CONDITIONAL PROBABILITY OF OCCURRENCE FOR VARIATIONS IN CLIMATE BASED ON WIDTH OF ANNUAL TREE-RINGS IN ARIZONA

CHARLES W. STOCKTON and HAROLD C. FRITTS

Laboratory of Tree-Ring Research
The University of Arizona

ABSTRACT

A method is presented for making probability statements about past climatic conditions for the state of Arizona given the corresponding relative width of tree rings. The probability statements about periods of extreme climate from 1650 to 1899 are based upon the joint occurrence of the state-wide average seasonal climate and ring widths during 1899-1957. The ring-width values used are index chronologies selected from four different areas within Arizona. Spatial homogeneity among the four chronologies is evaluated by using digital filtering and correlation techniques. The chronologies are then normalized, averaged to form a state-wide series, and the values of state-wide growth for each year placed into one of nine equally probable classes. Similarly, seasonal temperatures and seasonal precipitation are placed into three equally probable classes and the joint occurrences between temperature and precipitation become nine climatic classes. Contingency tables are used to establish the joint occurrence of the nine climatic classes and the nine ring-width classes. A number of 10 and 20 year intervals since 1650 are identified as periods of unusually high or low probability of occurrence of above or below normal precipitation for any season of the year. The period 1880-1889 is of special interest as it was a period when downcutting was initiated in many Arizona streams and is also one of the periods in which the probability for below normal seasonal precipitation was unusually high (p = 0.48).

INTRODUCTION

Modern statistical innovations have been incorporated into several recent analyses of tree-ring growth as related to climate. For example, Fritts (1962) used stepwise multiple regression techniques to study the systematic relationship of ring widths to climatic parameters in the southwestern United States; Bryson and Dutton (1962) and LaMarche and Fritts (in press) have utilized power and cross-power spectral analyses in analyzing tree-ring records for periodicities; Mitchell (1967) and LaMarche and Fritts (1971) applied factor analysis; Julian and Fritts (1967) introduced digital filter techniques as a means of appraising the systematic relations of tree growth to climatic variables; and Fritts and others (1971) describe a new multivariate technique.

None of these studies, however, has attempted to analyze the joint occurrence of specific ring widths with certain climatic types so that probability statements can be made about climate from ring widths. This study analyzes the joint occurrence of climate and relative width of tree rings for the state of Arizona. Conditional probabilities of occurrence are used to establish quantitative relevance of state-wide tree-ring growth from 1900 through 1957 to recorded climate for 1899 through 1957. The results are then utilized in making probability statements about climate from ring widths for the period 1650 through 1899.

CLIMATIC DATA

The study is based on total-precipitation and average-temperature data from all reporting stations in the state of Arizona for each season in the period starting with the summer of 1899 and ending with the summer of 1957. The seasonal data had been initially analyzed elsewhere (Dickson 1966) and the state-wide averages for all 232 seasonal occurrences in the 58-year period of record had been divided into three equally probably classes: below normal, near normal, and above normal. The limits for the individual seasonal classes differed for each season (Table 1) and were based on the assumption of normal distribution of the full frequency range.

The joint occurrences of the precipitation and temperature classes resulted in nine different combinations, each of which was assigned a numerical designation and termed a climatic class. The designations and frequencies of occurrence are shown in Table 2. The frequencies by season are also shown as probabilities. The distribution of seasonal climatic classes within any of the four seasons is not greatly skewed, but the combination of particular conditions (classes 1, 5, and 9) is dominant. This indicates a general inverse association between temperatures and precipitation where

Table 1. Seasonal Precipitation and Temperature Class Limits for the State of Arizona.

CLIMATIC CLASS	AUTUMN September-November	WINTER December-February	SPRING March-May	SUMMER June-August
	TOTAL PRECIPITATION (in.)			
BELOW NORMAL	$P \leq 2.06$	$P \leq 2.85$	$P \leq 1.37$	$P \leq 4.12$
NEAR NORMAL	$2.06 < P \leq 3.28$	$2.85 < P \leq 4.09$	$1.37 < P \leq 2.31$	$4.12 < P \leq 5.44$
ABOVE NORMAL	$3.28 < P$	$4.09 < P$	$2.31 < P$	$5.44 < P$
	AVERAGE TEMPERATURE (F.)			
BELOW NORMAL	$T \leq 61.07$	$T \leq 42.33$	$T \leq 57.47$	$T \leq 77.10$
NEAR NORMAL	$61.07 < T \leq 62.47$	$42.33 < T \leq 45.03$	$57.47 < T \leq 59.27$	$77.10 < T \leq 78.30$
ABOVE NORMAL	$62.47 < T$	$45.03 < T$	$59.27 < T$	$78.30 < T$

Period of Record is 1899 through 1957

Table 2. Combinations of Climatic Classes, Numerical Designations, Frequency, and Probability of Occurrence by Season for the State of Arizona.

Combinations		Numerical	Frequency of	Probability of Occurrence			
Precipitation	Temperature	Designation	Occurrence	Autumn	Winter	Spring	Summer
Below normal	Above normal	1	38	.172	.121	.172	.190
Below normal	Near normal	2	24	.120	.121	.086	.086
Below normal	Below normal	3	18	.052	.103	.086	.069
Near normal	Above normal	4	19	.052	.103	.086	.086
Near normal	Near normal	5	29	.138	.121	.103	.155
Near normal	Below normal	6	26	.138	.103	.121	.086
Above normal	Above normal	7	12	.069	.103	.017	.017
Above normal	Near normal	8	26	.103	.086	.155	.103
Above normal	Below normal	9	40	.155	.138	.172	.207

Period of Record is Summer 1899 through Spring 1957

conditions of high precipitation tend to occur most frequently with conditions of low temperature and conditions of low precipitation tend to occur with conditions of high temperature. The probability of occurrence of any of the 9 climatic classes did not differ markedly from season to season (Table 2).

A check for persistence of climatic conditions from season to season was made by comparing the occurrence of climatic classes in each of the 4 seasons with the occurrence of climatic classes in the consecutive season. The greatest tendency toward persistence is in climatic class 1, especially for the winter-spring and spring-summer periods. However, persistence of all classes between seasons was judged not to be sufficiently pronounced to invalidate the present analysis.

RING–WIDTH DATA

Since the climatic data are averages for the entire state, it was necessary to obtain a ring-width chronology which also was representative of the state. Four tree-ring chronologies from Arizona were chosen to provide the best available state-wide distribution and the greatest chronology length. Chronology 1 represents seven Douglas-fir (*Pseudotsuga menziesii* Mirb., Franco) from Tsegi Canyon, Navajo National Monument, which were sampled along four radii from each tree (Stokes, in press). Chronology 2 represents 10 ponderosa pine (*Pinus ponderosa* Laws.) northwest of the San Francisco Mountains (Fritts and others 1965, sites C and D), which were sampled along four radii from each tree. This sample was merged with the ponderosa pine chronology for Flagstaff described by Douglass (1947). Chronology 3 represents 10 Douglas-fir from Nantack Gap, sampled along two radii (Parker 1967). Chronology 4 represents 38 Douglas-fir in southern Arizona sampled along two radii; this sample comprises 10 trees from the Santa Catalina Mountains, nine trees from the Galiuro Mountains, and 19 trees from the Santa Rita Mountains (Stokes, in press; Drew, in press).

