HOW LOGICAL REASONING ABILITY AND EMPIRICAL KNOWLEDGE INTERACT IN THE PROCESS OF SOLVING PROBLEMS ABOUT LIGHT AND VISION AMONG TAIWANESE SECONDARY SCHOOL STUDENTS

by

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Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copy of the dissertation to the Graduate College.

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DEDICATION

This dissertation is dedicated to my family.

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ABSTRACT

Piagetian scholars argue that the effect of problem content, e.g., empirical knowledge, should decrease with age. Indeed, they believe that the empirical knowledge cannot affect human problem-solving after individuals approach the formal operation stage. In arguing this point, this study uses an A-AR model to address how empirical knowledge affects the problem-solving process among Taiwanese secondary students.

The A-AR model is borrowed from mathematics and the symbols, A, A, and R, represent Assumption, Answering, and Reasoning, respectively. Similar to solving mathematics problems, the A-AR model problems require participants to use the given assumptions by logical reasoning in order to respond to the problems. In this situation, the effect of empirical knowledge on problem-solving is easy to detect. There are three results about human problem-solving found in this study.

First, the empirical knowledge still affects human problem-solving at the formal operation stage. Not like the Piagetian scholars' assumption: the effect of empirical knowledge is decreasing with age, this study finds that the effect of empirical knowledge is S-shape. The S-shape is a result of academic training.

Second, the academic training, major, shapes human problem-solving strategies. For instance, the 12^{th} grade science students' problem-solving strategy is based on logical

reasoning ability by the given assumptions and the same grade social science students' strategy is according of their empirical knowledge.

Third, the interference of logical reasoning ability and empirical knowledge is a predictor of the empirical knowledge effect on human problem-solving. The relation between the empirical knowledge and interference can be characterized as: the more negative interference the participants have, the more of the empirical knowledge effect they will have in the next year.

This study does not agree with the Piagetian theory about human problem-solving: the effect of empirical knowledge should decrease with age. Indeed, this study argues that the problem content still affects human problem-solving after individuals move into the formal operation stage. The different kinds of academic training--science and social science major--shape human problem-solving strategies into either a logical reasoning base or an empirical knowledge stand, respectively.

CHAPTER 1

INTRODUCTION

Rationale

In Feynman's semi-autobiography, *Surely You're Joking, Mr. Feynman!,* there is an interesting episode about a conversation between a scientist and a professional painter. The painter told Feynman, a physicist, that he could make yellow paint by mixing red and white paint. Although strongly skeptical based on his scientific knowledge, Feynman respected the painter who had been painting all his life. Therefore, he decided to ask the painter to show him how. It turned out that the painter couldn't make yellow by simply mixing red and white paint. He used a little yellow, along with red and white, to make yellow paint. At the end of the story, an onlooker remarked that how could a painter challenge a physics professor who had been researching light all his life.

In my qualifying study (2000), there is an episode on knowledge of light and vision. One of the questions that the participants were asked to answer was "If blue light shines on red glass, what color you will see on the other side of the glass?" One participant in the study, Pete (who holds a master's degree in fine arts and is currently a professional potter), answered "purple" because, he said, although blue light was blocked by red glass, it could blend with red glass and therefore show purple. I told him that if the given assumptions in the previous section of the questionnaire were followed, the correct answer would be black. Then Pete responded that the answer of black existed only on paper. Unless the experiment was carried out in front of him such an answer of "black" is nothing more than an assumption, and he would insist on "purple"; such a definite answer was based on his empirical knowledge.

From the two episodes above, we can see that people responds to problem in different ways: some follow the professionals' opinions, like Mr. Feynman; some defy the professionals' opinions and only believe in what they see and understand, like Pete. What principles do people follow while solving problems? We know that problem-solving is a decoding and computing process (Liao, 2000; Mayer, 1992; Sternberg, 1977). It is a decoding process because the problem solver translates the problem into an internal representation, and this decoding process is influenced by memory, action, and sociocultural interaction (Mascolo & Fischer, 1999). It is a computing process because the problem solver maps, links, analyzes, and applies what s/he has known to figure out the results. Both processes reflect individual differences in cognitive competence and performance.

Problem-Solving Process

The problem-solving process is a computing process. The same person might react differently to the same problem presented in different situations (Liao, 2000). In my qualifying study research, there is a very good example to illustrate this point. When I asked Miko, a Japanese woman who got her Ph.D. in language education in the USA, how much two melons would cost if each melon cost three Japanese yens. Her response was "Are you kidding? It's impossible to buy a melon with 3 yens!" A little later in this conversation, Miko admitted that if this was a question from her elementary school teacher, she would simply answer 6 yens, although she thought it was a very odd question. Obviously, the same person is likely to respond differently in different situations, namely, the problem-solving process is deeply influenced by the situation in which a problem is presented and the content of the problem. As stated in the Chinese war strategy classic, *The Art of Wars*, people should act contingently according to the time, location, and rivals when a crisis comes up.

From the existing literature about problem-solving, we know that the process of problem-solving is affected by factors like people's age, the major field of study (hereafter major), whether a person is an expert or novice in the participant matter, and so forth (see, for example, Arenberg, 1982; Chi et al., 1981; Liao, 2000; Spivack et al. 1986). Moreover, a problem solver's thinking styles play an important role in the problem-solving process, and such thinking styles reflect the influences of age, culture, gender, parenting style, schooling and occupation (Sternberg, 1997, 2000). Both biological aspects and the social and cultural aspects of the problem solver influence the problem-solving process (Hunt, 1994; Serpell & Boykin, 1994).

Age is an index of the growth of problem-solving strategy, includinglogical reasoning ability and empirical knowledge, but this research does not focus on the development oflogical reasoning ability; rather it focuses on the effects oflogical reasoning ability, empirical knowledge, and their interference with the human problem-solving process versus age, gender, and the distinction of expert /novice. Especially, the current study tries to understand the effect of empirical knowledge on the problem-solving process. Light and vision are the chosen topics for the research of the problem-solving process for this study because knowledge on such topics can be obtained from everyday life and has a solid life explanation.

An Empirical Knowledge: Light and Vision

Although light and vision are well developed subjects of physics, most people reach an understanding about them from everyday life. The understanding of light and vision is an example of human empirical knowledge. When people are asked problems about light and vision, how are they going to respond? The episode about Pete shows that they might use empirical knowledge to solve the problems, although they are shown with a fully developed physical theory about light and vision. Or they may, like the foremost physicist Mr. Feynman, put aside their light and vision knowledge, but trust an expert's mystery. It is interesting to attempt to understand what different concepts and strategies are brought to the task of solving problems by different people.

Several studies investigate children's misconceptions about light and vision. It is observed that some children believe that humans can see because light shines upon the eyes (Saxena, 1991). Some students believe that cats can see in the dark because light is emitted from their eyes (Fetherstonhaugh et al., 1987, 1988; Wang, 1992). Other studies reveal children's misconceptions about light. Fetherstonhaugh et al. (1987, 1988) report that many children think light cannot move. All these studies investigate children's misconceptions about light and vision, but they fail to present how children achieve their misconception. Actually, these studies do not focus on light and vision problem-solving processes.

The current study is a follow up of my previous study (Liao, 2000). My qualifying study and current study both investigate people's conception of light and vision for the problem topic, but the participants are different. The participants for my qualifying study and current study are adults, graduate students or people with at least a Master's degree, and secondary school students (grade 7 to 12), respectively. The reason why these secondary school students were chosen for the current study is that they have some empirical knowledge about light and vision and the capability to follow some directions to solve problems. Moreover, why Taiwanese secondary students were chosen is they are taught abstract thinking using symbols and are asked to use abstract ideas to solve problems, especially in science courses. The current study focuses on how empirical knowledge andlogical reasoning ability affect the problem-solving process of answering questions on light and vision. The current study involves participants from the 7th to the 12th grades, who are generally from 12 to 18 years old.

Purpose of Study

The problem-solving process is shaped by the problem solver's knowledge about the content of the problem as well as his or her cognitive ability, especiallylogical reasoning ability and empirical knowledge. This study aims to investigate the problem-solving process of students of different grade levels (from the 7th to the 12th), genders, and majors (social science or science). Particularly, this study aims to increase the understanding of the interaction between empirical knowledge andlogical reasoning ability and whether this relationship changes among subgroups of different grade levels, genders, and majors. The investigation of how secondary school students answer questions on and solve problems about light and vision, how they elicit various sources (e.g., logic reasoning and empirical knowledge) to solve problems, and how they apply different strategies to solve problems will yield valuable information for science education and science curriculum design.

Research Questions

Based on its purpose, the current study tries to address the following questions:

Research Question-1. How dological reasoning ability and empirical knowledge influence Taiwanese secondary school students to solve problems concerning light and vision? Do these two factors interfere with each other in the problem-solving process? If yes, what are the relations of the interference versus ages, genders, and majors?

Research Question-2. Do participants who recognize that the structures of two problems are the same more often use the same strategy to solve these two problems than those who do not?

The first question tries to understand the effects oflogical reasoning ability, empirical knowledge, and their interference in the problem-solving process. The second helps to understand deeply how the problem-solving process is affected by these two factors, especially when the problem-solving process is content independent or content dependent.

CHAPTER 2

LITERATURE REVIEW

The purpose of the current study is to understand howlogical reasoning ability and empirical knowledge affect secondary Taiwanese school students solving light and vision problems. To this aim, the first part of the literature review discusses related researches on the humans' problem-solving process. Because the research of the human problem-solving process is a branch of the study of human development. Human development theory is discussed later to understand more deeply the fundamental issue concerning the human problem-solving process. Because most of the recent human development study is still under the Piagetian school shadow, Piagetian development theory is the main topic for this discussion. Finally, the possible outcomes about howlogical reasoning ability and empirical knowledge affect human problem-solving process are discussed.

What Is Problem-Solving

People face and solve problems everyday. What foods should I prepare for this evening's party? What will I wear? How is the seating going to be arranged? However, a problem to one person is not automatic to another. For example, a second-degree polynomial with one variable, $x^2 - 8x + 12 = 0$, is not a problem for mathematicians; it is a problem for beginning algebra learners. For a mathematician, s/he immediately finds the two roots of the above polynomial, which are 2 and 6. For a beginning learner, s/he might use trial and error, or perhaps the quadratic formula taught in algebra classes. When is a problem a problem for a person?

The definition of problem by Chi and Glaser (1985) is "a situation in which you are trying to reach some goal and must find a means for getting there" (p. 229). Anderson (1995) says that the three essential features of problems are (1) goal directedness---the behavior is organized toward a goal, (2) subgoal decomposition---the original goal is decomposed into subtasks or subgoals, and (3) operator application---the solution of the overall problem is a sequence of operators, which are the actions that transform one problem state into another problem state (pp. 237-238). Mayer (1992) describes the three certain characteristics of a problem as:

> *Given*---The problem begins in a certain state with certain conditions, objects, pieces of information, and so forth being present at the onset of work on the problem.
>
> *Goals*---The desired or terminal state of the problem is the goal state, and thinking is required to transform the problem from the given to the goal state.
>
> *Obstacles*---The thinker has at his or her disposal certain ways to change the given state or the goal state of the problem. The thinker, however, does not already know the correct answer; that is, the correct sequence of behaviors that will solve the problem is not immediately obvious. (p. 5)

Psychologists generally accept a four-part typology of problems that is suggested by Greeno (1978) and Greeno & Simon (1988): problems of transformation, problems of arrangement, problems of inductive structure, and problems of deductive structure. As for the structure of problems, there is a distinction between well-defined and ill-defined problems which is discussed by Howard (1983) and Chi and Glaser (1985). For Howard, two features set well-defined problems apart from ill-defined ones. The first is that well-defined problems state clearly the criteria that should be used when deciding whether the goal has been achieved or not. In ill-defined problems, the goals are vague. The second is that well-defined problems state precisely what information is needed to solve the problems while ill-defined problems do not. For Chi and Glaser, well-defined problems can be readily recognized when they are solved, and they must have a clear initial state, a set of permissible operators, and a goal state. To observe humans solving well-defined problems is a way to understand what factor(s) affected the humans' problem-solving process. The factors that affect humans' problem-solving processes can be categorized as universal and individual differences. Inside of any ethnic group, age and gender difference are universal differences for the problem-solving process, because these differences are based on nature. Individual difference contains the distinction of expert and novice and empirical knowledge. The reason of individual difference is individual, because the difference is caused by person's environment and it exists even if two individual share natural characteristics (e.g., age and gender). For instance, same aged people might have different experiences with the same problem. Therefore, they might have individual empirical knowledge. The following literature review focuses on humans' problem-solving processes on universal differences, such as age and gender.

Universal Difference on Humans' Problem-Solving

Age

A human's internal representation is very important for human problem-solving (Hunt, 1994; Mayer, 1992). Actually the development of representation strongly connects to human development. Mascolo & Fischer (1999) argue that human representation cannot be broken into distinct components for memory, action and socioculture interaction, but a combination of these three aspects. They try to use dynamic skill theory to describe the development of representation. Dynamic skill theory establishes a set of conceptual and empirical tools to understand development changes in children's cognitive, affective, and behavioral skills (Fischer, 1980; Mascolo & Fischer, 1999). Fischer & Rose (1994) and Mascolo & Fischer (1999) use EEG to measure the levels of skill development in terms of age.

Sternberg (1994) believes that brain activity is the most popular approach in the biological study of intelligence. EEG is the earliest measurement of brainwaves, which are electrical patterns created by the activities of neurons. Matousek and Peterson (1973) completed a classic study of cognition development using EEG. They demonstrated that the relative power of the EEG changed systematically over age. The most significant discovery from this study, as later shown by John (1977) and Hudspeth and Pribram (1992) who analyzed the original data, is that there are cycles of growth in relative energy in EEG across cortical regions. Hudspeth and Pribram describe three cycles: from 1 to 9 years old, from 9 to 19 or 20 years old, and the third cycle begins around 19 or 20 years old. Fischer and Rose (1994) used EEG to measure brainwaves to study cognitive development and categorized cognitive growth into 13 different levels that fall in 4 different stages.

The second cycle in Hudspeth and Pribram's (1992) study, the cycle that describes the participants of this study, corresponds approximately to the three skill levels of cognitive development in Fischer and Rose's (1994) and Mascolo and Fischer's (1999) studies: representational systems/single abstractions level (10-12 years of age), abstract mappings level (14-16 years of age), and abstract systems (from 18-20 years of age).

During secondary school, students develop from using representational systems/single abstractions to using abstract systems. Based on Fischer's studies on cognitive development, we know that cognitive ability is a very important factor in problem-solving. Fischer and Rose (1994) and Mascolo and Fischer (1999) use dynamic skill level as an indicator of cognitive ability. They also point out that before the age of 25, dynamic skill level increases with age. Human problem-solving is related to cognitive development, which can be categorized by dynamic skill level. And age is the index of dynamic skill levels; therefore, age can be a chart for the development of problem-solving. This indicates that biological factor (maturation) is one of the major influences of problem-solving. Because the dynamic skill levels are presented by answering oflogical reasoning ability tasks in Fischer's studies, thenlogical reasoning ability is used to describe the human ability to solve problems.

Gender

Human development is not only determined by biological factors, but also by socioculture. Piaget believed that maturation, experience, social transmission, and equilibrium together shape the process of human development. Vygotsky (1978) went one step further and claimed that social factors were at least as important as internal factors in shaping human growth. Social factors include interaction between the individual and the social community. From being in the community, people learn the rules, requirements, and expectations of the social group, and such experiences in turn assist in tackling problems in everyday life. Therefore, sociocultural interaction and experience also influence the problem-solving process.

That the two genders perform differently on math and verbal questions may be an evidence of sociocultural influence on problem-solving ability. Some psychologists think that males are genetically better at math while females better at language, but Richardson (1997) argues that sociocultural factors like "participants' expectation" cause such gender differences. The society as a whole expects males to be better at math and females better at language, and therefore, there are more math and science learning and experience provided for males and language learning and experience for females. Maccoby and Jacklin (1974) also argue that culture is the leading cause of gender differences in math, science, verbal, and spatial achievements.

From the above discussion about the universal differences on humans' problem-solving, there are two important considerations. First, age is an important index of the changing of the problem-solving process, because it is an index of the development oflogical reasoning ability andlogical reasoning ability strongly affects how humans solve problems. Second, the gender difference in achievements might be caused by environment, such as sociocultural but not by nature. After discussing the universal difference on problem-solving, the following discussion focuses on the individual difference.

Individual Difference on Humans' Problem-Solving

The Distinction Between Expert and Novice

Just as gender difference is a good example to show sociocultural influence on problem-solving, expert vs. novice distinction is a good example to show training influence on problem-solving. There are three main domains of study of the effect of distinction of the expert and novice in the problem-solving process. They are chess, computer programming, and physics tasks. Because the content of the problem for this study is physics (light and vision), then the research related to the distinct problem-solving processes of the expert and novice in physics task are discussed as follows.

Chi et al. (1981, 1989) conducted a seminal study about how different level students, graduate students for experts and undergraduate students for novices, solve physics problems by analogy between the problems and the examples. The comparison of the reason of category of physics problems by experts (graduate students) and novices (undergraduate students) is on the basis of fundamental physics laws and on the basis of the content of the problems, respectively. Moreover, Chi and her associates (1981, 1985) and VanLehn (1998) state that training makes physics graduate students (experts) more aware of the deep structure of physics questions than physics undergraduate students (novices) (pp. 392-397). Mayer (1992) points out four areas of differences between expert and novice in problem-solving: factual knowledge, semantic knowledge, schematic knowledge, and strategic knowledge. All of these four differences is the result of training.

Empirical Knowledge

If training can be labeled as formal education, then empirical knowledge can be thought as the life education that humans learn from everyday life. Research (Chi, Bassok, Lewis, Reimann, & Glaser, 1989; VanLehn, 1998) on the differences between expert and novice on solving physics problems indicates that experts, possessing more empirical knowledge, show better problem-solving strategies and analogical thinking ability. On the other hand, although training (formal education) might reduce the effect of empirical knowledge (life education) on the problem-solving process, empirical knowledge might still affect humans' problem-solving processes. For example, McGarrigle and Donaldson's studies (1974-1975) indicate that in a number of conservation experiments, the possibilities of children answering correctly were influenced by the empirical knowledge, such as the experiment contexts (stones or candies). Mehler and Dupoux (1994) have another explanation. They state that when answering test questions, children took into account both the research questions themselves and why the researchers asked and presented such questions in a particular way. They argue that children have enoughlogical reasoning ability to answer these problems, but use the wrong dimension of knowledge to answer questions. Does empirical knowledge have a positive effect for problem-solving? Based on Markman's (1979) and Baker and Anderson's (1982) studies of the comprehension monitoring tasks, the answer is yes. When humans can correctly transfer empirical knowledge to the facing problems, empirical knowledge can help to bring some strategies to solve problems.

Dominowski and Bourne (1994) argue that empirical knowledge and experience have both positive and negative effects on problem-solving.

From the above discussion, the distinction of expert and novice and empirical knowledge positively or negatively affecting problem-solving depends on whether or not humans can find the right strategy to solve the problem. Problem solvers bring their own past/empirical knowledge to bear in constructing problem spaces and in searching for solutions to problems. Transfer of learning research involves understanding how substance learned at one point in time affects the learning of other substance. If people can build problem representations based on the correct past/empirical knowledge, then s/he can find more and better strategies to solve the problem. Experience with earlier substances may either facilitate (positive transfer) or hinder (negative transfer) the solving of the related substance.

Representation Transfer

People may experience considerable difficulty in transferring what they know about solving a problem of a particular type when they are confronted with a new problem of the same type. For the purpose of understanding how representation transformation influences the problem-solving process, psychologists (Gick & Holyoak, 1983; Liao, 2000; Reed, Ernst & Banerji, 1974) asked participants to answer isomorphic/homomorphic problems. Isomorphic problems must be well-defined problems and the same isomorphic problems share common details (therefore, superficial similarity), and the relations of objects in one problem correspond to the relations of objects in another (therefore, structural similarity) (Gentner, 1989). My qualifying paper work (Liao, 2000) defines isomorphic problems as well-defined problems that have the same structures of initial state, goal state, and operators. It means the same problem-solving strategy can be used to answer the two isomorphic problems. For example, the Towers of Hanoi problem is isomorphic to the Chinese Tea Ceremony problem. If problems have similar but not same structures of initial and final stages, then these problems are homomorphic problems. The Missionaries and Cannibals problem is homomorphic to the Jealous Husbands problem which needs more problem-solving skills. Because problem-solving strategies can be analyzed in terms of the possible paths of moves taken by an individual solver through a state space, isomorphic and homomorphic problems can give hints to researchers to find out whether participants use the same problem-solving strategies for problems-solving, in order to learn how humans transfer representation. Therefore, we can detect the influences of the content of problems on the problem-solving process.

Problem-solving schema is the memory representations that contain knowledge based on former experiences with a particular type of problems. Schema learning is the process of building such a representation. Using isomorphic/homomorphic problems, Gick and Holyoak (1983) produced a series of experiments to investigate the strategy of analytical problem-solving and to find problem-solving schema in order to build a schema for learning. They found that solving one problem does help to solve isomorphic/homomorphic problems later. The participants may create some partial solutions that found at the earlier problem to solve the later isomorphic/homomorphic problems. Moreover, the more encounters with isomorphic problems, the better people become at solving them. They call this process solution development.

Reed et al. (1974) used Missionaries and Cannibals and Jealous Husbands problems to predict that participants who are given a homogeneous problem first would show improved performance on the second homogeneous problem. When participants were given the same problems, they showed improved performance, since fewer moves were required to solve the problem the second time. When participants were first presented with the Jealous Husbands problem, then the Missionaries and Cannibals problem, they showed improved performance only when the experimenter disclosed the relationship between the two problems. When participants were taken the Missionaries and Cannibals problem followed by the Jealous Husband problem, they did not display any improvement, even given the relationship between the problems. Reed's study shows that experience on a problem does not necessarily transfer to another problem of the same type unless people are aware of the similarity and the second problem is simpler than the first.

The above experimental studies on representation transformation show that participants might not receive any help from solving a problem to solve anther problem with the same problem strategy, such as isomorphic/homomorphic problems. In general, individual differences affect the problem-solving process. The same person might employ different problem-solving strategies and get different answers to the same problems that have the same problem structure but under different contents. In other words, humans' problem-solving process is content dependent. The definition of content independent is "if humans' problem-solving and reasoning within a domain and restricted to that domain contrast with problem-solving and reasoning of a more generic form, this kind of thinking is content independent" (Rosser, 1994).

The formal operations stage from the Piaget stage theory states that children develop the ability to think in terms of the hypothetical probabilities, and the possible rather than concrete here-and-now (Mayer, 1992). If this statement holds true, then humans should solve problems by using the deeper problem structure, such as hypotheses, not the problem content. This is contradicted by the fact that the content independence really exists in the problem-solving process. Because the study of the change in the problem-solving process versus age is a part of developmental psychology that is strongly affected by the Piagetian School, the last part of the literature review will focus on the Piagetain School's views on problem-solving.

Information Process Model, Human Development, and Problem-Solving Process

In Hunt and his colleagues' (Hunt, 1978, 1985; Hunt, Frost, & Lunneborg, 1973; Hunt, Lunneborg, & Lewis, 1975) series of papers, they attempted to determine whether there is correlation between information processing capacity and verbal ability. This new way of understanding intelligence, called the cognitive correlates approach or the information processing approach uses the idea of an information processing system which varies from one individual to another. The information processing system includes the capacity, storage, activation, and application of memory in information processing. Each of these components is susceptible to individual differences.

Contemporary cognitive research is under a great influence from Piaget's and his follower's thoughts. Anderson (1995) says, "Much of the recent information-processing work in cognitive development has been concerned with correcting and restructuring Piaget's theory of cognitive development" (p. 415). For example, the information process model is an extension of Piagetian stage theory. Bruner's (1964), Case's (1985) and Siegler's (1986) experimental researches that employ the information process model to explain human development are based on the observation of the performances of the human's problem-solving process to modify Piagetian stage theory. The purpose of this study is to discuss the humans' problem-solving process. The problem-solving process study is a branch of human development, then the following discussion focuses on human development, and then turns into Piagetian theory of problem-solving.

