PRINCIPAL COMPONENTS ANALYSIS OF TREE-RING SITES

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ABSTRACT

A principal components model for analyzing tree-ring data is presented which allows one to examine site heterogeneity and to compose chronologies of a new kind in a conceptually unified and computationally efficient manner. Using this model, one can develop chronologies that correlate better with local climate data than the standard chronology for a site and which can be tested for time stability within the framework of the model. The numerical procedures are described and applied to a specific tree site to illustrate their use and their usefulness.

The tree-ring width data tested is from a white spruce *(Picea glauca* [Moench.] Voss) site in the forest-tundra ecotone of Yukon Territory. The results from the study of this and other sites indicate that more climatic information can be extracted using these techniques.

Es wird ein Hauptkomponentenmodell zur Jahrringanalyse vorgestellt, mit dem die Standortsheterogenität geprüft werden kann und das die Bildung neuartiger Chronologien in konzeptionell vereinheitlichter und rechnerisch effizienter Weise ermöglicht. Mit Hilfe dieses Models können Chronologien entwickelt werden, die besser mit dem Lokalklima korrelieren als die Standardchronologie eines Standortes und die auf ihre zeitliche Homogenität him überprüfbar sind. Die numerischen Verfahren werden beschrieben und auf einen Baumstandort angewandt, um ihren Gebrauch und ihren Nutzen zu illustrieren. Die Jahrringdaten stammen von einem Weißfichten-Standort (Picea glauca [Moench.] Voss) in der Wald-Tundra des Yukon Gebietes. Die Ergebnisse für diese und andere Standorte zeigen, daß mit dem beschriebenen Verfahren mehr Klima-Information gewonnen werden kann.

Un modèle d'analyse en composante principale destiné à l'étude des données dendrochronologiques est présenté. Celui-ci permet d'examiner l'hétérogénéité du site et de composer des chronologies d'un nouveau type d'une mainére homogène sur le plan conceptuel et efficace sur celui des calculs. En utilisant ce modèle, on peut développer des chronologies qui se corrèlent mieux avec les données climatiques locales que les chronologies obtenues de la façon usuelle et qui peuvent être testées pour leur homogénéité temporelle dans le cadre du modèle. Les procédures numériques sont décrites et appliquées à un site particulier pour illustrer leur usage et leur utilité.

Les données dendrochronologiques testées proviennent d'une station de "white spruce" (Picea glauca [Moench.] Voss) dans le milieu de la forêt-tundra dans le territoire du Yukon. Les résultats de l'étude de ce site ainsi que d'autres indiquent que plus d'informations climatiques peuvent être retirées en utilisant ces techniques.

INTRODUCTION

Trees in closed-canopy forests and northern tree line environments experience more non-climatic variability in growing conditions both spatially and temporally than do trees in more open situations. As a result the yearly variation of the annual growth index for trees in closed-canopy forests has a proportionately weaker climatic component, and a sample of many radial cores taken from trees at such a site often have proportionately less annual growth index variation in common (Fritts 1976). If the spatial variability of the growing conditions is very pronounced, averaging over a large sample of cores will not necessarily yield a higher proportion of climatic variance than averaging over a small sample. The term site heterogeneity is used here to denote this condition, which is typical of sites in the eastern U.S. and along the northern tree line in Canada and Alaska. Heterogeneity when very pronounced causes a site to be non-averaging for the usual number of cores. The traditional solution is to increase the number of samples. We are suggesting in this paper that under such conditions it might be useful to try to sort out the heterogeneities and use smaller subsets of cores.

In developing a time series of composite annual growth indices, or a chronology (Fritts 1976), for a site from the individual cores in the site sample, the numerical procedures used affect the proportion of climatic variance in the composite chronology. In this paper we are concerned primarily with assessing site heterogeneity and looking for homogeneous subsets of cores within the complete site sample. The procedure of filtering the time series of raw ring widths of each core to remove non-climatic variance and form homoscedastic indices, called standardization (Fritts 1976), is not discussed except as it relates to Principal Components Analysis (PCA). In addition, PCA performed on a subset yields a Principal Components Chronology (PCC) which is composed differently from a standard chronology (SC) on the same subset. However we have not composed SC's on all the subsets. In this paper we are not primarily concerned with separating the effects of core selection and method of chronology composition on the resulting chronologies. Our application of PCA to tree sites performs both of these procedures within the context of one model. In the example, the SC based on all the usable cores at the site is used as a fiducial series. For this example we have chosen tree-ring data from a site in the forest-tundra ecotone of Yukon Territory.

To see how much climatic variance is in the PCC's and the SC in the example, we perform two suites of regressions. We have summarized the results of regressing each of the chronologies on the meteorological data from the nearest weather station (response function analysis [Fritts 1976]) to determine which climate variables most influence tree growth. We have also summarized the results of regressing specific climate variables on each of the chronologies lagged forward on itself (transfer function analysis [Fritts 1976]) to determine how the growth response is delayed with respect to climate. We have not attempted to reconstruct any climate variables from the chronologies for times when no meteorological data is available for reasons that are iterated later.

TREE SITES AND PCA

The application of PCA (Cooley and Lohnes 1971) to a tree site presents no formal difficulties. The data matrix consists of n annual tree-ring indices for each of N years in a contiguous sequence (the common period). It is assumed that climate is a common growth influence amongst all the trees of the site, and it is well known that index series of cores from the same tree and from neighboring trees usually correlate significantly and positively over time. Thus we seek modes of covariation amongst the index series. Linear PCA yields n new variables which are linear combinations of the n original variables. The latter are inter-correlated and each has been scaled to zero mean and unit variance in our procedure. The n new variables (amplitudes) are not intercorrelated and the total variance $(=n)$ has been redistributed as unevenly as possible. The coefficients (loadings) of each of these combinations are the elements of the corresponding principal components, which are orthogonal unit vectors. These manipulations are formally equivalent to determining the eigenstructure of the correlation matrix. We shall use the PCA terms "amplitude" and "loading" but substitute "eigenvector" for "principal component" and use "eigenvalue" to denote the variance of an amplitude over time. The first eigenvector can be considered as defining a direction in the n-dimensional space of the model along which the annual indices have maximum common variance (first eigenvalue). The higher order eigenvectors define other mutually orthogonal directions along which successively less important (smaller eigenvalues) modes of covariation amongst the indices occur. This geometrical interpretation will be used in the example.

