LOSSES IN TITANIUM-DIFFUSED LITHIUM NIOBATE
CHANNEL WAVEGUIDES DUE TO DIRECTIONAL CHANGES

by
Lynn Donald Hutcheson

A Dissertation Submitted to the Faculty of the
COMMITTEE ON OPTICAL SCIENCES (GRADUATE)
In Partial Fulfillment of the Requirements
For the Degree of
DOCTOR OF PHILOSOPHY
In the Graduate College
THE UNIVERSITY OF ARIZONA

1980
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ABSTRACT

The optical transmission characteristics of dielectric optical waveguides having directional changes is considered in this study. Experimental and theoretical loss results are presented for two types of waveguide bends. First is the corner bend where two straight waveguides are joined together at some angle. The second type is a curved waveguide having some radius of curvature. The loss mechanism is different for each type of bend. The corner bend is basically a scattering loss due to a mismatch of the modes of the two joining waveguides. The loss in the curved waveguide is due to radiation of the energy away from the waveguide as it propagates around the bend.

The waveguides were fabricated by diffusing 3 μm wide 200 Å thick titanium strips into LiNbO₃. All of the curved portions of waveguide were joined by straight waveguides at the input and output of the curved waveguides. Rayleigh scattering and absorption loss was measured in the straight waveguides to determine their optical quality which yielded about 1.4 dB/cm.

The loss due to corner bends was measured for angles from 0.1° to 3.0° in steps of 0.1°. The loss ranged from about 0.1 dB to 23 dB for 0.1° and 3.0° respectively. The
results are slightly dependent upon the polarization of the light and the orientation of the LiNbO$_3$ crystal.

The curved waveguides were fabricated in two different geometries. The first geometry was a straight waveguide joined by a curved portion and then joined by another straight waveguide. The second geometry is different from the first by another curved portion joining the first curved portion in between the two straight waveguides. The two curved portions are equal but have opposite curvature which have an S shape. The radiation loss was measured for radius of curvatures from 1.0 cm to 3.0 cm. The results ranged from 41 dB/cm to 1 dB/cm for 1.0 cm and 3.0 cm radius of curvatures respectively. At each of the straight to curved and curved to curved junctions there exists a mode mismatch loss. The straight to curved mode mismatch loss was 1.65 dB for R = 1 cm and 0.5 dB for R = 3 cm. The curved to curved mode mismatch loss was 6 dB for R = 1 cm and 0.5 dB for R = 3 cm.

The results for the corner bends and the curved bends were used to study the constraints on integrated optical devices. In many integrated optical devices, it is necessary for two different straight portions of a single mode channel waveguide to be connected with a given amount of transverse offset. The experimental and theoretical results showed that for small transverse offsets the corner bend approach yields smaller loss. The curved bend (S bend) approach was better for larger transverse offsets.
Theory was developed for this study of bending loss in titanium diffused LiNbO$_3$ waveguides. In general all of the experimental results agreed quite well with the theoretical predictions.
CHAPTER 1

INTRODUCTION

The invention of the laser has stimulated work in the entire field of optics as well as opened up new avenues of research. One of the new areas that has generated a great deal of interest is integrated optics. The term "integrated optics", coined by Miller (1969), came into being ten years ago. Since that time the number of governmental, industrial, university, and research laboratories involved in this field has grown rapidly. Light guiding was first observed in 1961 in optical fibers by Snitzer and Osterberg (1961) and by Kapany and Burke (1961).

Like integrated electronics, the promise of integrated optics (IO) lies in the realization of devices and systems that would be too cumbersome or expensive to be utilized in bulk form. Traditional optical apparatus must be aligned with extreme accuracy and is thus susceptible to the smallest amount of vibration and temperature change, whereas an IO circuit is relatively insensitive to vibrations and temperature changes. Integrated optic devices concentrate light in thin film waveguides that are deposited on the surface or inside a substrate. Because of the short wavelength of light, dielectric light waveguides can be
extremely small in their dimensions. The reduced size of IO circuits thus makes it possible to achieve a much higher density of components compared to conventional optical equipment aligned on steel rails or on heavy optical benches.

Integrated optics involves the technology of guiding and manipulating optical waves in dielectric waveguides, the generation and detection of optical waves, and the coupling of optical waves into and out of IO circuits. Integrated optics can be divided into two general areas. The first is passive IO, which involves coupling, guiding, and partitioning of optical waves in IO circuits without the application of external fields or acoustic waves. The second area, active IO circuits, makes use of electric fields or acoustic waves to provide for the modulation, deflection, variable coupling, etc., of optical waves. Several excellent review articles by Tien (1971), Taylor and Yariv (1974), and Kogelnik (1975a and b) as well as recent books by Kapany and Burke (1972), Marcuse (1974), and Tamir (1975) give detailed discussions of the concepts of optical waveguiding.

The type of waveguide to be investigated in this work is the embedded strip waveguide. A cross-sectional view depicting typical dimensions of the waveguide is shown in Fig. 1.1. The waveguides are fabricated (Schmidt and Kaminow, 1974) by diffusing titanium into lithium niobate (LiNbO$_3$). The width of the titanium strip before diffusion
Fig. 1.1. Embedded Strip Waveguide of the Type Used for Measuring Bending Losses.
is 3 \, \mu m \text{ and the diffusion depth is approximately } 3 \, \mu m. \text{ The diffusion depth depends on the particular orientation of the crystal. The diffusion process increases the index of refraction at the surface } \approx 0.005 \text{ to } 0.01. \text{ A more complete discussion of the waveguide fabrication process will be described in Chapter 3. The reason for choosing LiNbO}_3 \text{ is because of its excellent piezo-electric and electro-optic properties. The LiNbO}_3 \text{ waveguide is diffused at temperatures below the Curie temperature, it has tight optical confinement and low transmission loss. These properties make the LiNbO}_3 \text{ waveguide an excellent candidate for highly efficient modulation. Much research has been conducted on device fabrication using LiNbO}_3 \text{, but very little research involving some of its more fundamental properties, such as losses, has been reported. This study is an attempt to answer some of the loss problems associated with bends in LiNbO}_3 \text{ in-diffused waveguides.}

Integrated optics has not yet been developed to the same maturity as integrated electronics. The density of electronic components on a single substrate is quite large and is increasing all the time. The question of density of optical components on a single integrated optical chip is being asked. To achieve high component density the waveguides must bend to guide the light from one optical component to the next. The loss due to waveguide bending is one of the primary limitations of component density. In addition to
bending loss affecting component density on a single substrate, many of the individual integrated optical components have channel waveguides with bends in them; two examples are shown in Figs. 1.2 and 1.3. The first example in Fig. 1.2 is a switched directional coupler described by Kogelnik and Schmidt (1976). In this configuration light is coupled into one of the waveguides and by altering the electro-optic properties of the waveguide by applying a voltage across the electrodes all or part of the light can be coupled into the other waveguide. In Fig. 1.3 is shown an interferometric waveguide modulator developed by Burns et al. (1978). In this example coupled light from the input is split equally into the two branches. If nothing is done to either of the light beams in the two branches then the light will recombine and proceed down the waveguide. However, a phase shift can be applied to one of the branches by altering its electro-optic properties, and the light will interferometrically add to zero yielding a transmission minimum. The output is modulated by using an alternating voltage on the electrodes.

A theory was developed in this study to predict the loss due to waveguide bends. Measurements were made and compared to the theory. Two types of bends were investigated, the angle bend where two straight waveguides are joined together at some angle. The second type is a bend
Fig. 1.2. Switched Directional Coupler.

Fig. 1.3. Interferometric Waveguide Modulator.
which has a smooth curvature. The loss results from the two types of bends are then compared to determine which type of bend yields the lowest loss for different waveguide designs.
CHAPTER 2

EXPERIMENTAL APPARATUS AND PROCEDURE

Introduction

The apparatus used for the experimental study consisted of a HeNe laser, PIN structure photodiodes, variable reflector, beamsplitter, polarization rotator, lenses, rotation and translation stages, lock-in amplifiers, x-y recorder, chopper, and the coupling arrangement. A general experimental configuration is shown in Fig. 2.1 which will be described in further detail later in this chapter. This setup is only general and certain aspects of it are changed depending on the peculiarities of a given experiment.

Laser to Waveguide Coupler

In all of the experiments performed in this study the laser beam was coupled to the waveguide via a prism as first reported by Harris and Schubert (1969) and Tien, Ulrich, and Martin (1969). The principle of operation of the prism coupler can be understood by considering the illustrations in Fig. 2.2. In Fig. 2.2(a) the upper boundary has index \( n_p \) for the prism while the lower boundary has index \( n_a \) which is usually air. Consider a beam incident upon the interface at angle \( \theta_p \) greater than the critical angle where
Fig. 2.1. Experimental Configuration for Measuring Bending Loss.
Fig. 2.2. Coupled-mode Description of Prism Couplers.

a. Plane wave incident from a denser medium.
b. Field propagating along a waveguide.
c. Incident field couples to waveguide mode.
For this case the superposition of the incident wave and the totally reflecting wave yield a standing wave along the vertical y direction in the denser medium. Below the interface y < 0 the field is an evanescent exponentially decaying field. In Fig. 2.2(b) is shown the field amplitude in the waveguide. Inside the waveguide the field is sinusoidal while outside the waveguide the field is an exponentially decaying evanescent field. For the two cases in Fig. 2.2(a) and 2.2(b) the waves propagate along the z axis with propagation coefficients $n_p k \sin \theta_p$ and $n_g k \sin \theta_g$ respectively where $k = 2\pi/\lambda$, $\lambda$ is the free space wavelength, $n_g$ is the index of the guide, and $\theta_g$ is the propagation angle inside the waveguide, of the two plane waves which comprise a mode (Tamir, 1975).

If the two regions are brought close enough together as shown in Fig. 2.2(c), such that the evanescent fields from the prism and the waveguide overlap, then energy can be transferred to the waveguide from the prism. For such a condition to exist the two propagating modes must be phase matched. This condition is met if

$$n_p k \sin \theta_p = n_g k \sin \theta_g . \quad (2.2)$$

In all cases the propagation angle $\theta_g$ is greater than $\theta_p$. The reason is that since the substrate index $n_b > n_a$ the
critical angle for the guided mode is greater than the critical angle for the prism and $\sin \theta_g > \sin \theta_p$. Consequently, the prism must have an index larger than the waveguide index. For the case of LiNbO$_3$ waveguides the index is approximately 2.2; consequently, the prism index needs to be quite large.

Besides needing prisms with a large index of refraction, prism coupling techniques suffer from other disadvantages. The gap between the prism $h_a$ (shown in Fig. 2.2) and the waveguide is usually less than half a wavelength. The surface of the waveguide needs to be flat and extremely clean because any dust particles cause additional coupling losses. For waveguides with a large index of refraction such as LiNbO$_3$ the amplitude of the field at the waveguide-air interface is very small as will be discussed in Chapter 4. This causes the exponential tail of the evanescent wave extending outside the waveguide to be quite short and requires the gap between the prism and the waveguide to be smaller than for lower index waveguides.

Prism coupling does have a number of advantages that other coupling techniques do not. Other coupling techniques require additional processing steps of the integrated optical chip to achieve coupling. End fire coupling, which involves the focusing of the laser light onto the end of waveguide, requires either cleaving or polishing the end of the waveguide to minimize the scattering at the waveguide facet.
Another coupling technique, grating couplers fabricated directly on top of the waveguide, is very time consuming to realize. Prisms, on the other hand, are extremely simple to use; once the coupling apparatus has been machined then the only additional steps is to clean the sample and assemble the coupling apparatus in a clean hood to avoid dust particles. A sketch of the prism coupling apparatus is shown in Fig. 2.3.

The prism material used in these experiments is rutile (TiO₂). Rutile is a uniaxial negative anisotropic crystal which poses no problem as long as the optic axis is oriented in one particular direction. Figure 2.4 shows the correct orientation. If the prism's optic axis has the orientation shown in Fig. 2.4 then for TE polarization the electric field is polarized along the optical axis of the prism, regardless of the angle of incidence upon the prism, and has an extraordinary index of refraction \( n_e = 2.8672 \) at a wavelength of 0.6328 \( \mu \text{m} \). Also, TM polarized light is polarized perpendicular to the optical axis, regardless of the incidence angle upon the prism and has an ordinary index of refraction of \( n_o = 2.5848 \) at \( \lambda = 0.6328 \mu \text{m} \). The ordinary and extraordinary indices of refraction were measured using the angle of minimum deviation procedure, which is described in the next section of this chapter. If the prism did not have its optic axis oriented in this direction,
Fig. 2.3. Apparatus Used for Input and Output Prism Coupling.
Fig. 2.4. Orientation of the Optic Axis of the Rutile Coupling Prism.

The prism is a $30^\circ$-$60^\circ$-$90^\circ$ prism with the base $1 \times 1$ cm.
then the index of the prism would change with the rotation of the prism and it would be difficult to couple only TE or only TM modes into the waveguides. In addition, if the optic axis of the prism is not oriented properly, the output coupled beam would be split into two beams having orthogonal polarizations.

**Measurement of Prism Parameters**

The ordinary and extraordinary indices of refraction were measured using the angle of minimum deviation techniques as described by Bond (1965). The angles of the prism were also measured. The prism parameters are extremely important to the measurement of the waveguide parameters, which will be discussed later in this chapter. The measurements were made on a prism spectrometer. The spectrometer used is extremely accurate and can measure angles to ± 10 arcsecond accuracy.

The prism is placed on a tilting adjustable and rotatable platform. The geometry for measuring the prism angles is shown in Fig. 2.5. A slit is illuminated by a HeNe laser and collimated light therefrom is incident onto two sides of the prism simultaneously as shown in Fig. 2.5. The table is adjusted until the slit can be seen through the telescope in position 1. The telescope is rotated to position 2 and the table is adjusted until the slit can be seen.
Fig. 2.5. Apparatus Used to Measure the Prism Angles.
Then the telescope is rotated back to position 1 and the procedure is repeated until the slit can be seen in positions 1 and 2 with no adjustment of the prism table. The base about which the telescope rotates is marked in degrees and there is a vernier which allows the angle to be read to ten arcseconds. The angle is read at both telescope positions with the difference between the two angular readings being twice the prism angle. This procedure is repeated until all of the prism angles have been measured. A typical angle measurement of one of the nominal 30-60-90° rutile prisms yielded the three angles 29.9167° - 59.9842° - 90.0991°.

