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GAIN BANDWIDTH EFFECTS AND COMPENSATION IN TWO ACTIVE RC FILTERS

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GAIN BANDWIDTH EFFECTS AND COMPENSATION IN TWO ACTIVE RC FILTERS

by

John Sheridan Chaille

A Thesis Submitted to the Faculty of the

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In Partial Fulfillment of the Requirements
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Date

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

ABSTRACT

The subject of this thesis concerns the effects that the introduction of the concept of gain bandwidth into the modeling of an operational amplifier produces on the performance of two active RC filters. Further, it introduces solutions that compensate for these effects. The two filters under consideration are the second-order low pass Sallen and Key design and a second-order low pass infinite gain multiple path feedback design.

CHAPTER 1

INTRODUCTION

When operational amplifiers are used in standard analog signal electronics, some type of simplified mathematical description must be used to describe their performance. To use a complete description of an operational amplifier would greatly complicate the mathematics and, in most cases, add little to predicting its behavior. Indeed, operational amplifier designers have made it a goal to achieve a stable amplifier that can be simply modeled. In active filter design, it is highly desirable to know how the bandwidth of the filter can be affected by the operational amplifier. This is because a filter's purpose is usually to pass certain frequencies and reject others so an operational amplifier's frequency-dependent performance must be included.

It is the purpose of this thesis to first introduce a further degree of sophistication to the simple, or ideal, model of the operational amplifier and then to present a method for designing two different active low pass filters using this new model.

CHAPTER 2

GAIN BANDWIDTH EFFECTS

2.1 The Ideal Model

The operational amplifier in Figure 2.1 is assumed to be ideal if it meets the following three requirements.

1. The input impedance is so small it can be assumed to be zero.
2. The output impedance is so large that it can be assumed to be infinite.
3. The open loop voltage gain, $\frac{V_o}{V_i}$, is so large that it can be assumed to be infinite.

It could be added as a fourth requirement that the ideal operational amplifier not be used at too high a frequency. In active filter circuits frequencies are often high, so what is meant by "too high" must be clarified. Further, if there is some frequency that is too high, what would describe the operational amplifier as one approaches this frequency? What is needed is an operational amplifier description that includes its frequency response so one can know what limitations this will impose on the filter.

2.2 Gain Bandwidth in the Operational Amplifier

The actual frequency response of an operational amplifier is a complicated expression. Most operational amplifiers, before they can be

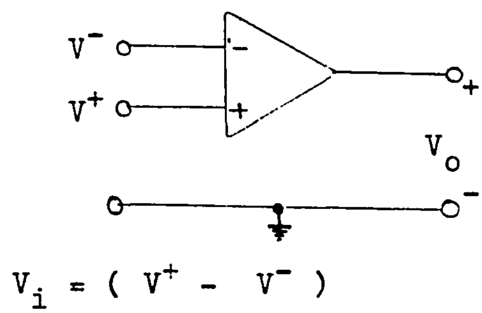

Figure 2.1. The operational amplifier.

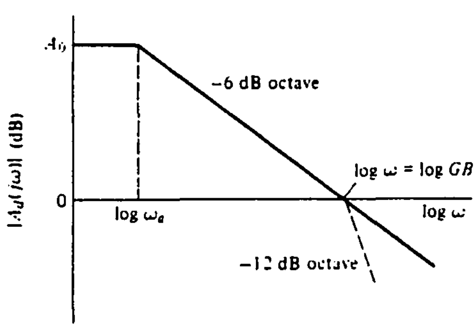

Figure 2.2. The frequency response for a dominant pole model [2].

used in active filters or most any other application, must be frequency compensated to produce stable operation. The objective of most compensation schemes is to produce an amplifier with a simple single pole characteristic. This is done by adding an internal, or in some cases external, capacitance to the amplifier so that the gain is one, or less, at a frequency lower, or equal to, the frequency where the second pole is located. This will insure stable operation of the operational amplifier regardless of any feedback scheme that it might be used in because the gain will be one, or less, before the phase can reach -180°. The frequency response of such a single or dominant pole operational amplifier is shown in Figure 2. This figure is a plot of the magnitude versus radian frequency of the following transfer function for a frequency compensated dominant pole model of an operational amplifier

$$A_d(s) = \frac{V_o}{V_i} = \frac{A_o \omega_a}{s + \omega_a} = \frac{GB}{s + \omega_a} \qquad (2.1)$$

where

A_o = the DC gain of the operational amplifier

ω_a = the 3 dB bandwidth of the operational amplifier

GB = the gain bandwidth product or the unity gain bandwidth of the operational amplifier.

Typical values are $A_o = 10^5$ and $\omega_a = 10$ rad/s.

Now we can clarify what was too high a frequency in Section 2.1 for the ideal operational amplifier model. The gain bandwidth frequency, labeled log GB in Figure 2.2, is the limit of the operational amplifier's performance. At the GB frequency or any lower frequency, the operational amplifier has a gain of one or greater, but at a higher frequency the gain is less than one and it no longer functions as a stable amplifier.

2.3 Active Filter Circuits

Traditional frequency-selective filters have been made using passive devices, resistors, capacitors, and inductors. One of the problems with this approach is that passive filters inherently attenuate the signal, and another is the problem of obtaining precise inductance values. Also there is the fact that inductors are bulky. Figure 2.3 shows a block diagram of an RC amplified filter configuration which uses no inductors. This circuit has none of the above problems associated with passive filters. For a low pass filter, one of the main design considerations is the cutoff frequency, or the -3 dB frequency (ω_c). It is this frequency that is used to define the -3 dB bandwidth, BW, of this filter. Two other criteria that are used to specify the performance of a filter are Q, the sharpness or quality factor, and ω_n, the resonant frequency or frequency of peak magnitude. These are defined as

$$Q = \frac{\omega_n}{BW} \tag{2.2}$$

In general we can define the bandwidth as

BW = the difference between the frequencies at which the magnitude of the transfer function is 3 dB down from its peak magnitude at ω_n.

As can be seen from Figure 2.2, the GB is of the order of 10^6 radians/s so the BW of a low pass filter can extend to quite a range.

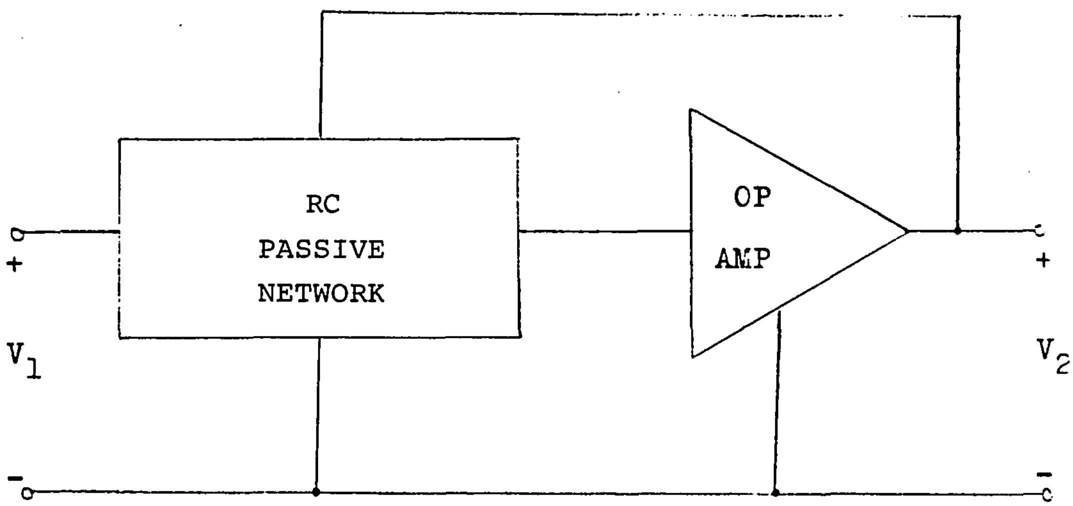

Figure 2.3. Block diagram of an RC amplifier filter.