Most of the climatic variance in the site sample is assumed to be in the first one or possibly two but no more than three amplitudes because the orthogonality constraints on the higher order eigenvectors preclude their being reliably determined by climate, and because the first few amplitudes usually contain all the variance that reasonably can be assigned to common climatic influence. It seems unlikely that all the climatic information recorded by a site is contained in one parameter alone (mean value ring-width index, or the standard chronology) and that differential growth amongst the cores is completely unaffected by climatic influences. By using a PCA model for a site, additional climate information can be retrieved from parameters other than mean index, e.g. the first amplitude, which is usually an unequally weighted average index.

PCA provides a convenient way of analyzing the site heterogeneity, by examining the loadings on the first eigenvector, and composing a chronology, by computing the first amplitude. One could of course analyze the site heterogeneity and compose chronologies by unrelated methods. Cluster analysis is one possibility. We assume it would yield comparable information if it were based on the correlation matrix. Discriminant analysis we've rejected because it requires a prior classification of the cores, and we disavow such knowledge. Also, while we are interested in differential growth, we do not want to emphasize it with respect to common growth. Analysis of variance and examination of the correlation matrix of the index series (Fritts 1976) can be considered in addition to the PCA. The latter has the advantage of reducing the number of parameters that need to be considered to a small multiple of n rather than n^2. The first eigenvector plus eigenvalue for instance are an $n + 1$ parameter partial summary of the correlation matrix. Indeed, the n eigenvalues and eigenvectors contain exactly the same information as the correlation matrix, but the most useful information is contained in the first few, whereas it is scattered throughout the correlation matrix.

PRINCIPAL COMPONENT CHRONOLOGIES

The preliminary procedures in developing a PCC are the same as those in developing a SC, through the rejection of obviously useless cores, e.g. those with distorted rings and/or reaction wood, those that cannot be crossdated, etc. (Stokes and Smiley 1968). Also, in both cases one must decide how to standardize each core series of raw ring widths. The method of standardization should be chosen independently of the method of chronology construction and quite separately from the problem of site heterogeneity. Choice of standardization method will naturally have a strong effect on the PCA via the correlation matrix. Ideally the data matrix should consist of N samples from an n-variate normal distribution. In this case the correlation matrix is a set of sufficient statistics for describing the stochastic interdependencies of the index series. Such is not generally the case. If in addition the N samples are autocorrelated over time, the precision of the cross-correlation estimates is reduced. Except for single-index series, we are not aware of any studies that would justify the multi-normality assumption, nor of any useful results regarding the robustness of PCA under departures from multi-normality. It is well known that the index series of a core can be highly autocorrelated. In fact, knowing that climatic variables exhibit long-period variations, one usually chooses a mode of standardization that preserves long-period variances in the indices. Even under these conditions, however, PCA usually yields useful results and we have not been standardizing our core series specifically for subsequent use in PCA.

At this point the two methods of chronology development diverge and one must choose a time interval common to all the cores which will be used for the PCA. One must compromise between a long chronology with a few cores and a shorter chronology with many cores. It has been our experience that PCC's are robust with respect to the number of cores in that a surprisingly small part of the complete usable inventory, say only 20%, can yield a chronology very similar to one based on all the cores or to the standard chronology. This necessity of having a common period for the PCA has emphasized the need for examining the time homogeneity of any kind of chronology, depending as they do on a dwindling subset of the full inventory of cores as one attempts to look farther into the past. If mean index is the only parameter examined and site inhomogeneity goes undetected, then systematic bias can be introduced into the standard chronology, especially in the older part. For a PCC, the same subset of cores is used over the whole length, thus avoiding one possible contribution to what the paleoclimatologists have called a "no-analogue" condition (Sachs 1977).

Having decided on standardization and a common period, we scale each core index series to a mean of zero and variance of 1. The resulting eigenanalysis is based on the cross-correlation matrix of the core series, not the covariance matrix, and thus is invariant with respect to a linear scale change in the original variables. This differs from the usual method of chronology development in that a standard chronology is dominated by the cores with larger variance, while a PCC is dominated by the cores that load most heavily. Gnanedesikan and Wilk (1977) report that there is little agreement on a simple general criterion for deciding whether to use the covariance matric or the correlation matrix for PCA. Our choice is largely experimental. We discuss it further in the last section of the paper.

These scaled variables are then submitted to PCA, and eigenvalues, eigenvectors and amplitudes are computed. We routinely check for significant structure both by examining the correlation matrix and the eigenvalues and by applying Bartlett's sphericity test (Cooley and Lohnes 1971). This test has never failed to reject the null hypothesis (no structure) at very high levels of confidence (usually $> .999$).

The next step is to examine the loadings on the first eigenvector. This can be done either by making a histogram or cumulative plot of these variables or simply by ordering them. Sometimes these histograms are quite smooth, assuming a log-normal or gamma shape. In this case we use the first amplitudes as the PCC and are finished. If the histogram is very lumpy (possibly multimodal), we divide the cores into the most obvious subsets. Results have indicated that this division should be done without regard for the tree from which a core was taken. The division is done by core, not by tree. Although in some cases cores from the same tree load very nearly equally, in other cases cores from different trees load much more nearly equally than do cores from the same tree.

Having done the first division of all the cores into subsets by the loading of each core on the first eigenvector, we repeat the PCA on each subset as a second step and examine the eigenvalues and the loadings on the first eigenvectors for each subset and if possible subdivide each subset, forming subsets of subsets. This procedure continues until the subsets cease to exhibit (possible) heterogeneity.

The end results of this procedure are twofold. First we have a heterogeneity analysis of the cores from the trees on the site. In conjunction with adequate field notes and other site information, it affords a rather complete assessment of possible site heterogeneity. Failure to subdivide at all is accepted as confirmation of site homogeneity. A first subdivision into a small number of subsets (2 or 3) is sometimes quite easy to explain in terms of factors such as changes in slope aspect and altitude. However, site information is usually not sufficient to explain subdivision into many small subsets.

Second, for a heterogeneous site, we have more than one chronology, each consisting of the first amplitude for subsets of different sizes (including the one consisting of all the cores) and characteristics. We feel that it is acceptable to select the ones that correlate best with the local climate data, since their multiplicity derives quite objectively from site heterogeneity which is usually present but goes undetected in less refined core selection procedures. In general, small subsets of cores tend to exhibit more variability of performance in regression with climate, and some of our most explainable chronologies are based on five and fewer cores. It is a moot point to say that these results are spurious *a priori* without reliable knowledge of the sampling characteristics of the population from which the samples are taken. The smaller subsets should be judged on the same criteria as the larger. On the other hand, one must consider that the multiplicity of the chronologies itself contributes a random component to their variable performance in regressions with climate.