Measurements of the prism index by the angle of minimum deviation is shown in Fig. 2.6. The experimental apparatus used for measuring the prism angle is the same for the prism index measurement except part of the collimated laser beam is deviated by the prism and the other portion passes undeviated to the telescope. In Fig. 2.7 is sketched an expanded view of the prism showing the total angle of deviation of the light ray traveling through the prism. Let the angle of incidence upon the prism be $\alpha_0$ and the angle of the ray from the exit face of the prism be $\beta_0$. As $\alpha_0$ is changed so is $\delta$, the total deviation angle. The total angle of deviation $\delta$ is a minimum when $\alpha_0 = \beta_0$ and $\alpha_1 = \beta_1 = \Delta/2$ where $\alpha_1$ and $\beta_1$ are the angles of the rays inside the prism and $\Delta$ is the prism angle. Then
Fig. 2.6. Apparatus Used to Measure the Index of Refraction of the Prism Using the Angle of Minimum Deviation Method.

Fig. 2.7. Expanded View of the Prism Showing the Total Angle of Deviation for Measuring the Prism Index Shown in Fig. 2.6.
\[ \delta_{\text{min}} = 2 \alpha_o - \Delta \] (2.3)

and

\[ n_p = \frac{\sin \alpha_o}{\sin \alpha_1} \] (2.4)

Using \( \alpha_1 = A/2 \) and the above two equations yields

\[ n_p = \frac{\sin \left( \delta_{\text{min}} + \frac{A}{2} \right)}{\sin \left( \frac{A}{2} \right)} \] (2.5)

The index of the prism \( n_p \) is now in terms of measurable quantities. The prism angle \( A \) is measured using the technique previously described. The angle \( \delta_{\text{min}} \) is measured by placing the prism in the collimated beam as shown in Fig. 2.6. The image of the slit is viewed through the telescope undeviated and then viewed after being deviated through the prism. The prism is rotated until the angle is a minimum. The difference between two angle readings of the telescope yields \( \delta_{\text{min}} \).

The accuracy to which the index of the prism \( n_p \pm \Delta n_p \) can be measured is determined by differentiating Eq. (2.5) with respect to the minimum deviation angle \( \delta_{\text{min}} \) which yields

\[ n_p = \frac{\cos \left( \delta_{\text{min}} + \frac{A}{2} \right) \Delta \delta_{\text{min}}}{\sin \left( \frac{A}{2} \right)} \] (2.6)

from Eq. (2.5) into Eq. (2.6) and rearranging; the minimum deviation angle \( \delta_{\text{min}} \) must be measured to an accuracy

Hence, for the rutile prisms used in these experiments, $A \approx 30^\circ$, $\delta_{\text{min}} \approx 65^\circ$, and $n_e \approx 2.8$, the value of $\delta_{\text{min}}$ needs to be measured to better than $0.0045^\circ$ or $16''$ in order to measure $n_p$ to the fourth decimal place. For $n_o \approx 2.6$ and $\delta_{\text{min}} \approx 53^\circ$, $\delta_{\text{min}}$ must be measured to better than $0.0039^\circ$ or $14''$ to achieve fourth decimal place accuracy in $n_p$.

Upon examination of Eq. (2.5) the prism angle $A$ must also be measured quite accurately which is determined by differentiating Eq. (2.5) with respect to $A$ and upon rearranging terms and using a trigonometric identity the value of $A$ must be measured to an accuracy

$$\Delta A < \frac{2 \sin^2 (A/2) \Delta n_p}{\sin (\delta_{\text{min}}/2)}.$$  

(2.9)

For the same values used above, in order to determine $n_p \pm 10^{-4}$, the prism angle must be measured to better than $5''$ and $6''$ for extraordinary and ordinary indices respectively. One should note that $A$ has the requirement

$$A < 2 \sin^{-1} (1/n)$$  

(2.10)

otherwise light will not pass through the prism, but will be totally reflected. Hence, $A$ must be less than $42^\circ$ and $45^\circ$ for extraordinary and ordinary indices respectively.
The prism, as previously discussed, is birefringent. Since the optic axis of the prism is oriented as shown in Fig. 2.4 the prism can be rotated with the light polarized parallel to the optic axis ($n_e$) or perpendicular to the optic axis ($n_o$) without changing the direction of polarization with respect to the optic axis. This allows $\delta_{\text{min}}$ to be measured for both polarizations yielding $n_e$ and $n_o$. The minimum deviation angle was measured for both polarizations for $\Delta = 29.9167^\circ$ and yielded, for light polarized parallel to the optic axis, $\delta_{\text{min}} = 65.5583^\circ$, and polarized perpendicular to the optic axis $\delta_{\text{min}} = 53.7833^\circ$. Thus Eq. (2.3) yields $n_e = 2.8672$ and $n_o = 2.5848$.

### Waveguide Coupling Efficiency

A technique described by Tangonan, Barnoski, and Lee (1977) for measuring the coupling efficiency of prism couplers is useful for planar or wide channel (multimode) waveguides, is sketched in Fig. 2.8. The system consists of a laser, polarization rotator (for choosing either TE or TM modes) beamsplitter, detector, lock-in amplifier, x-y recorder, and the prism coupler and waveguide arrangement mounted on a rotation stage with micro positioners. A 90° prism as is shown in Fig. 2.9 acts as a corner reflector in that the beam totally internally reflected from the base of the prism reflects back in the same direction as the incident
Fig. 2.8. Experimental Configuration for Measuring Coupling Efficiency into Planar or Wide Channel Waveguides.

\[ \text{COUPLING EFFICIENCY} \]

Fig. 2.9. Plot of Reflected Intensity as a Function of Incidence Angle upon the Prism.

The dip in the reflected intensity is the coupling efficiency.
beam, independent of the angle of incidence upon the prism. The reflected intensity from the base of the prism is measured as a function of the incidence angle. As the angle is varied through the resonance angle (angle of coupling into the waveguide) the reflected intensity is reduced by the amount coupled into the waveguide. The dip in the reflected intensity shown in Fig. 2.9 is a measure of the coupling efficiency. Routinely 50 to 60% of the input energy is coupled into the lowest order mode in a planar Ti in-diffused waveguide. The reason for the coupling efficiency not being larger for planar waveguides in LiNbO₃ is the evanescent tail extending outside the waveguide is quite small due to the large index difference between LiNbO₃ and air.

The above described technique produces accurate results provided the coupling efficiency is larger than a few percent. For smaller coupling efficiencies than a few percent it becomes difficult to measure changes this small in intensity to any degree of accuracy. For single mode channel waveguides extremely small coupling efficiencies on the order of 1% or less are typical. Channel widths for Ti diffused waveguides in LiNbO₃ are on the order of 3 μm for single mode operation. In this case the small coupling efficiency is due to the diameter of the input laser beam being much larger than the channel width.
The arrangement for measuring small coupling efficiencies is shown in Fig. 2.10. The input laser beam passes through the polarization rotator and is partially transmitted through a beamsplitter and is focused onto the base of the prism via a long focal length, 50 cm, lens. The portion of the input beam focused onto the base of the prism is reflected back out of the prism onto the beamsplitter and through the chopper and collected to be focused onto the detector. The chopper and the prism coupling arrangement are both very critical in their alignment in this experimental setup. The chopper is aligned such that the one beam from the variable reflector passes through the chopper while the beam from the prism is blocked by the chopper and vice versa. If this is accomplished then the two beams are exactly 180° electrically out of phase with respect to each other. Precise alignment of the prism coupling arrangement is necessary so that the beam reflected out of the prism coupling arrangement is not shifted as the input angle is varied. If the beam is shifted the two beams will no longer be 180° out of phase when the input angle is rotated. The reflection from the variable reflector is adjusted until the two beams incident on the collection lens are equal in magnitude. This is accomplished by adjusting the input angle on the prism such that no light is coupled into the waveguide and the chopper and variable reflectors are adjusted
Fig. 2.10. Experimental Arrangement for Measuring Small Coupling Efficiencies into Single Mode Channel Waveguides Using the Zero Modulation Technique.
until the output of the lockin amplifier is nulled to zero at a high gain setting. The two beams are now equal in intensity but are 180° out of phase, therefore, they produce no modulation. The alignment of the prism coupler arrangement is accomplished by rotating the prism-waveguide system to a few degrees from the resonance angle. The prism-waveguide system is adjusted by precision translation stages until the output from the lockin amplifier remains at zero while the prism is rotated through several degrees, but not through resonance. The collecting lens shown in Fig. 2.10 is used to assure that the two out-of-phase beams are focused onto the same spot on the detector. This eliminates any nonlinearities of the detector. Now as the angle of the input beam is rotated through resonance a change in reflected intensity yields an output from the detector.

The prism coupling arrangement is now rotated through resonance and the output from the lockin is a measure of the change in the reflected intensity. The gain in the amplifier is reduced and the variable reflector is moved from the system and the prism coupling arrangement is rotated slightly off resonance. The lockin now measures only the reflected intensity with nothing else in the system changed. The ratio of the two readings now gives the percentage of the light coupled into the waveguide. The dynamic range of the detector, lockin amplifier combination was measured to assure linearity. Calibrated neutral density filters for a
wavelength of 0.6328 μm were used to test the linearity over a range of 30 dB. The measurement yielded a linearity of better than 1% over the entire 30 dB of interest. Several coupling efficiency measurements were made by taking the waveguide out of the prism assembly and remounting it and measuring the coupling efficiency again. This was done several times and other waveguides were put into the system and also measured several times by removing and remounting several times. The results were averaged and yielded 0.96% ± 0.14% coupling efficiency for TE and 0.85% ± 0.13% for TM. All of the measurements were made on identically fabricated waveguides, i.e., same titanium thickness, same channel width before diffusion, same diffusion temperature, and diffusion time. The results of 0.95% and 0.85% are averages of all of the measurements while the ± 0.14% and ± 0.13% are the standard deviations from all of the measurements.

The reason for the coupling efficiency of the TE mode being larger than the TM mode is probably due to the difference of the angle of incidence on the base of the coupling prism. Since rutile is anistropic the prism coupling angles for the TE and TM modes are quite different. The prism used in these experiments was a 30°-60°-90° prism. The angle of incidence upon the base of the prism for TE is about 52° while it's only about 62° for TM.
A different technique of simply comparing the two signals, the constant intensity beam with the intensity of light reflected from the base of the coupling prism, was tried by using a differential preamp. The results using this technique were not reproducible. There are two possible explanations why this method does not work. First, the two signals needed to feed the differential preamp must come from two separate detectors. The detectors must be matched extremely close because we are subtracting two almost equal large signals to get a very small signal. Second, small instabilities in the preamp will give erroneous results because the two input signals are so close to each other. In the zero modulation technique described above the two signals are focused onto the same spot on one detector. The two beams can never be made to produce exactly zero modulation, however, it is easy to achieve a 25 to 30 dB reduction of the two signals below the level of the individual signals. This provides sufficient gain in the system to allow a very small change in intensity to be measured which is the advantage of the zero modulation technique. Once familiarity of the system is achieved the alignment procedure can be accomplished in a matter of a few minutes.
CHAPTER 3

WAVEGUIDE FABRICATION

Introduction

In the previous chapter a description of the experimental equipment used to measure bending loss was described. An explanation of the coupling of light into waveguides was given as well as techniques used to measure waveguide parameters. This chapter gives a detailed procedure for fabrication of the waveguides used in this experimental study. A description is given for the making of the mask used, cleaning and preparation of the substrate, delineation of the waveguide, and fabrication of the waveguide.

Mask Preparation

Before the optical waveguide can actually be fabricated a photographic mask of the necessary pattern needs to be prepared. All of the masks that were used in the loss measurement experiments were generated using a Gerber 733 Artwork Generator. The Gerber Plotter is an analog device controlled by instructions from a punched mylar tape generated by a computer. The Gerber Fortran Plotting Package was developed by the United States Atomic Energy Commission at Sandia Laboratories, Albuquerque, New Mexico. This plotting package was adapted for use on the 1110
Univac Computer at the Naval Weapons Center. The machine plotting mode is a "light head" that exposes a photosensitive emulsion on a 0.004 inch stable mylar back. The maximum size of film that can be used in the plotter is 24 x 24 inches, which is available in both positive and negative film. The light head has many different aperture sizes, but the size of interest for these experiments was the smallest one available, 0.002 inches. This means that a photoreduction (10 to 15 times) is necessary to achieve the narrow lines (3-5 micrometers) needed for single mode waveguide operation. The photoreduction, and thus the final mask formation, is performed on high resolution photographic glass plates.

Typically, industrial and other government laboratories use a standard pattern generator to make their masks. This type of pattern generator can only generate straight line segments. Therefore, if a curved line is needed, as is the case for these experiments, then several very short straight line segments have to be connected to form a curve. The Gerber Plotter has the advantage of being able to generate smooth curves, thus reducing the possibility of additional scattering losses caused by waveguide imperfections.

Substrate Preparation

The substrate material used in these experiments was select acoustic grade lithium niobate (LiNbO₃) purchased
from Crystal Technology. A few samples of optical grade LiNbO$_3$ were also purchased from the same supplier. Measurements performed on the optical grade samples indicated no increase in performance or reduction in loss over the acoustic grade samples. Due to the fact that the purchase price of the optical grade LiNbO$_3$ was three to four times more than the acoustic grade, all of the remaining measurements utilized the acoustic grade. The LiNbO$_3$ crystals were polished to a flatness of $\lambda/4$ on the waveguide surfaces. A surface roughness measurement was made on several samples with a Tallystep profile measuring device. The typical surface roughness profile is shown in Fig. 3.1. The measurement yielded a surface roughness of less than 5 micrometers. The stylus tip used for these measurements was spherical in shape and approximately 0.5 $\mu$m in diameter.

