INFORMATION TO USERS

This reproduction was made from a copy of a document sent to us for microfilming. While the most advanced technology has been used to photograph and reproduce this document, the quality of the reproduction is heavily dependent upon the quality of the material submitted.

The following explanation of techniques is provided to help clarify markings or notations which may appear on this reproduction.

1. The sign or "target" for pages apparently lacking from the document photographed is "Missing Page(s)". If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting through an image and duplicating adjacent pages to assure complete continuity.

2. When an image on the film is obliterated with a round black mark, it is an indication of either blurred copy because of movement during exposure, duplicate copy, or copyrighted materials that should not have been filmed. For blurred pages, a good image of the page can be found in the adjacent frame. If copyrighted materials were deleted, a target note will appear listing the pages in the adjacent frame.

3. When a map, drawing or chart, etc., is part of the material being photographed, a definite method of "sectioning" the material has been followed. It is customary to begin filming at the upper left hand corner of a large sheet and to continue from left to right in equal sections with small overlaps. If necessary, sectioning is continued again—beginning below the first row and continuing on until complete.

4. For illustrations that cannot be satisfactorily reproduced by xerographic means, photographic prints can be purchased at additional cost and inserted into your xerographic copy. These prints are available upon request from the Dissertations Customer Services Department.

5. Some pages in any document may have indistinct print. In all cases the best available copy has been filmed.

University Microfilms International
300 N. Zeeb Road
Ann Arbor, MI 48106

Adams, William Mark

APPLICATION OF THE VARIANCE-TO-MEAN RATIO METHOD FOR DETERMINING NEUTRON MULTIPLICATION PARAMETERS OF CRITICAL AND SUBCRITICAL REACTORS

The University of Arizona M.S. 1985

PLEASE NOTE:

In all cases this material has been filmed in the best possible way from the available copy. Problems encountered with this document have been identified here with a check mark __√__.

1. Glossy photographs or pages _____
2. Colored illustrations, paper or print _____
3. Photographs with dark background _____
4. Illustrations are poor copy _____
5. Pages with black marks, not original copy _____
6. Print shows through as there is text on both sides of page _____
7. Indistinct, broken or small print on several pages __✓__
8. Print exceeds margin requirements _____
9. Tightly bound copy with print lost in spine _____
10. Computer printout pages with indistinct print _____
11. Page(s) _____ lacking when material received, and not available from school or author.
12. Page(s) _____ seem to be missing in numbering only as text follows.
13. Two pages numbered _____. Text follows.
14. Curling and wrinkled pages _____
15. Dissertation contains pages with print at a slant, filmed as received _____
16. Other _____

_____

_____

University
Microfilms
International

APPLICATION OF THE VARIANCE-TO-MEAN RATIO METHOD FOR DETERMINING NEUTRON MULTIPLICATION PARAMETERS OF CRITICAL AND SUBCRITICAL REACTORS

by

William Mark Adams

A Thesis Submitted to the Faculty of the

DEPARTMENT OF NUCLEAR AND ENERGY ENGINEERING

In Partial Fulfillment of the Requirements
For the Degree of

MASTER OF SCIENCE
WITH A MAJOR IN NUCLEAR ENGINEERING

In the Graduate College

THE UNIVERSITY OF ARIZONA

1 9 8 5

STATEMENT BY AUTHOR

This thesis has been submitted in partial fulfillment of requirements for an advanced degree at The University of Arizona and is deposited in the University Library to be made available to borrowers under rules of the Library.

Brief quotations from this thesis are allowable without special permission, provided that accurate acknowledgment of source is made. Requests for permission for extended quotation from or reproduction of this manuscript in whole or in part may be granted by the head of the major department or the Dean of the Graduate College when in his or her judgment the proposed use of the material is in the interests of scholarship. In all other instances, however, permission must be obtained from the author.

SIGNED: W. Mark Adams

APPROVAL BY THESIS DIRECTOR

This thesis has been approved on the date shown below:

G W Nelson

George W. Nelson
Professor of Nuclear & Energy Engineering

10/15/85

Date

TABLE OF CONTENTS

TABLE OF CONTENTS--Continued

TABLE OF CONTENTS--Continued

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS--Continued

LIST OF TABLES

LIST OF TABLES--Continued

ABSTRACT

Reactor parameters were experimentally measured for the University of Arizona TRIGA reactor using the Feynman variance-to-mean ratio method, a zero-power reactor noise technique in which characteristics of a multiplying system are determined from the mean number of neutrons detected in a short time interval and the variance about that mean. The magnitude of the deviation of the variance-to-mean ratio from unity is a function of the reactor parameters. A BF_3 detector and multi-channel analyzer were used to collect counts in various width time intervals with the University of Arizona TRIGA reactor from critical to -$.30 subcritical. The measurements and analysis demonstrated several limitations of the method for accurate results. These include requiring a large number of data points ($\sim 10^5$ intervals per ratio), a high detector efficiency ($\sim 10^{-4}$ counts per fission) and a multiplying system which is near critical ($\rho/\beta > -1.0$).

CHAPTER 1

INTRODUCTION

This paper deals with an experimental method that can be used to determine a number of important nuclear reactor parameters. Probably the most useful information available from its use is the reactivity of the reactor under consideration. The ability to quickly and accurately determine the reactivity of a nuclear system would constitute a major step forward in the industry, especially with regard to the storage and shipping of nuclear materials.

The variance-to-mean ratio (VTMR) method is an experimental technique based on the theory of statistical fluctuations in the neutron population of a steady-state reactor. The term zero power reactor noise is often used to describe these fluctuations when they occur in an extremely low power system. The variance-to-mean ratio technique was one of the earliest zero power reactor noise methods developed. In this chapter, the method is introduced and the objectives and organization of this paper are discussed.

Background and Definitions

The variance-to-mean ratio method is also known as the Feynman method, the Feynman-α method, the Feynman-variance method, or simply the variance method. The theoretical basis for this method was suggested by Richard P. Feynman during the 1940's. The technique

consists of sampling the neutron population in a large number of identical, short time intervals, by using a detector placed inside or near a multiplying system. When a sufficient ensemble of data has been collected, the mean number of neutrons detected in each time interval, and the variance about this mean, are calculated. The ratio of the variance to the mean is dependent on the length of the time interval; by varying this length, certain reactor parameters can be determined.

A number of terms and abbreviations are used throughout this paper. First, the terms **fluctuation** and **noise** are often used and refer to zero power reactor noise. Also, the term **variance-to-mean ratio**, although formally introduced in Chapter 3, is abbreviated VTMR throughout the paper; however, its first occurance in each chapter is spelled out to assist the reader. This abbreviation is employed when referring to both the VTMR method and the quantity VTMR itself. The symbol β is used to represent the effective fraction of delayed neutrons consistantly. Finally, the terms **channel width** and **time interval** are used interchangeably when discussing the length of time during which the neutron population is sampled. These terms are both represented by the symbol t.

Table 1.1 lists a large number of noise techniques and is included to illustrate how the VTMR method fits into this area of study. Many of these other techniques have evolved from the study and application of the VTMR method simply because it was one of the earliest developed. As such, it is an excellent starting point for any reactor noise experiment. Theoretically, all these techniques are related, having a common origin in the complicated neutron noise distribution

Table 1.1. Partial List of Zero Power Reactor Noise Methods

Time Correlation Analysis
Rossi-Alpha Method
Endogenous Pulse Source Method
Autocorrelation and Cross-Correlation Methods
Neutron Noise Distribution Analysis
Variance-to-Mean Ratio Method
Z Parameter Method
Bennett Variance Method
Third-Order, Correlated Triplet Methods
Covariance Method
Probability of Neutron Detection Analysis
Zero-Count Probability Method
Polya-Model Method
Time Interval from Random Origin Method
Count-to-Count Time Interval Method
Correlation Among Signs of Fluctuation Analysis
Polarity Autocorrelation and Cross-Correlation Method
Covariance Method via Polarity Detection

generating function, which is mentioned in the next chapter. The techniques are based on the assumed correlation between neutrons due to the chain reactor that is occuring. Certain methods of noise analysis are, therefore, more broadly applicable to other correlated events, such as photons in a non-coherent beam of light and the switching of domains in a magnet (Teodosić, 1971). These techniques have found their major use in the field of nuclear physics and nuclear engineering, for which they were developed.

A number of noise techniques have been developed for use in power reactors to monitor fluctuations in neutron density due to global system fluctuations such as fluid flow, control rod and fuel rod vibrations, and temperature and moderator density changes. The VTMR method, along with the other techniques of Table 1.1, cannot be used for this purpose because the macroscopic noise sources just mentioned completely overshadow the microscopic noise due to the chain reaction.

Applying information theory to reactor noise techniques indicates that a method will generally provide more information if the detector efficiency (defined explicitly in Chapter 3) is increased, or if the system is brought closer to criticality (Albrecht, 1970). As will be seen, this is true for the VTMR method.

The VTMR method was first used to calculate $\overline{\nu^2}$, the average of the squared number of neutrons per fission. More recently it has been used to determine a number of other reactor parameters. Its most successful use has been in calculating β/ℓ.

Objectives and Organization of Paper

The objectives of this paper are to provide a complete picture of the VTMR method, and present results of its application to the University of Arizona TRIGA reactor. This method was chosen over other noise techniques primarily because it had the potential for simple determination of near critical reactivities. This was the major objective of the experiment. The ability to do this would be a positive development and could be used to monitor the reactivity of any near critical system. During the condensing or repackaging of fuel assemblies in commercial spent fuel storage pools, for instance, this would be a useful safety feature.

The paper is organized into nine chapters. In Chapters 2 and 3, relevant theory is discussed and the important equations are derived. Chapter 2 deals with the statistics involved in the VTMR method, while Chapter 3 covers the reactor parameters and derives the relationship between the statistics and these parameters. Extensions to the VTMR method are also discussed. Chapter 4 is centered on the behavior of the analytic expression, derived in Chapter 3, including a short section on the different parameters that can be experimentally obtained from it. Chapters 5 and 6 concern the analysis and collection of data, respectively. An experimental procedure is contained in the latter of these. In Chapter 7 possible sources of error in the VTMR are discussed. This is an important chapter because it establishes a number of limitations in the use of the VTMR technique. The results of applying the VTMR method are presented and discussed in Chapter 8. Finally, the conclusions of the paper are presented in Chapter 9.

CHAPTER 2

THEORY: STATISTICS

The variance-to-mean ratio method is a statistical method based on determining the first (average or mean) and second (variance) order moments of a distribution. The distribution is that of neutron counts in time intervals of a fixed length and is referred to as the neutron noise distribution. Because the technique is statistical in nature, a large number of intervals are necessary to ensure a suitable ensemble of data from which to calculate the moments. A relationship between these calculated moments and certain reactor parameters based on the point reactor kinetics model is derived in the next chapter. In this chapter, general concepts from statistics and probability theory which are needed to understand and use this method are first presented. This is followed by a section on the distributions inherent in the technique. The chapter is concluded with a note on statistical uncertainty and equations for the standard deviation of the VTMR. For additional information about statistics, the reader is referred to the references for this section (Young, 1962; Bevington, 1969) or any text on the subject.

General Concepts

Statistics involves drawing conclusions about a large population based on measurements from a relatively small sample. The theoretical foundation for statistics is probability theory and one of the most

important concepts from this theory is that of a probability distribution. A probability distribution is a function that gives the probability of a variable x taking on a specific value x_i, for each x_i value possible. If a continuum of x_i values exists, then the probability distribution is a continuous function; if only certain x_i values occur, it is a discrete function. Since neutrons are considered discrete particles in most nuclear engineering applications, discrete probability distribution functions are of primary interest. Summing all probabilities in such a function results in a value of unity by definition, since it is a certainty that any value x_i, once obtained, is possible.

Mean and Variance

The average or mean of a discrete probability distribution is given by Eq. (2.1) and the variance by Eq. (2.2). The subscript i has been dropped to simplify the expressions.

$$\mu = \sum_{\text{All } x} x\, P(x) \tag{2.1}$$

$$\sigma^2 = \sum_{\text{All } x} (x-\mu)^2\, P(x) = \sum_{\text{All } x} x^2\, P(x) - \mu \tag{2.2}$$

In these equations, N is the total number of values and P(x) is the probability distribution. The standard deviation, σ, is defined as the square root of the variance or, in other words, the root mean square of the deviations from the mean. This quantity is often used as a measure

of uncertainty in statistics, as discussed in the last section of this chapter.

For a series of N measurements ($N<<\infty$), the true probability distribution is not known. In this case the best estimate of μ is the sample mean given by

$$\bar{x} = \frac{1}{N} \sum_{x=1}^{N} x \, . \tag{2.3}$$

The best estimate of the variance is the sample variance:

$$s^2 = \frac{1}{N-1} \sum_{x=1}^{N} (x - \bar{x})^2 \approx \overline{x^2} - (\bar{x})^2 \, . \tag{2.4}$$

The N-1 in the denominator is present because one degree of freedom is lost while determining $\bar{x}$ from the N observations. In the limit as N goes to ∞, the difference between N and N-1 becomes numerically insignificant and the approximation in Eq. (2.4) becomes an equality. When this occurs, the sample variance and mean equal the actual variance and mean. Then the second equality in both Eqs. (2.2) and (2.4) can be proven as shown in Appendix A. The concept of the mean and variance of a distribution are essential for the use of this reactor noise method, as its name suggests.

The Binomial Distribution

Consider n trials of an experiment that has two possible outcomes, A and B, which are not influenced by earlier trial results. If p is the probability of outcome A occurring in each trial, then 1-p must be the probability that outcome B is the result. Let b (x; n,p)

represent the probability distribution of outcome A occurring x times in n independent trials. The binomial distribution is then written

$$b(x;\ n,p) = \binom{n}{x}\ p^x\ (1-p)^{(n-x)}; \qquad x = 0,1,2,\ldots n \tag{2.5}$$

where

$$\binom{n}{x} \equiv \frac{n!}{x!\ (n-x)!}\ . \tag{2.6}$$

Equation (2.6) is important because it gives the number of different combinations possible for n events taken x at a time. This is used in the derivation of the VTMR equation in the next chapter. The mean of a binomial distribution, μ, is equal to np and the variance, σ^2, is equal to np(1-p).

The Poisson Distribution

The Poisson distribution is an approximation of the binomial distribution in the special case where $\mu << n$ and $p << 1$. It is commonly used in counting experiments where the number of independent random events are recorded during time intervals of prescribed duration. If x is the random variable representing the number of events counted in each time interval and $\bar{x}$ is the sample average of x from all the time intervals, the Poisson distribution is written:

$$p(x,\bar{x}) = e^{-\bar{x}}\ \frac{(\bar{x})^x}{x!} \qquad x = 0,1,2,\ldots \tag{2.7}$$

To verify that $p(x,\bar{x})$ is in fact a correctly normalized probability distribution, one must show that all its probabilities add to unity.

$$\sum_{\text{All } x} p(x,\bar{x}) = \sum_{\text{All } x} e^{-\bar{x}}\ \frac{(\bar{x})^x}{x!}$$

A series expansion of $e^{\bar{x}}$ yields

$$e^{\bar{x}} = 1 + \bar{x} + \frac{(\bar{x})^2}{2!} + \ldots = \sum_{\text{All } x} \frac{(\bar{x})^x}{x!} \quad ,$$

and therefore

$$\sum_{\text{All } x} p(x,\bar{x}) = \sum_{\text{All } x} e^{-\bar{x}}\, e^{\bar{x}} = 1.0 \quad .$$

$p(x,\bar{x})$ is concluded to be a proper probability distribution, which it in fact is. The derivation of Eq. (2.7) from the binomial distribution, Eq. (2.5), is clearly performed by Young (1962) and Bevington (1969), and is too long to repeat here.

The mean of a Poisson distribution is the same as that for a binomial distribution, as it must be, since the first is just a special case of the second. Also, in the limit as p approaches zero, the variance of the binomial distribution goes to np, the mean. One of the fundamental concepts behind the variance-to-mean ratio method and the clue that may well have led to its discovery is the fact that for a Poisson distribution the variance is equal to the mean. This is clearly proven in Appendix B.

The Gauss or Normal Distribution

The Gauss or normal distribution is the most important probability distribution for use in data analysis, although it takes a secondary role here behind the Poisson distribution. The Gauss distribution is another special case of the binomial distribution where the number of possible events being measured goes to infinity. In other

words, it is a continuous probability distribution and therefore the discrete variable x in Eqs. (2.1) and (2.2) is replaced by a continuous variable and the summations in these equations are replaced by integrals for calculating the mean and variance. The importance of this distribution lies in its similarity to the distribution of estimations or measurements of the parameters from most other distributions about their actual values. It can therefore be used to define a probable error due to the random perturbations that occur in most systems. The Gauss distribution is written:

$$G(x, \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \tag{2.8}$$

where the quantity $1/\sqrt{2\pi}$ is a normalization constant. The probable error is defined as the absolute value of the deviation $|x-\mu|$ such that the probability for the deviation of a random observation $|x_i-\mu|$ is less or equal to 0.5. It is related to the standard deviation (Bevington, 1969) by

$$\text{Probable error} = 0.6745\sigma \ . \tag{2.9}$$

Theoretically, the Gaussian and Poisson distributions are indistinguishable when the Poisson distribution is symmetric. Unfortunately, many of the Poisson distributions experimentally obtained for this paper were not symmetric and were certainly not continuous. Therefore, the definition of probable error must be used with care.

The concept of a probability distribution is quite important for a complete understanding of reactor noise analysis. For the VTMR

method, this concept, along with many of the others presented here, is essential.

Neutron Distributions

All distributions observed in nature cannot be accurately represented by simple mathematical functions. In particular, two distributions which are not easily modeled pertain to the VTMR method and are discussed in this section. The first is the distribution in the number of neutrons, ν, resulting from a fission. This distribution is often represented by the Diven fission parameter which is a measure of its relative width (Thie, 1963). The second is the distribution of neutron fluctuations in short time intervals, or more simply, the neutron noise distribution.

The Diven Fission Parameter

The exact distribution of the value ν is almost impossible to measure experimentally from a reactor for a number of reasons. This distribution not only depends on the internal energy state of the fissile nucleus, it is a function of the incident neutron energy as well. In a multiplying system, neutrons of many different energies cause fissions so that any distribution measured would actually be that averaged over all energies present in the system.

Determining the energy-averaged result from such a system is also quite difficult, simply because there is no easy way to distinguish between fissions that result in one number of produced neutrons and those that result in a different number. However, it is possible to determine the ν distribution using much simpler systems. Diven (1956)

first experimentally obtained it from a small sample of U-235 using 80 keV incident neutrons. His results, which are accepted today, appear in Table 2.1.

The relative width of the above distribution was also calculated. This quantity, defined by Eq. (2.10), is called the Diven fission parameter.

$$D \equiv \frac{\overline{\nu^2} - \overline{\nu}}{(\overline{\nu})^2} \tag{2.10}$$

D is important because it is a parameter that can be experimentally obtained using the VTMR method and because its definition is used in deriving the fundamental equation for that same method. A typical value for U-235 systems is D = 0.795 (Thie, 1963). The Diven fission parameter is also described as a measure of the neutron dispersion in fission (Lewis, et. al.; 1975).

The Neutron Noise Distribution

The distribution in the number of neutrons observed in a series of time intervals of identical length is central to the VTMR experiment. The mean and variance used to determine reactor parameters are the first and second order moments calculated from a sampling of this distribution. Many attempts have been made at deriving an analytic probability distribution that accurately models the observed distribution. Knowing the correct distribution would be beneficial for many reasons. It would allow an accurate dead-time correction (Teodosić, 1971), as well as the development of simple and short-time probability methods. In fact the Polya model noise method mentioned in

Table 2.1. Dispersion of Fission Neutrons in U-235 Resulting From 80 keV Incident Neutrons

Number of Neutrons, ν	Probability (%)	Error (%)
0	2.7	0.4
1	15.8	1.0
2	33.9	1.4
3	30.5	1.5
4	13.3	1.3
5	3.8	0.9
6	-0.1*	0.3
7	0.1	0.2
8	0.0	0.0

$\overline{\nu} = 2.47 \pm 0.030$

$\overline{\nu^2} = 7.32 \pm 0.015$

$D = 0.795 \pm 0.007$

*As published

Table 1.1 is based on fitting an experimentally determined distribution to an approximate mathematical model to obtain reactor parameters (Pacilio, 1966).

The mathematical problem has been solved independently by at least five authors, including Pal and Babala who are referenced for different work in this paper. Each of the five has used independent assumptions to obtain the same result, known as the PMZBB distribution generating function. Successive derivatives of this function yield the probability distribution (Pacilio, et. al.; 1975). Unfortunately, the analytical expressions obtained are so complicated that "they are practically useless for any theory-experimental confrontation" (Pacilio and Jorio, 1975). However, this generating function has been used to directly derive the point-reactor kinetics equations, Eqs. (3.3 - 3.9) (Wang and Ruby, 1975).

A less complex negative binomial distribution has been used to model the neutron noise with some success, although it is not extremely accurate. This distribution, although similar, is flatter than the Poisson distribution with its mean value itself distributed (Lindman and Ruby, 1967). It is written in terms of recursion formulas as

$$p(k=0) = (1+Y)^{-\bar{x}/Y} \tag{2.11}$$

$$p(k) = \frac{\bar{x} + (k-1)Y}{k(1+Y)} p_{k-1} \quad . \tag{2.12}$$

The parameter Y in these expressions is introduced and derived in the next chapter. It is equal to the VTMR minus one and is, therefore, zero for purely random processes. In that case, Eqs. (2.11) and (2.12) reduce to a Poisson distribution as expected. These equations were used to

represent the theoretical noise distribution used in the polya model method (Pacilio, 1969). Table 2.2 compares an experimentally obtained noise distribution with one generated using the backward binomial distribution. The same Y value is used in both cases. Table 2.3 makes a similar comparison for random data and a Poisson distribution, both having the same mean value. The inaccuracy of the backward binomial model is clearly shown by Table 2.2. The Poisson distribution, though, is well matched experimentally, verifying to some extent that the equipment was working correctly; both experimental distributions were obtained using identical apparatus, but with different neutron sources.

The two neutron distributions discussed above play an important role in all zero power reactor noise analysis. In the VTMR method, the Diven fission parameter is used in deriving the fundamental equation and the approximate noise distribution, known as the backward binomial distribution, can be used to generate test data from one known Y value. More importantly, Eqs. (2.11) and (2.12) can be fit to an experimentally obtained noise distribution to directly obtain Y in an alternative data analysis method from the VTMR technique. With the development of better analytic noise distributions, this could become a useful method.

Statistical Uncertainty

The variance-to-mean ratio method is a statistical method and, as such, any parameter determined from it will have an associated statistical uncertainty. The standard deviation is the usual method of placing bounds on this uncertainty. If a number of measurements are made, the usual spread of results has the form of a Gaussian

Table 2.2. Comparison Between an Experimental Noise Distribution and One Obtained Using the Backward Binomial Probability Distribution

Number of Counts	Experimental Distribution(%)	Backward Binomial Distribution(%)
0	1.772	5.208
1	6.378	10.547
2	11.783	13.607
3	16.117	14.222
4	17.282	13.123
5	15.290	11.144
6	11.759	8.918
7	8.256	6.824
8	5.108	5.043
9	3.152	3.624
10	1.576	2.545
11	0.770	1.753
12	0.403	1.188
13	0.224	0.794
14	0.086	0.524
15	0.024	0.342
16	0.002	0.221
17	0.0	0.142
18	0.0	0.090
19	0.0	0.057
20	0.0	0.036

Mean of Experimental Distribution = 4.5532

Y for both Distributions = 1.2484

Table 2.3. Comparison Between Experimental Random Data and Data Generated Using the Poisson Distribution

Number of Counts	Experimental Distribution(%)	Poisson Distribution(%)
0	0.0	0.007
1	0.037	0.069
2	0.305	0.330
3	0.843	1.048
4	2.224	2.496
5	4.619	4.758
6	7.344	7.558
7	9.861	10.289
8	13.074	12.257
9	12.915	12.979
10	12.952	12.369
11	11.584	10.716
12	8.492	8.511
13	6.452	6.239
14	4.057	4.247
15	2.322	2.698
16	1.393	1.607
17	0.819	0.900
18	0.367	0.477
19	0.147	0.239
20	0.122	0.114
21	0.061	0.052
22	0.012	0.022
23	0.0	0.009
24	0.0	0.004
25	0.0	0.001

Mean of both distributions = 9.5301

distribution. The best estimate of the exact quantity being measured is then the first order moment or mean of this distribution. Therefore, the standard deviation of the mean is needed as an error bar for the final result. Equation (2.13) defines this result which is derived in Appendix C.

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}} \tag{2.13}$$

In general, this equation is not true for the data collected in this experiment, since the fluctuations of neutrons are not distributed in a Gaussian manner. However, for the sole purpose of obtaining an estimate of uncertainty on the results, the assumption is made that the data form an approximately symmetric Poisson distribution. As pointed out previously, this is then nearly identical to a Gauss distribution.

The general formula for the propagation of errors is given by Bevington (1969) as

$$\sigma_x^2 \simeq \sigma_u^2 \left(\frac{\partial x}{\partial u}\right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v}\right) + 2\, \sigma_{uv}^2 \left(\frac{\partial x}{\partial u}\right)\left(\frac{\partial x}{\partial v}\right) + \cdots \tag{2.14}$$

where

$$x = f(u,v,\ldots) \tag{2.15}$$

and σ_{uv}^2 is the covariance, defined as

$$\sigma_{uv} = \frac{1}{N} \sum_{i=1}^{N} (u_i - \bar{u})(v_i - \bar{v}) \,. \tag{2.16}$$

The covariance is often used in neutron noise experiments that employ two detectors.

Using Eq. (2.14) the following specific equations are obtained for error propagation.

$$x = au \pm bv \quad : \quad \sigma_x^2 = a^2\sigma_u^2 + b^2\sigma_v^2 + 2ab\,\sigma_{uv}^2 \tag{2.17}$$

$$x = \frac{au}{v} \quad : \quad \frac{\sigma_x^2}{x^2} = \frac{\sigma_u^2}{u^2} + \frac{\sigma_v^2}{v^2} - 2\,\frac{\sigma_{uv}^2}{uv} \tag{2.18}$$

$$x = au^{\pm b} \quad : \quad \frac{\sigma_x}{x} = b\,\frac{\sigma_u}{u} \tag{2.19}$$

These equations, along with Eq. (2.13), can then be used to calculate an approximation of the statistical uncertainty of the VTMR. Recalling that the quantity Y has been defined as VTMR-1, we can write

$$\text{VTMR} = 1 + Y = \frac{\sigma_x^2}{\bar{x}} = \frac{\overline{x^2}-(\bar{x})^2}{\bar{x}} \tag{2.20}$$

and define the following standard deviations. Here it is assumed that both the covariance between the variance and the mean, and between the mean squared and squared mean are negligible.

$$\sigma_{\bar{x}} = \left[\frac{\bar{x}(1+Y)}{N}\right]^{1/2} \tag{2.21}$$

$$\sigma_{(\bar{x})^2} = \sqrt{2}\;\bar{x}\;\sigma_{\bar{x}} \tag{2.22}$$

$$\sigma_{\overline{x^2}} = \left[2(\bar{x})^2 + (1+Y)^2\right]^{1/2}\sigma_{\bar{x}} \tag{2.23}$$

$$\sigma_{\overline{x^2}-(\bar{x})^2} = \left[\frac{2(N-1)}{N^2}\right]^{1/2}\bar{x}\,(1+Y) \tag{2.24}$$

$$\sigma_{\text{VTMR}} = \left[\frac{2}{N}\left(\frac{N-1}{N} + \frac{1+Y}{2\bar{x}}\right)\right]^{1/2}(1+Y) \tag{2.25}$$

assuming $N \gg 1$, allows the approximation $N-1/N \simeq 1$ and we have

$$\sigma_{\text{VTMR}} = \left[\frac{2}{N}\left(1 + \frac{1+Y}{2\bar{x}}\right)\right]^{1/2}(1+Y) \quad . \tag{2.26}$$

In the limit, as Y goes to zero, Eq. (2.26) reduces to

$$\sigma_{random} = \left[\frac{2}{N}\left(1 + \frac{1}{2\bar{x}}\right)\right]^{1/2} . \qquad (2.27)$$

This corresponds to the standard deviation of the VTMR for a perfect Poisson distribution, which, it will be recalled, is the standard deviation of unity when experimentally determined in this fashion. The variable Y will be discussed at length in the next chapter and it will become clear then that Y=0.0 represents random fluctuations or, in other words, a Poisson distribution. The standard deviation of Y = VTMR-1 can, after combining Eqs. (2.26) and (2.27) and performing some algebra, be written

$$\sigma_Y = \left[\frac{1}{N}\left\{\left(4 + \frac{3}{\bar{x}}\right) + \left(2 + \frac{3}{\bar{x}}\right) Y + \frac{Y^2}{\bar{x}}\right\}\right]^{1/2} \qquad (2.28)$$

Equations (2.26) through (2.28) are the desired result of this section. If random data were collected, the VTMR should equal unity with an approximate standard deviation given by Eq. (2.27); if non-random data were collected the VTMR can be calculated with an approximate standard deviation given by (2.26); and if the standard deviation of Y is desired, it is given approximately by Eq. (2.28). This derivation was first carried out and published in Italian by Pacilio (1965). The results have since been used by a number of authors in presenting VTMR experiments, but with no mention of the inherent approximations.

In Summary

The variance-to-mean ratio noise analysis method is a statistical method. The fluctuations in neutron density that are measured are based on the randomness of all nuclear reactions. They would occur even if ν

was not distributed and all fissions resulted in the same number of neutrons (Cohn, 1971). However, because this is not the case, the relative width of this distribution must be taken into account when dealing with the fluctuations. General statistical concepts helpful in understanding the VTMR experiment are first discussed in this chapter. The two important neutron distributions then are examined. Finally, a section on statistical uncertainty concludes the chapter.

CHAPTER 3

THEORY: VARIANCE-TO-MEAN RATIO

The variance-to-mean ratio method consists of analyzing fluctuations in the number of neutrons counted during a series of sequential time intervals. From this analysis, it is possible to determine parameters of the neutron chain reaction. In this chapter, a derivation of the equation which relates the VTMR to parameters of the neutron chain reaction is outlined. This theory and derivation were first proposed in the 1940's by Feyman, de Hoffmann, and Serber (de Hoffmann, 1949). A section titled General Concepts precedes the derivation and contains a qualitative description which helps to clarify the algebra. A section on more recent additions to the theory concludes the chapter.

General Concepts

If a neutron detector is used to count neutrons which are leaking out of a nuclear reactor and the resulting amplified signal is fed into a multi-channel analyzer (MCA), the number of counts in each channel varies considerably. To a certain extent, this fluctuation in counts-per-channel is caused by the neutron source, which in most cases randomly introduces neutrons to the system. Neutrons resulting from spontaneous fission, α-n reactions, and cosmic rays are also randomly introduced (Courant & Wallace, 1947) and add to the variations. The MCA

and the neutron detector contribute as well due to counts which are randomly (as far as the neutrons are concerned) lost between the MCA gates and unresolved by the detector (Ruby, 1981). The variation in counts per channel is too large to be accounted for by these statistically random effects; therefore, the nature of the chain reaction must be taken into account.

In order to understand the larger-than-expected fluctuations, one must consider what occurs between a primary fission, one that does not continue an already begun chain, and the detection of a daughter neutron that has leaked out of the system at a later time. Each neutron resulting from this initial fission can either carry on the chain by causing a subsequent fission, or disappear from consideration, through absorption in a non-fissile material or through leakage out of the system (possibly to be detected). In this manner, a chain may grow to any length; the probability of a certain length being determined by the multiplication factor of the overall reaction. It is this variation in chain length that causes fluctuations in the number of counts per channel beyond those that are statistically random in nature. In other words, the larger-than-expected variation in counts per channel is due to short bursts of fissions resulting from long chains.

The quantity $\frac{\overline{c^2}-(\overline{c})^2}{\overline{c}}$, which is defined as the ratio of the variance in counts-per-channel to the mean counts-per-channel, is an appropriate measure of the extent that the fluctuations differ from those of a statistically random or Poisson distribution. For a pure

Poisson distribution, this VTMR is equal to unity as was shown in the previous chapter. Therefore, for neutrons from a purely random source such as spontaneous fission

$$\frac{\overline{c^2}-(\overline{c})^2}{\overline{c}} = 1.0 \; . \tag{3.1}$$

As can be seen, the general variable x of Chapter 2 has been replaced with the specific variable c representing neutron counts. Taking into account the non-random behavior introduced by the fission chains, we expect that for neutrons resulting from a multiplying system

$$\frac{\overline{c^2}-(\overline{c})^2}{\overline{c}} = 1.0 + Y \tag{3.2}$$

where Y is a measure of the non-random behavior of the distribution. The existence of a relation between Y and parameters of the chain reaction enables one to determine information about the reaction using the easily calculated mean and variance.

Derivation of the Variance-to-Mean Ratio Equation

The fluctuations in counts per channel above those which are statistically random in nature depend on the detector/counter efficiency, the time interval during which counts are collected, the criticality of the system, and the variation in the number of neutrons per fission (Feynman, de Hoffmann, Serber; 1956). The derivation is simplified by assuming one group of identical neutrons. Therefore all space, energy, and delayed neutron effects are ignored, and we have a point-reactor

model. The seven point-reactor kinetics equations are clearly derived by Hetrick (1971). With no external source present, they can be written:

$$\frac{dn(t)}{dt} = \frac{\rho-\beta}{\ell}n(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \tag{3.3}$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\ell}n(t) + \lambda_i C_i(t); \qquad i = 1,2,\ldots 6 . \tag{3.4-3.9}$$

If a short time scale is being considered (i.e., less than 0.1 seconds), the assumption that no delayed neutrons are present appears valid. In this case the six delayed groups are neglected and Eqs. (3.3-3.9) reduce to a single first order, ordinary differential equation:

$$\frac{dn(t)}{dt} = \frac{\rho-\beta}{\ell}n(t) . \tag{3.10}$$

Solving Eq. (3.10) gives:

$$n(t) = n(0)e^{-\alpha t} . \tag{3.11}$$

Here, α is defined as $\frac{\beta-\rho}{\ell}$ and is referred to as either the prompt decay coefficient or the Rossi-α. It represents the overall prompt neutron time behavior of the system and is negative below prompt critical and positive above (Orndoff, 1957). Equation (3.11) allows a calculation of the number of neutrons present after time t due to a neutron at time zero. The Rossi-α noise method involves experimentally determining α based on this idea.

Now, define τ as the mean time between fissions, F as the average number of fissions per unit time or fission rate

$$F \equiv \int_E \int_r \Sigma_f(E,\vec{r})\, n(\epsilon,\vec{r})\, v\, d^3\vec{r}\, dE , \tag{3.12}$$

and ε as the detector efficiency

$$\varepsilon \equiv \frac{\int_E \int_r \Sigma_D(E,\vec{r})\, n(E,\vec{r})\, v\, d^3\vec{r}\, dE}{F} \tag{3.13}$$

where

$n(E,\vec{r})\, d^3\vec{r}\, dE$ ≡ the expected number of neutrons in $d^3\vec{r}$ about $\vec{r}$ with energy dE about E

$\Sigma_f(E,\vec{r})$ ≡ the probability per unit path length that a neutron with energy E at point $\vec{r}$ causes a fission

$\Sigma_D(E,\vec{r})$ ≡ the probability per unit path length that neutron with energy E at point $\vec{r}$ suffers a collision that results in detection

v ≡ neutron speed corresponding to energy E.

Using these definitions allows the following to be established.

$\frac{1}{\tau}dt$ ≡ probability that a given neutron causes a fission in time dt

$\frac{\varepsilon}{\tau}dt$ ≡ probability that a given neutron is detected in time dt

τF ≡ average number of neutrons in the system

εF ≡ average number of neutrons detected per unit time

Hence, the average number of detected neutrons in a time interval of length t can be written:

$$\overline{c} = \varepsilon t F \tag{3.14}$$

The next step is to select two arbitrary time intervals, dt_1 about t_1 and dt_2 about t_2 with $t_2 > t_1$, and calculate the number of neutron pairs (i.e., one neutron in each interval) that will be detected. As shown in Fig. 3.1, two kinds of neutron pairs are possible. In the first,

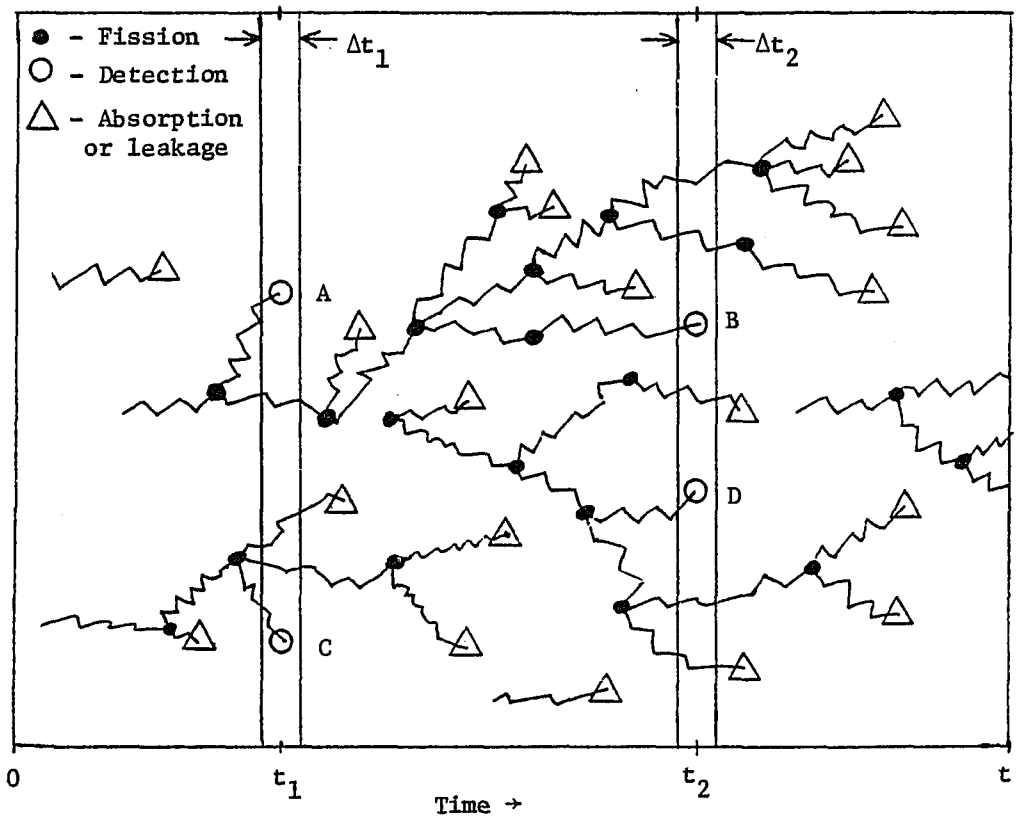

Figure 3.1. Neutron Chains Illustrating Related and Unrelated Neutron Pairs

each neutron is from a separate chain, while in the second both neutrons are members of the same chain. Neutrons A and B of Fig. 3.1 are an example of the related neutron pair. Neutrons A and C make up an unrelated pair. The sum of the related and unrelated pairs will equal the total number of neutron pairs, which is the desired result.

