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Application of Powell's conjugate direction method to slope stability analysis

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APPLICATION OF POWELL'S CONJUGATE DIRECTION METHOD TO SLOPE STABILITY ANALYSIS

by

Nassim Riyad Abifadel

A Thesis Submitted to the Faculty of the

DEPARTMENT OF CIVIL ENGINEERING AND ENGINEERING MECHANICS

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To my Father.

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ABSTRACT

Slope stability problems often arise in construction engineering projects. They are major problems in dams construction and mines excavation. For the purpose of improving the efficiency of slope stability analysis, the optimization method suggested by Powell (1964) is used to locate the critical failure surface. The reader should bear in mind the possibility of applying optimization to a wide variety of different civil engineering problems.

1. INTRODUCTION

The purpose of analyzing a slope stability problem is to identify the critical failure surface and estimate the associated minimum safety factor. Limit equilibrium procedures are commonly used for slope stability analysis, and many of these methods are available as computer programs. Most of these existing computer programs rely on simple searching techniques to direct the search for the minimum safety factor.

It is very likely that the search methods currently used in conjunction with computer-based stability analyses are not the most efficient. This thesis therefore is concerned with the application of Powell's (1964) conjugate direction method for unconstrained optimization to the problem of slope stability analysis. The conjugate direction procedure is discussed in detail, and the algorithm integrated into an existing slope stability computer program. The results obtained with the new search technique are compared with those of existing search techniques to demonstrate the accuracy, efficiency, and reliability of the new procedure.

2. LITERATURE SURVEY

2.1 Optimization Techniques

The choice of an "ideal" optimization strategy depends on the type of problem and the type of data available. Optimization algorithms are iterative and fall into one of the following categories, according to the type and amount of information they require:

1) Direct search methods.
2) Conjugate direction and conjugate gradient methods.
3) Restricted-step methods.
4) Discrete Newton methods.
5) Quasi-Newton methods.
6) Second-derivative methods.

The various classes of methods have been arranged in order of increasing efficiency. Table 1 (DeNatale, 1986) provides a summary of the different categories listed above. The relative applicability of these various classes of methods to the problem of slope stability analysis may now be considered.

2.2 Application of Optimization Techniques to Slope Stability Analysis

Several algorithms belonging to the class of direct search methods have been previously applied to slope stability analysis.

Table 1. A Comparison of the Various Classes of Optimization Techniques (Denatale 1986)

Class	Requirements	Advantages	Disadvantages
Second Derivative Methods	*function *gradient *curvature	*superlinear convergence *self-corrective *possible to distinguish between local minima and saddle points	*may not converge from poor initial guess *requires second derivative *requires solution of n-linear equations at each iteration
Discrete Newton Methods	*function *gradient	*same as for Second Derivative Methods	*inefficient for large-dimension problems *optimal differencing intervals must be determined
Quasi-Newton Methods	*function *gradient	*requires first derivatives only *no equation solving is required	*round-off errors can have large effect on performance
Restricted Step Methods	*function	*excellent convergence	*requires many arithmetic operations
Conjugate Direction and Conjugate Gradient Methods	*function *function *gradient	*requires little core memory *few arithmetic operations per iteration *excellent for large problems	*less efficient and robust than Newton-type methods
Direct Search Methods	*function	*extremely general and simple to code *immune from rounding errors and ill-conditioning *requires little core memory	*rather slow convergence *function evaluations increase exponentially with the dimension of the problem *a large number of user specified constants is required

Among these algorithms are the grid search technique, the pattern search technique, and the Simplex method (which is the most efficient of the direct search procedures). These procedures are advantageous for they:

1. Require only the evaluation of the merit function.
2. Involve simple numerical computations that eliminate rounding errors and ill-conditioning.
3. Require only modest amounts of computer storage.

Table 2 (DeNatale, 1986) shows that all available computer programs rely on direct search methods to identify the minimum safety factor and corresponding critical slip surface.

The conjugate direction method introduced by Powell (1964) also requires no gradient information. It is considered because it is applicable to a class of merit functions which have a shape similar to the shape of the safety factor function in slope stability analysis. The optimization methods listed in Table 1 that require the gradient of the function to be defined are not applicable in slope stability analysis because the factor of safety function is not differentiable. In the following section, Powell's conjugate direction algorithm is presented with the necessary information pertaining to its structure.

2.3 Powell's Algorithm

A general quadratic function $f(\hat{x})$ can be written as

$$f(\hat{x}) = \hat{x}^T \hat{A} \hat{x} + \hat{b}^T \hat{x} + c$$

Table 2. A Comparison of Available Stability Programs (Denatale 1986)

Program Name	References	Safety Factor Formulation	Search Technique
	Little & Price (1958)	Bishop's Modified Method	None
	Horn (1960)	Swedish Circle Method	Pattern Search
ICES-LEASE-1	Bailey & Christian (1969) Newman (1985)	Bishop's Modified Method	Grid Search
STABR	Lefebvre (1971)	Bishop's Modified Method and Ordinary Method of Slices	Pattern Search
MALE	Schiffman (1972) Schiffman & Jubenville (1975)	Morgenstern's Method	Grid Search
SSTAB1	Wright (1974)	Spencer's Method	Grid Search
SSTAB2	Chugh (1981)	Spencer's Method	Grid Search
SLOPE	Fredlund (1974) Fredlund & Krahn (1977)	All State-of-the-Art Methods	Grid Search
SLOPE-II	Fredlund & Nelson (1985) Geo-Slope (1985)	All State-of-the-Art Methods	Grid Search
PC-SLOPE	Fredlund & Nelson (1985) Geo-Slope (1985)	All State-of-the-Art Methods	Grid Search
STABL	Siegel (1975a) Siegel (1975b) Siegel et al. (1979)	Bishop's Modified Method and Janbu's Simplified Method	Randomly Generated Grid Search
STABL2	Boutrup (1977) Boutrup et al. (1979)	Bishop's Modified Method and Janbu's Simplified Method	Randomly Generated Grid Search
STABL3	Chen (1981)	Bishop's Modified Method and Janbu's Simplified Method	Randomly Generated Grid Search

Table 2.—Continued

Program Name	References	Safety Factor Formulation	Search Technique
STABL4	Lovell et al. (1984)	Bishop's Modified Method and Janbu's Simplified Method	Randomly Generated Grid Search
PCSTABL4	Carpenter (1985)	Bishop's Modified Method and Janbu's Simplified Method	Randomly Generated Grid Search
SSDP	Baker (1980)	Spencer's Method	Dynamic Programming
	Celestino & Duncan (1981)	Spencer's Method	Alternating Variable
REAME	Huang (1981) Huang (1983)	Bishop's Modified Method	Grid and Pattern Search
SWASE	Huang (1983)	Sliding Block	None
	Cross (1982)	Bishop's Modified Block	None
	Nguyen (1985)	Bishop's Modified Method & Morgenstern-rice Method	Simplex Method of Spendley et al.
SB-SLOPE	Von Gunten (1985)	Bishop's Modified Method	Grid Search
CSLIP2	Awad (1986)	Bishop's Modified Method	Simplex Method of Nelder and Mead

where $\hat{x}$ is a n-dimensional vector, $\hat{A}$ is a constant symmetric matrix, $\hat{b}$ is a constant vector and c is a scalar.

If $\hat{p}$ and $\hat{q}$ are two n-dimensional vectors that satisfy the condition

$$\hat{p}\hat{A}\hat{q} = 0$$

then $\hat{p}$ and $\hat{q}$ define two conjugate directions.

If $\hat{d}_1, \hat{d}_2, \hat{d}_3, \ldots, \hat{d}_n$ are n mutually conjugate directions in an n-dimensional space, then the minimum of the quadratic function $f(\hat{x})$ may be obtained by starting at a point $\hat{x}_1$ and performing a line search along each of the directions once only (Powell, 1964). The minimum is then located at the point:

$$\hat{x}_1 + \sum_{i=1}^{n} \alpha_i \hat{d}_i$$

where α_i are scalar step-lengths which minimize the function

$$f(\hat{x}_1 + \sum_{i=1}^{n} \alpha_i \hat{d}_i) = \sum_{i=1}^{n} [\alpha_i^2 \, \hat{d}_i^T \hat{A} \hat{d}_i + \alpha_i \hat{d}_i^T (2\hat{A}\hat{x}_1 + \hat{b})] + f(\hat{x}_1)] \quad (1)$$

Since no mixed terms in $\alpha_i \cdot \alpha_j$ $(i \neq j)$ exist, a line search in the direction $\hat{d}_i$ will obtain α_i. Hence it is possible to locate the minimum in exactly n line searches.

Contours of a two-dimensional quadratic function are shown in Figure 1. Here, $\hat{x}_1$ defines the initial user-specified starting point. Directions $\hat{d}_1$ and $\hat{d}_2$ represent the selected search directions. Initially, a line search is conducted along direction $\hat{d}_1$ in order to locate the function's minimum with respect to this direction. In Figure 1, this point is defined as $\hat{x}_2$. Then a second

