TREE-RING BULLETIN, *Vol. 43, 1983*

TESTING THE SIGNIFICANCE OF SUMMARY RESPONSE FUNCTIONS

B. M. GRAY

Climatic Research Unit
University of East Anglia
Norwich, U. K.

and

J. R. PILCHER

Palaeoecology Centre
The Queen's University of Belfast
Northern Ireland

ABSTRACT

A simple method of testing the statistical significance of the summary response function derived by Pilcher and Gray is given and applied to European oak data.

Es wird ein einfaches Verfahren zur Prüfung der'summary response function' nach Pilcher und Gray vorgestellt und auf Jahrringdaten von Eichen angewandt.

Une simple méthode pour tester la signification de la fonction-réponse résumée par Pilcher et Gray est fournie et appliquée à des données provenant de chênes européens.

INTRODUCTION

Fritts (1974) discussed the averaged response function of a group of clustered chronologies, and noted that, for the chronologies considered, the species differences were not as marked as the site differences. Such an averaged response function fulfills the same purposes as, but is different from, the summary response function derived by Pilcher and Gray (1982). The latter is a histogram composed of the counts of the numbers of significant positive and negative response function elements from a collection of response functions, in this case of oak chronologies. This paper describes the application of the binomial distribution to determine the significant months in a summary response function.

THE U. K. OAK SUMMARY RESPONSE

The application of the binomial distribution to determine the overall significance of an individual response function is described in Gray, Wigley, and Pilcher (1981). The method has been extended to consider the statistical significance of the summary response in any one month to either temperature or precipitation. To illustrate the method, it has been applied to the summary response in Pilcher and Gray (1982). Here, 16 site chronologies *(Quercus robur, Q. petraea,* and hybrids) in the United Kingdom are used to produce a summary response (Figure 1). The 16 chronologies exhibit 75 temperature and 77 precipitation elements which are significant at the 95% level, an average of 5 variables/response function.

To evaluate the significance of the summary function of these chronologies, consider the probable summary functions that would arise if the significant coefficients of one variable, say temperature, were randomly distributed across the 14 months in the response function. Each month can have either a positive or negative response, so there are five coefficient elements and 28 possibilities or slots.

The probability, p, of having one element in any one slot is found by adding the probabilities of the five elements arriving in turn at that slot.

$$p = p(E_1) + p(E_2) + p(E_3) + p(E_4) + p(E_5)$$

Now $p(E_1)$ is obviously 1/28. $p(E_2)$ is equal to $p(E_1)$ multiplied by the probability that the slot is still empty. Twenty seven slots are empty, but only 26 of these are acceptable, since in practice no month has both positive and negative significant coefficients. Hence

$$p(E_2) = p(E_1) \times 26/27$$
$$\text{similarly } p(E_3) = p(E_2) \times 24/26$$
$$p(E_4) = p(E_3) \times 22/25$$
$$p(E_5) = p(E_4) \times 20/24$$

$$\text{Then } p = .1531$$

The probability q of having a significant element not fall in a given slot is then

$$q = 1 - p = .8469$$

The summary function records the occurence or non-occurrence of a significant

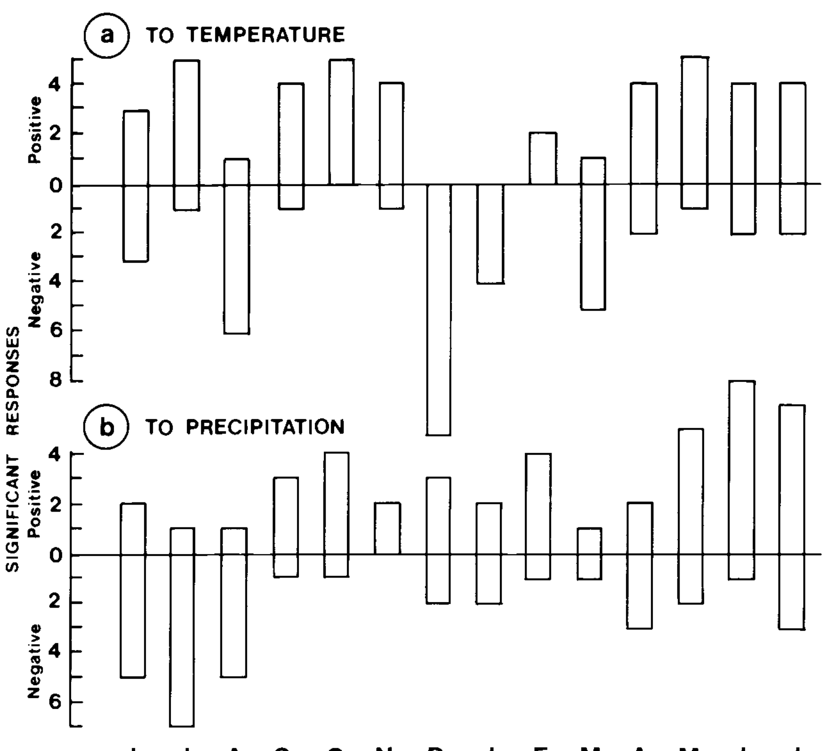

Figure 1. Summary response function for British Isles oak (n = 16).

coefficient in any month in the N samples of response function available.

