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Strengthening of concrete beams with composite plastic plates

An, Wei, M.S.
The University of Arizona, 1990
STRENGTHENING OF CONCRETE BEAMS WITH COMPOSITE PLASTIC PLATES

by

Wei An

A Thesis Submitted to the Faculty of the
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In Partial Fulfillment of the Requirements For the Degree of
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In the Graduate College
THE UNIVERSITY OF ARIZONA

1990
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The author greatly appreciates her parents’ constant giving, love and support. Special appreciation is due my father, Y. X. An, a professor in Transportation Engineering, for his understanding, love and education during my entire life.
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$A$  area under concrete stress-strain curve
$a$  distance between external loads
$A_{pl}$  area of plate section
$A_{si}$  area of steel rebars
$b$  beam width
$b_1$  T-beam web width
$c$  depth of neutral axis to top concrete compression fiber
$C_c$  concrete compression force for rectangular beam
$C_{c1}$  concrete compression force between neutral axis and junction of T-beam
$C_{c2}$  concrete compression force between junction and top of T-beam
$d_i$  depth of steel rebars to section top
$d_{pl}$  depth of plate to section top
$E_c$  elastic modulus of concrete
$E_{pl}$  elastic modulus of plate
$E_{si}$  elastic modulus of steel
$F_{pl}$  ultimate strength of plate
$f_c$  stress in concrete
$f_c'$  concrete cylinder strength
$f_c''$  strength of concrete in a member
$f_{pl}$  stress in plate
$f_{pli}$  initial stress in plate due to cambering
$f_{si}$  stress in steel rebar
LIST OF SYMBOLS... cont.

\( f_{sti} \)  initial stress in steel rebar due to cambering
\( f_u \)  ultimate stress in plate
\( f_y \)  steel yield stress
\( G \)  force in plate
\( h \)  beam section height
\( h_1 \)  T-beam flange thickness
\( k \)  factor for concrete strength in a member
\( l \)  clear span
\( M \)  internal moment
\( P \)  internal force
\( Q \)  first moment of area about origin of area under concrete stress-strain curve
\( S_i \)  force in steel rebar
\( V_{mid} \)  deflection at midspan
\( \alpha \)  mean stress factor under concrete stress-strain curve for rectangular beam
\( \alpha_1 \)  mean stress factor under concrete stress-strain curve for T-beam
\( \alpha_2 \)  mean stress factor under concrete stress-strain curve for T-beam
\( \gamma \)  centroid factor to determine distance between centroid of area and the origin under concrete stress-strain curve for rectangular beam
\( \gamma_1 \)  centroid factor to determine distance between centroid of area and the origin under concrete stress-strain curve for T-beam
\( \gamma_2 \)  centroid factor to determine distance between centroid of area and the origin under concrete stress-strain curve for T-beam
\( \phi \)  curvature
\( \phi_{mid} \)  curvature at midspan
\( \varepsilon_c \)  strain in concrete
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<td>$\varepsilon_{cf}$</td>
<td>strain in top concrete fiber</td>
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<tr>
<td>$\varepsilon_{h1}$</td>
<td>concrete strain at junction of web and flange of tee beam</td>
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<tr>
<td>$\varepsilon_o$</td>
<td>concrete strain at maximum stress</td>
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<td>$\varepsilon_{pl}$</td>
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<td>$\varepsilon_{pli}$</td>
<td>initial strain in plate due to cambering</td>
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<tr>
<td>$\varepsilon_{si}$</td>
<td>strain in steel rebar</td>
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<tr>
<td>$\varepsilon_{sti}$</td>
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<td>$\varepsilon_u$</td>
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<td>$\varepsilon_y$</td>
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This study investigates the feasibility of strengthening reinforced concrete beams with epoxy-bonded Glass-Fiber-Reinforced-Plastic (GFRP) plates. The composite plate is epoxy-bonded to the tension flange of the beam to increase its stiffness and strength. Seven rectangular and one T-beam, retrofitted with composite plates, were tested to failure under symmetrical 4-point bending. The load versus deflection and the load versus strain in the composite plate, steel rebar and the extreme compression fiber of concrete were measured and plotted for the midspan section throughout the entire range of loading up to failure. Analytical models based on the equilibrium of forces and compatibility of deformations were developed to predict the stresses and deformations of the beam in the linear and nonlinear regions. The predicted and measured results correlated well. The analytical models were used in a parametric study to investigate the effects of design variables such as, plate area, plate strength and stiffness, reinforcement ratio, etc., on the moment-curvature relationships of typical rectangular and T cross sections.
CHAPTER 1

INTRODUCTION

As a result of decades of neglect and inadequate maintenance, many of the bridges in the United States are either structurally deficient and/or functionally obsolete and require immediate attention. For example, according to the Eighth Annual Report of the Secretary of Transportation to the Congress of the United States on “Highway Bridge Replacement and Rehabilitation Program” (HBRRP), more than 40 percent of the nation’s 574,729 inventoried highway bridges are in need of replacement and/or rehabilitation [1]. In addition, many of these bridges were designed for lower traffic volumes and lighter loads than are common today. Therefore, strengthening must be considered to bring these bridges up to current standards.

Rehabilitation is more cost-effective than replacement. By accomplishing rehabilitation early and before deterioration reaches an advanced stage, more costly bridge replacement can be avoided. In fact, the Federal Highway Administration recommends that the states, in developing bridge projects, consider the rehabilitation alternative before deciding to replace a structure [1]. There are several different methods that can be used to increase the live load capacity of bridges such as, (1) external post-tensioning; (2) steel plates epoxy-bonded to tension flange; and (3) plastic plates epoxy-bonded to tension flange.

External post-tensioning has been successfully used to increase the strength of girders in existing bridges and buildings. However, this method has several practical difficulties such as providing anchorage for the post-tensioning strands, maintaining the lateral stability of the girders during post-tensioning, and protecting the exposed strands against corrosion.

Epoxy-bonded steel plate has been used effectively in Europe, South Africa and Japan to increase the load-carrying capacity of existing bridges. The principles
of this strengthening technique are fairly simple. Steel plates are epoxy-bonded to
the tension flange, increasing both the strength and stiffness of the girder. The
advantages of this structural system include the ease of application and the elim-
ination of special anchorages needed in the post-tensioning method. However, a
shortcoming of this method is the danger of corrosion at the epoxy/steel interface
which adversely affects the bond strength. An effective approach to eliminate the
corrosion problem is to replace the steel plates with corrosion-resistant synthetic
materials such as fiber composites. In addition to corrosion resistance, many fiber
composites have tensile and fatigue strengths that exceed those of steel. In this
study the feasibility of using Glass-Fiber-Reinforced-Plastics (GFRP) plates in lieu
of steel to strengthen existing concrete beams will be investigated. The need to
improve existing methods as well as to develop new procedures and materials for
increasing or restoring the load-carrying capacity of existing bridges was the reason
for this research.
CHAPTER 2

LITERATURE REVIEW

An extensive literature search revealed no published report on strengthening of concrete girders with epoxy-bonded fiber composite plates. However, several studies were found on strengthening of concrete girders with epoxy-bonded steel plates. Summaries of these studies and also several field applications of this strengthening technique are given in the following.

2.1 Research work

McDonald and Calder (1978) investigated the technique of bonding steel reinforcement externally to hardened concrete [2]. The laboratory work consisted of testing 4.9 m (16 ft) and 3.5 m (11.5 ft) long concrete beams with steel plates bonded to their tension flanges, in 4-point bending. The variables studied were two different adhesives (A and B) and a jointed plate. All mild steel plates were 140 mm (5.5 in.) wide, 4.2 m (13.8 ft) long and 10 mm (0.4 in.) thick. The average glue thickness was 1.5 mm (1/16 in.). The mechanical properties of adhesives were not given.

They concluded that: (1) a progressive, ductile failure could be achieved by using a wide plate; (2) to obtain the maximum increase in the serviceability limit conditions of stiffness and crack control, a fully-bonded plate using a stiff adhesive was necessary; (3) to combine the maximum increase in ultimate load with a progressive ductile failure, the plate width-to-thickness ratio had an optimum value of about 60 for the type of beams tested; (4) a significant corrosion at the steel/resin interface took place on all specimens which had been exposed to natural environments. This led to some reduction in strength of the exposed specimens as compared with specimens kept in the controlled laboratory environment where no corrosion had taken place. They also found out that the corrosion was fairly
uniformly distributed over the steel plate, implying that moisture had penetrated through the concrete and the resin rather than by seeping between the steel and the edge of the beam. The microstructure of the epoxy layer consisted of a labyrinth of very fine cracks which were likely to be inherent in the resin rather than caused by natural exposure or stressing of the systems. These cracks were likely to have provided routes for corrosive moisture to reach the steel plates. The application of a chromate primer to the surface of the steel plate appeared from the limited evidence to have been generally effective in limiting the corrosion of the steel without greatly affecting the structural strength of the member.

Meda et al. (1980) studied the deterioration and repairing of reinforced concrete slabs of highway bridges in Japan [3]. In the last decade, several deteriorations of reinforced concrete slabs of highway bridges were reported in Japan. The predominant pattern of such deteriorations was flexural cracking observed on the bottom surfaces of slabs leading to spalling of the concrete. Therefore, inspection and repairing works of cracked slabs were important for management and maintenance of highway bridges.

Among the many methods of reinforcing, the following three methods are common in Japan. The first one is the “stringer increasing method”. In this method, one or two intermediate stringers are added between existing girders to reduce the bending moments in the center of the original span by shortening the slab span. From the results of this method, the authors concluded that both bending and torsional moments were remarkably reduced by the reinforcing works, however, the method had no effect for vertical shear forces. Large slit-shaped cracks had to be simultaneously repaired by resin injection. The second method is the “steel plate attaching”, which takes advantage of composite action obtained by attaching a thin steel plate on the bottom surface of the cracked slab using resin bond and anchor bolts. They suggested that normal thickness of the resin layer should not be more than 5 mm (0.2 in.) at most and that anchor bolts should be used at maximum intervals of 50 cm (19.7 in.) to increase the bond strength. Before attaching the plates, the bottom surface of a slab should be cleaned with power brushes. The
minimum pulsating load was 1 ton, and the maximum load was changed from 11 tons to 17 tons. The authors concluded that in field applications, refailure of the repaired slab did not occur at the center of the plate but at the edge of the plate. The results of the maximum stress in the bond layer at the end of the attached plates indicated that extension of the plate end to lower edges of a haunch was very desirable.

The third method is the “slab thickness increasing” method, which places a new concrete layer on an existing slab. The different layer thicknesses were investigated in relation to shearing stresses. Cases 1, 2 and 3 corresponded to the thicknesses of 0 cm, 5 cm (2 in.) and 8 cm (3.1 in.), respectively, to study stress concentrations of both vertical and torsional shear stresses. Stress concentrations of both vertical and torsional shear stresses were reduced remarkably as the thickness of the new layer increased. However, the thickness of 8 cm (3.1 in.) seemed to be the upper limit, over which the dead weight became too heavy. They found that this method was advantageous in reinforcing for both shear and bending with regard to practicality and cost. However, an increase in the dead load and a rise of road surface were inevitable. Furthermore, closing of the bridge during rehabilitation was needed. So, a careful construction plan had to be prepared in advance. The result of a fatigue test demonstrated that reinforcing by this method led to an increased capacity to withstand both shear and bending due to composite action between the new layer and the existing slab.

Jones et al. (1980) studied the behavior and strength of plain and reinforced concrete beams strengthened with steel plates bonded by epoxy to the tension face of the beams [4]. Two types of glue and two types of steel plates with different yield stresses were used, and the effects of glue thickness, plate lapping, multiple plates and of precracking prior to bonding were investigated.

They concluded that the use of external reinforcement in the form of steel plates glued to the tension face of plain and reinforced concrete beams increases the range of elastic behavior; reduces the tensile strains in the concrete due to the composite action of the concrete, glue and steel plate; delays the appearance of
the first visual cracks with a resulting increase in service loads; increases flexural stiffness at all loads, and consequently reduces deflection at corresponding loads; and increases the ductility at flexural failure.

Mcdonald (1981) studied the possibility of a reduction in strength of a resin bonded joint due to relative movement of adherents during curing [5]. Two different resin systems were used, both of them were two component epoxy resin adhesives with inert mineral fillers having a paste-like consistency when mixed. Cyclic movement was applied to single-lap shear specimens, while the resin was curing, to reproduce a movement and frequency similar to that of a highway bridge. He concluded that with one adhesive, type A (no trade name was given), commonly used in practice, there was a reduction in shear strength in each of the tests of between 7 and 31 percent due to relative movement between the adherents while the adhesive was curing. The average reduction was 16 percent. For resin system B (no trade name was given), there was no reduction in strength. The different epoxy moduli might have caused this difference in performance. The mechanical properties of adhesives were not given.

Hoigard (1986) studied the effects of hostile environment on adhesively-bonded steel bridge connections [6]. He investigated the durability of steel-bonded joints when subjected to the harsh rain environment of bridges in service. The test environment was that of submerging in water of 4.4 pH combined with an applied load which created shear stresses in the adhesive layers of the specimens. The load applied to the specimens was determined by considering a 50-year service life. He concluded that (1) significant increase in bond strength could be achieved by curing the joints longer than the manufacturer's suggested time of 72 hours; (2) increase in bond strength due to increase in curing time could be achieved even when the joint was stressed; (3) significant losses in bond strength were observed as a result of the simulated rain environment.

Tests on seventeen 3.5 m (11.5 ft) long reinforced concrete beams strengthened with epoxy-bonded steel plates and subjected to 4-point bending were reported [7]. Also tested were beams of length 4.9 m (16 ft) with a single plate geometry. In
the final tests, the 4.9 m (16 ft) long beams were loaded to produce flexural cracks before plates were added. The results showed that there could be a significant effect on crack control and stiffness but a relatively small increase in ultimate strength [8,9,10]. They concluded that: (1) the type of failure associated with the earlier laboratory work, caused by rapid shear failure of the concrete, could be prevented by the selection of a plate with suitable width and thickness; (2) to gain the maximum increase in stiffness and crack control, a stiff adhesive should be used to bond the plate to the concrete to combine the maximum increase in ultimate load with a progressive ductile failure; (3) the plate proportions \(b/t\) had an optimum value of about 60 for these tests; (4) with the stiff adhesive, plating of the as-cast beams gave an increase of 100% in the load required to produce the first visible crack; (5) the load to produce a mean crack width of 0.1 mm was increased by 140%; (6) the addition of plates produced an average increase in maximum load of 40% for both as-cast and precracked conditions and an average increase in stiffness of 190% for as-cast and 250% for precracked beams.

Van Gemert and Bosch tested several reinforced concrete beams with epoxy-bonded steel plates [11]. Due to atmospheric corrosion, the beams had lost part of their prestressing. The loss of prestressing was partly compensated for by a glued reinforcement along the longitudinal axis of the beams and partly by a supplementary external prestressing. Cyclic loading tests were executed on two prestressed beams with a span of six meters, reinforced with a double layer of glued steel plates. The cross-sections of the beams were 300 mm × 200 (11.8 × 7.9 in.), and with lengths of 5 m (16.4 ft) and 4 m (13.1 ft). The beams were loaded in a 4-point bending test setup. The maximum load resulted in stress in the steel strips of 40 N/mm² (145 lb/in²). The load varied sinusoidaly between 5 kN (1.1 kips) and the maximum load with a frequency of 30 cycles per minute. The beams were subjected to 500 cycles. The tests showed that no redistribution of stresses took place by deformations in the glue or by any failure of the glued connection under the cyclic load.
Full-scale temperature loading tests in the temperature range from $-20^\circ$C to $+90^\circ$C were conducted on specimens glued with EPICOL U epoxy adhesive [11]. It was found that the cold-hardening epoxy glue had a poor thermal resistance. There was no decrease of the ultimate load for lower temperatures. At higher temperatures, however, the situation was different. At a temperature of about $65^\circ$C, the glue started to become weaker and more deformable. At lower temperatures, the crack always started at the end of the plate and moved into the concrete. At higher temperatures, the epoxy joint was not able to transfer the shearing stresses from the steel plate to the concrete, and a crack propagated through the epoxy joint, starting at the plate end. The performance of the epoxy joint was strongly reduced at high temperatures.

Warner surveyed the techniques which were commonly used for strengthening, stiffening and repairing concrete structures [12]. He described a systematic procedure for structural assessment and reviewed techniques for the repair of damage. He outlined several repair methods which involved the removal and replacement of damaged or deteriorated concrete, the treatment of excessively cracked regions of concrete, and the replacement of rusted or damaged steel reinforcement. Epoxy resin injection techniques were mentioned for repair of cracks. He concluded that epoxy injection repair of dormant cracks had poor fire resistance and tended to brittleness in cold weather; injection of cement grout was therefore a viable alternative to resin injection and might be preferred if the cracks were wide (e.g., in excess of 5 mm) or if fire resistance or resistance to large temperature fluctuations were important considerations.

2.2 Applications

One of the first documented cases of malleable steel plates adhesively bonded to concrete beams was in 1964 in Durban, South Africa, where reinforcing steel required in a concrete beam of a new apartment complex was accidentally omitted during construction [13]. Since then, epoxy bonded plates have been used to strengthen buildings and bridges in several other countries.
The technique of externally strengthening concrete bridges with bonded steel plates is used to improve the working load capacity of a group of bridges at the M5 motorway interchange at Quinton, England [13]. As many as 1376 steel plates were bonded to the curved under-surfaces of the four bridges at the interchange. The plates were mostly 254 mm (10 in.) wide and 6.5 mm (0.3 in.) thick, and had lengths of up to 3.6 m (11.8 ft). The designers of these bridges concluded that externally-bonded steel plates could be used successfully as additional reinforcement for defective concrete bridges or where there was a need to strengthen an existing bridge deck to carry loads greater than allowed for in the original design. Full-scale loading tests on the strengthened bridges demonstrated the effectiveness of the strengthening schemes in providing increased flexural stiffness and in reducing crack-opening under load. Early indications show that slight internal rusting may have developed at the interface between the steel and the resin adhesive, which might imply the need for additional protective treatment of the steel surface to avoid the danger of eventual failure due to corrosion.

