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Dynamic response of structures with geometrically softening components including foundation interaction

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DYNAMIC RESPONSE OF STRUCTURES WITH GEOMETRICALLY SOFTENING COMPONENTS INCLUDING FOUNDATION INTERACTION

by

NICOLE COLETTE STAPLETON - HART

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TO

QUINN

and

MY PARENTS

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ABSTRCT

The effects of material nonlinearity and geometric instabilities of a bracing system of a dynamically loaded steel frame, as well as nonlinear soil - structure interaction are studied in this thesis. To investigate the response of a frame subjected to severe dynamic loads, a model of the force - displacement relationship of the bracing system is developed to account for the inelastic, post - buckling behavior of a brace subjected to strong compressive loads. This thesis also develops a model of the force - deformation relationship of the foundation - soil interface, confining the study only to the slide mode of failure. From these models, a softening behavior for the bracing system and a hyperbolic load-deformation behavior of the interface are proposed. It is shown that the structural response of a frame is significantly affected when the analysis includes both the effects of softening of the bracing system and soil - structure interaction.

1. INTRODUCTION

The design of large structural buildings in seismically active areas is of great concern to engineers. Structures in these areas must withstand large lateral loads developed by seismic activity. For this purpose, certain components such as the use of shear walls in concrete structures and moment - resisting frames in steel structures are introduced to enhance the resistance to these loads. Another popular measure to counteract these forces for steel frames in seismic areas is the use of diagonal bracing elements. It has been found that the use of braces in moment - resisting frames is both economical and functional. Part of this thesis will concentrate on the use of bracing systems in steel buildings and the structural response of a brace due to large lateral loads.

In general, previous analyses have been elastic with solutions in the frequency domain, yet studies have found that structures behave inelastically during strong ground motions. Consequently, the nonlinear behavior of the bracing system is of primary concern in the dynamic analysis of structures. If the braces in the structure are introduced to minimize displacements, they should be examined as both tension and compression members. In previous studies, braces in structures have been examined under various assumptions. For some cases, they have been assumed to be linearly elastic truss members, as well as linearly elastic wires, and at other times as elastic perfectly - plastic bars or wires. Experimental work has also been conducted to observe the behavior of braces under both static and cyclic loading. Both material nonlinearities (elastoplastic behavior) and geometric nonlinearities (buckling and post - buckling behavior) have been examined.

However, failure of a member in a structure may not imply global structural failure. This fact has been recognized by the engineering community and has been examined in the academic environment, yet its implementation in practice has been delayed due to the theoretical and computational difficulties.

Another important aspect in the structural analysis of buildings is soil - structure interaction. Many designs neglect this effect and assume the structure to rest on a rigid foundation; yet the behavior of the foundation is greatly influenced by the behavior of the soil beneath it. Therefore, it is of importance that the structure and the foundation be treated as an interacting system during the design process, especially for structures subjected to large ground motions as seen in severe earthquakes.

The purpose of this thesis is to develop a simple method to account for the effects of material nonlinearity and geometric instabilities of the bracing system as well as nonlinear soil - structure interaction on a dynamically loaded structure . The value of this work is largely qualitative, although for certain problems its quantitative value is as important, as will be discussed later.

To develop the method presented here, the response of braced structures is based on both experimental observations and theoretical aspects of the axially loaded braces and foundation - soil interfaces.

2. SCOPE AND OBJECTIVES

The objective of this thesis is to study the effects of partial structural failure and soil - structure interaction on a dynamically loaded braced steel structure by the means of a reasonably accurate yet simple method. This method is implemented in a FORTRAN 77 program and computations are performed on a personal computer. The analysis features nonlinear force - displacement relationship for both the structural response and the interaction of the structure with the soil.

The above problems are difficult and have been investigated separately by many researchers. The scope of this study includes a simplified analysis where the structure is modeled by an assembly of masses and nonlinear springs. Only the lateral deflection is considered significant during seismic loading. In summary, this thesis includes:

1. The development of a simple model to study the effects of structural member failures and soil - structure interaction which includes the proper selection of the stiffness relationship for both the bracing system and the soil.
2. The development of a computer program to implement the above model.
3. The interpretation and evaluation of the results obtained.

The presentation of this work is structured as follows:

* Chapter Three presents a general review of studies that have examined either some type of structural response due to dynamic loading or the interface between soil and foundations for soil - structure interaction problems.
* Chapter Four discusses the general dynamic equation of motion emphasizing Multiple Degree of Freedom systems. The analysis includes structures subjected to base movements (earthquake loads, for example) and the Newmark Time

Integration scheme used to obtain the solution for nonlinear systems.

* Chapter Five presents the failure mechanism of braces and its effect on a structural cell (here, a portal frame). It also presents the interface modeling between the structure foundation and the underlying soil.
* Chapter Six first steps through a specific example of a one story frame to show the details on how the model is developed. These details include obtaining a relationship for the nonlinearity of both the bracing system and the soil. The frame is then subjected to a severe earthquake. Comparisons between elastic, elastic perfectly - plastic, and softening solutions are made both with and without soil - structure interaction. Next, the response of a ten story frame subjected to the same earthquake is presented.
* Chapter Seven discusses specific solutions obtained from the multistory frame and from the overall analysis. Finally, conclusions are summarized as to the specific assumptions and restrictions used in this thesis and suggestions for future improvements are made.
* The Appendix contains the program that was developed and the user manual for its operation.

3. LITERATURE REVIEW

This chapter is divided into two sections. The first section discusses the importance of obtaining the structural response of a frame subjected to strong earthquake loads, and then presents some specifics necessary to examine this response. The second section presents an overview of the importance of considering soil - structure interaction and lists several references for further information on the specifics of this concept.

3.1 Structural Response

The structural response of buildings during strong earthquakes is of great interest to engineers. It is well known that structures subjected to these large lateral loads enter inelastic deformations; therefore, their design must allow for both elastic and inelastic response. One common mode of analysis for nonlinear systems is the use of a numerical step - by - step integration scheme. Details of specific schemes are given in [1,3,6]. Although the results are fairly accurate, the rigorous computations that arise from this method have caused researchers to use simpler techniques, such as the use of an inelastic response spectrum, [15,21].

Another important consequence from the induced ground motions is the excessive deformations a frame undergoes which may lead not only to individual member failures, but the failure of the structure as a whole. One method to reduce these displacements is to introduce dampers to a system, [22], and another, more popular, method is to introduce braces to a system. Several studies have emphasized the importance of including braces to structure to help develop strength and stiffness necessary to resist large lateral forces caused by severe earthquakes [1,6,7,10,12,13].

A significant amount of experimental work has been performed to investigate the behavior of a brace under an axially applied cyclic load ([7,10,12,13]). A few of these studies have then developed a model of the response of the brace from the experimental data that was obtained in the given work, see Figures (3.1) and (3.2).

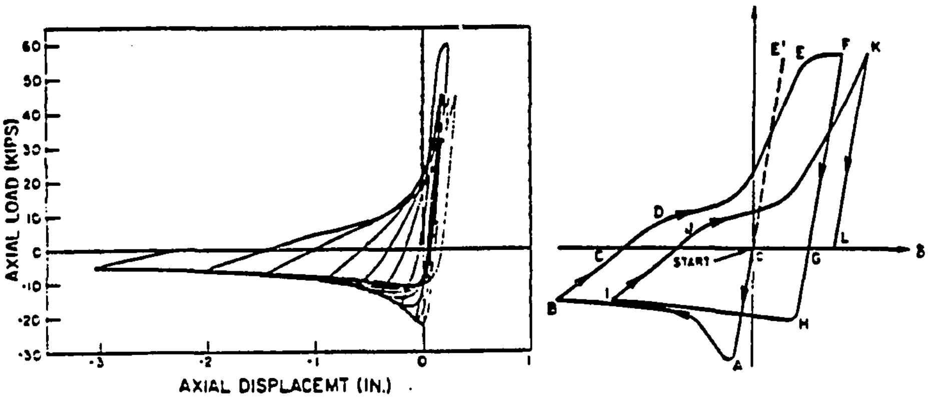

Figure 3.1: Hysteretic Loops for Brace - Actual and Idealized [10]

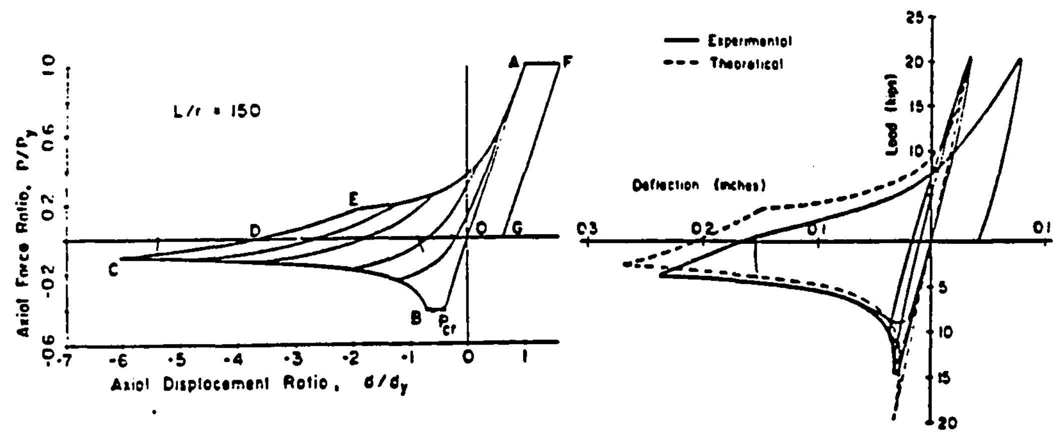

Figure 3.2: Theoretical Behavior of Brace and Comparison of Experimental and Theoretical Axial Load - Deflection Response of Brace [7]

There are several important results that were observed from the experimental work performed. First, it is found that the buckling load for a brace is less than the assumed Euler critical buckling load. This can be explained by the fact that structural members as well as the loading environment these members are subjected to, violate the assumptions used in determining the Euler load, such as a member must be of a truly homogeneous material and perfectly straight, and the applied load must be central and perfectly axial. Another important observation pertains to the response of the brace after elastic buckling. It was found that after the member buckles, deformations increase with no change in load until plastic yielding of the material occurs. Upon initiation of plastic deformation, the axial force decreases with an increase in displacement. The point of plastic yielding can be seen in figure (3.2a) as point B. From figures (3.1a) and (3.2b), it is also noticed that elastic reloading is evident with small deformations in compression, yet as these deformations increase, significant deterioration to the internal structure of the material occurs, and reloading becomes inelastic. Finally, these large displacements cause the tensile yield load and the compressive buckling load to decrease upon successive cycles.

There also exist several studies that have modeled theoretically the behavior of a brace subjected to dynamic (cyclic) loads. One common assumption used is the force - displacement relationship of the brace has an elastic perfectly - plastic response; however, these braces vary from being only tension members [6], to both tension and compression members [21], to a combination of both, specifically, the brace is modeled to be a truss member in tension, yet in compression, once the critical load is obtained, it is seen to displace freely (figure (3.3)), [1]. Other authors assume an elastoplastic force - displacement relationship, [15].

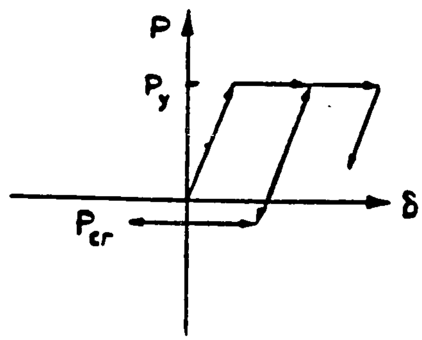

Figure 3.3: Hysteresis Characteristic of Truss Element [1]

The above models and studies are only a few examples pertaining to nonlinear analysis of structural members and seismic analysis of buildings. More studies in specified areas such as nonlinear analysis of concrete structures can be found in related journals and proceedings of conferences.

3.2 Soil - Structure Interaction

In all of the previously mentioned studies, the structural foundation was assumed to be rigidly connected to its base. However, soil is not rigid and the response of the soil - structure interface is a very serious interaction problem. The interface behavior is very complex for there are several modes of failure to consider - stick or no slip mode, slip or sliding mode, and separation or debonding mode. For the simplified analysis presented here, only the slide mode is considered significant, while the effects of actual separation are ignored. The interface problem is often modeled within the context of FEM by the use of an interface element called the "thin - layer" element. The thin-layer is a finite element with a very large length to width ratio. It is intended to model the behavior of a thin smeared interface zone that is formed between two bodies in contact. Its constitutive behavior is modeled by:

$$\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{Bmatrix} \tau \\ \sigma \end{Bmatrix}$$

where

D_{ij} are the constitutive matrix coefficients

u is the horizontal displacement of the "thin - layer"

v is the vertical displacement of the "thin - layer"

τ is the shear stress of the element

σ is the normal stress of the element

For the purpose of this study, the above relation is reduced to $K\ u = \tau$, thereby ignoring the effects of vertical interface displacement. The literature on this subject as well as the other above mentioned modes is very wide and cannot be covered in this thesis. Important studies on the field include [5,8,9,18,19,20] and may be referenced for further information.

4. FUNDAMENTALS OF THE THEORY OF DYNAMIC ANALYSIS

4.1 Equation of Motion

4.1.1 Structures Subjected to Externally Applied Loads

The Single Degree of Freedom (SDOF) equation of motion at time t can be expressed by:

$$m\ddot{u} + c\dot{u} + ku = p(t) \tag{4.1}$$

where m is the mass, c is the damping, k is the stiffness, p(t) is the externally applied load of the system and the initial conditions u_o and $\dot{u}_o$ are given. The equations of motion for a Multiple Degree of Freedom (MDOF) system can be expressed similarly by the semi - discrete equation:

$$\mathbf{M}\,\ddot{\mathbf{u}} + \mathbf{C}\,\dot{\mathbf{u}} + \mathbf{K}\,\mathbf{u} = \mathbf{p(t)} \tag{4.2}$$

where **M** is the mass matrix, **C** is the damping matrix, **K** is the stiffness matrix, $\ddot{\mathbf{u}}$, $\dot{\mathbf{u}}$, **u** are vectors for acceleration, velocity and displacement, respectively, and **p(t)** is a vector of the loading function with respect to time. Equation (4.2) can be developed for continuous systems using some discretization techniques such as FEM, or using simpler lumped schemes. To illustrate the development of (4.2), consider the dynamic system in figure (4.1a). Each node in figure (4.1a) follows its own independent movement described by u_i (i = 1,2,3); therefore, there are three degrees of freedom. At an arbitrary position, it is assumed that all displacements, velocities, accelerations, and external loads are positive as shown and that all springs are under tension. The free body diagrams of the three members are shown in Figure (4.1b).

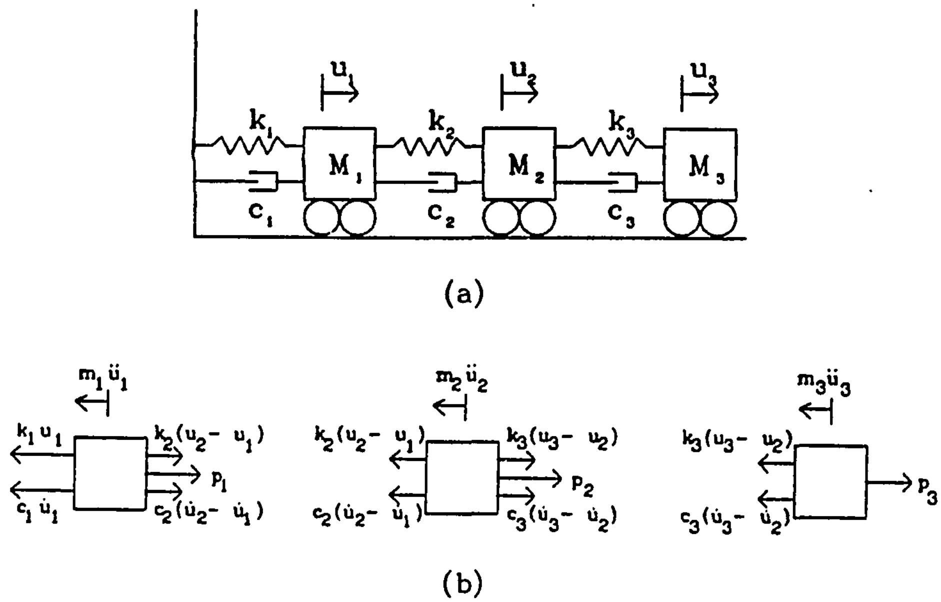

Figure 4.1: Multiple Degree of Freedom Dynamic System

The dynamic equilibrium of each system can be expressed by:

$$m_1 \ddot{u}_1 + c_1 \dot{u}_1 - c_2 (\dot{u}_2 - \dot{u}_1) + k_1 u_1 - k_2 (u_2 - u_1) - p_1 = 0$$

$$m_2 \ddot{u}_2 + c_2 (\dot{u}_2 - \dot{u}_1) - c_3 (\dot{u}_3 - \dot{u}_2) + k_2 (u_2 - u_1) - k_3 (u_3 - u_2) - p_2 = 0 \tag{4.3}$$

$$m_3 \ddot{u}_3 + c_3 (\dot{u}_3 - \dot{u}_2) + k_3 (u_3 - u_2) - p_3 = 0$$

Rearrangement of terms of equation (3) and expressing them in matrix form will give:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 & 0 \\ -c_2 & c_2+c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{u}_3 \end{Bmatrix}$$

$$+ \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} p_1 \\ p_2 \\ p_3 \end{Bmatrix} \tag{4.4}$$

or

$$\mathbf{M} \, \ddot{\mathbf{u}} + \mathbf{C} \, \dot{\mathbf{u}} + \mathbf{K} \, \mathbf{u} = \mathbf{p} \tag{4.5}$$

4.1.2 Structures Subjected to Base Movement

Structures subjected to base movements such as earthquake loads are analyzed in a similar manner as seen above with a few modifications. It can easily be shown that the equation of motion changes to:

$$\mathbf{M}\,\ddot{\mathbf{u}} + \mathbf{C}\,(\dot{\mathbf{u}} - \mathbf{I}\,\dot{\mathbf{a}}) + \mathbf{K}\,(\mathbf{u} - \mathbf{I}\,\mathbf{a}) = 0 \tag{4.6}$$

where a, $\dot{a}$, $\ddot{a}$, are the base displacement, velocity, and acceleration respectively, and **I** is the identity vector: $\mathbf{I}^T = [1, 1, 1, ..., 1]$. Now, setting $\mathbf{U} = \mathbf{u} - \mathbf{I}a$, $\dot{\mathbf{U}} = \dot{\mathbf{u}} - \mathbf{I}\dot{a}$, $\ddot{\mathbf{U}} = \ddot{\mathbf{u}} - \mathbf{I}\ddot{a}$, the new equation of motion will be

$$\mathbf{M}\,\ddot{\mathbf{U}} + \mathbf{C}\,\dot{\mathbf{U}} + \mathbf{K}\,\mathbf{U} = -\mathbf{M}\,\mathbf{I}\,\ddot{a} \tag{4.7}$$

where $-\mathbf{M}\,\mathbf{I}\,\ddot{a}$ is taken as the loading function with respect to time.