The binomial distribution is therefore suitable to derive the probability P_n of a summary function having n coefficients in any one month, above or below the zero line.

$$P_n = p^n q^{N-n} \frac{N!}{(N-n)!\ n!}$$

In the United Kingdom oak summary response, N = 16. From this distribution function we can calculate the probabilities of having 0, 1, 2, 3 . . . significant coefficients in a month arising by chance.

P_0 = .0700
P_1 = .2024
P_2 = .2745
P_3 = .2316
P_4 = .1361
P_5 = .0591
P_6 = .0195 * (significant at 5%)
P_7 = .0051 ** (significant at 1%)
P_8 = .0010 *** (significant at .1%)

If we apply the above probability table to Table 1, the significiant elements in the summary function can be identified. The previous August temperature and current July precipitation are significant at the 1% level. December temperatures are the most marked influence, being significant at the .1% level. Gray et al. (1981) and Pilcher and Gray (1982) commented that the only suggestion of difference between the two species of oak appears to be a slightly stronger positive response to high temperature in the growing season in *Q. robur*. Otherwise, the species and hybrids respond similarly. On these grounds the areal range of the summary function was extended to include oak chronologies from outside the British Isles.

EUROPEAN OAK SUMMARY RESPONSE

Fourteen additional response functions from 11 European sites and areas were added to the British Isles summary function. Two sites had long chronologies for which response functions could be calculated for two or three non-overlapping periods. The minimum period used to calculate a response function was 60 years, the average period being 100 years. The additional chronologies are listed in Table 2; one is from Sweden, two are from Germany, three from the British Isles and five from northern France, and they include both site and area chronologies.

The significant months in the summary function were identified as before (Figure 2). There were 151 temperature and 139 precipitation elements significant in the 30 cases, an average of 5 variables/response. Hence p and q have the same values as before, and, since we have N = 30 samples now,

$$P_n = p^n q^{30-n} \frac{30!}{(30-n!)\ n!}$$

In the N = 30 probability function, 9 and 10 elements in any month reach the 5% level of significant difference from chance expection, 11 and 12 indicate the 1% level, and 13 the .1% level. Statistically, a month with one or zero elements would indicate the 5% or 1% lower limits of significant difference from chance expectation. These values usually occur next to a significant coefficient on the other side of the zero line. They appear to have no physical meaning, and to be merely a reflection of the method used.