The expected number of unrelated neutron pairs is equal to the probability of a neutron being detected in dt_1 times the probability of a neutron being detected in dt_2, since these are independent events.

$$\left\{\begin{matrix}\text{number of}\\ \text{unrelated}\\ \text{neutron pairs}\end{matrix}\right\} = (\varepsilon F dt_1)(\varepsilon F dt_2) \tag{3.15}$$

The number of related pairs is more difficult to calculate since it depends on the four independent quantities mentioned at the beginning of this section. If T is defined as the time when the most recent fission common to both related neutrons has taken place ($-\infty<T<t_1$), the expected number of related neutron pairs can be written as the product of the following independent quantities summed over all T and ν:

FdT	$\equiv$	the probability that a fission took place in dT about T which resulted in such a related pair
$P(\nu)$	$\equiv$	the probability that ν prompt neutrons are emitted from this initial fission
$\nu e^{-\alpha(t_1-T)}$	$\equiv$	the expected number of neutrons present at time t_1 due to neutrons created at time T
$\frac{\varepsilon}{\tau}dt_1$	$\equiv$	the probability that a neutron present at time t_1 will be detected
$(\nu-1)e^{-\alpha(t_2-T)}$	$\equiv$	the expected number of neutrons present at time t_2 due to neutrons created at time T (ν-1 since one was detected at time t_1)

$\frac{\varepsilon}{\tau}dt_2$ ≡ the probability that a neutron present at time t_2 will be detected

$$\left\{\begin{matrix}\text{calculated number}\\ \text{of related}\\ \text{neutron pairs}\end{matrix}\right\} =$$

$$= \sum_{\nu=1}^{\nu_{max}} \int_{-\infty}^{t_1} FdTP(\nu)\nu e^{-\alpha(t_1-T)} \frac{\varepsilon}{\tau}dt_1(\nu-1)e^{-\alpha(t_2-T)} \frac{\varepsilon}{\tau}dt_2 \qquad (3.16)$$

After carrying out the integration and substituting in the definition of a mean:

$$\sum_{\nu=1}^{\nu_{max}} P(\nu)[\nu(\nu-1)] = \overline{\nu(\nu-1)} = \overline{\nu^2}-\overline{\nu} \qquad (3.17)$$

we can add in the number of unrelated pairs to get:

$$\left\{\begin{matrix}\text{calculated number}\\ \text{of neutron pairs}\\ \text{in } dt_1 \text{ and } dt_2\end{matrix}\right\} =$$

$$= \varepsilon^2F^2dt_1dt_2 + \frac{\varepsilon^2F}{2\alpha\tau^2}e^{-\alpha(t_2-t_1)}\, dt_1dt_2(\overline{\nu^2}-\overline{\nu}) \qquad (3.18)$$

Integrating this over the channel width, t, with respect to both t_1 and t_2 gives:

$$\left\{\begin{matrix}\text{calculated}\\ \text{number of}\\ \text{neutron pairs}\end{matrix}\right\} =$$

$$= \int_0^t dt_2 \int_0^{t_2} dt_1 \left[\varepsilon^2F^2 + \frac{\varepsilon^2F}{2\alpha\tau^2}e^{-\alpha(t_2-t_1)}\,(\overline{\nu^2}-\overline{\nu})\right]$$

$$= \frac{\varepsilon^2F^2t^2}{2} + \frac{\varepsilon^2F(\overline{\nu^2}-\overline{\nu})t}{2\alpha^2\tau^2}\left[1 - \frac{(1-e^{-\alpha t})}{\alpha t}\right] \qquad (3.19)$$

The number of neutron pairs in a time interval in which a total of c neutrons have been detected can also be obtained directly from probability theory. As explained in Chapter 2

$$\binom{c}{2} \equiv \frac{c!}{(c-2)!\,2!} = \frac{c(c-1)}{2} \quad . \tag{3.20}$$

The expected number of neutron pairs is then simply the average number of pairs, written $\overline{c(c-1)}/2$. After substituting this, along with our earlier result $\bar{c} = \varepsilon t F$ into Eq. (3.19), it is rewritten

$$\frac{\overline{c(c-1)}}{2} = \frac{\bar{c}^2}{2} + \frac{\varepsilon \bar{c}(\bar{\nu}^2 - \bar{\nu})}{2\alpha^2\tau^2}\left[1 - \frac{(1-e^{-\alpha t})}{\alpha t}\right] \tag{3.21}$$

which can be rearranged to read

$$\frac{\overline{c^2}-(\bar{c})^2}{\bar{c}} = 1 + \frac{\varepsilon(\bar{\nu}^2-\bar{\nu})}{\alpha^2\tau^2}\left[1 - \frac{(1-e^{-\alpha t})}{\alpha t}\right] \tag{3.22}$$

From Eq. (3.10), we know that at time t after a primary neutron enters the system, $e^{-\alpha t}$ daughter neutrons remain. Hence, the probability of a fission in time dt about t due to the primary neutron is $e^{-\alpha t}dt/\tau$. Integrating over all time gives the total number of fissions caused by the original neutron

$$\int_0^\infty \frac{e^{-\alpha t}}{\tau}\, dt = \frac{1}{\alpha\tau} \tag{3.23}$$

and each of these fissions produces $\bar{\nu}$ additional neutrons. In other words, one neutron causes a total of $\frac{1}{\alpha\tau}$ fissions which, combined, result in $\frac{\bar{\nu}}{\alpha\tau}$ neutrons. This quantity is the prompt multiplication of the primary neutron.

The prompt neutron multiplication can also be written in terms of k_p, the prompt multiplication factor. If we again consider one primary neutron, after the first generation or fission, k_p neutrons will be present; after the second k_p^2; and so on. The total number of neutrons produced can then be written as

$$1 + k_p + k_p^2 + k_p^3 + \ldots\ldots = \frac{1}{1-k_p} \tag{3.24}$$

using a Taylor series transformation, and a second definition of the prompt multiplication of a primary neutron is obtained. Equating the two gives:

$$\frac{\bar{\nu}}{\alpha\tau} = \frac{1}{1-k_p} \tag{3.25}$$

Now relax the assumption that no delayed neutrons exist if t, the channel width, is kept small compared to the half life of the fastest delayed neutron precursor group; $t < 0.23$ sec (Furuhashi and Sigeru, 1968). Then, by defining $k_p = (1-\beta)k$ where β is the fraction of delayed neutrons and $k = \frac{1}{1-\rho}$ where ρ is the reactivity, we can write Y in terms of the detector/counter efficiency and parameters of the chain reaction as we set out to do.

$$Y = \frac{\varepsilon(\bar{\nu}^2-\bar{\nu})}{(\bar{\nu})^2} \frac{(1-\rho)^2}{(\beta-\rho)^2} \left[1 + \frac{1-\exp\left(\frac{\rho-\beta}{\ell}t\right)}{\left(\frac{\rho-\beta}{\ell}\right)t} \right] \tag{3.26}$$

Defining

$$A \equiv -\alpha = \frac{\beta}{\ell}(\$-1) \tag{3.27}$$

where \$ is the reactivity in dollars, and recalling the Diven parameter defined in Chapter 2, Y can be written:

$$Y = \varepsilon D \frac{(1-\rho)^2}{(\beta-\rho)^2}\left[1 + \frac{1-e^{At}}{At}\right] \tag{3.28}$$

This equation can be found using a number of alternative approaches, although they are somewhat more involved. It has been obtained both from a finite reactor model (Natelson, Osborn, and Shure; 1965) and from the neutron noise probability distribution generating function introduced previously (Wang and Ruby, 1975). In these alternative derivations and in recent literature, the term $(1-\rho)^2$ in Eq. 3.28) is replaced by $(1-\beta)^2$. Numerically, this is a trivial difference for the reactivities close to delayed critical which are of interest here. The final VTMR equation is now written

$$Y = \varepsilon D \frac{(1-\beta)^2}{(\beta-\rho)^2}\left[1 + \frac{1-e^{At}}{At}\right] \tag{3.29}$$

$$\text{VTMR} = \frac{\overline{c^2}-(\overline{c})^2}{\overline{c}} = 1 + Y \,. \tag{3.30}$$

Equation (3.29) is sometimes expressed in terms of k_p or R rather than ρ. In these forms it appears as

$$Y = \varepsilon D \frac{k_p^2}{(1-k_p)^2}\left[1 + \frac{1-e^{At}}{At}\right] \tag{3.31}$$

and

$$Y = \frac{\varepsilon D}{(1-\$)^2}\left(\frac{1-\beta}{\beta}\right)^2\left[1 + \frac{1-e^{At}}{At}\right] \,. \tag{3.32}$$

As t becomes large, the bracketed quantity in these equations becomes saturated and essentially equals unity for all values of A. In other words, Y becomes independent of channel width. If this is true, and yet the t values are not so large that they require the

consideration of delayed neutrons, Eq. (3.29) can be written as

$$Y_{critical} = \varepsilon D \frac{(1-\beta)^2}{\beta^2} \tag{3.33}$$

$$Y_{non\text{-}critical} = \varepsilon D \frac{(1-\beta)^2}{(\beta-\rho)^2} \cdot \tag{3.34}$$

Dividing Eq. (3.32) by Eq. (3.33) gives

$$\frac{Y_{crit}}{Y} = \frac{(\beta-\rho)^2}{\beta^2} = (1-\$)^2 \tag{3.35}$$

which is a very convenient expression for determining the reactivity. Restrictions on the use of this equation are addressed in the next chapter when the behavior of the VTMR equation is discussed. This idea was first introduced by Lindeman and Ruby (1966).

As prompt-critical is approached, the quantity Y diverges in Eqs. (3.28) through (3.33). This reflects the possibility that the power level for a critical system has no upper limit, and is an expected result. At delayed-critical this does not occur unless delayed neutrons are taken into account. Physically the delayed neutrons and spontaneous fissions act as an internal source and measurements in delayed critical systems occur in the same nuclear environment that characterizes measurements in subcritical systems (Bergström, et. al.; 1967). From the prompt-neutron viewpoint, both systems appear to be subcritical (Pacilio, et. al.; 1976).

Modifications to the Variance-to-Mean Ratio Equation

A number of modifications and extensions have been suggested in attempts to make the VTMR equation more practical to use and more accurate in modeling neutron fluctuations. The first of these was made by Bennett (1960) who extended it to include the effects of delayed

neutrons. Six years later, the parameter $Z = \frac{Y}{\overline{c}}$ was introduced (Furuhashi, 1966). Although very similar to Y, Z has a number of advantages that are of interest. At nearly the same time, Dragt (1966) suggested including the third-order moment of neutron fluctuations, to increase the information available from the data. More recently, a large number of additional methods have been proposed, many of which utilize two or more detectors (Behringer and Peier, 1979; Hage, 1985).

As previously mentioned, the VTMR method was one of the first reactor noise techniques introduced; because of this, a large number of variations have been suggested. Some of these have evolved into independent noise techniques which are not related as closely to the VTMR method, and therefore are not discussed. Except for the Z parameter method, the following modifications were not used experimentally and are presented simply for completeness.

Delayed Neutrons

It has been found that when delayed neutrons are considered, the VTMR is decreased, suggesting that these neutrons tend to smooth out neutron fluctions (Wang and Ruby, 1975). Deriving the VTMR equation so that it includes delayed neutrons can be accomplished using the same method presented in the previous section; however, Eqs. (3.3) through (3.9) are not simplified to Eq. (3.10) in this case. The equation is written (Thie, 1963)

$$VTMR = 1 + 2\epsilon D \sum_{j=1}^{7} \frac{A_j}{\alpha_j} G_0(\alpha_j) \left[1 - \frac{1-e^{-\alpha_j t}}{\alpha_j t}\right] \tag{3.36}$$

where A_j and α_j are defined in terms of the zero power transfer function

$$G_0(w) = \sum_{j=1}^{7} \frac{A_j}{\alpha_j + iw} \tag{3.37}$$

For j=1, this reduces to Eq. (3.29), since $\alpha_1 = \alpha$, $A_1 = (1-\beta)/\ell$, and $G_0(\alpha_1) = (1-\beta)/2(\beta-\rho)$.

The problem with using all six delayed neutron groups is that fourteen independent parameters are present, making a curve fit to experimental data impractical. The use of one equivalent delayed neutron group has been suggested (Szeless and Ruby, 1971), but was found to be inaccurate (Albrecht, 1971; El-Zeftawy and Ruby, 1972). A two delayed group model, though, has been used with some success (Wang and Ruby, 1973).

It has been suggested that including delayed neutrons in the VTMR analysis may cause some correlation between successive time intervals. In order to avoid this problem, a waiting time of five seconds between each channel could be used (Pál, 1963). However, this would make data acquisition extremely slow and, if necessary, is a strong argument against including delayed neutrons.

If the channel width is kept small, the VTMR equation can be used without considering delayed neutrons. These neutrons, if detected in different time intervals than the prompt neutrons to which they are correlated, appear as random neutrons and do not affect the results.

However, delayed neutrons detected in the same channel contribute to the fluctuations in a non-random manner, causing errors if they are not accounted for. The channel width size at which delayed neutrons effects become apparent is a function of the reactivity, decreasing as criticality is approached (Pál, 1963). As long as the condition

$$t << \frac{1}{|\alpha_2|} \tag{3.38}$$

is maintained, delayed neutrons do not need to be considered (Pacilio, 1969). Since α_2 cannot be determined without including delayed neutrons, a conservative upper limit on channel width must be determined. Table 3.1 contains various values from the literature. Based on these values,

$$t_{max} = 60 \text{ msec.} \tag{3.39}$$

was chosen to replace the value of 0.23 seconds mentioned in the previous section in order to increase accuracy.

Z Parameter

Another modification of the VTMR equation has been suggested (Furuhashi, 1967) to allow a more accurate fit of the data. In certain far subcritical systems, such as the light-water moderated, natural uranium lattice at the University of Arizona, the parameter Y reaches a maximum on the order of 10^{-2}, which is too small to easily resolve. This system is approximately nineteen dollars subcritical and therefore the average chain length is quite short. The result is a small number of neutron pairs detected and a corresponding small Y value, which can only be improved by increasing the detector efficiency. However, by

Table 3.1. Maximum Channel Width for Neglecting Delayed Neutrons at Critical Conditions

Author	Year	Value (msec.)
Bennett	1960	$t < 10$
Albrecht	1962	$t < 100$
Pál	1963	$t < 120$
Furuhashi and Sigeru	1968	$t < 230$
Uhrig	1970	$t < 50$
Szeless and Ruby	1971	$t < 10$

dividing the VTMR equation by $\bar{c} = \varepsilon F t$, the efficiency cancels out and after rearranging terms, a new parameter, Z, is obtained.

$$\frac{\overline{c^2} - \bar{c}}{(\bar{c})^2} = 1 + Z \tag{3.40}$$

$$Z = \frac{D}{F} \frac{(\beta-\rho)^2}{(1-\rho)^2} \frac{1}{t} \left[1 + \frac{1-e^{At}}{At}\right] \tag{3.41}$$

After manipulating Eq. (3.38), Z may be rewritten

$$Z = \frac{\left(\frac{\bar{c}}{2}\right) - \frac{(\bar{c})^2}{2}}{\frac{(\bar{c})^2}{2}} \tag{3.42}$$

which is a physical parameter of the system; the ratio of the expected number of related neutron pairs to the expected number of random neutron pairs. In direct contrast to Y, Z increases as the channel width decreases since the probability of a correlated pair increases relative to the probability of a random pair. It reaches a maximum at t = 0 and has a stronger dependence on α than Y does (Pacilio, 1969). Using Z rather than Y for subcritical systems is generally better, but the best parameter to use is simply the largest.

Although Z's independence of detector efficiency has been verified experimentally, small detection efficiencies (i.e., those less than 1×10^{-4}) make its use impractical. This occurs because the fission rate, F, needed to maintain $\bar{n}$ at a suitable value makes Z too small for analysis (Pacilio, 1969).

Third-Order Moment

The possibility of reaching additional information from neutron fluctuations via a third-order moment analysis of the count distribution was first suggested by Dragt (1966) and further developed by Furuhashi and Sigeru (1968). Actually, several types of analysis are possible depending on the combinations of third order moments that are adopted in manipulating the data. Each type theoretically contains related neutron triplets as well as related neutron pairs.

The parameter X is usually used and is analogous to Y and Z. One equation may be written (Dragt, 1966)

$$\frac{\overline{c(c-1)(c-2)} - 3(\bar{c})\,\overline{c(c-1)} + 2(\bar{c})^3}{3\bar{c}} = X \tag{3.43}$$

$$X = \frac{\epsilon^2 D^2 k_p^4}{(1 - k_p)^4}\left(1 + e^{At} + \frac{2 - Ze^{-At}}{At}\right) . \tag{3.44}$$

From a strictly theoretical perspective, the above equations coupled with other third order formulas could yield β and ℓ separately, which would be very valuable. However, the detection of related neutron triplets is too infrequent to make the analysis possible. Experiments using this method yield no more information than the VTMR method, which utilizes the first and second order moments only (Pacilio, 1969).

Multiple Detectors

More recently, reactor noise techniques closely related to the VTMR method, but using more than one detector, have been suggested. Incentive for this work is provided by two independent objectives: the desire to better understand spatial effects, and the construction of an

accurate method for assay of fissile material. Harris, Natelson, and Schmidt (1969) proposed a modified coefficient of correlation, obtained experimentally with two detectors and used to measure the degree of reactor coupling in addition to kinetic parameters. They performed a number of experiments illustrating the usefulness of this coefficient. The positioning of the two detectors in terms of modal harmonics was quite important. Hage and Ciforelli (1985) derived a mathematical model for the non-destructive assay of fissile material using an extension of the VTMR technique to separate correlated neutron counts from those unrelated. They have experimentally tested the theory using 32 detectors surrounding the material to obtain extremely high efficiencies with good results.

In Summary

The theory behind the VTMR method has been explained and the equation derived. This equation (3.28) or one of its alternative forms (Eqs. (3.31) and (3.32)) is easier to use than the possible modifications and is a natural starting point for any experiment. Due to both equipment and time constraints, only the Z parameter was experimentally investigated as is discussed in the chapter on analysis of data. The modifications are presented in an attempt to provide a complete picture of the VTMR technique.

CHAPTER 4

BEHAVIOR OF THE VARIANCE-TO-MEAN RATIO EQUATION

The variance-to-mean ratio equation relates the first and second-order moments of the neutron noise distribution to the detector efficiency, the length of the sample time interval, and a number of reactor parameters. However, these quantities cannot all be obtained independently; additional information must be provided. In the first section of this chapter, the additional information needed to obtain each particular parameter is discussed. In the second, the results of a parametric study are presented. These results illustrate the effect that individual variables have on the behavior of the VTMR equation.

Variance-to-Mean Ratio Parameters

A number of useful reactor parameters can be experimentally determined using the VTMR method. These include β, the effective fraction of delayed neutrons; ℓ, the prompt neutron generation time; D, the Diven fission parameter; ρ, k_p or R, the reactivity of the system; and different combinations of these same quantities. In addition, the detector efficiency, ε, and the length of the time interval, t, can be determined. The easiest parameter to vary is t; hence in VTMR experiments, t is generally the independent variable and information on the other parameters is found by observing their dependence on the channel width. The detector efficiency could also be varied, by changing

the detector voltage. This would not yield nearly as much information because the VTMR is linearly dependent on ε, in contrast to its more complicated functional dependence on t.

Table 4.1 shows the additional information that must be provided to obtain each specific parameter experimentally . As can be seen, the method is best suited for determining β/ℓ from a critical system and $(\beta-\rho)/\ell$ from a non-critical system, since these quantities can be directly obtained by varying t. However, the method was first used to determine $\overline{\nu^2}$ (Feynman, de Hoffmann, and Serber; 1956), and has since been used primarily to measure ε and β/ℓ.

As shown in the next section, the VTMR at a particular channel width decreases as ρ decreases. Because of this, the VTMR method does not work well for far subcritical conditions, and the non-critical systems referred to in Table 4.1 must be relatively close to critical. Also, if the reactivity or multiplication factor is known, the same results can be obtained from the non-critical system as from the critical system. This table is a useful reference when first considering an experiment employing the VTMR method.

Parametric Study of the Variance-to-Mean Ratio Equation

A computer code called VMEX was written to calculate exact VTMR values using Eqs. (3.29) and (3.30), and reasonable numbers for the needed reactor parameters. A listing of the FORTRAN program can be found in Appendix D. The subroutines called in subroutine PLOT are part of an external plotting package which is discussed in Chapter 6; it was used to produce the figures in this section.

Table 4.1. Information Available from Experimental Use of the Variance-to-Mean Ratio Equation

Parameter Desired	System	Information Needed Prior to Experiment
ℓ	critical	β, t
β	critical	ℓ, t ϵ, D, t
ϵ	critical	β, D, t
t	critical	ϵ, β, D
$\frac{\beta}{\ell}$	critical	t
D	critical	ϵ, β, t
$\overline{\nu^2}$	critical	ϵ, β, t, $\overline{\nu}$
ρ, k_p, R	non-critical	β, ℓ, t β, ϵ, D, t β, t, critical result
$\frac{\beta-\rho}{\ell}$	non-critical	t
$\frac{1-\beta}{\beta-\rho}$	non-critical	ϵ, D, t

The Channel Width

The dependence of the VTMR on t, the channel width, is discussed first because it is used as the independent variable. Figure 4.1 shows a plot of the VTMR versus t for a critical reactor with $\beta = 0.007$, $\ell = 54$ μsec, D = 0.795, and a detector efficiency of 2.0×10^{-4}. These are the approximate values from the University of Arizona TRIGA reactor except for the detector efficiency, which is seldom this high even with the detector placed at an optimum position. The figure shows that as t approaches zero, the VTMR goes to unity. This is expected, since no related neutrons can be detected together in a time interval that approaches a length of zero. In addition, the plot clearly shows that the slope of the curve is greatest at t=0 and steadily decreases with increasing t until leveling off at approximately t = 75 msec. When the VTMR no longer varies with t, it is referred to as "saturated." Physically, the non-random effects have increased with an increasing time interval until reaching a maximum. At this point the variance in counts per channel no longer increases faster than the average counts per channel, and the VTMR remains constant. Analytically, this simply corresponds to the bracketed part of Eq. (3.29) asymptotically reaching its maximum value of 1.0.

The Reactivity

Determining the system reactivity using the VTMR method was one of the primary goals of this work; therefore, the effect of reactivity on the VTMR is considered next. Figure 4.2 contains the base-case curve along with additional plots generated using various subcritical

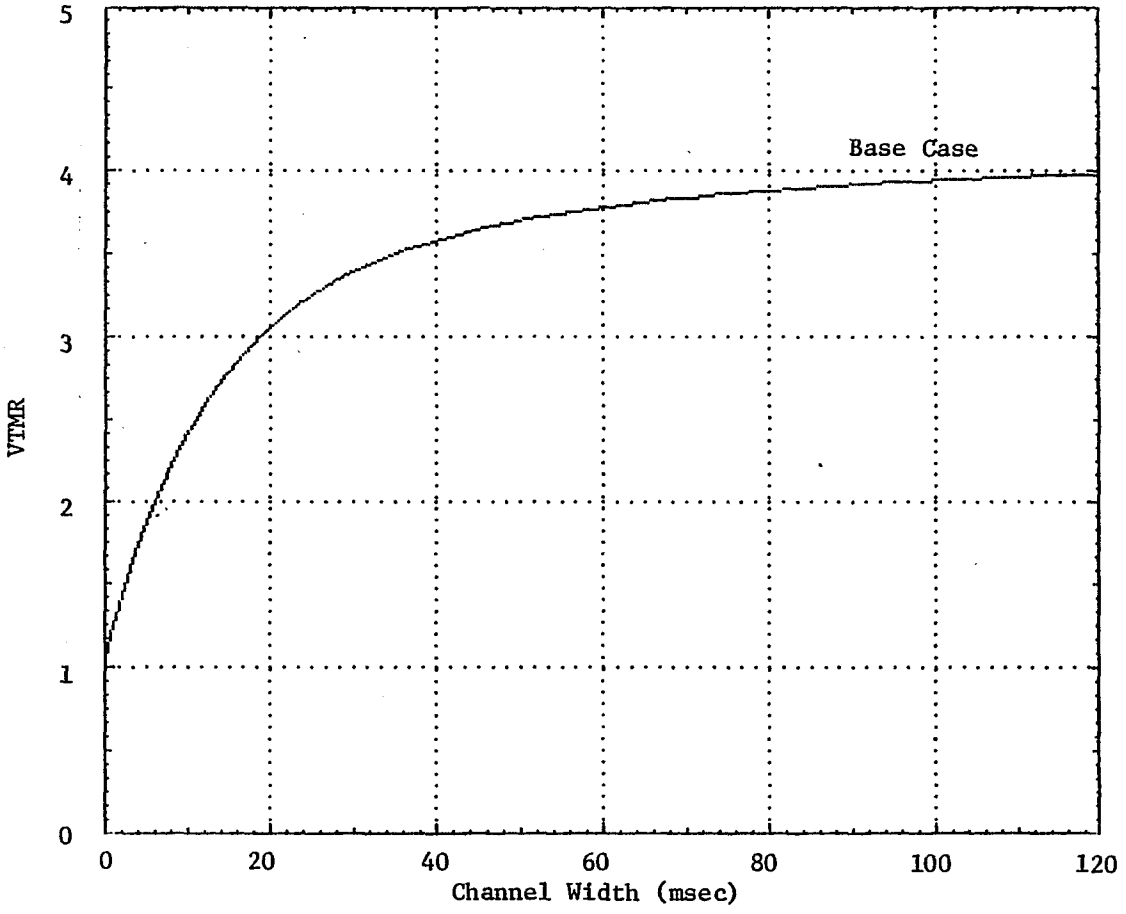

Figure 4.1. Exact Variance-to-Mean Ratio Versus Channel Width

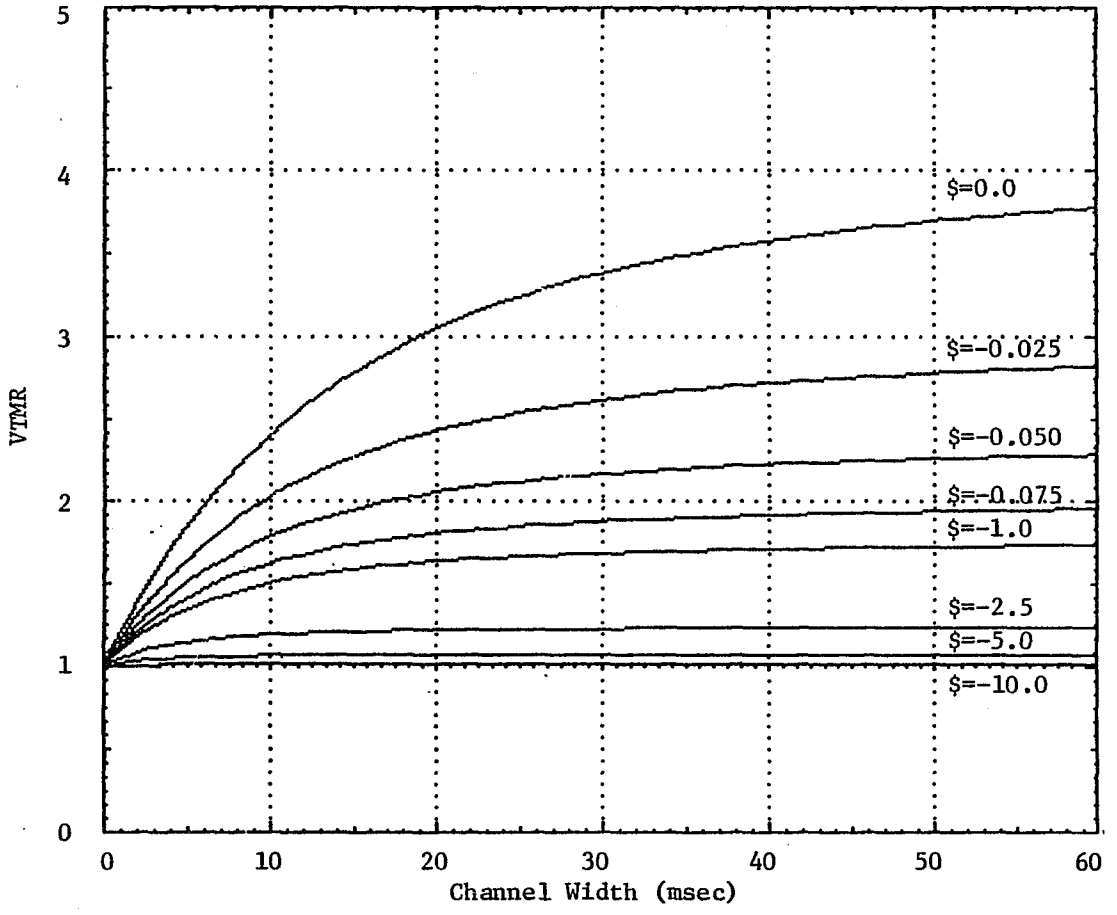

Figure 4.2. Exact Variance-to-Mean Ratio Versus Channel Width for Different Subcritical Reactivities

reactivities. The horizontal scale is twice that of Fig. 4.1 in order to more clearly show the region of greatest interest; $t \leq 60$ msec. These plots show that as the system becomes further subcritical, the VTMR for a particular channel width decreases. Physically this corresponds to a smaller degree of correlation between the neutrons, due to the shorter average chain length. It also makes the VTMR method impractical for far subcritical systems, since resolving information from such nearly straight lines with statistical variations is quite difficult. In fact, for one experiment, the method did not show agreement for reactivities below -30 cents (Wang and Ruby, 1975). However, it appears from Fig. 4.2 that reactivities as low as -$2.0 should provide reasonable results if detector efficiencies of this magnitude are available and enough data are collected.

Referring to Eqs. (3.27) and (3.32), a reactivity change from $=0.0 to $=-1.0 reduces Y by a factor of four after saturation. This is shown to be nearly true in Fig. 4.2, even though at t = 60 msec, the $=0.0 curve is only about 85% saturated while the $=-1.0 curve is about 93% saturated. For the same one-dollar reactivity change at a much smaller channel width of 10 msec, the bracketed part of Eq. (3.32) is increased from 0.4396 to 0.6431, decreasing the reduction in Y to approximately a factor of 2.8. Again, this can be seen in Fig. 4.2. Figure 4.3 is identical to Fig. 4.2 except that the reactivity changes are now positive. This figure shows that supercritical systems appear no differently than subcritical systems to the VTMR equation. At prompt critical, the VTMR equation diverges and it has no meaning for

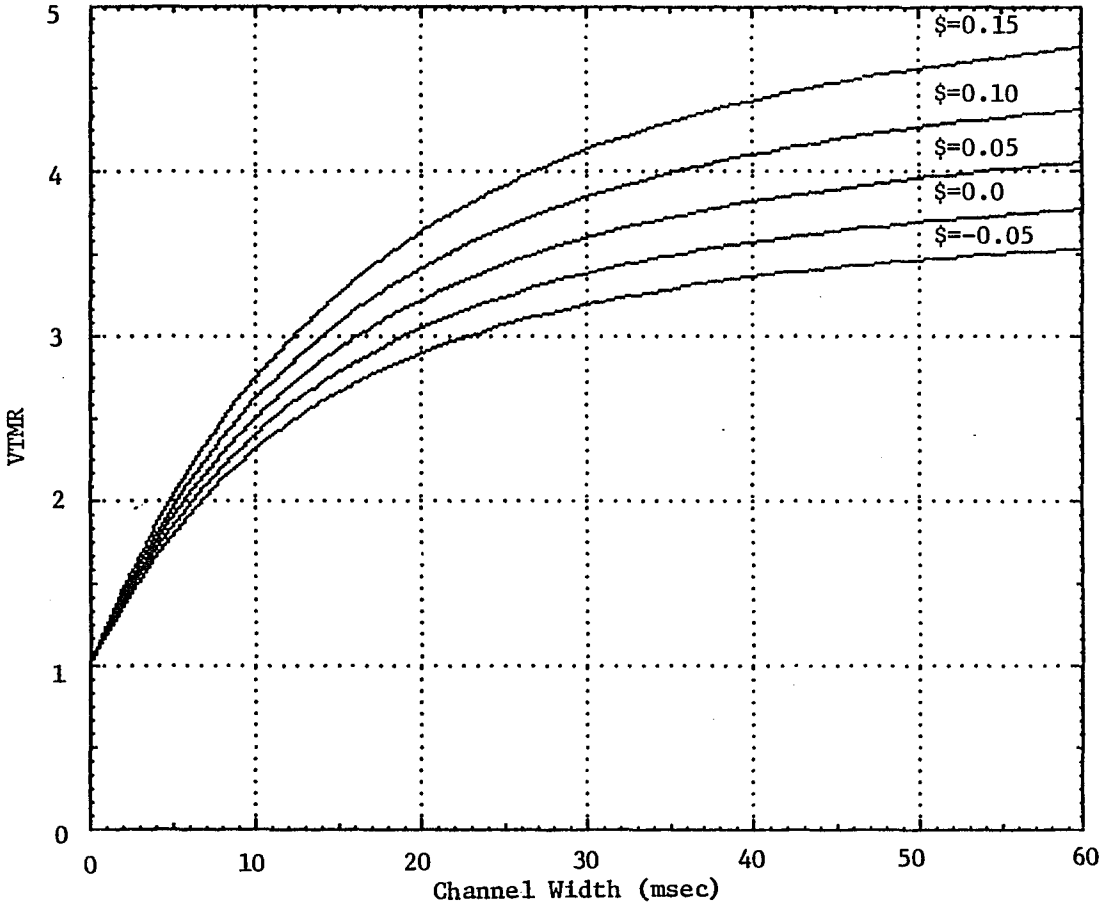

Figure 4.3. Exact Variance-to-Mean Ratio Versus Channel Width for Different Reactivities

reactivities above one dollar or super-prompt-critical. This is understandable because the system is completely unstable in this region.

In Chapter 3, Eq. (3.35) was derived, which is a convenient expression for determining the reactivity by comparing a critical and subcritical saturated Y value. The question of which channel width or widths optimizes the measurement of these Y values has not been answered. Too large a value of t leads to delayed neutron problems, while too small a value leads to unsaturated VTMR values. In Eq. (3.29) the quantity $(1-e^{At})/At$ is dominated by $1/At$ for the channel widths greater than 20 msec that are of interest here. As the reactivity is decreased, this domination increases.

Saturation occurs as the bracketed quantity in Eq. (3.31) approaches 1.0; this happens in the limit as $|At|>>1$, so that $1/At$ goes to zero. Recalling Eq. (3.38), the useful range of Eq. (3.35) is then written

$$\frac{1}{|\alpha_2|} >> t >> \frac{1}{|A|} \tag{4.1}$$

or in terms of Eq. (3.39)

$$60 \text{ msec} \geq t >> \frac{1}{|A|} \tag{4.2}$$

The above condition cannot be met for heavy water or graphite moderated systems where β/ℓ and therefore $|A|$ are on the same order as $|\alpha_2|$ (Pacilio, 1969). However, for the light water systems of interest, it can be approximately achieved for t between 50 and 60 msec. Some error will be incurred in this range since the critical VTMR values are only about 85% saturated, but these can easily be corrected for.

The Detector Efficiency and Diven Fission Parameter

The VTMR has a linear dependence on both the detector efficiency, ε, and the Diven fission parameter, D. Doubling either of these quantities will double the VTMR, while doubling both of them results in a factor of four increase. Actually, the Diven fission parameter is much less likely to vary because it is a physical constant for any particular system as long as the fuel composition remains approximately uniform. Even relatively large variations in fuel, such as those experienced over the core life, result in changes of less than 10% in D. On the other hand, ε can vary quite dramatically, depending on the detector position and the voltage used. Figure 4.4 shows the effect of changing either of these parameters, although ε was actually varied. These curves show the expected results. In the limit as ε goes to zero, the VTMR approaches unity because, physically, no correlated neutrons can then be detected.

If a channel width of $t = \ell/(\beta-\rho)$ is chosen, the bracketed quantity in Eq. (3.29) is equal to e^{-1}. For a critical system this channel width is equal to about eight milliseconds. In this case, it can be seen that an efficiency of 1.7×10^{-4} is needed to obtain a VTMR greater than two, corresponding to a saturated, critical VTMR of 3.72. This exercise shows the order of efficiency needed to obtain the magnitude of VTMR values desired for good data curve fits. The following two expressions are often used to set these requirements more generally .

