DESIGN, FABRICATION AND CONTROL OF HYBRID THERMAL/PIEZOELECTRIC MEMS ARRAY

By

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For my family, wife and son
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This dissertation describes the development of a hybrid actuation solution, which utilizes a micro-machined actuator array to provide switching of mechanical motion of a larger meso-scale piezo-electric actuator. The hybrid actuator approach alleviates the main shortcoming of the conventional micro-electro-mechanical systems (MEMS) – the limited stroke and force characteristics, and combines the high-density and batch processing of the MEMS components with the high force/stroke and efficiency of the macro-scale actuation. One motivating application of this technology is the development of a portable tactile display, where discrete mechanical actuators apply vibratory excitation on the skin. The tactile display consists of three major parts: (i) MEMS 4 x 5 actuator array of individual vibrating pixels, (ii) Macro-scale piezoelectric-actuator and mechanical assembly and (iii) Control electronics module. The MEMS chip is an array of micro folded beam thermal actuators used to redirect the displacement of the main piezoelectric actuator. The electronics module includes a microcontroller and specialized drivers to control the MEMS array and the piezoelectric actuator simultaneously, generating complex pixel sequences.

Optimization of the MEMS actuator performance required in-depth understanding of the underlying actuation principles, leading to the development of a set of analytical tools for steady state and transient analysis of the microactuators. Special attention was paid to improving the commonly used linear mechanical models, which do not produce accurate
results for large actuator displacements. The developed analytical models were compared against finite element simulations and showed very good agreement.

Another major contribution of this dissertation is the integration of micro fabrication, mechanical design and advanced embedded system design into a single device. This integration allowed significant decrease in size compared to the existing tactile displays. The developed prototype is completely self-contained, powered by one watch battery and approximately the size of a wristwatch. Despite its size, the device is ‘intelligent’ due to the use of onboard high performance microcontroller. The achieved reduction of size and power consumption is a very important step toward mainstream adoption of devices for tactile communication.
1. INTRODUCTION

Since their initial inception more than thirty years ago, micro-mechanical actuators have become the hallmark of micro-electro mechanical systems (MEMS). Initially, these were simple electrostatically driven cantilevers, fabricated using semiconductor processing [1]. With the maturation of this technology and the emergence of high aspect ratio micro-machining techniques such as LIGA [2], HEXSIL [3], soft-LIGA [4], and DRIE [5], these devices grew closer and closer to their macroscopic counterparts by extending in the third dimension. During this development, virtually all known actuation mechanisms have been explored in the construction of MEMS actuators. Most commonly, these devices utilized electrostatic [6], piezo-electric [7], electromagnetic [8], electro-thermal [9, 10], thermo-pneumatic [11], electrochemical [12], electro- and magnetostrictive [13], shape memory [14], and mass transport [15] effects. Among these, the electrostatic, piezo-electric, and electro-thermal are the most successful and widespread actuation modalities in use due to their compatibility with electronic circuit fabrication. Despite this significant progress, the energy output of MEMS-based micro-actuators remains quite small. For example, a linear electrostatic actuator with a “large” displacement of more than 100 μm generates up to 10 μN of force [6], amounting to 1 nano-Joule of energy output. For comparison, this is the energy required to lift a 4-mg ant by 1/1000 of an inch in the Earth’s gravitational field [16]. Thus, most successful applications of micro-actuators are in systems requiring minimal force, such as light steering by electrostatically actuated micro-mirrors [17], or applications requiring larger forces but small displacements, such as piezo-electric actuators [7].
1.1 Hybrid Actuation for Tactile Communications

Vibro-tactile displays generate a sensation of motion on the skin by a limited number of mechanical actuators applying light oscillatory pressure via an array of discrete vibrating points (Tactile Actuator or “tactors”). If the frequency and duration of each vibration is within a subject-dependent range, a feeling of continuous motion can be produced by a phenomenon known as tactile illusion [18-20]. This illusion produces the sense of a line being drawn on the skin, similar to graphesthesia (i.e., the tactual ability to recognize writing on the skin). The most serious limitations of the existing tactile display designs are the large size and high power consumption. Employing MEMS technology reduces the size, while decreasing the power consumption and cost, which allows tactile displays to be embedded in a wide variety of portable devices such as watches, pagers etc. Preliminary studies [18] of the tactile illusion effect using large solenoid-driven displays have established the physiological parameters required by such an actuator array and are listed in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of vibrations</td>
<td>Total number of protrusions of each solenoid while activated = 5</td>
</tr>
<tr>
<td>Required force</td>
<td>More than 10 mN</td>
</tr>
<tr>
<td>Required displacement</td>
<td>More than 20 μm</td>
</tr>
<tr>
<td>Solenoid on-time</td>
<td>Time of core protrusion for each vibration of each solenoid = 10 ms</td>
</tr>
<tr>
<td>Solenoid off-time</td>
<td>Time of no active core protrusion for each vibration cycle = 10 ms</td>
</tr>
<tr>
<td>Solenoid delay</td>
<td>Time between end of last solenoid vibration and onset of next solenoid vibration = 5 ms</td>
</tr>
</tbody>
</table>
The parameters listed in Table 1 were selected as target values in the design of the micro-
mechanical switch array. The required force and stroke of the actuators are significantly
larger than the typical values achieved by MEMS devices. To meet these requirements, a
hybrid solution, consisting of a meso-scale vibrating plate driven by a set of piezo-
actuators and mounted under an array of micromechanical switches (clutches) is
proposed.

![Micro switch open](image1)

Micro switch open  Micro switch closed

![Piezo bender not actuated](image2)

Piezo bender not actuated

No transmission of motion  The plate pushes the pixel up

Nickel  Silicon  Vibrating Plate

Figure 1.1. Principle of operation of the switching mechanism

The micro switches re-direct the vibrations of the plate to the individual protruding pins
(pixels). A pair of micro actuators, displacing laterally a lock mechanism as illustrated in
Figure 1.1, provides the engagement of each pixel. The array is mounted on top of the
vibrating plate assembly as shown in Figure 1.2. In order to provide out-of-plane
protrusion, the actuators need a second layer of metal rising above the surface of the
actuator (Figure 1.1).
Different actuator technologies were evaluated and folded beam actuators were found best suited for the micro clutch array. Their major advantages are the ease of fabrication, very rugged design and high stroke. There are two basic types of thermal actuators: thermal bimorphs, utilizing materials with two different thermal expansion coefficient are used, and homogeneous actuators, in which a temperature difference is generated between the narrower “hot” and the wider “cold” arm. In both devices, a bending moment is created in the two beams, and the two-arm structure deflects toward the beam with the smaller expansion. The chosen folded beam actuator (Figure 1.3) consists of two arms with different cross sections. When current is passed through the arms, a temperature difference is established between the narrower “hot” and the wider “cold” arm. The resulting thermal strain is responsible for the lateral motion.
The hybrid actuation system has two distinct components – the macro-scale piezoelectric actuator assembly and the MEMS switching array, specifically developed for this purpose by the author. The focus of this work is the design of the MEMS switching array, since single high quality piezoactuators are readily available. The next chapter is an overview of the basic building block of the MEMS array – the folded beam thermal actuator.

1.2. Overview Of Folded Beam Thermal Actuators

The folded beam thermal actuator (Figure 1.3) is a single material structure, which deflects as a result of unequal thermal expansion of its arms and was first introduced by Guckel et al [21]. The actuator has several functional parts: hot arm, cold arm, flexure and anchors. The structural material is conductive and the actuator is heated through Joule heating by applying voltage difference to the anchors. The anchors are attached to the underlying substrate while the rest of the actuator is free to move. When current is flowing through the device, the hot arm heats to a higher temperature because of its smaller cross-section resulting in higher resistance and Joule heating \( P = I^2 R \). The net effect of the actuator geometry is an induced temperature difference between the actuator...
arms. Since the hot arm is at higher temperature and expands more, the tip deflects towards the cold arm (Figure 1.3). The cold arm and the flexure of the actuator have two functions:

(i) To provide the return path for the electrical current

(ii) To amplify mechanically the net expansion of the hot arm, which is on the order of 0.1\textmu m

The net expansion of the hot arm is proportional to the temperature difference between the hot arm and the cold arm and is a function of the driving current and the geometry. The mechanical amplification is roughly equal to the ratio of the length of the actuator and the gap between the arms: \( L_a / \delta \) [22]. The tip displacement is linearly proportional to the dissipated power and is limited by the plastic deformation of the hot arm. The most serious limitations of the folded beam thermal actuator are the high power consumption and the low conversion efficiency. In order to keep the actuator at a prescribed position, a constant current has to be supplied to the actuator and all the input power is dissipated in the form of heat. Three heat transfer modes are at play: conduction, convection and radiation. At reasonable temperatures, for example up to the melting point of the material, the heat conduction dominates. There are two possible heat conduction paths: lateral conduction through the material of the beams to the device anchors and conduction through the air film directly to the substrate. The heat loss through the air film becomes significant when the actuator is suspended close to the surface. At high temperature the hot arm starts to glow emitting in the red/infrared. Even though the glow could be observed with a naked eye, it is estimated that less than 1\% of the energy is lost through
radiation [23]. The convective heat transfer has also been shown to be negligible for hot arms on the order of few microns [22].

Inducing larger temperature difference with the same dissipated power increases the efficiency of the thermal actuator. This has been demonstrated by etching a trench under the hot arm [24] thereby decreasing the heat flux to the substrate. Another approach is to widen the cold arm to enhance it’s cooling. If the cold arm is much wider than the hot arm, less heat is generated in it. In addition to that, the larger width of the cold arm decreases the thermal resistances for conduction through the air film and lateral conduction to the anchors, which helps maintain lower cold arm temperature.

After the excitation current is stopped, the actuator cools down uniformly to the substrate temperature and returns elastically to its original position. This operational mode is called forward deflection. The maximum obtainable tip deflection in this regime is limited by the plastic deformation of the hot arm, which occurs beyond a certain drive level. In the case of short overdriving, the actuator starts to bend backwards. During the overdriving phase, the hot arm is under compression ‘pushing’ the cold arm and the flexure, and the plastic deformation decreases its overall length. Upon cooling, the shortened hot arm bends the actuator past its starting position [25]. In this configuration, the actuator can deliver static force and operate in the forward direction from the new initial position (Figure 1.4).
The permanent deformation during overdriving has two possible reasons – exceeding the yield strength and electromigration of the hot arm material. Electromigration becomes important at very high current levels, rarely encountered during steady state operation. However, this failure mode may be significant if high pulsed currents are present, for example if pulse width modulation with small fill ratio is used. The high electrical current density removes material from certain locations along the crystal boundaries and decreases the cross section of the arm [26]. The thinning causes increase in the local electrical resistance, which in turn generates more heat and elevates the local temperature. As a result an initially thin segment will heat up and will have increased local current density because of the smaller cross-section. Both factors reduce the electromigration threshold and above certain current the system becomes unstable resulting in catastrophic failure (hot arm burns). Due to the unstable nature of the electromigration, it cannot be used for predictable backward deflection. Exceeding the yield strength on the other hand, is more controllable and has been successfully used for specific applications such as bringing separated parts in contact. It has been shown that plastic creep in polysilicon actuators occurs at the places of maximum temperature, not at the place of maximum stress in the hot arm [27]. This effect is a manifestation of the fact
that the yield strength is a strong function of the temperature, and local heating above the brittle-to-ductile transition (approximately 660°C) reduces the yield strength and causes plastic deformation. This makes prediction of the maximum tip displacement quite complicated, since both the stress and temperature profile must be accurately predicted. Additional problem is the need for reliable data for the yield strength as a function of temperature, which is not readily available.

1.3. Microactuator Fabrication Techniques

Very important advantage of the folded beam thermal actuator is that the extremely simple design could be fabricated in any MEMS process with only one releasable current-carrying layer, such as LIGA, MUMPs etc. In contrast with electrostatic MEMS, close dimensional tolerances are not required. Thermal actuators have been demonstrated on regular printed circuit boards (PCB) with a single lithography step [10]. The most common materials for thermal actuators are polysilicon [28], single crystal silicon [29] and nickel [21]. The different fabrication technologies impose different restrictions on the actuator size and power requirements. The next paragraph is a short overview of the three general fabrication techniques used commonly to fabricate thermal actuators.

Polysilicon thermal actuators are widely used because of the maturity and availability of commercial polysilicon MEMS foundry services such as MUMPs [30]. A simplified manufacturing process flow is shown in (Figure 1.5).
The fabrication process starts with the deposition of a thin silicon nitride layer used as an etch stop and electrical isolation. Alternating layers of sacrificial oxide and polysilicon are deposited and patterned lithographically to form the device. Finally, the sacrificial oxide is etched away in HF leaving only the polysilicon features as shown in Figure 1.5, Step 12. The thickness of the actuator beams is up to about 2µm and is limited by the intrinsic stress in the polysilicon film caused by the deposition method. The beam aspect ratio (the ratio thickness/width) has to be slightly larger than 1 for reliable in-plane bending. If the aspect ratio is smaller than 1, the actuator could buckle out-of-plane, which decreases the in-plane actuator stroke. The small thickness of the beam also limits the maximum beam length to about 200-300µm, because longer beams tend to stick to the substrate. The free tip displacement of these actuators is around 10µm and they can deliver forces on the order of few micro-newtons. A major advantage of the polysilicon actuators is that they have resistance on the order of 1kΩ, power consumption around 10mW and actuation voltages 1-20V. These power supply requirements are directly
compatible with CMOS circuitry. The small size of the actuators is beneficial for the speed of operation and they have time constants on the order of 200μs [22]. However, the small size is also a major limitation of the polysilicon thermal actuators if larger force or stroke is desired. Furthermore, the thin polysilicon device is very fragile and is not suitable for interaction in the macro domain.

In general, fabricating longer thermal actuators requires increased beam thickness to preserve the aspect ratio. Since polysilicon surface micromachining requires blanket deposition of structural and sacrificial materials, it becomes impractical for films above a few microns. In contrast with the surface micromachining, bulk micromachining is a general technique based on selective removal of substrate material and structure thicknesses in the range 10μm-300μm are common. The process is mature and is offered by commercial foundries such as SOIMUMPs from MEMSCAP [31]. A simplified fabrication flow for folded beam thermal actuators is shown in Figure 1.6

![Fabrication flow of single crystal thermal actuator](image)
The process starts with a silicon-on-insulator (SOI) wafer, which is a sandwich of single crystal silicon layers separated by silicon oxide. The SOI wafers for MEMS applications are manufactured by bonding two silicon wafers. Fine grinding and polishing is used to adjust the thickness of the structural single crystal silicon layer. Photoresist is spun and patterned with standard UV photolithography (Steps 2-3). The exposed silicon is then removed by deep reactive ion etching (DRIE), producing very high aspect ratio and vertical sidewalls. The oxide layer buried in the substrate serves as an etch stop for the DRIE. The arms of the actuator are released by removal of the photoresist and a timed etch of the oxide layer. The bonding pads are much wider and the oxide etch undercut is not sufficient to debond them from the substrate. Bulk micromachining of 100 µm silicon on SOI wafer was successfully used for thermal actuators as long as 14 mm [32]. The main advantage of the SOI processing for thermal actuators is the large actuator length and aspect ratio yielding stable in-plane displacement and large stroke. Besides, the fabrication has only a few steps and a single lithography mask is sufficient. However, this is at the expense of using costly wafers (SOI) and processing (DRIE). The SOI wafers impose additional restrictions since the thickness of the thermal actuator is equal to the thickness of the top silicon layer. In order to vary this thickness, different SOI wafers must be purchased. Since silicon is used as a structural element, the actuators are brittle and do not tolerate overstressing.

Metals are very desirable as structural material for thermal actuators since they are not brittle and have larger coefficient of thermal expansion (12.7 × 10⁻⁶ for nickel compared
to $2.63 \times 10^4$ for silicon). The metal actuators are typically built by electroplating in polymer molds as shown in Figure 1.7.

Figure 1.7. Fabrication flow of nickel folded beam actuators

Very important advantage of the metal actuators is that they could be built on any polished nonconductive surface and even on conventional printed circuit boards [10]. If silicon wafer is used as a substrate it is first oxidized for electrical insulation (Step 1). The next step is the deposition of a conductive seed layer providing a uniform low resistance electrical contact for the electroplating. Photoresist is then spun and patterned forming the mold for electroplating. Nickel is a very popular material for the electroplating because of its good mechanical properties and low cost. After the wafer is electroplated, the photoresist is stripped and the seed layer is undercut with a timed etch, leaving the narrow structures free and the anchors connected to the substrate. The process is very simple and inexpensive and requires a single lithography step. The critical part is the formation of the electroplating mold, which has to be thick for long actuators and have nearly vertical sidewalls. Using a regular spin-coater and a thick film photoresist
such as AZ4903, it is easy to achieve 25\(\mu\)m for a single coat. Thick photoresist molds up to 80\(\mu\)m have been fabricated using a spin-coater with rotating cover (Karl-Suss RC8 Gyrset) to minimize the air turbulence and by depositing two layers of AZ4562 photoresist [33]. Increasing the plating thickness further using conventional thick film photoresists introduces problems such as coating uniformity and process repeatability. The aspect ratio of the mold could be improved significantly by replacing the deep UV illumination with synchrotron radiation. The high particle energy minimizes the diffraction and scattering due to local resist non-homogeneities. Because the irradiating particles travel in a straight-line through the resist, the sidewalls are nearly vertical. However, technologies using high-energy radiation, such as LIGA, have high cost and limit many Microsystems laboratories from exploiting this process. The introduction of the thick near-UV resist called SU-8 by IBM created a simple solution for standard lithographic patterning of films few hundred microns thick [34]. Aside from using standard UV exposure tools and spin-coaters, SU-8 has the advantage of nearly vertical sidewalls, and easily achievable aspect ratio on the order of 20. Unfortunately, the SU-8 is a photosensitive epoxy, the exposure with UV creates highly stable cross-linked polymer, resistant to most solvents. As a result, if SU-8 is being used as an electroplating mold, the resist removal is quite challenging. Many different techniques have been studied, but so far the most successful one is to use a sacrificial layer under the SU-8 mold. This has been demonstrated using photoresist [35], or the specially formulated release layer OmniCoat from Microchem [36]. However, the technique is not perfect and
small SU-8 pieces remain lodged between the arms of the thermal actuator when the gap between the arms is small.

Each of the described fabrication technologies has its advantages and specific area of application. If small stroke/force is required, the well-developed polysilicon technology could be used. Longer actuators producing more displacement and force require thicker structural layers and are fabricated from single crystal on SOI wafers or are electroplated in resist mold. For electroplating thickness of up to 40μm, conventional photoresist are the most convenient. Typically, thicker films require special resist such as SU-8 or high energy radiation exposure. Combined, these techniques allow fabrication of thermal actuators with beam thicknesses in the range 2μm to 200μm and lengths from 200μm to 14000μm.

1.4. Long Term Performance Of Folded Beam Thermal Actuators

Thermal actuators have limitations for long-term use since the hot arm is subject to high temperatures during operation. The high temperature can decrease the yield strength and lead to plastic creep and change material structure. However, if the driving level is kept under a certain threshold, up to 980 million cycles have been demonstrated without loss of performance [25]. It is important to notice that increasing the driving level leads to worse long term displacement characteristics and early actuator failure. Conant et al [27] observed 41% drop in the actuator displacement over time for polysilicon actuators driven with 11.1mW and virtually no change in the actuator performance under 7.5mW.
excitation. Besides, the actuator driven with lower power outperformed the high power one after about a million cycles. In addition to that, Comtois et al [25] have shown that the resistance of the actuators drops with 5-10% over time due to annealing of the hot arm if the actuator is used at higher current level. Since both mechanical and electrical characteristics change during the life of the actuator, good long term positioning accuracy is difficult to achieve and requires close loop system.

The next chapter develops the analytical tools required for the design of the folded beam actuators.
2. FOLDED BEAM ACTUATOR DESIGN

The most important element in the design of the folded beam thermal actuator was the analytical modeling of the steady state and transient characteristics. The analytical analysis provided in-depth understanding of the underlying physical phenomena, and was the basis of the developed procedure for geometry optimization. Even though FEA has proven to yield very accurate solutions, it does not solve the problem of the initial design. The following chapters describe in detail the steady state electro-thermal, steady state mechanical, transient mechanical and transient thermal analyses. Throughout the chapter, the results from the analytical analyses are compared against the finite element simulation results and the measurements taken from the fabricated actuators.

2.1. Electro-Thermal Modeling Of The Folded Beam Thermal Actuator

The first step in the design of the folded beam thermal actuator is to analyze the electro-thermal conversion. The following chapters describe the analytical analysis, experimental measurements and finite element modeling of the microactuator.

2.1.1 Analytical Electro-Thermal Analysis

A thermal actuator has three modes of heat transfer: conduction, convection, and radiation. It has been estimated that the radiation and convection are relatively insignificant [23, 37], and almost all the heat is dissipated through conduction. The thermal actuators developed for the tactile display application are suspended in air, and the only heat dissipation path is conduction through the bonding pads, which simplified the solution of the thermal problem. A steady-state thermal analysis was conducted assuming a constant current excitation of the actuator from a current source. The
resistivity of the actuator material (nickel) was considered a linear function of the temperature \( T \), \( \rho = \rho_0 (1 + \beta (T - T_s)) \). The temperature coefficient of resistivity of the nickel is \( \beta \), and the substrate temperature, \( T_s \), was assumed constant. The differential equation describing the heat distribution along a homogeneous beam under these conditions is similar to that in [23], however, a temperature-dependent resistivity is included:

\[
\frac{d^2 T}{dx^2} + \rho_0 (1 + \beta (T - T_s)) \frac{I^2}{Kw^2 t^2} = 0 \tag{2.1}
\]

Huang and Lee [28] solved the thermal distribution problem along the unfolded actuator using similar assumptions; however, the actuators they fabricated were suspended over the substrate and the additional substrate heat flux changed the form of the solution [38]. The solution of (2.1) along the unfolded length of the actuator for the hot arm, cold arm, and flexure yields

\[
T_h(x) = A_1 \sin(\gamma_1 x) + B_1 \cos(\gamma_1 x) + \left( T_s - \frac{1}{\beta} \right) x \in [0, L_h], \gamma_1 = \sqrt{\frac{\rho_0 \beta}{Kt^2}} \frac{I}{w_h}
\]

\[
T_c(x) = A_2 \sin(\gamma_2 x) + B_2 \cos(\gamma_2 x) + \left( T_s - \frac{1}{\beta} \right) x \in [L_h, L_h + L_c], \gamma_2 = \sqrt{\frac{\rho_0 \beta}{Kt^2}} \frac{I}{w_c}\tag{2.2}
\]

\[
T_f(x) = A_3 \sin(\gamma_3 x) + B_3 \cos(\gamma_3 x) + \left( T_s - \frac{1}{\beta} \right) x \in [L_h + L_c, L_h + L_c + L_f], \gamma_3 = \sqrt{\frac{\rho_0 \beta}{Kt^2}} \frac{I}{w_f}
\]

In order to find the unknown coefficients \( A_i \) and \( B_i \), a set of six boundary conditions is used. The first two conditions fix the temperature at the bonding pads equal to the substrate temperature:

\[
T_h(0) = T_s \tag{2.3}
\]
The last four boundary conditions enforce continuity of the temperature and the heat flux across the hot arm-cold arm and cold arm-flexure junctions

\[ T_h(L_h + L_c + L_f) = T_s \]  
(2.4)

These six boundary conditions could be used to solve for the six unknowns \( A_1, B_1, A_2, B_2, A_3, \) and \( B_3 \). Furthermore, the system is linear and can be expressed in the following matrix form: \( \mathbf{A}[A_1, B_1, \ldots, B_3]^T = \mathbf{b} \). In order to simplify the expressions lets denote \( L_1 = L_h \), \( L_2 = L_h + L_c \), \( L_3 = L_h + L_c + L_f \).

**Boundary condition (2.3)**

\[ T_h(x) = A_1 \sin(\gamma x) + B_1 \cos(\gamma x) + T_s - \frac{1}{\beta}, \quad T_h(0) = B_1 + T_s - \frac{1}{\beta} = T_s \Rightarrow B_1 = \frac{1}{\beta} \]

The first boundary condition determines \( B_1 \). Therefore, the system of equations could be transformed to 5x5 and the new vector of unknowns is \( \mathbf{V}^T = [A_1, A_2, B_2, A_3, B_3] \).

