IMAGING EXO-SOLAR PLANETARY SYSTEMS

WITH

TERRESTRIAL PLANET FINDER

by

Andrew Lynn Eatchel

A Dissertation Submitted to the Faculty of the
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

In Partial Fulfillment of the Requirements
For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

The University of Arizona

2004
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John A. Reagan, Ph.D.  
Christopher K. Walker, Ph.D.  
Hal S. Tharp, Ph.D.  
Neville J. Woolf, Ph.D.  
Philip M. Hinz, Ph.D.

Final approval and acceptance of this dissertation is contingent upon the candidate’s submission of the final copies of the dissertation to the Graduate College.

I hereby certify that I have read this dissertation prepared under my direction and recommend that it be accepted as fulfilling the dissertation requirement.

John A. Reagan, Ph.D.  
Christopher K. Walker, Ph.D.
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ACKNOWLEDGEMENTS

Professors John Reagan and Hal Tharp made significant contributions to my graduate studies at the University of Arizona and served on my committee. Professors Nick Woolf, Chris Walker, Phil Hinz, and Keith Hege provided the framework for this research. Financial support and academic advice was given by professor Larry Schooley. Without his assistance, I would not have been able to complete the requirements for the degree. Important insights were obtained through conversations with Thangasamy Velusamy at JPL. Partial funding for this research was received in the form of an Imaging Science Fellowship from the people of Arizona through Proposition 301 and a federal research grant through NASA/JPL. I wish to express my gratitude to all for their help and support.
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ABSTRACT

The concept of building a space based telescope capable of directly imaging extra-solar planetary systems has been in existence for more than a decade. While the basic ideas of how such an instrument might work have already been discussed in the literature, specific details of the design have not been addressed that will enable a telescope of this class to be functionally realized. A straw man configuration of the instrument is examined here for its ability to acquire data of sufficient informational content and quality to produce images and spectra of distant planetary systems and to find what technical problems arise from analyzing the interferograms it delivers. Computer programs that simulate the signals expected to be produced by a structurally connected instrument (SCI) version of Terrestrial Planet Finder (TPF) and reconstruct images from those signals will be presented along with programs that extract planetary parameters. An abbreviated radiometric performance analysis will also be provided that will assist astronomers in designing an appropriate mission.
INTRODUCTION

Terrestrial Planet Finder (TPF) is related to the European Space Agency's Darwin mission in its fundamental goals of finding terrestrial planets and examining them for signs of life or conditions required to support life. By 2020, technological achievements may be sufficient to enable launch of the mission. Work on the TPF telescope began more than a decade ago and is broadly managed under NASA's Origins Program. Its primary purpose will be to directly image and characterize extra-solar planetary systems. Stated goals of the program and TPF are to answer the fundamental questions:

a. Are there Earth-like planets in the "habitable zones" around their parent stars where the surface temperature is capable of supporting liquid water over a range of surface pressures?

b. What are the compositions of the atmospheres of terrestrial planets orbiting nearby stars? Is water, carbon monoxide, or carbon dioxide present?

c. Are there atmospheric components or conditions attributable to primitive life, such as ozone or molecular oxygen, seen in the Earth's atmosphere?

d. How do planets form out of disks of solid and gaseous material around young stars?
To answer these questions, TPF will have to be sizeable to provide sufficient resolution, and glare from the parent star must be blocked in some fashion to enable any surrounding planets to be seen.

TPF is of Hubble Space Telescope (HST) lineage whose antecedents also include the Next Generation Space Telescope (NGST) with increased mirror sizes and improved optical precision, the Space Infrared Telescope Facility (SIRTF) designed specifically for long wavelength observations. Each progressive technological development in its genealogy is a prerequisite to TPF's success. Ground based interferometers such as the Large Binocular Telescope (LBT), the Very Large Telescope Interferometer (VLTI), and the Keck Interferometer will also provide hardware and software tools necessary for nulling interferometry which is projected to be the core technology for the TPF interferometer.

Interferometry is the method of choice for stellar suppression in this study because it potentially blocks more of the star's light than other methods being considered. Two dimensional imaging detectors such as CCDs will be ineffectual because the central peak of the point spread function (PSF) in the IR region of interest encompasses the field of view (FOV), so 2D images will have to be constructed from snapshots of the spatially integrated photon flux as the interferometer is rotated and/or
scanned in some fashion. Images must be artificially produced and numerically analyzed.
CHAPTER 1

ORGANIZATION AND PURPOSE

As will be discussed in the next chapter, some technologies and methodologies necessary to accomplish the mission have not been fully developed. This work addresses some of those deficiencies by:

a. Producing a computer program that simulates a set of spectral interferograms consistent with the physical layouts of the most likely structurally connected TPF candidates.

b. Developing a method of constructing images and identifying the positions and spectral intensities of planets and other features of extra-solar systems from the interferograms.

c. Examining the separability of contributing signal sources within the FOV through Fourier domain analyses.

d. Analyzing the sensitivity of the prime consolidated instrument configuration (straw man) to determine whether it is sufficient or if formation flying may be necessary.

e. Mapping the focal plane irradiance distribution to ensure sufficient coincidence of Airy patterns to avoid side lobe interference resulting in mutual cancellation or manufacturing of artificial features.
f. Addressing signal-to-noise ratio (SNR) requirements as determined by the image construction techniques employed.

g. Providing a brief radiometric performance analysis to assist in determining the efficiencies required of TPF optics and detectors, as well as the integration times needed to achieve the required SNRs, for the list of candidate stars, and to organize the list to show which ones would be expected to produce the best results.

Previous attempts to produce accurate images from one and two stages of light beam combiners from two to six telescopes have been made by other researchers but none were completely successful. A more thorough investigation was required to uncover the physical and mathematical reasons for the failures and to see if some simple solutions can be found to solve the problems and, if not, what compromises will have to be made in the science that can be accomplished with the instrument.

Presentation order of the material will be as follows:

a. A synopsis of the science objectives.

b. Some discussion of the principles of operation of the interferometer.
c. Modes of operation required for obtaining the desired scientific information.

d. Synthesizing the time series.

e. Proper application of simulated noise to the synthesized signals.

f. Fourier domain representations of the signals and implications for separability of independent signal sources.

g. Required sensitivity of the instrument and integration times to achieve acceptable SNRs for select targets.

h. Reconstructing images using crosscorrelation techniques.

i. Refining planetary parameters using non-linear optimization.

j. Evaluating the algorithm's performance through an independent examination conducted by JPL.

k. Summary of the applicability and limitations of the analytical methods developed herein and recommendations for further study.
CHAPTER 2

SCIENCE AND TECHNOLOGY OBJECTIVES

The primary purpose of this study is to address the technical issues of how to properly configure TPF so that it will provide data that can answer the basic science questions, and to formulate a method of analyzing that data to recover the parameters of an extra-solar planetary system. An important question to be considered is whether formation flying will be necessary or if a single consolidated structure will suffice. This depends on the required length, and the size, number and relative placements of the telescope primaries. Remaining chapters will be devoted to analyzing a proposed high resolution structurally connected straw man configuration.

2.1 Science Objectives and Technical Requirements

Interferometry is only one option for TPF and is the focus of the current Steward Observatory TPF development program. Coronography and differential band imaging, which will not be addressed here, are other options that will give different information about the system being observed (Woolf, 2002). Aversion to these methods is due to the very large aperture sizes, sophisticated occultations and related lagging technology
requirements (Angel, 1997 and Beichman, 1999). Additionally, the ratio of planet signal to starlight is expected to be orders of magnitude better for an instrument operating in the IR, rather than the visible portion of the spectrum, while allowing detection of faint companions in close proximity to the star.

Targets of observation may extend from ~10 to 15 pc or more. At these distances, an Earth-like planet orbiting at 1 AU will appear from ~100 milliarcseconds (mas) down to 67 mas from the star. Reflected visible light from a planet with the same size and albedo as Earth will be $10^{10}$ times less than the star, but the reradiated IR flux ratio is only $10^7$ at its emission peak of 10 μm (Angel, 1997). The contrast ratio is 1000 times better in the IR. Null depths depend on how beams from the telescopes are combined and are practically expected be in the range of $10^{-3}$ to $10^{-7}$. Compromises enabling background cancellation as discussed in the next chapter puts this figure near $10^{-5}$. Even with nulling, a planet's signal is expected to be two or three orders of magnitude less than the background sources as portrayed in Figure 2.1.1. Modulation of point sources, background subtraction and integration times on order of a month will be required for 3.2 m telescopes to bring SNRs to acceptable levels to measure planetary spectra.
Figure 2.1.1. Anticipated signal levels for sources within the FOV of TPF in the local and exo-solar system at ~10 pc (Beichman, 1999). $R = \lambda/\Delta \lambda$.

Biomarkers indicative of life or the conditions requisite for life will most likely come from planetary atmospheres that circulate globally and can produce measurable features that allow partial pressures to also be determined if sufficient spectral resolution is achieved. Ideally, the filter banks will span a region of the spectrum where water, oxygen, ozone, carbon dioxide, and methane absorption are found (ref. Figure 2.1.2). $O_2$ absorption occurs in the visible to near IR and will likely not be directly measurable with this platform, but its photolytic byproduct $O_3$ is
Figure 2.1.2. The spectral energy distribution of the Earth and Sun from 10 pc. The contrast ratio problem is evident for visible wavelengths with orders of magnitude improvement in the IR. The portion of the spectrum where atmospheric absorption features of interest appear extends from roughly 7-17 μm (Angel, 1986).

observable in the mid IR. TPF will also provide data on emissions from H₂O and CO₂ ices and dust, and perhaps CO and H₂ gas clouds. Temperature and mass distributions derived from the general shape of the curve (ref. Figure 2.1.2) will give clues to how planets form.

Spanning the 7-17 μm range would provide the basic information sought, but a narrower span might be possible. Spectral bins need not be contiguous or uniform in width, but the relative bandwidths would have to be characterized as the analytical functions have to scale interferograms
accordingly. The required number and width of the bins depends on the resolution requirement, which depends on the precision requirement for the linewidth measurements. The latter will be addressed by the science team once the image construction method is known. At minimum, one channel will be required for each absorption line to be measured.

A more detailed view of the 7-25 μm Earth’s spectral features is provided in Figure 2.1.3. Besides a planet’s temperature and size, which can be construed from a low resolution spectrum, temporal variability of spectral intensities would indicate large amounts of surface water and/or ice or changing weather conditions. Rotation rates might also be determined from periodicity exhibited in the spectra. Comparison of Earth, Venus, and Mars as per Figure 2.1.4 clearly shows each have some atmosphere, but only one has the coveted biomarkers.

2.2 Simplicity and Sensitivity Issues

Longer wavelengths in the mid-IR have better SNRs than shorter ones, so from a detectability standpoint, longer wavelengths are more desirable. However, shorter wavelengths better separate planets because the interference peaks are closer together and the first maximum is nearer the star, which positively affects the accuracy of position determinations.
Figure 2.1.3. A 7-25 μm Earth absorption spectrum. Surface temperature can be deduced from its general shape. An atmosphere with oxygen and large quantities of liquid water is indicated by the absorption features and spectral peak. Courtesy NASA/JPL-Caltech.

Figure 2.1.4. Example spectra of Earth and its nearest neighbors (Beichman, EX-NPS study 1997).
One goal of the project is to locate the planets with sub milliarcsecond accuracies and determine the intensities to within 1% amidst extremely poor SNRs. To accomplish this will require advanced analytical techniques. Moreover, if the number of filters can be limited to about five, and the telescopes connected along a single truss, implementation costs will be significantly reduced. The required number of bands is determined by the science objectives and the needed phase diversity for the analytical programs to work. The band question will be discussed in latter chapters.

2.3 Toward Reconstructing Images

Maximizing throughput and minimizing noise implies that the lower practical limit to an interferometer’s FOV be primarily determined by the smallest telescope in the set. Telescope beams overlap at the focal plane and the PSFs are superimposed. Ideally one would approximately match the FOV to the Airy disk produced by the smallest aperture at the longest wavelength. The focal plane irradiance distribution is found by convolving the superimposed PSF(λ) with the object irradiance distribution multiplied by the instrument transmission pattern. The resolution limit considering diffraction only for a static interferometer at wavelength λ is roughly

\[ \theta_{\text{min}} = \frac{1.22 \lambda}{D} \text{ rad}, \]

where D can be taken as the diameter of the smallest entrance aperture.
As will be explained later, modulation allows planet detection inside this limit. TPF Technology Working Group members under the direction of Brent Ware have decided that mirrors of 3.2 m diameter may be practically deployed. At 8, 10, and 12 μm, θ min ≥ 629, 786, and 944 mas, respectively. Initially, the Science Working Group (SWG) considered the prospect of limiting the sampled spectrum to 8-12 μm or possibly 14 μm for cost containment and other reasons. Superpositioning 8, 9, 10, 11, and 12 μm maps produces the combined PSF of Figure 2.3.1 which is attenuated by ~80% out to 500 mas. Fiber lenses or field stops should be used, or feed horns (if implemented with integrated optics), dimensioned such that all fields of view are matched between bands, or a planet may appear in some channels and not others.

Figure 2.3.1. Two views of superimposed and normalized 8, 9, 10, 11, and 12 μm PSFs for a set of telescopes of equal 3.2 m diameters.
For the purpose of establishing a method of image reconstruction, a star can be modeled as an unresolved point source at the origin with intensity equal to the integral of the product of the pupil plane intensity and the null pattern. Cosmic Background Explorer (COBE) data and associated models can be adapted to simulate the exo-zodiacal cloud (Kelsall, 1998) and to choose an appropriate value for the local zodiacal emission. A face view of a model disk on the same spatial scale as the combined PSF is presented in Figure 2.3.2. The static image extends over 1 arcsec in both dimensions (approximately matching the instrument FOV) and integration is from 8 to 12 μm. Intensities scaled for size, albedo, wavelength, and temperature (as a function of separation distance from the star) can be assigned for each planet and added, as for the star, by treating them as point sources (delta functions) at assigned grid positions. Since the Airy disk for a matched FOV is a mixture of flux from all sources within the FOV, and since their fluxes will be small by comparison to star

Convolving the PSF of Figure 2.3.1 with the residual star flux, the local and exo-zodiacal fluxes, and three planets at random positions, results in the image of Figure 2.3.3. The distribution was produced for a 1 arcsec FOV by summing the interferometer response (Beam) to the Signal over all wavelengths \( \lambda \) and rotation angels \( \phi \) of the interferometer for each grid position \( i \) and \( j \) according to the equation
Figure 2.3.2. Two views of a simulated smooth zodiacal cloud that follows the Kelsall model derived from COBE data integrated from 8 to 12 μm. Levels are measured in Janskys. Clumps, wakes, rings, etc., not included.

The existence of planets is not evident and what appears is the diffraction pattern that has been modified by the transmission pattern. If the planet signals were equal to or larger than the background, this method might be viable. Thus, there is a need to find other methods that work directly on the time series signals to extract the planets.

For imaging in which PSFs of all objects in the FOV overlap, the anti-symmetry produced by reflection of one beam which suffers a π phase reversal when two are superimposed by a semi-transparent material (see
Figure 2.3.4) is not a problem, because the integrated photon flux can be collected by a single detector (or pair of detectors) for each band.

Figure 2.3.3. Two views of a virtual image produced by a simple deconvolution from one rotation of a 4 telescope TPF interferometer with three planets, a nulled star, local, and exo-zodiacal dust emissions in the FOV.

A prism spectrograph or grating is assumed to spread the spectrum with a set of Si:As detectors similar to Ge:Ga SIRTF array shown in Figure 2.3.5. Note the depth of the cells required by long absorption depths. Crosstalk and other problems related to this kind of detector have been dealt with elsewhere and will not be addressed here.

The challenge is to find a method of reconstructing images from the time series and extracting planet locations and spectra which will be the subject of remaining chapters. The next chapter will address optimum configuration of the instrument to obtain the desired science data.
Symmetry by Reflection

Figure 2.3.4. Anti-symmetry produced by reflection at a simple beam combiner that plagues other nulling interferometers will not be an issue for TPF because of PSF overlap and spatial integration by the detector.

Figure 2.3.5. Detector testbed used in the SIRTF program. Detectors are mounted to match the curvature of the focal plane.
CHAPTER 3

NULLING INTERFEROMETERS

Nulling interferometry can be implemented many different ways. Space limitations will allow only a couple of basic configurations to be examined here. It is assumed that Bracewell combiners will be used in some fashion to produce 1D interferograms.

3.1 Basics of Interferometry

Null patterns come about by overlapping beams from two or more telescopes coherently at the focal plane. Referring to Figure 3.1.1, the wavefront from an on-axis source, such as a star at the center of the FOV, will arrive simultaneously at the apertures of a pair of telescopes. Identical images will be produced by a matched pair and if the two beams are added with a $\pi$ radian phase shift in one of them, they will cancel each other and no illumination of the focal plane will occur. The $\pi$ phase shift for one beam and superpositioning of both beams can be accomplished by reflection with a piece of semitransparent material shown schematically in Figure 3.1.2. Dr. Walker at the University of Arizona has demonstrated that a magic T waveguide structure can accomplish the same thing using integrated optics.
For off-axis sources in Figure 3.1.1, the wavefront will be tilted, arriving at the two apertures at different times. A phase difference of other than $\pi$ radians between the two beams will occur when they are superimposed by the beam combiner for off-axis sources. The phase difference depends on the angle the wavefront makes with respect to the baseline. The result will be a set of interference fringes appearing on the sky (or entrance pupil) running perpendicular to the baseline. Imagining an exo-zodiacal dust cloud surrounding a star as shown at the left in Figure 3.1.3, produces a fringe pattern like that shown at the right. Note that the on-axis star has been nulled out.

Figure 3.1.1. In the single Bracewell configuration, a wavefront arrives at two telescope apertures at different times for off axis sources. A $\pi$ phase delay is added to one beam, which is then superimposed on the other at the focal plane. Interference fringes will be superimposed on the sky.
The focal plane irradiance distribution is the product of the two dimensional null pattern and the point spread functions (PSF) of the telescope apertures convolved with the flux at the pupil plane. The resolution depends on wavelength $\lambda$ and aperture diameter $D$ as given earlier by $1.22 \frac{\lambda}{D}$. Fringe spacing is a function of the baseline length $B$. Constructive interference peaks occur at angles

$$\theta = \left(n + \frac{1}{2}\right) \frac{\lambda}{2B}.$$

Figure 3.1.2. Schematic representation of a single Bracewell beam combiner. One beam is transmitted and the other reflected by a semi-transparent beam splitter. Two beams are thus superimposed with a $\pi$ phase difference between them. Light collected from both sides increases the efficiency. Adjustment is required to match right and left phases.

For an object separated by an angle $\theta$ measured from the pointing vector, the transmission for a stationary interferometer matching Figure 3.1.1 is
\[ T(\theta) = \sin^2 \left( \frac{\pi \theta B}{\lambda} \right). \]

Figure 3.1.3. At the left is a resolved topographical face-on "image" of an exo-zodiacal dust cloud with a star at its center. On the right is an illustration of a fringe pattern superimposed on the image at the left that would result from a nulling interferometer (Beichman, 1999).

Telescope optics will be small enough to guarantee overlap of the PSFs from all sources within the FOV. Adding more sources to the FOV will only serve to increase the amplitude of the Airy disk. Rotating the baseline about its center modulates any features that are not uniformly radially symmetric and allows them to be separated from the background which is orders of magnitude stronger.
Any number and placement of telescopes and beam combiners can be employed with the result that different information about the source can be obtained from each configuration. Only a few of combinations of two and four telescopes will be examined in this chapter.

3.2 The Single Bracewell Interferometer

A single Bracewell interferometer only uses two telescopes. One artist's conception of how they might be connected along a truss that can be folded into the shroud of a Delta IV Heavy rocket is provided as Figure 3.2.1. Optics are thermally shielded from solar radiation and the beam combiner and electronics are underneath.

The mathematical description of a single Bracewell interferometer is quite simple. If the primaries have a baseline separation \( B \), the transmission at angle \( \theta \) is

\[
T(\lambda, \theta) = \sin^2 \left( \frac{\pi \theta B}{\lambda} \right),
\]

(Velusamy, 2003). If the baseline (truss) is rotated by an angle \( \phi \), the composite 1D interferogram function or pattern for equal telescope diameters becomes

\[
P(\alpha, \beta, \phi, \lambda) = S \times T \left( \lambda, \alpha k \cos \left( \frac{\pi}{180} \phi \right) + \beta k \sin \left( \frac{\pi}{180} \phi \right) \right).
\]
Here $S$ is the intensity of a signal source, $k = 4.84813689$ enables Cartesian angular coordinates $\alpha$ and $\beta$ to be input in mas, $\phi$ is measured in degrees and $\lambda$ in \( \mu \text{m} \). For a 36 m baseline, the transmission pattern for a stationary single Bracewell interferometer at 10 \( \mu \text{m} \) is shown in Figure 3.2.2. The corresponding 1D noiseless interferogram in which measurements are taken in $1^\circ$ increments for a planet at $X = 100$ and $Y = 0$ mas is shown in Figure 3.2.3.

![Figure 3.2.1. An artist's conception of a consolidated single Bracewell TPF interferometer. The instrument folds up for stowage during launch. Courtesy University of Arizona Steward Observatory.]

Single Bracewell interferometers provide no way of extracting planets from the background except by rotation. Multiple telescope pairs used in conjunction with a second stage beam combiner will provide
additional means of canceling zodiacal cloud and star light that are rotationally symmetric.

![Stationary Null Pattern](image1)

**Figure 3.2.2.** Stationary transmission pattern for a single Bracewell interferometer at 10 μm and a 36 m baseline.

![Interferogram Produced by Rotation](image2)

**Figure 3.2.3.** The 10 μm1D interferogram produced by rotation of the single Bracewell interferometer of 36 m length. In Cartesian coordinates, the planet producing the signal is located at α=100 mas and β=0 mas.
3.3 The Double Bracewell Interferometer

Double Bracewell interferometers involve three or four telescopes. The increased size may mean formation flying will be required in which individual telescopes are on separate platforms as depicted on the right in Figure 3.3.1. However, cost control measures give serious preference to single integrated structures such as the one shown on the left in this figure.

Figure 3.3.1. Artist's conceptions of two different TPF concepts. On the left is a 4 telescope structurally connected instrument. On the right are distributed platforms in which the telescopes fly in formation. Courtesy NASA/JPL-Caltech.
3.3.1 High Resolution Interleaved Baselines

Consider the interleaved baseline configuration of Figure 3.3.1.1. Assume, for example, a baseline of 27 m for each of two telescope pairs. The total truss length is 36 m. Let mirror positions be -18, -9, 9, and 18 m, respectively, from the center of the truss, and their diameters all equal to 3.2 m. Rotating the interferometer produces a 1D interferogram; the shape depends on whether or not chopping is employed and if the outputs of the detectors are added or subtracted.

