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ANALYTICAL STUDY OF REINFORCED CONCRETE BEAMS
STRENGTHENED WITH FIBER REINFORCED PLASTIC PLATES (FABRICS)

by

Amir Masoud Malek

A Dissertation Submitted to the Faculty of the
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1997
As members of the Final Examination Committee, we certify that we have read the dissertation prepared by Amir Masoud Malek entitled Analytical Study of Reinforced Concrete Beams Strengthened with Fiber Reinforced Plastic Plates (Fabrics) and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

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To:

My wife, Soheila
and
My daughter, Sahar

for their sacrifice and their patience
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ABSTRACT

Epoxy-bonding a composite plate to tension face, is an effective technique for repair and retrofit of reinforced concrete beams. Experiments have indicated local failure of the concrete layer between the plate and longitudinal reinforcement in retrofitted beams. This mode of failure is caused by local stress concentrations at the plate end, as well as at the flexural cracks. A method has been presented for calculating shear and normal stress concentrations at the cut-off point of the plate. Stress concentrations predicted by this method have been compared to both finite element method and experimental results. The analytical models provide closed form solutions for calculating stresses at the plate ends and can easily be incorporated in design equations.

The ultimate capacity of the reinforced concrete beams strengthened by composite plates bonded to the tension face, is controlled by either compression crushing of concrete, rupture of the plate, local failure of concrete at the plate end, or debonding of the plate. These failure modes have been considered in developing design guidelines for flexural strengthening of reinforced concrete beams using fiber composite plates.

Bonding composite plates (fabrics) to the web of reinforced concrete beams can increase the shear and flexural capacity of the beam. An analytical model has been developed to calculate the stress distribution in the strengthened beam, and the shear force resisted by the composite plate before cracking and also after formation of flexural cracks. Parametric study has been performed to reveal the effect of important parameters such as fiber orientation, and plate thickness. The ultimate shear capacity of reinforced concrete
beams is also increased by epoxy-bonding composite plates to the side faces of the beam. Truss analogy and compression field theory have been used to determine the effect of the composite plate on the crack inclination angle and the shear capacity of reinforced concrete beams at ultimate state. The effects of important parameters such as plate thickness and fiber orientation angle on the crack inclination angle and the shear capacity of the strengthened beam have been investigated through a parametric study.
CHAPTER 1
INTRODUCTION

1.1 General

In recent years, repair and retrofit of existing structures have been among the most important challenges in civil engineering. The primary reasons for strengthening of structures include: upgrading of resistance to withstand underestimated loads; increasing the load carrying capacity for higher permit loads; eliminating premature failure due to inadequate detailing; restoring lost load carrying capacity due to corrosion or other types of degradation caused by aging; etc.

Different techniques have been developed to retrofit a variety of structural deficiencies. The traditional methods include steel jacketing of concrete columns, and external post-tensioning or bonding steel plates to concrete beams.

For concrete columns, lateral confinement has been provided by means of steel jackets or fiber-reinforced-plastic (FRP) wraps (Priestly, et al., 1991) (Saadatmanesh, et al., 1993). External post-tensioning has been used since early 1950's (Lee, 1952). The main disadvantage of external post-tensioning is required periodic testing and inspection of cables and anchorages, as well as transferring high anchorage forces to a relatively small area of the anchorage. Flexural and shear strengthening have been also performed by epoxy bonding steel or FRP plates to the tension face and the web of the beams, as shown in Fig. 1.1. Steel plates have been used in many countries for flexural strengthening of concrete
beams for several years (Dussek, 1980) (Macdonald & Caldar, 1982) (Swamy, et al., 1987). The main disadvantage of using steel plate is corrosion of steel which adversely affects the bond at the steel concrete interface. The problem is more severe for bridges where deicing chemicals are commonly used during cold seasons. In order to eliminate the corrosion problem, steel plate has been replaced by FRP plate.

Fiber reinforced plastic materials have been used successfully in the aerospace industry for several decades. These materials can be made from different types of fibers and matrices. They have been used in several forms such as rebars, plates, and fabrics, in a variety of structural members. FRP plates are not prone to electrochemical corrosion as is steel. Furthermore, they can be formed, fabricated and bonded easier than steel plates. FRP's generally behave linearly elastic to failure if loaded in the fiber direction. The mechanical properties of FRP vary with the type and orientation of the reinforcing fibers. Therefore, the fibers can be placed in any orientation to maximize the strength in a desired direction.

1.2 Problem Statement

In strengthening reinforced concrete beams by bonding plates to the tension face, a specific type of failure has been reported. This failure which is generally referred as "Local Failure", results from high shear and normal (peeling) stress concentrations at the plate end. Local failure initiates by cracking of concrete beam at cut-off point of the plate. The cracks eventually propagate through midspan of the beam, leading in complete debonding of the
plate (Fig. 1.2). In this way, the strengthened beam can not reach to its ultimate flexural capacity, which is generally calculated based on plate rupturing, or crushing of concrete. Closed form solutions are required to calculate the stress concentrations, and to provide the necessary tools for proper designing of the strengthened beam.

There has been a large number of experimental and analytical studies demonstrating the effectiveness of flexural strengthening of reinforced concrete beams using composite plates. However, in order for practicing engineers to be able to use this retrofit system design guidelines are required. The second part of this study presents a comprehensive design methodology and guidelines for strengthening of concrete beams with epoxy bonded composite plates or fabrics.

Experimental study of reinforced concrete beams strengthened with web-bonded FRP plates has shown significant increase in bending and shear capacity of the beam. However, application of the method has been mostly limited to research activities. Analytical study of this type of strengthening provides the necessary tools for its field applications. The method used by American Concrete Institute (ACI) for calculating shear capacity of the reinforced concrete beams is based on truss analogy and assuming that shear cracks are formed in an inclination of 45 degrees. This assumptions should be modified in case of strengthening of the beam. Parameters such as fiber orientation angle and plate thickness affect the inclination angle, and must be considered in estimating the shear capacity of the beam.
1.3 Scope of the Study

Reinforced concrete beams strengthened with FRP plates bonded to the bottom face for flexural strengthening, or to the side faces for shear strengthening have been studied throughout the following sections.

1.3.1 Stress Concentrations in Reinforced Concrete Beams Strengthened with FRP Plates

In strengthening reinforced concrete beams with FRP plates, different failure modes have been reported (Ritchie, et al., 1991) (Saadatmanesh and Ehsani, 1991). These modes can be divided into two general categories of "flexural" and "local" failures. "Flexural failure" is defined as concrete crushing in compression or plate rupturing in tension. "Local failure" is defined as the peeling of the FRP plate at the location of high interfacial stresses and shear failure of the concrete layer between the plate and the longitudinal reinforcement, as shown in Fig. 1.2.

Flexural failure has been already investigated analytically (An, et al., 1991). This part of study has been concentrated on analytical modelling of "local failure." Since in many cases the failure of retrofitted beams is governed by the "local" failure, the investigation of the stresses at the concrete/FRP interface is an important issue in the analysis and design of this type of beam.

Closed form solutions have been presented for calculating the shear and normal interfacial stresses. In order to verify this method, the results have been compared to those of the finite element and experimental studies. Although the method has been developed
based on uncracked beam, its validity for cracked beams has been also investigated.

### 1.3.2 Design Guidelines for Flexural Strengthening of Reinforced Concrete Beams Using FRP Plates

The ultimate capacity of the reinforced concrete beams strengthened with FRP plates is controlled by either compression crushing of concrete, rupture of the plate, local failure of concrete at the plate end, or debonding of the plate. These failure modes have been considered in developing design guidelines for strengthening reinforced concrete beams using fiber composite plates. The effect of multi-step loading of the beam, before and after upgrading, has been also considered. Limit state design procedure has been followed in this study. Terms, definitions and notations compatible to ACI design guidelines for ordinary reinforced concrete beams have been utilized.

### 1.3.3 Analytical Study of Reinforced Concrete Beams Strengthened with Web-bonded FRP Plates

Epoxy-bonding composite plates (fabrics) to the web of the reinforced concrete beams can increase the shear and flexural capacity of the beam. This part of study presents an analytical model to calculate the stress distribution in the strengthened beam, as well as the shear force resisted by the composite plate, considering orthotropic behavior of the plate.

The method has been developed based on complete bonding, and using compatibility of the strains in the plate and the reinforced concrete beam. The validity of
the assumptions used in the method has been compared to the finite element method. A parametric study has been performed to reveal the effect of important parameters such as fiber orientation on the shear force resisted by the composite plate. The method has been developed for both uncracked and cracked beams, and it has been used for stress analysis of this type of beams.

1.3.4 Ultimate Shear Capacity of Reinforced Concrete Beams Strengthened with Web-Bonded FRP Plates

The ultimate shear capacity of reinforced concrete beams is increased by epoxy-bonding composite plates to the side faces of the beam. The crack inclination angle is changed as a result of bonding the plate. In this part of study truss analogy and compression field theory have been used to determine the effect of the plate on the inclination of the cracks in the reinforced concrete beams at ultimate case. Subsequent to calculation of the crack inclination angle, the equilibrium and compatibility equations have been used again to obtain the shear force resisted by the plate.

A parametric study has been carried out to reveal the effect of important parameters such as plate thickness and fiber orientation on the crack inclination angle. The upper bound of crack inclination angle found in this study can be used as a conservative value to determine the effectiveness of the plate. Knowing the inclination of the cracks, the shear force in the composite plate and concrete beam can be calculated and used to design this type of beam.
CHAPTER 2
LITERATURE REVIEW

Reinforced concrete beams strengthened with externally bonded steel or composite plates have been studied both experimentally and analytically by several researchers.

2.1 Concrete Beams Strengthened with Steel Plates

MacDonald and Calder (1982) tested 4.9 m and 3.5 m beams with steel plates bonded to their tension flange, in four point bending. Significant improvement in terms of ultimate load, crack control and stiffness was observed. The effect of several parameters such as type of adhesive, multi-layered plates, and plate geometry was investigated. Using a stiff adhesive resulted in an increase of about 100% in visible cracking load. In this case the crack pattern was changed to develop more cracks with closer spacing. The mean increase in the failure load of the beam plated as-cast and plated pre-cracked was 71%. The failure modes, were yielding of both internal and external reinforcement, and horizontal shear failure occurring in the concrete layer between internal and external reinforcement resulting in complete separation of the plate. The recent mode of failure was more often observed for cases that thicker and narrower plates were bonded to the concrete beam. The shear stress calculated based on maximum strain gradients could not provide a reasonable estimate of the shear stress concentration at the plate end.

The exposure tests were performed for three different climates of high rainfall, industrial, and marine environments. A significant amount of corrosion at the steel/resin
interface was observed. The interfacial corrosion led to strength reduction in the strengthened beam.

VanGemert and VandenBosch (1985) studied the effects of fatigue, long term exposure in the outside climate, and thermal loads on reinforced concrete beams strengthened with double layers of steel plates. In all the laboratory experiments and also practical applications the method was proven to be a reliable one. The fatigue tests showed that no redistribution of stresses took place by deformation in the glue or by any failure of the glued connection. Atmospheric conditions had no remarkable influence on the mechanical properties. The full-scale thermal loading tests showed that cold-hardening epoxy had a poor thermal resistance. For low temperatures there was no decrease in ultimate load, however at high temperatures (about 60°C) it was unable to transfer shear stresses properly, and a crack propagated through the epoxy joint, initiated at the plate cut-off point. At lower temperatures local shear failure in the concrete occurred at the end of the plate.

Swamy et al. (1987) investigated the effect of bonded steel plate on some of the important mechanical properties of the concrete beam such as initiation of cracking, postcracking behavior, deformations, and ultimate strength.

Twenty four beams were tested and the effects of adhesive and plate thicknesses were investigated. Several beams with lapped plates, double plates and variable thickness of adhesive along the beam were also tested for comparison. The results showed increase in flexural stiffness and ultimate capacity and decrease in cracking and structural
deformations at all levels of loading. Lapped plates, precracking prior to bonding the plate, and variable adhesive thickness had no adverse effect on the structural behavior of the strengthened beam.

Hamoush and Ahmad (1990) performed an analytical study on interfacial failure of damaged reinforced concrete beams strengthened with steel plates. They used linear fracture mechanics and finite element method to study a simply supported beam under four point bending. In order to simulate the damaged beam, they assumed a number of vertical flexural cracks in the beam. Furthermore, they assumed that as a result of opening of the flexural cracks, interfacial cracks are initiated and extended toward the supports. Eight-node isoparametric elements together with four node ones were used to model the beam. After applying the loads, the stress intensity factors for modes I and II, deflection of the beam, and strain energy release rate were obtained by using finite element method.

They also tested six beams, and the results of one of them which experienced interfacial failure was compared to finite element results. Based on their parametric studies, the length of the interfacial crack which maximized the strain energy release rate was obtained to be equal to the length of the flexural crack. They also concluded that undamaged strengthened beam has a very small strain energy release rate, resulting in a very high debonding load. The existence of large number of flexural cracks was observed to reduce the stress intensity factors and energy release rates. The effect of thickness of the adhesive layer was observed to be negligible.

Ziraba et al. (1994) combined principles developed by ACI for design of unplated
beams together with experimental and numerical results to develop design guidelines for reinforced concrete beams strengthened with steel plates. They carried out a parametric study using finite element method to investigate the possible modes of failure in this type of beam, and to develop appropriate expressions for interfacial stresses. They suggested using external jacketing in the shear span region to arrest concrete rip-off failure and to insure flexural mode of failure.

2.2 Concrete Beams Strengthened with Composite Plates

Meier (1987) studied the utilization of carbon reinforced plastic sheets in rehabilitation applications. He tested a series of twenty six reinforced concrete beams strengthened by CFRP plates bonded to the tension face. He showed that steel plates can be replaced by composite plates, which are more economical considering construction expenses. The addition of FRP plates doubled the strength of the beam, and reduced the deflection to half of the unplated beam.

Saadatmanesh and Ehsani (1990) studied the effect of chosen adhesive material on the performance of the concrete beams strengthened with composite plates. They constructed five reinforced concrete beams of 1675 mm length, four of them externally reinforced with glass fibers. Four different types of two-component epoxies were used.

The first beam (A) failed without any significant increase in strength comparing to the control beam. The reason of this ductile behavior was high flexibility of the adhesive that prevented any load transfer from concrete beam to the composite plate. The second
beam (B) failed in shear, initiated by peeling off of the concrete bonded to the plate at the location of shear cracks. The third beam (C) failed due to horizontal shear failure in the concrete layer between longitudinal steel rebars and the plate. The fourth beam (D) failed as a result of debonding of the plate following the initiation of the large flexural cracks in the concrete beam. They concluded that effectiveness of this technique highly depends on the mechanical properties of the adhesive material.

Saadatmanesh and Ehsani (1991) studied the static strength of reinforced concrete beams strengthened with glass fiber reinforced plastic (GFRP) plates bonded to the tension flange. Five rectangular beams (A through E) and a T-beam (F) were tested. The beams were 4.88 m long, and were tested under 4-point bending. Ready mixed concrete, and a two-component epoxy with a lap shear strength of 14-15 Mpa were used in making the specimens. The epoxy thickness was maintained on an average of 1.5 mm. The concrete compressive strain, plate tensile strain and deflection of the beam were measured during the test.

Failure in beam (A) was caused by crushing of concrete in compression zone. The load verses deflection curve showed reduction in stiffness following to the yielding of the tensile reinforcement.

Beam (B) failed as a result of debonding of the composite plate and the beam prior to reaching the crushing load of the concrete. The debonding occurred suddenly and in a brittle manner. However, there was no major damage in the beam and it could still resist loads after debonding of the plate.
Beam (C) was cambered before bonding the plate. The beam failed as a result of sudden failure of concrete between the plate and longitudinal steel rebars. Measured load-deflection curve showed some disagreement to the calculated one, which was due to local debonding of steel and concrete.

Beam (D) was precracked prior to bonding the plate. It was also cambered in the same manner of beam (C). Due to precracking, the load-deflection curve did not exhibit any reduction of stiffness as a result of cracking of concrete. The failure in the beam was caused by shear failure of the concrete layer below the steel rebars.

Beam (E) was reinforced only by composite plate bonded to the tension flange. Plating could only slightly increase the load carrying capacity of the beam, and it failed due to opening of the large cracks before concrete can reach to its ultimate compressive strength.

Beam (F) had a T-section and the composite plate bonded to the tension flange without cambering. It failed by premature separation of the plate and the beam, due to poor workmanship in surface preparation and bonding.

The overall result of their study indicated that significant increase in the flexural strength is achieved by bonding GFRP plates to the tension face. The increase is more significant for beams with lower steel reinforcement ratios. Plating reduces the crack size in the beam, and at the same time ductility of the beam. Their experiments also showed that additional analytical and experimental work were required to predict failure of the concrete layer between the tensile steel rebars and composite plate.
An et al. (1991) developed analytical models based on equilibrium and compatibility of deformations in the strengthened section, to predict the stresses and deformations of the strengthened beam. Moment-curvature diagram was developed for the strengthened beam and was compared to unplated beam, as well as to experimental results.

Their model was capable of analyzing strengthened beams with both rectangular and T-shape cross sections. The main assumptions in developing their model were: linear strain distribution through the full depth of the section, small deformations, no tensile stress in the concrete, no shear deformations, and no slip between composite plate and concrete beam. Furthermore, they assumed that cross section of the plate is small enough to avoid shear failure in the concrete layer between the plate and steel rebars.

A computer program was developed and used through a parametric study. The effect of design variables such as steel reinforcement, plate cross sectional area, plate ultimate strength and stiffness, and compressive strength of concrete was studied. Moment-curvature diagram was generated for strengthened beams with both rectangular and T-shape cross sections. The failure of the beam was defined as either reaching the ultimate compressive strain in concrete, or the plate tensile strength.

Their parametric study revealed that this technique is particularly effective for concrete beams with relatively low steel reinforcement ratio. Unlike ordinary reinforced concrete beams, increase in the compressive strength of concrete could appreciably increase the ultimate moment of the section.
Ritchie et al. (1991) tested a series of 16 under-reinforced concrete beams strengthened with glass, carbon, or aramid reinforced plastic plates. The plates used in this study were as follows:

- A molded standard pultruded fiberglass sheet (four beams).
- 0/90° molded fiber reinforced plastic (one beam).
- Molded fiberglass channel, splitted in two angles (one beam).
- 0/90° 65 percent glass/35 percent carbon reinforced plastic (one beam).
- A spring-orientation glass fiber reinforced plastic (one beam).
- 0/±60° carbon fiber reinforced plastic (one beam).
- 0/90° carbon fiber reinforced plastic (one beam).
- Unidirectional aramid fiber reinforced plastic (one beam).
- Mild steel plate (two beams).