For all values of ω_n greater than ω_a, Equation (2.1) can be approximated by

$$A_d(s) \simeq \frac{GB}{s} \qquad (2.3)$$

It is Equation (2.3) that will be used to model the effect GB has on operational amplifier performance in this paper.

2.4 Gain Bandwidth Effects Graph for a Sallen and Key Design

A second-order low pass Sallen and Key filter [1] design is shown in Figure 2.4. Considering the operational amplifier as ideal, the voltage transfer function is

$$\frac{V_2}{V_1} = \frac{K_i/N1}{s^2 + sD1 + DO} \qquad (2.4)$$

where

$$K_i = \frac{R_A + R_B}{R_A}$$

$$N1 = R_1R_3C_2C_4$$

$$D1 = \frac{1}{R_3C_4} + \frac{1}{R_1C_2} + \frac{1}{R_3C_2} - \frac{K_i}{R_3C_4}$$

$$DO = \frac{1}{R_1R_3C_2C_4}$$

If we include Equation (2.3) in this circuit, K_i changes to

$$K_i = \frac{1}{\frac{1}{A_d(s)} + \frac{R_A}{R_A + R_B}} = \frac{1}{\frac{s}{GB} + \frac{1}{K}}\ ; \quad K = \frac{R_A + R_B}{R_A}$$

and an analysis by Budak [3] makes the denominator of Equation (2.4)

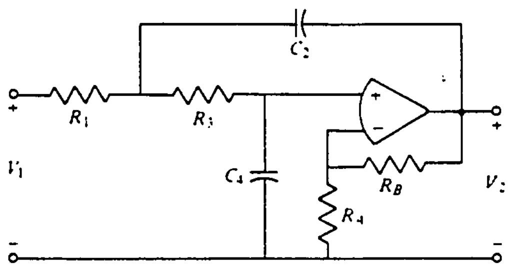

Figure 2.4. A second-order low pass Sallen and Key filter [2].

$$D(s) = \frac{C_2C_4}{GB}(s^3 + s^2 D2 + sD1 + DO) \qquad (2.5)$$

where for $R_1 = R_3 = R$ and $C_2 = C_4 = C$ we use[1]

$$\omega_n = \frac{1}{RC},$$

$$Q = \frac{1}{3 - K}$$

the coefficients for Equation (2.5) become

$$D2 = 3\omega_n + \frac{GBQ}{3Q - 1}$$

$$D1 = \omega_n^2 + \frac{GB\omega_n}{3Q - 1}$$

$$DO = \frac{GBQ\omega_n^2}{3Q - 1}$$

then using a frequency normalization by ω_n such that

$$s_n = s/\omega_n$$

and

$$GB_n = GB/\omega_n \qquad (2.6)$$

The denominator equation of (2.5) becomes

$$D(s_n) = \frac{C_2C_4\omega_n^2}{GB_n}(s_n^3 + s_n^2 D2_n + s_n D1_n + DO_n) \qquad (2.7)$$

where

---

1. These are the design parameters that would occur if the operational amplifier were ideal, as in Equation (2.4).

$$D2_n = 3 + \frac{GB_n Q}{3Q - 1}$$

$$D1_n = 1 + \frac{GB_n}{3Q - 1}$$

$$D0_n = \frac{GB_n Q}{3Q - 1}$$

Setting this denominator to zero and plotting the upper complex pole, as GB_n varies, using Q as a parameter we get Figure 2.5. The dashed lines are lines of constant GB_n, and the solid lines are lines of the design-for or intended Q and not lines of constant Q. Lines of constant Q would be straight and pass through the origin. The effect of GB can be seen to be a change in the value of Q that is realized and a lowering of the resonant frequency, ω_n. To illustrate this, assume that a normalized realization in which Q = 1.75 and $\omega_n = 1$ is desired. Also assume that the operational amplifier has a $GB_n = 2$. Drawing a radial line out from the origin that intersects the Q = 1.75 with $GB_n = \infty$ line gives a line of constant Q = 1.75. As can be seen in Figure 2.6, this line intersects the $GB_n = 2$ line with the Q = 10 line. Thus, due to the effects of GB, a realization designed for a Q = 10 and a $GB_n = 2$ has produced one with a Q of 1.75. Also note that the resonant frequency has dropped to approximately 0.43 rad/s. Although one can design for a different Q that will realize the desired or correct Q, it is not possible to frequency-denormalize the capacitors of Figure 2.4 to obtain the correct ω_n. This is because the GB of the chosen operational amplifier depends on capacitance and resistance internal to the semiconductor chip and therefore

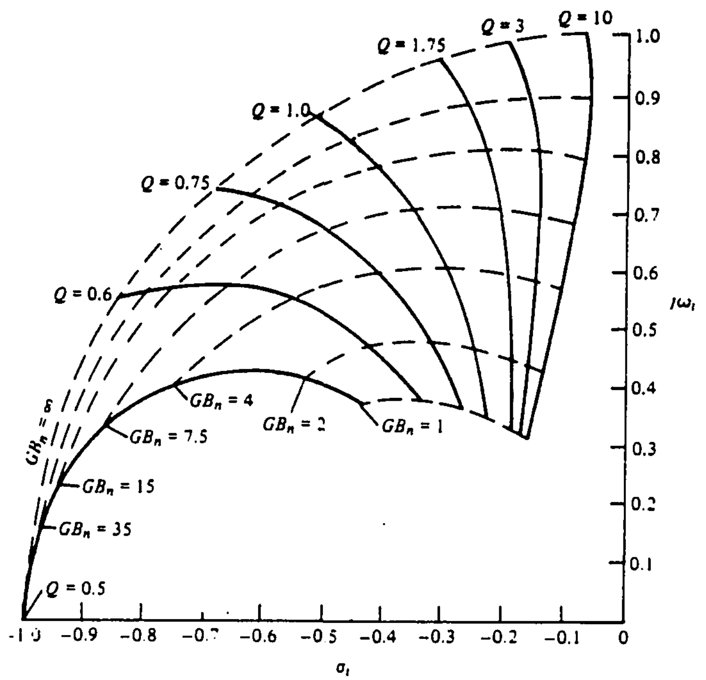

Figure 2.5. Effect of GB on the poles of a second-order low pass Sallen and Key equal R equal C filter design [3].

