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COUPLED THERMAL AND VIBRATION NUMERICAL ANALYSIS OF SOLDER JOINTS

by

Mark D. Nickerson

A Dissertation Submitted to the Faculty of the DEPARTMENT OF CIVIL ENGINEERING AND ENGINEERING MECHANICS In Partial Fulfillment of the Requirements For the Degree of DOCTOR OF PHILOSOPHY WITH A MAJOR IN ENGINEERING MECHANICS

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS ........................................................................................................... 9
LIST OF TABLES .......................................................................................................................... 9
ABSTRACT .................................................................................................................................. 20

CHAPTER:

1. Introduction ........................................................................................................................ 22
   1.1 General .......................................................................................................................... 22
   1.2 Background ................................................................................................................... 24
   1.3 Current Surface Mount Technology ........................................................................... 27
   1.4 63Sn/37Pb Package Failure Mechanisms ..................................................................... 28

2. Review OF Solder joint thermal fatigue ................................................................. 39
   2.1 General .......................................................................................................................... 39
   2.2 Review of representative models ................................................................................. 51

3. REVIEW OF SOLDER JOINT VIBRATION FATIGUE ............................................. 84
   3.1 Introduction .................................................................................................................... 84
   3.2 Experimental Testing .................................................................................................... 87
   3.3 Vibration ....................................................................................................................... 91
   3.4 PWB Experimental Testing ....................................................................................... 92
   3.5 BGA Vibration Fatigue Modeling .............................................................................. 96
   3.6 Non-BGA Vibration Fatigue Modeling .................................................................... 104
   3.7 Conclusion .................................................................................................................... 108

4. COUPLED VIBRATION AND THERMAL CYCLING .......................................... 119
   4.1 Introduction .................................................................................................................... 119

5. The Disturbed State Concept ...................................................................................... 121
   5.1 Introduction .................................................................................................................... 121
   5.2 Elastic-Viscoplastic Disturbance: Perzyna Formulation ........................................... 123
   5.3 Katona and Mular Formulation ............................................................................... 125
   5.4 Elastic Viscoplastic and HiSS Model ......................................................................... 128
   5.5 Elastic-Viscoplastic Temperature Disturbance Formulation ................................... 130
6. HEAT TRANSFER ANALYSIS SUBROUTINE .............................................. 151
   6.1 Introduction .................................................................................. 151
   6.2 H2D Subroutine Finite Element Procedure ..................................... 152
   6.3 H2D Subroutine Finite Element Formulation ................................. 153
   6.4 Time Integration Scheme ................................................................ 158
   6.5 H2D Subroutine - Introduction for Users ...................................... 160
   6.6 Parameters Required For H2D ...................................................... 161
   6.7 Steady State Heat Transfer - Static Temperature Load, Case 1 ...... 163
   6.8 Steady State Heat Transfer - Cyclic temperature Load, Case 2 ...... 163
   6.9 Transient Heat Transfer - Static Load, Case 3 .............................. 164
   6.10 Transient Heat Transfer - Cyclic Load, Case 4 ............................ 165

7. Heat Transfer subroutine Verification ................................................. 174
   7.1 Introduction .................................................................................. 174
   7.2 Heat Conduction Theory .............................................................. 175
   7.3 Heat Convection Theory .............................................................. 178
   7.4 Verification of H2D Heat Transfer Subroutine .............................. 180
   7.5 Conclusion ................................................................................... 182

8. MATERIAL PARAMETERS ...................................................................... 201
   8.1 Material Parameters .................................................................... 201
   8.2 Material Parameters - Discussion ................................................ 203

9. ACCELERATED ANALYSIS VERIFICATION ......................................... 206
   9.1 Introduction .................................................................................. 206
   9.2 Macro-Micro Mesh Technique ..................................................... 209
   9.3 Verification of the prediction method .......................................... 210
   9.4 Prediction Verification - Modified Raytheon Thermal Cycle .......... 212
   9.5 Prediction Verification - JPL Thermal Cycle ............................... 213
   9.6 Summary ..................................................................................... 217

10. CALIBRATION OF 313 BALL PBGA FEA MODEL .............................. 231
    10.1 Introduction ............................................................................... 231
    10.2 Calibration Procedure ............................................................... 235
    10.3 Full Package (2D Slice) Calibration Results ............................... 237
LIST OF ILLUSTRATIONS

Figure 1.1. The Forces Driving Electronic Package Miniaturization and Reliability......33

Figure 1.2. Comparison of Through Hole and Surface Mount Technology (not to scale), Showing Plan View, Side View, Package To Board Attachment Mechanism, and Electronic Package Mounted To PWB .................................34

Figure 1.3. Typical Overmolded PBGA Schematic (not to scale)..............................35

Figure 1.4. A: Amkor PBGA, FGA. B: Amkor PBGA FGA and PGA.........................36

Figure 1.5. Practical Components PBGA, FGA showing daisy chain..........................37

Figure 1.6. Voids in solder balls from thermal cycling [43]........................................38

Figure 1.7. Voids in solder balls from thermal cycling [43]........................................38

Figure 1.8. Voids in solder balls from thermal cycling [43]........................................38

Figure 1.9. Voids in solder balls from thermal cycling [43]........................................38

Figure 1.10. Voids in solder balls from thermal cycling [43]........................................38

Figure 1.11. Voids in solder balls from thermal cycling [43]........................................38

Figure 2.1. Weibull Plot [36]. ..................................................................................65

Figure 2.2. Fracture of Solder Ball [68]. ....................................................................66

Figure 2.3. Personal Communication PWB Showing Microvias [69]. ....................67

Figure 2.4. Temperature Dependence of Material Properties. (A) Relationship between various properties and temperature [137]. (B) Elastic modulus with temperature [70]. .................................................................68

Figure 2.5. SEM Photos Showing Cracks Occurring Away From Metallization Layer [36]....................................................................................................................69

Figure 2.6. SMT capacitor solder joint crack growth plots [110]. (A) Crack Length as a function of cycles. (B) Crack growth rate as a function of cycles.............70

Figure 2.7. Voids in BGA solder joints from thermal cycling [43]..............................71

Figure 2.8. Various style heat sinks [146]..................................................................72

Figure 2.9. MEMs examples. From Goodrich Advanced MicroMachines web-site, www.memslink.com..............................................................................................................73

Figure 2.10. (A) Non-ideal temperature cycle. (B) Examples of random vibration plots, Dallas Instruments www.dallasinstruments.com...............................................................74

Figure 2.11. FEA Mesh Incorporating Different Phases of Solder [138]..................75

Figure 2.12 Various experimental data from solder testing [102]. (A) True stress vs. strain plot at different strain rates. (B) Nominal stress vs. strain plot at
different strain rates. (C) Cycles to failure vs. velocity plot. (D) Log strain vs. Log stress at different temperatures. (E) Strain vs. time for differently cooled specimens.

Figure 2.13. Solder Joint Showing Coarsened Area After Thermal Cycling [105].

Figure 2.14. Various experimental data from solder testing [102]. (A) Cycles to failure versus frequency. (B) Strain range versus cycles to failure for distinct temperatures.

Figure 2.15. Various results using global-local model [12]. (A) Local-global model meshes. (B) Various solder joint shape as function of joint volume. (C) Comparison of shape effects on fatigue life. (D) Pad size effects on fatigue life. (E) Solder ball volume effects on fatigue life. (F) Joint height effect on fatigue life.

Figure 2.16. Distance to Neutral Point Comparisons between FEA and Experiment [77]. (A) Chip scale package. (B) DNP results from chip scale package analysis. (C) Flip chip. (D) DNP results from flip chip analysis. (E) CBGA package and DNP results.

Figure 2.17. Solder Joint Shape Results [20]. (A) Comparison of model with experiment. (B-C) Comparison of modeled steady state creep behavior with various experimental results.

Figure 2.18. Effect of Cooling Rate and Thermal Cycling on Solder. (A) R= 5.4°C/min, (B) R= 37.8°C/min, (C) R= 97°C/min, (D) Vickers micro-hardness as a function of cooling rate [14]. (E) Coarsening in band parallel to prescribed strain, (F) Fracture through coarsened band [40].

Figure 2.19. Fit of Model to Experimental Data Points [40].

Figure 3.1. Ball strength testing results [150]. (A) Strength test mechanism. (B) Ball strength versus storage time at 150°C. (C) Ball strength versus number of thermal cycles.

Figure 3.2. Experimental results for various solders: (A) Uniaxial tensile strength vs. temperature. (B) Elongation vs. temperature [61].

Figure 3.3. Experimental results regarding reflow temperature. (A) Test vehicle. (B) Percent failures vs. cycle time as a function of heating factor "Q". (C) Median vibration lifetime vs. heating factor "Q".
Figure 3.4: Experimental results for notched solder specimens [17, 18]. (A) V notch specimen. (B) U notch specimen. (E) Deflection vs. Number Cycles. (F) Crack Length vs. Number Cycles

Figure 3.5. Time series changed to frequency PSD. (A) Random acceleration time series data. (B) Random acceleration frequency series (PSD)

Figure 3.6. (A) Vibration apparatus. (B) Peak to peak vibration amplitude vs. frequency (free vibration). (C) Peak to peak vibration amplitude vs. frequency (forced vibration) [72]

Figure 3.7. FEA model and prescribed random vibration [141]. (A) Macro/Micro Model. (B) Table of Random Vibration. (C) Generic PSD for use with random vibration table

Figure 3.8: Model and prescribed loads from ref. 6. (A) Package and PWB schematic. (B) Package considered rigid compared to PWB. (C) Comparison of thermal and vibration strain time histories [6]

Figure 5.1. Schematic illustration of disturbed state concept (stresses) [28]

Figure 5.2. Schematic illustration of disturbed state concept: Balance of forces

Figure 5.3: Large displacements: 90° rotation causes incorrect stress accounting

Figure 5.4. Loading and unloading from yield surface

Figure 5.5: Plot of HiSS yield function for solder showing the effect of γ, R and α. 3R is 650MPa, n = 2.1, and α varies as shown

Figure 5.6. Normality of HiSS yield surface with J1 axis

Figure 5.7. Plot of yield function with different β, the yield surface shape parameter [134]

Figure 5.8: Plot of yield function with different values of n, the phase change parameter [134]

Figure 5.9: Schematic relation between hardening parameters a1 and η1 and deviatoric strain ξ

Figure 5.10: Disturbance parameters A and Z

Figure 5.11. Schematic illustration of the effect of parameters Z and A on disturbance
Figure 6.1. 4 Node Quadrilateral Discretization .................................................. 166
Figure 6.2. Time Dependent (Cyclic) Thermal Loading ....................................... 167
Figure 6.3. Quadrilateral Local Coordinates ....................................................... 168
Figure 6.4. Crank Nicholson time integration procedure (Equations 6.21) ............ 169
Figure 6.5. Case 1, steady state with applied temperature, convection and flux .... 170
Figure 6.6. Case 2, steady state, applied temperature cycle ............................... 171
Figure 6.7. Case 3, transient with applied temperature, convection and flux ........ 172
Figure 6.8. Case 4, transient with cyclic temperature, convection and flux .......... 173
Figure 7.1 For a steady state problem, the system is in equilibrium. The change in flux is zero ................................................................. 184
Figure 7.2 Steady state: Applied temperature. (A) ANSYS, (B) DSC-SST2D ......... 185
Figure 7.3 Steady state: Convection and temperature. (A) ANSYS, (B) DSC-SST2D 186
Figure 7.4 Steady state: Flux and temperature. (A) ANSYS, (B) DSC-SST2D ....... 187
Figure 7.5 Steady state: Temperature with flux and convection. (A) ANSYS, (B) DSC-SST2D ................................................................. 188
Figure 7.6 Transient: Temperature. Time = 100s. (A) ANSYS, (B) DSC-SST2D ... 189
Figure 7.7 Transient: Temperature. Time = 1000s. (A) ANSYS, (B) DSC-SST2D ... 190
Figure 7.8 Transient: Temperature and convection. Time = 1000s. (A) ANSYS, (B) DSC-SST2D ................................................................. 191
Figure 7.9 Transient: Temperature and convection. Time = 5000s. (A) ANSYS, (B) DSC-SST2D ................................................................. 192
Figure 7.10 Transient temperature with flux. Time = 1000s. (A) ANSYS, (B) DSC-SST2D ................................................................. 193
Figure 7.11 Transient: Temperature with flux. Time = 5000s. (A) ANSYS, (B) DSC-SST2D ................................................................. 194
Figure 7.12 Transient. Temperature with convection and flux. Time = 2000s. (A) ANSYS, (B) DSC-SST2D ................................................................. 195
Figure 7.13  Transient: Temperature with convection and flux. Time = 5000s. (A) ANSYS, (B) DSC-SST2D.

Figure 7.14  Mesh Used In FEA Verification Problems

Figure 7.15 - Continued. Nodal temperature output from H2D Subroutine

Figure 9.1  Accelerated analysis relation of deviatoric plastic strains and cycles.

Figure 9.2  Graphs of prescribed temperature cycles. (A) Raytheon* Cycle. (B) JPL Cycle.

Figure 9.3  Meshes used to verify accuracy of prediction method. (A) Coarsely meshed macro-mesh. (B) Micro-mesh used to determine disturbance.

Figure 9.4  Graphs of temperature and displacement. (A) Temperature profile applied to macro-mesh in Figure 9.2. (B) Relative displacements found at largest DNP solder ball (Raytheon* Cycle).

Figure 9.5  Graphs of relative displacements. (A) Relative displacements found at solder ball under die edge (JPL cycle). (B) Displacements applied to micro-mesh shown in Figure 9.2 for JPL cycle, viscoplastic analysis.

Figure 9.6  Contour plots: Full cycle plastic analysis, Raytheon* Cycle, largest DNP ball, at cycle 1,785. (A) Disturbance distribution. (B) Area above disturbance of 0.7.

Figure 9.7  Contour plots: Full cycle viscoplastic analysis, Raytheon* Cycle, largest DNP ball, cycle 1,892. (A) Disturbance distribution. (B) Area above disturbance of 0.7.

Figure 9.8  Contour plots for accelerated analysis Raytheon* cycle, plastic case. (A) Disturbance distribution. (B) Disturbance above 0.7.

Figure 9.9  Contour plots for accelerated analysis, Raytheon* cycle, viscoplastic case. (A) Disturbance distribution. (B) Disturbance above 0.7.

Figure 9.10  Contour plots: Full cycle plastic analysis, JPL cycle, ball under die edge, cycle 1,800. (A) Disturbance distribution. (B) Area above disturbance of 0.7.

Figure 9.11  Contour plots for full cycle viscoplastic analysis, JPL cycle, ball under die edge, cycle 1,900. (A) Disturbance Distribution. (B) Area above disturbance of 0.7.
Figure 9.12. Contour plots for accelerated analyses, JPL cycle. (A) Plastic disturbance at 1,800 cycles. (B) Viscoplastic disturbance at 1,900 cycles.

Figure 9.13: Graphs of disturbance plotted for every 100 cycles. Under die edge solder ball, element 100, JPL cycle. (A) Plastic analysis. (B) Viscoplastic Analysis.

Figure 10.1. Pictures of JPL test materials. Top: Depopulated PBGA. Bottom: Test Vehicle Board and 313 PBGA [111].

Figure 10.2. Graphs of JPL test data. (A) Testing cycle, (B) Failure distributions [45].

Figure 10.3. Graphs of failure data. Top: Experimental failure results for Raytheon cycle [111]. Bottom: JPL cycle failures, neglecting packages 7 and 8, with linear trend.

Figure 10.4. Mesh used in calibration analysis.

Figure 10.5. (A) Graph of disturbance density assuming linear accumulation. (B) Graph comparing of predicted package failures with JPL experimental results.

Figure 10.6. Graphs of disturbance density with cycles. (A) Viscoplastic case, (B) Plastic case.

Figure 10.7. Viscoplastic disturbance contour plots at cycle 1,000.

Figure 10.8. Viscoplastic disturbance contour plots for cycles 1,600 and 2,000.

Figure 10.9. Viscoplastic disturbance contour plots for cycle 2,400.

Figure 10.10. Plastic disturbance contour plots for cycles 1,000 and 1,600.

Figure 10.11. Contour plot of plastic disturbance for cycles 2,000 and 2,400.

Figure 10.12. (A) Graph of transient temperature in solder ball along horizontal centerline. (B) Element showing centerline where temperature was taken.

Figure 10.13. Failed BGA solder balls due to thermal cycling, (A) [111] (B) [44].

Figure 10.14. Flip chip failed solder bump [20].

Figure 11.1. Graph of temperature cycles. (A) JPL and Cycle A. (B) Cycle A.
Figure 11.2. Graph of cumulative distribution of failures for JPL and Cycle A conditions.

Figure 11.3. Cycle A package failures. (A) Graph of failure distributions for each solder ball. (B) Graph of predicted failures compared to experimental failures.

Figure 11.4 Contour plots of disturbance at various cycles.

Figure 11.5. Contour plots of various stresses $\sigma_x$, $\sigma_y$, $\sigma_{xy}$, $\tau_{oct}$ (MPa) at step 140, cycle 1.

Figure 11.6. Contour plot of disturbance in solder balls 1-6 at various cycles.

Figure 12.1. Graph of PCT temperature cycle used in FEA analysis. Temperature is assumed to be generated, i.e. constant throughout the die volume.

Figure 12.2. Mesh showing die, where temperature is applied for PCT analysis.

Figure 12.3. Contour plots comparing temperature (K) distributions for PWB with convection, and without convection.

Figure 12.4. Comparison of TCT and PCT deflections at step 74, with different Y fixity at 100 times actual deformation.

Figure 12.5. Graphical comparison of disturbance densities between PCT and TCT. (A) PTC disturbance density, (B) TCT disturbance density.

Figure 12.6. Contour plots of disturbance in solder balls for various TCT cycles, no Y fixity.

Figure 12.7. Contour plots of disturbance in solder balls for various TCT cycles, no Y fixity.

Figure 12.8. Contour plots comparing disturbance in solder ball 5 for TCT and PCT cycling, cycle 2,100, no Y fixity. (A) TCT disturbance, (B) PTC disturbance.

Figure 12.9. Contour plots of equivalent stress: Enlarged view of balls under die for TCT and PCT, first cycle, 64th step, no Y fixity. (A) TCT stress, (B) PCT stress.

Figure 12.10. Contour plots of various strains for PCT, first cycle, 64th step, no Y fixity.

Figure 12.11. Contour plots of various stresses for TCT-isothermal: $\sigma_x$, $\sigma_y$, $\sigma_{xy}$, and $\tau_{oct}$ (all in MPa), first cycle, 64th step, no Y fixity.
Figure 12.12. Contour plots of various stresses for PCT: $\sigma_x$, $\sigma_y$, $\sigma_{xy}$, and $\tau_{oct}$ (all in MPa), first cycle, 64th step no Y fixity. .................................................................297

Figure 12.13. (A) Von Mises stress in solder joint [121b]. (B) Crack in PTC experiment [36]. ........................................................................................................................................298

Figure 12.14. Disturbance density in solder balls with differing package convection and die flux. (A) $h = 27e-6W/(mm^2K)$, $q = 750mW$. (B) $h = 81e-7W/(mm^2K)$, $q = 250mW$ ..................................................................................................................299

Figure 12.15. Graph of relationship between die flux and package convection ..........300

Figure 13.1. (A) FEA model showing location of prescribed displacement. (B) Graph of prescribed displacement. ..................................................................................................................309

Figure 13.2. Graph of disturbance density in solder balls due to vibration .............310

Figure 13.3. Contour plots of vibration disturbance at various cycles.....................311

Figure 13.4. Contour plots of vibration disturbance in solder ball 13 at various cycles.
........................................................................................................................................312

Figure 13.5. Contour plots of various stresses (MPa) in critical solder ball, ball 13....313

Figure 13.6. Fatigue crack growth in solder ball [147]. ..............................................314

Figure 14.1. (A) Mesh used in analysis, showing void approximation. (B) Graph of Cycle A temperature cycle. ..................................................................................................................318

Figure 14.2. Graph of disturbance density in solder ball 5, containing void, versus cycle.
........................................................................................................................................319

Figure 14.3. Contour plot of disturbance in solder ball 5, with void, at various cycles.320

Figure 14.4. Contour plots of various stresses in solder ball 5, with void, cycle 1, step 28.
........................................................................................................................................321

Figure 14.5. Voids in solder balls of 313 PBGA package ............................................322

Figure 15.2. Contour plots of disturbance in solder ball 13 with void in bottom right at various cycles. ..................................................................................................................327

Figure 15.3. Graph of disturbance density in critical solder ball, ball 13 at various cycles ........................................................................................................................................328
Figure 15.4. Contour plots of various stresses (MPa) in critical solder ball, ball 13, (cycle 1, step 2).

Figure 16.1. Graph of disturbance density at various cycles.

Figure 16.2. Contour plots of disturbance at critical (11%) and ultimate (17%) cycles

Figure 16.3. Outline of area with disturbance > .7, taken from Figure 16.2 at $D_{dens} = 11\%$. Blue = 300°K, Green =330°K, Red = 390°K.

Figure 16.4. Solder ball showing fatigue crack.

Figure 17.1. Time to first failure comparison of PTC thermal cycle and vibration.
LIST OF TABLES

TABLE 3.1: Comparison of modal frequencies with and without a SMC on a PWB ..... 94
TABLE 7.4.1. Transient Analysis Parameters .......................................................... 180
TABLE 7.4.2. Steady State Analysis Parameters ....................................................... 181
TABLE 7.4.3 Comparison of Min and Max Temperatures ........................................... 182
TABLE 8.1.1 Thermal Material Parameters .............................................................. 201
TABLE 8.1.2 Elastic Material Parameters ................................................................ 201
TABLE 8.1.3 Plastic (HiSS Model) Material Parameters ........................................... 202
TABLE 8.1.4 Viscoplastic Parameters ........................................................ ............. 202
TABLE 8.1.5 Disturbance Material Parameters ......................................................... 202
TABLE 8.1.6 Temperature Dependent Property Exponents ....................................... 202
TABLE 9.1.1: Parameters Different From Chapter 8 .................................................... 206
TABLE 9.1.2 Testing Protocol .................................................................................. 208
TABLE 9.4.1 Comparison of Disturbance Density (Raytheon* Cycle) ....................... 213
TABLE 9.5.1 Comparison of Disturbance Density (JPL Cycle) ............................... 214
TABLE 10.1.1 Testing Protocol .............................................................................. 232
TABLE 10.3.1: Package Failure Prediction ................................................................. 242
TABLE 11.1.1: Comparison of JPL and Cycle A Temperature Cycles ..................... 261
TABLE 11.2.1: Package Failure Prediction ................................................................. 264
TABLE 16.3.1. Comparison of Results for Vibration Fatigue With Prescribed Temperature
ABSTRACT

A heat transfer subroutine has been implemented into an existing finite element code developed in the Civil Engineering and Engineering Mechanics Department at the University of Arizona by Dr. Desai and students. The code is capable of performing non-linear material and dynamic analysis. The heat transfer subroutine has been implemented such that any inelastic material behavior induced by a temperature increment is captured at every time step in a loading cycle. With the addition of the heat transfer routine, both thermal sources and sinks can be modeled. For example, power generating chips and power dissipating heat sinks, respectively. This will allow a more realistic representation of electronic packages under operational conditions.

A 313 ball PBGA staggered area array package was used in all the analyses performed in this dissertation. The calibration of the models was based on research performed by the JPL consortium which included members such as Raytheon, Boeing and Xilinx.

The focus of this dissertation was to determine the thermal and vibration fatigue lifetimes of electronic packages using the Disturbed State Concept. To achieve this goal, numerous analyses were performed, representing different test cases. The different test cases included thermal test chamber cycling (TCT), power cycling (PCT), vibration, thermal test chamber cycling with voids in solder balls, vibration with voids in solder balls, and coupled temperature with vibration.

Based on the results of these analyses, the Disturbed State Concept was found to predict the fatigue lifetimes of the 313 PBGA package with excellent accuracy, when test
results were available for comparison.
CHAPTER 1
INTRODUCTION

1.1 General

Modern society, in its daily activities, is constantly in contact with electronic devices. There is little that humans do today that is not directly facilitated by or concomitant with some type of electronic device (Figure 1.1). In some cases the electronic devices are complex, as in the case of computers, or very simple as in the case of LED pointers. In any case, it is certain that in the future humans will interact, directly or indirectly, with even more electronic devices. To achieve this goal, the semiconductor industry is continually decreasing the size and/or increasing the power of electronic packages, and the electronics industry is continually increasing the component densities of circuit cards. A number of packaging approaches are being investigated to meet these challenges, most of which are considered a variant of the ball grid array (BGA), such as DCA or direct chip attach, CSP or chip scale package, and flip chip [42]).

The demand for more economical electronic packages, in terms of size and cost, together with the demand for higher performance, has driven chip manufacturers to adopt surface mount technology (SMT) [48]. This includes BGA, CSP, DCA, and other related technologies. Presently, many military, aerospace, and portable electronics either contain these designs or are being forced to adopt the newer circuit card/electronic package designs [111]. In the past, these designs were rarely analyzed for thermomechanical stress or thermal fatigue because of the perceived inaccuracy and low analysis benefit to time ratio [152]. Thus, in the past, the majority of studies dealing with the reliability of
electronic packages were experimental [119]. However, this is changing as the power of commercial codes, especially finite element codes (FEM) and computers increase [12]. At present, there is some skepticism that large, general purpose FEM codes such as ANSYS, NASTRAN, and ABAQUS are sophisticated enough to provide meaningful thermal-fatigue and vibration-fatigue lifetime analysis of solder joints. This is primarily due to the limited constitutive equations available within the codes for fatigue modeling. However, this is changing as more constitutive relations are added to the codes, and as the codes adopt "user-input" constitutive relations. Researchers now regularly incorporate finite element analysis (FEA) into the design process, and many [73, 99, 76, 12, 140] have published data using ANSYS, NASTRAN and ABAQUS, some employing their own constitutive equations. The use of other non-commercial codes for thermal cycling of electronic packages is still common in academia, and includes codes written at Sandia National Laboratories (SNL), and the Jet Propulsion Laboratories (JPL) [42, 111]. The Disturbed State Concept (DSC), originally developed by Professor C. S. Desai [28], has been incorporated into the DSC-SST2D FEA code [33], and is capable of assessing the reliability of electronic packages in both thermal cycling and vibration environments. The DSC-SST2D code will be the code adopted in this research, although NASTRAN and ANSYS will be used to document certain findings, and generate the complex meshes used in later chapters.

As electronic packages become smaller, and circuit boards become more densely populated, understanding the relationship between thermomechanical and vibrational forces on solder interconnects is becoming more critical. It has been estimated that
eighty percent of electronic failures are due to thermal fatigue [116]. However, in certain industries, such as the aerospace and automotive industries, the primary cause of failure is mechanical vibration fatigue. The limits of solder design and reliability are especially at risk with the harsh operating environments encountered by military/space hardware and new portable electronics (PCs, wireless telecommunications equipment) [35, 36]. As electronics become more profuse, the environments in which they operate will certainly become more demanding.

1.2 Background

The use of the ball grid array style package is a relatively recent phenomenon, however, it is not a recent innovation. BGA packages are mounted on printed wiring boards (PWB) with a relatively recent advance in mounting called surface mount technology (SMT). In the early years of electronics, around the 1960's, devices were mounted directly on PWBs, however, the technology was quite different from what is now considered to be SMT. In the 1960's and 1970's the dual inline package (DIP) dominated the electronics industry as the package of choice. This chip was mounted on PWBs with a lead that extended through a hole in the PWB, and was then soldered to the board (through hole technology or THT). By the 1980's the DIP package had reached its limits. The package had become too large, requiring a large amount of the PWB, and the long wire lengths were affecting chip speed. Just prior to this time, in the late 1970's, two new types of packages had been developed, the leadless ceramic chip carrier (LCCC), and the small outline integrated ceramic package (SOIC). The LCCC was leadless, and the SOIC was leaded. However, the SOIC leads had a much finer pitch, and the leads
laid directly on the surface of the PWB, as opposed to THT. These new packages were soldered to the PWB using a technique called solder reflow. In this process, solder paste is deposited onto PWB lands, the packages placed on the lands, and the entire PWB heated until the solder melts and "reflows". These new packages were the forerunners of current leaded and leadless SMT. Figure 1.2 profiles the different package types.

Because there are no leads on an LCCC, the electrical performance of these packages is excellent. This is a result of the short distances between the board and die (silicon chip). As advances continued, quad flat packs (QFP) were developed. These were leaded packages that contained leads on all four sides of the package. These packages used "gull wing" style or "J" style leads. The QFPs were very reliable, using a pitch of .025 inches. As the semiconductor industry continued to strive for smaller, faster packages, die sizes continued to decrease in size. This allowed for more package interconnects (input-output or I/Os). In the 1990’s the pitch continued to decrease, and QFPs were available with over 132 leads. As the pin counts or I/O counts increased, the pitch between the leads became smaller. At a pitch of .02 inches, the pin count could be over 200. With the small pitch and chip size, problems arose in the assembly process which could not be overcome. Hence, the QFP had reached its limit, but the semiconductor industry’s desire for smaller, cheaper and more powerful packages had not.

With the QFP having reached its limit, the semiconductor industry began investigating other methods of designing electronic packages called BGA, DCA and CSP. The CSP is considered a package where the dimensions of the die or integrated
circuit (IC) and package are very similar. The most prominent DCA is flip-chip technology, and this technology is now becoming available in commercial packages. The BGA is very similar to flip-chip technology, but where the flip-chip had been primarily used for die attach, the BGA was now being used for package attachment to PWBs. The BGA, because of encapsulation, is capable of withstanding hostile environments where moisture, chemical, and particulates are present. Because of this, the BGA is robust and readily adaptable to military, aerospace, automobile and consumer products. The pitch for a BGA is typically .05 inches (but is decreasing), reducing manufacturing problems associated with small pitches. The large pitch is achievable because of the multiple rows of solder balls made possible by SMT. Also, because the chip is an area array package, its "footprint" is reduced, allowing a higher packaging density on circuit cards. In comparison with leaded SMT packages, the BGA offers faster circuitry speed due to shorter PWB terminations. However, even with the faster speeds, the heat dissipation can be better than leaded SMTs.

The greatest advantage of the BGA (Figure 1.3 and 1.4) is the robust design, which is critical during the manufacturing process [111]. In addition, during the manufacturing process, self alignment of the solder balls occurs during the reflow process [71]. This is just one more added benefit of BGAs. The drawback of BGAs is that solder joints on the interior of the package cannot be inspected after reflow because they are not visible. Inspection must be performed by non-visual methods such as acoustic, x-ray, or other means.
BGAs may be manufactured with the solder balls on the periphery of the package (PGA or peripheral grid array), over the entire plane of the package (FGA or full grid array), or in staggered rows over the package. In many cases, the solder balls are eliminated under the die (Figure 1.4), as these balls are most susceptible to failure by thermomechanical fatigue. During the BGA development, solder balls were distributed in a regular pattern over the bottom plane of the package (FGA). However, it was found that the solder balls under the die failed at a higher rate than the balls on the periphery [75]. It was determined that high stresses developed under the die due to a coefficient of thermal expansion (CTE) mismatch. The die (silicon) was not allowing the top boundary of the solder balls to expand relative to the bottom of the balls. However, even with the peripheral grid array (PGA), the highest stresses are still found in the solder balls at the corners of the die [153]. These balls are typically referred to as being under the "die shadow". Although the PGA, where balls are removed from under the die, is generally considered more reliable than the FGA, the FGA is still manufactured due to the high pin count (Figure 1.5).

1.3 Current Surface Mount Technology

The chip scale package (CSP) is the new generation of chip packaging methods. In general, a CSP is a BGA or BGA variant type package with a ratio of chip-area to package-area greater than 80% [6]. The CSP presents advantages over other packaging methods in terms of testing, standardization and handling durability [73] because of its small size, which leads to less warpage and co-planarity issues. Because of the general definition of a CSP, numerous package styles fall under the CSP category. These
include, but are not limited to, the slightly larger than IC carrier (SLICC) by Motorola, [148], the micro ball grid array (μBGA) by Tessera [65], the fine pitch BGA (FPBGA) family of packages by NEC [85], and the molded chip sides encapsulation (MCSP) family of packages manufactured by Nitto Denko [125].

While these types of chips, especially the BGA, CSP and flip chip are currently dominating the semiconductor industry's efforts, new chip designs are constantly being advanced. The most innovative of the new chip technologies is the three dimensional chip introduced by NEC. This chip is a "stack up" or vertical stack of CSP style chips. It is currently being introduced as a memory chip that will substantially increase memory size, while decreasing board area usage.

It is clear that surface mount technology will continue to evolve, and that the need to understand package and solder behavior must keep pace with these new designs. With the complex behavior of solders, and the rich environments to which they are subjected, the task of predicting package and solder reliability is challenging. However, with "complete" constitutive models such as the DSC, the physics of failure for these components can now be identified, and introduced into the design process.

1.4 63Sn/37Pb Package Failure Mechanisms

The mechanical behavior of Sn/Pb solder joints is complicated, and the mechanism(s) of failure under thermal cyclic loading are due to multiple factors. In general, the behavior, and subsequent failure of a solder joint, is related to the temperature difference of a cycle, the temperature ramp up time, the temperature ramp down time, the temperature dwell times, and the type and size of the solder joint [43].
The types of solder joints include leaded (J, gull wing, pin) and non-leaded (column, ball, pad). The size of a non-leaded solder joint can vary greatly. For example, from .1mm in LG Semicon's bottom leaded plastic (BLP) CSP assembly to .56mm in Motorola's SLICCC. If long dwell times and high homologous temperatures (>0.5 melting temperature or $T_m$) are prescribed, then creep deformation and relaxation may be a significant factor in joint failure. For lower temperatures and/or high loading rates as encountered in vibration, plastic deformation and/or fracture may be the significant mechanisms for joint failure. Knecht and Fox as reported in [66] have developed a constitutive model for eutectic solder, accounting for steady-state creep and time independent plasticity. Other researchers have used elasto-viscoplastic constitutive relations to model solder. However, the most important aspect of thermal fatigue in Sn/Pb solder joints appears to be micro-cracking. Indeed, it is the most dominant failure mechanism in Sn/Pb solder joints [73]. Therefore, a constitutive model attempting to describe Sn/Pb behavior under thermal cycling must be capable of describing micro-cracking and fracture. Using advanced continuum behavior (viscoelastic, viscoplastic or creep/relaxation) in a model is important, but it cannot predict the one aspect that is most critical: When will the joint open? This is where the DSC modeling approach is most advantageous. The DSC model is capable of predicting separation or fracture of a solder joint, in a holistic modeling approach, taking into account the many types of behavior the material may exhibit. However, unlike classical fracture mechanics and micro-mechanics which also describe fracture, the DSC approach does not require detailed knowledge of the number of cracks, locations, orientations or sizes of these cracks to perform an analysis.
1.5 Proposed Research

The research in this dissertation investigates the fatigue of electronic packages due to thermal cycling, mechanical vibration, and the combination of vibration under thermal loads. The thrust of the research is on the development of a physics of failure finite element code capable of predicting fatigue through damage mechanisms. In particular, the development of a heat transfer subroutine which allows temperature gradients, and heat sources and sinks to be modeled. This allows the heat source (die) and heat drain (heat sink or package exterior) to be modeled through their material parameters (specific heat, thermal conductivity, etc.). The parameters needed for the thermal analysis are the material parameters and boundary conditions, i.e., heat flux from the silicon chip, and the heat transfer coefficient of the package. Based on the temperatures within the chip, the thermomechanical stresses can then be calculated.

The model was first calibrated to thermal-experimental testing performed by JPL [42, 43], and vibrational-experimental testing performed by Raytheon [140]. In each case, the experiments were performed on a 313 ball, PBGA with a staggered ball arrangement. This particular PBGA is an exceptional case study because it is a full grid array (FGA), allowing a study of the fatigue of the solder balls directly under the die. It is also a rather typical PBGA with a medium ball count, and overmolded package (OMPAC) style. Also, experimental testing, both thermal and vibration, have been performed on this package, allowing comparisons with the DSC-SST2D FEA analyses.