Usually we use only the first amplitude as a PCC. One reason is its interpretability. If the loadings are about the same for all the cores as they will be in a homogeneous subset, the first amplitude is roughly a mean index. This is often the case and the first amplitude reflects common variation amongst the cores. Another more cogent reason is stability. It is necessary that a chronology represent a time-stable response of a tree site or subsite to climate. Thus at least the gross eigenstructure of a site should be time-stable. Since it appears unusual along the northern tree line and in the northeastern U.S. for climate alone to explain even half the variance of a chronology, it is unlikely that higher order amplitudes, even if they are time-stable, would be reflecting climate. Our experience indicates that for large number of cores (>15) only the first amplitude is time-stable. This stability can be assessed by comparing a first amplitude or eigenvector determined for part of the common period with the same object determined for the whole. On the other hand, if only a few cores are used, higher order eigenvectors and amplitudes can be time-stable also. In these cases all the stable amplitudes can be tried as predictors and searched for climatic information.

ILLUSTRATIVE EXAMPLE

Of the 10 sites for which we have developed PCC's, the one discussed here has been most thoroughly analyzed and illustrates well some of the more interesting results from PCA.

SITE DESCRIPTION: The site is along the Dempster Highway in the Yukon Territory of Canada at approximately 64°N and 137°W. This forest-tundra ecotone site is in the region of discontinuous permafrost (Oswald and Senyk 1977). The site (TT-HH) is a composite of trees from two subareas, one along the northeast-facing slope (Heartrot Hill = HH), the other from the contiguous east-facing slope (Twisted

Tree = TT). All the samples are from one species, white spruce *(Picea glauca* [Moench] Voss).

METEOROLOGICAL DATA: The Dawson weather station is about 125 km south of the site across some significant intervening mountains. Therefore the climatic regime should not be considered as being entirely homogeneous between the Dawson meteorological station and the tree-ring site. The mean annual precipitation at the site could be about 25% greater (Oswald and Senyk 1977) and the altitude difference (about 600 m) could result in average temperatures about 3.5°C lower at the site. The data used consists of average monthly temperatures and total monthly precipitation for the years 1902-1974 and monthly degrees days >10°C (>50°F) for the years 1921-1965.

CHRONOLOGIES:

1. Chronology Designations. These consist of two parts. The prefix is either "SC", which denotes a standard chronology, or "P", which denotes a Principal Components Chronology. The numeric part specifies the subset of cores from which the chronology is composed. This subset designation scheme is explained below with references to Table 1. A Roman suffix denotes a particular amplitude and its omission implies the first amplitude.
2. Standard Chronology (SC). The final standard chronology was based on 27 cores from 13 trees. Twenty-one of these were standardized by a horizontal line through the mean of the series and the other six using a modified negative-exponential (Fritts et al. 1969) to remove juvenile growth effects.
3. Principal Component Chronologies (PCC's). The interval 1750-1974 (225 years) was chosen for the common period. Five cores beginning after 1750 were deleted. Two of those deleted had been standardized using a negative exponential. Since 18 of the remaining 22 cores had been fitted with a horizontal line through the mean, we decided to use no standardization on any of the cores to preserve as much low-frequency variance as possible and to see how the four cores with possible juvenile effects would go through the analysis.

For all 22 cores over the common period, the first amplitude has 43% of the variance. Bartlett's sphericity test rejects the null hypothesis with a confidence level >.999 (χ^2 = 5444 with 231 degrees of freedom). Table 1 shows the analysis through step 2. The subset designation scheme is positional by step and should be apparent from examination of Table 1. Each subset at step 1 is designated by one digit; at step 2, by two digits, etc. For the common period, the loadings at step 1, for example, suggested two possible subdivisions of all the cores. These appear to the left (21 and 22) and to the right (23 and 24) of the step 1 data in Table 1. The number in parentheses beside the subset designation is the proportion of variance in the first amplitude, e.g. 23 (.55). The cores are arranged by loading. The asterisks denote the cores with possible juvenile effects. The sequence number refers to the order in which the samples were taken. Core designations beginning with 10 are from the site TT. Those beginning with 20 are from HH.

A casual inspection of the loadings confirms the correctness of the decision to combine the two original sites into one. Whatever differences in microclimatic effects may exist at the sites, they are not resolvable at this stage of the analysis. Within site TT, however, except for two cores (sequence numbers 5 and 6), there is a definite trend at step 1 between sequence number and loading or rank. The field notes indicate that the sampling proceeded along the hill in a generally southerly direction. Also, there is

Table 1. Principal component analysis (PCA) for common period (1750-1974)

Step 2 PCA on 21 and 22					Step 1 PCA on subset 2[1] (.43)					Step 2 PCA on 23 and 24					
	Loading Rank	Seq.#	Core #	Loading		Loading Rank	Seq.#	Core #	Loading		Loading Rank	Seq.#	Core #	Loading	
	1	19	2073	−.317		1	19	2073	−.297		1	19	2073	−.301	
	2	7	1071	−.315		2	10	1091	−.282		2	10	1091	−.287	
211(.86)	3	8	1072	−.314		3	8	1072	−.275		3	8	1072	−.283	
	4	10	1091	−.309		4	7	1071	−.274		4	7	1071	−.282	
	5	9	1073	−.303		5	15	2061	−.266		5	15	2061	−.271	
	6	15	2061	−.288	21(.67)	6	20	2081	−.264		6	9	1073	−.267	231(.21)
212(.72)	7	17	2071	−.288		7	9	1073	−.258		7	20	2081	−.266	
	8	21	2082	−.284		8	17	2071	−.256		8	17	2071	−.261	
	9	20	2081	−.277		9	21	2082	−.247	23(.55)	9	21	2082	−.253	
	10	4	1022	−.255		10	22	2083	−.247		10	22	2083	−.246	
213(.70)	11	11	1092	−.253		11	11	1092	−.232		11	4	1022	−.233	
	12	22	2083	−.250		12	4	1022	−.229		12	11	1092	−.233	
	1	14	2031*	−.435		13	16	2062	−.207		13	16	2062	−.206	−232
	2	6	1061*	−.401		14	3	1021*	−.175		14	3	1021*	−.170	
	3	12	2011	−.383		15	13	2023	−.167		15	13	2023	−.168	233(.46)
	4	18	2072*	−.352		16	1	1011	−.162		16	1	1011	−.160	
	5	3	1021*	−.347	22(.36)	17	18	2072*	−.161		17	18	2072*	−.159	
	6	5	1031	−.331		18	14	2031*	−.111		1	12	2011	−.527	
	7	1	1011	−.288		19	2	1012	−.090		2	5	1031	−.518	241(.68)
	8	16	2062	−.213		20	6	1061*	−.062	24(.55)	3	6	1061*	−.504	
	9	13	2023	−.133		21	12	2011	−.010		4	14	2031*	−.446	
	10	2	1012	−.047		22	5	1031	+.056		5	2	1012	−.039	−242