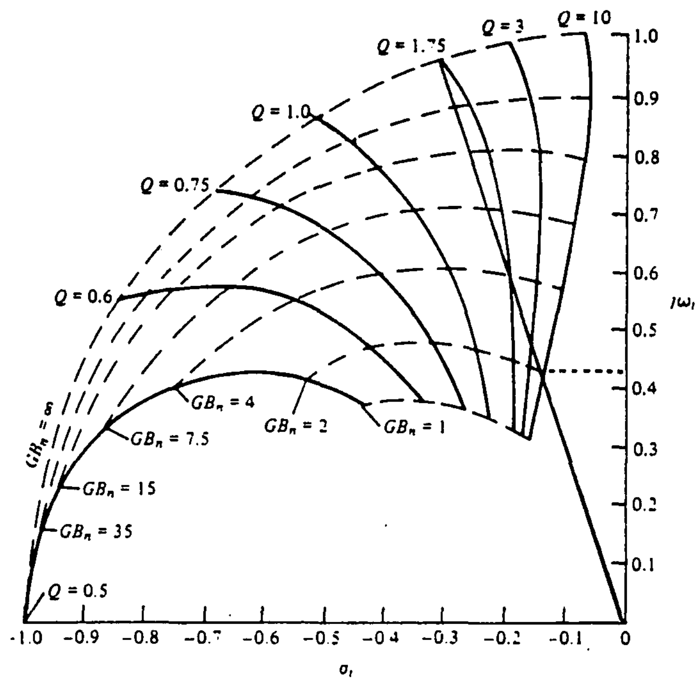

Figure 2.6. Effect of GB on a Q = 1.75 and $\omega_n = 1$ design in a low pass Sallen and Key equal R equal C filter [3].

cannot be frequency-denormalized. The dominant pole model for the operational amplifier that has given us a value for the GB cannot be changed.

2.5 Gain Bandwidth Effects Graph for an Infinite Gain Design

A second-order low pass infinite gain multiple path feedback filter is shown in Figure 2.7. Considering the operational amplifier as ideal, we get the following second-order voltage transfer function

$$\frac{V_2}{V_1} = \frac{N1}{s^2 + sD1 + DO} \tag{2.8}$$

where

$$N1 = -\frac{1}{R_1 R_3 C_5 C_6}$$

$$D1 = \frac{1}{C_5}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$DO = \frac{1}{R_2 R_3 C_5 C_6}$$

For this case, the design parameters Q and ω_n become

$$\omega_n = \sqrt{\frac{1}{R_2 R_3 C_5 C_6}}$$

$$Q = \frac{\sqrt{\frac{C_5}{C_6}}}{\sqrt{\frac{R_2}{R_3}} + \sqrt{\frac{R_3}{R_2}} + \sqrt{\frac{R_2 R_3}{R_1}}}$$

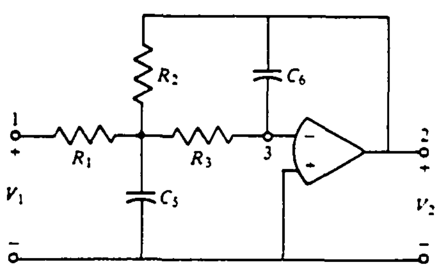

Figure 2.7. A second-order low pass infinite gain multiple path feedback design filter [2].

If we now include the dominant pole frequency response of Equation (2.3) for the operational amplifier and choose an equal R design, an analysis again done by Budak [4] makes the denominator of Equation (2.8)

$$D(s) = \frac{C_5 C_6}{GB} (s^3 + s^2 D2 + sD1 + D0) \qquad (2.9)$$

where for $R_1 = R_2 = R_3$ and incorporating the design parameters Q and ω_n above the coefficients for Equation (2.9) become

$$D2 = GB + \frac{\omega_n}{Q} + 3Q\omega_n$$

$$D1 = \frac{\omega_n GB}{Q} + 2\omega_n^2$$

$$D0 = \omega_n^2 GB$$

Using the frequency normalization of Equation (2.6), the denominator Equation (2.9) becomes

$$D(s_n) = \frac{C_5 C_6 \omega_n^2}{GB_n} (s_n^3 + s_n^2 D2_n + s_n D1_n + D0_n)$$

where (2.10)

$$D2_n = GB_n + \frac{1}{Q} + 3Q$$

$$D1_n = \frac{GB_n}{Q} + 2$$

$$D0_n = GB_n$$

If we set this denominator Equation (2.10) to zero and plot the upper complex pole as, GB_n varies, using Q as a parameter we get Figure 2.8.

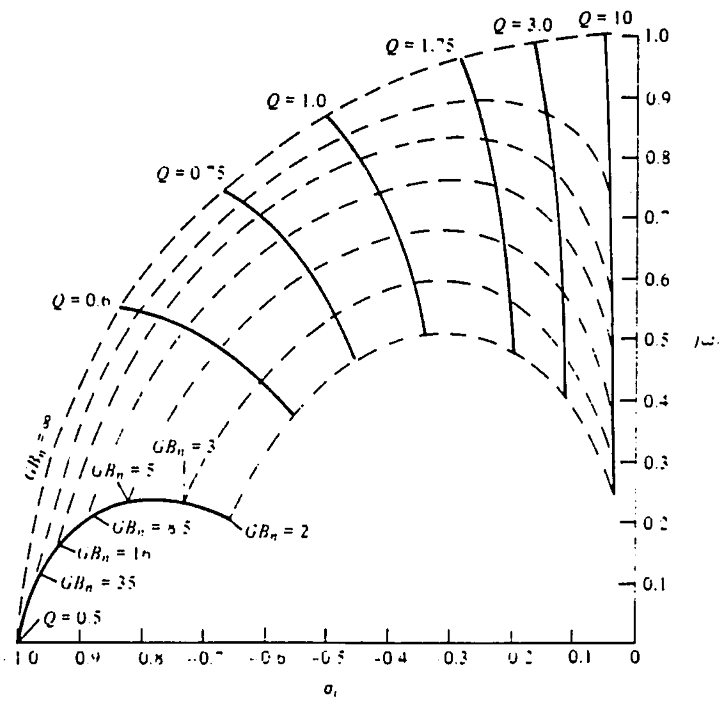

Figure 2.8. Effect of GB on the poles of a second-order low pass infinite gain multipath feedback filter design [4].

Again the dashed lines are lines of constant GB_n, and the solid lines are the designed-for or intended Q and not lines of constant Q. As was the case for the Sallen and Key design, the effect of the GB is a change in the value of Q that is realized and a lowering of the resonant frequency, ω_n. For example, assume that the operational amplifier has a $GB_n = 2$ and we want a filter with a normalized realization in which $Q = 1.75$ and $\omega_n = 1$. Drawing a straight radial line from the origin to the point where the $Q = 1.75$ line intersects the $GB_n = \infty$ line yields a line of constant $Q = 1.75$. As can be seen in Figure 2.9, this line intersects the $GB_n = 2$ line with (approximately) the $Q = 3.0$ line and the resonant frequency has dropped to approximately 0.42 rad/s. The effect that the GB has on this filter is similar to that given above for the Sallen and Key filter. And again, this chart cannot be used to effectively compensate for both the effects of GB. What is needed are new values for the filter's resistors and capacitors that will preserve the desired pole positions as specified by the design parameters Q and ω_n.