All area chronologies are from the mountainous regions in the eastern and central portions of the state. However, these regions are considered fairly representative of the climatic variation for the state as a whole (Green 1964; Hastings and Turner 1965), as they generally receive large amounts of precipitation and cover a north-south transect across the state.

All sites produced ring-width chronologies indicative of the lower forest border, where climate each year is most limiting to growth-controlling processes within the trees (Fritts 1966). All samples were continuous at least back to 1861; thus, this part of the chronology is based on 65 trees, representing a total of 164 measured stem radii. Because of varying tree ages, the chronology from 1860 back to 1650 was based upon decreasing numbers of trees. The chronology for the year 1650 was the minimum sized sample and included 40 stem radii representing at least one tree in every sampled site.

Although differences between the two species sampled could introduce some variation in the growth response, studies in Arizona and southwestern Colorado summarized by Fritts (1966, Fig. 3) show that, in both ponderosa pine and Douglas-fir, individual ring widths are related to the climate over the 15 months starting with June of the previous growing season and ending with the August that is concurrent with growth. For both species, the highest correlation between ring-width chronologies and climate was for the nine months (autumn, winter, and spring) immediately preceeding the period of growth. The only apparent difference is that the summer climate concurrent with growth is more highly correlated with ring widths of ponderosa pine, whereas the summer climate for the year prior to growth is more highly correlated with ring widths of Douglas-fir (Fritts 1965).

The trend in ring widths associated with tree age was measured by a computer technique which fits to each measured series a modified exponential equation of the form

$$\hat{Y}_t = ae^{-bt} + k \,, \tag{1}$$

where $\hat{Y}_t$ is the expected ring width, t is the sequence in years ranging from 1 to n for the first to last ring, and a, b, and k are constants determined by a least squares fit of the data (Fritts, Bottorff, and Mosimann 1968). This equation is used to obtain an expected growth value ($\hat{Y}_t$), which is divided into the measured ring-width value (Y_t) resulting in a ring-width index (I_t), where

$$I_t = \frac{Y_t}{\hat{Y}_t} \,. \tag{2}$$

This transforms the ring-width measurements into a series of values, or chronology, which has a homogeneous variance through time and a mean of 1 for the entire length of each measured radius. It eliminates the gradual changing trends due to growth or climate that operate over the entire length of the ring record and retains a greater amount of the variance as the frequency increases. A chronology for a site or area is obtained by averaging the indices (I_t) from all sampled radii for each year (t).

Although the above-described removal of growth trend insures stationarity for each measured radius, it does not produce equal variances for chronologies among different trees and sites, nor does it insure an exact mean of unity, either for group chronologies made up of many stem radii of varying ages or for segments of long chronologies. The variances among the four regional chronologies used in this study for the period 1900[1] through 1957 ranged from 0.33 in sample 4 to 0.48 in sample 1. The means for the same period ranged from 1.05 in sample 4 to 1.15 in sample 2. In order to insure equal weight in averaging all four area chronologies, they were individually transformed to standard normal variates (Z_t) in the following manner:

$$Z_t = \frac{I_t - \mu}{\sigma s}, \tag{3}$$

where μ is the estimate of the mean and σ is the estimate of the standard deviation of the indices for the period 1900 through 1957. Each of the four area chronologies was transformed for its entire length, starting with the ring-width indices for 1650, according to the above equation. The four standard normal variates were averaged each year to obtain a single statewide chronology ($\bar{Z}_t$). This chronology was then renormalized (equation 3) to insure that the period 1900 through 1957 had unit variance and a mean of zero (Fig. 1).

A chi-square (χ^2) test for goodness of fit on nine equally probable classes of the normalized tree-ring indices established that the data for the period 1900 through 1957 are normally distributed, giving a calculated χ^2 value of 7.80 with 6 degrees of freedom (9-1-2, where 2 degrees of freedom are lost because the expected frequencies are computed by estimating 2 population parameters – the mean and standard deviation – from sample statistics). The yearly standard normal deviates of the state-wide ring-width chronology were then classified into the nine equally probable classes based upon the area under the normal curve. These classes are defined in Table 3 and shown in Fig. 1.

A basic assumption underlying the use of the several ring-width chronologies in this study is that long-term as well as short-term changes in climate are accompanied by similar changes in the chronology as derived from tree rings. Variations due to nonclimatic causes are assumed to be removed or are similar in magnitude for the 1900 through 1957 standard interval and the earlier periods.

It could be argued that any of the four chronologies selected for this analysis might include aberrant portions resulting from such local conditions as fire or insect infestation. If this had occurred, the deviant chronology might include variance which would appear as long-term changes independent of the changes in climate shown by chronologies from neighboring

[1]The ring for 1900 is matched with the climate of June 1899 through July of 1900.

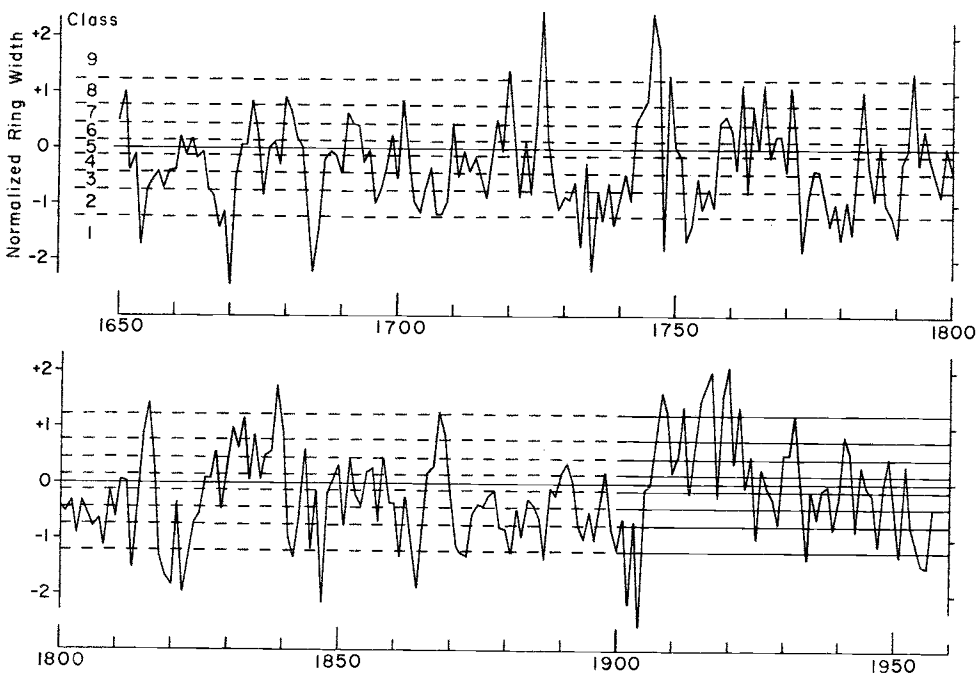

Figure 1. Ring-width chronology for the state of Arizona based upon the average of four area chronologies expressed as standard normal variates. Ring-width class limits are shown as horizontal solid lines for the standard interval from 1900 through 1957, and as dashed lines for the earlier record.