Two of the beams were tested unplated as control specimens. A two-part rubber-toughened epoxy was used for bonding the plates. The control beams were loaded to failure in one cycle, while strengthened beams were cycled up and down several times to determine permanent displacements. The loading was force-controlled as long as part of steel reinforcement remained elastic. For plastic range, and also for control beam and the beams strengthened with steel plates only deflection control was used.

In order to shift the mode of failure from local failure to flexural failure, as well as to increase the ultimate capacity of the strengthened beam, four types of modifications were applied.
The first modification was anchoring the end of the plate by using fiberglass angles with unequal legs. This method increased the ultimate load, but could not change the mode of failure.

The second method was bonding full-height FRP plates to side faces of the beam at the plate ends. They were connected to the longitudinal plate by using fiberglass angles. This technique led to higher load carrying capacity, and also shifted the mode of failure for one of the beams. While on the other two the failure occurred as a result of debonding of the plate followed by the local failure of the beam.

The third method was replacing the plate with a pair of angles bonded along the underside of the beam and extended above the longitudinal steel reinforcement. This method was also ineffective in shifting the mode of failure.

The last method was extending the plate right up to the supports. It was very successful in increasing the capacity of the beam, and also shifting the mode of failure. This method was more effective for the cases that the ratio of shear to moment was low.

The cracking pattern in most of the cases was shifted from several widely spaced and large cracks to many more closely spaced and narrower cracks. This is very advantageous from serviceability point of view. The strengthened beam did not demonstrate ductile behavior in the same manner of the unplated beams. However, the authors pointed out that ordinary heavily reinforced concrete beams would not be more ductile than strengthened beams. In all cases using FRP plates could appreciably increase stiffness and strength of the beam, that indicates the effectiveness of using the plates.
Uji (1992) performed experiments on a series of eight concrete beams 100\text{mm} \times 200\text{mm} in cross section and 3000\text{mm} long. Two of the beams were used as control beams without any plate, the remaining were strengthened with continuous fiber sheets wrapped around the beam, bonded to the sides, or passed across the cracks. The corners of the wrapped beams were rounded to avoid rupture of the plate.

The beams were tested in four point bending. Three of the strengthened specimens and one of the control specimens (containing stirrups) failed in flexural compression. However, the stirrups of the unplated beam yielded, while those of plated ones did not. This indicates that carbon fiber sheets have resisted a part of the applied shear force. For specimens without stirrups, the performance of the beam was highly improved by using composite sheets, however, this improvement was not as high as that resulted from using stirrups. In one of the wrapped beams the fibers ruptured at the corner of the beam, remarking the importance of rounding the corners with flatter curves before bonding the sheets. Debonding of the plate occurred for specimens strengthened with web-bonded plates. The resisted load was higher for 45° fiber orientation. Debonding began from diagonal shear cracks, and then propagated outward intersecting the edge of the sheet. The ratio of the transferred force to the debonded area was determined to be almost constant, giving the approximate shear strength of the bonded area. However, this conclusion has not been generalized to large beams.

In the specimens with both composite sheets and stirrups, the shear capacity of the beam was reported to depend on thickness and stiffness of the plate, as well as the length
of the debonded fibers.

A method was also developed based on compatibility of deformations between steel stirrups and debonded fibers to calculate the shear resisted by the composite sheet. For design purposes this method should be used iteratively, to find the debonding length that satisfies equilibrium equations, and to analyze the section consequently.

Triantafillou et al. (1992) bonded pretensioned CFRP sheets to the tension zone of concrete beams to study the behavior of the strengthened beam. They tested four plain concrete beams of 600 mm length. The specimens were cut out of a one-year-old concrete wall. They used drilled-out cylindrical specimens of the same wall to obtain the mechanical properties of the concrete. Torsion pendulum tests showed transition temperature of 60°C for the epoxy used for bonding the plates in their experiments.

The FRP ends were clamped by steel plates in order to minimize the stress concentrations at those points. The plates were bonded by using high-performance epoxy adhesive cured in a hydraulic press under constant pressure and temperature. The CFRP plates were pretensioned to the desired load before applying the adhesive on the surfaces of the concrete and the composite plate. Thereafter, the concrete beam was placed on the sheet. After curing the adhesive for three days, the pretensioning load was released gradually from one end, until the first visible cracks were appeared above the concrete-adhesive interface. In this way, the maximum achievable pretension level was computed as the difference between the load corresponding to first cracking, and the initial prestressing force. All of their beams failed by localized peeling-off of the plate due to
diagonal cracks, followed by slip of the plate. They also developed an analytical model to determine the maximum achievable prestressing force in the composite plate.

Dolan et al. (1993) investigated the effect of composite plates on the flexural and shear behavior of T-section concrete beams, when wrapped around the soffit of the beam.

A series of four T-beams were constructed and kevlar fibers were bonded to three of them. The surface of the beams were roughened by wire brushing, but sandblasting was reported as a better method to remove the loose surface materials. The composite wrap was clamped during the curing process of the adhesive.

Design equations developed by ACI for contribution of steel stirrups in shear capacity of ordinary reinforced concrete beams were modified to incorporate the effect of the plate. This modification was performed by replacing strength over spacing of the steel stirrups by tensile strength per unit length of the composite fabric. In this way the ultimate shear strength of the strengthened beam was approximated.

The unplated beam failed in flexural shear, while the strengthened beams could fully develop the flexural capacity of the beam showing enough shear capacity as a result of using the composite wraps. These beams showed higher strength and stiffness than the unplated ones, and the moment deflection curve was almost linear which was distinguished as a result of linear elastic behavior of the composite wraps. It was observed that 0° plies undergo larger strains and this is accompanied by local delamination of the fibers. Sharp corners of the beam soffit may lead to air pockets in the adhesive around these points and must be considered during the strengthening process.
Norris et al. (1994) tested a series of twenty one beams. strengthened with carbon fibers bonded by a two-part room temperature cured epoxy resin. The beams were of two different lengths of 1220 mm and 2440 mm. The shorter beams were used as shear specimens, and the longer ones as flexural specimens. Three different types of FRP systems, and four different types of fiber orientations were used.

The first system consisted of unidirectional continuous fiber sheets adhesively bonded to a fiberglass scrim and cross fibers. The bonding adhesive was used to maintain the geometry of the product, and not to surround the structural fibers as in a prepreg. The second system was a unidirectional stitched fabric with a two-part epoxy. The last system was a woven fabric.

FRP specimens were tested to determine the stiffness and strength of the plates. Test specimens were made in fiber orientations of 0°, 90°, and ±45° for unidirectional systems (A and B). Since the third system (C) was a balanced weave, three specimens in each of the fiber orientations of 0/90° and ±45° were made and tested. The elastic modulus in the longitudinal and transverse directions, as well as the shear modulus and poisson's ratio were outcome of these tests.

The composite plates were used to provide both flexural and shear strengthening for the concrete beams. Some of the beams were strengthened over the entire length, while for others web-strengthening was achieved only near the supports. Strain gages were assembled on different points and different orientations to allow better study of the strain variations along the beams. Some of the beams were precracked in order to simulate the
effect of cracks which are initiated due to loading of the beam prior to strengthening process.

They observed that the effect of the plates bonded on the web was not significant before yielding of the steel rebars. After that, the plate resisted against opening of the cracks and could appreciably affect the stiffness of the beam.

Different modes of failure were observed. Beams with longitudinal fibers bonded to the tension face failed due to local failure in the concrete layer located between steel rebars and the composite plate. Some of the beams with [0/90°] fiber orientation failed as a result of crushing of concrete or rupture of the plate. Beams with [45/-45°] fiber orientation, failed in a ductile manner with least improvement in bending. In all the studied cases the bonded composite had a significant effect on ultimate flexural and shear strength of the concrete beam.

Chajes et al. (1996) studied the bond strength and force transfer of graphite/epoxy composite plates adhered to concrete blocks. Single-lap shear test specimens were used. Composite plate of 25mm width was bonded to a concrete block with a 75mm bond length. The force was then applied to the top of the composite plate.

In the first set of their tests the effect of surface preparation, type of adhesive, and concrete strength on average bond strength was investigated. Based on their experiments, using a mechanically abraded or sandblasted concrete surface, applying a primer, and roughening of the composite plate surface before bonding was recommended. They also recommended a specific type of adhesive material that had a better performance.
In most of the cases the failure of the bond occurred as a result of shearing of the concrete directly beneath the bond surface. In this case the ultimate bond strength was observed to be proportional to $\sqrt{f_c}$ (where $f_c$ is the compressive strength of concrete).

The second set of their tests was carried out to investigate the effect of the bond length on the force transfer from the plate to concrete. It was concluded that there is a bond development for a joint beyond which no further increase in the failure load is achievable.

The importance of developing analytical models including the effects of concrete strength, adhesive layer thickness, and also mechanical properties of concrete was reminded.
CHAPTER 3

COMPOSITE MATERIALS CONSTITUENTS

3.1 Introduction

Progressive technology demands high-strength, light-weight and durable materials for construction applications. The newly developed composite materials were initially used in aircraft technology because of their high specific strength and stiffness. These materials have been used recently in civil engineering applications because of their superior mechanical properties as well as their resistance to aggressive environmental factors. These characteristics are what makes composites attractive as compared to conventional construction materials such as steel.

In general, a composite can be defined as a combination of two or more materials, essentially without any chemical interaction and insoluble into one another, such that some specific properties of the combination is better than that of the individual constituent. Fiber reinforced composites have a basic advantage in that they can be fabricated from a wide variety of reinforcement and matrix materials, so that a designer can choose constituents based on specific design considerations. The mechanical properties such as strength, stiffness, toughness and fatigue strength are generally high.

In the following sections, properties of typical fibers and resins will be discussed. This discussion is not exhaustive, but is limited to the types of fibers and resins that have potential for application in construction.
3.2 Fibers

The American Society for Testing Materials (ASTM). Committee D30, defines fibers as elongated materials with aspect ratio of at least 10:1, maximum cross-sectional area of $5 \times 10^{-2}$ mm$^2$, and maximum transverse dimension of 0.25 mm (Gill, 1972). Fibers, natural or synthetic, are the fundamental constituent of a fiber reinforced composite. They occupy the largest volume and are the major load carrying element of the composite. Proper selection of type, amount and orientation of fibers results in a composite with desired mechanical characteristics such as tensile and compressive strengths, elastic modulus, fatigue strength, and cost.

Fibers used in tensile elements in structural engineering must meet some basic requirements, such as: high strength; high stiffness, sufficient elongation at the tensile fracture; high toughness; durability; low cost; and availability in suitable forms. Furthermore, the diameter of the fibers must be small enough to provide a high specific area to develop the necessary bond for ensuring satisfactory transfer of shear stresses to the matrix. Smaller diameter fibers also reduce the possibility of surface flaws. Important parameters related to fibers that affect the engineering performance of a fiber reinforced composite are:

Length: Fibers are used in short and long forms. Short fibers are generally randomly distributed and oriented to result in almost homogeneous behavior. Long fibers are oriented in a specific direction to optimize the structural performance.

Cross sectional shape: In general, most fibers have circular cross sections. However,
hexagonal, rectangular, polygonal, annular (hollow circle) and irregular cross sections are used in some cases to improve certain mechanical properties. Circular fibers achieve a better interface behavior than the other types.

Type: The type and chemical composition of fibers affect several properties such as: durability, stress-strain relationship, toughness, and fatigue resistance.

The following sections describe basic characteristics of the most commonly used fibers in polymer composites.

3.2.1 Glass

Glass fibers are widely used for polymeric (plastic) matrix composites. Molten glass can be drawn into fine continuous filaments, which are bundled into yarns and rovings. These rovings can be fabricated into chopped fibers, continuous strands, chopped strand mats, woven fabrics and milled fibers, before using as reinforcement in composites. Fiber surfaces are coated during the manufacturing process to improve complete wetting by the resin, and provide better adhesion between matrix and fibers.

The strength reached with glass fiber is highly dependent on the form in which the fiber is used. Continuous fibers can reach the highest strength levels, whereas chopped fibers, despite easy manufacturing have the lowest specific strength. The average ultimate tensile strength of freshly drawn glass fibers may exceed 3500 MPa (500 ksi); however, surface flaws tend to reduce it to values in the range of 1750-2100 MPa (Mallik. 1993). Strength degradation is increased as the surface flaws grow under cyclic loads. This is one
of the major disadvantages of using these fibers in fatigue applications. Sustained loads also cause surface flaws to grow, resulting in reduced tensile strength (static fatigue). Figure 3.1 shows reduction of strength versus time for E-glass fibers under different temperatures (Mallik. 1993).

The behavior of the bond between glass fiber and plastic matrix highly affects the mechanical properties of the composite. The resistance of the composite against degradation upon aggressive environmental exposure also depends on the bond. Glass is a polar material, so when exposed to moisture, it is coated with a number of molecular layers of water that can adversely affect the bond. Coating with a coupling agent will provide a flexible layer at the interface. In this way, the strength of the bond is improved and the number of the voids in the material is reduced. The strong bond between glass fiber and the coupling agent prevents any undesired effect on the interface (Schwartz. 1992). Water presence also reduces the strength of fibers by deepening the surface flaws already present in the fibers. In spite of the aforementioned disadvantages, glass fibers are inexpensive to produce, they have a relatively high strength and good resistance to environmental factors when properly protected.

The internal structure of glass fiber is a three-dimensional network of different atoms. Therefore, glass fibers are amorphous and isotropic. The tensile modulus of these fibers is less than the other commonly used fibers such as carbon and Kevlar. They behave linearly elastic to failure. Glass filaments are typically round, having diameters ranging from 5 to 25 microns.
A variety of glass fibers have been produced for different applications. The most common are E-glass, S-glass, C-glass and D-glass. The following provides a brief description of properties of these glass fibers.

E-glass:

This type of glass was developed specifically for production of continuous fibers, for use in electrical applications. It has good insulation properties, and is the least expensive of all glass types. The low cost of this type of glass is the main reason for its wide application in fiber reinforced plastic (FRP) industry. It has been used in FRPs ranging from decorative to structural products.

S-glass:

The tensile strength and modulus of elasticity of this type of glass are about 25% and 20% greater than E-glass, respectively. Despite the higher tensile strength and modulus, the higher cost of S-glass makes it less popular than E-glass. Typical compositions of E-glass and S-glass are listed in Table 3.1 (Schwartz, 1992). Table 3.2 summarizes the physical and mechanical properties of E-glass and S-glass (Mallik, 1993).

C-glass:

This type of glass was developed for applications in corrosive environments, where chemical attack can preclude the use of E-glass. It is primarily used where fibers are
exposed to acidic environments.

Table 3.1 Composition of E-Glass and S-Glass Fibers

<table>
<thead>
<tr>
<th>Glass Type</th>
<th>S\text{O}_2</th>
<th>Al\text{2}O_3</th>
<th>CaO</th>
<th>MgO</th>
<th>B\text{2}O_3</th>
<th>Na\text{2}O_3</th>
<th>K\text{2}O</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>52-56</td>
<td>12-16</td>
<td>16-25</td>
<td>0-6</td>
<td>8-13</td>
<td>0-3</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>65</td>
<td>25</td>
<td>--</td>
<td>10</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2 Typical Properties of Glass Fibers

<table>
<thead>
<tr>
<th>Fiber Type</th>
<th>Virgin Tensile Strength (GPa)</th>
<th>Modulus of Elasticity (GPa)</th>
<th>Elongation at Failure (%)</th>
<th>Coefficient of Linear Thermal Expansion (10^{-6}/°C)</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-Glass</td>
<td>3.45</td>
<td>72.4</td>
<td>4.8</td>
<td>5</td>
<td>0.2</td>
</tr>
<tr>
<td>S-Glass</td>
<td>4.30</td>
<td>86.9</td>
<td>5</td>
<td>2.9</td>
<td>0.22</td>
</tr>
</tbody>
</table>

D-glass:

In applications where dielectric properties are important, D-glass is used. The lower density and dielectric constant of D-glass make it more attractive for certain applications.
3.2.2 Carbon

Although the terms "carbon" and "graphite" are used interchangeably for defining these types of fibers, there are some significant differences between these two. The difference between carbon and graphite comes from molecular structure. Carbon atoms are arranged in crystallographical parallel planes of regular hexagons to form graphite. In carbon, the bonding between layers is weak, so it has a two dimensional ordering.

Carbon and graphite fibers are produced by thermal decomposition of precursors such as rayon (cellulose), poly-acrylonitrile (PAN) and pitch (a by-product of petroleum distillation or coal coking). PAN type carbon has achieved market dominance due to its relatively low product cost and good physical properties. The manufacturing process for this type of fiber consists of several stages. These are oxidation (at 200-300°C), different stages of carbonization (at 1000-1500°C and 1500-2000°C), and finally graphitization (2500-3000°C). Materials with different properties result from different stages of this process. Graphite has a higher tensile modulus than carbon. Therefore, high-modulus fibers are produced by graphitization.

Carbon fibers are commercially available in three basic forms: long and continuous tow, chopped (6-50 mm long), and milled (30-300 μm long). The long and continuous tow, which is a bundle of 1000 to 160,000 parallel filaments, is used for high performance applications (Mallik, 1993), (Schwartz, 1992). The price of carbon fiber tow decreases with increasing filament count. Although high filament counts are desirable for improving productivity in continuous molding operations, it becomes increasingly difficult to wet
them with the matrix. Carbon fiber tows can also be woven into two dimensional fabrics of various styles.

For applications in high strength composites, graphite fibers are mainly considered. Graphite fibers show very high specific strength and stiffness. Generally as the modulus of elasticity increases, ultimate strength and elongation decrease. Therefore, high modulus graphite fiber exhibits a lower strain to failure than high strength carbon. The tensile strength and modulus of graphite fibers do not vary as temperature rises. They behave elastically to failure, and fail in a brittle manner. Carbon and graphite fibers are highly resistant to aggressive environmental factors. Their diameter falls in the range of 5 to 10 \( \mu m \). Table 3.3 shows some physical and mechanical properties for carbon and graphite fibers (Schwartz, 1992).

