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Design methods for focusing grating coupler using holographic optical elements

Cronkite, Patrick Joseph, M.S.

The University of Arizona, 1988

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DESIGN METHODS FOR FOCUSING GRATING COUPLER

USING HOLOGRAPHIC OPTICAL ELEMENTS

by

Patrick Joseph Cronkite

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A Thesis Submitted to the Faculty of the
COMMITTEE ON OPTICAL SCIENCES
In Partial Fulfillment of the Requirements
For the Degree of
MASTER OF SCIENCE
In the Graduate College
THE UNIVERSITY OF ARIZONA

1988
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G. L. Lawrence
Associate Professor of Optical Sciences
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ABSTRACT

Light can be coupled out of a waveguide to a focused point by a focusing grating coupler and has possible applications in optical data storage. The grating can be fabricated with either e-beam techniques or holographic techniques. Two design methods are demonstrated that model the focusing grating coupler with holographic optical elements. Both methods take a geometrical optics approach to designing the holographic optical elements and both methods make use of commercially available ray trace programs.

The first method uses complicated non-rotationally symmetric construction optics and requires either a modified ray trace program or special user defined surfaces.

The second method involves a much simpler approach which did not require any changes to an existing ray trace program and requires only rotationally symmetric elements to correct the aberrations.
1. INTRODUCTION

**Integrated optical head**

Recently much interest is being generated in optical data storage because of the capability to store large amounts of data on single removable disks. The information is written to and read from the disk via an optical system that usually consists of conventional glass optics. A new device has been demonstrated by S. Ura et al.\(^1\), see Figure 1.1. Instead of conventional optics the device uses integrated optics. Light is coupled from a laser diode into the slab waveguide where it propagates to the focusing grating coupler (FGC). The light is diffracted by the FGC and comes to a focus at the optical disk. Upon reflection it is coupled back into the waveguide by a second diffraction from the FGC. A linear grating diffracts part of the return light into two orders that focus on either side of the laser diode. Photodiodes are used to detect the light and generate the appropriate signals. Some of the advantages in optical data storage for this type of device include reduced size and weight (leading to faster access times), reduced cost and possibly manufacturing improvements.\(^1\)

The main component on this device is the focusing grating coupler, shown in detail in Figure 1.2.\(^2\) The device in Figure 1.1 was made by the Japanese by electron-beam direct-writing techniques. The electron-beam generation of the grating may be slow if a high numerical aperture is desired, because of the difficulty of producing high frequency gratings. However, this method has the capability of producing aberration free FGC's at any wavelength.

A grating such as the FGC is essentially a hologram and as such the FGC also lends itself to construction by holographic techniques where C1 and C2 in Figure 1.2 show possible construction points for the FGC. Holographic techniques for generating grating couplers have been demonstrated in the literature.\(^3-5\)
Figure 1.1. Optical disk pick-up device. Light is coupled from a laser diode into a waveguide and then propagates to the FGC and diffracted. Upon reflection from an optical disk the light is coupled back into the guide via the FGC. The light is then diffracted by another grating and propagates to two photodiodes.

Figure 1.2. Focusing grating coupler. The FGC could be fabricated with either e-beam techniques or holographic techniques where C1 and C2 could be sources to form the grating holographically.
Approach

The objective here has been to develop a method or methods using a geometrical optics approach to design a FGC that would be made using holographic techniques. This can be done by modeling the FGC using a holographic optical element (HOE). A specialized in-house ray trace program could have been developed to analyze the FGC. This approach would be time consuming to develop and because of the verification of the program that would be required. Also, the program would not be readily available to others interested in using the design method. It was decided, instead, that an existing, commercially available, ray trace program that analyzes holographical optical elements (HOE) would be used if possible. This approach has the advantage of universal availability, as well as having all the extensive modeling and optimization capabilities that are resident in most commercial programs.

Two design methods will be demonstrated that use commercially available ray trace programs. The first method evolved from original concepts of the problem and the solution involved either a modified ray trace program or special user defined surfaces. Both the HOE and construction optics are modeled as one integrated system on the computer which includes a reversal of the end-use and construction points to correctly do the model on the computer. In addition, the path of one of the construction beams is traced as a virtual optical system. The optics in each of the construction paths required complicated, non-rotationally symmetric systems. The second method was developed, chronologically, well into the development of the first method. The second method uses a much simpler approach which does not require any changes to an existing ray trace program. A phase matching technique is used on two pairs of beams and a fictitious glass is used with an index equal to the ratio of the construction and end-use wavelengths. In fact, the solution does not even require that HOE ray tracing be available in the program.
Special problems

A number of special problems arise when trying to use geometrical optics to model the FGC with an HOE. In general, the photosensitive material, perhaps photoresist, used to generate a grating coupler using holographic techniques is insensitive to near-infrared wavelengths. The wavelength of light of the laser diode that will be used in the final application of the device is in the near-infrared (≈800 nm). Consequently, the photoresist must be exposed at a different wavelength (≈480 nm) from the end-use wavelength of the FGC. This wavelength scaling will cause aberrations in the final spot of the FGC and must be compensated for in the design by pre-aberrating the construction beams of the HOE. In this thesis, the term wavelength scaling will be used instead of wavelength shift which is commonly used in the literature. The distinction has to be made because a wavelength shift in the optical data storage industry is considered to be a small change in the wavelength of the laser diode due to temperature changes, current changes, etc. The work presented does not compensate for wavelength shifts in laser diodes but does compensate for wavelength scaling due to changes in wavelength between construction and reconstruction of the HOE.

A straightforward and well known vector grating equation can be used to describe the geometrical optics properties of a conventional HOE, as will shown in the next chapter. In most standard applications both of the reconstruction points of the HOE are located in air, but in this particular application one of the reconstruction points will be located in the waveguide. The appropriate changes must be made to the grating equation to correct for this difference.

A geometrical optics approach to holograms and waveguides is not the complete picture. A physical optics analysis must be applied to these devices to determine such things as grating efficiency, i.e., coupling coefficients, and polarization considerations
(a very important consideration in magneto-optical data storage). Although not considered here, some of the physical optics problems are discussed in the literature. A geometrical approach can be used to determine ray angles, optical path differences and the Bragg condition. The use of geometrical optics to determine diffracted angles can be justified. The electric field distribution for a TE guided wave can be written as:

$$E_y = \text{Re} \{ E_r \cos(\kappa_f x) e^{-j\beta z} \} , \quad (1.1)$$

where $E_r$ is the amplitude coefficient in the guiding film, $\kappa_f$ is the transverse propagation constant in the film and $\beta$ is the propagation constant in the $z$-direction.

Figure 1.3 shows the configuration of the waveguide and the electric field distribution.

The propagation constant, $\beta$, can be written as:

$$\beta = n_{\text{eff}} k \equiv n_f \sin \phi , \quad (1.2)$$

where $k = \frac{2\pi}{\lambda}$, the wavenumber in free space, $n_{\text{eff}}$ is the effective index of the guiding layer and $n_f$ is the index of the guiding layer. Two approaches can be used to describe rays in a waveguide. In the most familiar bouncing ray method the rays are thought of as being total internally reflected at an angle of $\phi$ at both the top and bottom of the guiding layer hence the rays bounce down the waveguide in the $z$-direction, refer to Figure 1.4. In the second approach rays characterize the direction of propagation of the mode. From Eq. (1.1) the propagation of the mode is in the $z$-direction with a propagation factor $\beta$, hence the rays travel down the waveguide parallel to the $z$-axis in a medium with a refractive index equal to $n_{\text{eff}}$. The power flow of the mode is in the direction of propagation along the $z$-axis hence rays
travelling parallel to the waveguide are also parallel to the direction of power flow. The second method of having rays characterize the direction of propagation of the mode is most useful for defining the grating diffraction properties and will be used throughout this thesis for the mathematical development and numerical implementation.

![Figure 1.3. Asymmetric waveguide showing electric field distribution for a TE guided wave. The mode propagates in the z-direction and as such rays can be thought of as trajectories parallel to the direction of propagation.](image)

An expression for the diffracted angle of a beam exiting from a grating coupler will be used in the ray trace analysis and can be derived using the parallel ray approach described above and the Bragg condition.

\[ \hat{n} \times (k_i - k_c) = \hat{n} \times k_\sigma , \]  

where \( \hat{n} \) is the unit normal vector to the surface, \( k_i \) and \( k_c \) are the wavenumber vectors for the leaky mode leaving the grating and the guided mode, respectively, and
Figure 1.4. Bouncing rays in optical waveguide. Total internal reflection occurs at both the film-air interface and the film-substrate interface at an angle of $\phi$ with respect to the normal.

$k_e$ is the wavenumber vector for the grating. This approach to rays in waveguides is in full agreement with Tamir's wave approach in determining the diffracted angle of the leaky mode exiting from a grating coupler. He has shown that the diffracted angle, $\theta$, can be determined by phase matching the propagation factor of the guided wave with that in the diffracted wave:

$$k \sin \theta = \beta + \left( \frac{2\pi \nu}{d} \right). \quad (1.4)$$

where $k = \frac{2\pi}{\lambda}$, the wavenumber of the diffracted wave in air, $\beta = n_{\text{eff}} k$, the guided wave propagation factor, $\nu$ is an integer representing the order of the grating and $d$ is the grating spacing.

Incident rays from $c$ into the FGC will be parallel to the grating and hence strike it at grazing incidence as shown in Figure 1.5. Notice that there could be numerous intersections of the ray with the grating. This indeterminate intersection caused by the
grazing incidence is very difficult, if not impossible, for most ray trace codes to handle and hence, the design methods must work around it. One might suggest to trace the rays from $i$ to $c$ to eliminate this problem but, in fact, this will not work either. Consider a small change, $\Delta \theta_1$, in the angle of an incident ray onto the HOE, see Figure 1.6. The small change in angle would represent small aberrations in the HOE. The change in the exiting rays, $\Delta \theta_2$, due to $\Delta \theta_1$ will be relatively large. Therefore, small aberrations in the HOE would cause a large spread in the rays leaving the HOE and make it difficult to analyze.