For a general dynamic system of n degrees of freedom, it is not always convenient to carry out the complete analysis described above to develop the **M**, **C**, **K** matrices. The mass matrix in systems of lumped masses is always diagonal, and its formulation does not need much explanation. The damping matrix **C** and the stiffness matrix **K** also can be easily developed using the direct stiffness method if we consider the physical meaning of the generic stiffness element K_{ij} : *The stiffness element K_{ij} is defined as the force developed in the i^{th} degree of freedom due to a unit displacement of the j^{th} degree of freedom, while all other degrees of freedom are restricted from movement.*

Equation (4.7) can be solved analytically to obtain exact expressions for the displacement, velocity and acceleration functions for a limited number of simple problems. However, structures with nonlinear spring or damper responses or those subjected to a complex loading function p(t) do not have an easy analytical solution. This thesis concentrates on a step-by-step integration technique attributed to Newmark [11] to solve for systems with nonlinear spring and/or damping responses and time dependent loading functions.

4.2 Newmark Time Integration Scheme

In general, finite difference techniques are developed based on Taylor expansion approximations. An approximate expression for $\mathbf{u}_{i+1}$ i.e. $\mathbf{u}(t_i + \Delta t_i)$ can be written as:

$$\mathbf{u}\,(t+\Delta t) \simeq \mathbf{u}\,(t) + \dot{\mathbf{u}}\,(t)\,\Delta t + \frac{1}{2}\,\ddot{\mathbf{u}}\,(t)\,\Delta t^2 \tag{4.8}$$

Note that in equation (4.8) the approximation is kept to the level of acceleration (the highest derivative used in the dynamic equation of motion (4.2)). Of course, equation (4.8) includes an error since the remaining terms of the infinite series are neglected. To partially correct this error, Newmark modified (4.8) as:

$$\mathbf{u}\,(t+\Delta t) \simeq \mathbf{u}\,(t) + \dot{\mathbf{u}}\,(t)\,\Delta t + \left[\left(\frac{1}{2} - \beta\right)\ddot{\mathbf{u}}(t) + \beta\ddot{\mathbf{u}}\,(t+\Delta t)\right]\Delta t^2 \tag{4.9}$$

or in the more familiar difference form:

$$\mathbf{u}_{i+1} \simeq \mathbf{u}_i + \dot{\mathbf{u}}_i\,\Delta t + \left[\left(\frac{1}{2} - \beta\right)\ddot{\mathbf{u}}_i + \beta\,\ddot{\mathbf{u}}_{i+1}\right]\Delta t^2 \tag{4.10}$$

where $0 \leq \beta \leq \frac{1}{2}$.

Likewise, first order Taylor expansion of velocity $\dot{\mathbf{u}}\,(t+\Delta t)$ results into:

$$\dot{\mathbf{u}}\,(t+\Delta t) \simeq \dot{\mathbf{u}}\,(t) + \ddot{\mathbf{u}}\,(t)\,\Delta t \tag{4.11}$$

which Newmark modified as:

$$\dot{\mathbf{u}}\,(t+\Delta t) \simeq \dot{\mathbf{u}}\,(t) + [(1-\gamma)\,\ddot{\mathbf{u}}\,(t) + \gamma\,\ddot{\mathbf{u}}\,(t+\Delta t)]\,\Delta t \tag{4.12}$$

or, in the usual difference form:

$$\dot{\mathbf{u}}_{i+1} \simeq \dot{\mathbf{u}}_i + [(1-\gamma)\,\ddot{\mathbf{u}}_i + \gamma\,\ddot{\mathbf{u}}_{i+1}]\,\Delta t \tag{4.13}$$

where $0 \leq \gamma \leq 1$.

Solving equations (4.10) and (4.13) for $\ddot{\mathbf{u}}_{i+1}$ and $\dot{\mathbf{u}}_{i+1}$:

$$\ddot{\mathbf{u}}_{i+1} \simeq \frac{1}{\beta}\left[\frac{1}{\Delta t^2}(\mathbf{u}_{i+1} - \mathbf{u}_i) - \frac{1}{\Delta t}\,\dot{\mathbf{u}}_i - \left(\frac{1}{2} - \beta\right)\ddot{\mathbf{u}}_i\right] \tag{4.14}$$

$$\dot{u}_{i+1} \simeq \frac{\gamma}{\beta}\frac{1}{\Delta t}(u_{i+1} - u_i) + \left(1 - \frac{\gamma}{\beta}\right)\dot{u}_i + \left(1 - \frac{\gamma}{2\beta}\right)\Delta t\, \ddot{u}_i \tag{4.15}$$

The Newmark method equations presented above are in terms of total variables; however, in nonlinear problems, incremental formulations may be preferable. The increment of a variable will be defined as $\Delta x_i = x_{i+1} - x_i$. For example, equations (4.14) and (4.15) can be rearranged to obtain increments of acceleration and velocity as follows:

$$\Delta\ddot{u}_i = \ddot{u}_{i+1} - \ddot{u}_i \simeq \frac{1}{\beta}\left[\frac{1}{\Delta t^2}\Delta u_i - \frac{1}{\Delta t}\dot{u}_i - \frac{1}{2}\ddot{u}_i\right] \tag{4.16}$$

$$\Delta\dot{u}_i = \dot{u}_{i+1} - \dot{u}_i \simeq \frac{\gamma}{\beta}\frac{1}{\Delta t}\Delta u_i - \frac{\gamma}{\beta}\dot{u}_i + \left(1 - \frac{\gamma}{2\beta}\right)\Delta t\, \ddot{u}_i \tag{4.17}$$

It is interesting to note that for $\beta = \frac{1}{6}$ and $\gamma = \frac{1}{2}$, Newmark's method becomes the more commonly known linear acceleration method, and $\beta = \frac{1}{4}$ and $\gamma = \frac{1}{2}$ results in the constant (or average) acceleration method. To obtain an unconditionally stable algorithm, γ must be greater than or equal to $\frac{1}{2}$ and β must be greater than or equal to one half of γ. For other values of γ and β, the system becomes conditionally stable. This thesis will concentrate on the linear acceleration method given small time increments. By substituting (4.16) and (4.17) into the dynamic equation of motion in the incremental form one will obtain:

$$M\,\Delta\ddot{u}_i + \hat{C}_i\,\Delta\dot{u}_i + \hat{K}_i\,\Delta u_i = \Delta p_i \tag{4.18}$$

where $\hat{K}$ and $\hat{C}$ are "average" values of stiffness and damping for the interval Δt (which will be discussed later). Incorporating Equations (4.16) and (4.17) into (4.18) results in:

$$\tilde{K}_i\,\Delta u_i = \Delta\tilde{p}_i \tag{4.19}$$

where

$$\tilde{K}_i = \hat{K}_i + \frac{1}{\beta\,\Delta t^2}M + \frac{\gamma}{\beta\,\Delta t}\hat{C}_i \tag{4.20}$$

$$\Delta\tilde{p}_i = \Delta p_i + M\left(\frac{1}{\beta\Delta t}\dot{u}_i + \frac{1}{2\beta}\ddot{u}_i\right) + \hat{C}_i\left(\frac{\gamma}{\beta}\dot{u}_i + \left(\frac{\gamma}{2\beta} - 1\right)\Delta t\, \ddot{u}_i\right) \tag{4.21}$$

The solution of equation (19) follows an iterative procedure. Given below are the iterative steps used in this analysis. The steps with an * warrent further explanation.

1. Given $\mathbf{M}_i$, $\mathbf{C}_i$, $\mathbf{K}_i$, $\dot{\mathbf{u}}_{oi}$, $\mathbf{u}_{oi}$, $\mathbf{p}(t_j)$ where i = 1,2,..NDOF (number of degrees of freedom) and j = 1,2..m (number of external point loads with respect to time)
2. Solve for $\ddot{\mathbf{u}}_{oi}$ from Eq. (4.2)
3.* Input/Calculate Δt_j and $\Delta \mathbf{p}(t_j)$
4.* Form $\hat{\mathbf{K}}$ from Eq. (4.20)
5.* Calculate $\Delta \hat{\mathbf{p}}$ from Eq. (4.21)
6. Solve for $\Delta \mathbf{u}_j$ from Eq. (4.19)
7. Solve for $\Delta \ddot{\mathbf{u}}_j$, $\Delta \dot{\mathbf{u}}_j$, from Eq. (4.16), (4.17)
8. Solve for $\ddot{\mathbf{u}}_{j+1} = \ddot{\mathbf{u}}_j + \Delta \ddot{\mathbf{u}}_j$
   $\dot{\mathbf{u}}_{j+1} = \dot{\mathbf{u}}_j + \Delta \dot{\mathbf{u}}_j$
   $\mathbf{u}_{j+1} = \mathbf{u}_j + \Delta \mathbf{u}_j$
9. Calculate spring elongation, damper elongating velocity
10. Calculate Spring Force $(Fsp)_i$
11. Calculate Inertia Force $(Fi)_i$, Damping Force $(Fc)_i$
12. Calculate the internally equilibrated force $p_i = \mathbf{Fi}_i + \mathbf{Fc}_i + \mathbf{Fsp}_i$
13. Calculate the difference between p_i of step 12 and actual $p(t_i)$ of system
14. If difference is less than the tolerance, mark the convergence point for time, spring force, displacement, velocity and acceleration. Go back to step 3 for new time increment.
    If difference is more than tolerance, set $\Delta p(t_i)$ equal to the difference. Go back to step 4.

To determine the proper time step used in the above algorithm, several factors must be considered. For example, it is necessary that the time step used is smaller than one tenth of the smallest natural period of the system, and, for time - dependent loads, the time step must be smaller than one tenth the period of the load.

The smallest natural period of a structural system is T_{min} :

$$T_{min} = \frac{2\pi}{\text{natural frequency (highest mode)}} \tag{4.22}$$

To obtain the highest undamped natural frequency, the Stadola - Vianello Method [16] will be used. The Stadolla - Vianello procedure is as follows: Begin by assuming a shape and compute the inertial forces corresponding to this shape. Next, calculate the displacements corresponding to these forces. If the guessed and computed shapes agree, convergence is obtained. Starting with the stiffness form of the equation of motion, the method converges to the highest mode. This equation can be modified to include the fundamental modes as

$$[\mathbf{K} - \rho^2 \mathbf{M}][\phi] = 0$$

or

$$\mathbf{K}\,\phi = \rho^2 \mathbf{M}\,\phi$$

$$\mathbf{M}^{-1} \mathbf{K}\,\phi = \rho^2 \phi$$

which can be written as

$$\mathbf{H}\,\phi = \lambda\,\phi$$

The procedure is as follows:

1. Assume a trial value of ϕ
2. Compute $\mathbf{H}\,\phi$.
3. Approximate λ by dividing any element of the vector $\mathbf{H}\,\phi$ by the corresponding element of ϕ. If ϕ is the true eigenvector, all such ratios would be equal.
4. Normalize vector $\mathbf{H}\,\phi$ by dividing all elements by first element to obtain next trial of ϕ. Return to step 2.

Continue iterating until $|\phi|$ new - $|\phi|$ old < tolerance where $|\ |$ is any convenient vector norm.

The natural frequency of the system in the highest mode can now be obtained by taking the square root of λ at convergence, and T_{min} can be obtained. Note that in the above analysis, the maximum frequency of the system is found conservatively by taking into account only the stiffness of the system.

To determine $\hat{\mathbf{K}}$ from Eq. (4.19), it is first necessary to calculate the stiffness of each spring. For linear systems or multi-linear systems, this stiffness is simply the stiffness at that given displacement; however, for nonlinear stiffnesses, the procedure is more involved. The stiffness can be found by using either the secant method or the tangent method. For this analysis, the tangent method is used. Further details are presented in chapter 4. Once the stiffness for each spring is calculated, the matrix $\hat{\mathbf{K}}$ is formed as a symmetric banded matrix of NDOF equations with a halfbandwidth equal to 2 for systems of the form as that seen in Figure (4.1a).

Finally, an iterative procedure is used to obtain convergence of the system when analyzing the nonlinear reactions to the externally applied loads. The analysis presented is computed for each degree of freedom of the system. To calculate Δp, it is first necessary to define Δp. At the beginning of each successive convergence point, Δp is equal to the increment of external load. The velocity and acceleration in the mass and damping functions of Eq. (4.21) are the values of the previous equilibrated time step (initially, the values at time zero). The mass and damping functions are multiplied by a factor, F1, which is set equal to one at the beginning of the iteration, and set equal to zero during the iterations. This is due to the fact that after the first iteration, these functions will have been equilibrated. The incremental load of the system can be found using the inertia force, damping force, and spring force using the displacements, velocities and accelerations obtained from (4.19), (4.16), (4.17), respectively. The difference between this load and the actual external load is then calculated. If the difference is larger than a given tolerable value, the iterations begin or continue. F1 is set equal to zero, and the new Δp is now equal to the difference of the system and the actual load. When the difference is less than the tolerance, a new point of convergence has been found and Δp is the new increment of external load to be calculated.

Detailed explanation for this step is best seen graphically:

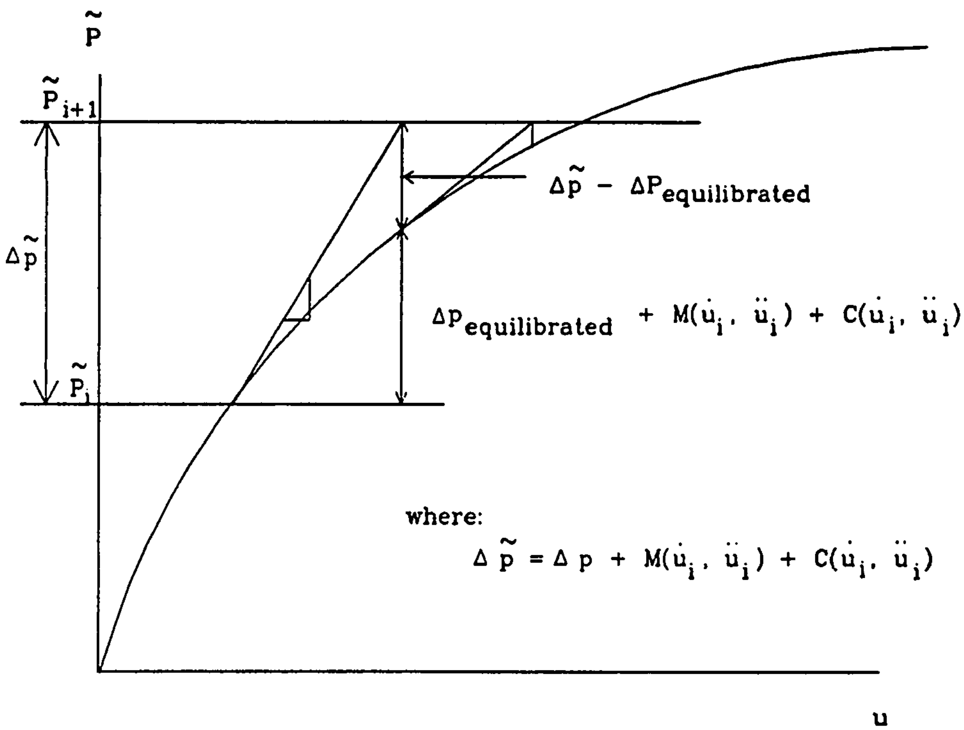

Figure 4.2: Tangent Step Method Used in Analysis

5. SPECIFICS ON SOFTENING AND SOIL - STRUCTURE INTERACTION

5.1 Buckling of Braces - Elastic and Inelastic

As stated previously, braces are used to minimize the lateral deflection of structures subjected to earthquake loads, so the behavioral pattern of these members must be examined. The failure of a brace under an applied compressive axial load is related to the extent of elastic deflection the member can undergo. The load that causes initial elastic buckling is commonly referred to as the critical load of the element. During elastic deformation, the deflections and stresses remain proportional to the applied loads; however, after buckling occurs, the deflections and stresses can increase rapidly, not proportional to the load. At this point, the member is said to become unstable because, even though the member may have the capability to carry more load, the ratio of the load to the deflection is much less than that seen before buckling. The buckling load is often defined using the assumptions that the slender members under consideration are perfectly straight, they are subjected to an applied load that is truly axial, and the material of the member is perfectly homogeneous. However, there exists deviations from the ideal conditions, and the calculated critical buckling load may never be reached.

Part of this thesis is concerned with the modeling of the post - buckling behavior of a brace. Figure (5.1a) shows a model for the relationship of an applied axial load versus the resulting displacement for a typical brace. Also presented is the internal stresses developed at certain stages of the loading (figure (5.1b)).

