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Parameter study for dynamic models of power bursts for the KEWB5, CRAC, and SILENE experiments

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Parameter Study for Dynamic Models of Power Bursts for the Kewb5, Crac, and Silene Experiments

By

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Statement By Author

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ABSTRACT

Two models were investigated to predict the power and pressure behavior of three liquid Uranium fueled reactors (KEWB5, CRAC, and SILENE) during fast power excursions. Both models use the point reactor kinetics equations to generate the power and energy distributions in time, and an equilibrium thermodynamic analysis to generate the pressure, temperature, and volume distributions. The first model uses equilibrium thermodynamics to generate an equation of state for bubble formation that is static in bubble radius versus time. The second model uses mass transport theory to generate an equation of state that is dynamic in bubble radius. Simulations of the power excursions were run for both models and the results were compared to the experimental data and to each other. The first model was analyzed by varying the threshold gas mass fraction and the static bubble radius verses a range of step reactivity inputs to determine the effect on peak power and pressure. The second model used a trial and error strategy to determine the adjustable parameters that best fit the experimental data, then was also analyzed against the same ranges of step reactivity input. In the comparison, the dynamic model predicts the power and pressure versus time traces better than the static model.

CHAPTER 1

INTRODUCTION

This thesis seeks to analyze various one-region, lumped parameter, dynamic computer models to predict experimental results which were obtained from three liquid solution Uranium-Water reactor experiments:

KEWB5 — **K**inetic **E**xperimental **W**ater **B**oiler Number **5** — An unreflected, cylindrical, liquid Uranium Sulphate fueled reactor. The power excursion is generated by a horizontal, air-ejected, pulse control rod.

CRAC — **C***onsequences* **R***adiologiques d'un* **A***ccident de* **C***ritcité* — A set of French experiments to measure the effects of accidental criticalities. This is an unreflected, cylindrical, liquid Uranium Nitrate fueled reactor. The power excursion is generated by pumping liquid fuel into the reactor vessel.

SILENE — A French experimental reactor for studying the effects of accidental criticalities. This is an unreflected, cylindrical, liquid Uranium Nitrate fueled reactor. The power excursion is generated by a vertical, air-ejected, pulse control rod.

The same dynamic model, based on physical theories, is used to analyze all three reactor types. Then, various models are analyzed to determine which physical characteristics are important and the time scales in which they are significant.

The analysis technique is based on the simulation language DARE/P. The models are programmed in DARE/P and the peak power and pressure are obtained using different initial reactivity steps, using various physical submodels, and varying adjustable parameters. The resulting peak powers and pressures are then compared to the experimental values. A few selected models are chosen and the full power and pressure time plots are compared to the experimental results. From these power and pressure plots, the accuracy and applicability of the model can be assessed.

The various models are assembled from submodels that describe the physical processes postulated to be important. These submodels include:

- Point reactor kinetics equations to describe the power and energy being generated by the fissile solutions;
- Equilibrium thermodynamic analysis to describe the transient pressure and temperature in the fuel;
- Equilibrium thermodynamics to describe an equation of state for the solution containing bubbles of radiolytic gas;
- Mass transport to describe a dynamic model for the radiolytic gas formation.

The mathematics of the submodels are explained in Section 3. Details of the computer implementation of the submodels are explained in the corresponding Appendices.

Chapter 2

Literature Survey

The British, French, and Japanese as well as the Americans have done theoretical and experimental research to generate models and data. A survey of this research and how it pertains to this thesis is included here.

The French and Americans lead in the generation of physical data. The French have generated data with the CRAC (Reference [1]) and SILENE (Reference [2]) sets of experiments. The Americans generated data with the KEWB (Reference [3]) set of experiments. The KEWB experiments were designed to test the safety of a research reactor. Every model used in this thesis must generally predict the peak power, peak pressure, power shape, and pressure shape for all three reactor experiments.

The major theoretical research that pertains to this thesis is the work performed by Dr. David L. Hetrick at the University of Arizona's Department of Nuclear and Energy Engineering. His paper given at the *International Topical Meeting on Safety Margins in Criticality Safety*, November 1989 (Reference [4]), gives the basis to this thesis. The first part of this thesis determines the accuracy of the model outlined in Dr. Hetrick's paper over various adjustable parameter ranges.

David Pribyl's Masters Thesis (Reference [5]) introduces two submodels: momentum dissipation and bubble growth during the power excursion. The momentum dissipation submodel adds a constant term to the fluid acceleration equation to dissipate some of the energy as the fluid expands. This submodel is used by Dr. Hetrick in his paper. The bubble growth submodel was not taken to a point of inclusion in any published computer model. The second part of this thesis expands and implements this bubble growth submodel and tests its applicability to the problem.

Adrienne Smith's Masters Thesis (Reference [6]) investigates a *moving* bubble front by introducing multiple regions to the problem. The bubble formation is still instantaneous and has a constant radius in each region, but the energy deposited in each region is different so a time delay in the bubble formation in each region occurs. This simulates a moving bubble front. The results of this investigation did not significantly improve the prediction of the pressure plots.

A. Bickley, D. J. Mather, and P. M. Shaw have created a computer simulation called CRITEX using bubble transport theory (Reference [7]) to predict the CRAC power excursions. The models predictsd the general shape of the power excursion, but makes no attempt to predict the pressure excursions.

R. Kato, T. Naramura. and Y. Senda of the Mitsubishi Atomic Power Industries, Inc. have created a computer simulation called CREST to simulate the multiple power excursions in the CRAC experiments. The results of their investigation were given at the *International Topical Meeting on Safety Margins in Criticality Safety*, November 1989 (Reference [8]). This model predicts the general shape of the power excursions, but makes no attempt to predict the pressure excursions.

The ultimate conclusion of this modeling process is to predict the multiple excursion CRAC experiments along with the other experiments generated to date. It has not been demonstrated in the literature that a single model has accomplished this conclusion. This thesis does not attempt to make this conclusion, but only to advance the knowledge of what physical characteristics are and are not important to the process.

CHAPTER 3

MODEL DESCRIPTION

In order for the analysis technique to be used, a physical submodel must be translated into a mathematical submodel, and then, into a computer submodel. The computer submodel must then be integrated into the larger system model. This section describes the mathematical submodels investigated.

The basis for the system model is a one-region, lumped parameter model. All temperatures and pressures are for the entire system. In the simulation, when gas bubbles form, they form everywhere instantaneously. This is a simple model used to determine which physical characteristics are more important than others for fast reactor power excursions. The major prediction problem is the pressure transient and volume reactivity feedback. These submodels are the latest attempt to accurately predict these physical characteristics.

The physical models used to predict the physical characteristics are:

Point Reactor Kinetics Model: This model describes the power and energy production in the fuel from the nuclear processes.

Equilibrium Thermodynamic Model: This model uses equilibrium thermodynamics (Reference [9]) to describe the relationships between the energy produced, the temperature, the pressure, and the volume of the fuel.

Equilibrium Radiolytic Bubble Model: This model uses equilibrium thermodynamics with a fixed radiolytic gas bubble size.

Mass Transfer Radiolytic Bubble Model: This model uses a mass transfer by diffusion relationship to describe the growth of gas bubbles in the liquid as a function of gas density, gas and liquid pressure, and solution temperature.

Momentum Dissipation Model: This model diminishes fluid energy content by momentum dissipation. This dissipation may be through the vessel walls and internals, moving bubbles, and internal fluid friction.

These physical characteristics are simulated by computer submodels with adjustable parameters. These adjustable parameters can represent unknown physical constants or just unknown physical information. The analysis technique discussed in Section 4 describes the controlling DARE/P logic programming to compare the peak power and pressure generated by the values of these adjustable parameters to the experimental peak power and pressure.

3.1 Point Reactor Kinetics Model

The energy and power production generated by this system is strictly due to the fissile material contained in the reactor. The point reactor kinetics model (Reference [10]) is used to describe this power and energy production versus time. A step input of reactivity is used to initiate the model's power excursion. For the KEWB5 and SILENE experiments being modeled, the step input is the actual physical mechanism used. For the CRAC experiments being modeled, the reactivity input is a fast ramp, but since the excursions were performed without external sources, they are effectively initiated as random reactivity steps. All of the power excursions are controlled by reactivity feedbacks. The major reactivity feedback is produced by volume changes due to voids in and thermal expansion of the fuel. The model described in this section details the physics and mathematics of the reactor power, energy deposited, and volumetric reactivity feedback.

The point reactor kinetics model describes the neutron diffusion equation as separable in space and time. This means that the spatial distribution of the neutrons (or any reactor property proportional to the neutron density) must be fixed during the time transient. The other major assumptions of this model are that the reactor be a homogeneous medium or can be represented as such and the neutrons are considered to have one effective speed or energy.

The limitations of this model include the inability to account for the effects of local perturbations to the reactor. This may be interpreted in terms of a time lag that occurs as the local perturbation's effects propagate throughout the reactor.

For this thesis, it is assumed that the KEWB5, CRAC, and SILENE experiments conform to these assumptions and limitations. The mathematical model is shown below:

$$\frac{dn}{dt} = \frac{(\rho - \beta)}{\ell} n + \sum_{i=1}^{m} \lambda_i c_i + q \tag{3-1}$$

$$\frac{dc_i}{dt} = \frac{\beta_i}{\ell} n - \lambda_i c_i \, , \qquad i = 1, m \tag{3-2}$$

where,

$n =$ The neutron power;

$c_i =$ The precursor latent power;

$t =$ Time;

$\rho =$ The reactivity, $\frac{k-1}{k}$, the fractional change in the neutron reproduction factor;

$\beta =$ The delayed neutron fraction $\sum_{i=1}^{m} \beta_i$;

$\ell =$ The neutron generation time;

$\lambda_i =$ The decay constant for the specific precursor power;

$q =$ The effective source strength;

$m =$ The total number of precursor *groups* (not physical precursor atom species).

For this thesis, the six group precursor model is used. The step input reactivity and the volume feedback are described in one equation:

$$\rho = \rho_0 - \beta\phi(V - V_0) \tag{3-3}$$

where,

$\rho_0 =$ The step input reactivity;

$\phi =$ The volume feedback reactivity coefficient;

$V =$ The system volume;

$V_0 =$ The initial system volume.

The energy deposited in the fuel is the time integral of the power generated by the fuel. The DARE/P computer package can only accept differential equations, so the integral power equation is differentiated. The mathematical model for this is:

$$E(t) = \int_0^t n(t')dt' \tag{3-4}$$

$$dE(t) = d\int_0^t n(t')dt' \tag{3-5}$$

$$\frac{dE(t)}{dt} = n(t) - n(0) \tag{3-6}$$

where,

$E =$ The energy deposited in the fuel up to time t;

Equations 3-1, 3-2, 3-3, and 3-6 define the power and energy model. This model is the same for all system models that occur in this analysis.

3.2 Equilibrium Thermodynamic Model

The model outlined in Section 3.1 determines the reactor energy as a function of time. Using equilibrium thermodynamics, this energy is used to generate the temperature, pressure, and volume as a function of time. The thermodynamic analysis uses definitions and nomenclature from Reynolds and Perkins (Reference [9]).

The analysis starts from the definition of the internal energy as a function of volume and temperature: the state postulate. The internal energy is also a function of the energy generated and the mechanical work done by the fluid: conservation of energy. The mathematical models are:

$$\frac{dU}{dt} = \left(\frac{\partial U}{\partial V}\right)_T \frac{dV}{dt} + \left(\frac{\partial U}{\partial T}\right)_V \frac{dT}{dt} \tag{3-7}$$

$$\frac{dU}{dt} = \frac{dE}{dt} - P\frac{dV}{dt} \tag{3-8}$$

where,

$U =$ The internal energy;

$T =$ The system temperature;

$P =$ The system pressure;

$E =$ The energy deposited in the system;

$V =$ The system volume;

$t =$ Time.

Eliminate $\frac{dU}{dt}$ from both equations, use the definition of the specific heat at constant volume, and solve for $\frac{dT}{dt}$:

$$\frac{dT}{dt} = \frac{1}{c_v}\left\{\frac{dE}{dt} - \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right]\frac{dV}{dt}\right\} \tag{3-9}$$

where,

$c_v =$ The specific heat at constant volume.

From the Gibbs Equation, a relationship between the internal energy and measurable quantities is derived (Reference [9, Equation 8-54, page 265-6]). From this relationship, Equation 3-9 can be rewritten as:

$$\frac{dT}{dt} = \frac{1}{c_v}\left[\frac{dE}{dt} - T\left(\frac{\partial P}{\partial T}\right)_V \frac{dV}{dt}\right] \tag{3-10}$$

From the state postulate for the pressure, and the definitions for the isobaric and isothermal compressibility, an expression is derived to relate the change in pressure with respect to the temperature at constant volume and a new equation that relates the change in pressure with respect to time to the change in temperature and volume with respect to time:

$$\left(\frac{\partial P}{\partial T}\right)_V = -\frac{\left(\frac{\partial V}{\partial T}\right)_P}{\left(\frac{\partial V}{\partial P}\right)_T} = \frac{\beta}{\kappa} \tag{3-11}$$

$$\frac{dP}{dt} = \frac{\beta}{\kappa}\frac{dT}{dt} - \frac{1}{\kappa V}\frac{dV}{dt} \tag{3-12}$$

where,

$\beta =$ The isobaric compressibility factor;

$\kappa =$ The isothermal compressibility factor.

Using Equation 3-11, Equation 3-10 is rewritten as:

$$\frac{dT}{dt} = \frac{1}{c_v}\left[\frac{dE}{dt} - \frac{\beta}{\kappa}T\frac{dV}{dt}\right] \tag{3-13}$$

Equations 3-12 and 3-13 give relationships between the pressure and the temperature with respect to the volume. An equation for the volume and how it changes with time is needed. The analysis starts with Newton's Second Law of Motion recast with the pressure gradient instead of force in the center of mass coordinate system:

$$\overline{\nabla P} = -\rho \frac{d^2 z}{dt^2} \tag{3-14}$$

where,

$z =$ The center of mass ordinate;

$\rho =$ The mass density.

If a linear pressure distribution is assumed along the length of the cylinder, then the average gauge pressure is:

$$\overline{P} = \frac{1}{h} \int_0^h P(z)dz \tag{3-15}$$

$$\overline{P} = \frac{1}{h} \int_0^h P_{max} \left(1 - \frac{z}{h}\right) dz \tag{3-16}$$

$$\overline{P} = \frac{P_{max}}{2} \tag{3-17}$$

where,

$h =$ The height of the fluid in the cylinder;

$P_{max} =$ The maximum pressure in the system which occurs at the bottom of the cylinder.

The average pressure gradient is:

$$\overline{\nabla P} = \frac{1}{h} \int_0^h \frac{\partial P}{\partial z} dz \tag{3-18}$$

$$\overline{\nabla P} = -\frac{P_{max}}{h} \tag{3-19}$$

Eliminate P_{max} between Equations 3-17 and 3-19 and assume that $\overline{P}$ is the system gauge pressure P_g:

$$\overline{\nabla P} = -\frac{2P_g}{h} \tag{3-20}$$

Assume that the center of mass is at the center of the cylinder, then the volume is equal to two times the height of the center of mass times the cross sectional area of the cylinder. Take this relationship's second time derivative, the definition of mass density, plus Equations 3-14 and 3-20 to get a relationship between the average system pressure and the second time derivative of the volume:

$$\frac{d^2V}{dt^2} = \frac{4A^2\,(P - P_0)}{M} \tag{3-21}$$

where,

$A =$ The cylinder's cross sectional area;

$P_0 =$ The reference pressure to convert gauge pressure to absolute pressure;

$M =$ The mass of the solution;

It can be shown that P_{max}, and therefore $\frac{d^2V}{dt^2}$, are independant of the pressure shape as long as the proper constants are used in both equations.

Equations 3-12, 3-13, and 3-21 describe the basic thermodynamic model that relates the generated power to the system temperature, pressure, and volume. These equations contain the isobaric and isothermal compressibility factors. An Equation of State will be needed to describe these factors in conjunction with radiolytic gas formation (See Sections 3.3 and 3.4). Also, Equation 3-21 has a momentum dissipation term added which is described in Section 3.5.

3.3 Equilibrium Thermodynamic Bubble Model

Section 3.2 describes a set of equations that relate pressure, temperature, and volume. These equations contain physical properties that must be related to the state of the system: formation of radiolytic gas bubbles. The model described in this section is a *static* model in which bubbles form with an average radius and do not grow or shrink. Section 3.4 describes a model that allows for bubble growth.

This model has two parts: how the isothermal and isobaric compressibility factors are influenced by the bubble formation, and the time and amount of the bubble formation.

3.3.1 Isothermal and Isobaric Compressibility Factors

The isothermal and isobaric compressibility factors are influenced by the state of the reactor. The definitions of these factors are (Reference [9, Equations 4-4 and 4-5, page 92]):

$$\beta \equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \tag{3-22}$$

$$\kappa \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \tag{3-23}$$

The total system volume is the sum of the volume taken up by the liquid and the volume taken up by the gas. Let the gas volume be a fraction, f, of the total volume. Then, the liquid volume is $(1-f)$ times the total volume. Using these definitions, Equations 3-22 and 3-23 are rewritten as:

$$\beta = (1-f)\beta_0 + \frac{f}{V_g}\left(\frac{\partial V_g}{\partial T}\right)_P \tag{3-24}$$

$$\kappa = (1-f)\kappa_0 - \frac{f}{V_g}\left(\frac{\partial V_g}{\partial P}\right)_T \tag{3-25}$$

where,

$\beta_0 =$ The isobaric compressibility factor for the liquid.

$\kappa_0 =$ The isothermal compressibility factor for the liquid;

$V_g =$ The volume of the gas generated.

From Equations 3-24 and 3-25, an expression for the change in the gas volume with respect to temperature and pressure is needed to evaluate explicit compressibility factors. This analysis starts with the determination of the pressure inside the gas bubble. From a force balance on one-half of a free bubble (Reference [11, Equation 11-3, page 297]), the pressure inside the bubble is:

$$P_i = P + \frac{2\sigma}{r_b} \tag{3-26}$$

where,

$P_i =$ the pressure inside the bubble;

$\sigma =$ the surface tension;

$r_b =$ the bubble radius.

If it is assumed that the gas is a perfect gas, then the perfect gas law relates the pressure inside the bubble to the equilibrium temperature and the total volume of the gas. The total volume of the gas is also the total number of bubbles times the volume of a sphere with radius r_b (the assumed shape of the bubble). This gives:

$$PV_g = NR_gT - 2\sigma\left(\frac{4\pi n}{3}\right)^{\frac{1}{3}} V_g^{\frac{2}{3}} \tag{3-27}$$

where,

$N =$ The number of kilograms of gas;

$R_g =$ The gas constant for $2H_2 + O_2$;

$n =$ The total number of gas bubbles.

Since the total number of bubbles and the total mass of the gas is unknown, the total number of bubbles is eliminated by the volume information, and the total mass of the gas is eliminated by the perfect gas law. Differentiating this new equation gives:

$$\left(P + \frac{4\sigma}{3r_b}\right)\frac{dV_g}{V_g} + dP = \left(P + \frac{2\sigma}{r_b}\right)\frac{dT}{T} \tag{3-28}$$

If dP is set to zero, then the unknown partial derivative in Equation 3-24 is given as:

$$\frac{1}{V_g}\left(\frac{\partial V_g}{\partial T}\right)_P = \frac{1}{T}\frac{P + \frac{2\sigma}{r_b}}{P + \frac{4\sigma}{3r_b}} \tag{3-29}$$

If dT is set to zero, then the unknown partial derivative in Equation 3-25 is given as:

$$\frac{1}{V_g}\left(\frac{\partial V_g}{\partial P}\right)_T = -\frac{1}{P + \frac{4\sigma}{3r_b}} \tag{3-30}$$

Therefore, the complete compressibility factors are:

$$\beta = (1 - f)\beta_0 + \frac{f}{T}\frac{P + \frac{2\sigma}{r_b}}{P + \frac{4\sigma}{3r_b}} \tag{3-31}$$

$$\kappa = (1-f)\kappa_0 + \frac{f}{P + \frac{4\sigma}{3r_b}} \tag{3-32}$$

Again, the volume fraction is not known in terms of volumes, so it is derived in terms of the known gas mass fraction. Start with these definitions:

$$f = \frac{V_g}{V} \tag{3-33}$$

$$x = \frac{N}{M} \tag{3-34}$$

$$V_g = \frac{NR_gT}{P + \frac{2\sigma}{r_b}} \tag{3-35}$$

$$V = v_s M \tag{3-36}$$

$$v_s = (1-x)v_l + xv_g \tag{3-37}$$

where,

$v_s =$ The specific volume of the solution;

$v_l =$ The specific volume of the liquid;

$v_g =$ The specific volume of the gas;

$x =$ The ratio of the mass of the gas to the total mass.

Substituting the last four equations into Equation 3-33 gives:

$$f = \frac{xR_gT}{(1-x)v_l(P + \frac{2\sigma}{r_b}) + xR_gT} \tag{3-38}$$

From Equations 3-31, 3-32, and 3-38, a complete model relating the pressure and temperature to the mass fraction is known. The only unknowns left are the time and amount of bubble formation.

3.3.2 Time and Amount of Static Bubble Formation

The CRAC experiments have measured the dissociation rate of H_2O per MegaJoule of energy deposited in the fuel. From this value, the mass fraction of radiolytic gas is calculated as a function of time. At some point in time, the amount of gas in solution reaches a critical value and gas bubbles start to form. In this model, the critical value is a threshold mass fraction. When the amount of gas is below this threshold, the gas mass fraction is set to zero — no gas bubbles. When the amount of gas is greater than the threshold, the gas mass fraction is set to the excess amount above the threshold. The mathematical model is:

$$x_E = \frac{GE}{M} \tag{3-39}$$

$$x_R = x_E - x_T \tag{3-40}$$

$$\text{IF } (x_E < x_T)\ x = 0.0 \text{ ELSE } x = x_R \tag{3-41}$$

where,

$x_E =$ The mass fraction of gas generated;

$x_R =$ The excess gas mass fraction;

$G =$ The dissociation rate of H_2O;

$x_T =$ The threshold mass fraction used to determine the mass of gas needed to generate bubbles.

3.4 Mass Transfer Bubble Model

In Section 3.3, a static bubble model was introduced. The bubble radius is either zero or a fixed positive constant throughout the power excursion. This gives a physical situation of all the bubbles appearing simultaneously and growing instantaneously to a fixed size. The mass transfer model uses mass diffusion (Reference [12]) to allow the gas bubbles to grow during the power excursion; however, the bubble still appear simultaneously and grow in unison.

This model has two parts: the determination of the bubble radius with time, and the time and amount of bubble formation.

