IDENTIFYING LOW-FREQUENCY TREE-RING VARIATION

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ABSTRACT

I propose an approach to provide 95% confidence intervals for a chronology of low-frequency tree-ring variation so that a level of significance or importance for trends can be inferred. The approach also visually reveals the portions of a chronology in which sample depth is so poor that low-frequency variation is not robustly estimated. A key characteristic of the approach is that it is essentially a reordering of the individual steps commonly used in constructing standard tree-ring chronologies; consequently, it is computationally simple for researchers who already routinely construct standard tree-ring chronologies. The most important ramification of the approach is that each year of the chronology has a distribution of smoothed index values with which to estimate confidence intervals around the chronology of low-frequency variation. It can be argued that the approach constitutes multiple significance testing of means, which causes the $\alpha$ level for the confidence interval to be unknown. Nonetheless, the approach is still useful in that it provides a way to evaluate the probable importance of low-frequency trends expressed in tree-ring chronologies.
INTRODUCTION

The dendrochronological literature is replete with studies that, in part, identify and interpret low-frequency variation, e.g., period length >10 years, as expressed by tree-ring chronologies (e.g., LaMarche 1974, Jacoby et al. 1985, Norton et al. 1989, Briffa et al. 1990). A common approach to identifying low-frequency tree-ring variation is (1) to construct a standard index chronology using several samples from a homogeneous stand of trees (Fritts 1976), (2) to generate a smoothed index series from that standard chronology, e.g., with a cubic smoothing spline (Cook and Peters 1981), and (3) to overlay plot the standard chronology with its smoothed index series. Trends in low-frequency variation, i.e., departures from a reference line, are commonly interpreted as possibly being caused by a biological or physical mechanism.

There are, however, two inadequacies with this approach as stated, both of which arise from the fact that the smoothed index series is generated from only a single time series. First, the smoothed index series does not have accompanying confidence intervals with which trends might be evaluated as being significant or important. Second, the smoothed index series does not reflect changing sample depth through time, even though sample depths of tree-ring chronologies commonly range from as low as a single sample at the beginning year to a maximum value sometime before the ending year. While it might be tempting to interpret low-frequency trends for the full length of a chronology, there should be some distinction between well-replicated portions of a chronology, where low-frequency variation is robustly estimated, and poorly replicated portions of a chronology, where low-frequency variation is weakly estimated (Shiyatov et al. 1990).

I propose an approach to overcome these two inadequacies. Specifically this approach provides approximate 95% confidence intervals for a chronology of low-frequency variation so that a level of significance or importance for trends may be inferred. The approach also visually reveals the portions of a chronology in which sample depth is so poor that low-frequency variation is not robustly estimated, i.e., when the mean value is potentially affected by the addition or deletion of a single sample. Essentially, this approach reorders the individual steps commonly used in constructing standard tree-ring chronologies. Consequently, it is computationally simple for researchers who already routinely construct standard tree-ring chronologies, especially those who have access to the library of tree-ring data-reduction programs compiled under the auspices of the International Tree-Ring Data Bank, Paleoclimatology Program, National Geophysical Data Center, National Oceanic and Atmospheric Administration.

METHODS

Constructing a Standard Tree-Ring Chronology

A standard tree-ring chronology is constructed by the method described by Fritts (1976) and outlined in Table 1, left column. The deterministic, series-length trend of each individual ring-width series is removed by fitting and dividing out either a modified negative exponential curve (Fritts et al. 1969) or a linear regression line. This step removes variation at frequencies longer than the fundamental frequency of 1/n, where n is the series length; frequencies longer than the series length are difficult to interpret with respect to causal mechanisms because they can not be differentiated from pure trend (Cook et al. 1990). Finally, the detrended index series are averaged to form a standard chronology.
Table 1. Steps in constructing a standard tree-ring chronology and a chronology of low-frequency variation with associated confidence intervals

<table>
<thead>
<tr>
<th>Standard Tree-Ring Chronology</th>
<th>Low-Frequency Chronology</th>
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<tbody>
<tr>
<td>1. Create index series by removing the series-length trend from each ring-width series.</td>
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<tr>
<td>2. Average the index series into a standard chronology.</td>
<td>2. Determine the variation frequency of interest (3).</td>
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<tr>
<td>3. Determine the variation frequency of interest.</td>
<td>3. Generate a smoothed low-frequency series for each index series (4).</td>
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<tr>
<td>4. Generate a smoothed low-frequency series for the single, standard chronology.</td>
<td>4. Average the smoothed index series into a chronology of low-frequency variation (2).</td>
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<tr>
<td>5. Overlay plot the standard chronology and the smoother low-frequency series.</td>
<td>5. Overlay plot the standard chronology and the chronology of low-frequency variation (5).</td>
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<tr>
<td>6. Calculate the 95% confidence interval for each mean value of the low-frequency chronology.</td>
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<tr>
<td>7. Overlay plot the 95% confidence intervals with the low-frequency chronology.</td>
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¹ Number in parentheses refers to corresponding step in constructing a standard tree-ring chronology (left column).