The most important disadvantage of carbon and graphite fibers is their high cost, which is not comparable to glass fibers. They are 20 to 50 times (by weight) more expensive than E-glass (Riewald, 1988), (Schwartz, 1992). The high price of raw materials, or precursors, and the long process of carbonization and graphitization is what contributes to their cost. These fibers (specifically graphite fibers) cannot be wetted by the matrix easily, so sizing is necessary before being used in the matrix. The impact resistance of these fibers is low.

Graphite and carbon fibers have been used more widely in aerospace and military applications. However, less expensive pitch-base fibers with improved properties are being developed for structural engineering applications. Some effort has also been concentrated
on using different types of fibers together to form hybrid composites.

Table 3.3 Typical Properties of Carbon, Graphite Fibers

<table>
<thead>
<tr>
<th>Fiber Type</th>
<th>Density kg/m$^3$</th>
<th>Tensile Strength MPa</th>
<th>Modulus of Elasticity GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon: Pan based</td>
<td>1700</td>
<td>827</td>
<td>41</td>
</tr>
<tr>
<td>Pitch based</td>
<td>2000</td>
<td>2413-2482</td>
<td>221-234</td>
</tr>
<tr>
<td>Graphite: Ultrahigh Modulus</td>
<td>2160</td>
<td>2240-2413</td>
<td>683-854</td>
</tr>
<tr>
<td>High Modulus</td>
<td>2000</td>
<td>1723-2413</td>
<td>342-546</td>
</tr>
<tr>
<td>Ultrahigh Strength</td>
<td>1800</td>
<td>4826-5516</td>
<td>236-287</td>
</tr>
<tr>
<td>High Tensile</td>
<td>1777</td>
<td>2758-4137</td>
<td>226-294</td>
</tr>
<tr>
<td>Strength/Intermediate Modulus</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.2.3 Aramid Fibers

Aramid stands for aromatic polyamide, and is a generic term for a group of fibers having the lowest specific gravity and highest specific tensile strength among the current reinforcing fibers (Mallik, 1993). The first aramid fiber was introduced commercially around 1965, and possessed outstanding heat resistance. The first organic fiber with advantageous mechanical properties, used as a reinforcing fiber, was introduced in 1970 under the trademark "Kevlar aramid" (De Wilde, 1988). After that time, due to advantages such as high tensile strength and modulus, and high impact damage resistance, Kevlar aramid (or Kevlar) fibers have been used extensively for engineering applications.

Kevlar fibers are produced by shearing a liquid crystalline solution of the polymer with partially oriented molecules. In the resulting fibers, rigid molecules are well aligned
along the fiber axis with few entanglements, resulting in high strength and high modulus (De Wilde, 1988).

There are several types of Kevlar fibers as follows:

Kevlar 29: Designed for ballistics, cut and slash resistant protective apparel, ropes and cables, coated fabrics, asbestos replacement, and for composites with maximum impact and damage tolerance.

Kevlar 49: Used in reinforced plastics. Its important properties will be described next.

Kevlar 129 (or HT): Used in ballistic application due to its higher strength and toughness.

Kevlar 149: Used in airplane, helicopter, and sporting goods applications. This type of Kevlar has the highest tensile modulus among all commercially available aramid fibers.

There are other types of Kevlar fibers such as Kevlar 69 and Kevlar 100, which are not commonly used in fiber reinforced plastics. Kevlar fibers have high specific strength and stiffness. Figure 3.2 shows the comparison of Kevlar fibers with other fibers and materials on a specific tensile strength specific-tensile modulus plot (De Wilde, 1988). (Schwartz, 1992). Kevlar values have been determined from resin impregnated strands (ASTM D2343). Specific tensile strength/modulus means tensile strength/modulus divided by density.

The compressive strength of Kevlar fibers is less than 20% of the tensile strength. Figure 3.3 shows behavior of epoxy reinforced with unidirectional Kevlar 49 fibers under tensile and compressive loading (De Wilde, 1988).

It can be seen that Kevlar 49 has brittle behavior under tension. Under
compression, it is ductile, metal like with yielding beginning at 0.3-0.5% strain and absorbing considerable energy. Also, it shows a high degree of yielding on the compression side when subjected to bending. This type of behavior, which is not observed in carbon or glass fibers, gives Kevlar composites better impact resistance (Mallik, 1993).

Kevlar fibers have an excellent tension-tension fatigue resistance and a low creep. Kevlar 149 has the lowest creep among all Kevlar fibers. Table 3.4 shows some typical mechanical properties of Kevlar aramid fibers (De Wilde, 1988). Fiber properties have been determined from untwisted epoxy impregnated strand (ASTM-D2343).

Kevlar fibers can withstand high temperatures. Kevlar 49 fiber, used most commonly for fiber reinforced plastics, does not melt or support combustion, but starts to carbonize at about 430°C (Mallik, 1993). The strength and modulus of Kevlar fibers decrease linearly when the temperature rises, but they retain more than 80% of their original strength at 180°C.

Table 3.4 Typical Mechanical Properties of Kevlar Fibers

<table>
<thead>
<tr>
<th>Fiber</th>
<th>Tensile Strength (MPa)</th>
<th>Modulus (GPa)</th>
<th>Elongation at Failure (%)</th>
<th>Density (kg/m^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kevlar and Kevlar 29</td>
<td>3620</td>
<td>82.7</td>
<td>4.4</td>
<td>1440</td>
</tr>
<tr>
<td>Kevlar 49</td>
<td>3620</td>
<td>124-131</td>
<td>2.9</td>
<td>1440</td>
</tr>
<tr>
<td>Kevlar 149</td>
<td>3450</td>
<td>172-179</td>
<td>1.9</td>
<td>1470</td>
</tr>
<tr>
<td>Kevlar 129</td>
<td>4210 (est.)</td>
<td>110 (est.)</td>
<td>--</td>
<td>1440</td>
</tr>
</tbody>
</table>
Kevlar fibers absorb some water. The amount of absorbed water depends on the type of the fiber. Kevlar 149 has an equilibrium moisture content of 1.2% at 65% humidity, which is the lowest rate among Kevlar fibers. Under similar conditions, equilibrium moisture content of Kevlar 49 is as high as 4%. At high moisture content, Kevlar fibers tend to crack internally at the preexisting microvoids and produce longitudinal splitting (De Wilde. 1988). (Mallik. 1993).

From a chemical point of view, Kevlar fibers are resistant to most solvents and chemicals, but they can be degraded by a few strong acids and alkalies. The chemical resistant properties decrease from Kevlar 149 to Kevlar 49 and from the latter to Kevlar 29. Ultraviolet radiation can also degrade Kevlar. The problem is less serious when they are used in fiber reinforced plastics where the matrix protects the fibers. The degree of degradation naturally depends on the thickness of the material.

3.3 Matrix

Matrix in a composite material can be regarded as a primary or secondary element. As the primary element, it is a structural material that is reinforced by fibers. As the secondary element, its major roles are transferring stresses between the fibers, protecting fibers against environmental attacks, and protecting the surface of the fibers from mechanical abrasion. The latter definition is the traditional one, especially for soft matrices. In such cases, the strength of the composite is almost entirely due to the fibers, and the fiber content should be chosen as high as possible.
The importance of the matrix in a composite is its influence on interlaminar and in-plane shear strengths. It also provides support against the buckling of the fibers under compression loads. If there are broken fibers in the composite, the load is transferred from broken fibers to the unbroken ones through the matrix. Therefore, the matrix design is an important issue in composite design. Physical and thermal characteristics of the matrix affect the processability and mechanical properties of the composite material. Besides acceptable mechanical properties, thermal stability and chemical/environmental resistance are important parameters in choosing a matrix. Usually the matrix weight in the composite is chosen as the minimum required for adequate shear strength and low void content.

Matrix materials, in the most general sense, can be classified as: polymers, metallic and ceramic types. Among these, polymeric matrices have the greatest commercial use due to ease of processing. Metallic and ceramic matrices are primarily considered for high temperature applications. Only polymeric matrices will be discussed in this paper because they have the highest potential applications in the construction industry.

3.3.1 Polymeric Matrices

Polymeric matrices are divided into two categories of thermoplastics and thermosets. In thermoplastic polymers, individual molecules are in a linear structural form and are held in place by weak secondary bonds. Applying heat or pressure to a solid thermoplastic polymer temporarily breaks these bonds, resulting in relative movement between molecules. Upon cooling, the molecules freeze in their new position resulting in
a new solid shape. Therefore, a thermoplastic polymer can be heated, softened, melted and reshaped for as many times as desired.

In a thermosetting polymer, also called resin, the molecules are chemically joined together by cross links, forming a three-dimensional rigid structure. These links are formed during the polymerization (curing) process, and as a result of the presence of these links, the thermosets cannot be reshaped by applying heat or pressure. Figure 3.4 shows the schematics of thermoplastics and thermosets (Mallik, 1993). These two types of polymeric matrices are briefly described here.

3.3.1.1 Thermoplastic Polymers

Thermoplastic matrices are amorphous and sometimes partially crystalline. There is a glass-rubber transition temperature ($T_g$) for these materials, at which the mechanical properties (such as modulus of elasticity) drop by several orders of magnitude. Thermoplastics have higher impact strength, fracture resistance and microcracking resistance as compared to thermosets. They need shorter fabrication time and provide post-formability. They can be repaired by welding and they are recyclable.

Thermoplastic matrices have some disadvantages if compared to thermosets. Incorporation of continuous fibers to thermoplastic matrices is difficult due to high melt or solution viscosities. They exhibit lower creep resistance and thermal stability. These properties have limited their application in fiber reinforced composites. They are mostly used in automotive industry, appliances and business machine fields. New types of
thermoplastics are under survey for further applications in advanced composites. Often, short glass fibers are added to these materials for reinforcing and providing an easy way of processing. Fillers may also be used to reduce the cost or to make special properties.

3.3.1.2 Thermosetting Polymers

Thermosetting polymers have been used as matrix materials more often. These matrices provide good wet-out between fibers and matrix, without applying high pressure or temperature. Other advantages are thermal stability and chemical resistance, low creep and stress relaxation as compared to thermoplastics. The main disadvantages of thermosetting polymers are the limited storage life before molding at room temperature, long required fabrication time, and low strain-to-failure, which results in low impact resistance. The most common thermosetting matrices used in advanced composites are: epoxy, polyester and vinyl ester, which are discussed here.

Epoxy: Epoxy resins are made of low-molecular weight organic liquid resins. These resins contain a number of epoxide groups. Each epoxide group contains two carbon atoms and one oxygen atom. Other ingredients are mixed to reduce its viscosity and improve the impact resistance of the cured epoxy. Just prior to adding fibers, small amounts of reactive curing agents are added to liquid resin to initiate polymerization (curing). During that process, cross links are formed and epoxy changes to a solid material (Mallik, 1993).

Epoxy resins have been used largely in high-performance composites. This is due
to several advantages such as: ease of processing, excellent mechanical properties, excellent resistance to chemicals and solvents, low shrinkage during cure, and good adhesion to a wide variety of fibers and fillers. Furthermore, they can be designed to have a wide variety of properties, since a large number of possible starting materials, curing agents and modifiers are available to make epoxies. Typical properties of cast epoxy resin are given in Table 3.5 (Mallik, 1993).

Table 3.5 Typical Properties of Cast Thermosetting Resins

<table>
<thead>
<tr>
<th>Resin</th>
<th>Specific Gravity</th>
<th>Tensile Strength (MPa)</th>
<th>Tensile Modulus (GPa)</th>
<th>Cure Shrinkage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epoxy</td>
<td>1.2-1.3</td>
<td>55-130</td>
<td>2.75-4.10</td>
<td>1-5</td>
</tr>
<tr>
<td>Polyester</td>
<td>1.1-1.4</td>
<td>34.5-103.5</td>
<td>2.1-3.45</td>
<td>5-12</td>
</tr>
<tr>
<td>Vinyl Ester</td>
<td>1.12-1.32</td>
<td>73-81</td>
<td>3-3.5</td>
<td>5.4-10.3</td>
</tr>
</tbody>
</table>

The main disadvantage of epoxy resins are their relatively high cost and long curing period. They must be processed carefully to obtain moisture resistance. The cross links which are formed during polymerization (curing) process and change the initial liquid resin to a solid epoxy resin play a major role in changing the properties of the solid epoxy. Tensile modulus, glass transition temperature ($T_g$), thermal stability and chemical resistance are improved as the density of these cross links increases. On the other hand, strain-to-failure and fracture toughness are reduced. The cross-link density is a function of the chemical structure of the starting liquid resin, curing agent and reaction conditions such as time and temperature.
Polyester: The starting material for a polyester matrix is unsaturated polyester resin. Unlike epoxy resins, this starting material contains only carbon atoms (without any oxygen atoms). Other chemical agents are added to modify the chemical structures between cross-links to reduce its viscosity and to prevent premature polymerization during storage. Small amounts of catalyst are required for initiating the curing reaction. Curing temperature for polyester is higher than epoxy and it exhibits more shrinkage during cure.

Polyester resins are manufactured with a wide variety of properties, from hard and brittle to soft and flexible. Their advantages are low viscosity, fast cure time and low cost. As can be seen in Table 3.5, their properties are generally lower than epoxies. The most important disadvantage of polyester resins is high volumetric shrinkage. By adding a thermoplastic component this volumetric shrinkage can be reduced. The interface bond between epoxy and fiber is better than that of polyester and fiber.

Cross-link density can change the properties of polyester resins in the same manner as explained for epoxy resins. Cross-link density depends on the weight ratio of different ingredients used for making unsaturated polyesters.

Vinyl Ester: In this type of resin, the starting materials are produced by the reaction of an unsaturated carboxylic acid and an epoxy resin. Because, due to the chemical structure of these materials, there are fewer cross links, they are more flexible and have a higher fracture toughness than polyester resins. Furthermore, they exhibit excellent wet-out and
good adhesion when used with glass fibers. Vinyl ester exhibits good characteristics of epoxy resins such as chemical resistance and tensile strength, as well as those of unsaturated polyester resins such as viscosity and fast curing, but the volumetric shrinkage is higher than epoxy. It exhibits only moderate adhesive strength compared to epoxy resins. Typical properties of cast vinyl ester resin are also given in Table 3.5.
CHAPTER 4

STRESS CONCENTRATIONS AT THE PLATE END

4.1 Introduction

In flexural strengthening of reinforced concrete beams with epoxy-bonded FRP plates, local failure in the concrete layer between the steel reinforcement and the composite plate has been observed in experiments. This type of failure prevents the strengthened beam from reaching its ultimate flexural capacity, and therefore it must be included in design considerations. This failure mode is unique to plated beams and is caused by shear and normal stress concentrations at the plate end and at the flexural cracks present along the beam. Closed form solutions of stress concentrations are developed in this chapter. Using these equations, design guidelines for strengthening reinforced concrete beams with FRP plates are developed in the next chapter.

4.2 Analytical Models

In this section, analytical models are developed for predicting the shear and normal stresses at the concrete/FRP interface. The following assumptions are made: linear elastic and isotropic behavior for FRP, epoxy, concrete and steel reinforcement; complete composite action between plate and concrete (no slip); and linear strain distribution through the full depth of the section. The above assumptions do not oversimplify the behavior of this system since the plate cut off point is usually taken near the inflection or points of zero
moments where the normal stresses are generally low and justify the assumptions of linear elastic for the materials.

4.2.1 Shear Stress

The interfacial shear stress between FRP plate and epoxy can be calculated by considering the equilibrium of an infinitesimal part of the FRP plate, as shown in Fig. 4.1. In this figure, \( \tau(x) \) and \( f_n(x) \) are shear and normal stresses, respectively. The shear stress can be defined by:

\[
\tau(x) = f_n(x) \frac{df_n(x)}{dx} t_p
\]  

(4.1)

where: \( f_p(x) \) = tensile stress in FRP plate; and \( t_p \) = thickness of plate.

Assuming linear elastic behavior, Eq. (4.1) can be rewritten as:

\[
\frac{df_p(x)}{dx} = \frac{G_a}{t_p} (\frac{du}{dx} \cdot \frac{dv}{dx})
\]  

(4.2)

where: \( u \) and \( v \) = horizontal and vertical displacements in the adhesive layer, respectively; \( G_a \) = shear modulus of elasticity of the adhesive layer; \( x \) and \( y \) = measured along the longitudinal axis and perpendicular to the longitudinal axis of FRP plate, respectively.

Differentiating Eq. (4.2) with respect to \( x \), results in:

\[
\frac{d^2f_p(x)}{dx^2} = \frac{G_a}{t_p} \left( \frac{d^2u}{dx^2} \cdot \frac{d^2v}{dx^2} \right)
\]  

(4.3)

The relationship between the bending moment, \( M \), and the flexural deflection is given by:
where: $E_c$ = elastic modulus of concrete in tension; and $I_r$ = moment of inertia of transformed section based on concrete. Furthermore, $du/dxdy$ can be expressed as:

$$\frac{d^2 u}{dx \ dy} = \frac{1}{t_a} (\epsilon_p - \epsilon_c) \quad (4.5)$$

where: $\epsilon_p$ and $\epsilon_c$ = interfacial strains in the lower and upper faces of the epoxy layer; and $t_a$ = thickness of epoxy layer. Therefore, Eq. (4.2) can be written as:

$$\frac{d^2 f_p(x)}{dx^2} = \frac{G_s}{t_p} (\epsilon_p - \epsilon_c) \quad (4.6)$$

The magnitude of the third term in the parenthesis is relatively small as compared to the other terms and therefore it can be neglected. Eq. (4.6) is then reduced to:

$$\frac{d^2 f_p(x)}{dx^2} = \frac{G_s}{t_s t_p} (\epsilon_p - \epsilon_c) \quad (4.7)$$

where: $\epsilon_p = f_p(x)/E_p$ and $\epsilon_c = f_c(x)/E_c$, assuming uncracked section and using the corresponding stress-strain relationships for concrete and FRP plate: $E_p$ = elastic modulus of plate; $f_c(x)$ = tensile stress in the bottom of the concrete beam. The governing differential equation for the tensile stress in the plate can be expressed as:

$$\frac{d^2 f_p(x)}{dx^2} = \frac{G_s}{t_s t_p} \frac{f_p(x)}{E_p} \frac{f_c(x)}{E_c} \quad (4.8)$$
The solution of the above equation is given by:

\[ f_p(x) = C_1 \sinh(\sqrt{A} \ x) \cdot C_2 \cosh(\sqrt{A} \ x) \cdot b_1 \ x^2 \cdot b_2 \ x \cdot b_3 \]  

(4.9)

where: 

\[ A = \frac{G_a}{t_p \frac{E_p}{ya \frac{E_p}{E_c}}} \]

and: 

\[ b_1 = \frac{\bar{y}E_p}{I_p E_c} \]

\[ b_2 = \frac{\bar{y}E_p}{I_p E_c} (2 \ a_1 L_o \ a_2) \]

\[ b_3 = E_p \left[ \frac{\bar{y}}{I_p E_c} (a_1 L_o^2 \ a_2 L_o \ a_3) \right. \]

In developing the above solution, the origin of \( x \) has been assumed at the cut-off point of the plate. Furthermore, bending moment can be expressed by:

\[ M(x_o) = a_1 \ x_o^2 \cdot a_2 \ x_o \cdot a_3 \]

(4.10)

where the origin of \( x_o \) is arbitrary, and can be assumed at any convenient point at a distance \( L_o \) from the cut-off point. In other words, \( x_o = x + L_o \cdot \bar{y} = \) distance from neutral axis of the strengthened section to center of FRP plate; and \( C_1, C_2 \) = integration constants.