In the early to mid-1970's, epoxy-bonded steel plates were used for several buildings in Switzerland [14]. Additional live load had to be carried on reinforced concrete floor slabs and supporting reinforced concrete beams in a building in Zurich. Steel shear-reinforcing plates were also bonded to the sides of the beams to help carry the additional live load. This method was also used to strengthen a critical foundation of a building and to strengthen the industrial floor of a factory.

In 1972, an excessive deflection of 8 cm (3.1 in.) in a skewed highway T-beam bridge constructed in 1960 was found in France [15]. To reduce the deflection caused by fatigue, steel plates were bonded to both the sides and bottoms of the beams.

In 1974, a highway bridge crossing the Saint-Denis canal in France was repaired by bonding epoxy-bonded steel plates for both flexure and shear [16].

In the USSR city of Karastan, a reinforced concrete continuous bridge built in 1912 was repaired with bonded steel plates in 1974 [17]. The bridge was open to traffic one-lane-at-a-time during reinforcing.
By 1975, over 200 bridges on an elevated motorway in Japan were reinforced with bonded steel plates and anchor bolts [18]. Two erection techniques were used. Either the adhesive was applied to the steel and concrete surfaces prior to setting, or the plate was first set in place and then a liquid resin was injected between the concrete and steel surfaces.

In 1975, Isnard and Thomasson reported that a frame constructed with reinforced concrete T-beams for an overhead crane in a paper mill in France was cracking severely near the supports and was deflecting on the spans sufficiently enough to interfere with the operation of the crane [19]. Maximum deflections of 3 mm (1/8 in.) were measured in the beams. In order to reinforce the beams, steel plates were bonded to the tops of the beams to provide negative moment strengthening at the supports and they were also bonded to the webs and bottom of the T-beams to provide shear and positive moment strengthening, respectively.

In 1978, epoxy-bonded steel plates were used to strengthen the cantilevered portions of the piers of the Shelley Bridge in Perth, Australia [20]. Plates were adhesively bonded and bolted in three layers on top of each pier.

In the late 1970s, two bridges over the Netekanaal in Belgium were rehabilitated [21]. The bridges were three-span, continuous, prestressed concrete built in 1955. Steel plates were bonded to the bottom of the concrete stringers in the center span. Additional steel plates were bonded around the webs and lower flanges of the stringers near the post-tensioning anchorages in the end spans.

The application of adhesively-bonded plates has been found to be one of the most economical and practical methods of strengthening bridges in Poland [22]. Some repair-work included the bonding of flat strips to the lower surface of the concrete bridge decks. The strips were also fastened at the ends with high-strength bolts.

There are other applications of epoxy-bonded steel plates to concrete girders in other countries. They all attest to the economy and effectiveness of strengthening by plating.
CHAPTER 3

FIBER COMPOSITES

3.1 Introduction

Reinforced plastics and fiber composites are a large, complex, and constantly developing group of materials which present the structural engineer with a combination of often-unfamiliar and unique advantages and limitations that can challenge his engineering ability to the utmost.

Composites are made of synthetic, organic, high polymers which range in strength and stiffness from soft and flexible to hard and rigid. On a strength-to-weight basis, composites can provide the most efficient structural materials available to the engineer. Conversely they may easily fail if not properly designed and employed.

3.2 Important Characteristics for Civil Engineering Applications

In addition to structural qualities, reinforced plastics and fiber composites often possess characteristics that make them desirable from an engineering standpoint by reducing cost, promoting ease of fabrication, simplifying installation, reducing weight, and many others. The most important features of these materials are [23]:

**Formability:** Plastics have no inherent form of their own. They must be shaped. This provides an opportunity to select forms most efficient for the desired application. Shape can help to reduce cost and to meet the imposed stresses.

**Consolidation of parts:** The formability of plastics frequently leads to one-piece construction to eliminate joints, fasteners, sealants, and other joining problems.

**Corrosion Resistance:** Plastics are largely immune to the electrochemical corrosion to which metals are often susceptible. Consequently, they can frequently be used profitably to contain water and corrosive chemicals that would attack metals.
Examples in construction are chemical tanks, water treatment plants, and piping to handle drainage, sewage, and water supply. Structural shapes for corrosive conditions may be made of fiber-reinforced plastics.

**Light-Weight, Good Strength-to-Weight Ratio:** The generally low specific gravity (approximately 0.9 to 1.7) of plastics and their high strength, result in a high ratio of strength-to-weight. Thin, light-weight rocket motor cases, aircraft components, sports equipment, and shells and plates have been made using reinforced plastics.

**Wear Resistance, Smoothness, Surface Texture:** Depending upon the type of wear or abrasion involved, plastics may have good-to-excellent resistance. The smooth surface provided by polished molds and extrusion dies results in smooth-surfaced pipe with low resistance to the flow of water and other liquids. Surface textures can be provided by molding, embossing, printing, milling and other means, as required.

**Flexibility:** Plastics such as polyethylene are inherently soft and flexible, even at temperatures below freezing. Others are usually stiff or rigid at normal temperatures, but can be made flexible by adding plasticizers. Structural and semi-structural applications taking advantage of flexibility include plain or reinforced membrane skins for air-supported structures and flexible underground pipes.

**Ductility:** This generally implies substantial yielding when loaded beyond the limit of approximate proportionality of stress to strain. Most plastics and plastic matrix composites do not exhibit such behavior and are not considered to be ductile materials. However, some plastics such as polycarbonates and polyethylene do yield ductilely prior to failure, exhibiting considerable similarity in stress-strain behavior to mild steel.

Absence of ductility need not result in brittleness or lack of flexibility. Glass-fiber-reinforced-plastics do not exhibit ductility in their stress-strain behavior, yet they are not brittle and they have good flexibility. Because of their generally high ratio of strength to stiffness as reinforced composites, energy absorption is accomplished by high elastic deflection prior to failure.
Thermal Expansion: Plastics generally have much higher coefficients of thermal expansion than conventional metal, wood or concrete materials, and this characteristic frequently becomes an important consideration in structural design.

3.3 Glass-Fiber-Reinforced-Plastics (GFRP)

Some plastic materials can be molded or otherwise processed to obtain products with useful structural qualities without reinforcement or fillers. However, these unreinforced plastic materials exhibit creep and loss in strength under long-term load and this problem is accentuated by increasing temperature. Thus, glass fiber reinforcement is incorporated with plastic materials to improve both short-term and long-term mechanical properties, to reduce creep and to improve dimensional stability. Incorporation of reinforcement reduces mold shrinkage and lowers coefficients of thermal expansion.

Glass-Fiber-Reinforced-Plastics are the least-expensive types of composite materials. GFRP plate costs about $2.00 per pound [24]. The tensile strength of such composites exceeds that of the structural steel. However, their moduli of elasticity is about one-fourth that of steel. In this study, GFRP plates were epoxy-bonded to the tension face of reinforced concrete beams to increase their strengths and stiffnesses.

3.4 Applications of Glass-Fiber-Reinforced-Plastics

For structural purposes, plastics are most commonly utilized in the form of composites. Of these, the reinforced plastics or fibrous composites are the most widespread. In these composites, the plastic is strengthened and stiffened with high-strength fibers, most commonly glass. Fibers may range from extremely short (1.3 \text{mm} or 0.05 \text{in}, or less) to continuous filaments of indefinite length. Glass-Fiber-Reinforced-Plastic has been successfully employed in major structural components of aircraft, ships, automobiles, etc. for many years.

Other examples include: boats and ships up to 25 \text{m} (80 \text{ft}) long; sewage tank domes up to 34 \text{m} (110 \text{ft}) diameter; tanks up to 380,000 liters (100,000 gals.) capacity; standard I, wide-flange, tubular, and channel structural shapes up to 30.5
cm (12 in) deep; molded shell-roofed vacation houses; automobile and truck bodies; standard cargo containers for ocean, rail and truck shipment; large size pipe, ducts, and fume stacks; helicopter rotor blades; radomes and antennas.

The economy, resistance to electrochemical corrosion, and high strength-to-cost ratio of GFRP will make this material an ideal candidate for strengthening of existing bridges by plating.
CHAPTER 4

EXPERIMENTAL STUDY

The static strength of concrete beams externally reinforced with epoxy-bonded Glass-Fiber-Reinforced-Plastic (GFRP) plates was investigated with tests of eight beams. The experimental part of this study was performed by Saadatmanesh and Ehsani at the Structural Engineering Laboratory of the Department of Civil Engineering and Engineering Mechanics, University of Arizona. The author analysed the data and used it to verify the analytical models developed in Chapter 5. The materials and test procedure of these beams are explained in the following.

4.1 Specimens

Seven $457 \times 203 \text{ mm (18} \times \text{8 in)}$ rectangular beams and one T-beam were tested. The flange width and thickness of the T-beams were $610 \text{ mm (24 in)}$ and $76 \text{ mm (3 in),}$ respectively. The total height of the T section was $457 \text{ mm (18 in)}$; thickness of the web was $203 \text{ mm (8 in).}$ Each beam was $4.9 \text{ m (16 ft)}$ long and was supported on a clear span of $4.6 \text{ m (15 ft)}$. The beams were strengthened with $150 \text{ mm (6 in)}$ wide by $6 \text{ mm (1/4 in)}$ thick and $4.3 \text{ m (14 ft)}$ long GFRP plates bonded to the tension flange. The design details of all beams are summarized in Table 4-1.

4.2 Materials

Concrete: Ready-mixed concrete was used for all the beams. Nine $150 \times 300 \text{ mm (6} \times \text{12 in)}$ concrete cylinders were cast and tested at the same time as beams to determine the compressive strength of the concrete. The average compression strength was $35 \text{ MPa (5044 psi).}$
Steel: Three samples of the steel rebar were tested under uniaxial tension. The average measured yield stress of the bar was 456 MPa (66.2 ksi). The elastic modulus was 29,000 ksi. Figure 4.1 shows the stress-strain curve of a typical specimen.

GFRP plate: Three samples of GFRP plate were tested under uniaxial tension. The GFRP exhibited a linear elastic behavior up to failure with a modulus of elasticity of 37,231 MPa (5,400 ksi) and an ultimate strength of 400 MPa (58 ksi). Figure 4.1 shows the measured stress-strain curve of a typical sample.

Epoxy: Two different types of two-component epoxies were used. These were selected after consultation with a large number of manufacturers about this particular application. In order to prevent brittle failure of the bond after cracking of concrete in tension, it is desirable to have a ductile epoxy. The shear strength of the epoxy used for the first beam test (beam B) was 13,100 KPa (1,900 psi) with a maximum elongation of 1%. This beam failed in a brittle manner. To prevent the brittle failure of other beams, a more ductile epoxy, FUSOR 320/322, was recommended by the manufacturer with shear strength of 15,168 KPa (2,200 psi), and maximum elongation at failure of 40%. The required curing time for this epoxy was 4 hours at room temperature. In addition, the manufacturer reported that this particular epoxy had a good resistance to high humidity, salt spray, and cold and hot environments. This epoxy was used in all beams and it was found to be satisfactory and could fully develop the strengths of concrete and plate without any bond distress or failure.

4.3 Fabrication

All beams were cast, strengthened, and tested in the Structural Engineering Laboratory of the University of Arizona. After the beams were cured for at least 28 days, the tension face of each beam was sand-blasted clear to aggregate as shown in Figs. 4.2 and 4.3. Next, a layer of epoxy (2 mm thick) was applied on the tension face of each beam. The plates were then placed on the epoxy (Fig. 4.4). Weights in the form of concrete cylinders were placed on the plates during the curing time.
After the epoxy cured, the weights were removed, the beams turned so that the plates were on the under-(tension-) side, and then each beam was tested to failure.

4.4 Instrumentation

The strains in the concrete, steel rebars and plastic plate were measured by electric resistance strain gauges. For measuring the steel strain, three strain gauges were mounted on each tension rebar in the beam at midspan. The concrete strain was measured by means of three 76 mm (3 in) long electric resistance strain gauges placed on the compression face of each beam at midspan. For measuring the plate strain, two gauges were placed on the plate surface at midspan. Deflection at midspan was measured by means of two Linear Variable Differential Transformers (LVDT’s). Load was measured by a load cell.

4.5 Test Procedure

All beams were simply supported on a clear span of 4.6 m (15 ft) and they were subjected to two concentrated loads symmetrically placed about the midspan. The loading points were placed 0.6 m (2 ft) apart. The beams were incrementally loaded to failure. After each increment of the load, the strains in the concrete, steel rebars, plastic plate and the deflection at midspan were measured and recorded. The data were recorded by means of an automatic data acquisition system. The same loading rate was used for all beams.

4.6 Test Results

The measured load vs. deflection and the load vs. strain in the concrete, steel rebar and GFRP plate are discussed for each beam in the subsequent sections. In addition to the curves shown with solid lines, two other curves are shown on each figure. The dashed curve represents the predicted (calculated) curve for the same beam; the dotted curve represents the same curve for a conventional beam with no GFRP plate. The dashed and the dotted curves are generated using a computer program based on the analytical models explained in the next chapter. All
calculated curves are terminated when the strain in concrete reaches 0.003 and/or when the composite plate reaches its ultimate strength.

**Beam A:** Beam A was a rectangular control beam without any bonded plastic plate. The beam had 2 No. 4 rebars for compression steel, 3 No. 8 rebars for tension steel and 14 No. 3 stirrups at 33 cm (13 in) spacing. Fig. 4.5a shows the beam placed in the test frame. Fig. 4.5b shows the load vs. deflection curve. As shown on the plot, the behavior was initially linear elastic. After concrete cracked at a load of about 40 kN (9.4 kips), the stiffness of the beam reduced, resulting in larger deflections. Further increments of load resulted in a progressively nonlinear load-deflection response to failure. The calculated plot shows yield load clearly. After the yield load, the deflection increases dramatically as the external load increases. The ultimate load of the calculated plot is a bit lower than that of the measured curve. This could be due to the strain hardening of steel rebars which was neglected in the analytical models. Fig. 4.5c shows the load vs. strain in steel. In the measured plot, the curve terminated shortly after the steel reached its yield strain. No experimental data are available afterwards. The calculated curve shows that the yield strain closely matches the measured yield strain. Fig. 4.5d shows the load vs. strain in concrete. The measured plot terminated before the concrete strain reached 0.003. The beam failed when the top concrete fiber crushed at midspan. The calculated curve terminated at concrete strain of 0.003 which indicates that the beam failed due to concrete failure. The two curves correlate very well. Fig. 4.5e shows the beam at failure.

**Beam B:** Beam B was a rectangular concrete beam with 2 No. 4 bars for compression steel, 3 No. 8 bars for tension steel and 14 No. 3 stirrups spaced at 33 cm (13 in). Fig. 4.6a shows the test setup of beam B. Fig. 4.6b shows the load vs. deflection. The behavior was similar to that of beam A. The measured and calculated loads correlated well. The ultimate load of the calculated curve is very close to the experimental one. The curve calculated for the beam with no plate has lower stiffness and ultimate load since the beam is not strengthened; but its ductility is higher, indicating that the failure mode of the unbonded beam is more ductile than the bonded beam. Fig. 4.6c shows the load vs. strain in steel.
At a load of 35 kN (7.9 kips), the first crack occurred in the beam; this is not shown in the calculated curves because concrete tension strength is not considered. The calculated curve for bonded beam predicts the stiffness satisfactorily. The calculated curve for the beam with no plate shows lower stiffness and ultimate load, but higher ductility. The yield strains for the calculated curves closely match the measured result. Fig. 4.6d shows the load vs. strain in plate. The plate did not yield when the beam failed. In the plot, a small difference between the measured and calculated curves is noticed, which might indicate an imperfect composite action of the beam and the plate. The strain in plate increased at a higher rate when the beam cracked at a load of 35 kN (7.9 kips). The plate did not reach its ultimate strain when the beam failed. Fig. 4.6e shows the load vs. strain in concrete. The experimental curve terminated before concrete strain reached 0.003. The stiffness for the theoretical curve of the bonded beam is fairly close to the experimental. The curve for beam with no plate has lower stiffness and ultimate load. The beam failed when concrete crushed in compression at the top midspan fiber. Fig. 4.6f shows the beam at failure.

Fig. 4.7a shows the comparison of measured load-deflection curves for beams A and B. The ultimate load of beam B is 27% higher than the control beam (beam A). Fig. 4.7b shows the comparison of strains in steel rebars for the two beams. The bars in beam B yielded at a higher load than that in beam A. Fig. 4.7c shows the comparison of strains in concrete. The load causing crushing of concrete increased in beam B as a result of bonding composite plate to the beam.

**Beam C:** Beam C was a rectangular beam with 2 No. 4 bars for compression steel, 2 No. 4 bars for tension steel and No. 3 stirrups spaced at 15.2 cm (6 in). Fig. 4.8a shows the beam setup. Fig. 4.8b shows load vs. deflection curve. The cracking load, as can be seen from the plot, is 22.8 kN (5.1 kips). The beam was under-reinforced and failed under a small applied load. The calculated curve shows a higher ultimate load than that of the experimental curve. The beam did not reach its full ultimate capacity due to premature bond failure at the plate/concrete interface caused by large tension cracks in the beam. The calculated curve for the beam
with no plate shows more ductility but much smaller ultimate load. It can be seen from the figure that plating can increase a beam’s load capacity dramatically when the steel reinforcement ratio is relatively low. Fig. 4.8c shows the load vs. strain in steel rebar. The strain at cracking load had a sudden increase. The measured and calculated curves correlate well throughout loading, except at the ultimate load, because the beam did not reach the predicted ultimate load due to the premature bond failure. Fig. 4.8d shows the load vs. strain in plate. The measured results show higher stiffness than calculated values before cracking. This is due to the tensile strength of concrete which was neglected in the analytical models. The two curves correlate well beyond cracking load. Fig. 4.8e shows the beam at failure.