Before titanium deposition the LiNbO$_3$ surface must be thoroughly cleaned. The crystal is cleaned in a series of hot solvent baths. At 55°C the LiNbO$_3$ is cleaned in trichlorethylene, acetone, and methanol. Cotton swabs are used to scrub the surface in each bath. LiNbO$_3$ has a thermal expansion coefficient of $16.7 \times 10^{-6}$/°C so care must be taken to avoid thermal shock to the crystal when moving it between baths.

The cleaned LiNbO$_3$ crystal is put into a vacuum chamber so that a thin layer of titanium can be deposited
Fig. 3.1. Surface Roughness Measurement of LiNbO₃ Used in This Study.
on the surface. Two methods were used for Ti deposition; evaporation and sputtering. The evaporation system consisted of a vacuum chamber, an e-beam gun, and a quartz crystal thickness monitoring device. The chamber pressure was about $10^{-7}$ Torr. The sample is not heated and is approximately 12 inches from the Ti target. The resonant frequency of the quartz crystal changes as the Ti is evaporated. The frequency change for a particular material is a linear function of the thickness of the deposited material. For a given thickness, the frequency change is also dependent on the particular material. Consequently, the system needs to be calibrated for each material. Several test runs were made to get a calibration of the Ti thickness as a function of frequency. A LiNbO$_3$ sample with a small diameter wire blocking a portion of the surface is mounted beside the resonant cavity. The step in the Ti film on the target is needed to measure the thickness with the Tallystep. Several measurements yielded the calibration curve shown in Fig. 3.2. The frequency meter has a nulling capability so that before each Ti evaporation the frequency is nulled to zero. The Ti target is heated via the e-beam gun which bombards the target with high energy electrons. A shutter between the Ti target and the sample is kept closed until the target reaches its melting temperature of 1675°C. This is necessary to avoid uneven deposition thickness. At 1675°C the Ti target gets white hot, as can be
Fig. 3.2. Calibration Curve for Titanium Evaporation.

SLOPE = 0.5 Å/Hz
seen through the glass vacuum chamber. The shutter is left open until the desired thickness is obtained by monitoring the resonant frequency of the quartz crystal. The shutter is then closed and the system turned off. About 4 minutes evaporation time was needed to get a Ti thickness of 200 Å. The disadvantage of the evaporation process is that the Ti film evaporated onto the LiNbO₃ surface has a larger surface roughness than the LiNbO₃ surface itself. Tallystep measurements indicated 30 to 35 Å rms roughness. This is another reason for using a sputtering system.

Sputtering, like evaporation, is a coating process, but unlike evaporation, sputtering can transport almost any material from a source, called a target, to a substrate of almost any other material. The ejection of the source material is accomplished by the bombardment of the surface of the target with gas ions accelerated by a high voltage. Particles on the order of atomic dimension from the target are ejected as a result of momentum transfer between the incident ions and the target. The target-ejected particles are deposited on the substrate as a thin film. Sputtering has certain advantages over evaporation that make it more desirable. It is easier to control and to repeat accurately the deposition of thin films. It is very important to be able to control the film thickness accurately for reproducibility fabricating Ti in-diffused waveguides. In addition, the sputtered film follows more closely the
surface on which it is being deposited, thus there is no increase in surface roughness. The LiNbO$_3$ sample was placed about 10 inches from the titanium target. The vacuum chamber was evacuated to about 10$^{-7}$ torr. Argon gas was then purged into the chamber at such a rate that the pressure of the chamber was reduced to about 10$^{-2}$ torr. The sample was not heated so it remained at room temperature. Titanium was sputtered at the rate of 1.8Å/sec. The thickness of the Ti film was monitored in the same manner as the evaporation system, i.e., a quartz crystal monitoring device.

**Waveguide Pattern Preparation**

Once the desired thickness of titanium is deposited on the surface of the LiNbO$_3$ then the desired waveguide pattern can be etched out of the titanium. The technique used to accomplish this is called photolithography. It is much the same as that used in integrated circuits except that the dimensions are much smaller and the tolerances are stricter for integrated optics.

A positive photoresist is deposited on top of the titanium. Typically, a layer of photoresist is deposited by putting a couple of drops on the surface and spinning the sample at a high rpm. The photoresist ends up as a thin coat, on the order of 1 µm, on the sample, with the excess photoresist spun off the sample. This technique
yields a buildup of excess photoresist on the edges. Consequently the pattern definition is washed out near the edges. This problem is avoided by dipping the sample in the photoresist with a very slow rpm motor. The slower the sample is dipped the more uniform the photoresist on the sample. As the sample is slowly brought out of the photoresist the surrounding photoresist in the container tends to drag off the excess photoresist to yield a thin, even distribution. The sample with photoresist is then put under an IR lamp to harden and dry. All of this is done in an ultra-violet free room to avoid exposure of the photoresist. After the photoresist is dried the mask is placed on the sample and exposed with a UV lamp. The sample is then developed in a special photoresist developer. After developing, the desired waveguide pattern of photoresist remains.

Once the waveguide pattern exists in the photoresist there remains the task of removing the unwanted titanium. Due to the extremely narrow lines and high resolution needed for the integrated optics applications the commercially available titanium etchant is undesirable. This etchant consists of diluted hydrofluoric acid in deionized water. It etched the titanium away much too rapidly and consequently tended to undercut the titanium under the photoresist which yielded either broken lines or very ragged edges. If the HF acid was diluted further with DI water
to slow the etching process then inconsistent and uneven etching resulted. To alleviate this problem a much slower working titanium etchant was developed. It consisted of 1.05 grams sodium fluoride plus 5.7 grams of ammonium persulfate per 50 milliliters of deionized water. The disadvantage of this titanium etch is that it has a very short shelf life (i.e., on the order of 1 or 2 days), therefore, a new batch has to be mixed up each time titanium needs to be etched. Etching the titanium is a fairly simple process. The sample is placed in the etchant and one can see when the titanium starts etching away. The sample is pulled out of the solution periodically and rinsed off with deionized water and blown dry with dry nitrogen. The sample is placed under a microscope which has a red filter under the lamp to avoid exposure of the remaining photoresist. This process is repeated until all of the undesired portion of the titanium is completely removed. The sample can now be cleaned with acetone followed by methanol to remove the remaining photoresist so that the only thing that is left on the sample is the titanium waveguide pattern.

Waveguide Formation

The sample is placed in a furnace at 500°C in a flowing O₂ atmosphere for two hours to completely oxidize the titanium. The oxidation process can be eliminated but it has been found that the waveguide properties are more
reproducible if the titanium is totally oxidized first. The waveguides also tended to be lossier without totally oxidizing the sample first. Titanium tends to oxidize fairly rapidly but if the oxidation takes place at temperatures greater than 700°C then the TiO₂ film looks cloudy, indicating that scattering and absorption are going to increase. If the oxidation takes place at lower temperatures the TiO₂ film is clear; in fact, the film cannot be seen with the naked eye.

After oxidation, the sample is heated to 1000°C for 4 to 8 hours to diffuse the titanium into the LiNbO₃ sample. All of the photolithographic and diffusion processes are summarized in Fig. 3.3. The length of time for diffusing depends on titanium film thickness, waveguide channel width, and the number of modes desired. In the region where the titanium is diffused into the LiNbO₃ the index of refraction is increased by 0.005 to 0.01. For single mode operation, the smaller the thickness of titanium and the narrower the channels the longer the diffusion time. The reason is that a certain waveguide width is necessary to achieve mode confinement. Consequently, for narrower channel widths longer diffusion times are required to achieve the minimum channel width. Almost all of the samples in these experiments had 200 Å thick Ti layers and were diffused for 6 hours, yielding single mode waveguides in both dimensions.
Fig. 3.3. Steps Used to Fabricate Titanium Diffused LiNbO₃ Waveguides.
Care must be taken in both heating and cooling the sample. The sample must be changed in temperature at a very slow pace due to its fairly large thermal expansion coefficient \(16.7 \times 10^{-6}/°C\). The mistake was made a couple of times of heating and cooling the LiNbO\(_3\) so fast that the sample cracked in several places. Due to the constraints of the diffusion furnace, the furnace has to be kept on at all times. The furnace consists of a 6 foot long by 4 inch diameter quartz tube. There are three heating coils which are controlled to \(±1°C\). These coils keep the center three feet of the tube at 1000°C. The temperature inside the tube falls off sharply from 1000°C to 220°C at the very end of the tube. The reason the furnace has to be kept on is because the quartz tube will crack and break if it is made to cycle through the temperature swing by turning the furnace on and off. With such a procedure the tube is only good for about 4 or 5 cycles. If the furnace is kept on, the tube will last indefinitely.

A temperature profile was taken from the center of the furnace tube to the end using a thermocouple. The temperature profile is shown in Fig. 3.4. The temperature profile was used to determine the position in the tube where the temperature was 500°C. This presented the problem of somehow pushing the sample into the furnace tube at a very slow rate. A pushrod was connected to a slow rpm motor.
Fig. 3.4. Temperature Profile of Diffusion Furnace.
This setup pushed the sample at a rate of 0.5 inches/minute. The sample was stopped at the 500°C position for two hours then pushed at the same rate to the center of the tube for the period of time required for diffusing the sample. The sample was then pulled out at 0.5 in/min. No problems were encountered by using this rate of temperature change. The waveguides produced under these circumstances were low loss and very reproducible.
CHAPTER 4

EVALUATION OF WAVEGUIDE AND MODAL PARAMETERS

Introduction

In this chapter is presented a complete analysis of the waveguide and mode field parameters which best describe the waveguides used in the experimental study in Chapter 6. The waveguides used in this study consisted of Z-cut, Y-propagating TE polarized input light and Y-cut, Z-propagating with TE and TM polarized input light. These two particular cuts are shown in a perspective sketch shown in Fig. 4.1. The optic axis of the crystal is parallel to the Z axis and "Z-cut" means the Z axis is perpendicular to the surface of the crystal; similarly for "Y-cut". Since the mode angle is almost parallel to the waveguide axis the only index of concern for both TE and TM waves is the ordinary index. The effective index method is briefly described in the next section, since this is the method used for the evaluation of the parameters.

A complete experimental study of the waveguide parameters was performed by Burns et al. (1979) for the particular cuts and fabrication parameters of the LiNbO₃ used in our study. Several of the experiments performed in the study by Burns et al. (1979) were repeated to verify that the
Fig. 4.1. Perspective Sketch of the Two Cuts of LiNbO₃ Used in This Study.

a. Y-cut, Z-propagating,
b. Z-cut, Y-propagating.
waveguides used in our experimental loss study did have the same values. Indeed they agreed quite well. The values of diffusion depth, surface index, and index profiles reported in Burns' study are therefore used, in conjunction with published dispersion curves for these index profiles, to determine the mode effective index.

The fundamental mode field parameters are then evaluated. The field profile in the plane of the bend is well known for both straight waveguides (Burns and Hocker, 1977) and curved waveguides (Peterman, 1976). However, the field profile in the direction perpendicular to the waveguide surface had to be evaluated independently because published results for this field profile were not precise enough for this study. This is presented in the last section of this chapter.

**Effective Index Method**

Kogelnik and Ramaswamy (1974) have shown that, by defining a set of three normalized parameters which describe a dielectric optical waveguide, the modes of any optical waveguide can be determined from universal curves. The parameters are normalized waveguide thickness, $V$, normalized effective mode index, $b$, and the asymmetry parameter $a$ of the waveguide and its surrounding media. The mode dispersion equation gives the propagation constant $\beta$, or the effective index $n_{eff}$, for each mode in terms of five
independent quantities, the refractive index of the waveguide, substrate and media above the waveguide, the waveguide depth and the optical wavelength. The effective index method reduces the number of parameters needed to evaluate the characteristic waveguides. Furthermore, plotting the universal curve of b vs. V with a as a parameter describes any dielectric optical waveguide.

The effective index method was adapted to a two-dimensional diffused waveguide by Hocker and Burns (1977). Consider the rectangular waveguide shown in Fig. 4.2(a). In a rectangular waveguide the areas outside the waveguide at the corners can be assumed to have a negligible effect (Marcatilli, 1969) on the dispersion parameters of the waveguide itself. By allowing the waveguide dimension in the x direction to go to infinity the effective index $n_{\text{eff}}$ of the waveguide shown in Fig. 4.2(b) can be calculated from the mode dispersion curves for a diffused waveguide from Hocker and Burns (1975). This calculated $n_{\text{eff}}$ is used to form a waveguide in the x direction as shown in Fig. 4.2(c). Again using the dispersion curves of Hocker and Burns (1975) the effective index $n'_{\text{eff}}$ of the mode is calculated.

