherical aberration
is less than ¼-wave over the mirror portion tested. Open-loop feedback, as defined in Section 4.3.3, is where the actuators are manually dithered as the user observes the real-time effect on the wavefront.

Figure 56. Open-loop wavefront maps for (a) $\Delta\text{ROC} = 0.0$ mm, (b) $\Delta\text{ROC} = 48.7$ mm, and (c) $\Delta\text{ROC} = 97.4$ mm. The scales on the right are in waves.

Three WFEs are displayed in Figure 56 – a corrected version of Figure 54(b), an ROC change of 48.7mm (1.5% increase), and an ROC change of 97.4mm (3.0% increase) in parts (a), (b), and (c), respectively. The dominate aberration is astigmatism, with coma and spherical aberration less so as indicated in Table 16. We see that open-loop iteration was able to decrease the static error by 52%, a sizeable decrease in error. However, the PV error quickly becomes large as the mirror is deformed, resulting in 19.7wv PV at a 3.0% increase in ROC. Even larger ROC changes were
attempted, but the large phase differences within the clear aperture pushed the WFE beyond the Zygo’s measurement limit. The large aberration spike in Figure 55(b) was not observed since the ROC increases are much smaller than was modeled.

Table 16. Aberration values for the three tested states with the open-loop mirror. All coefficients are listed in waves. Labels: ΔROC = change in radius of curvature, SA = spherical aberration, and PV = peak-to-valley.

<table>
<thead>
<tr>
<th>ΔROC</th>
<th>0.0 mm</th>
<th>48.7 mm</th>
<th>97.4 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Astigmatism</td>
<td>3.43</td>
<td>9.16</td>
<td>14.28</td>
</tr>
<tr>
<td>Coma</td>
<td>0.14</td>
<td>4.45</td>
<td>9.65</td>
</tr>
<tr>
<td>SA</td>
<td>0.64</td>
<td>-2.92</td>
<td>5.10</td>
</tr>
<tr>
<td>PV</td>
<td>4.20</td>
<td>14.57</td>
<td>19.76</td>
</tr>
</tbody>
</table>

Regardless of the errors found in Figure 56 and Table 16, the results suggest that two annular rings are a viable actuation method for active CFRP mirrors, a proportion further demonstrated in Section 6.4.

6.3 Annular Ring Opto-Mechanical Apparatus
This section describes the opto-mechanical apparatus for the active CFRP mirror. The apparatus is based upon the open-loop setup above but makes many improvements, such as edge constrain and mirror alignment, that are detailed below.

6.3.1 CFRP Mirror
As stated earlier, the active CFRP mirror is mechanically identical to the tested coupons in Figure 27(b) and is the exact same mirror used in Ref. 79. The mirror, fabricated by CMA, is constructed of 24 layers of prepreg material and uses a similar lay-up scheme to the quasi-isotropic scheme on the right in Figure 30 (the exact lay-up scheme is proprietary). The mirror weighs 91.5g which ~20X less than a glass mirror of similar ROC, a far greater reduction in weight than predicted in Section 4.4 since the CFRP mirror is much thinner. The CFRP mirror is pictured next to a glass mirror in Figure 57 for ease in comparison.
The unactuated WFE of the CFRP mirror was measured with the Zygo interferometer and is found in Figure 58. The mirror was mounted vertically and was taped to a mirror mount to remove any mounting effects. The WFE is 13.9wv PV (2.58wv RMS) with 12.4wv of that being astigmatism; other aberrations are spherical aberration at 0.4wv PV and coma at 0.5wv PV. The large astigmatism is most likely gravitational sag, as predicted in Section 4.3.1, but could also be due to CFRP mirrors not being completely isotropic (see next paragraph). Regardless of the source of the error, the active CFRP mirror starts from a fairly aberrated. This non-ideal initial state decreases the possibility of a diffraction limited active CFRP mirror.

Figure 58 shows that astigmatism is the dominant aberration in the CFRP mirror. As described in Section 4.3.1, Refs. 61 and 67 purport that astigmatism in thin-shelled CFRP mirrors is mainly caused by gravitationally-induced sag. However, Figure 59 seems to refute that claim, instead showing that the astigmatism is inherent to the CFRP mirror. Here, the WFE of a CFRP mirror is displayed as it is rotated by -45° from Figure 59(a) to (b), with all aberrations except for astigmatism removed. (Note that this is mirror is not the same mirror as the active CFRP mirror but is identical in all respects.) The astigmatic axis would not move with a -45° rotation if gravity
is the dominant cause of astigmatism. Figure 59, however, clearly shows that the axis tracks the mirror’s rotation, implying that the astigmatism is not caused by gravity. Thus, the astigmatism is most likely caused by the non-isotropic nature of CFRP mirrors (see Section 4.2.1). The non-circular pupil in Figure 59 is due to measurement noise since the mirror was taped in place.

![Figure 59. A second CFRP mirror rotated by 45° from (a) to (b). Aberrations except for astigmatism were removed from the WFE.](image)

### 6.3.2 General Description

Figure 60(a) displays a schematic of the actuator and annular ring layout. Three PI actuators push upon the inner ring and six upon the outer ring. An extra actuator was added to the outer ring (because Table 14 shows that only five are required) to ensure that the actuators would not be force-limited and to enable hexapolar control. Notice that the inner three actuators form a triangle that is ‘flipped’ with respect to Figure 55(a). The open-loop increase with the parabolic CFRP mirror showed that gravitational sag was difficult to control with only one actuator on the lower half of the mirror; thus, the final design placed two actuators in the lower half to increase sag control. The outer six actuators are arranged in a hexapolar fashion, giving them good control over trefoil and coma, less so with astigmatism since the spokes are not orthogonal. Also, the outer ring defines the clear aperture of the mirror at 160mm (the outer diameter of the actuators), as discussed in Section 5.3.1.