![Figure 1.5. Illustration of grazing incidence. Within a ray trace program light from $c$ incident upon the grating at grazing incidence may have a number of possible intersection points. This indeterminate intersection must be overcome in the design.](image)

**Background**

A number of papers in the literature address hologram imagery and aberrations\(^8-10\) and also the problem of wavelength scaling and ways to correct it.\(^11-14\) In addition, a few papers have been written on holographic grating couplers.\(^3-5\) A number of these
approaches are discussed briefly here to indicate the reasoning for the method described in this thesis.

In Reference 10, Latta describes hologram imagery and monochromatic aberrations based on the development of Champagne\(^9\). The uniqueness of this approach is that the analysis is not confined to the paraxial region as with much of the previous work\(^8\). Latta shows that holograms suffer from severe aberrations when the reconstruction geometry deviates only slightly from the construction geometry. He also explores the effects of hologram bending to modify the aberration characteristics similar to the approach that is taken with conventional optical elements.

In a follow-up paper\(^11\), Latta looks at aberrations and aberration balancing of holograms induced by wavelength scaling. The aberration balancing is done in a closed analytical form considering only third order aberrations. Although conceptually a clean approach, for complicated systems and higher order aberrations the algebra would essentially be unsolvable.

In addition, Latta also developed a computer-based ray tracing analysis for
holograms. Although a general program that considers different geometries, thick holograms and hologram shrinking, it would have to be developed from square one by the user. Another "in-house" developed ray trace program utilizes a method called k-vector closure. Along the lines of ray tracing, Sweatt has developed an exact lens analogy that can be performed with standard lens design programs. Sweatt showed that a planer HOE can be represented with two plano-convex lenses having very high indices of refraction and very small thicknesses and curvatures such that the power of the elements remained finite.

Lin and Doherty demonstrated a method for generating an aberration-free wavefront reconstruction when the construction and reconstruction wavelengths were different. Their procedure still requires that a hologram can be made at each of the two wavelengths.

All of the above mentioned techniques do not consider the more specific case of a waveguide hologram as is described in References 3, 4 and 5. Wüthrich and Lukosz describe a method that uses a guided wave as one of the construction beams to form the hologram but the hologram was recorded and read out at the same wavelength. The methods described in References 4 and 5 form the hologram on the waveguide with two beams external to the guide at the construction wavelength and then reconstruct the image with a guided mode at the second wavelength. Although Heitmann and Ortiz did an "in-house" ray trace analysis, it did not involve the optimization and analysis of complicated optical systems which, as will be shown, are required to correct the aberrations due to the wavelength scaling particularly for a high numerical aperture device.
2. MATHEMATICAL DESCRIPTION

Closed form

The first order properties of an HOE are helpful in initially setting up a system to determine the location of the image. Champagne\(^9\) demonstrates the nonparaxial imaging properties of an HOE where as a majority of the literature only considers the paraxial case. The hologram is formed or constructed by an object beam (designated \(o\)) and a reference beam (designated \(r\)) at the wavelength \(\lambda_c\). The hologram is then read out or reconstructed with a beam (designated \(c\)) at a different wavelength, \(\lambda_u\). A number of different HOE systems will be described in Section 3. By the use of a wavefront matching technique an expression for the distance and direction of the image (designated \(i\)) can be generated. The form of the four beams is arbitrary, they can be converging, diverging or planer. Although the subscript notation is often cumbersome, it has historical reasons and is used in most of the literature hence it will be used throughout this thesis.

The distance from the origin of the HOE to the image, \(R_i\), is given by:\(^9\)

\[
\frac{1}{R_i} = \frac{1}{R_c} \pm \left(\frac{\mu}{m^2}\right) \left(\frac{1}{R_0} - \frac{1}{R_r}\right),
\]

(2.1)

where: \(R_q\) = the radial distance from the origin of the HOE to the corresponding source \(i, c, o,\) or \(r\). Refer to Figure 2.1,

\(\mu = \frac{\lambda_u}{\lambda_c}\),

\(m'\) = an enlargement of the HOE,

\(\pm\) denotes: primary image for + sign

conjugate image for - sign.\(^{10}\)
Figure 2.1. Definition of \( R_q, \alpha_q \) and \( \beta_q \). \( R_q \) is the distance from the center of the HOE which is in the x-y plane to the corresponding point source \( i, c, o, \) or \( r \). \( \alpha_q \) is the angle between the chief ray and the y-z plane. \( \beta_q \) is the angle between the projection of the chief ray on the y-z plane and the z-axis.

The direction of the image with respect to the origin of the HOE is given by either form of two equations:

\[
\frac{x_i}{R_i} = \frac{x_c}{R_c} \pm \left( \frac{\mu}{m'} \right) \left( \frac{x_o}{R_o} - \frac{x_r}{R_r} \right) \quad (2.2)
\]

or

\[
\sin \alpha_i = \sin \alpha_c \pm \left( \frac{\mu}{m'} \right) (\sin \alpha_o - \sin \alpha_r) \quad , \quad (2.3)
\]

and

\[
\frac{y_i}{R_i} = \frac{y_c}{R_c} \pm \left( \frac{\mu}{m'} \right) \left( \frac{y_o}{R_o} - \frac{y_r}{R_r} \right) \quad (2.4)
\]

or

\[
\cos \alpha_i \sin \beta_i = \cos \alpha_c \sin \beta_c \pm \left( \frac{\mu}{m'} \right) (\cos \alpha_o \sin \beta_o - \cos \alpha_r \sin \beta_r) \quad , \quad (2.5)
\]

where: \( x_q, y_q, z_q \) = the x, y, and z locations of the sources \( i, c, o, r \).

\( \alpha_q, \beta_q \) = the angles as shown in Figure 2.1.

In addition, the coefficients of spherical aberration, coma and astigmatism can be
written in terms of the variables used above:

(a) spherical aberration

\[ S = \frac{1}{R^3_c} \pm \left( \frac{\mu}{m^2} \right) \left( \frac{1}{R^3_o} - \frac{1}{R^3_r} \right) - \frac{1}{R^3_i} \]  

(2.6)

(b) coma

\[ C_x = \frac{x_c}{R^3_c} \pm \left( \frac{\mu}{m^2} \right) \left( \frac{x_o}{R^3_o} - \frac{x_r}{R^3_r} \right) - \frac{x_i}{R^3_i} \]  

(2.7)

\[ C_y = \frac{y_c}{R^3_c} \pm \left( \frac{\mu}{m^2} \right) \left( \frac{y_o}{R^3_o} - \frac{y_r}{R^3_r} \right) - \frac{y_i}{R^3_i} \]  

(2.8)

(c) astigmatism

\[ A_x = \frac{x_o^2}{R^3_c} \pm \left( \frac{\mu}{m^2} \right) \left( \frac{x_o^2}{R^3_o} - \frac{x_r^2}{R^3_r} \right) - \frac{x_i^2}{R^3_i} \]  

(2.9)

\[ A_y = \frac{y_o^2}{R^3_c} \pm \left( \frac{\mu}{m^2} \right) \left( \frac{y_o^2}{R^3_o} - \frac{y_r^2}{R^3_r} \right) - \frac{y_i^2}{R^3_i} \]  

(2.10)

\[ A_{xy} = \frac{x_o y_c}{R^3_c} \pm \left( \frac{\mu}{m^2} \right) \left( \frac{x_o y_c}{R^3_o} - \frac{x_r y_r}{R^3_r} \right) - \frac{x_i y_i}{R^3_i} \]  

(2.11)

**Sign conventions**

The sign conventions used for these first order equations can often be confusing hence they are listed here as an aid to first order layout:

1. Usually the + sign is used in the equations above, indicating the primary image. The primary image or the +1 diffraction order can be defined as the image that reproduces the original object when no wavelength scaling is present. The conjugate image (-1 diffraction order) or the image obtained using the - sign is distorted and often the opposite sense (virtual or real) from the primary image. Note that the primary image is not termed virtual.
as is often the case, because the primary image can be virtual or real
depending upon the geometries of the construction beams.

2. Referring to Figure 2.1, the sign of \( R_q \) is positive if the source is diverging
and \( R_q \) is negative if the source is converging.

3. The angles \( \alpha_q \) and \( \beta_q \) can be determined from any convenient reference such
as the positive z-axis. Whatever convention is adopted for the sign of the
angle it must be used consistently for all the angles. If the source \( q \) is
converging then \( 180^\circ \) must be added to the angle.

Ray tracing

As mentioned in the Introduction, use of these equations and higher order
equations to balance aberrations for complicated systems would be mathematically
impossible in closed form. An easier approach to determine the aberrations of a
system is to do a raytrace analysis. The approach presented here was developed by
Welford.\(^{16} \) In this raytrace as with all raytraces of HOE's in lens design programs,
only the direction of the diffracted ray is determined. Physical optics must be used to
determine the relative intensities of each of the diffracted beams. The HOE's will be
considered as an infinitely thin phase-changing layer.

A vector grating equation can be used to describe the raytrace properties of an
HOE (refer to Figure 2.2):\(^{16} \)

\[
\hat{n} \times (\hat{r}_i - \hat{r}_c) = m \frac{\lambda_u}{\lambda_c} \hat{n} \times (\hat{r}_o - \hat{r}_r),
\]

(2.12)

where: \( \hat{n} \) = surface normal vector.

\( m \) = diffraction order.

\( \lambda_u, \lambda_c \) = end-use and construction wavelengths, respectively.