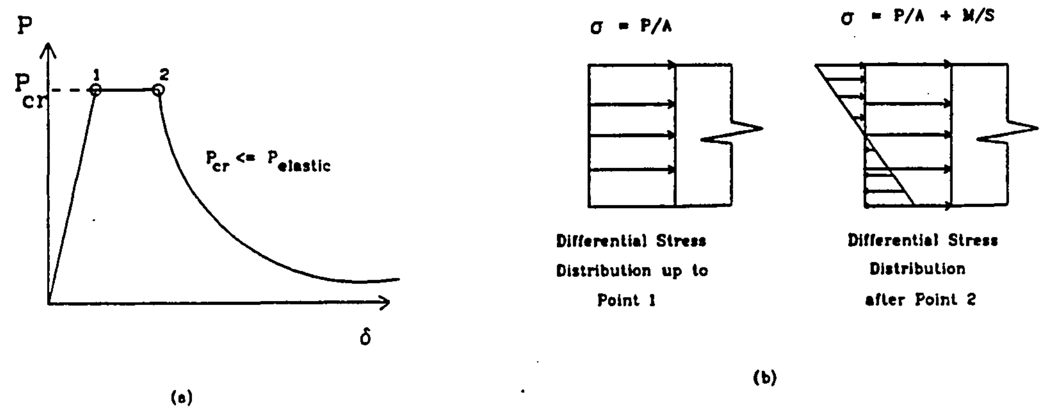

Figure 5.1: Behavior of Brace Subjected to an Increasing Axial Load

Before the critical load on a member is reached (point 1), an increase in axial load does not cause buckling to occur. However, once this critical load is reached, elastic buckling occurs and continues up to point 2. The length of line 1-2 can be shown to be quite small for most steel cross sections. At this point, the proportional limit of the material is exceeded and the maximum resisting bending moment is developed, and the brace is said to enter into the plastic range and this, combined with geometric instabilities, causes the load carrying capacity of the brace to decrease with an increase in deflection. It has been shown in previous studies that elastic buckling is usually followed by plastic deformations which result in significant losses of structural strength. The value of residual strength is not a constant value, but depends on the geometric characteristics of the structural element as well as on the material properties of the element such as strength and ductility. In general, post - buckling behavior is very difficult to model. The author of this thesis is aware of only two studies that attempt this, [7] and [10], yet these models are highly specialized (see figures (3.1a),(3.2a)).

Due to the lack of reliable information, for the purpose of this thesis, it is assumed that a structural element loses all its compressive strength after buckling. It is also assumed that the post - buckling deformation is not large enough to cause excessive distortion of the element geometry, and thus there is no loss in tensile stiffness upon reversal of the load. This can be seen to be experimentally reasonably accurate in figures (3.1a) and (3.1b) for the first few cycles of loading.

Experimental evidence shows that violation of the small deformation assumption leads to very different load - deflection curves for cyclic loads. As is shown in figure (3.1a), unloading followed by reverse loading after significant geometric distortion of the compressed member result in a compliant load - deflection curve, since the load application initially straightens the deformed bar. Only after this has been obtained to a considerable degree can the relation become stiff again. Also, the large deflections may cause permanent damage of the member, which will result in a smaller critical load upon repeated compressive loading.

The response of a braced structural system subjected to lateral loading is affected significantly by post - buckling behavior of its braces. The analysis of this behavior for a simple frame is presented in the following:

Figure 5.2a: The braced frame is initially subjected to a lateral force, P.

Figure 5.2b: The force increases to a point of incipient buckling of the compressed brace, and reaches its critical buckling load at P_1.

Figure 5.3c: The brace in compression is assumed to no longer contribute to the strength of the system. Force P_2 is the load that causes the brace in tension to yield.

Figure 5.3d: There are three specific cases to analyze post buckling effects of P_2:

Line 1. The critical buckling load is less than one half the yield load of the brace. A slight increase in load with an increase in deflection up to the yield point is then observed.

Line 2. The critical buckling load is approximately equal to one half the yield load of the brace. Perfect plasticity is then observed as the load does not change with an increase in deflection.

Line 3. The critical buckling load is greater than one half the yield load. At this point, there may exist an elastic buckling range for a given amount of deflection, yet for this study, it is assumed to be negligible, and the load is seen to decrease with an increase in deflection.

Note that the horizontal load of the column is also assumed to be negligible.

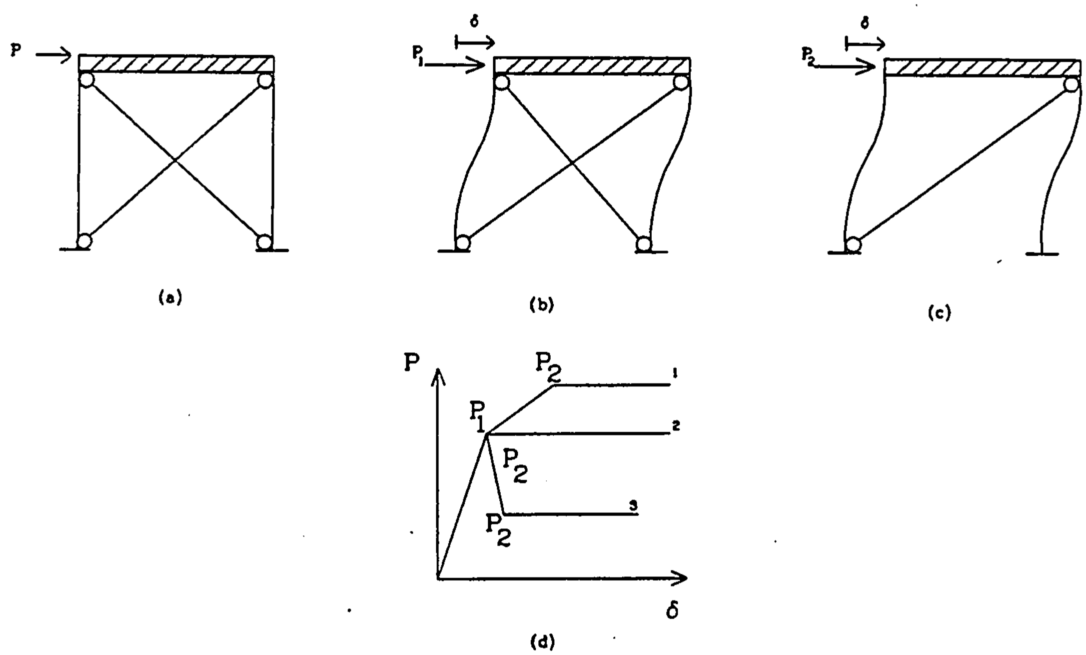

Figure 5.2: Frame Subjected to Lateral Force With Various Failure Modes

5.2 Soil - Structure Interaction

Traditionally, the design of structural systems is based on assumed displacement fixity of the foundation supports. The support reactions are calculated and then applied to the foundations to calculate the respective settlements. This is inconsistent with the original assumption of zero settlements used to solve the structure. Although large factors of safety are included in the foundation design, the error increases as the applied load increases, and the need for soil - structure interaction becomes evident to reduce both the error and the construction costs.

Important aspects regarding soil - structure interaction are the properties of the foundation, the properties of the soil median, the interface reactions, and a proper estimation of the friction between the foundation and soil. The test most commonly used to determine the friction at the interface is the Direct Shear test; however, deviations from this test are becoming more popular ([20]). For this analysis, experimental results from [5] are used as an example to analyze the interface and friction between concrete and Ottawa sand. A short description of their testing device is given here. For more details, the reader is referenced to the original study, [5]. The testing device used is called the "Cyclic Multi - Degree - of - Freedom" (CYMDOF) Device, and was designed to subject interfaces and joints to loads and displacements in all six degrees - of - freedom occurring at one point. The nature of the applied stresses and displacements can be static or time dependent (cyclic harmonic, triangular, step function). A translational sample box and a torsional sample box are used to determine the interface properties for the various degrees of freedom. A specified degree - of - freedom may be tested in either a displacement - controlled mode or a load - controlled mode. The dynamic load (time dependent) was sinusoidal in nature to determine the cyclic behavior of the concrete - sand interface.

From the cyclic interface tests, plots of the shear stress versus the relative displacement were developed. From these plots, the initial shear stiffness, K_i, and the maximum shear stress, τ_m during a given cycle of loading can be determined. The initial stiffness of the interface is defined as the slope of the initial loading curve, and the maximum shear stress corresponds to the maximum stress measured. Figure (5.3) shows this relationship for a relative density of 80 percent, and a normal stress of 28 psi, and where N is the number of loading cycles.

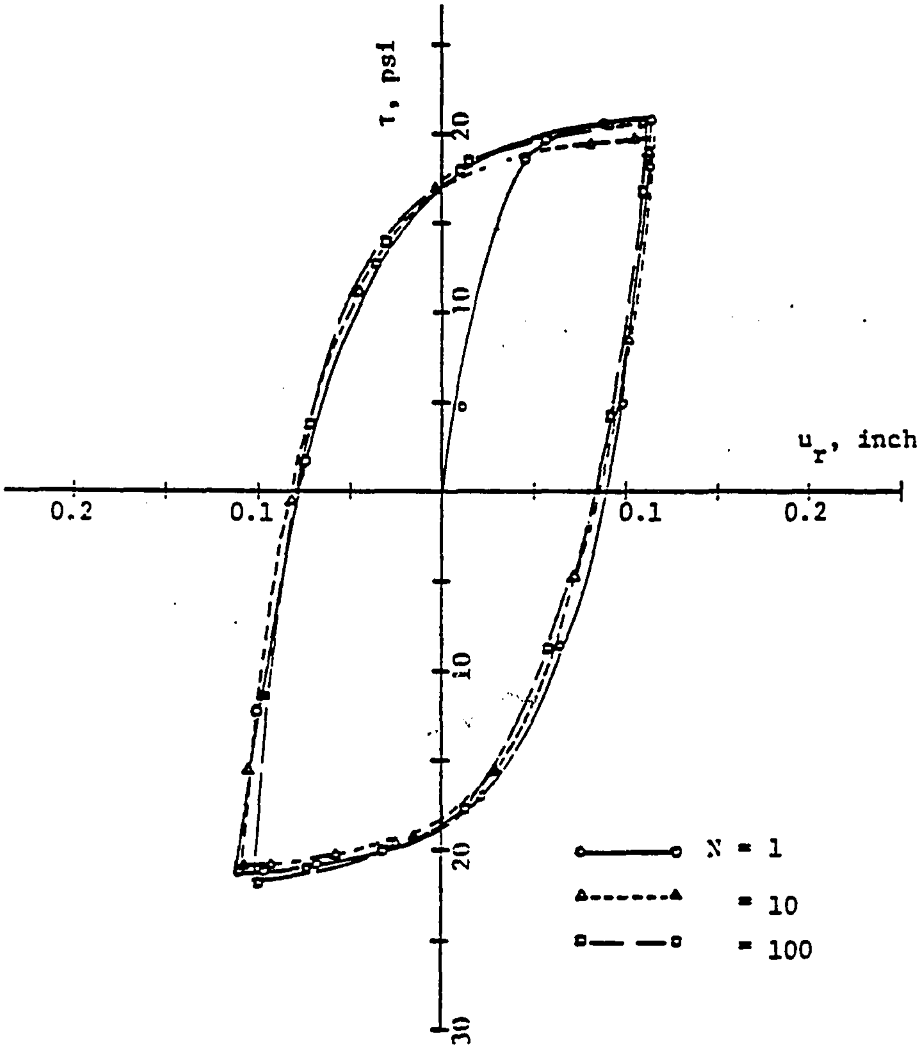

Figure 5.3: Shear Stress versus Displacement - D_r = 80, σ_n = 28 psi, u_r^m = 0.10 in. [5]

The shape of the curve presented in figure (5.3) is quite typical of sand - concrete interface. For the purposes of this study, the foundation is a mass block, the soil is the base, and their interface is modeled by a spring with a hyperbolic load deformation behavior:

$$F = \frac{e}{\frac{1}{k_i} + \frac{e}{F_{ult}}} \tag{5.1}$$

or

$$\bar{F} = \frac{\bar{e}}{\frac{1}{k_i} + \frac{\bar{e}}{\bar{F}_{ult}}} \tag{5.2}$$

which can be seen graphically as

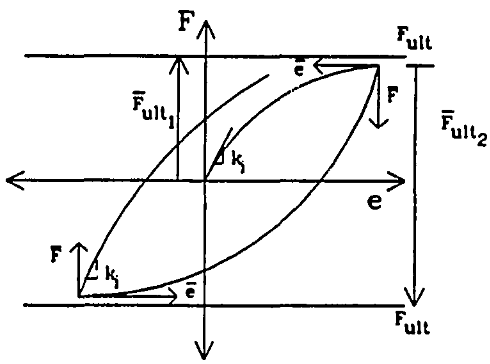

Figure 5.4: Hyperbolic Representation of Force - Elongation Relationship

where:

F is the actual spring force
e is the elongation of the spring
k_i is the initial tangent stiffness of the force - elongation diagram.
F_{ult} is the ultimate force that can be obtained.
$\bar{F}$ is the force from each reversal point due to change in velocity or

$$\bar{F} = (F - F_{max}) * \text{sign of velocity}_i$$

where F_{max} is the spring force at the most resent reversal point due to a change in the velocity

$\bar{e}$ is the elongation from each reversal point due to change in velocity, or

$$\bar{e} = (e - e_{max}) * \text{sign of the velocity}$$

where e_{max} is the elongation at the most resent reversal point due to change in velocity$_i$

$\bar{F}_{ult}$ is the total force from the point of reversal to the actual ultimate force.

The capability of this model to describe the interface behavior is demonstrated in figure (5.5) where the experimental curve from figure (5.4) and the theoretical curve (Eq. 5.1) are plotted together with satisfactory comparisons.

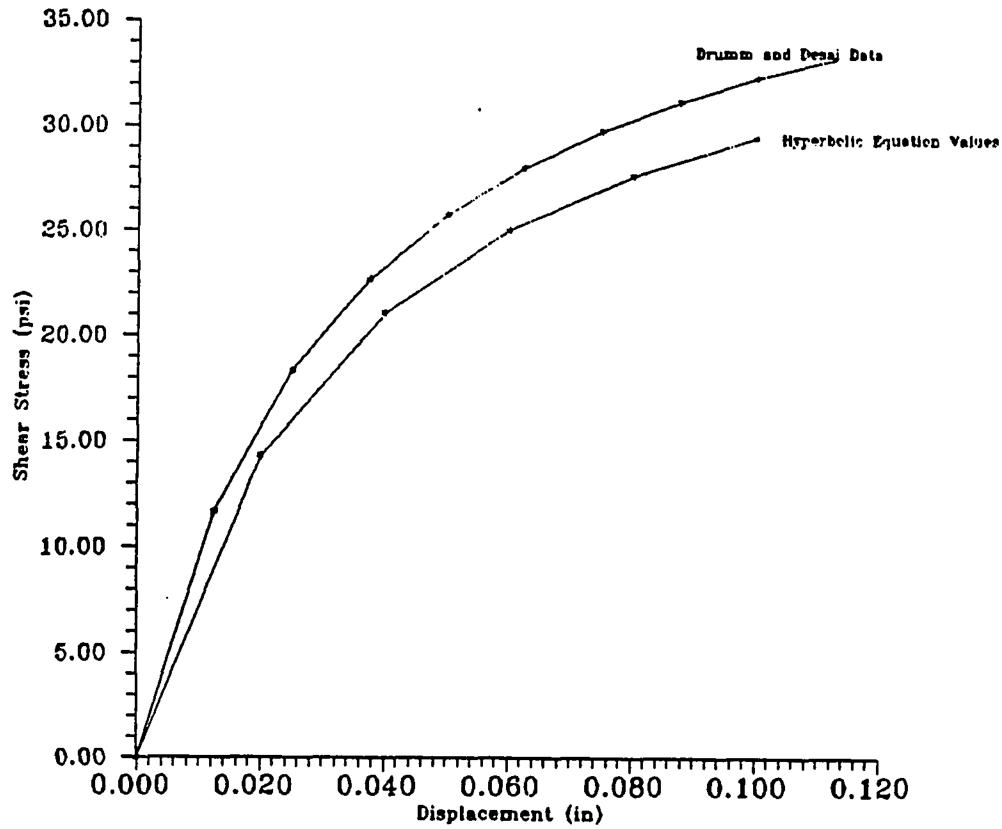

Figure 5.5: Shear Stress vs Displacement for Eq. (5.1) and Measured Values from [5]

6. SPECIFIC DEVELOPMENT OF PROPOSED MODEL

6.1 Illustration of Analysis for Single Story Frame

For specific details on how the model is developed, an example of a simplified single story structure will be presented. Assume, for this analysis, a structural design of the building recommended the columns to be W14 X 34, the girders to be W18 X 50, the area of the braces to be 19.097 cm^2, the mass of the roof to be 60,000 kg, and the dimensions as seen on figure (6.1a). The structure is then modeled as the dynamic mass - spring system as seen on figure (6.1b).