The opto-mechanical mount is pictured in Figure 60(c). The blue-colored component has six spokes to hold the actuators that push upon the two annular rings to enact the annular ring actuation method described above. The CFRP mirror rests upon a silver trapezoidal ‘plunger’ in Figure 60(b). When the mirror is vertical, the mirror/plunger combination is held in place by a snap ring from the front and the actuators pressing against the annular rings on the back.
6.3.3 Effect of Edge Constrain

The mirror was initially held in place via an L-shaped rubber gasket on the circumferential edge. It was thought that if the gasket had a low durometer value the edge could expand radially as the ROC increases, just as the FEM assumed. Unfortunately, the rubber gasket did not allow the edge to expand and, in fact, over-constrained the system, as is seen in the Hartmanngram in Figure 61(a). The stretching of the square grid pattern along a -45° line is due to the rubber gasket applying a radial inward force on the mirror, causing significant astigmatism. Larger diameter gaskets exhibited a similar effect, albeit less pronounced. (Note: the ‘keyhole’ shadow observed in Figure 61 is due to a fold mirror and mount that was used in an earlier version of the AMT.)
The solution was to allow the circumferential edge to be completely unencumbered, as in Figure 60(b). The effect on the Hartmanngram in Figure 61(b) is stark compared to (a), where the square grid of lenslet spots is clearly visible. Now, the edge is under-constrained and free to expand radially, thus matching the FEM. The plunger is described further in Section 6.3.4.

![Figure 61. Hartmanngrams displaying effect of edge constrain. (a) Mirror over-constrained by rubber gasket and (b) mirror under-constrained by plunger.](image)

A second cause of the error in Figure 61(a) is the boundary condition between the mirror and the mirror mount. Portions of the apparatus that make direct contact with the mirror surface will add their imperfections to the WFE, in a similar fashion to the glass mandrel adding aberrations in Figure 29. The mirror’s circumferential edge does not have optical tolerances; in fact, the diameter differs by ~3mm in orthogonal directions. Thus, the plunger contacts the CFRP on the reflecting surface of the mirror, which does have optical tolerances, to ensure a clean boundary condition between the plunger and mirror.

### 6.3.4 Plunger Mount

The trapezoidal shape of the plunger, as seen in the computer aided design (CAD) drawing in Figure 60(b) and Figure 62(a), allows the edge to be completely unconstrained and makes contact with the CFRP mirror on the reflecting surface. The plunger was fabricated with tight mechanical tolerances from aluminum. Specifically, the plunger’s top surface, that contacts the mirror, has a surface roughness of ~25μm. Ideally, the top surface of the plunger would be diamond-turned to provide optical tolerances on both sides of the boundary condition, an action not taken due to budgetary concerns. Regardless, the effect of the plunger’s error is alleviated by careful alignment of the mirror and plunger, as described in Section 6.3.5.
The CFRP mirror is affixed to the plunger via magnetic attraction with neodymium magnets. Thirty-two magnets are equally spaced at 11.25° angles around the circumferential edge of the plunger, with another 32 magnets epoxied to the mirror’s backside. The circular magnets are 3/16” in diameter and 1/32” thick. The magnets provide enough attractive force to hold the mirror in place when mounted vertically while allowing the mirror’s circumferential edge to displace radially during actuation. This is demonstrated by Figure 60(b) where the mirror stays affixed to the mount even when turned upside down. The uniformity of the magnetic attraction also causes
the mirror to self-center on the plunger, thereby removing the need for a specific mirror/plunger alignment procedure.

The magnets are easily aligned on the plunger due to the countersunk holes seen in Figure 62(a). Aligning the magnets affixed to the mirror’s back is a more complicated and manual process. Figure 63 displays the magnet alignment guide created in CAD. The outer edge is 200mm in diameter, so that it is just outside the mirror’s diameter, and the black spots represent the 32 magnets to be affixed. Then, the mirror’s edge is aligned to the alignment guide and taped in place. Finally, the magnets are aligned with the spots and epoxied in place. Care was given to maintain correct magnet orientation so that the force between mirror and mount is attractive and not repulsive. This process has a relatively low accuracy, compared with the countersunk holes on the mount, and is thus a potential source of error.

![Figure 63. Neodymium magnet alignment guide. The outside ring is 200mm in diameter, so that it is outside the CFRP mirror’s edge, and the black dots represent magnet placement.](image)

### 6.3.5 Mirror & Plunger Alignment

The mirror/plunger combination is not rotationally symmetric, partially due to the plunger itself as explained above. It is also due to the mirror itself not being rotationally symmetric, as can be seen in Figure 64. (Note that Figure 64 is the same mirror as Figure 59.) Here, the mirror is taped to a standard mirror mount to remove the effect of the plunger and rotated 360° in four 90° steps. The comma-esque bulge on the bottom of (a) clearly rotates by 90° along with the mirror through
(b)-(d), meaning that this error is inherent to the mirror and not a product of the opto-mechanical mount. The comma-esque bulge is most likely due to fabrication errors.

Regardless of the cause, the WFE can be mitigated by finding a mirror/plunger combination that result in minimum WFE, a possibility since the plunger surface is not optically flat and may be complementary in WFE at certain orientations. A schematic of this test is shown in Figure 65(a), where the back of the CFRP mirror is colored in tan, 8 of the 32 neodymium magnets are displayed as black dots, and the plunger (colored in gray) extending beyond the diameter of the mirror. First, the plunger angle was fixed with respect to an origin, delineated by the red rectangle, while the mirror rotated by 45° from another origin, delineated by the red dot. WFE was measured by the Zygo at each step, with Figure 65(b) displaying WFE vs. CFRP angle. Two minimums in WFE are observed at 180° (7.05 wv PV) and 315° (6.45 wv PV). The sharp
The difference between 270° and 315° is likely due to the mirror/plunger WFE being additive instead of complementary.
Next, the mirror angle was held fixed (with respect to the plunger) and the mirror/plunger combination was rotated by 45° steps. Since two minimums were observed in Figure 65(b), both orientations were tested with the results in Figure 65(c). The mirror orientation of 315° clearly produces a lower WFE across all tested rotations. Wavefronts for four of the eight data points from Figure 65(c) are found in Figure 66; WFE for (a)-(d) are 7.2wv, 6.3wv, 6.3wv, and 5.8wv for plunger orientations of 90°, 135°, 180°, and 315°, respectively, for a 190mm clear aperture. Notice that the plunger has caused a decrease in WFE from Figure 58.

However, minimum WFE is not the only concern. The AMT uses a SHWS to measure the wavefront, which has a fixed range of measurement. SHWS’ specifically are limited by local wavefront curvature, since the wavefront is assumed to be flat over the diameter of a single lens in the lenslet array. Thus, the reflected wavefront of the CFRP needs to not only be low error but also smoothly varying to ensure accurate measurement by the AMT.
Figure 66. Zygo wavefront maps while CFRP angle is held fixed with respect to the plunger. Displayed are plunger angles of (a) 90º, (b) 135º, (c) 180º, and (d) 315º.