**Boundary condition (2.4)**

\[ A_3 \sin(\gamma_3 L_3) + B_3 \cos(\gamma_3 L_3) + \left( T_s - \frac{1}{\beta} \right) = T_s \]

\[ [0, 0, 0, \sin(\gamma_3 L_3), \cos(\gamma_3 L_3)]^T \mathbf{V} = \frac{1}{\beta} \]
Boundary condition (2.5)

\[ A_1 \sin(\gamma_1 L_1) - B_1 \cos(\gamma_1 L_1) = A_2 \sin(\gamma_2 L_1) - B_2 \cos(\gamma_2 L_1) \]

\[ [- \sin(\gamma_1 L_1), \sin(\gamma_2 L_1), \cos(\gamma_2 L_1), 0, 0] V = \frac{1}{\beta} \cos(\gamma_1 L_1) \]

Boundary condition (2.6)

\[ w_{h_1} A_1 \cos(\gamma_1 L_1) - w_{h_2} B_1 \sin(\gamma_1 L_1) = w_{c_2} A_2 \cos(\gamma_2 L_1) - w_{c_3} B_2 \sin(\gamma_2 L_1) \]

\[ [w_{h_1} \gamma_1 \cos(\gamma_1 L_1), - w_{h_2} \gamma_2 \cos(\gamma_2 L_1), w_{c_2} \gamma_2 \sin(\gamma_2 L_1), 0, 0] V = w_{h_1} \frac{1}{\beta} \sin(\gamma_1 L_1) \]

Boundary condition (2.7)

\[ A_2 \sin(\gamma_2 L_2) + B_2 \cos(\gamma_2 L_2) = A_3 \sin(\gamma_3 L_2) + B_3 \cos(\gamma_3 L_2) \]

\[ [0, \sin(\gamma_2 L_2), \cos(\gamma_2 L_2), - \sin(\gamma_3 L_2), - \cos(\gamma_3 L_2)] V = 0 \]

Boundary condition (2.8)

\[ w_{c_2} A_2 \gamma_2 \cos(\gamma_2 L_2) - w_{c_3} B_2 \sin(\gamma_2 L_2) = w_{f_1} A_3 \gamma_3 \cos(\gamma_3 L_2) - w_{f_2} B_3 \sin(\gamma_3 L_2) \]

\[ [0, w_{c_2} \gamma_2 \cos(\gamma_2 L_2), - w_{c_3} \gamma_3 \sin(\gamma_2 L_2), - w_{f_1} \gamma_3 \cos(\gamma_3 L_2), w_{f_2} \gamma_3 \sin(\gamma_3 L_2)] V = 0 \]

Assembling boundary conditions (2.4) through (2.8) results in the linear system \( A V = b \):

\[ \Delta = \begin{bmatrix}
0 & 0 & 0 & \sin(\gamma_3 L_3) & \cos(\gamma_3 L_3) \\
-\sin(\gamma_1 L_1) & \sin(\gamma_3 L_3) & \cos(\gamma_3 L_3) & 0 & 0 \\
w_{h_1} \gamma_1 \cos(\gamma_1 L_1) & - w_{h_2} \gamma_2 \cos(\gamma_2 L_2) & w_{c_2} \gamma_2 \sin(\gamma_2 L_2) & 0 & 0 \\
0 & \sin(\gamma_2 L_2) & \cos(\gamma_2 L_2) & - \sin(\gamma_3 L_3) & - \cos(\gamma_3 L_3) \\
0 & w_{c_2} \gamma_2 \cos(\gamma_2 L_2) & - w_{c_3} \gamma_3 \sin(\gamma_2 L_2) & - w_{f_1} \gamma_3 \cos(\gamma_3 L_2) & w_{f_2} \gamma_3 \sin(\gamma_3 L_2)
\end{bmatrix} \]

\[ b^T = \frac{1}{\beta} [1, \cos(\gamma_1 L_1), w_{h_1} \gamma_1 \sin(\gamma_1 L_1), 0, 0], \ V^T = [A_1, A_2, B_2, A_3, B_3] \]
Once the unknown coefficients are found, the average temperatures of the beams required for the mechanical analysis are

\[ \bar{T}_i = \frac{1}{L_i} \int_{L_{i-1}}^{L_i} (A_i \sin(\gamma_i x) + B_i \cos(\gamma_i x)) \, dx + T_s - \frac{1}{\beta} \] (2.10)

Evaluating this integral for the three beams yields:

\[ \bar{T}_h = \frac{1}{L_h \gamma_1} \left( A_1 \left( 1 - \cos(\gamma_1 L_h) \right) + B_1 \sin(\gamma_1 L_h) \right) + T_s - \frac{1}{\beta} \] (2.11)

\[ \bar{T}_c = \frac{1}{L_c \gamma_2} \left( A_2 \left( \cos(\gamma_2 L_2) - \cos(\gamma_2 L_1) \right) + B_2 \left( \sin(\gamma_2 L_2) - \sin(\gamma_2 L_1) \right) \right) + T_s - \frac{1}{\beta} \]

\[ \bar{T}_f = \frac{1}{L_f \gamma_3} \left( A_3 \left( \cos(\gamma_3 L_2) - \cos(\gamma_3 L_3) \right) + B_3 \left( \sin(\gamma_3 L_2) - \sin(\gamma_3 L_3) \right) \right) + T_s - \frac{1}{\beta} \]

Finally, the actuator resistance, voltage drop and power consumption are

\[ R_{\text{act}} = \frac{\rho_h (L_h + L_s)}{t w_h} \left( 1 + \beta (T_h - T_s) \right) + \frac{\rho_{L_e}}{t w_c} \left( 1 + \beta (T_e - T_s) \right) + \frac{\rho_{L_f}}{t w_f} \left( 1 + \beta (T_f - T_s) \right) \] (2.12)

\[ U_{\text{act}} = IR_{\text{act}} \] (2.13)

\[ P_{\text{act}} = I^2 R_{\text{act}} \] (2.14)

This concludes the analytical electro-thermal analysis and supplies the average beam temperatures required for the mechanical deflection analysis. The developed electro-thermal model was compared against results obtained using finite element simulations and measurements of the fabricated devices. Since the author did not have access to an infrared microscope, direct comparison of the thermal distribution along the unfolded actuator length was not possible. However, by using the current-voltage (I-V)
characteristics generated from the analytical model, the FEA and the measured from the fabricated devices, the average actuator temperature can be compared. Even though this results in an indirect thermal model validation, the additional benefit is that the obtained I-V curve is required for the design of the electronics control module and is important from electrical point of view. The experimental I-V measurements and the finite element simulations are presented in the following chapters.

2.1.2. Experimental Measurements

The fabricated thermal actuators were powered by a constant current source, with currents ranging between 110 mA and 230 mA. Since the resistance of the actuators is very small and on the order of 1Ω, the current–voltage measurements were conducted using a four-probe technique. Two probes were used to power the actuator, while additional two contacted the actuator close to the hot arm and flexure ends, in order to measure the actuator voltage drop accurately. The position of the first two probes is not critical since the additional parasitic resistance in series with the actuator does not change the supplied current. For each value of the supply current, both the voltage drop and the actuator displacement were recorded and compared with the theoretical estimates. The tip displacement was measured optically by processing images captured by a CCD camera mounted on the microscope probe station.
2.1.3 Finite Element Electro-Thermal Modeling

The finite element model of the folded beam thermal actuator was developed in ANSYS. The electro-thermal simulation is both nonlinear and coupled field, with the source of nonlinearity being the change in material resistivity with temperature, according to (2.1). The material properties of the thermal actuator material (electroplated nickel) used in the finite element analysis and the analytical solution are listed in Table 2.

Table 2. Nickel Material Properties

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ₀</td>
<td>Electrical resistivity at Tₛ</td>
<td>15×10⁻⁸</td>
<td>Ωm</td>
</tr>
<tr>
<td>β</td>
<td>Temperature coefficient of resistivity</td>
<td>2×10⁻³</td>
<td>K⁻¹</td>
</tr>
<tr>
<td>α</td>
<td>Coefficient of thermal expansion</td>
<td>13.6×10⁻⁶</td>
<td>K⁻¹</td>
</tr>
<tr>
<td>K</td>
<td>Thermal conductivity of nickel</td>
<td>90.9</td>
<td>W/(m.K)</td>
</tr>
<tr>
<td>ρ̃</td>
<td>Nickel density</td>
<td>8900</td>
<td>kg/m³</td>
</tr>
<tr>
<td>c</td>
<td>Specific heat of nickel</td>
<td>444</td>
<td>J/(kg.K)</td>
</tr>
<tr>
<td>E</td>
<td>Young's modulus of nickel</td>
<td>177.3×10⁹</td>
<td>Pa</td>
</tr>
<tr>
<td>Tₛ</td>
<td>Substrate temperature</td>
<td>18</td>
<td>°C</td>
</tr>
</tbody>
</table>

The material resistivity was specified in ANSYS in the form of a look-up table as shown in Figure 2.1 (the SI resistivity value is multiplied by 10⁻⁶ because the simulations were carried out using μm as units of length). The microactuator was discretized with SOLID98 10-node tetrahedral coupled field elements with VOLT and TEMP degrees of freedom activated (KEYOPT(1)=1, the MAG degree of freedom was not used). The ANSYS thermal model assumes that the entire heat flux is dissipated through the bonding pads, which were held at a constant temperature equal to the substrate temperature. Voltage was applied to the bonding pads as a boundary condition and the corresponding
steady state temperature distribution was obtained. The current consumption was
determined as a nodal reaction solution at the bonding pads.

![Figure 2.1. ANSYS lookup table for nickel resistivity as a function of temperature](image)

The analysis was performed with the geometrical data listed in Table 3, matching the fabricated actuators.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>Actuator thickness</td>
<td>20.6</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$w_h$</td>
<td>Hot arm width</td>
<td>17.5</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$w_c$</td>
<td>Cold arm width</td>
<td>105</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$w_f$</td>
<td>Flexure width</td>
<td>17.5</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$L_h$</td>
<td>Hot arm length</td>
<td>1368</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$L_c$</td>
<td>Cold arm length</td>
<td>953</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$L_f$</td>
<td>Flexure length</td>
<td>415</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Separation between the hot and cold arm</td>
<td>30</td>
<td>$\mu m$</td>
</tr>
</tbody>
</table>
The simulation was run under ANSYS 7.0 and one example of the temperature distribution for applied voltage \( U=0.2V \) is shown in Figure 2.2.

The maximum temperature is reached in the hot arm as shown in Figure 2.2. Since the actuator dissipates heat only through bonding pads, the cold arm stays at elevated temperature and this offsets the spot with maximum temperature towards the actuator tip. If the actuator is suspended in a close proximity above an isothermal substrate such as a silicon wafer, the wide cold arm dissipates significant amount of the heat directly to the substrate. This reduces the cold arm temperature and shifts the maximum temperature spot closer to the middle of the hot arm.
2.1.4. Electro-Thermal Model Comparison

A comparison between the current-voltage characteristics calculated analytically according to (2.12) and (2.13), the finite element simulation results and the measured data is presented in Figure 2.3. The analytical calculations were performed using MATLAB, and the source code is included in Appendix F. For reference, the ANSYS input files are also included in Appendix G.

Two important observations could be made:

- The resistivity changes significantly due to the actuator heating - the I-V curve is not a straight line.
- The analytical model is in good agreement with the finite element simulations.

The experimental data however, does not agree so well with the simulations.
The divergence between the models and the measurement could be attributed to the additional heat flux from parts of the hot arm and the flexure directly to the substrate. In the initial design, the silicon substrate was removed selectively under the whole actuator, which is consistent with the assumption of thermally isolated beams used in the analytical and FEM models. However, in the next generation of mask layout, some of the substrate under the actuator was left to provide mechanical guiding in the case of larger vertical tip displacement. This additional flux through the thin air film reduces the beam temperatures and the actuator resistance, which explains why the experimental data shows smaller voltage drop for the same actuator current.

Once the current-voltage characteristic of the actuator is computed, it is later used in the design of the power supply and the driving electronics for the MEMS array. This completes the electro-thermal modeling and validation, and the next chapter describes a nonlinear mechanical model for prediction of the free tip displacement, utilizing the average temperatures supplied by the thermal model.

2.2 Nonlinear Mechanical Analysis Of The Folded Beam Thermal Actuator

The mechanical analysis of the folded beam thermal actuator is essentially a two-dimensional problem. Three separate beams need to be analyzed: the hot arm, the cold arm and the flexure. The bending of each of the beams can be described by a second order differential equation according to Euler-Bernoulli beam theory. The resulting system of equations becomes quite complex when nonlinear geometric effects are
considered. However, the equations could be simplified greatly if we note that typically the width of the cold arm is several times larger than the width of the hot arm and the flexure. This results in cold arm cross-sectional moment of inertia two or three orders of magnitude larger, since the moment of inertia is proportional to the third power of the beam width. The hot and the cold arms are subjected to similar bending moments and the significantly larger moment of inertia of the cold arm results in its negligible bending. By assuming that the cold arm is rigid one second-order differential equation is replaced with a linear algebraic equation, thus simplifying the analysis. The linear elongation on the other hand is proportional to the cross-sectional area of the beam, which scales linearly with the beam width. Therefore, the extension of the cold arm is the same order of magnitude and has to be accounted for. To summarize, the working assumptions for the mechanical analysis could be formulated as:

(1) Only the bending of the hot arm and the flexure is considered.

(2) All three beams change length due to the axial loads applied to them.

Under these assumptions, the system of equations consists of two second-order differential equations and three equations for the axial displacement of the beams. The coordinate system used for the bending of the hot arm and flexure and the deflection convention is shown in Figure 2.4.
The directions of the axial reaction force $P$ and the transversal force $T$ are reversed for the hot arm and the flexure (Figure 2.4) in order to satisfy balance of forces. The pair of forces $P$ separated by the distance between the arms $\delta$ generates additional moment and the moment balance yields

$$M_1 = P\delta - M \quad (2.15)$$

The general nonlinear beam equations including large displacement effects of the hot arm and the flexure are

$$I_1 E \, w'_1(x) = T x - P \, w_1(x) - M \quad I_1 = tw^3_1/12$$

$$I_2 E \, w'_2(x) = -T x + P \, w_2(x) + M - P\delta \quad I_2 = tw^3_2/12 \quad (2.16)$$
Here \( w_1(x) \) and \( w_2(x) \) are the deflections of the hot arm and the flexure respectively, as shown in Figure 2.4. The built-in ends of the beams impose the following boundary conditions

\[
\begin{align*}
  w_1(0) &= 0 & \text{(2.17)} \\
  w_1'(0) &= 0 & \text{(2.18)} \\
  w_2(0) &= 0 & \text{(2.19)} \\
  w_2'(0) &= 0 & \text{(2.20)}
\end{align*}
\]

The two second-order differential equations (2.16) require total of four unknown integration constants that could be found from the boundary conditions (2.17-2.20) The remaining unknowns are the reactions \( T, P \) and \( M \). The axial load \( P \) causes beam deflection \( u \) in the \( x \) direction shown in Figure 2.4. In order to accommodate large strain effects, the mechanical strain in the beam is approximated as

\[
\varepsilon_x = \frac{du}{dx} + \frac{1}{2} \left( \frac{dw}{dx} \right)^2
\]

(2.21)

Using the strain defined in (2.21) results in the following equation for the axial force \( P \)

\[
-P = \int \sigma dA = \int E(\varepsilon_x - \alpha \Delta T) dA = \int E \left( \frac{du}{dx} + \frac{1}{2} \left( \frac{dw}{dx} \right)^2 \right) - \alpha \Delta T dA
\]

(2.22)

Integrating across the cross-section of the beam yields

\[
-P = EA \frac{du}{dx} + \frac{1}{2} EA \left( \frac{dw}{dx} \right)^2 - \alpha \Delta TEA
\]

(2.23)
Here \( A \) is the cross-sectional area of the beam, \( E \) is the Young's modulus and \( \Delta T \) is the difference between the average beam temperature and the substrate temperature \( T_s \).

Integrating once again along the length of the beam results in

\[
- \frac{PL}{EA} = \Delta u - \alpha \Delta TL + \frac{1}{2} \int_0^L \left( \frac{d\psi}{dx} \right)^2 dx \tag{2.24}
\]

Substituting the values for the hot arm, cold arm and flexure using \( P \) with the appropriate sign (positive for the hot arm and negative for the flexure and the cold arm) results in a system of three nonlinear equations

\[
- \frac{PL_h}{EA_h} = \Delta u_h - \alpha \Delta T L_h + \frac{1}{2} \int_0^L \left( \frac{d\psi_h}{dx} \right)^2 dx \tag{2.25}
\]

\[
\frac{PL_f}{EA_f} = \Delta u_f - \alpha \Delta T L_f + \frac{1}{2} \int_0^L \left( \frac{d\psi_f}{dx} \right)^2 dx \tag{2.26}
\]

\[
\frac{PL_c}{EA_c} = \Delta u_c - \alpha \Delta T L_c + \frac{1}{2} \int_0^L \left( \frac{d\psi_c}{dx} \right)^2 dx \tag{2.27}
\]

The integral of the square of the derivative of the deflection with respect to \( x \) in (2.25) and (2.26) is a known function of the reactions \( T, P \) and \( M \) according to (2.16-2.20). The similar term in (2.27) is also easy to calculate since we assumed that the cold arm is rigid and the cold arm rotation angle \( \theta = d\psi_c / dx \) is constant and a known function of the reactions (not a new unknown). Equation (2.27) can therefore be rewritten in the form

\[
\frac{PL_c}{EA_c} = \Delta u_c - \alpha \Delta T L_c + \frac{1}{2} \theta^2 L_c \tag{2.28}
\]
The three newly introduced equations (2.25-2.27) include the additional unknowns $\Delta u_h, \Delta u_f$ and $\Delta u_c$, which are needed for the boundary conditions. To complete the system of equations and solve for the remaining unknown reactions, it is necessary to define three additional boundary conditions. The first boundary condition is the previously mentioned cold arm rigidity condition. The assumption that the cold arm does not bend forces the slope at the end of the hot arm and the slope at the end of the flexure to be equal, which can be expressed as

$$w_1'(L_h) = w_2'(L_f) = \theta \quad (2.29)$$

Since the hot arm is connected to the cold arm and the flexure, the transversal tip displacement $w$, calculated from the hot arm must be equal to the deflection calculated from the flexure and the cold arm. The second boundary condition is therefore

$$w_1(L_h) = w_2(L_f) + w_1'(L_h)L_c \quad (2.30)$$

The last boundary condition states that extension of the hot arm must be equal to the extension of the cold arm and the flexure

$$\Delta u_h - \Delta u_f = \Delta u_f + \Delta u_c \quad (2.31)$$

The additional term $\Delta u_s$ is the geometric displacement caused by the rotation of the small beam with length $\delta$ connecting the hot arm with the cold arm. This geometric factor translates the displacement of the hot arm along its axis to the axis of the flexure and the cold arm and is

$$\Delta u_s = -\delta \sin(\theta) \approx -\delta \theta \approx -\delta w_2'(L_f) \quad (2.32)$$
Substituting (2.25), (2.26), (2.28) and (2.32) in (2.31) yields

\[
\frac{P}{E} \left( \frac{L_f}{A_f} + \frac{L_e}{A_e} + \frac{L_h}{A_h} \right) = \alpha \left( L_a \Delta T_h - L_f \Delta T_f - L_e \Delta T_e \right) - \frac{1}{2} \int_0^L w_1'' dx + \frac{1}{2} \int_0^L w_2'' dx + \frac{1}{2} L_c \theta^2 + \delta \theta \tag{2.33}
\]

The last condition (2.33) completes the system of equations. The final set of equations consists of the two second-order differential equations for the deflection (2.16), the four boundary conditions enforced by the built-in ends of the beams (2.17-2.20) and the three boundary conditions (2.29), (2.30) and (2.33). The first step in the solution is finding \( w(x) \) and \( w'(x) \) as a function of the reactions \( T, P \) and \( M \) using (2.16-2.20). The deflections and their derivatives are then substituted in the boundary conditions (2.29), (2.30) and (2.33) defining three equations for the three unknown reactions. This approach will be applied to solve the mechanical model in three different cases with varying complexity as shown in the following.

2.2.1 Nonlinear Mechanical Solution Of Folded Beam Thermal Actuator

The differential equations describing the deflection of the hot arm and the flexure could be transformed as

\[
w_1''(x) + \frac{P}{I_1 E} w_1(x) = -\frac{M}{I_1 E} + \frac{T}{I_1 E} x \tag{2.34}
\]

\[
w_2''(x) - \frac{P}{I_2 E} w_2(x) = -\frac{P \delta}{I_2 E} + \frac{M}{I_2 E} - \frac{T}{I_2 E} x \tag{2.35}
\]
In these equations the axial reaction force $P$ is positive since the hot beam is under compression (Figure 2.4) when deflecting the cold arm and flexure. Furthermore, the moments of inertia and the Young’s modulus are also positive quantities. Let’s define

$$k_1 = \sqrt{\frac{P}{I_1E}} = \sqrt{\frac{12P}{tw_1^3E}}, k_2 = \sqrt{\frac{P}{I_2E}} = \sqrt{\frac{12P}{tw_2^3E}} \quad (2.36)$$

The type of the solution is determined by the sign of the term in front of $w(x)$:

$$w_1(x) = A_1 \sin(k_1x) + B_1 \cos(k_1x) + \frac{T}{P} x - \frac{M}{P}$$ \quad (2.37)

$$w_2(x) = A_2 \sinh(k_2x) + B_2 \cosh(k_2x) + \delta \frac{M}{P} + \frac{T}{P} x$$ \quad (2.38)

The four unknown integration constants are found by applying the built-in ends boundary conditions (2.17-2.20)

$$B_1 = \frac{M}{P}, A_1 = -\frac{T}{k_1P} \quad (2.39)$$

$$B_2 = \frac{M}{P} - \delta, A_2 = -\frac{T}{P} \frac{1}{k_2} \quad (2.40)$$

The necessary equations of the deflections and their derivatives are obtained after substituting (2.39) and (2.40) in (2.37) and (2.38) resulting in

$$w_1(x) = \frac{T}{P} \left( x - \frac{1}{k_1} \sin(k_1x) \right) + \frac{M}{P} (\cos(k_1x) - 1) \quad (2.41)$$

$$w'_1(x) = \frac{T}{P} (1 - \cos(k_1x)) - \frac{M}{P} k_1 \sin(k_1x) \quad (2.42)$$

$$w_2(x) = \frac{T}{P} \left( x - \frac{1}{k_2 \sinh(k_2x)} \right) + \frac{M}{P} (\cosh((k_2x) - 1) + \delta (1 - \cosh(k_2x))) \quad (2.43)$$
Using the above equations, the boundary conditions (2.29) and (2.30) can be rewritten in the form

\[
\frac{T}{P} \left( 1 - \cos(k_1 L_h) \right) - \frac{M}{P} k_k \sin(k_1 L_h) = \frac{T}{P} \left( 1 - \cosh(k_1 \ell_f) \right) + \frac{M}{P} k_2 \sinh(k_1 \ell_f) - \delta k_2 \sinh(k_2 \ell_f) \quad (2.45)
\]

\[
\frac{T}{P} \left( L_h - \frac{1}{k_1} \sin(k_1 L_h) \right) + \frac{M}{P} \left( \cos(k_1 L_h) - 1 \right) = \frac{T}{P} \left( L_f - \frac{1}{k_2} \sinh(k_2 \ell_f) \right) + \frac{M}{P} \left( \cosh(k_2 \ell_f) - 1 \right) + \delta \left( 1 - \cosh(k_2 \ell_f) \right) + \frac{T}{P} \left( L_c - L_c \cos(k_1 L_h) \right) - \frac{M}{P} L_c k_1 \sin(k_1 L_h) \quad (2.46)
\]

Selecting \( T/P \) and \( M/P \) as unknowns and arranging (2.45) and (2.46) in a matrix form yields

\[
\begin{bmatrix}
  f_{11} & f_{12} \\
  f_{21} & f_{22}
\end{bmatrix}
\begin{bmatrix}
  T/P \\
  M/P
\end{bmatrix} = \begin{bmatrix}
  b_1 \\
  b_2
\end{bmatrix} \quad (2.47)
\]

The coefficients of the linear system are:

\[
f_{11} = L_h - L_f - L_c + \frac{1}{k_2} \sinh(k_2 \ell_f) + L_c \cos(k_1 L_h) - \frac{1}{k_1} \sin(k_1 L_f)
\]

\[
f_{12} = \cos(k_1 L_h) - \cosh(k_2 \ell_f) + L_c k_1 \sin(k_1 L_h)
\]

\[
f_{21} = \cos(k_1 L_h) - \cosh(k_2 \ell_f)
\]

\[
f_{22} = k_1 \sin(k_1 L_h) + k_2 \sinh(k_2 \ell_f)
\]

\[
b_1 = \delta \left( 1 - \cosh(k_2 \ell_f) \right)
\]

\[
b_2 = \delta k_2 \sinh(k_2 \ell_f)
\]
In order to use the last boundary condition (2.33) the integrals of the deflection derivatives must be evaluated

\[
\frac{1}{2} \int_0^{L_h} \omega_1^2 \, dx = \frac{1}{2} \int_0^{L_h} \left( \frac{T}{P} \right) \left( 1 - \cos(k_1 x) \right) - \left( \frac{M}{P} \right) k_1 \sin(k_1 x) \right)^2 \, dx = \\
= \left( \frac{T}{P} \right)^2 \left( \frac{3}{4} L_h - \frac{1}{k_1} \sin(k_1 L_h) + \frac{1}{8k_1} \sin(2k_1 L_h) \right) + \\
+ \left( \frac{T}{P} \right) \left( \frac{M}{P} \right) \cos(k_1 L_h) - \frac{1}{4} \cos(2k_1 L_h) - \frac{3}{4} + \left( \frac{M}{P} \right)^2 \left( \frac{k_1^2 L_h}{4} - \frac{k_1}{8} \sin(2k_1 L_h) \right)
\]

(2.48)

\[
\frac{1}{2} \int_0^{L_f} \omega_2^2 \, dx = \frac{1}{2} \int_0^{L_f} \left( \frac{T}{P} \right) \left( 1 - \cosh(k_1 x) \right) + \left( \frac{M}{P} - \delta \right) k_2 \sinh(k_2 x) \right)^2 \, dx = \\
= \left( \frac{T}{P} \right)^2 \left( \frac{3}{4} L_f - \frac{1}{k_2} \sinh(k_2 L_f) + \frac{1}{8k_2} \sinh(2k_2 L_f) \right) + \\
+ \left( \frac{T}{P} \right) \left( \frac{M}{P} - \delta \right) \left( \cosh(k_2 L_f) - \frac{1}{4} \cosh(2k_2 L_f) - \frac{3}{4} \right) + \left( \frac{M}{P} - \delta \right)^2 \left( \frac{k_2^2 L_f}{8} \sinh(2k_2 L_f) - \frac{k_2^2 L_f}{4} \right)
\]

(2.49)

The deflection angle of the cold arm used in the last two terms in (2.33) is

\[
\theta = \left( \frac{T}{P} \right) \left( 1 - \cos(k_1 L_h) \right) - \left( \frac{M}{P} \right) k_1 \sin(k_1 L_h)
\]

(2.50)

For clarity, equation (2.33) will be rewritten as a function of the ratio \( P/E \)

\[
\frac{P}{E} = \frac{1}{L_f + \frac{L_c}{A_c} + \frac{L_h}{A_h}} \left( \alpha \left( L_h \Delta T_h - L_f \Delta T_f - L_c \Delta T_c \right) - \frac{1}{2} \int_0^{L_h} \omega_1^2 \, dx + \frac{1}{2} \int_0^{L_f} \omega_2^2 \, dx + \frac{1}{2} L_c \theta^2 + \delta \theta \right)
\]

(2.51)

The full nonlinear system of equations consists of (2.47) and (2.51), with the last four terms substituted from (2.48-2.50). The linear system (2.47) calculates the ratios \( T/P \)
and $M/P$ as a function of $k_1$ and $k_2$, which in turn are function of the ratio $P/E$ according to (2.36). The last four terms in (2.51) are also functions of $T/P$ and $M/P$ as shown in (2.48-2.50) and therefore functions of $P/E$. The conclusion is that equation (2.51) is a nonlinear transcendental equation for the ratio $P/E$. Once this equation is solved, the values of $T/P$ and $M/P$ are calculated to find the tip displacement. Another important observation is that the tip displacement does not depend on the Young’s modulus of the thermal actuator material and is only a function of the geometry and the thermal coefficient of expansion. This is critical for accurate displacement prediction since the Young’s modulus of electroplated material is a very strong function of the electroplating conditions, and can vary significantly.