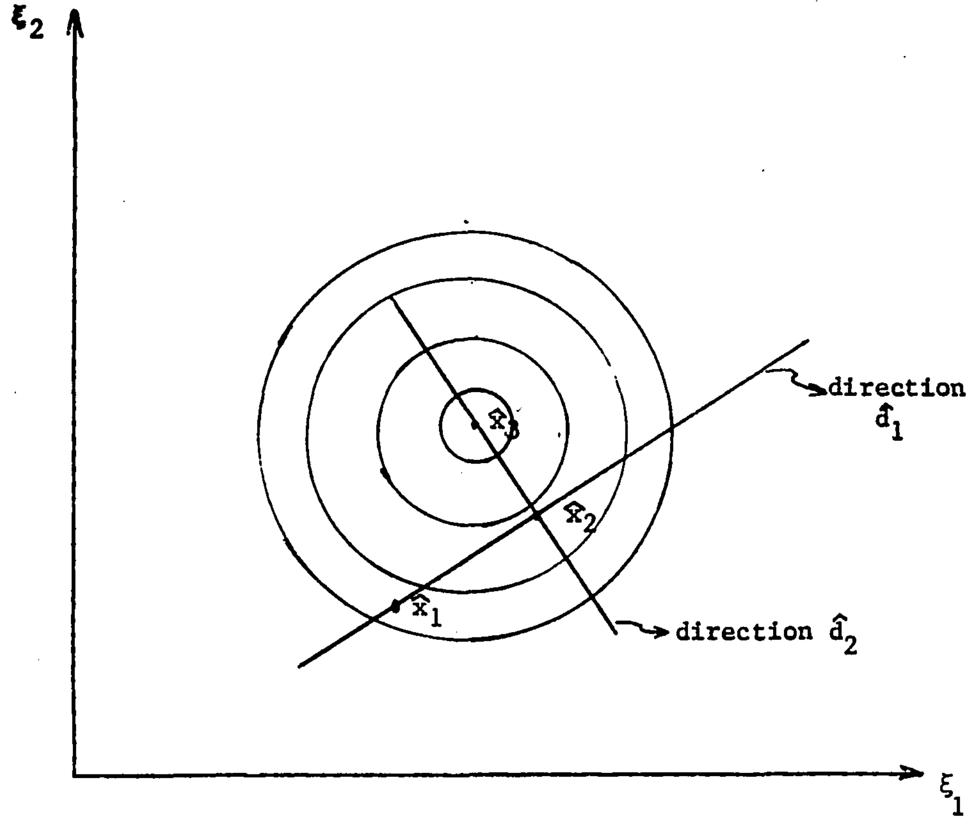


Figure 1. The Minimum of a Two-Dimensional Quadratic Function Found by Performing Two Line Searches in Directions $\hat{d}_1$ and $\hat{d}_2$

line search is conducted along direction $\hat{d}_2$, beginning at point $\hat{x}_2$. The minimum in this direction is defined as $\hat{x}_3$ and this point is also the merit function's global minimum. Note that directions $\hat{d}_1$ and $\hat{d}_2$ need not correspond to the coordinate directions ξ_1 and ξ_2. Note also that the two-dimensional quadratic function could thus be minimized exactly in two line searches.

For functions that are not exactly quadratic, Powell's algorithm requires more than one cycle of n line searches to converge to the true minimum. One cycle of the basic algorithm is as follows (Powell, 1964):

1. Begin at some point $\hat{x}_1$.
2. For i = 1, . . . , n move $\hat{x}_i$ to $\hat{x}_{i+1}$, the minimum along direction $\hat{d}_i$.
3. Reset the search directions: set $\hat{d}_i \leftarrow \hat{d}_{i+1}$ and $\hat{d}_n \leftarrow \hat{x}_{n+1} - \hat{x}_1$
4. Move $\hat{x}_{n+1}$ to the minimum along $(\hat{x}_{n+1} - \hat{x}_1)$, and call the new starting point $\hat{x}_1$.

This strategy was modified by Powell (1964) to improve its rate of convergence. The improved conjugate direction algorithm is as follows (Powell, 1964).

1. Begin at some point $\hat{x}_1$.
2. For i = 1, . . . , n use a line search to find the minimum $\hat{x}_{i+1}$ in each direction $\hat{d}_i$.
3. Find m such that $f(\hat{x}_{m-1}) - f(\hat{x}_m)$, is a maximum.

4. Let $r = (\hat{x}_{m-1} - \hat{x}_m)$, $2 \leq m \leq (n + 1)$

Let $f_0 = f(\hat{x}_1)$

$f_1 = f(\hat{x}_{n+1})$

$f_2 = f(2\hat{x}_{n+1} - \hat{x}_1)$

5a. If $f_2 \geq f_0$

or

$(f_0 - 2f_1 + f_2) \cdot (f_0 - f_1 - r^2) \geq 1/2 \cdot r \cdot (f_0 - f_2)^2$

Let $\hat{x}_1 \leftarrow \hat{x}_{n+1}$ for next iteration and use the same directions.

5b. Otherwise, move $\hat{x}_{n+1}$ to the minimum along the direction $(\hat{x}_{n+1} - \hat{x}_1)$ and let $\hat{x}_1 \leftarrow$ the new point obtained. Then, change directions by eliminating $\hat{d}_{m-1}$ and adding $(\hat{x}_{n+1} - \hat{x}_1)$. The new directions are.

$\hat{d}_i \leftarrow \hat{d}_i \qquad i = 1, m - 2$

$\hat{d}_i \leftarrow \hat{d}_{i+1} \qquad i = m, n$

$\hat{d}_n = (\hat{x}_{n+1} - \hat{x}_1)$

A FORTRAN coding of Powell's algorithm is provided in Appendix A.

2.4 The Line Search

The line search implemented in Powell's algorithm is based on work presented by Fletcher (1980) and DeNatale (1983). The purpose of the line search is to identify the scalar step length α_i which locate the minimum of the function in the specified search direction. The line search algorithm is therefore a type of one-dimensional minimization algorithm. It is based on (1) bracketing α in an interval (α_1, α_2) of acceptable values, and (2) sectioning to decrease the size of the current search interval. The accuracy of the line search is defined by a user specified constant σ. A value of σ = 0 will cause the line search algorithm to find the precise location of the minimum in the current search direction. The search becomes less and less exact as σ is increased from $\sigma = 0$ to $\sigma = 1$. Complete discussions of this line search strategy are provided by Fletcher (1980) and DeNatale (1983). It should be mentioned that other line searches have also been used in conjunction with the conjugate direction method. Two of these alternate line search strategies are discussed by Powell (1964) and Brent (1973).

3. MATERIALS AND METHODS

3.1 Structure of CSLIP2

Powell's conjugate direction algorithm is incorporated into an existing slope stability program STABR (Lefebvre, 1971) to perform the search for the critical failure surface. The resulting program is named CSLIP2. Program CSLIP2 begins with two calls to data-reading subroutines DATAIN and SSDATA. Subroutine DATAIN reads all information required by Powell's minimization procedure. The input to DATAIN is arranged as follows:

1. First line variables

   NDIM = Number of independent directions (2 or 3)

   2 for a base or toe circle

   3 for a general circular failure surface

   IMAX = maximum number of iterations permitted

   FERR = maximum permissible error in computing F

   XERR = maximum permissible error in the X coordinate location

   DX = differencing interval (0.05 is recommended)

   σ = line search accuracy parameter (10^{-3} to 10^{-1} is recommended).

2. Second line variables

   if NDIM = 2, input the (x, y) coordinates of the center of the starting slip circle

NDIM = 3, input the (x, y) coordinates and the radius of the starting slip circle

Subroutine SSDATA reads all information required by the STABR program. The input is arranged as described in the STABR User's Manual (Lefebvre, 1971). Therefore, a CSLIP2 data file looks identical to a STABR data file except for the presence of the two additional lines described above. After calling DATAIN and SSDATA, Program CSLIP2 calls subroutine POWELL to perform the search according to the flow chart presented earlier in Figure 2. A complete listing of Program CSLIP2 is provided for reference in Appendix A.

3.2 Description of Example Problems

CSLIP2 was used to analyze several slope stability cases that have been previously examined by other researchers. Among these problems are:

1. Steep slope in a homogeneous soil.
2. Mild slope in a homogeneous soil.
3. Mild slope in a stratified soil deposit.
4. Soil with a shear strength that increases with depth.
5. Stepped slope in homogeneous soil.
6. Several case histories reported in geotechnical literature.

The cases mentioned above were studied earlier by Gillett (1987) using the STABR program which relies on a pattern search method. In order to compare the results from STABR and CSLIP2, the same starting points used by Gillett (1987) are used in the present work. The choice of the starting point qualifies the initial guess as poor, fair, good or expert, depending on its distance from the true global minimum (Gillett, 1987). The purpose of the comparison is to establish the validity of the conjugate direction method as both efficient and reliable in slope stability analysis.

4. PRESENTATION OF RESULTS

4.1 Problems

Problem 1 involves the steep slope in homogeneous soil illustrated in Figure 2. The critical failure surface is a circle passing through the toe, with its center at (x, y) = (110, 91). The effect of the line search parameter σ on the overall accuracy and efficiency of the analysis was considered by starting the search at a given point (139.00, 62.00) and varying the value of σ while keeping all other parameters constant. Table 3 summarizes the results obtained.

As σ is increased from 0 to 1, the line search becomes less exact. A less exact line search is generally more efficient when quasi-Newton methods are being used (Fletcher, 1980). However, Powell (1964) recommends that a relatively precise line search be used with the conjugate direction algorithm. As the results of Table 3 indicate, the overall efficiency of the search increases as σ decreases. However, when a relatively large value of σ is used (σ = 0.4), the search terminates prematurely, before reaching the minimum. Function values of 1.0928 and 1.0936 may be regarded as minima because a rather loose termination criterion was used in this study. A value of σ = 0.10 (which corresponds to a fairly precise line search) appears to result in an accurate and efficient search. This