Piagetian Theory About Human Development

Piagetian theory about human development can be roughly categorized into two parts: stage theory and development functions. Stage theory describes the development of human cognitive ability. Development functions characterize the internal abilities that help human cognition development. Stage theory states that these functions are independent of stages and that two functions are common to development: adaptation (the need to stay alive and survive in the environment) and organization (the need to have well-organized and orderly internal structure) (Mayer, 1992). Because schema is the smallest unit of the mental organization and the equilibration is used to describe the organization process, then both of them will be introduced later.

Adaptation. The environment that any organism lives in is ever-changing, and adaptation to the environment is necessary and part of the development. It has to be emphasized that Piaget believes that the learner assumes an active role in the process of assimilation and accommodation. In other words, the learner cannot be separated from the process of mental growth. There is no total objectivity in the development of knowledge in the Piagetian theory. The external events in the mental environment are always understood through the internal mental structures of the learner.

Organization. The other principle common to all biological development, according to Piaget, is organization, which refers to the organization of the internal mental structure. The research of genetic epistemology, as represented by Piaget's work, studies how knowledge is constructed. People understand and interpret their experiences through the internal mental organization, which is composed of schemas. From the biological perspective, an organism becomes more and more complex due to consistent assimilation and accommodation processes (e.g., from single-cell to multi-cell). From the intellectual perspective, mental organization becomes more and more complex also due to the processes of assimilation and accommodation. Therefore, the structure of the mental organization is varying, not static. People construct knowledge through the mental organization of the outside world, and this mental organization is constantly changing due to the adaptation process.

Schema. Schema is the smallest unit of the mental organization. It does not simply stand for one single act or event. Schema refers primarily to an abstract structure which covers a whole class of similar behaviors. Indeed, schema is an internal design for action.

Equilibration. Moessinger (1978) says that "At equilibrium, the system reacts to perturbations and resists to some of them" (p. 256). People use assimilation and accommodation processes to adapt to the outside world, and the interaction and balancing process between assimilation and accommodation is called equilibration. Facing a new event in the environment, a person adapts to it by assimilating or accommodating. The schema will become more complex, and so does the mental organization. The mental structure changes from one state to another, and so does equilibration. The change in equilibration happens when the existing equilibration is not able to deal with the new event and especially when the new event causes a conflict or contradiction in the existing equilibration.

The old equilibration and new equilibration are related in that the new equilibration is not totally new. It is a refined version of equilibration based on the old one, and it makes possible a broader application and use of the old equilibration. Piaget also thinks that new equilibration makes the old equilibration more stable and active.

Through re-equilibration and auto-regulation, the subject's cognitive system becomes more stable, more resistant to perturbations (Moessinger, 1978). Although new equilibration starts with the encounter of a new event, it has to be noted that the person takes on an active role in recognizing the new event, adapting to it, and self-reorganizing his or her mental structure.

Functions are the internal cognitive abilities to help human development, while Piaget uses stage theory to describe the process of human development. The method how Piaget and his colleagues characterized the development stages is observing thehuman's problem-solving process.

Stage theory. The stage theory of children's development has been the most well known theory in Piaget's work. Piaget thinks that infants are born lacking virtually all the cognitive competencies that adults have, and they develop these competencies gradually going through different stages. The stages are defined by two aspects of cognitive life: (1) structure---how the child represents the world, and (2) operation---how the child can act upon this representation (Mayer, 1992, p. 291). As children gradually develop their cognitive abilities, they have better cognitive structure and more power operations.

Operation is an important defining component in Piaget's stage theory. Piaget (1999) says,

> An operation is an internalized action, but it is also a reversible action. But an operation is never isolated; it is always subordinated to other operations; it is part of a more inclusive structure. Consequently, we define intelligence in terms of operations, coordination of operations. (p. 36)

and q, not-p and q, and not-p and not-q, or s/he can examine for the contradiction in the falsehood of p and not q. Moreover, as Inhelder and Piaget (1958) point out,

> . . . Formal thought is more than verbal reasoning (prepositional logic). It also entails of operational schemata which appear along with it; these include combinatorial operations, proportions [sic], double systems of reference, a scheme of mechanical equilibrium (equality between action and reaction), multiplicative probabilities, correlations, etc. (p. xxii)

At the formal operation stage, children create a mental operation system, an INRC group, to solve problems. This operation system helps children to solve problems by focusing on the deeper problem structure, such as proportional relations, not on the problem content. And, children can directly transfer the representation of problems into corresponded isomorphic/homormophic problems.

Summarizing Piagetian theory, the characteristic of human problem-solving strategy is content independent, when humans turn into formal operation stage. Their problem-solving strategies are based on the problem's hypothesis and proposition, not on the problem's content. At this stage, the problem-solving strategies only depend on humans' mental ability, especially forlogical reasoning ability. Moreover, humans can apply theirlogical reasoning ability in any field without caring the problem's content. It is the concept that human problem-solving is content independent.

Domain –Specific Thinking Versus Domain-General Thinking

The different problem-solving performance between expert and novice is evidence that the training causes the different searching of the problem-solving strategy. And, the different performances between expert and novice show that human problem-solving is a computing and decoding process. The comparison between Domain-Specific Thinking and Domain-General Thinking is evidence that human problem-solving is a computing and decoding process, too. Indeed, the comparison of these two scientific thinking describes how the problem content affects human problem-solving.

Domain-specific thinking (hereafter, DST) is one of the two types of scientific thinking, the other type being domain-general thinking (DGT). DGT is the type of scientific thinking that characterizes intelligence and rationality. It is the cognitive activity that involves logical, rational, conscious, and deliberate working of the mind, and it can be applied to any subjects and phenomenon. DST, on the other hand, is intuitive and constrained to specific subject and phenomenon. It is pre-conscious and not demanding of cognitive working. DGT usually does not emerge until cognitive maturation while DST appears early in development (Rosser, 2001)

DGT is involved in the scientific thinking of any subject domain while DST is applied in specific subject field. Principles of one domain cannot be applied in another domain. The knowledge, strategies, theories, and principles used for solving problems of a specific field is DST, while the general logic and rational thinking used for solving problems of any field is DGT.

Rosser (1994) argues that there is a unique set of problem-solving principles for humans from DGT view. From DST, each different domain has its own principles that cannot be reduced to or directly transform across different domains. Moreover, the scholars who believe DGT think that there is a universal approach to problem-solving. On the other hand, the scholars who believe DST trust there exist some domain-general aspects of cognition, but a lot of important activity that goes on in the mind is modular (Rosser, 2001). The factor driving human scientific thinking into DST or DST is the problem content.

DST appears earlier in human development than DST (Rosser, 2001). Humans organize the experiences, including successful and unsuccessful ones, about a specific domain into a specific knowledge. This organization process of knowledge is how humans build a DST. When a human faces a problem, s/he decodes the problem into the corresponding domain/content and recalls the related experiences to help her/him to solve the problem. Therefore, human problem-solving is content dependent.

The Relation Between Problem Content and Problem-Solving Process

Individuals develop different DST knowledge early in their lives in response to their environment. A child that grows up in the market place will have an early knowledge about money-merchandise trade. Furthermore, for the purpose of survival, individuals display precocious DST in certain domains but not others (Rosser, 1994).

Observation is the major tool for gathering knowledge, but observation without complete organization is inconsistent, full of bits and pieces, and sometimes contradictory to facts. For example, seeing a moving toy car that is remote-controlled, a child might think that a tiny driver is inside the car to make it move. This child is yet to development the knowledge that there are no humans small enough to fit into the toy car. Such inconsistent bits and pieces of intuitive knowledge gathered from observation is what forms DST (Keil & Springer, 1991). Different types of DST are developed from observations of different types of domains. This is why DST is DST--domain specific. Intuitive knowledge of one domain does not apply to other domains.

The development of DST is often data-driven. That means the DST knowledge of a certain domain is constructed from the situation, not from a higher level of cognitive or rational activity. The development of DST is also often achieved from personal successful experience in a certain domain.

Humans might not apply the same strategies to isomorphic/homormophic problems, even though these problems have the same problem structures. The reason is that these problems are under different content areas and the same people might have different personal experiences in different content areas. Therefore, the problem-solving process is content dependent for the same person.

Summary of Literature Review

From the above discussion about related research, we know the individual differences, such as the distinction of expert and novice and empirical knowledge yield different problem-solving processes. Gender difference, a universal difference, seems to impact problem-solving strategy through social expectation rather than nature. Age is not only an index of human mature development, but also a guide to the changing/growth of empirical knowledge. And, based on the discussion of the relation between the development of DST and problem-solving process, humans' problem-solving still depends on the content, not just on theirlogical reasoning ability.

The above literature can be summarized into three conclusions that seem relevant to the current study:

1. Before the age of 25, the development oflogical reasoning ability is strongly influenced by age. Before 25,logical reasoning ability can be roughly divided into 3 stages: from birth to age 9, from age 9 to 19, and from age 19 to 25.
2. The problem solver's age, gender, and expert/novice distinction on the participant matter will affect the problem-solving process and strategies.
3. The studies of representational transfer show that humans cannot directly transfer their problem-solving strategies to corresponding isomorphic problems. Under different contexts, the same person might employ different problem-solving strategies and yield different answers to the same question. Moreover, the same person might employ different problem-solving strategies and yield different answers to two isomorphic questions that are asked in different ways or terms.

If Piagetian theory on the problem-solving process on the formal operation stage were right, then humans solve problem not only on the basis of objects, but also on the basis of hypotheses, or the basis of propositions when they approach formal operation stage; while the major influence factor of the problem-solving process should be theirlogical reasoning ability after the age of 12. Therefore, humans can transfer problem representation and solving strategies directly to other isomorphic/homomorphic problems. It means that humans solve problems by considering the deeper structure, not depending on the problem content. The idea of this kind of problem-solving absolutely is DGT after the age of 12. Unfortunately, it is the wrong approach, because humans still cannot directly transfer their problem-solving strategies to other isomorphic/homomorphic problems at the formal operation stage and human problem-solving still depends on the problem content.

To support the above idea and to answer the study research questions, the research hypotheses and corresponded methodology will be stated in the next chapter.

CHAPTER 3

RESEARCH METHODOLOGY

Methodology

The goal of the current study is to understand the problem-solving process and strategies that Taiwanese students from the 7^{th} to the 12^{th} grades take. Particularly I want to investigate how the problem-solving process and strategies are influenced by empirical knowledge,logical reasoning ability, and their interference, which in turn are shaped by possible factors like age/grade level, gender, and major. The distinction between science major and social science major will be considered a distinction between expert and novice. In Taiwan, 11^{th} and 12^{th} grade science majors take six more hours a week of physics and chemistry than their social science counterparts; 11^{th} and 12^{th} grade social science majors take six more hours a week of history and geography than their science counterparts. Furthermore, the content of math courses for science majors are more advanced than that for social science majors. It is predicted that in this study the 11^{th} and 12^{th} grade science majors will be more flexible and strategic than their social science counterparts in answering questions about light, vision, and logic. Therefore, the science majors can be categorized as experts in the content area of the questionnaire while the social science majors, novices.

Hypotheses

Based on the purpose of this study, related literature that is reviewed above, and the research questions, the following hypotheses are drawn:

1. Thelogical reasoning ability of Taiwanese secondary school students is affected by age/grade level but not by major or gender.
2. In the process of problem-solving, the effect of empirical knowledge will be affected by major and age/grade level but not by gender. Specifically, science majors will use less empirical knowledge to solve problems than social science majors of the samelogical reasoning ability.
3. Generally, in the problem-solving process, the use of empirical knowledge will be affected bylogical reasoning ability. Specifically, the abler the participants are within logic, the less empirical knowledge they use.
4. Science majors in this study will show a higher tendency of detecting the relations between problem structures, especially for isomorphic problems, than social science majors. In other words, science majors more often than social science majors use the same problem-solving process and strategies to answer the isomorphic questions.
5. Higher-grade level in this study will show a higher tendency of detecting the relations between problem structures, especially for isomorphic problems, than lower-grade level. In other words, higher-grade level more often than lower-grade level uses the same problem-solving process and strategies to answer the isomorphic questions.

Participants

As I discussed earlier, the participants of this study can be categorized into three dynamic skill levels according to the study of Mascolo and Fischer (1999) and Fischer and Rose (1994). As mentioned in chapter 2, because the dynamic skill levels are categorized bylogical reasoning ability tasks, these levels can be described as the differentlogical reasoning ability levels. The twological reasoning ability levels discussed by these researchers are the abstract mapping level, the level of students from the 8^{th} to the 10^{th} grades, and the abstract systems level, the level of students from the 11^{th} to the 12^{th} grades. For convenience, the current study will recruit participants from the 7^{th} to the 12^{th} grades since in Taiwan, junior high schools include the 7^{th}, 8^{th}, and the 9^{th} grades and senior high schools cover the 10^{th}, 11^{th}, and the 12^{th} grades. At each of these twological reasoning ability levels, there are two subgroups--male and female students. The 11^{th} and 12^{th} graders are further divided into social science versus science majors. Therefore, in total there are 16 subgroups (see Table 3.1).

According to Fischer and Rose (1994) and Mascolo and Fischer (1999), the participants of this study can be divided into three cognitive ability groups, as shown in Table 3.2.

Development of the Questionnaire

Since this study is a continuation of my qualifying study work (Liao, 2000), the questionnaire used in that study is used as a basis in this one, too. However, my qualifying study recruited adults with master's degree or above and adopted one-on-one

Table 3.1

*The Distribution of Participants**

Class ID	Sample Size	Gender	Grade	Major
1	28	F	7	No
2	30	M	7	No
3	28	F	8	No
4	25	M	8	No
5	28	F	9	No
6	30	M	9	No
7	30	F	10	No
8	28	M	10	No
9	30	F	11	Social
10	30	F	11	Science
11	30	M	11	Social
12	30	M	11	Science
13	30	F	12	Social
14	29	F	12	Science
15	30	M	12	Social
16	30	M	12	Science
Total	466	F:233 M:233		Social: 120 Science:119

*There are 466 participants in this study and 233 participants in each gender subgroup. For 11th and 12th grades, there are 120 participants in social major and 119 in science major. The missing data contains 13 participants.

Table 3.2

Cognitive Ability Groups

Cognition Level	Age (Years)	Grade
Systems of Representational Systems/Single Abstractions	10-12	7
Abstract Mapping	14-16	8,9,10
Abstract Systems	18-20	11,12

format of data collection. The current study recruits both junior and senior high school students and adopts a whole-group questionnaire taking as the data collection method. Therefore, the questionnaire needs to be adjusted to a certain extent so as to match the student's level.

The content of the questionnaire comes from a physics textbook (Feynman et al., 1963). To ensure content validity, two things are done: (1) a senior high school science teacher in Taiwan is asked to read the questionnaire to make sure the questions are appropriate for the level of high school students, and (2) questions in the questionnaire will be reviewed by a physics professor.

Due to the wide age range of the participants, I (1) asked one elementary school teacher who has more than 10 years elementary teaching experience and one high school teacher who has 6 years elementary teaching and another 6 years high school teaching experience to read the questionnaire first to ensure that the language of the questionnaire is appropriate for the participants, and (2) before starting the test, I asked three junior high school students and three senior high school students to read the questionnaire and explain what each question is asking about. Such procedures are meant to reduce test bias caused by the language difficulty of the questionnaire.

Compared to the questionnaire from my qualifying study work (Liao, 2002), the current questionnaire is somewhat different. Seven binary questions in my earlier study are deleted from the current study because these questions are of a quite advanced level in content for junior high school students. In Part II, 5 new questions are added while 4 questions from the earlier study are deleted. In Part IV, 4 new questions are added while 4 questions from the earlier study are deleted. The questions that are deleted are either too difficult in content or ambiguous in language. The current questionnaire contains a cover page and 5 parts. The cover page gives instructions of taking the questionnaire, explains the rights of the participant, and asks information about gender, age, and major. Part I Facts (A) contains scientific facts about light and vision. Part II Question (A) contains 9 questions that should be answered correctly if the facts described in Part I are understood. Part III Facts (B) consists of abstract statements about filtering and straining. Part IV Questions (B) is composed of 8 questions that should be answered correctly if the facts described in Part III are understood.

The facts described in Parts I and III actually involve the same physics concepts. A difference between them is that Part I facts are explained in more concrete objects like blue light and colored glass while Part III facts are explained in more abstract terms like X object and A filter. The significance of this difference is that light and colored glass are part of most people's empirical knowledge while X object and A filter are only symbols. Therefore, when solving problems in Parts II and IV, participants will more probably resort to their previous empirical knowledge on light and colored glass to answer questions in Part II while less empirical knowledge is available to answer questions in

Part IV. At this point, it is interesting to ask the participants whether they see the relations between questions in Parts II and IV and observe whether those participants that see such relations have more consistent problem-solving performance on isomorphic questions in both Parts II and IV.

As designed in the questionnaire of my qualifying work, Part I and II and Part III and IV are designed by the A-AR model. The A-AR model is borrowed from mathematics. In mathematics, it requires participants to use logical thinking and computing by following given assumptions in order to solve problems.

The A-AR Model

In Part I, some scientific facts about light and vision are listed in the form of assumptions. In Part II, subjects are asked to answer questions about light and vision by following the assumptions in Part I. The questions are developed according to the assumptions in Part one, and the assumptions are sufficient for solving these questions. Subjects do not need any more information to solve the problems. In Part III, subjects are asked to explain and justify their answers.

The problems in the two sections are isomorphic. The difference between the two sets of problems is that in the first set, assumptions are explained with real life examples. For example, red glass is red because only red light can pass through the glass. In the second set, assumptions and problems are presented in abstract symbols. For example, the assumption about red light in the first section is represented by "only X can go through A" in the second section. Red light is replaced by X and red glass replaced by A.

While every subject has some experience and prior knowledge with red light and red glass, the assumption of "only X can go through A" is only a formula to all subjects.

The A-AR model looks like a mathematical model in the sense that it requires logical thinking and computing by following given assumptions in order to find the answer. In mathematics, it is not uncommon to give an unsolvable problem to the student in order to test their understanding of a certain theorem or concept. It would be interesting to observe how the subjects in this study react to a question that is intentionally designed to be unsolvable.

In the A-AR model, rules and conditions for problem-solving are itemized and given to the problem solver. In real life, however, it is rare that people are asked to follow itemized assumptions to solve problems. An important aspect of the methodology in scientific research is that some variables are controlled in order to understand the effects of others. In earlier studies of problem-solving, the subjects were given the problems presented in the form of a passage or text. In such cases, it is questionable whether the problem solvers constructed the same problem representation, including initial and goal state and the conditions of the problems. In the A-AR model, it is made sure that the problem solvers will have the same initial and goal state and the conditions of the problems. That is because the conditions and rules for solving the problems are presented in items and assumptions. The subjects' answers and reasons and my interviews with them provide a support to my claim about the A-AR model.

One advantage of the A-AR model over multiple-choice questions is that it reveals both qualitative and quantitative information. The participants' reasoning of their responses provides a chance to catch a glimpse of their problem-solving process and how they decode and understand the questions in the questionnaire. In short, it is from the A-AR model and the design of isomorphic pairs of questions that the influence of empirical knowledge in the process of problem-solving will be found and understood. In Part II questions on light and vision are asked. The questions in Part IV require the same logical reasoning as those in Part II but are more abstract. Four pairs of isomorphic problems are designed and placed in Parts II and IV for the purpose of understanding whether participants show different problem-solving performance when answering these isomorphic problems.

Part V contains only one question. It asks the participants whether there are any connections or relations between Parts I and II and Parts III and IV and why or why not. There are two purposes for this question. The first purpose is to understand whether 11^{th} and 12^{th} grade science majors (experts) are better at detecting the deep structures of problems and go beyond the surface level of problems than their social science counterparts (novices). The second purpose is that this question can help to know to understand the effect of problem representation, transformation on human problem-solving. Indeed, this question can address whether the understanding of the problem structure affects the participant problem-solving. Table 3.3 outlines the overall structure of the questionnaire.

Table 3.3

Test Construction

Question	Items	Isomorphic Questions
Part II	A-1, A-2, ..., A-9	
Part IV	B-1, B-2, ..., B-8	A-1 and B-2, A-2 and B-1, A-4 and B-4, A-7 and B-7
Part V	C	

Procedures

Either the researcher or the science teacher of each class of participants administered the questionnaire-taking process. Considering the fact that my earlier study participants finished the questionnaire in 50 minutes, I predicted that 50 minutes should be sufficient for the participants. As a precaution, 6 students were asked to answer the questionnaire to make sure of the appropriateness of the questionnaire language before I formally started to collect data, I timed them and made sure 50 minutes would be enough. A period of 55 minutes was asked to be reserved for this study.

The first few minutes are spent explaining the purpose and the rules of the study to the participants. The following messages are made clear to the participants before they begin. (1) It is emphasized that this is a questionnaire, not a test. (2) The questions are designed to understand their logic thinking ability, not physics knowledge. (3) Participants are not required to put their names or any personal identification information on the questionnaire. (4) All the information collected is confidential and only the researchers have access to the data. In short, these messages are to ensure the participants that this questionnaire is a survey, not an exam, and their performance wouldn't affect their school grade or future study. (5) The participants have the right to stop answering at any point they wish. (6) The participants have the right to change their answer any time. They may cross out the original answers, but not erase them. The participants are given a chance to bring up questions about the questionnaire or the study itself. These questions are answered in front of the whole class. Please see the cover page of the questionnaire in Appendix A.

Rating

The purpose of this study is to ascertain how empirical knowledge andlogical reasoning ability affect participants' problem-solving processes. Therefore, for quantitative analysis, each question in Part IV is given two grades: answering grade and reasoning grade, and each question in Part II is given one grade: reasoning grade. There are three different possible grades: 1 is given for an answer that shows that the participant had a right answer with the right problem-solving process, 0 for the participant's answer is wrong or the right answer comes from an incorrect problem-solving process, and 3 for missing data, including blank and answer without reasoning. For the reasoning grade, there are 4 different possible grades: 2 is given for a reasoning that shows the participant uses the given facts only to solve the problem, 0 for a reasoning that shows the participant uses empirical knowledge only to solve the problem, 1 for partially facts and partially empirical knowledge, and 3 for missing data. Where there is a score of 3 (missing data), in reasoning grade, this item will be removed from data analysis.

Part V contains only a question, item C, which is about whether there are relations between the two sets of questions in Parts II and IV. One score is assigned to the answer: reasoning score. For the reasoning score, 3 types of scores are given: 0 for not detecting the correct relations, 1 for detecting the correct relations, and 3 for missing data. For reasoning to be counted as correct, it must point out either the comparable relations between light and substances, glass and filter, or non-transparent object and strainer or the fact that both sets of questions are about filtering and passing through. Table 3.4 outlines the overall structure of the questionnaire.

Table 3.4

Grading Construction

Item	Rating	Range
Part II	Reasoning Grade	0,1,2,3
Part IV	Answering Grade	0,1,2
	Reasoning Grade	0,1,2,3
Part V	Reasoning Grade	0,1,2

Two judges grade the questionnaires. One of them is a researcher in this study, who has a strong background in math and psychometrics. The other judge is from outside the study who has a background in linguistics and language education. These two judges have worked together before in my qualifying study. For the current study, since the questionnaire has been adjusted and changed to a certain extent, the questionnaires from the six sample students were used for the two judges to practice grading. Any divergence in their grading was brought up and discussed together. If no agreement could be made, this grade was removed from the database.

Scale

There are four kinds of scale used for data analysis of the research questions of this paper.

Thelogical reasoning ability scale. This scale is based on the answer grading on Part IV items and is used to understand thelogical reasoning ability development of Taiwanese secondary school students. For each sub-group, there are two parts for this scale:logical reasoning ability item scales andlogical reasoning ability total scale. logical reasoning ability item scales are denoted by B_i, where i is from 1 to 8. B_i is equal to the i-item's answering grade in Part V. Because item B_8 does not have good item discrimination, B_5 is not counted in the totallogical reasoning ability. Therefore, in thelogical reasoning ability total scale, B_{av} is the average of B_i, i is from 1 to 7.

The effects of empirical knowledge scale. The effects of empirical knowledge scales show how participants use empirical knowledge when they solve the problems. Because there are two resources: the reasoning grades in Parts II and IV questions, to detect the empirical knowledge effects, the effects of empirical knowledge scales are the difference between the using of empirical knowledge in solving Parts II and IV questions.