Let the dielectric waveguide material occupy the half space $y > 0$ and $x = 0$ be the center of the channel. The index profile can be written as
Fig. 4.2. Illustrating the EFFECTIVE INDEX METHOD.

a. Configuration of the rectangular dielectric waveguide;
b. A 1-D guide with confinement in the y direction,
c. An equivalent 1-D guide with confinement in the x direction having effective index $n_{eff}$ of the mode in guide (b) (Hocker and Burns, 1977).
\[ n(x,y) = n_b + \Delta n f(y/D) g(2x/W) \quad (4.1) \]

where \( n_b \) is the bulk index of the substrate, \( \Delta n \) is the amount of index increase at the surface \( y = 0 \) due to diffusion, \( f(y/D) \) and \( g(2x/W) \) are the index profiles in the \( y \) and \( x \) directions respectively, with \( D \) the diffusion depth and \( W \) the waveguide channel width before diffusion. Assuming \( \Delta n \) to be much smaller than \( n_b \) then the index profile becomes

\[ n^2(x,y) = n_b^2 + (n_s^2 - n_b^2) f(y/D) g(2x/W) \quad (4.2) \]

where \( n_s \) is the index at the surface of the waveguide \( (n_s = n_b + \Delta n) \). The index profile is now in a form convenient for the use of the effective index method where

\[ \nu = \frac{2\pi D}{\lambda} (n_s^2 - n_b^2)^{1/2} \quad (4.3) \]

\[ b = (n_{\text{eff}}^2 - n_b^2)/(n_s^2 - n_b^2) \quad (4.4) \]

and

\[ a = (n_b^2 - n_a^2)/(n_{\text{eff}}^2 - n_b^2). \quad (4.5) \]

The waveguide asymmetry parameter \( a \) is assumed to be infinity for all of the following analysis because for Ti-diffused LiNbO\(_3\) waveguides \( a > 1000 \), and Kogelnik (1975a) claims that for \( a > 10 \) the dispersion curves are degenerate.
The shape of the index profile has been measured by Burns et al. (1979) in the y direction to be

\[ f(y/D) = \exp\left(-\frac{y^2}{D^2}\right) \] (4.6)

and in the x direction to be

\[ g\left(\frac{2x}{W}\right) = \frac{1}{2} \left\{ \text{erfc}\left[\frac{W}{2D}\left(\frac{2x}{W} - 1\right)\right] + \text{erfc}\left[\frac{W}{2D}\left(\frac{2x}{W} + 1\right)\right]\right\} \] (4.7)

The index profile in the y direction has the same Gaussian shape as if the diffusion was one-dimensional. However, the index change at the surface is reduced by the factor \( g(2x/W) \) as shown in Fig. 4.3. If the width of the waveguide channel is large then the surface index is unchanged, however, for narrow channels, as is the case in this study, the index at the surface is less than \( n_s \).

**Waveguide Parameters**

The waveguide parameters are determined for y and z cut Ti diffused channel waveguides in LiNbO₃. All of the following analysis is for a 200 Å thick titanium layer, diffusion time 6 hours, diffusion temperature of 1000°C, a channel width of 3 μm and a wavelength of 0.6328 μm. Burns et al. (1979) measured the diffusion coefficients in LiNbO₃, at 1000 C to be \( d_y = 0.34 \mu m/h \) and \( d_z = 0.52 \mu m/h \).

The diffusion depth is

\[ D = 2(dt)^{1/2} \] (4.8)
Fig. 4.3. The Shape in the x Direction of the 2-D Diffused Index Profile for Various Values of the Ratio of Strip Width to Diffusion Depth W/D (Hocker and Burns, 1977).
where \( d \) is the diffusion coefficient and \( t \) is the diffusion time; this yields \( D_y = 2.86 \, \mu m \) and \( D_z = 3.53 \, \mu m \). Burns et al. (1979) also measured the index change, \( \Delta n \), at the surface. For Z-cut, \( \Delta n_o = 0.0057 \) and \( \Delta n_e = 0.0081 \) for ordinary and extraordinary polarization respectively, the values are the same for Y-cut. These values are for planar diffused waveguides or large channel widths in which there is no sideways diffusion. To calculate \( \Delta n \) and \( n_s \) for the narrow channel waveguide having sideways diffusion \( g(2x/W) \) must be evaluated at \( x = 0 \). For Z-cut and Y-cut respectively, \( g(0) = 0.542 \) and \( g(0) = 0.452 \) for \( W = 3 \, \mu m \); this yields \( \Delta n_o = 0.0031 \) and \( \Delta n_e = 0.0044 \) for Z-cut and \( \Delta n_o = 0.0026 \) and \( \Delta n_e = 0.0037 \) for Y-cut. A plot of the index versus depth for Z-cut is shown in Fig. 4.4(a) and for Y-cut in Fig. 4.4(b).

The normalized diffusion depth \( V \) can be calculated using Eq. (4.3). From the dispersion curves for a Gaussian index profile shown in Fig. 4.5 and taken from Hocker and Burns (1975) the normalized effective index can be obtained. The effective index can now be calculated using Eq. (4.4) which yields \( n_{\text{eff}} = 2.2912 \) and \( n_{\text{eff}} = 2.2908 \) for Z and Y-cuts respectively. The bulk index \( n_b \) of \( \text{LiNbO}_3 \) used in the above calculations is \( n_b = 2.2885 \) for ordinary polarized light. The waveguide parameters for extraordinary polarized light can be evaluated using the same procedure,
Fig. 4.4. Plot of Index Versus Depth in the Waveguide for (a) Z-cut and (b) Y-cut.
Fig. 4.5. Normalized Dispersion Curves for a Gaussian Index Profile (Hocker and Burns, 1975).
but since no experimental measurements were made with extra-
ordinary polarized light no calculations are made using \( n_e \).

To evaluate the effective index \( n_{\text{eff}}' \) of the mode the
above evaluated \( n_{\text{eff}} \) is used in Eqs. (4.3) and (4.4) for
the normalized waveguide width \( V' \) and normalized effective
index \( b' \) which gives

\[
V' = \frac{2\pi W}{\lambda} \left( n_{\text{eff}} - n_b \right)^{1/2} \quad (4.9)
\]

\[
b' = \frac{\left( n_{\text{eff}}'^2 - n_b^2 \right)}{\left( n_{\text{eff}}^2 - n_b^2 \right)} \quad (4.10)
\]

Using the same procedure as before, but employing the two
dimensional dispersion curves of Hooker and Burns (1977)
shown in Fig. 4.6, we obtain the mode effective index of
\( n_{\text{eff}}' = 2.2888 \) and \( n_{\text{eff}}' = 2.2887 \) for \( Z \) and \( Y \)-cut respec-
tively. All of these waveguide parameters, as well as the
field profile parameters, are repeated in Table 1 at the
end of the next section, page 64.

**Waveguide Mode Field Parameters**

The modal field in the waveguide takes on \( x \) and \( y \)
dependent profiles which also varies with waveguide curva-
ture. It is important to know the field profile as exactly
as possible for the evaluation of the theory in Chapter 5.
The field profile in the plane of the waveguide is well
documented, so it will be considered first. Due to the
Fig. 4.6. Two-dimensional Dispersion Curves for a Gaussian Index Profile in Depth and a Complimentary Error Function Index Profile for Sideways Diffusion (Hocker and Burns, 1977).
waveguide symmetry in the x direction the field profile can be very closely approximated by a Gaussian such as

\[ E(x) = f(x) \exp\left(-\frac{x^2}{X_0^2}\right) \]  \hspace{1cm} (4.11)

where \( f(x) = 1 \) for a straight waveguide and \( X_0 \) is the half width of the field. If the waveguide is curved then the fundamental mode field is deformed in the plane of the bend from that of the straight waveguide (Peterman, 1976). The mode field of the curved waveguide takes on the shape defined by Eq. (4.11) with

\[ f(x) = (1 + dx) \]  \hspace{1cm} (4.12)

where, for constant radius of curvature \( R_0 \),

\[ d = \frac{(2\pi n_b X_0/\lambda_0)^2}{2R_0} \]  \hspace{1cm} (4.13)

where \( n_b \) is the bulk substrate refractive index and \( \lambda_0 \) is the free space wavelength.

The exact field profile in the y direction (plane perpendicular to the surface of the waveguide) is a solution to the wave equation (Burns and Hocker, 1977)

\[ \frac{\delta^2 E(u)}{\delta u^2} = V^2 \left[b - f(u)\right] E(u) \]  \hspace{1cm} (4.14)

where

\[ u = \frac{y}{D} \]  \hspace{1cm} (4.15)
and $f(u)$ is the index profile given by Eq. (4.6). Typically the wave equation is solved numerically and a Gaussian fit is matched to the $e^{-1}$ points and used for the field profile. Due to the strong asymmetry of the waveguide in the $y$ direction this Gaussian approximation is not good enough. Instead the wave equation is solved numerically and a weighted Gaussian function is fit to the numerical solution (White, Hutcheson, and Burke, 1979) as shown in Figs. 4.7 and 4.8. The parameters $V$, $b$ and $D$ are as defined earlier in this chapter. Figure 4.7 is for $Z$-cut TE modes; Fig. 4.8 is for $Y$-cut TE and TM. The weighted Gaussian takes on the form

$$E(y) = y \exp \left[ - \frac{(y-y_0)^2}{Y_0^2} \right] \quad (4.16)$$

where $y_0$ is a parameter of the weighted Gaussian function and $Y_0$ is the half width. From Eqs. (4.5) and (4.6) the values of $y_0$ and $Y_0$ are obtained. These values are used in conjunction with the plots of geometric aspect ratio and modal aspect ratio, both as a function of the ratio of indiffused channel width $W$ to the diffusion length $D$ shown in Figs. 4.9 and 4.10 taken from Burns and Hocker's (1977) study. The values obtained for the modal field parameters are summarized in Table 1 along with the waveguide parameters. These values are now used in the theoretical study presented in the next chapter.
Fig. 4.7. Field Profile Perpendicular to the Surface of the Waveguide for Z-cut LiNbO$_3$. 
Fig. 4.8. Field Profile Perpendicular to the Surface of the Waveguide for Y-cut LiNbO$_3$. 
Fig. 4.9. The Geometric Aspect Ratio of an Isotropically Diffused Channel Waveguide Plotted Versus the Ratio of the Undiffused Channel Width W to the Diffusion Depth D.

(Burns and Hocker, 1977).
Fig. 4.10. The Ratio of the Modal Aspect Ratio to the Geometric Aspect Ratio for the First Order Mode in Several Channel Waveguides is Plotted Versus the Geometric Aspect Ratio.

(Burns and Hocker, 1977).
Table 1. Waveguide and Modal Field Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Z-cut TE</th>
<th>Y-cut TE, TM</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>3 µm</td>
<td>3 µm</td>
</tr>
<tr>
<td>D</td>
<td>3.53 µm</td>
<td>2.86 µm</td>
</tr>
<tr>
<td>Δn</td>
<td>0.0031</td>
<td>0.0026</td>
</tr>
<tr>
<td>g(0)</td>
<td>0.542</td>
<td>0.452</td>
</tr>
<tr>
<td>n_b</td>
<td>2.2885</td>
<td>2.2885</td>
</tr>
<tr>
<td>n_s</td>
<td>2.2916</td>
<td>2.2911</td>
</tr>
<tr>
<td>n_{eff}</td>
<td>2.2912</td>
<td>2.2908</td>
</tr>
<tr>
<td>n'_{eff}</td>
<td>2.2888</td>
<td>2.2887</td>
</tr>
<tr>
<td>X_o</td>
<td>3.56 µm</td>
<td>3.82 µm</td>
</tr>
<tr>
<td>Y_o</td>
<td>2.08 µm</td>
<td>2.22 µm</td>
</tr>
<tr>
<td>λ_o</td>
<td>0.63 µm</td>
<td>0.63 µm</td>
</tr>
</tbody>
</table>
CHAPTER 5

ANALYTICAL ASPECTS OF BENDING LOSSES

Introduction

The fabrication of the waveguides used in this study was described in Chapter 3. The previous chapter described the method to obtain the waveguide and modal field parameters used in the theoretical analysis. In this chapter the theory is developed to predict the loss due to directional changes in the waveguide.

There have been basically two different philosophies for calculating bending losses in waveguides. The first applies coupled mode theory (Marcuse, 1976) which spectrally decomposes the radiation field into an orthogonal set of fields to determine the coupling into each component. The second utilizes antenna theory (White, 1979) to evaluate the radiation for known current distributions of current sources and is known as the "volume current method". Both methods have been used to calculate bending losses in fibers and dielectric waveguides but it was not until recently (White, Hutcheson, and Burke, to be published) that the volume current method was employed to calculate the radiation loss in diffused LiNbO₃ waveguides.

The basic notion of the volume current method, which is
the method used in this chapter, is that under the influence of an electric field, a dielectric imperfection in an infinite homogeneous dielectric can be interpreted as a volume-current density. The core of the waveguide can be considered as a radiating antenna, with a distribution of induced electric currents over its volume excited by an incident electromagnetic field. The volume current method is used in this study to calculate the radiation loss caused by a constant curvature of the waveguide.

The loss mechanisms discussed in this study are two-fold, the first is a power loss due to a constant curvature of the waveguide. The second is the power scattered from the fundamental mode when incident upon a junction between two non-identical guides. Most integrated optical circuits have either one or both of the above described waveguide loss mechanisms incorporated into their design. In the next section of this chapter the radiation loss due to curvature of the waveguide is calculated. In the third section the mode mismatch loss at waveguide junctions is calculated. These theoretical results are compared to experimental data in Chapter 6. Then the results of the theoretical calculations, in conjunction with the experimental results, are utilized in Chapter 7 to predict optimum waveguide design for lowest loss.
Calculation of Power Guided in Diffused Channel Waveguides

In an electromagnetic analysis such as this a description of the modal fields is extremely important. The diffused waveguide complicates the analysis somewhat because of the asymmetric nature of the waveguide cross section and the inhomogeneous index profile caused by diffusion. For convenience the dielectric and field profiles from Chapter 4 are repeated here. For the indiffused waveguide the dielectric profile $\varepsilon(x,y)$ is

$$\varepsilon(x,y) = \varepsilon_b + \Delta \varepsilon(x) \varepsilon(y) \quad y > 0 \quad (5.1)$$

$$\quad = \varepsilon_o \quad y < 0$$

where $\Delta = \varepsilon_o (n_s^2 - n_b^2) \ll 1$. The subscripts $b$, $s$, and $o$ represent the bulk substrate, the surface of the waveguide, and free space respectively. The directions $x$ and $y$ represent the width and depth respectively. Also, $\varepsilon(x)$ and $\varepsilon(y)$ are given by

$$\varepsilon(x) = \frac{1}{2} \left[ \text{erfc} \left( \frac{X_D - W}{2D} \right) - \text{erfc} \left( \frac{X_D + W}{2D} \right) \right] \quad (5.2)$$

and

$$\varepsilon(y) = \exp \left( -y^2/D^2 \right) \quad (5.3)$$

where $D$ is the diffusion depth. Also, from Chapter 4 the field profiles in straight waveguides are

$$E(x) = \exp \left( -x^2/X_o^2 \right) \quad (5.4)$$

$$E(y) = y \exp \left[ - (y - y_o)^2/Y_o^2 \right] \quad (5.5)$$
the electromagnetic field \((\mathbf{E}, \mathbf{H})\) is given by (Burns and Hocker, 1977)

\[
\mathbf{E} = \hat{\mathbf{a}}_n E(x) E(y) \exp(-i\beta z) \quad (5.6)
\]

\[
\mathbf{H} = \left(\frac{\varepsilon_s}{\mu}\right)^{1/2} \mathbf{\hat{z}} \times \mathbf{E} \quad (5.7)
\]

where \(\hat{\mathbf{a}}_n\) is the polarization vector of the field. In a bent waveguide the fundamental mode field is deformed in the plane of the bend from that of the straight waveguide (Peterman, 1976). The bent waveguide deforms the field given in Eq. (5.4) by

\[
E(x) = (1 + d_1 x) \exp(-x^2/X_o^2) \quad (5.8)
\]

where for a constant radius of curvature \(R_o\)

\[
d_1 = \frac{(2\pi n_b X_o/\lambda)^2}{2R_o} \quad (5.9)
\]