Determining the variation frequency of interest can be accomplished in various ways. In some cases, researchers may hypothesize a priori the existence of a specific causal mechanism, with a known frequency of variation, that might affect tree growth. Such mechanisms include regimes of periodic wildfire, cyclic budworm infestations, or any of the many solar cycles that might affect climate. In these cases, researchers have essentially already determined the frequency of variation that is pertinent to their study.

Lacking an a priori hypothesis, researchers could use the standard chronology itself to determine the frequency of variation to analyze. If a chronology expresses a sine wave pattern, researchers could simply approximate the frequency of variation visually, as observed in a time-series plot of the chronology, or they could analyze the time series for its peak spectral density value to determine the frequency more precisely (Mazepa 1990).

Having determined the variation frequency of interest, a smoothed index series to represent that frequency is generated by fitting an appropriate cubic smoothing spline to the standard chronology. I fit the spline with a frequency response retention of 90% of the variation frequency of interest. Finally, the smoothed index series of low-frequency variation is plotted over the standard chronology.
Reordered Approach

The reordered approach uses essentially the same steps used to construct the standard tree-ring chronology, but the steps are executed in a different order (Table 1, right column). Indeed, as a result of having constructed the standard tree-ring chronology, the individual index series are already created, and the pertinent frequency of variation is already determined. Smoothed index series to represent that frequency are generated by fitting to all individual index series a cubic smoothing spline with the same rigidity (frequency response retention) that was used on the standard chronology. Fitting the spline to all index series produces a distribution of smoothed index values for each year of the chronology. Averaging the smoothed index values for each distribution, i.e., for each year of the chronology, creates a chronology of low-frequency variation.

The most important implication of the fact that each year of the chronology now has a distribution of smoothed index values is that confidence intervals around the chronology of low-frequency variation can be estimated. One method for estimating confidence intervals around tree-ring data is bootstrap sampling, i.e., sampling the distributions of values for up to hundreds — or even thousands — of times, as demonstrated by Cook (1990) for smoothed ring-width chronologies. Cook opted for bootstrap estimation of confidence intervals primarily because distributions of smoothed ring-width values may not be normal.

However, a useful side effect of standardizing the ring-width series into index series is that distributions of smoothed index values are less likely to violate the assumption of normality. Consequently, I estimate confidence intervals by using the usual parametric equation (Sokal and Rohlf 1981), which is computationally less arduous than bootstrap estimation. Thus, given a distribution of smoothed index values for each year, the 95% confidence intervals are calculated as follows:

\[
95\% \text{ confidence interval}_t = \bar{x}_t \pm s_{\bar{x}(t)} \cdot t_{0.05}[d.f.(t)]
\]

where \(\bar{x}_t\) is the mean smoothed index value of the \(t^{th}\) year, \(s_{\bar{x}(t)}\) is the standard error of the mean of the \(t^{th}\) year, and \(t_{0.05}[d.f.(t)]\) is the critical value of the Student’s \(t\) distribution for \(\alpha = 0.05\) and degrees of freedom (sample depth - 1) of the \(t^{th}\) year (Rohlf and Sokal 1981). Finally, the associated 95% confidence intervals are plotted over the low-frequency chronology, and the presence or absence of departures from a reference line is interpreted.

RESULTS

Coddington Lake, Minnesota (47°44’N, 94°03’W, 128 m), White Oak (Quercus alba)

The Coddington Lake oak standard tree-ring chronology clearly expresses sine wave variation (Figure 1a). Indeed, this chronology demonstrates a strong low-frequency signal, i.e., variation held synchronously across samples (Cook et al. 1990), as evidenced by a representative subset of three of the site’s individual detrended index series (Figure 1b). Spectral analysis was used to determine that the prominent frequency of variation in this standard chronology is at 74 years, and the 43-year cubic smoothing spline was generated to retain 90% of the 74-year-period variation (Figure 1a).

Each individual index series was also fit with a 43-year spline (Figure 1b), i.e., the same spline that was used with the standard chronology, and these spline series were averaged into a chronology of low-frequency variation. This low-frequency chronology (Figure 1c) appears
Figure 1. Coddington Lake, Minnesota, white oak time series: (a) standard index chronology with a 43-year spline series, (b) three representative individual index series, each with a 43-year spline series, (c) low-frequency chronology (solid line) with its associated 95% confidence intervals (dashed lines), and (d) chronology sample depth through time.
to be very similar to the 43-year spline generated from the standard chronology (Figure 1a) with differences restricted to the first few decades of the chronology.