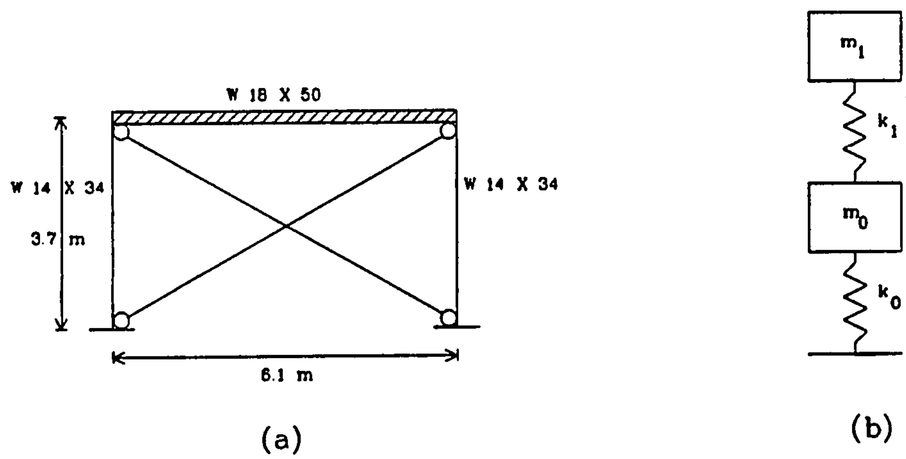

Figure 6.1: Example of Single Story Building and a Model of the Dynamic System

where
m_o: mass of the foundation
k_o: stiffness of the soil
m_1: mass of the roof
k_1: stiffness of the column - girder system

To begin the analysis, the force - displacement diagram for the bracing system must be created, figure 6.2:

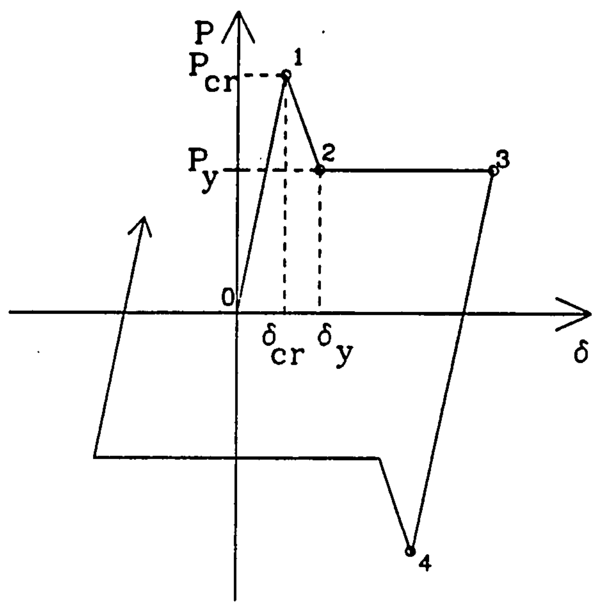

Figure 6.2: Force - Displacement Diagram for Bracing System

Step 1: From figure (6.2), the slope of line 0-1 must be obtained. First, calculate the critical load, F_{cr}, for the brace under compression using Euler's equation:

$$F_{cr} = \frac{\pi^2 E I}{L^2} \tag{6.1}$$

which assumes the brace is simply supported and elastic. From this equation, F_{cr} is equal to 497 kn. Note that the yield load of the brace in compression is F_y = 477 kn. This force is obtained by the relationship:

$$F_y = \frac{\sigma_y}{A} \tag{6.2}$$

where the σ_y is assumed to be 250,000 $\frac{kn}{m^2}$ for steel. It has been shown that the theoretical critical load is rarely reached in practice due to manufacturing and geometric imperfections of the brace, eccentricity of the loads, and residual stresses. Therefore, it shall be assumed that the critical load coincides with the yield load, i.e $F_y = F_{cr} = 477$ kn. Now that the critical buckling force is known, the external lateral force, P_{cr}, that causes this buckling load must be computed using the following ratio:

$$\frac{P_{cr}}{F_{cr}} = \frac{P}{F} \tag{6.3}$$

where P can be any lateral load applied to the frame and F is the resulting axial force on the brace due to that load. To determine this axial force, the structure is evaluated using Program PC Frame [14] developed by Dr. R. Richard. Finally, Program PC Frame is once again used to determine the critical lateral displacement, δ_{cr}, of the frame under the critical lateral load. Now point 1 is determined as (δ_{cr}, P_{cr}).

Step 2: Next, the slope of line 1-2 must be calculated. The structure is assumed to have lost all the strength of the buckled brace that was in compression. To obtain point 2, the yield elongation of the brace in tension in the modified steel structure (see figure 5.3c) must be calculated:

$$\epsilon_y = \frac{\sigma_y}{E} \tag{6.4}$$

where $\sigma_y = 250{,}000 \frac{kn}{m^2}$, and the modulus of elasticity, E, of steel used in this example is assumed to be equal to 200,000,000 $\frac{kn}{m^2}$. The yield elongation of the brace can now be calculated as $e_y = L\,\epsilon_y$, and assuming small displacement theory, the yield displacement, δ_y, can be determined. Finally, the yield load can be obtained by a direct ratio as:

$$\frac{P_y}{\delta_y} = \frac{P}{\delta} \tag{6.5}$$

where, once again, P can be any lateral load applied to the frame and δ is the resulting displacement of the frame acquired by running Program PC Frame with load P. Point 2 is determined as (δ_y, P_y).

Step 3: After the tensile brace yields, the system is assumed to be perfectly plastic upon further loading. Point 3 occurs whenever the system is subjected to unloading followed by reverse loading.

Step 4: Immediately following unloading, the system is assumed to be subjected to small displacements. This implies that there is no loss of strength to the system and the slope of line 3-4 is equal to the slope of line 0-1. The point of critical buckling for the brace in reverse loading is assumed to be equal to the above calculated P_{cr}, and, using the slope of the line and - P_{cr}, point 4 can be obtained.

Once the force - displacement diagram for the bracing system is created, it is necessary to calculate the force - displacement diagram for the soil. From the results obtained from [5], assuming a concrete foundation resting on Ottawa sand with an internal friction angle of 31 degrees, the ultimate shear force can be calculated given the relationship:

$$\tau = \sigma_n \tan \delta + \alpha \tag{6.6}$$

where

σ_n is the normal stress
δ is the interface friction coefficient
α is the apparent adhesion described by the figure below
- Note that this equation is only valid for $\sigma > \sigma^*$, while for small values of σ, $\tau = \sigma_n \tan \delta_o$ is more appropriate where δ_o is the initial internal friction coefficient

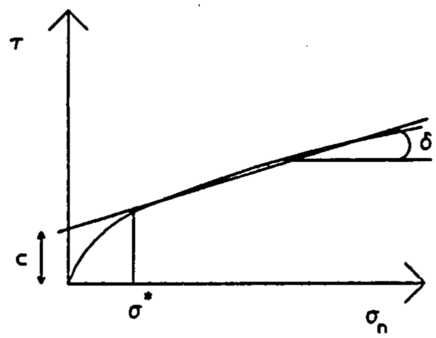

Figure 6.3: Relationship between Normal Stress and Shear Stress for Sand

From [5], for a friction angle of 31 degrees, $\alpha \simeq 10.34\ \frac{kn}{m^2}$. Given $\sigma_n = \frac{P}{A}$, where P equals the total load on the foundation and A equals the area of the foundation, the ultimate shear stress can be found. However, the area of the foundation is unknown. From the general bearing capacity (assuming c = 0 for Ottawa sand and horizontal load is applied):

$$q_{ult} = \frac{P_{ult}}{A} = q\ N_q\ F_{qs}\ F_{qd} + \frac{1}{2}\gamma\ B\ N_\gamma\ F_{\gamma s}\ F_{\gamma d}$$

where:

- P_{ult} = ultimate load allowed on the foundation $\left(= \frac{P}{FS}\right)$
- FS = factor of safety (assume = 3)
- B = width of footing
- L = length of footing (= B for square footing)
- γ = unit weight of soil
- ϕ = friction angle of soil
- D_f = Depth of embedment of foundation
- q = equivalent surcharge of the soil above the bottom of the foundation (= $\gamma\ D_f$)
- N_q, N_γ = bearing capacity factors

  $$N_q = \tan^2\left(45 + \frac{\phi}{2}\right)e^{\pi \tan\phi}$$

  $$N_\gamma = (N_q - 1)\cot\phi$$

- F_{qs}, $F_{\gamma s}$ = shape factors

  $$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi$$

  $$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right)$$

F_{qd}, $F_{\gamma d}$ = depth factors

$$F_{qd} = 1 + 2\tan\phi\,(1 - \sin\phi)^2 \left[\frac{D_f}{B}\right] \text{ for } \frac{D_f}{B} \leq 1$$

$$F_{qd} = 1 + 2\tan\phi\,(1 - \sin\phi)^2 \tan^{-1}\left[\frac{D_f}{B}\right] \text{ for } \frac{D_f}{B} > 1$$

$$F_{\gamma d} = 1$$

From reference [5], the soil unit weight is calculated by the given minimum and maximum densities:

$$D_r = \frac{\rho_f - \rho_1}{\rho_2 - \rho_1}\,\frac{\rho_2}{\rho_1}$$

where

D_r = relative density of the soil (= 65%)

ρ_1 = minimum density $\left(= 1.47\ \frac{gm}{cc}\right)$

ρ_2 = maximum density $\left(= 1.76\ \frac{gm}{cc}\right)$

ρ_f = natural density of soil

From the above equation, ρ_f is found to be 1.63 $\frac{gm}{cc}$ and from $\gamma = \rho\ 9.807\ \frac{kn}{m^3}$, $\gamma = 16\ \frac{kn}{m^2}$). Therefore, the only unknown in the above equation is B with the assumption of a square footing and is then calculated to be 1.9 meters. From Eq. (6.2) and (6.6), the ultimate shear stress is determined and the ultimate force for the force - displacement diagram is computed as $P_{ult} = \tau_{ult} B^2$. To determine the initial stiffness of the soil, an equation was obtained from the results given by [5] as:

$$\log k_i = \log \sigma_n + \log(\text{int}) \tag{6.7}$$

where

int is the intercept of the line and equal to 430.94 for ϕ = 31 degrees.

Now that the initial stiffness and ultimate load for the soil is known, the force - displacement diagram for the soil can be developed. Figure (6.4) shows the two force - displacement diagrams just developed:

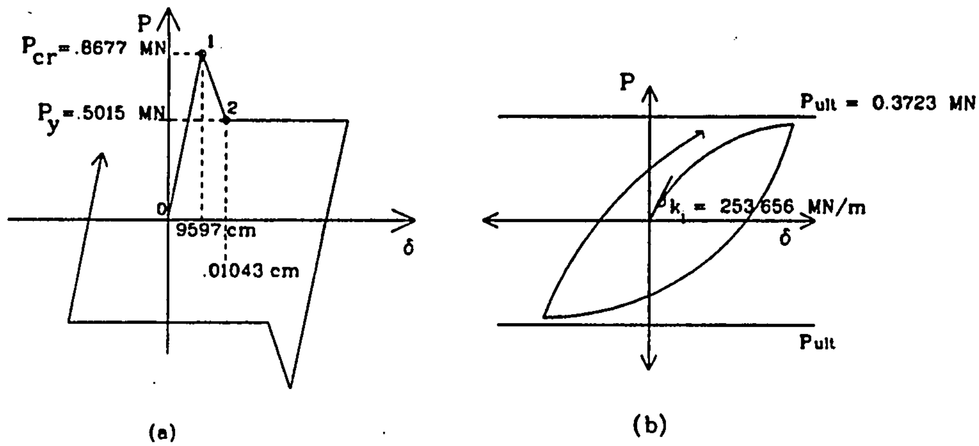

Figure 6.4: Force - Displacement Diagrams for the Bracing System for the Single Story and Ottowa Soil.

The last step in the analysis is to subject the structure to the earthquake load and analyze it under various conditions. The accelerogram used in this thesis is the N-S component of the May 18, 1940 El Centro Earthquake multiplied by a factor of 1.5 to represent a severe earthquake as seen in reference [6]. The first 6 seconds of this accelerogram are used in the response computation because the peak acceleration impulses in the ground motion occur during this time.

Figures (6.5 - 6.7) show the results of the structural response of the frame not assuming soil-structure interaction. This frame will be referred to as frame1.

Figures (6.8 - 6.11) show the results of the structural response of the frame assuming soil - structure interaction. This frame will be referred to as frame2.

Specifically, figures (6.5) and (6.8) show the comparison of the floor displacements and time for the frame with braces having an elastic, elastic perfectly - plastic, and softening force - displacement relationship. Figures (6.6) and (6.10) show the spring force verses elongation of the bracing system having an elastic perfectly - plastic relationship, whereas figures (6.7) and (6.11) show the spring force versus elongation of the bracing system having a softening relationship.

It is interesting to note that the frame geometry and its material properties can significantly affect the response of the structure. For certain problems, the results of all three cases may vary by a large degree as seen from figure (6.5) for frame1. On the other hand, the results presented in figure (6.8) for frame2 are very similar. Also, upon analyzing figures (6.6 - 6.7) and figures (6.10 - 6.11), the extent into which the frame enter the plastic zone varies. Frame1 is seen to enter and stay in the plastic zone for a greater range of displacements than does frame2.

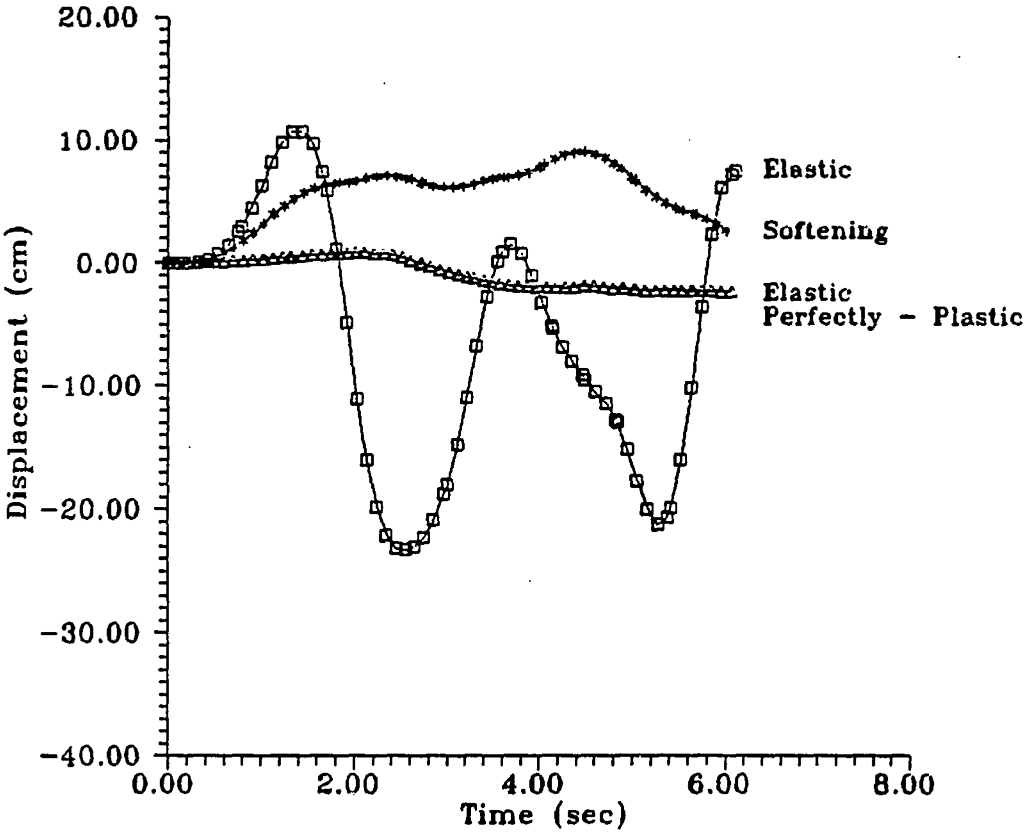

Figure 6.5: Displacement vs. Time for Roof Response, No Soil - Structure Interaction

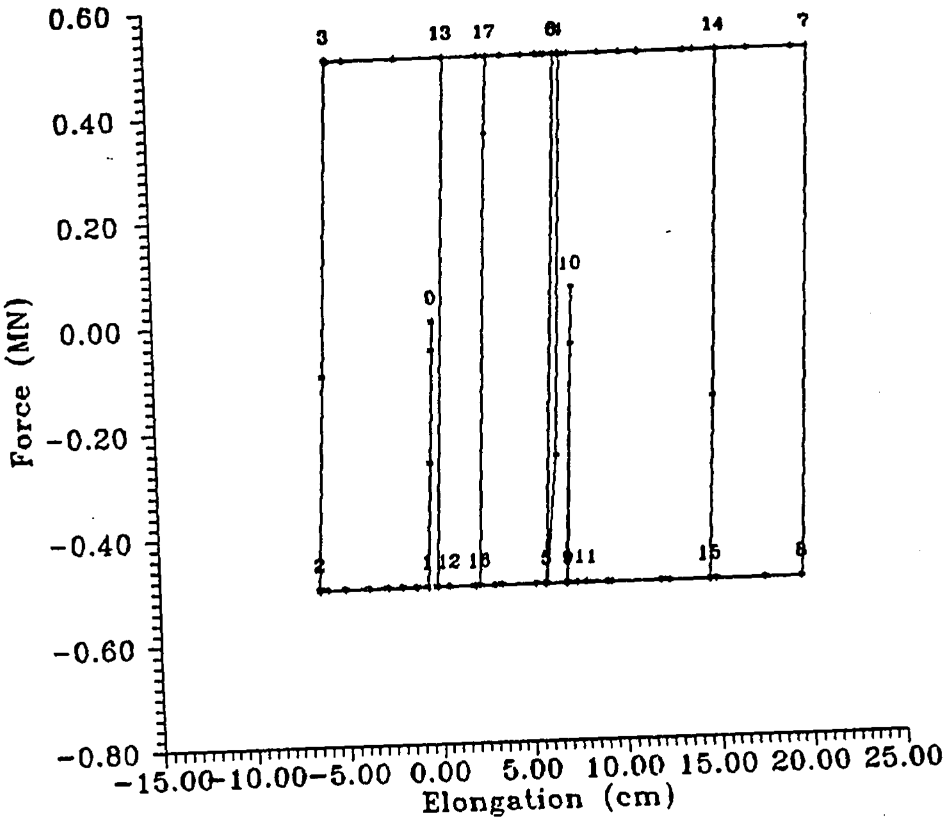

Figure 6.6: Force vs. Elongation - Elastic Perfectly-Plastic Case, No Soil - Structure Interaction