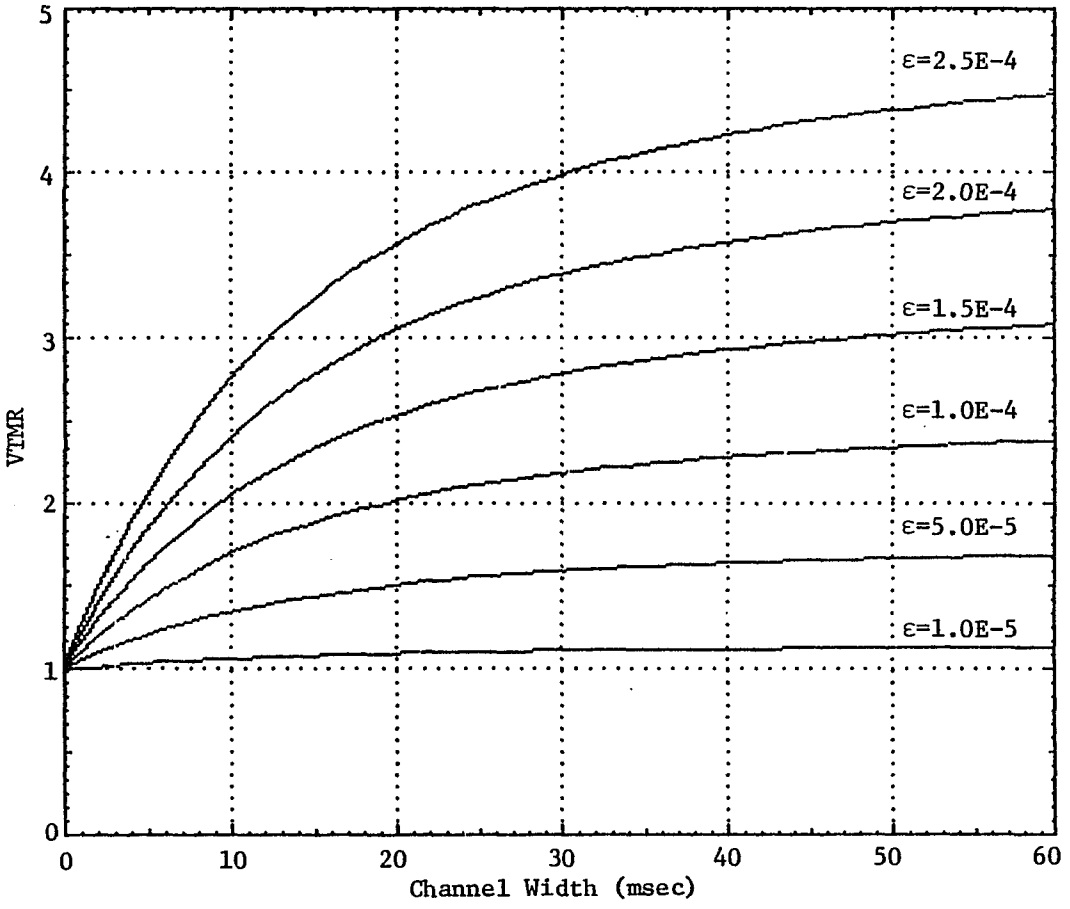

Figure 4.4. Exact Variance-to-Mean Ratio Versus Channel Width for Different Detector Efficiencies

$$\varepsilon \geq (1 - k_p)^2$$

$$\varepsilon \geq 0\ (\beta^2)$$

These are often quite difficult to meet without serious deadtime problems.

The Quantity β/ℓ

Experimental determination of β/ℓ is quite important in small thermal reactors moderated by hydrogeneous materials. In these systems, the values of ℓ and β are sensitive to composition and geometry (Pál, 1963). The VTMR method is well suited for determining this ratio as mentioned in the previous section. For these reasons, the effect on the VTMR of varying β/ℓ has also been considered. Figure 4.5 presents the results. It shows that decreasing β/ℓ increases the VTMR. Table 4.2 shows the individual β and ℓ values used to vary the ratio β/ℓ. The primary cause of the observed behavior is the effect of the quantity $1/\beta$, which when squared, has a large influence. For systems in which the geometry is not varied and fuel composition is fairly uniform, the ratio β/ℓ is expected to remain constant. This was the case for the University of Arizona TRIGA reactor during these experiments.

Simultaneous Variations in Reactivity and Efficiency

A Comparison of Figs. 4.2 and 4.4 illustrates that, despite the difference in functional dependence, varying the reactivity has approximately the same effect on the VTMR as varying the efficiency. This is due to the VTMR being more strongly dependent on the reactivity which explicitly appears in Eq. (3.32) than it is on the reactivity contained in the quantity A. Because of this, nearly identical curves

Table 4.2. Individual β and ℓ Values Used When Varying β/ℓ

ℓ (msec)	β (%)	β/ℓ
50.00	0.6	120
52.00	0.65	125
54.62	0.71	130
55.71	0.78	140
56.67	0.85	150

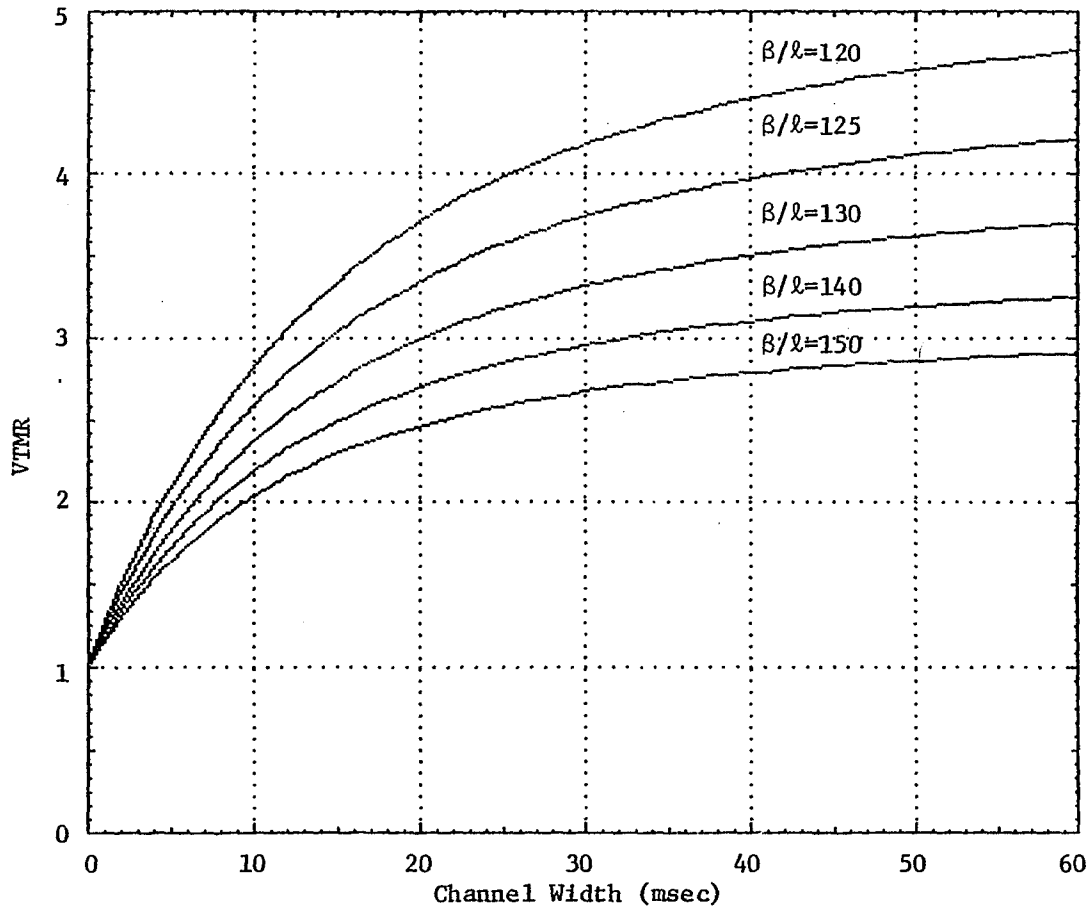

Figure 4.5. Exact Variance-to-Mean Ratio Versus Channel Width for Different β/ℓ Values

with slight statistical variations could produce quite different efficiency and reactivity results after an appropriate curve fit. Figure 4.6a clearly shows this effect. Although the curves are nearly indistinguishable, one was generated with $\varepsilon = 2 \times 10^{-4}$ and $\rho = 0.0$ (i.e., the base case), while the other was generated with $\varepsilon = 2.19 \times 10^{-4}$ and $\rho = -5$ ¢. Figures 4.6b and 4.6c illustrate additional, but more extreme, examples of this effect. These were generated so that differences between the effects of ρ and ε on the VTMR can be more easily observed. The similarity of the VTMR for different ρ and ε cannot always be accurately resolved, leading to potential problems. It will be discussed further in Chapter 7.

In Summary

In this chapter the behavior of the VTMR equation was discussed. The first section contains a useful table that summarizes the parameters which can be experimentally measured using the VTMR equation. It also lists any additional information that must be provided to obtain these parameters. The second section displays the results of a parametric study on the VTMR equation. β and ℓ were not individually perturbed in this study because they are nearly constant for a particular system and because considering other than their ratio is extraneous. An important result of this study was the observation that the reactivity and detector efficiency affect the VTMR in a similar manner. This makes some VTMR curves generated with different values of ε and ρ appear nearly identical.

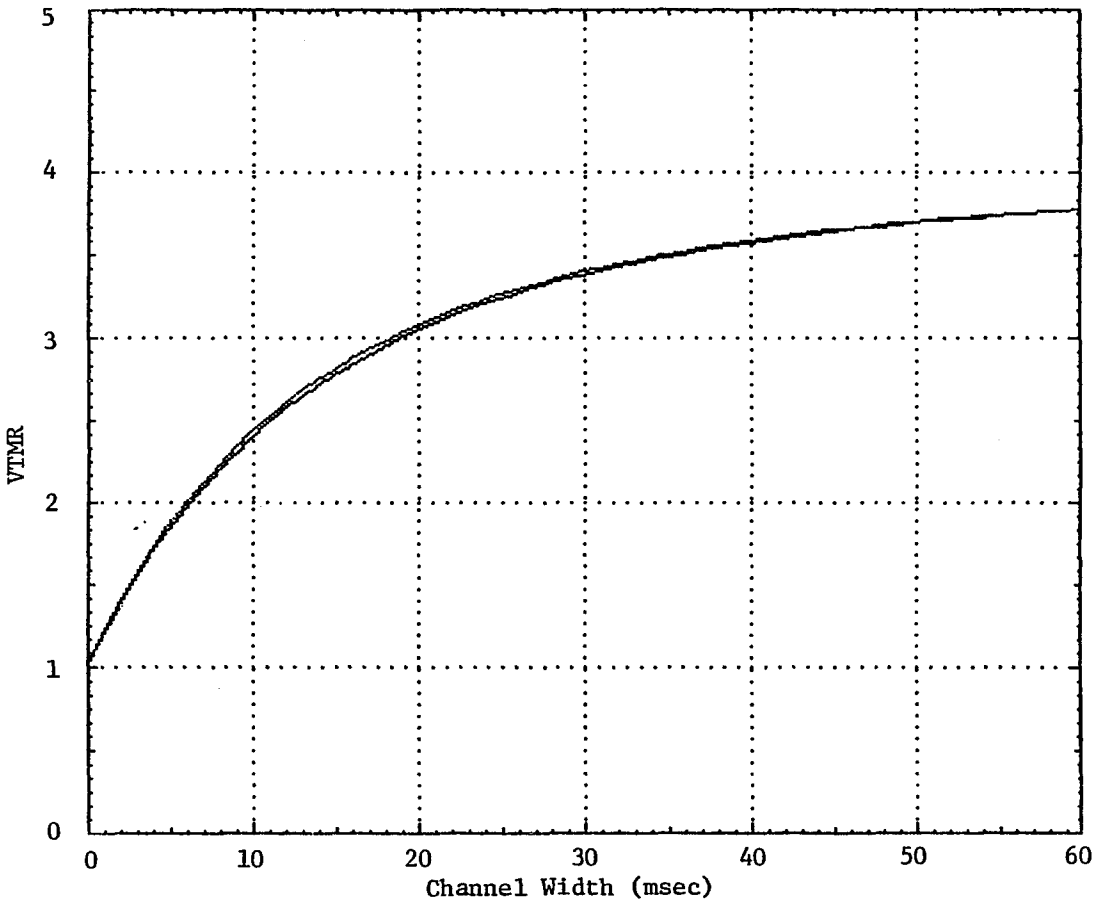

Figure 4.6a. Exact Variance-to-Mean Ratio Versus Channel Width for Simultaneous Variations of the Reactivity and Detector Efficiency

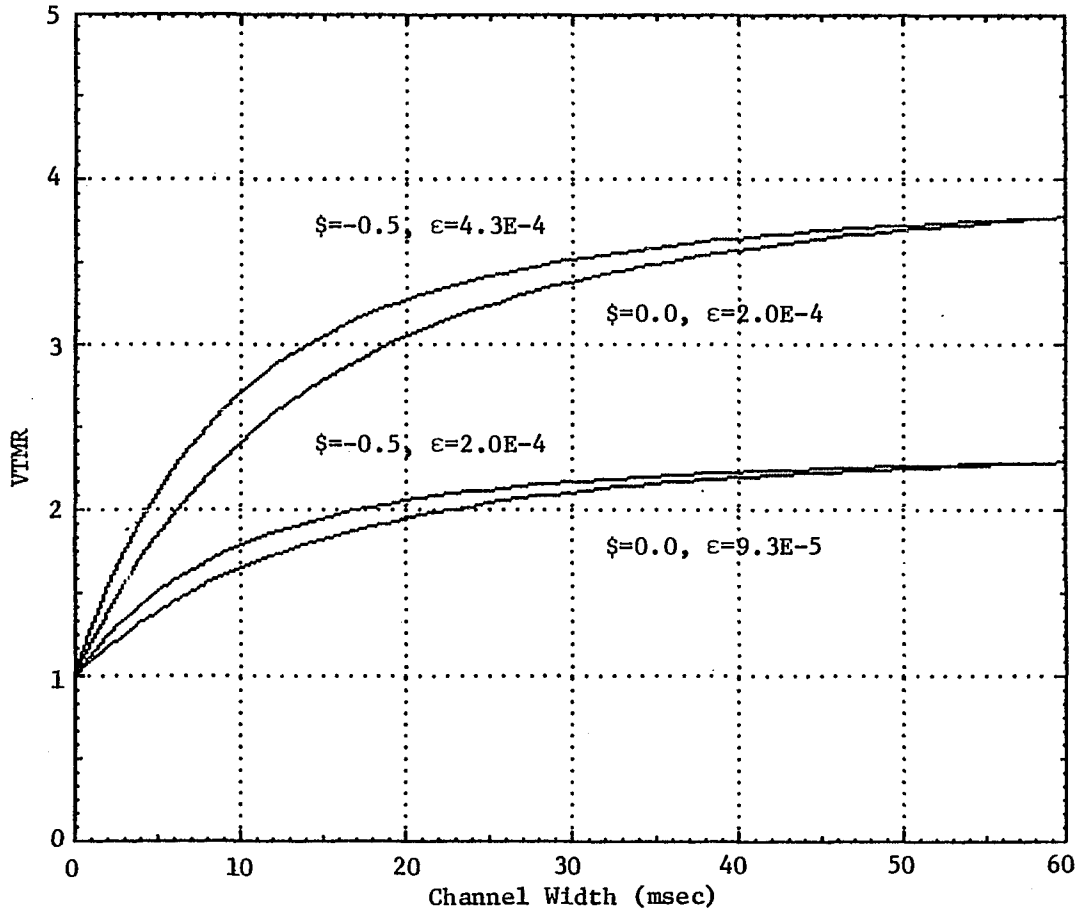

Figure 4.6b. Exact Variance-to-Mean Ratio Versus Channel Width for Simultaneous Variations of the Reactivity and Detector Efficiency

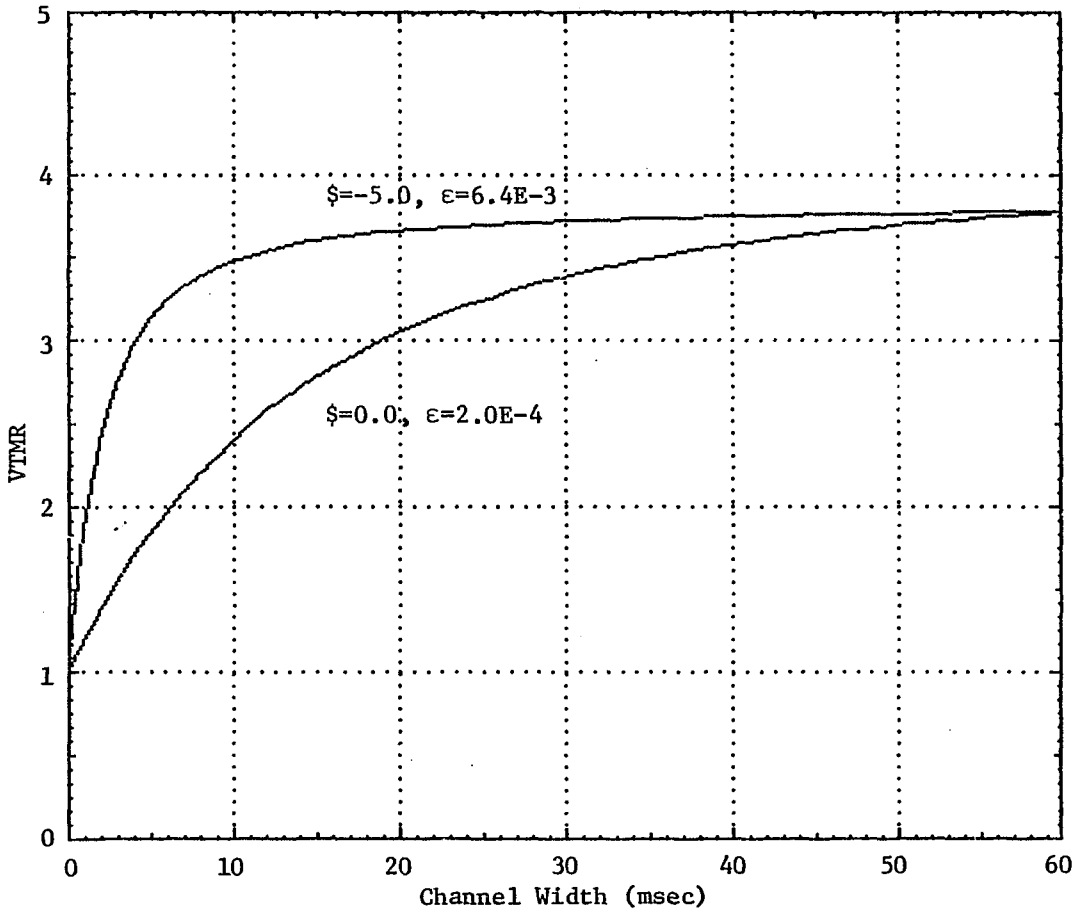

Figure 4.6c. Exact Variance-to-Mean Ratio Versus Channel Width for Simultaneous Variations of the Reactivity and Detector Efficiency

CHAPTER 5

COLLECTION OF DATA

Most experiments performed in the field of nuclear engineering are composed of two primary parts. The first of these is the collection of data: setting up the equipment, testing it, making the measurements, and then recording the raw data in some manner. The second part is the analysis of the data in order to obtain the desired result. This is often done with the assistance of a digital computer to increase speed and to remove the errors that inevitably occur when performing long strings of hand calculations. Chapter 6 deals with the analysis of the collected data, while this chapter considers the collection itself.

Methods of Data Collection used in Variance-to-Mean Ratio Experiments

At least four different, but related, data collection methods are available when carrying out a variance-to-mean ratio experiment. All methods utilize a neutron detector of some sort coupled with a preamplifier, an amplifier, and a discriminator known as a PAD. The techniques are differentiated by the manner in which the resulting signals are dealt with. Each method is discussed briefly in this section. Figure 5.1 illustrates the different methods, and is borrowed from a similar diagram by Pacilio (1969). Many of the ideas in this section have their origins in this reference as well.

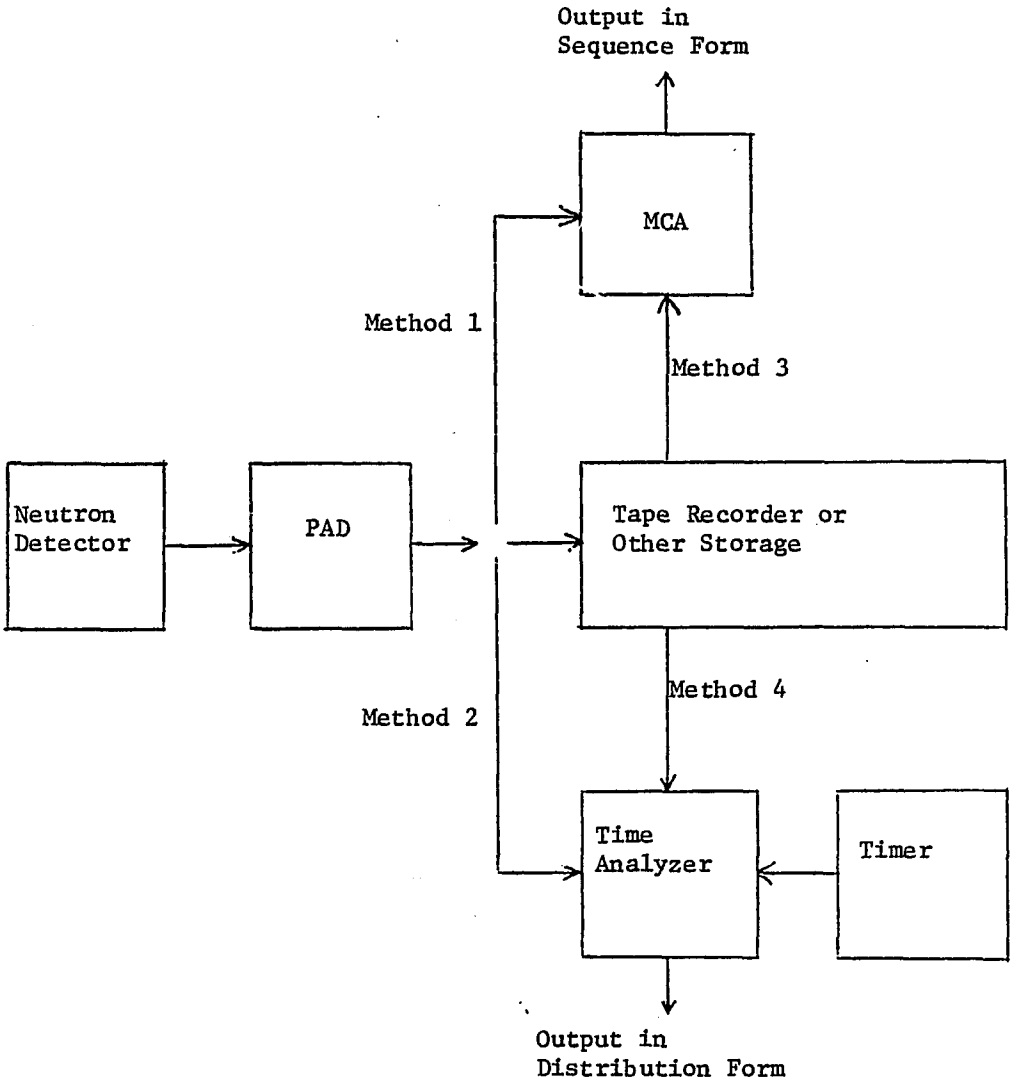

Figure 5.1. Block Diagram of the Different Measuring Apparatus Used in Variance-to-Mean Ratio Experiments

Method 1: Sequential Collection

In method one, the output pulses from the PAD are counted for a large series of sequential time intervals of length t and then stored in that order. This method was used in the experiments performed by Feynman, de Hoffmann and Serber (1956), and Lindeman and Ruby (1967). For these measurements, only one value of t was used; however, other authors (Pál, 1963) have used several different values.

A short **deadtime** interval occurs between two consecutive observations that last a few microseconds in modern MCAs, and does not pose a problem. However, obtaining enough gates in a reasonable time period is a limitation, since many MCAs have a maximum of 8192 channels. The major criticism of this technique is that the time sequence of the channels is unused information; more data is collected than needed. Alternative methods of analysis which make use of the time ordering are possible and will be discussed in the next chapter. However, from a data collection point of view, more compact and time-saving methods are possible.

Method 2: Distribution Collection

In this method the neutron-induced pulses are recorded directly in the form of the neutron noise distribution, discussed in Chapter 2. The PAD output is fed into an address register that advances one step for each arriving pulse. At the end of the time interval t, signaled by an external timer, the final number in the address register is fed into a data register in the form of one count corresponding to that number. The address register then resets to zero and the process is repeated.

This results in channel i+1 of the data register recording the number of time intervals in which i pulses were counted. The two registers combine to form the time analyzer on Fig. 5.1.

Using this method, the data collection can proceed continuously until the desired number of channels has been collected. The data are stored in a much more compact form; instead of transferring and working with thousands of numbers, a maximum of between twenty and two hundred numbers are used, depending on the channel width chosen.

One drawback of both this and the first method is that data can be collected using only one channel width at a time. Since the VTMR experimental technique analyzes the VTMR as a function of the channel width, many independent sets of data must be collected. This is not the case with the next method.

Method 3: Storage and Sequential Collection

In this data collection method, the neutron pulses are continuously recorded using either a tape recorder or perhaps the memory of a digital computer. The recorded pulses are then repeatedly "replayed" once for each desired channel width, as in method one. This procedure avoids the discrepancy possible when the VTMR(t) is determined using sets of data collected at different times. Albrecht (1962) used this method to obtain good results.

Method 4: Storage and Distribution Collection

This last method combines the distribution collection technique of method two and the storage concept of method three. Here, the neutron pulses are "replayed" for the time analyzer, so that the same

data can be utilized in determining many different VTMR values--one for each time interval. This is probably the most effective, rapid, and compact data collection technique for VTMR experiments (Pacilio, 1969). Use of this method is conspicuously lacking in the literature, at least to the best of the author's knowledge.

Method Used in Collection of Experimental Data

Of the four methods described, only the first was easily used at the University of Arizona due to equipment restrictions. A circuit board in the process of being designed and built to interact with an AT&T or IBM personal computer would have allowed use of the third or fourth method. Unfortunately, this was not completed in time to be employed in this experiment. Modifications were made to the first method in order to utilize the available time ordered nature of the data. These included combining short channels to form longer ones and, in some cases, doing this in a number of different ways. The modifications made the sequential collection method of data acquisition more attractive, with its major limitation becoming the inability to obtain large enough amounts of data in a reasonable period of time. The 8192 channel MCA, when used in this experiment, took between one second and two minutes to fill with data, depending on the channel width. A three minute data transfer is then necessary before additional data can be gathered. This makes collecting more than 5×10^4 channels of data impractical, even for the shortest channel widths. Obtaining the 1×10^6 channels of information suggested by some as necessary (Pacilio, 1969) would take over eight hours when using short channels. Holding a critical reactor

at steady state for this period of time is also quite difficult, since as the reactivity approaches zero, the system becomes much more susceptible to power level drifts resulting from small changes in the multiplication (Harris, et. al.; 1970). Because of this, a maximum of between 8,000 and 50,000 channels of data were collected while performing this experiment.

The technique which was used, although actually a sequential collection method, could be considered an early attempt at either of the two storage methods, since the data were saved in the memory of a digital computer and analyzed later. There are two reasons it was not classified in this fashion. First, only a small amount of data can be collected at one time. However, if the MCA had 1×10^6 channels instead of less than 1×10^4 channels this would not be the case. Secondly, the data are collected for just one channel width. These channels can be combined to form larger ones, but the number of new channels is quickly reduced as more and more of the original channels are combined to form them. Also, only multiples of the original channel length can be obtained in this method. The time-ordered data could then be transformed into a distribution if desired.

Description and Discussion of the Apparatus

Many different sets of data were collected for analysis during the course of this experiment. Figure 5.2 is a block diagram that shows how the equipment was connected when acquiring the data. The setup was used consistantly throughout the experiment; however, some pieces of equipment were replaced with identical pieces between measurements. This was unavoidable due to the maintenance, testing, and lending of

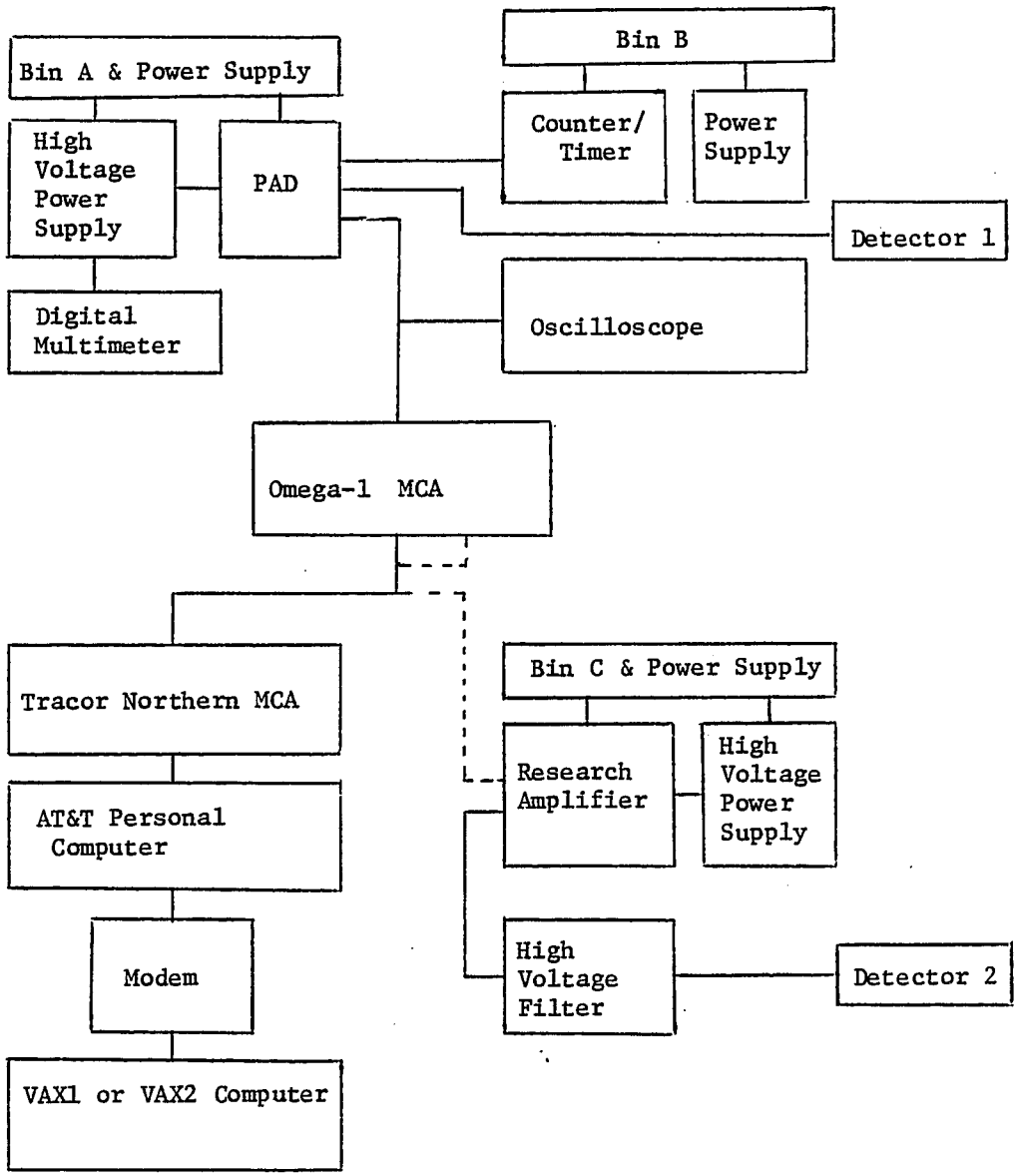

Figure 5.2. Block Diagram of the Equipment Setup for the Variance-to-Mean Ratio Experiment

equipment which naturally takes place. Table 5.1 lists the equipment used in the most recent sets of experiments. The bin power supplies, the counter/timer, and the PAD were the only pieces which differed from those in the earlier experiments.

The neutron detector used is a BF_3 gas-filled detector having an internal pressure of 20 cm of mercury. It has a maximum operating voltage of 3,000 volts and is cylindrical in shape measuring 27 cm in length by 2.8 cm in diameter. The High Voltage Power Supply was set to provide 2,300 volts across the detector; however, this level was consistantly measured as 2,260 ± 10 volts by the digital multimeter. In one test, this voltage drifted about 30 volts during the first hour of operation, but while monitored in use, it remained stationary at the measured level.

Four different PADs were tested using an electronic pulse generator. All four showed double pulses on the oscilloscope under certain conditions, which indicated that perhaps the pulse generator had a problem. However, the same double pulses were observed using a second generator. These incorrect results were a function of both the input pulse amplitude and the gain of the amplifier. If the 64 db coarse gain setting on the PADs was not used, the double pulses did not appear, except when very large pulses (much bigger than those generated by neutron detection) were input and the fine gain was set at its maximum. For all experimental measurements, a coarse gain setting of 32 db was utilized, and in the most recent measurements, after the double pulsing had been observed, a fine gain setting of 7 db was used. Prior to this the fine gain was set to its maximum value.

Table 5.1. List of Apparatus Used in the Variance-to-Mean Ratio Experiment

Apparatus	Model Number	Serial Number	Manufacturer
BF_3 Detector	RSN-7A	K-479	---
High Voltage Power Supply	456	5077 33	Ortec
Bin A & Power Supply	---	A173475	Ortec
Oscilloscope	2213A	A192039	Tektronix
PAD	814	A164748 281922	Canberra
Omega-1 Multichannel Analyzer	4001	276100	Canberra
Multichannel Analyzer	TN-1710	A149656	Tracor Northern
Digital Multimeter	8050	83038404	Soar
Counter/Timer	1772	280761	Canberra
Bin B	---	A158596	Canberra
Power Supply	402M	1786 66	Ortec
Personal Computer	---	A191578	AT&T
Modem	LD5125	A172677	Gandalf
Germanium Detector	8011- 23250-NS	A158742 20-9091	Ortec
High Voltage Power Supply	3002	A188945	Canberra
Bin C & Power Supply	---	A107300	Canberra
Research Amplifier	1412	A107307	Canberra
High Voltage Filter	119	2788-24	Ortec
VAX1 and VAX2 Computers			
Coaxial Cable and Connectors			
University of Arizona TRIGA Reactor Facility			

The detector, PAD, and high voltage source were tested in conjunction by observing the amplified neutron and gamma pulse shapes on the oscilloscope. Figure 5.3 shows a sketch of the oscilloscope screen. The large dark peak is comprised of the neutron pulses on which the oscilloscope sweeps are triggered; the occasional large light peaks are additional neutrons detected during the sweep; and the continuous small peaks are those resulting from gamma ray detection. This test illustrated that the detector was pulsing correctly, although double pulsing would not necessarily be evident.

A resolving time measurement was also performed to determine the length of time required for the detector to recover from one pulse sufficiently to detect the next. This was performed by using two Plutonium Beryllium neutron sources and indicated that the detector was performing correctly. The results are shown in the next chapter.

The pulse discriminator on the PAD was not used in these experiments because the discriminator built into the Omega had previously proven more reliable. The discriminator setting was determined by observing the pulse energies using the Omega's pulse height analysis (PHA) mode, and then adjusting the level until the large number of small gamma pulses were removed. Figure 5.4 shows a sketch of this energy spectrum, as well as the approximate discriminator setting. For all the measurements performed, the maximum pulse allowed to pass the discriminator was between 0.4 and 0.8 volts depending mainly on the PAD setting, and to a lesser extent on the detector's position, relative to the reactor.