Following the calibration of the FEM model, the heat transfer characteristics, and their effect on fatigue was investigated. Of interest was the correlation between
experimental thermal accelerated testing, and operational thermal fatigue. The investigation of the correlation between thermal accelerated testing and operational fatigue is very sparse in the literature. Numerous articles regarding the mechanisms of failure due to external thermal cycling have been found, but few regarding the differences between internal heating (power cycling testing or PCT) and external heating (thermal cycling test or TCT). It is well known that for an experimental investigation to be accurate, the mechanisms of failure between the experiment and actual operation must be the same. However, there appears to be a contradiction between experiment and operational use, as typical experiments heat the entire package externally in a thermal chamber (TCT). This is a contradiction to operational use, where the package is heated internally by a power flux from the die (PCT). Also, TCT neglects the heat transfer characteristics of the package, as externally heating a package investigates the opposite conditions of an electronic package in operational use. That is, heat is transferred from the package boundary to the interior, where the external perimeter acts as a source, and the internal acts as a thermal sink. In operation, the die acts as a thermal source, and the package perimeter acts as a thermal sink. Also, due to time constraints and cost, experiments are accelerated. This neglects, to some extent, the well known anelastic, viscoelastic and viscoplastic properties of solders.

Voids within solder balls has been well documented (Figure 1.6). However, voids have been shown to have little to no impact on solder ball life. This appears to be a contradiction, based on the fact that solder joint shape has been documented to have a significant impact on fatigue life. Analyses were performed, where solder balls with
voids were subjected to both thermal cycling and vibration, to understand how voids are able to modify solder joint geometry, yet not degrade the joint life.

Mechanical vibration fatigue is a common cause of electronic package failures in aerospace and automotive environments. As more devices become portable (telephones, laptops, etc.), this type of failure will become a serious problem for commercial product reliability. To investigate the effect of vibration fatigue, the 313PBGA package used in the thermal analyses was subjected to a quasi-dynamic environment. In this study, the worst displacement found from a random vibration test was applied to the PBGA. A calibration was not necessary, as it was assumed that the calibration done for the thermal analyses would be adequate. A comparison was then made with a combined thermal and vibration loading. Studies of this type were infrequently found in the literature.
Figure 1.1. The Forces Driving Electronic Package Miniaturization and Reliability
Figure 1.2. Comparison of Through Hole and Surface Mount Technology (not to scale), Showing Plan View, Side View, Package To Board Attachment Mechanism, and Electronic Package Mounted To PWB
Figure 1.3. Typical Overmolded PBGA Schematic (not to scale)
Figure 1.5. Practical Components PBGA, FGA showing daisy chain
Figure 1.6. Voids in solder balls from thermal cycling [43]
CHAPTER 2
REVIEW OF SOLDER JOINT THERMAL FATIGUE

2.1 General

Numerous texts [71, 73, 75, 38] have discussed the thermal cyclic fatigue behavior of solder joints in electronic packages, and numerous articles regarding solder joint reliability have been published in the many electronic-packaging journals. One method of describing solder joint reliability is to use empirically derived statistical relationships between thermal cycles and number of solder joint failures, rather than constitutive or physics based methods. A commonly used statistical measure found in the literature is the Weibull distribution seen in Figure 2.1 [73, 42, 43]. Researchers who have employed physics based methods to describe solder joint failure have used classical elasticity [81], variants of classical fracture mechanics [90] and plasticity/viscoplasticity [112, 27, 22]. These material behaviors may not be sufficient, independently, for solder fatigue analysis. It is well known that solder exhibits elastic-viscoplastic-fracture behavior as shown in Figure 2.2 [68]. Therefore, a more “complete” model is crucial to completely capture and describe solder behavior. By complete, it is meant that constitutive relations are available in conjunction with fracture or damage procedures. The FEM code proposed for this dissertation employs such a “complete” constitutive model, and is a strong reason for its success in modeling a wide range of materials and loading conditions.

With the increasing power of personal computers, FEA is becoming the dominant research tool in this area. Numerous examples of this abound in the literature. This is a
natural evolution of research in electronic packaging, taking advantage of the power of the FEA to create better, faster packages, which in turn allows FEA models to become more sophisticated and accurate. However, there are a number of considerations which must be addressed before an FEA model can be properly implemented. These considerations are addressed in references [73] and [92]. While there are numerous examples of electronic packaging FEA models in the literature, the dominant material behavior considered appears to be plastic or viscoplastic (creep). No evidence of complete constitutive models including thermo-elastic-visco-plastic-fracture behavior were found, although some researchers have investigated fracture based methods [78].

A number of physics based approaches are present in the literature regarding the prediction of fatigue life of electronic packages. These include fracture mechanics, energy, damage, and micromechanical approaches [32]. There are advantages and disadvantages to each approach, and these must be evaluated on a case-by-case basis. However, if the life prediction is to be based on the results of FEA (or any other numerical method), these approaches will generally involve a separate code for the life cycle prediction. This is a great advantage of the DSC-SST2D FEA code used in this research, where a complete constitutive model is available, coupled with a fully integrated "fracture" or damage capability.

The greatest adversary to solder joint fatigue life, when subjected to thermal loads, is the stress caused by the global CTE mismatch between the silicon die and the PWB [32, 38]. However, the local CTE mismatch at the solder joint level cannot be ignored, especially when materials of similar CTEs are attached [36, 12]. It has been
widely documented that the solder joints most likely to fail under CTE mismatch are the joints furthest from the neutral point, sometimes referred to as the distance to neutral point (DNP), and the joints under the die [22, 137, 103, 77]. Because of this wide discrepancy, one cannot know a-priori which balls are most likely to fail, and hence experimental testing or FEA methods must be employed. However, as the cost of experimental testing increases, FEA is becoming the method of choice for reducing company expenditures. With this push towards modeling, comes a greater need for more accurate life prediction methods.

As noted in Chapter 1, solder bump type packages are not new, having been developed in the early to mid 1980's [36]. However, their upsurge in use is a relatively recent phenomenon. This is due to a number of factors, including manufacturing process improvement, and materials innovations. However, there has been less innovation in the area of lead-free solders. Currently, the most widely used solder is still eutectic and near-eutectic Sn/Pb. The United States Congress and European Union are currently making plans to ban lead solders (perhaps as soon as 2004). However, Nortel has already made the switch to unleaded solder. Nortel developed the first lead free telephone in 1997 [127]. While this may be encouraging to environmentalists, the process of finding a satisfactory replacement for 63Sn/37Pb and 60Sn/40Pb has been costly to industry. As such, no clear replacement has been found. For example, Hitachi is favoring a Sn/Bi/Ag solder, NEC is leaning towards Sn/Ag/Cu variants, Sony is considering Sn/2Ag/4Bi/.5Cu/.1Ge, and Matsushita prefers Sn/Ag/Bi/x [76]. The lack of concensus will surely lead to problems in repairing and rework, as mixing of solders will lead to
serious problems. Interestingly, Harman [55] reported that 99.3Sn/.7Cu and 98Sn/2Ag were the most promising unleaded solders. Clearly, there is no consensus. As unleaded solders will soon become the standard, problems associated with the higher reflow temperatures of these materials will need to be addressed, as the industry standard FR-4 board may not be suitable for these temperatures [76, 103], with a glass transition temperature ($T_g$) of 120°C.

In general, it is impractical to duplicate the complexity of a PWB in a numerical model (Figure 2.3). A PWB is a complex system of ground planes and PTVs (plated through hole vias) with a myriad of differently shaped components mounted on it. Usually, to avoid the complexity in a FEA model, simplifications are made. One of the most common simplifications is to model the board as a homogeneous material. However, doing so distorts the results. It has been shown, for instance, that the PTVs in PWBs cause an increase in the CTE of the PWB, and hence a decrease in the reliability of solder joints [36]. These simplifications are necessary though, and with proper engineering judgment, an analyst can attenuate the results based on the fidelity of the model. There is no doubt, however, that solder material parameters are temperature dependent (Figure 2.4). Thus, for models undergoing temperature cycling, the material parameters must include their temperature dependence, as solder joints are subject not only to internal temperature sources (chip), but also environmental temperature fluctuations [36, 102]. Therefore, the use of isothermal material parameters in predicting solder joint reliability is suspect [102].
extension. As the number of cycles increased, the size of the crack extension decreased. However, examining Figure 2.6, it can be seen that the cracks he measured are very large. For today's CSP and BGA packages, with very small ball sizes, these crack sizes are basically meaningless, since the first increment of crack growth is larger than the ball diameters.

Voiding in solder balls is a well documented [96, 58] and intriguing phenomenon. Some researchers have shown that voids cause failures [83], and others show experimental research documenting that voids do not increase or decrease the likelihood of failure in a solder joint, i.e., voids do not cause failures [111]. This finding appears to contradict the premise that the shape of a solder joint plays a key role in the fatigue life of solder. When voids were initially discovered, it was assumed that they were the culprit of early fatigue failures, and should be eliminated. However, as more research data is accumulated, it is unclear whether voids have an impact on failure or not, i.e. there is no clear evidence that they are always a problem. Because of this, from the mechanics-of-failure point of view, interest in them has declined. It may be, however, that on a case by case basis, certain package configurations, and/or solder ball configurations, are sensitive to certain void shapes. Although interest in voids has declined, they may still have research potential, even if they are found not to present a problem for the package of interest. For example, it is commonly assumed that the life of a solder joint (ball) is highly dependent upon it's geometry. If this is the case, then how can a void be present in a solder ball without affecting the fatigue characteristics? If this can be understood, then perhaps more economical solder joints can be developed. For example, a reduction in the
amount of solder volume, based on the size of typical voids. Note that some voids are very large (Figure 2.7).

Fatigue life modeling of electronic packages is a complicated task, and simplifications must be made, as was previously discussed for PWBs. If the simplifications are made with good engineering judgment, then the results should be reasonable. For thermal cyclic loading, determining whether a steady state or transient analysis is appropriate is one such simplification decision. For vibrational loading, determining whether a normal mode, direct transient method, or PSD approach is one such simplification decision. In fact, for dynamic analysis, if the frequency of the applied loading is 1/3 or lower than the first mode of the structure, a dynamic analysis is unnecessary. This is also the case if the applied loading frequency is 3 times or greater than the first mode of the structure. If either of these two cases is present, then the analysis can generally be performed as a static displacement or force analysis if either are known [4]. Depending on the accuracy needed or how quickly a load is varying, a transient analysis may be necessary. However, based on computer power, a transient analysis may take a very long time to calculate. Therefore, a judgement needs to be made regarding speed versus accuracy. Experimentally, it has been found that steady-state thermal conditions generally do not exist [36]. However, steady state conditions are assumed in many cases, as in [109] where researchers used a steady state analysis while incorporating creep. Generally speaking, TCT analysis is done assuming isothermal conditions in the model, and therefore no heat transfer analysis is done. For power cycling (PCT) conditions, heat transfer must be included. However, it will be shown in
later chapters that a transient analysis is not always necessary, and steady state heat transfer can be used as a reasonable approximation.

Air-heat-sink cooling is the primary cooling method for electronic packages because of its numerous advantages (Figure 2.8). The advantages of using air as the coolant include no cost (surrounding air is free), requiring no containers (except in space or underwater), and being non-toxic. However, air is a poor thermal conducting fluid, and is thus limited to low power applications. At present, devices are currently being manufactured with waste heat fluxes greater than 300W/cm² [109]. Greater advances will be necessary in heat sink technology for these devices to be reliable using air cooling. Research into heat sinks for electronic packages is a very active research area, and has grown into more elaborate cooling methods such as heat pipes, glycol tubes, evaporation liquids, etc. However, for the near future, air flow through heat sink fins will be the primary cooling method.

With the increasing demands being placed on solder joints, performing electrical as well as mechanical functions, the design of electronic packages is now incorporating approaches used in aerospace (structural, thermal) and power generation industries [102]. Also, with computing power increasing, and commercially available FEA codes offering more advanced constitutive models, the complexity of FEA models continues to increase, both geometrically, and constitutively. Viscoplastic models are commonly seen in the literature now. Many different viscoplastic models can be found in the literature, and there has been some discussion regarding the "artificial" separation of the viscoplastic and plastic components of stress and strain found in some of the models [20]. Some
researchers might argue that this is an “academic” point as any continuum theory will always be an approximation, since a material is not a continuum. Therefore, from the initial formulation, any continuum theory is immediately an approximation. While the approximation is negligible for most engineering structures, MEMs technology will challenge this negligibility. The accuracy of continuum theories will continue to be challenged as future technologies design and produce smaller devices. Presently, MEMs (micro-electronic machines) technology is producing working machines designed from the atomic and elemental level. Devices such as gears several angstroms (1x10^{-10}m) in thickness are routinely made (Figure 2.9) [49]. It is not clear that continuum mechanics will still be accurate at this nanometer level. A separate mechanics approach called "discrete mechanics" or nanomechanics is being developed to attack these challenges [37]. If continuum theories are to be used, i.e., phenomenological models, then material parameters will need to be ascertained at the micro level [32]. This area is only in its infancy, but the challenges are enormous.

Along with the decreasing size of MEMs machines, solder joints are also shrinking in size. With the decreasing size of the solder joint, it is becoming increasingly difficult to measure strain experimentally. Thus, FEA is rapidly becoming the only method available to understand joint behavior, and gather stress-strain data [80]. Because of this, accuracy and robustness of FEA models will become critical in the future, as experimental data will only provide gross stresses and strains. FEA models will be used to determine micro-level stresses and strains, and will be verified only through the gross strains at the package level determined through testing.
One of the challenges of FEA is the determination of what material parameters to use, and how to determine what constitutes failure. A comparison of experimentally determined material parameters shows that the value of a "simple" parameter like elastic modulus varies widely from researcher to researcher, and test to test. One explanation for this is that solder is geometrically sensitive, strain rate sensitive, and cooling rate sensitive. Therefore, even for the exact same test conditions, a specimen cooled faster or slower than another will give different material parameters. The same is true for specimens of the exact same geometry. Any variation in strain-rate, or solidification rate of the solder joint will cause a difference in material parameters. A comparison of fatigue life data from various sources is also difficult, since the definition of a failure is variable from source to source [80].

Mechanical vibration fatigue is generally considered stress based, where a load is placed on a specimen/model, and thermal fatigue is generally considered strain based, where a strain is placed on a specimen through a CTE mismatch [80]. It is not clear that a model used for thermal cycling can be used for vibration or vice-versa. No research has been found which has investigated this. However, for low cycle fatigue (thermal cycling is considered low cycle), the Coffin-Manson equation is the most widely used analytical model [20, 80]. A common form found throughout the literature is:

$$\frac{\Delta \varepsilon_p}{2} = \varepsilon'_f \left( 2N_f \right)$$

(2.1)

where \(N_f\) is the total number of cycles to failure, \(\Delta \varepsilon_p\) is the plastic strain amplitude, \(\varepsilon'_f\) is the fatigue ductility coefficient, \(c\) is the fatigue ductility exponent. Briefly, it is assumed
in the Coffin-Manson equation that the plastic strain drives fatigue, and hence the elastic strains are completely ignored. For high cycle fatigue (mechanical vibration is high cycle), Basquin's equation is more appropriate [80], and is given by:

$$\frac{\Delta \varepsilon}{2} = \frac{\sigma_f}{E}(2N_f)$$

(2.2)

Basquin's equation is basically the opposite of the Coffin-Manson equation, in that only the elastic strain is considered, the plastic strain being completely neglected. In most cases, apparently, Basquin's equation is limited to mechanical vibration. No use of it for thermal cycling has been found. From these two equations, a generalization can then be made, about whether a thermal fatigue model can be used for a mechanical vibration problem or vice-versa. If the fatigue model does not include elastic strains, then it will not accurately model vibration fatigue. If the model does not include plastic strains, then it will not accurately model thermal cyclic fatigue. Hence, again, it is clear that a proper (and complete) constitutive model be used, such as the DSC model.

In operation, actual packages are subjected to non-ideal temperature cycles [80] and random vibration (Figure 2.10). However, it is impossible to model the time-temperature, and random vibration loads a package will encounter, as they are unknown. Most general-purpose FEA commercial codes have random vibration generators which are based on PSDs (power spectral density) distributions and can be used to model mechanical vibration. When a random vibration analysis is not available (for modeling) there are methods to approximate this type of loading [117]. Random thermal cycling, has not been seen in the literature, but is arguably not as important. This is because
during operation, an electronic package may indeed experience randomness during operation, but frequency and magnitude differences in temperature are likely to be mild due to the thermal inertial of the package, board, and enclosure.

Creep is a complicated phenomena, and because of the high homologous temperature of eutectic solder, must be included in a reliability model if accurate life prediction is to be determined. The $T_m$ (melting point) of 63Sn/37Pb is 183°C, which gives a homologous temperature of about $300K / 456K = .6$, when operating in a room temperature ambient condition. As a rule of thumb, creep is expected to occur at $.5T_m$ and higher. Creep is a result of grain boundary sliding, matrix creep (dislocation movement), screw dislocations, cross-slip, climb of edge dislocations and other mechanisms [80, 100]. At lower temperatures, and/or faster loading, and/or stiff assemblies, the creep mechanism is primarily matrix creep. At higher temperatures, and/or slower loading, and/or softer assemblies the creep mechanism is primarily grain boundary sliding [122]. Based on the mechanisms of creep, it can be understood why the grain size (from solidification rate) and type/amount of impurities in solder have such an impact on fatigue life. Research incorporating different material phases of solder in FEA is not common. However, Winter and Wallach [137] have investigated FEA models implementing structural changes in solder balls, i.e., incorporating different solder phases (Figure 2.11). Because of the slow thermal loading rate of solder joints in service, it is likely that stress relaxation is dominant [102]. However, it is likely that stress relaxation is accompanied by creep, since relaxation can occur by the material creeping. It is also believed that the two (creep and relaxation) are mechanistically equivalent [102].
Experimental results for thermomechanical fatigue under both strain (displacement) cycling and concurrent temperature cycling are not common. However, Wang [133] has presented results from a newly designed testing apparatus specifically designed for solder materials, which can incorporate both displacement and temperature cycling. During strain cycling, it is likely that compressive dwell times help offset tension dwell damage, and thus a "healing" of the solder occurs [102]. That is, it has been experimentally shown that tension induces more damage than compression. The DSC constitutive model presented in this research has the capability of modeling this "healing" behavior [28]. There appears to be evidence that increasing the thermal cycling frequency reduces solder joint life, possibly due to the reduction in annealing time at the high temperature dwell [102]. For mechanical cycling, increasing the frequency also reduces solder joint life, which may be due to the reduced time for stress relaxation to occur [16].

While there are many 3D models present in the literature, 2D models continue to be used, and have been used with great success due to the reduction in model development and solution time [73]. All of the analyses performed in this research use 2D plane stress models.

2.2 Review of representative models

Plumbridge [102], has presented an overview of monotonic as well as cyclic experimental results for eutectic solder, primarily focusing on creep behavior. The results are summarized here (Figure 2.12 and 2.14). Utilizing two loading rates spanning two orders of magnitude ($7 \times 10^{-3}$ and $7 \times 10^{-5}$), the true-stress versus true-strain plots show
a nearly linear relationship, with the lower strain rate producing lower stress levels throughout the strain range. Plotting engineering-stress versus engineering-strain, the results are highly non-linear, and any relationship would be difficult to determine. Plotting log-strain-rate versus log-stress for samples at different temperatures shows a slightly bi-modal relationship. However, the effect of temperature is not clearly evident. At lower strain rates \((\varepsilon < 1 \times 10^{-4})\) stresses are higher at higher temperatures. At faster strain rates \((\varepsilon > 1 \times 10^{-4})\) stresses are higher at lower temperatures. For constant stress tests, using bulk specimens, the effect of temperature is more clear. At 20\(^\circ\)C, 60Sn/40Pb solder displays a bimodal stress-time relationship. At about 3MPa, a clear discontinuity in the stress-time relationship is present, indicating a possible change in the failure mechanism. This discontinuity is not as prevalent in samples tested at 100\(^\circ\)C, indicating that the failure mechanism may be constant at higher temperatures.

The effect of cooling rate on molten solder has been observed to change the response of solders, and Plumbridge [102] shows data indicating this effect. As cooling rate increases, the stiffer a solder specimen becomes. It is also well known that the solder joints of BGA type packages are stiffer than leaded packages, and this leads to high stresses in the BGA solder joints [137]. Under a constant stress of 8.25MPa, a slowly cooled bulk solder specimen (cooled in a furnace) reached .18% strain in approximately 6.25 hours. However, for water quenched specimens, the time was about 8.75 hours to reach the same strain. It might be speculated that the fine matrix of a water cooled specimen is the likely cause. It is well known that a slowly cooled material develops large internal crystalline structure, and it is possible that these crystals are more prevalent
to sliding along adjacent crystal boundaries. Frear [40] and Qun [105] have shown that
during thermal cycling, the microstructure of solder evolves from a finer structure to a
highly coarsened form prior to fracture (Figure 2.13).

For cyclic temperature tests, the Coffin-Manson equation has been shown to
provide reasonable results for varying temperatures and strain rates. Plumbridge [102]
shows that for temperature cycling, an increase in total strain range decreases cycles to
failure, and increasing the cycle ramp time increases the \( N_f \) (cycles-to-failure).
Interestingly, the higher the strain range, the flatter the solder response in terms of \( N_f \)
versus frequency. However, there appears to be a clear discontinuity in the \( N_f \) versus
frequency at approximately \( 10^{-3.8} \) creating a distinctly bimodal relationship between \( N_f \)
and frequency.

Lau and Pan [76] performed finite element analysis on a flip chip using 3 solders:
96.5Sn/3.5Ag, 100In, and 62Sn/36Pb/2Ag. They performed the analysis using ANSYS
and the Garofalo creep constitutive equation. The Garofalo creep equation is given by:

\[
\frac{d\gamma}{dt} = C \left( \frac{G}{\theta} \right)^n \sinh \left( \omega \frac{\tau}{G} \right) \exp \left( -\frac{Q}{k\theta} \right)
\]

\[ (2.3) \]

Where \( \gamma \) is the creep strain, \( t \) is time, \( C \) is a material constant, \( G \) is shear modulus, \( \theta \) is the
temperature in Kelvin, \( \omega \) is the stress where the power law stress dependence is no longer
valid, \( \tau \) is the shear stress, \( n \) is a stress exponent, \( Q \) is the diffusion activation energy, and
\( k \) is Boltzman’s constant. It is interesting to note that Darveaux and Banerji [26]
proposed a similar creep equation:
Which has been used by a number of researchers [12, 137]. It appears that this is a simplified form of the Garofalo equation with the following simplifications:

\[
\frac{dy}{dt} = A \left[ \sinh \left( \frac{\tau}{\beta} \right) \right]^{m} \exp \left( - \frac{Q}{k\theta} \right) \quad (2.4)
\]

\[
A = \frac{CG}{6} \quad \text{and} \quad \beta = \frac{G}{\omega} \quad (2.5)
\]

Bingzhi et. al. [12] have used the Darveaux and Banerji creep equation to model a 540 ball PBGA, and will be discussed later. Lau and Pan applied a 60 minute thermal cycle, cycling between -20°C and 110°C with a 20 minute dwell at 110°C, and a 10 minute dwell at -20°C. For all solders considered, the Garofalo equation predicted that the creep strain stabilized after the first thermal cycle. That is, the rate of increase in creep strain became a constant after the first cycle. While the authors did not point this out, the 62Sn/36Pb/2Ag and 96.5Sn/3.5Ag solders displayed a negative creep-strain versus time slope, while the indium solder displayed a positive creep-strain versus time slope. No attempt was made to explain why this would be the case. The authors did not compare their results with test data.

As previously mentioned, Bingzhi et. al. [12] used Equation 2.4, implemented in an ANSYS 3D FEM model of a 540 pin PBGA. They used a thermal cycle of -60°C to 130°C with 4 minute dwells, and 1.2°C/s ramp rates. Although the authors did not discuss this, 1.2°C/s is very fast, and is more of a thermal shock than a realistic temperature rate in service. However, they did not appear to take heat transfer into account, and thus their model is isothermal which alleviates temperature gradients, and
thus transient stresses. The authors used a submodeling technique where a global
(macro) model is used to determine the largest displacements, and a local (micro) model
to determine detailed stresses based on the displacements from the macro model. This
technique is quite prevalent in the literature, and appears to produce good results [24, 21].
The primary focus of their research was on pad size, solder shape, and solder size (Figure
2.15). They found that the solder joint fatigue life was dominated by local (solder to pad)
CTE mismatch, rather than global (substrate, chip, etc to PWB) CTE mismatch. This has
also been pointed out by Engelmaier [36]. To determine solder joint geometry, they used
the Surface Evolver program which determines the final solidified geometry of a solder
joint from the molten state. Using a number of solder ball shapes, volumes, and pad
sizes, they concluded that cycles-to-failure is affected by all these parameters. They
report that the joint fatigue life increases as pad diameter is increased, possibly due to the
solder ball morphing into a thin column geometry as the pad size is increase. They found
that as the volume of the solder ball is increased, the fatigue life decreases. This is
probably due to an increased stiffness of the joint, due to the increase in diameter to
height ratio. In general, the underlying increase in fatigue life appears to be the reduction
of joint stiffness. This may be why voids are not always deleterious to joint life. The
decreased stiffness of the joint (due to voids) may offset the stress concentration they
cause. The data seems to suggest that if the stiffness of the joint is greater than the global
CTE mismatch, then the fatigue will be locally dominated. If the joint is soft, then the
global CTE mismatch will dominate the fatigue life. As a side note, the Surface Evolver
program is a public domain computer code [135]. The Evolver program works on the
principle of minimum energy, calculating the surface tension of a liquid (in this case molten solder), the forces acting on it, and the shape which produces an equilibrium of forces.

As packages become smaller, it is likely that the shape of area-array solder joints will become less spherical due to wicking action between the package pads and the PWB pads. When this critical dimension is reached, it may become necessary to use programs such as the Surface Evolver to determine the solder shape. At present, however, area array solder joints take on fairly regular shapes which can be reasonably described by a semi-spherical or semi-column geometry.

Lau [77] performed a FEA study using ABAQUS to determine the accuracy of the simple DNP (distance to neutral point) formula. Although Lau did not provide the formula for shear strain using the DNP, it may be found as:

$$\gamma = \frac{DNP(\alpha_c \Delta T_c - \alpha_{si} \Delta T_{si})}{H}$$

(2.6)

where DNP is the distance from the center of the package to the outermost solder joint, $\alpha_c$ is the CTE of the chip carrier (substrate), $\alpha_{si}$ is the CTE of silicon, $H$ is the solder joint height, and $T$ is the temperature. If it is assumed that the each layer achieves the same temperature (as in isothermal modeling), the equation simplifies to:

$$\gamma = \frac{DNP(\alpha_c - \alpha_{si}) \Delta T}{H}$$

(2.6a)

Lau used a 2D plane strain idealization for all the FEA models, based on a diagonal cross section from the center of the package to the outermost corner. He modeled a flip chip, a CBGA, and a PBGA, all mounted to a PCB. In all cases he used 63Sn/37Pb solder. The
temperature cycle he prescribed was from $-40^\circ C$ to $125^\circ C$ with 20 minute dwells and 16.5°C/minute ramp rates. All material behavior was considered elastic except the solder which was assumed viscoplastic. For the viscoplastic model, he used Norton's steady state creep equation. Lau, again, did not provide the equation for Norton's creep equation, but it may be found as [73]:

$$\frac{d\gamma}{dt} = B \exp\left(\frac{-\Delta H}{kT}\right)\tau^n$$  \hspace{1cm} (2.7)

Here, $n$ is a stress exponent (fitting parameter), $\Delta H$ is the activation energy, $k$ is Boltzmann's constant, $T$ is temperature in Kelvin, $\tau$ is the shear stress, and $B$ is a material constant. Note the similarity of Norton's equation to the previous Darveaux and Garofalo equations. Lau found that for each package type, the shear strain determined using the DNP formula was always approximately twice as large as the shear strain using FEA and Norton's creep equation (Figure 2.16). Lau surmised that the DNP equation, taking into account only the outermost ball, will always give shear strain results that are too high. The DNP equation neglects all the other solder joints which resist shear strain. Based on these results, it might be possible to get a rough idea of whether or not a FEA model is returning reasonable results by comparing the shear strain at a corner ball with a quick DNP shear strain calculation. If the DNP $\gamma$ is about twice that of the FEA $\gamma$, then the model is probably correct. If it is not, the material parameters may be incorrect, or the analysis type (plastic, elastic, etc.) may need to be upgraded to higher constitutive behavior, such as viscoplastic.
Englemaier [36], performed low acceleration testing of CBGA (ceramic ball grid array) and CCGA (ceramic column grid array) packages attached to modified PWBs. The PWB modifications included designs to enhance the CTE mismatch between the electronic package and PWB, thus reducing the time to failure. It was Englemaier's supposition that highly accelerated testing was likely to induce failures not seen in actual service conditions. However, non-accelerated testing would take years, and was thus unfeasible. Therefore, increasing the dominant failure mechanism (CTE mismatch), and utilizing a low acceleration thermal cycle would produce failures found in actual conditions. The prescribed thermal cycle during testing was accompanied by internal power generation (PTC) in the chips (1 watt), such that a junction temperature of 105°C was achieved during the heating portion of the temperature cycle. This appears to be a common practice, i.e., prescribing a $T_j$ (junction temperature) of around 100°C. Guenin, et. al. [49], used a $\theta_{ja}$ (junction to ambient) of $60^\circ$C, assuming an ambient temperature of $40^\circ$C, thus giving a $T_j$ of 100°C.

$$\theta_{ja} = T_j - T_a$$

The method of prescribing one variable to determine another, can be used to determine the heat transfer coefficient of the package, or the power dissipation of the chip, knowing one or the other. For example, Dean [27] has shown that the heat transfer coefficient of an electronic package is between .0005W/cm$^2$K and .0025W/cm$^2$K. Knowing this, the power of the chip can be varied in the FEA model such that the $T_j$ does not exceed 100°C.
The model Englemaier employed is based on a damage parameter, and is given by:

\[ N_f(50\%) = \frac{1}{2} \left[ \frac{2\varepsilon_f}{\Delta D} \right]^{-\frac{1}{c}} \]  

Equation 2.8

According to Englemaier, Equation 2.8 relates the cyclic viscoplastic strain energy to the median thermal cyclic fatigue life through the damage term \( \Delta D \). \( \varepsilon_f \) is the fatigue ductility exponent, and \( c \) is a parameter which compensates for incomplete stress relaxation. The development of this equation is involved, and will not be attempted here. Englemaier did not back predict the experimental results, but he did predict reliability results for a 1 cycle per day thermal load, typical of a product in service. The results are summarized as follows. Both the CBGA and CCGA packages displayed a bimodal shape in Weibull plots of percent failed solder joints versus cycles-to-failure (Figure 2.1). The reliability of the CCGA packages was approximately an order of magnitude better than the CBGA packages when attached to FR-4 PWBs. Nearly all of the failures (fatigue cracks) were found on the PWB side of the solder balls. This was assumed to be caused by the use of SMD (solder mask defined) pads. SMD pads cause a reduction in cross-sectional area, and a SCF (stress concentration factor) within the solder joint. Voids were found in some of the failed solder joints, but appeared to have no affect on the location or initiation of fatigue crack growth. Increasing the height of the solder joints increased the reliability of the solder joints.

Cheng et. al. [20] performed testing on 92.5Pb/5Sn/2.5Ag, and presented results using the nine parameter, Anand viscoplastic model found in ANSYS. They further
derived Anand model parameters from testing done elsewhere for 60Sn/40Pb, 62Sn/36Pb/2Ag and 96.5Sn/3.5Ag. They present results for a SMT capacitor, QFP (quad flat pack) and flip chip models. The SMT capacitor and QFP were modeled using 3D bricks, and the flip chip was modeled using axisymmetric plane 2D elements. It is not clear why axisymmetric elements were chosen, as plane strain or plane stress elements are the typical choice. The testing was performed on dog-bone shaped specimens. The geometry (size) of the specimens was not divulged. It is unclear why they opted for this specimen geometry, as it is well known that solder parameters derived from testing are highly sensitive to size and shape. Thus, it is more common for test samples to have geometries similar to the actual solder joint in service. The authors chose the Anand model based on the coupling of plasticity and viscoplasticity in the formulation. That is, the plastic and viscoplastic strains are not separated as in some formulations.

The Anand model is given as [20]:

$$\dot{\varepsilon}_p = A \exp\left(-\frac{Q}{RT}\right) \sinh \left(\frac{\varepsilon \sigma_s}{s}\right)^\frac{1}{m}$$  \hspace{1cm} (2.9)

Where $A$ is fitting factor, $Q$ is the activation energy, $m$ is the strain-rate sensitivity, $c$ is a stress multiplier, $R$ is the gas constant, and $T$ the temperature in Kelvin. One internal variable is used in the formulation, and is given by:

$$\dot{s} = \left[h_0 \left|1 - \frac{s}{s^*}\right|^a \text{sign} \left(1 - \frac{s}{s^*}\right)\right] \dot{\varepsilon}_p$$  \hspace{1cm} (2.10)

where

$$s^* = \frac{\dot{s}}{A} \exp \left(\frac{Q}{RT}\right)^a$$  \hspace{1cm} (2.11)
and $h_0$ is a hardening/softening parameter, $a$ is a strain rate sensitivity parameter, $s^*$ is the saturation value of $s$, "$s^*$" is a fitting parameter, and $n$ the strain rate sensitivity for $s^*$.

Using the Anand equation, the authors back predicted results from a number of other researchers with reasonable results. The results for the SMT capacitor mounted on a FR-4 board with 60Sn/40Pb cycled at -55°C to 125°C (ramp rate 18°C/min, 15min. dwells) showed that the solder shape was inconsequential to fatigue reliability (Figure 2.17). This is in contrast to BGA type joints where it is well documented that shape and size play a significant role in the fatigue reliability. The results for the QFP and flip chip were reported as satisfactory. The QFP was modeled using $\frac{1}{4}$ symmetry, 3D elements, and the flip chip using plane axisymmetric elements along the diagonal. It is interesting that the axisymmetric model produced reasonable results, since a flip chip does not have axisymmetry.

The authors note that the limitations of the Anand model include the inability to account for kinematic hardening or recovery in the evolution equation.

Popelar [99] has presented research on the reliability of underfilled and non-underfilled flip chips attached to organic PWBs (FR-4), soldered with 63Sn/37Pb solder. ANSYS was used for the FEA, and 2D plane strain elements were used, based on the diagonal line from the center of the package to the outermost solder ball. A viscoplastic equation was implemented in ANSYS, and utilized to characterize the creep behavior. It has the form of:

$$
\dot{\varepsilon} = \exp\left(\frac{Q}{KT}\right) \left[ B_1 \left(\frac{\sigma}{E}\right)^n + B_2 \left(\frac{\sigma}{E}\right)^{n_1} \right]
$$

(2.12)
where \( E \) is the elastic modulus, \( T \) is the temperature in Kelvin, \( k \) is the Boltzmann constant. \( n_1, n_2, B_1 \) and \( B_2 \) are material parameters. Popelar also analyzed a 256 pin PBGA, and a 361 PBGA under thermal cycles of \(-40^\circ C\) to \(125^\circ C\) for comparison to the flip chip results. For the flip chip models, he used a \(-50^\circ C\) to \(150^\circ C\) temperature range with 15 minute dwells and \(8^\circ C/\text{min}\) ramp rates. Based on the FEM analysis, the following results were found. Die thickness showed a marginal effect on fatigue life. As the die thickness increased, the fatigue life decreased slightly. Substrate thickness had nearly no effect. The underfill modulus, however, had a significant impact. In all cases, underfilling provided higher fatigue lives over non-underfilled flip chips. However, the relationship between underfill modulus and fatigue life was found to be highly non-linear. For the models examined, a nearly quadruple increase in fatigue life could be found by optimizing the underfill modulus. A highly non-linear relationship between underfill CTE and fatigue life was also found. A nearly five-fold increase in fatigue life could be achieved using the appropriate underfill CTE. The results also corroborated the finding that taller solder joints are likely to have longer fatigue lives. It was also pointed out that although underfilling generally increases flip-chip life, it can also create problems due to the curing process. After curing, it has been found that flip-chips may have a slight curvature due to the CTE mismatch of materials. The curvature produces a tension in the die (which is a brittle material) and makes it susceptible to fracture. It may be noted that the curvature of a package is a significant problem for the opto-electronics industry where LED semiconductors are manufactured. The curvature, also called "smile" in the industry, causes the optical beam to be offset from the calculated
trajectory. Thus, the smile needs to be calculated into the manufacturing process, which is a difficult task. In addition to causing a smile, the underfill has the capacity to delaminate and trap contaminants. Thus, underfilling helps alleviate certain problems, but also creates others.