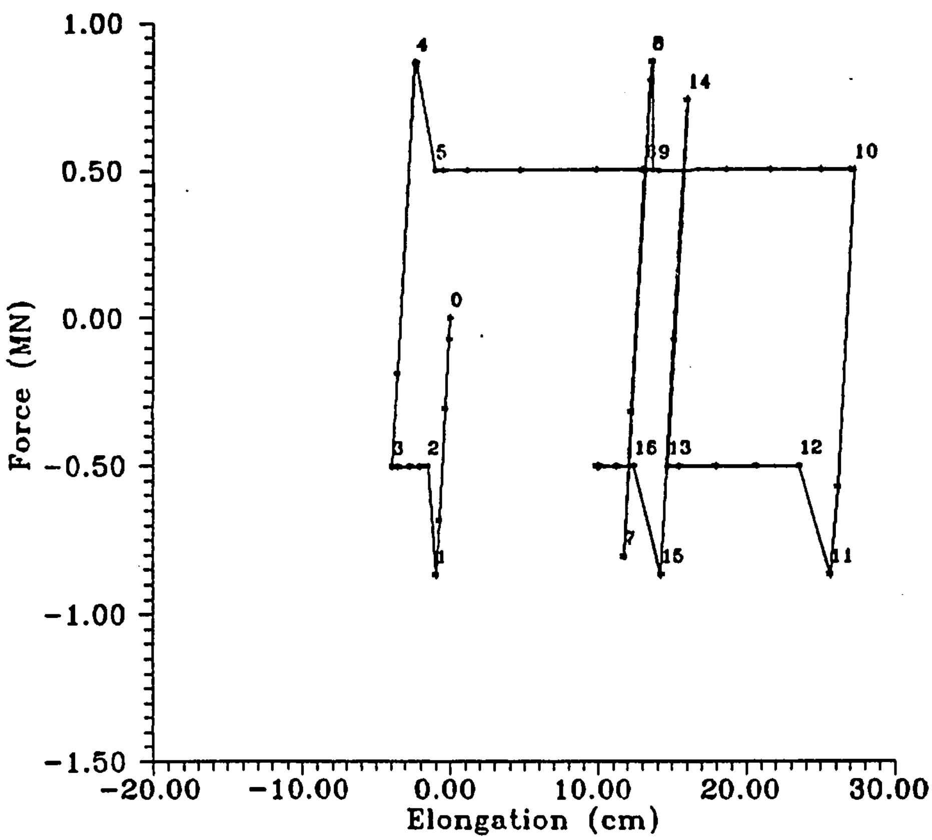

Figure 6.7: Force vs. Elongation - Softening Case, No Soil - Structure Interaction

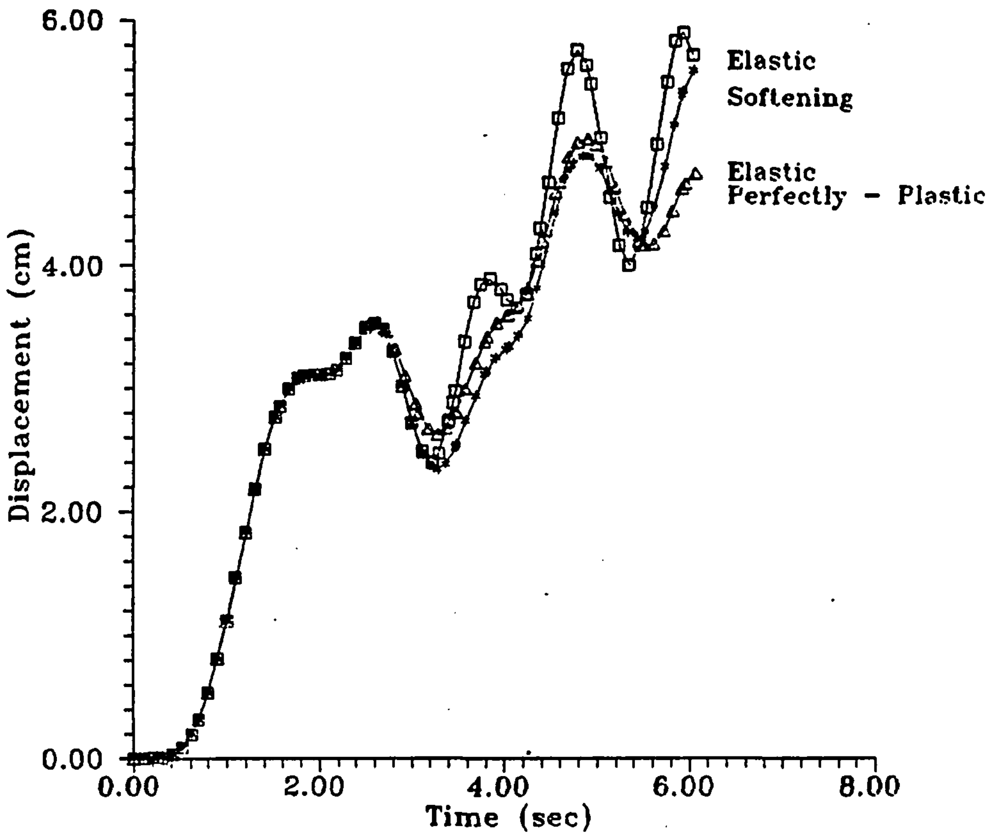

Figure 6.8: Displacement vs. Time for Roof Response, Including Soil - Structure Interaction

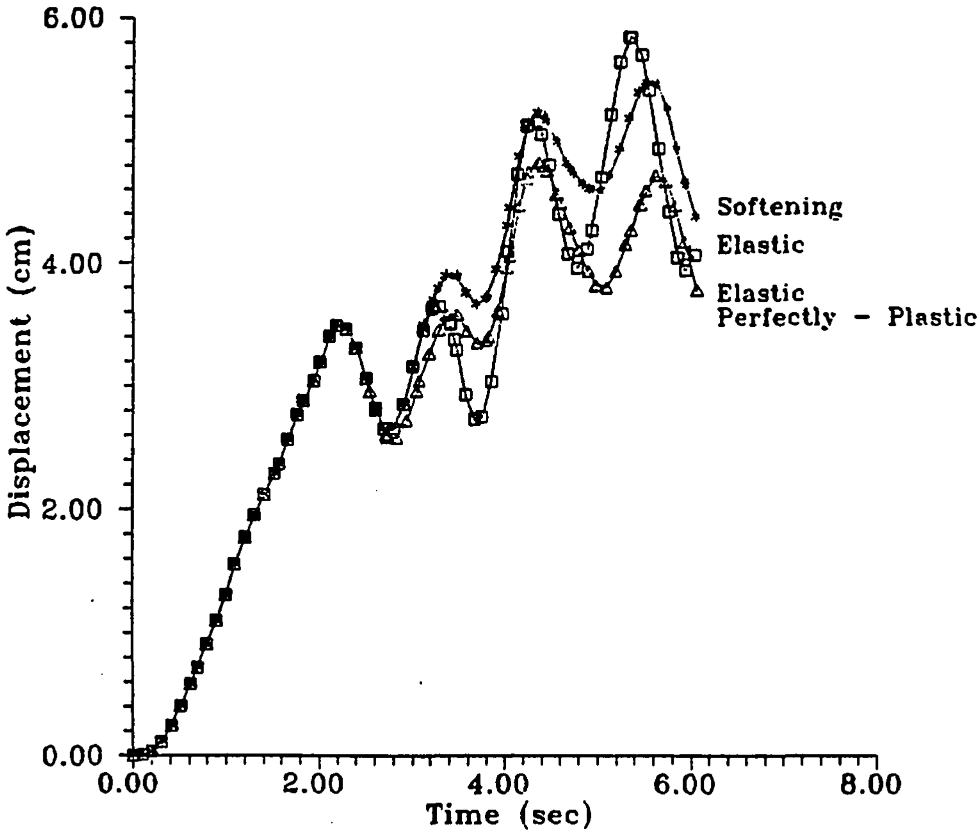

Figure 6.9: Displacement vs. Time for Foundation Response, Including Soil - Structure Interaction

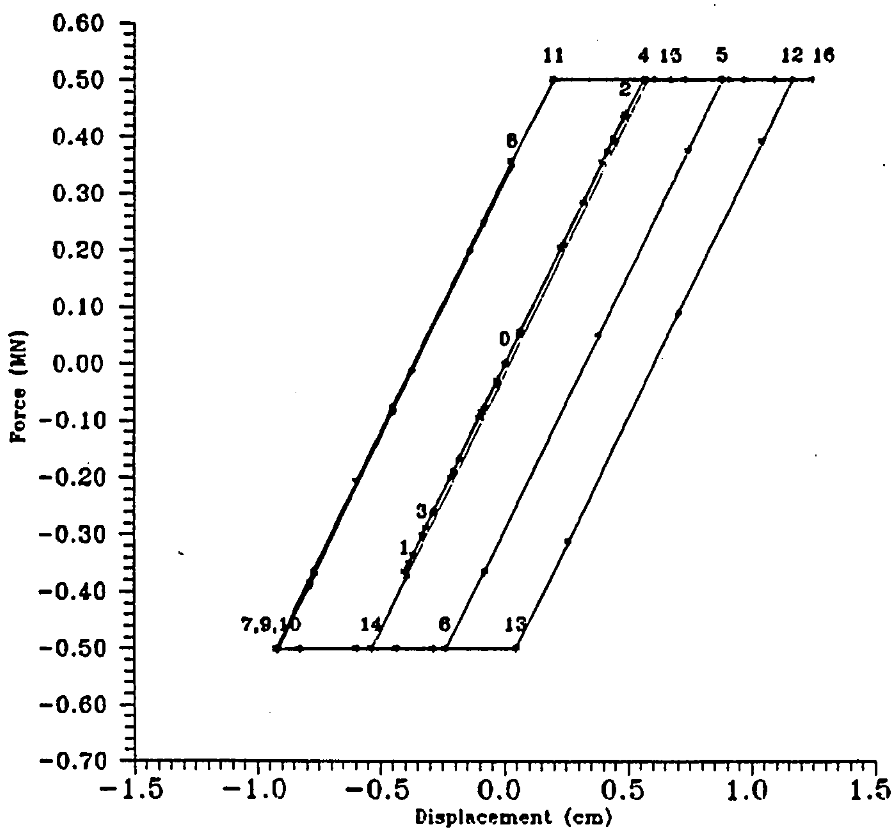

Figure 6.10: Force vs. Elongation - Elastic Perfectly-Plastic Case, Including Soil - Structure Interaction

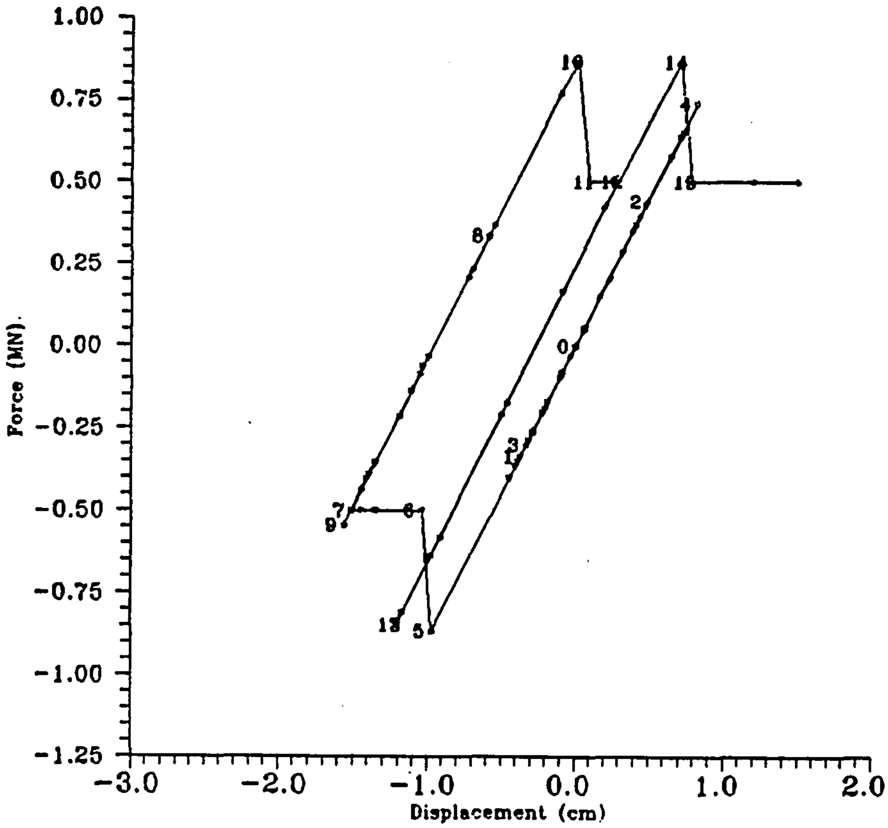

Figure 6.11: Force vs. Elongation - Softening Case, Including Soil - Structure Interaction

6.2 Analysis of Multistory Frame

To expand the above illustration, a multistory structure will be subjected to the first six seconds of the modified El Centro earthquake. The structure considered is ten stories high with a uniform story height of 3.7 meters and a bay width of 6.1 meters (see figure (6.1a)). The properties of the columns and girders were obtained from [6] and are presented in Table (6.1).

Table 6.1: Structural Properties of 10 Story Frame

Floor Level	Section	
	Girders	Columns
10	W18 X 50	W14 X 34
9	W18 X 50	W14 X 53
8	W18 X 50	W14 X 53
7	W18 X 50	W14 X 78
6	W18 X 50	W14 X 78
5	W18 X 60	W14 X 103
4	W18 X 60	W14 X 103
3	W18 X 60	W14 X 119
2	W18 X 60	W14 X 119
1	W18 X 60	W14 X 136

The bracing area is constant throughout the frame 19.097 cm^2. Finally, a uniform floor weight was given as approximately 43,000 kg which was increased to 60,000 kg for this study. To examine the effects bracing has on the structure to reduce lateral deflection, two different layouts of the diagonal bracing will be considered - fully braced and alternate stories open (figure (6.12)).

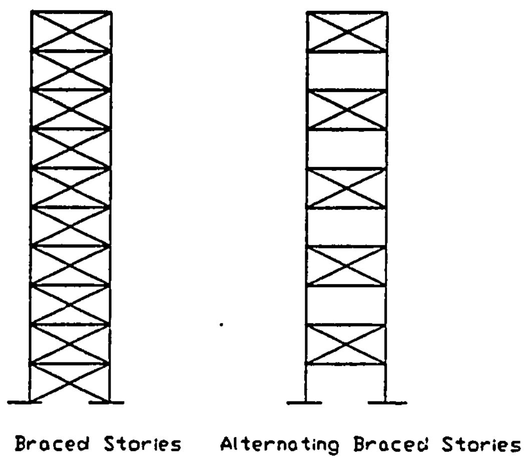

Figure 6.12: Specific Layouts of Multistory Structure

The force - displacement diagram for all ten bracing systems and the soil are obtained following the exact analysis given for the single story frame. The structural response of the top floor is presented in figures (6.13 - 6.16). For a specific discussion on the several responses given, Table (6.2) summarizes some important values acquired from the multistory frame illustration:

Table 6.2: Displacement Summary of 10 Story Frame

	FBN		FB		ABN		AB	
	MDD	MD	MDD	MD	MDD	MD	MDD	MD
Elastic	11.2	37.5	21.2	7.3	22.4	4.3	19.2	6.3
Elastic Perfectly Plastic	20.6	4.4	16.0	4.95	31.9	6.35	17.6	7.8
Softening	20.3	5.4	16.2	6.25	33.2	6.66	17.2	8.3

where

FBN = Fully Braced Frame Without Soil - Structure Interaction
FB = Fully Braced Frame With Soil - Structure Interaction
ABN = Alternating Braced Frame Without Soil - Structure Interaction
AB = Alternating Braced Frame With Soil - Structure Interaction
MDD = Maximum Differential Displacement (cm)
MD = Maximum Displacement of 10th Floor (cm)

The first thing to notice is the importance of bracing in the structure. Although the maximum displacements for the tenth floor vary only by a couple of centimeters between the two configurations (except for the elastic case not considering interaction), a more important factor to examine is the maximum differential displacement between two floors. For the frames assuming no interaction, an increase of up to 13 centimeters is seen when changing the structural configuration from a fully braced system to an alternating braced system. However, when soil - structure interaction is taken into account, the difference in the displacement is not so severe, and only changes by one centimeter.

Finally, an observation is noted on the effect of soil - structure interaction. When interaction is considered, the displacements of the tenth floor increased in both configurations, yet the maximum differential displacement decreases.