Figure 3.3.1.1. Interleaved double Bracewell interferometer. Three beam combiners are arranged in two stages. The second stage also serves as a chopper by dithering \( \pi/2 \) “reference” wavelengths to either side of the achromatic null position or phase chopping can be implemented by adding or subtracting detector outputs appropriately.
An analytical function that replicates the pattern $P$ of the interferograms is given by

$$P(\alpha, \beta, \varphi, \lambda) = S \frac{1}{2} \left( 1 + C + \cos[(\lambda_0 - 36 \theta)(\frac{\pi}{2\lambda})] + C \cos[(\lambda_0 - 36 \theta)(\frac{\pi}{2\lambda})] \right) \sin\left(\frac{27 \theta \pi}{\lambda}\right)^2$$

where

$$\theta = \alpha \frac{\pi}{180} + \beta \frac{\pi}{180}.$$

Again, $S$ is the source intensity and $k$ enables Cartesian coordinate angles $\alpha$ and $\beta$ to be input in mas, $\varphi$ in degrees and $\lambda$ in μm. The value of $C$ selects the chop mode. A value of $+1$ produces symmetrical chopping and is the result of adding the outputs of the detectors at the two chop positions. If $C=-1$, anti-symmetrical chopping is implemented.

Asymmetrical chopping is produced by making $C = 0$, which adds one detector and subtracts the other during one half the chop cycle or using only one chop position. A reference wavelength $\lambda_0$ compensates for a phase shift increase as wavelength decreases for path length chopping and, if made equal to zero, simulates no chopping. Experiments show 8 μm to be optimum as providing the best accuracy for analytical methods, but 10 μm gives a better null depth. Implementing phase rather than pathlength chopping is accommodated by making $\lambda_0 = \lambda$. 

3.3.2 Low Resolution Interleaved Baselines

Changing the combination order in Figure 3.3.1.1 so that M1 and M2 form a pair and M3 and M4 make another will produce similar stationary patterns as with the high resolution case, but a broader central null will block more of the star light. This is better for nearby stars around 10 pc where planets close to the star will have a relatively larger separation angle than say 15 or 20 pc where such a broad null is not required, but the closer fringe spacing is essential to separate planets. Mathematics of the high resolution case can be applied directly by keeping the same mirror designations and swapping the mirror 2 and 3 positions.

3.3.3 Double Bracewell Beam Combination

Whether or not a system of planets can be adequately resolved depends on the diversity of frequency and phase information, the null depth, and the noise level. Only the high resolution straw man will be examined here to find the performance limits for distant stars.

*High resolution configuration*

  a. Four telescopes mounted with collinear baselines on a truss.

  b. Unequally spaced mirrors of diameter 3.2 m at positions -18, -9, 9, and 18 m from truss center.
c. Double Bracewell beam combination with mirror pair A and C and pair B and D superimposed in the first stage as shown in Figure 3.3.3.1.

Stage 1 Beam Combination

![Diagram of Stage 1 Beam Combination](image)

Figure 3.3.3.1. First stage of a double Bracewell beam combiner illustrating an interleaved configuration.

Whether or not chopping is employed, detector outputs at the two positions are summed or differenced in some fashion with different information contained in the interferograms. Star leakage, SNRs, background observability, and planet detectability will all be affected by these factors. Chopping modes will have to be dealt with as separate cases to understand the merits of each. It is necessary to examine the different interference patterns produced in each case.
3.3.4 Stationary Transmission Patterns

Radiometric calculations for the list of prime candidate stars indicate detectability must be achieved for SNRs down to \(-2.5\) or less depending on the mission phase. A deep central null is critical to noise reduction which depends on the chop mode. For the instrument under consideration, the pattern is shown in Figure 3.3.4.1 at the achromatic position in which the outputs of two detectors are summed (the system is assumed to have four detectors, but two can only be used to analyze the star light). It gets better by a factor of two for a \(\pi\) chop excursion and reference wavelength of \(8\ \mu\text{m}\), and another factor of two improvement is achieved if the reference wavelength is \(10\ \mu\text{m}\). Figure 3.3.4.2 shows a factor of two reduction in the null depth for a two detector readout when symmetrical nulling is employed. Two orders of magnitude improvement over the no chop condition can be gained using anti-symmetrical nulling as demonstrated in Figure 3.3.4.3. For illustration and reference purposes, extended views of the no-chop, symmetrical, anti-symmetrical, and asymmetrical patterns are provided in Figures 3.3.4.4 through 3.3.4.7 respectively.

With symmetrical patterns, mirror images of features in the FOV appear. Anti-symmetrical patterns cause anti-symmetrical images but real
features are distinguishable by their positive peaks. Anti-symmetry also causes radially symmetric structures such as uniform zodiacal clouds to disappear (as with anti-symmetrical planets of equal intensity) and as already observed, produces the best reduction of the star light. Symmetrical planets of unequal intensity will extract as a single planet equal to their difference. Asymmetrical patterns reduce mirror images and are most useful for characterizing large scale features such as clumps, wakes, and rings. The background will not be sufficiently reduced, though, to enable planet detection.

Figure 3.3.4.1. Central null produced by double Bracewell combination with 27 m interleaved baselines, a 36 m truss and no chopping.
Figure 3.3.4.2. Central null produced by double Bracewell combination with 27 m baselines interleaved, a 36 m truss and symmetrical chopping.

Figure 3.3.4.3. Central null produced by double Bracewell combination, interleaved 27 m baselines, a 36 m truss and anti-symmetrical chopping.
Figure 3.3.4.4. Stationary null pattern produced by a double Bracewell instrument with 27 m baselines, a 36 m truss and no chopping.

Figure 3.3.4.5. Stationary null pattern produced by a double Bracewell instrument with 27 m baselines, a 36 m truss and symmetrical chopping.
Stationary Transmission Pattern

Figure 3.3.4.6. Stationary pattern from a double Bracewell instrument with 27 m baselines, a 36 m truss and anti-symmetrical chopping.

Key:
8\(\mu\)m ___
9\(\mu\)m ___
10\(\mu\)m ___
11\(\mu\)m ___
12\(\mu\)m ___

Stationary Transmission Pattern

Figure 3.3.4.7. Stationary pattern from a double Bracewell instrument with 27 m baselines, a 36 m truss and asymmetrical chopping.
3.3.5 Rotational Interferograms

The more distinct the interferograms are for any system of planets, the easier it will be to extract them. The most fundamental limitation to planet extraction is SNR. The noise is almost entirely derived from the background. As will be discussed hereafter, a planet does not need to be very far inside the first peak for its signal level to drop far enough to preclude detection at any acceptable level of uncertainty. The location of the first peak is a critical piece of information for the designers and will set a lower limit to the size of the instrument. The most important case is the anti-symmetrical one for size determination because it is this mode that can best be used for planet detection.

3.3.5.1 Anti-symmetrical Pattern Advantage

Planet detection can be accomplished with anti-symmetrical chopping which produces rotationally symmetric interferograms with positive and negative values. At the 40 mas first peak of the stationary pattern, the corresponding anti-symmetric interferogram is shown in Figure 3.3.5.1.1. Asymmetrical chopping will also produce a bipolar signal, but the symmetrical cases will yield only unipolar interferograms.
A simple prototype program for generating interferograms in any of the chop modes along with instructions for its use may be found in Appendix B. It has a noise feature, as discussed in the next section, and can be modified to suit the experiment at hand.

3.4 Addition of Noise

Correct understanding of how noise is simulated in context of the signal is important to accurately predict instrument performance. If signals from all channels are combined, expected SNRs for Earth-like planets around stars in the 15 pc neighborhood will likely be ~2.5 when

Figure 3.3.5.1.1. Noiseless spectral interferograms produced by a 40 mas planet of unit intensity by anti-symmetrical chopping.
integration times are long enough to compute spectra. Astronomical and instrument sources produce additive white Gaussian noise when integration times are long, and substantial photon flux (signal plus background) is collected. The distribution is actually Poisson, but the probability of an incoming photon \( p \) per measurement \( n \) will be large (i.e., the mean photon flux \( np \gg 1 \)), indicating appropriate application of Gaussian statistics. The SNR cannot be strictly defined as the ratio of RMS values of signal and noise because the RMS signal for a given planet flux will fluctuate due to nulling according to Figure 3.4.1 at the focal plane as the planet moves outward. A more useful definition, and one that corresponds well with the radiometric performance calculations, is to

![Figure 3.4.1. RMS signal variation for a planet of unit intensity as it moves away from the star.](image)
estimate the signal as the peak value of the planet flux and the noise to be the square root of the background plus planet flux. Signal and noise are both reduced by nulling, but the exo-zodiacal emission will be reduced more proportional to the planet’s emission because its flux concentrates near the star where the nulling is most efficient.

Assume a transmission of one for a planet in the vicinity of a null pattern peak as just characterized (no loss due to nulling) and generate a noiseless interferogram; the result is a signal that has an RMS value equal to the RMS noise for a SNR of 1 if the following formula is applied in generating the interferograms:

$$\text{Signal}(\lambda, \varphi) + \text{Noise}(\lambda, \varphi) = \text{Pattern}(\lambda, \varphi) + \frac{1}{\text{SNR}} \text{Gaussian}(\mu=0, \sigma=1)$$

The zero mean is well adapted to anti-symmetrical cases where chop phases are differenced but may be adjusted to allow for a positive offset where detectors and chop phases are summed. The noise is independent of the signal. It is also uniform for all points across the FOV (since the PSF when convolved with the flux at the entrance aperture and multiplied by the transmission pattern throws all photons into the same bucket at the focal plane). The planet signal level at any point in the interferogram depends on its sky position as illustrated in Figure 3.4.1 and will therefore scale the SNR as previously discussed.
An example of a 10 μm interferogram from one planet at the first stripe of the stationary null pattern at 40 mas with noise added according to the prescription just given for a SNR of 1.0 can be found in Figure 3.4.2. JPL personnel have developed a signal generator that adds noise with a Poisson distribution. It will be used at the end of this study to test the analytical programs discussed in remaining chapters. The code in Appendix B should be equivalent, though the amplitudes (intensities) of the signals scaled somewhat differently, requiring normalization of the JPL interferograms before processing.

![10 μm Interferograms w/ and w/o Noise](image)

Figure 3.4.2. Normalized signal with Gaussian noise added according to the prescription given in the narrative at an SNR of 1.0.
CHAPTER 4

FOURIER DOMAIN ANALYSIS

A look in the frequency domain can give important insights to problematic aspects of analyzing signals from rotational interferometers. Overlapping frequency components aliases contributing signal sources causing false planet production in correlation and spectral energy distribution maps and intensity and position errors in real ones as will be demonstrated in material that follows. Extractions from composite interferograms require more spectral diversity for different planetary arrangements than exists in the example of Figure 4.1. The upper chart is an 8 \( \mu \)m power spectrum of four planets in one dimension with 100 mas separations and the lower chart applies to the same system except an additional planet has been added 100 mas further out. Elements added at the higher frequencies by the fifth planet contribute little to the composite signal; a more serious difficulty lies in the fact that another arrangement of planets nearer the star mimics the lower frequency portion. The phase spectrum seems to predict features of correlation maps better than the amplitude spectrum. Emerging patterns indicate that feature abstractions nearer the star, producing phase plots equivalent to more distributed real systems, tend to match those of correlator outputs.
Figure 4.1. Power spectra for four (top) and five (bottom) planets separated by 100 mas. Planets are located at X = 100, 200, 300, 400, and at the bottom 500 mas.

Correlation maps often exhibit false topographical features owing to crosstalk between real planet signals. Understanding their origin is facilitated in part by comparing the combined spectra of mapped planets (including false ones) with the actual two planet system spectra as in Figure 4.2. The upper chart in part a shows phase elements from the two planet spectrum and the lower chart shows those for the three main peaks and two subsidiaries whose contributions were too small to appear in the correlation maps computed from the five band composite signals. Negative versions of each of the above are also present that always exist in
diametric opposition through the star. Relative peak values used to produce the spectra for the lower parts of the figure came from map topography. Lower frequency phase elements match fairly well and the differences at the higher end of the spectrum are due to exclusion from the transformation of all outside features on the map and some subliminal ones at mid positions. Previous experiments demonstrated that inside planets dominate the phase spectrum. Part b of the figure shows some

![10 um Phase Spectrum](image)

Figure 4.2 a. Comparison of phase spectra from a two planet system with that corresponding to a combination of one large abstracted peak and several subsidiary ones scattered inside and out of the FOV including the two real peaks and positive and negative versions of all.
power transfer from higher order elements to the fundamental between the source and image spectra. This type of behavior has been frequently observed. In some cases where planets are further out, thereby producing even higher frequency components, some of the energy from the higher frequencies is transferred to the second and sometimes the third harmonics. This is a type of "intermodulation distortion" (though no

Figure 4.2 b. Amplitude spectrum associated with the phase charts of part a. Some power is transferred from each of the higher harmonics to the fundamental.
heterodyning is implied here). The phase spectrum appears to dominate the amplitude spectrum at the lower frequencies. Sorting out the complexities presents a challenging task.

Although the power spectrum contains only odd harmonics, the phase spectrum has both odd and even elements as was demonstrated in Figure 4.2. Therefore, noise will be added to all phase elements, contrary to the argument put forth that only odd harmonics will be affected by noise if full rotations are always used. Beside that, the extent of the power spectrum shrinks by half while the number of harmonics remains the same as shown in Figure 4.3 when only half rotations are used. Harmonic and crosstalk problems remain when using half rotations, but if the noise spectrum is flat, the SNR should not change. Using only half rotations increases the speed of the analysis significantly. Separate halves of a 360° interferogram could be used for noise reduction by inverting and overlapping one side with the other and summing each pair of points, constituting an additional benefit to the data reduction. The outcome of implementing such measures will be reported on in the final chapter. The effect of noise is to distort the underlying waveform and change its spectrum. Restoration of the original signal by averaging both redundant halves of the interferograms can be enhanced if signals from multiple rotations are used in the analysis.
4.1 Properties of the Fourier Transform Method

The cross-power spectrum of a pair of functions $f_1(t)$ and $f_2(t)$ is calculated as

$$F_{1,2}(\omega) = \overline{F_1(\omega)} \cdot F_2(\omega)$$

where the bar indicates the complex conjugate of $F_1(\omega)$ and $F_1(\omega)$ denotes the Fourier transform of $f_1(t)$. The magnitude spectrum of a signal source from an instrument with a uniform symmetric response does not change
as a function of rotation (clock) angle of the interferometer as long as its angular distance from the rotational axis remains fixed. Its rotational position (clock angle) can be determined by the phase of the zero crossings of the time series signal or alternatively from the phase spectrum. Determination of the angular distance ($\theta$) of a planet from the axis of rotation can be accomplished by multiplying the magnitude spectrum of the composite signal with that of a test planet at angles $\theta = 0 \rightarrow \frac{1}{2}$ FOV. This process will produce a set of $m$ vectors of length $n$ equal to the number of samples taken during one rotation that can each be summed over all $n$ elements to produce a one dimensional vector $X(t)$ of length $m$ (a function of the increment of phase delay $t$), which when plotted, peaks at the positions of the planets. This process produces a map of the spectral energy distribution. Overlapping spectral elements of component signals reduces the effectiveness of this process because of aliasing.

Insight can be gained by examining energy distributions for a set of planets in one dimension as changes are made to the number and relative positions of planets. Spectral energy distributions are examined one at a time in Figure 4.1.1 to see what significant harmonics are being generated as a function of radial distance from the star. Maps were made by summing the energy in the cross-power spectrum at each delay point (separation angle). Computations were at 10 $\mu$m and the actual location of
Figure 4.1.1. 1D spectral energy distributions for five planets at different distances from the star. The planet progresses inward, incrementally from 500 to 400, then 300, 200, and 100 mas separation angles.
Figure 4.1.1 cont. 1D spectral energy distributions for five planets at different distances from the star. Planets progress inward, incrementally from 500 to 400, then 300, 200, and 100 mas separation angles.

The planet is marked. Though the source was of fixed intensity, the total energy in the signal varied over a few orders of magnitude as the planet moved inward. Peak misregistration is a good indicator of insufficient frequency discrimination provided by the present baseline for close-in planets. Registration can be improved by lengthening the interferometer. Harmonic distortion of significant proportion occurs for planets closer than ~250 mas. False peaks at 40 mas associated with the 200 mas
planet and at 60 mas associated with the 100 mas planet also appear in
the correlation maps, thereby smearing the image. These peaks can only
be attributed to non exclusive harmonic content because a single planet
produces the signal. Abscissa in the figures are unitless because inputs
were normalized.

Crosstalk effects can be further examined by mixing various
combinations of planets in the 1D FOV. A few 10 μm examples are given in
Figure 4.1.2. Build-up of cross-terms in the complex multiplications
causes peaks in the energy distribution maps to move from their original
positions and merge in some instances reducing their total number. In the
first example (at the top), all five planets are present. The second
eliminates the innermost (100 mas) planet and so on. Actual planet
positions are indicated in each figure. Close-in planets produce more
harmonic distortion than distant ones.

Figure 4.1.3 reveals the mechanism of crosstalk for the same
system. Overlapping elements from real parts of the complex amplitude
spectra of contributing sources to the aggregate signal at 12 μm are shown
to cancel each other in some cases and add up in others in a way that
creates a composite spectrum representing a different planetary system.
Figure 4.1.2. 1D spectral energy distributions for some combinations of planets (see text). Significant interference effects are evident.
Figure 4.1.2 cont. 1D spectral energy distributions for some combinations of planets (see text). Significant interference effects are evident.
Figure 4.1.2 cont. 1D spectral energy distributions for some combinations of planets (see text). Significant interference effects are evident.
Figure 4.1.3. For the same planetary system that was the subject of the energy distribution maps, only two out of three spectrum elements from P1 are represented in the real part of the amplitude cross-power spectrum. Additionally, the missing element at a frequency of about 170 rot$^{-1}$ in P1's amplitude is not represented in the aggregate amplitude spectrum. It was cancelled by one of equal amplitude but opposite sign in P2's amplitude spectrum at that same frequency.
Figure 4.1.3 cont. For the same planetary system that was the subject of the energy distribution maps, only two out of three spectrum elements from P2 are represented in the real part of the amplitude cross-power spectrum. Additionally, the missing element at a frequency of about 170 rot\(^{-1}\) in P2’s amplitude is not represented in the aggregate amplitude spectrum. It was cancelled by one of equal amplitude but opposite sign in P1’s amplitude spectrum at that same frequency.
Figure 4.1.3 cont. All frequency elements stemming from P3's signal are accounted for in the cross-power spectrum. Comparing P3's amplitude spectrum with the aggregate spectrum at the top, a clear separation of is evident making P3 a relatively unambiguous detection.
Figure 4.1.3 cont. P1, P2, and P3 all have an element in their spectra at a frequency of 100 rot\(^{-1}\). The aggregate spectrum is the sum of the three in which P2 cancels with P4 at this frequency. The cross-power spectrum has no representation of P4 at the 100 rot\(^{-1}\) frequency because it is nullified by P2. This accounts for the translation of the corresponding peak in Figure 4.1.1 from its nominal position.
Figure 4.1.3 cont. Enough separation of P5’s spectral elements from those of its siblings allows each to be represented in the cross-power spectrum. Proximity to major components from other siblings has altered the amplitudes of the two lowest frequency elements of P5’s cross-power spectrum. The effect is to change the amplitude of the spectral energy function peak for that planet and corresponding correlation map peaks.
Potential exists to use energy distribution maps to find the number and locations of planets if the instrument is configured properly and of sufficient length to reduce interplanetary interference in the amplitude spectra. The highest peak from a full set of linear extractions at each measurement angle would be assumed to correspond with the brightest planet. A corresponding signal could be modeled for each rotation angle which can then be proportionally subtracted from each interferogram and the process repeated as necessary so as to implement a so called “CLEAN procedure” that will be explained in the next chapter. Some interpolation may be necessary for larger fields of view which would complicate the process considerably.

By expanding the baseline 1.5 to 2 times the straw man length, initial extractions can be accomplished with one or a few wavelengths. Figure 4.1.4 represents a 1D CLEAN operation of sorts applied to a 10 μm noiseless interferogram from a 54 m baseline instrument. The largest peak corresponds with the brightest feature in the FOV in each cycle. Cross-terms are deemphasized by raising the energy function to an even power for display. False peaks appear at the same places as in the time domain correlation maps showing they have a basis in systematically produced harmonics and crosstalk.
Figure 4.1.4. 1D frequency domain CLEAN maps show the positions of planets as each is extracted sequentially. Five real planets are at locations marked.
Figure 4.1.4 cont. 1D frequency domain CLEAN maps show the positions of planets as each is extracted sequentially. Five real planets are at locations marked.
Figure 4.1.4 cont. 1D frequency domain CLEAN maps show the positions of planets as each is extracted sequentially. Five real planets are at locations marked.

A grid of point values representing a correlation map in the frequency domain can be produced by making a polar plot of the linear functions, or performing the linear computations repeatedly along the orthogonal dimension. Alternatively, the information could be presented as a waterfall diagram to show the two dimensional structure. CLEANing has a natural end when the peak being subtracted can no longer be removed. The final peak is often found inside the first stripe of the stationary interference pattern and is composed purely of distortion elements. Continued attempts at removal fail. As with the time domain maps, revisitation to previously CLEANed features is often observed, especially for cases in which map peaks are not well registered with real planet locations.
Energy distribution maps can be useful for comparing instrument performance for different lengths of the interferometer. Convergence failures for the optimizer can be eliminated when initial conditions are sufficiently close the solution if the interferogram SNR is $\geq 5$. Accuracy and precision of the optimizations also depend on the starting points. Figure 4.1.5 shows that alignment and definition of the peaks can be

![1D Spectral Energy Distribution Map](chart)

Figure 4.1.5. The chart on the bottom is the energy distribution for an instrument twice the size of the one at the top (straw man). Peaks are now correctly registered and much better defined but subsidiary peaks equivocating to false planets are still present.
improved so that a factor of \~2.5 increase in the output SNR can be expected from the optimizer. The bottom chart is for an instrument twice the size of the straw man at the top. Peaks corresponding to separate targets must be distinguishable in a composite energy distribution map or convergence will fail. The accuracy and precision of the solutions will also be unacceptable. Target separations of 10 mas require a 5X or 6X increase in length for peak identifications as seen in Figure 4.1.6.

Figure 4.1.6. 5X (top) or 6X (bottom) lengthening of the interferometer allows objects separated by 10 mas to be resolved in the energy distribution maps, distinguished in the correlation maps and convergence of the optimizer.
4.1.1 Consequences of Time Delay

A single planet beyond the first peak of the null pattern is easier to detect than multiple planets since aliasing is not a problem, although harmonics may be. In this case, the magnitude spectrum does not change with position angle as already mentioned. When two or more planets enter the FOV, their individual signals are superimposed and relative phases become important. The frequency-phase relationship is given by

\[ \Im \{ f(t - t_o) \} = F(\omega)e^{-j\omega t_o}. \]

Thus, if a time domain signal \( f(t) \) is delayed by \( t_o \) relative to another, its magnitude spectral density remains fixed and a negative phase \(-\omega t_o\) is added to each frequency component. If \( t_o \) is negative, the time series is advanced and the phase spectrum moves outward. Since the phase shift is frequency dependant and, therefore, different for each element, and since relative phase shifts for some spectral components of a signal from a planet exceed \( 2\pi \), no useful patterns emerged during experimentation to assist in decomposing the composite spectrum. Apparent motion of a planet in a correlation map when a sibling is introduced is accounted for in part by the relative phase shifts of elements in the two spectra.
An unfortunate consequence of non-uniformity of the instrument response across the FOV and anti-symmetric null pattern for the double Bracewell chopped beam is a partial or complete cancellation of one planet signal by another at certain position angles and angular distances from the star. Anti-symmetry of the null pattern causes a reduction in the composite signal strength and amplitudes of the spectral components. Figure 4.1.1.1 reveals the dependency of the spectral energy on the

Figure 4.1.1.1. Spectral energy function for a 2 planet system in which P1 and P2 are both at 50 mas (upper left), 100 mas (upper right), 250 mas (lower left), and 500 mas (lower right) from center. P1 is fixed and P2 revolves about the origin.
relative positions for a two planet system. Polar plots were created by integrating the power spectrum over all components for each increment of time in which one planet was fixed to the positive X axis (0 deg) at the specified angular distance and another planet of equal intensity was allowed to revolve at the same distance around the clock to produce the patterns shown. They are similar to antenna patterns and give the total spectral energy, or equivalently the signal strength produced by the planetary system as a function of their relative azimuth angles. When diametrically opposed, the two planet system is seen to disappear. At any other angle $\Delta \geq \pm 3^\circ$ from opposition, sufficient energy exists to enable discrimination, aliasing conditions notwithstanding.