\( \hat{r}_q \) = unit vectors from the incident point on the HOE to the \( i, c, o \)
or \( r \) sources, respectively.
The direction of an exiting ray leaving the HOE can be found by solving for the vector \( \hat{r}_i \) in terms of the known vectors \( \hat{n} \), \( \hat{r}_c \), \( \hat{r}_o \), and \( \hat{r}_r \). Although the expression for \( \hat{r}_i \) in Reference 16 is correct, the derivation has a number of typo-graphical errors. A corrected derivation is presented here. First, take the cross product of Eq. (2.12) and \( \hat{n} \):

\[
\hat{n} \times (\hat{n} \times \hat{r}_i) = \hat{n} \times (\hat{n} \times U),
\]

where

\[
U = \hat{r}_c + m \frac{\lambda_n}{\lambda_c} (\hat{r}_o - \hat{r}_r).
\]

Using the vector triple product expansion on Eq. (2.13) gives:
\[
(n \cdot \hat{f}_i)\hat{n} - \hat{f}_i = (\hat{n} \cdot U)\hat{n} - U .
\]

(2.15)

Taking the dot product of Eq. (2.15) with itself and solving for \(\hat{n} \cdot \hat{f}_i\) gives:

\[
\hat{n} \cdot \hat{f}_i = \sqrt{1 - U^2 + (\hat{n} \cdot U)^2} .
\]

(2.16)

Substituting Eq. (2.16) back into Eq. (2.15) gives:

\[
\hat{f}_i = U - (\hat{n} \cdot U)\hat{n} + \hat{n}\sqrt{1 - U^2 + (\hat{n} \cdot U)^2} .
\]

(2.17)

If \(\hat{n}\) is along the \(z\)-axis of the coordinate system of the HOE Eq. (2.12) can be written as two direction cosine scalar equations:

\[
K_i - K_c = m \frac{\lambda_i}{\lambda_c} (K_o - K_r) ,
\]

(2.18)

\[
L_i - L_c = m \frac{\lambda_i}{\lambda_c} (L_o - L_r) .
\]

(2.19)

where,

\[
K_q = \frac{x_s - x_q}{\sqrt{(x_s - x_q)^2 + (y_s - y_q)^2 + (z_s - z_q)^2}} ,
\]

(2.20)

\[
L_q = \frac{y_s - y_q}{\sqrt{(x_s - x_q)^2 + (y_s - y_q)^2 + (z_s - z_q)^2}} .
\]

(2.21)

In Eqs. (2.20) and (2.21) \(q\) corresponds to the source index \(i, c, o,\) or \(r\) while \(x_s, y_s\) and \(z_s\) are the \(x, y\) and \(z\) coordinates on the HOE surface, respectively.
3. HOE SYSTEMS

Simple systems

A number of simple systems are described by Harihan\textsuperscript{17} and Collier et. al.\textsuperscript{18} that illustrate reconstruction of both virtual and real images. The first part of this section starts with one of these simple systems and walks through the logic used to get the basic end-use system of Figure 3.5 used to model the FGC. Once the end-use system is developed then the methods used to solve the problems specific to modeling the FGC (refer to the Introduction) will be described.

The goal is to eventually model the FGC therefore the systems described here will restrict themselves to point sources for the construction and reconstruction sources instead of planer or some complicated sources. In addition, only transmission HOE's will be considered and not reflection HOE's. For a transmission HOE the construction sources are on the same side of the recording medium where as for the reflection HOE the construction sources are on opposite sides.\textsuperscript{17}

The simplest and most familiar system is one that reconstructs an exact but virtual image of the object point source. Figure 3.1 shows such a system. In these systems the construction beams are represented by a dashed line and the reconstruction beams are represented by solid lines. To get an exact replica of the object the reference beam (r) and the reconstruction beam (c) must have the same geometries and the construction wavelength $\lambda_c$ must equal the reconstruction or end-use wavelength $\lambda_u$.

For any useful optical system such as the FGC the image must be real and not virtual. A real image can be generated in two ways:

1. If the reconstruction beam is a conjugate to the reference beam then a real image is produced at the location of the original object. A conjugate beam is one that reverses all the rays of the original beam. See Figure 3.2 for such a system. Note that in this system the reconstruction beam is a
converging beam. To generate a converging beam an optical system would be required and in the case of the FGC the reconstruction beam is in the waveguide. Therefore, to eliminate designing a waveguide lens system the HOE system in Figure 3.2 was not considered.

2. A real image can be generated by using one diverging beam and one converging beam for the two construction beams, see Figure 3.3. Note that the systems in Figures 3.1 and 3.2 use diverging beams for the construction optics. The reconstruction beam, \(c\), is now diverging as required. Although one of the construction beams is converging, it will not pose a problem like the one encountered with the reconstruction beam. Recall from the Introduction that aberrations generated from the wavelength scaling will be compensated by pre-aberrating the construction beams which will be done with an optical system. The optical system can easily be designed to obtain a converging beam for one of the construction beams.

System to model FGC

An additional change must be made to get the basic end-use system used to model the FGC. The system in Figure 3.3 has the two construction beams and the final image on opposite sides of the HOE. The design would be simplified slightly if the construction beams and the final image where on the same side of the HOE, then the construction beams will not have to pass through the waveguide substrate to expose the photoresist. A way to meet this condition is to reconstruct the HOE shown in Figure 3.3 with a beam conjugate to the object beam. This system is shown in Figure 3.4.

The basic end-use system is shown in Figure 3.5 now with the construction and end-use wavelengths different. As discussed, the image or end-use spot will be aberrated due to the wavelength scaling and will be represented by the jagged line at \(i\). In the actual designs point \(c\) will be parallel to the HOE surface to represent the beam
in the waveguide but in this and following systems it is drawn away from the surface for ease in visualization.

**System for correct computer analysis**

The end-use system shown in Figure 3.5 cannot be analyzed and optimized directly in the ray trace program because of the grazing incidence problem discussed in the Introduction. As mentioned, \( c \) will eventually be parallel to the HOE surface and, as such, rays cannot be traced at grazing incidence (refer to Figure 1.5). Also, as discussed in the Introduction, the possible solution to eliminate grazing incidence by analyzing the system backwards from \( i \) to \( c \) is not viable. Pre-aberrating the construction paths \( o \) and \( r \) with conventional optical systems can be done to help reduce the aberrations at \( i \). In general, the construction points \( o \) and \( r \) are defined as perfect point sources in the definition of the HOE in a lens design program, as such, conventional optics cannot be put into the construction beams in the end-use configuration. The lens design programs SYNOPSIS\(^{19} \) and CODE \( V^{20} \) do allow the construction beams to be aberrated with either polynomial coefficients describing the aberrations or with a user defined optical system stored in the library. With either of these approaches the grazing incidence problem is still present.

A solution to pre-aberrating the construction beams or modelling the construction optics has been described by Hayford.\(^{21} \) Fortunately, this method may be extended to solve the grazing incidence problem. Consider the end-use point sources as ideal and reverse the roles of the construction and end-use points to create a new HOE system that will be used in the lens design program to model the FGC. The new system is illustrated in Figure 3.6. The \( o \) and \( r \) point sources in Figure 3.6 are now the end-use point sources where \( o \) is the end-use final spot and \( r \) is the point source for the beam in the waveguide and both are ideal point sources in the definition of the HOE in the lens design program. The \( c \) and \( i \) point sources in Figure 3.6 are the end-use
construction points where \( i \) is again an aberrated spot. In the lens design program rays will be traced from \( c \) to \( i \) hence optics can now be inserted to generate the appropriate aberrations at \( i \). Referring to Figure 3.6, when point \( r \) is parallel to the HOE surface representing the waveguide beam both of the points \( c \) and \( i \) are away from the surface, hence the grazing incidence problem is eliminated. Table 3.1 summarizes the labelling scheme used in Figures 3.5 and 3.6.

Reversing the roles of the construction and end-use points requires that a restriction be put on the diffraction order that can be used. The grating equation for the end-use system in Figure 3.5 can be written as:

\[
\hat{n} \times (\hat{r}_i - \hat{r}_c) = m \frac{\lambda_u}{\lambda_c} \hat{n} \times (\hat{r}_o - \hat{r}_r). \tag{3.1}
\]

Reversing the roles of \( i \) and \( c \) with \( o \) and \( r \), respectively, gives:

\[
\hat{n} \times (\hat{r}_o - \hat{r}_r) = m \frac{\lambda_c}{\lambda_u} \hat{n} \times (\hat{r}_i - \hat{r}_c). \tag{3.2}
\]

Expression (3.1) and (3.2) will be equivalent only when \( m \equiv \frac{1}{m^*} \). Reversing the roles of the construction and end-use points can only be done when the diffraction order \( m \) is equal to \( \pm 1 \).

When an end-use HOE is constructed with a point source and an appropriate aberrated beam aberration free imaging will occur between the end-use conjugate points. The task has been reduced to designing construction optics using a conventional optical system that will create an aberrated image or spot (\( i \) in Figure 3.6). This approach is somewhat novel in that the optical system is designed to create aberrations and not eliminate them.
Figure 3.1. Ideal virtual image formed by a diverging reconstruction point source $c$. The HOE is formed by two diverging point sources $r$ and $o$. Both construction and reconstruction wavelengths are the same.

Figure 3.2. Ideal real image formed by a converging reconstruction point source $c$. The HOE is formed by two diverging point sources $r$ and $o$. Both construction and reconstruction wavelengths are the same.
Figure 3.3. Ideal real image formed by a diverging reconstruction point source \( c \). The HOE is formed by a diverging point source \( r \) and a converging point source \( o \). Both construction and reconstruction wavelengths are the same.

\[ \lambda_c = \lambda_u \]

Figure 3.4. Ideal real image formed by a converging reconstruction point source \( c \) but now the image and construction sources are on the same side of the HOE. The HOE is formed by a diverging point source \( r \) and a converging point source \( o \). Both construction and reconstruction wavelengths are the same.

\[ \lambda_c = \lambda_u \]
Figure 3.5. End-use system. The image is now aberrated because the construction and reconstruction wavelengths are different. The image is formed by the reconstruction point source $c$ which is shown away from the surface for ease in visualization. The HOE is formed by a diverging point source $r$ and a converging point source $o$ where optics will be inserted in these beams to correct for the aberrations at $i$.

Figure 3.6. New HOE system that can be used to model the FGC on the computer and correct the grazing incidence problem. Points $o$ and $r$ now represent the ideal end-use points where $r$ is shown away from the surface for ease in visualization. Rays are now traced from $c$ to $i$ which are the construction points.
Table 3.1. Labelling scheme used in Figures 3.5 and 3.6.