Table 3. Upper Class Limits Used to Seperate Normalized Tree-Ring Data for Arizona into 9 Equally Probable Classes.

CLASS NUMBER	VALUE ≤
1	−1.2200
2	−0.7643
3	−0.4306
4	−0.1395
5	0.1395
6	0.4306
7	0.7643
8	1.2200
9	∞

regions. Since such nonclimatic changes could affect estimates of past climate from ring-width indices, the four chronologies were tested in the following manner.

The six linear correlation coefficients for all combinations of the four ring-width chronologies were calculated and averaged for overlapping 20-year periods lagged every five years starting with 1650 and ending in 1959. In addition, each chronology was modified by use of two reciprocal digital filters, designed by Paul R. Julian of the National Center for Atmospheric Research, Boulder, Colorado. The two filters separate the chronology variance with a frequency of less than 10 years (high-pass filter) from the variance with a frequency greater than 10 years (low-pass filter). These filters are essentially a series of specially designed weights symmetric to a central weight (Table 4), which are multiplied against the original chronology in the following way

$$\overline{x}_t = \sum_{i=-6}^{i=+6} w_i \, z_{t+i} \, , \qquad (4)$$

where $\overline{x}_t$ is the filtered value of the series corresponding to the t th term and w_i is the weight (Table 4) by which the value of the series i units removed from t is multiplied. The filter weights are symmetric so the weight value of $w_1 = w_{-1}$, $w_2 = w_{-2}$, etc. (Mitchell and others 1966). The high-frequency components of each chronology and then the low-frequency components expressed as yearly values were used to calculate linear correlations between each combination of the four chronologies in the same manner as described for the original unfiltered series. Since each filter involved 13 weights (six on each side of the central weight), the filtered series were shorter by 12 years, and correlations were obtained for 20-year periods starting with 1660 and ending with 1954.

The means of the correlation coefficients, first using the 4 original chronologies, then using their high-frequency component, and finally using their low-frequency component, are plotted in Fig. 2. It is apparent from the figure that the high-frequency variance component exhibits mean correlations among the four Arizona stations ranging from 0.27 to 0.83, while the average of the low-frequency variance correlations ranges more widely, from a minimum of −0.23 to a maximum of 0.93. It may be inferred that the long-term changes or low-frequency component is responsible for the greatest amount of heterogeneity among the four regional chronologies. The greatest heterogeneity for tree-ring widths within Arizona occurred for

Table 4. The Sets of Weights Representing Digital Filters Designed to Separate the Variance with Frequencies less than 10 Years (high-pass) from the Variance with Frequencies greater than 10 Years (low-pass). Weight Number Expressed as Position Relative to Central Weight.

WEIGHT NUMBER	WEIGHT VALUE	
	High-Pass Filter	Low-Pass Filter
0 (central weight)	.7744	.2256
−1 and +1	−.1933	.1933
−2 and +2	−.1208	.1208
−3 and +3	−.0537	.0537
−4 and +4	−.0161	.0161
−5 and +5	−.0030	.0030
−6 and +6	−.0003	.0003

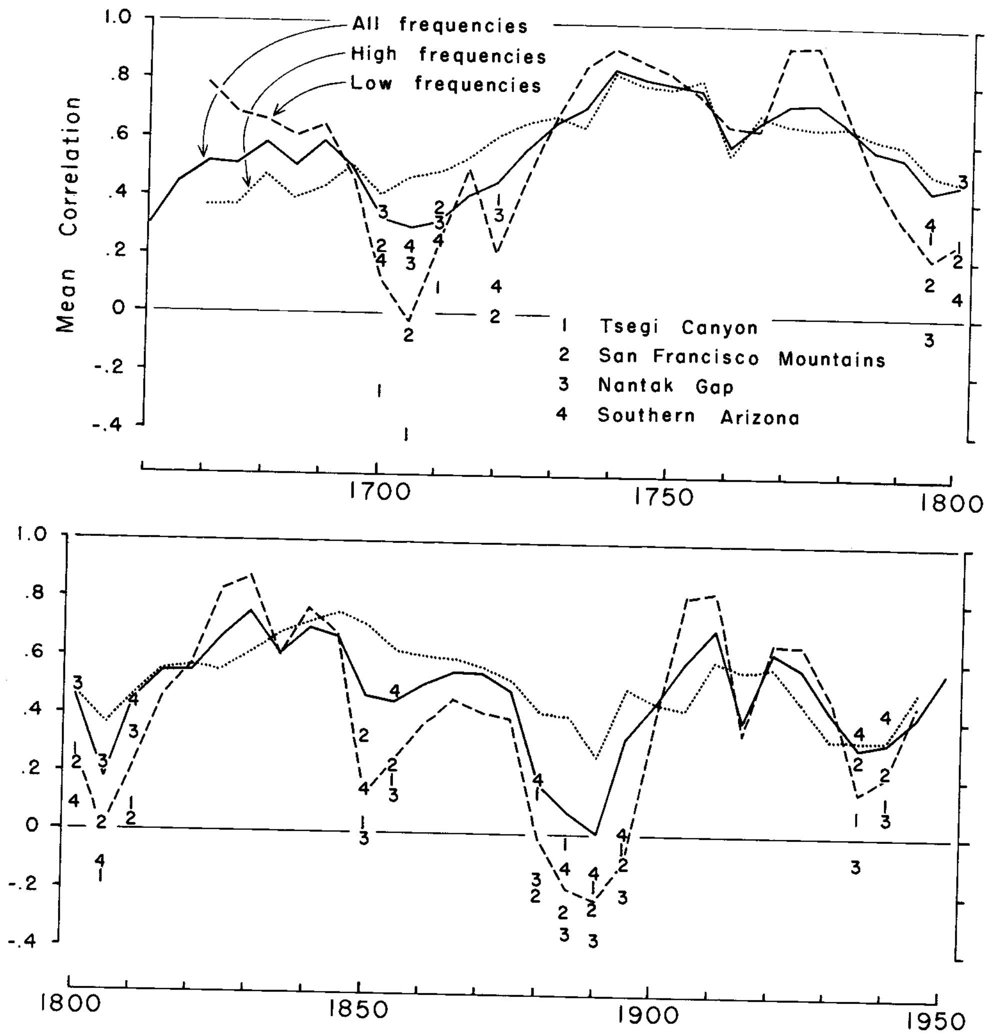

Figure 2. Mean of all correlation coefficients for each chronology with each of the other three chronologies for 20-year overlapping intervals lagged every five years and plotted as the central year. Solid line connects correlations of full frequency variances; dotted line connects the correlations of the frequencies less than 10 years; dashed line connects the correlations of frequencies greater than 10 years. The location of the numbers shows the relative positions of the low-frequency correlation for each chronology (numbers 1 to 4) during intervals when the average of the coefficients was less than 0.30.

intervals centering in 1700-1710, 1720, 1795-1810, 1850-1855, 1886-1895, and 1935-1940. During these periods of time the average growth at any one site is most likely to depart from the state-wide average shown in Fig. 1.