The disappearing planets will generally not be a serious problem because of the unlikely situation of finding two planets of equal intensity at the same distance but $180 \pm 3^\circ$ out of phase in their orbits. Figure 4.1.1.2 demonstrates the effect of position angle on signal strength in cases of different planet separation distances.

The dual of the delay property is the frequency-translation property,

$$\Im\{f(t)e^{j\omega_0 t}\} = F(\omega - \omega_0)$$

that causes a frequency translation of $\omega_0$ rad/sec. Phase delays of $\omega_0 t$ can increase overlap of spectral components and worsen the aliasing problem.
They also introduce additional position errors for objects in motion which may be an issue for long dwell times associated with spectrum collections following detection, and will be the subject of future investigation. A planet moving counter to the direction of the interferometer's rotation will appear further from the star than it really is for slow rotations and month long data collections.

Figure 4.1.1.2. Spectral energy function for a 2 planet system in which P1 is fixed at 0° and P2 is allowed to revolve about the origin at distances of 20 and 50 mas (upper left), 60 and 30 mas (upper right), 50 and 40 mas (lower left), and 500 and 490 mas (lower right). Asymmetry is evident and less of an issue at wider separations.
4.1.2 Complex Addition of Signals

The algebraic sum of two complex spectra $F_1$ and $F_2$, where

$$F_n = a_n + ib_n,$$

is governed by superposition

$$\alpha f_1 + \beta f_2 \equiv \alpha F_1 + \beta F_2$$

where $\alpha$ and $\beta$ are constants. The rather insidious problem of harmonic structure appearing in the correlation and cross power spectrum maps may be partially understood by examining the sums of products at each grid point. Imaginary terms of

$$F_{1,2} = a_1 a_2 - b_1 b_2$$

cancel, leaving the difference of two product terms to be summed over all points of the time series and all wavelengths. $F_1$ is the composite signal and $F_2$ is the model or vice versa. The $a_i$ terms are the real products and the $b_i$ terms are the imaginary ones. At any grid point, more than one planet may contribute energy to the spectrum. An inter-mixing (summing) of spectral elements occurs. If a sufficient match of the low frequency components of the composite signal to one that would be produced by a planet at that grid position exists, a peak in the map will
appear at that point. Errors due to cancellations or enhancements of spectral elements from aliasing will cause artifacts to appear in the energy distribution functions and correlation maps. These types of errors are inherent to rotational interferometers.

Time and frequency domain analyses are equivalent and no fundamental benefit is incurred by working in one domain over the other. Thus no further development of Fourier transform methods will be done for the purpose of this preliminary study to allow for characterization of time domain methods.
CHAPTER 5

RADIOMETRIC PERFORMANCE ANALYSIS

The most fundamental limiting factor for any of the analyses discussed herein is noise. The success or failure of TPF will lie in its ability to extract the modulated signal from background and instrument noise.

Table A3 in Appendix A gives the data point SNRs for the closest stars from the Yale Bright Star and Gliese Catalogues and NASA-ADC data bases which include potential candidates for TPF study. An Earth like planet orbiting each star was assumed. Distances were calculated from parallax measurements. Bolometric magnitudes in the table were found from given magnitudes and standard correction factors (Johnson, 1965). Exo-earth flux was derived from the Rayleigh-Jeans' law in good agreement with the Plank function. Distance scaling was applied to account for wavefront spreading and an appropriate solid angle as a function of distance computed for each star in the list to find the expected planet flux at TPF. All signals and noise were converted to numbers of electrons in order to add in with detector noise features which are specified in electrons per read and electrons per second for the read noise and dark current, respectively. Flux from the nulled star is then
\[ F^* = F^\odot \left( \frac{L^*}{L^\odot} \right) \left( \frac{T^*}{T^\odot} \right) N^* , \]

the product of solar flux \( F^\odot \) scaled for distance, the ratio of angular
diameters of the star \( L^* \) and Sun \( L^\odot \), the temperatures of the star \( T^* \) and
Sun \( T^\odot \), and the null depth for the star \( N^* \) which can be closely
approximated for the high resolution TPF configuration by

\[ N^* = 0.1383 \pi^2 \frac{b^2}{64} \left( \frac{L^* 2\pi}{\lambda} \right)^2 . \]

Here \( b \) is the baseline length between telescope pairs combined in the first
stage and \( \lambda \) the wavelength at which evaluation is taking place. Units
must be the same for \( b \) and \( \lambda \), and \( L^* \), the star's angular diameter
converted to radians. \( N^* \) differs from a single Bracewell approximation
(Woolf 1998) of the null depth only by the correction factor 0.1383 which
was empirically derived by the author for the straw man configuration.
Differences between application of the exact integral (which is a complex
formula and will not be given here) and the simple small angle
approximation above are extremely minor and can be neglected.

Simplified local zodiacal background flux formulas have been
provided by Wesley Traub (personal communication) at JPL that closely
match the COBE data (Kelsall, 1998). The COBE face-on optical thickness $t(r)$ and grain Temperature $T(r)$ as functions of radial distance $r$ in the ecliptic plane of the cloud are

$$t(r) = 0.71 \times 10^{-7} r^{-0.39} \quad \text{and} \quad T(r) = 286 r^{-0.42}.$$ 

Thermal emission in terms of the Plank function $B$ is then

$$F_{\text{therm}}(r) = t(r) B(T(r)).$$

For the solar system at 10 μm and 1 AU, the emission is 19 MJ/sr. For other stars, the temperature scales as the fourth root of its luminosity. Given an albedo of $a = 0.16$, Sun temperature $T_\odot$ and radius $r_\odot$, the reflection is

$$F_{\text{refl}}(r) = t(r) a B(T_\odot) \left( \frac{r_\odot}{r} \right)^2.$$

At any wavelength, the edge view is approximately five times brighter in the plane than perpendicular to it and varies as

$$\exp\left(-3.26 \frac{z}{r}\right)$$
out of the plane, z being the vertical displacement normal to the plane. In the spreadsheets, a 10-zodi (60° inclination) emission is assumed for the exo-systems and the local zodi was calculated at 1 AU. Fifty thousand seconds of integration were assumed, consistent with detection intervals during the mission planet finding sequence. Traub mentions (personal communication) that the factor of two which is built in to the exo-zodiacal disk thickness is lost but gained back again because the average view angle near Earth is about 60° from the normal to the ecliptic giving a slant factor of \( \frac{1}{\cos(60°)} = 2 \).

The composite signal consists of flux from the planets, the local and exo-zodiacal clouds, and the nulled star among other interstellar sources. The noise at the entrance pupil is roughly equal to the square root of the composite signal which is almost entirely background. Instrument noise is then added in a root-sum-squared (RSS) fashion because the two are independent. JPL engineers estimate the instrument noise to be about half of the photon noise for middle bands (personal communication).

Photon shot noise can be reduced by decreasing the chop cycle (assuming one readout per chop position). Noise reduction is proportional to \( 1/\sqrt{N} \), N being the number of values summed at each rotation angle.
The signal will also be reduced by a factor of $N$, thereby canceling the $\sqrt{N}$ improvement in the SNR as related by the equation

$$\text{SNR} = \frac{\frac{\text{planet signal}}{\sqrt{N \text{ improvement}}} \cdot \frac{\text{planet signal}}{\sqrt{\text{background signal}}}}{\sqrt{N}} = \frac{\frac{\text{planet signal}}{\sqrt{\text{background signal}}}}{\sqrt{N}}$$

with the result that the only way to increase the SNR for a given target being to increase the integration time or use signal averaging techniques. The chop cycle is arbitrary as long as it does not hinder photon accumulation. Fewer read cycles will reduce the read noise component.

Some assumptions had to be made in computing the SNRs for each target because the design has not advanced to a point where certain characteristics of the instrument can be known. SNRs were calculated at the focal plane which takes advantage of both exo-zodiacal and stellar nulling. Beam transfer efficiencies and pattern transmissions were estimated and included in the calculations. In applying the null depth, it is assumed that there are practical engineering limits below which the star flux will not fall, regardless of the computed value. The null depth was therefore truncated at $3.0 \times 10^{-5}$. Instrumental (systematic) noise is estimated at half the photon shot level for middle bands. Local and exo-
zodiacal fluxes were verified as being consistent with outputs of the previously mentioned JPL signal generator code.

Only the 10 μm band calculations were included as an example in Appendix A for brevity. The SNRs peak at 12-14 μm. The signal is expected to be relatively strong near 10 μm with significant roll-off for adjacent shorter wavelength bands. Table A3 should be representative and provide pertinent information for identifying and locating each star and associated ratios of incoming flux from the planet, stellar leakage, and zodiacal clouds along with the band SNR. A 10-zodi EZ dust cloud produces about as much flux and sometimes more than the star leak. SNRs are very low and confirmation of detections will have to wait until spectra are collected in phase 2 when integration times are long enough to boost point values to ≥ ~1.0.
Chapter 6

DATA ANALYSIS – CORRELATION MAPS

As already mentioned, because the PSF occupies the FOV, imaging arrays will be useless for spatially separating targets within the FOV. Images must be artificially produced. A so called “maximum correlation method” (MCM) has been proposed (Velusamy, 2001) in which an image is created by matching amplitude coefficients (peak intensities for each point on a grid) of a model function, to produce a time series equivalent to the set of spectral interferograms. Intensity values at each grid point in the “map” are iteratively refined over a specified number of cycles, each time better matching the spectral interferograms to the model.

This author has developed a similar method that is seemingly more accurate in providing initial values for the nonlinear optimization. It is referred to here as the standard (STD) correlation method because it tracks rather closely textbook crosscorrelation procedures. Both methods will be briefly outlined and some outcomes presented for comparison.

6.1 Maximum Correlation Method

In this method, the interferogram from the interferometer (instrument response) is given by
\[ R_o(\varphi, \lambda) = \sum_x \sum_y I(x, y, \lambda) P(x, y, \varphi, \lambda) \]

where \( I \) is the irradiance in the object plane, \( P \) is the stationary null pattern, \( x \) and \( y \) specifies a grid point, \( \varphi \) is the angle of the baseline from the reference, and \( \lambda \) is the wavelength. Crosscorrelation inverts the process and delivers the synthetic image

\[ I(x, y) = \sum_{\varphi} \sum_{\lambda} R_o(\varphi, \lambda) P(x, y, \varphi, \lambda). \]

In the first step, the image matrix is initialized to unity

\[ I_o(x, y, \lambda) = 1 \]

and the 2D null pattern is computed as

\[ R_c(\varphi, \lambda) = \sum_x \sum_y I_o(x, y, \lambda) P(x, y, \varphi, \lambda). \]

A correction factor for each pixel is obtained by comparing the measured intensity with that computed in the current iteration

\[ c(x, y, \varphi, \lambda) = (R_o(\varphi, \lambda) - R_c(\varphi, \lambda)) P(x, y, \varphi, \lambda) \]

which is then averaged over all rotation angles to get \( c_{av}(x, y, \lambda) \). This correction is added to the value in the image pixel

\[ I_1(x, y, \lambda) = c_{av}(x, y, \lambda) + I_o(x, y, \lambda). \]

For detection only, \( c(x, y, \lambda) \) is averaged over all wavelengths to get \( c_{av}(x, y) \) and \( I(x, y) \) used instead or \( I_1(x, y, \lambda) \) averaged to get \( I_{av}(x, y) \) in the end. A number of variants have produced similar results.
Carrying out this procedure for a variety of one and two planet cases resulted in broader features and less accurate peak position determinations than a modified standard correlation method. Peak intensities required scaling which depended on the number of iterations and wavelengths, etc. Without regard to modifications that can be made to improve the performance of the MCM, the standard correlation method became the focus of this study as it seemed to produce the best initial results and because it was known that JPL personnel were focused on developing the MCM and a duplication of effort seemed unnecessary. Illustrations will be given shortly that compare the performances of each method. Some example Matlab compatible C code is provided in Appendix C to do the MCM computations and produce a level 1 correlation map.

6.2 Standard Correlation Method

Crosscorrelation is a standard method of estimating the degree to which pairs of time series are alike. Let model and data time series be denoted \( p(i) \) and \( q(i) \) where \( i = 0, 1, 2, ..., N-1 \). The crosscorrelation \( r \) at delay \( d \) is defined as

\[
r(d) = \frac{\sum_i [(p(i) - mp)(q(i - d) - mq)]}{\sqrt{\sum_i (p(i) - mp)^2 \sum_i (q(i - d) - mq)^2}},
\]
where \( m_p \) and \( m_q \) denote the means of the corresponding series. The coefficients \( r(d) \) are then computed and mapped out for every grid point. Correlation coefficients are calculated for each wavelength. Summing over all wavelengths and/or baselines enhances the images.

Once the location of the brightest peak in the map is identified, spectral intensities are determined by least-squares fitting of the model to the interferograms for a planet at that grid point. Knowing the parameters of the brightest planet facilitates its subtraction from the composite interferograms and creation of a new map emphasizing the next brightest planet and so forth. The process continues until the peak drops below the RMS background level. This iterative process is referred to as a CLEAN algorithm and is reminiscent of ones used in radio astronomy.

Least squares fitting solves the matrix

\[
\begin{align*}
a_0 n + a_1 \sum I_i + a_2 \sum I_i^2 &= \sum I_i \\
a_0 \sum I_i + a_1 \sum I_i^2 + a_2 \sum I_i^3 &= \sum I_i M_i \\
a_0 \sum I_i^2 + a_1 \sum I_i^3 + a_2 \sum I_i^4 &= \sum I_i^2 M_i
\end{align*}
\]

where the \( I_s \) are trial intensity values and \( M_s \) are residuals of the squared differences between the interferograms and model function evaluated at the trial intensity and corresponding wavelength, position, and rotation.
angle. The best estimate is where the residual is minimized and the derivative of the quadratic equals zero. Then

\[ I_{\min} = -\frac{a_1}{2a_2}. \]

The crosscorrelation examples that follow assume anti-symmetrical chopping, the mode that best cancels the background. Spectral intensities can be off by a considerable amount due to the fact that the function used in the least squares fitting often has a minimum far from where it should be due to small errors in position estimates for some wavelengths. However, the average intensity provides a value close enough for initialization of the optimizer (within ~20%). The average value may then be used for all wavelengths. Positions of the peaks produced during the CLEAN process are usually the best estimates of each planet's position for the purpose of nonlinear optimization initialization. Matlab compatible C code is provided in Appendix C for standard crosscorrelation computations that also produce a set of CLEAN maps.

6.2.1 Performance of the STD Method

The ultimate value of a correlation map depends on its ability to accurately approximate the locations and intensities of the planets. Figure 6.2.1.1 compares the two methods discussed previously. The map on the
right was produced by the STD algorithm and the one on the left by the MCM. Interferograms were created for a single planet at $X = 100$ mas. Peaks were reported at $X = 90$ mas for the STD and at $X = 60$ mas for the MCM methods. Scales were matched for comparability. Clearly the best results are from the STD method.

Figure 6.2.1.2 deals with the case of two planets of unit intensity separated by 50 mas. P1 was placed at $X = 40$ mas and P2 at $X = 100$ mas. In both cases, the main peak was identified by each program to be at 50 mas (a 20% error) but the STD map showed more structure and at least some indication of a two planet system.

Figure 6.2.1.1. Both correlation maps are a response to a single planet at $X = 100$ mas. Scaling was identical in both cases.
Figure 6.2.1.2. Both correlation maps are a response to a two planet system with P1 at \(X = 40\) mas and P2 at \(X = 100\) mas. Planets were of unit intensity. Scaling was identical in both cases.

The same experiment was repeated for P1 at \(X = 80\) and P2 at \(Y = 80\) mas with the results shown in Figure 6.2.1.3. The STD procedure reported the principal (false) peak to be at \(X = 30, Y = 30\) mas. The MCM claimed it to be at \(X = 40, Y = 30\) mas. Once again, the STD process showed more structure and an indication of the two real planets.

For close-in planets the STD method gave consistently better results for a variety of experiments. For this reason, the STD procedure became the method of choice for further developmental work to obtain initial conditions for the optimizer.
Figure 6.2.1.3. Both correlation maps are a response to a two planet system with P1 at \( X = 80 \) mas and P2 at \( Y = 80 \) mas. Both planets were of unit intensity. Scaling was identical in both cases.

6.2.2 STD Correlation Resolution

Resolution is determined by how close a pair of planets can be in proximity (in the neighborhood of the first peak of the stationary null pattern) and still be distinguished as separate bodies and the accuracy by how close the peaks are to their expected locations. RMS variation about the parametric mean in the presence of noise determines the precision. Experiments showed that two planets must be at least 40 mas apart to be separable with a large background. Peaks in the correlation maps can be off by tens of mas. Fortunately, the optimizer (which is quite sensitive to initial conditions near the first peak) is able to converge on a solution from
these distances but the accuracy and precision depends on the initial values and noise levels. Particularly notable is the fact that measured peak amplitudes and deciphered locations depend on map locations and other planets in proximity for a fixed set of source intensities. Two planets of unit intensity at the same distance from the star but angularly separated will appear at the same intensity on the map, but as one moves radially outward, it alternately disappears and then reappears. Inner planets are usually emphasized. To show up, outer planets have to be somewhat brighter than inner ones. Some quantification of these behaviors will now be attempted.

Examination of Figure 6.2.2.1 will demonstrate how three planets tend to merge as they are brought closer together. All three are of unit intensity but the closer-in planet appears brighter. Less than 40 mas separation results in a single blur spot. A topographical ridge usually appears between proximal planets. Frequently, the main peak occurs near the center of the system and can masquerade as a bright planet where there may be none (referred to herein as a false planet). Figure 6.2.2.2 compares a two and three planet system in which the third planet was located at the center of the system coincident with the false planet. The false planet remained when the central planet was removed but the peak was decreased by the amount contributed by the real planet. This result
Figure 6.2.2.1. Correlation maps for three planets with separations of 80, 60, 50, and 40 mas as indicated. All three planets were of unit intensity.

punctuates the need to solve for the intensities using nonlinear optimization. If a peak in the map represents a false planet, the optimizer
must show it has zero intensity. Figures 6.2.2.3 and 6.2.2.4 demonstrate what occurs for various arrangements of two and three planets. In each case, a false planet appears at the centroid of the star and planet cluster. When two planets are aligned with the centroid, it can make outer ones disappear unless they are of somewhat greater intensity. In the case of Figure 6.2.2.3, the outer planet had to be twice as bright as the closer ones to be seen. With three planets in alignment through the centroid, outer ones had to be tripled in brightness to appear on the map.

Peak values in a map depend on what is in the neighborhood as well as distance from the star. Close in, a system of planets appears as a spot at the centroid with spokes projecting toward each planet. Visibility of the spokes is a function of relative planet brightness's. P2 disappeared in Figure 6.2.2.5, when P5 and P6 were added at unit intensity, so P2's intensity was doubled to make it visible again. The star is at the left and center in this and remaining figures unless otherwise noted. Crosstalk and distance factors separately require adjustment of the intensity scale for good visibility. The further away a planet is from the star, the brighter it needs to be if it is to be seen. Figure 6.2.2.6 shows what happens as planets move even closer together.
Figure 6.2.2.2. Correlation maps for two planets separated by 120 mas and three planets separated as indicated. All were of unit intensity.

Figure 6.2.2.3. Correlation maps for two planets separated by 120 mas and three planets separated as indicated. The intensity of P3 was made twice the other two so the peaks would be visible and of nearly equal amplitude.
Figure 6.2.2.4. Correlation maps for two planets of unit intensity separated by 80 mas and three planets separated as indicated.

Figure 6.2.2.5. Correlation maps for 4 and 6 planet systems at marked positions. P2 at the left was made ~2.5 times P1 and P3 (both unity) in order to be seen. Intensities on the right are P1=1, P2=4, P3=P4=1.8, and P5=P6=3.1.
Figure 6.2.2.6. Correlation maps for systems of 6 planets at positions marked. Intensities are $P_1=1$, $P_2=4$, $P_3=P_4=1.8$, and $P_5=P_6=3.1$.

Stringing the planets out in a line as in Figure 6.2.2.7 produces a somewhat different picture. A characteristic splitting or fork in the ridge shows up on approach of the star. This is a bit deceptive as the three planets at the left appear as two planets separated in the vertical plane at the same average radial distance. Once again, intensities of the planets were adjusted as indicated in the caption so that details of the topographical structure could be seen. Targets at 15 pc may require an increased baseline to establish the number of planets and initial positions for edge viewed systems.
Figure 6.2.2.7. Six planets at positions marked are mapped. Intensities are $P_1=P_3=P_4=2$, $P_2=P_5=1$, and $P_6=3$. Above, SNR=$10^6$, below, SNR=1.

While a few wavelengths may be sufficient to map widely dispersed targets, some systems require more. This study showed that a half dozen
or so bins in the range of 8-14 μm will solve most of the two planet cases. Spreading the bin selection over a wider range did not improve map definition because the peaks were moved radially outward and expanded (magnified) causing some smearing by including the longer wavelengths. This despite the fact that SNRs are expected to be better at the longer wavelengths. Magnification of the map outweighs feature enhancement as a determent to the extraction process. Another point worthy of mention is that the grid resolution should be determined by the requirements of the optimizer as the image will look about the same for a narrow FOV whether, for example, a 1, 5, or 10 mas grid is used but the numerical outputs become less accurate with a sparser grid.

6.2.2.1 Two Planet Interactions

The effect of relative planet positions will now be addressed. In Figure 6.2.2.1.1, one planet remains fixed at 40 mas while the other one is moved outward from 80 to 360 mas in 40 mas increments. Both were of unit intensity. The more distant planet shows considerable variability as it moves outward. At some locations, it seems to disappear altogether, only to reappear later as it continues in its course. This is primarily due to a change in relative signal strengths. The CLEAN process will in some way have to compensate for the signal loss.
Figure 6.2.2.1.1. Two planets of unit intensity with increasing separation. P2 shows considerable variability depending on its distance from the star.
Figure 6.2.2.1.1 cont. The effect of increasing planet separation. P2 shows considerable variability depending on its distance from the star.
Figure 6.2.2.1.1 cont. The effect of increasing planet separation. P2 shows considerable variability depending on its distance from the star.

6.2.3 The CLEAN Process

Radio astronomers use a CLEAN algorithm to dissect images and extract features from the background. A similar approach can be applied to TPF. In each step, the largest peak is assumed to always correspond with the strongest feature in the FOV (from which its intensities and
position may be derived). As will be seen, this is a flawed assumption due to accumulation of cross-products between siblings and noise. Modeling a signal after a source of spectral amplitudes and position corresponding to the brightest peak in the FOV and subtracting it from the composite signal provides a new time series from which another map may be produced. The strongest peak should then belong to the next brightest source and so forth. Repeated subtractions are made until the largest peak does not rise appreciably above the background. Each extraction is associated with a probability that the associated peak value (PV) represents a statistically significant feature such as a planet, and appropriate error bars can be attached to each measurement. The probability is given by

\[ p(PV) = \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{PV - \mu}{\sigma \sqrt{2}} \right) \]

where \( \mu = 0 \) for anti-symmetrical chopping and \( \sigma \) is measured during image construction along with the peak value. Probabilities calculated this way are valid only if the surrounding field is composed of random noise. Mathematical artifacts and other systemic variations will have to be eliminated for this to work in determining confidence levels.