<table>
<thead>
<tr>
<th></th>
<th>construction points (complicated optics)</th>
<th>waveguide source</th>
<th>end-use spot</th>
</tr>
</thead>
<tbody>
<tr>
<td>End-use system</td>
<td>r o</td>
<td>c</td>
<td>i</td>
</tr>
<tr>
<td>(Figure 3.5)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>System used in lens</td>
<td>c i</td>
<td>r</td>
<td>o</td>
</tr>
<tr>
<td>design program</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Figure 3.6)</td>
<td></td>
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</tr>
</tbody>
</table>
4. INTEGRATED SOLUTION

Integration of construction optics and HOE system

One solution to designing the construction optics is to perform the design with both the construction optics and the HOE as one integrated system in the ray trace program.\textsuperscript{22-23} A sketch of the integration of possible construction optics and an HOE system is illustrated in Figure 4.1. Unfortunately, it is impossible for the ray trace program to start at two point sources (c and \( i' \) in Figure 4.1) and trace rays to a single plane (the HOE in this case). There is still a way to trace rays through the system. Start at point \( c \) in Figure 4.1 and trace rays through the HOE and to the aberrated spot \( i \). Trace the rays backwards past the HOE (the program does not know it is there) through some conventional but now virtual optical system (still tracing backwards) to an image \( i' \).

![Figure 4.1. Integration of new HOE system and optical system used to correct the aberrations at \( i \). Rays are traced from \( c \) to the HOE and then diffracted and traced to the aberrated image \( i \). The rays are then traced backwards through a virtual optical system to the final image \( i' \).](image-url)
All the extensive modeling and optimization capabilities are now available to minimize the aberrations at the final image $i'$ for a given pair of end-use point sources. Some special considerations are necessary when using a conventional lens design program to design a virtual optical system:

1. When tracing rays backwards through a virtual system the rays are in effect being traced through negative glass and air thicknesses. In some lens design programs, such as CODE V, switches are set in the optimization routines to prevent negative thicknesses. These switches must be turned off for the virtual part of the system. Care must also be taken to see that the virtual thicknesses do not become positive during optimization.

2. As with all tilted systems, provisions must be provided so that the chief ray passes through the center of the image plane, i.e., the chief ray and the optical axis must coincide for a point source object. For a system such as that shown in Figure 4.1 this can be accomplished by inserting a dummy surface at the HOE and tilting it such that the coordinate system (optical axis) coincides with the chief ray diffrated by the HOE.

**Implementing the correct FGC model**

As noted in the Introduction, most standard applications assume that all of the $\hat{f}$ vectors in Eq. (2.12) are in air. To correctly model the FGC using an HOE, Eq. (2.12) must be modified to account for the fact that one of the vectors ($\hat{f}_r$ in the case of the system in Figure 3.6) is in the slab waveguide with an effective index, $n_{\text{eff}}$. The modified grating equation can be written as follows:

$$\hat{n} \times (\hat{f}_i - \hat{f}_c) = m \frac{\lambda_{\text{air}}}{\lambda_{c}} \hat{n} \times (\hat{f}_o - n_{\text{eff}} \hat{f}_r) \quad (4.1)$$
The aberration coefficients, Eq. (2.6)-(2.11), must also be modified to account for the effective index of the waveguide:

(a) spherical aberration

\[
S = \frac{1}{R_c^3} \pm \left( \frac{\mu}{m^2} \right) \left( \frac{1}{R_o^3} - n_{\text{eff}} \frac{1}{R_r^3} \right) - \frac{1}{R_i^3} \quad (4.2)
\]

(b) coma

\[
C_x = \frac{x_c}{R_c^3} \pm \left( \frac{\mu}{m^2} \right) \left( \frac{x_o}{R_o^3} - n_{\text{eff}} \frac{x_r}{R_r^3} \right) - \frac{x_i}{R_i^3} \quad (4.3)
\]

\[
C_y = \frac{y_c}{R_c^3} \pm \left( \frac{\mu}{m^2} \right) \left( \frac{y_o}{R_o^3} - n_{\text{eff}} \frac{y_r}{R_r^3} \right) - \frac{y_i}{R_i^3} \quad (4.4)
\]

(c) astigmatism

\[
A_x = \frac{x_c^2}{R_c^3} \pm \left( \frac{\mu}{m^2} \right) \left( \frac{x_o^2}{R_o^3} - n_{\text{eff}} \frac{x_r^2}{R_r^3} \right) - \frac{x_i^2}{R_i^3} \quad (4.5)
\]

\[
A_y = \frac{y_c^2}{R_c^3} \pm \left( \frac{\mu}{m^2} \right) \left( \frac{y_o^2}{R_o^3} - n_{\text{eff}} \frac{y_r^2}{R_r^3} \right) - \frac{y_i^2}{R_i^3} \quad (4.6)
\]

\[
A_{xy} = \frac{x_c y_c}{R_c^3} \pm \left( \frac{\mu}{m^2} \right) \left( \frac{x_o y_o}{R_o^3} - n_{\text{eff}} \frac{x_r y_r}{R_r^3} \right) - \frac{x_i y_i}{R_i^3} \quad (4.7)
\]

where

\[
\frac{1}{R_i} = \frac{1}{R_c} \pm \left( \frac{\mu}{m^2} \right) \left( \frac{1}{R_o} - n_{\text{eff}} \frac{1}{R_r} \right) \quad (4.8)
\]
The exiting ray direction from an HOE surface can be calculated via Eq. (2.17):

$$\hat{r}_i = \mathbf{U} - (\hat{n} \cdot \mathbf{U})\hat{n} + \hat{n} \sqrt{1 - \mathbf{U}^2 + (\hat{n} \cdot \mathbf{U})^2}$$

(2.17)

but now,

$$\mathbf{U} = \hat{r}_c + m \frac{\lambda n}{\lambda c} (\hat{r}_o - n_{\text{eff}} \hat{r}_r)$$

(4.9)

In addition, if $\hat{n}$ is along the z-axis of the coordinate system of the HOE Eq. (2.18) and (2.19) change to:

$$L_i - L_c = m \frac{\lambda n}{\lambda c} (L_o - n_{\text{eff}} L_r)$$

(4.10)

$$M_i - M_c = m \frac{\lambda n}{\lambda c} (M_o - n_{\text{eff}} M_r)$$

(4.11)

A FORTRAN program was written by the author to calculate the exiting ray angles for a flat HOE surface using these two equations as well as Eqs. (2.20) and (2.21). The source code for this program is given in Appendix 1.

A simple system shown in Figure 4.2 is used to illustrate the change brought about by inserting one of the beams into a medium. Figure 4.3 is the output from the program in Appendix 1 for the configuration in Figure 4.2 and shows the exiting chief ray angle as a function of the angle of the incoming chief ray for the cases of $n_{\text{eff}} = 1.0$ and $n_{\text{eff}} = 1.5$. Note that for a given incoming ray angle the exiting ray angle is larger when $n_{\text{eff}} = 1.5$. A cutoff angle exists for both cases of $n_{\text{eff}}$ such that the incoming ray is totally internally reflected. Also, for the case of $n_{\text{eff}} = 1.0$, perfect imagery is obtained when the same geometry is used for construction and reconstruction (this was done in Figure 3.4) but this is not the case when $n_{\text{eff}} = 1.5$. 
A number of different lens design programs were investigated as to their feasibility in performing these designs. Unfortunately, all the commercially available programs require the two construction vectors to be in the same medium and the two end-use vectors to be in the same medium. No provision is made for direct insertion of just one beam into a different index of refraction. All the programs investigated do allow for tracing rays backwards through a virtual optical system.

**SUPEROSLO**

The lens design program SUPEROSLO allows for user-defined surfaces where ray data are passed to a user-written routine. The routine could then do similar calculations as the program in Appendix 1. The new ray data is passed back to the
main program and the ray trace is continued. In this manner the integrated design of
the FGC can be done with SUPEROSLO.

SYNOPSISYS

Although SYNOPSISYS does not support user-defined surfaces, the source code was
modified to accommodate the modified grating equation. To protect the proprietary
nature of the SYNOPSISYS code the subroutine that was modified cannot be included
here. Unlike Appendix I, the approach used to change SYNOPSISYS allows for an
arbitrary surface shape and follows the method described by Welford (Eq. (4.9)). An
outline of the procedure used will be described here. Ray parameters in SYNOPSISYS
are not given in terms of the direction cosines but are given in terms of two
normalized direction cosines ZZ and HH:
where,

\[ K^2 + L^2 + M^2 = 1 \]  \hspace{1cm} (4.14)

Given the normalized direction cosines, ZZ and HH, the direction cosines can be calculated with the following formulas:

\[ M = \frac{1}{\sqrt{1 + ZZ^2 + HH^2}} \]  \hspace{1cm} (4.15)

\[ K = (ZZ)(M) \]  \hspace{1cm} (4.16)

\[ L = (HH)(M) \]  \hspace{1cm} (4.17)