In order to provide a means of further examining this heterogeneity, the mean value of the three correlation coefficients for the low-frequency variance component of each chronology with the other three chronologies was calculated. These values are shown by the location of numerals in Fig. 2 for those periods when the mean of all low-frequency correlations was less than 0.30. The Nantak Gap chronology (3) from east-central Arizona most commonly departed from the others in the state. The lowest mean correlation representing chronologies other than 3 occurred 8 out of 16 times. The southern Arizona chronology which was obtained from trees in three separate sites exhibited the lowest mean correlation only 1 out of 16 times.

These results may also be compared with the mapped tree-ring data presented by Fritts (1965). Fig. 2 in this paper and the corresponding maps of Fritts show that the divergent trends among the four chronologies may be accompanied by similar trends in adjacent chronologies of neighboring states, and by steep gradients across Arizona. This would indicate that at least a portion of the low correlation among the Arizona chronologies for certain intervals before 1900 is partly a result of contrasting climate conditions over the state and not entirely due to aberrant nonclimate changes resulting from conditions of the site. The above analysis also indicates that all four chronologies contribute some independent information on the mean growth for the entire state, and samples from additional areas within the state, if available, would probably improve the relationship between tree growth and the state-wide climate.

Joint Occurrence of Climatic Classes with Ring-Width Classes

The climatic classes for each year were matched with the ring-width classes according to the model described by Fritts (1966). The association assumed a relationship between climate for the 15 months of June through December for the year previous to growth and the January through August for the year of growth. The joint occurrence of ring-width classes with the climatic classes for each of the five seasons within the 15-month period was determined and the data are arranged in contingency tables (Tables 5 through 9).

Table 5. Joint Frequency Distribution between Ring-Width Classes and Climate of Arizona for the Summer prior to Ring Growth.

CLIMATIC CLASS	RING-WIDTH CLASS 1	2	3	4	5	6	7	8	9	Total
1	3	1	3	1	2	0	0	1	0	11
2	0	1	2	0	1	0	1	0	0	5
3	0	0	1	0	1	1	1	0	0	4
4	0	2	0	0	2	0	1	0	0	5
5	1	2	1	1	3	1	0	0	0	9
6	1	0	0	1	0	0	1	0	2	5
7	0	0	0	0	0	0	0	0	1	1
8	0	0	0	1	1	1	2	0	1	6
9	1	0	0	1	0	2	1	1	6	12
Total	6	6	7	5	10	5	7	2	10	58

Table 6. Joint Frequency Distribution between Ring-Width Classes and Climate of Arizona for the Autumn prior to Ring Growth.

CLIMATIC CLASS	RING-WIDTH CLASS 1	2	3	4	5	6	7	8	9	Total
1	4	2	1	1	2	0	0	0	0	10
2	1	1	0	1	2	1	1	0	0	7
3	0	1	0	0	0	0	0	0	2	3
4	0	1	2	0	0	0	1	0	0	4
5	0	0	2	0	2	2	0	0	1	7
6	0	0	1	1	1	1	2	1	1	8
7	1	0	1	1	1	0	0	0	0	4
8	0	0	0	1	2	0	0	1	2	6
9	0	1	0	0	0	1	3	0	4	9
Total	6	6	7	5	10	5	7	2	10	58

Table 7. Joint Frequency Distribution between Ring-Width Classes and Climate of Arizona for the Winter prior to Ring Growth.

CLIMATIC CLASS	RING-WIDTH CLASS 1	2	3	4	5	6	7	8	9	Total
1	4	2	0	0	0	0	1	0	0	7
2	1	1	2	0	1	2	0	0	0	7
3	1	1	2	1	0	0	0	0	1	6
4	0	1	2	1	1	0	0	0	1	6
5	0	0	1	2	2	0	2	0	0	7
6	0	0	0	1	1	2	1	0	1	6
7	0	0	0	0	3	0	1	2	0	6
8	0	0	0	0	0	0	1	0	4	5
9	0	1	0	0	2	1	1	0	3	8
Total	6	6	7	5	10	5	7	2	10	58

Table 8. Joint Frequency Distribution between Ring-Width Classes and Climate of Arizona for the Spring Concurrent with the Beginning of Ring Growth.

CLIMATIC CLASS	RING-WIDTH CLASS 1	2	3	4	5	6	7	8	9	Total
1	3	2	1	2	0	1	0	0	0	9
2	0	0	1	0	0	1	2	0	1	5
3	1	0	1	1	2	0	0	0	0	5
4	1	2	1	1	0	0	1	0	0	6
5	0	0	1	0	2	0	1	0	2	6
6	0	0	1	1	2	0	1	0	2	7
7	0	2	0	0	0	0	0	0	0	2
8	1	0	1	0	3	1	0	1	1	8
9	0	0	0	0	1	2	2	1	4	10
Total	6	6	7	5	10	5	7	2	10	58

Table 9. Joing Frequency Distribution between Ring-Width Classes and Climate of Arizona for the Summer Concurrent with Ring Growth.

CLIMATIC CLASS	RING-WIDTH CLASS 1	2	3	4	5	6	7	8	9	Total
1	2	2	2	1	3	0	1	0	0	11
2	0	0	0	1	1	2	1	0	0	5
3	0	0	0	1	0	1	1	0	1	4
4	0	0	2	1	0	1	0	0	1	5
5	1	3	1	0	3	0	0	0	1	9
6	0	0	1	0	0	0	0	1	3	5
7	0	0	0	0	0	0	1	0	0	1
8	1	0	1	0	1	0	1	0	2	6
9	2	1	0	1	2	1	2	1	2	12
Total	6	6	7	5	10	5	7	2	10	58

These data were then combined into a single table for (a) all five seasons, (b) four seasons including previous summer, autumn, winter, and spring, (c) four seasons including autumn, winter, spring, and concurrent summer, and (d) three seasons including autumn, winter, and spring. Chi-square was used to test the significance of joint occurrence for the nine climate classes with the nine ring-width classes as follows

$$\chi^2 = \sum^{kr} \frac{(f_{mn} - h_{mn})^2}{h_{mn}} , \qquad (5)$$

where f_{mn} is an observed number of occurrences in a particular cell $(k_m r_n)$ representing ring-width class k_m and climatic class r_n and h_{mn} is the corresponding hypothetical frequency (Li 1964). The frequency for h_{mn} is computed using the number of occurrences for all nine cells within the k_m ring-width class, the number of occurrences for all nine cells within the r_n climatic class and the total number of occurances for all classes, Σr, where

$$h_{mn} = k_m \frac{r_n}{\Sigma r} . \qquad (6)$$

For example, using equation 6 the value of h_{15} for ring-width class 1 and summer climatic class 5 would be calculated from the totals in Table 9 as follows

$$h_{15} = 6 \times \frac{9}{58} = \frac{54}{58} = 0.93 .$$

The ring-width classes may be considered as nine random samples drawn from nine multinomial populations. The hypothesis is that the climatic classes have the same set of relative frequencies; i.e., the distribution of the climatic classes is independent of the ring-width classes. If the value of the computed χ^2 (column 1, Table 10) exceeds the 95 percent value of the χ^2 distribution, the conclusion is that the distribution of climatic classes within each ring-width class is not the same, and the occurrence of the

Table 10. Results of the X^2 tests for Independence, Number of Occurrences, and the Coefficient of Contingency (C) for the Association of Ring-Width Classes with Climatic Classes.