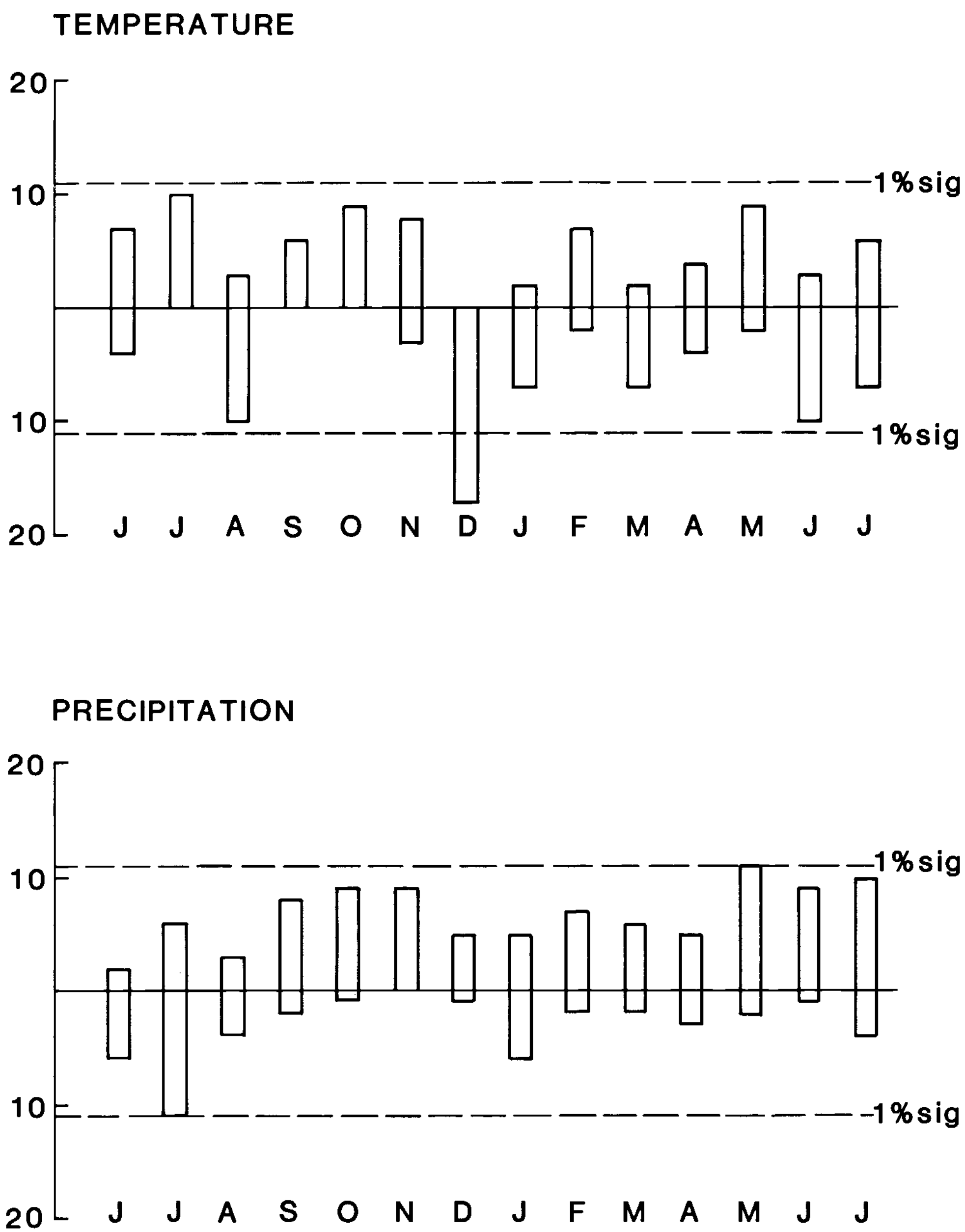

Figure 2. Summary response function for European oak (n = 30).

Table 3 lists the 30 sample European oak summary function. Comparison of Tables 1 and 3 show that all the months identified as significant in the 16 case summary remain significant in the 30 case summary. In addition, the temperature results show the July previous, October, May and June to be significant. The result for precipitation now include October, November, and May as significant months. Overall, May to July and October to November are the most important months in the growth response to precipitation. Strong responses to both variables occur outside the expected growing season April to August.

