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EVENNESS INDICES MEASURE THE SIGNAL STRENGTH OF BIWEIGHT SITE CHRONOLOGIES

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ABSTRACT

The signal strength of a biweight site chronology is properly viewed as an outcome of analysis rather than as a property of the forest-climate system. It can be estimated by the evenness of the empirical weights that are assigned to individual trees. The approach is demonstrated for a 45-year biweight chronology obtained from 40 jack pine (*Pinus banksiana* Lamb.) trees. The annual evenness of the empirical weights is calculated by indices derived from the Shannon and Simpson diversity indices, and the variances are found by the jackknife procedure. The annual estimates are then averaged to find an overall estimate of biweight signal strength for the 45-year period. These techniques are most useful for determining sample sizes for the biweight procedure, and for comparing different methods of detrending and standardizing data sets prior to applying the biweight mean-value function.

Die Signalstärke einer durch 'robuste Mittelung' gebildeten Standortchronologie ('biweight site chronology') ist eher eine Folge der Berechnung als eine Eigenschaft des Systems aus Wald und Klima. Die Signalstärke läßt sich anhand der Gleichheit der den Einzelbäumen zugeordneten empirischen Gewichte abschätzen. Dies wird für eine 45jährige 'robust gemittelte' Chronologie von 40 Kiefern (*Pinus banksiana* Lamb.) gezeigt. Die jährliche Gleichheit der empirischen Gewichte wird anhand von Indices berechnet, die aus den Shannon- und Simpson-Diversitäts-Indices abgeleitet sind; die Varianzen werden mit der 'jackknife'-Prozedur gefunden. Danach werden die jährlichen Schätzungen gemittelt, um eine Schätzung der Signalstärke für die gesamte 45jährige Periode zu erhalten. Dies ist wichtig für die Bestimmung der Stichprobengröße für die 'robuste Mittelung,' aber auch für den Vergleich verschiedener Verfahren der Trendeleminierung und Standardisierung von Datensätzen vor Anwendung der sog. 'biweight-mean value function', d.h. der 'robusten Mittelung'.

La force du signal d'une chronologie de site bipondérée est correctement considérée comme un résultat d'analyse plutôt que comme une propriété du systême forêt-climat. Elle peut être estimée par l'équitabilité des poids empiriques qui sont attribués aux arbres individuels. Cette approche est démontrée pour une chronologie bipondérée de 45 ans obtenue à partir de 40 *Pinus banksiana* Lamb. (jack pine). L'équitabilité annuelle des poids empiriques est calculée par des indices dérivés des indices de diversité de Shannon et Simpson et les variances sont obtenues par la procédure dite "jacknife". Les estimées annuelles sont alors moyennées pour trouver une estimée d'ensemble de la force du signal bipondére pour une période de 45 ans. Ces techniques sont les plus utiles pour déterminer les dimensions de l'échantillonnage par la procédure par bipondération et pour comparer les différentes méthodes utilisées pour supprimer les tendances et standardiser les jeux de données avant d'appliquer la fonction de la valeur moyenne bipondérée.

INTRODUCTION

It is often useful to know the strength of the common climate signal carried by the ring-width indices from a sample of trees. This information quantifies the similarity among the growth responses of individual trees and can be used to optimize sample designs. The classical "signal-to-noise ratio" measures signal strength in terms of the between-year and within-year variance components of ring-width indices, obtained by an analysis of variance (Fritts 1976). Another measure is based on a resampling of cross-correlations of ring-width indices among individual trees (Wigley *et al.* 1984).

Each individual tree in a biweight (Mosteller and Tukey 1977) chronology is assigned an empirical weight that depends on its similarity to the common biweight signal. The biweight signal is a transformation of the biological signal, where the operator is the biweight function. Information about the biweight signal is carried by the empirical weights, and so signal strength should be measured with reference to them. This paper demonstrates one approach based on the evenness of the empirical weights.

INDICES USED TO MEASURE SIGNAL STRENGTH

Diversity and Evenness Indices of Empirical Weights

Pielou (1975), Washington (1984), and Magurran (1988) review diversity indices and their applications in statistical ecology. Historically, diversity indices have been developed to summarize in a single number the regularity of an assemblage of species, considering their variety, equitability, and abundance. For most diversity indices, an index of evenness may be obtained by scaling the calculated value to the maximum value that is possible for the number of species in the sample (e.g., Lloyd and Ghelardi 1964). Such indices have been applied in other contexts to estimate, for example, niche breadth (Feinsinger *et al.* 1981) and variety of recreation supply (Saunders and Burnett 1983).