CONTINGENCY TABLE ANALYZED	COMPUTED X^2	OCCURENCES	C
Previous Summer Climate (Table 5)	65.8	58	—
Autumn Climate (Table 6)	72.9	58	—
Winter Climate (Table 7)	100.0[1]	58	.80[2]
Spring Climate (Table 8)	74.2[1]	58	.75[2]
Concurrent Summer Climate (Table 9)	63.3	58	—
Five Seasons Combined (Tables 5-9)	132.9	290	.56
Four Seasons Combined (Tables 5-8)	132.4	232	.60
Four Seasons Combined (Tables 6-9)	123.1	232	.59
Three Seasons Combined (Tables 6-8)	124.7	174	.65

[1] is greater than 73.6 which is the 95% confidence limit for 64 degrees of freedom.

[2] value questionable because of small sample size.

climatic classes is significantly associated with the occurrence of the ring-width classes.

If the above tests indicate dependence of the classification within the contingency table, the degree of dependence is measured by the coefficient of contingency,

$$C = \left[\frac{\chi^2}{\chi^2 + N}\right]^{1/2},$$

where χ^2 is computed from equation (5) and $N = \Sigma r$ (Spiegel 1961). The larger the value of C, the greater the degree of dependence. The number of rows and columns in the contingency table determines the maximum value of C, which is never greater than 1. In this case, since the number of rows and columns in the contingency table are equal, the maximum value of C is given by $\left[(k-l)/k\right]^{1/2}$ where k is the number of columns and rows (Spiegel 1961). For a 9 by 9 contingency table the maximum C is .943. Table 10 summerizes the results of this analysis.

These data confirm the model for tree-ring growth and climate of Arizona described by Fritts (1965,1966). The χ^2 analyses for association of ring-width classes with climatic classes for each season were based upon 58 occurrences, which is a small sample for an 81 cell contingency table. By pooling the classes for several seasons (Table 10) there are a sufficient number of occurrences to obtain realistic chi-square and coefficient of contingency results.

The degree of dependence as measured by the coefficient of contingency was highest for winter climate and second highest for spring climate, but these values are questionable because of the small sample size. Dependency of ring widths with climatic classes for several seasons was most marked when the autumn, winter, and spring classes were used. The coefficient of contingency using four seasonal classes was higher for the period including the previous summer than for the period including the summer concurrent with growth. These data indicate a primary dependence of ring-width growth on the climate during the autumn, winter, and spring periods. However, subsequent analyses utilize the relationship with four seasons (including the previous summer) as well as the more significant relationship for three seasons, so that probability statements concerning four as well as three seasons can be made.

The final objective of this study is to write a probability statement such that the probability of occurrence of one of the nine climatic classes can be expressed for a given ring-width class. Using the notation of the contingency tables, they may be expressed as

$$Pr\ (r_n | k_m) = \frac{k_m r_n}{k_m}\ , \qquad (8)$$

where $Pr\ (r_n|k_m)$ is the probability of occurrences of climatic class r_n given the ring-width class k_m has occurred and $k_m r_n$ is the number of occurrences in a given cell.

The conditional probabilities for the three seasons including autumn, winter, and spring and for the four seasons including the previous summer, autumn, winter, and spring are presented in Tables 11 and 12. Figs. 3 and 4 express these probabilities as the areas of circles. The figures show the association between the two variables as high probabilities near the diagonal elements of each table, especially near the upper left and lower right hand corners.

The probability distributions within each ring-width class (Tables 12 and 13) are plotted as cumulative distribution functions in Figs. 5 and 6. The cumulative distribution function is

$$F(x) = Pr\ (X \leqslant x)\ , \qquad (9)$$

where X is a random variable, x a real number, and F(x) the probability that X takes an integer value less than or equal to x.

The distribution function allows a more graphic comparison of the probability of occurrence of a particular climatic class or classes among the various ring-width classes. The greater the spread between cumulative distribution functions for each of the tree-ring classes, the greater the information in tree-ring widths on climatic classes. If one observes the $Pr\ (X \leqslant 3)$ in Figs. 5 and 6 and compares the values for various ring-width classes, the decrease in probability of occurrence from ring-width class 1 to ring-width class 9 is most marked. This is indicative of the greater precision of narrow rings as opposed to wide rings on conditions of climate.

Figs. 3, 4, 5, and 6 also show the predominance of the below-normal precipitation classes (1, 2, 3) associated with the lower ring-width classes (1, 2, 3) while the above-normal precipitation classes (7, 8, 9) are associated to a lesser degree with the higher ring-width classes (7, 8, 9). These results provide the basis for assigning probabilities of occurrences for the past climate in Arizona based upon corresponding occurrences of ring-width classes. In addition, they allow for differences in precision in the various ring-width classes and for correlation between temperature and precipitation classes.

Table 11. Conditional Probability for Each Climatic Class Given a Ring-Width Class, using the Joint Occurrence of Ring-Width and Climatic Classes for Autumn, Winter and Spring.

CLIMATIC CLASS	RING-WIDTH CLASS 1	2	3	4	5	6	7	8	9
1	.611	.333	.095	.200	.067	.067	.048	.000	.000
2	.111	.111	.143	.067	.100	.267	.143	.000	.033
3	.111	.111	.143	.133	.067	.000	.000	.000	.100
4	.056	.222	.238	.133	.033	.000	.095	.000	.033
5	.000	.000	.190	.133	.200	.133	.143	.000	.100
6	.000	.000	.095	.200	.133	.200	.190	.167	.133
7	.056	.111	.048	.067	.133	.000	.048	.333	.000
8	.056	.000	.048	.067	.167	.067	.048	.333	.233
9	.000	.111	.000	.000	.100	.267	.286	.167	.367

