APPLICATION OF THE HEAT ENGINE FRAMEWORK TO MODELING OF LARGE-SCALE ATMOSPHERIC CONVECTION

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Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copy of the dissertation to the Graduate College.

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DEDICATION

To Allan Taini

And to those who struggle for human freedom in this world plagued by the evils of religion, capitalism and nationalism.

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ABSTRACT

The heat engine framework is examined in terms of large-scale atmospheric convection in order to investigate several theoretical and modeling issues related to the steady-state convecting atmosphere. Applications of the heat engine framework to convective circulations are reviewed. It is shown that this framework provides fundamental insights into the nature of various atmospheric phenomena and estimates of their potential intensity. The framework is shown to be valid for both reversible and irreversible systems; the irreversible processes' sole effect is to reduce the thermodynamic efficiency of the convective heat engine. The heat engine framework is then employed to demonstrate that the two asymptotic limits of quasi-equilibrium theory are consistent. That is, the fractional area covered by convection goes to zero, $\sigma \rightarrow 0$, as the ratio of the convective adjustment to large-scale time scale (e.g. radiative time scale) go to zero, $\frac{\tau_{ADJ}}{\tau_{LS}} \rightarrow 0$, despite recent arguments to the contrary. Furthermore, the heat engine framework is utilized to develop a methodology for assessing the strength of irreversibilities in numerical models. Using the explicit energy budget, we derive thermodynamic efficiencies based on work and the heat budget for both open (e.g., the Hadley circulation) and closed (e.g., the general circulation) thermodynamic systems. In addition, the Carnot efficiency for closed systems is calculated to ascertain the maximum efficiency possible. Comparison of the work-based efficiency with that of the efficiency based on the heat budget provides a gauge for assessing how close to reversible model-generated circulations

are. A battery of experiments is carried out with an idealized GCM. The usefulness of this method is demonstrated and it is shown that an essentially reversible GCM is sensitive (i.e., becomes more irreversible) to changes in numerical parameters and horizontal resolution.

INTRODUCTION

1.1 Research Problem

The treatment of atmospheric motion, from small to large-scale, as a simple thermodynamic heat engine has had a long history in meteorology (Brunt 1941; Riehl 1950; Kleinschmidt 1951; Lorenz 1967; Emanuel 1986). In natural convective systems, air is heated at high and cooled at lower pressure resulting in work that is available to drive atmospheric motions. This is the essence of the atmospheric convective heat engine. How efficiently the atmospheric heat engine drives the motion has been the source of a great deal of debate within meteorology. Some authors (e.g., Lorenz (1967), Peixoto and Oort (1992)) have argued that the thermodynamic efficiency of the general-circulation heat engine is of the order of a few per cent. Others have claimed that this estimate is too low and the actual atmospheric efficiency is greater (e.g., Wulf 1952, Michaud 1996a,b). The accurate determination of this efficiency is one of the fundamental observational and theoretical problems of atmospheric energetics, according to Lorenz (1967).

How thermodynamically efficient atmospheric motions are is intimately tied to how close to reversible these motions are. The heat engine framework applied to many atmospheric phenomena assumes reversible thermodynamics (Emanuel 1986, 1988 and 1999; Emanuel and Bister 1996; Rennó and Ingersoll 1996). It has been used to examine large-scale tropical convection in quasi-equilibrium and to derive theoretical values for parameters governing planetary-scale convection (Michaud 1995a,b; Emanuel and Bister 1996; Rennó and Ingersoll 1996; Craig 1996). It has also been employed for small-scale convective vortices such as dust devils and waterspouts to estimate their maximum wind speeds (Rennó et al. 1998; Rennó and Bluestein 2001).

In this dissertation, the heat engine framework is applied to large-scale atmospheric circulations. To begin with, applications in the literature of the heat-engine framework to convective circulations and small-scale convective vortices are reviewed. It is shown that this framework provides fundamental insights into the nature of atmospheric circulations and gives estimates of their maximum potential intensity. Both reversible and irreversible processes are demonstrated to be valid within the heat engine framework. The work carried out in this dissertation is motivated by several objectives, both theoretical and applied. The first objective, principally within the theoretical realm, is to demonstrate that the heat-engine framework is entirely consistent with the time/space scale separation in "quasi-equilibrium theory" as originally proposed in the seminal work of Arakawa and Schubert (1974). A second objective, which has both theoretical and applied aspects, is to employ the heat engine framework to develop a methodology for assessing the strength of irreversibilities in numerical models. This methodology is based on comparison of thermodynamic efficiencies which are developed, from first principals, for *both* open and closed atmospheric systems. As an example or "proof of concept", this methodology is applied to an idealized general circulation model.

1.2 Review of Relevant Literature

1.2.1 The Heat Engine Framework and Quasi-Equilibrium Theory

The notion that the production of convective instability by large-scale forcing is balanced by its consumption in an ensemble of convective clouds has had a long and very important influence in the atmospheric sciences. It has typically been referred to as the "quasi-equilibrium theory" and has been most closely associated with the work of Arakawa (1969) and Arakawa and Schubert (1974). However, it has also served as the basis for theoretical work (Emanuel et al. 1994, Rennó and Ingersoll 1996), numerical/modeling studies (Ogura and Kao 1987; Kao and Ogura 1987) and has been tested in observational studies (Lord and Arakawa 1980; Lord 1982; Xu and Emanuel 1989; Brown and Bretherton 1997).

The underlying principle of quasi-equilibrium theory is that the time scale of convection motions is much shorter than that of the large-scale forcing with which it interacts. If the time scale for large-scale forcing, τ_{ls}, due to changes in sensible or latent heat fluxes, radiative forcing, large-scale uplift, or frictional heating etc., is much longer than the time scale, τ_{adj}, for an ensemble of cumulus clouds to adjust the atmosphere back towards a neutral state, then the statistics of the cumulus ensemble can be described by the large-scale forcing at the same instant. This, in fact, is the essence of the cumulus parameterization problem (Arakawa and Schubert 1974). If the cumulus ensemble cannot be described in terms of the properties of the large-scale environment, then there can be no parameterization of sub-grid scale cumulus convection in terms of the resolved large-scale variables.

Rennó and Ingersoll (1996) utilized quasi-equilibrium theory in their development of the heat engine theory for large-scale tropical convection. They argued that in steady state, the convective motions generated by work from the convective heat engine are just strong enough to balance mechanical dissipation. Therefore, knowing the rate of dissipation of large-scale convective circulations provides a measure of the amount of work produced by the convective heat engine in steady state. Locally, the energy available for convection is related to the convective available potential energy (CAPE). Following the time/space scale separation arguments of Arakawa and Schubert (1974), Rennó and Ingersoll (1996) assumed a state of radiative-convective equilibrium in which an ensemble of cumulus clouds is in near balance with the large-scale radiative forcing. This implies that there is near balance between the rate of production of convective instability by radiative forcing and its consumption by the cumulus ensemble. The authors used this fact to estimate the statistical equilibrium quantity of CAPE present in the Earth's atmosphere among other parameters governing large-scale convection.

There have been authors who have taken issue with Arakawa and Schubert's quasi-equilibrium theory. Mapes (1997) has argued that the space/time separation of the quasi-equilibrium theory is artificial. He believes that it is not correct to view ensembles of cumulus activity as necessarily small scale responding to some poorly defined "large-scale" forcing. There is, he argues, a great deal of interaction between spatial scales. The rigid separation into large-scale and small scale can be partially attributed to the "functional definition of large and small scales as gridscale and subgridscale" associated with general circulation models and do not necessarily have any bearing on physical reality.

More recently, Yano (1999), Yano et al. (2001) and Yano (2003) have also called into question the validity of the time-space scale separation of quasi-equilibrium theory. Specifically, Yano (1999) has attempted to show that the asymptotic limits ($\sigma \rightarrow 0$ as $\tau_{adj}/\tau_{ls} \rightarrow 0$) put forward by Arakawa and Schubert (1974) are inconsistent for large-scale tropical convection. He argues that contrary to what would be expected, smaller scale motions are not characterized by shorter time scales. And, therefore, the quasi-equilibrium principle is least satisfied as the fractional area covered by convective drafts, σ, becomes very small. Yano (1999) arrives at this conclusion by following the scaling arguments used in the original Arakawa and Schubert (1974) article. His results suggest that the time scale of the convective ensemble, τ_{adj}, varies proportionally to the inverse of the fractional area covered by convection, σ. The physical reasoning, he argues, is that at smaller scales, (σ small), the cumulus ensemble becomes less efficient in adjusting to large-scale forcing. He makes the argument that the inconsistency in the space/time scale separation is "formally established" by the heat engine theories put forth by Rennó and Ingersoll (1996) and Emanuel and Bister (1996).

In the papers contained in Appendix B and Appendix C, we present theoretical arguments from the heat engine framework which contradict the arguments put forth by Yano (1999). We work through the definitions of the various time-scales in accordance with the heat engine framework for large-scale tropical convection. We demonstrate that the quasi-equilibrium theory as originally framed by Arakawa and Schubert (1974) is perfectly consistent with the heat engine theory for large-scale tropical convection.

1.2.2 The Heat Engine Framework and Thermodynamic Efficiency of a General Circulation Model

A question to investigate in studies of atmospheric motions is just how much of the available work is used to generate kinetic energy; that is, what portion of the heat input is converted into atmospheric motions of various scales? Much of the previous work, which is briefly reviewed here, has considered this question for a steady-state atmosphere and principally for the general circulation of the atmosphere. This question is directly related to the efficiency of the atmospheric heat engines or, stated in another way, how irreversible the atmospheric heat engines are.

In theoretical studies by Rennó and Ingersoll (1996), Emanuel and Bister (1996), Michaud (1995a,b) and Craig (1996), large-scale, steady-state tropical convection is idealized as a reversible heat engine. This framework permits the prediction of buoyancy, vertical velocity and fractional area covered by convection for steady-state, large-scale convection. On the mesoscale, Emanuel (1986, 1988, 1999) used the reversible heat engine to predict successfully the maximum intensity of hurricanes. A fundamental assumption underlying these studies was that dissipation of kinetic energy is the dominant irreversible source of entropy. Other sources of entropy such as those due to the water cycle (e.g., evaporation, diffusion and dissipation due to precipitation) were ignored.

Several recent studies have called into question the use of reversible thermodynamics for determining the atmospheric entropy budget (Paulius et al., 2001; Paulius and Held a,b). Specifically, Paulius et al., (2001) have argued that the heat engine theories of Rennó and Ingersoll (1996) and Emanuel and Bister (1996) greatly overestimate the available work generated by the atmosphere's convective heat engine. Their studies, discussed in detail below, has focused on the role of irreversibilities associated with the water substance in decreasing the available work produced by the reversible atmospheric heat engine.

Paulius et al., (2000) and Pauluis and Held (2001 a,b) argue that, in addition to dissipation of kinetic energy in turbulent cascades to smaller scales, there is a substantial fraction of dissipation of available work from the convective heat engine that occurs around falling hydrometeors. The work of Rennó and Ingersoll (1996) and Emanuel and Bister (1996) assume that the turbulent cascade of energy is the dominant irreversible process and, hence, the principal source of entropy in determining the entropy budget. By neglecting sources of entropy related to the diffusion of heat, water vapor and other irreversible sources in the entropy budget, Rennó and Ingersoll (1996) and Emanuel and Bister (1996) were able to calculate the strength of large-scale, steady-state tropical convection as noted above.

Pauluis et al. (2000) argue from theory and numerical modeling that the work generated by the reversible heat engine is not solely dissipated in convective-scale turbulence, but that frictional dissipation around hydrometeors may consume a great deal of the available work. In addition, they state that processes associated with irreversible phase changes and diffusion of water vapor greatly limit the amount of work available for driving convective motions. They estimate, from a cloud ensemble model, that the maximum work for generating kinetic energy from the tropical convective heat engine is approximately 10 W/m^2. This value comes from assuming a thermodynamic efficiency of the atmosphere of 0.1 and a heat flux of

100 W/m^2.

To estimate the amount of work consumed by frictional dissipation around falling hydrometeors, Paulius et al., (2000) determine a precipitation pathlength of 5 to 10 km. Assuming the total rate of dissipation due to precipitation is proportional to the rate of precipitation reaching the surface, they estimate the dissipation due to hydrometeors as 2-4 W/m^2. Although this is small compared to their assumed tropical value of 100 W/m^2 from latent heating, it is very close to previously estimated values of kinetic energy dissipation due to turbulent cascades of energy (Lorenz 1967; Peixoto and Oort 1992). Their conclusions imply that dissipation due to turbulence should not be considered the principal irreversible source of entropy. Only a small fraction of the available work from the convective heat engine is dissipated in turbulent motions. These results call directly into question the reality of treating steady-state convection as a reversible heat engine.

Emanuel and Bister (1996) hold that the irreversible sources of entropy neglected in the reversible heat engine are small. They, in fact, estimate the neglected frictional dissipation due to falling hydrometeors as less than 5% of the work generated by the reversible convective heat engine. This is a much smaller percentage than the approximately 30% of available work consumed by falling hydrometeors found in the modeling study of Pauluis et al. (2001). The implication that precipitating convective systems are very irreversible is also not supported by the work of Emanuel (1986, 1988 and 1999) in which he idealized hurricanes as reversible convective heat engines. The results from these studies predict, reasonably well, the intensity of highly precipitating hurricanes suggesting that dissipation in the vicinity of hydrometeors and irreversible processes associated with water and its phase changes are not critical.

The determination of the work dissipated around falling hydrometeors may also not be a trivial matter (Rennó 2001). It has been pointed out that viscous dissipation acting on hydrometeors is a function of the Reynolds number (Pruppacher and Klett 1997; Rennó 2001; Lorenz and Rennó 2002). Lorenz and Rennó (2002) and Rennó (2001) note that at low Reynolds numbers, $Re \leq 1$ (in the cloud droplet regime), viscous forces become important. The kinetic energy associated with slowly falling cloud droplets is dissipated and converted into heat by molecular viscosity in adjacent shear zones. This momentum dissipation adds no velocity to the cloudy air parcel and, therefore, does not contribute to accelerating convective downdrafts. For high Reynolds number regime, $Re \gg 1$, the hydrometeors continually add momentum to the air parcel, accelerating the downdrafts. These downdrafts are then dissipated through convective turbulence; that is, in the manner suggested by the reversible heat engine framework of Rennó and Ingersoll (1996) and Emanuel and Bister (1996).

The work of Pauluis et al. (2000) and Pauluis and Held (2001 a,b) does not distinguish in any way between the low and high Reynolds number regimes. Moreover, the cloud ensemble model employed in their studies does not explicitly calculate the amount of frictional dissipation that goes into localized heating and that which goes into accelerating the downdrafts. Furthermore, the resolution of their model is only 2km, while convective drafts are typically on the order of 100m with only a small percentage on the order of 2km (Zipser and LeMone 1980; LeMone and Zipser 1980). Given these important limitations in their modeling study, it is very difficult to assess the veracity of their conclusions.

Rennó (2001) has suggested that the strength of the irreversibilities emphasized in these studies may be due to the dissipative nature of numerical models, particularly of moist convection, and not to the irreversibility of natural convective heat engines. This provides a strong motivation for investigation of the nature of irreversibilities in numerical models. In addition, it is known that the results from numerical models are very sensitive to model resolution. In Appendix D, we develop a framework for examining the strength of irreversibilities in an idealized general circulation model in order to gain insight into the question of how reversible numerical models are. Several experiments are performed in order to examine the sensitivity of a numerical model to changes in different parameters. Included in these experiments is an examination of how horizontal resolution impacts the strength of irreversibility generated in model circulations.

1.3 Dissertation Format and Contents

In accordance with the policy established by The University of Arizona Graduate Council in January, 1992, published and submitted manuscripts serve as the primary component of this dissertation. The four manuscripts, the first three of which have been published, and the fourth submitted for publication, are attached as appendices A, B, C and D. Authorship is shared with Nilton Rennó presently of the Department of Atmospheric, Oceanic and Space Sciences at the University of Michigan. Dr. Rennó has been the primary advisor for this research and has provided guidance for the direction the research has taken. Nevertheless, all other aspects of this research including the writing of the manuscripts and dissertation represent the original work and contribution of David K. Adams.

The published manuscript in appendix A represents a formal literature review of applications of the heat engine framework to meteorological phenomena. These phenomena range in size from small-scale convective vortices, such as dust devils, to planetary-scale convection. Topics relevant to the work of this dissertation are also reviewed. A more detailed review, however, of the work relevant to the large-scale convection investigated in this dissertation has been covered in the Introduction to this dissertation. Appendices B and C are two published manuscripts based on the application of the heat engine framework to large-scale, steady-state tropical convection. The work described in these manuscripts examines the consistency between the time/space scale separation in the quasi-equilibrium theory of Arakawa and Schubert (1974) and the time/space scale separation of the heat engine theory proposed by Rennó and Ingersoll (1996). The submitted manuscript in appendix D applies the heat engine framework in the context of an idealized general circulation model. Thermodynamic efficiencies are developed for both open and closed systems which are used to determine the strength of irreversible processes in an idealized general circulation model.

PRESENT STUDY

2.1 Summary

The complete theory, methods, results and conclusions are found in the four manuscripts appended to this dissertation. In this section, the most important findings are summarized.

The heat engine framework has been applied to large-scale atmospheric convection for probing into both theoretical and modeling questions in the atmospheric sciences. Firstly, arguments are presented to demonstrate that the heat engine framework is entirely consistent with the time/space scale separation between convective motions and large-scale forcings as originally developed in Arakawa and Schubert (1974). The heat engine framework for large-scale, steady-state convecting atmospheres as proposed by Rennó and Ingersoll (1996) demonstrates that the fractional area covered by convection is proportional to the ratio of the convective adjustment time scale to that of the large-scale forcing; that is, $\sigma \approx \frac{\tau_{adj}}{\tau_{ls}}$. Therefore, Yano's (1999) argument that τ_{adj} increases with decreasing fractional area covered by convection is not correct in terms of the heat engine theory. Instead, the adjustment time scale, τ_{adj}, decreases with decreases in the fractional area covered by convection due to the increased energy flux per convective draft.

The results of the heat engine framework follow from the assumption of an atmosphere in radiative-convective equilibrium. This leads to the use of a radiative relaxation time for the large-scale time scale, τ_{ls}. Arakawa and Schubert (1974) make no estimate of τ_{ls} other than to assume it is much larger than the adjustment time scale, τ_{adj}, of an ensemble of cumulus clouds. In using the radiative relaxation time scale, the space/time scale separation as proposed by Arakawa and Schubert (1974) clearly follows and, therefore, the heat engine framework does in no way "formally establish" the inconsistency in scale separation investigated by Yano (1999).

A second application of the heat engine framework in this dissertation is to develop an original methodology for gauging the strength of irreversibilities in a general circulation model (GCM). This methodology is based on the comparison of three thermodynamic efficiencies: one based on work, one on the heat budget and the third, the Carnot efficiency. Development of this methodology is for both open and closed thermodynamic systems, a novel approach in the atmospheric sciences.

As a proof of concept, this method is applied to an idealized GCM. A series of experiments are presented in order to demonstrate the efficacy of this method. The results show that the GCM is sensitive to changes in model parameters and, in particular, is sensitive to changes in horizontal resolution. The model sensitivity is reflected in changes in irreversibility; that is, separation between the work-based and reversible thermodynamic efficiencies. For example, there is a measurable increase in reversibility with decreases in model resolution. The motivation for developing this method has been to help us to understand how reversible, even very simply forced, models can be.

For future work, this methodology is to be employed for a moist model in order to ascertain the strength of irreversible entropy sources associated with the water substance. In addition, this framework could easily be adapted for use with assimilated data in order to compare directly models with data from the real world.

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APPENDIX A

THE CONVECTIVE HEAT ENGINE

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Abstract

We review recent applications of the heat-engine framework to convective circulations and small-scale convective vortices. We show that this framework provides fundamental insights into the basic nature of various atmospheric phenomena and provides estimates of their potential intensity. We show that the framework is valid for both reversible and irreversible processes, and that irreversible processes solely reduce the thermodynamic efficiency of the reversible convective heat engine. Results from this framework applied to convective vortices, both dust devils and waterspouts, show very good agreement with observational data when the heat engine is assumed to be reversible. Application to larger scale convective circulations, such as the Hadley Cell, also support the validity of the heat-engine framework, even when irreversible processes are neglected.

A.1 Introduction

Convection in the atmosphere gives rise to many natural phenomena, ranging from planetary (*e.g.*, the Hadley circulation) to microscale turbulence. Convection also plays an extremely important role in the global energy budget and the general circulation of the atmosphere. One approach to examining convection in the atmosphere is to utilize the heat-engine framework. In this paper, we review a body of literature that applies the heat-engine framework to various atmospheric phenomena. We begin with an examination of the heat-engine framework applied to large-scale convecting atmospheres in quasi-steady state or quasi-equilibrium. We show how the heat-engine framework is relevant to quasi-equilibrium theory and provides a theoretical estimate for the values of convective available potential energy (CAPE) and the fractional area of convection. In addition, in light of the results from the heat-engine framework, we address recent controversies related to the original formulation of the quasi-equilibrium theory (Arakawa and Schubert, 1974, hereinafter A-S, 1974). We then focus on several smaller scale phenomena to which the heat-engine framework has been applied, including dust devils, topography-induced circulations, and waterspouts. Estimates of the strength of the convective vortices and circulations attest to the generality of this framework. Finally, we address irreversible processes in natural convection and comment on a recent controversy related to the validity of the heat-engine framework for precipitating systems.

A.2 Large-scale Atmospheric Convection

A.2.1 Observations

The heat-engine model is not new to the atmospheric sciences. Brunt (1941) and Lorenz (1967) used the heat-engine concept to explain the Earth's general circulation. They argued that the thermodynamic efficiency of the general-circulation heat engine is of the order of one per cent. The accurate determination and explanation of this efficiency constitutes the fundamental observational and theoretical problem of atmospheric energetics, according to Lorenz. The heat-engine framework has been applied to monsoon circulations (Shuleikin, 1953) and also to the steady-state hurricane (Riehl, 1950; Kleinschmidt, 1951; Emanuel, 1986).

