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FEASIBILITY STUDY FOR A PULSED COLD NEUTRON SOURCE

AT THE

UNIVERSITY OF ARIZONA TRIGA REACTOR

BY

Malek Ahmad Chatila

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A Thesis Submitted to the Faculty of the
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In the Graduate College
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1995
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Date 11/21/95
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I wish to express my gratitude to my Professor J.G. Williams for his support, advice, patience and understanding, which made it possible to complete this work.

I also wish to express my utmost appreciation to my father Ahmad, and my mother Miryal whose financial and emotional support made every thing seem achievable.
DEDICATION

To my father Ahmad, my mother Miryal, my sisters Dunia and Eva, and my brother Wissam, who form the backbone of my life.
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A computer code that utilizes the consistent energy dependent P1 equations and the multigroup diffusion equations was developed to calculate the cold neutron flux that could be generated if a cold neutron source was built at the University of Arizona TRIGA reactor. The cold neutron current density at the surface of the cold neutron source and the energy deposition in the cold neutron source due to neutron-proton interactions were also calculated.

The computer code models the cold neutron source as a spherically shaped hydrogen medium embedded in the center of a cylindrical shaped homogenous TRIGA core. The outputs of the computer code were compared for various cold neutron source dimensions.
INTRODUCTION

In recent years, there has been an increasing interest in the use of slow neutrons and a growing importance of those neutrons in the fields of physics, chemistry, material science and even biology. Slow neutrons, neutrons with energies below 0.005 eV, are divided into two main categories; cold and ultra-cold neutrons. Cold neutrons are defined as those neutrons with energies less than 5 meV (velocity < 1000 m/s), 60 °K, while ultra-cold neutrons characterize neutrons with energies less than 10^-7 eV (velocity < 5 m/s), 10^-3 °K. The main reason for dividing slow neutrons into two different categories is due to their different reflection properties. Cold neutrons can undergo total reflections when incident on a surface of many materials at a certain angle, while ultra-cold neutrons can undergo total reflections from many materials at arbitrary angles of incidence.

The objective of this paper, is to estimate the cold neutron flux that can be obtained if a spherically shaped cold neutron source, made of hydrogen, is used in the University of Arizona TRIGA reactor. The motivation of such work arises from the fact that even though the U of A TRIGA reactor is licensed to operate at a power of only 100 KW at steady state, it can reach peak power of 650 MW in the pulse
mode. So when it operates in the pulse mode, it may be competitive with the more powerful research reactors that are used to generate cold neutrons such as the 20 MW research reactor at the National Institute of Standards and Technology. The TRIGA will offer three major advantages. First, its relative small size, safety and ease of operation, make it ideal for conducting research. Second, in some experiments, it is more convenient to use pulses of cold neutrons than a steady source. Third, it can generate large pulses of cold neutrons while not overheating the cold source because of the time between pulses. Also, if the cold source can be treated as being adiabatic during a pulse, than it may be possible to use a transfer system for the cold source, hence it may be simpler, and require less space in the reactor core. What is meant by a transfer system is a vessel that can be filled outside of the reactor with cold moderator, than placed in the reactor, without requiring continuous cooling of the moderator.

This paper is divided into two main parts. In the first part, a brief overview of the physics behind the reflection properties of slow neutrons and some of the different methods that can be used to produce and extract them are discussed. In the second part, the energy dependent P1 equations and the three group diffusion equations are utilized to calculate the cold neutron flux
that can be obtained and the simultaneous heating of the hydrogen medium for various pulse sizes, including pulses at and above the maximum pulse size for the U of A TRIGA, and for different source dimensions. Calculations are also used to test whether the cold source can be treated adiabatically during a pulse or whether the temperature rise during the slowing down of neutrons needs to be accounted for in the calculations. The performance characteristics and the parametric studies are then analyzed as a preliminary to a possible pulsed cold source design.
LITERATURE REVIEW

2.1 Reflection Of Neutrons

The reflection of a neutron from a surface can be explained by the use of the Schrodinger equation

\[ -\frac{\hbar^2}{2m} \nabla^2 \psi(r) + V(r) \psi(r) = E \psi(r) \]

(2.1)

where

\[ \psi(r) = \text{Neutron wave function} \]
\[ E = \text{Energy of the neutron} \]
\[ V(r) = \text{Neutron-Nucleus interaction potential} \]
\[ \hbar = \text{Planck's constant divided by } 2\pi \]

If the reflection of a neutron from a surface that occupy the semi-space \( x > 0 \) is treated as a reflection from a potential step, than in equation (2.1), the potential \( V(r) \) can be written as

\[ V(r) = V_0 \delta(x>0) \]

where \( \delta(x>0) \) is a step function which is equal to one when \( x \) is greater than zero, and equal to zero when \( x \) is less than zero. The potential that describes the interaction of a neutron with matter is given by Fermi potential

\[ V_0 = \left( \frac{\hbar^2}{2m} \right) 4\pi N_0 b \]
where $N_0$ is the atomic density and $b$ is the coherent scattering amplitude. So the Schrödinger equation becomes

$$-\frac{\hbar^2}{2m} \nabla^2 \psi(x) + \left(\frac{\hbar^2}{2m}\right) 4\pi N_0 b \delta(x>0) \psi(x) = E \psi(x)$$  \hspace{1cm} (2.2)

From the previous equation, it can be shown that if the total energy of the neutron is less than the interaction potential, then total reflection occurs irrespective of the angle of incidence. Since $b$, the coherent scattering amplitude of a neutron on a nucleus, depends on the material, than the limiting energy ($E_{lim} = V_0$) will differ for different materials. Neutron-nucleus interaction potentials for some selected materials are given in table 3 (2.1).

**TABLE 2.1: Neutron-Nucleus Interaction Potentials**

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>V (neV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu</td>
<td>168</td>
</tr>
<tr>
<td>Fe</td>
<td>210</td>
</tr>
<tr>
<td>Ni</td>
<td>252</td>
</tr>
<tr>
<td>Al</td>
<td>54</td>
</tr>
<tr>
<td>H2O</td>
<td>-14.7</td>
</tr>
<tr>
<td>He</td>
<td>11</td>
</tr>
</tbody>
</table>

Thus Ni can be used to contain neutrons of energies below $2.5 \times 10^{-7}$ eV.

Due to wave properties, neutrons can undergo total reflections at surfaces that have different refraction indices. The critical angle $\theta_c$, angle at which total reflection occurs if the neutron incident on a surface with
an angle smaller than $\theta_c$, for a vacuum-material interface is given by

$$\theta_c = \lambda \cdot \left( \frac{N \cdot b}{\pi} \right)^{1/2}$$  \hspace{1cm} (2.3)

where $\lambda$ is the neutron wavelength, $N$ is the atomic density, and $b$ is the coherent neutron scattering length.

### 2.2 Production Of Slow Neutrons

When terms such as cold neutron source or production of cold neutrons are used, it is not meant that cold neutrons are generated by some source; rather the "source" is some type of moderator that slows existing neutrons to lower energies. The concept of a cold neutron source can be best understood by examining the Maxwell energy distribution function for atoms or molecules in a gas, which is given by:

$$N(E) = \frac{2\pi N}{(\pi kT)^{3/2}} \sqrt{E} \exp\left( \frac{-E}{kT} \right)$$  \hspace{1cm} (2.4)

where

- $N(E)$ = Number of atoms or molecules with energy $E$ per unit volume
- $N$ = Total number of atoms or molecules per unit volume
- $T$ = Temperature of the gas in Kelvin
- $K$ = Boltzmann constant, $8.6170 \times 10^{-5}$ eV/°K
The average energy of the gas molecule can be found from the relation

\[ \bar{E} = \frac{1}{N_0} \int dE f(E) E \]

which gives

\[ \bar{E} = \frac{3}{2} kT \] (2.5)

From equation (2.5), it is observed that the lower the temperature of the gas, the lower the average energy of the gas molecules. Figures (2.1) and (2.2) shows the graph of the Maxwellian distribution function, equation (2.4), at two different temperatures, \( T = 293^\circ K \), and at \( T = 20^\circ K \). As can be seen in those two figures, lowering the temperature results in shifting the spectrum down to lower energies, and hence a cold medium will be able to moderate a larger fraction of the neutrons down to lower energies than it is possible with a higher temperature medium. Another important fact that can be recognized by comparing figures (2.1) and (2.2) is that neutron slowing down models can be utilized down to lower energies (0.008 eV, assuming 5 kT cutoff between thermal and slowing down regions) in a moderator that is at 20 \( ^\circ K \) than in a moderator at 296 \( ^\circ K \) (0.126 eV) since the overwhelming majority of atoms have lower energies, and
FIGURE 2.1: Maxwellian distribution at 293.61 K
MAXWELLIAN DISTRIBUTION AT 20 K

FIGURE 2.2: Maxwellian distribution at 20 K
hence upscattering can be ignored as long as the neutron energies are above 0.008 eV.

Before the process of moderating neutrons can begin, it is necessary first to produce "free" neutrons. Some of the methods that are currently being used to produce free neutrons includes: (1) (d,t) reaction, in which a deuterium beam strikes a tritium target resulting in the production of a number of free neutrons. (2) Spallation sources, where energetic protons (800-1200 MeV) collide with heavy nuclei, and as a result several neutrons are liberated, and (3) in fission processes which occur in nuclear reactors. Since the interest of this paper lies in studying the feasibility of building a cold neutron source, an overview of the current production methods for cold and Ultra-cold neutrons is given, (references 2 and 3 gives a more detailed discussion).

One of the most commonly used source of free neutrons is the fission reactor. Most of the current cold sources utilize the fission reactor as a source of free neutrons. In most instances, the cold source, a vessel containing cold moderator (around 20 °K), usually filled with liquid hydrogen, deuterium or helium, is placed in the center of the reactor. Cold and Ultra-cold neutrons are extracted from the cold source by the use of neutron guides (equation 2.3). For an infinite, non-absorbing moderator, with a
steady state moderation, the neutron would eventually come into thermal equilibrium with the moderator according to equation (2.4). It is not difficult to show that in this case, the density of neutrons (ρ) with velocities between v and v+dv is given by

$$\rho(v)dv = \frac{2\Phi_0 v^2}{a^3} \exp(-\frac{v^2}{a^2}) \frac{dv}{a}$$  \hspace{1cm} (2.6)$$

where $\Phi_0$ is the total thermal flux, \(a=(2kT_n/m)^{1/2}\), and \(T_n\) is the neutron temperature. If equation (2.6) is integrated up to energy \(E\), than

$$\rho_E = \frac{2\Phi_0}{3a} \left(\frac{E}{kT} \right)^{3/2}$$  \hspace{1cm} (2.7)$$

where \(\rho_E\) is the density of neutrons that have energy lower than \(E\). If \(E\) is chosen to be \(10^{-7}\) eV, then equation (2.7) gives the density of Ultra-cold neutrons for a given thermal flux in a non absorbing infinite medium. Since it is possible to obtain higher peak thermal fluxes with a pulsed source than with a steady source, it is only logical that pulsed sources be used to "generate" cold and Ultra-cold neutrons. Pulse sources are extremely attractive especially while working with Ultra-cold neutrons. In principle, with the aid of a shutter, it is possible to accumulate Ultra-cold neutrons in a trap up to the impulse density. This can
be done by making the shutters open to the Ultra-cold neutron source during the pulse, and then close between the bursts. Some other methods that can be used in the production of cold neutrons include: the use of solid state, liquid and freeze-deposited converters, and by the reflection of neutrons from moving mirrors (neutron turbine).

2.3 Application Of Slow Neutrons

The interest in cold neutrons arises from the fact that they can be used in a wide variety of applications. Those applications are divided into two main categories; (1) The study of the neutron properties and (2) The determination of fundamental physical interactions. A brief summary of some of those applications will be discussed in this section, a more complete and comprehensive discussion is given in the references that were mentioned in the previous section.

One of the usefulness of using cold neutrons in the study of the neutron properties arises from the fact that under controlled conditions, Ultra-cold neutrons can be stored in material bottles for times approaching the neutron lifetime. This fact was and still is used to measure the most accurate values for the neutron lifetime³ (887.6 ± 3 sec). An accurate value for the neutron lifetime is essential for the theory of Semi-Leptonic weak interactions.
Bottled Ultra-cold neutrons are also being used to measure the neutron electric dipole moment. The main advantage of using Ultra-cold neutrons in those experiments results from Ultra-cold neutrons insensitivity to the relativistic effects which limit the precision of the conventional beam experiments. Cold neutrons have also been used in a lot of important experiments including: the measurement of the electric charge of the neutron ($<4.3 \times 10^{-21}$ e), the study of bound neutron states and various quantum mechanical problems.

Another class of applications has utilized cold neutrons in the study of matter. To just mention a few of those studies, cold neutrons are being used to investigate inhomogeneities in non-homogenous substances, to study the roughness and excitations of various surfaces, and to measure the limiting energies for various substances. The attraction of using cold neutrons as a probe in some of those experiments is due mainly to the nondestructive nature of cold neutrons.
3.1 Overview Of The System Model

Analysis of neutron slowing down in a cold source within a TRIGA core was performed using the following steps:

I. Treating the reactor core as a bare cylindrical homogenous medium made of Hydrogen, Oxygen, Zirconium, Uranium-235 and Uranium-238, 23-energy-groups slowing down calculations in the energy range between 10.5 MeV to 1 eV were performed to find:

   A) Neutron spectrum (\(\phi(E)\))
   B) Slowing down densities (\(q(E)\)).

Leakage effects from the core were accounted for by the use of geometrical buckling in the calculation, and a distributed fission source was assumed. Since the mean free path of fast neutrons is comparable to, or larger than the size of the spherically shaped, Hydrogen filled cold source, it was assumed that in the fast energy range, the cold source spectrum is equal to the core spectrum.

II. The fast neutron spectrum that was found in part I was then used to find:

   A) Fast group constants for the core
   B) Fast group constants for the cold source.

III. For the cold source, using 18 energy groups in the energy range from 1 eV to 0.01 eV, with a monoenergetic
neutron source of intensity equal to the core slowing down density at 1 eV in the highest energy group, slowing down calculations were utilized in order to find the energy spectrum.

IV. The energy spectrum that was found in step III was then used to calculate "thermal" (0.01 < E < 1) group constants for the cold source, while a Maxwellian spectrum was used for the cold group (0.001 < E < 0.01).

V. The group constants that were found in steps II and IV, were used to couple the core and the cold source calculations by using them in a three group, 2 region diffusion theory calculation. The value for cold source geometrical buckling was found by \( B^2 = -\langle \nabla^2 \phi(x) / \phi(x) \rangle \), then the previous steps were repeated until the geometrical buckling converged.

VI. Using the converged value for the geometrical buckling, slowing down calculations for the cold source were repeated in order to obtain

A) Neutron flux

B) Energy deposition in the cold source due to neutrons in the energy range: 0.01 < E < 10.5 MeV
Most of the theoretical work that is presented in sections 3.2, 3.3 and 3.4, especially the material that is concerned with the derivation of the Pl equations and the various expressions for the slowing down densities could be found in reference 4. The numerical methods that were utilized in those sections are based on the information that is given in references 4 and 5, while the materials that is related to the physical states of the hydrogen molecule were obtained from references 7 and 8. A complete list of the references is given at the end of this report. The information that is presented in the references was merged and modified as needed in order to construct the model that was described in the previous section.

3.2 Neutron Transport Equation

The neutron transport equation (Boltzmann equation) is given by:

\[ \frac{1}{\Theta} \frac{\partial \phi}{\partial t} = \int_0^\infty \int_0^{2\pi} \int_0^\pi \Sigma_s(E' \rightarrow E, \Omega' \rightarrow \Omega) \phi(r, E', \Omega', t) \cdot S(r, E, \Omega, t) \]

\[ -\Sigma_s(E) \phi(r, E, \Omega, t) - \Omega \cdot \nabla \phi(r, E, \Omega, t) \]  

(3.1)

If we expand the angular flux in terms of spherical harmonics up to the first order, using only the first four harmonics, we get:
\[ \psi(r,E,\Omega,t) = \frac{1}{4\pi} \phi(r,E,t) + \frac{3}{4\pi} J(r,E,t) \cdot \Omega \] (3.2)

It is important to point out that by neglecting all terms of higher than linear order in \( \Omega \), we are assuming that the angular flux is linearly anisotropic, hence it only depends linearly on the cosine of the angle. This expansion enables the development of the P1 equations and the diffusion equation which will be used later.

