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Zhao, Zhengwei, M.S.
The University of Arizona, 1991
RELIABILITY ANALYSIS OF FATIGUE UNDER RANDOM LOADING CONSIDERING MODELING UPDATING THROUGH INSPECTION IN THE MARTA BRIDGE SYSTEM

by

ZHENGWEI ZHAO

A Thesis Submitted to the Faculty of the DEPARTMENT OF CIVIL ENGINEERING & ENGINEERING MECHANICS In Partial Fulfillment of the Requirements For the Degree of MASTER OF SCIENCE WITH A MAJOR IN CIVIL ENGINEERING In the Graduate College THE UNIVERSITY OF ARIZONA

1991
STATEMENT BY AUTHOR

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Professor of Civil Eng. and Eng. Mechanics  Date
TO MY DEAR WIFE

YUHONG WU
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<td>$A_i$</td>
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<td></td>
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<tr>
<td>ASMM</td>
<td>Advanced Second Moment Method</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>Fatigue Strength Exponent in S-N Curve</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Fatigue Growth Parameter in Paris Equation</td>
<td></td>
</tr>
<tr>
<td>COV</td>
<td>Coefficient of Variation</td>
<td></td>
</tr>
<tr>
<td>$C_X$</td>
<td>COV of X</td>
<td></td>
</tr>
<tr>
<td>COV[X,Y]</td>
<td>Covariance of Random Variables between X and Y</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Miner's Damage</td>
<td></td>
</tr>
<tr>
<td>$D \leq 0$</td>
<td>Event of Crack Detection</td>
<td></td>
</tr>
<tr>
<td>$D = 0$</td>
<td>Event with Crack Size Measurement</td>
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<td>$d\alpha/dN$</td>
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<td>$E(X)$</td>
<td>Expected Value of Random Variable X</td>
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<tr>
<td>$E_g$</td>
<td>Gross Error</td>
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<tr>
<td>$E_r$</td>
<td>Residual Random Error</td>
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</tr>
<tr>
<td>erfc()</td>
<td>Complementary Error Function</td>
<td></td>
</tr>
<tr>
<td>$E(S^m)$</td>
<td>Mean Stress Effect</td>
<td></td>
</tr>
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<td>$F(\alpha,Y)$</td>
<td>Geometry Function</td>
<td></td>
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<td>Cumulative Distribution Function of X</td>
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<td>$f_X(x)$</td>
<td>Probability Density Function of X</td>
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</tr>
<tr>
<td>FORM</td>
<td>First-Order Reliability Method</td>
<td></td>
</tr>
<tr>
<td>$g(Z) \leq 0$</td>
<td>Failure Function</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>Section Property</td>
<td></td>
</tr>
</tbody>
</table>
LIST OF NOTATIONS continued

I≤0  Event of No Crack Detection
K  Stress Intensity Factor
KC  Fracture Toughness
ΔK  Stress Intensity Range
ΔKth  Stress Intensity Threshold Range
LEFM  Linear Elastic Fracture Mechanics
M  Limit State Function of Fatigue Reliability
m  Fatigue Growth Exponent
n'  Cyclic Strain Hardening Exponent
N  Number of Load Cycles
N0  Crack Initiation Period
NDT  Non-destructive Testing
NDE  Non-destructive Evaluation
NDI  Non-destructive Inspection
Pf  Probability of Failure
Pf,sys  System Failure Probability
Pf,up  Updated Failure Probability
R=0  Event of Repair
Re[ ]  The Real Part
(τ,θ)  Polar Coordinate
S  Stress
ΔS  Stress Range
S0  Stress Range Parameter in Rayleigh Loading Model
LIST OF NOTATIONS continued

\( S_{th} \) Stress Range Threshold
\( \bar{\sigma} \) Mean Stress Effect
SORM Second Moment Reliability Method
\( U_i \) Standard Normal Variable
\( w \) Width of Specimen
\( z^* \) Design Value of Z
\( \alpha \) Crack Size
\( \alpha_0 \) Initial Crack Size
\( \alpha_d \) Crack Detectability
\( \alpha_c \) Critical Crack Size
\( \beta \) Reliability Index
\( \beta_M \) Initial Fatigue Reliability Index
\( \beta_{M,up} \) The Updated Fatigue Reliability Index
\( \beta_c \) The Critical Reliability Index
\( \Gamma() \) Gamma Function
\( \Delta \) Miner’s Sum at Failure
\( \Delta \varepsilon \) Strain Range
\( \lambda \) Statistic Parameter in Model of Detectability
\( \mu_X \) Statistic Parameter in Model of Detectability
\( \rho \) Correlation Coefficient
\( \sigma \) Stress
\( \sigma_{max} \) Maximum Stress Range
\( \sigma_X \) Standard Deviation of X
LIST OF NOTATIONS continued

\( \Delta \sigma \)  Stress Range
\( \tau \)  Shear Stress
\( \phi() \)  Probability Density Function of Normal Distribution
\( \Phi() \)  Normal Distribution Function
\( \Phi_k() \)  Multinormal Distribution Function
\( \Psi(\alpha_2, \alpha_1) \)  Damage Accumulation Function
ABSTRACT

A reliability-based inspection and maintenance procedure is proposed here for the fatigue damage potential evaluation of the full-penetration butt welds in MARTA's bridges. A stochastic fatigue crack growth model is formulated to account for the uncertainties in the material properties and environmental conditions.

Several methods of fatigue reliability analysis are identified. The sensitivity and capability of some of the inspection methods including ultrasonic inspection are investigated. The reliability of inspections for various events are formulated. A risk-based model of crack size measurement is proposed to account for the uncertainty in the equipment as well as the human error in interpreting the inspection signals.

Efficient algorithms for updating the reliability model through inspection are proposed. The fatigue safety index can be updated using the information obtained from the inspection.

A case study is conducted for MARTA's steel bridges. Fatigue reliability, inspection, repair, updating of random variables and inspection plan are investigated.
Chapter 1
Introduction

1.1 Problem Statement

MARTA (The Metropolitan Atlanta Rapid Transit Authority), created in 1965 by the Georgia General Assembly, is one of the best bus and railway public transportation systems in North America. As a major transportation service system, MARTA plays an important role in the economic expansion of the metro Atlanta area. It is essential that MARTA continue to provide reliable and dependable service in the future.

In an earlier study, Haldar and Josey (1988) identified all the major aerial structures in MARTA's railway system. Several hundred bridges with spans of varying length constitute MARTA's aerial structures; about 70 percent of these bridges are simply supported steel box girders with full-penetration butt weld connection in their tension flanges. These butt joints in steel bridges are subjected not only to constant forces due to the gravity load, but also to the reversal of forces due to the travel of trains on the rail producing repeated cyclic tensile loading on the connections.

MARTA's structures, as built, are in excellent condition, and there is no reason to believe that their condition has deteriorated appreciably since their construction. However, the structures are being loaded continuously with the cyclic tensile load. Furthermore, with time the material is being degraded and cracks, even in a very minute form, are expected to grow with time. In such a complicated case, fatigue, the process which causes premature failure or damage to a component subject to repeated tensile loading, may become the major cause of structural failure. In a study under the sponsorship of the American Society of Civil Engineers (1982), it was indicated that 80 to 90 percent of the failures in metallic
structures are related to fatigue and fracture, and most of them result from poor design. Thus, it is essential to develop a method to prevent these modes of failure.

In order to study and control the fatigue and fracture problem in MATRA's aerial structures, a thorough understanding of the factors that lead to these failures is necessary. Furthermore, for existing structures such as MARTA's aerial structures, inspection and maintenance are necessary and important tools to prevent failure due to fatigue and fracture. As will be discussed in detail in Chapter 4, a deterministic evaluation may not be adequate to address the problem. Since the uncertainties in the fatigue and fracture problem are numerous, a systematic evaluation of these uncertainties is essential in the overall reliability of bridge structures.

Nondestructive inspection and maintenance methods can be used to prevent fatigue and fracture failures. However, the currently available nondestructive inspection methods have yet to be perfected. Thus, imperfect inspection will add another source of uncertainty to the overall evaluation process.

It is clear that the fatigue and fracture problems of full-penetration butt welds in the tensile flange of box girders in MARTA's aerial structures are multifaceted. It is, indeed, very challenging to develop a basic understanding of the fatigue and fracture problem, to evaluate all the methods available to address the problem, to develop a risk-based model considering various sources of uncertainty and to use information from imperfect inspections to develop a bridge management program for MARTA's aerial structures.

In the following chapters of this study, all these items are addressed. Procedures are developed, and inspection schemes are suggested to maintain the overall integrity of MARTA's aerial structures.

1.2 Objectives

Since the problem of fatigue and fracture is extremely complicated and imperfect inspection, maintenance and repair procedures are essential tools to ensure the integrity of the system, the research must be conducted in several steps. The major objectives of this study can be identified as:
(1) To conduct a comprehensive literature review of structural fatigue and fracture to identify the major parameters and the methods used at present to address these problems.

(2) To develop a theoretical and feasible reliability-based method to assess and predict the structural fatigue reliability under cyclic loading, since the sources of uncertainty are numerous.

(3) To evaluate non-destructive inspection schemes that can be applied to inspect full-penetration butt welds in MARTA’s aerial structures and their corresponding accuracy and reliability.

(4) To introduce the imperfect inspection results into the risk model developed in Objective 2 and to reassess the reliability of structures; and

(5) To develop a strategy of inspection, maintenance and repairs to maintain the reliability of MARTA’s aerial structures above an acceptable level and to update the reliability model as additional information is generated during the inspection and maintenance.

1.3 Scope of the Research

The MARTA system consists of many types of structures ranging from tunnels to aerial structures. In this study, only the fatigue and fracture of full-penetration butt welds of steel bridges in MARTA’s aerial structures are studied. This type of study is particularly important since extensive evidence indicates that fatigue and fracture are the most common modes of failure for steel bridges. Furthermore, the consequence of failure could be catastrophic since MARTA is a public transportation system.

A typical full-penetration butt weld is shown in Figure 1.1. Initially, a simply supported span with two full-penetration butt welds in the tension flange are considered and a reliability-based algorithm is developed to study the fatigue and fracture problem. Based on this study of simply supported bridges and using the system reliability theory, the algorithm is extended to consider continuous span bridges.
Figure 1.1 Typical Full-Penetration Butt Weld
1.4 Organization

This study is presented in eight chapters. The first chapter is a brief introduction to this study. The second chapter gives a brief review of the literature on fatigue study, design and maintenance of steel bridges. The third chapter describes a stochastic model for fatigue crack growth considering the basic variables, including material parameters, crack initiation and load parameters. Chapter Four deals with the first-order and second-order reliability methods and the system reliability methods. Reliability of crack growth is also discussed, including crack growth to brittle fracture under random load process. Chapter Five reviews nondestructive inspection methods and their corresponding probabilistic models for crack detectability for inspections considering events with or without crack detection. Events with crack size measurement are also considered. Chapter Six proposes analytical methods of updating the reliability and reassessing uncertainties in the basic variables. Chapter Seven gives the description of some basic parameters in the fatigue risk assessment for MARTA's bridge system, including initial crack size, geometry, loading and material parameters. A case study is conducted, considering fatigue reliability, inspection, repair and updating of random variables, and inspection plan for MARTA's steel bridges is conducted. Chapter Eight contains the summary, conclusions and recommendations for future work.
Chapter 2

Literature Review

2.1 Introduction

For steel bridges with full-penetration butt weld, the failure modes can be classified as:

(1) strength failure, including fracture due to overload and excessive plastic deformation;
(2) serviceability failure, such as excessive deformation or vibration under service load conditions; and
(3) fatigue failure.

The strength and serviceability modes of failure are well understood in the profession. However, metal fatigue is a principal failure mode for many structures, such as bridges, aircraft, components in power plants, and offshore structures. This failure mode is still understood less than any other modes of metal failure.

It is essential at this stage to discuss the basic concepts of fatigue and fracture, since such understanding is necessary for the completion of this study. As mentioned earlier, fatigue is a process causing premature failure or damage to a component subjected to cyclic tensile loading. The physical process of fatigue is shown in Figure 2.1. According to ASCE(1982), the fatigue process can be divided into three phases:

(1) the crack initiation phase;
(2) the subcritical crack propagation phase; and
(3) the fracture stage.

Phase 3 results in unstable crack growth. Brittle failure occurs when the crack grows beyond a critical size, and the member separates into two parts, resulting in the loss of its load carrying capacity and possible collapse of the structure.
Direction of Loading

General Direction of Crack Growth

Stage I: Slip Plane Crack

Brittle Cleavage
Dominated Striaeions:
Found Mainly Under
Corrosion Fatigue
Conditions

Stage II

Fatigue "striaeions" are
smaller parallel lines,
revealed by microscopic
examination. One stri-
ation represents one
cycle of loading.

Stage III: Final Fracture

Crack Nucleation Usually
Occurs Along Planes of
Max. Shearing Stress

Visible "beach markings"
Attributed to different
Periods of growth.

Crack Initiation Phase

Subcritical crack propagation

High Strain Shear Deformation Behavior

Figure 2.1 An Illustration of General Behavior of Fatigue in Metal
2.2 Fatigue and Fracture

The first fatigue investigation appears to have been reported by a German mining engineer, Albert (Trong, 1989), who in 1829 performed repeated loading tests on iron chains. Some of the earliest in-service fatigue failures occurred in the axles of stagecoaches. With the rapid development of the railway system in the middle of the nineteenth century, fatigue failure in railway axles became a widespread problem and began to draw serious attention to cyclic loading effects. As is often done in the case of unexplained service failure, effects were made to reproduce the failure in the laboratory. Between 1852 and 1870, the German railway engineer, Wuhler (Rofle, 1987) set up and conducted the first systematic fatigue investigation. He conducted the tests on full-scale railway axles, and also conducted some small-scale bending, torsion and axial cyclic loading tests on several different materials. These data for Krupp axle steel were plotted for stress range versus cycles to failure in logarithmic scale. The resulting plot is widely known as the S-N diagram.

In the early twentieth century, with the development of higher-speed machinery and the aviation industry, much more attention was paid to the fatigue problem. Also, as the final result of fatigue, the theory of fracture became a very hot topic at that time. The first attempt to employ a mathematical approach to fracture mechanics was made by Inglis (1913), who analyzed the case of an elliptical hole in a plate under uniform tensile stress. His results were, of course, at variance with practical experience. This study was rationalized later by Griffith (1924) who applied energy conservation principles to the case of the centrally cracked plate.

By the mid-twentieth century, more widespread fatigue investigation was undertaken worldwide at both the microscopic and macroscopic levels. At the microscopic level, physicists and metallurgists attempted to explain the basic phenomena. At the macroscopic level, engineers attempted to design components and systems by employing semi-empirical design theories. The development of dislocation theory by Heages and Mitecell (1956) contributed much to the understanding of fatigue at the microscopic level. Development of the electron microscope by Hirsch,
Horne and Whelan (1956) greatly contributed to the direct observation of fatigue processes. Also, the invention of high-speed computers gave designers a powerful computational tool for making better estimates of fatigue life and strength. Finally, the improvement in fracture mechanics concepts in the past decade gave a better understanding of crack propagation behavior as well as providing the basis for a practical new failure prediction tool useful to a designer faced with fatigue loading conditions. The theory and applications of the fatigue and fracture mechanics equations are now well documented in a wide variety of references (Hertzberg, 1976; Gurney, 1979; Fuchs, 1980 and Broek, 1984); as well as the numerous works of the American Association of Testing and Material (ASTM), such as ASTM STP 738. For bridge design, the American Association of State Highway and Transportation Officials' (AASHTO) Fatigue Specification played an important role. All these studies can be classified into two groups: the studies based on experimental data which led to the development of the S-N Curves, including AASHTO's Specification; and the studies based on crack propagation theory, such as the model proposed by Paris (1964), Miller (1981), Ortiz (1985) and Veers (1987). AASHTO's specification will be discussed in detail in Section 2.4 and the crack propagation theories will be discussed in Chapter 3.

The fracture mechanics approach to fatigue has been discussed by Barsom (1972). Recently, fatigue has drawn much more attention than in the past. Because of the large uncertainties involved in the fatigue damage accumulation process, the probabilistic methods were proposed to study the problem of fatigue and how to control it. A number of fatigue reliability methods are now available to study fatigue of airplanes, ships and offshore structures. Some of the important works were reported by Wirsching (1984), Wu (1984), Yao (1986), Moan (1987), Madsen (1987), Schueller (1988) and Jiao (1989a).

2.3 Inspection and Maintenance

Inspections are frequently made for in-service structures such as MARTA's bridge structures. Since the fatigue process is very slow, it is possible and feasible
to use periodic inspection and maintenance to control this process. Two basic types of scheduled maintenance procedures have been proposed:

1. scheduled inspection and repair or replacement maintenance, and
2. scheduled proof test maintenance.

For many fatigue critical components, scheduled maintenance is an effective way to ensure their structural safety. In addition to the inspection, proof testing is another quality assurance technique, which can be performed prior to service or under in-service conditions in order to screen out the weakest component (Rackwitz, 1985).

To measure the integrity of the structure, non-destructive inspection can be used to detect the growth of fatigue cracks. Crack sizes are measured either directly or indirectly through a non-destructive inspection method. The detectability of an inspection can be defined as the probability of detection of a crack with a given length. It varies with the types of the equipment used as well as the environmental conditions during the testing. If a potentially damaging crack is detected during a service inspection, the cracked member may be repaired or even replaced in some very serious cases. After a component is replaced or repaired, its fatigue strength will be improved. Based on the results of the in-service inspection, the reliability of the joints or the system can be updated and the uncertainties in the initial modeling of the basic variables can be reduced. The details of non-destructive inspections will be discussed further in Chapter 5.

**2.4 S-N Curve and Miner's Damage Accumulation Rule**

**2.4.1 S-N Curve**

Based on extensive test data, the classical approach to fatigue investigation has focused on the S-N curve (Figure 2.2), which relates the fatigue life to the cyclic stress range and can be expressed as:

\[ N = AS^{-B} \]  

(2.1)
Figure 2.2 Illustration of S-N Curve
where \( N \) is the number of cycles to failure, \( S \) is the fatigue stress range, \( A \) is the fatigue strength coefficient, and \( B \) is the fatigue strength exponent. Taking the log of both sides of Eq. 2.1, one obtains:

\[
\log N = \log A - B \log S
\]  

(2.2)

Note that Eq. 2.2 will result in a straight line when plotted (Figure 2.2). S-N curves would be appropriate to describe either the crack initiation period or the total fatigue life of a structural component.

2.4.2 Miner’s Damage Accumulation Rule

The linear damage accumulation hypothesis, commonly known as Miner’s Rule (Miner, 1945), is widely used in fatigue design. The rule as described in Figure 2.3, (ASCE, 1982) predicts damage using the following expression:

\[
D = \sum_{i=1}^{k} \Delta D_i = \sum_{i=1}^{B} \frac{n_i}{N_i}
\]  

(2.3)

where \( n_i \) is the number of cycles under stress range level \( S_i \), \( N_i \) is the total number of cycles to failure under stress range level \( S_i \) and \( D \) is the Miner’s damage accumulation index.

Combining the S-N curve and the mean stress effect, Miner’s damage index can be expressed as discussed in the following sections:

2.4.2.1 Discrete Distribution of Stress

If the distribution of stress range is discrete and \( f_i \) is the frequency of stress range at level \( S_i \) (Figure 2.4), then the number of cycles at level \( S_i \) in the lifetime:

\[
n_i = f_i n
\]  

(2.4)

where \( n \) is the total number of fatigue stress cycles applied in the lifetime. The Miner’s damage accumulation index can be rewritten as:

\[
D = \sum_{i=1}^{k} \frac{n_i}{N_i} = \frac{n}{A} \sum_{i=1}^{k} f_i S_i^B
\]  

(2.5)
The S-N Curve Obtained from Constant Amplitude Fatigue Tests

Failure Defined When \( \sum_{i=1}^{n} \frac{N_i}{N_i^*} = 1 \)

Figure 2.3 Miner Hypothesis (ASCE, 1982)
Figure 2.4 Probability Density Function for Discrete Stress Range
where A and B are the fatigue strength coefficient and exponent of S-N curves, respectively. The expected value or mean value of $S^B$ is defined as:

$$E(S^B) = \sum_{i=1}^{k} f_i S_i^B$$

(2.6)

Thus,

$$D = \frac{n}{A} E(S^B)$$

(2.7)

### 2.4.2.2 Continuous Distribution of Stress

If the distribution of stress range is continuous and represented by the probability density function $f_S(s)$ (Figure 2.5), then the fraction of stress range in the range $(s, s+\delta s)$ is:

$$f_i = f_S(s) \delta s$$

(2.8)

The Miner’s damage accumulation index can be defined as:

$$D = n \sum_{i=1}^{k} \frac{f_S(s) \Delta s}{N(s)}$$

(2.9)

As $\Delta s \to 0$, Eq. 2.9 becomes:

$$D = n \int_0^{\infty} \frac{f_S(s)ds}{N(s)} = \frac{n}{A} \int_0^{\infty} S^B f_S(s)ds$$

(2.10)

where A and B are the fatigue strength coefficient and the exponent of the S-N curves, respectively. Using Eq. 2.6, Eq. 2.10 can be rewritten as:

$$D = \frac{n}{A} E(S^B)$$

(2.11)

According to Miner’s rule, the failure due to fatigue occurs when $D \geq 1.0$. Wirsching (1984) extended the concept of Miner’s rule and defined failure as:

$$D \geq \Delta$$

(2.12)

where $\Delta$ is a random variable with mean value of 1.0.
Figure 2.5 Probability Density Function for Continuous Stress Range
It is generally thought that Miner's rule more accurately describes fatigue damage of structural components whose life consists mainly of crack initiation (ASCE, 1982). As a damage accumulation model, Miner's rule does not consider crack sizes. Thus, the limitation of this model is that it can not predict crack growth. To overcome this disadvantage, a more sophisticated analytical approach using the linear fracture mechanics theory has been developed to predict the fatigue crack growth. In this approach, the crack size and the structural geometry are treated as important parameters to predict the crack growth in addition to the load effect and material properties. In this case, the failure can be defined as the maximum size of a crack that will lead to fracture or a tolerance level that is acceptable for practical designs.