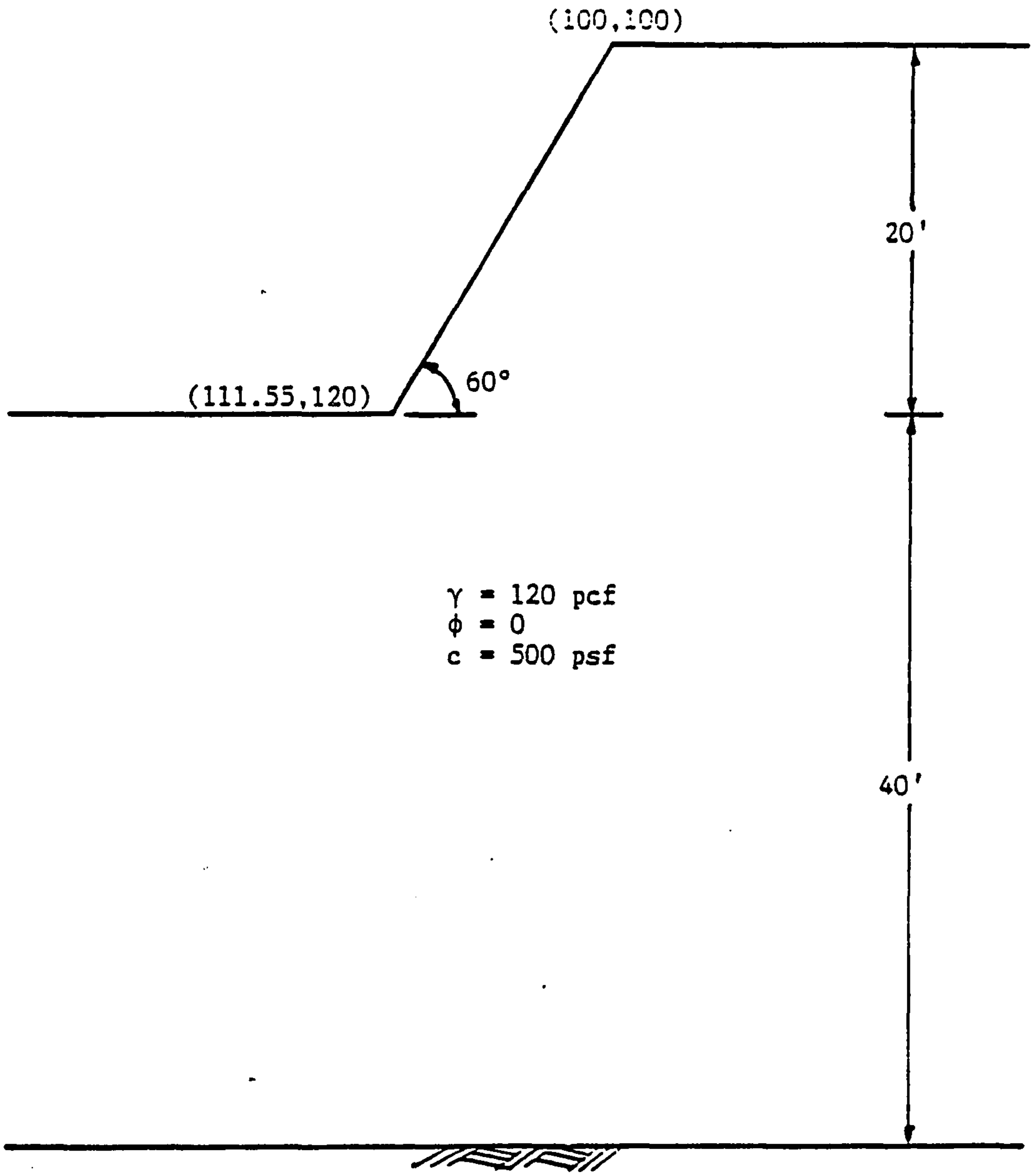


Figure 2. Problem 1--Steep Slope in Homogeneous Soil

Table 3. Effect of Varying the Line Search Accuracy Parameter σ

Line Search Accuracy Parameter σ	Minimum Safety Factor F	Number of Function Evaluations	(x,y) Location of Critical Toe Circle	
0.4	1.1627	20	124.70	59.84
0.3	1.0928	42	111.37	90.52
0.2	1.0928	42	111.37	90.52
0.1	1.0928	44	111.13	90.89
0.01	1.0936	55	112.19	89.19
0.001	1.0936	55	112.19	89.19
0	1.0936	55	112.19	89.19

value will therefore be used throughout the remainder of this research project.

It is also of interest to investigate the influence of the termination criterion FERR. The search for the minimum safety factor stops when the safety factors differ by less than FERR between two subsequent iterations. The results of this investigation are shown in Table 4. Since there will always be some uncertainty regarding the soil's shear strength parameters c and ϕ it would be impractical to define the safety factor to more than three significant digits. Therefore a small value of FERR (such as FERR = 0.0001) is unnecessarily precise. In addition, a greater number of function evaluations (or safety factor evaluations) is required as FERR is decreased. However, if FERR is made too large (say FERR = 0.1), the search terminates before the true minimum is reached, as shown in Table 4. A value of FERR = 0.01 results in an accurate yet efficient search. This value will therefore be used throughout the remainder of this project.

Using values of $\sigma = 0.1$ and FERR = 0.01, CSLIP2 was used to locate the critical slip circle for Problem 1 (Figure 2). The searches were begun at points far from the critical center, and in subsequent runs the starting point was moved closer to the true minimum. The results of these analyses are summarized in Table 5. As the starting point moves closer to the critical center, the number of function evaluations decreases. It is also observed that for a given starting guess, the number of function evaluations is larger if the

Table 4. Effect of Varying the Search Termination Criterion FERR

FERR Maximum Function Error	Minimum Safety Factor F	Number of Function Evaluations
0.1	1.1226	26
0.01	1.0928	44
0.009	1.0928	44
0.008	1.0928	44
0.007	1.0928	44
0.006	1.0928	44
0.005	1.0928	44
0.004	1.0928	44
0.003	1.0930	50
0.002	1.0930	50
0.0001	1.0928	109

Table 5. Effect of Varying Starting Point in Problem 1

Starting Guess	Starting Point (x,y)	F_{min}	Number of F Evaluations
Poor	189, 62	1.09	44
Poor	81, 62	1.09	46
Fair	129, 72	1.09	40
Fair	91, 72	1.09	43
Good	119, 82	1.09	23
Good	101, 82	1.09	24
Expert	114, 87	1.09	12
Expert	96, 87	1.09	22

starting point is to the right of the critical center (Figure 3). As an example, the number of function evaluations for the starting point (81, 62) is larger than for (189, 62). However, the difference is small for all starting guesses except for the expert starting guess.

The results of the conjugate direction based analyses with Program CSLIP2 are compared with STABR pattern search analyses performed by Gillett (1987) and CSLIP1 simplex-based analyses performed by Awad (1986) in Table 6. The conjugate direction algorithm used in CSLIP2 is far more efficient than the pattern search method used in STABR, since the number of safety function evaluations is considerably reduced. The conjugate direction analyses are somewhat more efficient than the simplex analyses for expert starting guesses, but somewhat less efficient for good, fair and poor starts. For the present problem, Program CSLIP2 is about 50% more efficient than STABR. However, the efficiency of CSLIP2 does not depend only on the starting guess but also on the choice of the orientation of the coordinate directions since these define the initial conjugate directions (which Powell's algorithm depends on). Therefore, if the initial directions used in CSLIP2 were more nearly conjugate, the efficiency of the search would increase, and CSLIP2 would be superior to CSLIP1 for all starting guesses.

Problem 2 involves the mild slope in homogeneous soil illustrated in Figure 4. The critical failure surface is a circle tangent to the firm base layer and centered at (126, 75). Table 7 summarizes the effect of varying the starting point. As observed in

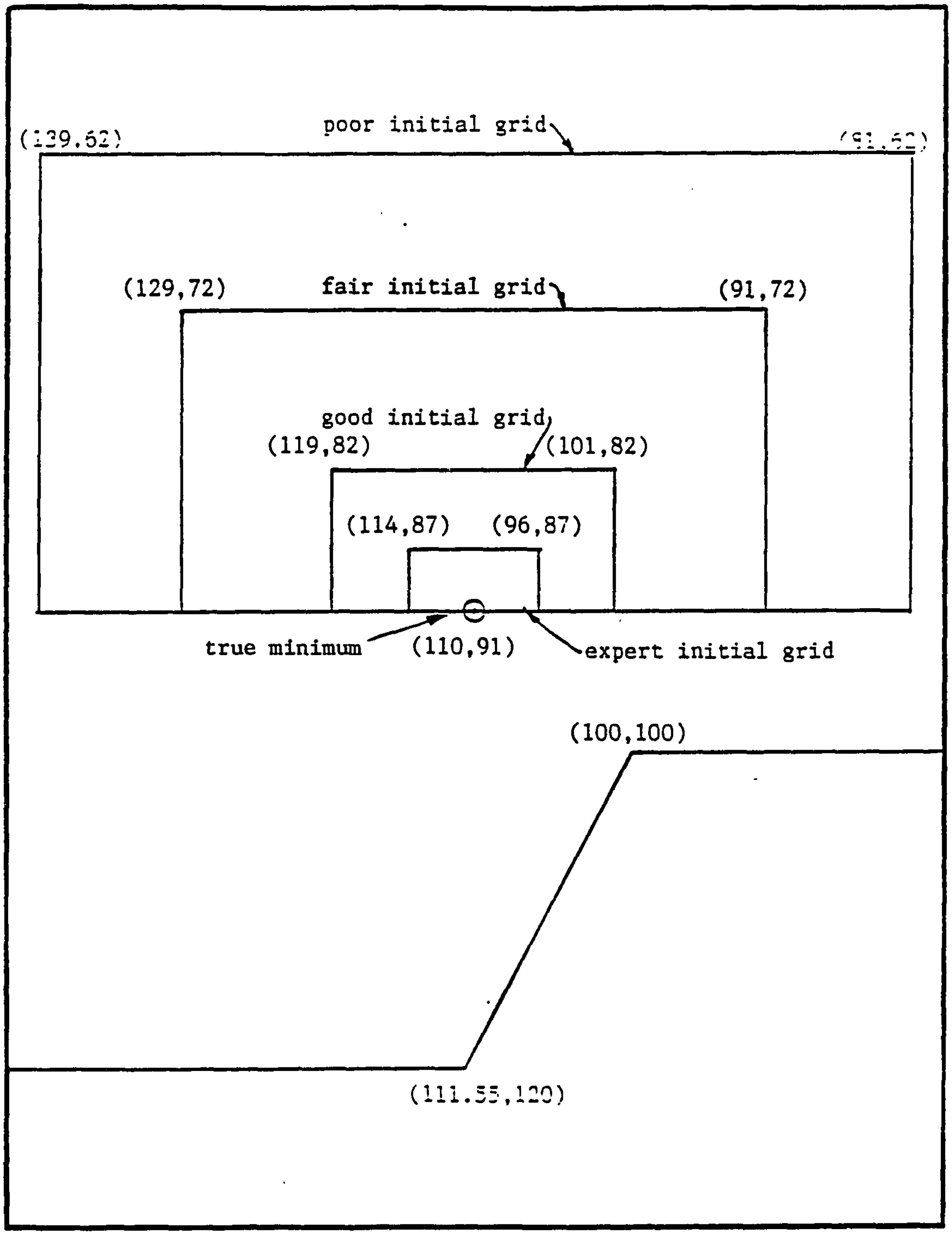


Figure 3. Illustration of Starting Guess as Applied to Problem 1

Table 6. Relative Efficiency of CSLIP2 and STABR

Initial Guess	Average Number of Function Evaluations CSLIP2	CSLIP1	STABR
Poor	45	39	76
Fair	42	34	52
Good	24	25	39
Expert	17	20	33