The nonlinear transcendental equation (2.51) could be solved numerically through minimization of the quadratic function

$$W(P) = P - \frac{E}{\left(\frac{L_f}{A_f} + \frac{L_c}{A_c} + \frac{L_h}{A_h}\right)} \left(\alpha \left(L_h \Delta T_h - L_f \Delta T_f - L_c \Delta T_c\right) - \frac{1}{2} \int_0^{l_f} w_i^2 \, dx + \frac{1}{2} \int_0^{l_c} w_c^2 \, dx + \frac{1}{2} L_c \theta^2 + \delta \theta\right)^2$$

The function is minimized for the exact solution of the axial load $P_{exact}$ and $\min(W(P_{exact})) = 0$. The minimization could be realized with Newton-Raphson iterations until the difference between the tip displacements calculated from two consecutive values of the axial load $P$ is within a specified tolerance. In order to achieve faster convergence, a suitable initial condition for $P$ is required, which is the subject of the next topic.
2.2.2 Simplified Quadratic Solution Of The Nonlinear Mechanical Problem

The need for iterations in the full nonlinear model comes from the fact that \( T/P \) and \( M/P \) are transcendental functions of \( P \) through \( k_1 \) and \( k_2 \). If the term \( Pw \) is neglected in the differential equations describing the beam bending (2.16), the ratios \( T/P \) and \( M/P \) become a function of the geometry only and the solution of the new linear system is explicit as shown in the following paragraphs. The simplified differential equations are

\[
I,E \, w_1''(x) = T \, x - M
\]

\[
I,E \, w_2''(x) = -T \, x + M - P\delta
\]

Integrating twice using the built-in end boundary conditions (2.17-2.20) yields

\[
w_1'(x) = \frac{T}{I,E} \frac{x^2}{2} - \frac{M}{I,E} x
\]

\[
w_1(x) = \frac{T}{I,E} \frac{x^3}{6} - \frac{M}{I,E} \frac{x^2}{2}
\]

\[
w_2'(x) = -\frac{T}{I,E} \frac{x^2}{2} + \frac{M - P\delta}{I,E} x
\]

\[
w_2(x) = -\frac{T}{I,E} \frac{x^3}{6} + \frac{M - P\delta \, x^2}{2}
\]

Substituting the deflections and their derivatives in boundary conditions (2.29) and (2.30) generates a 2x2 linear system of equations similar to (2.47) with

\[
\begin{bmatrix}
    f_{11} & f_{12} \\
    f_{21} & f_{22}
\end{bmatrix}
\begin{bmatrix}
    (T/P) \\
    (M/P)
\end{bmatrix}
= \begin{bmatrix}
    b_1 \\
    b_2
\end{bmatrix}
\]

(2.58)

Here, the coefficients of the system are function of the geometry only as follows
Using the derivatives defined in (2.54) and (2.56) the last four terms in boundary condition (2.33) are evaluated as in the previous method

\[ \frac{1}{2} \int_0^{L_h} \omega_1^2 \, dx = \frac{1}{2} \int_0^{L_h} \left( \frac{T}{I_E} \frac{x^2}{2} - \frac{M}{I_E} x \right)^2 \, dx = \left( \frac{P}{E} \right)^2 \frac{L_h^3}{2I_1^2} \left( \frac{T}{P} \right)^2 \frac{L_f^2}{20} - \left( \frac{T}{P} \right) \frac{M}{P} \frac{L_h}{4} + \frac{1}{3} \left( \frac{M}{P} \right)^2 \] (2.59)

\[ \frac{1}{2} \int_0^{L_h} \omega_2^2 \, dx = \frac{1}{2} \int_0^{L_h} \left( - \frac{T}{I_E} \frac{x^2}{2} + \frac{M - P \delta}{I_E} x \right)^2 \, dx = \left( \frac{P}{E} \right)^2 \frac{L_f^3}{2I_1^2} \left( \frac{T}{P} \right)^2 \frac{L_f^2}{20} - \left( \frac{T}{P} \right) \frac{M}{P} \frac{L_f}{4} + 1 \left( \frac{M}{P} - \delta \right)^2 \] (2.60)

\[ \delta \theta = \delta \omega_1 (L_h) = \frac{P}{E} \frac{\delta}{I_1} \left( \left( \frac{T}{P} \right) \frac{L_h^2}{2} - \left( \frac{M}{P} \right) L_h \right) \] (2.61)

\[ \frac{1}{2} L_c \theta^2 = \left( \frac{P}{E} \right)^2 \frac{L_f^2 L_c}{2I_1} \left( \left( \frac{T}{P} \right)^2 \frac{L_h^2}{4} - L_h \left( \frac{T}{P} \right) \frac{M}{P} + \left( \frac{M}{P} \right)^2 \right) \] (2.62)

Substituting (2.59-2.62) in boundary condition (2.33) results in a quadratic equation for \( P/E \) in the form

\[ a \left( \frac{P}{E} \right)^2 + b \left( \frac{P}{E} \right) + c = 0 \] (2.63)

The coefficients of the quadratic equation are as follows
The coefficient in front of the quadratic term (2.64) accounts for the nonlinear strain effects caused by the large deformations. The linear term $b$ represents the actuator deformation due to the axial force $P$ and the difference between the extensions of the cold arm and the flexure caused by the actuator rotation. The last term $c$ is the thermally induced net expansion. The quadratic equation can be solved explicitly since its coefficients are a function of $T/P$ and $M/P$, which in turn are function of the geometry. The solution for $P/E$ is therefore

$$ \frac{P}{E} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} $$

This completes the analytical solution for this simplified nonlinear model, and the value for $P$ obtained from (2.67) could be used as initial condition of the full nonlinear model.
2.2.3 Linear Mechanical Model Of Folded Beam Thermal Actuator.

The mechanical model can be simplified further if the large strain effects are neglected, resulting in a linear mechanical model. Since the large strain effect generate the quadratic term in (2.63), this model results in the linear equation

$$b\left(\frac{P}{E}\right) + c = 0$$  \hspace{1cm} (2.68)

Substituting the values of $b$ and $c$ from (2.65) and (2.66) yields

$$\frac{P}{E} = \frac{\alpha(L_h \Delta T_h - L_f \Delta T_f - L_c \Delta T_c)}{\delta \left(\frac{T}{P}\right) \left(\frac{M}{P}\right) \left(\frac{L_h}{I_1}\right) - \left(\frac{L_f}{A_f} + \frac{L_c}{A_c} + \frac{L_h}{A_h}\right)}$$ \hspace{1cm} (2.69)

The ratios $T/P$ and $M/P$ are found from the linear system (2.58). Finally, the tip displacement is

$$w_t(L_h) = \frac{\alpha(L_h \Delta T_h - L_f \Delta T_f - L_c \Delta T_c)}{\delta \left(\frac{T}{P}\right) \left(\frac{M}{P}\right) \left(\frac{L_h}{I_1}\right) - \left(\frac{L_f}{A_f} + \frac{L_c}{A_c} + \frac{L_h}{A_h}\right)} \left(\frac{T}{P}\right) \frac{L_h^3}{6I_1} - \left(\frac{M}{P}\right) \frac{L_h^2}{2I_1}$$ \hspace{1cm} (2.70)

2.2.4 Finite Element Validation Of The Mechanical Model

The large deflection effects were implemented in ANSYS by discretizing the actuator with SOLID92 10-node 3-D tetrahedral structural solid element with UX, UY and UZ degrees of freedom. The mesh as well as the deformed actuator shape is shown in Figure 2.5. In order to compare the models with the ANSYS simulation, the tip displacement was evaluated at different hot arm temperatures, while keeping the cold arm and the flexure temperatures equal to the substrate temperature. Fixing the temperature of the
cold arm and the flexure is not overly restrictive since the thermal loading appears as a
difference in the thermal strains in a single term such as (2.66). Temperature is not a
degree of freedom for the SOLID 92 element, and the three different temperatures were
applied as body loads.

Figure 2.5. ANSYS displacement simulation utilizing large deflection effects

Two different sets of ANSYS simulations were run: one with NLGEOM=ON, which
includes the geometric nonlinearity and one set with NLGEOM=OFF, without geometric
nonlinearity. The analytical results for the tip deflection $w_i(L_y)$ evaluated for the three
models – full nonlinear, quadratic and linear is compared with the simulations in Figure
2.6. Furthermore, the axial and the transversal reaction forces $P$ and $T$ are shown in
Figures 2.7-2.8.
Figure 2.6. Tip displacement results for ANSYS and analytical models

Figure 2.7. Axial force $P$ obtained from ANSYS and the analytical models
As evident from Figure 2.6, the analytical results are in good agreement with the ANSYS simulations. The results are separated in two groups – the three analytical models have a slightly different initial slope than the linear and nonlinear ANSYS simulations. This is probably due to the replacement of the solid connection between the hot arm and the cold arm with the two boundary conditions (2.29) and (2.30). The nonlinear analytical model accurately captures the drop in the tip displacement due to the large strain effects, and the difference between the linear and the nonlinear analytical models is very close to the difference between the linear ANSYS (NLGEOM=OFF) and the nonlinear case NLGEOM=ON. Finally, the tip displacement obtained from the simplified nonlinear quadratic model is very close to the result of the full nonlinear model, suggesting that the
assumption of nonlinear strain (2.21) is the most important nonlinearity in the folded beam actuator. The results for the axial force $P$ shown in Figure 2.7 are again arranged in two groups, with the results of the full nonlinear model closely following the shape of the nonlinear ANSYS. The comparison of the transversal reaction force $T$ is shown in Figure 2.8. The results of the full nonlinear model match closely the nonlinear ANSYS solution, while the rest of the models and the linear ANSYS simulations show reaction force in the opposite direction.

In summary, the free tip displacement obtained from the three analytical models is in very good agreement with the ANSYS simulations. If knowledge of the reaction forces is not necessary, the full nonlinear mechanical model requiring iterative solution could be replaced with the explicit nonlinear quadratic model. However, if the reaction forces are of interest, the full nonlinear model must be used, and large strain effects have to be included in the finite element simulations.

2.3. Validation Of The Coupled Electro-Mechanical Model

The analytical models developed in the previous chapters were combined and tested against a coupled field finite element solution and the experimental measurements. Since the folded beam thermal actuator performs a nonlinear conversion from electrical energy to temperature distribution to mechanical displacement, the coupled model verification is a two-step process. In the first step, the nonlinear electro-thermal model is solved both analytically and in ANSYS and the temperature distribution is obtained. The analytical
solution then averages the beam temperatures and supplies them to the analytical mechanical model. The finite element model uses the temperature distribution from the first simulation to as a body force for the following steady state thermo-mechanical analysis, where the actuator is discretized with SOLID92 element with large strain effects included. In both cases, the final outcome is the free tip displacement as a function of the electrical current flowing through the actuator. The coupled models show good agreement with the experimental data as shown in Figure 2.9.

Figure 2.9. Current-Displacement characteristic of the thermal actuator

2.4. Procedure for Geometry Optimization

The most important benefit form the described analytical analyses were the additional insights gained, resulting in a simple procedure for the MEMS layout optimization. The
design starts with choosing the actuator length, based on known layout constraints – for example the pixel-to-pixel separation. The actuator thickness is then chosen such that the actuator is sufficiently stiff in the out-of-plane direction. The width of the hot arm and the flexure must be smaller than the actuator thickness and is determined by the aspect ratio (the ratio thickness/width) of the fabrication process. Using conventional UV photolithography and electroplating yields aspect ratio approximately one. The gap between the cold arm and the hot arm must be small since the free tip displacement is approximately inversely proportional to it. Using a fabrication process with a line/space ratio of one fixes the gap to be equal to the hot arm width. The last two parameters to be specified are the cold arm width and the flexure length. Contrary to the commonly analyzed case of sacrificially released actuators, if the actuators are suspended in air, the cold arm remains at elevated temperature. The significance of the wider cold arm is therefore two-fold – to decrease the electrical resistance and generate less heat, and to decrease the thermal resistance by increasing the conduction cross-section. Ideally, the cold arm will be at a temperature close to the substrate temperature, hence maximizing the thermal difference between the arms. If all the heat is dissipated through the bonding pads, increasing the cold arm width above \( w_c \sim 5w_h \) is not beneficial since at this point the thermal 'bottleneck' is the cooling through the flexure. By fixing the cold arm width to \( w_c \sim 5w_h \), the last parameter to be selected is the flexure length, which has to be optimized to accommodate the conflicting thermal and mechanical requirements:

- Long flexure provides mechanically softer structure and produces larger free tip displacement for a given thermal strain. However, the added thermal resistance of
the flexure keeps the cold arm at high temperature and decreases the temperature difference.

- Short flexure enhances the cold arm cooling and increases the temperature difference, but makes the actuator mechanically stiffer.

Following the outlined guideline resulted in rapid design optimization and supplied the parameters needed for the MEMS layout.

2.5 Transient Response Of The Folded Beam Actuator
In order to quantify the transient behavior of the folded beam thermal actuator, time dependent term has to be included in (2.1) and the mechanical model has to be augmented with terms accounting for the actuator inertia. As we saw previously, due to the nonlinear electro-thermo-mechanical coupling, obtaining even steady state solution is challenging. However, significant simplification could be achieved if we notice that the thermal and mechanical subsystem and the equations associated with them have quite different time constants. This allows us to find the characteristic times for the thermal and the mechanical transients separately. As shown in the following chapters, the bandwidth limiting process is the thermal cooling and it determines the overall device time constant.

2.5.1 Mechanical Transient
Even though accurate solution for the mechanical resonance of the folded beam thermal actuator has been published by Enikov et al [10]. Here a simplified model is used since the mechanical transient time does not need to be computed very accurately as long as it is much smaller than thermal transient. Therefore, we shall develop a simplified solution,
which calculates the resonance based on the cold arm inertia and actuator stiffness, neglecting the inertia effects in the hot arm and the flexure. The main advantage of this approach is that we get an explicit solution for the actuator stiffness, which is quite useful in the mechanical design. The actuator stiffness is found by calculating the resulting tip displacement when an external force $F$ is applied at the actuator tip as shown in Figure 2.10.

![Figure 2.10. Folded beam deflection under external force $F$](image)

The resulting force and moment diagrams are presented in Figure 2.11.

![Figure 2.11. Reaction forces with external force applied at the actuator tip](image)
Neglecting the additional bending moment caused by the axial force and following the previously described steps (2.52-2.57), the modified beam bending equations and their solutions for the hot arm and flexure are

\[ I_1 E \ w_1'(x) = T \ x - M \]  
(2.71)

\[ I_2 E \ w_2'(x) = (F - T) \ x + (M + P \delta - F L_h) \]  
(2.72)

\[ w_1'(x) = \frac{T}{I_1 E} \ x^2 - \frac{M}{I_1 E} \ x \]  
(2.73)

\[ w_2'(x) = \frac{(F - T) \ x^2}{I_2 E} + \frac{(M + P \delta - F L_h)}{I_2 E} \]  
(2.74)

\[ w_1(x) = \frac{T}{I_1 E} \ x^3 - \frac{M}{I_1 E} \ x^2 \]  
(2.75)

\[ w_2(x) = \frac{(F - T) \ x^3}{I_2 E} + \frac{(M + P \delta - F L_h)}{I_2 E} \]  
(2.76)

In the absence of thermal strain and neglecting the large strain effects, (2.33) is transformed to

\[ \frac{P \left( \frac{L_f}{A_f} + \frac{L_c}{A_c} + \frac{L_h}{A_h} \right)}{E} = -\delta \theta \]  
(2.77)

Substituting the deflection angle at the end of the beam, \( \theta \), (2.77) is transformed to express the reaction force \( P \) as a function of the reaction \( T \), the bending moment \( M \) and the known applied force \( F \).
The cold arm rigidity boundary condition (2.29) yields

\[ \frac{T}{I_1 E} \frac{L_h^2}{2} - \frac{M}{I_1 E} L_h = \frac{(F - T) L_f^2}{2} + \frac{(M + P\delta - FL_h)}{I_2 E} L_f = \theta \]  

(2.79)

Rearranging the terms of (2.79) such that the two unknowns \( T \) and \( M \) are on the left hand side results in

\[ T \left( \frac{L_h^2}{2I_1} + \frac{L_f^2}{2I_2} \right) - M \left( \frac{L_h}{I_1} + \frac{L_f}{I_2} \right) = F \left( \frac{L_f^2}{2I_2} - \frac{L_f L_h}{I_2} \right) + \frac{P\delta}{I_2} L_f \]  

(2.80)

Substituting \( P \) from (2.78) in (2.80) generates a linear equation for the two unknowns \( T \) and \( M \)

\[ Tf_{11} + Mf_{12} = b_1 F \]  

(2.81)

Here, the coefficients of the linear system are

\[ f_{11} = \frac{L_f^2}{2I_2} + \frac{L_h^2}{2I_1} + \frac{\delta^2 L_f t}{I_2} \left( \frac{L_f}{w_f} + \frac{L_c}{w_c} + \frac{L_h}{w_h} \right) \]  

(2.82)

\[ f_{12} = -\frac{L_f}{I_2} - \frac{L_h}{I_1} + \frac{\delta^2 L_f t}{I_2} \left( \frac{L_f}{w_f} + \frac{L_c}{w_c} + \frac{L_h}{w_h} \right) \]  

(2.83)
Similarly, evaluating the beam deflection boundary condition (2.30) results in
\[
\frac{T}{I_1E} T h^3 - \frac{M}{I_1E} T h^2 = \frac{(F-T) T f^3}{I_2E} + \left( M + P \delta - F L h \right) \frac{T f^2}{2} + L_c \left( \frac{T}{I_1E} \frac{T h^2}{2} - \frac{M}{I_1E} T h \right)  
\]
(2.85)

Regrouping this equation yields
\[
T \left( \frac{L_h^3}{6I_1} + \frac{L_f^3}{2I_1} - \frac{L_h^2 L_c}{2I_1} \right) - M \left( \frac{L_h^2}{2I_1} + \frac{L_f^2}{2I_1} - \frac{L_h L_c}{I_1} \right) = F \left( \frac{L_f^3}{6I_2} - \frac{L_f^2 L_h}{2I_2} \right) + P \delta \frac{T_f^2}{2I_2}  
\]
(2.86)

Finally, substituting the axial force P from (2.78) defines the second linear equation
\[
T f_{21} + M f_{22} = b_2 F  
\]
(2.87)

The coefficients of this equation are a function only of the geometry as follows
\[
f_{21} = \frac{L_h^3}{6I_1} + \frac{L_f^3}{2I_1} - \frac{L_h^2 L_c}{2I_1} + \frac{\delta^2 L_f^2 L_h}{4I_1 I_2} \left( \frac{L_f}{w_f} + \frac{L_c}{w_c} + \frac{L_h}{w_h} \right)  
\]
(2.88)

\[
f_{22} = \left\{ \frac{L_h^2}{2I_1} + \frac{L_f^2}{2I_1} - \frac{L_h L_c}{I_1} + \frac{\delta^2 L_f^2 L_h}{2I_1 I_2} \left( \frac{L_f}{w_f} + \frac{L_c}{w_c} + \frac{L_h}{w_h} \right) \right\}  
\]
(2.89)

\[
b_2 = \left( \frac{L_f^3}{6I_2} - \frac{L_f^2 L_h}{2I_2} \right)  
\]
(2.90)

Assembling the two linear equations (2.81) and (2.87) generates a 2x2 linear system
Solving the system for the two unknown reactions yields

\[
\begin{bmatrix}
T \\
M
\end{bmatrix} = \begin{bmatrix}
f_{11} & f_{12} \\
f_{21} & f_{22}
\end{bmatrix}^{-1} \begin{bmatrix}
b_1 \\
b_2
\end{bmatrix} F = \begin{bmatrix}
d_1 \\
d_2
\end{bmatrix} F
\] (2.92)

Finally, the actuator tip displacement is found after substituting \(T\) and \(M\) in (2.75)

\[
w_{\text{tip}} = T h' M W_1 L C M_1 E_1 E_2 E_1 I_2
\] (2.93)

As evident from (2.93), there is a linear relation between the applied external force and the tip deflection, which defines the actuator stiffness

\[
k = E \left\{ \left[ \frac{L_h^3}{6 I_1}, \frac{L_h^2}{2 I_1} \right]^{-1} \begin{bmatrix}
f_{11} & f_{12} \\
f_{21} & f_{22}
\end{bmatrix}^{-1} \begin{bmatrix}
b_1 \\
b_2
\end{bmatrix} \right\}^{-1}
\] (2.94)

The next step in solving for the natural frequency of the actuator is the addition of inertia effects. To simplify the notation \(q = w_{\text{tip}}\) will be used as a generalized coordinate. Specifying the kinetic and potential energy of the actuator as a function of \(q\) and \(\dot{q}\) generates the equations of motion and will be used to solve for the lowest resonant frequency. The potential energy of the deflected actuator is

\[
V = \frac{1}{2} k q^2
\] (2.95)
As mentioned earlier, for simplicity, only the inertia of the cold arm will be considered. The kinetic energy of the cold arm consists of terms coming from the translation of the center of mass and the rotation of the cold arm. The moment of inertia of the cold arm with respect to the center of mass is

\[ I_{zz} = \frac{\rho L_c w_c t}{12} \left( v_c^2 + w_c^2 \right) \]  

(2.96)

If we neglect the deflection at the end of the flexure (it is much smaller than the hot arm bending), a simplified expression for the deflection angle as a function of the tip deflection is obtained

\[ \theta = \frac{w_{0p}}{L_c} = \frac{q}{L_c} \]  

(2.97)

The velocity of the center of mass can be approximated with

\[ V_c = \frac{q}{2} \]  

(2.98)

The factor of two in (2.98) comes from the assumption that the end of the flexure does not move, and the center of mass is in the middle of the cold arm. Finally, the kinetic energy is obtained as

\[ T = \frac{1}{2} I_{zz} \dot{\theta}^2 + \frac{m}{2} V_c^2 = \frac{1}{2} \dot{\tilde{m}} q^2 \]  

(2.99)

Here, the equivalent mass \( \tilde{m} \) is defined as

\[ \tilde{m} = \rho L_c w_c t \left( \frac{1}{3} + \frac{w_c^2}{12 L_c^2} \right) \]  

(2.100)

Using the analogy with a harmonic oscillator, the resonant frequency is
Substituting the actuator geometry and nickel material properties in (2.94), (2.100) and finally in (2.101) yields

\[ f_{\text{res}} = 10199\text{Hz} \]  \hspace{1cm} (2.102)

The mechanical rise-time can be estimated as: \[ 39 \]

\[ \tau_{\text{mech}} = \frac{1.8}{2\pi f_{\text{res}}} = 25\mu s \]  \hspace{1cm} (2.103)

Accurate finite element modal analysis of the actuator conducted with ANSYS resulted in \[ f_{\text{res}} = 6939\text{Hz} \]. As expected, the ‘real’ resonance frequency is lower, since the ANSYS solution takes into account the inertia of the hot arm and the flexure. However, since the bandwidth limiting process is the thermal transient, which is on the order of few milliseconds (as shown in the next section), the developed simple solution is adequate.