3.4.1 Bubble Radius Determination Model

This analysis starts by assuming that at time t_N, a gas bubble of radius r_0 nucleates. This bubble grows in a solution that is supersaturated in gas. Since the solution is supersaturated, it is assumed that the concentration at the bubble's surface is at saturation. If the solution is not moving with respect to the system, then *Fick's Second Law of Diffusion* can be used as a model basis. The mathematical model for this is:

$$\frac{\partial c}{\partial t} = \frac{D}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial c}{\partial r}\right) \tag{3-42}$$

with these initial and boundary conditions:

$$
\begin{aligned}
c(r,t_N) &= c_{super} \\
c(r_b,t) &= c_{sat} \\
c(\infty,t) &< \infty
\end{aligned}
\tag{3-43}
$$

where,

$c(r,t) =$ The concentration distribution at any bubble radius r and any time t;

$D =$ The gas diffusivity constant;

$c_{super} =$ The supersaturation concentration;

$r_b =$ The radius of the gas bubble;

$c_{sat} =$ The saturation concentration.

This set of equations has been solved by David Pribyl (Reference [5]). The resulting solution and its gradient at the bubble surface for $t > t_N$ is:

$$
c(r,t) = c_{super} - \frac{r_b}{r}(c_{super} - c_{sat})\ erfc(\frac{(r-r_b)}{2\sqrt{D(t-t_N)}})
\tag{3-44}
$$

and,

$$
\left.\frac{\partial c}{\partial r}\right|_{r_b} = (c_{super} - c_{sat})\left[\frac{1}{\sqrt{\pi D(t-t_N)}} + \frac{1}{r_b}\right]
\tag{3-45}
$$

The mass flow rate, $\frac{dm}{dt}$, of gas into the bubble at the surface is defined as the bubble surface area times the concentration gradient:

$$
\frac{dm}{dt} \equiv 4\pi r_b^2 D\left.\frac{\partial c}{\partial r}\right|_{r_b}
\tag{3-46}
$$

where,

$m =$ The mass flow.

The mass of gas in the bubble is defined as the density of gas in the bubble times the volume of the bubble (assumed to be a sphere). Differentiating this with respect to time gives:

$$\frac{dm}{dt} = 4\pi\rho_g r_b^2 \frac{dr_b}{dt} + \frac{4}{3}\pi r_b^3 \frac{d\rho_b}{dt} \tag{3-47}$$

where,

$\rho_g =$ The density of the gas inside the bubble.

As a model simplification, the bubble density rate of change term is negligible compared to the bubble radius rate of change term. This has been verified numerically over important ranges of the parameters invloved.

As time increases and at small bubble radii, the square root term in Equation 3-45 can be neglected to give:

$$\left.\frac{\partial c}{\partial r}\right|_{r_b} = \frac{(c_{super} - c_{sat})}{r_b} \tag{3-48}$$

Equation 3-46 is set equal to Equation 3-47, and Equation 3-48 is used. The resulting equation is solved for $\frac{dr_b}{dt}$:

$$\frac{dr_b}{dt} = \frac{D}{\rho_g}\frac{(c_{super} - c_{sat})}{r_b} \tag{3-49}$$

In Equation 3-49, the density and concentration terms must be transformed into mass fractions. The concentration terms are straight forward: $c = \rho_l x$. This, however, leaves the ratio of the liquid to gas densities which is unknown. The analysis starts with the definition of the specific volume of the solution and Equation 3-38. Two assumptions are made:

$$v_s = xv_g + (1-x)v_l \approx v_l = \frac{1}{\rho_l} \tag{3-50}$$

$$f = \frac{xR_gT}{(1-x)v_l(P+\frac{2\sigma}{r_b}) + xR_gT} \tag{3-51}$$

$$f \approx \frac{xR_gT}{v_l(P+\frac{2\sigma}{r_b})} \tag{3-52}$$

These approximations are valid if x is small. From the perfect gas law:

$$v_g = \frac{R_gT}{P+\frac{2\sigma}{r_b}} \tag{3-53}$$

From Equations 3-53 and 3-50, Equation 3-52 is rewritten as:

$$f \approx x\frac{\rho_l}{\rho_g} \tag{3-54}$$

Substituting this into Equation 3-49 gives the final equation for this model:

$$\frac{dr_b}{dt} \approx \frac{Df}{r_b x}(x_{super} - x_{sat}) \tag{3-55}$$

Equations 3-52, and 3-55 determine the mathematical model for the variable radius bubble as long as the bubble radii are small, the gas mass fraction is small, and the time after bubble nucleation is large.

3.4.2 Time and Amount of Dynamic Bubble Formation

The determination of the time of bubble formation is the same as for the static model described in Section 3.3.2. However, the arbitrary threshold mass fraction is replaced with the supersaturation mass fraction. This supersaturation mass fraction is not quite as arbitrary, and it does define the driving force for bubble growth. It is set equal to an adjustable parameter times the saturation mass fraction which is a quanity that can be approximated from the literature (Reference [5]). From the CRAC and SILENE data, the adjustable parameter could have a range of 3.5 to 50.0. The excess gas mass fraction available for bubbles is determined from a homogeneous differential equation that reduces to the difference between the amount of gas generated and the amount needed for supersaturation for steady state conditions. This allows for a time lag for gas diffusion before it appears in the bubbles. The mathematical model is:

$$x_{super} = FP \times x_{sat} \tag{3-56}$$

$$x_E = \frac{GE}{M} \tag{3-57}$$

$$\frac{dx_R}{dt} = GMS(x_E - x_R - x_{super}) \tag{3-58}$$

$$\text{IF } (x_E < x_{super})\ x = 0.0 \text{ ELSE } x = x_R \tag{3-59}$$

where,

$FP =$ An adjustable parameter;

$GMS =$ An adjustable parameter.

3.5 Momentum Dissipation Model

David Pribyl in his Masters Thesis outlined a momentum dissipation model (Reference [5, pages 22-4]). This model is used in this analysis unmodified. The model analysis starts from the modified Newton's Second Law of Motion outlined in Equation 3-21. When the fluid expands, energy is lost to the vessel walls, intrafluid friction, etc.... Since the overall model is a lumped-parameter model, an average rate of momentum loss is subtracted from the fluid momentum. The new Equation of Motion is:

$$\frac{d^2V}{dt^2} = \frac{4A^2(P - P_0)}{M} - \theta\frac{dV}{dt} \tag{3-60}$$

where,

$\theta =$ An adjustable parameter.

Chapter 4
Parameter Survey Procedure

The analysis tool used to evaluate a proposed model is the DARE/P differential equation computer package (Reference [13]). This package solves first-order ordinary differential equations with an array of different integration rules. The package input allows the direct usage of the differential equation format. For this analysis, the peak power and pressure are calculated, saved, and plotted against the experimental values. For the static model, these peak values are calculated over ranges of the model's adjustable parameters (See Table 1). For the dynamic model, the adjustable parameters are those that best fit the power and pressure versus time plots for the fastest available experimental data (See Table 2). From these plots, the applicability of the proposed model is evaluated.

To achieve these results, a logic block is created for use by DARE/P. This logic block is the controlling programming for the execution of the simulation. For this thesis, all data storage is turned off by the `CALL STROF` command. Loops are setup to loop over the major analysis parameter. The maximum pressure and power files are opened. The minor loops are setup. The simulation is reset to initial conditions with the `CALL RESET` command. The simulation is run with the `CALL RUN` command. The minor loops are closed, the output files are closed, and the major loop is closed. The program is then exited. For the dynamic model, the only loops were over the step input reactivity. The DARE/P code to perform this logic exists in the programs listed in Appendix D.

To find the best fit parameters for the dynamic model, a trial and error strategy is employed. Using this strategy: The volumetric feedback and the momentum dissipation terms regulate the peak power and pressure values. The saturation mass fraction drives the time of nucleation. The FP factor is a fine adjustment for the time of nucleation and drives the amount of gas available for bubble formation and growth. The GMS factor determines the shape of the tail of the pressure plot after nucleation.

Table 1: Adjustable Parameter Ranges for Static Bubble Model

Adjustable Parameter	Units	KEWB5	CRAC	SILENE
Inverse Period	sec^{-1}	273 – 1902	200 – 1500	150 – 1000
FeedBack Coefficient	$\frac{\$}{m^3}$	12000	14000	5600
Momentum Dissipation	sec^{-1}	3000	5000	2700
Static Bubble Radius	*meters*	1.0E-7 – 3.0E- 5		
Threshold Mass Fraction	—	1.0E-6 – 1.0E- 2		

Table 2: Best Fit Adjustable Parameters for Dynamic Bubble Model

Adjustable Parameter	Units	KEWB5	CRAC	SILENE
Volumetric Feedback	$\frac{\$}{m^3}$	12370.0	10900.0	6140.0
Momentum Dissipation	sec^{-1}	3987.0	3200.0	2590.0
FP Factor	—	16.5	7.5	16.0
GMS Factor	sec^{-1}	25.0	30.0	12.5
Gas Diffusivity	sec^{-1}	0.5E-8		
Saturation Fraction	—	1.0E-5		

Chapter 5
Results and Conclusions

5.1 Results

The major objective of this thesis is to model and simulate the KEWB5, CRAC, and SILENE experiments. Two models were analyzed: a static bubble radius model, and a dynamic bubble radius model. Each of these models use an equilibrium thermodynamic model to describe the pressure, temperature, and volume of the system.

The static bubble radius model was analyzed by varying the threshold gas mass fraction and the static bubble radius versus the step input reactivity after the reactivity feedback and momentum dissipation parameters were chosen to fit the fastest available excursion for all three reactor types (Reference [4]). Figures 1 through 6 show how the peak reactor power changes with respect to changing threshold gas mass fraction and step input reactivity for all the interesting bubble radii in the KEWB5 reactor. When the bubble radius was expanded beyond the values shown, the plots did not change. Figures 7 through 12 show the same information for the peak pressure. As expected, these plots show that when the step input reactivity gets larger, the peak power and pressure get larger. However, the peak pressures are generally under-predicted for conditions were there is no bubble formation before peak pressure, and over-predicted when bubble formation occurs before peak pressure. For the peak powers, as long as bubble formation occurs after peak power, the predictions are accurate over the entire investigated step reactivity range. This pattern is consistent over all three reactor types as shown by Figures 13 through 36.

The dynamic bubble radius model was analyzed from the beginning: the best fit parameters for the fastest available power excursion needed to be found for each reactor type. After a trial and error strategy, the resulting power and pressure traces for the reactor types are shown in Figures 37 through 42. These plots consistently show that the constant reactivity feedback is not strong enough to match the experimental power data. The pressure traces show that the pressure can be brought down fast enough to generally match the experimental data, but does not have the correct shape. The KEWB5 and SILENE models predict the approximate time for peak pressure, but the CRAC model does not. Using these best fit parameters, a survey of the peak power and pressure versus the step input reactivity was generated. Figures 43 through 48 show the results. The peak powers and pressures were predicted well by the dynamic model, but not any better than the static model.

5.2 Conclusions and Limitations

From the above results, the first conclusion is to abandon the static bubble radius model. The pressure traces were predicted better with the dynamic bubble radius model than with the static bubble radius model. The power traces were about the same. However, the dynamic bubble radius model only predicts the general shape and not the exact shape of both traces. It still cannot shut any of the reactors down fast enough. This might be investigated with a variable reactivity feedback coefficient. A preliminary analysis using perturbation theory was investigated by the author, but has not been included in a simulation as of this date. Some kinks in the KEWB5 and SILENE ascending pressure traces still need to be modeled. These kinks may be due to the reflected pressure wave from the closed top of the reactors. The CRAC experiments were performed with an open top and do not show these kinks. The thermodynamic model used is based on an open top (zero gauge pressure) at the reactor fuel surface. The SILENE experiment is using an ultrasonic transducer system to determine when bubble formation occurs. From their preliminary results, bubble formation occurs much later than the dynamic model needs to match the pressure tail shape. If bubble nucleation is delayed more than it is, then the pressure is brought down too quickly. A mechanism for this phenomenon is not offered.

The dynamic bubble radius model still needs in-depth investigation into the various approximations made in the mathematical theory and in the choosing of the parameters and physical constants. The bubbles are still generated at the same time and all grow with the same radius. Most of the solution constants are for water and not water containing very heavy molecules (uranium sulphate or uranium nitrate). The problem is that some of the physical constants simply do not exist in the literature. These constants could be analyzed for best fit in a manner resembling the static bubble radius analysis or determined experimentally.

In trying to extend this type of model to slower transients and to multiple excursions, other mechanisms will have to modeled. However, the limitations of the lumped-parameter type of model may be reached and a finite difference or finite element method will need to be used.

In conclusion, the dynamic bubble radius model makes better predictions of the power and pressure over time and the peak power and pressure over step input reactivity than the static bubble radius model. Therefore, the dynamic model should be investigated further and in more detail. The adjustable parameters and physical constants should be scrutinized, analyzed, or outright replaced with physically based models.

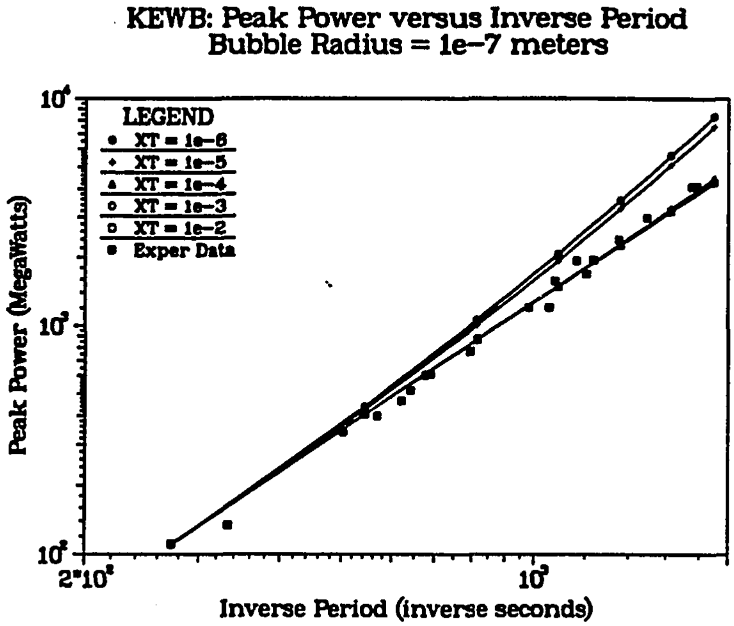

Figure 1: Peak Power versus Inverse Period for a Bubble Radius of 1E-7 meters for the KEWB5 Reactor

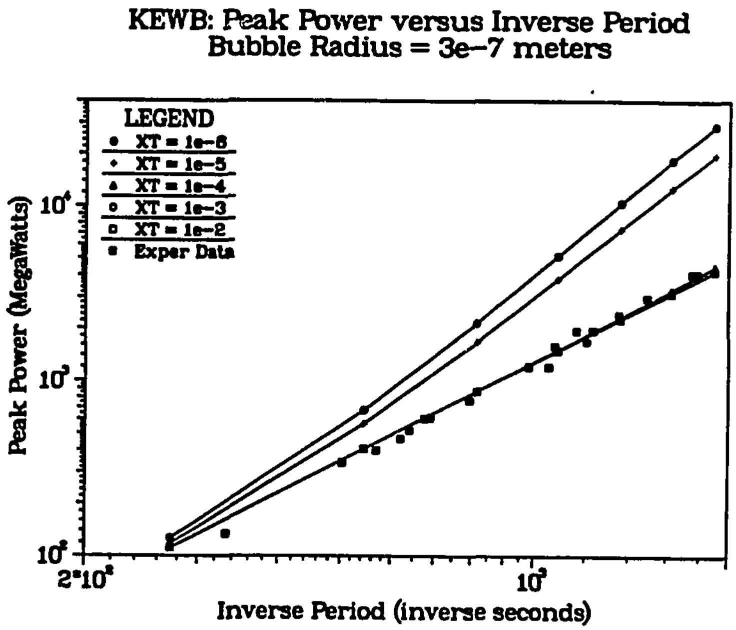

Figure 2: Peak Power versus Inverse Period for a Bubble Radius of 3E-7 meters for the KEWB5 Reactor

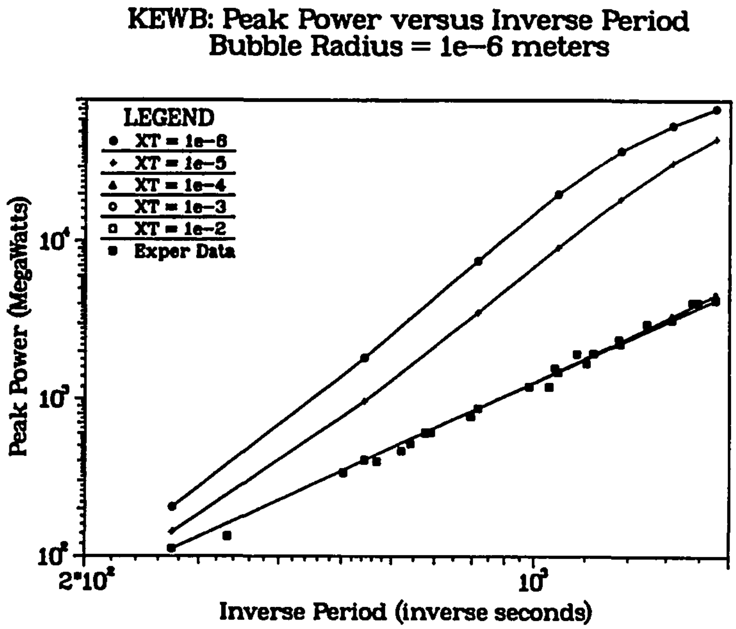

Figure 3: Peak Power versus Inverse Period for a Bubble Radius of 1E-6 meters for the KEWB5 Reactor

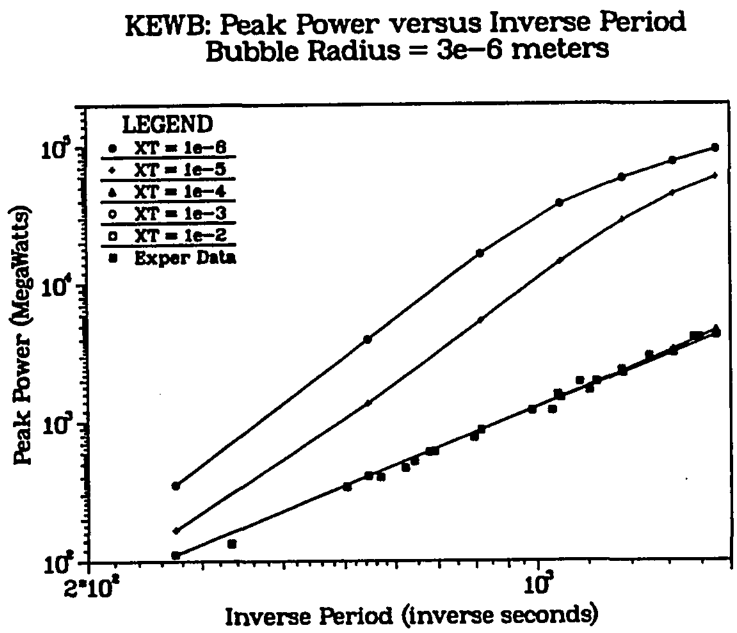

Figure 4: Peak Power versus Inverse Period for a Bubble Radius of 3E-6 meters for the KEWB5 Reactor

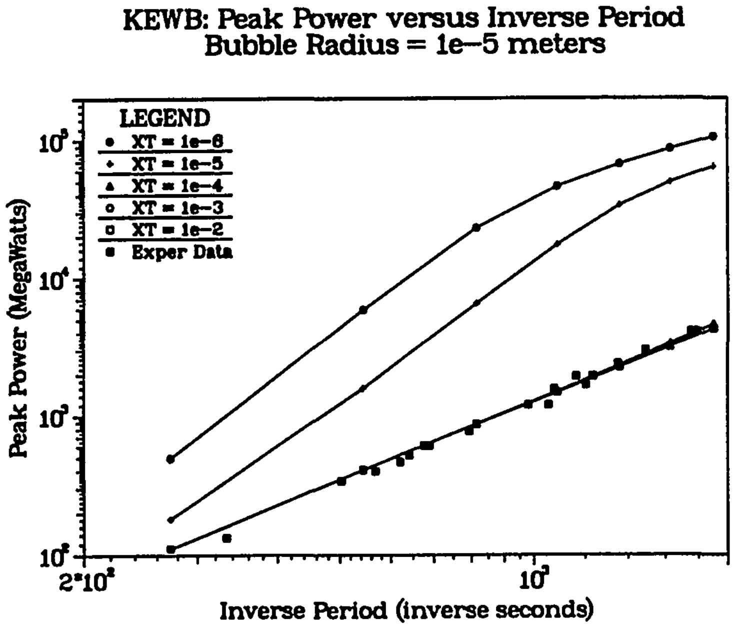

Figure 5: Peak Power versus Inverse Period for a Bubble Radius of 1E-5 meters for the KEWB5 Reactor

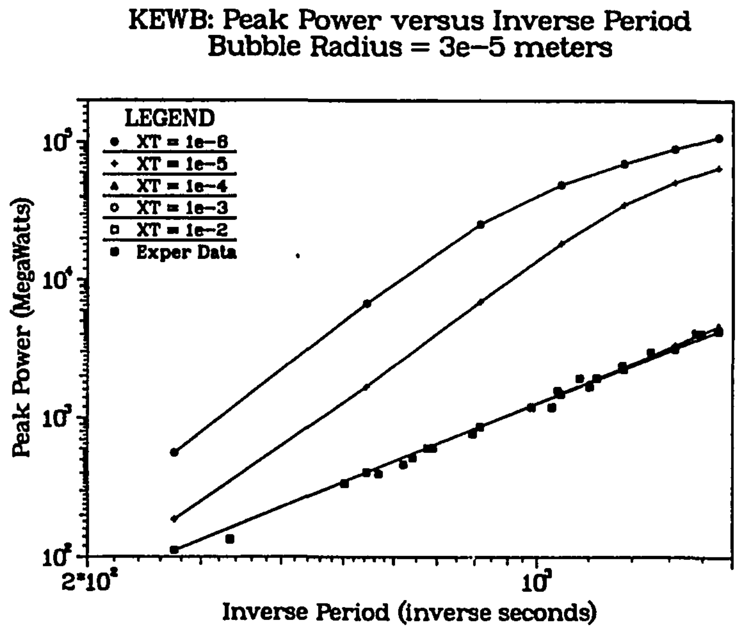

Figure 6: Peak Power versus Inverse Period for a Bubble Radius of 3E-5 meters for the KEWB5 Reactor

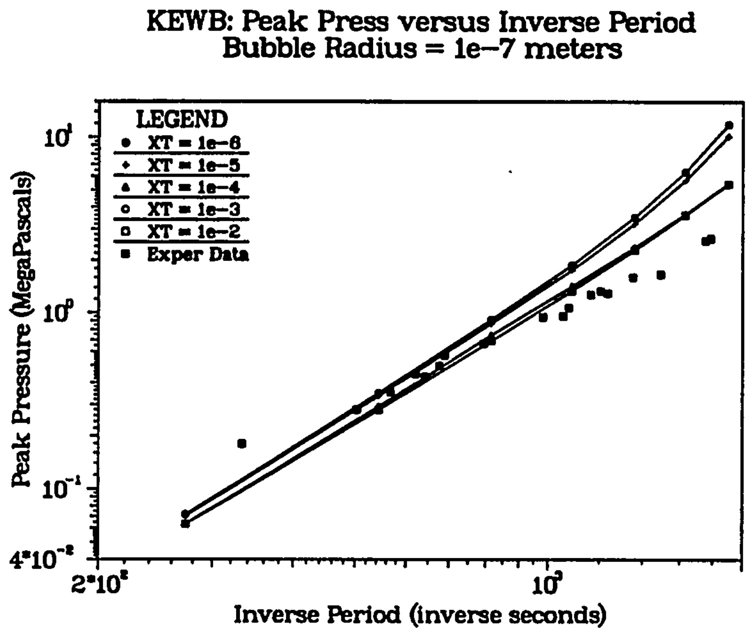

Figure 7: Peak Pressure versus Inverse Period for a Bubble Radius of 1E-7 meters for the KEWB5 Reactor