Substituting the expression for \( f_p(x) \) given by Eq. (4.9) into Eq. (4.1), results in:

\[ \tau(x) = t_p \left[ C_1 \sqrt{A} \cosh(\sqrt{A} \ x) \cdot C_2 \sqrt{A} \sinh(\sqrt{A} \ x) \cdot 2b_1 x \cdot b_2 \right] \]

(4.11)

Constants of integration \( C_1 \) and \( C_2 \) are evaluated using the following boundary conditions:
the first boundary condition is evaluated at \( x = 0 \) where the plate ends. At this point \( f_a(x) = 0 \). The second boundary condition is evaluated at the point where shear force in the beam is zero, i.e.:

\[
\tau(L_e) = 0 \quad \text{or} \quad \frac{df_a(x)}{dx} \bigg|_{x=L_e} = 0
\]

where: \( L_e = \) distance to the point of zero shear force measured from the plate end.

Using the above boundary conditions the following expressions for \( C_1 \) and \( C_2 \) can be obtained:

\[
C_1 = \frac{b_1 \sqrt{A} \sinh(\sqrt{A} L_e) - 2 b_1 L_e \cdot b_2 \cosh(\sqrt{A} L_e)}{b_1}
\]
\[
C_2 = b_1
\]

A parametric study of variables in Eq. (4.12) revealed that generally, \( \sinh(\sqrt{A} L_e) \) and \( \cosh(\sqrt{A} L_e) \) are equal and have very large values compared to the other terms in the numerator. Therefore, \( C_1 \) can be simplified to:

\[
C_1 = b_1
\]

Using \( C_1 \) and \( C_2 \) in Eq. (4.11), the shear stress is expressed by:

\[
\tau(x) = \tau_p \left[ b_1 \sqrt{A} \cosh(\sqrt{A} x) - b_1 \sqrt{A} \sinh(\sqrt{A} x) - 2 b_1 x \cdot b_2 \right]
\]

The maximum shear stress occurs at the cut-off point (\( x = 0 \)):

\[
\tau_{\text{max}} = \tau_p (b_1 \sqrt{A} \cdot b_2)
\]
4.2.2 Normal (Peeling) Stress:

Considering concrete beam and FRP plate as two isolated beams ("concrete beam" and "plate beam") connected together by the adhesive layer as shown in Fig. 4.2, the fourth order differential equation for each beam can be expressed as:

\[- K \frac{d^4v}{dx^4} = E_i I \frac{d^4v}{dx^4} - q \cdot b_p f_n(x)\]  

\[- E_p I_p \frac{d^4v}{dx^4} = b_p f_n(x)\]  

where: \(v_p\) and \(v_c\) = deflection of FRP plate and concrete beam, respectively; \(I_p, I_c\) = moments of inertia of plate and concrete beam; \(b_p\) = width of FRP plate; \(q\) = distributed load on the concrete beam; and \(f_n(x)\) = normal stress in the epoxy layer. Considering deformation in the epoxy layer, \(f_n(x)\) can be expressed as:

\[f_n(x) = K_n (v_p - v_c)\]  

where: \(K_n = E_a/t_a; E_a\) = modulus of elasticity of adhesive; and \(t_a\) = thickness of adhesive.

Differentiating Eq. (4.17) four times results in:

\[\frac{d^4 f_n(x)}{dx^4} = K_n \left( \frac{d^4 v_p}{dx^4} \cdot \frac{d^4 v_c}{dx^4} \right)\]

Solving Eqs. (4.15) and (4.16) for \(d^4v_p/dx^4\) and \(d^4v_c/dx^4\) and substituting the corresponding
values in Eq. (4.18) gives the governing differential equation of the normal stress:

\[
\frac{d^4 f(x)}{dx^4} = \frac{K_n}{E_J} \cdot \frac{b_p}{E_p} \cdot f(x) \cdot \frac{K_n}{E_I} \cdot q
\]  

(4.19)

The solution of this fourth order linear differential equation is the summation of the homogeneous and particular solutions as given below:

\[
f(x) = e^{\beta x} \left[ D_1 \cos (\beta x) - D_2 \sin (\beta x) \right]
\]  

\[
\cdot e^{\beta x} \left[ D_3 \cos (\beta x) - D_4 \sin (\beta x) \right] \cdot \frac{qE_J}{b_pE_I}
\]

(4.20)

where: \( \beta = (K_n b_p/4E_p J_p)^{0.25} \); and \( D_1 \) to \( D_4 \) = constants of integration. The term \( b_p/E_I \) is relatively small compared to \( b_p/E_J \) and has been neglected in Eq. (4.19). For large values of \( x \), i.e., for the points far from the cut off point, the normal stress and its derivatives approach zero. Since \( \beta \) is a positive number, the coefficient of \( e^{\beta x} \) must be zero to satisfy the above condition, that is, \( D_3 = D_4 = 0 \). Eq. (4.20) is reduced to:

\[
f(x) = e^{\beta x} \left[ D_1 \cos (\beta x) - D_2 \sin (\beta x) \right] \cdot \frac{qE_J}{b_pE_I}
\]

(4.21)

Constants of integration \( D_1 \) and \( D_2 \) are calculated using the appropriate force boundary conditions at the plate cut-off point. Differentiating Eq. (4.17) results in:

\[
\frac{d^2 f(x)}{dx^2} = K_s \left( \frac{d^2 \gamma_L}{dx^2} - \frac{d^2 \gamma_L}{dx^2} \right)
\]

(4.22)

Considering the isolated "concrete beam" and the "plate beam" and using the moment-curvature relationships for these beams, Eq. (4.22) can be rewritten as:
where: $M_p(x)$ and $M_c(x)$ = bending moments in the "plate beam" and "concrete beam." respectively. Differentiating Eq. (4.22) once more, and substituting third derivatives of displacements by the corresponding shear forces results in:

$$\frac{d^3 f_0(x)}{dx^3} = \frac{K_n}{E_p I_p} M_p(x) - \frac{K_n}{E_c I_c} M_c(x)$$  \hspace{1cm} (4.23)

where: $V_p(x)$ and $V_c(x)$ = shear forces in the plate and concrete beams, respectively. The effect of the interfacial shear stress must be considered in defining the bending moment and shear forces in the isolated beams. Shear stress given by Eq. (4.13) multiplied by the width of the plate can be assumed as a distributed load per unit length (shear flow) along the interface of each of the beams with the adhesive layer, as shown in Fig. 4.3(a). The static equivalent of these distributed loads at the centroid of the beams are distributed loads plus distributed moments as shown in Fig. 4.3(b). Therefore, the equation of bending moment in concrete and plate beams due to this distributed loads or shear flow at the interface can be written as:

$$M'_c(x) = - \frac{l_p}{2} \left[ b_1 \sinh(\sqrt{A}x) \cdot b_4 \cosh(\sqrt{A}x) \cdot b_2 x^2 \cdot b_6 \right]$$  \hspace{1cm} (4.25)

$$M'_p(x) = - \frac{l_p}{2} \left[ b_1 \sinh(\sqrt{A}x) \cdot b_4 \cosh(\sqrt{A}x) \cdot b_2 x^2 \cdot b_6 \right]$$  \hspace{1cm} (4.26)
where: $M_p(x)$ and $M_c(x) = \text{bending moments due to shear flow at the interface of concrete and plate beams, respectively.}$ At the end of the plate where $x = 0$: both of the above moments are zero. Therefore, the bending moment in each of the beams is only due to the externally applied loads, and is expressed by:

$$
\frac{M_c - M_p}{M_p} = 0
$$

(4.27)

where: $M_p = \text{bending moment in the concrete beam at the plate-end due to externally applied load.}$ In the above expressions, it is assumed that the external load is applied to the concrete beam only.

Differentiating Eqs. (4.25) and (4.26), and substituting $x = 0$, results in the shear forces in the isolated beams at the plate end:

$$
V_c' = b_p \bar{y}_c \ell_p (b_1 \sqrt{A} - b_2)
$$

(4.28)

$$
V_p' = b_p \ell_p \frac{1}{2} (b_1 \sqrt{A} - b_2)
$$

(4.29)

where $V_c'$ and $V_p'$ = \text{shear forces at the plate end, in the concrete and plate beams due to interfacial shear stresses, respectively.}$ The total shear force in the concrete and plate beams are calculated as:
where $V_\alpha = \text{shear force in the concrete beam at the plate end due to externally applied loads.}$

Here, again it is assumed that the concrete beam alone takes the full shear due to the externally applied loads. Inserting the corresponding values given by Eqs. (4.27) and (4.30) into the right side of Eqs. (4.23) and (4.24), $D_1$ and $D_2$ are obtained:

$$D_1 = \frac{K_n}{E_p I_p} \cdot \frac{V_\alpha}{2\beta^3} \cdot \frac{K_n}{E_c I_c} \cdot \frac{V_c \cdot \beta M_\alpha}{2\beta^3}$$

$$D_2 = \frac{K_n}{E_c I_c} \cdot \frac{M_\alpha}{2\beta^3}$$

Considering the fact that $e^{\beta x}$ approaches zero for large values of $x$, the maximum normal stress occurs at the cut-off point and is expressed by:

$$f_{\text{um}} = \frac{K_n}{2\beta^3} \left( \frac{V_\alpha}{E_p I_p} - \frac{V_c \cdot \beta M_\alpha}{E_c I_c} \right) \cdot \frac{q E_p I_p}{b_p E_c I_c}$$

Eqs. (4.14) and (4.33) express the maximum shear and normal interfacial stresses, respectively, and can provide the necessary tools for designing strengthened beam against local failure. The parameters in these equations can be simply calculated based on mechanics of materials.
4.2.3 Effect of Shear Stress Concentration on Flexural Stresses:

In order to highlight the effect of interfacial shear stresses at the concrete/plate interface on the flexural stresses in the concrete beam, the "plate beam" and the "concrete beam" are considered without externally applied loads but with self equilibrating interfacial shear stresses as shown in Fig. 4.4.

Isolating elements ABCD from the "concrete beam" and element A'B'C'D' from the "plate beam", as shown in Fig. 4.5(a), one can see that for equilibrium of these elements, generally the existence of internal forces shown on Fig. 4.5(b) is necessary.

Writing the equilibrium equation for elements in Fig. 4.5(b) results in:

\[
\frac{dM_I}{dx} = (V_c \cdot \tau \cdot b_p \cdot \bar{y}) \tag{4.34}
\]

\[
\frac{dM_p}{dx} = (V_p \cdot \tau \cdot b_p \cdot \frac{l_p}{2}) \tag{4.35}
\]

The strengthened beam, where the plate is attached to the concrete is shown in Fig. 4.6. Considering the equilibrium of this beam, shows that when no external load is applied to the beam the internal forces acting on any element of the beam will be zero. In other words, the superposition of the internal forces of the concrete beam and the plate beam should add up to zero at any location along the beam, that is:

\[
M_p \cdot M_c = 0 \tag{4.36}
\]
Differentiating Eq. (4.36) results in:

\[ \frac{dM_y}{dx} \cdot \frac{dM_z}{dx} = 0 \]  \hspace{1cm} (4.38)

Considering Eqs. (4.34) and (4.35), one may write:

\[ \frac{dM_y}{dx} \cdot \frac{dM_z}{dx} \cdot (V_y - V_z) = - \tau \cdot b_p \cdot (\tilde{y}_c - \frac{l_p}{2}) \]  \hspace{1cm} (4.39)

According to Eqs. (4.37) and (4.38), left-hand side of the above equation is zero, so must be the right-hand side. This can only be true if \( \tau \) is zero. This is a trivial solution of the above equation, and is not of concern in this analysis. The other approach to look at this term is as an error term which must be eliminated. This elimination is performed analytically by imposing a shear force at the cut-off point in the opposite direction. However, this free body diagram will not be in equilibrium unless we have a set of forces as shown in Fig. 4.7. This requires a moment at the cross-section expressed by:

\[ M_m \cdot L_o \cdot \tau \cdot b_p \cdot (\tilde{y}_c - \frac{l_p}{2}) \]  \hspace{1cm} (4.40)

In Eq. (4.40), the value of \( \tau \) is obtained from Eq. (4.14), and \( l_p/2 \) can be neglected since its value is small compared to \( \tilde{y}_c \). Therefore:

\[ M_m \cdot L_o \cdot \tau \cdot b_p \cdot \tilde{y}_c \cdot (b_1 \sqrt{A} \cdot b_2) \]  \hspace{1cm} (4.41)
This moment is characteristic of the cut-off point in plated beam due to high shear stresses at this location. The magnitude of this moment rapidly decreases as the distance from the cut-off point increases. This moment is added to the moment from externally applied loads for the flexural design of the section at the cut-off point.

4.2.4 Effect of Flexural Cracks

Cracks play a significant role in the redistribution of the shear stresses. The same procedure to calculate shear stresses can be followed when cracks are present along the beam as shown in Fig. 4.8. Using Eq. (4.11) between two successive cracks, and assuming axial stresses in the plate at crack locations as known boundary conditions, constants $C_1$ and $C_2$ can be calculated:

$$C_1 = \frac{C_1 \left( (b_1 x_2^3 - b_2 x_3) \cdot f_1 \right) - C_2 \left( (b_1 x_1^3 - b_2 x_2) \cdot f_1 \right)}{S_1 C_2 - S_2 C_1}$$  \hspace{1cm} (4.42)

$$C_2 = \frac{f_2 \cdot C_1 \frac{\bar{S}_2 - b_1 x_2^3}{\bar{C}_2} \cdot b_2 x_3 \cdot b_3}{\bar{C}_2}$$  \hspace{1cm} (4.43)

where: $x_1$ and $x_2$ = coordinates of two successive cracks; $f_1$ and $f_2$ = longitudinal stress of plate at the location of the cracks; $\bar{S}_1 = \sinh (\sqrt{\lambda} x_1)$; $\bar{S}_2 = \sinh (\sqrt{\lambda} x_2)$; $\bar{C}_1 = \cosh (\sqrt{\lambda} x_1)$; and $\bar{C}_2 = \cosh (\sqrt{\lambda} x_2)$.

Defining the origin of $x$ at the first crack, Eq. (4.42) is reduced to:
where; \( l \) = distance between cracks. Generally, \( \overline{C}_2 \) and \( \overline{S}_2 \) are equal and have large values, so the final expression for the shear stress at the crack can be simplified to:

\[
C_1 = \frac{b_1 l^2 - b_2 l + b_3 f_2 - \overline{C}_2 f_1 - \overline{C}_2 b_3}{\overline{S}_2}
\]

(4.44)

Therefore, by knowing the longitudinal stress in the FRP plate, one can find the shear stress in the adhesive layer at the same location. Considering the fact that due to opening of the cracks usually there is debonding in the adhesive layer at the crack. Eq.4.45 is an important equation from the design point of view. In this equation, \( f_2 \) is predominant compared to the other terms. In other words, any approximation used in defining the tensile stress at the bottom of the concrete beam such as linear elastic behavior, has negligible effect and can be ignored.

4.3 Verification of the Method

The method has been verified by comparing it to both finite element analysis and experimental results. Several researchers have reported local failure in concrete beams strengthened with FRP plates (Ritchie, et al., 1991) (Saadatmanesh, and Ehsani, 1991). In this study, the beams tested by Saadatmanesh and Ehsani (1991) were analyzed by using both the method described in this paper and the finite element method. For brevity, only the results of one of these beams which has failed due to local failure of concrete at the cut-
off point is discussed here. The general view and also the cross section of this beam are shown in Fig. 4.9.

The mechanical properties of the materials used in the construction of the test beam are listed in Table 4.1.

Table 4.1 - Mechanical Properties of Materials Used in the Test Beam

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus of Elasticity, Mpa</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>27,990</td>
<td>0.18</td>
</tr>
<tr>
<td>Steel</td>
<td>200,000</td>
<td>0.3</td>
</tr>
<tr>
<td>FRP</td>
<td>37,230</td>
<td>0.35</td>
</tr>
<tr>
<td>Adhesive</td>
<td>814</td>
<td>0.37</td>
</tr>
</tbody>
</table>

In order to compare the results of the present method with the finite element and experimental results, it is first necessary to calculate the shear and normal stresses at the cut-off point for the test beam using the present method.