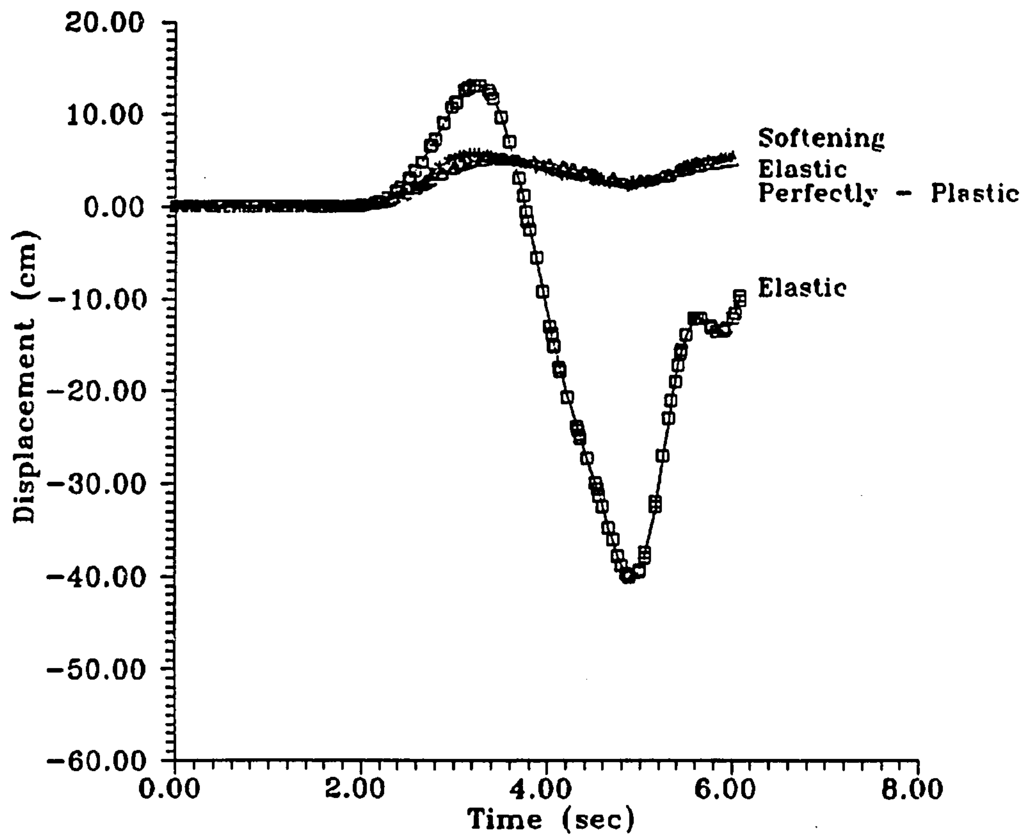

Figure 6.13: Displacement vs. Time for 10th Floor Response of 10 Story Fully Braced Frame, No Soil - Structure Interaction

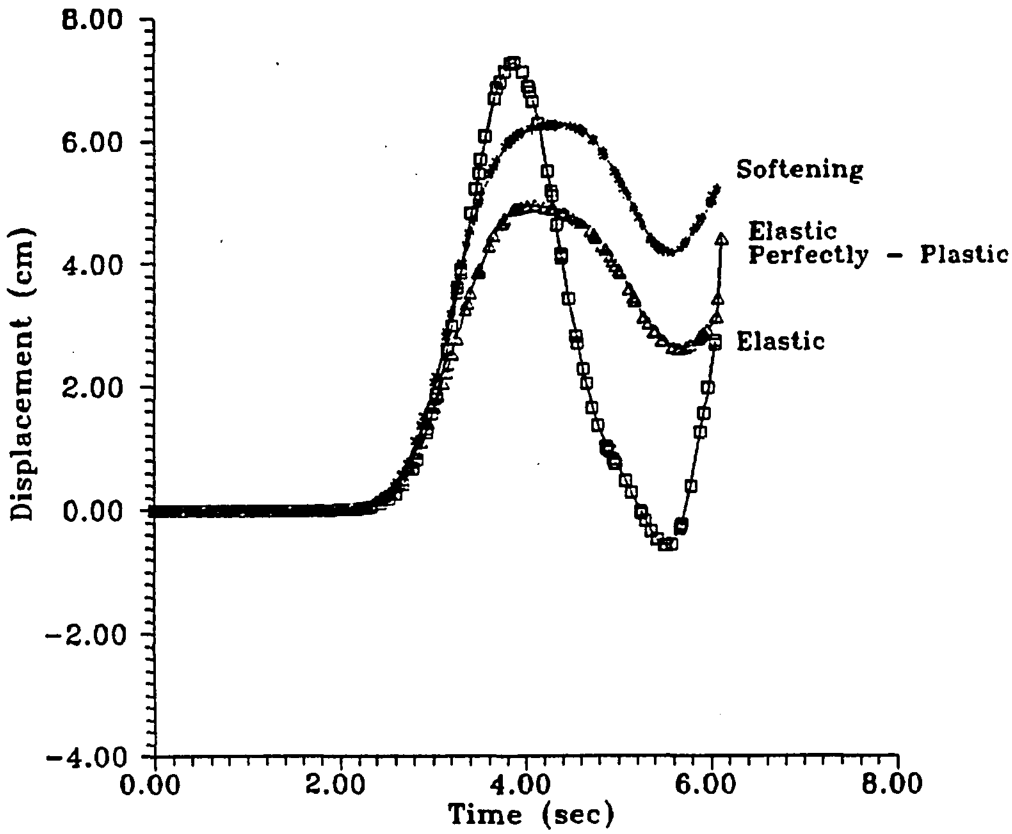

Figure 6.14: Displacement vs. Time for 10th Floor Response of 10 Story Fully Braced Frame, Including Soil - Structure Interaction

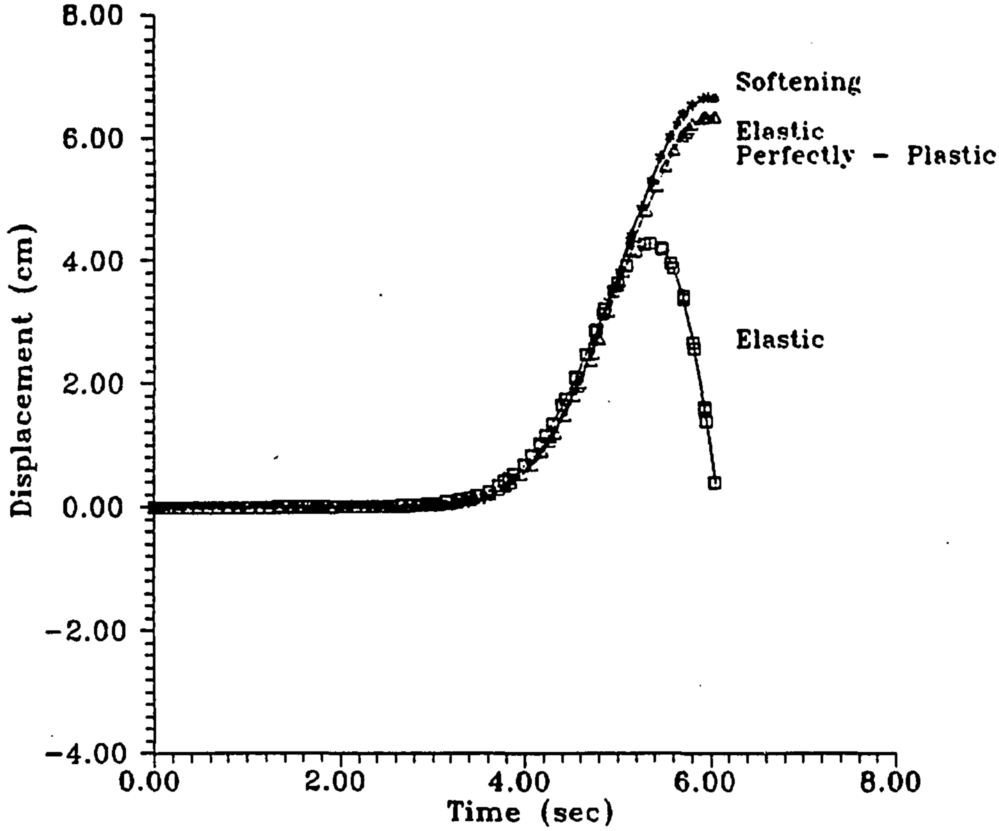

Figure 6.15: Displacement vs. Time for 10th Floor Response of 10 Story Alternating Braced Frame, No Soil - Structure Interaction

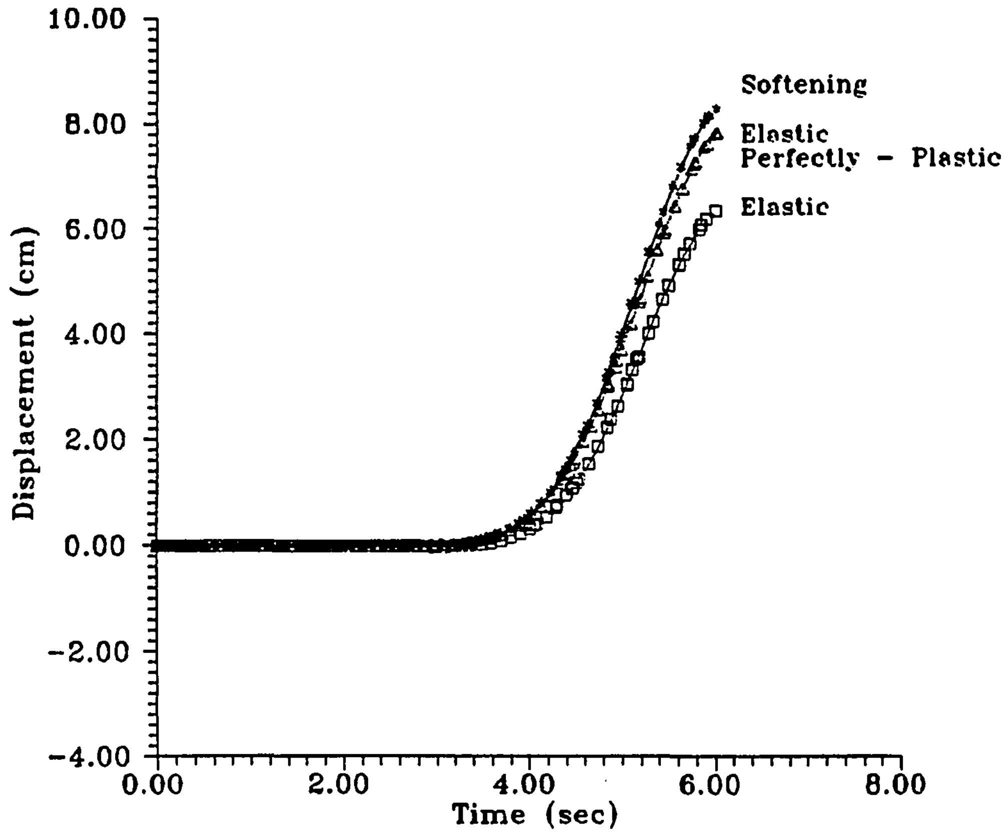

Figure 6.16: Displacement vs. Time for 10th Floor Response of 10 Story Alternating Braced Frame, Including Soil - Structure Interaction

7. DISCUSSION AND CONCLUSIONS

7.1 Discussion

This thesis presents a theoretical study of the seismic behavior of structures emphasizing member failure and soil - structure interaction. Although this thesis is limited in scope, some deductions can be made upon examining the various results obtained. First, the response of frames used in this analysis depends on both the force - displacement relationship of the bracing system and the force - displacement relationship of the soil - structure interface. Consideration of soil - structure interaction in this problem can cause significant reduction in deformation. This happens because the transfer of deformation from the soil to the foundation through the interface layer is damped due to interface slipping which causes energy dissipation. However, examination of the response of a similar structure, but with reduced stiffness (alternating braced with unbraced stories) results in an increase of deflections when soil - structure interaction is considered. It is hypothesized that the reduction in deformation is dependent on the relative stiffness of the structural system to the stiffness of the foundation - soil interface. The structural response of the frame with alternating braced stories was again analyzed using a soil interface with smaller strength, and reduced deformations are observed when considering soil - structure interaction (see figure (7.1). Therefore, it is important to design a structural system with a strong stiffness when compared to the stiffness of the soil interface. The deformations are also reduced, compared to the elastic solution, when either an elastic perfectly - plastic response or a softening response is assumed.

Although not enough data has been produced to generalize the results of this thesis, it has been shown that there is a significant need to consider both nonlinear bracing responses and nonlinear soil - structure interaction. For a thorough investigation, further numerical (and experimental) studies as well as comparisons with other methods are required.

7.2 Conclusions

Some assumptions and restrictions used in this thesis are summarized here, and future improvements are suggested. First, only one degree of freedom per floor (lateral deflection) is assumed. This can be expanded to include both the rotation and the vertical displacement at the expense of memory and time requirements for the computer solution. This is a serious matter to consider for the program presently takes approximately thirty minutes to run a ten degree of freedom system including soil - structure interaction on a 25 MHz 386 IBM compatible machine.

Another important matter to consider is the development of the model for the force - displacement softening relationship. This model is mainly valid for small plastic deformations. As was seen earlier, the buckling load is less than the Euler critical buckling load, and if the member is subjected to large plastic deformations, this load is further reduced upon successive cyclic loading. Also, the buckled brace does not lose all its strength as was assumed. One way to improve the model would be to obtain the values needed experimentally. Specifically, the values obtained should include the initial and successive buckling loads, the yield load following compressive loading, and the rate of loss of strength for the braces used in the analysis, if possible. Experimental tests can also be run on specific foundation - soil interfaces to determine the values needed to develop the force - displacement relationship.

This model can be expanded theoretically as well. For example, the analysis can include concrete frames and other types of bracing systems, as well as other types of foundation - soil interfaces. It would also be beneficial to this study to compare these results with results obtained by other methods; however, a study developed using similar models could not be found that included the necessary numbers needed for comparison.

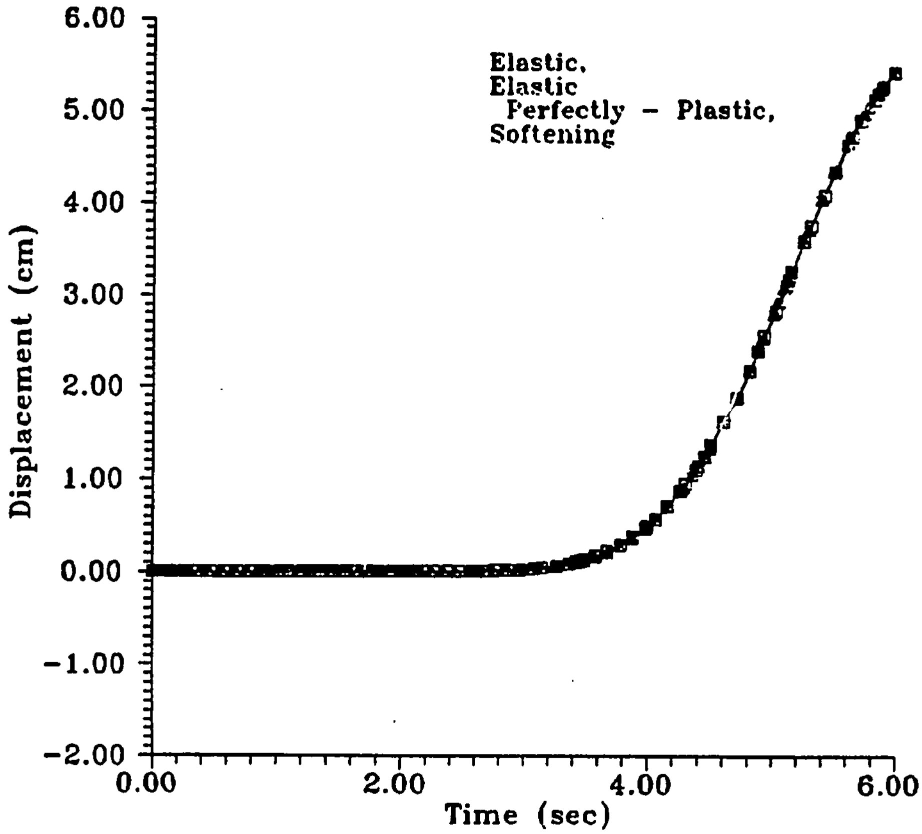

Figure 7.1: Displacement vs. Time for 10th Floor Response of 10 Story Alternating Braced Frame, Including Soil - Structure Interaction

APPENDIX A

USERS MANUAL

This program calculates the response of a dynamic mass - spring - damper system. An input file must be created to run the program. Upon running the program, the user will be asked to specify the input file name and the name for the output file that will be created. The program calculates the dynamic response of the structure and outputs the displacement of each degree of freedom and the respective spring force with time. The format of the input file must be created as follows (* is for actual line inputs)

Set 1: * ndof, tolp, tolv

where: ndof = number of degrees of freedom

tolp = tolerance used for convergence

tolv = tolerance used to determine approximately zero velocity

* β, γ, t_o, t_f, timedi

where: β, γ = variables in Newmark Time Integration Scheme

t_o = initial time

t_f = final time to run program

timedi = variable used in defining increment in time i.e.,

$\Delta t = \frac{T_{min}}{timediv}$ (see Eq. 4.22 for definition of T_{min})

Set 2: The following group of lines is necessary for each degree of freedom: Beginning of loop

* M, dis, vel

where: M = mass

dis = initial displacement

vel = initial velocity

* CODE, KODE

where: CODE = 1 for linear damping coefficient

KODE = 1 for linear stiffness coefficient or varying linear stiffnesses

KODE = 2 for nonlinear stiffness coefficient

* C

where: C = damping coefficient (if there is no damping, input CODE = 1 and C = 0)

If KODE = 1 then

* nk

where: nk = number of stiffnesses

* (loop from 1, nk) kk, fm

where: kk = stiffness

fm = ultimate force of that stiffness or the point of changing stiffness

If KODE = 2 then

* k_i, fult

where k_i = initial stiffness

fult = ultimate force for the stiffness system

End of loop

The program presently assumes the last degree of freedom to be attached to a wall by a spring and damper. Repeat lines beginning at CODE, KODE. If there is no wall, input CODE = KODE = 1 and C = 0, nk = 1, kk = 0, fm = 0.