In this paper, we review various studies that describe atmospheric convection as a simple heat engine operating in planetary atmospheres at several different scales, ranging from global scale to that of the dust devil. At the large scale (sections A.2 and A.3), the heat-engine framework provides theoretical estimates of convective available potential energy, in addition to fractional area covered by convection. We discuss this in the context of the quasi-equilibrium framework put forth by A-S (1974). In section A.4, we examine the application of the heat-engine framework to natural heat engines, including convective vortices (*e.g.*, dust devils) and terrain-induced circulations. In section A.5, we conclude with thoughts on the irreversible heat engine and the dissipative nature of numerical models.

A.2.2 The Heat-Engine Framework

For non-equilibrium thermodynamic conditions, an idealized air parcel will either expand or contract. This will result in work done on or by the environment. In any atmospheric system, in general, the work due to expansion is the most relevant kind of work, apart from mechanical or frictional dissipation. In areas where convection is occurring, heated air parcels expand, thereby performing work on the environment. Further work is done at the expense of the thermal energy of the updraft air during its moist adiabatic expansion. Energy is then expended in compressing the mixture of cooled air and condensate, while a smaller amount of heat is rejected to the environment.

The net result of this convective cycle is that heat is taken from the surface layer (the hot source) and a portion of it is rejected to the free troposphere (the cold sink), while the balance is transformed into mechanical work. This mechanical work is expended in the maintenance of the convective motions against mechanical dissipation. The energy dissipated by mechanical friction is ultimately converted to heat, some of which is radiated to space, while the remainder is recycled by the convecting parcels. In the case of the large scale (e.g. synoptic scale), the convective motions are just strong enough so that the net work done by the convective heat engine is used exclusively to overcome the mechanical dissipation of energy. Under these conditions, a statistical steady state or quasi-equilibrium state exists (see Rennó and Ingersoll, 1996).

The nature of the mean state of a convectively adjusted atmosphere is not clearly understood. In the case of the Earth's atmosphere, in both theoretical and modeling studies, it has been assumed that superadiabatic temperature lapse rates are, to a first-order approximation, adjusted to dry adiabatic lapse rates. That moist convection adjusts moist unstable lapse rates to moist neutral ones is far from clear. In fact, the nature of the mean atmospheric state resulting from moist adjustment is controversial. Rennó and Ingersoll (1995, 1996), Michaud (1995), and Emanuel and Bister (1996) looked at planetary convection from a large-scale perspective, that is, in quasi-equilibrium, in order to shed light on the nature of the convectively adjusted atmosphere. By describing atmospheric convection as a natural heat engine, they focused on the thermodynamical process by which convection adjusts unstable atmospheres. Their main objective was to present a framework useful for the basic conceptual understanding of the equilibrium state of convecting atmospheres. In the next several sections, we examine the nature of convecting atmospheres in quasi-equilibrium.

A.2.3 The Quasi-Equilibrium Idea

The quasi-equilibrium theory of A-S (1974) for large-scale convection has provided the basis for many theoretical and modeling studies. The essence of this theory is that the production of convective instability by large-scale forcing (*e.g.*, large-scale sensible or latent heat flux, radiative heating/cooling, *etc.*) is balanced by its consumption by an ensemble of convective clouds. On the convective scale, the energy available for atmospheric convection is proportional to the local value of the convective available potential energy (CAPE). Quasi-equilibrium theory states that an ensemble of convective clouds over a large area consumes CAPE at such a rate as to maintain the large-scale tendency of CAPE at approximately zero.

Although the time rate-of-change of CAPE is nearly zero, the absolute value of CAPE is non-zero. Some authors have argued that the tropical atmosphere is very nearly convectively adjusted, that is, neutral to the ascent of moist parcels (Betts, 1982; Xu and Emanuel, 1989). This implies near-zero values of CAPE. A-S (1974) also make this assumption in their formulation of the quasi-equilibrium theory. We argue in section A.3.2 that this is due solely to *their* original formulation of the quasi-equilibrium theory and not to the theory itself. The heat-engine framework, on the other hand, provides an estimate of the value of CAPE that is necessary in steady state, in order to overcome mechanical dissipation.

A.3 The Heat Engine Framework for Large-Scale Convection

In this section, we demonstrate that the heat-engine framework provides theoretical estimates of several important parameters in large-scale convection. For steady-state conditions, Rennó and Ingersoll (1996) use the heat-engine framework to analyze the energy cycle of convective systems on the large scale. Their approach was simply to follow a hypothetical convecting air parcel around a streamline in a closed cycle, assuming that convection is in statistical equilibrium. Therefore, the closed circulation around the streamline exists only in a statistical sense (see Figure A.1). Upon integration around the streamline, the energy cycle of the air parcel's convective heat engine is obtained. For a convecting air parcel, the energy equation may be arrived at by simply taking the dot product of the velocity vector and the equation of motion (Haltiner and Martin, 1957). With the aid of the ideal gas law, and the first and second laws of thermodynamics applied to moist air, we have that

$$Tds - d\left(\frac{1}{2}|\vec{v}|^2 + c_pT + L_vr + gz\right) - \vec{f}\cdot d\vec{l} = 0, \tag{A.1}$$

along a streamline, where T is the absolute temperature, s the specific entropy, $\vec{v}$ the vector velocity, c_p the heat capacity at constant pressure per unit mass, L_v the latent heat of vaporization of water per unit mass, r the water vapor mixing ratio, g the gravity acceleration, z the height above a reference level, $\vec{f}$ the frictional force per unit mass, and $d\vec{l}$ the incremental distance along the streamline. We assume the processes to be thermodynamically reversible. The assumptions of steadiness and closed circulation are irrelevant to the energetics of the heat engine.

Integrating Eq. (A.1) around a closed cycle, we get

$$\oint T ds - \oint \vec{f} \cdot d\vec{l} = 0, \tag{A.2}$$

which asserts that in steady state, frictional dissipation is balanced by the net heat input. Integrating the first law of thermodynamics, we get

$$\oint T ds = \oint p\, d\alpha, \tag{A.3}$$

where p is the pressure and α is the specific volume. Thus, the first term in Eq. (A.2) represents the net work done by the heat-engine cycle. Since the net work done by one cycle of the heat engine is equal to the total mechanical energy available for convection over one cycle, we define

$$TCAPE \equiv \oint T ds, \tag{A.4}$$

where TCAPE is the total convective available potential energy from a *reversible* heat engine over one cycle. This is the energy that can be converted to kinetic energy by a *reversible* heat engine. It includes the available energy that is converted to kinetic energy by both the updraft and the downdraft.

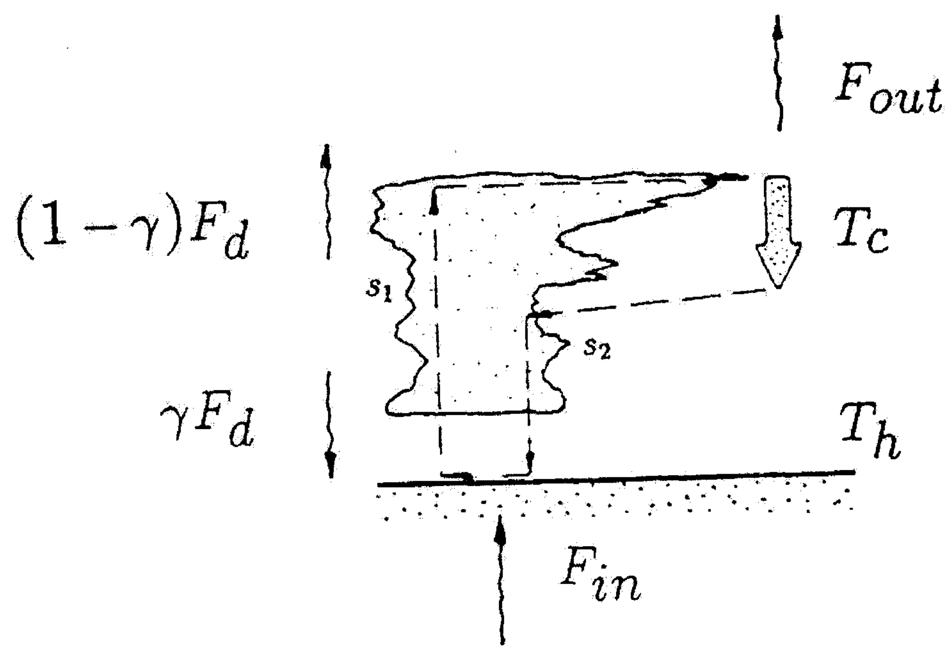

Figure A.1: An idealization of a large-scale convective system as a Carnot heat engine. The Carnot cycle consists of the heat input branch (isothermal expansion at T_h), the cold sink (subsidence region at T_c), and the adiabatic updraft and downdraft, S_1 and S_2, respectively. $F_{\rm in}$ is the heat input, $F_{\rm out}$ is the heat rejected, and $F_{\rm d}$ is the heat flux from mechanical dissipation. The cycle is closed only in a statistical sense for an ensemble of these convective systems (adapted from Rennø' and Ingersoll 1996).

If the convective circulation is idealized as a Carnot cycle, then Eq. (A.4) represents the area enclosed by the hot and cold adiabatics, s_1 and s_2, and the hot and cold isotherms, T_h and T_c. The isothermal expansion takes place at the bottom of the convective layer (the surface) and the isothermal compression at the top. The convective updrafts and downdrafts are represented by adiabats (see Figure A.1). The net work available for moving an air parcel around the convective cycle is represented by the area on a temperature-entropy diagram. This is the definition of TCAPE. This quantity is the energetically meaningful one in the convective circulation and should not be confused with CAPE, which represents only the work done by buoyancy forces in the updraft.

For a deep convective cloud, the cold-sink temperature, T_c, is approximated as the entropy-weighted mean temperature of the layer emitting infrared radiation. This can be expressed as follows:

$$T_c = \frac{1}{\Delta s} \int_{\text{out}} T ds, \tag{A.5}$$

where $\Delta s = \int_{\text{out}} ds = s_1 - s_2$ accounts for the excess entropy of the updrafts over the downdrafts.

It follows from Eqs. (A.2) and (A.4) that, for a general convective heat engine, we must have

$$\text{TCAPE} - \oint \vec{f} \cdot d\vec{l} = 0. \tag{A.6}$$

This asserts that, in quasi-steady state, friction is balanced by TCAPE. This statement is quite relevant for quasi-equilibrium theory, for it definitively states that zero total CAPE atmospheres are an impossibility unless convective motions are inviscid.

In order to understand the nature of this dissipation of mechanical energy, we can define a dimensionless coefficient of dissipation as

$$\mu \equiv \frac{\oint \vec{f} \cdot d\vec{l}}{w^2}, \tag{A.7}$$

where w is the magnitude of the convective velocity. Thus, from Eqs. (A.6) and (A.7), we get

$$w = \{\mu^{-1}\,\mathrm{TCAPE}\}^{\frac{1}{2}}. \tag{A.8}$$

From this it is clear that large μ will result in small convective velocities. For convective circulations, we can assume that the mechanical dissipation essentially results from turbulent viscosity $\nu_{\mathrm{turb}} \sim l\,\Delta w$, l and Δw being characteristic length and velocity scales of the eddy. As such, the coefficient of dissipation can be written as

$$\mu \sim \frac{(\nu_{\mathrm{turb}}\,\nabla^2\,w)\,l_{\mathrm{path}}}{w^2} \sim \frac{(\Delta w)^2\,l_{\mathrm{path}}}{w^2\,l.} \tag{A.9}$$

For a simple order-of-magnitude calculation of μ in large-scale convective circulations, we assume that the convective up- and down-drafts can be idealized as turbulent eddies in homogeneous and isotropic turbulence. In this case, a characteristic velocity scale is $\Delta w \sim 2\,w$. If we further assume a pathlength of $l_{\mathrm{path}} \gtrsim 4\,l$ (a lower bound for a shallow atmosphere), then the coefficient of dissipation of mechanical energy is given by $\mu \sim 20 - 50$.

In a rotating planet, the length of the integral path must be a function of the Rossby radius of deformation. Given that the present goal is solely to present an order-of-magnitude calculation, the above estimate suffices. A more sophisticated parameterization for μ is possible, and hence μ is the potentially tunable parameter of this scaling analysis.

A.3.1 CAPE and Fractional Area Covered by Convection

The heat-engine framework provides us with theoretical estimates of several important parameters for quasi-steady state convecting atmospheres or atmospheres in quasi-equilibrium. In this section, we look at convective available potential energy, CAPE, and fractional area covered by convective drafts, σ. In the previous section, we demonstrated that the mechanical dissipation of energy around a convective cycle in quasi-steady state is simply the value of TCAPE. From the value of TCAPE, we can easily estimate the order of magnitude of CAPE.

The quasi-equilibrium principle, originally elaborated by A-S (1974), can be applied to the simple case of a steady-state convecting atmosphere on the large scale, in which radiative heating/cooling is essentially the main forcing. Therefore, the production of TCAPE by radiative forcing is nearly the same as its rate of conversion to kinetic energy by an ensemble of convective clouds and the rate of dissipation of this kinetic energy. Thus, in quasi-equilibrium, all energy available from the convective heat engine is used for overcoming frictional dissipation of convective motions. For a grid-cell ensemble of convective clouds, this may be expressed as

$$M\ \overline{\mathrm{TCAPE}} - F_d = 0, \tag{A.10}$$

where M is the total convective mass flux, $\overline{\mathrm{TCAPE}}$ is a *mean* value total CAPE, and F_d is the total flux of energy mechanically dissipated by the ensemble of convective drafts.

For quasi-steady state, the near equality of energy dissipation by convection to that available from the heat engine gives

$$F_d = F_{\mathrm{av}}, \tag{A.11}$$

where F_{av} is the heat flux available for mechanical work by the ensemble of convective heat engines. Under this assumption, the energy available for the convective heat engine is entirely converted into mechanical energy. Ultimately, this mechanical energy is dissipated by mechanical friction.

In a thermodynamically reversible heat engine, we have that

$$F_{\mathrm{av}} = F_{\mathrm{in}} - F_{\mathrm{out}}, \tag{A.12}$$

where F_{in} and F_{out}, are, respectively, the heat fluxes *in* and *out* of the convective heat engine. F_{av} can be written as

$$F_{\mathrm{av}} = \eta F_{\mathrm{in}}, \tag{A.13}$$

where η is the mean thermodynamic efficiency of the ensemble of convective heat engines. The thermodynamic efficiency of a reversible heat engine is defined as

$$\eta \equiv \left(\frac{\int_{\mathrm{in}} T ds - \int_{\mathrm{out}} T ds}{\int_{\mathrm{in}} T ds} \right), \tag{A.14}$$

where the subscripts *in* and *out* represent, respectively, integration at the heat intake and at the heat output branches of the cycle. For the ensemble of convective heat engines in statistical equilibrium, we have that

$$\mathrm{TCAPE} \approx \left(\frac{1}{M} \right) \eta F_{\mathrm{in}}. \tag{A.15}$$

Given parameter values typical of the tropics, Rennó and Ingersoll (1996) arrive at a CAPE value ($\approx$ 1000 Jkg^{-1}) that is of the same order of magnitude as the observed value of CAPE in the tropics (Williams and Rennó, 1993).

In addition to estimates of a global value for CAPE, the heat-engine framework also provides an estimate for the proportion of the large scale that is covered by convective drafts. Observations of tropical convection suggest that only a small fractional area is covered by active updrafts. Using very simple physical arguments, the heat-engine framework provides a theoretical value of σ for quasi-equilibrium conditions.

To a first order, we can assume that radiative cooling of the troposphere is approximately balanced by the warming due to compression of subsiding air parcels. Assuming that both the temperature lapse rate and radiative cooling rate are approximately constant in the troposphere, the subsidence mass flux, M_R, is also approximately constant throughout the troposphere. Given these arguments, the temperature at which heat is rejected by the ensemble of convective systems is the entropy-averaged tropospheric temperature. Consequently, Eq. (A.15) represents the integral effect of an ensemble of convective systems of various depths extending throughout the troposphere. Mass conservation requires that the convective mass flux, M, be equal to the subsidence mass flux, M_R. Assuming Newtonian cooling, we get

$$M = M_R \approx \left(\frac{\Delta p}{g\tau_R}\right), \tag{A.16}$$

where τ_R is the radiative time scale and Δp is the troposphere's thickness. In steady-state conditions, the entropy excess of the convective updrafts over the downdrafts must be lost by the emission of infrared radiation to space by the subsiding air parcels [in between the top of convective updrafts and the root of the convective downdrafts (see Figure A.1)]. From the equation above, the subsidence is estimated as,

$$w_R \approx \left(\frac{\Delta p}{\rho g \tau_R}\right). \tag{A.17}$$

A rough estimate of τ_R can be obtained by considering a slab of atmosphere with pressure thickness Δp and uniform density radiating like a gray body (Houghton, 1986). In this case, one gets

$$\tau_R \approx \left(\frac{c_p \Delta p}{8 g \epsilon \sigma_R {T_c}^3}\right), \tag{A.18}$$

where ϵ is the slab's emissivity and σ_R is the Stefan-Boltzmann constant. Consistent with the derivations earlier in this section, the radiative cooling term must be computed using the entropy-weighted mean temperature of the subsidence region. Thus, T_c is the entropy-weighted mean temperature of the subsidence region. Note that, for the entire ensemble of convective drafts in quasi-equilibrium conditions, Δp is the thickness of the troposphere, and T_c is the entropy-weighted mean tropospheric temperature [see Eq. (A.5)].

It follows from Eq. (A.15) that

$$\eta F_{\text{in}} \approx \rho \sigma w \, \text{TCAPE}. \tag{A.19}$$

The mass continuity equation can be written as

$$\begin{aligned} \rho \, \sigma \, w &= \rho_R \, (1 - \sigma) \, w_R \\ w_R &\approx \sigma \, w, \end{aligned} \tag{A.20}$$

where we have assumed $\sigma \ll 1$, and that the density of the subsiding air parcel is $\rho_R \approx \rho$. From Eqs. (A.8), (A.19), and (A.20), we get

$$\eta F_{\text{in}} \approx \rho \, w_R (\mu \, w^2)$$

$$\eta F_{\text{in}} \approx \rho\,\mu \left(\frac{{w_R}^3}{\sigma^2}\right). \tag{A.21}$$

Substituting Eq. (A.17) into Eq. (A.21), we get

$$\sigma \approx \left(\frac{\mu}{\eta}\right)^{\frac{1}{2}} \left(\frac{\Delta p}{\rho\, g\, \tau_R}\right)^{\frac{3}{2}} \left(\frac{F_{\text{in}}}{\rho}\right)^{-\frac{1}{2}}. \tag{A.22}$$

For tropical values of the various parameters ($\eta \approx 0.1$, $F_{\text{in}} \approx F_L \approx 155\,\mathrm{W\,m^{-2}}$, $\Delta p \approx 8 \times 10^4\,\mathrm{Pa}$, $\rho \approx 1\,\mathrm{kg\,m^{-3}}$, $g = 9.8\,\mathrm{m\,s^{-2}}$, $\tau_R \approx 1.3 \times 10^6\,\mathrm{s}$, and $\mu \sim 20$), we get $\sigma \approx 5.0 \times 10^{-4}$. This appears to be a reasonable value, conforming with previously given estimates (Cotton and Anthes, 1989, Black *et al.*, 1994). With the above parameters, TCAPE $\approx$ 2500 $\mathrm{Jkg^{-1}}$, thus CAPE $\approx$ 1250 $\mathrm{Jkg^{-1}}$, which is of the order of magnitude of the observed value (Williams and Rennó, 1993).

Defining $\tau_c \equiv \left(\frac{\Delta p}{\rho\, g\, w}\right)$, we get

$$\sigma \approx \frac{\tau_c}{\tau_R}, \tag{A.23}$$

where for physically possible solutions, we must have $0 < \sigma < 1$. Since in the derivation of the above equation we assumed $(1-\sigma) \approx 1$, Eq. (A.23) is valid only for $\sigma \ll 1$ (that is, for $\tau_c \ll \tau_R$). Note that this theory predicts decreases in the fractional area covered by convective drafts with decreases in the convective timescale and increases in the radiative timescale. In the following section, we demonstrate that, for the steady-state convecting atmosphere, this formulation of σ is consistent with the original quasi-equilibrium framework.

A.3.2 Yano's Critique

The quasi-equilibrium theory proposed by A-S (1974) has proved useful in theoretical and modeling studies; however, a problem in the original formulation has come to light, indirectly, through a recent article by Yano (2000). Yano's critique of the A-S (1974) cumulus parameterization scheme states that the two asymptotic limits under which the A-S (1974) scheme is formulated are in contradiction; that is, both the fractional area covered by convective drafts, σ, and the ratio of the timescale of the large-scale forcing, τ_{LS}, and that of the convective adjustment, τ_{ADJ}, are small. That is, Yano argues that the two asymptotic limits in which $\sigma \to 0$ and $\tau_{ADJ}/\tau_{LS} \to 0$, are inconsistent. We show that this inconsistency that Yano has discovered results from the manner in which A-S (1974) chose to define the large-scale timescale and the convective timescale. We show, however, that for steady-state convecting atmospheres, Eq. (A.23) is consistent with the quasi-equilibrium theory.

The quasi-equilibrium idea is expressed in equation 140 from A-S (1974) as

$$\frac{dA}{dt} = \left(\frac{dA}{dt}\right)_{\mathrm{C}} + \left(\frac{dA}{dt}\right)_{\mathrm{LS}}, \tag{A.24}$$

which simply states that the time rate of change of the cloud work function, A, which for this discussion can be thought of as Convective Available Potential Energy (CAPE), is a balance between production of potential energy by large-scale forcing, $(dA/dt)_{\mathrm{LS}}$, and its removal by the ensemble of cumulus clouds, $(dA/dt)_{\mathrm{C}}$. Since quasi-equilibrium merely indicates that the large-scale flow is in quasi-steady state, in quasi-equilibrium, dA/dt must be approximately zero.