**Beam D:** Beam D was a rectangular beam with 2 No. 4 bars for compression steel, 2 No. 8 bars for tension steel, and No. 3 stirrups spaced at 15.2 cm (6 in). Fig. 4.9a shows the beam setup. Fig. 4.9b shows load vs. deflection. The beam cracked at a load of 49.4 kN (11.1 kips). The calculated deflection curves do not show the cracking load because the tensile strength of concrete was ignored in the analytical models. The measured and calculated yield and ultimate loads are fairly close. The curve for a beam with no plate gives a lower yield load, lower ultimate load and higher ductility. Fig. 4.9c shows load vs. strain in steel rebars. The strain in rebar suddenly increased when the first concrete tension crack occurred. The calculated yield strain is slightly smaller than the measured value. The calculated yield load of the retrofitted beam is 24% higher than the one having no plate. Fig. 4.9d shows the load vs. strain in composite plate. After concrete cracked, the strain in the plate suddenly increased. The measured and calculated curves correlate well beyond cracking load. Fig. 4.9e shows the load vs. strain in concrete. The concrete cracked at a load of 49.4 kN (11.1 kips). The concrete strain in top compression fiber reached its ultimate strain 0.003 when the failure occurred. Fig. 4.9f shows the beam at failure. From the beam failure picture it can be seen that the plate was separated from the beam at ultimate load.

**Beam E:** Beam E had a rectangular cross section with 2 No. 4 compression rebars, 2 No. 4 tension rebars and No. 3 stirrups spaced at 15.2 cm (6 in). This beam was cambered before the plate was attached to induce initial stresses in the beam
that oppose stresses caused by gravity loads. Fig. 4.10a shows the beam setup during cambering operation. Fig. 4.10b shows the test setup of beam. Fig. 4.10c shows load vs. deflection. The beam had an initial upward deflection due to cambering. The calculated curve of the plated beam matches perfectly with the measured curve. For this cambered beam no sudden deflection increase occurred as a result of cracking of concrete, indicating good composite action of plate and concrete. The theoretical curve for the beam with no plate starts from origin since no cambering was applied to it. Fig. 4.10d shows the load vs. strain in steel rebar. The calculated and measured curves match very well in the elastic region. At higher values of load, the agreement between the two curves is not very good, this is perhaps due to local bond failure between concrete and steel rebar. For the beam without plate the strain in steel rebar increased a great amount after the steel yielded. The yield load of the bonded beam is 95% higher than that of the unbonded beam. Fig. 4.10e shows load vs. strain in plate. The measured strains are smaller than the calculated values. This can be attributed to relative slip at the bond line between plate and concrete. Fig. 4.10f shows the load vs. strain in concrete. The calculated curve of the bonded beam matches fairly well with the measured curve in the elastic range. In the inelastic range, the difference between the two curves is more significant due to the relative slip between the plate and concrete. Fig. 4.11g shows the beam at failure.

**Beam F:** Beam F was a rectangular beam with 2 No. 4 bars for compression steel, 2 No. 8 bars for tension steel and No. 3 stirrups spaced at 15.2 cm (6 in). Fig. 4.11a shows the beam setup. Fig. 4.11b shows the load vs. deflection. The cracking occurred at a load of 14.2 kN (3.2 kips). The calculated curve of the bonded beam shows no cracking load because of the assumption made. The deflection increased rapidly after steel yielded. The agreement between the calculated and measured results is reasonably good. The calculated curve of the beam with no plate shows smaller yield and ultimate loads, but higher ductility as compared to the retrofitted beam. Fig. 4.11c shows the load vs. strain in steel rebar. The calculated and measured curves match well before the yielding point. From the calculated curve for an unbonded beam it can be seen that after steel yielded, the
strain in rebar increased dramatically until the beam failed. Almost no more load could be carried by the unbonded beam after yielding while the bonded beam could carry more load even after the steel yielded. Fig. 4.11d shows the load vs. strain in plate. The strain in plate increased when the first crack occurred. The plate did not reach its ultimate strain at failure of the beam. The measured ultimate load is smaller than the predicted value. This can be caused by imperfect composite action at the plate/concrete interface. Fig. 4.11e shows the load vs. strain in concrete. The strain in concrete increased a great deal after the rebar yielded. It can be seen from the plots that after the steel yielded the strain in the bonded beam did not increase as much as that in the unbonded beam for the same load increment. Fig. 4.11f shows the beam at failure.

**Beam F':** Beam F' was similar to beam F except that this beam was cambered. Fig. 4.12a shows the beam test setup. Fig. 4.12b shows the load vs. deflection. The deflection increased linearly in the elastic region as the external load increased. The beam deflected rapidly after steel rebars yielded. The calculated and measured curves of the bonded beam correlate well in the elastic range. The calculated curve of the unbonded beam shows lower yield and ultimate load. It can be seen that after yield load the bonded beam still had some stiffness while the unbonded beam had very little stiffness until it failed. Fig. 4.12c shows the load vs. strain in steel rebars. The measured strains are larger than calculated values. This could be attributed to imperfect composite action at the plate/concrete interface. Fig. 4.12d shows the load vs. strain in plate. The strain in the plate increased at a higher rate after steel rebar yielded. It is seen that the ultimate load of the plate is smaller than the predicted value due to the imperfect composite action. Fig. 4.12e shows the load vs. strain in concrete. In the elastic range, the measured and predicted values correlate well. The difference in ultimate loads between the calculated and the measured curves is due to premature failure of the concrete layer between composite plate and steel rebars, as shown in Fig. 4.12f.

**Beam G:** Beam G had a T cross section with 3 No. 4 bars for compression steel, 2 No. 8 bars for tension steel and No. 3 stirrups spaced at 15.2 cm (6 in). Fig. 4.13a shows the beam in the test frame. Fig. 4.13b shows the load vs. deflection.
The beam cracked at a load of 31.9 kN (7.2 kips). The calculated curve of the bonded beam matches very well with the experimental curve. The ultimate load is increased by 86% as compared to the beam with no plate; Fig. 4.13c shows the load vs. strain in steel rebar. In the elastic range, the calculated and measured values correlate perfectly. However, the calculated curve predicts a larger ultimate load than the measured result. This is due to premature separation of the composite plate and concrete beam. From the plot, it can be seen that the calculated curve for the unbonded beam shows a smaller ultimate load but higher ductility as compared to the same curve for the bonded beam. Fig. 4.13d shows the load vs. strain in plate. After the steel rebars yielded, the strains in the plate increased at a higher rate. The agreement between the measured and predicted result is reasonably good. Fig. 4.13e shows the load vs. strain in concrete. The concrete strain for the bonded beam did not reach its ultimate value when the beam failed. Failure was due to separation of plate from the beam as shown in Fig. 4.13f.

**Beam H:** This beam did not have any steel longitudinal reinforcement. The plastic plate was bonded at the tension face of the beam to see whether the plate can be used in lieu of steel to reinforce the beam. Fig. 4.14a shows the beam in the test frame. Fig. 4.14b shows the load vs. deflection curves. From the measured curve it can be seen that the beam failed at a load of 65 kN (14.6 kips), while the unbonded beam has no tensile strength at all. The measured ultimate load did not reach the predicted value. It was perhaps caused by the separation of the plate and the concrete beam due to the severe crack. Fig. 4.14c shows the curves for load vs. strain in plate. The beam failed as soon as the plate separated from the beam. The plate reached its ultimate strain when the beam failed. Fig. 4.14d is the load vs. strain in concrete. The beam bonded with plate failed prematurely before concrete top fiber reached its ultimate strain. Large tension cracks were observed in the beam close to the failure load. Fig. 4.14e shows the beam at failure.
In this chapter analytical models and computer programs are developed for the analysis of rectangular and T-beams. The models predict strains, stresses, and curvature at midspan of the beam throughout the entire range of loading up to failure.

5.1 Rectangular beam

Analytical models are developed to calculate deflection and curvature at midspan, strains in concrete, tension steel and plastic plate for single-span, reinforced concrete beams strengthened with fiber composite plate epoxy-bonded to the tension face.

The strain compatibility method is used to calculate the strains across the depth of the cross section. Given the value of concrete strain in the extreme compression fiber, the depth of the neutral axis, c, is obtained from equilibrium of forces across the depth of the cross section. Knowing the location of the neutral axis, the strains in the concrete, steel rebar and plastic plate can be calculated from similar triangles in the strain diagram. The stresses in concrete, steel rebar and plastic plate may then be found from the stress-strain curves of concrete, steel and plastic plate, respectively. The forces in the concrete, rebars and plate are calculated by multiplying their stresses by their corresponding areas. The deflection at midspan is calculated from the first moment of area of curvature diagram about the support. The external load corresponding to the internal forces is obtained from the equilibrium of internal and external moments. The steps involved in the analysis are described in more detail in the following sections.
5.1.1 Assumptions

The following assumptions are made in the analysis:
1. Linear strain distribution through full depth of beam;
2. Small deformation;
3. No creep and shrinkage deformations;
4. Complete composite action between GFRP plate and concrete beam, i.e., no slip;
5. No shear deformations; and
6. No tensile strength for concrete.

5.1.2 Stress-Strain Relationships of Materials

Concrete: Hognestad's Parabola of the idealized stress-strain curve of concrete in uniaxial compression is used [25]. Fig. 5.1 shows the stress-strain curve of concrete, where \( f'_c \) = strength of concrete in a member, \( f''_c = kf'_c, k = 0.92 \) for cylinder strength \( f'_c = 5000 \text{ ksi} \); \( f_c \) = stress in concrete; \( \epsilon_c \) = strain in concrete; and \( \alpha \) = angle between tangent line at origin and x-axis.

Steel: The stress-strain behavior of steel is idealized as elastic-perfectly plastic, as shown in Fig. 5.2, where \( f_y \) = yield stress; and \( \epsilon_y \) = yield strain.

Plastic plate: The GFRP plate behaves linearly elastic to failure. A typical stress-strain relationship for the GFRP plate is shown in Fig. 5.3, where \( f_u \) = stress at failure; and \( \epsilon_u \) = strain at failure.

5.1.3 Calculation of Strains and Stresses

For convenience of calculation, strain in the extreme compression fiber of concrete, \( \epsilon_{cf} \), rather than the load, is increased to generate the curvature and the load-deflection curves. For a given concrete strain in the extreme compression fiber, \( \epsilon_{cf} \), and an unknown neutral axis depth, \( c \), the steel strains, \( \epsilon_{si} \), and the plate strain, \( \epsilon_{pl} \), can be determined from similar triangles of the strain diagram as shown in Fig. 5.4, in which \( h \) = section height; \( b \) = section width; \( c \) = depth of the neutral
axis measured from top concrete fiber; \( \epsilon_{si} = \) strain in steel; \( \epsilon_{pl} = \) strain in plate; 
\( \epsilon_{cf} = \) strain in concrete top compression fiber; \( d_i = \) distance from top concrete fiber to centroid of steel rebar; \( d_{pl} = \) distance from top concrete fiber to centroid of plate; 
\( f_{si} = \) stress in steel rebar; \( f_{pl} = \) stress in plastic plate; \( \gamma_c = \) distance from top concrete fiber to resultant of compression force in concrete; \( S_i = \) force in steel rebar; \( G = \) force in plate; \( C_c = \) compression force in concrete; \( M = \) internal moment.

\[
\begin{align*}
\epsilon_{cf} &= \text{assumed} \quad (5.1) \\
\epsilon_{si} &= \epsilon_{cf} \frac{c - d_i}{c} \quad (5.2) \\
\epsilon_{pl} &= \epsilon_{cf} \frac{c - d_{pl}}{c} \quad (5.3)
\end{align*}
\]

The steel stresses, \( f_{si} \), and plate stress, \( f_{pl} \), corresponding to strains, \( \epsilon_{si} \) and \( \epsilon_{pl} \), are found from the stress-strain curves for steel and plate:

\[
\begin{align*}
f_{si} &= E_{si} \epsilon_{si} \quad \text{for } \epsilon_{si} \leq \epsilon_y, \quad (5.4a) \\
f_{si} &= f_y \quad \text{for } \epsilon_{si} > \epsilon_y, \quad (5.4b) \\
f_{pl} &= E_{pl} \epsilon_{pl}. \quad (5.5)
\end{align*}
\]

The steel forces, \( S_i \), and plate force, \( G \), are found by multiplying the steel stresses by their corresponding areas and the plate stress by plate area, respectively:

\[
\begin{align*}
S_i &= f_{si} A_{si}, \quad (5.6) \\
G &= f_{pl} A_{pl}. \quad (5.7)
\end{align*}
\]

The distribution of concrete stresses over the compressed part of the section is found from the stress-strain curve for concrete. The concrete stress-strain relationship is expressed as follows:

\[
\begin{align*}
f_c &= f''_c \left[ 2 \frac{\epsilon_c}{\epsilon_o} - \left( \frac{\epsilon_c}{\epsilon_o} \right)^2 \right] \quad \text{for } 0 \leq \epsilon_c < \epsilon_o, \quad (5.8)
\end{align*}
\]
and
\[ f_c = f_c'' \left[ 1 - \frac{0.15}{0.004 - \varepsilon_o} (\varepsilon_c - \varepsilon_o) \right] \quad \text{for} \quad \varepsilon_o \leq \varepsilon_c \leq 0.003 \]  

(5.9)

For any given concrete strain in the extreme compression fiber, \( \varepsilon_{cf} \), the concrete compression force, \( C_c \), and its position measured from the top concrete fiber, may be defined in terms of parameters \( \alpha \) and \( \gamma \), respectively, as follows:

\[ C_c = \alpha f_c'' b_c. \]  

(5.10)

The parameter \( \alpha \) (mean stress factor) is used to convert the nonlinear, stress-strain relationship of concrete into an equivalent rectangular stress-strain curve; it is calculated by equating the area under the curve to the equivalent rectangular area (see Fig. 5.1).

\[ A = \int_0^{\varepsilon_{cf}} f_c d\varepsilon_c = \alpha f_c'' \varepsilon_{cf} \]  

(5.11)

Then \( \alpha \) is obtained from Eq. (5.11) as:

\[ \alpha = \frac{\int_0^{\varepsilon_{cf}} f_c d\varepsilon_c}{f_c'' \varepsilon_{cf}} \]  

(5.12)

After integration, \( \alpha \) is obtained as:

\[ \alpha = \frac{\varepsilon_{cf}}{\varepsilon_o} - \frac{\varepsilon_{cf}^2}{3 \varepsilon_o^2} \quad \text{for} \quad 0 \leq \varepsilon_{cf} < \varepsilon_o \]  

(5.13)

\[ \alpha = -\frac{1}{3} \varepsilon_o + \varepsilon_{cf} \left[ 1 - \frac{0.15}{0.004 - \varepsilon_o} \left( \frac{\varepsilon_{cf}}{2} - \varepsilon_o \right) \right] - \frac{0.075}{0.004 - \varepsilon_o} \times \left( \frac{\varepsilon_o^2}{\varepsilon_{cf}} \right) \]  

for \( \varepsilon_o \leq \varepsilon_{cf} \leq 0.003 \)  

(5.14)

The parameter \( \gamma \) (centroid factor) is obtained as follows. The first moment of area under the concrete stress-strain diagram about the origin, \( Q \), is defined as:

\[ Q = \int_0^{\varepsilon_{cf}} f_c \varepsilon_c d\varepsilon_c = \varepsilon_c A \]  

(5.15a)

where \( A = \text{area under stress-strain curve} \); \( \varepsilon_c = \text{strain at centroid of area under stress-strain diagram} \).
Strain at centroid, $\varepsilon_c$, can be defined in terms of $\varepsilon_{cf}$, as:

$$\varepsilon_c = (1 - \gamma)\varepsilon_{cf}$$  \hspace{1cm} (5.15b)

and therefore,

$$Q = \varepsilon_c A = (1 - \gamma)\varepsilon_{cf} \int_0^{\varepsilon_{cf}} f_c d\varepsilon_c.$$  \hspace{1cm} (5.15c)

The parameter $\gamma$ is obtained by equating (5.15a) and (5.15c):

$$\gamma = 1 - \frac{\int_0^{\varepsilon_{cf}} \varepsilon_c f_c d\varepsilon_c}{\varepsilon_{cf} \int_0^{\varepsilon_{cf}} f_c d\varepsilon_c}.$$  \hspace{1cm} (5.15d)

After integration, $\gamma$ is obtained as:

$$\gamma = \frac{\frac{1}{3} - \frac{\varepsilon_{cf}}{12\varepsilon_o}}{1 - \frac{\varepsilon_{cf}}{3\varepsilon_o}} \quad \text{for} \quad 0 \leq \varepsilon_{cf} < \varepsilon_o$$  \hspace{1cm} (5.16a)

and

$$\gamma = 1 - \frac{(0.7\varepsilon_{cf}^3 - 5.1\varepsilon_o\varepsilon_{cf}^2 - 0.6\varepsilon_{cf} - 0.004\varepsilon_o^2 + 0.024\varepsilon_{cf}^2)}{\varepsilon_{cf}(3.925\varepsilon_o^2 - 10.2\varepsilon_o\varepsilon_{cf} - 0.9\varepsilon_{cf} - 0.016\varepsilon_o + 0.048\varepsilon_{cf})} \quad \text{for} \quad \varepsilon_o \leq \varepsilon_{cf} < 0.003$$  \hspace{1cm} (5.16b)

Hence, if the stress in the concrete, $f_c$, can be written in terms of the strain, $\varepsilon_c$ (i.e., if the stress-strain curve is known), the concrete force and its line of action can be determined from Eqs. (5.10) through (5.16).

The internal force and moment equilibrium equations can be written as:

$$P = \alpha f_c'' h c + \sum_{i=1}^{n} f_{si} A_{si} + f_{pl} A_{pl} = 0,$$  \hspace{1cm} (5.17)

and

$$M = \alpha f_c'' h c (\frac{h}{2} - \gamma c) + \sum_{i=1}^{n} f_{si} A_{si} (\frac{h}{2} - d_i) + f_{pl} A_{pl} (\frac{h}{2} - d_{pl}),$$  \hspace{1cm} (5.18)

respectively.

The curvature is given by

$$\phi = \frac{\varepsilon_{cf}}{c}.$$  \hspace{1cm} (5.19)
The external load, $P$, can be found from equilibrium of internal and external moments. For a beam loaded by two concentrated loads, $P$, placed symmetrically about the midspan, the external load is calculated in terms of internal resisting moment from the following equation:

$$P = \frac{2M}{l - a}.$$  \hspace{1cm} (5.20)

where

- $l$ = clear span;
- $a$ = distance between two symmetrical loads;
- $M$ = internal resisting moment.