There exist two kinds of scale in the effects of empirical knowledge scale for Part II and IV: item reasoning scale and average reasoning scale. A_i and R_j are denoted the reasoning item scales in Part II and Part IV, respectively, where A_i , $i = 1,2,\cdots,9$, is equal to i^{th} item's reasoning grade in Part II and R_j , $j = 1,2,\cdots,8$, is equal to j^{th} item's reasoning grade and i is from 1 to 9 and j is from 1 to 8.

The average reasoning scales for Parts II and IV are A_{av} and R_{av}, respectively. Based on item A_5 is neither a filter problem nor container problem and item A_5 and B_8 without good item latent plots, none of them is counted in A_{av} or R_{av}. Therefore, A_{av} is the average of A_i, where $i = 1,\cdots,4,6,\cdots,9$, and R_{av} is the average of R_j, where $j = 1,2,\cdots,7$. The rationale for eliminating item A_5 and B_8 from this study will be presented in a later section.

The different scales between isomorphic problems. Four pairs of isomorphic problems are designed to understand whether participants have different problem-solving performances when answering these isomorphic problems. In this study, different problem-solving performances of isomorphic problems are based on the use of empirical knowledge. Therefore, there are four different scales for the four pairs of isomorphic problems: Δ_1, Δ_2, Δ_3, and Δ_4 to find out how empirical knowledge influences the participants' problem-solving process. The computing processes of Δ_1, Δ_2, Δ_3, and Δ_4 are as follows:

Δ_1 = the reasoning grade of item B2 –the reasoning grade of item A1

$= R_2 - A_1$

Δ_2 = the reasoning grade of item B1 – the reasoning grade of item A2

$= R_1 - A_2$

Δ_3 = the reasoning grade of item B4– the reasoning grade of item A4

$= R_4 - A_4$

Δ_4 = the reasoning grade of item B7– the reasoning grade of item A7

$= R_7 - A_7$

Overall, positive or negative or 0 are three possible outcomes for the computing of the effects of empirical knowledge scale. The positive sign in this scale shows that the participant used more empirical knowledge in Part IV questions than in those of Part II. The negative sign in this scale shows that the participant uses more empirical knowledge in Part II questions than in Part IV. Zero means that the participant's use of empirical knowledge in Part II and Part IV is even.

The understanding of problem structure scale. This scale is used to describe whether a participant's understanding of the structures of Parts I and II questions, and Parts III and IV questions are the same. It is the grading of item C and denoted by S. Table 3.5 outlines the overall structure of the scales.

To make sure the data analysis is reliable, the item properties and the reliability of questionnaire are checked first.

Table 3.5

The Structures of Scales

Scale	Notation	Category	Range	Type
Thelogical reasoning ability scale	B_i	logic ability item scale	0, 1, 2	Discrete
	B_{av}	logic ability total scale	From 0 to 2	
The effects of empirical knowledge scale	A_i (for Part II)		0, 1 ,2	
	R_i (for Part IV)	Item scale	0,1,2	Discrete
	A_{av} and R_{av}	total scale	From 0 to 2	
The different scale for Isomorphic problems	Δ_i	The different scale for isomorphic problems	-2, -1, 0, 1 ,2	Discrete
The understanding of problem Structure scale	S	The understanding of problem structure scale	0,1	Discrete

Item Properties

The major target of item properties is to find and delete the irrelevant item(s) and to improve the test reliability. There are two resources to detect item properties: classical item analysis and item response theory (IRT). The mean, standard deviation, and item-total correlation of each item can be found from classical item analysis. IRT gets the latent trait model item plots for each item. Tables 3.6 and 3.7 are the items' mean, standard deviation, and item-total correlation.

Based on Tables 3.6 and 3.7, items A_3 and A_1 are the easiest one and the hardest one for item A's, respectively. Similarly, items B_1 and B_6 are the easiest one and the hardest one for item A's, respectively. The latent trait model item plots of item A's and B's are shown in Figure 3.1.

Table 3.6

Mean, Standard Deviation, and Item-Total Correlation for Item A's

Item	Mean	Standard Deviation	Difficulty	Discrimination	Item-Total Correlation
A1	.322	.467	.788	.973	.714
A2	.761	.426	-1.701	.973	.708
A3	.768	.422	-1.770	.973	.705
A4	.425	.494	.254	.973	.754
A5	.415	.493	.301	.973	.562
A6	.415	.493	.301	.973	.735
A7	.420	.494	.278	.973	.733
A8	.647	.478	-.868	.973	.664
A9	.217	.412	1.459	.973	.600

Table 3.7

Mean, Standard Deviation, and Item-Total Correlation for Item B's

Item	Mean	Standard Deviation	Difficulty	Discrimination	Item-Total Correlation
B1	.916	.278	-1.719	1.832	.666
B2	.877	.328	-1.229	2.991	.759
B3	.872	.334	-1.199	2.988	.776
B4	.863	.344	-1.144	2.984	.778
B5	.598	.490	-.292	3.000	.675
B6	.316	.465	.657	1.551	.502
B7	.740	.439	-1.133	.672	.636
B8	.417	.493	.642	.038	.487

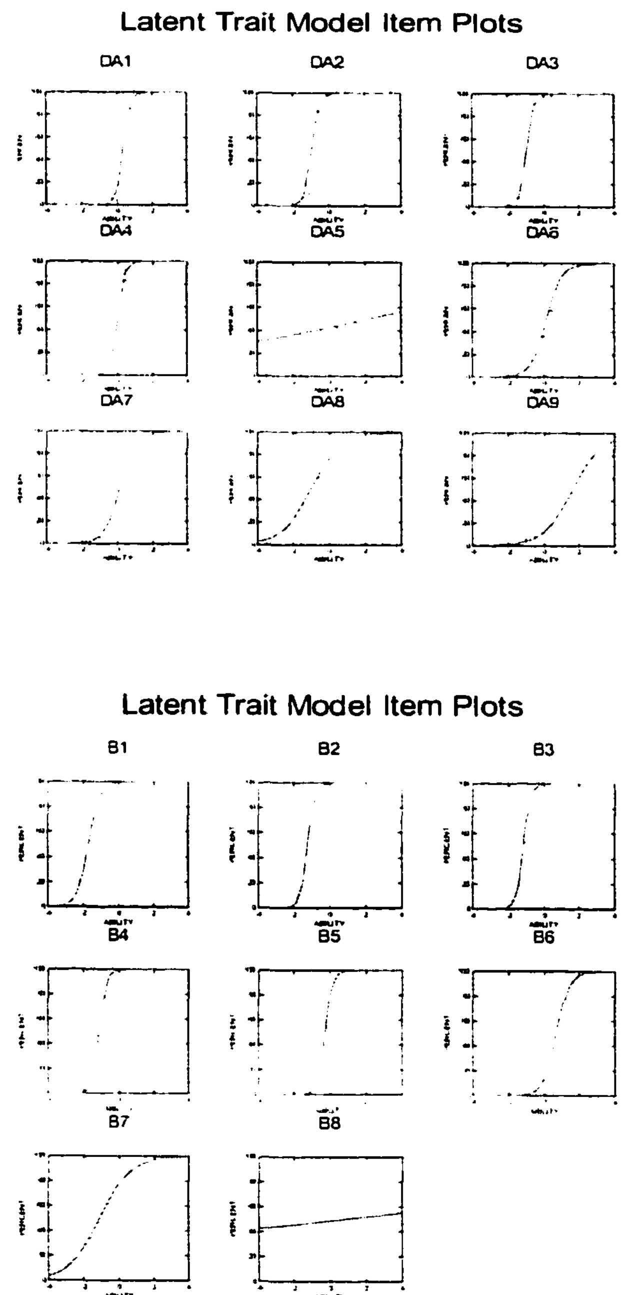

Figure 3.1. Latent trait model item plots of item A's and B's.

The latent trait model item plots of item A_5 and B_8 are linear. It means that both of these two items do not have good item discrimination. Moreover, their item-total correlations, .562 and .487, are the lowest for item A's and B's, respectively.

Reliability

Cronbach's alpha is computed for the internal consistency estimate of reliability of the total items, items in Part II (light and vision question), and items in Part IV (abstract question). Because both of the alphas (see Table 3.8) of the items in Parts II and IV are increasing after removing A_5 and B_8, respectively, and both of the item latent trait plots of them are linear, then A_5 and B_8 are removed from the following computing. Therefore, item A_5 and B_8 are removed from the items. The overall, Part II, and Part IV reliabilities are .886, .912, and .804, respectively.

Table 3.8

The Cronbach's Alpha for Questionnaire

Item	All Item	All Item except A5 and B8	All A Item	All A Item except A5	All B Item	All B Item expect B8
Cronbach's alpha	.889	.886	.908	.912	.785	.804

Data Analysis Methods

Factor analysis, analysis of variance (ANOVA), and regression are used in data analysis for the current study. Factor analysis is used to investigate the relations among items to figure out the main factors that influence participants in their problem-solving process in various sub-groups. Specifically, factor analysis provides information about how many groups of homogeneous questions there are in the questionnaire and whether the same pair of isomorphic questions falls into the same group of homogeneous questions.

To answer the research questions and examine the hypothesis, there are three different of ANOVA tests. The first ANOVA test focuses on the performances of problem-solving on age/grade level, gender, and major variance in all items. It contains three subparts: the development oflogical reasoning ability, the effect of empirical knowledge on solving Part IV problems, and the effect of empirical knowledge on solving Part II problems. The second ANOVA test still focuses on the performances of problem-solving over age/grade level, gender, and major variance. Here the dependent variable is the pairs of isomorphic problems instead of all items, because the problem-solving process and strategy of the same pair of isomorphic problems is assumed to be the same. The third ANOVA test is used to examine the relation between the understanding of the structure of two problem sets and the problem-solving strategy. The data for the three ANOVA tests are collected from all of the 466 participants except 13 participants with missing data.

Linear regression is used to examine howlogical reasoning ability, empirical knowledge, and the interference with each other in the problem-solving process of Taiwanese secondary students. First, linear regression checks the individual effects oflogical reasoning ability and empirical knowledge with the problem-solving process on different grade level and major subgroups. Second, linear regression helps to describe the total effects oflogical reasoning ability and empirical knowledge with the problem-solving process with different grade levels and major subgroups. For the targets, there are two bivariate linear regression tests and one multiple linear regression test with three independed variables. The predicted variable for these regression tests is the effect of empirical knowledge on questions in Part IV. logical reasoning ability scale and the effect of empirical knowledge on Part II are the independent variables for the two bivariate linear regression tests and both of them and their interference are the independent variables for the multiple regression. Because the linear regression tests describe the effects oflogical reasoning ability and the influence of empirical knowledge on problem-solving process at different grade levels and major subgroups, the row scores of thelogical reasoning ability, and the effect of empirical knowledge of Partss II and IV problems are replaced by their standard scores ($z-scores$) for these regression tests. The equations for these regression tests are as follows:

$$\hat{y}_i = \beta_{LA-1i} z_{LA} + c_{1i}$$

$$\hat{y}_j = \beta_{EK-1j} z_{EK} + c_{2j}$$

$$\hat{y}_k = \beta_{LA-k} z_{LA} + \beta_{EK-k} z_{EK} + \beta_{I-k} (z_{LA} \cdot z_{EK}) + c_{4-k},$$

where z_{LA} and z_{EK} are the $z-scores$ of thelogical reasoning ability and the effect of empirical knowledge on the problem-solving process in Part II with different subgroups, respectively, $z_{LA} \cdot z_{EK}$ is the interference oflogical reasoning ability (LA) and the effect of empirical knowledge (EK) and equal to the product of the z-scores oflogical reasoning ability and the effect of empirical knowledge on light and vision questions, and i, j and k are the index of the subgroups.

CHAPTER 4

DATA ANALYSIS AND RESULTS

Data Analysis

The process of data analysis follows from the methods developed in chapter 3. The first, factor analysis is used to understand the relationships among items.

Relations Among Items: Factor Analysis

Factor analysis tries to probe into whether the main factors that are combined by items on Parts II and IV are significantly different among various subgroups. For the present target, the logical reasoning ability scales and the effect of empirical knowledge scale on Part II, except items 5 and 8 in Parts II and IV, respectively, are separately used for the data of factor analysis on various subgroups. There are three rules for picking the main factors from Factor Analysis. First rule, the factor with eigenvalue which is greater than 1 should be picked up. Second, the factors above the elbow on the screen plot should be picked up. And, the third is based on my personal opinion about the questionnaire. Finally, the comparisons of all of the main factors in the various subgroups are made.

Factor analysis of logical reasoning ability. The dimensionality of the 7 items from the scales of logical reasoning ability of Part IV (except item 8) was analyzed using maximum likelihood factor analysis on different subgroups. For the whole group, there exits only one main factor: the knowledge of light and vision. The factor contains items

B5, B6, and B7 and accounted for 47.4% of the item variance. The corresponding correlation matrix is shown in Table 4.1.

Table 4.1

Item Correlation Matrix of Logical Reasoning Ability on Whole Group

	B1	B2	B3	B4	B5	B6	B7
B1	1.000						
B2	.668	1.000					
B3	.653	.694	1.000				
B4	.526	.705	.751	1.000			
B5	.307	.368	.379	.430	1.000		
B6	.151	.203	.194	.222	.363	1.000	
B7	.328	.385	.468	.469	.313	.175	1.000

For the female subgroup, there are two main factors: the knowledge of light and the knowledge of vision. The first factor contains items B5, B6, and B7 and accounted for 48.7% of the item variance. The second factor contains items B2, B3 and B4 and accounted for 19.0% of the item variance. The corresponding correlation matrix is shown in Table 4.2.

Table 4.2

Item Correlation Matrix of Logical Reasoning Ability on Female Subgroup

	B1	B2	B3	B4	B5	B6	B7
B1	1.000						
B2	.642	1.000					
B3	.642	.642	1.000				
B4	.568	.730	.769	1.000			
B5	.262	.384	.412	.434	1.000		
B6	.119	.172	.201	.213	.340	1.000	
B7	.388	.402	.527	.497	.333	.219	1.000

For the female subgroup, there are two main factors: the knowledge of light and the knowledge of vision. The first factor contains items B5, B6, and B7 and accounted for 46.5% of the item variance. The second factor contains items B2, B3 and B4 and accounted for 20.0% of the item variance. The corresponding correlation matrix is shown in Table 4.3.

Table 4.3

Item Correlation Matrix of Logical Reasoning Ability on Male Subgroup

	B1	B2	B3	B4	B5	B6	B7
B1	1.000						
B2	.705	1.000					
B3	.670	.757	1.000				
B4	.487	.674	.729	1.000			
B5	.349	.352	.345	.427	1.000		
B6	.184	.238	.186	.231	.387	1.000	
B7	.271	.369	.406	.441	.293	.131	1.000

For the social science major group, there exits only one main factor. This factor contains items B5, B6, and B7 and accounted for 42.8% of the item variance. The corresponding correlation matrix is shown in Table 4.4.

Table 4.4

Item Correlation Matrix of Logical Reasoning Ability on Social Science Major Subgroup

	B1	B2	B3	B4	B5	B6	B7
B1	1.000						
B2	.788	1.000					
B3	.758	.704	1.000				
B4	.467	.429	.660	1.000			
B5	.336	.290	.309	.384	1.000		
B6	.155	.166	.143	.177	.339	1.000	
B7	.236	.291	.371	.421	.117	.206	1.000

For the science major group, there exits only one main factor. This factor contains items B5, B6, and B7 and accounted for 48.9% of the item variance. The corresponding correlation matrix is shown in Table 4.5.

Table 4.5

Item Correlation Matrix of Logical Reasoning Ability on Science Major Subgroup

	B1	B2	B3	B4	B5	B6	B7
B1	1.000						
B2	.414	1.000					
B3	.503	.782	1.000				
B4	.383	.931	.840	1.000			
B5	.135	.399	.379	.429	1.000		
B6	.029	.220	.220	.236	.311	1.000	
B7	.272	.572	.510	.524	.407	.212	1.000

For 7^{th} grade subgroup, there are two main factors. The first factor contains items B5, B6, and B7 and accounted for 45.8% of the item variance. The second factor contains items B2 and B3 and accounted for 19.3% of the item variance. The corresponding correlation matrix is shown in Table 4.6.

Table 4.6

Item Correlation Matrix of Logical Reasoning Ability on 7^{th} Grade Subgroup

	B1	B2	B3	B4	B5	B6	B7
B1	1.000						
B2	.615	1.000					
B3	.932	.558	1.000				
B4	.557	.615	.509	1.000			
B5	.320	.400	.356	.320	1.000		
B6	.284	.289	.305	.284	.417	1.000	
B7	.371	.201	.411	.264	.194	.006	1.000

For the 8th group, there exits only one main factor. This factor contains items B1, B2, B3, B4, B5, B6, and B7 and accounted for 68.2% of the item variance. The corresponding correlation matrix is shown in Table 4.7.

Table 4.7

Item Correlation Matrix of Logical Reasoning Ability on 8th Grade Subgroup

	B1	B2	B3	B4	B5	B6	B7
B1	1.000						
B2	.874	1.000					
B3	.936	.936	1.000				
B4	.819	.941	.881	1.000			
B5	.693	.585	.649	.633	1.000		
B6	.397	.291	.372	.320	.574	1.000	
B7	.650	.650	.713	.702	.535	.337	1.000

For the 9th group, there exits only one main factor. The factor contains items B3, B4, B5, B6, and B7 and accounted for 47.3% of the item variance. The corresponding correlation matrix is shown in Table 4.8.

Table 4.8

Item Correlation Matrix of Logical Reasoning Ability on 9th Grade Subgroup

	B1	B2	B3	B4	B5	B6	B7
B1	1.000						
B2	.398	1.000					
B3	.229	.341	1.000				
B4	.384	.632	.824	1.000			
B5	.121	.297	.443	.470	1.000		
B6	-.033	.162	.123	.142	.176	1.000	
B7	-.071	.152	.523	.469	.404	.007	1.000

For the 10th group, there exits only one main factor. This factor contains items B2, B3, B4, B5, B6, and B7 and accounted for 43.6% of the item variance. The corresponding correlation matrix is shown in Table 4.9.

Table 4.9

Item Correlation Matrix of Logical Reasoning Ability on 10th Grade Subgroup

	B1	B2	B3	B4	B5	B6	B7
B1	1.000						
B2	.675	1.000					
B3	.473	.851	1.000				
B4	.404	.735	.735	1.000			
B5	.180	.267	.156	.357	1.000		
B6	.010	.140	.015	.190	.357	1.000	
B7	.327	.360	.235	.384	.201	.152	1.000

For the 11th group, there exits only one main factor. This factor contains items B5, B6, and B7 and accounted for 48.9% of the item variance. The corresponding correlation matrix is shown in Table 4.10.

Table 4.10

Item Correlation Matrix of Logical Reasoning Ability on 11th Grade Subgroup

	B1	B2	B3	B4	B5	B6	B7
B1	1.000						
B2	.690	1.000					
B3	.539	.780	1.000				
B4	.344	.600	.803	1.000			
B5	.290	.355	.420	.465	1.000		
B6	164	.238	.238	.263	.412	1.000	
B7	.248	.359	.442	.536	.259	.245	1.000

For the 12th group, there exits only one main factor. The factor contains items B5, B6, and B7 and accounted for 42.4% of the item variance. The corresponding correlation matrix is shown in Table 4.11.

Table 4.11

Item Correlation Matrix of Logical Reasoning Ability on 12th Grade Subgroup

	B1	B2	B3	B4	B5	B6	B7
B1	1.000						
B2	.582	1.000					
B3	.659	.659	1.000				
B4	.582	.791	.659	1.000			
B5	.230	.322	.227	.322	1.000		
B6	.056	.147	.097	.147	.253	1.000	
B7	.260	.495	.391	.378	.235	.194	1.000

Factor analysis of empirical knowledge. The dimensionality of the eight items from the scales of empirical knowledge of Part II (except item 5) was analyzed using maximum likelihood factor analysis on different subgroups. For the whole group, there is one main factor: the knowledge of light and vision. This factor contains items A1, A2, A3, A4, A6, A7, A8, and A9 and accounted for 62.3% of the item variance. The corresponding correlation matrix is shown in Table 4.12.

For the female group, there is one main factor: the knowledge of light and vision. The factor contains items A1, A2, A3, A4, A6, A7, A8, and A9 and accounted for 63.5% of the item variance. The corresponding correlation matrix is shown in Table 4.13.

Table 4.12

Item Correlation Matrix of Empirical Knowledge on Whole Group

	A1	A2	A3	A4	A6	A7	A8	A9
A1	1.000							
A2	.700	1.000						
A3	.671	.902	1.000					
A4	.781	.710	.715	1.000				
A6	.501	.448	.464	.567	1.000			
A7	.537	.512	.522	.582	.702	1.000		
A8	.463	.522	.528	.537	.712	.586	1.000	
A9	.465	.402	.379	.457	.533	.571	.461	1.000

Table 4.13

Item Correlation Matrix of Empirical Knowledge on Female Subgroup

	A1	A2	A3	A4	A6	A7	A8	A9
A1	1.000							
A2	.681	1.000						
A3	.634	.873	1.000					
A4	.790	.706	.710	1.000				
A6	.494	.495	.530	.590	1.000			
A7	.500	.515	.536	.594	.737	1.000		
A8	.458	.574	.568	.522	.736	.626	1.000	
A9	.462	.433	.412	.430	.589	.592	.550	1.000

For the male group, there is one main factor: the knowledge of light and vision. The factor contains items A1. A2. A3. A4. A6. A7. A8. and A9 and accounted for 62.3% of the item variance. The corresponding correlation matrix is shown in Table 4.14.

Table 4.14

Item Correlation Matrix of Empirical Knowledge on Male Subgroup

	A1	A2	A3	A4	A6	A7	A8	A9
A1	1.000							
A2	.719	1.000						
A3	.708	.934	1.000					
A4	.775	.715	.720	1.000				
A6	.505	.395	.396	.547	1.000			
A7	.572	.506	.509	.572	.663	1.000		
A8	.468	.465	.487	.552	.688	.544	1.000	
A9	.470	.372	.347	.482	.483	.555	.373	1.000

For the social science major subgroup, there is one main factor: the knowledge of light and vision. The factor contains items A1, A2, A3, A4, A6, A7, A8, and A9 and accounted for 58.9% of the item variance. The corresponding correlation matrix is shown in Table 4.15.

Table 4.15

Item Correlation Matrix of Empirical Knowledge on Social Science Major Subgroup

	A1	A2	A3	A4	A6	A7	A8	A9
A1	1.000							
A2	.657	1.000						
A3	.608	.877	1.000					
A4	.689	.704	.700	1.000				
A6	.403	.366	.401	.563	1.000			
A7	.479	.477	.515	.495	.634	1.000		
A8	.389	.426	.472	.515	.708	.539	1.000	
A9	.429	.345	.343	.391	.510	.472	.400	1.000

For the social science major subgroup, there is one main factor: the knowledge of light and vision. The factor contains items A1, A2, A3, A4, A6, A7, A8, and A9 and accounted for 64.9% of the item variance. The corresponding correlation matrix For the social science major subgroup, there is one main factor: the knowledge of light and vision. The factor contains items A1, A2, A3, A4, A6, A7, A8, and A9 and accounted for 58.9% of the item variance. The corresponding correlation matrix is shown in Table 4.16.

Table 4.16

Item Correlation Matrix of Empirical Knowledge on Science Major Subgroup

	A1	A2	A3	A4	A6	A7	A8	A9
A1	1.000							
A2	.781	1.000						
A3	.710	.910	1.000					
A4	.829	.783	.739	1.000				
A6	.596	.591	.509	.513	1.000			
A7	.580	.641	.570	.561	.598	1.000		
A8	.571	.692	.600	.568	.686	.593	1.000	
A9	.452	.445	.406	.481	.514	.540	.454	1.000

For 7^{th} grade subgroup, there is one main factor: the knowledge of light and vision. The factor contains items A1, A2, A3, A4, A6, A7, A8, and A9 and accounted for 52.8% of the item variance. The corresponding correlation matrix is shown in Table 4.17.