Since the fracture mechanics model describes the crack growth process in detail and makes it possible to follow the propagation of cracks, the linear fracture mechanics model is used as a basis for the design and inspection plan in this study.

2.5 AASHTO Fatigue Design Provision

2.5.1 Introduction

Bridge engineers have recognized the effect of cyclic loading on the structural fatigue of welded bridge components for a long time. The American Welding Society's (AWS) Specification For Welded Highway And Railway Bridges (AWS, 1941) was based on fatigue tests of welded details conducted in 1940s. Based on extensive fatigue test results and the regression analysis technique, S-N curves were developed for different types of materials. In the late 1950s, AASHTO conducted studies on fatigue crack growth at welded details. These studies indicated the need for further study of welded details and for modification of the existing specifications.

In order to study the influence of stress concentration and discontinuities on the fatigue strength, a series of fatigue test programs were undertaken with the
sponsorship of the National Cooperative Highway Research Program (NCHRP) from 1970 to 1986. About 500 beams and plate girders, each with 2 or more welded details, were tested to simulate actual bridge details with or without attachments. Some of these test results have been reported by Fisher (1974, 1980), Keating (1986), and Moses (1987). In order to fulfill the need for fatigue evaluation and repair of some old steel bridges without protection against erosion, some recent projects under the sponsorship of NCHRP focused on the fatigue behavior of weathered steel bridges. (Albrecht, 1982; Yamada, 1984 and Fisher, 1988).

From the analysis of NCHRP fatigue test data, several important conclusions were made (National Highway Institute, 1990):

1. The stress range, defined as the algebraic difference between the maximum stress and the minimum stress at the location under consideration, is the primary parameter controlling the fatigue life (Fisher, 1970). Since the dead load does not contribute to the stress range, the stress range is due only to the live load. As illustrated in Figure 2.6, other stress parameters, such as the minimum stress, the maximum stress and the stress ratio, do not play an important role.

2. As indicated by Schilling (1974), the steel type (yield strength) does not have a significant effect on the fatigue strength (Figure 2.7); and

3. The fatigue strength of a girder with welded attachments is strongly governed by the length of the attachment in the direction of the stress (Fisher, 1980). For longer attachments, the higher stress concentration is expected at the toe of the weld. Consequently, it will lower the fatigue strength.

2.5.2 The AASHTO Fatigue Design Curves

As mentioned earlier, the AASHTO Fatigue Design Specification uses S-N curves. In order to get the best fit curve from the test data, AASHTO used the regression technique. These regression equations give the mean or expected value (Figure 2.7). For the practical design purposes, S-N curves representing a 95 percent confidence limit were selected. These design S-N curves can be obtained
Figure 2.6 Effect of Minimum Stress and Stress Range on the Cycles (NCHRP Report 102)

Log $N = 10.870 - 3.372 \log S$
Stress Range (ksi)  90  40  90  23H  20  19  K)

OI Log N = 10.870 - 3.372 Log S

Figure 2.7 Effect of Stress Range and Type of Steel on the Cycles (NCHRP Report 102)
by drawing lines two standard deviations below and parallel to the mean curves, as shown in Figure 2.8. The probability that the test results for a given detail will fall below this design line is only 2.5 percent.

Based on the test results, the current AASHTO Fatigue Design Curves (AASHTO, 1988) are classified into seven categories: Category A through Category F. The complete set of current S-N curves, commonly referred to as the AASHTO Fatigue Design Curves, is shown in Figure 2.9. With an identical slope constant of 3 for each curve, the fatigue strength could be represented by a single equation, i.e., Eq. 2.2. The logN intercept, A, is given in Table 2.1 for each category.

It is important and necessary at this stage to discuss some of the basic ideas of these classifications, although they are defined in detail in the current AASHTO code (AASHTO, 1988). A brief description of each category is given below.

**Category A**

Category A defines the fatigue strength of rolled shapes and plates without welded or bolted details or attachments. The category provides the maximum fatigue strength of any bridge detail. Fatigue cracks initiate at surface discontinuities or at sharp edge indentations. Generally, this category does not control the design unless some unusual condition exists.

**Category B**

Category B applies to a variety of welded details such as: longitudinal fillet and full penetration groove welds, transverse welds ground flush, and transitioned flange splices. Bolted details are also classified as Category B.

**Category B′**

Category B′ is a relatively new detail classification introduced in the 1988 AASHTO specifications. Category B′ applies to longitudinal partial penetration groove welds, full penetration groove welds with backing bars left in place, and straight transition splices with A514/A517 steel.

**Category C**

Category C is primarily applicable to stiffeners and short attachments as well as unimproved transverse groove welds. The category marks the transition between
Figure 2.8 Lower Bound Fatigue Life Curve for Fatigue Test Data
Table 2.1 Log-N Intercept Coefficients for AASHTO

<table>
<thead>
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<th>Category</th>
<th>A constant</th>
</tr>
</thead>
<tbody>
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<td>A</td>
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</tr>
<tr>
<td>B</td>
<td>11,920,000,000</td>
</tr>
<tr>
<td>B'</td>
<td>6,109,000,000</td>
</tr>
<tr>
<td>C</td>
<td>4,446,000,000</td>
</tr>
<tr>
<td>D</td>
<td>2,183,000,000</td>
</tr>
<tr>
<td>E</td>
<td>1,072,000,000</td>
</tr>
<tr>
<td>E'</td>
<td>390,800,000</td>
</tr>
</tbody>
</table>
the initial discontinuity and the geometrical stress concentration influencing the fatigue strength.

**Category D**

Category D represents a transition zone between the high and low strength details. The fatigue strength of these details is influenced by the attachment length and any improvement to reduce the geometrical stress concentration.

**Category E**

Category E represents a wide variety of details, all exhibiting low fatigue strength. These details include: coverplated beams, gusset plates, and attachments with lengths greater than 4 in. or transition radius of less than 2 in. The fatigue strength is governed by the stress concentration at the weld toe.

**Category E'**

Category E' represents the lowest fatigue strength detail allowed by the AASHTO specifications. Types of details included in this classification are similar to those found in Category E, but they exhibit reduced strength due to the thickness effect which results in a higher geometrical stress concentration.

**Category F**

Category F represents the fatigue strength of welds subject to shear and is used to prevent fatigue cracks in the weld metal, initiating at the weld root. Normally, this type of stress condition will not control the fatigue design unless the weld is required to carry a relatively large force through the throat.

For full-penetration butt welds in the MARTA's bridges, it appears that Category B can be used. In this study, these welds are conservatively considered to be in Category E. Of course, they can be considered in any category by simply changing the constants A given in Table 2.1.
Chapter 3

Fatigue and Crack Growth Model

3.1 Introduction

The prediction of fatigue crack growth in bridge structures is very important since the consequence of fatigue could be catastrophic. The crack growth stage of the fatigue process is often the most important stage since cracks are generally present for a major fraction of the useful life of the structures. In this chapter, the most commonly used fatigue crack growth model is discussed and the important parameters involved in the model are identified. The statistical uncertainties in the estimation of these variables are also discussed. The crack propagation law presented in the following section is only one of several available models (Miller, 1979), although it is perhaps the one most commonly used. But the general methodology presented here will apply to any other models equally well.

According to Miller (1981), metal fatigue can be divided into the following three stages:

1. initiation of a small crack or a crack-like defect;
2. the growth of the crack due to cyclic loading or environmental effects; and
3. final fracture when the length of the crack exceeds a critical value.

Thus, the fatigue life then can be divided into two phases: the crack initiation period corresponding to stage 1 and the crack propagation period corresponding to stages 2 and 3.

As illustrated in Figure 3.1, the general case of cracking in solid can be divided into three different loading modes (Paris, 1965). They are Mode 1: the opening mode corresponding to tension loading; Mode 2: the sliding mode corresponding to in-plane shear loading; and Mode 3: the tearing mode corresponding to out-of-plane shear loading. Since the shear loading modes (mode 2 and mode 3)
Figure 3.1  The Three Basic Modes of Crack Surface Displacement
mainly contribute to crack initiation, the opening mode, which controls the crack propagation stage, is the most important one in this study.

3.2 Fracture Mechanics Solution for Cracks

The crack growth rate, generally defined as $\frac{d\alpha}{dN}$, in which $\alpha$ is the length of the crack and $N$ is the number of cycles, is the extension of a crack with length $\alpha$ in one stress cycle. It depends on the stress and the elastic-plastic response of the material at the tip area, and also on the fracture criterion. According to the loading modes discussed earlier, Irwin (1957) established a method for cracks in elastic solid. As illustrated in Figure 3.2, a typical Mode I crack, with length $2\alpha$ in an infinite plate, is subject to a tensile global load of $\sigma$. The corresponding elastic stresses near the crack tip are given as follows:

$$\sigma_x = \frac{K_1}{(2\pi r)^{\frac{1}{2}}} \cos \frac{\theta}{2} (1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2})$$  \hspace{1cm} (3.1)

$$\sigma_y = \frac{K_1}{(2\pi r)^{\frac{1}{2}}} \cos \frac{\theta}{2} (1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2})$$  \hspace{1cm} (3.2)

$$\tau_{xy} = \frac{K_1}{(2\pi r)^{\frac{1}{2}}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}$$  \hspace{1cm} (3.3)

$$\tau_{xz} = \tau_{yz} = 0$$  \hspace{1cm} (3.4)

For plane stress condition:

$$\sigma_z = 0$$  \hspace{1cm} (3.5)

For plane strain condition:

$$\sigma_z = \nu(\sigma_x + \sigma_y)$$  \hspace{1cm} (3.6)

where the stress components and the corresponding $r$ and $\theta$ are shown in Figure 3.2. $K_I$, the parameter reflecting the redistribution of stresses in Mode I due to the presence of cracks, is the crack tip stress intensity factor and is given by:

$$K_I = F(\alpha, Y)\sigma \sqrt{\pi \alpha}$$  \hspace{1cm} (3.7)
Figure 3.2 Coordinate System and Stress Components in Crack Tip
where \( F(\alpha, Y) \) is a geometry function, \( Y \) is a set of random parameters describing the geometry and \( \sigma \) is the global tensile stress.

### 3.3 Fatigue Crack Growth Model

#### 3.3.1 Crack Growth Model

In order to understand the fatigue crack growth behavior under the cyclic tensile loading, a number of experiments were conducted in the 1960s. Based on test data and the elastic theory mentioned in section 3.2, Paris and Erdogan (1964) suggested a model for predicting fatigue crack growth. The model, namely the Paris Equation, is described as:

\[
\frac{da}{dN} = C(\Delta K)^m
\]

(3.8)

where \( \alpha \) is the crack size, \( N \) is the number of stress cycles, \( C \) and \( m \) are material parameters, and \( \Delta K \) is the stress intensity range. \( \Delta K \) is computed by the linear elastic fracture mechanics (LEFM) theory. Using Eq. 3.7, \( \Delta K \) can be estimated as:

\[
\Delta K = K_{max} - K_{min} = F(\alpha, Y)S\sqrt{\pi \alpha}
\]

(3.9)

where \( S \) is the tensile stress range and \( F(\alpha, Y) \) is the geometry function to account for the possible stress concentration.

If the Paris-Erdogan Equation was correct, then a plot of \( \log(da/dN) \) versus \( \log\Delta K \) would show a straight line. In Figure 3.3, a schematic plot of typical fatigue crack growth data is shown.

It can be seen from Figure 3.3 that the fatigue crack propagation curve is composed of three regions: Region I defines the threshold value \( \Delta K_T \), below which a crack will not propagate; Region II, where the crack growth curve is modeled as a straight line on a log-log scale, agrees with the description of the Paris Equation; and Region III is defined by the fracture toughness, \( K_C \), of the material. Failure
Figure 3.3 A description of the Behavior of the Material
occurs when the magnitude of the cyclic stress intensity factor $\Delta K$ is greater than $K_C$.

### 3.3.2 Threshold Effect

The threshold effect can be described by the threshold stress range level. From Eq. 3.9, this threshold value $S_T(\alpha)$ is obtained as:

$$S_T(\alpha) = \frac{\Delta K_T}{F(\alpha, Y)\sqrt{\pi\alpha}}$$

(3.10)

Thus, $\Delta K_T$ filters out non-damaging stress cycles from the total stress process.

Also, note that $S_T(\alpha)$, a function of $\alpha$, changes as the crack grows. This implies that the sequence of stress range affects the crack growth. Small stress ranges will contribute to the crack growth only when the crack size is considerably large.

For practical problems, structures experience many cycles at the lower stress range. Hudak (1981) studied the problem for small cracks and suggested that the use of a crack initiation model coupled with the Paris propagation model without threshold produced accurate life predictions in real structures. In this study, threshold effect is not considered.

### 3.3.3 Fatigue Damage Accumulation Function

Integrating Eq. 3.8 with respect to $\alpha$ and $N$ from $\alpha(N_1) = \alpha_1$ to $\alpha(N_2) = \alpha_2$, one obtains:

$$\int_{\alpha_1}^{\alpha_2} \frac{d\alpha}{[F(\alpha, Y)\sqrt{\pi\alpha}]^m} = \int_{N_1}^{N_2} CS^m dN$$

(3.11)

According to Madsen (1985), a function reflecting the damage accumulation from crack size $\alpha_1$ to $\alpha_2$ can be defined as:

$$\Psi(\alpha_2, \alpha_1) = \int_{\alpha_1}^{\alpha_2} \frac{d\alpha}{[F(\alpha, Y)\sqrt{\pi\alpha}]^m}$$

(3.12)

This damage accumulation function is related to the load accumulation by

$$\Psi(\alpha_2, \alpha_1) = CS^m(N_2 - N_1)$$

(3.13)
where \( S^m \) is the mean stress range. All other parameters were defined earlier. \( S^m \) can be calculated as:

\[
S^m = \begin{cases} 
S^m & \text{constant amplitude loading;} \\
\int_0^\infty S^m \tilde{f}_s(S) dS & \text{stationary ergodic loading;} \\
\frac{1}{N_2-N_1} \sum_i S_i^m & \text{otherwise.}
\end{cases}
\]  

(3.14)

3.3.4 Mean Stress Effect

Because of the random nature of the load process, the reliability of predicting fatigue under a variable amplitude stress process has been studied extensively. Schijve (1975) provided an overview of various approaches including: (1) cycle by cycle counting in which crack growth for each cycle is computed; and (2) the characteristic stress method.

Due to the cyclic nature of the loading, the fatigue crack growth process has a relatively long propagation period. Thus, the characteristic stress method may be the most appropriate one and the fatigue damage accumulation can be evaluated using the mean stress effect. It is necessary at this stage to define the mean stress effect under different loading distributions.

According to Eq. 3.14, the mean stress effect \( \tilde{S}^m \) of a total number of cycles \( N_0 \) is defined as:

\[
\tilde{S}^m = \frac{1}{N_0} \sum_{i=1}^N S_i^m = \int_0^\infty S^m \tilde{f}_s(S) dS
\]  

(3.15)

where \( \tilde{f}_s(s) \) is the stress range density function if the stress history is a stationary, ergodic process. The stress range density function, \( \tilde{f}_s(s) \), is available only for some special loadings. Closed-form expressions for the mean stress effect can be obtained for the following special cases.
Weibull Loading

If the stress range follows a Weibull distribution, the density function of $S$, $f_S(s)$, can be expressed as (Jiao, 1989a):

$$F_S(s) = 1 - \exp\left[-\left(\frac{s}{\theta}\right)^\xi\right]$$

(3.16)

The mean stress effect, therefore, is:

$$S_m^m = \int_0^\infty S^m f_S(s)ds = \theta^m \Gamma(1 + \frac{m}{\xi})$$

(3.17)

where $\Gamma()$ is the gamma function. This distribution especially satisfies the case of ocean waves (Figure 3.4a). A Weibull distribution becomes an exponential distribution when $\xi=1$, and it becomes a Rayleigh distribution when $\xi=2$, which is applicable for narrow-banded Gaussian loading.

Lognormal loading

If the stress range follows a lognormal distribution, then its distribution function can be represented as:

$$F_S(s) = \Phi\left(\frac{\ln s - \mu_{\ln s}}{\sigma_{\ln s}}\right)$$

(3.18)

The mean stress effect is then:

$$S_m^m = \int_0^\infty S^m f_S(s)ds = \mu_{\ln s}^{-m}(1 + \delta_s^2)^{\frac{m-1}{2}}$$

(3.19)

where $\delta_s$ is the coefficient of variation.

Rayleigh loading

Fatigue loads experienced by bridges, cranes, pressure vessels, etc. can be modeled as a sequence of discrete variable amplitude loadings occurring randomly in time (Figure 3.4b) rather than by a continuous function (ASCE,1982). Based on extensive observations, Schilling (1974) indicated that the Rayleigh distribution would be appropriate for the loading of bridges. Albrecht (1974) suggested that the distribution functions of Polynomial could be used to model the loading conditions on bridges. Ang(1974) indicated that the Beta distribution could be used for the loading on highway bridges.
Figure 3.4 List of Stress Histories for Fatigue Failure
Since the railway bridge is the only type of bridge involved in this study, the Rayleigh distribution is used in the fatigue study of MARTA’s bridge structures.

If the stress range follows a Rayleigh distribution, then the distribution function of the stress range can be expressed as:

$$F_S(s) = 1 - \exp\left[-\frac{1}{2}\left(\frac{s}{S_0}\right)^2\right]$$  \hspace{1cm} (3.20)

where $S_0 = \sqrt{\frac{2}{\pi}}\mu_s$ is a statistical parameter and $\mu_s$ is the mean value of $S$. The mean stress effect is then

$$S_m = \int_0^\infty S^m f_S(s)ds = (\sqrt{2}S_0)^m \Gamma\left(\frac{m}{2} + 1\right)$$  \hspace{1cm} (3.21)

### 3.3.5 Random Modeling of Material Properties

As pointed out earlier, the material property in the crack propagation model is characterized by two parameters, $C$ and $m$. Due to the large uncertainties in the experiment as well as in the process of fabrication, these two parameters should be treated as random variables.

An empirical relationship between material parameters $C$ and $m$ is described by Veers (1987) as:

$$C = AB^m$$  \hspace{1cm} (3.22)

where $A$ and $B$ are constants.

Eq. 3.22 can be rewritten in logarithmic form as:

$$\ln C = \ln A - m \ln B$$  \hspace{1cm} (3.23)

Five possible assumptions about the distribution of $C$ and $m$ can be made as follows:

1. $C$ and $m$ are constant;
2. $C$ is a constant and $m$ follows a normal distribution;
3. $\ln C$ follows a normal distribution and $m$ is a constant;
4. $\ln C$ and $m$ are uncorrelated normal variables;
(5) $\ln C$ and $m$ are correlated normal variables.

In the last case, a joint normal distribution with a strongly negative correlation coefficient $\rho_{\ln C,m} \rightarrow -0.95$ (Yao, 1986) can be used to model the joint distribution of $\ln C$ and $m$. The conditional distribution function of $\ln C$ and $m$ can be represented as:

$$F_{\ln C|m}(\ln C|m) = \Phi\left[\frac{1}{\sqrt{1 - \rho_{\ln C,m}^2}} \left(\frac{\ln C - \ln \mu_{\ln C}}{\sigma_{\ln C}} - \rho_{\ln C,m} \frac{m - \mu_m}{\sigma_m}\right)\right] \quad (3.24)$$

### 3.4 Descriptions of Crack Initiation

Crack initiation is defined as the development of an initial flaw from which a crack may grow. It takes place entirely at the microscopic level. There are many possible origins for fatigue crack initiation, such as during fabrication and construction. Mohaupt (1987) suggested a method to model crack initiation using S-N curves. However, the initiation stage is less important than the crack growth stage because cracks are present for a major fraction of the total fatigue life for most welded structures. Two aspects of crack initiation are of interest: initial crack size and crack initiation period.