3.3 Slowing Down Theory

If we integrate the transport equation over all \( \Omega \), and use the relation \( \Omega \cdot \nabla = \nabla \cdot \Omega \), since \( \Omega \) is independent of \( r \), we obtain:

\[ \frac{1}{\Theta} \frac{\partial \phi}{\partial t} \int dE' \Sigma_{\text{st}}(E' \to E) \phi(r,E',t) - \nabla \cdot J(r,E,t) \]

\[ - \Sigma_{\text{c}}(E) \phi(r,E,t) + S_0(r,E,t) \] (3.3)

where

\[ \Sigma_{\text{st}}(E' \to E) = \int d\Omega \Sigma_{\text{st}}(E' \to E, \Omega' \to \Omega) \] (3.4)
\[ S_0(r,E,t) = \int_{4\pi} d\Omega S(r,E,\Omega,t) \] (3.5)

\( \phi(r,E,t) = \text{Neutron flux} \)

\( J(r,E,t) = \text{Current density} \)

\( \Sigma_{\text{sc}}(E' \rightarrow E) = \text{Macroscopic isotropic differential scattering cross section (Elastic + Inelastic)} \)

\( S_0(r,E,t) = \text{Isotropic component of the source.} \)

As can be seen from equation (3.3), when the angular dependence was removed, we ended up with one equation and two unknowns, \( \phi \) and \( J \). A second equation relating the neutron flux and the current density can be obtained by multiplying the transport equation by \( \Omega \), substituting equation (3.2) for the angular flux in the leakage term, integrating over all \( \Omega \), and using

\[ \int_{4\pi} d\Omega \Omega_i \Omega_j = \frac{4\pi}{3} \text{ if } i=j, \text{ or } = 0 \text{ if } i\neq j \]

and

\[ \int_{4\pi} d\Omega \Omega_i \Omega_m \Omega_n = 0 \text{ if } l,m, \text{ or } n \text{ is odd} \]
then we obtain:

\[
\frac{1}{\delta} \frac{\partial J}{\partial t} = \int_{E_0}^{\infty} dE' \Sigma_{s_{1e}}(E' \rightarrow E) J(r, E', t) - \frac{1}{3} \nabla \cdot J(r, E, t)
\]

\[-\Sigma(E) J(r, E, t) + S_1(r, E, t) \quad (3.6)\]

where

\[
\Sigma_{s_{1e}}(E' \rightarrow E) = \int_{4\pi} d\Omega d\Omega' \Sigma_{s}(E' \rightarrow E, Q \rightarrow Q')
\]

\[
S_1(r,E,t) = \int_{4\pi} d\Omega Q S(r,E,Q,t)
\]

\[
S_{1e}(E' \rightarrow E) = \text{Linearly anisotropic scattering component of the differential cross section (Elastic + inelastic)}
\]

\[
S_1(r,E,t) = \text{Linearly anisotropic component of the source.}
\]

Equations (3.3) and (3.6) are known collectively as the energy dependent Pl equations and they can be used to solve for the neutron flux and current density.

As was pointed out before, the only essential assumption used in the Pl equations is that the neutron angular flux is well approximated by only a linearly anisotropic dependence on the angular variable, i.e. equation (3.2). Equations (3.4) and (3.7) give the isotropic and linearly anisotropic scattering components of the
differential cross section. These include both elastic and inelastic scattering, since the scattering cross section consists of an elastic and inelastic components. If we separate the scattering cross section into its two different components, and include the inelastic scattering in the source terms, then equations (3.3) and (3.6) become, respectively:

\[
\frac{1}{\phi} \frac{\partial \rho}{\partial t} + \nabla \cdot \Sigma_t(E) \phi(r, E, t) = \int_0^E dE' \Sigma_{s_0}(E' \rightarrow E) \phi(r, E', t) + S_0(r, E, t)
\]

(3.8)

\[
\frac{1}{\phi} \frac{\partial J}{\partial t} + \frac{1}{3} \nabla \cdot \Sigma_t(E) J(r, E, t) = \int_0^E dE' \Sigma_{s_1}(E' \rightarrow E) J(r, E', t) + S_1(r, E, t)
\]

(3.9)

where

\[
\Sigma_{s_0}(E' \rightarrow E) = \text{Elastic isotropic scattering component of the differential cross section}
\]

\[
\Sigma_{s_1}(E' \rightarrow E) = \text{Elastic linearly anisotropic scattering component of the differential cross section}
\]

\[
S_0(r, E, t) = \text{Isotropic component of the source, includes isotropic inelastic scattering}
\]

\[
S_1(r, E, t) = \text{Linearly anisotropic component of the source, includes linearly anisotropic inelastic scattering}
\]

It is usually more convenient to use the logarithm of the neutron energy $E$, lethargy, as an independent variable. So we define lethargy, $u$, as:
where $E_0$ is chosen to be the maximum energy that neutrons can have in a given problem. In terms of lethargy, equations (3.8) and (3.9) become:

$$u = \ln \left( \frac{E_0}{E} \right) \quad (3.10)$$

To proceed further, we need to introduce a very important concept, which is the slowing down density. Since we are interested in calculating the number of neutrons that slow down past some energy, $E$, we will introduce the slowing down density, $g(r,E)$, characterizing elastic scattering as:

$$\frac{1}{3} \frac{\partial \phi}{\partial t} + \nabla J_s \Sigma_t (u) \phi(r, u, t) = \int_{E_0}^{E} du' \Sigma_{g_0} (u' \to u) \phi(r, u', t) + S_0 (r, u, t) \quad (3.11)$$

$$\frac{1}{3} \frac{\partial J}{\partial t} + \frac{1}{3} \nabla \phi + \sum_t (u) J(r, u, t) = \int_{E_0}^{E} du' \Sigma_{g_1} (u' \to u) J(r, u', t) + S_1 (r, u, t) \quad (3.12)$$

where upscattering was ignored. The physical interpretation of the slowing down density is such that it is equal to the number of neutrons that slow down past energy $E$, per second per unit volume. In term of lethargy, equation (3.13)
becomes:

\[
q(r, u) = \int_{u}^{\infty} du'' \int_{0}^{u} du' \Sigma_{q}(u'\rightarrow u'') \phi(r, u')
\]  

(3.14)

By defining \( q_{o}(r, u) \) as the slowing down density characterizing isotropic elastic scattering from each isotopic species \( j \), then:

\[
j q_{o}(r, u) = \int_{u}^{\infty} du'' \int_{0}^{u} du' \Sigma_{q_{o}}(u'\rightarrow u'') \phi(r, u')
\]  

(3.15)

Similarly, \( q_{l}(r, u) \) can be defined as the linearly anisotropic scattering contribution to the slowing down density, hence

\[
j q_{l}(r, u) = \int_{u}^{\infty} du'' \int_{0}^{u} du' \Sigma_{q_{l}}(u'\rightarrow u'') \phi(r, u')
\]  

(3.16)

where:

\[
j \Sigma_{q_{o}}(u'\rightarrow u) = \begin{cases} 
\frac{j \Sigma_{u}(u') e^{u'-u}}{1-\alpha_{j}}, & u-\ln\left(\frac{1}{\alpha_{j}}\right) \leq u' \leq u \\
0, & \text{otherwise}
\end{cases}
\]  

(3.17)

\[
j \Sigma_{q_{l}}(u'\rightarrow u) = \begin{cases} 
\frac{j \Sigma_{s}(u') e^{u'-u}}{1-\alpha_{j}} \left[ \frac{A+1}{2} e^{\frac{u'-u}{2}} - \frac{A-1}{2} e^{-\frac{u'-u}{2}} \right], & u-\ln\frac{1}{\alpha_{j}} \leq u' \leq u \\
0, & \text{otherwise}
\end{cases}
\]  

(3.18)
\[ j_{E_0}(u' \rightarrow u) = \text{Elastic isotropic scattering} \]
\[ j_{E_1}(u' \rightarrow u) = \text{Linearly anisotropic scattering} \]
\[ \Sigma_e(u) = \text{Elastic scattering cross section}. \]
\[ \alpha = \left( \frac{(A-1)}{(A+1)} \right)^2. \]

For the case of hydrogen, \( A = 1 \), \( \Rightarrow \alpha = 0 \), then if equations (3.17) and (3.18) are substituted into (3.15) and (3.16) respectively, and then differentiating those expressions with respect to lethargy, we obtain:

\[ \frac{\partial q_0}{\partial u} + q_0(r,u) = \Sigma_e(r,u)\phi(r,u) \quad (3.19) \]

\[ \frac{\partial q_1}{\partial u} + \frac{3}{2} q_1(r,u) = \frac{2}{3} \Sigma_e(r,u)J(r,u) \quad (3.20) \]

If we differentiate equations (3.15) and (3.16) with respect to lethargy and note that

\[ \Sigma_e(u) = \int_0^\infty du'' \Sigma_{E_0}(u'' \rightarrow u) \]

\[ \overline{\mu_0}\Sigma_e(u) = \int_0^\infty du'' \Sigma_{E_1}(u'' \rightarrow u) \]
where

\[ \bar{\mu}_0 = \text{Average scattering angle cosine in the lab frame.} \]

we obtain:

\[
\int_0^u du' j \Sigma(u' \rightarrow u) \phi(r, u') = - \frac{\partial j}{\partial u} + \bar{\mu}_0 j \Sigma(u) \phi(r, u)
\]  \hspace{1cm} (3.21)

\[
\int_0^u du' j \Sigma_1(u' \rightarrow u) J(r, u') = - \frac{\partial j}{\partial u} + \bar{\mu}_0 j \Sigma(u) J(r, u)
\]  \hspace{1cm} (3.22)

If equations (3.21) and (3.22) are substituted into (3.11) and (3.12) respectively, then the PI equations become:

\[
\frac{1}{3} \frac{\partial J}{\partial t} + \nabla \cdot (\Sigma_t(u) - \Sigma_s(u)) \phi(r, u, t) = - \sum_{j=1}^N \frac{\partial j}{\partial u} + S_0(r, u, t)
\]  \hspace{1cm} (3.23)

\[
\frac{1}{3} \frac{\partial J}{\partial t} + \frac{1}{3} \nabla \cdot (\Sigma_t(u) - \bar{\mu}_0 \Sigma_s(u)) J(r, u, t) = - \sum_{j=1}^N \frac{\partial j}{\partial u} + S_1(r, u, t)
\]  \hspace{1cm} (3.24)

By considering equations (3.23) and (3.24), it is observed that in order to solve for the unknown variables, i.e., the flux, current density and the various slowing down densities, it is essential to find expressions for the various slowing down densities in terms of either the flux or the current density. For the case of hydrogen, this was already done, and is given by equations (3.19) and (3.20).
It is still needed to develop expressions similar to equations (3.19) and (3.20) for nonhydrogen species. The method that will be chosen is the consistent PI approximation. In this method, Taylor expansion is used to expand the collision densities $\Sigma_{a}(u')\phi(r,u')$ and $\Sigma_{a}(u')J(r,u')$ about $u'=u$.

\[ j\Sigma_{a}(u')\phi(r,u) = j\Sigma_{a}(u)\phi(r,u) + (u'-u) \frac{\partial}{\partial u} [ j\Sigma_{a}(u)\phi(r,u) ] \] (3.25)

\[ j\Sigma_{a}(u')J(r,u') = j\Sigma_{a}(u)J(r,u) + (u'-u) \frac{\partial}{\partial u} [ j\Sigma_{a}(u)J(r,u) ] \] (3.26)

Then equations (3.17), (3.25) and (3.18), (3.26) are substituted into equations (3.15) and (3.16) respectively to give

\[ jq_{0}(r,u) = \xi_{j} j\Sigma_{a}(u)\phi(r,u) \] (3.27)

\[ jq_{1}(r,u) = \zeta_{j} j\Sigma_{a}(u)J(r,u) \] (3.28)

where

\[ \xi_{j} = 1 - \frac{\alpha_{j} \ln \left( \frac{1}{\alpha_{j}} \right)}{1 - \alpha_{j}}, \quad \zeta_{j} = A_{j}^{-1} \] (3.29)
\[
\gamma_j = \left(\frac{1 + \gamma_j}{\gamma_j}\right)^2 \left[\frac{1 + \gamma_j}{9} \left(1 - \alpha_j^{3/2} \left(\frac{3}{2} \ln \frac{1}{\alpha_j} + 1\right)\right)\right] - \left(1 - \gamma_j\right) \left(1 - \alpha_j^{3/2} \left(\frac{1}{2} \ln \frac{1}{\alpha_j} + 1\right)\right) \tag{3.30}
\]

In obtaining equations (3.25) and (3.26), only low order terms were retained, while in equations (3.27) and (3.28) only the first term was retained. Equations (3.19), (3.20) and (3.27), (3.28) can be used to supplement equations (3.23) and (3.24) to solve for the flux, current density, and the various slowing down densities.

If the time dependence is neglected in the PI equations then equations (3.23) and (3.24) can be written as

\[
\nabla J^+ \Sigma_{ne}(u) \phi(\mathbf{r}, u) = -\sum_{j=1}^{N} \frac{\partial^3 q_0}{\partial u^3} - \frac{\partial^h q_0}{\partial u} \cdot S_0(\mathbf{r}, u) \tag{3.31}
\]

\[
\frac{1}{3} \nabla \phi + \Sigma_{tr}(u) J(\mathbf{r}, u) = -\sum_{j=1}^{N} \frac{\partial^3 q_1}{\partial u^3} - \frac{\partial^h q_1}{\partial u} \tag{3.32}
\]

Where \(N\) corresponds to the numbers of nonhydrogen species in the medium, \(\Sigma_{ne}(u)\) and \(\Sigma_{tr}(u)\) are the nonelastic and transport cross sections respectively, and are given by:

\[
\Sigma_{ne}(u) = \Sigma_{t}(u) - \Sigma_{e}(u) = \Sigma_{t}(u) - \sum_{j=1}^{N} j \Sigma_{e}(u) \tag{3.33}
\]
In equations (3.33) and (3.34), the N+1 term arises due to the fact that hydrogen effects must be included in the calculations of the nonelastic and transport cross sections, since these cross sections characterize the entire medium. It is important to restate that in equation (3.31), the inelastic contribution to scattering has been included in $S_0(r,u)$:

$$S_0(u) = S_{\text{Fission}} + \sum_{j=1}^{N} \int_{0}^{u} d\alpha' j \Sigma_{\text{in}}(\alpha' \rightarrow u) \phi(u)$$ \hspace{1cm} (3.35)$$

while the anisotropic components of inelastic scattering and neutron sources have been explicitly ignored by setting $S_s(r,u)$ equal to zero in equation (3.32).

The spatial dependence in the P1 slowing down equations can be treated by assuming that the lethargy and spatial dependence are separable. Also, the spatial dependence can be characterized by a single Fourier mode, $\exp(iBz)$, where $B$ is the buckling, and $B^2$ can be regarded as characterizing the average core leakage. It is important to note that the above process is just a crude method to account for gross leakage effects. Equations (3.31),(3.32),(3.19),(3.20),
(3.27) and (3.28) can then be written in the cylindrical coordinates as:

\[ i(B_z + B_r)J(u) + \Sigma_{ne}(u)\phi(u) = -\sum_{j=1}^{N} \frac{jdq_0(u)}{du} - \frac{d^2q_0}{du^2} + S_0(u) \]  
\[ \frac{1}{3} i(B_z + B_r)\phi(u) + \Sigma_{\ell r}(u)J(u) = -\sum_{j=1}^{N} \frac{jdq_1(u)}{du} - \frac{d^2q_1}{du^2} \]  
\[ \frac{d^2q_0}{du^2} + \frac{d^2q_0}{du^2} + \frac{d^2q_1}{du^2} + \frac{d^2q_1}{du^2} = \frac{2}{3}\Sigma_s(u)J(u) \]  
\[ jq_0(u) = \xi_j j\Sigma_s(u)\phi(u) \]  
\[ jq_1(u) = \zeta_j j\Sigma_s(u)J(u) \]

where \( \nabla = \frac{\partial}{\partial z} \hat{e}_z + \frac{\partial}{\partial r} \hat{e}_r \) in cylindrical coordinates, and lethargy-spatial dependence is assumed to be separable in the form \( \phi(u, r, z) = \phi(u)e^{(ib_r r)}e^{(ib_z z)} \). For a bare cylindrical core, \( B_r \) and \( B_z \) are given by:
Equations (3.36) through (3.41) were solved numerically by discretizing them over a fine group mesh. The group averages of dependent variables such as \( \phi, J, \Sigma \) and \( S \), were defined as:

\[
\Sigma_n = \frac{1}{\Delta u_n} \int_{u_{n-1}}^{u_n} du \Sigma(u)
\]

\[
\phi_n = \frac{1}{\Delta u_n} \int_{u_{n-1}}^{u_n} du \phi(u)
\]

\[
\Sigma_{n'n} = \frac{1}{\Delta u_n} \frac{1}{\Delta u_{n'}} \int_{u_{n'-1}}^{u_{n'}} du \int_{u_{n'-1}}^{u_{n'}} du' \Sigma_{nn'}(u'u) \]

and so on. The slowing down densities were approximated by:

\[
\frac{\nu q_{n+n-1}}{2} = \frac{1}{\Delta u_n} \int_{u_{n-1}}^{u_n} du \nu q(u) \quad (3.43)
\]

\[
\nu q_n = \frac{1}{\Delta u_n} \int_{u_{n-1}}^{u_n} du \nu q(u) \quad (3.44)
\]

while for derivatives, simple difference expressions were used:
In equation (3.43), the Hydrogen slowing down density is assumed to vary linearly in the lethargy interval, while in equation (3.44), the slowing down densities of non-hydrogen species are assumed to be constant and equal to their end values. The need to use this inconsistent approach is in order to avoid negative values for the slowing down densities of non-hydrogen species which would otherwise arise if it was assumed that the slowing down density varies linearly in the interval. This problem arises because we used only 23 energy groups, and hence, the energy width of these groups is large.