Set 4: * eqk

where eqk = 1 for earthquake loads, ≠ 1 for other loads

If eqk = 1

* naccp

where naccp = number of acceleration points to be input

* for 1 to naccp: gac, tac

where gac = earthquake acceleration value

tac = time at each acceleration point

If eqk ≠ 1, Loop for each degree of freedom: Beginning of loop

* LODE

where LODE = 1 for sinusoidal load

LODE = 2 for point loads with respect to time

If LODE = 1

* A, fq

where A = amplitude for external loads

fq = frequency for external loads, i.e., load = A sin(fq t)

If LODE = 2

* np, A

where np = number of point loads to be input

* from 1 to np: ta, fa

where ta = time for each input load

fa = point load

As an example, consider the one story frame example including soil - structure interaction given in Chapter 6:

Sample input:

```
2,.1,.1
.1667,.5,0,6,150
.046,0,0
1,2
0
2.536,.3723
.06,0,0
1,1
0
3
.9042,.8677
-4.3965,.5015
0,.5015
1,1
0
1
0,0
1
66
0.0,0.0
18.81,.02
.
.
.
16.665,6.12
```

PROGRAM

PROGRAM DYNA

c * Program DYNA approximates the force - displacement relationship
c * for nonlinear multiple degree of freedom systems

DEBUG
DECLARE

```
      IMPLICIT DOUBLE PRECISION (a-h,o-z)
      DOUBLE PRECISION  nfq,ki(11),M(10),C(11),Fspb(11),factor(11),
     +kk(11,5),fm(11,5),dis(10),vel(10),acc(10),disi(10),phn(10),dn(11),
     +veli(10),acci(10),umx(11),fq(10),oldumx(11),np(10),disb(10),
     +ta(10,80),fa(10,80),pi(10),Kh(10,2),u(11,0:5),delP(10),phl(10),
     +Fspi(11),Fsp(11),dsi(10),ck(11),ckv(11),dtP(10),Dp(10),pp(10),
     +M1(10),C1(10),Kw(10,2),KwI(10,2),deldis(10),delvel(10),delacc(10),
     +elgb(11),xfsp(11),pck(10),A(10),fmx(11),fult(11),sgn(11),Fi(10),
     +Fc(10),pwfac(10),va(11),elg(11),CN(10,2),elgmax(11),egv(11),
     +gac(0:80),tac(0:80),slope(0:80),xi(0:80),Ci(0:80),Di(0:80),
     +zacc(10),zvel(10),zdis(10),xo(0:80),vo(0:80),ndis(10),nvel(10),
     +nacc(10),phi(10),vn(11),Dpw(10),nk(11),egvi(11),Eig(10,2)
      integer CODE(11),KODE(11),LODE(10),stf(11),fl
      character40 idata,odata

      write(,800)
      write(,810)
      read(,820)idata
      open(unit=65,file=idata,status='old')
      write(,830)
      read(,820)odata
      open(unit=75,file=odata,status='unknown')

      read(65,)ndof,tolp,tolv
      CALL DATAIN(ndof,bta,gam,dis,vel,A,fq,C,M,ki,fult,umx,Fsp,time,
     +     acc,ti,ta,fa,np,CODE,KODE,LODE,KK,FM,nk,delt,Kh,elgmax,CN,
     +     naccp,gac,tac,eqk,slope,vo,xo)

      write(75,70)
      delti = delt
      fl = 1
      t2 = 0.0
      diffmx = 0.0
      do 1 i = 1,ndof+1
        if(i.ne.ndof+1) factor(i) = 1
        u(i,0) = 0.0
        oldumx(i) = umx(i)
        fmx(i) = Fsp(i)
```

```
      stf(i) = 1
      egv(i) = 0.0
  1 continue
100 delt = delti
    do 2 i = 1,ndof
      pwfac(i) = 1.0
      disi(i) = dis(i)
      veli(i) = vel(i)
      acci(i) = acc(i)
      Fspi(i) = Fsp(i)
      egvi(i) = egv(i)
      dsi(i) = 0.0
      ck(i) = 1.
      ckv(i) = 1.
  2 continue
      wflag = 1.0
      Fspi(ndof+1) = Fsp(ndof+1)
      egvi(ndof+1) = egv(ndof+1)
      ck(ndof+1) = 1.
    CALL LOAD(LODE, A, fq, ti, ta, fa, np, pi, ndof)
    t = ti + delt
    CALL LOAD(LODE, A, fq, t, ta, fa, np, pp, ndof)
    do 3 i = 1,ndof
      dtP(i) = pp(i) - pi(i)
      Dp(i) = dtP(i)
      if(egv(i).ge.0.0) sgn(i) = 1.0
      if(egv(i).lt.0.0) sgn(i) = -1.0
      xfsp(i) = fult(i) - dabs(Fsp(i))
      if (i.eq.1) elg(i) = dis(i)
      if (i.ne.1) elg(i) = dis(i) - dis(i-1)
      elgb(i) = (elg(i) - elgmax(i))sgn(i)
  3 continue
    if(egv(ndof+1).ge.0.0) sgn(ndof+1) = 1.0
    if(egv(ndof+1).lt.0.0) sgn(ndof+1) = -1.0
    xfsp(ndof+1) = fult(ndof+1) - dabs(Fsp(ndof+1))
    elg(ndof+1) = -dis(ndof)
    elgb(ndof+1) = (elg(ndof+1) - elgmax(ndof+1))sgn(ndof+1)
200 CALL STIFF(sgn, Fsp, fmx, fult, ki, elgb, KODE, Kh, kk, fm, stf, nk, ndof)
    Call STFMAT(C, CN, ndof)
    do 4 i = 1,ndof
    do 4 j = 1,2
```

$$Kw(i,j) = Kh(i,j) + \left(\frac{gam}{btadelt}\right)CN(i,j)$$

$$4 \quad \text{if (j.eq.1) } Kw(i,j) = Kw(i,j) + 1.\frac{}{btadelt^2}M(i)$$

```
    do 5 i = 1,ndof
```

5 $M1(i) = M(i)\left[\left(\frac{1.}{bta\,delt}\right)veli(i) + \left(\frac{1.}{2.bta}\right)acci(i)\right]$

do 6 i = 1,ndof

6 $va(i) = \left(\frac{gam}{bta}\right)veli(i) + \left[\left(\frac{gam}{2.bta}\right) - 1.\right]deltacci(i)$

c * do if nband is greater than two

```
c     do 7 k = 1,ndof
c       C1(k) = 0.0
c       do 8 i = 1,nband
c 8      C1(k) = CN(k,i)va(k+i-1) + C1(k)
c       do 9 i = 1,nband-1
c         if((k-i).eq.0.0) go to 70
c 9      C1(k) = C1(k) + CN(k-1,i+1)va(k-1)
c 7 continue
```

c * do if nband equals two

```
      do 7 i = 2,ndof
        if (i.eq.ndof) va(ndof+1) = 0.0
 7    C1(i) = CN(i-1,2)va(i-1) + CN(i,1)va(i) + CN(i,2)va(i+1)
        C1(1) = CN(1,1)va(1) + CN(1,2)va(2)
      do 8 i = 1,ndof
 8    Dpw(i) = Dp(i) + pwfac(i)(M1(i) + C1(i))

      Call UDU(Kw,Dpw,1,ndof,2,ndof,2,40)

      do 9 i = 1,ndof
        deldis(i) = Dpw(i) + dsi(i)
```

$delvel(i) = \left(\frac{gam}{bta}\right)\left(\frac{1.}{delt}\right)deldis(i) - \left(\frac{gam}{bta}\right)veli(i)$

$+ \;+ \left[1. - \left(\frac{gam}{2.bta}\right)\right]deltacci(i)$

$delacc(i) = \left(\frac{1.}{bta}\right)\left(\left(\frac{1.}{delt^2}\right)deldis(i)\right.$

$+ \qquad - \left(\frac{1.}{delt}\right)veli(i) - 0.5acci(i))$

9 dsi(i) = deldis(i)

c * Initially set sign functions for first loop

```
      if (fl.gt.0.0) then
        do 10 i = 1,ndof
          if (delvel(i).gt.0.0.or.delvel(i).eq.0.0) sgn(i) = 1.
          if (delvel(i).lt.0.0) sgn(i) = -1.
 10     continue
        if (delvel(ndof).gt.0.0) sgn(ndof+1) = -1.
        if (delvel(ndof).lt.0.0.or.delvel(ndof).eq.0.0) sgn(ndof+1)=1.
        fl = -1
      end if

c * Define total elongation

      do 11 i = 1,ndof
        dis(i) = disi(i) + deldis(i)
        if (i.eq.1) elg(i) = dis(i)
 11     if (i.ne.1) elg(i) = dis(i) - dis(i-1)
      elg(ndof+1) = -dis(ndof)
      do 12 i = 1,ndof+1
 12     elgb(i) = (elg(i) - elgmax(i))sgn(i)

c * Must check if the change in spring force brought the force
c * over the ultimate force allowed for either the softening
c * case or the linear perfectly plastic case.

      do 13 i = 1,ndof+1
        if (KODE(i).EQ.1) then
          if (Fsp(i).eq.fm(i,stf(i))sgn(i)) go to 13
          if (stf(i).eq.1) then
            Fspb(i) = elgb(i)kk(i,stf(i))
          else
            Fspb(i) = 0.0
            do 14 j = 1, stf(i)-1
 14           Fspb(i) = (u(i,j) - u(i,j-1))kk(i,j) + Fspb(i)
            Fspb(i) = Fspb(i) + (elgb(i) - u(i,stf(i)-1))kk(i,stf(i))
          end if
          Fsp(i) = sgn(i)Fspb(i) + fmx(i)
          xfsp(i) = fm(i,stf(i)) - (Fsp(i))sgn(i)
          if (kk(i,stf(i)).lt.0.0) xfsp(i) = -xfsp(i)
          if (xfsp(i).lt.0.0.and.dabs(xfsp(i)).lt.tolP) then
            Fsp(i) = fm(i,stf(i))sgn(i)
            ck(i) = -1.
            go to 13
          elseif (xfsp(i).lt.0.0) then
            delt = 0.5delt
            t = ti + delt
            CALL LOAD (LODE,A,fq,t,ta,fa,np,pp,ndof)
            do 15 k = 1,ndof
```

```
          if (ck(k).lt.0.0) ck(k) = 1.0
          if (ckv(k).lt.0.0) ckv(k) = 1.0
         Dp(k) = pp(k) - pi(k)
         Fsp(k) = Fspi(k)
         dis(k) = disi(k)
         vel(k) = veli(k)
         if (k.eq.1) then
          elg(k) = dis(k)
          egv(k) = vel(k)
         else
          elg(k) = dis(k) - dis(k-1)
          egv(k) = vel(k) - vel(k-1)
         end if
         elgb(k) = (elg(k) - elgmax(k))sgn(k)
         acc(k) = acci(k)
         dsi(k) = 0.0
         pwfac(k) = 1.0
15       continue
        elg(ndof+1) = -dis(ndof)
        elgb(ndof+1) = (elg(ndof+1) - elgmax(ndof+1))sgn(ndof+1)
        Fsp(ndof+1) = Fspi(ndof+1)
        egv(ndof+1) = -vel(ndof)
        go to 200
       end if
      elseif (KODE(i).EQ.2) then
```

$$\text{Fspb(i)} = \text{elgb(i)}/\left(\left[1.\frac{}{\text{ki(i)}}\right]\right.$$

$$+ \qquad + \left.\left[\frac{\text{elgb(i)}}{\text{fult(i) - fmx(i)sgn(i)}}\right]\right)$$

```
       Fsp(i) = sgn(i)Fspb(i) + fmx(i)
      end if
 13 continue

    do 16 i = 1,ndof
      vel(i) = veli(i) + delvel(i)
      acc(i) = acci(i) + delacc(i)
      if(i.eq.1) egv(i) = vel(i)
      if(i.ne.1) egv(i) = vel(i) - vel(i-1)
 16 continue
    egv(ndof+1) = -vel(ndof)

c * Check if velocity has reached zero or has changed signs.

    do 17 i = 1,ndof+1
      if (dabs(egv(i)).lt.tolV.and.(egv(i)egvi(i)).lt.0.0)then
```

```
c * When resonance is reached or near, the displacements become very
c * large and therefore, the delt becomes very large. The following
c * five lines account for this and subsequently reduce the delt

        oldumx(i) = dmax1(dabs(umx(i)),dabs(oldumx(i)))
        if (dabs(oldumx(i)).gt.1.0d-8) then
          factor(i) = dabs[umx(i)/oldumx(i)]
        end if
        if (factor(i).gt.1)  delti = delti/factor(i)
        ckv(i) = -1.0
      elseif ((egvi(i)egv(i)).lt.0.0) then
        delt = 0.5delt
        t = ti + delt
        CALL LOAD (LODE,A,fq,t,ta,fa,np,pp,ndof)
         do 18 k = 1,ndof
           if (ck(k).lt.0.0) ck(k) = 1.0
           if (ckv(k).lt.0.0) ckv(k) = 1.0
           Dp(k) = pp(k) - pi(k)
           Fsp(k) = Fspi(k)
           dis(k) = disi(k)
           vel(k) = veli(k)
           if (k.eq.1) then
             elg(k) = dis(k)
             egv(k) = vel(k)
           else
             elg(k) = dis(k) - dis(k-1)
             egv(k) = vel(k) - vel(k-1)
           end if
           elgb(k) = (elg(k) - elgmax(k))sgn(k)
           acc(k) = acci(k)
           dsi(k) = 0.0
           pwfac(k) = 1.0
18        continue
         elg(ndof+1) = -dis(ndof)
         elgb(ndof+1) = (elg(ndof+1) - elgmax(ndof+1))sgn(ndof+1)
         Fsp(ndof+1) = Fspi(ndof+1)
         egv(ndof+1) = -vel(ndof)
         go to 200
      end if
17 continue

    delP1 = 0.0
    do 19 i = 1,ndof
19  Fi(i) = M(i)acc(i)
```

```
      do 20 i = 1,ndof
 20     vn(i) = vel(i)
      do 21 i = 2,ndof
        if (i.eq.ndof) vn(ndof+1) = 0.0
 21     Fc(i) = CN(i-1,2)vn(i-1) + CN(i,1)vn(i) + CN(i,2)vn(i+1)
        Fc(1) = CN(1,1)vn(1) + CN(1,2)vn(2)
      do 22 i = 1,ndof
        pck(i) = Fi(i) + Fc(i) + Fsp(i) - Fsp(i+1)
        delP(i) = pp(i) - pck(i)
 22     delP1 = delP(i)delP(i) + delP1
      adelP = dsqrt(delP1)
      if (adelP.GT.tolP) then
        do 23 i = 1,ndof
          Dp(i) = delP(i)
          pwfac(i) = 0.0
          if (ck(i).lt.0.0) ck(i) = 1.0
          if (ckv(i).lt.0.0) ckv(i) = 1.0
 23     continue
          go to 200
      else
        do 24 i = 1,ndof
          if (eqk.eq.1) then
            do 25 j = 1,naccp
              k = j-1
              if (t.lt.tac(j)) then
                tp = t - tac(k)
                call acalc(gac(k),slope(k),tp,zacc(i))
                call vcalc(gac(k),slope(k),tp,vo(k),zvel(i))
                call xcalc(gac(k),slope(k),tp,vo(k),xo(k),zdis(i))
                go to 150
              end if
 25         continue

c * Calculate actual displacements

 150      continue
          end if
          nacc(i) = acc(i) + zacc(i)
          nvel(i) = vel(i) + zvel(i)
          ndis(i) = dis(i) + zdis(i)
          ti = t
          if (ck(i).lt.0.0) then
              if (KODE(i).eq.1) then
                if (stf(i).eq.1) then
                      Fspb(i) = (Fsp(i) - fmx(i))sgn(i)
```

$$elgb(i) = \frac{Fspb(i)}{kk(i,stf(i))}$$

```
          else
                    elgb(i) = u(i,stf(i)-1) +
     +                          fm(i,stf(i)) - fm(i,stf(i)-1)
                                ------------------------------
                                        kk(i,stf(i))
           end if
          end if
         u(i,stf(i)) = elgb(i)
         stf(i) = stf(i) + 1
         wflag = -1.
        end if
        if (ckv(i).lt.0.0) then
         stf(i) = 1
         umx(i) = dis(i)
         fmx(i) = Fsp(i)
         elgmax(i) = elg(i)
         wflag = -1.
        end if
         if (elg(i).gt.diffmx) then
          diffmx = elg(i)
          imx = i
          tmx = t
        end if
 24    continue
        if (t.ge.(t2+.1).or.wflag.lt.0.0) then
c          write(75,75) t,(ndis(i),i = 1,ndof),(Fsp(j),j = 1,ndof)
c          write(75,75) t,(elg(i),i = 1,ndof),(Fsp(j),j = 1,ndof)
         write(75,75) t,ndis(10),Fsp(10)
         write(,)'time = ',t
         t2 = t
         wflag = -1.
        end if
       if (t.lt.time) go to 100
      end if
      imx2 = imx - 1
      write(75,76)imx,imx2,diffmx,tmx
 70 Format(,3X,'Time, Displacements from 1 to ndof and Forces from
  +1 to ndof', )
 75 Format(23(e11.3))
 76 Format('Maximum difference occurs between floors ',i4,'& ',i4,,
   + 'The difference and time are:  ',e11.3,e11.3, )
800 Format(T10,'   Step-by-step Integration   ',T10,
   +  '   Dynamic Equation of Motion   ',,T10'
```

```
     + '* Written by Nicole Stapleton *',-)
810 Format(/,5X,'Enter input data Filename:  ')
820 Format(A40)
830 Format(/,5X,'Enter output data Filename:  ')
     STOP
     END
```

```
      SUBROUTINE DATAIN(ndof,bta,gam,dis,vel,A,fq,C,M,ki,fult,umx,Fsp,
     + time,acc,ti,ta,fa,np,CODE,KODE,LODE,kk,fm,nk,delt,Kh,elgmax,CN,
     + naccp,gac,tac,eqk,slope,vo,xo)

c * Subroutine DATAIN reads in the neccessary parameters to begin
c * the step - by - step iteration scheme of mdof system
DEBUG
DECLARE
      IMPLICIT DOUBLE PRECISION (a-h,o-z)
      DOUBLE PRECISION nfq,ki(ndof+1),M(ndof),C(ndof+1),kk(ndof+1,5),
     +fm(ndof+1,5),dn(11),vn(11),dis(ndof),vel(ndof),acc(ndof),C1(10),
     +umx(ndof+1),A(ndof),fq(ndof),Phl(10),Phn(10),np(10),Phi(10),
     +ta(ndof,150),fa(ndof,150),pi(10),Kh(ndof,1),K1(10),Eig(10,2),lam,
     +fult(ndof+1),elgmax(ndof+1),Fsp(ndof+1),nk(ndof+1),CN(ndof,2),
     +gac(0:80),tac(0:80),slope(0:80),xi(0:80),Ci(0:80),Di(0:80),
     +vo(0:80),xo(0:80)

      INTEGER CODE(ndof+1),KODE(ndof+1),LODE(ndof)
      pii = 3.141592654
      tolp = .000001

c * Read in Newmark coefficients and initial conditions

      read(65,)bta,gam,ti,time,timediv
      do 5 i = 1,ndof
        read(65,) M(i),dis(i),vel(i)
        umx(i) = dis(i)
        if (i.eq.1) elgmax(i) = dis(i)
        if (i.ne.1) elgmax(i) = dis(i) - dis(i-1)

c       * Determine function of K and C
c       *     CODE, KODE = 1 :  Varying linear functions of C,K
c       *                       NK = number of stiffnesses
c       *                       KK(i) = siffness(j) of elem i
c       *                       FM(i,j) = fult(j) of elem i
c       *                       UMX(i) = beginning point for disp
c       *     CODE, KODE = 2 :  Nonlinear C, K

        read(65,)CODE(i),KODE(i)
        if (CODE(i).EQ.1) read(65,) C(i)
        if (KODE(i).EQ.1) then
          read(65,)nk(i)
          do 10 j = 1,nk(i)
            read(65,)kk(i,j),fm(i,j)
 10         if (j.eq.1) ki(i) = kk(i,j)
          if (i.eq.1) Fsp(i) = ki(i)dis(i)
```

```
      if (i.ne.1) Fsp(i) = ki(i)(dis(i)-dis(i-1))
    end if
    if (KODE(i).EQ.2) then
      read(65,) ki(i),fult(i)
```