Widely separated sources in a large FOV are decipherable at moderate resolutions with a 5 mas grid. Not withstanding the effects of
crosstalk and noise, numerical outputs are generally accurate enough for convergence of the optimizer if features are spaced by at least the resolution limit. The first steps of the CLEAN method are applied to a system of three planets with an SNR of 2.5 located as marked in Figure 6.2.3.1. Numerical outputs of the program are provided in Table 6.2.3.1.

If the solution for a given feature is slightly off the mark, the effect is to leave a residual in the neighborhood after subtraction as demonstrated between the first and second iterations. In each cycle, the planet was much brighter than the background. The fourth cycle (not shown) produced a uniform speckling of approximately equal amplitude peaks, marking an end to the CLEAN process. Noise peaks can be discriminated from crosstalk by their immobility through multiple subtractions. The above maps were made with a 5 mas grid. A 10 mas grid produced basically the same images.

Returning to the two planet systems analyzed in the earlier discussion on resolution, the CLEAN procedure will now be applied. Figure 6.2.3.2 is the outcome of this process for a planet at $X = 40$ mas and another at $X = 80$ mas. Iterations 3 and 4 are revisitations due to incomplete subtraction of the primary target in iteration 1 stemming from errors in the position and amplitude determinations of the principal peak.
Figure 6.2.3.1. Wide field CLEAN maps for a three planet system.
Table 6.2.3.1. Feature values for the CLEAN maps of Figure 6.2.3.1.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Peak Intensity</th>
<th>X Position (mas)</th>
<th>Y Position (mas)</th>
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</thead>
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<tr>
<td>1</td>
<td>1.0</td>
<td>700</td>
<td>-20</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>110</td>
<td>180</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>340</td>
<td>-90</td>
</tr>
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</table>

Figure 6.2.3.2. Outputs from four iterations of the CLEAN process.
Figure 6.2.3.2 cont. Outputs from four iterations of the CLEAN process.

Intensities and locations of planets extracted here would be sufficiently close for nonlinear optimization to complete the sorting process and identify false peaks. Iterations 3 and 4 contain artifacts of crosstalk that would be rejected by the optimizer. Table 6.2.3.2 is the numerical output of the program.
Table 6.2.3.2. Feature values for the CLEAN maps of Figure 6.2.3.1.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Peak Intensity</th>
<th>X Position (mas)</th>
<th>Y Position (mas)</th>
</tr>
</thead>
<tbody>
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<td>70</td>
<td>0</td>
</tr>
<tr>
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<td>5</td>
</tr>
<tr>
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<td>45</td>
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<tr>
<td>4</td>
<td>0.3</td>
<td>80</td>
<td>5</td>
</tr>
</tbody>
</table>

Discrimination of false objects is left to the optimizer, but an experienced analyst will observe structural trends that indicate the two off-axis peaks at 40 mas are due to crosstalk and harmonics and might interpolate numerical outputs of iterations 2 and 3 to find initial optimization parameters. Figure 6.2.3.3 repeats the above except that P2 is moved off line so that the planets are at positions X and Y (40, 0) and (40, 40) mas respectively. Numerical outputs are likewise found in Table 6.2.3.3. Figure 6.2.3.3a is a noiseless slightly magnified image for a system in which P1 at (40, 0) and P2 at (40, 60) mas has an expanded amplitude scale to enhance the feature at (40, -60) owing to crosstalk.

Occurrences of some kinds of false planets can be predicted by paying attention to symmetries (from harmonics) and intersections (from crosstalk) of topographical ridgelines where several planets are in the FOV. In these examples, additional phase diversity helps in the discrimination process.
Figure 6.2.3.3. Outputs from four iterations of the CLEAN process.
Figure 6.2.3.3 cont. Outputs from four iterations of the CLEAN process.

Table 6.2.3.3. Feature values for the CLEAN maps of Figure 6.2.3.2.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Peak Intensity</th>
<th>X Position (mas)</th>
<th>Y Position (mas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6</td>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>30</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>0.3</td>
<td>40</td>
<td>5</td>
</tr>
</tbody>
</table>

Figure 6.2.3.3a. Crosstalk produces a false planet at (30, -50) mas.
Harmonics and crosstalk are due to degeneracy in signals from proximal planets near the star and overlap of frequency components from multiple sources along with the inherent non-linearity of the instrument response. Spectral amplitudes computed by least squares fitting from composite interferograms of multi-planet systems using a least squares fitting process were an astounding 33 to 75 times larger in preliminary tests than the amplitudes from any of the contributing sources. This caused an outright failure of the CLEAN procedure and subsequent abandonment of the least squares amplitude approximations in favor of spectrally averaged peak values from the map to do the subtractions.

Sources strung out in a line are of particular interest as some systems are expected to have a large inclination. Consider the four planet arrangement of Figure 6.2.3.4 and numerical output of Table 6.2.3.4. Planet spacings are 40 mas. The characteristic fork described earlier complicates the extraction of $P_2$. Iterations 3, 4, and 5 can be combined by the analyst to identify the planet at $X = 150$ mas. Linear systems require a broader spectrum to compensate for the lack of phase diversity. Spectral bins in this example range from 7-12 $\mu$m.

In general, the definition gets worse when more (longer) wavelengths were used. A 15-17 $\mu$m range tends to produce the same images as 7-9
Figure 6.2.3.4. Six stage CLEAN process for four planets in a line.
Figure 6.2.3.4 cont. Six stage CLEAN process for four planets in a line.
Table 6.2.3.4. Feature values for the CLEAN maps of Figure 6.2.3.4.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Peak Intensity</th>
<th>X Position (mas)</th>
<th>Y Position (mas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>255</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>0.6</td>
<td>345</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>0.6</td>
<td>85</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>0.7</td>
<td>90</td>
<td>-20</td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>345</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.5</td>
<td>40</td>
<td>0</td>
</tr>
</tbody>
</table>

μm, but enlarged and shifted (as if looking at the shorter wavelength case through a tilted magnifying lens). Image shifting, as already mentioned, decreases the accuracy of the peak location estimations. Reported peak values are usually averaged over all wavelengths.

Addition of noise to composite signals is the subject of Figures 6.2.3.5 (the noiseless case), 6.2.3.6 (noise added at a SNR of 2.5), and the accompanying Tables 6.2.3.5 and 6.2.3.6. The most common effect is to move the peaks around and increase the variability of their amplitudes. Noise may require additional iterations to solve for all planets during the CLEAN procedure because false planets composed of noise sometimes appear brighter than, and are indistinguishable from real ones, thereby confounding them. More than one cycle may also be needed to CLEAN a given peak. In some cases, noise can shift real ones far enough (30 mas or more) to disrupt the CLEAN process. Site revisitations are indicative of
Figure 6.2.3.5. Four stage CLEAN process for a noise free system.
Figure 6.2.3.5 cont. Four stage CLEAN process for a noise free system.

Table 6.2.3.5. Feature values for the CLEAN maps of Figure 6.2.3.5.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Peak Intensity</th>
<th>X Position (mas)</th>
<th>Y Position (mas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6</td>
<td>195</td>
<td>-10</td>
</tr>
<tr>
<td>2</td>
<td>0.7</td>
<td>25</td>
<td>-90</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>350</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>100</td>
<td>65</td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>195</td>
<td>-15</td>
</tr>
</tbody>
</table>
Figure 6.2.3.6. Six stage CLEAN process for a noisy system.
Figure 6.2.3.6 cont. Six stage CLEAN process for a noisy system.
Table 6.2.3.6. Feature values for the CLEAN maps of Figure 6.2.3.6.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Peak Intensity</th>
<th>X Position (mas)</th>
<th>Y Position (mas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>25</td>
<td>-90</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>195</td>
<td>-10</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td>355</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>0.3</td>
<td>25</td>
<td>-90</td>
</tr>
<tr>
<td>5</td>
<td>0.3</td>
<td>195</td>
<td>-15</td>
</tr>
<tr>
<td>6</td>
<td>0.3</td>
<td>100</td>
<td>60</td>
</tr>
<tr>
<td>7</td>
<td>0.007</td>
<td>20</td>
<td>100</td>
</tr>
</tbody>
</table>

Poor location approximations that get worse with increasing noise levels. Being subjugated to the other planets and noise, it took six cycles to find the fourth planet from the noisy interferograms.

Using peak values from the maps to do the CLEAN subtractions underestimates the contributions to the composite signal from a set of planets because they are averaged over several wavelengths. While this adds robustness to the procedure, additional cycles are often needed to complete the subtractions. Successive subtractions in the presence of a strong noise component either causes convergence toward points of real origin, or chaos ensues, and after several cycles, the peaks wander outside the FOV. In the first case, latter position and intensity estimates are better and in the second case, first estimates are closest to the actual values.

Figure 6.2.3.7 and Table 6.2.3.7 are the outputs of the first 4 cycles of a
Figure 6.2.3.7. Typical movements of peaks around the true points of origin are illustrated for four CLEAN cycles.
Figure 6.2.3.7 cont. Typical movements of peaks around the true points of origin are illustrated for four CLEAN cycles.

Table 6.2.3.7. Feature values for the CLEAN maps of Figure 6.2.3.7.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Peak Intensity</th>
<th>X Position (mas)</th>
<th>Y Position (mas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.51</td>
<td>36</td>
<td>-12</td>
</tr>
<tr>
<td>2</td>
<td>0.44</td>
<td>70</td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>0.38</td>
<td>52</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>0.27</td>
<td>76</td>
<td>34</td>
</tr>
</tbody>
</table>

typical case in which peaks circle the true points of origin and eventually diverge. This configuration of planets will be encountered again in the next chapter upon which nonlinear optimizations will be performed.

Architectural changes such as increasing the length and number of baselines will help deal with the degeneracies of the system. Behaviors just described will improve and require less intervention from the analyst
and better facilitate automation. Adjustments such as interferogram normalizations have been shown to improve the accuracy and efficiency of the process and will be the subject of further experimentation.

Additional examples of correlation maps will be presented in the final chapter where the results of a blind test conducted by JPL are presented. Normalization of interferograms was found to reduce the number of intermediary "noise planets" in that study. Comparisons between noisy and noiseless datasets will show differences between false planets due to noise and crosstalk.
As already discussed, correlation maps are useful for estimating the number, locations, extent, and intensities of features within the FOV. A means of refining these estimates and eliminating artifacts of correlation is needed. Least squares fitting of the interferograms to a model is an obvious choice, but can be prohibitively time consuming when a large number of parameters are solved for simultaneously. Six planets measured in 20 bands will require 12 position parameters and 120 spectral intensities to be found. Since all planets will have to be solved for at once, and if all wavelengths are used simultaneously in the analysis, even the fastest super computers will not solve the problem in anyone's lifetime for a high resolution, 1600 mas FOV. Substantial nesting of loops is required, limiting the applicability of large scale parallel computing resources.

It will be shown later that interferograms may be analyzed separately, but with lesser accuracy and precision than obtained with a combined approach. Then the number of parameters solved for from a six planet system will be eight at a time. Parallel computing will then be of considerable value if each processor is assigned the task of solving the
planetary system for one data channel. On shared memory parallel computing platforms, many C/C++ and FORTRAN compilers support OpenMP multiprocessing directives based on the API standard. Compilation on a supporting C platform requires a -mp specification on the cc command line. The simple structure of Figure 7.1 can be applied.

```c
#pragma omp parallel default(shared)
{
    #pragma omp sections private(‘list of variables’) nowait
    {
        #pragma omp section
        {
            <structured block for channel 1>
        }

        #pragma omp section
        {
            <repeated block for channel 2>
        }

        ...
    }
}
```

Figure 7.1. Code frame for analyzing data channels in parallel on OpenMP supported platforms.

Optimization algorithms have proven very helpful in speeding the analysis. Three categories are discussed in the literature. The first deals
with linear systems and is not useful for TPF applications. The second applies to nonlinear systems that can be linearized. The least squares equation requiring optimization takes the form

$$\text{residual}_{x,y} = \sum_{\lambda} \sum_{\phi} (\text{data}(\lambda, \phi) - \text{model}(\lambda, \phi)_{x,y})^2$$

where the model function is a trigonometric series for which small angle approximations cannot be made. The third category applies to inherently non-linear functions typical of the TPF problem. Chapter 10 of *Numerical Recipes* (Press, 1992) provides a sampling. Prime candidates include gradient and conjugate gradient algorithms because of their simplicity and speed. Gradient methods use only first derivative information to find a solution and are too inaccurate. Newton or quasi-Newton methods including Levenberg-Marquardt and Gauss Newton use second derivatives to provide better accuracy with some sacrifice of speed. For extremely noisy data sets, application of the Newton method proved necessary to achieve good convergence. Newton, and quasi-Newton including Levenberg-Marquardt optimization routines were all found to give similar results, though in certain cases, one or another method showed superior convergence. Newton was the most accurate where only short jumps from the initial values were needed. Less accurate first guesses required that
Levenberg-Marquardt be used. Gauss-Newton often provided a good compromise where runtimes were critical.

Newton methods seek to find a minimum of an error function $F(v)$ that is the sum of squares of nonlinear functions of the parameter vector $v$

$$F(v) = \frac{1}{2} \sum_{\lambda} \sum_{\varphi} [f_{x,y}(v)]^2,$$

where $f_{x,y}(I,x,y) = \text{data}(\lambda, \varphi) - I \cdot \text{normmodel}(\lambda, \varphi)_{x,y}$.

Let the Jacobian of $f(v)$ be denoted $J(v)$. Different approaches give similar results in finding a solution for $v$. Several Newton variants have been successfully used.

Newton's approach starts from the initial parameter vector $v_0$ and refines it using the assumption that $f(v)$ is locally linear. A first order approximation of $f(v_0 + \Delta)$ is

$$f(v_0 + \Delta) = f(v_0) + J(v_0)\Delta$$

with $\Delta$ being a small displacement scalar. Minimizing the error,

$$\varepsilon = \varepsilon_0 - J\Delta$$

is solved through linear least squares yielding

$$J^TJ\Delta = J^T\varepsilon.$$
This is the normal equation. The solution is found by starting from initial values and refining them in successive iterations

$$v_{i+1} = v_i + \Delta_i$$

with $\Delta_i$, the solution of the normal equation evaluated at $v_i$. The algorithm may only find a local minimum, so good initial values $v_0$ are important.

Levenberg-Marquardt is a variation of the Newton method but the normal equations

$$N\Delta = J^TJ\Delta = J^T\epsilon$$

are augmented to

$$N'\Delta = J^T\epsilon$$

where

$$N'_{i,j} = (1 + \delta_{i,j}\gamma)N_{i,j}$$

with $\delta_{i,j}$, the Kronecker delta. The value $\gamma$ is initialized to $10^{-3}$. If $\Delta$ reduces the error, the increment is accepted and $\gamma$ is divided by 10 before the next iteration. If $\epsilon$ increases, then $\gamma$ is multiplied by 10 and the augmented normal equations are solved again until an increment is obtained that reduces the error. This is bound to happen, since for large $\gamma$ the method approaches steepest descent.
7.1 Uncertainty in the Outcome of Optimization

Consider an exo-Earth at various separation angles from its parent star. How well can non-linear optimization programs solve for its spectral intensities and position? To answer this, 25 ten μm interferograms of unit peak intensity were generated (each having a different Gaussian noise component of zero mean and unit standard deviation) for a single planet at each of four separation angles with the SNR of the input signal set to 0.9, 2.5, 10, and 20, for each set of 25. The planet positions were Y = 0, and X = 20, 30, 40, and 420 mas. Ten μm was chosen to avoid expected attenuation of the twin Earth's signal at shorter wavelengths and loss of resolution at longer wavelengths. A measure of the uncertainty in each of the parameters was found by calculating the standard deviation of the sample set. The results are graphed on a log-log scale in Tables 7.1.1 through 7.1.4. Measurements were the outcomes of the optimization.

Clearly a serious degradation in the uncertainty occurs inside the first peak (at 40 mas for this instrument). The RMS uncertainty in the measurement is proportional to the interferogram SNR. For planets at or beyond the first peak of the null pattern, intensities and positions relative to the peak will generally be accurate to within ~6% at the 1σ level for an interferogram (input) SNR of 5. Similar performance inside the first peak at 30 mas would require an interferogram SNR of ~20. At a SNR of 2.5,
Table 7.1.1. Position and intensity STDEVs from a normalized signal of a planet at 20 mas from its star for signal SNRs of 2.5, 5, 10, and 20.

Table 7.1.2. Position and intensity STDEVs from a normalized signal of a planet at 30 mas from its star for signal SNRs of 2.5, 5, 10, and 20.
Table 7.1.3. Position and intensity STDEVs from a normalized signal of a planet at 40 mas from its star for signal SNRs of 0.9, 2.5, 5, 10, and 20.

Table 7.1.4. Position and intensity STDEVs from a normalized signal of a planet at 420 mas from its star for signal SNRs of 0.9, 2.5, 5, 10, and 20.
convergence was not always achieved for the 30 mas planet and at closer
points, fails outright. Mercury would be found at ~26 mas at a distance of
15 pc. At that separation angle it would only be detectable with very long
integration times or if data smoothing or prefiltering was employed.

Would applying an ideal low pass filter in this type of analysis help?
To answer this, datasets for a 500 mas planet were generated and Fourier
transformed to find the highest frequency content in the noiseless signal
and establish a cutoff frequency for an ideal filter. A brick wall filter was
then applied to each of several datasets for planets inside and beyond the
first stripe before doing the optimization. Use of the filter increased the
uncertainties in position and intensity in most all cases.

7.2 Sibling Effects

The next logical step is to consider the effect of adding siblings. An
experiment was conducted in which three siblings were introduced and
the effect on the uncertainty of planet parameters examined. Actual
intensities and positions of the four planet system are given in Table 7.2.1.
Table 7.2.3 indicates that for an interferogram SNR of 2.5, the variation of
the spectral intensity at 10 μm was worsened with introduction of the
siblings by a factor of ~25 and the 1σ variation in the positions by about
~3.5 for the 40 mas planet. Figures 7.2.2 through 7.2.5 compare
variations in the uncertainties as a product of sibling introduction. In all cases, the input SNR was 2.5. The largest effect is observed for planets at or beyond the first stripe of the stationary transmission pattern, but outward of the first stripe, the difference does not change considerably.

Table 7.2.1. Input parameters for the sibling experiment.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Intensity</th>
<th>X Position (mas)</th>
<th>Y Position (mas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>42.43</td>
<td>42.43</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>63.64</td>
<td>-63.64</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>420</td>
</tr>
</tbody>
</table>

Differences in the uncertainties were significant enough to warrant a closer look at some two planet only cases. Tables 7.2.5 through 7.2.7 compare the measurement uncertainties at 10 µm for a single planet of unit intensity at X = 40 mas and input SNR of 2.5 to those where a second is added at Y = 420 mas who's intensity is allowed to fluctuate. Intensities were in order 0.5, 1, and 10 for the 420 mas planet. Differences are statistically insignificant. Figures 7.2.9 through 7.2.11 demonstrate the 420 mas planet parameters track variations in intensity as expected. Similar experiments showed the output variability for one of two widely separated planets to be independent of the location and intensity of its sibling.
Sibling Effect on SNR for 20 mas Planet

Table 7.2.2. Parametric uncertainties for a 20 mas separation. One planet in the FOV is compared with four as described in Table 7.2.1.

Sibling Effect on SNR for 30 mas Planet

Table 7.2.3. Parametric uncertainties for a 30 mas separation. One planet in the FOV is compared with four as described in Table 7.2.1.
Table 7.2.4. Parametric uncertainties for a 40 mas separation. One planet in the FOV is compared with four as described in Table 7.2.1.

Table 7.2.5. Parametric uncertainties for a 420 mas separation. One planet in the FOV is compared with four as described in Table 7.2.1.
Table 7.2.6. Uncertainties in parameters for a unit intensity planet at $X = 40$ mas with a sibling at $Y = 420$ mas of intensity 0.5.

Table 7.2.7. Uncertainties in parameters for a unit intensity planet at $X = 40$ mas with a sibling at $Y = 420$ mas of intensity 1.
Table 7.2.8. Uncertainties in parameters for a unit intensity planet at \(X = 40\) mas with a sibling at \(Y = 420\) mas of intensity 10.

Table 7.2.9. Uncertainties in parameters for a 0.5 intensity planet at \(Y = 420\) mas with a sibling at \(X = 40\) mas of unit intensity.
Sibling Effect on SNR for 420 mas Planet

Table 7.2.10. Uncertainties in parameters for a unit intensity planet at Y = 420 mas with a sibling at X = 40 mas also of unit intensity.

Table 7.2.11. Uncertainties in parameters for a planet of intensity 10, at Y = 420 mas with a sibling at X = 40 mas of unit intensity.
7.3 Decipherability Constraints

As objects move closer together, crosstalk becomes more of a problem, and for close-in non-symmetrical features, overlapping harmonics play an increasingly larger role in confounding sources. Such factors contribute to the increased variability seen in the four planet sibling experiment to the extent that false peaks in correlation maps stemming from the crosstalk coincides with those from natural objects. Additional trials and analyses are needed to increase understanding of these issues.