Figure 67 displays 1D cuts in the WFE plots from Figure 66. The cuts were manually chosen to be the WFE gradient in each wavefront while keeping the secant cut as close to a diameter cut as possible. Each wavefront cut moves through ~1.5 cycles of wavefront variation. Each 0.5 cycle portion was approximated as linear and the slope calculated between the trough and peak or vice versa. The average of the three slope measurements (in arbitrary units of waves/pixel) is displayed in Table 17, along with PV and RMS WFE values re-calculated with the correct 160mm clear aperture. A plunger angle of 135º displays the lowest mean slope and lowest coefficient of variation, making it the most smoothly varying, and also has the lowest WFE. Thus, the plunger angle was set to 135º. Note that the coefficient of variation measures the population variability relative to the mean, given as 83.
\[ CV = \frac{\sigma}{\mu}, \]  

(73)

where \( CV \) is the coefficient of variation, \( \sigma \) is the sample standard deviation, and \( \mu \) is the sample mean.

Figure 67. 1D cuts of the WFE from Figure 66. Displayed are plunger angles of (a) 90º, (b) 135º, (c) 180º, and (d) 315º, i.e., correlated to Figure 66. Note that the wavefront diameter was masked to the proper 160mm clear aperture.

Table 17. Slope and WFE data for the four examined wavefronts in Figure 66 and Figure 67. The slope values have arbitrary units of waves/pixels. The given PV and RMS data is for the correct 160mm clear aperture. Labels: STD = standard deviation and CV = coefficient of variation.

<table>
<thead>
<tr>
<th>Plunger Angle</th>
<th>Mean Slope</th>
<th>STD Slope</th>
<th>CV Slope</th>
<th>PV (wv)</th>
<th>RMS (wv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90º</td>
<td>1.32</td>
<td>0.60</td>
<td>46%</td>
<td>5.56</td>
<td>0.81</td>
</tr>
<tr>
<td>135º</td>
<td>0.99</td>
<td>0.33</td>
<td>34%</td>
<td>4.29</td>
<td>0.65</td>
</tr>
<tr>
<td>180º</td>
<td>1.34</td>
<td>0.97</td>
<td>73%</td>
<td>4.51</td>
<td>0.65</td>
</tr>
<tr>
<td>315º</td>
<td>1.29</td>
<td>0.70</td>
<td>54%</td>
<td>4.24</td>
<td>0.69</td>
</tr>
</tbody>
</table>
Figure 68. CFRP WFE following mirror and plunger alignment – (a) WFE measured with the Zygo, (b) WFE measured with the AMT, and (c) Zernike coefficients of CFRP mirror. Focus was removed throughout this figure. Labels: ‘F’ is focus, ‘A0’ is astigmatism at 0°, ‘A45’ is astigmatism at 45°, ‘C0’ is coma at 0°, ‘C90’ is coma at 90°, ‘SA’ is spherical aberration, ‘T0’ is trefoil at 0°, and ‘T90’ is trefoil at 90°.

The mirror/plunger WFE following the placement test is pictured in Figure 68(a) and (b) with the Zernike coefficients in (c). The WFE as measured by the Zygo in Figure 68(a) is 4.75wv PV (0.735wv RMS); the dominant aberrations are trefoil and coma, with astigmatism and spherical aberration less so. Again, we see good agreement between the Zygo and AMT in Figure 68(b), where the AMT measured WFE is 4.87wv PV (0.796wv RMS) a mere 2.5% larger than the Zygo.
measurement and identical in functional form. Comparing Figure 68(b) to the WFE of just the mirror in Figure 58 we see a marked decrease in total WFE (13.9wv PV to 4.87wv PV), but especially so with astigmatism (12.4wv PV to 1.23wv PV). Thus, it appears that the uniform and rotationally symmetric holding force created by the plunger greatly reduces the inherent astigmatism. Note that focus was removed throughout Figure 68.

6.3.6 Inner Ring Alignment

Astigmatism is not just caused by gravity or the mirror, but by the weight of the annular rings themselves. Figure 69(a) and (b) shows an increase in WFE with the rings attached (via epoxy) and not attached, respectively. The astigmatism increases by 67% with both rings attached versus without the rings. Note that all other aberrations were removed in the below wavefronts and that the astigmatism axis rotation is due to a rotation of the mirror, as we saw with Figure 59.

![Figure 69. Increased WFE caused by weight of rings – (a) with rings attached and (b) without rings.](image)

![Figure 70. Inner ring alignment. (a) CAD picture of ring with magnet holes and (b) ring in place on mount and actuators.](image)
To combat the sag and minimize the weight, the inner ring is not attached to the mirror but to the PI actuators via neodymium magnets. Figure 70(a) shows a close-up of the inner ring and magnets. The magnets are countersunk for ease in alignment, as with the plunger. Then, the metal tips of the PI actuators magnetically attract the magnets and hold it in place, as in Figure 70(b). This has the added benefit of correctly aligning the inner ring to the mirror’s axis.

6.3.7 Outer Ring Alignment
Testing showed that the outer ring is too heavy to be accurately held in place by magnets. Thus, the outer ring must be epoxied to the mirror. A custom tool was designed and fabricated to align the outer ring to the mirror; it is pictured in Figure 71(a). The fork-like end of the tool is the exact width of the outer ring. To align the outer ring, the mirror/plunger combination established in Section 6.3.5 is first placed into the spoke mount (of Figure 60(c)) with the PI actuators removed. Then, two of the custom tools are inserted into the mount holes for the PI actuators. Next, the outer ring is placed into the fork of the two tools, as in Figure 71(b), to align the ring to the mount. Finally, the ring is carefully epoxied in place. A minimum amount of adhesive is used in order to avoid the ‘dead zones’ observed in Figure 38(b).

6.3.8 Actuator Influence Function
The PI actuators essentially apply point-load forces, which Figure 50(a) shows produces a large, undesirable spike in the reflected wavefront. To decrease this effect, a thin layer of neoprene was placed between the rings and mirror surface in an effort to ‘spread out’ and decrease the actuator influence function. The rubber layer, visible in Figure 72, is 1/32” thick and has a fairly high
durometer of 70A to maintain force absorption fidelity during actuation. Figure 69 shows that any additional weight on the mirror further increases astigmatism. As such, the rubber layer was nearly form-cut to the ring diameter to minimize the extra weight due to the rubber layer.