The quasi-equilibrium theory may be expressed qualitatively as the fact that, at large scales, the effective adjustment timescale of the ensemble of convective systems is nearly equal to the timescale of the large-scale processes. The heat-engine framework demonstrates clearly this relationship in Eq. (A.23) above. Given the near equality of the timescales for the adjustment of the large-scale fields by the ensemble of convection and that of the large-scale forcing, it is clear that

$$\tau_{\mathrm{ADJ}} \approx \sigma \tau_{\mathrm{LS}}. \tag{A.25}$$

Written in this form, $\tau_{\mathrm{ADJ}}/\sigma$ may be thought of as an effective adjustment timescale, the timescale for which the large scale is adjusted by the ensemble of convective clouds. A-S (1974) give the order-of-magnitude calculation

$$\left| \left(\frac{dA}{dt} \right)_{\mathrm{C}} \right| \sim \frac{A}{\tau_{\mathrm{ADJ}}}, \tag{A.26}$$

for the cumulus ensemble terms in their equation 146. This implies that the large scale is adjusted on the same timescale as that of a single deep convective updraft, τ_{ADJ}, defined by A-S as:

$$\tau_{\mathrm{ADJ}} \equiv H/w, \tag{A.27}$$

where H is the depth of the convective layer and w is a typical vertical velocity. In order for there to be equality between the right and left sides of Eq. (A.24), given A-S (1974) order-of-magnitude argument, they additionally assume that A itself must be nearly zero.

Since the convective drafts cover only a small fraction of the grid, the order-of-magnitude calculation from the convection ensemble term is

$$\left(\frac{dA}{dt} \right)_{\mathrm{C}} \sim \frac{-\sigma A}{\tau_{\mathrm{ADJ}}}. \tag{A.28}$$

The above equation is consistent with the prediction of the heat-engine framework (Rennó and Ingersoll, 1996). The use of the above equation in the quasi-equilibrium theory also eliminates the inconsistency that Yano (2000) points out. It follows from Eq. (A.23) that, in radiative-convective equilibrium,

$$\lim_{\tau_{\mathrm{ADJ}} \to 0} \sigma \approx \lim_{\tau_{\mathrm{ADJ}} \to 0} \frac{\tau_{\mathrm{ADJ}}}{\tau_{\mathrm{LS}}} = 0, \tag{A.29}$$

where $\tau_{\mathrm{ADJ}} = \tau_{\mathrm{c}}$ and $\tau_{\mathrm{LS}} = \tau_{\mathrm{R}}$. Therefore, the two asymptotic limits discussed in A-S (1974) are not in contradiction.

A.4 Natural Heat Engines

The heat-engine framework provides not only insights into fundamental physical processes of atmospheric convection, but also estimates of quantities of interest, such as, intensity of convective circulations. In this section, we examine the heat-engine framework for natural atmospheric phenomena, such as convective vortices and surface-heterogeneity induced circulations.

For the natural convective heat engine, we assume that the convective vortices (dust devils and waterspouts) are in steady-state, cyclostrophic balance and represent thermodynamically reversible processes. Using this framework offers the possibility of estimating the limiting potential intensity of convective vortices. A great deal of literature has already focused on the dynamics of these vortices (*e.g.*, Sinclair, 1969; Golden, 1974a,b). Here, we examine them solely from the thermodynamic perspective.

A.4.1 Convective Vortices

For a convecting air parcel, the energy equation may be arrived at by simply taking the dot product of the velocity vector and the equation of motion [see Eq. (A.1)]. For the steady-state convecting parcel, we have

$$d\left(\frac{1}{2}|\vec{v}|^2 + gz\right) + \alpha dp - \vec{f} \cdot d\vec{l} = 0 \ , \tag{A.30}$$

where the variables are as defined in Eq. (A.1), α is the specific volume, and p is the pressure.

For steady-state, conservation of mass requires that the circulation contained in a material volume be closed when moving with the material volume. Integrating the above equation over mass in this material volume, we have

$$\int_{\mathrm{m}} \alpha dp = \int_{\mathrm{m}} \vec{f} \cdot d\vec{l}, \tag{A.31}$$

where the integral is over mass for the entire material volume occupied by the convective system (normalized by its mass). As in the previous sections, we have, using the first and second laws of thermodynamics and defining the frictional dissipation as negative,

$$\int_{\mathrm{m}} T ds = - \int_{\mathrm{m}} \vec{f} \cdot d\vec{l}, \tag{A.32}$$

which again simply states that the net work available from the convective heat engine balances frictional dissipation of the convective motions. Equation (A.32) can also be expressed as a line integral around the convective cycle; that is,

$$\oint T ds = - \oint \vec{f} \cdot d\vec{l}. \tag{A.33}$$

In this analysis of convective vortices, we can make some simplifications. We assume a flat surface across which the air is flowing. Hence, we are neglecting any changes in potential energy along the surface inflow branch of the convective cycle. Furthermore, changes in kinetic energy are neglected as the parcels move in from large radius, ∞, to the center of the convective vortex, 0 (Figure A.2). This is justified because there is a stagnation point at the center of the dust devil. So for a streamtube, which idealizes flow near the surface, we have

$$\int_{\infty}^{0} \alpha dp \approx \int_{\infty}^{0} \vec{f} \cdot d\vec{l}. \tag{A.34}$$

Employing the ideal gas law, we get

$$\int_{\infty}^{0} RTdlnp \quad \approx \quad \gamma \int_{\mathrm{m}} \vec{f} \cdot d\vec{l},$$

where γ is the fraction of total dissipation of mechanical energy consumed at the surface

$$\gamma \equiv \frac{\int_{\infty}^{0} \vec{f} \cdot d\vec{l}}{\int_{\mathrm{m}} \vec{f} \cdot d\vec{l}}. \tag{A.35}$$

From Eq. (A.32), we then have

$$-\int_{\infty}^{0} RTdlnp \quad \approx \quad \gamma \int_{\mathrm{m}} Tds, \tag{A.36}$$

where $\int_{\mathrm{m}} Tds$ is the amount of energy available to do work. A thermodynamic efficiency can also be easily formulated for the convective vortex. Assuming the heat input for the convective vortex is that absorbed at the surface level, we have for a reversible heat engine

$$\eta \quad \equiv \quad \frac{\int_{m} Tds}{\int_{\infty}^{0} Tds}. \tag{A.37}$$

Assuming a mean surface-layer air temperature $\overline{T_s}$ (the overbar denotes a horizontal average from ∞ to 0) at which heat is input into the heat engine, we can rewrite Eq. (A.36) as

$$-R\overline{T_s} \ln \frac{p_0}{p_\infty} \quad \approx \quad \gamma\eta \int_{\infty}^{0} Tds. \tag{A.38}$$

From this equation, it is apparent that the net work of the convective vortex heat engine will depend on the pressure drop between large radius and the vortex center. Using the first and second laws of thermodynamics and the ideal gas law, we can rewrite the heat input term on the right-hand side of Eq. (A.38) as

$$\begin{aligned} \int_{\infty}^{0} Tds \approx c_p(T_0 \quad &-T_\infty) + \ L_v(r_0 - r_\infty) \\ &-R\overline{T_s} \ln \left(\frac{p_0}{p_\infty}\right). \end{aligned} \tag{A.39}$$

Substituting this into Eq. (A.38),

$$\Delta p \equiv (p_\infty - p_0) \approx p_\infty \exp\left\{\left(\frac{\gamma\eta}{\gamma\eta - 1}\right)\left[\left(\frac{c_p}{R}\right)\right.\right.$$
$$\left.\left.\left(\frac{T_0 - T_\infty}{\overline{T_s}}\right) + \left(\frac{L_v}{R}\right)\left(\frac{r_0 - r_\infty}{\overline{T_s}}\right)\right]\right\}. \tag{A.40}$$

This expression gives the pressure drop across the convective vortex. We demonstrate in the following sections that, using this simple analysis including reversible thermodynamics, the potential maximum intensity of both dust devils and water spouts can be estimated.

Dust devils

Dust devils are warm-core, convective vortices with typical diameters of the order of 1 to 10 meters. They are most frequently observed in hot deserts, although they have been observed in other regions. It is generally believed that the vortex develops on the bottom of convective plumes. The near-surface, radial inflow into the convective plume may result in weak vortex motion. Generation of the vorticity in the convective updraft may be attributed to local wind shears. As the plume continues to rise, the pressure drop near the surface increases. This results in greater inflow, which spirals in towards the low pressure. Surface friction plays a role in convergence of the near-surface air. The tangential velocity profile resembles that of a Rankine vortex and, to a first appromation, the above-surface winds are in cyclostrophic balance.

Dust devils are convective vortices that lend themselves to analysis within the heat-engine framework. This framework assumes that the dust devil is in steady state; that is, work done by the convective heat engine is balanced by frictional dissipation. The heat input for the dust devil heat engine is derived from the surface heat flux into the atmospheric surface layer. The rejection of heat into the cold sink comes through emission of thermal radiation in the subsiding air parcels at the top of the convective plume. The convective drafts are again assumed adiabatic and the heat engine reversible. Given these assumptions, this analysis aims at estimating the maximum bulk thermodynamic intensity of the dust devil.

The heat input into the dust devil heat engine arises solely from surface sensible heat flux into the near-surface air. This means that we can neglect the term in Eq. (A.40) that is associated with changes in water-vapor content (latent heat). The radial pressure drop between large radius ∞ and the low-pressure center can then be approximated as

$$\Delta p \approx p_s \left\{ 1 - \exp\left[\left(\frac{\gamma\eta}{\gamma\eta - 1} \right) \left(\frac{1}{\chi} \right) \left(\frac{T_0 - \overline{T_s}}{\overline{T_s}} \right) \right] \right\}, \tag{A.41}$$

where $\chi \equiv R/c_p$ (we have assumed $T_\infty \approx \overline{T_s}$ and $p_\infty \approx p_s$).

Due to the relationship between the pressure drop and the net work performed by the convective heat engine, the pressure drop is a good measure of the intensity of the dust devil. The intensity of the dust devil also depends on the surface temperature increase along the inflow path. Because the temperature over the desert regions is essentially controlled by surface sensible heat flux, the ground temperature provides an upper bound for the temperature in the center of the dust devil, T_0. The greatest temperature increase would most likely occur when there is a local gradient in temperature along the inflow path. This would suggest that dust devils would be most likely to occur in areas where warm and cooler surfaces lie juxtaposed (*e.g.*, an irrigated field next to a dry field). These are also areas where the baroclinic source of vorticity would be largest. There is some observational evidence to support this (Georgii, 1952).

From Eq. (A.41), it can be seen that the intensity of the dust devil is also a function of the thermodynamic efficiency. For the purpose of estimating both the vertical velocity and the tangential velocity of the dust devil vortex in a simple way, a vertical and a horizontal thermodynamic efficiency can be estimated. The vertical efficiency can be derived for a convective boundary layer where entropy is nearly constant with height. The temperature of the heat source is, to a first approximation, the average surface temperature, $\overline{T_{\mathrm{s}}}$ (*i.e.*, the value integrated along the inflow path from large radius). The temperature of the cold sink is simply equal to the pressure-averaged temperature across the depth of the convective layer. For a dry adiabatic layer, the temperature profile can be deduced from the surface temperature, $\overline{T_{\mathrm{s}}}$, in this case. As a result, the thermodynamic efficiency may be estimated solely from the surface temperature, $\overline{T_{\mathrm{s}}}$, and the pressure thickness of the convective boundary layer. Because the change in temperature of the inflowing air parcel is large only near the vortex, $T_\infty \approx \overline{T_{\mathrm{s}}}$ ambient surface air temperature, and we have

$$\eta \approx 1 - b. \tag{A.42}$$

b is defined as

$$b \equiv \frac{(p_{\mathrm{s}}^{\chi+1} - p_{\mathrm{top}}^{\chi+1})}{(p_{\mathrm{s}} - p_{\mathrm{top}})(\chi + 1)p_s^{\chi}},$$

where p_{s} is the ambient surface pressure and p_{top} is the ambient pressure at the top of the convective boundary layer.

A "horizontal thermodynamic efficiency" can be defined as

$$\eta_H \equiv \frac{T_0 - \overline{T_s}}{\overline{T_s}}, \tag{A.43}$$

where again we have assumed $T_\infty \approx \overline{T_s}$. Given these two efficiencies, Eq. (A.41) becomes

$$\Delta p \approx p_s \left\{ 1 - \exp\left[\left(\frac{\gamma\eta}{\gamma\eta - 1} \right) \left(\frac{\eta_H}{\chi} \right) \right] \right\}. \tag{A.44}$$

This result shows that, to a first order, the intensity of the dust devil is function of surface pressure, the vertical and horizontal efficiencies, and mechanical dissipation of energy near the surface (via γ). Making use of the cyclostrophic balance assumption, the tangential velocity can be estimated as

$$v \approx \left\{ R\overline{T_s} \left\{ 1 - \exp\left[\left(\frac{\gamma\eta}{\gamma\eta - 1} \right) \left(\frac{\eta_H}{\chi} \right) \right] \right\} \right\}^{\frac{1}{2}}. \tag{A.45}$$

From the derivation in section A.3 of this paper, the vertical velocity of the dust devil may be estimated as

$$w \approx \left\{ \left(\frac{c_p}{8\epsilon\sigma_R {T_c}^3} \right) \frac{\eta F_{\text{in}}}{\mu} \right\}^{\frac{1}{2}}. \tag{A.46}$$

If the dimensionless coefficient of mechanical dissipation μ (as defined in section A.3) is assumed to be 20 to 50, we can estimate the vertical velocity. Given typical conditions for Tucson, Arizona (Sinclair, 1973), the tangential and vertical velocities are estimated to fall between 10 and 15 ms^{-1}. Using Sinclair's values, $T_\infty \approx 319\,\mathrm{K}$, $T_0 \approx 324\,\mathrm{K}$, and $\Delta p \approx 3.0\,\mathrm{hPa}$, and assuming typical summertime pressure values for the surface layer and the boundary layer top of 925 mb and 650 mb, respectively, we get a horizontal and vertical efficiency of $\eta \approx 0.050$, $\eta_H \approx 0.016$. By taking

$\gamma \approx 0.5 - 1.0$ and these numbers above, Eqs. (A.44), (A.45) and (A.46) give $\Delta p \approx 1.3 - 2.7\,\mathrm{hPa}$, $v \approx 11 - 16\,\mathrm{ms}^{-1}$, and $w \approx 8 - 16\,\mathrm{ms}^{-1}$. The values are quite close to those given by Sinclair, thereby supporting the validity of the use of the reversible heat-engine framework.

Waterspouts

Waterspouts, like dust devils, are convective vortices that lend themselves to examination under the heat-engine framework. Again, the basic assumptions of steady state and near cyclostrophic balance are made. As with the dust devils, the concern here is focusing solely on the thermodynamic processes that maintain the pressure depression within the waterspout. Equation (A.40) can be used to estimate the intensity of the waterspout. Unlike the dust devil, the term involving changes in the water vapor content cannot be neglected. Equation (A.45), also rewritten to include the effects of water vapor content, gives the tangential wind speed as

$$v_a \approx \left\{RT_\infty \left\{1 - \exp\left\{\left(\frac{\gamma\eta}{\gamma\eta - 1}\right)\left[\left(\frac{c_p}{R}\right)\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\left(\frac{T_0 - T_\infty}{T_\infty}\right) + \left(\frac{L_v}{R}\right)\left(\frac{r_0 - r_\infty}{T_\infty}\right)\right]\right\}\right\}\right\}^{\frac{1}{2}}. \tag{A.47}$$

Again, this equation demonstrates that the maximum intensity depends only on the thermodynamics of the heat engine. Both the updrafts and downdrafts associated with waterspouts are saturated to a first approximation. Observations (Golden, 1974a; Simpson *et al.*, 1986; Golden and Bluestein, 1994) show that, for a weak waterspout, $T_\infty \approx 299.5\,\mathrm{K}$, so that $r_\infty \approx 0.022$; $T_0 \approx 300.5\,\mathrm{K}$, so that

$r_0 \approx 0.024$; and the top of the convective layer is at $z_{\text{top}} \approx 3\,\text{km}$, which corresponds to $\eta \approx 0.1$. Assuming that most of the frictional dissipation of energy takes place at the surface, *i.e.*, $\gamma \approx 1$, we have $\Delta p \approx 6.5\,\text{hPa}$, and $v_a \approx 25\,\text{ms}^{-1}$. For a strong waterspout, we have $T_\infty \approx 298\,\text{K}$, so that $r_\infty \approx 0.020$; $T_0 \approx 302\,\text{K}$, so that $r_0 \approx 0.025$; and $z_{\text{top}} \approx 10\,\text{km}$, which corresponds to $\eta \approx 0.2$. It follows from Eqs. (A.40) and (A.47) that $\Delta p \approx 40\,\text{hPa}$, and $v_a \approx 60\,\text{ms}^{-1}$. These results are consistent with the observations.

A.4.2 Meso- and Large-Scale Circulations

The effectiveness of the heat-engine framework for convective vortices has been demonstrated in the previous section. We now apply the theory to other natural convective heat engines: circulations induced by surface inhomogeneities (Souza *et al.*, 2000). Sea breezes, mountain-valley circulations, and circulations forced by changes in surface characteristics (*e.g.*, forested versus deforested land) can all be idealized as natural heat engines. The intensities of these circulations follow from Eq. (A.40) in the last section. However, in the derivation of that equation, changes in potential energy were neglected. In many of these surface-forced circulations, the potential energy changes along the flowpath cannot be neglected (*e.g.*, the mountain-valley circulation).

Following the same general framework for the convective vortex as a heat engine, the net heat input into the surface-induced circulation occurs along a near-surface streamline. Integrating Eq. (A.32) along a near-surface streamline going from the ocean, valley, or forest (point a) to the continent, mountain top, or deforested area (point b), we get

$$\int_a^b \alpha dp \approx \int_a^b \vec{f} \cdot d\vec{l} - \int_a^b g dz \ , \tag{A.48}$$

where we have neglected changes in the kinetic energy that, in this study, are two orders-of-magnitude smaller than changes in the other terms.

The fraction of the total dissipation of mechanical energy consumed by friction near the surface and the thermodynamic efficiency of the induced circulation are calculated in exactly the same manner as the convective vortices, except that the limits on integration along surface streamline represent two points differing in surface characteristics and possibly elevation. Rewriting Eq. (A.48), we have

$$\int_a^b \alpha dp \approx -\gamma\eta \int_a^b T ds - \int_a^b g dz. \tag{A.49}$$

The total heat input along the near surface path is

$$\int_a^b T ds = \int_a^b d(c_p T + L_v r) - \int_a^b \alpha \ dp. \tag{A.50}$$

Combining the two previous equations, integrating along the path, and neglecting changes in c_p and L_v, we arrive at

$$\begin{aligned} p_b \approx p_a \exp \Bigg\{ & \frac{\gamma\eta}{(\gamma\eta - 1)R} \left[\frac{c_p \Delta T}{\bar{T}_s} + \frac{L_v \Delta r}{\bar{T}_s} \right] \\ & + \frac{1}{(\gamma\eta - 1)R} \left[\frac{g \Delta z}{\bar{T}_s} \right] \Bigg\} \ , \end{aligned} \tag{A.51}$$

where $\Delta T = T_b - T_a$, $\Delta r = r_b - r_a$, and $\Delta z = z_b - z_a$, and where $\bar{T}_s$ is the mean surface air temperature between points a and b.

The first term on the right-hand side of Eq. (A.51) is due to the absorption of sensible heat between the two points. This term represents the main physical process for producing sea-land breezes (*e.g.*, Atkinson, 1981; Pielke, 1984) and dust devils (Rennó *et al.*, 1998). The second term represents the absorption of latent heat. This term is not important in dry convective systems, but it is the most important term in moist convective systems, such as waterspouts (Rennó and Bluestein, 2000) or hurricanes (Emanuel, 1986). The third term is a combination of the hydrostatic pressure drop due to the difference in height between points a and b (following an adiabat) and an upslope nonadiabatic expansion. This term will be discussed in more detail below. When the surface is assumed to be flat, Eq. (A.51) is identical to Eq. (A.40).

In order to better understand the physical mechanisms responsible for the upslope pressure drop, we divide the temperature difference between points a and b into adiabatic and nonadiabatic parts. We assume that no condensation occurs near the surface and take

$$\Delta T = \Delta T_{\mathrm{ad}} + \Delta T_{\mathrm{na}} = -\frac{g}{c_p}\Delta z + \Delta T_{\mathrm{na}} \ , \tag{A.52}$$

where the subscripts *ad* and *na* stand for adiabatic and nonadiabatic parts, and where the temperature drop following a dry adiabat is given by $\Delta T_{\mathrm{ad}} = -(g/c_p)\Delta z$. Substituting Eq. (A.52) into Eq. (A.51), we get a simple expression for the pressure drop between points a and b, that is,

$$\Delta p \approx p_a \left\{ 1 - \exp\left[\frac{\gamma\eta}{(\gamma\eta - 1)R} \left(\frac{c_p \Delta T_{na}}{\bar{T}_s} + \frac{L_v \Delta r}{\bar{T}_s} \right) - \frac{\Delta z}{H_s} \right] \right\} , \tag{A.53}$$

where $\Delta p \equiv p_a - p_b$, $\Delta T_{\mathrm{na}} = T_b - T_a + (g)/(c_p)\Delta z$, and $H_s \equiv (R\bar{T}_s)/g$ is a scale height.

Equation (A.53) predicts that the nonhydrostatic pressure drop, and therefore the intensity of a convective circulation in sloping terrain, depends on the thermodynamic efficiency of the circulation, the nonadiabatic temperature difference, and the difference in water vapor content between points a and b. Equation (A.53) suggests that topographical features induce convective circulations because they lead to nonadiabatic heating of the air parcels moving upslope.

The intensity of the thermally direct component of the circulation can be determined independently of the pressure drop between points a and b. In the tropics, this is the main component of the circulation. Thus, following Rennó and Ingersoll (1996), the intensity of the convective circulation (forest-pasture breeze) is given by

$$| \vec{v} | \approx \left\{ \frac{\eta}{\mu}(c_p \Delta T_{na} + L_v \Delta r) \right\}^{\frac{1}{2}}, \tag{A.54}$$

where μ is a dimensionless coefficient of dissipation of mechanical energy. Note that since the circulation is from higher to lower pressure, it is directed from the regions of lower (colder) to higher (warmer) entropy. Thus, during the day, the direction of a forest-pasture breeze is from the forest to the pasture. Equation (A.54) states that the intensity of the breeze is a function of the thermodynamic efficiency of the circulation, the nonadiabatic temperature and humidity differences between points a and b, and the magnitude of the coefficient of mechanical dissipation of energy.