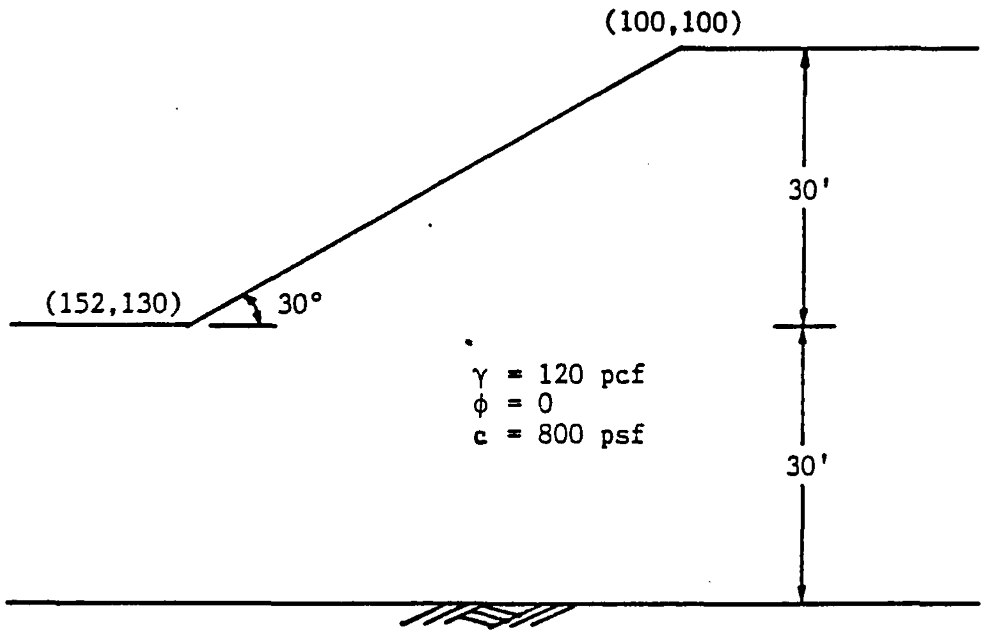


Figure 4. Problem 2--Mild Slope in Homogeneous Soil (After Gillett, 1987)

Table 7. Effect of Varying Starting Point in Problem 2

Starting Guess	Starting Point (x,y)	F_{min}	Number of F Evaluations
Poor	210.5, -9.0	1.29	63
Poor	41.5, -9.0	1.29	42
Fair	181.0, 20.5	1.29	35
Fair	71.0, 20.5	1.29	31
Good	151.0, 50.5	1.29	22
Good	101.0, 50.5	1.29	23
Expert	138.5, 63	1.29	18
Expert	113.5, 63	1.29	18

Problem 1, the number of function evaluations decreases when the starting point gets closer to the critical center. However, the trend of variation is different, especially in that the location of the expert starting guess to the left or right of the critical center does not effect the number of function evaluations. But as the starting guess changes from expert to poor, the difference in number of function evaluations between left and right starting points increases. This is due to the shape of the safety factor function which is very flat near the critical center and becomes steeper further away from the minimum.

Table 8 compares results obtained from CSLIP2 (conjugate direction), CSLIP1 (simplex) and STABR (pattern). The conjugate direction method is once again clearly superior to the pattern search method. Once again, the conjugate direction based search is more efficient than the simplex-based search for some starting points (fair and good) but not for others (poor and expert). No specific trend in percentage decrease is observed as the initial guess changes from poor to expert except that it is largest for the expert initial guess.

Problem 3 involves the stratified deposit illustrated in Figure 5. Theoretically, the critical surface is tangent to one of the soil interfaces. The minimum factor of safety for circles tangent to the interface of the top and second layers is 1.27. The minimum factor of safety for circles tangent to the interface of the second and third layers is 2.22. The minimum factor of safety for circles tangent to the base is 1.41. These values are obtained by running

Table 8. Comparison of Line and Pattern Searches in Problem 2

Initial Guess	Average Number of Function Evaluations CSLIP2	CSLIP1	STABR
Poor	52	38	159
Fair	33	43	94
Good	22	30	60
Expert	18	16	43

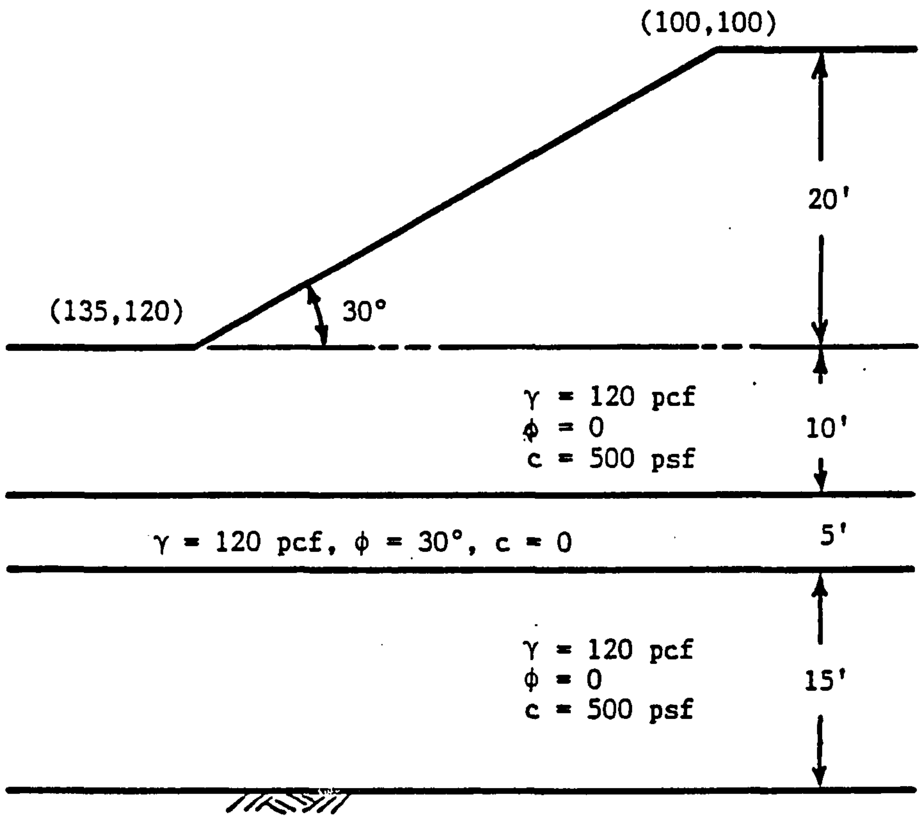


Figure 5. Problem 3--Stratified Soil Deposit (after Gillett, 1987)

CSLIP2 for the corresponding tangent elevations. Table 9 summarizes the results of varying the starting point for F = 1.27. The number of function iterations decreases as the starting point gets closer to the critical center. Similarly to Problem 2, the safety factor function being very flat in the region close to the minimum, the number of function evaluations is about the same for good and expert guesses.

Table 10 compares results from STABR and CSLIP2 to prove that searching efficiency is improved. However, for the present problem the percentage decrease in the number of function evaluations is smaller than for Problems 1 and 2. No analyses were done by Awad (1986) with CSLIP1.

Problem 4 involves the soil with shear strength that increases with depth as shown in Figure 6. After performing the search for the critical circle at different tangent depths, the minimum safety factor obtained is F = 0.781 for a circle tangent to a depth of 4 ft below the toe level and centered at (118, 85).

Table 11 summarizes the results obtained by varying the starting point. The number of function evaluations decreases as the starting guess changes from poor to expert. The safety factor function is also flat as in Problems 2 and 3 resulting in a very slight difference in the number of function evaluations for a certain initial guess. Table 12 compares results obtained from CSLIP2, CSLIP1 (Awad, 1986), and STABR (Gillett, 1987). For all initial guesses, a percentage decrease in the number of function evaluations is brought up by CSLIP2. In addition, CSLIP2 is superior to CSLIP1 in all cases.

Table 9. Effect of Varying Starting Point in Problem 3

Initial Guess	Initial Point (x,y)	F	Number of F Evaluations
Poor	161.0, 43.0	1.27	67
Poor	74.0, 43.0	1.27	45
Fair	145.5, 58.5	1.27	38
Fair	89.5, 58.5	1.27	42
Good	130.5, 73.5	1.27	22
Good	104.5, 73.5	1.27	22
Expert	124 , 80	1.27	19
Expert	111 , 80	1.27	22

Table 10. Comparison of Line and Pattern Searches in Problem 3

Initial Guess	Average Number of Function Evaluations CSLIP2	STABR
Poor	56	74
Fair	40	59
Good	22	44
Expert	20	33

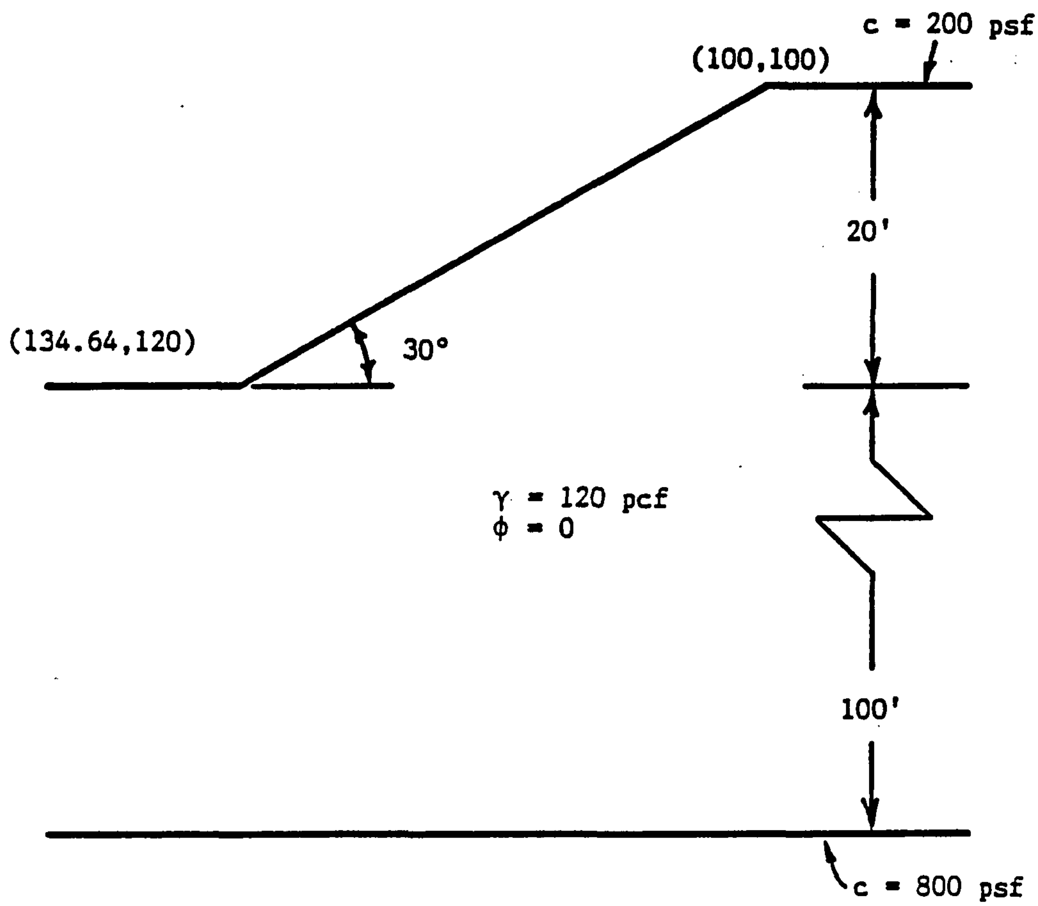


Figure 6. Problem 4--Soil with Shear Strength that Increases with Depth (after Gillett, 1987)