Figure 72. Close-up of rubber gasket attached to outer ring.

6.3.9 Final Apparatus

The final opto-mechanical apparatus is pictured from the front in Figure 73(a), with the mirror’s reflective surface visible, and from the back in Figure 73(b), with the PI actuators protruding. The large black mirror mount is a COTS part from AeroTech to hold the active CFRP mirror assembly. The AeroTech mount has tip/tilt for fine mirror adjustment. The entire apparatus is
placed upon a linear translation stage and graduated rail so that it can be displaced along the optical axis when the ROC is altered, as described in Section 5.1.3.

6.4 Closed-Loop ROC Increase

6.4.1 Actuator Engagement

The distance between the PI actuator tips and the annular rings is only known to ~0.1mm, an immense distance in terms of optical wavelengths. Thus, the actuators need to be slowly moved forward until they engage the annular rings; in other words, ‘just touching’ the rings. This process is highly manual as the determination of ‘just touching’ is made by real-time observation of the WFE. The procedure, created through iteration, is explained below:

1. Set the inner three actuators to 1.5mm and the outer six to 4.5mm.
2. Move a single actuator by 0.05mm. Do the same with each of the other 8 actuators.
3. Repeat Step 2 until movement is observed in the WFE for a single actuator, as in Figure 74(a) to (b). The WFE change should be ~0.4wv PV. Move that actuator back by 0.05mm and do not change its position with repeats of Step 2.
4. Continue Steps 2 and 3 until movement has been observed in all 9 actuators.
5. Repeat Steps 2-4 with 0.025mm displacements. The WFE change will be smaller in magnitude than in Figure 74; it should be ~0.2wv PV.
6. Repeat Steps 2-4 with 0.0125mm displacements. The magnitude should be ~0.1wv PV.
7. Repeat Steps 2-4 with 0.00625mm displacements. This step is not always achievable as the motion is minute. In that case, move actuator forward by 0.00625mm to place it in the middle of the displacements determined in Step 6.

Engaging the actuators adds some WFE despite the slow, methodical process outlined above. Referring to Figure 75, the functional form of the WFE does not change from (a) to (b) when the actuators move from not being engaged to engaged, respectively. The PV error, however, increases by 14% mainly due to slight increases in astigmatism at both 0º and 45º, as evidenced by Figure 75(c). The WFE increase is expected, since the engagement process is highly manual, and is relatively small compared to the overall error.
Figure 74. Screenshots from HASO of wavefronts during actuator engagement – (a) actuator not engaged, and (b) actuator moved forward by 0.05mm. The circles highlights the change in the wavefronts and PV error due to actuator displacement.
Figure 75. Effect of PI actuator engagement – (a) WFE with no actuators engaged, (b) WFE with engaged actuators, and (c) change in low-order Zernike coefficients. Labels: ‘F’ is focus, ‘A0’ is astigmatism at 0º, ‘A45’ is astigmatism at 45º, ‘C0’ is coma at 0º, ‘C90’ is coma at 90º, ‘SA’ is spherical aberration, ‘T0’ is trefoil at 0º, and ‘T90’ is trefoil at 90º.

6.4.2 ROC Change Procedure

The ROC of the active CFRP mirror was slowly increased by physically displacing the mirror along the optical axis. First, the mirror is shifted backwards (i.e., away from the AMT) 2mm using the linear stage, which equates to a 2.52wv of defocus using.
where $\delta_z$ is the axial mirror displacement, $W_{020}$ is the Seidel coefficient for focus, and $F_\#$ is the system focal ratio. Since the AMT tests the active mirror in ROC space, $\delta_z$ equals the change in ROC which, in this case, is a 0.1% increase. Then, closed-loop correction is run with PI actuators attached to the CFRP mirror to correct the defocus and re-collimate the return beam in the AMT. Note that CASAO can control the PI actuators using a custom DLL similar to the DLL employed in Ref. 79. Finally, residual aberrations are corrected with the mirao using closed-loop control.

### 6.4.3 Experimental Results

Closed-loop correction with the CFRP mirror and mirao was performed at ROC increases of 0mm, 2mm, 4mm, 6mm, 8mm, and 10mm; WFE for increases of 0mm, 6mm, and 10mm are displayed in Figure 76, Figure 77, and Figure 78, respectively. In all three below figures, part (a) displays the uncorrected WFE before any closed-loop correction, part (b) displays the WFE after correction with the CFRP mirror, part (c) displays the correction of part (b) with the mirao, and part (d) shows the aberration coefficients from (a)-(c). Note that Figure 76(b) is blank since no correction with the CFRP mirror at $\Delta$ROC = 0mm could be realized. This is due to the CFRP mirror being most adept at correcting focus, as is observed in Figure 77 and Figure 78, which is negligible in Figure 76(a).

First, let’s discuss the ability of the active CFRP to correct low-order aberrations. The annular ring actuation modality was designed to mitigate low-order aberrations, particularly focus but potentially astigmatism, coma, and trefoil as well, as described in Section 6.1.4. Figure 77 and Figure 78 demonstrate that the active CFRP mirror is very good at correcting focus since it greatly decreases during closed-loop control (moving from wavefronts (a) to (b)). However, other low-order aberrations are either not affected or mildly increased during closed-loop control, as seen in the (d) plots and in Figure 79.
Figure 76. WFE results for ΔROC = 0mm - (a) no correction, (b) no wavefront graph, (c) correction of (a) with mirao, and (d) aberration coefficients for (a) and (c).

(a) 5.86wv PV, 0.912wv RMS

(b) No CFRP Correction

(c) 0.22wv PV, 0.043wv RMS

(d)
Figure 77. WFE results for $\Delta ROC = 6\text{mm}$ - (a) no correction, (b) correction of (a) with CFRP mirror, (c) correction of (b) with mirao, and (d) aberration coefficients for (a) and (c).
Figure 78. WFE results for ΔROC = 10mm - (a) no correction, (b) correction of (a) with CFRP mirror, (c) correction of (b) with mirao, and (d) aberration coefficients for (a) and (c).