The optimizer’s resolution depends on how well it separates features despite components of noise and distortion. Sticking with the two planet cases, an experiment was devised to see how well the optimizer performs given initial conditions provided by the correlation mapper. Correlation maps will be of little use in separating planets less than 40 mas apart. A good test is the case where one planet is at X and Y (40, 0) and the other 40 mas away at 45°, e.g., (68, 28) mas (ref. Figure 6.2.3.7). Table 7.3.1 is the result of a sample size of 25. Optimizer outputs were recorded for a-priori initial conditions and ones derived from numerical outputs of the
Table 7.3.1. Comparative outputs of the STD correlation and optimization processes.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Correlation Map Results</th>
<th>Optimizer Output</th>
<th>Optimizer Using STD Map ICs</th>
<th>Optimizer Using a-priori ICs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P1 @ X = 0.040 arcsec</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P2 @ X = 0.068 arcsec</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y = 0.028 arcsec</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Noiseless</td>
<td>Mean</td>
<td>STDEV</td>
<td>Noiseless</td>
</tr>
<tr>
<td>P1 8 μm Intensity</td>
<td>0.8</td>
<td>0.4250</td>
<td>0.0683</td>
<td>1</td>
</tr>
<tr>
<td>P1 9 μm Intensity</td>
<td>0.8</td>
<td>0.4250</td>
<td>0.0683</td>
<td>1</td>
</tr>
<tr>
<td>P1 10 μm Intensity</td>
<td>0.8</td>
<td>0.4250</td>
<td>0.0683</td>
<td>1</td>
</tr>
<tr>
<td>P1 11 μm Intensity</td>
<td>0.8</td>
<td>0.4250</td>
<td>0.0683</td>
<td>1</td>
</tr>
<tr>
<td>P1 12 μm Intensity</td>
<td>0.8</td>
<td>0.4250</td>
<td>0.0683</td>
<td>1</td>
</tr>
<tr>
<td>P1 13 μm Intensity</td>
<td>0.8</td>
<td>0.4250</td>
<td>0.0683</td>
<td>1</td>
</tr>
<tr>
<td>P1 14 μm Intensity</td>
<td>0.8</td>
<td>0.4250</td>
<td>0.0683</td>
<td>1</td>
</tr>
<tr>
<td>P1 X Position (arcsec)</td>
<td>0.050</td>
<td>0.0518</td>
<td>0.0056</td>
<td>0.040</td>
</tr>
<tr>
<td>P1 Y Position (arcsec)</td>
<td>-0.002</td>
<td>0.0013</td>
<td>0.0083</td>
<td>0.000</td>
</tr>
<tr>
<td>P2 8 μm Intensity</td>
<td>0.8</td>
<td>0.3188</td>
<td>0.0544</td>
<td>1</td>
</tr>
<tr>
<td>P2 9 μm Intensity</td>
<td>0.8</td>
<td>0.3188</td>
<td>0.0544</td>
<td>1</td>
</tr>
<tr>
<td>P2 10 μm Intensity</td>
<td>0.8</td>
<td>0.3188</td>
<td>0.0544</td>
<td>1</td>
</tr>
<tr>
<td>P2 11 μm Intensity</td>
<td>0.8</td>
<td>0.3188</td>
<td>0.0544</td>
<td>1</td>
</tr>
<tr>
<td>P2 12 μm Intensity</td>
<td>0.8</td>
<td>0.3188</td>
<td>0.0544</td>
<td>1</td>
</tr>
<tr>
<td>P2 13 μm Intensity</td>
<td>0.8</td>
<td>0.3188</td>
<td>0.0544</td>
<td>1</td>
</tr>
<tr>
<td>P2 14 μm Intensity</td>
<td>0.8</td>
<td>0.3188</td>
<td>0.0544</td>
<td>1</td>
</tr>
<tr>
<td>P2 X Position (arcsec)</td>
<td>0.072</td>
<td>0.0724</td>
<td>0.0021</td>
<td>0.068</td>
</tr>
<tr>
<td>P2 Y Position (arcsec)</td>
<td>0.034</td>
<td>0.0337</td>
<td>0.0026</td>
<td>0.028</td>
</tr>
</tbody>
</table>
correlation mapper. These results are summarized in Table 7.3.2. Column 3, estimates the (40, 0) mas planet to be 12 mas and the (68, 28) mas planet to be 7 mas off their marks. Average values tabulated in column 3 and associated RMS variations in column 4 (both of which include the effects of noise) should be compared to the no-added-noise case in column 2. Columns 5 through 9 are the result of using the correlation map values (columns 6 and 7) and the a-priori planet positions (columns 8 and 9) to do the extractions with noisy signals. Noiseless data was used to produce column 5. Whether a-priori starting points or mapper outputs were used, the noise free case converged to the same predicted values. More distant planets are always better resolved as might be expected due to additional frequency content. Comparing ratios of noise free positions (column 2) from the mapper to RMS variations (column 4) in these parameters due to noise inclusion for inner and outer planets, the outer one shows a six fold increase in output SNR (OSNR) over the inner planet. The same trend persists with the optimizer outputs. Table 7.3.2 also compares the OSNRs among processes. In this experiment, the improvement in OSNR achieved by using a-priori ICs rather than mapper ICs is a factor of ~5 for the intensities and ~2.5 for the positions. Precision of the measurements are a sensitive function of the ICs. Further refinement of the ICs provided by the mapper is needed to improve the OSNRs of the optimizer. Such might
include scaling of peak amplitudes used in the CLEAN process and averaging their positions over several CLEAN cycles.

Table 7.3.2. Average parametric SNR comparisons of process outputs.

<table>
<thead>
<tr>
<th>P1 @ X = 0.040 arcsec</th>
<th>P2 @ X = 0.068 arcsec</th>
<th>Y = 0.028 arcsec</th>
<th>Mapper Output OSNR</th>
<th>Optimizer Output w/Map ICs OSNR</th>
<th>Optimizer Output w/a-priori ICs OSNR</th>
<th>ΔSNR Optimizer w/Map ICs</th>
<th>ΔSNR a-priori ICs Map ICs</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1 Intensity</td>
<td>11.7</td>
<td>2.5</td>
<td>16.0</td>
<td>0.2</td>
<td>6.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P1 Position</td>
<td>3.6</td>
<td>9.4</td>
<td>25.5</td>
<td>2.6</td>
<td>2.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P2 Intensity</td>
<td>14.7</td>
<td>6.0</td>
<td>19.2</td>
<td>0.4</td>
<td>3.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P2 Position</td>
<td>22.9</td>
<td>50.2</td>
<td>113.7</td>
<td>2.2</td>
<td>2.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Experiments with other two-planet systems produced similar results and this example can be considered typical. A note of importance must be made here. Values used in the preceding calculations came from reasonably well behaved trials. An interferogram SNR of 2.5 became the standard for these experiments as a borderline case. About 20% of attempts failed due to noise effects. When convergence fails, intensities often exhibit chaotic behavior yielding values in the ± millions, and sometimes, positions that are near zero or way off the map. A small fraction of convergence failures were in a range of contestability. Significant amounts of work will be needed to properly characterize
convergence behavior and should be postponed until the FFI initial design
decision has been made, as the behavior will change significantly with
alteration of the instrument. Another crucial point is that a remarkable
performance improvement of the mapping and optimization algorithms
was observed with the inclusion of the 7 \( \mu \text{m} \) band. This band was not
used in the experiment just described because it is expected to produce
little signal from an Earth-like planet. However, it indicates that a baseline
increase will have a marked affect on overall performance.

7.4 Validity of Feature Extractions

As discussed in the previous chapter, statistical measures can be
applied to extracted parameters of the correlation maps to estimate the
number of true features and determine error bars or confidence levels for
detections, if mathematical artifacts can be made negligible. The optimizer
provides the possibility of a more certain way of deciding the correct
number of planets in the FOV. Since the optimizer implements a least-
squares fitting procedure, a cumulative residual is calculated and is one of
the outputs of the program. Underestimating the number of planets, or
significantly the contribution (spectral intensities) of a planet, for example,
produces an unexpectedly large residual. Expected values of the aggregate
residual depend on the dimensions of the search space and interferogram
SNRs. Selected combinations of extracted planets from a comprehensive set can be run again to find which yields the minimum residual just beyond the "ledge" where a large drop occurs. This concept is illustrated in Figure 7.4.1 for experimental 1, 2, and 3 planet extractions. Consistency should be the subject of further study. Multiple runs in which the Levenberg-Marquardt algorithm is used first, followed by a Newton variant may improve the final outcome a bit. Least-squares fitting about the optimized parameter set using direct substitution should always be the final step in the process if the number of parameters is small enough to allow such fitting.

Figure 7.4.1. A significant drop in the residual of the least squares fit occurs when the correct number of targets has been solved for.
The fundamental limit of the algorithms presented in this work in their ability to detect and characterize planetary systems, is integration time, with lesser dependencies on aperture size, baseline length, number of baselines, bandwidths, number of bands, and spectral range. All but the integration times were constrained in this study by the straw man configuration. Integration time allowances are determined by the mission parameters. No degrees of freedom were delegated to the analyst in the evaluation phase. The objective was to find whether developed procedures would be able to correctly extract positions and spectra from multi-planet systems in the presence of worst case background conditions, given the straw man and mission constraints. Correlation mapping has proven useful as a first step in approximating planet positions given phase 2 integration times that elevate data point SNRs to greater than unity with the high resolution straw man configuration. Integration times to accomplish the requisite SNRs for easy targets are on the order of a month. This is much longer than the half day allotments for the observations in the planet finding phase. Spectral intensities can be approximated by least-squares fitting of the model to the interferograms,
using peak positions extracted during the correlation CLEAN process from data acquired during mission phase two. Parameters obtained in this way are not accurate enough for scientific purposes so other operations are needed to refine them. Artifacts of crosscorrelation depend on the noise level, and the number, intensities, and absolute and relative positions of real objects in the FOV. Artifacts from crosstalk move when planets move relative to one another. Nonlinear optimizations separate real objects from artifacts by showing zero intensity at the false peaks, and further refine parameter values derived from the maps given sufficiently high data point SNRs. Direct substitution in a least-squares fitting can then be used to polish the outcomes of non-linear optimization resulting in milliarcsecond position accuracies and 1% amplitude errors for low noise conditions. Accuracies of the extractions diminish rapidly when the noise approximately equals or exceeds the signal. Smoothing interferograms and averaging redundant halves of a 360° rotation partially corrects the badly distorted signals due to extremely high noise levels for half day integration times and move data point SNRs toward a workable range of ≥ 2.5. Operations on the signal waveforms must be mirrored in the model. This significantly increases the programs complexity and computing time. Distortion of the model functions occurs when running averages are performed on them if many points are used in the averaging.
Planet finding has a different set of constraints than spectrum gathering. In the first case, only a half day of examination will be allotted for each star. Radiometric performance calculations in Appendix A were computed assuming ~50,000 second integrations and can serve as a guide for selecting targets based on detectability. The descending SNR target selection order roughly tracks that computed for phase 2 measurements, but with somewhat improved SNR values.

The aforementioned examination conducted by JPL, to see how well algorithms being developed by Ball Aerospace, JPL, and the University of Arizona might work for deciphering planets among high levels of shot and systematic noise, showed that none of the procedures developed by any of these organizations are sufficient by themselves to do reliable extractions from half-day observations. The other two groups used enhanced mapping techniques without further parametric refinements. All of the correlation processes suffered from similar difficulties in dealing with mathematical artifacts and noise features indistinguishable from planets. Data point SNRs for the blind trials ranged between 1 and 0.01.

Smoothing the time series with a five to nine point circular boxcar average, cutting the interferograms in half, inverting and shifting one side to overlap the other, and summing each overlapping pair of points
produced half rotation interferograms with much better noise characteristics than the original time series. Unfortunately, noise levels associated with 50,000 s detections were high enough to warp the underlying waveforms in many cases, thereby preventing a complete restoration that accurately represents the original system. A small mismatch between the JPL generator signal waveforms and the analytical model functions was also discovered that is probably a result of approximation and/or photon counting errors. Noted discrepancies alter the ratios of sibling spectral intensities with a smaller effect on the positions.

Differences between smoothed and normalized noisy interferograms and model functions for high noise cases are often a large percentage of the normalized signal. Consider a Sun-like star at 10 pc with one planet having a spectrum matching Earth’s but twice its brightness located at (-100, 0) mas. The second is a twin Earth at (0, 200) mas. Assume a 10 EZ plus LZ background. As demonstrated in Figure 8.1, peak values in the difference between the normalized smoothed noisy data stream and model function at 7 μm are on the order of 50% of the normalized model peak values. The large difference produces artifacts in correlation maps and detours for the optimizer. The situation improves with the noise free case provided in Figure 8.2. Here, the difference peaks are about 7% of the
Figure 8.1. The difference between a normalized smoothed interferogram (top) and a normalized model function (center) is shown at the bottom.
Figure 8.2. The difference between a normalized smoothed interferogram (top) and normalized model function (center) is shown at the bottom.
normalized model function. The difference depends on the relative intensities, number, and positions of planets composing the signal, which are parameters that cannot be known a-priori. For the five point running average used, the differences should be quite small and can be reduced with modification to the transmission module in the JPL signal generator or adjustments to the model.

Further testing showed that local minima exist everywhere along LS fitting lines, even with boxcar averaging. Because of this, the optimizer will be ineffective without the aid of a working correlation mapper or some other means of obtaining ICs. Smoothing increased the complexity of the derivative functions. During initial runs, they were averaged at each point in the same manner as the model to form the Jacobian. Line searches are sensitive to interferogram scaling which proved difficult with limited target system information. Comparing midpoint values with five-point slope averages revealed equivalence of waveforms allowing single point derivatives to be used instead. Finite differencing proved more accurate than analytical Jacobians because of the mismatch between the model and generator. Results of the JPL blind test evaluations will be presented in the next section. Problems encountered along the way will be discussed with each test case.
8.1 Blind Test Evaluation

Fourteen test cases were used to judge the effectiveness of the Ball Aerospace, JPL, and University of Arizona algorithms in a competitive survey. All of the algorithms performed similarly as would be expected since they are not fundamentally different. Only products of correlation mapping will be given because the differences between the JPL generator and the University of Arizona models have not been sufficiently worked out to enable proper functioning of the optimizer.

Case 1. No planets in the FOV.

This dataset was properly judged in all tests to be void of planets. It was clear that the field was entirely composed of noise. Whenever negative band amplitudes are computed, the likelihood of a false detection is high. Outputs from the correlation mapper for the first two CLEAN cycles using the three longer wavelength bands were:

Planet 1
Location: X = -0.195 arcsec, Y = -0.135 arcsec
Image SNR: 3.37

Band 3 amplitude: 0.8653
Band 4 amplitude: -0.2226
Band 5 amplitude: 1.5602

Planet 2
Location: X = -0.180 arcsec, Y = 0.055 arcsec
Image SNR: 3.20
Case 2. A single planet.

Figure 8.1.1 is from the original dataset. Noise peaks dominated the FOV. Regenerating the noisy interferograms using JPL's generator and input parameters but a 10 EZ background produced results different from the original as shown in Figure 8.1.2. Now the planet appears in the first iteration and others composed of noise follow. The second planet is false because of negative band amplitudes calculated in the second cycle.

---

Figure 8.1.1. Maps from the original dataset. The planet's expected location is marked. Noise forms the strongest features in the images.
Figure 8.1.2. The first two CLEAN images from regenerated interferograms with additive noise.

**Case 3. Two planets – one very dim inside the first stripe.**

The second planet was not identified in the blind trials as evidenced in Figure 8.1.3. Planet 1 was noise shifted to the left. A single “noise planet” was found in iteration 2 at (-150, 200) mas.

Figure 8.1.4 demonstrates that with noise free signals, the planets can be separated and both are detectable. The planet inside the first stripe is more susceptible to noise because its signal strength is reduced by the central null and can disrupt the entire CLEAN procedure if the noise leads to a false detection at any point. The planet inside the first stripe has a broader peak indicative of its greater uncertainty.
Figure 8.1.3. The first two CLEAN images obtained during the blind trials.

Figure 8.1.4. The first two CLEAN cycles using noiseless interferograms to produce the images. Both planets are now observable.
Case 4. Generic two planet case.

Both planets were sufficiently located in the correlation maps for refinement with the optimizer once the signal generator issues have been worked out. The first two cycles are presented in Figure 8.1.5.

Figure 8.1.5. The first two CLEAN images from the original dataset.

Case 5. Dynamic range test.

Figure 8.1.6 is an image set from the blind test. Maps were scaled to show the second planet is present, but subjugated by crosstalk and noise. The dimmer planet was not the brightest peak in any of several cycles with the noisy signals. In Figure 8.1.7, made from noiseless interferograms, the dimmer planet appeared in the first cycle as the brightest peak. Planet 2 was slightly off the mark because of the waveform-model mismatch.
University of Arizona generator signals produced a map with a perfect match in positions. Figure 8.1.8, derived using a different noise stream, bares too much resemblance to Figure 8.1.6 to be mostly a result of noise, showing there is an even more significant contribution from crosstalk. The second iteration peak is indistinguishable from a real planet, but the third iteration yielded negative band amplitudes and would therefore be judged as false. Image SNR and band amplitudes were significantly reduced in the second iteration from the first. The probability of a correct detection is proportional and should be reduced accordingly.

![STD Correlation Map](Iteration 1)

![STD Correlation Map](Iteration 2)

Figure 8.1.6. The first two CLEAN images from the original dataset scaled to show subjugation of the real planet to a "crosstalk planet" emphasized by the noise.
Figure 8.1.7. The first two CLEAN images produced from noiseless signals. Both planets are now observable.

Figure 8.1.8. The first two iterations using a different noise stream from that of Figure 8.1.6. Since different noise streams produce similar images, the false peaks are shown to be a result of crosstalk.
Case 6. One Earth temperature planet and one at 180K.

It was clear from the blind test that the first planet was false (a "noise planet") because of its negative band 5 amplitude. The warmer planet was missed entirely but the larger cooler one was somewhat close, being noise shifted to the lower right. The false planet found in the first cycle of Figure 8.1.9 derailed the CLEAN process and all subsequent finds are suspect.

Figure 8.1.9. The first two CLEAN images from the original dataset.

Maps of Figure 8.1.10 exhibit several artifacts from noise free datasets, which are the result of crosstalk. The peak belonging to the larger cooler planet is subjugated by other cross-products. Cross-products can be identified in some cases where they appear along a line through
Figure 8.1.10. First two CLEAN images from noiseless interferograms.

the real planets and the star. Maps were scaled to show subordinate structure.

Some insight can be gained by looking at maps with only the dimmer planet in the FOV as per Figure 8.1.11 that shows where harmonics are available for mixing with those of the brighter planet to form artifacts. The image at the left is scaled to show only the brightest peak. The threshold for the image on the right is lowered to reveal the broad structure corresponding to phase equivalent spectra. Noise and crosstalk both play a role in masking the real planet, making a CLEAN extraction difficult where they work together to confound some of the
siblings. Under certain circumstances, a brighter planet can be masked by crosstalk or harmonic suppression from dimmer ones.

Figure 8.1.11. Maps of the dimmer planet clipped at two different thresholds.

**Case 7. Generic three planets I.**

Only two of three planets were found from the original time series of Figure 8.1.12. The planet near the first stripe has a greater uncertainty in its position and exhibits a broad peak. The "planet" in iteration 3 may only be attributed to noise because significant structure was not found in the noiseless maps of Figure 8.1.13. Figure 8.1.14 was a rerun with a 10 EZ noise component added. This time, all three planets appear in proper sequence.
Differences between Figures 8.1.12 and 8.1.14 might be explained by assuming a different azimuth and inclination for the exo-zodiacal cloud. Noise properties, however, would not be different enough to create the discrepancies exhibited. In any event, the only sensible explanation for failure to find the third planet is noise, unless an error occurred when inputting parameters of the original dataset or if a glitch occurred (thought to happen on occasion) in the pattern generator. Iteration 4 produced negative band amplitudes so planet 4 would not be considered real. A finer grid resolution might help to improve the accuracy somewhat.

Figure 8.1.12. The first tree CLEAN images from the original dataset. The third planet was not found. Instead, a residual from incomplete subtraction of planet 1 was the largest feature.
Figure 8.1.12 cont. The first three CLEAN images from the original dataset.
The third planet was not found. Instead, a residual from incomplete subtraction of planet 1 was the largest feature.

Figure 8.1.13. Noiseless interferograms produced a near match of the first planet and good matches with the other two.
Figure 8.1.13 cont. Noiseless interferograms produced a near match of the first planet and good matches with the other two.

Figure 8.1.14. With a 10 EZ face-on noise component, the positions do not change appreciably.
Figure 8.1.14 cont. With a 10 EZ face-on noise component, the positions do not change appreciably.

**Case 8. Generic three planets II.**

The first three iterations in Figure 8.1.15 reveal the three planets

Figure 8.1.15. Original extraction in which all three planets were found.
Figure 8.1.15 cont. Original extraction in which all three planets were found.

but the fourth iteration (not shown) produced a planet that could not be discounted, because no criterion for elimination was met.

**Case 9. Five planets.**

Figure 8.1.16 shows that if only the two planets nearest the star are in the FOV, they are easy to find and with high accuracy. From the original time series of the five planet system, the first two were found in proximity, the same for the next two that appear in iteration 3 of Figure 8.1.17. The fourth and fifth iterations make planets out of noise with non-physical band amplitudes. The sixth goes back to finish cleaning up planet 1. Planet 5 was completely missed.
Figure 8.1.18 is comprised of CLEAN images from a noiseless dataset. The first four planets are found with better accuracy in the first four cycles, but the process gets hung up on a set of cross-products for the next several cycles preventing the fifth planet from being found. The problem gets worse as the FOV increases, providing additional opportunities for large amplitude cross-products at higher frequencies.

This example also illustrates one effect of noise, which is to offset the peaks from their points of origin in random directions. The higher the noise level, the greater will be the uncertainty in the positions extracted from the maps. The same trend persists when non-linear optimizations are performed.

Figure 8.1.16. The first two CLEAN images from noiseless composite signals, with only the two most difficult to find of the five planets in the FOV.
Figure 8.1.17. The first six CLEAN images from the original data. Four out of five planets were identified. The last planet was masked by crosstalk.
Figure 8.1.18. The first eight CLEAN images from noiseless signals.
Figure 8.1.18 cont. The first eight CLEAN images from noiseless signals.

**Case 10. High zodi.**

The only problem incurred with this dataset was deciding when to stop iterating. A false planet in the third iteration looked as real as the others. Maps from the original dataset are shown in Figure 8.1.19.

Figure 8.1.19. The first two CLEAN images from the original dataset.
Case 11. One planet inside and one beyond the first stripe.

Planets inside the first stripe are extremely susceptible to noise and increase the uncertainties of all other measurements when aggravated by crosstalk. Both planets are observed in sequence, but noise shifted in Figure 8.1.20.

Figure 8.1.20. The first two CLEAN images from the original dataset.

Maps from noiseless interferograms are provided in Figure 8.1.21. Peak spreading indicates an increased uncertainty for the planet inside the first stripe. Past experiments showed that the distance separating two siblings made a big difference in the positional accuracy of the one closest to the star when it was located within the first stripe.
Figure 8.1.21. Maps from noiseless interferograms showing significant measurement uncertainties for close planets with one inside the first stripe.

**Case 12. Four planets – face on solar system.**

Correlation maps for this system were similar whether or not noise was present in the interferograms. In each instance, three planets were proximally located and one was not. This is due in part to the high intensity of planet 1 relative to the other three that were clustered. The fourth planet never appeared. The problem is attributable to artifacts that are the result of crosstalk. Once a large amplitude false planet signal is subtracted, the original waveform is distorted enough to preclude further extractions of dim siblings. Figure 8.1.22 shows the first four maps from the original dataset. Figure 8.1.23 provides noiseless versions. The tallest peak in iteration 2 of the noiseless set is a cross-product with the real
planet being shifted and subjugated by the artifact. Noise in the original dataset caused the real planet to be observable. Iteration 4 in the noisy set found a "noise planet" and in the noiseless data, went back to finish cleaning the residual of planet 1.

Figure 8.1.22. The first four CLEAN images from the original dataset.
Figure 8.1.23. The first four CLEAN images from a noiseless dataset.

Problems of crosstalk and “noise planets” are ubiquitous, plaguing all crosscorrelation procedures used by each of the groups involved in TPF re-imaging. Nonlinear optimization is also sensitive to noise, but even
more sensitive to errors in the waveform from the signal generator. The more noise, the closer the initial conditions have to be to get good convergence from the optimizer.

**Case 13. Two planet non-blind test case.**

No difficulties were experienced in finding the planets in this non-blind exercise. Maps from the original dataset can be found in Figure 8.1.24. Locations of the two planets were determined with good enough accuracy for the optimizer to finish the job within the process tolerance as determined in previous studies.

---

Figure 8.1.24. The first two CLEAN images from the original dataset.
Case 14. Three planets in a line.

Here the results are somewhat ambiguous. The three planets were found but mixed in with false ones produced from noise and crosstalk. Figure 8.1.25 provides maps from the original dataset, and Figure 8.1.26 from noiseless versions. Iteration 2 produced a "crosstalk planet" and

Figure 8.1.25. Images from the original dataset.
iteration 3, a "noise planet" from the noisy dataset, before resuming real planet extractions. The third iteration found a "crosstalk planet" from the noiseless dataset followed by residuals of only proximal subtractions.