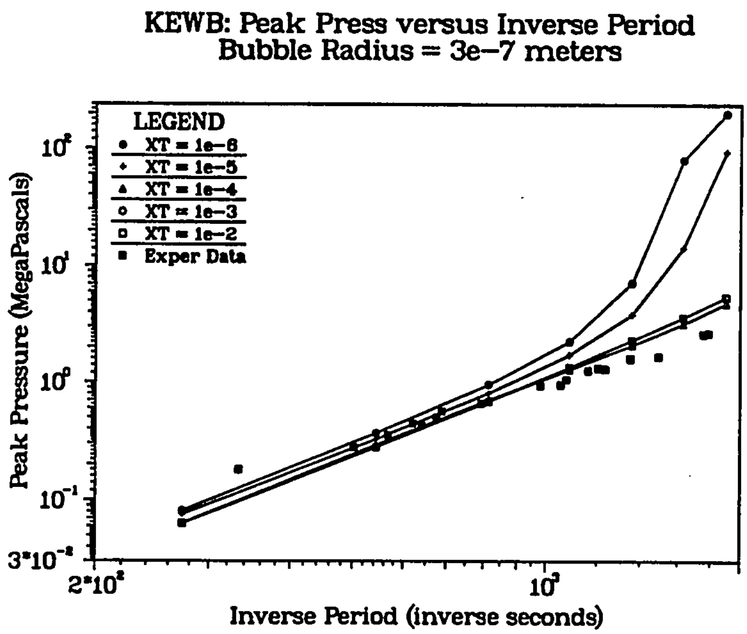

Figure 8: Peak Pressure versus Inverse Period for a Bubble Radius of 3E-7 meters for the KEWB5 Reactor

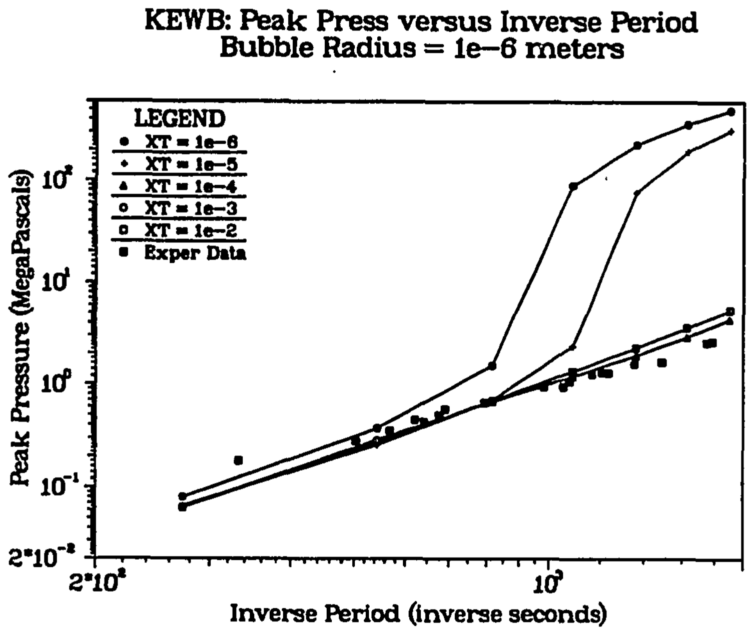

Figure 9: Peak Pressure versus Inverse Period for a Bubble Radius of 1E-6 meters for the KEWB5 Reactor

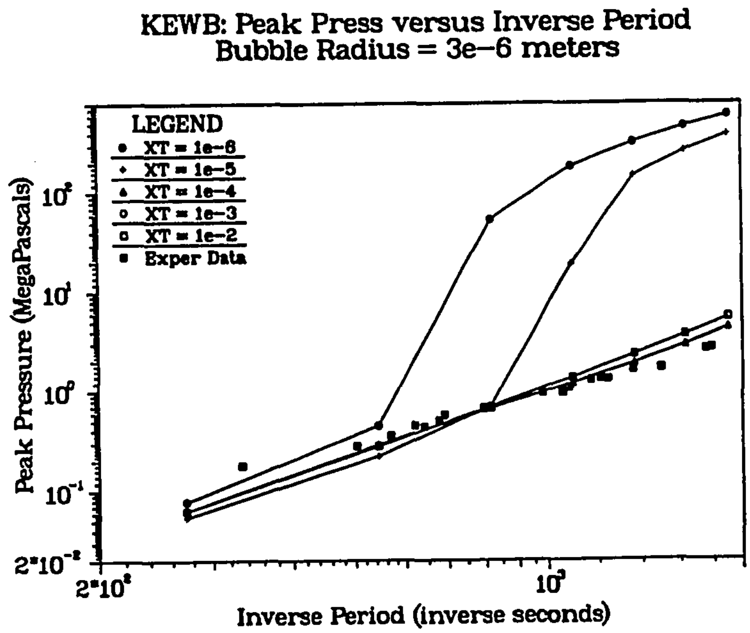

Figure 10: Peak Pressure versus Inverse Period for a Bubble Radius of 3E-6 meters for the KEWB5 Reactor

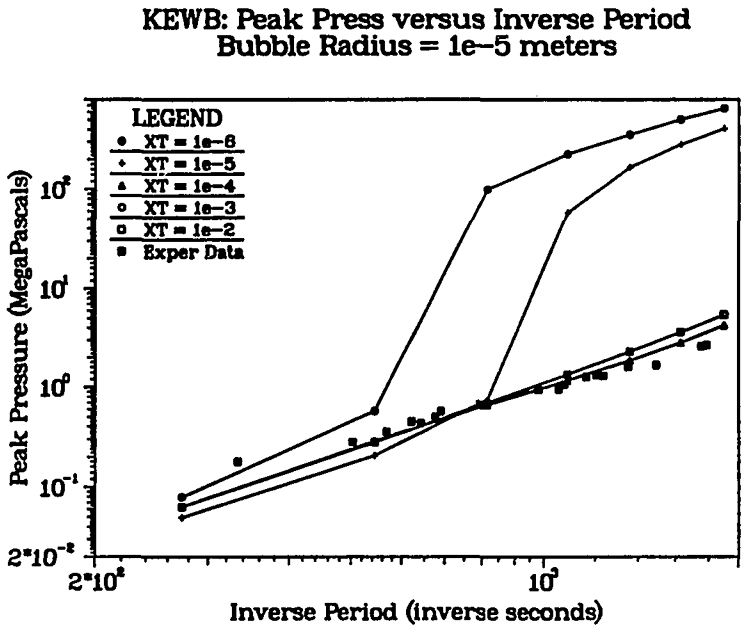

Figure 11: Peak Pressure versus Inverse Period for a Bubble Radius of 1E-5 meters for the KEWB5 Reactor

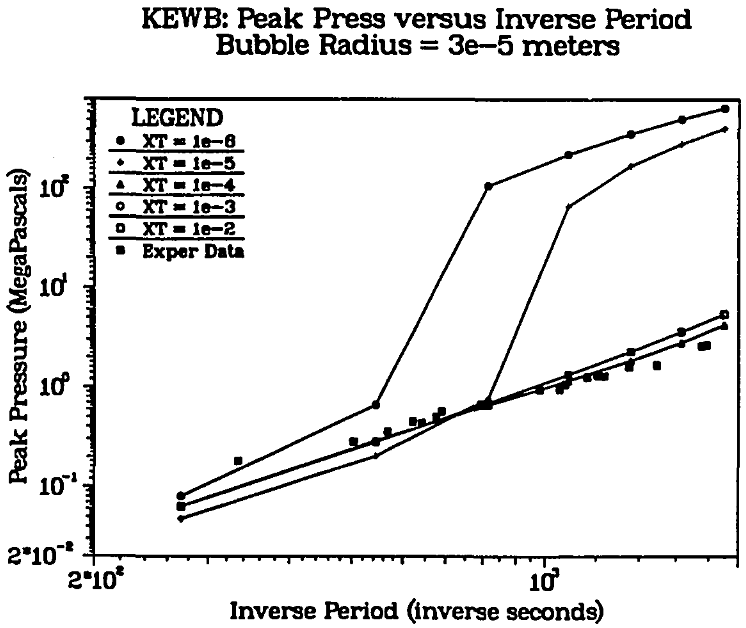

Figure 12: Peak Pressure versus Inverse Period for a Bubble Radius of 3E-5 meters for the KEWB5 Reactor

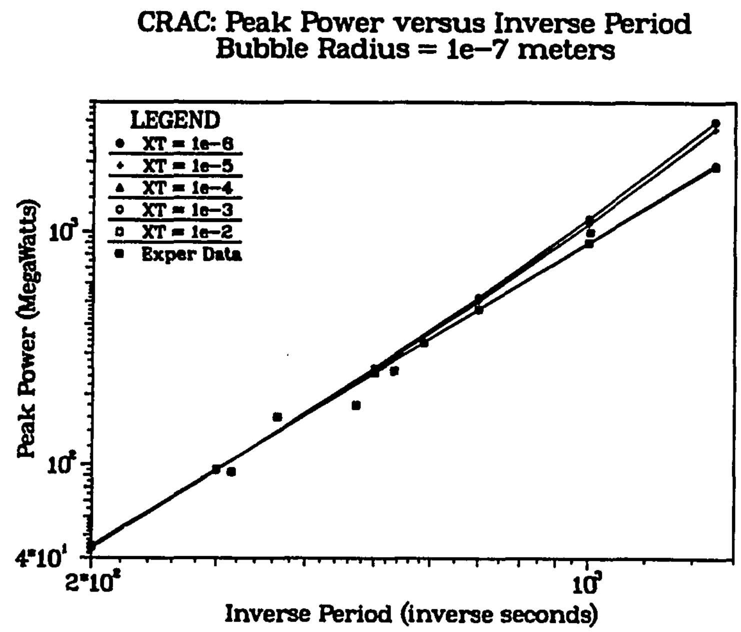

Figure 13: Peak Power versus Inverse Period for a Bubble Radius of 1E-7 meters for the CRAC Reactor

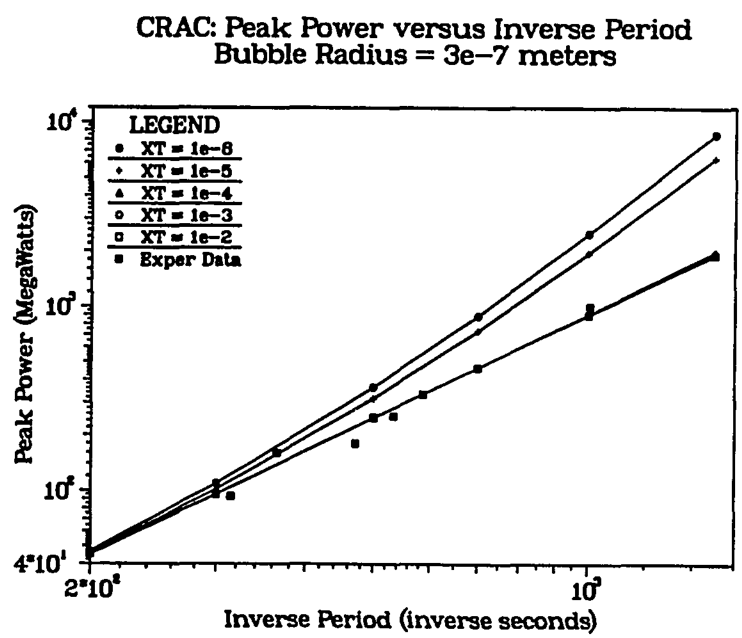

Figure 14: Peak Power versus Inverse Period for a Bubble Radius of 3E-7 meters for the CRAC Reactor

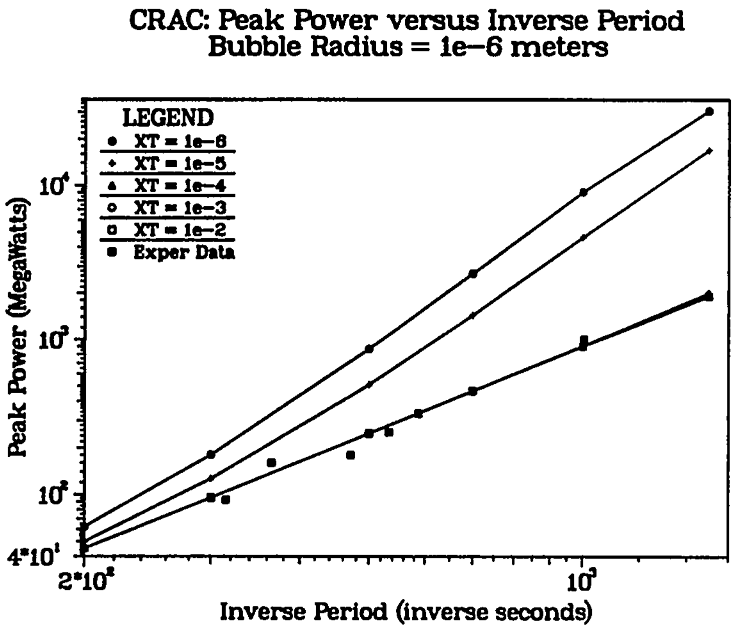

Figure 15: Peak Power versus Inverse Period for a Bubble Radius of 1E-6 meters for the CRAC Reactor

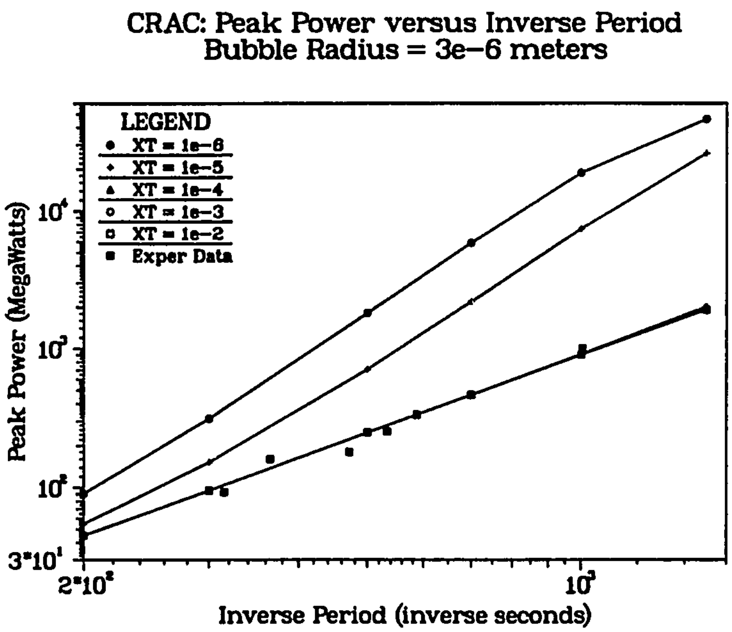

Figure 16: Peak Power versus Inverse Period for a Bubble Radius of 3E-6 meters for the CRAC Reactor

CRAC: Peak Power versus Inverse Period
Bubble Radius = 1e–5 meters

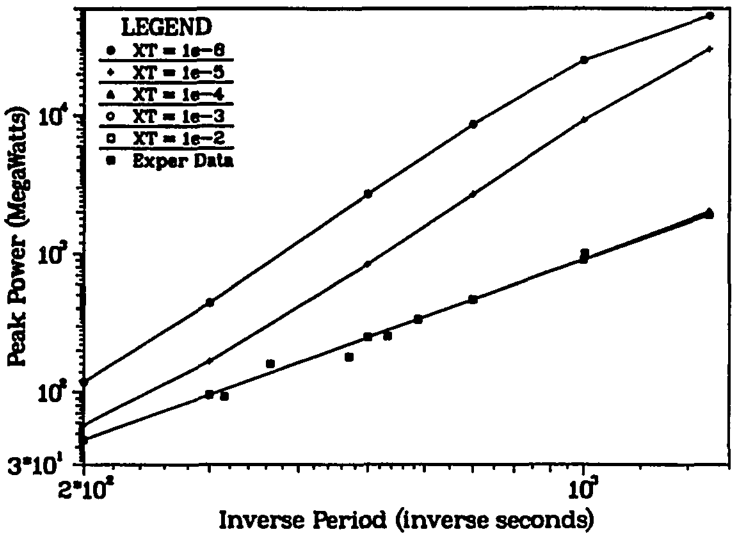

Figure 17: Peak Power versus Inverse Period for a Bubble Radius of 1E-5 meters for the CRAC Reactor

CRAC: Peak Power versus Inverse Period
Bubble Radius = 3e–5 meters

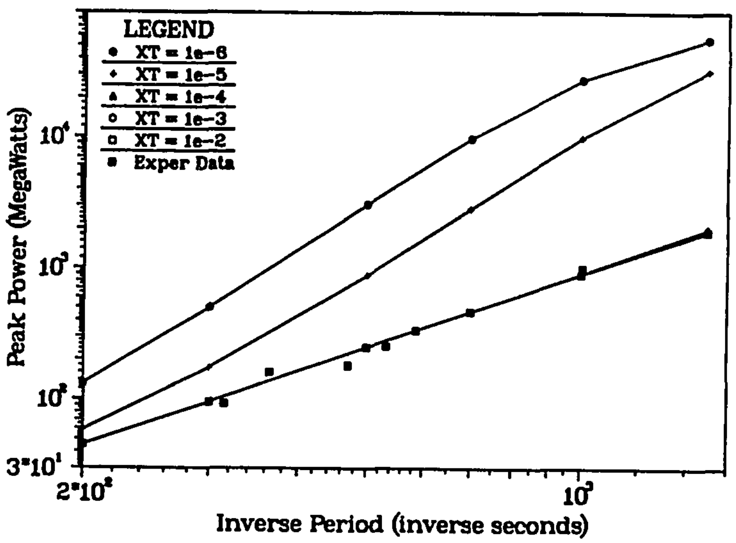

Figure 18: Peak Power versus Inverse Period for a Bubble Radius of 3E-5 meters for the CRAC Reactor

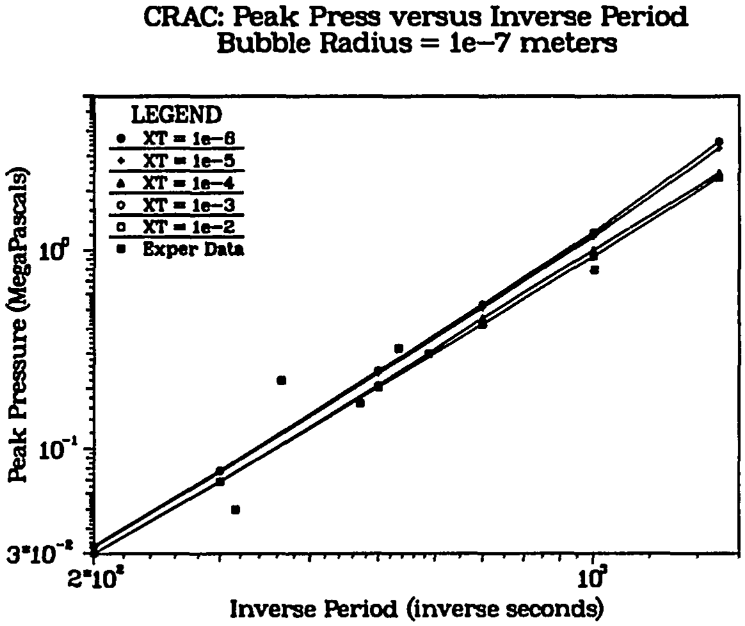

Figure 19: Peak Pressure versus Inverse Period for a Bubble Radius of 1E-7 meters for the CRAC Reactor

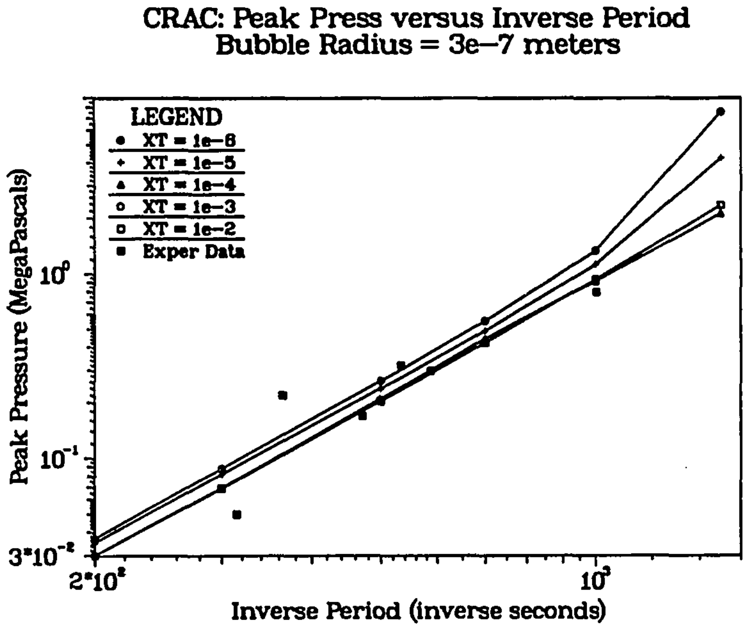

Figure 20: Peak Pressure versus Inverse Period for a Bubble Radius of 3E-7 meters for the CRAC Reactor

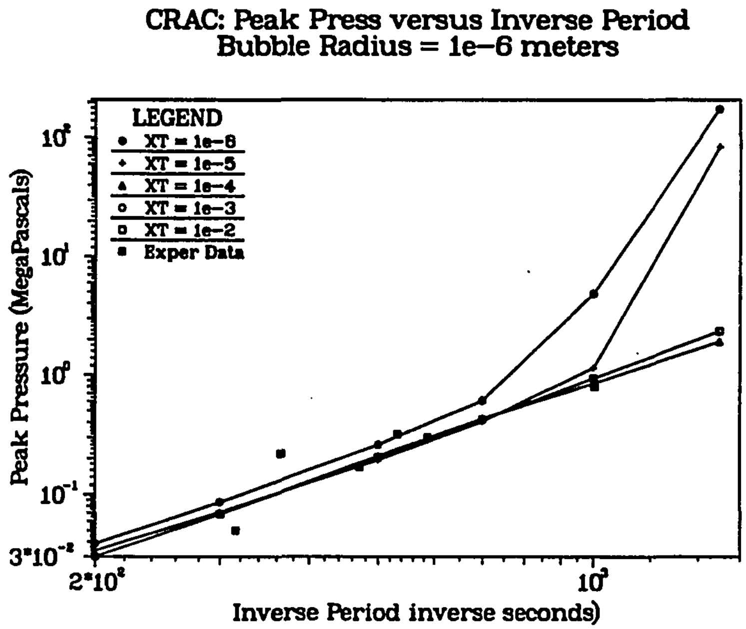

Figure 21: Peak Pressure versus Inverse Period for a Bubble Radius of 1E-6 meters for the CRAC Reactor