[1]Subset 1 is all 27 cores used in the standard chronology. Number in parentheses after the subset designation is the proportion of variance accounted for by the first amplitude. Cores numbered 1000 are from site TT and 2000 are from HH. The third digit is the tree number and the last digit is the core. The sequence number refers to the order in which the cores were taken. The starred cores exhibited possible junvenile effects.

grouping within the site HH. Field notes indicate sequence numbers 15 through 22 are cores from trees close to each other, and with one exception (# 2072), they have the highest loadings for the HH cores. Field notes do not help in explaining any more details of the heterogeneity indicated by the PCA.

The loadings on the first eigenvector do not show a strong or definite heterogeneity and thus one possible chronology is the first amplitude at step 1. If one proceeds to subdivide, this can be done in several ways. We chose two ways of subdividing (each resulting in two subsets) as suggested by the loadings. Rounding to the nearest one-hundredth, the loadings for the cores ranked 1 through 12 and 14 through 17 form separate blocks on the histogram. The first of these blocks contain none of the starred (asterisk) cores and should be considered for separate analysis for this reason. The cores ranked 13 through 22 contain all four of the starred cores and form the complement to the block of twelve. Thus we consider the subsets 21 and 22 as one possible subdivision. On the other hand, the cores ranked 18 through 22 load very weakly and for this reason the subdivision 23 and 24 might also be considered. This subdivision gains support from the observation that at step 2 the ranking of the cores contained in 23 is nearly the same as at step 1, implying that the subset 24 is roughly independent of 23.

Submitting these four step 2 subsets to PCA yields a very definite triple regrouping within 21 and no apparent regrouping within 22. 23 yields the two subsets 231, which is 21, and 233, which appeared at step 1, and one core, #2062, as an isolated case. The core #1012 is also isolated at step 3. These two cores were isolated at step 1 but not as obviously.

At this point the starred cores become a problem because the PCA results do not allow us to simply discard them as pathological. Within the subset 22, for instance, they are not isolated from the rest of the cores by their loadings. Of course the juvenile effects would not involve the whole length of the cores. On the other hand, there may be no juvenile effects. Standardizing these cores using modified negative exponentials and rerunning PCA did not change their loadings sufficiently to change the grouping. We considered it advisable to use the suggested subsets containing these cores in some of the subsequent stages of the analysis.

To investigate the time stability of the site structure, we performed PCA on three of the subsets suggested by the loadings in Table 1 over the calibration period for which Dawson meteorological data is available (1902-1973). These particular subsets 2, 21, 213 were chosen because they consisted of a large, a medium, and a small number of cores respectively, and they all correlated well with the meteorological data. This look at the problem of time stability, while obviously incomplete, serves our purposes in this paper, namely to determine how the site structure during the calibration period differs from the long-term structure for subsets which could be used for reconstruction.

The PCA for the calibration period is shown in Table 2. At step 1 the sphericity test again rejects the null hypothesis at a confidence level $>.999$ ($\chi^2 = 1438$ at 231 degrees of freedom). A casual inspection reveals some similarities with Table 1 and some differences. For instance, at step 1 the gross homogeneity of the two sites TT and HH is reconfirmed. Cores #2073 and #1031 loaded first and last respectively and the starred cores loaded below the median in both cases. Cores #1091, #1072, #1071, and # 2061 maintain their relative order and all the signs of the loadings are the same. In a very general way, the ranks of the loadings agree, especially amongst the more strongly loading cores. (The Spearman rank correlation coefficient for all the cores is

Table 2. Principal components analysis (PCA) for calibration period (1902-1973)

Step 1 PCA on 2 (.41)				Step 2 PCA on 21 (.54)				Step 3 PCA for Calb. Period −213(.58)			
Loading Rank	Seq. #	Core #	Loading	Loading Rank	Seq. #	Core #	Loading	Loading Rank	Seq. #	Core #	Loading
1	19	2073	−.309	1	19	2073	−.355	1	11	1092	−.618
2	22	2083	−.295	2	22	2083	−.346	2	22	2083	−.558
3	20	2081	−.286	3	20	2081	−.324	3	4	1022	−.554
4	10	1091	−.269	4	7	1071	−.311	and for common period −213 (.70)			
5	8	1072	−.269	5	8	1072	−.302	1	11	1092	−.604
6	7	1071	−.253	6	15	2061	−.302	2	4	1022	−.567
7	15	2061	−.249	7	9	1073	−.283	3	22	2083	−.561
8	1	1011	−.239	8	10	1091	−.280				
9	13	2023	−.233	9	17	2071	−.249				
10	9	1073	−.213	10	21	2082	−.246				
11	16	2062	−.197	11	11	1092	−.235				
12	14	2031*	−.195	12	4	1022	−.181				
13	12	2011	−.189								
14	6	1061*	−.172								
15	21	2082	−.168								
16	3	1021*	−.168								
17	18	2072*	−.163								
18	11	1092	−.158								
19	17	2071	−.151								
20	4	1022	−.135								
21	2	1012	−.043								
22	5	1031	+.123								

0.68.) However the correlation of loadings and rank with sequence numbers disappears. Also, if one were to subdivide the site by calibration period loadings, the results would be somewhat different. Generally, no subdivision is suggested at step 1 or 2. The loadings are more homogeneous. Note also that all the first eigenvalues account for a smaller proportion of the total variance. In one sense this can be attributed to the shortness of the analysis period (72 years). The eigenstructure of the site is not resolvable at shorter periods because of "noise" in the system. On the other hand, it is known that climate at this site and other northern tree line sites has been changing slowly over the last several centuries. The warming trend beginning around 1850 and ending around 1940 is one aspect of this change. It is apparent in most of our northern chronologies. The calibration period is warmer at least, and thus we might expect the site structure to be different in general for this period. It is one of the purposes of this analysis to determine whether within the site there may be subsets of cores that do have time-stable structure.