The slight flaring of the 95% confidence intervals in the late part of the chronology (after 1970, Figure 1c) occurs because smoothing spline values are successively less influenced at that point by future index values, which do not exist. This flaring illustrates the end-effect phenomenon, whereby values at the end of a time series exert more influence on an estimated fit line than do values in the middle of a time series. The much wider flaring of the 95% confidence intervals in the early part of the chronology (Figure 1c) occurs because of low sample depth (Figure 1d), which leads to poor estimation of the standard error of the mean and to large critical values of the Student's t distribution that are associated with few degrees of freedom (Rohlf and Sokal 1981). The flaring of 95% confidence intervals with low sample depth should occur in most applications of this approach.

The low-frequency chronology and its 95% confidence intervals permit visual identification of important low-frequency tree-ring variation (Figure 1c). Negative departures from 1825 to 1855 and from 1890 to 1935 are significant and worthy of interpretation, as are positive departures from 1855 to 1890 and from 1935 to 1980. Furthermore, by the erratic nature of the low-frequency chronology and its wide 95% confidence intervals prior to 1820 (Figure 1d), it is apparent that low-frequency variation is not robustly estimated during the chronology's first 50 years, primarily due to poor sample depth (Figure 1d). Interpreting low-frequency variation of this chronology prior to 1820 is not prudent.

Gaylor Lakes, California (37°55'N, 119°16'W, 3,100 m), Lodgepole Pine (Pinus contorta)

Because lodgepole pines at Gaylor Lakes are growing at the upper treeline, I suspected that their growth might be sensitive to variation in ambient temperature (Fritts 1976). Therefore, I hypothesized a priori that tree growth at this site might reflect influences of the climatic anomaly referred to as the Little Ice Age (~1550 to the late 1800s), which is presumed to have been a period of below-average temperatures (Lamb 1977). Consequently, for this chronology I decided to investigate low-frequency tree-ring variation at the 100-year period. This variation frequency was represented by the 58-year spline, which retained 90% of the 100-year-period variation expressed by the standard chronology (Figure 2a). Individual index series were also fit with a 58-year spline, i.e., the same spline that was used with the standard chronology, and these spline series were averaged into a chronology of low-frequency variation. This low-frequency chronology (Figure 2b) appears to be very similar to the 58-year spline generated from the standard chronology (Figure 2a); as with the Coddington Lake oak site, differences between these two smoothed series are restricted to the first several decades of the chronology.

The low-frequency chronology and its 95% confidence intervals (Figure 2b) show that relatively little of the 100-year-period variation departs significantly from the 1.0 reference line. Only the negative departure at Gaylor Lakes is growing at the upper treeline, I suspected that their growth might be sensitive to variation in ambient temperature (Fritts 1976). Therefore, I hypothesized a priori that tree growth at this site might reflect influences of the climatic anomaly referred to as the Little Ice Age (~1550 to the late 1800s), which is presumed to have been a period of below-average temperatures (Lamb 1977). Consequently, for this chronology I decided to investigate low-frequency tree-ring variation at the 100-year period. This variation frequency was represented by the 58-year spline, which retained 90% of the 100-year-period variation expressed by the standard chronology (Figure 2a). Individual index series were also fit with a 58-year spline, i.e., the same spline that was used with the standard chronology, and these spline series were averaged into a chronology of low-frequency variation. This low-frequency chronology (Figure 2b) appears to be very similar to the 58-year spline generated from the standard chronology (Figure 2a); as with the Coddington Lake oak site, differences between these two smoothed series are restricted to the first several decades of the chronology.

The low-frequency chronology and its 95% confidence intervals (Figure 2b) show that relatively little of the 100-year-period variation departs significantly from the 1.0 reference line. Only the negative departure from 1785 to 1870 and the positive departure from 1870 to 1910 are strong enough so that the associated 95% confidence intervals do not enclose the 1.0 reference line (Figure 2b). If the low-frequency variation of this chronology were associated with low-frequency variation in ambient temperature, the negative departure might represent a response to the latter third of the Little Ice Age. However, by the erratic nature of the low-frequency chronology and its wide 95% confidence intervals prior to 1680 (Figure 2b), it is apparent that low-frequency variation is not robustly estimated during the chronology's first 170 years, primarily due to poor sample depth (Figure 2c). Interpreting low-frequency variation of this chronology prior to 1680 is not prudent.
Figure 2. Gaylor Lakes, California, lodgepole pine time series: (a) standard index chronology with a 58-year spline series, (b) low-frequency chronology (solid line) with its associated 95% confidence intervals (dashed lines), and (c) chronology sample depth through time.
DISCUSSION