Table 11. Effect of Varying Starting Point in Problem 4

Starting Guess	Starting Point (x,y)	Safety Factor	Number of Function Evaluations
Poor	150.5, 49	0.78	55
Poor	79.5, 49	0.78	56
Fair	142 , 62.5	0.78	32
Fair	93 , 62.5	0.78	29
Good	124 , 75.5	0.78	24
Good	106 , 75.5	0.78	25
Expert	123 , 81.5	0.78	20
Expert	112 , 81.5	0.78	23

Table 12. Comparison of Line and Pattern Searches in Problem 4

Initial Guess	Average Number of Function Evaluations CSLIP2	CSLIP1	STABR
Poor	55	67	78
Fair	30	65	54
Good	25	59	43
Expert	22	36	33

However, no specific trend in the percentage decrease is observed except that it is smallest for the poor initial guess.

Problem 5 involves the stepped slope in homogeneous soil illustrated in Figure 7. If the firm stratum is at the level of the toe the minimum safety factor is F = 1.24 for a circle tangent to the base with coordinates (134, 47) and a radius of 84. The factor of safety decreases, as the firm stratum goes deeper.

4.2 Case Histories

Krahn and Fredlund (1977) proposed some case histories to verify slope stability computer programs. These case histories are used to check the validity of CSLIP2. They are illustrated in Figures 8 to 16, as presented by Gillett (1987) with the critical failure surface determined using program STABR. Table 13 summarizes a review of these case histories using program CSLIP2. All the factors of safety listed in this table compare with the corresponding factors of safety in Figures 8 to 16, and bring additional proof to the reliability of the conjugate direction method in slope stability analysis.

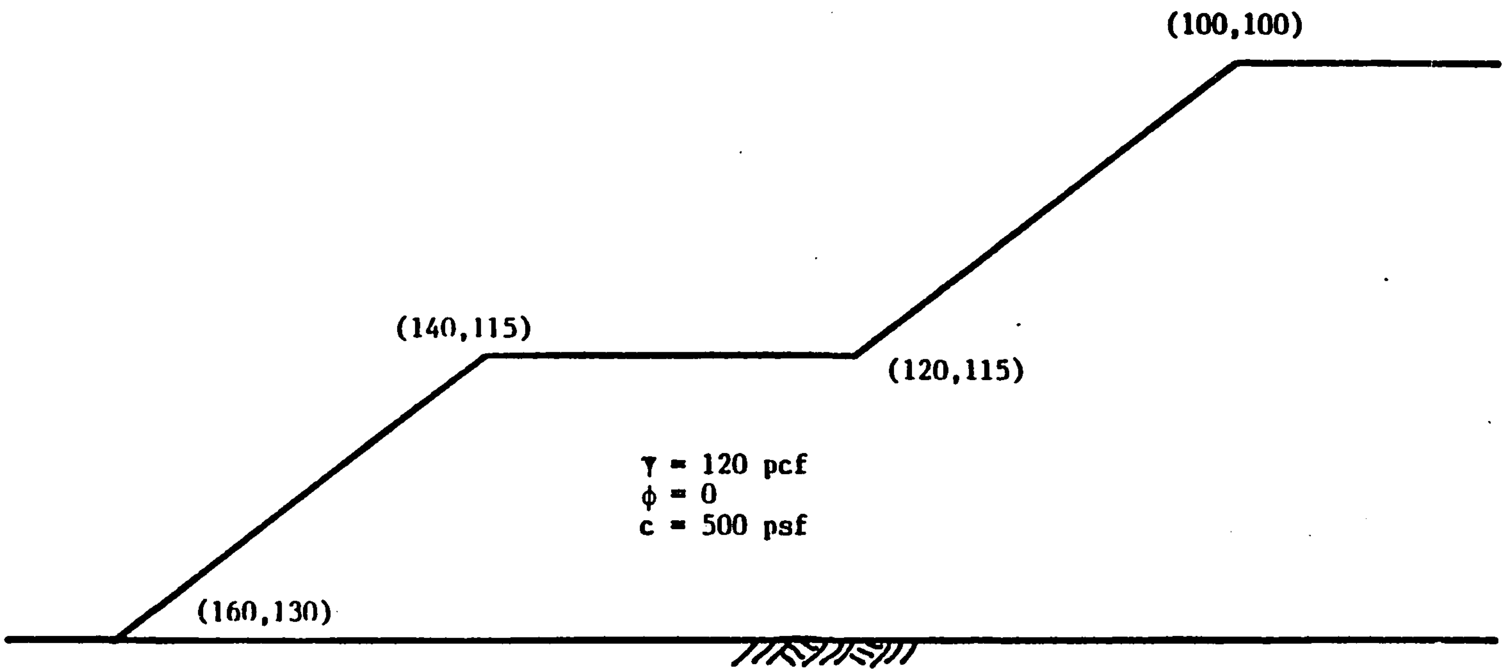


Figure 7. Problem 5--Benched Slope Face
(after Gillett, 1987)

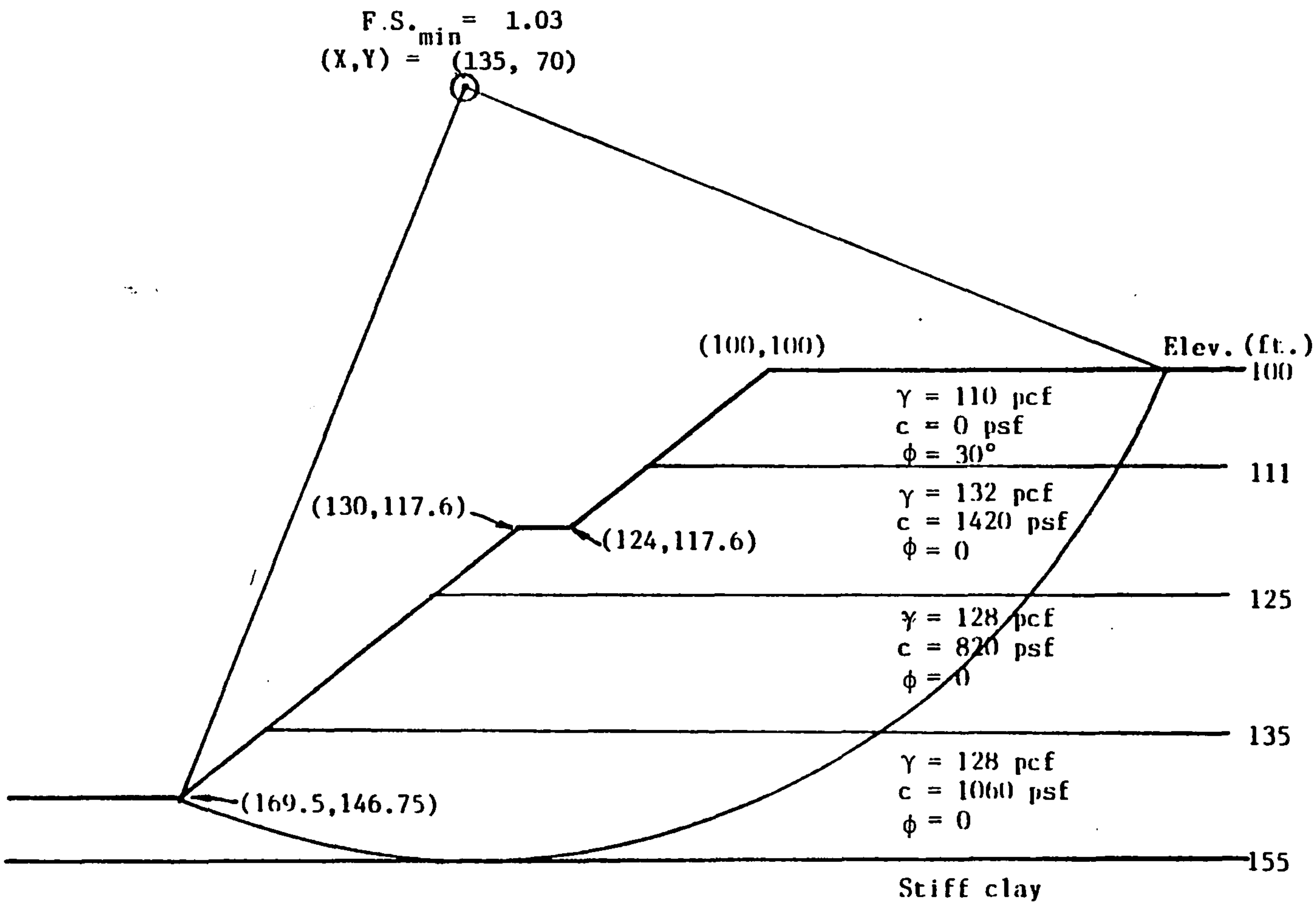


Figure 8. Congress Street Open Cut (after Gillett, 1987)