Figure 79 shows the percent changes of the largest five aberration coefficients caused by closed-loop correction with the CFRP mirror (the other aberrations were not included for brevity). Focus (F) experiences the largest change, decreasing by an average of 96.3% over the five tested ROCs. Astigmatism at 0° (A0) and 45° (A45) experiences mean increases of +30.0% and +39.3%, respectively. These are significant changes in astigmatism that could limit the ability of the CFRP
mirror in closed-loop. Trefoil at $0^\circ$ (T0) and $90^\circ$ (T90) experience small changes of -8.9% (a correction) and +18.1% (an increase), respectively. Regardless, Figures 75-78 demonstrate that the active CFRP mirror can be controlled in closed-loop analysis and functions as a low-order aberration corrector, exactly as designed.

Figure 79. Plot of corrective ability of active CFRP mirror at all tested ROC increases. Only the largest aberration coefficients are included.

Second, let’s discuss the corrective ability of the CFRP and mirao system, thus including both the active CFRP mirror and the mirao within the AMT. Figure 80(a) and (b) shows the PV and RMS errors, respectively, for all five ROC increases as the error moves from not corrected, to corrected by the active CFRP mirror, and finally corrected by the mirao. The CFRP closed-loop causes PV error increases for ROC increases larger than 6mm; this is most likely due to the increased astigmatism. In terms of RMS, which can be a better description of wavefront quality, the WFE decreases at every corrective step regardless of the ROC change. Most of the WFE decrease is provided by the mirao since its stroke is very large, as described in Section 1.4.3. Thus, the active system of the CFRP/mirao combination fully functions as a low-order corrector, as designed. The closed-loop settings for both the CFRP and mirao are found in Table 18.
Third, the dominant aberration at all ROC increases is astigmatism, with astigmatism at 45° (A45) being the most severe. A large amount of astigmatism is expected since the CFRP mirror itself is limited by astigmatism, as described in Section 6.3.1. Some of the observed astigmatism, however, could be caused by the mirror and plunger combination. If the mirror is not completely centered on the plunger, the mirror will exhibit astigmatism as the mirror is actuated since orthogonal directions will see different amounts of sag change. Also, it is possible that the mirror shifted on the plunger during closed-loop control, as evidenced by the dramatic shift in aberration values between Figure 76(d), and Figure 77(d) and Figure 78(d). The values of coma at 0° (C0) and trefoil (T0 and T90) are near parity with astigmatism at no ROC increase and much lower in value after the ROC increase, suggesting that the mirror shifted on the plunger between ROC changes inducing an increase in astigmatism. Regardless of the cause, the observed astigmatism is too large for good imaging.

Table 18. Closed-loop parameter settings for correction with the CFRP mirror and mirao. The parameters are described in Section 5.2.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CFRP Value</th>
<th>Mirao Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain</td>
<td>0.05</td>
<td>1</td>
</tr>
<tr>
<td>Averaging</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Actuator Displacement</td>
<td>±0.02mm</td>
<td>±0.2</td>
</tr>
<tr>
<td>Target Function</td>
<td>Plane wave</td>
<td>Plane wave</td>
</tr>
<tr>
<td># of Modes</td>
<td>5</td>
<td>44</td>
</tr>
</tbody>
</table>
6.4.4 Repeatability between ROCs

The goal of the active CFRP mirror is be the primary mirror in an experimental AOZ system. In that respect, system complexity could be greatly reduced if the ROC change could occur open-loop, i.e., without the active control of the SHWS and CASAO. To test this functionality, the active CFRP mirror moved between different ROCs by merely changing PI actuator positions. These positions were determined during the closed-loop analysis in Section 6.4.3. The mirror was placed at the position for $\Delta\text{ROC} = 8\text{mm}$ and moved between $\Delta\text{ROC} = 6\text{mm}$ and $10\text{mm}$, making the focus coefficient negative for $6\text{mm}$ and positive for $10\text{mm}$.

Table 19. Repeatability of WFE between ROC states of $\Delta\text{ROC} = 6, 10\text{mm}$. The left section shows results when only the CFRP mirror moves between states and the right section shows results when both the CFRP and mirao move between states. Note that CV = coefficient of variation, and other labels are as previously defined.

<table>
<thead>
<tr>
<th>CFRP Only</th>
<th>CFRP/Mirao System</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>$\Delta\text{ROC} = 6.0\text{mm}$</strong></td>
<td><strong>$\Delta\text{ROC} = 10.0\text{mm}$</strong></td>
</tr>
<tr>
<td><strong>Mean (wv)</strong></td>
<td><strong>CV</strong></td>
</tr>
<tr>
<td>F</td>
<td>-4.95</td>
</tr>
<tr>
<td>A0</td>
<td>-1.70</td>
</tr>
<tr>
<td>A45</td>
<td>2.40</td>
</tr>
<tr>
<td>C0</td>
<td>0.47</td>
</tr>
<tr>
<td>C90</td>
<td>0.12</td>
</tr>
<tr>
<td>SA</td>
<td>-0.19</td>
</tr>
<tr>
<td>T0</td>
<td>-1.06</td>
</tr>
<tr>
<td>T90</td>
<td>0.66</td>
</tr>
<tr>
<td>PV</td>
<td>1.276</td>
</tr>
<tr>
<td>RMS</td>
<td>7.52</td>
</tr>
</tbody>
</table>

The repeatability of the CFRP mirror by itself and the CFRP/mirao system was tested over five cycles of the two states, with the aberration coefficients and WFE displayed in Table 19. CV again stands for coefficient of variation as defined in Eq. 73. The results for the CFRP mirror by itself are in the left-hand section in Table 19. Overall, the CFRP mirror is a very repeatable device as evidenced by its low CV values, where a low CV value is generally less than 5%. CV values at 10mm ROC increase are slightly higher due to the higher WFE. Most important for an AOZ system is the repeatability of the focus term since that describes how accurately the ROC is maintained between states. Here, the mirror performs well in both states with CVs of -1.5\% and 4.9\%. The higher CV at 10mm is again most likely due to the larger aberration content.
The right-hand column of Table 19 displays the results for the CFRP/mirao system. The CFRP/mirao system is a likely combination for an experimental AOZ system since WFE values in Figure 77 and Figure 78 can be too large for certain applications. In general, the CV values are larger than the CFRP mirror by itself but this could be caused by the low numeric values. In fact, some of the CV numbers are non-sensical due to the values being extremely low. Furthermore, a larger CV value has a lower impact when the numbers are small, thus mitigating the effect of the larger CVs. Therefore, we conclude that both the CFRP mirror and the CFRP/mirao combination are highly repeatable devices thereby creating the possibility of the entire system functioning in open-loop.