If we assume that the only source of neutrons is due to fission, then we can write

\[ S(E) = C \cdot P_{\nu} \cdot \chi(E) / V \]  \hspace{1cm} (3.45)

Where

\[ S(E) = \text{Rate of neutron produced with energy } E \]
\[ \chi(E) = \text{Fission spectrum} \]
C = Conversion factor relating power in Kw to total number of neutrons from fission, \(7.64578 \times 10^{13}\) neutron/(sec-kw)

\[ P_w = \text{Operating power in Kw} \]

\[ V = \text{Volume of the core.} \]

If we take only prompt neutrons into consideration, then the fission spectrum can be approximated by:

\[ \chi(E) = 0.453e^{-0.036E}\sinh\sqrt{2.29E} \]

where \(E\) is in MeV. It then follows that

\[ S(E) = C \cdot P_w \cdot 0.453e^{-0.036E}\sinh\sqrt{2.29E} / V \]

As before, we can define \(S_n\) as:

\[ S_n = \frac{1}{\Delta E_n} \int_{E_n}^{E_n+1} dE S(E) = C \cdot P_w \cdot 0.453e^{-0.036E}\sinh\sqrt{2.29E_n} / V \quad (3.46) \]

Equation (3.46) can be converted from energy into the lethargy variable by the use of the relation

\[ S(E) \, dE = - S(u) \, du \]

where \(dE/du\) is equal to minus \(E\).

The last group average that still needs to be defined is the average scattering angle cosine, \(\mu_0\). In the center of mass system, the angular distribution is assumed to have a mean cosine of scattering equal to:

\[ \mu' = 0.07 \cdot A^{1/3} \cdot E \quad (3.47) \]
where

\[ A = \text{Atomic mass} \]

\[ E = \text{Neutron energy in MeV} \]

provided that the energy is far from the scattering resonance. If we assume that the scattering is linearly anisotropic in the center of mass system, then the average cosine of the scattering angle in the lab system \( (\bar{\mu}_0)_{\text{aniso}} \) can be written as:

\[
(\bar{\mu}_0)_{\text{aniso}} = (\bar{\mu}_0)_{\text{iso}} + \bar{\mu}(1 - \frac{2}{5A^2})
\]

(3.48)

Where

\[ (\bar{\mu}_0)_{\text{iso}} = \text{Average cosine of the scattering angle in the lab system, assuming isotropic scattering in center mass system, } = \frac{2}{3A}. \]

In term of lethargy groups, equation (3.48) is written as:

\[
(\bar{\mu}_0)_{\text{aniso}} = \frac{2}{3A} + 0.07A^{2/3}E_n(1 - \frac{3}{5A^2})
\]

(3.49)

where \( E_n \) is the average energy of group \( n \). If we now integrate equations (3.36) through (3.41) from \( u_{n-1} \) to \( u_n \), and then use the above expressions for group averages, we get:
\[ i(B_x + B_z) \Delta u_n \mathbf{J}_n + \Sigma_{ne} \phi_n \Delta u_n = \sum_{j=1}^{N} \left( j q_{0n} - j q_{0n-1} \right) - \left( \frac{\partial q_{0n}}{\partial n} - \frac{\partial q_{0n-1}}{\partial n} \right) + S_{\phi n} \Delta u_n \]

\[ + \Delta u_n \sum_{j=1}^{N} \sum_{n'=1}^{n-1} \frac{j \Sigma_{n'n} \Delta u_{n'} \phi_{n'}}{3} \quad (3.50) \]

\[ \frac{i}{3} \left( B_x + B_z \right) \Delta u_n \phi_n + \Sigma_{tr} \Delta u_n \mathbf{J}_n = \sum_{j=1}^{N} \left( j q_{1n} - j q_{1n-1} \right) - \left( \frac{\partial q_{1n}}{\partial n} - \frac{\partial q_{1n-1}}{\partial n} \right) \quad (3.51) \]

\[ \left( \frac{\partial q_{0n}}{\partial n} - \frac{\partial q_{0n-1}}{\partial n} \right) + \frac{1}{2} \left( \frac{\partial q_{1n}}{\partial n} - \frac{\partial q_{1n-1}}{\partial n} \right) \Delta u_n = \frac{\Sigma_{s n} \Delta u_n \phi_n}{3} \quad (3.52) \]

\[ \left( \frac{\partial q_{1n}}{\partial n} - \frac{\partial q_{1n-1}}{\partial n} \right) \cdot \frac{3}{4} \left( \frac{\partial q_{2n}}{\partial n} - \frac{\partial q_{2n-1}}{\partial n} \right) \Delta u_n = \frac{2}{3} \frac{\Sigma_{s n} \Delta u_n \mathbf{J}_n}{3} \quad (3.53) \]

\[ j q_{0n} \Delta u_n = \xi_j \frac{j \Sigma_{s n} \Delta u_n \phi_n}{3} \quad (3.54) \]

\[ j q_{1n} \Delta u_n = \xi_j \frac{j \Sigma_{s n} \Delta u_n \mathbf{J}_n}{3} \quad (3.55) \]

Solving for the flux, current density and the slowing down densities yields:
\[ \phi_n = S_0 \left( \frac{Y_\theta}{1 + Y_{\theta n} Y_5 n} \right)^r \sum_{j=1}^N j^q_{0_{n-1}} \left( \frac{Y_{\theta n} Y_{6 n}}{1 + Y_{\theta n} Y_5 n} \right) \sum_{j=1}^N j^q_{1_{n-1}} \left( \frac{iY_{6 n}}{1 + Y_{\theta n} Y_5 n} \right) \sum_{j=1}^N j^\Sigma_{n'n} \phi_n' \]

\[ j^q_0 = \xi_j \Sigma_{s_a} \phi_n \]

\[ j^q_1 = \zeta_j \Sigma_{s_a} J_n \]

where

\[ Y_1 = \frac{2 H_{s_a} \Delta u_n}{2 + \Delta u_n} \quad Y_3 = \frac{8 H_{s_a} \Delta u_n}{9 \Delta u_n + 12} \]
Equations (3.56) through (3.61) can be solved by recognizing that for the uppermost group, \( \phi_0 \), \( \psi_1 \), \( q_0 \), \( q_1 \) are equal to zero since neutrons cannot slow down into the highest group. It is then possible to solve for \( \phi_1 \) and \( J_1 \), then use those results to find \( \phi_0 \), \( \psi_1 \), \( q_0 \), \( q_1 \). The slowing down densities of group 1 can then be used to find \( \phi_2 \) and \( J_2 \), and then the above process is repeated for the remaining groups.
Even though hydrogen exists as a diatomic molecule, so far we have been treating the neutron-proton scattering not as an interaction between a neutron and a hydrogen molecule, rather as neutron scattering of a single hydrogen atom. So, in effect, we have been neglecting the chemical binding between the hydrogen atoms. The neglect of the chemical binding can be justified by noting that since the chemical binding energies are on the order of 0.5 eV, if the incident neutron have energy greater than 1 eV, when it collide with a proton, the proton recoil will be so large that it result in the dissociation of the hydrogen molecule into its two components, so that the protons can be considered as if they were completely free particles. From the above discussion, it becomes clear that the chemical binding effects of the hydrogen molecules need to be accounted for when we are dealing with slow neutrons. In order to take into consideration the chemical binding effects of the hydrogen molecule, $A_{molecule}$ rather than $A$ is used in calculating $\alpha$ and $\mu_o$. Since we are interested in calculating the slowing down density in hydrogen down to 0.01 eV, in the region below 1 eV, neutrons collisions with the hydrogen molecule can be modeled as neutrons collisions with a nuclear species with $A = 2$, i.e. with Deuterium.
So far we have given expressions for the slowing down density in hydrogen and in heavy species (consistent P1 approximation). Neither of these two approaches adequately describe light species, since in the case of Hydrogen it was treated exactly, while in keeping only the first terms in the expansion of the collision densities, i.e, equations (3.27) and (3.28), it is assumed that the neutrons lose only a small amount of their energies in each collision, which implies that the neutrons are colliding with heavy species. In order to develop expressions for the slowing down densities in light species, the Grueling-Goertzel approximation will be used. In this approximation the same procedure that led to equations (3.27) and (3.28) is followed, except that low order terms are kept, not just the first term. Then, the resulting expressions for the slowing down densities are cast into a form similar to the one for Hydrogen, i.e:

\[
\lambda_0 \frac{\partial j_0}{\partial u} + j_0(r,u) = \beta_0 \phi(r,u) \tag{3.62}
\]

\[
\lambda_1 \frac{\partial j_1}{\partial u} + j_1(r,u) = \beta_1 J(r,u) \tag{3.63}
\]

where
\( \lambda_{0j} = \frac{1 - \alpha_j \left(1 + \ln \frac{1}{\alpha_j} + \frac{1}{2} \ln^2 \frac{1}{\alpha_j} \right)}{1 - \alpha_j \left(1 + \ln \frac{1}{\alpha_j} \right)} , \quad \beta_{0j} = \zeta_j \Sigma_g(u) \quad (3.64) \)

\( \lambda_{1j} = \left( \frac{(1 + \gamma_j)^2}{4 \gamma_j} \left[ \frac{(1 + \gamma_j)}{3} \left[ \frac{8}{9} - \alpha_j^{3/2} \left( \ln^2 \frac{1}{\alpha_j} - \frac{4}{3} \ln \frac{1}{\alpha_j} + \frac{8}{9} \right) \right] \right] \right) \zeta_j^1 \)

\( + \left( \frac{(1 - \gamma_j)}{4 \gamma_j} \left[ -\frac{8}{9} - \alpha_j^{3/2} \left( \ln^2 \frac{1}{\alpha_j} - \frac{4}{3} \ln \frac{1}{\alpha_j} + \frac{8}{9} \right) \right] \right) \zeta_j^1 , \quad \beta_{1j} = \zeta_j \Sigma_g \quad (3.65) \)

So for the region between 1 and 0.01 eV, the energy spectrum and the slowing down densities can be found by following a similar procedure as before to get

\( \phi_n = \left( \frac{X_{5n}}{BX_{7n}} S_{0n} \right) S_{0n} + \left( \frac{X_{6n}}{X_{7n}} q_{0n+1} - \left( \frac{iX_{6n}}{X_{7n}} \right) q_{1n+1} \right) \quad (3.66) \)

\( J_n = (iX_{5n}) \phi_n - \left( \frac{i}{B} \right) S_{0n} - (iX_{10n}) q_{0n+1} \quad (3.67) \)

\( q_{0n} = (X_{1n}) \phi_n + (X_{2n}) q_{0n+1} \quad (3.68) \)

\( q_{1n} = (X_{3n}) J_n + (X_{4n}) q_{1n+1} \quad (3.69) \)

with
There are three important topics that are concerned with the above discussion that still need to be examined. First, we neglect the intermolecular forces during the scattering by H\textsubscript{2}. This can be justified by assuming that the molecules’ frequency at which they vibrate independently about the positions of equilibrium is given by: \( \nu = \frac{3kT}{4\hbar} \), where \( T = 105^0K \) is the characteristic Debye temperature. It then follows that the molecules can be treated as if they are free, if the neutron energy is large comparing to 0.007 eV. Second, since the neutron wavelength at low energies is comparable to the interatomic spacing of the scattering medium, the neutron will undergo diffraction. It is the norm in nuclear engineering to neglect neutron diffraction.
in order to simplify calculations, as it has been done in this paper. Third, neutron inelastic scattering from the hydrogen molecules was also neglected. In this context, by inelastic scattering, what is meant is the excitation of the internal states of the Hydrogen molecule, such as molecular vibrational and rotational states. The main reason for not considering internal mode excitation is in order to simplify calculations. The neglect of inelastic scattering by the Hydrogen molecules should not affect the final conclusions of this work for the following reason. Around 20 °K, Hydrogen molecules will consist mostly of parahydrogen (anti-parallel proton spins), in its lowest possible energy state. When low energy neutrons collide with those molecules, they might undergo either elastic or inelastic scattering with the parahydrogen molecule being excited to either orthohydrogen (parallel proton spins), or to an excited energy state of para-hydrogen, depending on the incident neutron energy. In either case, the end result is the down scattering of the neutrons. So by neglecting inelastic scattering, we underestimate the down scattering power of the medium, and hence our results will be somewhat conservative, which is important for this type of work.
3.4 Diffusion Equation

In order to derive the multigroup diffusion equation, we need to use Fick's law, which basically states that the current density is approximately proportional to the negative spatial gradient of the density.

\[ \mathbf{J}(r,t) = -D(r) \nabla \rho(r,t) \]  

(3.70)

Substituting equation (3.70) into equation (3.3), and assuming that the diffusion coefficient \( D \), is independent of position (homogenous medium) gives

\[ \frac{1}{\delta} \frac{\partial \phi}{\partial t} = \int dE' \Sigma_a(E' \rightarrow E) \phi(r,E',t) + S(r,E,t) + D \nabla^2 \phi(r,E,t) \]

\[ -\Sigma_t(E) \phi(r,E,t) \]  

(3.71)

Equation (3.71) is known as the energy dependent diffusion equation. The multigroup diffusion equation is obtained by integrating the energy dependent diffusion equation over the interval \( E_g < E < E_{g-1} \) (discretizing it into \( G \) energy groups) and using:

\[ \phi_g(r,t) = \int_{E_g}^{E_{g-1}} dE \phi(r,E,t) \]
$$\Sigma_{t_g} = \frac{1}{\phi_g} \int_{E_g}^{E_{g-1}} dE \Sigma_l(E) \phi(\mathbf{r}, E, t)$$

$$\Sigma_{a_{g'g}} = \frac{1}{\phi_g} \int_{E_g}^{E_{g'-1}} dE \int_{E_{g'}}^{E_{g'-1}} dE' \Sigma_a(E' \rightarrow E) \phi(\mathbf{r}, E', t)$$

$$\frac{1}{\mathcal{S}_g} = \frac{1}{\phi_g} \int_{E_g}^{E_{g-1}} dE \frac{1}{\mathcal{S}} \phi(\mathbf{r}, E, t)$$

$$D_g = \frac{1}{\phi_g} \int_{E_g}^{E_{g-1}} dE D(E) \phi(\mathbf{r}, E, t)$$

$$\mathcal{S}_g \cdot \frac{\partial \phi_g}{\partial t} = D_g \nabla^2 \phi_g - \Sigma_{t_g} \phi_g(\mathbf{r}, t) + \sum_{g' \leq 1} \Sigma_{a_{g'g}} \phi'_{g'} S_g \quad g = 1, 2, \ldots, G \quad (3.72)$$

Equation (3.72) represents \( G \) coupled diffusion equations which can be solved for the \( G \) unknown group fluxes provided that the group constants for each group are known.
3.5 Energy Deposition

In order to maintain a given medium at a certain temperature, it is necessary to know the amount of energy deposited in the medium as a result of the various nuclear reactions. For the case of slowing down in Hydrogen medium, there are only three reactions that need to be considered; elastic scattering, absorption, and gamma ray heating. We point out that Hydrogen does not undergo inelastic scattering in the energy range of our interest.

3.5.1 Neutron Elastic Scattering

So far, when we have been talking about elastic scattering, we mentioned isotropic and linearly anisotropic scattering. In the formalism that we are using, the energy dependent Pl equations, the elastic differential scattering cross section is equal to the elastic isotropic differential scattering cross section. So only isotropic scattering needs to be considered in calculating the various nuclear reactions and in generating few group constants.