Table 4.17

Item Correlation Matrix of Empirical Knowledge on 7^{th} Grade Subgroup

	A1	A2	A3	A4	A6	A7	A8	A9
A1	1.000							
A2	.507	1.000						
A3	.502	.863	1.000					
A4	.659	.583	.580	1.000				
A6	.395	.351	.415	.347	1.000			
A7	.314	.314	.311	.278	.587	1.000		
A8	.345	.475	.508	.291	.616	.515	1.000	
A9	.282	.329	.245	.186	.459	.521	.495	1.000

For 8^{th} grade subgroup, there is one main factor: the knowledge of light and vision. The factor contains items A1, A2, A3, A4, A6, A7, A8, and A9 and accounted for 69.1% of the item variance. The corresponding correlation matrix is shown in Table 4.18.

Table 4.18

Item Correlation Matrix of Empirical Knowledge on 8^{th} Grade Subgroup

	A1	A2	A3	A4	A6	A7	A8	A9
A1	1.000							
A2	.806	1.000						
A3	.828	.931	1.000					
A4	.910	.782	.804	1.000				
A6	.672	.548	.590	.765	1.000			
A7	.627	.510	.542	.696	.810	1.000		
A8	.613	.556	.640	.706	.795	.663	1.000	
A9	.478	.334	.333	.492	.574	.518	.556	1.000

For 9th grade subgroup, there is one main factor: the knowledge of light and vision. The factor contains items A1, A2, A3, A4, A6, A7, A8, and A9 and for 57.5% of the item variance. The corresponding correlation matrix is shown in Table 4.19.

Table 4.19

Item Correlation Matrix of Empirical Knowledge on 9th Grade Subgroup

	A1	A2	A3	A4	A6	A7	A8	A9
A1	1.000							
A2	.748	1.000						
A3	.700	.862	1.000					
A4	.614	.554	.611	1.000				
A6	.283	.288	.342	.587	1.000			
A7	.354	.426	.486	.705	.758	1.000		
A8	.390	.339	.333	.570	.773	.576	1.000	
A9	.383	.460	.397	.464	.463	.556	.368	1.000

For 10th grade subgroup, there is one main factor: the knowledge of light and vision. The factor contains items A1, A2, A3, A4, A6, A7, A8, and A9 and accounted for 70.9% of the item variance. The corresponding correlation matrix is shown in Table 4.20.

Table 4.20

Item Correlation Matrix of Empirical Knowledge on 10th Grade Subgroup

	A1	A2	A3	A4	A6	A7	A8	A9
A1	1.000							
A2	.784	1.000						
A3	.767	.987	1.000					
A4	.885	.838	.850	1.000				
A6	450	.517	.532	.523	1.000			
A7	.605	.640	.651	.625	.865	1.000		
A8	.444	.638	.632	.518	.757	.690	1.000	
A9	.612	.562	.570	.555	.639	.831	.575	1.000

For 11th grade subgroup, there is only one main factor: the knowledge of light and vision. The factor contains items A1, A2, A3, A4, A6, A7, A8, and A9 and accounted for 63.4% of the item variance. The corresponding correlation matrix is shown in Table 4.21.

Table 4.21

Item Correlation Matrix of Empirical Knowledge on 11th Grade Subgroup

	A1	A2	A3	A4	A6	A7	A8	A9
A1	1.000							
A2	.760	1.000						
A3	.654	.869	1.000					
A4	.788	.761	.707	1.000				
A6	.532	.522	.459	.526	1.000			
A7	.477	.550	.525	.468	.589	1.000		
A8	.539	.635	.586	.576	.689	.559	1.000	
A9	.359	.454	.379	.316	.411	.461	.459	1.000

For 12th grade subgroup, there is one main factor: the knowledge of light and vision. The factor contains items A1, A2, A3, A4, A6, A7, A8, and A9 and accounted for 61.2% of the item variance. The second factor contains items A6, A7, A8, and A9 and accounted for 13.3% of the item variance. The corresponding correlation matrix is shown in Table 4.22.

Table 4.22

Item Correlation Matrix of Empirical Knowledge on 12th Grade Subgroup

	A1	A2	A3	A4	A6	A7	A8	A9
A1	1.000							
A2	.683	1.000						
A3	.679	.916	1.000					
A4	.745	.728	.742	1.000				
A6	.495	.436	.475	.538	1.000			
A7	.597	.560	.571	.586	.642	1.000		
A8	.414	.448	.491	.502	.689	.548	1.000	
A9	.484	.359	.388	.513	.562	.557	.392	1.000

Summary for factor analysis. There does exist one factor for participants' logical reasoning ability, even though there might exist two main factors for participants' logical reasoning ability for some subgroups. If the second main factor exists for a subgroup, this factor might contain item B2 or B3 or B4. Due to the fact that both of the item difficulty (see Table 3.7) of combining items and the corresponding item variance is very low, this factor is minor and should be ignored. The main factor for participants' logical reasoning ability for all subgroups must contain items B5, B6, and B7. The item difficulties for these three items are higher than other items, except item B8 that has the highest one. The factor shows the advanced logical reasoning ability to solve these problems. For all of subgroups, there is one main factor for their empirical knowledge for using solving problems. It contains all of the items on Part II but A5.

The following ANOVA test shows that, there does not exist any difference on the participants' logical reasoning ability; but the effects of using empirical knowledge have statistical significance among various subgroups, although there just exists one main factor for their empirical knowledge.

First ANOVA Test: Performance on All Items

The first ANOVA test is conducted to understand the development of participants' logical reasoning ability and whether grade level, major, gender, and their interactions lead to significant differences in the participants' performance in answering questions and justifying their answers, and whether there are any interactions among the three factors of grade, major, and gender for the scale of each question in Parts II and IV. The design for this test is described as three $6 \times 2 \times 3$ ANOVA tests (the number of grade level by the gender difference by the major difference). It contains six variables: a factor classifying the grade levels of participants, a second factor classifying the gender of participants, a third factor classifying the major of participants, and one dependent variable: the performance (the effect of empirical knowledge of light and vision problems, the average reasoning grade for Part II problems, or the logical reasoning ability, the average answering grade for Part IV problems, or the effect of empirical knowledge of abstract problems, the average reasoning grade for Part IV problems) of participants' problem-solving process.

For the development of logical reasoning ability. From the ANOVA test for participants' logical reasoning ability, the logical reasoning ability (hereafter, LA) of Taiwanese secondary school students is not affected neither by main factors: grade level ($p= .772$), gender ($p=.993$), and major ($p=.183$) nor by interactions of: gender by grade ($p=.425$), major by grade ($p=.788$), gender by major ($p=.455$), and gender by major by grade ($p=.419$). The corresponding means and mean plots are shown in Tables 4.23, 4.24, and 4.25 and Figures 4.1, 4.2, 4.3, 4.4, and 4.5.

Table 4.23

Summary Statistics of Logical Reasoning Ability Effect on Part II Problem Solving for Each Gender

	Mean	Std. Error	95% Confidence Interval	
Gender			Lower Bound	Upper Bound
Female	.731	.017	.698	.765
Male	.730	.017	.696	.765

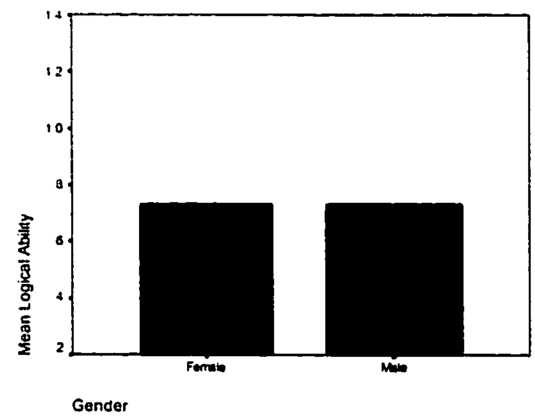

Figure 4.1. Graph of meaning for Table 4.23.

Table 4.24
Summary Statistics of Logical Reasoning Ability Effect on Part II Problem Solving for Each Grade

	Mean	Std. Error	95% Confidence Interval	
grade			Lower Bound	Upper Bound
7	.635	.034	.568	.702
8	.686	.037	.613	.759
9	.677	.035	.609	.746
10	.685	.036	.614	.755
11	.781	.024	.733	.828
12	.801	.024	.754	.848

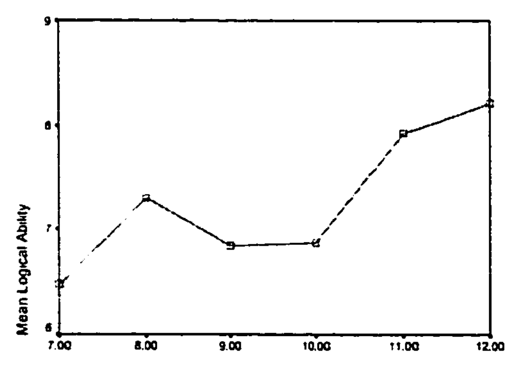

Figure 4.2. Graph of meaning for Table 4.24.

Table 4.25

Summary Statistics of Logical Reasoning Ability Effect on Part II Problem Solving for Each Major

	Mean	Std. Error	95% Confidence Interval	
major			Lower Bound	Upper Bound
no difference	.671	.018	.636	.706
social science	.768	.024	.721	.815
science	.814	.024	.766	.861

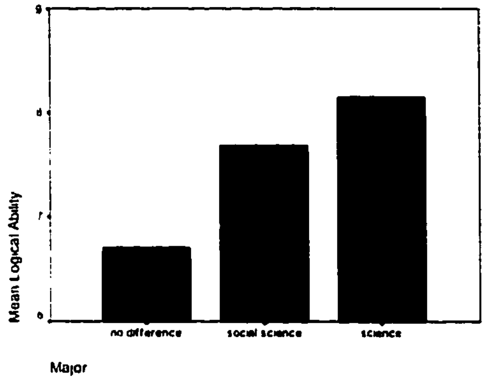

Figure 4.3. Graph of meaning for Table 4.25.

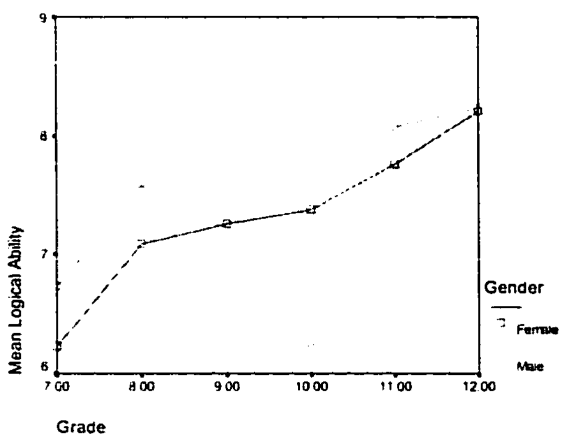

Figure 4.4. Mean graph of logical reasoning ability by grade for both genders on Part II problem solving.

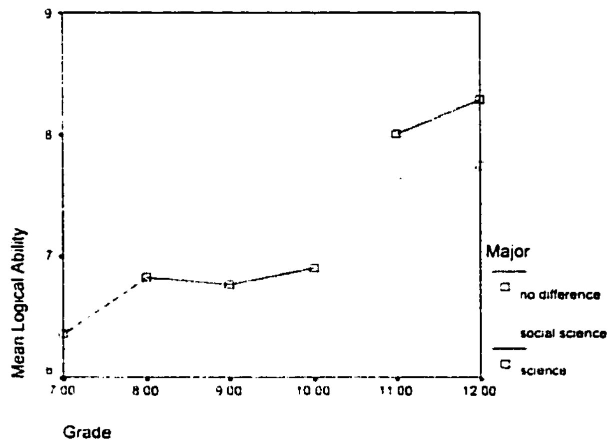

Figure 4.5. Mean graph of logical reasoning ability by grade for both majors on Part II problem solving.

The effect of empirical knowledge on solving Part II problems. The effect of empirical knowledge (hereafter, EK) on solving Part II problems is affected by main factor age and major, but not by gender. Their p-values are .001, .005, and .656, respectively. Using Fisher's Least Significant Difference (hereafter, LSD) for the performance of the effect of empirical knowledge on solving Part IV problems on different grades, there is statistical significance between 11th grade and 7th grade, and between 12th grade and 7th, 8th, 9th, and 10th grade. Their p-values are $< .05$. Their means and means plots are shown in Tables 4.26, 4.27, and 4.28 and Figures 4.6, 4.7, and 4.8.

Table 4.26.

Summary Statistics of Empirical Knowledge Effect on Part II Problem Solving for Each Gender

	Mean	Std. Error	95% Confidence Interval	
Gender			Lower Bound	Upper Bound
Female	1.201	.040	1.122	1.280
Male	1.218	.040	1.139	1.297

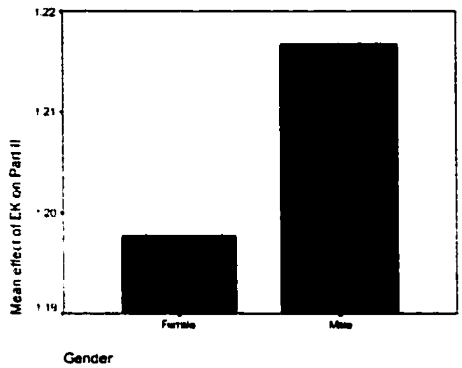

Figure 4.6. Graph of meaning for Table 4.26.

Table 4.27

Summary Statistics of Empirical Knowledge Effect on Part II Problem Solving for Each Grade

	Mean	Std. Error	95% Confidence Interval	
grade			Lower Bound	Upper Bound
7	.996	.080	.839	1.154
8	1.300	.084	1.135	1.466
9	1.400	.081	1.240	1.559
10	1.246	.080	1.088	1.404
11	1.086	.056	.977	1.196
12	1.281	.056	1.171	1.391

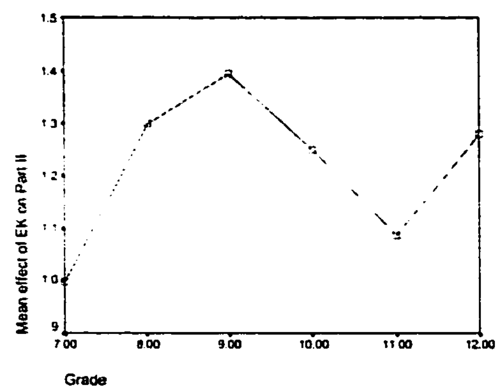

Figure 4.7. Graph of meaning for Table 4.27.

Table 4.28

Summary Statistics of Empirical Knowledge Effect on Part II Problem Solving for Each Major

	Mean	Std. Error	95% Confidence Interval	
major			Lower Bound	Upper Bound
no difference	1.236	.041	1.156	1.316
social science	1.072	.056	.962	1.182
science	1.295	.056	1.185	1.406

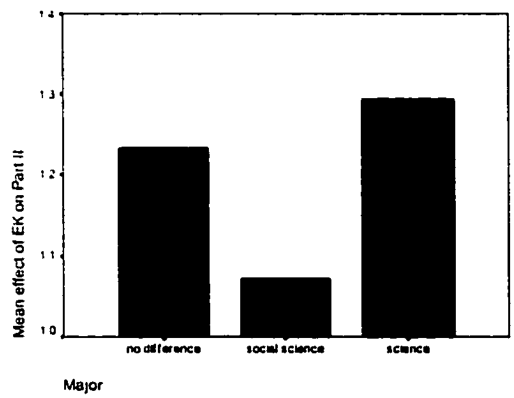

Figure 4.8. Graph of meaning for Table 4.8.

The interactions of gender by grade, gender by major, and gender by grade by major are not statistically significant for the logical reasoning ability of Taiwanese secondary school students. The *p*-values are .349, .058, and .848, respectively. The interactions of grade by major are statistically significant for the logical reasoning ability of Taiwanese secondary school students with *p*-values .045. The corresponding means plots are shown in Figure 4.9 and 4.10.

The effect of empirical knowledge on solving Part IV problems. The effect of empirical knowledge on solving Part IV problems is not affected neither by main factors: grade level (p=.453), gender (p=.270), and major (p=.237) nor by interactions: gender by grade (p=.353), major by grade (p=.402), gender by major (p=.981), and gender by major by grade (p=.988). The corresponding means and mean plots are shown in Tables 4.29, 4.30, and 4.31 and Figures 4.11, 4.12, 4.13, 4.14, and 4.15.

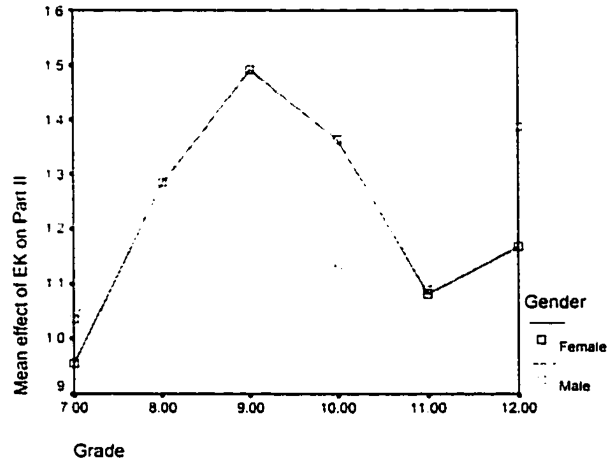

Figure 4.9. Mean graph of empirical knowledge by grade for both genders on Part IV problem solving.

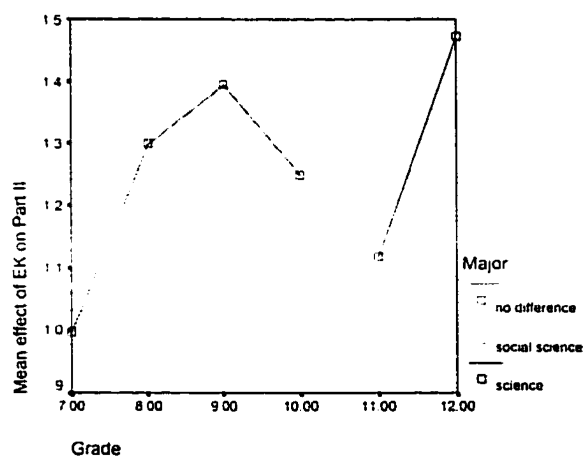

Figure 4.10. Mean graph of empirical knowledge by grade for both majors on Part IV problem solving.

Table 4.29

Summary Statistics of Empirical Knowledge Effect on Part IV Problem Solving for Each Gender

	Mean	Std. Error	95% Confidence Interval	
Gender			Lower Bound	Upper Bound
Female	.152	.029	.009	.209
Male	.209	.029	.152	.266

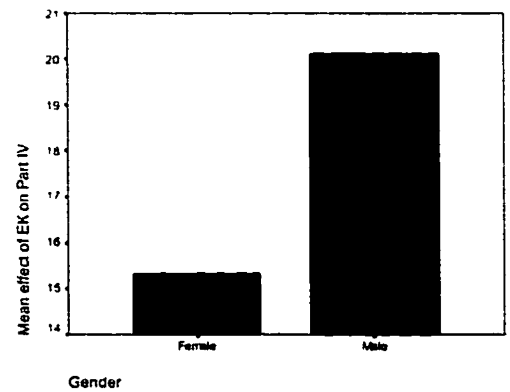

Figure 4.11. Graph of meaning for Table 4.29.

Table 4.30.

Summary Statistics of Empirical Knowledge Effect on Part IV Problem Solving for Each Grade

	Mean	Std. Error	95% Confidence Interval	
Grade			Lower Bound	Upper Bound
7	.285	.056	.174	.396
8	.192	.065	.006	.319
9	.217	.057	.104	.330
10	.284	.059	.167	.401
11	.154	.040	.008	.233
12	.078	.040	.0002	.156

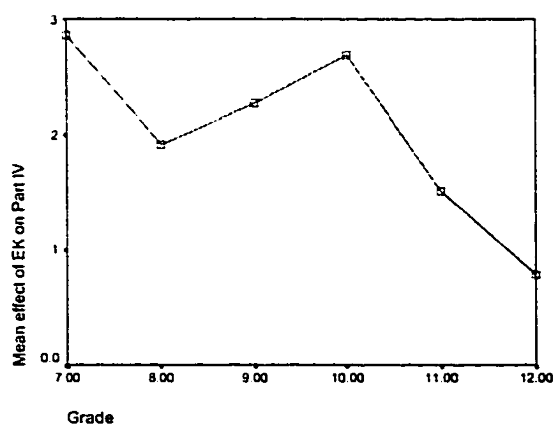

Figure 4.12. Graph of meaning for Table 4.30.

Table 4.31

Summary Statistics of Empirical Knowledge Effect on Part IV Problem Solving For Each Major

	Mean	Std. Error	95% Confidence Interval	
major			Lower Bound	Upper Bound
no difference	.244	.030	.186	.303
social science	.083	.040	.005	.161
science	.149	.040	.071	.228

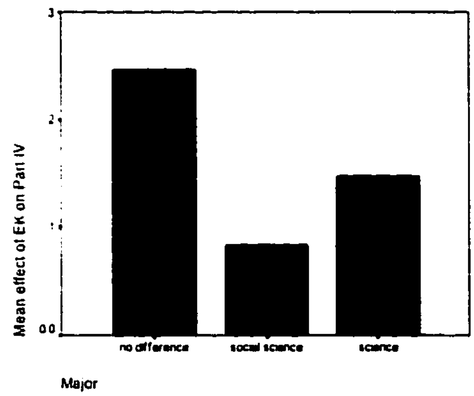

Figure 4.1. Graph of meaning for Table 4.31.

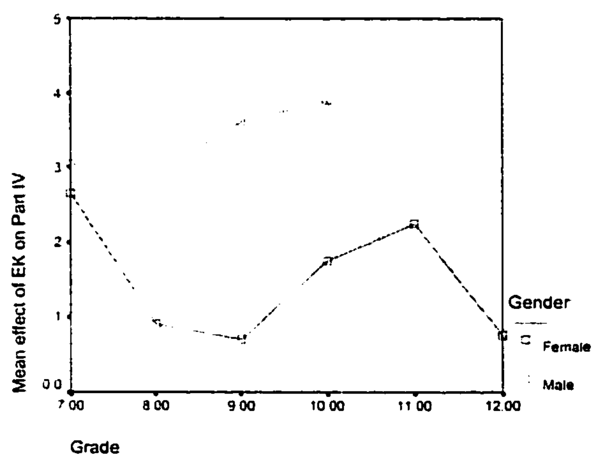

Figure 4.14. Mean graph of empirical knowledge by grade for both genders on Part IV problem solving.

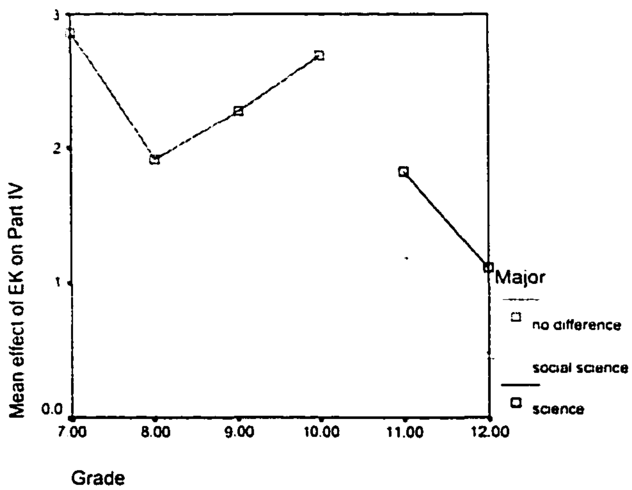

Figure 4.15. Mean graph of empirical knowledge by grade for both majors on Part IV problem solving.

Summary for first ANOVA test. This ANOVA test identifies that there does not exist any difference for participants' logical reasoning ability used to solve the problems, but different subgroups might have a different degree of using EK. Particularly, the effects of EK on the problem solving process are statistically different among different grades or majors. The curve of the effect of EK is S-shape and peak point is 9^{th} grade and through valley point is 11^{th} grade.

The different problem solving performance on isomorphic problems can address the effect of the representation transformation in order to figure out the use of empirical knowledge. The dependent variable for the next ANOVA test is the four pairs of isomorphic problems.