The calculations of the Shannon (1948) and Simpson (1949) diversity indices, and evenness indices derived from them, are described below for the empirical weights derived from a biweight analysis. If the annual weight for tree i ($i = 1,...,n$) from the biweight procedure is r_i, the relative or proportional weight may be calculated as follows:

$$w_i = r_i \;/\; \Sigma^n_{i=1}\; r_i$$

The number of trees and their proportional weights are used in place of the number of species and their relative abundances to calculate the diversity and evenness indices.

The Shannon index of the diversity of the w_i is defined as follows:

$$H = -\,\Sigma^n_{i=1}\; w_i \log_e w_i$$

Because a zero w_i has meaning, the summation term is set equal to zero in those cases. The Simpson index of the relative weights is as follows:

$$D = 1 - \Sigma^n_{i=1}\; w^2_i$$

D is actually in the form $\Sigma^n_{i=1}\; w^2_i$ in Simpson's paper, but this reexpression is easier to compare to the Shannon index.

The evenness indices are more convenient than the diversity indices because they normalize the estimates with respect to the maximum value that is possible for a given sample size. The evenness index derived from the Shannon index is (Lloyd and Ghelardi 1964) as follows:

$$E_H = H \;/\; H_{max} = H \;/\; \log_e n$$

The evenness index derived from the above formulation of the Simpson index is as follows:

$$E_D = D \; / \; D_{max} = D \; / \; 1 - n^{-1}$$

Variances of the Evenness Indices

With simple random sampling and large sample sizes, the variances of E_D and E_H can be derived from the formulas in Simpson (1949) and Basharin (1959) (see also Tong 1983). In tree-ring studies, however, nonrandom sampling and small sample sizes may require other methods. The jackknife procedure (e.g., Efron 1982) has been used to estimate the variances of diversity indices (e.g., Heltshe and Forrester 1983, Zahl 1977). Applied to this study, the concept is to divide the ring-width indices for each year into groups, calculate the w_i, E_D, and E_H for each group and use the between-group variance of E_D and E_H as an estimate of their sampling variance.

The jackknife computational algorithm starts by dividing the n ring-width index observations into n groups of size n-1 each, by leaving out a different ring-width index for each group. The biweight procedure is then applied to each of the n groups to obtain n sets of w_i, where i = 1,...,n-1 in each set. The evenness indices are then calculated for each of the n sets of w_i.

Let ϕ be the value of an evenness index for all n trees, and let $\phi_k^{(-k)}$ be the value obtained from the set that excludes the k^{th} ring-width index. Define a new value,

$$\hat{\phi}_k = n\phi - (n-1)\, \phi_k^{(-k)}$$

for k = 1,...,n. The $\hat{\phi}_k$ values may be assumed to be independently and identically distributed, normal variables for any distribution of the r_i or w_i (Tukey 1958). With that assumption, the jackknife estimates of the variances of E_D and E_H are the usual sample variances of the means of the corresponding $\hat{\phi}_k$ values (e.g., Efron 1982). The sample average of the $\hat{\phi}_k$ is the jackknife estimate of ϕ, and is used in the calculation of the sample variance. The variance obtained applies also to the nonjackknife estimate of ϕ (Efron 1982).

Measures of Signal Strength

The biweight signal strength is estimated on a year-by-year basis by the evenness indices. Biweight signal strengths of different years or of different chronologies may be compared by constructing confidence intervals or tests of significance based on the jackknife estimates of variance (see Efron 1982). There is also a need for a summary statistic that measures the average signal strength over an entire chronology. An average measure provides a quick comparison of biweight signal strengths obtained for different methods of detrending, standardization, or sampling. Let ϕ_j be the evenness index for the j^{th} year. A simple measure of average signal strength is the mean index over all m years:

$$\phi^* = \Sigma_{j=1}^{m} \; \phi_j \; / \; m$$

The variance of ϕ^* is as follows:

$$V(\phi^*) = m^{-2} \; [\Sigma_{j=1}^{m} \; \mathrm{Var}(\phi_j) + 2 \, \Sigma_{j<j'}^{m} \mathrm{Cov}(\phi_j \; , \; \phi_{j'}) \;]$$