A.5 Irreversible Heat Engines

In this section, we show that irreversible processes can easily be included in the heat-engine framework. Rennó and Ingersoll (1996) assume steady-state and reversible processes, which imply that their theory provides an upper bound for the intensity of the convective circulations. Relaxing the assumption that the convective processes are reversible, the first and second laws of thermodynamics applied to moist air can be written as

$$Tds \geq dQ \geq d(c_pT + L_v r) - \alpha dp \ , \tag{A.55}$$

where dQ is the heat absorbed by the system. The inequalities apply when the changes are irreversible. Thus, we can write

$$\begin{aligned} Tds = (Tds)_{\text{rev}} + (Tds)_{\text{irr}} = d(c_pT + L_v r) \\ -\alpha dp + (dw)_{\text{irr}} \ , \end{aligned} \tag{A.56}$$

where $(Tds)_{\text{rev}}$ and $(Tds)_{\text{irr}}$ are associated with reversible and irreversible entropy sources, and $(dw)_{\text{irr}}$ is the irreversible work (*e.g.*, the work done by friction forces opposing the expansion). Note that, in this case, the heat absorbed by the system is $dQ = (Tds)_{\text{rev}} = Tds - (Tds)_{\text{irr}}$.

It follows from the above that

$$(Tds)_{\text{rev}} - (dw)_{\text{irr}} = d(c_pT + L_v r) - \alpha dp \ , \tag{A.57}$$

which shows that, in a real (irreversible) heat engine, a portion of the heat input is used to do irreversible work. Substituting Eq. (A.55) into Eq. (A.30), we get

$$Tds \geq dQ \geq d\left(\tfrac{1}{2}|\vec{v}|^2 + gz + c_pT + L_v r\right) - \vec{f} \cdot d\vec{l} \ . \tag{A.58}$$

Integrating Eqs. (A.55) and (A.58) along the convective circulation, we get

$$\oint Tds \geq \oint dQ \geq -\oint \alpha dp = \oint p d\alpha \ , \tag{A.59}$$

and

$$\oint T ds \geq \oint dQ \geq -\oint \vec{f} \cdot d\vec{l}\,, \tag{A.60}$$

where in the presence of irreversibility the *total* work done by the convective system is $W = \oint dQ > \oint p d\alpha$. For example, in the presence of friction, the total work of expansion is greater than it would be in the absence of this irreversibility. However, even in the presence of irreversibility, it is convenient to define $TCAPE$ as the work available from the convective heat engine, that is $TCAPE \equiv \oint p d\alpha$.

Recall that Rennó and Ingersoll (1996) assume that

$$\eta = \eta_{\mathrm{rev}} \equiv \frac{\oint p d\alpha}{\int_{\mathrm{in}} (T ds)_{\mathrm{rev}}}\,, \tag{A.61}$$

where the subscript *in* denotes an integration along the heat-input branch of the convective circulation. The thermodynamic efficiency of an irreversible convective heat engine should be similarly defined, that is

$$\eta = \eta_{\mathrm{irr}} \equiv \frac{W - \oint (dw)_{\mathrm{irr}}}{\int_{\mathrm{in}} dQ} = \frac{\oint p d\alpha}{\int_{\mathrm{in}} (T ds)_{\mathrm{rev}}}\,. \tag{A.62}$$

Equation (A.57) shows that only a fraction of the heat input $[\int_{\mathrm{in}} dQ = \int_{\mathrm{in}} (T ds)_{\mathrm{rev}}]$ into a real heat engine is used to do reversible work ($\oint p d\alpha$). Therefore, the thermodynamic efficiency of any real heat engine is smaller than that of a reversible heat engine, that is $\eta_{\mathrm{irr}} < \eta_{\mathrm{rev}}$. Except for this reduction in the thermodynamic efficiency, the reversible heat-engine framework is unchanged when the heat engine is irreversible.

In a recent critique, Pauluis *et al.* (2000) have questioned the validity of using the entropy budget from the reversible heat-engine framework for estimating the strength of convective motions in precipitating atmospheres. These authors argue that frictional dissipation around falling hydrometeors is actually a large source of entropy that is neglected in the reversible heat-engine framework. In neglecting entropy sources other than those of frictional dissipation, Pauluis *et al.* (2000) argue that the heat-engine framework greatly overestimates the order of magnitude of the mechanical work available to drive convective motions. We argue that, from a theoretical perspective and through consideration of observational data, the neglect of entropy sources due to falling precipitation is quite justifiable.

The calculations given by Pauluis *et al.* (2000) suggest that about 1/3 of the work available from the convective heat engine is used in frictional dissipation around falling hydrometeors. Further reductions in available mechanical work from the convective heat engine are attributed by Pauluis *et al.* (2000) to the diffusion of water vapor and irreversible phase changes of the water substance. In essence, they argue that only a small fraction (less than 10%) of the available work from an irreversible convective heat engine can be used to drive convective motions.

For the idealized model of large-scale convective motions proposed by Rennó and Ingersoll (1996), precipitation falling within convective downdrafts does reversible work through pressure drag. Frictional drag around hydrometeors is at least an order of magnitude smaller. In this sense, work performed by falling hydrometeors goes mostly into accelerating the convective downdrafts. Rennó (2000) made simple order-of-magnitude calculations to show that frictional dissipation around hydrometeors is not dominant, as assumed by Pauluis *et al.* (2000).

We demonstrated in the previous section that irreversible processes can easily be included within the reversible heat-engine framework. Contrary to Pauluis et al. (2000), we argue that the only important effect of irreversibility is to reduce the efficiency of the heat engine. It is, therefore, appropriate to examine some applications of the heat-engine framework to precipitating convective systems in order to assess the validity of the Pauluis *et al.* argument.We now present observational evidence in precipitating convective systems that appears to contradict their conclusions.

Equation (A.40) with $\gamma = 1$ predicts the maximum possible intensity of real convective circulations. Equation (A.40) was already used to show that the predictions of reversible thermodynamics are consistent with the observations of systems ranging in size from small- to meso-scale. Here we look at some of these calculations in more detail. Golden and Bluestein (1994) observed waterspouts under precipitating cumulus clouds with tops at about 3.0 km, which corresponds to $\eta_{\text{rev}} \approx 0.1$. Based on the thermodynamic sounding at Key West displayed in their article, we estimate that the convective updrafts are about 1 K warmer than the downdrafts. Assuming that both are saturated, that $\eta = \eta_{\text{rev}}$, that $\gamma = 1$, and that the wind is in cyclostrophic balance, the predicted pressure drop is $\Delta p \approx 6.5\,\text{hPa}$, and the tangential windspeed is $v \approx 25\,\text{ms}^{-1}$. Since the minimum windspeed necessary to create a spray ring is 20 m s^{-1}, this prediction is in good agreement with observations. If the irreversible processes associated with water substance were as important as suggested by Pauluis *et al.* (2000), the thermodynamic efficiency would be at least an order of magnitude smaller than the values used in section A.4.1. Thus, the suggestion by Pauluis *et al.* (2000) that irreversible processes are of paramount importance, is not consistent with observations of waterspouts. The same argument holds for the prediction of the maximum intensity of hurricanes. However, the reader might argue that convective vortices belong to a special class of convective systems that are thermodynamically efficient. Since, Eq. (A.40) applies to any moist convective circulation, next we apply it to a large-scale tropical circulation, that is, the Hadley circulation.

Taking $\eta = \eta_{rev} \approx 0.1$, the approximate value of the thermodynamic efficiency of the circulation, Rennó (2000) shows that Eq. (A.41) predicts $\Delta p \approx 10$ hPa when we take $\gamma \approx 0.5$, indicating that half of the dissipation of mechanical energy occurs near the surface. Assuming that $\eta = \eta_{irr} \leq 0.01$, as suggested by Pauluis *et al.* (2000), we get $\Delta P \leq 2$ hPa. This result is inconsistent with observations. It is interesting that data from one general circulation-model atlas also shows that the Hadley cell extends from 5°N to about 30°S, and $\Delta p \approx 10$ hPa. However, in contrast with observations, this model gives $\Delta T \approx 15$ K, and $\Delta r \approx 9 \times 10^{-3}$ kg/kg. These values are more than double the observed values. Applying Eq. (A.41) to the model data with $\gamma\eta \approx 0.05$, we get $\Delta p \approx 24$ hPa. In order to obtain the model pressure drop, we have to assume $\gamma\eta \approx 0.025$. We found a similar result when trying to compute the pressure drop across the Walker circulation. Thus, the numerical model is much more dissipative than nature.

A.6 Summary and Conclusions

The heat-engine framework has a history in atmospheric sciences. It has been applied to the atmospheric circulation, global convection, hurricanes, and smaller scale phenomena. In addition to providing fundamental insights into the basic nature of atmospheric phenomena, it also provides theoretical estimates of the intensity of atmospheric circulations. The framework is valid for both reversible and irreversible processes; irreversible processes solely reduced the thermodynamic efficiency of the convective heat engine. Results from this framework applied to convective vortices, both dust devils and waterspouts, show very good agreement with observational data. Application to larger scale convective circulations such as the Hadley Cell, also support the validity of the heat-engine framework, even when irreversible processes such as frictional dissipation around hydrometeors are neglected.

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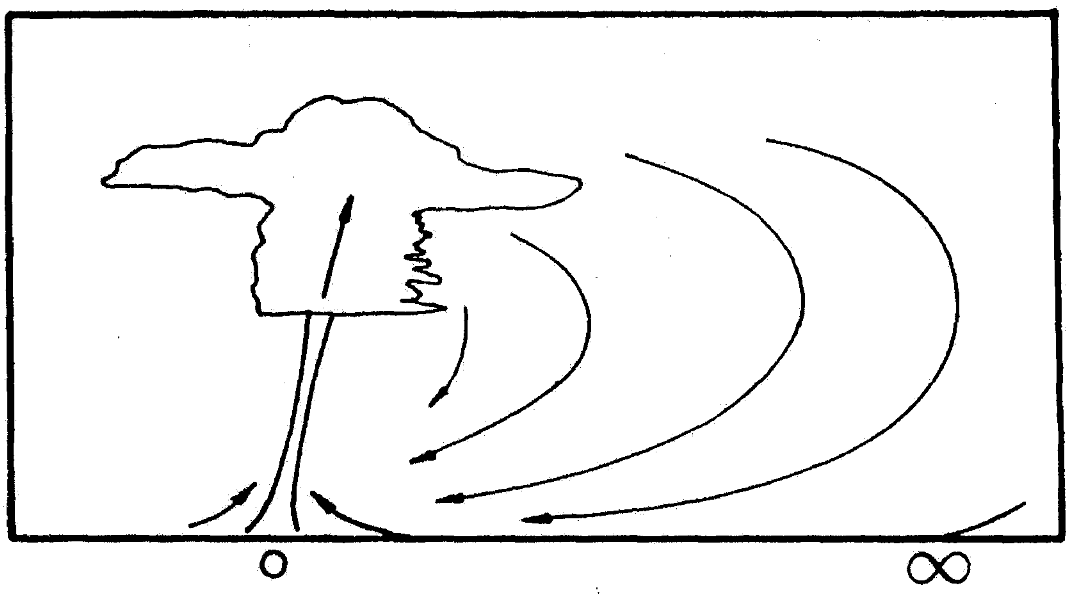

Figure A.2: Idealization of convective vortex. Parcels move in from large radius (infinity) to the center of the convective vortex (0). The surface is assumed flat, so changes in potential energy are ignored..

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APPENDIX B

REMARKS ON QUASI-EQUILIBRIUM THEORY

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Abstract

A recent article by Yano (1999) has indicated that there is a contradiction in the original formulation of the quasi-equilibrium theory of Arakawa and Schubert (1974). He argues that this contradiction in the two asymptotic limits of the theory; that is, the fractional area covered by convection and the ratio of the convective adjustment and large-scale time scales cannot simultaneously go to zero, $\sigma \rightarrow 0$ and $\frac{\tau_{ADJ}}{\tau_{LS}} \rightarrow 0$. Yano (1999) cites the heat engine theory proposed by Rennó and Ingersoll (1996) as "formally establishing" this contradiction. We demonstrate in this note that the quasi-equilibrium framework originally developed by Arakawa and Schubert (1974) is perfectly consistent with the heat engine theory for steady-state convection; that is, when the time scale associated with the large-scale forcing, τ_{LS}, approximates the effective adjustment time scale of the large-scale ensemble of convective clouds, τ_{EFF}. Indeed, the quasi-equilibrium framework states that, on the large-scale, the atmosphere is in quasi steady-state.

B.1 Introduction

Quasi-equilibrium theory has provided the basis for many theoretical and modeling studies of large-scale atmospheric convection (e.g., Emanuel et al. 1994; Randall et al. 1997). Originally proposed by Arakawa (1969), applied to shallow non precipitating convection by Betts (1973) and to deep atmospheric convection by Betts (1974) and Arakawa and Schubert (1974) (hereinafter AS), this theory essentially states that, over large areas, the production of instability by large-scale forcing (e.g., large-scale sensible and latent heat fluxes, radiative cooling, etc.) is balanced by its removal through cumulus convection. Although quasi-equilibrium theory has proved useful for both theoretical and modeling studies, Yano (1999) has recently pointed out that there is a contradiction in the quasi-equilibrium theory put forth by AS. In this note, we demonstrate that this contradiction results solely from a confusion in the definition of the convective and effective adjustment time scales, τ_{ADJ} and τ_{EFF}. Examining the quasi-equilibrium theory in terms of the heat engine framework for steady-state convecting atmospheres, in which the large-scale time scale, τ_{LS}, is the radiative relaxation time scale, we show that the contradiction described by Yano disappears when the adjustment time scales are properly defined.

We turn first to a review of AS quasi-equilibrium theory, including its mathematical formulation and scaling analysis. This is followed by an examination of the Rennó and Ingersoll (1996) formulation of quasi-equilibrium under the heat engine framework. Finally, we review Yano's critique of the quasi-equilibrium theory and demonstrate that the "contradiction" in this theory results from using AS's ambiguous scaling arguments and not from the theory itself.

B.2 The Quasi-Equilibrium Theory

B.2.1 Arakawa-Schubert's Mathematical Model

The fundamental equation underlying quasi-equilibrium theory, equation (140) in AS,

$$\frac{dA}{dt} = \left(\frac{dA}{dt}\right)_C + \left(\frac{dA}{dt}\right)_{LS}, \tag{B.1}$$

simply states that the time rate of change of the "cloud work function," A, which for this discussion can be thought of as Convective Available Potential Energy (CAPE), is a balance between the production of instability by large-scale forcing, $(dA/dt)_{LS}$, and its removal by the ensemble of cumulus clouds at the sub-grid cell scale $(dA/dt)_C$. In "quasi-equilibrium," that is, in quasi steady-state, dA/dt must be approximately zero. AS argue that the foundation upon which quasi-equilibrium theory lies is the separation of two time scales; that of the large-scale forcing, τ_{LS}, and that of the convective adjustment, τ_{ADJ}. This separation of time scales allows cumulus convection to quickly respond to changes in the large-scale forcing, maintaining quasi-equilibrium. We show, in section B.2.2, that Arakawa and Schubert (1974) define an adjustment time scale, τ_{ADJ}, which has the same order of magnitude as that of a *local* convective adjustment, H/w. We demonstrate that estimating the first term on the right-hand side of equation (B.1) with this adjustment time scale leads to the contradition noted by Yano(1999). We define, in section B.3, an effective adjustment time scale τ_{EFF}, which balances the terms on the right-hand side of equation (B.1) resulting in quasi-equilibrium; i.e, $\frac{dA}{dt} \sim 0$. This effective adjustment time scale also preserves the time scale separation which serves as the basis for quasi-equilibrium according to Arakawa and Schubert (1974);

i.e, $\tau_{ADJ} << \tau_{LS}$.

On page 691 of their influential article Arakawa and Schubert (1974) state:

> "The problem we are considering is a problem with two time scales: τ_{ADJ}, the adjustment time scale, and τ_{LS}, the time scale of the large-scale processes. Quasi-equilibrium exists when $\tau_{ADJ} << \tau_{LS}$. When the adjustment process is filtered out, we obtain a sequence of quasi-equilibria. In such a sequence, the large-scale forcing and the cumulus ensemble vary in time in a coupled way and, therefore, the time scale of the statistical properties of the ensemble is equal to the time scale of the large-scale processes, τ_{LS}."

Mathematically, AS define the convective adjustment time scale, τ_{ADJ}^{AS} (superscripts refer to time scale definitions appropriate to each author), and the large-scale time scale, τ_{LS}^{AS}, by their equations (146) and (147) respectively,

$$\left|\left(\frac{dA}{dt}\right)_C\right| \sim \frac{A}{\tau_{ADJ}^{AS}}, \tag{B.2}$$

and

$$\left|\left(\frac{dA}{dt}\right)\right| \sim \frac{A}{\tau_{LS}^{AS}}. \tag{B.3}$$

Their adjustment time scale, τ_{ADJ}^{AS}, can be conceptualized as follows. Given the presence of CAPE (we are using CAPE interchangeably with the cloud-work function of AS) and some triggering mechanism, cumulus activity will develop. If there is no large-scale forcing, the instability present in the large-scale "grid-box" will be consumed by the cumulus ensemble thereby bringing the atmosphere to a neutral state. τ_{ADJ}^{AS} is a measure of the time needed to reach this neutral state. In this sense, τ_{ADJ}^{AS} is really the adjustment time scale of the large-scale (grid-box) ensemble of cumulus clouds (the time scale of the "statistical properties of the ensemble" mentioned in the quote above); that is, it is the effective adjustment time scale that brings the entire grid-box to a neutral state. The large-scale time scale, τ_{LS}^{AS}, is defined by AS as being, not a relaxation time scale, but a time scale on which the cloud-work function varies (an externally imposed time scale).

AS argue that quasi-equilibrium exists when $\tau_{ADJ}^{AS} << \tau_{LS}^{AS}$. Yano (1999) follows AS employing these order of magnitude arguments to demonstrate that a contradiction exists in the asymptotic limit of quasi-equilibrium theory; namely, $\sigma \rightarrow 0$ and $\tau_{ADJ}^{AS}/\tau_{LS}^{AS} \rightarrow 0$ contradict each other, where σ is the fractional area of the grid-box covered by convective updrafts. We will show in section B.3 that this contradiction disappears when the convective adjustment and effective time scales are defined in a manner consistent with heat engine framework for steady-state convecting atmospheres. We turn first to AS's scaling arguments to reveal exactly how they define their convective adjustment and effective time scales.

B.2.2 Arakawa-Schubert's Scaling Analysis

By employing AS's order of magnitude analysis of the quasi-equilibrium theory, given on page 692 of their original article, we demonstrate that their effective convective adjustment time scale, τ_{ADJ}^{AS}, turns out to be on the same order of magnitude as the *local* convective adjustment time scale; i.e, the time scale for a deep cumulus updraft to travel from the surface to its level of neutral buoyancy. We show in section B.3 that it is the use of this order of magnitude estimation for τ_{ADJ}^{AS} over the entire grid-box, instead of an effective adjustment time scale, *not* quasi-equilibrium theory per se, that leads to the contradiction pointed out by Yano (1999).

AS state that $\tau_{ADJ}^{AS} \sim 10^3 - 10^4\, s$ [their equation (154)], assume a value of the vertical velocity $w \sim 1 - 10\, ms^{-1}$, and of the depth of the convective layer $H \sim 10^4\, m$. This implies that AS's convective adjustment time scale has the same order of magnitude as that of the local convective adjustment, H/w. It then follows that AS assume that rate of change of A by the large-scale ensemble of convective clouds is given by

$$\left(\frac{dA}{dt}\right)_C \sim \frac{A}{\tau_{ADJ}^{AS}}, \tag{B.4}$$

where $\tau_{ADJ}^{AS} \sim H/w$. This equation is identical to Equation (B.2). Therefore, this scaling argument implies that the ensemble of convective elements in the grid-box is adjusted on the same time scale as a single convective updraft (i.e., the time scale for local convective adjustment, H/w). In the next section, we derive an effective adjustment time scale, τ_{EFF} for the cumulus ensemble in the grid-box. We demonstrate that this effective adjustment time scale is consistent with the asymptotic limits originally proposed by AS.

B.3 Heat Engine Theory and the Steady-State Convecting Atmosphere

Another way to analyze the quasi-equilibrium theory is to examine the steady-state convecting atmosphere as described by the heat engine theory proposed by Rennó and Ingersoll (1996) (RI). As with AS, the basis for quasi-equilibrium in a convecting atmosphere is that there is near equality between the production of

CAPE by large-scale processes and its consumption by a large-scale ensemble of convective systems. That is, over large-scales the atmosphere is in quasi steady-state. The amount of CAPE present in the quasi steady-state convecting atmosphere is then a measure of the amount of mechanical dissipation of energy present. Yano (1999) uses estimates of the fractional area covered by convective drafts, σ, derived from RI's heat engine theory to "formally establish" the "contradiction" in the asymptotic limits of AS's derivation. We demonstrate in this section that the effective adjustment time scale, τ_{EFF} is estimated differently in RI from that of AS and the contradiction cited by Yano results from these different estimates.