Second ANOVA Test: Performance on Isomorphic Items

Special attention is given to the four pairs of isomorphic questions in Parts II and IV. The problem solving process ought to be the same or similar in the same pair of isomorphic questions. To examine the isomorphic problems is one more way to understand whether empirical knowledge is involved in answering questions. For this purpose, ANOVA examines the scales of the difference answering between the same pair of isomorphic problem, Δ_1, Δ_2, Δ_3, and Δ_4. The design for this test is described as four $6 \times 2 \times 3$ ANOVA tests (the number of grade level by the gender difference by the major difference). It contains six variables: a factor classifying the grade levels of participants, a second factor classifying the gender of participants, a third factor classifying the major of participants, and one depended variable: the difference performance on the same pair of isomorphic problems: Δ_1, Δ_2, Δ_3, and Δ_4.

The first pair of isomorphic problems. From the ANOVA test, the different answering between first isomorphic problems (Δ_1) is affected by age/grade level (.002) and major (.001) but not by gender (.353). The corresponding mean plotting is shown in Figures 4.16, 4.17, and 4.18.

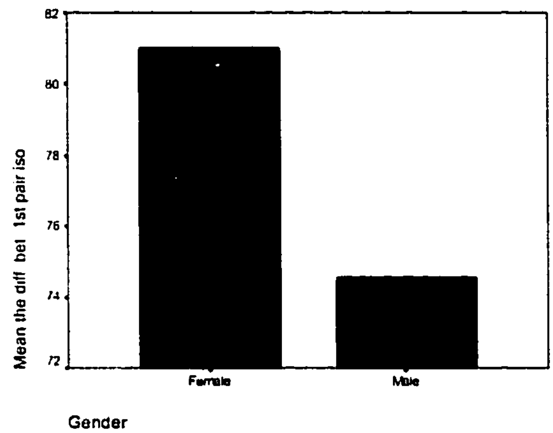

Figure 4.16. Mean of empirical knowledge different effect between the first pair of isomorphic problems for each gender.

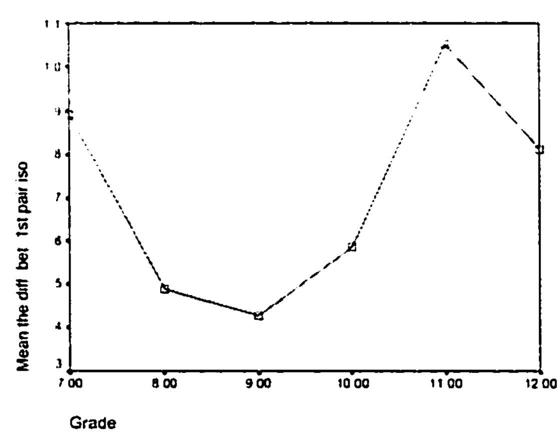

Figure 4.17. Mean of empirical knowledge different effect between the first pair of isomorphic problems for each grade.

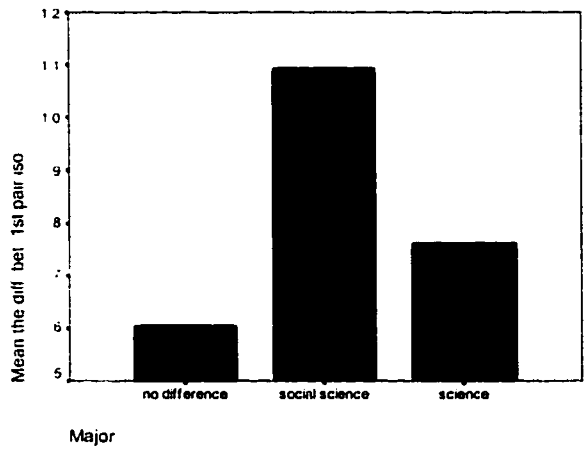

Figure 4.18. Mean of empirical knowledge different effect between the first pair of isomorphic problems for each major.

Using LSD, there are statistical contract significances between 7th grade and 8th, 9th, and 10th grade and the p-values are .008, .001, and .034, respectively, 8th grade and 11th grade and the p-value is < .005, 9th grade and 11th and 12th grade and the p-values are <.005 and .002, respectively, and 10th grade and 11th grade and the p-value is <.005.

The interactions of gender by grade, major by grade, grade by major, and gender by grade by major are not statistical significance for different answering of the first pair of isomorphic problems. The p-values are .589, .130, .206, and .564, respectively. The corresponding plots are seen in Figures 4.19 and 4.20.

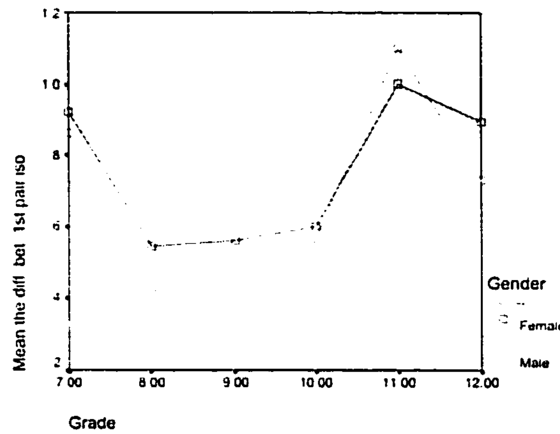

Figure 4.19. Mean of empirical knowledge different effect between the first pair of isomorphic problems for grade by gender.

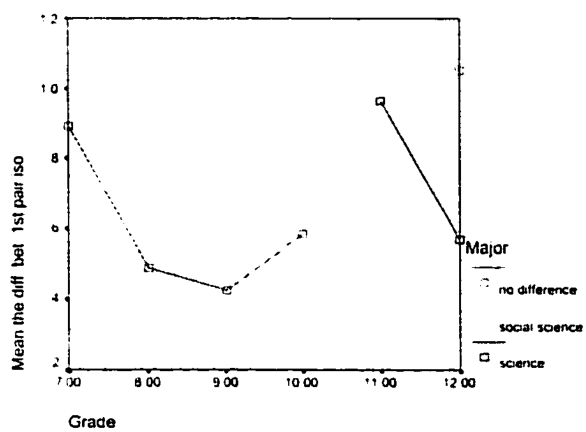

Figure 4.20. Mean of empirical knowledge different effect between the first pair isomorphic problem for grade by major.

The second pair of isomorphic problems. From the ANOVA test, the different performance between second isomorphic problems (Δ_2) are not affected by any of the main factors: age/grade level (p=.722), major (p=.511), gender (p=.461) or interactions: gender by grade (p=.157), gender by major (p=.124), grade by major (p=.172), and gender by major by grade (p=.598). The corresponding mean plotting and histogram are shown in Figures 4.21, 4.22, 4.23, 4.24, and 4.25.

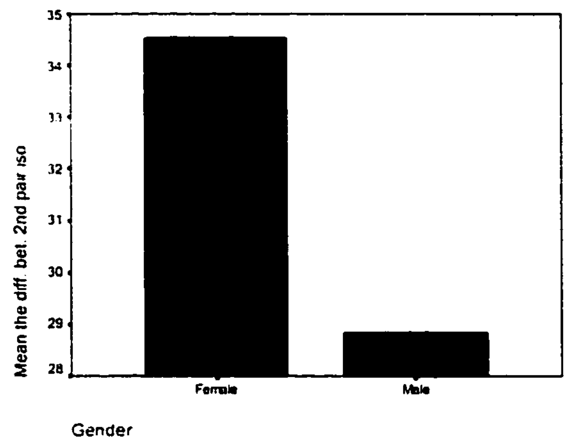

Figure 4.21. Mean of empirical knowledge different effect between the second pair of isomorphic problems for each gender.

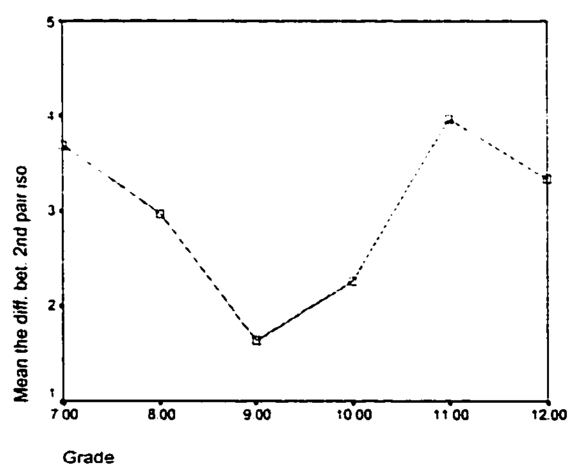

Figure 4.22. Mean of empirical knowledge different effect between the second pair of isomorphic problems for each grade.

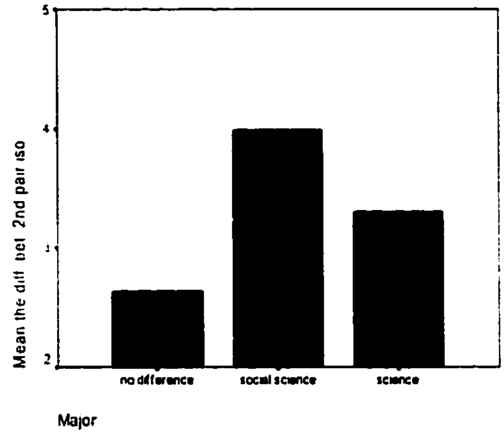

Figure 4.23. Mean of empirical knowledge different effect between the second pair of isomorphic problems for each major.

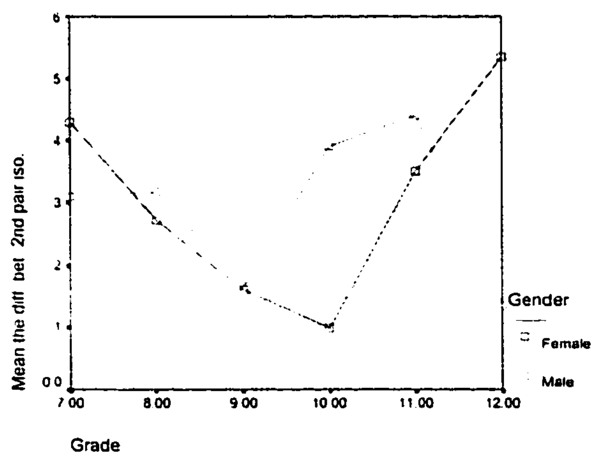

Figure 4.24. Mean of empirical knowledge different effect between the second pair of isomorphic problems for grade by gender.

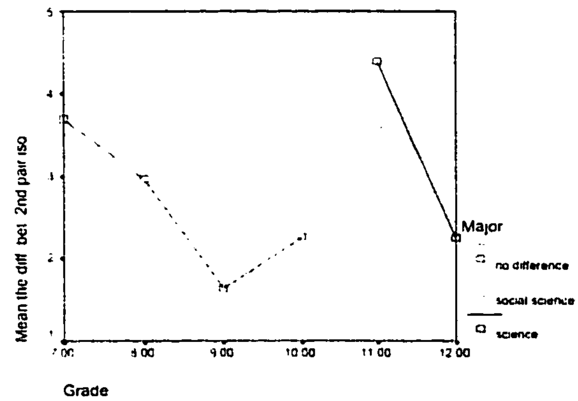

Figure 4.25. Mean of empirical knowledge different effect between the second pair of isomorphic problems for grade by major.

The third pair of isomorphic problems. From the ANOVA test, the different answering between the third pair of isomorphic problems (Δ_3) are affected by age/grade level (<.001) and major (.005) but not by gender (.694). The mean plots and histograms of the mean of empirical knowledge different effect for the third pair of isomorphic problems are shown in Figures 4.26, 2.27, and 4.28.

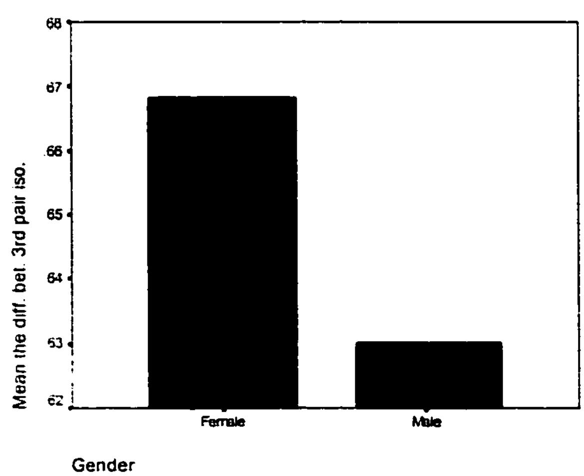

Figure 4.26. Mean of empirical knowledge different effect between the third pair of isomorphic problems for each gender.

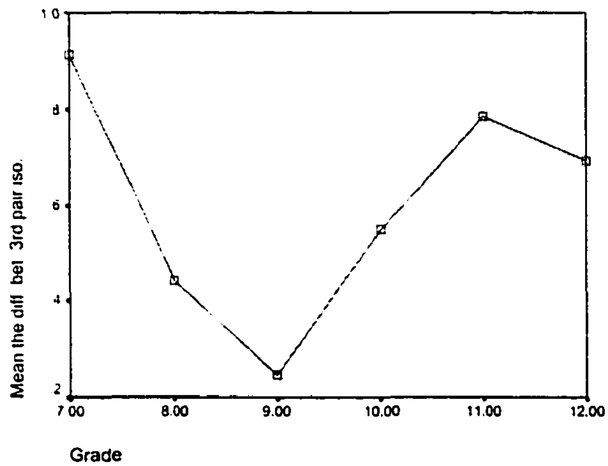

Figure 4.27. Mean of empirical knowledge different effect between the third pair of isomorphic problems for each grade.

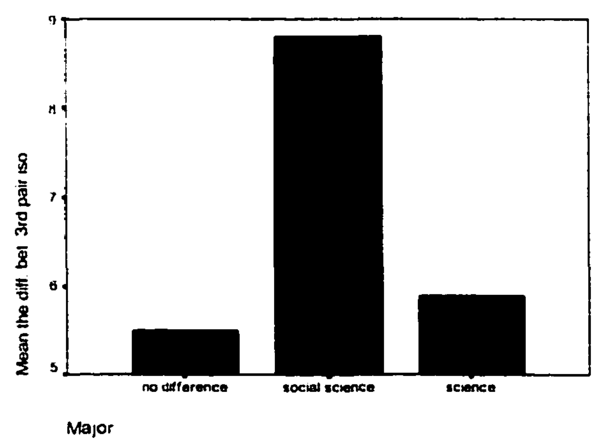

Figure 4.28. Mean of empirical knowledge different effect between the third pair of isomorphic problems for each major.

Using LSD, there are statistical contract significances between 7th grade and 8th, 9th, and 10th grade and the *p*-values are .004, <.001, and .019, respectively, 8th grade and 11th and 12th grade and the *p*-value is .017 and .001, respectively, and 9th grade and 11th and 12th grade and the *p*-values are <.001 and .001, respectively.

The different answering of the third pair of isomorphic problems is affected by grade by major but not by gender by grade or major by grade or gender by grade by major. The p-values are .046, .649, .618, and .922, respectively. The mean plots of the mean of empirical knowledge different effect for the third pair of isomorphic problems are shown in Figures 4.29 and 4.30.

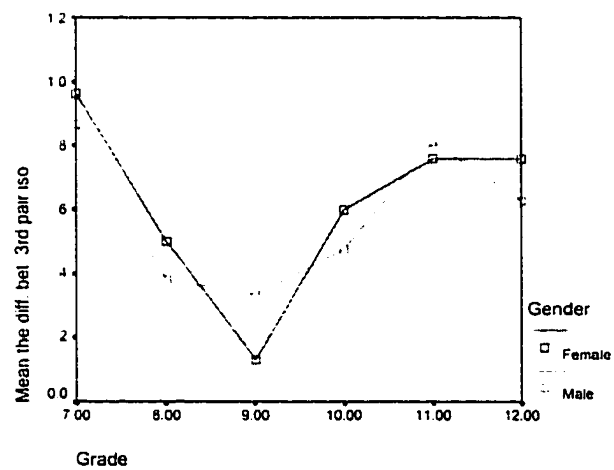

Figure 4.29. Mean of empirical knowledge different effect between the third pair of isomorphic problems for grade by gender.

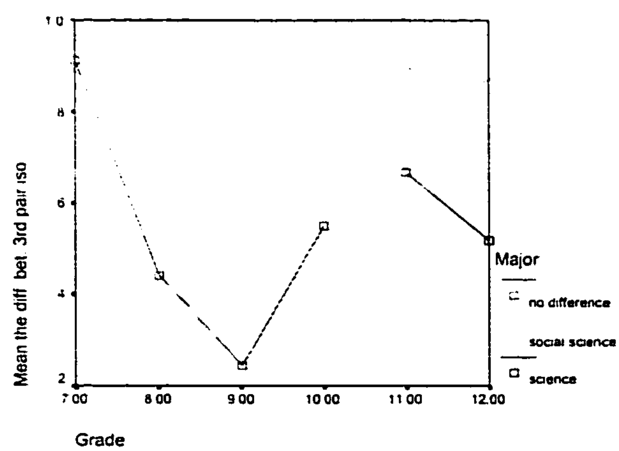

Figure 4.30. Mean of empirical knowledge different effect between the third pair of isomorphic problems for grade by major.

The fourth pair of isomorphic problems. From the ANOVA test, the different answering between the fourth pair of isomorphic problems (Δ_4) is affected by major (p=.007), but neither by grade (p=.073) nor by gender (p=.092). The mean plots and histograms of the mean of empirical knowledge different effect for the fourth pair isomorphic problems are shown in Figures 4.31, 4.32, 4.33.

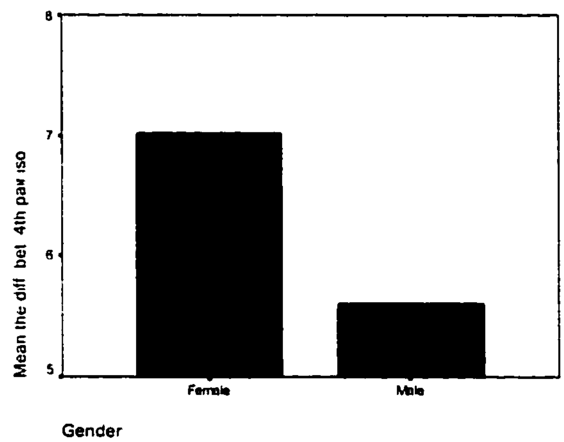

Figure 4.31. Mean of empirical knowledge different effect between the fourth pair of isomorphic problems for each gender.

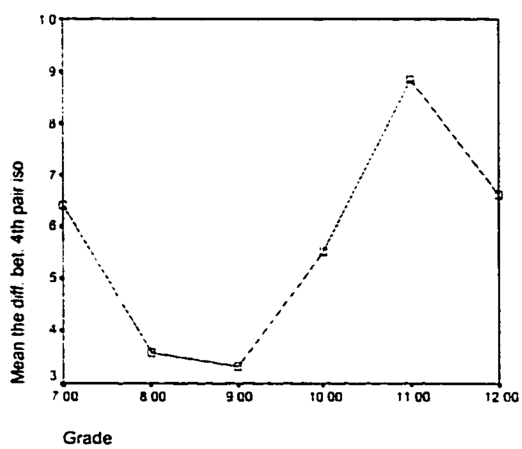

Figure 4.32. Mean of empirical knowledge different effect between the fourth pair of isomorphic problems for each grade.

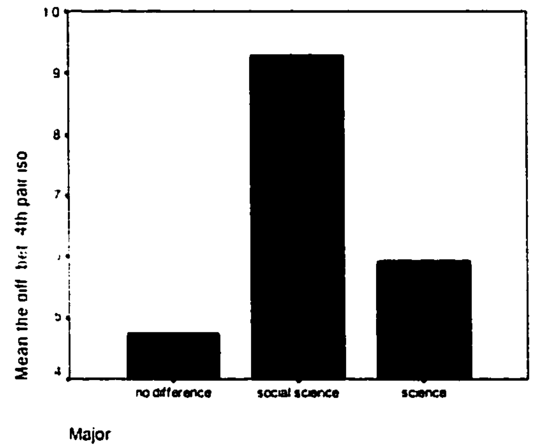

Figure 4.33. Mean of empirical knowledge different effect between the fourth pair of isomorphic problems for each major.

The different answering of the third pair of isomorphic problems is affected by grade by major but not by gender by grade or major by grade or gender by grade by major. The p-values are .001, .972, .124, and .618, respectively. The mean plots of the mean of empirical knowledge different effect for the fourth pair of isomorphic problems are shown in Figures 4.34 and 4.35.

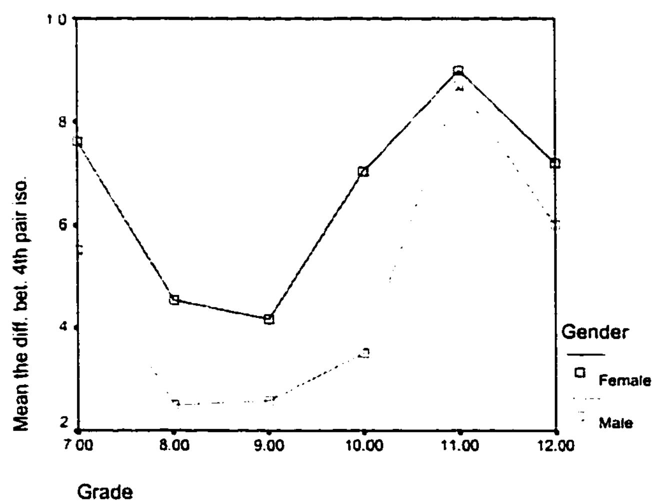

Figure 4.34. Mean of empirical knowledge different effect between the fourth pair of isomorphic problems for grade by gender.

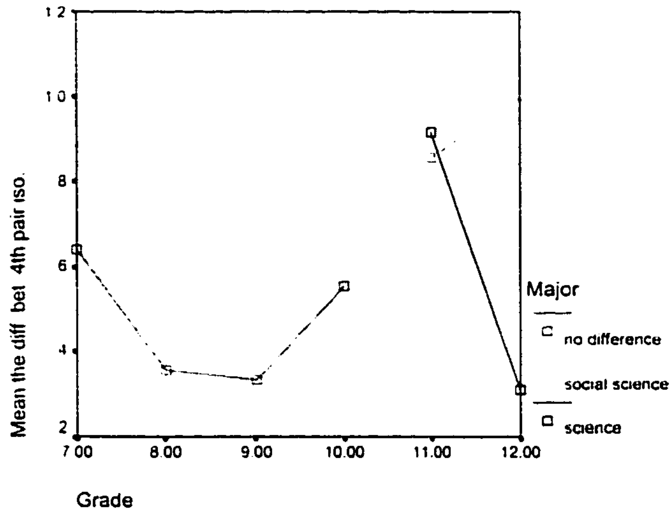

Figure 4.35. Mean of empirical knowledge different effect between the fourth pair of isomorphic problems for grade by major.

Summary of second ANOVA test. For the above ANOVA tests on the four pairs of isomorphic problems, there are statistical significances among different grade subgroups, but neither on different gender nor major subgroups. The curves of the different scale mean for four pairs of isomorphic problem are S-shape, and the 9th and 11th grades are at the critical points, minimum and maximum, respectively. Moreover, according the absolute values of different scale mean for the same isomorphic problems, the 9th grade have the most consistent performance on the isomorphic problem, while the 11th grade performance on the isomorphic problem are the most inconsistent.

Third ANOVA Test: Understanding of the Problem Structures Versus Performance

The dependent variables for the second ANOVA test are the four pairs of isomorphic problems. It helps to understand whether participants using the same problem strategy to solve the same structure problems. The next ANOVA test examines the relation between the understanding of the structure between two parts of the problem and the corresponding performance. Therefore, it can help to figure out whether understanding problem structure affects problem solving process from global view.

ANOVA tests on the understanding problem structure. There are three one-way ANOVA conducted to evaluate the relationships between the understanding of the structure of two-part problems and the difference of gender, major, and grade.

Based on F (1, 421) =5.737, p= .017, and η^2= .013, there exists within comparisons of the scores of understanding of the structure of two-part problems (S) between different gender subgroups (the mean of S for female and male are .624 and .510, respectively.) Based on F (1, 213) =8.666, p=.004, and η^2=.039, there exists within comparisons of the scores of understanding of the structure of two-part problems between different major subgroups on 11^{th} and 12^{th} grades (the mean of S for social science and science are .575 and .761, respectively.) For grade, F (5, 417) =3.992, p=.002, and η^2=.046, the different grade subgroups had statistical significance on the scores of understanding of the structure of two-part problems. There exist statistical significances between grade 7^{th} and 11^{th} and 12^{th} and both *p*-values are .007. The means of S on different grades and corresponded plots are shown in Table 4.32 and Figures 4.36, 4.37, and 4.38).