#### 3.4.1 Initial Crack Size

It is well established in the profession that all welding processes result in discontinuities at or near the weld. It is these discontinuities that may be susceptible to the unstable crack growth characteristic leading to fracture. Typical fabrication discontinuities include lack of fusion, slag inclusion, porosity, arc strikes, and stop/start positions (ASCE, 1982; Jiao, 1989a).

In addition to the discontinuities arising from fabrication, cracks that may lead to fracture can occur during transportation, construction, and assembly (ASCE, 1982). These include extraneous tack welds or arc strikes, nicks, and gouges. Also, cracks may occur during the service life of the structure. These
cracks result from fatigue crack growth, severe section loss due to corrosion and collision or other types of damage.

Three primary factors control the crack propagation of structural members (National Highway Institute, 1989):

1. crack size;
2. stress level; and
3. fracture toughness.

All three factors interact to determine whether or not fracture will occur.

In crack propagation modeling, the most sensitive variable may be the initial crack size $\alpha_0$ because a considerable amount of uncertainty is expected in its estimation. The distribution types used most frequently in modeling variability in initial crack size are lognormal distribution and Weibull distribution (Jiao, 1989a). If the underlying distribution is lognormal, it can be expressed as:

$$F_{\alpha_0}(\alpha_0) = \Phi\left(\frac{\ln \alpha_0 - \mu_{\ln \alpha_0}}{\sigma_{\ln \alpha_0}}\right)$$

(3.25)

If the underlying distribution is Weibull, it can be represented as:

$$F_{\alpha_0}(\alpha_0) = 1 - exp\left[-\left(\frac{\alpha_0}{\theta}\right)^\xi\right]$$

(3.26)

where $\theta$ and $\xi$ are scale and shape parameters, respectively.

Experimental data is very scarce in this area. Besides the model uncertainty, the variability in the basic parameters are also significant. It is noted by Madsen (1985) that the fatigue life is highly dependent on the initial crack size and the large variability in the fatigue life estimation is attributed to the uncertainty in the initial crack size.

Jiao (1989a) also observed that there was not much difference between Weibull and lognormal representation of the initial crack size. According to the tail behavior of these two distributions, lognormal distribution tends to be more conservative, since it predicts a higher probability of large cracks.

In welded structures, an exponential distribution for the initial crack size, which is a special case of Weibull distribution with $\theta=1$, is very common. Karlsen
(1982) measured undercuts in production butt welds of plates with thicknesses from 10 to 25 mm. He found an exponential distribution with a mean crack length of 0.11 mm in describing the initial crack size. Mohaupt (1987) also suggested the mean initial crack size \( \alpha_0 \) to be 0.02 inch corresponding to the smallest crack length that can be measured reliably. In this study, for illustration purpose, a lognormal distribution with mean crack length of 0.02 inch will be used in the random modeling of initial crack size. Of course, any other reasonable value or model can be used for this purpose.

3.4.2 Crack Initial Period

Another important aspect of crack initiation is the crack initial period \( N_0 \). Although it seems that no reasonable theory exists for the crack initiation, a number of models have been developed to address the problem. Based on the stress-strain response, Lawrence (1978) proposed a crack initiation period model for high cycle fatigue \( (N_0 \geq 10^4 \text{ cycles}) \) of welded structures. Jiao (1989a) also suggested a model of crack initiation period combining the S-N data and the linear elastic fracture mechanics theory.

For the welded bridge details under consideration and as discussed earlier, the welding process inherently results in initial flaws from which crack growth may occur. So the crack growth stage is of primary interest in the fatigue life estimation of welded joints. Thus, it might be reasonable to assume \( N_0 = 0 \) for the welded structures under consideration (Jiao, 1989b).

The crack initiation period, however, may not be negligible when the fatigue life has been improved by post-weld methods. If it is assumed that S-N curves are available for the period of crack initiation, the initiation period can be expressed as

\[
N_0 = K_0 S^{-m_0} \tag{3.27}
\]

Thus, in this case \( N_0 \) is a random variable described by the stress variable and the S-N parameters.
3.5 Description of the Geometry Function

A considerable amount of model uncertainty in describing the geometry function of cracks, as defined by Eq. 3.10, is expected. The structural details and loading conditions will also alter the type of geometry function. It is generally recognized that $F(a, \mathbf{Y})$ is a function of the crack size $a$, and a set of the other random variables $\mathbf{Y}$, such as stress concentration coefficient and dimensions of the specimen under consideration.

Closed-form models to express the geometry function are available only for special details. Almar (1985) proposed a model to describe the geometry function for crack growth through the stress concentration zone.

For bridge components, Fisher (1970) suggested the geometry function for side-notched specimens as:

$$F(\xi) = 2 \left[ \sin^2 \left( \frac{\pi \xi}{2} \right) + \sec^2 \left( \frac{\pi \xi}{2} \right) \right]$$

(3.28)

where $\xi = 2a/w$ and $w$ is the width of specimen. Also, Paris (1963) proposed a geometry function to treat the case of center-notched specimens, which can be expressed as:

$$F(\xi) = \frac{1 - 0.5\xi + 0.37\xi^2 - 0.044\xi^3}{\sqrt{1 - \xi}}$$

(3.29)

where $\xi$ is defined as in Eq. (3.28).

Hudak (1981) suggested a geometry function to represent real structures, which can be expressed as:

$$F(\alpha) = \lambda \alpha^{-\theta}$$

(3.30)

where $\lambda$ and $\theta$ are the statistical parameters.

For full-penetration butt welds, the most probable location of an initial crack is in the middle of the specimen, so Eq. 3.29 will be used as the geometry function in this study.
3.6 Nonlinear Fracture Mechanics Approach

Most of the preceding discussion has focused on the high-cycle fatigue in which nominal stresses are restricted to the elastic range of the material and the cycles to failure are typically greater than $10^4$. However, in many cases it may be good design practice to allow some cyclic plasticity, for example, in weld undercut notches or in rivet holes.

Also, almost all the low to medium-strength structural steel sections used for large complex structures such as bridges, ships, pressure vessels, etc., are of insufficient thickness to maintain plane-strain conditions under slow-loading conditions at normal service temperatures (Atkins, 1985). Thus, for many structural applications, the linear elastic analysis used to calculate $K$ in Eq. 3.7 is invalid due to the formation of large plastic zone and the elastic-plastic behavior. So, it is necessary to develop an elastic-plastic fracture mechanics analysis procedure as an extension to the linear elastic analysis.

3.6.1 Low Cycle Fatigue

For many materials, such as steel, the relationship between the cyclic stress range $\Delta \sigma$ and the plastic component of cyclic strain $\Delta \epsilon_p$ is given by:

$$\Delta \sigma = \sigma'_0 \Delta \epsilon_p^{n'}$$

where $n'$ is the strain hardening parameter. This is obviously a direct parallel to the Ludwik stress-strain relationship for monotonic plastic flow (Figure 3.5).

3.6.2 $da/dN$ in Low cycle Fatigue

During the process of low cycle fatigue, cracks propagate within plastic stress strain fields. This means that either the stress amplitude will exceed the yield stress or strain amplitudes will exceed the yield strain. Thus, instead of the linear elastic
Figure 3.5  Hysteresis Loop for Failure in the Plastic Strain Range
fracture mechanics approach, the nonlinear fracture mechanics approach would be an appropriate alternative to study the low fatigue problem.

Although $K_c$, as defined previously, is no longer an appropriate parameter to describe the crack tip behavior in low cycle fatigue, the idea of critical absorbed energy in the compression zone may still apply. Thus, instead of the Paris Equation in terms of $\Delta K$, Eq.3.8 can be rewritten in terms of $\Delta J$ as:

$$\frac{da}{dN} = \tilde{C}(\Delta J)^m$$  \hspace{1cm} (3.32)

where, $\Delta J$ is the energy absorbed in the presence of remote plastic flow.

A number of researchers (Solomon, 1972; Wareing et al., 1973 and Ibrahim and Miller, 1980) observed that the low fatigue crack propagation law can be expressed as:

$$\frac{da}{dN} = \bar{C}(\Delta \epsilon_p)^q$$  \hspace{1cm} (3.33)

where $\bar{C}$ and $q$ are material constants and $q$ is a number between 1 and 2. Table 3.1 gives values of $q$ for a variety of steels.

Using the experimental results, Tomkins (1980) observed that $\bar{C}$ varies in proportion to $(\Delta \sigma/T)^2$ rather than $(\Delta \sigma/T)$, and

$$T = \left(\frac{\sigma_0}{2}\right)(n')^n$$  \hspace{1cm} (3.34)

where $T$ is the cyclic analog of the monotonic true stress equivalent $(\sigma_0 n^n)$ of the ultimate tensile strength.

Combining Eqs. 3.33 and 3.34, one obtains:

$$\frac{da}{dN} = 2^{-\frac{\alpha}{n'}}(\Delta \sigma)^{\left(2+\frac{\alpha}{n'}\right)}(T)^{\left(2+\frac{\alpha}{n'}\right)\left(n'-1\right)}(n')^\alpha$$  \hspace{1cm} (3.35)

For simplification, defining:

$$A = 2 + \frac{\alpha}{n'}$$  \hspace{1cm} (3.36)

and

$$B = 2^{-\frac{\alpha}{n'}}(T)^{-A}(n')^\alpha$$  \hspace{1cm} (3.37)
Table 3.1  The Parameters in the Lower Cycle Behavior

<table>
<thead>
<tr>
<th>Steel</th>
<th>0.2% proof stress (MPa)</th>
<th>(\Delta TS)</th>
<th>(n)</th>
<th>(T) (MPa)</th>
<th>(\alpha)</th>
<th>(1+2n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EN 1A</td>
<td>241</td>
<td>483</td>
<td>0.20</td>
<td>575</td>
<td>1.47</td>
<td>1.40</td>
</tr>
<tr>
<td>Ducol W30</td>
<td>545</td>
<td>690</td>
<td>0.095</td>
<td>710</td>
<td>1.18</td>
<td>1.19</td>
</tr>
<tr>
<td>304</td>
<td>276</td>
<td>630</td>
<td>0.15</td>
<td>793</td>
<td>1.23</td>
<td>1.30</td>
</tr>
<tr>
<td>20/25 Nb (750°C)(^2)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.53</td>
<td>1.54</td>
</tr>
<tr>
<td>316 (625°C)(^2)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.30</td>
<td>1.48</td>
</tr>
<tr>
<td>1018</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.86</td>
<td>1.52</td>
</tr>
<tr>
<td>'Mild steel'</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2.00</td>
<td>-</td>
</tr>
</tbody>
</table>

\(^2\)Refers to test temperature (°C).
Eq. 3.34 can be rewritten as:

$$\frac{d\alpha}{dN} = B(\Delta \sigma)^A \alpha$$  \hspace{1cm} (3.38)$$

Integrating Eq. 3.38 from $\alpha_0$ to $\alpha_N$, one obtains:

$$Ln\alpha_N = B(\Delta \sigma)^A(N - N_0) + Ln\alpha_0$$  \hspace{1cm} (3.37)$$

Once the initial crack size is determined, the corresponding crack initiation period can be found.

Since the test data for nonlinear fracture mechanics is limited, the LEFM approach will be used in this research.
4.1 Introduction

It is generally accepted that the initiation of fatigue cracks and their subsequent growth in structures is, in part, a random phenomenon (Freudenthal, 1972; Yao, 1972). Numerous sources of uncertainties exist in a structural fatigue analysis. For the fracture mechanics approach, some of these uncertainties can be identified as (Wirsching, 1987):

1. Scatter in the data;
2. Model error; and
3. Human error

Scatter exists in the material parameters as well as in extrapolating laboratory data to predict behavior in practice. Also, loading and environmental effects are typical random processes. Thus, the parameters in the fatigue problem are random variables. This source of uncertainty can be important if the sample size is small, since it is expensive to conduct fatigue tests and in many cases available test data are extremely limited, it is a major source of uncertainty in the fatigue problem.

Many assumptions are made in computing fatigue stress in a component of a bridge structure. The geometry factor in the model is another important source of model uncertainties.

Inability to obtain and express the material properties accurately, error in interpreting the inspection results and possible failure of inspection to detect cracks contribute to the other source of uncertainties.

Due to the presence of these amounts of uncertainties, the absolute safety of bridge structures can not be guaranteed. However, the risk of unacceptable
consequences can be limited to a reasonable level. Estimation of this risk is a necessity for practical engineering problems.

Before a risk-based crack growth model is developed to study the fatigue reliability, it is essential to discuss several basic concepts of a risk-based model. All the discussion made here is in the context of the structural reliability analysis.

The area of structural reliability has grown at a tremendous rate in the last two decades (Ang and Conell, 1974; Hasofer and Lind, 1974; Moses, 1974; Ditelvson, 1981; Ang and Tang, 1984; and Haldar and Ayyub, 1984). Many methods have been proposed addressing this type of problem, the parameters involved and the uncertainty associated with these parameters. The statistical description of a random variable is made in terms of its mean (central tendency), the variance (the dispersion about the mean), and the probability density and distribution functions. Various risk estimation techniques use part or all of this information in different ways to estimate the corresponding risk. The limitations of a particular method depend on the assumptions made in developing the algorithm.

4.2 Limit State Function and Probability of Failure

Let $X = [X_1, X_2, X_3, \ldots, X_n]^T$ denote a vector of design variables, which are in general random, with joint distribution function $f_X(x)$. For each set of values of $X$, it is possible to state whether it has failed or not. This leads to a unique division of the $X$-space into two sets, called the safe set $S$ and the failure set $F$, respectively. As shown in Figure 4.1, the two sets are separated by the failure surface, or limit state surface $L_x$.

A failure function, $Z = g(X)$, of random design variables can be formulated so that the event of failure can be defined as:

$$Z = g(X) \leq 0$$

(4.1)

The corresponding limit state function is expressed as:

$$Z = g(X) = 0,$$

(4.2)
Figure 4.1 Illustration of Limit State Function
And the event of safety can be identified as:

\[ Z = g(\mathbf{X}) \geq 0 \]  

(4.3)

Thus, the structural failure probability \( P_f \) can be defined as:

\[ P_f = P(g(\mathbf{X}) \leq 0) = \int_{g(\mathbf{x}) \leq 0} f_X(\mathbf{x}) d\mathbf{x} \]  

(4.4)

where \( f_X(\mathbf{x}) \) is the joint probability density function for \( X_1, X_2, ..., X_n \).

The structural reliability, \( P_r \), can similarly be defined as:

\[ P_r = 1 - P_f = P(g(\mathbf{X}) > 0) \]  

(4.5)

In general, due to lack of statistical data, the joint probability density function for the basic variables vector \( \mathbf{X} \) is difficult to obtain. Even if this density function is available, the evaluation of the multiple integral in Eq. 4.4 can be extremely complicated. Thus, the exact method of numerical integration is practical for only a very limited class of problems of low dimensions. Consequently, various approximate and exact methods have been developed. Among approximate methods, the First-Order Second-Moment (FOSM), First-Order Reliability Method (FORM) and Second-Order Reliability Method (SORM) are regarded as acceptable considering the accuracy and efficiency of the algorithms, and they are widely used in practical problems. To estimate probabilities associated with complicated functions of random variables, the Monte Carlo simulation method can be used. This method is easy to apply and capable of solving complicated problems, and is therefore a valuable tool for design and research. However, it can be very expensive to obtain results with acceptable accuracy.
4.3 Methods for Component Reliability Analysis

4.3.1 First-Order Second-Moment Reliability Method

Element or component reliability and system reliability are two aspects of structural reliability. The term component reliability refers to the probability of survival or failure of individual elements of a structure corresponding to a performance criterion. The term system reliability refers to the probability of survival or failure of the structure as a whole. The failure of the entire structural system may occur in one of the several failure modes; each of them may consist of failure of several components. Thus, in any structural reliability analysis, the component or element reliability is estimated first, and the structural reliability is then estimated by considering the joint probability of failure of several elements.

In order to avoid some computational difficulties in the evaluation of risk using Eq. 4.4, the First-Order Second-Moment (FOSM) Reliability method was proposed for structural reliability analysis (Cornell, 1969; Ang and Cornell, 1974). The basic idea behind this method is based on the first-order Taylor series approximation of the limit state function using only the second-order statistics (mean and covariance) of the random variables. Expanding \( g( ) \) in Eq. (4.1) in a Taylor series about the mean values of the \( X_i \) and truncating the series at the linear terms, the first-order approximate mean and variance of \( Z \) can be shown to be

\[
\bar{Z} \approx g(\bar{X}_1, \bar{X}_2, ..., \bar{X}_n) \tag{4.6}
\]

and

\[
\sigma_z^2 \approx \sum_{i=1}^{n} \sum_{j=1}^{n} \left( \frac{\partial g}{\partial X_i} \right) \left( \frac{\partial g}{\partial X_j} \right) \text{Cov}(X_i, X_j) \tag{4.7}
\]

The partial derivatives of \( g( ) \) are evaluated at the mean values of all parameters, and \( \text{Cov}(X_i, X_j) \) is the covariance of \( X_i \) and \( X_j \).
The second-order mean (considering the square term in the Taylor series) can be used to improve the accuracy of the estimation of the mean and is shown to be

\[ \tilde{Z} \approx g(\bar{X}_1, \bar{X}_2, ..., \bar{X}_n) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left[ (\frac{\partial^2 g}{\partial X_i \partial X_j}) \text{Cov}(X_i, X_j) \right] \]  

(4.8)

Again, the partial derivatives are evaluated at the mean values of all parameters. For practical purposes, the use of the second-order mean and the first-order variance are adequate for most engineering applications (Haldar, 1981).

A measure of risk can be estimated by introducing a parameter \( \beta^* \) (Cornell, 1969) as:

\[ \beta^* = \frac{\tilde{Z}}{\sigma_z} \]  

(4.9)

This definition is illustrated geometrically in Figure 4.2. In this one-dimensional case, the concept behind the reliability index definition is the distance from the mean, \( \mu_z \), to the limit state surface measured in units of the uncertainty scale parameter \( \sigma_z \). It provides a good measure of reliability.

If \( Z \) is assumed to be a normal variable, the probability of failure, \( P_f \), can be given by

\[ P_f = 1 - \Phi(\beta^*) \]  

(4.10)

in which \( \Phi \) is the cumulative probability distribution function of the standard normal variate.

This formulation has several shortcomings (Haldar and Ayyub, 1984): (1) The function \( g(\cdot) \) in Eq. 4.2 is linearized at the mean values of the \( X_i \) variables. When \( g(\cdot) \) is nonlinear, significant error may be introduced by neglecting higher order terms; (2) it also may yield different results for different mechanically equivalent formulations of the same problem, for example, stress and strain formulations (Ditlevsen, 1973; Lind, 1973); and (3) most importantly, the approximate method completely ignores the information on distributions of the random variables.
Figure 4.2 Geometrical Illustration of Cornell Reliability Index
4.3.2 Advanced Second-Moment Method

To overcome the shortcomings of the First-Order Second-Moment Method, the Advanced Second-Moment Method (ASMM) was proposed by Hasofer and Lind (1974), in which Taylor series expansion of \( g() \) is linearized at some point on the failure surface rather than at the mean, say at point \( (X_1^*, X_2^*, ..., X_n^*) \). The linearizing point is called the design point or checking point. According to the approximation of the tangent hyperplane or hyperparaboloid to represent the limit state function at the checking point, the ASMM can be classified as the First-Order Reliability Method (FORM) or the Second-Order Reliability Method (SORM), respectively.

First-Order Reliability Method

In the FORM algorithm, the selection procedure for the design point can be explained as follows. With the limit state function and its variables as given by Eq. 4.2, the random variables \( X_i \)'s are first transformed to reduced uncorrelated variables with zero mean and unit variance. If the original basic variables \( X_i \)'s are uncorrelated, the transformation is given by:

\[
U_i = \frac{X_i - \mu_i^N}{\sigma_i^N} \tag{4.11}
\]

If \( X_i \)'s are correlated, the covariance matrix should be evaluated.