In the heat engine theory for steady-state convection, RI assume that the large-scale atmosphere is in radiative-convective equilibrium. We can think of the steady-state convecting atmosphere in the following simple, qualitative way. Given a fixed value of the surface temperature, creation of instability ($A \sim CAPE$) by radiative cooling of the atmosphere results in increased convective activity. This convective activity, realized through an ensemble of cumulus clouds, then forces large-scale subsidence which pushes the atmosphere towards a convectively adjusted state. Assuming that this perturbation to the atmosphere's radiative equilibrium is small, RI use the Newtonian cooling approximation to estimate the radiative time scale (the time scale for the atmosphere to radiatively relax back to the unadjusted state once convection has been "turned off"). In steady state, this time scale must be equal to the effective adjustment time scale, τ_{EFF}; that is, to the time scale over which the cumulus ensemble will bring the atmosphere from the unstable equilibrium to a convectively neutral state if the large-scale forcing is "turned off." Given this linear approach taken by RI (see sections 6 and 7 of their article), the large-forcing term in equation (B.1) can be estimated as;

$$\left|\left(\frac{dA}{dt}\right)_{LS}\right| \sim \frac{g}{\Delta p}\eta F_{\text{in}}, \tag{B.5}$$

where Δp is the thickness of the radiating layer (i.e., the troposphere), η is the thermodynamic efficiency of the convective heat engine and F_{in} is the heat flux into the convective heat engine (i.e., sensible, latent and radiative heat fluxes in to the near surface air). RI show in their equation (39) that this large-scale forcing is estimated as,

$$\frac{g}{\Delta p}\eta F_{\text{in}} \sim \frac{A}{\tau_{LS}^{RI}}, \tag{B.6}$$

where $\tau_{LS}^{RI} \sim \tau_R$ is the radiative relaxation time, and $A \sim CAPE$ is the total amount of work done by buoyancy forces. In quasi steady-state, the magnitude of this large-scale term, $\left(\frac{dA}{dt}\right)_{LS}$, must be nearly equal to the magnitude of the cumulus ensemble term, $\left(\frac{dA}{dt}\right)_{C}$. Rearranging terms from RI's equations (34) and (39), we have,

$$\frac{A}{\tau_{LS}^{RI}} \approx \frac{g}{\Delta p}\rho\sigma w A, \tag{B.7}$$

from which it follows

$$\frac{A}{\tau_{LS}^{RI}} \approx \frac{\sigma A}{\tau_{ADJ}^{RI}}, \tag{B.8}$$

where $\tau_{ADJ}^{RI} \approx H/w$ and $H \sim \frac{\Delta p}{\rho g}$. The effective adjustment time scale can be defined as $\tau_{EFF} \equiv \frac{\tau_{ADJ}^{RI}}{\sigma}$ and under radiative-convective equilibrium conditions equation (B.1) can be rewritten as

$$\frac{dA}{dt} \sim \frac{-A}{\tau_{EFF}} + \frac{A}{\tau_{LS}^{RI}}. \tag{B.9}$$

Because in quasi-equilibrium conditions there is a nearly balance between the terms on the r.h.s. of equation (B.9), we have that

$$\tau_{LS}^{RI} \approx \tau_{EFF} \equiv \frac{\tau_{ADJ}^{RI}}{\sigma}. \tag{B.10}$$

Thus, in quasi-equilibrium the effective adjustment time scale is approximately equal to the large-scale time scale. Since, on the large-scale, the fractional area covered by convective updrafts is much smaller than one ($\sigma << 1$), then $\tau_{EFF} >> \tau_{ADJ}^{RI} \approx \tau_{ADJ}^{AS}$.

B.4 Yano's Critique

We now turn to Yano's critique of the quasi-equilibrium theory. Yano states that the two asymptotic limits under which the AS scheme is formulated, $\sigma \rightarrow 0$ and $\tau_{ADJ}^{AS}/\tau_{LS}^{AS} \rightarrow 0$, are in contradiction to each other and that "the smallness of these two quantities is established only under a compromise." We argue that this contradiction is due solely to AS's ambiguous definition of τ_{ADJ}^{AS}. Following AS, Yano expresses the adjustment time scale [his equation (1)] in the same way as AS; that is, as

$$\tau_{ADJ}^{AS} \sim \left| \frac{A}{(\frac{dA}{dt})_C} \right|. \tag{B.11}$$

This equation implies that the convective adjustment time scale is equal to the effective time scale, that is $\tau_{EFF} = \tau_{ADJ}^{AS}$. This result is inconsistent with AS's scaling analysis described in section (B.2.2), which implies that τ_{ADJ}^{AS} is of the same order of magnitude as a time scale for local convective adjustment. Our expression for estimating the convective adjustment time scale [from equations (B.1, B.8, B.9)], in turn, is given by

$$\tau_{ADJ}^{RI} \sim \sigma \left| \frac{A}{(\frac{dA}{dt})_C} \right|, \tag{B.12}$$

which shows that the convective adjustment time scale is equal to the product of the fractional area covered by convection with the effective time scale, that is $\tau_{ADJ}^{RI} = \sigma\tau_{EFF}$.

The contradiction that Yano points to in the AS scheme results directly from their failure to include σ in equation (B.11). If the equation for the magnitude of the convective time scale we derived in section (B.3) is used, then the "contradiction" in AS's quasi-equilibrium theory disappears as we demonstrate below.

It follows from equation (B.8) and the quasi-equilibrium assumption that

$$\tau_{ADJ}^{RI} \approx \sigma \tau_{LS}^{RI}, \tag{B.13}$$

where, again, τ_{LS}^{RI} depends only on the values of the large-scale forcing (e.g., sensible and latent heat fluxes, radiative cooling rate, etc.). Equation (B.13) shows that the convective time scale, τ_{ADJ}^{RI}, decreases with decreases in the fractional area covered by cumulus convection. This happens because, in steady state, the energy flux per convective draft increases as the fractional area covered by them decreases. Therefore, Yano's suggestion that τ_{ADJ} increases with decreasing fractional area covered by cumulus convection is incorrect in terms of the quasi-equilibrium idea defined by equation (B.9). Furthermore, it follows from equation (B.13) that

$$\lim_{\tau_{ADJ}^{RI} \to 0} \sigma \approx \lim_{\tau_{ADJ}^{RI} \to 0} \frac{\tau_{ADJ}^{RI}}{\tau_{LS}^{RI}} = 0, \tag{B.14}$$

demonstrating that the two asymptotic limits $\tau_{ADJ}^{RI}/\tau_{LS}^{RI} \to 0$ and $\sigma \to 0$ do not contradict each other when quasi-equilibrium is expressed in terms of the radiative-convective equilibrium atmosphere.

B.5 Conclusions

The point of departure between the quasi-equilibrium theory expressed by AS and that by RI results from the estimation of the large-scale forcing term $\left(\frac{dA}{dt}\right)_{LS}$

in equation (B.1). AS estimate the *total* time rate of change of the cloud work function (CAPE) over the grid cell [equation (B.3)] and, in doing so, introduce the characteristic large-scale time scale, τ_{LS}^{AS}. This time scale characterizes net changes in CAPE over the grid cell. RI, on the other hand, assume radiative-convective equilibrium and that in the linear regime of small perturbations to this equilibrium state, the large-scale term can be approximated by a Newtonian cooling rate, as in equation (B.5) through equation (B.9). Following the arguments of RI, it is evident that the asymptotic limits of quasi-equilibrium theory do not lead to the contradiction in time/space scale separation mentioned by Yano. In this sense, RI's framework should not be cited as "formally establishing" this contradiction when, in fact, it is entirely consistent with the quasi-equilibrium time/space scale separation as originally proposed by AS.

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APPENDIX C

REPLY

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Yano (2002) argues that the paper by Adams and Rennó (2003) can lead to potential misunderstandings of Yano (1999) because it does not "clearly distinguish" between multiple definitions for quasi-equilibrium. We do not agree because the definition of quasi-equilibrium is unique. Recall that on page 691 of their article Arakawa and Schubert (1974) state that quasi-equilibria exists when the large-scale forcing and the cumulus ensemble vary in time in a coupled way, so that the time scale of the statistical properties of the ensemble is equal to the time scale of the large-scale processes. Indeed, this simply states that over larger areas the atmosphere is in quasi steady-state. This is also our definition of quasi-equilibrium. Next, we reply to some of Yano's specific comments.

Yano's (1999) original objective was to demonstrate that Arakawa and Schubert's (1974) quasi-equilibrium principle is least satisfied for large-scale tropical circulations. He argues that this is due to a contradiction in the separation of the space-scale and timescale as originally put forth by Arakawa and Schubert (1974). He cites the heat engine framework (Rennó and Ingersoll 1996) as formally establishing this contradiction.

The main objective of our original article was to show that the time/space scale separations of the heat engine framework are entirely consistent with quasi-equilibrium as defined by Arakawa and Schubert (1974). Indeed, the heat engine framework as formulated by Rennó and Ingersoll (1996) starts with the assumption of quasi steady-state. Therefore, the heat engine theory should not have been cited by Yano (1999) as formally establishing a contradiction in the quasi-equilibrium framework.

Adams and Rennó (2003) show that Arakawa-Schubert quasi-equilibrium idea for large scale convection simply means quasi steady-state, and that mathematically it must be defined as

$$\frac{dA}{dt} = \left(\frac{dA}{dt}\right)_C + \left(\frac{dA}{dt}\right)_{LS} \approx 0, \tag{C.1}$$

where A is the cloud work function, and the subscripts C and LS refers to convective and large-scale adjustments over the grid-scale. It follows from the above that, in quasi steady-state (quasi equilibrium),

$$\left(\frac{dA}{dt}\right)_C \sim \left(\frac{dA}{dt}\right)_{LS} \tag{C.2}$$

and that since cumulus convection covers only a fraction σ of the large-scale, we get

$$\sigma\frac{A}{\tau_C} \sim \frac{A}{\tau_{LS}}, \tag{C.3}$$

where τ_C is the adjustment time scale of a single convective draft. Thus, the necessary condition for quasi-equilibrium is that $\sigma << 1$ when $\tau_C << \tau_{LS}$. It follows from the above that the quasi-equilibrium framework originally developed by Arakawa and Schubert (1974) is consistent with theories for steady-state convection such as developed with the heat engine framework. Indeed, the fact that quasi steady-state occurs when the time scale of the statistical properties of the ensemble, $\tau_{EFF} \equiv \tau_C/\sigma$, is equal to the time scale of the large-scale processes, τ_{LS}, is consistent with Arakawa and Schubert (1974) physical interpretation of quasi-equilibrium (see quote in our paper). Finally, it is important to state that Yano's (Yano 1999) suggestion that the two asymptotic limits ($\tau_C/\tau_{LS} \to 0$ and $\sigma \to 0$) in Arakawa Schubert (1974) "tend to contradict each other" is merely a consequence of the ambiguous definition of τ_C in their original article.

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APPENDIX D

THE THERMODYNAMIC EFFICIENCY OF AN IDEALIZED GENERAL CIRCULATION MODEL

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Abstract

Using the heat engine framework, we derive a simple method for assessing the strength of irreversible processes, both numerical and physical, in general circulation models (GCMs). Using the explicit energy budget, we derive thermodynamic efficiencies based on work and net heating for both open (e.g., the Hadley circulation) and closed (e.g., the general circulation) thermodynamic systems. In addition, the Carnot efficiency for closed systems is calculated to assess the maximum efficiency possible of the model. Comparison of the work-based efficiency with that based on the heat budget provides a gauge for assessing how close to reversible model-generated circulations are. A battery of experiments are carried out with an idealized GCM. We demonstrate the usefulness of this method and show the sensitivity of an essentially reversible model to changes in parameters and resolution.

D.1 Introduction

The earth's atmosphere absorbs energy at a higher temperature and pressure than it emits to space. As a result, the atmospheric system is capable of doing work. The work generated by this atmospheric heat engine is used to generate convective motions which transport energy from the heat source (the low-latitude, near surface region) to the cold sink (higher latitudes and the upper troposphere). In steady-state, the kinetic energy resulting from the work generated by the atmospheric heat engine is balanced by frictional dissipation. How efficiently the general circulation converts the heat into the work that maintains atmospheric motions against dissipation is an open question in the atmospheric sciences. Lorenz (1967), in fact, states that "the determination and explanation of the efficiency η constitute the fundamental observational and theoretical problems of atmospheric energetics".

In addition to treating the planetary-scale circulation within the heat engine framework, atmospheric phenomena at smaller scales have also been idealized as heat engines. Rennó and Ingersoll (1996), Emanuel and Bister (1996) and Craig (1996) have idealized steady-state tropical convection as a natural heat engine in order to estimate convective available potential energy (CAPE), convective velocities, and the fractional area covered by convection. Emanuel (1986) has estimated the maximum intensity of hurricanes by treating them as heat engines. At the smaller end of the spectrum, the heat engine framework provides estimates of the intensity of convective vortices such as dust devils (Rennó et al., 1998) and waterspouts (Rennó and Bluestein 2001). In each of the above studies, these heat engines were considered to be thermodynamically reversible.

In a series of recent studies, Pauluis et al., (2000), Pauluis and Held (2002a,b) have called into question idealizing steady-state tropical convection as a thermodynamically reversible heat engine. In the studies of Rennó and Ingersoll (1996) and Emanuel and Bister (1996), ensembles of moist convective systems are treated as reversible heat engines. The primary source of irreversibility in the entropy budget for large-scale tropical convection is assumed to be the frictional dissipation of the convective motions. Because the work required to maintain convective motions must equal the rate of frictional dissipation, Rennó and Ingersoll (1996) and Emanuel and Bister (1996) were able to estimate values of CAPE, among other parameters, for the steady-state convective atmosphere.

Pauluis et al., (2000) and Pauluis and Held (2002a,b) suggest that the heat engine framework proposed by Rennó and Ingersoll (1996) and Emanuel and Bister (1996) greatly overestimates the available work from the convective heat engine. Their modeling studies suggest that irreversibilities associated with falling hydrometeors and with diffusion of water vapor greatly reduce the available work from the reversible convective heat engine. Their results suggest that nature's thermodynamic efficiency is much smaller than the reversible heat engine framework predicts.

Rennó (2001) has argued that it may actually be the numerical models that are highly irreversible since predictions from the heat engine framework are close to what is observed in convective circulations in nature. A number of other authors have also examined the irreversibilities and their effects on the entropy budgets in numerical models (e.g., Johnson 1997; Egger 1999, Johnson et al., 2000). Johnson (1997) argued that positive definite aphysical source of entropy in general circulation models can lead to a "general coldness" in their steady-state climates. Likewise, Egger (1999) showed that aphysical sources of entropy associated with numerical schemes are always present in numerical models. In this study, we present a simple method for assessing the strength of irreversibilities in a general circulation model (GCM) based solely on the model's explicit energy budget. Treating the model circulations as heat engines, our methodology lies in determining and comparing three thermodynamic efficiencies: a work-based efficiency, a reversible efficiency and the Carnot efficiency.

The work-based efficiency is simply the ratio of the work performed by the heat engine to the heat input. The work in this case is the steady-state frictional dissipation of circulations generated in the model. The reversible efficiency is calculated from the model heat budget and is based on the ratio of the net heating available to do work to that of the heat input. The Carnot efficiency is the third efficiency calculated and it represents the highest efficiency possible for given heat source and sink temperatures. The details of these three efficiency calculations are presented in section D.3.2. Assessing the strength of irreversibilities comes from comparing the work-based efficiency with the reversible efficiency. For a thermodynamically reversible model, free of error or spurious numerical energy sources/sinks, the efficiency based on the kinetic energy budget and that based on the heat budget should be equal. And, hence, comparison of these efficiencies provides a measure of the irreversibility associated with model physics and/or numerics. The greater the separation between the two efficiencies, the greater the irreversibility in model circulations.

For this study, we derive the three different thermodynamic efficiencies for the general circulation, a closed thermodynamic system. In addition, we derive the work-based and reversible thermodynamic efficiencies for an open thermodynamic system and apply it to the Hadley cell. In this case, the thermodynamic system exchanges mass and energy with its surroundings. The open-system Carnot efficiency is not derived due to the inherent difficulties in defining the temperatures associated with the net fluxes of energy in and out of the system; i.e., the difficulty in defining heat source and cold sink temperatures for open systems.

Given the uncertainty of how reversible nature/model circulations are, we provide a methodology for evaluating their degree of irreversibility. In the present study, application of this methodology to a highly idealized GCM is an attempt to provide a "proof of concept" for the heat engine framework in assessing model irreversibility. We carry out a series of experiments in which numerical and physical parameters are varied in order to determine how they affect model reversibility. One particularly important experiment is the variation of horizontal resolution and its effect on reversibility. This specific test also demonstrates that our thermodynamic framework can also be used for determining minimum horizontal resolution in modeled climate studies.

The organization of the paper is outlined here. We first present a brief overview of the model utilized in this study. Next, the error in the model energy budget is evaluated for both the Hadley Circulation and global general circulation. The derivations of the different thermodynamic efficiencies are then presented. This is followed by an examination and discussion of the results from a series of experiments in which the thermodynamic efficiency is evaluated after changes in model parameters.

D.2 Model Formulation

The numerical model employed in this study is the Geophysical Fluid Dynamics Laboratory Idealized GCM (Held and Suarez 1994). This model is highly idealized in the spirit of using simplified physical parameterizations. It consists of an ideal gas atmosphere (dry air) on a rotating sphere with optional topography (not employed in our experiments). The model numerical formulation is spectral, semi-implicit, hydrostatic, hybrid sigma-pressure coordinates in the vertical, and employs a leap-frog time-stepping arrangement with a time filter to damp the computational mode. A spectral transform method is used in order to evaluate grid-point physical processes and non-linear dynamics. The forcing is particularly simple (see Appendix E). It consists of a Newtonian relaxation, whereby the atmospheric temperature field is driven towards a prescribed "radiative equilibrium temperature" which is a function of both latitude and pressure level. Dissipation consists of Rayleigh damping of low-level winds in order to simulate boundary layer friction. The specified frictional dissipation of the horizontal winds is linear and is only a function of the pressure level. No explicit diffusion is included in the model formulation. Nevertheless, a scale-selective horizontal numerical mixing is included, but vertical mixing (diffusion or convection) is left out altogether.

D.3 Heat Engine Framework Applied to Simple GCM

One of the first tasks in the investigation is to calculate the model's energy budget. Accurate assessment of the model's energy budget and its error is critical to the application of the heat engine framework. For the present study, we consider only the model's explicit energy budget. We do not explicitly calculate energy losses due to the numerical damping associated with horizontal mixing nor losses due to the numerical scheme (i.e., time marching scheme). Furthermore, the total model energy is not conserved since the frictional dissipation of the mechanical energy is not converted into internal energy; it is simply lost. Our calculations, therefore, are based solely on the explicit energy budget of the model and the unaccounted for energy terms are lumped into an "error" term. For the global heat engine, in steady state, the energy budget is determined solely by the heat input and frictional dissipation. In order to properly determine the energy budget for the steady-state Hadley circulation, on the other hand, care must be taken to determine the fluxes of energy across the system's boundaries. We derive, below, the model energy budget which takes a simple form given the forcing present in the idealized GCM employed in this study.

We obtain the energy equations beginning with the Navier-Stokes equations for a rotating fluid parcel under the influence of gravity,

$$\frac{d\vec{v}}{dt} = -R_f\vec{v} - \rho^{-1}\nabla p - 2\vec{\Omega} \times \vec{v} + \vec{g}. \tag{D.1}$$

R_f is the Rayleigh friction coefficient. The other quantities follow standard meteorological conventions. By taking the dot product of $\vec{v}$ and equation (D.1) and using the identity, $\nabla \cdot (p\vec{v}) = p\nabla \cdot \vec{v} + \vec{v} \cdot \nabla p$, we have,

$$\rho\frac{d}{dt}(\frac{v^2}{2} + gz) = -\rho R_f v^2 - \nabla \cdot p\vec{v} + p\nabla \cdot \vec{v}. \tag{D.2}$$

Using the mass conservation equation in flux form; that is,

$$\frac{\partial\rho}{\partial t} = -\nabla \cdot (\rho\vec{v}), \tag{D.3}$$

and the fact that for any scalar quantity γ, we can write

$$\rho\frac{d\gamma}{dt} = \frac{\partial}{\partial t}(\rho\gamma) + \nabla \cdot (\rho\gamma\vec{v}), \tag{D.4}$$

equation (D.2) can be rewritten as

$$\frac{\partial}{\partial t}[\rho(\frac{v^2}{2} + gz)] + \nabla \cdot [(\rho\frac{v^2}{2} + \rho gz + p)\vec{v}] = -\rho R_f v^2 + p\nabla \cdot \vec{v}. \tag{D.5}$$

Employing the first law of thermodynamics and the mass conservation equation, the last term in equation (D.5) can be rewritten as,

$$p\nabla \cdot \vec{v} = \rho\dot{Q} - \rho c_v\frac{dT}{dt}. \tag{D.6}$$

where $\dot{Q}$ represents the heating rate. Substituting this into equation (D.5) and using equation (D.4) to rewrite the scalar c_vT, the energy equation can be rewritten as

$$\frac{\partial}{\partial t}[\rho(\frac{v^2}{2} + c_vT + gz)] + \nabla \cdot [(\rho\frac{v^2}{2} + \rho c_vT + \rho gz + p)\vec{v}] = \rho\dot{Q} - \rho R_f v^2. \tag{D.7}$$

Integrating this equation over the volume of the circulation of interest, we have

$$\begin{aligned}\int\int\int \frac{\partial}{\partial t}[\rho(\frac{v^2}{2} + c_vT + gz)]dV + \int\int(\rho\frac{v^2}{2} + \rho c_vT + \rho gz + p)\vec{v}\cdot\hat{n}dA = \\ \int\int\int \rho\dot{Q}dV - \int\int\int \rho R_f v^2 dV,\end{aligned} \tag{D.8}$$

where the divergence equation has been used and $\hat{n}$ is the unit vector normal to the boundaries of the volume containing the circulation. This equation can be applied to the Hadley cell where the advective fluxes of energy are measured along latitudinal and vertical boundaries of the circulation. For the global integral, the energy flux term simply disappears as there are no advective fluxes across, or pressure work on, the boundaries.

In our assessment of the model energy budget, we consider only steady-state values. This implies that the first term in equation (D.8) is negligible. From this simplification, it can be seen that for the Hadley circulation, the heating and dissipation within the cell are balanced by the energy fluxes (potential, internal and kinetic) across, and pressure work, on the boundaries. For the global energy balance in steady state, the net heating simply balances the energy dissipated by friction.

A further simplification arises from the fact that the model is hydrostatic. This means that we can rewrite the flux of internal energy and pressure work term in equation (D.8) in terms of an enthalpy flux ($\rho c_p T\vec{v}$); that is,

$$\int\int \rho(\frac{v^2}{2} + c_pT + gz)\vec{v}\cdot\hat{n}dA = \int\int\int \rho\dot{Q}dV - \int\int\int \rho R_f v^2 dV. \qquad \text{(D.9)}$$

The hydrostatic assumption also has implications for the thermodynamic analysis which will be discussed in section (D.3.2).