### 6.4.5 Annular Ring Sag Change

The sag of a spherical surface is briefly explained in Section 2.4.3 and is given as

\[
Sag = R - \sqrt{R^2 - y^2},
\]

(75)

where \(R\) is ROC and \(y\) is the position on the \(y\)-axis. For the active CFRP mirror, \(y\) is merely the radii of the two annular rings. Eq. 75 shows that an ROC change induces a sag change; thus, the sag changes of the annular rings due to the ROC increases in Section 6.4.3 should be easily determined.

This presumption is complicated by factors – a basic assumption of Eq. 75 and the neoprene layer placed between the ring and mirror. The basic assumption is that the origin of the sag change is at the mirror apex, which is not the case with the active CFRP mirror since the mirror is held at the edge via the plunger. This problem is solved with a slight re-derivation of Eq. 75 found in Appendix C, culminating in Eq. 81. The second is more complicated. The neoprene layer serves to decrease the actuator influence function, as described in Section 6.3.8, and could also decrease the correlation between the predicted and measured annular ring sag changes. The data in Appendix C shows that the neoprene layer decreases the correlation to nearly zero, meaning that the ring sag changes cannot be predicted by the derived theory.

### 6.4.6 FEM Predicted WFE

Finally, FEM was used to design and predict the WFE with the annular ring actuation method in Figure 52(b). We can check the predictive ability of the FEM using the experimental results in Section 6.4.3. Figure 81(a) displays the FEM-predicted wavefront for an ROC increase of 100mm and Figure 81(b) displays the largest experimental ROC increase of 10mm. The FEM assumes
that the CFRP mirror is a perfect sphere initially; as such, the initial WFE in Figure 76(b) is subtracted from the final WFE in Figure 78(b). Since the two ROC changes differ by an order of magnitude, the two plots will only be compared qualitatively.

![Figure 81](image1.png)

Figure 81. FEM-predicted and actual WFE of the active CFRP mirror – (a) FEM-predicted for ΔROC = 100.0mm and (b) experimental WFE for ΔROC = 10.0mm.

As can be seen above, the FEM and experimental wavefronts correlate very poorly in functional form. The near rotational symmetry observed with FEM is not found with the measured wavefront mainly due to the large astigmatism present. Removing astigmatism is code does not greatly improve the correlation as other aberrations, such as coma and trefoil, also remove rotational symmetry. Further experimentation to improve the overall wavefront or more accurately modeling the apparatus may increase the correlation, but at this time FEM cannot accurately predict the resulting WFE.
CHAPTER 7: POTENTIAL APPLICATIONS

The active CFRP mirror developed in Chapter 6 could potentially enhance a wide-range of disciplines. Below are three applications we are actively pursuing.

7.1 Active Secondary Mirror

All major ground-based telescopes use some form of adaptive optics to correct for atmospheric turbulence, usually with additional optics beyond the telescope. The past 15 years have seen the development of active primary and secondary mirrors, where aberrations are corrected within the telescope itself to eliminate the need for additional optics. To date, active secondaries on new telescopes, such as the Multiple Mirror and the Large Binocular Telescopes in Arizona, use a thin-shelled glass mirror around 2 mm thick. The active secondary on the Multiple Mirror Telescope is pictured in Figure 82.

![Figure 82: Active secondary mirror and assembly in place on the MMT.](image)

The thinness allows deformation but also increases the complexity of control due to the “floppiness” of thin glass, a consequence of its low stiffness. Since CFRP has a much higher stiffness than glass, while maintaining low weight, a CFRP active secondary mirror could reduce system complexity without reducing performance. Also, CFRP material has a near-zero coefficient of thermal expansion, making it an ideal candidate for the harsh environments found in space-based and ground-based applications.

While astronomers at the University of Arizona are pursuing CFRP active secondary mirrors for ground-based telescopes, Sandia National Laboratories is investigating applications for space or airborne systems. With sufficient dynamic range, misalignments or other unforeseen
aberrations could be corrected in situ, providing risk mitigation for deployed systems. By changing the ROC, enhanced imaging is also possible.

7.2 Phase Diversity
Phase diversity increases an image’s resolution by combining multiple images of a single scene. Known phase aberrations are optically added to the images, typically by moving the focal plane array along the optical axis to introduce defocus. Then, the unknown aberrations in any single image can be calibrated with the known aberrations from the multiple images. Finally, software optimization techniques recover and reduce the unknown aberrations to produce an image with lower aberration content and higher resolution than any single observable image (known as a ‘super resolution’ image). A schematic of the concept is shown in Figure 83, where two ray bundles have varied in size between the two defocused planes.

![Figure 83. Schematic of phase diversity. Notice how the two ray bundles vary with defocus.](image)

Phase diversity could also be implemented with an active CFRP mirror. The ROC change realized in Section 6.4.4 would produce the necessary defocus to calibrate the image aberrations. Phase diversity could now be implemented on already existing elements, since every system needs a focusing element, thus removing the need for focal plane movement and simplifying the system.

7.3 Adaptive Optical Zoom
Adaptive optical zoom, thoroughly described in Section 2.4, utilizes active elements to produce the necessary magnification change for zoom. One possible design for an AOZ system using an active CFRP primary is found in Figure 84. This design, created by Dr. David Wick of Sandia National Laboratories, produces a 3X zoom ratio with a 200mm diameter CFRP primary mirror in the upper right of Figure 84. Then, a flat fold mirror diverts the beam toward a concave static secondary mirror. Next, the mirao, a commercially-available deformable mirror, functions
as both the tertiary mirror and the second deformable element, changing from -1235mm to 3700mm ROC. Finally, a double Gauss lens images the beam onto the image plane.