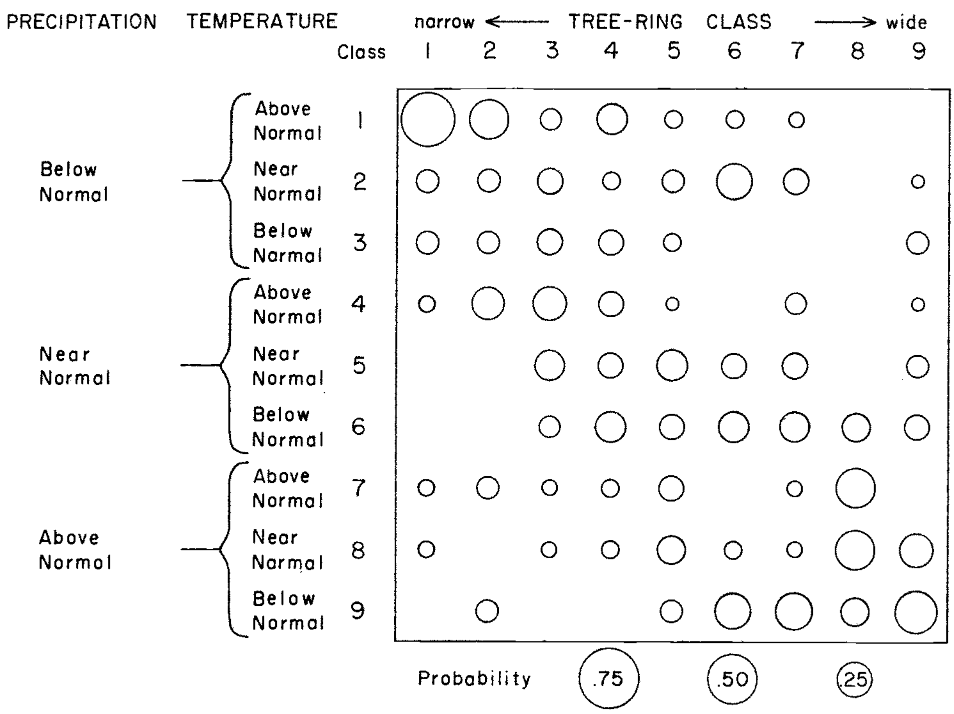

Figure 3. Conditional probability for each climatic class (expressed as area of a circle) given a ring-width class using joint occurrence of ring-width and seasonal climatic classes for autumn, winter, and spring (Table 11).

Table 12. Conditional Probability for Each Climatic Class Given a Ring-Width Class using the Joint Occurrence of Ring-Width and Seasonal Climatic Classes for Previous Summer, Autumn, Winter and Spring.

CLIMATIC CLASS	RING-WIDTH CLASS 1	2	3	4	5	6	7	8	9
1	.583	.292	.179	.200	.100	.050	.036	.125	.000
2	.083	.125	.179	.050	.100	.200	.143	.000	.025
3	.083	.083	.143	.100	.075	.050	.036	.000	.075
4	.042	.250	.179	.100	.075	.000	.107	.000	.025
5	.042	.083	.179	.150	.225	.150	.107	.000	.075
6	.042	.000	.071	.200	.100	.150	.179	.125	.150
7	.042	.083	.036	.050	.100	.000	.036	.250	.025
8	.042	.000	.036	.100	.150	.100	.107	.250	.200
9	.042	.083	.000	.050	.075	.300	.250	.250	.425

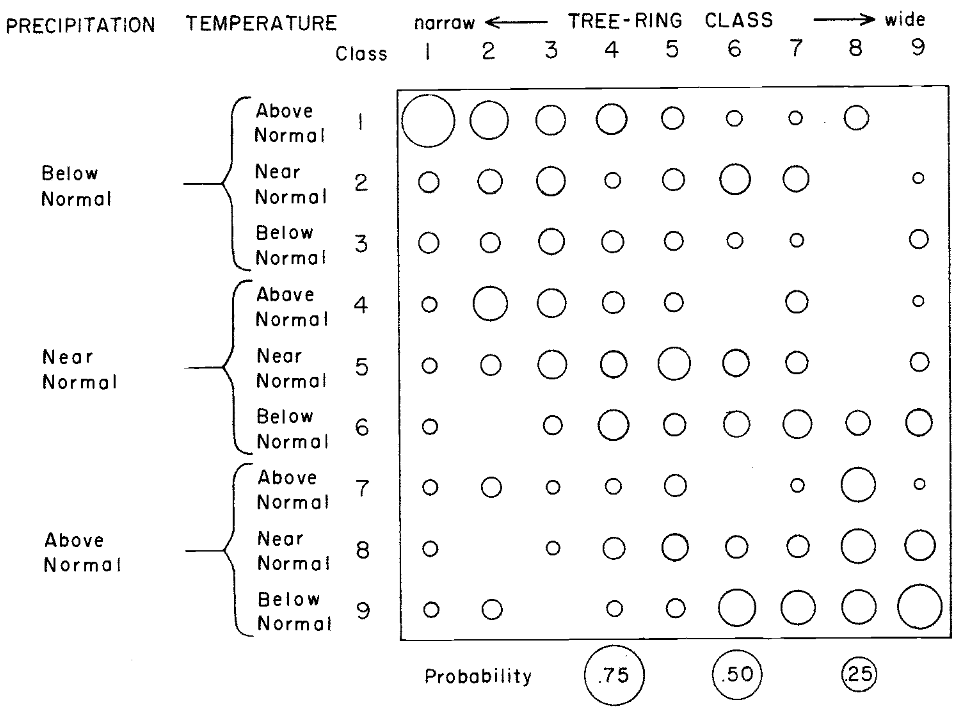

Figure 4. Conditional probability for each climatic class (expressed as area of a circle) given a ring-width class using joint occurrence of ring-width and seasonal climatic classes for previous summer, autumn, winter, and spring (Table 12).