Deflection at midspan, $V_{\text{mid}}$, is calculated from the first moment of area of curvature diagram about the support. The curvature diagram for the elastic range of the behavior is shown in Fig. 5.5. It is assumed that the curvature varies linearly from zero at supports to the maximum value at load point. Accordingly,

$$V_{\text{mid}} = 3036\phi_{\text{mid}},$$  \hspace{1cm} (5.21)

where

- $\phi_{\text{mid}}$ = curvature at midspan.

5.1.4 Computer Program

A computer program was developed in FORTRAN for the VAX/VMS computer system to carry out the numerical calculations involved in the analysis. The flow chart of the program and a listing of the program are given in Appendix A.

5.2 T-Beam

Analytical models based on the same concepts as those outlined for rectangular beams are developed for the analysis of T-beams. The models can be used to calculate deflection and curvature at midspan, strain in the extreme compression
fiber of concrete, strains in the plastic plate and steel rebars at midspan. Fig. 5.6 shows the stresses, strains, and forces across the cross section of a typical T-beam.

5.2.1 Calculation of Parameters $\alpha$ and $\gamma$

In order to extend the analytical method of section 5.1 for T-beam analysis, it is necessary to modify the parameters $\alpha$ and $\gamma$ to reflect the change in width at the junction of flange and web. Similar steps as those for rectangular beams are taken to calculate $\alpha$ and $\gamma$. The parameters $\alpha_1$, $\gamma_1$ and $\alpha_2$, $\gamma_2$ are calculated for the web and flange, respectively, as shown in the following:

$$\alpha_1 = \frac{\int_{\epsilon_{h1}}^{\epsilon_{f}} \epsilon_c f_c d\epsilon_c}{f''_{c} \epsilon_{cf}},$$  \hspace{1cm} (5.22)

$$\alpha_2 = \frac{\int_{\epsilon_{h1}}^{\epsilon_{f}} \epsilon_c f_c d\epsilon_c}{f''_{c} \epsilon_{cf}},$$  \hspace{1cm} (5.23)

where $\epsilon_{h1} = $ concrete strain at junction of flange and web.

The parameter $\gamma_1$ is calculated for the web from a similar equation as that for a rectangular beam:

$$\gamma_1 = 1 - \frac{\int_{\epsilon_{h1}}^{\epsilon_{f}} \epsilon_c f_c d\epsilon_c}{\epsilon_{h1} \int_{\epsilon_{h1}}^{\epsilon_{f}} \epsilon_c f_c d\epsilon_c}.$$  \hspace{1cm} (5.24)

The parameter $\gamma_2$ is calculated for the flange from the following equation:

$$Q = \bar{\epsilon}_c A,$$  \hspace{1cm} (5.25a)

where

$$Q = \int_{\epsilon_{h1}}^{\epsilon_{f}} \epsilon_c f_c d\epsilon_c$$  \hspace{1cm} (5.25b)

$$\bar{\epsilon}_c = \epsilon_{h1} + (1 - \gamma_2)(\epsilon_{cf} - \epsilon_{h1}),$$  \hspace{1cm} (5.25c)

$$A = \int_{\epsilon_{h1}}^{\epsilon_{f}} f_c d\epsilon_c.$$  \hspace{1cm} (5.25d)

Substituting the above relations in Eq. (5.25a), $\gamma_2$ is obtained as:

$$\gamma_2 = 1 - \frac{\int_{\epsilon_{h1}}^{\epsilon_{f}} \epsilon_c f_c d\epsilon_c}{(\epsilon_{cf} - \epsilon_{h1}) \int_{\epsilon_{h1}}^{\epsilon_{f}} f_c d\epsilon_c} + \frac{\epsilon_{h1}}{\epsilon_{cf} - \epsilon_{h1}}.$$  \hspace{1cm} (5.26)
The parameters $\alpha_1$, $\alpha_2$, $\gamma_1$ and $\gamma_2$ are obtained for the following three cases, depending on the magnitude of strain in the extreme compression fiber of concrete.

**Case 1:** $0 \leq \epsilon_{cf} < \epsilon_o$

Fig. 5.7 shows the state of stress and strain corresponding to this case. Performing the integration in Eq. (22) yields the following expression for $\alpha_1$:

$$\alpha_1 = \frac{\epsilon_{h1}^2}{\epsilon_{cf} \epsilon_o} \left(1 - \frac{\epsilon_{h1}}{3 \epsilon_o}\right). \quad (5.27)$$

Similarly from Eq. (5.24) the integration of $\gamma_1$ is obtained as:

$$\gamma_1 = 1 - \frac{\frac{2}{3} - \frac{\epsilon_{h1}}{4 \epsilon_o}}{1 - \frac{\epsilon_{h1}}{3 \epsilon_o}}. \quad (5.28)$$

For $\alpha_2$ and $\gamma_2$, the same procedure is used. The following expressions are obtained for $\alpha_2$ and $\gamma_2$:

$$\alpha_2 = \frac{1}{\epsilon_{cf}} \left[ \frac{\epsilon_{cf}^2}{\epsilon_o} \left(1 - \frac{\epsilon_{cf}}{3 \epsilon_o}\right) - \frac{\epsilon_{h1}^2}{\epsilon_o} \left(1 - \frac{\epsilon_{h1}}{3 \epsilon_o}\right) \right], \quad (5.29)$$

$$\gamma_2 = 1 - \frac{\epsilon_{cf}^2 \left(\frac{2}{3} - \frac{\epsilon_{cf}}{4 \epsilon_o}\right) - \epsilon_{h1}^3 \left(\frac{2}{3} - \frac{\epsilon_{h1}}{4 \epsilon_o}\right)}{(\epsilon_{cf} - \epsilon_{h1}) \left[\epsilon_{cf}^2 \left(1 - \frac{\epsilon_{cf}}{3 \epsilon_o}\right) - \epsilon_{h1}^2 \left(1 - \frac{\epsilon_{h1}}{3 \epsilon_o}\right) \right]} + \frac{\epsilon_{h1}}{\epsilon_{cf} - \epsilon_{h1}}. \quad (5.30)$$

**Case 2:** $0 \leq \epsilon_{h1} < \epsilon_o$ and $\epsilon_o \leq \epsilon_{cf} \leq 0.003$

Fig. 5.8 shows the stress-strain diagram corresponding to this case. From Eqs. (5.27) and (5.28), $\alpha_1$ and $\gamma_1$ are obtained as:

$$\alpha_1 = \frac{\epsilon_{h1}^2}{\epsilon_{cf} \epsilon_o} \left(1 - \frac{\epsilon_{h1}}{3 \epsilon_o}\right),$$

$$\gamma_1 = 1 - \frac{\frac{2}{3} - \frac{\epsilon_{h1}}{4 \epsilon_o}}{1 - \frac{\epsilon_{h1}}{3 \epsilon_o}}. \quad \text{Case 2}$$

The parameters for $\alpha_2$ and $\gamma_2$ are obtained from Eqs. (5.23) and (5.26) as follows:

$$\alpha_2 = \frac{1}{\epsilon_{cf}} \left[ \frac{\epsilon_{h1}^3}{3 \epsilon_o} - \frac{\epsilon_{h1}^2}{\epsilon_o} - \frac{1}{3 \epsilon_o} + \epsilon_{cf} - \frac{0.15}{0.004 - \epsilon_o} \left(\frac{1}{2} \epsilon_{cf}^2 - \epsilon_{cf} \epsilon_o + \frac{1}{2} \epsilon_o^2\right) \right], \quad (5.31)$$
\[
\gamma_2 = 1 + \frac{(e_0^4 + 8e_0^2e_{h1}^2 - 3e_0^4 - 6e_0^2e_{cf}^2) + A(4e_{cf}^3 - 6e_0e_{cf}^2 + 2e_0^3)}{(e_{cf} - e_{h1})B - A(6e_{cf}^2 - 12e_0e_{cf} + 6e_0^2)} + \frac{e_{h1}}{e_{cf} - e_{h1}} \quad (5.32a)
\]

where
\[
A = \frac{0.15e_0^2}{0.004 - e_0}, \quad (5.32b)
\]
\[
B = 12e_0^2e_{cf}^2 + 4e_0^3e_{cf} - 4e_0^3e_{cf} - 12e_0^2e_{cf}^2. \quad (5.32c)
\]

**Case 3:** \(\epsilon_c \geq \epsilon_o\)

The stress-strain diagram for this case is shown in Fig. 5.9. The parameter \(\alpha_1\) is calculated from the following expression:

\[
\alpha_1 = \frac{\int_0^{\epsilon_0} f_cde_c + \int_{\epsilon_0}^{\epsilon_{h1}} f_cde_c}{\int_0^{\epsilon_0} f_cde_c + \int_{\epsilon_0}^{\epsilon_{h1}} f_cde_c}. \quad (5.33a)
\]

The evaluation of the above expression results in:

\[
\alpha_1 = \frac{1}{\epsilon_{cf}} \left[ \left( \epsilon_{h1} - \frac{1}{3}\epsilon_o \right) - \frac{0.15}{0.004 - \epsilon_o} \left( \frac{1}{2}\epsilon_{h1}^2 - \epsilon_0\epsilon_{h1} + \frac{1}{2}\epsilon_o^2 \right) \right]. \quad (5.33b)
\]

Similarly, \(\gamma_1\) is calculated from the following expression:

\[
\gamma_1 = 1 - \frac{\int_0^{\epsilon_0} f_cde_c + \int_{\epsilon_0}^{\epsilon_{h1}} f_cde_c}{\epsilon_{h1} \left\{ \int_0^{\epsilon_0} f_cde_c + \int_{\epsilon_0}^{\epsilon_{h1}} f_cde_c \right\}}, \quad (5.34a)
\]

which results in

\[
\gamma_1 = 1 - \frac{\left( \frac{1}{2}\epsilon_{h1}^2 - \frac{1}{12}\epsilon_o^2 \right) - \frac{0.15}{0.004 - \epsilon_o} \left( \frac{1}{3}\epsilon_{h1}^3 - \frac{1}{2}\epsilon_0\epsilon_{h1}^2 + \frac{1}{6}\epsilon_o^3 \right)}{\epsilon_{h1} \left[ \left( \epsilon_{h1}^2 - \frac{1}{3}\epsilon_o^2 \right) - \frac{0.15}{0.004 - \epsilon_o} \left( \frac{1}{2}\epsilon_{h1}^2 - \epsilon_0\epsilon_{h1} + \frac{1}{2}\epsilon_o^2 \right) \right]}. \quad (5.34b)
\]

In the same manner, the parameters \(\alpha_2\) and \(\gamma_2\) are calculated from:

\[
\alpha_2 = \frac{\int_{\epsilon_{h1}}^{\epsilon_{cf}} f_cde_c}{\int_0^{\epsilon_{cf}} f_cde_c}, \quad (5.35a)
\]

and

\[
\gamma_2 = 1 - \frac{\int_{\epsilon_{h1}}^{\epsilon_{cf}} f_cde_c}{(e_{cf} - e_{h1}) \int_{\epsilon_{h1}}^{\epsilon_{cf}} f_cde_c} + \frac{e_{h1}}{e_{cf} - e_{h1}}. \quad (5.35b)
\]

After integration:

\[
\alpha_2 = \frac{1}{\epsilon_{cf}} \left[ \left( e_{cf} - e_{h1} \right) - \frac{0.15}{0.004 - \epsilon_o} \left( \frac{1}{2}e_{cf}^2 - \epsilon_0e_{cf} + \epsilon_0e_{h1} - \frac{1}{2}\epsilon_o^2 \right) \right], \quad (5.36a)
\]
and

\[ \gamma_2 = 1 - \frac{3(\varepsilon_f^2 - \varepsilon_h^2)}{(\varepsilon_f + \varepsilon_h)} - \frac{0.15}{0.004 - \varepsilon_f} \left( 2\varepsilon_f^3 - 3\varepsilon_f \varepsilon_h^2 + 3\varepsilon_h^2 - 2\varepsilon_f^3 \right) \]

\[ + \frac{\varepsilon_h}{\varepsilon_f - \varepsilon_h}. \]

(5.36b)

Hence, the internal force and moment equilibrium equations can be written as follows:

\[ P = \alpha f''c + \sum_{i=1}^{n} f_{si}A_{si} + f_{pl}A_{pl} = 0 \quad c \leq h_1 \]

(5.37a)

\[ P = \alpha_1 f''_c(c - h_1)b_1 + \alpha_2 f''_h h_1 b + \sum_{i=1}^{n} f_{si}A_{si} + f_{pl}A_{pl} = 0 \quad c > h_1 \]

(5.37b)

\[ M = \alpha f''_c bc \left( \frac{h}{2} - \gamma c \right) + \sum_{i=1}^{n} f_{si}A_{si} \left( \frac{h}{2} - d_i \right) + f_{pl}A_{pl} \left( \frac{h}{2} - d_{pl} \right) \quad c \leq h_1 \]

(5.38a)

\[ M = \alpha_1 f''_c b_1 \left( c - h_1 \right) \left[ \frac{h}{2} - h_1 - \gamma_1 \left( c - h_1 \right) \right] + \alpha_2 f''_h h_1 b \left( \frac{h}{2} - \gamma_2 h_1 \right) + \]

\[ + f_{pl}A_{pl} \left( \frac{h}{2} - d_{pl} \right) + \sum_{i=1}^{n} f_{si}A_{si} \left( \frac{h}{2} - d_i \right) \quad c > h_1 \]

(5.38b)

Equations (5.37a) and (5.38a) are the same as those for a rectangular section. These equations pertain to cases where the neutral axis falls within the flange thickness. The strains in steel and plate can be found from Eqs. (5.2) and (5.3). Curvature at midspan can be found from Eq. (5.19), the external load from Eq. (5.20) and the deflection from Eq. (5.21).

### 5.2.2 Computer Program

The computer program discussed in section 5.2 was modified for the analysis of T-beams. The flow chart for this program and a listing of the program are given in Appendix B.

### 5.3 Cambered Beam

Two rectangular beams were cambered (externally prestressed) to examine the effects of initial stresses on the behavior of beams. The same theory as that
explained in section 5.1 is used in the analysis of these beams, except that the initial values of deflection, and stresses in steel rebars and composite plate are taken into account in the program to calculate the final strains and stresses across the depth of the section due to applied loads. The effects of initial stresses and strains are incorporated in the stress-strain relationships of the materials as explained in the following.

**Concrete**: Hognestad's Parabola of the idealized stress-strain curve for concrete in uniaxial compression is used (Fig. 5.1). Note that the tensile strength of concrete is ignored in this model.

**Steel**: Considering the initial values of stress and strain due to the counter-acting negative moment caused by the upward jacking forces, the stress-strain curve for steel is modified as in Fig. 5.10, where $\varepsilon_{sti}$ and $f_{sti}$ are the initial strain and stress in steel rebar.

**Plate**: Considering the initial values of stress and strain in the plate for cambered beam, the stress-strain curve for the plate is modified as shown in Fig. 5.11, where $\varepsilon_{pli}$ and $f_{pli}$ are the initial strain and stress in the plate.

The prestressing is accomplished by cambering the beam using two concentrated load points spaced one foot apart. The loads are removed after the epoxy is cured. This results in initial stresses in the beam which oppose stresses caused by gravity loads on the cambered beams. Parameters $\alpha$ and $\gamma$ are the same as those for rectangular beams. Deflection and curvature at midspan, and strains in steel and plate are calculated by taking into account the initial values. Accordingly,

$$
\varepsilon_{si} = \varepsilon_{sti} + \varepsilon_{cf} \frac{c - d_{st}}{c}, \quad (5.39)
$$

$$
\varepsilon_{pl} = \varepsilon_{pli} + \varepsilon_{cf} \frac{c - d_{pl}}{c}, \quad (5.40)
$$

$$
V_{mid} = V_{i} + 2874\phi_{mid}. \quad (5.41)
$$

The rest of the calculations are the same as those in section 5.1.

Appendix C shows a listing of the program that includes the effects of initial strains and stresses in cambered beams.
6.1 Introduction

In this chapter a parametric study is conducted on the behavior of rectangular and T-beams strengthened with epoxy-bonded fiber composite plates. The parameters studied are concrete compressive strength, $f'_c$, reinforcement ratio, $\rho$, ratio of plate area to gross concrete area, $A_{pl}/bh$, ultimate strength of plate, $F_{yp}$, and modulus of elasticity of plate, $E_{yp}$. Accordingly, the following values are used: $f'_c = 20.6 \text{ MPa (3,000 psi)}$, and $41.4 \text{ MPa (6,000 psi)}$; $\rho = 0.005$, and $0.015$; $F_{yp} = 413 \text{ MPa (60 ksi)}$, and $E_{yp} = 34,474 \text{ MPa (5,000 ksi)}$; $F_{yp} = 827.4 \text{ MPa (120 ksi)}$, and $E_{yp} = 68,948 \text{ MPa (10,000 ksi)}$; and $A_{pl}/bh = 0.0025, 0.005, \text{ and } 0.015$. Plots of moment vs. curvature are generated using different combinations of these variable parameters. In the plots, each curve is identified with an acronym, where the first symbol stands for the cross section type, i.e., R = rectangular beam and T = T-beam; the second symbol stands for compressive strength of concrete, i.e., 3 indicates 20.6 MPa (3,000 psi) and 6 indicates 41.4 MPa (6,000 psi); the third symbol stands for reinforcement ratio, i.e., L indicates the low reinforcement ratio or $\rho = 0.005$ and H indicates the high reinforcement ratio or $\rho = 0.015$; the fourth symbol stands for the combination of modulus of elasticity and ultimate strength of plate, i.e., 5 indicates $E_{yp} = 34,474 \text{ MPa (5,000 ksi)}$, $F_{yp} = 413 \text{ MPa (60 ksi)}$ and 10 indicates $E_{yp} = 68,948 \text{ MPa (10,000 ksi)}$, $F_{yp} = 827.4 \text{ MPa (120 ksi)}$; finally, the last symbol indicates the ratio of plate area to gross area of concrete, i.e., L indicates $A_{pl}/bh = 0.0025$; M indicates $A_{pl}/bh = 0.005$; and H indicates $A_{pl}/bh = 0.015$. 
6.2 Rectangular Beam

Figs. 6.1 through 6.11 show the effects of variable parameters on the moment vs. curvature behavior of a 457 x 43 mm (18 x 8 in) rectangular section throughout the entire range of loading up to failure.