### 2.5.2 Thermal Transient

The time-dependent temperature distribution along a homogeneous beam is described by

\[
\frac{\partial T(x,t)}{\partial t} = \frac{K}{\rho c} \frac{\partial^2 T(x,t)}{\partial x^2} + \frac{I^2 \rho R (1 + \beta(T(x,t)-T_s))}{\rho c \omega_s^2 t^2}
\]  \hspace{1cm} (2.104)

Here \( \rho R \) is the electrical resistivity, \( \rho \) is the density and \( c \) is the specific heat of nickel.

If we are to solve the transient problem accurately, (2.104) needs to be applied to the hot arm, cold arm and flexure separately. As we saw previously, this yields a 5×5 linear
system and the solution is not explicit. A reasonably accurate solution can be developed if the cold arm is removed from the unfolded thermal actuator. The rationale behind this is based on our previous assumptions

(i) There is no conductive heat transfer from the wide cold arm directly to the substrate. Therefore, the heat flux that enters the cold arm through the hot arm junction is the same as the heat flux exiting through the flexure-cold arm junction.

(ii) The cold arm is much wider which results in negligible self-heating and small thermal resistance.

Furthermore, the width of the hot arm and the cold arm are assumed equal, which is consistent with the fabricated actuators and simplifies the solution. Under these assumptions, the unfolded thermal actuator is represented by a homogeneous beam with length $L_{eq}$ as shown in Figure 2.12.

![Figure 2.12. Geometry used for the thermal analysis](image_url)
Our primary goal is to determine the characteristic thermal time constant of the actuator
when the driving current is switched from $I_0$ to $I_1$. During the switching, the
temperature distribution transitions from the steady state solution corresponding to $I_0$, to
the steady state distribution for $I_1$. The steady state solution of \((2.104)\) under the
excitation with current $I_1$ according to \((2.2)\) is

$$T_1(x) = \frac{1}{\beta} \cos(\gamma_1 x) + \left( T_s - \frac{1}{\beta} \right) x \in \left[ 0, L_{eq} \right], \gamma_1 = \sqrt{\frac{\rho_0 \beta}{K l^2}} \frac{I_1}{w_h} \quad (2.105)$$

Applying the fixed temperature boundary condition at the bonding pads determines the
two unknown coefficients $A$ and $B$

$$B = \frac{1}{\beta}, \quad A = \frac{1 - \cos(\gamma_1 L_{eq})}{\beta \sin(\gamma_1 L_{eq})} \quad (2.106)$$

Substituting \((2.106)\) into \((2.105)\) yields

$$T_1(x) = \frac{\sin(\gamma_1 x) - \sin(\gamma_1 L_{eq}) + \sin(\gamma_1 (L_{eq} - x))}{\beta \sin(\gamma_1 L_{eq})} + T_s \quad (2.107)$$

The steady state solution could be used to simplify the transient problem \((2.104)\) if the
$T(x,t)$ is expressed as a sum of the steady state solution \((2.107)\) and a time–dependent
term

$$T(x,t) = T_1(x) + \tilde{T}(x,t) \quad (2.108)$$

Substituting $T(x,t)$ in \((2.104)\) results in
\[
\frac{\partial \tilde{T}(x,t)}{\partial t} = \frac{K}{\rho c} \frac{\partial^2 \tilde{T}(x,t)}{\partial x^2} + \frac{K}{\rho c} \frac{\partial^2 T_1(x)}{\partial x^2} + \frac{I_1^2 \rho_R}{\rho c w^2 t^2} (1 + \beta (T_1(x) - T_s)) + \frac{I_1^2 \rho_R \beta}{\rho c w^2 t^2} \tilde{T}(x,t) \quad (2.109)
\]

The sum of the second and the third terms on the right hand side of (2.109) is zero since \(T_1(x)\) is the steady state solution corresponding to the excitation current \(I_1\). Therefore, equation (2.109) is simplified to

\[
\frac{\partial \tilde{T}(x,t)}{\partial t} = \frac{K}{\rho c} \frac{\partial^2 \tilde{T}(x,t)}{\partial x^2} + \frac{I_1^2 \rho_R \beta}{\rho c w^2 t^2} \tilde{T}(x,t) \quad (2.110)
\]

Let's denote

\[
\tau_\beta = \frac{\rho c w^2 t^2}{I_1^2 \rho_R \beta} \quad (2.111)
\]

Applying separation of variables, the time dependent temperature distribution is expressed as a sum of modal components

\[
\tilde{T}(x,t) = \sum_n C_n T_n(x,t) = \sum_n C_n G_n(t) \Phi_n(x) \quad (2.112)
\]

Substituting (2.112) in (2.110) results in a series of equations for the modal eigenfunctions \(\Phi_n(x)\) and \(G_n(t)\)

\[
\Phi_n(x) \frac{\partial G_n(t)}{\partial t} = \frac{K}{\rho c} G_n(t) \frac{\partial^2 \Phi_n(x)}{\partial x^2} + \frac{1}{\tau_\beta} G_n(t) \Phi_n(x) \quad (2.113)
\]

Dividing both sides of the equation by \(G_n(t) \Phi_n(x)\) yields

\[
\frac{1}{G_n(t)} \frac{\partial G_n(t)}{\partial t} = \frac{K}{\rho c} \frac{1}{\Phi_n(x)} \frac{\partial^2 \Phi_n(x)}{\partial x^2} + \frac{1}{\tau_\beta} = \frac{1}{\tau_x} \quad (2.114)
\]
The time dependent part of the solution is an exponential decay with a characteristic time \( \tau_n \), which is a function of the mode number \( n \)

\[
G_n(t) = e^{-\frac{t}{\tau_n}}
\]  
(2.115)

The eigenfunctions \( \Phi_n(x) \) and the time constants \( \tau_n \) are determined from

\[
\frac{1}{\Phi_n(x)} \frac{\partial^2 \Phi_n(x)}{\partial x^2} = \left( \frac{1}{\tau_n} + \frac{1}{\tau_\beta} \right) \frac{\rho c}{K}
\]  
(2.116)

The boundary conditions for \( \Phi_n(x) \) are

\[
\Phi_n(0) = 0
\]  
(2.117)

\[
\Phi_n(L_{eq}) = 0
\]  
(2.118)

The eigenfunction determined from (2.116-2.118) is

\[
\Phi_n(x) = \sin \left( \frac{n\pi x}{L_{eq}} \right)
\]  
(2.119)

Substituting the explicit form of \( \Phi_n(x) \) in (2.116) allows us to determine \( \tau_n \)

\[
\frac{1}{\Phi_n(x)} \frac{\partial^2 \Phi_n(x)}{\partial x^2} = \frac{n^2 \pi^2}{L_{eq}^2} = \left( \frac{1}{\tau_n} + \frac{1}{\tau_\beta} \right) \frac{\rho c}{K}
\]  
(2.120)

Equation (2.120) expresses the modal time constant \( \tau_n \) as a function of the mode number \( n \) and the time constants \( \tau_0 \) and \( \tau_\beta \)

\[
\frac{n^2}{\tau_0} = \frac{1}{\tau_n} + \frac{1}{\tau_\beta}, \quad \tau_0 = \frac{\rho c L_{eq}^2}{\pi^2 K}
\]  
(2.121)
Here, $\tau_0$ is the thermal time constant when the resistivity is assumed constant ($\beta = 0$), while $\tau_\beta$ is the additional current-dependent time constant arising from the temperature dependence of the resistivity. The modal time constant is therefore

$$\tau_n = \frac{\tau_0 \tau_\beta}{n^2 \tau_\beta - \tau_0}$$

(2.122)

Once the eigenfunctions are found, the initial condition, which is the difference between the steady state distributions for $I_0$ to $I_1$, needs to be expressed as a combination of eigenfunctions. However, the first eigenfunction ($n=1$) is very close to the normalized steady state distribution, as shown in Figure 2.13, and in the light of the assumptions at the beginning of the transient thermal analysis, they can be assumed identical.

Figure 2.13. Comparison between the normalized steady state temperature distribution and the first eigenfunction
Equation (2.122) was evaluated for \( n=1 \) and different values of the supply current, while keeping the values of the thermal conductivity and the specific heat constant, and the results are shown in Figure 2.14. The equivalent length of the thermal actuator was assumed to be \( L_{eq} = L_h - L_f \), which is to match the second generation of thermal actuators, in which case the silicon substrate is in close proximity to the flexure and part of the hot arm. If the substrate is removed under the whole length of the device, its equivalent length becomes \( L_{eq} = L_h + L_f \).

![Figure 2.14. Nonlinear thermal time constant \( \tau_1 \) as a function of the supply current](image)

As evident from Figure 2.14, the increase in the time constant determined by the temperature dependence of the resistivity is significant at higher current levels. In addition to the time constant change due to the resistivity increase at higher temperatures,
the thermal conductivity and the heat capacity also change, which affects the time constant according to (2.121). For temperatures in the range of 223°K to 599°K (-50°C to 326°C), the thermal conductivity, density and the specific heat of bulk nickel can be approximated as [40]:

\[
K(T) = 1.131921 \times 10^{-9} \times T^4 - 2.193597 \times 10^{-6} \times T^3 + 1.648640 \times 10^{-3} \times T^2 - 0.6437058 \times T + 185.3755
\]

\[
\rho(T) = -1.117427 \times 10^{-10} \times T^4 + 4.097446 \times 10^{-7} \times T^3 - 5.582955 \times 10^{-4} \times T^2 - 0.1034728 \times T + 8.969428 \times 10^3
\]

\[
c(T) = -1.187679 \times 10^{-4} \times T^4 + 2.223124 \times 10^{-5} \times T^3 + 1.467413 \times 10^{-2} \times T^2 + 4.504691 \times T - 90.38582
\]

In these expressions the temperature \( T \) is in °K. Substituting \( K(T), \rho(T) \) and \( c(T) \) in (2.121) gives us the time constant \( \tau_0 \) as a function of the average actuator temperature as shown in Figure 2.15.

![Figure 2.15. Cooling time constant \( \tau_0 \) as a function of the temperature](attachment:figure_2_15.png)
The temperature-dependent time constant $\tau_0(T)$ shown in Figure 2.15 represents the thermal transient time for small deviations around the average actuator temperature $T$. In the previous calculations, data for bulk nickel was used, which is not necessarily close to that of the electroplated material. Furthermore, variations in the electroplating process, for example different current densities, have been shown to influence the material properties significantly. It is clear that an experimental measurement technique needs to be developed to test the response time of the microactuators, and one simple solution is outlined in the following chapter.

2.5.3 Transient Time Measurement

The transient response of the folded beam actuator is a very important parameter and needs to be kept less than 10ms. A simple way to measure the transient time is to take a sequence of images with a high-speed camera and determine the actuator tip deflection as a function of time. This method has the advantage of measuring the overall transient time (thermal and electrical) directly, but requires a high-speed camera, which is quite expensive and was not available to the author. However, we can exploit the fact that the mechanical response is much faster and measure the thermal transient time only. As shown previously, the thermal response time is in the millisecond range. Using an infrared microscope to measure the temperature transition is possible, but the necessary equipment is even more expensive. A very simple solution of this problem was developed by the author and measures the thermal transient indirectly by the time variation of the actuator resistance. The measurement is straightforward and requires a
controlled current source and a digital storage oscilloscope. The thermal transient is extracted from the time variation of the total actuator resistance, $R_{\text{tot}}$, which is the sum of the constant series resistance, $R_s$, and the variable actuator resistance, $R_a(1 + \beta(T - T_0))$.

Assuming that the heating and cooling cycles have a thermal time constant, $\tau_{th}$, the average temperature change during cooling is

$$T - T_0 = \Delta T e^{-\frac{t}{\tau_{th}}}, \Delta T = T_{\text{max}} - T_0$$  \hspace{1cm} (2.100)

The voltage drop across the thermal actuator is then

$$\Delta V = IR_{\text{tot}} = I \left( R_s + R_a \left( 1 + \beta \Delta T \right) \right) = I \left( R_s + R_a \right) + I\beta R_a \Delta T e^{-\frac{t}{\tau_{th}}}$$  \hspace{1cm} (2.101)

The two-level current source developed for this experiment is shown in Figure 2.16. The high-current pulse heats the device to its working temperature, and then the low current is used to monitor the change of the actuator resistance during the cooling cycle.

Figure 2.16. Two level current source used in the thermal transient measurements.
The main part of the two-level current source is the three terminal linear regulator LM317, which is used in a current source configuration. The regulator maintains 1.25V difference between \( V_{out} \) and \( Adj \) terminals, and the resistance between them \( (R_i) \) sets the output current. A function generator produces a 5V digital signal, connected to JP1, which controls the MOSFET switch transistor. The input voltage from the signal generator is monitored with JP2. When the gate voltage is 5V, the transistor is conducting and connects \( R_2 \) in parallel with \( R_i \), thus increasing the output current. The capacitor \( C_i \) filters the supply voltage (5-9V) connected to pin 3 on JP1. The thermal actuator is connected to JP3, while JP4 is used to measure the actuator voltage drop. In this case four probes are not necessary, since the additional series resistance does not change with temperature and is included in the term \( R_s \) in (2.101). The bottom copper layer and the top silkscreen of the printed circuit board layout are shown in Figure 2.17. Using the described current source, the voltage drop across the thermal actuator during current switching was recorded with a digital storage oscilloscope Tektronix TDS3012B, and the result is presented in Figure 2.18.

![Figure 2.17. PCB layout of the current source (bottom copper layer and top silkscreen)](image)
The heating and cooling regions of the actuator are clearly visible on the rising and falling edges when the data is compared against the voltage drop across a large 1Ω discrete resistor, which could be assumed to stay at a constant temperature. The actuator resistance increases during the heating cycle (rising edge) and decreases during the cooling cycle (falling edge), generating the small exponential transients. Since it is always possible to decrease the time for reaching the operating temperature, it is important to find the cooling time, which is a function of the system itself and cannot be reduced by any other means.

![Figure 2.18. Actuator voltage drop during current switching (black), compared to the voltage drop across a large 1Ω discrete resistor (gray)](image)

Assuming an exponential temperature decay as described by (2.100), a nonlinear curve fit of the falling edge yields $\tau_a \approx 6.6 \text{ ms}$ (Figure 2.19), which is less than the required time of 10 ms according to Table 1.
Figure 2.19. Exponential curve fit

The measured time constant is in good agreement with the analytically determined one according to Figure 2.15. This completes the analytical and experimental study of the folded beam thermal microactuators, and the next chapter describes the fabrication of the MEMS array.
3. FABRICATION OF THE THERMAL ACTUATOR ARRAY

The operation of the clutch mechanism described in Figure 1.1 requires that the thermal actuators have displacement on the order of 50 μm. Since there is no external force applied to the actuator and the small in-plane actuator stiffness is not a problem, using long actuators improves the energy efficiency. The actuator length for this design was determined by the pixel-to-pixel separation, which is 2000 μm. Allowing additional space for power lines and bonding pads for the springs supporting the circular pixel button fixes the maximum actuator length to approximately 1400 μm. The fabrication process was chosen to accommodate these geometrical requirements.

Conventional polysilicon micromachining is not suitable for such long actuators, due to the thin polysilicon structural layer (~2μm). The small thickness makes the actuators very soft for in the out-of-plane deflection and often causes adhesion of the actuator to the substrate. Increasing the thickness of the polysilicon layer further is not practical, since the deposition process is conformal. Substantially thicker sacrificial layers can be realized using silicon on insulator (SOI) and deep reactive ion etching, but the processing and the SOI wafers are quite expensive. An excellent alternative for high aspect ratio structures is LIGA, but it requires synchrotron radiation and is expensive. However, the advancements in thick resist chemistry have made it possible to produce high quality photoresist molds using conventional near UV photolithography, thus avoiding the costly high-energy wafer exposure. When combined with nickel electroplating, this common process results
in very rugged structures and excellent yield. The details of the fabrication flow are described in the following.

The process flow is shown in Figure 3.1, with each step labeled separately. The fabrication starts by oxidizing a \(<100>\)-silicon wafer in oxygen atmosphere with water vapor. A thin seed layer of Ti/Cu is evaporated using electron beam, followed by spin coating of one layer of AZ4903 approximately 20 \(\mu m\) thick (Steps 1-3). The titanium is used as an adhesion layer, while the higher conductivity copper layer provides an excellent base for the electroplating. The chosen thick-film photoresist AZ4903 is routinely used for plating of structures up to 20 \(\mu m\) high, and produces very good sidewall profiles.

![Figure 3.1(a). Fabrication sequence (Steps 1-16)](image)

After the photoresist is patterned with UV and developed (Step 4), the freshly exposed seed layer is used to electroplate 20 \(\mu m\) of nickel (Step 5). A commercial nickel
sulfamate plating bath, Microfab Ni100 (Enthone, RI), is used at 150 mA (0.4 amps per square decimeter). So that a raised segment of the actuator contacts the vibrating plate, a second layer of photoresist is spun over the first nickel layer and used as a mold to electroplate a second layer of nickel (Steps 6-10). The old photoresist is then removed using acetone and a fresh layer is spun (Steps 11-12) to protect the structure during the back-side oxide etch (Step 13) with buffered oxide etch (BOE). Next, the Ti/Cu seed layer is etched between the plated structures since it is opaque to the IR used for back-to-front alignment (Steps 14-15). Photoresist is spun on the top and bottom of the wafer, and the bottom layer is patterned with UV (Steps 16-17).

![Fabrication sequence](image)

**Figure 3.1(b). Fabrication sequence (Steps 17-21)**

The 3” MEMS wafer is then flipped and bonded with photoresist to a 4” carrier wafer for the following deep reactive ion etching (DRIE). After the etching, the 3” MEMS wafer is released from the carrier wafer with acetone and the oxide for the etch-stop is removed (Steps 18-19). The final step of the process includes etching of the copper seed-layer and the titanium adhesion layer to separate the micro actuators electrically (Steps 20-21). The
selective removal of copper with respect to the nickel structures is achieved by timed wet etching in a (NH₄)₂S₂O₈ solution [41]. During this step, the wider nickel features are not completely undercut and are therefore fixed to the underlying substrate in order to anchor the device. The complete processing sequence including all fabrication parameters (layer thicknesses, spin rates, exposure times etc.) is included in Appendix A. The parameters of the DRIE are also included in Appendix B. A pair of fabricated devices is shown in Figure 3.2. The cold arms have release holes, allowing the Cu etchant to undercut them.

Figure 3.2. Fabricated MEMS switch: (a) Pair of two thermal actuators (b) Close view of the actuator switching end

One of the most challenging steps in the fabrication of the MEMS tactile array was the deep reactive ion etching. Two major problems were encountered in the DRIE step: (1) non-uniform etching and (2) misalignment between the plated features on the top of the
wafer with respect to the etched pattern. Prior to etching, the MEMS wafer is bonded to the 4" carrier wafer with photoresist to protect the electroplated structures and provide polished surface for the electrostatic gripper in the DRIE chamber. During the DRIE the photoresist heats to a high temperature and starts to ‘boil’, which results in varying elevation of the MEMS wafer from the carrier and consecutively, different cooling rates. The regions of the MEMS wafer that are separated from the carrier wafer with bubbles are hotter and the etch rate is faster. Aside from the different etch rate, the photoresist bubbles cause an angular wafer-to-wafer misalignment, which effectively shifts the lateral alignment of the MEMS wafer with up to 5 microns. The effect of different etch rates is illustrated in Figure 3.3 showing the etch results at two different wafer locations.

![Figure 3.3: DRIE progress after 2 hours etching](image)

(a) Left side of the wafer (b) Right side of the wafer
The absence of photoresist bubbles under the portion of the wafer shown in Figure 3.3(a), resulted in good cooling, lower local temperature and therefore lower etch rate. The other side of the wafer was relatively isolated from the cooled carrier wafer due to the photoresist boiling, resulting in faster etching. Another cause for etch nonuniformity is the loading effect, which slows down the etching of small features. Because of this effect, the small gap between the cylindrical tactor button and the rest of the die etches slower than the wide openings under the thermal actuators, as shown in Figure 3.4(a). Therefore, controlled over etching is required to separate the tactor pixel completely - Figure 3.4(b). It is important to notice that the rounded corners in the Figure 3.4(a) are due to under etching, and once the etching is complete, the corners become sharp as designed.

Figure 3.4. Under etched part of the wafer (a), compared to completely etched through part (b)

Unfortunately, the over etching required to compensate for the thermally induced etch rate nonuniformity and the loading effect increases the separation between the protruding pixel and the thermal actuator as shown in Figure 3.5.
The larger actuator-to-pixel separation requires larger actuator stroke, which increases the power consumption. Besides, since the stroke is limited, if the separation is above a certain amount, the device is nonfunctional. Changing the geometry of the DRIE mask as shown in Figure 3.6 significantly reduced the problems arising from the over etching.

Figure 3.5. Close view of the overetched portion near the pixel

Figure 3.6. Change in DRIE mask geometry (a) First iteration (b) Final DRIE geometry
The two important changes made in the DRIE mask layout were the more uniform feature size and the extension of the protruding pixels under the ends of the thermal actuator. The similar feature size results in almost uniform etch rate, thus neutralizing the loading effect. The button extension, on the other hand, compensates for the over etching and decreases the pixel-to-actuator separation. If we overcompensate, the ends of the thermal actuators will stay above the protruding pixel, which will be always engaged. However, it was found that applying controlled back bending of the thermal actuators could rectify this situation. This is accomplished by overheating the actuators for a short period of time, resulting in retraction of the actuators from their original position.

Finally, the problem of photoresist ‘boiling’ was alleviated by removing the photoresist layer from the center of the 4” carrier wafer, leaving a small bonding rim with the diameter of the MEMS wafer. The smaller photoresist area released less gas and made the wafer debonding after DRIE much easier.

The last step in the fabrication of the MEMS array is the soldering of the 30-pin connector directly to the die. The soldering proved to be quite challenging partly because of the 500-micron pin-to-pin spacing and partly because of the high thermal conductivity of silicon. The small spacing makes the manual alignment quite difficult and facilitates short circuit connections. The high thermal conductivity of the substrate prohibits the use of point contact soldering iron and necessitates uniform heating on all the pins.
simultaneously. A hot air soldering station Weller WHA-300 was bought specifically for this purpose and yielded excellent results as shown in Figure 3.7.

Figure 3.7. Connector soldering (a) Manual with a soldering iron (b) Using WHA hot air soldering station

The solder uniformity was greatly improved by using preformed Tin-Lead solder sheets approximately 2 mils thick. Special flux MSF-003-NI from Piezo Systems Inc. was used to wet the surface of the nickel traces, after which the solder sheets were re-flown. After the MEMS array was connected to the power source, the thermal actuators were energized to verify that they could reach over the protruding pixels. Figure 3.8(a) shows one thermal actuator pair prior to turning the thermal actuators on, and Figure 3.8(b) shows the pair in the final engaged position for a 240 mA actuation current.
Figure 3.8. Thermal clutches in ‘off’ position (a) engaged thermal clutches (b).

The test was successful, showing that the actuators have sufficient stroke for the pixel switching. The next chapter describes the design of the electronic control module.
4. DESIGN OF THE ELECTRONIC CONTROL MODULE

In order to demonstrate the hybrid actuation approach, the thermal clutches and the piezoactuators must be engaged simultaneously, which required the development of a custom embedded control module. The main objective in the electronics design was to create a low power microcontroller-based system, which interfaces to the MEMS array and the piezo actuators, capable of generating pre-programmed pixel sequences. In addition to that, the tactile display proof-of-concept required a very small (approximately 1 inch square), self-contained unit powered by a single watch battery.

While designing the control electronics, special attention was paid to the power efficiency of the system, since battery life is critical in portable applications. The high power consumption of the folded beam microactuators makes the optimization of their driving central in the electronics development. During the electro-thermal analysis, it was assumed that a constant current source powers the folded beam thermal actuators. However, implementing current excitation in the electrical schematic requires the addition of analog constant current source connected in series with the actuator, resulting in a significant power loss. In this work, pulse width modulation technique (PWM) is used to supply the thermal actuators with the exact amount of power required for a fixed beam deflection. This solution eliminates the constant current source and the power loss associated with it, and simplifies the schematic. Furthermore, since the PWM is entirely under software control, the system is more flexible and could easily vary the power supplied to different pixels, accommodating the fabrication tolerances across the MEMS
array. The PWM technique could also be used to compensate the variation of the battery voltage by adjusting the fill ratio of the driving signal, avoiding actuator overheating and thus increasing the device reliability. A serious challenge in the electrical design is the low resistance of the folded beam thermal actuator pair – around 1.5 \( \Omega \), which requires low-resistance connection between the MEMS array and the control module. The connection resistance consists of the resistance of the printed circuit board traces, the contact resistance between the die connector and the metal conductors on the MEMS layout and the ‘on’ resistance of the driving transistors. In this design, special MOSFET switches with \( R_{on} = 33 \text{m\Omega} \) were selected and the resistance of the PCB traces and connectors was kept to a minimum. Finally, the small footprint of the module makes its layout design and final assembly difficult, and necessitates the use of the smallest commercially available packages. A complete block diagram of the hybrid actuator array and the control electronics is shown in Figure 4.1.