In an elastic collision, total kinetic energy is conserved. For the case of elastic isotropic scattering between an incident neutron with initial kinetic energy $E_i$ and a nucleus at rest, the average energy lost by the neutron, $\Delta E_n$, in the collision is very well known, and is given by:
\[ \Delta E_0 = \left( \frac{1-a}{2} \right) E_i \] (3.73)

where \( a = \left( \frac{A-1}{A+1} \right)^2 \) and \( A \) is the Nuclear mass number. For hydrogen, \( a = 0 \), so on the average, a neutron will lose half of its energy when it undergoes an elastic isotropic scattering with hydrogen, which is equal to the average kinetic energy gained by the hydrogen nucleus. The hydrogen nucleus will then dissipate its energy in the medium through various processes.

From the above discussion, it follows that when a neutron undergoes an elastic isotropic scattering, on the average, the amount of energy deposited in the medium will be given by equation (3.73). Since the expected rate at which a particular interaction occurs in a unit volume is given by \( F = \phi \Sigma \), where \( F \) is the Reaction rate density, \( \phi \) is the flux and \( \Sigma \) is the macroscopic cross section, then, the average energy deposited in the medium per unit volume due to elastic isotropic scattering, \( E_{s_0} \), could be written as:

\[ E_{s_0} = \int dE_i \phi(E_i) \Sigma_{s_0}(E_i) \Delta E_0(E_i) \] (3.74)

When doing a fine group spectrum calculation, then for each group, equation (3.74) can be approximated by:
\[
E_{s_{on}} = \frac{1}{\Delta E_n} \int \frac{dE_i \phi(E_i) \Sigma_{sn}(E_i) \Delta E_0(E_i)}{\Sigma_{sn}(E_i) \Delta E_0(E_i)} = \phi_n \Sigma_{sn} \Delta E_0(E_n) \tag{3.75}
\]

where:

- \(\phi_n\) = Average Neutron flux of group \(n\)
- \(\Sigma_{sn}\) = Average macroscopic scattering cross section of group \(n\)
- \(E_n\) = Average energy of group \(n\)
- \(\Delta E_n\) = Energy width of the \(n'\)th group.

By summing \(E_{s_{on}}\) over all different fine groups, then a value for \(E_{s_o}\) is obtained.

\[
E_{s_o} = \sum_n (E_{s_{on}})(\Delta E_n) \tag{3.76}
\]

### 3.5.2 Neutron Absorption

When a nucleus absorbs a neutron, the resulting compound nucleus will be in an excited energy state that is equal to the kinetic energy of the incident neutron, \(E_i\), plus the binding energy of the neutron in the compound, \(E_o\). The binding energy of the neutron or the separation energy \(S_n\), is given by:

\[
S_n = \left[ m^{A-1} \chi_{N-1} - m^{A} \chi_{N} + m \right] C^2 \tag{3.77}
\]
where:  

\[ m(^{Z}X_{N}) = \text{Mass of an atom with } Z \text{ proton and } N \text{ neutrons} \]

\[ m(^{Z-1}X_{N-1}) = \text{Mass of an atom with } Z \text{ proton and } N-1 \text{ neutrons} \]

\[ m_{n} = \text{Mass of a neutron} \]

\[ C = \text{Speed of light} \]

For deuteron:  

\[ ^{1}H + n \rightarrow ^{2}H \rightarrow ^{2}H + \gamma \]

\[ S_{n} = (1.007825 - 2.014102) \times (931.5 \text{ MeV/amu}) + 939.573 \text{ MeV} \]

\[ S_{n} = 2.225 \text{ MeV} \]

Now, let \( E_{is} \) be equal to the initial excited state of the compound nucleus, i.e \( E_{is} = E_{i} + E_{b} \), while \( E_{fs} \) is the final state of the nucleus after undergoing gamma decay. After decaying, the nucleus must have a recoil momentum, \( P_{r} \) and a corresponding recoil kinetic energy, \( E_{r} \), in order to conserve linear momentum. Applying conservation of total energy and momentum to the decay process gives:

\[ E_{is} = E_{fs} + E_{r} + E_{\gamma} \quad (3.78) \]

\[ 0 = P_{R} + P_{\gamma} \quad (3.79) \]

where \( E_{\gamma} \) and \( P_{\gamma} \) are the energy and momentum of the gamma ray. From (3.79), it is observed that the nucleus will recoil with a momentum equal and opposite to that of the gamma ray. Equation (3.78) can be rewritten as:
\[ E_{fs} - E_{fs} = E_y + E_r \]  

(3.80)

Since \[ E_y = C \cdot P \], \[ E_r = \frac{P_s^2}{2M} \] (nonrelativistic), and defining

\[ \Delta E = E_{fs} - E_r \]

Then

\[ E_y^2 + 2M \cdot C^2 \cdot E_y - 2M \cdot C^2 \cdot \Delta E = 0 \]

Which has the solution

\[ E_y = MC^2 \left( -1 \pm \sqrt{1 + \frac{2\Delta E}{MC^2}} \right) \]

Expanding the square root and keeping the first three terms:

\[ E_y = \Delta E - (\Delta E)^2 / 2MC^2 \]

\[ E_y = (E_i + E_b - E_{fs}) - (E_i + E_b - E_{fs})^2 / 2MC^2 \]

where:

- \( E_i \) = Kinetic energy of the incident neutron
- \( E_b \) = Binding energy of neutron in deuteron, 2.223 MeV
- \( E_{fs} \) = Final energy state of the deuteron
- \( MC^2 \) = Mass energy of the deuteron, 1876.28 MeV.

If the final energy state of the deuteron is the ground state, i.e. \[ E_{fs} = 0 \], then:

\[ E_y = (E_i + E_b) - (E_i + E_b)^2 / 2MC^2 \]  

(3.81)

By looking at equation (3.81), we can associate the second term on the right hand side as the energy of the recoiling nucleus. The same result can be obtained by substituting equation (3.81) into (3.78) and then solving for \( E_r \). So
when a nucleus undergoes gamma decay, it will recoil with a kinetic energy \( E_R \), given by:

\[
E_R = \frac{(\Delta E)^2}{2MC^2}
\]

\[
E_R = \frac{(E_{1e} - E_{re})^2}{2MC^2}
\]

If the decaying nucleus ends up in its ground state after undergoing gamma decay, \( E_{re} = 0 \), then:

\[
E_R = \frac{(E_{1e})^2}{2MC^2}
\]

\[
E_R = \frac{(E^2 - E_{fb})^2}{2MC^2}
\]

(3.82)

For the case of deuteron, equation (3.82) becomes

\[
E_R(\text{MeV}) = \frac{(E_i + 2.223)}{3752.56}
\]

(3.83)

The total energy deposited in the medium per unit volume, due to the deposition of the kinetic energy of the recoiling deuteron nucleus when it undergoes a gamma decay \( (E_{dm}) \) is given by

\[
E_{dn} = \int dE_i \phi(E_i) \Sigma_a(E_i) E_R(E_i)
\]

(3.84)

As in the case of equation (3.75), equation (3.84) can be written for each energy group as:

\[
E_{dn} = \frac{1}{\Delta E_{En}} \int dE_i \phi(E_i) \Sigma_a(E_i) E_R(E_i) = \phi \Sigma_a E_{Rn}
\]

(3.85)
Then the total energy deposited due to the recoil of the deuteron nucleus is found by summing equation (3.85) over all energy groups.

\[ E_{dr} = \sum_n (E_{dr_n}) (\Delta E_n) \]  

(3.86)

3.5.3 Gamma Ray Heating

Gamma heating of the cold source arises due to the gamma-proton interaction in the cold source and the subsequent energy deposition due to this interaction. Even though gamma ray heating can play an important role in heating the source, it was not given significant consideration in this report; rather, it was estimated by the following expression:

\[ T_v(0K) = 5 \times 10^{-14} \frac{KJ}{kg-n-cm^2} \left( \frac{1}{C_p} \right) \phi_{fluence} \]  

(3.87)

where \( C_p \) is the hydrogen specific heat and \( \phi_{fluence} \) is the neutron fluence. Equation (3.87) was obtained from an approximation that relates the gamma dose to the neutron fluence. This is expected to provide an upper bound for gamma doses within the TRIGA core (J. Williams-private communication to M. Chatila).
It will be mentioned that gamma ray heating can be measured by first using an ionization chamber to measure exposure rate to gamma rays, and then by using the appropriate mass absorption coefficient of the medium in question (Hydrogen), the energy deposition can be calculated.

3.6 Problem Modeling And Coupling Of Calculations

I. The reactor core was modelled as a cylindrical homogenous medium of height $H$ and radius $R$, made of Hydrogen, Oxygen, Zirconium, Uranium-235 and 238, with the fission sources being distributed according to the fundamental buckling mode (equation (3.45)). Leakage effects from the core were accounted for by the use of the geometrical buckling (equation (3.42)). By the use of equations (3.56) through (3.61), in the fast energy range, i.e., between 10.5 MeV to 1 eV, a 23-energy-group slowing down calculation was performed on the core to obtain the energy spectrum ($\phi_e(E)$), current density spectrum ($J_e(E)$), and the slowing down density spectrum ($q_e(E)$). The core energy spectrum was then used to generate fast group constants for the core.
\[
\Sigma_{C_{s_{1}}} = \frac{\int_{u_{1}}^{u_{2}} du \Sigma_{C_{c}}(u) \phi_{c}(u)}{\int_{u_{1}}^{u_{2}} du \phi_{c}(u)} = \sum_{i=1}^{23} \Delta u_{n} \Sigma_{C_{c_{n}}} \phi_{n} / \sum_{i=1}^{23} \Delta u_{n} \phi_{n}
\]

\[
D_{C_{1}} = \frac{\int_{u_{1}}^{u_{2}} du \frac{1}{3 \Sigma_{C_{tr}}} \phi_{c}(u)}{\int_{u_{1}}^{u_{2}} du \phi_{c}(u)} = \sum_{i=1}^{23} \Delta u_{n} \frac{1}{3 \Sigma_{C_{tr_{n}}}} \phi_{n} / \sum_{i=1}^{23} \Delta u_{n} \phi_{n}
\]

\[
\Sigma_{C_{s_{11}}} = \Sigma_{C_{s_{2}}} - \Sigma_{C_{s_{12}}}
\]

\[
\Sigma_{C_{s_{12}}} = \Sigma_{C_{s_{1}}} + \sum_{j=1}^{N} j \Sigma_{s_{12}}
\]

where

\[
\frac{\int_{E_{1}}^{E_{0}} \int_{E_{1}}^{E_{0}} dE \frac{dE'}{E'} \Sigma_{s}(E') \phi(E')}{\int_{E_{1}}^{E_{0}} dE' \phi(E')} = (E_{0} - E_{1}) \sum_{i=1}^{23} \frac{\Delta E_{n} \Sigma_{s_{n}} \phi_{n}}{\Sigma_{E_{n}}} / \sum_{i=1}^{23} \Delta E_{n} \phi_{n}
\]
II. The neutron cold source was modelled as spherically-shaped Hydrogen medium of radius $R$ at a temperature of 20 K. Since the mean free path of fast neutrons would be comparable to or even larger than the size of the source, it was assumed that in the fast energy range, the cold source spectrum was equal to the core spectrum. The fast energy spectrum of the core was then used to generate fast group constants for the cold source.
III. Using a monoenergetic neutron source (neutrons with energies that fall in the top thermal group (n= 24)) with a magnitude equal to the core slowing down density at 1 eV, an 18-energy-groups slowing down calculation in the cold source
in the energy range from 1 eV to 0.01 eV was performed to find the energy spectrum, current density, and the slowing down densities (equations (3.66) through (3.69) for the cold source. This energy spectrum was then used to find the single "thermal" group (0.01 < E < 1) constants.

IV. In the energy region between 0.01 and 0.001 eV, the Maxwellian spectrum (T = 20 °K) was used to generate cold group constants for the cold source. A procedure similar to step II was followed except that in this energy range, the differential scattering cross section is given by:

\[
\sigma_{\delta}(E_i \rightarrow E_f) = \frac{\sigma_{\delta}}{E_i} \text{erf} \sqrt{\frac{E_f}{kT}}, \quad E_f \leq E_i
\]

V. The group constants that were found in the previous steps, were then utilized in the diffusion equation to couple the core and the cold source calculations. In these calculations, the cold source is modeled to be embedded in the center of an infinite core. For the cold source, three groups: fast, thermal and cold are considered, while for the core, only the fast and the thermal groups are considered.
The model utilized the following boundary conditions:

A) Continuity of the fast and the thermal fluxes and the current densities at the core-cold source boundary.

1. \( \phi_{s1}(R) = \phi_{c1}(R) \)
2. \( J_{s1}(R) = J_{c1}(R) \)
3. \( \phi_{s2}(R) = \phi_{c2}(R) \)
4. \( J_{s2}(R) = J_{c2}(R) \)

B) Flux must be nonnegative, real and finite.

5. \( \phi_{c1}(r), \phi_{c2}(r) < \infty \) as \( r \to \infty \)

C) Due to symmetry, the cold source current densities must be equal to zero at the center of the source.

6. \( J_{s1}(0) = J_{s2}(0) = J_{s3}(0) = 0 \)

D) The inward directed partial cold current density for the cold source is zero (Black boundary condition).

7. \( -J_{s3}(R) = 0 \) \quad \left( -J(r) = \frac{1}{4} \phi(r) + \frac{D}{2} \frac{d\phi}{dr} \right) \)

Solving the multigroup diffusion equation (3.72) subjected to the above boundary conditions yields:

\[
\phi_{s1}(r) = G_1 \frac{1}{r} \sinh \left( \frac{r}{L_{s1}} \right) \quad 0 < r < R \quad (3.89)
\]

\[
\phi_{s2}(r) = G_2 \frac{1}{r} \sinh \left( \frac{r}{L_{s2}} \right) - G_3 \frac{L_{s12}}{L_{s1}} \sinh \left( \frac{r}{L_{s1}} \right) \quad 0 < r < R \quad (3.90)
\]
\[ \phi_s(r) = G_3(r) \frac{1}{r} \sinh \left( \frac{r}{L_{s_3}} \right) + G_4 \frac{1}{r} \sinh \left( \frac{r}{L_{s_1}} \right) + G_5 \frac{1}{r} \sinh \left( \frac{r}{L_{s_2}} \right) \quad 0 < r < R \quad (3.91) \]

\[ \phi_{c1}(r) = G_6 \frac{e^{-r/L_{c1}}}{r} + \frac{S_c}{L_{c_1}} \quad R < r < \infty \]

\[ \phi_{c2}(r) = G_7 \frac{1}{r} e^{-r/L_{c2}} - G_6 \frac{\Sigma c_{s12} \frac{e^{-r/L_{c1}}}{r}}{R D_{c2} \left[ \frac{1}{L_{c_1}^2} - \frac{1}{L_{c_2}^2} \right]} + \frac{S_c \Sigma c_{s12}}{D_{c2} \Sigma c_{s_1}} \quad R < r < \infty \]

where

\[ G_1 = \frac{Z_2 Z_{10}}{Z_1 Z_{10} - Z_2 Z_9} , \quad G_2 = G_7 \frac{Z_{13}}{Z_{11}} - Z_{21} , \quad G_3 = G_7 \frac{Z_{18}}{Z_{15}} - G_1 \frac{Z_{16}}{Z_{15}} \]

\[ G_4 = G_1 \frac{Z_{16}}{Z_{15}} , \quad G_5 = -G_2 \frac{Z_{18}}{Z_{15}} , \quad G_6 = \frac{Z_3 Z_9}{Z_1 Z_{10} - Z_2 Z_9} , \quad G_7 = \frac{Z_3 Z_{11}}{Z_1 Z_4 - Z_2 Z_6} \]

\[ Z_1 = \frac{1}{R} \sinh \left( \frac{R}{L_{s_1}} \right) , \quad Z_2 = \frac{1}{R} \left( e^{-R/L_{s_1}} \right) , \quad Z_3 = \frac{S_c}{\Sigma c_{s_1}} \]

\[ Z_4 = \frac{1}{R} \sinh \left( \frac{R}{L_{s_2}} \right) , \quad Z_5 = \frac{\Sigma c_{s12}}{R D_{s_4} \left[ \frac{1}{L_{s_1}^2} - \frac{1}{L_{s_2}^2} \right]} \sinh \left( \frac{R}{L_{s_1}} \right) \]
\[ Z_6 = \frac{e^{-R/L_{c_2}}}{R}, \quad Z_7 = \frac{\Sigma_{c_{s12}} e^{-R/L_{c_1}}}{RDc_2 \left( \frac{1}{L_{c_1}} - \frac{1}{L_{c_2}} \right)}, \quad Z_8 = \frac{\Sigma_{c_{s11}} L_{c_2}}{\Sigma_{c_{s1}} D_{c_2}} \]

\[ Z_9 = \frac{D_{s_1}}{R^2} \sinh \left( \frac{R}{L_{s_1}} \right) - \frac{D_{s_2}}{RL_{s_1}} \cosh \left( \frac{R}{L_{s_1}} \right), \quad Z_{10} = \frac{D_{c_1}}{R} \left( \frac{1}{R} + \frac{1}{L_{c_1}} \right) e^{(R/L_{c_1})} \]