Fig. 6.1 shows the plots of moment vs. curvature for the section with three different ratios of plate area to gross concrete section. The remaining parameters are held constant at the following values: \( f'_c = 20.7 \text{MPa (3,000 psi)} \); \( \rho = 0.005 \); \( F_{yp} = 413.7 \text{ MPa (60 ksi)} \); and \( E_{yp} = 34,474 \text{ MPa (5,000 ksi)} \). In addition, the moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments of beams R3L5H, R3L5M and R3L5L are increased by 147.4%, 100.5% and 59.4%, respectively as compared to that of beam R3L with no plate. However, the ultimate curvatures are lowered by 58%, 48% and 43%, respectively. Additionally, the ultimate moment of beam R3L5H is 23.4% higher than that of beam R3L5M, while the ultimate curvature is 19.9% lower. The ultimate moment of beam R3L5M is 25.6% higher than that of beam R3L5L, while the ultimate curvature is 8.6% lower.

Fig. 6.2 shows the plots of moment vs. curvature for the section with three different ratios of plate area to gross concrete section. The remaining parameters are held constant at the following values: \( f'_c = 41.4 \text{MPa (6,000 psi)} \); \( \rho = 0.015 \); \( F_{yp} = 827.4 \text{ MPa (120 ksi)} \); and \( E_{yp} = 68,948 \text{ MPa (10,000 ksi)} \). In addition, the moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments of beams R6H10H, R6H10M and R6H10L are increased by 73.1%, 47.5% and 29% as compared to that of beam R6H with no plate. However, the ultimate curvatures are lowered by 42%, 31.7% and 21.9%, respectively. Additionally, the ultimate moment of beam R6H10H is 17.3% higher than that of beam R6H10M, while the ultimate curvature is 15.2% lower. The ultimate moment of beam R6H10M is 14.3% higher than that of beam R6H10L, while the ultimate curvature is 12.7% lower. Comparing Fig. 6.2 with Fig. 6.1, it can be seen from the plots that the plate area plays an important role in increasing the beam's ultimate strength when the steel reinforcement ratio
is relatively low. The higher the steel reinforcement ratio, the smaller the effect of the plate area on the ultimate capacity.

Fig. 6.3 shows the plots of moment vs. curvature for the section with three different ratios of plate area to gross concrete section. The remaining parameters are held constant at the following values: $f'_c = 41.7 \text{ MPa (6,000 psi)}$; $\rho = 0.005$; $F_{yp} = 413.7 \text{ MPa (60 ksi)}$; and $E_{yp} = 34,474 \text{ MPa (5,000 ksi)}$. In addition, the moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments of beams R6L5H, R6L5M and R6L5L are increased by 228%, 119% and 57.6% as compared to that of beam R6L with no plate. However, the ultimate curvatures are lowered by 67.8%, 68.9% and 70.6%, respectively. Also, the ultimate moment of beam R6L5H is 49.7% higher than that of beam R6L5M, while the ultimate curvature is 3.6% lower. The ultimate moment of beam R6L5M is 39% higher than that of beam R6L5L, while the ultimate curvature is 5.8% lower. Comparing Fig. 6.3 with Fig. 6.1, it can be noticed that higher concrete strength increases the load capacity of beam.

Fig. 6.4 shows the plots of moment vs. curvature for the section with three different ratios of plate area to gross concrete section. The remaining parameters are held constant at the following values: $f'_c = 20.7 \text{ MPa (3,000 psi)}$; $\rho = 0.015$; $F_{yp} = 413.7 \text{ MPa (60 ksi)}$; and $E_{yp} = 34,474 \text{ MPa (5,000 ksi)}$. In addition, the moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moment of beams R3H5H, R3H5M and R3H5L are increased by 23.6%, 13.9% and 7.7% as compared to that of beam R3H with no plate. However, the ultimate curvatures are lowered by 20.1%, 12.7% and 7.4%, respectively. Additionally, the ultimate moment of beam R3H5H is 8.5% higher than that of beam R3H5M, while the ultimate curvature is 8.5% lower. The ultimate moment of beam R3H5M is 5.7% higher than that of beam R3H5L, while the ultimate curvature is 5.7% lower. Comparing Fig. 6.4 with Fig. 6.1, in which the only difference between these two graphs is the values of the steel reinforcement ratio, it can be further proved that the bonding of the plate for reinforcing beam is
more effective when the beam has a relatively low steel reinforcement ratio. High reinforcement ratio increases a beam’s capacity significantly.

Fig. 6.5 shows the plots of moment vs. curvature for the section with three different ratios of plate area to gross concrete section. The remaining parameters are held constant at the following values: $f'_c = 20.7 \, MPa \, (3,000 \, psi); \, \rho = 0.015; \, F_{yp} = 827.4 \, MPa \, (120 \, ksi); \, \text{and} \, E_{yp} = 68,948 \, MPa \, (10,000 \, ksi)$. In addition, the moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moment of beams R3L10H, R3L10M and R3L10L are increased by 203.5%, 146.4% and 100.1% as compared to that of beam R3L with no plate. However, the ultimate curvatures are lowered by 66.9%, 58.3% and 48%, respectively. Also, the ultimate moment of beam R3L10H is 23.2% higher than that of beam R3L10M, while the ultimate curvature is lower by 20.6%. The ultimate moment of beam R3L10M is 23.1% higher than that of beam R3L10L, while the ultimate curvature is 19.8% lower. Comparing Fig. 6.5 with Fig. 6.1, it can be noticed that when the only difference among the parameters is the plate properties (plate ultimate strength and modulus of elasticity), the higher the ultimate strength of the plate, the higher the ultimate moment of the beam. By increasing the elastic modulus and ultimate strength of the plastic plate, the capacity of beams can be increased. The higher the area of the plate, the higher beam capacity can be obtained, but the moment capacity cannot be increased indefinitely as it can be noticed in Fig. 6.5.

The following paragraphs study the effect of different plate elastic moduli and ultimate strengths on the behavior of rectangular beams. Two plate ultimate strengths and two elastic moduli are used. These are $F_{pl} = 413.7 \, MPa \, (60 \, ksi)$ and $E_{pl} = 34,474 \, MPa \, (5,000 \, ksi); \, F_{pl} = 827.4 \, MPa \, (120 \, ksi)$ and $E_{pl} = 68,948 \, MPa \, (10,000 \, ksi)$.

Fig. 6.6 shows the plots of moments vs. curvature for the section with different plate elastic moduli and ultimate strengths. The remaining parameters are held constant at the following values: $f'_c = 20.7 \, MPa \, (3,000 \, psi); \, \rho = 0.005; \, \text{and} \, A_{pl}/bh = 0.0025$. In addition, the moment vs. curvature for the same section without plate
is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments of beams R3L10L and R3L5L are 100.1% and 59.6% higher than that of beam R3L with no plate. However, the ultimate curvatures are lowered by 48% and 43.2%, respectively. The ultimate moment of beam R3L10L is 25.4% higher than that of beam R3L5L, while its ultimate curvature is 8.4% lower.

Fig. 6.7 shows the plots of moment vs. curvature for the section with different plate elastic moduli and ultimate strengths. The remaining parameters are held constant at the following values: $f'_c = 41.4 \text{ MPa (6,000 ksi)}$; $\rho = 0.015$; and $A_{pl}/bh = 0.015$. In addition, the moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moment for beams R6H10H and R6H5H are 73.1% and 47.9% higher than that of beam R6H with no plate. However, the curvatures are 42.1% and 31.7% lower, respectively. Also, the ultimate moment of beam R6L10H is 17.1% higher than that of beam R6L5H, while its ultimate curvature is 15.2% lower. Comparing Fig. 6.7 with Fig. 6.6, it can be seen that for beams having higher values of concrete cylinder strength, $f'_c$, steel reinforcement ratio, $\rho$, and plate area, $A_{pl}$, bonding plate to increase the ultimate load capacity is not very efficient.

Fig. 6.8 shows the plots of moments vs. curvature for section with different plate elastic moduli and ultimate strengths. The remaining parameters are held constant at the following values: $f'_c = 6,000 \text{ ksi (41.4 MPa)}$; $\rho = 0.005$; and $A_{pl}/bh = 0.0025$. In addition, the moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments for beams R6L10L and R6L5L are 118.6% and 57.6% higher than that of beam R6L with no plate. However, the curvatures are 68.9% and 70.6% lower, respectively. Additionally, the ultimate moment of beam R6L10L is 38.7% higher than that of beam R6L5L, while the ultimate curvature is 5.8% lower. Comparing Fig. 6.8 with Fig. 6.6, it can be seen that higher concrete compression strength increases the ultimate capacity of the beam.

Fig. 6.9 shows the plots of moment vs. curvature for section with different plate elastic moduli and ultimate strengths. The remaining parameters are held constant at the following values: $f'_c = 20.7 \text{ MPa (3,000 psi)}$; $\rho = 0.015$; and $A_{pl}/bh$
= 0.0025. In addition, the moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments for beams R3H10L and R3H5L are 13.8% and 7.7% higher than that of beam R3H with no plate. However, the curvatures are 12.7% and 7.4% lower, respectively. The ultimate moment of beam R3H10L is 5.7% higher than that of beam R3H5L, while its ultimate curvature is 5.7% lower. Comparing Fig. 6.9 with Fig. 6.6, it can be seen that high steel reinforcement ratio significantly increases the load capacity and that bonding the plate will not greatly increase the ultimate capacity when the steel reinforcement ratio is relatively high.

Fig. 6.10 shows the plots of moments vs. curvature for the section with different plate elastic moduli and ultimate strengths. The remaining parameters are held constant at the following values: \( f'_c = 20.7 \text{ MPa (3,000 psi)} \); \( \rho = 0.005 \); and \( A_{pl}/bh = 0.005 \). The moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments for beams R3L10M and R3L5M are 146.4% and 100.5% higher than that of beam R3L with no plate. However, the curvatures are 58.3% and 48% lower, respectively. The ultimate moment of beam R3L10M is 22.9% higher than that of beam R3L5M, while its ultimate curvature is 19.7% lower. By comparing Fig. 6.10 with Fig. 6.6, it can be noticed that when the steel reinforcement ratios remain the same, the ultimate capacity of beam can be increased significantly by increasing the plate area.

Fig. 6.11 shows the plots of moments vs. curvature for the section with different plate elastic moduli and ultimate strengths. The remaining parameters are held constant at the following values: \( f'_c = 20.7 \text{ MPa (3,000 psi)} \); \( \rho = 0.005 \); and \( A_{pl}/bh = 0.015 \). The moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments for beams R3L10H and R3L5H are 203.5% and 147.4% higher than that of beam R3L, while the curvatures are 66.9% and 58.4% lower, respectively. The ultimate moment of beam R3L10H is 22.7% higher than that of beam R3L5H, while the ultimate curvature is 20.5% lower. By comparing Fig. 6.11 with Fig. 6.6,
it can be seen that the ultimate capacity of the beam is increased significantly by increasing the plate area when the steel reinforcement ratio is held constant.

From this study it is concluded that plating plays an important role in increasing beam’s ultimate strength when steel reinforcement ratio is low; for beams having higher values of concrete compression strength and steel reinforcement ratio, plating is not very efficient. The higher the steel reinforcement ratio, the smaller the effect of plating on ultimate load capacity; by increasing the elastic modulus and the ultimate strength of plate, the load capacity of the beam can be increased significantly while the beam’s other properties are held constant; the load capacity can be increased significantly by increasing the elastic modulus and ultimate strength of the plate and the plate area.

6.3 T-Beam

Figs. 6.12 through 6.22 show the effects of variable parameters on the moment vs. curvature behavior of a T-beam. The cross-sectional properties of the T-beam are as follows: 457 mm (18 in) high; 813 mm (32 in) wide by 102 mm (4 in) thick flange; and 8 in (203 mm) thick web.

Fig. 6.12 shows the plots of moment vs. curvature for the section with three different ratios of plate area to gross concrete section. The remaining parameters are held constant at the following values: \(f'_c = 20.7 \text{ MPa} \) (3,000 psi); \(\rho = 0.005\); \(F_y = 413.7 \text{ MPa} \) (60 ksi); and \(E_y = 34,474 \text{ MPa} \) (5,000 ksi). The moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments of beams T3L5H, T3L5M and T3L5L are increased by 247.5%, 121.3% and 60.1% as compared to that of beam T3L with no plate. However, the ultimate curvatures are lowered by 74.3%, 75.4% and 75.9%, respectively. Also, the ultimate moment of beam T3L5H is 57% higher than that of beam T3L5M, while the ultimate curvature is 4.8% lower. The ultimate moment of beam T3L5M is 38.2% higher than that of beam T3L5L, while the ultimate curvature is 1.8% lower.
Fig. 6.13 shows the plots of moment vs. curvature for the section with three different ratios of plate area to gross concrete section. The remaining parameters are held constant at the following values: $f'_c = 41.4 \text{ MPa (6,000 ksi)}$; $\rho = 0.015$; $F_{yp} = 827.4 \text{ MPa (120 ksi)}$; and $E_{yp} = 68,948 \text{ MPa (10,000 ksi)}$. In addition, the moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments of beams T6H10H, T6H10M and T6H10L are increased by 165.8% and 84.8% and 39.1% as compared to that of beam T6H with no plate. However, the ultimate curvatures are lowered by 74.6%, 74.7% and 76.6%, respectively. The ultimate moment of beam T6H10H is 43.8% higher than that of beam T6H10M. The ultimate moment of beam T6H10M is 32.9% higher than that of beam T6H10L. The ultimate curvatures of beams T6H5H, T6H10M and T6H10L are approximately the same. Comparing Fig. 6.13 with Fig. 6.12, it can be seen from the plots that the plate area plays an important role in increasing a beam’s ultimate strength when the steel reinforcement ratio is relatively low. The higher the steel reinforcement, the smaller the effect of the plate area on the ultimate capacity.

Fig. 6.14 shows the plots of moment vs. curvature for the section with three different ratios of plate area to gross concrete section. The remaining parameters are held constant at the following values: $f'_c = 41.7 \text{ MPa (6,000 ksi)}$; $\rho = 0.005$; $F_{yp} = 413.7 \text{ MPa (60 ksi)}$; and $E_{yp} = 34,474 \text{ MPa (5,000 ksi)}$. The moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments of beams T6L5H, T6L5M and T6L5L are increased by 240.8% and 128.5% and 59.2% as compared to that of beam T6L with no plate. However, the ultimate curvatures are lowered by 78.7%, 77.6% and 79.4%, respectively. The ultimate moment of beam T6L5H is 49.1% higher than that of beam T6L5M. The ultimate moment of beam T6L5M is 43.5% higher than that of beam T6L5L. The ultimate curvatures of beams T6L5H, T6L5M and T6L5L are approximately equal. Comparing Fig. 6.14 with Fig. 6.12, it is seen that higher concrete strength increases some load capacity of the beam.

Fig. 6.15 shows the plots of moment vs. curvature for the section with three different ratios of plate area to gross concrete section. The remaining parameters
are held constant at the following values: $f'_c = 20.7 \text{ MPa (3,000 psi)}$; $\rho = 0.015$; $F_{yp} = 413.7 \text{ MPa (60 ksi)}$; and $E_{yp} = 34,474 \text{ MPa (5,000 ksi)}$. The moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments of beams T3H5H, T3H5M and T3H5L are increased by 82.6%, 41.4% and 20.2% as compared to that of beam T3H with no plate. However, the ultimate curvatures are lowered by 46.3%, 47.7% and 48.7%, respectively. Additionally, the ultimate moment of beam T3H5H is 29.2% higher than that of beam T3H5M. The ultimate moment of beam T3H5M is 17.6% higher than that of beam T3H5L. The ultimate curvatures of beams T3H5H, T3H5M and T3H5L are approximately equal. Comparing Fig. 6.15 with Fig. 6.12, in which the only difference is the values of steel reinforcement ratio, it can be further proved that the bonding of the plate for reinforcing the beam is more effective when the beam has relatively low steel reinforcement ratio.

Fig. 6.16 shows the plots of moment vs. curvature for section with three different ratios of plate area to gross concrete section. The remaining parameters are held constant at the following values: $f'_c = 20.7 \text{ MPa (3,000 psi)}$; $\rho = 0.015$; $F_{yp} = 827.4 \text{ MPa (120 ksi)}$; and $E_{yp} = 68,948 \text{ MPa (10,000 ksi)}$. The moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments of beams T3L10H, T3L10M and T3L10L are increased by 500.12% and 246.1% and 121% as compared to that of beam T3L with no plate. However, the ultimate curvatures are lowered by 71.7%, 74.2% and 75.4%, respectively. The ultimate moment of beam T3L10H is 73.4% higher than that of beam T3L10M. The ultimate moment of beam T3L10M is 56.6% higher than that of beam T3L10L. The ultimate curvatures of beams T3L10H, T3L10M and T3L10L are approximately the same. Comparing Fig. 6.16 with Fig. 6.12, it is seen that when the only difference among the parameters is the plate properties (plate ultimate strength and modulus of elasticity), the higher the ultimate strength of the plate, and the greater the ultimate moment of the beam. By increasing the elastic modulus and ultimate strength of the plastic plate, the capacity of beams can be increased significantly. The higher the area of the plate, the higher the load capacity.
In the following paragraphs, the effect of different plate elastic moduli and ultimate strengths on the behavior of T-beams is studied. The two ultimate strengths used are $F_{pl} = 413.7 \text{ MPa} (60 \text{ ksi})$ and $E_{pi} = 34,474 \text{ MPa} (5,000 \text{ ksi}); F_{pl} = 827.4 \text{ MPa} (120 \text{ ksi})$ and $E_{pi} = 68,948 \text{ MPa} (10,000 \text{ ksi})$.