If it is important, the covariance term is likely to be manifested as autocorrelation among the ϕ_j values. This would imply that the biweight signal strength was not independent from year to year. There may also be trends in signal strength over time. The best investigative approach is probably Box-Jenkins modeling (Box and Jenkins 1970). If it can be demonstrated that the covariance term is not important, the variance of ϕ^* may be estimated by a linear combination of the jackknife estimates of the variances of the ϕ_j values. If the covariance is important, Box-Jenkins models can be used to account for autocorrelation or for trends in biweight signal strength over time.

AN APPLICATION

Ring-width indices were calculated for data from large jack pine (*Pinus banksiana* Lamb.) trees from an even-aged stand near Fort Smith, Northwest Territories, Canada. The data are from site CM8 (Cherry Mountain plot number 8) in Sweda and Umemura (1979). Riitters (1990) provides more detailed descriptions of the statistical procedures. A gamma-type detrending model (Monserud 1986) was fitted to 45 years of ring-width data for each of 40 trees, and the residuals were standardized by the usual procedure (Fritts 1976). The biweight was implemented with an iteratively reweighted, least-squares algorithm (Goodall 1983), which yielded the biweight weights, r_i. The evenness indices, E_D and E_H were estimated for each year. The variances, $V(E_D)$ and $V(E_H)$, were estimated for each year by the jackknife procedure and, for comparison, by the large-sample formulas.

The time trends of E_D and E_H are plotted with confidence intervals (plus and minus two standard errors) in Figure 1. Except for scale differences, E_D and E_H appear to give similar estimates of signal strengths over time. For both indices, the differences in signal strengths among years are generally insignificant according to the jackknife standard errors. Confidence intervals based on the large-sample variances are unrealistically narrow, and are shown for comparison only. The confidence intervals based on the jackknife variances vary markedly in width, and there is a tendency for years with low E_D or E_H to be associated with relatively wide intervals. Yet when evenness is high, some intervals are wider than others, and so the variances are not strictly functions of the point estimates.

Box-Jenkins modeling techniques (Box and Jenkins 1970) were used to test autocorrelations and trends in E_D and E_H. Plots of the autocorrelation function, and the insignificance of the Q-statistic (Ljung and Box 1978), indicated that the time series were stationary and white noise. From this evidence it was concluded that the covariance terms in $V(\phi^*)$ were unimportant. The standard errors of the average signal strength were then calculated from the $V(\phi_j)$:

$$SE(\phi^*) = [\ m^{-2} \sum_{j=1}^{m} V(\phi_j)\ ]^{0.5}$$

The average signal strengths (and standard errors) obtained for E_D and E_H are 0.9990 (0.0017) and 0.9925 (0.0133), respectively. Not much can be made of the result that the average signal strengths do not differ significantly from a complete evenness (i.e., 1.0000), because that result can change depending on the formulation of the biweight algorithm. Similar results (not shown) were obtained from four other similar sets of data.