\[ Z_{11} = \frac{D_{s_2}}{R^2} \sinh \left( \frac{R}{L_{s_1}} \right) - \frac{D_{s_1}}{RL_{s_2}} \cosh \left( \frac{R}{L_{s_1}} \right) \]

\[ Z_{12} = \frac{\Sigma_{s_{s12}}}{\frac{R}{L_{s_1}} - \frac{1}{L_{s_1}^2}} \left( \frac{1}{L_{s_1}} \cosh \left( \frac{R}{L_{s_1}} \right) - \frac{1}{R} \sinh \left( \frac{R}{L_{s_1}} \right) \right) \]

\[ Z_{13} = \frac{D_{c_2} e^{-R/L_{c_1}}}{R^2} \left( 1 + \frac{R}{L_{c_1}} \right), \quad Z_{14} = \frac{\Sigma_{c_{s12}} e^{-R/L_{c_1}}}{R^2 \left( \frac{1}{L_{c_1}^2} - \frac{1}{L_{c_2}^2} \right)} \left( 1 + \frac{R}{L_{c_1}} \right) \]

\[ Z_{15} = \frac{1}{R} \sinh \left( \frac{R}{L_{s_1}} \right) \left( 1 - \frac{2D_{s_3}}{R} \right) + \frac{2D_{s_2}}{RL_{s_2}} \cosh \left( \frac{R}{L_{s_2}} \right) \]
\[
\frac{\left(\frac{\zeta^6 Z - \zeta^{-1} Z^0_t Z}{\zeta^6 Z + \zeta^{-1} Z^0_t Z}\right)}{\left(\frac{\zeta^6 Z - \zeta^{-1} Z^0_t Z}{\zeta^6 Z + \zeta^{-1} Z^0_t Z}\right)} = \zeta^2 Z
\]

\[
\left[\frac{\zeta^T}{R}\cos\frac{R s T}{s^2} + \left(\frac{R}{s^2} - 1\right)\left(\frac{\zeta^T}{R}\sin\frac{R}{s}\right)\right] = \zeta^6 Z
\]

\[
\frac{\left(\frac{\zeta^T}{s^2} - \frac{\zeta^T}{s^2}\right)}{\zeta^6 Z} = \zeta^2 Z
\]

\[
\left[\frac{\zeta^T}{R}\cos\frac{R s T}{s^2} + \left(\frac{R}{s^2} - 1\right)\left(\frac{\zeta^T}{R}\sin\frac{R}{s}\right)\right] = \zeta^6 Z
\]

\[
\left(\frac{\zeta^T}{1} - \frac{\zeta^T}{s}\right)\frac{ds}{\zeta^6 Z} = \zeta^2 Z
\]
A new value for the geometrical buckling for the thermal region of the cold source was calculated according to

$$B^2 = -\left(\frac{1}{\phi_s(r)} \nabla^2 \phi_s(r)\right)$$  \hspace{1cm} (3.92)

where $\phi_s$ is the slow flux in the cold source. Numerically, this was done by the use of finite difference forms for the derivatives

$$\frac{d^2 \phi}{dr^2}\bigg|_{r=r_i} = \frac{\phi_{i-1} - 2\phi_i + \phi_{i+1}}{\Delta^2}, \quad \frac{d\phi}{dr}\bigg|_{r=r_i} = \frac{\phi_{i+1} - \phi_i}{\Delta}$$  \hspace{1cm} (3.93)

where $\phi_i$ is the value for $\phi_s$ at $r = r_i$. With the application of equation (3.93), equation (3.92) becomes

$$B^2 = -\frac{3}{R^3} \sum_{i=1}^{N-1} \frac{1}{\phi_i} \left[ \phi_{i-1} \left(\frac{r_i^2}{\Delta}\right) - \phi_i \left(\frac{2r_i^2}{\Delta} + 2r_i\right) + \phi_{i+1} \left(2r_i + \frac{r_i^2}{\Delta}\right) \right]$$  \hspace{1cm} (3.94)

The boundary conditions were also satisfied by the use of the two point difference formula for the derivatives.

VI. Steps III, IV and V were then repeated until the difference between two consecutive iterations for the buckling satisfied a preset conditions ($< 10^{-3}$ cm$^{-1}$). Then, the actual energy spectrum of the cold source was used to calculate the energy deposition in the cold source according to equations (3.76), (3.86) and (3.87).
VII. The average cold flux and the total leakage of cold neutrons form the cold source were found from

\[ \langle \phi(r) \rangle = \frac{1}{V} \int dV \phi(r) = \frac{3}{R^3} \sum_{i=1}^{N-1} \phi_i r_i^2 \Delta r \]

\[ \text{TOTAL LEAKAGE} = 4\pi \int_0^R drr^2 (\nabla \cdot J) = 4\pi \sum_{i=0}^{N} (\nabla \cdot J)_i r_i^2 \Delta r \]

3.7 Computer Code DUNIA

To implement the model that was discussed in the previous section, a computer code that utilizes Microsoft FORTRAN as a language was written. The user has the choice to choose whether to use the University of Arizona TRIGA reactor data or his/her own. If the latter was chosen, then the user needs to enter the radius and height of the core, as well as the atomic densities of hydrogen, oxygen, zirconium, uranium-235 and 238. In either case, the user still needs to enter the radius of the cold source, the operating power and the number of intervals that would be used in discretizing the core and the cold source fluxes. Then, the computer code would read the various cross section
data from different data files and write the output into three different files. The three output files are: CSLDOP.DAT, SSLDOP.DAT and GFLUXS.DAT. The first two files consist of the core and the cold source energy spectrum respectively, while the third file contains the fast, "thermal" and cold fluxes of the cold source. A flow chart as well as a listing of the code is given in Appendix A.

3.8 Testing Of Code

The testing of the computer code was divided into 2 segments, one was concerned with the core and the second was concerned with the cold source. The core segment of the calculations was tested by comparing the neutron energy spectrum that was generated by the computer code (DUNIA) to the measured spectrum of the University Of Illinois Advanced TRIGA\textsuperscript{10} (figure 3.1). As can be seen in figure (3.1), the patterns of both spectra are extremely similar, and hence the computer code core generated spectrum is adequate to represent the core. Since there isn't a neutron spectrum for the cold source to compare to, the cold source spectrum was tested by assuming an infinite nonabsorbing medium. In that case, it was found that the slowing down density was constant and is equal to the source intensity, as it must be.
FIGURE 3.1: Reactor core neutron spectrum
NUMERICAL EXPERIMENTS AND RESULTS

4.1 Description Of Cases

The computer code was run for various cold neutron source dimensions. Special emphasis was made in comparing the average cold flux, total leakage of cold neutrons, current density of cold neutrons at the surface of the cold source, and the energy deposition in the cold source for different radius dimensions.

4.2 Input Data

The cross sections data for energies above 1 eV that were utilized in the program were obtained from Group Constants For Nuclear Reactor Calculations\[1\], which gives the average values of cross sections of various material in a 25 energy groups. Hydrogen cross sections data for energies below 1 eV were obtained from references 12 and 13. Hydrogen cross sections data below 1 eV were given as a point wise values for selected energies. Linear interpolation was carried out to find the values of the cross sections at the boundaries of the selected energy groups, then those values were averaged by using inverse energy as weight, in order to give an average value for the cross sections for each energy group. A listing of the cross sections data is giving in Appendix B in a tabulated form.
A standard TRIGA fuel element and coolant composition values for hydrogen, oxygen, zirconium, uranium-235 and uranium-238 atomic densities were used\textsuperscript{14}. These values are given in table 4.1. Table 4.2 gives the values of those atomic densities after taking into considerations that 33.3 volume % of a standard cell is water.

\begin{table}[h]
\centering
\caption{Standard TRIGA fuel and coolant composition}
\begin{tabular}{llll}
\hline
Fuel Region & Nuclei/cm\textsuperscript{3} & H\textsubscript{2}O & Nuclei/cm\textsuperscript{3} \\
\hline
U\textsuperscript{238} & 9.32x10\textsuperscript{20} & H & 6.68x10\textsuperscript{22} \\
U\textsuperscript{235} & 2.36x10\textsuperscript{20} & O & 3.34x10\textsuperscript{22} \\
Zr & 3.6x10\textsuperscript{22} & & \\
H & 5.94x10\textsuperscript{22} & & \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{Homogenized core composition}
\begin{tabular}{lll}
\hline
Species & Nuclei/cm/barn & \\
\hline
U\textsuperscript{238} & 6.213x10\textsuperscript{-04} & \\
U\textsuperscript{235} & 1.573x10\textsuperscript{-04} & \\
Zr & 0.024 & \\
O & 1.113x10\textsuperscript{-02} & \\
H & 6.187x10\textsuperscript{-02} & \\
\hline
\end{tabular}
\end{table}

For liquid Hydrogen at 20 degrees Kelvin and 1 atm, the physical density and the specific heat are equal to 71.1039 Kg/m\textsuperscript{3} and 12.6 KJ/Kg/°K.
4.3 Results And Discussion

The results are divided into two parts. The first part contains the raw outputs obtained by running the computer code for various cold source dimensions, while the second part consist of data analysis.

Part I. Results

Tables (4.3) through (4.12) give the cold source fast, thermal, and cold fluxes for different cold source dimensions at an operating power of 100 kW in the reactor core. The columns labeled FLUXS1, FLUXS2, and FLUXS3 are respectively the fast (10.5 MeV to 1.0 eV), thermal (1.0 to 0.01 ev), and cold (0.01 to 0.001 ev) fluxes. There are 26 entries in each of those tables because we chose to discretize equations (3.89), (3.90) and (3.91) over 25 intervals. The radial position from the center of the cold source is given by r, with the last entry for r being equal to the cold source radius.
TABLE 4.3: Cold source fast, thermal and cold fluxes at 100 kw with 0.5 cm radius

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AVERAGE COLD FLUX (n/cm^2 \cdot sec) = 5.821388E+10
TEMP. INCREASE DUE TO NEUTRONS (K/sec) = 1.794583E-02
TEMP. INCREASE DUE TO GAMMA (K/sec) = 3.853407E-03
TOTAL TEMPERATURE INCREASE (K/sec) = 2.179924E-02
POWER (KW) = 100.000000
TABLE 4.4: Cold source fast, thermal and cold fluxes at 100 kw with 1.0 cm radius

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LEAKAGE OF COLD NEUTRONS (n/sec)= 1.772118E+11
COLD CURRENT DENSITY (n/cm^2/sec)= 1.410207E+10
AVERAGE COLD FLUX (n/cm^2/sec)= 9.444623E+10
TEMP. INCREASE DUE TO NEUTRONS (K/sec)= 1.794583E-02
TEMP. INCREASE DUE TO GAMMA (K/sec)= 3.503167E-03
TOTAL TEMPERATURE INCREASE (K/sec)= 2.144900E-02
POWER (KW)= 100.000000
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<th>r(cm)</th>
<th>FLUXS1 (n/cm²·sec)</th>
<th>FLUXS2 (n/cm²·sec)</th>
<th>FLUXS3 (n/cm²·sec)</th>
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Leakage of Cold Neutrons (n/sec) = 5.139603E+11
Cold Current Density (n/cm²·sec) = 1.817763E+10
Average Cold Flux (n/cm²·sec) = 1.117124E+11
Temp. Increase due to Neutrons (K/sec) = 1.794583E-02
Temp. Increase due to Gamma (K/sec) = 3.174664E-03
Total Temperature Increase (K/sec) = 2.112049E-02
Power (KW) = 100.00000
TABLE 4.6: Cold source fast, thermal, and cold fluxes at 100 kw with 2.0 cm radius

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LEAKAGE OF COLD NEUTRONS (n/sec)= 1.069480E+12
COLD CURRENT DENSITY (n/cm^2/sec)= 2.127662E+10
AVERAGE COLD FLUX (n/cm^2/sec)= 1.181631E+11
TEMP. INCREASE DUE TO NEUTRONS (K/sec)= 1.794583E-02
TEMP. INCREASE DUE TO GAMMA (K/sec)= 2.901295E-03
TOTAL TEMPERATURE INCREASE (K/sec)= 2.084712E-02
POWER (KW)= 100.000000
TABLE 4.7: Cold source fast, thermal, and cold fluxes at 100 kw and 2.5 cm radius

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COLD CURRENT DENSITY (n/cm^2/sec) = 2.359565E+10
AVERAGE COLD FLUX (n/cm^2/sec) = 1.191788E+11
TEMP. INCREASE DUE TO NEUTRONS (K/sec) = 1.794583E-02
TEMP. INCREASE DUE TO GAMMA (K/sec) = 2.60553E-03
TOTAL TEMPERATURE INCREASE (K/sec) = 2.06238E-02
POWER (KW) = 100.000000
### TABLE 4.8: Cold source fast, thermal, and cold fluxes at 100 kW and 3.0 cm radius

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**LEAKAGE OF COLD NEUTRONS (n/sec)** = 2.865313E+12

**COLD CURRENT DENSITY (n/cm^2 \cdot sec)** = 2.533493E+10

**AVERAGE COLD FLUX (n/cm^2 \cdot sec)** = 1.177428E+11

**TEMP. INCREASE DUE TO NEUTRONS (K/sec)** = 1.794583E-02

**TEMP. INCREASE DUE TO GAMMA (K/sec)** = 2.500133E-03

**TOTAL TEMPERATURE INCREASE (K/sec)** = 2.044596E-02

**POWER (KW)** = 100.000000
TABLE 4.9: Cold source fast, thermal, and cold fluxes at 100 kw and 3.5 cm radius

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<th>FLUX3(n/cm²·sec)</th>
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AVERAGE COLD FLUX (n/cm²·sec) = 1.152134E+11
TEMP. INCREASE DUE TO NEUTRONS (K/sec) = 1.794583E-02
TEMP. INCREASE DUE TO GAMMA (K/sec) = 2.351607E-03
TOTAL TEMPERATURE INCREASE (K/sec) = 2.029743E-02
POWER (KW) = 100.000000
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AVERAGE COLD FLUX (n/cm^2·sec) = 1.122889E+11
TEMP. INCREASE DUE TO NEUTRONS (K/sec) = 1.794583E-02
TEMP. INCREASE DUE TO GAMMA (K/sec) = 2.226909E-03
TOTAL TEMPERATURE INCREASE (K/sec) = 2.017274E-02
POWER (KW) = 100.000000
TABLE 4.11: Cold source fast, thermal, and cold fluxes at 100 kw and 4.5 cm radius

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AVERAGE COLD FLUX (n/cm^2*sec) = 1.092918E+11
TEMP. INCREASE DUE TO NEUTRONS (K/sec) = 1.794583E-02
TEMP. INCREASE DUE TO GAMMA (K/sec) = 2.102207E-03
TOTAL TEMPERATURE INCREASE (K/sec) = 2.006604E-02
POWER (KW) = 100.000000
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Leakage of Cold Neutrons (n/sec) = 9.125785E+12
Cold Current Density (n/cm²·sec) = 2.904827E+10
Average Cold Flux (n/cm²·sec) = 1.063392E+11
Temp. Increase Due to Neutrons (K/sec) = 1.794583E-02
Temp. Increase Due to Gamma (K/sec) = 2.028562E-03
Total Temperature Increase (K/sec) = 1.997439E-02
Power (KW) = 100.000000
PART II. Data Analysis

Data analysis was centered around four significant quantities. These four quantities were: average cold flux, total leakage of cold neutrons, current density of cold neutrons at the cold source surface, and the heating of the cold source. The values of these quantities for various cold source radius dimensions at 100 kW are given in the previous tables. Figure (4.1) shows the result of graphing the average cold flux. As can be seen in the figure, the average cold flux reaches a maximum value for a cold source radius around 2.5 cm. The total leakage of cold neutrons from the cold source and the cold current density at the surface of the cold source versus different radius values are given in figures (4.2) and (4.3) respectively. The observation that can be made by examining those figures is that for a cold source radius less than 5 cm, total leakage, and the cold current density of cold neutrons at the surface increases with increasing radius. The cold fluxes for a cold source dimensions of 1 and 5 cm at 100 kw are given in figures (4.4) and (4.5). By comparing figures (4.4) and (4.5), or the above tables, we can conclude that as the radius of the cold source becomes larger, then as is expected, the cold flux in the cold source will not be overwhelmed entirely by the leakage process.
FIGURE 4.1: Average cold flux versus various cold source dimensions at 100 KW
FIGURE 4.2: Total leakage of cold neutrons versus various cold source dimensions at 100 kW
FIGURE 4.3: Cold current density at the surface versus various cold source dimensions at 100 KW
COLD FLUX AT 100 KW FOR 1 CM RADIUS

FIGURE 4.4: Cold flux at 100 kW for 1 cm radius cold source
FIGURE 4.5: Cold flux at 100 kW for 5 cm radius cold source
From tables (4.3) through (4.12) it is observed that heating of the source decreases slightly with the increase of the cold source radius. As was mentioned before, there are two components that contribute to the heating of the source. These two components are neutron-proton interactions and gamma ray heating. From the previous tables, we see that the heating due to neutron-proton interactions is independent of the cold source radius. This independence implies that fast neutrons dominate the heating process, since during modelling, we assumed that in the fast energy region, the cold source spectrum is equal to the core spectrum and hence it is independent of the cold source dimensions. This assumption was necessary in order to avoid negative spectrum values in the fast energy region of the cold source which otherwise would have arisen if we had attempted a 23-energy-group slowing down calculations as was done for the reactor core. This problem is due to the small size of the cold source that is being analyzed. From those tables, it is also observed that gamma ray heating decreases slightly with the increase of the radius. This decrease is due to the way that gamma ray heating was approximated, equation (3.87). The value that was used for the fluence in that equation was the average thermal fluence of the cold source, i.e. the average value of FLUXS2, which decreases with the radius.
CONCLUSIONS AND RECOMMENDATIONS

In figure (4.1), we saw that an optimum value for the cold flux is obtained for a cold source radius of 2.5 cm. If a cold neutron source is to be built for the University of Arizona TRIGA reactor, the maximum radius it could have, within the core, will be approximately 2 cm, which is the size of the fuel element inserts of the core. Even then, the cold flux will be 99% of the optimum value.