Equation (4.13) was used to predict the interfacial shear stress. Based on an elastic analysis, which is reasonable for the end regions of the beam, the cross-sectional properties were calculated as:

\[ \bar{y} = 232 \text{ mm}, \quad I_y = 1.77 \times 10^6 \text{ mm}^4 \]

The expression for the bending moment at the ultimate load of 100 kN is given by:

\[ M(x) = 100,000 x_n \]
where the origin of $x_0$ is defined at the left support. Therefore, the coefficients of the polynomial given in Eq. (4.10) are:

$$a_0 = 0, \quad a_1 = 100,000, \quad \text{and} \quad a_2 = 0$$

Using Eq. (4.9) the following parameters are calculated:

$$b_1 = 0, \quad b_2 = 0.0174, \quad \text{and} \quad b_3 = 2.70$$

Subsequently, $C_1$ and $C_2$ are obtained as:

$$C_1 = 2.70, \quad \text{and} \quad C_2 = -2.70$$

Knowing $G_u = 297$ MPa, $t_u = 1.5$ mm, and $t_p = 6$ mm, the constant $A$ is calculated:

$$A = 8.96 \times 10^4$$

The equation of shear stress distribution along the interface can now be expressed by:

$$\tau(x) = 0.4825 \cosh (0.0298x) - 0.4825 \sinh (0.0298x) + 0.1045$$

and the tensile stress in the FRP plate can be written as:

$$f_p(x) = 2.7 \sinh (0.0298x) - 2.7 \cosh (0.0298x) + 0.0174x + 2.7$$

The maximum shear stress at the cut-off point is calculated by evaluating shear stress at $x = 0$: $\tau_{\text{max}} = 0.587$ MPa (0.085 ksi).

The cross-sectional properties of "concrete beam" and "plate beam" are as follows:

- Concrete beam:
  $$\bar{y}_c = 227.5 \text{ mm}, \quad I_c = 1.61 \times 10^9 \text{ mm}^4$$

- Plate beam:
  $$\bar{y}_p = 3 \text{ mm}, \quad I_p = 2736 \text{ mm}^4$$

The following can also be obtained:

$$K_n = 542.3 \text{ Mpa/mm}, \quad \beta = 0.1192 \text{ mm}^{-1}.$$
\[ V_c = 79.667 \text{ kN} : V_p = -0.268 \text{ kN} : \]

\[ D_l = -0.427 \text{ Mpa} : D_r = 0.00656 \text{ Mpa}. \]

The equation for the normal stress is obtained as:

\[ f_n(x) = e^{-0.1192x} \left[ -0.427 \cos(0.1192x) + 0.00656 \sin(0.1192x) \right] \]

At the cut-off point \((x = 0)\) the maximum value of normal stress is obtained as:

\[ f_{n,\text{max}} = -0.427 \text{ Mpa} \]

The negative sign shows tensile stress.

4.3.1 Comparison with Finite Element Analysis

The "ABAQUS" Finite Element program was also used to analyze the test beam (ABAQUS, Version 5.4, 1994).

Due to the symmetry of the beam, only half of the beam was analyzed with appropriate constraints at the centerline, as shown in Fig. 4.10. Rebars were modeled as one dimensional bar elements. Different meshes were used for the analysis and the results of three typical cases are discussed here.

Case I: 4-node elements - with one layer of elements in the adhesive.

Case II: 8-node serendipity elements - with one layer of elements in the adhesive.

Case III: 8-node serendipity elements - with five layers of elements in the adhesive.

The mesh definition around the cut-off point for Case III is shown in Fig. 4.11.

The results of the finite element analysis together with the closed form solution (present method) for interfacial shear and normal stresses as well as the longitudinal
stresses of the plate, are shown in Figs. 4.12(a), 4.12(b) and 4.12(c), respectively. It can be concluded that only a very fine mesh can show the descending branch in the shear stress very close to the cut-off point. However, the maximum shear stress predicted by this method is in good agreement to the results of the finite element analysis. It is also concluded that shear stress concentration at the cut-off point rapidly vanishes when moving toward the center of the beam.

The results of the normal stress show more deviation from the finite element results at the cut-off point. However, at the location of maximum stresses, which is used for design, the agreement between the finite element results and the present method is good as can be seen from Fig. 4.13.

4.3.2 Effects of Flexural Cracks:

Cracking is one of the major characteristics of concrete that affects analysis and design procedures. In order to investigate the effect of large flexural cracks on the distribution of stresses in a beam strengthened with FRP plates, the beam was analyzed assuming that two cracks were present. Mesh definition and location of the predefined flexural cracks are shown in Fig. 4.14.

Eight-node elements were used, and intermediate nodes of the elements around the crack tip were defined at quarter points to simulate the stress singularity at this point (Cook, 1981). The average plate tensile stress at cracks 1 and 2 was 700 MPa and 726 Mpa based on the finite element analysis. Using Eq. (4.44), the calculated shear stresses are 112.50
and 116.84 Mpa, respectively. The maximum shear stress in the adhesive layer around these cracks obtained by the finite element analysis was 105 Mpa and 112 Mpa, which shows a good agreement. Finite element results also showed that there is compressive normal stress accompanied by shear stress, but normal stress does not show high concentration like that at the cut-off point. According to this analysis the increase in the shear stress at the cut-off point due to cracks was negligible.

4.3.3 Parametric Study for Isotropic and Orthotropic Behaviors of FRP Plate:

A parametric study was carried out to investigate the effect of unisotropy of the plate on the shear and normal stress concentrations. The test beam had unidirectional FRP plate which results in orthotropic behavior of the plate. This study showed that the variation of elastic modulus in transverse direction does not have a significant effect on shear and normal stresses. The variation of the shear modulus of elasticity, however, can somewhat change these stresses as shown in Table 4.2. Assuming isotropic behavior for the plate (based on longitudinal direction) results in an upper bond on the magnitude of shear stress which is a conservative solution. The variation of normalized shear and normal stress (with respect to isotropic behavior) against normalized shear modulus (with respect to isotropic case) are shown in Table 4.2. Where in this table subscripts I and o refer to isotropic and orthotropic.
Table 4.2 - Variation of stresses due to orthotropic behavior of the plate

<table>
<thead>
<tr>
<th>Normalized Shear Modulus $(G_c/G_f)$</th>
<th>Normalized Shear Stress $(\tau_c/\tau_f)$</th>
<th>Normalized Normal Stress $(f_c/f_f)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.942</td>
<td>1.07</td>
</tr>
<tr>
<td>0.2</td>
<td>0.967</td>
<td>1.10</td>
</tr>
<tr>
<td>0.4</td>
<td>0.984</td>
<td>1.08</td>
</tr>
<tr>
<td>0.6</td>
<td>0.993</td>
<td>1.05</td>
</tr>
<tr>
<td>0.8</td>
<td>0.998</td>
<td>1.03</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

4.3.4 Comparison with Experimental Results

According to the method presented in this paper, the shear and normal stresses at the cut-off point of the test beam are calculated as $0.586\, \text{Mpa}$ and $0.427\, \text{Mpa}$, respectively. The flexural stress in the concrete at the end of the plate, considering the increase in moment, $M_m$, is calculated as $2.545\, \text{Mpa}$. Using these values, the principal stresses are obtained as $2.696\, \text{Mpa}$ and $0.276\, \text{Mpa}$, respectively. A biaxial failure model for concrete (Kupfer and Grestle, 1973), shows a tensile strength of $3.11\, \text{Mpa}$ for the concrete used in making the test beam $(f'_c = 34.32\, \text{Mpa})$. Comparing these results ($2.696\, \text{MPa}$ vs. $3.11\, \text{MPa}$) shows 13 percent difference which is due to the approximations made in the model as well as the linear elastic behavior assumed in the method presented in this paper.
CHAPTER 5
DESIGN GUIDELINES FOR FLEXURAL STRENGTHENING

There has been a large number of experimental and analytical studies demonstrating the effectiveness of bonding composite plates to the tension face of reinforced concrete beams. However, in order to enable the practicing engineers to use this retrofitting system, design guidelines are required. A comprehensive design methodology and guidelines for strengthening of concrete beams with epoxy bonded composite plates or fabrics is presented in this chapter.

5.1 Effect of Initial Stresses on Ultimate Strength:

The effect of the stresses that the reinforced concrete beam undergoes before bonding the plate, must be considered in calculation of the ultimate flexural capacity of the strengthened beam. In order to simplify the design process, it is assumed that the strain in the plate and the concrete at the interface are equal. However, it is noted that in reality the strain in the concrete at the interface is higher due to the initial stresses. In order to account for this discrepancy, the axial strain in the composite plate is reduced to include the effect of initial strains, and to allow direct application of linear strain variation along the section of the strengthened beam. If the service moment (without any load factors) acting on the reinforced concrete beam before upgrading is defined by $M_{ser}$, then strain at the top of the concrete beam can be expressed by:
Where: $c_n$ is the depth of neutral axis (Fig. 5.1); $I_{mn}$ is the moment of inertia of the transformed cracked section based on concrete, and $E_c$ is the modules of elasticity of the concrete. Considering linear variation of strains along the section of the beam, an equivalent uniform tensile strain at the centroid level of the composite plate is calculated from:

$$
\varepsilon_{e*} = \frac{M_{a}c_{e}}{E_{c}I_{mn}}
$$

In the above expression, the effect of the thickness of the composite plate has been ignored to simplify the design equations. It is noted that the plate thickness is generally much smaller than the height of the beam, justifying this approximation.

In stress calculating the stress in the composite plate, the above strain is subtracted from the total strain calculated assuming a linear variations of strains in the cross section. Therefore, the actual stress in the composite plate is calculated from:

$$
f_{p} \cdot E_{p} (\varepsilon_{p} - \varepsilon_{e*})
$$

Where: $f_p$ is axial stress in the composite plate; $E_p$ is modules of elasticity of the plate in the fiber direction; and $\varepsilon_p$ is axial strain at the level of the plate based on linear strain variation.
in the strengthened beam.

Furthermore, in all derivations that follow it is assumed that full composite action exists between the composite plate and the beam, i.e., no slip at the interface level.

5.2 Balanced Plate Ratio for Steel Yielding:

This ratio gives the maximum cross sectional area of the plate to assure yielding of the steel reinforcement at the time of concrete crushing. Under balanced condition, where steel yields at the same time that concrete crushes, and based on linear strain variation, the balanced plate ratio is obtained from:

\[
\rho_{p,\beta} = \frac{\rho f_c \cdot 0.85 f_c \beta \eta_1 \cdot \rho f_y}{h \cdot \eta_1 d \left( \frac{e_s}{\eta_1 d} \cdot e_{\gamma} \right) E_p}
\]

If compression steel has yielded

\[
\rho_{p,\beta} = \frac{\left( \frac{\eta_1 d \cdot d}{\eta_1 d} \right) E \cdot \rho \cdot 0.85 f_c \beta \eta_1 \cdot \rho f_y}{h \cdot \eta_1 d \left( \frac{e_s}{\eta_1 d} \cdot e_{\gamma} \right) E_p}
\]

If compression steel has not yielded

In the above equations:

\[
\eta_1 = \frac{e_s}{e_s \cdot e_{\gamma}}
\]

Where: \( e_s \) is the ultimate strain in the concrete (generally = 0.003); \( e_{\gamma} \) is the yield strain of...
the steel reinforcement; \( f_y \) is yield stress of steel reinforcement; \( E_s \) is modulus of elasticity of steel; \( f_c \) is compressive strength of concrete; and \( \beta_i \) is parameter of rectangular stress block (Nilson and Winter, 1991). Furthermore:

\[
\rho_p \cdot \frac{A_p}{b \cdot d} \quad \rho \cdot \frac{A_c}{b \cdot d} \quad \beta \cdot \frac{\dot{A}_c}{b \cdot d}
\]

Where: \( A_p \), \( A_c \), and \( \dot{A}_c \) are the cross sectional areas of the composite plate, tension reinforcement and compression reinforcement, respectively.

In the balanced condition, compression reinforcement will yield provided that the following condition is satisfied:

\[
d \leq \left( \frac{\varepsilon_{y,c} - \varepsilon_r}{\varepsilon_{y,c} - \varepsilon_r} \right) d \quad (5.7)
\]

The maximum plate ratio is limited to the following amount in order to avoid over reinforcing of the section:

\[
\rho_{p,\text{max}} = 0.75 \rho_{p,b} \quad (5.8)
\]

5.3 Yielding of the Compression Steel at the Ultimate case:

Using linear strain diagram and also the corresponding stress diagram (Fig. 5.2), the minimum plate ratio for yielding of the compressive steel is calculated. Based on the mode of failure one of the following equations are used to find the critical plate ratio:
5.3.1 Composite Plate Rupture:

\[ \rho_{p,cr} = \frac{0.85 f_z \beta_1 \frac{c}{d} \cdot f_r \cdot \rho f_c}{f_p} \]  \hspace{1cm} (5.9)

Where: \( c \cdot \frac{e_y \cdot h \cdot e_r \cdot d}{e_r \cdot e_r} \cdot f_p = \) composite plate stress at rupture, and \( \frac{f_p}{E_p} \) is the ultimate strain in the composite plate at rupture.

If the plate ratio exceeds the value of \( \rho_{p,cr} \) the compressive steel reinforcement will yield at the ultimate load level of the strengthened beam.

5.3.2 Concrete Crushing:

\[ \rho_{p,cr} = \frac{0.85 f_z \beta_1 \eta \frac{d}{d} \cdot (\rho \cdot \rho) f_r}{E_p \left( \frac{e_p}{e_r} \cdot e_p \right)} \] \hspace{1cm} (5.10)

Where: \( \eta = \frac{e_u}{e_r} \), and \( e_p = \frac{h \cdot \eta \cdot d}{\eta \cdot d} \)

5.4 Ultimate Capacity of the Strengthened Beam

The failure of the strengthened beam may result from crushing of concrete or rupture of the plate. The condition at which the compressive stress in concrete, and tensile stress in the composite plate reach their ultimate values at the same time is here referred to as "balanced failure". Defining the required plate ratio for this mode of failure as \( \rho_{p,bb} \) and using the linear strain diagram, balanced plate ratio is calculated:
Where: $\eta_1 = \frac{\epsilon_s}{\epsilon_s - \epsilon_p}$

In the above equation, $\epsilon_s$ is strain in the compression steel reinforcement and is calculated using the following equation:

$$\epsilon_s \cdot \left(1 - \frac{d}{\eta_1 h}\right) \epsilon_s \leq \epsilon_p$$  (5.12)

In cases where $\rho_p \leq \rho_{pb}$, failure of the strengthened beam is caused by rupture of the plate. Otherwise, it is caused by crushing of concrete in the compression zone. Based on the above failure modes, the nominal flexural capacity of the strengthened beam, $M_n$, is calculated using one of the following equations:

5.4.1 Rupture of the Plate

- Compression steel will yield at ultimate load ($\rho_p \geq \rho_{p0}$):

$$M_n = A_t f_y \left(\frac{\rho_p c}{2} - \hat{d}\right) \cdot A_s f_p \left(\frac{\rho_p c}{2}\right) \cdot A_p f_p \left(h - \rho_p c\right)$$  (5.13)

Where: $c \cdot A_t f_y \cdot A_s f_p \cdot A_p f_p \cdot \dot{A}_s f_s \cdot \frac{.85 f_c b \rho_p}{.85 f_c b \rho_p}$

- Compression steel will not yield at ultimate load ($\rho_p \leq \rho_{p0}$):
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\[ M_n = \left( \frac{c-d}{h-c} \right) (e_r - e_{p_r}) \dot{A}_s E_s \left( \frac{\beta_1 c}{2} - d \right) \cdot A_i f_y (d - \frac{\beta_1 c}{2}) \cdot A_p f_p (h - \frac{\beta_1 c}{2}) \]  \hspace{1cm} (5.14)

Where depth of neutral axis, \( c \), is calculated using the following quadratic equation:

\[ \overline{A} c^2 - \overline{B} c + \overline{C} = 0 \]  \hspace{1cm} (5.15)

\[ \overline{A} = .85 \dot{f}_c \beta_1 b \]
\[ \overline{B} = .85 \dot{f}_c \beta_1 h b \cdot (e_r - e_{p_r}) \dot{A}_s E_s \cdot A_i f_y A_p f_p \]
\[ \overline{C} = (e_r - e_{p_r}) E_s \dot{A}_s \frac{d}{(A_i f_y A_p f_p) h} \]

5.4.2. Crushing of Concrete in the Compression Zone

- **Compression steel will yield at ultimate load** \( (\rho_p \geq \rho_{p,y}) \):

\[ M_n \cdot \dot{A}_s f_y \left( \frac{\beta_1 c}{2} - d \right) \cdot A_i f_y (d - \frac{\beta_1 c}{2}) \cdot \left( \frac{h - c}{c} e_r - e_{p_r} \right) E_s A_p (h - \frac{\beta_1 c}{2}) \]  \hspace{1cm} (5.16)

Where \( c \) is found using Eq. 5.15 with following parameters:

\[ \overline{A} = .85 \dot{f}_c \beta_1 b \]
\[ \overline{B} = (\dot{A}_s A_i) f_y (e_r - e_{p_r}) E_s A_p \]
\[ \overline{C} = e_r h A_p E_p \]

- **Compression steel will not yield at ultimate load** \( (\rho_p < \rho_{p,y}) \):

\[ M_n \cdot (e_r \frac{c-d}{c}) E_s \dot{A}_s \left( \frac{\beta_1 c}{2} - d \right) \cdot A_i f_y (d - \frac{\beta_1 c}{2}) \cdot \left( \frac{h - c}{c} e_r - e_{p_r} \right) E_s A_p (h - \frac{\beta_1 c}{2}) \]  \hspace{1cm} (5.17)

Where \( c \) is calculated using Eq. 5.15 and following parameters:
\[ \tilde{A} = 0.85 f_c \beta h \]
\[ \tilde{B} = E_v \varepsilon_v \hat{A}_v \cdot A_f \cdot (e_u \cdot e_p) E_p A_p \]
\[ \tilde{C} = e_u h A_p E_p \cdot e_u d \hat{A}_v E_v \]

5.5 Local Failure at the Cut-off Point of the Composite Plate:

The interfacial shear and normal stresses in the concrete beam at the cut-off point may lead to premature local failure in the concrete beam and separation of the plate, therefore they must be considered in design procedures. The analytical closed form solution of chapter 4. is modified for cases that strengthened beam undergoes uniformly distributed loads. For this type of loading, the maximum shear stress is calculated using the following equation:

\[ \tau_{\text{max}} = \tau_s \left( 1 + \gamma_s \right) \]

Where: \( \tau_s = \frac{t_s E_p \bar{y} L q}{2 I_c E_c} \); \( \gamma_s = L_o \sqrt{\frac{G_o}{t_s E_p}} \); \( \bar{y} \) is the distance between the composite plate and the neutral axis of the strengthened beam based on elastic analysis of uncracked section; \( L \) is the length of the beam; \( q \) is the uniformly distributed load acting on the beam; \( L_o \) is distance between cut-off point and adjacent support as shown in Fig. 5.4; \( G_o \) and \( t_s \) are shear modules and thickness of adhesive layer, respectively.