Figure 8.1.26. Images from noiseless interferograms.
Figure 8.1.26 cont. Images from noiseless interferograms.

_Evaluation Summary_

Results of working with the blind test data are summarized in Table 8.1. Judgments of proximity were based on the ability of the optimizer to converge from those initial conditions. The outcome of this exercise is typical of previous internal experiments in which roughly 75% of planets
in the FOV are found, with 25% “false negative” and 25% “false positive” error rates. False positives (mis-finds) are defined as planets made of noise, crosstalk, etc. False negatives are real planets that were never identified in the extraction process, being suppressed by noise and crosstalk.

Table 8.1. Summary of findings using only correlation maps for planet identification.

<table>
<thead>
<tr>
<th>#</th>
<th>Case</th>
<th>planets in FOV</th>
<th>on target finds</th>
<th>proximal finds</th>
<th>mis-finds</th>
<th>cause of missed ID</th>
<th>noise</th>
<th>X-talk</th>
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<tr>
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<td>1</td>
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<td>0</td>
<td>1</td>
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<td></td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1 of 2 distant planets inside 1st stripe</td>
<td>2</td>
<td>1</td>
<td>0</td>
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<td></td>
<td></td>
<td>1</td>
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<td>11</td>
<td>1 of 2 close planets inside 1st stripe</td>
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<td>0</td>
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<td></td>
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<td>1</td>
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<tr>
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<td>face-on developmental</td>
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<td>2</td>
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</tr>
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<td>14</td>
<td>3 planets in a line</td>
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<td>3</td>
<td>0</td>
<td>2</td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Totals  33  19  7  8  10  5

8.2 Recommended Procedure

Following is a brief summary of the procedure being developed to reconstruct images and extract parameters from planets in the FOV.
This process is graphically summarized in Figure 8.2.1.

Figure 8.2.1. Flow chart summarizing the analytical process to produce the final image and extract the parameters.

The steps are:

1. Create a data file composed of the sine chopped interferograms.

2. Compute running averages for each interferogram.
   - Band 1 is a 5 point average.
   - Band 2 is a 7 point average.
   - Bands 3, 4, and 5 are 9 points averaged.

3. Cut the interferograms in half and slide the second over the first.

4. Subtract the second half from the first.

5. Repeat these steps for the model and register the two.
6. Normalize the registered interferograms.

7. Crosscorrelate the normalized interferograms with normalized model equations at each grid point of a corresponding map for each band and sum the maps point-by-point over all bands to get a composite map.

8. Locate the strongest peak in the composite map and compute an image peak to RMS background ratio as a first cut at setting a detection threshold.

9. Use least-squares fitting to approximate the normalized spectral amplitudes of the brightest planet.

10. Fashion signals from the location and amplitude parameters just found and subtract them from the composite interferograms.

11. If the peak to background ratio is above some threshold T.B.D., and if all band amplitudes for the current iteration are positive, use the new composite interferograms to repeat the process and find the next brightest planet and so forth.

12. Once a full set of planets have been identified, use the derived band amplitudes and locations as initial guesses for a Levenberg-Marquardt nonlinear optimization. When all goes well, artifacts of the correlation process are removed by showing they have zero valued band amplitudes.
13. Determine peak photon rates from the un-normalized interferograms to be used as scale factors for finding the photon flux from the original interferograms.

14. Use planet locations and normalized planet flux ratios multiplied by the peak photon rates from the optimization as initial values for a straight forward least squares fitting to get the final spectral intensities and positions of each planet.

15. Subtract scaled noiseless modeled planet signals from cosine chopped interferograms to create a new set containing only broad features and noise.

16. Create first level correlation maps using cosine chopping from the output of the previous step.

17. Compute two dimensional running averages in each spectral map to smooth the topography and reduce the noise, with the number of points determined by the map resolution and wavelength. A few percent of the total grid dimension should give good results.

18. Model a set of signals corresponding to the intensities and positions at each grid point and subtract them from the output of step 15 to produce a "picture" of the noise that remains. This image should be checked for Gaussian characteristics and provides a statistical baseline for setting the threshold of detection.
19. Sum the images from all bands containing planets, broad features, and statistical noise (if desired) to produce the final image.

8.3 Summary and Findings

Programs have been written to simulate the signals expected to be produced by the TPF SCI straw man, extract a full set of parameters for each planet in the FOV, and create synthetic images in the form of correlation maps. A few simple code modules that can be used as a basis for further development are located in the appendices.

The Airy disk from the telescopes roughly coincides with the FOV in all bands and manufactured artifacts or mutual cancellation is minimized. Simple deconvolution reveals a ring of reduced sensitivity extending from ~100-200 mas. The sensitivity of the instrument is in general a strong function of position on the sky. The resolution limit of the interferometer was determined to be ~40 mas. Planets closer than that, or within the 40 mas first stripe of the null pattern will have very large uncertainties and are considered inseparable or undetectable.

To get the most accurate results, image reconstruction and planet extractions will involve many operations accomplished in several stages.
Crosscorrelations and nonlinear optimizations form the basis of the extractions. In order for the optimizer to do its job, a complete set of planets must first be identified by the correlation mapper. The optimizer can eliminate false planets produced by the correlator if interferograms are of sufficient quality, i.e., waveforms closely match the model functions and noise components are sufficiently retractable to provide data point SNRs ≥ 2.5 for at least five bands or with two to three times as many, an SNR ≥ 1.5. Since the noise in each band is independent, more bands are favored over wider bandwidths when data smoothing is applied, allowing additional spectral maps to be overlaid and a further reduction in the size of the noise peaks and number of false detections, but the gains will be limited because narrower bandwidths will mean reduced band SNRs. Noise at a data point SNR of 2.5 was shown to cause 7% amplitude and 10% position variations from the mean in the correlation maps. Optimization variations were 3% in position and including outliers, up to 150% in amplitude, punctuating the need to finish the process with substitutional LS fitting that brings the position errors down to around 1 mas and amplitude errors to ≤ 5%.

Mathematical artifacts are ubiquitous among crosscorrelation methods, and with the very high noise levels associated with TPF interferograms (especially in the planet finding sequence), will be very hard
to get rid of. While deciphering “noise planets” may be possible with repeat visits or measurements, “crosstalk planets” are more difficult to eliminate because their positions change when the planets responsible for the crosstalk change locations.

With lower noise in signals from phase 2, the optimizer can eliminate the crosstalk for planets no closer than ~40 mas with the straw man configuration. The problem of artifacts will have to be dealt with using waveform matching algorithms that by their nature require long processing times. Alternatively, the FOV can be changed in a stepwise fashion to allow subtraction of signals from close-in features from the wider FOVs, thereby replacing the missing spectral components in the composite interferograms. Analysis would proceed as normal for each onion layer but there would be more datasets to work with.

Most of the developments discussed herein have focused on anti-symmetrical nulling (sine chopping). Near edge-view systems have the problem of confounded or disappearing planets, especially when pairs are of equal intensity and diametrically opposed in the FOV. The problem of disappearing planets may be ameliorated using cosine chopped signals in the analysis and broad features such as zodiacal cloud structures must be examined with cosine or no chopping. For point sources, performance is
degraded because the star leak and uniform symmetric zodiacal cloud will not be cancelled, and any planets detected will have a twin. All of the algorithms and code developed here can be adapted to cosine chopping with appropriate sign changes in the transmission functions and analytical equations.

The two most important concerns for image reconstruction are lack of ample integration time and the degeneracies stemming from lack of frequency and phase discrimination for close-in planets. FFL instruments enable significant increases in the baselines with substantial improvement in data reduction capability. If the baselines were extended by a factor of about five, spectral energy distribution functions which are more easily calculated and potentially more accurate may supplant correlation maps in finding planet locations, and the ring of reduced sensitivity mentioned earlier is smaller. Relative spectral intensities can then be found directly from peak values of the energy distribution plots. However, the correct baseline length is not just a matter of setting some minimum value. Characteristics of the analysis are determined by several factors that depend on the length of the instrument. If the instrument is too long, it will produce results just as bad as one that is too short. More data points in running averages provide better noise reduction capability but increase the waveform distortion, especially at shorter wavelengths. Shorter truss
lengths produce interferograms with gentler slopes that are easier to smooth without causing severe deformities of the waveforms, and, as already discussed, longer wavelengths have the same affect as shortening the truss with the added benefit of better SNRs. Previous experiments with a 20 meter truss gave about the same results as the 36 meter. If detection in proximity of the star is the most critical factor, then the optimal baseline is decided by the distance to the first stripe of the stationary null pattern but integration times will have to be increased significantly to counter the increase in noise from an increase in the star leak. The instrument length is best optimized in the frequency domain where harmonics and cross-products are observable in conjunction with RPA calculations to see the effect on SNR when the null width changes.

8.4 Topics for Future Study

Analytical methods studied here are widely used to solve a variety of scientific problems, among them, matching parameters of time series signals. Further development is required and other types of procedures will be examined as progress continues. The scheduled launch date is 2020 leaving time for further exploration of alternative methodologies.
Only a few truly independent methods are amenable to analyzing TPF data. Crosscorrelations or frequency domain equivalent spectrum energy distributions have provided the best initial means of finding planets. Any roadmap for future work must address the problem of harmonics, crosstalk, and noise artifacts that appear in images formed by either approach.

Differences of scaling and normalization cause artifacts to appear in different places and at different stages of a CLEAN procedure. An aggregate approach using combined maps from each algorithm developed by the three institutions involved may help solve the puzzle.

Underlying waveforms of sine and cosine chopped signals are different and have different amplitude and phase spectra. This may lead to a different set of artifacts mixed in with physical features that can potentially provide a means of sorting out the real planets. Maps should be produced by each method and compared to see what they reveal.

Altered mathematical methods failing, changes to the instrument design should be considered. If in addition to rotation, lateral scanning mechanisms are employed, degeneracies associated with close-in planets may be slightly reduced. Rather than relying on differences in wavelength to provide frequency diversity, longer wavelengths where SNRs are better
may be used exclusively in the plant finding phase of the mission. Once again, increased observation times, complexity of the design, and computation times may prohibit such a modification.

Changing the FOV in an onion layer approach as previously described and solving each layer separately holds considerable promise. The major benefit is in the ability to solve for close-in planets separately against the larger regional zodiacal emission. The drawback is added complexity of the instrument. Observation times will also be negatively impacted.

Increasing the number of telescopes in a 2D array and thus separating the imaging resolution from the null width may provide additional frequency diversity but spectral overlap will still be a problem.

If the degeneracies, aliasing, and inadequate observation times cannot be overcome within available schedule and cost constraints, artifacts from harmonics, crosstalk, and noise will have to be well characterized through extensive experimentation to set the limits of TPF's science capability.
APPENDIX A

RADIOMETRIC CALCULATIONS

Properties of the TPF SCI used for radiometric performance calculations are summarized in tables A1 and A2. Spreadsheets immediately follow in Table A3 that give predicted SNRs for a list of candidate stars during the detection phase of the mission. The first set are those for which a short baseline is deemed adequate and the second set may require a longer baseline.

Table A1. Synopsis of instrument imaging parameters.

- Number of Telescopes = 4
- Baselines = 2 @ 27 m
- Mirror diameters = 3.2 m
- Integration time = 50000 s / rotation
- Data points / rotation = 360
- Wavelength = 10 um
- Spectral resolution = 5.04x10^12 Hz (R=6)
- Background = 1 LZ and 10 EZ
- Rotations / image = 1
- FOV = 1572 mas
- Configuration = hi resolution

Table A2. Instrument throughput.

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Table A3. Short baseline target list. SNRs are for a 50000 s rotation of the interferometer. The image is constructed from one rotation.

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Wavelength: 9.9E-06 m
Table A3 cont. Short baseline target list. SNRs are for a 50000 s rotation of the interferometer. The image is constructed from one rotation.

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Table A3 cont. Long baseline target list. SNRs are for a 50000 s rotation of the interferometer. The image is constructed from one rotation.

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Table A3 cont. Long baseline target list. SNRs are for a 50000 s rotation of the interferometer. The image is constructed from one rotation.

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APPENDIX B

SIGNAL GENERATOR PROGRAM

This Matlab script used in conjunction with the mex file gen.c will generate five interferograms for wavelengths as specified for up to six planets and place them in the working directory.

Interferogram Generator Matlab Script

Program name: generator.m

% Executable Matlab script for generating spectral interferograms
% last updated 3/20/04

% compile C program
mex gen.c

% planet Intensities matrix (photon flux rates) – [I1,I2,I3,I4,I5,I6]
I=[1.0,0.0,0.0,0.0,0.0,0.0];

% planet Positions matrix - [x1,y1,x2,y2,x3,y3,x4,y4,x5,y5,x6,y6]
% specify values in arc sec
P=[0.100,0.0,0.0,-0.200,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0];

% create spectral interferograms - specify instrument configuration as
% follows: (mirror locations) p1,p2,p3,p4, (aperture areas) a1,a2,a3,a4,
% Intensities array, Positions array – repeat for each wavelength to get
% independent noise streams for each band

gen(-18.0,-9.0,9.0,18.0,3.2,3.2,3.2,3.2,2.5,1,P); copyfile sig1_ sig1;
gen(-18.0,-9.0,9.0,18.0,3.2,3.2,3.2,3.2,2.5,1,P); copyfile sig2_ sig2;
gen(-18.0,-9.0,9.0,18.0,3.2,3.2,3.2,3.2,2.5,1,P); copyfile sig3_ sig3;
gen(-18.0,-9.0,9.0,18.0,3.2,3.2,3.2,3.2,2.5,1,P); copyfile sig4_ sig4;
gen(-18.0,-9.0,9.0,18.0,3.2,3.2,3.2,3.2,2.5,1,P); copyfile sig5_ sig5;
% import spectral interferograms for plotting, etc.
fid = fopen('sig1','r'); one = fscanf(fid, '%g'); status = fclose(fid);
 fid = fopen('sig2','r'); two = fscanf(fid, '%g'); status = fclose(fid);
 fid = fopen('sig3','r'); three = fscanf(fid, '%g'); status = fclose(fid);
 fid = fopen('sig4', 'r'); four = fscanf(fid, '%g'); status = fclose(fid);
 fid = fopen('sig5','r'); five = fscanf(fid, '%g'); status = fclose(fid);

% combine them into a single file for later mapping and optimization
 fid = fopen('COMBINED', 'w');
 for i = 1:360
  fprintf(fid, '%f %f %f %f %f
', one(i)/max(one), two(i)/max(two), ...
 three(i)/max(three), four(i)/max(four), five(i)/max(five));
end
 status = fclose(fid);

% plot spectral interferograms
 x = 1:1:360;
 y0 = one; y1 = two; y2 = three; y3 = four; y4 = five;
 plot(x, y0, 'r', x, y1, 'k', x, y2, 'b', x, y3, 'g', x, y5, 'm');
 legend('7.44 um', '8.5 um', '9.92 um', '11.9 um', '14.9 um', 0);
 title('Spectral Interferograms', 'FontSize', 18);
 xlabel('rotation angle $\phi$', 'FontSize', 14);
 ylabel('transmssion', 'FontSize', 14);
 axis tight; line(x, 0);
 end

Program name: gen.c

/**************************************************************************/
/* $Revision: 1.0 $ */

Composite signal generator for up to 6 planets at various positions x & y,
intensities, and wavelengths. Gaussian noise is added to the signals at a
ratio specified by SNR. The instrument is a dual Bracewell interferometer.
Zero or +/- Pi phase chopping has been employed for a set of 4 telescopes.
Updated 3/20/04.
*************************************************************************/

/)*/ $Revision: 1.0 $ */
#include "mex.h"
#include "stdio.h"
#include "stdlib.h"
#include "math.h"
#include "float.h"
#include "time.h"

#define pi 3.14159265
#define kappa 4.84813689

int i, j, cnt = 0, phi;
double SNR = 1000000.0;
double apos = -18.0, bpos = -9.0, cpos = 9.0, dpos = 18.0;
double aamp = 3.2, bamp = 3.2, camp = 3.2, damp = 3.2;
double A[6] = {1.0, 0.0, 0.0, 0.0, 0.0, 0.0};
double P[6][2] = {0.100, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0};
double *Amp, *Pos, lambda;

double e(S1, S2)
int S1, S2;
{
    return(S1 * 3.0 * pi / 2.0 + S2 * pi);
}

double f(S1, S2)
int S1, S2;
{
    return(S1 * pi + S2 * 3.0 * pi / 2.0);
}

double g(S1, S2)
int S1, S2;
{
    return(S1 * pi / 2.0);
}
double h(S1, S2)
int S1, S2;
{
    return(S2*pi/2.0);
}

double t(lambda, theta, S1, S2)
int S1, S2;
double lambda, theta;
{
    double aph = 2*pi*aupos*theta/lambda+e(S1,S2);
    double bph = 2*pi*bpos*theta/lambda+f(S1,S2);
    double cph = 2*pi*cpos*theta/lambda+g(S1,S2);
    double dph = 2*pi*dpos*theta/lambda+h(S1,S2);
    double Real, Imagt;

    Realt = aamp*cos(aph)+bamp*cos(bph)
    +camp*cos(cph)+damp*cos(dph);

    Imagt = aamp*sin(aph)+bamp*sin(bph)
    +camp*sin(cph)+damp*sin(dph);

    return((Realt*Realt+Imagt*Imagt) / pow(aamp+bamp+camp+damp,2.0));
}

double T(lambda, theta)
double lambda, theta;
{
    return (t(lambda,theta,0,1) - t(lambda,theta,1,0));
}

double pattern(alpha, beta, phi, lambda)
int phi;
double alpha, beta, lambda;
{ 
    return(T(lambda, alpha*kappa*sin(phi*pi/180.0) 
        + beta*kappa*cos(phi*pi/180.0))); 
}

double noise() 
{
    int i;
    double X = 0.0, m=0.0, s=1.0;

    for (i=1; i<=12; i++) {
        x += ((double)rand()/(double)RAND_MAX);
    }

    return(s*(x-6.0)+m);
}

double PATTERN(phi, lambda) 
int phi;
double lambda; 
{
    return((A[0]*pattern(P[0][0],P[0][1],phi,lambda) 
            + A[1]*pattern(P[1][0],P[1][1],phi,lambda) 
            + A[2]*pattern(P[2][0],P[2][1],phi,lambda) 
            + A[3]*pattern(P[3][0],P[3][1],phi,lambda) 
            + A[4]*pattern(P[4][0],P[4][1],phi,lambda) 
            + A[5]*pattern(P[5][0],P[5][1],phi,lambda)) 
        + 0.9/SNR*noise()); 
}

void gen(void) {
    FILE *ofp, *fopen();
    cnt=0.0;
for (i = 0; i <= 5; i++) {
    A[i] = *(Amp+i);
    for (j = 0; j <= 1; j++) {
        P[i][j] = *(Pos+cnt);
        cnt++;
    }
}
srand(time(0));

ofp = fopen("sig1_","w");
    for (phi = 0; phi <= 359 ; phi++) {
        fprintf(ofp,"%f\n",PATTERN(phi,7.44));
    }
fclose(ofp);

ofp = fopen("sig2_","w");
    for (phi = 0; phi <= 359 ; phi++) {
        fprintf(ofp,"%f\n",PATTERN(phi,8.50));
    }
fclose(ofp);

ofp = fopen("sig3_","w");
    for (phi = 0; phi <= 359 ; phi++) {
        fprintf(ofp,"%f\n",PATTERN(phi,9.92));
    }
fclose(ofp);
ofp = fopen("sig4_", "w");

    for (phi = 0; phi <= 359 ; phi++) {
        fprintf(ofp, "%f\n", PATTERN(phi, 11.9));
    }

fclose(ofp);

ofp = fopen("sig5_", "w");

    for (phi = 0; phi <= 359 ; phi++) {
        fprintf(ofp, "%f\n", PATTERN(phi, 14.9));
    }

fclose(ofp);

printf("Five spectral interferogram files have been created. \n");

}

void mexFunction(int nlhs, mxArray *plhs[], int nrhs, const mxArray *
prhs[])
{
    apos = mxGetScalar(prhs[0]);
    bpos = mxGetScalar(prhs[1]);
    cpos = mxGetScalar(prhs[2]);
    dpos = mxGetScalar(prhs[3]);
    aamp = mxGetScalar(prhs[4]);
    bamp = mxGetScalar(prhs[5]);
    camp = mxGetScalar(prhs[6]);
    damp = mxGetScalar(prhs[7]);
    SNR = mxGetScalar(prhs[8]);
    Amp = mxGetPr(prhs[9]);
    Pos = mxGetPr(prhs[10]);
    gen();
}

/* end */
The intensities and positions of the six planets are adjustable. The program will work for low and high resolution cases by locating the mirrors appropriately. To use the code in Matlab, copy the files to the working directory and run the script (issue the command "generator"). A mex file will be compiled from gen.c. A single plot of the overlaid interferograms will then be displayed. Modifications and additions to the code can be made to suit the experiment.

Arguments of the gen() call provide the instrument parameters. The first four parameters are the mirror positions and the next four are the aperture areas (or diameters if equal) which are all initialized to the configuration under study. It is initially set up for the high resolution case. For the low resolution situation, just reverse the signs of the two inside mirror positions.

To change the intensities for the 6 planets, modify the values in the I[] matrix. Use unity values to test the code. X and Y placements of the 6 planets are specified in arcsecond separation angles on in the P[] matrix. Measured from the origin at the center of the FOV, position angles can take on positive or negative values. The choice of wavelengths was made to correspond with the JPL specification for the straw man.
Anti-symmetrical phase chopping is assumed since it is the preferred mode of operation for planet detections. The signal-to-noise ratio parameter is initially set to a very high number to produce a noiseless signal. Typical values will be ~2.5 or so.

Signal files produced by this program will have two columns. The first is the rotation angle; the second is the photon count at that angle. All bands are collected into another datafile for importation by the analytical programs. The code can be run without the Matlab interface if desired. Just compile gen.c by itself.
APPENDIX C

CORRELATION MAPPING PROGRAMS

This Matlab script used in conjunction with the mex file corHalf.c will produce crosscorrelation maps (cmp1 ... up to ... cmp8) from a set of five interferograms of wavelengths specified by the straw man instrument for up to six planets and place them in the working directory. The program will also locate the peak in X and Y and estimate its intensity (amplitude) for each wavelength and iteration. An average across wavelengths is computed for the peak intensity which can be used as input to nonlinear optimization programs. A CLEAN process is applied to get cmp1 ... cmp8 in which the position and spectral intensities for each planet are solved for in turn. The final results are written to a file called “out” which can be imported by the optimizer.