2.6 Compensation Using Optimization Techniques

For the Salley and Key low pass equal R equal C filter considered above, a procedure which preserves the desired pole positions has been done [5]. In this technique an optimization program is applied to the dominant pole transfer function, Equation (2.4). Using this approach, values for K_i, R and C that take account of their interaction with the GB so as to preserve the desired complex pole positions may be found. Table 2.1 is a partial list of these results. For example, for a

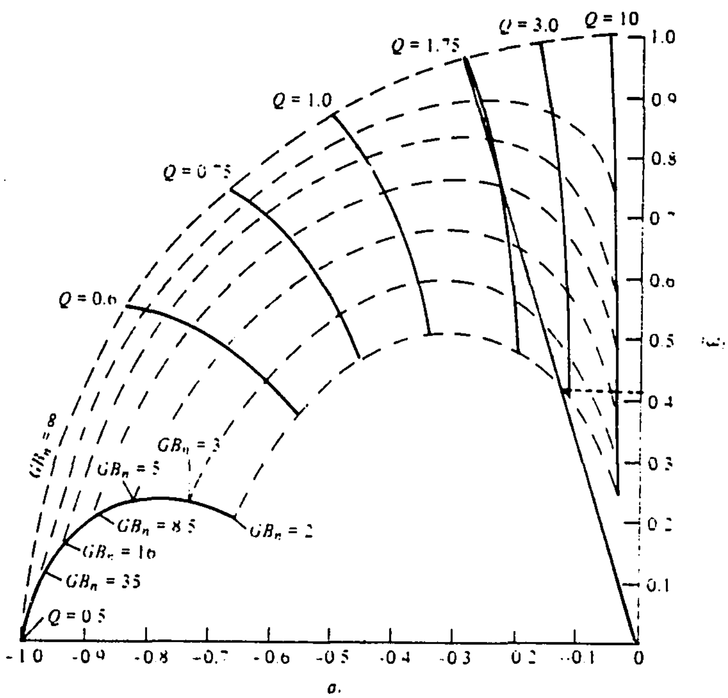

Figure 2.9. Effect of GB on a Q = 1.75 and $\omega_n = 1$ design in low pass infinite gain design [4].

Table 2.1. Compensation for GB effects using an optimization strategy for a low pass Sallen and Key design, $\omega_n = 1$ [5].

Q	GB_n	K	C
0.707	∞	1.586	1.000
	10	1.430	0.875
	4	1.275	0.735
	2	1.152	0.547
1.75	∞	2.429	1.000
	10	2.375	0.741
	8	2.397	0.682
	6	2.490	0.590
3.0	∞	2.667	1.000
	10	2.724	0.692
	8	2.810	0.623
	6	3.098	0.508
10.0	∞	2.900	1.000
	10	3.132	0.641
	8	3.332	0.560
	6	4.207	0.404

normalized realization where $Q = 3.0$, $GB_n = 10$ and $\omega_n = 1$ requires a $K_i = 2.724$ and a $C = 0.692$.

In the next two chapters it will be shown that the results obtained by the optimization approach can be attained by an exact closed-form solution for both the Sallen and Key and the infinite gain design. These solutions do not require sophisticated optimization techniques that need a large digital computer to handle the software for solving an equation in several unknowns. Instead, these closed-form solutions require only a pocket calculator to solve an equation in one unknown.

CHAPTER 3

GAIN BANDWIDTH IN A SALLEN AND KEY FILTER

3.1 The Gain Bandwidth Model

For the second-order low pass Sallen and Key filter of Figure 2.4, the voltage transfer function of Equation (2.4) becomes, for the equal R equal C case (using an ideal operational amplifier),

$$\frac{V_2}{V_1} = \frac{K_i/N1}{s^2 + sD1 + DO} \tag{3.1}$$

where for $R_1 = R_3 = R$ and $C_2 = C_4 = C$ the coefficients become

$$K_i = \frac{R_A + R_B}{R_A}$$

$$N1 = R^2C^2$$

$$D1 = \frac{3}{RC} - \frac{K_i}{RC}$$

$$DO = \frac{1}{R^2C^2}$$

If we now include the frequency response described by the dominant pole equation (2.3) for the operational amplifier, the gain constant K_i around the operational amplifier becomes

$$K_i \simeq \frac{GB}{s + GB/K}$$

where

$$K = \frac{R_A + R_B}{R_A} \tag{3.2}$$

Including this new frequency-dependent K_i in Equation (3.1) yields the following GB-dependent transfer function

$$\frac{V_2}{V_1} = \frac{N1}{s^3 + s^2 D2 + sD1 + DO} \tag{3.3}$$

for

$$K = \frac{R_A + R_B}{R_A}, \quad R_1 = R_3 = R \quad \text{and} \quad C_2 = C_4 = C$$

$$N1 = \frac{GB}{R^2C^2}$$

$$D2 = \frac{GB}{K} + \frac{3}{RC}$$

$$D1 = \frac{3GB}{KRC} - \frac{GB}{RC} + \frac{1}{R^2C^2}$$

$$DO = \frac{GB}{KR^2C^2}$$

3.2 A Solution for Fixing Pole Positions

To preserve the pole positions, a general third-order polynomial is constructed to match with the denominator in the above transfer function. This new denominator is factored into a first-order term and a quadratic term. The quadratic term incorporates the design parameters Q and ω_n. These two parameters fix the complex poles in their desired positions. The location of the real pole is unimportant so long as it remains in the left half plane. This new denominator polynomial is

$$D(s) = (s + g)(s^2 + \frac{\omega_n}{Q}s + \omega_n^2) \qquad (3.4)$$

Multiplying this out, we obtain

$$D(s) = s^3 + s^2(g + \frac{\omega_n}{Q}) + s(\omega_n^2 + \frac{g\omega_n}{Q}) + g\omega_n^2$$

If we normalize ω_n to equal one, then

$$D(s) = s^3 + s^2(g + \frac{1}{Q}) + s(1 + \frac{g}{Q}) + g \qquad (3.5)$$

for $\omega_n = 1$

Matching coefficients from the denominator of the transfer function to those of this new polynomial, we obtain the following set of equations:

1. $g = \frac{GB}{KR^2C^2}$

2. $g + \frac{1}{Q} = \frac{GB}{K} + \frac{3}{RC}$

3. $1 + \frac{g}{Q} = \frac{3GB}{KRC} - \frac{GB}{RC} + \frac{1}{R^2C^2}$

Using impedance scaling, we can pick one of the circuit elements to equal one. Choosing R = 1, the above three independent equations have only three unknowns: the real pole, g; the gain constant for the operational amplifier, K; and the capacitor, C. The GB is determined once the operational amplifier is chosen, and the value for Q fixes the complex poles position for the desired filter characteristic, i.e., Butterworth, elliptic, etc. Thus an exact solution exists for this filter for a given set of design parameters. Solving for C gives the fourth-order solution

$$C^4 + C^3A3 + C^2A2 + CA1 + 1 = 0 \qquad (3.6)$$

using the same frequency normalization by ω_n as was done in Equation (2.6), $GB_n = GB/\omega_n$

$$A1 = GB_n - \frac{3}{Q}$$

$$A2 = 7 + \frac{1}{Q^2}$$

$$A3 = -GB_n - \frac{3}{Q}$$

Although the solution for Equation (3.6) is fourth-order, only two roots were found that were not negative. Further, the two valid solutions produce values for K that are an order of magnitude or more apart. A value for K can be found from

$$K = \frac{GB_n(1 - 3QC)}{Q(1 - GB_nC - C^2)} \qquad (3.7)$$

and

$$R = 1$$

completes the solution.