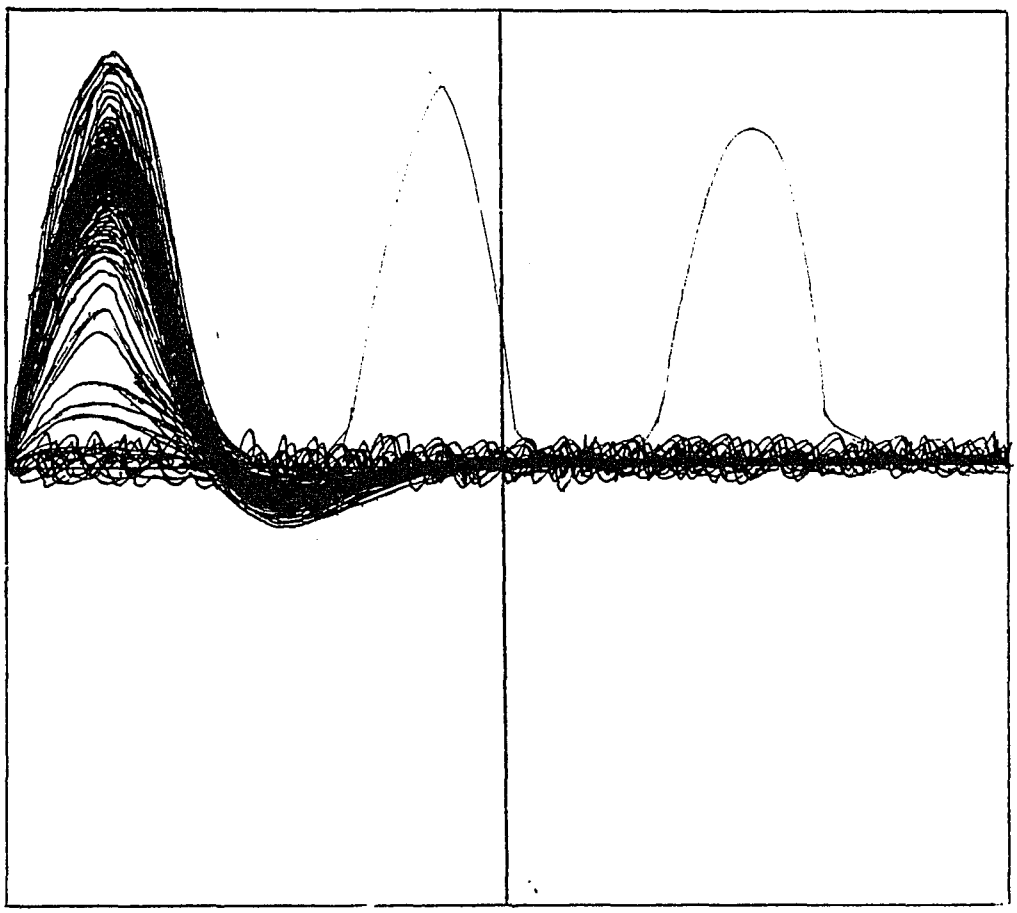

Figure 5.3. Sketch of Neutron and Gamma Pulses As Seen On Oscilloscope Screen

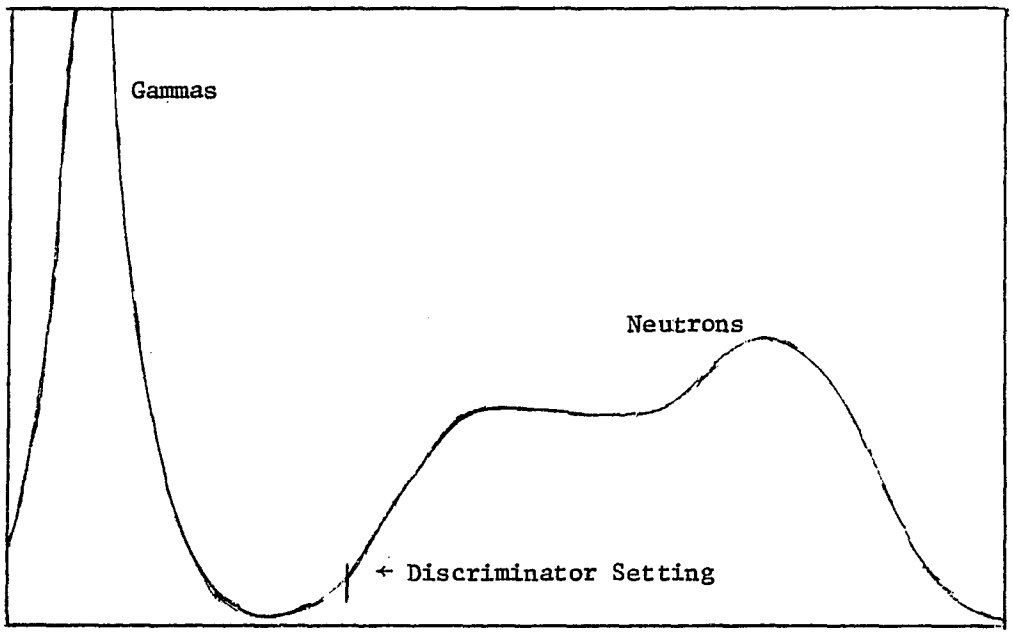

Figure 5.4. Sketch of the Reactor Neutron and Gamma Spectrum (no source)

In order to use the Omega's discriminator output as input for the Tracor Northern MCA, the SCA OUT port was disconnected from the COUNT IN port on the Omega and connected to coaxial line 121, which carried the signal under the floor from the reactor room to the room next door (Engineering Room 120). In this room, where the Tracor Northern is located, the other end of line 121 was connected as input to the MCA. The rationale for using the Tracor Northern MCA rather than the Omega MCA is that the former has 8192 channels, while the latter has only 1024 channels. Some earlier investigations used the Omega, until it became clear that much more information was necessary.

The method of data transfer between the Omega and the VAX did not work for the older Tracor Northern because it could not communicate with the computer during the transfer. The Computer Center at the University of Arizona makes extensive use of time-sharing and when the computer sent signals indicating that the MCA was to pause momentarily, they were ignored and data were lost. To avoid this problem, a short Basic program was written for use on the AT&T personal computer to transfer the data to that machine and store it on floppy disk. At a later time the KERMIT software package was employed to complete the transfer onto the VAX. A listing of the Basic program, called TRANS, is included in Appendix D. Appendix E lists the detailed steps involved in transferring data from either the Omega or the Tracor Northern to the VAX computer.

To test the equipment while working in unison a large number of data sets were collected using random sources; either gamma rays from reference gamma sources or neutrons from a Plutonium-Beryllium source.

Both sources are produced by the radioactive decay of long half-lived isotopes, and this decay is established as occurring randomly. In other words, if everything is working correctly, the distribution of counts should be an exact Poisson distribution and the VTMR should equal unity. The detection of gamma rays required the use of a germanium detector, with its own independent PAD and power supply.

Experimental Collection of Data

Although experimental measurements were made numerous times, no formal procedure was followed, since each group of measurements was generally different from the other groups. For instance, the subcritical data were initially collected from the reactor with a negative period. When it was realized that "the first fundamental assumption for all the neutron noise analysis techniques is that the reactor is in steady-state operating conditions, i.e., subcritical in equalibrium with a source" (Pacilio, et. al.; 1976), the measurements were repeated using a different procedure. Also, the equipment checks described in the previous section were not all performed prior to every experiment. The idea that the PAD might be double pulsing did not arise prior to the penultimate set of measurements, and so this test was not conducted until prior to the final experiment. Table 5.2 lists the date of each measurement, the information collected, and the equipment tests that were performed during or prior to the measurement. The remainder of this section describes a typical procedure, basically that used in collecting the last two sets of data.

Table 5.2. Date, Information, and Equipment Tests for Each Experimental Measurement

Date	Information	Tests
Fall 1984	5 files; A,D	1,2,3,4
Spring 1985	10 files; A,C,D	1,2,3,4
6-15-85	10 files; D	1,2
6-26-85	4 files; B	3,4
6-27-85	7 files; A,C	2,3,4
7-2-85	30 files; A,C,D	1,3,4
7-11-85	24 files; A,B,D	1,3,4
7-26-85	33 files; A,B	3,4,5,6

A - Critical Reactor
B - Subcritical Reactor in S.S
C - Subcritical Reactor with Period
D - Random Source
E - Subcritical Pile

1 - Random Source
2 - Resolving Time
3 - Check Pulse Shape
4 - Check Energy Spectrum
5 - Check for Double Pulses
6 - Monitor High Voltage Level

After connecting the equipment listed in Table 5.1 in the configuration shown by Fig. 5.2, the detector is lowered down the lazy Susan access tube until coming to rest on the rim, at the level of the top of the graphite reflector. Figure 5.5 is a sketch of the University of Arizona Mark V TRIGA nuclear reactor showing this position. The Omega is used in PHA mode to obtain a 512 channel spectrum and the discriminator level is determined. Also, the neutron and gamma pulses are viewed on the oscilloscope. A resolving time or dead-time experiment has already been performed with this detector and so is not repeated. The reactor is taken critical at a power level of 0.1 watts and the neutron source is removed. Having obtained the critical rod position, the power is reduced to about 0.7 milli-watts, a level difficult to measure precisely because it is so low, and criticality is again established.

At this point, or perhaps earlier, the moderator thermostat is set to a higher temperature to eliminate the possibility of the refrigeration system abruptly starting during the data collection. After waiting approximately fifteen minutes to allow the transients time to die away and to ensure that the system is as nearly critical as possible, the acquisition of data begins. This is a cyclic operation in which the MCA records the amplified and discriminated neutron signal for 41 seconds or less (with the channel widths used here, often much less), after which the data transfer occurs, taking more than three minutes. When an adequate amount of data has been collected, the reactor is shut down and the source is again moved, this time close enough to the core to provide the required signal allowing the control rods to be pulled

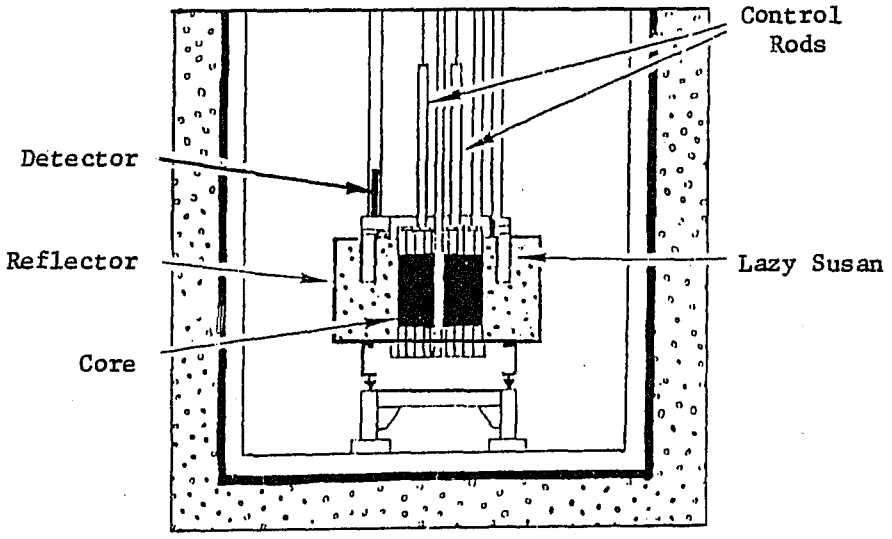

Figure 5.5. Sketch of the University of Arizona TRIGA Reactor

out. The critical rod position is used to determine a new rod position that corresponds to the desired degree of subcriticality. When this position is obtained the source is once again moved, now to a position on the side of the core opposite the detector. This is done to minimize the number of unrelated neutrons detected from the source. The system is allowed to stabilize, again for 15 minutes, and data are collected.

To conclude the experiment, the detector is pulled about half way up the lazy Susan access tube and the reactor neutron source is repositioned near by the detector. Only the completely unrelated neutrons are detected and the so called random data are collected. Meanwhile, the reactor is shut down and after completing this datum collection, the equipment is disconnected and the moderator thermostat restored to its original setting.

In Summary

This chapter dealt with the collection of data for use in VTMR experiments. In the first section, four different techniques for acquiring the data were presented. The method used in this experiment and the reason behind choosing it were then discussed. The available equipment primarily dictated the decision. In the third section, the equipment was described as were the tests utilized to ensure its proper performance. Finally, in the last section, the actual procedure employed while conducting the most recent measurements was outlined.

CHAPTER 6

ANALYSIS OF DATA

After collection and storage of the data, it is necessary to analyze them to obtain the desired result. In this chapter the methods of analysis along with the computer programs used to assist in the analysis are discussed. The data collected are either in the form of a string of sequential counts-per-channel values, or a neutron noise distribution. These two forms correspond to the first two methods of data collection discussed in the previous chapter. The first can easily be obtained from the second, but the reverse is not possible, since the time-ordered information is not present in the distribution form. In the limit as the channel width goes to zero, the first form also approaches a continuous storage of the pulses. Using a digital computer, these data could be analyzed directly or resolved into a distribution and then analyzed. These two possibilities correspond to the third and fourth methods outlined in Chapter 5.

Because no equipment was available to continuously store the data, it was generally not possible to calculate the VTMR for a particular group of channel widths without using data from a variety of measurements. Once the channel width on the MCA had been selected, only exact multiples of that time interval could be obtained by combining sequential channels. If a VTMR was desired at other than an

exact multiple of the original setting, a different set of data were needed. Two computer programs were written which, when used in conjunction, automatically analyzed data from a large number of different data sets to produce every possible VTMR value. A third program could then be employed to obtain an optimal fit to either the calculated Y or Z values. These computer codes are discussed in the second section of this chapter. In the first section, three methods of analyzing the VTMR values to obtain reactor parameters are discussed. They are the saturated Y method, the Z parameter fit, and the VTMR parameter fit.

Methods of Analysis Employed in the Variance-to-Mean Ratio Experiment

The first method is the saturated Y method which employes the average VTMR value in the range of t given by Eq. (4.2). After this quantity has been determined from critical and subcritical data, Y is obtained by subtracting one from it. The Y values are then corrected for the degree to which the curve is not saturated in this range, and Eq. (3.35) is used to calculate the reactivity. Table 6.1 lists the correction factors for different reactivities. In this table, B represents the average bracketed part of Eq. (3.29) for t between 50 msec and 60 msec.

The second method of data analysis consists of using a non-linear least squares fit of the analytic VTMR expression to the calculated VTMR values. The quantity A is determined from this fit and by assuming a value for β/ℓ, the reactivity is calculated using Eq. (3.27). If the system is critical, then A is equal to $-\beta/\ell$ and this

Table 6.1. Correction Factors Used in the Saturated Y Analysis Method

Reactivity ($)	A	B	Correction Factor (1/β)
0.0	-129.63	0.860	1.163
-0.05	-136.11	0.866	1.154
-0.10	-142.60	0.873	1.146
-0.15	-149.07	0.878	1.139
-0.20	-155.56	0.883	1.132
-0.25	-162.04	0.888	1.126
-0.30	-168.52	0.892	1.121
-0.40	-181.48	0.900	1.111
-0.50	-194.44	0.906	1.103
-0.75	-226.85	0.920	1.087
-1.0	-256.26	0.930	1.075
-1.5	-324.07	0.944	1.059
-2.0	-388.89	0.953	1.049
-2.5	-453.70	0.960	1.042

quantity is obtained directly. The quantity $\varepsilon D\ (1-\beta)^2/(\beta-\ell)^2$ is also determined from the fit and by providing necessary information any of the reactor parameters listed in Table 4.1 can be calculated.

The third method for analyzing the data is identical to the second, except the parameter Z of Eq. (3.41) replaces Y. This method did not work because the detector efficiency was too low, but it provided valuable information.

Regardless of the method used, the VTMR and Z parameter were initially calculated as a function of t. Saturated Y values could then be determined or either of the curve fits could be performed. The computer codes used to assist in these computations are discussed in the next section.

Computer Programs Used in the Variance-to-Mean Ratio Experiment

With the equipment that was employed, the VTMR experiment would have been impossible without a digital computer. For each set of 8192 channels, of which there were more than 150 collected over several months, on the order of 10^6 calculations were performed. The University of Arizona's VAX-1 and VAX-2 were the computers used. Each of the three programs that ran on these computers is discussed below. The first two were written as part of this experiment. The third is borrowed from Los Alamos National Laboratory. The language of all three codes is VAX-2 FORTRAN, although the borrowed program originally was in a different version of FORTRAN and had to be converted for use on the VAX. The plotting package used in the VMEX code (see Chapter 4) was also used with these codes. This library of subroutines was written by a

University of Arizona Computer Center employee, although it is not supported by this computer center. It is written in FORTRAN and assembly language and is quite useful for producing simple graphics output. A complete listing of each piece of software can be found in Appendix D.

VTMR Computer Program

A program called VTOM was first written which performed the tasks outlined in Fig. 6.1. This program was used in conjunction with the LALS program (discussed later in this section) in calculating the primary experimental results presented at the 1985 American Nuclear Society Western Regional Student Conference by the author. The output of this code was a large number of VTMR values calculated for every multiple of the original channel width. The name of the code was changed to VTMR after the plotting package and further data analysis features were added. These included calculating every possible combined channel of a particular size and averaging the results to obtain a statistically more accurate VTMR value; allowing any portion of a data file to be easily analyzed, rather than just the entire file; and calculation of the standard deviations given by Eqs. (2.26) through (2.28). The first change mentioned is the most significant, and Fig. 6.2 (an extension of Fig. 6.1) outlines how the calculations are now performed. This figure clearly illustrates that much more computer time is required when this option, referred to as varying channel combinations, is used. Figure 6.3 shows a plot of VTMR values versus the channel width calculated from experimental data without using the

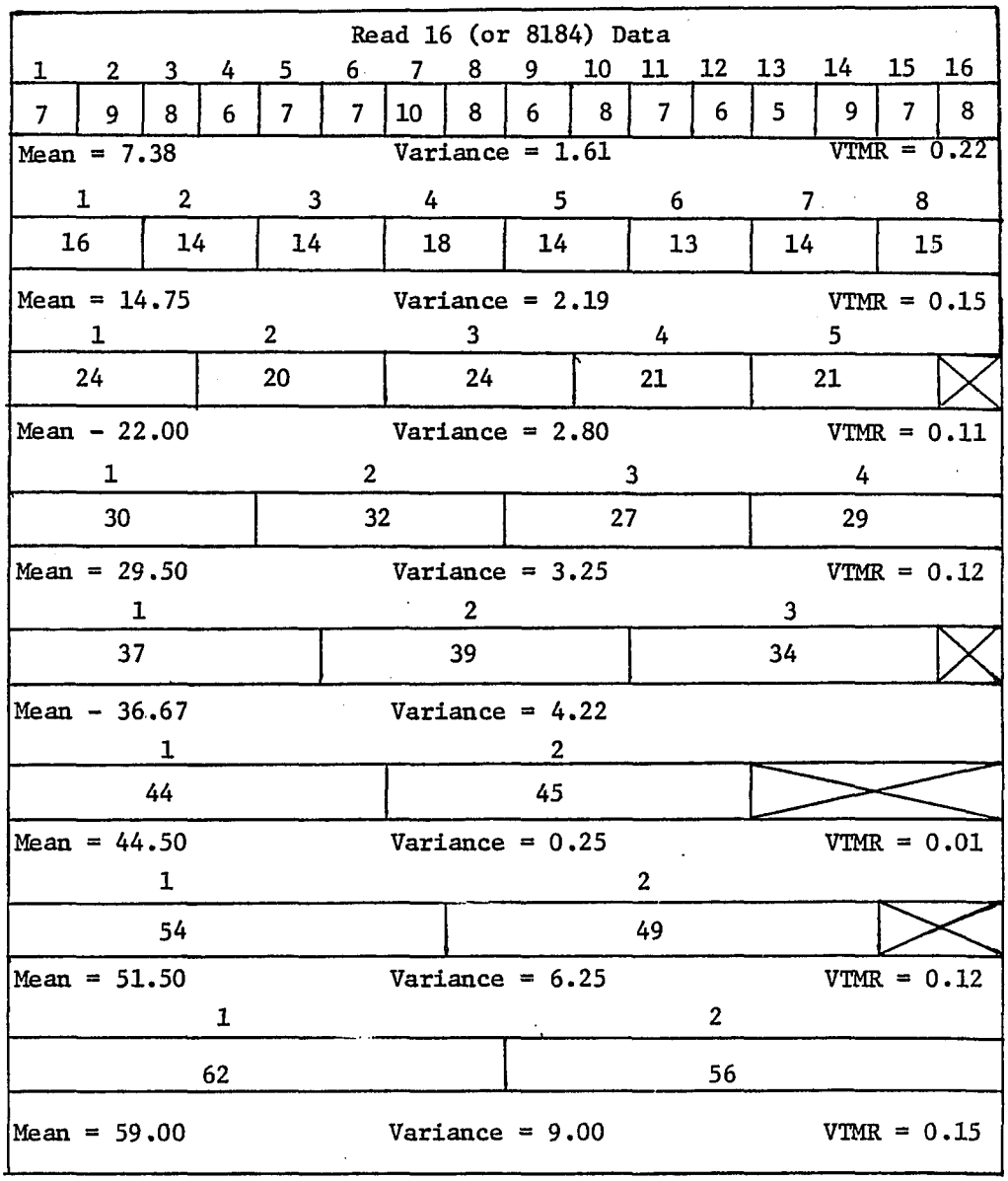

Figure 6.1. Diagram of the VTOM Computer Code Computation

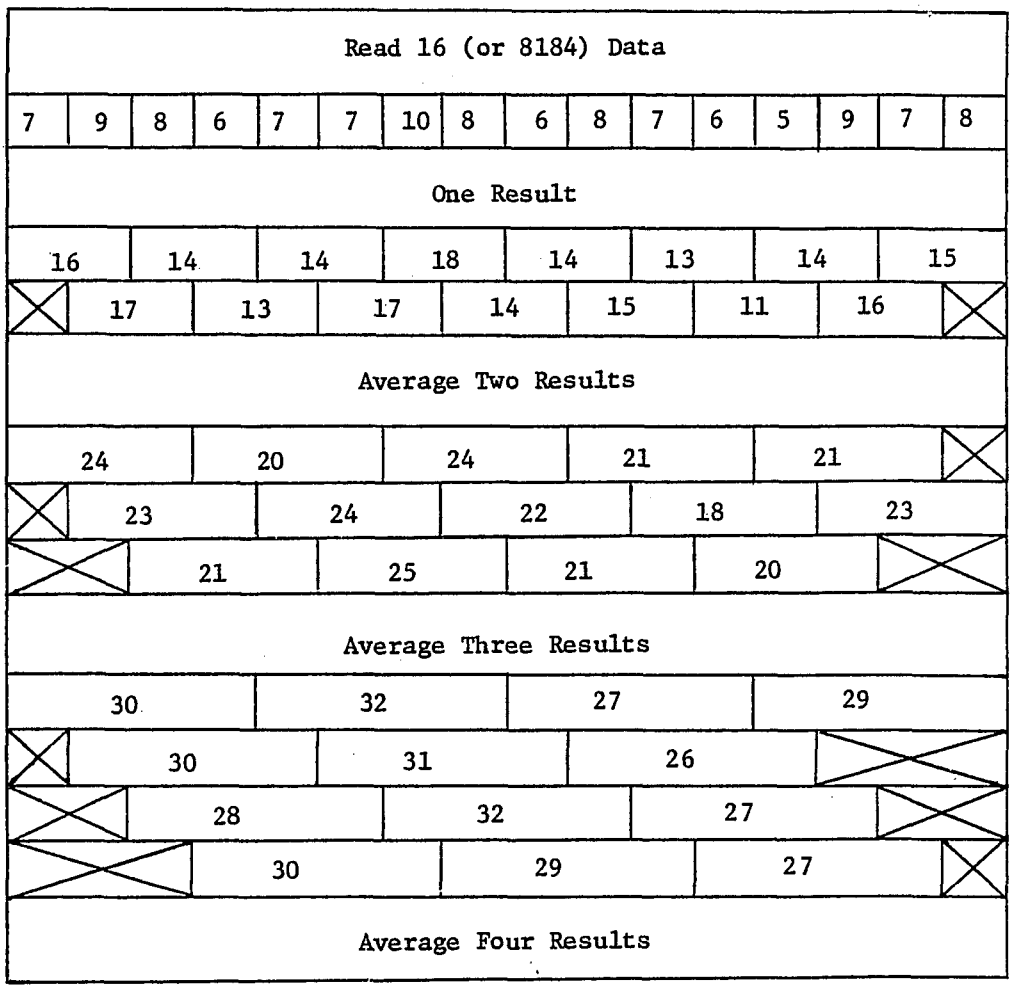

Figure 6.2. Diagram of VTMR Computer Code Computation Illustrating Variation of Channel Combinations

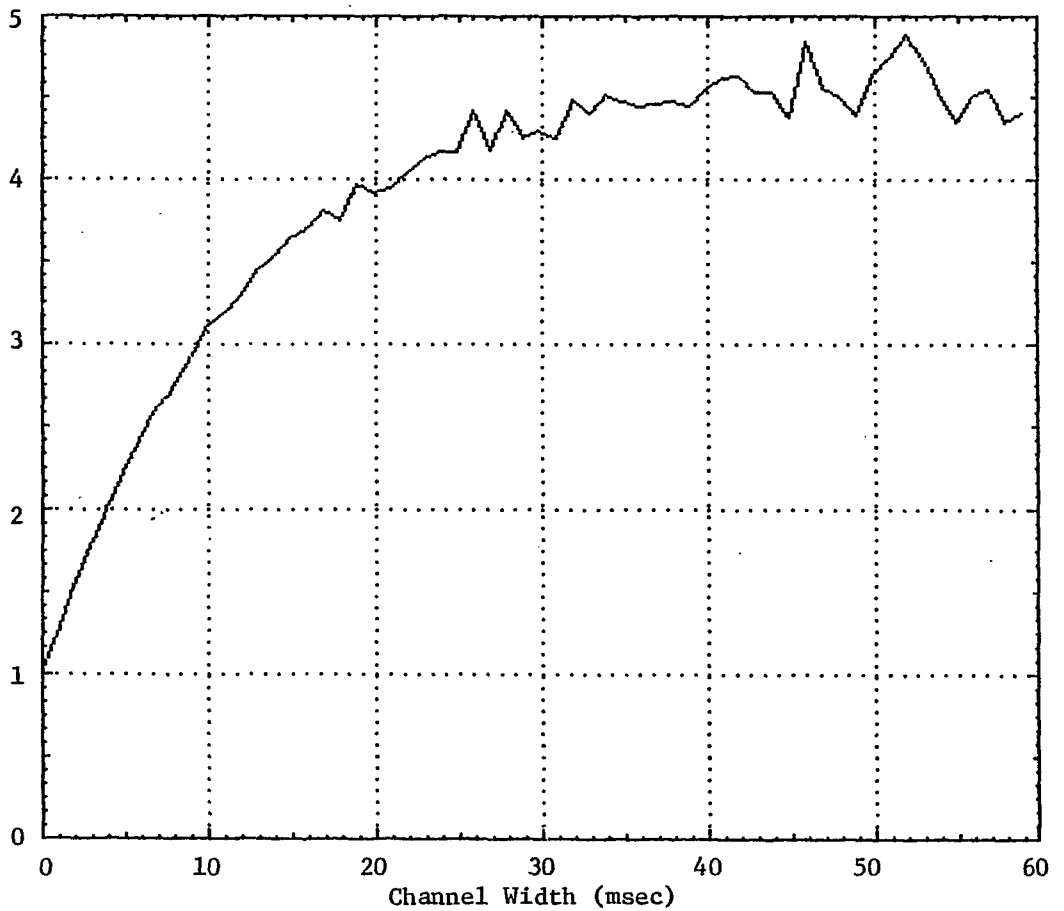

Figure 6.3. Experimental Variance-to-Mean Ratio Versus Channel Width Without Varying Channel Combinations

addition. Figure 6.4 is the same curve when the channel combinations are varied.

The averaging process is seen to substantially smooth the statistical fluctuations which occur as fewer and fewer channels are present. However, it becomes much more difficult to approximate the standard deviation of the results, because the different VTMR values that are averaged appear to be related in a subtle manner.

Figure 6.5 illustrates the behavior of the Z parameter calculated from the same data used to generate Figs. 6.3 and 6.4. This quantity hardly varies at all because of the low detector efficiency and is much too smooth to be accurately fit. The curve is identical whether the channel combinations are varied or not. This is expected because of the scale used. Because the Z parameter represents the ratio of correlated to random counts detected, this curve contains useful information. From it we can see that the vast majority of neutrons collected are random because the detector efficiency is low.

The data used in generating these plots had the highest efficiency obtained during this experiment. It can be estimated as approximately 1.5×10^{-4} from Fig. 6.4. Many of the results from this experiment were obtained with efficiencies of approximately one fifth this size.

When calculating the variance and mean from either the original data or a subsequent combination of it, the VTMR program uses Eqs. (2.3) and (2.4). If the data were resolved into a distribution, Eqs. (2.1) and (2.2) could be used. For a calculation of the first- and second-order moments involving just one channel width, these equations would require

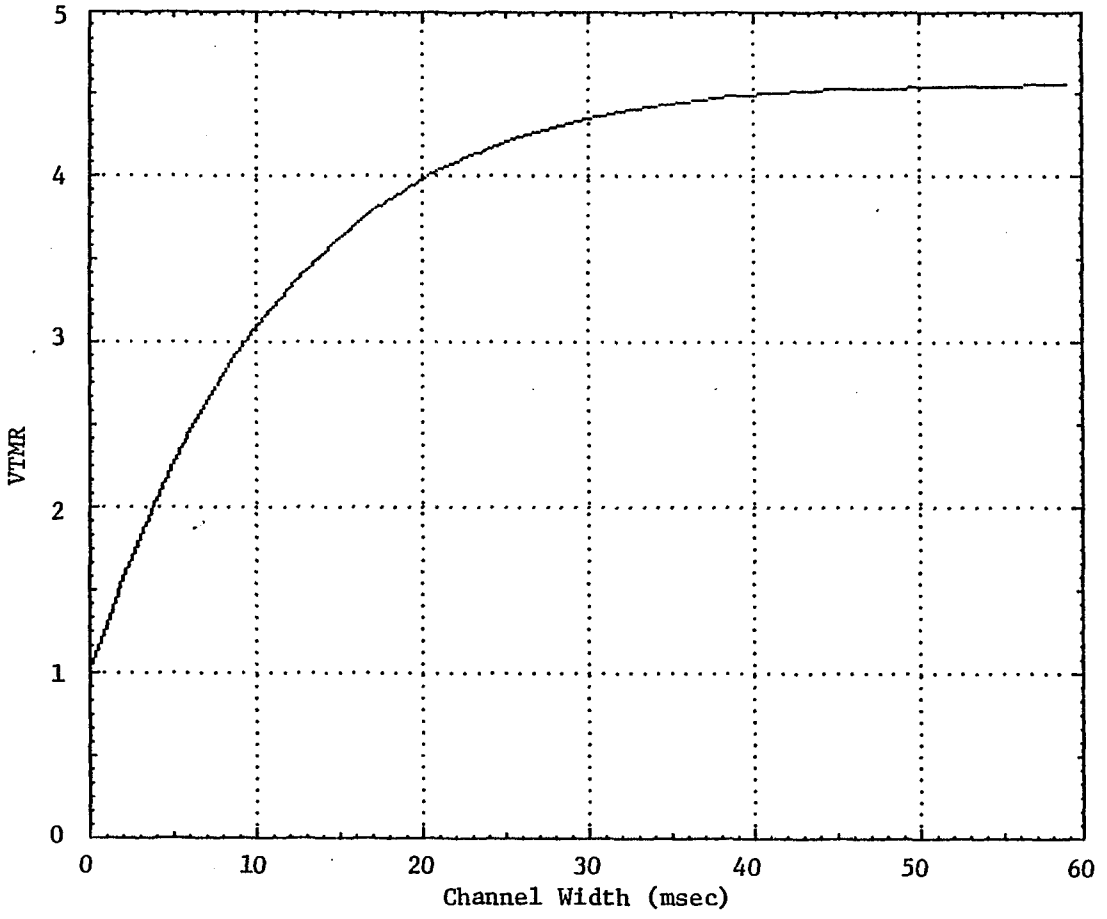

Figure 6.4. Experimental Variance-to-Mean Ratio Versus Channel Width While Varying Channel Combinations

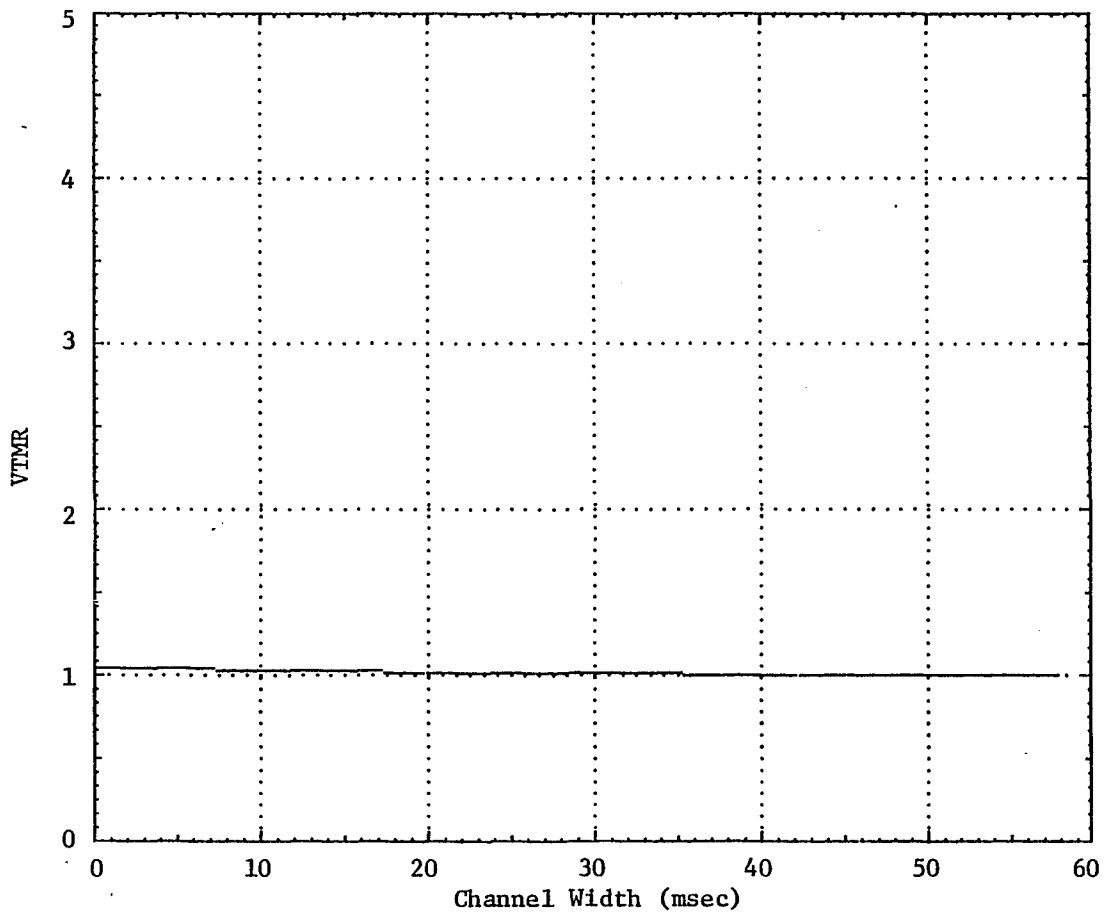

Figure 6.5. Experimental Quantity Z Versus Channel Width

much less computer time. Rather than summing every number and every number squared, they sum the product of the numbers and their squares times the frequency that they occur. However, because the time ordering of the channels is being used when combining sequential channels, this would require resolving the data into distribution form repeatedly, reducing the benefit. Equations (2.3) and (2.4) were used initially because the possibility of employing Eqs. (2.1) and (2.2) was not realized, and more recently because the coding already in use appeared more straightforward.

MERGE Computer Program

This program was written to combine output from the VTMR code flexibly. Figure 6.6 illustrates its flow chart. The program first reads the desired output files, which in most cases have overlapping values (i.e., when a certain combined channel width is the multiple of more than one initial MCA channel widths). It then sorts all the data into order by channel width and averages the overlapping values, producing an output file identical to that of the VTMR program, but containing many more results. In addition, this program automatically calculates the mean VTMR quantity in the range given by Eq. (4.2), to more accurately approximate the saturated Y value.

LALS Computer Program

LALS, an acronym for Los Alamos Least Square, is a computer code that performs a non-linear least square fit of an analytic expression to experimental data points. It has extremely fast convergence and is an essential part of this experiment. A number of input and output

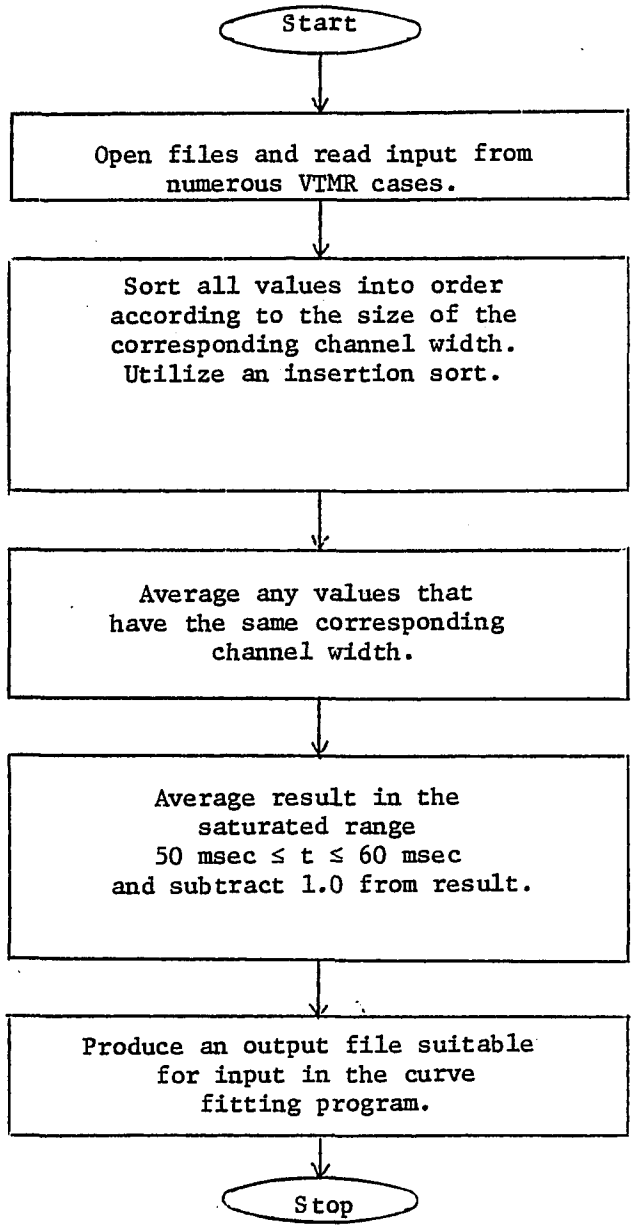

Figure 6.6. Flow Chart for Computer Code MERGE

modifications were made to this program when it was converted for use on the VAX computer. The modified code, which is called MLALS, automatically obtains and fits the most recent results of the VTMR or MERGE programs. To use the MLALS program, one must provide a subroutine containing the analytic expression to be fit, the independent and dependent variables, and the derivatives of the expression with respect to the dependent variables. Table 6.2 shows this subroutine as it was used with the VTMR equation.

Combined Use of the Computer Programs

These three programs, when run together, automatically performed all the data analysis for the VTMR experiment. Figure 6.7 is a flow chart that clearly summarizes how these computer codes function in concert. The VMEX program is included in this figure because it may be used as a test of the LALS code by producing exact results to be fit. Where necessary, each program can directly access the output of the other programs.

In Summary

The analysis of data is accomplished with the use of the VTMR, MERGE, and MLALS computer programs. In the VTMR code, Eqs. (2.3) and (2.4) are used to calculate the mean and the variance using many different combinations of the data. The MERGE program combines the output from different runs of the VTMR code and also averages VTMR values in the saturated range. The LALS code then fits these intermediate results to the VTMR equation providing the final result.