Fig. 6.17 shows the plots of moments vs. curvature for the section with different plate elastic moduli and ultimate strengths. The remaining parameters are held constant at the following values: $f'_{c} = 20.7 \text{ MPa} (3,000 \text{ psi}); \rho = 0.005; \text{ and } A_{pl}/bh = 0.0025$. The moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moment of beams T3L10L and T3L5L are 121% and 60.1% higher than that of beam T3L with no plate. However, the ultimate curvatures are lowered by 75.4% and 75.9%, respectively. Additionally, the ultimate moment of beam R3L10L is 38% higher than that of beam T3L5L. The ultimate curvatures of beams T3L10L, T3L5L are approximately equal. From the plots, it can be seen that the ultimate moment is doubled when the elastic modulus of the plate is doubled if the plate area is held constant.

Fig. 6.18 shows the plots of moments vs. curvature for section with different plate elastic moduli and ultimate strengths. The remaining parameters are held constant at the following values: $f'_{c} = 6,000 \text{ ksi} (41.4 \text{ MPa}); \rho = 0.015; \text{ and } A_{pl}/bh = 0.015$. The moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. The ultimate moments for beams T6H10H and T6H5H are 165.8% and 85.4% higher than that of beam T6H with no plate. However, the curvatures are 74.6% and 74.8% lower, respectively. Additionally, the ultimate moment of beam T6L10H is 43.4% higher than that of beam T6L5H, while the ultimate curvature is approximately equal. Comparing Fig. 6.18 with Fig. 6.17, it can be seen that for beams having higher values of concrete compression strength, $f'_{c}$, steel reinforcement ratio, $\rho$, and plate area, $A_{pl}$, the ultimate load capacity can be up to 3.5 times higher than that of beams having lower values.

Fig. 6.19 shows the plots of moment vs. curvature for section with different plate elastic moduli and ultimate strengths. The remaining parameters are held
constant at the following values: \( f'_c = 6,000 \text{ ksi (41.4 MPa)}; \rho = 0.005; \) and \( A_{pl}/bh = 0.0025 \). The moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments for beams T6L10L and T6L5L are 128.1% and 59.2% higher than that of beam T6L with no plate. However, the curvatures are 77.6% and 79.4% lower, respectively. Additionally, the ultimate moment of beam T6L10L is 43.3% higher than that of beam T6L5L, while the ultimate curvature is 5.8% lower. Comparing Fig. 6.19 with Fig. 6.17, it can be seen that higher concrete compression strength gives higher load capacity.

Fig. 6.20 shows the plots of moments vs. curvature for section with different plate elastic moduli and ultimate strengths. The remaining parameters are held constant at the following values: \( f'_c = 20.7 \text{ MPa (3,000 psi)}; \rho = 0.015; \) and \( A_{pl}/bh = 0.0025 \). The moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments for beams T3H10L and T3H5L are 41.2% and 20.2% higher than that of beam T3H with no plate. However, the ultimate curvatures are 47.7% and 48.8% lower, respectively. The ultimate moment of beam T3H10L is 17.5% higher than that of beam T3H5L. The ultimate curvatures of beams T3H10L and T3H5L are almost the same. Comparing Fig. 6.20 with Fig. 6.17, for beams having higher value of steel reinforcement ratio, \( \rho \), in Fig. 6.20, it is seen that high steel reinforcement ratio significantly increases the load capacity, but the ultimate capacity is not greatly increased by bonding the plate when the steel reinforcement ratio is relatively high.

Fig. 6.21 shows the plots of moment vs. curvature for section with different plate elastic moduli and ultimate strengths. The remaining parameters are held constant at the following values: \( f'_c = 20.7 \text{ MPa (3,000 psi)}; \rho = 0.005; \) and \( A_{pl}/bh = 0.005 \). The moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments for beams T3L10M and T3L5M are 246.5% and 121.3% higher than that of beam T3L with no plate. However, the curvatures are 74.2% and 75.4% lower, respectively. Additionally, the ultimate moment of beam T3L10M is 16% higher than that of beam T3L5M, while the ultimate curvature is 4.9% lower. Comparing
Fig. 6.21 with Fig. 6.17 it can be calculated that the ultimate load capacities of beams T3L5M and T3L10M are 56.6% and 38.2% higher than those of beams T3L5L and T3L10L. Higher plate area gives higher load capacity.

Fig. 6.22 shows the plots of moments vs. curvature for section with different plate elastic moduli and ultimate strengths. The remaining parameters are held constant at the following values: $f'_c = 20.7 \text{ MPa (3,000 psi)}$; $\rho = 0.005$; and $A_{pl}/bh = 0.015$. The moment vs. curvature for the same section without plate is plotted with dashed-dotted lines. From the plots, it can be seen that the ultimate moments of beams T3L10H and T3L5H are 500.2% and 247.5% higher than that of beam T3L with no plate. However, the curvatures are 71.7% and 74.3% lower, respectively. Additionally, the ultimate moment of beam T3L10H is 72.7% higher than that of beam T3L5H, while the ultimate curvature is 10% lower. Comparing Fig. 6.22 with Fig. 6.17, it can be calculated that the ultimate capacities of beams T3L5H and T3L10H are 171.6% and 117% higher than those of beams T3L5L and T3L10L. When the steel reinforcement ratio and concrete compression strength are relatively low, the load capacity of beams can be increased up to five times by bonding a plate having high ultimate strength, elastic modulus and plate area.

From the study, it is concluded that plating plays an important role in increasing a beam’s ultimate strength when steel reinforcement ratio is low. The higher the steel reinforcement ratio, the smaller the effect of plating on the ultimate load capacity; the load capacity can be increased significantly by increasing ultimate strength and elastic modulus of the plate; the higher the plate area, the higher the beam ultimate capacity; when steel reinforcement ratio and concrete compression strength are low, a much greater increase in load capacity can be obtained by bonding plate to the tension flange.

Plating has a greater effect on increasing the ultimate capacity of T-beams than its does on rectangular beams; it decreases T-beams’ ductility more than it does that of rectangular beams.
CHAPTER 7

CONCLUSIONS

The experimental and analytical studies performed on concrete beams strengthened with epoxy-bonded fiber composite plates indicate that this method can be used to effectively and economically increase the load carrying capacity of structurally deficient concrete beams. The following conclusions are drawn from the results of the study.

1. The bonding of external reinforcement in the form of fiber composite plates glued to the tension face of reinforced concrete beams enlarges the elastic range of behavior; increases the stiffness of the beam at all load levels; delays the appearance of the first visual cracks; and increases the ultimate capacity.

2. An ideal epoxy for the plating application would be one that has sufficient strength to transfer the shear forces between the concrete beam and composite plate. At the same time, the epoxy should be ductile enough to prevent a brittle failure after concrete cracks.

3. The analytical models, based on the compatibility of deformations and equilibrium of forces, predict well the stresses, deflection, and curvature throughout the entire range of loading up to failure.

4. The results of the analytical studies indicate that plating is more efficient for under designed reinforced concrete beam. Therefore, this method is particularly desirable for structurally deficient girders. Plating is not very effective for girders with high steel reinforcement ratio because, in these types of girders, failure will reach by premature crushing of concrete without the full utilizations of steel and composite plate.
Fig. 4.1 Measured Stress-Strain Relationship of Steel Rebar and GFRP Plate
Fig. 4.2 Preparation of Concrete Surface
Fig. 4.3 Concrete Surface After Sandblasting
Fig. 4.4 Curing of Epoxy
Fig. 4.5a Test Set-Up (Beam A)

Fig. 4.5b Load vs. Deflection at Midspan (Beam A)
Fig. 4.5c Load vs. Strain in Steel Rebar at Midspan for (Beam A)

Fig. 4.5d Load vs. Strain in Concrete at Midspan for (Beam A)
Fig. 4.5e Beam A at Failure
Fig. 4.6a Test Set-Up (Beam B)

Fig. 4.6b Load vs. Deflection at Midspan (Beam B)
Fig. 4.6c Load vs. Strain in Steel Rebar at Midspan (Beam B)

Fig. 4.6d Load vs. Strain in Composite Plate at Midspan (Beam B)
Fig. 4.6e Load vs. Strain in Concrete at Midspan (Beam B)

Fig. 4.6f Beam B at Failure
Fig. 4.7a Load vs. Deflection at Midspan (Beam A & B)

Fig. 4.7b Load vs. Strain in Steel Rebar at Midspan (Beam A & B)
Fig. 4.7c Load vs. Strain in Concrete at Midspan (Beam A & B)
Fig. 4.8a Test Set-Up (Beam C)

Fig. 4.8b Load vs. Deflection at Midspan (Beam C)
Fig. 4.8c Load vs. Strain in Steel Rebar at Midspan (Beam C)

Fig. 4.8d Load vs. Strain in Composite Plate at Midspan (Beam C)
Fig. 4.8e Beam C at Failure
Fig. 4.9a Test Set-Up (Beam D)

Fig. 4.9b Load vs. Deflection at Midspan for Beam D
Fig. 4.9c Load vs. Strain in Steel Rebar at Midspan (Beam D)

Fig. 4.9d Load vs. Strain in Composite Plate at Midspan (Beam D)
Fig. 4.9e Load vs. Strain in Concrete at Midspan (Beam D)

Fig. 4.9f Beam D at Failure
Fig. 4.10a Beam E During Post-Tensioning
Fig. 4.10b Test Set-Up (Beam E)

Fig. 4.10c Load vs. Deflection at Midspan (Beam E)
Fig. 4.10d Load vs. Strain in Steel Rebar at Midspan (Beam E)

Fig. 4.10e Load vs. Strain in Composite Plate at Midspan (Beam E)
Fig. 4.10f Load vs. Strain in Concrete at Midspan (Beam E)

Fig. 4.10g Beam E at Failure
Fig. 4.11a Test Set-Up (Beam F)

Fig. 4.11b Load vs. Deflection at Midspan (Beam F)
Fig. 4.11c Load vs. Strain in Steel Rebar at Midspan (Beam F)

Fig. 4.11d Load vs. Strain in Composite Plate at Midspan (Beam F)
Fig. 4.11e Load vs. Strain in Concrete at Midspan (Beam F)

Fig. 4.11f Beam F at Failure
Fig. 4.12a Test Set-Up (Beam F')

Fig. 4.12b Load vs. Deflection at Midspan (Beam F')
Fig. 4.12c Load vs. Strain in Steel Rebar at Midspan (Beam F')

Fig. 4.12d Load vs. Strain in Composite Plate at Midspan (Beam F')
Fig. 4.12c Load vs. Strain in Concrete at Midspan (Beam F')

Fig. 4.12f Beam F' at Failure
Fig. 4.13a Test Set-Up (Beam G)

Fig. 4.13b Load vs. Deflection at Midspan (Beam G)
Fig. 4.13c Load vs. Strain in Steel Rebar at Midspan (Beam G)

Fig. 4.13d Load vs. Strain in Composite Plate at Midspan (Beam G)
Fig. 4.13e Load vs. Strain in Concrete at Midspan (Beam G)

Fig. 4.13f Beam G at Failure
Fig. 4.14a Test Set-Up (Beam H)

Fig. 4.14b Load vs. Deflection at Midspan (Beam H)
Fig. 4.14c Load vs. Strain in Composite Plate at Midspan (Beam H)

Fig. 4.14d Load vs. Strain in Concrete at Midspan (Beam H)
Fig. 4.14e Beam H at Failure
Fig. 5.1 (Hognestad's Parabola)

Idealized Stress-Strain Curve for Concrete in Uniaxial Compression

\[ f_c = f_c'' \left[ 1 - \frac{0.15}{0.004 - \epsilon_o} (\epsilon_c - \epsilon_o) \right] \]

\[ f_c = f_c'' \left[ \frac{2\epsilon_c}{\epsilon_o} - \left( \frac{\epsilon_c}{\epsilon_o} \right)^2 \right] \]

\[ \epsilon_o = \frac{2f_c''}{E_c} \]

\[ E_c = \tan \alpha \]
Fig. 5.2 Stress-Strain Curve for Steel
Fig. 5.3 Stress-Strain Curve for Composite Plate
Fig. 5.4 Rectangular Section with Strain, Stress and Force Distribution
Fig. 5.5 Curvature Diagram
Fig. 5.6 Section with Strain, Stress and Force Distribution (T-Beam)
Fig. 5.7 Stress-Strain curve for Concrete in Case 1
Fig. 5.8 Stress-Strain curve for Concrete in Case 2
Fig. 5.9 Stress-Strain Curve for Concrete in Case 3
Fig. 5.10 Stress-Strain Curve for Steel (Cambered Beam)
Fig. 5.11 Stress-Strain Curve for Plate (Cambered Beam)
Fig. 6.1 Moment vs. Curvature (Beams R3L, R3L5L, R3L5M and R3L5H)

Fig. 6.2 Moment vs. Curvature (Beams R6H, R6H10L, R6H10M and R6H10H)
Fig. 6.3 Moment vs. Curvature (Beams R6L, R6L5L, R6L5M and R6L5H)

Fig. 6.4 Moment vs. Curvature (Beams R3H, R3H5L, R3H5M and R3H5H)
Fig. 6.5 Moment vs. Curvature (Beams R3L, R3L10L, R3L10M and R3L10H)

Fig. 6.6 Moment vs. Curvature (Beams R3L, R3L5L and R3L10L)
Fig. 6.7 Moment vs. Curvature (Beams R6H, R6H5H and R6H10H)

Fig. 6.8 Moment vs. Curvature (Beams R6L, R6L5L and R6L10L)
Fig. 6.9 Moment vs. Curvature (Beams R3H, R3H5L and R3H10L)

Fig. 6.10 Moment vs. Curvature (Beams R3L, R3L5M and R3L10M)
Fig. 6.11 Moment vs. Curvature (Beams R3H, R3L5H and R3L10H)
Fig. 6.12 Moment vs. Curvature (Beams T3L, T3L5L, T3L5M and T3L5H)

Fig. 6.13 Moment vs. Curvature (Beams T6H, T6H10L, T6H10M and T6H10H)
Fig. 6.14 Moment vs. Curvature (Beams T6L, T6L5L, T6L5M and T6L5H)

Fig. 6.15 Moment vs. Curvature (Beams T3H, T3H5L, T3H5M and T3H5H)
Fig. 6.16 Moment vs. Curvature (Beams T3L, T3L10L, T3L10M and T3L10H)

Fig. 6.17 Moment vs. Curvature (Beams T3L, T3L5L and T3L10L)
Fig. 6.18 Moment vs. Curvature (Beams T6H, T6H5H and T6H10H)

Fig. 6.19 Moment vs. Curvature (Beams T6L, T6L5L and T6L10L)
Fig. 6.20 Moment vs. Curvature (Beams T3H, T3H5L and T3H10L)

Fig. 6.21 Moment vs. Curvature (Beams T3L, T3L5M and T3L10M)
Fig. 6.22 Moment vs. Curvature (Beams T3H, T3L5H, and T3L10H)
### TABLE 4-1 DESIGN DETAILS OF TEST SPECIMENS

<table>
<thead>
<tr>
<th>BEAM</th>
<th>COMPRESSION STEEL</th>
<th>TENSION STEEL</th>
<th>NO. OF STIRRUPS***</th>
<th>PLATE LENGTHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2 No. 4</td>
<td>3 No. 8</td>
<td>14@13&quot;</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td>4267</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>2 No. 4</td>
<td>34@6&quot;</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>2 No. 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E*</td>
<td></td>
<td>2 No. 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F*</td>
<td></td>
<td>2 No. 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G**</td>
<td>3 No. 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>2 No. 4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Cambered
** T-Beam
*** No.3 Stirrups were used for all beams
APPENDIX A
Rectangular Beam

Read
Iter, Nst, Axial
H, Wide, Ulim, Llim
Fprco
Yield, Elmst, Ystr
Ast, Dst
Apl, Dpl
Elmpl, Upstn, Upsts