3.3 Design Graphs

By using a root-solving routine [7] combined with a plotting program [8], design graphs for the two valid solutions are presented in Figures 3.1 and 3.2. Figure 3.2 represents the higher valued solution, and Figure 3.1 the lower valued. For example, an operational amplifier with a $GB_n = 15$ and a design calling for a $Q = 4$ would require a K of approximately 75, a C of 1/15, g = 45.6, from Figure 3.2, whereas Figure 3.1 gives a value of K near 2.7, and a C of approximately 1/1.3, g = 9.0.

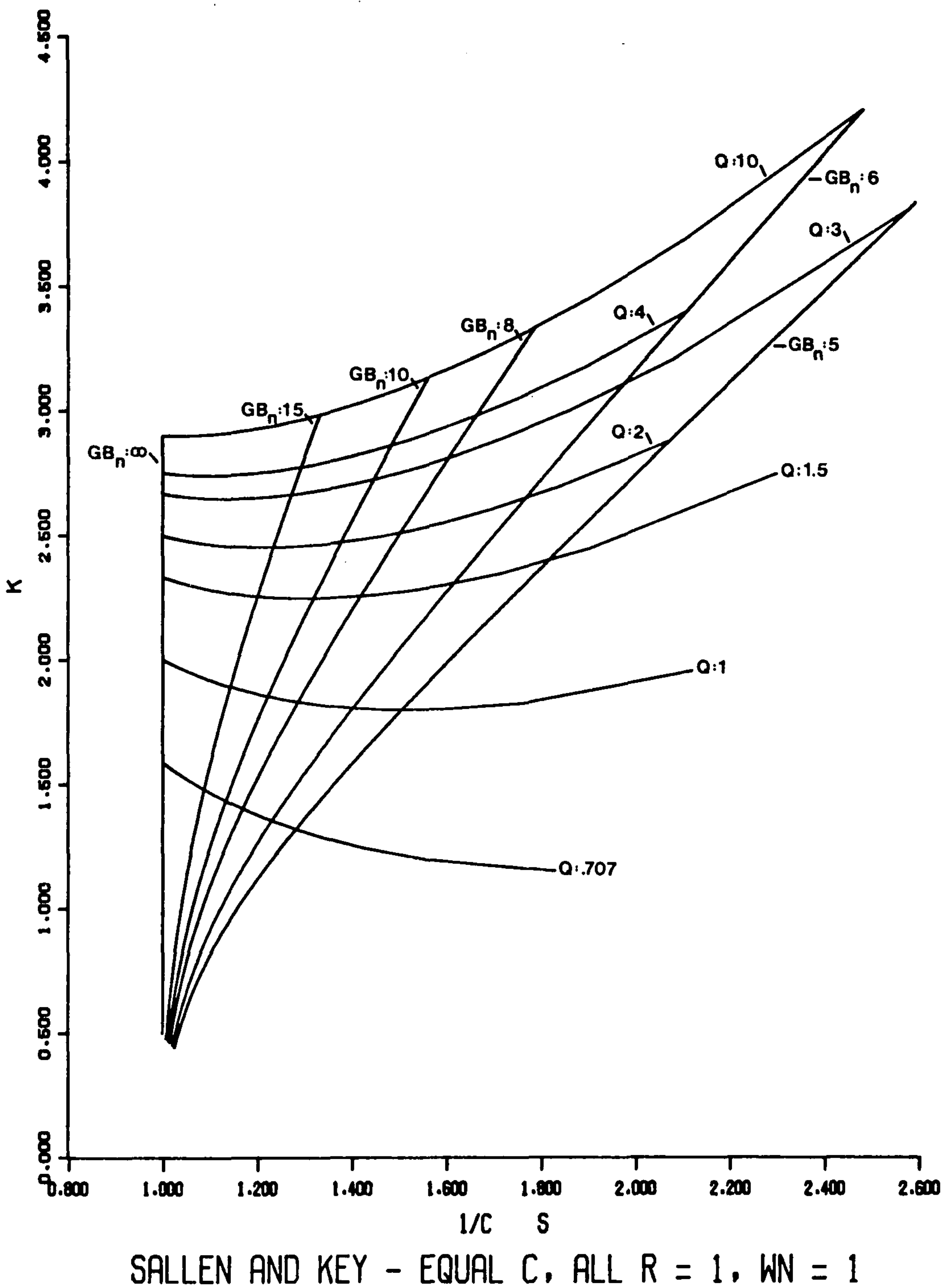

Figure 3.1. Design graph for a second-order low pass Sallen and Key filter design, lower valued solution.