In many structural engineering problems, the design variables are non-normal. According to Rackwitz and Fiessler (1976), non-normal probability distributions can be incorporated in the reliability analysis by transforming the non-normal variables into equivalent normal variables at the checking point. In order to estimate the parameters of the equivalent normal distribution, \( \mu_i^N \) and \( \sigma_i^N \), two conditions can be imposed (Rackwitz and Fiessler, 1978). The cumulative distribution functions and the probability density functions of the actual variables and the approximate normal variables should be equal at the checking point \( (x_1^*, x_2^*), \)
on the limit state surface. The corresponding equivalent mean value and standard deviation of the original basic variables are shown to be:

\[ \sigma_{x_i}^N = \frac{\phi(\{F_i(X_i^*)\})}{f_i(X_i^*)} \]  

(4.12)

and

\[ \mu_{x_i}^N = X_i^* - \Phi^{-1}[F_i(X_i^*)]\sigma_{x_i}^N \]  

(4.13)

where \( F_i \) and \( f_i \) are non-normal distribution and density functions of \( X_i \), respectively, and \( \Phi(\cdot) \) and \( \phi(\cdot) \) are the cumulative distribution and density function for the standard normal variate, respectively.

The reliability index, \( \beta \), according to the Advanced Second-Moment Method (ASMM), is defined as the shortest distance to the limit state surface from the origin in the reduced U-space. As illustrated in Figure 4.3, \( \beta \) is the distance OA, when the limit state surface is represented by two variables.

The reliability index \( \beta \) can be calculated in both the reduced space or the original space. If the reduced space is use, the design points \( u_i^* \) can be calculated as:

\[ u_i^* = -\beta \alpha_i^* \]  

(4.14)

where \( \alpha_i^* \) is the unit normal vector to the failure surface at design points \( u_i^* \) and can be calculated as:

\[ \alpha_i^* = -\frac{\nabla g(u_i^*)}{|\nabla g(u_i^*)|} \]  

(4.15)

where \( \nabla g(u) \) is the gradient vector. \( \nabla g(u) \) can be expressed as

\[ \nabla g(u) = \left[ \frac{\partial g(u)}{\partial u_1}, \frac{\partial g(u)}{\partial u_2}, \ldots, \frac{\partial g(u)}{\partial u_n} \right] \]  

(4.16)

The actual limit state surface \( g(u) = 0 \) is then approximated by its tangent hyperplane, which at the design point \( u_i^* \) is the equation

\[ g(u) = \beta + \alpha^T u_i^* = 0 \]  

(4.17)
Figure 4.3 Hasofer-Lind Reliability Index: Nonlinear Case
The corresponding first order approximate limit state function $M$ can be defined as:

$$M = g(u) = \beta + \alpha^T u$$  \hspace{1cm} (4.18)

Also, the design point should satisfy the new limit state as:

$$g(u_1^*, u_2^*, \ldots, u_n^*) = 0$$  \hspace{1cm} (4.19)

Thus, $\beta$ can be obtained iteratively and the corresponding approximate failure probability can be estimated as:

$$P_f \simeq \Phi(-\beta)$$  \hspace{1cm} (4.20)

An algorithm proposed by Rackwitz (1976) to compute $\beta$ and $x_i^*$ can be summarized as follows:

1. Assume initial values of the design point $x_i^*$, i=1,2,...,n. Typically the initial design point may be assumed to be the mean values of the random variables. Using Eq.4.11, the corresponding reduced design point $u_i^*$ can be obtained.
2. Using Eqs. 4.12 and 4.13, obtain the equivalent mean values and standard deviations for the original basic non-normal variable.
3. Evaluate $\alpha_i^*$ at $u_i^*$ according to Eqs. 4.15 and 4.16.
4. Obtain the new design point $u^*$ in terms of $\beta$, as in Eq. 4.14.
5. Substitute the above $u_i^*$ in the limit state function, as expressed as Eq. 4.19, and solve for $\beta$.
6. Using the $\beta$ thus obtained in step 5, re-evaluate the reduced design point $u_i^*$ according to Eq. 4.14; and
7. Repeat steps 2 through 5 until convergence is achieved.

The algorithm is shown in Figure 4.4. The algorithm constructs a linear approximation to the limit state surface at every search point and finds the shortest distance from the origin to the linearized limit state surface as defined in Eq. 4.17.
Figure 4.4 The Algorithm by Rackwitz for Finding H-F Index
Also, note that the mean and standard deviation of $M$, as defined in Eq. 4.18, are $\beta$ and 1, respectively. Thus, the first order reliability index can be expressed as:

$$\beta = \frac{\mu M}{\sigma M} \quad (4.21)$$

This implies that the important difference between ASMM and FOSM is that the limit state function is finally linearized at the most probable failure point rather than at the mean values of the random variables.

**Second-Order Reliability Method**

If the limit state function is highly nonlinear, the First-Order Reliability Method (FORM), discussed above, may not converge. In such a situation, the Second-Order Reliability Method (SORM) would be a good alternative. In a SORM approach, the limit state surface is approximated by a hyperparaboloid which has the same tangent hyperplane and the main curvatures at the design point as FORM. An approximation to the failure probability is then (Madsen, 1985; Jiao, 1989a):

$$P_{f,SORM} \approx \Phi(-\beta)\prod_{j=1}^{n-1}(1 - \beta \kappa_j)^{-\frac{1}{2}}$$

$$+ \sum_{k=1}^{n} [\beta \Phi(-\beta) - \phi(\beta)] \prod_{j=1}^{n-1}(1 - \beta \kappa_j)^{-\frac{1}{2}} - \prod_{j=1}^{n-1} \left[1 - \left(\beta + 1\right) \kappa_j\right]^{-\frac{1}{2}}$$

$$+ (\beta + 1)[\beta \Phi(-\beta) - \phi(\beta)] \left[\prod_{j=1}^{n-1}(1 - \beta \kappa_j)^{-\frac{1}{2}} - \text{Re}[\prod_{j=1}^{n-1}(1 - (\beta + i) \kappa_j)^{-\frac{1}{2}}]ight] \quad (4.22)$$

where $i$ in the third term is the imaginary unit, $\text{Re}[\ ]$ denotes the real part, and $\kappa_i$ $(i=1,2,...,n-1)$ are the principal curvatures at the design point. The first term is the asymptotic result for $\beta \to \infty$.

Using SORM, an equivalent hyperplane can be defined as a linear approximation to the true failure surface with a reliability index

$$\beta_{SORM} = -\Phi^{-1}(P_{f,SORM}) \quad (4.23)$$

The unit normal vector $\alpha_{SORM}$ is, in practice, set approximately equal to that obtained by FORM.
For practical structural engineering problems, there is no significant difference between FORM and SORM approaches for solution of $\beta$ (Jiao, 1989b). FORM is used in this study.

### 4.3.3 Monte Carlo Simulation

#### 4.3.3.1 Introduction

Simulation is the process of replicating the real world based on a set of assumptions and conceived models of reality. It can be performed theoretically or experimentally. In practice, theoretical simulation is usually performed numerically; this has become a much more practical tool since the invention of computers. As with experimental methods, numerical simulation may be used to generate (simulate) data, either in lieu of or in addition to actual real-world data. In effect, theoretical simulation is a method of numerical or computer experimentation.

Monte Carlo Simulation is often used for some complicated problems which involved random variables with known (or assumed) probability distributions. It consists of the following steps:

1. Generate the input random variables according to their probability distribution functions.
2. Perform the deterministic analysis using the simulated numbers and check if the system has failed or not (i.e. $g(X) \leq 0$).
3. Repeat steps 1 and 2 a number of times ($N$) and count the number of failures ($N_f$).
4. Obtain the estimate of the mean probability of failure as

   $\hat{P}_f = \frac{N_f}{N}$

Since a sample from a Monte Carlo simulation is similar to a sample of experimental observation, Monte Carlo simulation is also a sampling technique, and as such shares the same problems of sampling theory; namely, the result is also subject to sampling errors.
In order to reduce the sampling error and ensure the accuracy of the solution, a large sample size is necessary. However, with the increase of sample size, the requirements of computer time and storage will also increase significantly. So it is necessary to determine a minimal sample size without compromising the accuracy. Generally, in structural reliability problems, the reliability index $\beta$ can be expected to be around 4.0 and the corresponding probability of failure will be no less than $3 \times 10^{-5}$. Thus, a reasonable sampling size would be between 10,000 and 500,000 depending upon the problem under consideration.

4.3.3.2 Generation of Random Number

One of the main tasks in a Monte Carlo simulation is the generation of random numbers from prescribed distributions. Thus, for a given set of generated random numbers, the simulation process is deterministic. With the help of a computer, the automatic generation of the requisite random numbers with specified distributions can be accomplished systematically for each variable by first generating a uniformly distributed random number between 0 and 1.0, and then, through appropriate transformations, obtaining the corresponding random number with the specified probability distribution.

Consider a random variable $X$ with a cumulative density function (CDF) $F_X(x)$. Then, at a given cumulative probability $F_X(x) = u$, the value of $X$ is

$$x = F_X^{-1}(u) \quad (4.25)$$

Now suppose that $u$ is a value of the standard uniform variate, $U$, with a uniform PDF between 0 and 1.0; then (as shown in Figure 4.5):

$$F_U(u) = u \quad (4.26)$$

Equation 4.26 denotes that the cumulative probability of $U \leq u$ is equal to $u$. 
Figure 4.5  PDF and CDF of Standard Uniform Variable U
Therefore, if \( u \) is a realization of \( U \), the corresponding value of the variate obtained through Eq. 4.25 will have a cumulative probability:

\[
P(X \leq x) = P[F_X^{-1}(U) \leq x] = P[U \leq F_X(x)] = F_X(x)
\]

(4.27)

which means that if \( (u_1, u_2, ..., u_n) \) is a set of values for \( U \), the corresponding set of values obtained using Eq. 4.25 is:

\[
x_i = F_X^{-1}(u_i); \quad i = 1, 2, ..., n
\]

(4.28)

\( x_i \) will have the desired CDF \( F_X(x) \). The relationship between \( u \) and \( x \) is presented graphically in Figure 4.6.

Using the Eq. 4.25, random numbers corresponding some important distributions can be obtained as discussed in the following sections.

**Exponential Distribution**

For the exponential distribution, the CDF is

\[
F_X(x) = 1 - \exp(-\lambda x); \quad x \leq 0
\]

(4.29)

The inverse function is

\[
x = F_X^{-1}(u) = -\frac{1}{\lambda} \ln(1 - u)
\]

(4.30)

**Normal Distribution**

If \( u_1 \) and \( u_2 \) are a pair of independent uniformly distributed random numbers, a pair of independent random numbers from a normal distribution \( N(\mu, \sigma) \) can be generated by (Ang and Tang, 1984):

\[
x_1 = \mu + \sigma \sqrt{-2\ln u_1 \cos 2\pi u_2}
\]

\[
x_2 = \mu + \sigma \sqrt{-2\ln u_2 \sin 2\pi u_2}
\]
Figure 4.6 Relation Between U and X.
**Lognormal Distribution**

For a lognormal variate $X$ with parameters $\lambda$ and $\xi$, it can be shown that $\ln X$ has a normal distribution with mean $\lambda$ and standard deviation $\xi$. Therefore, if $x'$ is a value from a normal distribution $N(\lambda, \xi)$, then

$$x = \exp(x')$$

(4.33)

will be a random number from the lognormal distribution with parameters $\lambda$ and $\xi$.

Once the random numbers corresponding to the different distributions have been generated, Monte Carlo simulation can be conducted easily using the steps discussed in Section 4.3.3.1.

### 4.4 Reliability Analysis of In-service Crack Growth

In a fatigue reliability analysis, it is of particular importance to estimate the probability that a fatigue crack will exceed a preassigned value during the service life of the structure.

In order to develop a risk-based fatigue evaluation model, it is necessary to have the critical crack length $\alpha_c$, which can be either a value which will cause failure or the design value according to the serviceability requirement.

When the critical crack size $\alpha_c$ is specified, the fatigue failure criterion at the $N$th stress cycles can be defined as:

$$\alpha_c - \alpha_N \leq 0$$

(4.34)

The in-service crack size $\alpha_N$, which propagated from its initial values $\alpha_0$ at $N_0$ to $\alpha_N$ at $N$ cycles, can be predicted by a crack growth law, such as the Paris Equation. Once $\alpha_N$ is greater than the critical crack size, failure occurs. Thus, Eq. 4.34 represents the limit state of interest.
As the function $\psi(\alpha_2, \alpha_1)$, as defined in Eq. 3.12, is monotonically increasing with crack size $\alpha$, the above failure criterion can be replaced equivalently by:

$$\psi(\alpha_c, \alpha_0) - \psi(\alpha_N, \alpha_0) \leq 0$$  \hspace{1cm} (4.35)

It is known from Eq. 3.13 that:

$$\psi(\alpha_N, \alpha_0) = C\tilde{S}^m (N - N_0)$$  \hspace{1cm} (4.36)

The corresponding limit state function is

$$M = \psi(\alpha_c, \alpha_0) - C\tilde{S}^m (N - N_0)$$  \hspace{1cm} (4.37)

The failure event can then be expressed as:

$$g(Z) = \psi(\alpha_c, \alpha_0) - C\tilde{S}^m (N - N_0) \leq 0$$  \hspace{1cm} (4.38)

And the failure probability can be calculated by:

$$P_f = P(g(z) \leq 0) \cong \Phi(-\beta)$$  \hspace{1cm} (4.39)

If the distribution function of $\alpha_N$, $F_{\alpha_N}(\alpha_N)$, is known, the failure probability can be estimated alternatively as:

$$P_f = P(\alpha_N \leq \alpha_c) = 1 - F_{\alpha_N}(\alpha_c)$$  \hspace{1cm} (4.40)

For practical engineering problems, the theoretical expression for the distribution function of crack size, $F_{\alpha_N}(\alpha_N)$, is very difficult to obtain. So, Eq. 4.39 is used to evaluate the failure probability of the event.
4.5 Methods for System Reliability Analysis

4.5.1 Intersections and Unions

A real structure, consisting of a number of components, can be represented by several configurations whose failure will lead to the failure of the system. Failure of all the components in a particular configuration will result in the failure of the system, and this type of configuration is termed a cutset (Madsen, 1986; Jiao, 1989a). Therefore, a cutset is the intersection of all the failures of the components in the configuration. System failure occurs when any one of the cutsets is realized. That is, the system failure is the union of all these cutsets.

As illustrated in Figure 4.7, a structural system consists of \( n_1 \) cutset, each of which is composed of \( n_2 \) components. If the first-order approximation of the limit state function of the \( i \)th component, according to Eq. 4.16, can be represented as:

\[
M_i = \beta_i + \alpha_i^T U
\]

then the corresponding probability of failure of the system, \( P_{f,sys} \), can be expressed as (Jiao, 1989a):

\[
P_{f,sys} = P(\cup^{n_1}(\cap^{n_2}M_i \leq 0))
\]

where \( n_1 \) is the total number of cutsets in the system and \( n_2 \) is the number of components in a given cutset.

Probability Evaluation of Intersection of Events

In a cutset with \( n_2 \) components, the component safety index corresponding to the first-order limit state can be evaluated by Eq. 4.15. Thus, the reliability indices vector and correlation vector of the components are given by:

\[
\beta = \{\beta_i\} = \{\alpha_i^T u^*\}
\]

and

\[
\rho = \{\rho_{ij}\} = \{\alpha_i^T \alpha_j\}
\]
Figure 4.7 Illustration of Structural Configurations
So the first-order failure probability of the intersection (parallel system) is a multiple normal integral given by:

\[ P(\cap^{n_2} M_i \leq 0) = \Phi_{n_2}(-\beta; \rho) \]  

(4.45)

**Probability Evaluation of Union of Events**

To compute the probability of a union of events (series system), two approaches are available. The first approach is to express the union of events in terms of an intersection of complementary events, that is

\[ P(U^{n_1} M_i \leq 0) = 1 - P(\cap^{n_1} M_i \geq 0) = 1 - \Phi_{n_1}(\beta; \rho) \]  

(4.46)

Using Eq. 4.45, the probability of a union of events can be further simplified.

The second approach is based on the exact expression as (Jiao, 1989a):

\[ P(U^{n_1} M_i \leq 0) = \sum_i P(M_i) - \sum_i \sum_{j<i} P(M_i \cap M_j) + \sum_{i<j<k} \sum_{j<i} P(M_i \cap M_j \cap M_k) - \ldots + (-1)^{n_1+1} P(\cap^{n_1} M_i) \]  

(4.47)

which leads to an upper and lower bound to the system failure probability (Ditlevsen, 1986). They can be expressed as:

\[ P(U^{n_1} M_i) \leq P(M_1) + \sum_{i=2}^{n_1} -\max\{P(M_i \cap M_j)\} \]  

(4.48)

and

\[ P(U^{n_1}) \geq P(M_1) + \sum_{i=2}^{n_1} [\max\{0, P(M_i) - \sum_{j<i} P(M_i \cap M_j)\}] \]  

(4.49)
4.5.2 Formulation of System Reliability

In a system with $n$ components or details, the limit state function of fatigue safety of the $i$th component is defined as

$$M_i = \Psi_i(\alpha_{c,i}, \alpha_{0,i}) - C_i \hat{S}_i^{m_i}(N - N_{0,i}) \quad i = 1, 2, ..., n$$  (4.50)

The basic variables involved may differ from one component to another due to possible differences in structural details, material properties, environmental conditions, and responses to random loadings. The component limit states are usually neither identical nor independent. This leads to a complicated system whose components are generally partly correlated.

In cutset expression, the system failure events can thus be expressed in terms of its safety margin as in Eq. 4.21. The probability of system failure can be evaluated by Eqs. 4.48 and 4.49.
Chapter 5

Reliability of Non-destructive Inspection

5.1 Introduction

The purpose of maintenance, as discussed earlier, is to control and improve the structural safety and reliability under in-service condition. To obtain an acceptable level of fatigue reliability in a structural component or system, two strategies can be selected (ASCE, 1982). The first strategy completely depends on the careful selection of design loads, methods of analysis, quality control procedures for material properties and fabrication, and a critical reliability assessment of the completed structure before it is put into service. The second strategy is partial dependence on a plan of periodic reassessing of the reliability of the structure through inspections with repair if required. This may allow a more economical design and may help to extend the service life of the structure. The selection of the second strategy is more economical in general, if the selection is not dominated by an operating environment or a structural configuration which makes one or more critical components non-inspectable.

In this chapter, some of the very important aspects of the reliability of non-destructive inspection are discussed in detail. The focus is on the selection of inspection methods, the crack detectability in terms of its size and the corresponding accuracy. Also, a risk-based model of in-service inspection is proposed.

5.2 Methods of Non-destructive Inspection

Non-destructive Inspection (NDI), also called Non-destructive Testing (NDT) or Non-destructive Evaluation (NDE), is a technique to detect the defects in engineering materials and components. The term non-destructive means that any satisfactory specimen examined remains fit for service after the inspection. During
the NDT, the material properties could change, but the change will be within the allowable level.

Any non-destructive inspection method consists of several distinct elements. Some of the most important elements are: (1) components to be inspected; (2) type of cracks; (3) inspection technique, inspection equipment and inspection procedure; and (4) interpretation of the results.

There are several factors that would affect inspection results (Engesvik, 1985). They are: (1) Modeling effects; (2) human factors; and (3) inspection factors.

The modeling effects are composed of material characteristics, types of defects, component configuration and surface condition including thickness, presence of abrupt geometry changes, and accessibility of critical regions.

The human factors include variation in inspector skill, interpretation of results, variations in calibration of equipment, variations in inspection procedures and sequence of operations.

The inspection factors are attributed to different inspection environments including laboratory, factory and field conditions, and the corresponding detectability.

Various NDI techniques have been used for the purpose of detecting cracks, such as visual inspection, ultrasonic inspection, liquid penetrant inspection, magnetic particle inspection, magnetic field inspection, and radiographic inspection (Yee, 1976). In the following sections, the first two inspection techniques will be discussed in detail since they are the most basic and important, and are commonly used methods in practice to detect cracks in welded connections of steel structures.

5.2.1 Visual Inspection

Visual inspection is the most basic, but perhaps the most disregarded approach to NDT. It covers a wide field, ranging from sophisticated TV monitoring system to the simple hand operation. It improves the detectability considerably when used in conjunction with magnetic inks and penetrant dyes.
In most cases, visual inspection with either the unaided eye or magnifiers is the most economical method available for detecting surface cracks in structural components. It can be used for all materials, but the reliability of results depends on the skill of the inspector (ASCE, 1982).

Whether a TV system, microscope or the unaided eye is used, it is clear that a visual examination of the surface of a specimen can locate defects and give warning of changes in general condition.

The procedure of visual inspection includes: (1) to determine the critical location where defects are most likely to occur; (2) to completely remove dirt, rust, paint and other surface coatings before the examination; and (3) to use instrument or hand operations to examine the cracks in the critical locations.