D.3.1 Analysis of the Energy Budget

In order to properly calculate the thermodynamic efficiencies, it is necessary to calculate explicitly the energy budget (See Appendix F). For the steady-state global energy balance, equation D.9 becomes

$$\int\int\int \rho\dot{Q}dV = \int\int\int \rho R_f v^2 dV. \qquad \text{(D.10)}$$

The net heating term on the left-hand side of equation D.10 is balanced by the frictional dissipation of the kinetic energy. The global error is simply the difference between these two terms and can be written in a short-hand form as,

$$\widehat{Q}_{net} - \widehat{D} = \widehat{Err}, \qquad \text{(D.11)}$$

where

$$\widehat{Q}_{net} = \int\int\int_g \rho\dot{Q}dV = \int\int\int_g \rho(\dot{Q}_{(+)} - \dot{Q}_{(-)})dV \tag{D.12}$$

is the net heating integrated over the volume of the atmosphere. $\dot{Q}_{(+)}$ represents the positive heating and $\dot{Q}_{(-)}$ the cooling.

$$\widehat{D} = \int\int\int_g \rho R_f v^2 dV \tag{D.13}$$

is the frictional dissipation integrated over the volume of the global atmosphere (indicated by the subscript g). Figure D.1 is a plot of a 1000-day, global running mean at T42 resolution of the net heating, $\widehat{Q}_{net}$, versus the frictional dissipation, $\widehat{D}$. The error in the 1000-day, global running mean between the net heating and frictional dissipation is approximately 20% of the value of the frictional dissipation or the work that is done by the general circulation.

In the case of the steady-state Hadley cell energetics, a similar approach is taken. However, in this case, the advective fluxes through the boundaries must be taken into account. From equation D.9, it is clear that the difference between the net heating and frictional dissipation should equal the sum of the advective fluxes through the boundaries plus the pressure work term. The energy budget for the Hadley cell, including the error, $\widetilde{Err}$, can then be written in short-hand form as

$$(\widetilde{Q}_{net} - \widetilde{D}) = \widetilde{E}_{(f)} + \widetilde{Err}, \tag{D.14}$$

where

$$\widetilde{Q}_{net} = \int\int\int_{hc} \rho\dot{Q}dV \tag{D.15}$$

is the net heating over the volume of the Hadley cell,

$$\widetilde{D} = \int\int\int_{hc} \rho R_f v^2 dV \tag{D.16}$$

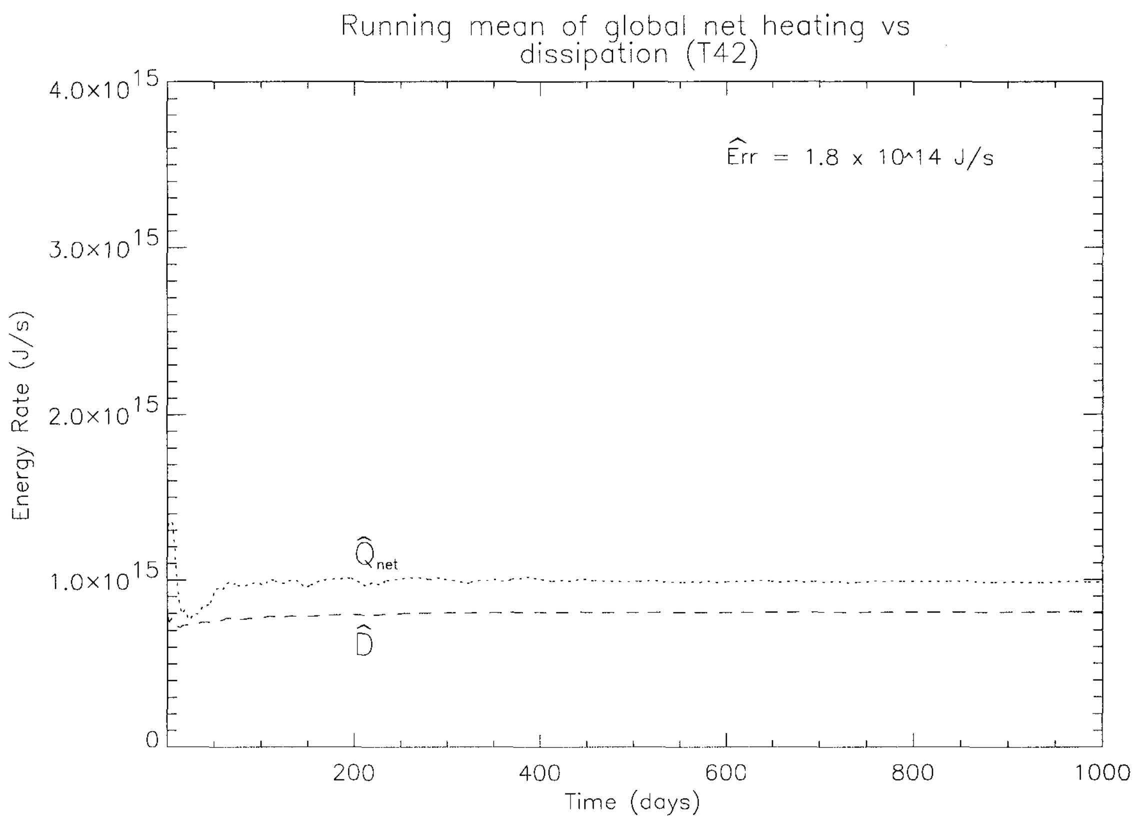

Figure D.1: 1000-day running mean of global net heating, $\widehat{Q}_{net}$, and dissipation, $\widehat{D}$ at T42 resolution. $\widehat{Err}$ is the global error term measured at the end of the 1000-day running mean.

is the frictional dissipation within the Hadley cell volume (indicated by the subscript hc). $\widetilde{E}_{(f)}$, the left-hand side of equation D.9, represents the advective flux across the boundaries, while $\widetilde{Err}$ is the error in the Hadley cell budget. Figure D.2 is a plot of the $\widetilde{Q}_{(net)} - \widetilde{D}$ vs. $\widetilde{E}_{(f)}$ for the southern hemisphere Hadley cell with the latitudinal boundaries at 29S and the equator. The Hadley cell error, $\widetilde{Err}$, is about 5% of the value of $\widetilde{Q}_{(net)} - \widetilde{D}$.

D.3.2 Thermodynamic Efficiency

By examining the steady-state thermodynamic efficiencies, the irreversibility due to changes in model physical parameterizations and/or numerics can be assessed. In this section, we derive the thermodynamic efficiencies for both open and closed systems. We calculate three thermodynamic efficiencies to gauge the strength of irreversibilities in model-generated circulations. The first method for calculating the efficiency is based on work. For closed systems, the work performed by the heat engine is equal to the work consumed by mechanical dissipation of energy. For open systems, work exported into and out of the control volume must be taken into account. Thus, advective fiuxes of kinetic energy must be considered. The second is based on the heat budget. It is the efficiency based on the net energy available to do work for a reversible heat engine. The third is the Carnot efficiency, or maximum attainable efficiency. In a reversible convective cycle in which the heat budget is correct, the efficiency based on mechanical dissipation and heat budget would be identical. This is because in the reversible heat engine, the net heat supplied to the engine is convertly entirely into work due to the absence of irreversible processes (e.g., diffusion of heat and water vapor). The magnitude of the difference

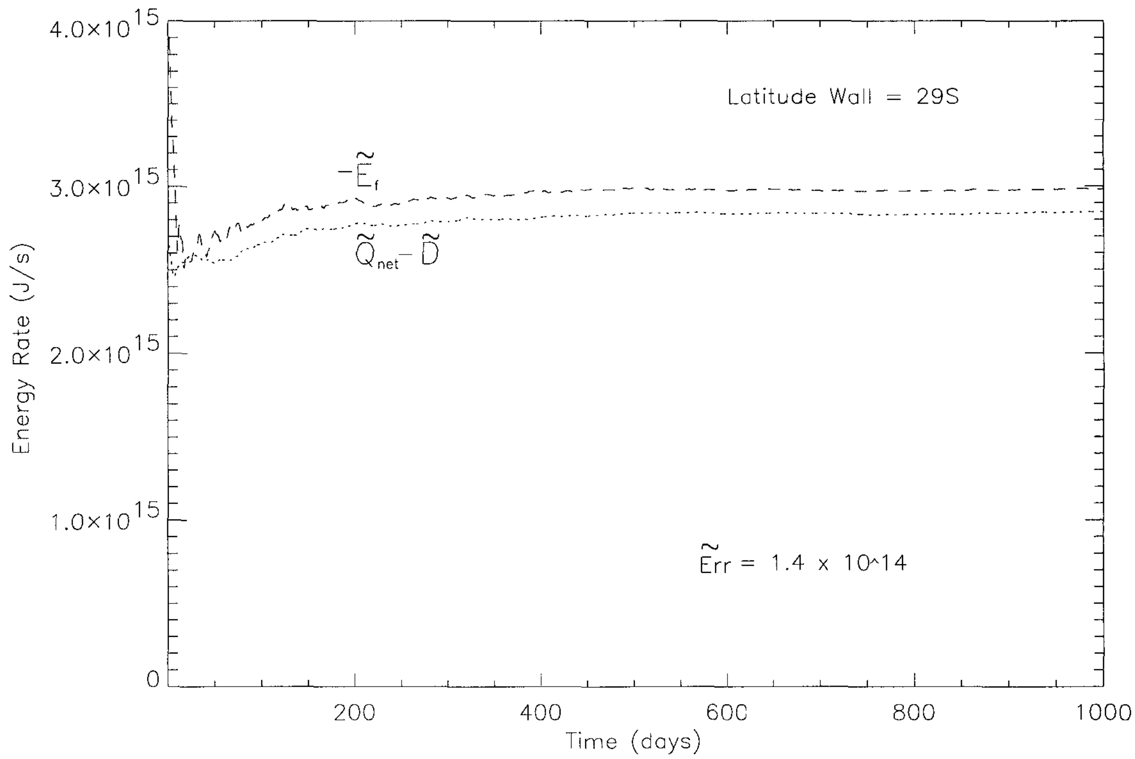

Figure D.2: 1000-day running mean of energy flux, $\widetilde{E}_{(f)}$, into Hadley cell versus net heating, $\widehat{Q}_{net}$, minus dissipation, $\widehat{D}$ at T42 resolution. $\widetilde{Err}$ is the Hadley cell error term measured at the end of the 1000-day running mean.

between these two efficiencies reflects the strength of irreversibilities generated by the numerical model.

The heat engine framework for the open and closed steady-state circulations are developed as follows. Beginning with the first law of thermodynamics for a material element, we have

$$\dot{Q}_M = (c_v \frac{dT}{dt} + p\frac{d\alpha}{dt})_M \tag{D.17}$$

where $\dot{Q}_M$ is the heating rate and α is the specific volume. The subscript M refers to a given material element. Employing the ideal gas law, we can rewrite equation (D.17) as

$$\dot{Q}_M = (c_v \frac{dT}{dt} + R\frac{dT}{dt} - \alpha\frac{dp}{dt})_M, \tag{D.18}$$

where R is the dry air gas constant. Now employing the hydrostatic equation and using $c_v + R = c_p$, we have

$$\dot{Q}_M = (c_p \frac{dT}{dt} + g\frac{dz}{dt})_M. \tag{D.19}$$

Employing equations D.3 and D.4, assuming steady state, and integrating over the control volume of interest, we have

$$\int\int\int \rho\dot{Q}_M dV = \int\int\int \nabla \cdot \rho(c_pT + gz)_M \vec{v} dV. \tag{D.20}$$

Applying the divergence theorem gives

$$\int\int\int \rho\dot{Q}_M dV = \int\int \rho(c_pT + gz)_M \vec{v} \cdot \hat{n} dA \equiv \int\int \vec{Q_f} \cdot \hat{n} dA. \tag{D.21}$$

Therefore, the sum of the enthalpy and geopotential flux; that is, the dry static energy flux, $(\rho(c_pT + gz)_M\vec{v} \equiv \vec{Q_f})$, can be identified as the advective *"effective" heat flux* across the control volume boundaries. It follows from equation (D.9) that

$$\int\int \rho(\frac{v^2}{2})\vec{v} \cdot \hat{n} dA + \int\int\int \rho R_f v^2 dV = \int\int\int \rho\dot{Q} dV - \int\int \vec{Q_f} \cdot \hat{n} dA, \tag{D.22}$$

The difference between the integrated heating rate over the control volume and the "effective" heat flux is attributed to the flux of kinetic energy through the boundaries and the mechanical dissipation of kinetic energy within the volume. This approach permits the separation of mechanical energy (l.h.s.) from an effective heating term (r.h.s.) for an open heat engine such as the Hadley cell; thereby, making it possible to derive the thermodynamic efficiencies based on work and heat budgets.

The thermodynamic efficiency of a general heat engine is defined as the fraction of the heat input which is converted to mechanical energy. In the case of an open system in steady-state, the mechanical energy generated (i.e., bulk fluid motions) is dissipated by friction. However, mechanical energy may be generated in the volume and exported through its boundaries in the form of kinetic energy flux (the first term in equation D.22). In addition, in order to isolate the work done solely by the heat engine, it is necessary also to consider the dry static energy flux across the boundaries of the control volume. This is because this term represents fluxes of enthalpy through, and and pressure work on, the boundaries which modify the heat input to the open heat engine (for inward directed fluxes only). Using the convention that fluxes into the system are negative and outward fluxes are positive, we calculate the thermodynamic efficiency in two ways. In the case where the dry static energy flux is directed into the system; i.e., $\int\int \rho(\vec{Q_f}) \cdot \hat{n} dA \leq 0$, we have

$$\eta = \frac{\int\int\int \rho R_f v^2 dV + \int\int \rho(\frac{v^2}{2})\vec{v} \cdot \hat{n} dA}{\int\int\int \rho \dot{Q}_{(+)} dV - \int\int \rho(\vec{Q_f}) \cdot \hat{n} dA}. \tag{D.23}$$

From this calculation, it is apparent that large fluxes of dry static energy into the volume of interest tend to decrease the overall efficiency as do influxes of kinetic energy. This is due to the fact that, for a given quantity of work performed, there is an increase in the heat input to the heat engine thereby making it less efficient (the denominator becomes larger). Likewise, increases in kinetic energy fluxes to the heat engine, for a given heat input, decrease the overall efficiency of the open heat engine. This is because influxes of kinetic energy does not represent motions generated directly from the heat input to the heat engine. In the case in which the fluxes of dry static energy are outward from the system, these fluxes are no longer part of the heat input into the heat engine. As a result, the thermodynamic efficiency, η, is calculated as

$$\eta = \frac{\int\int\int \rho R_f v^2 dV + \int\int \rho(\frac{v^2}{2})\vec{v} \cdot \hat{n} dA}{\int\int\int \rho \dot{Q}_{(+)} dV}. \tag{D.24}$$

In order to calculate the thermodynamic efficiency for the global circulation (a closed system), the only modification is to eliminate the advective fluxes. As a result, the global efficiency based on frictional dissipation of energy is calculated as

$$\eta = \frac{\int\int\int \rho R_f v^2 dV}{\int\int\int \rho \dot{Q}_{(+)} dV} \tag{D.25}$$

The above derivations are based on work; that is, the frictional dissipation on the bulk fluid motion generated by the heat engine. By comparing these work-based efficiencies with the reversible efficiency based on net heating, we can assess how close model-generated circulations are to reversible. The thermodynamic efficiency of an open reversible heat engine based on net heating is

$$\eta_{rev} = \frac{\int\int\int \rho \dot{Q}_{(+)} dV - \int\int\int \rho \dot{Q}_{(-)} dV - \int\int (\rho(\vec{Q_f}) \cdot \hat{n} dA}{\int\int\int \rho \dot{Q}_{(+)} dV - \int\int (\rho(\vec{Q_f}) \cdot \hat{n} dA}, \tag{D.26}$$

where the same sign convention for fluxes applies. For the case when the fluxes of dry static energy are directed outward from the open system, the reversible efficiency maybe be written as

$$\eta_{rev} = \frac{\int\int\int \rho\dot{Q}_{(+)}dV - \int\int\int \rho\dot{Q}_{(-)}dV - \int\int(\rho(\vec{Q_f}) \cdot \hat{n}dA}{\int\int\int \rho\dot{Q}_{(+)}dV}. \tag{D.27}$$

For the closed system the advective heat fluxes across the boundaries vanish and we have,

$$\eta_{rev} = \frac{\int\int\int \rho\dot{Q}_{(+)}dV - \int\int\int \rho\dot{Q}_{(-)}dV}{\int\int\int \rho\dot{Q}_{(+)}dV}. \tag{D.28}$$

Since $\eta_{rev} \geq \eta$, the difference between the two numbers provides a measure of just how large irreversibilities are. This is discussed in the next section in the context of the GFDL GCM.

For a final comparison, the Carnot efficiency for the global circulation is calculated. This efficiency is the highest efficiency possible and, therefore, provides an upper bound on the efficiencies of model-generated circulations. In order to properly calculate the Carnot efficiency, it is necessary to weight the temperature of the heat source and sink by the heating and cooling rates, respectively. This gives

$$\eta_c = \frac{[T_h] - [T_c]}{[T_h]}, \tag{D.29}$$

where $[T_h]$ and $[T_c]$ are calculated in the following manner:

$$[T_h] = \frac{1}{[\dot{Q}_{(+)}]}(\int\int\int \rho T\dot{Q}_{(+)}dV), \tag{D.30}$$

where $[\dot{Q}_{(+)}]$ is the heating integrated over the volume. In the case of $[T_c]$, we have

$$[T_c] = \frac{1}{[\dot{Q}_{(-)}]}(\int\int\int \rho T\dot{Q}_{(-)}dV). \tag{D.31}$$

We turn now to applications of these efficiencies to the GFDL idealized GCM.

D.4 Applications of Thermodynamic Efficiencies

In this section, a series of experiments are carried out in which various physical/numerical parameters are modified. These experiments are designed to provide a "proof of concept" demonstrating the value of comparing model efficiencies for assessing the strength of irreversibilities. The different experiments can be classified for convenience under these categories: (1) model numerics, (2) model forcing, and (3) model parameters. For the three efficiency calculations, experiments were run to steady-state and at a resolution of T30. T30 appears to be sufficient resolution for the efficiency calculations (see discussion below). Specifically, the experiments were started from an isothermal state at rest with small perturbations added to break the symmetry. Integrations were carried out for 1000 model days (54000 timesteps). Model statistics approach a steady-state around 1000 days (see Figure D.1 and Figure D.2). The 1000-day mean taken from these runs and was used to identify the latitudinal boundaries of the Hadley cell. The determination of the latitudinal boundaries was based upon the latitude of maximum vertical velocity and convergence of the meridional wind. The 1000-day mean was then used to restart the experiment and the model was integrated for 1000 more days in order to calculate the necessary energetic quantities.

D.4.1 Modification of Numerical Parameters

A series of experiments were carried out in which the values of several numerical parameters were modified. These experiments provide a sensitivity test for the model's steady-state solutions to changes in model numerics. The measure of this sensitivity is reflected in the behavior of the thermodynamic efficiencies. We have carried out two experiments: modification of the spectral damping coefficient and the model's spectral resolution. The results are presented for both the general circulation (a closed system) and the Hadley cell (an open system).

Spectral Damping Coefficient

In the model no explicit diffusion is specified and only a very scale-selective horizontal mixing is included. The form of this mixing of vorticity, divergence and temperature is a Laplacian raised to the fourth power whose strength is set to that of an e-folding time of 0.1 days for the highest frequency wave. The purpose of this mixing is to prevent the accumulation of energy at high frequencies and is unrelated to controlling numerical instabilities. The spectral damping coefficient was varied by modifying the e-folding time for the smallest wave from between 5.0 days^{-1}; that is, weaker damping, to values of 200 days^{-1}; that is, stronger damping. The net effect of varying this parameter leads to a more energetic global circulation (weaker damping) and an increase in the thermodynamic efficiency based on work, η. Frictional dissipation increases on the order of 30% between the experiments with the strongest and weakest damping, while increases in positive heating are only approximately 10%. Hence, the heat-based and Carnot efficiencies, η_{rev} and η_c, respectively, remain close to constant as relative changes in Q_{net} and Q_+ are small. This implies that the model becomes more irreversible as spectral damping strength is increased; that is, as horizontal mixing is increased. This can be seen by comparing the three global efficiencies (Figure D.3).

In the case of the Hadley cell efficiencies, the increase in irreversibility as a

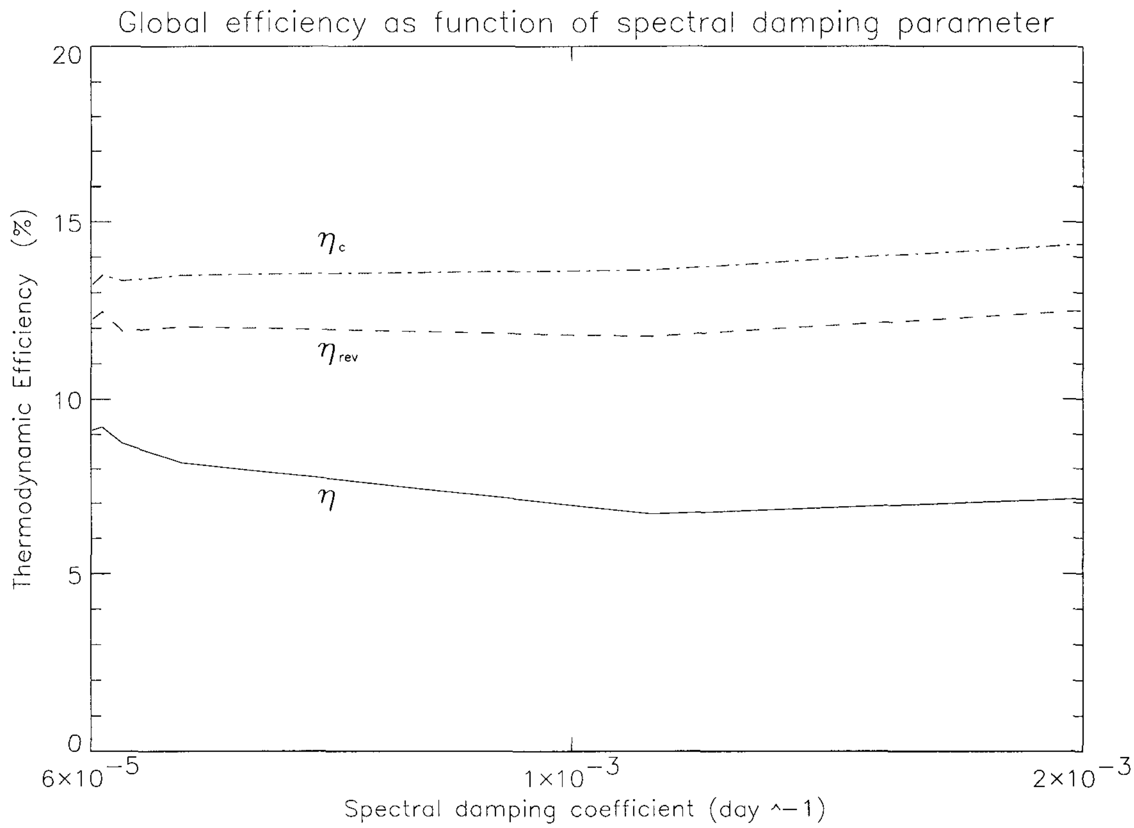

Figure D.3: A plot of the three global thermodynamic efficiencies versus the spectral damping coefficient.

function of increased spectral damping is not apparent (Figure D.4). The relative differences in positive heating and dissipation within the Hadley cell change little leading to only small differences in η. Likewise, the relative changes in the heat flux versus net heating within the Hadley cell are also small. As a result, there is no obvious trend towards increased irreversibility with strengthened spectral damping.