Frear et al. [40] performed testing and FEM analysis on the microstructural evolution of near eutectic 60Sn/40Pb solder. Based on experimental evidence, they found that the failure of a solder joint, under thermal cycling, occurs through the following sequence. During solidification of the solder joint, a composite structure of lamellae develop. These lamellae tend to grow towards one another, forming colonies of similar phases. When dissimilar colonies contact one another, they form a coarsened area. During low temperature swings in the thermal cycle, damage is initiated at the coarsened areas. Upon heating, and especially during dwell times at elevated temperatures, the damage anneals through mass diffusion, but further coarsens the previously coarsened areas. This cycle of damage-anneal-coarsening continues until the grain size in the coarsened areas reaches a critical size, and intergranular cracks initiate. The authors propose that the cracks are caused by strain concentrations at these coarsened areas (Figure 2.18).

A 3D FEM model was developed using the Garofalo minimum creep strain rate equation [40]:

$$\dot{\varepsilon}_{\text{min}} = A[\sinh(\alpha \sigma_c)]^n \exp\left(-\frac{Q}{k\theta}\right)$$  \hspace{1cm} (2.13)
where $\alpha$ is a fitting parameter, $\sigma_c$ is the applied stress, $R$ is the gas constant, $T$ is the temperature in Kelvin, $Q$ is the activation energy for creep, and $A$ and $n$ are material constants. They also employed a microstructurally based internal state variable model into the FEM model to determine the effect of microstructural changes. The state variable was used to account for isotropic hardening and recovery, and a second order tensor was used to account for kinematic hardening and recovery. The model is somewhat involved, requiring eleven material parameters, and will not be discussed here, other than the results obtained. The authors found that the minimum creep strain rate equation gave a 91.7% accuracy against their experimental data, and that the evolution of the coarsened regions in the model followed the experimental results where cracks appeared (Figure 2.19).

The authors pointed out that the Sherby-Dorn equation, used elsewhere in the literature, gave results which did not match their experimental results for 60Sn/40Pb. Based on the Sherby-Dorn equation, as the grain size increases (coarsening), the lifetime should increase. The Sherby-Dorn equation for inelastic strain rate is given as [40]:

$$
\dot{\varepsilon} = \gamma n = n\varepsilon_0\left(\frac{b}{\gamma}\right)^p\left(\frac{\tau}{\mu}\right)^m
$$

(2.14)

where $\varepsilon_0$ is a scalar function of absolute temperature, $b$ is the Burger's vector magnitude, $\gamma$ is the grain size, $\tau$ is a deviatoric stress measure, $\mu$ is the shear modulus, and $m$ and $p$ are material parameters. Observing equation 2.14, it can be seen that the grain size is in the denominator, clearly indicating that as grain size increases, strain decreases.
Figure 2.1. Weibull Plot [36].
Figure 2.2. Fracture of Solder Ball [68].
Figure 2.3. Personal Communication PWB Showing Microvias [69].
Temperature Dependence of Properties of Eutectic Tin-lead Solder:

<table>
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<tr>
<th>Temp. (°C)</th>
<th>Young's Modulus (GPa)</th>
<th>Poisson's Ratio</th>
<th>Thermal Expansion (x10⁻⁶ °C⁻¹)</th>
</tr>
</thead>
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<td>0.354</td>
<td>24.6</td>
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<tr>
<td>0</td>
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<td>0.360</td>
<td>25.2</td>
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</tr>
<tr>
<td>125</td>
<td>34.6</td>
<td>0.384</td>
<td>27.9</td>
</tr>
</tbody>
</table>

Material Properties at Room Temperature

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<th>Ref.</th>
<th>Young's Modulus (GPa)</th>
<th>Poisson's Ratio</th>
<th>Thermal Expansion (x10⁻⁶ °C⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>385.0</td>
<td>0.30</td>
<td>6.0</td>
</tr>
<tr>
<td>Copper</td>
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<td>126.0</td>
<td>0.37</td>
<td>13.0</td>
</tr>
<tr>
<td>Cu₃Sn₅</td>
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<td>85.6</td>
<td>0.31</td>
<td>16.3</td>
</tr>
<tr>
<td>Epoxy</td>
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<td>8.4</td>
<td>0.35</td>
<td>29.0</td>
</tr>
<tr>
<td>Lead</td>
<td>6</td>
<td>19.0</td>
<td>0.35</td>
<td>28.0</td>
</tr>
<tr>
<td>Silicon</td>
<td>4</td>
<td>188.0</td>
<td>0.30</td>
<td>2.8</td>
</tr>
<tr>
<td>Tin</td>
<td>5</td>
<td>41.6</td>
<td>0.33</td>
<td>23.5</td>
</tr>
</tbody>
</table>

Figure 2.4. Temperature Dependence of Material Properties. (A) Relationship between various properties and temperature [137]. (B) Elastic modulus with temperature [70].
Figure 2.5. SEM Photos Showing Cracks Occurring Away From Metallization Layer [36].
Figure 2.6. SMT capacitor solder joint crack growth plots [110]. (A) Crack Length as a function of cycles. (B) Crack growth rate as a function of cycles.
Figure 2.7. Voids in BGA solder joints from thermal cycling [43]
Figure 2.8. Various style heat sinks [146].
Figure 2.9. MEMs examples. From Goodrich Advanced MicroMachines web-site, www.memslink.com.
Figure 2.10. (A) Non-ideal temperature cycle. (B) Examples of random vibration plots,
Figure 2.11. FEA Mesh Incorporating Different Phases of Solder [138].
Figure 2.12 Various experimental data from solder testing [102]. (A) True stress vs. strain plot at different strain rates. (B) Nominal stress vs. strain plot at different strain rates. (C) Cycles to failure vs. velocity plot. (D) Log strain vs. Log stress at different temperatures. (E) Strain vs. time for differently cooled specimens.
Figure 2.13. Solder Joint Showing Coarsened Area After Thermal Cycling [105].
Figure 2.14. Various experimental data from solder testing [102]. (A) Cycles to failure versus frequency. (B) Strain range versus cycles to failure for distinct temperatures.
Figure 2.15. Various results using global-local model [12]. (A) Local-global model meshes. (B) Various solder joint shape as function of joint volume. (C) Comparison of shape effects on fatigue life. (D) Pad size effects on fatigue life. (E) Solder ball volume effects on fatigue life. (F) Joint height effect on fatigue life.
Figure 2.16. Distance to Neutral Point Comparisons between FEA and Experiment [77].

(A) Chip scale package. (B) DNP results from chip scale package analysis. (C) Flip chip. (D) DNP results from flip chip analysis. (E) CBGA package and DNP results.
Figure 2.17. Solder Joint Shape Results [20]. (A) Comparison of model with experiment. (B-C) Comparison of modeled steady state creep behavior with various experimental results.
Figure 2.19. Fit of Model to Experimental Data Points [40].
CHAPTER 3
REVIEW OF SOLDER JOINT VIBRATION FATIGUE

3.1 Introduction

The vibration fatigue of solder joints, especially ball grid array (BGA) type solder joints, has not been researched as extensively as thermal cycling fatigue [82]. However, vibration fatigue of electronic packages is of great interest to a number of industries, including the automotive, military, aerospace, and increasingly, personal electronics manufacturers [139]. All of these industries face great challenges designing products which will face uncertain environments. This is especially true for the personal electronics industries. The electronics of today are expected to be small, lightweight, portable, and robust. Cell phones, portable computers, electronic personal assistants (PDA), etc., are being used by an increasingly broad market base, and being placed in increasingly harsh environments. For example, it is common to see portable computers on construction sites, and PDAs in the backpacks of students.

The electronics used in the military, aerospace and automotive industries have always faced exceedingly harsh environments. However, as military hardware, avionics, and automotive vehicles become increasingly more sophisticated, the requirements placed on the electronics become even greater. Packages mounted directly on jet engines, and automotive engines are problems that engineers must address today. To accomplish the "faster, smaller, cheaper" product, all industries are embracing the surface mount technology (SMT) revolution. The surface mount component (SMC) is smaller, lighter, and more powerful than its leaded counterpart. This is especially true for the
BGA type SMC. However, without a lead to absorb strain, the second level solder balls become the primary mechanical failure point [79]. Compared to leads, solder balls contain more variance in terms of shape, size, and microstructure; they have a more complicated phenomenological behavior, and thus, require a more sophisticated modeling approach. Many researchers have adopted the finite element method (FEM) to determine the frequencies of the PWB the package is mounted on, and the stress within the solder balls. However, this still leaves open the question of when the solder balls will fail. Presently, the most common method of predicting the cycles to failure is the Manson equation and Miner equation. These two equations are often used in conjunction, and produce reasonable results (although they are considered conservative). However, with the increasing need to reduce cost, and the speed and low cost of computers, physics of failure (FEA) models are becoming more popular. These codes can be used to predict when and where failures occur for changing designs, and determine if the failure is a material, geometry, and /or environmental issue. The codes can then be used to examine an alternative material or geometry, and create a failure free environmental envelope. A number of models are presented in this chapter, along with experimental data from a number of researchers.

The vibration fatigue failure of BGA type solder bumps has not received the same attention as thermal cycling fatigue failure [82]. However, fatigue failure due to vibration is a serious problem for both automotive and aerospace/military applications [139]. With the current emphasis on reducing cost, many military applications are transitioning to the use of commercial off the shelf (COTS) electronic packages [106].
This presents a problem, as COTS packages have not been designed specifically for military/aerospace applications. Yet, the military/aerospace environment is known to be significantly more demanding than the environments typically encountered in the "commercial" sector. As the semiconductor industry continues to reduce the size of the SMT solder joint, applying BGA, CSP and DCA technologies, the reliability of these packages will require quantification. It is well known that these types of packages are more susceptible to thermally induced strain due to the lack of leads to absorb the motion [79]. It seems likely that this will also be the case for vibration fatigue. Generally speaking, vibration failure of solder joints is considered a high cycle fatigue, whereas thermal cycle fatigue is considered low cycle fatigue [54]. Therefore, the failure mechanisms within the solder bumps may not be the same for the two cases. No work has been found which specifically addresses this issue for solder bumps.

Searching the EI compendex, the Applied Science and Technology, the Web of Science, the IEEE, the Aerospace and High Technology, the Engineered Materials, the Mechanical Engineering, the Metadex, and the Weldsearch databases with a search query of (BGA or CSP or DCA) and (vibration or vibrational), revealed a total of 88 papers. Of these papers, many were duplicates from database to database, some were not relevant, and one was not available at the University of Arizona. Out of the papers that were relevant and available, i.e., having to do with vibration fatigue of solder joints in electronic packages, only four presented models which predicted when failure occurred in a solder bump. The majority of the articles described vibrational testing, and the remainder described leaded surface mount joints. It appears that most of the research
effort has been placed on experimental testing rather than model development. It was also found that many, if not most, of the papers were written by industry, indicating the importance of this subject to practicing engineers.

3.2 Experimental Testing

Experimental testing is equally important as modeling, as calibration of models, and accuracy of models can only be verified through experimental data [31]. In addition, physics-of-failure models need material properties which can only be attained through testing. Researchers have shown that the material properties of solder are size and geometry dependent, and also temperature dependent [129, 144]. Hence, material properties for physics-of-failure models are most accurately obtained through experiments which use specimen geometries similar to the geometries in the models. Due to the large number of papers devoted to experimental testing of solder joints, only a small "cross section" of the papers are presented below.

Zhong and Yi [150] performed mechanical shear testing of 63Pb/37Sn solder balls while considering intermetallic thickness, pad metallurgy and previous thermal cycling on shear strength (Figure 3.1). While they did not vibrate the samples, they did present a coupled experimental test, and hence their research is considered relevant for this paper. Their results indicated that thermal cycling reduced shear strength, and was proportional to the number of thermal cycles. As the number of thermal cycles increased, the shear strength decreased. However, the relationship is not linear. Zhong is referenced to AMD (American Micro Devices).
Jones et. al. [61] performed uniaxial tensile tests on eight Sn/Pb, Pb/In and Sn/In formulations while varying temperature from -250°C to 200°C (Figure 3.2). The test samples were not solder bumps, but 15mm (gage) length by 2mm thick cast specimens. They determined ultimate tensile strength (UTS) and elongation at a given temperature for each solder formulation. Commenting only on eutectic solder (63Sn/37Pb), their results showed that eutectic solder has a nearly linear UTS vs. temperature response from -100°C to 150°C. At 150°C, the UTS is about 5MPa, whereas at -100°C, the UTS is approximately 92MPa. Clarke is referenced to Teledyne Technologies.

Tu et. al. [128, 15] investigated the effect of intermetallic compounds on bending and vibration fatigue, while varying the reflow temperature profiles (Figure 3.3). Their test specimens consisted of dummy versions of Tessera's microBGA electronic package, using eutectic Pb/Sn solder. Their results indicate that the temperature reflow profile has a substantial effect on vibration cycles to failure, and that the relationship is not simple. They quantify their results in terms of a heating factor "Q_n", which is defined as the integral of the measured dwell time above the liquidus temperature in the reflow profile. They found that the vibration lifetime increases from Q_n = 33 until Q_n = 682, then begins decreasing with increasing Q_n.

Barr and Mehta [7] investigated vibration, shock and thermal cycling on the reliability of a 472 pin DBGA (dimpled BGA) for space flight applications. The test specimens were actual packages assembled on PWBs of "flight like configurations." The solder balls were primarily Sn60/Pb40 (some packages used Sn/Pb/Bi) and two PWBs were used, one a polyimide-glass, and the other an Aramid polyimide. The tests were not
frequency range of 10-500Hz, and 10G for the frequency range of 501-1500Hz, with a duration of 1 hour. The authors do not present any data gathered from the mechanical testing, but report that all samples passed the qualification test. The work was performed at Hyundai Electronics Industries.

Solomon [114] has presented a paper on low-cycle fatigue shear cycling on a range of solder compositions. The tests were done in simple shear, under displacement control, with solder joints of .19mm thick, at a frequency of .3Hz. Although this is almost quasi static, i.e. not vibration, the paper presents some interesting statements. As an example, the author states that Sn60/Pb40 solder is "highly strain rate dependent; in fact, the flow stress is only a function of the strain rate and is not also a function of the plastic strain range (i.e., it is non-Newtonian viscous). This means that curves generated at different cycling frequencies all fall on a single stress strain rate curve." This is an interesting statement, because if a material is strain rate dependent, then it would also seem that the material should be cycling frequency dependent (since the rise time of the cycle is a strain rate). The author points out that for low cycle fatigue, the Coffin-Manson low cycle fatigue law is usually accurate. He presents the equation as:

\[ N_f^a \Delta_y = \theta \]  

(3.1)

Where \(N_f\) is the cycles to failure, \(a\) is the Coffin-Manson low cycle fatigue exponent, \(\Delta y\) is the plastic strain range, and \(\theta\) is a constant. Solomon is referenced to General Electric Research and Development center.

An interesting feature of almost all the papers cited in this review, is that at least one of the authors was referenced to industry. This indicates that vibration fatigue of
electronic packages is extremely important to a number of industries. Yet, judging by the number of university authored papers, this interest has not yet spilled over into the academic community. This could be due to a number of issues, including funding, resources, etc. However, it seems that this is a very ripe area for university-industry partnership.

3.3 Vibration

The subject of vibration theory has been addressed copiously, in texts and in journal papers. Manson [84], indicates that over 4000 papers had been published between 1950 and 1962. Electronic apparatus response has been addressed by Steinberg [116, 117, 118], Sloan [113], Dally [25] and Suhir [121a]. The available texts provide an excellent general background, but do not explore the mechanisms of failure or offer a physics of failure approach to solder fatigue.

A few of the articles discussed in this paper have presented results from random vibration testing. Therefore, a very brief description will be given regarding the differences between random and deterministic vibration. In general, there are two forms of vibration, deterministic and random. The deterministic form of vibration is described by periodic motion, which may be harmonic or non-harmonic. Random vibration is non-harmonic and non-periodic, and can only be described through statistical or stochastic means. Examining a previous time history of a random vibration signal can only provide estimates or probabilities of occurrence of the next amplitudes. It cannot forecast a time series of amplitudes as would be the case if the signal was periodic. The complexity of random vibration has necessarily led to the development of simplification schemes. Most
notably, the power spectral density (PSD) function which has the units of \((\text{acceleration})^2/\text{frequency}\). The PSD changes the time series input to a frequency series input (Figure 3.5). PSD curves are an entire subject by themselves, and Steinberg [117] gives a good introduction. Periodic vibration is commonly observed in unbalanced rotating equipment, such as winches, conveyor belts, turntables, shafts, etc. Random vibration is commonly observed when an external boundary is subject to "natural" disturbances, i.e., wind and air variations (missiles, rockets, planes), road variations (cars, trucks, tractors), buildings (traffic vibration, earthquakes, wind), etc.

3.4 PWB Experimental Testing

PWB testing is important to understand the interaction between the board and package. It is commonly understood that for vibration induced fatigue, the flexing or bowing of the PWB contributes the most damage to the solder joints. Bump type solder joints are especially susceptible to this since there is no lead to absorb any strain. Many researchers have attempted to quantify the flexing of PWBs based on the fundamental frequency of the board. However, determining the fundamental frequency of a PWB (through modeling) is difficult. For example, if the frequency is going to be determined through FEM, the fully populated PWB will need to be tested for the elastic modulai and Poisson's ratios. However, if this testing is going to be done, then it may be just as easy to vibrate a fully populated PWB under the same boundary conditions as the service conditions.

If modeling is going to be used to determine frequencies, it is extremely important to obtain the correct frequencies, as displacement is a second order function of frequency:
Therefore, for FEM models, incorrect material parameters may give significant errors in terms of displacements of the PWB and stresses in the solder joints. However, although incorrect material parameters may give poor results, it is the incorrect modeling of boundary conditions which generally produce the most significant errors in FEM modeling, especially PWBs subjected to vibration. Steinberg [117, 118] gives some good analytical examples of how changing the boundary conditions can have a large impact on the derived frequencies.

Roberts and Stillo [107] performed a FEM analysis of a PWB and 20 pin SMT capacitor, and compared the results against experimental data for the same board. The board was subjected to a two hour random vibration test. The random vibration levels were $0.04 G^2/Hz$ from 1 to 300Hz, $0.2 G^2/Hz$ from 500Hz to 1000 Hz, and a ramp down from $0.2 G^2/Hz$ at 1000Hz to $0.04 G^2/Hz$ at 2000Hz. They found significant differences between the FEA results and the experimental results. They report a 10% difference in the first frequency and a 40% difference in the accelerations. They attribute this to the assumed boundary conditions in the FEM model and some shifting of the PWB connections in the experiment. No failures were reported during the test for correctly soldered joints. Stillo is referenced to Westinghouse Electric Corporation.

Wong et al. [141] investigated the modal response of PWBs with SMT components mounted to them. Their procedure was to experimentally test a bare PWB, then test the same PWB with a single SMT component mounted in the center. They then
used a commercially available modal analysis software to compare the results. They found that adding a single SMC (surface mounted component) changed the natural frequencies, but not exactly as one might expect. The frequencies are listed below in Table 3.1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>PWB (Hz)</th>
<th>PWB with SMC (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32.41</td>
<td>31.22</td>
</tr>
<tr>
<td>2</td>
<td>98.77</td>
<td>102.68</td>
</tr>
<tr>
<td>3</td>
<td>168.99</td>
<td>163.19</td>
</tr>
<tr>
<td>4</td>
<td>324.38</td>
<td>313.79</td>
</tr>
</tbody>
</table>

Examining Table 3.1, it can be seen that the first mode is hardly changed, and the second mode actually increased when the SMC was added. This is in contrast to the simple formula shown in Equation 3.2, where one would expect as the mass is increased, the modal frequencies would go down. The authors do not comment on why their results show this. The modeling showed that incorporating an SMC as a point mass was not a good idea, as the point mass may fall on a modal node line. Gang Wang, a co-author is referenced to Dyna-Air Corp.

Lau [72] has presented results on the reliability of PBGA packages subjected to vibration (Figure 3.6). Eighty, 225 I/O PBGA packages were tested. The experiment was designed such that there was only one package per PWB. The boards were vibrated (sinusoidal) at an average peak to peak displacement of 3.05mm, for 50 minutes, with a frequency sweep between 80Hz, and 120Hz (the average board first resonance was
100Hz). This gives an approximate number of total-cycles of: \( 50 \text{min} \times 60 \text{s/min} \times 100 \text{cyc/s} = 300,000 \) cycles. None of the PBGA solder joints failed, and no model is presented specifically for vibration fatigue. However, a model is presented for thermal fatigue, as the packages were subjected to thermal cycling also (not concurrently). Lau is referenced to the Hewlett-Packard Company in this paper.

Yang et al. [147] tested four PBGA packages, on a single PWB, for reliability under constant amplitude, sinusoidal vibration. Each package was a 256 I/O full array package. They measured deflections, input accelerations and output accelerations. They then compared their results with Steinberg's [117] equations:

Deflection at center of board: \( Z = Z_0 \sin(\Omega t) \)

Maximum deflection: \( Z_{\text{max}} = \frac{9.8 G_{\text{out}}}{f_n^2} \) \hspace{1cm} (3.3a-d)

\( G_{\text{out}} \) is given by: \( G_{\text{out}} = G_{\text{in}} Q \)

Where \( Q \) (transmissibility) is approximately: \( Q = \sqrt{f_n} \).

They found that Steinberg's equations were conservative. They show that using these equations, a \( G_{\text{in}} \) of 6.026G should give a displacement of 1.5mm. However, their test results showed that a \( G_{\text{in}} \) of 2.5G would give a displacement of 1.8mm. This indicates that Steinberg's equations are fine for obtaining a rough idea of what the numbers will be, but should not be used for design. The experimental results also show that the corner solder balls were the first to fail, and that the balls always failed on the PWB side of the solder ball.
3.5 BGA Vibration Fatigue Modeling

Papers which presented models for predicting cycles to failure of BGA type solder bumps are outlined below. A number of papers were found which presented methods for determining stresses in solder joints, but did not present models which predicted cycles to failure. One might assume that the authors expect readers to use a standard $S$ vs. $N$ type fatigue curve to determine whether the joint will fail, knowing the stress ($S$). However, this may not be as simple as it appears. It is well known that solder behavior under vibration is highly influenced by temperature, thickness, geometry, past strain history, strain rate, aging, reflow profiles, microstructure, and concurrent thermal cycling. Therefore, a particular $S$ vs. $N$ curve may not be applicable for the solder joints/package being modeled. This is where damage type physics of failure (POF) models appear to be most advantageous, as they only require the material parameters from testing. Once the parameters for a particular POF model have been determined, different designs can be explored, and their relative cost/benefits in terms of actual cost, design complexity, reliability, etc. can be determined.

When reading papers regarding material reliability, cycles to failure, durability, etc., it is common to see references to "Miner's rule", and the "Coffin-Manson high/low cycle fatigue equation." Usually, the Coffin-Manson low cycle fatigue equation is associated with thermal cycling fatigue, and the high cycle equation to vibration fatigue. High cycle fatigue is often cited as vibrational fatigue. Because these two equations are so commonly cited in the literature, a brief review of these equations is given below.
Miner's Rule. Miner's rule [89] was developed by Milton Miner, while working at Douglas Aircraft Company in 1945. It is a very simple formula which accounts for the cumulative damage in a material based on the number of stress cycles it has endured. To simplify the formula, Miner allowed a number of assumptions. These include that the material does not work harden, the loading is sinusoidal, the initiation of an observable crack will be considered failure, and that it is the amount of work a material undergoes that produces failure. Miner then wrote the following equation:

\[
\sum_{i=1}^{n} \frac{w_i}{W} = \sum_{i=1}^{n} \frac{n_i}{N_i} = 1
\]  

Where \( w_i \) is the work done at \( n_i \) cycles, and \( W \) is the total work done, \( N_i \) is the number of cycles for failure at a stress \( S_i \), and failure occurs when the summation equals one. Miner checked his equation against tests performed on Alclad 24S-T. He found that, although there was modest scatter in the results, the average cumulative damage was near 1. For experiments where two or more stress levels were used, the minimum, maximum and average cumulative damage were .61, 1.45, and .98, respectively. For experiments where two or more stress ratios (\( \sigma_{\text{min}}/\sigma_{\text{max}} \) of vibration cycle) were used, the minimum, maximum, and average cumulative damage were .79, 1.49, and 1.05, respectively. Therefore, based on the average cumulative damage, Miner concluded that the method was satisfactory in determining the number of cycles to failure for aluminum. Miner further points out that to use the equation judiciously, accurate values of the working stress, cycles of operation, and S-N curves are needed, "but may be difficult to obtain."
Coffin-Manson high cycle fatigue equation. The "Coffin-Manson" equation has been cited in many journal articles, and has been cited in many forms, below are three common variations:

\[
\frac{\Delta \varepsilon_T}{2} = \varepsilon_r (2N_f)^{\frac{1}{2}}
\]

\[
\Delta \varepsilon_p = \varepsilon_o N_f^c
\] (3.5a-c)

\[
\frac{\Delta \varepsilon_T}{E} = \frac{\sigma_f}{E} (2N_f)^{\frac{1}{2}} + \varepsilon_r (2N_f)^{\frac{1}{2}}
\]

Two of the most cited articles for these and other forms of the equation are Manson [84] and Coffin [23]. However, Coffin does not present any such equation in reference. The equation actually proposed by Manson [84] is:

\[
\Delta \varepsilon_T = \varepsilon_e + \varepsilon_p = 3.5 \frac{\sigma_u}{E} N_f^{-1.2} + D^6 N_f^{-6}
\] (3.6)

Where \(\Delta \varepsilon_T\) is the total strain, \(\varepsilon_e\) is the elastic strain, \(\varepsilon_p\) is the plastic strain, \(\sigma_u\) is the ultimate tensile strength, \(E\) is the elastic modulus, \(N_f\) is the cycles to failure, and \(D\) is the ductility given by \(1/(1-RA)\), where \(RA\) is the reduction in area. Manson arrived at this formula by testing 29 materials, to failure, under vibration. His material set consisted of two types of titanium, one magnesium, five types of aluminum, one silver, one beryllium, and five types of steel. He found that when \(\Delta \varepsilon_p/D\) versus \(N_f\) was plotted in log-log space, all of the data points fell reasonably close to a straight line. He also found this to be the case when \(\Delta \varepsilon_e/(\sigma_u/E)\) versus \(N_f\) was plotted in log-log space. Hence, he concluded that a reasonably accurate cycles to failure prediction could be made by using a combination of the elastic and plastic strains.
GDI is the accumulated damage, \( n_i \) is the number of cycles multiplied by the \( \sigma \) level (for random vibration). For example, \( n_1 = (1000\text{cycles}) \times 0.6831 \). When GDI equals 1, failure is predicted to occur. Inverting Equation 3.8, and substituting into Equation 3.10:

\[
\sum_{k=1}^{3} \frac{n_k}{N_{k,j}} = 1
\]

where

\[
N_{k,j} = \left( \frac{1}{k\varepsilon_j} \right)^{\frac{1}{A_j}} A_D \ldots j = 1..m
\]

These equations can then be solved for \( A_i \) and \( A_D \) by minimizing Equation 3.11a.

Many details have been left out of the model development. One must note, however, that the method requires a macro and micro FEA model to be developed, and the authors used a number of ancillary FORTRAN codes in the model creation process. In the macro model, beams are used for all of the joints (except the four corner joints) and, grouping of solder balls is used (i.e. representing 5 solder balls as one beam). Also, FORTRAN codes are used in developing PSD curves and strain transfer functions for some of the calculations.

They have compared the model to test results with reasonable success. The work was carried out at Raytheon.

Leicht and Skipor [81] have presented two (square plate and circular plate) analytical models based on plate theory to determine the force (normal to PWB) at failure on PWBs with mounted PBGA packages. Although their paper describes how the theory
can be used for PBGA packages, the theory appears to be applicable for any type of package, since the theory does not take into account the geometry of the solder interconnects. The circular plate theory equation is given as:

\[
\begin{align*}
  w = \frac{P}{8\pi D} \left[ (a^2 - b^2) \left( 1 + \frac{1 - \nu}{2(1 + \nu)} \frac{a^2 - b^2}{b^2} \right) + 2b^2 \log \left( \frac{b}{a} \right) \right]
\end{align*}
\] (3.12)

In this equation, \( w \) represents deflection of the PWB, \( P \) is the load, \( D \) is the flexural rigidity, \( a \) is the distance from the center of the package to the supports, \( b \) is the distance from the center of the package to outer edge of the package, \( \nu \) is the Poisson ratio of the PCB. This is the same classical solution of a circular plate, with a concentric circular load applied at the center given by Timoshenko and Krieger [126] in 1959 as cited by the authors.

The authors have compared the formula against mechanical bending of a 196 pin PBGA with success. However, they do not outline their method of determining the stress in a solder joint based on the deflection of the PWB. The work was carried out at Motorola.

Basaran et. al. [16, 149, 9] outline a method of determining fatigue failure in solder bumps based on a damage mechanics approach under combined (superposition) thermal cycling and vibration. Their method is based on the Disturbed State Concept (DSC) developed by Desai [22, 28, 32, 34]. The method is somewhat complex, and is best suited for a numerical approach such as the finite element method. However, it can be stated succinctly as an approach which measures disturbance (damage) within a material. The disturbance is measured as any material behavior which deviates from an
assumed undamaged behavior. As an example, if the material is assumed to behave elastically in an undamaged state, then the damaged part may be considered to behave plastically. In their paper, the authors assume the damaged material to behave viscoplastically.

Basaron et. al., assumed an isotropic material, such that the incremental average stress was described as:

$$d\sigma_{ij}^a = D C_{ijkl}^p d\varepsilon_{kl}$$  \hspace{1cm} (3.13)

where $D C_{ijkl}^p$ is the total constitutive stress tensor, and $d\varepsilon_{kl}$ is the incremental strain tensor. The total constitutive tensor may be expanded as:

$$D C_{ijkl}^p = [(1-D) C_{ijkl}^p + D (1+\alpha) C_{ijkl}^{rp} + (\sigma_{ij}^c - \sigma_{ij}^d) R_{kl}]$$  \hspace{1cm} (3.14)

Where, $D$ is the damage, $C_{ijkl}^p$ is the constitutive tensor for the intact material, $C_{ijkl}^{rp}$ is the constitutive tensor for the damaged material, $\sigma_{ij}^d$ is the total stress for the intact material, $\sigma_{ij}^c$ is the total stress for the damaged material, $R_{kl}$ represents a "material moment arm"(see [8] for a description), and $\alpha$ is a strain coefficient.

The damage $D$ (considered a scalar here) is described as:

$$D = \left(1 - e^{-A \xi_D}\right)$$  \hspace{1cm} (3.15)

Here $A$ and $Z$ are material constants, and $\xi_D$ is the trajectory of deviatoric plastic strain.

The viscoplastic model used is based on the Perzyna model [101]:

$$\dot{\varepsilon}_{ij}^{vp} = \Gamma(\theta) \left(\frac{F}{F_0}\right) > \frac{\partial F}{\partial \sigma_{ij}}$$  \hspace{1cm} (3.16)
$F$ is the yield surface function, $F_0$ is the uniaxial yield stress, and $\Gamma$ is a material parameter.

The above equations lay the foundation for a solution based on the finite element method. The authors have implemented a solution for a LLCC similar to one experimentally tested by Hall and Sherry [52] for thermal cycles to failure. They show good correlation with their model based on thermal cycling. Because the experiment by Hall and Sherry did not include vibration, they were unable to make a comparison with any vibration data. However, based on the damage incurred from vibration in their model, they concluded that vibration is a significant contributor to damage, and cannot be neglected.

Miner's rule was used to superpose the damage caused by thermal cycling and vibration.

Lee and Ham [65] have presented a model for fatigue vibration based on Von Mises equivalent stress:

$$\sigma_{eq} = \frac{1}{\sqrt{2}} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{\frac{1}{2}} \quad (3.17)$$

They conducted vibration tests on a simulated BGA package under force control. They used two thicknesses of 1.2mm and 1.6mm. Both the PWB and simulated BGA package were given the same thickness. They measured the cycles to failure, and noted that the load amplitude vs. cycles to failure was linear in log-log space. They then created two finite element models, one macro and one micro. The macro model contained the PWB, package and solder balls modeled as equivalent beams. They showed that using the
PATRAN equivalent beam modeler, the results between an equivalent beam and actual solder ball are nearly identical. The micro model consisted of a single solder ball finely meshed. The micro model was then subjected to the reaction forces found from the macro model. When this was done, they found that the equivalent Von Mises stress appeared to produce high stresses where the solder balls were known to crack (near the interface of the solder ball and PWB). They then correlated the force vs. cycles to failure as Von Mises equivalent stress vs. cycles to failure. This again produced a linear curve in log-log space. However, the two test variables (1.2mm and 1.6mm thickness) now collapsed onto one line, indicating that the Von Mises equivalent stress may be an independent failure criteria.

3.6 Non-BGA Vibration Fatigue Modeling

Because so few papers regarding solder bump vibration fatigue models could be found, SMT models of non-bump solder joints will also be discussed. Although the geometry of these solder joints differs significantly from BGA type bumps, there appears to be significantly more papers addressing these types of joints. However, only a few will be discussed, as the primary focus of this paper is solder bump fatigue modeling.

McKeown [86] has investigated the fatigue of LLCC (leadless chip carriers) under vibration and thermal cycling (not concurrent). McKeown presents a model based on the traditional S-N (stress vs. number of cycles to failure) curve. The test configuration consisted of two PWBs bonded to a central heat sink, forming a sandwich. The modeling utilized a number of "factors" which are determined from tables, as outlined below.

For sinusoidal vibration, the "uncorrected" cycles to failure is given as:
Here, $K'$ is a vibration life multiplier and $\lambda$ is a vibration life exponent. The "corrected" cycles to failure is then:

$$n_f = \frac{K'}{\varepsilon^\lambda}$$  \hspace{1cm} (3.18)

Where $K_R$ is a reliability correction factor. To determine the stress, they use the following procedure:

$$\tau = \frac{G^2}{S}$$  \hspace{1cm} (3.20)

Here, $S$ is a strain factor and:

$$\gamma = \sqrt{\gamma_x^2 + \gamma_y^2 + \varepsilon_z^2}$$  \hspace{1cm} (3.21)

$$\gamma_{x,y} = \varepsilon_{x,y} \frac{L_{x,y}}{2h}$$  \hspace{1cm} (3.22)

$L_{x,y}$ is the package length in the X and Y directions, respectively, and:

$$\varepsilon_z = \frac{\delta_z}{h}$$  \hspace{1cm} (3.23)

$h$ represents the solder joint height, and $\delta_z$ is the out-of-plane deflection given by:

$$\delta_z = \max \left( \frac{L_x^2}{16 \rho_x}, \frac{L_y^2}{16 \rho_y} \right)$$  \hspace{1cm} (3.24)

Where $\rho_{x,y}$ is the radius of curvature in the X and Y directions respectively, and is given by:
Here $\sigma_c$ is the in-plane surface stress on the component side of the PWB, and $\sigma_b$ is the in-plane surface stress on the bond side of the PWB.

He then combines the stresses from thermal and vibration linearly using Miner's rule:

$$D = \sum_{i=1}^{n} \left( \frac{n_i}{N_{F_i}} \right)$$

The author does not present any experimental evidence to compare the model with. However, he does show that the model produces a linear curve in log-log space of life versus surface strain for both a random vibration and a sinusoidal vibration.

McKeown is referenced to Martin Marietta Aircraft Controls.