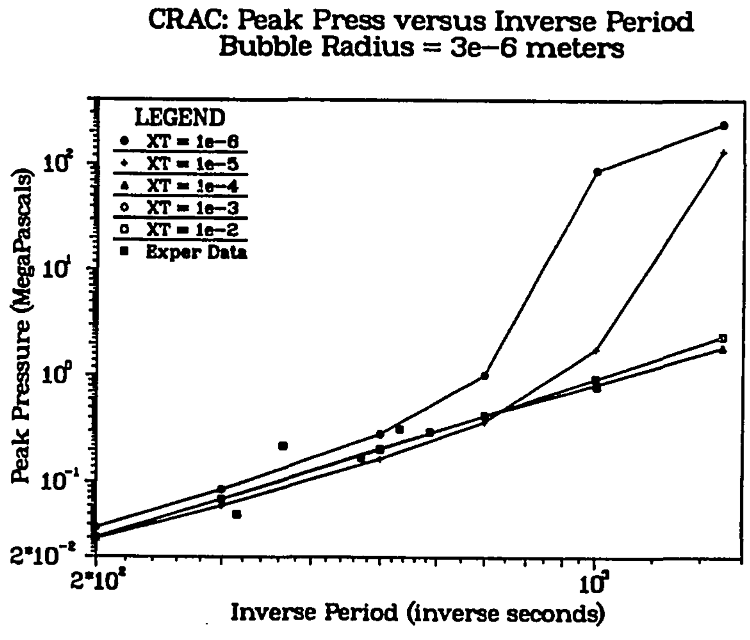

Figure 22: Peak Pressure versus Inverse Period for a Bubble Radius of 3E-6 meters for the CRAC Reactor

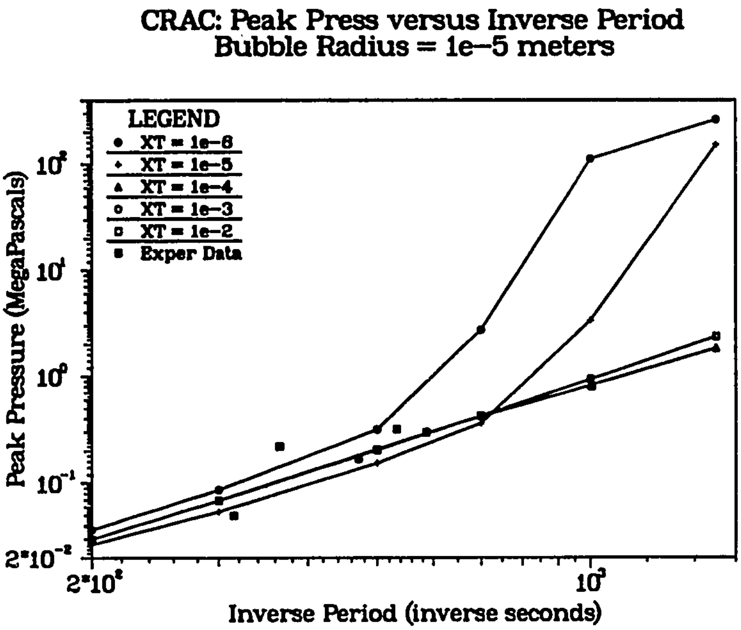

Figure 23: Peak Pressure versus Inverse Period for a Bubble Radius of 1E-5 meters for the CRAC Reactor

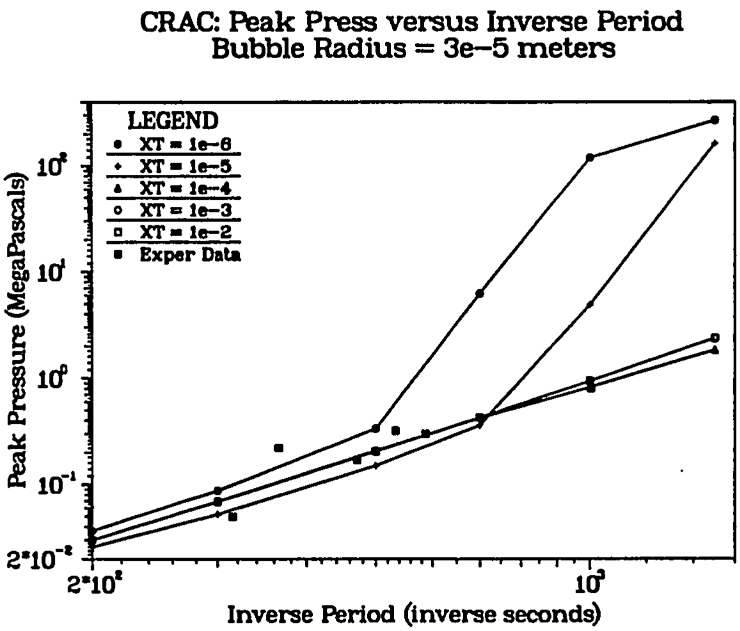

Figure 24: Peak Pressure versus Inverse Period for a Bubble Radius of 3E-5 meters for the CRAC Reactor

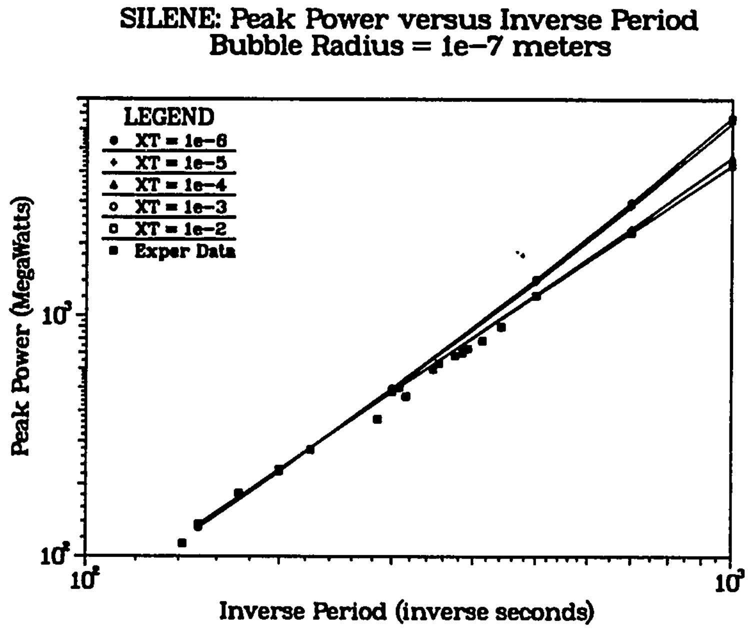

Figure 25: Peak Power versus Inverse Period for a Bubble Radius of 1E-7 meters for the SILENE Reactor

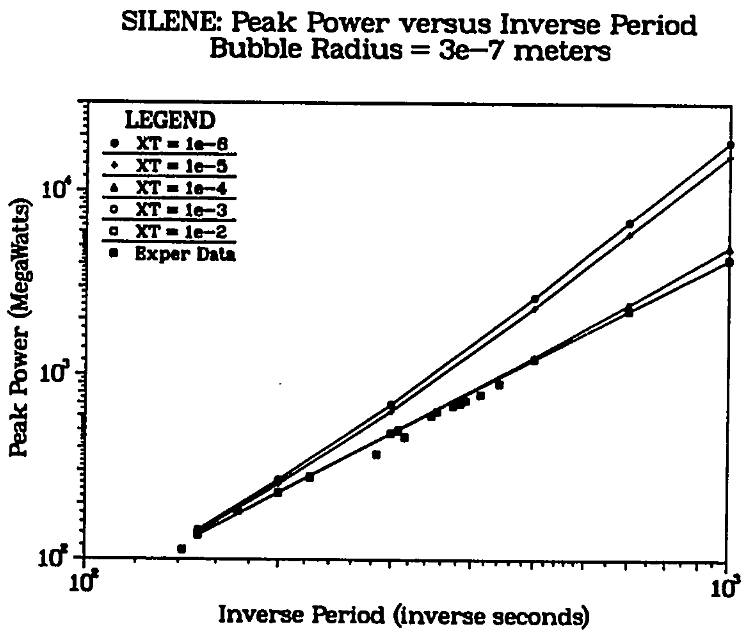

Figure 26: Peak Power versus Inverse Period for a Bubble Radius of 3E-7 meters for the SILENE Reactor

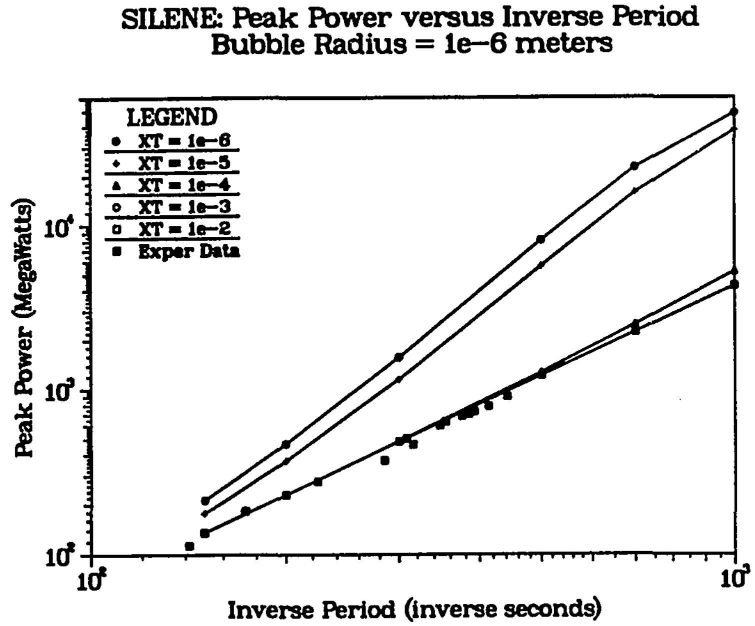

Figure 27: Peak Power versus Inverse Period for a Bubble Radius of 1E-6 meters for the SILENE Reactor

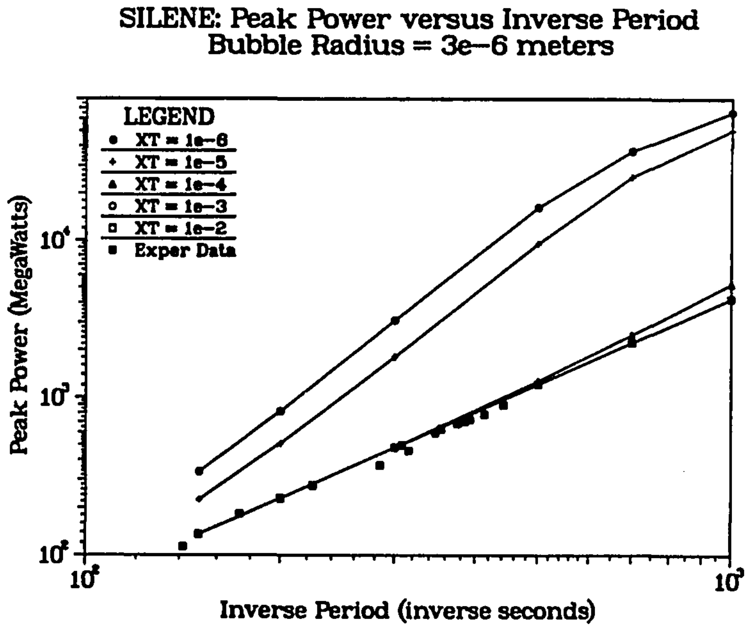

Figure 28: Peak Power versus Inverse Period for a Bubble Radius of 3E-6 meters for the SILENE Reactor

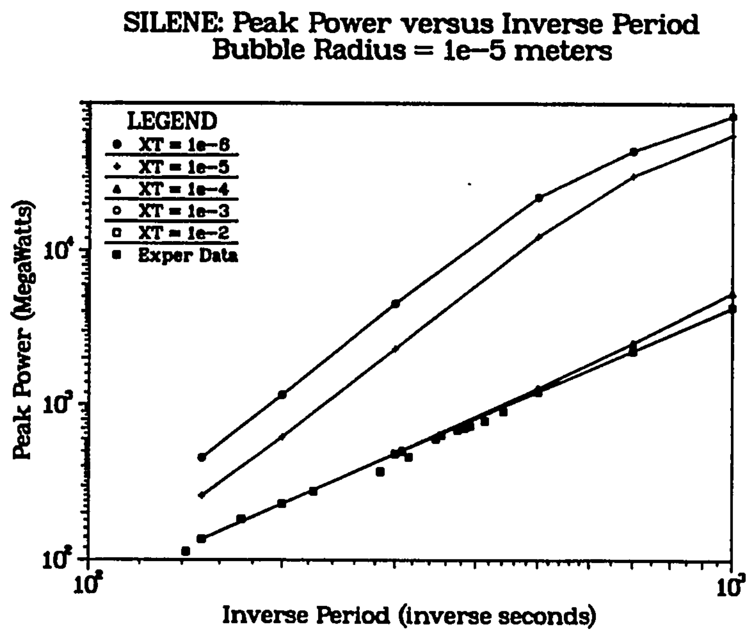

Figure 29: Peak Power versus Inverse Period for a Bubble Radius of 1E-5 meters for the SILENE Reactor

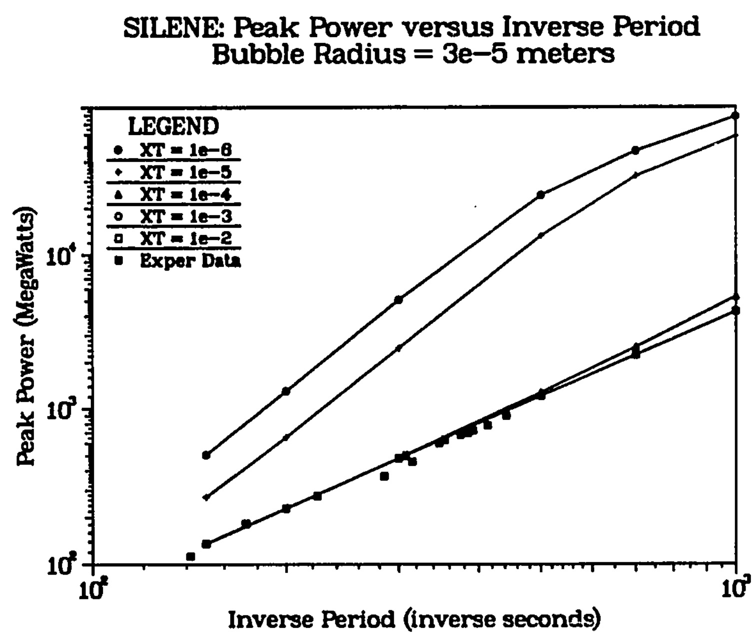

Figure 30: Peak Power versus Inverse Period for a Bubble Radius of 3E-5 meters for the SILENE Reactor

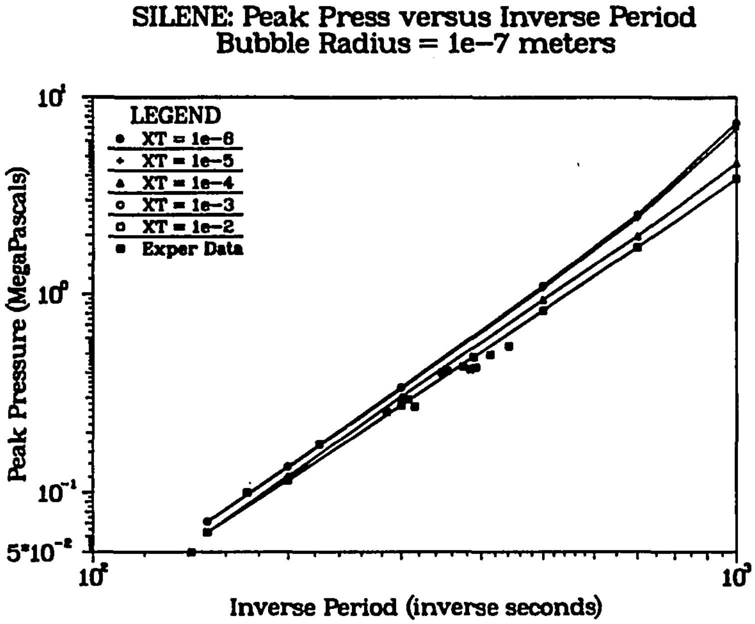

Figure 31: Peak Pressure versus Inverse Period for a Bubble Radius of 1E-7 meters for the SILENE Reactor

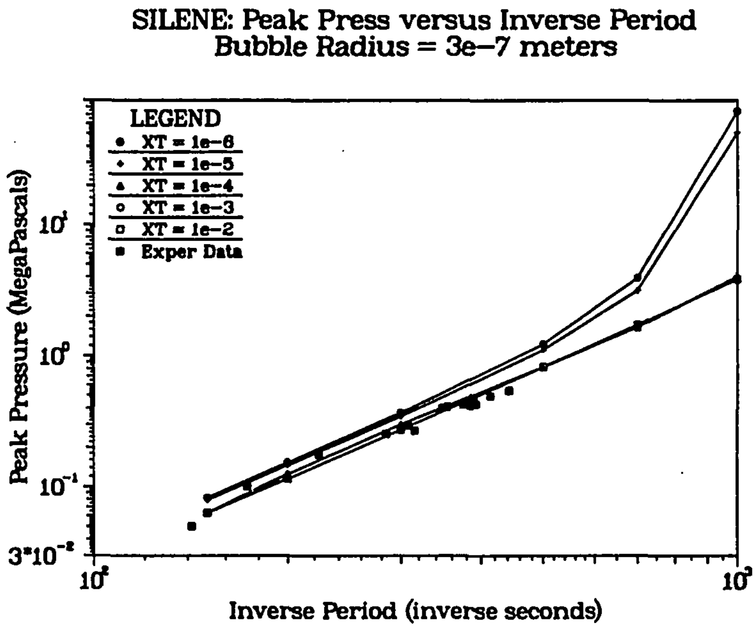

Figure 32: Peak Pressure versus Inverse Period for a Bubble Radius of 3E-7 meters for the SILENE Reactor

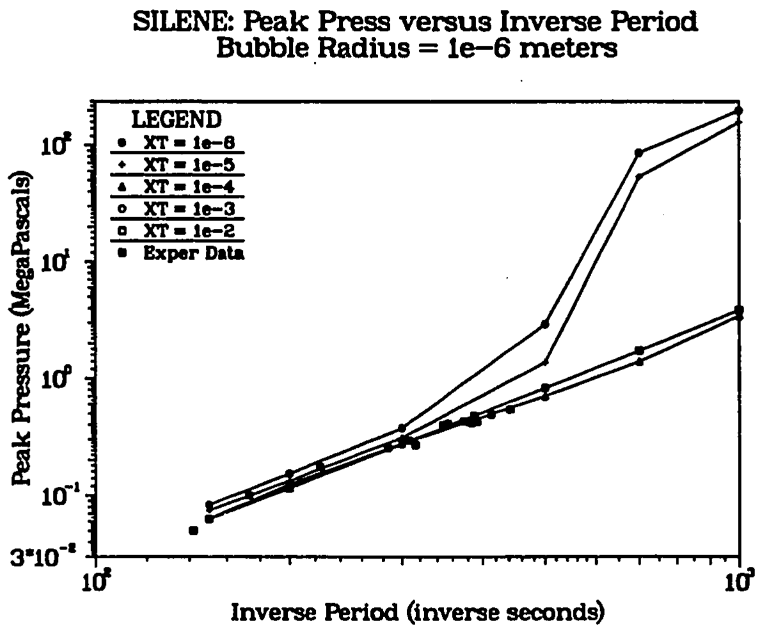

Figure 33: Peak Pressure versus Inverse Period for a Bubble Radius of 1E-6 meters for the SILENE Reactor

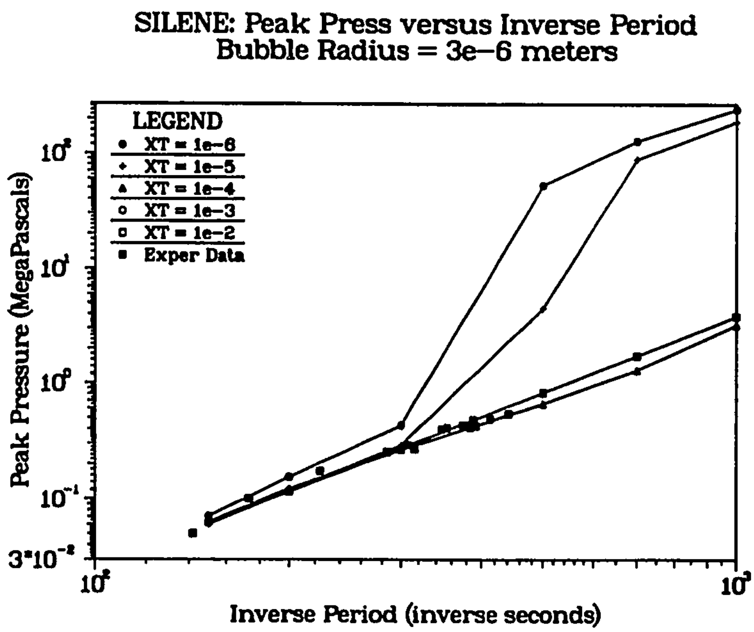

Figure 34: Peak Pressure versus Inverse Period for a Bubble Radius of 3E-6 meters for the SILENE Reactor

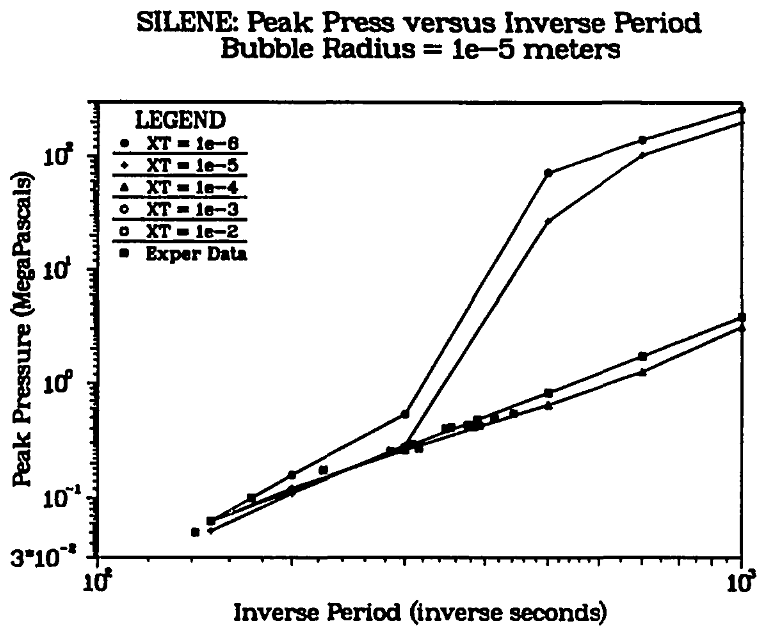

Figure 35: Peak Pressure versus Inverse Period for a Bubble Radius of 1E-5 meters for the SILENE Reactor