Correlation Map Matlab Script

Program name: correlate.m

% Executable Matlab script for producing correlation CLEAN maps
% last updated 3/20/04
% compile C program
mex corHalf.c

% (mirror locations) p1,p2,p3,p4, (aperture areas) a1,a2,a3,a4,
% first band, last band, one sided grid dimension (e.g., number of points),
% grid resolution (mas), number of iterations, intensity range.
corHalf(-18.0,-9.0,9.0,18.0,3.2,3.2,3.2,3.2,1,5,80.0,0.005,6,30);

% display level 1 map, replace cmp1 with cmp2, etc., to display the others
load cmp1;
maxvalue=max(max(cmp1));
cmp=cmp1/maxvalue;
contour3(cmp1,200);
view(0,-90);
whitebg([0 0 .3]);
xlabel('X position (mas)','FontSize',14);
ylabel('Y position (mas) ','FontSize',14);
title('Correlation Map','FontSize',18,'color','white');
axis square;
grid off;
box on;

% end

Crosscorrelation C Code

Program name: corHalf.c

/*%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%***/
This program crosscorrelates composite signals from up to 8 planets with
model functions at five wavelengths and all grid points and combines the
results to form a composite set of images. A CLEAN subtraction process is
implemented to extract locations and amplitudes of planets one at a time.
Anti-symmetrical chopping and normalized and superpositioned half
rotations are assumed. Updated 3/15/04.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%***/

/* $Revision: 1.0 $ */

#include "mex.h"
#include "stdio.h"
#include "stdlib.h"
#include "math.h"
#include "float.h"
#include "time.h"
# define pi 3.14159265
# define kappa 4.84813689
# define numdatapoints 361
# define numamplitudes 101
# define maxplanets 6
# define planets 2
# define bins 5
# define griddimension 100
# define gridscale 0.001
# define halfrotation 180

double WAVELENGTH=0.0;
int gridsize=griddimension;
int gridpoints=(2*griddimension+1)*(2*griddimension+1);
int datapoints=numdatapoints;
int numplanets=planets, amplitudes=numamplitudes, planet=0;
int firstwavelength=1, lastwavelength=5;
double scalefactor=gridscale;
double aposn=-18.0, bposn=-9.0, cposn=9.0, dposn=18.0;
double asize=3.2, bsize=3.2, csize=3.2, dsize=3.2;

double a(D1,D2)
int D1, D2;
{
    return(D1*3.0*pi/2.0+D2*pi);
}

double b(D1,D2)
int D1, D2;
{
    return(D1*pi+D2*3.0*pi/2.0);
}
double c(D1,D2)
int D1, D2;
{
    return(D1*pi/2.0);
}

double d(D1,D2)
int D1, D2;
{
    return(D2*pi/2.0);
}

double t(lambda,theta,D1,D2)
int D1, D2;
double theta, lambda;
{
    double aph = 2*pi*aposn*theta/lambda+a(D1,D2);
    double bph = 2*pi*bposn*theta/lambda+b(D1,D2);
    double cph = 2*pi*cposn*theta/lambda+c(D1,D2);
    double dph = 2*pi*dposn*theta/lambda+d(D1,D2);
    double Realt, Imagt;
    Realt = asize*cos(aph)+bsize*cos(bph)+csize*cos(cph)+dsize*cos(dph);
    Imagt = asize*sin(aph)+bsize*sin(bph)+csize*sin(cph)+dsize*sin(dph);
    return((Realt*Realt+Imagt*Imagt)/pow(asize+bsize+csize+dsize,2.0));
}

double T(lambda,theta)
double theta, lambda;
{
    return (t(lambda,theta,0,1) - t(lambda,theta,1,0));
}
double pattern(alpha, beta, phi, lambda)
int phi;
double alpha, beta, lambda;
{
    return(T(lambda, alpha * kappa * sin(phi * pi / 180.0) + beta * kappa * cos(phi * pi / 180.0)));}

double correlate(sig1, sig2)
double sig1[], sig2[];
{
int i, j, delay;
double sum1=0.0, sum2=0.0, sum12=0.0;
double mean1=0.0, mean2=0.0, result, denom=0.0;

mean1=0.0; mean2=0.0;
for (i=0; i<datapoints; i++) {
    mean1 += sig1[i];
    mean2 += sig2[i];
}
mean1 /= datapoints;
mean2 /= datapoints;
sum1=0.0; sum2=0;
for (i=0; i<datapoints; i++) {
    sum1 += (sig1[i] - mean1) * (sig1[i] - mean1);
    sum2 += (sig2[i] - mean2) * (sig2[i] - mean2);
}
denom = sqrt(sum1*sum2);
for (delay = -halfrotation; delay<halfrotation; delay++) {
    sum12=0.0;
    for (i=0; i<datapoints; i++) {
        j = i + delay;
while(j < 0)

    j += datapoints;
    j %= datapoints;
    sum12 += (sig1[i] - mean1) * (sig2[j] - mean2);

if(denom<=0.000001)

    result=0.0;

else

    result = sum12/denom;

return(result);

}

double lsfit(X, Y)
double X[], Y[];
{
    int j;
    float min=0.0,a=0.0,b=0.0,c=0.0,d=0.0,e=0.0;
    float f=0.0,g=0.0,h=0.0,i=0.0,X = 0.0,Y = 0.0,z=0.0;

    for (j=0; j<amplitudes; j++) {

        b = b + X[j];
        x = x + Y[j];
        c = c + X[j]*X[j];
        f = f + X[j]*X[j]*X[j];
        i = i + X[j]*X[j]*X[j]*X[j];
        y = y + X[j]*Y[j];
        z = z + X[j]*X[j]*Y[j];

    }

    a = amplitudes;
\[
\min = \frac{0.5(-a*z*f + a*i*y + c*b*z - c*y*c + x*c*f - x*i*b)}{(b*b*z - b*y*c - c*a*z - b*f*x + c*x*c + y*f*a)};
\]

```c
return(min);
}
void cor(void) {
    int phi, i, x, y, wavelength;
    double sx1, sx2, STDEV, sumampl, peak;
    double signal[numdatapoints], tstptrn[numdatapoints];
    double sum[2*griddimension+1][2*griddimension+1];
    double xcor[2*griddimension+1][2*griddimension+1][2*griddimension+1];
    double compsig[2*griddimension+1][numdatapoints], diff[2*griddimension+1][numdatapoints];
    double X[2*griddimension+1][2*griddimension+1];
    double xp[5][numamplitudes], residual[5];
    double ampl[5] = {1, 1, 1, 1, 1};
    W[3] = {9.92, 11.9, 14.9};
    float b1, b2, b3, b4, b5;
    static char out[1];
    char *strcpy();
    FILE *ifp, *ofp, *afp, *fopen();

    printf("Calculating planet positions and normalized relative amplitudes...
    ");

    /* clear output files */
    ofp = fopen("cmp1","w"); fcloze(ofp);
    ofp = fopen("cmp2","w"); fcloze(ofp);
    ofp = fopen("cmp3","w"); fcloze(ofp);
    ofp = fopen("cmp4","w"); fcloze(ofp);
    ofp = fopen("cmp5","w"); fcloze(ofp);
    ofp = fopen("cmp6","w"); fcloze(ofp);
    ofp = fopen("cmp7","w"); fcloze(ofp);
    ofp = fopen("cmp8","w"); fcloze(ofp);
    ofp = fopen("stats","w"); fcloze(ofp);
    ofp = fopen("locations","w"); fcloze(ofp);
    ofp = fopen("planet_id","w"); fcloze(ofp);
    ofp = fopen("locations","w"); fcloze(ofp);

    //*************************************************************************/
```
/* read composite signals */

ifp = fopen(".. / generator / allbands","r");

    for (phi = 0; phi <= halfrotation-1; phi++) {
        fscanf(ifp,"%f %f %f %f %f
",&b1,&b2,&b3,&b4,&b5);
        compsig[0][phi]=2.0*b1;
        compsig[1][phi]=2.0*b2;
        compsig[2][phi]=2.0*b3;
        compsig[3][phi]=2.0*b4;
        compsig[4][phi]=2.0*b5;
    }

fclose(ifp);

/***************************
/* create model interferograms and cross correlate with composite signal
at each grid point */

planet=0;
printf("\n Planet %d \n",planet+1);

findaplanet:
for (wavelength=firstwavelength; wavelength<=lastwavelength;
wavelength++) {
    ampl[wavelength-firstwavelength]=1.0;
    for (phi = 0; phi <= halfrotation-1; phi++) {
        signal[phi]=pow(compsig[wavelength-firstwavelength][phi],1.0);
    }
    for (y=-gridsize; y<=gridsize; y++) {
        for (x=-gridsize; x>gridsize; x--)

            for (phi = 0; phi <= halfrotation-1; phi++) {
$$tstptrn[\phi] = \text{pow}(\text{pattern}(x \times \text{scalefactor}, y \times \text{scalefactor}, \phi, W[\text{wavelength} - \text{firstwavelength}]), 1.0);$$

$$\text{xcor[\text{wavelength-firstwavelength}][\text{gridsize+y}] [\text{gridsize-x}]=correlate(tstptrn, signal);}$$

/* initialize the correlation matrix */

for (\text{y}=-\text{gridsize}; \text{y}\leq\text{gridsize}; \text{y}++) {
    for (\text{x}=\text{gridsize}; \text{x}>=-\text{gridsize}; \text{x}--) {
        \text{sum[gridsize+y][gridsize-x]}=0.0;
    }
}

/* sum correlation coefficients over all wavelengths for each grid point */

\text{gridpoints}=(2*\text{gridsize}+1)*(2*\text{gridsize}+1);

for (\text{wavelength}=\text{firstwavelength}; \text{wavelength}\leq\text{lastwavelength}; \text{wavelength}++) {
    \text{peak}=0.0, \text{xp[planet]}=0.0, \text{yp[planet]}=0.0, \text{sx1}=0.0, \text{sx2}=0.0;
    for (\text{y}=-\text{gridsize}; \text{y}\leq\text{gridsize}; \text{y}++) {
        for (\text{x}=\text{gridsize}; \text{x}>=-\text{gridsize}; \text{x}--) {
            \text{sum[gridsize+y][gridsize-x]}+=\text{xcor[\text{wavelength}
if (sum[gridsize+y][gridsize-x] > peak) {
    peak = sum[gridsize+y][gridsize-x];
    xp[planet] = x*scalefactor;
    yp[planet] = y*scalefactor;
}

sx1 += sum[gridsize+y][gridsize-x];
sx2 += sum[gridsize+y][gridsize-x] * sum[gridsize+y][gridsize-x];

STDEV = sqrt((gridpoints*sx2-sx1*sx1)/(gridpoints*(gridpoints-1)));

/*******************************
/* print results */
/*****************************/

printf(" Location: X = %f, Y = %f \n", xp[planet], yp[planet]);
printf(" PEAK amplitude: %f \n", peak);
printf(" STDEV: %f \n", STDEV);
printf(" SNR: %f \n\n", peak/STDEV);

afp = fopen("stats", "a");
    fprintf(afp," Planet: %d \n", planet);
    fprintf(afp," Location: X = %.3f, Y = %.3f \n", xp[planet], yp[planet]);
    fprintf(afp," PEAK: %f \n", peak);
    fprintf(afp," STDEV: %f \n", STDEV);
    fprintf(afp," SNR: %f \n\n", peak/STDEV);
fclose(afp);

afp = fopen("locations", "a");
fprintf(afp, "%.3f%.3f\n", xp[planet], yp[planet]);

fclose(afp);

/* dump correlation matrix to file for plotting in Cartesian coordinates*/

if (planet == 0)
    strcpy(out, "cmp1");
if (planet == 1)
    strcpy(out, "cmp2");
if (planet == 2)
    strcpy(out, "cmp3");
if (planet == 3)
    strcpy(out, "cmp4");
if (planet == 4)
    strcpy(out, "cmp5");
if (planet == 5)
    strcpy(out, "cmp6");
if (planet == 6)
    strcpy(out, "cmp7");
if (planet == 7)
    strcpy(out, "cmp8");
if (planet == 8)
    strcpy(out, "cmp9");
if (planet == 9)
    strcpy(out,"cmp10");

afp = fopen(out,"a");

for (y=-gridsize; y<=gridsize; y++) {
    for (x=gridsize; x>=-gridsize; x--) {
        fprintf(afp,"%f ",sum[gridsize-y][gridsize+x]);
    }
    fprintf(afp,"
");
}
fclose(afp);

/*****************************/
/* calculate residuals and amplitude for each wavelength at Xp and Yp */
sumampl=0.0;

for (wavelength=firstwavelength; wavelength<=lastwavelength;
wavelength++) {
    for (i=0; i<=amplitudes-l; i++) {
        residual[i]=0.0; X[i]=i;
    }
    for (phi = 0; phi <= halfrotation-l; phi++) {
        diff[wavelength-firstwavelength][phi]= X[i]
        *pattern(xp[planet],yp[planet],phi, W[wavelength-
        firstwavelength])-compsig[wavelength-firstwavelength]
        [phi];
residual[i]+=pow(diff[wavelength-firstwavelength][phi],2);
}
}

/* least squares fit to find approximate amplitudes for each wavelength at Xp and Yp */

ampl[wavelength-firstwavelength] = lsfit(X,residual);

printf(" Band %d amplitude: %.4f \n",wavelength,ampl[wavelength-firstwavelength]);

/* subtract current primary signal source from the composite signal and repeat all of the above */

for (wavelength=firstwavelength; wavelength<=lastwavelength; wavelength++) {
    for (phi = 0; phi <= halfrotation-1; phi++) {
        compsig[wavelength-firstwavelength][phi]-=ampl[wavelength-firstwavelength]*pattern(xp[planet],yp[planet],phi,
        W[wavelength-firstwavelength]);
    }
}

planet++;

    if (planet <= numplanets-1) {
        printf(" \n Planet %d \n",planet+1);
goto findaplanet;
}

printf(" ... finished.\n\n");
}

/***************************************************************************/

void mexFunction(int nlhs, mxArray *plhs[], int nrhs, const mxArray *
*prhs[])
{
    aposn = mxGetScalar(prhs[0]);
    bposn = mxGetScalar(prhs[1]);
    cposn = mxGetScalar(prhs[2]);
    dposn = mxGetScalar(prhs[3]);
    asize = mxGetScalar(prhs[4]);
    bsize = mxGetScalar(prhs[5]);
    csize = mxGetScalar(prhs[6]);
    dsize = mxGetScalar(prhs[7]);
    firstwavelength = mxGetScalar(prhs[8]);
    lastwavelength = mxGetScalar(prhs[9]);
    gridsize = mxGetScalar(prhs[10]);
    scalefactor = mxGetScalar(prhs[11]);
    numplanets = mxGetScalar(prhs[12]);
    amplitudes = mxGetScalar(prhs[13]);

cor();

    /* end */
Instrument parameters such as mirror positions and sizes must match that of the interferogram generator program. Once again, the change from a high to a low resolution configuration is made by swapping signs of the inside mirror positions. The first four parameters in the cor() function call are the mirror locations, the next four are the aperture areas (diameters can be used if sizes are identical). The wavelength range is specified in the next two parameters as the number of contiguous bands (e.g., 1, 5 meaning 1 through 5). Parameter eleven is the number of points measured from the center of the grid. The next parameter is the grid resolution. If for example, a 200 x 200 mas FOV is implemented in 1 mas increments, values of 100 and 0.001 would be input. The final two parameters are the number of planets (up to six) that are to be solved for in the CLEAN process and the upper limit to the intensities. For normalized interferograms a value of ~30 is appropriate. The code can be run without the Matlab interface if desired. Just compile cor.c by itself.

As discussed in Chapter 6, the maximum correlation method is used elsewhere so it is useful to have a block of code that can be used for comparing the different methods and will be given next. A CLEAN process has not been implemented for reasons discussed in Chapter 6.
Maximum Correlation Method (MCM) Matlab Script

Program name: maximum.m

% Maximum correlation prototype program
% last updated 3/24/04
% compile C program
mex mcm.c

% (mirror locations) p1,p2,p3,p4, (aperture areas) a1,a2,a3,a4, start
% wavelength, end wavelength, grid dimension, grid resolution, number
% of planets, amplitude scale
mcm(-18.0,-9.0,9.0,18.0,3.2,3.2,3.2,3.2,8,12,30,0.01,1,5);

% display correlation map
load cmp0;
contour3(cmp0,200);
view(0,-90);
whitebg([0 0 .3]);
title('MCM Map','FontSize', 18,'color','white');
xlabel('X position (mas)','FontSize',14);
ylabel('Y position (mas)','FontSize',14);
axis([0 62 0 62 16 22 16 22]);
set(gca,'XTick',[0 10 20 30 40 50 60]);
set(gca,'YTick',[0 10 20 30 40 50 60]);
axis square;
grid off;
box on;

% end
Maximum Correlation Method (MCM) C Code

Program name: max.c

MCM algorithm. This program correlates a composite signal from up to 6 planets with model functions for a number of grid positions specified by the square of "griddimension" and for 5 wavelengths and combines the results to form a single image. The FOV is determined by the resolution "gridsize" and "griddimension". The output is written to a numerical text file. Intensities may vary over 2 orders of magnitude. The configuration is a dual Bracewell. High and low resolutions are implemented by appropriate choice of sign for the inner mirror positions. Anti-symmetrical chopping is assumed. Updated 3/20/04.

/* $Revision: 1.0 $ */

#include "mex.h"
#include "stdio.h"
#include "stdlib.h"
#include "math.h"
#include "float.h"
#include "time.h"

#define pi 3.14159265
#define kappa 4.84813689
#define numdatapoints 361
#define numamplitudes 101
#define maxplanets 6
#define planets 2
#define bins 5
#define griddimension 30
#define gridscale 0.01
#define halfrotation 180
#define ci(ii, jj, kk, ll) (((ii) * bins * (2*griddimension+1)*(2*griddimension+1)) \ + ((jj) * (2*griddimension+1)*(2*griddimension+1)) + ((kk) * \ (2*griddimension+1)) + (ll))

int gridsize=griddimension;
int gridpoints=(2*griddimension+1)*(2*griddimension+1);
int datapoints=numdatapoints;
int numplanets=planets, amplitudes=numamplitudes, planet=0;
int firstwavelength=8, lastwavelength=12;
double scale=gridscale;
double refwavelength=8.0;
double apos=-18.0, bpos=-9.0, cpos=9.0, dpos=18.0;
double aamp=3.2, bamp=3.2, camp=3.2, damp=3.2;

double del1(lambda)
int lambda;
{

    /* return((pi/2.0)*(refwavelength/lambda)); */
    return((pi/2.0)*(lambda/lambda));
}

double del2(lambda)
int lambda;
{
    return(-del1(lambda));
}

double e(S1,del1)
int S1;
double del1;
{
    return(S1*pi+del1);
}

double f(S1,del2)
int S1;
double del2;
{
    return(S1*pi+del2);
}
double g(S2, del1)
int S2;
double del1;
{
    return(S2 * pi + del1);
}

double h(S2, del2)
int S2;
double del2;
{
    return(S2 * pi + del2);
}

double t(lambda, theta, S1, S2, del1, del2)
int S1, S2, lambda;
double theta, del1, del2;
{
    double aph = 2 * pi * aps * theta / lambda + e(S1, del1);
    double bph = 2 * pi * bps * theta / lambda + f(S1, del2);
    double cph = 2 * pi * cps * theta / lambda + g(S2, del1);
    double dph = 2 * pi * dps * theta / lambda + h(S2, del2);
    double Realt, Imagt;

    Realt = aamp * cos(aph) + bamp * cos(bph) +
             camp * cos(cph) + damp * cos(dph);
    Imagt = aamp * sin(aph) + bamp * sin(bph)
+camp*\sin(cph)+damp*\sin(dph);

return((\text{Realt}^{2}+\text{Imagt}^{2})/\text{pow}(aamp+bamp+camp+damp,2));

}\)

double T(lambda,theta)
double theta;
int lambda;
{
    return (0.5*(t(lambda,theta,1,0,del1(lambda),0.0)
        + t(lambda,theta,0,1,0.0,del2(lambda))
        - t(lambda,theta,1,0,-del1(lambda),0.0)
        - t(lambda,theta,0,1,0.0,-del2(lambda))));
}

double pattern(alpha,beta,phi,lambda)
int phi, lambda;
double alpha, beta;
{
    return(T(lambda,alpha*\kappa*\cos(\phi*\pi/180.0)
        beta*\kappa*\sin(\phi*\pi/180.0)));
}

void mcm(void) {
    int phi, x, y, itteration, intin, lambda;
double sx1, sx2, peak;
double I[bins][2*griddimension+1][2*griddimension+1];
double sum[2*griddimension+1][2*griddimension+1];
double compsig[bins][numdatapoints], xp[planets], yp[planets];
double cav[bins][2*griddimension+1][2*griddimension+1];
double Rc[bins][numdatapoints], Ro[bins][numdatapoints];
  //double c[numdatapoints][bins][2*griddimension+1][2*griddimension+1];
double* c = malloc(numdatapoints * bins * (2*griddimension+1) * 
  (2*griddimension+1)* sizeof(double)); // free(c);

float floatin;
static char out[1];
char *strcpy();

FILE *ifp, *ofp, *afp, *fopen();

printf(" Calculating planets positions ...
");
  /* clear output files */
  ofp = fopen("cmp0","w");
  fclose(ofp);
  ofp = fopen("cmp1","w");
  fclose(ofp);
  ofp = fopen("cmp2","w");
  fclose(ofp);
  ofp = fopen("cmp3","w");
  fclose(ofp);
  ofp = fopen("cmp4","w");
  fclose(ofp);
  ofp = fopen("cmp5","w");
  fclose(ofp);
  ofp = fopen("stats","w");
  fclose(ofp);
fclose(ofp);
ofp = fopen("planet_id","w");
fclose(ofp);
ofp = fopen("locations","w");
fclose(ofp);

/* get composite signals */
ifp = fopen("../generator/comp_signal_8","r");
for (phi = -180; phi <= 180; phi++) {
    fscanf(ifp," %d %f\n",&intin,&floatin);
    compsig[0][phi+180]=floatin;
}
fclose(ifp);

ifp = fopen("../generator/comp_signal_9","r");
for (phi = -180; phi <= 180; phi++) {
    fscanf(ifp," %d %f\n",&intin,&floatin);
    compsig[1][phi+180]=floatin;
}
fclose(ifp);
ifp = fopen("../generator/comp_signal_10","r");
    for (phi = -180; phi <= 180; phi++) {
        fscanf(ifp," %d %f
",&intin,&floatin);
        compsig[2][phi+180]=floatin;
    }
    fclose(ifp);

ifp = fopen("../generator/comp_signal_11","r");
    for (phi = -180; phi <= 180; phi++) {
        fscanf(ifp," %d %f
",&intin,&floatin);
        compsig[3][phi+180]=floatin;
    }
    fclose(ifp);

ifp = fopen("../generator/comp_signal_12","r");
    for (phi = -180; phi <= 180; phi++) {
        fscanf(ifp," %d %f
",&intin,&floatin);
        compsig[4][phi+180]=floatin;
    }
    fclose(ifp);

/*******************************************************************************/
/* correlate composite signals with a model function at each position */

printf("\n Creating correlation maps with the MCM algorithm... \n \n Planet %d \n", planet);

findlocations:

    for (lambda=firstwavelength; lambda<=lastwavelength; lambda++) {
        for (phi=-halfrotation; phi<=lastwavelength; phi++) {
            for (Y = -gridsize; y<=gridsize; y++) {
                for (X = gridsize; x>=-gridsize; x--) {
                    I[lambda-firstwavelength]
                    [gridsize+y][gridsize-x]=1.0;
                    Rc[lambda-firstwavelength]
                    [phi+halfrotation]=0.0;
                    Ro[lambda-firstwavelength]
                    [phi+halfrotation]=0.0;
                    sum[gridsize+y][gridsize-x]=0.0;
                }
            }
        } for (itteration=0; itteration<=1; itteration++) {
...
for (lambda=firstwavelength; lambda<=lastwavelength; lambda++) {
    for (phi=-halfrotation; phi<=lastwavelength; phi++) {
        for (Y = -gridsize; y<=gridsize; y++) {
            for (X = gridsize; x>=-gridsize; x--) {
                cav[lambda-firstwavelength]
                    [gridsize+y][gridsize-x]=0.0;
                /* c[phi+halfrotation][lambda-firstwavelength][gridsize+y]
                    [gridsize-x]=0.0; */
                c[ci(phi+halfrotation,
                    lambda-firstwavelength,
                    gridsize+y, gridsize-x)]=0.0;
                Rc[lambda-firstwavelength]
                    [phi+halfrotation]
                    +=I[lambda-firstwavelength]
                    [gridsize+y][gridsize-x]
                    * pattern(x*scale,y*scale,phi,lambda);
            }
        }
    }
}
Ro[lambda-firstwavelength]
$$[\phi + \text{halfrotation}] = \text{compsig} [\lambda - \text{firstwavelength} ][\phi + \text{halfrotation}] ;$$