Figure 4.1. Hybrid actuator block diagram.
The heart of the electronic circuit is an 8-bit RISC microcontroller with advanced power-saving features - PIC16F819 from Microchip. A single 1.5V watch battery type S76 is used as a power source. The microcontroller engages the individual pixels of the MEMS array through a low resistance MOSFET multiplexer, and deflects the piezo-actuators by turning them on and off via a high-voltage driver. A micro-power switch mode power supply provides the power for the microcontroller, converting the low battery voltage to stable 5V. The individual functional blocks are described in detail in the following.

4.1 Micropower DC-DC Converter

The DC-DC boost converter powers the PIC microcontroller with 5V using the 1.5V supplied by the battery and is built around UCC39411 – a low power synchronous boost converter manufactured by Texas Instruments. The main advantage of this circuit is that it handles a wide range of input voltages – from 1V to 3.2V, and once the converter starts, it operated down to 0.5V thereby maximizing the battery usage. Given the low
power consumption of the PIC (1.7mA at 5V for 8MHz clock) additional advantage is the low 35μA quiescent current improving the overall conversion efficiency. The complete schematic of the converter is shown in Figure 4.2 and is designed according to TI data sheet [42].

4.2 Multiplexing the MEMS Thermal Actuator Array

The MEMS thermal actuator array is arranged electrically in five columns and four rows for a total of 20 pixels as shown in Figure 4.3. The thermal actuators for each column share a common electrode, which appears to the left of the actuators in the figure. The MEMS array is connected to the electronics control module with a flexible 0.5mm pitch cable, inserted in a 30-pin connector soldered directly to the MEMS array as shown in Figure 3.7.

Figure 4.3. MEMS thermal actuator array
The pixels are energized sequentially by applying voltage difference between the corresponding common electrodes (column select) and the individual contacts (row select). A partial schematic of the multiplexer is presented in Figure 4.4 with only two actuator columns shown for clarity (the thermal actuators are drawn as resistors). The battery voltage is redirected to the chosen pixel by addressing the corresponding row and column MOSFET switches. Four switches connect the selected row electrodes to the battery and additional 5 switches connect the common electrodes to ground.

![Figure 4.4. Multiplexer partial schematic showing](image)

The multiplexer serves two purposes – to switch from pixel to pixel and to vary the power supplied to the selected pixel. The pixel switching is achieved via addressing the new pixel with the corresponding row and column, while the variable power output is generated through pulse width modulation (PWM). Once the pixel is selected, one of the gates (column of row) is driven with a square wave with variable fill ratio.
\[ T_{on} / (T_{on} + T_{off}) \], where \( T_{on} \) is the time when the MOSFET switch is conducting and \( T_{off} \) is the time when the switch is ‘off’. The power dissipated in the thermal actuator is linearly proportional to the fill ratio of the driving square wave. The microcontroller measures the input battery voltage, which is in the range of 1.0V to 1.6V and adjusts the fill ratio such that the power consumption of the thermal actuator remains constant. This feed-forward control helps to keep the displacement of the thermal clutches constant, regardless of the battery voltage, improving the energy efficiency. Finally, the microcontroller produces complex sequence of pixel patterns according to a software-programmed sequence. In order to achieve pixel protrusion, the piezo actuator must be engaged simultaneously with the thermal actuator. The next paragraph describes the high voltage driver used for the piezo actuators.

4.3. High Voltage Piezo Actuator Driver

The designed high-voltage driver (Figure 4.5) has two control inputs – one for charging the piezo actuators \( (HV_{on}) \) and one for piezo discharging \( (HV_{off}) \). When the microcontroller applies logic high (5 V) to \( HV_{off} \), the piezo actuator capacitance is discharged to ground through \( R_s \) and \( T_2 \). When \( HV_{on} \) is at 5V, part of the collector current of \( T_1 \) flows through the base of \( T_3 \) and its collector current charges the piezoelectric actuator capacitance.
The high voltage steady state current consumption required to keep the piezoactuator engaged is determined by $I_{HV} = V_{HV} / R_6 = 120V / 1M\Omega = 120\mu A$. The Zener diode $D_2$ protects the base-emitter junction of $T_3$ and ensures that the maximum voltage difference across this junction is equal to the Zener voltage (2.4 V), which is less than the base-emitter breakdown voltage of $T_3$. Finally, the resistor $R_5$ discharges the base capacitance of $T_2$ and increases the switching speed. The value of $R_5$ must be chosen such that the voltage drop across it with $T_3$ disconnected is larger than $V_{BE}$ of $T_3$ (approximately 0.7 V). With the current values of $R_5$ and $R_6$, this voltage drop is $V_{R5} = 120V \times R_5 / (R_5 + R_6) = 2.05V$. The microcontroller moves the piezo actuators up by setting $HV_{on} = 5$ V and $HV_{off} = 0$ V, returns them to the initial position with $HV_{on} = 0$ V and $HV_{off} = 5$ V and leaves the positive lead disconnected if $HV_{on} = 0$ V and $HV_{off} = 0$ V. The combination when both control lines are held high is to be avoided since $T_2$ and $T_3$ are conducting, which results in unnecessary energy dissipation.
In order to test the high voltage driver, the microcontroller was programmed to charge, discharge and disconnect the piezo actuators for 20ms, and the results are shown in Figures 4.6 and 4.7. The voltage waveform across the piezo actuators shown in Figure 4.6 reveals very good driver performance with rise time approximately 84µs, which is much shorter than the required switch time of 5ms. The experiment was performed for two sets of resistor values: $R_S = 17.4k\Omega, R_6 = 1M\Omega$ and $R_S = 510k\Omega, R_6 = 10M\Omega$. The voltage waveforms in both cases are similar. The high voltage current consumption however, is quite different as shown in Figure 4.7.
The current consumption in the case of the lower value resistors could be divided clearly in several sections. First, $HV_{ON}=5V$ and $HV_{OFF}=0V$ and the piezo actuator is charged with higher current, represented by a peak in the power consumption. Few milliseconds after the start of the charging, the piezo actuators reach the supply voltage, and the consumed current equilibrates to $I_{HV} = V_{HV} / R_6$. The microcontroller then turns off $HV_{ON}$ and applies logic high to $HV_{OFF}$, discharging the piezo capacitor. After the initial transient, the consumed current reaches zero. Finally, both control inputs are at logic low, and the charged piezo actuator is disconnected from the high-voltage power source. The current consumption in the case of $R_5 = 510k\Omega$, $R_6 = 10M\Omega$ has similar structure, but is affected by the significantly larger time constant $R_6C_{par}$. The achieved lower average
high-voltage power consumption is at the expense of the increased rise time, which could be noticed in Figure 4.6.

4.4. Physical Layout Of The Control Module

The complete layout of the control module including the DC-DC converter, microcontroller, low voltage MEMS multiplexer and the high voltage driver for the piezo actuators is shown in Figure 4.8.

Figure 4.8. Layout of the electronics control module including the DC-DC converter, multiplexer, microcontroller and high voltage driver
All the components are surface mounted on one side of the printed circuit board, while the bottom side is stacked against the piezoassembly discussed in Chapter 5. The detailed schematic is included in Appendix C, the top and bottom PCB layouts could be found in Appendix D, and the bill of materials is in Appendix E. The board requires 1.5V and 120V external supplies, and controls the MEMS array and the piezoactuators according to the software programmed in the microcontroller. Since the microcontroller is soldered directly to the board, a 10-pin connector (right side of the board in Figure 4.8) is provided for in-circuit programming and software updating. This connector also supplies 5V from the DC-DC converter and is connected to the unused I/O pins of the microcontroller.

For microcontroller programming, the board is connected to the programming adapter shown in Figure 4.9, which is inserted in the zero-insertion-force (ZIF) socket of PICSTART development programmer from Microchip.

The MEMS actuator array is powered via a flexible 30-pin cable, inserted in the top connector of the controller board and in a matching connector soldered directly to the
MEMS array. Special attention was paid to place the connector at the exact same position as the end of the MEMS array to avoid cable twisting, which puts stress on the MEMS die and may lead to trace debonding from the silicon substrate. The piezoactuators are connected to the module by the two bonding pads under the 10-pin connector. The initial software development was performed using an array of light emitting diodes, replacing the MEMS array and is described in the following.

4.5 MEMS Multiplexer Test

The testing of the MEMS multiplexer was greatly simplified by substituting the array of thermal actuators with a printed circuit board with light emitting diodes (LEDs). Each LED represents one vibrating pixel of the tactile display. The schematic and the board layout are shown in Figure 4.10.
This visual pixel representation proved to extremely helpful in identifying bad solder joints of the 30-pin connector and the MOSFET switches. Using this board as a visual aid, the firmware for the microcontroller was developed and debugged. The final version of the source code written in Microchip assembly (MPASM) is included in Appendix H. The program displays the numbers from 0-9 in a sequence by drawing them on the LED display one pixel at a time, emulating the operation of the tactile display. If the delay between the pixels is chosen appropriately, the human observer gets a sensation of a continuous drawing, a visual phenomenon similar to the tactile illusion. To complete the tactile display prototype, the control board needs external 1.5V and 120V, supplied by the separate board described in the following.

4.6 High Voltage Generator
The high voltage generator is based on a commercially available electroluminescent lamp (EL) driver HV803, manufactured by Supertex Inc. This device is typically used to provide the high voltage necessary for screen backlight of portable devices. The generator block diagram provided by Supertex [43] is shown in Figure 4.11.

![Block diagram of the HV803 EL driver](from Supertex datasheet)

Figure 4.11. Block diagram of the HV803 EL driver (from Supertex datasheet)
As shown in the Figure 4.11, the EL driver consists of two major parts – a high voltage generator and a lamp driver, operating in H-bridge mode to double the voltage swing across the EL lamp. The custom high voltage generator utilizes the HV803 and provides DC 110V for the piezo actuators. The board schematic is shown in Figure 4.12.

![Figure 4.12. Schematic of the high voltage generator](image)

The design in Figure 4.12 is based on Supertex application note. The HV803 is supplied with 5V provided by the DC-DC converter on the controller board. To save power, the converter is turned off when not needed by applying logic low on RB2 pin of the microcontroller. This pin is connected to the forth lead of the programming connector. The output DC voltage is adjusted by varying the resistor $R_1$, which is connected in series with the voltage divider in the HV803 chip. This design does not use the H-bridge driver inside HV803, and soldering the jumper $R_4$ ($R_4=0\Omega$) connecting $R_4$ to ground can turn off its oscillator to save power.
In addition to the high voltage generator, this second board includes a battery holder for the main 1.5V battery. The final printed circuit board is shown in Figure 4.13.

![Figure 4.13. PCB layout of the high voltage generator](image)

The layout in Figure 4.13 has the same dimensions as the control board, as well as matching holes for the alignment pins. The board is stacked on top of the control module and is connected to it via a small 10-pin flexible cable and four solder joints providing connections for the battery, 120V and GND. The 10-pin cable supplies the high voltage generator with 5V and allows software on/off control.

The high voltage source for the piezoactuators is the last part in the hybrid actuator block diagram as shown in Figure 4.1. The next chapter describes the building of the mechanical assembly and the mounting of the piezoelectric actuators.
5. PIEZOELEMENT DESIGN AND ACTUATOR SWITCHING TESTS

5.1 Piezoelectric Actuators

The macro-scale piezo actuator consists of three piezoelectric bimorphs with length $l = 16\text{mm}$ and width $w = 5\text{mm}$. A schematic diagram illustrating the operation of one piezo beam is shown in Figure 5.1.

Each piezoelectric beam consists of two layers of piezo ceramic material separated with a central conductive brass layer. The two ceramic layers are poled in opposite directions, which generates strain with different polarity under the same applied voltage. In this configuration, the lateral extension of the piezoelectric material is used, described by the piezoelectric coefficient $d_{31}$. When the electric field is perpendicular to the beam, as shown in Figure 5.1, the lateral extension of the bottom piezoceramic layer and the lateral contraction of the top layer cause the whole structure to bend upwards as shown in the figure. In this work, commercially available double-layer piezo material T215-H4CL-503X from Piezo Inc. was used. A summary of the piezo beam characteristics is compiled in Table 4. Note that 1mm is assumed bonded to the substrate, which reduces the useful
length of the actuator. Variation of the bonding length is the main cause of variation in the tip deflection, and has to be minimized.

### Table 4. Piezoactuator summary (single beam)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>$l = 16\text{mm}$</td>
</tr>
<tr>
<td></td>
<td>(1mm for bonding)</td>
</tr>
<tr>
<td>Width</td>
<td>$w = 5\text{mm}$</td>
</tr>
<tr>
<td>Thickness</td>
<td>$t = 380\mu\text{m}$</td>
</tr>
<tr>
<td>Tip displacement</td>
<td>$\delta = 67\mu\text{m}$ (Free end)</td>
</tr>
<tr>
<td>Maximum force</td>
<td>$F = 133\text{mN}$</td>
</tr>
<tr>
<td>Driving voltage</td>
<td>$U = 120\text{V}$</td>
</tr>
<tr>
<td>Resonant frequency</td>
<td>$f = 1.2\text{kHz}$</td>
</tr>
<tr>
<td>Capacitance</td>
<td>$C = 4.5\text{nF}$</td>
</tr>
</tbody>
</table>

5.2 Piezoactuator Wiring Board

The wiring board for the piezoactuators is the base of the hybrid actuator assembly, as shown in Figure 1.2. A standard FR4 epoxy-fiberglass substrate was used because of the low cost and ease of fabrication. Besides, the low thermal conductivity of the board makes the soldering easier. The layout of the piezo wiring board is shown in Figure 5.2.

![Figure 5.2. Layout of the piezo wiring board](image-url)
The board was fabricated using conventional photolithography and copper etching and has bonding pads for the three piezoactuators as well as alignment marks and holes for the final assembly. All three piezoactuators are connected in parallel to the external voltage, supplied by the connector shown on the left in Figure 5.2. The fabricated board is shown in Figure 5.3, and includes the vibrating plate fixed at the tip of the actuators.

![Image](image.png)

Figure 5.3. Piezoelectric assembly

Initially, the piezoactuators were soldered to the board, but this technique was found ineffective, because of the actuator degradation caused by the heating. Two processes take place when the piezo material is heated:

- The higher kinetic energy of the atoms caused by the temperature rise results in piezo-depolarization. The material used in these actuators has low Curie temperature (around 300°C) and is very sensitive to overheating.
During heating, the pyroelectric effect causes charge build-up. This charge creates a field in the direction of the polarization and is not detrimental. However, during cooling the field generated by this charge changes direction and depolarizes the piezo ceramic.

The second effect can be avoided by shorting the piezo electrodes to prevent the charge build-up. In our case shorting of the electrodes can be done only after they are connected (soldered), and this method is not applicable. To assess the degree of piezo depolarization, the actuator capacitance can be measured. The following table shows the experimental results of a soldering test and indicates 60% displacement drop due to the overheating.

Table 5. Piezo Soldering Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Before soldering</th>
<th>After soldering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitance</td>
<td>4.68nF</td>
<td>3.44nF</td>
</tr>
<tr>
<td>Displacement</td>
<td>75 microns</td>
<td>25 microns</td>
</tr>
</tbody>
</table>

The piezoactuator soldering was abandoned due to the displacement degradation and replaced with conductive epoxy bonding. Since the conductive epoxy is mechanically weak, conventional epoxy was added for mechanical strength. The following procedure proved to be reliable and resulted in excellent bonding:

- The bonding pad of the wiring board is surrounded with scotch tape as a mask.
- A drop of conductive epoxy is placed in the center of the PCB bonding pad.
- Two drops of conventional epoxy are put on both ends of the pad.
• The excess epoxy and the scotch tape mask are removed, leaving a very thin epoxy film on the bonding pad.
• The piezo beam is aligned and pressed for 15 minutes.
• The top piezo electrode is contacted with a thin wire soldered to the PCB and connected to the piezo beam with conductive epoxy.

The next stage in the stack fabrication is the fabrication and attachment of the vibrating plate described in the following chapter.

5.3 Vibrating Plate Fabrication

The vibrating plate was fabricated from a silicon wafer using bulk anisotropic etching in KOH with silicon oxide utilized as a masking material. The bottom side of the diced vibrating plate is shown in Figure 5.

![Vibrating plate](image)

Figure 5.4. Vibrating plate

The three spheres at the tips of the piezo actuators rest in the pyramidal pits etched in the silicon plate. After the etching, the mask oxide is stripped and a new oxide layer is
regrown. The oxide insulates the tactile array from the top electrode of the piezoelectric actuators, which is connected to the positive terminal of the high voltage supply.

5.4 Displacement And Vibration Analysis Of The Hybrid Actuator.

The interaction between the piezoactuators and the vibrating pixels affects both the steady state and the dynamic performance of the hybrid actuator array and is discussed in detail in the following.

5.4.1 Steady State Displacement Analysis.

The stroke of the tactile pixels is determined by the height of the second electroplating layer of the MEMS array. In order to maximize the pixel protrusion, the plating thickness should be approximately the same as the displacement of the vibrating plate. However, the pixel stroke is reduced due to the stiffness of the springs supporting the pixel. The spring design needs to satisfy the following conflicting requirements:

(1) The springs need to be stiff enough to follow the vibrating plate closely.

(2) The springs must be soft not to reduce the displacement of the piezoactuators.

A finite element model of one of the springs was created to determine the stiffness accurately. The resulting deflection of the spring for 4.075 mN applied at the end is shown in Figure 5.5. The tip deflection at the tactile pixel was found to be 20 μm, resulting in spring stiffness $k_{spring} = 203.7 ~ N/m$. 
Figure 5.5. Support spring deflection for 4.075mN applied at the end

According to the data provided by the piezoactuator manufacturer, the stiffness of one beam is $k_{\text{piezo}} = 1985 \, N/m$. Therefore, the steady state deflection of the protruding pixel can be found from

$$\delta_{\text{pixel}} = \delta_{\text{piezo}} - \frac{F}{3k_{\text{piezo}}}, \quad F = 4\delta_{\text{pixel}}k_{\text{spring}}$$

(5.1)

The numerical factors three and four in (5.1) reflect the fact that there are three piezoactuators and each pixel is supported by four identical springs. Simplifying (5.1) yields
As expected, the decrease in the displacement due to the spring stiffness can be tolerated. The next chapter analyzes the transient response of the system.

### 5.4.2 Vibration Analysis

The natural frequency of the moving pixel suspended by the four springs is

$$f_{\text{pixel}} = \frac{1}{2\pi} \sqrt{\frac{k_{\text{pixel}}}{m_{\text{pixel}}}} = \frac{1}{2\pi} \sqrt{\frac{4k_{\text{spring}}}{\pi d^2}} = 6.9\text{kHz}$$ \hfill (5.3)

The self-resonant frequency of the piezoactuator is much lower: $f_{\text{piezo}} = 1.2\text{kHz}$ (Table 5).

The following transient analysis assumes that there is certain small separation between the MEMS pixel and the vibrating plate. Upon application of the high voltage to the piezo actuators, the vibrating plate accelerates and hits the pixel. The voltage pulse may be assumed close to an ideal step function since the electrical raise time of the driver described in chapter 4.3 is only $84\mu s$. Since the mass of the vibrating plate is much lower than the mass of the pixel, conservation of energy and momentum (assuming perfectly elastic collision) results in

$$V_{\text{pixel}} = 2V_{\text{plate}}$$ \hfill (5.4)
Here, $V_{\text{pixel}}$ is the velocity of the pixel after the impact, and $V_{\text{plate}}$ is the velocity of the vibrating plate before the impact. The maximum amplitude of the induced oscillation of the pixel, $A_{\text{pixel}}$, can be found from

$$\frac{m_{\text{pixel}} (2V_{\text{plate}})^2}{2} = \frac{k_{\text{pixel}} A_{\text{pixel}}^2}{2}$$

(5.5)

Substituting (5.3) in (5.5) yields

$$\frac{2V_{\text{plate}}}{A_{\text{pixel}}} = w_{\text{pixel}}$$

(5.6)

The maximum velocity of the vibrating plate is

$$V_{\text{plate}} = A_{\text{piezo}} w_{\text{piezo}}$$

(5.7)

Substituting (5.7) in (5.6) determines the oscillation amplitude of the pixel

$$A_{\text{pixel}} = A_{\text{piezo}} \frac{2w_{\text{piezo}}}{w_{\text{pixel}}} = 19 \mu m$$

(5.8)

A sketch of the pixel response is presented in Figure 5.6. Note that the vertical velocity of the pixel after the impact is twice the velocity of the plate, which is shown with the different slope. In addition to that, the pixel is reflected from the vibrating plate and follows the plate oscillation. It is important to notice that the described simple vibration analysis does not include the finger pressing on the tactile display. The added viscous damping and stiffness can change the described picture significantly. The next chapter describes the final assembly of the hybrid actuator and the switching tests.
5.5 Final Mechanical Assembly and Actuator Switching Tests

The final design of the mechanical assembly consisted of spaced and a top plate, which are shown in the following figure.

![Image of mechanical components]

Figure 5.7. Mechanical components (left) Spacer (right) Top Plate

The individual components are assembled with a resistive slip fit using stainless steel dowel pins. The final assembly of the tactile array is shown in Figure 5.8.
After bonding of the piezo beams and the vibrating plate, the plate vertical displacement was measured to be in the range of 50 μm to 60 μm. During the testing of the stack, it was established that the tolerances in the spacer geometry and the piezo beam bonding are larger than the thickness of the second electroplating layer (11 μm), making proper alignment and switch demonstration very difficult. In order to test the tactile perception, one actuator pair was deformed plastically above the center of the pixel, increasing the effective stroke from 11 μm to approximately 30 μm and alleviating the alignment problem. With this new protrusion height, distinct tactile perception was produced with the piezo actuators.
6. CLOSING REMARKS

This dissertation presents the complete design of a micromechanical switch array for tactile communication. The device utilizes a custom developed MEMS actuator array for switching of the mechanical motion of a piezoelectric actuator. This hybrid approach alleviates the main shortcoming of the conventional micro-electro-mechanical systems – the limited stroke and force characteristics, and combines the high-density and batch processing of the MEMS components with the high force/stroke and efficiency of the macro-scale actuation. During the testing of the prototype, reliable tactile perception was established for a protrusion of 30 μm. A major contribution of this dissertation is development of a very small self-contained prototype through integration of MEMS, mechanical design and electronics design into a single device. This integration allowed significant decrease in size compared to the existing tactile displays.

The main component of the hybrid actuator – the folded beam microactuator, was studied analytically, both for steady state performance, as well as transient response. The developed analytical models showed good agreement with the finite element simulations and the experimental measurements. Special attention was paid to the improvement of the existing mechanical models for the folded beam actuator, and the importance of the nonlinear mechanical effects, such as large strain effects and buckling effects, was studied extensively. The insights gained from the analytical analysis resulted in the development of a simple procedure for geometry optimization of the folded beam actuator.
One of the major problems addressed by this dissertation is the development of a reliable, high-yield MEMS fabrication process, compatible with the requirements of the MEMS tactile display. The process is a combination of surface micromachining and bulk micromachining, and uses conventional near-UV photolithography, nickel electroplating and deep reactive ion etching. The utilization of conventional equipment makes the process low-cost and allowed in-house manufacturing.

A custom electronic module controlled the MEMS array and the piezoelectric actuator. The combination of a high performance microcontroller and the smallest commercially available surface mount parts allowed significant size reduction, and the prototype was approximately the size of a wristwatch.