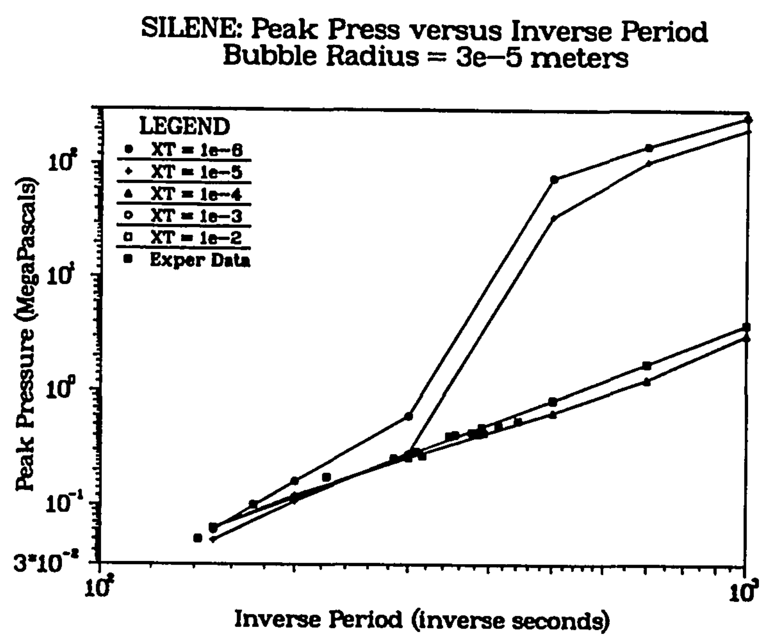

Figure 36: Peak Pressure versus Inverse Period for a Bubble Radius of 3E-5 meters for the SILENE Reactor

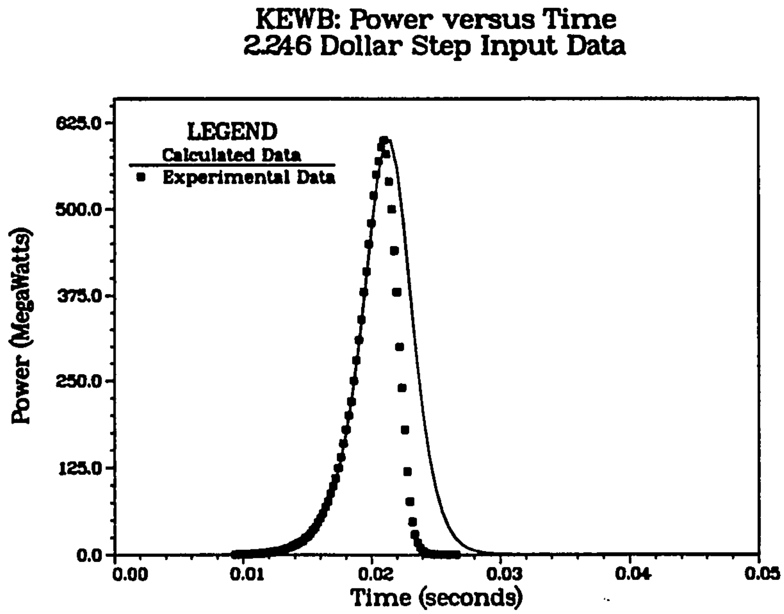

Figure 37: Power versus Time for a 2.246$ Step Reactivity Input for the KEWB5 Reactor

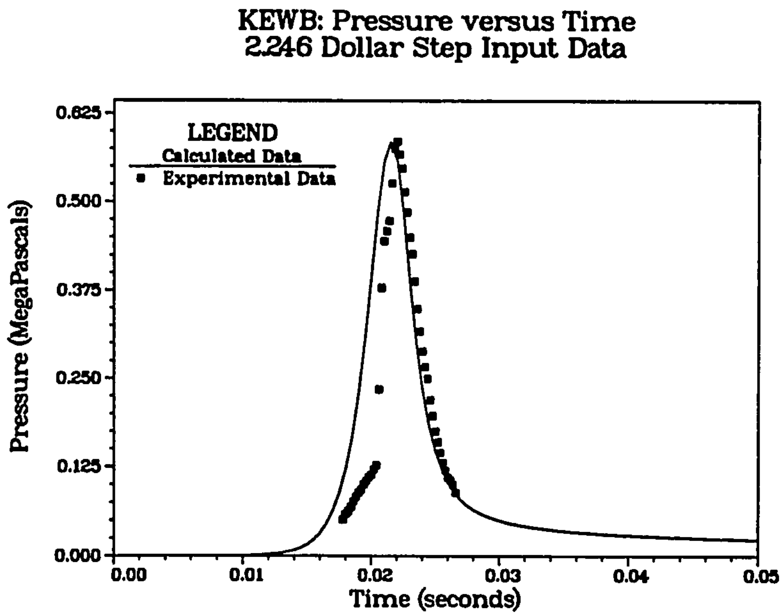

Figure 38: Pressure versus Time for a 2.246$ Step Reactivity Input for the KEWB5 Reactor

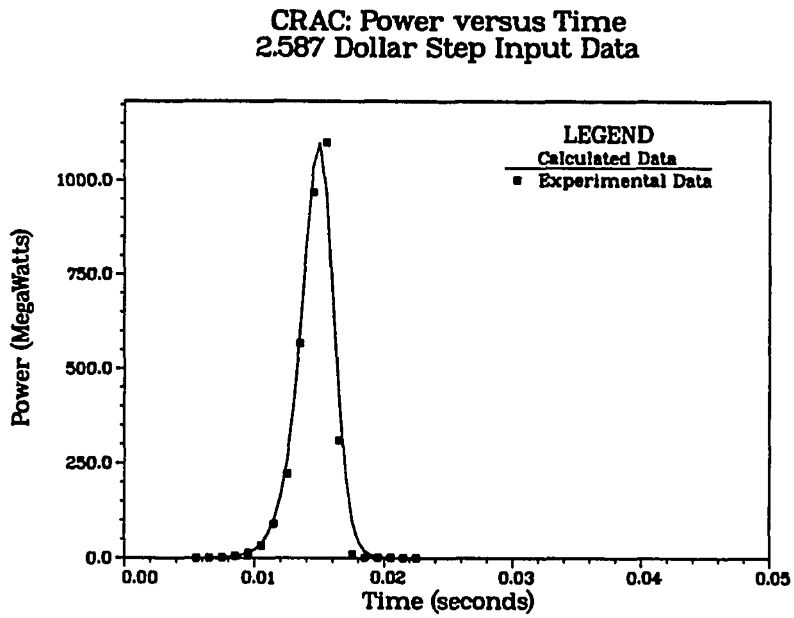

Figure 39: Power versus Time for a 2.587$ Step Reactivity Input for the CRAC Reactor

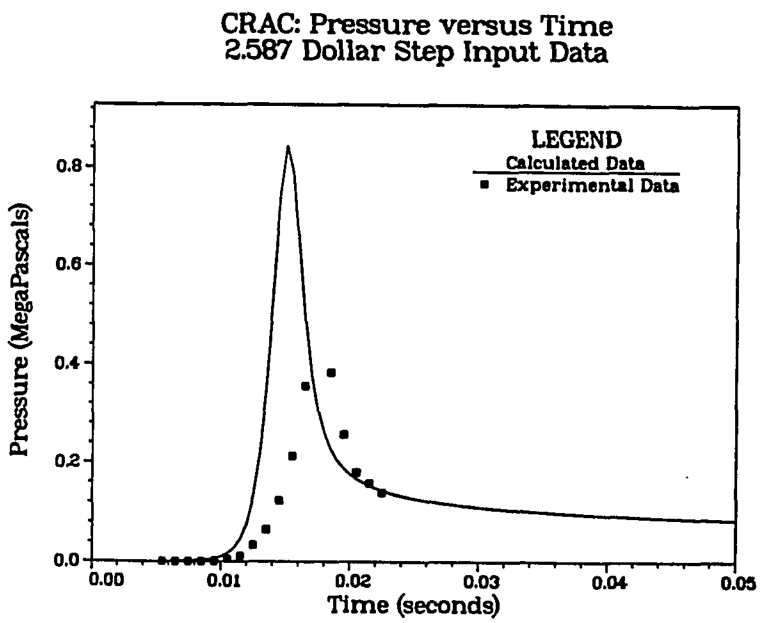

Figure 40: Pressure versus Time for a 2.587$ Step Reactivity Input for the CRAC Reactor

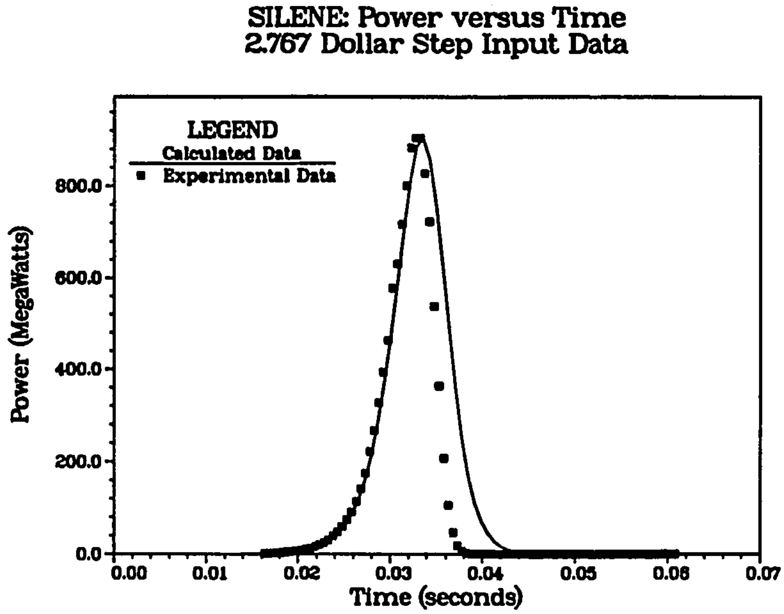

Figure 41: Power versus Time for a 2.767$ Step Reactivity Input for the SILENE Reactor

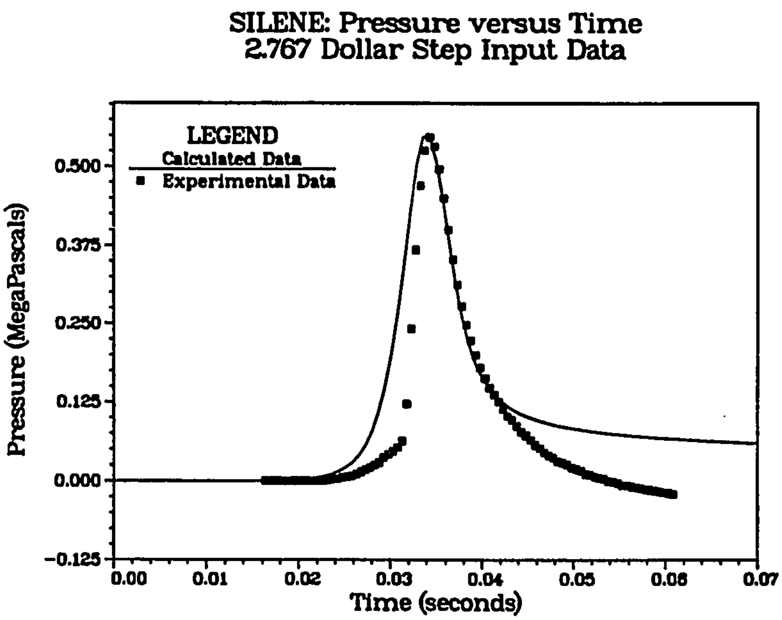

Figure 42: Pressure versus Time for a 2.767$ Step Reactivity Input for the SILENE Reactor

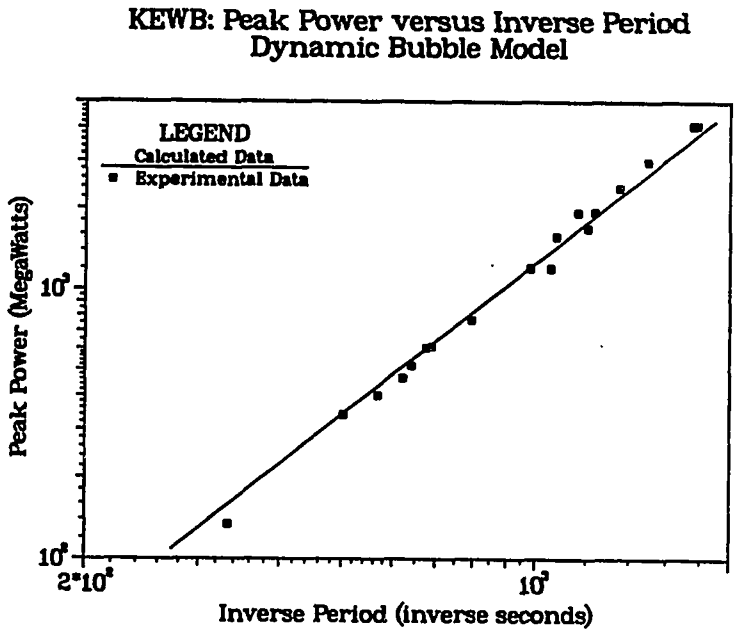

Figure 43: Peak Power versus Inverse Period for the Dynamic Bubble Radius Model for the KEWB5 Reactor

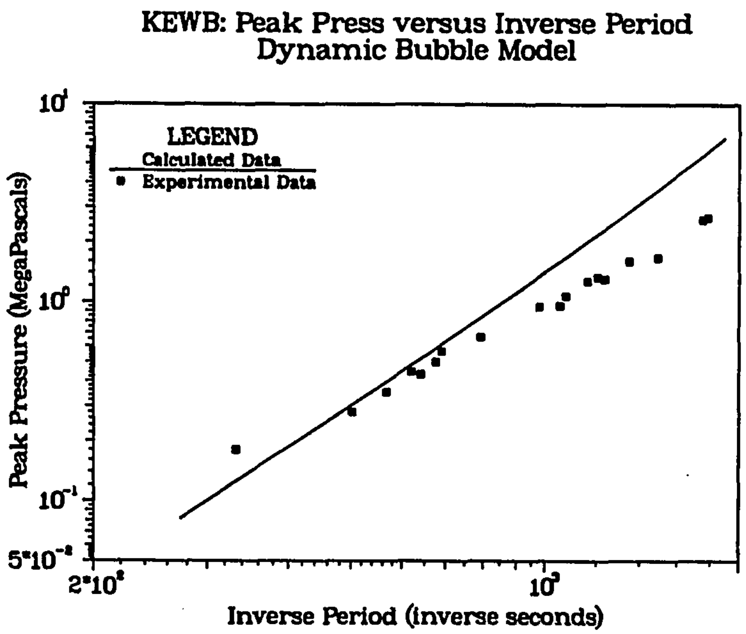

Figure 44: Peak Pressure versus Inverse Period for the Dynamic Bubble Radius Model for the KEWB5 Reactor

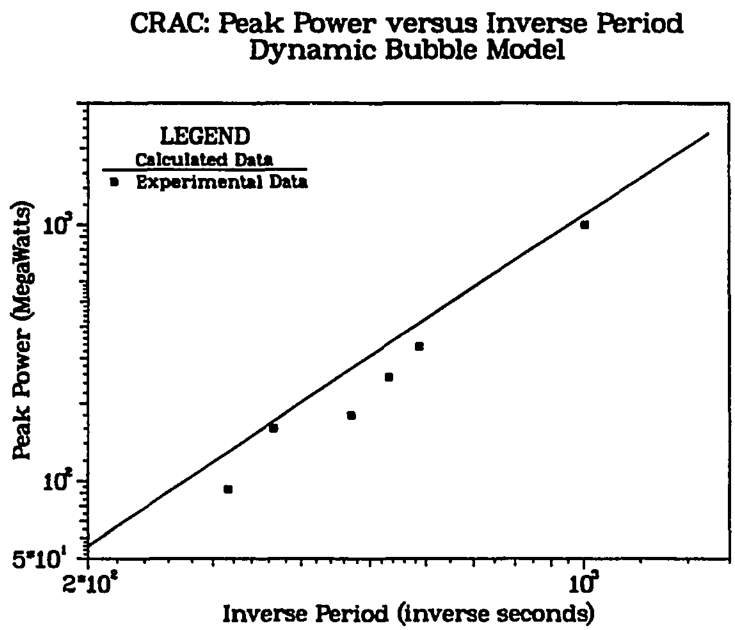

Figure 45: Peak Power versus Inverse Period for the Dynamic Bubble Radius Model for the CRAC Reactor

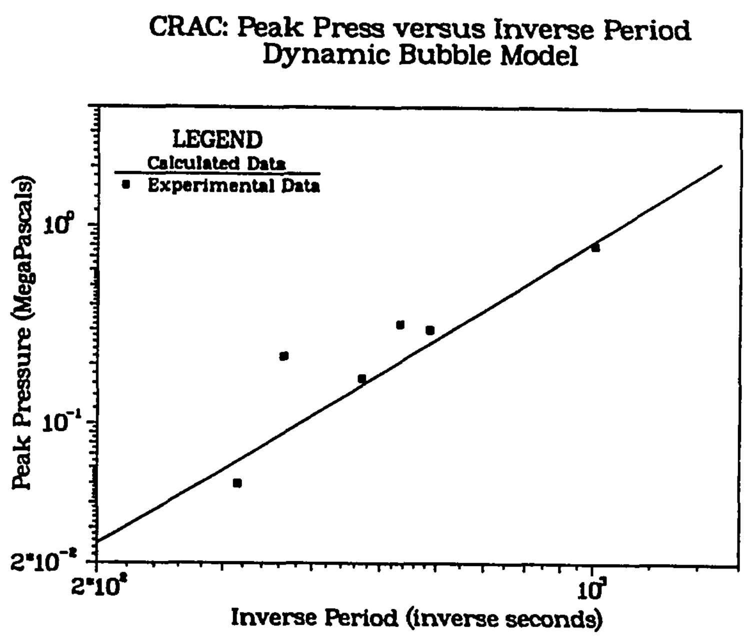

Figure 46: Peak Pressure versus Inverse Period for the Dynamic Bubble Radius Model for the CRAC Reactor

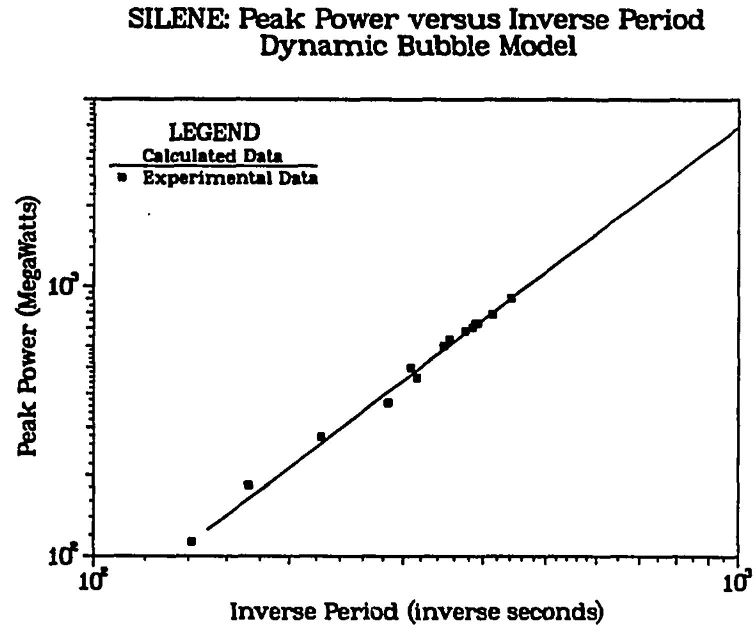

Figure 47: Peak Power versus Inverse Period for the Dynamic Bubble Radius Model for the SILENE Reactor

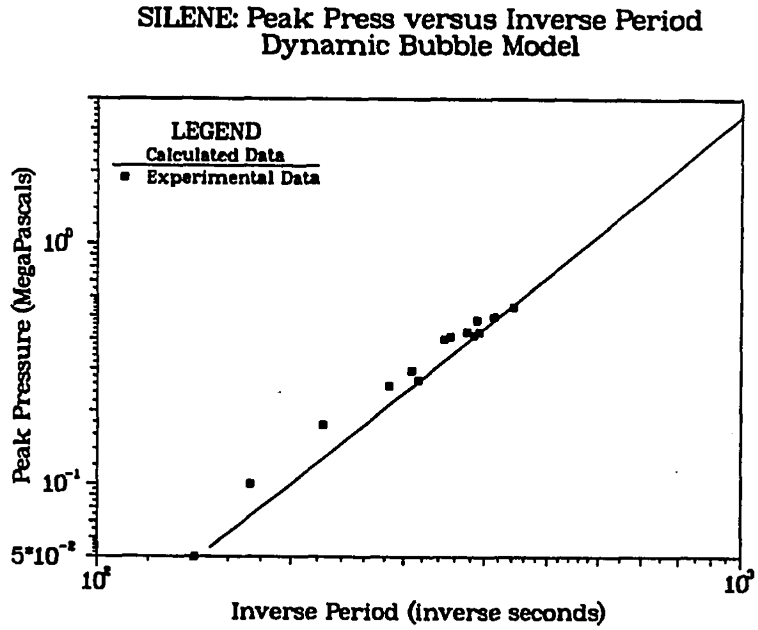

Figure 48: Peak Pressure versus Inverse Period for the Dynamic Bubble Radius Model for the SILENE Reactor

Appendix A

Neutronic Model Details

This Appendix details the computer submodel for the point reactor kinetics submodel that generates the power and energy in the system as described in Section 3.1. The parameters β_i, λ_i, and ℓ are assumed to be constant and reactor type dependent for this analysis. A step reactivity input is the initiating event in this analysis. The mathematical model is rewritten from Equations 3-1, 3-2, 3-3 and 3-6 as:

$$\frac{dN}{dt} = \frac{\beta}{\ell}\left(\frac{\rho}{\beta} - 1\right)N + \frac{\beta}{\ell}\sum_{i=1}^{m}\lambda_i D_i \tag{A-61}$$

$$\frac{dD_i}{dt} = \frac{\beta_i}{\beta}N - \lambda_i D_i\,, \quad i = 1, m \tag{A-62}$$

$$\frac{\rho}{\beta} = \frac{\rho_0}{\beta} - \phi(V - V_0) \tag{A-63}$$

$$\frac{dE}{dt} = N \times n_0 \tag{A-64}$$

where,

$N =$ The relative neutron power $= \frac{n}{n_0}$;

$D_i =$ The relative precursor latent power $= \frac{\ell c_i \lambda_i}{\beta_i n_0}$;

$E =$ The deposited energy;

$n_0 =$ The initial reactor power;

$t =$ Time;

$\frac{\rho}{\beta} =$ The reactivity;

$\frac{\rho_0}{\beta}$ = The initial step reactivity;

$\frac{\beta}{\ell}$ = The delayed neutron fraction over the neutron generation time;

$\frac{\beta_i}{\beta}$ = The relative neutron yield;

λ_i = The decay constant for the specific precursor power;

ϕ = The volume feedback reactivity coefficient;

V = The system volume;

V_0 = The initial system volume;

m = The total number of precursor *groups* (not physical precursor atom species).