More formally, we have considered two indices of stability for comparing the two periods. The first is the angle between eigenvectors of the same order which has an obvious geometric meaning. If amplitudes were formed from sets of uncorrelated variables (correlation matrix = identify matrix), their crosscorrelation would be equal to the cosine of this angle. The second is the crosscorrelation between corresponding amplitudes formed from the sets of core series. This depends not only on the angle between the corresponding eigenvectors but also on the inter-correlations of the cores over the period being considered.

Letting primes denote chronologies developed using data from the calibration period only (1902-1974), we find at step 1 (P2 vs. P2') that the angle between the first eigenvectors is about 19°. At step 2 (P21 vs. P21') it is 9° and at step 3 (P213I vs. P213I') it is 0° (to three places in the loadings). At steps 1 and 2 the second eigenvectors differ from orthogonality by about 10° while the third eigenvectors are even more nearly orthogonal. At step 3, however, the angular changes are only 4° and 5° for the second and third eigenvectors respectively. It is clear that while at steps 1 and 2 the second and third (and higher) eigenvectors are obviously not time stable, at step 3 they are nearly so, in the sense used in this paper.

The cross-correlations of the chronologies appear in Table 3. For the unprimed chronologies these cross-correlations have been computed for both the calibration period (above the main diagonal) and the 225-year common period. The boxed cells contain the second of the stability indices. Their high values result from both the alignment of the eigenvectors (stable site structure) and the positive intercorrelations of the cores' responses. Clearly P2, for instance, will yield nearly the same calibration with the climate data as P2'. On the other hand, the second and higher order amplitudes for the subsets 2 and 21 are not time stable (according to the first stability indices) and the information they contain should not be used in reconstructions; while all the amplitudes for the subset 213 could be used.

The standard chronology (SC1) and the first amplitude chronologies P2, P21 and P213I are shown in Figure 1. Also shown are the second and third amplitudes for the subset 213 (P213II and P213III). They have all been scaled to zero mean and unit variance to facilitate comparison. The first three of these look very similar at all frequencies. P213I differs in several interesting ways. In the interval (1750-1800) P213I does not show the same small change in level between the first 20 years and the last 30 years, but shows larger relative variance with the same signs for the first differences. During the intervals (1935-1960) and possibly (1810-1830) P213I shows a dip

Table 3. Cross-correlations of several chronologies

	SC1	P2	P21	P213I	P213II	P213III	P2'	P21'	P213I'	P213II'	P213III'
SC1	1.0000	.9502	.8502	.6908	.3822	−.3027	.9586	.8745	.6813	.3523	−.3061
P2	.9762	1.0000	.9633	.7996	.4139	−.2568	$\boxed{.9916}$	.9745	.7917	.3886	−.2586
P21	.9457	.9825	1.0000	.8676	.3470	−:1431	.9409	$\boxed{.9941}$	.8636	.3270	−.1370
P213I	.8205	.8624	.8574	1.0000	.0249	−.0673	.7774	.8526	$\boxed{.9995}$	.0097	−.0310
P213II	.0816	.0772	.0021	.0004	1.000	.0122	.4282	.3743	.0228	$\boxed{.9888}$	−.0649
P213III	−.1224	−.0803	−.0623	.0000	−.0107	1.0000	−.2644	−.1697	−.0359	.0663	$\boxed{.9963}$
P2'							1.0000	.9615	.7692	.3391	−.2681
P21'								1.0000	.8478	.3507	.1663
P213I'									1.0000	.0093	.0005
P213II'										1.0000	−.0105
P213III'											1.0000

The prefix "SC" denotes a standard chronology, the prefix "P" a principal components chronology. The numeric part of the chronology designation specifies the subset of cores used (see text). Primes indicate that the PCA was limited to the calibration period (1902-1973). The values in boxes are for the second stability index (see text).

Table 4. Summary of response function analyses

			No prior growth		Prior growth	
Chro.	#Cores	r^2	Rank	R^2adj.	Rank	R^2adj.
SC1	27	.38	6.2	.34	5.0	.46
P2	22	.31	5	.35	6	.45
P21	12	.24	7	.30	8	.41
P22	10	.42	1	.42	2	.56
P23	17	.28	4	.37	5	.46
P24	5	.23	9	.27	11	.36
P211	5	.34	6	.35	3	.50
P212	4	.33	8	.27	7	.42
P213	3	.12	10	.26	9	.39
P232	1	.82	2	.42	1	.73
P233	4	.33	3	.38	4	.49
P241	4	.27	11	.26	10	.37
P242	1	.00	12	.12	12	.11

Note: r^2 denotes the square of the autocorrelation of the chronology at lag 1.

that is not seen in the other three. The response in 1923 is more exaggerated. Finally, the response to global warming beginning circa 1850 and maximizing circa 1940 is not as pronounced. It is not certain that these differences imply more sensitivity rather than less reliability. While the stability indices and the wide separation of the three cores imply the former (since it is unlikely that widely separated trees would experience the same non-climatic microsite effects), we cannot completely dismiss the latter.

P213II and P213III are shown because they enter the analysis and discussion.

RESPONSE FUNCTION ANALYSES: Each of the first amplitudes for the subsets shown in the Figure 1 and the standard chronology were regressed on eigenvectors of Dawson climate data computed for the period 1902-1973. Analyses were done without and with prior growth included as a predictor. The dendroclimatic year extended from previous March through current September. The results are summarized in Table 4. Lag-one autocorrelation is denoted by *r*. Only climate amplitudes with eigenvalues greater than one were used and maximum adjusted R^2 (Neter and Wasserman 1974) was used as a selection criterion for the response functions summarized in Table 4. (The PCA of the climate variables has nothing to do with the PCA of the tree-ring data. It is convenient to have uncorrelated predictors for the regression and it is thought that by using only the stronger amplitudes one emphasizes important climatic variations.)