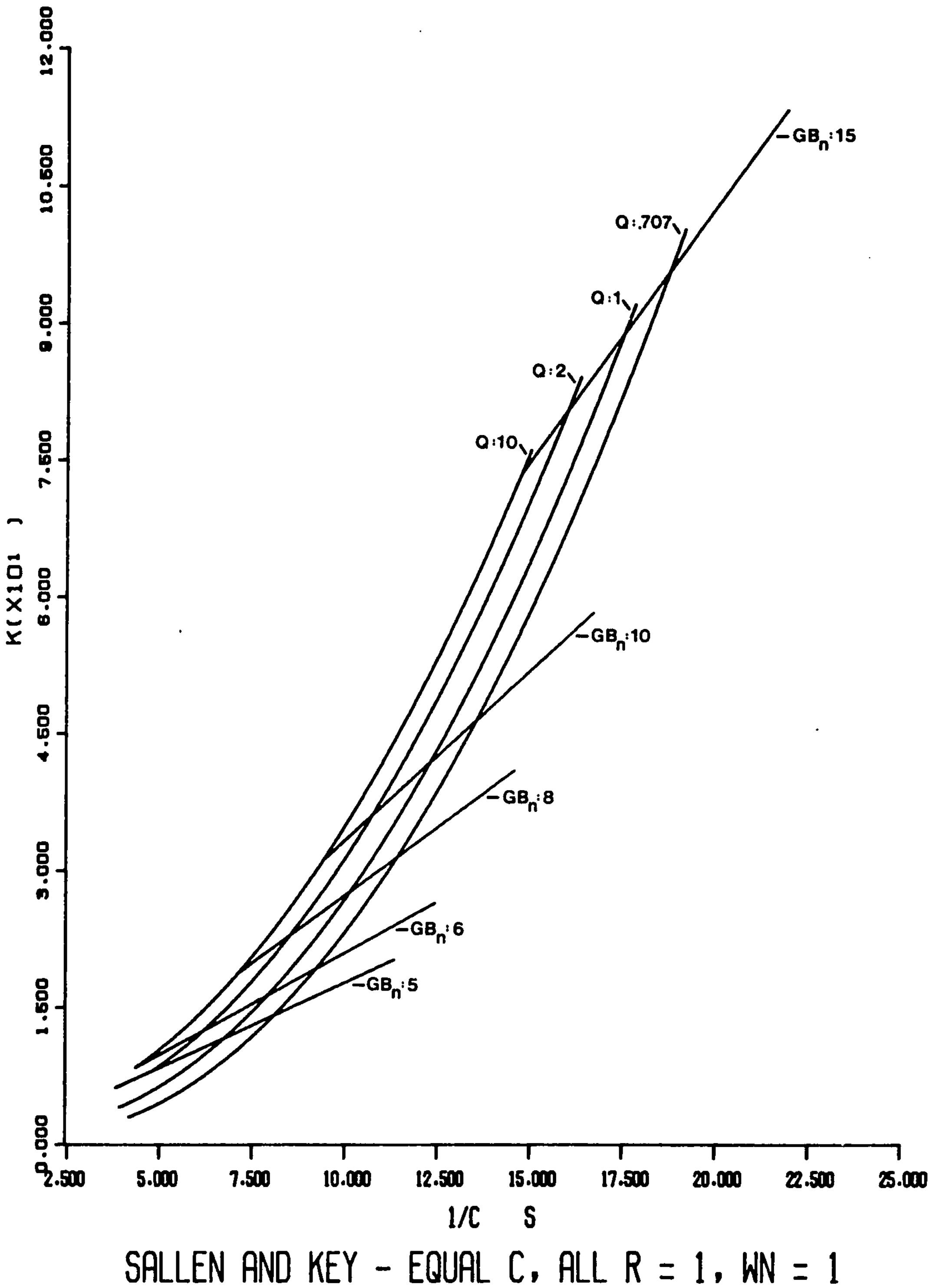

Figure 3.2. Design graph for a second-order low pass Sallen and Key filter design, higher valued solution.

These solutions are shown in Figures 3.3 and 3.4, respectively. As another example, for a $GB_n = 500$, $Q = .707$, and $\omega_n = 1$, the respective solution values are approximately $K = 84{,}000$, $C = 1/500$, $g = 1488.1$, and the lower root gives $K = 1.6$ and $C = 1$, $g = 312.5$. Obviously the lower value of gain is the preferred one.

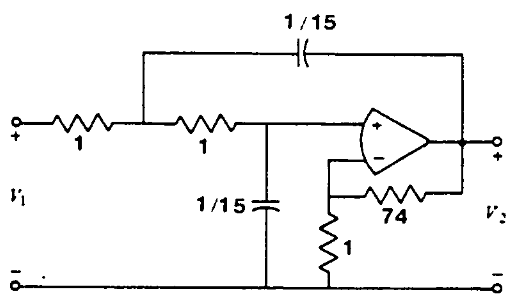

Figure 3.3. Higher valued solution for a low pass Sallen and Key design with a normalized $GB_n = 15$, $\omega_n = 1$ and $Q = 4$ [2].

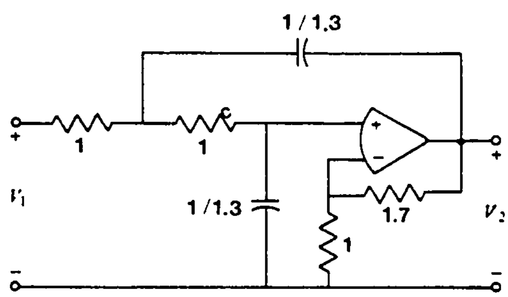

Figure 3.4. Lower valued solution for a low pass Sallen and Key design with a normalized $GB_n = 15$, $\omega_n = 1$ and $Q = 4$ [2].

CHAPTER 4

GAIN BANDWIDTH IN AN INFINITE GAIN DESIGN

4.1 The Gain Bandwidth Model

A second-order low pass infinite gain multiple path feedback filter is shown in Figure 2.7. Considering the operational amplifier as ideal, we get the following second-order voltage transfer function

$$\frac{V_2}{V_1} = \frac{N1}{s^2 + sD1 + DO} \qquad (4.1)$$

where

$$N1 = -\frac{1}{R_1R_3C_5C_6}$$

$$D1 = \frac{1}{C_5}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$DO = \frac{1}{R_2R_3C_5C_6}$$

If we now model the operational amplifier to include its frequency response, as specified by Equation (2.3), the new transfer function becomes third-order, as follows:

$$\frac{V_2}{V_1} = \frac{N1}{s^3 + s^2D2 + sD1 + DO} \qquad (4.2)$$

where

$$N1 = \frac{-GB}{R_1R_3C_5C_6}$$

$$D2 = \frac{1}{C_5}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) + \frac{1}{R_3C_6} + GB$$

$$D1 = \frac{1}{R_3C_5C_6}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{GB}{C_5}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$D0 = \frac{GB}{R_2R_3C_5C_6}$$

4.2 $R_1 = R_3$ Solution that Preserves Pole Positions

We choose the same general third-order polynomial that was used in the Sallen and Key solution to preserve pole positions (Equation (3.4)).

$$D(s) = s^3 + s^2\left(g + \frac{\omega_n}{Q}\right) + s\left(\omega_n^2 + \frac{g\omega_n}{Q}\right) + g\omega_n^2 \tag{4.3}$$

Matching coefficients from the denominator of the transfer function (Equation (4.2)) to those of this new polynomial (Equation (4.3)), we obtain the following set of three equations for $R_1 = R_3$:

1. $g\omega_n^2 = \dfrac{GB}{R_1R_2C_5C_6}$ (4.4)

2. $\omega_n^2 + \dfrac{g\omega_n}{Q} = \dfrac{1}{R_1C_5C_6}\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right) + \dfrac{GB}{C_5}\left(\dfrac{2}{R_1} + \dfrac{1}{R_2}\right)$

3. $g + \dfrac{\omega_n}{Q} = \dfrac{1}{C_5}\left(\dfrac{2}{R_1} + \dfrac{1}{R_2}\right) + \dfrac{1}{R_1C_6} + GB$

With $R_1 = R_3$, the following set of parameters is now available for designing the filter:

$|H_o| = R_2$, the magnitude of the dc gain of the filter

$Q = \dfrac{BW}{\omega_n}$, the sharpness or quality factor

ω_n = the resonant frequency

$GB_n = \frac{GB}{\omega_n}$, the frequency-normalized gain bandwidth

By using impedance scaling for R_1 and using the design parameters, the above set of three equations can be solved for the element values that will preserve the desired pole positions just as was done for the Sallen and Key design. The solution is:

$$C_5^2 + C_5 A1 + A0 = 0 \qquad (4.5)$$

where

$$A1 = \frac{GB_n Q\omega_n(\frac{1}{Q^2} - |H_o| - 1) - \omega_n(1 + |H_o| + GB_n^2)}{Q\omega_n^2|H_o|}$$

$$A0 = \frac{(1 + |H_o|)(2|H_o| + 1) + GB_n(2|H_o| + 1)(GB_n - \frac{1}{Q})}{\omega_n^2|H_o|^2}$$

By using the quadratic equation, the solution for C_5 becomes

$$C_5 = -\frac{A1}{2} - \sqrt{\frac{A1^2}{4} - A0} \qquad (4.6)$$

The rest of the solution becomes

$$C_6 = \frac{|H_o| + 1 - GB_n/Q}{\omega_n(|H_0|C_5\omega_n - GB_n(2|H_o| + 1))} \qquad (4.7)$$

$$R_2 = |H_o|$$

$$R_1 = R_3 = 1$$

4.3 $R_1 = R_2 = R_3$ Solution that Preserves Pole Positions

In this solution, impedance scaling is used to set all resistance values to one and frequency scaling is used to set ω_n equal to one. Then by matching the denominator coefficients of the transfer function (Equation (4.2)) to the same polynomial used above (Equation (4.3)), the following set of equations is obtained.

$$1. \quad g = \frac{GB}{C_5 C_6} \qquad (4.8)$$

$$2. \quad 1 + \frac{g}{Q} = \frac{2}{C_5 C_6} + \frac{3GB}{C_5}$$

$$3. \quad g + \frac{1}{Q} = \frac{3C_6 + C_5}{C_5 C_6} + GB$$

As in the previous two cases, this set of equations can be solved for the element values that will yield the desired pole positions. The solution for C_5 is the quadratic equation

$$C_5{}^2 + C_5 A1 + A0 = 0 \qquad (4.9)$$

using the same frequency normalization by ω_n, as was done in Equation (2.6), $GB_n = GB/\omega_n$

$$A1 = -\, 4GB_n - \frac{1}{Q^2}\,(QGB_n - 1)(GB_n - 2Q)$$

$$A0 = 3(GB_n{}^2 + 2 - \frac{GB_n}{Q})$$

By using the quadratic equation, the complete solution becomes

$$C_5 = -\frac{A1}{2} - \sqrt{\frac{A1^2}{4} - A0} \quad (4.10)$$

$$C_6 = \frac{GB_n/Q - 2}{3GB_n - C_5}$$

$$R_1 = R_2 = R_3 = 1$$

$$\omega_n = 1$$

After the element values have been found, they can be impedance-denormalized to any desired level and the resonant frequency moved where desired by frequency denormalizing to the actual GB frequency.

4.4 $R_1 = R_2 = R_3$ Design Graph

Figure 4.1 shows the design graph for the equal R solution. For example, an operational amplifier with a GB_n of 15 and a design calling for a Q of 4 with a normalized ω_n of one would require a C_5 of approximately the anti log of 0.93 or 8.5 and a C_6 of 1/anti log of 1.3 or 0.05, g = 35.3. This solution is shown in Figure 4.2. For a Q design of .707 and a $\omega_n = 1$ (a Butterworth filter), changes in the GB produce little change in element values. In this case, a $GB_n = 5{,}000$ will require a C_5 of near 2.1 and a C_6 of 0.47, g = 5,066.0, whereas a GB_n of 10 needs a C_5 of 1.8 and a C_6 of 0.43, g = 12.9.

4.5 Optimization Search for a Better Solution

Although the solutions presented by Equation (4.6) and Equation (4.10) are of a closed form and are simple to use, they produce results with rather a wide variation in capacitor values. For example, for a

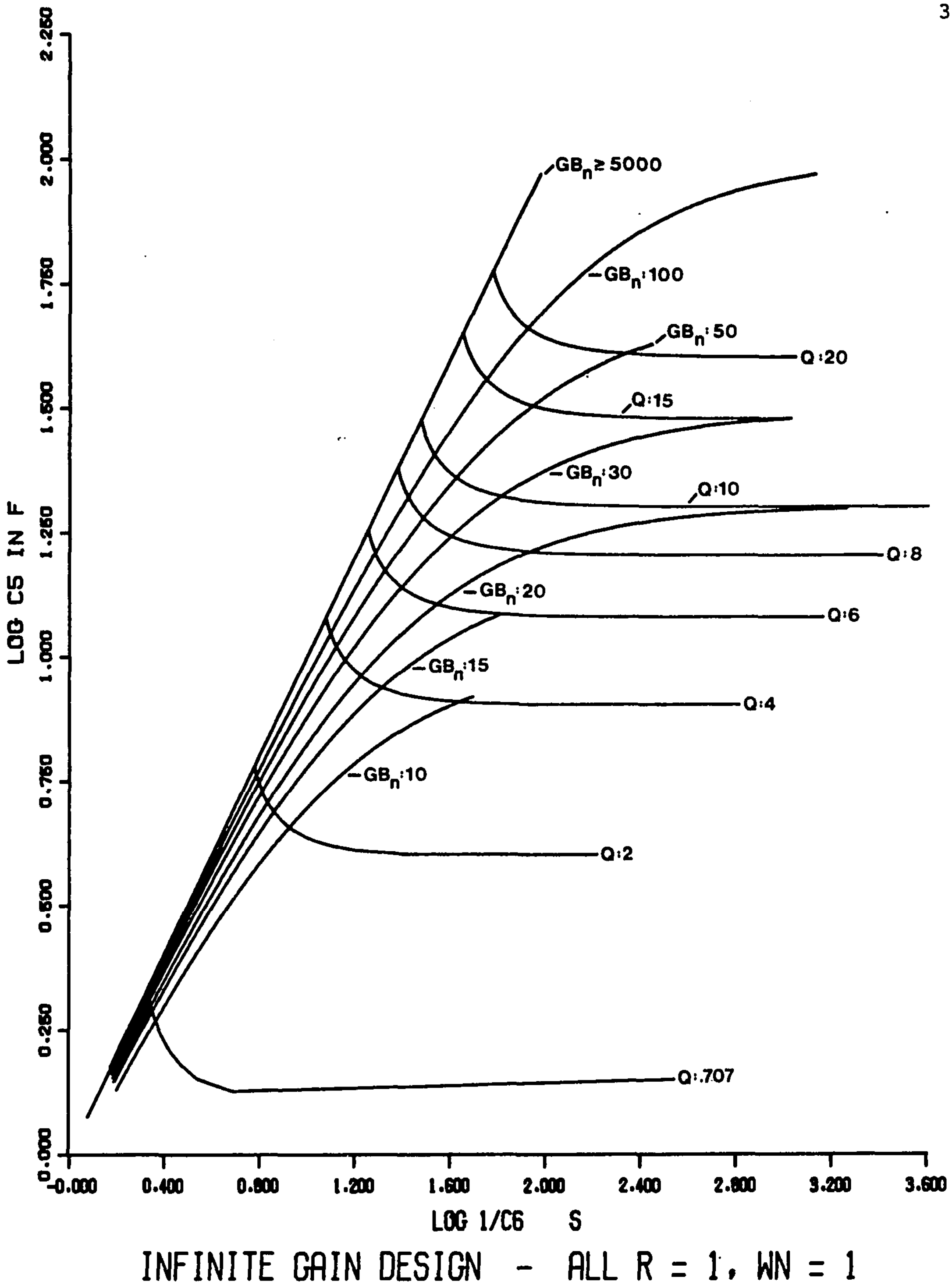

Figure 4.1. Design graph for a second-order low pass infinite gain multiple path feedback filter design.