As was stated in the previous section, the temperature increase in the cold neutron source changed only slightly for various cold source dimensions. For an operating power of 100 kw, the total temperature increase was approximately 2.1E-02 °K/sec.

Even though the results in the previous section were carried out for an operating power of 100 kW, the same conclusions can be reached for any other power since the nature of the equations used in this model are linear. As an example, the biggest pulse that can be generated by the University of Arizona TRIGA reactor is 20 MJ. Then, for a 2 cm radius cold source, the average cold fluence would be 2.36E13 n/cm², the cold current at the surface would be 4.26E12 n/cm², while the temperature increase would be equal to 4.2 °K. So even for the biggest pulse, no significant heating occurs and hence a transfer system can be used.
For more accurate results, especially concerning energy depositions, a fast energy spectrum for the cold source must be generated. In order to avoid the problems that were discussed in the previous section, finer energy intervals must be used.

In conclusion, we see that the results that are obtained are highly motivating and reassuring with respect to the objective of this work, and strongly suggest that it will be worthwhile to carry on a more accurate and detailed analysis such as Monte-Carlo Calculations.
APPENDIX A

This appendix contains a flow chart and the listing of the computer code DUNIA.
READ INPUT DATA

23-ENERGY GROUPS SLOWING DOWN
CALCULATION (P1 EQUATIONS) TO
GENERATE CORE/SOURCE FAST SPECTRUM

18-ENERGY-GROUPS SLOWING DOWN
CALCULATION (G.G. APPROXIMATION) TO
GENERATE SOURCE "THERMAL" SPECTRUM

MAXWELLIAN SPECTRUM
AT 20 K AS SOURCE COLD SPECTRUM

CROSS SECTIONS COLLAPSING
INTO FEW GROUPS CONSTANTS

FAST AND THERMAL FLUXES FOR CORE
FAST, THERMAL AND COLD FLUXES
FOR SOURCE

BUCKLING CONVERGE ?

NO

YES

ENERGY DEPOSITION

END

FIGURE A.1: Flow chart for the computer code DUNIA
PROGRAM DUNIA

***THIS PROGRAM UTILIZES THE ENERGY DEPENDENT P1 EQUATIONS TO SOLVE***
***FOR THE ENERGY SPECTRUMS, CURRENT DENSITIES, AND THE SLOWING DOWN***
***DENSITIES OF A SPHERICALLY SHAPED HYDROGEN MEDIUM EMBEDDED IN THE **
***CENTER OF A CYLINDRICALLY SHAPED HOMOGENEOUS CORE MADE OF HYDROGEN, **
***OXYGEN, ZIRCONIUM, URANIUM-235 AND URANIUM-238. THE ENERGY SPEC-**
***TRUM IS THEN UTILIZED TO GENERATE FEW GROUP CONSTANTS, WHICH ARE **
***THAN USED IN TWO REGIONS, THREE GROUPS DIFFUSION EQUATION. FINALLY* **
***ENERGY DEPOSITION IN THE "COLD SOURCE" DUE TO NEUTRON-PROTON** *
*******INTERACTIONS, AND GAMMA HEATING IS CALCULATED. ***************

************************************************************************
************************************************************************
************************************************************************

************************** *DOCUMENTATIONS* **************************
************************************************************************

* H= HYDROGEN, O= OXYGEN, Z= ZIRCONIUM, U5= URANIUM-235, U8= URANIUM-238.
*IICS(I)= MICROSCOPIC TOTAL CROSS SECTION OF THE i’TH SPECIE IN THE *
*I’TH ENERGY GROUP.
* iSCS(I)= MICROSCOPIC SCATTERING CROSS SECTION OF THE i’TH SPECIE.
* iCCS(I)= MICROSCOPIC CAPTURE CROSS SECTION OF THE i’TH SPECIE.
* iMTCS(I)= MACROSCOPIC TOTAL CROSS SECTION OF THE i’TH SPECIE.
* iIS(I)= MICROSCOPIC INELASTIC SCATTERING CROSS SECTION OF THE i’TH *
* SPECIE.
* IMIS(I)= MACROSCOPIC ............
* MTCS(I)= MACROSCOPIC TOTAL CROSS SECTION OF THE CORE.
* MSCS(I)= MACROSCOPIC SCATTERING CROSS SECTION OF THE CORE.
* MNECS(I)= MACROSCOPIC NONELASTIC CROSS SECTION OF THE CORE.
* MTRCS(I)= MACROSCOPIC TRANSPORT CROSS SECTION OF THE CORE.
* MTCSC1= MACROSCOPIC TOTAL CROSS SECTION OF THE CORE IN THE FAST *
* ENERGY REGION (10.5 MeV TO 1.0 eV)
* MSC12= MACROSCOPIC TRANSPORT CROSS SECTION OF THE CORE FROM THE *
* FAST TO THE THERMAL ENERGY GROUP (BELOW 1 eV).
* MACSC2= MACROSCOPIC ABSORPTION CROSS SECTION OF THE CORE FOR THE *
* THERMAL ENERGY GROUP.
* MTCSS1= MACROSCOPIC TOTAL CROSS SECTION FOR THE COLD SOURCE IN THE *
* FAST ENERGY GROUP (10.5 MeV TO 1.0 eV).
* MRCSS2= MACROSCOPIC REMOVAL CROSS SECTION FOR THE COLD SOURCE IN *
* THE THERMAL ENERGY GROUP (1.0 eV TO 0.01 eV).
* MOSS23= MACROSCOPIC TRANSFER CROSS SECTION FROM THE SOURCE FROM *
* THERMAL TO COLD (0.01 TO 0.001 eV) ENERGY GROUP DUE TO *
* ELASTIC ISOTROPIC SCATTERING.
* MOSIl2= MACROSCOPIC TRANSFER CROSS SECTION FROM FAST TO THERMAL *
* ENERGY GROUP AS A RESULT OF NEUTRON INTERACTION WITH THE *
* i’TH SPECIE.
* DC1= DIFUSION COEFFICIENT OF THE CORE IN THE FAST ENERGY REGION.
* LS2= DIFUSION LENGTH OF THE SOURCE IN THE THERMAL ENERGY REGION.
* SHAD= ATOMIC DENSITY OF THE HYDROGEN IN THE COLD SOURCE.
* iAD= ATOMIC DENSITY OF THE i’TH SPECIE.
* ISANG(I)= AVERAGE COSINE OF THE SCATTERING ANGLE OF THE i’TH ENERGY *
* GROUP, IN THE LAB SYSTEM, ASSUMING LINEARLY ANISOTROPIC *
* SCATTERING IN THE CENTER OF MASS SYSTEM.
* DELU(I)= LETHARGY INTERVAL WIDTH OF THE i’TH GROUP.
* DELE(I)= ENERGY INTERVAL WIDTH OF THE i’TH GROUP.
AVERAGE ENERGY OF THE I'TH GROUP.

CONSTANT FOR THE I'TH SPECIE GIVEN BY EQUATION

RADIUS OF THE CORE.

HEIGHT OF THE CORE.

RADIUS OF THE COLD SOURCE.

NUMBER OF INTERVALS THAT ARE USED IN DISCRETIZING THE SPATIAL FLUX OF THE CORE.

CORE BUCKLING IN THE RADIAL DIRECTION.

BUKLING OF THE COLD SOURCE.

OPERATING POWER OF THE CORE.

FISSION SPECTRUM OF THE I'TH GROUP.

NEUTRON SOURCE FROM FISSION INTO THE I'TH GROUP.

E0, E1, E2 AND E3 ARE ENERGY BOUNDARIES. E0= 10.5 MeV, E1= 1.0 eV
E2= 0.01 eV, E3= 0.001 eV.

ELOSSO(I) = AVERAGE ENERGY LOSS BY A NEUTRON WHEN IT UNDERGO ELASTIC ISOTROPIC SCATTERING WITH HYDROGEN IN THE I'TH GROUP.

ER(I) = RECOIL ENERGY OF DEUTERIUM AFTER IT UNDERGO GAMMA DECAY IN THE I'TH ENERGY GROUP.

ESO(I) = ENERGY DEPOSITION IN THE COLD SOURCE, IN THE I'TH GROUP, DUE TO ELASTIC ISOTROPIC SCATTERING.

ESO = TOTAL ENERGY DEPOSITION IN THE COLD SOURCE DUE TO ELASTIC ISOTROPIC SCATTERING.

FLUXU(I) = LETHARGY SPECTRUM OF THE CORE IN THE I'TH GROUP (FLUX(U)).

CD(I) = CURRENT DENSITY OF THE CORE IN THE I'TH GROUP (J(U)).

iQ0(I) = SLOWING DOWN DENSITY INTO THE I'TH GROUP DUE TO ELASTIC ISOTROPIC SCATTERING WITH THE I'TH ELEMENT.

INELST(I) = TRANSFER CROSS SECTION INTO THE I'TH GROUP DUE TO INELASTIC SCATTERING.

SFLUX(I) = LETHARGY SPECTRUM OF THE COLD SOURCE IN THE I'TH GROUP.

FLUXS1 = SPATIAL FLUX OF THE COLD SOURCE IN THE FAST REGION.

FLUXC2 = SPATIAL FLUX OF THE CORE IN THE THERMAL REGION.

********************************************************************

***********DECLARATIONS***********

REAL HTCS(55), HSCS(55), HCCS(55), OTCS(25), OSCS(25), ZTCS(25)
REAL U5CS(25), U5SCS(25), U6CS(25), U6SCS(25), U7CS(25), U7SCS(25)
REAL HMTCS(55), HMCS(55), HMCCS(55), OMTC5(25), OMCS(25)
REAL ZMTCS(25), ZMC5(25), ZMCS(25), ZMCC5(25)
REAL U8MTCS(0:25), U8MCS(0:25), U8MC5(25), U8MCC5(25)
REAL O1S(25,25), Z1S(25,25), U1S(25,25), U6S(25,25)
REAL O1MIS(25,25), Z1MIS(25,25), U1MIS(25,25), U6MIS(25,25)
REAL MTC5, M5CS(25), M5CC5, M5CS(25), M5CC5
REAL MTCC5, MTC5S2, MTCSS3, MRCS31, MRCS32, MRCS33, MRCS34
REAL MTRCC2, MAC5C2, MOSS11, MOSS22, MOSS33, MOSS12, MOSS23
REAL MOSS12, MOSS12, MOSS12, MOSS12, MOSS12, MOSS11
REAL MOSS812, MOSS11, MOSS12, MOSS13, MOSS34, DC1, DC2, DS1
REAL DS2, DS3, LC1, LC2, LS1, LS2, LS3, SHAD, HAD, OAD, ZAD, U5AD
REAL U8AD, HSANG(55), OSANG(25), ZSANG(25), U5SANG(25)
REAL U5SANG(25), HMTCS(55), BELS(55), DELE(55), AGE(55), AAO
REAL AA2, AAS, AAB, ABG, ABZ, AB5, AB8, RADIUS, HEIGHT, TGAMMA
REAL AAD, ABD, ACD, ADD, R, BZ, BR, INTS, INTC, RS(0:52), V
REAL RC(0:52), SO(25), FS(25), POWER, SS(24:40), E0, E1, E2, E3
REAL ELOSSO(45), HMNC5(25), Y1(25), Y2(25), Y3(25), Y4(25)
REAL Y5(25), Y6(25), Y7(25), Y8(25), Y9(25), Y10(25), CDS3SA
REAL NN, EN1, EN2, EN3, ALPHAO, ALPHAZ, ALPHA5, ALPHA8
REAL EN4, EN5, EN6, EN7, EN8, EN9, EN12, EN13, EN14, EN15
REAL EN16, EN17, EN18, DEN1, DEN3, EDRN(45), S1, TEMPIN
REAL ER(45), ES0, EDR, TED, ESUN(45), Z1, Z2, Z3, Z4, Z5, DEN4
REAL Z6, Z7, Z8, Z9, Z10, Z11, Z12, Z13, Z14, Z15, Z16, Z17, Z18
REAL Z19, Z20, Z21, Z22, Z23, Z24, Z25, Z26, SUM2, TNEUTRN, CP
REAL FLUXS1(0:55), FLUXS2(0:55), FLUXS3(0:55), FLUXCL(55), AVEFS3
REAL FLUXC2(55), INELST(26), INL(26), AFLUX2, PI, TLEKG3
REAL FLUXU(0:25), HQ0(0:25), OQ0(0:25), ZQ0(0:25), U5Q0(0:25)
REAL USQ0(0:25), FLUXE(0:25), GRADJ3(0:55), VS, DENSITY
REAL EN21, EN22, EN23, EN24, DEN5, M(42:50)
COMPLEX SFLUXU(45), SFLUXE(45)
COMPLEX XI(24:45), X2(24:45), X3(24:45), X4(24:45), X5(24:45)
COMPLEX X6(24:45), X7(24:45), X8(24:45), X9(24:45), X10(24:45)
COMPLEX CD(0:25), HQ1(0:25), OQ1(0:25), ZQ1(0:25), U5Q1(0:25)
COMPLEX U8Q1(0:25), B22, B2, SCD(23:45)
COMPLEX SQO(23:45), SQ1(23:45)
INTEGER I, J, JJ, K, NINTS, NINTC, XXX

************************************************************************
*  *
************************************************************************
********THIS SEGMENT READ THE REQUIRED DATA FROM DATA FILES, THAN*******
********USES THOSE DATA IN MAKING ARRAYS FOR THE VARIOUS CONSTANTS******
************************************************************************
*
WRITE (*,*) 'ENTER 1 TO USE U OF A TRIGA DATA, 0 OTHERWISE'
READ (*,*) XXX
IF (XXX.EQ.1) THEN
   HAD= 0.06195855
   OAD= 0.01154805
   ZAD= 0.023553
   U5AD= 1.54403E-04
   U8AD= 6.09761E-04
   RADIUS= 22.86
   HEIGHT= 38.1
ELSE
   WRITE (*,*) 'ENTER THE VALUES FOR HYDROGEN, OXYGEN, ZIRCONIUM,'
   WRITE (*,*) 'U235 AND U238 ATOMIC DENSITIES IN THE CORE'
   READ (*,*) HAD, OAD, ZAD, U5AD, U8AD
   WRITE (*,*) 'ENTER RADIUS AND HEIGHT OF THE CORE IN CM'
   READ (*,*) RADIUS, HEIGHT
ENDIF
WRITE (*,*) 'ENTER RADIUS OF THE COLD SOURCE'
READ (*,*) R
WRITE (*,*) 'ENTER THE NUMBER OF INTERVALS TO BE USED IN'
WRITE (*,*) 'DISCRETIZING THE SPATIAL FLUX OF THE CORE AND'
WRITE (*,*) 'THE COLD SOURCE (MAXIMUM OF 50 EACH)'
READ (*,*) NINTC, NINTS
WRITE (*,*) 'ENTER THE OPERATING POWER IN KW'
READ (*,*) POWER

* 
PI= 3.141592654
SHAD= 0.04253
B2= (PI/(HEIGHT+12.0))
BR= (2.405/(RADIUS+7.14))
INTS= R/NINTS
INTC= R*20.0/NINTC
V= (3.1416*RADIUS**2.0)*HEIGHT
VS= 4.0/3.0*PI*(R/100)**3