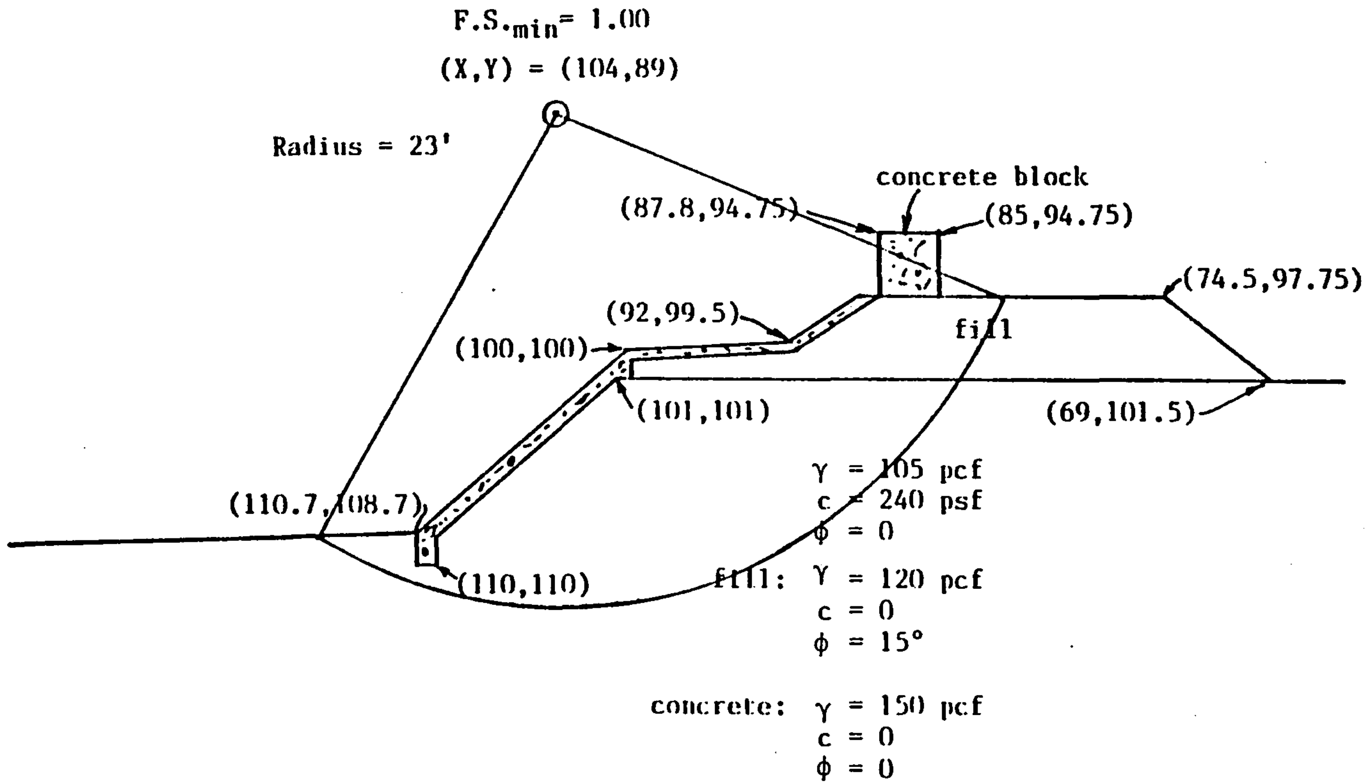
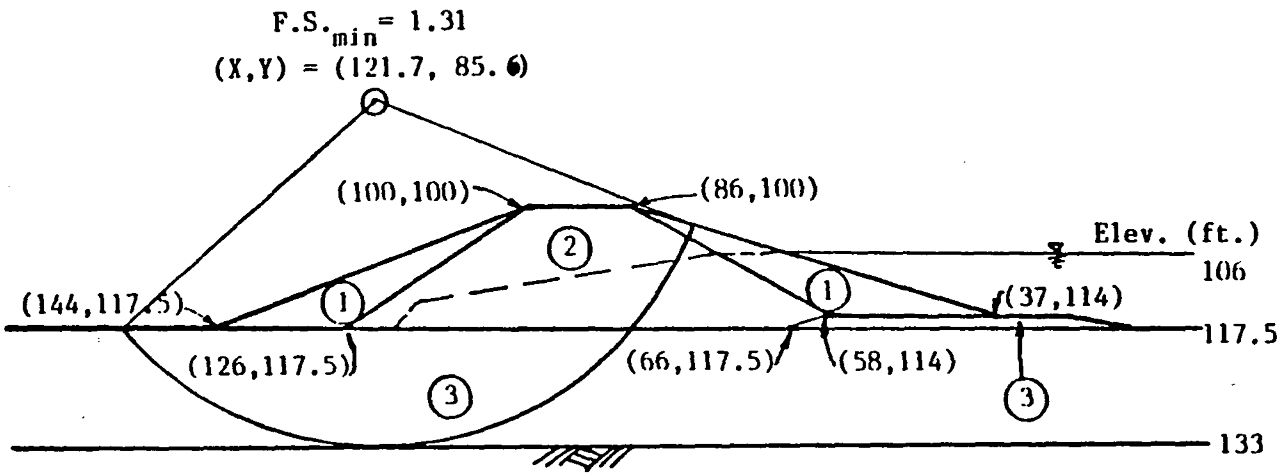


Figure 9. Brightlingsea Slide
(after Gillett, 1987)



1 Rock:
γ = 115 pcf
c = 0
ϕ = 0

2 γ = 121 pcf
c = 518 psf
ϕ = 13°

3 γ = 105 pcf
c = 432 psf
ϕ = 0

Figure 10. Seven Sisters' Slide
(after Gillett, 1987)

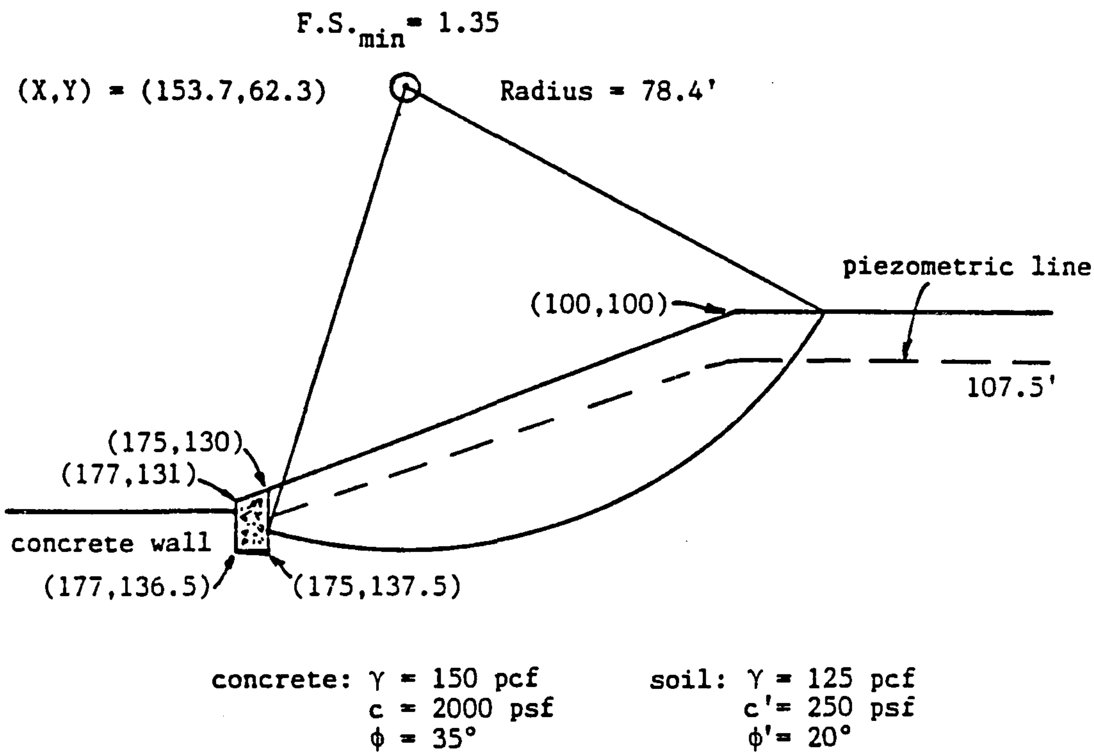


Figure 11. Northolt Slide
(after Gillett, 1987)

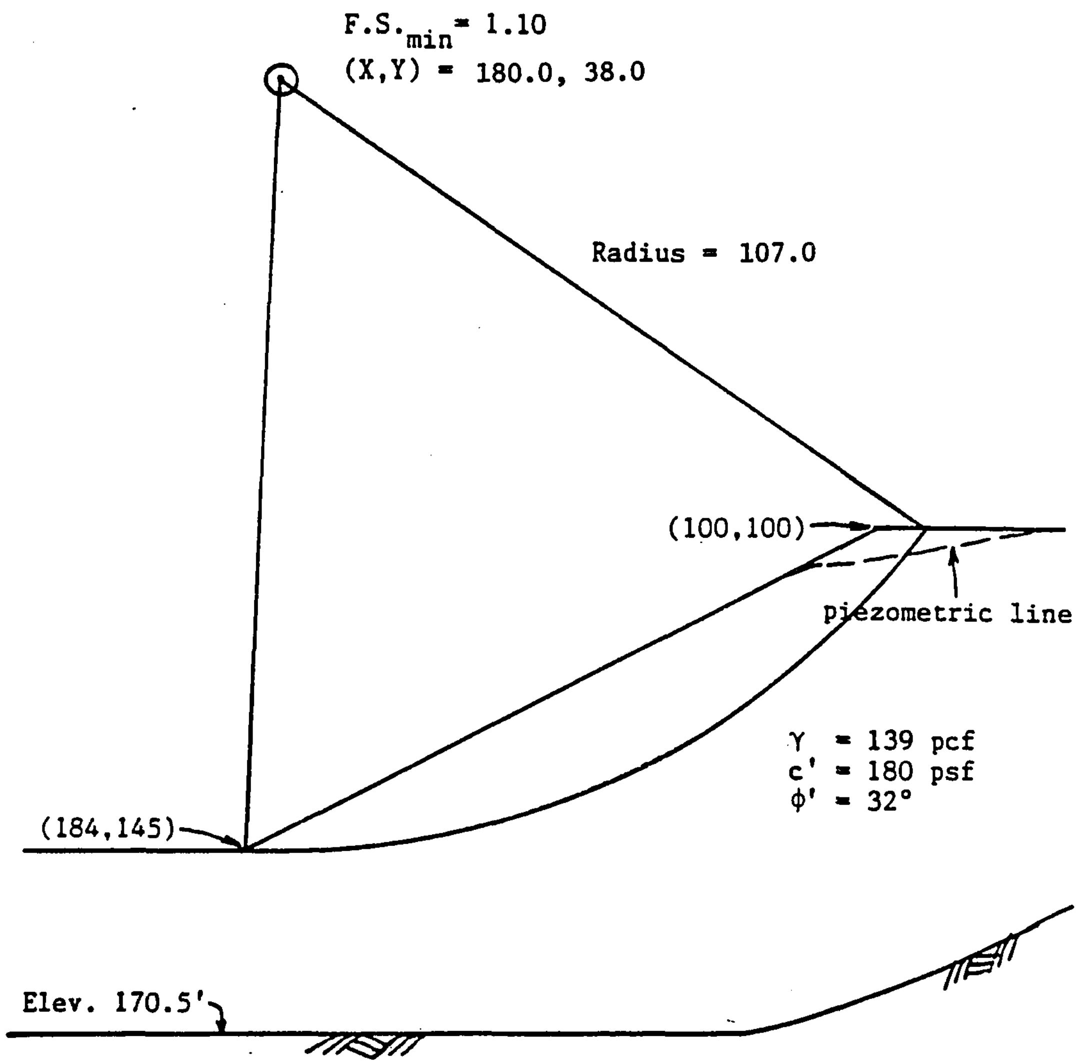


Figure 12. Selset Landslide
(after Gillett, 1987)