At first glance, this approach appears to provide a basis for testing the significance of low-frequency departures from the 1.0 reference line. It could be argued, however, that this approach also constitutes multiple significance testing of means, which causes the \( \alpha \) level for the confidence interval to be unknown (Sokal and Rohlf 1981); because of this, it is difficult to know the level of significance for low-frequency tree-ring trends. Assuming that a chronology constitutes a single experiment of multiple tests (Miller 1981), i.e., a collection of as many tests of significance as the chronology is long, a new error level \( (\alpha') \) would be necessary for each test in order to yield a known error level \( (\alpha) \) for the single experiment. The new error level could be determined as follows:

\[
\alpha' = 1 - (1 - \alpha)^{1/k}
\]

where \( \alpha' \) is the error level for each of the \( k \) tests and \( \alpha \) is the error level of the single experiment (Sokal and Rohlf 1981). In the case of the Coddington Lake chronology, for an experiment with \( \alpha = 0.05 \) and \( k = 215 \) tests, \( \alpha' \) must be \( \approx 0.0002 \). Regardless of sample depth for any year of the chronology, the critical values of the Student’s \( t \) distribution that correspond to an \( \alpha' \) of \( 0.0002 \) are very large. Such a conservative adjustment would make it highly unlikely that any departure could be deemed significant at the experiment \( \alpha \) level of 0.05.

However, adjusting the \( \alpha \) level to account for multiple significance testing assumes that the \( k \) tests are independent, which clearly is not true in this case; indeed, the low-frequency chronology is an example of a time series with extreme autocorrelation. Because of this, the number of actual tests \( (k) \) should be adjusted downward to a number of effective, independent tests \( (k') \) as follows:

\[
\frac{k'}{k' - 1} = \left[ 1 - \frac{1 - r^2}{k(1 - r)^2} + \frac{2r(1 - r^k)}{k^2 (1 - r)^2} \right]^{-1}
\]

where \( k' \) is the number of effective, independent tests, \( r \) is the estimated AR(1) coefficient of the low-frequency chronology, and \( k \) is the number of actual tests, i.e., the length of the chronology (Dawdy and Matalas 1969, modified from equations 8-III-41 and 8-III-42). In the case of the Coddington Lake low-frequency chronology with an AR(1) coefficient estimated to be 0.9977, the \( k \) of 217 actual tests is reduced to a \( k' \) of \( \approx 1 \) effective, independent test, which nullifies the adjustment of \( \alpha \) levels to account for multiple significance testing. Thus, the desired experiment \( \alpha \) level of 0.05 is approximately obtained even when the \( \alpha' \) level of 0.05 is used for the calculation of the confidence interval around each mean. Because any low-frequency chronology constructed using this approach will be extremely autocorrelated, i.e., AR(1) \( \rightarrow 1.0 \), this correspondence between \( \alpha \) and \( \alpha' \) will be true in general when using this approach. Thus, given that the experiment error level is considered to be known, i.e., \( \alpha = 0.05 \), this approach provides the basis for identifying periods of a chronology where low-frequency variation departs significantly from the 1.0 reference line.

However, if the experiment error level is considered to be known only approximately, this approach remains useful in that it still provides the means to evaluate the probable importance of low-frequency departures expressed in tree-ring chronologies. In the Coddington Lake oak chronology (Figure 1c), the approximate 95% confidence intervals associated with trends since 1790 clearly do not enclose the 1.0 reference line; it cannot be argued reasonably
that these trends do not exist. Thus, this approach demonstrates that the sine wave variation expressed by the Coddington Lake oak chronology is real and important, and an investigation into the mechanism that caused this low-frequency variation is warranted.

Similarly, even if a level of significance cannot be stated precisely for the trends expressed by the Gaylor Lakes lodgepole pine chronology (Figure 2b), it still appears that this chronology expresses an important negative departure from 1785 to 1870. Thus, this approach demonstrates that this chronology may confirm the influence of the Little Ice Age at Gaylor Lakes, California. However, this approach also demonstrates that the sample depth of the Gaylor Lakes lodgepole pine chronology is not adequate from its beginning year to 1680, and it is not prudent to interpret the effects of the Little Ice Age in this part of the chronology until its sample depth is improved.

CONCLUSIONS

I recommend that this approach be used especially in studies that have the primary objective of identifying and interpreting low-frequency tree-ring variation. By merely reordering the individual steps used to construct the standard tree-ring chronology, this approach provides a low-frequency chronology with accompanying confidence intervals. Regardless of whether an exact level of significance can be stated for low-frequency tree-ring variation, this approach provides the means to evaluate the importance of trends expressed by standard chronologies.

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