Future device improvements include the increase of the thickness of the second electroplating layer and the introduction of intermediate gold plating. The thicker second nickel layer will make the device less sensitive to mechanical misalignment during the final assembly. The addition of a thin gold plating layer will provide cleaner base for the second plating and will ease the connector soldering.
APPENDICES
APPENDIX A: DETAILED FABRICATION PROCESS

Step 1a – Wafer Oxidation
1. Oxidize the wafers for 4 hrs @1080 C with steam (water heater turned to max)
2. Immediately after oxidation put the warm wafers in the E-beam. If the oxidized wafers stay in a box for more than a day, proceed to Step 1b - Wafer cleaning

Step 1b – Wafer Cleaning
1. Clean the oxidized wafers:
   150ml Hydrogen Peroxide 37%
   500ml Sulfuric Acid
   Total time: 15 min
2. Rinse in DI water for 5 min
3. Spin dry @3000 PRM
4. Dehydration bake 40 min @ 200deg hotplate

Step 2 – Ti/Cu Electron Beam Deposition
1. Ti deposition:
   • Ramp 5mA/min from 0 to 51 mA
   • Soak @ 51mA 3 min
   • Deposit @ 51mA
   • Initial conditions: 0.3A/sec p=9e-7 Torr
   • Deposition time 7 min 45 sec
   • Final conditions: p=5e-7 Torr
   • Total thickness: 10nm
   • Ramp down 20mA/min
   Note: The pressure goes down when the shutter is opened.
2. Wait 5 min to cool down the crucible
3. Cu deposition:
   • Ramp 5mA/min from 0 to 50 mA
   • Soak @ 50mA 5 min
   • Deposit @50mA
   • Initial conditions: 1.8 A/sec p=2.7e-6 Torr
   • Deposition time: 20 min
   • Final conditions: N/A Torr
   • Total thickness: 200nm
   • Ramp down 20mA/min

Step 3 – First AZ4903 Lithography – MASK1
1. Dispense AZ4903 photo resist with the pipette and remove the bubbles.
2. Spin @ 2500 RPM 60 sec (include 5 sec spreading @ 500RPM)
   Note: The resist thickness should be around 13 microns
3. Wait 10 min for the resist to relax.
4. Soft bake 3 min @110deg on the hot plate full contact (Al foil)
5. Remove the edge bead with Acetone @ 3000 RPM
6. Clean the backside of the wafer with acetone and a cotton swab
7. Wait 60 min
8. Expose 5.2 min @~ 3.2 mW/cm²
9. Wait 30 min
10. Develop 3 min in AZ400K diluted developer
11. Rinse with DI water and dry with nitrogen (or spin dry)
12. Inspect under the microscope

**Step 4 –First Nickel electroplating**
1. Plating settings:
   - Temperature: 50deg
   - Distance between the anode and cathode: 7 hollow slots
   - Agitation: Magnetic stirrer (Big hot plate dial setting – 7)
   - Solution: Nickel solution with wetting agent
   - Current: 80mA
   - Voltage: 1.4V
   - Plating time: 2hr 30 min (low current density to reduce stress)
   - Deposited thickness: 10.5 microns
2. Clean the wafer with Acetone @ 5000 RPM
   **Note:** If the adhesion is not sufficient, at this speed the structures will detach
3. Clean the PR residue with PG remover
4. Rinse in DI water for 10 min
5. Bake 15 min @60 deg hotplate

**Step 5 – Second AZ4903 Lithography – MASK2**
1. Dispense AZ4903 photo resist with the pipette and remove the bubbles.
2. Spin @ 2000 RPM 60 sec (include 5 sec spreading @ 500RPM),
   **Note:** The resist thickness should be around 15 microns
3. Wait 10 min for the resist to relax.
4. Soft bake 3 min @110deg on the hot plate full contact (Al foil)
5. Wait 5 min
6. Repeat steps 1-4 to get two layers of PR
7. Remove the edge bead with Acetone @ 3000 RPM
8. Clean the backside of the wafer with acetone and a cotton swab
9. Measure the thickness of the photoresist
   **Note:** 22 microns above the Ni and additional 13 microns, total thickness 35 microns after the second coat
10. Wait 60 min
11. Expose 15 min @ ~3.2 mW/cm²
12. Wait 30 min
13. Develop 3 min in AZ400K
14. Rinse with DI water and dry with nitrogen (or spin dry)
15. Inspect under the microscope
Step 6 - Second Nickel electroplating
1. Width of the electroplating rim – 3mm average. Total plating area 0.071 Dm^2
2. Reverse plate for 30 sec at 200mA
   Note: 1min 30 sec completely etches the copper seed layer
3. Plating settings: Temperature: 50deg
   - Distance between the anode and cathode: 7 hollow slots
   - Agitation: Magnetic stirrer (Big hot plate dial setting – 7)
   - Solution: Nickel solution with wetting agent
   - Current: 40mA
   - Voltage: 1.4V
   - Plating time: 1h 35 min
   - Deposited thickness: 18 microns (brakes only) and 10.5 microns beams from the previous plating
4. Rinse in DI water and spin-dry
5. Clean the PR with acetone @5000 RPM (the high RPM will remove lose features)
6. Clean the PR residue with PG remover
7. Rinse in DI water

Step 7 - Seed layer etch
1. Etch the copper with: 20g Ammonium Persulfate @30 deg 60g DI water
2. Rinse in DI water
3. Etch the Ti adhesion layer with 10:1 BOE for 20 sec
4. Rinse in DI water for 10 min
5. Spin-dry @3000 RPM
6. Inspect under the microscope
7. Label the wafer with a diamond scribe
8. Test the plating adhesion with scotch tape
9. Measure:  Plating thickness
     Brake thickness
     Oxide thickness
     Wafer thickness (for the DRIE)
10. Dehydration bake @ 60 deg for 15 min

Step 8 - Etch the oxide from the back side
1. Spin AZ4903 @2000 RPM (include 5 sec spreading @ 500RPM) – front side
2. Wait for 10 min
3. Soft Bake @100 for 3 min on hotplate full contact (Al foil used)
4. Wait 10 min for the wafer to cool down
5. Clean the backside of the wafer with acetone (use wipes and cotton swab)
6. Dip in BOE for 25 min to etch the back side oxide
   Note: If the BOE has surfactant it will wet the silicon so for endpoint detection dip in DI
7. Remove the front side photoresist with acetone and PG
8. Rinse with DI water for 10 min

Step 9 – DRIE Photolithography – MASK3
1. Spin AZ4903 photo resist @ 2500 RPM 60 sec (include 5 sec spreading @ 500RPM)
2. Wait 10 min for the resist to relax.
3. Soft bake 3 min @105deg on the hot plate full contact
4. Wait 5 min to cool down
5. Remove the edge bead from the front side with Acetone @ 3000 RPM
6. Spin resist on the back side repeating steps 1-4
7. Remove the edge bead from the back with Acetone @ 3000 RPM
8. Wait 60 min
9. Expose the back side 5.2 min @ ~3.2 mW/cm² – Mask 3 (UofA DRIE-2)
10. Wait 30 min
11. Develop 4 min in AZ400K developer
12. Rinse with DI water, spin-dry and dry with nitrogen

Step 10 – Wafer bonding
1. Choose a 4” wafer without a notch (flat only) to avoid He leakage during DRIE clamping
2. Put dicing saw tape on the front side of the wafer
3. Remove the tape leaving 2.5” circle in the center using a razor blade
4. Spin AZ4903 @ 2500 RPM on the 4” wafer with the tape
5. Remove the tape leaving the center with no resist
6. Flip the 3” Tactor Wafer and bond it to the 4” carrier
7. Bake 3 min on the hot plate full contact (Al foil). During baking press the 3” wafer firmly on the 4” to remove the PR bubbles
8. Package the carrier wafer in a holder with a spring (4”)

Step 11 – DRIE
1. Etch the wafer according to the recipe in the appendix for 2h00m
2. Inspect under the microscope (Probably not etched through)
3. Etch the wafer according to the recipe in the appendix for 30m
4. Inspect under the microscope. If necessary repeat 15 min etch, inspection until the wafer is completely etched.

Notes:
- It took 163min (2h43min) for a complete etch
- The small test dies do not show anything – inspect the real dies instead
- The bonding is an issue. If there is a PR bubble it will lift the 3” wafer from the carrier and the lifted spot will etch MUCH faster - up to 15%. This is due to the elevated temperature away from the He cooled carrier wafer. Another problem due to the bonding is the misalignment due to the 3” wafer not being parallel to
the carrier causing non-vertical etch trenches. Angles of 50mil/3” were observed corresponding to 5um alignment shift.

- Because of the temperature variation due to the bonding ALL DIES MUST BE CHECKED!!
- The lines separating the dies can be a good indication for etch stop
- After the etching, DO NOT release immediately. Put some cleanroom rags between the spring and the wafer and package the wafer carrier
APPENDIX B: DEEP REACTIVE ETCHING PARAMETERS

Step 1,2,3. Pump down, wafer transport, and helium-cooling stabilization. Steps 4,5,6 are repeated to etch the deep trenches

Step 4. Deposition step

<table>
<thead>
<tr>
<th>Duration</th>
<th>Pressure</th>
<th>Power RF1</th>
<th>Power RF2</th>
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<tbody>
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<td>22 mtorr</td>
<td>1 watt</td>
<td>825 watts</td>
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<th>Flow Rates</th>
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</thead>
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<tr>
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<td>Ar</td>
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<tr>
<td>O₂</td>
<td>0 sccm</td>
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Step 5. Etch Step A

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<tr>
<th>Duration</th>
<th>Pressure</th>
<th>Power RF1</th>
<th>Power RF2</th>
</tr>
</thead>
<tbody>
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<td>9 watt</td>
<td>825 watts</td>
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<table>
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<tr>
<th>Gases</th>
<th>Flow Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₄F₈</td>
<td>0.5 sccm</td>
</tr>
<tr>
<td>SF₆</td>
<td>50 sccm</td>
</tr>
<tr>
<td>Ar</td>
<td>40 sccm</td>
</tr>
<tr>
<td>O₂</td>
<td>0 sccm</td>
</tr>
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</table>

Step 5. Etch Step A

<table>
<thead>
<tr>
<th>Duration</th>
<th>Pressure</th>
<th>Power RF1</th>
<th>Power RF2</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0 sec</td>
<td>23 mtorr</td>
<td>9 watt</td>
<td>825 watts</td>
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<table>
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<th>Gases</th>
<th>Flow Rates</th>
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<td>SF₆</td>
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<tr>
<td>O₂</td>
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Figure C1. Detailed schematic of the electronic control module
Figure E1. Top copper and silkscreen of the control module layout (not to scale)

Figure E2. Bottom copper layer (not to scale)
### APPENDIX E: CONTROL MODULE BILL OF MATERIALS

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<thead>
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<th>Part</th>
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<tr>
<td>C6</td>
<td>150u</td>
<td>PCE2113CT-ND</td>
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<td>BZT52C2V4-7DICT-ND</td>
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<tr>
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<td>FMMT596</td>
<td>FMMT596CT-ND</td>
</tr>
</tbody>
</table>
APPENDIX F: MATLAB SOURCE CODE

MATLAB input file ‘mech_model.m’
Used to generate Figures 2.6, 2.7 and 2.8

close all; clear all; clc;
format long e;

% Font Size and Line Width for all the plots
Fsize=13;
Axis_lineWidth=2;
Plot_lineWidth=2;

global t w_c w_h w_f L_h L_c L_f L_e delta alpha E NL;

% Nickel Material Properties
ro=15e-8; % Electrical resistivity [Ohm.m]
betha=2.0e-3; % Temperature coefficient of resistivity
K=90.9; % Thermal conductivity [W/(mK)]
Ts=18; % Substrate temperature [C]
alpha=13.3e-6; % Coefficient of thermal expansion
ro=8900; % Density [kg/m^3]
c_thermal=444; % Specific heat [J/(kg.K)]
E=177.3e9; % Young's modulus [Pa]

% Geometry description
w_h=17.5e-6; % Width of the hot arm [um]
w_f=17.5e-6; % Width of the flexure [um]
w_c=105e-6; % Width of the cold arm [um]
L_h=1368e-6; % Length of the hot arm [um]
L_f=415e-6; % Length of the flexure [um]
L_c=L_h-L_f; % Length of the cold arm [um]
L_e=15e-6; % Length of the extension [um]
delta=30e-6; % Separation between the hot and the cold arm (center-to-center) [um]

% ANSYS Simulations 03/24/2004
T_ANS=[0,10,20,30,40,50,60,70,80,90,100];

% ANSYS 7.0 Simulations NLGEOM=ON
UX_NL=[0,5.105,10.194,15.254,20.275,25.246,30.155,34.991,39.745,44.407,48.969];
FX_NL=[0,-8.5823,-9.9165,-4.2899,7.9227,26.263,50.210,79.189,112.60,149.82,190.25];
FY_NL=[0,939.88,1861.3,2760.1,3632.6,4475.1,5284.7,6058.8,6795.8,7494.4,8154.2];

% ANSYS 7.0 Simulations NLGEOM=OFF
UX_L=[0,5.111,10.221,15.332,20.443,25.553,30.664,35.775,40.885,45.996,51.106];
FX_L=[0,-12.281,-24.561,-36.842,-49.122,-61.403,-73.683,-85.964,-98.244,-110.52,-122.81];
% Thermal Loading
Ts=0;
T_c=0;
T_f=0;

% Mechanical Model - Linear
for i=1:length(T_ANS);
    T_h=T_ANS(i);
end

% Nonlinear Mechanical Model - Kalin Lazarov 03/24/2004
if i>=2
    p=mech_nonlin(T_h,T_c,T_f,Ts);
    UX_KNL(i)=abs(1e6*p(4));
    P_KNL(i)=p(1);T_KNL(i)=p(2);M_KNL(i)=p(3);
else
    UX_KNL(i)=0;
    P_KNL(i)=0;T_KNL(i)=0;M_KNL(i)=0;
end

% Linear Mechanical Model - Kalin Lazarov 03/24/2004
I1=(t*w_h'^3)/12;
I2=(t*w_f'^3)/12;
c11=(L_h'*2)/(2*I1)+(L_f'*2)/(2*I2);
c12=-(L_h/I1+L_f/I2);
c21=L_h'^3/(6*I1)+L_f'^3/(6*I2)+(L_f'*2*L_c)/(2*I2);
c22=-(L_f'^2/(2*I1)+L_f'^2/(2*I2)+L_f'*L_c/I2);
z1=-delta*L_f/I2;
z2=-delta*(L_f'^2/(2*I2)+(L_f'*L_c/I2);
F=inv([c11,c12;c21,c22])*[z1;z2];
Tp=F(1);Mp=F(2);

% Quadratic Mechanical Model - Kalin Lazarov 03/24/2004
% Hot arm extension
a1=((L_h'^3)/(2*I1'*2*E^2))*((Tp'^2)*(L_h'^2)/20-(L_h/4)*Tp*Mp+(Mp'^2)/3);
% Flexure extension
a2=((L_f'^3)/(2*I2'^2*E^2))*((Tp'^2)*(L_f'^2)/20-(L_f/4)*Tp*Mp+(Mp-delta'^2)/3);
% Cold arm extension
a3=((L_c*L_h'^2)/(2*I1'^2*E^2))*((Tp'^2)*(L_h'^2)/4-L_h*Tp*Mp+Mp'^2);
\[ a = a_1 - a_2 - a_3; \]
\[ P_{Q(i)} = \frac{-b + \sqrt{b^2 - 4ac}}{2a}; \]
\[ T_{Q(i)} = T_p P_{Q(i)}; M_{Q(i)} = M_p P_{Q(i)}; \]
\[ W_{Q(i)} = \left( \frac{T_{Q(i)}}{(6L^1 * E) * L_h^3} - \frac{(M_{Q(i)} * (2L^1 * E)) * L_h^2}{2} \right); \]

\[ d = \text{delta}; I_h = t * (w_h^3) / 12; I_c = t * (w_c^3) / 12; \]
\[ f_{11} = (1/(3E*I_h)) *(L_h^3 + L_f^3) + \left(1/(3E*I_c)\right)* (3L_h^2 + L_h^3 - L_f^3); \]
\[ f_{12} = \left(1/(2E*I_h)\right)* (L_f^2 + d - (1/(2E*I_c)) * (d * L_i + L_h^2 - L_f^2)); \]
\[ f_{21} = f_{12}; \]
\[ f_{13} = (1/(2E*I_h)) *(L_h^2 * L_f^2) - \left(1/(2E*I_c)\right)* (L_h^2 + L_c^2 - L_f^2); \]
\[ f_{31} = f_{13}; \]
\[ f_{22} = (1/(3E*I_h)) * (d^3 + 3L_c^2 (d^2)) + (1/(E*I_h)) * (d^4 + L_f^2); \]
\[ f_{23} = (1/(2E*I_c)) * (d^2 + 2L_c^2); \]
\[ f_{32} = f_{23}; \]
\[ f_{33} = (1/(E*I_h)) * (L_f^3 + L_h^3); \]
\[ F = [f_{11}, f_{12}, f_{13}, f_{21}, f_{22}, f_{23}, f_{31}, f_{32}, f_{33}]; \]
\[ v = [0; \alpha; (L_h^2 - L_c^2 + L_f - L_f^2)]; \]
\[ X = \text{inv}(F)*v; \]
\[ X_1 = X(1); X_2 = X(2); X_3 = X(3); \]
\[ w = (L_h^2 / (6E*I_h)) * (X_1 * L_h^3 - X_3); \]
\[ UX_{JM(i)} = \text{abs}(1e6*w); \]

end

figure(1); hold on; grid on;
plot(T_ANS, UX_L, 'k-o', 'LineWidth', Plot_LineWidth);
plot(T_ANS, UX_NL, 'k-', 'LineWidth', Plot_LineWidth);
plot(T_ANS, UX_KNL, 'k-', 'LineWidth', Plot_LineWidth);
plot(T_ANS, 1e6*W_Q, 'k-', 'LineWidth', Plot_LineWidth);
plot(T_ANS, 1e6*W, 'k-', 'LineWidth', Plot_LineWidth);
legend('ANSYS NLGEOM=OFF', 'ANSYS NLGEOM=ON', 'Nonlinear Model', 'Quadratic Model', 'Linear Model', 0)
xlabel('Hot arm temperature [C]', 'FontSize', Fsize);
ylabel('Displacement [\mu m]', 'FontSize', Fsize);
h = gca; set(h, 'FontSize', Fsize, 'LineWidth', Axis_LineWidth);

figure(2); hold on; grid on;
plot(T_ANS, 1e-6*FX_L, 'k-o', 'LineWidth', Plot_LineWidth);
plot(T_ANS, 1e-6*FX_NL, 'k-', 'LineWidth', Plot_LineWidth);
plot(T_ANS, T_KNL, 'k-', 'LineWidth', Plot_LineWidth);
plot(T_ANS, T_Q, 'k-', 'LineWidth', Plot_LineWidth);
plot(T_ANS, T, 'k-', 'LineWidth', Plot_LineWidth);
legend('ANSYS NLGEOM=OFF', 'ANSYS NLGEOM=ON', 'Nonlinear Model', 'Quadratic Model', 'Linear Model', 0)
xlabel('Hot arm temperature [C]', 'FontSize', Fsize); ylabel(T [N], 'FontSize', Fsize);
h = gca; set(h, 'FontSize', Fsize, 'LineWidth', Axis_LineWidth);

figure(3); hold on; grid on;
plot(T_ANS,1e-6*FY_L,'k-o','LineWidth',Plot_LineWidth);  
plot(T_ANS,1e-6*FY_NL,'k-', 'LineWidth',Plot_LineWidth); 
plot(T_ANS,P_KNL,'k--','LineWidth',Plot_LineWidth);  
plot(T_ANS,P_Q,'k-','LineWidth',Plot_LineWidth);  
plot(T_ANS,P,'k-v','LineWidth',Plot_UncWidth); 
legend('ANSYS NLGEOM=OFF','ANSYS NLGEOM=ON','Nonlinear Model','Quadratic Model','Linear Model',0) 
xlabel('Hot arm temperature [C]','FontSize',Fsize); ylabel('P [N]','FontSize',Fsize); 
h(gca;set(h,'FontSize',Fsize,'LineWidth',Axis_LineWidth));  

% Stiffness analysis of the folded beam thermal actuator,  

% ANSYS simulation results for stiffness calculation k=17.86 N/m 

ANSYS=[50,2.714,-1385.0;100,5.428,-2765.7;150,8.087,-62.636, 
-4142.9;200,10.734,-5517.8;250,13.318,-6891.1;300,16.017,-176.95,- 
8263.6;350,18.647,-234.53,9417.7;400,21.275,-408.66,- 
-13360.7;450,23.901,-35.04,29611.7;500,26.529,-95.066,- 
-16522.6;550,29.160,-62.636,-73311.7;600,31.799,- 
-48382.9;650,34.446,-281.44,-9636.0;700,37.104,-8263.6, 
-11009.4;750,39.778,-20697.8;800,42.468,-932.04,- 
-22100.8;850,45.18,-1041.2,-23511.9;900,47.917,- 
-1157.2,-24931.3;950,50.682,-1280.1,-26359.4;1000,53.482,- 
-1410.5,-27798.5;1050,56.321,-1548.5,-29249.7]; 

% Analytical Model 
I1=(t*w_h^3)/12; 
I2=(t*w_f^3)/12; 
f11=(L_f*2)/(2*I2)+(L_h^2)/(2*I1)+(L_f*(delta*2))/I2)*t/(L_f/w_f+L_c/w_c+L_h/w_h); 
f12=-((L_f/I2+L_h/I1)+(L_f*(delta*2))/I2)*t/(L_f/w_f+L_c/w_c+L_h/w_h));  
f21=(L_h^3)/(6*I1)+(L_f*3)/(6*I2)-( 
(L_h^2)*L_c)/(2*I1)+(delta*2*L_f*2*L_h^2)/(4*I1*I2))*t/(L_f/w_f+L_c/w_c+L_h/w_h);  
f22=-((L_h^2)/(2*I1)+L_f*2)/(2*I2)- 
L_c^*L_h/w/h1+(delta*2*L_f*2*L_h^2)/(2*I1*I2))*t/(L_f/w_f+L_c/w_c+L_h/w_h));
b1=L_f^2/(2*I2)-L_f*L_h/I2;
b2=L_f^3/(6*I2)-L_h*(L_f^2)/(2*I2);
D=inv([f11,f12,f21,f22])*[b1;b2];

F=50e-6:50e-6:1050e-6;
w=(1/E)*[L_h^3/(6*I1),-L_h^2/(2*I1)]*D*F;
theta=(1/E)*[L_h^2/(2*I1),-L_h/I1]*D*F;
T=D(1)*F;
P=-E*delta*theta*t/(L_f/w_f+L_c/w_c+L_h/w_h);

figure(4);hold on;
plot(F*1e6,-w*1e6,'k','LineWidth',Plot_LineWidth);legend('ANSYS', 'Linear Model', 0);
figure(5);hold on;
plot(F*1e6,T*1e6,'k','LineWidth',Plot_LineWidth);legend('ANSYS', 'Linear Model', 0);
figure(6);hold on;
plot(F*1e6,P*1e6,'k','LineWidth',Plot_LineWidth);legend('ANSYS', 'Linear Model', 0);

k=-E/([L_h^3/(6*I1),-L_h^2/(2*I1)]*D);
d3=([L_h^2/(2*I1),-L_h/I1]*D)/([L_h^3/(6*I1),-L_h^2/(2*I1)]*D);

% ANSYS MODAL Analysis f=6939 Hz
m_tild=(ro*L_c*w_c*t)*(l/3+w_c'^2/(12*L_c'^2));
f_res=(1/(2*pi))'*sqrt(k/m_tild);
MATLAB input file 'factor.m'
Used to generate Figures 2.3, 2.9, 2.13, 2.14, 2.15. Requires the function described in mech.nonlinear.m

clear all; clc; close all; format long e;

% Font Size and Line Width for all the plots
Fsize=13;
Axis_LineWidth=2;
Plot_LineWidth=2;

% Nickel Material Properties
r0=15e-8;  % Electrical resistivity [Ohm.m]
betha=2.0e-3;  % Temperature coefficient of resistivity
K=90.9;  % Thermal conductivity [W/(mK)]
Ts=18;  % Substrate temperature [C]
alpha=13.3e-6;  % Coefficient of thermal expansion
ro=8900;  % Density [kg/m^3]
c__thermal=444;  % Specific heat [J/(kg.K)]
E=177.3e9;  % Young's modulus [Pa]

% Geometry description
 t=20.6e-6;  % Actuator thickness [um]
w__h=17.5e-6;  % Width of the hot arm [um]
w_f=17.5e-6;  % Width of the flexure [um]
w_c=105e-6;  % Width of the cold arm [um]
L__h=1368e-6;  % Length of the hot arm [um]
L_f=415e-6;  % Length of the flexure [um]
L_c=L__h-L_f;  % Length of the cold arm [um]
L__e=15e-6;  % Length of the extension [um]
deita=30e-6;  % Separation between the hot and the cold arm (center-to-center) [um]

% Experimental Data
I=1e-3*[10, 15, 20, 25, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170, 180,
190, 200, 205, 210, 215, 220, 225, 230, 235, 240, 245, 250];
V=0.5*1e-3*[17, 25.7, 33.6, 42.3, 50.3, 67.4, 84.6, 102.2, 120.1, 139.4, 158.1, 177.7, 197.2, 218,
240, 263, 287, 313, 341, 371, 404, 441, 464, 486, 512, 539, 571, 606, 654, 731, 831, 895];

clear I_ro, clear V_ro;  % Calculate the resistance of the actuator
for i=1:length(I);
    if I(i)<60e-3;
        I_ro(i)=I(i);
        V_ro(i)=V(i);
    end

end

p=polyfit(I_ro,V_ro,1);
P_act=p(1);
r0=R_act/((L_h/(t*w_h))+(L_c/(t*w_c))+(L_f/(t*w_f)))

for i=1:length(I);

% Solution of the thermal problem
Im=I(i);
g1=sqrt(r0*betha/(t^2*K))*Im/w_h;
g2=sqrt(r0*betha/(t^2*K))*Im/w_c;
g3=sqrt(r0*betha/(t^2*K))*Im/w_f;
L1=L_h;
L2=L_h+L_c;
L3=L_h+L_c+L_f;