Table 4.32

Summary Statistics of Understanding Problem Structure for Each Grade

	Mean	Std. Error	95% Confidence Interval	
Grade			Lower Bound	Upper Bound
7	.429	.065	.301	.557
8	.444	.073	.302	.587
9	.472	.067	.340	.603
10	.500	.066	.370	.630
11	.670	.047	.577	.763
12	.670	.047	.578	.762

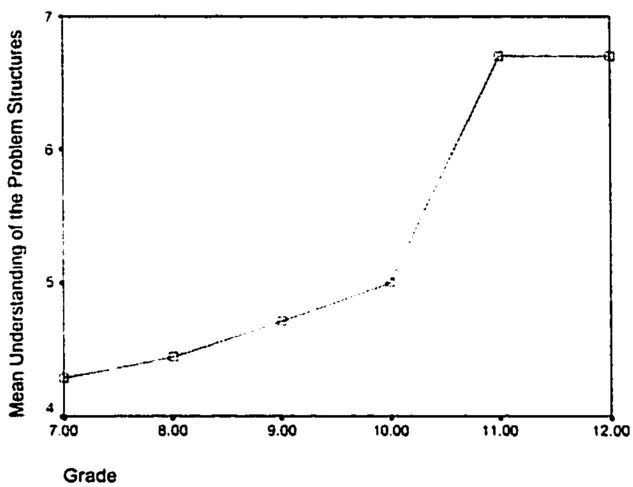

Figure 4.36. Graph of mean for Table 4.32.

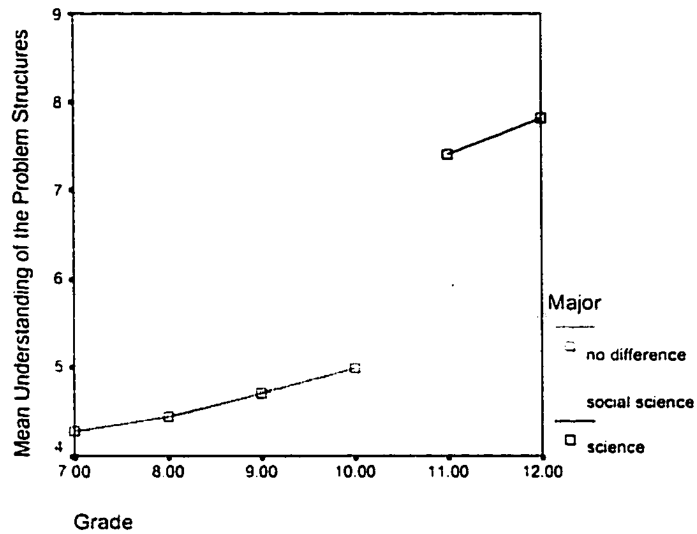

Figure 4.37. Mean of understanding about problem structure for grade by major.

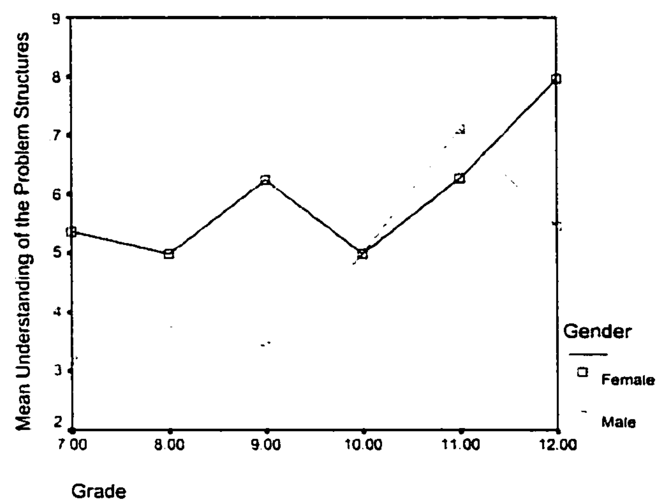

Figure 4.38. Mean of understanding about problem structure for grade by gender.

I focus on those participants who recognize the structure of the two parts of "same structure" problems. To what degree do they employ the same problem solving strategy in each of the two problems, particularly the pairs of isomorphic problems? The tests used to approach this aim are two-way ANOVA tests with dependent variables: the different effect of empirical knowledge on Parts II and IV, Δ_1, Δ_2, Δ_3, and Δ_4, (see Table 3.5) and independent variables S and grade and S and major, and S and gender.

ANOVA tests for interaction of S by grade. Based on the p-values of the interaction of S by grade for the dependent variables: the different effect of empirical knowledge on Parts II and IV, Δ_1, Δ_2, Δ_3, and Δ_4, there exists no statistically significant difference caused by the interaction of S and grade for these dependent variables. The corresponded p-values are .305, .592, .567, .578, and .124, respectively. The mean plots and histograms of the mean of empirical knowledge different effect for the isomorphic problems are shown in Figures 4.39, 4.40, 4.41, 4.42, and 4.43.

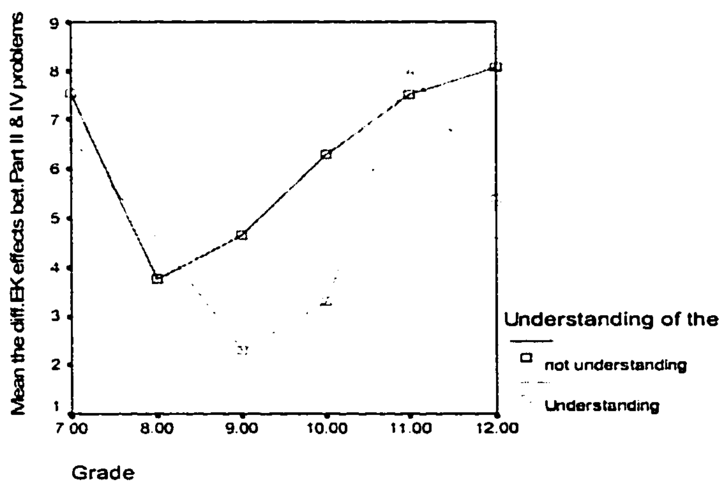

Figure 4.39. Mean of empirical knowledge different effect between the two pairs of problems for the understanding about problem structure by gender.

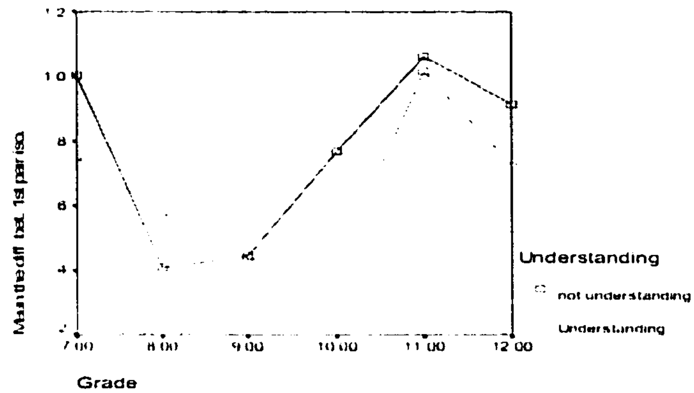

Figure 4.40. Mean of empirical knowledge different effect between the first pair of isomorphic problems for the understanding about problem structure by gender.

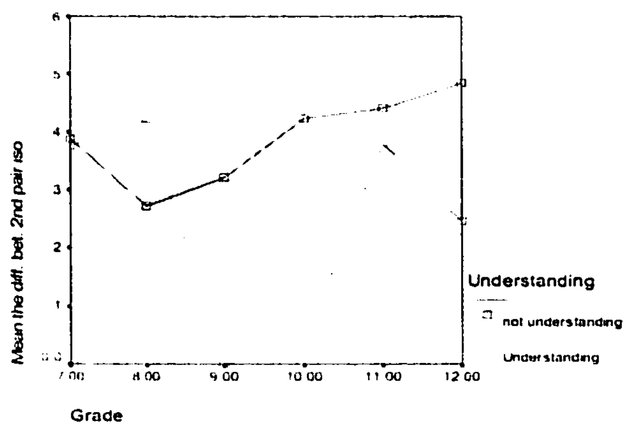

Figure 4.41. Mean of empirical knowledge different effect between the second pair of isomorphic problems for the understanding about problem structure by gender.

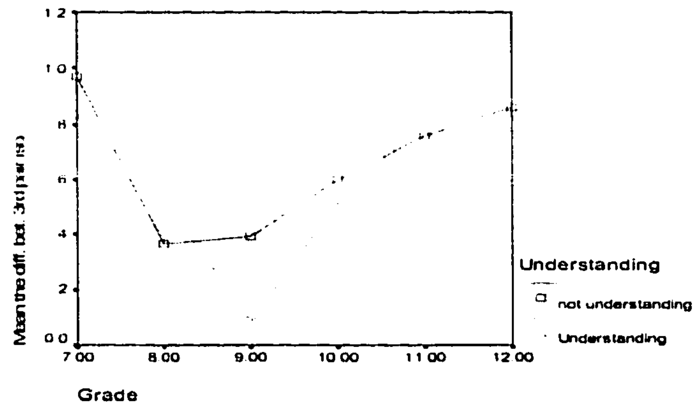

Figure 4.42. Mean of empirical knowledge different effect between the third pair of isomorphic problems for the understanding about problem structure by gender.

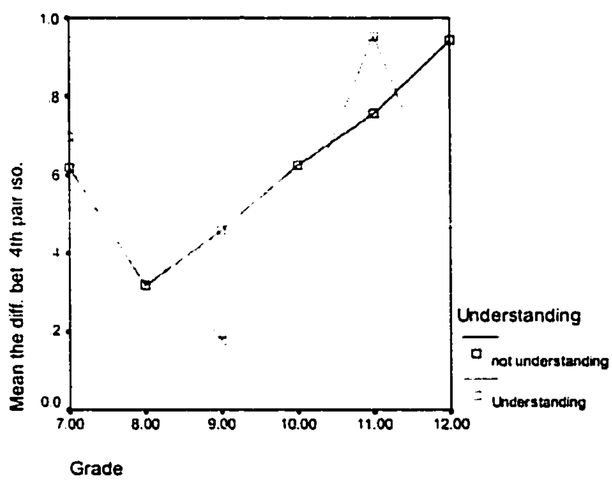

Figure 4.43. Mean of empirical knowledge different effect between the fourth pair of isomorphic problems for the understanding about problem structure by gender.

ANOVA tests for interaction of S by major. Based on the p-values of the interaction of S by grade for the dependent variables: the different effect of empirical knowledge on Parts II and IV. Δ_1, Δ_2, Δ_3, and Δ_4, there exists no statistically significant difference caused by the interaction of S and grade for these dependent variables except Δ_4. The corresponded p-values are .352, .212, .719, .318, and .007, respectively. The mean plots and histograms of the mean of empirical knowledge different effect for the isomorphic problems are shown as following (Figure 4.44, 4.45, 4.46, 4.47, and 4.48):

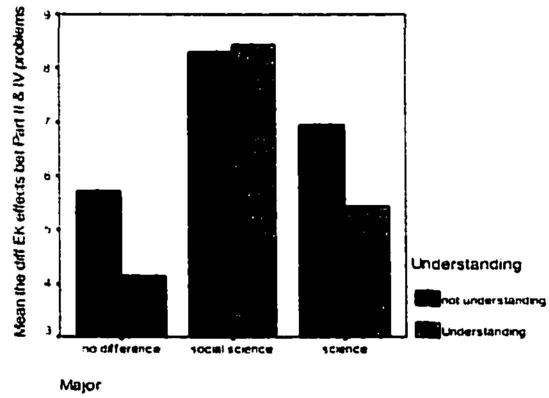

Figure 4.44. Mean of empirical knowledge different effect between the two pairs of problems for the understanding about problem structure by major.

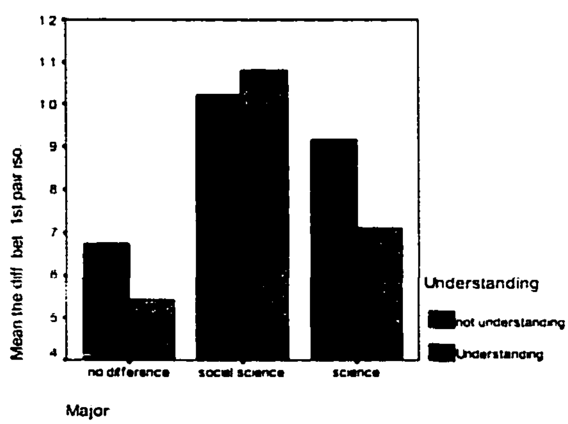

Figure 4.45. Mean of empirical knowledge different effect between the first pair of isomorphic problems for the understanding about problem structure by major.

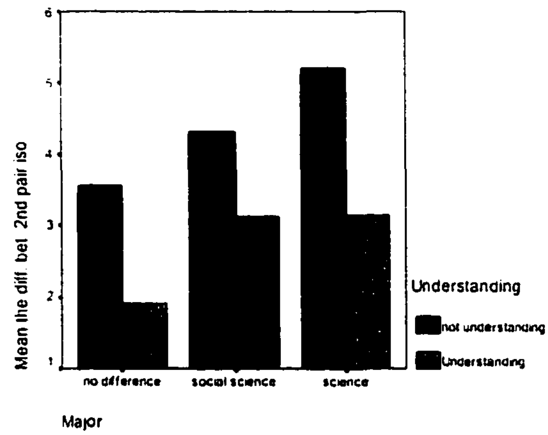

Figure 4.46. Mean of empirical knowledge different effect between the second pair of isomorphic problems for the understanding about problem structure by major.

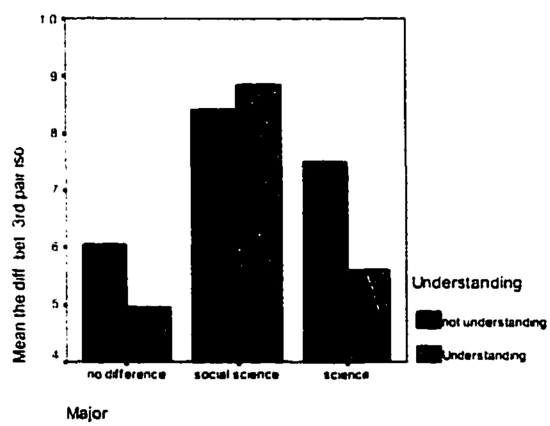

Figure 4.47. Mean of empirical knowledge different effect between the third pair of isomorphic problems for the understanding about problem structure by major.

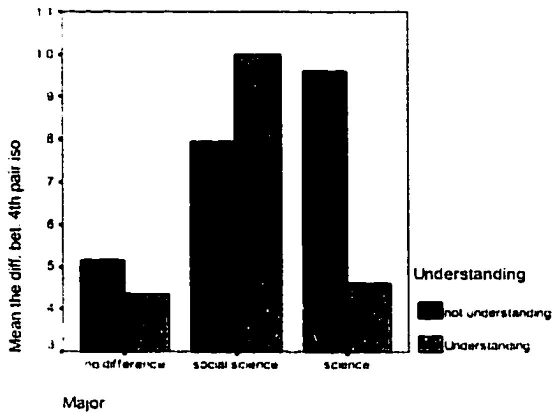

Figure 4.48. Mean of empirical knowledge different effect between the fourth pair of isomorphic problems for the understanding about problem structure by major.

ANOVA tests for interaction of S by gender. Based on the *p*-values of the interaction of S by grade for the dependent variables: the different effect of empirical knowledge on Parts II and IV, Δ_1, Δ_2, Δ_3, and Δ_4, there exists no statistically significant difference caused by the interaction of S and grade for these dependent variables. The corresponded *p*-values are .962, .694, .363, .892, and .112, respectively. The mean plots and histograms of the mean of empirical knowledge different effect for the isomorphic problems are shown in Figures 4.49, 4.50, 4.51, 4.52, and 4.53.

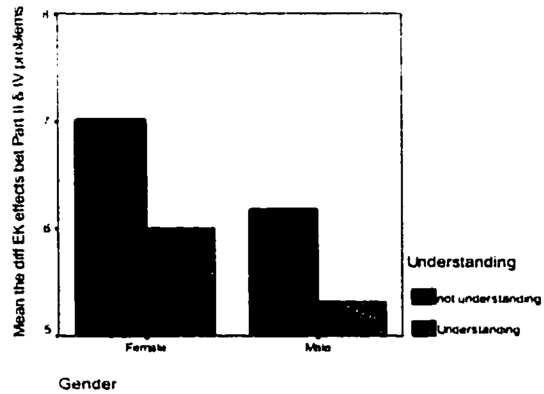

Figure 4.49. Mean of empirical knowledge different effect between the two pairs of problems for the understanding about problem structure by gender.

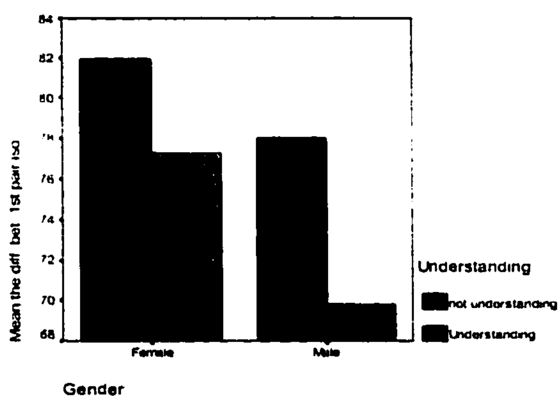

Figure 4.50. Mean of empirical knowledge different effect between the first pair of isomorphic problems for the understanding about problem structure by gender.

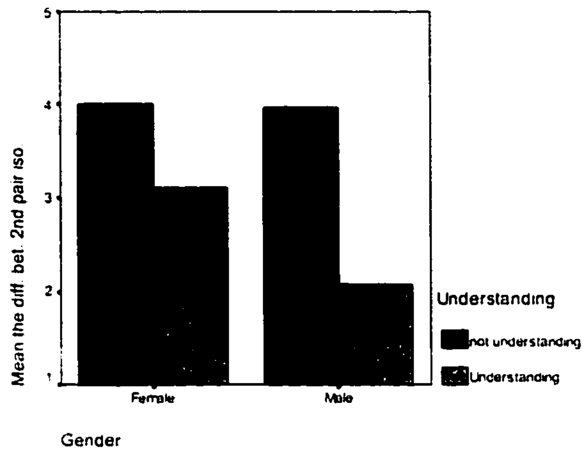

Figure 4.51. Mean of empirical knowledge different effect between the second pair of isomorphic problems for the understanding about problem structure by gender.

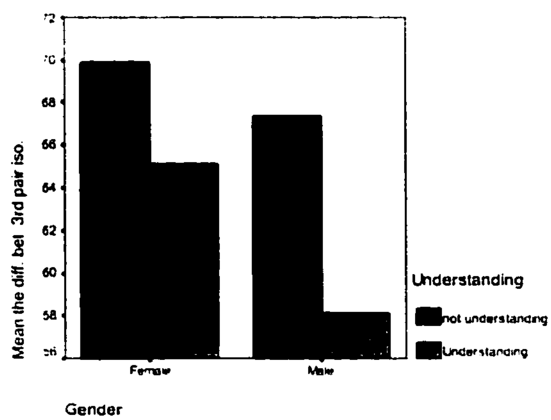

Figure 4.52. Mean of empirical knowledge different effect between the third pair of isomorphic problems for the understanding about problem structure by gender.

Linear Regression Analysis: The Effects of Logical Reasoning Ability and Empirical Knowledge

Two bivariate linear regression tests and one multiple linear regression test with three independent variables are used to understand the participants' performance on the problem-solving process. The score of effect of empirical knowledge on Part IV is the prediction variable explanatory for these three linear regression tests and the logical reasoning ability scale, the effect of empirical knowledge on Part II, and their interference are the independent variables for the three bivariate linear regression tests and all of them are the independent variables for the multiple regression.

The effect of logical reasoning ability on problem solving. The linear regression test that was conducted to evaluate the prediction of the participants' performance on the problem-solving process from the logical reasoning ability on different grade level and major subgroups shows that the curve of coefficients of logical reasoning ability versus age is not monotone. All of the coefficients are negative and with the biggest absolute value at the 8^{th} grade and minimum one at the 11^{th} science major subgroup. The summary of this analysis and corresponding curve are shown in Figure 4.54 and Table 4.33.

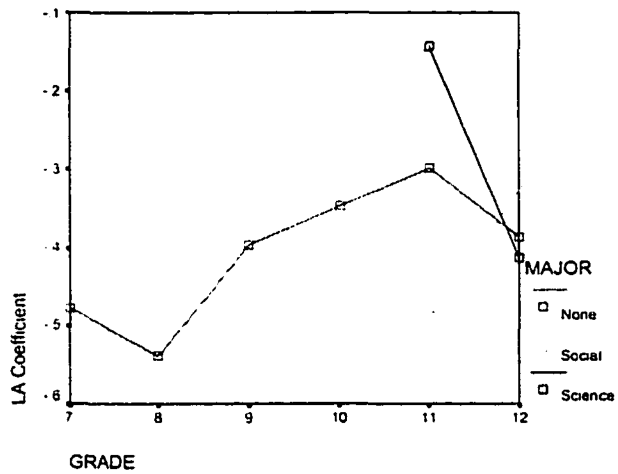

Figure 4.54. The coefficient curve of linear regression for predicting the empirical knowledge effect on Part IV problem solving from the logical reasoning ability effect.

Table 4.33

Summary of Linear Regression for Predicting the Empirical Knowledge Effect On Part IV Problem Solving from the Effect of Logical Reasoning Ability

Subgroup	Beta*	Std. Error	p-value	R^2	95% Confidence Interval for LA Lower Bound	Upper Bound
Whole	-.388	.043	.000	.151	-.471	-.301
7th grade	-.477	.115	.000	.227	-.697	-.236
8th grade	-.539	.119	.000	.291	-.766	-.288
9th grade	-.396	.124	.003	.157	-.640	-.141
10th grade	-.347	.135	.010	.103	-.631	-.089
11th grade whole	-.299	.090	.001	.089	-.482	-.124
Social	-.487	.109	.000	.237	-.679	-.241
Science	-.143	.142	.285	.020	-.437	.131
12th grade whole	-.386	.083	.000	.149	-.542	-.212
Social	-.327	.115	.011	.107	-.535	-.079
Science	-.413	.115	.001	.171	-.623	-.163

* Beta is the coefficient of effect of EK of the regression.

The effect of empirical knowledge on problem solving. The curve of coefficients of the effect of empirical knowledge versus age is not monotonical can be founded from the linear regression test that is conducted to evaluate the prediction of the participants' performance on problem-solving process from the effect of empirical knowledge on different grade level and major subgroups. All of the coefficients are positive and with the biggest absolute value at the 9th grade and minimum one at 12th social science major subgroup. The information about this analysis is shown in Figure 4.55 and Table 4.34.

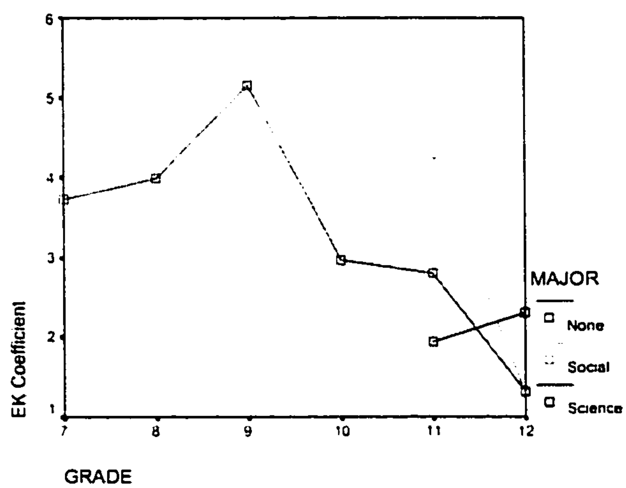

Figure 4.55. The coefficient curve of linear regression for predicting the empirical knowledge effect on Part IV problem solving from the empirical knowledge effect on Part II problem solving.