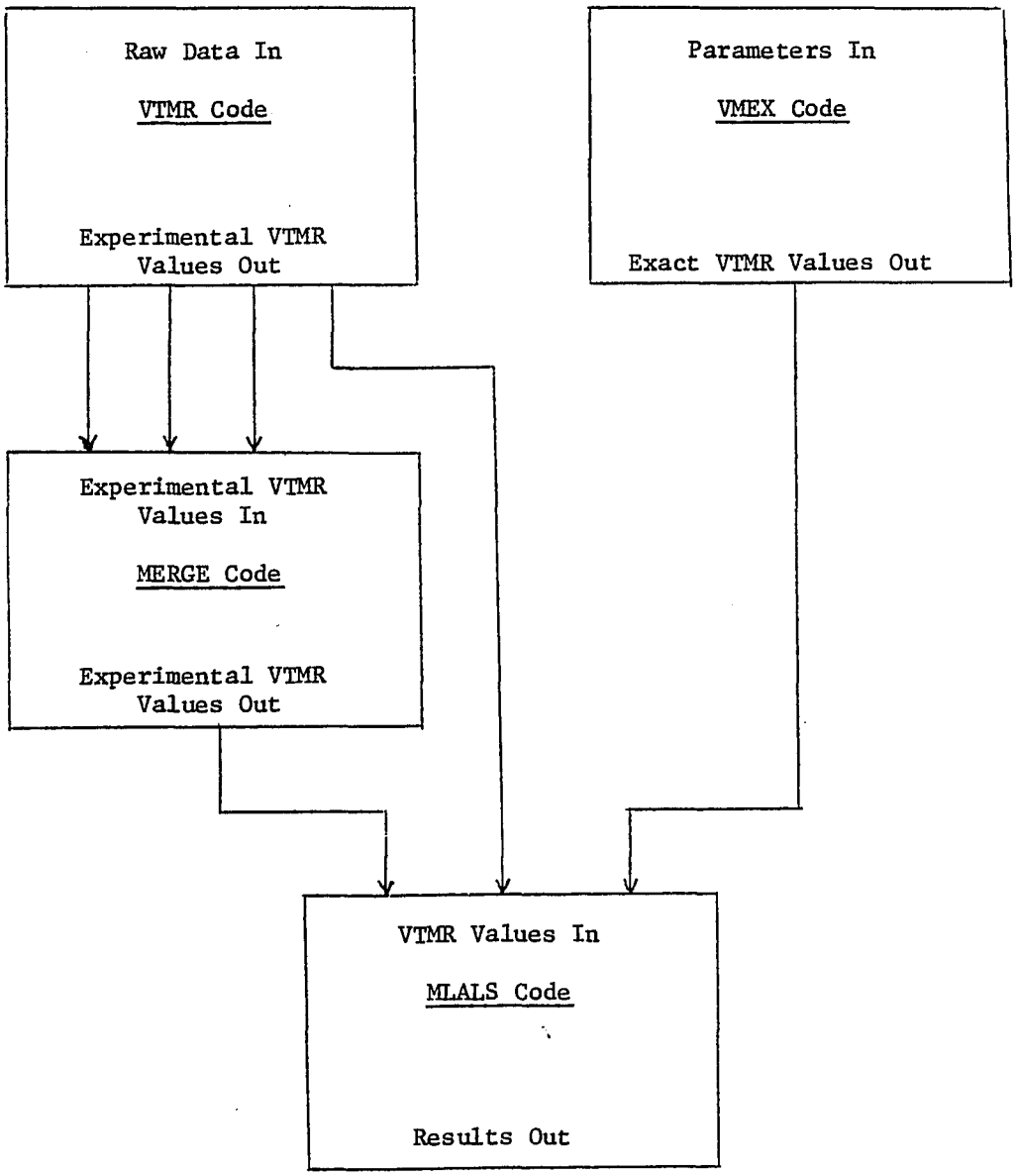

Figure 6.7. Flow Chart for Integrated Use of Computer Codes

Table 6.2. Subroutine for Use of the Variance-to-Mean Ratio Equation in the MLALS Computer Code

```
C ************************************************************
      SUBROUTINE YPS(I)
      COMMON Y(500), X(10,500), W(500), IX(50), PG(50), P(50),
     1 YC(500), DY(500), BM(20,21), TITLE(80), Z(50), PART(50)
      COMMON / INDEX / N, IK, IM, IW, M, IB, ITEST, IPR, IFG
     1 , TEST, WVAR, SSQ, IDF, DET, YT, INFIL
      IF(I.GT.0)  GO TO 3
      WRITE(9,2)
 2    FORMAT(10X,'ADAMS')
      GO TO 4
 3    YT = 0.0
C
C         THE FOLLOWING PORTION OF SUBROUNTINE YPS IS SUPPLIED
C         BY THE USER.  "YT" IS THE FUNCTION TO BE FITTED.  Z(I)
C         IS THE I TH INDEPENDENT VARIABLE.  P(I) IS THE I TH
C         PARAMETER TO BE DETERMINED BY LASL.  PART(I) IS THE
C         PARTIAL DERIVATIVE WITH RESPECT TO THE I TH PARAMETER.
C
      C = P(2) * Z(1)
      D = 1.0 + (1.0 - EXP(C))/C
      YT = 1.0 + P(1)*D
      PART(1) = D
      PART(2) = -1.*P(1)/P(2)*(EXP(C)+(1.-EXP(C))/C)
C
C           END OF USER SUPPLIED SECTION
C
 4    RETURN
      END
```

CHAPTER 7

ERRORS IN THE VARIANCE-TO-MEAN RATIO EXPERIMENT

With the possible exception of the conclusion, this chapter is the most important of the paper. Understanding the sources of error and limitations when using the variance-to-mean ratio technique is as essential as a thorough understanding of the method itself. In section one, general concepts are discussed which help to clarify and introduce this topic. Section two considers the assumptions made while deriving the VTMR equation and how these could lead to errors in its use. Errors that occur during data collection are regarded in the third section. These include systematic errors resulting from incorrect equipment usage, malfunctioning apparatus, or poor preparation of the reactor, in addition to numerous others, both systematic and random. The fourth section contains a brief discussion of possible sources of error during the analysis of the collected data.

General Concepts

In all branches of physical science and engineering, experimental observations are made and used to identify correlations related to the phenomena being observed. Inaccuracies are never absent from these measurements and understanding their extent is almost always an important part of the procedure. If a result based on a theoretical prediction is being compared to one experimentally obtained, some

information about the accurancy of both is needed before any conclusions can be drawn about their agreement or disagreement. This is exactly the case for the VTMR technique, which compares the VTMR(t) determined by measurement and analysis to that predicted by a theoretical expression in order to determine reactor parameters. Any discussion of the errors associated with this technique must consider the accuracy of both the theoretical equation and the experimental measurements.

When discussing the errors inherent in all observations, the terms **systematic** error and **random** error are usually introduced. A systematic error is one that consistantly occurs with the same magnitude unless corrected. Examples from the VTMR method primarily deal with the instrumentation, including such things as: incorrect equipment setup, detector deadtime after a pulse, poor choice of discriminator level, or the malfunction of a certain instrument. Random error, on the other hand, is produced by unpredictable and usually small variations in the experimental situation. For the VTMR method, these might include fluctuations in moderator temperature and density; slight vibrations in the control rod, detector, or neutron source positions; and spurious electronic noise, occasionally large enough to be counted as a neutron by the equipment. Of course, many other possibilities exist.

As mentioned in Chapter 2, random experimental errors are generally distributed in a Gaussian distribution and can then be described by the probable error as defined in Eq. (2.9). However, the statistical fluctuations that are being measured in this experiment are not distributed in this fashion, and therefore a different variance is being considered. In this case, the best way to handle the probable

error is by using the approximate expression derived at the end of Chapter 2.

To complete this section, the difference in meaning of the terms **accuracy** and **precision** is briefly considered. An observation has a high degree of accuracy if there are few systematic errors; the average of a number of repeated measurements will be exactly the best estimate of the measured quantity possible. In contrast, an observation that has a high degree of precision has small random errors. A repeated measurement will produce very similar results, but the average of these may differ considerably from the correct result. Observations of high accuracy and high precision are desirable, but if one must be sacrificed for the other, it should in most instances, be the precision.

Accuracy of the Theoretical Variance-to-Mean Ratio Equation

A number of significant assumptions were made during the derivation of the VTMR expression in Chapter 3. By far the most important of these is the assumption that the system can accurately be modeled using one group of identical neutrons, essentially ignoring all space, energy, and delayed neutron effects. The majority of this section will address this approximation and the limitations it imposes on the use of the VTMR method. Another assumption that is discussed is that of considering only correlated neutron pairs, while ignoring higher numbers of related neutrons. This is closely related to the number of moments of the neutron noise distribution that are considered.

Delayed Neutrons

The analytic expression for the VTMR including delayed neutrons is given in the final section of Chapter 3 by Eq. (3.36). In that section it was noted that including delayed neutron precursors resulted in a reduction of the VTMR (Wang and Ruby, 1975). Therefore, any delayed neutrons which are both detected in a time interval and related to earlier neutrons detected in that same interval, are expected to result in a lower than predicted VTMR. In other words, if the maximum t, set in Eq. (3.39), is too large, the VTMR equation will over predict the measured results. Because 60 msec is much shorter than the half-life of the shortest delayed neutron precursor group, very few correlated delayed neutrons will be observed. However, a finite number will certainly be seen and this will lead to some inaccuracy in the prediction of the VTMR equation. This error is introduced sooner (i.e., for a smaller t value) and to a larger extent as the system approaches a critical condition (Pal, 1963; Pacilio, 1965). Although difficult to determine, it is not expected to be larger than a few percent of Y for a critical system, with the limitation that a t value less than 60 msec is used.

Space and Energy

Nearly all reactor noise experiments assume space and energy independence from the outset, even though the assumptions underlying this one energy group, point reactor model are not valid for most reactors (Cohn, 1964; Sheff and Albrecht, 1966). A number of papers presenting results of noise methods simply mention this approximation,

but do not consider its effect. One exception is Gotoh (1964), who used the VTMR method to measure the neutron life in a reflected D_2O system. He acknowledges that the results are only approximate due to the inability of the space-energy independent theory to accurately model the system considered.

The point reactor kinetic model assumes a space-time decoupling of the mean neutron noise distribution in a finite reactor. Although often a valid approximation, there is no reason to believe that this is correct for higher order moments of this distribution such as the variance used in the VTMR method (Natelson, Osborn, and Shure; 1966). If the capture and production of neutrons were a continuous phenomena, this approximation would be exact (Current and Wallace, 1947). More confidence would be possible in the present reactor noise methods if a space and energy dependent model were available for comparison with the space and energy independent model. Pál wrote the first paper on spatial effects in reactor noise methods in 1958. His theory reduces to that of neutron transport, but is too general to analytically solve for comparison (Sheff and Albrecht, 1966).

Actually, the assumption of spatial independence is much more serious than that of energy independence. The VTMR equation has been derived assuming a continuum of neutron energies by Feynman (Feynman, de Hoffmann, and Serber; 1956) with an identical result. More recently Cohn (1962) utilized a simple two group model and found that it predicted the same results as the one group model.

Although Feynman also performed a calculation that predicted an error due to spatial effects of about 1%, it must be stressed that this

was based on a very small, spherical, and nearly homogeneous "water boiler" reactor. Every neutron born in this system had a good chance of traversing the sphere during its lifetime (Feynman, de Hoffmann, and Serber, 1956). More recent models which account to some degree for spatial effects predict more serious errors depending on the reactor configuration, the detector configuration, and the detector position within the system.

For small reactors that are water moderated such as the University of Arizona TRIGA reactor, fast leakage is the primary leakage from the core. Because of this, a reasonable treatment of fast diffusion is needed to understand the VTMR equation and predict its accuracy. When this is included, it is found that the equation is reasonable if "the ratio of the migration length to the greatest linear dimension is large enough to render the nonsingular part of the doublet density a factorizable function of its two spatial variables" (Natelson, Osborn, and Shure; 1966). In other words, the VTMR technique is invalid for

$$\frac{M}{R} \ll 1 \tag{7.1}$$

where M is the neutron migration length and R is the greatest dimension of the core. This result was dependent on the reactor being clean or unreflected, and cold or operating at a very low power level when no recent high power use has occurred.

The effect of a symmetric reflector were found to be insignificant in situations of experimental interest by Cohn (1962, 1964), who employed an elementary reflected-reactor model. Experimental

results have shown distortion when an asymmetric reflector is used, but these can be corrected (El-Zaftawy and Ruby, 1971). Therefore, the symmetric graphite reflector used on the TRIGA reactor during this experiment is not expected to affect Eq. (7.1).

The study resulting in Eq. (7.1) did consider the position and size of the detector, but no specific results were formulated. The point-reactor model assumes the use of an infinite detector which does not influence the system in any manner (Sheff and Albrecht, 1966). As the detector becomes smaller, the predictions of this model are therefore expected to become less accurate. Also, since the detector absorbs all detected neutrons, it causes a local flux depression. This is one possible explanation of why the VTMR has had little success in power level measurements. As noted by Pacilio (1969), "The flux level is probably seriously altered by the presence of the instrument, and the consequent power-level determination is derived from an incorrect evaluation of the detector efficiency." In experiments performed by Harris, et. al. (1970), the size and the position of the detector were chosen to perturb the reactor as little as possible. In effect, they were able to eliminate the influence of all odd transverse modes by accurately positioning their detector. As a system approaches critical, the fundamental mode becomes extremely dominant (Otsuka and Saito, 1965) and for this reason the VTMR experiment has generally not been very sensitive to higher mode eigenvalue presence (Pacilio, 1969).

For the University of Arizona TRIGA reactor

$$\frac{M}{R} \approx 0.4 \quad . \tag{7.2}$$

Therefore, Eq. (7.1) predicts that the VTMR equation is valid for this reactor. The presence of a symmetric reflector and higher order modes is not expected to seriously affect the method. However, the use of a small detector, located in an arbitrary position adjacent to the reactor in the lazy Susan access tube, could introduce some problems. Unfortunately, the extent of this accuracy is beyond the scope of the paper, but it would be foolish to consider it less than ± 5%.

Neutron Pairs

The assumption that a maximum of two correlated neutron pairs will appear in a channel is actually quite accurate for the time intervals and detector efficiencies being considered. If every neutron leaking out of the core were detected, the precision of the VTMR method could certainly be increased by considering correlated triplets, but with the detector efficiencies available and the short channel widths necessary to avoid delayed neutrons, no difference is expected. The probability of three related neutrons all interacting with a small detector in a 50 msec time interval is extremely small. Therefore, this assumption is expected to have negligible effect on the VTMR expression.

Errors in the Collection of Data

Numerous errors, both systematic and random, are possible during the collection of data. All of these are associated with either the equipment used or the preparation of the system being observed. This

section is split into two parts. In the first, systematic errors are considered, while in the second, random errors are discussed.

Systematic Errors During Data Collection

Table 7.1 lists a large number of systematic errors that are possible during the collection of data for a VTMR experiment. As can be seen, these errors all pertain to the equipment being used and the system under observation. Unlike most observations, it is fairly easy to verify that the equipment is functioning properly with regard to systematic error in a VTMR experiment. This is accomplished with the use of a random neutron source. As mentioned in Chapter 5, if VTMR values obtained from a random source are equal to unity, within the standard deviation derived in Chapter 2, the equipment is almost certainly functioning as desired regarding systematic errors. If the consistent double pulses observed at some amplifier settings were present during this test, the VTMR would not equal unity, since every pulse would then be correlated to its double. If pulses were routinely lost, the VTMR would again deviate from unity. However, when the simple, random neutron source is replaced by a complex multiplying system, a number of changes occur. First a huge number of gamma rays are now present. These attenuate more slowly than neutrons in water and so the ratio of gamma induced pulses to neutron induced pulses increases as the detector is moved further from the core. The gamma rays could possibly hinder the detector's ability to resolve neutrons, although for the type of detector used here, this is not expected to be significant. However, the numerous small gamma pulses must be

Table 7.1. Possible Systematic Errors Associated with Data Collection for the Variance-to-Mean Ratio Experiment

Equipment
- Double pulsing by amplifier or other equipment
- Voltage drifts by detector voltage source
- Neutrons lost due to discriminator setting
- Detector deadtime
- Inaccurate control rod level indicator
- Faulty connections
Reactor Preparation
- Higher gamma background due to recent use of the reactor → higher discriminator setting
- Higher radiation of all types due to fission product decay after recent use → smaller percentage of pulses detected are correlated neutrons
- Source too close to detector
- Inaccurate control rod calibration

electronically discriminated against so that they are not counted as neutrons. In doing this, low energy neutrons are lost as well. If no correlation exists between a detected neutron's energy and the probability of a related neutron being detected, this represents a linear decrease in the efficiency of the detector system and is not an error. However, suppose that a smaller percentage of these low energy neutrons are correlated due to the greater average distance they must travel to obtain such low energy. In this case, the systematic loss of all extremely low energy neutrons would artifically increase the VTMR and represent an error. Luckily, very few neutrons detected (far less than 1%) are lost due to the gamma discrimination, and therefore this possible source of error, although important to consider, is expected to be insignificant. Interestingly, the discriminated gammas are also correlated, although to a smaller extent than the neutrons and some noise methods consider this.

Another change that occurs when the reactor replaces the random source is a higher count rate. The deadtime between channels of an MCA are random, as far as the neutrons are concerned, and therefore represents a linear decrease in the detector/counter system. This is also true when a number of channels are combined, and therefore is not an error in the VTMR method. However, the deadtime of the detector is correlated to the instantaneous count rate and does represent an error. Correction for the detector deadtime presents a special problem for reactor fluctuation measurements such as the VTMR method. First, during the very bursts of correlated neutrons which one wishes to detect, the detector is dead for the largest fraction of the time. Second, because

Y is a function of the second moment as well as the first, most count loss analysis methods, which assume a Poisson distribution for moments higher than the first, will not work (Harris, et. al.; 1970). At least six different corrections for deadtime in the VTMR method have been proposed (Pacilio, 1969). All six of these methods assume that a maximum of one count can be lost during each dead period. Obviously, for extremely high count rates, none of these corrections will be accurate. The counting loss can be made negligible so that none of the corrections are necessary by employing low count rates. For BF_3 detectors, counting rates less than 1000 counts per second introduce an error to neutron fluctuation experiments of less than 1%, usually much less (Harris, et. al.; 1970).

VTMR volumes less than unity have often been observed for very short channel widths ($t<1$ msec). Since in order to obtain good statistics, a high counting rate is needed for time intervals of this length, the deadtime becomes significant. The values less than one are therefore explained by the deadtime seriously altering the neutron noise distribution (Pacilio, 1969).

Systematic errors during data collection are not expected to exceed one or two percent under most conditions when a random neutron source consistantly produces VTMR values of unity, within the expected standard deviation. If the random VTMR results are other than unity, either the source is not actually random or the apparatus is malfunctioning.

Random Errors During Data Collection

Table 7.2 lists a number of random errors that could take place during the collection of data in a VTMR experiment. These sources of error are generally minor compared to systematic errors. Although, random in nature, these errors are distributed according to a Gaussian distribution whose mean and variance are not usually equal. For this reason, random errors do contribute to VTMR measurements. However, as can be seen by Table 7.2, by their very nature these errors are small and largely uncontrollable. As such, they are usually ignored after taking all possible precautions and predicting their expected magnitude using Eq. (2.9).

It is interesting to note, that under certain conditions, it might be beneficial to decrease the precision of the equipment so that the variance of the random error distribution is more closely equal to the mean. Under these conditions the random errors could be approximately described by a Poisson distribution, and would simply contribute to the value of unity such as neutrons from spontaneous fission do.

The random errors during data collection are expected to amount to less than a one percent error in the calculation of the VTMR. Of course, under certain circumstances this could be higher.

Errors During Data Analysis

Because all analysis of the data is accomplished with the use of digital computers, the only errors possible during this phase of the experiment must take place in the computer or the programs employed. Roundoff error in the computer is negligible when compared to the other

Table 7.2. Possible Random Errors Associated with Data Collection for the Variance-to-Mean Ratio Experiment

- Local temperature fluctuations in the moderator
- Local temperature fluctuations in the fuel
- Local density fluctuations in the moderator
- Small vibrations of control rod
- Small vibrations of fuel
- Small vibrations of neutron source
- Electronic noise - occasional neutron-like pulse
- Any occasional double pulse by the equipment
- Power surge

errors that have been discussed. This is even more true, because most of the operations performed in the VTMR code are integer operations. The two computer programs written for this project have been extensively tested, and since they are generally simple codes, it is expected that no errors are present. For these reasons, no significant error should be present during the analysis of the data. As discussed in the next section, if the experimental values are not predicted theoretically, the MLALS code will not produce a good fit. This is an indication of inaccuracies in the VTMR equation or the experimental data, though, and does not imply any error in the data analysis.

Statistical Error

In Chapter 2, approximate standard deviations for Y, the general VTMR, and the VTMR of a Poisson distribution were derived. In obtaining these equations, it was assumed that no covariance occurred between the quantities in the VTMR and that the neutron noise distribution was symmetric. Neither of these assumptions are correct, but they are valid in approximation. It is easy to show using these results that a large number of time intervals are needed to maintain a small standard deviation for the VTMR. Because channels were combined in the data analysis, the number of channels was often relatively small. For instance, if a data set initially consisted of 8192 channels, each 1 msec wide, only 136 channels will be available to calculate the VTMR at a channel width of 60 msec. This was the motivation behind both varying the channel combinations and developing the MERGE program. However, in general, if a large number of channels (at least 10^4) is not available to

calculate the VTMR at every channel width, significant statistical deviations will occur. These deviations, in turn, can cause disaster when the MLALS code attempts to fit the VTMR values to the theoretical equation. As Fig. 4.6a shows, VTMR(t) curves that are only slightly different in shape can produce quite different results. This can lead to extremely large error in the reactor parameters calculated using this method. The system reactivity is particularly sensitive to the statistical deviations.

In Summary

Errors in the VTMR method were discussed in this chapter. Generally, suggestions on how to reduce or eliminate these errors have not been provided, since they will be discussed in the next chapter. In the first section of this chapter a number of general concepts were introduced. This was followed by a large section where the inherent accuracy of the VTMR expression was considered. An important conclusion from this section is that the VTMR equation itself has a ± 5-10% error associated with it. Error during data collection and data analysis were discussed in the next two sections. From these sections, we can conclude that although error in the data analysis is negligible, significant error (on the order of at least 5%) is possible during data collection. In the final section, the statistical variations in the VTMR were pointed out as the cause of serious error, quite possibly greater than any other source, when determining the reactor parameters using the MLALS code.

CHAPTER 8

RESULTS AND DISCUSSION

In this chapter results of the experimental VTMR method are presented and discussed. The first section contains published results from a number of VTMR experiments other than that performed here. In the second section, results of the experiment completed for this paper are presented. Both are discussed in the final section, as are methods of reducing error.

Selected Results from the Literature

The VTMR technique was first used experimentally in the 1940's. The result was classified until 1956, when it was published (Feynman, de Hoffmann, and Serber; 1956). Figure 8.1 is borrowed directly from this paper and shows the Y value calculated using 2200 time intervals, each 283 msec in length. The Y values for shorter time intervals were obtained using an alternative method of data collection, and were not used in the final result. The dotted curve in this figure was normalized to exactly fit the saturated Y value, using the best information available for the other parameters. The solid curve gives a reasonable fit to all the data. The result of this experiment was

$$\overline{\nu^2} = 7.8 \pm 0.6 \qquad (8.1)$$

for U-235. This compares fairly well with Diven's (1956) result using a different method. He obtained

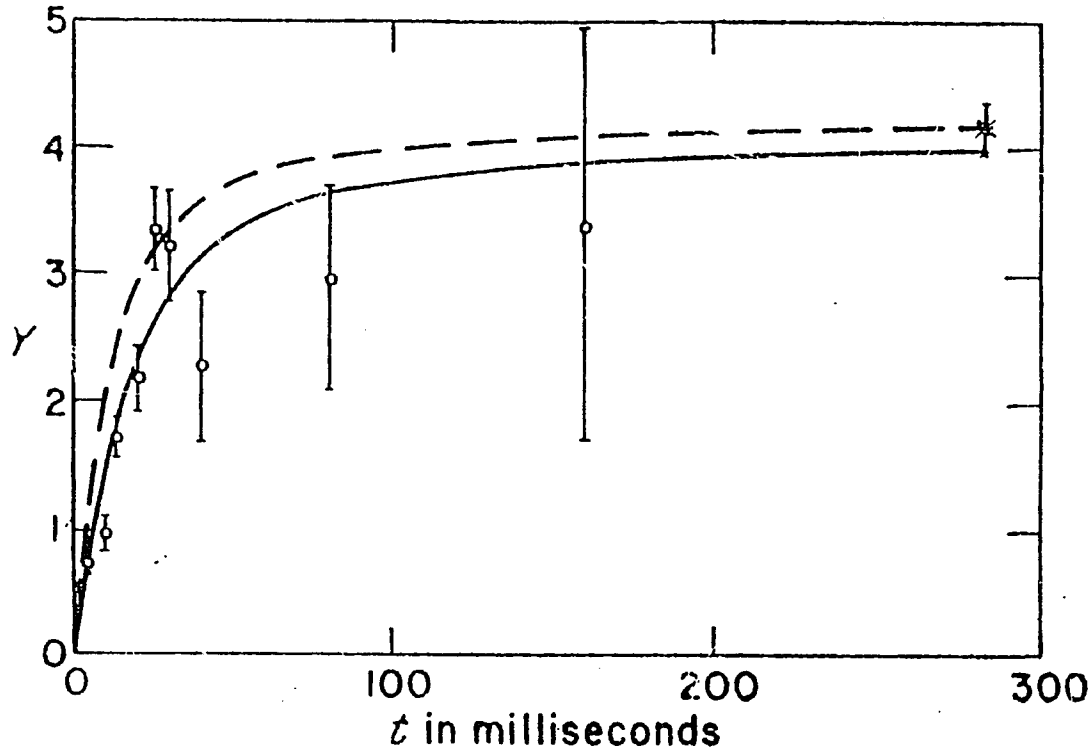

Figure 8.1. Results of the First Variance-to-Mean Ratio Experiment

$$\overline{\nu^2} = 7.32 \pm 0.15 \qquad (8.2)$$

for 80 kev neutron induced U-235 fission. The detector efficiency for this first VTMR experiment was approximately 3.5×10^{-4}.

In 1965, the VTMR was employed to determine β/ℓ using 1.3×10^5 time intervals and a detector efficiency of 1.3×10^{-3}. Although the final result is not available, it had an assigned accuracy of ± 0.5% (Pacilio, 1966).

The VTMR was first used to determine reactivity by Lindeman and Ruby (1967) using Eq. (3.35). A summary of their results is given in Table 8.1. Although these appear to be fairly good results, a number of criticisms are warranted. First, a time interval of 0.125 seconds was used, which is much to long to ignore delayed neutrons as they did. Second, the fact that Y was not completely saturated, even at this large time interval, was not corrected for, causing an error of between one and seven percent in each Y value depending on the reactivity. Finally, no estimation is made on the accuracy of their results for either of the experimental methods compared. It should also be noted that they are comparing reactivities from two experimental techniques, without providing the actual reactivity of the system. Hopefully, the two techniques are not biased in the same direction.

Wang and Ruby (1973, 1975) also published results of reactivity measurements using the VTMR method. These were obtained using a two delayed neutron extension of the VTMR equation and are summarized in Table 8.2. Their results seem to be more realistic than the earlier ones of Table 8.1. In this experiment, no reactivities below -$0.3 could be determined. This was explained as the result of too small a detector

Table 8.1. Results of Lindeman and Ruby's Variance-to-Mean Ratio Experiment

Number of Time Intervals	Average Counts per Channel	$\$_1$*	$\$_2$**
4095	1.39	-3.38	-3.42
4095	3.72	-1.25	-1.44
8183	2.05	-1.25	-1.44
4094	5.39	-0.81	-1.04
4095	8.87	-0.51	-0.56
8183	2.31	-0.20	-0.33
4094	24.37	-0.15	-0.14
4095	849.41	0.0	0.0

* reactivity in dollars using VTMR method

** reactivity in dollars using a pulsed neutron method

Table 8.2. Results of Wang and Ruby's Variance-to-Mean Ratio Experiment

Experimental Reactivity ($)	Theoretical Reactivity ($)
-0.05 ± 0.02	-0.03 ± 0.01
-0.07 ± 0.02	-0.05 ± 0.01
-0.09 ± 0.02	-0.12 ± 0.03
-0.15 ± 0.02	-0.15 ± 0.03
-0.20 ± 0.03	-0.27 ± 0.05
-0.22 ± 0.03	-0.28 ± 0.04

efficiency and too high a background gamma level. 4096 channels were used in obtaining the experimental results, and both period measurements and control rod calibrations were used in determining the theoretical values. The theoretical errors were estimated, while those for the experimental results were approximated using Eq. (2.28).

Although the VTMR method has been used on numerous other occasions, these selected experiments give a flavor for the kinds of results expected. They include all published reactivity results that the author is aware of.

Results of the Experiment

In this section, the results of the VTMR are presented. Because varying channel combinations required substantially more computer time, a comparison between results with and without its use was first performed. Table 8.3 summarizes these results. File 5123 consists of three data sets collected four minutes apart while the reactor was operating at a critical level. File 2728 contains two sets of data collected four minutes apart from a random neutron source. The error bars on these results are those from the fit of the data only, except for those on the random data which were calculated using Eq. (2.27). This is the only error important for this comparison.

Table 8.4 summarizes the results obtained from the VTMR experiments. The quantities A and C represent the two parameters fit to the curve and are defined on the table. The value in parentheses is the reactivity in dollars determined from A. It was generally not possible to determine the reactivity from C alone because the detector efficiency

Table 8.3. Comparison Between Results Obtained Using Varied Channel Combinations and Those Obtained Without Using This Feature

File	Combinations Varied	Channels Analyzed	Average Saturated Y	Result Fitting All Data $(\frac{\beta-\rho}{\ell})$	Results Fitting Half the Data $(\frac{\beta-\rho}{\ell})$
5123	no	all	3.568	167.78±6.46 (-29.4¢)	141.00±4.95 (-8.8¢)
5123	yes	all	3.560	170.79±3.37 (-31.7¢)	143.65±1.82 (-10.8¢)
5123	no	first half	3.984	154.28±6.51 (-19.0¢)	139.32±5.65 (-7.5¢)
5123	yes	first half	3.873	162.39±3.08 (-25.3¢)	135.68±1.78 (-4.7¢)
5123	no	second half	3.305	177.19±8.31 (-36.7¢)	154.90±8.41 (-19.5¢)
5123	yes	second half	3.232	182.33±3.73 (-40.7¢)	155.38±2.14 (-19.9¢)
2728	no	all	0.9884±0.082	---	---
2728	yes	all	0.9912±0.011	---	---
73	no	all	0.9467±0.058	---	---
73	yes	all	0.9512±0.016	---	---

Table 8.4. Summary of Results From the Variance-to-Mean Ratio Experiment

File	t(msec)	N	$\bar{c}$	$	$Y_{average}$	$Y_{corrected}$	Fit (All)*+	Fit (Half)
91	5000	8184	2.70	0.0±0.01	0.686±0.142	0.798±0.166	-129.0±12.2 (0.005) 0.753±0.013	-128.5±16.0 (0.009) 0.751±0.026
91 V**	5000	8184	2.70	0.0±0.01	0.664±0.043	0.772±0.050	-147.1±3.47 (0.071) 0.703±0.007	-120.4±5.1 (-0.134) 0.771±0.010
96	5000	8184	2.93	0.0±0.01	0.623±0.145	0.724±0.168	-117.4±18.6 (0.095) 0.687±0.034	-125.8±19.3 (0.029) 0.672±0.047
96 V	5000	8184	2.93	0.0±0.01	0.601±0.041	0.699±0.048	-112.1±3.21 (0.135) 0.711±0.007	-128.8±2.5 (0.006) 0.645±0.006
99	5000	8184	3.13	0.0±0.01	0.592±0.144	0.668±0.167	-124.1±8.8 (0.043) 0.684±0.015	-100.7±6.2 (0.223) 0.763±0.024
99 V	5000	8184	3.13	0.0±0.01	0.569±0.041	0.661±0.048	-119.4±3.71 (0.079) 0.664±0.007	-139.0±3.6 (-0.072) 0.619±0.007
75	5000	8184	4.55	-0.15±0.02	0.422±0.127	0.480±0.145	-165.5±23.8 (-0.277) 0.499±0.018	-136.1±24.1 (-0.050) 0.541±0.042
75 V	5000	8184	4.55	-0.15±0.02	0.453±0.039	0.516±0.044	-142.4±7.0 (-0.098) 0.528±0.007	-124.8±10.8 (0.037) 0.561±0.022
79	5000	8184	4.41	-0.15±0.02	0.446±0.128	0.508±0.146	-203.1±35.0 (-0.566) 0.497±0.020	-149.6±15.2 (-0.154) 0.567±0.029
79 V	5000	8184	4.41	-0.15±0.02	0.437±0.039	0.487±0.044	-209.6±9.9 (-0.617) 0.485±0.005	-180.0±9.3 (-0.391) 0.516±0.010

Table 8.4. Summary of Results From the Variance-to-Mean Ratio Experiment--continued

File	t(msec)	N	$\bar{c}$	$	$Y_{average}$	$Y_{corrected}$	Fit (All)*+	Fit (Half)
84	5000	8184	4.47	-0.15±0.02	0.495±0.131	0.564±0.149	-131.0±13.2 (-0.10) 0.568±0.017	-112.3±23.6 (0.133) 0.618±0.063
84 V	5000	8184	4.47	-0.15±0.02	0.466±0.039	0.531±0.045	-148.3±4.2 (-0.144) 0.538±0.004	-135.4±2.0 (-0.045) 0.560±0.004
51	1000	8184	4.53	0.0±0.01	3.140±0.703	3.652±0.818	-135.1±7.2 (-0.042) 4.811±0.078	-115.4±4.5 (0.110) 5.210±0.100
52	1000	8184	4.57	0.0±0.01	3.262±0.603	3.794±0.701	-119.1±11.9 (-0.536) 3.728±0.054	-167.2±5.5 (-0.290) 4.025±0.054
53	1000	8184	4.57	0.0±0.01	3.508±0.632	4.080±0.736	-165.5±8.9 (-0.277) 3.952±0.057	-145.2±9.5 (-0.120) 4.198±0.12
55	250	8184	1.13	0.0±0.01	---	---	-119.7±6.2 (0.077) 6.611±0.136	-86.6±8.4 (0.332) 8.340±0.629
57	5000	8184	21.25	0.0±0.01	3.915±0.361	4.554±0.420	-148.4±6.5 (-0.145) 5.628±0.069	-140.1±1.00 (-0.081) 5.77±0.179
58	2500	8184	11.23	0.0±0.01	3.889±0.429	4.523±0.499	-116.4±5.6 (0.102) 4.552±0.072	-112.5±3.4 (0.132) 4.666±0.070
5123 V	1000	24552	4.55	0.0±0.01	3.580±0.374	4.164±0.435	-167.8±6.5 (-0.294) 4.110±0.43	-141.0±5.0 (-0.088) 4.450±0.071
51,52,53, 54,55,58; MERGE	---	49104	---	0.0±0.01	4.187	4.869	-132.3±6.5 (-0.021) 4.789±0.076	------ -- ------

Table 8.4. Summary of Results From the Variance-to-Mean Ratio Experiment--continued

File	t(msec)	N	$\bar{c}$	$	$Y_{average}$	$Y_{corrected}$	Fit (All)*+	Fit (Half)
61	500	8184	1.19	-0.05±0.02	2.722±0.726	3.141±0.838	-187.1±11.5 (-0.443) 2.786±0.048	-177.8±13.8 (-0.372) 2.846±0.093
62	500	8184	1.38	-0.05±0.02	3.224±0.810	3.720±0.935	-169.1±7.9 (-0.305) 3.678±0.051	-172.4±8.6 (-0.330) 3.650±0.080
63	1000	8184	2.76	-0.05±0.02	3.537±0.638	4.082±0.736	-151.9±7.9 (-0.172) 4.074±0.060	-127.7±8.2 (0.015) 4.443±0.13
64	1000	8184	2.61	-0.05±0.02	3.404±0.623	3.928±0.719	-171.8±8.0 (-0.325) 3.785±0.047	-155.9±8.3 (-0.203) 3.961±0.09
65	2500	8184	6.53	-0.05±0.02	3.952±0.435	4.561±0.502	-115.2±4.3 (0.112) 4.644±0.066	-122.3±6.5 (0.057) 4.489±0.119
66	2500	8184	6.69	-0.05±0.02	3.701±0.416	4.271±0.481	-134.7±8.1 (-0.039) 4.134±0.079	-157.5±10.5 (-0.215) 3.845±0.110
6566	2500	16368	6.61	-0.05±0.02	3.873±0.303	4.469±0.350	-117.3±3.3 (0.095) 4.546±0.042	-113.6±2.8 (-0.031) 4.265±0.042
6768	5000	16368	12.56	-0.05±0.02	3.082±0.184	3.557±0.212	-158.2±5.1 (-0.221) 3.502±0.031	-160.0±4.7 (-0.234) 3.478±0.09
222	1000	40920	0.90	0.0±0.01	0.453±0.118	0.526±0.137	-110.1±6.5 (0.150) 0.549±0.011	-91.2±7.7 (0.297) 0.615±0.029
333	5000	81840	4.47	-0.15±0.02	0.468±0.038	0.534±0.043	-138.3±9.8 (-0.067) 0.546±0.011	-115.8±8.0 (0.107) 0.596±0.02

Table 8.4. Summary of Results From the Variance-to-Mean Ratio Experiment--continued

File	t(msec)	N	$\bar{c}$	$	$Y_{average}$	$Y_{corrected}$	Fit (All)*+	Fit (Half)
444	1000	65472	0.60	-0.15±0.02	0.447±0.093	0.509±0.106	-190.4±11.4 (-0.469) 0.509±0.007	-160.1±12.7 (-0.241) 0.549±0.01
111	5000	79000	2.94	0.0±0.01	0.708±0.043	0.824±0.050	-104.7±5.6 (0.192) 0.840±0.016	-126.2±7.7 (0.027) 0.7681±0.02
73	2000	8184	9.44	random	0.9467±0.0680			
73 V	2000	8184	9.44	random	0.9512±0.0157			
73	2000	1-4092	9.49	random	0.0184±0.0814			
73	2000	4092-8184	9.40	random	0.9896±0.0814			
74	5000	8184	23.4	random	0.9504±0.0510			

* This curve fit employs all points available for t≤60 msec. The other fit employs the first half of these points.