In the curved waveguide the phase change along the axis (see Fig. 5.1) is \(\exp(-im\phi)\) where

\[
m = \beta R_o \quad (5.10)
\]

In a weakly guiding dielectric waveguide, the total power \(P_o\) guided along the waveguide axis in a mode is (Snyder, 1969)

\[
P_o = \frac{1}{2} (\varepsilon_b/\mu)^{1/2} \int_{A_{\infty x}} |\mathbf{E}|^2 \, dA \quad (5.11)
\]

where \(A_{\infty x}\) is the infinite transverse cross section. Using
Fig. 5.1. Configuration of Curved Waveguide with Bent Axis.
the fields defined in Eqs. (5.5) and (5.8) and well known
tables of Gaussian integrals (Gradsteyn and Ryzhik, 1965)

\[
P_0 = \frac{1}{2} \left( \varepsilon_b/\mu \right)^{1/2} P_x P_y
\]

with

\[
P_x = \left( \pi/2 \right)^{1/2} x_0 \left[ 1 + \left( d_1 x_0/2 \right)^2 \right]
\]

\[
P_y = \left( \pi/8 \right)^{1/2} y_0^3 \left[ \left( y^2/y_0^2 + .25 \right) \left( 1 + \text{erf}(\sqrt{2}y_0/y_0) \right) \\
+ \left( y_0/y_0 \sqrt{2} \pi \right) \exp(-2y^2/y_0^2) \right]
\]

for straight waveguides \( R_0 = \infty \) and thus \( d_1 = 0 \).

**Curvature Induced Radiation Loss**

From Maxwell's equations for source free media in
which the dielectric permittivity tensor is inhomogeneous,
\( \varepsilon = \varepsilon(\vec{r}) \) we have for time harmonic fields proportional to
\( \exp(i\omega t) \)

\[
\nabla \times \vec{H} = i \omega \varepsilon(\vec{r}) \cdot \vec{E}.
\]

Writing the permittivity tensor in the form

\[
\varepsilon(\vec{r}) = \varepsilon_b + \delta \varepsilon(\vec{r}) - \varepsilon_b
\]

\[
= \varepsilon_b + \delta \varepsilon(\vec{r})
\]

and substituting Eq. (5.16) into Eq. (5.15)

\[
\nabla \times \vec{H} = i \omega \varepsilon_b \cdot \vec{E} + i\omega \delta \varepsilon(\vec{r}) \cdot \vec{E}(\vec{r})
\]

\[
= i \omega \varepsilon_b \cdot \vec{E} + \vec{J}_e
\]
where $\mathbf{J}_e$ is the pseudo-volume current density. In this the inhomogeneous permittivity profile is represented as an induced volume distribution of electric dipoles.

In the present application of the volume current method to radiation from a curved segment of a single mode waveguide the bent core of the waveguide is treated as the volume of the antenna. Peterman (1976) has shown that the fundamental mode of such a curved structure can be well approximated near the waveguide axis. Peterman (1976) showed that for isotropic dielectrics and for a symmetric waveguide if the electric field of a straight waveguide is Gaussian then the electric field of a bent waveguide is simply that of the straight waveguide modified by a multiplicative factor as shown in Eqs. (5.8) and (5.9). Because the fundamental mode field is confined near the guide axis, the distribution of induced electric dipoles can be determined. The time averaged radial Poynting vector of the radiation field, $S^R$, far from the waveguide axis as given by Snyder (1970) is

$$S^R = \frac{\varepsilon_0}{\varepsilon_b} \frac{k_b^2}{32(\pi R)^2} \left| \hat{R} \times \int_{\text{core}} \mathbf{J}_e \exp \left( ik_b \hat{R} \cdot \mathbf{R}' \right) dV' \right|^2$$

(5.18)

where $\varepsilon_b$ is the unperturbed permittivity of the substrate, $\mathbf{R}'$ is the position vector of a source point in the waveguide core, and $\hat{R}$ is the unit vector in the direction of
the observation point in the radiation field a distance \( R \) from the origin near the waveguide core. Since we are interested in the total power radiated instead of power density we can assume that the waveguide is in a homogeneous and isotropic medium. This does not seem to be an unreasonable assumption since the theoretical predictions agreed so well with experimental results.

The total amount of power \( P^R \) radiated is

\[
P^R = R^2 \int S^R d\Omega \quad (5.19)
\]

where \( d\Omega \) is an elemental solid angle. The power attenuation coefficient \( \gamma \) from a guide of length \( L \) is given by

\[
\gamma = \frac{P^R}{L} P_o \quad (5.20)
\]

where

\[
P(z) = P_o \exp(-\gamma z) \quad (5.21)
\]

with \( z \) an axial distance along the waveguide, \( P_o \) the incident guided power. A complete annular waveguide, as shown in Fig. 5.2, is considered so that the radiation due to curvature of the waveguide axis is isolated. For this configuration

\[
L = 2\pi R_o \quad (5.22)
\]

where \( R_o \) is the radius of curvature. Assuming a symmetric structure and small bending loss, the radiation field should
Fig. 5.2. Complete Annulus of Curved Waveguide.
be symmetric and uniform in the plane of the bend, so that
the radiation field and Poynting vector need only be evalu-
ated at one point in this plane. For simplicity choose
\[ \phi = \pi/2 \] (5.23)
which yields
\[ P^R = 2\pi R^2 \int S^R \sin \theta d\theta \] (5.24)

The coordinate system for the bent waveguide is
shown in Fig. 5.3. From Eqs. (5.18) and (5.24) the power
attenuation coefficient \( \gamma \) can be calculated. The mathemati-
cal details, shown in Appendices A, B, and C yield the
result
\[
\gamma = \left[ (\pi Q)^3 R_0 \right]^{-1/2} \left( V^2 Y_1 \right)^2 \left| P_y \right|^2 \exp\left\{ \frac{(QX_o)^2}{2} - \frac{2y_o^2}{(Y_o^2 + D^2)} \right\} \left\{ 8X_o Y_o \left[ 1 + \left( \frac{d_1 X_o}{2} \right)^2 \right] \gamma \right\} \left[ (1 + d_1 X_1) F_x^{(1)} + d_1 F_x^{(2)} \right]^2 \exp\left[ -\frac{4Q^3}{3\beta^2} \frac{R_o}{2} \right] \left\{ 8X_o Y_o \left[ 1 + \left( \frac{d_1 X_o}{2} \right)^2 \right] \gamma \right\}
\] (5.25)

where
\[
V^2 = k_o^2 \left( n_s^2 - n_o^2 \right) \] (5.26)
\[
Q^2 = (\beta^2 - n_o^2 k_o^2) \] (5.27)
\[
Y_1 = Y_o D / (Y_o + D^2)^{1/2} \] (5.28)
\[
d_1 = (k_o n_b X_o)^2 / 2R_o \] (5.29)
Fig. 5.3. Coordinate System of a Curved Waveguide.
where $X_o, Y_o$ are the parameters of the fundamental mode field profile discussed in Chapter 4. $d_1$ is the correction term for the field in a curved waveguide described by Eq. (5.9) and $D$ is the diffusion depth. Also,

$$S_y = Y_o^2 \left[ (y_o^2/Y_o^2 + 0.25) \left( 1 + \text{erf} \left( \sqrt{2} \frac{y}{Y_o} \right) \right) + \frac{y_o}{Y_o} \sqrt{2\pi} \exp \left( -2\frac{y_o^2}{Y_o^2} \right) \right]$$  \hspace{1cm} (5.30)$$

$$F_y = Y_1 \left[ Y_1 \frac{Y_o}{Y_o} \left( 1 + \text{erf} \left( \frac{Y_1 Y_o}{Y_o} \right) \right) + \exp \left[ -\left( \frac{Y_1 Y_o}{Y_o} \right)^2 \right] \right] / \sqrt{\pi}$$ \hspace{1cm} (5.31)$$

Finally,

$$F_x^{(1)} = X_o \left[ \text{erf} \left( \frac{\omega + D}{X_o} \right) + \text{erf} \left( \frac{\omega - D}{X_o} \right) - \text{erf} \left( \frac{\omega}{p} \right) - \text{erf} \left( \frac{\omega}{p} \right) \right]$$ \hspace{1cm} (5.32)$$

and

$$F_x^{(2)} = \sqrt{\pi} (X_o^3/pD) \sinh(WQX_o^2/2D^2p^2) \exp \left\{ - \frac{(W/2D)^2 + (X_1/D)^2}{p^2} \right\}$$ \hspace{1cm} (5.33)$$

where $W$ is the width of the channel before diffusion and

$$P^2 = 1 + (X_o/D)^2$$ \hspace{1cm} (5.34)$$

$$X_1 = QX_o^2/2$$ \hspace{1cm} (5.35)$$

$$\omega_{\pm} = W/2D \pm (X_1/d)$$ \hspace{1cm} (5.36)$$

The result of Eq. (5.24) shows the strong exponential dependence on the radius of curvature. The attenuation
coefficient $\gamma$ is plotted as a function of radius of curvature $R_0$ in Fig. 5.4. The waveguide and field parameters evaluated in Chapter 4 are used to obtain the plot of Fig. 5.4.

**Scattering Loss at Waveguide Junctions**

The scattering loss at waveguide junctions can be divided into two categories. First, the loss at a junction where two straight waveguides are joined together at some discrete angle, in which case there exists a discontinuity in the direction of the waveguide axis. Second, the waveguide junction is continuous (first derivative is continuous), such as occurs when a straight section is joined with a curved section continuously or a curved section is joined continuously with another curved section of opposite curvature. In both the continuous and discontinuous junctions, the loss is due to the mode field profile mismatch on opposite sides of the junction.

In practice, the reason for considering the two different types of junctions is due to the directional change requirement in an integrated optical waveguide. This study considers the discontinuous directional change with only straight waveguides and the continuous directional change with a combination of straight and curved waveguides. In Chapter 7 we compare the two types to determine the conditions under which one type has a definite advantage over the other.
Fig. 5.4. Attenuation Coefficient, $\gamma$, as a Function of Radius of Curvature, $R_o$, for a Curved Waveguide.
If we ignore the power scattered back away from a junction between two weakly guiding waveguides, then the incident electromagnetic field at the junction is the excitation field for the next section of the guide. Therefore, for single mode waveguides, to calculate the amount of power coupled from one guide to the next, only one integral needs to be solved for the relative modal amplitude of the fundamental mode of the second guide, \( a_{\infty} \), excited by the incident mode from the first guide:

\[
a_{\infty} = \frac{1}{2} \left( \frac{\varepsilon_b}{\mu} \right)^{1/2} \int_{A_{\infty}} \bar{E}_{02} \cdot \bar{E}_{01}^* \, dA \quad (5.37)
\]

where \( \bar{E}_{01} \) is the electric field of the fundamental mode of the \( i \)th guide normalized to unit power and \( A_{\infty} \) is the infinite cross sectional area of the junction between the two guides. For an incident field with a total guided power \( P_{\text{inc}} \), the power excited in the next guide is

\[
P_{\text{ex}} = |a_{\infty}|^2 P_{\text{inc}} \quad . \quad (5.38)
\]

Because the waveguides are single mode, any power not coupled into the fundamental mode is scattered into the leaky or radiation mode spectrum and rapidly lost from the waveguide core. Therefore, the power lost from the guide to a junction is

\[
\Delta P^R = P_{\text{inc}} (1 - |a_{\infty}|^2) \quad . \quad (5.39)
\]
For a junction joining two straight, in-diffused, channel waveguides whose axes are inclined at an angle $\theta_T$ with respect to each other, and whose axes are curved with radii of curvature $R_1$ and $R_2$, the overlap integral Eq. (5.37) can be solved. With the fields described by Eqs. (5.4), (5.5), and (5.8), Eq. (5.37) reduced to evaluation of the integral over the width only, because the fields in depth do not change at the junctions but do change in the plane of the waveguide. We obtain from Marcuse (1978)

$$|a_{\infty}|^2 = \frac{2W'^2 [(1+i\theta_1 b)(1+i\theta_2 b) + \frac{1}{2} W'^2 \exp \left\{ -\beta_1 W^2 \sin^2 \theta_T/2 \right\}]}{(X_{01}X_{02}) \left[ 1 + \left( \frac{d_{1X_{01}}}{2} \right)^2 \right] \left[ 1 + \left( \frac{d_{2X_{02}}}{2} \right)^2 \right]} \quad (5.40)$$

where

$$W'^2 = \frac{(X_{01}X_{02})^2}{(X_{01}^2 + X_{02}^2)} \quad (5.41)$$

and

$$b = \beta_1 W'^2 \sin \theta_T/2 \quad (5.42)$$

and $X_{0i}$ is the width of the fundamental mode fields. $\beta_1$ is the longitudinal propagation constant of the first waveguide from which the mode is incident on the junction and $d_1$ and $d_2$ are the field deformations due to curvature of the waveguide axes

$$d_i = \frac{(k_{n_0} X_{0i})^2}{2R_0} \quad (5.43)$$

For a straight waveguide $d_i = 0$ since $R = \infty$. For a curved waveguide $d_1$ and $d_2$ may be opposite sign if the two curved waveguides have opposite curvature.
For two straight identical waveguides \((X_{01} = X_{02})\) inclined at angle \(\theta_T\), the power coupling coefficient becomes

\[
|a_{oo}|^2 = \exp \left[ -\beta^2 \frac{W^2}{2} \sin^2 \frac{\theta_T}{2} \right]. \tag{5.44}
\]