The CFRP mirror in Figure 84 changes ROC by 20%, from 2000mm to 2400mm, a change not fully realized in Section 6.4. As shown in Figure 11, there is a correlation between the ROC changes of the two active elements – namely, a small ROC change on one element means a larger change on the other. Thus, the reduced ROC change on the primary must be accounted for by the tertiary, i.e., the mirao must deform further than originally designed. Realistically, this is a difficult conclusion since the mirao is already nearing its ROC stroke limits of [-1200, +1200]mm. However, improvements made to the active CFRP mirror, detailed in Section 8.2, could shift the ROC change to 20% to realize this system.
CHAPTER 8: CONCLUSIONS

8.1 Summary of Research

This dissertation presented the theoretical and experimental exploration of active reflective components for adaptive optical zoom systems (AOZ). A novel optical design theory for AOZ systems was derived using existing theories for static objectives and aberration simulation. The theory enabled a large-scale tradespace analysis to ease the burden on the optical designer by simulating nearly 265 million AOZ systems and allowing designers to choose a particular design based on certain criteria. For this dissertation, the theory lead to the design of a diffraction limited AOZ Cassegrain objective with a 3.3X zoom ratio and a 375mm diameter primary.

A large-aperture active mirror was designed and constructed using a mirror fabricated of the composite material carbon fiber reinforced polymer (CFRP). Finite element modeling (FEM) was employed to design and simulate the annular ring actuation modality, a novel actuation scheme where the radius of curvature (ROC) is increased through force applied to two annular rings attached to the mirror’s backside. Engineering issues such as mirror edge constrain and opto-mechanical assembly design were solved in order to increase the ROC by 10mm from 2000mm to 2010mm over a 160mm diameter clear aperture. Closed-loop control to correct aberrations kept the peak-to-valley error to 1.05waves (with respect to a HeNe laser) at 2010mm ROC using the annular ring actuators and a commercially-available deformable mirror. Finally, the active CFRP mirror was able to accurately change ROC between 6mm and 10mm, thereby creating the potential for open-loop control of the active CFRP mirror.

8.2 Final Remarks

As stated in Section 1.2, the underlying goal of this research was the realization of an experimental AOZ system which would require an ROC increase of 10% or greater. Several improvements could be made to the active CFRP mirror assembly in order to push the current ROC increase of 0.5% towards 10%:

- Diamond-turn plunger
  - The plunger is not optically flat and so contributes to the overall error. Diamond-turning the plunger to meet optical tolerances should remove this effect.
Add two actuators to the outer ring

- Astigmatism is the most prevalent aberration with the CFRP mirror both statically and dynamically. However, astigmatism is tough to address with the current actuation scheme since the outer ring actuators are assembled in a hexapolar fashion. Adding two actuators to the outer ring, for a total of eight at separation angles of 45°, places two orthogonal axes of actuators to better control astigmatism.

- Improve alignment strategies

- The mirror, annular rings, and actuators are all aligned mechanically. Improving the alignment procedure and decreasing the fabrication tolerances should remove alignment errors which may contribute to the observed astigmatism.

However, the main improvement for the active CFRP mirror is the CFRP mirror itself. The current mirror is essentially a static mirror that is forced to become an active mirror through actuation. The large stiffness found with static CFRP mirrors is very likely not advantageous for active CFRP mirrors. The stiffness can be decreased by tailoring the mechanical properties through either fewer layers of prepreg material or with different lay-up schemes. Both of these can be studied with FEM, but only experimentation with CFRP mirror fabrication will find a solution since the fabrication process is fickle. Finally, the initial wavefront error of the CFRP mirror must be decreased because achieving a diffraction limited active mirror is nearly impossible when the mirror is ~5 waves peak-to-valley before any ROC change. This can only be accomplished with an improved fabrication process.

Regardless of the above delineated improvements, this dissertation has demonstrated the viability of a large-aperture mirror to change ROC by an appreciable extent. Future work will entail pushing the ROC change to greater than 100mm, and the design and construction of an experimental AOZ system using the active CFRP mirror.
APPENDIX A: AOZ THEORY CODE

The following code solves the system of equations detailed in Chapter 3 in order to derive Eqs. 47 -51. The code was written in Mathematica.

(* Wetherell & Rimmer, App. Opt. Vol.11 No.12. List of equations used; they are altered from their published forms to accommodate for a two-state system. Note that equation number is the number used in the original publication. Also note that first number is state number (1=unzoom, 2=zoom) and the second number is the element number (1=primary, 2=secondary). If only one number is listed, it is the state number.*)

Clear[Fn1, Fn2, f11, f22, Dp, f12];

(* Eq. 1 - Magnification of secondary. Fn = f-number *)
m1 == Fn1/Fp1;
m2 == Fn2/Fp2;

(* Eq. 2 – Normalized back focal distance *)
eta == WD/Dp;

(* Eq. 4 - Focal ratio of primary *)
f11 == Dp Fp1;
f12 == Dp Fp2;

(* Eq. 6 - Axial separation between primary and secondary *)
d == Dp*(Fn1 - eta)/(m1 + 1);
d == Dp*(Fn2 - eta)/(m2 + 1);

(* Eq. 10 - Curvature of secondary *)
1/R21 == (1 - m1 m1)/(2 m1 Dp (Fp1 + eta));
1/R22 == (1 - m2 m2)/(2 m2 Dp (Fp2 + eta));
(* Now, we take all above listed equations and solve. Independent parameters are [Fn1,Fn2,f11,f12,Dp], while dependent parameters are [R21,R22,d,WD,m1,m2,Fp1,Fp2,eta]. *)

\[
\text{Solve}\left\{m_1 = \frac{F_{n1}}{F_{p1}}, \ m_2 = \frac{F_{n2}}{F_{p2}}, \ \eta = \frac{W_D}{D_p},
\begin{align*}
\ d & = D_p\left(F_{n1} - \eta\right)/(m_1 + 1), \ d & = D_p\left(F_{n2} - \eta\right)/(m_2 + 1), \\
1/R_{21} & = \left(1 - m_1 m_1\right)/(2 m_1 D_p \left(F_{p1} + \eta\right)), \\
1/R_{22} & = \left(1 - m_2 m_2\right)/(2 m_2 D_p \left(F_{p2} + \eta\right)), \\
\ f_{11} & = D_p \ F_{p1}, \ f_{12} = D_p \ F_{p2}, \ \{R_{21}, \ \ R_{22}, \ d, \ \ WD, \ m_1, \ m_2, \ F_{p1}, \ F_{p2}, \ \eta\}\right\} //
\text{FullSimplify}
\right.
\]