Barker et al. [6] present a model that uses superposition to sum the fatigue due to thermal cycling and vibrational cycling (Figure 3.8). In their model, they assume that the package is a rigid member, and that the PWB bends around the package. They also assume that all forces other than the out of plane forces are negligible. From a finite element analysis, they determine the displacement of each solder joint. From this they determine the force in each solder joint as:

$$\sum_{i=1}^{n} F_{yi} = 0 = K_{yi} (\delta_i - \delta_o)$$

Here $F_{yi}$ is the out of plane force, $K_{yi}$ is the out of plane stiffness and $\delta_o$ is the starting deflection and $\delta_i$ is final deflection. They then calculate an average stress in the solder joints as:
\[ \sigma = \frac{F}{A} \quad (3.28) \]

Where \( F \) is the out of plane force, and \( A \) is the nominal cross sectional area. The fatigue life can then be determined using Basquin’s high cycle fatigue equation [93]:

\[ \sigma^b N_f = \text{const.} \quad (3.29) \]

To combine the effects of thermal cycling fatigue and vibration fatigue, they use Miner’s rule to superpose the two. They assume Miner’s relation, in that damage is the ratio of cycles experienced \( n_i \) to the fatigue life \( N_i \), and that when \( R=1 \), complete failure has occurred:

\[ R = \sum \frac{n_i}{N_i} \quad (3.30) \]

The thermal damage and vibration damage are then superposed:

\[ R = R_{th} + R_v = \frac{n_{th}}{N_{th}} + \frac{n_v}{N_v} \quad (3.31) \]

Here the subscript \( th \) represents thermal, and \( v \) represents vibration. They then introduce a ratio to equivalence the cycles between the vibration frequency and the thermal frequency, as the vibration frequency is generally much higher:

\[ R_v = \frac{n_{th}}{f_{th}} \frac{f_v}{N_v} \quad (3.32) \]

Placing Equation 3.32 into Equation 3.31, and then solving for \( n_{th} \), the cycles to failure can be found, assuming a suitable value for \( R \) (1 is generally assumed, but a more conservative value such as .9 could be assumed):
They also present the generalized Coffin-Manson equation as being a more accurate method of calculating fatigue life if plasticity is present. By using Miner’s rule, it is assumed that the fatigue is a function of total strain. They present the generalized Coffin-Manson equation as:

\[
N_f = n_{th} = \frac{R}{f_v/f_{th} + \frac{1}{N_v} + \frac{1}{N_{th}}} \tag{3.33}
\]

Experimental data was not presented to verify the model.

3.7 Conclusion

The academic community has not addressed vibration fatigue failure of solder joints in electronic packaging in the same volume as it has addressed thermal cycling fatigue. No papers could be found which actually coupled thermal cycling fatigue and vibration fatigue. A few papers were found that linearly superposed the two, but none which actually coupled the two, i.e., loading the joints/package with thermal cycling and vibration cycling at the same time. This may be important if non-linear material behavior is to be accounted for. Judging by the amount of vibration fatigue papers authored or co-authored by industry, there appears to be a disconnect between industry and the academic community regarding this topic. Clearly, the vibration fatigue of solder joints is an
important topic to industry. The high ratio of industry authored to academic authored papers indicates that this is an area where the academic community could become more involved.

The focus of this chapter was on the vibration fatigue of bump type solder joints found in BGA, CSP and DCA type electronic packages. Because the fatigue of solder joints is complex, a number of types were examined. As an example, the modal response of PWBs is an important topic when discussing fatigue. However, due to space limitations, this topic, and others, were only brought to the readers attention, and not pursued in depth. The least common approach of determining fatigue life appears to be modeling the entire package. If modeling the entire package is undertaken, two models are generally created; a macro and a micro. The macro model, in most cases, uses beams substituted for all the solder balls. The reactions (worst case) at the beam supports are then applied to the micro model which consists of a single solder ball. This approach is primarily taken to reduce model development time and solution time. 2D models can be helpful in this respect, as they drastically reduce model size and computer run time. Although there are a number of 2D slices which could be made, the most effective appears to be the diagonal slice passing through the center of the package (note due to symmetry only one half of the diagonal slice is needed). Note that a method of determining what constitutes failure must still be decided upon with this approach.

The most common "modeling" practices, when considering vibration fatigue of solder joints, appear to be the stress based approach, the board deflection approach, and the damage approach. The stress based approach generally uses either the Manson
equation, or the Miner equation. The deflection approach is similar to the stress based approach, but uses the deflection or curvature of the PWB as the primary unknown. The damage approach utilizes a "damage parameter" indicating how much damage a solder joint has accumulated. Miner's rule is not really a damage approach (in current terminology), since there is no damage parameter in the formulation. Thus, one is forced to assume that damage accumulates at areas of high stress, but at an unknown rate. In essence, Miner's rule is an empirically-derived ratio of cycles rule, that provides reasonable results. It does not provide information on the physics underlying the failure.

Because of the exploding use of BGA type packages in electronic hardware, it is necessary for these packages to be understood in terms of their mechanisms of failure. This will only be possible through POF models and/or extensive testing. With the current power of desktop computers, it seems that POF models are beginning to make their way into daily engineering calculations. As POF models become standard electronic packaging analysis tools, it seems likely that the performance and reliability of these packages will certainly increase.
Figure 3.1. Ball strength testing results [150]. (A) Strength test mechanism. (B) Ball strength versus storage time at 150°C. (C) Ball strength versus number of thermal cycles.
Figure 3.2. Experimental results for various solders: (A) Uniaxial tensile strength vs. temperature. (B) Elongation vs. temperature [61].
Figure 3.3. Experimental results regarding reflow temperature. (A) Test vehicle. (B) Percent failures vs. cycle time as a function of heating factor "Q". (C) Median vibration lifetime vs. heating factor "Q".
Figure 3.4: Experimental results for notched solder specimens [17, 18]. (A) V notch specimen. (B) U notch specimen. (E) Deflection vs. Number Cycles. (F) Crack Length vs. Number Cycles
Figure 3.5. Time series changed to frequency PSD. (A) Random acceleration time series data. (B) Random acceleration frequency series (PSD).
Figure 3.6. (A) Vibration apparatus. (B) Peak to peak vibration amplitude vs. frequency (free vibration). (C) Peak to peak vibration amplitude vs. frequency (forced vibration) [72].
Figure 3.7. FEA model and prescribed random vibration [141]. (A) Macro/Micro Model. (B) Table of Random Vibration. (C) Generic PSD for use with random vibration table.
Figure 3.8: Model and prescribed loads from ref. 6. (A) Package and PWB schematic. (B) Package considered rigid compared to PWB. (C) Comparison of thermal and vibration strain time histories. [6].
CHAPTER 4
COUPLED VIBRATION AND THERMAL CYCLING

4.1 Introduction

This chapter will be very short for a number of reasons. First, very few researchers have investigated this topic. Second, thermal cycling and vibration cycling occur over very different time scales, and it is difficult to correlate the two.

Very few researchers have investigated combined thermal and vibration cycling in PBGA packages. Those that have investigated this area [9], did not use a coupled approach, but a superposition of the results from each loading. That is, superposing the damage from the thermal cycling, and the vibration. Other researchers [6] have investigated combined thermal cycling and vibration for non-PBGA packages, but again used a superposition principal to combine the effects.

Because the time scales over which temperature cycling and vibration occur are so different, it is difficult to develop accelerated testing, and FEA models to account for both in a coupled manner. That is, applying vibration and thermal cycling at the same time. For example, in an operational environment, an electronic package might be subjected to one thermal cycle per day. However, over the course of the day, the package might be subjected to one million vibration cycles or more. If the package is expected to have an operational life of ten years, then 3,650 thermal cycles and 3,650,000,000 vibration cycles might be expected. To model this by FEA would be very difficult. However, a realistic approximation can be made, which disposes of the time scale difference. It is well known that solder accumulates more damage at elevated
temperatures. Although temperature gradients may induce damage as the package heats up or cools down, this only occurs once per day. Thus, it is more likely that the package will accumulate the greatest damage at elevated temperatures during vibration. Therefore, an FEA model which treats the temperature as constant (isothermal) and applies a vibration concurrently appears to be a valid approximation. This is the approach taken in Chapter 16.
CHAPTER 5
THE DISTURBED STATE CONCEPT

5.1 Introduction

The disturbed state concept is a general constitutive modeling concept, and has found applications for a diverse range of materials. This is notable, since most constitutive models find limitations based on material characterization (brittle, ductile), homogeneity, material behavior (elastic, plastic) and time dependency (static, dynamic, viscous). The DSC, because of its generality, has been applied successfully to interfaces [30], brittle materials [28], viscoplastic materials [8, 22, 28, 153], and inhomogeneous viscoplastic materials [28]. The DSC is developing as a reliable method of predicting solder behavior in electronic packaging and has been used in modeling a variety of packages and solders [8, 22, 34, 153].

The DSC theory has been discussed in detail in other papers and texts [8, 22, 28, 30, 34,], and only a brief examination will be given here. The DSC may be generalized as follows. Materials, even prior to loading, contain imperfections, i.e. inhomogeneities. These inhomogeneities may be as conspicuous as large voids or cracks as in rock and concrete, or as inconspicuous as grain and slip boundaries within metals. During loading, materials experience many forms of weakening, including dislocations, slip, and microfracture and also strengthening and healing. The DSC groups all heterogeneous properties of a material into a class called the "fully adjusted (FA) state". This is a state which weakens the material from a reference state called the "relative intact (RI) state".
Together, the RI and FA states represent the "currently observed (CO)" state (Figure 5.1). The intact state is called "relative" intact, because the intact state is really fictitious, i.e. all materials contain imperfections even prior to loading. The RI state is only intact relative to the CO state, and therefore, the term relative. The CO state is called current because it is continuously changing as the material is loaded (Figure 5.1). The response of a material, which is the currently observed state, represents the coupling (\( \Theta \)) of the relative intact and fully intact states. The coupling is more than a simple sum, it is a coupled integration of the two parts:

\[
CO = RI \Theta FA
\]  

In general, as a material is loaded, the RI state decreases and the FA state increases. For force equilibrium (tensor notation), the forces of the states must balance (Figure 5.2):

\[
F_i^{CO} = F_i^{RI} + F_i^{FA}
\]  

The disturbance \( (D) \) is considered a scalar quantity in this research, and in simple terms, may be related to the ratio of the volume or area of the material in the FA state, to the total volume or area of the material:

\[
D = \frac{A^{FA}}{A^{tot}}
\]  

Expressing the ratio of the area of the RI state relative to the total area, the following relationship is found:

\[
\frac{A^{RI}}{A^{tot}} = \frac{A^{tot} - A^{FA}}{A^{tot}} = 1 - D
\]  

From Equations 5.2-5.4, the currently observed stress may be written as:
This formulation is based on describing the Green’s deformation tensor $C$, as a function of time. The deformation tensor has the capability of describing finite changes in orientation, as well as changes in geometry. Further expanding on this, Green’s deformation tensor may be given as:

$$C = \gamma \left( \frac{f(T, t)}{X_0} - 1 \right) \frac{\partial f(T, t)}{\partial t}$$  \hspace{1cm} (5.9)

The tensor in Equation 5.10 is second order, and when used to describe strains becomes:

$$L_{ij} = \frac{1}{2} \left( C_{ij} - \delta_{ij} \right)$$  \hspace{1cm} (5.11)

Where $L_{ij}$ is the Green finite strain tensor and $\delta_{ij}$ is Kronecker’s tensor. Equation 5.11 is the “measure of deformation” occurring between two particles based on the initial and final configurations. Based on this formulation, a particle may experience rotations or finite displacements (without strains) such that the deformation is a rigid body displacement. For large rotations this formulation is optimal since no confusion between coordinates can occur. As an example, a $90^\circ$ rotation may have the effect of changing the orientation of a body such that, in global coordinates, the $\sigma_x$ is now $\sigma_y$ (Figure 5.3). In the formulation of Green’s deformation tensor, it is assumed that a distinction between the Lagrangian (material) and Eulerian (local) must be made to account for finite displacements and strains. This is not the case for infinitesimal small deformation theory.

The deformation tensor is further defined by the loading history, in a manner analogous to associative plasticity, where $f(T)$ represents the static yield surface or
potential function for associative plasticity. In this formulation, it is assumed that the material behavior is time independent in elastic response, but time and history dependent in plastic response. Here, $\gamma$ represents a fluidity parameter, $\chi_0$ is a normalizing parameter, and $T_t$ is a flow stress. Perzyna envisioned the normalizing parameter as a yield constant, such as the yield stress. The flow stress is the stress above the yield stress such that $T_t = \sigma - \sigma_y$.

5.3 Katona and Mulert Formulation

Katona and Mulert [62] based their formulation on Perzyna's elastic-viscoplastic formulation, applying it to geologic materials. They assume a form for the viscoplastic response as:

$$\dot{\varepsilon}_{vp} = \gamma \phi(f)f'$$

(5.12)

where:

$$\phi(f) = \begin{cases} \phi(f) & f > 0 \\ 0 & f \leq 0 \end{cases}$$

$$f = f(\sigma, \kappa)$$

(5.13)

$$f' = \frac{df}{d\sigma}$$

Here, $f$ is the plastic yield function, and $f'$ is the gradient or normal to the yield surface. The deformation tensor $C_{ij}$ (Perzyna’s work) has now been replaced by the viscoplastic strain tensor ($\varepsilon_{vp}$). The definition or type of plastic yield surface (which is arbitrary in Perzyna’s work) was chosen as the cap model, which has found significant success modeling highly discontinuous and granular materials. No attempt is made here to
describe the cap model, except to state that the cap model shares similarities with the HiSS plastic model used in this dissertation, and hence, the reason for briefly discussing Katona and Mulert’s work.

The form chosen for the flow function was taken as:

$$\phi(f) = \left(\frac{f}{f_0}\right)^N$$  \hspace{1cm} (5.14)

based on previous work [1, 151] performed elsewhere. The authors fit the model to two different materials, sand and rock (limestone). They used material parameters from testing for the elastic parameters, but "fit" the viscoplastic parameters to their data, based on trial and error. For sand, they found $N = 1$ and $f_0 = 250\text{psi}$. For rock, they found $N = 1$, and $f_0 = 1,000\text{psi}$. For a predictive model, it is not sufficient to leave $f_0$ as an unknown, since $f_0$ should be a material parameter. While Perzyna suggested that $f_0$ may be the yield stress, based on simple calculations, it appears that $f_0$ may be calculated directly from the elastic modulus. This may be helpful for materials such as solder, which do not have a clearly defined yield point. Examining Katona and Mulert’s work, a simple formula has been devised in the following paragraphs.

Based on Goodman [47], the onset of viscous behavior in geologic materials (using a Berger’s body viscoplastic formulation, assuming deviatoric stress) occurs at a strain given by:

$$\varepsilon_0 = \sigma_i \left( \frac{2}{9K} + \frac{1}{3G} \right)$$  \hspace{1cm} (5.15)
Applying this formula to Katona and Mulert's data, it was found that $\sigma_1$ is approximately 2,500psi for sand. This is almost identical to the fitted $f_0$ (250psi) upon dividing by 10. Therefore, it is proposed that $f_0$ is simply:

$$f_0 = \frac{\sigma_1}{10} = \frac{\varepsilon_0}{10} \left( \frac{1}{2} \frac{1}{9K} + \frac{1}{3G} \right)$$  \hspace{1cm} (5.16)

To further simplify the calculation, the well known relationships between $G$, $K$, and $E$ can be employed:

$$K = \frac{E}{3(1-2\nu)}$$  \hspace{1cm} (5.17)

$$G = \frac{E}{2(1+\nu)}$$  \hspace{1cm} (5.18)

and, Equation 5.16 simplifies to:

$$f_0 = \frac{3E\varepsilon_0}{20(2-\nu)}$$  \hspace{1cm} (5.19)

For the sand of Katona and Mulert's test, $E$, $\nu$ and $\varepsilon_0$ are approximately 100ksi., .376 and .03 respectively. Applying these, $f_0$ is found as:

$$f_0 = 257 \text{ psi}$$  \hspace{1cm} (5.20)

This is almost exactly the $f_0$ of 250psi Katona and Mulert found from fitting the data. For solder materials, where $\nu$ is approximately .4, equation 5.19 simplifies to:

$$f_0 = .094E\varepsilon_0$$  \hspace{1cm} (5.21)

The Berger's formulation is a general viscoplastic formulation, and is not tuned to any particular material. Therefore, applying equation 5.21 for eutectic solder, where the onset
of plastic strain is approximately .2% [73], and $E$ is about 25GPa, gives an $f_0$ of about 4.7MPa. This is approximately the yield stress found by Wang [133] for a loading rate of $2.18 \times 10^{-4}$.

It is noted that this is a simple procedure for determining a rough value of the normalizing constant $f_0$, as Perzyna only suggested that it might be the yield stress, not that it specifically was the yield stress. The determination of $f_0$ is a topic of debate and research, and the method presented above for determination its value should be viewed as an estimate only, but supports Perzyna's vision of using the yield stress as the normalizing constant.

The value of $\gamma$ has been shown to be a significant factor in regulating the amount of viscoplasticity in the material behavior. The results of Katona and Mulert show that as $\gamma$ approaches 1, the material approaches the elastoplastic response. The best fit value used in the modeling of limestone was $\gamma = .2 \times 10^{-11}$. This is a very small number, but apparently should be small to account for viscous effects. For solder, the value has been found to be $2.4 \times 10^{-11}$ through back predictions of solder stress-strain curves.

5.4 Elastic Viscoplastic and HiSS Model

The elastic-viscoplastic constitutive model used in this dissertation is based on a form of Perzyna's viscoplastic formulation where it is assumed that the material behavior is time and temperature dependent. It is well known that some materials display creep and stress relaxation over time, and this behavior may be enhanced by increased temperature, a so called "thermal activation." Eutectic solder is one such material,
displaying creep and stress relaxation at temperatures below the peak temperatures found in operation.

The viscoplastic model has the form:

\[ \dot{\varepsilon}_{ij}^{vp} = \Omega \] (5.22)

where:

\[ \Omega = \Gamma \left\{ \frac{F}{F_0} \right\}^N \frac{\partial Q}{\partial \sigma_{ij}} \] (5.23)

Here the strain is now a function of time. The exponent \( N \) is a power law parameter found from testing, \( \Gamma \) is the fluidity parameter found from testing, and \( F \) is the yield surface function where \( \alpha \) is a hardening parameter:

\[ F = F(\sigma_{ij}, \alpha) \] (5.24)

\( F_0 \) is the yield stress, and \( Q \) becomes \( F \) for associative plasticity. The brackets represent the Macauley Function, and have the following meaning:

\[ \left\{ \frac{F}{F_0} \right\} = 0 \rightarrow \text{for} F \leq 0 \] (5.25)

\[ \left\{ \frac{F}{F_0} \right\} = \frac{F}{F_0} \rightarrow \text{for} F > 0 \]

Thus, when the current state of stress is on the yield surface, and the increment of stress or stress trajectory is normal or tangent to the yield surface, viscoplastic strains will be activated (Figure 5.4). The Perzyna viscoplastic formulation has the benefit of accounting for plastic as well as viscoplastic strains. That is, the plastic and viscoplastic strains are not separated. This is important, since for many materials, the rate of loading
has a significant impact on the material parameters, and delineation of time independent and time dependent properties is difficult to establish. The total strain is given by:

$$\dot{\varepsilon}_{ij} = \varepsilon_{ij}^e + \Gamma \left( \frac{F}{F_0} \right) \frac{\partial F}{\partial \sigma_{ij}}$$  \hspace{1cm} (5.26)

Where the superscript "e" denotes elastic strain.

5.5 Elastic-Viscoplastic Temperature Disturbance Formulation [28]

If temperature is included in the formulation, then the fluidity parameter $\Gamma$ and the yield function $F$ may become temperature dependent. They are then given by:

$$F = F(\sigma_{ij}, \alpha, \theta)$$  \hspace{1cm} (5.27)

$$\Gamma = \Gamma(\theta)$$  \hspace{1cm} (5.28)

In this case, the total strain is given by:

$$\dot{\varepsilon}_{ij} = \varepsilon_{ij}^e + \Gamma \left( \frac{F}{F_0} \right) \frac{\partial F}{\partial \sigma_{ij}} + \alpha \dot{\theta} \delta_{ij}$$  \hspace{1cm} (5.29)

where the last term accounts for thermal strain ($\alpha$ here is the coefficient of thermal expansion). Note that the temperature is assumed to be time dependent.

Based on the total strain, the total stress can then be found as:

$$\dot{\mathbf{\sigma}}_{ij} = C^{\text{ep}}_{ijkl} \left[ \varepsilon_{kl}^e - \dot{\Omega}_{kl} - \alpha \dot{\theta} \delta_{kl} \right]$$  \hspace{1cm} (5.30)

Here it can be seen that the time dependent stress is a function of three distinct strains, elastic, plastic-viscoplastic, and temperature. If the material has not yielded, then $\Omega_{kl}$ is zero, and the stress is found as:
\[
\dot{\sigma}_{ij} = C^{\text{VP}}_{ijkl} \left[ \epsilon_{kl}^e - \alpha \dot{\theta} \delta_{kl} \right] \tag{5.31}
\]

This has the same form as the standard (classical) time independent elastic stress constitutive formulation for a material under both force and thermal loads [13]:

\[
\sigma_{ij} = C_{ijkl}^{\text{e}} \left[ \epsilon_{kl}^e - \alpha \dot{\theta} \delta_{kl} \right] \tag{5.32}
\]

or:

\[
\sigma_{ij} = \delta_{ij} \lambda \dot{\epsilon}_{kk} + 2 \mu \epsilon_{ij} - \delta_{ij} (3 \lambda + 2 \mu) \alpha \dot{\theta} \tag{5.33}
\]

For the current research, the plastic potential function (yield surface) is taken from the Hierarchical Single Surface (HiSS) associative plasticity model [28]:

\[
F = \frac{J_{2D}}{P_a} - \left[ -\alpha \left( \frac{\dot{J}_1}{P_a} \right)^a + \gamma \left( \frac{\dot{J}_1}{P_a} \right)^2 \right] \left( 1 - \beta S_r \right)^m = 0 \tag{5.34}
\]

where \(\beta\) and \(m\) are material parameters determined from testing, and \(P_a\) is the atmospheric pressure. \(J_{2D}\) is the second invariant of the deviatoric stress:

\[
J_{2D} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \tag{5.35}
\]

\(J_1\) is the first invariant of the stress tensor:

\[
J_1 = \sigma_1 + \sigma_2 + \sigma_3 \tag{5.36a}
\]

\[
\dot{J}_1 = 3 \dot{R} + J_1 \tag{5.36b}
\]
$\alpha$ is a hardening function defined as:

$$\alpha = \frac{a_1}{\xi^n} \quad (5.37)$$

Here, $a_1$ and $\eta_1$ are material parameters, and $\xi$ is the trajectory of total (both volumetric and deviatoric) plastic strain given by:

$$\xi = \int \sqrt{\text{det} \mathbf{E} \text{det} \mathbf{E}^p} \quad (5.38)$$

For eutectic solder, research has shown that both $\beta$ and volumetric strain ($\varepsilon_v$) are approximately equal to zero. With this knowledge, the plastic potential function can be simplified as:

$$F = \frac{J_{2D}^2}{P_a^2} \left[ -\alpha \left( \frac{J_1}{P_a} \right)^n + \gamma \left( \frac{J_1}{P_a} \right)^2 \right] \quad (5.39)$$

and the total plastic strain trajectory simplifies to the deviatoric plastic strain trajectory:

$$\xi = \xi_D = \int \sqrt{\text{det} \mathbf{E} \text{det} \mathbf{E}^p} \quad (5.40)$$

where $\text{det} \mathbf{E}$ is the increment of deviatoric plastic strain.

A full derivation of the material parameters are described elsewhere [28], however, a brief description of their effects (for solder) are described as follows.

$R$ is the bonding stress, and determines at what $J_1$ stress the deviatoric stress $J_{2D}$ is zero (Figure 5.5). $R$ has the effect of sliding the yield surface along the $J_1$ axis, determining the effect of compressive (negative) stresses. Based on the yield function, the value of $R$ can be determined from a uniaxial test, at the ultimate state. At this state, $\alpha = 0$ and the yield function becomes:
\[ F = \frac{J_{2D}}{P_n^2} \left[ \gamma \left( \frac{J_1}{P_n} \right)^2 \right] = 0 \]  

and thus:

\[ J_{2D} = \gamma (J_1 + 3R)^2 \]  

(5.42)

giving:

\[ 3R = \sqrt[\gamma]{\frac{J_{2D}}{\gamma}} - J_1 \]  

(5.44)

For a uniaxial stress state:

\[ \sqrt{J_{2D}} = \frac{\sigma_f}{\sqrt{3}} \quad \text{and} \quad J_1 = \sigma_f \]  

(5.45)

However, because \( \gamma \) is small, the \( J_{2D} \) term will be much larger than the \( J_1 \) term in Equation 5.44. This is equivalent to making the assumption that solder is independent of the confining pressure, so \( J_1 \), which is equivalent to the confining pressure can be assumed as small. With this assumption, 3R is reduced to:

\[ 3R = \sqrt[\gamma]{\frac{J_{2D}}{\gamma}} \]  

(5.46)

Finally, combining Equations 5.42 and 5.46, 3R is found to be:

\[ 3R = \frac{\sigma_f J_1}{\sqrt{3}J_{2D} - \sigma_f} \]  

(5.47)

This equation can also be found by geometrical methods, using Figure 5.5, where the slope of the line yield surface is given by:

\[ \sqrt{J_{2D}} = \frac{\sigma_f}{3R\sqrt{3}} (3R - J_1) \]  

(5.48)
The slope of the yield surface is denoted as small $\gamma$, and has the effect of increasing the final elastic range (Figure 5.5). The ultimate yield surface is denoted as big $\gamma$, and determines the stress state at which no further hardening occurs (Figure 5.5). This is the value of $\gamma$ for $\alpha = 0$. Note that it is difficult to see the intercept of the yield surfaces with the $J_1$ axis in Figure 5.5. Figure 5.6 shows a single yield surface, for $\alpha = .00045$, which clearly shows that the yield surface intersects the $J_1$ axis at a right angle. That is, the yield surface is normal to the $J_1$ axis.

The parameter $\beta$ determines the shape of the yield surface (Figure 5.7) and therefore, controls the level of anisotropy. A circular yield surface indicates an isotropic material (common for metals), and a non-circular shape indicates anisotropic material behavior (common for non-metallic materials). Again, starting from the yield function, a formula for $\beta$ can be derived. However, it is much easier to start by simply assuming that $\beta$ is zero. This is appropriate, as it has been verified, experimentally, that solder is isotropic.

If $\beta$ is assumed zero, the yield function becomes:

$$F = J^{*2D} - \left( \sqrt{\beta} J^{*2} - \alpha \right)$$  \hspace{1cm} (5.49)

Where the star indicates that the parameter is normalized to $P_a$. For purposes of illustration, $n$ will be approximated as 2 (this is not unreasonable as $n$ has been found to be approximately 2.07 for eutectic solder.

$$F = J^{*2D} - J^{*2} (\alpha - \gamma)$$  \hspace{1cm} (5.50)

Assuming the quantity $(\alpha - \gamma) \ll 1$, then Equation 5.50 can be approximated as:
\[ F = J_{2D} \] (5.51)

This, again, is not an unreasonable approximation for illustration purposes, as \( \alpha \) and \( \gamma \) have been found to be 3e-3\( \rightarrow \)8e-7 and 1e-3\( \rightarrow \)8e-4, respectively, for solder [22, 133]. It can be seen that Equation 5.51 is approximately the Von Mises yield surface formulation. Thus, for \( \beta = 0 \), and \( n = 2 \), the yield formulation becomes the Von Mises formulation. It will later be shown that \( n \) is 2.07, indicating the above simplifications are reasonably approximate. A schematic illustration of \( \beta \) is shown in Figure 5.7.

The parameter \( n \) is the phase change parameter, and determines when the material changes from compaction to dilation (Figure 5.8). For solders, and metals in general, the change in volume under loading is negligible, and assumed zero during plastic strain. Beginning from the yield function, \( n \) can be derived by taking the derivative of Equation 5.49 (where \( \beta \) is assumed zero) with respect to \( J_1^* \):

\[
\frac{\partial F}{\partial J_1^*} = c n J_1^{*n-1} - 2 \gamma J_1^{*2} = 0
\] (5.52)

then:

\[
n = 2 \gamma J_1^{*2} \]

or:

\[
n = 2
\] (5.53)

This is a theoretical formulation for \( n \), but is a reasonable value, as it has been shown by other means that the value is 2.07 as stated earlier.
The parameter \( a_i \) is a hardening parameter, and is directly proportional to the hardening factor \( \alpha \). As \( a_i \) increases, hardening of the initial yield surface occurs (Figure 5.5). The parameter \( \eta_1 \) is the strain hardening exponent and is inversely proportional to hardening, based on strain trajectory. As \( \eta_1 \) increases, hardening of initial yield surface occurs (Figure 5.5). Both \( a_i \) and \( \eta_1 \) are related to the hardening factor \( \alpha \), where \( \alpha \) is defined as:

\[
\alpha = \frac{a_i}{\varepsilon^{\eta_1}} \quad (5.55)
\]

Both \( a_i \) and \( \eta_1 \) can be found beginning from the yield function. Assuming \( \beta \) is zero, then:

\[
F = J^*_{2D} - \gamma J_1^{*2} + \alpha J_1^{*n} = 0 \quad (5.56)
\]

or:

\[
\alpha J_1^{*n} = \gamma J_1^{*2} - J^*_{2D}
\]

which leads to:

\[
\alpha = \frac{\gamma J_1^{*2} - J^*_{2D}}{J_1^{*n}} = \gamma J_1^{*2-n} - J^*_{2D} \quad (5.58)
\]

If the approximation is made that \( n = 2 \), Equation 5.58 can be simplified to:

\[
\alpha = \gamma - \frac{J^*_{2D}}{J_1^{*2}} = \gamma - \frac{J^*_{2D}}{(J_1^{*} - 3R)^2} \quad (5.59)
\]

Taking the natural logarithm of Equation 5.59 gives:

\[
Lna = Lna_i - \eta_1 Lne \quad (5.60)
\]

Figure 5.9 shows Equation 5.60 with both \( a_i \) and \( \eta_1 \) shown.
Examining Equation 5.60, it can be seen that this equation represents a straight line in ln(\(\alpha\)) vs. ln(\(\varepsilon\)) space, with Ln\((a_1)\) representing the Ln(\(\alpha\)) intercept, and \(\eta_1\) representing the slope of the line (Figure 5.10). With \(\alpha\) known from Equation 5.59, \(\alpha\) can be plotted as Ln(\(\alpha\)) vs. Ln(\(\varepsilon\)), and thus \(a_1\) and \(\eta_1\) can be determined.

The parameter \(D_u\) determines the maximum disturbance (damage) a material can accumulate prior to complete "local failure" (Figure 5.1). \(D_u\) is similar to a macro crack, but may still contain shear and/or hydrostatic strength. The value of \(D_u\) is an asymptotic value, and, in the limit, \(D_u = 1\). However, this limit cannot be achieved in normal engineering practice, and is thus approximated as .85 to .95 for most materials. .9 is assumed in this research for the ultimate value of eutectic solder.

The parameter \(A\) can be considered (loosely) as determining the damage level at zero deviatoric strains. That is, \(A\) has the effect of increasing the "initial" damage level in a material (Figure 5.11), and may be important for preloaded or materials containing significant heterogeneities. Examples might include geologic materials such as rock, brick and concrete. As a comparison, the value for \(A\) has been shown to be 668 for plain concrete [33], and .5 for solders [133]. Figure 5.11 shows plots of Equation 5.61 for varying values of \(A\) and \(Z\). It can be seen that both \(A\) and \(Z\) work in conjunction, but for the same \(Z\) value, a higher value of \(A\) will result in a higher initial value of disturbance.

The parameter \(Z\) determines the rate of disturbance increase due to deviatoric plastic strains (Figure 5.11). In other words, it determines how quickly a material will accumulate damage, and hence fail. As noted above both \(A\) and \(Z\) work in conjunction, but for the same values of \(A\), a higher value of \(Z\) will result in a faster rise in disturbance.
The parameters \( A \) and \( Z \) are thus important for determining the life of a structure. If \( A \) and \( Z \) are not determined correctly, the accumulated damage will not represent the actual damage in a structure over repeated loadings or time.

Both \( A \) and \( Z \) are determined from the disturbance function:

\[
D = D_u (1 - e^{-A\xi^j})
\]  

(5.61)

Which leads to:

\[
\frac{D_u - D}{D_u} = e^{-A\xi^j}
\]  

(5.62)

Taking the natural logarithm of both sides gives:

\[
\ln\left[-\ln\left(\frac{D_u - D}{D_u}\right)\right] = \ln A + Z \ln\xi
\]  

(5.63)

Examining Equation 5.63, it can be seen that this is the equation of a straight line in \( \ln\{-\ln[(D_u-D)/D_u]\} \) vs. \( \ln(\xi) \) space, where \( \ln(A) \) is the Y axis intercept, and \( Z \) is the slope of the line. This can be seen in Figure 5.10, where \( \ln\{-\ln[(D_u-D)/D_u]\} \) vs. \( \ln(\xi) \) is plotted.

All of the parameters can be found from uniaxial testing of suitable samples. For example, from uniaxial testing, a plot of \( J_{2D} \) versus \( J_1 \) can be constructed assuming a very small confining pressure [28]. The slope of the curve will represent the parameter \( \gamma \), the intersection of the curve with the \( J_{2D} \) axis is the tensile strength, and the intersection with the \( J_1 \) axis is the parameter \( 3R \) (Figure 5.5). The phase change parameter can be found by using Equation 5.53, with the variables set to values found at the point where the material
begins to yield. The hardening parameters can be found by plotting $\alpha$ and the deviatoric plastic strains found from testing. The parameter $\alpha$ can be found by using the yield function (Equation 5.34), where all the other terms will be known. By plotting as shown in Figure 5.9, $\eta_1$ and $a_1$ can be found, satisfying the hardening function (Equation 5.55).
Figure 5.1. Schematic illustration of disturbed state concept (stresses) [28]
Figure 5.2. Schematic illustration of disturbed state concept: Balance of forces
Figure 5.3: Large displacements: 90° rotation causes incorrect stress accounting
Figure 5.4. Loading and unloading from yield surface
Figure 5.5. Plot of HiSS yield function for solder showing the effect of $\gamma$, $R$ and $\alpha$. $3R$ is 650MPa, $n = 2.1$, and $\alpha$ varies as shown.
Figure 5.6. Normality of HiSS yield surface with $J_1$ axis.
Figure 5.7. Plot of yield function with different $\beta$, the yield surface shape parameter [134].
Figure 5.8: Plot of yield function with different values of $n$, the phase change parameter [134].
Figure 5.9: Schematic relation between hardening parameters $a_i$ and $\eta_i$ and deviatoric strain $\xi$. 
Figure 5.10: Disturbance parameters $A$ and $Z$
Figure 5.11. Schematic illustration of the effect of parameters $Z$ and $A$ on disturbance.
CHAPTER 6
HEAT TRANSFER ANALYSIS SUBROUTINE

6.1 Introduction

This chapter documents the heat transfer subroutine (subroutine H2D) added to the DSC-SST2D FEM code, and is divided into the following chapter sections: H2D Theory Manual (6.2, 6.3, 6.4), H2D Parameters (6.5, 6.6) and H2D Example Decks (6.7-6.10).

The H2D subroutine is used for heat transfer analysis, and has been adapted from the heat transfer code HEAT-2DFE [29]. The subroutine has been added to the DSC-SST2D FEM program, and significantly increases the power and problem solving capability of the code. At present, the H2D subroutine has been verified for plane elements, although axisymmetric elements are available.