+ The quantities in this columm are: A, ($), and $\epsilon D \frac{(1-\beta)^2}{(1-\rho)^2}$.

** Channel combinations are varied.

is not accurately known. The error bars on A and C were obtained from the curve fit, and those on saturated Y were obtained using the expression derived in Chapter 2. These results are discussed in the next section.

Discussion of Results

The result of the first VTMR experiment performed by Feynman has an accuracy of ± 7.7% which is fairly small, considering the number of channel values and the equipment used. The quantity $\overline{\nu^2}$ was calculated from D by providing $\overline{\nu}$. The detector efficiency and the fraction of delayed neutrons also were provided. The fact that these quantities were known quite accurately helped lead to an error bar of this size. The method of calculation is not known, but it does not seem overly conservative.

The results of Pacilio were obtained using a much higher detector efficiency and a much larger number of channels. The accuracy of this result was calculated using the expression derived in Chapter 2. This does not take into account the error inherent in the model, though, which must be considered; but it does provide an idea of the statistical uncertainty.

Lindeman and Ruby's results were discussed in the last section. No accuracy was assigned or calculated for their values, and their findings seem unlikely because of the channel width chosen, which would lead to a large error due to delayed neutrons. The results obtained by Wang and Ruby seem much more plausable.

The results listed in Table 8.3 show good agreement between the two methods of calculating the VTMR values. The error associated with the curve is generally much smaller for the cases where the channel combinations were varied as expected. However, the magnitude of the results are close. In every case except one, the result obtained while varying the channel combinations is further from the correct result, even though the VTMR(t) curves appear to be more correct. The reason for this is not clear, and because of it much of the analysis was performed without using this option.

In Table 8.4, the cases in which the combinations were varied are marked with a V. This table summarizes the experimental results and shows a favorable correlation between the obtained and expected values. However, a wide range of error bars and magnitudes exist. Also, these error bars do not take into account the inaccuracy of the VTMR expression and this should be kept in mind while viewing the results. From the corrected saturated Y values it is possible to obtain the reactivity using the first method of analysis mentioned in Chapter 6. Any combination of a critical and non-critical value can be used. Table 8.5 lists a few of the better results contained in Table 8.4.

Although some of the numbers in Table 8.4 are quite close to the expected result, this table does not imply good results from the method; there are at least as many results in Table 8.4 which do not agree at all. The error bars which are located in Table 8.4 are not included in this table to underscore this fact.

Based on these results and knowledge about the behavior of the VTMR equation, it seems that the VTMR technique has little chance for

Table 8.5. Review of Best Results

	File	Expected Result	Experimental Result
Curve Fits	91	\$0.0 $\frac{\beta}{\ell} = 129.63$	\$0.005 $\frac{\beta}{\ell} = 129.00$
	99	\$0.0 $\frac{\beta}{\ell} = 129.63$	\$0.043 $\frac{\beta}{\ell} = 124.09$
	75	-\$0.15 $\frac{\beta-\rho}{\ell} = 149.07$	-\$ = 0.098 $\frac{\beta-\rho}{\ell} = 142.39$
	84	-\$0.15 $\frac{\beta-\rho}{\ell} = 149.07$	-\$0.144 $\frac{\beta-\rho}{\ell} = 148.28$
	66	-\$0.05 $\frac{\beta-\rho}{\ell} = 136.11$	-\$0.039 $\frac{\beta-\rho}{\ell} = 134.73$
$Y_{critical}/Y$	75 and 99	-\$0.15	-\$0.17
	79 and 96	-\$0.15	-\$0.19
	53 and 62	-\$0.05	-\$0.05
	57 and 66	-\$0.05	-\$0.03

success with system reactivities below -$1.0, even under ideal conditions. The possibility of accurately determining reactivities above this value exists. In order to accomplish this, the sources of error must be reduced. The random errors can be minimized only to a point and do not appear to be a major source. Also, by testing with a random source, the systematic errors are observed to be relatively minor. Decreasing the size of either of these sources further than achieved during this experiment becomes quite difficult and has a diminishing return. The statistical uncertainty, on the other hand, can be significantly reduced by collecting more data and by using an increased detector efficiency. These are the changes necessary to obtain better results.

In Summary

In this chapter results from VTMR experiments were presented, including a sampling of those from the literature and a summary of those from the experiment performed here. The method is shown to be relatively inaccurate from these results. To a certain extent this inaccuracy is inherent in the technique, but significant error reduction is possible in the experiment performed for this paper. This reduction can be achieved by improving the statistics, either through increased detector efficiency, increased collection of data, or for best results, both.

CHAPTER 9

CONCLUSIONS

The variance-to-mean ratio method has been extensively researched through this study using both experimental techniques and numerous articles, reports, and books. One of the major objectives of the paper was to provide a complete picture of this method, including the theory behind it, the manner in which it can be employed, the results of its application, and the errors associated with its use. This objective has been met. The second major objective was to apply this method to the University of Arizona TRIGA reactor to test its actual performance, especially regarding a simple determination of near critical reactivities. This objective has also been accomplished.

The VTMR equation, which relates reactor noise to specific system parameters, is derived using statistical theory and the point reactor kinetics model. The well established statistical theory is quite helpful in understanding the VTMR method, in addition to other noise methods. From it, an approximate standard deviation of the VTMR is calculated. For the number of channels obtained in the experiment, this standard deviation was often relatively large in the region where the VTMR no longer varied excessively with t. This led to difficulty when fitting the experimental results to the analytic expression. The point reactor kinetics model is also well established. However, it does not

accurately model most reactors in use today with regard to higher order moments of the noise distribution. This is primarily caused by its neglect of spatial effects such as the detector location and size, the core geometry and homogenity, and the higher modes of the neutron flux.

The VTMR method is a useful experimental technique that is valid in approximation when more than 10^5 time intervals are collected using a detector with an efficiency of at least 1×10^{-4} on certain systems. These conditions, especially the high detector efficiency, are difficult to achieve using less than the best apparatus. Even using ideal equipment, the system must be small enough so that the maximum linear dimension does not greatly exceed the neutron migration length, and have a reactivity not less than -$1.0, for meaningful results. Under the best conditions, the VTMR has been employed with a published accuracy of ± 0.5%, but this was accomplished using an efficiency of 1.3×10^{-3} and more than 3×10^5 time intervals. The inaccuracy of the analytical expression does not seem to have been considered when assigning this relative error. Most VTMR results published have error bars of at least 5% and those over 10% are not uncommon. For this reason, the technique is used frequently to determine if fission neutrons are present in an unknown source of nuclear radiation, but not to determine any other information about the source (Brunson, 1983).

Almost all noise methods are based on the point reactor model and therefore are expected to have an error of a few percent for reactors such as the University of Arizona TRIGA. Many alternative noise techniques do not directly use higher order moments of the noise distribution, though, and this may result in greater accuracy than

possible with the VTMR method. However, no noise method is expected to work for systems such as a commercial spent fuel storage pool, which is simply too disperse for modeling by the point reactor equations.

From a parametric study of the VTMR expression and the equation for its approximate standard deviation, the following conclusions can be drawn.

1. The dependence of Y on A is sensitive for $At_{\bullet}1$, but decreases as t is increased.
2. The statistical uncertainty of Y or the VTMR depends weakly on $\bar{c}$, mildly on the value itself, and most strongly on N, the number of channels used. For this reason, many short samples are superior to fewer long samples.
3. The VTMR decreases linearly with decreasing detector efficiency.
4. The VTMR decreases in a nearly similar manner with decreasing activity.
5. Close to identical VTMR(t) curves can occur due to different combinations of the efficiency and reactivity. Slight statistical variations can cause an experimental curve to appear quite different with regards to these two parameters.
6. The VTMR behaves approximately as $1/\beta^2$ for changes in β/ℓ.

From the experiment performed, a number of conclusions can also be drawn. Because of limitations of the experiment, such as the inability to accurately vary parameters such as β/ℓ, ε, and D, these conclusions are different in nature.

1. The most basic behavior of the VTMR(t) predicted by the theory

is correct; it increases with increasing channel width until leveling off.

2. The accuracy of either the VTMR or Y increases with increasing N.
3. Analysis based on fitting the curve does incur problems due to slight statistical changes in the shape of the curve. This is shown by the significantly different results produced by two halves of the same data set.
4. Analysis based on the comparison of nearly saturated subcritical and critical Y values is correlated to the expected results, but is not accurate.
5. Analysis based on the Z paraneter is not feasible with the detector efficiencies that were obtained. However, calculation of this value did show that between one twentieth and one one hundredth of the neutrons detected were correlated, depending on the channel width and the reactivity.
6. When calculating the VTMR values, the use of the time ordered nature of the data to vary channel combinations increases the accuracy of the result but does not significantly change its magnitude.
7. With regard to relative error, β/ℓ is more accurately determined by the VTMR method than the reactivity.

With these conclusions in mind, a number of suggestions for future work can be made. First, larger changes in the degree of subcriticality could be attempted. Actually \$=-0.5, -1.0, and -1.5 were

used earlier in this experiment, but the data were collected while the reactor operated at non-steady-state, and contained just 1024 channels, causing ridiculous results. Second, the fitting program MLALS might be modified so that the efficiency was fixed for all cases. The code would then determine the best reactivity independent of the efficiency, after which the results could all be normalized to the critical case. The saturated analysis would follow. Third, the model could be extended to include two groups of delayed neutrons and significantly larger time intervals could then be employed. Finally, and probably most fruitful, the collection of much larger groups of data using a larger, more efficient detector could be carried out.

The VTMR method is an interesting technique based on the correlation of neutrons due to the chain reaction. Its most valuable use today is in determining if fission neutrons are present in an unknown sample. It can also be employed as an excellent and worthwhile laboratory experiment for teaching students. If an efficient enough detector is used and enough data are collected, this method can be used as an accurate experimental technique on certain small systems. However, in most cases more accurate means are available.

APPENDIX A

PROOF THAT THE VARIANCE EQUALS THE MEAN OF THE SQUARES MINUS THE SQUARED MEAN

For any distribution P(x):

$$\text{Variance} \equiv \sum_{\text{All } x} (x-\bar{x})^2 P(x)$$

$$\sum_{\text{All } x} (x-\bar{x})^2 P(x) = \sum_{\text{All } x} (x^2 - 2\bar{x}x + \bar{x}^2) P(x)$$

$$= \sum_{\text{All } x} \bar{x}^2 P(x) - 2\bar{x} \sum_{\text{All } x} x P(x) + \bar{x}^2 \sum_{\text{All } x} P(x)$$

but P(x) is a normalized probability distribution

$$\therefore \sum_{\text{All } x} P(x) = 1$$

$$\text{and the mean} \equiv \bar{x} \equiv \sum_{\text{All } x} xP(x)$$

$$= \sum_{\text{All } x} x^2P(x) - 2\overline{x\bar{x}} + (\bar{x})^2$$

$$= \overline{x^2} - (\bar{x})^2$$

$$\text{Variance} \equiv \sum_{\text{All } x} (x-\bar{x})^2 P(x) = \overline{x^2} - (\bar{x})^2$$

APPENDIX B

PROOF THAT THE VARIANCE EQUALS THE MEAN IN A POISSON DISTRIBUTION

Poisson Distribution $\quad P(x,\bar{x}) = \dfrac{e^{-\bar{x}}(\bar{x})^x}{x!}$

$$\text{Mean} \equiv \sum_{\text{All } x} x\, P(x)$$

$$\text{Variance} \equiv \sum_{\text{All } x} (x-\bar{x})^2\, P(x)$$

i) Mean:

$$\sum_{\text{All } x} \frac{x\, e^{-\bar{x}}(\bar{x})^x}{x!} = e^{-\bar{x}}(\bar{x}) \sum_{x=1}^{\infty} \frac{(\bar{x})^{x-1}}{(x-1)!}$$

$$\text{but, } e^{\bar{x}} = \sum_{\text{All } x} \frac{(\bar{x})^x}{x!} = \sum_{x=1}^{\infty} \frac{(\bar{x})^{x-1}}{(x-1)!}$$

$$\sum_{\text{All } x} \frac{x\, e^{-\bar{x}}(\bar{x})}{x!} = e^{-\bar{x}}(\bar{x})\, e^{\bar{x}} = \bar{x}$$

ii) Variance:

$$\sum_{\text{All } x} (x-\bar{x})^2 \frac{e^{-\bar{x}}(\bar{x})^x}{x!} = \sum_{\text{All } x} \left[x^2 - 2x\bar{x} + (\bar{x})^2\right] \frac{e^{-\bar{x}}(\bar{x})^x}{x!}$$

$$= e^{-\bar{x}}\left[\sum_{\text{All } x} \frac{x^2(\bar{x})^x}{x!} - 2(\bar{x}) \sum_{\text{All } x} \frac{x(\bar{x})^x}{x!} + (\bar{x})^2 \sum_{\text{All } x} \frac{(\bar{x})^x}{x!}\right]$$

$$= e^{-\bar{x}}\left[\sum_{\text{All } x} \frac{[x(x-1)+x](\bar{x})^x}{x!} - 2(\bar{x})^2 \sum_{x=1}^{\infty} \frac{(\bar{x})^{x-1}}{(x-1)!} + (\bar{x})^2 e^{\bar{x}}\right]$$

$$= e^{-\bar{x}}\left[\sum_{\text{All } x} \frac{x(x-1)(\bar{x})^x}{x!} + \sum_{\text{All } x} \frac{x(\bar{x})^x}{(x-1)!} - 2(\bar{x})^2 e^{\bar{x}} + (\bar{x})^2 e^{\bar{x}}\right]$$

$$= e^{-\bar{x}}\left[\sum_{x=1}^{\infty} \frac{(x-1)(\bar{x})^x}{(x-1)!} + \bar{x} \sum_{x-1}^{\infty} \frac{(\bar{x})^{x-1}}{(x-1)!} - (\bar{x})^2 e^{-\bar{x}}\right]$$

$$= e^{-\bar{x}}\left[\bar{x} \sum_{x=1}^{\infty} \frac{(\bar{x})^{x-1}}{(x-1)!} + \bar{x}\, e^{\bar{x}} - (\bar{x})^2 e^{\bar{x}}\right]$$

$$= e^{-\bar{x}}\left[(\bar{x})^2 \sum_{x=2}^{\infty} \frac{(\bar{x})^{x-2}}{(x-2)!} + \bar{x}\, e^{\bar{x}} - (\bar{x})^2 e^{\bar{x}}\right]$$

$$= e^{-\bar{x}}\left[(\bar{x})^2 e^{\bar{x}} + \bar{x}\, e^{\bar{x}} - (\bar{x})^2 e^{\bar{x}}\right]$$

$$= e^{-\bar{x}}\, \bar{x}\, e^{\bar{x}} = \bar{x}$$

APPENDIX C

DERIVATION OF THE STANDARD DEVIATION OF THE MEAN FOR A GAUSSIAN DISTRIBUTION

$M \equiv$ number of sets of data

$N \equiv$ number of measurements in each set

$j \equiv$ 1 to M indicates particular data set

$k \equiv$ 1 to N indicates particular measurement

$x_{jk} \equiv$ data point k in data set j

$\bar{x}_j \equiv$ average of data set j

$\bar{x} \equiv$ average of all data

$$\sigma^2_x = \frac{1}{MN} \sum_{j=1}^{M} \sum_{k=1}^{N} (x_{jk} - \bar{x})^2$$

$$\sigma^2_{\bar{x}} = \frac{1}{M} \sum_{j=1}^{M} (\bar{x}_j - \bar{x})^2$$

$$= \frac{1}{M} \sum_{j=1}^{M} \left[\frac{1}{N} \sum_{k=1}^{N} x_{jk} - \bar{x} \right]^2$$

$$= \frac{1}{M} \sum_{j=1}^{M} \left[\frac{1}{N} \sum_{k=1}^{N} (x_{jk} - \bar{x}) \right]^2$$

$$= \frac{1}{MN^2} \sum_{j=1}^{M} \left[\sum_{k=1}^{N} (x_{jk} - \bar{x}) \right]^2$$

due to the symmetry of the Gauss distribution, the cross term cancels out.

$$= \frac{1}{MN^2} \sum_{j=1}^{N} \sum_{k=1}^{N} (x_{jk} - \bar{x})^2$$

$$\sigma^2_{\bar{x}} = \frac{\sigma^2_x}{N}$$

APPENDIX D

The computer codes that were utilized during the variance-to-mean ratio experiment are listed in this Appendix. The listings occur in the following order:

VMEX Code

TRANS Code

VTMR Code

MERGE Code

MLALS Code

PLOT Subroutines

This is the order in which the codes were used during the experiment. The programming language is FORTRAN for all these codes except TRANS, which is written in BASIC. Also, one routine in the PLOT package is in assembly language.

Listing of VMEX Code

```
C
C PROGRAM VMEX.  WRITTEN BY W. M. ADAMS.  SPRING 1985
C
C THIS  CODE  CALCULATES  THE  EXACT  VARIANC-TO-MEAN RATIO
C VALUES  FROM THE  VARIANCE-TO-MEAN  EQUATION.  IT IS USED
C FOR COMPARISON TO EXPERIMENTAL RESULTS AND FOR PERFORMING
C PARAMETRIC STUDIES ON THE VTMR EQUATION.
C
      PROGRAM VMEX
      DIMENSION VTMR(100), T(100), SLOPE(100)
      OPEN(UNIT=1,NAME='PLOTEX.OUT',STATUS='NEW')
      OPEN(UNIT=2,NAME='VMEX.OUT',STATUS='NEW')
      L = 0
    5 IF(L.EQ.0) GO TO 10
      WRITE(5,*) 'RUN AGAIN?  (YES = 1)'
      READ(5,*)   IFLAG
      IF(IFLAG.EQ.1) GO TO 10
C COMPLETE PLOT, CLOSE FILES, AND PRINT RESULTS
      CALL  ENDPLOT
      WRITE(1,111)
  111 FORMAT(//,10X,'VTMR VERSUS TIME.  EXACT ')
      CLOSE(1)
      CLOSE(2)
      IF(IOUT.EQ.1) CALL LIB$SPAWN('PRINT/NOFEED VMEX.OUT,
     *PLOTEX.OUT')
      STOP
C PROMPT USER FOR INPUT INFORMATION
   10 WRITE(5,*) 'WHAT REACTIVITY?  (DOLLARS) '
      READ(5,*)   REAC
      WRITE(5,*) 'CONSTANT EFFICIENCY  (1=YES)'
      READ(5,*)   IEFF
      WRITE(5,*) 'IF CONSTANT, WHAT EFFICIENCY? (E-5)'
      READ(5,*)   EFF
      WRITE(5,*) 'DO YOU WISH TO VARY B/L? (1=YES)'
      READ(5,*)   IV
      EFF = EFF * 1.0E-05
      IF(IV.NE.1) GO TO 15
      WRITE(5,*) 'WHAT IS BETA?'
      READ(5,*)   BETA
      WRITE(5,*) 'WHAT IS L? (E-6)'
      READ(5,*)   ELLL
      ELLL = ELLL * 1.0E-06
   15 IF(L.NE.0)  GO TO 20
      WRITE(5,*) 'RESULTS PLOTTED? (1=YES)'
      READ(5,*)   IPLOT
      WRITE(5,*) 'AUTOMATIC PRINT OUT? (1=YES)'
      READ(5,*)   IOUT
```

```
C CALCULATE EFFICIECY AS A FUNCTION OF REACTIVITY
      IF(IEFF.EQ.1) GO TO 20
      EFF = (1.0 - REAC) * 5.0E-05
C CALCULATE CONSTANTS
   20 A = 0.795 * EFF
      IF(IV.NE.1) THEN
      BM1OB = 141.857
      BOL   = 129.6296
      ELSE
      BM1OB = (BETA - 1.0) / BETA
      BOL   =  BETA / ELLL
      ENDIF
      B = (BM1OB / (1.0 - REAC) ) **2
      C = (1.0 - REAC) * BOL
C WRITE HEADINGS
      WRITE(2,1111) REAC
 1111 FORMAT(1H1,'THESE VTMR VALUES WERE CALCULATED USING


     *VMEX.',/,2X,'60',/,2X,'1.  -130.',/,1X,
     *'REACTIVITY = ',F13.7)
      IF(IEFF.EQ.1)WRITE(2,2222) EFF
 2222 FORMAT(1X,'EFFICIENCY = ',F13.7,//,
     *21X,'CHANNEL WIDTH',5X,'VTMR',9X,'SLOPE',//)
      L = 1
C CALCULATE THE VTMR AND SLOPE
   25 IF(L.EQ.1) GO TO 30
      T(L) = T(L-1) + 0.001
      GO TO 35
   30 T(L) = 0.001
   35 EX  = EXP(-C*T(L))
      VTMR(L)  = 1.0 + A*B*(1.0 - (1.0 - EX) / (C*T(L)) )
      SLOPE(L) = A*B/C * (1.0/T(L)**2 - 1.0 / T(L)**2 *
     *EX-C/T(L)*EX)
CCC      WRITE(2,3333) T(L), VTMR(L)
CCC      WRITE(6,3333) T(L), VTMR(L)
      WRITE(2,3333) T(L), VTMR(L), SLOPE(L)
      WRITE(6,3333) T(L), VTMR(L), SLOPE(L)
 3333 FORMAT(21X,3(F12.8,2X))
      IF(T(L).GE.0.06) THEN
      IF(IPLOT.EQ.1) THEN
      N = L + 1
      CALL PLOT1(T,VTMR,N)
      END IF
      GO TO 5
      END IF
      L = L + 1
```

```
      GO TO 25
      END
C
C PLOT VTMR VERSUS TIME
C
      SUBROUTINE PLOT1(X,Y,N)
      DIMENSION X(N), Y(N)
C COUNT THE NUMBER OF TIMES THIS ROUTINE IS CALLED
      ICOUNT = ICOUNT + 1
C MOVE ALL VALUES OVER ONE IN THEIR ARRAYS
      DO 5 I = 1,99
      J = 100 - I
      X(J+1) = X(J)
      Y(J+1) = Y(J)
    5 CONTINUE
C SPECIFY FIRST DATA POINT (HERE ALWAYS 1.0)
      X(1) = 0.0
      Y(1) = 1.0
C DEFINE PLOT PARAMETERS IF THIS THE FIRST CALL
      IF(ICOUNT.NE.1) GO TO 15
C (XMAX-XMIN) / HSIZE EQUALS THE MAJOR DIVISION DIVISION
C (YMAX-YMIN) / VSIZE EQUALS THE MAJOR VERTICAL DIVISION
      XMIN  = 0.0
      XMAX  = 0.06
      YMIN  = 0.0
      YMAX  = 5.0
      HSIZE = 6.0
      VSIZE = 5.0
      CALL INITPLOT(HSIZE,VSIZE,XMIN,XMAX,YMIN,YMAX)
      CALL FRAME
      CALL GRID
C PLOTLINE IS CALLED ONCE FOR EACH CURVE ON THE PLOT
   15 CALL PLOTLINE(X,Y,N)
CCC      CALL  EZ_PLOTLINE(X,Y,N)
      RETURN
      END
```

Listing of TRANS Code

```
10 REM     PROGRAM TO RECEIVE DATA FROM THE TRACOR
20 REM     NORTHERN MCA IN MULTISCALE MODE.
30 REM     G. NELSON    6/25/85
40 DIM D(8200)
50 CLS: KEY OFF: PRINT: PRINT: PRINT
60 PRINT"PROGRAM TO RECEIVE AND WRITE MCS DATA"
70 PRINT" FROM THE TRACOR-NORTHERN TO DISKETTE"
80 PRINT
90 INPUT"PLEASE INPUT 3-CHAR. EXTENSION ";EXT$
100 COMM$="COM1:9600,E,7,1,CS,DS"
110 NC%=8191
120 NL=NC%/8: NL%=INT(NL)
130 HEAD$(1)=DATE$:HEAD$(2)=TIME$
140 OPEN COMM$ FOR INPUT AS #1
150 PRINT COMM$
160 PRINT"WAITING TO RECEIVE DATA."
170 PRINT"PLEASE PRESS I/O ENABLE ON MCA."
180 D$=INPUT$(24,1)
190 PRINT D$
200 E$=INPUT$(19,1)
210 PRINT E$
220 CYCLES =VAL(MID$(E$,4,5))
230 DELTAT=VAL(MID$(E$,13,4))
240 F$=INPUT$(46,1)
250 J%=0
260 WHILE (LOC(1)>65) OR (J%<NL%)
270 D$=INPUT$(65,1)
280 D(8*J%)=VAL(MID$(D$,9,6))
290 D(8*J%+1)=VAL(MID$(D$,16,6))
300 D(8*J%+2)=VAL(MID$(D$,23,6))
310 D(8*J%+3)=VAL(MID$(D$,30,6))
320 D(8*J%+4)=VAL(MID$(D$,37,6))
330 D(8*J%+5)=VAL(MID$(D$,44,6))
340 D(8*J%+6)=VAL(MID$(D$,51,6))
350 D(8*J%+7)=VAL(MID$(D$,58,6))
360 J%=J%+1
370 WEND
380 PRINT FRE(0)
390 PRINT FRE(D$)
400 ROBOT$ = LEFT$(HEAD$(1),2)
410 DROID$ = MID$(HEAD$(1),4,2)
420 F$="B:MA"+ROBOT+DROID+"."+EXT$
430 OPEN F$ FOR OUTPUT AS #2
440 PRINT #2, HEAD$(1)
450 PRINT #2, HEAD$(2)
460 PRINT #2, DELTAT
```

```
470 PRINT #2, CYCLES
480 FOR I%=0 TO NC%
490 PRINT #2, STR$(D(I%));
500 IF (I%+1) MOD 8 =0 THEN PRINT #2, ""
510 NEXT I%
520 PRINT
530 PRINT NC%+1;" CHANNELS OF DATA WRITTEN
540 PRINT" TO DISKETTE AS FILE ";F$
```