From this mathematical model, a DARE/P based computer program is created. Since DARE/P algebraically sorts the equations, they are in no specific order as listed here or in the program. The submodel equations are grouped for easy understanding of the program. Also, for ease of reading and inputing the equations, some temporary variables are used. The variable names closely match the mathematical names except where the FORTRAN V default variable name convention gets in the way. All mathematical variables that start with an integer specifier (I–N) have a real specifier as the first letter (A–H, and O–Z). The mathematical model above reduces to the computer algorithm below:

```
RFB = PHI * ( V - VO )
R = RO - RFB
D1.  = AL1 * ( EN - D1 )
D2.  = AL2 * ( EN - D2 )
D3.  = AL3 * ( EN - D3 )
D4.  = AL4 * ( EN - D4 )
D5.  = AL5 * ( EN - D5 )
D6.  = AL6 * ( EN - D6 )
SUM = F1*D1 + F2*D2 + F3*D3 + F4*D4 + F5*D5 + F6*D6
```

```
EN. = BOL * ((R-1) * EN + SUM )
ENP = ENO * EN
E. = ENP - ENO
```

This set of equations is contained in the DARE/P programs (See Appendix D).

Table 3 contains the six group precurser model constants. Tables 4 and 5 contain the constants and initial conditions used in the static and dynamic bubble models. These constants represent the best fit of peak pressure and power data, and the pressure and power versus time traces.

Table 3: Six Group Precursor Constants

Group Number	Decay Constant sec^{-1}	Relative Yield
1	0.0124	0.003
2	0.0305	0.219
3	0.111	0.196
4	0.301	0.395
5	1.14	0.115
6	3.01	0.042
Total	—	1.000

Table 4: Static Bubble Model Constants and Initial Conditions

Parameter	Symbol	Units	KEWB5	CRAC	SILENE
$\frac{\beta}{\ell}$	BOL	sec^{-1}	554	633	250
Initial Reactor Volume	V0	m^3	0.024	0.018	0.037
Feedback Coefficient	PHI	$\frac{\$}{m^3}$	12000	14000	5600
Initial Power	N0	MW	0.001		
Initial Relative Power	N	—	1.0		
Relative Precurser Power	Di	—	0.0		

Table 5: Dynamic Bubble Model Constants and Initial Conditions

Parameter	Symbol	Units	KEWB5	CRAC	SILENE
$\frac{\beta}{\ell}$	BOL	sec^{-1}	554	633	250
Initial Reactor Volume	V0	m^3	0.024	0.018	0.037
Feedback Coefficient	PHI	$\frac{\$}{m^3}$	12370	10900	6140
Initial Power	N0	MW	0.001		
Initial Relative Power	N	—	1.0		
Relative Precurser Power	Di	—	0.0		

Appendix B

Equilibrium Thermodynamic Model Details

This appendix details the computer submodels for the equilibrium thermodynamic based submodels for the temperature and pressure relationships to time, the equilibrium thermodynamic model for radiolytic bubble formation and the resulting equation of state, and the momentum dissipation term in the equation of state. The mathematical submodel is:

$$\frac{dT}{dt} = \frac{1}{c_v}\left[\frac{dE}{dt} - \frac{\beta}{\kappa}T\frac{dV}{dt}\right] \tag{B-65}$$

$$\frac{dP}{dt} = \frac{\beta}{\kappa}\frac{dT}{dt} - \frac{1}{\kappa V}\frac{dV}{dt} \tag{B-66}$$

$$\beta = (1-f)\beta_0 + \frac{f}{T}\frac{P+\frac{2\sigma}{r_b}}{P+\frac{4\sigma}{3r_b}} \tag{B-67}$$

$$\kappa = (1-f)\kappa_0 + \frac{f}{P+\frac{4\sigma}{3r_b}} \tag{B-68}$$

$$f = \frac{xR_gT}{(1-x)v_l(P+\frac{2\sigma}{r_b}) + xR_gT} \tag{B-69}$$

$$\frac{d^2V}{dt^2} = \frac{4A^2(P-P_0)}{M} - \theta\frac{dV}{dt} \tag{B-70}$$

$$x_E = \frac{G \times E}{M} \tag{B-71}$$

$$x_R = x_E - x_T \tag{B-72}$$

$$\text{IF } (x_E < x_T)\ x = 0.0 \text{ ELSE } x = x_R \tag{B-73}$$

where,

$T =$ The calculated system lumped parameter temperature;

$P =$ The calculated system lumped parameter absolute pressure;

$V =$ The calculated system liquid and gas mixture volume;

$E =$ The calculated energy deposited in the solution due to nuclear fission;

$v_l =$ The specific volume of the liquid;

$x =$ The calculated gas mass fraction;

$r_b =$ The radiolytic gas bubble radius;

$c_v =$ The specific heat at constant volume for the solution;

$\beta =$ The calculated isobaric compressibility factor for the liquid and gas mixture;

$\kappa =$ The calculated isothermal compressibility factor for the liquid and gas mixture;

$P_0 =$ The atmospheric pressure to convert P to gauge pressure;

$\beta_0 =$ The isobaric compressibility factor for liquid only;

$\kappa_0 =$ The isothermal compressibility factor for liquid only;

$R_g =$ The ideal gas constant for $2H_2$ and O_2;

$f =$ The calculated gas volumetric fraction;

$\sigma =$ The surface tension of the solution;

$\theta =$ The momentum dissipation parameter;

$A =$ The cross sectional area of the reactor cylinder;

$M =$ The mass of the total volume;

$x_E =$ The calculated gas mass fraction generated by nuclear fission;

$x_T =$ The threshold gas mass fraction to determine when gas bubbles form;

$x_R =$ The excess gas mass fraction over the threshold;

$G =$ The mass of gas produced per MegaJoule of energy deposited in the solution.

The isobaric compressibility factor for liquid water varies widely over the temperature range of interest. The treatment for this thesis is to fit a third degree polymonial to the line that describes the standard relationship between β_0 for water and temperature. It is assumed that the liquid water factors are the same as the solution factors. The coefficients are listed in Table 6.

From this mathematical model, a DARE/P based computer program is created. The same naming conventions are used as described in Section A.

The Second Law and Energy equations including the momentum dissipation term are programmed as follows:

```
V. = VDOT
VDOT. = 1.0E6 * 4.0 * A * A * (P-PO) / EM - THETA * VDOT
TABS. = TDOT
TDOT = ( ENP - ENO - (BETA/AK)*TABS*VDOT ) / ( EM*CV )
P. = PDOT
PDOT = ( BETA * TDOT - VDOT/V ) / AK
```

The Equations of State and bubble nucleation time determination are programmed as follows:

```
XE = G * E / EM
XR = XE - XT
PROCED X = XE,XT,XR
```

```
IF ( XE .LT. XT ) THEN
X = 0.0
ELSE
X = XR
ENDIF
ENDPRO
PA = P + 2.0 * SIGMA / RB
PB = P + (4.0/3.0) * SIGMA / RB
FA = X * RG * TABS
FB = FA + VL * (1.0-X) * PA
F = FA / FB
TEMP = TABS - 273.15
BETAO = AO + A1*TEMP + A2*TEMP*TEMP + A3*TEMP*TEMP*TEMP
BETA = 1.0E-6 * BETAO * (1.0-F) + F * PA / ( TABS * PB )
AK = AKO * (1.0-F) + F / PB
```

These sets of equations are contained in the DARE/P programs (See Appendix D).

Table 6 contains the solution properties. Table 7 contains the equilibrium thermodynamic constants and initial conditions needed by the model. Some of these constants represent a best fit of the peak power and pressure, and the power and pressure versus time traces.

Table 6: Solution Properties for the Reactor Types

Parameter	Symbol	Units	KEWB5	CRAC	SILENE
Specific Volume	VL	$\frac{m^3}{kg}$	8.3E-4	8.3E-4	8.6E-4
Specific Heat	CV	$\frac{MJ}{kg}$	3.6E-3	3.6E-3	3.4E-3
Total Mass of Solution	M	kg	29	21	43
Surface Tension	SIGMA	$\frac{MPa}{m}$	7.5E-8	7.5E-8	7.5E-8
Gas Constant	RG	$\frac{MPa}{°K}$	6.89E-3	6.89E-3	6.89E- 3
Liquid Isothermal Factor	AK0	$\frac{1}{MPa}$	4.5E- 4		
Liquid Isobaric Factor 0	A0	$\frac{1}{°K}$	- 23.10932		
Liquid Isobaric Factor 1	A1	$\frac{1}{°K\,°C}$	13.15603		
Liquid Isobaric Factor 2	A2	$\frac{1}{°K\,°C^2}$	- 0.086950906		
Liquid Isobaric Factor 3	A3	$\frac{1}{°K\,°C^3}$	0.00032788806		

Table 7: Equilibrium Thermodynamic Model Constants and Initial Conditions

Parameter	Symbol	Units	KEWB5	CRAC	SILENE
Reactor Cross Section	A	m^2	0.07	0.07	0.10
Momentum Dissipation	THETA	sec^{-1}	3000	5000	2700
Initial Volume	V	m^3	0.024	0.018	0.037
Initial Volume Derivative	VDOT	$\frac{m^3}{sec}$	0.0		
Initial Energy	E	MJ	1.0E-6		
Initial Temperature	TABS	$^\circ K$	293.15		
Initial Pressure	P	MPa	0.1		
Gas Generation	G	$\frac{kg_{gas}}{MJ}$	2.0E-3		
Reference Pressure	P0	MPa	0.1		

Appendix C

Bubble Growth Model Details

This appendix details the computer submodels for the mass diffusion based relationships between the radiolytic gas bubble radius and time. The equilibrium thermodynamic submodels that describe the temperature, pressure, and volume relationships to time (Equations B-65, B-66 and B-70) are used and are described in detail in Appendix B. The mass diffusion mathematical model is:

$$\frac{dr_b}{dt} \approx \frac{Df}{r_b x}(x_{super} - x_{sat}) \qquad t > t_N \tag{C-74}$$

$$f \approx \frac{xR_gT}{v_l(P + \frac{2\sigma}{r_b})} \tag{C-75}$$

$$v_s \approx v_l \approx \frac{1}{\rho_l} \tag{C-76}$$

$$x_{super} = FP \times x_{sat} \tag{C-77}$$

$$x_E = \frac{GE}{M} \tag{C-78}$$

$$\frac{dx_R}{dt} = GMS(x_E - x_R - x_{super}) \tag{C-79}$$

$$\text{IF } (x_E < x_{super})\ x = 0.0 \text{ ELSE } x = x_R \tag{C-80}$$

where,

$r_b =$ The dynamic bubble radius;

$D =$ The diffusion coefficient for this process;

$f =$ The calculated gas volumetric fraction;

$x =$ The calculated gas mass fraction;

$x_{super} =$ The supersaturated gas mass fraction that exists in the liquid;

$x_{sat} =$ The saturation gas mass fraction that exists at r_b;

$R_g =$ The ideal gas constant for $2H_2$ and O_2;

$T =$ The calculated system temperature;

$v_s =$ The specific volume for the solution;

$v_l =$ The specific volume for the liquid;

$P =$ The calculated system pressure;

$\sigma =$ The surface tension of the solution;

$x_E =$ The amount of gas created by nuclear reactions;

$x_R =$ The excess gas available to form bubbles;

$E =$ The energy deposited in the solution;

$G =$ The dissociation rate of H_2O;

$M =$ The mass of the solution;

$FP =$ An adjustable parameter;

$GMS =$ An adjustable parameter.

From this mathematical model, a DARE/P based computer program is created. The same naming conventions described in Section A are used here. The mathematical model above reduces to the computer algorithm below:

```
PA = P + 2.0 * SIGMA / RB
FA = RG * TABS
FB = VL * PA
F = X * FA / * FB
```

```
RB. = STEP * ( D * F ) * (XSUPER-XSAT) / ( RB * X )
XE = G * E / M
XSUPER = FP * XSAT
XR. = STEP * GMS * ( XE - XR - XSUPER )
PROCED STEP,X=XE,XR,XSUPER
IF ( XE .LT. XSUPER ) THEN
X = 0.0
STEP = 0.0
ELSE
X = XR
STEP = 1.0
ENDIF
ENDPRO
```

These sets of equations are contained in the DARE/P programs (See Appendix D).

Table 6 contains the properties of the fuel solutions. Table 7 contains the constants used in this model except for the momentum dissipation which are listed in Table 2.

APPENDIX D

PROGRAM LISTINGS

This appendix lists the DARE/P source code for the tasks performed:

- Parameter survey for the Reference [4] KEWB5 model;
- Parameter survey for the Reference [4] CRAC model;
- Parameter survey for the Reference [4] SILENE model;
- Peak pressure and peak power versus inverse period for the dynamic bubble growth submodel for the KEWB5 model;
- Peak pressure and peak power versus inverse period for the dynamic bubble growth submodel for the CRAC model;
- Peak pressure and peak power versus inverse period for the dynamic bubble growth submodel for the SILENE model.

D.1 KEWB5 Models

This section contains the DARE/P programs used to evaluate the KEWB5 reactor for the static and dynamic models:

D.1.1 Static Bubble Model

```
*****************************************************************
*                                                               *
*     FISSILE SOLUTION CONTAINING GAS (ONE-REGION FEEDBACK)     *
*                                                               *
*        SYSTEM            V        EM        VL        A       *
*                                                               *
```

```
*         CRAC              0.018     21.0     8.3E-4     0.07          *
*         SILENE            0.037     43.0     8.6E-4     0.10          *
*         KEWB 5            0.024     29.0     8.3E-4     0.07          *
*                                                                       *
*************************************************************************
*
*************************************************************************
***********************  INTEGRATION RULES
*************************************************************************
*
 $M11,7,3,1
*
*************************************************************************
***********************  LOGICAL STATEMENTS SECTION
*************************************************************************
*
 $L
*
*************************************************************************
* TURN OFF STORING ROUTINES
*************************************************************************
*
      CALL STROF
*
*************************************************************************
* SETUP DATA CONSTANTS
*************************************************************************
*
      IPOW = 99
      IPRS = 98
      ISAVE = 1
*
*************************************************************************
* LOOP OVER THE MAGNITUDE OF THE CONSTANT BUBBLE RADIUS
*************************************************************************
*
      DO IRB = 7, 5, -1
*
*************************************************************************
* LOOP OVER THE MANTISSA OF THE CONSTANT BUBLE RADIUS
*************************************************************************
*
        DO IRB2 = 1, 2
*
*************************************************************************
* DETERMINE ACTUAL BUBBLE RADIUS
*************************************************************************
*
          IF ( IRB2 .EQ. 1 ) THEN
            RB = 1.0 * ( 10.0 ** (-IRB) )
          ELSE
            RB = 3.0 * ( 10.0 ** (-IRB) )
          END IF
*
*************************************************************************
* OPEN THE POWER AND PRESSURE DATA FILES AND WRITE THE
*    BUBBLE RADIUS
*************************************************************************
*
          OPEN ( UNIT=IPOW, FILE='POWER.DAT', STATUS='NEW' )
          OPEN ( UNIT=IPRS, FILE='PRESS.DAT', STATUS='NEW' )
*
*************************************************************************
* LOOP OVER THE MAGNITUDES OF THE THRESHOLD MASS FRACTION
*************************************************************************
*
          DO IXT = 6, 2, -1
*
*************************************************************************
* DETERMINE THE ACTUAL THRESHOLD MASS FRACTION
*************************************************************************
*
```

```
          XT = 1.0 * ( 10.0 ** (-IXT) )
*
*****************************************************************
* LOOP OVER ALL THE INITIAL REACTIVITY ADDITIONS
*****************************************************************
*
          DO RO = 1.5, 4.5, 0.5
*
*****************************************************************
* WRITE A MESSAGE TO THE OPERATOR
*****************************************************************
*
            WRITE(*,*)'RUN ', ISAVE, ' WITH RO, XT, RB = ',
$                RO, XT, RB
*
*****************************************************************
* RUN THE SIMULATION
*****************************************************************
*
            CALL RESET
            CALL RUN
            WRITE(*,*)'RUN ', ISAVE, ' WITH OMEGA = ', OMEGA
*
*****************************************************************
* INCREMENT THE RUN NUMBER
*****************************************************************
*
            ISAVE = ISAVE + 1
*
*****************************************************************
* WRITE THE MAXIMUM POWER AND PRESSURE DATA TO THEIR
*     RESPECTIVE FILES
*****************************************************************
*
            WRITE ( IPOW, 200 ) OMEGA, POWMAX
            WRITE ( IPRS, 200 ) OMEGA, PREMAX
*
*****************************************************************
* END OF REACTIVITY LOOP
*****************************************************************
*
          END DO
*
*****************************************************************
* WRITE THE DATA TERMINATION VALUES FOR RPLOT PACKAGE
*****************************************************************
*
          WRITE ( IPOW, 200 ) -1.0, -1.0
          WRITE ( IPRS, 200 ) -1.0, -1.0
*
*****************************************************************
* END OF THERSHOLD LOOP
*****************************************************************
*
        END DO
*
*****************************************************************
* CLOSE THE DATA FILES
*****************************************************************
*
        CLOSE ( UNIT=IPOW )
        CLOSE ( UNIT=IPRS )
*
*****************************************************************
* END THE BUBBLE LOOPS
*****************************************************************
*
      END DO
    END DO
*
*****************************************************************
* FORMAT STATEMENTS
```

```
*****************************************************************
*
200   FORMAT ( T1, G15.6, ',', T25, G15.6 )
*
*****************************************************************
**************************  OMEGA/RO TABLE BLOCK
*****************************************************************
*
 $T
      TOMEGA,7
      1.5, 273.
      2.0, 544.
      2.5, 815.
      3.0, 1087.
      3.5, 1359.
      4.0, 1630.
      4.5, 1902.
*
*****************************************************************
**************************  DERIVATIVE BLOCK 1
*****************************************************************
*
 $D1
*
*****************************************************************
* NEUTRON DYNAMICS
*****************************************************************
*
      R      = RO+B*T-RFB
      RFB    = PHI*(V-VO)
      SUM    = F1*D1+F2*D2+F3*D3+F4*D4+F5*D5+F6*D6
      ENDOT  = BOL*((R-1.0)*EN+SUM-RS)
      EN.    = ENDOT
      D1.    = AL1*(EN-D1)
      D2.    = AL2*(EN-D2)
      D3.    = AL3*(EN-D3)
      D4.    = AL4*(EN-D4)
      D5.    = AL5*(EN-D5)
      D6.    = AL6*(EN-D6)
      ENP    = ENO*EN
      E.     = ENP
*
*****************************************************************
* GAS AND MOMENTUM PRODUCTION
*****************************************************************
*
      XE     = G*E/EM
      XR     = XE-XT
      PROCED X=XE,XT
         IF (XE.LT.XT) THEN
            X = 0.0
         ELSE
            X = XR
         END IF
      ENDPRO
      PROCED TAU=X,THETA1,THETA2
         IF (X.LE.0.0) THEN
           TAU = THETA1
         ELSE
           TAU = THETA2
         END IF
      ENDPRO
*
*****************************************************************
* EQUATION OF STATE
*****************************************************************
```

```
*
      TABS  = TEMP+273.15
      PA    = P+2.*SIGMA/RB
      PB    = P+(4./3.)*SIGMA/RB
      FA    = X*RG*TABS
      FB    = FA+VL*(1.0-X)*PA
      F     = FA/FB
      BETA0 = 1.0E-6*(A0+A1*TEMP+A2*TEMP*TEMP+A3*TEMP*TEMP*TEMP)
      BETA  = BETA0*(1.0-F)+F*PA/(TABS*PB)
      AK    = AK0*(1.0-F)+F/PB
      PC    = BETA/AK
*
*****************************************************************
* ENERGY AND STATE EQUATIONS
*****************************************************************
*
      V.    = VDOT
      VDOT. = 1.0E6*FP*A*A*(P-P0)/EM-TAU*VDOT
      TEMP. = TDOT
      TDOT  = (ENP-EN0-(BETA/AK)*TABS*VDOT)/(EM*CV)
      P.    = PDOT
      PDOT  = (BETA*TDOT-VDOT/V)/AK
      PRESS = P-P0
      PRBOT = PRESS*2
*
*****************************************************************
* FIND THE PERIOD DUE TO THE INITIAL REACTIVITY
*****************************************************************
*
      OMEGA = TOMEGA(R0)
*
*****************************************************************
* FIND MAXIMUM POWER AND PRESURE
*****************************************************************
*
      PROCED PREMAX=PRBOT
         IF ( PREMAX .LT. PRBOT ) THEN
           PREMAX = PRBOT
         END IF
      ENDPRO
      PROCED POWMAX=ENP
         IF ( POWMAX .LT. ENP ) THEN
           POWMAX = ENP
         END IF
      ENDPRO
*
*****************************************************************
* RUNTIME DISPLAY COMMAND
*****************************************************************
*       DISPLAY P,ENP,PREMAX,POWMAX
END
*****************************************************************
************************  CONSTANT AND INITIAL CONDITION SECTION
*****************************************************************
*
      TMAX  = 0.1
      NPOINT= 2501
      DTMAX = 5.0E-6
      DTMIN = 5.0E-8
*
*****************************************************************
* COEFFICIENTS IN EQUATION OF STATE
*****************************************************************
*
      RG    = 6.89E-4
      SIGMA = 7.5E-8
      A0    = -23.10932
      A1    = 13.15603
```

```
      A2     = -8.6950906E-2
      A3     = 3.2788806E-4
      AKO    = 4.5E-4
*
*******************************************************************
* INITIAL VALUES FOR THE EQUATIONS OF STATE
*******************************************************************
*
      P      = 0.1
      TEMP   = 20.0
*
*******************************************************************
* SOLUTION PROPERTIES
*******************************************************************
*
      V      = 0.024
      EM     = 29.0
      VL     = 8.3E-4
      A      = 0.07
      FP     = 4.0
      THETA1= 3000.0
      THETA2= 3000.0
      CV     = 3.6E-3
      PO     = 0.1
      G      = 2.0E-3
*
*******************************************************************
* INITIAL VALUES FOR NUCLEAR DYNAMIC AND POWER EQUATIONS
*******************************************************************
*
      RS     = 0.0
      B = 0.0
      PHI    = 12000.
      VO     = 0.024
      BOL    = 543.5
      E      = 1.E-6
      ENO    = 0.001
      EN     = 1.0
      AL1    = 0.0124 , F1 = 0.033 , D1 = 0.0
      AL2    = 0.0305 , F2 = 0.219 , D2 = 0.0
      AL3    = 0.111  , F3 = 0.196 , D3 = 0.0
      AL4    = 0.301  , F4 = 0.395 , D4 = 0.0
      AL5    = 1.14   , F5 = 0.115 , D5 = 0.0
      AL6    = 3.01   , F6 = 0.042 , D6 = 0.0
*
*******************************************************************
* INITIAL VALUES FOR MAXIMUM POWER AND PRESSURE CALCULATIONS
*******************************************************************
*
      POWMAX= 0.0
      PREMAX= 0.0
*
END
*
*******************************************************************
*********************************  OUTPUT BLOCK
*******************************************************************
*
* USE ANOTHER PACKAGE BESIDE DARE TOPLOT VALUES UNLESS THERE IS
* SOMEWAY TO PLOT MULTIPLE SAVE FILES AGAINST ONE ANOTHER?
*
END
```