There is strong agreement between the rankings in columns four and six. The Spearman rank correlation coefficient is 0.93. Also, the values in columns three, five and seven correlate strongly. The chronologies based on small numbers of cores tend

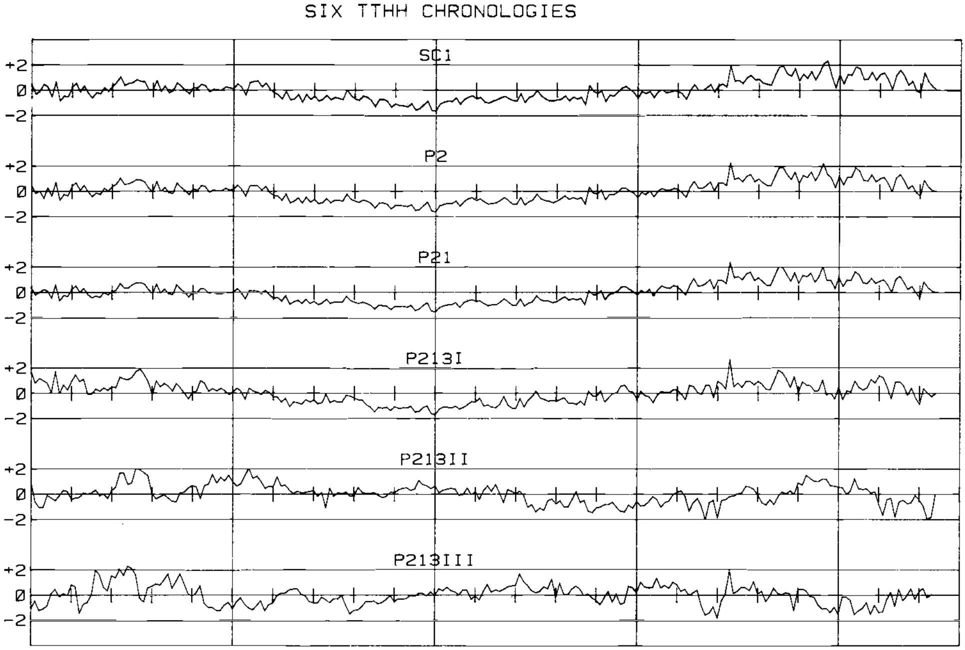

Figure 1. Six of the 15 chronologies presented in this paper are shown for the interval 1750-1974. SC1, P2, P21 and P213 I, II, III are based on 27, 22, 12 and 3 cores respectively. All six chronologies have been scaled to zero mean and unit variance for comparison.

to show a wider range of performance, especially when prior growth is included, than the many-core chronologies. Thus, for instance, the adjusted R^2 for SC1, P2 and P23 is near the median value while it is extreme for the two one-core chronologies P232 and P242. These results show that larger samples do not necessarily yield chronologies that correlate better with climate. It is likely that there are several different responses to climate in these different core series and averaging tends to obscure any particular one of them, whether it is a SC or first amplitude.

The response functions (regression coefficients) for the standard chronology and the 12 PCC's are shown in Figure 2 for the no-prior-growth predictors. The response functions with prior growth are essentially the same and are not shown. Notice first that amongst the PCC's the partitioning suggested by loadings in Table 1 produces different response functions. Thus, while all of them (except that for P242) are roughly similar in shape, the main partitioning into the disjoint subsets 21 and 22 results in

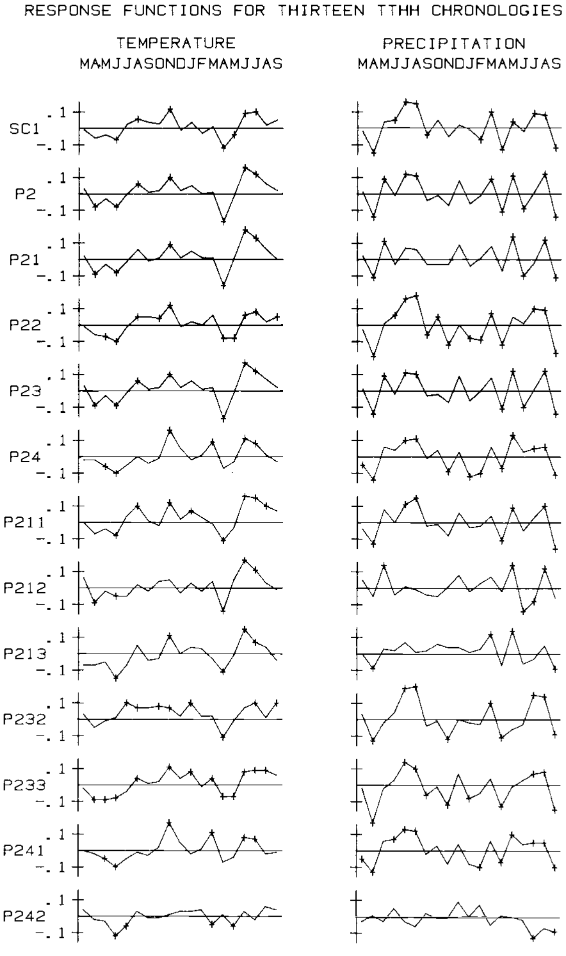

Figure 2. Response functions for 13 chronologies are based on the period 1902 to 1973. The meteorological data consists of monthly average temperature and total precipitation at Dawson. The crosses denote regression coefficients which differ from zero by more than two standard deviations.

somewhat different response functions, particularly for precipitation. (P242 is a one-core chronology that appears as an outlier on a plot of mean ring width vs. ring-width variance. It can be thought of as a noise sample for the site and used for comparison. Note that it loaded very lightly in both Table 1 and 2.) Next, notice that amongst the response functions for the subsets of 21 (211, 212, 213) there is a strong similarity, and also for the subsets of 22 (24, 241, 232, and 233). Consider, for example, for temperature the response functions for (P2, P21, P211, or P213) and (P22, P24, P241) based on (22, 12, 5, or 3) and (10, 5, 4) cores respectively; and for precipitation the response functions for (P2, P21, P211, or P212), (P22, P24, P241) and (P22, P233, or P232) based on (22, 12, 5, or 4), (10, 5, 4) and (10, 4, or 1) cores respectively. This robustness of the response functions with respect to the number of cores used is not peculiar to this species or site. These results show clearly one of the effects of averaging over heterogeneous subsets. There is a dichotomy of response exhibited by the disjoint subsets 21 and 22. The climate variables explain considerably less of the variance in the chronology 21, and the precipitation response for this subset is difficult to interpret because of the strong oscillation about zero over the interval (March-August) in the current year and the more subdued but similar behavior for the same interval in the previous year. By contrast, the response of the subset 22 over these intervals is smoother and in the subsets 232 and 233 even becomes monotonic. In terms of precipitation not much is gained by going from one core in 232 to 22 cores in 2 which is almost completely dominated by 21. In fact, just the opposite. Adding five more cores and computing a standard chronology, SC1, yields a response function that is almost identical to that for 22. One can question what would be the effect of five more cores, or 10, or 50? Without understanding the cause of the apparent dichotomy in terms of microsite conditions, it is not certain that a much larger number of cores would yield a chronology that is better explained by climate variables and that is more interpretable in terms of tree physiology and site conditions.