Table 4.34

Summary of Linear Regression for Predicting the Empirical Knowledge Effect on Part IV Problem Solving from the Empirical Knowledge Effect on Part II Problem Solving

					95% Confidence Interval for LA	
Subgroup	Beta*	Std. Error	*p*-value	R^2	Lower Bound	Upper Bound
Whole	.295	.045	.000	.087	.205	.383
7th grade	.374	.123	.004	.140	.125	.617
8th grade	.399	.133	.007	.159	.106	.644
9th grade	.516	.114	.000	.267	.273	.732
10th grade	.297	.137	.031	.088	.030	.580
11th grade						
whole	.281	.091	.002	.079	.105	.466
Social	.430	.127	.001	.185	.202	.710
Science	.194	.131	.148	.038	-.070	.453
12th grade						.311
whole	.132	.091	.157	.017	-.051	
Social	.130	.221	.325	.017	-.223	.662
Science	.231	.091	.081	.053	-.021	.344

*: Beta is the coefficient of effect of EK of the regression.

The effect of interference of logical reasoning ability and empirical knowledge on problem solving. Three variables, Aav, Bav, and their interference, linear regression is used to predict Rav in order to understand the effect of interference of logical reasoning ability and empirical knowledge on problem solving. The following tables show the coefficients of Aav, Bav, and their interaction on whole item and four pairs of isomorphic problems for each subgroup. The information about this analysis and the corresponding curve are shown in Figure 4.56 and Table 4.35.

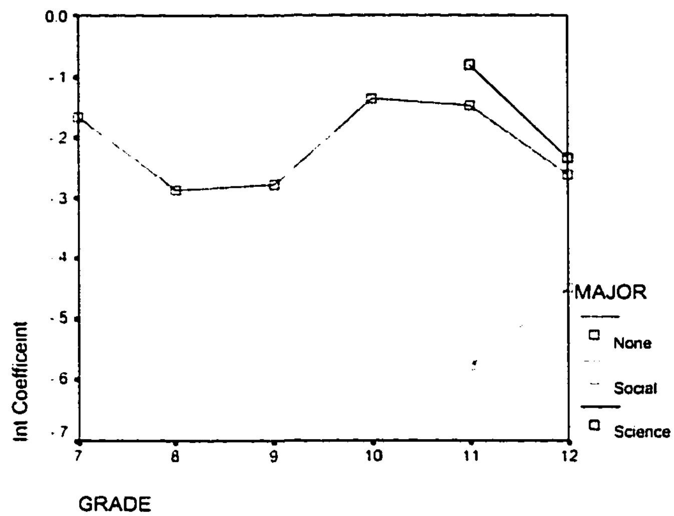

Figure 4.56. The coefficient curve about the interference of logical reasoning ability and the empirical knowledge effect on Part II to predict the empirical knowledge effect on Part IV problem solving.

Table 4.35

Summary of Linear Regression for Predicting the Empirical Knowledge Effect on Part IV Problem Solving from the Logical Reasoning Ability Effect, Empirical Knowledge Effect on Part II Problem Solving, and Their Interference

Subgroup	β_1^*	β_2	β_3	p-value	R^2
7th grade	-.555	.070	-.167	<.05	.431
8th grade	-.717	.106	-.287	<.05	.830
9th grade	-.639	.153	-.279	<.05	.689
10th grade	-.772	.053	-.137	<.05	.706
11th grade	-.634	.022	-.149	<.05	.506
Social	-.139	.091	-.577	<.05	.521
Science	-.855	.005	-.080	<.05	.756
12th grade	-.484	-.031	-.262	<.05	.378
Social	-.240	.050	-.450	<.05	.371
Science	-.720	-.022	-.233	<.05	.637

*: β_1, β_2, and β_3 are the coefficients of Aav, Bav, and ABav, and the constant of the regression, respectively.

Summary of regression analysis. The logical reasoning ability has negative correlation with the effect of empirical knowledge on problem solving, because the sign of the coefficients of effect of logical reasoning ability for predicting using empirical knowledge on solving Part II problems is negative. It means that the more logical reasoning ability the participant has, the less empirical knowledge s/he uses. Based on the absolute values of the coefficients, the logical reasoning ability has it strongest affect on the 8th grade, and its least effect on the 11th grade. On the other hand, the effect of empirical knowledge on Part IV problem solving has positive correlation with the effect of empirical knowledge on Part II problem solving, because there is a positive sign for the coefficients of predicting the use of empirical knowledge on solving Part II problems by the effect of empirical knowledge on Part IV problem solving. By the value of these coefficients, 9th grade has the strongest empirical knowledge effect and 12th has the least effect.

The coefficient signs of the LA and EK effects on the third linear regression are negative and positive, respectively, are the same as the signs of the first and second linear regression, respectively. The sign of the LA and EK interference coefficient from the same linear regression is negative. The curve of the interference versus age is S-shape and peak point is 8th grade and through valley point is 10th grade.

Hypothesis Checking

Based on the result of data analysis, we can address the basic hypotheses for this study.

Hypothesis 1

The logical reasoning ability of Taiwanese secondary school students is affected by age/grade level but not by major or gender.

From the first ANOVA analysis, the logical reasoning ability of Taiwanese secondary school students is not affected either by main factors: age/grade level, gender, and major or by interactions of gender by major, grade by major, and grade by gender by major. The p-values are >.05.

Hypothesis 2

In the process of solving problems, the effect of empirical knowledge will be affected by major and age/grade level but not by gender. Specifically, science majors use less empirical knowledge to solve problems than social science majors of the same logical reasoning ability.

From the first ANOVA analysis, the effect of empirical knowledge in the process of solving Part II problems is affected by major and grade level but not by gender. The p-values are .001, .005, and .656, respectively. Science majors use less empirical knowledge to solve problems than social science majors do on Part II problems. The means of the effect of empirical knowledge on Part II problems of science and social science major are .149 and .000, respectively. The interactions of grade by major are statistically significant for the logical reasoning ability of Taiwanese secondary school students with p-values .045. The interactions of gender by grade, gender by major, and gender by grade by major are not statistically significant for the logical reasoning ability of Taiwanese secondary school students. The p-values are .349, .058, and .848, respectively.

From the same analysis, the effect of empirical knowledge in the process of solving Part IV problems is not affected either by any of the main factors: age/grade level, gender, and major or by the interactions of gender by major, grade by major, and grade by gender by major. The corresponding p-values are $>.05$.

Because of considering the next two hypotheses simultaneously, we discuss them together.

Hypothesis 3

Generally in the process of problem solving, the use of empirical knowledge will be affected by logical reasoning ability. Specifically, the higher logical reasoning ability participant has, the less empirical knowledge the participant uses.

From the linear regression analysis, the all sign of coefficient of logical reasoning ability for predicting the effect of empirical knowledge is negative with p-values $<.05$, except 11^{th} grade science subgroup with $p = .285$. The use of empirical knowledge is affected by logical reasoning ability with negative association.

Hypothesis 4

Science majors in this study will show a higher tendency of detecting the relations between problems in Parts II and IV than social science majors. In other words, science majors more often than social science majors use the same problem solving process and strategies to answer the four pairs of isomorphic questions.

Hypothesis 5

Higher-grade level in this study will show a higher tendency of detecting the relations between problems in Parts II and IV than lower-grade level. In other words, higher-grade level more often than lower-grade level use the same problem solving process and strategies to answer the four pairs of isomorphic questions.

According to the third ANOVA test, there exists a statistically significant different within comparisons of the understanding scores of two-part problem structures between different major, grade level subgroups with p-values .004 and .002, respectively. Therefore, science major participants have a higher tendency of detecting the relationship between problems in Parts II and IV than social science majors and higher-grade level major participants have higher tendency of detecting the relations between problems in Parts II and IV than lower-grade level participants do. One more finding from this ANOVA test is that female participants have higher ability to detect the relationship than the male participants, with $p= .017$.

From the same test, the interaction of the understanding of the structure between two-part problems by different grade level does not have any statistical significance for the scores of the different effect of empirical knowledge in Parts II and IV, Δ_1, Δ_2, Δ_3, and Δ_4 with p-values are >.05. It means that the problem–solving strategies for two-part problems and pairs of isomorphic problems are affected by the understanding of problem structures on different grade level subgroups. The interaction of the understanding of the structure between two-part problems by different major does not have any statistically significance in the scores of the different effect of empirical knowledge on Parts II and IV and pairs of isomorphic problems, except the fourth pair. The p-values are .352, .212, .719, .318, and .007, respectively. Therefore, understanding the structure causes a significant to impact on the problem–solving strategy between two-part problems and pairs of isomorphic problems on different major subgroups for the fourth pair only.

Moreover, the rate of understanding of the structure by the female participants is statistically significantly higher than male participants. As before, this interaction does not cause any significant impact on the scores of the different effect of empirical knowledge on Parts II and IV, Δ_1, Δ_2, Δ_3, and Δ_4 with p-values > .05. It means that understanding the structure does not have any statistically significant impact on the problem solving strategy between two-part problems and pairs of isomorphic problems on different gender subgroups.

Results

There are two important findings from the results of data analysis. First, the participants' problem-solving strategy is affected by the empirical knowledge. Second, the effect of empirical knowledge depends both on grade level and on major. The curve of effect of empirical knowledge is not monotonically increasing or decreasing versus grade level, but S-shape. Both of the results need to be evaluated.

First, both of the two parts of the problem are constructed in the A-AR model. Under light and vision problems, the assumptions and problems of Part II are composed in reference to tangible objects, such as red light and blue glass. The problems of Part IV are written by using symbolic/abstract objects such as container X and filter A. The data analysis results in this chapter show that there does not exist any statistically significant difference in logical reasoning ability among different subgroups. Grade level and academic major but not gender affect the use of empirical knowledge on the problem-solving process. One more important finding is that the participant understanding of the structure of the two problem sets does not affect the problem-solving strategy. Therefore, the different contents of two problem sets cause the different effects in the use of empirical knowledge for different grade levels and with different majors, but not for different genders. Particularly, the different problem-solving strategies between the same pair of isomorphic problems for the same participant is strong evidence that neither the participants' logical reasoning ability nor the understanding of the problem structure affects the problem-solving process, but the content of the problem does.

Second, the effect of empirical knowledge changes among different grade levels and majors. The Piagetian viewpoint would argue that empirical knowledge from successful experiences helps humans search for a useful problem-solving strategy. In addition, older humans focus more on the structure of the problem than the content.

Consequently, the older a person is, the less empirical knowledge s/he uses for problem-solving process. Therefore, from the Piagetian view, the curve of effect of empirical knowledge should be monotonically decreasing versus grade level. From the result of data analysis, the curve of effect of empirical knowledge is not monotonical but S-shape. After a general discussion, I will use these two findings to address the research questions in this dissertation.

As mentioned in chapter 2, the Piagetian scholars typically argue that after individuals approach the formal operation stage, human problem-solving is based on the hypotheses and propositions of problems, not on the superficial aspects of problems. If the view of the Piagetian school about human problem-solving was correct, then the following two statements would be true: first, human problem-solving should not depend on the problem content, it should be content independent, after about age 11; second, any empirical knowledge effect should decrease with age.

If the first statement about human problem-solving were true, then human problem-solving should depend on their logical reasoning ability, not on the problem content, after age 11. In the other words, domain-general thinking should play the major part in human problem-solving at the formal operation stage. The above discussion about different performance on the two parts of the problem provided a counterexample to the first statement. The following discussion, based on the first and second ANOVA tests, provides counter evidence for the second statement.

For the second statement about human problem-solving, the Piagetian scholars hypothesize that the domain-general thinking becomes more important for human problem-solving; while the effect of empirical knowledge should decrease with age. It can be deduced, too, that the curve of empirical knowledge effect versus age should be monotonically decreasing. From the first and second ANOVA test, there are differences between the empirical knowledge effect on the two parts of the problem and the pairs of isomorphic problem. Moreover, the curve of empirical knowledge effect on Part II problems versus age is S-shape with peak point at 9th grade and valley point at 11th grade, (see Figure 4.7).

The S-shape is evidence that empirical knowledge still plays the major role for human problem-solving after age 11. Mehler and Dupoux (1994) argue that children who can't solve a conservation problem use the wrong dimension of knowledge to solve the problem. Because each part of the problem is designed using on the A-AR model, the empirical knowledge implies participants use the wrong dimension of knowledge also. Therefore, it would appear that human problem-solving is still content dependent after age 11. In other words, human problem-solving still depends on the problem content, not just on domain-general thinking.

Based on the first ANOVA test, logical reasoning ability does not become more important for human problem-solving with growth. Actually, the effect of logical reasoning ability and empirical knowledge interfere with each other and the interference depends on age. This issue will be discussed later. The statistical significance for the empirical knowledge effect is not only on different grade levels, but also on the different major. Because the different majors reference to the curriculums and they are directly connected with the school training, the training effect on human problem-solving is discussed next.

Training Affects Problem-Solving

This research has the central idea: Training affects human problem-solving (Chi et al., 1981, 1985, 1989; Vanlehn, 1998). The first ANOVA test shows that two different majors have the statistically significant different empirical knowledge effects on Part II problem-solving. The social science major students show more effect of empirical knowledge than the science major students do. And the 12th grade social science and science have the most and least effect of empirical knowledge, respectively. The different empirical knowledge effects on Part II problem-solving for each major are the results in the curriculums; moreover, the school training is.

The curriculums are different between two majors on 11th and 12th grades of Taiwanese students. The comparison of the Taiwanese secondary school training between different majors on 11th and 12th grade is that science major students take six more units of science courses (e.g., physics and chemistry) than social science major students do, and the topics of the mathematics class for science major students is more advanced than social science major students, and the social science students take six more units of social science courses (e.g. history and geometry) than the science students do. The science major students have more training in scientific thought than the social science students do. Moreover, by the 12th grade, social science majors have the most training in social science thinking and science majors have the most training in science thought, because they have longer training in different curriculums than the 11th grade students do.

Although the student's personal background, such as IQ, might affect a Taiwanese secondary student choosing her/his academic major, the training still is one of the factors that result in the major affecting the participant's problem-solving. According to the same test, the effects of empirical knowledge on Part II problems depend on the interaction of grade by major. Because the 12^{th} grade social science students have the most effect of empirical knowledge and the 12^{th} grade science major students have the least effect of empirical knowledge on Part II problems (see Figure 4-10), the more school science education training students have, the less effect of empirical knowledge they use for problem-solving. Training affects human problem-solving.

Training affects individual differences in human problem-solving strategy. On the other hand gender does not appear to have an effect. The following discussion shows that gender difference does not affect human problem-solving.

Gender Does Not Affect Problem-Solving

Based on the first and second ANOVA test, gender is not associated with any statistically significant difference in the effects of empirical knowledge on either part II or IV problem-solving or on the different performance on pairs of isomorphic problem. Both genders display same problem-solving strategy on the A-AR model problem. Therefore, both genders have the same ability to use the given assumptions to solve the problems on questionnaire.

According to the first and third ANOVA test, the female participants in performance aspects the same effect of empirical knowledge as the male participants' performance was, but the female participants have more knowledge/understanding about the two parts of the problem having the same problem structure than the male participants do. Also, higher grade or science major students have more understanding about the problem structures than lower grade or social science major students do, respectively.

Because the effect of empirical knowledge on the Part II problems is statistically different on both of grades and majors but not on the different genders, then it rises whether the understanding about the two parts of the problem having the same structure (hereafter, S) affects problem-solving or not. If S really did affect the participants' problem-solving, i.e. the participants' problem-solving were not contained by the problem content, then the problem-solving was dependent on the representation transformation. It means that the problem-solving should be domain–general thinking, not bounded by problem content.

The next discussion addresses the relation between the understandings about the structures between two parts of the problem versus the corresponding problem-solving performance to see whether the problem representation transformation affects human problem-solving or not. The aim for the following section is to suppose that human problem-solving is content dependent after age 11.

Representation Transformation

To address whether the understanding of problem structure improves human problem-solving or not, researchers use pairs of isomorphic/homomorphic problems to perceive the problem representation transformation. For instance, giving a pair of isomorphic problems to a participant, if human problem-solving were not restricted by the problem content and the problem solver understands that the pair of problems have the same problem structure, i.e., s/he can transfer the representation of problem to anther directly, then the problem solver can perform the same problem-solving strategy for these two problems.

Reed et al. (1974) argue that although the two problems have the same problem structure and can be solved by the same problem strategy, participants still do not get any help from the first problem-solving experience to solve the second one, unless the second one needs less moves, and the participants need to be told the corresponding structures are the same. The homomorphic problems for Reed's study .are the Missionaries and Cannibals and Jealous Husbands problems. The statements of these two problems are story-telling. It means that the problem solvers must find the problem rules by themselves. In order to help the participants understand the problem structure, the questionnaire is designed using on A-AR model and the two parts of problems have the same problem structure. Moreover, there are four pairs of isomorphic problem on Parts II and IV and the question on the Part V asks the participants whether there are any connections between Parts I and II and Parts III and IV and why or why not.

Based on the third ANOVA, female students appear to have greater understanding about the problem structures, but their performance on pairs of isomorphic problem is the same as male students. The female students do not show that they have more consistent performance on the same isomorphic problems than male students do. Moreover, neither the effect of empirical knowledge on Part II problem-solving nor the different performance on any of isomorphic problem is statistically different on any of the interactions of S by grade, S by major, and S by gender, even though S is statistically different on grade, major, and gender. Therefore, understanding that the two parts of the problem that have the same problem structure does not directly affect the participants' problem-solving process.

If a participant has better problem representation transformation, then s/he has greater understanding that the two parts of the problem have the same structure. Because the understanding of problem structure for a participant does not have any relation to her/his problem-solving process, then the representation transformation is not the only factor that affects the participants' problem-solving. Actually, some factors have more power than the understanding of problem structures to affect the participant problem-solving. The effect of empirical knowledge is one such factor, because the topical reason for a participant, who, knowing the two parts of the problem have the same problem structure but have different performance on the same pair of isomorphic problem, is that: "We are required to answer the problems by using the given assumptions and I know the two parts of the problem are the same, but one part problem is symbolic/logic problem and the other part is light and vision problem. They are not really the same."

Based on the above discussions, the understanding of the problem structure might not directly affect the participant's problem-solving, but the effect of empirical knowledge may. This example demonstrates that after human approach formal operation stage, human problem-solving still depends on the problem content, not just on problem structure. Moreover, this example is contrary to the Piagetian typical argument that human problem-solving should be content independent after they approach formal operational stage. Actually, domain-general thinking does not play the major part for human problem-solving after age 11. For instance, a participant with logical reasoning ability to solve a pair of isomorphic problems by using given assumptions might have a different performance on them, because each problem is under different content. The

Seeing these two line segments, most people would think the top line is longer than the bottom one. The fact is that they are of equal length. The arrows on the ends of the lines elicit the perception of one line being longer than the other. This represents a phenomenon that Piattelli-Palmarini (1994) calls "cognitive illusions." Cognitive illusions make us reach a quick but wrong solution. They are the results of the work of heuristics and biases, although heuristics and biases do not always end with cognitive illusions.

Piattelli-Palmarini explains that ". . . heuristics are specific mental strategies used to solve specific problems and biases are the mental constraints on our reasoning in a certain class of problems" (1994, p. 19). Heuristics result from earlier successful experiences with specific types of problems. Such experience helps to develop quick and easy problem-solving strategies that are usually successful pragmatically but difficult to justify theoretically and logically. When humans face a certain class of problems, their biases make them categorize the problem and information in a certain way and assign them cognitive value (that depends on the individual). In other words, biases make humans tend to come to certain solutions when solving certain types of problems and, therefore, restrict their possible responses. Heuristics and biases are related in that:

1. Humans do not necessarily have a one-to-one correspondence.
2. One bias can "suggest" more than one heuristic.
3. And a given heuristic can be activated by different biases.
4. Two biases can combine and "force" a certain heuristic. (Piattelli-Palmarini, 2001)

From the discussion at this chapter beginning, the empirical knowledge affects the participants' problem-solving. The relation among empirical knowledge, heuristics, and bias on human problem-solving is discussed as follows.

The Relation Among Empirical Knowledge, Heuristics, and Bias

In this study, empirical knowledge of light and vision does affect the participants' problem-solving. The empirical knowledge might drive/constrain the participants to think Part II problems are light and vision problems. Based on the first ANOVA test, all of participants have the same logical reasoning ability to solve the two parts of the problem, but there exists different problem performance among different grade levels and different major. If the participant just thinks the Part II problems are everyday light and vision problems and combines her/his empirical knowledge to answer these problems, then her/his problem-solving might be different between these two parts of the problems. The problem content, but neither their logical reasoning ability nor the understanding of the problem structures causes the difference; moreover, it is caused by how the participant decoded the problems and searched her/his problem strategy between two problem contents.

The problem content drives some participants to use the correlated empirical knowledge to answer Part II problems, even though they know the Part II problems have the same problem structure as the Part IV problems. It is evident that human problem-solving is constrained by problem content. Therefore, human problem-solving is content dependent. Empirical knowledge may function as a bias to help some participants to search the problem-solving strategies. Actually, the empirical knowledge helps some participants to find the shortcut to "successfully" solve the problem.

According to the linear regression test, the effects of logical reasoning ability and empirical knowledge and their interference affect the participants' problem-solving, although the participants' logical reasoning ability is not statistically different among different subgroups. Based on the same test, the weights of these three factors are changing on different grade levels and majors. It shows that the effects of logical reasoning ability and empirical knowledge affect human problem-solving. Based on the comparison of the curves of the effect of empirical knowledge on Part II problems and the effect of interference (see Figures 4.10 and 4.56), the interference is a predictor of the change in heuristics. To support this concept and to deeply understand how the effect of empirical knowledge drives human problem-solving into content dependent, the curves about the logical reasoning ability and empirical knowledge effects and their interference are discussed next.

The Curves of the Effects of LA and EK and Interference

Because the first ANOVA test shows that the participants have the same logical reasoning ability to solve the problems and the difference of the empirical knowledge effect are existing on different grade levels and majors, then the following discussion about the curves are separated into two parts: on different grade levels and majors.

Grade Level

From the linear regression, the effects of logical reasoning ability and the effect of empirical knowledge on Part II problems make negative and positive prediction of the effect of empirical knowledge on Part IV problems, respectively. From the curve of logical reasoning ability effect on Part IV problems, it has an S-shape with peak point at 8^{th} grade through the valley point at 11^{th} grade (see Figure 4.54). The shape of the curve of empirical knowledge effect on Part IV problems is a U-shape with the peak point at 9^{th} grade. The characteristics from these two curves are the 8^{th} and 11^{th} grades have the most and the least logical reasoning ability effect on the Part IV problem-solving, respectively; and the 9^{th} grade has the most empirical knowledge effect on the Part IV problem-solving. Based on the linear regression test, the effects of empirical knowledge has a statistically significant affect on Part IV problem-solving with different weights for each of the grade levels. Then there might exist different effects of empirical knowledge on Part IV problem-solving by grade levels. According to the first ANOVA test, the effect of empirical knowledge on Part IV problem-solving is not affected by grade level. The reason why the Part IV problem-solving cannot detect the different effect of empirical knowledge on the grade levels is that Part IV problems are abstract and symbolic. On the other hand, based on the same ANOVA test, there exist different effects of empirical knowledge on the grade levels, because the Part II problems are tangible.

There are two pieces of information of the empirical knowledge effect about Part II problem-solving; one is from the first ANOVA test, the empirical knowledge effect of Part II problem-solving, and the other one is from the linear regression test, the interference of the effect of empirical knowledge on Part II problem-solving and the logical reasoning ability to predict the effect of empirical knowledge on Part IV problem-solving. Because the signs of the interference coefficient for all grade levels are negative, then the interference is negatively predicting the effect of empirical knowledge. For convenience in writing, the corresponding curves for the empirical knowledge effect on Part II problem-solving and the interference are denoted by curve-1 and curve-2, respectively.