The plane elements have been formulated as QUAD4 (four node) solid elements, and may take thickness into account, i.e. plane stress. QUAD4 elements are the standard thermal element, as temperature in an element does not vary quadratically. In terms of a structural formulation, temperature can be thought of as a load, and the temperature gradient as a strain. Solid, plane elements are not the same as shell elements. Solid, plane elements are 2D elements with two degrees of freedom at each node. Shells are plane 2D elements as well, but with six degrees of freedom at each node. This distinction is important if results from the DSC-SST2D code are going to be compared with a NASTRAN, ABAQUS or ANSYS solution. Each of these commercial codes make a
distinction between shells and plane, solid elements. In general, shell elements are used in structures where significant bending is likely to occur.

6.2 H2D Subroutine Finite Element Procedure

The H2D subroutine is a transient and steady state heat transfer routine. It allows thermal sinks and sources to be modeled in the general DSC-SST2D FEM code, where the associated thermal stresses and strains produced by the thermal gradients and CTE mismatches are calculated. The code is very concise, and is based on the two dimensional conduction equation:

$$\nabla^2 T + \frac{\partial^2 T}{\partial y^2} + Q = \rho c \frac{\partial T}{\partial t}$$

Where $\kappa$ is the thermal conductivity, $\rho$ is the mass density, $c$ is the specific heat and $Q$ is the internal heat flux. It is assumed that the thermal conductivities in the X and Y directions may be different, but that each is constant in it's own relative direction. To increase the capability of the code, thermal convection has been added to Equation 6.1 so that convection may considered as a boundary condition in the model:

$$\kappa_x \frac{\partial^2 T}{\partial x^2} + \kappa_y \frac{\partial^2 T}{\partial y^2} + Q = \rho c \frac{\partial T}{\partial t} + h(T - T_\infty)$$

Where $h$ = convection heat transport coefficient, $T$ = temperature in the body, and $T_m$ = ambient temperature.

Examining Equation 6.2, it may be seen that the convection is modeled using Newton's law of cooling, i.e., $h(T - T_m)$. Note that Equation 6.2 requires two spatial conditions on $T$, and one temporal condition (time is only considered for the transient
case). Also note that the sides of the body in the Z direction are considered adiabatic, and is a standard 2D assumption for thermal problems. The boundary conditions may be satisfied by any one or any combination of two of the following cases: Temperature is prescribed and/or flux is prescribed and/or convection is prescribed. The initial condition is found from an initial temperature which must be prescribed. These boundary conditions may be conveniently written as:

\[ T = T^*_{(x,y,t)} \quad * \text{indicates prescribed} \quad (6.3) \]

\[ K_x \frac{\partial T}{\partial x} + K_y \frac{\partial T}{\partial y} + Q^* + h^* (T_{in} - T) = 0 \]

\[ T(x,y,0) = T_i^* \]

where \( T_i \) represents the initial temperature in the body, which may be non isothermal. Note that \( T_i \) must always be prescribed, for both steady state and transient solutions. For a transient solution, \( T_i^* \) represents the temperature solution at time zero.

6.3 H2D Subroutine Finite Element Formulation

A variational procedure will be adopted in outlining the finite element formulation. By considering the potential energy functional:

\[ \Pi_p = U + \Sigma_Q + \Sigma_q + \Sigma_h \quad (6.4) \]

a finite element formulation can be derived using a minimization procedure. In Equation 6.4, \( U \) represents the potential energy, \( \Sigma_Q \) represents the internal energy, \( \Sigma_q \) represents the conduction energy, and \( \Sigma_h \) the convection energy. These may be expanded as follows:
This is in contrast to a QUAD4 structural element which will be an eight DOF solid element, or 24 DOF shell element.

The discretization scheme may be expressed as:

\[
T_{(x,y)} = \sum_{i=1}^{n} N_i T_{i(0)}
\]

\[
T_{(x,y)} = [N] T_{n(0)}
\]

Equations 6.7 have been shown in matrix notation, where [ ] represents a row vector, and \{ \} represents a column vector. \( T_n \) are the nodal temperatures, \( n \) is the number of degrees of freedom (number of nodes for a thermal element), and \( N \) is the matrix of interpolation functions.

Using the quadrilateral element shown in Figure 6.3, the interpolation functions may be found by using the Lagrange polynomial formula:

\[
N_k = \prod_{m=0, k \neq m}^{n} \frac{x - x_m}{x_k - x_m}
\]

(6.8)

The interpolation functions may then be succinctly written as:

\[
N_i = \frac{1}{4} (1 + s_i)(1 + t_i)
\]

\[
s_i = (-1,1,1,-1)
\]

\[
t_i = (-1,-1,1,1)
\]

(6.9)

Where \( s \) and \( t \) are the local coordinates of the element (Figure 6.3). Because the procedure is based on the isoparametric formulation, the shape functions are used to define the element geometry, as well as the value of the primary unknown (displacement for structural elements, temperature for thermal elements). Therefore, the global coordinates of the element may be expressed as:
\[ X = \sum_{i=1}^{4} N_i X_i \]
\[ Y = \sum_{i=1}^{4} N_i Y_i \]  
(6.10)

The temperature gradients can then be expressed as:

\[ \frac{\partial T}{\partial x} = \frac{\partial}{\partial x} (N_i T) = [B_x] \{T_n\} \]
\[ \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} (N_i T) = [B_y] \{T_n\} \]  
(6.11)

Combining Equations 6.11, it may be shown that the temperature gradients are found as shown in Equation 6.12, where \([B]\) is a square diagonal matrix.

\[ \{g\} = [B] \{T_n\} \]  
(6.12)

Note that Equation 6.12 will be convenient for determining the flux within the body, as shown later.

A time derivative is necessary to determine the transient heat transfer. The actual time integration will be described in detail later, where the time dependent function will be a function of the shape function:

\[ \frac{\partial T}{\partial t} = [N] \{\dot{T}_n\} \]  
(6.13)

In Equation 6.13, it is noted that the shape functions are independent of time (see Equation 6.9). Combining the preceding equations with the potential energy equation functional of Equation 6.6, the following equation may be written:
The matrix \([D]\) in Equation 6.14 is the matrix of thermal conductivities, and is analogous to the \([E]\) or elasticity material matrix in structural formulations:

\[
[D_e] = \begin{bmatrix} K_{xx} & 0 \\ 0 & K_{yy} \end{bmatrix}
\]  
(6.15)

Where the subscript \(e\) indicates that the matrix is for an arbitrary element. The stationary value of the potential energy function \(\Pi_p\) can be found by taking the derivative (variation) as:

\[
\frac{\partial \Pi_p}{\partial T} = 0
\]  
(6.16)

The derivative of Equation 6.14 can be simplified by directly calculating the derivative using matrix calculus, i.e.:

\[
\frac{\partial \Pi_p}{\partial T} = \frac{\partial \Pi_p}{\partial \{T\}^T}
\]  
(6.17)

After taking the derivative with respect to the primary unknown \((T)\), then equating to zero, the potential function becomes:

\[
[K\{T\}] + [P_t]\{\dot{T}\} = \{f\}
\]  
(6.18)

In Equation 6.18, \([K]\) is the thermal conductivity matrix, \([P_t]\) is the time dependent property matrix, and \(\{f\}\) is the forcing function vector. These can be expanded as:
Assembling Equations 6.19 will result in an unsolvable system of equations, i.e., there are more unknowns than there are equations. However, if the boundary and initial conditions are prescribed as outlined in Equations 6.3, then Equations 6.19 are solvable as:

$$\begin{align*}
\{ \dot{T} \} = \begin{bmatrix} \mathbf{K} \end{bmatrix} \{ T \} + \begin{bmatrix} \mathbf{P} \end{bmatrix} \begin{bmatrix} \dot{T} \end{bmatrix} = \{ \dot{f} \}
\end{align*}$$

provided a suitable time integration is available. The overbar indicates that the matrices have been suitably modified with respect to the boundary conditions. The time integration scheme utilized is outlined in the following section.

### 6.4 Time Integration Scheme

The time integration scheme employed is based on the Crank-Nicholson procedure. In this procedure, the time domain is divided into small, equal divisions of $\Delta t$. Therefore, each time step “marches forward” or propagates the solution in time by $\Delta t$. The Crank-Nicholson scheme is based on the following procedure, where it is assumed that the average temperature value between $t$ and $t + \Delta t$ is a linear average (Figure 6.4).

$$\begin{align*}
\{ \ddot{T} \}_k &= \frac{\{ T \}_k + \{ T \}_k}{2} \\
\{ \dot{T} \}_k &= \frac{\{ T \}_k + \{ T \}_k}{\Delta t} \\
\{ f \}_k &= \frac{\{ f \}_k + \{ f \}_k}{2}
\end{align*}$$

(6.21)
Note that the time "derivative" is simply the elementary description of rise over run or slope of a function. Substituting Equations 6.21 into Equation 6.20:

\[
[K][T]_{t+\Delta t} + [P]_{t}\left[\frac{T_{t+\Delta t} - T_{t}}{\Delta t}\right] = \frac{\{f\}_{t+\Delta t} + \{f\}_{t}}{2}
\]  

(6.22)

Using the formulation for \( T_t \) as

\[
\{T\}_{t+\Delta t} = 2\{T\}_{t+\Delta t/2} - \{T\}_t
\]  

(6.23)

Equation 6.22 becomes:

\[
\left[K\right][T]_{t+\Delta t} + \left(P\right)_{t}\left[\frac{2\{T\}_{t+\Delta t/2} \left(\frac{T_{t+\Delta t} - T_{t}}{\Delta t}\right) \left\{T\}_{t+\Delta t} - \left\{T\right\}_t\right]}{2} = \frac{\{f\}_{t+\Delta t} + \{f\}_{t}}{2}
\]  

(6.24)

\[
\left(\left[K\right] + \frac{2}{\Delta t}\left[P\right]\right)\left\{T\right\}_{t+\Delta t/2} = \frac{\{f\}_{t+\Delta t} + \{f\}_{t}}{2} + \frac{2}{\Delta t}\left[P\right]\left\{T\right\}_t
\]  

(6.25)

Collecting terms,

\[
\left[K\right]_{t+\Delta t} = \left\{\frac{f}{T}\right\}_t
\]  

(6.26)

The subscripts in Equation 6.26 illustrate that the matrices are now fully integrated with respect to time, and have been minimized with respect to temperature.

The general solution sequence may be outlined as follows:

1.) At elemental level, determine \([K]\), \([P]\), and \(\{f\}\).

2.) Apply time integration.

3.) Assemble element equations (Equation 6.26).

4.) Apply initial conditions, set \(t = 0\).

5.) Apply boundary conditions.

6.) Solve Equation 6.26 for \(\{T\}_{0+\Delta t/2}\), noting that \(T_0\) is known from initial conditions.
7.) Solve for $T_{0+\Delta t}$ using Equation 6.23.

7.) Solve for secondary components such as flux: $q = -[K][g]$ which follows directly from the definition of flux:

$$q_i = -K_i \frac{dT}{dx_i} \quad (6.27)$$

8.) Increment $\Delta t$, repeat steps 5 → 7.

6.5 H2D Subroutine - Introduction for Users

The H2D heat transfer subroutine allows the calculation of heat transfer within a solid body. This is of considerable importance when calculating stresses due to time varying (cyclic) temperature loads. In general, if a body is subjected to both sources (applied temperature ($T_1$) or flux) and sinks (convection or applied temperature ($T_2$)) such that $T_1 \neq T_2$, then a temperature gradient will exist in the body. Temperature gradients will also exist if the applied temperature is transient, for small times, even if $T_1 = T_2$. If any of these conditions are present, then it may be insufficient to assume that the body is in an isothermal state. Therefore, a heat transfer calculation may be necessary. With the use of the H2D heat transfer subroutine, a realistic approximation to the temperature profile within a solid body can be obtained, and therefore, a realistic approximation to the actual thermal stresses.

Within the DSC-SST2D FEM code, a number of methods are available to the user for calculating thermal stresses. Those that do not use the H2D heat transfer subroutine are described elsewhere. The possible permutations of a heat transfer analysis are outlined as follows, where each case is described in detail in sections 6.7-6.10:
Steady state

Static (Case 1)

Cyclic (Case 2)

Transient

Static (Case 3)

Cyclic (Case 4)

The outline of each case is presented in the following order:

1.) Description of case, which includes the relevant parameters and an example problem

2.) Example problem input deck

6.6 Parameters Required For H2D

The parameters unique to the H2D subroutine, and necessary for a heat transfer analysis to run are described below. All other parameters are input as usual within the DSC-SST2D FEM code, and are described elsewhere. Note that IdcTmp, NTYP and Tmp_TC are not unique to the H2D subroutine, but are necessary for the subroutine to function correctly.

1.) IdcTmp: 3 for heat transfer analysis, 2 for non-uniform Ti, 1 for uniform Ti, 0 for non-thermal problem

2.) H2D: 2 for cyclic H2D problem, 1 for non-cyclic H2D problem

3.) NOPT: 2 for axisymmetric, 1 for plane

4.) NCASE: 1 for steady state, 0 for transient

5.) NFLX: Number of element sides with flux prescribed
6.) NCONV: Number of element sides with convection prescribed
7.) NTIME: Number of time steps to skip between printing of output
8.) DT: Time step size
9.) TOTM: Total time of analysis
10.) Tmp_TC: Initial temperature
11.) PERX: Thermal conductivity in the X direction
12.) PERY: Thermal conductivity in the Y direction
13.) C: Specific heat
14.) NT: Number of nodes with prescribed temperature
15.) TEMP: Prescribed temperature
16.) NF1: Start node for prescribed flux
17.) NF2: End node for prescribed flux
18.) FLX: Prescribed flux
19.) NC1: Start node for prescribed convection
20.) NC2: End node for prescribed convection
21.) H: Convection coefficient
22.) TAMB: Ambient temperature
23.) NTYP: Temperature change. Set to 0 (uniform load increment) when using H2D

The following are a few general comments for the successful use of the H2D subroutine. Because the routine has been integrated into the DSC-SST2D code in a simple manner, there are only a few rules required for its use:
1.) To enable a heat transfer analysis set IdcTmp = 3

2.) For cyclic thermal cases, set H2D = 2, NTYP = 0

3.) For static thermal cases, set H2D = 1, NTYP = 2

4.) Only QUAD4 elements are supported

6.7 Steady State Heat Transfer – Static Temperature Load, Case 1

In this case, the application of the load (flux or applied temperature) is constant, and the time at which the temperatures are to be calculated is sufficiently long such that the body has achieved thermal equilibrium i.e., a steady state. The simplest example of this is a bar under adiabatic conditions along the sides, and two differing temperatures applied at opposite ends. Note that under steady state conditions, for this simple geometry, the temperature gradient within the bar will be a simple linear dependence from $T_1$ to $T_2$. A sample input deck is shown in Figure 6.5.

6.8 Steady State Heat Transfer – Cyclic temperature Load, Case 2

In this case, the applied load is cyclic, but the heat transfer calculations are assumed steady state. It is assumed that the temperature increments occur over a sufficiently long time, such that the body achieves thermal equilibrium between temperature increments. Figure 6.2 represents a prototypical thermal half cycle. A sample input deck is shown in Figure 6.6.

This may be a contradictory load case if the cyclic frequency results in a change in temperature faster than the body can reach equilibrium. For example, if it takes one minute for a bar to reach steady state, then the period of the temperature increment should
be approximately one minute for steady state conditions to approximate the actual conditions. For the temperature cycle shown in Figure 6.2, if $t_1$ is 15 seconds, then it is clear that the bar will not have reached steady state by $t_1$ and a steady state analysis may not be appropriate. However, if $t_1$ was 60 seconds, then a steady state analysis would be appropriate.

In some situations a steady state heat transfer analysis may be necessary even though the steady state assumption is probably incorrect. These situations primarily result from a lack of computer processing speed. Although a transient case may be more appropriate, the cost (in time) to run such a case may be prohibitive. Depending on the computer speed, the time step size, thermal parameters, number of degrees of freedom in the model and total time of analysis, a transient heat transfer model may take many hours to days for each cycle to run. This does not include the calculations necessary for nonlinear material behavior such as plasticity or viscoplasticity. If viscoplasticity and disturbance calculations are added, a model will require significantly longer run times compared to a thermal transient elastic model. The longer run times may be time prohibitive.

In these situations, it may be necessary to assume steady state conditions, knowing that they are indeed incorrect. However, assuming a steady state condition will always be more accurate than assuming isothermal conditions.

6.9 Transient Heat Transfer – Static Load, Case 3

For this case, the heat transfer within the body is now time dependent, but the thermal load is static. A sample input deck is shown in Figure 6.7. In this case, the
progression of heat energy through the body is now tracked as a function of time. This case will be appropriate for determining how long it takes for a body to achieve equilibrium, i.e. steady state. Knowing this, a determination can be made as to whether a transient simulation (for cyclic loading) is necessary. In most cases, a transient static simulation is not necessary for determining stresses, unless the analysis is a thermal shock case. Thermal shock is the rapid application of thermal energy to a body. Thermal shock causes high temperature gradients and can cause thermal cracking. Generally speaking, a thermal shock is considered to be a rise in temperature greater than 30°C/minute.

6.10 Transient Heat Transfer – Cyclic Load, Case 4

This approximation should be used when the temperature cycle changes so rapidly that the body can not achieve steady state equilibrium from one temperature step to another. This is the most time intensive case. For large models and nonlinear material behavior, this case may be time prohibitive. A sample input deck is shown in Figure 6.8.
Figure 6.1. 4 Node Quadrilateral Discretization.
Figure 6.2. Time Dependent (Cyclic) Thermal Loading.
Figure 6.3. Quadrilateral Local Coordinates.
Figure 6.4. Crank Nicholson time integration procedure (Equations 6.21).
Case 1
Steady state, applied temp, convect & flux, no cycle
75 96 2 0 1 1 1 20 2 0.0 0.1013 .1 0.0
0 0 1.0E-4 1.0E-3 0 0.0 20 4 4 0 0 0 0 0 0 1 ← (NTEMP R)
0 2 15700 0.3 2707.0 0.0
0.00000 0.0 0.0 2.10 0.0000078 0.46 0.0 1000.0 0.0 0.0
624.0 1 0.0 3.0e-6 0 0.0 0.0 0.0 0
2
Element connectivity should go here (Not included to shorten length)
Nodal coordinates should go here (Not included to shorten length)
6 1 1
1 1 1
204 204, 896 ← (PERX, PERY, C)
1, 50 (NT, TEMP)
2, 50
3, 50
4, 50
5, 50
6, 50
1, 7, 5000 (NF1, NF2, FLX)
7, 13, 5000
13, 19, 5000
19, 25, 5000
25, 31, 5000
6, 12, 10, 20 (NC1, NC2, H, TAM B)
12, 18, 10, 20
18, 24, 10, 20
24, 30, 10, 20
30, 36, 10, 20
0 (TEMPL)
8 0 0 0 50 0 2 0 0 1 10. Comment 1
0 0 1 1 1

Comment 1:
(8 indicates thermal loading, 1 indicates 1 total load cycle, 10 is for total time)

Figure 6.5. Case 1, steady state with applied temperature, convection and flux.
Case 2

Steady state. DSC provides temp cycle. Calculates stresses and strains

75 96 12 0 1 1 1 20 2 0.0 0.1013 0.0 0.0 0 0 20 4 4 0 0 0 0 0 0 0 0 1
0 2 10e6 0.3 3707.0 0.0 0.00081 0.0 0.0 2.10 0.0000078 0.46 0.0 1000.0 0.0 0.0
624.0 1 0.0 1.0e-6 0 0.0 0.0 0.0

Element connectivity should go here (Not included to shorten length)
Nodal coordinates should go here (Not included to shorten length)

1,1,0
2,1,0
3,1,0
4,1,0
5,1,0
6 1 0
91,1,0
92,1,0
93,1,0
94,1,0
95,1,0
96,1,0
96 75
1
1 4 8
0

2 (H2D)
1.1,6,0,0,500,.1,100.
204,204,896

1.50 The value of TEMP is arbitrary here. The actual value of TEMP is given by the
temperature cycle below. The NT values are the nodes where the cycled temperature
values will be applied. If nodes are to be kept at a prescribed temperature, i.e.
are independent of the cycled temperature, prescribe a convection on these nodes
with a value of H very high, and the value of TAMB at the temperature
desired.

100 Initial temperature

8 0 0 0 5 0 2 2 1 1 10. Comment 1
0 0 1 1 1
0 (NTYP)

400 Temperature change

5 The cycle has been set to a trapezoidal shape
0 2 1 2 0 0 The cycle has been prescribed to start at the initial temperature (100),
two steps to reach the max temperature (500), 1 step to stay at this
temperature, 2 steps to ramp down to the initial temperature. The
temperature cycle will look like the one shown in Figure 6.2.

Comment 1:
(8 indicates thermal loading, 5 indicates number of steps is broken into, 1 indicates 1 total
load cycle, 10 is for total time)

Figure 6.6. Case 2, steady state, applied temperature cycle.
Figure 6.7. Case 3, transient with applied temperature, convection and flux.
Case 4

Transient, cycle
75 96 12 0 1 1 1 20 2 0.0 0.1013 0.1 0.0
0 1.0E-4 1.0E-3 0 0.0 20 4 4 0 0 0 0 0 0 0 1
0 2 10e6 0.3 2707.0 0.0
0.00081 0.0 0.0 2.10 0.0000078 0.46 0.0 1000.0 0.0 0.0
624.0 1 0.0 1.0e-6 0 0.0 0.0 0.0 0.0
2
Element connectivity should go here (Not included to shorten length)
Nodal coordinates should go here (Not included to shorten length)

1.0
2.1.0
3.1.0
4.1.0
5.1.0
6 1 0
91.1.0
92.1.0
93.1.0
94.1.0
95.1.0
96.1.0
96 75
1
1 4 8
0
2 (H2D)
1.0,6.0,0,500,.1,100.
204,204,896

1.50 The value of TEMP is arbitrary here. The actual value of TEMP is given by the
temperature cycle below. The NT values are the nodes where the cycled temperature
values will be applied. If nodes are to be kept at a prescribed temperature, i.e.
are independent of the cycled temperature, prescribe a convection on these nodes
with a value of H very high, and the value of TAMB at the temperature
desired.

100 Initial temperature
8 0 0 0 5 0 2 2 1 1 10.
0 0 1 1 1
0 (NTYP)
400 Temperature change

5 The cycle has been set to a trapezoidal shape
0 2 1 2 0 0 The cycle has been prescribed to start at the initial temperature (100),
two steps to reach the max temperature (500), 1 step to stay at this
temperature, 2 steps to ramp down to the initial temperature. The
temperature cycle will look like the one shown in Figure 6.2.

Figure 6.8. Case 4, transient with cyclic temperature, convection and flux.
7.1 Introduction

A heat transfer subroutine, based on the heat conduction equation, has been written in FORTRAN90 and documented. The subroutine is called H2D, and is capable of solutions for multiple materials with differing thickness, steady state and transient, as well as convection, flux and temperature boundary conditions. All cases have been verified and documented using the general purpose commercial code ANSYS as a reference.

The subroutine was developed for implementation into the general FEM code DSC-SST2D, allowing thermal sinks and sources to be modeled, as well as the associated thermal stresses and strains produced by the thermal gradients and CTE mismatches. The code is very concise, and is based on the two dimensional conduction equation which was shown previously:

\[ \kappa_x \frac{\partial^2 T}{\partial x^2} + \kappa_y \frac{\partial^2 T}{\partial y^2} + Q = \rho c \frac{\partial T}{\partial t} \]  \hspace{1cm} (7.1)

where \( \kappa \) is the thermal conductivity, \( \rho \) is the mass density, \( c \) is the specific and \( Q \) is the internal heat flux. However, thermal convection has been added to Equation 7.1 so that convection may considered as a boundary condition in the model:

\[ \kappa_x \frac{\partial^2 T}{\partial x^2} + \kappa_y \frac{\partial^2 T}{\partial y^2} + Q = \rho c \frac{\partial T}{\partial t} + h(T - T_\infty) \]  \hspace{1cm} (7.2)

Where \( h \) = convection heat transport coefficient and \( T_\infty \) is the ambient temperature.
7.2 Heat Conduction Theory

No attempt is made here to outline the theory of finite elements in heat transfer. This has been documented in [59], as well as other texts. The only theory presented is for heat transfer.

The heat equation is based on the Fourier conduction law. The heat equation can be found by simply placing the Fourier conduction law into the conservation of thermal energy equation. Therefore, the three dimensional Fourier law may be thought of as a thermal diffusion, and thus similar to Darcy's permeability theory. Even when flux and convection boundary conditions are added to the heat conduction equation, similar analogies can be found in permeability and diffusion problems.

The two dimensional heat conduction equation is expressed as:

$$\rho c \frac{\partial T}{\partial t} = Q + \frac{k_x}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2}$$  \hspace{1cm} (7.3)

where $Q$ = internal energy source, $\rho$ = mass density, $c$ = specific heat, $T$ = temperature, $t$ = time, $k_i$ = conductivity, where the thermal conductivities are assumed independent of their primary direction, i.e. constants. This is the equation used in the H2D subroutine.

If the problem is independent of time, the problem is steady state. In this case, Equation 7.1 simplifies to:

$$k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} + Q = 0$$  \hspace{1cm} (7.4)

Note that in a steady state problem, time, density and specific heat are not considered. This is apparent from the definition of thermal capacity (specific heat * mass). Thermal
Capacity is the amount of heat required to raise the temperature of one mole of material by one degree Kelvin. Because it takes time to raise the temperature of a material, it is clear that specific heat cannot be involved in a steady state problem. This shows the interesting phenomena that two different materials, having the same thermal conductivity, will achieve the same steady state temperature distribution under the same boundary conditions. The thermal conductivity of a material is defined as the amount of heat (thermal energy) that must flow through a unit length of material in a unit time to raise the temperature one unit. The greater the thermal conductivity, the greater the heat energy flow. Metals generally have higher conductivities compared to other materials such as geomaterials, foams and wood products. For example, the thermal conductivity of common low carbon steel is 60 W/mK, the conductivity of pure copper is 401 W/mK, the conductivity of diamond is 2300 W/mK, the conductivity of cement is .72 W/mK, and the conductivity of cork is .039 W/mK. Noting the conductivities of diamond and copper, it is evident why copper is such a common material in heat sinks, and why diamond is gaining attention as a heat spreader for semiconductor applications.

If the material is thermally isotropic, and under steady state conditions, Equation 7.4 can be further simplified to:

\[
k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q = 0
\]

(7.5)

For one dimensional, steady state conditions, with no sources or sinks, the heat equation is reduced to its simplest form:

\[
k \frac{d^2 T}{dx^2} = 0
\]

(7.6)
For a one dimensional problem, Fourier's Law is:

\[ q = -k \frac{dT}{dx} \]  

(7.7)

Equation 7.7 represents a heat flux (watts/area\(^2\)). A heat rate or flow (watts) can be found by multiplying the heat flux by the plane area the heat flux passes through. Note that the derivative in Equations 7.6 and 7.7 is no longer a partial derivative, and that the derivative has been reduced from a second order to a first order in equation 7.7. If the thermal conductivity and flux is constant, then Equation 7.6 can be obtained by taking the derivative of Equation 7.7. Thus, Equation 7.6 states that the change in flux must be zero in a steady state problem (\(dq/dx = 0\)), and that Equation 7.7 is a boundary condition to be satisfied (Figure 7.15). Equation 7.7 is analogous to the structural law relating stress and strain: \(\sigma = E\frac{du}{dx}\). The negative sign in Equation 7.7 indicates that heat flows in the direction opposite of the temperature increase, thus defining the direction of heat flow as positive.

It is interesting to note that Equation 7.7 is the similar to the standard equation shown in text books describing one dimensional permeability [41, 56]. In the case of Darcy's law, the \(k\) term is defined as the coefficient of permeability, and \(T\) is the fluid "head". Also, if heat transfer is analogous to the diffusion of energy, then the most general form of heat transfer becomes the diffusion equation [51]:

\[ Q + \frac{\partial}{\partial x}\left(k_x \frac{\partial U}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial U}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial U}{\partial z}\right) = \chi \frac{\partial U}{\partial t} \]  

(7.8)
where $\chi$ represents material parameters. It can be seen that Equation 7.8, a form of the diffusion equation, can be found by placing Fourier's Law (Equation 7.7) into the conservation of thermal energy equation:

$$c_p \frac{\partial U}{\partial t} = -\frac{\partial q}{\partial x} + Q \quad \text{(7.9)}$$

and expanding to 3 dimensions.

7.3 Heat Convection Theory

Convection is a boundary condition for heat flow in solid bodies, and occurs when a body is in contact with a fluid (gas or liquid). Two types of convection are recognized, forced, where the fluid is moved by external means (fan, pump) and natural where buoyancy forces move the liquid. Buoyancy forces are created as the liquid gains thermal energy, causing the density to decrease. The liquid with a reduced density rises, and higher density (cooler) liquid takes its place, thus cooling the solid body. Natural convection is dependent upon the surface area of the solid body, and is very inefficient at high temperatures and/or small areas.

Convection theory is extremely involved, primarily due to the complex nature of the convection coefficient. In general, convection problems are not amenable to generalized solutions, because the convection coefficient is highly dependent upon the environmental conditions as well as the surface conditions of the body. Thus, there can be no "standard" convection coefficient for a material, as there is for elastic modulus or specific heat. The convection of heat from or to a surface is dependent upon the
geometry, surface smoothness, surrounding air temperature, and velocity (speed and direction) of the surrounding fluid. The simplest convection equation is Newton's law of cooling:

\[ q = h(T_\infty - T_s) \]  

(7.10)

where \( h \) is the convection coefficient, \( T_\infty \) is the ambient temperature, and \( T_s \) is the surface temperature. The convection coefficient \( h \) is defined as the amount of heat-energy that must move across a unit surface area, in a unit-time, to raise the temperature of the surface one unit. Thus, the convection coefficient \( (h) \) is similar to the conductivity \( (k) \), where both are plane phenomenon, except that \( q(k) \) flows through the plane, and \( q(h) \) flows from the plane. While Equation 7.10 is algebraically simple, its actual use is complex. The complexity results in determining the value of \( h \). This value must be determined by testing, and the testing must be performed under conditions similar to the operating environment of the specimen. In general, when the convection coefficient is unknown, and the problem is reasonably complex, the problem degenerates into a fluid flow problem. Indeed, a primary complication in determining the convection coefficient is determining the thickness of the boundary layers at the surface of the specimen. For thermal problems there are two boundary layers at the surface of a body. The velocity boundary layer of the moving fluid and the thermal boundary layer of the temperature.

In summary, if the \( h \) coefficient is unknown, the problem must be solved using computational fluid dynamics (CFD) codes. However, if the convection coefficient is known, then the solution of Equation 7.6 becomes a simple algebraic manipulation.
The H2D subroutine treats the convection as a boundary condition, hence, the convection coefficient must be known.

7.4 Verification of H2D Heat Transfer Subroutine

Identical models were solved using ANSYS, a general purpose commercial FEA code and the DSC-SST2D FEA code. The parameters used in the analysis are listed in Tables 7.4.1 and 7.4.2. The parameters are summarized as follows. The density used is the mass density, and is mass divided by volume. Specific heat is a measure of the energy required to raise the temperature of the material, and how well it stores the heat. The higher the specific heat, the longer the material will hold the thermal energy. Hence, metals have comparatively low values of specific heat relative to wood or foams. Specific heat is given in energy per mass where Joules is the fundamental unit of energy. The conduction coefficient is a measure of the how easily thermal energy moves through a material. The conduction coefficient is given in watts/(length*temperature). Watts is a measure of power, and has the units of J/s (joules per second).

<table>
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<th>Table 7.4.1. Transient Analysis Parameters</th>
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<td>Parameter</td>
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<td>Density</td>
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<td>Specific Heat</td>
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<td>Conduction Coefficient</td>
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<td>Time Step Size:</td>
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<td>Initial Temperature</td>
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<tr>
<td>Convection</td>
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<td>Flux</td>
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Table 7.4.2. Steady State Analysis Parameters

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<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Specific Heat</td>
<td>896 J/kg</td>
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<td>Conduction Coefficient</td>
<td>204 W/m$^*K$</td>
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<tr>
<td>Time Step Size:</td>
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<tr>
<td>Initial Temperature</td>
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<tr>
<td>Convection$^1$</td>
<td>10 W/m$^2*K$, 20K</td>
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<tr>
<td>Convection$^2$</td>
<td>100 W/m$^2*K$, 20K</td>
</tr>
<tr>
<td>Flux</td>
<td>5000 W/m$^2$</td>
</tr>
</tbody>
</table>

$^1$ used in temperature with convection and flux analysis
$^2$ used in temperature with convection analysis

Table 7.2 Parameters used in steady state analyses.

A time step is necessary in transient analysis, and in general, as the time step size is decreased, the approximate finite element solution converges to the analytical solution. However, this also increases solution time. Because some of the transient analysis test cases used a total time of 5000s, a time step of .1 was considered reasonable. The convection parameter is listed as a pair of values. The first number is the amount of power transferred between the specimen and the surrounding fluid, and the second parameter is the ambient temperature of the surrounding fluid. The flux parameter represents the power transfer per area of the specimen. In terms of structural analysis, flux and convection can be thought of as surface loads or pressures, whereas applied temperatures can be thought of as point loads applied on nodes.

Figure 7.2 through Figure 7.13 document and verify the test cases comparing the DSC-SST2D FEM solutions and ANSYS solutions. Specifically, Figures 7.2-7.4 show the steady state results, and Figures 7.5-7.13 show the transient results. Table 7.3 shows the minimum and maximum temperatures, for each test case, found by ANSYS and the
DSC-SST2D. The mesh used in the solutions is shown in Figure 7.14, and the actual nodal values from the DSC-SST2D solutions are shown in Figure 7.15. It can be seen

<table>
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<tr>
<th>Test Case</th>
<th>DSC-SST2D</th>
<th>ANSYS</th>
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</thead>
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<td>0 K, 50 K</td>
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<td>27.19 K, 50 K</td>
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<td>50 K, 168.38 K</td>
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<td>Steady State Flux and Convection</td>
<td>50K, 147.59K</td>
<td>50 K, 147.57 K</td>
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<td>Transient Temp (1000s)</td>
<td>297K, 325K</td>
<td>297 K, 325 K</td>
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<td>298K, 423K</td>
<td>298.6 K, 423</td>
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<td>300K, 432K</td>
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<tr>
<td>Transient Flux and Conv (5000s)</td>
<td>308K, 432K</td>
<td>308.93K, 432 K</td>
</tr>
</tbody>
</table>

that the DSC-SST2D solutions compare well with the ANSYS solutions, matching six of the eight cases within .02K. The other two cases (transient convection and transient flux and convection) are within .93K of the ANSYS solution at a time of 5000 seconds. These errors are most probably due to numerical calculation differences/round off errors in the solutions. However, a maximum difference of .93K between two numerical codes, at a time of 5000 seconds, using a time step of .1s is very reasonable.