Under the same type of loading, maximum normal (peeling) stress at cut-off point is calculated as:
f_{a, max} \cdot \frac{K_n q L}{4 \beta^4} \left( \frac{\alpha I_p (1 - \gamma)}{E_p I_p} \cdot \frac{1}{2} \cdot \frac{\alpha h (1 - \gamma) \cdot \beta L_o}{E_c I_c} \right) \tag{5.19}

Where: \( \alpha \), \( \beta \), \( \frac{b_p t_e E_p \gamma}{I_p E_c} \), \( K_n \), \( \frac{E_s}{t_o} \), \( \frac{K_s b_p}{4 E_p I_p} \); \( b_p \) is width of the composite plate; \( I_p \) and \( I_c \) are moment of inertia of the isolated concrete beam and composite plate (Fig. 5.4), and are calculated as:

\[
I_p = \frac{b_p t_e^3}{12}, \quad I_c = \frac{b h^3}{12}
\]

Bending moment in the concrete beam at the cut-off point is magnified as a result of shear stress concentration at this point. The amount of this magnification is calculated using the following equation:

\[
M_m = \frac{\alpha L_o h L q}{4} (1 - \gamma) \tag{5.20}
\]

This moment is added to the bending moment calculated based on statical equilibrium equations.

It is reminded that the load used in calculation of shear and normal stress concentrations, as well as moment magnification amount is only the extra load which is superimposed on the beam after strengthening, and includes the corresponding load factors.
5.5.1 Failure Criterion of Concrete under Biaxial Stresses

At the cut-off point the concrete beam undergoes biaxial stresses. In this case, three components of stress are present: $\sigma_\varepsilon$ is calculated from flexural analysis, $\sigma_p$ and $\tau_\omega$, peeling and shear stresses are calculated based on preceding discussion. Only superimposed live load is used in calculating $\sigma_\varepsilon$ and $\tau_\omega$, while $\sigma_p$ consists of two components. The first component is obtained by considering unretrofitted beam under dead and live loads applied on the beam before upgrading. The second component is calculated using the magnified moment described above based on superimposed live load. These two components are added together to obtain the total axial stress ($\sigma_\varepsilon$).

The failure model presented by Kupfer and Gerstle (1973), for behavior of concrete under biaxial state of stresses can be used to check the local failure of the concrete beam. According to this model the strength of concrete under different combinations of stresses is approximated by:

a) Under compression-compression

$$\left( \frac{\sigma_1}{f_m} \cdot \frac{\sigma_2}{f_m} \right)^2 \cdot \frac{\sigma_1}{f_m} \cdot 3.65 \frac{\sigma_2}{f_m} = 0$$

(5.21)

b) Under compression-tension

$$\frac{\sigma_2}{f_m} \cdot 1 \cdot 0.8 \frac{\sigma_1}{f_m}$$

(5.22)
c) Under tension-tension

\[ \sigma_1 = f_u + 0.64 \left( f_{cu} \right)^{0.2} \cdot \text{Constant} \]  \hspace{1cm} (5.23)

Where: \( \sigma_1 \) and \( \sigma_2 \) are principal stresses in the concrete (\( \sigma_1 \geq \sigma_2 \)), positive if tensile; \( f_u \) and \( f_{cu} \) are ultimate tensile and compressive strengths of concrete (\( \text{kg/cm}^2 \)). The principal stresses are calculated using stress transformation relations under plane stress conditions and are compared to the above strengths.

Shear stress concentration around flexural cracks may also lead to local debonding of the plate. Maximum shear stress in adhesive layer ignoring the minor terms is obtained from the following approximate equation:

\[ \tau_{\text{max}} = \sqrt{\frac{G_s t_p}{E_p t_a}} f_p \]  \hspace{1cm} (5.24)

Where: \( f_p \) is the axial stress in FRP plate. This shear stress is compared to allowable interfacial shear stress.

The following design example illustrates the application of the above design guidelines for strengthening of a typical reinforced concrete beam using epoxy bonded fiber composite plate.

5.6 Design Example:

The reinforced concrete beam shown in Fig. 5.5 has been primarily designed for
a dead load of 40 kN/m and live load of 75 kN/m. The live load applied on the beam is supposed to be increased to 120 kN/m, which equals to upgrading of 60%. The service bending moment in midspan of the beam before upgrading is calculated as: $M_i=180 \text{ kN.m}$ and $M_j=337.5 \text{ kN.m}$. Therefore the total factored moment in the midspan of the beam is 825.75 kN.m which is close to flexural capacity of the reinforced concrete beam ($\phi M_u = 847 \text{ kN.m}$). The factored moment in the midspan of the beam considering the superimposed live load is 1170 kN.m which indicates the necessity of strengthening. The mechanical properties of the material used in construction of the beam as well as the composite plate are listed in Table 5.1. Furthermore, poisson's ratio of the adhesive is assumed 0.37, resulting in shear modules ($G_a$) equal to 751 Mpa.

<table>
<thead>
<tr>
<th>Material</th>
<th>Modules of Elasticity (Gpa)</th>
<th>Other Properties (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>200</td>
<td>$f_y = 470$</td>
</tr>
<tr>
<td>Concrete</td>
<td>27.900</td>
<td>$f'_c = 35$</td>
</tr>
<tr>
<td>FRP</td>
<td>37.230</td>
<td>$f'_c = 390$</td>
</tr>
<tr>
<td>Adhesive</td>
<td>2.058</td>
<td>$f'_c = 36$</td>
</tr>
</tbody>
</table>

Using an elastic analysis for cracked section of reinforced concrete beam (without plate), the location of neutral axis and also moment of inertia of the transformed section based on concrete are calculated:
Using service loads before upgrading, tensile and compressive stresses in steel rebars and also maximum compressive stress in concrete are calculated:

\[ f_s = 220 \text{ Mpa} \quad f_c = 121 \text{ Mpa} \quad f_{c_s} = 21.57 \text{ Mpa} \]

It is concluded that both tension and compression reinforcement are not yielded at this stage of loading. Using Eq. 5.2 initial strain at the location of the plate is obtained:

\[ \epsilon_{\text{in}} = 1.27 \times 10^{-1} \]

Assuming \( \epsilon_s = 0.003 \) and \( \epsilon_{c_s} = 0.00235 \), and using Eq. 5.7 indicates that at balanced condition the compression reinforcement will yield. Therefore, assuming \( \beta = 0.8 \) and using Eq. 5.4 results in:

\[ \rho_{ps} = 0.0867 \quad \rho_{p_{cmax}} = 0.065 \]

Choosing a plate with nominal dimensions of \( t_p = 10 \text{ mm} \), \( b_p = 360 \text{ mm} \) (\( \rho_p = 0.0164 \)), and using Eqs. 5.11 and 5.12, \( \rho_{p_{ab}} \) is obtained a negative number, indicating that crushing of concrete is the predominant mode of failure. Since failure of the strengthened beam is resulted from crushing of concrete, \( \rho_{p_{cmax}} \) is calculated as 0.0128, when using Eq. 5.10. Comparing the recent value to \( \rho_p \) shows that compression steel reinforcement yields at ultimate case. Considering crushing of concrete, and also yielding of compression reinforcement, and using Eq. 5.15 with corresponding parameters, \( c \) is calculated as 238.47 mm. Consequently using Eq. 5.16 results in: \( M_s = 1294 \text{ kN.m} \) and \( \phi M = 1164 \text{ kN.m} \). The maximum moment resulted from factored loads, including superimposed live load is 1170 kN.m which is very close to the ultimate capacity of the strengthened beam. Therefore using the selected plate...
is adequate from flexural point of view.

Local shear failure:

The properties of the strengthened beam are calculated assuming uncracked section. Location of the neutral axis and moment of inertia are calculated as:

\[
\bar{y} = 283 \text{ mm} \quad I_y = 9.75 \times 10^5 \text{ mm}^4
\]

In calculating maximum shear and normal interfacial stresses, only that part of the load which is applied after strengthening (superimposed live load) is taken into account. In other words:

\[
q \cdot (120 / 75) \times 1.7 = 76.5 \text{ kN/m}
\]

Replacing \(q, \bar{y}, I_y\) in Eq. 5.18, and assuming \(L_o = 10 \text{ cm}\) results in: \(\tau_{\text{max}} = 0.371 \text{ Mpa}\).

Parameters used in Eq. 5.19 are calculated:

\[
K_o = 1029 \frac{\text{ Mpa}}{\text{ mm}}, \beta = 0.095 \frac{1}{\text{ mm}}, \gamma = 3.17
\]

Consequently the maximum normal (peeling) stress is calculated as: \(f_{n,\text{max}} = 0.72 \text{ Mpa}\). Using Eq. 5.20 the amount of magnification of bending moment is obtained: \(M_m = 4 \text{ kN.m}\).

The bending moment in the concrete beam at the location of the plate-end due to factored loads which are applied before strengthening is 76.7 kN.m. Therefore total factored bending moment of 80.7 kN.m is used to find flexural stresses in the concrete beam at this
point. This moment results in tensile stress of 2.26 Mpa in the concrete beam. The state of stresses acting in concrete beam at the plate-end is shown in Fig. 5.6. Using conventional stress transformation relations maximum principal stress is calculated 2.35 Mpa. Based on Eq.5.23. Ultimate capacity of concrete ($f_t \cdot 350 \frac{kg}{cm^2}$) under biaxial tensile stresses is 3.23 Mpa. The resulting strength reduction factor is obtained .72 which provides adequate margin of safety. Therefore using the above plate with a curtailment point 10cm away from the support is acceptable.

Maximum shear stress in the adhesive layer around the flexural cracks is estimated by obtaining maximum axial stress in the FRP plate. The depth of the neutral axis and moment of inertia of the strengthened beam are calculated, assuming that the section is cracked:

$$\bar{y} = 240 \text{ mm} \quad I_x = 6.11 \times 10^6 \text{ mm}^4$$

Axial stress in the FRP plate under superimposed factored live load is 27.33 Mpa. Using Eq.5.24 maximum shear stress in the adhesive layer is 8.679 Mpa. This shear stress is compared to the interfacial shear strength of the adhesive, which depends on type and conditions of the adhesive as well as physical properties of the bonded surfaces.
CHAPTER 6

ANALYTICAL STUDY OF REINFORCED CONCRETE BEAMS STRENGTHENED WITH WEB-BONDED FRP PLATES

This chapter presents an analytical method to investigate the stress distribution and the shear force resisted by the composite plate in reinforced concrete beams strengthened with web-bonded FRP plates before cracking and also after formation of flexural cracks. The next chapter extends this discussion to post cracking behavior at the ultimate load level, where the diagonal shear cracks are formed. The method discussed here, has been developed based on strain compatibility between the reinforced concrete beam and the composite plate, and considering the orthotropic behavior of the FRP plate. Equations have been developed for two different cases of uncracked beam, and the beam with flexural cracks. The assumptions made in developing the method have been verified by comparing to finite element method. The results of this method have been compared to experimental results. An extensive parametric study has been carried out to show the effect of some important parameters such as fiber orientation and geometric properties of the plate on the shear force resisted by the plate.

6.1 Analytical Model

The stress-strain relationship in an orthotropic lamina (plate) in any arbitrary system of coordinates such as $x$-$y$ under plane stress conditions is written as (Jones, 1975) (Mallik, 1993):
\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{xy}
\end{bmatrix} = \begin{bmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{13} \\
\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{23} \\
\bar{Q}_{13} & \bar{Q}_{23} & \bar{Q}_{33}
\end{bmatrix} \begin{bmatrix}
e_x \\
e_y \\
e_{xy}
\end{bmatrix} \tag{6.1}
\]

Where \(\bar{Q}\) represents the transformed stiffness matrix of the lamina, and its elements are written as:

\[
\begin{align*}
\bar{Q}_{11} &= \bar{Q}_{11} + \frac{2}{\bar{Q}_{12} \cdot \bar{Q}_{33}} \bar{Q}_{12}^2 \bar{Q}_{33} \\
\bar{Q}_{12} &= \bar{Q}_{12} + \frac{2}{\bar{Q}_{12} \cdot \bar{Q}_{33}} \bar{Q}_{12}^2 \bar{Q}_{33} \\
\bar{Q}_{13} &= \bar{Q}_{13} + \frac{2}{\bar{Q}_{12} \cdot \bar{Q}_{33}} \bar{Q}_{12}^2 \bar{Q}_{33} \\
\bar{Q}_{22} &= \bar{Q}_{22} + \frac{2}{\bar{Q}_{12} \cdot \bar{Q}_{33}} \bar{Q}_{12}^2 \bar{Q}_{33} \\
\bar{Q}_{23} &= \bar{Q}_{23} + \frac{2}{\bar{Q}_{12} \cdot \bar{Q}_{33}} \bar{Q}_{12}^2 \bar{Q}_{33} \\
\bar{Q}_{33} &= \bar{Q}_{33} + \frac{2}{\bar{Q}_{12} \cdot \bar{Q}_{33}} \bar{Q}_{12}^2 \bar{Q}_{33}
\end{align*}
\tag{6.2}
\]

In the above expressions, \(S_\theta = \sin(\theta)\) and \(C_\theta = \cos(\theta)\), where \(\theta\) is the angle between the x axis and the longitudinal axis (fiber direction) of the composite plate, measured counterclockwise as shown in Fig.6.1. Furthermore:

\[
\begin{align*}
\bar{Q}_{11} &= \frac{E_{11}}{1 - v_{12} v_{21}} \\
\bar{Q}_{22} &= \frac{E_{22}}{1 - v_{12} v_{21}} \\
\bar{Q}_{12} &= v_{21} \frac{E_{11}}{1 - v_{12} v_{21}} \\
\bar{Q}_{12} &= G_{12}
\end{align*}
\tag{6.3}
\]

Where: 1 and 2 refer to the longitudinal (fiber) and transverse directions in the composite
plate respectively, as shown in Fig. 6.1. Therefore, $E_{11}$ and $E_{22}$ are the elastic moduli of the plate in the longitudinal and transverse directions. $G_{12}$ is the shear modulus of the plate, and $\nu$ is the poisson's ratio.

In order to show the effect of the plate on the stress distribution in the reinforced concrete beam strengthened with web-bonded FRP plates, two different cases of uncracked and cracked beam are considered, and equations are developed for each case.

6.1.1 Uncracked Beam

The following assumptions are made in developing this method:

- There is complete composite action between the plate and the reinforced concrete beam, that is, the strains in the plate and the reinforced concrete beam are identical.
- Plane sections remain plane after applying the loads.
- All materials behave linearly elastic both in tension and compression.

It is also assumed that under pure bending we may write:

$$\sigma_{yy} = 0$$
$$\gamma_{xy} = 0$$

(6.4)

Where: $x$ is the longitudinal axis of the beam, and $y$ is perpendicular to the $x$ in the plane of the beam. Therefore $\sigma_{yy}$ and $\gamma_{xy}$ are transverse normal stress and shear strain, respectively. The validity of these assumptions will be verified by the finite element analysis.
Expanding the second row of Eq. 6.1 to find $\sigma_{yy}$ results in:

$$\sigma_{yy} = \overline{Q}_{12} \cdot \overline{e}_{xx} \cdot \overline{Q}_{12} \cdot \overline{e}_{yy} \cdot \overline{Q}_{22} \cdot \gamma_{yy}$$ \hspace{1cm} (6.5)

According to Eq. 6.4 this stress as well as $\gamma_{yy}$ are negligible in the plate. Therefore $e_{yy}$ is written as:

$$e_{yy} = \frac{\overline{Q}_{12}}{Q_{22}}$$ \hspace{1cm} (6.6)

Replacing $e_{yy}$ from Eq. 6.6 and $\gamma_{yy}$ from Eq. 6.4 in to the first and the third rows of Eq. 6.1, the stresses in the composite plate are written as:

$$\sigma_{xx} = \overline{Q}_{s_{11}} \cdot \overline{e}_{xx}$$ \hspace{1cm} (6.7)

$$\tau_{xy} = \overline{Q}_{s_{13}} \cdot \overline{e}_{xx}$$ \hspace{1cm} (6.8)

Where, $Q_{s_{11}}$ and $Q_{s_{13}}$ are given by:

$$Q_{s_{11}} = \overline{Q}_{11} \cdot \frac{\overline{Q}_{12}^2}{\overline{Q}_{22}}$$ \hspace{1cm} (6.9)

$$Q_{s_{13}} = \overline{Q}_{13} \cdot \frac{\overline{Q}_{12} \overline{Q}_{22}}{\overline{Q}_{22}}$$ \hspace{1cm} (6.10)
The cross section of the strengthened beam as well as the assumed linear strain variations through the section of the beam are shown in Fig. 6.2. The strains in the steel rebars located in tension and compression zones (ε_t and ε_c), the strain at the top of the composite plate (ε_p), and also the maximum tensile strain in the concrete beam (ε_t) can be related to the maximum compressive strain in the concrete (ε_c) using the following expressions:

\[
\begin{align*}
\epsilon_t &= \epsilon_c \left( \frac{d \bar{y}}{y} \right) \\
\epsilon_c' &= \epsilon_c \left( \frac{y - d}{y} \right) \\
\epsilon_p &= \epsilon_c \left( \frac{y - d_p}{y} \right) \\
\epsilon_c &= \epsilon_c \left( \frac{h - \bar{y}}{y} \right)
\end{align*}
\] (6.11)

These strains as well as \( \bar{y} \) (location of the neutral axis) are shown in Fig. 6.2. Based on the strain variation and using the stress-strain relationship of the materials, the stresses acting on the section of the composite beam are calculated. Using the equilibrium equation of the horizontal forces resulting from the stresses acting on the section of the strengthened beam, and also considering Eq. 6.11 to relate all the strains to the corresponding terms including \( \epsilon_c \), the location of the neutral axis is obtained:

\[
\frac{Q_{s_{11}} t_p d_p^2 \cdot E_s b h^2 \cdot 2 A_t E_t d \cdot 2 A_c h \cdot Q_{s_{11}} t_p h^2}{2 ( d_p t_p Q_{s_{11}} \cdot E_s b h^2 \cdot E_t A_t A_c h \cdot Q_{s_{11}} t_p h)}
\] (6.12)
Where, $t_p$ = summation of the thicknesses of the plates used on both faces of the beam; $E_c$ = elastic modules of the concrete; $b$ = width of the concrete beam; $A_t$ = area of the reinforcement in the tension zone of the concrete; and $A_s$ = area of the steel reinforcement in the compression zone of the concrete. The bending moment caused by the stress distribution about the neutral axis of the beam is equal to the internal bending moment which results in an equation including $\varepsilon_c$. Using this equation $\varepsilon_c$ is calculated from the following equation:

$$
\varepsilon_c = \frac{M}{S'}.
$$

(6.13)

Where: $M$ is the internal bending moment acting on the section of the strengthened beam and $S'$ is defined as:

$$S' = \bar{y} (t_p \sum (d_p \cdot h) \cdot E_c A_t \cdot E_s A_s \cdot E_h b h) \cdot Qs_{11} t_p (d_p^2 \cdot h^2) \cdot 2 d E_s A_s \cdot 2 \bar{d} E_s A_s \cdot h^2 b E_c \cdot \left( \frac{1}{3} Qs_{11} t_p (h^3 \cdot d_p^3) \cdot E_c A_t d^2 \cdot \frac{1}{3} E_s b h^3 \cdot E_s A_s d^2 \right) \frac{1}{y}.
$$

(6.14)

Knowing $\bar{y}$ and $\varepsilon_c$, the normal strain at any point on the plate is calculated from:

$$
\varepsilon_n = \frac{y}{\bar{y}} \varepsilon_c.
$$

(6.15)

Where: $y$ is the distance of the point to the neutral axis of the strengthened beam (positive if the point is above the neutral axis).