The ray parameters for the incoming ray, ZZ\textsubscript{c} and HH\textsubscript{c}, and the parameters for the normal to the surface, ZZ\textsubscript{n} and HH\textsubscript{n}, are given to the subroutine by SYNOPSYS. In addition, the ray parameters for both of the construction points are also given to the subroutine: ZZ\textsubscript{0}, HH\textsubscript{0}, ZZ\textsubscript{r} and HH\textsubscript{r}. Note that all of these parameters are only for one particular point on the HOE and for one particular ray. The calculations shown are carried out for each ray that is traced. The direction cosines for the incoming ray, the normal to the surface and the construction points were calculated from the ray parameters given by SYNOPSYS via Eqs. (4.15), (4.16) and (4.17). Each of the unit
vectors in the expression for $U$ (Eq. (4.9)) were written in terms of the axis unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$:

$$\hat{f}_q = K_q \hat{i} + L_q \hat{j} + M_q \hat{k} \quad (4.18)$$

By collecting like terms in the three axis unit vectors and letting:

$$A = K_c + m \frac{\lambda_{uu}}{\lambda_c} (K_o - n_{eff} K_r) \quad (4.19)$$

$$B = L_c + m \frac{\lambda_{uu}}{\lambda_c} (L_o - n_{eff} L_r) \quad (4.20)$$

$$C = M_c + m \frac{\lambda_{uu}}{\lambda_c} (M_o - n_{eff} M_r) \quad (4.21)$$

then,

$$U = A \hat{i} + B \hat{j} + C \hat{k} \quad (4.22)$$

Also,

$$\hat{n} \cdot U = K_n A + L_n B + M_n C \quad (4.23)$$

$$U^2 = A^2 + B^2 + C^2 \quad (4.24)$$
The exiting ray vector was computed from the above expressions:

\[
\hat{r}_i = \left( A - K_n(\hat{a} \cdot \hat{U}) + K_n\sqrt{1 - \hat{U}^2 + (\hat{a} \cdot \hat{U})^2} \right) \hat{i} \\
+ \left( B - L_n(\hat{a} \cdot \hat{U}) + L_n\sqrt{1 - \hat{U}^2 + (\hat{a} \cdot \hat{U})^2} \right) \hat{j}
\]

\[
+ \left( C - M_n(\hat{a} \cdot \hat{U}) + M_n\sqrt{1 - \hat{U}^2 + (\hat{a} \cdot \hat{U})^2} \right) \hat{k},
\]

or

\[
\hat{r}_i = K_i\hat{i} + L_i\hat{j} + M_i\hat{k},
\]

where,

\[
K_i = A - K_n(\hat{a} \cdot \hat{U}) + K_n\sqrt{1 - \hat{U}^2 + (\hat{a} \cdot \hat{U})^2}
\]

\[
L_i = B - L_n(\hat{a} \cdot \hat{U}) + L_n\sqrt{1 - \hat{U}^2 + (\hat{a} \cdot \hat{U})^2}
\]

\[
M_i = C - M_n(\hat{a} \cdot \hat{U}) + M_n\sqrt{1 - \hat{U}^2 + (\hat{a} \cdot \hat{U})^2}
\]

The normalized direction cosines, \(ZZ_i\) and \(HH_i\), were then calculated from Eqs. (4.12) and (4.13) and passed back to the SYNOPSYS program. The changes made to the subroutine were verified with both the program in Appendix 1 and CODEV (but only for the case of \(n_{\text{eff}} = 1.0\), see below). A SYNOPSYS listing, single ray traces and a *.RLE file for a sample system similar to the one in Figure 4.2 is given in Appendix 2.

\textit{CODE V}
Neither of the above solutions could be applied to CODE V. Hence, it was impossible to properly model the FGC with CODE V. A number of preliminary designs were done with CODE V to illustrate the merits of the virtual optical system approach, these will be discussed in Chapter 6.

**Drawbacks**

A number of problems are associated with using the integrated approach to design the construction optics for the FGC. The designs that will be shown in Chapter 6 include non-rotationally symmetric lenses such as cylindrical and toroidal elements. Aspheric elements are also included in these designs. As well as being difficult to fabricate, these elements would also be difficult to test.

The systems to be discussed have been optimized to minimize the aberrations at \( i' \) in Figure 4.1. Note that this is one of the end-use construction point sources which is used to create one of the aberrated construction beams. Ultimately, the performance of the end-use final spot is desired. The residual aberrations that exist at point \( i' \) can be transferred to the end-use final spot for evaluation in the following manner. Eliminate the residual aberrations at \( i' \) by freezing all the previous variables in the system and optimize by varying only the coefficients of a wavefront polynomial that can be defined on the HOE surface. The result is an HOE system with two perfect construction point sources, some conventional construction optics, two perfect end-use point sources and a wavefront polynomial at the surface of the HOE. The wavefront polynomial can then be transferred to the end-use final spot. The result is the aberrations that exist at the end-use final spot with an HOE system that has two perfect construction point sources, some intervening construction optics and a perfect reconstruction point source.
5. ROTATIONALLY SYMMETRIC SOLUTION

Changes to original system

The major problem with the solution discussed in Chapter 4 is the complicated non-rotationally symmetric systems required for the construction optics. A method, using only rotationally symmetric construction optics, of producing non-waveguide HOE’s when wavelength scaling is present has been demonstrated.\textsuperscript{25–26} A similar approach but for waveguide HOE’s, i.e. FGC’s, has also been described.\textsuperscript{27–28} This second method will be discussed here.

A couple of changes to the original end-use system, Figure 3.5, will simplify the problem and allow this approach to be used. The beam in the waveguide, \( c \), should now be collimated which can be done with a waveguide collimating lens as shown in Figure 5.1. Although not necessary, as will be illustrated, the following development assumes that the chief ray of the end-use beam is perpendicular to the grating surface, also shown in Figure 5.1.

Mathematical justification

The correct interference pattern will be generated and aberration-free imaging will be accomplished if the two construction beams \( o \) and \( r \) are phase matched to the two end-use beams \( c \) and \( i \), respectively. Matching the phase of the two collimated beams \( c \) and \( o \) gives:

\[
\frac{2\pi n_{\text{eff}}}{\lambda_u} \sin(\theta_c) = -\frac{2\pi}{\lambda_c} \sin(\theta_o) , \tag{5.1}
\]

where \( \lambda_u \) is the end-use wavelength, \( \lambda_c \) is the construction wavelength, \( n_{\text{eff}} \) is the effective index of the waveguide and \( \theta \) is the angle from the normal to the collimated beam as shown in Figure 5.1. Hence, the required angle for the collimated construction beam is:
Figure 5.1. Configuration used in rotationally symmetric design. $o$ and $r$ are the contraction beams and $c$ and $i$ are the end-use beams. Both $r$ and $i$ lie on a normal from the grating surface. The end-use beam $c$ is collimated before the grating with a waveguide collimating lens.

$$\theta_o = - \sin^{-1} \left( n_{\text{eff}} \frac{\lambda_c}{\lambda_u} \right).$$  \hspace{1cm} (5.2)

The phase at the surface of the grating from converging beam $i$ is:
\[ \phi_i = \frac{R_i}{\lambda_u} \left[ \sqrt{1 + \frac{\rho^2}{R_i^2}} - 1 \right], \quad (5.3) \]

and the phase at the surface from the diverging beam at \( r \) is:

\[ \phi_r = -\frac{R_r}{\lambda_c} \left[ \sqrt{1 + \frac{\rho^2}{R_r^2}} - 1 \right]. \quad (5.4) \]

where \( R_i \) is the distance from the grating to the end-use point \( i \), \( R_r \) is the distance from the grating to the construction point \( r \) and \( \rho^2 = x^2 + y^2 \), the radial coordinate on the grating surface. Note that the negative signs in Eq. (5.1) and (5.4) are used to account for the reversal of direction in the construction beams.

The phase difference at the surface between \( i \) and \( r \) can be found by expanding Eq. (5.3) and (5.4) in a power series and taking the difference:

\[ \phi_i - \phi_r = 0 = \]

\[ 2\pi \left[ \frac{\rho^2}{2} \left( \frac{1}{\lambda_u R_i} + \frac{1}{\lambda_c R_r} \right) - \frac{\rho^4}{8} \left( \frac{1}{\lambda_u R_i^3} + \frac{1}{\lambda_c R_r^3} \right) + \frac{\rho^6}{16} \left( \frac{1}{\lambda_u R_i^5} + \frac{1}{\lambda_c R_r^5} \right) \cdots \right]. \quad (5.5) \]

To the second order, the zero phase condition can be satisfied by letting

\[ \lambda_u R_i = \lambda_c R_r. \quad (5.6) \]

The \( \rho^4 \) and higher-order terms represent spherical aberration where the fourth-order term is
Hence, to meet the zero phase condition for the two sets of beams the proper angle must be chosen via Eq. (5.2) and the spherical aberration via Eq. (5.5) must be corrected. The spherical aberration can be corrected with known techniques using only rotationally symmetric elements. Note that all the spherical aberration terms are independent of the effective index of the waveguide. The same optical system used to correct the spherical aberration could then be used with waveguides that had different refractive indices. A different effective index in the waveguide would only require a change in the angle of the collimated construction beam. Some examples of systems to correct the spherical aberration will be discussed in Chapter 6.

Computer modelling without HOE's

The design could be performed by defining an HOE in a lens design program with the two end-use points and then tracing rays through the system as described in Chapter 5. As discussed in Chapter 4, this requires either a modified lens design code or a special user defined surface. A much more convenient method has been described\textsuperscript{27} that allows the construction optics to be designed as a conventional optical system without the use of holographic elements. The phase of the end-use beam $\phi_i$ at wavelength $\lambda_u$ will be identical to the phase $\phi'_i$ at wavelength $\lambda_c$ as long as the beam is now in a fictitious medium of index $n_r$, where $n_r$ is less than one, i.e.,

$$\phi'_i = \frac{n_r R_i}{\lambda_c} \left[ \sqrt{1 + \frac{\rho^2}{R_i^2}} - 1 \right].$$ \hspace{1cm} (5.8)

where
In summary, the rays for beam \( i \) can be traced at wavelength \( \lambda_c \) instead of wavelength \( \lambda_u \) if they are in a fictitious medium with a refractive index of \( \frac{\lambda_c}{\lambda_u} \).

Figure 5.2 shows a schematic of a system to model the FGC by using only rotationally symmetric construction optics. Rays are started at end-use point \( U_2 \) with wavelength \( \lambda_c \) and traced backwards through a fictitious medium with index \( n_f \). The numerical aperture of the end-use system is determined by this cone of rays. The rays are refracted at the fictitious medium - air interface which represents the grating surface. The tracing is continued forward, now in air, through an optical system to point \( S \). The optical system can now be designed to minimize the spherical aberration at point \( S \). In addition, gratings that produce non-symmetric end-use beams can still be generated with rotationally symmetric construction optics as illustrated in Figure 5.3.