Spectral Resolution

For climate studies, it is critical to understand model sensitivity to horizontal resolution. Models that demonstrate no tendency to converge towards a solution as resolution is increased would certainly be mathematically inconsistent. Work by Boer and Denis (1997) suggests that the resolution requirement for properly capturing the *dynamical* aspects of climate is not that large (T30 or greater). Using a simply forced model similar to the model used in our study, they state that T32 is sufficient resolution for the convergence of model dynamics. We have carried out a series of experiments in which the sensitivity of the thermodynamic efficiency is observed as a function of changing horizontal resolution. Our approach is similar to theirs in that we assume known and "correct" physical parameterizations. However, our results focus on the strength of irreversibilities as a function of resolution.

Figure D.5 gives the global efficiencies as a function of resolution. In agreement with Boer and Denis (1997), there appears to be convergence at resolution $\geq T30$. In terms of η, there is a greater than two-fold increase in its value from resolution $T15$ to $T63$. The changes in η between $T30$ and $T63$ are much less pronounced. This decreasing rate of convergence agrees with the results of Boer

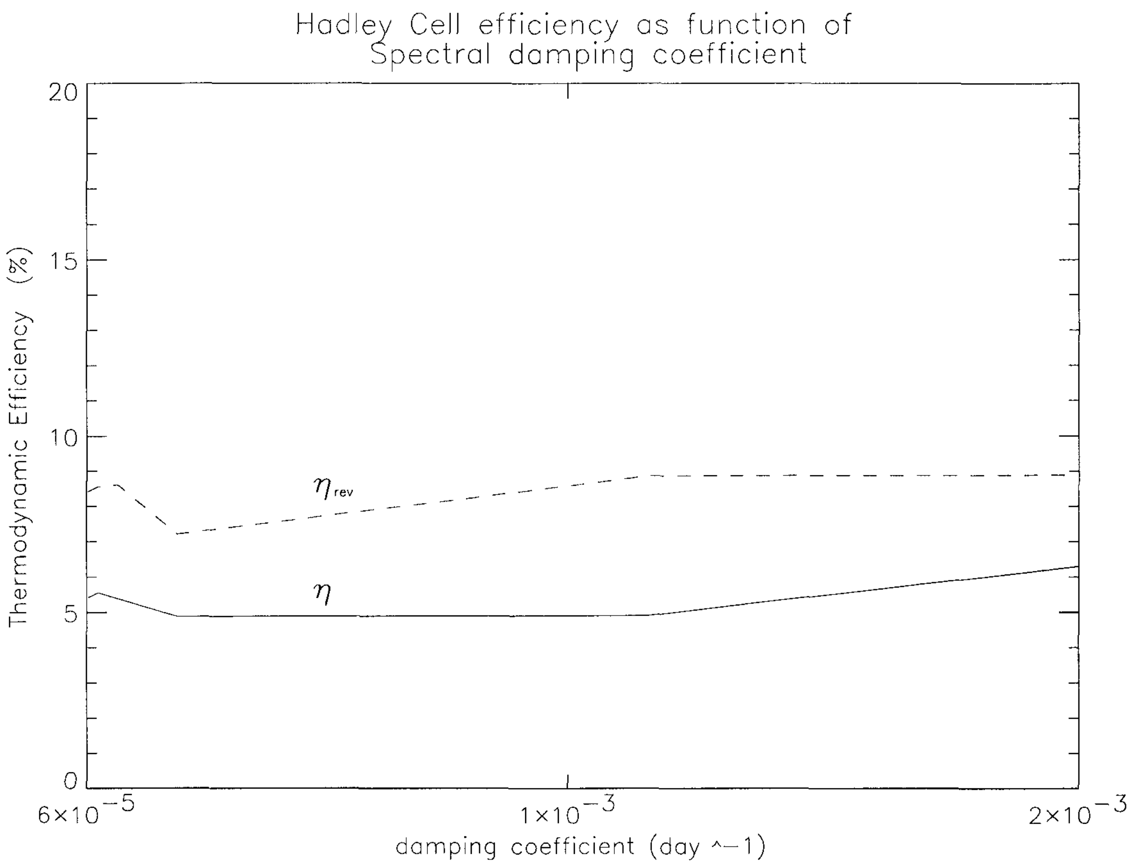

Figure D.4: A plot of the three thermodynamic efficiencies versus the spectral damping coefficient for the Hadley cell.

and Denis (1997) and others who have examined the effects of horizontal resolution on dynamical variables (e.g., eddy fluxes) (e.g., Boyle 1993; Boyville 1991; Senior 1995).

For the Hadley cell, the variation of η is small with respect to resolution changes (Figure D.6). As in the case of spectral damping, the relative differences in positive heating and dissipation within the Hadley cell change little resulting in a near constant η. η_{rev}, on the other hand, varies greatly over the range of spectral resolution. As resolution decreases, there is a relative decrease in the heat flux out of the Hadley cell. Referring to equation (D.27), it can be seen that decreases in outward heat fluxes lead to increase in η_{rev}. These results suggest that model energetics are sensitive to resolution, with coarser resolution leading to greater irreversibility. These results most likely result from the increase in the importance of the horizontal mixing term which acts on the highest frequency waves. As the resolution decreases, this horizontal mixing term becomes relatively more important leading to greater numerical heat source/sinks which is reflected in the model energy budget and, hence, the efficiency based on the heat budget.

D.4.2 Modification of model forcing

Frictional damping coefficient

The form of frictional dissipation in the model is a simple linear damping of the velocities, which is solely a function of pressure (see Appendix E). Several experiments were conducted in which the strength of the frictional damping coefficient is varied. The standard value for the frictional damping time, k_f, is given

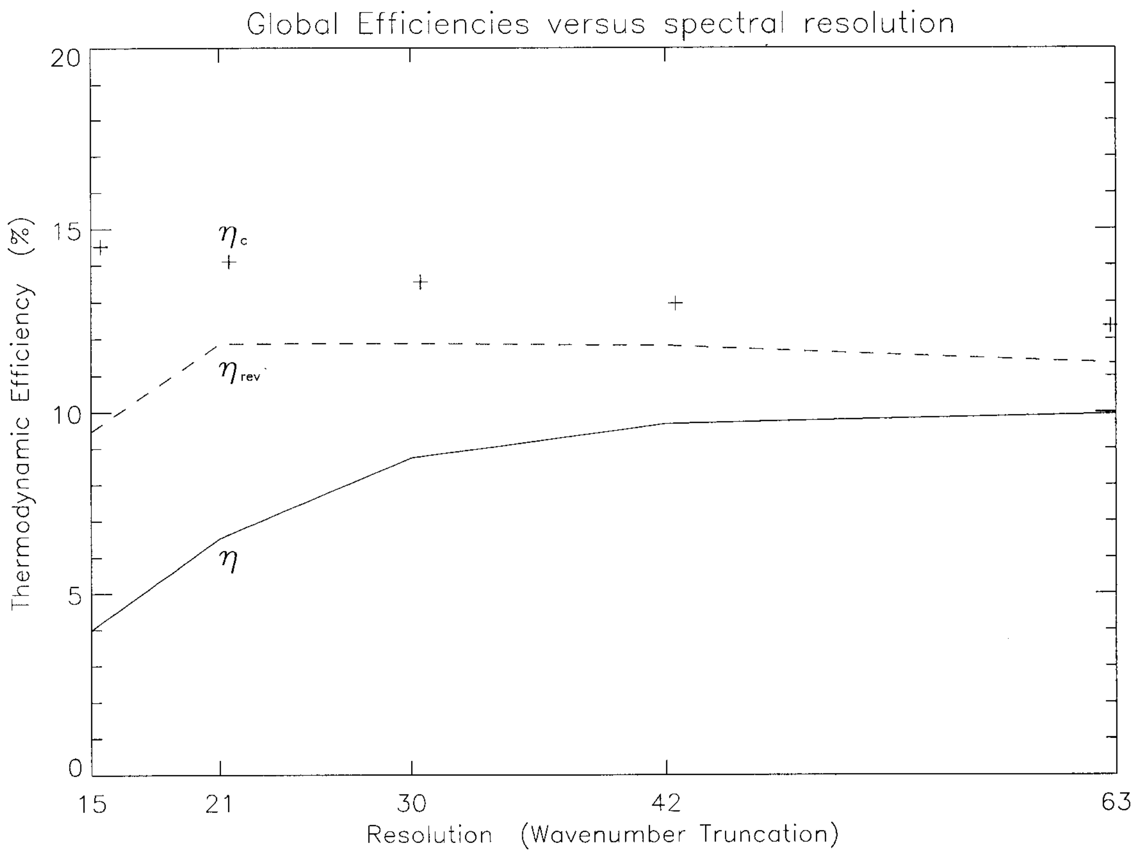

Figure D.5: A plot of the three thermodynamic efficiencies versus spectral resolution.

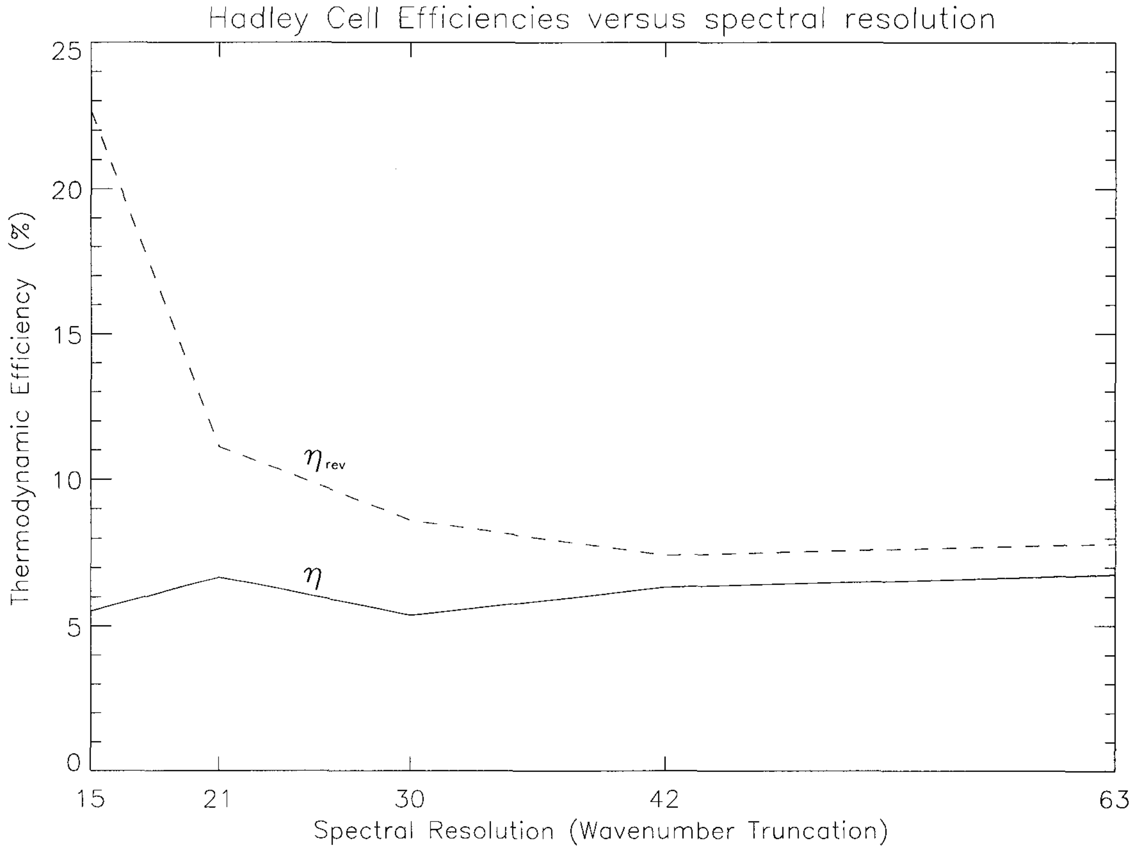

Figure D.6: A plot of the three Hadley cell thermodynamic efficiencies versus spectral resolution.

as 1 day^{-1}. This value was varied over a range from strong damping, 10 day^{-1}, to weaker damping, 0.5 day^{-1}. Global efficiencies show a slight tendency for increased irreversibility with a decrease in the frictional damping timescale (Figure D.7). For the Hadley cell, similar behavior is also observed (Figure D.8).

Newtonian damping

The model temperature fields are relaxed towards a prescribed "radiative equilibrium" through a newtonian damping term. One of the damping coefficients, k_s (see Appendix E), was varied between 0.1 day^{-1} to 1 day^{-1} to modify the relaxation time. The experiments for both the global circulation (Figure D.9) and Hadley circulation (Figure D.10) reveal no clear trend in irreversibility due to modifications of the relaxation time. There is only a tendency towards decreases in the three efficiencies as the relaxation time is increased.

D.4.3 Modification of model parameters

Modification of Rotation Rate

The model rotation rate was modified in several experiments. The changes in rotation rate were varied between 2 Earth days on the slow end and $\frac{1}{4}$ Earth days on the fast end. There are interesting features observable in the variation of the global efficiencies (Figure D.11). The work-based efficiency, η, increases from rapid rotation to slower rotation. This agrees with the theory for rapidly rotating atmospheres which are known to inhibit the conversion of potential into kinetic energy (Emanuel 1994). η_{rev} and η_c show a somewhat different behavior. η_{rev} and

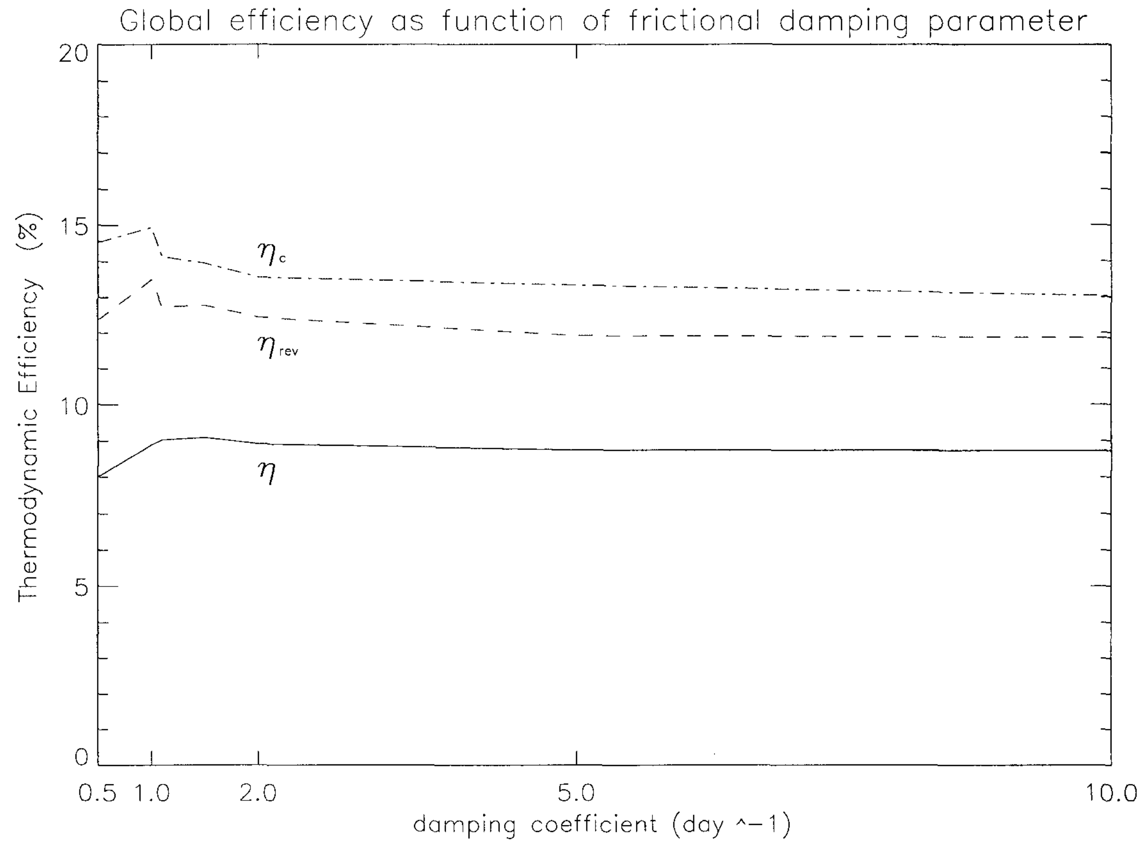

Figure D.7: A plot of the three global thermodynamic efficiencies versus frictional damping coefficient.

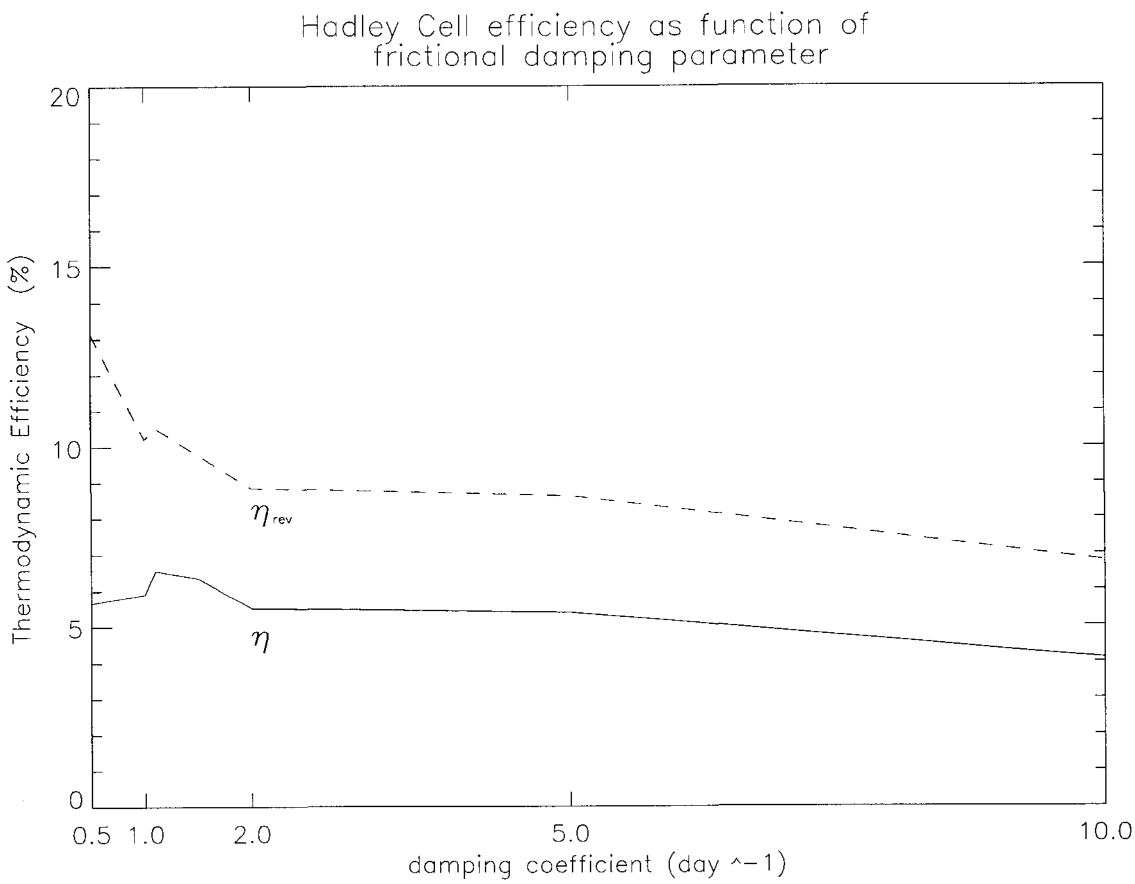

Figure D.8: A plot of the three Hadley cell thermodynamic efficiencies versus frictional damping coefficient.