Using Eq.6.15 the shear stress at any point of the plate is obtained:
The shear force in the composite plate is calculated by integrating the above shear stress over the section of the plate, as given by the following equation:

\[ V' = \frac{c_0 Q_1}{2 \gamma} \left( d^2 - h^2 - 2 \tilde{y} h - 2 \tilde{y} d_p \right) \]  

(6.17)

The superscript \( I \) has been used to indicate that the above shear force is caused by the pure bending of the beam. The conventional positive sign of this force is shown in Fig. 6.3.

Generally, each section of the beam undergoes bending moment and shear force at the same time. To find the other part of the shear force in the composite plate which is caused by the variations of the bending moment in the beam (shear force), we may consider two successive sections of the strengthened beam with an infinitesimal distance \( dx \) as shown in Fig. 6.4. The same procedure is followed to find \( \varepsilon_y \) for each of the sections, based on the bending moment acting on the section. The free-body diagram for an isolated part of the plate between the bottom face and a section at distance \( y \) from the neutral axis is shown in Fig. 6.5. The resulting forces applied on the sides this section due to normal stresses are shown by \( F \) and \( F + dF \). The increment of the force \( (dF) \) is calculated by integrating the difference of the normal stresses acting on the two sides of the plate over the area of this part. Therefore \( dF \) is expressed by:

\[ dF = \int_{b}^{c} \left( \sigma_{1x} \cdot t_p \right) dy \]  

(6.18)
Where.

\[ d\sigma_y = Qs_\text{ill} \left( \cdot \frac{y}{y} \right) \frac{dM}{S} \]  

(6.19)

The horizontal shear stress acting on the plate at distance \( y \) from the neutral axis is calculated by dividing the above force to the area of the plate, as shown below:

\[ \tau = \frac{dF}{t_y \, dx} \]  

(6.20)

Replacing Eq.6.19 in Eq.6.18. performing the integration to find \( dF \) and then substituting \( dF \) in Eq.6.20. the shear stress in the plate at any point located at distance \( y \) from the neutral axis is obtained from:

\[ \tau = \frac{Qs_\text{ill}}{2\, y} \left( -y^3 - (h \cdot y)^3 \right) \]  

(6.21)

The total shear force acting on the section of the plate due to unequal bending moments is then written as:

\[ V_p^u = \int_{y_b}^{y} \tau \cdot t_y \, dy \]  

(6.22)

Superscript II indicates that this component of shear force is due to increment in the bending moment of the beam. Substituting the shear stress given in Eq.6.21 in the above equation and integrating, the shear force is obtained:

\[ V_p^u = \frac{Qs_\text{ill} \cdot t_y \cdot V}{2\, y \, S} \left( \frac{2}{3} \: h^3 \cdot \frac{d^3}{3} \cdot \bar{y} \cdot h^2 \cdot \bar{y} \cdot d_\text{p} \cdot \bar{y} \cdot d_\text{p} \cdot 2 \cdot d_\text{p} \cdot \bar{y} \cdot h \right) \]  

(6.23)
The conventional positive sign for the above shear force is shown in Fig. 6.6. This convention is selected for consistency of signs in beam theory and signs assumed for developing the stress-strain relationship in the orthotropic composite plate. The total shear force in the plate \( V_p \) is the algebraic sum of the forces given by Eqs. 6.17 and 6.23:

\[
V_p = V'_p - V''_p
\]  

(6.24)

It is noted that generally the numerical value of \( V'_p \) will be negative according to the above assumed sign convention, resulting in addition of \( V'_p \) and \( V''_p \). The shear force resisted by the reinforced concrete beam alone is given by:

\[
V_c = V - V_p
\]  

(6.25)

6.1.2 Reinforced Concrete Beam with Flexural Cracks

The procedure explained for uncracked beam can be followed for the strengthened cracked beam. It is assumed that concrete does not resist any tensile stress after cracking, therefore the strain and stress diagrams of a typical section of the strengthened beam under pure bending are as shown in Fig. 6.7. Using a procedure similar to uncracked beam and writing the equilibrium equation of the horizontal forces acting on the section of the beam, the following equation for \( \text{\overline{y}} \) is obtained:
\[ A \bar{y}^2 \cdot B \bar{y} \cdot C = 0 \]  
(6.26)

Where,

\[
A = \frac{1}{2} E_c b \cdot \frac{1}{2} t_p \bar{Q}_{11}'', \\
B = E_s \bar{A}_s \cdot E_s A_s \cdot h t_p \bar{Q}_{11}'', \\\nC = E_s \bar{A}_s \bar{d} \cdot E_s A_s \bar{d} \cdot \frac{1}{2} h^2 t_p \bar{Q}_{11}''
\]  
(6.27)

The parameters used in the above equations have been already defined. Similar to the uncracked beam, the maximum compressive strain at the top of the section \( \epsilon_c \) is obtained by writing the moment equilibrium equation of the section as:

\[
\epsilon_c = \frac{M}{S_c'}
\]  
(6.28)

Where,

\[
S_c' = \frac{1}{3} E_c b \bar{y}^2 \cdot \frac{1}{y} (E_s \bar{A}_s (\bar{y} - \bar{d})^2 \cdot E_s A_s (d - \bar{y})^2 \cdot \frac{1}{3} \bar{Q}_{11}'', t_p (h - \bar{y})^2)
\]  
(6.29)

Using the procedure same as the uncracked beam, \( V_p' \) which is the shear force in the plate due to pure bending of the beam is obtained from:

\[
V_p' = \frac{t_p}{2y} \bar{Q}_{11} \epsilon_c (h - \bar{y})^2
\]  
(6.30)

For this case, \( V_p'' \) is not different from the uncracked beam and Eq.6.23 is still valid.
Therefore, Eqs. 6.24 and 6.25 are used to obtain the shear force carried by the reinforced concrete beam alone.

The validity of the basic assumptions used in developing this method has been verified by comparing its results to the finite element method. Thereafter the method has been used in an extensive parametric study to investigate the effects of the important parameters such as fiber orientation and the thickness of the plate on the shear force carried by the reinforced concrete beam.

6.2 Verification of the Method

In order to demonstrate the application of the method and also to verify the validity of the assumptions used in this method, a reinforced concrete beam strengthened with FRP plates bonded to the side faces was studied. The geometry of this beam and the location of the plates are shown in Fig. 6.8. The mechanical properties of the materials are given in Table 6.1. Furthermore the shear modules of the composite plate \( G_{12} \) was assumed as 6.3 Gpa.

Using Eq. 6.3 the elements of the stiffness matrix of the plate are obtained:

\[ Q_{11} = 34664 \text{ Gpa}; \quad Q_{12} = 1484 \text{ Gpa}; \quad Q_{22} = 4124 \text{ Gpa}; \quad Q_{33} = 6300 \text{ Gpa}. \]
Table 6.1 - Mechanical Properties of the Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Elastic Modules (Gpa)</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>27.9</td>
<td>0.18</td>
</tr>
<tr>
<td>Steel</td>
<td>200</td>
<td>0.3</td>
</tr>
<tr>
<td>FRP</td>
<td>$E_{11} = 34.13$</td>
<td>$\nu_{12} = 0.36$</td>
</tr>
<tr>
<td></td>
<td>$E_{22} = 4.06$</td>
<td></td>
</tr>
</tbody>
</table>

6.2.1 Comparison with Finite Element Method

Finite element analysis (ABAQUS, version 5.4) was used to verify the validity of the assumptions used in developing the analytical model. Considering the lack of the external lateral forces, a two dimensional analysis (plane stress) was used for this study. A mesh of four-node elements was used to model the concrete beam. Rebars were modeled as one-dimensional bar elements in the computer program. Four-node composite membrane elements were used to model the FRP plate (ABAQUS/Standard User's Manual, 1994). The same nodes used for concrete elements were used to define the membrane elements. In this way the stiffness of the membrane elements was added to the stiffness of the concrete elements appropriately. The mesh definition and the deflected shape of the beam under the applied loads are shown in Fig.6.9.

Fiber orientation was assumed 45 degrees and the variation of $\left| \frac{\varepsilon_{xy}}{\varepsilon_{xx}} \right|$ and $\left| \frac{\sigma_{xy}}{\sigma_{xx}} \right|$ along section 2 shown in Fig.6.8 were studied. The node numbering along this section of the beam is also shown in Fig.6.8. The finite element results are listed in Table 6.2.
Table 6.2 - Variation of $\frac{e_{\sigma}}{\varepsilon_{\sigma}}$ and $\frac{\sigma_{\sigma}}{\sigma_{\sigma}}$ along section 2

<table>
<thead>
<tr>
<th>Node Numbers</th>
<th>$\frac{e_{\sigma}}{\varepsilon_{\sigma}}$</th>
<th>$\frac{\sigma_{\sigma}}{\sigma_{\sigma}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>0.063</td>
<td>0.058</td>
</tr>
<tr>
<td>230</td>
<td>0.079</td>
<td>0.037</td>
</tr>
<tr>
<td>330</td>
<td>0.059</td>
<td>0.039</td>
</tr>
<tr>
<td>430</td>
<td>0.071</td>
<td>0.043</td>
</tr>
<tr>
<td>530</td>
<td>0.070</td>
<td>0.050</td>
</tr>
<tr>
<td>730</td>
<td>0.070</td>
<td>0.028</td>
</tr>
<tr>
<td>830</td>
<td>0.070</td>
<td>0.034</td>
</tr>
<tr>
<td>930</td>
<td>0.059</td>
<td>0.036</td>
</tr>
<tr>
<td>1030</td>
<td>0.079</td>
<td>0.036</td>
</tr>
<tr>
<td>1130</td>
<td>0.063</td>
<td>0.058</td>
</tr>
</tbody>
</table>

It is observed that both of the ratios given in table 6.2 are negligible along the section, which confirms the accuracy of the assumptions used in developing this method. The variation of the maximum compressive strain in the concrete along the beam has been also compared to the finite element results in Fig.6.10. This figure shows close agreement between the results of this method and the results of finite element analysis.

### 6.2.2 Comparison with Experimental Results

This method has been applied to precracked flexural specimens IA and IIA tested
by Norris et. al. (1994). The geometry of these beams is shown in Fig.6.8. The ultimate loads of these beams have been reported as \( P=68.97 \text{ kN} \) and \( 62.29 \text{ kN} \), respectively. The fiber orientation angle for both of the beams has been \( (0) \), and the thickness of the composite plate has been \( .33 \text{ mm} \) and \( .66 \text{ mm} \), respectively. The mechanical properties of the composite used in beam IA are as given in Table 6.1, and for beam IIA are slightly different. At ultimate load, the axial strain in the composite plate at the center of the beam was calculated \( 0.00781 \) and \( 0.00608 \) in beams IA and IIA, respectively. The corresponding reported experimental results are \( 0.0072 \) and \( 0.0056 \), respectively. This shows an agreement between the experimental and theoretical results. The difference is due to assumptions made in analysis, such as ignoring slip and linear elastic assumption.

6.3 Parametric Study

The effect of important parameters such as fiber orientation was studied through a parametric study. This study was carried out for both uncracked and cracked beams, and the variation of the shear force resisted by the composite plate was the main concern.

6.3.1 Uncracked Beam

The effect of the fiber orientation was studied for section 1 shown in Fig.6.8. At this section of the beam, the shear force and bending moment are \( V=136.2 \text{ KN} \); and \( M=54.48 \text{ KN.m} \) respectively. It is assumed that the composite plates have a uniform thickness of \( 4 \text{ mm} \) and that they cover all the depth of the concrete beam, that is \( d_c = 0 \). The
variation of the shear force resisted by the plate ($V_p$) against fiber orientation is shown in Fig.6.11. It is observed that the maximum shear force in the plate is only about 6% of the shear force acting on the section of the strengthened beam. This force is mainly caused by the internal shear force ($V_p''$) and its maximum value occurs when $\theta=0$.

The beam was also studied with four different fiber orientation angles of 0, 45, 90, and 135 degrees and different plate thicknesses. The variation of the shear force in the plate is shown in Fig.6.12. As can be seen from this figure, the contribution of the plate to the shear capacity of the beam increases almost linearly with the increase in the plate thickness. However, the shear force resisted by the plate is not considerable within the range of the practical thicknesses.

The third part of this parametric study was concentrated on the effect of the plate height on the shear force in the composite plate. The thickness of the plates was assumed 4 mm, and two different fiber orientations of 45, and 135 degrees were studied. The effect of $d_p$, which inversely corresponds to the height of the composite plate was investigated. The variation of $V_p$ against $d_p$ is shown in Fig.6.13. As shown in this figure, for both of the fiber orientations the maximum shear force occurs when the plate stops at a point near to the neutral axis of the strengthened beam. The significant effect of fiber orientation in the shear force resisted by the plate is observed in this figure.

6.3.2 Beam with Flexural Cracks

The geometry of the beam was assumed the same as the uncracked beam. It was
assumed that due to high bending moment in the section there are flexural cracks in the reinforced concrete beam. The variation of the shear force against fiber orientation angle, as well as the thickness of the plate are shown in Figs. 6.14 and 6.15, respectively. In this case concrete does not resist any tensile stress due to cracking, which results in higher tensile stresses in the plate as well as steel rebars. Both normal and shear stress in the plate are proportional to the maximum strain in the concrete beam. Therefore shear stress and consequently shear force in the composite plate are considerably higher than uncracked beam. Furthermore, due to cracking, the relationship between the thickness of the plate and the contribution of the plate to the shear resistance of the beam is no longer linear. The maximum effect of the plate occurs where fiber orientation angle is about 135 degrees. At this angle the two components used in calculating $\nu_p$ are added, resulting in the highest effect of the plate.
CHAPTER 7
ULTIMATE SHEAR CAPACITY OF REINFORCED CONCRETE BEAMS STRENGTHENED WITH WEB-BONDED FRP PLATES

The ultimate shear capacity of reinforced concrete beams can be increased by epoxy-bonding fiber-reinforced-plastic (FRP) plates to the side faces of the beam. The shear crack inclination angle is changed as a result of bonding the plate. In this chapter truss analogy and compression field theory are used to determine the effect of the FRP plate on the shear capacity and crack inclination angle of reinforced concrete beams at ultimate case. Subsequent to calculation of the crack inclination angle, the equilibrium and compatibility equations are used to obtain the shear force resisted by the plate. A parametric study has been carried out to reveal the effect of important parameters such as plate thickness and fiber orientation on the crack inclination angle and shear capacity. The upper bound value of crack inclination angle found in this study has been suggested as a conservative value to determine the shear capacity of the retrofitted beam. Knowing the inclination angle of cracks, the shear force in the composite plate and concrete beam are calculated and used to design this type of beams.

7.1 Ultimate Shear Capacity of Reinforced Concrete Beams

Ultimate shear capacity of reinforced concrete beams is calculated based on truss analogy developed by Ritter and Morsch about a century ago. The schematics of a small
part of the reinforced concrete beam with inclined cracks propagated as a result of pure shear force is shown in Fig. 7.1. Assuming that concrete can resist only compressive stresses, the system is replaced by an analogous truss. The truss consists of upper and lower horizontal longitudinal chords, representing the compressive strength of the reinforced concrete section and the longitudinal steel reinforcement, respectively, and a web composed of diagonal concrete struts and transverse steel stirrups. The analogous truss is determinate, and the forces and stresses in all the members are calculated using conventional truss analysis.

In most of the well-known reinforced concrete design codes such as ACI, the angle of inclination of the concrete struts ($\theta_c$) is assumed 45 degrees, conservatively (Nilson, and Winter, 1991). Tension field theory developed by Wagner (1929) provides the necessary tools in determining the inclination angle (Collins, and Mitchell, 1980) (Vecchino, and Collins, 1988). In studying the post-buckling behavior of plate girders, Wagner assumed that after buckling, the thin web can not resist compressive stresses, and the shear would be carried by a field of diagonal tension. He assumed that the angle of inclination of the diagonal tensile members is the same as principle tensile strain. This approach is known as tension field theory. Similarly, the compression field theory assumes that after cracking, concrete will not carry any tensile stress, and it acts as a series of struts as described earlier. Furthermore, the inclination of the concrete struts is the same as the principle compressive strain. Using Mohr’s circle for strains and assuming that the cracks are initiated and propagated perpendicular to the maximum principle stress, the following equation is
developed to relate the crack inclination angle ($\theta_c$) to normal strains:

$$\tan(\theta_c) = \frac{\epsilon_x - \epsilon_y}{\epsilon_1 - \epsilon_2}$$

Where; $x$ refers to the longitudinal axis of the beam; $y$ is perpendicular to $x$; and 1 and 2 are principle directions which are perpendicular to the crack and along the crack respectively, as shown in Fig.7.1.