D.1.2 Dynamic Bubble Model

```
*****************************************************************
***********************  INTEGRATION RULES
*****************************************************************
*
 $M11
*
*****************************************************************
***********************  LOGICAL STATEMENTS SECTION
*****************************************************************
*
 $L
*
*****************************************************************
* TURN OFF STORING ROUTINES
*****************************************************************
*
      CALL STROF
*
*****************************************************************
* SETUP DATA CONSTANTS
*****************************************************************
*
      IPOW = 99
      IPRS = 98
      ISAVE = 1
*
*****************************************************************
* OPEN THE POWER AND PRESSURE DATA FILES
*****************************************************************
*
      OPEN ( UNIT=IPOW, FILE='POWER.DAT', STATUS='NEW' )
      OPEN ( UNIT=IPRS, FILE='PRESS.DAT', STATUS='NEW' )
*
*****************************************************************
* LOOP OVER ALL THE INITIAL REACTIVITY ADDITIONS
*****************************************************************
*
      DO RO = 1.5, 4.5, 0.5
*
*****************************************************************
* RUN THE SIMULATION
*****************************************************************
*
           PREMAX = 0.0
           POWMAX = 0.0
           TMAXMW = 0.0
           TMAXPR = 0.0
           TNUC   = 0.0
           CALL RESET
           CALL RUN
           WRITE (*,100) ISAVE, RO, OMEGA, POWMAX, TMAXMW, PREMAX,
$            TMAXPR, TNUC
*
*****************************************************************
* INCREMENT THE RUN NUMBER
*****************************************************************
*
           ISAVE = ISAVE + 1
*
*****************************************************************
* WRITE THE MAXIMUM POWER AND PRESSURE DATA TO THEIR
*     RESPECTIVE FILES
*****************************************************************
*
           WRITE ( IPOW, 200 ) OMEGA, POWMAX
           WRITE ( IPRS, 200 ) OMEGA, PREMAX
*
```

```
*******************************************************************
* END OF REACTIVITY LOOP
*******************************************************************
*
       END DO
*
*******************************************************************
* WRITE THE DATA TERMINATION VALUES FOR RPLOT PACKAGE
*******************************************************************
*
       WRITE ( IPOW, 200 ) -1.0, -1.0
       WRITE ( IPRS, 200 ) -1.0, -1.0
*
*******************************************************************
* CLOSE THE DATA FILES
*******************************************************************
*
       CLOSE ( UNIT=IPOW )
       CLOSE ( UNIT=IPRS )
*
*******************************************************************
* FORMAT STATEMENTS
*******************************************************************
*
100   FORMAT(' RUN = ',I3.3,T12,' RO  = ',G12.5,' OMG = ',G12.5,/,
$                           T12,' POW = ',G12.5,' TMW = ',G12.5,/,
$                           T12,' PRS = ',G12.5,' TPR = ',G12.5,/,
$                           T12,' TN  = ', G12.5)
200   FORMAT ( T1, G15.6, ',', T25, G15.6 )
*
*******************************************************************
**************************  OMEGA/RO TABLE BLOCK
*******************************************************************
*
 $T
      TOMEGA,7
      1.5, 273.
      2.0, 544.
      2.5, 815.
      3.0, 1087.
      3.5, 1359.
      4.0, 1630.
      4.5, 1902.
*
*******************************************************************
**************************  DERIVATIVE BLOCK 1
*******************************************************************
*
 $D1
*
*******************************************************************
* NEUTRON DYNAMICS
*******************************************************************
*
      R     = RO + B*T - RFB
      RFB   = PHI * (V-VO)
      SUM   = F1*D1 + F2*D2 + F3*D3 + F4*D4 + F5*D5 + F6*D6
      ENDOT = BOL * ( (R-1.0)*EN + SUM - RS )
      EN.   = ENDOT
      D1.   = AL1*(EN-D1)
      D2.   = AL2*(EN-D2)
      D3.   = AL3*(EN-D3)
      D4.   = AL4*(EN-D4)
      D5.   = AL5*(EN-D5)
      D6.   = AL6*(EN-D6)
      ENP   = ENO*EN
```

```
      E.     = ENP - EN0
*
******************************************************************
* GAS AND MOMENTUM PRODUCTION
******************************************************************
*
      XE     = G * E / EM
      XR.    = STEP * GMS * ( XE - XR- XSUPER )
      XSUPER= FP * XSAT
      PROCED TNUC,STEP,X=XE,XSUPER,XR,T
         IF (XE.LT.XSUPER) THEN
            X = 0.0
            STEP = 0.0
         ELSE
            X = XR
            IF ( STEP .LT. 0.5 ) THEN
               TNUC = T
            END IF
            STEP = 1.0
         END IF
      ENDPRO
*
******************************************************************
* EQUATION OF STATE
******************************************************************
*
      PA     = P + 2.0 * SIGMA / RB
      PB     = P + (4.0/3.0) * SIGMA / RB
      FA     = RG * TABS
      FB     = VL * PA
      F      = X * FA / FB
      RB.    = STEP * ( D * FA ) * ( XSUPER - XSAT ) / ( RB * FB )
      TEMP   = TABS - 273.15
      BETA0  = 1.0E-6*(A0+A1*TEMP+A2*TEMP*TEMP+A3*TEMP*TEMP*TEMP)
      BETA   = BETA0 * (1.0-F) + F*PA / (TABS*PB)
      AK     = AK0 * (1.0-F) + F / PB
*
******************************************************************
* ENERGY AND STATE EQUATIONS
******************************************************************
*
      V.     = VDOT
      VDOT.  = 1.0E6 * 4.0 * (A*A) * (P-P0) / EM - THETA*VDOT
      TABS.  = TDOT
      TDOT   = ( ENP - EN0 - (BETA/AK)*TABS*VDOT ) / ( EM * CV )
      P.     = PDOT
      PDOT   = ( BETA*TDOT - VDOT/V ) / AK
      PG     = P - P0
      PRBOT  = 2*PG
*
******************************************************************
* FIND MAXIMUM POWER AND PRESSURE
******************************************************************
*
      PROCED TMAXPR,PREMAX=PRBOT,T
        IF ( PREMAX .LT. PRBOT ) THEN
           PREMAX = PRBOT
           TMAXPR = T
        END IF
      ENDPRO
*
      PROCED TMAXMW,POWMAX=ENP,T
        IF ( POWMAX .LT. ENP ) THEN
           POWMAX = ENP
           TMAXMW = T
        END IF
      ENDPRO
```

```
*
********************************************************************
* FIND THE INVERSE PERIOD DUE TO THE INITIAL REACTIVITY
********************************************************************
*
      OMEGA = TOMEGA(RO)
*
********************************************************************
* RUNTIME DISPLAY COMMAND
********************************************************************
*
*      DISPLAY ENP, PRBOT, EXDATA, X
*
END
********************************************************************
************************  CONSTANT AND INITIAL CONDITION SECTION
********************************************************************
*
      TMAX  = 0.1
      NPOINT= 2501
      DTMAX = 5.0E-6
      DTMIN = 5.0E-8
*
********************************************************************
* COEFFICIENTS AND CONSTANTS IN THE EQUATION OF STATE
********************************************************************
*
      RG    = 6.89E-4
      SIGMA = 7.5E-8
      A0    = -23.10932
      A1    = 13.15603
      A2    = -8.6950906E-2
      A3    = 3.2788806E-4
      AK0   = 4.5E-4
*
********************************************************************
* INITIAL VALUES FOR THE EQUATIONS OF STATE
********************************************************************
*
      TABS  = 20.0 + 273.15
      RB    = 5.0E-8
      VDOT  = 0.0
      V     = 0.024
      P     = 0.1
      XR    = 0.0
*
********************************************************************
* SOLUTION AND REACTOR VESSEL PROPERTIES
********************************************************************
*
      EM     = 29.0
      VL     = 8.3E-4
      A      = 0.07
      CV     = 3.6E-3
      PO     = 0.1
      G      = 2.0E-3
      VO     = 0.024
      XSAT   = 1.0E-5
      D      = 0.5E-8
      FP     = 16.5
      GMS    = 35.0
*
********************************************************************
* CONSTANTS FOR NUCLEAR DYNAMIC AND POWER EQUATIONS
********************************************************************
*
      PHI   = 12370.0
      THETA = 3987.0
      RS    = 0.0
      B     = 0.0
      BOL   = 543.5
      ENO   = 0.001
```

```
      AL1    = 0.0124 , F1 = 0.033
      AL2    = 0.0305 , F2 = 0.219
      AL3    = 0.111  , F3 = 0.196
      AL4    = 0.301  , F4 = 0.395
      AL5    = 1.14   , F5 = 0.115
      AL6    = 3.01   , F6 = 0.042
*
*****************************************************************
* INITIAL VALUES FOR NUCLEAR DYNAMIC AND POWER EQUATIONS
*****************************************************************
*
      E      = 1.E-6
      EN     = 1.0
      D1     = 0.0
      D2     = 0.0
      D3     = 0.0
      D4     = 0.0
      D5     = 0.0
      D6     = 0.0
*
END
*
*****************************************************************
********************************  OUTPUT BLOCK
*****************************************************************
*
*
END
```

D.2 CRAC Models

This section contains the DARE/P programs used to evaluate the CRAC reactor for the static and dynamic models:

D.2.1 Static Bubble Model

```
****************************************************************
*                                                              *
*     FISSILE SOLUTION CONTAINING GAS (ONE-REGION FEEDBACK)    *
*                                                              *
*         SYSTEM          V        EM        VL        A       *
*                                                              *
*         CRAC          0.018     21.0     8.3E-4     0.07     *
*         SILENE        0.037     43.0     8.6E-4     0.10     *
*         KEWB 5        0.024     29.0     8.3E-4     0.07     *
*                                                              *
****************************************************************
 $M11,7,3,1
*
****************************************************************
***********************  LOGICAL STATEMENTS SECTION
****************************************************************
*
 $L
*
****************************************************************
* GET OMEGA'S
****************************************************************
*
REAL*4 OMEGA(6)
DATA OMEGA / 200.0, 300.0, 500.0, 700.0, 1000.0, 1500.0 /
*
****************************************************************
* TURN OFF STORING ROUTINES
****************************************************************
*
      CALL STROF
*
****************************************************************
* SETUP DATA CONSTANTS
****************************************************************
*
      IPOW = 99
      IPRS = 98
      ISAVE = 1
*
****************************************************************
* LOOP OVER THE MAGNITUDE OF THE CONSTANT BUBBLE RADIUS
****************************************************************
*
      DO IRB = 7, 5, -1
*
****************************************************************
* LOOP OVER THE MANTISSA OF THE CONSTANT BUBBLE RADIUS
****************************************************************
*
        DO IRB2 = 1, 2
*
****************************************************************
```

```
* DETERMINE ACTUAL BUBBLE RADIUS
*****************************************************************
*
         IF ( IRB2 .EQ. 1 ) THEN
           RB = 1.0 * ( 10.0 ** (-IRB) )
         ELSE
           RB = 3.0 * ( 10.0 ** (-IRB) )
         END IF
*
*****************************************************************
* OPEN THE POWER AND PRESSURE DATA FILES AND WRITE THE
*   BUBBLE RADIUS
*****************************************************************
*
         OPEN ( UNIT=IPOW, FILE='POWER.DAT', STATUS='NEW' )
         OPEN ( UNIT=IPRS, FILE='PRESS.DAT', STATUS='NEW' )
*
*****************************************************************
* LOOP OVER THE MAGNITUDES OF THE THRESHOLD MASS FRACTION
*****************************************************************
*
         DO IXT = 6, 2, -1
*
*****************************************************************
* DETERMINE THE ACTUAL THRESHOLD MASS FRACTION
*****************************************************************
*
           XT = 1.0 * ( 10.0 ** (-IXT) )
*
*****************************************************************
* LOOP OVER ALL THE PERIODS
*****************************************************************
*
           DO IOMEGA = 1, 6, 1
*
*****************************************************************
* WRITE A MESSAGE TO THE OPERATOR
*****************************************************************
*
             WRITE(*,*)'RUN ', ISAVE, ' WITH OMEGA, XT, RB = ',
     $                  OMEGA(IOMEGA), XT, RB
*
*****************************************************************
* RUN THE SIMULATION
*****************************************************************
*
             OM = OMEGA(IOMEGA)
             CALL RESET
             CALL RUN
             WRITE(*,*)'RUN ', ISAVE, ' WITH R0 = ', R0
*
*****************************************************************
* INCREMENT THE RUN NUMBER
*****************************************************************
*
             ISAVE = ISAVE + 1
*
*****************************************************************
* WRITE THE MAXIMUM POWER AND PRESSURE DATA TO THEIR
*   RESPECTIVE FILES
*****************************************************************
*
             WRITE ( IPOW, 200 ) OMEGA(IOMEGA), POWMAX
             WRITE ( IPRS, 200 ) OMEGA(IOMEGA), PREMAX
*
*****************************************************************
* END OF PERIOD LOOP
*****************************************************************
*
           END DO
*
```

```
*****************************************************************
* WRITE THE DATA TERMINATION VALUES FOR RPLOT PACKAGE
*****************************************************************
*
            WRITE ( IPOW, 200 ) -1.0, -1.0
            WRITE ( IPRS, 200 ) -1.0, -1.0
*
*****************************************************************
* END OF THERSHOLD LOOP
*****************************************************************
*
          END DO
*
*****************************************************************
* CLOSE THE DATA FILES
*****************************************************************
*
          CLOSE ( UNIT=IPOW )
          CLOSE ( UNIT=IPRS )
*
*****************************************************************
* END THE BUBBLE LOOPS
*****************************************************************
*
        END DO
      END DO
*
*****************************************************************
* FORMAT STATEMENTS
*****************************************************************
*
200   FORMAT ( T1, G15.6, ',', T25, G15.6 )
*
*****************************************************************
**************************  OMEGA/RO TABLE BLOCK
*****************************************************************
*
 $T
      TOMEGA,6
       200.0, 1.314
       300.0, 1.473
       500.0, 1.790
       700.0, 2.106
      1000.0, 2.580
      1500.0, 3.371
*
*****************************************************************
**************************  DERIVATIVE BLOCK 1
*****************************************************************
*
 $D1
*
*****************************************************************
* NEUTRON DYNAMICS
*****************************************************************
*
      R=RO+B*T-RFB
      RFB=PHI*(V-VO)
      SUM=F1*D1+F2*D2+F3*D3+F4*D4+F5*D5+F6*D6
      ENDOT=BOL*((R-1.0)*EN+SUM-RS)
      EN.=ENDOT
      D1.=AL1*(EN-D1)
      D2.=AL2*(EN-D2)
      D3.=AL3*(EN-D3)
      D4.=AL4*(EN-D4)
      D5.=AL5*(EN-D5)
      D6.=AL6*(EN-D6)
      ENP=ENO*EN
```

```
      E.=ENP
* GAS PRODUCTION
   XE=G*E/EM
   XR=XE-XT
  PROCED X=XE,XT
    IF (XE.LT.XT) THEN
    X=0.0
    ELSE
    X=XR
    END IF
  ENDPRO
  PROCED TAU=X,THETA1,THETA2
    IF (X.LE.0.0) THEN
    TAU=THETA1
    ELSE
    TAU=THETA2
    END IF
  ENDPRO
* EQUATION OF STATE
   TABS=TEMP+273.15
   PA=P+2.0000000000*SIGMA/RB
   PB=P+1.3333333333*SIGMA/RB
   FA=X*RG*TABS
   FB=FA+VL*(1.0-X)*PA
   F=FA/FB
   BETA0=1.0E-6*(A0+A1*TEMP+A2*TEMP*TEMP+A3*TEMP*TEMP*TEMP)
   BETA=BETA0*(1.0-F)+F*PA/(TABS*PB)
   AK=AK0*(1.0-F)+F/PB
   PC=BETA/AK
* ENERGY AND STATE EQUATIONS
   V.=VDOT
   VDOT.=1.0E6*FP*A*A*(P-P0)/EM-TAU*VDOT
   TEMP.=TDOT
   TDOT=(ENP-EN0-(BETA/AK)*TABS*VDOT)/(EM*CV)
   P.=PDOT
   PDOT=(BETA*TDOT-VDOT/V)/AK
   PRESS=P-P0
   PRBOT=PRESS*2
*
*****************************************************************
* FIND THE INITIAL REACTIVITY DUE TO THE PERIOD
*****************************************************************
*
      R0 = TOMEGA(OM)
*
*****************************************************************
* FIND MAXIMUM POWER AND PRESURE
*****************************************************************
*
      PROCED PREMAX=PRBOT
         IF ( PREMAX .LT. PRBOT ) THEN
           PREMAX = PRBOT
         END IF
      ENDPRO
      PROCED POWMAX=ENP
         IF ( POWMAX .LT. ENP ) THEN
           POWMAX = ENP
         END IF
      ENDPRO
*
*****************************************************************
* RUNTIME DISPLAY COMMAND
*****************************************************************
*       DISPLAY P,ENP,PREMAX,POWMAX
END
  TMAX=0.1
```

```
  NPOINT=2501
  DTMAX=5.0E-6
  DTMIN=5.0E-8
* COEFFICIENTS IN EQUATION OF STATE
  RG=6.89E-4
  SIGMA=7.5E-8
  A0=-23.10932
  A1=13.15603
  A2=-8.6950906E-2
  A3=3.2788806E-4
  AK0=4.5E-4
* INITIAL VALUES
  P=0.1
  TEMP=20.0
  V=0.018
* SOLUTION PROPERTIES
  EM=21.0
  VL=8.3E-4
  A=0.07
  FP=4.0
  THETA1=5000.
  THETA2=5000.
  CV=3.6E-3
  P0=0.1
  G=2.0E-3
* POWER GENERATION
*   PARAMETERS
  B=0.0
  RS=0.0
  PHI=14000.
  V0=0.018
  BOL=632.6
  E=1.E-6
  EN0=0.001
  EN=1.0
  AL1=0.0124, F1=0.033, D1=0.0
  AL2=0.0305, F2=0.219, D2=0.0
  AL3=0.111 , F3=0.196, D3=0.0
  AL4=0.301 , F4=0.395, D4=0.0
  AL5=1.14  , F5=0.115, D5=0.0
  AL6=3.01  , F6=0.042, D6=0.0
*
  POWMAX = 0.0
  PREMAX = 0.0
END
*
END
```

D.2.2 Dynamic Bubble Model

```
*******************************************************************
************************  INTEGRATION RULES
*******************************************************************
*
 $M11
*
*******************************************************************
************************  LOGICAL STATEMENTS SECTION
*******************************************************************
*
 $L
*
*******************************************************************
* GET OMEGA'S
*******************************************************************
*
REAL*4 OMEGA(6)
DATA OMEGA / 200.0, 300.0, 500.0, 700.0, 1000.0, 1500.0 /
*
*******************************************************************
* TURN OFF STORING ROUTINES
*******************************************************************
*
      CALL STROF
*
*******************************************************************
* SETUP DATA CONSTANTS
*******************************************************************
*
      IPOW = 99
      IPRS = 98
      ISAVE = 1
*
*******************************************************************
* OPEN THE POWER AND PRESSURE DATA FILES
*******************************************************************
*
      OPEN ( UNIT=IPOW, FILE='POWER.DAT', STATUS='NEW' )
      OPEN ( UNIT=IPRS, FILE='PRESS.DAT', STATUS='NEW' )
*
*******************************************************************
* LOOP OVER ALL THE INVERSE PERIODS
*******************************************************************
*
      DO IOMEGA = 1, 6, 1
*
*******************************************************************
* RUN THE SIMULATION
*******************************************************************
*
           PREMAX = 0.0
           POWMAX = 0.0
           TMAXMW = 0.0
           TMAXPR = 0.0
           TNUC   = 0.0
           OM     = OMEGA(IOMEGA)
           CALL RESET
           CALL RUN
           WRITE (*,100) ISAVE, R0, OM, POWMAX, TMAXMW, PREMAX,
$            TMAXPR, TNUC
*
*******************************************************************
* INCREMENT THE RUN NUMBER
*******************************************************************
*
           ISAVE = ISAVE + 1
*
```

```
*****************************************************************
* WRITE THE MAXIMUM POWER AND PRESSURE DATA TO THEIR
*    RESPECTIVE FILES
*****************************************************************
*
           WRITE ( IPOW, 200 ) OMEGA(IOMEGA), POWMAX
           WRITE ( IPRS, 200 ) OMEGA(IOMEGA), PREMAX
*
*****************************************************************
* END OF INVERSE PERIOD LOOP
*****************************************************************
*
       END DO
*
*****************************************************************
* WRITE THE DATA TERMINATION VALUES FOR RPLOT PACKAGE
*****************************************************************
*
       WRITE ( IPOW, 200 ) -1.0, -1.0
       WRITE ( IPRS, 200 ) -1.0, -1.0
*
*****************************************************************
* CLOSE THE DATA FILES
*****************************************************************
*
       CLOSE ( UNIT=IPOW )
       CLOSE ( UNIT=IPRS )
*
*****************************************************************
* FORMAT STATEMENTS
*****************************************************************
*
100   FORMAT(' RUN = ',I3.3,T12,' R0  = ',G12.5,' OMG = ',G12.5,/,
$                           T12,' POW = ',G12.5,' TMW = ',G12.5,/,
$                           T12,' PRS = ',G12.5,' TPR = ',G12.5,/,
$                           T12,' TN  = ', G12.5)
200   FORMAT ( T1, G15.6, ',', T25, G15.6 )
*
*****************************************************************
*************************  OMEGA/R0 TABLE BLOCK
*****************************************************************
*
 $T
      TOMEGA,6
       200.0, 1.314
       300.0, 1.473
       500.0, 1.790
       700.0, 2.106
      1000.0, 2.580
      1500.0, 3.371
*
*****************************************************************
*************************  DERIVATIVE BLOCK 1
*****************************************************************
*
 $D1
*
*****************************************************************
* NEUTRON DYNAMICS
*****************************************************************
*
      R      = R0 + B*T - RFB
      RFB    = PHI * (V-V0)
      SUM    = F1*D1 + F2*D2 + F3*D3 + F4*D4 + F5*D5 + F6*D6
      ENDOT  = BOL * ( (R-1.0)*EN + SUM - RS )
      EN.    = ENDOT
      D1.    = AL1*(EN-D1)
```

```
      D2.    = AL2*(EN-D2)
      D3.    = AL3*(EN-D3)
      D4.    = AL4*(EN-D4)
      D5.    = AL5*(EN-D5)
      D6.    = AL6*(EN-D6)
      ENP    = ENO*EN
      E.     = ENP - ENO
*
*******************************************************************
* GAS AND MOMENTUM PRODUCTION
*******************************************************************
*
      XE     = G * E / EM
      XR.    = STEP * GMS * ( XE - XR- XSUPER )
      XSUPER = FP * XSAT
      PROCED TNUC,STEP,X=XE,XSUPER,XR,T
         IF ( XE .LT. XSUPER ) THEN
            X = 0.0
            STEP = 0.0
         ELSE
            X = XR
            IF ( STEP .LT. 0.5 ) THEN
               TNUC = T
            END IF
            STEP = 1.0
         END IF
      ENDPRO
*
*******************************************************************
* EQUATION OF STATE
*******************************************************************
*
      PA     = P + 2.0 * SIGMA / RB
      PB     = P + (4.0/3.0) * SIGMA / RB
      FA     = RG * TABS
      FB     = VL * PA
      F      = X * FA / FB
      RB.    = STEP * ( D * FA ) * ( XSUPER - XSAT ) / ( RB * FB )
      TEMP   = TABS - 273.15
      BETA0  = 1.0E-6*(A0+A1*TEMP+A2*TEMP*TEMP+A3*TEMP*TEMP*TEMP)
      BETA   = BETA0 * (1.0-F) + F*PA / (TABS*PB)
      AK     = AK0 * (1.0-F) + F / PB
*
*******************************************************************
* ENERGY AND STATE EQUATIONS
*******************************************************************
*
      V.     = VDOT
      VDOT.  = 1.0E6 * 4.0 * (A*A) * (P-P0) / EM - THETA*VDOT
      TABS.  = TDOT
      TDOT   = ( ENP - ENO - (BETA/AK)*TABS*VDOT ) / ( EM * CV )
      P.     = PDOT
      PDOT   = ( BETA*TDOT - VDOT/V ) / AK
      PG     = P - P0
      PRBOT  = 2*PG
*
*******************************************************************
* FIND THE MAXIMUM POWER AND PRESSURE
*******************************************************************
*
      PROCED TMAXPR,PREMAX=PRBOT,T
        IF ( PREMAX .LT. PRBOT ) THEN
           PREMAX = PRBOT
           TMAXPR = T
        END IF
      ENDPRO
```

```
*
      PROCED TMAXMW,POWMAX=ENP,T
        IF ( POWMAX .LT. ENP ) THEN
           POWMAX = ENP
           TMAXMW = T
        END IF
      ENDPRO
*
*****************************************************************
* FIND THE INITIAL REACTIVITY DUE TO THE INVERSE PERIOD
*****************************************************************
*
      RO = TOMEGA(OM)
*
*****************************************************************
* RUNTIME DISPLAY COMMAND
*****************************************************************
*
*      DISPLAY ENP, PRBOT, EXDATA, X
*
END
*****************************************************************
************************  CONSTANT AND INITIAL CONDITION SECTION
*****************************************************************
*
      TMAX  = 0.1
      NPOINT= 2501
      DTMAX = 5.0E-6
      DTMIN = 5.0E-8
*
*****************************************************************
* COEFFICIENTS AND CONSTANTS IN THE EQUATION OF STATE
*****************************************************************
*
      RG    = 6.89E-4
      SIGMA = 7.5E-8
      A0    = -23.10932
      A1    = 13.15603
      A2    = -8.6950906E-2
      A3    = 3.2788806E-4
      AK0   = 4.5E-4
*
*****************************************************************
* INITIAL VALUES FOR THE EQUATIONS OF STATE
*****************************************************************
*
      TABS  = 20.0 + 273.15
      RB    = 5.0E-8
      VDOT  = 0.0
      V     = 0.018
      P     = 0.1
      XR    = 0.0
*
*****************************************************************
* SOLUTION AND REACTOR VESSEL PROPERTIES
*****************************************************************
*
      EM     = 21.0
      VL     = 8.3E-4
      A      = 0.07
      CV     = 3.6E-3
      P0     = 0.1
      G      = 2.0E-3
      V0     = 0.018
      XSAT   = 1E-5
      D      = 0.5E-8
      FP     = 7.5
      GMS    = 30.0
*
*****************************************************************
* CONSTANTS FOR NUCLEAR DYNAMIC AND POWER EQUATIONS
*****************************************************************
```

```
*
       RS    = 0.0
       B     = 0.0
       BOL   = 632.6
       EN0   = 0.001
       AL1   = 0.0124 , F1 = 0.033
       AL2   = 0.0305 , F2 = 0.219
       AL3   = 0.111  , F3 = 0.196
       AL4   = 0.301  , F4 = 0.395
       AL5   = 1.14   , F5 = 0.115
       AL6   = 3.01   , F6 = 0.042
*
*****************************************************************
* INITIAL VALUES FOR NUCLEAR DYNAMIC AND POWER EQUATIONS
*****************************************************************
*
       PHI   = 10900.0
       THETA = 3200.0
       E     = 1.E-6
       EN    = 1.0
       D1    = 0.0
       D2    = 0.0
       D3    = 0.0
       D4    = 0.0
       D5    = 0.0
       D6    = 0.0
*
END
*
*****************************************************************
********************************  OUTPUT BLOCK
*****************************************************************
*
*
END
```