A=[0,0,0,sin(g3*L3),cos(g3*L3);-sin(g1*L1),sin(g2*L1),cos(g2*L1),0,0;
    w_h*g1*cos(g1*L1),-w_c*g2*cos(g2*L1),w_c*g2*sin(g2*L1),0,0;
    0,w_c*g2*cos(g2*L2),-w_c*g2*sin(g2*L2),-w_f*g3*cos(g3*L2),
    w_f*g3*sin(g3*L2));
b=(w_h/g1*[1;cos(g1*L1);w_h*g1*sin(g1*L1);0;0];
v=inv(A)*b;
A1=v(1);B1=1/betha;A2=v(2);B2=v(3);A3=v(4);B3=v(5);
T_h=(A1*(1*cos(g1*L_h)+B1*sin(g1*L_h)))/(L_h*g1)+(Ts-1/betha);
T_c=(A2*(cos(g2*L_l)-cos(g2*L2))+B2*(sin(g2*L2)-sin(g2*L_l)))/(L_c*g2)
    +(Ts-1/betha);
T_f=(A3*(cos(g3*L2)-cos(g3*L3))+B3*(sin(g3*L3)-sin(g3*L2)))/(L_f*g3)
    +(Ts-1/betha);
R=((r0*L1)/((t*w_h)))*(1+betha*(T_h-Ts))+(r0*(L2-L1)/(t*w_c))*(1+betha*(T_c-Ts))
    +(r0*(L3-L2)/(t*w_f))*(1+betha*(T_f-Ts));
Vm(i)=Im*R;

% Linear Mechanical Model - Kalin Lazarov 03/24/2004
L1=(t*w_h^3)/12;
L2=(t*w_f^3)/12;
c11=(L_h^2)/(2*(L_f^2)+(L_h^2)/2);
c12=(L_h^2)/(L_f^2);
c21=(L_h^2*(6*11)+L_f^2*(6*12)+(L_f^2*L_c))/(2*L2);
c22=(L_h^2*(2*11)+L_f^2*(2*12)+(L_f^2*L_c))/(2*L2);
z1=-delta*L_f/l2;
z2=-delta*(L_f^2*(2*l2)+(L_f^2*L_c)/l2);
F=inv([c11,c12;c21,c22])*[z1;z2];
Tp=F(1);Mp=F(2);

b1=(L_f/l2+c12*L_c/w_c+L_h/w_h)/(E*t);
b2=-(delta*(11*E))*(L_h^2*2*tp/2-Mp*L_h); % Elastic stretch
% Delta geometric term
\[ c = \alpha (L_c \cdot T_c + L_f \cdot T_f - L_h \cdot T_h); \quad \text{% Thermal strain} \]
\[ b = b_1 + b_2; \]
\[ P = -\frac{c}{b}; \]
\[ T = T_P \cdot P; M = M_P \cdot P; \]
\[ W(i) = \left( \frac{T}{6 \cdot I_l \cdot E} \right) \cdot L_h^3 - \left( \frac{M}{2 \cdot I_l \cdot E} \right) \cdot L_h^2; \]

% Quadratic Mechanical Model - Kalin Lazarov 03/24/2004
% Hot arm extension
\[ a_1 = \left( \frac{L_h^3}{2 \cdot I_1 \cdot E^2} \right) \cdot \left( \frac{\left( T_P^2 \right) \cdot (L_h^2)}{20} - \frac{L_h \cdot T_P \cdot M_P}{4} + \frac{M_P^2}{3} \right); \]
% Flexure extension
\[ a_2 = \left( \frac{L_f^3}{2 \cdot I_2 \cdot E^2} \right) \cdot \left( \frac{\left( T_P^2 \right) \cdot (L_f^2)}{20} - \frac{L_f \cdot T_P \cdot M_P}{4} + \frac{(M_P - \delta)^2}{3} \right); \]
% Cold arm extension
\[ a_3 = \left( \frac{L_c \cdot L_h^2}{2 \cdot I_1 \cdot E^2} \right) \cdot \left( \frac{\left( T_P^2 \right) \cdot (L_h^2)}{4} - \frac{L_h \cdot T_P \cdot M_P}{2} + \frac{M_P^2}{2} \right); \]
\[ a = a_1 - a_2 - a_3; \]
\[ P = -\frac{b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}; \]
\[ T = T_P \cdot P; M = M_P \cdot P; \]
\[ W_{Q(i)} = \left( \frac{T}{6 \cdot I_l \cdot E} \right) \cdot L_h^3 - \left( \frac{M}{2 \cdot I_l \cdot E} \right) \cdot L_h^2; \]
end

figure(1);
% I-V Characteristic of the thermal actuator
plot(I, V, 'kd'); % 'LineWidth', Plot_LineWidth); hold on; grid on;
plot(I, Vm, 'k-'); % 'LineWidth', Plot_LineWidth);
xlabel('Actuator Current [A]', 'FontSize', Fsize);
ylabel('Actuator Voltage [V]', 'FontSize', Fsize);
h = gca; set(h, 'FontSize', Fsize, 'LineWidth', 'Axis_Line Width');

Id = l:3 * [110, 120, 130, 140, 150, 160, 170, 180, 190, 200, 210, 220, 230, 240, 250];
D = [484, 295, 474, 284; 482, 293, 468, 278; 482, 292, 465, 276; 483, 293, 462, 273; 483, 293, 459, 269; 483, 293, 455, 264; 483, 293, 450, 259; 482, 292, 443, 253; 483, 293, 436, 246; 483, 293, 427, 238; 485, 295, 421, 231; 490, 299, 141, 222; 493, 303, 398, 208; 501, 311, 370, 180; 537, 347, 377, 187];

for i = 1:length(D);
    g = 90/(0.5 * (D(i, 1) - D(i, 2) + D(i, 3) - D(i, 4)));
    UX(i) = 0.5 * g * (D(i, 1) + D(i, 2) - D(i, 3) - D(i, 4));
end

figure(2);
% Current-Displacement characteristic of the thermal actuator
plot(Id, UX, 'kd'); % 'LineWidth', Plot_LineWidth); hold on; grid on;

plot((1e6 * abs(W), 'k-'), 'LineWidth', 'Plot_LineWidth');
plot((1e6 * abs(W_Q), 'k-'), 'LineWidth', 'Plot_LineWidth');
xlabel('Actuator Current [A]', 'FontSize', Fsize);
ylabel('Actuator Displacement [microns]', 'FontSize', Fsize);
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\begin{verbatim}
% ANSYS Data
V_Ans=[0.025,0.050,0.075,0.1,0.11,0.12,0.13,0.14,0.15,0.16,0.17,0.18,0.19,0.2,0.21,0.22
0.23,0.24,0.25,0.26,0.27,0.28,0.29,0.30,0.31,0.32,0.33,0.34,0.35,0.36,0.37,0.38,0.39,0.40
];
I_Ans=[0.028706,0.056134,0.081523,0.1043,0.11267,0.1206,0.1281,0.13482,0.1414,0.1
476,0.1538,0.1594,0.16468,0.16967,0.17391,0.17826,0.18237,0.18624,0.19045,0.1940,0.
19739,0.2,0.20293,0.20572,0.20838,0.2116,0.214,0.21648,0.21801,0.22008,0.22206,0.22
396,0.22663,0.22845];
U_Ans=[0.3,1.211,2.686,4.66,5.579,6.566,7.615,8.642,9.778,10.956,12.276,13.566,14.89
4,34.225,35.822,37.426,38.762,40.335,41.907,43.485,45.387,46.998];

figure(1);hold on;plot(I_Ans,V_Ans,'k','LineWidth',Plot_LineWidth);
figure(2);hold on;plot(I_Ans,U_Ans,'k','LineWidth',Plot_LineWidth);
figure(1);axis([0,0.23,0,0.4]);legend('EXPERIMENT', 'ANALYTICAL', 'FEA',0);
figure(2);axis([0,0.23,0,46]);legend('EXPERIMENT', 'ANALYTICAL', 'FEA',0);

load transient;
figure(3);
plot(t_res,ch1_res,'k', 'LineWidth',Plot_LineWidth/4);
hold on;grid on;
plot(t_tact,ch1_tact,'k', 'LineWidth',Plot_LineWidth);
title('Voltage drop across the thermal actuator and a reference resistor','FontSize',Fsize);
xlabel('Time [ms]','FontSize',Fsize);ylabel('Voltage Drop [V]','FontSize',Fsize);
h=gca;set(h,'FontSize',Fsize,'LineWidth',Axis_LineWidth);

j=1;
for i=1:length(t_tact_1);
    if ((t_tact_1(i)>0)&(t_tact_1(i)<25)
        t_trans(j)=t_tact_1(i);
        volt(j)=ch1_tact_1(i);
        j=j+1;
    end
end
end
\end{verbatim}
n=16; j=1; % Filter the signal by averaging
for i=n:length(volt)-n;
    t_filt(j)=t_trans(i);
    y_filt(j)=sum(volt(i-n/2:i+n/2))/(n+l);
    j=j+i;
end

figure(4);plot(t_filt,y_filt,'k:','LineWidth',Plot_LineWidth);hold on;grid on;
A=0.1035;B=0.0045;tau=6.6;
y=A+B*exp(-t_trans/tau);
plot(t_trans,y,'kVLineWidth',Plot_LineWidth);
xlabel('Time [ms]VFontSize',Fsize');ylabel('Voltage Drop [V]VFontSize',Fsize');
title(strcat('Cooling of the thermal actuator \tau=',num2str(tau),'ms'),'FontSize',Fsize');
h=gca;set(h,'FontSize',Fsize,'LineWidth',Axis_LineWidth);

% Cooling time of the thermal actuator - Analytical prediction
T_c=0:1:400; % Temperature range in degree C
T0=273.15; % Absolute zero
T=T_c+T0;

% Nickel density ro(T)
ro_T=le3*(-1.117427e-013.*T.*T.*T.*T +4.097446e-010.*T.*T.*T -
      5.582955e-007.*T.*T -1.034728e-004.*T+8.969428e+000);

% Nickel specific heat c(T)
for i=1:length(T);
    if(T(i) >= 100.0 && T(i) < 599.0)
        c_T(i) = -1.187679e-008*T(i)^4+2.223124e-005*T(i)^3 -
               1.467413e-002*T(i)^2 +4.504691e+000*T(i)-9.038582e+001;
    elseif(T(i) >= 599.0 && T(i) < 631.0)
        c_T(i) = 1.809085e-004*T(i)^3-2.972179e-001*T(i)^2 +
               1.625617e+002*T(i)-2.926106e+004;
    elseif(T(i) >= 631.0 && T(i) < 700.0)
        c_T(i) = 1.341446e-002*T(i)^2-2.008609e+001*T(i)+8.011774e+003;
    elseif(T(i) >= 700.0 && T(i) <= 1728.0)
        c_T(i) = -5.186512e-008*T(i)^3 +2.399215e-004*T(i)^2 -
                2.122174e-001*T(i)+5.725331e+002;
    else
        c_T(i)=le100;
    end
end
for i=1:length(T);
    if(T(i) >= 0.0 && T(i) < 18.0)
        k_T(i) = 1.304208e-003*T(i)^5-2.700268e-002*T(i)^4-
                 4.089829e-001*T(i)^3+2.199849e+000*T(i)^2+2.140436e+002*T(i);
    elseif(T(i) >= 18.0 && T(i) < 30.0)
        k_T(i) = 3.189215e-004*T(i)^5-5.321568e-002*T(i)^4+3.410781*T(i)^3
                 -1.039216e+002*T(i)^2+1.431939e+003*T(i)-5.208712e+003;
    elseif(T(i) >= 30.0 && T(i) < 90.0)
        k_T(i) = 6.040699e-005*T(i)^5-2.121476e-002*T(i)^4+7.61625*T(i)^3
                 -1.610480e+002*T(i)^2+3.817001e+003;
    elseif(T(i) >= 90.0 && T(i) < 223.0)
        k_T(i) = 3.139035e-007*T(i)^5-2.776075e-002*T(i)^4+8.276592e-002*T(i)^3
                 -1.121132e+001*T(i)^2+7.007906e+002;
    elseif(T(i) >= 223.0 && T(i) < 630.0)
        k_T(i) = 1.313912e-009*T(i)^5-2.193597e-006*T(i)^4+8.67396e-002*T(i)^3
                 +1.648640e-003*T(i)^2-6.437085e-001*T(i)+1.853755e+002;
    elseif(T(i) >= 630.0 && T(i) <= 1500.0)
        k_T(i) = 2.143115e-002*T(i)^5+5.042098e+001;
    else
        k_T(i)=1e100;
    end
end

figure(5);
L_eq=L_h-L_f;
I=100e-3;
x=0:L_eq/100:L_eq;
gamma=sqrt(rO*betha/(K*t*t))*I/w_h;
A1=(1-cos(gamma*L_eq))/(betha*sin(gamma*L_eq));
T_eq=(sin(gamma*x)-sin(gamma*L_eq)+sin(gamma*(L_eq-x)))/
     (betha*sin(gamma*L_eq));
plot(x,T_eq/max(T_eq),k,'LineWidth',Plot_LineWidth);hold on;grid on;
plot(x,sin(pi*x/L_eq),k,'LineWidth',Plot_LineWidth);
xlabel('Unfolded length x [m]',FontSize,Fsize');ylabel(T_eq/max(T_eq)',FontSize,Fsize');
legend('Steady state solution', 'Eigenfunction \Phi(x)=sin(\pi \it{x}/L_{eq})',0);
h=gca;set(h,FontSize,Fsize',LineWidth,Axis_LineWidth);
figure(6);
t_cooling=ro_T.*(L_h-L_f)^2.*c_T./(k_T*pi*pi);
plot(T_c,1e3*t_cooling,k,'LineWidth',Plot_LineWidth);grid on;
xlabel('Temperature [C]','FontSize',Fsize');
ylabel('Cooling time $\tau$ [ms]','FontSize',Fsize');
h=gca;set(h,'FontSize',Fsize,'LineWidth',Axis_LineWidth);

figure(7);
I=10e-3:5e-3:250e-3;
tau_b=l./(I.*I.*r0*betha/(c_thermal*ro*w_h^2*t^2));
tau_0=ro*(L_h-L_f)^2*c_thermal/(K*pi*pi);
tau_eff=tau_0.*(tau_b./tau_b-tau_0);
plot(I,1e3*tau_0*ones(length(I),1),k,'LineWidth',Plot_LineWidth);hold on;grid on;
plot(I,1e3*tau_eff,k,'LineWidth',Plot_LineWidth);
xlabel('Current [A]','FontSize',Fsize');ylabel('Time constant [ms]','FontSize',Fsize');
legend('Time constant $\beta=0$', 'Nonlinear time constant',0);
axis([min(I) max(I) 3.5 5]);
h=gca;set(h,'FontSize',Fsize,'LineWidth',Axis_LineWidth);
MATLAB input file 'mech_nonlin.m'

function f=mech_nonlin(T_h,T_c,T_f,Ts)

global t w_c w_h w_f L_h L_c L_f delta alpha E;

% Start with initial condition from Linear model
I1=(t*w_h^3)/12;
I2=(t*w_f^3)/12;
c11=(L_h^2)/((2*I1)+(L_f^2)/(2*I2));
c12=-((L_h/L1)+L_f/I2);
c21=L_h^3/(6*I1)+L_f^3/(6*I2)+((L_f^2*L_c)/(2*I2));
c22=-((L_h^2)/(2*I1))+(L_f^2)/(2*I2)+((L_f*L_c)/I2);
z1=-delta*L_f/I2;
z2=-delta*L_f^2/(2*I2)+((L_f*L_c)/I2);
F=inv([c11,c12;c21,c22])*[z1;z2];
Tp=F(1);Mp=F(2);

b1=(L_f/w_f+L_c/w_c+L_h/w_h)/(E*t);
b2=-((delta/(I1*E)))^*(L_h^2*2*tp-2*Mp*L_h);
c=alpha*(L_c*T_c+L_f*T_f-L_h*T_h);
b=b1+b2;
Pc=-c/b;

% Solution of the nonlinear mechanical problem
% Iteration Parameters
h=0.5e-5;
tol=0.1e-6;
W=[1,0];

% Newton-Raphson
while (abs(W(1)-W(2))>tol);
    for j=1:2;
        for i=1:2;
            Pi=[Pc;Pc*(1+h)];
            P0=Pi(i);
            c11i=cos(k1*L_h)-cosh(k2*L_f);
            c12i=(k2*sinh(k2*L_f))+(1/k2)*sinh(k2*L_f)+L_c*cos(k1*L_h); 
            c21i=(L_h-L_f+L_c)/(1/k2)*sinh(k2*L_f)+L_c*cos(k1*L_h)-(1/k1)*sin(k1*L_h);
            c22i=(cos(k1*L_h)-cosh(k2*L_f)+L_c*k1*sin(k1*L_h));
            d1=delta*k2*sinh(k2*L_f);
        end
    end
end

MATLAB input file 'mech_nonlin.m'

function f=mech_nonlin(T_h,T_c,T_f,Ts)

global t w_c w_h w_f L_h L_c L_f delta alpha E;

% Start with initial condition from Linear model
I1=(t*w_h^3)/12;
I2=(t*w_f^3)/12;
c11=(L_h^2)/((2*I1)+(L_f^2)/(2*I2));
c12=-((L_h/L1)+L_f/I2);
c21=L_h^3/(6*I1)+L_f^3/(6*I2)+((L_f^2*L_c)/(2*I2));
c22=-((L_h^2)/(2*I1))+(L_f^2)/(2*I2)+((L_f*L_c)/I2);
z1=-delta*L_f/I2;
z2=-delta*L_f^2/(2*I2)+((L_f*L_c)/I2);
F=inv([c11,c12;c21,c22])*[z1;z2];
Tp=F(1);Mp=F(2);

b1=(L_f/w_f+L_c/w_c+L_h/w_h)/(E*t);
b2=-((delta/(I1*E)))^*(L_h^2*2*tp-2*Mp*L_h);
c=alpha*(L_c*T_c+L_f*T_f-L_h*T_h);
b=b1+b2;
Pc=-c/b;

% Solution of the nonlinear mechanical problem
% Iteration Parameters
h=0.5e-5;
tol=0.1e-6;
W=[1,0];

% Newton-Raphson
while (abs(W(1)-W(2))>tol);
    for j=1:2;
        for i=1:2;
            Pi=[Pc;Pc*(1+h)];
            P0=Pi(i);
            c11i=cos(k1*L_h)-cosh(k2*L_f);
            c12i=(k2*sinh(k2*L_f))+(1/k2)*sinh(k2*L_f)+L_c*cos(k1*L_h); 
            c21i=(L_h-L_f+L_c)/(1/k2)*sinh(k2*L_f)+L_c*cos(k1*L_h)-(1/k1)*sin(k1*L_h);
            c22i=(cos(k1*L_h)-cosh(k2*L_f)+L_c*k1*sin(k1*L_h));
            d1=delta*k2*sinh(k2*L_f);
        end
    end
end
\[ d_2 = \Delta * (1 - \cosh(k_2 L_f)) \];
\[ F = \text{inv} \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} * \{ d_1; d_2 \}; \]
\[ T_p = F(1); M_p = M_p * P_0; T = T_p * P_0; \]
\[ \text{Ex}_1 = 1/4 * (6 * T^2 * k_1 L_h - 8 * T * \sin(k_1 L_h) + 8 * M * k_1 T * \cos(k_1 L_h) + T^2 * \sin(2 * k_1 L_h) - 2 * M * k_1 T * \cos(2 * k_1 L_h) - 6 * M * k_1 T - M^2 * 2 * k_1 T * \sin(2 * k_1 L_h) + 2 * M^2 * 2 * k_1 L_h)/k_1/P_0^2; \]
\[ g = (M - P_0 * \Delta)/P_0; \]
\[ \text{Ex}_2 = 1/4 * (6 * T^2 * k_2 L_f - 8 * T * \sinh(k_2 L_f) + 8 * g * k_2 T * \cosh(k_2 L_f) * P_0 + T^2 * \sinh(2 * k_2 L_f) - 2 * g * k_2 T * \cosh(2 * k_2 L_f) - 6 * g * k_2 T * P_0 + 2 * g * k_2 P_0^2 * \sinh(2 * k_2 L_f) - 2 * g^2 * k_2^3 * P_0^2 * L_f)/k_2/P_0^2; \]
\[ \theta = (T_p * (1 - \cos(k_1 L_h)) - k_1 M_p * \sin(k_1 L_h)); \]
\[ \text{Ex}_3 = \theta * \Delta; \]
\[ \text{Ex}_4 = L_c * (1 - \cos(\theta)); \]
\[ P = (E T)/(L_h/w_h + L_c/w_c + L_f/w_f); \]
\[ (\alpha * (L_h T_h - L_c T_c - L_f T_f) - 0.5 * \text{Ex}_1 + 0.5 * \text{Ex}_2 + \text{Ex}_3 + \text{Ex}_4); \]
\[ M = M_p * P; T = T_p * P; \]
\[ w = (T/P) * (L_h - (1/k_1) * \sin(k_1 L_h)) + (M/P) * (\cos(k_1 L_h) - 1); \]
\[ V(i) = (P - P_0)^2; \]
end
\[ P_c = P(i) - V(1)/((V(2) - V(1))/(P(2) - P(1))); \]
\[ W(j) = w; \]
end
end

f = [P, T, M, w];
APPENDIX G: ANSYS INPUT FILES

ANSYS input file ‘coupled.txt’
Used to generate Figures 2.1, 2.2, 2.3 and 2.9

/CLEAR,START
/filename, Thermal
/title, Thermal Actuator
/units, SI

VDD=0.075
Ts=0

! Material properties - Nickel

Exx_met=177.3e9*le-6
rho_met=8.9e3*le-18
c_met=444*le12
k_met=90.9*le6
res_met=15.6e-8*le-6
nu_met=0.312
alph_met=13.3e-6
alph_r=2.0e-3

! Geometry description
! All units are in microns!

tol=0.5
t=20.6 ! Actuator thickness [um]
w_h=17.5 ! Width of the hot arm [um]
w_f=17.5 ! Width of the flexure [um]
w_c=105 ! Width of the cold arm [um]
L_h=1368 ! Length of the hot arm [um]
L_f=415 ! Length of the flexure [um]
L_c=L_h-L_f ! Length of the cold arm [um]
L_e=15 ! Length of the extension [um]
d=30 ! Separation between the hot and the cold arm (center-to-center) [um]

/PREP7
ET,1,SOLID98,1 ! Define VOLT;TEMP,MAG DOF
UIIMP,1,EX, ,Exx_met
UIIMP,1,ALPX, ,alph_met
UIIMP,1,PRXY, ,nu_met
UIIMP,1,KXX, ,k_met
UIIMP,1,C, ,c_met
UIIMP,1,DENS, ,rho_met
UIIMP,1,RSVX, ,res_met

MPTEMP,1,0 ! Define the resistance change
MPTEMP,2,1000
MPDE,RSVX,1
MPDE,RSVY,1
MPDE,RSVZ,1
MPDATA,RSVX,1,"res_met"
MPDATA,RSVX,1,"(res_met*(1+alph_r*1000))"

! Geometry description

x0=-w_h/2-d+w_f/2
g=d-w_h/2-w_f/2

NUMSTR,VOLU,1
blc4,x0,0,w_h,L_h,t ! Hot arm
blc4,0,L_h,-g,L_e,t
blc4,x0,L_h,w_h,L_e,t

blc4,0,L_h,w_f,L_e,t ! Cold arm
blc4,w_f,L_h,(w_c-w_f),L_e,t
blc4,0,L_f,w_f,L_c,t
blc4,w_f,L_f,(w_c-w_f),L_c,t

blc4,0,0,w_f,L_f,t ! Flexure

! Eng of geometry

!******************************************************************************
! Meshing
!******************************************************************************

ALLSEL,ALL
VATT,1
NUMMERG,ALL
SMRT,10
MSHAPE,1,3D
MSHKEY,0
VMESH,ALL
ALLSEL,ALL

/SOLU

ANTYPE,STATIC,NEW  ! Define static analysis
NSUB,10,100,5      ! Dividing the load step to 10 subsets
NSUBST,20,5,50

ALLSEL,ALL
ASEL,S,LOC,Y,-tol,tol
NSLA,R,1
D,ALL,TEMP,Ts       ! Fix the bottom temperature of the Bonding pads

ALLSEL,ALL
ASEL,S,LOC,Y,-tol,tol
NSLA,R,1
ASEL,S,LOC,X,-tol,w_f+tol
NSLA,R,1
D,ALL,VOLT,0         ! Apply VOLTAGE to the Cold arm - GND

ALLSEL,ALL
ASEL,S,LOC,Y,-tol,tol
NSLA,R,1
ASEL,S,LOC,X,(x0-tol),(x0+w_h+tol)
NSLA,R,1
D,ALL,VOLT,VDD       ! Apply VOLTAGE to the Hot arm - VDD

ALLSEL,ALL
NCMIT
SOLVE
ALLSEL,ALL
SAVE,thermal_temp,db,,ALL
FINISH

/PREP7

ALLSEL,ALL
ET,1,SOLID92        ! Define UX,UY,UZ DOF
ALLSEL,ALL
FINISH

/SOLU
ANTYPE,STATIC,NEW  ! Define static analysis
NLGEOM,ON
NSUB,10,20,5        ! Dividing the load step to 10 subsets
                   ! with maximum number of load subsets
ALLSEL,ALL
ASEL,S,LOC,Y,-tol,tol
NSLA,R,1
D,ALL,UX,0,,UY,UY

! Fix the bottom of the Bonding pads

ALLSEL,ALL
LDREAD,TEMP,LAST,,,,thermal,rst,,
ALLSEL,ALL
SOLVE
ALLSEL,ALL
SAVE,thermal_ux,,db,,ALL
FINISH