The dichotomy in response exhibited by the trees in 21 and 22 and their respective subsets obviously cannot be attributed to juvenile effects. Perhaps the four starred cores and the others in 22 are pathological in some other way, but the additional variance removed from P22 over P21 and the shape of its response function for precipitation recommend that it be considered further.

TRANSFER FUNCTIONS: Previous work with the standard chronology had shown that the most explainable of many simple combinations of monthly climate variables were June and July average temperatures (T_{JJ}) and June and July total degree days $> 10°C$ (D_{JJ}). This is also the case for the PCC's, and the results which will be discussed here are summarized in Table 5. The climate variables were regressed stepwise on the chronologies lagged forward 14 years in the case of T_{JJ} and nine years in the case of D_{JJ}. Thus the interval for T_{JJ} is (1902-1959) and for D_{JJ} (1921-1965). Many lags were included because there is some evidence of long needle retention in this species (Fraser et al. 1964) and we wanted to see which lags would be selected if there were many available. The stopping criterion used was maximum adjusted R^2 and these values are listed in column 8. The number in parentheses after the chronology name (column 3) is the number of cores used for that chronology. The superscript[1] denotes restriction to current and lag-one growth. L(1) and R^2 adj. (1) refer to the first lag selected and the variance explained by this variable respectively. Similarly for L(2) and R^2 adj. (2). The number in parentheses after R^2 adj. (Max) is the number of lags finally retained and these are listed in order by the associated Student t value. The variance explained by those to the right of the slash (/) can be considered not signifi-

Table 5. Summary of transfer function analysis

Variable	Case	Chronology Designation	L(1)	R^2adj(1)	L(2)	R^2adj(2)	R^2adj(Max)	Lags retained
T_{JJ}	1	SC1(27)	0	.108	1	.209	.320(6)	0,1,6/8,11,10
T_{JJ}	2	P2(22)	0	.129	1	.272	.433(8)	0,1,8,6,11/9,2,10
T_{JJ}	3	P21(12)	0	.136	1	.285	.479(7)	0,1,11,8,6/9,10
T_{JJ}	4	P22(10)	0	.100	1	.223	.398(10)	1,0,8,6/10,12,9,3,2,5
T_{JJ}	5	P23(17)	0	.134	1	.286	.448(8)	0,1,8,6,11/9,2,10
T_{JJ}	6	P24(5)	0	.125	1	.215	.352(9)	0,1,8,12/6,10,5,2,3
T_{JJ}	7	P211(5)	0	.145	10	.244	.360(9)	0,1,8/10,14,9,11,6,7
T_{JJ}	8	P212(11)	0	.103	1	.255	.462(8)	0,1,11,8,6/9,12,2
T_{JJ}	9	P213I(3)	1	.099	0	.277	.493(8)	1,0,8,11,6/10,5,7
T_{JJ}	10	P232(1)	2	.055	1	.089	.195(5)	1,2,0/12,10
T_{JJ}	11	P233(4)	8	.064	6	.133	.442(9)	1,8,0,6,10/12,3,2,5
T_{JJ}	12	P241(4)	0	.126	1	.214	.343(9)	0,1,8,12/6,10,5,3,2
T_{JJ}	13	P242(1)	14	.029	1	.056	.064(4)	/14,1,2,11
T_{JJ}	14	P213I,II(3)	1(II)	.106	1(I)	.203	.553(11)	1(I),0(I),8(I),11(I),0(II),7(I)/6(I),6(II),5(I),10(I),9(II)
T_{JJ}	15	P213I,II'(3)	1(II)	.106	1(I)	.203	.338(3)	1(I),0(I),0(II)/
D_{JJ}	16	SC1(27)	0	.221	4	.263	.304(4)	0/1,8,4
D_{JJ}	17	SC1(27)'	0	.220	1	.249	.249(2)	0/1
D_{JJ}	18	P2(22)	0	.310	1	.361	.413(4)	0,1/8,4
D_{JJ}	19	P21(12)	0	.291	1	.374	.411(3)	0,1/8
D_{JJ}	20	P213I(3)	1	.145	0	.322	.458(5)	1,0,8/4,6
D_{JJ}	21	P213I,II(3)	1(II)	.183	0(I)	.338	.611(11)	0(I),1(I),8(I),3(II),9(III),1(II),0(II),6(II)/6(I),2(I),8(II)
D_{JJ}	22	P213I,II'(3)	1(II)	.183	0(I)	.338	.475(4)	0(I),1(I)/0(II),1(II)

Column 1 contains the dependent variable. It is either average June and July temperature (T_{JJ}) or total June-July degree days >10°C. The regressions are numbered in column 2 for ease of reference in the text. The number in parentheses after the chronology designation (column 3) is the number of cores in the corresponding subset. Note that in cases 14, 15, 21 and 22 two amplitudes (I,II) were used as predictors. The superscript' indicates that only lags 1 and 2 were allowed to enter. L(1) and L(2) specify the first and second lags entered into the regression. R^2adj(1) and R^2adj(2) denote the variance (adjusted for degrees of freedom) explained after 1 and 2 lags respectively have been entered. R^2adj(Max) denotes the maximum adjusted variance, which is attained after (Max) lags have been entered. The lags finally retained in the regression after (Max) have been entered are listed in the last columns. Those to the left of the "/" have t values greater than 2.0.

cant at $>95\%$ ($t<2.0$). They are listed because there is some agreement in their order between the various chronologies even when the subsets used had no cores in common.

Generally, for this calibration period, the high lags contribute significantly as predictors. This has been found to be the case at other northern tree line sites we have studied. At TT-HH this dependence is more pronounced for the temperature data than for degree-day data. We have degree-day data only for Dawson and cannot make this comparison for other sites. This dependence is more variable for chronologies based on small subsets. It is well known that individual cores vary widely in their persistence and thus in the encoding of climate in their higher lags. Averaging will affect the recovery of this part of the climate signal.