The above development has restricted itself to collimated end-use beams \( c \) and \( o \) in Figure 5.1. If this is not the case the construction wavelength \( \lambda_c \) can be used as another degree of freedom to solve the problem. It is still desirable to use rotationally symmetric elements in only one of the construction paths hence we still want to match the phases in beam \( c \) and \( o \) but now they are not collimated. Phase matching will occur approximately when a converging beam \( o \) is near grazing incidence to the surface and comes to a focus at the origin of the guided wave beam \( c \). The correct phase matching can be accomplished when \( \theta_o \) in Eq. (5.2) is equal to \( 90^\circ \). When \( \theta_o \) is approximately \( 90^\circ \):

\[
\lambda_c \approx \frac{\lambda_u}{n_{\text{eff}}}.
\]
Therefore, by proper selection of $\lambda_c$ a non-collimated construction beam can be used near grazing incidence with no optics required in that path. Rotationally symmetric optics are still needed in the other construction beam path to correct the spherical aberration as described above.

Figure 5.2. System to model the construction optics for the FGC using fictitious glass with an index of $n_f = \frac{\lambda_c}{\lambda_u}$. The end-use spot $i$ is in the fictitious glass but the rays are traced at the wavelength $\lambda_c$. The end-use N.A. is determined by this cone of rays. Upon refraction at the "grating surface" an optical system can be designed with rotationally symmetric optics to eliminate the spherical aberration at point $S$. 

Figure 5.3. Similar layout to that shown in Figure 16 but here a non-symmetric end-use beam can be formed. Note that rotationally symmetric optics are still used but, by an appropriately decentered aperture, the desired end-use beam can be generated.

\[ n_f = \frac{\lambda_c}{\lambda_u} \]
6. DESIGNS/STUDIES

Integrated designs

A number of designs using the integrated solution of Chapter 4 have been completed on the CODE V lens design program but as mentioned in Chapter 4 the modified grating equation, Eq. (4.1), could not be used. In addition to the designs in Reference 22 two additional higher numerical aperture designs have been completed and will be discussed here.

The end-use point in Figure 3.6 has now been shifted such that it is parallel to the HOE surface and the spot o is at an angle of 20 degrees above the center line as shown in Figure 3.6. Due to the anamorphic nature of the beam converging to the point i in Figure 4.1, it may be appropriate to use non-axially symmetric elements such as cylindrical lenses in the construction optics. As used by Heitmann and Ortiz, a cylindrical lens and a spherical lens in each construction beam can be used as a starting point in eliminating the aberrations at i'. A CODE V layout in the y-z plane for one design is shown in Figure 6.1 along with the same system but in the x-z plane shown in Figure 6.2. A CODE V listing and sequence file for this design are included in Appendix 3. In this configuration surfaces S3, S4, and S9 are cylindrical in the x direction. It was also found that the elimination of the aberrations was helped by making one of the surfaces torodial, surface S8, and one of the surfaces aspheric, surface S10. This design has an end-use (point o) numerical aperture of approximately 0.4 and a Strehl ratio of 0.84. Figures 6.3 and 6.4 show the wavefront aberration plot and point spread function, respectively, at point i' in Figure 6.1.

The complicated beams needed to correct the aberrations could be constructed with other point-source-constructed HOE's instead of conventional optics. These other HOE's would be easier to fabricate and also easier to test than the complicated conventional optics designs. One such design is shown in Figure 6.5 which has an
end-use numerical aperture of approximately 0.3 and a Strehl ratio of 0.83. The CODE V listing and sequence file are included in Appendix 4. The wavefront aberration plot and point spread function are given in Figures 6.6 and 6.7, respectively.

Unlike the designs shown, to correctly model the FGC the modified grating equation, Eq. (4.1), must be used. Designs similar to the ones described above were being developed with the SYNOPSIS lens design program using the modified grating equation when the much simpler rotationally-symmetric design approach was developed. Consequently, the more difficult and higher numerical aperture designs using SYNOPSIS were not continued.
Figure 6.1. Design done by integrated method with conventional optics in the construction beams. The design is shown in the y-z plane. Surfaces S3, S4 and S9 are cylindrical, surface S8 is toroidal and surface S10 is aspheric. The design has an end-use numerical aperture of approximately 0.4 and a Strehl ratio of 0.84.

Figure 6.2. Same design as that in Figure 6.1 but now shown in the x-z plane. Note the size of the beam at the HOE compared to that in Figure 6.1, i.e., the beam at the HOE is elliptical.
Figure 6.3. Wavefront aberration plot at point $i'$ for the design shown in Figure 6.1.
Figure 6.4. Point spread function at point $i'$ for the design shown in Figure 6.1.
Figure 6.5. Design done by integrated method with point-source-constructed HOE's in the construction beams.
Figure 6.6. Wavefront aberration plot at point \( i' \) for the design shown in Figure 6.5.
Figure 6.7. Point spread function at point $i'$ for the design shown in Figure 6.5.
A design using the rotationally symmetric solution of Chapter 5 has been demonstrated and will be discussed here. Table 1 gives some key parameters for this particular design. To correct for the negative fourth order spherical aberration, Eq. 5.7, a plane parallel plate could be used. If used on the air side of the grating between $r$ and the grating (refer to Figure 5.2), the plane parallel plate would produce negative fourth order spherical aberration and hence not help in the correction. The plane parallel plate could be used on the substrate side of the grating to produce positive spherical aberration but as discussed in Chapter 2 this is an undesirable configuration. The plane parallel plate or, in fact, a simple lens could be used before point $r$ to generate spherical aberration with the correct sign. For the system in Table 1 a significant amount of higher order spherical aberration is present and three plano-convex elements as shown in Figure 6.8 were found to be necessary to correct the aberrations. The CODE V listing and sequence file for this system are given in Appendix 5. The design in Figure 6.8 is very similar to the Offner field lens design where a field lens is place at or near the intermediate focus to generate higher order spherical aberration. Figure 6.9 shows the wavefront aberration at point $S$ and Figure 6.10 shows the spot diagram at point $S$. The circle drawn in Figure 6.10 represents the first dark ring in the Airy disk pattern at $\lambda_u$, hence the image at point $S$ is diffraction limited.

Table 1. Key parameters of design.

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>End-use wavelength</td>
<td>0.78 $\mu$m</td>
</tr>
<tr>
<td>Construction wavelength</td>
<td>0.48 $\mu$m</td>
</tr>
<tr>
<td>End-use numerical aperture</td>
<td>0.65</td>
</tr>
<tr>
<td>Diameter of grating</td>
<td>3 mm</td>
</tr>
<tr>
<td>Working distance</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>Theoretical spot size ($0.78 \mu$m)</td>
<td>1.46 $\mu$m</td>
</tr>
<tr>
<td>Fictitious glass index</td>
<td>0.61538</td>
</tr>
<tr>
<td>Glass for construction optics</td>
<td>BK-7, Schott</td>
</tr>
</tbody>
</table>
Figure 6.8. Rotationally symmetric design that has grating parameters as given in Table 1. The source point is the end-use point \( i \). The diverging light from \( i \) is in a fictitious medium with refractive index \( n_f = 0.615 \). At the grating surface (fictitious medium - air interface) the light is refracted and proceeds through the system to the image point \( S \). Both an enlarged view of the elements and a full layout are shown.
Figure 6.9. Wavefront aberration plot at point S for the system shown in Figure 6.8.
Figure 6.10. Spot diagram at point S for the system shown in Figure 6.8. The circle is the first dark ring of the Airy pattern at the end-use wavelength.
7. CONCLUSION

Two design methods have been developed to minimize the aberrations of holographic optical elements when they undergo a wavelength scaling. The particular configuration developed can be used to model focusing grating couplers that have possible applications in optical data storage. Both methods take a geometrical optics approach to designing the holographic optical elements and both methods make use of commercially available ray trace programs. The geometrical analysis cannot be used solely to completely characterize these devices. A physical optics analysis must also be performed. Physical optics can be used to calculate such parameters as coupling coefficients, diffraction effects of a finite size grating, and polarization effects. In fact, situations could exist where the geometrical optics predict perfect imaging but the physical optics would show intensity and polarization variations across the pupil of the radiated mode. The geometrical optics is an essential first step in the full evaluation and is a more efficient and insightful approach for understanding the devices.

The first method investigated models both the holographic optical element and the construction optics as one integrated system and requires either a modified ray trace program or special user defined surfaces. One of the designs demonstrated required complicated construction optics even for a modest numerical aperture. The construction optics for this system included non-rotationally symmetric optics such as toroids and cylindrical elements. This method proved to be feasible but difficult to implement.

The second method involved a much simpler approach which did not require any changes to an existing ray trace program. The solution required a fictitious medium with an index equal to the ratio of the construction and end-use wavelengths and did not even require that HOE ray tracing be available in the program. A design with an end-use numerical aperture of 0.65 was demonstrated and required only three
rotationally symmetric elements to correct the aberrations.