Calculate Strcm

Strcm = 0

If N .Gt. 1

No

Yes

Calculate Strcm

Calculate Elmco, Strco

Calculate α, γ

If Strcm .Lt. Strco

No

Yes

Calculate α, γ

Calculate Stfor, Forpl, Px

Calculate C

If Px .Lt. Tol

No

Yes

Calculate Stns, Strap, Xmom, Cur, Defl, Zload

Write Xmom, Cur, Strcm, Stns, Strap, Zload, Defl

Stop
This program is to calculate the deflection, moment, and curvature at midspan, strains in top concrete fiber, tension rebars, and plate at midspan and the external loads for the rectangular beam under 4-point loading.

```
IMPLICIT REAL*16(A-H,O-Z)
DIMENSION XMOM(IOOO),CUR(1000)
integer nfiles
character*80 filel,file2,file3
COMMON/MOHAN/ ZLOAD,STNS,STRAP,STRS
COMMON/STRAIN/ STRCO,STRCM,YSTR1,YSTR2,USTRA,UPSTN
COMMON/STRESS/ ELMST,ELMPL,YIELD,USTRE,UPSTS,FPRCO
COMMON/FORCE/ FORST(1000),FORPL,AXIAL,STFOR,ALPHA,GAMMA
COMMON/DISTNCE/ DST(IOOO),DPL,WIDE
COMMON/AREA/ AST(IOOO),APL,NST
COMMON/XFORC/ PX,inp,n,iter
open (unit=20,file='ins.dat',status='old')
read (20,'(i2)') nfiles
do 25 i=1,nfiles
read (20,'(a80/a80/a80)') filel,file2,file3
OPEN(UNIT=5,FILE='AS1.DAT',TYPE='OLD')
OPEN (UNIT=6, FILE='MOMT.AS1', TYPE='NEW')
OPEN(UNIT=7,FILE='CURV.AS1',TYPE='NEW')
OPEN(UNIT=8,FILE='CON.AS1',TYPE='NEW')
OPEN(UNIT=9,FILE='STEL.AS1',TYPE='NEW')
OPEN(UNIT=10,FILE='PLAT.AS1',TYPE='NEW')
OPEN(UNIT=11,FILE='KIP.AS1',TYPE='NEW')
OPEN(UNIT=12,FILE='DEF.AS1',TYPE='NEW')
READ(5,*) ITER,NST,AXIAL
iteration no., nst°1 if no comp. steel; =2 with comp. steel, axial load
READ(5,*) H,WIDE,ULIM,LLIM
section height, width, upper limit, lower limit
READ(5,*) FPRCO
concrete strength in a member
READ(5,*) YIELD,ELMST,YSTR1
eyield strength, elastic modulus of steel, yield strain
DO L = 1,NST
   n = 2
   READ(5,*) AST(L),DST(L)
   area of steel, distance from steel rebar to top comp. fiber
END DO
READ(5,*) APL,DPL
area of plate, distance from centroid of plate to top comp. fiber
READ(5,*) ELMPL,UPSTN,UPSTS
elastic modulus of plate, ultimate strain of plate, ultimate stress of plate
TOL = 1.E-3
```
STRCM = 0.0
STRIN = 0.003/ITER
A = ULIM
B = LLIM
C = (A+B)/2.
DO N = 1,(ITER+1)
  INF = 0
  IF(N.GT.1) THEN
    STRCM = STRCM+STRIN
  ELSE
    END IF
  print*,'strcm =',strcm
  ELMCO = 57.*(FPRCO*1000.)**0.5
  STRCO = 2.*FPRCO/ELMCO
  IF (STRCM.LT.STRCO) THEN
    ALPHA = (STRCM/STRCO)-(1./3.)*(STRCM/STRCO)**2
    GAMMA = (1./3.-STRCM/(12.*STRCO))/(1.-STRCM/(3.*STRCO))
  ELSE
    AX = 0.004 - STRCO
    BX = STRCM/2. - STRCO
    ALPHA = (-STRCO/3. + STRCM*(1.-(0.15/AX)*BX)-
    (0.075/AX)*(STRCO**2))/STRCM
    CX = (STRCM**3/3.-STRCO*STRCM**2/2.)
    DX = (STRCM**2/2.-STRCO*STRCM)
    EX = -STRCO**2/12.+STRCM**2/2.-(0.15/AX)*CX-(0.15/AX)*
    STRCO**3/6.
    FX = -STRCO/3.+STRCM-0.15/AX*DX-0.075/AX*STRCO**2
    GAMMA = 1.-EX/(STRCM*FX)
  END IF
  CALL BYSEKT(CiTOL)
  CALL STEEL (C)
  CALL PLATE (C)
  TSTEEL=0.0
  DO R = 1,NST
    TSTEEL=TSTEEL+FORST(K)*(H/2.-DST(K))
  END DO
  XMOMENT = TSTEEL+ALPHA*FPRCO*WIDE*C*(H/2.-GAMMA*C) +
  +FORPL*(H/2.-DPL)
  XMOM(N) = XMOMENT
  CUR(N) = STRCM/C
  DEFL = 3036.*CUR(N)
  ZLOAD = XMOM(N)/39.
  WRITE(6,12) XMOM(N)
  WRITE(7,12) CUR(N)
  WRITE(8,12) STRCM
  WRITE(9,12) STNS
  WRITE(10,12) STRAP
  WRITE(11,12) ZLOAD
  WRITE(12,12) DEFL
12 FORMAT(F13.6)
  END DO
  CLOSE (5)
  CLOSE (6)
  CLOSE(7)
  CLOSE(8)
CLOSE(9)
CLOSE(10)
CLOSE(11)
CLOSE(12)

C25 continue

STOP
END

SUBROUTINE BYSENT(C,TOL)
IMPLICIT REAL*16(A-H,O-Z)
COMMON/MOHAN/ ZLOAD,STNS,STRAP,STRS
COMMON/STRAIN/ STRCO,STRCM,YSTR1,YSTR2,USTRA,UPSTN
COMMON/STRESS/ ELMST,ELMPL,YIELD,USTRE,UPSTS,FPRCO
COMMON/FORCE/ FORST(1000),FORPL,AxIAL,STFOR,ALPHA,GAMMA
COMMON/DISTANCE/ DST(1000),DPL,WIDE
COMMON/AREA/ AST(1000),APL,NST
COMMON/XFORCE/ PX,inp,n,iter

1 CALL STEEL (C)
CALL PLATE (C)
CALL FUNC(C)
IF(QABS(PX).LT.TOL) GO TO 10
IF(PX.LT.0.0) THEN
  C = C + 1.E-05
ELSE
  C = C-1.E-05
END IF
GO TO 1
10 RETURN
END

SUBROUTINE FUNC(C)
IMPLICIT REAL*16(A-H,O-Z)
COMMON/MOHAN/ ZLOAD,STNS,STRAP,STRS
COMMON/STRAIN/ STRCO,STRCM,YSTR1,YSTR2,USTRA,UPSTN
COMMON/STRESS/ ELMST,ELMPL,YIELD,USTRE,UPSTS,FPRCO
COMMON/FORCE/ FORST(1000),FORPL,AxIAL,STFOR,ALPHA,GAMMA
COMMON/DISTANCE/ DST(1000),DPL,WIDE
COMMON/AREA/ AST(1000),APL,NST
COMMON/XFORCE/ PX,inp,n,iter

PX = ALPHA*FPRCO*WIDE*C*STFOR+FORPL

PRINT*,PX=' ',PX
RETURN
END
SUBROUTINE STEEL (CENTR)
IMPLICIT REAL*16(A-H,O-Z)
COMMON/MOHAN/ ZLOAD,STNS,STRAP,STRS
COMMON/STRAIN/ STRCO,STRCM,YSTR1,YSTR2,USTRA,UPSTN
COMMON/STRESS/ ELMST,ELMPL,YIELD,USTRE,UPSTS,FPRCO
COMMON/FORCE/ FORST(1000),FORPL,AXIAL,STFOR,ALPHA,GAMMA
COMMON/DISTNCE/ DST(1000),DPL,WIDTH
COMMON/AREA/ AST(1000),A_PL,NST
COMMON/XFORC/ PX,inp,n,iter
DIMENSION STRAS(1000),STRES(1000)
STFOR = 0.0
DO J = 1,NST
  STRAS(J) = STRCM* (CENTR -DST(J))/CENTR
  IF (QABS(STRAS(J)).LT.YSTR1) THEN
    STRES(J) = ELMST*QABS(STRAS(J))
  ELSE
    STRES(J) = YIELD
  END IF
  IF (STRAS(J).LT.0.0) THEN
    STRES(J) = -STRES(J)
    FORST(J) = STRES(J)*AST(J)
    STFOR = STFOR + FORST(J)
  ELSE
    FORST(J) = STRES(J)*AST(J)
    STFOR = STFOR + FORST(J)
  END IF
END DO
STNS = STRAS(NST)
RETURN
END

SUBROUTINE PLATE (CENTR)
IMPLICIT REAL*16(A-H,O-Z)
COMMON/MOHAN/ ZLOAD,STNS,STRAP,STRS
COMMON/STRAIN/ STRCO,STRCM,YSTR1,YSTR2,USTRA,UPSTN
COMMON/STRESS/ ELMST,ELMPL,YIELD,USTRE,UPSTS,FPRCO
COMMON/FORCE/ FORST(1000),FORPL,AXIAL,STFOR,ALPHA,GAMMA
COMMON/DISTNCE/ DST(1000),DPL,WIDTH
COMMON/AREA/ AST(1000),A_PL,NST
COMMON/XFORC/ PX,inp,n,iter
STRAP = STRCM* (CENTR -DPL)/CENTR
IF (QABS(STRAP).GT.UPSTN) THEN
  STREP = 0.0
ELSE
  STREP = ELMPL*STRAP
END IF
FORPL = STREP * A_PL
RETURN

END
Tee Beam

Read
Iter, Nat, Axial
H, B, Bl, H1, Ulim, Llim
Fprco
Yield, Elmst, Ystr
Ast, Dst
ApI, Dpi
Elmpl, Upstn, Upsts

Calculate Strcm

If N . Gt. 1

Calculate Elmco, Strco

Calculate Stfor, Forpl, Px

Calculate C

If Px . Lt. Tol

If C . Lt. H1

If Px1 . Lt. Tol

Calculate Stns, Strap, Xmom, Cur, Defl, Zload

Write Xmom, Cur, Strcm, Stns, Strap, Zload, Defl

Stop
The program is to calculate the deflection, moment, curvature at midspan, strains in top concrete fiber, tension rebars, and plate at midspan and the external load for T-beam under 4-point loading.

**--------------------------**

**--------------------------**

```fortran
IMPLICIT REAL*16(A-H, O-Z)
COMMON/MOHAN/STNS, STNS1, STRAP, STRS, ALPHA1, ALPHA2, GAMMA1, GAMMA2
COMMON/STRAIN/STNCO, STRCM, YSTR1, YSTR2, USTR, USTP1, STRH1
COMMON/STRESS/ELMST, ELMPL, YIELD, USTRE, USTPS, FPRCO
COMMON/FORCE/ FORST(IOOO), FORPL, AXIAL, STFOR, ALPHA, GAMMA
COMMON/DISTANCE/DST(1000), DPL, B1, H1
COMMON/AREA/AST(1000), APL, NST
COMMON/XFORC/ PX, PX1, INP, N

DIMENSION XMOM(1000), CUR(1000), CSTR(1000), SSTR(1000), PSTR(1000)

DIMENSION DEFL(1000), XLOAD(1000), SSTR1(1000)

OPEN(UNIT=5, FILE=FILE1, TYPE='OLD')
OPEN(UNIT=6, FILE=FILE2, TYPE='NEW')
OPEN(UNIT=7, FILE=FILE3, TYPE='NEW')
C OPEN(UNIT=8, FILE='CONC.OUT', TYPE='NEW')
C OPEN(UNIT=9, FILE='STEEL.OUT', TYPE='NEW')
C OPEN(UNIT=10, FILE='PLATE.OUT', TYPE='NEW')
C OPEN(UNIT=11, FILE='KIP.OUT', TYPE='NEW')
C OPEN(UNIT=12, FILE='DEFL.OUT', TYPE='NEW')

READ(5, *) ITER, NST, AXIAL
C iteration no., nst=1 if no comp. steel, = 2 with comp. steel
READ(5, *) H, B, B1, H1, ULIM, LLIM
C section height, flange width, web width, flange thickness, upper limit,
c lower limit
READ(5, *) FPRCO
C concrete strength in a member
READ(5, *) YIELD, ELMST, YSTR1
C yield strength, elastic modulus of steel, yield strain
DO L = 1, NST
C no. of steel layer
READ(5, *) AST(L), DST(L)
C area of steel, distance from rebar to top comp. fiber
END DO
READ(5, *) APL, DPL
C area of plate, distance from plate centroid to top concrete comp. fiber
READ(5, *) ELMPL, USTP1, USTPS
C elastic modulus of plate, ultimate strain of plate, ultimate stress of plate
CLOSE (5)

STRCM = 0.0
STRIN = 0.003/ITER
TOL = 1.0E-3
XT = ULIM
YT = LLIM

DO N = 1, (ITER+1)
C = (XT+YT)/2.

INP = 0
```
IF(N.GT.1) THEN
  STRCM = STRCM+STRIN
ELSE
END IF

ELMCO = 57.*(FPRCO*1000.)*0.5
STRCO = 2.*FPRCO/ELMCO
CALL BYSEKT(C,TOL)

IF (C.LT.H1) THEN
  C=H1
  CALL BYSEKTL(C,TOL)
  CALL STEEL (C)
  CALL PLATE (C)
  TSTEEL = 0.0
  DO K = 1,NST
    TSTEEL=TSTEEL+FORST(K)*(H/2.-DST(K))
  END DO
  XMOMENT = TSTEEL+ALPHA*FPRCO*C*B*(H/2.-GAMMA*C)+FORPL*(H/2.-DPL)
ELSE
  CALL STEEL (C)
  CALL PLATE (C)
  TSTEEL = 0.0
  DO K = 1,NST
    TSTEEL = TSTEEL+FORST(K)*(H/2.-DST(K))
  END DO
  XMOMENT = TSTEEL+FPRCO*
1 (ALPHA1*(C-H1)*B1*(H/2.-H1-GAMMA1*(C-H1))+
2 ALPHAT2*B1*(H/2.-GAMMA1*H1))
2 +FORPL*(H/2.-DPL)
END IF

XMOM(N) = XMOMENT
CUR(N) = STRCM/C
CSTR(N) = STRCM
SSTR(N) = STNS
SSTR1(N) = STNS1
FSTR(N) = STRAP
DEFL(N) = 3036.*CUR(N)
XLOAD(N) = XMOM(N)/39.

WRITE(*,*)'n=',n
WRITE(*,*)'c=',c
WRITE(*,*)'px=',px
WRITE(*,*)'pxl=',pxl
WRITE(*,*)'alpha=',alpha1
WRITE(*,*)'alpha=',alpha2
WRITE(*,*)'gamma=',gamma1
WRITE(*,*)'gamma=',gamma2
WRITE(*,*)'load=',xload(n)
WRITE(*,*)'concrete=',strcm
WRITE(*,*)'alpha=',alpha
WRITE(*,*)'gamma=',gamma
c write(*,*) 'steel=',stns
  
c write(*,*) 'steel=',stns1

END DO

KX = ITER+1

DO MX = 1,KX

IF(MX.EQ.1) THEN
  WRITE(6,12) XMOM(MX)
  WRITE(7,12) CUR(MX)
  WRITE(8,12) CSTR(MX)
  WRITE(9,12) SSTR(MX)
  WRITE(10,12) PSTR(MX)
  WRITE(11,12) XLOAD(MX)
  WRITE(12,12) DEFL(MX)
  WRITE(13,12) SSTR1(MX)
ELSE
  DIFF = XMOM(MX)-XMOM(MX-1)
  IF(DIFF.LT.0.) GO TO 100
  WRITE(6,13) XMOM(MX)
  WRITE(7,13) CUR(MX)
  WRITE(8,13) CSTR(MX)
  WRITE(9,13) SSTR(MX)
  WRITE(10,13) PSTR(MX)
  WRITE(11,13) XLOAD(MX)
  WRITE(12,13) DEFL(MX)
  WRITE(13,13) SSTR1(MX)
END IF
END DO

100 CLOSE(6)
CLOSE(7)
CLOSE(8)
CLOSE(9)
CLOSE(10)
CLOSE(11)
CLOSE(12)
STOP

END

SUBROUTINE BYSEKT(C,TOL)

IMPLICIT REAL*16(A-H,O-Z)
COMMON/MOHAN/STNS,STNS1,STRAP,STRS,ALPHA1,ALPHA2,GAMMA1,GAMMA2
COMMON/STRAIN/ STRCO,STCRM,YSTR1,YSTR2,USTRA,UPSTN,STRH1
COMMON/STRESS/ ELMST,ELMPL,YIELD,USTRE,UPSTS,FPRCO
COMMON/FORCE/ FORST(1000),FORPL,AXIAL,STFOR,ALPHA,STRE
COMMON/DISTANCE/ DST(1000),DPL,B,B1,H,H1
COMMON/AREA/ AST(1000),APL,NST
COMMON/XFORC/ PX,PX1,INF,N
CALL STEEL (C)
CALL PLATE (C)
CALL FUNC(C)

IF(QABS(PX).LT.TOL) GO TO 10
  IF(PX.LT.0.0) THEN
    C = C +1.E-05
  ELSE
    C = C -1.E-05
  END IF
GO TO 1

10 RETURN
END

SUBROUTINE BYSEKT(C,TOL)
IMPLICIT REAL*16(A-H,O-Z)

COMMON/MOHAN/STNS,STNS1,STRAP,STRS,ALPHA1,ALPHA2,GAMMA1,GAMMA2
COMMON/STRAIN/ STRCO,STRCM,YSTR1,YSTR2,USTRA,UPSTN,STRH1
COMMON/STRESS/ ELMST,ELMPL,YIELD,USTRE,UPSTS,FPRCO
COMMON/FORCE/ FORST(1000),FORPL,AXIAL,STFOR,ALPHA,GAMMA
COMMON/DISTANCE/ DST(1000),DPL,B,B1,H,H1
COMMON/AREA/ AST(1000),APL,NSST
COMMON/XFORCE/ PX,PX1,INP,N

CALL STEEL (C)
CALL PLATE (C)
CALL FUNC1(C)

IF(QABS(PX1).LT.TOL) GO TO 10
  IF(PX1.LT.0.0) THEN
    C = C +1.E-05
  ELSE
    C = C -1.E-05
  END IF
GO TO 1

10 RETURN
END

SUBROUTINE FUNC(C)
IMPLICIT REAL*16(A-H,O-Z)

COMMON/MOHAN/STNS,STNS1,STRAP,STRS,ALPHA1,ALPHA2,GAMMA1,GAMMA2
COMMON/STRAIN/ STRCO,STRCM,YSTR1,YSTR2,USTRA,UPSTN,STRH1
COMMON/STRESS/ ELMST,ELMPL,YIELD,USTRE,UPSTS,FPRCO
COMMON/FORCE/ FORST(1000),FORPL,AXIAL,STFOR,ALPHA,GAMMA
COMMON/DISTANCE/ DST(1000),DPL,B,B1,H,H1
COMMON/AREA/ AST(1000),APL,NSST
COMMON/XFORCE/ PX,PX1,INP,N

IF(STRCM.EQ.0.) GO TO 2000
  STRH1=(1.-H1/C)*STRCM
  IF (STRCM.LT.STRCO) THEN
    ALPHA1 = STRH1**2/(STRCM*STRCO)**(1.-STRH1/(3.*STRCO))
  ELSE
    ALPHA1 = 1. - Z1/Z2
  END IF

  Z1 = (2./3.)-(.25*STRH1/STRCO)
  Z2 = 1.-STRH1/(3.*STRCO)
  GAMMA1 = 1. - Z1/Z2

  X1 = STRCM**2/STRCO
  X2 = 1.-STRCM/(3.*STRCO)
  X3 = STRH1**2/STRCO

  IF(STRCM.EQ.0.) GO TO 2000
  STRH1=(1.-H1/C)*STRCM
  IF (STRCM.LT.STRCO) THEN
    ALPHA1 = STRH1**2/(STRCM*STRCO)**(1.-STRH1/(3.*STRCO))
  ELSE
    ALPHA1 = 1. - Z1/Z2
  END IF