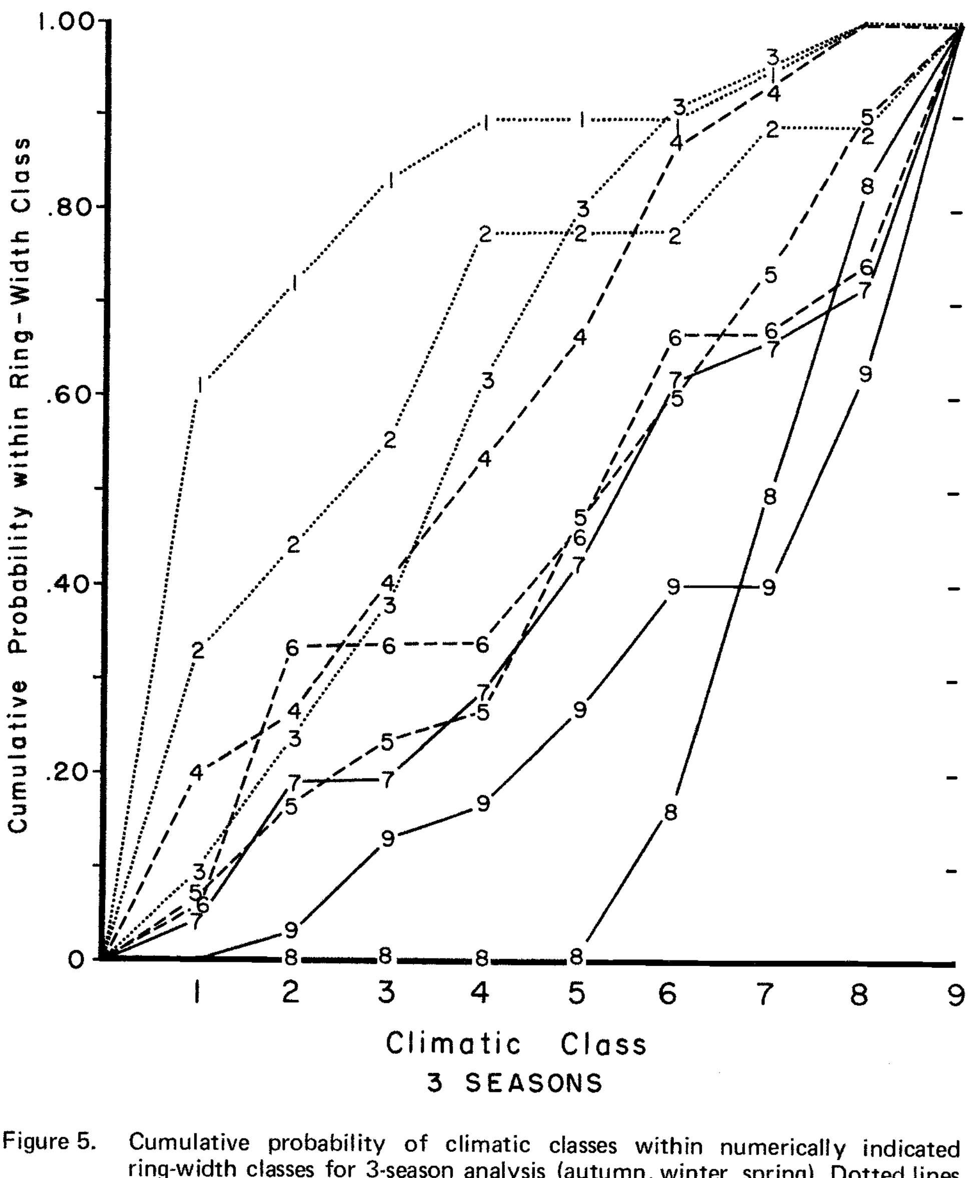

Figure 5. Cumulative probability of climatic classes within numerically indicated ring-width classes for 3-season analysis (autumn, winter, spring). Dotted lines indicate below normal ring-width classes, dashed lines near normal ring-width classes, and solid lines above normal ring-width classes.

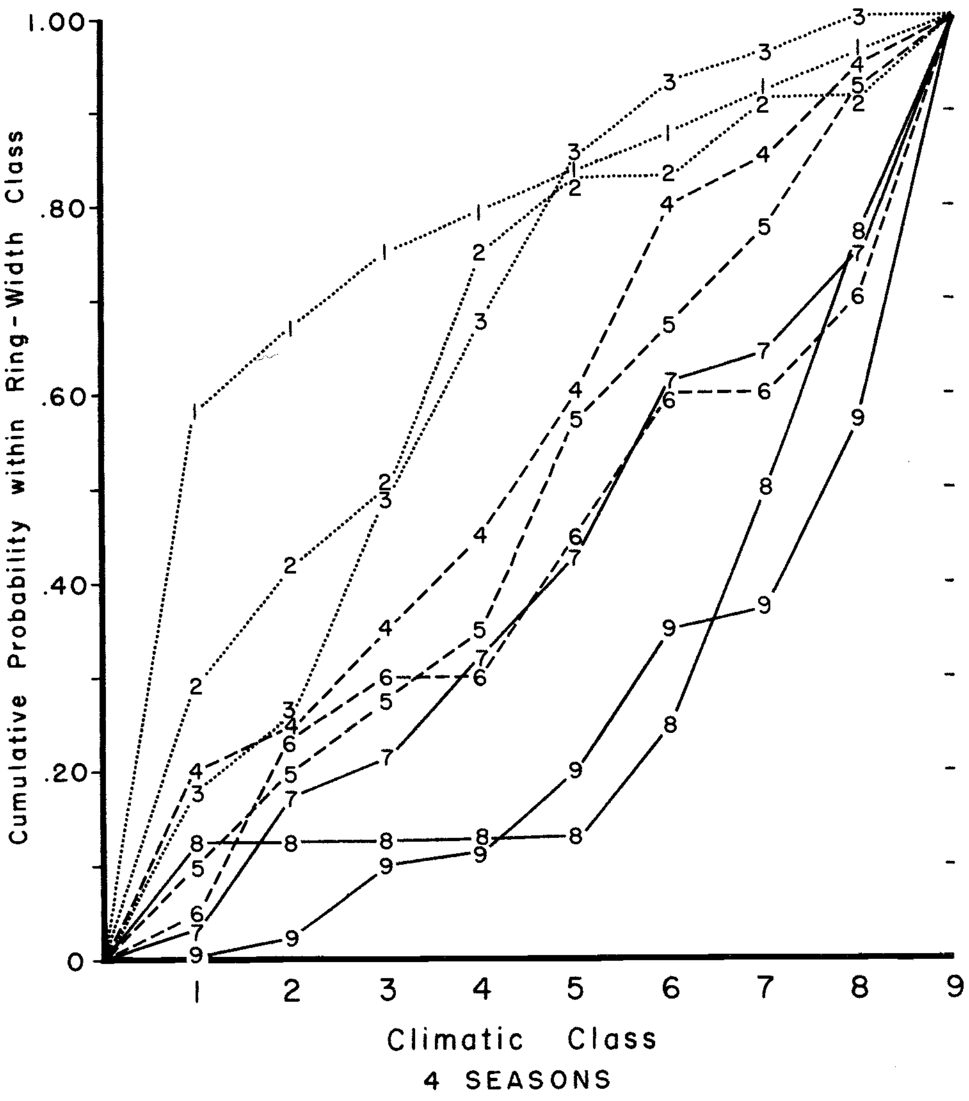

Figure 6. Cumulative probability of climatic classes within numerically indicated ring-width classes for 4-season analysis (previous summer, autumn, winter, spring). Dotted lines indicate below normal ring-width classes, dashed lines near normal ring-width classes, solid lines above normal ring-width classes.

Applications of Results

The classes and probabilities obtained for the 1899 through 1957 interval of climatic record comprise the normal or standard period. This interval is indicated in Fig. 1 by continuous lines designating ring-width class limits. The ring-width class limits extended back to 1650 (dashed lines, Fig. 1) are used to assign ring-width class. Tables 11 and 12 are then used to assign probabilities for occurrences of the 9 climatic classes relative to the standard period.

For example, the Arizona ring widths for 1872 shown in Fig. 1 fall in class 1. Entering Table 12 under ring-width class 1, the probabilities for climatic class 1, 2. . .9 are 0.583, 0.083. . .0.042. The interpretation is that any one of the 4 seasons for that year has a 0.583 probability of being in climatic class 1 (below normal precipitation and above normal temperature), a 0.083 probability of being in climatic class 2, and a 0.042 probability of being in climatic class 9. Applying the cumulative distribution function shown in Fig. 6 for ring-width class 1 at climatic class 3, $Pr(X \leq 3)$ is nearly 0.75 and at climatic class 6, $Pr(X \leq 6)$ is 0.88. These are interpreted as a 0.75 probability that the precipitation for any season of that year was below normal and a 0.88 probability that precipitation was not above normal. The probabilities for any combination of climatic classes are obtained by summing the corresponding values shown in the table. Probabilities concerning the climate of autumn, winter, or spring are found in Table 11 and Fig. 5.

The probable occurrence of climatic classes within a period of years may also be obtained. Let E_n be the event of the occurrence of one of the nine ring-width classes within the time interval being considered (t), where t is substantially less than the period from 1650 through 1957. Then

$$Pr(E_n) = \frac{N_n}{\Sigma N}, \tag{10}$$

where N_n is the number of occurrences of the ring-width class being considered and ΣN is the total number of occurrences (years) within that time increment being considered.