To simplify the studies a number of details of actual optical data storage configurations were omitted. One in particular is an addition of a cover layer between the device and the optical disk which would introduce spherical aberration. When tracing rays for end-use a thin glass plate can be inserted to accommodate the design methods, remembering that the index has to be scaled by the ratio of the wavelengths if the rotationally symmetric design method is used. The solution is not as simple when considering the definition of the HOE, i.e., the construction point sources. The departure from perfect two-point construction, i.e., spherical aberration, when a glass substrate is included would have to be calculated in terms of a polynomial expansion and the coefficients added to the definition of the HOE.
Appendix 1: FORTRAN program for calculating HOE ray angles.

```fortran
program hoeang
implicit double precision (a-h,k-z)
dimension ys(3), kangle(3), ktan(3), langle(3), ltan(3),
zz(3), hh(3)
character*12 input, output1, output2

calculate the direction of the exiting ray at the +/- edge and center of the
aperture (y coordinate only, x coord = 0.0) of an HOE grating.
Angle (alpha) of incoming ray varies from -90 to +90.

c input file format:
alpha = rotation of surface about x axis (in degrees)
beta = rotation of surface about y axis (in degrees)
(note alpha rotation is first)
dis = axial distance from object point source to hoe
x1, y1, z1 = coordinates of first construction point ("object")
x2, y2, z2 = coordinates of second construction point ("reference")
iv1, iv2 = virtual or real? for each construction point
  1 = real
  -1 = virtual
order = diffraction order
endlam = end-use wavelength
conlam = construction wavelength
gin = waveguide index
size = aperture size (diameter)
inum = number of points
filename = graphics output file

c read input
write(6,900)
format(' Enter input filename: ',) 
read(5,910) input
format(a)
open (unit=20, file=input, status='old', err=1000)
go to 916
1000 write(6,915)
format(' File does not exist')
go to 899
915 read (20,*) alpha,beta,dis
read (20,*) x1,y1,z1
read (20,*) x2,y2,z2
read (20,*) iv1,iv2
read (20,*) order, endlam, conlam, gin, size, inum

c graphics output files
write(6,920)
format(' Enter output filename (angles): ',)
read(5,910) output1
open(unit=10, file=output1, status='old', err=1010)
```

go to 930
write(6,915)
go to 919
930 write(6,931)
931 format('Enter output filename (ratio of dirn. cosines):','
read (5,910) output2
open(unit=11, file=output2, status='old', err=1011)
go to 935
1011 write(6,915)
go to 930
C C write input to output file
935 open(unit=9, file='disk2:[cronkite.hoedrn]output.dat',
status='old')
write(9,200) alpha, beta, dis, gin
200 format ('05x,'alpha=',f8.4,5x,'beta=',f8.4,5x,'dis=',f8.4,5x,'gin=',f6.3)
write(9,201)x1,y1,z1
201 format ('05x,'construction 1:*3f 15.5)
write(9,202)x2,y2,z2
202 format ('05x,'construction 2:*3f 15.5)
write(9,203)endlam,conlam
203 format ('05x,'endlam=',f5.1,5x,'conlam=',f5.1)
write(9,204)size, inum
204 format ('05x,'aperture size=',f5.2,5x,'number=',i5)
C C

step = 180 / inum
ys(1)=size/2
ys(2)=0.0
ys(3)=-size/2
C C main loop for number of data points
do 100 i=1,inum
C alpha = -90.0 + step*(i-1)
xs = 0.0
C C main loop for three y coordinates
do 700 j=1,3
C C calculate coordinates (in hoe system) of object point source
x = dis*dcosd(alpha)*dsind(beta)
y = dis*dsind(alpha)
z = -dis*dcosd(alpha)*dcosd(beta)
C C do calculation for each y coordinate w/ x = 0.0
C C C dirn cosines of in coming ray
xdiff = (xs - x)
ydiff = (ys(j) - y)
mag = dsqrt( xdiff**2 + ydiff**2 + z**2 )
kin = xdiff / mag
lin = ydiff / mag
min = dsqrt( 1 - kin**2 - lin**2 )
if (min.ne.0.0) go to 1100
zzin = 100.0
hhin = 100.0
go to 1101
1100
zzin = kin / min
hhin = lin / min

C
dirn cosine of construction optics

1101
xdiff = (xs - x1)
ydiff = (ys(j) - y1)
mag = dsqrt( xdiff**2 + ydiff**2 + zl**2 )
k1 = xdiff / mag
l1 = ydiff / mag
m1 = dsqrt( 1 - k1**2 - l1**2 )
zzl = k1 / m1
hhl = l1 / m1

C
if (iv1.ne.-1) go to 20
k1 = -k1
l1 = -l1
zzl = -zzl
hhl = -hhl
20
continue

xdiff = (xs - x2)
ydiff = (ys(j) - y2)
mag = dsqrt( xdiff**2 + ydiff**2 + z2**2 )
k2 = xdiff / mag
l2 = ydiff / mag
m2 = dsqrt( 1 - k2**2 - l2**2 )
zz2 = k2 / m2
hh2 = l2 / m2

C
if (iv2.ne.-1) go to 30
k2 = -k2
l2 = -l2
zz2 = -zz2
hh2 = -hh2
30
mu = order*endlam/conlam

C
dirn cosines of exiting ray

kout = (kin + mu*( k1 - gin*k2 ))/gin
lout = (lin + mu*( l1 - gin*l2 ))/gin
if (kout.ge.1.0) kout=1.0
if (kout.le.-1.0) kout=-1.0
if (lout.ge.1.0) lout=1.0
if (lout.le.-1.0) lout=-1.0
mout = dsqrt( 1 - kout**2 - lout**2 )
if (mout.ne.0.0) go to 1110
zzout = 100.0
hhout = 100.0
go to 1111

1110
zzout = kout / mout
hhout = lout / mout

C
C angles and tan of angles
1111
kangle(j) = dasind(kout)
langle(j) = dasind(lout)

C
ktan(j) = dtand(kangle(j))
ltan(j) = dtand(langle(j))

C
zz(j) = zzout
hh(j) = hhout

700 continue

c
C output to graphic file
write(10,600)alpha, langle(1), langle(2), langle(3)
write(11,600)alpha, hh(1), hh(2), hh(3)

600
format( '4f15.5')

C
C output
write(9,298)xs,ys,alpha
format( '0',5x,3fl2.4)
write(9,301) kin,lin
format( '10x,2f12.5)
write(9,306)kl,li
format( '10x,k1, l1 =',t30,2f12.5)
write(9,308)k2,l2
format( '10x,k2, l2 =',t30,2f12.5)
write(9,305)kout,lout
format( '10x,kout, lout =',t30,2f12.5)
write(9,302)kout,lout
format( '10x,kout, lout =',t30,2f12.5)
write(9,302)kout,lout
format( '10x,angles: z30,2f12.5)
format( '10x,zzout, hhout =',t30,2f12.5)
continue
close(unit=9)
close(unit=10)
500
close(unit=20)
stop
end
Appendix 2: SYNOPSYS listing, single ray traces and .RLE file for a sample system with waveguide effective index equal to 1.5.

CALL SPE
LENS SPECIFICATION
ID R(R) = 30 W/O OPTICS

| Obj. Dist.     | 45.7470 | Focal Length | 19.1803 |
| Obj. Height    | 0.0000  | Back Focal Dist. | 24.4294 |
| Marg. Ray Height | 0.0000  | Paraxial Focal P. | 33.0278 |
| Chief Ray Height | 0.0000  | Overall Length | 0.0000  |
| Marg. Ray Angle | 6.2375  | Entr. Pupil Pos. | 0.0000  |
| Chief Ray Angle | 0.0000  | Exit Pupil Pos. | 0.0000  |
| F/Number       | 3.3028  | Gaussian Im. HT. | 0.0000  |

WAVELENGTHS 0.48800 0.48800 0.48800
UNITS MM
STOP IS ON SURF. NO. 2
LENS IS FOCAL, MAGNIFICATION -0.534010
GLOBAL OPTION IS ON
POLARIZATION AND COATINGS ARE IGNORED.

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<td>AIR</td>
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<td>INFINITE</td>
<td>0.00000</td>
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</tr>
<tr>
<td>5</td>
<td>INFINITE</td>
<td>0.00000</td>
<td>AIR</td>
</tr>
</tbody>
</table>

NOTE: ITEMS MARKED "P" OR "S" ARE SUBJECT TO PICKUPS OR SOLVES

DEFORMATION COEFFICIENTS
2 HOLOGRAPHIC ELEMENT OF ORDER 1.
INDEX AVERAGE, MODULATION 0.100000E+01 0.100000E+01
HOE THICKNESS 0.100000E-04
CONSTRUCTION WAVELENGTH 0.780000E+00
POINT ONE AT (X,Y,Z) 0.000000E+00 0.684040E+01 -0.187938E+02 -1.
POINT TWO AT (X,Y,Z) 0.000000E+00 -0.300000E+02 0.100000E-06 1.

TILTS AND DECENTERS ALPHA, BETA, GAMMA, AXIS
X-DECN, Y-DECN, Z-DECN
1 TDC 10 SURFACES
0.969802E+01 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00 0.000000E+00 0.000000E+00
4 TDC 10 SURFACES
-0.632353E+02 0.000000E+00 0.000000E+00 0.000000E+00
Single ray traces

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ENTER FILE NAME>RAY

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<th>Z</th>
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<tr>
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\[rk, rl = 0.00000 -0.27456\]

\[zza, hha = 0.00000 -0.63901\]

\[ak, al = 0.00000 0.53846\]

\[zzb, hhb = 0.00000 -1000.00000\]

\[bk, bl = 0.00000 1.00000\]

\[zzout, hhout = 0.00000 -1.81744\]

\[exitk, exitl = 0.00000 -0.87613\]

\[3 0.00000E+00 -0.516904E+01 0.00000E+00 0.00000E+00\]

\[-0.181744E+01\]

\[4 0.00000E+00 -0.249344E+01 0.00000E+00 0.00000E+00\]

\[0.358992E-01\]

\[0.00000000E+00 -0.16164502E+01 0.00000000E+00 0.00000000E+00\]