7.5 Conclusion

The heat transfer subroutine H2D has been successfully written into the DSC-SST2D FEM general finite element code, documented and verified against ANSYS solutions. Besides the insertion of the H2D subroutine into the DSC-SST2D code, general upgrades of the H2D subroutine include the dynamic allocation of arrays, removal of common blocks, capability of including thickness, addition of cyclic thermal
load capability and upgrade to FORTRAN90. Comparison between the ANSYS and DSC-SST2D solutions show an extremely good correlation.
Figure 7.1 For a steady state problem, the system is in equilibrium. The change in flux is zero.
Figure 7.2 Steady state: Applied temperature. (A) ANSYS, (B) DSC-SST2D.
Figure 7.3 Steady state: Convection and temperature. (A) ANSYS, (B) DSC-SST2D
Figure 7.4 Steady state: Flux and temperature. (A) ANSYS, (B) DSC-SST2D.
Figure 7.5 Steady state: Temperature with flux and convection. (A) ANSYS, (B) DSC-SST2D.
Figure 7.6 Transient: Temperature. Time = 100s. (A) ANSYS, (B) DSC-SST2D.
Figure 7.7 Transient: Temperature. Time = 1000s. (A) ANSYS, (B) DSC-SST2D.
Figure 7.8 Transient: Temperature and convection. Time = 1000s. (A) ANSYS, (B) DSC-SST2D.
Figure 7.9 Transient: Temperature and convection. Time = 5000s. (A) ANSYS, (B) DSC-SST2D.
Figure 7.10 Transient temperature with flux. Time = 1000s. (A) ANSYS, (B) DSC-SST2D.
Figure 7.11 Transient: Temperature with flux. Time = 5000s. (A) ANSYS, (B) DSC-SST2D.
Figure 7.12 Transient. Temperature with convection and flux. Time = 2000s. (A) ANSYS, (B) DSC-SST2D.
Figure 7.13 Transient: Temperature with convection and flux. Time = 5000s. (A) ANSYS, (B) DSC-SST2D.
Figure 7.14 Mesh Used In FEA Verification Problems
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<th>Steady State</th>
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Figure 7.1S Nodal temperature output from heat transfer subroutine
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3.09E+02

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2.67E+01
2.67E+01
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2.00E+01
2.00E+01
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2.00E+01
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Figure 7.15 - Continued. Nodal temperature output from H2D Subroutine


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Figure 7.15 - Continued. Nodal temperature output from H2D Subroutine
Table 8.1.3 Plastic (HiSS Model) Material Parameters [22, 28, 133]

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<th>β</th>
<th>m</th>
<th>n</th>
<th>a₁</th>
<th>η₁</th>
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Table 8.1.4 Viscoplastic Parameters

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<th>Fluidity Exponent</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>

* From discussion with Dr. C. S. Desai

Table 8.1.5 Disturbance Material Parameters

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<th>Dₙ</th>
<th>A</th>
<th>Z</th>
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</thead>
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</tbody>
</table>

* From discussion with Dr. C. S. Desai

Table 8.1.6 Temperature Dependent Property Exponents [133]

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<th>E</th>
<th>ν</th>
<th>CTE</th>
<th>A</th>
<th>Z</th>
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<td>1.9</td>
<td>2.45</td>
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8.2 Material Parameters - Discussion

Table 8.1.1 lists the heat transfer material parameters, and have been taken from sources as noted. The values for the density and specific heat of the BT substrate could not be found, and were estimated based on the values for FR-4. These values should be reasonably close. For an isothermal case, these are not needed. For a steady state thermal analysis, only the conductivity \( (K_i) \) is required. For a transient analysis, the density \( (\rho) \) and specific heat \( (C) \) are required. The power listed is taken from [47], and is based on a junction temperature at 100°C in non-forced ambient air flow. However, as will be outlined in Chapter 12, this parameter will not be necessary. The convection coefficient is listed as a range, as given by [48] for typical package geometries. Note that the convection coefficient for the PWB (FR-4) is listed as a range beginning from zero. It has been shown that the convection from a FR-4 PWB is negligible for certain situations. This can result from a horizontal orientation of the PWB and/or the attachment of the PWB to a plate style PWB carrier. For instance, in this dissertation, the PWB is located on the bottom of the assembly. The electronic package is located on top, and the solder balls are in the middle. In this case, the natural convection of heat is severely limited from the bottom of the PWB, since buoyant forces rise. Neither the power or convection coefficients are used in the calibration (Chapter 9), as the tests were performed in thermal chambers without operating the package (no internal heating from the die). However, the convection parameter shown for the molding is used in Chapter 12 where a power cycling analysis is performed. It is important to emphasize that the convection coefficient is not a material parameter. It has only been listed for the molding
because it is applied on the perimeter of molding. The convection coefficient is a complex function relating surface characteristics, geometry and fluid velocity (air usually).

Table 8.1.2 lists the elastic parameters, and have been taken from sources as noted. The elastic modulus shown for eutectic solder is based on engineering judgment, noting the values reported in the literature, and the values obtained from testing of shear samples [22, 133].

Table 8.1.3 lists the plastic parameters for the HiSS model used in this research, as described in Chapter 5. The values shown represent the culmination of testing, and back-prediction performed elsewhere [22, 28, 133], and discussions with Dr. C. S. Desai. The meaning of the parameters was outlined in Chapter 5, and will not be discussed here. When performing a plastic analysis, only the solder is considered to be a plastic material, all other materials are assumed to behave elastically.

Table 8.1.4 lists the viscoplastic parameters, as described in Chapter 5. The values were taken from discussions with Dr. C. S. Desai, and back predictions performed by Russell Whitenack. The parameters are based on the Perzyna viscoplastic formulation. For a viscoplastic analysis, only the solder is considered to behave viscoplastically, all other materials are assumed to behave elastically.

Table 8.1.5 lists the disturbance (damage) parameters as described in Chapter 5. The values shown represent the culmination of testing and back-prediction performed elsewhere [22, 133]. Only the solder is assumed to accumulate damage. All other materials are assumed to remain in an undamaged state.
Table 8.1.6 lists the temperature dependent material properties. Only the solder was considered temperature dependent. The exponents listed are used in the following equation:

\[ \text{parameter}_{(\tau)} = \text{parameter}_{(300)} \left( \frac{T}{300} \right)^{\text{exp.}} \]  

(8.1)

For example, the elastic modulus at any temperature is calculated as:

\[ E_{(\tau)} = E_{(300)} \left( \frac{T}{300} \right)^{-29} \]  

(8.2)
CHAPTER 9
ACCELERATED ANALYSIS VERIFICATION

9.1 Introduction

The calibration of the model was separated into a progression of phases in order to insure completeness and accuracy. The actual calibration of the model will be performed in Chapter 10. This chapter documents the first phase, which was to verify the accuracy of the accelerated analysis method. Note that in all subsequent chapters, the accelerated analysis method will be used for all the modeling. A macro-micro or submodeling technique was used to verify the accelerated analysis method. This procedure was adopted so that a full cycle analysis could be performed on the micro model and compared with the results of the accelerated analysis performed on the micro model.

This chapter uses the material parameters outlined in Chapter 8, with the following exceptions:

<table>
<thead>
<tr>
<th>Table 9.1.1: Parameters Different From Chapter 8</th>
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<tbody>
<tr>
<td>Parameter</td>
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<tr>
<td>A (disturbance parameter)</td>
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<tr>
<td>Z (disturbance parameter)</td>
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<tr>
<td>CTE_{silicon}</td>
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</tbody>
</table>

The differing material parameters (compared to the ones used in the remaining chapters) will have no effect on the benefit of this chapter, as the only purpose here is to verify the accelerated analysis method. All further chapters use the parameters as outlined in
Chapter 8. The test vehicle used in the JPL and Raytheon experiment is shown in Chapter 10 as Figure 10.1.

The accelerated analysis is based on work developed by Desai and Whitnack [32]. It is based on the result that after a certain number of cycles \( N_r \), a reference cycle, the relationship between deviatoric plastic strain and number of cycles becomes linear in log-log space (Figure 9.1). The number of cycles \( N_r \) where the linear relation begins can be found by plotting \( \ln(\varepsilon_D) \) versus number of cycles. Because the relation is linear after \( N_r \), the plastic deviatoric strain can be determined for any cycle, and the disturbance can be calculated based on the calculated deviatoric plastic strain using Equation 5.61. The benefit of the accelerated analysis is that it requires only a fraction of the time compared to a full cycle analysis. As a general rule of thumb, based on previous work performed elsewhere [32], the prediction analysis is very accurate if the number of reference cycles are approximately \( 1/400 \) the number of cycles expected for failure. For example, in a thermal analysis where the expected cycles to failure might be 4000 cycles, then 10 cycles in the accelerated analysis will provide accurate results. For vibration, if the expected cycles to failure is 2,000,000, then 5,000 cycles in the accelerated analysis will provide accurate results. This is important for large mesh sizes, reducing the time of the analysis. This chapter shows that the accelerated analysis provides results which are comparable to the results from a full analysis, and indicates that the accelerated analysis may be used with confidence.

It is important to reiterate that this chapter is used only to verify the accelerated analysis procedure. Although actual thermal-cycle test profiles from JPL and Raytheon
experiments were used, the results from this chapter were not used for calibration purposes. The calibration of a separate, very large degree of freedom (DOF) model, was performed in Chapter 10.

The verification of the accelerated analysis employed the test profiles of a JPL and a modified Raytheon thermal cycle. Both the JPL and unmodified Raytheon cycle were used in the experimental thermal cycling of 313 ball depopulated (staggered pin) PBGA packages [111, 42]. The test protocols were designed for military and aerospace applications, and are summarized in Table 9.1.1, and Figure 9.2. Note that Figure 9.2 shows the modified Raytheon cycle. For the modified Raytheon cycle, the cycle time has been reduced to 92 minutes, whereas the original experimental cycle was 168 minutes as shown in Table 9.1.2. This was done to shorten the run times. However, because no calibration of the model was done using the Raytheon cycle, shortening the cycle is inconsequential. The only information necessary (from the results) is that the accelerated analyses match the full cycle analyses. From here on, the modified Raytheon cycle will be denoted as Raytheon* so that no confusion can be made.

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<th>Table 9.1.2 Testing Protocol [42, 111]</th>
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9.2 Macro-Micro Mesh Technique

For the submodeling technique, two models were created. The first was a macro model which is comprised of a coarsely meshed 2D slice of the 313 ball PBGA shown in Figure 9.1. The slice was taken diagonally from the center of the package through the furthest DNP solder ball (Figure 9.1). The second model was a micro model, and is comprised of a finely meshed single solder ball. The two models together constitute the so-called macro-micro or submodeling meshes.

A common analysis technique found in the literature is the well documented submodeling technique (macro-micro FEA technique), where a macro FEA model is created of the package with a coarse mesh, and displacements from the first cycle are obtained from the worst case solder ball (Figure 9.3a). The displacements are then applied to a micro-model of the solder ball (Figure 9.3b), and cycled until failure. As noted previously, this technique was adopted only to determine the efficacy of the accelerated analysis procedure. For the actual calibration, a large mesh of the package was created, with finer meshes at the solder balls (Chapter 10). The material parameters used for the verification of the accelerated analysis method were shown in Chapter 8, except as noted in Table 9.1.1.

For verification of the accelerated analysis method, both the JPL and modified Raytheon test protocols and two material cases were investigated, one viscoplastic, and one plastic. This gives a total of four analyses: Raytheon* cycle, plastic case, Raytheon* cycle viscoplastic case, JPL cycle plastic case, and JPL cycle viscoplastic case. All cases were isothermal. However, all models included temperature dependence of the elastic
were then applied to the micro model shown in Figure 9.3. The displacements were applied at the top of the solder joint, assuming the base as fixed. An example of the applied displacement profile is shown in Figure 9.5b for the JPL cycle. An accelerated analysis was then performed using the micro model for both plastic and viscoplastic cases using both the JPL and Raytheon* temperature cycles and derived displacements (giving four analyses). The disturbance values from the full analyses (performed on the micro model) were then compared with the disturbance values from the accelerated analyses, based on a 20 cycle reference. That is, running the accelerated analysis for 20 cycles, and predicting what the disturbance will be at any given cycle.

Note that the displacements were applied to the micro model in conjunction with the corresponding temperature cycle. The temperature cycle was applied because the elastic and disturbance solder parameters were assumed to be temperature dependent. The displacements were applied so that they corresponded to the temperature profile, as shown in Figure 9.5b. Note that in applying the temperature cycle, the solder CTE (coefficient of thermal expansion) was set to zero. This was necessary because the effect of the CTE was already taken into account by the displacements found from the macro model. Examining Figures 9.4b and 9.5a, it can be seen that there is a significant Y displacement. This is caused by the silicon die which has a much lower CTE, and results in "bowing" of the package. This result indicates that a realistic stress and fatigue life prediction analysis should include the silicon die in the FEA mesh. If the die is not included, the Y component of displacement will be reduced, possibly leading to erroneous results.
9.4 Prediction Verification - Modified Raytheon Thermal Cycle

The results for the full cycle Raytheon* plastic case are now described. For this chapter, a $D_e$ of 0.7 was chosen, however, analysts may chose other values. Examining Figure 9.6, after 1,785 cycles, 12% of the solder ball has reached a disturbance of 0.7 or greater. The maximum values of disturbance are found at the corners of the solder ball, locations typically found to be the areas first to fracture in experimental testing. The progression of disturbance is from the two diagonal corners (top right and bottom left) towards the center of the solder ball. This may be due to the boundary conditions imposed on the macro mesh. It is more likely that the package should be free to move in the Y direction at the center of the package (Figure 9.3). However, the primary purpose of this chapter is to verify the accuracy of the accelerated analysis, and not to calibrate the model. Therefore, the error in boundary conditions is not important here, and is corrected in Chapter 10, where the calibration is performed.

The full cycle viscoplastic results for the Raytheon* cycle are shown Figure 9.7. This result, again shows that damage to the solder joint is accumulated along the diagonal of the solder joint. Because this behavior was also shown in the plastic case, it can be assumed that this is caused by an incorrect boundary condition. Examining Figure 9.7, it can be seen that 12% of the solder ball has reached a disturbance of 0.7 or greater at cycle 1,892. Comparing the levels of disturbance between the plastic and viscoplastic full cycle analyses, it can be seen that the viscoplastic case predicts failure at higher cycles than the plastic case. Based on a 12% disturbance density (12% of the solder area
achieving a disturbance level > 0.7), the viscoplastic case predicts failure at 1,892 cycles, whereas the plastic case predicts failure at 1,785 cycles.

The results for the plastic and viscoplastic accelerated analyses are shown in Figure 9.8 and Figure 9.9 respectively. Comparing the results from the accelerated analysis (Figures 9.8, 9.9) with the full cycle analysis (Figures 9.6, 9.7), it can be seen that the accelerated analysis method provides very good results. At 1,785 cycles the accelerated plastic case shows a disturbance density of 10%, and at 1,892 cycles the viscoplastic case shows a disturbance density of 10%. There are slight differences comparing elemental disturbances, but the overall disturbance trends are very similar. A comparison of the accelerated and full cycle results are shown in Table 9.4.1.

<table>
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<tr>
<th>Table 9.4.1 Comparison of Disturbance Density (Raytheon* Cycle)</th>
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<td>Plastic Case (1,785 cycles)</td>
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<tr>
<td>Viscoplastic Case (1,892 cycles)</td>
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<tr>
<td>Full cycle</td>
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</table>

9.5 Prediction Verification - JPL Thermal Cycle

The results for the full cycle plastic analysis (JPL cycle) are now described. Examining Figure 9.10, it can be seen that at cycle 1,800, 28% of the solder ball has a disturbance of .7 or greater. A value of 0.7 indicates well developed fatigue fractures. This level of disturbance is greater than the plastic case using the Raytheon* cycle, and indicates that the JPL cycle is more harsh than the Raytheon* cycle.
The results for the full cycle viscoplastic analysis (JPL cycle) are now described. Examining Figure 9.11, after cycle 1,900, it can be seen that 22% of the solder ball has reached a disturbance of 0.7 or greater. In contrast to the plastic case, the values of disturbance are more uniform, with the maximum values of disturbance slightly away from the corners. This is similar to the experimental results where fatigue fractures initiated at the corners of the solder balls, slightly away from the interface, and progressed towards the center of the solder ball parallel to the interface between the solder ball and package/PWB. The viscoplastic disturbance density for the JPL cycle is higher than the disturbance density for the Raytheon* cycle, again indicating that the JPL cycle is more damaging than the Raytheon* cycle. This was also found for the plastic case as mentioned previously.

<table>
<thead>
<tr>
<th>Table 9.5.1 Comparison of Disturbance Density (JPL Cycle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic Case (1,800 cycles)</td>
</tr>
<tr>
<td>Full cycle</td>
</tr>
<tr>
<td>28%</td>
</tr>
</tbody>
</table>

The accelerated analysis results for the JPL cycle plastic and viscoplastic cases are shown in Figure 9.12. Comparing the full cycle plastic results (Figure 9.10) with the plastic accelerated analysis results (Figure 9.12a) it can be seen that the general trend in disturbance is almost identical. There is a slight difference in the elemental disturbances, as the full cycle analysis predicts a disturbance density of 28%, whereas the accelerated analysis predicts a value of 26%. Comparing the full cycle viscoplastic results with the
accelerated analysis results, it can be seen that the disturbance distribution, again is almost identical. Comparing the elemental disturbance, the results are identical. The full cycle analysis predicts a disturbance density of 22%, and the accelerated analysis a value of 22%. This result shows that the accelerated analysis is nearly 100% accurate predicting the response for a viscoplastic analysis. This is probably due to the more "regular" behavior of damage accumulation in the viscoplastic case as compared to the plastic damage accumulation. The differences in damage accumulation can be seen in Figure 9.13, and are described below.

Figure 9.13 shows plots of the disturbance accumulation for the plastic (top) and viscoplastic (bottom) analyses for the JPL cycle. A full cycle analysis using 2,000 cycles was performed using the coarse mesh macro model shown in Figure 9.3-top. The results for the JPL cycle are shown, because a copious amount of failure data for this cycle are available to make some general comments. The disturbance is taken from element 100, which is at the top, right corner of solder ball 6 (under die edge). Examining Figure 9.13 (top), it can be seen that the trend in plastic disturbance accumulation is non-linear, displaying a nearly bimodal shape. The trend in disturbance follows a steep slope from cycle 1 to cycle 500, and a reduced slope from cycle 500 to cycle 2,000. This indicates that the plastic formulation predicts rapid damage accumulation in the early cycles, and by cycle 500, most of the damage has been created. Based on experimental observations for the JPL cycle, this was not verified. The JPL results indicated that first failures generally occurred around cycle 2,000 (recall that in all of the analyses, the plastic formulation results showed a higher disturbance density). While the purpose of this
chapter is only to verify the accuracy of the accelerated analysis method, an observation should be made regarding this result. This finding indicates that it is unlikely a plastic analysis will be as accurate as a viscoplastic analysis when describing solder thermal fatigue, as solder is known to behave in a viscoplastic manner, showing both creep and stress relaxation phenomenon. This result has also been found elsewhere, where the authors state that "Time independent plasticity leads to an early saturation of the irreversibly accumulated strains...it is not appropriate to assess the lifetime behavior [2]." This is easily explained by noting that time independent plasticity is equivalent to viscoplasticity at steady state. This means that the accumulated plastic strains will be less in a viscoplastic model than in a plastic model. This is because the yield surface in a viscoplastic model grows with time towards the inviscid plastic yield surface. Whereas the plastic yield surface immediately achieves the final surface. Thus, the accumulated plastic strains, over time, will be less in a viscoplastic model compared to a plastic model. Examining Figure 9.13 (bottom), it can be seen that the disturbance accumulation for a viscoplastic analysis is significantly different than that for the plastic analysis. The viscoplastic analysis shows that the average disturbance accumulation is a relatively steady increase, which is verified experimentally based on fatigue crack growth. Noting that disturbance tracks microcrack growth, it should be expected that the rate of increase in disturbance will be steady, rather than that predicted by the plastic analysis. This is documented in SEM photos of solder balls, where the fatigue cracks start at the corners of the solder ball and grow at a relatively steady rate towards the center. The somewhat "steady" microcrack growth and damage found experimentally in solder balls, might be
attributed to the blunting of microcrack tips, due to the extreme ductility of solder. By contrast, for brittle materials, the onset of microcrack growth is followed by a rapid increase in damage accumulation due to the high stress concentrations at the microcrack tips. The blunting of microcrack tips may help explain why voids in solder balls do not lead to a drastic reduction in fatigue life.

9.6 Summary

The accelerated analysis method provides an accurate method of predicting disturbance. It reduces the number of cycles required to predict the number of cycles to failure. Instead of performing an analysis for thousands of cycles, which may take days, weeks or months for large DOF meshes, only a fraction of the cycles are required, which reduces the analysis times significantly. Because the accelerated analysis method was found to provide good results, the accelerated analysis method will be adopted in all subsequent chapters.
Figure 9.1. Accelerated analysis relation of deviatoric plastic strains and cycles.
Figure 9.2. Graphs of prescribed temperature cycles. (A) Raytheon* Cycle. (B) JPL Cycle.
Figure 9.3: Meshes used to verify accuracy of prediction method. (A) Coarsely meshed macro-mesh. (B) Micro-mesh used to determine disturbance.
Figure 9.4. Graphs of temperature and displacement. (A) Temperature profile applied to macro-mesh in Figure 9.2. (B) Relative displacements found at largest DNP solder ball (Raytheon* Cycle).
Figure 9.5. Graphs of relative displacements. (A) Relative displacements found at solder ball under die edge (JPL cycle). (B) Displacements applied to micro-mesh shown in Figure 9.2 for JPL cycle, viscoplastic analysis.
Figure 9.6. Contour plots: Full cycle plastic analysis, Raytheon* Cycle, largest DNP ball, at cycle 1,785. (A) Disturbance distribution. (B) Area above disturbance of 0.7.
Figure 9.7. Contour plots: Full cycle viscoplastic analysis, Raytheon* Cycle, largest DNP ball, cycle 1,892. (A) Disturbance distribution. (B) Area above disturbance of 0.7.
Figure 9.8. Contour plots for accelerated analysis Raytheon® cycle, plastic case. (A) Disturbance distribution. (B) Disturbance above 0.7.
Figure 9.9. Contour plots for accelerated analysis, Raytheon* cycle, viscoplastic case. (A) Disturbance distribution. (B) Disturbance above 0.7.
Figure 9.10. Contour plots: Full cycle plastic analysis, JPL cycle, ball under die edge, cycle 1,800. (A) Disturbance distribution. (B) Area above disturbance of 0.7.
Figure 9.11. Contour plots for full cycle viscoplastic analysis, JPL cycle, ball under die edge, cycle 1,900. (A) Disturbance Distribution. (B) Area above disturbance of 0.7.
Figure 9.12. Contour plots for accelerated analyses, JPL cycle. (A) Plastic disturbance at 1,800 cycles. (B) Viscoplastic disturbance at 1,900 cycles.
Figure 9.13: Graphs of disturbance plotted for every 100 cycles. Under die edge solder ball, element 100, JPL cycle. (A) Plastic analysis. (B) Viscoplastic Analysis.
CHAPTER 10
CALIBRATION OF 313 BALL PBGA FEA MODEL

10.1 Introduction

The calibration of the model was separated into two phases, in order to insure completeness and accuracy of the calibration. The two phases were:

I. Verification of accelerated analysis method using sub-modeling technique (Chapter 9).
II. Calibration of large DOF, 2D slice mesh of 313 PBGA electronic package using accelerated analysis method.

The calibration of the large DOF, 2D slice model was based on the results of JPL experimental thermal cycling of 313 ball depopulated (staggered pin) PBGA packages [44]. The JPL test vehicle is shown in Figure 10.1. The test protocol was designed for military and aerospace applications, and is summarized in Table 10.1.1, and Figure 10.2a. Table 10.1.1 also shows a Raytheon temperature cycle which was used in verifying the accuracy of the accelerated analysis method. The JPL cycle was chosen to calibrate the large mesh, 2D slice model because there is an abundance of data regarding this experiment, including the complete test setup, solder balls which failed, determination of what was considered failure, which balls failed first, and SEM photos of the failed balls. However, the Raytheon cycle is a more typical test profile, and was used in conjunction with the JPL cycle to verify the accelerated analysis. Although the JPL temperature cycle rise and fall ramp rates are very high (12°C/min), it is not a temperature shock. A
temperature shock is considered to be a temperature rise of greater than 30°C/min [108].

The Raytheon temperature cycle has a more modest ramp rate of 5°C/min. The JPL cycle has a cycle time of 68 minutes, and the Raytheon cycle has a cycle time of 160 minutes. Both cycles have a temperature swing of 180°C. A total of nine 313-PBGA packages were tested for the JPL cycle, and six for the Raytheon cycle. The test vehicle is shown in Figure 10.1. Both the Raytheon and JPL test vehicles were identical. The cumulative distribution of the package failures, for the JPL cycle, is shown in Figure 10.2b. Examining Figure 10.2, it can be seen that packages one through eight failed in the cycle range of 1,650 to 2,015 cycles. Package nine failed at 2,400 cycles, and may be an anomaly since it failed at a cycle well above the other eight packages. However, it is more likely that packages seven and eight failed "prematurely". This conclusion is arrived at based on the Raytheon cycle failures (Figure 10.3a) which show a linear trend in failures, and that a reasonably linear trend can be extrapolated in the JPL failures if packages seven and eight are neglected (Figure 10.3b).

<table>
<thead>
<tr>
<th>Table 10.1.1 Testing Protocol</th>
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<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>Maximum T°(C)</td>
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<tr>
<td>Minimum T°(C)</td>
</tr>
<tr>
<td>Cycle Time</td>
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<tr>
<td>Total ΔT°</td>
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</tbody>
</table>
During the JPL experiments, failure of the packages was based on the electrical resistivity of daisy chained solder ball loops. The loops were arranged such that failures could be identified in the balls under the die, and successive loops around the die. It should be noted that the failure of a package was assumed to occur when a daisy chain electrical signal was interrupted a set number of times.

The calibration analysis used a 8,034 DOF mesh (Figure 10.4). The solder balls were more finely meshed than the surrounding materials, as the main purpose of the calibration was to determine the fatigue life of the second level package balls. The elastic and disturbance solder parameters were considered temperature dependent, however, only the solder balls were considered viscoplastic materials. All other materials were modeled as elastic with no temperature dependence. The solder balls were the only material modeled with disturbance parameters. The analysis was performed as plane stress, with a thickness of 0.57mm. This thickness was arrived at such that the solder balls would have the same volume as their 3D counterpart. The boundary conditions were prescribed in accordance with the test conditions, as closely as possible. Examining Figure 10.1, it can be seen that the 313 PBGA is located in the bottom right corner of the test vehicle, approximately 7mm from the PWB mounting screw. Thus, this boundary condition was prescribed in the mesh as a fixity in Y, but free in X. This is a standard fixity condition for screws, as there is slop in the PWB hole/screw interface, allowing X translation. The centerline of the package was assumed to be free in the Y direction, but fixed in the X direction (Figure 10.4). This is a standard fixity condition, and allows the structure to bow or flex about the centerline.
The FEA was performed using a sequential analysis formulation, which has been suggested as a more reliable method of thermomechanical analysis [93, 136]. This procedure has been implemented into the DSC-SST2D FEM code, and is transparent to the user. At each step during a thermomechanical analysis, the temperature variation in the structure is calculated, and then the stress and strain resulting from that temperature change is calculated. This process is repeated for each step in the analysis. For example, at time step one, a temperature load is applied to the mesh. The heat transfer within the mesh is then calculated such that each node element may have a different temperature. The change in temperature at the nodes is then calculated, and using the CTE of the materials, a strain is calculated. Using the strain, the stress and resulting damage can then be determined. This process is repeated at each load/time step.

The calibration analysis was performed using an isothermal assumption, which is typical for modeling temperature cycling or TCT tests [126, 132]. To test this common approach, a transient analysis was performed, and showed that during temperature cycling, the entire package achieved the varying ambient temperature (chamber temperature) in approximately 24 seconds (Figure 10.12). This rapid response is primarily a result of the very small dimensions of the package, as the entire thickness (PWB bottom edge to molding top edge) is only 3.85mm (Figure 10.4). The sides of the model, normal to the slice (Z direction), are considered adiabatic, and is a standard assumption [108]. Based on this result, the isothermal approximation appears reasonable for TCT analysis.
10.2 Calibration Procedure

Calibration of the model is done by simply modeling the JPL experiment, and determining the value of disturbance at each experimental failure. It is a method of correlating the model to the experiment. The calibration procedure employed a 2D slice of the full package (Figure 10.4), with refined meshes at the solder balls. The slice was taken diagonally from the center of the package through the furthest DNP solder ball (Figure 10.1). This is a standard method when using a plane 2D analysis, since it incorporates the solder balls under the die, and the furthest DNP solder ball. Because the DOF of the mesh was relatively large, the accelerated analysis method, documented in Chapter 9, was used instead of running a full cycle analysis. Using the accelerated analysis method, the 20th cycle was used as the reference cycle.

A common analysis technique found in the literature is the well documented macro-micro FEM technique or submodeling technique. In this method a macro FEM model is created of the package with a coarse mesh, and displacements from the first cycle are obtained from the worst case solder ball. The displacements are then applied to a micro-model of the solder ball, and cycled until failure. As noted previously, this technique was adopted only to determine the efficacy of the accelerated analysis procedure. For the calibration performed in this chapter, a large DOF mesh of the package was created, with finer meshes at the solder balls. This technique has several advantages compared to the submodeling technique [108]. The most important of these is that the plastic behavior of the solder balls is retained throughout the loading. This means that the solder balls, as they deform plastically, are never forced to return to an
original starting position. This is not the case for micro models with applied
displacements. For applied displacement loadings, the displacement is prescribed, which
forces the solder ball to deform in a prescribed manner. This is not correct, and may lead
to erroneous results. For example, during operation, a solder ball may develop plastic
displacements such that it deforms with a permanent X displacement. However, if a
displacement cycle is applied to the solder ball, the ball will always be cycled through the
original starting position, and the plastic displacements are never retained. In addition,
researchers [50] have shown that the submodeling approach is likely to overestimate the
fatigue life. The material parameters used for the calibration of were shown in Tables
8.3.1 - 8.3.6 of Chapter 8.

For the calibration, all cases were isothermal. However, all models included
temperature dependence of the elastic and disturbance properties of the solder material.
In general, it is uncommon to include heat transfer when performing stress or fatigue
analysis, even when the stress is thermomechanical. It is not completely understood how
this affects fatigue calculations, and no clear guidelines as to when heat transfer should
be taken into account have been found. However, some researchers suggest that TCT
analyses can be done isothermally, but PCT (power cycling testing) analyses should be
done with a heat transfer analysis [108, 132, 2]. Because the temperature distribution in a
body is dependent upon many factors (applied temperature rise/reduction and hold time,
specific heat of material, ambient temperature, etc.) it is not well established when an
isothermal model is acceptable, especially if sources and sinks are present. For example,
when slow temperature rises and reductions are present, the material has time to achieve
a relatively uniform temperature distribution. However, for fast temperature swings, a similar situation may occur away from the point of temperature application. In other words, if the temperature swings are rapid enough, the applied temperature (away from the point of application) becomes an average of the temperature swings, and the body may achieve a quasi-isothermal distribution as if a constant temperature was being applied. Thus, the problem of temperature cycling is complex, and guidelines may not be possible, in a general sense, and may need to be evaluated on a case by case basis. However, when analyzing TCT (packages in thermal chambers), where there are no clear sources and sinks, an isothermal model is probably reasonable for FEA analysis. This was shown using a transient analysis, where the temperature along the centerline of the solder ball reached ambient in approximately 24 seconds in Chapter 9.

Because the DOF for the mesh used in the calibration procedure was relatively large (8,034 degrees of freedom), it was not convenient to run the analysis for the required number of cycles. Having reduced the bandwidth of the model to approximately 300, the model still required nearly 10 days to complete 2000 cycles. Therefore, the accelerated analysis method for determining the disturbance at any number of cycles was adopted. Using the accelerated analysis method, only 20 cycles were used to predict the disturbance, and calibrate the model.

10.3 Full Package (2D Slice) Calibration Results

The accelerated analysis method was used to calibrate the full package, by determining the disturbance within the solder balls at various cycles. The mesh used for the calibration is shown in Figure 10.4 and was subjected to the JPL cycle. The results
are shown in Figure 10.5, and Figures 10.6 - 10.11. Figure 10.5 shows a plot of disturbance density \( D_{\text{density}} \), which is calculated as:

\[
D_{\text{density}} = \frac{\left( \sum_{\text{Gauss Points}} D_{\text{GP}} \right) > 0.8}{4 \cdot \text{Ball Area}}
\]  

Where \( D_{\text{GP}} \) is the disturbance value at a gauss point (four per element), and 0.8 is the critical disturbance in the solder ball.

Figures 10.7 - 10.9 show disturbance results for the viscoplastic case. Figure 10.7 shows the disturbance in the solder balls at cycle 1,000. Examining Figure 10.7, it can be seen that the solder balls under the die (balls 1-6) accumulate more damage than the solder balls away from the die. Figure 10.8 shows disturbance in the solder balls at 1,600 cycles and 2,000 cycles. Figure 10.9 shows disturbance in the solder balls at cycle 2,400. Comparing cycle 1,000 (Figure 10.7) with cycle 2,400 (Figure 10.9), the accumulation of damage with cycling is apparent. Examining the progression of cycles, it can be seen that the disturbance is accumulated primarily along the interface between the solder ball and PWB/substrate. This has been verified by experiment.

Figures 10.10 and 10.11 show disturbance results for the plastic case. Figure 10.10 shows the disturbance in the solder balls for cycles 1,000 and 1,600, and Figure 10.11 shows the disturbance in the solder balls for cycles 2,000 and 2,400. Examining Figure 10.10, it can be seen that the plastic case also predicts disturbance to initially accumulate along the interface between the solder ball and PWB/substrate. However, comparing the plastic and viscoplastic results for later cycles, it can be seen that the plastic case predicts significantly more damage than the viscoplastic case. Specifically,
Examining the plastic disturbance results in Figure 10.6, it can be seen that the solder balls are grouped according to whether they are under the die or die edge (balls 1-6) or away from the die (balls 7-13). The disturbance accumulation is not linear for the balls under the die. There is a reduction in disturbance accumulation from approximately cycle 1,400 to cycle 1,800. The disturbance accumulation then increases again up to cycle 2,400. Examining the disturbance values for the plastic case, it can be seen that they are very high. For example, balls 1-5 are predicted to accumulate nearly 55% damage ($D > .8$) at cycle 2,400. This would correspond to nearly 55% of the solder ball volume as being heavily fractured. This was not verified during the JPL experiment. Of the solder balls that failed, each failed by a single fracture at the top and or bottom of the solder ball interface. Thus, the plastic case is predicted to over-predict damage accumulation.

Because the location of failures (known solder balls to fail) and location of fatigue cracks within the solder balls (cracks at corners) have been established by experiment, the determination of what constitutes a failed solder ball (by analysis) has been correlated to experiment, based on the following information:

1.) Fatigue fractures are known to occur at the corners of solder balls.
2.) The cracks initiate from the boundary of the solder joint and PWB/package and migrate horizontally, along the interface towards the center of the solder ball.
3.) During the JPL experiment, the failure of a solder joint was determined by electrical resistance. Therefore, a solder joint will not necessarily be completely fractured to cause an elevated resistance.
4.) The first package failure was observed at approximately 1,650 cycles, and the final package failure was observed at approximately 2,420 cycles.

5.) Plastic material behavior over-predicts damage accumulation.

Table 10.3.1 shows the viscoplastic disturbance density in the solder balls, and is plotted in Figure 10.5a. The disturbance density is calculated as shown in Equation 10.1.

Based on the above information, the distribution of viscoplastic disturbance in Figures 10.7 - 10.9, and the disturbance densities in Table 10.3.1, the following criteria for a failed solder ball are stated. A solder ball is considered to have a 100% chance of being measured as an open (cycle 2,400) when the disturbance density in the solder balls reaches 17%. At approximately 2,400 cycles, it was shown experimentally that all packages had failed. Therefore, the disturbance density at 2,400 cycles represents the maximum disturbance a solder ball can accumulate before being measured as an open. Note that the disturbance distribution in the solder balls (Figures 10.7-10.9) matches very well with Figure 10.13 and Figure 10.14, which show fatigue cracks in a solder ball from experimental thermal cycling.

When the disturbance density in a solder ball reaches 11%, there is a 1/9 or 11% chance (cycle 1,650) that the solder ball will be measured as an open (Figure 10.2). It is important to note that while a disturbance above .8 represents well formed fatigue fractures, microcracking is assumed to begin at a disturbance of .5. Therefore, the electrical conductivity of a solder ball is reduced at areas with a disturbance over .5. This indicates that there are two mechanisms by which the electrical conductivity of the solder ball is reduced. The first is the reduction in cross sectional area, the area which current
can flow, caused by gross fracturing of the solder ball ($D > .8$). The second is the reduction in conductivity by microfracturing ($0.5 > D < 0.7$). The reduction in cross-sectional area may be visualized as fracturing at the top and bottom of the solder joint. The reduction in conductivity by microfracturing is a porosity or Swiss-cheese effect. As the number of microfractures grow and increase in volume, the solder becomes less homogeneous, and the conductivity is reduced. However, because it is difficult to correlate the "Swiss-cheese" microfracturing with failure, it will be assumed that gross fracturing ($D > .8$) of the solder ball is the primary cause of failure. Thus, the disturbance density is only calculated based on areas with $D > .8$.