Considering the stresses acting on different parts of the strengthened beam shown in Fig.7.2, and using the equilibrium equations, the normal stress in the concrete struts ($f_x$) and the axial force generated in the longitudinal steel reinforcement ($\Delta N$) are expressed by:

$$f_x = \frac{V}{b_x h_x \sin(\theta_c) \cos(\theta_c)}$$

$$\Delta N = \frac{V}{\tan(\theta_c)}$$

Where: $V$ is the internal shear force in the section; and $b_x$ and $h_x$ are as shown in Fig.7.2.

7.2 Reinforced Concrete Beams with Web-Bonded Plates

To analyze reinforced concrete beams with web-bonded composite plates, in addition to the assumptions used for ordinary reinforced concrete beams the following assumptions are also made:

- Complete composite action between the plate and the concrete beam, i.e. no slip between
FRP and concrete.

- Linear elastic behavior for composite plate.
- Elastic-plastic steel behavior.
- No stress concentration effect.

The stress-strain relationship in a composite plate are expressed by using Eqs.6.1 to 6.3. The stress-strain relationship of the composite plate in a system of coordinates coinciding with the principle directions (1 and 2), can be written by replacing \( x \) and \( y \) by 2 and 1, respectively. In this case, \( \theta \) which shows the angle between \( x \) and \( l \) axes, should be replaced by \( \theta - \theta_0 \), as shown in Fig.7.3. Since \( l \) and 2 refer to principle axes of strain, then \( \gamma_{l2} = 0 \), and Eq.6.1 is simplified to the following equations which are used in order to calculate the stresses in the composite plate:

\[
\begin{align*}
\sigma_{11} &= \overline{Q}_{11} \varepsilon_1 \cdot \overline{Q}_{12} \varepsilon_1 \\
\sigma_{22} &= \overline{Q}_{12} \varepsilon_2 \cdot \overline{Q}_{12} \varepsilon_2 \\
\sigma_{12} &= \overline{Q}_{13} \varepsilon_1 \cdot \overline{Q}_{12} \varepsilon_1 
\end{align*}
\]  
(7.3)

The vertical force in the composite plate, \( F_p \) along the crack is the resultant of the shear and normal stresses acting in the plate as shown in Fig.7.4. This force is calculated using the following equation:

\[
F_p = h t \left( \overline{Q}_{11} \varepsilon_1 \cdot \overline{Q}_{12} \varepsilon_1 \cdot \overline{O}_{12} \varepsilon_2 \cdot \overline{O}_{22} \varepsilon_2 \right) / \tan(\theta_0) 
\]  
(7.4)

Where: \( t_p \) shows the total thickness of the composite plates on the two sides of the
reinforced concrete beam. \( \sigma_{11} \) and \( \sigma_{12} \) are calculated from Eq. 7.3.

Along the same crack, the vertical force carried by steel stirrups is calculated using the corresponding stress-strain relationship of the stirrup:

\[
F_v = E_s E_s A_s \frac{h_v}{\tan(\theta)} S \quad \text{where} \quad \epsilon_s \leq \frac{F_v}{E_s}
\]

Where: \( A_s \) is the cross sectional area of the two legs of the stirrups; \( S \) is the spacing of the stirrups; \( F_v \) is the yield stress of the steel stirrups; and \( E_s \) is the elastic modules of the steel stirrups.

The total shear force resisted by the composite plate and the steel stirrups is assumed equal to the shear force acting in the section of the strengthened beam. The contribution from interlocking of aggregates is ignored. Therefore, the following equation is applicable based on the equilibrium of the forces along the crack:

\[
F_v = F_p \cdot V_i \quad (7.6)
\]

Where \( V_i \) is the total shear force resisted by the composite plate and the steel stirrups.

7.2.1 The Inclination of the Cracks:

The preceding discussion provides the necessary tools to determine the inclination of the shear cracks in the strengthened beam. The following steps are followed in order to obtain the correct inclination angle that satisfies both the equilibrium and compatibility
requirements:

1. Assuming that the internal shear force is given, the crack inclination angle may be chosen as any arbitrary value. (Generally $\vartheta$ is between 20 and 60 degrees.)

2. The axial force developed in the longitudinal steel reinforcement and the composite plate ($\Delta N$) is calculated using Eq.7.2.

3. The axial force, $\Delta N$, is distributed between the composite plate and the longitudinal reinforcement in proportion to their respective stiffnesses. Therefore, the uniform axial strain in the composite plate and the longitudinal rebars ($\varepsilon_x$) is calculated using the following equation:

$$
\varepsilon_x = \begin{cases} 
\frac{\Delta N}{(A_t \cdot A_c) E_t \cdot \bar{Q}_{II} t_y h} & \text{if } \varepsilon_x \leq \frac{F_y}{E_t} \\
\frac{\Delta N \cdot (A_t \cdot A_c) F_y}{\bar{Q}_{II} t_y h} & \text{otherwise}
\end{cases}
$$  

(7.7)

Where: $A_t$ and $\bar{A}_c$ are the cross sectional area of the longitudinal tensile and compressive steel reinforcements, respectively. Generally, the effect of the plate is not considerable and the above equation can be simplified to:

$$
\varepsilon_x \leq \frac{\Delta N}{A_t \cdot \bar{A}_c} \leq \frac{F_y}{E_t}
$$  

(7.8)
4. The compressive stress in the concrete struts is calculated using Eq.7.2. The normal force resisted by the composite plate bonded to the concrete struts is negligible, if compared to the normal force resisted by the concrete struts. This conclusion is reached by considering the small cross sectional area of the plate comparing it to the cross sectional area of concrete struts, as well as the fact that the composite plate is usually bonded in a manner to provide the maximum resistance against opening of the cracks, that is fibers perpendicular to the crack. This results in the lowest modules of elasticity \(E_{c2}\) along the crack and therefore, minimum stiffness along the compressive concrete struts as shown in Fig.7.5. Therefore, it is assumed that compressive force is totally resisted by the concrete struts. The compressive stress in the concrete struts \(f_c\) is still calculated using Eq.7.2:

\[
\begin{align*}
\sigma_c &= \frac{f_c \cdot f_{c,\text{max}}}{2} \cdot \frac{\varepsilon_{c,\text{max}}}{\varepsilon_{c,\text{cu}}} \\
\text{Where:} & \quad f_{c,\text{max}} = \frac{f_c}{0.8 \cdot 0.34} \cdot \frac{\varepsilon_{c,\text{cu}}}{\varepsilon_{c,\text{cu}}} 
\end{align*}
\]

5. The compressive strain in the concrete struts \(\varepsilon_c\) is determined using the appropriate stress-strain relationship of the concrete. The model suggested by Vecchino et al. (1986) has been used in this study. According to this model, the following equations are used to obtain the compressive strain in the concrete:

\[
\varepsilon_c = \frac{\varepsilon_{c,\text{mix}}}{\varepsilon_{c,\text{cu}}} \cdot \frac{\varepsilon_{c,\text{cu}}}{\varepsilon_{c,\text{cu}}} \cdot \varepsilon_c
\]

In the above equations, \(f_c\) is the maximum compressive stress of the concrete, and \(\varepsilon_{c,\text{cu}}\) is the corresponding compressive strain.
6. Knowing $e_x$ and $e_z$, $e_1$ and $e_3$, are obtained using the following transformation equations:

$e_1 \cdot (1 - \cot^2(\theta)) \cdot \cot(\theta)$

$e_3 \cdot e_1 \cdot (1 - \cot^2(\theta)) \cdot e_3 \cot(\theta)$

(7.10)

7. The total shear force resisted by the beam is calculated based on Eq. 7.6.

Steps 1-7 are repeated for different values of $\theta$, until a close agreement between $V'$ and $V$ is reached. In that case, $\theta_c$ is the correct inclination angle of the cracks, and the stresses in the stirrups, the composite plate, and the concrete beam are calculated using the equations. The strengthened beam discussed before, can be designed to avoid any mode of failure.

7.2.2 Application of the Method and Parametric Study:

A computer program was developed to analyze the strengthened reinforced concrete section and to find the crack inclination angle that satisfies both equilibrium and compatibility equations according to the seven steps outlined above. Thereafter the shear forces resisted by the plate, steel stirrups, and steel reinforcement and also the compressive stress in the concrete struts were calculated. This program was used in performing a parametric study which is discussed subsequently. The cross section of the beam used in parametric study is shown in Fig. 7.6. The mechanical properties of the materials used in the strengthened beam are as follows:
Concrete: $f_c = 39.7 \text{ Mpa. } \varepsilon_{cu} = .003$.

Steel reinforcement and stirrups: $E_s = 200 \text{ Gpa. } F_y = 470 \text{ Mpa.}$

Composite Plate: $E_{11} = 34 \text{ Gpa, } E_{22} = 4 \text{ Gpa (var.), } G_{12} = 6.3 \text{ Gpa. } v_{12} = .36$.

The cross sectional area of steel reinforcement both in tension and compression ($A_t$ and $A_c$) were assumed $142 \text{mm}^2$. The cross sectional area of stirrups (two legs) was assumed $142 \text{mm}^2$. The spacing of the stirrups was one of the parameters altered in the study.

7.2.2.1 Effect of the Plate Thickness and the Applied Shear Force

The stirrups spacing ($S$ ) was assumed $100 \text{mm}$. The variation of the crack inclination angle versus the shear force acting on the section of the strengthened beam for four different plate thicknesses (0.1, 1.2, and 4 mm) are shown in Fig.7.7. The initiation of the stirrup yielding has been shown by a solid circle in each case. It is concluded that the crack inclination angle is constant prior to yielding of the stirrups. Thereafter the angle drops to provide a longer crack to compensate for the drop in the shear load resulting from yielding of the stirrups. The inclination angle prior to yielding of stirrups, however, increases as the thickness of the plates increase (Fig.7.8). It is observed that adding the composite plate to the reinforced concrete beam, will increase the inclination angle before yielding of stirrups as was shown in Fig.7.8, and after yielding of stirrups as can be seen by the reduction in the slope of the curves shown in Fig.7.8a through d beyond the elastic limit of stirrup. Therefore, the inclination angle of 45 degrees is conservative only for
ordinary reinforced concrete beam \((t_p = 0\)). It is also interesting to note how the shear capacity of the beam increases with the addition of the composite plate, by observing that the load causing the first yielding of stirrups increases as the plate is added and its thickness is increased, as shown in Fig.7.9.

The ratio of the shear force resisted by the composite plate to the total shear force acting in the section, is shown in Fig.7.10. This ratio is also constant as long as the stirrups have not yielded. Following the yielding of the stirrups the relative shear force in the plate increases, to compensate the loss of shear force resisted by stirrups. The considerable effect of the plate thickness on the shear force resisted by the plate is evident in this figure.

7.2.2.2. Effect of the Fiber Orientation Angle

In this part of the study, the total plate thickness was assumed 4mm (2mm on each side). The stirrups spacing was assumed 100mm, and the shear force of 200KN was used in the analysis. The effects of fiber orientation on the crack inclination angle and the shear force in the plate are shown in Fig.7.11. It is observed that both the crack inclination angle and the relative shear force in the composite plate oscillate as a function of the fiber orientation angle. The maximum shear force in the plate occurs at fiber orientation angle of 135 degrees. In this case, the crack inclination angle is about 50 degrees, indicating that the fibers are almost perpendicular to the crack, and minimizing the shear force resisted by the stirrups. The least effective case happens when fiber orientation angle is about 20-30 degrees, which the fibers are almost parallel to the crack, and the plate resists the minimum
shear force.

7.2.2.3 Effect of the Transverse Modules of Elasticity of the Plate

In this part of study the thickness of the plate, the stirrup spacing, and the shear force acting on the beam were assumed as those in part-II above. The effects of transverse modules of elasticity on the crack inclination angle and the shear force in the plate for different fiber orientation angles are shown in Figs.7.12 and 7.13, respectively. The variation of the crack inclination angle $\theta_i$ for fiber orientation angles of 90 and 135 degrees is not appreciable. However, for 0 and 45 degrees, the transverse modules can somehow change the crack inclination. In practical applications, and based on the discussion given in the previous part, the plate is usually bonded in a way that the fibers are perpendicular to the crack (i.e. $\theta$=135 degrees), therefore the effects of transverse modules of elasticity on the shear force in the plate and fiber orientation angle are negligible.

7.2.2.4 Effect of Stirrup Spacing

Fiber orientation angle was assumed 135 degrees and the effect of stirrup spacing was studied. The variations of crack inclination angle and the shear force in the plate are shown in Fig.7.14. As expected, by increasing the stirrup spacing crack inclination angle decreases, while the shear force in the plate increases.
7.2.3 Comparison with Experimental Results

One of the beams (shear beam IF) tested by Norris, et al. (1996) was used to verify the method. The cross section of this beam is shown in Fig. 7.7. The fiber orientation angle in this beam is 90 degrees. The total thickness of the fabric bonded to the side faces was 0.66 mm. The mechanical properties of the material are the same as those used in parametric study. The cross sectional area of steel stirrups was 56.5 mm² (two legs). The crack inclination angle is calculated 38 degrees for the strengthened beam. Assuming that steel stirrups have yielded and also ignoring the effect of shear stresses in the plate, the following equation can be used to determine the ultimate shear capacity of the strengthened beam:

\[ V_s \cdot \frac{h_s}{S \tan \theta_e} F_s A_s \cdot \frac{h}{\tan \theta_e} \sigma_s' \]

(7.11)

In the above equation, \( \sigma_s' \) is the ultimate strength of the composite plate in the direction normal to the crack. This strength can be approximated by the axial strength of the plate when the inclination of the fibers with respect to the loading direction is 45 degrees. For the composite plate used in experimental study, this strength has been reported 67.78 Mpa (Norris, et al., 1996). Therefore the ultimate shear capacity of the strengthened beam is calculated 63.69 kN. The experimental shear capacity of this beam has been 68.43 kN, which shows agreement between the experimental and theoretical results. The difference between the results is due to approximations such as ignoring the
shear force resisted by aggregate interlocking action.
CHAPTER 8
CONCLUSIONS

Shear and normal stress concentrations near the cut-off point of the FRP plate and also flexural cracks must be considered in the design of reinforced concrete beams strengthened with epoxy bonded FRP plates. These stresses may lead to failure modes such as peeling and debonding of the plate or local failure in the concrete layer between the FRP plate and longitudinal reinforcements of the beam. The method presented in chapter 4, can be used to predict the distribution of shear and normal stress at the interface of the plate and the adhesive throughout the entire length of the plate and particularly the location of the cut-off point. The maximum values of these stresses which are important from design point of view are given by the following simple equations:

\[
\tau_{\text{max}} = f_p \left( b_1 \sqrt{A} \cdot b_2 \right)
\]

\[
f_{n,\text{max}} = \frac{K_n}{2b^2} \left( \frac{V_p}{E_p I_p} - \frac{V_c \cdot \beta M_0}{E_c I_c} \right) \cdot \frac{qE_p l_p}{b_p E I_c}
\]

The method was developed based on linear elastic behavior of the concrete. However, the effect of flexural cracks has been investigated and included in this study. The effect of anisotropic behavior of FRP plate on stress distribution were studied as well. It was concluded that the isotropic assumption for the behavior of FRP plate, used in developing this method, is acceptable. The method was applied to a beam that had been tested and had
failed due to local failure of the concrete layer between the FRP plate and longitudinal reinforcement. The results of the presented method indicate a good agreement to both finite element and experimental results.

Design guidelines that can be followed for strengthening reinforced concrete beams using fiber reinforced plastic plates have been developed in chapter 5. The effect of the stresses that concrete beam undergoes before upgrading was considered. Rupture of the plate and crushing of concrete are the major modes of failure which are considered in estimating the ultimate strength of the plated beam. Based on modes of failure, and condition of the tensile and compressive steel reinforcement at ultimate case, different equations were developed to calculate the ultimate capacity of the strengthened beam. Local failure of concrete beam at the plate end, and debonding of the plate due to shear stress concentration at the flexural cracks were also considered in developing these guidelines.

The effect of the composite plates bonded to the side faces of the reinforced concrete beams, on the shear force carried by the concrete beam was studied in chapter 6. Closed form solutions were developed based on strain compatibility of the plates and the beam, assuming that the materials behave linearly elastic, and there is complete bonding between the plate and the beam. The shear force in the plate is composed of two different components. The first one is caused due to orthotropic behavior of the plate. This component is present even if the beam resists only pure bending. The second component, which is caused by variations of the bending moment (internal shear force on the section).
is calculated in a manner similar to the isotropic plate. Two different cases of uncracked and flexurally cracked beams were studied. It was observed that for the uncracked beam the shear force resisted by the composite plate is negligible. However, for beam with flexural cracks this force is considerably higher and depends on the thickness and fiber orientation of the plate. In general, the maximum tensile stress in a strengthened reinforced concrete beam can be calculated by using the present method and assuming that the reinforced concrete beam is uncracked. This stress can be compared to the modules of rupture of concrete to investigate if the concrete beam is cracked. Then by using the corresponding equations presented in this chapter, maximum compressive strain in concrete, stresses acting at different points of the section, and the shear force in the concrete beam are calculated.

The effect of the FRP plate on the ultimate shear capacity of reinforced concrete beams was studied in chapter 7. Truss analogy and compression field theory were used to develop the necessary equilibrium and compatibility equations. These equations were combined to provide the necessary tools for calculating crack inclination angle which is different from ordinary reinforced concrete beams. A parametric study showed that several parameters such as plate thickness, fiber orientation angle, and stirrup spacing can considerably change this angle and the behavior of the retrofitted beam. Following determination of the crack inclination angle, the stresses in the stirrups, composite plate and concrete struts are calculated using the equations presented in this chapter. These stresses can be compared to the ultimate values for design purposes, to calculate the shear capacity
of the strengthened beam.

Assuming that the steel stirrups yield before the failure of the plate, and also ignoring the effect of the shear stresses in the composite plate along the crack, the following simplified equation can be used to calculate the nominal shear strength of the beam:

\[ V_n = \frac{h_n}{S \tan(\theta_p)} F_y A_y \cdot \frac{h}{\tan(\theta_n)} \sigma_n t_r \]  \hspace{1cm} (8.2)

In the above equation \( \sigma_n \) is the ultimate strength of the composite plate in the direction normal to the crack.
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REFERENCES