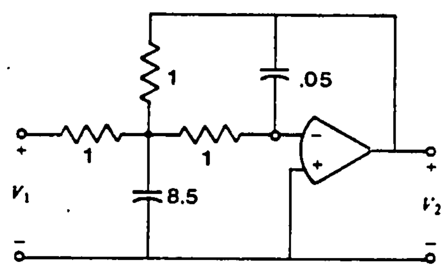

Figure 4.2. Solution for a low pass infinite gain design with a normalized $GB_n = 15$, $\omega_n = 1$ and $Q = 4$ [2].

normalized $GB_n = 500$ and a $Q = 2$, Equation (4.10) yields approximately a $C_5 = 6.0$ and a $C_6 = 0.17$. These are nearly reciprocal values. Equation (4.6), with a normalized $\omega_n = 1$ and $|H_o| = 1$ reduces to the Equation (4.10) solution. If, however, we choose a $|H_o| = 10$ with a normalized $\omega_n = 1$, Equation (4.6) produces (approximately) a $C_5 = 4.0$ and a $C_6 = 0.02$ for a $Q = 2$ and a $GB_n = 500$. These values are even further apart than those from Equation (4.10). In an effort to produce a better capacitance ratio, a solution to the general transfer function, Equation (4.2), was sought using optimization techniques [6].

The first approach was the most general. Using a $GB_n = 15$, a $Q = 4$, and a normalized $\omega_n = 1$, a solution that would preserve the pole positions of Equation (4.2) was followed that treated all elements, R_1, R_2, R_3, C_5, and C_6 as unknowns. It was soon discovered that such an attempt produced too many solutions. Table 4.1 is a partial list of the results. The fact that there did not appear to be a unique solution and, indeed, several solutions were close in value to each other, is the result of too many degrees of freedom in this approach. That is, the solution to fixing the pole positions of Equation (4.2) does not require that all of the elements be treated as unknowns. What was needed was to reduce the number of unknowns from five to four. The first attempt was to make $C_5 = C_6$, but no solution could be found for this case.

The number of unknowns was next reduced by equating the resistors in three combinations, first $R_1 = R_2$ was tried, then $R_1 = R_3$ and finally $R_2 = R_3$. Table 4.2 is a partial list of all of these resistor relations for a $GB_n = 15$, a $Q = 4$, and a normalized $\omega_n = 1$. Again, solutions are

Table 4.1. Optimization compensation for GB effects, most general solution for Equation (4.2). -- All elements are treated as unknowns. Design values are for a $GB_n = 15$, $Q = 4$, and $\omega_n = 1$. These results were obtained in as few as 7 and as great as 200 iterations for each solution. The minimum least square error obtained ranged from 4.4×10^{-14} to 4.8×10^{-21}.

R_1	R_2	R_3	C_5	C_6	C_5/C_6
2.330	5.707	5.236	2.426	.001	2,426.0
1.528	3.955	2.440	3.653	.001	3,653.0
1.043	1.001	1.000	8.547	.050	170.0
.805	1.441	1.066	8.087	.018	449.0
.716	1.734	1.424	7.948	.004	1,987.0
.997	2.678	1.161	5.530	.001	5,530.0
9.944	20.000	14.130	.621	.001	621.0

found in each strategy that are close to each other, indicating that there are still too many unknowns. To reduce Equation (4.2) to three unknowns means that we would have the case of $R_1 = R_2 = R_3$ which was the technique that led to the closed-form solutions of Equations (4.5) and (4.10).

For a comparison of the capacitor ratio C_5 / C_6, the solution Equation (4.10) produces a $C_5 / C_6 = 181$ for the same design parameters of $GB_n = 15$, $Q = 4$, and $\omega_n = 1$. A comparison of element values in Table 4.2 for all three resistor pair equalization strategies shows that they can yield nearly identical results amongst themselves and with the values from the closed-form solution of Equation (4.10), i.e., $C_6 = 0.048$ and $C_5 = 8.68$. The fact that they are not exactly equal again reveals that there are too many unknowns in the resistor pair strategy. For the $R_2 = R_3$ case, only one solution could be found, and it is approximately the closed-form solution. As can be seen from these two tables, no solution was found that significantly improved the capacitor ratio.

Table 4.2. Optimization compensation for GB effects, resistors pairs equated in Equation (4.2). -- Design values are for a $GB_n = 15$, $Q = 4$, and $\omega_n = 1$. These results were obtained in as few as 14 and as great as 200 iterations for each solution. The minimum least square error obtained ranged from 1.9×10^{-11} to 8.4×10^{-16}.

Strategy	$R_i=R_{ii}$	R_n	C_5	C_6	C_5/C_6
$R_1 = R_2$	1.566	.825	6.203	.043	144.0
$n = 3$	1.364	.744	7.077	.049	144.0
	1.203	.650	8.040	.056	143.0
	1.110	.595	8.724	.060	145.0
	.990	.994	8.700	.048	181.0
$R_1 = R_3$	1.500	1.658	5.528	.028	197.0
$n = 2$	1.312	1.455	6.310	.032	197.0
	1.189	1.272	7.188	.038	189.0
	1.066	1.089	8.065	.044	183.0
	.998	.998	8.698	.048	181.0
$R_2 = R_3$	1.023	.964	8.686	.044	197.0
$n = 1$					

CHAPTER 5

SUMMARY AND CONCLUSIONS

This thesis has dealt with a study in the effects and compensation techniques for these effects in two active RC filters that are produced when the frequency characteristics of an operational amplifier are included in the filter's circuit analysis. The frequency response of the operational amplifier was included by describing it as a single dominant pole amplifier. Previous attempts at compensation for the gain bandwidth effects produced by the dominant pole model were discussed and a simpler exact closed-form solution for two low pass designs were derived. Design graphs for these simpler solutions were presented that give a quick idea of approximate element values that are required to fully compensate for the effects of gain bandwidth. Finally, it was shown that by the use of optimization techniques [6] it is possible to find additional solutions for the infinite gain multiple path feedback design. These solutions, although valid, could not improve the capacitor ratio and require a large digital computer that will accommodate the software, whereas the simple solution this paper presents requires only a pocket calculator to solve.

Future work in finding GB compensation schemes for other active RC filters, such as the state variable and resonator filters, should be undertaken. Also the sensitivities to GB should be investigated as well as the effect the real pole, g, has on overall filter performance.

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