<table>
<thead>
<tr>
<th>Package Failure</th>
<th>Cycle Predicted</th>
<th>Disturbance</th>
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<tbody>
<tr>
<td>1</td>
<td>1,650</td>
<td>11%</td>
</tr>
<tr>
<td>2</td>
<td>1,680</td>
<td>11.75%</td>
</tr>
<tr>
<td>3</td>
<td>1,730</td>
<td>12.5%</td>
</tr>
<tr>
<td>4</td>
<td>1,860</td>
<td>13.25%</td>
</tr>
<tr>
<td>5</td>
<td>1,930</td>
<td>14%</td>
</tr>
<tr>
<td>6</td>
<td>1,990</td>
<td>14.75%</td>
</tr>
<tr>
<td>7</td>
<td>2,010</td>
<td>15.5%</td>
</tr>
<tr>
<td>8</td>
<td>2,015</td>
<td>16.25%</td>
</tr>
<tr>
<td>9</td>
<td>2,400</td>
<td>17%</td>
</tr>
</tbody>
</table>

Correlating the experimental results with the full package (large DOF mesh) analysis, it was found that when a solder ball has $D_{density} = 11\%$, there is an 11\% chance of the solder ball being measured as an open. When a solder ball has a $D_{density} = 17\%$, there is a 100\% chance that the solder ball will be measured as an open. Using these two values as the end points of a failure spectrum, it is now possible to determine when the analysis predicts the failures of the remaining seven packages. Assuming a linear
relationship between the disturbance density and failure (Table 10.3.1) a comparison can be plotted with the experimental results (Figure 10.5b). It can be seen in Figure 10.5, that the prediction is reasonably accurate, especially at cycles less than 2,200. The experimental results are skewed from the analysis results because three packages (six, seven and eight) failed around 2,000 cycles, and package nine failed at 2,400 cycles. It is unclear whether packages seven and eight failed prematurely or package nine was an anomaly, failing at a higher cycle than should be expected. However, examining the results for the Raytheon cycle (Figure 10.3a) it can be seen that the package failures follow a nearly linear failure progression, without any anomalous package failures. Based on the Raytheon result, it seems more likely that packages seven and eight, in the JPL experiment, failed prematurely. Thus, the prediction results appear satisfactory.

10.4 Summary

The micro-macro method of analyzing solder joints using prescribed displacements was not used. The prescribed displacements force the solder joint to deform in a manner inconsistent with the plastic deformations of the package-solder ball-PWB system. This may cause erroneous conclusions to be drawn regarding failure prediction in both viscoplastic and plastic cases. The plastic material behavior was found to over-predict the amount of disturbance in the solder joints, compared to the viscoplastic case (see also Chapter 9). This result is consistent with other research, as it is well documented that solder is a viscoplastic material.

The correlation of the experimental and FEA analysis results was found by deriving a disturbance density, using a critical disturbance of $D_C = .8$. The disturbance
density at first package failure and final package failure was used to predict the failure of the intermediate packages based on a linear progression of disturbance density. This method is consistent with other damage prediction research, and the results are reasonable compared with the experimental results. The disturbance density at first package failure (1,650 cycles) was found to be 11%, and at last package failure (2,400 cycles) 17%. Thus, the disturbance density correlation indicates that a value of 11% is the critical disturbance density, where packages will begin failing, and 17% the ultimate disturbance density, where all packages will have failed. The main points of this chapter are stated as follows:

1.) The macro-micro mesh technique does not capture the effect of the package system, and may not be as reliable as a macro mesh (with finer meshes at the critical solder balls) in predicting fatigue failures.

2.) The plastic material case over-predicts the amount of damage accumulated in the solder joints, and under predicts the cycles to failure.

4.) The disturbance density (Equation 10.1) of the solder joint provides a satisfactory means of predicting cycles to failure.

5.) The critical disturbance density is 11%, and the ultimate disturbance density is 17%.
Figure 10.1. Pictures of JPL test materials. Top: Depopulated PBGA. Bottom: Test Vehicle Board and 313 PBGA [111].
Figure 10.2. Graphs of JPL test data. (A) Testing cycle, (B) Failure distributions [45]
Figure 10.3. Graphs of failure data. Top: Experimental failure results for Raytheon cycle [111]. Bottom: JPL cycle failures, neglecting packages 7 and 8, with linear trend.
Figure 10.4. Mesh used in calibration analysis.
Figure 10.5. (A) Graph of disturbance density assuming linear accumulation. (B) Graph comparing of predicted package failures with JPL experimental results.
Figure 10.6. Graphs of disturbance density with cycles. (A) Viscoplastic case, (B) Plastic case.
Figure 10.7. Viscoplastic disturbance contour plots at cycle 1,000.
Figure 10.8. Viscoplastic disturbance contour plots for cycles 1,600 and 2,000.
Figure 10.9. Viscoplastic disturbance contour plots for cycle 2,400.
Figure 10.10. Plastic disturbance contour plots for cycles 1,000 and 1,600.
Figure 10.11. Contour plot of plastic disturbance for cycles 2,000 and 2,400.
Figure 10.12. (A) Graph of transient temperature in solder ball along horizontal centerline. (B) Element showing centerline where temperature was taken.
Figure 10.13. Failed BGA solder balls due to thermal cycling, (A) [111] (B) [44].
Figure 10.14. Flip chip failed solder bump [20].
CHAPTER 11
INDEPENDENT THERMAL CYCLE FATIGUE LIFE ANALYSIS

11.1 Introduction

This chapter predicts (independently) the results for the "Cycle A" temperature cycle (Figure 11.1) and previously shown in Figure 10.2a. What this means is that the Cycle A experimental results were not used in the calibration of the model. Thus, if the calibration was accurate (Chapter 10), then the model should predict the Cycle A experimental results accurately. The Cycle A temperature cycle was a separate condition designed to determine the life of the 313 PBGA electronic package under different conditions from the JPL temperature cycle. The experiment was managed by the same authority (JPL consortium) which designed and oversaw the JPL temperature cycle testing. The nomenclature "Cycle A" is retained from the references [42, 43] which describe the experiment setup, and experimental results so that no confusion can be made regarding what temperature cycles are being used in this dissertation. The test vehicle was identical to the one used for the JPL temperature cycle (Figure 10.1), and the method of determining failure (described in Chapter 10) during the experiment remains unchanged. Based on the calibration performed in Chapter 10, where the JPL temperature cycle was used, a fatigue life prediction is made using a different temperature cycle - the Cycle A condition. The Cycle A temperature cycle is significantly different from the JPL temperature cycle, and the differences can be seen in Table 11.1, and Figure 11.1. It can be expected that the fatigue life of the package will be greater under the Cycle A conditions, compared to the JPL cycle. This is due to several
reasons. The first is that the mechanism of fatigue failure (strain) is driven by the CTE of the materials, and the change in temperature. For the same materials, a higher temperature will induce greater strains. For temperature cycling, the material will experience stresses proportional to the CTE mismatch between materials. In other words, even if the package is not constrained, stresses will still develop due to the different materials, and their respective differences in CTE values. The second reason is that viscoplastic effects will likely be greater for the Cycle A condition due to the slower ramp rates. Even though the JPL cycle has a higher temperature difference, the JPL cycle ramp rates are high enough such that viscoplastic effects are likely to be negligible during the ramp up and down. The viscoplastic behavior will have the effect of relaxing the stresses in the solder balls, and thus reducing the accumulated stress, and hence damage. Even though stress relaxation may be accompanied by creep, the net effect will result in lower damage. The combined phenomena of stress relaxation and creep is often seen in telephone cables strung between telephone poles. Over many years the stress in the telephone cable relaxes, and the cable stretches by creep. It is unlikely that relaxation and creep can occur separately, except under imposed boundary conditions which specifically prevent displacements. This is one benefit of using Perzyna's theory, which makes no distinction between creep and stress relaxation.
Table 11.1.1: Comparison of JPL and Cycle A Temperature Cycles

<table>
<thead>
<tr>
<th></th>
<th>JPL Temperature Cycle</th>
<th>Cycle A Temperature Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise and Fall Time</td>
<td>12°C/min</td>
<td>5°C/min</td>
</tr>
<tr>
<td>High Temp Dwell Time</td>
<td>30 min</td>
<td>20 min</td>
</tr>
<tr>
<td>Low Temp Dwell Time</td>
<td>30 min</td>
<td>5 min</td>
</tr>
<tr>
<td>Cycle Time</td>
<td>68 min</td>
<td>82 min</td>
</tr>
<tr>
<td>Total Temperature Change</td>
<td>180°C</td>
<td>130°C</td>
</tr>
<tr>
<td>Min Temp / Max Temp</td>
<td>-55°C / 125°C</td>
<td>-30°C / 100°C</td>
</tr>
</tbody>
</table>

11.2 Results of Fatigue Life Prediction

Based on the analysis performed in Chapter 10, the life of the package is given as a failure distribution, instead of an actual failure cycle. Using the percentages of failed packages found in the calibration of the package in Chapter 10, Table 11.2.1 has been constructed. Again, a disturbance density \( D > .8/A_{ball} \) is used as the criteria for a failed solder ball, where a \( D_{dens} \) of 11% represents the critical disturbance density, and a \( D_{dens} \) of 17% represents the ultimate disturbance density. At a disturbance density of 11%, it was found that there is an 11% probability that a solder ball (and hence package) will fail. At a disturbance density of 17%, the probability of a solder ball failing is 100%.

For the Cycle-A temperature cycle, the number of packages experimentally tested was not presented in the data, only the cumulative packages failed. However, examining
Figure 11.2, it appears that fourteen packages were tested, thirteen of them failed. Plotting the predicted failure distribution against the experimental failure distribution (Figure 11.3), it can be seen that the prediction is conservative. However, even the worst case prediction, the first package to fail, is $\frac{3,130}{3,500} = 89.4\%$ accurate. Based on a cumulative distribution, the first experimental package to fail represents $\frac{1}{14}$ $\%$, or 7.1% of the package failures (see Figure 11.2). This is a very small percentage, and the analysis would predict that $7.1 \times 89.4\% = 6.4\%$ of the packages will have failed. This is a very close approximation. The approximation becomes even better with higher cycles, and larger cumulative failures.

Examining Figure 11.3a, it can be seen that the disturbance density in the solder balls is grouped in a regular pattern, as it was in the calibration performed in Chapter 10. The grouping is seen to be related to the distance from the package center or silicon die. Solder balls 1-5 comprise one group, and solder balls 7-13 comprise the other group. Solder ball 6, which has a disturbance density in between the two groups, approximately represents the halfway point from the center of the package, and is located at the edge of the silicon die. It is also furthest solder ball from the center of the package that is still under the die. Observing Figure 11.3, it is predicted that the solder balls closest to the die center will fail first, with the solder ball at the furthest DNP failing last. This pattern has been verified experimentally, and is the primary reason why numerous packages are designed without solder balls under the die. The reason for this pattern of solder ball failures can be understood in the following rationale. The silicon die represents the stiffest component of the package, and for the solder balls under the die, prevents the top
of the solder balls from undergoing similar displacements as the bottom of the solder balls. This is in contrast to the solder balls away from the die, which experience similar (though different) displacements at their respective top and bottom. Because the primary mechanism of thermal fatigue is driven by strain, it should be expected that the solder balls under the die will fail first. This explains why solder ball 6 develops damage in between the solder balls immediately under the die, and those away from the die. Because solder ball 6 is located under the die edge, it is influenced by the die, but not to the extent that balls 1-5 are.

It can be seen in Figure 11.3 that solder ball 1 (center of package), under the die, initially accumulates more damage than the other solder balls. Thus, this solder ball is predicted to fail first. However, due to randomness and uncertainty, it may not fail. If it does not fail, solder balls 2 through 4, which have similar disturbance densities, may fail. In fact, up to cycle 3,800 it appears that balls 1-5 are all (approximately) equally likely to fail. However, after cycle 3,800, ball 5 begins to accumulate more damage than solder balls 1-4. Thus, it is most likely that first package failures will be caused by the fatigue failure of solder ball 1. However, packages that are able to survive longer than the first package failure are likely to fail by any of the balls 1-5. After the median package failures, it is most likely that the packages fail by fatigue of solder ball 5. This was found in the Cycle-A experiment, where the comment was made that the packages failed at the innermost daisy chains, and progressed outward away from the die [42]. Note that the disturbance is clustered near the solder ball package/PWB interface (Figure 11.4 and Figure 11.6). This is where the failures were found following the experiment.
Examining Figure 11.4, the progression of disturbance in the solder balls can be observed. Because the analysis was performed as a TCT, i.e., a thermal chamber test, the temperature was assumed isothermal. Because of this, all of the solder balls display some disturbance. However, examining Figure 11.4 carefully, it can be seen that the solder balls under the die show greater values of disturbance than the solder balls away from the die. This pattern of disturbance was commented on previously, and is plotted in Figure 11.3 as a disturbance density. Figure 11.6 shows an enlarged view of the disturbance in solder balls 1-6. Examining Figure 11.6, it can be seen that as the cycles progress from cycle 3,000 to cycle 4,000, the area of disturbed material changes only slightly. It is the value of disturbance that is changing, which can be seen by the brighter reds in cycle 4,000 compared to cycle 3,000. This is precisely what should be expected, based on experimental results. The disturbance is tracking microcrack growth. Based on experimental findings, the solder balls experience very localized fracture at the solder

<table>
<thead>
<tr>
<th>Package Failure</th>
<th>Cycle Predicted</th>
<th>Cycle Experiment</th>
<th>Disturbance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3,130</td>
<td>3,500</td>
<td>11%</td>
</tr>
<tr>
<td>2</td>
<td>3,227</td>
<td>3,722</td>
<td>11.5%</td>
</tr>
<tr>
<td>3</td>
<td>3,323</td>
<td>3,777</td>
<td>12%</td>
</tr>
<tr>
<td>4</td>
<td>3,420</td>
<td>3,777</td>
<td>12.5%</td>
</tr>
<tr>
<td>5</td>
<td>3,517</td>
<td>3,944</td>
<td>13%</td>
</tr>
<tr>
<td>6</td>
<td>3,613</td>
<td>3,970</td>
<td>13.5%</td>
</tr>
<tr>
<td>7</td>
<td>3,710</td>
<td>4,055</td>
<td>14%</td>
</tr>
<tr>
<td>8</td>
<td>3,807</td>
<td>4,110</td>
<td>14.5%</td>
</tr>
<tr>
<td>9</td>
<td>3,903</td>
<td>4,140</td>
<td>15%</td>
</tr>
<tr>
<td>10</td>
<td>4,000</td>
<td>4,165</td>
<td>15.5%</td>
</tr>
<tr>
<td>11</td>
<td>4,097</td>
<td>4,222</td>
<td>16%</td>
</tr>
<tr>
<td>12</td>
<td>4,193</td>
<td>4,277</td>
<td>16.5%</td>
</tr>
<tr>
<td>13</td>
<td>4,290</td>
<td>4,333</td>
<td>17%</td>
</tr>
</tbody>
</table>
ball and package/PWB interface.

11.3 Stress and Strain Results

Figures 11.5 shows the stresses developed during cycle 1, at step 140. It can be seen that the highest stresses in the X direction are located in and around the silicon die. This is because the package, as a whole, is in bending due to the mismatch in CTE. Because the die is the stiffest component in the package, it develops the largest X stress. The largest stress in the Y direction is in the top left corner of solder ball 6. This may be a result of the die edge directly above it. The largest shear stresses are located in the top left and bottom right of the solder balls, and at the die edge above solder ball 6. The location of the highest shear stresses is where the fractures were found to initiate in the JPL experiments. The highest octahedral shear stresses are found in and around the die. Examining the values of $\sigma_x$, $\sigma_y$, and $\sigma_{xy}$, it can be seen that it is the $\sigma_x$ stress that is driving the high values of $\tau_{oct}$ in the die. It is interesting to note that the solder balls, which are most likely to fail, do not necessarily correlate with the solder balls with the highest stresses. This can be explained by the distribution of stress in the balls. For example, a high stress located in the corner of a solder ball is not as damaging as a lower stress spread over a larger area. This is due to the ductility of solder. For materials which are more brittle, it may be expected that cracks will grow at the point of high stress, and thus lead to failure. However, because solder has a high ductility, a small area of high stress will only lead to a localized crack. The crack is immediately blunted due to the ductility, and hence stops growing. If a high global stress, greater than the fracture stress, is not present around the blunted crack tip, it will cease to grow. For lower global
stresses (high enough to initiate a crack) blunting will still occur, but because the stress is
global around the crack tip, the crack will still be able to generate high local stresses and
continue growing. Because of the high ductility of solder, the rules of elastic fracture are
not applicable, and hence the formation of a crack does not necessarily lead to failure. It
is the global stress which primarily drives the crack growth, in combination with the
localized stress concentration at the crack tips. If the combination of global stress and
localized stress are not sufficient, a crack will not grow. This is in contrast to brittle
materials where the crack tips are sufficiently sharp such that it is the localized stress (at
the crack tip) which drives crack growth. Although there must still be a global stress to
drive the crack growth in a brittle material, a small stress will be sufficient, since the
stress concentration at the crack tip is very high. With solder, the cracks blunt to such an
extent, that the localized stress concentration is subordinate to the global stress.
Figure 11.1. Graph of temperature cycles. (A) JPL and Cycle A. (B) Cycle A.
Figure 11.2. Graph of cumulative distribution of failures for JPL and Cycle A conditions.
Figure 11.3. Cycle A package failures. (A) Graph of failure distributions for each solder ball. (B) Graph of predicted failures compared to experimental failures.
Figure 11.4 Contour plots of disturbance at various cycles.
Figure 11.5. Contour plots of various stresses $\sigma_x$, $\sigma_y$, $\sigma_{xy}$, $\tau_{oct}$ (MPa) at step 140, cycle 1.
Figure 11.6. Contour plot of disturbance in solder balls 1-6 at various cycles.
CHAPTER 12

POWER CYCLING OF 313 BALL PBGA WITH HEAT TRANSFER

12.1 Introduction

Power cycling testing (PCT) and analysis is performed by assuming that the die generates the heat, and the ambient temperature remains relatively constant. Because the die is assumed to generate the heat, the die becomes a source, and the ambient air and (package boundary) becomes a sink. This is in contrast to thermal cyclic testing (TCT) where the surrounding air is heated, and cycled through a minimum and maximum temperature. The purpose of TCT testing is mainly to compare different packages and determine wear-out (failure) distributions [94]. PCT cycling is used to determine the life expectancy of a package under typical operational conditions. However, PCT is not as prevalent as TCT [132, 124], and is therefore more difficult to correlate with FEA analysis due to lack of experimental data. The modeling of PCT requires a heat transfer analysis prior to any structural calculations (stress, damage, etc.), whereas TCT analysis can be performed under isothermal conditions. In this chapter, power cycling analyses are undertaken to verify some general findings reported in the literature, and show that the heat transfer subroutine developed in this dissertation can be used to model complex package boundary conditions requiring heat transfer analysis.

No papers were found which performed PCT on PBGA packages similar to the one analyzed in this dissertation. All of the papers found, which performed PCT and FEA analysis, used CCGA, CBGA, flip chip or CSP packages. The results for CCBA and CBGA packages are not useful, as the chip carrier is ceramic, and is much stiffer than
the BT substrate found in the 313 PBGA package used in this dissertation. However, the CSP and flip chip packages will provide some insight, as they are BGA type packages with similar substrate materials as the 313 PBGA package used in this dissertation. Of the articles which presented FEA, none of them presented complete descriptions regarding the FEA procedure used. Either the boundary conditions prescribed were not given, the material properties were not stated or the solder balls which failed first were omitted in the discussions. Only four articles [124, 2, 108, 104] were found which gave enough detail to make reasonable comparisons with the results of this chapter.

For the CSP analyses [2, 53], it was generally found that under PCT, the outer ball failed first. This is the solder ball furthest away from the package center (largest DNP). However, for CSP packages, because of the reduced area of the package, all of the solder balls are still influenced by the die. This is because the package area is only slightly larger than the area of the die itself, and the outer solder ball may still be in the die shadow. This is an important point, as the furthest DNP solder joint in a typical BGA package is not influenced by the die. For the package used in this research, there are six balls between the last ball under the die shadow (ball 6) and the furthest DNP solder ball (ball 13). Thus, the furthest DNP solder ball has a different connotation in CSP packages compared to BGA packages. In effect, the furthest DNP solder ball in a CSP package probably corresponds best with solder ball 6 in the 313 PBGA package used in this research.

For the flip chip analysis [104], the authors state that the location of the highest Von Mises stress is found in the corner joints under the die shadow (ball 6 in the
313PBGA package in this dissertation). This was the only comment about the FEA analysis. There was no mention of the dimensions of the package or any information regarding the FEA analysis. The authors used a power cycling method such that the die approached a temperature of 110°C, 48 cycles/day, in an ambient environment with natural convection. As will be shown later, this thermal cycle was similar to the one used in this chapter.

12.2 Power Cycling of 313 PBGA

Having performed a calibration of the model for the 313 PBGA under TCT in Chapter 10, and an independent life prediction in Chapter 11, a PCT analysis is now undertaken. A review of the literature found no articles which have investigated power cycling of PBGA packages. However, numerous articles were found on power cycling of CBGA packages, CSP and flip chip packages. Based on these articles, a power cycle was prescribed as shown in Figure 12.1. The power cycle was not prescribed as an actual power, i.e. a flux, but as an actual temperature applied at the die. This was done for several reasons. The first is that the actual power a chip may generate is operationally dependent. That is, it depends on how it is being used, and how hard it is being driven. This will vary from package to package, for natural convection or forced convection conditions, and operational conditions. The second is that the environmental conditions the package will be subjected to are variable. This means that the convection condition on the chip surface will vary, not only due to the ambient condition, but also with the materials ability to convect heat as it itself is heated. The third is that it is easier to apply a temperature cycle, since the maximum and minimum temperature will be constants.
This allows a better correlation with other research, as it is typical to drive chips at a junction temperature around 100°C for research purposes. A steady state heat transfer was used in the analysis. Although this is an approximation, it is a good approximation since it was found from a previous transient analysis (Chapter 10), that a steady state temperature is reached (for a temperature increment) in about 24 seconds. It is also a good approximation since the majority of the damage will be caused by the time the package is in the "on" or peak temperature condition. This approximation does not assume that it is the high temperature alone that induces fatigue failure, since the package is still subjected to a temperature transition from the on and off states. That is, the fatigue induced by heating a cold package and cooling a hot package is still present in the analysis.

12.3 FEA of 313 PBGA

The analysis was performed using a steady state heat transfer approximation. The solder was the only material modeled as viscoplastic with disturbance. All other materials were considered elastic. The package mesh and structural boundary conditions are shown in Figure 12.2, and the material properties used are as shown in Chapter 8. The prescribed temperature is shown in Figure 12.1, and is applied at the die (Figure 12.2). It is assumed that the heat is generated uniformly throughout the die volume. The package outer surface is assumed to have a convection coefficient of 27W/m²K. The PWB is assumed to have negligible heat transfer. This is because the analysis assumes natural convection. It has been shown that under natural convection, with the electronic package mounted over the PWB, the PWB does not dissipate significant heat [49]. This
will be discussed later, where a comparison between PWB convection and PWB adiabatic conditions was performed. The total cycle time was 39 minutes, with a 30 minute hold time at the "on" temperature, and a 5 minute hold time at the "off" temperature. This cycle was chosen because it allows the cold package to experience temperature turn on effects, the hot package to experience turn off effects, and for viscoplastic behavior to take effect during the 30 minute high temperature dwell. Note that there is no significant "off" time or low temperature dwell. This is because the low temperature is room temperature, and damage at room temperature is assumed to be zero.

12.4 Results of PCT

Figure 12.3 shows a comparison of the temperature distributions when the bottom edge of the PWB is considered adiabatic, and when a convection is applied to it. For the condition where a convection was applied, the convection coefficient \((h)\) was given the same value as that of the convection coefficient on the molding compound \((27 \text{W/m}^2\text{K})\). Examining Figure 12.3, it can be seen that the two cases are very similar. This is due to several reasons. The first is that the die, which is generating the heat, is very close to the top surface of the molding compound where there is a prescribed convection. Secondly, for the case where a convection is applied to the bottom of the PWB, the heat must flow through the solder balls to reach the bottom of the PWB. This is a very inefficient path to take, as the solder ball cross sectional areas are small, and there is more air space between the electronic package and PWB than there is solder area connecting the two. The path is also very inefficient due to the thermal conductivities of the materials between the die and the bottom of the PWB. Although solder has a thermal conductivity of 0.051W/mK,
the bismaleimide triazine resin (BT) substrate has a conductivity of .00019 W/mK, and the FR-4 PWB itself has a conductivity of .00026 W/mK. Thus, the thermal energy would rather flow out the top of the molding compound which has a thermal conductivity of .00067 W/mK. For all the subsequent analyses, the bottom of the PWB was assumed as adiabatic.

Figure 12.4 shows a comparison of the displacements between PCT, TCT and a Y fixity on the PWB and no fixity on the PWB at time step 74. Examining Figure 12.4, it can be seen that the overall gross deformation is similar in all cases, with the package and PWB bowing up. It has been reported [53] that for CSP assemblies, there is a difference in the deformation caused by PCT and TCT. Reportedly, a TCT bends the assembly in the opposite direction as a PCT. Examining Figure 12.4, it can be seen that this is not the case for the 313 PBGA assembly used in this study. The reason CSP packages may exhibit this phenomenon, is that the die is nearly the same size as the package. When the die heats up, nearly the entire top of the package will achieve the die temperature, while the bottom of the assembly (the PWB) will be at much lower temperature. Because of this, there will be a significant expansion on top of the assembly, causing the assembly to bend in a concave down deformation. When the assembly is heated isothermally, all levels of the assembly achieve the same temperature, and the package bends in a concave up deformation. This will not be the case for the 313 PBGA used in this dissertation, because the die is much smaller than the overall size of the package and PWB. Thus, even though there is a significant difference in the CTE’s of the die and PWB, the die is too small to control the deformation of the assembly.
The disturbance results are shown in Figures 12.5-12.12. Figure 12.5 shows the disturbance density in the solder balls for both PCT (12.5-top) and TCT (12.5-bottom). Comparing the two plots, it can be seen that the disturbance densities are nearly the same for the solder balls under the die (balls 1-5), but very different for the solder under the die edge (ball 6), and the remaining balls away from the die. The similarity in disturbance density for balls 1-5 should be expected, since the temperature in the assembly around these balls basically follows the temperature of the die (Figure 12.3). Therefore, these balls will experience stresses and strains similar to the TCT analysis, as well as experiencing the same temperatures. This is not true for solder ball 6 under the die edge. Examining Figure 12.3, it can be seen that this is the first solder ball to experience a different temperature than that of the die. Further examination of Figure 12.3 shows that the temperature gradually decreases away from the die, until reaching the lowest temperature in the region of solder ball 13. Compared to a TCT analysis, balls 6-13 should display different disturbances. For a TCT analysis, all of the solder balls will experience the same temperature. Whereas for a PCT analysis, it can be seen that balls 1-5 will experience the die temperature, and balls 6-13 will experience a temperature proportional to the distance from the die.

A close inspection of the PCT analysis (Figure 12.5a), shows that initially, all of the balls under the die have about the same disturbance density, except ball 6 under the die edge. However, with increasing cycles, ball 5 accumulates more disturbance density until cycle 2,250. At this point, all the balls under the die, except ball 6, have approximately the same disturbance density. Based on a critical disturbance density of
11%, ball 5 is predicted to fail at about 2,100 cycles, and an ultimate disturbance density of 17%, all balls 1-5 are about equally likely to fail. This means that the first package failures (about cycle 2,100) will likely be a result of ball 5 failing. However, of the packages that do not fail at this cycle, ball 5 will still be the likely ball to fail up until cycle 2,250, where any of the balls are likely to be the ones that fail.

For the TCT analysis, the disturbance results are about the same for the balls 1-5, compared to the PCT results, but very different for the remaining balls. This is caused by the temperature being applied isothermally. Examining Figure 12.5b, it can be seen that the disturbance results, described above for the PCT analysis, are similar to the TCT analysis, with the exception of ball 6. Initially, ball 6 has the highest disturbance density, but flattens out around cycle 1,500 to cycle 1,950. At this point the disturbance density in the solder ball increases greatly, and becomes about the same as the rest of the solder balls under the die at cycle 2,400. However, based on a critical disturbance density of 11%, the first package failures are predicted to be caused by fatigue failure of solder ball 5 at cycle 2,040. At an ultimate disturbance density of 17%, all balls 1-5 are about equally likely to fail at cycle 2,220. This means that the first package failures (about cycle 2,040) will likely be a result of ball 5 failing. However, of the packages that do not fail at this cycle, ball 5 will still be the likely ball to fail up until cycle 2,220. If a package survives past this cycle, any of the balls are likely to be the ones that fail.

The biggest difference between PTC and TCT can be seen in disturbance density accumulation in the solder balls away from the die (balls 6-13). However, none of these balls are likely to be the balls which will lead to failure. Thus, although it is interesting to
note the differences, a properly designed TCT experiment is likely to be as accurate in predicting the critical solder balls as a PCT test for PBGA packages. As noted previously, PCT tests are not common because they are much more difficult and expensive to design and perform. However, when they are performed, it is perceived that they provide a more accurate view of how the package will fail in operation. From the results of this chapter, it appears that a TCT test will provide the same information as a PCT test for the critical solder balls in a PBGA package. However, this may not be true for other style packages.

Figure 12.6 shows the disturbance in the solder balls for various PCT cycles. Examining Figure 12.6, the accumulation of disturbance can be clearly seen as the number of cycles increase. Figure 12.7 shows the disturbance in the solder balls for various TCT cycles. Comparing Figure 12.7 with Figure 12.6, it can be seen the solder balls under the die accumulate damage at about the same rate. However, comparing the disturbance of the solder balls away from the die, it can be seen that the TCT and PCT cases are very different. This should be expected, as the TCT case is isothermal, so the entire assembly is heated. Thus, the solder balls away from the die will experience the same temperature as the solder balls underneath the die. This will not occur for the PCT case, as only the die is heated, and the heat diffuses through the assembly by thermal conductivity.

Examining Figure 12.8, it can be seen that the disturbance accumulation in the critical ball (ball 5), is different compared to the disturbance distributions seen in previous chapters. This is due to the temperature cycle having only a "positive"
temperature. That is, the previous thermal cycles used in Chapters 9, 10 and 11, had thermal cycles which had a maximum temperature above room temperature and a minimum temperature below room temperature. Thus, the package was placed in concave up bending at the maximum temperature, and in concave down bending at the minimum temperature. This is not the case for the half cycle used in this chapter or any typical operational conditions a package might encounter. For the half cycle, the minimum temperature is room temperature, and at room temperature the assembly is not in bending. For a half cycle typical of PCT analyses, all of the accumulated damage is caused by the temperature above room temperature. Observing Figure 12.8, it can be seen that there are high levels of disturbance at the left and right perimeter of the solder ball. This is caused by high $\varepsilon_y$ and $\varepsilon_x$ strains in this area of the solder ball, and can be seen in Figure 12.10. These strains lead to high values of $\sigma_x$ and $\sigma_y$ in the same locations as shown in Figure 12.11 and 12.12 for the TCT and PCT case respectively. These high stresses are repeated for every cycle, and are never reversed by a temperature swing below room temperature, as was the case in Chapters 9, 10, and 11. Therefore, the damage accumulation in a half cycle, commonly used for PTC tests, will be different compared to a typical full cycle where there are temperature swings above and below room temperature. The high levels of disturbance at the sides of the solder ball are areas of highly plastically deformed material. This corresponds to Figure 12.13, where a SEM photograph of a solder ball is shown. Because JPL did not perform PCT experiments, no correlation with the FEA analysis can be made. However, Sur and Tarlik [121b] performed PCT on a multi-chip module, and found similar results (Figure 12.13b). They
comment that "It can be seen that the two diagonally opposite sides of the bump curvature and the two opposite sides of the bump-interfaces experience the maximum Von-Mises stress. This is because the x-normal and shear stress, respectively, are a maximum at those locations. These are the most likely areas for crack growth."

12.5 Heat Sink Results

Figures 12.14 and 12.15 show results from analyses simulating heat sink convection. It is assumed that a heat sink is attached to the top of the molding compound. To simulate the heat sink, an increased convection coefficient was prescribed on the top package boundary, and a temperature cycle prescribed at the die. The temperature cycle was the same as used previously. Thus, it is assumed that the die will be kept below 100°C by the heat sink, which is a typical assumption for FEA analyses using heat transfer. For a package convection (heat sink) of $h = 8.1\times10^{-6}$, the temperature cycle corresponds to a die flux of 250 milliwatts (mW). For a package convection of $h = 2.7\times10^{-5}$, the temperature cycle corresponds to a die flux of 750 mW. Without a heat sink, the package convection was $h = 2.7\times10^{-6}$. Without a heat sink the temperature cycle corresponds to a die flux of 100mW. These are all typical power levels for low power electronic packages.

Examining Figure 12.14, it can be seen that the disturbance densities are nearly identical to the disturbance densities found without a heat sink (Figure 12.5). This is because each heat sink and die flux were chosen so that the temperature in the die's were identical. Thus, the packages experience approximately the same temperature distribution, and thus develop about the same stresses and strains. This analysis shows
that the power in the package can be increased 2.5 times by increasing the convection at the top boundary three times using a heat-sink. The power in the package can be increased 7.5 times by using a heat-sink which increases the convection on the top of the package by ten times. In each of these cases, the packages are all expected to fail at about the same cycles, yet the power in the package has been increased. This shows the benefit of using a heat sink. Figure 12.15 shows the relationship between heat sink convection, and die flux, when the die temperatures for all three cases are the same. In other words, the heat sink and power in the die are matched such that in all three cases, the package experiences the same temperature distribution. Because the temperature distributions will be nearly identical, the cycles to failure will be nearly identical, as well. Examining Figure 12.15, it can be seen that the relationship between die flux and heat sink convection is linear. Thus, for this package, it would be easy to determine what heat sink would be necessary for any particular die flux to give the same cycles to failure.

12.5 Summary

A power cycle test (PCT) was performed on the 313 PBGA electronic package used in previous chapters. A heat transfer analysis was performed, as it is standard practice to use a heat transfer analysis with PCT. The PTC cycle does not have a minimum temperature below the ambient temperature, as the minimum temperature the die can achieve during operation is the ambient temperature. An isothermal analysis (TCT) was also performed to compare with the PCT results. The following results are summarized.

1. Solder ball 5 is the critical ball for both the TCT and PCT analyses.
2. The failure of the solder balls can be grouped in terms of solder balls under the die (balls 1-5), solder ball at die edge (ball 6), and solder balls away from die (balls 7-13).

4. The disturbance accumulation is different for the isothermal (TCT) case compared to the power cycling (PCT) case for the solder balls away from the die (balls 7-13).

5. The damage (disturbance) accumulation in the solder balls is different for a PCT half cycle thermal loading as compared to the full cycle thermal loadings used in previous chapters. This was also verified by other researchers [121b].