/POST1

/EFACE,1
/SHRINK,0
/ESHAPE,1
/EFACET,1
/RATIO,1,1,1
/PLOPTS,INFO,2
/PLOPTS,LEG1,1
/PLOPTS,LEG2,1
/PLOPTS,LEG3,1
/PLOPTS,FRAME,1
/PLOPTS,TITLE,1
/PLOPTS,MINM,1
/PLOPTS,LOGO,1
/PLOPTS,WINS,1
/PLOPTS,WP,0
/PLOPTS,DATE,2
/TRIAD,ORIG

PLNSOL,UX,0,

/EOF
ANSYS input file 'mechanical.txt'
Used to generate Figures 2.6, 2.7 and 2.8

/CLEAR,START
/filename, Thermal
/title, Thermal Actuator
/units, SI

!********************************************************************************
! Thermal Loading
!********************************************************************************

T_h=100
T_c=0
T_f=0
Ts=0

!********************************************************************************
! Material properties - Nickel
!********************************************************************************

Exx_met=177.3e9*1e-6 ! Constants converted for microns
rho_met=8.9e3*1e-18
c_met=444*1e12
k_met=90.9*1e6
res_met=15.6e-8*1e-6
nu_met=0.312 ! Unitless constants
alph_met=13.3e-6
alph_r=2.0e-3

!********************************************************************************
! Geometry description
! All units are in microns!
!********************************************************************************

tol=0.5

t=20.6 ! Actuator thickness [um]
w_h=17.5 ! Width of the hot arm [um]
w_f=17.5 ! Width of the flexure [um]
w_c=105 ! Width of the cold arm [um]
L_h=1368 ! Length of the hot arm [um]
L_f=415 ! Length of the flexure [um]
L_c=L_h-L_f ! Length of the cold arm [um]
L_e=15 ! Length of the extension [um]
d=30 ! Separation between the hot and the cold arm (center-to-center) [um]

!********************************************************************************
/PREP7

ET,1,SOLID95
UIIMP,1,EX,,Exx_met
UIIMP,1,ALPX,,alph_met
UIIMP,1,PRXY,,nu_met
UIIMP,1,KXX,,k_met
UIIMP,1,C,,c_met
UIIMP,1,DENS,,rho_met
UIIMP,1,RSVX,,res_met

MPTEMP,1,0
MPTEMP,2,1000
MPDE,RSVX,1
MPDE,RSVY,1
MPDE,RSVZ,1
MPDATA,RSVX,1,,res_met
MPDATA,RSVX,1,,(res_met*(1+alph_r*1000))

! Geometry description

x0=-w_h/2-d+w_f/2
g=d-w_h/2-w_f/2

NUMSTR,V0LU,1
blc4,x0,0,w_h,L_h,t ! Hot arm
blc4,0,L_h,-g,L_e,t
blc4,x0,L_h,w_h,L_e,t
blc4,0,L_h,w_f,L_e,t ! Cold arm
blc4,w_f,L_h,(w_c-w_f),L_e,t
blc4,0,L_f,w_f,L_c,t
blc4,w_f,L_f,(w_c-w_f),L_c,t
blc4,0,0,w_f,L_f,t ! Flexure

! Eng of geometry

!********************************************************************************
! Meshing !********************************************************************************

ALLSEL,ALL
VATT,1
NUMMERG,ALL
SMRT,10
MSHAPE,1,3D
MSHKEY,0
VMESH,ALL
ALLSEL,ALL

***************************************************************
! Nonlinear solution for the displacement
***************************************************************

/SOLU
ANTYPE,STATIC,NEW                     ! Define static analysis
NLGEOM,ON                             ! Define nonlinear geometry analysis
NSUB,10,20,5                          ! Dividing the load step to 10 subsets
                                       ! with maximum number of load subsets step 20

ALLSEL,ALL
ASEL,S,LOC,Y,-tol,tol
NSLA,R,1
D,ALL,UX,0,,UY,UY,UY

ALLSEL,ALL
VSEL,ALL
VSEL,S,VOLU,,1,3,,
NSLV,R,1
BF,ALL,TEMP,T_h                     ! Fix the bottom of the Bonding pads

ALLSEL,ALL
VSEL,ALL
VSEL,S,VOLU,,4,7,,
NSLV,R,1
BF,ALL,TEMP,T_c                     ! Apply thermal load to hot arm

ALLSEL,ALL
VSEL,ALL
VSEL,S,VOLU,,8,,
NSLV,R,1
BF,ALL,TEMP,T_f                     ! Apply thermal load to cold arm

ALLSEL,ALL
SOLVE
ALLSEL,ALL
SAVE,thermal,db,,ALL
FINISH

/POST1

/EFACE,1
/SHRINK,0
/ESHAPE,1
/EFACET,1
/RATIO,1,1,1
/PLOPTS,INFO,2
/PLOPTS,LEG1,1
/PLOPTS,LEG2,1
/PLOPTS,LEG3,1
/PLOPTS,FRAME,1
/PLOPTS,TITLE,1
/PLOPTS,MINM,1
/PLOPTS,LOGO,1
/PLOPTS,WINS,1
/PLOPTS,WP,0
/PLOPTS,DATE,2
/TRIAD,ORIG

PLNSOL,UX,0,

ALLSEL,ALL ! Select the nodes of the base of the hot arm
ASEL,S,LOC,Y,-tol,tol
ASEL,R,LOC,X,(x0-tol),(x0+w_h+tol)
NSLA,R,1

/EOF
ANSYS input file ‘spring.txt’  
Used to generate Figure 5.5

!/filename, spring  
/title, Tactor spring  
/units, SI

Force=4.073e-3 ! Applied force in N
N_nodes=13 ! Number of nodes for force application

! All units are in microns !
! Geometry definitions

tol=0.5
l0=15
l1=170
l2=30
l3=370
w1=40
w2=15
w3=20
thk=20

! Material properties

Exx_met=177.3e9*1e-6 ! Constants converted for microns
rho_met=8.9e3*1e-18
c_met=444*1e12
k_met=90.9*1e6
res_met=15.6e-8*1e-6
nu_met=0.312
alph_met=13.3e-6
alph_r=2.0e-3

/PREP7

ET,1,SOLID92 ! Define UX,UY,UZ DOF
UIMP,1,EX, ,Exx_met
UIMP,1,ALPX, ,alph_met
UIMP,1,PRXY, ,nu_met
UIMP,1,KXX, ,k_met
UIMP,1,C, ,c_met
UIMP,1,DENS, ,rho_met
UIMP,1,RSVX, ,res_met

! Geometry description
! Define the rounded regions

\[ \pi = \text{ACOS}(-1) \]  \hspace{1cm} ! \pi = \text{arc cosine of -1, calculated to machine accuracy}

\[ R_1 = \frac{12}{2} + w_2 \]

\[ R_2 = \frac{12}{2} \]

\[ x_0 = -\frac{(11-w_1)}{2} \]

\[ y_0 = 10 + w_2 + \frac{12}{2} \]

\[ \theta = 0 \]

\[ K, 1, x_0 - R_1 \sin(\theta), y_0 + R_1 \cos(\theta), 0 \]

\[ \theta = \frac{\pi}{6} \]

\[ K, 2, x_0 - R_1 \sin(\theta), y_0 + R_1 \cos(\theta), 0 \]

\[ \theta = \frac{2\pi}{6} \]

\[ K, 3, x_0 - R_1 \sin(\theta), y_0 + R_1 \cos(\theta), 0 \]

\[ \theta = \frac{3\pi}{6} \]

\[ K, 4, x_0 - R_1 \sin(\theta), y_0 + R_1 \cos(\theta), 0 \]

\[ \theta = \frac{4\pi}{6} \]

\[ K, 5, x_0 - R_1 \sin(\theta), y_0 + R_1 \cos(\theta), 0 \]

\[ \theta = \frac{5\pi}{6} \]

\[ K, 6, x_0 - R_1 \sin(\theta), y_0 + R_1 \cos(\theta), 0 \]

\[ \theta = \frac{6\pi}{6} \]

\[ K, 7, x_0 - R_2 \sin(\theta), y_0 + R_2 \cos(\theta), 0 \]

\[ \theta = \frac{\pi}{6} \]

\[ K, 8, x_0 - R_2 \sin(\theta), y_0 + R_2 \cos(\theta), 0 \]

\[ \theta = \frac{2\pi}{6} \]

\[ K, 9, x_0 - R_2 \sin(\theta), y_0 + R_2 \cos(\theta), 0 \]

\[ \theta = \frac{3\pi}{6} \]

\[ K, 10, x_0 - R_2 \sin(\theta), y_0 + R_2 \cos(\theta), 0 \]

\[ \theta = \frac{4\pi}{6} \]

\[ K, 11, x_0 - R_2 \sin(\theta), y_0 + R_2 \cos(\theta), 0 \]

\[ \theta = \frac{5\pi}{6} \]

\[ K, 12, x_0 - R_2 \sin(\theta), y_0 + R_2 \cos(\theta), 0 \]

\[ \theta = \frac{6\pi}{6} \]

\[ K, 13, x_0 - R_2 \sin(\theta), y_0 + R_2 \cos(\theta), 0 \]

\[ \theta = 0 \]

\[ K, 14, x_0 - R_2 \sin(\theta), y_0 + R_2 \cos(\theta), 0 \]

\[ \theta = -\frac{\pi}{6} \]

\[ A, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 1 \]

\[ x_0 = w_1 + \frac{(11 - w_1)}{2} \]

\[ y_0 = 10 + w_2 + \frac{12}{2} \]

\[ \theta = 0 \]

\[ K, 15, x_0 - R_1 \sin(\theta), y_0 + R_1 \cos(\theta), 0 \]

\[ \theta = -\frac{\pi}{6} \]

\[ K, 16, x_0 - R_1 \sin(\theta), y_0 + R_1 \cos(\theta), 0 \]

\[ \theta = -\frac{\pi}{6} \]

\[ K, 17, x_0 - R_1 \sin(\theta), y_0 + R_1 \cos(\theta), 0 \]

\[ \theta = -\frac{3\pi}{6} \]
K, 18, x0-R1\sin(\theta), y0+R1\cos(\theta), 0  
theta = -4\pi/6

K, 19, x0-R1\sin(\theta), y0+R1\cos(\theta), 0  
theta = -5\pi/6

K, 20, x0-R1\sin(\theta), y0+R1\cos(\theta), 0  
theta = -6\pi/6

K, 21, x0-R1\sin(\theta), y0+R1\cos(\theta), 0  
theta = -6\pi/6

K, 22, x0-R2\sin(\theta), y0+R2\cos(\theta), 0  
theta = -5\pi/6

K, 23, x0-R2\sin(\theta), y0+R2\cos(\theta), 0  
theta = -4\pi/6

K, 24, x0-R2\sin(\theta), y0+R2\cos(\theta), 0  
theta = -3\pi/6

K, 25, x0-R2\sin(\theta), y0+R2\cos(\theta), 0  
theta = -2\pi/6

K, 26, x0-R2\sin(\theta), y0+R2\cos(\theta), 0  
theta = -\pi/6

K, 27, x0-R2\sin(\theta), y0+R2\cos(\theta), 0  
theta = 0

K, 28, x0-R2\sin(\theta), y0+R2\cos(\theta), 0

A, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 15

VEXT, ALL,, 0, 0, thk,

BLC4, 0, 0, w1, l0, thk
BLC4, 0, l0, w1, w2, thk
BLC4, 0, l0, -((l1-w1)/2), w2, thk
BLC4, w1, l0, ((l1-w1)/2), w2, thk

delta = (l1-w3)/2

BLC4, -((l1-w1)/2), (l0+w2+l2), delta, w2, thk
BLC4, delta -((l1-w1)/2), (l0+w2+l2), w3, w2, thk
BLC4, (delta -((l1-w1)/2)+w3), (l0+w2+l2), delta, w2, thk
BLC4, delta -((l1-w1)/2), (l0+w2+l2+w2), w3, l3, thk

! Eng of geometry

ALLSEL, ALL
NUMMERGE, ALL
VSEL, ALL
VATT, 1
SMRT, 10
MSHAPE, 1, 3D
MSHKEY, 0
VMESH, ALL

ALLSEL, ALL  ! Fix the bottom of the Bonding pads
ASEL,S,LOC,Y,-tol,tol
NSLA,R,1
D,ALL,UX,0,,UY,UY,UY

YL=10+w2+l2+w2+l3 ! Apply force at the tip
ALLSEL,ALL ! and constrain UX=UY=0
ASEL,S,LOC,Y,YL-tol,YL+tol
NSLA,R,1
D,ALL,UX,0
D,ALL,UY,0
F,ALL,FZ,(Force*1e6/N_nodes)

/SOLU
ALLSEL,ALL
SOLVE
ALLSEL,ALL
SAVE,spring,db,,ALL
FINISH

/POST1
/EFACE,1
AVPRIN,0,0,
PLNSOL,U,Z,2,1
/PLOPTS,INFO,2
/PLOPTS,LEG1,1
/PLOPTS,LEG2,1
/PLOPTS,LEG3,1
/PLOPTS,FRAME,1
/PLOPTS,TITLE,1
/PLOPTS,MINM,1
/PLOPTS,LOGO,1
/PLOPTS,WINS,1
/PLOPTS,WP,0
/PLOPTS,DATE,2
/TRIAD,ORIG
/REPLOT

FINISH

/EOF
APPENDIX H: PIC MICROCONTROLLER SOURCE CODE

The program displays the numbers from 0-9 in an endless loop

Author: Kalin Lazarov - kalin@email.arizona.edu

Program words: 381
RAM location: 4

#include <p16f818.inc>
errorlevel -302

_CONFIG CP_OFF & CCP1_RB3 & _DEBUG_OFF & _WRT_ENABLE_OFF &
_CPD_OFF & _LVP_OFF & _BODEN_OFF & _MCLR_OFF & _PWRTE_ON & _WDT_OFF &
_INTRC_IO

抬成#define

#define ROW1 PORTB,5
#define ROW1_TRIS TRISB,5
#define ROW2 PORTA,6
#define ROW2_TRIS TRISA,6
#define ROW3 PORTA,7
#define ROW3_TRIS TRISA,7
#define ROW4 PORTA,0
#define ROW4_TRIS TRISA,0
#define COL1 PORTA,1
#define COL1_TRIS TRISA,1
#define COL2 PORTB,3
#define COL2_TRIS TRISB,3
#define COL3 PORTB,4
#define COL3_TRIS TRISB,4
#define COL4 PORTA,2
#define COL4_TRIS TRISA,2
#define COL5 PORTA,3
#define COL5_TRIS TRISA,3
#define HV_OFF PORTB,0
#define HV_OFF_TRIS TRISB,0
#define HV_ON PORTB,1
#define HV_ON_TRIS TRISB,1

抬成
org 0x0000
goto Start

Symbol Writing Database

The symbol definition database contains the sequence of the pixels required to display one symbol. The end of the record for each symbol is at 0x00. Valid pixel addresses start from 1.

Usage: 1. Load the sum of the addresses of the symbol and the pixel offset in W
2. Call Read_Symbol
3. The returned value in W is the number of the pixel to be displayed

Memory locations: 0
Program words : 136

if (Symbol_End > 0xFF)
  error "Symbol Database crosses page boundary"
endif

Decode_Symbol
  clrf PCLATH
  movwf PCL

ASCII_0
  retlw d'8'
  retlw d'3'
  retlw d'2'
  retlw d'5'
  retlw d'9'
  retlw d'13'
  retlw d'18'
  retlw d'19'
  retlw d'16'
  retlw d'12'
  retlw d'8'
  retlw 0x00

ASCII_1
  retlw d'4'
  retlw d'8'
  retlw d'12'
  retlw d'16'
  retlw d'20'
  retlw 0x00

ASCII_2
  retlw d'5'
  retlw d'2'
  retlw d'3'
  retlw d'8'
  retlw d'11'
<table>
<thead>
<tr>
<th>Array_Test</th>
<th>ASCII_7</th>
<th>ASCII_8</th>
<th>ASCII_9</th>
</tr>
</thead>
<tbody>
<tr>
<td>retlw d'16'</td>
<td>retlw d'16'</td>
<td>retlw d'8'</td>
<td>retlw d'1'</td>
</tr>
<tr>
<td>retlw d'11'</td>
<td>retlw d'11'</td>
<td>retlw d'3'</td>
<td>retlw d'2'</td>
</tr>
<tr>
<td>retlw d'10'</td>
<td>retlw d'10'</td>
<td>retlw d'5'</td>
<td>retlw d'3'</td>
</tr>
<tr>
<td>retlw d'9'</td>
<td>retlw d'9'</td>
<td>retlw d'4'</td>
<td>retlw d'6'</td>
</tr>
<tr>
<td>retlw 0x00</td>
<td>retlw 0x00</td>
<td>retlw d'5'</td>
<td>retlw d'7'</td>
</tr>
<tr>
<td>retlw d'11'</td>
<td>retlw d'11'</td>
<td>retlw d'10'</td>
<td>retlw d'8'</td>
</tr>
<tr>
<td>retlw d'16'</td>
<td>retlw d'16'</td>
<td>retlw d'11'</td>
<td>retlw d'12'</td>
</tr>
<tr>
<td>retlw d'19'</td>
<td>retlw d'19'</td>
<td>retlw d'12'</td>
<td>retlw d'16'</td>
</tr>
<tr>
<td>retlw d'18'</td>
<td>retlw d'18'</td>
<td>retlw d'16'</td>
<td>retlw d'20'</td>
</tr>
<tr>
<td>retlw d'13'</td>
<td>retlw d'13'</td>
<td>retlw 0x00</td>
<td>retlw d'20'</td>
</tr>
<tr>
<td>retlw d'10'</td>
<td>retlw d'10'</td>
<td></td>
<td></td>
</tr>
<tr>
<td>retlw d'11'</td>
<td>retlw d'11'</td>
<td></td>
<td></td>
</tr>
<tr>
<td>retlw d'8'</td>
<td>retlw d'8'</td>
<td></td>
<td></td>
</tr>
<tr>
<td>retlw d'16'</td>
<td>retlw d'16'</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Subroutines Set_Pixel and Clear_Pixel

Usage: 1. Load the pixel number in W (Valid pixels 1-20)
   offset in W
   2. Call Set_Pixel/ Clear_Pixel to Set/Clear the pixel

Memory locations : 1
Program words : 134

Register definitions

CBLOCK 0x20
Pixel_Addr ENDC

;***** Subroutine Set_Pixel *****
Set_Pixel movwf Pixel_Addr
            bcf STATUS,C
            rrf Pixel_Addr,f
            addwf Pixel_Addr,f ; Multiply the Pixel offset by 3
            movlw High Pixel_0_Set
            movwf PCLATH
            movf Pixel_Addr,w
            addlw Low Pixel_0_Set
            btsc STATUS,C
            incf PCLATH,f
            movwf PCL ; Jump to the Pixel location

Pixel_0_Set nop
            nop return

Pixel_1_Set bsf ROW1
            bsf COL1 return

Pixel_2_Set bsf ROW2
            bsf COL1 return

Pixel_3_Set bsf ROW3
            bsf COL1 return
Pixel_4_Set    bsf ROW4
        bsf COL1
        return
Pixel_5_Set    bsf ROW4
        bsf COL2
        return
Pixel_6_Set    bsf ROW3
        bsf COL2
        return
Pixel_7_Set    bsf ROW2
        bsf COL2
        return
Pixel_8_Set    bsf ROW1
        bsf COL2
        return
Pixel_9_Set    bsf ROW1
        bsf COL3
        return
Pixel_10_Set   bsf ROW2
        bsf COL3
        return
Pixel_11_Set   bsf ROW3
        bsf COL3
        return
Pixel_12_Set   bsf ROW4
        bsf COL3
        return
Pixel_13_Set   bsf ROW4
        bsf COL4
        return
Pixel_14_Set   bsf ROW3
        bsf COL4
        return
Pixel_15_Set   bsf ROW2
        bsf COL4
        return
Pixel_16_Set   bsf ROW1
        bsf COL4
        return
Pixel_17_Set   bsf ROW1
        bsf COL5
        return
Pixel_18_Set   bsf ROW2
        bsf COL5
        return
Pixel_19_Set   bsf ROW3
        bsf COL5
        return
Pixel_20_Set   bsf ROW4
        bsf COL5
        return
Set_Pixel_End

;****** Subroutine Clear_Pixel ******
Clear_Pixel

movwf Pixel.Addr
bcf STATUS,C
rrf Pixel.Addr,f
addwf Pixel.Addr,f ; Multiply the Pixel offset by 3
movlw High Pixel_0_Clear
movwf PCLATH
movf Pixel.Addr,w
addlw Low Pixel_0_Clear
btfsc STATUS,C
incf PCLATH,f
movwf PCL ; Jump to the Pixel location

Pixel_0_Clear

nop
nop
return

Pixel_1_Clear

bcf ROW1
bcf COL1
return

Pixel_2_Clear

bcf ROW2
bcf COL1
return

Pixel_3_Clear

bcf ROW3
bcf COL1
return

Pixel_4_Clear

bcf ROW4
bcf COL1
return

Pixel_5_Clear

bcf ROW4
bcf COL2
return

Pixel_6_Clear

bcf ROW3
bcf COL2
return

Pixel_7_Clear

bcf ROW2
bcf COL2
return

Pixel_8_Clear

bcf ROW1
bcf COL2
return

Pixel_9_Clear

bcf ROW1
bcf COL3
return

Pixel_10_Clear

bcf ROW2
bcf COL3
return

Pixel_11_Clear

bcf ROW3
bcf COL3
return

Pixel_12_Clear

bcf ROW4
bcf COL3
return

Pixel_13_Clear

bcf ROW4
Subroutine Display_Symbol

Usage: 1. Load the symbol address in W (Should be in the first bank)
2. Call Display_Symbol to write the symbol with LED

Memory locations: 2
Program words: 24

Register definitions

CBLOCK
Pixel_Offset
Pixel_Number
ENDC

Display_Symbol
Display_Loop

movwf Pixel_Offset ; Get the pixel value
call Decode_Symbol
movwf Pixel_Number
andlw 0xFF         ; Test the pixel number
btfsc STATUS,Z    ; If the pixel number is 0
return            ; Exit the subroutine
call Set_Pixel
call Pixel_Wait_High
movf Pixel_Number,W
call Clear_Pixel
call Pixel_Wait_Low
incf Pixel_Offset,f ; Move to the next record
movf Pixel_Offset,W ; Move the current Pixel address in W
goto Display_Loop

Pixel_Wait_High movlw d'192'
Wait_Loop1 addlw 0x01
btfss STATUS,C
goto Wait_Loop1
return

Pixel_Wait_Low movlw d'254'
Wait_Loop2 addlw 0x01
btfss STATUS,C
goto Wait_Loop2
return

******************************************************************************

Subroutine Clear_MUX

The subroutine clears the multiplexer pins (ROW1-ROW4, COL1-COL5)

Usage: 1. Call Clear_MUX

Memory locations : 0
Program words : 10

******************************************************************************

Clear_MUX bcf ROW1 ; Clear MUX pins
bcf ROW2
bcf ROW3
bcf ROW4
bcf COL1
bcf COL2
bcf COL3
bcf COL4
bcf COL5
return

******************************************************************************

Subroutine Init_MUX

The subroutine initializes the multiplexer pins defining them as outputs.
Special care must be taken to initialize PORTA properly (specific to PIC16F818)

Usage: 1. Call Init_MUX

******************************************************************************
Memory locations: 0
Program words: 18

Init_MUX
    call Clear_MUX ; Clear MUX latches
    clrf PORTA ; Initialize PORTA
    bsf STATUS,RP0
    movlw 0x06 ; Configure all pins
    movwf ADCON1 ; as digital inputs
    movlw 0xFF ; Value used to
    movwf TRISA ; Set RA<7:0> as inputs
    bcf ROW1_TRIS ; Define individual pins
    bcf ROW2_TRIS
    bcf COL1_TRIS
    bcf COL2_TRIS
    bcf COL3_TRIS
    bcf COL4_TRIS
    bcf COL5_TRIS
    bcf STATUS,RP0
    return ; End of initialization

Main Program

The main program displays the numbers from 0-9 in an endless loop

Memory locations: 1
Program words: 41

Register Definitions

CBLOCK
Wait_Cnt
ENDC

Start
    call Init_MUX

Loop
    movlw ASCII_0
    call Display_Symbol
    call Symbol_Wait
    movlw ASCII_1
    call Display_Symbol
    call Symbol_Wait
    movlw ASCII_2
    call Display_Symbol
call   Symbol_Wait
movlw  ASCII_3
call   Display_Symbol
call   Symbol_Wait
movlw  ASCII_4
call   Display_Symbol
call   Symbol_Wait
movlw  ASCII_5
call   Display_Symbol
call   Symbol_Wait
movlw  ASCII_6
call   Display_Symbol
call   Symbol_Wait
movlw  ASCII_7
call   Display_Symbol
call   Symbol_Wait
movlw  ASCII_8
call   Display_Symbol
call   Symbol_Wait
movlw  ASCII_9
call   Display_Symbol
call   Symbol_Wait

goto   Loop

;***** Subroutine Symbol_Wait *****

Symbol_Wait  movlw  '3'
             movwf  Wait_Cnt ; 350ms wait @ 31250 Hz
             movlw  '29'
             addlw  0x01
             btfss  STATUS,C
             goto   L2
             decfsz  Wait_Cnt,f
             goto   L2
             decfsz  Wait_Cnt,f
             goto   L1
             return

;*****************************************************************************

end
REFERENCES