$$\text{if(i.eq.1) Fsp(i)} = \frac{\text{dis(i)}}{\left(1.\frac{}{\text{ki(i)}}\right) + \left(\frac{\text{dis(i)}}{\text{fult(i)}}\right)}$$

$$\text{if(i.ne.1) Fsp(i)} = \text{dis(i)-dis(i-1)/(}\left(1.\frac{}{\text{ki(i)}}\right)$$

$$+ \qquad\qquad + \left(\frac{\text{dis(i)-dis(i-1)}}{\text{fult(i)}}\right))$$

```
    end if
  5 continue
   read(65,)CODE(ndof+1),KODE(ndof+1)
   if (CODE(ndof+1).EQ.1) read(65,) C(ndof+1)
   if (KODE(ndof+1).EQ.1) then
     read(65,)nk(ndof+1)
     do 15 j = 1,nk(ndof+1)
       read(65,)kk(ndof+1,j),fm(ndof+1,j)
 15     if (j.eq.1) ki(ndof+1) = kk(ndof+1,j)
     Fsp(ndof+1) = ki(ndof+1)(-dis(ndof))
   end if
   if (KODE(ndof+1).EQ.2) then
     read(65,) ki(ndof+1),fult(ndof+1)
```

$$\text{Fsp(ndof+1)} = \text{dis(ndof+1)-dis(ndof)/(}\left(1.\frac{}{\text{ki(ndof+1)}}\right)$$

$$+ \qquad\qquad + \left(\frac{\text{dis(ndof+1)-dis(ndof)}}{\text{fult(ndof+1)}}\right))$$

```
   end if

c * Read in force function
c *   nkl =     number of external loads
c *   nl  =     element load is applied on
c *   LODE = 1: sinusoidal load
c *             read in parameters A, w of p(t)=Asin(wt)
c *   LODE = 2: point load w.r.t. time
c *             read in:  of pts,np; time array ta(i,j);
c *                       force array fa(i,j)

c * Determine whether load is earthquake load
c *       eqk = 1 earthquake loads
c             = 0 externally applied loads
c         naccp = number of acceleration points
c *    gac(i) = gravitational acceleration due to earthquake
```

```
c *   tac(i) = time step for acceleration point loads

      read(65,) eqk
      if (eqk.eq.1) then
        read(65,) naccp
        do 20 i = 1,ndof
          LODE(i) = 2
20        np(i) = naccp
        do 25 i = 1,naccp
          j = i-1
          read(65,) gac(j), tac(j)
          do 30 k = 1,ndof
            ta(k, i) = tac(j)
            fa(k, i) = -M(k)gac(j)
30        continue
          CALL INTERP(ta, fa, np, ti, pi, ndof)
          if (i.ne.1) then
                         gac(j) - gac(j-1)
            slope(j-1) = -----------------
                         tac(j) - tac(j-1)
          end if
25      continue
        vo(0) = 0.0
        xo(0) = 0.0
        do 31 i = 1,naccp-1
          tp = tac(i) - tac(i-1)
          CALL VCALC(gac(i-1),slope(i-1),tp,vo(i-1),vo(i))
          CALL XCALC(gac(i-1),slope(i-1),tp,vo(i-1),xo(i-1),xo(i))
31      continue
      else
        do 35 i = 1,ndof
          read(65,) LODE(i)
          if (LODE(i).EQ.1) then
            read(65,)A(i), fq(i)
            pi(i) = A(i)dsin(fq(i)ti)
          elseif (LODE(i).EQ.2) then
            read(65,)np(i), A(i)
            do 40 j = 1,np(i)
40            read(65,)ta(i, j), fa(i, j)
            CALL INTERP(ta, fa, np, ti, pi, ndof)
          end if
35      continue
      end if

c * Calculate initial acceleration

      call STFMAT(ki, Kh, ndof)
```

```
      call STFMAT(C,CN,ndof)

c* Calculate initial acceleration

      do 45 i = 1,ndof
 45   dn(i) = dis(i)
      do 50 i = 2,ndof
        if (i.eq.ndof) dn(ndof+1) = 0.0
 50   K1(i) = Kh(i-1,2)dn(i-1) + Kh(i,1)dn(i) + Kh(i,2)dn(i+1)
        K1(1) = Kh(1,1)dn(1) + Kh(1,2)dn(2)
      do 55 i = 1,ndof
 55   vn(i) = vel(i)
      do 60 i = 2,ndof
        if (i.eq.ndof) vn(ndof+1) = 0.0
 60   C1(i) = CN(i-1,2)vn(i-1) + CN(i,1)vn(i) + CN(i,2)vn(i+1)
        C1(1) = CN(1,1)vn(1) + CN(1,2)vn(2)
      do 70 i = 1,ndof
 70   acc(i) = 1/M(i) (pi(i)-C1(i)-K1(i))

c * Calculate delt by finding optimum frequency

      call STFMAT(Ki,Kh,ndof)
      do 75 i = 1,ndof
      do 75 j = 1,2
 75   Eig(i,j) = Kh(i,j)/M(i)
      if (ndof.eq.1) then
        Phi(1) = 1.
        go to 200
      end if
      do 80 i = 1,ndof-1,2
        Phi(i) = 2./3.
 80   Phi(i+1) = 1./3.
100 do 85 i = 2,ndof-1
        Phl(i) = Eig((i-1),2) M(i-1)/M(i) Phi(i-1) + Eig(i,1)Phi(i)
   +           + Eig(i,2)Phi(i+1)
 85 continue
      Phl(ndof) = Eig(ndof-1,2) M(ndof-1)/M(ndof) Phi(ndof-1)
   +           + Eig(ndof,1)Phi(ndof)
200 Phl(1) = Eig(1,1)Phi(1) + Eig(1,2)Phi(2)
      ckph = 0.0
```

```
    lam = Phl(1)
    do 90 i = 1,ndof
      Phn(i) = Phl(i) / lam
 90   ckph = ckph + (Phn(i) - Phi(i))^2
    if (dsqrt(ckph).lt.tolp) then
      go to 150
    else
      do 95 i = 1,ndof
 95   Phi(i) = Phn(i)
      go to 100
    end if
150 nfq = dsqrt(lam)
    amp = 2 pii / nfq
    delt = amp / timediv
    return
    end
```

```
      SUBROUTINE STIFF(sgn,fsp,fmx,fult,ki,elgb,KODE,Kh,kk,fm,stf
     +                ,nk,ndof)

c * Subroutine STIFF calculates the stiffness of the system
c * whether the system is linear or nonlinear
DEBUG
DECLARE
      IMPLICIT DOUBLE PRECISION (a-h,o-z)
      DOUBLE PRECISION ki(ndof+1),Kh(ndof,2),kk(ndof+1,5),elgb(ndof+1)
     +,fm(ndof+1,5),Kh1(11),sgn(ndof+1),fmx(ndof),f1(11),fsp(ndof+1)
     +,fult(ndof+1),nk(ndof+1)
      INTEGER KODE(ndof+1),stf(ndof+1)

      do 10 i = 1,ndof+1
        if (KODE(i).EQ.1) then
          if (fsp(i).eq.fm(i,nk(i))sgn(i).and.fl.lt.0.0) then
            Kh1(i) = kk(i,nk(i))
            go to 10
          else
            Kh1(i) = kk(i,stf(i))
            go to 10
          end if
        elseif (KODE(i).EQ.2) then
          f1(i) = fult(i) - fmx(i)sgn(i)
```

$$Kh1(i) = ki(i)\frac{f1(i)}{f1(i)+ki(i)elgb(i)}$$

$$+ \qquad - (ki(i)^2.)f1(i)\frac{elgb(i)}{(f1(i)+ki(i)elgb(i))^2}.$$

```
          go to 10
        end if
   10 continue
      call stfmat(Kh1,Kh,ndof)
      return
      end
```

```
      SUBROUTINE STFMAT(Kh1,Kh,ndof)

c * Subroutine STFMAT calculates the stiffness matrix of the system
c * given the various stiffnesses of all the degrees of freedom
c *      Kh(i,j j=1,2) banded form of Stiffness matrix

DEBUG
DECLARE
      IMPLICIT DOUBLE PRECISION (a-h,o-z)
      DOUBLE PRECISION Kh(ndof,2),Kh1(ndof+1)

      Do 10 i = 1,ndof-1
        Kh(i,1) = Kh1(i) + Kh1(i+1)
   10   Kh(i,2) = -Kh1(i+1)
      Kh(ndof,1) = Kh1(ndof) + Kh1(ndof+1)
      Kh(ndof,2) = 0.0

      return
      end
```

```
      SUBROUTINE LOAD(LODE,A,fq,t,ta,fa,np,p,ndof)

c * Subroutine LOAD calculates the load for a given case;
c * either for sinusoidal loads or given point loads
DEBUG
DECLARE
      IMPLICIT DOUBLE PRECISION (a-h,o-z)
      DOUBLE PRECISION ta(ndof,80),fa(ndof,80),p(ndof),np(ndof)
     +                 ,A(ndof),fq(ndof)
      INTEGER LODE(ndof)

      do 10 i = 1,ndof
        if (LODE(i).EQ.1) then
          p(i) = A(i)dsin(fq(i)t)
       elseif(LODE(i).EQ.2) then
          CALL INTERP(ta,fa,np,t,p,ndof)
        end if
   10 continue

      return
      end
```

```
      SUBROUTINE INTERP(ta,fa,NP,T,P,ndof)

c * Subroutine INTERP uses linear interpolation to determine
c * the load given different time increment from given
c * forces verses given times
DEBUG
DECLARE
      IMPLICIT DOUBLE PRECISION (a-h,o-z)
      DOUBLE PRECISION ta(ndof,80),fa(ndof,80),p(ndof),NP(ndof)

      do 10 i = 1,ndof
        if (T.GT.ta(i,NP(i))) then
          P(i) = 0.0
        else
          flag = 0.0
          x = 0.0
          do 20 j = 1,NP(i)
            if (T.LT.ta(i,j)) then
              if (flag.GT.0.0) then
                P(i) = ( fa(i,x+1) - fa(i,x) )
                       ( ta(i,x+1) - ta(i,x) )
     +                 (T-ta(i,x)) + fa(i,x)
              go to 10
              else
                P(i) = 0.0
                go to 10
              end if
            elseif (T.EQ.ta(i,j)) then
                P(i) = fa(i,j)
                go to 10
            elseif (T.GT.ta(i,j)) then
                flag = 1.
                x = j
            end if
 20       continue
        end if
 10   continue
      return
      end
```

```
C*****************
C
C         SUBROUTINE  UDU
C
C
C      A SUBROUTINE FOR THE FACTORIZATION OF A SYMMETRIC
C      BANDED MATRIX AND THE SOLUTION OF THE MATRIX EQUATION
C
C              [A](X) =(F)
C
C
C     A        = NAME OF MATRIX (UNFACTORED IF LU=1, FACTORED
C               IF LU=0).  NOTE:  THE A-MATRIX MUST BE IN THE
C               PROPER BANDED FORM USED FOR SYMMETRIC MATRICES.
C     Y        = RIGHT HAND SIDE WHEN SUBROUTINE IS CALLED FROM
C               MAIN PROGRAM.  (I.E. Y = F)
C             = SOLUTION OF MATRIX EQUATION WHEN SUBROUTINE IS
C               RETURNED TO MAIN PROGRAM.  (I.E. Y = X)
C     LU       = 1  MEANS THAT THE A-MATRIX IS IN OGIGINAL FORM.
C                 FACTORIZATION IS DESIRED AS WELL AS THE
C                 SOLUTION VECTOR, Y = X.
C             = 0  MEANS THAT THE A-MATRIX AS ENTERED IS ALREADY
C                 FACTORED AND ALL THAT IS DESIRED IS THE
C                 SOLUTION VECTOR, Y = X.
C        NUMEQ      = THE NUMBER OF EQUATIONS REPRESENTED BY THE
MATRIX
C               EQUATION.  NOTE: THIS MUST BE EQUAL TO OR LESS
C               THAN THE FIRST DIMENSION OF THE A-MATRIX.
C     IB       = THE 'HALF' BAND WIDTH OF THE SET OF EQUATIONS.
C               NOTE : THIS VALUE MUST BE EQUAL TO OR LESS THAN
C               THE SECOND DIMENSION OF THE A-MATRIX.
C     IDIM      = FIRST DIMENSION OF THE A-MATRIX AS SPECIFIED
C               IN THE DIMENSION STATEMENT IN THE CALLING
C               PROGRAM.
C     JDIM      = SECOND DIMENSION OF THE A-MATRIX AS SPECIFIED
C               IN THE DIMENSION STATEMENT OF THE CALLING
C               PROGRAM.
C     LLPT      = TAPE (OR DISK) NUMBER USED FOR OUTPUT.
C               (USUALLY, THIS IS EQUAL TO 6)
C
C
C*****************

DEBUG
DECLARE
   SUBROUTINE UDU(A,Y,LU,NUMEQ,IB,IDIM,JDIM,LLPT)
```

```
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION A(IDIM,JDIM), Y(IDIM)

c     IF(NUMEQ.GT.IDIM) GO TO 7001
c     IF(IB   .GT.JDIM) GO TO 7002

      NEM1 = NUMEQ-1

C     BEGIN FORWARD ELIMINATION

      DO 450 I = 1,NEM1
        JEND = NUMEQ-1+1
        IF(JEND.GT.IB) JEND = IB

        DO 440 J = 2,JEND
          J1 = I+J-1
          IF(LU.EQ.0) GO TO 435
```

$$FAC = \frac{A(I,J)}{A(I,1)}$$

```
          K1 = 0
          DO 430 K = J,JEND

            K1 = K1 + 1
            A(J1,K1) = A(J1,K1) - A(I,K)FAC
430       CONTINUE
        A(I,J) = FAC

435     CONTINUE
        Y(J1) = Y(J1) - Y(I)A(I,J)
440     CONTINUE
450   CONTINUE

C     BEGIN BACK SUBSTITUTION
```

$$Y(NUMEQ) = \frac{Y(NUMEQ)}{A(NUMEQ,1)}$$

```
      DO 470 I = 1,NEM1
        I1 = NUMEQ - I
```

```
      Y(I1) = Y(I1) / A(I1,1)
      JEND = NUMEQ -I1 + 1
      IF(JEND.GT.IB) JEND = IB
      DO 460 J = 2,JEND
        J1 = I1 + J -1
        Y(I1) = Y(I1) - A(I1,J)  Y(J1)
460    CONTINUE
470  CONTINUE

     RETURN

C    ERROR MESSANGES

c7001 WRITE(LLPT,11) NUMEQ,IDIM
c    STOP

c7002 WRITE(LLPT,12) IB, JDIM
c    STOP

c11  FORMAT('NUMBER OF EQUATIONS EXCEEDS IDIM.', ,'NUMEQ =',I5,
c   &'  IDIM = ',I5)
c12  FORMAT('BAND WIDTH EXCEEDS JDIM.', ,'IB =',I5, '   JDIM = ',I5)

     END
```

```
      SUBROUTINE ACALC(gp,xm,tp,za)

c * Subroutine ACALC calculates the acceleration at a given time t
c * Typical values:  gp = acceleration at point passed
c *                  xm = slope at point passed
c *                  tp = time at point passed

      IMPLICIT DOUBLE PRECISION (a-h,o-z)

      za = gp + xmtp
      return
      end

      SUBROUTINE VCALC(gp,xm,tp,vp,v)

c * Subroutine VCALC calculates the velocity at a given time t

      IMPLICIT DOUBLE PRECISION (a-h,o-z)

      v = gptp + 0.5xmtp^2 + vp
      return
      end

      SUBROUTINE XCALC(gp,xm,tp,vp,xp,x)

c * Subroutine VCALC calculates the velocity at a given time t

      IMPLICIT DOUBLE PRECISION (a-h,o-z)

      x = 0.5gptp^2 + (1./6.)xmtp^3 + vptp + xp

      return
      end
```

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