Listing of VTMR Code

```
C
C PROGRAM VTMR.  WRITTEN BY W. M. ADAMS, 11/84 - 7/85.
C
C THIS CODE CALCULATES VTMR VALUES FORM EXPERIMENTAL DATA.
C IT HAS A LARGE NUMBER OF OPTIONS FOR FLEXIBILITY.
C
      PROGRAM VTOMR
      COMMON/INPUT/ IST,NFIL,NCH,ACHW,MAX,IOPT,IPL2,
     *IDT,IMIN,IMAX,IPL1,IPRT,MCA,KMIN,KMAX,NRAD
      COMMON/CHANNEL/ KCHAN(82000),ICHAN(82000)
      DIMENSION ITOT(82000), VTMR(500), WIDTH(500), TIME(500),
     *SD(500), KT(500)
      INTEGER CCS, CCS1
      OPEN(UNIT=2,NAME='VTOMR.OUT',  STATUS='NEW')
      OPEN(UNIT=3,NAME='INMLAL1.DAT',STATUS='NEW')
      OPEN(UNIT=4,NAME='INMLAL2.DAT',STATUS='NEW')
      OPEN(UNIT=7,NAME='INPUT.DAT',  STATUS='OLD')
C
C                          MAIN PROGRAM
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C NFIL   = NUMBER OF DATA FILE BEING ANALYSED     (CONSTANT)
C IDIST  = IDIST = 1 PROB. DIST.;OTHERWISE, NONE  (CONSTANT)
C IMIN   = PROB. DIST. MINIMUM                    (CONSTANT)
C IMAX   = PROB. DIST. MAXIMUM                    (CONSTANT)
C NCH    = NUMBER OF CHANNELS USED                (CONSTANT)
C MAX    = MAXIMUM NUM. OF CHANNELS COMBINED      (CONSTANT)
C MIN    = MINIMUM NUM. OF VALUES FOR A MEAN      (CONSTANT)
C CCS    = COMBINED-CHANNEL SIZE                  (VARIABLE)
C NUMCC  = NUMBER OF COMBINED-CHANNELS            (VARIABLE)
C NLOST  = NCH NOT USED IN CCS DUE TO FRAC. CHS.  (VARIABLE)
C ITOT() = TOTAL NUM. OF COUNTS IN COMBINED-CH.      (ARRAY)
C ICHAN()= INDIVIDUAL CHANNEL VALUES FROM M.C.A.     (ARRAY)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C CALL SUBROUTINE TO OBTAIN INPUT INFORMATION.
      CALL INOUT
C COMBINE CHANNELS INTO A NEW COMBINED-CHANNEL SIZE.
      JJOT  = KMAX - KMIN + 1
      IF(((IPL2.OR.IPL1).EQ.1).OR.(IPL2.EQ.3))
     *OPEN(UNIT=1,FILE='PLOTV.OUT',STATUS='NEW')
CCC       IF(JFLAG.EQ.2) CALL DEADT
      IF(IDT.EQ.1) CALL PROB(1)
      IF((IDT.EQ.1).AND.(JJOT.NE.NCH)) CALL PROB(2)
C MESSAGE TO SCREEN
      WRITE(6,*) 'BEGIN DATA ANALYSIS'
      VTMRM  = 0.0
      WIDTHM = 0.0
      WRITE(6,111)
```

```
  111 FORMAT(//,25X,'SUMMARY OF RESULTS',/,12X,
     *'RESULTS ARE PRINTED AS THEY ARE CACULATED',
     *//,12X,'TIME',10X,'VTMR',10X,'WIDTH',//)
C LOOP OVER ALL POSSIBLE COMBINED CHANNEL SIZES.
      DO 25 CCS = 1 , MAX
      CCS1  = CCS - 1
      TIME(CCS) = ACHW * REAL(CCS) * 1.0E-06
      NUMCC = JJOT / CCS
      NLOST = JJOT - NUMCC*CCS
CCC       WRITE(2,1111) CCS, NUMCC, NLOST
CCC 1111 FORMAT(/,10X,'CHANNEL SIZE = ',I5,5X,
CCC      *'NUM. OF CHANNELS = ',I5,5X,'NUM. OF',
CCC      *' CHANNELS LOST = ',I4)
      J = 1
      JMAX = CCS
C LOOP OVER ALL POSSIBLY COMBINATIONS FOR THIS


C COMBINED CHANNEL SIZE.
      IF(IOPT.NE.1) JMAX = 1
CCC       IF(JMAX.GE.30) JMAX = 30
      DO 20 J = 1, JMAX
      NUMCC1 = NUMCC - 1
      IF(J.LE.(NLOST+1)) NUMCC1 = NUMCC
C INITIALIZE ITOT TO ZERO
      IT = NUMCC + 100
      DO 5 K = 1, IT
      ITOT(K) = 0
    5 CONTINUE
C LOOP OVER EACH OF THE COMBINED-CHANNELS.
      DO 10 K = 1, NUMCC1
      L = K + (K-1)*CCS1 + J - 1
C SUM COUNTS IN ORIGINAL CHANNELS TO OBTAIN
C COMBINED-CHANNEL VALUE.
      DO 10 M = 1, CCS
      ITOT(K) = ITOT(K) + ICHAN(L+M-1)
   10 CONTINUE
CCC       WRITE(2,2222) (ITOT(K),K=1,NUMCC1)
CCC 2222 FORMAT(1X,'CHANNEL VALUES:',1X,20(I4,1X),
CCC     */,410(17X,20(I4,1X),/))
C CALCULATE TOTAL COUNTS AND TOTAL COUNTS SQUARED
C FOR ALL COMBINED-CHANNELS.
      DO 15 K = 1, NUMCC1
      KTOT    = KTOT + ITOT(K)
      ATOT    = ATOT + REAL(ITOT(K))**2
      ITOT(K) = 0
   15 CONTINUE
```

```
      ANUM     = REAL(NUMCC1)
      AMEAN    = REAL(KTOT) / ANUM
      AMEANS   = ATOT / ANUM
      VAR      = AMEANS - AMEAN*AMEAN
      TEMP1    = VAR / AMEAN
      TEMP2    = (AMEANS-AMEAN)/(AMEAN*AMEAN)
C STAMDARD DEVIATION OF VTMR FOR RANDOM VARIABLE
      IF(NRAD.EQ.1)SDA = SQRT( 2.0/ANUM +
     *1.0/(ANUM*AMEAN) )
C STANDARD DEVIATION OF VTMR FOR CORRELATED VARIABLE
      IF(NRAD.NE.1)SDA = SQRT(2.0/ANUM+(1.0+TEMP1)/
     *(ANUM*AMEAN) )*(1.0 + TEMP1)
C STANDARD DEVIATION OF Y (NOT USED)
C       IF(NRAD.NE.1)SDA=TEMP1*SQRT(1.0/ANUM*(4.0/TEMP1+
C      *3.0/(AMEAN*TEMP1)+2.0+3.0/AMEAN+TEMP1/AMEAN))
      TOTAL1   = TOTAL1 + TEMP1
      TOTAL2   = TOTAL2 + TEMP2
      SDTOT    = SDTOT + SDA
      KTOT  = 0
      ATOT  = 0.0
CCC       WRITE(2,3333) AMEAN,AMEANS,VAR,TEMP1,SDA
CCC 3333 FORMAT(1X,'MEAN =',F10.4,3X,'MEAN OF SQUARES =',
CCC      *F13.4,3X,'VARIANCE =',F10.4,3X,'VTOMR =',F10.4,
CCC      *1X,'+/-',F8.5)
   20 CONTINUE
      AB = REAL(JMAX)
      VTMR(CCS)   = TOTAL1 / AB
      SD(CCS)     = (SDTOT/AB) / (SQRT(AB))
      KT(CCS) = 0
      IF (ABS(1.0-VTMR(CCS)).GT.SD(CCS)) KT(CCS) = 1
      MBUTT = MBUTT + KT(CCS)
      IF ((NRAD.EQ.1).AND.(ABS( 1.0-VTMR(CCS))).GT.
     *(2.0*SD(CCS)))THEN
      KT(CCS) = 2



      NBUTT = NBUTT + 1
      END IF
      WIDTH(CCS) = TOTAL2 / AB
      IF(VTMRM.LT.VTMR(CCS))  VTMRM  = VTMR(CCS)
      IF(WIDTHM.LT.WIDTH(CCS))WIDTHM = WIDTH(CCS)
      WRITE(6,3344) TIME(CCS), VTMR(CCS) ,WIDTH(CCS)
 3344 FORMAT(5X,3(1X,F13.6))
      TOTAL1 = 0.0
      SDTOT  = 0.0
      TOTAL2 = 0.0
```

```
CCC       WRITE(2,4444) VTMR(CCS)
CCC 4444 FORMAT(/,20X,'AVERAGE OF THE VARIANCE TO',
CCC      *' MEAN RATIOS =',F13.6,/)
   25 CONTINUE
      WRITE(2,5555) MAX
 5555 FORMAT(5(/),20X,'SUMMARY OF VTOMR VALUES',//,
     *20X,'NUMBER OF VTOMR VALUES = ',I3,3(/),2(12X,
     *'TIME',10X,'VTMR',14X,'S.D.',5X),//)
      WRITE(3,6666)NFIL,MAX,1.0,-130.0,0.000001,1.0
      WRITE(4,6666)NFIL,MAX,1.0,-130.0,0.000001,1.0
 6666 FORMAT(2X,'DATA FROM DATA FILE ',I5,/,2X,I7,
     *' DATA POINTS',/,2X,F3.1,5X,F7.2,/,
     *2(5X,F13.6,1X,F13.6))
C WRITE RESULTS TO OUTPUT FILES
      DO 30 II = 1, MAX
      WRITE(3,7777) TIME(II), VTMR(II)
      WRITE(4,7777) TIME(II), WIDTH(II)
 7777 FORMAT(5X,F13.6,1X,F13.6)
   30 CONTINUE
      NN = MAX/2 + 1
      DO 35 II = 1,NN
      JJ = II + NN
      WRITE(2,9999)TIME(II),VTMR(II),SD(II),KT(II),
     *             TIME(JJ),VTMR(JJ),SD(JJ),KT(JJ)
   35 CONTINUE
      BG=REAL(MAX)
      ASS1=(BG-REAL(MBUTT))/BG*100.0
      ASS2=(BG-REAL(NBUTT))/BG*100.0
      IF(NRAD.NE.1)GO TO 41
      WRITE(2,8899) ASS1,1
      IF(ASS1.NE.100.0) WRITE(2,8899) ASS2,2
 8899 FORMAT(//,20X,F8.3,1X,'PERCENT OF THE VTMR',
     *' VALUES ARE WITHIN ',I2,' STANDARD DEVIATION')
   41 WRITE(2,8888)
 8888 FORMAT(5(/),20X,'SUMMARY OF WIDTH VALUES',3(/),
     *2(12X,'TIME',5X,'WIDTH VALUE'),//)
      DO 40 II = 1,NN
      JJ = II + NN
      WRITE(2,11111)TIME(II),WIDTH(II),TIME(JJ),WIDTH(JJ)
   40 CONTINUE
 9999 FORMAT(2(5X,F13.7,1X,F13.7,2X,'+/-',F13.7,2X,I1))
11111 FORMAT(2(5X,F13.7,1X,F13.7))
C PRODUCE A PLOT OR PLOTS IF THEY ARE DESIRED
      IF(IPL2.NE.3)  GOTO 45
      CALL PLOT2(TIME,VTMR,VTMRM)
      CALL PLOT2(TIME,WIDTH,WIDTHM)
   45 IF(IPL2.EQ.1)  CALL PLOT2(TIME,VTMR,VTMRM)
      IF(IPL2.EQ.2)  CALL PLOT2(TIME,WIDTH,WIDTHM)
```

```
C CLOSE ALL FILES USED DURING EXECUTION
      DO 50 I = 1,4
      CLOSE(I)
   50 CONTINUE
      CLOSE(7)


C AUTOMATIC PRINT OUT IF DESIRED
      IF((IPRT.EQ.1).AND.((IPL2.NE.4).OR.(IPL1.EQ.1)))
     *CALL LIB$SPAWN('PRINT/NOFEED VTOMR.OUT,PLOTV.OUT')
      IF((IPRT.EQ.1).AND.(IPL2.EQ.4))
     *CALL LIB$SPAWN('PRINT/NOFEED VTOMR.OUT')
      WRITE(6,22222)
22222 FORMAT(10X,'COMPLETE')
      STOP
      END
C
C SUBROUTINE INOUT - THIS SUBROUTINE PROMPTS THE
C USER FOR ALL NECESSARY INFORMATION, READS IN
C M.C.A. CONTENTS, AND PRINTS OUT THIS INFORMATION.
C
      SUBROUTINE INOUT
      COMMON/INPUT/ IST,NFIL,NCH,ACHW,MAX,IOPT,IPL2,
     *IDT,IMIN,IMAX,IPL1,IPRT,MCA,KMIN,KMAX,NRAD
      COMMON/CHANNEL/ KCHAN(82000),ICHAN(82000)
C WRITE TITLE AND HEADING TO SCREEN
      WRITE(6,*) 'BEGIN SUBROUTINE INOUT'
      WRITE(6,1111)
 1111 FORMAT(3(/),30X,'PROGRAM VTMR',//,25X,
     *'WRITTEN BY M. ADAMS',//,34X,'1985',5(/),
     *1X,'PLEASE ANSWER THE FOLLOWING QUESTIONS:',///)
C WRITE INPUT INFORMATION
      WRITE(6,*) 'IS THIS A STANDARD RUN? (1=YES)'
      READ(5,*)   IST
      WRITE(6,*) 'WHAT DATA FILE ? (NFIL)'
      READ(5,*)   NFIL
      WRITE(6,*) 'IS THIS A RANDOM DATA FILE? (1=YES)'
      READ(5,*)   NRAD
      WRITE(5,2222)
 2222 FORMAT(5X,'THIS DATA WAS OBTAINED FROM:',/,
     *10X,'THE OMEGA M.C.A.                (ENTER 1)',/,
     *10X,'THE TRAKOR NORTHERN M.C.A.      (ENTER 2)',/,
     *10X,'IBM OR ATT PERSONAL COMPUTER (ENTER 3)',/)
      READ(5,*) MCA
      IF(IST.EQ.1) GO TO 10
      WRITE(6,*) 'HOW MANY CHANNELS IN INPUT FILE? (NCH)'
      READ(5,*)   NCH
```

```
      WRITE(6,*) 'WHICH CHANNELS ANALYZED? (MIN,MAX)'
      READ(5,*)   KMIN, KMAX
      WRITE(6,*) 'WHAT IS THE CHANNEL WIDTH ? (MICRO-SEC)'
      READ(5,*)   ACHW
      WRITE(6,*) 'MAXIMUM NUMBER OF CHANNELS COMBINED?'
      READ(5,*)   MAX
      MIN=NCH/MAX
      WRITE(6,*) 'VARY COMBINATIONS? (1=YES)'
      READ(5,*)   IOPT
      IF( (IOPT.EQ.1).AND.(MIN.LT.10) ) THEN
      WRITE(6,*) 'WARNING...NCH/MAX MUST BE .GT. 10.',
     *' RE-ENTER MAX.'
      READ(5,*)   MAX
      END IF
      WRITE(6,3333)
 3333 FORMAT(1X,'WHICH RESULTS PLOTTED?',/,
     *5X,'1  -  PLOT OF VTMR  VERSUS CHANNEL WIDTH',/,
     *5X,'2  -  PLOT OF WIDTH VERSUS CHANNEL WIDTH',/,
     *5X,'3  -  BOTH 1 AND 2',/,
     *5X,'4  -  NO PLOT',/)
      READ(5,*)   IPL2
      WRITE(6,*) 'DO YOU WANT A DISTRIBUTION? (1=YES)'
      READ(5,*)   IDT


      IF(IDT.NE.1) GO TO 5
      WRITE(6,*) 'WHAT CHANNELS IN DISTRIBUTION? (MIN,MAX)'
      READ(5,*)   IMIN,IMAX
      WRITE(6,*) 'IS A PLOT OF THE DIST. DESIRED? (1=YES)'
      READ(5,*)   IPL1
    5 WRITE(6,*) 'ALL CHANNELS WRITTEN IN OUTPUT? (1=YES)'
      READ(5,*)   IW
      WRITE(6,*) 'AUTOMATIC PRINT OUT? (1=YES)'
      READ(5,*)   IPRT
      GO TO 15
C STANDARD CASE VALUES
   10 NCH  = 8184
      NRAD = 1
      KMIN = 1
      KMAX = 8184
      ACHW = 1000.0
      MAX  = 60
      IOPT = 1
      IPL2 = 3
      IDT  = 1
      IMIN = 0
      IMAX = 20
```

```
      IPL1 = 1
      IW   = 2
      IPRT = 1
C MESSAGE TO SCREEN
   15 WRITE(6,4444)
 4444 FORMAT(///,1X,'NOW READING DATA INTO CODE')
      IF(MCA.EQ.1) READ(7,5555) (KCHAN(I),I=1,NCH)
      IF(MCA.EQ.2) READ(7,6666) (KCHAN(I),I=1,NCH)
      IF(MCA.EQ.3) THEN
      READ(7,7777)
      READ(7,*) (KCHAN(I),I=1,NCH)
      ENDIF
C OMEGA (MCA=1)
 5555 FORMAT(5(/),I6,8(/),10(100(I6,/),3(/)),24(I6,/))
C TRAKO-NORTHERN (MCA=2)
 6666 FORMAT(//,1024(11X,8(I3,4X),/))
C IBM OF ATT PERSONAL COMPUTER
 7777 FORMAT(4(/))
      WRITE(6,*) 'DATA HAS ALL BEEN READ'
C PLACE DATA FOR ANALYSIS IN ICHAN
      DO 16 II = KMIN, KMAX
      JJ = II - KMIN + 1
      ICHAN(JJ) = KCHAN(II)
   16 CONTINUE
C
      JJOT = KMAX - KMIN + 1
      MIN  = JJOT / MAX
C WRITE OUT INPUT INFORMATION
      WRITE(2,8888)
 8888 FORMAT(//,50X,'PROGRAM VTOMR',/,45X,'WRITTEN BY',
     *' W. M. ADAMS',/,54X,'11/84',//)
      IF(IST.EQ.1) WRITE(2,9999)
 9999 FORMAT(20X,'THIS IS A STANDARD RUN',/)
      CALL IDATE(K,L,M)
      WRITE(2,11111)K,L,M,NFIL,NCH,KMIN,KMAX,ACHW,MIN,MAX
11111 FORMAT(3(/),
     *20X,'TODAYS DATE IS:  ',30X,I2,'/',I2,'/',I2,/,
     *20X,'THIS DATA IS FROM DATA FILE:      ',15X,I5,/,
     *20X,'NUMBER OF CHANNELS IN INPUT FILE:',15X,I6,/,
     *20X,'CHANNELS USED IN ANALYSIS:',19X,I6,' - ',I6,/,
     *20X,'CHANNEL WIDTH IS (MICRO-SECONDS):',15X,F7.1,/,


     *20X,'MIN. NUMBER OF VALUES FOR A MEAN: ',15X,I5,/,
     *20X,'MAX NUMBER OF CHANNELS COMBINED:  ',15X,I5)
      IF(IOPT.EQ.1) WRITE(2,22222)
22222 FORMAT(20X,'THE COMBINATIONS ARE VARIED',/)
```

```
      IF(IDIST.EQ.1)  WRITE(2,33333)
33333 FORMAT(20X,'A DISTRIBUTION IS PRINTED',3(/))
      DO 20 I = 1, JJOT
      MTOT = MTOT + ICHAN(I)
   20 CONTINUE
      WRITE(2,44444) JJOT, MTOT
44444 FORMAT(20X,'TOTAL COUNTS IN THE ',I5,
     *' CHANNELS:',13X,I7)
      IF(IW.NE.1) GOTO 25
      WRITE(2,55555) JJOT, (ICHAN(I),I=1,JJOT)
55555 FORMAT(///,20X,I4,' CHANNEL VALUES FOLLOW',//,
     *410(5X,20(I4,1X),/),//)
C
   25 RETURN
      END
C
C SUBROUTINE DEADT - CURRENTLY NOT USED. THIS ROUTINE
C CORRECTS FOR DEADTIME BY ADJUSTING THE CHANNEL AVERAGE
C COUNTS UPWARD BY THE FRACTION:
C
C   CH. WIDTH / (CH. WIDTH - TOTAL DEADTIME IN CH.).
C
C THIS METHOD IS NOT GENERALLY VALID FOR NOISE TECHNIQUES
C AND THEREFORE NOT USED.
C
      SUBROUTINE DEADT
      COMMON/INPUT/ IST,NFIL,NCH,ACHW,MAX,IOPT,IPL2,
     *IDT,IMIN,IMAX,IPL1,IPRT,MCA,KMIN,KMAX,NRAD
      COMMON/CHANNEL/ KCHAN(8195),ICHAN(8195)
C
      WRITE(6,*) 'WHAT IS THE DEAD TIME ? (MICRO-SEC)'
      READ(5,*)   DEAD
      ACHW=ACHW*1.0E-6
      DEAD=DEAD*1.0E-6
      WRITE(2,333) ACHW,DEAD
  333 FORMAT(20X,'A CHANNEL WIDTH OF ',F7.5,' SECONDS WAS',
     *' USED',/,20X,'DETECTOR DEAD TIME WAS',E14.4,
     *' SECONDS',//)
C
C DEAD TIME CORRECTION
C
      DO 2 I=1,NCH
      ICHAN(I)=IFIX(REAL(ICHAN(I)) /
     *(1.0-(DEAD*REAL(ICHAN(I))/TIME)))
    2 CONTINUE
      RETURN
      END
```

```
C
C SUBROUTINE PROB - THIS SUBROUTINE CALCULATES THE
C  NEUTRON NOISE DISTRIBUTION FROM TIME ORDERED DATA.
C
      SUBROUTINE PROB(IJFLAG)
      COMMON/INPUT/ IST,NFIL,NCH,ACHW,MAX,IOPT,IPL2,
     *IDT,IMIN,IMAX,IPL1,IPRT,MCA,KMIN,KMAX,NRAD
      COMMON/CHANNEL/ KCHAN(82000),ICHAN(82000)
      DIMENSION IDIST(100)
C MESSAGE TO SCREEN
      WRITE(6,*) 'BEGIN SUBROUTINE PROB'
      JMIN = IMIN + 1



      JMAX = IMAX + 1
C
      ITOT = 0
      JJOT = KMAX - KMIN + 1
      IF(IJFLAG.EQ.1) THEN
      IJK  = 1
      IJKL = NCH
      DIV  = REAL(NCH)
      WRITE(2,1111) NCH
      ELSE
      IJK  = KMIN
      IJKL = KMAX
      DIV  = REAL(JJOT)
      WRITE(2,1111) JJOT
      ENDIF
 1111 FORMAT(5(/),20X,'OUTPUT FORM SUBROUTINE PROB',
     *3(/),20X,'DISTRIBUTION  OF ',I6,' CHANNELS',/)
C
    2 DO 10 I =  IJK, IJKL
      IDIST(KCHAN(I)+1) = IDIST(KCHAN(I)+1) + 1
      ITOT = ITOT + KCHAN(I)
   10 CONTINUE
      AVEG = REAL(ITOT)/DIV
      WRITE(2,2222) IMIN,IMAX,AVEG
 2222 FORMAT(
     *20X,'MINIMUM CHANNEL IN DISTRIBUTION IS:',12X,I4,/,
     *20X,'MAXIMUM CHANNEL IN DISTRIBUTION IS:',12X,I4,/,
     *20X,'AVERAGE CHANNEL VALUE IS:      ',15X,F8.4,//,
     *20X,'THE PROBABILITY DISTRIBUTION FOLLOWS',//,
     *20X,'CHANNEL VALUE',10X,'NUMBER OF CHANNELS',//)
C PLOT DISTRIBUTION IF IT IS DESIRED
      IF(IPL1.EQ.1)  CALL PLOT1(IDIST)
C PRINT OUT DISTRIBUTION AND SET ARRAY BACK TO ZERO
```

```
C FOR FUTURE USE
      DO 15 I = JMIN, JMAX
      J1 = I - 1
      WRITE(2,3333) J1, IDIST(I)
      IDIST(I) = 0
   15 CONTINUE
      AVEG = 0.0
 3333 FORMAT(23X,I4,21X,I5)
      RETURN
      END
C
C PLOT1 - PLOT OF THE PROBABILITY DISTRIBUTION
C
      SUBROUTINE PLOT1(IY)
      COMMON/INPUT/ IST,NFIL,NCH,ACHW,MAX,IOPT,IPL2,
     *IDT,IMIN,IMAX,IPL1,IPRT,MCA,KMIN,KMAX,NRAD
      COMMON/CHANNEL/ KCHAN(8195),ICHAN(8195)
      DIMENSION X(100), IY(100), Y(100)
C MESSAGE TO SCREEN
      WRITE(6,*) 'BEGIN SUBROUTINE PLOT1'
C COPY IY ARRAY INTO Y ARRAY FOR PLOTTING. ALSO DETERMINE
C THE MAXIMUM VALUE IN THE Y ARRAY.
      DMAX = 0.0
      JMIN = IMIN + 1
      JMAX = IMAX + 1
      DO 5 I = JMIN, JMAX
      J1 = I - 1
      X(I)   = J1
      Y(I)   = REAL( IY(I) )
      TEST = Y(I)


      IF(TEST.GE.DMAX) DMAX = TEST
    5 CONTINUE
C SET PLOT PARAMETERS
      XMIN  = 0.0
      XMAX  = REAL(IMAX) + 2.0
      YMIN  = 0.0
      YMAX  = DMAX + 50
      HMD   = XMAX/10.0
      HSIZE = 6.0
      VSIZE = 5.0
      CALL  INITPLOT(HSIZE,VSIZE,XMIN,XMAX,YMIN,YMAX)
      CALL  FRAME
      CALL  GRID
      CALL  PLOTLINE(X,Y,MAX)
CCC      CALL EZ_PLOTLINE(X,Y,MAX)
```

```
      CALL  ENDPLOT
C
      WRITE(1,1111) NFIL
 1111 FORMAT(//,10X,'NUMBER OF CHANNELS WITH VALUE X ',
     *'VERSUS X. ',/,10X,'DATA FILE ',I5,/)
      RETURN
      END
C
C PLOT2 - PLOT OF VTOMR VERSUS TIME
C
      SUBROUTINE PLOT2(X,Y,VMAX)
      COMMON/INPUT/ IST,NFIL,NCH,ACHW,MAX,IOPT,IPL2,
     *IDT,IMIN,IMAX,IPL1,IPRT,MCA,KMIN,KMAX,NRAD
      COMMON/CHANNEL/ KCHAN(8195),ICHAN(8195)
      DIMENSION X(MAX), Y(MAX)
C MESSAGE TO SCREEN
      IFLAG = IFLAG + 1
      WRITE(6,123) IFLAG
  123 FORMAT(/,1X,'BEGIN SUBROUTINE PLOT2 ----',I2)
C MOVE ALL VALUES OVER ONE IN THEIR ARRAYS FOR PRINTING
      DO 5 I = 1,99
      J = 100 - I
      X(J+1) = X(J)
      Y(J+1) = Y(J)
    5 CONTINUE
C INPUT FIRST POINT (HERE ALWAYS 1.0)
      X(1) = 0.0
      Y(1) = 1.0
C SET PLOT PARAMETERS
      XMIN  = 0.0
      XMAX  = 0.06
CCC      XMAX  = REAL(MAX) * ACHW * 1.0E-06
      YMIN  = 0.0
      YMAX  = 5.0
CCC      YMAX  = VMAX + 1.0
CCC      HMD   = MAX*0.0001
CCC      VMD   = 0.4
      HSIZE = 6.0
      VSIZE = 5.0
CCC      HSIZE = (XMAX-XMIN) / HMD
CCC      VSIZE = (YMAX-YMIN) / VMD
C SET UP PLOT
      CALL  INITPLOT(HSIZE,VSIZE,XMIN,XMAX,YMIN,YMAX)
      CALL  FRAME
      CALL  GRID
      CALL  PLOTLINE(X,Y,MAX)
C
CCC      CALL  EZ_PLOTLINE(X,Y,MAX)
```

```
      CALL  ENDPLOT
      IF(IFLAG.EQ.1) WRITE(1,1111) NFIL
 1111 FORMAT(//,10X,'VTOMR VRS. CH. WIDTH. DATA FILE ',I4,/)
      IF(IFLAG.EQ.2) WRITE(1,2222) NFIL
 2222 FORMAT(//,10X,'WIDTH VRS. CH. WIDTH. DATA FILE ',I4,/)
      RETURN
      END
```

Listing of MERGE Code

```
C
C PROGRAM MERGE.  WRITTEN BY W. M. ADAMS.  JULY 1985
C
C THE PURPOSE OF THIS CODE IS TO READ IN A NUMBER OF
C DIFFERENT SETS OF DATA, SORT THESE DATA INTO ORDER,
C AND AVERAGE ANY DUPLICATE POINTS.  THE RESULTING
C YTOMR VALUES ARE AVERAGED IN THE SATURATED RANGE.
C
      PROGRAM MERGE
      DIMENSION X(2,2000),BB(2,1000),TEMP(50),MAX(10),
     *PG1(10),PG2(10)
      CHARACTER*12 ANAME(10)
      CHARACTER*1 TITLE(80)
C INITIALIZE ARRAYS TO ZERO
      DO 5 J = 1, 1000
      DO 5 I = 1, 2
      X(I,J)      = 0.0
      X(I,J+1000) = 0.0
      BB(I,J)     = 0.0
    5 CONTINUE
      OPEN(UNIT=2,NAME='INMLAL1.DAT',STATUS='NEW')
      WRITE(6,*) 'HOW MANY FILES? (MAXIMUM OF 10)'
      READ(5,*)   NFIL
      LL = 1
C LOOP OVER EACH INPUT FILE
      DO 10 I = 1, NFIL
      WRITE(6,*) 'WHAT IS THE NAME OF FILE ',I,
     *'? (INCLOSE IN SINGLE QUOTES)'
      READ(5,*)   ANAME(I)
C THIS IS ANOTHER METHOD WHICH IS SUPERIOR,
C REQUIRERING NO QUOTES AROUND THE FILE NAME.
C     DOUBLE PRECISION ANAME
C     TYPE*,'ENTER FILE NAME'
C     ACCEPT 5, ANAME
C   5 FORMAT(A10)
      WRITE(6,*) 'HOW MANY VALUES IN FILE  ',I,
     *'? (MAX. OF 100)'
      READ(5,*)   NVAL
      OPEN(UNIT=1,NAME=ANAME(I),STATUS='OLD')
      READ(1-1111) (TITLE(KK), KK=1,80)
 1111 FORMAT(80A1)
      READ(1,*) MAX(I)
      READ(1,*) PG1(I), PG2(I)
      READ(1,*) ( X(1,J),  X(2,J),  J=LL,(NVAL+LL) )
CCC      WRITE(2,2222)( X(1,J),X(2,J),J=LL,(NVAL+LL))
      CLOSE(1)
      ITOT = ITOT + NVAL
      LL = ITOT + 1
```

```
   10 CONTINUE
C SORT ALL VTMR VALUES INTO ORDER BY CHANNEL WIDTH
      DO 20 K = 1, (ITOT-1)
      IF(X(1,K).GT.X(1,K+1)) THEN
         TEMP1 = X(1,K+1)
         TEMP2 = X(2,K+1)
         J=K
   15    IFLAG = 0
         IF((J.GE.1).AND.(X(1,J).GT.TEMP1)) THEN
            X(1,J+1) = X(1,J)
            X(2,J+1) = X(2,J)
            J = J - 1
            IFLAG=1
         ENDIF
         IF(IFLAG.EQ.1) GO TO 15
         X(1,J+1) = TEMP1



         X(2,J+1) = TEMP2
      ENDIF
   20 CONTINUE
CCC      WRITE(2,2222) (X(1,J), X(2,J), J=1,ITOT)
CCC 2222 FORMAT(/,500(2(5X,F13.7),/))
C AVERAGE VALUES WITH THE SAME CHANNEL WIDTH
      J=1
      DO 35 K = 1, ITOT
      I = 1
      KK = 0
      IF(X(1,J).EQ.0.0) GOTO 30
   25 IF(X(1,J).EQ.X(1,J+I)) THEN
      TOT = TOT + X(2,(J+I))
      I = I + 1
      ELSE
      AVG = ( TOT + X(2,J) ) / REAL(I)
      KK = 1
      TOT = 0.0
      ENDIF
      IF(KK.EQ.0) GOTO 25
      BB(1,K) = X(1,J)
      BB(2,K) = AVG
      J  = J + I
      JJ = JJ + I - 1
      GO TO 35
   30 IDIP = 1.0
   35 CONTINUE
C AVERAGE VTMR VALUES AFTER SATURATION
C (i.e., IN THE 50-60 mSEC RANGE)
```

```
      TOTAL = 0.0
      DO 40 L = 1, ITOT-JJ+1
      IF((BB(1,L).GE.0.05).AND.(BB(1,L).LE.0.06)) THEN
      TOTAL = TOTAL + BB(2,L)
      LJ = LJ + 1
      ENDIF
   40 CONTINUE
      AVERAGE = TOTAL/REAL(LJ) - 1.0
C
C WRITE RESULTS TO OUTPUT FILE AND SCREEN
C
      WRITE(2,3333) (ANAME(KK),KK=1,NFIL)
 3333 FORMAT(12X,10(A12))
      WRITE(2,3344) AVERAGE
 3344 FORMAT('+',1X,F8.6)
      WRITE(2,4444) (ITOT-JJ+1), PG1(NFIL), PG2(NFIL)
 4444 FORMAT(1X,I5,/,1X,F10.4,2X,F10.4)
      WRITE(2,5555) (BB(1,K), BB(2,K), K = 1,ITOT-JJ+1)
 5555 FORMAT(1000(2(5X,F13.7),/))
      IF(IDIP.EQ.1) WRITE(6,6666) (ITOT-JJ+1)
 6666 FORMAT(//,1X,I4,' DATA POINTS ARE OUTPUT',
     *' BY THIS RUN',//)
      WRITE(6,7777) AVERAGE
 7777 FORMAT(/,1X,'AVGERAGE SATURATED Y VALUE IN THE',
     *' 50-60 mSEC RANGE = ',F13.7,/)
      STOP
      END
```

Listing of MLALS Code

```
      PROGRAM MLALS
C
C LALS IS A GENERAL NON-LINEAR LEAST SQUARES
C FITTING CODE WRITTEN AT LOS ALAMOS NATIONAL LAB.
C
C SAME....MODIFIED  FOR USE BY MARK ADAMS 2/85.
C
      COMMON Y(500), X(10,500), W(500), IX(50),
     1 PG(50), P(50), SP(50), YC(500), DY(500),
     2 BM(20,21), TITLE(80), Z(50), Q000FL(50)
      DIMENSION PART(50)
      COMMON / INDEX / N,IK,IM,IW,M,IB,ITEST,
     1 IPR,IFG,TEST,WVAR,SSQ,IDF,DET,YT,INFIL
      OPEN(UNIT=9,NAME='OUTLAL1.DAT',STATUS='NEW')
      OPEN(UNIT=10,NAME='INMLAL1.DAT',STATUS='OLD')
    1 IDF = 0
      DET = 0.0
      SSQ = 0.0
      WVAR = 0.0
      CALL INPUT1
   60 CONTINUE
      CALL P S PAK
   70 CONTINUE
      CALL R S PAK
   80 CONTINUE
      Z(1)=8.75
      DO 99 K=1,200
      Z(1)=Z(1)+.25
      YT=0.0
      H=68.
      DO 15 J=2,IK
      L=J+3
      X1=J+3
      C1=X1*(X1-3.)/4.
      C2=X1*(X1-4.)/3.
      PART(J)=Z(1)**L - C1*(H**(L-4))*Z(1)**4 +
     * C2*(H**(L-3))*Z(1)**3
        DRDZ=X1*Z(1)**(L-1)-4.*C1*(H**(L-4))*Z(1)**3
        DRDZ=DRDZ + 3.*C2*(H**(L-3))*Z(1)**2
        DRDZ=P(J)*DRDZ
        DRDV=2.19*DRDZ*100.
 15   YT=YT+PART(J)*P(J)
      YT=(YT + P(1))*100.
 99   CONTINUE
CCC       GO TO 1
C    NO STACKED INPUT NOW
      STOP
      END
```

```
C ***************************************************
      SUBROUTINE INPUT1
      COMMON Y(500), X(10,500),W(500),IX(50),PG(50),
     1 P(50), SP(50), YC(500), DY(500), BM(20,21),
     2 TITLE(80), Z(50), QOOOFL(50)
      COMMON / INDEX / N, IK, IM, IW, M, IB, ITEST,
     1 IPR,IFG ,TEST,WVAR, SSQ, IDF, DET, YT, INFIL
      READ(10,200,END=410) (TITLE(I),I=1,80)
  200 FORMAT(80A1)
      READ(10,*) MAX
      GO TO 400
  410 WRITE(9,22222)
22222 FORMAT(10X,'TERMINATION NORMAL ')
      STOP
  400 CONTINUE



      WRITE(5,*)'HOW MANY VTMR VALUES ARE TO BE FIT?'
      READ(5,*) N
      WRITE(5,*) 'ARE INPUT VALUES FROM VTMR',
     *' OF VMEX CODE? (1=VTMR)'
      READ(5,*) INFIL
      IK = 2
      IM = 0
      IW = 0
      M = 1
      IB = 1
      ITEST = 0
      IPR = 0
      IFG = 0
      DO 110 I=1,N
      Y(I)=0.0
      W(I)=0.0
      YC(I)=0.0
  110 DY(I)=0.0
      DO301I=1,IK
      IX(I)=0
      PG(I)=0.0
      P(I)=0.0
  301 SP(I)=0.0
  604 LK=IK+1
      DO28I=1,IK
      DO28J=1,LK
   28 BM(I,J)=0.0
  790 DO305I=1,M
      DO305J=1,N
  305 X(I,J)=0.0
```

```
      READ(10,*)(FG(I),I=1,IK)
   20 IF(INFIL-1) 346,344,346
C
  344 DO 23 I = 1, MAX
      READ(10,*) (X(J,I),J=1,M), Y(I)
      W(I)    = 1.0
   23 CONTINUE
      GO TO 348
C
  346 CONTINUE
      DO 24 I = 1,MAX
      READ(10,*) (X(J,I),J=1,M), Y(I)
   24 W(I) = 1.0
  348 CONTINUE
    6 TEST = 0.000001
C
C           PRINT OUT INPUT DATA
C
      WRITE(9,9)
    9 FORMAT(1H1,47X,'LOS ALAMOS NON-LINEAR',
     *' LEAST SQUARES FIT',
     C  ///,57X,'REVIEW OF INPUT DATA',//)
      WRITE(9,10) TITLE
   10 FORMAT(10X,80A1,/)
      WRITE(9,11) N, IK, IM, IW, M, TEST
   11 FORMAT(/,10X,'NUMBER OF DATA POINTS',19X,I5,/,
     C 10X,'NUMBER OF PARAMETERS',20X,I5,/,
     C 10X,'NUMBER OF PARAMETERS HELD CONSTANTS',5X,I5,/,
     C 10X,'WEIGHTING OPTION',24X,I5,/,
     C 15X,'0 -- W(I) = 1.0',/,
     C 15X,'1 -- W(I) = (SUPPLIED BY USER)',/,
     C 15X,'2 -- W(I) = 1.0/Y(I)',/,
     C 15X,23H3 -- W(I) = 1.0/Y(I)''2,/,


     C 10X,'NUMBER OF INDEPENDENT VARIABLES',9X,I5,/,
     C 10X,'CONVERGENCE TEST',19X,1PE10.2,//)
      WRITE(9,12)
   12 FORMAT(22X,'I',8X,'W(I)',8X,'Y(I)',8X,'X(1)',8X,
     C'X(2)',8X,'X(3)',8X,'X(4)',8X,'X(5)',//)
      DO 14 I=1,N
      WRITE(9,13) I,W(I),Y(I),(X(J,I),J=1,M)
   13 FORMAT(20X,I3,3X,7(1PE12.4))
  14  CONTINUE
  100 RETURN
      END
```

```
C ******************************************************
      SUBROUTINE P S PAK
      COMMON Y(500), X(10,500), W(500), IX(50), PG(50),
     1 P(50),SP(50),YC(500),DY(500),BM(20,21),TITLE(80),
     2 Z(50), QOOOFL(50)
      COMMON / INDEX / N, IK, IM, IW, M, IB, ITEST, IPR,
     1 IFG, TEST, WVAR, SSQ, IDF, DET, YT, INFIL
      DIMENSION AN(50),TEMP(41),AM(20,20),DP(50),PC(50)
 1005 FORMAT(///3X,'K',39X,'A(K,L)',65X,'B(K)',//)
 1006 FORMAT(I5,1P5E17.7/(1PE21.7,1P4E17.7))
 1007 FORMAT(1H+1P1E119.7//)
 1008 FORMAT(24HOVALUE OF DETERMINANT = 1P1E14.7///)
 1009 FORMAT(4X,'K',35X,'INVERSE OF A(K,L)',5X,'1',//)
 1010 FORMAT(1HO)
 1014 FORMAT(1H1)
 1019 FORMAT(///,I6,  ' ITERATIONS ')
      ITS=0
      IHSP=0
      M25C=0
      KOOOFX = 2
      VAR = 0.0
       SS = 0.0
      LIES=0
 1100 DO 1101 I=1,IK
      DP(I)=0.0
      PC(I)=PG(I)
      BM(I,1) = 0.0
      SP(I)=0.0
 1101 P(I)=PG(I)
      LIT = 0
      LIE = 0
      IF(IPR) 51, 51, 21997
21997 WRITE(9,28000) TITLE
28000 FORMAT(1H1,27X,40A1,//)
   51 LICK = 0
      K=IK-IM
 1103 IT=0
      KP=K+1
      IF(K) 1102, 11011,11032
11011 LIES = 1
      GO TO 11087
11032 DO1108I=1,K
      DO1108J=1,KP
      IF(J-KP)1104,1105,1105
 1104 AM(I,J)=0.0
 1105 IF(I+1-J)1106,1107,1106
 1106 BM(I,J)=0.0
      GOTO1108
```

```
 1107 BM(I,J)=1.0
 1108 CONTINUE
      H = 1.0
      IT=IT+1



11087 DO21122L=1,N
      DO 11084 J = 1, M
11084 Z(J) = X(J,L)
   66 CALLYPS(L)
11085 IF(LIES)11086,11086,1115
11086 JACK=0
      DO9003JUK=1,IK
      IF(IM)1102,9002,9001
 9001 DO1110JOKE=1,IM
      IF(JUK-IX(JOKE))1110,11131,1110
 1110 CONTINUE
 9002 JAKE=JUK-JACK
      AN(JAKE) = QOOOFL(JUK)
      GO TO 9003
11131 JACK=JACK+1
 9003 CONTINUE
 1115 YC(L) = YT
      DY(L) = Y(L) - YC(L)
      IF(LIES)21116,21116,31117
31117 VAR=VAR+W(L)*DY(L)**2
      SS=SS+DY(L)**2
      GO TO 21122
21116 IF(K)1102,21122,1117
 1117 DO1122I=1,K
      DO 1122 J=I,KP
      IF(J-KP)1118,1119,1119
 1118 AM(I,J)=AM(I,J)+AN(I)*AN(J)*W(L)
      GO TO 1122
 1119 BM(I,1)=BM(I,1)+AN(I)*DY(L)*W(L)
 1122 CONTINUE
21122 CONTINUE
      V = 0.0
      DO 60 L = 1,N
   60 V = V + W(L) *DY(L) **2
      IF (LIES) 21128, 21128, 1153
21128 IF (K - 1) 21129, 21129, 31123
31123 DO31124 J=2,K
      JIG = J - 1
      DO 31124 I=1,JIG
31124 AM(J,I) = AM(I,J)
21129 IF (K) 1102, 1123, 11221
```

```
11221 IF(KOOOFX.EQ.2)     GO TO 11251
 1123 KOOOFX = 1
      IF (K) 1102, 1152, 21123
21123 IF(IPR)11251,11251,21148
21148 WRITE(9,1005)
      DO1124I=1,K
      WRITE(9,1006)I,(AM(I,J),J=1,K)
 1124 WRITE(9,1007)BM(I,1)
11251 IF(LIE) 1102, 6000,7000
 6000 KT = 1
      GO TO 8000
 7000 KT=KP
 8000 IF (K - 1) 1152, 3000, 4000
 3000 DET = AM(1,1)
      BM(1,1) = BM(1,1)/AM(1,1)
      BM(1,2) = 1.0/AM(1,1)
      GOTO1131
 4000 KPL=K+1
      CALL LSS(K,KPL,20,AM,BM,TEMP,DET)
 1131 IF(KOOOFX.EQ.1)     WRITE(9,1008) DET
 1133 JUK = 0
      DO 11351 I=1,IK


14332 IF (IM) 1102, 11343, 11331
11331 DO1134JOKE=1,IM
      IF(I - IX(JOKE)) 1134, 11352, 1134
 1134 CONTINUE
11343 JAKE = I - JUK
      IF(IFG-1)27002,22353,6661
27002 IF(IT-5)6661,6661,22353
 6661 DP(I) = BM(JAKE,     1)
16666 PC(I) = P(I) + H*      DP(I)
      GO TO(7777, 26666), KOOOFX
26666 IF(IFG-1)27000,11351,6667
27000 IF(IT-5)6667,6667,11351
 7777 KOOOFX = 1
      GOTO11351
 6667 IF(P(I)*PC(I))6668,11351, 11351
 6668 H = H/2.0
      IF (H - .1E-09) 12352,  16666, 16666
11352 JUK = JUK + 1
11351 CONTINUE
      GO TO (1147,1142)  , KOOOFX
22353 IF(IT-25)6661,6661,13000
13000 ITS=IT
11341 IF(ABS(DP(I)) - ABS(BM(JAKE,1)))
```

```
     *       11342,11342,6661
11342 H = H/2.0
      IF(H-.1E-09)12352,6661,6661
12352 IHSP=1
      GOTO1102
 1142 IF(IPR)1145,1145,1143
 1143 WRITE(9,13100)IT,H,V
13100 FORMAT(1H0I3,1P2E17.7)
      DO1144I=1,IK
 1144 WRITE(9,1006)I,PG(I),P(I),PC(I),DP(I)
 1145 JERK = 0
      DO 1146 I = 1,IK
      IF (P(I)) 11451, 11452, 11451
11451 IF(ABS((PC(I)-P(I))/P(I))- TEST )
     *1146,1146,1148
11452 JERK = JERK + 1
 1146 CONTINUE
      IF (JERK - IK) 1147, 1148, 1148
 1147 KOOOFX = 1
      LIE = 1
      M25C=1
 1148 DO1149I=1,IK
 1149 P(I)=PC(I)
      IF (LICK) 1102, 1150, 1152
 1150 IF(KOOOFX.EQ.1)   LICK = 1
 1151 IF(M25C)29768,29768,11032
29768 IF(IT-25)11032,11512,11032
11512 KOOOFX = 1
      IT=0
      ITS=26
      GO TO 1147
 1152 VAR=0.0
      SS=0.0
      LIES = 1
       GO TO 11087
 1153 DF=N-K
      IDF=N-K
      IF(K) 1102, 21155, 21154
21154 IF(IPR)21998,21998,21189
21189 WRITE(9,1009)


      DO 1155 I=1,K
      WRITE(9,1006)I,(BM(I,J),J=2,KP)
 1155 WRITE(9,1010)
21998 IF(ITS)13002,13004,13002
13002 IT=ITS
```

```
13004 WRITE(9,1019)IT
21155 WVAR = VAR/DF
      SSQ = SS
       JACK = 0
      DO 21160 I = 1,IK
      IF(IM)1102,1158,1156
 1156 DO 1157 J=1,IM
      IF(I-IX(J))1157,11591,1157
 1157 CONTINUE
 1158 JAKE = I - JACK
      J = JAKE + 1
 1159 SP(I) = SQRT(BM(JAKE,J)*WVAR)
      GO TO 21160
11591 JACK = JACK + 1
21160 CONTINUE
 1102 KOOOFX = 2
      IF(IHSP)22100,22108,22100
22100 WRITE(9,22102)
22102 FORMAT(1HO,'THE PROGRAM QUIT ITERATING BECAUSE',
     C' THE PARAMETER(S) INSIST ON CHANGING SIGNS')
22108 RETURN
      END
C ******************************************************
      SUBROUTINE R S PAK
      COMMON Y(500), X(10,500), W(500), IX(50), PG(50),
     1 P(50),SP(50),YC(500),DY(500),BM(20,21),TITLE(80),
     2 Z(50), QOOOFL(50)
      COMMON / INDEX / N, IK, IM, IW, M, IB, ITEST, IPR,
     1 IFG , TEST, WVAR, SSQ, IDF, DET, YT, INFIL
      DIMENSION RP(50),AN(20,20)
      WRITE(9,28000)  TITLE
28000 FORMAT(1H1,27X,40A1,///)
      ITCHY = 0
      DO 198 I = 1,IK
      IF (IX(I) - I) 199, 198, 199
  198 CONTINUE
      ITCHY = 1
  199 I = -1
      CALLYPS(I)
   68 IF (IM) 1, 1, 2
    1 IM = 0
    2 WRITE(9,1011)N,M,IK,IM,WVAR,SSQ
 1011 FORMAT(////23H THIS PROBLEM CONTAINS I3,
     114H DATA POINTS, I2,
     230H INDEPENDENT VARIABLE(S), AND I2,
     315H PARAMETER(S) (I2,
     424H OF THEM HELD CONSTANT).//
     526H THE WEIGHTED VARIANCE IS 1PE14.7,
```

```
     6' AND THE UNWEIGHTED SUM OF SQUARES OF THE',
     7' DEVIATIONS IS',1PE14.7,1H./////)
      WRITE(9,1015)
       JACK = 0
      DO 1164 I=1,IK
      IF(IM)1158,1158,1156
 1156 DO 1157 J=1,IM
      IF(I-IX(J))1157,11591,1157
 1157 CONTINUE
 1158 JAKE = I - JACK


      J = JAKE + 1
 1159 CONTINUE
      GO TO 21160
11591 JACK = JACK + 1
21160 WRITE(9,1006)I,PG(I),P(I),SP(I)
      IF (ITCHY) 1160, 1160, 11621
 1160 IF (IM) 11621, 11621, 1161
 1161 DO1162J=1,IM
      IF(I-IX(J))1162,1163,1162
 1162 CONTINUE
11621 A = 0.0
      B = 0.0
      DO 901 JOE = 1,N
      DO 900 JUMP = 1,M
  900 Z(JUMP) = X(JUMP,JOE)
  706 CALLYPS(JOE)
      A= A+ W(JOE) * YC(JOE) * Q000FL(I)
  901 B= B+ W(JOE) * Y(JOE) * Q000FL(I)
      WRITE(9,902)A,B
  902 FORMAT(1H+1PE102.7, 1PE17.7)
      IF (ITCHY) 1164, 1164, 1163
 1163 WRITE(9,1016)
 1164 CONTINUE
C
      REACT = P(2) * 54.E-06 / 0.007  +  1.0
C
 1006 FORMAT(I4,1P5E17.7/(1PE21.7,1P4E17.7))
  209 FORMAT(9H PARTIALS 1PE14.7,
     C1P4E17.7/(1PE23.7,1P4E17.7))
 1015 FORMAT(7X,'GUESSTIMATE OF   FINAL VALUE OF',
     1'       S.D.  OF                          ',
     2'        EXACT LEAST SQUARES EQUATIONS',/,3X,
     3'K   K-TH PARAMETER   K-TH PARAMETER',
     4'   K-TH PARAMETER                        ',
     5'   FITTED FUNCTION    INPUT DATA',//)
```

```
1016 FORMAT(1H+,'                                                ',
     1'                          ',
     2'THIS PARAMETER WAS HELD FIXED.')
      K=IK-IM
      IF(K)7008,7008,7000
7000  WRITE(9,1012)
1012  FORMAT(/////,' MATRIX OF CORRELATIONS',
     1' BETWEEN FREE PARAMETERS',/)
      DO 11  I=1,K
      DO 10  J=1,K
   10 RP(J)=BM(I,J+1)/SQRT(BM(I,I+1)*BM(J,J+1))
   11 WRITE(9,1013)I,(RP(J),J=1,K)
1013  FORMAT(1H0,I4,14F8.3/(F12.3,13F8.3))
      WRITE(9,7094) REACT
      WRITE(5,7094) REACT
7094  FORMAT(////,10X,'** REACTIVITY CALCULATED',
     *' FROM FIT = ',F12.3,//,10X,'   IF THIS DATA',
     *' IS FROM CRITICAL PILE, REACTIVITY = 0.0',/,
     *10X,'   AND THE ABOVE NUMBER HAS NO MEANING.')
7008  WRITE(9,28000)  TITLE
      WRITE(9,1014)
1014  FORMAT(23X,'INDEPENDENT',7X,'DEPENDENT',7X,
     1'CALCULATED',34X,'STD. DEV. OF',/,20X,'I',7X,
     2'WEIGHT',11X,'VARIABLE','  FUNCTION',9X,
     3'DEVIATION',16X,'PREDICTED MEAN')
      DO 20   I = 1,N
      DO 200 J=1,M
  200 Z(J)=X(J,I)



      CALLYPS(I)
      A=0.0
      JACK=0
      DO 205 JUK=1,IK
      IF(IM)203,203,201
  201 DO 202 JOKE=1,IM
      IF(JUK-IX(JOKE))202,204,202
  202 CONTINUE
  203 JAKE=JUK-JACK
      AN(JAKE,1)=Q000FL(JUK)
      GO TO 205
  204 JACK=JACK+1
  205 CONTINUE
      K=IK-IM
      IF(K)7012,7012,7016
7012  A=0.
      GOTO7020
```

```
 7016 DO 206 J=1,K
      DO 206 JJ=1,K
  206 A=A+AN(J,1)*AN(JJ,1)*BM(J,JJ+1)
      A=SQRT(A*WVAR)
 7020 J=1
      IF(M-2)300,312,312
  300 WRITE(9,1020) I,W(I),X(1,I),Y(I),YC(I),
     *DY(I),A
 1020 FORMAT(1HOI5,1PE17.7,1PE18.7,1P3E17.7,1PE27.7)
      GO TO 2070
  312 WRITE(9,302)I,W(I),J,X(1,I),Y(I),YC(I),DY(I),A
  302 FORMAT(1HOI5,1PE17.7,I3,1PE15.7,1P3E17.7,1PE27.7)
 2072 DO 207 J = 2,M
  207 WRITE(9,1021)J,X(J,I)
 1021 FORMAT (I26, 1PE15.7)
 2070 IF (ITCHY) 20, 20, 208
  208 WRITE(9,209)(QOOOFL(J),J=1,IK)
   20 CONTINUE
      RETURN
      END
C ******************************************************
      SUBROUTINE LSS (N,M,I,A,B,D,DET)
      DIMENSION A(I,N),B(I,N),D(N)
      DOUBLE PRECISION S1,S2
      CHARACTER COM1*48, COM2*48, COM3*48
      DATA COM1/'LSS NEAR SING. SYSTEM. CALC. CONT.'/
      DATA COM2/'LSS SING.SYSTEM-NO RESULT-INPUT GONE.'/
      DATA COM3/'LSS N IS ZERO. INPUT DATA NOT LOST.'/
      NN = N
      IF(NN.EQ.0)  GO TO 20
      MM = M
      X = 0.
      DO 1 J = 1,NN
      DO 1 K = 1,NN
      T = ABS(A(K,J))
      IF(T.GT.X) X = T
    1 CONTINUE
      IF (X.EQ.0.) GO TO 19
       IF (X.GT.1.E-15) GO TO 2
      PRINT 200, COM1
  200 FORMAT(// 10X, 8A10, //)
    2 SN = 1.
      DO 14 J =1,NN
      L = J - 1
      IF (J.EQ.NN) GO TO 11
      T = ABS(A(J,J))
```

```
      M1 = J
      M2 = J + 1
      DO 3 K = M2,NN
      X = ABS(A(M2,J))
      IF (X.LE.T) GO TO 3
      T = X
      M1 = K
    3 CONTINUE
      IF (M1.EQ.J) GO TO 6
      DO 4 K = 1,NN
      T = A(J,K)
      A(J,K) = A(M1,K)
    4 A(M1,K) = T
      DO 5 K = 1,MM
      T =B(J,K)
      B(J,K) = B(M1,K)
    5 B(M1,K) = T
      SN = -SN
    6 IF (A(J,J).EQ.0.) GO TO 19
      DO 10 K = M2,NN
      S1 = 0.
      S2 = 0.
      IF (L.EQ.0) GO TO 8
      DO 7 M3 =1,L
    7 S1 = S1 + A(J,M3)*A(M3,K)
    8 A(J,K) = (A(J,K) - S1)/A(J,J)
      DO 9 M3 = 1,J
    9 S2 = S2 + A(K,M3)*A(M3,M2)
   10 A(K,M2) = A(K,M2) - S2
   11 DO 13 K = 1,MM
      S1 = 0.
      IF (L.EQ.0) GO TO 13
      DO 12 M3 = 1,L
   12 S1 = S1 + A(J,M3)*B(M3,K)
   13 B(J,K) = (B(J,K) - S1)/A(J,J)
   14 CONTINUE
      DET = A(1,1)*SN
      DO15 J = 2,NN
   15 DET = DET*A(J,J)
      IF (DET.EQ.0.) GO TO 19
      IF (MM.EQ.0) GO TO 21
      M3 = NN-1
      DO 18 J = 1, MM
      DO 17 L = 1,M3
      M1 = NN - L
      S1 = 0.
      M2 = M1 + 1
      DO 16 K = M2,NN
```

```
   16 S1 = S1 + A(M1,K)*B(K,J)
   17 B(M1,J) = B(M1,J) - S1
   18 CONTINUE
      GO TO 21
   19 PRINT 200, COM2
      GO TO 21
   20 PRINT 200, COM3
   21 RETURN
      END
C ***************************************************
      SUBROUTINE YPS(I)
      COMMON Y(500), X(10,500), W(500), IX(50), PG(50),
     1 P(50),SP(50),YC(500),DY(500),BM(20,21),TITLE(80),
     2 Z(50), PART(50)
      COMMON / INDEX / N, IK, IM, IW, M, IB, ITEST, IPR,


     1 IFG , TEST, WVAR, SSQ, IDF, DET, YT, INFIL
      IF(I.GT.0)  GO TO 3
      WRITE(9,2)
 2    FORMAT(10X,'ADAMS')
      GO TO 4
 3    YT = 0.0
C
C THE FOLLOWING PORTION OF SUBROUNTINE YPS IS SUPPLIED
C BY THE USER.  "YT" IS THE FUNCTION TO BE FITTED.  Z(I)
C IS THE I TH INDEPENDENT VARIABLE.  P(I) IS THE I TH
C PARAMETER TO BE DETERMINED BY LASL.  PART(I) IS THE
C PARTIAL DERIVATIVE WITH RESPECT TO THE I TH PARAMETER.
C
      C = P(2) * Z(1)
      D = 1.0 + (1.0 - EXP(C))/C
      YT = 1.0 + P(1)*D
      PART(1) = D
      PART(2) = -1.*P(1)/P(2)*(EXP(C)+(1.-EXP(C))/C)
C
C          END OF USER SUPPLIED SECTION
C
 4    RETURN
      END
```