Some of the cracks can be missed during a simple visual inspection. To improve the detectability of the visual inspection, two techniques are commonly used. Dye penetrants can be used to stain the crack line. The magnetic particle inspection technique can also be used in which the stray field attracts suitably dyed magnetic ink to the crack region. These techniques are briefly discussed below.

5.2.2 Dye Penetrant Enhancement

Certain liquids can penetrate into the space between two surfaces separated by a very narrow gap, such as tight cracks. They will also enter cracks which are open to the surface. If these liquids are applied to the surface of the specimen, the crack becomes filled or partially filled with the liquid. If the liquid is colored, or carries a brightly colored dye, once the surplus liquid is removed, the crack opening stands out more clearly than when the crack was in its original state (Figure 5.1). An alternative approach to this process is the use of a liquid which fluoresces under ultraviolet light, so that the crack becomes clearly visible when examined under ultraviolet radiation.

The main disadvantage of the dye penetrant technique is that it relies on
Figure 5.1 Illustration for Penetrant and Magnetic Inspection
the ability of the penetrant liquid to enter the crack, and this will be affected by the crack condition. Thus, cracks already filled with liquid or corrosion products, or cracks which are very tight, may not allow the ingress of the penetrant, and so may not show up. On the other hand, shallow surface features, such as scratches, may allow some ingress of penetrant, and these would appear on the record to be important defects, creating an interpretation problem.

5.2.3 Magnetic particle inspection

If a tangential magnetic field is applied to the surface regions of a ferromagnetic specimen, it will normally lie totally within the specimen. However, if the specimen surface is cracked, a portion of the field is forced to leave the specimen locally, forming a stray field on the surface of the specimen. Magnetic particles will be particularly attracted to these regions of stray field. The magnetic particle technique relies on the application of a magnetic ink. Assuming that the ink is clearly visible, aided possibly by the addition of a dye, the regions which are defective will be clearly determined, as shown in Figure 5.1.

The technique has the advantage that the ink is easily attracted by the stray field, and that the magnetic field is affected by subsurface defects too, although the magnitude of the stray field generated falls off rapidly if the defect does not break the surface. On the other hand, the technique is limited in application to ferromagnetic materials. Also, tight cracks will not provide a large break in the magnetic circuit, and will thus not exclude the field so effectively. Thus, the result of magnetic particle techniques will be less sensitive to tight cracks, just like the dye penetrant method.

It is important to note that the surface condition has a very significant effect on the detectability of defects using visual inspection (Silk, 1987). If the surface is clean, crack length is of the order of 0.25 inch and with a depth of 0.01 inch, it can be detected with confidence in metals other than rough castings and some welds in metals under conditions of good illumination (daylight or strong flashlight) with the unaided eye (ASCE, 1982). By using a magnifier under laboratory conditions,
fatigue cracks of about one tenth of this length can be detected on polished surfaces (Pettit, 1974). Due to the effect of environmental conditions, the detectability under field conditions can be expected to be much poorer than for laboratory conditions.

5.2.4 Ultrasonic Inspection

Ultrasonic inspection is, perhaps, the most popular NDT technique in current use. It employs either the resonance method or the reflection method, which uses pulsed longitudinal waves. This method has been used to detect cracks in railroad rails since the late 1940s (Code, 1952). It is an accepted method for welding inspection (ASTM Designation) and the basis for a crack detection system being developed for steel bridges (Barton, 1973).

In any ultrasonic method, four components must be considered: First, the way in which the specimen is simulated, whether by piezoelectric transducer, hammer blow, heating, or by internal motion of dislocations under applied stress. Secondly, the nature, selection and various interactions of the elastic waves in the specimen which determine the response to the probe. Thirdly, a detector, which monitors the response and which incorporates any special post-processing required to extract information from received signals. Finally, it is important to arrange and select the probe and the detector so that they produce the most effective result.

The pulsed wave method is used to detect defects that are essentially normal to the surface being tested. A wave is reflected alternately between the surface and back to a receiving transducer from a defect, or from an edge of the element being tested. The echoes provide an indication of both the presence of the defect and its size. Readout can be in the form of an oscilloscope display, an audible signal of varying pitch, or digital display (ASCE, 1982).

The main advantages of the ultrasonic method include (Leonard, 1973; Silk 1987):

(1) the relative ease of penetration into materials of engineering significance such as steel and aluminium;
(2) the ability to test from only one surface and to detect defects at substantial depths;
(3) the sensitivity and comparative accuracy; and
(4) no significant radiation hazard requiring operational precautions.

The major restrictions are (ASCE, 1982; Silk, 1987):
(1) a number of materials rapidly attenuate an elastic wave; the method thus can not be used to inspect plastics, some heavy metals, and some composite materials; and
(2) it is also limited to small area coverage.

The reliability of crack detection and the accuracy of the measurement depend on the equipment used and the distance from the probe to defects, the surface condition of the material to be inspected, and the types of defects (Jiao, 1989a). In general, buried defects can be less readily detected. Weld defects may be more readily detected than cracks in situations where it is impossible to place probes optimally.

Equipment capable of detecting 0.75 inch long cracks at a distance from 3-10 ft is reported to be available (ASCE, 1982). According to Packman, et al. (1969), there is over a 50 percent chance of detecting cracks longer than 0.2 inch in aluminum and 0.1 inch in steel. Cracks longer than 0.25 inch in aluminum and 0.35 inch in steel can be detected with certainty. It is agreed that the minimum size detectable for in-service inspections may be much smaller than that under laboratory conditions. Although, fatigue cracks with lengths of only 0.25 inch have been detected under field conditions (Baldwin, 1978), the detection of crack size is very inconsistent.
5.2.5 Summary

As a summary, the sensitivities of different NDT methods are listed in Table 5.1. The detectabilities of the different NDT methods are given in Table 5.2. Based on the discussions in previous sections and information provided in Table 5.1 and Table 5.2, several important observations can be made:

1. Among a number of NDT techniques, no single NDT technique is unquestionably superior to others for all cases;
2. both the ultrasonic method and magnetic particle method are sensitive enough to detect small cracks and the results obtained are consistent; but the ultrasonic method may be more sensitive than the magnetic particle method for detecting cracks in steel specimens; and
3. the dye penetrant method is more sensitive to detect small fatigue cracks in aluminum than in steel, but it is poorer than the ultrasonic method.

MARTA has access to ultrasonic equipment that can be used to inspect cracks in full-penetration butt welds in the bridges. The USK 7S, Portable Ultrasonic Flow Detector, can detect cracks with a minimum size of 0.05 inch. Since the USK 7S is lightweight (only about 11.5 lbs), it is very easy to use for the field inspections for MARTA's bridge structures. The details of this equipment are given in Appendix B.

5.3 Probabilistic Model of Inspection Capability

The capability of an inspection technique can be defined in terms of two criteria:

1. the sensitivity of the method in detecting cracks; and
2. the accuracy of the method in interpreting detected cracks.

A considerable amount of uncertainties is expected in the evaluation of these two criteria. As discussed earlier, the material properties, human error and inspection environment are major sources of uncertainties.
Table 5.1  Sensitivity of Nondestructive Test Methods

<table>
<thead>
<tr>
<th>Actual Crack Length Range</th>
<th>FRACTION OF CRACKS DETECTED (SENSITIVITY)*</th>
<th>Magnetic Particle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Penetrant</td>
<td>Ultrasonics</td>
</tr>
<tr>
<td></td>
<td>Steel (3)</td>
<td>Aluminum (4)</td>
</tr>
<tr>
<td>Inches (1)</td>
<td>Millimeters (2)</td>
<td></td>
</tr>
<tr>
<td>0.001-0.050</td>
<td>0.01-1.27</td>
<td>0.1111</td>
</tr>
<tr>
<td>0.051-0.100</td>
<td>1.28-2.54</td>
<td>0.4000</td>
</tr>
<tr>
<td>0.101-0.150</td>
<td>2.55-3.81</td>
<td>0.3333</td>
</tr>
<tr>
<td>0.151-0.200</td>
<td>3.82-5.08</td>
<td>0.3000</td>
</tr>
<tr>
<td>0.201-0.250</td>
<td>5.09-6.35</td>
<td>0.6000</td>
</tr>
<tr>
<td>0.251-0.300</td>
<td>6.36-7.62</td>
<td>0.7978</td>
</tr>
<tr>
<td>0.301-0.350</td>
<td>7.63-8.89</td>
<td>0.6250</td>
</tr>
<tr>
<td>0.351-0.400</td>
<td>8.90-10.16</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.401-0.450</td>
<td>10.17-11.43</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.451-0.500</td>
<td>11.44-12.70</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

*Values reported by Packman et al. (56) for laboratory inspection of fatigue cracked steel and aluminum cylinders.
Table 5.2 Crack Size in Perspective

<table>
<thead>
<tr>
<th>INCHES</th>
</tr>
</thead>
<tbody>
<tr>
<td>.001</td>
</tr>
<tr>
<td>.01</td>
</tr>
<tr>
<td>0.1</td>
</tr>
<tr>
<td>1.0</td>
</tr>
<tr>
<td>MILLIMETERS</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
</tbody>
</table>

- **Range of Grain Size in Steels**
- **Average Flaw Size Observed in Welds (Albrecht)**
- **Median Flaw Size that Can be Detected by Ultrasonics (Harris)**
- **USN: Transition between crack initiation and propagation**
- **Median Flaw Size that Can be Detected by Ultrasonics (Harris)**
- **Average total defect size due to both undercut and slag inclusions (Himpey)**
- **Lower limit for the application of LEFM for fatigue crack growth (Engesvik)**
- **Average Flaw Size Observed in Welds (Albrecht)**
- **Range of Values for which detection is "almost certain" for production NDI, i.e., liquid penetrant, ultrasonics, eddy current and magnetic particles (Packman)**
- **"Engineering" cracks present in tubular welded joint data at 10% of fatigue (SAF Fatigue Design and Evaluation Committee "Crack Initiation")**
- **Median Flaw Size that Can be Detected by Ultrasonics (Harris)**

---

1 lattice spacing

$\equiv 4 \times 10^{-7} \text{ mm}$
5.3.1 Crack Detectability

From the previous discussion, it is clear that there is always a critical crack size for a given NDT technique below which a crack can not be detected. Due to the large uncertainties involved in the NDT techniques, the detection of cracks should be treated as a random event. The probability of detecting a crack, termed the detectability, depends on the size of the crack and the resolution capability of the particular NDT technique employed.

Defining the crack detectability as $\alpha_d$, the results of an inspection can be classified into one of the following cases:

1. crack detection
   \[ \alpha > \alpha_d \] (5.1)

2. crack non-detection
   \[ \alpha < \alpha_d \] (5.2)

3. limit state of crack detection (crack size is exactly equal to $\alpha_d$)
   \[ \alpha = \alpha_d \] (5.3)

Experimental results by Packman, et. al. (1969) have been used as the basis for establishing the distribution function of crack detection. Some of the commonly used models for crack detectability under laboratory conditions are discussed below (ASCE, 1982):

**Exponential Model**

An exponential model has been proposed by different researchers (Davidson, 1975; Ichikawa, 1984 and Silk, 1987) to represent the crack detectability.

If the actual crack size is smaller than the lower limit of detectability, $\alpha_1$, then the probability of crack detection is zero; mathematically it can be expressed as:

\[ P_D(\alpha) = 0 \]
otherwise

\[ P_D(\alpha_d) = C_1(1 - \exp[-\lambda(\alpha_d - \alpha_1)]) \]  \hspace{1cm} \text{(5.4)}

Where \( C_1 \) is a value slightly smaller than unity, indicating that even a very large crack may not be detected with 100% confidence, and \( \alpha_1 \) is the lower limit of \( \alpha_d \) below which the crack would not be detected. In Eq. 5.4, \( \lambda \) should be chosen to best fit the experimental results. By selecting the confidence level corresponding to a given crack size, the value of \( \lambda \) can be determined using Eq. 5.4.

This model was fitted by Urabe and Yoshitake (1981) to the data of Packman (1969) on the magnetic particle inspection method. Jiao (1989b) also suggested that this model can be used for the underwater magnetic particle inspection for offshore structures.

Goranson and Rogers (1983) proposed a three-parameter Weibull type distribution. It can be expressed as:

\[ P_D(\alpha_d) = 1 - \exp\left[-\left(\frac{\alpha_d - \alpha_1}{\lambda}\right)^\xi\right] \]  \hspace{1cm} \text{(5.5)}

Eq. 5.5 becomes an exponential distribution when \( \xi = 1 \). Also, \( \lambda \) is expected to be around 0.1.

**Polynomial Modeling**

Based on the experimental results by Packman, et. al. (1969), Yang and Trapp (1975) proposed the following polynomial model:

\[ P_D(\alpha_d) = \left(\frac{\alpha_d - \alpha_1}{\alpha_2 - \alpha_1}\right)^m \]  \hspace{1cm} \text{(5.6)}

where \( \alpha_1 \) is the minimum crack size below which the crack can not be detected; \( \alpha_2 \) is the maximum crack size beyond which the crack can be detected with certainty. If \( \alpha_d < \alpha_1 \), then the detectability is 0; if \( \alpha_d > \alpha_2 \) then the detectability is 1. In Eq. 5.6, \( m \) is the parameter to be determined empirically and values of \( \alpha_1, \alpha_2 \) and \( m \) should be chosen to best fit the experimental results of a particular NDI method.
This model gives a reasonable approximation of the dye penetrant method and the upper bound of the ultrasonic method (ASCE, 1982).

**Lognormal Model**

Based on the results of the ultrasonic method, Harris (1977) observed that the probability of detection of a crack size $\alpha$ follows a lognormal distribution and it can be expressed as:

$$P_D(\alpha) = 1 - \frac{1}{2} \text{erfc}(\nu \ln \frac{\alpha}{\alpha_{50}})$$  \hfill (5.7)

where $\alpha_{50}$ is the crack size that has a 50% chance of being detected, and $\text{erfc}(\cdot)$ is the complementary error function. It is observed that $\alpha_{50}$ varies from about 1.6 to 9.0 mm, with $\nu$ ranging from 1.33 to 3.0 mm.

The complementary error function can be expressed as:

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-t^2) \, dt$$  \hfill (5.8)

Jiao (1989a) introduced the relationship between the complementary error function and standard normal distribution, which can be represented as:

$$\text{erfc}(x) = 2(1 - \Phi(\sqrt{2}x))$$  \hfill (5.9)

Thus, the Eq. 5.7 can be expressed alternatively as:

$$P_D(\alpha_d) = \Phi\left(\frac{\ln \alpha_d - \ln \alpha_{50}}{\sigma_{ln \alpha}}\right)$$  \hfill (5.10)

where $\sigma_{ln \alpha} = \frac{1}{\sqrt{2\nu}}$, is in the range of 0.23 to 0.53 for $\nu = 1.33$ to 3.0.

**The Field Condition Model**

Information on the effectiveness of field inspection appears to be scare (Johnson, 1973). The general consensus is that the effectiveness of service inspection is much lower than that of controlled laboratory conditions. Nakatsuji et al. (1982)
proposed the following model based on their own data on the manual ultrasonic inspection of welds in steel building as:

\[ P_D(\alpha) = (1 - \frac{\alpha_1}{\alpha})^\delta; \quad (\alpha \geq \alpha_1) \]

(5.11)

where \( \alpha_1 \) is the lower limit of crack detection and parameter \( \delta \) should be determined to best fit the inspection results.

5.3.2 Measurement Accuracy

Generally, in the case of crack detection, the crack size will also be measured. Due to the large uncertainty and error in the interpretation of the measured signal, the accuracy of the measurement can be treated as a random variable. A possible relationship between a real crack size \( A \) and the signal magnitude \( X \) was suggested by Jiao (1989a) as:

\[ A = c_1 + c_2 X + E_r \]

(5.12)

where \( c_1 \) and \( c_2 \) are random regression coefficients and \( E_r \) is a residual random error with zero mean value.

Madsen (1987) suggested that the normal distribution would be a good approximation to address the randomness in the measurement accuracy.

Silk (1987) indicated that the errors in size measurement are not necessarily normally distributed, since a crack is more likely to be oversized and a large scatter is expected in the crack size measurement.

In this study, the normal distribution model is used, since this model is commonly accepted and can be used easily in the variable updating model which will be discussed in Chapter 6.
5.4 Reliability of In-service Inspection

A structure is expected to be inspected several times during its lifetime. As discussed earlier, the reliability of an inspection depends on the detectability of a given inspection technique and the actual crack distribution at the time of inspection.

The possible results of an inspection are expected to be of three types:
1. no crack is detected;
2. a crack is detected but the crack size is unknown; and
3. a crack is detected and the crack size is measured.

5.4.1 Event Without Crack Detection

During an inspection, if the actual crack size at the time of inspection is smaller than the crack detectability for a given technique, the crack is not expected to be detected during the inspection. During the kth NDT inspection at the Nth stress cycles, with the detectability \( \alpha_{d,k} \), if no crack is detected, the event of without crack detection can be can be expressed as:

\[
\alpha_N - \alpha_{d,k} \leq 0
\]  

(5.13)

As discussed earlier, the exact expression of the actual crack size \( \alpha_N \) is very difficult to obtain for practical engineering problems since the geometry factors are unknown. However, as an alternative, the damage accumulation function can be used for this purpose as:

\[
\Psi(\alpha_N, \alpha_0) = C \tilde{S}^m(N - N_0)
\]  

(5.14)

So, the event of non-detection during the kth inspection can then be defined as (Madsen, 1986; Jiao, 1989a):

\[
I_k = \Psi(\alpha_N, \alpha_0) - \Psi(\alpha_{d,k}, \alpha_0) = C \tilde{S}^m(N - N_0) - \Psi(\alpha_{d,k}, \alpha_0) \leq 0
\]  

(5.15)
Usually, all these inspection events $I_k$'s are partly positively correlated and the degree of the correlation is highly dependent on the correlation between $\alpha_{d,k}$'s. In some special cases, the correlation between the $I_k$'s will be weaker, if the $\alpha_{d,k}$'s are mutually independent due to the difference in the inspection methods, equipments used and operators. The probability of no crack detection is thus expected to be smaller.

The probability of no crack detection in the $k$th inspection given no detections in the previous $k-1$ inspections can be defined and calculated as

$$P_{ND,k|k-1} = P(I_k \leq 0|\bigcap_{i=1}^{k-1} I_i \leq 0) = \frac{P(I_1 \leq 0 \cap \ldots \cap I_k \leq 0)}{P(I_1 \leq 0 \cap \ldots \cap I_{k-1} \leq 0)}$$  \hspace{1cm} (5.16)

### 5.4.2 Event With Crack Detection

If the actual crack size at the time of inspection is larger than the crack detectability of the inspection procedure, then the crack can be observed and it is termed the event of crack detection.

#### 5.4.2.1 Event Of Crack Detection But Without Size Measurement

Sometimes, a crack is detected but the crack size is not measured during a NDT inspection. Such an event, with crack detection but without size measurement, can be defined as:

$$\alpha_{d,i} - \alpha_{N_i} \leq 0$$  \hspace{1cm} (5.17)

It is obvious that the event is complementary to the event without crack detection. Using the damage accumulation function, this event can thus be defined as:

$$D_i = -I_i = \Psi(\alpha_{d,i}, \alpha_0) - C(S^m(N_i - N_0) \leq 0)$$  \hspace{1cm} (5.18)

The probability of crack detection after $k$ successive inspections is

$$P_D = 1 - P_{ND} = 1 - P(\bigcap_{i=1}^{k} I_i \leq 0) = P(\bigcap_{i=1}^{k} D_i \leq 0)$$  \hspace{1cm} (5.19)
Considering both the detection and the non-detection events, the probability of crack detection at the \( k \)th inspection given that no crack detections occurred in the previous \( k-1 \) inspections can be defined as:

\[
P_{D,k|k-1} = P(D_k \leq 0 | \bigcap_{i=1}^{k-1} I_i \leq 0) = \frac{P(D_k \leq 0 \cap \bigcap_{i=1}^{k-1} I_i \leq 0)}{P(\bigcap_{i=1}^{k-1} I_i \leq 0)}
\]  \(\text{(5.20)}\)

### 5.4.2.2 Event With Crack Detection And Size Measurement

Generally, when a crack is detected, its size is also measured. As discussed earlier, because of human error in interpreting the measurement signals, the measured crack size should be treated as a random variable. If the actual crack size at the time of inspection is equal to the measured crack size, then the event of crack size measurement can be defined as (Madsen, 1985):

\[
\alpha_{N_i} - A_i = 0
\]  \(\text{(5.21)}\)

or equivalently, it can be defined as:

\[
D_i = \Psi(A_i, A_{i-1}) - C \bar{S}^m (N_i - N_{i-1}) = 0
\]  \(\text{(5.22)}\)

### 5.5 Summary

As discussed in Sections 5.3 and 5.4, three possible results of inspection were well defined. Different detectability models are also discussed. Since the results of NDT inspections are also used as additional information to address the integrity of the structures, the updating of the reliability and the randomness in the parameters are two very important aspects of any reliability-based inspection procedure. They are discussed in Chapter 6.
Chapter 6
Model Updating through Inspection

6.1 Introduction

During the lifetime of a structure, inspection could be conducted several times to insure the integrity of the structure. Whether any crack is detected or not, each inspection provides additional information and results in changes of prior estimated reliability and uncertainty of basic variables. After an inspection decisions, such as modification of inspection plans, changes in inspection methods, or the time to repair or replace the damaged components, should be based on the updated reliability or the distributions of the basic variables (Jiao, 1989).