D.3 SILENE Models

This section contains the DARE/P programs used to evaluate the SILENE reactor for the static and dynamic models:

D.3.1 Static Bubble Model

```
******************************************************************
*                                                                *
*     FISSILE SOLUTION CONTAINING GAS (ONE-REGION FEEDBACK)      *
*                                                                *
*         SYSTEM          V        EM        VL        A         *
*                                                                *
*         CRAC           0.018    21.0     8.3E-4     0.07       *
*         SILENE         0.037    43.0     8.6E-4     0.10       *
*         KEWB 5         0.024    29.0     8.3E-4     0.07       *
*                                                                *
******************************************************************
*
 $M11,7,3,1
*
******************************************************************
************************ LOGICAL STATEMENTS SECTION
******************************************************************
*
 $L
*
******************************************************************
* GET OMEGA'S
******************************************************************
*
REAL*4 OMEGA(7)
DATA OMEGA / 150.0, 200.0, 300.0, 500.0, 700.0, 1000.0 /
*
******************************************************************
* TURN OFF STORING ROUTINES
******************************************************************
*
      CALL STROF
*
******************************************************************
* SETUP DATA CONSTANTS
******************************************************************
*
      IPOW = 99
      IPRS = 98
      ISAVE = 1
*
******************************************************************
* LOOP OVER THE MAGNITUDE OF THE CONSTANT BUBBLE RADIUS
******************************************************************
*
      DO IRB = 7, 5, -1
*
******************************************************************
* LOOP OVER THE MANTISSA OF THE CONSTANT BUBBLE RADIUS
******************************************************************
*
        DO IRB2 = 1, 2
*
```

```
*****************************************************************
* DETERMINE ACTUAL BUBBLE RADIUS
*****************************************************************
*
          IF ( IRB2 .EQ. 1 ) THEN
            RB = 1.0 * ( 10.0 ** (-IRB) )
          ELSE
            RB = 3.0 * ( 10.0 ** (-IRB) )
          END IF
*
*****************************************************************
* OPEN THE POWER AND PRESSURE DATA FILES AND WRITE THE
*    BUBBLE RADIUS
*****************************************************************
*
          OPEN ( UNIT=IPOW, FILE='POWER.DAT', STATUS='NEW' )
          OPEN ( UNIT=IPRS, FILE='PRESS.DAT', STATUS='NEW' )
*
*****************************************************************
* LOOP OVER THE MAGNITUDES OF THE THRESHOLD MASS FRACTION
*****************************************************************
*
          DO IXT = 6, 2, -1
*
*****************************************************************
* DETERMINE THE ACTUAL THRESHOLD MASS FRACTION
*****************************************************************
*
            XT = 1.0 * ( 10.0 ** (-IXT) )
*
*****************************************************************
* LOOP OVER ALL THE PERIODS
*****************************************************************
*
            DO IOMEGA = 1, 7, 1
*
*****************************************************************
* WRITE A MESSAGE TO THE OPERATOR
*****************************************************************
*
              WRITE(*,*)'RUN ', ISAVE, ' WITH OMEGA, XT, RB = ',
     $                   OMEGA(IOMEGA), XT, RB
*
*****************************************************************
* RUN THE SIMULATION
*****************************************************************
*
              OM = OMEGA(IOMEGA)
              CALL RESET
              CALL RUN
              WRITE(*,*)'RUN ', ISAVE, ' WITH RO = ', RO
*
*****************************************************************
* INCREMENT THE RUN NUMBER
*****************************************************************
*
              ISAVE = ISAVE + 1
*
*****************************************************************
* WRITE THE MAXIMUM POWER AND PRESSURE DATA TO THEIR
*    RESPECTIVE FILES
*****************************************************************
*
              WRITE ( IPOW, 200 ) OMEGA(IOMEGA), POWMAX
              WRITE ( IPRS, 200 ) OMEGA(IOMEGA), PREMAX
*
*****************************************************************
* END OF PERIOD LOOP
*****************************************************************
*
            END DO
```

```
*
*****************************************************************
* WRITE THE DATA TERMINATION VALUES FOR RPLOT PACKAGE
*****************************************************************
*
            WRITE ( IPOW, 200 ) -1.0, -1.0
            WRITE ( IPRS, 200 ) -1.0, -1.0
*
*****************************************************************
* END OF THERSHOLD LOOP
*****************************************************************
*
          END DO
*
*****************************************************************
* CLOSE THE DATA FILES
*****************************************************************
*
          CLOSE ( UNIT=IPOW )
          CLOSE ( UNIT=IPRS )
*
*****************************************************************
* END THE BUBBLE LOOPS
*****************************************************************
*
        END DO
      END DO
*
*****************************************************************
* FORMAT STATEMENTS
*****************************************************************
*
200   FORMAT ( T1, G15.6, ',', T25, G15.6 )
*
*****************************************************************
**************************  OMEGA/RO TABLE BLOCK
*****************************************************************
*
 $T
      TOMEGA,7
       100.0, 1.396
       150.0, 1.597
       200.0, 1.798
       300.0, 2.199
       500.0, 2.999
       700.0, 3.799
      1000.0, 5.000
*
*****************************************************************
**************************  DERIVATIVE BLOCK 1
*****************************************************************
*
 $D1
*
*****************************************************************
* NEUTRON DYNAMICS
*****************************************************************
*
      R=R0+B*T-RFB
      RFB=PHI*(V-V0)
      SUM=F1*D1+F2*D2+F3*D3+F4*D4+F5*D5+F6*D6
      ENDOT=BOL*((R-1.0)*EN+SUM-RS)
      EN.=ENDOT
      D1.=AL1*(EN-D1)
      D2.=AL2*(EN-D2)
      D3.=AL3*(EN-D3)
      D4.=AL4*(EN-D4)
      D5.=AL5*(EN-D5)
```

```
      D6.=AL6*(EN-D6)
      ENP=ENO*EN
      E.=ENP
* GAS PRODUCTION
   XE=G*E/EM
   XR=XE-XT
  PROCED X=XE,XT
    IF (XE.LT.XT) THEN
    X=0.0
    ELSE
    X=XR
    END IF
  ENDPRO
  PROCED TAU=X,THETA1,THETA2
    IF (X.LE.0.0) THEN
    TAU=THETA1
    ELSE
    TAU=THETA2
    END IF
  ENDPRO
* EQUATION OF STATE
   TABS=TEMP+273.15
   PA=P+2.0000000000*SIGMA/RB
   PB=P+1.3333333333*SIGMA/RB
   FA=X*RG*TABS
   FB=FA+VL*(1.0-X)*PA
   F=FA/FB
   BETA0=1.0E-6*(A0+A1*TEMP+A2*TEMP*TEMP+A3*TEMP*TEMP*TEMP)
   BETA=BETA0*(1.0-F)+F*PA/(TABS*PB)
   AK=AK0*(1.0-F)+F/PB
   PC=BETA/AK
* ENERGY AND STATE EQUATIONS
   V.=VDOT
   VDOT.=VDDOT
   VDDOT=1.0E6*FP*A*A*(P-P0)/EM-TAU*VDOT
   TEMP.=TDOT
   TDOT=(ENP-ENO-(BETA/AK)*TABS*VDOT)/(EM*CV)
   P.=PDOT
   PDOT=(BETA*TDOT-VDOT/V)/AK
   PRESS=P-P0
   PRBOT=PRESS*2
*
****************************************************************
* FIND THE INITIAL REACTIVITY DUE TO THE PERIOD
****************************************************************
*
      R0 = TOMEGA(OM)
*
****************************************************************
* FIND MAXIMUM POWER AND PRESURE
****************************************************************
*
      PROCED PREMAX=PRBOT
         IF ( PREMAX .LT. PRBOT ) THEN
           PREMAX = PRBOT
         END IF
      ENDPRO
      PROCED POWMAX=ENP
         IF ( POWMAX .LT. ENP ) THEN
           POWMAX = ENP
         END IF
      ENDPRO
*
****************************************************************
* RUNTIME DISPLAY COMMAND
****************************************************************
```

```
*       DISPLAY P,ENP,PREMAX,POWMAX
END
  TMAX=0.1
  NPOINT=2501
  DTMAX=5.0E-6
  DTMIN=5.0E-8
* COEFFICIENTS IN EQUATION OF STATE
  RG=6.89E-4
  SIGMA=7.5E-8
  A0=-23.10932
  A1=13.15603
  A2=-8.6950906E-2
  A3=3.2788806E-4
  AK0=4.5E-4
* INITIAL VALUES
  P=0.1
  TEMP=20.0
  V=0.037
* SOLUTION PROPERTIES
  EM=43.0
  VL=8.6E-4
  A=0.10
  FP=4.0
  THETA1=2700.
  THETA2=2700.
  CV=3.4E-3
  P0=0.1
  G=2.0E-3
* POWER GENERATION
*   PARAMETERS
      B=0.0
      RS=0.0
      PHI=5600.
      V0=0.037
      BOL=250.0
      E = 1.0E-6
      EN=1.
      EN0 = 0.001
      AL1=0.0124, F1=0.033, D1=0.0
      AL2=0.0305, F2=0.219, D2=0.0
      AL3=0.111 , F3=0.196, D3=0.0
      AL4=0.301 , F4=0.395, D4=0.0
      AL5=1.14  , F5=0.115, D5=0.0
      AL6=3.01  , F6=0.042, D6=0.0
*
      PREMAX = 0.0
      POWMAX = 0.0
*
END
*
END
```

D.3.2 Dynamic Bubble Model

```
*********************************************************************
***********************  INTEGRATION RULES
*********************************************************************
*
 $M11
*
*********************************************************************
***********************  LOGICAL STATEMENTS SECTION
*********************************************************************
*
 $L
*
*********************************************************************
* GET OMEGA'S
*********************************************************************
*
REAL*4 OMEGA(6)
DATA OMEGA / 150.0, 200.0, 300.0, 500.0, 700.0, 1000.0 /
*
*********************************************************************
* TURN OFF STORING ROUTINES
*********************************************************************
*
      CALL STROF
*
*********************************************************************
* SETUP DATA CONSTANTS
*********************************************************************
*
      IPOW = 99
      IPRS = 98
      ISAVE = 1
*
*********************************************************************
* OPEN THE POWER AND PRESSURE DATA FILES
*********************************************************************
*
      OPEN ( UNIT=IPOW, FILE='POWER.DAT', STATUS='NEW' )
      OPEN ( UNIT=IPRS, FILE='PRESS.DAT', STATUS='NEW' )
*
*********************************************************************
* LOOP OVER ALL THE INVERSE PERIODS
*********************************************************************
*
      DO IOMEGA = 1, 6, 1
*
*********************************************************************
* RUN THE SIMULATION
*********************************************************************
*
           PREMAX = 0.0
           POWMAX = 0.0
           TMAXMW = 0.0
           TMAXPR = 0.0
           TNUC   = 0.0
           OM     = OMEGA(IOMEGA)
           CALL RESET
           CALL RUN
           WRITE (*,100) ISAVE, RO, OM, POWMAX, TMAXMW, PREMAX,
$            TMAXPR, TNUC
*
*********************************************************************
* INCREMENT THE RUN NUMBER
*********************************************************************
*
           ISAVE = ISAVE + 1
*
```

```
*****************************************************************
* WRITE THE MAXIMUM POWER AND PRESSURE DATA TO THEIR
*     RESPECTIVE FILES
*****************************************************************
*
            WRITE ( IPOW, 200 ) OMEGA(IOMEGA), POWMAX
            WRITE ( IPRS, 200 ) OMEGA(IOMEGA), PREMAX
*
*****************************************************************
* END OF INVERSE PERIOD LOOP
*****************************************************************
*
        END DO
*
*****************************************************************
* WRITE THE DATA TERMINATION VALUES FOR RPLOT PACKAGE
*****************************************************************
*
        WRITE ( IPOW, 200 ) -1.0, -1.0
        WRITE ( IPRS, 200 ) -1.0, -1.0
*
*****************************************************************
* CLOSE THE DATA FILES
*****************************************************************
*
        CLOSE ( UNIT=IPOW )
        CLOSE ( UNIT=IPRS )
*
*****************************************************************
* FORMAT STATEMENTS
*****************************************************************
*
100   FORMAT(' RUN = ',I3.3,T12,' RO  = ',G12.5,' OMG = ',G12.5,/,
$                          T12,' POW = ',G12.5,' TMW = ',G12.5,/,
$                          T12,' PRS = ',G12.5,' TPR = ',G12.5,/,
$                          T12,' TN  = ', G12.5)
200   FORMAT ( T1, G15.6, ',', T25, G15.6 )
*
*****************************************************************
*************************  OMEGA/RO TABLE BLOCK
*****************************************************************
*
 $T
      TOMEGA,7
       100.0, 1.396
       150.0, 1.597
       200.0, 1.798
       300.0, 2.199
       500.0, 2.999
       700.0, 3.799
      1000.0, 5.000
*
*****************************************************************
*************************  DERIVATIVE BLOCK 1
*****************************************************************
*
 $D1
*
*****************************************************************
* NEUTRON DYNAMICS
*****************************************************************
*
      R      = RO + B*T - RFB
      RFB    = PHI * (V-VO)
      SUM    = F1*D1 + F2*D2 + F3*D3 + F4*D4 + F5*D5 + F6*D6
      ENDOT  = BOL * ( (R-1.0)*EN + SUM - RS )
      EN.    = ENDOT
```

```
      D1.    = AL1*(EN-D1)
      D2.    = AL2*(EN-D2)
      D3.    = AL3*(EN-D3)
      D4.    = AL4*(EN-D4)
      D5.    = AL5*(EN-D5)
      D6.    = AL6*(EN-D6)
      ENP    = ENO*EN
      E.     = ENP - ENO
*
*****************************************************************
* GAS AND MOMENTUM PRODUCTION
*****************************************************************
*
      XE     = G * E / EM
      XR.    = STEP * GMS * ( XE - XR- XSUPER )
      XSUPER= FP * XSAT
      PROCED STEP,TNUC,X=XE,XSUPER,XR,T
         IF (XE.LT.XSUPER) THEN
            X = 0.0
            STEP = 0.0
         ELSE
            X = XR
            IF ( STEP .LT. 0.5 ) THEN
               TNUC = T
            END IF
            STEP = 1.0
         END IF
      ENDPRO
*
*****************************************************************
* EQUATION OF STATE
*****************************************************************
*
      PA     = P + 2.0 * SIGMA / RB
      PB     = P + (4.0/3.0) * SIGMA / RB
      FA     = RG * TABS
      FB     = VL * PA
      F      = X * FA / FB
      RB.    = STEP * ( D * FA ) * ( XSUPER - XSAT ) / ( RB * FB )
      TEMP   = TABS - 273.15
      BETAO  = 1.0E-6*(A0+A1*TEMP+A2*TEMP*TEMP+A3*TEMP*TEMP*TEMP)
      BETA   = BETAO * (1.0-F) + F*PA / (TABS*PB)
      AK     = AKO * (1.0-F) + F / PB
*
*****************************************************************
* ENERGY AND STATE EQUATIONS
*****************************************************************
*
      V.     = VDOT
      VDOT.  = 1.0E6 * 4.0 * (A*A) * (P-PO) / EM - THETA*VDOT
      TABS.  = TDOT
      TDOT   = ( ENP - ENO - (BETA/AK)*TABS*VDOT ) / ( EM * CV )
      P.     = PDOT
      PDOT   = ( BETA*TDOT - VDOT/V ) / AK
      PG     = P - PO
      PRBOT  = 2*PG
*
*****************************************************************
* FIND MAXIMUM POWER AND PRESSURE
*****************************************************************
*
      PROCED TMAXPR,PREMAX=PRBOT,T
        IF ( PREMAX .LT. PRBOT ) THEN
           PREMAX = PRBOT
           TMAXPR = T
        END IF
```

```
      ENDPRO
*
      PROCED TMAXMW,POWMAX=ENP,T
        IF ( POWMAX .LT. ENP ) THEN
           POWMAX = ENP
           TMAXMW = T
        END IF
      ENDPRO
*
*****************************************************************
* FIND THE INITIAL REACTIVITY DUE TO THE INVERSE PERIOD
*****************************************************************
*
      RO = TOMEGA(OM)
*
*****************************************************************
* RUNTIME DISPLAY COMMAND
*****************************************************************
*
*      DISPLAY ENP, PRBOT, EXDATA, X
*
END
*****************************************************************
**********************  CONSTANT AND INITIAL CONDITION SECTION
*****************************************************************
*
      TMAX  = 0.1
      NPOINT= 2501
      DTMAX = 5.0E-6
      DTMIN = 5.0E-8
*
*****************************************************************
* COEFFICIENTS AND CONSTANTS IN THE EQUATION OF STATE
*****************************************************************
*
      RG    = 6.89E-4
      SIGMA = 7.5E-8
      A0    = -23.10932
      A1    = 13.15603
      A2    = -8.6950906E-2
      A3    = 3.2788806E-4
      AKO   = 4.5E-4
*
*****************************************************************
* INITIAL VALUES FOR THE EQUATIONS OF STATE
*****************************************************************
*
      TABS  = 20.0 + 273.15
      RB    = 5.0E-8
      VDOT  = 0.0
      V     = 0.037
      P     = 0.1
      XR    = 0.0
*
*****************************************************************
* SOLUTION AND REACTOR VESSEL PROPERTIES
*****************************************************************
*
      EM     = 43.0
      VL     = 8.6E-4
      A      = 0.10
      CV     = 3.4E-3
      PO     = 0.1
      G      = 2.0E-3
      VO     = 0.037
      FP     = 16.0
      GMS    = 12.5
      XSAT   = 1.0E-5
      D      = 0.5E-8
*
*****************************************************************
* CONSTANTS FOR NUCLEAR DYNAMIC AND POWER EQUATIONS
```

```
*****************************************************************
*
      RS    = 0.0
      B     = 0.0
      BOL   = 250.0
      ENO   = 0.001
      AL1   = 0.0124 , F1 = 0.033
      AL2   = 0.0305 , F2 = 0.219
      AL3   = 0.111  , F3 = 0.196
      AL4   = 0.301  , F4 = 0.395
      AL5   = 1.14   , F5 = 0.115
      AL6   = 3.01   , F6 = 0.042
*
*****************************************************************
* INITIAL VALUES FOR NUCLEAR DYNAMIC AND POWER EQUATIONS
*****************************************************************
*
      PHI   = 6140.0
      THETA = 2590.0
      E     = 1.E-6
      EN    = 1.0
      D1    = 0.0
      D2    = 0.0
      D3    = 0.0
      D4    = 0.0
      D5    = 0.0
      D6    = 0.0
*
END
*
*****************************************************************
*******************************  OUTPUT BLOCK
*****************************************************************
*
*
END
```

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