  Z1 = (2./3.)-(.25*STRH1/STRCO)
  Z2 = 1.-STRH1/(3.*STRCO)
  GAMMA1 = 1. - Z1/Z2

  X1 = STRCM**2/STRCO
  X2 = 1.-STRCM/(3.*STRCO)
  X3 = STRH1**2/STRCO
X4 = 1. - STRH1/(3.*STRCO)

ALPHA2 = ((X1*X2-X3*X4)/STRCM)

Y1 = 2*STRCM**3/(3*STRCO)-STRCM**4/(4.*STRCO**2)
Y2 = 2*STRH1**3/(3*STRCO)-STRH1**4/(4.*STRCO**2)
Y3 = STRCM**2*(STRCO)-STRCM**3/(3.*STRCO**2)
Y4 = STRH1**2/(STRCO)-STRH1**3/(3.*STRCO**2)
Y5 = STRH1/(STRCM-STRH1)

GAMMA2 = 1.-(Y1-Y2)/((STRCM-STRH1)*(Y3-Y4))+Y5

ELSE

IF (STRH1.LE.STRCO) THEN

ALPHA1 = STRH1**2/(STRCM*STRCO)*(1.-STRH1/(3.*STRCO))

W1 = (2./.3.)-(.25*STRH1/STRCO)
W2 = 1.-STRH1/(3.*STRCO)
GAMMA1 = 1. - W1/W2

A1 = 2.*STRCO/3.
A2 = STRH1**2/STRCO
A3 = STRH1**3/(3*STRCO**2)
A4 = .15/(0.004-STRCO)
A5 = .5*STRCM**2-STRCM*STRCO+.5*STRCO**2

ALPHA2 = (A1-A2+A3*STRCM-STRCO-A4*A5)/STRCM

E1 = STRCM**4+(.5*STRCO*STRH1)**3-3.*STRH1**4-6.*STRCO*STRCM**2
E2 = .15*STRCM**3/(.004-STRCO)
E3 = 4.*STRCM**3-6.*STRCO*STRCM**2+2.*STRCO**3
E4 = E1+E2+E3
E5 =STRCM-STRH1
E6 = 12.*STRCO**2*STRCM+4.*STRH1**3-4.*STRCO**3-12.*STRCO*STRH1**2
E7 = 6.*STRCM**2-12.*STRCO*STRCM+6.*STRCO**2
E8 =E5*(E6-E2*E7)
GAMMA2 = 1.**E4/E8**/STRH1/E5

ELSE

F1 = -STRCO/3.+STRH1
F2 = .15/(0.004-STRCO)
F3 = .5*STRH1**2-STRCO**2/12.
ALPHA1 = (F1-F2+F3)/STRCM

Q1 = .5*STRH1**2-STRCM**2/12.
Q2 =.15/(0.004-STRCO)
Q3 = STRH1**3/3.-.5*STRCO*STRH1**2
Q4 = .025/(0.004-STRCO)
Q5 = Q1-Q2+Q3-Q4*STRCO**2
Q6=STRH1-STRCM/3.
Q7 = .5*STRH1**2-STRH1
Q8 = 0.075/(.004-STRCO)
Q9 = Q6-Q7-Q8*STRCO**2
GAMMA1 = 1.-Q5/(STRH1*Q9)

R1 = STRCM-STRH1
R2 = .15/(.004-STRCO)
R3 = .5*STRCM**2-STRCO*STRCM+STRCO*STRH1-0.5*STRH1**2
ALPHA2 = (R1-R2*R3)/STRCM
S1 = .5*STRCM**2
S2 = .15/(.004-STRCO)
S3 = STRCM**3/3.-STRCO*STRCM**2/2.
S4 = STRH1**2/2.
S5 = STRH1**3/3.-.5*STRCO*STRH1**2
S6 = S1-S2*S1-(S4-S2*S5)
S7 = .5*STRCM**2-STRCO*STRCM
S8 = .5*STRH1**2-STRCO*STRH1
S9 = STRCM -S2*S7-(STRH1-S2*S8)
S10= S6/(STRCM-STRH1)**1.5*STRH1/(STRCM-STRH1)
GAMMA2 = 1.-S10
END IF
END IF
GO TO 3000
2000 STRH1=0.
ALPHA1=0.
ALPHA2=0.
3000 CONTINUE

PX = ALPHA1*FPRCO*(C-H1)*B1+ALPHA2*FPRCO*H1*B+STFOR+FORPL
RETURN
END

SUBROUTINE FUNCl(C)
IMPLICIT REAL*16(A-H,O-Z)
COMMON/MOHAN/STNS, STNS1, STRAP, STRS, ALPHA 1, ALPHA2, GAMMA1, GAMMA2
COMMON/STRAIN/ STRCO, STRCM, YSTR1 , YSTR2 , USTRA, UPSTN, STRH1
COMMON/STRESS/ ELMST, EMLPL,YIELD,USTRE,UPETS,FPRCO
COMMON/FORCE/ FORST(1000),FORPL,AXIAL,STFOR,ALPHA,GAMMA
COMMON/DISTNCE/ DST(1000),DPL,B,B1,B1,H1
COMMON/AREA/ AST(10 0 0),APL,NST
COMMON/XFORC/ PX,PX1,INP,N
IF (STRCM.LT.STRCO) THEN
ALPHA = (STRCM/STRCO)-1./3.*(STRCM/STRCO)**2
GAMMA = 1./3.-STRCM/(12.*STRCO)/(1.-STRCM/(3.*STRCO))
ELSE
AX = 0.004 - STRCO
BX = STRCM/2. - STRCO
ALPHA = (-STRCO/3. + STRCM*(1.-(0.15/AX)*BX)-
1
0.075/AX)**(STRCM**2)))/STRCM
CX = (STRCM**3/3.-STRCO*STRCM**2/2.)
DX = (STRCM**3/3.-STRCO*STRCM)
EX = -STRCO**2/12.+STRCM**2/2.-0.15/AX)*CX-(0.15/AX)*
1
STRCO**3/6.
FX = -STRCO/3.+STRCM-0.15/AX*DX-0.075/AX*STRCO**2
GAMMA = 1.-EX/(STRCM*FX)
END IF

PX1 = ALPHA*FPRCO*C*B+STFOR+FORPL
SUBROUTINE STEEL (CENTR)

IMPLICIT REAL*16(A-H,O-Z)
COMMON/MOHAN/STNS, STNS1, STRAP, STRS, ALPHA1, ALPHA2, GAMMA1, GAMMA2
COMMON/STRAIN/ STRCO, STRCM, YSTR1, USTR, UPSTN, STRH1
COMMON/STRESS/ ELMST, ELMPL, YIELD, USTRE, UPSTS, FPRCO
COMMON/FORCE/ FORST(1000), FORPL, AXIAL, STFOR, ALPHA, GAMMA
COMMON/DISTANCE/ DST(1000), DPL, B1, H1, H1
COMMON/AREA/ AST(1000), APL, NST
COMMON/XFORC/ PX, PX1, INF, N

DIMENSION STRAS(1000), STRES(1000)
STFOR = 0.0
DO J = 1, NST
STRAS(J) = STRCM* (CENTR - DST(J))/CENTR
IF(QABS(STRAS(J)).LT.YSTR1) THEN
   STRES(J) = ELMST*QABS(STRAS(J))
ELSE
   STRES(J) = YIELD
END IF
IF(STRAS(J).LT.0.0) THEN
   STRES(J) = -STRES(J)
   FORST(J) = STRES(J)*AST(J)
   STFOR = STFOR + FORST(J)
ELSE
   FORST(J) = STRES(J)*AST(J)
   STFOR = STFOR + FORST(J)
END IF
END DO
STNS = STRAS(NST)
STNS1 = STRAS(1)
RETURN
END

SUBROUTINE PLATE(CENTR)

IMPLICIT REAL*16(A-H,O-Z)
COMMON/MOHAN/STNS, STNS1, STRAP, STRS, ALPHA1, ALPHA2, GAMMA1, GAMMA2
COMMON/STRAIN/ STRCO, STRCM, YSTR1, USTR, UPSTN, STRH1
COMMON/STRESS/ ELMST, ELMPL, YIELD, USTRE, UPSTS, FPRCO
COMMON/FORCE/ FORST(1000), FORPL, AXIAL, STFOR, ALPHA, GAMMA
COMMON/DISTANCE/ DST(1000), DPL, B1, H1, H1
COMMON/AREA/ AST(1000), APL, NST
COMMON/XFORC/ PX, PX1, INF, N

STRAP = STRCM* (CENTR - DPL)/CENTR
IF(QABS(STRAP).GT.UPSTN) THEN
   STREP = 0.0
ELSE
   STREP = ELMPL* STRAP
END IF
FORPL = STREP * APL
RETURN
END
APPENDIX C
This program is to calculate the deflection, moment, and curvature at midspan, strains in top concrete fiber, tension rebars, and plate at midspan and the external loads for cambered beam under 4-point loading.

IMPLICIT REAL*16(A-H,O-Z)
DIMENSION XMOM(IOOO),CUR(1000),CURV(IOOO)
COMMON/MOHAN/ ZLOAD, STNS, STRAP, STRS
COMMON/STRAIN/ STRCO, STRCM, YSTR1, YSTR2, USTRA, UPSTN
COMMON/STRESS/ ELMST, ELMPL, YIELD, USTRE, UPSTS, FPRCO
COMMON/FORCE/ FORST(1000), FORPL, AXIAL, STFOR, ALPHA, GAMMA
COMMON/DISTNCE/ DST(1000), DPL, WIDE
COMMON/AREA/ AST(1000), APL, NST
COMMON/XFORC/ PX, INP, N

OPEN(UNIT=5, FILE='EFL2.DAT', TYPE='OLD')
C OPEN(UNIT=6, FILE='MOMT.FH2', TYPE='NEW')
C OPEN(UNIT=8, FILE='CON.FL2', TYPE='NEW')
OPEN(UNIT=9, FILE='STEL.FL2', TYPE='NEW')
OPEN(UNIT=10, FILE='PLAT.FL2', TYPE='NEW')
OPEN(UNIT=11, FILE='KIP.FL2', TYPE='NEW')
OPEN(UNIT=12, FILE='DEF.FL2', TYPE='NEW')

READ(5,*) ITER, NST, AXIAL
C ITERO,NST=1 if no comp. steel; = 2 with comp. steel, axial force
READ(5,*) H, WIDE, ULIM, LLIM
C section height, width, upper limit, lower limit
READ(5,*) FPRCO
C concrete strength in a member
READ(5,*) YIELD, ELMST, YSTR1
C yield strength, elastic modulus of steel, yield strain
DO L = 1, NST
READ(5,*) AST(L), DST(L)
C area of steel, distance of steel rebar to top comp. fiber
END DO
READ(5,*) APL, DPL
C area of plate, distance from centroid of the plate to top comp. fiber
READ(5,*) ELMPL, UPSTN, UPSTS
C elastic modulus of plate, ultimate strain of plate, ultimate stress of plate
READ(5,*) ANDEF, ANSTNS, ANSTRAP
C initial deflection, initial strain in rebar, initial strain in plate
STRCM = 0.0
STRIN = 0.003/ITER
TOL = 1.E-3
A = ULIM
B = LLIM
C = (A+B)/2.
ANCURV = ANDEF/2874.
DO N = 1,(ITER+1)
IF(N.GT.1) THEN
  STRCM = STRCM+STRIN
ELSE
ENDIF

ELMCO = 57.*(FPRCO*1000.)*0.5
STRCO = 2.*FPRCO/ELMCO
IF (STRCM.LE.STRCO) THEN
  ALPHA = (STRCM/STRCO)**2
  GAMMA = (1./3.-STRCM/(12.*STRCO))/(1.-STRCM/(3.*STRCO))
ELSE
  AX = 0.004 - STRCO
  BX = STRCM/2. - STRCO
  ALPHA = (-STRCO/3. + STRCM*(1.-(0.15/AX)*BX)-
           (0.075/AX)*STRCM**2)/STRCM
  DX = (STRCM**2/2.-(STRCO*STRCM)
  EX = -STRCO**2/12.+STRCM**2/2.-((0.15/AX)*CX-(0.15/AX)**2)
  FX = -STRCO/3.+STRCM-0.15/AX*DX-0.075/AX*STRCM**2
  GAMMA = 1.-EX/(STRCM*FX)
END IF
CALL BYSEKT(C,TOL)
CALL STEEL (C)
CALL PLATE (C)
TSTEEL=0.0
DO K = 1,NST
  TSTEEL=TSTEEL+FORST(K)*(H/2.-DST(K))
END DO
XMOMENT = TSTEEL+ALPHA*FPRCO*WIDE*C*(H/2.-GAMMA*C)+
           FORPL*(H/2.-DPL)
STRAP1 = STRAP+ANSTRAP
STNS1 = STNS+ANSTNS
XMOM(N) = XMOMENT
CUR(N) = STRCM/C
CURV(N)=ANCURV+CUR(N)
DEFL = 2874.*CUR(N)+ANDEF
ZLOAD = XMOM(N)/42.
C print*,cur(n),curv(n)
C print*,stns, stns1
C print*,strap, strap1
C WRITE(6,12) XMOM(N)
C WRITE(7,12) CURV(N)
C WRITE(8,12) STRCM
C WRITE(9,12) STNS1
C WRITE(10,12) STRAP1
C WRITE(11,12) ZLOAD
C WRITE(12,12) DEFL
12 FORMAT(F12.6)
END DO
C CLOSE (5)
C CLOSE (6)
C CLOSE(7)
CLOSE(8)
SUBROUTINE BYSEKT(C,TOL)
IMPLICIT REAL*16(A-E,0-Z)
COMMON/MOHAN/ ZLOAD,STNS,STRAP,STRS
COMMON/STRAIN/ STRCO,STRCM,YSTR1,YSTR2,USTRA,UPSTN
COMMON/STRESS/ ELMST,ELMPL,YIELD,USTRE,UPSTS,FPRCO
COMMON/FORCE/ FORST(1000),FORPL,AXIAL,STFOR,ALPHA,GAMMA
COMMON/DISTANCE/ DST(1000),DPL,WIDE
COMMON/AREA/ AST(1000),APL,NST
COMMON/XFORC/ PX,INP,N
1 CALL STEEL(C)
CALL PLATE(C)
CALL FUNC(C)
IF(QABS(PX).LT.TOL) GO TO 10
   IF(PX.LT.0.0) THEN
     C = C +1.E-05
   ELSE
     C = C -1.E-05
   END IF
GO TO 1
10 RETURN
END

SUBROUTINE FUNC(C)
IMPLICIT REAL*16(A-H,0-Z)
COMMON/MOHAN/ ZLOAD,STNS,STRAP,STRS
COMMON/STRAIN/ STRCO,STRCM,YSTR1,YSTR2,USTRA,UPSTN
COMMON/STRESS/ ELMST,ELMPL,YIELD,USTRE,UPSTS,FPRCO
COMMON/FORCE/ FORST(1000),FORPL,AXIAL,STFOR,ALPHA,GAMMA
COMMON/DISTANCE/ DST(1000),DPL,WIDE
COMMON/AREA/ AST(1000),APL,NST
COMMON/XFORC/ PX,INP,N
INP=INP+1
PX = ALPHA*FPRCO*WIDE*C+STFOR+FORPL
RETURN
END

SUBROUTINE STEEL(CENTR)
IMPLICIT REAL*16(A-E,0-Z)
COMMON/MOHAN/ ZLOAD,STNS,STRAP,STRS
COMMON/STRAIN/ STRCM,YSTR1,YSTR2,USTRA,UPSTN
COMMON/STRESS/ ELMST,ELMPL,YIELD,USTRE,UPSTS,FPRCO
COMMON/FORCE/ FORST(IOOO),FORPL,AXIAL,STFOR,ALPHA,GAMMA
COMMON/DISTANCE/ DST(IOOO),DPL,WIDE
COMMON/AREA/ AST(IOOO),APL,NST
COMMON/XFORCE/ PX,INP,N
DIMENSION STRAS(IOOO),STRES(IOOO)
STFOR = 0.0
DO J = 1,NST
STRAS(J) = STRCM* (CENTR -DST(J)/CENTR
IF(QABS(STRAS(J)).LT.YSTR1) THEN
STRES(J) = ELMST*QABS(STRAS(J))
ELSE
IF(QABS(STRAS(J)).GE.YSTR1. AND. QABS (STRAS (J)). LT. YSTR2 ) THEN
STRES(J) = YIELD
ELSE
STRES(J) = YIELD + (QABS(STRAS(J))J-YSTR2)/(USTRA-YSTR2)*
1(USTR-YIELD)
END IF
END IF
IF(QABS(STRAS(J)).GT.UPSTN) THEN
FORST(J) = 0.0
STFOR = STFOR +FORST(J)
ELSE
IF(STRAS(J).LT.0.0) THEN
STRES(J) = -STRES(J)
FORST(J) = STRES(J)*AST(J)
STFOR = STFOR + FORST(J)
ELSE
FORST(J) = STRES(J)*AST(J)
STFOR = STFOR + FORST(J)
END IF
END IF
END DO
STNS = STRAS(NST)
RETURN
END
SUBROUTINE PLATE(CENTR)
IMPLICIT REAL*16(A-H,0-Z)
COMMON/MOHAN/ ZLOAD,STNS,STRAP,STRS
COMMON/STRAIN/ STRCM,YSTR1,YSTR2,USTRA,UPSTN
COMMON/STRESS/ ELMST,ELMPL,YIELD,USTRE,UPSTS,FPRCO
COMMON/FORCE/ FORST(IOOO),FORPL,AXIAL,STFOR,ALPHA,GAMMA
COMMON/DISTANCE/ DST(IOOO),DPL,WIDE
COMMON/AREA/ AST(IOOO),APL,NST
COMMON/XFORCE/ PX,INP,N
STRAP = STRCM* (CENTR -DPLJ/CENTR
IF (QABS(STRAP).GT.UPSTN) THEN
STREP = 0.0
ELSE
    STREP = ELMLP*STRAP
END IF
FORPL = STREP * APL
RETURN
END
REFERENCES


7. "The Flexural Performance of 3.5 m Concrete Beams with Various Bonded External Reinforcements." (Anon.).