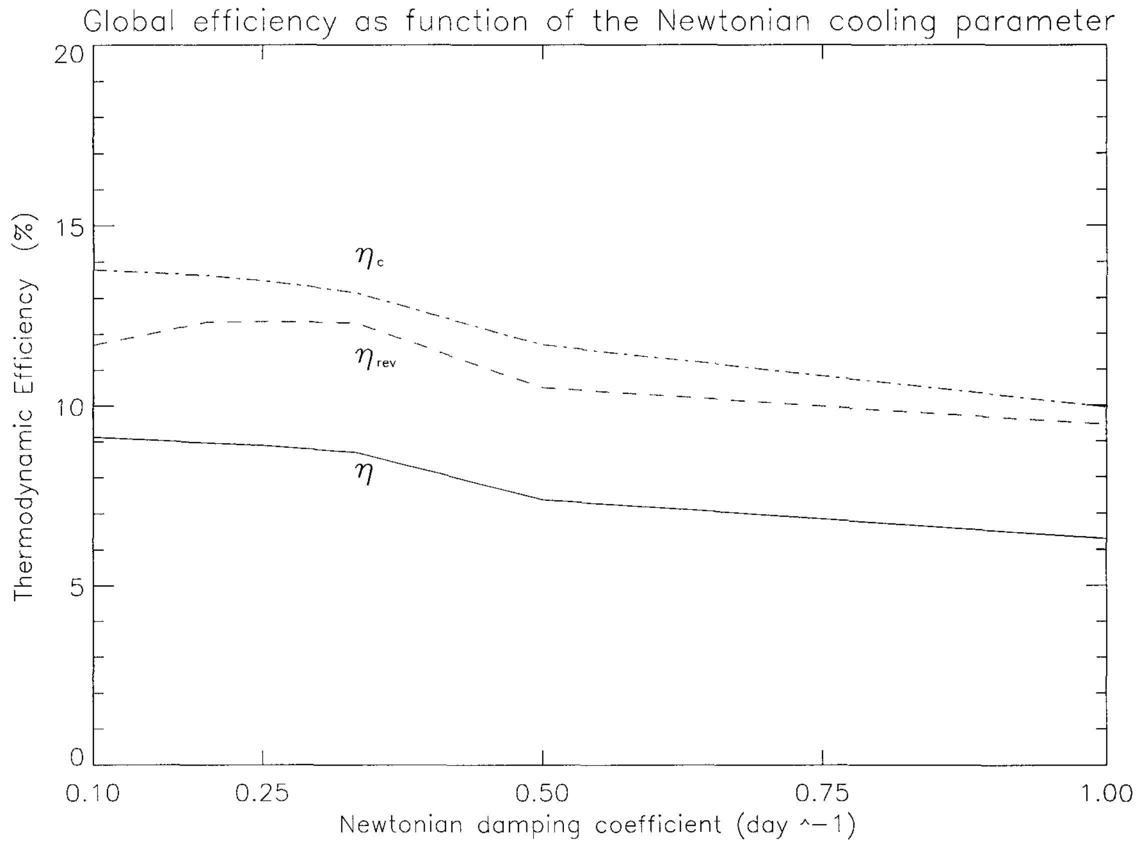

Figure D.9: A plot of the three global thermodynamic efficiencies versus newtonian cooling.

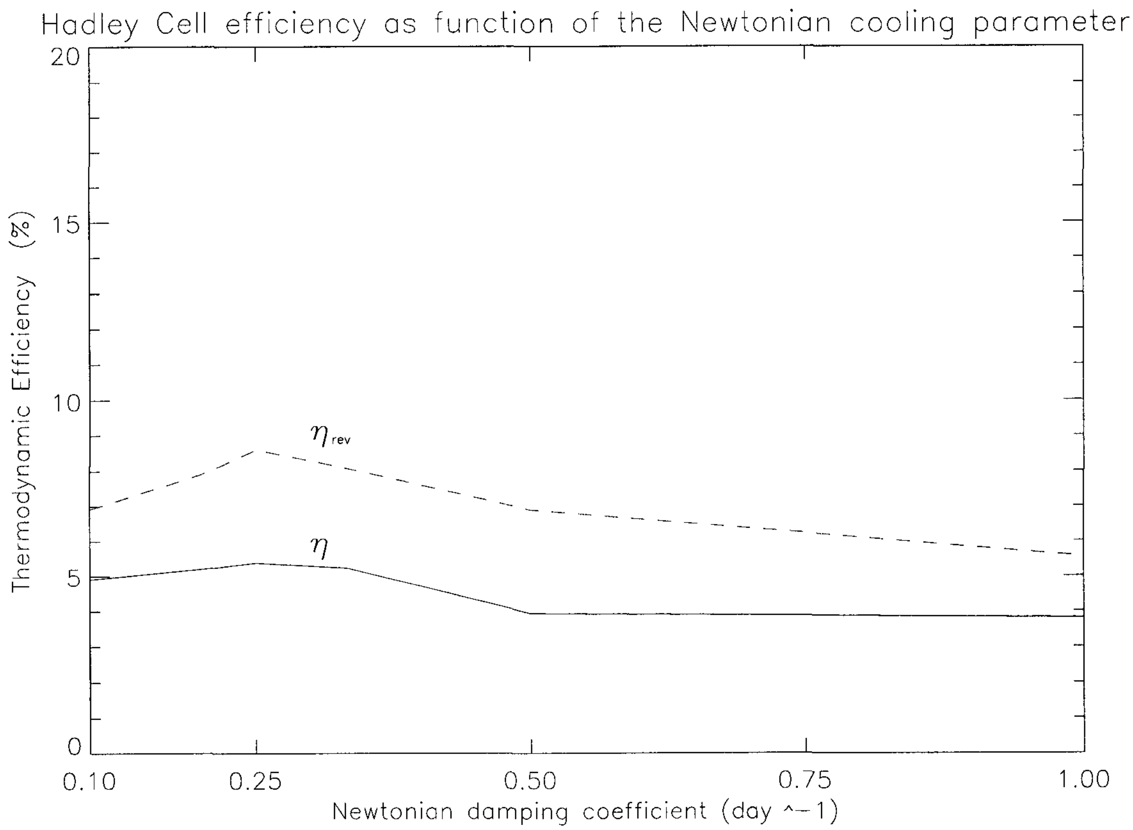

Figure D.10: A plot of the three Hadley cell thermodynamic efficiencies versus newtonian cooling.

η appear to converge at the most rapid rotation rate, 6hr, but then have maximum divergence (greatest irreversibility) at 12hr rotation. The explanation appears to lie in the weakness of meridional transport of energy in the 6hr rotation case. The rapidly rotating regime results in a heating distribution that is weaker (less heat input) at low latitudes and higher pressures and less cooling at higher latitudes and lower pressures (Figure D.12). This results in a marked decrease in the reversible and Carnot efficiencies on the global scale for the 6hr rotation rate.

For the Hadley cell in the regime of rapid rotation, there are very small fluxes of energy out of the cell and decreases in the positive heat input which lead to strong increases in the reversible efficiency, η_{rev}, (See figure D.11). As the rotation rate increases, there are increases in the positive heat input, through changes in the equilibrium temperature field (i.e., the newtonian relaxation). There are also increased heat fluxes out of the Hadley cell. The combined effect of the increase in positive heat input and increased heat fluxes out of the cell is to decrease η_{rev} to approximately 10%.

D.5 Summary

A simple framework for gauging the strength of irreversibilities for large-scale hydrostatic models has been presented. By properly defining the work and heat input terms, the heat engine framework permits the calculation of the thermodynamic efficiency for both closed and open circulations. By comparing thermodynamic efficiencies based on work, the heat budget and the Carnot efficiency, one can gauge the sensitivity that numerical models have with respect to changes in a given model parameter. Our application of this method to this dry, simply forced GCM

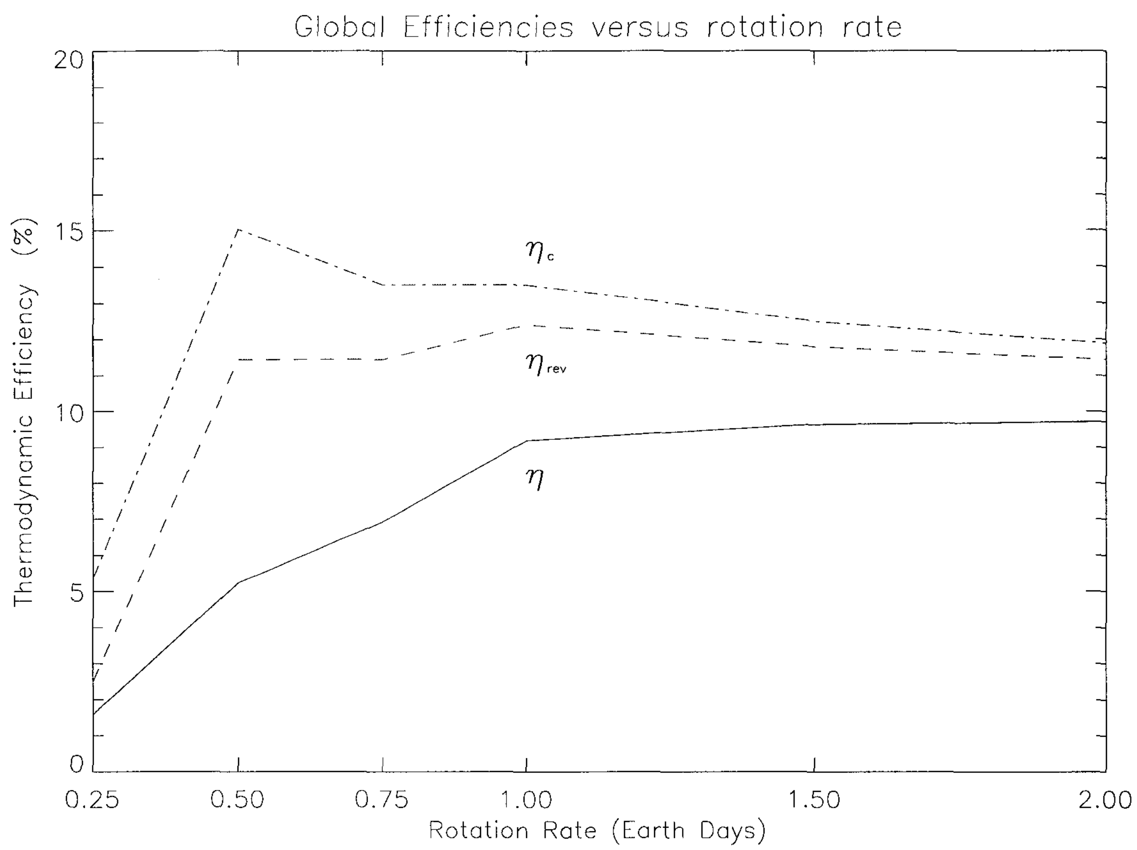

Figure D.11: A plot of the three global thermodynamic efficiencies versus planetary rotation rate.

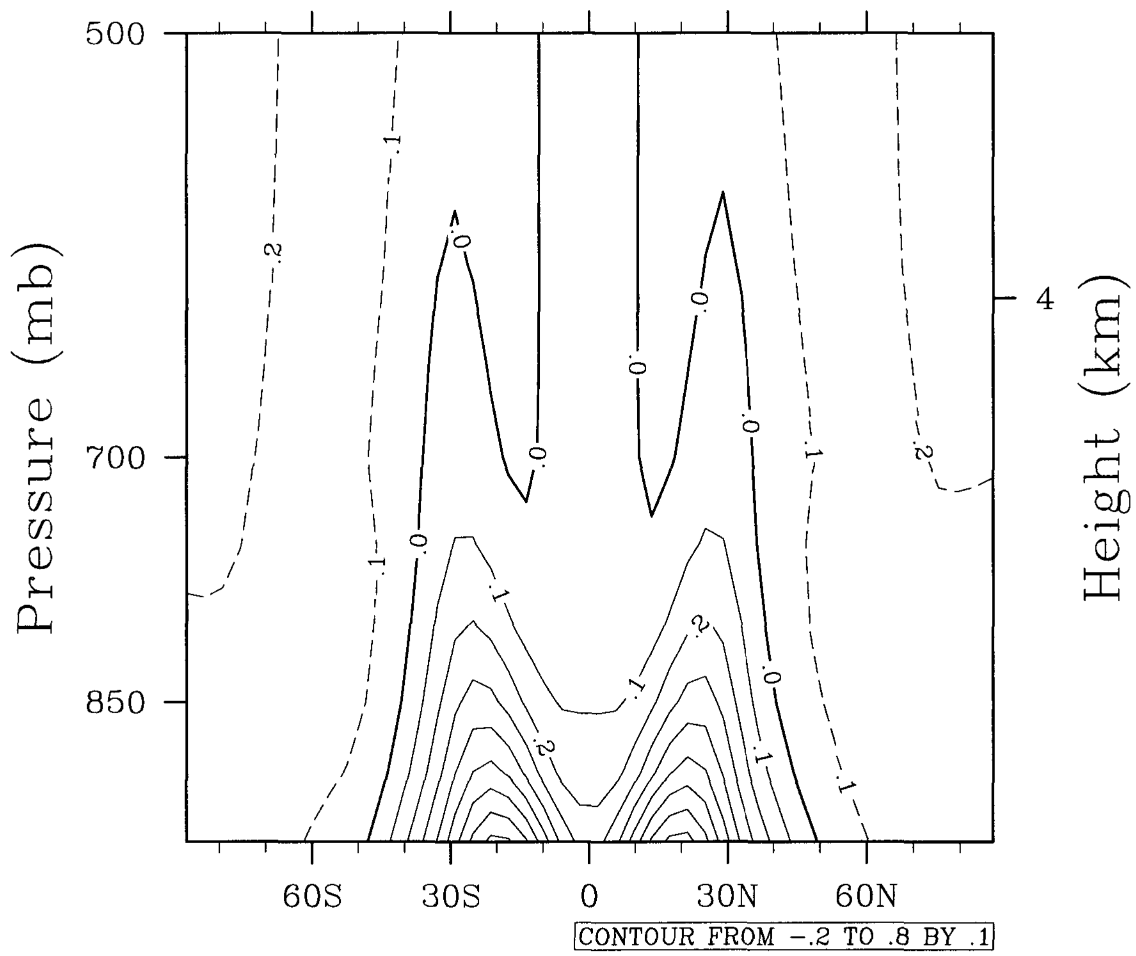

Figure D.12: Plot of difference in heating rate ($deg\ C\ day^{-}1$) between 12 and 6 hour rotation rates. Solid contours represent regions of larger heating rates while dashed contours represent regions of greater cooling.

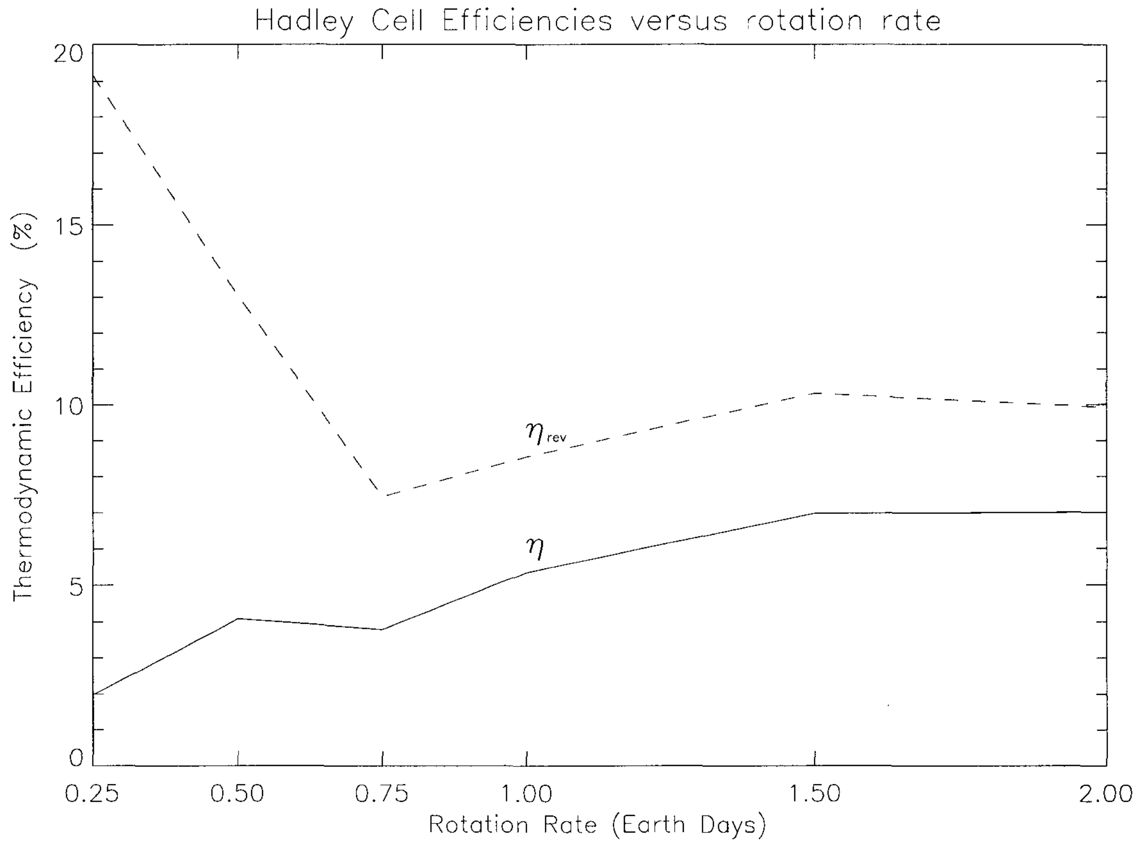

Figure D.13: A plot of the three thermodynamic efficiencies versus planetary rotation rate for the Hadley Cell.

demonstrates that model irreversibility can increase with modification of different parameters, such as spectral damping, or with coarser spectral resolution. In future work, we intend to apply this framework to a moist model, in order to ascertain the strength of irreversible entropy sources associated with the water substance. In addition, this framework could easily be adapted for use with assimilated data in order to compare directly models and data from observations.

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APPENDIX E

FORCING IN AN IDEALIZED GENERAL CIRCULATION MODEL

The following equations give the functional forms of the coefficients in the Newtonian cooling and Rayleigh damping terms and the equilibrium temperature. The frictional dissipation coefficient has the form

$$k_v = k_f \max(0, \frac{\sigma - \sigma_b}{1 - \sigma_b}), \tag{E.1}$$

where k_f is 1 day^{-1} and σ_b is 0.7. σ is the normalized pressure. The Newtonian cooling coefficient is expressed as

$$k_T = k_a + (k_s - k_a) \max(0, \frac{\sigma - \sigma_b}{1 - \sigma_b}) \cos^4 \phi, \tag{E.2}$$

where k_a is $\frac{1}{40}$ day^{-1} and k_s is $\frac{1}{4}$ day^{-1}. The equilibrium temperature, T_{eq}, is determined as the maximum between the baseline temperature of 200K and one determined as a function of pressure and latitude as follows,

$$T_{eq} = \max\{200, [315 - (\Delta T)_y \sin^2 \phi - (\Delta\Theta)_z \log(\frac{p}{p_0}) \cos^2 \phi](\frac{p}{p_0})^\kappa\}. \tag{E.3}$$

$(\Delta T)_y$ and $(\Delta\Theta)_z$ have values of $60K$ and $10K$, respectively. p_0 is a reference pressure of $1000mb$. ϕ represents the angle of latitude. κ has a value of 0.285.

APPENDIX F

EQUATIONS FOR CALCULATION OF MODEL ENERGETICS

The calculations of model dissipation and heating are as follows. Dissipation (D) is calculated as follows

$$D = k_v(u^2 + v^2), \tag{F.1}$$

where k_v is defined in Appendix E and u, v represent the horizontal velocity components. The model heating $\dot{Q}$ is defined as

$$\dot{Q} = -c_p k_T (T - T_{eq}) \tag{F.2}$$

where the variables are as defined in Appendix A. The calculation of global energetic quantities are derived form model output in the following manner. For global quantities, we have the following discrete forms,

$$\text{Mass} = \frac{1}{g} \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{i=1}^{I} {R_k}^2 \cos\Theta_j \Delta\Theta_j \Delta\lambda_i \Delta p_{ijk} \tag{F.3}$$

$$\text{Kinetic Energy} = \frac{1}{g} \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{i=1}^{I} {R_k}^2 (\frac{1}{2})(u_{ijk}^2 + v_{ijk}^2 + w_{ijk}^2) \cos\Theta_j \Delta\Theta_j \Delta\lambda_i \Delta p_{ijk} \tag{F.4}$$

$$\text{Internal Energy} = \frac{c_v}{g} \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{i=1}^{I} {R_k}^2 T_{ijk} \cos\Theta_j \Delta\Theta_j \Delta\lambda_i \Delta p_{ijk} \tag{F.5}$$

$$\text{Potential Energy} = \frac{1}{g} \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{i=1}^{I} {R_k}^2 \Phi_{ijk} \cos\Theta_j \Delta\Theta_j \Delta\lambda_i \Delta p_{ijk} \tag{F.6}$$

$$\text{Dissipation} = \frac{1}{g} \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{i=1}^{I} {R_k}^2 D_{ijk} \cos\Theta_j \Delta\Theta_j \Delta\lambda_i \Delta p_{ijk} \tag{F.7}$$

$$\text{Heating} = \frac{1}{g} \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{i=1}^{I} {R_k}^2 \dot{Q}_{ijk} \cos\Theta_j \Delta\Theta_j \Delta\lambda_i \Delta p_{ijk} \tag{F.8}$$

The indices in each of the summations are as follows: k is the full pressure-levels indice, K is the maximum number of full pressure-levels. i is the longitude indice, I is the maximum longitude, j is the latitude indice, J is the maximum latitude. $\Delta\Theta_j = |\Theta_{j+1} - \Theta_j|$, $\lambda = |\lambda_{i+1} - \lambda_i|$ and $\Delta_{p_ijk} = p_{ijk} - p_{ijk-1}$.

In order to isolate the energetics of the Hadley cell, it is necessary to calculate fluxes through the poleward and equatorward boundaries. In the case, of vertical fluxes, they can be ignored because there are no surface fluxes of energy and the upper boundary of the atmosphere is formally at 0 pressure. The horizontal fluxes into/out of the Hadley cell are calculated in the following manner.

$$\text{Kinetic Energy Flux} = \frac{1}{g}\sum_{k=1}^{K}\sum_{i=1}^{I} R_k(v_{iJ_Bk})(\frac{1}{2})(u^2_{iJ_Bk}+v^2_{iJ_Bk}+w^2_{iJ_Bk})\cos\Theta_{J_B}\Delta\lambda_i\Delta p_{iJ_Bk} \tag{F.9}$$

$$\text{Internal Energy Flux} = \frac{c_v}{g}\sum_{k=1}^{K}\sum_{i=1}^{I} R_k(v_{iJ_Bk})T_{iJ_Bk}\cos\Theta_{J_B}\Delta\lambda_i\Delta p_{iJ_Bk} \tag{F.10}$$

$$\text{Pressure Work} = \frac{1}{g}\sum_{k=1}^{K}\sum_{i=1}^{I} R_k(v_{iJ_Bk})T_{iJ_Bk}\cos\Theta_{J_B}\Delta\lambda_i\Delta p_{iJ_Bk} \tag{F.11}$$

$$\text{Geopotential Flux} = \frac{1}{g}\sum_{k=1}^{K}\sum_{i=1}^{I} R_k(v_{iJ_Bk})\Phi_{iJ_Bk}\cos\Theta_{J_B}\Delta\lambda_i\Delta p_{iJ_Bk} \tag{F.12}$$

J_B is the index of the latitude at either the poleward or equatorward boundary.

APPENDIX G

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