6. For a PCT thermal half cycle, the final fatigue fracture is predicted to occur at the interface of the solder ball and PWB/package, but slightly further away from the boundary as compared to a full cycle thermal loading (as shown in Chapters 9, 10, 11). This has been seen in other work [36], and is probably due to the massive deformation the joints accumulate. Because a PCT is only a half thermal cycle, the deformations are never reversed by a temperature below the ambient temperature. Thus, the strains accumulate in the same direction over every cycle. This is in contrast to the typical full cycle (TCT) where the temperature cycles above and below the ambient temperature. This has the effect of partially reversing the strains accumulated at either temperature extreme.
Figure 12.1. Graph of PCT temperature cycle used in FEA analysis. Temperature is assumed to be generated, i.e. constant throughout the die volume.
Figure 12.2. Mesh showing die, where temperature is applied for PCT analysis.
Figure 12.3. Contour plots comparing temperature (K) distributions for PWB with convection, and without convection.
Figure 12.4. Comparison of TCT and PCT deflections at step 74, with different Y fixity at 100 times actual deformation.
Figure 12.5. Graphical comparison of disturbance densities between PCT and TCT. (A) PTC disturbance density, (B) TCT disturbance density.
Figure 12.6. Contour plots of disturbance in solder balls for various TCT cycles, no Y fixity.
Figure 12.7. Contour plots of disturbance in solder balls for various TCT cycles, no Y fixity.
Figure 12.8. Contour plots comparing disturbance in solder ball 5 for TCT and PCT cycling, cycle 2,100, no Y fixity. (A) TCT disturbance, (B) PTC disturbance.
Figure 12.9. Contour plots of equivalent stress: Enlarged view of balls under die for TCT and PCT, first cycle, 64th step, no Y fixity. (A) TCT stress, (B) PCT stress.
Figure 12.10. Contour plots of various strains for PCT, first cycle, 64th step, no Y fixity.
Figure 12.11. Contour plots of various stresses for TCT-isothermal: $\sigma_x$, $\sigma_y$, $\sigma_{xy}$, and $\tau_{oct}$ (all in MPa), first cycle, 64th step, no Y fixity.
Figure 12.12. Contour plots of various stresses for PCT: $\sigma_x$, $\sigma_y$, $\sigma_{xy}$, and $\tau_{oct}$ (all in MPa), first cycle, 64th step no Y fixity.
Figure 12.13. (A) Von Mises stress in solder joint [121b]. (B) Crack in PTC experiment [36].
Figure 12.14. Disturbance density in solder balls with differing package convection and die flux. (A) $h = 27 \times 10^6 \text{W/(mm}^2\text{K)}$, $q = 750 \text{mW}$. (B) $h = 81 \times 10^7 \text{W/(mm}^2\text{K)}$, $q = 250 \text{mW}$
Figure 12.15. Graph of relationship between die flux and package convection
CHAPTER 13

VIBRATION FATIGUE ANALYSIS

13.1 Introduction

As commented on in Chapter 3, vibration fatigue analysis of electronic packages has not received the same attention as thermal cyclic fatigue. The industries most concerned with vibration fatigue are the automotive and military industries, as their products are exposed to severe vibration environments. The 313 PBGA package analyzed in the previous chapters for thermal fatigue, has also been experimentally tested for vibration fatigue [140, 141, 142, 143].

In general, the FEA of vibration fatigue is extremely difficult. This is partly due to the generally random nature of the vibration loading, and the complexity of the dynamic analysis. A dynamic analysis is complicated by the structure being analyzed. For a dynamic analysis to be accurate, the entire structure must be included in the analysis (generally), unless the boundary conditions are sufficiently simple, and vibration loads are known at the sub-component being analyzed. For an electronic package, this means that the PWB must be included in the analysis to capture the deflections of the PWB, and the resulting stresses induced in the solder joints due to the deflections. Because a dynamic analysis, by definition, incorporates the masses of the components, a 2D analysis is generally not applicable for a PWB - electronic package structure. However, if the deflections at locations in the electronic package are found from experimental testing, then a 2D analysis may be possible. However, knowing the displacements, a dynamic analysis may not be necessary if the loading frequency is
sufficiently higher or lower than the natural frequency of the package [2]. This is generally the case, as an electronic package is much stiffer than the PWB, and hence has a higher natural frequency compared to the PWB. In general, an applied random vibration will contain the natural frequency of the PWB, and excitations will occur in PWB, and hence cause deflections leading to stresses in the solder joints of the electronic package.

13.2 Development of Applied Displacement for the 313 PBGA Vibration Analysis

Wong, et. al., [140, 141, 142, 143], applied a random vibration load on a test vehicle containing the 313 PBGA previously analyzed in Chapters 11, 12, and 13. They applied a random vibration with the prescribed conditions shown previously in Figure 3.7. The experimental testing revealed that the furthest DNP solder ball (ball 13) was the critical ball, and out of four packages tested, the average failure cycle was 2,325,000. After the random vibration testing, they performed FEA analysis and determined a fatigue life based on a volume averaged equivalent strain at the top layer of elements of the worst case solder ball. The process they used was very involved, and was described in Chapter 3. Because they used a 3D ANSYS model, and two in house FORTRAN programs to develop their model, their formulation will not be used. Instead, it will be easier to calculate an approximate Z displacement in the PWB, which will correspond to a Y displacement in the 2D FEA model used here. The Y displacement will then be used for the approximate displacement model in this analysis.
The following formulation for determining the PWB Z displacement, based on a random vibration input, follows Steinberg [117]. Based on the input PSD, the chassis RMS acceleration is calculated as:

\[ G_{RMS} = \sqrt{\frac{\pi}{2} PQf_n} = 35.56 \quad (13.2) \]

\[ Q = \frac{f_{pwb}}{f_{chassis}} \sqrt{\frac{f_{chassis}}{f_{chassis}}} = 13.6 \]

\[ P = 0.16 \]

Where \( P \) is the PSD input to the chassis holding the PWB, \( Q \) is the estimated chassis transmissibility, and \( f_n \) is the measured natural frequency of the PWB (370Hz). It is assumed that the chassis natural frequency is twice that of the PWB (370Hz), and follows the octave rule generally used by designers when engineering a PWB chassis. The chassis RMS displacement is then calculated as:

\[ Z_{RMS} = \frac{9.8 \times G_{RMS}}{f_n^2} = 0.025in \text{ or } 0.065mm \quad (13.3) \]

Because there will be coupling between the chassis and PWB, two coupled transmissibilities will need to be determined, one at 370Hz, and one at 740Hz:

\[ Q = \frac{1}{1 - \left( \frac{f_{pwb}}{f_{chassis}} \right)} = 1.33 \quad (13.4) \]

\[ Q = \frac{1}{1 - \left( \frac{f_{chassis}}{f_{pwb}} \right)} = 0.333 \quad (13.5) \]
Because of this coupling, the PWB transmissibility at its own natural frequency becomes:

\[ Q_1 = 1.33 \times \sqrt{370} = 25.6 \]  \hspace{1cm} (13.6)

and, the coupling at the chassis' natural frequency becomes:

\[ Q_2 = .333 \times \sqrt{740} = 4.5 \]  \hspace{1cm} (13.7)

The RMS acceleration can now be determined as:

\[ G_{\text{RMS}} = \sqrt{\frac{\pi}{2} P(f_{\text{pwb}}Q_1 + f_{\text{chassis}}Q_2)} = 55.72 \]  \hspace{1cm} (13.8)

and the RMS displacements are:

\[ Z_{\text{RMS}} = \frac{9.8 \times G_{\text{RMS}}}{f_n^2} = .004\text{in or } .101\text{mm} \]  \hspace{1cm} (13.9)

Where \( G_{\text{RMS}} \) is now the coupled value found from Equation 13.8. Because the displacement in Equation 13.9 was calculated from a random vibration input, the displacement amplitude of .101mm is expected to occur 68.3% of the time, assuming a Gaussian distribution of amplitudes. However, the highest amplitudes and greatest damage is known to occur at the PWB natural frequency. Assuming a "3σ" relationship, which is typically used in the literature, the displacement becomes \( 3 \times Z_{\text{RMS}} \), or approximately .3mm. This is the single amplitude displacement occurring at the center of the PWB. Examining the test vehicle, it can be seen that the edge of the 313 PBGA is approximately 1/3 of the distance from the center of the PWB. Assuming a linear displacement function, it can be assumed that the 2/3 of .3mm displacement occurs at this
edge. i.e. approximately .2mm. Because the package is assumed to be significantly more rigid than the PWB, this is the displacement applied as shown in Figure 13.1.

13.3 FEA of 313 PBGA

The package analyzed is the same package analyzed previously for thermal cycling (Figure 13.1). The structural boundary conditions remain the same as for the thermal cycling cases in Chapters 10 and 11, and follows the boundary conditions used by Wong et. al. [143]. Because a dynamic analysis was not necessary, and indeed dubious based on a 2D half slice model, no new material parameters were necessary. All previous parameters shown in Chapter 8 were used. The analysis follows the work of Wong et. al. [140, 141, 142, 143], who used a submodeling technique to determine the fatigue life of the same 313 PBGA package.

The accelerated analysis method described in Chapter 9 was used for the analysis, with a reference cycle of 5,000. The solder was the only material modeled as viscoplastic with disturbance. No temperature was applied in the analysis.

13.4 Results of Vibration Analysis

The results of the vibration analysis are shown in Figures 13.2 - 13.4. The researchers who performed the experimental work [140, 141, 142, 143], stated that four packages were tested, and failed at minute 56, 64, 65, and 88. This corresponds to cycle 1,888,320, 2,158,080, 2,191,800, and 2,967,360, based on an approximate sinusoidal frequency of 562Hz. The approximate frequency is based on the "number of zero crossings" caused by the random frequency loading. Examining Figure 13.2, it can be
seen that the analysis predicts failure (ball 13) at 2,580,000 cycles, using the critical disturbance of $D_{dens} = 11\%$. This disturbance density corresponds to the first package failure found in the calibration performed in Chapter 10. The failure cycle at the ultimate disturbance density, $D_{dens} = 17\%$ was found to be 2,640,000, again in ball 13. This disturbance density corresponds to the last package to fail in Chapter 10. Refreshing the findings of Chapter 10, it was found that 1/9 packages failed at a $D_{dens} = 11\%$, and all packages had failed at a $D_{dens} = 17\%$. Examining Figure 13.2, it can be seen that the difference in cycles between $D_{dens} = 11\%$ and $D_{dens} = 17\%$ is very small (about 60,000 cycles). Observing the trend in disturbance density in solder ball 13 (Figure 13.2), it can be seen that the solder ball accumulates disturbance very quickly after cycle 2,300,000. It can be seen in Figure 13.2, that the analysis predicts failure between the lowest experimental failure cycle, and the highest failure cycle. However, experimentally, three of the four packages failed around cycle 2,200,000, which indicates that this may be the critical cycle. Assuming that the critical cycle is 2,200,000, the analysis is 2.2/2.58 or 85% accurate. Assuming that the critical cycle is the average of the experimental failure cycles (2,301390), then the analysis is approximately 2.3/2.58 = 89% accurate.

Observing Figure 13.3, it can be seen that the critical solder ball is ball 13. In fact, there is relatively little disturbance accumulation in any of the other solder balls. This is consistent with experimental evidence for vibration (across all journal articles), where the furthest DNP solder ball is always the first ball to fail. All the research found (Chapter 3) reported that the furthest DNP solder ball was the critical solder ball. For
vibration fatigue analysis, it is a rule of thumb that the furthest DNP solder ball is the most critical, and many times the only solder ball monitored during experiments.

Figure 13.4 shows enlarged views of the disturbance in the critical solder ball, ball 13. Examining Figure 13.4, it can be seen that the disturbance accumulation is different than the disturbance accumulation in the critical solder ball (ball 5) for thermal cycling. For vibration, the highest disturbance is found slightly away from the corner of the ball, and accumulates as a globular mass. This is in contrast to thermal cycling, where the disturbance accumulates as a well defined region at the interface of the solder ball and PWB/package. The researchers who performed the experimental testing on the 313 PBGA package did not present any pictures of the failed solder balls, so a comparison cannot be made with the DSC analysis. However, Yang et. al. [147] performed vibration fatigue on a PBGA assembly, and provided photographs of one of the test results [Figure 13.6]. Comparing the locations of the cracks in their results with the areas of high disturbance in Figure 13.4, a very good correlation is seen.

Figure 13.5 shows the stresses in the critical solder ball (ball 13). Examining Figure 13.5, it can be seen that the highest magnitudes of the \(\sigma_x\) and \(\sigma_y\) stresses occurs in the bottom right corner. Both stresses are negative. The highest shear stress occurs in the bottom right corner as well, and is positive. Observing the disturbance distribution in solder ball 13 (Figure 13.4), it can be seen that the disturbance follows the \(\tau_{xy}\) stress, although not as strongly as it does in a thermal loading. This indicates that vibration fatigue is affected more strongly by the normal stresses compared with thermal fatigue. This result has been reported by other researchers as well.
13.5 Summary

This is one of the first applications of the DSC in the analysis of electronic packages under vibration fatigue. Because of the limited availability of test data, especially for the package used in this research, a number of assumptions were made in the analysis presented in this chapter. However, the results are very encouraging, and shows that the DSC is applicable to vibration fatigue as well as thermal fatigue analysis of electronic packages.
Figure 13.1. (A) FEA model showing location of prescribed displacement. (B) Graph of prescribed displacement.
Figure 13.2. Graph of disturbance density in solder balls due to vibration
Figure 13.3. Contour plots of vibration disturbance at various cycles.
Figure 13.4. Contour plots of vibration disturbance in solder ball 13 at various cycles.
Figure 13.5. Contour plots of various stresses (MPa) in critical solder ball, ball 13.
Figure 13.6. Fatigue crack growth in solder ball [147].
CHAPTER 14

THERMAL CYCLING WITH VOIDS

14.1 Introduction

As described in Chapter 2, the effect of voids in solder balls is not clearly understood. In some cases, it has been reported that voids lead to premature failure of solder balls, and in other cases it has been reported that they have no effect on failure. To understand the effect of voids on the thermal cycling fatigue life of solder balls, a simple analysis was performed in this chapter. A void was introduced into the critical solder ball, and the effect of the void noted.

14.2 FEA Setup

The thermal cycle "Cycle-A" was used as the loading, with the mesh as shown in Figure 14.1. Four elements, comprising the void, have been weakened, and are located in the bottom right of the solder ball. Otherwise, this is the same mesh as used in Chapter 11, with the same boundary conditions. The accelerated analysis method was used in the analysis, as described in Chapter 9, with viscoplastic and disturbance behavior in the solder only. The material parameters are as shown in Chapter 8. The only exception is the elements that comprise the void. These elements are assumed to have an elastic modulus of 16MPa, a Poisson's ratio of .001, and a CTE of 0. A small elastic modulus was chosen to account for any bridging between the void walls. However, the modulus is approximately 1,500 times less than that of the solder, so in effect acts as an air pocket.
14.3 Thermal Cycling with Void: Results

The results are shown in Figures 14.2 and 14.3. Observing Figure 14.3, it can be seen that the critical cycle (at \( D_{\text{dens}} = 11\% \)) is approximately 3,000, where the first packages are expected to fail. The ultimate cycle (at \( D_{\text{dens}} = 17\% \)) is about 4,500, where all the packages are expected to have failed. Comparing this result to the result where no void was present (Chapter 11), it can be seen that there is a reduction of approximately 150 cycles for the critical disturbance density of 11\%, and 250 cycles for the ultimate disturbance density of 17\%. This indicates that voids do have an impact, reducing the fatigue lifetime. However, as a percentage, the fatigue lifetime is reduced by only \( \frac{3,150}{3,000} - 1 \), or 5\%. Therefore, in terms of statistical outcomes, taking into account the randomness of loadings, manufacturing processes and other factors, this is a negligible effect. This may explain, to some extent, why some researchers report voids as having an effect, and others reporting that voids do not have an effect. While this FEA analysis shows that voids do have an effect, experimentally, it would be very difficult to determine that a void was responsible for reducing the lifetime of a package by only 5\%. This is because there is always scatter in experimental results, and the scatter is difficult to correlate. Thus, numerically, voids cause a reduction in fatigue lifetime. Whether this could be proven experimentally is dubious due to the uncertainty and randomness of experimental results.

Examining Figure 14.3, the disturbance accumulation does not appear to be significantly altered by the presence of the void. This can be seen by examining Figure 11.6, where no void was present. This is an interesting finding, as it is typically assumed
that inhomogeneities are stress raisers, and precursors to fracture. Examining SEM photographs of voids (14.5), it can be seen that they are rather smooth in shape, and thus may not develop the high stresses that lead to fracture. Examining Figure 14.4, this appears to be the case. Figure 14.4 shows the $\sigma_x$, $\sigma_y$, and $\sigma_{xy}$ stresses in the critical solder ball, containing the void, at step 28 during the first cycle. Observing the stresses around the void, it can be seen that the void does not act as a stress raiser. It might be postulated that solder, because of its ductility, is not as susceptible to fracture by inhomogeneities as are other more brittle materials. This may be a result of severe blunting at the tips and edges of any microcracks that form. Because solder yields at very low stresses, this might be expected. At the onset of crack formation, the cracks blunt due to plasticity, and never cause the high stresses around the crack tip as is found in more brittle materials.
Figure 14.1. (A) Mesh used in analysis, showing void approximation. (B) Graph of Cycle A temperature cycle.
Figure 14.2. Graph of disturbance density in solder ball 5, containing void, versus cycle.
Figure 14.3. Contour plot of disturbance in solder ball 5, with void, at various cycles.
Figure 14.4. Contour plots of various stresses in solder ball 5, with void, cycle 1, step 28.
Figure 14.5. Voids in solder balls of 313 PBGA package
CHAPTER 15
MECHANICAL VIBRATION WITH VOIDS IN CRITICAL SOLDER BALL

15.1 Introduction

No papers have been found which investigated the effect of voids on the vibration fatigue of solder joints. For thermal cycling, the effect of voids has been investigated, although the research is ambiguous. Some researchers have shown that voids have an effect, and others have shown that voids have no effect. To investigate the effect of vibration on the fatigue lifetime of solder balls with voids, the same analysis used in Chapter 13 was undertaken, except that a void was introduced into the bottom right corner of the critical solder ball (ball 13). The analysis undertaken is very simple, but investigates an area which has not been seen in the literature.

15.2 FEA Setup

The same cycle used in Chapter 13 was used as the loading, with the mesh as shown in Figure 15.1. Four elements have been weakened, and are located in the bottom right of the solder ball. Otherwise, this is the same mesh as used in Chapter 13, with the same boundary conditions. The accelerated analysis method was used in the analysis, as described in Chapter 9, with viscoplastic and disturbance behavior in the solder balls only. The material parameters are as shown in Chapter 8. The only exception is the elements that comprise the void. These elements are assumed to have an elastic modulus of 16MPa, a Poisson’s ratio of .001, and a CTE of 0. A small elastic modulus was chosen
to account for the any bridging between the void walls. However, the modulus is approximately 1,500 times less than that of the solder, so in effect acts as an air pocket.

15.3 Vibration with Void: Results

The results are shown in Figures 15.2 - 15.4. Examining Figure 15.2, it can be seen that the voids do not concentrate damage (disturbance) around the void. A similar disturbance distribution is seen in the critical solder ball without a void, as shown in Chapter 13 (Figure 13.4). The disturbance accumulates in a somewhat amorphous shape, with a slight tendency to form diagonally from the upper left corner of the solder ball to the lower right. Further examination of the area around the void shows that the void has actually shifted the disturbance accumulation away from the lower right corner of the solder ball. In other words, not only does the void not act as damage concentrator, it actually seems to reduce the damage in the lower right corner. This is an interesting finding, and is supported by some researchers [111] who have stated that voids are not likely to have an effect, but if they do have an effect, the effect is positive. This positive effect may be a result of the reduced compliance of the solder joint. It has been well documented that the fatigue lifetime of a solder ball is increased by reducing the width to height ratio. Most researchers believe that this increase in lifetime is caused by the increased compliance of the solder ball. In other words, as the width to height ratio is reduced, the solder ball becomes more flexible and able to accommodate the imposed strains. Observing Figure 15.3, it can be seen that the critical disturbance density ($D_{\text{dens}} = 11\%$) is approximately 2,650,000 cycles, and the ultimate disturbance density ($D_{\text{dens}} = 17\%$) is 2,780,000. Comparing this to the results of Chapter 13 where no void was
present (2,570,000 cycles at $D_{\text{dens}} = 11\%$), it can be seen that the void actually increases the fatigue lifetime. However, the increase is only marginal. The increase is $2.65/2.57 - 1 = 3\%$. Thus, based on the scatter of experimental testing, operational use, etc., the increase is negligible. This may explain, to some extent, why some researchers have stated that voids have no effect on fatigue life. Experimentally, it would be difficult to correlate a 3% increase in life with the solder balls that failed. As an example, observing Figure 13.2 which shows package failures under vibration, it can be seen that there is a wide scatter in the results. The lowest cycles to failure was approximately 1.9 million cycles, while the highest cycles to failure was about 3 million cycles. This gives a variance of about 1 million cycles or 25% assuming a mean failure of 2.5 million cycles ($2.5/2 - 1 = 25\%$). Correlating a 3% increase in life, when there is a 25% scatter in mean life would no be possible. Thus, numerically, it is possible to show that voids marginally increase solder ball life for vibration cycling. However, this would not be possible to show under experimental conditions.

Examining Figure 15.4, it can be seen that the stresses in the solder ball with a void have significantly different distributions, compared to a solder ball without a void (Figure 13.5). The stress in the X direction is higher in the area around the void compared to the stress in the same area in the solder ball without a void. However, the peak stress in the Y direction is about the same. The greatest difference is in the shear stress, where the peak stresses are reduced in the ball with the void. Noting that damage appears to accumulate in a similar shape or distribution as the shear stresses, it can be surmised that the lower shear stresses are what lead to an increased lifetime in the ball.
with the void. This conclusion, that the shear stresses are what drive fatigue, has been pointed out in previous chapters. Although normal stresses have an impact, the shear stresses are more strongly correlated with the damage. This can be seen by simply examining the shear stress distributions and the disturbance distributions. In general, the distributions are similar. This is not the case for either the X or Y normal stresses. Therefore, it can be concluded that the shear stress is the driving stress in both thermal and vibration fatigue.
Figure 15.2. Contour plots of disturbance in solder ball 13 with void in bottom right at various cycles.
Figure 15.3. Graph of disturbance density in critical solder ball, ball 13 at various cycles
Figure 15.4. Contour plots of various stresses (MPa) in critical solder ball, ball 13, (cycle 1, step 2).
CHAPTER 16

VIBRATION AND CONSTANT TEMPERATURE

16.1 Introduction

This chapter introduces the effects of combined vibration and temperature. Based on the previous results in Chapters 10 through 15, and a literature search, it was found that the application of a thermal cycle in conjunction with vibration had no immediate applications. This is due to several reasons. The primary reason is that vibration has a very short period or high frequency compared to thermal frequencies. For example, the vibration frequency used in Chapter 13 was 562Hz, and the thermal frequencies used in Chapters 10, 11, 12 were on the order of one cycle per hour or .00028Hz. A literature search was unable to uncover engineering devices where a thermal cycle in the 100's of Hz was used. Because the vibration fatigue occurred in approximately one hour, the package would only experience one thermal cycle, using the previously defined thermal cycles. Or, approaching this from another perspective, consider a missile application, for which the 313 PBGA has applications. If an analysis considered operational lifetimes, a device in a JSOW missile would experience (for approximation purposes) a vibration of 570Hz for approximately 3 hrs/day [115]. The electronic package would, primarily, only experience a change in temperature due to the change in altitude of the jet aircraft. This is because the missile is only "turned on" prior to firing. Thus, over one day, the package would experience one thermal cycle, and approximately 6.2 million vibrational cycles. Extending this out to a year, assuming the jet flies 365 days a year, the package would experience only 365 thermal cycles, and approximately 2.2 billion vibrational cycles.
This type of approximation can also be applied to automotive electronic packages mounted under the hood. For automobiles, the vibration frequency might be on the order of 100Hz [116]. Assuming that the car is operated, on average, two times a day, for one hour at a time, the electronic package would be subjected to 720,000 vibration cycles and only two thermal cycles. However, the thermal cycles are not really cycles, as the engine is hot when the vehicle is parked. Thus, the package is not subjected to a cool down while under vibration.

Based on the two scenarios presented above, it can be seen that an electronic package primarily experiences vibration at a constant temperature, not a cyclic temperature. Therefore, the analyses performed in this chapter assume an isothermal temperature condition in the package. This, as described above, is more typical for automotive and military systems. Even if the package is in operation, the failures will primarily be driven by a constant temperature. This has been shown by Englemaier [36] and Rodgers et. al. [108] by testing and analyzing packages under external heating (TCT) with and without internal heating (PCT).

16.2 FEA Setup

The same package and boundary conditions used in the previous vibration analysis, where no temperature effects were considered (Chapter 13), are used again. The only difference is that an isothermal temperature condition is prescribed. Two temperatures were used in the analysis, 330K and 390K, to clearly show the effect on fatigue as temperature is raised.
16.3 Results

The results are shown in Figure 16.1-16.2. Examining Figure 16.1, it can be seen that temperature has a significant impact on vibration fatigue life. However, solder ball 13 is still the critical solder joint, as it was when the temperature was not considered, i.e., 300K or room temperature. In Chapter 13, it was shown that the critical cycle, where $D_{dens} = 11\%$, was 2.57 million cycles. The ultimate cycle, where all packages are predicted to fail ($D_{dens} = 17\%$) was found to be 2.64 million cycles. Comparing this to the results in Figure 16.1, it can be seen that for a temperature of 330K, the fatigue life has been reduced by $2.49/2.57 - 1 = 3.2\%$. At 390K, the fatigue life has been reduced by $2.32/2.57 - 1 = 10.8\%$. Table 16.3.1 has been constructed which tabulates the results from Chapter 13, and Figure 16.1.

| Table 16.3.1. Comparison of Results for Vibration Fatigue With Prescribed Temperature |
|---|---|---|
| | $T = 300K$ | $T = 330K$ | $T = 390K$ |
| Cycle at $D_{dens}=11\%$ | 2,580,000 | 2,480,000 | 2,320,000 |
| Cycle at $D_{dens}=17\%$ | 2,640,000 | 2,550,000 | 2,415,000 |

The differences in fatigue life can clearly be seen in Table 16.3.1, and Figure 16.1. Examining Figure 16.1, it can be seen that there are subtle differences in disturbance accumulation as the temperature is increased. There does not appear to be any clear relationship between temperature, and the disturbance curves. For example, at cycle 2.4 million, the disturbance density at 300K (4\%) is approximately the same as the disturbance density at 330K (6\%), whereas at 390K, the disturbance density is about 16\%. One can conclude, that not only is the disturbance accumulation non-linear with
respect to cycles, but also non-linear with respect to temperature. However, in general, it is clear that temperature has a marked effect on damage accumulation.

Figure 16.2 shows the disturbance distributions in the critical solder ball (ball 13) at the critical and ultimate cycles for temperatures of 330K and 390K. Comparing the disturbance distributions, it can be seen that an increase in temperature does not significantly change the area in the solder ball where disturbance accumulates. Although the cycles to critical and ultimate disturbance densities are reduced as temperature is increased, the disturbance accumulation is similar. As found in Chapter 13, for vibration when temperature is not considered (room temperature), the disturbance accumulates in a globular blob at the bottom right corner of the solder ball. The accumulation is located slightly away from the PWB/solder-ball interface. This indicates that a fatigue crack would initiate and grow similar to that shown in Figure 16.4. Further examination of Figure 16.2 shows that a significant area of the solder ball is above a disturbance of .7. This is significantly different from the disturbance accumulation found in the thermal analyses done previously, where the disturbance in the body of the solder ball was very low. This indicates that under vibration fatigue, the electrical conductivity of the critical solder ball (ball 13), is reduced by microfracturing in addition to a well defined macro crack. This is important, since during experimental testing, the failure of a package is determined by an increase in resistivity or a set number of electrical interruptions. As discussed in Chapter 10, microfractures will elevate the resistance of a solder ball, and are correlated to occur at a disturbance of .5 to .7. Based on the analyses performed for both thermal and vibration loadings, microfractures will only be significant for vibration
fatigue. This may be a result of the different loadings. For example, vibration fatigue (high cycle) is loosely regarded as more of an elastic fracture compared with thermal cycling (low cycle). Because of this, microfractures are more likely to be generated in a vibrational environment. This is because the material is in a relatively more elastic state compared with thermal cycling. During thermal cycling, the solder ball is more likely to be in a highly ductile state because of the increased temperature. This can be seen by comparing the areas in the solder ball at a disturbance of .7. As the temperature is increased, the area of disturbance equal to .7 actually decreases. This is shown in Figure 16.3 where the outline of areas with a disturbance greater than .7 are shown. It can be seen that the largest area, the blue line, corresponds to the vibration at room temperature. The green line corresponds to 330K, and the red line corresponds to 390K. In summary, it may be expected that vibration might cause more microfractures in room temperature environment (300K) as compared to an elevated temperature environment. However, because solder is so ductile, these microfractures do not have the opportunity to grow into large macro fractures easily. Therefore, a larger area of disturbance is seen at lower temperatures compared to higher temperatures. However, the highest levels of disturbance are seen at the highest temperatures. This is caused by the reduction in material properties and CTE mismatch strains.
Figure 16.1. Graph of disturbance density at various cycles.
Figure 16.2. Contour plots of disturbance at critical (11%) and ultimate (17%) cycles
Figure 16.3. Outline of area with disturbance > .7, taken from Figure 16.2 at $D_{\text{dens}} = 11\%$. Blue = 300°K, Green = 330°K, Red = 390°K
Figure 16.4. Solder ball showing fatigue crack.
17.1 Contributions

The contributions of this research include the following. A heat transfer code has been written and implemented into the DSC-SST2D FEA code. This significantly increases the power of the code, as previously, only isothermal temperature problems could be analyzed. With the heat transfer capability, a new class of problems can be analyzed using the DSC method, namely, non-isothermal temperature problems.

With the addition of heat transfer to the DSC-SST2D code, non-isothermal temperature loadings on an electronic package were investigated. This had not been done before using the DSC-SST2D code, as all previous electronic package investigations assumed isothermal conditions.

A coupled analysis, where a temperature was applied concurrently with a quasi-dynamic vibration, was also investigated. This had not been done before using the DSC-SST2D code. In fact, this coupled type of investigation could not be found in the literature. However, two papers [6, 16] were found which analyzed this type of behavior by an uncoupling of the loads. That is, the applied temperature and vibrational loadings were not applied concurrently. Both papers applied the loads separately, then used Miner’s rule to superpose the damage from the temperature and vibration loadings. This is not correct, since Miner’s rule is based on a linear superposition principle.
17.2 Conclusions

The disturbed state concept has been utilized in the finite element analysis of a 313 PBGA electronic package. Specifically, the second level solder balls have been analyzed for fatigue failure under thermal cycling (isothermal and power cycling), quasi-vibration, and coupled quasi-vibration with different constant temperatures. In addition, voids were introduced into the critical solder balls and analyzed for fatigue under quasi-vibration and thermal cycling.

A heat transfer subroutine was written and incorporated into a preexisting finite element code, DSC-SST2D [33]. The subroutine allows thermal cycling with heat transfer, including steady state and transient cases. In addition, temperature, flux and convection may be prescribed concurrently in the model. The stress and damage are calculated at each step in the temperature cycle, allowing accurate non-linear analyses to be performed.

The package used in the research was extensively tested under thermal chamber temperature cycling by the JPL consortium, which consisted of numerous commercial, military, and governmental agencies. Based on the extensive experimental testing performed by the JPL consortium, a high quality calibration of the model was attained. The calibration of the model (Chapter 10) showed that the package failures were best presented as a statistical percentage. Based on a disturbance density of 11% (critical) in the solder ball, there would be an 11% chance of the solder ball failing. With a disturbance density of 17% (ultimate), the chance of failure is 100%. This gives electronic package analysts a safety range to work with. Applying the critical and
ultimate disturbance densities to a thermal cycle not used in the calibration (Chapter 11), it was found that the analysis was nearly 90% accurate compared with the experimental failures. The critical solder ball was found to be ball five, but that balls one through four were also almost as likely to fail. This result was experimentally verified by the JPL testing. The experimental results showed that the balls under the die were most likely to fail. The FEA analysis also showed that under thermal chamber temperature cycling, it is not necessary to include heat transfer. This is primarily a result of the extremely small dimensions of the package. Because the package is heated on all boundaries, it only takes approximately 24 seconds to achieve thermal equilibrium. This result has also been found by other researchers, and hence FEA thermal chamber analyses are always performed without heat transfer. This is not the case for FEA of testing done with internal heating, or power cycling tests (PCT). In this case, the die is powered on and off, and due to inefficient internal current flow, the die heats up. For this situation, it is necessary to perform a heat transfer analysis in addition to damage calculations. An analysis was performed (Chapter 12) on the 313 PBGA package under PCT conditions, with and without external heat-sinks. Because the package was not experimentally tested under these conditions, the FEA results could not be compared to experimental results. However, general conclusions can be made. It was found that the solder balls under the die were again most likely to fail. This is due to the inefficient heat transfer from the die. In other words, the balls under the die experience the highest temperatures. Heat-sinks were added to the top of the package, and showed that with heat sinks, the package was capable of much higher power levels in the chip. This has been well documented, and is
the reason heat-sinks are used. Keeping the cycles to failure constant, it was found that a power level of 2.5 times could be achieved with a heat-sink which removed 3 times the amount of heat as a bare package. With a heat-sink which removed 10 times the amount of heat, the power level could be increased 7.5 times.

A simplified vibration analysis was performed on the 313 PBGA package in Chapter 13, based on results from Raytheon testing. A very close agreement was found comparing the FEA results and the experimental results. It was found that the critical solder ball was the ball with the highest distance to neutral point (DNP). This was also found in the Raytheon experimental testing.

In Chapter 14, a void was introduced in the critical solder ball (ball 5) and thermally cycled under isothermal conditions. It was found that the void had a very slight impact on the cycles to failure. This has been documented by the JPL consortium, as well as other researchers who have performed experimental testing.

In Chapter 15, a void was introduced in the critical solder ball (ball 13) and vibrated without any temperature influence. It was found that the void had very little impact on the cycles to failure. Again, this has been documented by other researchers who have performed experimental vibration testing. The voids are not likely to cause a decrease in fatigue life because they increase the compliance of the solder joint. It has been well documented that as the compliance of the joint increases, the cycles to failure also increases.

In Chapter 16, the 313 PBGA package was subjected to quasi-vibration with different isothermal temperatures. It was found that as the temperature increases, the
cycles to failure decreases. This corresponds well other researchers results. Although there was no data to compare with, general conclusions can be made. Because the model corresponded well with independent back predictions for thermal cycling, and vibration testing, it seems likely that the results are accurate. Ball 13, the furthest DNP solder ball is the most likely to fail, even under high temperature excursions. This indicates that when a package is subjected to high levels of vibration, it is the vibration that will dominate the fatigue process. It also shows that it is the cycling of temperature that creates the fatigue mechanism when temperature is applied. In other words, simply applying a high temperature to a package does not, in itself, lead to failure of the solder balls.

A summary of the effect of vibration and temperature is shown in Figure 17.1. It is difficult to make comparisons between TCT and PTC analyses (or testing) because in a TCT test the temperature is varied between both (+) and (-) temperature extremes, whereas in a PTC test, the minimum temperature is room temperature. It is also difficult to compare vibration and thermal testing in terms of cycles to failure, because vibration failures occur in the millions of cycles, whereas thermal cycling occurs in the thousands of cycles. However, a comparison can be made using the results of Chapters 12, 13, and 16 in terms of hours to failure. The results are shown in Figure 17.1. It can be seen that vibration is more severe than the thermal testing. However, the vibration levels were extreme and represent values designed to accelerate failures in a testing environment. Thus, they are not typical. In contrast, the temperature extremes used in the PTC and TCT testing shown in Figure 17.1 are probably realistic of an actual operational
environment. Thus, Figure 17.1 is not an accurate comparison. For future work, a typical vibration environment should be compared with the thermal cycle used in Chapter 12.

17.3 Recommendations

Because this is the first application of the DSC to vibrational modeling of solder (at the University of Arizona), the disturbance parameters used were obtained from thermal cycling tests. Although the FEA analysis showed a very good correlation with experimental testing, more research should be performed to ascertain disturbance parameters for solder under vibration. This will be very difficult, as vibration data for solder is very difficult to obtain. As mentioned in Chapter 3, vibration fatigue of solder has not been addressed in nearly the same volume as thermal fatigue.

Power cycling has also not been addressed in the literature as much as thermal chamber cycling. Because of this, it is very difficult to compare FEA analysis with PCT experimental testing. This is primarily due to the cost of equipment and labor. However, it is a very effective method of determining the failure of a package under operational use. Therefore, it is recommended that more research be performed, specifically PCT testing, so that the FEA results can be verified against experimental results.
Figure 17.1. Time to first failure comparison of PTC thermal cycle and vibration
CHAPTER 17

REFERENCES


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