**RLE file:**

```
RLE P
ID R(R) = 30 W/OUT OPTICS 12
WAVL 0.48800000 0.48800000 0.48800000
APS 2
GLOBAL
UNITS MM
CBA 45.747 1.00000E-05 5.00000 0.00000E+00 0.00000E+00
0.00000E+00
0 AIR
1 CV 0.00000000E+00 TH 0.00000000E+00 AIR
1 DEC 0.00000000E+00 0.00000000E+00 0.00000000E+00 10
1 AT 9.69802 0.00000000E+00 10
2 PIN 1
2 HOE
HIN 1.00000 1.00000
HTH 0.00000000E-04
CWAV 0.780000
P1 0.00000000E+00 6.84040 -18.7938 -1.
P2 0.00000000E+00 -30.0000 0.000000E+00 0.00000000E-06 1.
ORDER 1.00000
3 CV 0.00000000E+00 TH 0.00000000E+00 AIR
4 CV 0.00000000E+00 TH 24.429558 AIR
4 DEC 0.00000000E+00 0.00000000E+00 0.00000000E+00 10
4 AT -63.2353 0.00000000E+00 10
5 CV 0.00000000E+00 TH 0.00000000E+00 AIR
END```

Appendix 3: CODE V listing and sequence file for integrated system design using conventional optics.

**CODE V> lis**

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<td><strong>YTO:</strong></td>
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<tr>
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<tr>
<td><strong>A:</strong></td>
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<td><strong>AC:</strong></td>
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<td>9:</td>
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<tr>
<td><strong>CYL:</strong></td>
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STO:  -22.07415  -4.000000  BK7_SCHOTT  0  0
ASP:
  K :  -1.000000  KC :  100
  IC :  YES  CUF:  0.000000  CCF:  100
  A :  -0.646823E-06  B :  -0.130984E-07  C :  0.318118E-10  D :  0.187701E-12
  AC :  0  BC :  0  CC :  0  DC :

11:  60.70812  -75.000000  0  0
CYL:
  RDX:  INFINITY  CCX:  100

> IMG:  INFINITY  0.000000  100  100

SPECIFICATION DATA
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  DIM  MM
  WL  488.00
  REF  1
  WTW  1
  XAN  0.00000
  YAN  0.00000
  VUX  0.00000
  VUX  0.00000
  VUX  0.00000
  VUX  0.00000

REFRACTIVE INDICES
  GLASS CODE  488.00
  BK7_SCHOTT  1.522243

No solves defined in system

This is a decentered system. If elements with power are
decentered or tilted, the first order properties are probably
inadequate in describing the system characteristics.

INFINITE CONJUGATES
  EFL  -30.9613
  BFL  -233.2796
  FFL  7.0267
  FNO  -4.4761

AT USED CONJUGATES
  RED  -1.1456
  FNO  -5.0000
  OBJ DIS  20.0000
  TT  -87.1488
  IMG DIS  -75.0000
  OAL  -32.1488
PARAXIAL IMAGE
Sequence file

RDM;LEN
TITLE 'Construction optics using lenses'
NA 0.1
DIM M
WL 488.
REF 1
WTW 1
INI ' ' 
CA
XAN 0.
YAN 0.
VUX 0.
VLX 0.
VUY 0.
VLY 0.
SO 0. 20.
S -4.60744549642 1. BK7_SCHOTT
   CCY 0 ; THC 0
S -20.972905501 0.25
   CCY 0 ; THC 0
S 0. 1. BK7_SCHOTT
   THC 0
   CYL
   CUX 0.02221779596 ; CCX 0
S 0. 16.2961688329
   THC 0
   CYL
   CUX -0.071115816218 ; CCX 0
S 0.1E+14 0.
HOE
   CUX 0.1E-12 ; CCX 100 ; HV1 VIR ; HV2 REA
   HOR -1 ; HTH 0. ; HIN 0. ; HDI 0.
   HX1 0. ; HY1 6.840403; HZ1 -18.793852
   HX2 0. ; HY2 -50. ; HZ2 0.0000001
   HWL 780.00 ; HTO SPH ; HNO 0
   XDE 0. ; YDE 0.
   ADE 24.6650055557 ; BDE 0. ; CDE 0.
ADC 0 ; BDC 100 ; CDC 100
S 0.1E+13 25.
XDE 0. ; YDE 0.
ADE -55.989959849 ; BDE 0. ; CDE 0.
ADC 0 ; BDC 100 ; CDC 100
S 0. -50.
THC 0
S 9.14868490713 -4. BK7_SCHOTT
CCY 0 ; THC 0
YTO
CUX -0.038323073102 ; CCX 0
K -1. ; IC Yes
A 0.000523555267; B 0.39523732E-05; C 0.59953844E-15; D
-0.11146489E-17
AC 0 ; BC 0 ; CC 0 ; DC 0
S 0. -17.69499872211
THC 0
CYL
CUX -0.04593179616 ; CCX 0
S -22.0741479359 -4. BK7_SCHOTT
CCY 0 ; THC 0
STO
ASP
K -1.
IC Yes ; CUF 0.
A -0.64682275E-06; B -0.13098354E-07; C 0.31811833E-10; D
0.18770073E-12
AC 0 ; BC 0 ; CC 0 ; DC 0
S 60.7081178587 -75.
CCY 0 ; THC 0
CYL
CUX 0.
SI 0. 0.
GO
Appendix 4: CODEV listing and sequence file for integrated system design using holographic optical elements.

```
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Construction optics using HOE's

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No refractive materials defined in system

No solves defined in system

This is a decentered system. If elements with power are decentered or tilted, the first order properties are probably inadequate in describing the system characteristics.

INFINITE CONJUGATES

| EFL  | -7.7981 |
| BFL  | -25.9039 |
| FFL  | 7.1755  |
| FNO  | -0.9748 |
| IMG DIS | -25.0476 |
| OAL  | -10.0084 |
| PARAXIAL IMAGE |
| HT   | 0.0000  |
| ANG  | 0.0000  |
| ENTRANCE PUPIL |
| DIA  | 8.0000  |
Sequence file

RDM;LEN
TITLE "Construction optics using HOE's"
EPD 8.
DIM M
WL 488.
REF 1
WTW 1
INI '
CA
XAN 0.
YAN 0.
VUX 0.
VLX 0.
VUY 0.
VLY 0.
SO 0. 152109481646.
THC 0
S 0. 0.
HOE
CUX 0.; CCX 100.; HV1 REA.; HV2 REA
HOR -1.; HTH 0.; HIN 0.; HDI 0.
HX1 0.; HY1 -0.565710199173; HZ1 -11.7739934063
CY1 0.; CZ1 0.
HX2 0.; HY2 6.7145977148; HZ2 -34.2564675997
CY2 0.; CZ2 0.
HWL 488.00.; HTO SPH.; HNO 0
XDE 0.; YDE 0.
ADE -1.146447535.; BDE 0.; CDE 0.
ADC 0.; BDC 100.; CDC 100
S 0.18.3334706393
THC 0
XDE 0.; YDE 0.
ADE 5.88114050527.; BDE 0.; CDE 0.
ADC 0.; BDC 100.; CDC 100
S 0.1E+14 0.
STO
HOE
CUX 0.1E-12; CCX 100.; HV1 VIR.; HV2 REA
HOR -1.; HTH 0.; HIN 0.; HDI 0.
HX1 0.; HY1 -40.; HZ1 0.00000001
HX2 0.; HY2 8.5505.; HZ2 -23.4923
HWL 632.00.; HTO SPH.; HNO 0
XDE 0.; YDE 0.
ADE 27.0103561401; BDE 0.; CDE 0.
ADC 0.; BDC 100.; CDC 100
S 0.1E+14 21.5724974
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ADE -51.0780736377; BDE 0.; CDE 0.
ADC 0.; BDC 100.; CDC 100
S 0. -49.9143211654
THC 0
S 1000000000. 0.
HOE
CUX -0.002034902575; CCX 0.; HV1 REA; HV2 VIR
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HX1 0.; HY1 54.572453184; HZ1 -39.7942714638
CY1 0.; CZ1 0
HX2 0.; HY2 -4.7235349622; HZ2 26.2319382437
CY2 0.; CZ2 0
HXL 488.00; HTO XTO; HNO 0
XDE 0.; YDE 0.
ADE -60.3497275435; BDE 0.; CDE 0.
ADC 0.; BDC 100.; CDC 100
S 0. -25.0475529275
THC 0
XDE 0.; YDE 0.
ADE 8.50174425171; BDE 0.; CDE 0.
ADC 0.; BDC 100.; CDC 100
SI 0. 0.
GO
Appendix 5: CODEV listing and sequence file for rotationally symmetric design.

CODE V> lis
   HOLO: holographic lens

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> IMG: INFINITY

SPECIFICATION DATA
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DIM MM
WL 480.00
REF 1
WTW 1
INI gnl
XAN 0.00000
YAN 0.00000
VUY 0.00000
VLY 0.00000

APERTURE DATA/EDGE DEFINITIONS
CA
CIR S1 0.375000
CIR S2 0.150000
CIR S3 1.500000
CIR S4 0.150000
CIR S5 0.150000
CIR S6 1.000000
CIR S7 1.000000
CIR S8 1.200000
CIR S9 1.200000

PRIVATE CATALOG
PWL 480.00
'negn' 0.615380
'ainr' 1.000000
'gls' 1.517000

REFRACTIVE INDICES
GLASS CODE 480.00
'negn' 0.615380
No solves defined in system

INFINITE CONJUGATES
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  BFL    -1.1101
  FFL    -0.2482
  FNO     5.1301

AT USED CONJUGATES
  RED     6.6922
  FNO     8.3652
  OBJ DIS 0.7500
  TT      27.4718
  IMG DIS 21.2735
  OAL     5.4482

PARAXIAL IMAGE
  HT      0.0000
  THI     21.3651
  ANG     0.0000

ENTRANCE PUPIL
  DIA     0.6547
  THI     0.0000

EXIT PUPIL
  DIA     8.8599
  THI    -46.5626

CODE V> out t

Sequence file

RDM;LEN
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NAO  0.4
DIM M
WL  480.
REF  1
WTW  1
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CA
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YAN  0.
VUY  0.
VLY  0.
PRV
PWL  480.
'negn' 0.61538
'airn' 1.
'gls'  1.517
END
SO  0. 0.75
S   0. -0.75
STO
CIR 0.375
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CIR 0.15
S 0. 1.99314814413
THC 0
CIR 1.5
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CCY 0
CIR 0.15
S 0.75E+11 2.279012206
THC 0
CIR 0.15
S 0.1E+12 1. BK7_SCHOTT
CIR 1.
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CCY 0; THC 0
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CCY 0
CIR 1.2
S 0.1E+12 21.2735435646
THC 0
CIR 1.2
S 0. 0.
GO
REFERENCES


20. CODE V is a proprietary product of Optical Research Associates, Pasadena, California.


24. SUPEROSLO is a proprietary product of Sinclair Optics, Inc., Fairport, New York.