If the conditional probabilities from either Table 11 or Table 12 are equal to $r_n|k_n$, then the probability of occurrence of both the ring-width class and a climatic class, r_{nm}, is

$$Pr(r_{nm}) = Pr(r_n|k_m)\, Pr(E_n) \tag{11}$$

and the probability of occurrence for any climatic class (r_n) for the time interval considered is

$$Pr(r_n) = \Sigma Pr(r_{nm}), \tag{12}$$

where n is constant and m = 1, . . . ,9.

The entire period of record of the chronology was analyzed using this technique for 10-year intervals lagged by 1 year so that the 10-year interval of greatest probable occurrence of a year of below-normal or above-normal seasonal precipitation could be determined. The values for the most extreme periods are shown in Table 13.

The decade with the greatest probability of below-normal precipitation for any season is 1730 through 1739. This probability (the sum of climatic classes 1, 2, 3) is 0.59. The period 1773 through 1782 is comparable with a probability of below-normal seasonal precipitation of 0.57. Conversely, the period of most probable occurrence of a season with above-normal precipitation was 1831 through 1840, for which the sum of the probabilities for classes 7, 8, 9 is 0.55. The period of 1908 through 1917 is comparable with a probability of 0.53 for a season with above-normal precipitation. However, the chance of occurrence of a season with both above-normal precipitation and below-normal temperature is far greater (0.34) during 1908 through 1917 than for any other 10-year period during the interval covered in this analysis.

Intervals of 20 years were analyzed in a similar manner. The extreme 20-year intervals with greatest probability of below-normal precipitation and greatest probability of above-normal precipitation for any seasonal occurrence are shown in Table 14. Although the greatest probability of below-normal precipitation during any season is 0.49 for the period 1722-1741, the probabilities for below-normal precipitation during the other four extreme periods are almost as high, as they range between 0.47 and 0.48.

The interval of 1870-1889 is of particular interest as it was the period when down-cutting was initiated in many Arizona streams. Occurrences of ring widths indicate a probability of 0.48 for below-normal precipitation and

Table 13. Extreme 10-year Intervals Selected for Greatest Probability of Occurrence for any Season of Below Normal and Above Normal Precipitation Based upon the Ring-Width Chronology (Fig. 1) and Conditional Probabilities in Table 12.

INTERVALS WITH BELOW NORMAL PRECIPITATION	PROBABILITY OF OCCURRENCE OF CLIMATIC CLASS								
	1	2	3	4	5	6	7	8	9
1664-1673	.261	.112	.094	.126	.143	.073	.067	.070	.052
1730-1739	.365	.117	.103	.130	.102	.058	.049	.037	.038
1748-1757	.289	.108	.093	.116	.117	.062	.059	.070	.087
1773-1782	.390	.093	.087	.137	.080	.057	.060	.037	.060
1818-1827	.367	.102	.095	.081	.116	.075	.053	.068	.041
1872-1881	.321	.093	.093	.118	.114	.090	.058	.061	.052
1880-1889	.281	.102	.099	.131	.128	.093	.057	.060	.047
1895-1904	.283	.133	.099	.147	.117	.065	.049	.039	.068
1947-1956	.288	.130	.086	.116	.113	.070	.050	.055	.092
INTERVALS WITH ABOVE NORMAL PRECIPITATION									
1742-1751	.127	.077	.061	.073	.101	.117	.072	.141	.230
1831-1840	.081	.065	.033	.050	.085	.139	.133	.182	.232
1908-1917	.032	.069	.067	.046	.096	.161	.027	.161	.340
1916-1925	.068	.074	.072	.066	.109	.138	.039	.146	.288

Table 14. Extreme 20-year Intervals Selected for Greatest Probability of Occurrence for any Season of Below Normal and Above Normal Precipitation based upon the Ring-Width Chronology (Fig. 2) and Conditional Probabilities in Table 13.

INTERVALS WITH BELOW NORMAL PRECIPITATION	PROBABILITY OF OCCURRENCE OF CLIMATIC CLASS								
	1	2	3	4	5	6	7	8	9
1722-1741	.269	.128	.095	.138	.115	.064	.051	.050	.091
1729-1748	.274	.107	.086	.127	.093	.074	.059	.066	.114
1773-1792	.314	.087	.082	.130	.100	.077	.075	.063	.072
1804-1823	.290	.095	.088	.093	.111	.094	.060	.080	.087
1870-1889	.282	.096	.097	.128	.127	.099	.058	.064	.050
INTERVALS WITH ABOVE NORMAL PRECIPITATION									
1907-1926	.059	.079	.064	.054	.103	.148	.043	.151	.299

a probability of 0.17 for above-normal precipitation during any season of the period. Regrouping the probabilities in Table 14 for this period, it can also be shown that there is a probability of 0.47 for above-normal temperature (classes 1, 4, 7) and a probability of 0.25 for below-normal temperature (classes 3, 6, 9) during any season of the period. The tree-ring data indicate that the state-wide climate of this period was most likely unusually dry and warm. However, the correlation analyses in Fig. 2 show that marked differences occurred among the chronologies within the state (especially for 2 and 3, which were on the central plateaus). These differences appear to be due to steep climatic gradients which occurred through the state. However, changes in site factors perhaps attributable to man's activities may have occurred also during this period. Additional tree-ring data should be examined for the presence of low-frequency variations similar to chronologies used here to check the presence of these apparent climatic gradients during this period of time.

Table 14 and Fig. 1 also show that the interval 1907-1926 was substantially wetter and cooler than any other 20-year period since 1650. This undoubtedly biased the standard climatic interval so that it departs from a longer-term climatic mean. This accounts for the above-average ring-widths indices that were observed for the standard interval from 1900-1957.

Conclusions

The analyses presented in this study outline several objective methods leading to quantitative evaluation of past climate by employing the relative widths of annual rings in trees. Climatic data for the state, including both precipitation and temperature for each of four seasons are placed into three equally probable classes. Several ring-width chronologies from the state are standardized and normalized before combining them into a single state-wide series. Correlations of high-frequency and low-frequency variance among the four chronologies are employed to evaluate intervals of heterogeneity among ring-width records in the state for 20-year intervals of time.

The joint occurrences of the three precipitation and three temperature classes produce a multinomial population with 9 categories which are combined with nine equally probable ring-width classes. The joint occurrences of the ring-width and seasonal climatic classes for the period 1900-1957 are used to establish the probabilities for each climatic class given a ring-width class. A χ^2 test for independence and coefficients of contingency confirm that significant relationships exist. The climatic probabilities associated with ring-width classes for 1650 through 1899 are applied to write probability statements regarding the occurrences of seasonal climàtic classes prior to the 1900-1957 record.

The present approach employed only the average seasonal climate for the state of Arizona and several available ring-width chronologies from the state. Similar investigations have been made of single climatic stations near tree stands yielding especially good and long records in their rings. The approach shows considerable merit in that it expresses the relationship between ring-widths and climate in a form such that tree-ring chronologies may be utilized to make probability statements about climatic and hydrologic conditions in the past.

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