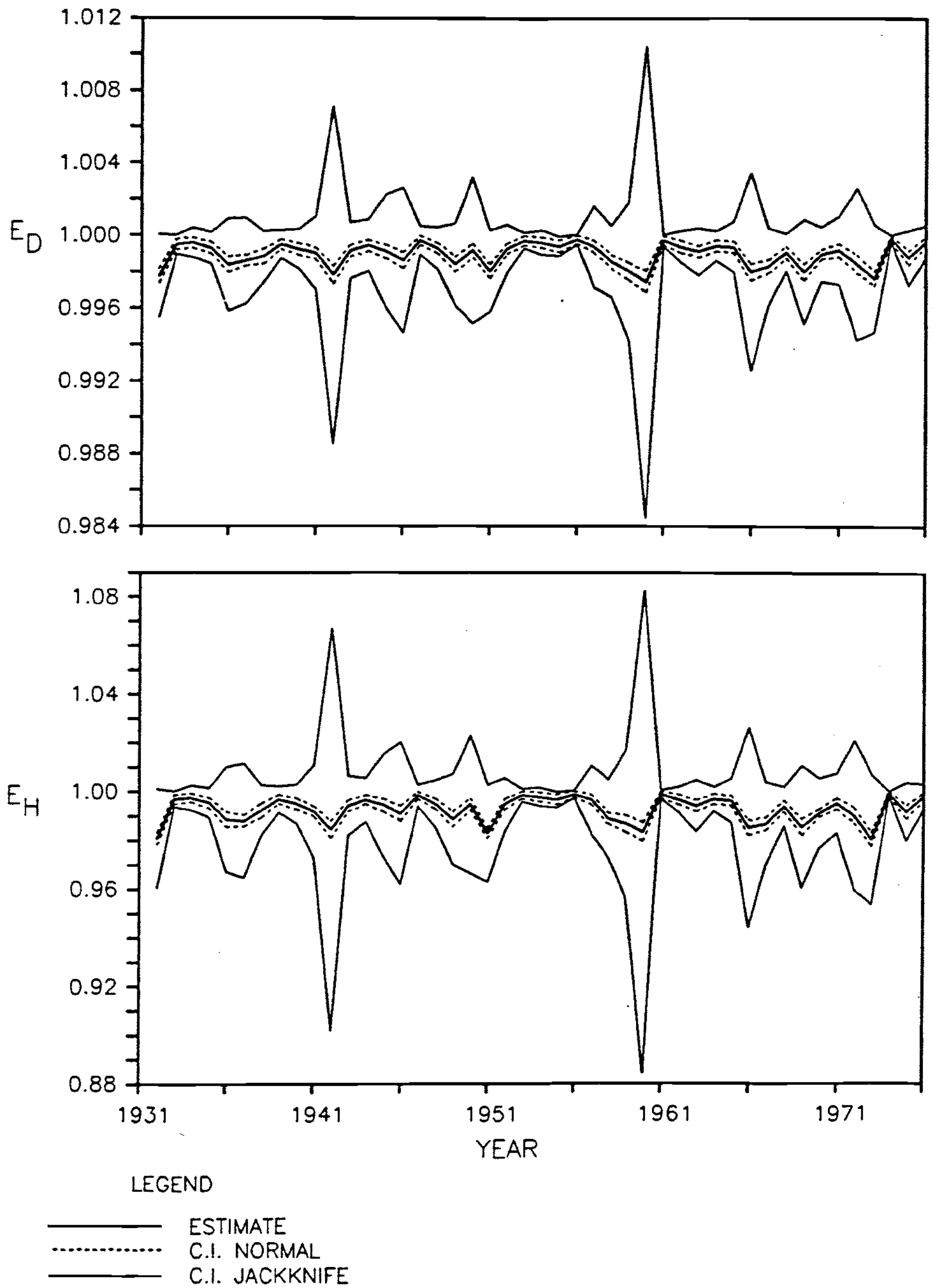

Figure 1. Time trends of biweight signal strengths for 40 jack pine trees from site CM8 and confidence intervals based on the normal approximation and on the jackknife procedure.

DISCUSSION AND SUMMARY

The evenness indices proposed to measure the signal strengths of biweight chronologies measure the regularity of the relative weights assigned by the biweight procedure. Calculated in this way, these indices do not necessarily reflect the regularity of the ring-width indices themselves. For example, it is possible to obtain nearly equal relative weights from a sample of ring-width indices that are, in fact, quite variable. This is acceptable because the objective is to measure the signal strength obtained from the biweight; the classical procedures may always be applied to the ring-width indices themselves. The proposed measures are most useful for comparing the signal strengths obtained by different methods of detrending and standardizing ring-width data prior to applying the biweight, or by different formulations of the biweight. Modifications in the proposed measures would allow inferences to be made about the underlying variability of ring widths.

The procedure of estimating signal strengths based on evenness indices may be applied to mean-value functions other than the biweight, provided that the input numbers can be expressed in proportional terms. For an average ring-width index chronology, the proportions could be the percentage of the total within-year sum of squares associated with each individual tree. Such an approach would more clearly measure the signal strength as reflected in the ring-width indices. In addition, other diversity indices could be considered. Some practical experience could be gained by comparing results obtained by the classical methods with those obtained by the methods proposed here.

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