* 

KK= 0
JJ= 0

* 

DO 5 I=1,23
   DO 7 J=1,23
      OMIS(I,J)= 0.0
      ZMIS(I,J)= 0.0
      USMIS(I,J)= 0.0
    7 CONTINUE
  5 CONTINUE

***********

OPEN (UNIT=1000, FILE='H1DATA.DAT', STATUS='OLD')
DO 10 1=1,23
   READ (1000,*) HTCS(I), HCCS(I), HSCS(I), DELU(I), AGE(I), DELE(I)
   HMTCS(I)= SHA*HTCS(I)
   HMCCS(I)= SHA*HCCS(I)
   HMSCS(I)= SHA*HSCS(I)
   HCMCS(I)= HAD*HTCS(I)
   HCMCCS(I)= HAD*HCCS(I)
   HCMSCS(I)= HAD*HSCS(I)
10 CONTINUE
CLOSE (UNIT=1000)

* 

OPEN (UNIT=1500, FILE='H220K.DAT', STATUS='OLD')
DO 15 I=24,50
   READ (1500,*) HTCS(I), HCCS(I), HSCS(I), DELU(I), AGE(I), DELE(I)
   HMTCS(I)= SHA*HTCS(I)
   HMSCS(I)= SHA*HSCS(I)
   HMCCS(I)= SHA*HCCS(I)
15 CONTINUE
CLOSE (UNIT=1500)

***********

OPEN (UNIT=2000, FILE='ODATA.DAT', STATUS='OLD')
DO 20 I=1,23
   READ (2000,*) OTCS(I), OSCS(I), OSANG(I)
   OMTCS(I)= OAD*OTCS(I)
   OMSCS(I)= OAD*OSCS(I)
20 CONTINUE
CLOSE (UNIT=2000)

* 

OPEN (UNIT=2100, FILE='OIND.DAT', STATUS='OLD')
READ (2100,*) (OIS(I,J), J=1,7)
DO 25 J=1,7
   OMIS(1,J)= OAD*OIS(1,J)
25 CONTINUE
CLOSE (UNIT=2100)

***********

OPEN (UNIT=3000, FILE='ZRDATA.DAT', STATUS='OLD')
DO 30 I=1,23
   READ (3000,*) ZTCS(I), ZSCS(I), ZSANG(I)
   ZMTCS(I)= ZAD*ZTCS(I)
   ZMSCS(I)= ZAD*ZSCS(I)
30 CONTINUE
CLOSE (UNIT=3000)

* 

OPEN (UNIT=3100, FILE='ZRIND.DAT', STATUS='OLD')
K = 0
DO 40 I=1,5
    READ (3100,*) (ZIS(I,J), J=K+1,K+9)
    DO 45 J=K+1,K+9
        ZMIS(I,J) = ZAD*ZIS(I,J)
    45 CONTINUE
    K = K+1
40 CONTINUE
CLOSE (UNIT=3100)
*************
OPEN (UNIT=5000, FILE='U5DATA.DAT', STATUS='OLD')
DO 70 I=1,23
    READ (5000,*) U5TCS(I), U5SCS(I), U5SANG(I)
    U5MTCS(I) = U5AD*U5TCS(I)
    U5MSCS(I) = U5AD*U5SCS(I)
70 CONTINUE
CLOSE (UNIT=5000)
*
OPEN (UNIT=5100, FILE='U5IND.DAT', STATUS='OLD')
K = 0
DO 80 I=1,10
    READ (5100,*) (U5IS(I,J), J=K+1,K+10)
    DO 85 J=K+1,K+10
        U5MIS(I,J) = U5AD*U5IS(I,J)
    85 CONTINUE
    K = K+1
80 CONTINUE
CLOSE (UNIT=5100)
*************
OPEN (UNIT=6000, FILE='U8DATA.DAT', STATUS='OLD')
DO 90 I=1,23
    READ (6000,*) U8TCS(I), U8SCS(I), U8SANG(I)
    U8MTCS(I) = U8AD*U8TCS(I)
    U8MSCS(I) = U8AD*U8SCS(I)
90 CONTINUE
CLOSE (UNIT=6000)
*
OPEN (UNIT=6500, FILE='U8IND.DAT', STATUS='OLD')
K = 0
DO 100 I=1,9
    READ (6500,*) (U8IS(I,J), J=K+1,K+10)
    DO 105 J=K+1,K+10
        U8MIS(I,J) = U8AD*U8IS(I,J)
    105 CONTINUE
    K = K+1
100 CONTINUE
CLOSE (UNIT=6500)
*
*************
DO 110 I=1,23
    HSANG(I) = 2.0/3.0
    MTCS(I) = HCMTCS(I)+OMTCS(I)+2MTCS(I)+U5MTCS(I)+U8MTCS(I)
    MSCS(I) = HCMSCS(I)+OMSCS(I)+2MSCS(I)+U5MSCS(I)+U8MSCS(I)
    MNECS(I) = MTCS(I)-MSCS(I)
    MTRCS(I) = MTCS(I)-HSANG(I)*HCMSCS(I)+OMSANG(I)*OMSCS(I)+ZSANG(I)*ZMSCS(I)
+ ZANG(I)*ZMSCS(I)+U5SANG(I)*U5MSCS(I)+U8SANG(I)*U8MSCS(I)
    HMTRCS(I) = HMTCSCS(I) = MTCS(I)-HSANG(I)*HCMSCS(I)
    HMNECS(I) = HMTCSCS(I)-HMSCS(I)
110 CONTINUE
*
DO 115 I=2,50
   HSANG(I)= 1.0/3.0+AGE(I)*0.09445
   HMTRCS(I)= HMTCS(I)-(HSANG(I)*HMSCS(I))
   HMNECS(I)= HMTCS(I)-HMSCS(I)
115 CONTINUE
*
************************************************************************
************************************************************************
***THIS SEGMENT CALCULATE THE SLOWING DOWN DENSITIES AND THE LETHARGY***
******SPECTRUM IN THE CORE FOR ENERGIES BETWEEN 10.5 MeV AND 1 eV******
************************************************************************
*
OMSCS(0)= 0.0
ZMSCS(0)= 0.0
U5MSCS(0)= 0.0
U8MSCS(0)= 0.0
AAO= 0.119945
AAZ= 0.02182
AAS= 0.00848789
AAB= 0.0083817
ABO= -0.0731566
ABZ= -0.014340
ABS= -0.005630928
AB= -0.005562573
DO 120 I=1,23
   Y1(I)= 2.0*HCMSCS(I)*DELU(I)/(2.0+DELU(I))
   Y2(I)= (DELU(I)-2.0)/(DELU(I)+2.0)
   Y3(I)= 8.0*HCMSCS(I)*DELU(I)/(9.0*DELU(I)+12.0)
   Y4(I)= (3.0*DELU(I)-4.0)/(3.0*DELU(I)+4.0)
   Y5(I)= +1.0/(DELU(I)*((BZ+BR)+MNNECS(I)*DELU(I)+(AAO*OMSCS(I)+
          AAZ*ZMSCS(I)+AAS*U5MSCS(I)+AAB*U8MSCS(I)))+Y1(I))
   Y6(I)= +1.0/(DELU(I)*(BZ+BR))
   Y7(I)= +2.0/((DELU(I)+2.0)*(BZ+BR))
   Y8(I)= +3.0/(DELU(I)*((BZ+BR)+(MT RCS(I)*DELU(I)+Y3(I)+(ABO*
          OMSCS(I)+ABZ*ZMSCS(I)+AB5*U5MSCS(I)+AB8*U8MSCS(I))))
   Y9(I)= +18.0/((3.0*DELU(I)+4.0)*(BZ+BR))
   Y10(I)= 1.0+Y5(I)*Y8(I)
   FS(I)= AGE(I)*0.453*EXP(-1.036*AGE(I))*SINH((2.29*AGE(I))**0.5)
   S0(I)= 3.12072850286E13*POWER(2.45*FS(I)/V
120 CONTINUE
**********
DATA HQO(0), HQ1(0), OQO(0), QO1(0), ZQO(0), ZQ1(0) /6*0.0/
DATA U5Q0(0), U5Q1(0), U8Q0(0), U8Q1(0), FLUXU(0), CD(0) /6*0.0/
DO 125 I=1,23
   INELST(I) = 0.0
125 CONTINUE
DO 130 I=1,23
   IF (1.0.E-2) THEN
      DO 133 J=1,I-1
         INL(J)= (OMIS(J, I)+ZMIS(J, I)+U5MIS(J, I)+U8MIS(J, I))*DELU(J)*
            FLUXU(J)
      ENDIF
   CONTINUE
ENDF
FLUXU(I)= +(Y8(I)/((BR+BL)*Y10(I)))*S0(I)+
         (Y8(I)*Y7(I)/Y10(I))*HQD(I-1)+(Y8(I)*Y6(I)/Y10(I))*
         (OQO(I-1)+ZQO(I-1)+U5QO(I-1)+U8QO(I-1))-(Y9(I)/Y10(I))*
         HQ1(I-1))CMPLX(0.0,1.0)-(3.0*Y6(I)/Y10(I))*OQ1(I-1)+
102

+ ZQ1(I-1)+U5Q1(I-1)+U8Q1(I-1))*CMPLX(0.0,1.0)+INELST(I)*
+ (Y8(I)/(BR+BZ)*Y10(I))
CD(I)= (Y5(I)*FLUXU(I)-Y7(I)*HQ0(I-1)-Y6(I)*(OQ0(I-1)+ZQ0(I-1)+
+ U5Q0(I-1)+U8Q0(I-1)-1.0/(BR+BZ)*S0(I)-1.0/(BR+BZ)*
+ INELST(I))*CMPLX(0.0,1.0)
HQ0(I)= Y1(I)*FLUXU(I)-Y2(I)*HQ0(I-1)
HQ1(I)= Y3(I)*CD(I)-Y4(I)*HQ1(I-1)
OQ0(I)= AAO*OMSCS(I)*FLUXU(I)
OQ1(I)= ABO*OMSCS(I)*CD(I)
ZQ0(I)= AAZ*ZMSCS(I)*FLUXU(I)
ZQ1(I)= ABZ*ZMSCS(I)*CD(I)
U5Q0(I)= AA5*U5MSCS(I)*FLUXU(I)
U5Q1(I)= AB5*U5MSCS(I)*CD(I)
U8Q0(I)= AA8*U8MSCS(I)*FLUXU(I)
U8Q1(I)= AB8*U8MSCS(I)*CD(I)
130 CONTINUE
*
DO 132 I=1,23
FLUXE(I)= FLUXU(I)/AGE(I)
132 CONTINUE
*
************************************************************************
************************************************************************
******THIS SEGMENT CALCULATE THE SLOWING DOWN DENSITIES, CURRENT******
*******DENSITY, AND THE LETHARGY SPECTRUM IN THE COLD SOURCE**********
************************************************************************
************************************************************************
B2= PI/R*CMPLX(0.0,1.0)

***************FAST ENERGY RANGE (10.5 MeV TO 1 eV)***************
***************THERMAL ENERGY RANGE (1 eV TO 0.01 eV)***************
SS(24) = HQO(23)+OQO(23)+ZQO(23)+U5QO(23)+U8QO(23)
SQ0(23)= 0.0
SQ1(23)= 0.0
DO 145 I=25,41
SS(I)= 0.0
145 CONTINUE
*
AAD= 0.7253488
ABD= -0.4236122
ACD= 0.5840117
ADD= -6.81027711
137 DO 150 I=24,41
X1(I)= 2.0*AAD*HMSCS(I)*DELU(I)/(2.0*ACD+DELU(I))
X2(I)= (2.0*ACD+DELU(I))/(2.0*ACD+DELU(I))
X3(I)= 2.0*ABD*HMSCS(I)*DELU(I)/(DELU(I)+2.0*ADD)
X4(I)= (2.0*ADD-DELU(I))/(2.0*ADD+DELU(I))
X5(I)= 3.0/(B2*DELU(I))*(HMTRCS(I)*DELU(I)+X3(I))
X6(I)= 6.0/(B2*(2.0*ADD+DELU(I)))
X7(I)= 1.0+X5(I)*HMNECS(I)/B2+X5(I)*X1(I)/(B2*DELU(I))
X8(I)= 2.0*X5(I)/(B2*(2.0*ACD+DELU(I)))
150 CONTINUE
$X9(I) = \frac{1.0}{B2*DELU(I)} \times (HMNECS(I)*DELU(I)+X1(I))$

$X10(I) = \frac{2.0}{B2*(2.0*ACD+DELU(I))}$

$SFLUXU(I) = \frac{X5(I)/(B2*X7(I))}{X7(I)}*SS(I)+\frac{X8(I)}{X7(I)}*SQ0(I-L)-X6(I)$

$\frac{1}{X7(I)}*SQ1(I-L)*CMPLX(0.0,1.0)$

$SCD(I) = (X9(I)\times SFLUXU(I)-SS(I)/B2-X10(I)*SQ0(I-L))$  

$SQ0(I-L) = X1(I)\times SFLUXU(I)+X2(I)\times SQ0(I-L)$  

$SQ1(I-L) = X3(I)\times SCD(I)+X4(I)\times SQ1(I-L)$

150 CONTINUE

*DO 153 I=24,41
SFLUXE(I) = SFLUXU(I)/AGE(I)
153 CONTINUE

************************************************************************

********THIS SEGMENT GENERATE A MAXWELLIAN SPECTRUM TO BE USED********
**********LATER ON IN GENERATING COLD GROUP CONSTANTS*****************
************************************************************************

NN= 1.0
DO 210 I=42,50
$M(I) = \frac{2.0*PI*NN}{(5.414220779E-09)^{3/2.0}} \times AGE(I)^{0.5}$

$+ \times EXP(-AGE(I)/1.7234E-09)$

210 CONTINUE

************************************************************************

******THIS SEGMENT CALCULATE FEW GROUP CONSTANTS FOR THE CORE AND******
*******THE COLD SOURCE TO BE USED IN THREE GROUPS, TWO REGIONS***********
************************************************************************

EN1= 0.0
EN2= 0.0
EN3= 0.0
EN4= 0.0
EN5= 0.0
EN6= 0.0
EN7= 0.0
EN8= 0.0
EN9= 0.0
EN10= 0.0
EN11= 0.0
EN12= 0.0
EN13= 0.0
EN14= 0.0
EN15= 0.0
EN16= 0.0
EN17= 0.0
EN18= 0.0
EN19= 0.0
EN20= 0.0
EN21= 0.0
EN22= 0.0
EN23= 0.0
EN24= 0.0
DEN1= 0.0
DEN2= 0.0
DEN3= 0.0
DEN4= 0.0
DEN5 = 0.0
E0 = 10.5
E1 = 1.0E-06
E2 = 1.0E-08
E3 = 1.0E-09
ALPHAO = 0.7785467
ALPHA2 = 0.9569943
ALPHA5 = 0.9831227
ALPHA8 = 0.9833336

* DO 200 I = 1, 23
  EN1 = DELE(I) * HMSCS(I) * SFLUXE(I) / AGE(I) + EN1
  EN18 = DELE(I) * HMSCS(I) * SFLUXE(I) + EN18
  EN2 = DELE(I) * HMTCS(I) * SFLUXE(I) + EN2
  EN3 = (DELE(I) * SFLUXE(I)) / (3.0 * HMTRCS(I)) + EN3
  EN4 = DELE(I) * MTCS(I) * FLUXE(I) + EN4
  EN5 = (FLUXE(I) * DELE(I)) / (3.0 * MTRCS(I)) + EN5
  EN6 = DELE(I) * OMSCS(I) * FLUXE(I) + EN6
  EN7 = DELE(I) * U5MSCS(I) * FLUXE(I) + EN7
  EN8 = DELE(I) * U8MSCS(I) * FLUXE(I) + EN8
  EN9 = DELE(I) * HCMSCS(I) * FLUXE(I) + EN9
  EN16 = DELE(I) * HMCS(I) * FLUXE(I) / AGE(I) + EN16
  EN17 = DELE(I) * HMCS(I) * FLUXE(I) + EN17
  DEN1 = SFLUXE(I) * DELE(I) + DEN1
  DEN4 = FLUXE(I) * DELE(I) + DEN4
200 CONTINUE

* DO 225 I = 24, 41
  EN21 = DELE(I) * HMTCS(I) * SFLUXE(I) + EN21
  EN22 = (DELE(I) * SFLUXE(I)) / (3.0 * HMTRCS(I)) + EN22
  EN23 = DELE(I) * HMSCS(I) * SFLUXE(I) / AGE(I) + EN23
  EN24 = DELE(I) * HMSCS(I) * SFLUXE(I) + EN24
  DEN5 = SFLUXE(I) * DELE(I) + DEN5
225 CONTINUE