For an identical straight and curved waveguide the power coupling coefficient is

\[
|a_{oo}|^2 = \frac{1}{\left[ 1 + \left( \frac{dX_2}{2} \right)^2 \right]}, \tag{5.45}
\]

Two identical curved waveguides, with opposite curvature yield the power coupling coefficient

\[
|a_{oo}|^2 = \frac{\left[ 1 - (dX_2/2)^2 \right]^2}{\left[ 1 + (dX_2/2)^2 \right]^2}, \tag{5.46}
\]

The power lost at the straight to straight junction is plotted in Fig. 5.5 as a function of \(\theta_T\), the power lost at the curved to curved and curved to straight junctions are plotted in Figs. 5.6 and 5.7 respectively. All three curves are for Z cut y propagating TE modes. The curved to curved mode mismatch can be seen to be quite lossy because the field deformation of the two waveguides are in opposite directions. These results as well as the results from curvature loss are compared to experimental results in the next chapter. The use of these results in the design of integrated optical devices is discussed in Chapter 7.
Fig. 5.5. Corner Bend Loss as a Function of Bend Angle.
Fig. 5.6. Mode Mismatch Loss for a Curved to Curved Junction for Equal but Opposite Curvature as a Function of Radius of Curvature.
Fig. 5.7. Mode Mismatch Loss for a Straight to Curved Junction as a Function of Radius of Curvature.
CHAPTER 6

EXPERIMENTAL MEASUREMENT OF WAVEGUIDE LOSSES

Introduction

The previous chapters developed procedures for measuring and predicting bending loss in waveguides which is experimentally verified in this chapter. The experimental results presented in this chapter are loss measurements in both straight waveguides having no bends and waveguides with bends. At the outset of this experimental study, it was intended to measure only the losses in waveguides having bends. However, there has been very little data reported on losses in single mode, in-diffused, LiNbO$_3$, straight channel waveguides. It is also of interest to determine the quality of the waveguides fabricated for this study; therefore experimental loss measurements on straight waveguides are presented in this chapter. Two types of bends have been investigated in this study. First are the corner bends where two straight waveguides are joined together at some angle. The second type are bends having smooth curvature (a portion of a circle) with a specified radius of curvature.
Losses in Straight Waveguides

The optical propagation loss in a straight portion of a channel waveguide was measured by two different techniques. The first technique utilizes the coupling efficiency measurement described in Chapter 2. After the coupling efficiency is measured the detector and chopper shown in Fig. 2.10 are moved to a position between the beamsplitter and input coupling prism. The laser intensity is measured and the Fresnel reflection off the front face of the prism is calculated and subtracted from the reading. The detector is then placed behind a slit and lens combination where the light coupled out of the waveguide passes through the slit and is imaged onto the detector. The slit is used to block the background light scattered by the prisms and waveguides. The output power of the waveguide was measured for various separation distances between the input and output coupling prisms. The measurement was repeated for many samples all fabricated identically, i.e., same titanium thickness 200 Å, same diffusion temperature 1000°C, and same diffusion time 6 hours. The results are shown in Fig. 6.1 for both TE and TM lowest order modes. A coupling efficiency of 0.96% was used for the TE mode and 0.85% for the TM mode. This yields a loss of 1.59 dB/cm ± 0.18 dB/cm and 1.66 dB/cm ± 0.15 dB/cm for TE and TM respectively. The results of 1.59 dB/cm and 1.66 dB/cm are the slopes of the best fit to the data and the
Fig. 6.1. Rayleigh Scattering and Absorption Loss for 3 μm Wide Channel Waveguides Which is Obtained by Measuring the Input Coupling Efficiency and Measuring the Output at Various Distances Down the Waveguide.
\[ \pm 0.18 \text{ dB/cm} \text{ and } 0.15 \text{ dB/cm} \] are the standard deviations from all of the measurements. The measurements were made on \( y \)-cut \( z \)-propagating \( \text{LiNbO}_3 \) where both the \( \text{TE} \) and \( \text{TM} \) modes are polarized perpendicular to the optic axis. The \( \text{TM} \) mode is not exactly polarized perpendicular to the optic axis but very close (approximately 89°) to it. The same loss measurement was made on \( z \)-cut \( y \)-propagating for the \( \text{TE} \) (ordinary polarized) mode and yielded the same result. The reason why \( \text{TM} \) measurements were not made on \( z \)-cut crystals is because the wave is extraordinary polarized and an out-diffused planar waveguide is formed at the same time the in-diffused channel waveguide is formed. \( \text{LiO}_2 \) diffuses out of the surface of the \( \text{LiNbO}_3 \), which causes an increase in the extraordinary index and not the ordinary index. Therefore, only those modes extraordinary polarized (\( \text{TM} \) in \( z \)-cut) are affected. It is impossible to couple light only into the channel for the \( \text{TM} \) mode and not into the planar \( \text{TM} \) out-diffused waveguide mode. This method does have the disadvantage that the output coupling efficiency is assumed to be 100%.

The second method used to measure the propagation loss does not depend on knowing the input coupling efficiency or assuming the output coupling efficiency to be 100%. It does, however, require the input and output coupling efficiency to remain constant for each data point.
Light is coupled into the waveguide and at known distances from the input coupling prism, light is coupled out with another prism and measured with the same slit, lens and detector as previously described. The output is plotted as a function of known prism separation in Fig. 6.2. The slope of these curves yields $1.34 \text{ dB/cm} \pm 0.22 \text{ dB/cm}$ and $1.38 \text{ dB/cm} \pm 0.22 \text{ dB/cm}$ for TE and TM respectively. When the two coupling prisms are closer than about 0.5 cm, the clamping of the output prism affects the input coupling efficiency. This is why the data in Fig. 6.1 does not have distances less than 0.8 cm. Therefore, for separations less than 0.5 cm, every time the output coupling prism was moved the input coupling efficiency had to be checked. For distances larger than 0.5 cm, the input coupling efficiency was checked a number of times and always remained constant. One can be reasonably sure that the output coupling efficiency is constant for each measurement because the prism pressure is adjusted to maximize the output for each reading. The disadvantage of this technique is that many data points need to be measured to get good results. With the former method, once the input coupling efficiency is measured, only one or two output measurements are required to get accurate results.

Even though the results achieved by the two different techniques are quite close, there is a possible
Fig. 6.2. Rayleigh Scattering and Absorption Loss for 3 μm Wide Channel Waveguides Which is Obtained by Measuring the Prism Output Coupled Light as a Function of Distance Down the Waveguide.
explanation for the slight difference. The output coupling efficiency is not 100% as was assumed in the first technique. A simple calculation shows that if the output coupling efficiency is 94% in the first technique then the results would agree exactly for both TE and TM modes. These results agree quite well with results measured by a destructive technique by Kaminow and Stulz (1978). Their technique was to focus the laser beam onto a cleaved edge of the waveguide. Length was changed by successively cleaving the crystal and measuring the output for each length. Their results was for 4 μm wide single mode channels and was 1.5 dB/cm.

It is interesting to compare the above results with loss measurements for planar waveguides. These measurements were performed on samples with the same titanium thickness, diffusion times and temperatures. Measurements showed a coupling efficiency of 51.5% for TE and 56.7% for TM and a loss of 1.02 dB/cm and 1.04 dB/cm for TE and TM modes respectively. Only two samples were measured, one with a prism separation of 1 cm and the second with a prism separation of 2 cm. The loss in the planar waveguide was measured again by using the second technique described above where the output of the waveguide is measured as a function of prism separation. This measurement is shown in Fig. 6.3. This yields 0.97 dB/cm for both TE and TM lowest order modes. The best results reported in the
Fig. 6.3. Rayleigh Scattering and Absorption Loss for a Planar Waveguide Measured Using the Same Technique as for Fig. 6.2.
literature for planar indiffused LiNbO$_3$ waveguides is 1 dB/cm. Therefore, the waveguides fabricated for use in this study are state of the art waveguides.

**Losses Due to Corner Bends**

On each substrate several waveguides separated by 0.5 mm were fabricated; some had bends and the rest were straight. The configuration for each LiNbO$_3$ sample is shown in Fig. 6.4. The straight waveguides were interspersed among the waveguides with the bends with typically no more than two bent waveguides beside each other. The pattern usually had one straight waveguide, then two bent waveguides, then one straight waveguide, then two bent waveguides and so on. The pattern was such that all of the waveguides fit under the base of both the input and output coupling prisms simultaneously. The width of the prisms were 1 cm, consequently, many waveguides could be fabricated simultaneously. Usually no more than seven waveguides, as shown in Fig. 6.4, were fabricated on any one substrate. This is due to the extreme difficulty of achieving uniform coupling over the entire base of the 1 cm prism. Seven waveguides require only about 3 mm of the prism base and it was possible to achieve uniform coupling into each of the waveguides. The input prism was placed over the portion of the waveguides which are
Fig. 6.4. Waveguide Geometry Used for Measuring Loss Due to Corner Bends.
parallel to each other (see Fig. 6.4). The output prism was placed 1 cm from the input prism with the bend approximately half way between the two prisms.

The straight waveguides were used to assure uniform coupling over the entire base of the prism. A 50 cm focal length lens focused the laser beam onto the base of the input coupling prism as shown in Fig. 2.10. A 50 cm focal length lens was used so that the optical density was not too large so that optical damage would not occur. The spot size needed to be small enough so that light would be coupled into only one waveguide at a time. Light was coupled out of the waveguide via another prism and passed through a slit and imaged onto a detector by a lens. The detector output was connected to a lock-in amplifier. The output was measured for each of the straight waveguides and coupling was adjusted until the outputs were equal. In addition, the input coupling efficiency was measured for each of the waveguides. As a consequence of the input coupling efficiency measurement and the equal output measurement from the straight waveguides, it was assumed that both the input and output coupling efficiencies were equal for all of the waveguides. The output from the straight waveguides was used as a baseline for the input into the bent waveguides. By measuring the output from the waveguides having bends the loss in decibels can be calculated by
\[ \alpha = -10 \log \left( \frac{P_B}{P_S} \right) \]  

(6.1)

where \( \alpha \) is the loss in dBs, \( P_S \) is the output from the straight waveguides, and \( P_B \) is the output from the bent waveguides. Figures 6.5 and 6.6 show the results obtained for corner bends for both TE and TM modes respectively for y-cut z-propagating. The results shown in Fig. 6.7 is for TE modes and z-cut y-propagating. All of the corner bend results were measured for angles ranging from 0.1° to 3.0° in steps of 0.1°. The results are compared to those predicted by the theory of Chapter 5. For comparison purposes measurements were made by measuring the coupling efficiency into each of the waveguides having corner bends. The outputs were measured and the loss calculated and plotted in Fig. 6.8. This measurement was for y-cut z-propagating and TE polarized light only. These results include loss due to Rayleigh scattering and absorption in addition to the losses caused by bends in the waveguides. The best fit to the data shown in Fig. 6.5 and 6.6 goes to zero loss for a zero bend angle. The data in Fig. 6.8 does not because the loss at zero bend angle is a measure of the Rayleigh scattering and absorption loss. The zero intercept of the ordinate axis yields approximately 1.4 dB loss. The zero intercept of the ordinate axis yields approximately 1.4 dB loss. For this measurement the input and output prisms were separated by 1 cm, therefore, the
Fig. 6.5. Experimental Corner Bend Scattering Loss as a Function of Bend Angle for TE Polarized Light for Y-cut, Z-propagating LiNbO₃ and Compared to Theory.
Fig. 6.6. Experimental Corner Bend Scattering Loss as a Function of Bend Angle for TM Polarized Light for Y-cut, Z-propagating LiNbO$_3$ and Compared to Theory.
Fig. 6.7. Experimental Corner Bend Scattering Loss as a Function of Bend Angle for TE Polarized Light for Z-cut, Y-propagating LiNbO$_3$ and Compared to Theory.
Fig. 6.8. Experimental Results Showing the Combination Loss Due to Corner Bends, Rayleigh Scattering, and Absorption for TE Polarized Light for Y-cut, Z-propagating LiNbO$_3$. 
1.4 dB loss agrees quite well with the results of the previous section. Also, if 1.4 dB is subtracted from the results shown in Fig. 6.8 the bend loss results agree quite well with those in Fig. 6.5 for TE modes in y-cut z-propagating LiNbO₃.

**Radiation Loss in Smooth Curved Bends and Mode Mismatch Loss**

Two types of curved waveguide geometries were investigated, both types are shown in Figs. 6.9 and 6.10. At all of the junctions where straight to curved and curved to curved sections are joined together there exists a smooth transition, i.e., the first derivative is continuous. All of the curves are portions of a circle with a given radius of curvature. Straight waveguides are interspersed among the curved waveguides as in the corner bend measurements and for the same reason. For the configurations shown in Fig. 6.9 and 6.10 there are two different mechanisms which contribute to the total loss. First, there is the loss due to radiation of the energy away from the waveguide as the energy propagates around the bend. The second loss is caused by a mode mismatch at each of the straight to curved, curved to curved, and curved to straight junctions. Figure 6.11 depicts the field profile of the lowest order mode for a straight and curved waveguide in the plane of the substrate. In the straight waveguide the
Fig. 6.9. One of the Curved Waveguide Configurations Used to Measure Radiation Loss and Straight to Curved Junction Mode Mismatch Loss.
Fig. 6.10. A Second Curved Waveguide Configuration (S Shaped) Used to Measure Radiation Loss and Straight to Curved and Curved to Curved Mode Mismatch Loss.
Fig. 6.11. Field Profiles for Straight and Curved Waveguides.