Except for the one core chronologies, all the PCC's explain summer temperature better than the SC. Since we have never tried one-core chronologies before, we don't know whether this performance is typical or peculiar to the dependent variables we've tried. Obviously, if monthly climate data for the whole dendroclimatic year weighted by the response function coefficients were chosen as the dependent variable, at least 70% (see Table 4) of the climate variance could be explained by 232. On the other hand, 242 exhibits its unique uselessness as a climate recorder. Of course a single core would have to be examined for time stability by a method other than that described in this paper. The chronologies from the 3-core subset 213 explain the most variance in both T_{JJ} and D_{JJ} (cases 14, 21). Note that for these cores the second amplitude is as important as the first as a predictor of Dawson summer temperature and degree days. The loadings in the second eigenvector are not all of the same sign (.68, .04, -.73); thus the second amplitude is a measure of differential growth. As for the response functions, the R^2 values are near the median for the chronologies based on the largest subsets, except for SCI for which it is anomalously low.

A preliminary look at the residuals shows that they are better for the D_{JJ} variable, but only in cases 1, 12 and possibly 2 is there evidence of inadequacy of the lagged linear model. We have not done a thorough analysis of the residuals for these fits because it is not our intent in this paper to do a reconstruction, nor are we ready to do one yet. There is some doubt regarding the time homogeneity of the Dawson meteorological data and the apparent dependence on higher lags needs to be examined more thoroughly. The most serious problem, however, is the anomalousness of the calibration period. Not only will the reconstructed values have large errors of estimate but the matter of verification will not lend itself to routine treatment. Also, as noted, the relationship between the climate at the site and at Dawson might not be constant in time, and this coupled with the possibility that the site structure might be evolving with the secular climate changes suggests that a more thorough assessment of overall time stability for the transfer function should be undertaken now that more responsive chronologies are available.

DISCUSSION AND SUMMARY

The model underlying a SC entails the assumption that the climate signal is reliably recorded only in mean index and its autocorrelation function. The more structured model underlying a PCC admits the possibility that climate information is also encoded in differential growth, which appears in several contexts. For instance, within the first eigenvector the loadings will usually have different values, even if they all have the same sign, and they define a direction of maximum common variance. In the context of site heterogeneity, the subsets 21 and 22 have different response functions and the lag dependence of T_{JJ} on them is different. The first amplitudes from these subsets might be tried as joint predictors. Finally, the second eigenvector for a particular subset is usually polar (both + and - loadings are present). It is interesting that P213II (lagged one year) correlates best with both T_{JJ} and D_{JJ} (it is selected first in the stepwise regression), while this second amplitude accounts for only 18% of the variance in the subset. A graphical comparison shows that the low-frequency agreement is particularly good. The fact that it is not the most important of the retained predictors suggests that the same climate information can be encoded in both the higher lags of the first amplitude and the higher order amplitudes when these are time stable.

It should be noted that the procedure described in this paper for analyzing site heterogeneity is not exhaustive. It seeks homogeneity within heterogeneity whereas subsets other than those suggested by Table 1 could be found that correlate better with the climate data. Some of these would yield polar first eigenvectors. On the other hand, a procedure that simply optimized climatic correlation over all possible partitionings of the cores would require a less intuitive underlying model.

In conclusion we would like to clarify the sense in which we have been comparing the SC with the PCC's. In this paper we have used the SC for the site as a basis for comparison as the fiducial chronology. Thus, for instance, we expected that the PCC's (first amplitude) based on a smaller number of cores would look similar to the SC and indeed lack of such similarity would be unacceptable. We have intentionally avoided comparing SC's and PCC's based on the same subset of cores. An SC is an arithmetic average over indices with different variances. Optimization is achieved when there is a positive correlation between index variance and index mean sensitivity under the assumption that the more sensitive trees respond to climate more reliably. We have been computing PCC's (first amplitude) as weighted averages over index series each of which has been scaled to unit variance (and zero mean). Optimization is achieved on the assumption that the most reliable response to climate is along the direction of the first eigenvector (i.e., that the most sensitive trees load most heavily on the first eigenvector). There are other ways of computing PCC's. One could attempt a double optimization by computing the first amplitude as a product of the first eigenvector of the correlation matrix and the matrix of unscaled index series; or one could rotate the covariance matrix of the unscaled indices. These and other methods of computing a PCC (or an SC) are related to the method of analyzing site heterogeneity discussed in this paper but need to be considered by themselves, so that the effects of site heterogeneity can be separated from effects attributable to different schemes for constructing chronologies from the same subset of indices.

It is expected that in general these latter effects will depend on species and on site conditions and location relative to the weather station used for calibration. For the site discussed in this paper we have computed standard chronologies for the subsets 2, 21, and 213. For these subsets the regressions for the variable D_{JJ} show that all the improvement in the transfer function for the subset 2 can be attributed to the different method of computing the PCC, while for the subsets 21 and 213 the PCC's and the SC give nearly identical performance. The SC for the subset 2 of 22 cores, SC2, performs no better than the original standard chronology based on 27 cores (SC1) while P2 does. Three overall characteristics of the indices are involved here. First, there is no correlation between index variance and mean sensitivity at this site and thus no optimization for the SC's. Second, the angles between the first eigenvector for each subset and the vector of mean-index standard deviations which weights the cores in computing the corresponding SC's, are 32°, 13°, and 6° respectively. Third, the average correlations between cores are .34, .63, and .53 respectively for these subsets. These values suggest that for the subset 2 the SC and the PCC are quite different in detail, while for the subsets 21 and 213 they are nearly identical.

The possibility of using higher-order amplitudes as additional predictors gives the PCC's a unique advantage. In the case of subset 213, inclusion of the second amplitude as a predictor gives a further increase in the removed variances. This increase is as large as the others being considered here and it has been adjusted for the increase in the number of predictors, suggesting that similar improvements might be expected in other cases and thus that the matter of PCC's on small subsets deserves special attention.

Principal component analysis of the site TT-HH and analyses of other northern and eastern sites have shown that small subsets of a complete site inventory of cores can yield chronologies and response functions very similar to those based on larger numbers of cores, and in general at least some of these will correlate much better with local climate data. We suggest that at least part of this improvement derives from the analysis of site heterogeneity and that under this latter condition a larger sample will not necessarily yield a better chronology. The PCC's are based on the same cores over their whole length, and the procedure for developing them allows one to assess the time stability of the eigenstructure of a subset. Only time-stable amplitudes should be used as predictors in transfer functions and reconstructions.

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