Considering the probabilistic crack growth model discussed in Chapter 4, the outcome of an inspection can be classified as no crack detection, crack detection without size measurement, and crack detection with size measurement. When no crack is detected, the target reliability of the structure is expected to increase and an extended service life can be expected. If a crack is detected, the target reliability is expected to change. Based on the amount of change in the reliability index, several decisions can be made, e.g., to repair a damaged element or replace it immediately, not to repair but to inspect more frequently, do nothing etc.

Since an inspection reflects the safety of the structure at the time of inspection, the limit state function of the structure as well as the distributions of all the random design variables need to be updated after an inspection. Thus, an applicable method of model updating is very important from a practical point of view.
6.2 Model Updating without Crack Detection

6.2.1 Reliability Updating

If the $i^{th}$ inspection is made at $N_i$th stress cycles and no crack is detected; it can be mathematically expressed as:

$$\alpha_N > \alpha_d$$

(6.1)

As proposed by Madsen (1987), the limit state function of target safety will remain the initial one and it can be written as:

$$M = \Psi(\alpha_c, \alpha_0) - \Psi(\alpha_N, \alpha_0) = \Psi(\alpha_c, \alpha_0) - C\tilde{S}_m(N - N_i)$$

(6.2)

The corresponding limit state function of the $i^{th}$ inspection event is defined as:

$$I_i = \Psi(\alpha_{N_i}, \alpha_0) - \Psi(\alpha_{d_i}, \alpha_0) = C\tilde{S}_m(N_i - N_0) - \Psi(\alpha_{d_i}, \alpha_0) \leq 0$$

(6.3)

Using the Bayesian Approach, the probability of failure can be obtained as:

$$P_f = \frac{P(M \leq 0 \cap I_i \leq 0 \cap \cdots \cap I_i \leq 0)}{P(I_i \leq 0 \cap \cdots \cap I_i \leq 0)}$$

(6.4)

In order to achieve an analytical solution, an equivalent limit state function can be defined as (Jiao, 1989a):

$$P(I_i \leq 0) = P(I_i \leq 0 \cap \cdots \cap I_i \leq 0)$$

(6.5)

Thus, the updated failure probability can be expressed as:

$$P_{f,up} = \frac{P(M \leq 0 \cap I_i \leq 0)}{P(I_i \leq 0)} = \frac{\Phi_2(-\beta_M, -\beta_I; \rho)}{\Phi(-\beta_I)}$$

(6.6)

where $\beta_M$ is the safety index corresponding to the limit state function of the fatigue safety and $\beta_I$ is the safety index of the equivalent limit state function with no crack detection, and the correlation coefficient of $M$ and $I$ can be expressed as:

$$\rho = \alpha_M^T \alpha_I$$

(6.7)
The CDF of bi-normal function for M and I can be expressed as:

\[ \Phi_2(-\beta_M, -\beta_I; \rho) = \Phi(\beta_M)\Phi(\beta_I) + \int_{0}^{\rho} \phi(-\beta_M, -\beta_I; z)dz \]  

(6.8)

where \( \phi(-\beta_M, -\beta_I; z) \) is the bi-normal probability density function and can be expressed as:

\[ \phi(x, y; \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp \left[ -\frac{x^2 + y^2 - 2\rho xy}{2(1-\rho^2)} \right] \]  

(6.9)

Using numerical methods, Eqs. 6.6, 6.8 and 6.9 can be evaluated.

To evaluated Eq. 6.6, another approximate solution algorithm was suggested by Terada and Takahashi (1988) and can be expressed as:

\[ P_f,up = P(M \leq 0 | I \leq 0) \approx \Phi(-\beta_{up}) \]  

(6.10)

where \( \beta_{up} \) is the updated reliability index. It can be estimated as:

\[ \beta_{up} = \frac{\beta_M - \rho A}{\sqrt{1-\rho^2}B} \]  

(6.11)

where

\[ A = \frac{\phi(-\beta_I)}{\Phi(-\beta_I)} \]  

(6.12)

and

\[ B = A(A - \beta_I) \]  

(6.13)

Such a simple solution gives a good approximation when the correlation between M and I is not very strong.

6.2.2 Updating the Distribution of the Random Variables

According to Madsen (1987), the distribution of a basic variable \( Z_i \) conditioned on a series of events of no crack detection can be updated as:

\[ F_{j,up}(z_j) = \frac{P(Z_j - z_j \leq 0 \cap I_1 \leq 0 \cap \cdots \cap I_i \leq 0)}{P(I_1 \leq 0 \cap \cdots \cap I_i \leq 0)} \]  

(6.14)
with the equivalent limit state function defined in Eq. 6.5, the distribution of $Z_j$ can be updated alternatively as:

$$ F_{j,up}(z_j) = P(Z_j - z_j \leq 0 | I \leq 0) = \frac{\Phi_2(-\beta_j, -\beta_I; \alpha_j)}{\Phi(-\beta_I)} $$  \hspace{1cm} (6.15)

where $\alpha_j$ is the $j$th component in $\alpha_I$, the design gradient vector of the equivalent event, and $\beta_j$ is defined as:

$$ \beta_j = -\Phi^{-1}(F_j(z_j)) $$  \hspace{1cm} (6.16)

Applying Eqs. 6.8, 6.14, and 6.16, the updated distribution becomes:

$$ F_{j,up}(z_j) = F_j(z_j) + \int_{-\beta_I}^{\alpha_j} \frac{\phi(\Phi^{-1}(F_j(z_j)), -\beta_I; z)dz}{\Phi(-\beta_I)} $$  \hspace{1cm} (6.17)

It could be observed that the system reliability computation can be avoided when the equivalent event margin ($I \leq 0$) is available.

When $\alpha_j$ is not close to 1.0, Eq. 6.10 can be alternatively evaluated as:

$$ F_{j,up}(z_j) = \Phi\left(\frac{\Phi^{-1}(F_j(z_j)) + \alpha_jA}{\sqrt{1 - \alpha_j^2B}}\right) $$  \hspace{1cm} (6.18)

For normal or lognormal variables, Eq. 6.18 can be simplified as described in the following paragraphs.

**Normal Variable**

For a normal variable $Z_j$ with mean $\mu_j$ and standard deviation $\sigma_j$,

$$ F_j(z_j) = \Phi\left(\frac{z_j - \mu_j}{\sigma_j}\right) $$  \hspace{1cm} (6.19)

The updated distribution becomes:

$$ F_{j,up}(z_j) = \Phi\left(\frac{z_j - \mu_{j,up}}{\sigma_{j,up}}\right) $$  \hspace{1cm} (6.20)

where

$$ \mu_{j,up} = \mu_j - \alpha_jA\sigma_j $$  \hspace{1cm} (6.21)
and

$$
\sigma_{j,up} = \sigma \sqrt{1 - \alpha_j^2 B}
$$

(6.22)

where \(A, B\) and \(\alpha_j\) are defined as in the previous sections.

**Lognormal Variable**

For a lognormal distribution:

$$
F_{j,up}(z_j) = \Phi\left(\frac{\ln z_j - \ln \tilde{z}_j}{\sigma_{\ln z_j}}\right)
$$

(6.23)

The updated distribution becomes:

$$
F_{j,up}(z_j) = \Phi\left(\frac{\ln z_j - \ln \tilde{z}_{j,up}}{\sigma_{\ln z_{j,up}}}\right)
$$

(6.24)

where

$$
\ln \tilde{z}_{j,up} = \ln \tilde{z}_j - \alpha_j A \sigma_{\ln z_j}
$$

(6.25)

and

$$
\sigma_{\ln z_{j,up}} = \sigma_{\ln z_j} \sqrt{1 - \alpha_j^2 B}
$$

(6.26)

It can be seen that a normal or lognormal distribution is invariant through updating, but its mean and standard deviation are updated.

### 6.3 Model Updating with Crack Detection

As mentioned earlier, the event of crack detection can be classified into two types of basic observations (Madsen, 1985):

1. crack detection with size measurement; and
2. crack detection without size measurement.

The corresponding mathematical expression for these events are:

$$
\alpha_{Ni} = A_i
$$

(6.27)

and

$$
\alpha_{Ni} \leq \alpha_d
$$

(6.28)
In the first case, a crack size $A_i$ is observed after $N_i$ stress cycles. $A_i$ is a random variable due to the measurement error and uncertainties in the interpretation of a measured signal.

In the second case, the crack is detected but the size is not observed. $\alpha_d$ represents a lower limit of detectability and is also a random variable because of the limitations of the equipment used.

6.3.1 Crack Size Measurement

6.3.1.1 Reliability Updating

If a crack is measured with sizes $A_1, A_2, \ldots, A_i$ at $N_1, N_2, \ldots, N_i$ cycles respectively, the limit state function of the $i$th inspection event is defined as:

$$D_i = \Psi(A_i, A_{i-1}) - CS^m(N_i - N_{i-1}) = 0 \quad (6.29)$$

where $A_0 = \alpha_0$ for $i = 1$. After the last crack size $A_i$ is measured, failure will occur if the crack size develops from $A_i$ to the critical value $\alpha_c$ within a given time period $(N - N_i)$. The limit state function of the re-defined target safety should be re-defined as:

$$M_{red} = \Psi(\alpha_c, A_i) - CS^m(N - N_i) \quad (6.30)$$

According to Madsen (1986), the updated failure probability is expressed as:

$$P_{f, up} = P(M \leq 0 | D_1 = D_2 = \cdots = D_i = 0) = \Phi(-\beta_{up}) \quad (6.31)$$

where the updated reliability index $\beta_{up}$ is given by (Madsen, 1985):

$$\beta_{up} = \frac{E(M | D_1 = D_2 = \cdots = D_i = 0)}{D(M | D_1 = D_2 = \cdots = D_i = 0)} \quad (6.32)$$

Because the joint distribution of the set of all first-order approximate limit functions is a normal distribution, the conditional reliability index follows from linear regression results as (Jiao, 1989a):

$$\beta_{up} = \frac{\beta_M - \rho_{MD}^{-1} \rho_{DD}^{-1} \beta}{\sqrt{1 - \rho_{MD}^{-1} \rho_{DD}^{-1} \rho_{MD}}} \quad (6.33)$$
where \( \rho_{MD} = \rho[M,D] = \{a^T \alpha_i \} \), \( \rho_{DD} = \rho[M,M] = \{a^T \alpha_j \} \), and \( \alpha \) is the design unit gradient vector of the limit state function of target safety, \( \alpha_i \) is the design unit gradient vector of limit state of the \( i \)-th event, \( \beta_M \) is the safety index of the limit state of interest, and \( \beta_i \) is the safety index corresponding to the limit state function of the \( i \)-th event.

For one observation, Eq. 6.33 reduces to:

\[
\beta_{up} = \frac{\beta_M - \rho_{MD} \beta_D}{\sqrt{1 - \rho_{MD}^2}}
\]  

(6.34)

### 6.3.1.2 Updating Distribution of Random Variables

Using the Bayesian theorem, the distribution of the basic variable \( Z_i \) conditioned on \( i \) successive detection events can be updated as follows:

\[
F_{j,up}(z_j) = P(M_j = Z_j - z_j \leq 0 | Z_1 = D_1 = \cdots = D_i = 0)
\]  

(6.35)

Since first-order approximation of the limit state follows a normal distribution, it can be shown that:

\[
F_{j,up}(z_j) = \Phi(-\beta_{j,up}(z_j)) = \Phi(-\frac{\beta_j(z_j) - \rho^T_{MD} \rho^{-1}_{DD} \beta_D}{\sqrt{1 - \rho_{MD}^2 \rho_{DD}^{-1} \rho_{MD}}})
\]  

(6.36)

Combining Eqs. 6.35 and 6.36, one obtains:

\[
F_{j,up}(z_j) = \Phi(\frac{\Phi^{-1}(F_j(z_j)) + \rho^T_{MD} \rho_{DD}^{-1} \beta_D}{\sqrt{1 - \rho_{MD}^2 \rho_{DD}^{-1} \rho_{MD}}})
\]  

(6.37)

In practical engineering problems, the updating is conducted right after the inspection. Based on the updated distribution of random variables, the next inspection will be conducted. So the actual updating is done only after one inspection.

If there is only one measurement, the updated distribution becomes:

\[
F_{j,up} = \Phi(\frac{\Phi^{-1}(F_j(z_j)) + \alpha_j \beta_D}{\sqrt{1 - \alpha_j^2}})
\]  

(6.38)
Note that the design value $z_j^*$ is related to the transformed value $u_j^*$ in the U-space by:

$$\Phi^{-1}(F_j(z_j^*)) = u_j^* = -\alpha_j\beta_D$$

(6.39)

The updated distribution in the original variable space is then:

$$F_{j,up}(z_j) = \Phi\left(\frac{\Phi^{-1}(F_j(z_j)) - \Phi^{-1}(F_j(z_j^*))}{\sqrt{1 - \alpha_j^2}}\right)$$

(6.40)

All the variables in Eqs. 6.35, 6.36, 6.37, 6.38, 6.39 and 6.40 were defined earlier.

In reliability analysis using the Advanced Second Moment Method, the limit state function is always linearized by its hyperplane $D_1 = 0$. Once the safety index and the gradient vector are obtained, the design values of the random variables can be easily evaluated by Eq. 4.17. Thus, the updating of the distribution of $Z_j$ becomes simple for one measurement.

The updated distribution can be further simplified for normal or lognormal variables as discussed below.

**Normal Variable**

If $Z_j$ is a normal variable with mean value $\mu_j$ and standard deviation $\sigma_j$, then it can be shown that:

$$\Phi^{-1}(F_j(z_j)) = \frac{z_j - \mu_j}{\sigma_j}$$

(6.41)

After introducing the inspection information, the distribution of $Z_i$ can be updated. According to Madsen (1985), the updated distribution of $Z_j$ is still normal and can be expressed as:

$$F_{j,up}(z_j) = \Phi\left(\frac{z_j - \mu_{j,up}}{\sigma_{j,up}}\right)$$

(6.42)

where $\mu_{j,up}$ and $\sigma_{j,up}$ are the updated mean and standard deviation of $Z_j$, and they can be calculated (for the case: $i \geq 2$) by:

$$\mu_{j,up} = \mu_j - \rho_{MD}\rho_{DD}\beta_D\sigma_j$$

(6.43)
and
\[ \sigma_{j,up} = \sigma_j \sqrt{1 - \rho_{MD}^{-1} \rho_{DD} \rho_{MD}} \] (6.44)

For a single measurement, \( D_1 = 0 \), one obtains:
\[ \Phi^{-1}(F_j(z_j^*)) = \frac{z_j^* - \mu_j}{\sigma_j} \] (6.45)

The updated characteristics \( \mu_{j,up} \) and \( \sigma_{j,up} \) become:
\[ \mu_{j,up} = z_j^* = \mu_j - \alpha_j \beta_D \sigma_j \] (6.46)
and
\[ \sigma_{j,up} = \sigma_j \sqrt{1 - \alpha_j^2} \] (6.47)

where \( \beta_D \) is the reliability index corresponding to event \( D_1 = 0 \), and \( \alpha_j \) is the ith component of the design gradient vector (\( \alpha_D \)). It can be seen that the updated mean of a normal variable \( Z_j \) is exactly its design value \( z_j^* \) corresponding to the hyperplane \( D_1 = 0 \).

**Lognormal Variable**

If \( Z_j \) is lognormally distributed, then \( \ln Z_j \) is normal with mean \( \ln z_j \) and standard deviation \( \sigma_{\ln Z_j} \). In this case:
\[ \Phi^{-1}(F_j(z_j)) = \frac{\ln z_j - \ln \tilde{z}_j}{\sigma_{\ln Z_j}} \] (6.48)

The updated distribution of \( Z_j \) is still lognormal and it can be expressed as:
\[ F_{j,up}(z_j) = \Phi\left(\frac{\ln z_j - \ln \tilde{z}_{j,up}}{\sigma_{\ln Z_{j,up}}}\right) \] (6.49)

where the characteristic values, \( \ln \tilde{z}_{j,up} \) and \( \sigma_{\ln Z_{j,up}} \), can be expressed as:
\[ \ln \tilde{z}_{j,up} = \ln \tilde{z}_j - \rho_{MD}^{-1} \rho_{DD} \beta_D \sigma_{\ln z_j} \] (6.50)

and
\[ \sigma_{\ln Z_{j,up}} = \sigma_{\ln Z_j} \sqrt{1 - \rho_{MD}^{-1} \rho_{DD} \rho_{MD}} \] (6.51)
In the case of one measurement, \( D_1 = 0 \), one obtains:

\[
\ln z_{j,up} = \ln z^*_j - \alpha_j \beta_j \sigma_{\ln Z_j} \tag{6.52}
\]

and

\[
\sigma_{\ln Z_j,up} = \sigma_{\ln Z_j} \sqrt{1 - \alpha_j^2} \tag{6.53}
\]

From the above discussion, the updating of a normal or lognormal variable is actually the updating of its characteristic values (mean and standard deviation) since its basic distribution is invariant through updating. When only one measurement is considered, it is important to note that the design value \( z^*_j \) is actually the updated mean value for a normal variable, while \( z^*_j \) is the updated median value for a lognormal variable.

### 6.3.2 Model Updating with Crack Detection

If the additional information is just crack detection without crack size measurement, then this event can be defined as:

\[
\alpha_{N_i} \geq \alpha_{d_i} \tag{6.54}
\]

The corresponding limit state of the event \( D_i \) is defined as:

\[
D_i = \Psi(\alpha_{d_i}, \alpha_0) - \Psi(\alpha_{N_i}, \alpha_0) = \Psi(\alpha_{d_i}, \alpha_0) - C \mathcal{S}_m(N_i - N_0) \leq 0 \tag{6.55}
\]

Thus, the updated failure probability can be expressed as:

\[
P_{f,up} = P(M \leq 0|D_1 \leq 0 \cap \cdots \cap D_i \leq 0) \tag{6.56}
\]

where \( M \) represents the initial limit state function of the fatigue safety. The updated distribution of a basic variable \( Z_j \) can be expressed as:

\[
F_{j,up} = P(Z_j - z_j \leq 0|D_1 \leq 0 \cap \cdots \cap D_i \leq 0) \tag{6.57}
\]
The event of non-detection \( (I_i \leq 0) \) and detection without size measurement \( (D_i \leq 0) \), at a stress cycle \( N \), are mutually exclusive and collectively exhaustive. Thus, the probability of the crack detection can be expressed as:

\[
P(D_i \leq 0) = 1 - P(I_i \leq 0)
\]  

(6.58)

According to the total probability theorem:

\[
P(M \leq 0 | D_i \leq 0) = \frac{P(M \leq 0) - P(M \leq 0 \cap I_i \leq 0)}{1 - P(I_i \leq 0)}
\]  

(6.59)

With the help of the technique discussed in Section 6.2.1, Eq. 6.59 can be evaluated easily.

**6.4 Model Updating After Repair**

As discussed earlier, to insure the integrity of the structure, a periodic inspection scheme is necessary. Based on the outcome of the inspection, the updated target safety index of the structure can be evaluated. If the updated target safety index is smaller than the prior selected critical safety index, a repair or replacement should be made immediately or in a short time period. If a repair takes place after \( N_{rep} \) stress cycles which leads to crack size \( A_{rep} \), the new safety margin can be established by considering the new material properties:

\[
M_{new} = \Psi(\alpha_c, c_{new}, \alpha_0, c_{new}) - C_{new}S\bar{m}_{new}(N - N_{rep})
\]  

(6.60)

The new failure probability after repair is simply:

\[
P_{f, up} = P(M_{new} \leq 0)
\]  

(6.61)

**6.5 Illustration**

Before proceeding further, it is important to summarize the information presented so far with the help of an illustration.
6.5.1 Reliability Analysis

Consider a bridge component shown in Figure 6.1. It consists of a large panel with a center crack and loaded uniaxially perpendicular to the crack. It is subjected to a constant-amplitude loading. For simplification, the geometry function $G(Y)$ is assumed to be 1.0. All lengths are measured in inches and the stress is expressed in ksi. The distribution of the basic variable are taken as:

\[
\begin{align*}
\ln \alpha_0 & \text{ follows } N(-3.912, -1.956) \\
\ln C & \text{ follows } N(-21.139, -10.569) \\
M & \text{ follows } N(2.5, 0.002)
\end{align*}
\]

where, $N(s, t)$ denotes a normal distribution with mean value $s$ and the standard deviation $t$.