The peak of the lowest order mode shifts from the center of the axis in the straight waveguide to the outside of the curve in the curved waveguide.
mode is symmetric in the plane of the waveguide while in the curved waveguide the curvature causes the mode to be nonsymmetric. The peak of the lowest order mode shifts from the center of the waveguide towards the outer edge of the curved waveguide. In the S geometry shown in Fig. 6.10 the curved to curved mode mismatch is much larger than for the straight to curved mode mismatch because the curvature of one is the negative of the other. The reason for measuring the S type geometry is twofold. First, as can be seen in Fig. 6.10, the output of the straight portions of the waveguides are all parallel to each other and also parallel to the input straight portions. It is much easier to align the prisms properly so that the polarization of the waveguide mode is parallel (TE) or perpendicular (TM) to the optic axis of the prism. The output straight portions of the non S geometry are not parallel or perpendicular to the waveguide modes for all of the waveguides simultaneously. This causes the output beam from the prism to be split into two orthogonally polarized spatially separated beams, causing an ambiguity in the measurement. The second reason for investigating the S bend geometry is because many configurations of integrated optical switches, modulators, and other devices require variable separations between straight waveguides. One can use the results of this section and of the previous section on corner bends to determine whether a two corner bend transition or an S
curve transition provides the lower loss for a given particular device design. This is discussed in greater detail in Chapter 7.

Measurements were made for both configurations shown in Fig. 6.9 and 6.10 for many different radii of curvature and several different arclengths for each radius of curvature. The measurements were made with the same experimental setup and procedure as was used with the corner bends. The straight waveguides are used to assure equal coupling in all waveguides and the output of the straight waveguides is used as the baseline for the input into the curved waveguides. The loss as a function of radius of curvature is shown in Figs. 6.12 and 6.13 with arclength as a parameter. The results in Fig. 6.12 and 6.13 are for the non S type geometry. All of the arclengths were chosen for the angle of the arc to be 10°, 30°, and 50° for Fig. 6.12 and 20° and 40° for Fig. 6.13 for a radius of curvature of 1 cm. The experimental results are compared to theory for each of the arclengths. Only the averages of the experimental results are shown so that the figure would not be too confusing.

The results for the S type geometry are shown in Figs. 6.14 and 6.15, again as a function of radius of curvature with arclength as the parameter. In this case the angle of the arc was 5°, 15°, and 25° for Fig. 6.14 and 10° and 20° for Fig. 6.15 for a radius of curvature of 1 cm;
Fig. 6.12. Experimental Loss Results for the Non S Bend Geometry Shown in Fig. 6.9 for Three Different Arclengths.

These results include both the radiation loss and the straight to curved mode mismatch loss and compared to theory.
Fig. 6.13. Experimental Loss Results Measured Under the Same Conditions as Fig. 6.12 Except for Two Different Arclengths.
Fig. 6.14. Experimental Loss Results for the S Bend Geometry Shown in Fig. 6.10 for Three Different Arclengths.

These results include the radiation loss, straight to curved mode mismatch loss, and curved to curved mode mismatch loss and compared to theory.
Fig. 6.15. Experimental Loss Results Measured Under the Same Conditions as Fig. 6.14 Except for Two Different Arc_lengths.
however, the total arclengths are the same as in the non S geometry because there are two equal arcs of opposite curvature. As is expected the losses are greater for the S curves than for the non S curves because of the additional loss at the curved to curved junction of the S curve. Also, the experimental results for the S curves have closer agreement with theory than does the non S curves. The reason for this, as explained earlier in this section, is the fact that the output coupling prism cannot be aligned with all of the straight output portions of the non S curves, whereas alignment is the same for all S curves. While the losses plotted in Fig. 6.12 through 6.15 are a function of radius of curvature with the arclength as a parameter, the losses can be plotted with the radius of curvature as a parameter as shown in Fig. 6.16. For each radius of curvature several loss measurements were made with different arclengths. The results shown in Fig. 6.16 are obtained by subtracting the loss at different arclengths for each radius of curvature. The loss is compared to theory for a few radii of curvature; the solid line represents theory and the data points are shown for the different radii of curvature. The slope of the line yields the loss in dB/cm for each radii of curvature. Table 2 gives a more complete statement of the results. Since the loss as a function of arclength is a straight line one can use the data in Table 2 to scale the
Fig. 6.16. Experimental Radiation Loss Due to Curvature Plotted as a Function of Arc Length for Several Radii of Curvatures and Compared to Theory.
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</tbody>
</table>
results to any arclength desired and calculate the loss. The results shown in Table 2 and Fig. 6.16 are the losses due only to the radiation of the light away from the waveguide as it propagates around the curve. These results can now be scaled to the proper arclengths and subtracted from the total loss to account for the total mode mismatch loss for all junctions. This is plotted in Fig. 6.17 as a function of radius of curvature. The results shown in Fig. 6.17 are the sum of all the mode mismatch losses at all three joints (see Fig. 6.10). Now the results from the non-S geometry are needed to separate the straight to curved mode mismatch loss from the curved to curved mode mismatch loss. Since the measurements were made for equal arclengths for both waveguide geometries the results of the non-S geometry can be subtracted from the results of the S curves to yield the loss due to curved to curved mode mismatch. This of course presupposes the straight to curved mode mismatch loss is the same for both geometries. There is no reason to think this is not the case so that assumption will be made. The results are plotted in Fig. 6.18 and were obtained by averaging all of the values for each arclength measured. The curved to curved mode mismatch loss in Fig. 6.18 can be subtracted from the total mode mismatch loss shown in Fig. 6.17 to get the straight to curved mode mismatch loss. This result is twice the mode mismatch
Fig. 6.17. Experimental Mode Mismatch Loss Plotted as a Function of Radius of Curvature.

These results include the mode mismatch loss at the curved to curved junction and both of the straight to curved junctions.
Fig. 6.18. Experimental Mode Mismatch Loss at the Curved to Curved Junction Plotted as a Function of Radius of Curvature and Compared to Theory.
loss for a single straight to curved junction because the S curves have two straight to curved junctions as can be seen in Fig. 6.10. The single straight to curved junction mode mismatch loss is shown in Fig. 6.19 and is obtained by dividing the above result by two.
Fig. 6.19. Experimental Mode Mismatch Loss at a Single Straight to Curved Junction Plotted as a Function of Radius of Curvature and Compared to Theory.
CHAPTER 7

RAMIFICATIONS OF BENDING LOSSES
WHEN DESIGNING INTEGRATED OPTICAL CIRCUITS

Introduction

Bending loss was measured which verified the theoretical predictions of Chapter 5. This chapter describes how those results are used in practice. When designing integrated optical circuits, it is important to consider the amount of losses induced by directional changes in the waveguides. In many applications of integrated optics, i.e., modulators, couplers, A/D converters, etc., it is necessary for two different straight portions of a single mode channel waveguide to be connected with a given amount of transverse offset. To date, in almost all of the integrated optical devices which have an offset, it is achieved by using two corner bends as shown in Fig. 7.1. This study shows that there are many useful cases where an S bend, as shown in Fig. 7.2 and studied in the previous two chapters, exhibits less loss than two corner bends. Comparisons are made between the two cases using the theory of Chapter 5 and the experimental results of Chapter 6.
Fig. 7.1. Two Straight Waveguides having Transverse Offset $x_s$ and Longitudinal Offset $z_s$ Joined Together by Another Straight Waveguide at Angle $\theta_T$. 
Fig. 7.2. Two Straight Waveguides having Transverse Offset $x_s$ and Longitudinal Offset $z_s$ Joined Together by Curved Waveguides having an S Shape.
Comparisons of Connecting Two Parallel Waveguides

Figure 7.1 illustrates the two corner bend approach, where the power out of the second corner bend, $P_c$, is given by

$$P_c = P_i |a_{12}|^2 |a_{23}|^2 \exp (-\gamma_o L_o), \quad (7.1)$$

where $P_i$ is the power incident upon the first bend, $\gamma_o$ is the attenuation coefficient due to absorption and Rayleigh scattering, and $L_o$ is the length of the joining segment. We assume that the input and output waveguides are parallel. Then, from Eq. (5.44)

$$|a_{12}|^2 = |a_{23}|^2 = \exp \left[ - \left( \beta^2 W^2 \sin^2 \theta_T \right) / 2 \right]$$

(7.2)

where $\theta_T$ is the angle of each bend.

The power coming out of the S bend, $P_s$, is given by

$$P_s = P_i |a_{45}|^2 |a_{56}|^2 |a_{67}|^2 \exp \left[ -\gamma_o L_1 - \gamma_2 L_2 - \gamma_3 L_3 \right]; \quad (7.3)$$

here $L_1$ is the total length of the two arcs $L_2$ and $L_3$ ($L_1 = L_2 + L_3$), $\gamma_2$ and $\gamma_3$ are the attenuation coefficients due to radiation in a curved waveguide [see Eq. (5.25)]. Again, assuming the waveguides to be parallel, we have, from Eqs. (5.45) and (5.46),
\[ |a_{45}|^2 = |a_{67}|^2 = \frac{1}{1 + \left( \frac{d X_0}{2} \right)^2} \] (7.4)

and

\[ |a_{56}|^2 = \frac{\left[ 1 - \left( \frac{d X_0}{2} \right)^2 \right]^2}{\left[ 1 + \left( \frac{d X_0}{2} \right)^2 \right]^2} \] (7.5)

where \( d \) is the field deformation term due curvature and \( X_0 \) is the field width in the x dimension as defined in Chapter 5. The attenuation coefficient, \( \gamma_0 \), due to absorption and Rayleigh scattering are the same in both cases.

The two waveguide axes are assumed parallel, separated by a transverse distance \( x_s \). The ends are separated axially by a distance \( z_s \) (See Figs. 7.1 and 7.2.) For an angle bend, with one straight connecting element between the waveguides, the axes are inclined at an angle \( \theta_T \) where

\[ \theta_T = \tan^{-1} \left( \frac{x_s}{z_s} \right) \] (7.6)

For two curved segments used as an S shape to join the two parallel waveguides (the connections at each of the three junctions are tangential) the radius of curvature is

\[ R_0 = \frac{z_s}{4} \left[ \frac{1 + \left( \frac{x_s}{z_s} \right)^2}{\left( \frac{x_s}{z_s} \right)} \right] \] (7.7)
where each of the curved segments is an arc of a circle subtending an angle \( \phi \) given by

\[
\phi = \cos^{-1}\left\{ \frac{1 - (x_s/z_s)^2}{1 + (x_s/z_s)^2} \right\}.
\]  

(7.8)

As shown in Appendix D the loss in the waveguides due to absorption and Rayleigh scattering can be ignored. Therefore, the following calculations include only the losses due to bends. The plots of the total loss associated with the connection of two such guides is plotted in Fig. 7.3. The loss is plotted as a function of the ratio of the transverse separation to the axial separation for both corner and S bends. There is only one curve for the corner bends while there exists a family of curves for the S bends with the radius of curvature \( R_o \) as the parameter. From these curves one can design integrated optical devices with lowest loss. Having selected an appropriate ratio of transverse to axial separation, the designer uses these curves along with Eqs. (7.6), (7.7), and (7.8), to establish that the coupling between the two parallel, colinear waveguides exhibits the least amount of loss. The results show in general that the corner bend approach is better for a very small transverse separation. As the transverse separation increases the S bend yields the lowest loss. The axial separation also gets larger which may be impractical in some applications.
Two Corner Bends

- $R_q = 1$ cm
- $R_q = 2$ cm
- $R_n = 3$ cm

Fig. 7.3. Experimental Loss as a Function of $x_s/z_s$ for the Two Waveguide Configurations Shown in Figs. 7.1 and 7.2.
CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

Our initial motivation for studying bending losses was to learn how densely integrated optical components could be placed on a single chip. The motivating factor for studying these losses has changed somewhat as the study progressed because of questions posed by concurrent work in other laboratories. Many integrated optical devices, such as modulators, switches, directional couplers, A/D converters, etc., require directional changes between passive waveguides. These directional changes, as can be seen from this study, can cause significant losses. Since integrated optical devices will probably use diode lasers, which typically emit an average power of 10 mw and must be operated over a large dynamic range (greater than 50 dB), there is not much allotment for power losses. To date most integrated optical devices having directional changes use the two corner bend approach. This study has described the conditions under which a particular device design can keep the losses to within a tolerable level.

The experimental bending loss results obtained in this study were initially intended to be compared with
published already existing theoretical results. The theoretical results were to be modified to fit the particular waveguide parameters encountered in the experimental study. However, we were unable to find a published theoretical model that could provide a quantitative comparison for experiments with Ti diffused LiNbO$_3$ waveguides. All of the analysis of bending loss reported in the literature used step index waveguides. Diffused waveguides have a graded index profile. In addition, the reported analysis did not take into account field deformation of the mode as it propagates around the bend. Therefore, a new, more exact theoretical model was developed for the Ti diffused LiNbO$_3$ waveguides, a model which takes into account the graded index profile and the deformation of the field as it propagates around the bend. The accuracy of the theoretical results depends on how precisely the mode field profiles are known. Typically a Gaussian approximation is fit to numerically derived solutions of the wave equation. This is a good approximation in the plane of the waveguide; however, it is not very good in the perpendicular direction. Therefore, a weighted Gaussian function that more accurately described the field profile was developed. In addition to developing theory for radiation loss in bends, a theory was developed for mode mismatching at junctions where the waveguide axis changes directions.
There were very few results for absorption and Rayleigh scattering in narrow single mode channel waveguides. Consequently, a technique was developed to measure these results. In conjunction with the loss measurements, a novel technique for measuring the coupling efficiency into narrow single mode channel waveguides was developed. Knowing the coupling efficiency is very important for keeping the optical intensity in the waveguide below the optical damage threshold, which is very low for LiNbO$_3$ as described by Schein, Cressman, and Tesche (1977).

The bending loss for curved bends as well as corner bends was measured and showed great reproducibility. In general the loss measurement results agreed quite well with theory. The loss measurements were used to predict design constraints for integrated optical devices. For small transverse separations the two corner bend approach appears to have lower loss than the S bend approach.