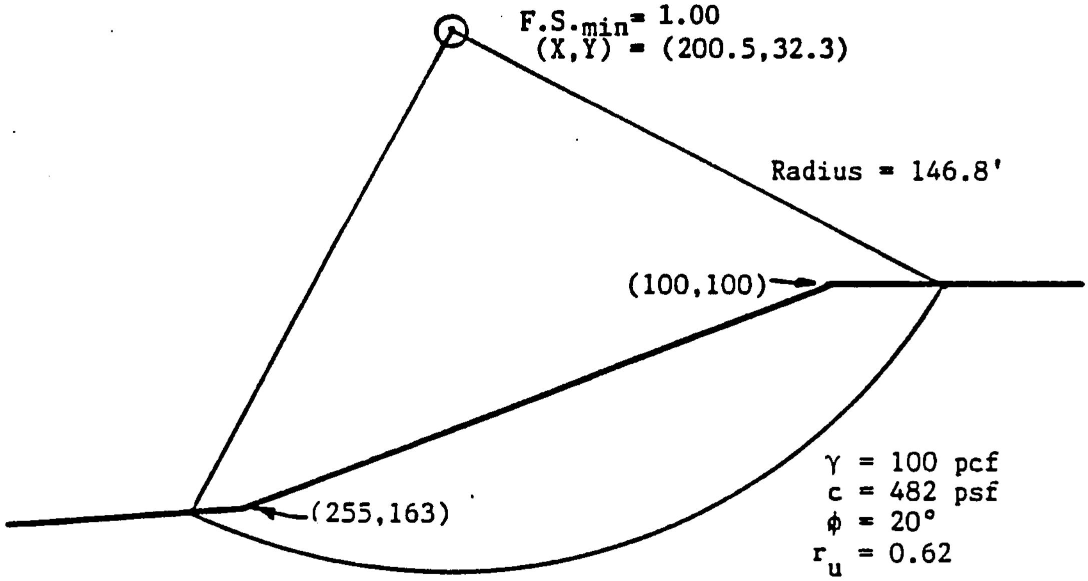


Figure 13. Green Creek Slide

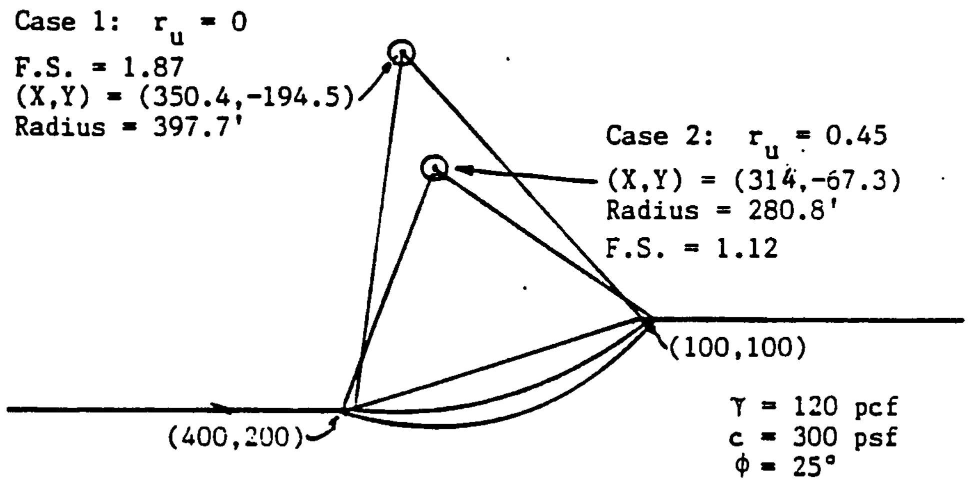


Figure 14. Bishop and Morgenstern's Example (after Gillett, 1987)

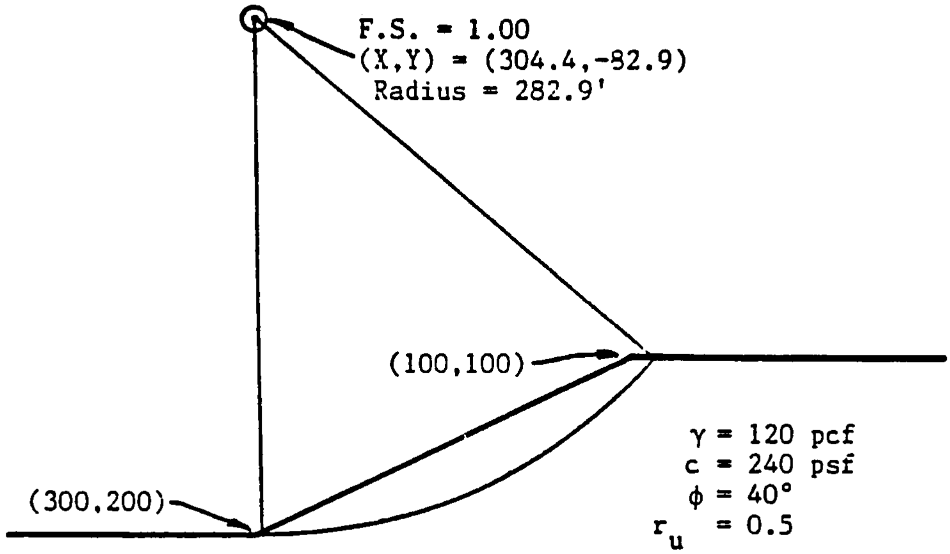


Figure 15. Spencer's Example Problem

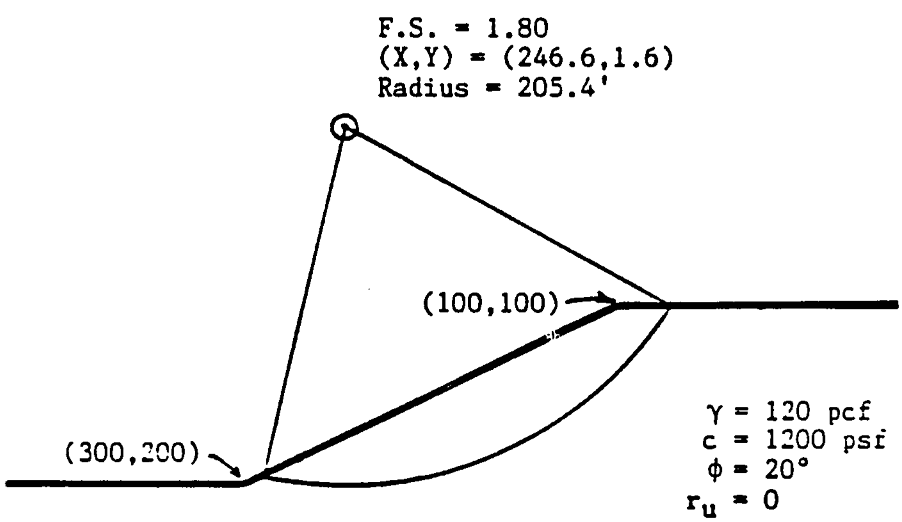


Figure 16. Morgenstern and Price's Example Problem (after Gillett, 1987)

Table 13. Results Obtained by Running CSLIP2 for Some Case Histories

Case History	Number of Directions	X, Y, R of Circle Starting	X, Y, R of Circle Critical	F	Number of Function Evaluations
Congress Street open cut	2	150, 60, 95	138, 73, 82	1.12	25
Brightlingsea slide	2	100, 90, 22	101, 94, 18	0.99	29
Seven Sisters' slide	2	100, 60, 73	122, 89, 43	1.34	36
Northolt slide	3	140, 50, 90	153, 51, 89	1.36	47
Selset landslide	2	160, 25, 122	183, 31, 114	1.10	24
Green Creek slide	3	180, 20, 160	203, 20, 160	1.00	22
Bishop and Morgenstern's example	3	300, -60, 280	314, -67, 281	1.12	33
Spencer's example problem	2	250, -50, 255	294, -60, 261	1.00	27
Morgenstern-Price example	3	200, 20, 200	239, 10, 200	1.80	31

5. SUMMARY AND CONCLUSIONS

5.1 Summary

For the purpose of improving the efficiency of slope stability programs, an optimization algorithm based on Powell's conjugate direction method is implemented into an existing computer code to search for the critical circle. This new program CSLIP2 is applied to a series of example problems and case histories analyzed previously using the STABR program of Lefebvre (1971) and a modified version CSLIP1 (Awad, 1986) which uses a simplex-based search technique. The results are compared and the following conclusions are drawn.

5.2 Conclusions

Powell's conjugate direction method algorithm is reliable in slope stability analysis for it is applicable to a class of functions that have a shape similar to the shape of the safety factor function. The method stated above has also proved to be more efficient than the pattern search techniques since it has decreased the number of function evaluations required to locate the global minimum. The method is generally as good or better than the simplex method, even though the initial search directions used in the present study were not "optimal."

5.3 Suggestions

It is suggested that the initial search directions be varied as a means of studying their effect on overall efficiency. Powell's conjugate direction method may be adapted to slope stability methods which use piecewise-linear slip surfaces. This will cause the dimension of the problem to increase beyond the two- and three-dimensional problems studied herein.