The comparison of these two curves is that both of them have S-shape curve but different peak and valley points. Curve-1, the curve of empirical knowledge effect versus age, has the peak point at 9th grade through the valley point at 11th grade (see Figure 4.7). It means that the 9th grade has the least empirical knowledge effect on their Part II problem-solving and the 11th grade has the most effect. This curve is evidence against the Piagetian scholars' concept about human problem-solving. They hypothesize the curve should be monotonically decreasing. Curve-2, the curve of the interference of empirical knowledge and logical reasoning ability versus age, has the peak point at 8th grade through the valley point at 10th grade (see Figure 4.56). The curve shows that the 8th grade has the most interference of empirical knowledge and logical reasoning ability and the 10th grade has the least interference.

Because the interference negatively predicts a negative empirical knowledge effect, the more interference the participants have, the less of the empirical knowledge effect they have. Moreover, the peak and valley points of curve-2 occur one year earlier than the corresponding peak and valley points of curve-1. The interference of empirical knowledge and logical reasoning ability appears to be a predictor of the empirical knowledge effect. There is one more proof from the major to identify the above statement.

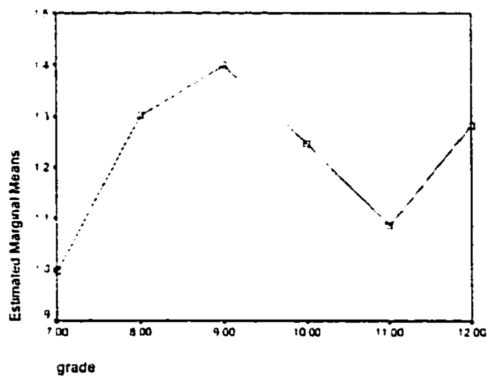

Figure 4.7. Graph of meaning for Table 4.26 (the effect of empirical knowledge on Part II problem-solving).

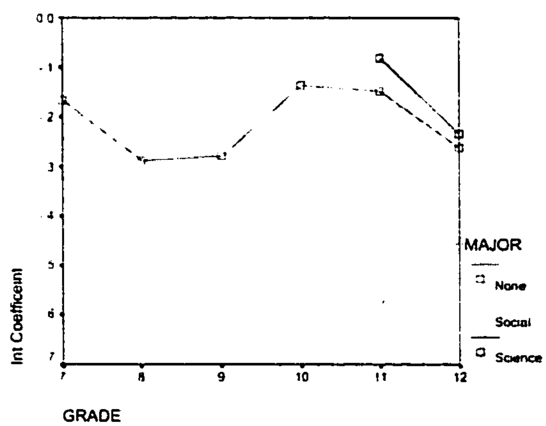

Figure 4.56. The coefficient curve about the interference of logical reasoning ability and the empirical knowledge effect on Part II to predict the empirical knowledge effect on Part IV problem-solving.

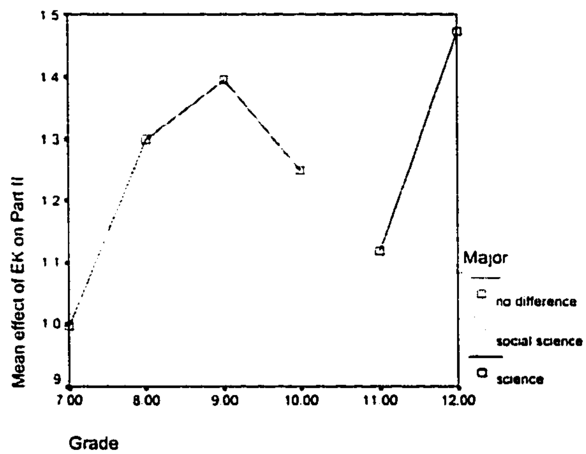

Figure 4.10. Mean graph of empirical knowledge by grade for both majors on Part II problem-solving.

Major

From Figure 4.56, it is apparent that the 11th grade social science majors have the biggest negative interference of any subgroups. The 12th grade social science majors have the largest effect of empirical knowledge on their Part II problem-solving (see Figure 4.10). This indicates that the interference of empirical knowledge and logical reasoning ability is a predictor of the empirical knowledge effect.

Consistent with this discussion, the social science students have more opportunities to use their own empirical knowledge to respond to the problems than the science students do. The major difference not only affects the participants' problem-solving strategies, but also affects the interference of empirical knowledge and logical reasoning ability for their problem-solving process, (e.g., the 11th grade social science students have more negative interference than other subgroups). The biggest negative interference for the 11th grade social science students can be characterized as a struggle between using either empirical knowledge or the given assumptions. The factor causing the 11th grade social science students to have confusion about the problem-solving strategy choosing is the academic training.

Before the 11th grade, there is no difference in the courses; during the 11th grade, the curriculum diverges by major. Therefore, before the 11th grade, the 11th grade social science students have as much training in scientific thought as science students. Beginning at 11th grade, the 11th grade social science students choose more training in social science thinking, but not in scientific thought. Their responses indicate they begin to use their empirical knowledge, not on the logical reasoning ability indicated by use of assumptions. When the experiment was administered in March, the 11th social science students began to choose empirical knowledge to respond to the question, but also used scientific thought. This indicates that the 11th grade social science majors had not abandoned their prior training in science thought, but were beginning to adopt a more empirical knowledge approach. This is the reason why the major causes the biggest negative interference for the 11th grade social science students.

Moreover, the 12th grade social students have another year training in social science thinking more than the 11th grade social science students do. This is the reason why during the 11th grade, social science students demonstrate less consistency in their problem-solving strategy choices than their 12th grades counterparts. Given their grade consistency in use of problem-solving strategies, it can be concluded that the 12th grade social science students are more comfortable using their own empirical knowledge to answer the problems.

The academic training affects the interference. For instance, the 11^{th} grade social science students have the biggest negative interference; they are inconsistent in their choices of problem-solving strategies. At the same time, the 11^{th} grade science majors do not have the confusion; they consistently choose the logical reasoning strategy. Thus, after another year of training (12^{th} grade), major difference drives the participants to demonstrate more stable problem-solving strategy usage. For instance, the 12^{th} grade social science majors have the biggest empirical experience effect in Part II problem-solving and the same grade science majors consistently use the logical reasoning strategy. Therefore, we can conclude that the academic training, major, affects the interference which, in turn, affects the empirical knowledge effect. Moreover, the interference is a temporary characteristic that can predict the effect of empirical knowledge over time.

The Roles of Heuristics and Biases in the Problem-Solving Changing

Certain domains in the everyday empirical knowledge are given higher cognitive values, and different problem-solving strategies of these specific domains are therefore formed. These domains are usually relevant to survival and physical/emotional needs and are strong in the environment. For instance, the corresponding empirical knowledge for this research questionnaire is the light and vision knowledge that is from everyday life and given higher cognitive values for solving the same content (light and vision) problems. The process of assigning higher cognitive values to these domains is the function of biases. Therefore, at the very beginning of problem-solving strategy choosing, problem content is already playing a role. Actually, the problem content plays the container function for human problem-solving the same as bias does.

For these domains of higher cognitive values, humans keep track of successful and unsuccessful experiences and come to some intuitive knowledge of why they are successful or unsuccessful (although they are not always correct). Relations among the items of a certain domain are organized in a certain manner. Later, when a problem in this specific domain is encountered, a shortcut can be made between the problem at hand and the certain problem-solving strategy of this domain. Following the organization and strategies of the specific domain and earlier successful experiences can solve the problem. This is the function of heuristics. Heuristics provide simple, quick, and usually successful solutions because heuristics rely on earlier successful experiences with the specific domain. However, such solutions are hard to justify rationally and theoretically. Moreover, such solutions restrict the range of possible answers and the use of other cognitive strategies for solving problems.

The light and vision knowledge (the empirical knowledge) and the given assumptions have higher cognitive values for solving the problems from this research questionnaire. For instance, the social science and science major students of the 12th grade students choose the light and vision knowledge (the effect of the empirical knowledge) and the given assumptions (the scientific problem-solving strategy) to respond to the problems, respectively. As mentioned before, the problem content plays the function of bias; on the other hand, the effect of empirical knowledge and the scientific problem-solving run the function of heuristics to find a shortcut to search the corresponding strategy in order to solve the problem. The heuristics is a shortcut for humans searching the corresponding problem-solving strategy and can be predicted by the interference of the logical reasoning ability and the effect of empirical knowledge.

Because the interference is a predictor of the effect of empirical knowledge that is a phenomenon of the participant's problem-solving strategy, then the interference is a predictor of the changing of the participants' problem-solving strategy searching. From Figures 4.7 and 4.56, the more interference a participant has now, the less effect of empirical knowledge s/he has in the next year. According to these two statements, the interference predicts the heuristics changing.

The school training separates the 11^{th} and 12^{th} grade students into the social science or science majors; moreover, the training shapes the students into different problem-solving performance. Although the interference for the 10^{th} and the 11^{th} grade are almost equal, there exists a big gap between of the two majors at the 11^{th} grade. The school training causes the bigger different interference gap between of two majors of the 11^{th} grade students. After the interference gap showing at the 11^{th} grade, the 12^{th} grade students have the two most consistent problem-solving strategies; as the 12^{th} grade social science and science major students have the strongest and weaker effects of empirical knowledge, respectively (see Figures 4.10 and 4.56). The bigger interference gap drives the participants into two more consistent problem-solving strategies in next year. The most consistent problem-solving performances of the 12^{th} grade students show that the 12^{th} grade students' problem-solving strategy turns into "the New Heuristics and No Bias."

As mentioned before, the 12^{th} grade students have the most consistent problem-solving performance. Either they use the empirical knowledge or the given assumptions to solve the problems. For both cases, the students directly respond to the problem content to answer the problems. The new heuristics is appearing at the 12^{th} grade students' problem-solving strategy searching. The interference for the 12^{th} grade social science major students is less than the 11^{th} grade, same major students. It shows that with the 12^{th} grade social science major students there are fewer struggles for choosing either the empirical knowledge or the given assumptions to response the problems. No more bias is existing for the 12^{th} grade students' problem-solving. The new problem-solving process, the new heuristics and no bias, for the 12^{th} grade students describes how the empirical knowledge and interference affect human problem-solving.

Implications for Research Questions

Based on the above data analysis and discussion, the research questions of this dissertation are answered as follows:

Research Question-1

How do logical reasoning ability and empirical knowledge influence Taiwanese secondary school students to solve problems concerning light and vision? Do these two factors interfere with each other on the problem-solving process? If yes, what are the relations of the interference versus ages, genders, and majors?

Taiwanese secondary students' problem-solving is influenced by their logical reasoning ability and empirical knowledge. Most of them have the corresponding logical reasoning ability to solve two parts of problem, but the effect of empirical knowledge causes different performance between the two parts of the problem. The empirical knowledge effect depends on the grade level and major, but not on gender. Social science major students have more effect of empirical knowledge than science students do. The curve of the empirical knowledge versus age is S-shape. The 9th and 11th grade students have the least and most effect of empirical knowledge, respectively.

Because the empirical knowledge and logical reasoning ability do not have statistical significance on gender, then we do not need to test for interference based gender difference. The interference between logical reasoning ability and empirical knowledge exists and the corresponding curve is S-shape, too. The 8th and 10th grade students have the most and least interference, respectively. It can be seen by comparing the above curves in Figures 4.7, 4.10, and 4.56 that interference is a predictor of the effect of empirical knowledge. At the same grade level, the social science major students have more interference than science major students do.

Research Question-2

Do participants who recognize that the structures of two problems are the same more often use the same strategy to solve these two problems than those who don't?

The understanding of the problem structure does not affect participants' problem-solving. For instance, the female participants have more understanding of the two parts of problems having the same structure, but they do show more frequently the same problem strategy than male participants do. Actually, recognizing the problem structure does not affect the problem-solving strategy, but the problem content does.

Conclusion

The manner in which humans solve problems depends on the problem content. It means that human problem-solving is content dependent, although logical reasoning ability affects human problem-solving. The understanding that problem structure does not affect human problem-solving is evidence that human problem-solving is content dependent. Some participants understand that two parts of the problem have the same problem structure, while they still are constrained by the problem content and have different performance on two parts of the problem.

The effect of empirical knowledge is additional evidence of human problem-solving being content dependent. The curve of the effect of empirical knowledge on Part II problems versus age is not monotonically decreasing as Piagetian scholars' typical expectation, but S-shape. Human problem-solving cannot be reduced to domain-general thinking, because the problem content causes the effect of empirical knowledge and it plays as the function of bias to drive the heuristics of problem-solving. Actually, the problem content helps humans to find a shortcut to solve problems, whether the heuristics/strategy based on the shortcut is right or not. The effect of empirical knowledge depends on the grade level and major. Science major students have less effect of empirical knowledge than social science major students do. The 9th grade and the 11th grade have the least and most effect of empirical knowledge respectively.

The effect of empirical knowledge is a fact of the participant searching her/his problem-solving strategies and this is her/his heuristics to solve the problems. The relationship between the interference and the empirical knowledge versus grade level is that the more interference the participant has, the less the effect of empirical knowledge s/he has for next year. Therefore, the changing of heuristics is caused by the interference between the effects of logical reasoning ability and empirical knowledge.

The different school training (the different curriculums) of the 11th and 12th grade shapes the students into different problem-solving strategies, such as the using either the empirical knowledge or the given assumptions to respond to the problems. Moreover, the school training causes the 12th grade students in both majors to have the most consistent problem-solving strategies. At the 12th grade, the students' problem-solving process turns into "the new heuristics with no bias."

APPENDIX A

QUESTIONNAIRE

Dear Parents:

My name is Shih-Chieh Liao. I am a doctoral student at the University of Arizona in the United States of America. This questionnaire is part of my dissertation research, and this research has been approved by the Human Subjects Committee of the University of Arizona, 1622 E. Mabel Street, PO BOX 245137, Tucson, Arizona 85724-5137 (phone number 002-1-520-626-6271.)

This questionnaire will not affect any of your current grades or any of your future studies. Please relax and do your best.

1. You don't need to put down your name or any other personal identification information. Please answer honestly and without worry.
2. You may mark or write on this questionnaire if you need to do so.
3. When you need to make changes, please do not erase or white-out. Simply cross out the original answer and put the new answer next to it.
4. Your participation is voluntary and you have the right to stop answering this questionnaire anytime you wish, but your help and cooperation is highly appreciated.

If you need any additional information about this study, you may contact either Professor Shou-Yong Chiu at the National Changhua University of Educaiton at (04) 723-2105 ext. 3217 or the researcher at 002-1-520-621-1837.

Now you may start. Raise your hands if you have any questions. Thanks!

I am a _____ female _____ male.

I am on the _____ grade.

(If you are on the 11th or 12th grade, please indicate your study major _____ social science _____ science.)

Date of birth: _____ month _____ year.

Part I. Facts (A)

The following statements are some scientific facts in physics. In Part II, you will be asked to answer questions using these facts.

According to well-established scientific facts in the subject of physics, light and vision have the following properties:

1. All visible light is perceived as a combination of three colors: red, green, and blue.
2. A red transparent object (for example, red glass) is red because only red light can go through the object. Green light and blue light are blocked. This also explains why a green transparent object is green and a blue transparent object is blue.
3. A red non-transparent object (for example, red paint) is red because only red light is reflected. Green light and blue light are absorbed by the object. This also explains why a green non-transparent is green and a blue non-transparent object is blue.
4. If your eyes receive more than one type of light within one tenth second, we will just assume that your eyes receive them at the same time.
5. When your eyes do not receive any light, you will see only black.
6. White light is a composition of all three colors equally bright.

If you have any questions of these assumptions, please ask!

Part II. Questions (A)

Note:

1. **Use the facts in Part I to answer the following questions.**
2. **If you need more space to answer the questions, please write on the back and mark what number it is.**

1. If blue light shines on red glass, what color will you see on the other side of the glass (see Figure 1)?
   Answer: ____________________

   Why: ____________________

   ____

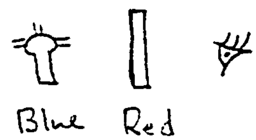

Figure 1

2. If blue light shines on blue glass, what color will you see on the other side of the glass (see Figure 2)?
   Answer: ____________________

   Why: ____________________

   ____

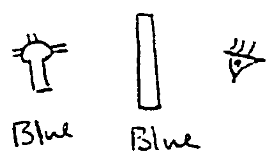

Figure 2

3. If white light shines on red glass, what light will you see on the other side of the glass (see Figure 3)?
   Answer: ____________________

   Why: ____________________

   ____

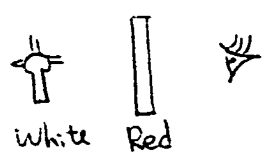

Figure 3

7. There are several light bulbs in a transparent glass box. We know that these light bulbs are on and the glass box looks green. What color(s) might these light bulbs are (see Figure 7)? (Please note that red bulb emits only red light, green bulb emits only green light, and blue bulb emits only blue light.)
   Answer: ____

   Why: ____

   ____

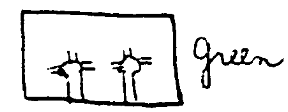

Figure 7

8. In a dark and closed room, a beam of white light is projected on a blue brick. What will you see (see Figure 8)?
   Answer: ____

   Why: ____

   ____

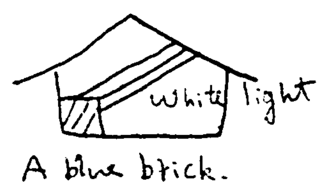

Figure 8

9. There are several light bulbs in a transparent glass box (not black glass). We know that these light bulbs are on and the glass box looks black. What color(s) might these light bulbs are (see Figure 9)? (Please note that red bulb emits only red light, green bulb emits only green light, and blue bulb emits only blue light.)
   Answer: ______________________________________

   Why: ______________________________________

   ____

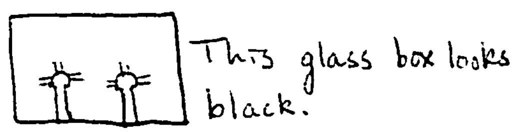

Figure 9

Now you can change any of the answers on this part, if you need to, but please do not white out your original answers. Cross out your original answers and mark what is original and what is new. Thanks!

Part III. Facts (B)

We have three substances X, Y, and Z. We have three filters A, B, and C. We also have three strainers E, F, and G.

1. (a) Filter A allows substance X to pass through, and only substance X can pass through filter A. Substances Y and Z will be caught by filter A when poured over it. Please see Figure 10.

(b) Filter B allows substance Y to pass through, and only substance Y can pass through filter B. Substances X and Z will be caught by filter B when poured over it.

(c) Filter C allows substance Z to pass through, and only substance Z can pass through filter C. Substances X and Y will be caught by filter C when poured over it.

2. (a) Strainer E catches substance X and allows substances Y and Z to pass through.

Only strainer E will catch substance X. Please see Figure 11.

(b) Strainer F catches substance Y and allows substances X and Z to pass through.

Only strainer F will catch substance Y.

(c) Strainer G catches substance Z and allows substances X and Y to pass through.

Only strainer G will catch substance Z.

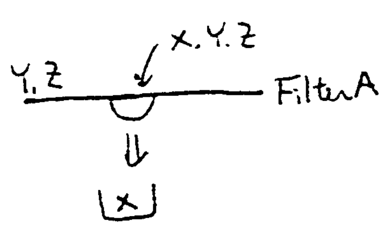

Figure 10

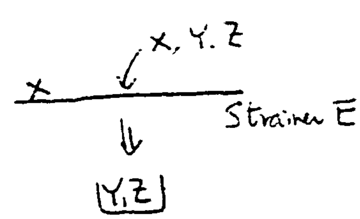

Figure 11

If you have any questions about these facts, please ask!

Part IV. Questions (B)

Note

1. **Use the facts in Part III to answer the following questions.**
2. **If you need more space to answer the questions, please write on the back and mark what number it is.**

1. **An empty box is placed under filter C (see Figure 12). Substance Z is poured over filter C. What substance(s) will you probably find in the box?**
   Answer: ____

   Why: ____

   ____

   ____

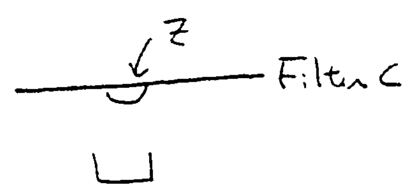

Figure 12

2. **An empty box is placed under filter A (see Figure 13). Substance Y is poured over filter A. What substance(s) will you probably find in the box?**
   Answer: ____

   Why: ____

   ____

   ____

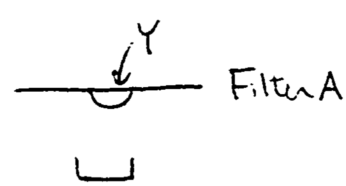

Figure 13

3. **An empty box is placed under filter A (see Figure14). Substances X, Y, and Z are poured over filter A. What substance(s) will you probably find in the box?**
Answer: ______________________________

Why: ______________________________

____

____

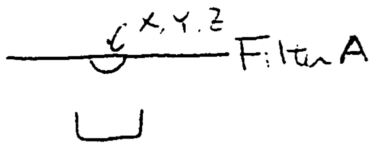

Figure 14

4. **The head of Filter B is attached to the bottom of filter A and an empty box is placed under filter B (see Figure 15). Substances X, Y, and Z are poured over filter A and any substances that pass through filter A will go to filter B. What substance(s) will you probably find in the box?**
Answer: ______________________________

Why: ______________________________

____

____

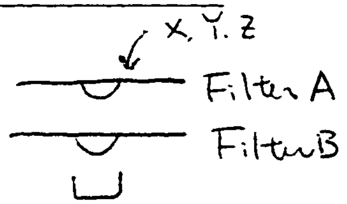

Figure 15

5. **A material is made of substances X, Y, and Z. That means it might be made of X, Y, or Z only, X and Y, Y and Z, or X and Z only, or X, Y, and Z all together. This material is poured over a filter and when examined substance Y is found in the filter (see Figure 16). What filter is it?**

Answer: ______________________________

Why: ______________________________

____

____

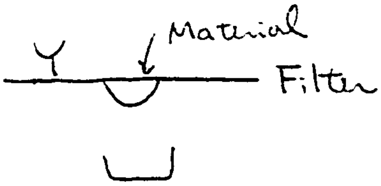

Figure 16

6. **A material is made of substances X, Y, and Z. That means it might be made of X, Y, or Z only, X and Y, Y and Z, or X and Z only, or X, Y, and Z all together. This material is poured over two filters that are attached together head to bottom. When examined, substance Y is found in the first filter and substance Z is found in the second filter (see Figure 17). What is the second filter?**

Answer: ______________________________

Why: ______________________________

____

____

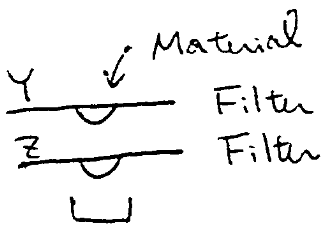

Figure 17

7. **A material is made of substances X, Y, and Z. That means it might be made of X, Y, or Z only, X and Y, Y and Z, or X and Z only, or X, Y, and Z all together. An empty box is placed under a filter, and this material is poured over the filter. When examined, substance Z is found in the box (see Figure 18). What substance(s) is this material made of?**

Answer: ______________________________

Why: ______________________________

____

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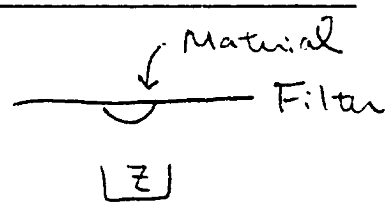

Figure 18

8. **A material is made of substances X, Y, and Z. That means it might be made of X, Y, or Z only, X and Y, Y and Z, or X and Z only, or X, Y, and Z all together. An empty box is placed under strainer G. This material is poured over strainer G. When examined, substance Z is found in the box (see Figure 19). What substance(s) is this material made of?**

Answer: ______________________________

Why: ______________________________

____

____

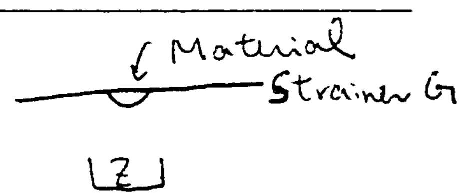

Figure 19

Note: Now you can change any of the answers on Part VI, if you need to, but please keep your original answers and mark what is original and what is new for me.

*** *** ***   *** *** ***   *** *** ***   *** *** ***

Part V.

Are there any connections or relations between Part I (Facts A) and Part III (Facts B)? If yes, please explain such relations that you observe. If no, please also explain why.

_____ **Yes. Reasons:** ____

____

_____ **No. Reasons:** ____

____

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