Listing of PLOT Subroutines

```
      SUBROUTINE INITPLOT(HZ_LIMINCH,VT_LIMINCH,
     *                    XXMIN,XXMAX,YYMIN,YYMAX)
*.....................................................
* Blank out page buffer and set physical limits for
* plot.  Set scaling constants so that XMIN = left
* border of plot.     XMAX = right "      "  "
*                     YMIN = lower "      "  "
*                     YMAX = top   "      "  "
* .....................................................
      INCLUDE 'PLTPAK.CMN/LIST'

      REAL HZ_LIMINCH,VT_LIMINCH
*23456
      INTEGER BUFFERLEN
      SAVE
* Set physical dimensions of the plot window..
      XMIN = XXMIN
      XMAX = XXMAX
      YMIN = YYMIN
      YMAX = YYMAX
      HZ_FRAME=AMIN1((HZ_LIMINCH*10),FLOAT(HORIZSIZ))
* HZ_ dim. converted to bytes @ 10 bytes(=60 dots)per in.
      MAX_XPOS = 6 * HZ_FRAME

      VT_FRAME=AMIN1((VT_LIMINCH*72),FLOAT(VERTSIZ))
* VT_ dimension in lines 72 lines per inch
      MAX_YPOS = VT_FRAME
* Blank out Page buffer.............................
      BUFFERLEN=VERTSIZ*HORIZSIZ
      CALL BLANKPAGE(%REF(PAGE(1,1)),BUFFERLEN)
* Find the transformation constants s.t.:...........
*               XPOS = X * HZ_SCALE + HZ_OFFSET
*               YPOS = Y * VT_SCALE + VT_OFFSET
      IF(XMAX -XMIN .NE.0)THEN
               HZ_SCALE = HZ_LIMINCH*60/(XMAX - XMIN)
      ELSE
               HZ_SCALE = 1.0E+06
      ENDIF
      HZ_OFFSET= - (XMIN * HZ_SCALE)
      IF(YMAX-YMIN.NE.0)THEN
           VT_SCALE = VT_LIMINCH * 72/(YMAX - YMIN)
      ELSE
           VT_SCALE = 1.0E+06
      ENDIF
      VT_OFFSET= - (YMIN * VT_SCALE)
      RETURN
      END
```

```
      SUBROUTINE EZ_PLOTLINE(X,Y,NO_POINTS)
*.....................................................
* Automatic scaling of continuous line plot to fit in an
* 8 x 8 inch plot window. If EZ_PLOTLINE is called, only
* ENDPLOT need be called. INITPLOT should not be called.
* X,Y are real vectors containing the coordinates of
* NO_POINTS points to be plotted

      INCLUDE  'PLTPAK.CMN'
      INTEGER NO_POINTS
      REAL X(NO_POINTS),Y(NO_POINTS),
     *     XXMIN,XXMAX,YYMIN,YYMAX

      PARAMETER(scl=1.2)

*23456
*     Find the maxima and minima of the plot.........

      XXMIN = X(1)
      XXMAX = X(1)
      YYMIN = Y(1)
      YYMAX = Y(1)

      DO J = 2,NO_POINTS
       IF(X(J).GT.XXMAX) XXMAX = X(J)
       IF(X(J).LT.XXMIN) XXMIN = X(J)
       IF(Y(J).GT.YYMAX) YYMAX = Y(J)
       IF(Y(J).LT.YYMIN) YYMIN = Y(J)
      END DO

      CALL INITPLOT(8.0,8.0,scl*XXMIN,scl*XXMAX,
     *              scl*YYMIN,scl*YYMAX)
      CALL FRAME
      CALL PLOTLINE(X,Y,NO_POINTS)

      RETURN
      END
```

```
      SUBROUTINE EZ_PLOTPOINTS(X,Y,NO_POINTS)
*.....................................................
* Automatic scaling of POINT  plot to fit in an
* 8 x 8 inch plot window. If EZ_PLOTLINE is called, only
* ENDPLOT need be called. INITPLOT should not be called.
* X,Y are real vectors containing the coordinates of
* NO_POINTS points to be plotted

      INCLUDE  'PLTPAK.CMN'
      INTEGER NO_POINTS
      REAL X(NO_POINTS),Y(NO_POINTS),
     *     XXMIN,XXMAX,YYMIN,YYMAX

      PARAMETER(scl=1.013)

*23456
*     Find the maxima and minima of the plot.........

      XXMIN = X(1)
      XXMAX = X(1)
      YYMIN = Y(1)
      YYMAX = Y(1)

      DO J = 2,NO_POINTS
       IF(X(J).GT.XXMAX) XXMAX = X(J)
       IF(X(J).LT.XXMIN) XXMIN = X(J)
       IF(Y(J).GT.YYMAX) YYMAX = Y(J)
       IF(Y(J).LT.YYMIN) YYMIN = Y(J)
      END DO

      CALL INITPLOT(8.0,8.0,scl*XXMIN,scl*XXMAX,
     *              scl*YYMIN,scl*YYMAX)
      CALL FRAME
      CALL PLOTPOINTS(X,Y,NO_POINTS)

      RETURN
      END
```

```
      SUBROUTINE PLOTPOINTS(X,Y,NO_POINTS)
*.....................................................
* Plot as individual points the coordinates in vectors
*               X & Y

      INCLUDE 'PLTPAK.CMN'
      INTEGER NO_POINTS,XPOS,YPOS
      REAL X(NO_POINTS), Y(NO_POINTS)
      SAVE

      DO J = 1, NO_POINTS
*       Transform to PAGE coordinates.....
        XPOS = NINT(HZ_SCALE * X(J) + HZ_OFFSET)
        YPOS = NINT(VT_SCALE * Y(J) + VT_OFFSET)
        CALL PUTPOINT(XPOS,YPOS)
      END DO

      RETURN
      END
```

```
        SUBROUTINE PUTPOINT(XPOS,YPOS)
*.................................................
* Set the bit corresponding to (X,Y) in PAGE

        INCLUDE 'PLTPAK.CMN'

        INTEGER*2 XPOS,YPOS
        INTEGER*4 HORIZPOS,BITPOS
        BYTE NEWBIT
        LOGICAL INRNGE
        SAVE

        HORIZPOS(0)=(XPOS/6+1)
* Find the HZ_ element in PAGE(?,YPOS)
* Find the position of the bit
* in PAGE(HORIZPOS,YPOS) to be set
*       BITPOS(0)=(2**(MOD(XPOS,6)))

*       Test if the data is within the plot frame.....
        INRNGE(0)=((XPOS.LE.MAX_XPOS .AND. XPOS.GE.FIRST)
     *       .AND.(YPOS.LE.MAX_YPOS.AND.YPOS.GE.FIRST))
*23456
        IF(INRNGE(0))THEN
* Set the bit
           CALL ADBITS(%REF(PAGE(HORIZPOS(0),YPOS)),XPOS)
        ENDIF

        RETURN
        END
```

```
      SUBROUTINE ENDPLOT
*.....................................................
* Write contents of PAGE buffer to disk file adding
* Plot mode code.
*.....................................................
      INCLUDE 'PLTPAK.CMN'
      SAVE

      INTEGER LENGTH
      BYTE PLOTCOD
      CHARACTER*40 BLANK
      PARAMETER (BLANK='@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@',
     *                 '@@@@@@@@@@@@@@@@')
      PARAMETER(PLOTCOD='005'O)
* Pad 2 blank lines before...
      WRITE(1,'(1HO)')
      DO J= VT_FRAME,FIRST, -1
       WRITE(1,'(132A1)')BLANK,PLOTCOD,
     *       (PAGE(I,J),I=FIRST,HZ_FRAME+1)
      END DO
      WRITE(1,'(1HO)')!Pad 2 blank lines after...

      WRITE(1,100)'Upper right corner  ',XMAX,YMAX
      WRITE(1,100)'Lower left corner   ',XMIN,YMIN

100   FORMAT(T12,A,1H(,E10.3,1H,E10.3,1H) )


      RETURN
      END
```

```
      SUBROUTINE PLOTLINE(X,Y,NO_POINTS)
*.....................................................
*  Take the coordinates in vectors   X & Y and plot a
*  continuous line using straight line interpolation


      INCLUDE 'PLTPAK.CMN'

      INTEGER NO_POINTS
     * ,XPOS,YPOS             ! Coordinates of this point
     * ,PREV_XPOS,PREV_YPOS    ! Coordinates of last point
     * ,PRES_XPOS,PRES_YPOS    ! Inter. coord. for interpol.
     * ,X_STEP, Y_STEP         ! Direction of increment

      REAL X(NO_POINTS), Y(NO_POINTS)
      REAL*8 DY_by_DX,D_x,DELTA_X

      SAVE



      DO J = 1, NO_POINTS
*       Transform to PAGE coordinates.............
        XPOS = NINT(HZ_SCALE * X(J) + HZ_OFFSET)
        YPOS = NINT(VT_SCALE * Y(J) + VT_OFFSET)

        IF(J.EQ.1) THEN     !Initialize for 1st point
           PREV_XPOS = XPOS
           PREV_YPOS = YPOS
        ENDIF

*       Test for a break and interpolate if necessary..
*       Select Vertical line, Horiz. line or Y = aX + b
        IF(     (ABS(XPOS - PREV_XPOS).GT.1)
     *  .OR.    (ABS(YPOS - PREV_YPOS).GT.1)    )THEN
          IF(XPOS.GT.PREV_XPOS)THEN
              X_STEP = +1
          ELSE
              X_STEP = -1
          ENDIF

          IF(YPOS.GT.PREV_YPOS)THEN
              Y_STEP = +1
          ELSE
              Y_STEP = -1
          ENDIF

        IF(     (ABS(XPOS - PREV_XPOS).GE.1)
     *  .AND.   (ABS(YPOS - PREV_YPOS).GE.1) )THEN
```

```
*          Interpolate using Y = (dy/dx)Dx + Yo..

                DY_by_DX=ABS(DBLE(YPOS-PREV_YPOS)/
     *                     (XPOS-PREV_XPOS))
*          Select D_x s.t PRES_YPOS and PRES_XPOS do
*          not increment in steps .GT. 1.0..
                IF(DY_by_DX .GT. 1.0)THEN
                        D_x =(1.0/DY_by_DX)*X_STEP
                ELSE
                        D_x = X_STEP
                ENDIF

*               Follow the interpolating line.....
                DO DELTA_X = 0.0, (XPOS-PREV_XPOS),D_x
                  PRES_YPOS=Y_STEP*NINT(ABS(DY_by_DX*
     *                    DELTA_X))+PREV_YPOS
                  PRES_XPOS= NINT(DELTA_X) + PREV_XPOS
                  CALL PUTPOINT(PRES_XPOS,PRES_YPOS)
                END DO

              ELSEIF( ABS(XPOS - PREV_XPOS).GT.1 )THEN
*             Horizontal line needed.....................
                DO PRES_XPOS = (PREV_XPOS),XPOS,X_STEP
                        CALL PUTPOINT(PRES_XPOS,YPOS)
                END DO

              ELSEIF( ABS(YPOS - PREV_YPOS).GT.1 )THEN
*             Vertical line needed.......................
                DO PRES_YPOS = (PREV_YPOS),YPOS,Y_STEP
                        CALL PUTPOINT(XPOS,PRES_YPOS)
                END DO
              ENDIF

     *
              ELSE !No interpolation.....................
                CALL PUTPOINT(XPOS,YPOS)
              ENDIF
*             Record for next time round.................
              PREV_XPOS = XPOS
              PREV_YPOS = YPOS
            END DO

            RETURN
            END
```

```
      SUBROUTINE FRAME
*.....................................................
* Draw frame around borders of the plot window
* Mark one tenth, one half and one inch intervals

      INCLUDE 'PLTPAK.CMN'

*     Top border.....................................
      CALL HZ_LINE(MAX_YPOS,1)     !Top border
      CALL HZ_LINE(MAX_YPOS-1,5)   !Ticks every 12th inch
      CALL HZ_LINE(MAX_YPOS-2,30)  !Half inch marks
      CALL HZ_LINE(MAX_YPOS-3,60)  !One inch marks
      CALL HZ_LINE(MAX_YPOS-4,60)

*     Bottom border..................................
      CALL HZ_LINE(FIRST,1)
      CALL HZ_LINE(FIRST+1,6)
      CALL HZ_LINE(FIRST+2,30)
      CALL HZ_LINE(FIRST+3,60)
      CALL HZ_LINE(FIRST+4,60)

*     Left border....................................
      CALL VT_LINE(FIRST,1)     !Left border
      CALL VT_LINE(FIRST+1,6)   !Ticks every 12th in.
      CALL VT_LINE(FIRST+2,36)  !Half inch marks
      CALL VT_LINE(FIRST+3,72)  !One inch marks

*     Right border ..................................
      CALL VT_LINE(MAX_XPOS,1)
      CALL VT_LINE(MAX_XPOS-1,6)
      CALL VT_LINE(MAX_XPOS-2,36)
      CALL VT_LINE(MAX_XPOS-3,72)

*     .....................................
      WRITE(1,'(1HO)')
      WRITE(1,100)'Vertical scale - Major div. =',
     *     72./VT_SCALE,'Minor div. =',6./VT_SCALE
      WRITE(1,100)'Horizon. scale - Major div. =',
     *     60./HZ_SCALE,'Minor div. =',5./HZ_SCALE
100   FORMAT(T12,A,E10.3,4X,A,E10.3)
*23456

      RETURN
      END
```

```
        SUBROUTINE HZ_LINE(ROW,INC)
*.....................................................
* Insert a horizontal line of dots every INC, at
* height ROW across width of PLOT window

        INCLUDE 'PLTPAK.CMN'

        INTEGER ROW,XPOS
        DO XPOS=FIRST,MAX_XPOS,INC
                CALL PUTPOINT(XPOS,ROW)
        END DO

        RETURN
        END
```

```
        SUBROUTINE VT_LINE(COL,INC)
*.....................................................
* Insert a vertical line of dots every INC,
* at position COL along height of PLOT window

        INCLUDE 'PLTPAK.CMN'

        INTEGER COL,YPOS

        DO YPOS = FIRST,MAX_YPOS,INC
                CALL PUTPOINT(COL,YPOS)
        END DO

        RETURN
        END
```

```
        SUBROUTINE PLOTDIMOND(X,Y,NO_POINTS)
*.........................................
* Plot as diamonds the coordinates in vectors
*               X & Y
* Requires calling INITPLT or EZ_PLOTLINE or
* EZ_PLOTPOINTS   first

        INCLUDE 'PLTPAK.CMN'
        INTEGER NO_POINTS,XPOS,YPOS
        REAL X(NO_POINTS), Y(NO_POINTS)
        SAVE

        DO J = 1, NO_POINTS
*         Transform to PAGE coordinates.....
          XPOS = NINT(HZ_SCALE * X(J) + HZ_OFFSET)
          YPOS = NINT(VT_SCALE * Y(J) + VT_OFFSET)
*         Draw diamond centered on XPOS,YPOS......
          DO IX = -2,0
                DO IY = (YPOS-2-IX),(YPOS+2+IX)
                 CALL PUTPOINT(XPOS+IX,IY)
                 CALL PUTPOINT(XPOS-IX,IY)
                END DO
          END DO

        END DO

        RETURN
        END
```

```
        SUBROUTINE PLOTCROSS(X,Y,NO_POINTS)
*.....................................
* Plot as crosses the coordinates in vectors
*               X & Y

        INCLUDE 'PLTPAK.CMN'
        INTEGER NO_POINTS,XPOS,YPOS
        REAL X(NO_POINTS), Y(NO_POINTS)
        SAVE

        DO J = 1, NO_POINTS
*         Transform to PAGE coordinates.....
          XPOS = NINT(HZ_SCALE * X(J) + HZ_OFFSET)
          YPOS = NINT(VT_SCALE * Y(J) + VT_OFFSET)

*         Draw Crosses centerd on XPOS,YPOS.....
          DO IX = (XPOS-1),(XPOS+1) !Horizont. line
                CALL PUTPOINT(IX,YPOS)
          END DO

          DO IY = (YPOS-1),(YPOS+1) !Vertical line
                CALL PUTPOINT(XPOS,IY)
          END DO

        END DO

        RETURN
        END
```

```
;.....................................
;
;       PROCEDURE ADBITS
;.....................................
; Perform bitwise OR on two bytes


        .ENTRY  ADBITS,^M()

;       Find the bit position to set.......
;       Bit pos = Mod( Horizontal pos,6)
        DIVL3   #6,@8(AP),R0 ;Horizontal pos/6
        MULL3   #6,R0,R1        ;
        SUBL3   R1,@8(AP),R0    ;Bit pos in R0
        BBCS    R0,@4(AP),FINISH
;                   Set this bit in PAGE(J,K)

FINISH: RET
;.....................................
;       PROCEDURE BLANKPAGE
;.....................................
;       Blank out the entire array


BLANK:  .QUAD   ^X4040404040404040
                ; 1 2 3 4 5 6 7 8
        .ENTRY  BLANKPAGE,^M(IV,R2)

        MOVL    4(AP),R1
;           base address of array PAGE(1,1)
        MOVL    @8(AP),R2
;           Length of array in bytes

        SUBL2   #8,R2 ;Adjust loop limit
        CLRL    R0   ;Initialize loop counter


LOOP1:  MOVQ    BLANK,(R1)+
;             Blank out 8bytes pointed to
              ;by R1 then inc R1+8
        ACBL    R2,#8,R0,LOOP1
;             Step in incs. of 8
              ;Thru array
        RET
        .END
```

APPENDIX E

Transferring Data From the MCAs to the VAX Computer

Omega MCA

1. Collect data on the MCA
2. Log onto VAX
3. Type: SET TERM/PASSALL
4. Type: CREATE filename
5. Disconnect terminal and connect MCA to modem
6. Dump data
7. Reconnect terminal
8. Use EDT to modify file

Note: Every line of data will contain a line feed character which can be deleted as follows:

Type: SUB/line feed//**

Type: Cntr. O

Also, the first 22 lines of data will be blank.

Tracor Northern MCA

1. Collect Data on the MCA
2. Connect MCA to personal computer
3. Run program TRANS on personal computer

4. Run KERMIT package on personal computer

   a. Type: KERMIT

   b. Type: SET BAUD 9600

   c. Type: CONNECT

   d. Log on to VAX computer

   e. Type: R LIB_EXE:KERMIT

   f. Type: SERVER

   g. Type: Cntr. C

   h. Type: SEND filename

   i. Type: Bye

5. Data file can now be edited on the VAX computer using EDT.

LIST OF REFERENCES

Albrecht, Robert W. "The Measurement of Dynamic Nuclear Reactor Parameters Using the Variance of the Number of Neutrons Detected," Nuclear Science and Engineering, Vol. 14, 1962.

Albrecht, Robert W. "The Information in Neutron Fluctuations," Nuclear Science and Engineering, Vol. 41, 1970.

Albrecht, Robert W. "Comment on Subcritical Reactivity from Reactor Noise," Nuclear Science and Engineering, Vol. 44, 1971.

Babala, Dušan. Interval Distributions of Reactor Neutron Counts, NORA Project, Institutt for Atomenergi, Kjeller, Norway, 1966.

Babala, Dušan. "On the Theory of Rossi-Alpha Experiment in Reactor Noise Studies," Nuclear Science and Engineering, Vol. 26, 1966.

Bärs, B. "On the Variety and Hierarchy of Data Extraction Methods in Nuclear Reactor Noise Analysis," Nuclear Instruments and Methods, 98, 1972.

Behringer, K. and W. Peier. "Theoretical Comparison Between Direct and Polarity Correlation Techniques for a Projected Gauss-Markovian Process as Related to an On-Line Reactivity Meter," Nuclear Instruments and Methods, 161, 1979.

Bennett, Edgar F. "The Rice Formulation of Pile Noise," Nuclear Science and Engineering, Vol. 8, 1960.

Bennett, Edgar F. and Robert L. Long. "Precision Limitations in the Measurement of Small Reactivity Changes," Nuclear Science and Engineering, Vol. 17, 1963.

Bergström, A., B. Brunfelter, J. Kockum and S. Söderberg. "Pulsed-source, Rossi-α and Variance-to-mean Measurements Performed at the FRO Reactor," Proceedings of the International Conference on Fast Critical Experiments and Their Analysis, USAEC Report ANL-7320, 1967.

Bevington, Philip R. Data Reduction and Error Analysis for the Physical Sciences, McGraw-Hill Book Company, New York, 1969.

Brunson, Glenn S. "An Extension to the Feynman Method of Detecting Fission Neutrons," Nuclear Instruments and Methods, 214, 1983.

Cohn, Charles E. "Determination of Reactor Kinetic Parameters by Pile Noise Analysis," Nuclear Science and Engineering, Vol. 5, 1959.

Cohn, Charles E. "A Simplified Theory of Pile Noise," Nuclear Science and Engineering, Vol. 7, 1960.

Cohn, Charles E. "Reflected-Reactor Kinetics," Nuclear Science and Engineering, Vol. 13, 1962.

Cohn, Charles E. "Applicability of Simple Reactor Kinetics to the Interpretation of Reactor Noise Experiments," Journal of Nuclear Engineering Abstracts, Vol. 18, No. 31, 1964.

Cohn, Charles E. "Book Review: Random Noise Techniques in Nuclear Reactor System, by Robert E. Uhrig," Nuclear Science and Engineering, Vol. 45, 1971.

Courant, E. D. and P. R. Wallace. "Fluctuations of the Number of Neutrons in a Pile," Physical Review, Vol. 72, No. 11, 1947.

de Hoffmann, Frederic. "Statistical Aspects of Pile Theory," The Science and Engineering of Nuclear Power, Vol. 2, 1949.

Diven, B. C., H. C. Martin, R. F. Taschek and J. Terrell. "Multiplicities of Fission Neutrons," Physics Review, Vol. 101, 1956.

Dragt, J. B. "Threefold Correlations and Third-Order Moments in Reactor Noise," Nuklconik, Vol. 10, No. 7, 1966.

El-Zeftawty, Medhat M. and Lawrence Ruby. "Kinetic Distortion in a TRIGA Reactor with an Asymmetric Reflector," Nuclear Science and Engineering, Vol. 45, 1971.

El-Zeftawy, Medhat M. and Lawrence Ruby. "Addendum to Note on Subcritical Reactivity from Reactor Noise," Nuclear Science and Engineering, Vol. 47, 1972.

Feynman, R. P., F. de Hoffmann and R. Serber. "Dispersion of the Neutron Emission in U-235 Fission," Journal of Nuclear Energy, Vol. 3, 1956.

Furuhashi, Akira and Sigeru Izumi. "A Proposal on Data Treatment in the Feynman-α Experiment," Journal of Nuclear Science and Technology, Vol. 4, No. 2, 1967.

Furuhashi, Akira and Sigeru Izumi. "Third Moment of the Number of Neutrons Detected in Short Time Intervals," Journal of Nuclear Science and Technology, Vol. 5, No. 2, 1968.

Gotoh, Yorio. "Measurement of Neutron Life in a D_2O-System by Neutron Fluctuation," Journal of Nuclear Science and Technology, Vol. 1, No. 6, 1964.

Hage, W. "Correlation Analysis with Neutron Count Distributions in Randomly or Signal Triggered Time Intervals for Assay of Special Fissile Materials," Nuclear Science and Engineering, Vol. 89, 1985.

Harris, D. R., M. Natelson, J. A. Galey and E. Schmidt. "Measurements and Detailed Model Analysis of Space-Correlated Reactor Fluctuations," Nuclear Science and Engineering, Vol. 40, 1970.

Hetrick, David L. Dynamics of Nuclear Reactors, University of Chicago Press, Chicago, 1971.

Iijima, Tsutomu, Teruo Sugi, Hideo Ezure and Mineo Kasai. "Space-Dependent Effects in Rossi-Alpha Experiments," Nuclear Science and Engineering, Vol. 21, 1968.

Lewis, R. A., E. F. Bennett, S. G. Carpenter, C. E. Cohn and C. C. Price. "Survey of Recent Applications of Reactor Noise Techniques at Argonne National Laboratory," Annals of Nuclear Energy, Vol. 2, 1975.

Lindeman, Arthur J. and Lawrence Ruby. "Subcritical Reactivity from Neutron Statistics," Nuclear Science and Engineering, Vol. 29, 1967.

Natelson, M., R. K. Osborn and F. Shure. "Space and Energy Effects in Reactor Fluctuation Experiments," Journal of Nuclear Energy (Parts A/B), Vol. 20, 1966.

Orndoff, John D. "Prompt Neutron Periods of Metal Critical Assemblies," Nuclear Science and Engineering, Vol. 2, 1957.

Otsuka, Masuhiko and Keiichi Saito. "Neutron Fluctuations in a Multi-Point Reactor," Journal of Nuclear Science and Technology, Vol. 2, 1965.

Pacilio, N. "The Polya Model and the Distribution of Neutrons in a Steady-State Reactor," Nuclear Science and Engineering, Vol. 26, 1966.

Pacilio, N. "Short Time-Variance Method for Prompt-Neutron-Lifetime Measurements," Nuclear Science and Engineering, Vol. 22, 1965.

Pacilio, N. Determination of Reactor Kinetic Parameters by Statistical Analysis of Prompt Neutron Distribution, Italian Report RT/FI-(65)49, 1965.

Pacilio, N. "On the Composition of the Counting-Loss Correction Formula in Nuclear Particle Detection Experiments," Nuclear Instruments and Methods, 73, 1963.

Pacilio, N. Reactor Noise in the Time Domain, USAEC Critical Review Series, USAEC Report TID-24512, 1969.

Pacilio, Nicola and Vincenzo M. Jorio. "Discrete Probability Distributions in Nuclear Reactor Noise," Annals of Nuclear Energy, Vol. 2, 1975.

Pacilio, Nicola, Vincenzo M. Jorio, Francesco Norelli, Raffaele Mosiello, Andrea Colombino and Emilio Zingoni. "Toward A Unified Theory of Reactor Neutron Noise Analysis Techniques," Annals of Nuclear Energy, Vol. 3, 1976.

Pál, L. "Determination of the Prompt Neutron Period from the Fluctuations of the Number of Neutrons in a Reactor," Reactor Science and Technology, Vol. 17, 1963.

Pál, L., L. Turi and J. Vigassy. "Experimental Determination of the Ratio β^{eff}/ℓ on the ZR-1 Critical Assembly," Reactor Science and Technology, Vol. 17, 1963.

Ruby, Lawrence. "Statistics of the Detection Process in Radioactive Decay," American Journal of Physics, Vol. 49, 1981.

Saito, Keiichi and Masuhiko Otsuka. "Theory of Statistical Fluctuations in Neutron Distributions," Journal of Nuclear Science and Technology, Vol. 2, No. 8, 1965.

Saito, Keiichi and Yukichi Taji. "Theory of Branching Processes of Neutrons in a Multiplying Medium," Nuclear Science and Engineering, Vol. 30, 1967.

Saito, Keiichi. "Some Basic Relations in the Waiting Time Analysis of Neutron Counting Statistics," Journal of Nuclear Science and Technology, Vol. 6, No. 10, 1969.

Sheff, James R. and Robert W. Albrecht. "The Space Dependence of Reactor Noise I-Theory," Nuclear Science and Engineering, Vol. 24, 1966.

Sheff, James R. and Robert W. Albrecht. "The Space Dependence of Reactor Noise-II Calculations," Nuclear Science and Engineering, Vol. 26, 1966.

Szeless, Andreas and Lawrence Ruby. "Subcritical Reactivity from Reactor Noise," Nuclear Science and Engineering, Vol. 44, 1971.

Szeless, Andreas and Lawrence Ruby. "The Exact Probability Distribution of Reactor Neutron Noise," Nuclear Science and Engineering, Vol. 45, 1971.

Theodosić, V. "Dead-Time Correction for any Distribution of Input Events," Nuclear Instruments and Methods, 92, 1971.

Thie, Joseph A. Reactor Noise, Rowman and Littlefield, Inc., New York, 1963.

Uhrig, Robert E. Random Noise Techniques in Nuclear Reactor Systems, The Ronald Press Company, New York, 1970.

Wang, Hsiao-Kang and Lawrence Ruby. "Subcritical Reactivity as Determined from Reactor Noise," Nuclear Science and Engineering, Vol. 56, 1975.

Wang, Hsiao-Kang and Lawrence Ruby. "Determination of Reactivity from Noise Data," Transactions of the American Nuclear Society, Vol. 17, 1973.

Wang, Hsiao-Kang and Lawrence Ruby. "Some Additions to Reactor Noise Theory Which Include Delayed Neutrons," Nuclear Science and Engineering, Vol. 57, 1975.

Young, Hugh D. Statistical Treatment of Experimental Data, McGraw-Hill Book Company, New York, 1962.

Zolnay, A. S., C. S. Barnett and H. P. Spracklen. "Passive Neutron Multiplication Measurements," IEEE Transactions on Nuclear Science, Vol. NS-30, No. 1, 1983.