APPENDIX A

FORTRAN LISTING OF CSLIP2

```
C     PROGRAM CSLIP2
C     FOR SLOPE STABILITY ANALYSIS BY
C     BISHOP'S SIMPLIFIED METHOD WITH
C     A SEARCH ROUTINE BASED ON POWELL'S
C     (1964) CONJUGATE DIRECTION METHOD
C
C     WRITTEN BY DR. J.S. DE NATALE
C     DEPT OF CIVIL ENGINEERING AND
C             ENGINEERING MECHANICS
C     THE UNIVERSITY OF ARIZONA
C     TUCSON, ARIZONA     85721
C
C     VERSION 1.0
C     MAY    1987
C
      PROGRAM CSLIP2
      CALL OPEN
      CALL DATAIN
      CALL SSDATA
      CALL POWELL
      CALL EXIT
      END
C
C
C
      SUBROUTINE OPEN
      CHARACTER*20 NAME
      WRITE (*,100)
  100 FORMAT(1X,'INPUT  DATA FILENAME = ?',$)
      READ  (*,110) NAME
  110 FORMAT(A20)
      OPEN  (UNIT=5,FILE=NAME,STATUS='OLD'   )
      WRITE (*,120)
  120 FORMAT(1X,'OUTPUT DATA FILENAME = ?',$)
      READ  (*,110) NAME
      OPEN  (UNIT=6,FILE=NAME,STATUS='NEW'   )
      OPEN  (UNIT=2,STATUS='NEW',FORM='UNFORMATTED')
      RETURN
      END
      SUBROUTINE EXIT
      CLOSE (UNIT=2)
      CLOSE (UNIT=5)
      CLOSE (UNIT=6)
      STOP
      END
C
C
C
      SUBROUTINE DATAIN
      PARAMETER  (ND=3)
      COMMON/BLK1/NDIM,FNUM,LNUM,INUM,DLIM,C2
```

```
      COMMON/BLK2/IMAX,FERR,XERR,XGIV(ND)
      INTEGER      FNUM
      CHARACTER    TITL(20)*4
  800 FORMAT(20A4)
  804 FORMAT(2I5,7E10.2)
  808 FORMAT(8E10.2)
  900 FORMAT(1H1,4X,20A4)
  904 FORMAT(//5X,'INPUT DATA:'/5X,11('-')//
     *         10X,'# DIMENSIONS =',I4/
     *         10X,'# ITERATIONS =',I4/
     *         10X,'MAX  F-ERROR =',1PE10.2/
     *         10X,'MAX  X-ERROR =',  E10.2/
     *         10X,'DX  INTERVAL =',  E10.2/
     *         10X,'LS  ACCURACY =',  E10.2)
  908 FORMAT(//5X,'INPUT DATA:'/5X,'INITIAL VALUES OF THE ',
     *         'INDEPENDENT VARIABLES X1,X2,...,XN:'/5X,57('-'))
  912 FORMAT(8X,1P10E12.3)
      READ  (5,800) TITL
      READ  (5,804) NDIM,IMAX,FERR,XERR,DLIM,C2
      READ  (5,808) (XGIV(I),I=1,NDIM)
      WRITE (6,900) TITL
      WRITE (6,904) NDIM,IMAX,FERR,XERR,DLIM,C2
      WRITE (6,908)
      WRITE (6,912) (XGIV(I),I=1,NDIM)
      RETURN
      END
C
C
C
      SUBROUTINE POWELL
      PARAMETER  (ND=3)
      COMMON/BLK1/NDIM,FNUM,LNUM,INUM,DLIM,C2
      COMMON/BLK2/IMAX,FERR,XERR,XGIV(ND)
      COMMON/BLK3/AA,DD,F1,F2,XCUR(ND),XTEM(ND),XFIN(ND),DCUR(ND)
      COMMON/BLK4/FMIN,XMIN(ND)
      INTEGER      FNUM
      DIMENSION    DSET(ND,ND)
      FNUM = 0
      LNUM = 0
      INUM = 0
      FA = FFUN(XGIV)
      F2 = FA
      DO 100 I=1,NDIM
      DO 100 J=1,NDIM
  100 DSET(I,J) = 0.0
      DO 110 I=1,NDIM
      XFIN(I) = XGIV(I)
  110 DSET(I,I) = 1.0
  200 INUM = INUM + 1
      IF (INUM.GT.IMAX) GO TO 300
      IBIG = 0
```

```
      FBIG = 0.0
      DO 220 I=1,NDIM
      DO 210 J=1,NDIM
  210 DCUR(J) = DSET(I,J)
      CALL LINMIN
      IF ((F1-F2).GT.FBIG) IBIG = I
  220 IF ((F1-F2).GT.FBIG) FBIG = F1-F2
      IF ((FA-F2).LE.FERR) GO TO 320
      DELX = 0.0
      DO 230 I=1,NDIM
  230 DELX = DELX + (XFIN(I)-XGIV(I))**2
      IF (SQRT(DELX).LE.XERR) GO TO 320
      DO 240 I=1,NDIM
  240 XTEM(I) = 2.0*XFIN(I) - XGIV(I)
      FC = FFUN(XTEM)
      IF (FC.GE.FA) GO TO 270
      IF ((FA+FC-2.0*F2)*((FA-F2-FBIG)**2).GE.
     *   ((FA-FC)**2)*FBIG/2.0) GO TO 270
      DO 250 I=1,NDIM
  250 DCUR(I) = XFIN(I)-XGIV(I)
      FC = 0.0
      DO 252 I=1,NDIM
  252 FC = FC + DCUR(I)**2
      FC = SQRT(FC)
      DO 260 I=1,NDIM
      DCUR(I) = DCUR(I)/FC
  260 DSET(IBIG,I) = DCUR(I)
      CALL LINMIN
  270 FA = F2
      DO 280 I=1,NDIM
  280 XGIV(I) = XFIN(I)
      GO TO 200
C
C     TERMINATION CRITERION SATISFIED
C
  300 WRITE (6,310)
  310 FORMAT(/5X,10('*'),' ADDITIONAL ',
     *        'ITERATIONS ARE NOT PERMITTED ',10('*'))
  320 WRITE (6,330) FMIN,(XMIN(I),I=1,ND)
  330 FORMAT(//5X,'THE MINIMUM SAFETY FACTOR BY BMM IS F =',
     *        F8.4/5X,'THE CENTER COORDINATES AND RADIUS OF ',
     *        'THE CRITICAL CIRCLE ARE (X,Y,R) = (',
     *        F9.4,',',F9.4,',',F9.4,')')
      RETURN
      END
C
C
C
      SUBROUTINE LINMIN
      PARAMETER  (ND=3)
      COMMON/BLK1/NDIM,FNUM,LNUM,INUM,DLIM,C2
```

```
      COMMON/BLK2/IMAX,FERR,XERR,XGIV(ND)
      COMMON/BLK3/AA,DD,F1,F2,XCUR(ND),XTEM(ND),XFIN(ND),DCUR(ND)
      INTEGER     FNUM
      LNUM =LNUM+1
      C1 = 1.0E-01
      C3 = 1.0E-01
      F1 = F2
      DD = 1.0
      A1 = 0.0
      AA = 1.0
      A2 = 1.0E+06
      S1 = FUN1(1)
      IF (S1.GT.0.0) DD = -1.0
      IF (S1.GT.0.0) S1 = -S1
  100 IF ((A2-A1).LE.1.0E-01) GO TO 130
      IF ((A2-A1)*ABS(S1).LE.FERR) GO TO 130
      F2 = FUN1(0)
      IF ((F1-F2).GE.(-C1*AA*S1)) GO TO 110
      AH = A1 + ((AA-A1)**2)*S1/(2.0*((AA-A1)*S1+(F1-F2)))
      C4 = C3*(AA-A1)
      IF (AH.LT.(A1+C4)) AH = A1+C4
      IF (AH.GT.(AA-C4)) AH = AA-C4
      A2 = AA
      AA = AH
      GO TO 100
  110 S2 = FUN1(2)
      IF (S2.GE.(C2*S1)) GO TO 120
      AH = AA + (AA-A1)*S2/(S1-S2)
      C4 = C3  *(AA-A1)
      IF (AH.LT.(AA+C4)) AH = AA+C4
      C4 = 9.0 *(AA-A1)
      IF (AH.GT.(AA+C4)) AH = AA+C4
      C4 = 0.5 *(A2-AA)
      IF (AH.GT.(AA+C4)) AH = AA+C4
      A1 = AA
      AA = AH
      F1 = F2
      S1 = S2
      GO TO 100
  120 IF (ABS(S2).LE.(-C2*S1)) GO TO 130
      C4 = AA-A1
      T1 = (2.0*(F1-F2)+C4*(S1+S2))/(C4**3)
      T2 = (S2-S1+3.0*T1*(A1**2-AA**2))/(2.0*C4)
      T3 = (AA*S1-A1*S2+3.0*T1*A1*AA*C4)/C4
      C4 = T2*T2-3.0*T1*T3
      IF (C4.LT.0.0) C4 = 0.0
      AH = T3/(-T2-SQRT(C4))
      C4 = C3*  (AA-A1)
      IF (AH.LT.(A1+C4)) AH = A1+C4
      IF (AH.GT.(AA-C4)) AH = AA-C4
      A2 = AA
```

```
      AA = AH
      GO TO 100
  130 IF (F2.GT.F1) RETURN
      DO 140 I=1,NDIM
  140 XFIN(I) = XCUR(I)
      RETURN
      END
C
C
C
      FUNCTION FUN1(IC)
      PARAMETER  (ND=3)
      COMMON/BLK1/NDIM,FNUM,LNUM,INUM,DLIM,C2
      COMMON/BLK3/AA,DD,F1,F2,XCUR(ND),XTEM(ND),XFIN(ND),DCUR(ND)
      INTEGER     FNUM
      GO TO (100,120,140),IC+1
  100 DO 110 I=1,NDIM
  110 XCUR(I) = XFIN(I) + AA*DD*DCUR(I)
      FUN1 = FFUN(XCUR)
      RETURN
  120 DO 130 I=1,NDIM
  130 XTEM(I) = XFIN(I) + DD*DLIM*DCUR(I)
      FUN1 = (FFUN(XTEM)-F2)/DLIM
      RETURN
  140 DO 150 I=1,NDIM
  150 XTEM(I) = XCUR(I) + DD*DLIM*DCUR(I)
      FUN1 = (FFUN(XTEM)-F2)/DLIM
      RETURN
      END
```

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