A reliability-based LEFM approach is used in this example to evaluate the safety index. Two inspections are conducted at the two different times. The updating is made on the basis of the inspection information. The details are shown below.

The example has three basic variables and the corresponding standardized independent normal variables $U_1$, $U_2$ and $U_3$ can be obtained as:

\[
\begin{align*}
U_1 &= \Phi^{-1}(F_{\alpha_0}(\alpha_0)) = \frac{\ln \alpha_0 + 3.912}{-1.956} \\
U_2 &= \Phi^{-1}(F_C(c)) = \frac{\ln C + 21.139}{-10.569} \\
U_3 &= \Phi^{-1}(F_M(m)) = \frac{m - 2.5}{0.002}
\end{align*}
\]

$S$ and $\alpha_c$ are taken as 7.5 ksi and 2.5 inches respectively. The first-order reliability index is shown in Table 6.1 together with the design gradient vectors $\alpha$ and the first-order approximation to the failure probability for various values of $N$. 
Figure 6.1  Illustration for the Geometry of Example
Table 6.1 The Reliability Index and Design Gradient Vectors

<table>
<thead>
<tr>
<th></th>
<th>2,000,000</th>
<th>200,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Cycles</td>
<td>2,000,000</td>
<td>200,000</td>
</tr>
<tr>
<td>Safety Index</td>
<td>1.56</td>
<td>1.78</td>
</tr>
<tr>
<td>Design Gradient Vectors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial Crack Size</td>
<td>-0.221650</td>
<td>-0.108440</td>
</tr>
<tr>
<td>Fatigue Growth Parameter</td>
<td>-0.003896</td>
<td>-0.007450</td>
</tr>
<tr>
<td>Fatigue Growth Exponential</td>
<td>-0.000016</td>
<td>-0.000032</td>
</tr>
<tr>
<td>Detability of Inspection</td>
<td>0.975110</td>
<td>0.994075</td>
</tr>
</tbody>
</table>
6.5.2 Crack Detection with Size Measurement

If, in the above case, the design life is taken as $N=2 \times 10^6$ stress cycles and the first-order safety index $\beta$ is 2.84. During service, assume the first inspection is made after $5 \times 10^4$ stress cycles. The design point for this inspection becomes:

$$u_1^* = \beta_1 \alpha_1 = 1.56 (-0.22165, -0.0038958, -0.0000162, 0.97511)$$  \hspace{1cm} (6.68)

with a detection length $\alpha_I$ follows $N(0.1,0.05)$ inch. The correlation coefficient $\rho_1$ becomes:

$$\rho_1 = \alpha^T \alpha_1 = 0.0818$$  \hspace{1cm} (6.69)

with $\alpha$ from Table 6.1, the updated reliability index from $2 \times 10^6$ stress cycles follows:

$$\beta_{up} = \frac{\beta_M - \rho_1 \beta_I}{\sqrt{1 - \rho_1^2}} = 2.70$$  \hspace{1cm} (6.70)

With the second observation $\alpha_I \subseteq N(0.2,0.1)$ inches after $2 \times 10^5$ stress cycles, the reliability index can be further updated. The design point for this second observation is:

$$u_2^* = \beta_2 \alpha_2^T = 1.78 (-0.10844, -0.00745, -0.00003184, 0.9940747)$$  \hspace{1cm} (6.71)

and the correlation coefficients $\rho_2$ and $\rho_{12}$ are

$$\rho_2 = \alpha^T \alpha_2 = 0.0452$$  \hspace{1cm} (6.72)

$$\rho_{12} = \alpha_1^T \alpha_2^T = 0.993405$$  \hspace{1cm} (6.73)

The updated value of the reliability index is given by Eq.(6.7) as:

$$\beta_{up} = 3.537$$  \hspace{1cm} (6.74)

The relatively small changes in the reliability index after the first inspection is explained by a relatively large initial crack size. Therefore, the crack growth rate is very similar as predicted by the Paris law. With the second inspection it is, however, predicted that the crack grows slowly enough that it does not become very critical before the design life of $2 \times 10^6$ stress cycles.
Chapter 7

Fatigue Reliability Analysis of MARTA’s Bridges

7.1 Introduction

The full penetration butt welds connections of the MARTA’s steel bridges are subjected to cyclic tensile stresses due to the train loads. Under the action of oscillatory tensile stresses, the small defects in the welds will propagate with time. Thus, the material properties will be degraded. As defined earlier, this type of physical phenomenon is called fatigue. Obviously, the consequence of the fatigue could be catastrophic unless proper repairs are made in time.

In recent decades, more and more attention has been drawn to the fatigue and fracture problems. A series of extensive researches have been conducted in several areas including aircraft, offshore structures, nuclear plants, naval vessels and bridge structures. A number of codes have been developed to fulfill the requirements of fatigue design and control.

In the area of transportation engineering and structural engineering, the AASHTO Fatigue Design Specifications (AASHTO, 1988) provide a guideline for fatigue design of steel bridges. S-N curves, classified by the different connection details, are very easy to apply and are used by engineers.

In order to insure the fatigue reliability of MARTA’s bridge structures, various strategies can be adopted as mentioned earlier. Among these strategies, periodic in-service inspections by non-destructive testing (NDT) will be the most economical and practical alternative. In such a strategy, the integrity of the structures can be updated through inspections.

S-N curves approach may not be appropriate to evaluate the reliability of full penetration butt welds. Although the crack size is an important parameter, this information can not be utilized in the S-N curve approach. This is the major limitation of using S-N curves in the inspection strategy.
To overcome the shortcomings of the S-N curve approach, a more comprehensive and reasonable model, the Paris law, is used to describe the fatigue crack growth. The crack size is considered in this approach. Based on the Paris law, the different limit state functions (such as the limit state function of fatigue safety, the limit state function of inspection events, etc.) can be established and the corresponding reliability can be estimated. Updating of the stochastic crack growth model and the distributions of the basic variables can also be performed using the results obtained from the inspection.

7.2 Description of MARTA’s Bridge System

There are over 500 spans of varying lengths in the MARTA system, in which about 45 percent are steel. Table 7.1 (Haldar and Josey, 1988) gave the steel spans according to span type, support type and length. Approximately 74 percent of the steel spans are box-girder, in which 99 percent are simply supported.

The most common steel bridge span in the MARTA system is the simply supported box-girder with length between 70 and 80 feet. Thus, a steel box-girder 70 feet long is selected as a typical example for fatigue reliability analysis in this study (Figure 7.1). Other steel spans with different structural types and lengths can also be analyzed since the method developed in this study is very general in nature.

7.3 Parameters in Fatigue Reliability Analysis

7.3.1 Geometry Function

The distribution of crack size existing in the structures under in-service conditions includes the defects considered acceptable according to the specification as well as the those defects which are unacceptable but remain undetected during the fabrication inspection. Surface defects are usually more dangerous than embedded
Table 7.1 The Major Steel Spans

<table>
<thead>
<tr>
<th>Type of Span</th>
<th>Number of Spans per Support type</th>
<th>Length (FT)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simple</td>
<td>Continuous</td>
</tr>
<tr>
<td>Box Girder</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>52</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>96</td>
<td>0</td>
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<tr>
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<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>SUB - TOTAL</td>
<td>187</td>
<td>2</td>
</tr>
<tr>
<td>Plate Girder</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
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<tr>
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<td>2</td>
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</tr>
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</tr>
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<td></td>
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<td>2</td>
</tr>
<tr>
<td>SUB TOTAL</td>
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<td>19</td>
</tr>
<tr>
<td>TOTAL</td>
<td>215</td>
<td>21</td>
</tr>
</tbody>
</table>
Figure 7.1  Structural Configuration of Design Example in MARTA Bridge
defects since they are often located at the stress concentration zone and are oriented normally to the principal stress. Thus, in this study, only surface defects are considered.

The geometry function gives a quantitative measurement of the stress concentration effect near the crack tip in the welds. In this study, the geometry function is selected to represent a typical crack condition, i.e., the crack is present at the center of the welds.

According to Eq. 3.29 and considering the width of the full penetration butt weld (Figure 7.1) to be 84 inches, the following geometry function is assumed in this study:

\[ Y(\alpha) = \frac{1 - 0.0119\alpha + 0.0002098\alpha^2 - 0.000006\alpha^3}{\sqrt{1 - 0.02381\alpha^2}} \]  \hspace{1cm} (7.1)

where \( \alpha \) is the crack size.

### 7.3.2 Material Properties

It is generally accepted that for steel, the fatigue growth parameter \( C \) approximately follows a lognormal distribution while the fatigue growth exponent \( m \) follows a normal distribution.

From the literature review (Fuchs and Stephens, 1980; Fisher, 1984; Wirsching, 1988), the mean value of \( C \) is assumed to be \( 2.05 \times 10^{-10} \) ksi and its coefficient of variation is 0.63 for the steel used in MARTA bridges. Usually, the mean of \( m \) is considered to be 3.0.

After introducing the above values, the mean value of the Paris equation can be expressed as:

\[ \frac{da}{dN} = 2.05 \times 10^{-10}(\Delta K)^{3.0} \]  \hspace{1cm} (7.2)

### 7.3.3 Load Effect

For design purposes, MARTA has its own standard loading. A train may consist of any number of cars from one to eight. The axle spacing, axle loading and car spacing are shown in Figure 7.2. The 30,500 pound axle loads reflect the weight of the car and a full load of passengers but do not include the effects of
Figure 7.2 The Standard MARTA Live Loading
impact. The Standard MARTA loading, increased by the appropriate impact factor, is used for all fatigue calculations.

As discussed in Section 3.3.4, Rayleigh distribution would be a good approximation for the loading of railway bridges.

If the stress range follows the Rayleigh distribution, the probability density function can be expressed as:

\[ f_S(s) = \frac{s}{S_0^2} \exp\left[-\frac{1}{2} \left(\frac{s}{S_0}\right)^2\right] \] (7.3)

where \( S_0 \) is a statistical parameter. The relationship between \( S_0 \) and mean of \( s \), \( E(s) \), can be defined as:

\[ S_0 = \sqrt{\frac{2}{\pi}} E(s) \] (7.4)

The mean value of the maximum moment at the location of welds can be obtained by influence line analysis. The section properties are given in Appendix A. The corresponding mean of stress range, \( E(s) \), can be evaluated as:

\[ E(s) = \frac{M}{I y} = \frac{2635.222 \times 12 \times 38.01}{151421.54} = 7.938 \text{ ksi} \] (7.5)

According to Eq. 7.4, \( S_0 \) can be estimated to be 6.334 ksi.

**7.3.4 Initial Crack Size**

As indicated in Section 3.4.1, initial crack size plays a very important role in the fatigue reliability analysis since a large amount of uncertainty is involved in the statistical model.

Among a number of the models discussed previously, the lognormal distribution is used to model the real condition of the butt welds of plates. In this study, the mean of the initial crack size is assumed to be 0.02 inch and the COV is considered to be 0.5.
7.4 Reliability Analysis by S-N Approach

In order to compare the results between the currently used AASHTO method and the FEFM approach, the reliability analysis using the S-N approach is conducted first. The S-N curves are of the form:

\[ N = AS^{-B}, \quad S > S_0 \]  

(7.6)

where \( A \) and \( B \) are the fatigue strength coefficient and exponent, respectively. \( S_0 \) is the endurance limit.

To account for the uncertainty in the material parameters, \( A \) is modeled as a lognormal variable in this study. For the AASHTO category E curve, the mean of \( A \) can be obtained as follows. From Table 2.1, the design value of \( A \), \( A_d \), is:

\[ A_d = 1.072 \times 10^9 \]  

(7.7)

Since S-N curves with a 95 percent confidence limit was selected by AASHTO, \( A_d \) can be considered to be the mean minus two standard deviations, i.e.,

\[ A_d = \mu_A - 2\sigma_A = \mu_A(1 - 2\delta_A) \]  

(7.8)

where \( \delta_A \) is the coefficient of variation (COV) of \( A \) and it is suggested that the COV of \( A \) can be assumed to be 0.45 (Wirsching, 1988). Thus, the mean of \( A \), \( \mu_A \), can be obtained from Eq. 7.8 as \( 1.072 \times 10^{10} \).

In order to consider the effect of the mean stress, Miner's rule is used in combination with the S-N curve. According to Eqs. 2.11 and 2.12, the limit state function for this case can be defined as:

\[ M = \frac{N}{A} E(S^B) - \Delta = 0 \]  

(7.9)

where \( N \) is the total number of stress cycles in the lifetime of the welds. \( A \) and \( B \) are defined earlier. \( \Delta \) is a tolerance which follows a lognormal distribution with mean of 1.0 and COV of 0.3 (Wirsching, 1988). \( E(s^B) \) is the mean stress effect. If
the stress range follows the Rayleigh distribution, then the mean stress effect can be written as:

$$E(S^B) = (\sqrt{2}S_0)^B \Gamma(1 + \frac{B}{2})$$  \hspace{1cm} (7.10)

From Eq. 7.9, the mean of $N$ can be estimated as:

$$\bar{N} = \frac{\Delta A}{(\sqrt{2}S_0)^B \Gamma(1 + \frac{B}{2})} = 1.1223 \times 10^7 \text{ cycles}$$  \hspace{1cm} (7.11)

Assuming that $N$ follows a lognormal distribution, the standard variation of $\ln N$ can be expressed as:

$$\sigma_{\ln N} = \sqrt{\ln[(1 + C_A^2)(1 + C_\Delta^2)]} = 0.5202$$  \hspace{1cm} (7.12)

where $C_A$ and $C_\Delta$ are the COV of $A$ and $\Delta$, respectively.

According to Wirsching (1988), the corresponding safety index can be evaluated as:

$$\beta = \frac{\ln(\frac{\bar{N}}{N_s})}{\sigma_{\ln N}}$$  \hspace{1cm} (7.13)

where $N_s$ is the mean of the design life in terms of number of cycles.

With a different design life, $N_s$, the corresponding safety index can be obtained by using Eq. 7.13. The results are listed in Table 7.2 and are plotted in Figure 7.3.

7.5 Verification of Fracture Mechanics Approach

In order to verify the accuracy of the fracture mechanics approach, the Monte Carlo simulation technique is used. An illustrative example is considered for this purpose. Three parameters are considered to be deterministic. They are:

1. stress range parameter, $S_0 = 8.92$ ksi;
2. specimen width, $w = 42$ inches; and
3. critical crack size, $\alpha_c = 2$ inches

Three variables are considered to be random with the following statistical characteristics.
Table 7.2 Results of S-N Approach

<table>
<thead>
<tr>
<th>Number of Cycles</th>
<th>Beta (S-N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000,000</td>
<td>4.648</td>
</tr>
<tr>
<td>1,500,500</td>
<td>3.869</td>
</tr>
<tr>
<td>2,000,000</td>
<td>3.316</td>
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<tr>
<td>2,500,000</td>
<td>2.887</td>
</tr>
<tr>
<td>3,000,000</td>
<td>2.536</td>
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<tr>
<td>3,500,000</td>
<td>2.536</td>
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<tr>
<td>4,000,000</td>
<td>2.240</td>
</tr>
<tr>
<td>4,500,000</td>
<td>1.983</td>
</tr>
<tr>
<td>5,000,000</td>
<td>1.757</td>
</tr>
<tr>
<td>5,500,000</td>
<td>1.554</td>
</tr>
</tbody>
</table>
Figure 7.3 The Reliability Index of S-N Approach
1. initial crack size, $L_{n0}$ follows $N(-4.274, 0.833)$;
2. fatigue growth parameter, $LnC$ follows $N(-24.048, 0.5)$; and
3. fatigue growth exponent, $m$ follows $N(3.0, 0.01)$

Results of the fracture mechanics approach and the Monte Carlo simulation approach are listed in Table 7.3 and are plotted in Figure 7.4.

It can be observed that the results of the two different approaches are very similar, with a maximum error of about 0.2 percent. So the fracture mechanics approach gives very accurate results.

7.6 Fracture Mechanics Approach to MARTA Bridges

Using the Paris equation, the failure criterion for propagation of the crack to a critical crack length ($\alpha_c = 2$ inches) under Rayleigh loading can be expressed as:

$$M = \int_{\alpha_0}^{\alpha_c} \frac{d\alpha}{[Y(\alpha)\sqrt{\pi\alpha}]^m} - C(\sqrt{2S_0})^m\Gamma(1 + \frac{m}{2})N$$  \hspace{1cm} (7.14)

Design Example 1

As a comparison to the reliability analysis using the S-N approach discussed in Section 7.4, the reliability analysis using the fracture mechanics approach is conducted in this section. Updating the inspection results is carried out in the next sections.

The design parameters in Eq. 7.14 are listed below:

**Deterministic Parameters:**
1. stress range parameter, $S_0 = 6.334$ ksi
2. critical crack size, $\alpha_c = 2$ inches
3. geometry function, $Y(\alpha)$ is defined in Eq. 7.1
Table 7.3 The Results of Different Methods

<table>
<thead>
<tr>
<th>Number of Cycles</th>
<th>Beta (FORM)</th>
<th>Beta (Monte-Carlo)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,500,000</td>
<td>4.600</td>
<td>4.680</td>
<td>1.701</td>
</tr>
<tr>
<td>3,000,000</td>
<td>4.065</td>
<td>4.000</td>
<td>1.625</td>
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<tr>
<td>3,500,000</td>
<td>3.915</td>
<td>3.860</td>
<td>1.425</td>
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<tr>
<td>4,000,000</td>
<td>3.702</td>
<td>3.650</td>
<td>1.425</td>
</tr>
<tr>
<td>4,500,000</td>
<td>3.501</td>
<td>3.448</td>
<td>1.537</td>
</tr>
<tr>
<td>5,000,000</td>
<td>3.338</td>
<td>3.392</td>
<td>1.560</td>
</tr>
</tbody>
</table>
Figure 7.4  The Difference between FORM and Monte-Carlo Approaches
Random Variables:
1. initial crack size, $\ln a_0$ follows $N(-4.024, 0.472)$
2. fatigue growth parameter, $\ln C$ follows $N(-22.475, 0.578)$
3. fatigue growth exponential, $m$ follows $N(3.0, 0.01)$

Analyses were carried out for different values of design life $N$ and the results are shown in Table 7.4 and plotted in Figure 7.5. It is found that the results obtained by the LEFM approach match closely those from the S-N curve approach when the design life $N$ is 2,000,000 stress cycles. The reliability indeces for the two approaches are 3.353 and 3.316, respectively.

The error is about 1.1 percent. The results of the two approaches are shown in Figure 7.6 for comparison.

7.7 Inspection and Updating

7.7.1 Introduction

To ensure the safety of MARTA's bridges, an acceptable fatigue reliability should be established. An acceptable level of fatigue reliability in a structural component or system can be obtained either by properly designing them, or by partial dependence on a periodic inspection plan and updating to keep track of the amount of degradation.

For the bridges in the MARTA system, it is assumed that all the butt welds are accessible for inspection. Thus, a plan of periodic inspections of butt welds is possible. To prevent a crack from growing to a critical size, regular inspection is necessary. The reliability-based inspection plan proposed here can be used for this purpose.
<table>
<thead>
<tr>
<th>Number of Cycles</th>
<th>Beta (LEFM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000,000</td>
<td>4.216</td>
</tr>
<tr>
<td>1,500,500</td>
<td>3.715</td>
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<tr>
<td>2,000,000</td>
<td>3.353</td>
</tr>
<tr>
<td>2,500,000</td>
<td>3.066</td>
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<tr>
<td>3,000,000</td>
<td>2.832</td>
</tr>
<tr>
<td>3,500,000</td>
<td>2.632</td>
</tr>
<tr>
<td>4,000,000</td>
<td>2.456</td>
</tr>
<tr>
<td>4,500,000</td>
<td>2.166</td>
</tr>
<tr>
<td>5,000,000</td>
<td>2.038</td>
</tr>
<tr>
<td>5,500,000</td>
<td>1.922</td>
</tr>
</tbody>
</table>
Figure 7.5 The Reliability Index of LEFM Approach
Figure 7.6 The Difference between S-N and LEFM Approaches
7.7.2 Ultrasonic Inspection

The reliability and sensitivity of different inspection strategies greatly affect the updating and thus affect the decisions to be made after the inspection. As discussed earlier, the decision alternatives may include doing nothing, inspecting the weld more frequently or replacing it if the crack size is unacceptable.

Since the ultrasonic equipment described in Appendix B will be used for inspection of MARTA's bridges, all subsequent discussion will be made for this NDT method.