* DO 220 I = 42, 50
  EN13 = DELE(I) * HMSCS(I) * M(I) + EN13
  EN14 = (DELE(I) * M(I)) / (3.0 * HMTRCS(I)) + EN14
  EN15 = DELE(I) * HMSCS(I) * M(I) + EN15
  EN12 = DELE(I) * HMSCS(I) * M(I) / AGE(I) + EN12
  DEN3 = M(I) * DELE(I) + DEN3
220 CONTINUE

* *************************************************COLD SOURCE (HYDROGEN) FAST GROUP CONSTANTS************************************************
* M0SS12 = (E1-E2) * EN1 / DEN1
M0SS13 = (E2-E3) * EN1 / DEN1
M0SS11 = EN18 / DEN1 - (M0SS12 + M0SS13)
MTCSS1 = EN2 / DEN1
MRCSS1 = MTCSS1 - M0SS11
DS1 = EN3 / DEN1
LS1 = (DS1 / MRCSS1) ** 0.5

* *************************************************COLD SOURCE (HYDROGEN) THERMAL GROUP CONSTANTS************************************************
* M0SS23 = (E2-E3) * EN23 / DEN5
M0SS22 = EN24 / DEN5 - M0SS23
MTCSS2 = EN21 / DEN5
MRCSS2 = MTCSS2 - M0SS22
DS2 = EN22 / DEN5
LS2 = (DS2/MRCSS2)**0.5

***********************COLD SOURCE (HYDROGEN) COLD GROUP CONSTANTS**********************

M0SS34 = 0.5536*E3*EN12/DEN3
M0SS33 = EN15/DEN3 - M0SS34
MTCSS3 = EN13/DEN3
MRCSS3 = MTCSS3 - M0SS33
DS3 = EN14/DEN3
LS3 = (DS3/MRCSS3)**0.5

***********************HYDROGEN (CORE) GROUP CONSTANTS***********************

M0SH12 = (E1-E2)*EN16/DEN4
M0SH11 = EN17/DEN4 - M0SH12

*************************OXYGEN GROUP CONSTANTS*************************

M0SO12 = E1*((E1/ALPHAO-E1)*OMSCS(23)*FLUXE(23)/((E1/ALPHAO+E1)/2.0 + ))/DEN4
M0SO11 = EN6/DEN4 - M0SO12

**************************ZIRCONIUM GROUP CONSTANTS************************

M0SZ12 = E1*((E1/ALPHAZ-E1)*ZMSCS(23)*FLUXE(23)/((E1/ALPHAZ+E1)/2.0 + ))/DEN4
M0SZ11 = EN7/DEN4 - M0SZ12

**************************URANIUM 235 GROUP CONSTANTS*************************

M0S512 = E1*((E1/ALPHA5-E1)*U5MSCS(23)*FLUXE(23)/((E1/ALPHA5+E1)/2.0 + ))/DEN4
M0S511 = EN8/DEN4 - M0S512

**************************URANIUM 238 GROUP CONSTANTS**************************

M0S812 = E1*((E1/ALPHA8-E1)*U8MSCS(23)*FLUXE(23)/((E1/ALPHA8+E1)/2.0 + ))/DEN4
M0S811 = EN9/DEN4 - M0S812

***********************CORE FAST GROUP CONSTANTS***********************

MTCSC1 = EN4/DEN4
DC1 = EN5/DEN4
MRCSC1 = MTCSC1-(MOSH11+MOSO11+MOSZ11+M0S511+M0S811)
LC1 = (DC1/MRCSC1)**0.5
MSC12 = MOSH12+MOSO12+MOSZ12+M0S512+M0S812
MACSC2 = 0.08674
MTRCC2 = 2.010
DC2 = 1.0/(3.0*MTRCC2)
LC2 = (DC2/MACSC2)**0.5

*************************************************************************

***THIS SEGMENT CALCULATE THE SPATIAL FAST AND THERMAL FLUXES IN THE***
***CORE, THE FAST, THERMAL AND COLD FLUXES IN THE HYDROGEN COLD SOURCE***

*************************************************************************


\[ S_1 = 3.12072850286E13 \times \text{POWER} \times 2.45/V \]

\[ Z_1 = 1.0/R \times \text{SINH}(R/LS1) \]

\[ Z_2 = \text{EXP}(-R/LC1)/R \]

\[ Z_3 = S_1/MRCSC1 \]

\[ Z_4 = 1.0/R \times \text{SINH}(R/LS2) \]

\[ Z_5 = \text{M0SS1} \times \text{SINH}(R/LS1)/(R \times DS2 \times (1.0/LS1**2 - 1.0/LS2**2)) \]

\[ Z_6 = \text{EXP}(-R/LC2)/R \]

\[ Z_7 = \text{MSC12} \times \text{EXP}(-R/LC1)/(R \times DC2 \times (1.0/LC1**2 - 1.0/LC2**2)) \]

\[ Z_8 = S_1 \times \text{MSC12} \times LC2**2/(MRCSC1 \times DC2) \]

\[ Z_9 = (DS1/R**2) \times \text{SINH}(R/LS1) - DS1/(R \times LS1) \times \text{COSH}(R/LS1) \]

\[ Z_{10} = DC1/R \times \text{EXP}(-R/LC1)/(1.0/R + 1.0/LC1) \]

\[ Z_{11} = DS2/(R \times (1.0/R \times \text{SINH}(R/LS2) - 1.0/LS2 \times \text{COSH}(R/LS2))) \]

\[ Z_{12} = \text{M0SS12}/(R \times (1.0/(LS1**2) - 1.0/(LS2**2))) \times (1.0/LS1* \text{COS}(R/LS1)) \]

\[ Z_{13} = DC2 \times \text{EXP}(-R/LC2)/(1.0/R + 1.0/LC2)/R**2 \]

\[ Z_{14} = \text{MSC12} \times \text{EXP}(-R/LC1)/(1.0/R + 1.0/LC1)/(R**2 \times (1.0/LC1**2 - 1.0/LC2**2)) \]

\[ Z_{15} = 1.0/R \times \text{SINH}(R/LS3)/(1.0 - 2.0*DS3/R) + 2.0*DS3/(R \times LS3) \times \text{COSH}(R/LS3) \]

\[ Z_{16} = \text{M0SS12} \times \text{M0SS23} / (DS2 \times DS3 \times (1.0/(LS1**2) - 1.0/(LS2**2)) \times (1.0/(LS3**2))) \]

\[ Z_{17} = Z_{16} \times (Z_{10} \times Z_{3} \times Z_{5} - Z_{3} \times Z_{7} \times Z_{9}) + Z_{4} \times (Z_{10} \times Z_{3} \times Z_{12} + Z_{14} \times Z_{3} \times Z_{9}) \]

\[ RS(0) = 0.0 \]

\[ RS(NINTS) = R \]

\[ \text{DO } 230 \ I = 1, \text{NINTS-1} \]

\[ RS(I) = RS(I-1) + \text{INTS} \]

\[ FLUXS1(I) = Z_{22}/RS(I) \times \text{SINH}(RS(I)/LS1) \]

\[ \text{DO } 240 \ I = 1, \text{NINTC} \]

\[ RC(I) = RC(I-1) + \text{INTC} \]

\[ FLUXC1(I) = Z_{23}/RC(I) \times \text{EXP}(-RC(I)/LC1) + S_1/MRCSC1 \]

\[ \text{CONTINUE} \]

\[ \text{END} \]
FLUXS1(0) = FLUXS1(1)
FLUXS1(NINTS) = FLUXS1(NINTS-1) * (DS1/INTS) / ((DS1/INTS) + (DC1/INTC))
+ FLUXC1(1) * (DC1/INTC) / ((DS1/INTS) + (DC1/INTC))
FLUXS2(0) = FLUXS2(1)
FLUXS2(NINTS) = FLUXS2(NINTS-1) * (DS2/INTS) / ((DS2/INTS) + (DC2/INTC))
+ FLUXC2(1) * (DC2/INTC) / ((DS2/INTS) + (DC2/INTC))
FLUXS3(0) = FLUXS3(1)
FLUXS3(NINTS) = FLUXS3(NINTS-1) * 2.0 * DS3 / (INTS + 2.0 * DS3)

*************** LEAKAGE AND AVERAGE FLUX OF COLD SOURCE ***********************

TLEKG3 = 0.0
DO 245 I = 1, NINTS
  GRADJ3(I) = Z26 * DS3 * (2.0 / (RS(I)**2 * LS3) * COSH(RS(I) / LS3) -
  (2.0 / RS(I)**3 + 1.0 / (RS(I) * LS3**2)) * SINH(RS(I) / LS3)) + Z16 * Z22 * DS3
  + Z18 * Z25 * DS3 * (2.0 / (RS(I)**2 * LS2) * COSH(RS(I) / LS2) -
  (2.0 / RS(I)**3 + 1.0 / (RS(I) * LS2**2)) * SINH(RS(I) / LS2))
  TLEKG3 = TLEKG3 + 4.0 * PI * GRADJ3(I) * RS(I)**2 * INTS
245 CONTINUE

CDS3SA = TLEKG3 / (4.0 * PI * R**2)

AVEFS3 = 0.0
DO 247 I = 0, NINTS
  AVEFS3 = AVEFS3 + 3.0 / R**3 * INTS * RS(I)**2 * FLUXS3(I)
247 CONTINUE

****************************************** NUMERICAL CALCULATIONS OF B^2 ***********************

SUM2 = 0.0
DO 250 I = 1, NINTS-1
  SUM2 = SUM2 + 1.0 / FLUXS2(I) * (FLUXS2(I-1) * (RS(I)**2 / INTS) -
  FLUXS2(I) * 2.0 * (RS(I)**2 / INTS + RS(I)) + FLUXS2(I+1) *
  (2.0 * RS(I) + RS(I)**2 / INTS))
250 CONTINUE

IF (SUM2 .LT. 0.0) THEN
  B22 = SQRT(-3.0 / R**3 * SUM2)
ELSE
  B22 = SQRT(3.0 / R**3 * SUM2) * CMPLX(0.0, 1.0)
ENDIF

IF (ABS(B22 - B2) .LT. 1.0E-03) THEN
  GO TO 255
ELSE
  B2 = B22
  JJ = JJ + 1
  GO TO 137
ENDIF

****************************************** THIS SEGMENT CALCULATE ENERGY DEPOSITION (MeV) IN THE COLD SOURCE ********

** THIS SEGMENT CALCULATE ENERGY DEPOSITION (MeV) IN THE COLD SOURCE********
*** DUE TO FAST (I=1,23) AND THERMAL (I=24,41) NEUTRONS. ALSO, IT ********
**** CALCULATE AN ESTIMATE VALUE FOR GAMMA HEATING *********************
DO 160 I=1,23
   ELOSSO(I) = 1.0/2.0*AGE(I)
   ER(I) = (AGE(I)+2.223)/3752.56
160 CONTINUE
DO 165 I=24,41
   ELOSSO(I) = (1.0-(1.0/3.0)**2)/2.0*AGE(I)
   ER(I) = (AGE(I)+2.223)/3752.56
165 CONTINUE
DO 170 I=1,41
   ES0(N)= SFLUXE(I)*HMSCS(I)*ELOSSO(I)
   EDRN(I)= SFLUXE(I)*HMCCS(I)*ER(I)
170 CONTINUE
ES0= 0.0
EDR= 0.0
DO 190 I=1,41
   ES0= ESO+ES0N(I)*DELE(I)
   EDR= EDR+EDRN(I)*DELE(I)
190 CONTINUE
TED= ESO+EDR
CP= 12.60
DENSTY= 71.1039
TNUTRN= TED/CP/DENSTY*1.602190E-10
* AFLUX2= 0.0
DO 285 I=0,NINTS
   AFLUX2= 3.0/R**3 *INTS*RS(I)**2*FLUXS2(I)+AFLUX2
285 CONTINUE
TGAMMA= (5.OE-14)/CP*AFLUX2
TEMPIN= TNUTRN+TGAMMA
* **************
OPEN (UNIT=7000, FILE='CSLDOP.DAT', STATUS='NEW')
WRITE (7000,*) ' I FLUX(U)'
DO 260 I=1,23
   WRITE (7000,*) I, FLUXU(I)
260 CONTINUE
WRITE (7000,*) 'SCS12 (b)=', MOSH12/HAD, ' SCS12 (b)='
WRITE (7000,*) 'SCS02 (b)='
WRITE (7000,*) 'SCS22 (b)='
WRITE (7000,*) 'SCS12 (b)='
WRITE (7000,*) 'SCS52 (b)=', M0S5212/USAD, ' SC52 (b)='
WRITE (7000,*) 'SCS82 (b)='
WRITE (7000,*) 'RADIUS (cm)=', RADIUS
WRITE (7000,*) 'HEIGHT (cm)='
WRITE (7000,*) 'POWER IN KW='
CLOSE (UNIT=7000)
* OPEN (UNIT=7500, FILE='SSLDP.DAT', STATUS='NEW')
WRITE (7500,*) ' I SFLUX(U)'
DO 270 I=1,41
   WRITE (7500,*) I, SFLUXU(I)
270 CONTINUE
WRITE (7500,*) 'B (1/cm)='
WRITE (7500,*) B22
WRITE (7500,*) 'DS1 (cm)=', DS1, ' LSI (cm)=', LS1
WRITE (7500,*) 'DS2 (cm)=', DS2, ' LS2 (cm)=', LS2
WRITE (7500,*) 'DS3 (cm)=', DS3, ' LS3 (cm)=', LS3
WRITE (7500,*) 'SCSS11 (b)=', MOSS11/SHAD, ' SCSS22=', MOSS22/SHAD
WRITE (7500,*) 'SCSS12 (b)=', MOSS12/SHAD, ' SCSS23=', MOSS23/SHAD
WRITE (7500,*) 'SCSS13 (b)=', MOSS13/SHAD, ' SCSS33=', MOSS33/SHAD
WRITE (7500,*) 'TEMP. INCREASE DUE TO NEUTRONS (K/sec)=', TNUTRN
WRITE (7500,*) 'TEMP. INCREASE DUE TO GAMMA (K/sec)=', TGAMMA
WRITE (7500,*) 'RADIUS IN CM=', R
CLOSE (UNIT=7500)

OPEN (UNIT=8000, FILE='GFLUXS.DAT', STATUS='NEW')
WRITE (8000,*) 'R FLUXS1 FLUXS2 FLUXS3'
DO 275 I=0, NINTS
    WRITE (8000,*) RS(I), FLUXSl(I), FLUXS2(I), FLUXS3(I)
275 CONTINUE
WRITE (8000,*) 'TOTAL LEAKAGE OF COLD NEUTRONS (n/sec)=', TLEKG3
WRITE (8000,*) 'COLD CURRENT DENSITY (n/cm^2/sec)=', CDS3SA
WRITE (8000,*) 'AVERAGE COLD FLUX (n/cm^2/sec)=', AVEFS3
WRITE (8000,*) 'TEMP. INCREASE DUE TO NEUTRONS (K/sec)=', TNUTRN
WRITE (8000,*) 'TEMP. INCREASE DUE TO GAMMA (K/sec)=', TGAMMA
WRITE (8000,*) 'TOTAL TEMPERATURE INCREASE (K/sec)=', TEMPIN
WRITE (8000,*) 'POWER (KW)=', POWER
CLOSE (UNIT=8000)

WRITE (*,*) 'NUMBER OF ITERATIONS:', JJ
END
This appendix contains the cross sections data for hydrogen, oxygen, zirconium, uranium-235 and uranium-238 that were used by the computer code DUNIA.
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**TABLE B.2:** Hydrogen total, capture, and scattering cross sections (b), lethargy interval, average energy (MeV) and energy interval of each group (from 1 to 0.001 ev).
### TABLE B.3: Oxygen total and elastic scattering cross sections (b)

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### TABLE B.4: Oxygen inelastic scattering cross sections (b) from one group to the others.

\[ \sigma_{in}(i, i+k) \] at \( k \) equal to

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**TABLE B.5:** Zirconium total and elastic scattering cross sections (b).

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**TABLE B.6:** Zirconium inelastic scattering cross sections (b) from one group to the others.

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TABLE B.7: Uranium-235 total and elastic scattering cross sections (b).

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TABLE B.8: Uranium-235 inelastic scattering cross sections (b) from one group to the others.

\[ \sigma_{in}(i,i+k) \] at k equal to

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TABLE B.9: Uranium-238 total and elastic scattering cross (b) sections.

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TABLE B.10: Uranium-238 inelastic scattering cross sections (b) from one group to the others. 

\[ \sigma_{in}(i,i+k) \text{ at } k \text{ equal to} \]

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REFERENCES