Each time the loss measurements were repeated new waveguides were fabricated. The success to achieving highly reproducible results for both the curved bends and corner bends was the high degree of quality control at each processing step. If any waveguide had small breaks or narrowing of the lines, that waveguide was not used in the measurement. Every waveguide was very carefully scrutinized before proceeding to the next processing step.
There is still more work that could be done on the bending loss problem. This study was appropriate for an isotropic crystal. Even though LiNbO₃ is anisotropic the curved bends were fabricated in orientations of LiNbO₃ that exhibit isotropy, i.e., the optic axis is perpendicular to the surface of the crystal. As a "follow-on" to this work a study of the bending losses with the optic axis in the plane of the crystal would be appropriate. Another interesting study would be to see how the mode mismatch loss at the two curved junctions could be reduced. Possibly a parabolic curve or some other smooth change in curvature would help reduce the mode mismatch loss. Another technique to reduce radiation loss would be to increase mode confinement at the outer edge of the curved waveguide where radiation loss occurs. This could be achieved by ion implanting beside the waveguide to reduce the index of refraction thus increasing the index difference which would increase mode confinement. Another method to increase mode confinement would be to grade the titanium thickness across the width of the waveguide with the thickest portion at the outer edge of the curve. This would increase the index difference at that boundary and yield a tighter bound mode much the same as in the ion implanting technique.
APPENDIX A

RADIATION FROM A CURVED WAVEGUIDE

From Fig. 5.3 writing the coordinates of a point in the core of the guide
\[ \bar{R}' = \hat{x}^G (R_0 + x) \cos \phi' + \hat{y}^G (R_0 + x) \sin \phi' + \hat{z}^G y \]  
(A.1)

where the superscript refers to the global coordinates relative to the center of curvature. The unit vector in the direction of the radiation field at \( \phi = \pi/2 \) is
\[ \hat{R} = \hat{y}^G \sin \theta + \hat{z}^G \]  
(A.2)

and Eq. (5.18) can then be written (with the definitions of the fields and the profile defined in Chapter 5)
\[ S^R = C(R_0 \omega \Delta)^2 |I_{\phi}^x|^2 |I_y|^2 \]  
(A.3)

where
\[ I_y = \int_0^\infty E(y) \varepsilon(y) \exp [ik_s y \cos \theta] dy \]  
(A.4)

and
\[ I_{\phi}^x = 2\pi \int_0^\infty J_m(k_s \sin \theta (R_0 + x)) E(x) \varepsilon(x) dx \]  
(A.5)

where
\[ m = \beta R_0 \]  
(A.6)

and
\[ C = (\mu/\epsilon_0)^{1/2} k_b^2 / (32 (\pi R)^2) \]  
(A.7)
where the integral around the axis in Eq. (A.5) has been written down by inspection, using the integral representation of Bessel Functions (Abramowitz and Stegun, 1972) \( J_m(x) \), where the principal of matching the phase along the center of energy of the curved guide to the straight guide phase change to obtain Eq. (A.6) was used.

From Appendix B, \( I_y \) is found to be

\[
I_y = F_y \frac{\sqrt{\pi}}{2} Y_1 \exp \left[ - \left( \frac{Y_0}{Y_0'} \right)^2 - \left( \frac{Y_1Y_0}{Y_0^2} \right) \right] \tag{A.8}
\]

where

\[
F_y = Y_1 \left\{ \frac{\sqrt{\pi}}{Y_0^2} \left( 1 + \text{erf} \left( \frac{Y_1Y_0}{Y_0^2} \right) \right) + \exp \left[ - \frac{Y_1Y_0}{Y_0^2} \right] \right\} \right/ \sqrt{\pi} \tag{A.9}
\]

where \( Y_1 \) is given by Eq. (5.28) and \( Y_0 \) and \( Y_0' \) are the parameters of the fundamental mode described in Chapter 4.

From Appendix C, evaluation of Eq. (A.5) yields

\[
I_{\phi'} = \left( \frac{2}{QR_0} \right)^{1/2} \left[ (1 + d_1X_0') F_x (1) + d_1F_x (2) \right] \exp \left[ -h(\theta)R_o + \left( \frac{QX^2}{2} \right) \right] \tag{A.10}
\]

where the parameters are defined in Appendix C.

Now using Eqs. (A.4) and (A.10) in Eq. (A.3) along with Eq. (5.24)

\[
P_R = 2\pi R^2 \int_0^\pi S_R \sin \theta \, d\theta \tag{A.11}
\]
where
\[ S^R = f(\cdot \exp(-2h(\theta) R_0)) \] (A.12)

for \( R_0 >> 1 \) the strong exponential dependence of \( \theta \) is isolated in comparison to the slowly varying term \( f(\theta) \) where
\[
f(\theta) = \frac{2c\omega^2 \Delta^2}{Q} R_0 |I_y|^2 |(1+i\Delta_{\perp}X_0)F_\perp(1) + d_{\perp}F_\perp(2)|^2 \exp\left(\frac{(QX_0)^2}{2}\right)
\] (A.13)

with
\[
h(\theta) = \left[ \beta \tanh^{-1}(Q/\beta) - Q \right]. \quad (A.14)
\]

With this form for \( S^R \), Eq. (A.11) can be evaluated using Laplace's method (Carrier, Krook, and Pearson, 1966). Using the definitions of Eqs. (A.12) and (A.14) is found that the maximum contribution of the integral occurs for
\[
\theta = \pi/2 \quad (A.15)
\]
where the width of the Gaussian approximation to the integral, \( \Delta\theta \) is for
\[
\Delta\theta = 1/2R_0 \quad Q << 1 \quad (A.16)
\]
i.e., the radiator is strongly confined to the plane of the bend, which gives
\[
P^R = 2\pi R^2 \left(\frac{\pi}{QR_0}\right)^{1/2} S^R \bigg|_{\theta = \pi/2}
\] (A.17)

and the power attenuation coefficient \( \gamma \) is
\[
\gamma = \frac{P^R}{2\pi R_0 P_0} \quad (A.18)
\]
which from Chapter 5 and Eqs. (A.12) and (A.14) yields Eq. (5.25) when Eq. (B.8) of Appendix B is used for $\theta = \pi/2$. 
APPENDIX B

EVALUATION OF $I_Y$

From Eq. (A.4) along with Eqs. (5.3) and (5.5)

$$I_Y = \int_{y_0}^{\infty} \left\{ y \exp \left[ - \left( \frac{y-y_0}{Y_o} \right) - \left( \frac{y}{Y_1} \right)^2 + i a y \right] \right\} dy$$  \hspace{1cm} (B.1)

where

$$a = k_s \cos \theta.$$  \hspace{1cm} (B.2)

Rewriting the exponent, and changing variables to

$$y' = \frac{(y-y_1)}{Y_1}$$  \hspace{1cm} (B.3)

where

$$Y_1 = \frac{Y_o D}{(Y_o^2 + D^2)^{1/2}}$$  \hspace{1cm} (B.4)

$$Y_1 = Y_1 \left\{ \frac{Y_o}{Y_1^2} + \frac{i a}{2} \right\}$$  \hspace{1cm} (B.5)

this yields

$$I_Y = \frac{\sqrt{\pi}}{2} Y_1 \left\{ \frac{y_1}{Y_1} \left( 1 + \text{erf} \left( \frac{y_1}{Y_1} \right) \right) + \exp \left[ - \frac{y_1^2}{Y_1^2} \right] \right\} \exp \left[ \frac{Y_o^2}{Y_1^2 + D^2} \right]$$  \hspace{1cm} (B.6)

and defining

$$I_Y = \frac{\sqrt{\pi}}{2} Y_1 F_Y \exp \left[ \frac{Y_o^2}{Y_1^2 + D^2} \right]$$  \hspace{1cm} (B.7)
where for \( a = 0 \) since \( \theta = \pi/2 \)

\[
F_Y = Y_1 \left\{ \frac{Y_0 Y_1}{Y_0^2} \left( 1 + \text{erf} \left( \frac{Y_0 Y_1}{Y_0^2} \right) \right) + \exp \left[ - \left( \frac{Y_0 Y_1^2}{Y_0^2} \right) \right] \right\}^{\frac{1}{\sqrt{\pi}}}
\]

(B.8)
APPENDIX C

EVALUATION OF $I_{\phi',x}$

Using the near Debye Asymptotic (Abramowitz and Stegun, 1972) form for $J_n(k_b \sin \theta (R_o + x))$ and since

$$\delta R_o = m > k_b R_o \quad (C.1)$$

for bound modes a Taylor series expansion of the exponent in powers of $x/R_o \ll 1$ and from Eqs. (5.2), (5.8), and (A.5) yields

$$I_{\phi',x} = \left(\frac{\pi}{2Q' R_o}\right)^{1/2} \left[\exp \left(-h(\theta) R_o\right)\right]$$

$$\times \int_{-\infty}^{\infty} \left(1 + d_1 x\right) \exp \left(-\left(\frac{x}{x_o}\right)^2 + Q' x\right) \left[\text{erfc}\left(\frac{x}{D}\right) + \omega'\right] \left[\text{erfc}\left(\frac{x}{D}\right) - \omega'\right] dx \quad (C.2)$$

where

$$Q' = (\beta^2 - k^2_b \sin^2 \theta)^{1/2} \quad (C.3)$$

and

$$h(\theta) = \left[\beta \tanh^{-1}\left(\frac{Q'}{\beta}\right) - Q'\right] \quad (C.4)$$

and

$$\omega' = W/2D \quad (C.5)$$
where Eq. (C.2) is valid in retaining only terms to order \(x/R_0\) for
\[
(\delta x_o)^2/2R_0 \ Q' \ll 1
\]  
(C.6)
because of the Taylor series expansion of the exponent
where the characteristic dimension of the guide is assumed
to be \(x_o\), the field width.
Completing the square of the exponent and defining
\[
\omega_\pm = \omega' \pm \frac{X_1}{D}
\]  
(C.7)
with
\[
X_1 = Q' x_o^2/2
\]  
(C.8)
Eq. (C.2) can be written as
\[
I_{Q'x} = \left[\frac{2}{Q'R_0}\right]^{1/2} \left[(1+d_1 X_1)F_x^{(1)} + d_1 F_x^{(2)}\right]
\]
\[
\exp \left\{ -h(\theta) R_0 - \left(\frac{Q'x_o}{2}\right)^2 \right\}
\]
where
\[
F_x^{(1)} = -\frac{\sqrt{\pi}}{2} \int_{-\infty}^{\infty} \left\{ \exp \left( -\frac{x^{'2}}{x_o^2} \right) \right\} \left[ \text{erfc} \ (\omega_- - \frac{x'}{D}) - \text{erfc} \ (-\omega_+ - \frac{x'}{D}) \right] \ dx'
\]  
(C.9)
and
\[
F_x^{(2)} = \frac{\sqrt{\pi}}{2} \int_{-\infty}^{\infty} \left\{ \exp \left( -\frac{x^{'2}}{x_o^2} \right) \right\} \left[ \text{erfc} \ (\frac{x'}{D} - \omega_-) - \text{erfc} \ (\frac{x'}{D} + \omega_+) \right] \ dx'
\]  
(C.10)
where
\[ x' = (x - X_1) \]  \hspace{1cm} (C.12)

Direct integration by parts for \( F_x^{(2)} \) yields
\[
F_x^{(2)} = \frac{X_0 \sqrt{\pi}}{D \rho} \sinh \left[ \frac{2w' X_1}{D \rho^2} \right] \exp \left\{ \frac{-(\omega^2 + X_1^2/D^2)}{\rho^2} \right\}
\]  \hspace{1cm} (C.13)

where
\[ \rho^2 = 1 + (X_0/D)^2 \]  \hspace{1cm} (C.14)

The equation for \( F_x^{(1)} \) is in the form of a convolution integral of complementary error functions and Gaussians. Thus, using the convolution theorem for Fourier Transform (Bracewell, 1978) pairs and solving the inverse Fourier Transform (Erdelyi, 1954) yields
\[
F_x^{(1)} = X_0 \left\{ \text{erf} \left( \frac{D\omega^+}{X_0} \right) + \text{erf} \left( \frac{D\omega^-}{X_0} \right) - \text{erf} \left( \frac{\omega^+}{\rho} \right) - \text{erf} \left( \frac{\omega^-}{\rho} \right) \right\}
\]  \hspace{1cm} (C.15)
In this appendix the losses due to bends for two different cases were compared to each other under the assumption that $\gamma_0 L_0$ and $\gamma_0 L_1$ may be ignored. We now examine this assumption. We have

$$L_0 = z_s \left[ 1 + \left( \frac{x_s}{z_s} \right)^2 \right]^{1/2}$$

and

$$L_1 = 2R_0 \phi .$$

The value for $x_s/z_s$ will usually satisfy

$$\frac{x_s}{z_s} < 0.1$$

therefore, $L_0$ becomes

$$L_0 = z_s + \frac{x_s^2}{2z_s} .$$

Substituting Eqs. (7.7) and (7.8) for $R_o$ and $\phi$, we find

$$L_0 = \left[ \frac{x_s^2 + z_s^2}{2x_s} \right] \cos^{-1} \left\{ \frac{z_s^2 - x_s^2}{z_s^2 + x_s^2} \right\} .$$
Using $\cos \phi = (1 - \sin^2 \phi)^{1/2}$ and $\sin \phi = \phi$ since $\phi$ will typically be $10^\circ$ or less, we have

$$|L_O - L_1| \approx x_s^2/2z_s . \quad \text{(D.6)}$$

A conservative estimate of $x_s$ is approximately $1$ mm, so that, combining this with Eq. (D.3), we have

$$|L_O - L_1| < 0.05 \text{ mm} . \quad \text{(D.7)}$$

From the experimental results of Chapter 5 the absorption and Rayleigh scattering losses yield $1.5 \text{ dB/cm or } \gamma_o = 0.345 \text{ cm}^{-1}$. Therefore,

$$10 \log [\exp (-\gamma_o |L_O - L_1|)] = 0.0075 \text{ db} \quad \text{(D.8)}$$

and the errors involved with ignoring $\gamma_o L_O$ and $\gamma_o L_1$ are negligible for purposes of comparison.
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