The detectable crack size, \( \alpha_d \), is a random variable for the inspection welds in bridges. It is assumed that a lognormal distribution, as defined by Eq. 5.9, can be used to consider the uncertainty in \( \alpha_d \). Referring to Tables 4.1 and 4.2, the characteristic values in Eq. 5.9 can be determined as follows. \( \text{Ln}\alpha_{50} \) is assumed to be -2.370 and \( \sigma_{\text{Ln}\alpha} \) is 0.38. Thus, Eq. 5.9 can be simplified as:

\[
P_D(\alpha_d) = \Phi\left(\frac{\text{Ln}\alpha_d + 2.37}{0.38}\right)
\]  
(7.15)

If a crack is detected after \( N_i \) stress cycles, the inspection event with crack detection can be expressed as:

\[
I_i = C(\sqrt{2}S_0)^m \Gamma\left(1 + \frac{m}{2}\right)N - \int_{\alpha_o}^{\alpha_d} \frac{d\alpha}{[\Gamma(\alpha)\sqrt{\pi}\alpha]^m}
\]  
(7.16)

Based on the parameters given in the example in Section 7.5 and the detectability as defined in Eq. 7.15, Monte Carlo simulation is used to verify the accuracy of the fracture mechanics approach (Eq. 7.16). As a comparison, the results obtained from Eq. 7.16 and from Monte Carlo simulation are shown in Figure 7.7. It is found that the difference between the two different approaches is very small with a maximum error of about 1.5 percent. Thus, it can be concluded that the FORM method gives very accurate results.
Figure 7.7 The Comparison of Detectability by FORM and M-C Methods
7.7.3 Updating with Crack Size Measurement

When a crack size $A_i$ is measured for the first time during an in-service inspection, the limit state function of the inspection event can be defined as:

$$D_1 = \int_{0}^{A_1} \frac{d\alpha}{[Y(\alpha)\sqrt{\pi\alpha}]^m} - C(\sqrt{2}S_0)^m \Gamma(1 + \frac{m}{2}) N_1 = 0$$  \hspace{1cm} (7.17)$$

Using this detection event, both the reliability and the distribution of the basic variables can be updated.

When a crack size $A_i$ is measured in the $i$th in-service inspection, the limit state of this inspection event is defined as:

$$D_i = \int_{A_{i-1}}^{A_i} \frac{d\alpha}{[Y(\alpha)\sqrt{\pi\alpha}]^m} - C(\sqrt{2}S_0)^m \Gamma(1 + \frac{m}{2})(N_i - N_{i-1}) = 0$$  \hspace{1cm} (7.18)$$

where $A_{i-1}$ is the crack size measured at the $(i-1)$th inspection, corresponding to $N_{i-1}$ stress cycles. It is important to note that all the basic variables are already updated based on the information of the $(i-1)$th inspection.

Based on Eq. 7.18 and the updated information on basic variables, the limit state function of target safety can be represented as:

$$M = \int_{A_i}^{\alpha_c} \frac{d\alpha}{[Y(\alpha)\sqrt{\pi\alpha}]^m} - C(\sqrt{2}S_0)^m \Gamma(1 + \frac{m}{2})(N - N_i) = 0$$  \hspace{1cm} (7.19)$$

Using the FORM technique, the updated target safety index, $\beta_{M,up}$, can be obtained. The updated probability of failure is then:

$$P_{f,up} \approx \Phi(-\beta_{M,up})$$  \hspace{1cm} (7.20)$$

Design Example 2

As an illustration, a practical example of updating for MARTA's steel bridges is given. The design parameters are listed below:
Deterministic Parameters
1. stress range parameter, \( S_0 = 6.334 \text{ ksi} \)
2. critical crack size, \( a_c = 2 \text{ inches} \)
3. geometry function, \( Y(\alpha) \) is defined in Eq. 7.1
4. initial target safety index, \( \beta_M \), is 3.35 which is obtained in Section 7.6 corresponding to 2,000,000 stress cycles of design life.

Random Variables
1. initial crack size, \( L_n a_0 \) follows N(-4.024, 0.472)
2. fatigue growth parameter, \( L_n C \) follows N(-22.475, 0.578)
3. fatigue growth exponent, \( m \) follows N(3.0, 0.01)
4. measured crack size, \( A_i \) follows normal distribution with mean, \( \mu_A \), which is measured by the ultrasonic inspection and the corresponding COV is 0.25.

(1) Detectability of crack size measurement

First, the interval of inspection is set to be 91980 stress cycles which corresponds to one year of operation of MARTA’s rail system based on the present load frequency. When \( N \) is 91980 stress cycles, the reliability of crack size measurement can be evaluated by Eq. 7.18. The results are plotted in Figure 7.8.

If the time of first inspection is postponed to the end of the second or third year, then the corresponding \( N \) values are 183960 and 275940, respectively. In both cases, reliability analyses of crack size measurement are conducted. The corresponding results are plotted in Figures 7.9 and 7.10.

For comparison, the results for 91980 and 183960 stress cycles are plotted in Figure 7.11. Based on the observations, the following conclusions can be made:

1) The trend in the crack size measurement at 91980 cycles and 183960 cycles is very similar;
2) as the measured crack size increases, the detectability index, \( \beta_D \), is also increased; and
3) the rate of increase of \( \beta_D \) decreases as the detected crack size increases.
Figure 7.8  The Detectability of Inspection (N = 91980 Cycles)
DEB OF CRACK SIZE AT SECOND INSPECTION

Figure 7.9  The Detectability of Inspection (N = 183960 Cycles)
Figure 7.10 The Detectability of Inspection (N = 275940 Cycles)
Figure 7.11 The Comparison of Detectability at Different Cycles
(2) Updating of the Basic Variables

Once the inspection is conducted, the information on basic variables can be updated using the information obtained from the inspection. In this study, all the basic variables follow either normal or lognormal distribution. Thus, only the characteristic values, mean and standard deviation, need to be updated. Eqs. 6.42, 6.43, 6.44, 6.49, 6.50, and 6.51 can be used for the purpose.

It is also important to note that:

(1) From Eqs. 6.46 and 6.52, it can be seen that the updated mean or median are exactly the design values for the normal or lognormal distribution, respectively; and

(2) from Eqs. 6.47 and 6.52, it can be observed that the updated standard deviations for both normal and lognormal distributions are reduced. This implies that the results of inspection always reduce the scatter of the basic variables.

(3) Updating the Fatigue Safety Index

The updated fatigue safety index, $\beta_{M, up}$, can be obtained by the FORM analysis based on the new limit state function defined by Eq. 7.19. The updated $\beta_{M, up}$, corresponding to the inspections occurring at the end of the first, second, and third years are plotted in Figures 7.12, 7.13 and 7.14, respectively. It can be observed that the updated safety index, $\beta_{M, up}$, increases as the measured crack size increases.

(4) The Influence of COV of the Measured Crack Size

As discussed in Section 5.3, the measured crack size, $A_i$, is a random variable. It is modeled here as a normal variable.

Due to the lack of test data, the COV of $A_i$ is very difficult to estimate. Thus, it is important at this stage to study the influence of the COV of $A_i$ on
Figure 7.13 The Updated Fatigue Safety Index
(N=91980 Cycles)
Figure 7.13 The Updated Fatigue Safety Index
(N=183960 Cycles)
Figure 7.14 The Updated Fatigue Safety Index
(N=275940 Cycles)
the detectability of the inspection and on the updated reliability. The two different
COV values of 0.1 and 0.25 are considered in this study to evaluate the sensitivity
of this parameter on the updated reliability index. The results are plotted in
Figures 7.15 and 7.16. It can be observed from these figures that the effect of COV
on the detectability of the inspection and the updated reliability index is small,
particularly when the measured crack size is less than about 0.2 inch.

The critical safety index, below which the structure needs immediate repair,
is assumed to be 3.0 in this study. From Figure 7.16, it can be observed that the
updated fatigue safety index is smaller than 3.0, when the measured crack size is
larger than about 0.05 inch.

Thus, the influence of the COV of $A_i$ can be neglected in this study.

7.8 Repair and Updating

In order to insure the integrity of MARTA’s bridges, a critical safety index
$\beta_c$, should be selected. If the updated safety index $\beta_{M,\text{up}}$ is smaller than $\beta_c$, repair
of damaged component must be made as soon as possible. If $\beta_{M,\text{up}}$ is greater than
$\beta_c$, then the component should be inspected during the next scheduled inspection.

In the Load Resistant Factor Design (LRFD), the critical safety index is
assumed to be 3.0 and the corresponding probability of failure is $1.3 \times 10^{-3}$. Thus,
$\beta_c$ is considered to be 3.0 in this study.

After the repair, the structural integrity is improved and the new limit state
function of target reliability can be expressed as:

$$M_{\text{new}} = \int_{\alpha_0,\text{new}}^{\alpha_c} \frac{d\alpha}{[Y(\alpha)\sqrt{\pi \alpha}]^{m_{\text{new}}}} - C_{\text{new}}(\sqrt{2S_0})^{m_{\text{new}}} \Gamma(1 + \frac{m_{\text{new}}}{2})(N - N_{\text{rep}}) = 0$$

(7.21)

where $\alpha_{0,\text{new}}$, $C_{\text{new}}$ and $m_{\text{new}}$ are the new design parameters to account for the
improvement in the structural integrity. $N$ and $N_{\text{rep}}$ are the design life and the
stress cycles at the time of repair, respectively.

Usually, the material properties are not expected to change during the re­
pair. Thus, the statistical properties of the material parameters, $C_{\text{new}}$ and $m_{\text{new}}$,
Figure 7.15  The Effect of COV of Crack Size on Detectability
Figure 7.16 The Effect of COV of Crack Size on Updated Safety Index
can be assumed to remain the same. However, defects may occur during the repair procedure as in the fabrication process. So, $\alpha_{0,\text{new}}$ is considered to be as the same as $\alpha_0$. After introducing these assumptions into Eq. 7.21, it can be simplified as:

$$M_{\text{new}} = \int_{\alpha_0}^{\alpha_{\text{c}}} \frac{d\alpha}{[Y(\alpha) \sqrt{\pi \alpha}]^m} - C(\sqrt{2S_0})^m \Gamma(1 + \frac{m}{2})(N - N_{\text{rep}}) \quad (7.22)$$

Based on the limit state function defined by Eq. 7.22, the FORM technique can be used to calculate the updated fatigue safety index. The corresponding probability of failure can be evaluated by Eq. 7.20.

**Design Example 3**

As a continuation of Design Example 2, it is assumed that the first inspection is conducted at the end of third year ($N_{\text{rep}} = 275940$) and the crack size of 0.1 inch is measured. Based on this information, the updated fatigue safety index, $\beta_{M,\text{up}}$, is estimated to be 2.96. Since $\beta_{M,\text{up}}$ is smaller than 3.0 (the critical safety index), the decision to repair can be made immediately. After the repair, the new target safety index, $\beta_{\text{new}}$, is final to be 3.29 (Figure 7.17). The updated fatigue safety index versus the detected crack size is plotted in Figure 7.17. As the detected crack size increases, the corresponding safety index drops sharply.

**7.9 Practical Fatigue Control Curves**

To fulfill the practical fatigue design requirements of MARTA’s bridges, the relationship between the number of cycles at the time of inspection and the updated fatigue safety index under the different measured crack sizes is studied.

From the results plotted in Figures 7.18 and 7.19, it can be found that a linear relationship exists between the updated fatigue safety index and the numbers of stress cycles at the time of inspection, when the measured crack size is smaller than 0.1 inch. The mathematical explanation behind this observation is obvious.

As mentioned earlier, the updated fatigue safety index can be geometrically expressed as the shortest distance from the origin to the updated limit state surface defined by Eq. 7.19. When the measured crack size is very small, the changes in
Figure 7.17 The Updated Fatigue Safety Index After Repair
Figure 7.18 Relationship between Stress Cycles and Updated Fatigue Safety Index (A)
Figure 7.19 Relationship between Stress Cycles and Updated Fatigue Safety Index (B)
the updated basic variables are also expected to be very small. Thus, the first term in Eq. 7.19 can be treated as a constant and the second term varies linearly with the numbers of stress cycles. Therefore, a series of parallel curves can be drawn to represent the different limit state surfaces corresponding to the different number of stress cycles. This implies that the shortest distance is the linear function of the number of stress cycles.

From the Paris equation, it can be observed that crack growth rate drops sharply when the propagated crack becomes large. Relatively small cracks are expected to be present during most of the design life of the structures. Thus, the linear behavior of these curves provides a basis of quick interpolation as shown in Figures 7.18 and 7.19. Once a series of control curves are plotted, the inspection results can be interpreted immediately to the updated target safety index and decisions can be made at once.

7.10 Updating of Inspection Plan

As mentioned earlier, a periodic NDT inspection plan can be used by MARTA to insure the integrity of the bridges. The interval of inspections is about two years. Once an NDT inspection is conducted, the interval until the next inspection could be updated using the information on the outcome of the current inspection. Since the fatigue safety index represents not only the current integrity of the structures but also a measure of the crack growth rate, the safety index can be related to the inspection interval.

As discussed earlier, based on the outcome at an inspection, e.g., detection of no crack, detection of crack but without size measurement and detection with crack size measured, the reliability index can be updated. If the updated fatigue safety index remains the same (3.35 in the example considered in this study), then no changes are required in the inspection schedule. If the updated fatigue safety index is smaller than the initial safety index but bigger than the critical safety index, then it can be concluded that the crack is propagating more quickly than
the design prediction and it should be inspected more frequently than originally scheduled.

Based on the above discussion, the model for updating the inspection schedule can be expressed as:

\[ T_{up} = \frac{\beta_M}{\beta_{M,up}} T_{int} \]  

(7.23)

where \( \beta_M \) and \( \beta_{M,up} \) are the initial and the updated fatigue safety indices, respectively, and \( T_{int} \) and \( T_{up} \) are the initial and the updated inspection intervals, respectively.
Chapter 8
Summary and Conclusions

8.1 Summary

A reliability-based inspection and maintenance procedure is proposed here for the fatigue damage potential evaluation of the full-penetration butt welds in MARTA's bridges. Major factors affecting the fatigue damage accumulation are identified and discussed in detail. A stochastic fatigue crack growth model is formulated to account for the uncertainties in the material properties and environmental conditions. Probabilistic models are proposed for the mean stress effect ($E(S^m)$), the initial crack size ($a_0$), crack growth parameters ($C$ and $m$) and S-N parameters ($A$ and $B$).

To illustrate the application potential of the proposed method, full-penetration butt welds in a simply supported steel box-girder with a 70 ft span are considered. An appropriate geometry function is proposed to consider the stress concentration near the crack tip.

Currently available structural reliability analysis methods are reviewed. The fundamental concepts of each method are presented in detail. Their advantages and limitations are also discussed. Several methods of fatigue reliability analysis are identified.

The sensitivity and capability of some of the inspection methods including ultrasonic inspection are investigated. The available different probabilistic models for crack detectability are described. The reliability of inspections for various events are formulated. A risk-based model of crack size measurement is proposed to account for the uncertainty in the equipment as well as the human error in interpreting the inspection signals.

Efficient algorithms for updating the reliability model through inspection are presented. The fatigue safety index can be updated using the information
obtained from the inspection. The inspection schedule can also be updated using this information.

Several computer programs are developed in this study to implement the proposed algorithm. These computer programs include the FORM-based fatigue reliability analysis, crack growth model using Monte Carlo simulation and updating the safety indices and the statistical information on the basic random variables through inspection.

The fatigue reliability of the full-penetration butt welds in MARTA's bridges are carried out in several ways. These include the traditional S-N approach and the reliability-based linear fracture mechanics approach. Strategies for periodic inspection and possible repair are studied to improve the integrity of MARTA's bridge.

8.2 Conclusions

Based on the results of the present study, the following conclusions can be made:

1. The Linear Elastic Fracture Mechanics (LEFM) approach appears to be a simple and robust model for fatigue analysis of full-penetration butt welds in bridges. The material properties, loading effect, geometry function and crack size are some of the important parameters in the proposed model. Since the crack size can be related directly to the fatigue safety index of the structure, model updating through inspection becomes feasible and practical.

2. The Advanced Second Moment Method is found to be conceptually simple and computationally efficient in the fatigue reliability analysis. By comparing results obtained by the FORM approach and Monte Carlo simulation techniques, it is observed that the FORM approach is very efficient and provides very accurate results.

3. The reliability-based fracture mechanics approach is a good alternative to the S-N curve approach. The two approaches give very similar results.
4. A periodic NDT inspection scheme can be used as an additional tool to
insure the fatigue reliability of the full-penetration butt welds in bridges. As
discussed in this study, the detectability of a crack varies with the measured
crack size. As crack size increases, the detectability also increases. Also, as
expected, as the crack size increases, the safety index decreases.
5. The updated safety index varies linearly with the time of inspection, par­
ticularly when the detected crack size is relatively small. Therefore, the
interpolation approach can be used to assess the inspection results more
quickly and efficiently.
6. Repair is an essential part of periodic inspections. When the current safety
index is smaller than the critical value, repairs should be made to improve
the integrity of the bridge. After repair, the fatigue safety index increases.
Thus, the design fatigue life of the bridges can be extended by periodic
inspections and repairs.

8.3 Suggestions for Further Work

Based on the results of the present study, the following topics need to be
addressed in further studies:
1. In the numerical section of this study, only a 70 foot simply supported
bridge is considered and the geometry function of the crack growth model
is considered to belong to category E of the AASHTO specifications. This
study can be extended further to consider different categories. The model
should be extended to considered continuous span bridges.
2. A nonlinear fracture mechanics model for crack growth is discussed in this
study. A detailed study in this area would be very challenging and interest­
ing.
3. Methods for model updating are considered in this study. However, all the
discussions are limited to variables with normal or lognormal distribution.
The results obtained are found to be very accurate. For other distributions,
the proposed model needs some modification.
Appendix A

Design Calculations

Design Assumptions

1. A 572 Grade 50 Steel
2. Dead Load: steel box-girder, concrete slab and 2nd pour
3. Live Load: two moving cars, the rear wheels of one car and the front wheels of an adjacent car, both cars are side by side going in opposite direction
4. Impact: according to AASHTO specification (1989)
5. Concrete strength: 5,000 psi
6. Simply supported box-girder span with length of 70 feet, the welds locate 15 feet away from the ends of the span

Section Properties

The section properties are shown in Figure A.1 and the major parameters are listed in Table A.1.

The y value can be calculated as:

\[ y = \frac{(96 \times 52) + (22.5 \times 47.33) + (35.88 \times 23.75) + (31.5 \times 0.38)}{175.88} \]  \hspace{1cm} (A.1)

and the section modulous can be obtained as:

\[ I = 151,421 \text{ in}^4 \]  \hspace{1cm} (A.2)

Loading

The stress range for the fatigue crack growth is contributed only by the live load and the impact effect. The Standard MARTA live load is used in this study. To consider the worst case, it is assumed that two trains travelling in opposite directions act on the girder at the same time.
Figure A.1 Illustration of Design Section
<table>
<thead>
<tr>
<th>Section</th>
<th>Area</th>
<th>Arm</th>
<th>I0</th>
<th>Ad2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>96.00</td>
<td>52.00</td>
<td>512.00</td>
<td>259584.00</td>
</tr>
<tr>
<td>2</td>
<td>22.50</td>
<td>47.38</td>
<td>2.93</td>
<td>50509.45</td>
</tr>
<tr>
<td>3</td>
<td>25.88</td>
<td>23.75</td>
<td>4562.63</td>
<td>14597.94</td>
</tr>
<tr>
<td>4</td>
<td>31.50</td>
<td>0.38</td>
<td>1.48</td>
<td>4.55</td>
</tr>
</tbody>
</table>
In order to determine the mean effect of the maximum moment acting on the welds, the influence line technique is used in the analysis. The critical location is shown in Figure C.2. As a verification, the following expressions is validated:

\[ \frac{R_t}{a} = 0 \leq \frac{P_c + R_r}{b} = \frac{4 \times 61}{55} = 4.436 \]  
(A.3)

\[ \frac{R_t + P_c}{a} = \frac{61}{15} = 4.067 \geq \frac{R_r}{b} = \frac{3 \times 61}{55} = 3.327 \]  
(A.4)

The mean of the maximum moment can be expressed as:

\[ M_{c_{\text{max}}} = 61 \times (11.786 + 10.179 + 6.964 + 5.357) = 2091.446 \]  
(A.5)

To consider the impact effect, 26 percent of the live load effect should be added according to AASHTO (1989). So, the fatigue stress range can be evaluated as:

\[ \Delta S = \frac{2091.446 \times 1.26 \times 38.0}{151421.54} \]  
(A.6)

and the corresponding stress range parameter is:

\[ S_0 = \sqrt{\frac{2}{\pi} \Delta S} = 6.334 \]  
(A.7)
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