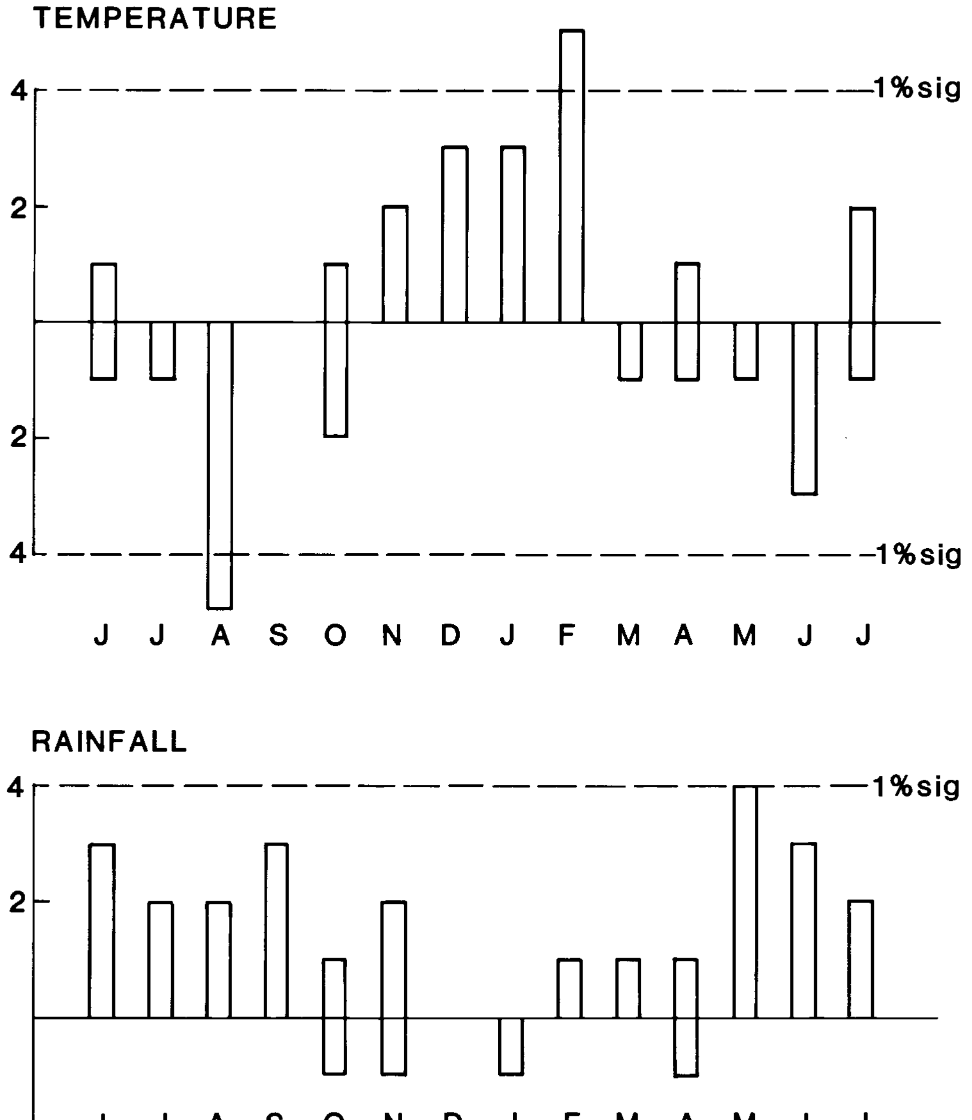

Figure 3. Summary response function for European fir (n = 6).

EUROPEAN FIR SUMMARY RESPONSE

In order to compare species over the same period, a summary function was constructed for six fir chronologies drawn from the German archaeological dating master curve (Becker and Giertz-Siebenlist 1970). There are three area chronologies and three site chronologies with climatic response varying from 32% to 70% of the tree-ring variance (Figure 3). The results of the significance test on this sample indicated August and February temperatures to be significant at the .1% level, and March precipitation to be significant at the 1% level. It is not possible to distinguish a clear species difference from the European oak summary from such a small sample.

CONCLUSIONS

A method of establishing the significant months of growth response to climate has been devised and applied to European oak summary response functions. This method should assist in determining which months are suitable for the reconstruction of climatic data from tree-ring chronologies.

ACKNOWLEDGEMENTS

We thank all our colleagues, too numerous to mention, who supplied the original tree-ring data.

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Table 1. The summary response function histogram for 16 oak chronologies in the British Isles (N = 16).

Months	Temperature Pos.	Temperature Neg.	Precipitation Pos.	Precipitation Neg.
June	3	3	2	5
July	5	1	1	7**
Aug.	1	6*	1	5
Sep.	4	1	3	1
Oct.	5	0	4	1
Nov.	4	1	2	0
Dec.	0	10***	3	2
Jan.	0	4	2	2
Feb.	2	0	4	1
Mar.	1	5	1	1
Apr.	4	2	2	3
May	5	1	5	2
June	4	2	7**	1
July	4	2	6*	3

Table 2. Additional oak chronologies used in summary response function.

Chronology	Type	Location	% Variance Explained by Climate
Halate	site	49.2N 2.6E	26,36,37*
Guines	S	50.5N 1.8E	21
Chambord	S	47.6N 1.5E	70
Chinon	S	47.2N 0.4E	67
Fontainebleau	S	48.5N 2.7E	38,38
Rothenburg	area	50.2N 10.1E	28
Ebrach	S	49.8N 10.5E	22
Hunneberg	A	58.0N 13.0E	61
Winchester	A	51.0N 1.3E	35
Belfast	A	54.6N 5.9W	42
Hereford — Cumberland	A	53.5N 2.5W	22

*Long chronologies divided respectively into 3 or 2 segments to make response function comparisons.

Table 3. European oak summary response (30 cases).

Months	Temperature Pos.	Temperature Neg.	Precipitation Pos.	Precipitation Neg.
June	7	4	2	6
July	10*	0	2	6
Aug.	3	10*	3	4
Sep.	6	1*	8	2
Oct.	9*	0**	9*	1*
Nov.	8	3	9*	0**
Dec.	1	17***	5	1*
Jan.	2	7	5	6
Feb.	7	2	7	2
Mar.	2	7	6	1*
Apr.	4	4	5	3
May	9*	2	11**	2
Jun.	3	10*	9*	1
Jul.	6	7	10*	4