} 

for (\lambda = \text{firstwavelength} ; \lambda <= \text{lastwavelength} ; \lambda++) {
for (\phi = -\text{halfrotation} ; \phi <= \text{lastwavelength} ; \phi++) {
for (Y = -\text{gridsize} ; y <= \text{gridsize} ; y++) {
for (X = \text{gridsize} ; x >= -\text{gridsize} ; x--) {
  /* c[\phi + \text{halfrotation}][\lambda - \text{firstwavelength}][\text{gridsize} + y][\text{gridsize} - x] = (\text{Ro}[\lambda - \text{firstwavelength}][\phi + \text{halfrotation}] - \text{Rc}[\lambda - \text{firstwavelength}][\phi + \text{halfrotation}]) \times \text{pattern}(x \times \text{scale}, y \times \text{scale}, \phi, \lambda) ; */
  c[ci(\phi + \text{halfrotation}, \lambda - \text{firstwavelength}, \text{gridsize} + y, \text{gridsize} - x)] = (\text{Ro}[\lambda - \text{firstwavelength}][\phi + \text{halfrotation}] - \text{Rc}[\lambda - \text{firstwavelength}][\phi + \text{halfrotation}]) \times \text{pattern}(x \times \text{scale}, y \times \text{scale}, \phi, \lambda) ; */
}
for (lambda=firstwavelength; lambda<=lastwavelength; lambda++) {
    for (y = -gridsize; y<=gridsize; y++) {
        for (x = gridsize; x>=-gridsize; x--) {
            for (phi=-halfrotation; phi<=lastwavelength; phi++) {
                cav[lambda-firstwavelength]
                [gridsize+y][gridsize-x]
                +=1.0/361.0*c[ci(phi+halfrotation,
                lambda-firstwavelength, gridsize+y,
                gridsize-x)];
            }
        }
    }
}
for (lambda=firstwavelength; lambda<=lastwavelength; lambda++) {
    for (Y = -gridsize; y<=gridsize; y++) {
        for (X = gridsize; x>=-gridsize; x--) {
            I[lambda-firstwavelength][gridsize+y][gridsize-x]+=cav[lambda-firstwavelength][gridsize+y][gridsize-x];
        }
    }
}

/* sum image over all wavelengths */
for (lambda=firstwavelength; lambda<=lastwavelength; lambda++) {
    for (Y = -gridsize; y<=gridsize; y++) {
        for (X = gridsize; x>=-gridsize; x--) {
            sum[gridsize+y][gridsize-x]+=l[lambda
/* find peak position for each planet */
peak=0.0, xp[planet]=0.0, yp[planet]=0.0, sx1=0.0, sx2=0.0;
for (Y = -gridsize; y<=gridsize; y++) {
    for (X = gridsize; x>=-gridsize; x--) {
        if (sum[gridsize+y][gridsize-x] >= peak) {
            peak=1.0*sum[gridsize+y][gridsize-x];
            xp[planet]=x*-scale;
            yp[planet]=y*-scale;
        }
    }
}

printf(" Location is x = %.2f, y = %.2f \\
        \n",xp[planet],yp[planet]);
printf(" PEAK is %f\n",peak);

/* write output files */
afp = fopen("stats","a");
    fprintf(afp," Planet %d \n",planet);
    fprintf(afp," Location is x = %.3f , y = %.3f \n",xp[planet],yp[planet]);
    fprintf(afp," PEAK is %f \n",peak);
fclose(afp);

afp = fopen("locations","a");
    fprintf(afp," %.3f %.3f \n",xp[planet],yp[planet]);
fclose(afp);

afp = fopen("out","a");
    for (Y = -gridsize; y<=gridsize; y++) {
        for (X = gridsize; x>=-gridsize; x--) {
            fprintf(afp,"%f ",sum[gridsize+y][gridsize-x]);
        }
        fprintf(afp,"\n");
    }
fclose(afp);

/*******************************************************************************/
printf(" ... done.\n\n");
void mexFunction(int nlhs, mxArray *plhs[], int nrhs, const mxArray *prhs[])
{
    apos = mxGetScalar(prhs[0]);
    bpos = mxGetScalar(prhs[1]);
    cpos = mxGetScalar(prhs[2]);
    dpos = mxGetScalar(prhs[3]);
    aamp = mxGetScalar(prhs[4]);
    bamp = mxGetScalar(prhs[5]);
    camp = mxGetScalar(prhs[6]);
    damp = mxGetScalar(prhs[7]);
    firstwavelength = mxGetScalar(prhs[8]);
    lastwavelength = mxGetScalar(prhs[9]);
    gridsize = mxGetScalar(prhs[10]);
    scale = mxGetScalar(prhs[11]);
    numplanets = mxGetScalar(prhs[12]);
    amplitudes = mxGetScalar(prhs[13]);

    mcmO;
}

/* end */
The front end matches for the most part that of the standard correlation algorithm and the setup is the same. The only difference being the addition of one more parameter in the call to mcm() that specifies the number of iterations which should produce convergence by ~40. Only one correlation map will be produced by the program. The code does not include a CLEAN algorithm because substantial work is required to determine a scheme for scaling the peaks in order to systematically subtract each planet due to the nonlinearity of peak amplitudes between wavelengths and across the amplitude scale, and for reasons explained in the text, the MCM algorithm is not useful for providing ICs to the optimizer. JPL personnel are also working on the MCM method and therefore a duplication of effort is not indicated at this time.
APPENDIX D

NONLINEAR OPTIMIZATION PROGRAMS

The following Matlab and Mathematica program modules provide only a general framework and will have to be altered to suite requirements of the experiment at hand. Written for a three planet system, they can be expanded to include any number of planets and wavelengths, etc., by adding additional terms to the interferogram function and/or deleting or adding lines in each block as desired by following the pattern presented. First derivatives have been included with the Matlab code and can be adapted for use in Mathematica. To use explicit derivatives in lieu of finite differences, the appropriate software switch must be flipped by setting the Jacobian parameter to on. Medium scale problems generally produce more accurate results using the Jacobian. Turning Levenberg-Marquardt off in Matlab selects the Gauss-Newton method of solution. Specification of QuasiNewton or Newton as the method in Mathematica gives about the same results. One or the other method may converge better depending on the topography. The user supplied difference function is specific to the straw man configuration and must be derived separately for different instruments. Filenames for the Matlab code are referenced as such in the programs. The command "nloptimize" will invoke execution in Matlab.
Program name: nloptimize.m

% This program does a least-squares minimized fit of model parameters to
% spectral interferograms generated with the JPL code. Three planets are
% assumed but the program can be expanded to any number of planets.
% Updated 6/4/04.

global signal
global band
global lambda

% read the time series
signals

% provide initial conditions [A1 X1 Y1 A2 X2 Y2]. Positions are in arcsec.
p = [ Initial values separated by spaces go here. ]

% output format
format short
% format short e

% set optimization parameters
options = optimset('TolX',1e-4,'TolFun',1e-4,'MaxFunEvals',1e3, ...
'LineSearchType','quadcubic','Jacobian','off', 'MaxIter',1e3, ...
'LevenbergMarquardt','on','LargeScale','off','DerivativeCheck','off');

% do LS fitting

lambda=7.44;
band=1;
[P,resnorm]=lsqnonlin(@planets,p,[],[],options)
q(5)=P(1); r(5)=P(2); s(5)=P(3); t(5)=P(4); u(5)=P(5); v(5)=P(6); x(5)=P(7);
y(5)=P(8); z(5)=P(9);

lambda=8.5;
band=2;
[P,resnorm]=lsqnonlin(@planets,p,[],[],options)
q(4)=P(1); r(4)=P(2); s(4)=P(3); t(4)=P(4); u(4)=P(5); v(4)=P(6); x(4)=P(7);
y(4)=P(8); z(4)=P(9);
\[
\lambda = 9.92; \quad \text{band} = 3;
\]
\[
[P, \text{resnorm}] = \text{lsqnonlin}(@\text{planets}, p, [], [], \text{options})
\]
\[
q(3) = P(1); \quad r(3) = P(2); \quad s(3) = P(3); \quad t(3) = P(4); \quad u(3) = P(5); \quad v(3) = P(6); \quad x(3) = P(7);
\]
\[
y(3) = P(8); \quad z(3) = P(9);
\]

\[
\lambda = 11.9; \quad \text{band} = 4;
\]
\[
[P, \text{resnorm}] = \text{lsqnonlin}(@\text{planets}, p, [], [], \text{options})
\]
\[
q(2) = P(1); \quad r(2) = P(2); \quad s(2) = P(3); \quad t(2) = P(4); \quad u(2) = P(5); \quad v(2) = P(6); \quad x(2) = P(7);
\]
\[
y(2) = P(8); \quad z(2) = P(9);
\]

\[
\lambda = 14.9; \quad \text{band} = 5;
\]
\[
[P, \text{resnorm}] = \text{lsqnonlin}(@\text{planets}, p, [], [], \text{options})
\]
\[
q(1) = P(1); \quad r(1) = P(2); \quad s(1) = P(3); \quad t(1) = P(4); \quad u(1) = P(5); \quad v(1) = P(6); \quad x(1) = P(7);
\]
\[
y(1) = P(8); \quad z(1) = P(9);
\]

% Approximation of the composite signal strength
fprintf(' Output parameters: 

');
allbands = 0;
for band = 1:5
    peak = max(bigsignal(:, band));
    fprintf(' Band %d photon rate = %.3f peak ...
photons/sec 
', band, peak / 50000);
    allbands = allbands + peak;
end
fprintf(' All band peak photon rate = %.3f photons/sec , allbands/50000);
fprintf(' A1 = %.2f XEF, X1 = %.3f arcsec, Y1 = %.3f arcsec 
 ...
A2 = %.2f XEF, X2 = %.3f arcsec, Y2 = %.3f arcsec 
 A3 = %.2f XEF, ...
X3 = %.3f arcsec, Y3 = %.3f arcsec 
 ', mean(q), mean(r), mean(s), ...
mean(t), mean(u), mean(v), mean(x), mean(y), mean(z));

% end

************************************************************************************
Program name: signals.m
% This file provides the raw data (interferograms) in column format. It also
% includes scripts to normalize them, do a 5 point boxcar average, and
% sum overlapped redundant halves. The variable “bigsignal” contains the
% interferograms in 5 columns of 180 elements (rows).

global signal

bigsignal = [ Paste one 360 X 5 raw data matrix here. Each column
contains one spectral interferogram (one for each of 5 bands). Modify the
code to include additional bands or change the number of datapoints
averaged to suit. Columns are delimited by a tab or spaces. ];

for j=1:5
expanded(1,j)=bigsignal(357,j);
expanded(2,j)=bigsignal(358,j);
expanded(3,j)=bigsignal(359,j);
expanded(4,j)=bigsignal(360,j);

for i=5:364
expanded(i,j)=bigsignal(i-4,j);
end
expanded(365,j)=bigsignal(1,j);
expanded(366,j)=bigsignal(2,j);
expanded(367,j)=bigsignal(3,j);
expanded(368,j)=bigsignal(4,j);
end

k=0:8;
for j=1:5
for i=1:360
smoothed(i,j)=sum(expanded(i+k,j))/9;
end
end

for j=1:5
peak = max(abs(bigsignal(:,j)));
for i=1:360
signal(i,j)=bigsignal(i,j)/peak;
end
end
Program name: planets.m

% The functions in this file are called when doing the least squares
% minimization by lsqnonlin.

function [F,dF] = planets(P)

global signal
global band
global lambda

for phi=1:180
    pk(phi) =pattern(P(1),P(2),P(3),phi+1,lambda) + pattern(P(4),P(5),P(6), ...
    phi+1,lambda) + pattern(P(7),P(8),P(9),phi+1,lambda);
end

peak=max(abs(pk));

for phi=1:180
    FM(phi) = 1./peak.*(pattern(P(1),P(2),P(3),phi+1,lambda) + pattern(P(4), ... 
    P(5),P(6),phi+1,lambda) + pattern(P(7),P(8),P(9),phi+1,lambda));
end

FS(phi) = 1./2.*(signal(phi,band) - signal(phi+180,band));

end

F=FS' - FM';

for phi=1:180
    dF(phi,1) = -(dapattern(P(1),P(2),P(3),phi+1,lambda))./peak;
    dF(phi,2) = -(dxpattern(P(1),P(2),P(3),phi+1,lambda))./peak;
    dF(phi,3) = -(dypattern(P(1),P(2),P(3),phi+1,lambda))./peak;
    dF(phi,4) = -(dapattern(P(4),P(5),P(6),phi+1,lambda))./peak;
    dF(phi,5) = -(dxpattern(P(4),P(5),P(6),phi+1,lambda))./peak;
    dF(phi,6) = -(dypattern(P(4),P(5),P(6),phi+1,lambda))./peak;
end
Program name: dapattern.m

% Derivative of pattern function with respect to amplitude

function da = dapattern(A,X,Y,phi,lambda)

k = 4.84813689;

da = -(1./2).*((cos((pi.*(lambda - 36.*k.*Y.*cos((phi.*pi)/180) + X. ...
*sin((phi.*pi)/180)))./(2.*lambda)) - cos((pi.*(lambda + 36.*k. ...
*(Y.*cos((phi.*pi)/180) + X.*sin((phi.*pi)/180))))./(2.*lambda)))).* ...
* (27.*k.*pi.*Y.*cos((phi.*pi)/180) + X.*sin((phi.*pi)/180)))./lambda).^2;

% end

***********************************************************************************

Program name: dxpattern.m

% Derivative of pattern function with respect to X position

function dx = dxpattern(A,X,Y,phi,lambda)

k = 4.84813689;

dX = -(1./lambda).*((27.*A.*k.*pi.*cos((27.*k.*pi.*Y.*cos((phi.*pi)/180) ...
+ X.*sin((phi.*pi)/180)))./lambda).*((cos((pi.*(lambda-36.*k.*Y. ...
*cos((phi.*pi)/180) + X.*sin((phi.*pi)/180)))./(2.*lambda)) - cos((pi.* ...
*(lambda + 36.*k.*Y.*cos((phi.*pi)/180) + X.*sin((phi.*pi)/180))))./(2.*lambda)))).* ...
* (27.*k.*pi.*Y.*cos((phi.*pi)/180) + X.*sin((phi.*pi)/180)))./lambda).^2.* ...
* (18.*k.*pi.*sin((phi.*pi)/180).*(lambda - 36.*k.*Y.*cos((phi.*pi)/180) ...
+ X.*sin((phi.*pi)/180)))./(2.*lambda))./lambda -18.*k.*pi.* ...
* sin((phi.*pi)/180).*(lambda - 36.*k.*Y.*cos((phi.*pi)/180) ...
+ X.*sin((phi.*pi)/180)))./(2.*lambda))/lambda; 

% end

***********************************************************************************

Program name: dypattern.m
% Derivative of pattern function with respect to Y position

function dy = dypattern(A,X,Y,phi,lambda)

k = 4.84813689;

dy = -((1./lambda).*(27.*A.*k.*pi.*cos((phi.*pi)./180).*cos((27.*k.*pi.*
(Y.*cos((phi.*pi)./180) + X.*sin((phi.*pi)./180)))./lambda).*
(cos((pi.*(lambda - 36.*k.*(Y.*cos((phi.*pi)./180) + X.*sin((phi.*pi)./180))).
/(2.*lambda)) - cos((pi.*(lambda + 36.*k.*(Y.*cos((phi.*pi)./180) + X.*
*sin((phi.*pi)./180)))./(2.*lambda)))).*sin((27.*k.*pi.*
(Y.*cos((phi.*pi)./180) + X.*sin((phi.*pi)./180)))./lambda).^2
-((18.*k.*pi.*cos((phi.*pi)./180).*sin((pi.*(lambda - 36.*k.*(Y.*
*cos((phi.*pi)./180) + X.*sin((phi.*pi)./180)))./(2.*lambda)))).
/lambda - (-18.*k.*pi.*cos((phi.*pi)./180).*sin((pi.*(lambda + 36.*k.*
Y.*cos((phi.*pi)./180) + X.*sin((phi.*pi)./180)))./(2.*lambda)))).
/lambda - (1./2).*A.*sin((27.*k.*pi.*
(Y.*cos((phi.*pi)./180) + X.*sin((phi.*pi)./180)))./lambda).^2

% end

*****************************************************************************

Program name: LMoptimization.nb

Comment: This code is compatible with Mathematica 5.0. A numerical
text file named signals.dat in the working directory is assumed. Five
columns tab or space delimited containing 5 spectral interferograms of
360 points (rows) provide the raw data (interferograms).

fp=OpenRead["c:/work/signals.dat"];
bigsignal=Table[Read[fp,{Number,Number, Number, Number, Number}],
{i,1,360}];
Close[fp];

Comment: create circular interferograms for boxcar averaging
pre=Take[bigsignal,\{359,360\}];
post=Take[bigsignal,\{1,2\}];
expanded=Join[pre,bigsignal,post];

s1=Table[expanded[[i,1]],{i,1,364}];
s2=Table[expanded[[i,2]],{i,1,364}];
s3=Table[expanded[[i,3]],{i,1,364}];
s4=Table[expanded[[i,4]],{i,1,364}];
s5=Table[expanded[[i,5]],{i,1,364}];

Comment: do boxcar average – create smoothed interferograms
sms1=Table[Mean[{s1[[i]],s1[[i+1]],s1[[i+2]],s1[[i+3]],s1[[i+4]]}],{i,1,360}];
sms2=Table[Mean[{s2[[i]],s2[[i+1]],s2[[i+2]],s2[[i+3]],s2[[i+4]]}],{i,1,360}];
sms3=Table[Mean[{s3[[i]],s3[[i+1]],s3[[i+2]],s3[[i+3]],s3[[i+4]]}],{i,1,360}];
sms4=Table[Mean[{s4[[i]],s4[[i+1]],s4[[i+2]],s4[[i+3]],s4[[i+4]]}],{i,1,360}];
sms5=Table[Mean[{s5[[i]],s5[[i+1]],s5[[i+2]],s5[[i+3]],s5[[i+4]]}],{i,1,360}];

Comment: normalize interferograms
normsig1=sms1/Max[Abs[sms1]];
normsig2=sms2/Max[Abs[sms2]];
normsig3=sms3/Max[Abs[sms3]];
normsig4=sms4/Max[Abs[sms4]];
normsig5=sms5/Max[Abs[sms5]];

Comment: overlap and sum opposite sides of the interferograms
signal[1]:=Take[normsig1,180]-Take[normsig1,-180];
signal[2]:=Take[normsig2,180]-Take[normsig2,-180];
signal[3]:=Take[normsig3,180]-Take[normsig3,-180];
signal[4]:=Take[normsig4,180]-Take[normsig4,-180];
signal[5]:=Take[normsig5,180]-Take[normsig5,-180];

Comment: specify wavelengths for 5 bands
W[1]:=7.44;
W[2]:=8.50;
W[3]:=9.92;
W[4]:=11.9;
W[5]:=14.9;

chopmode:=-1;
k:=4.84813689;

Comment: compute stationary transmission function to match JPL code
Trans[lambda_,theta_]:=1/2*(1+chopmode+Cos[(lambda-36*theta)*Pi/(2*lambda)]+chopmode*Cos[(lambda+36*theta)*Pi/(2*lambda)])*Sin[27*theta*Pi/lambda]^2;

Comment: compute rotational pattern to match JPL code
Pat[Ampl_, X_, Y_, phi_, lambda_] :=
Ampl*Trans[lambda, k*{X*Sin[phi*Pi/180]+Y*Cos[phi*Pi/180]}];

Comment: generate model functions
model[vl_, v2_, v3_, v4_, v5_, v6_, lambda_] := Table[Pat[vl, v2, v3, phi, lambda] + Pat[v4, v5, v6, phi, lambda], {phi, -2, 181}];
m1[vl_, v2_, v3_, v4_, v5_, v6_] := model[vl, v2, v3, v4, v5, v6, W[1]];
m3[vl_, v2_, v3_, v4_, v5_, v6_] := model[vl, v2, v3, v4, v5, v6, W[3]];
m5[vl_, v2_, v3_, v4_, v5_, v6_] := model[vl, v2, v3, v4, v5, v6, W[5]];

Comment: smooth the model functions to match smoothed interferograms
smm1[vl_, v2_, v3_, v4_, v5_, v6_] := Table[Mean[{m1[vl, v2, v3, v4, v5, v6][[i]], m1[vl, v2, v3, v4, v5, v6][[i + 1]], m1[vl, v2, v3, v4, v5, v6][[i + 2]], m1[vl, v2, v3, v4, v5, v6][[i + 3]], m1[vl, v2, v3, v4, v5, v6][[i + 4]]}, {i, 1, 180}];
smm2[vl_, v2_, v3_, v4_, v5_, v6_] := Table[Mean[{m2[vl, v2, v3, v4, v5, v6][[i]], m2[vl, v2, v3, v4, v5, v6][[i + 1]], m2[vl, v2, v3, v4, v5, v6][[i + 2]], m2[vl, v2, v3, v4, v5, v6][[i + 3]], m2[vl, v2, v3, v4, v5, v6][[i + 4]]}, {i, 1, 180}];
smm3[vl_, v2_, v3_, v4_, v5_, v6_] := Table[Mean[{m3[vl, v2, v3, v4, v5, v6][[i]], m3[vl, v2, v3, v4, v5, v6][[i + 1]], m3[vl, v2, v3, v4, v5, v6][[i + 2]], m3[vl, v2, v3, v4, v5, v6][[i + 3]], m3[vl, v2, v3, v4, v5, v6][[i + 4]]}, {i, 1, 180}];
smm5[vl_, v2_, v3_, v4_, v5_, v6_] := Table[Mean[{m5[vl, v2, v3, v4, v5, v6][[i]], m5[vl, v2, v3, v4, v5, v6][[i + 1]], m5[vl, v2, v3, v4, v5, v6][[i + 2]], m5[vl, v2, v3, v4, v5, v6][[i + 3]], m5[vl, v2, v3, v4, v5, v6][[i + 4]]}, {i, 1, 180}];

Comment: normalize model functions to match normalized interferograms
Comment: the error function used to do the LS minimization
error[a_,x_,y_] := Sum[diffsqr[a[1], x[1], y[1], a[2], x[2], y[2], phi, band], {phi, 1, 180}, {band, 1, 5}];

Comment: optimizer setup parameters
SetOptions[FindMinimum, MaxIterations -> 1000, Method -> Levenberg Marquardt];

Comment: do LS minimization and print results to screen – replace parm’s by initial values
FindMinimum[error[a, x, y], {a[1], parm}, {x[1], parm}, {y[1], parm}, {a[2], parm}, {x[2], parm}, {y[2], parm}]
REFERENCES


Woolf, N.J. Science Purposes, Technology Disposes or Why We