\[
\begin{align*}
R_{21} & \to \frac{\left(2 \ D_p \ f_{11} \left(f_{11} - f_{12}\right) F_{n1} F_{n2}\right)}{\left(f_{11} - D_p \ F_{n1}\right) \left(-f_{12} \ F_{n1} + f_{11} \ F_{n2}\right)}, \\
R_{22} & \to \frac{\left(2 \ D_p f_{12} \left(-f_{11} + f_{12}\right) F_{n1} F_{n2}\right)\left(f_{12} F_{n1} - f_{12} F_{n2}\right)}{\left(f_{12} F_{n1} - f_{12} F_{n2}\right)}, \\
d & \to \frac{\left(f_{11} f_{12} \left(-F_{n1} + F_{n2}\right)\right)}{\left(-f_{12} F_{n1} + f_{11} F_{n2}\right)}, \\
W_D & \to \frac{\left(D_p f_{12} F_{n1} F_{n2} - f_{11} \left(f_{12} \left(F_{n1} - F_{n2}\right) + D_p F_{n1} F_{n2}\right)\right)}{\left(f_{12} F_{n1} - f_{12} F_{n2}\right)},
\text{m}_1 & \to \frac{D_p F_{n1}}{f_{11}}, \ \text{m}_2 & \to \frac{D_p F_{n2}}{f_{12}}, \ F_{p1} & \to \frac{f_{11}}{D_p}, \\
F_{p2} & \to \frac{f_{12}}{D_p}, \ \eta & \to \frac{\left(f_{11} f_{12} F_{n1} - f_{11} f_{12} F_{n2} + D_p f_{11} F_{n1} F_{n2} - D_p f_{12} F_{n1} F_{n2}\right)}{D_p f_{12} F_{n1} - D_p f_{11} F_{n2}}.
\end{align*}
\]
APPENDIX B: ANNULAR RING SAG CHANGE

The sag of a spherical surface is briefly explained in Section 2.4.3 and is given as

\[ \text{Sag} = R - \sqrt{R^2 - y^2} , \]  

(76)

where \( R \) is ROC and \( y \) is the position on the \( y \)-axis referring to Figure 85. Eq. 76 shows that an ROC change induces a sag change. For the active CFRP mirror, the height \( y \) equals 17.5mm and 80mm, i.e., the radii of the two annular rings. Thus, we can calculate the necessary sag changes between ROC states of the active CFRP mirror.

Eq. 76 assumes that all spherical surfaces touch on the optical axis. The active CFRP mirror, however, is held at the edge of the mirror, requiring a slight redefinition of Eq. 76. First, we define the sag difference between the two curves in Figure 85,

\[ A = \Delta \text{Sag} = \text{Sag}_1 - \text{Sag}_2 , \]  

(77)

where \( \text{Sag}_1 \) is the sag of the more curved surface, \( \text{Sag}_2 \) is the sag of the less curved surface, and \( \Delta \text{Sag} \) is the sag difference between the two curves. Inserting Eq. 76, we have

\[ \Delta \text{Sag} = \left( R_1 - \sqrt{R_1^2 - y^2} \right) - \left( R_2 - \sqrt{R_2^2 - y^2} \right) , \]  

(78)

where \( R_1 \) is the ROC of the more curved surface and \( R_2 \) is the ROC of the less curved surface. Now, we need to shift the point where the two spherical surfaces meet, as in Figure 85, and find the sag change at the edge of the curves,

\[ \Delta \text{Sag}_{\text{edge}} = \left( R_1 - \sqrt{R_1^2 - y_{\text{edge}}^2} \right) - \left( R_2 - \sqrt{R_2^2 - y_{\text{edge}}^2} \right) , \]  

(79)

where \( \Delta \text{Sag}_{\text{edge}} \) is given by

\[ \Delta \text{Sag}_{\text{edge}} = A + B = \Delta \text{Sag} + \Delta \text{Sag}_{\text{CFRP}} , \]  

(80)

and \( \Delta \text{Sag}_{\text{CFRP}} \) is the sag change of the active CFRP mirror as it increases ROC. Finally, solving for \( \Delta \text{Sag}_{\text{CFRP}} \), we have
\[ \Delta \text{Sag}_{CFRP} = \Delta \text{Sag}_{\text{edge}} - \Delta \text{Sag}. \]  

(81)

Figure 85. Re-definition of two different spherical surfaces being held at the edge.

Eq. 81 calculates the distance each of the two annular rings must displace between ROC states. It does not, however, account for the neoprene layer placed between the mirror and rings, as described in Section 6.3.8, which may increase the distance traveled by each actuator for an equivalent ROC change.

Before calibrating the experimental sag change results, we first need to determine whether the actuators move as a group by ring or individually. The results of Section 6.4.3 imply that the two rings are coupled, thus moving as a group, since only focus can be corrected during closed-loop control. Figure 86 supports this claim where the movements of the 3 inner ring actuators and 6 outer ring actuators are displayed in (b) and (c), respectively. The actuators are labeled by capital letter as found in Figure 86(a). Both the inner and outer ring actuators have roughly the same functional form, compared by ring, although the correlation is much higher with the outer ring. The large spike in Actuator C in Figure 86(b) is most likely an anomaly. Therefore, it seems that the actuators are coupled by ring. Note that the non-optical tolerances of the plunger and neoprene layer cause the actuators to be at different heights, as observed in (b) and (c).
To see how the position of the inner and outer rings varies with ROC increase, we take the average of the actuator positions on each ring. This data is graphed along with the theoretical sag changes for each ring, determined with Eq. 81, in Figure 86(d) and (e) for the inner and outer rings, respectively. The sag change of the inner ring in Figure 86(d) experiences a wild variation at ROC changes of 4mm and 6mm, due to the non-linearties observed in all three actuators in Figure 86(b), and does not correlate well with the theoretical prediction. The outer ring in Figure 86(e) tracks the predicted values through the 4mm ROC increase, but falls off as the ROC is increased further. Thus, neither sag change correlates with the predicted values. Therefore, the neoprene layer obviates the opportunity to predict the ring sag changes using the derived theory.
Figure 86. Sag changes during the ROC increase in Section 6.4.3 – (a) layout of actuator names, (b) inner ring actuators, (c) outer ring actuators, (d) average of inner ring actuators versus predicted, and (e) average of outer ring actuators versus predicted.
REFERENCES


Notes: