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OPTIMIZATION IN THE DISTURBED STATE CONCEPT
CONSTITUTIVE MODELING
AND APPLICATION IN FINITE ELEMENT ANALYSIS

by
Joseph Yongxiang Chen

A Dissertation Submitted to the Faculty of the
DEPARTMENT OF CIVIL ENGINEERING AND
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THE UNIVERSITY OF ARIZONA

1997
As members of the Final Examination Committee, we certify that we have read the dissertation prepared by Joseph Yongxiang Chen entitled Optimization in the Disturbed State Concept Constitutive Modeling and Application in Finite Element Analysis and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

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Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copy of the dissertation to the Graduate College.

I hereby certify that I have read this dissertation prepared under my direction and recommend that it be accepted as fulfilling the dissertation requirement.

Dissertation Director, Chandrakant S. Desai Date 8/15/97
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SIGNED: [Signature]
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In constitutive modeling, one of important tasks is to calibrate the model. To calibrate a model is to find out the values of the model parameters for a material whose stress-strain behavior is to be simulated by the model. Conventional approach is to find certain well-defined states in certain tests where behavior of a material is controlled by those parameters and then the stress and strain and other history parameters at those states can be used to find them. However, as the model evolves more sophisticated, such as the Disturbed State Concept Model (DSC), in which a greater number of parameters are introduced to account for behavior of the material under various stress conditions, it is not possible to find an easy way to calibrate, mainly due to certain stress-strain states are difficult to be isolated out. In this study an optimization approach is proposed by using quasi-Newton method with BFGS updating scheme. Contrary to the conventional approach which determines parameter values by averaging values of laboratory tests or by simple data fitting of the assumed parameter relations, the optimization approach is to find the best agreement of the model simulation with the experimental observation, then gives a set of parameter values for the best agreement which is quantitatively measured by the least error residual. Weight is used in the optimization procedure to emphasize on better simulation agreement with the observation for certain stress path conditions. This weight can be decided based on the engineering judgment for certain practical problems. By using the DSC model to simulate stress-strain response of various laboratory tests of sands, and by using the DSC model in a finite element analysis to simulate dynamic soil-structure interaction response of a shaking table test for saturated soil, it is shown that the optimization approach yields closer agreement with the observation. Based on the proposed optimization approach, a computer program DSCOPT is developed for the DSC model. The program takes the laboratory test data as input and outputs the
model parameter values by the conventional and optimized approaches, and graphics plots of the model simulation.
Chapter 1
INTRODUCTION

Modern design of majority of man-made civil engineering structures involves solutions of problems involving soil, concrete, rock and the complex material interfaces in those structures. Examples of such civil engineering structures include buildings, bridges, roadways, canals, dams, offshore oil platforms, and underground caverns. One of the main concerns is to obtain an approximation or estimation of displacements, stresses and strains for the problem at hand. Despite the fact that many numerical solution techniques, e.g., the finite element method, have been developed for the solutions of the problems in civil engineering, many of those techniques utilize some idealized constitutive models based on elasticity or simple elasto-plasticity. Unfortunately, most engineering materials, especially geologic, do not obey the theory of elasticity or elasto-plasticity.

To fully understand the behavior of the materials, many research studies have been performed for constitutive modeling for their mechanical response. As a result, additional models have been proposed and developed based on the theory of elasticity and plasticity. The models have evolved to be more sophisticated, more capable of accounting for diverse stress-strain response exhibited by materials such as soil, rocks, concrete, joints and interfaces. Among those models, the most promising new models are models in the Hierarchical Single Surface (HiSS ) family and the Disturbed State Concept (DSC ) family, developed by Desai and his co-workers (Desai et al., 1986; Desai, 1992, 1995).
1.1 Statement of the Problem and Motivation

As constitutive models evolve, the models themselves become more sophisticated. This usually is due to that the theory of plasticity requires definitions of a yield surface, or a plastic potential surface, and a hardening law. In new models those yield surface and hardening law become more complicated, as a greater number of model parameter are introduced. Thus it is quite complicated or difficult to calibrate the model with those material parameters. Here calibration of a constitutive model means the determination of appropriate values for the material parameters so that the mechanical behavior predicted or simulated by the constitutive model matches as closely as possible the observed.

Before a constitutive model can be used in the numerical simulation, it needs to be calibrated based on laboratory tests. To calibrate a less complex constitutive model which has only a few material parameters, traditional approach is to find certain well-defined states in certain tests where behavior of a material is controlled by only those a few material parameters. Then the stress and strain and other history parameters at those states can be used to find them. However, it is harder to calibrate more complicated constitutive models in which a greater number of material parameters are introduced. To overcome this difficulty and to improve the constitutive model simulation, a better approach such as optimization is needed.

Using optimization approach to calibrate the constitutive model will not only overcome the difficulties discussed above but will also deal better with the complexity of material parameter couplings and constraints, which in most cases is neglected by the traditional (averaging) approach.

The motivation of the proposed research herein is to develop an optimization procedure to find best parameters in the advanced DSC model.
1.2 Scope of the Research and Objectives

In the present research, the objectives are:

1. Investigate into how the optimization technique can be used to significantly improve the constitutive modeling, particularly modeling by the DSC model.

2. Develop a general procedure to optimize the DSC model so that the optimized DSC model will be capable of better predicting the stress-strain response of materials.

3. Verify that the optimization of the DSC model is effective in that the prediction or simulation to stress-strain responses by the optimized DSC model matches better with the observed responses from laboratory tests, compares to the result by the conventional averaging procedure. This is done through using the readily available laboratory test data and simulating response of a soil-structure system in saturated soil subject to dynamic load using finite element analysis.

4. Implement computer programs based on the proposed optimization procedures so that they will provide general purpose tools for calibration of DSC model. The computer program can be able to integrate all possible stress paths to account for behavior of materials under different stress paths. The implementation also integrates engineers' options to specify the weight, based on their engineering judgment, for the stress-strain responses under different stress paths.

5. Finally, incorporate the constitutive modeling optimization into a improved computer finite element procedure, for solutions of geomechanical problems.

1.3 How the Text Is Organized

In Chapter 2 a review of literature on optimization in the constitutive modeling is presented, along with a review of constitutive models.
Chapter 3 describes how the DSC model which has been proposed by Desai (1992), (Desai, 1995) and subsequently his co-workers is derived in a form of incremental stress-strain relation. In addition, some technical issues are addressed in numerical implementation of the DSC.

In Chapter 4 the detail procedure for calibrating the DSC model is discussed, along with some technical issues which have not been found to be addressed in previous works.

Chapter 5 first discusses the constitutive optimization, including mathematical optimization strategy and objective functions for the DSC model. Then the effectiveness of the DSC optimization is verified by comparing the model prediction results between the the optimized and averaged DSC model, including the results from laboratory tests from the conventional cylindrical device and true triaxial device, and the results from simulation of a soil-structure system subjected to dynamic loads by using finite element analysis.

Chapter A presents the implementation of computer program DSCOPT based on the conventional procedure of model calibration and the optimization procedure discussed in Chapter 5.

In Chapter 7 the summary and conclusions inferred from the results of this study as well recommendations for future research are presented.
Chapter 2

LITERATURE REVIEW ON CONSTITUTIVE MODELING
OPTIMIZATION AND DISTURBED STATE CONCEPT (DSC) MODELING

This chapter presents literature review in two major parts – review on optimization applied in constitutive modeling and review on constitutive modeling based on the Disturbed State Concept (DSC).

2.1 Optimization In Constitutive Modeling

Only a limited number of published papers are available on the topic of constitutive optimization. This section reviews work of these articles.

An optimization procedure for the Bounding surface model was presented by De-Natale et al. (1983). The model to be optimized was called Bounding Surface model developed by Dafalias and Herrmann (1980). The optimization optimization strategy was based on the Quasi-Newton method. The objective function was defined as the total error between the experimental observations and model predictions, and the error was expressed by summing up the discrete residuals at each of the experimental data points. Then the optimized model was verified by predicting the response under undrained triaxial compression tests for an over consolidated soil. It was reported that the optimization was effective in terms of better simulation the stress-strain responses. However, the authors did not give any comparison with non-optimized models. So it was not clear that how much improvement was achieved by the optimization. Anandarajah and Agarwal (1991) applied the optimization technique to a modified bounding surface model for anisotropic soil material.

Using an optimization scheme called "Genetic Algorithm" originally developed in field of biology and artificial intelligence by Holland (1975), Salami et al. (1994)
performed research on constitutive modeling optimization of the HiSS $\delta_0$ (Desai et al., 1986) model for concrete materials.

Pal et al. (1996) applied the genetic algorithm to HiSS $\delta_1$ (Desai et al., 1986) model for sands. While it was reported that the improvements were achieved considerably, the objective function (or penalty function, as it was called in the article) was simply the error residual of the ultimate values between the predicted and the observed data. The optimization scheme was only to optimize the simulation of the ultimate stress-strain response as closely as possible.

The work described above dealt majorly with plastic hardening constitutive models. These are limited and are not capable to describe general behavior were reasonably good for simulating response of materials exhibited majorly hardening behavior. However, the models which were optimized were incapable for describing the observed behavior of material exhibited both hardening and softening and a better model is needed. DSC model is such a model that is capable of describing the sophisticated behavior of hardening and softening.

2.2 Constitutive Modelings based on Disturbed State Concept

The idea in the theory of the Disturbed State Concept (DSC) model was first introduced by Desai (1974) to characterize the behavior of over-consolidated clays exhibiting softening behavior. Desai postulated that the response of an over-consolidated soil can be expressed in terms of its response in its normally consolidated state (Curve 1, Fig. 2.1) as reference state, with influence of over-consolidation (Curve 2, Fig. 2.1) being treated as disturbance. This idea was later developed as the Disturbed State Concept (DSC) for behavior of geo-material, and interfaces and joints.

"The disturbed State Concept is a unified modeling theory for the characterization of mechanical behavior of material and interfaces (joints)" (Desai, 1992, 1995).
Figure 2.1. Observed Stress-Strain Behavior As composed of Behavior of Normal Consolidating and of Part Causing Overconsolidation
This theory allows for the incorporation of the internal microstructural changes and the resulting mechanisms in a deforming material. When a material is subjected to external excitation or loading, microstructural changes take place inside the material. Initially, the material is for the most part in relative intact (RI) state. As the external disturbances increase, the material transforms from the intact state, through a process of conscious self-adjustment, to the fully adjusted or critical state (Desai, 1992, 1995). Essentially, externally applied forces cause transformation inside the material so that the material transforms from the relative intact state to the fully adjusted (FA) (critical) state. Henceforth, at any given time the material is composed of randomly distributed clusters of the material in relative intact and in fully adjusted state. Consequently, the observed response of the material is defined by a combination of the response of the RI part and the response of the FA part of the material. Then response of the intact part and the response of the fully adjusted part are the reference responses of the material. An analogy given by Desai explains the main idea behind the DSC very clearly: "Consider a cube of ice which melts under a given temperature and includes a mixture of ice and water. The fully adjusted state is analogous to water. The behavior of the mixture can be expressed in terms of its two reference responses for ice and for water" (Desai, 1992). The deviation of the observed response from the reference response constitutes the disturbance that is caused due to applied forces and is affected by factors such as friction, anisotropy, microcracking, damage, fracture, and creep (Desai, 1992, 1995).

Engineering materials are usually inhomogenous and often involve flaws and discontinuities. When a material is subjected to an external disturbance such as a load, a temperature change, or an environmental load, the response of the material is not the same at every point in the material. This is due to the discontinuities within the material. The stress and strain will be different at different parts of the material. Therefore, the material cannot be treated as though it were a continuum. And the continuum mechanics laws can only be applicable for the continuum parts such
as the intact part, rather than for the entire domain of the material. To be able to use the continuum mechanics laws for a discontinuous medium with scattered continuum parts, certain variables have to be introduced and be defined to account for the discontinuous parts. It is necessary to introduce the discontinuous nature of the material by incorporating "finite" or nonlocal zones as influenced by a characteristic dimension which is dependent on the properties of particles or clusters of particles that constitute the material. The transformation of the material from the intact state to the fully adjusted state is defined through a disturbance function \( D \) as illustrated in Fig. 2.2. As the volume of the fully adjusted material increases the volume of the intact part decreases.

At any given time and in any given direction the ratio of the area of the material in the fully adjusted state to the total area of the material, in a given direction is equal to the disturbance \( D \) for that direction. Now the reference states and the disturbance function will be explained in following section.

### 2.2.1 Relative Intact State of the Material

The relative state (RI) of the material is one that excludes the influence of the factors that are considered as causing the disturbance. For example, if microcracking and subsequent softening is considered as the disturbance, the response of the material without microcracking can be treated as the intact state (Desai, 1992). The relative intact state of the material may vary with respect to the initial hydrostatic pressure, the initial density, and some other intrinsic factors. Thus the relative intact state is just a relative definition of the reference. The relative intact state can be determined from laboratory experiments or by approximation. If the relative intact response is obtained by experiments the material needs to be tested under different conditions such as initial density and initial pressure.
FIGURE 2.2. Schematic Plot of Stress-Strain Curve for Disturbance Function

(From Desai, 1992)
Figure 2.3. Schematic Plot of Growth of fully Adjusted State

(From Desai, 1992)
2.2.2 Fully Adjusted State of Material

The fully adjusted state (FA) of the material is an asymptotic state in which the material may no longer be further disturbed. At this state the disturbance or disorder is at its maximum and the material cannot transform into another state. In the fully adjusted state the material may be assumed:

(A): to carry no load and it has zero strength like a void as in the classical continuum damage model (Kachanov, 1986)

(B): carry only hydrostatic stress but no shear stress at all (like a constrained liquid), and

(C): can carry hydrostatic stress and shear stress that is reached up to that state, but cannot carry any additional shear stress. The material will deform in shear with zero volume change.

The transition of a material from the relative state to the fully adjusted state is illustrated in Fig. 2.3.

2.3 Hierarchical Single Surface (HiSS) Modeling

Since this model will be adopted for describing the relative state behavior of a material, this section briefly reviews the HiSS model.

The Hierarchical Single Surface (HiSS) Modeling approach was proposed by Desai et al. (1986). This modeling approach allows for progressive development of models of higher orders corresponding to different levels of complexities – perfect plasticity, hardening and continuous yielding, etc. The basic form in the HiSS family is $\delta_0$ model. The single yield surface function $F$ in a non-dimensional form, as used by Desai and Wathugala (1987), is given by

$$F = J_{2D} - (-\alpha J_1^p + \gamma J_1^p)(1 - \beta S)^m = 0 \quad (2.1)$$
where

\[ J_1 = \frac{J_1 + J_2}{p_a} \]  
\[ J_{2D} = \frac{J_{2D}}{p_a} \]  
\[ S_r = \frac{\sqrt{27} J_{3D}}{2 J_{2D}^{3/2}} \]  

\[ J_1 \] is the first invariant of stress tensor, \( J_{2D} \) and \( J_{3D} \) the second and third invariants of deviatoric stress tensor, respectively. \( \gamma, \beta, m, n, J_s \) are material parameters. \( p_a \) is the atmospheric pressure in unit of stress, and \( \alpha \) is a hardening function given by

\[ \alpha = \alpha (\xi, \xi_v, \xi_D) \]  

where \( \xi, \xi_v \) and \( \xi_D \) are trajectories of total, volumetric, and deviatoric plastic strains, respectively. Several forms of hardening functions are proposed for different materials (Desai and Hashmi, 1987; Wathugala and Desai, 1990). Here, \( F_b \) describes the shape of \( F \) in the \( J_1 - \sqrt{J_{2D}} \) space. Figure 2.4 shows the shape of \( F \) with different values of \( \alpha \). \( F_s \) is the function describing the shape of the yield function \( F \) in a octahedral plane as shown in Figure 2.5.

Based on the basic \( \delta_0 \)-Hi\( \text{ISS} \) model, several other models in the Hi\( \text{ISS} \) family have been proposed in order to simulate observed behavior of soils. Among them are \( \delta_i \) model for non-associative flow rule, \( \delta_2 \) for anisotropic hardening, \( \delta_i+r \) for damage state, and \( \delta_i+v \) for viscoplasticity. In this study, the \( \delta_0 \) model is used to describe the RI behavior in the DSC model.

The DSC approach has been successfully verified with respect to various laboratory test data, including rock joint interfaces (Ma, 1990; Desai and Ma, 1992), cohesion-less soils (Armaleh and Desai, 1990), undrained clay (Katti and Desai, 1992), solders in chip-substrate system in electronic packaging (Chia and Desai, 1994).
ceramic composites (Toth and Desai, 1994), finite element thermomechanical analysis for electronic packaging (Basaran and Desai, 1994), saturated clay-steel joint interfaces (Rigby, 1996), and sand-steel joint interfaces under static and cyclic loading (Alanazy, 1996).

"The disturbed state concept (DSC) is a unified and powerful modeling approach, contains the traditional damage model as a special case, and is based on the idea that at any stage during loading, the material is composed of two parts, relative intact (RI) and fully adjusted (FA) or critical state. The observed behavior of the material is then expressed in terms of its behavior under the RI and FA states by using the disturbance function." (Desai, 1996).
FIGURE 2.4. Shapes of Yield Surface in $J_1 - \sqrt{J_{2D}}$ Plane
FIGURE 2.5. Shapes of Yield Surface on Octahedral Plane
Chapter 3

DERIVATION AND NUMERICAL IMPLEMENTATION OF DSC MODEL

This chapter covers the steps in the mathematical derivation of Disturbed State Concept (DSC). Some technical issues in numerical implementation of DSC are addressed.

The DSC model has been successfully applied for predicting the constitutive behavior of various material such as rock joints, interfaces, sands and clays. In the DSC, the material is assumed to transform continuously from relative intact (RI) state to fully adjusted (FA) state. Thus the observed response is based on behavior of material parts in the two states:

- Response of relatively intact state which excludes the effects of disturbance
- Response of fully adjusted state

The transformation of the materials from the RI state into that in the FA state is illustrated in Fig. 2.3 and defined by the disturbance function $D$ (illustrated in Fig. 2.2).

3.1 Relative Intact (RI) State

The relatively intact (RI) state is an idealized state in which the disturbance is not taken into account. Many elasto-plastic models can be used to describe the materials for RI state. Here, HiSS model $\delta_0$, the basic model in HiSS family of constitutive models, is used to describe the behavior of the materials in the RI state. HiSS $\delta_0$ model has been successfully used to predict responses of various cohesive and cohesionless materials and interfaces.

HiSS $\delta_0$ model is based on associative plasticity theory and isotropic hardening. The behavior of stress-strain response deviating from that predicted by HiSS $\delta_0$ model
can be considered as caused by the disturbance in the material. The yield function $F$ in the HiSS $\delta_0$ model is stated in Equation 2.1. The basic form of the hardening function $\alpha$ is

$$\alpha = \frac{a_1}{\xi^m}$$

(3.1)

where $a_1$ and $\eta_1$ are material parameters and $\xi$ is trajectory of total plastic strains which is defined as

$$\xi = \int \sqrt{\det \varepsilon_p} \, d\varepsilon_p$$

(3.2)

and $d\varepsilon_p_{ij}$ is the incremental plastic strain tensor. Another form of hardening function (Katti and Desai, 1992) used for saturated clays is

$$\alpha = \frac{h_1}{(\xi_v + h_3 \xi_v^2)^{h_2}}$$

(3.3)

where $h_1, h_2, h_3$ and $h_4$ are material parameters, $\xi_v$ is the volumetric plastic strain defined by

$$\xi_v = \int \frac{1}{\sqrt{3}} |d\varepsilon_v|$$

(3.4)

and $\xi_D$ is the trajectory of the deviatoric plastic strain tensor defined as

$$\xi_D = \int \sqrt{\det \varepsilon_p} \, d\varepsilon_p$$

(3.5)

where $d\varepsilon_p_{ij}$ is deviatoric plastic strain increment tensor. In the theory of plasticity, for small strains, the total strain increment is decomposed into two parts: elastic strain increment $d\varepsilon^e_{ij}$ and plastic strain increment $d\varepsilon^p_{ij}$, that is,
The stress is assumed to be related to the elastic strain given by the elastic constitutive relation

\[ d\sigma_{ij} = C_{ijkl}^e \, d\varepsilon_{kl} \]  

(3.7)

where \( C_{ijkl}^e \) is the elastic constitutive tensor.

The plastic stress-strain relation can be derived by using the consistency condition, \( dF = 0 \), and the normality rule as

\[ d\sigma_{ij} = C_{ijkl}^p (d\varepsilon_{kl} - d\varepsilon_{kl}^p) = C_{ijkl}^{ep} \, d\varepsilon_{kl} \]  

(3.8)

where \( C_{ijkl}^{ep} \) is the elastic-plastic constitutive tensor as expressed below

\[ C_{ijkl}^{ep} = C_{ijkl}^e - \frac{C_{ijrs}^e n_{rs} n_{pq} C_{pqkl}^e}{n_{mn} C_{mnqp} n_{pq} - \frac{\partial F}{\partial \varepsilon} \left( \frac{\partial F}{\partial \sigma_{pq}} \frac{\partial F}{\partial \sigma_{pq}} \right) \frac{1}{2}} \]  

(3.9)

and \( n_{ij} \) is a unit tensor defined as

\[ n_{ij} = \frac{\partial F}{\partial \sigma_{ij}} \left( \frac{\partial F}{\partial \sigma_{ki}} \frac{\partial F}{\partial \sigma_{kl}} \right) \frac{1}{2} = \left\| \frac{\partial F}{\partial \sigma_{kl}} \right\| \]  

(3.10)

### 3.2 Incremental Stress-Strain Relation of Relative Intact State

#### 3.2.1 Given Strain to Calculate Stress

The yield function is expressed as

\[ F = F(\sigma, \xi) \equiv 0 \]  

(3.11)
\[
dF = \frac{\partial F}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial F}{\partial \xi} d\xi = 0
\] (3.12)

The flow rule is
\[
d\varepsilon_{ij}^p = \lambda \frac{\partial Q}{\partial \sigma_{ij}}
\] (3.13)

Hooke's Law
\[
d\sigma_{ij} = C_{ijkl} (d\varepsilon_{kl} - d\varepsilon_{kl}^p)
\] (3.14)

Substituting Eq. 3.13 into Eq. 3.14 we get
\[
d\sigma_{ij} = C_{ijkl} (d\varepsilon_{kl} - \lambda \frac{\partial Q}{\partial \sigma_{ij}})
\] (3.15)

By the definition of the trajectory of plastic strain
\[
d\xi = \sqrt{d\varepsilon_{ij}^p d\varepsilon_{ij}^p}
\] (3.16)

Substitute Eq. 3.13 into Eq. 3.16 we get
\[
d\xi = \lambda \sqrt{\frac{\partial Q}{\partial \sigma_{ij}} \frac{\partial Q}{\partial \sigma_{ij}}}
\] (3.17)

Substituting Eqs. 3.15, 3.17 into Eq. 3.12 leads to
\[
\frac{\partial F}{\partial \sigma_{ij}} C_{ijkl}^e d\varepsilon_{kl} - \lambda \frac{\partial F}{\partial \sigma_{ij}} C_{ijkl}^e \frac{\partial Q}{\partial \sigma_{kl}} + \lambda \frac{\partial F}{\partial \xi} \sqrt{\frac{\partial Q}{\partial \sigma_{ij}} \frac{\partial Q}{\partial \sigma_{ij}}} = 0
\] (3.18)

Then we can get \( \lambda \) as
\[
\lambda = \frac{\frac{\partial F}{\partial \sigma_{ij}} C_{ijkl}^e d\varepsilon_{kl}}{\frac{\partial F}{\partial \sigma_{ij}} C_{ijkl}^e \frac{\partial Q}{\partial \sigma_{kl}} - \frac{\partial F}{\partial \xi} \sqrt{\frac{\partial Q}{\partial \sigma_{ij}} \frac{\partial Q}{\partial \sigma_{ij}}}}
\] (3.19)

By rearranging the index we get
\[
\lambda = \frac{\frac{\partial F}{\partial \sigma_{mn}} C_{mpq}^e d\varepsilon_{pq}}{\frac{\partial F}{\partial \sigma_{rs}} C_{rstu}^e \frac{\partial Q}{\partial \sigma_{tu}} - \frac{\partial F}{\partial \xi} \sqrt{\frac{\partial Q}{\partial \sigma_{rs}} \frac{\partial Q}{\partial \sigma_{rs}}}}
\] (3.20)
Substituting Eq. 3.20 into Eq. 3.15 gives

$$d\sigma_{ij} = C_{ijkl}^e d\varepsilon_{kl} - \frac{\frac{\partial F}{\partial \sigma_{mn}} C_{njpq}^e d\varepsilon_{pq} \frac{\partial Q}{\partial \sigma_{kl}}}{\frac{\partial F}{\partial \sigma_{rs}} C_{rs}^e \frac{\partial Q}{\partial \sigma_{st}} - \frac{\partial F}{\partial \xi} \sqrt{\frac{\partial Q}{\partial \sigma_{rs}} \frac{\partial Q}{\partial \sigma_{rs}}}}$$  \hspace{1cm} (3.21)

By exchanging the index of the second term in Eq. 3.21 the way of $kl \rightarrow pq$ and $pq \rightarrow kl$

$$d\sigma_{ij} = \left( C_{ijkl}^e - \frac{\frac{\partial F}{\partial \sigma_{mn}} C_{jipq}^e \frac{\partial Q}{\partial \sigma_{pq}} \frac{\partial F}{\partial \sigma_{mn}} C_{nijkl}^e}{\frac{\partial F}{\partial \sigma_{rs}} C_{rstu}^e \frac{\partial Q}{\partial \sigma_{st}} - \frac{\partial F}{\partial \xi} \sqrt{\frac{\partial Q}{\partial \sigma_{rs}} \frac{\partial Q}{\partial \sigma_{rs}}}} \right) d\varepsilon_{kl}$$  \hspace{1cm} (3.22)

### 3.2.2 Given Stress to Calculate Strain

$$d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p$$

$$= D_{ijkl}^e d\sigma_{kl} + \lambda \frac{\partial Q}{\partial \sigma_{ij}}$$  \hspace{1cm} (3.23)

$$d\sigma_{ij} = C_{ijkl}^e (d\varepsilon_{kl} - d\varepsilon_{kl}^p)$$  \hspace{1cm} (3.24)

$$C_{ijkl}^e d\varepsilon_{kl} = d\sigma_{ij} + \lambda C_{ijkl}^e \frac{\partial Q}{\partial \sigma_{kl}}$$  \hspace{1cm} (3.25)

Substituting Eq. 3.25 into Eq. 3.20, then $\lambda$ can be expressed as

$$\lambda = -\frac{\frac{\partial F}{\partial \sigma_{mn}} d\sigma_{mn}}{\frac{\partial F}{\partial \xi} \sqrt{\frac{\partial Q}{\partial \sigma_{rs}} \frac{\partial Q}{\partial \sigma_{rs}}}}$$  \hspace{1cm} (3.26)

$$d\varepsilon_{ij} = \left( D_{ijkl}^e - \frac{\frac{\partial F}{\partial \sigma_{kl}} \frac{\partial Q}{\partial \sigma_{ij}}}{\frac{\partial F}{\partial \xi} \sqrt{\frac{\partial Q}{\partial \sigma_{rs}} \frac{\partial Q}{\partial \sigma_{rs}}}} \right) d\sigma_{kl}$$  \hspace{1cm} (3.27)

### 3.3 Fully Adjusted (FA) State

The fully adjusted (FA) state of the material is an "asymptopic" state. In this research, the critical state can be used to define fully adjusted state. For soils, the void ratio $e^c$ the material attains in a shear test is only related to hydrostatic stress $J_1$ as
\[ e^c = e_0^c - \lambda \ln \left( \frac{J_1^c}{3p_a} \right) \]  
(3.28)

or

\[ J_1^c = 3p_a \exp \left( \frac{e_0^c - e^c}{\lambda} \right) \]  
(3.29)

where \( e_0^c \) is the critical void ratio under atmospheric pressure \( J_1 = 3p_a \), and \( \lambda \) is a material parameter. At this critical state, the shear stress the material can carry is given by

\[ \sqrt{J_2^c} = \tilde{m}J_1^c \]  
(3.30)

where \( \tilde{m} \) is a material parameter.

Equations 3.29 and 3.30 are used to define the critical response of material at fully adjusted state. Those may be reduced to some special cases as:

- Letting \( \tilde{m} = 0 \) leads to \( \sqrt{J_2^c} = 0 \) and further leads to \( J_2^c = 0 \), meaning the material (like water) in FA state can only carry the hydrostatic stress.

- Letting both \( \tilde{m} = 0 \) and \( \lambda \rightarrow \infty \) means the material in FA state can neither carry shear stress nor hydrostatic stress. Such material having zero stress capacity behaves like a "void".

### 3.4 Disturbance Function \( D \)

In general, the disturbance function \( D \) can be defined as

\[ D = \frac{M^c_s}{M_s} \]  
(3.31)

where \( M^c_s \) is the mass of material in the critical state, and \( M_s \) the total mass of the material. Thus here the disturbance function is a scalar. However, to describe
micro-cracking like in concrete $D$ may be defined as a tensor (Desai and Toth, 1996). The disturbance increases as the plastic deformation increases. At the beginning of the loading, $D$ is zero, that is, the initial disturbance $= 0$. As the load increases, the deformation increases and consequently the disturbance increases. When the value of $D$ becomes 1, the material reaches the FA state.

Based on the observation of laboratory tests, various forms of disturbance functions have been proposed, for example, Frantziskonis and Desai (1987), Wathugala and Desai (1990). The one used by Armaleh and Desai (1990) is adopted here and defined as

$$D = D_u \left[1 - \exp \left(-A\frac{\varepsilon^{\varepsilon}}{D_u}\right)\right] \quad (3.32)$$

where $A$, $Z$ and $D_u$ are material parameters. As a simplification, $D_u$ may be adapted as unity. if test data is available to define the residual state, $D_u < 1$ can be defined.

### 3.5 Stress-Strain Relation of the DSC

Based on the theory of the disturbed state concept, the expression for the observed or average stress is derived as (Desai, 1995; Desai and Toth, 1996):

$$\sigma_{ij}^a = (1 - D) \sigma_{ij}^t + D\sigma_{ij}^c \quad (3.33)$$

Since $J_i^a = \sigma_{ii}^a$, $J_i^t = \sigma_{ii}^t$, and $J_i^c = \sigma_{ii}^c$,

the above equation can lead to the following equation

$$J_i^a = (1 - D) J_i^t + D J_i^c \quad (3.34)$$

Multiply both sides of Eq. (3.34) by $\delta_{ij}$ and subtract the result from Eq. (3.33), the following equation can be obtained

$$S_{ij}^a = (1 - D) S_{ij}^t + D S_{ij}^c \quad (3.35)$$
where $S_i^\epsilon$, $S_i^\alpha$, and $S_i^\sigma$ are the deviatoric stress tensor of the averaged, intact, and critical stress, respectively.

Now assume that $S_i^\epsilon$ is related to $S_i$, that is, $S_i^\epsilon = k(D) S_i$, the relation of $S_i^\epsilon$ and $S_i$ can be expressed as

$$
S_i^\epsilon = \frac{\sqrt{J_{2D}^c}}{\sqrt{J_{2D}}} S_i
$$

(3.36)

Thus $\sigma_i^\epsilon$ can be expressed as

$$
\sigma_i^\epsilon = S_i^\epsilon + \frac{1}{3} J_i^c \delta_{ij}
$$

(3.37)

Substituting Equation 3.36 into Equation 3.37 leads to

$$
\sigma_i^\epsilon = \frac{\sqrt{J_{2D}^c}}{\sqrt{J_{2D}}} S_i + \frac{1}{3} J_i^c \delta_{ij}
$$

(3.38)

And further substituting Equation 3.30 into Equation 3.38, and rearranging leads to

$$
\sigma_i^\epsilon = \frac{\bar{m} J_i^c}{\sqrt{J_{2D}}} S_i + \frac{1}{3} J_i^c \delta_{ij} = J_i^c \left( \frac{\bar{m} S_i}{\sqrt{J_{2D}}} + \frac{1}{3} \delta_{ij} \right)
$$

(3.39)

Now to find the incremental form expression of stress, differentiate Equation 3.33

$$
d\sigma_i^\epsilon = (1 - D) d\sigma_i^\alpha + D d\sigma_i^\epsilon + dD (\sigma_i^\epsilon - \sigma_i^\alpha)
$$

(3.40)

Using the incremental formulation, the increment of stress can be written in the following form:

$$
d\sigma_i^\epsilon = C_i^{ijkl} e_{kl}
$$

(3.41)

where $C_i^{ijkl}$ is the elasto-plastic constitutive tensor for the RI part and $d e_{kl}$ is the incremental total strain tensor for the intact part. For the FA part

$$
d\sigma_i^\epsilon = C_i^{ijkl} e_{kl}
$$

(3.42)
where $C_{ijkl}^e$ is the elasto-plastic constitutive tensor for the FA part and $d\varepsilon_{kl}^e$ is the incremental total strain tensor for the FA part.

Assume that the strains in the RI and the FA part are different and has the following relation (Basaran and Desai, 1994):

$$d\varepsilon_{kl}^e = (1 + \alpha)d\dot{\varepsilon}_{kl}$$

(3.43)

where $\alpha$ is a scalar coefficient. Substituting the above equation into Eq. (3.40) gives

$$d\sigma_{ij}^a = [(1-D)C_{ijkl}^i + D(1+\alpha)C_{ijkl}^e]d\dot{\varepsilon}_{kl} + dD(\sigma_{ij}^e - \sigma_{ij}^t)$$

(3.44)

In order to express Eq. (3.44) in the conventional constitutive equation for, $dD$ needs to be expressed in terms of $d\varepsilon_{kl}^e$. Since the disturbance function $D$ is a function of the trajectory of the deviatoric plastic strain, hence

$$dD = \frac{dD}{d\xi_D}d\xi_D$$

(3.45)

and since $\xi_D$ is a function of $\varepsilon_{kl}^p$, the deviatoric plastic strain, $d\xi_D$ can be expressed as

$$d\xi_D = \frac{d\xi_D}{d\varepsilon_{ij}^p}d\varepsilon_{ij}^p$$

(3.46)

Using the associative flow rule

$$d\varepsilon_{ij}^p = \lambda \frac{\partial F}{\partial \sigma_{ij}}$$

(3.47)

$d\varepsilon_{ij}^p$ can be further expressed as

$$d\varepsilon_{ij}^p = \lambda \left[ \frac{\partial F}{\partial \sigma_{ij}} - \frac{1}{3} \frac{\partial F}{\partial \sigma_{kk}} \delta_{ij} \right]$$

(3.48)

Since $d\xi_D$ is defined as

$$d\xi_D = \sqrt{d\varepsilon_{ij}^p d\varepsilon_{ij}^p}$$

(3.49)
Substituting Eq. (3.49) into Eq. (3.48) yields

\[ d\xi_D = \lambda \sqrt{\left(\frac{\partial F}{\partial \sigma_{ij}} - \frac{1}{3} \frac{\partial F}{\partial \sigma_{kk}} \delta_{ij}\right) \left(\frac{\partial F}{\partial \sigma_{ij}} - \frac{1}{3} \frac{\partial F}{\partial \sigma_{kk}} \delta_{ij}\right)} \]  

(3.50)

Expanding and simplifying the above equation yields

\[ d\xi_D = \lambda \sqrt{\left(\frac{\partial F}{\partial \sigma_{ij}} \frac{\partial F}{\partial \sigma_{ij}} - \frac{1}{3} \frac{\partial F}{\partial \sigma_{mn}} \frac{\partial F}{\partial \sigma_{nn}} \delta_{ij}\right)} \]  

(3.51)

The derivative of the disturbance function with respect to the trajectory of the deviatoric plastic strain is

\[ \frac{dD}{d\xi_D} = \frac{d}{d\xi_D} D_u \exp(-A\xi_D^2) = D_u AZ \xi_D^{-1} \exp(-A\xi_D^2) \]  

(3.52)

From Equations 3.20, 3.52, the following equation can be obtained:

\[ dD = \left[D_u AZ \xi_D^{-1} \exp(-A\xi_D^2)\right] \frac{\partial F}{\partial \sigma_{uv}} C_{ij}^i \sqrt{\frac{\partial F}{\partial \sigma_{uv}} \frac{\partial F}{\partial \sigma_{ij}} - \frac{1}{3} \frac{\partial F}{\partial \sigma_{ii}} \frac{\partial F}{\partial \sigma_{kk}}} \]  

(3.53)

\[ dD = R \frac{d\xi_D}{d\xi_D} \]  

(3.54)

where \( R \) is given by

\[ R = \left[D_u AZ \xi_D^{-1} \exp(-A\xi_D^2)\right] \frac{\partial F}{\partial \sigma_{uv}} C_{ij}^i \sqrt{\frac{\partial F}{\partial \sigma_{uv}} \frac{\partial F}{\partial \sigma_{ij}} - \frac{1}{3} \frac{\partial F}{\partial \sigma_{ii}} \frac{\partial F}{\partial \sigma_{kk}}} \]  

(3.55)

Hence from the above equations, the following can be obtained (Basaran and Desai, 1994)

\[ d\sigma_{ij}^a = [(1 - D)C_{ijkl} + D(1 + \alpha)C_{ijkl}^e + (\sigma_{ij}^r - \sigma_{ij}^r)R_{kl}] \]  

(3.56)
or the above equation can be written as

$$d\sigma_{ij}^a = C_{ijkl}^{DSC} \, d\epsilon_{kl}$$

(3.57)

where $C_{ijkl}^{DSC}$ is given by

$$C_{ijkl}^{DSC} = (1 - D)C_{ijkl}^i + D(1 + \alpha)C_{ijkl}^c + (\sigma_{ij}^c - \sigma_{ij}^t) R_{kl}$$

(3.58)

### 3.6 Numerical Issues

#### 3.6.1 Drift Correction in Numerical Implementation of DSC

One of important steps in the formulation of the elasto-plastic constitutive law is to enforce the statement of the consistency conditions. The consistency condition requirements that plastic loading paths must begin at the current yield surface and end on a subsequent yield surface, and that changes in the size and location of the yield surface must be consistent with the hardening rule which is adopted in the constitutive law. During numerical integration of the constitutive equations, it is necessary to insure that the consistency condition is satisfied so that "drift" away from the yield surface does not occur. Such drift would accumulate and could potentially introduce significant errors in numerical analysis. The drift from the yield surface is due to: linearization of the constitutive equations during solution process, the use of finite rather than infinitesimal strain increments, and numerical error introduced during the solution of boundary value problems. Thus the drift correction must be carried out in the process of numerical integration. There are a number of techniques for correcting such drift. Two of the more popular correction algorithms are discussed below.

#### 3.6.2 Correction With Stress Normal to the Plastic Yield Surface

The method of applying a correction stress normal to the plastic yield surface as described by Chen (1994), assumes that the length of the stress increment parallel to the
yield surface has been computed accurately, but error has caused a drift perpendicular from the yield surface. Let \( \sigma_{ij} \) be the current stress state which is erroneous, \( \sigma'_{ij} \) the corrected stress state, \( \Delta \sigma_{ij} \) the stress increment needed to adjust the current stress state, and then there exists

\[
\sigma'_{ij} = \sigma_{ij} + \Delta \sigma_{ij} \tag{3.59}
\]

When "drift" occurs, the value of the yield function becomes \( F(\sigma_{ij}) > 0 \). To make a correction is to add a stress change and therefore to ensure the value of the yield function becomes

\[
F(\sigma'_{ij}) = F(\sigma_{ij} + \Delta \sigma_{ij}) = 0 \tag{3.60}
\]

Take

\[
\Delta \sigma_{ij} = c \frac{\partial F(\sigma_{ij})}{\partial \sigma_{ij}} \tag{3.61}
\]

where \( c \) is a scalar quantity. Eq.3.61 means that \( \Delta \sigma_{ij} \) is in the direction of the normal of the yield surface. By applying Taylor's theorem and taking the first order approximation, Eq. 3.60 becomes

\[
F(\sigma_{ij} + \Delta \sigma_{ij}) \approx F(\sigma_{ij}) + \frac{\partial F(\sigma_{ij})}{\partial \sigma_{ij}} \Delta \sigma_{ij} = 0 \tag{3.62}
\]

and by substituting Eq. 3.61 into Eq. 3.62 and by some rearranging the scalar \( c \) can be obtained as

\[
c = -\frac{F(\sigma_{ij})}{\frac{\partial F}{\partial \sigma_{mn}} \frac{\partial F}{\partial \sigma_{mn}}} \tag{3.63}
\]

Combining Eqs. 3.59 and 3.63 leads to the corrected stress \( \sigma'_{ij} \) as

\[
\sigma'_{ij} = \sigma_{ij} - \frac{F(\sigma_{ij})}{\frac{\partial F}{\partial \sigma_{mn}} \frac{\partial F}{\partial \sigma_{mn}}} \tag{3.64}
\]

This method of drift correction assumes that changes in the plastic strains and the hardening parameters are negligible. Some iteration may be required to find \( c \), since \( F \) is not generally linear.
3.6.3 Correction With Both Strain and Stress Normal To the Plastic Potential Surface

In this method (Potts and Gens, 1985) of the drift correction, both stress and strain are corrected and therefore the hardening parameter is changed. The stress change $\sigma'_{ij}$ will cause an associated change in the elastic strain given by

$$\Delta \varepsilon^e_{ij} = D^e_{ijkl} \Delta \sigma_{kl}$$

(3.65)

where $D^e_{ijkl}$ is the elastic transfer tensor of stress to strain. Assuming no change in the total strains during the correction process implies that the elastic strain change must be balanced by an equal and opposite change in the plastic strains, that is

$$\Delta \varepsilon^p_{ij} = -\Delta \varepsilon^e_{ij} = -D^e_{ijkl} \Delta \sigma_{kl}$$

(3.66)

Take that the plastic strain change is proportional to the gradient of the plastic potential $Q(\sigma_{ij}\alpha)$, that is, the flow rule, then the plastic strain change $\Delta \varepsilon^p_{ij}$ is given by

$$\Delta \varepsilon^p_{ij} = c \frac{\partial Q}{\partial \sigma_{ij}}$$

(3.67)

where $c$ is a scalar quantity. Rearranging Eqs. 3.66 and 3.67 leads to the stress change $\Delta \sigma_{ij}$ as

$$\Delta \sigma_{ij} = -C^e_{ijkl} \Delta \varepsilon^p_{kl} = -cC^e_{ijkl} \frac{\partial Q}{\partial \sigma_{kl}}$$

(3.68)

There will also be a change to the hardening parameter $\alpha$:

$$\Delta \alpha = \alpha(\varepsilon^p_{ij} + \Delta \varepsilon^p_{ij}) - \alpha(\varepsilon^p_{ij})$$

(3.69)

By applying the changes of both stress and hardening function, the value of the yield function becomes zero, that is,

$$F(\sigma_{ij} + \Delta \sigma_{ij}, \alpha + \Delta \alpha) = 0$$

(3.70)
Since the hardening function is expressed as a function of $\xi$, that is, $\alpha = \alpha(\xi)$, Eq. 3.70 can also be expressed as

$$F(\sigma_{ij} + \Delta \sigma_{ij}, \xi + \Delta \xi) = 0$$  \hspace{1cm} (3.71)$$

where

$$\xi = \int \sqrt{de_{ij}^p de_{ij}^p}$$

and

$$\Delta \xi = \sqrt{\Delta e_{ij}^p \Delta e_{ij}^p} = c \sqrt{\frac{\partial Q}{\partial \sigma_{ij}} \frac{\partial Q}{\partial \sigma_{ij}}}$$  \hspace{1cm} (3.72)$$

Again applying Taylor's theorem and by taking the first order of approximation, Eq. 3.71 can be expressed as

$$F(\sigma_{ij} + \Delta \sigma_{ij}, \xi + \Delta \xi) \approx F(\sigma_{ij}, \xi) + \frac{\partial F}{\partial \sigma_{ij}} \Delta \sigma_{ij} + \frac{\partial F}{\partial \xi} \Delta \xi = 0$$  \hspace{1cm} (3.73)$$

Combining Eqs. 3.68, 3.67 and 3.73 gives

$$F(\sigma_{ij}, \xi) + \frac{\partial F}{\partial \sigma_{ij}} \left(-cC_{ijkl}^e \frac{\partial Q}{\partial \sigma_{kl}}\right) + \frac{\partial F}{\partial \xi} c \sqrt{\frac{\partial Q}{\partial \sigma_{ij}} \frac{\partial Q}{\partial \sigma_{ij}}} = 0$$  \hspace{1cm} (3.74)$$

$$c = \frac{F(\sigma_{ij}, \xi)}{\frac{\partial F}{\partial \sigma_{ij}} C_{ijkl}^e \frac{\partial Q}{\partial \sigma_{kl}} - \frac{\partial F}{\partial \xi} \sqrt{\frac{\partial Q}{\partial \sigma_{ij}} \frac{\partial Q}{\partial \sigma_{ij}}}}$$  \hspace{1cm} (3.75)$$

Then the corrected stress $\sigma'_{ij}$ is

$$\sigma'_{ij} = \sigma_{ij} - \frac{\frac{\partial F}{\partial \sigma_{rs}} C_{rstu}^e \frac{\partial Q}{\partial \sigma_{tu}} - \frac{\partial F}{\partial \xi} \sqrt{\frac{\partial Q}{\partial \sigma_{ij}} \frac{\partial Q}{\partial \sigma_{ij}}}}{\frac{\partial F}{\partial \sigma_{ij}} C_{ijkl}^e \frac{\partial Q}{\partial \sigma_{kl}}} \frac{\partial Q}{\partial \sigma_{ij}}$$  \hspace{1cm} (3.76)$$

and the corrected trajectory of plastic strain $\xi'$ is

$$\xi' = \xi + \frac{\frac{\partial F}{\partial \sigma_{rs}} C_{rstu}^e \frac{\partial Q}{\partial \sigma_{tu}} - \frac{\partial F}{\partial \xi} \sqrt{\frac{\partial Q}{\partial \sigma_{mn}} \frac{\partial Q}{\partial \sigma_{mn}}}}{\frac{\partial F}{\partial \sigma_{ij}} C_{ijkl}^e \frac{\partial Q}{\partial \sigma_{kl}} \frac{\partial Q}{\partial \sigma_{ij}}} \frac{\partial Q}{\partial \sigma_{mn}}$$  \hspace{1cm} (3.77)$$
3.6.4 Drift Correction with Boundary Condition Correction

In a laboratory test problem when there exists a force boundary condition, simply applying the drift correction, as described in Section 3.6.2 and Section 3.6.3, will likely in most cases result in that the corrected stress violates the boundary conditions. This kind of problems have been observed during numerical implementation in the computer program DSCOPT. For example, in model prediction of CTC test, the stress state is as

\[
\sigma_{ij} = \begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_0 & 0 \\
0 & 0 & \sigma_0
\end{bmatrix}
\]

(3.78)

When the drift correction is applied, the stress correction is

\[
\begin{bmatrix}
\Delta\sigma_{11} & \Delta\sigma_{12} & \Delta\sigma_{13} \\
\Delta\sigma_{21} & \Delta\sigma_{22} & \Delta\sigma_{21} \\
\Delta\sigma_{31} & \Delta\sigma_{31} & \Delta\sigma_{33}
\end{bmatrix}
\]

(3.79)

while the boundary condition allows only increment of \(\Delta\sigma_1 = \Delta\sigma_{11}\) and others should be zero, that is,

\[
\Delta\sigma_{12} = \Delta\sigma_{21} = 0
\]

(3.80)

\[
\Delta\sigma_{13} = \Delta\sigma_{31} = 0
\]

(3.81)

\[
\Delta\sigma_{23} = \Delta\sigma_{32} = 0
\]

(3.82)

\[
\Delta\sigma_{22} = \Delta\sigma_{33} = 0
\]

(3.83)

It is observed that the boundary conditions above are changed after a number of drift correction iterations. To solve this problem, the following steps are proposed:

1. Apply a strain increment, \(\Delta\varepsilon_{ij}\).
2. Calculate $\Delta \sigma_{ij} = C_{ijkl}^{ep} \Delta e_{kl}$.

3. Update stress state, $\sigma'_{ij} = \sigma_{ij} + \Delta \sigma_{ij}$

4. Calculate value of the yield function $F = F(\sigma_{ij})$

5. if $F < \epsilon$ (tolerance $\epsilon = 1.0 \times 10^{-12}$, for example), stop.

6. Otherwise, perform drift correction to get $\Delta \sigma'_{ij}$

7. update stress, $\sigma'_{ij} = \sigma_{ij} + \Delta \sigma_{ij}$

8. Check $\Delta \sigma_{ij}$ against the boundary conditions.
   for all elements in $\Delta \sigma_{ij}$, if
   $|\Delta \sigma'_{ij} - \Delta \sigma_{ij}^b| < \epsilon$, $b$ denotes boundary, (tolerance, $\epsilon = 1.0 \times 10^{-12}$, for example),
   Stop.

9. Otherwise, apply a compensation stress $\Delta \sigma_{ij} = -\Delta \sigma'_{ij}$

10. update stress $\sigma_{ij} = \sigma_{ij} + \Delta \sigma_{ij}$

11. Calculate new state of strain, $\Delta \epsilon_{ij} = D_{ijkl}^{ep} \Delta \sigma_{kl}$


### 3.7 Algorithm In Matrix Notation

In the numerical implementation, it is convenient that all algorithms are carried out in matrix notation.
3.7.1 Given Strain to Calculate Stress

Now in matrix notation, Eq. 3.22 can be expressed as

\[
\{d\sigma\} = \left( [C^e] - \frac{[C^e]}{\left[ \frac{\partial F}{\partial \sigma} \right]^T [C^e] \left[ \frac{\partial F}{\partial \sigma} \right] - \frac{\partial F}{\partial \epsilon} \right) \{de\} \tag{3.84}
\]

\[
= \{C_{\sigma\sigma}\}\{de\}
\]

The stress and strain components are defined by the vectors \(\{\sigma\}\) and \(\{\epsilon\}\), respectively, which are given by

\[
\{\sigma\} = [\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6]^T
\]

\[
= [\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{31}]^T
\]

\[
= [\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{zx}]^T
\]

\[
\{\epsilon\} = [\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5, \epsilon_6]^T
\]

\[
= [\epsilon_{11}, \epsilon_{22}, \epsilon_{33}, 2\epsilon_{12}, 2\epsilon_{23}, 2\epsilon_{31}]^T
\]

\[
= [\epsilon_x, \epsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}]^T
\]

\( [C^e] \) is the elastic constitutive matrix, which is given by

\[
[C^e] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix}
(1-\nu) & \nu & \nu & 0 & 0 & 0 \\
\nu & (1-\nu) & \nu & 0 & 0 & 0 \\
\nu & \nu & (1-\nu) & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2}
\end{bmatrix}
\]

\[
\left\{ \frac{\partial F}{\partial \sigma} \right\} = \left[ \frac{\partial F}{\partial \sigma_x}, \frac{\partial F}{\partial \sigma_y}, \frac{\partial F}{\partial \sigma_z}, \frac{\partial F}{\partial \tau_{xy}}, \frac{\partial F}{\partial \tau_{yz}}, \frac{\partial F}{\partial \tau_{zx}} \right]^T
\]

\[
\left\{ \frac{\partial Q}{\partial \sigma} \right\} = \left[ \frac{\partial Q}{\partial \sigma_x}, \frac{\partial Q}{\partial \sigma_y}, \frac{\partial Q}{\partial \sigma_z}, \frac{\partial Q}{\partial \tau_{xy}}, \frac{\partial Q}{\partial \tau_{yz}}, \frac{\partial Q}{\partial \tau_{zx}} \right]^T
\]

\[
\left\| \frac{\partial Q}{\partial \sigma} \right\| = \sqrt{\left\{ \frac{\partial Q}{\partial \sigma} \right\}^T \left\{ \frac{\partial Q}{\partial \sigma} \right\}}
\]
\[ F = \tilde{J}_D^2 - (-\alpha \tilde{J}_l^m + \gamma \tilde{J}_l^2)(1 - \beta S_r)^m \]  
(3.91)

\[ \alpha = a\xi^{-m} \]  
(3.92)

\[ \frac{\partial F}{\partial \xi} = \frac{\partial F}{\partial \alpha} \frac{\partial \alpha}{\partial \xi} = -\tilde{J}_l^m (1 - \beta S_r)^m a\eta_1 \xi^{-(m+1)} \]  
(3.93)

\[ \xi = \int \sqrt{\{de\}^T \{de\}} = \int \|de\| \]  
(3.94)

### 3.7.2 Given Stress to Calculate Strain

\[ \{de\} = \left( [D^e] - \left\{ \frac{\partial F}{\partial \sigma} \right\} \left\{ \frac{\partial Q}{\partial \sigma} \right\}^T \right) \{d\sigma\} \]  
(3.95)

\[ = [D^{ep}] \{d\sigma\} \]

Where

\[ [D^e] = [C^e]^{-1} \]  
(3.96)

The matrix of \([D^{ep}]\) can also be computed as

\[ [D^{ep}] = [C^{ep}]^{-1} \]  
(3.97)

### 3.7.3 Stress and Its Derivatives

\[ \{\sigma\} = [\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6]^T \]  
(3.98)

Where

\[ [\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6]^T = [\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{31}]^T \]  
(3.99)

\[ \left\{ \frac{\partial F}{\partial \sigma} \right\} = \left[ \frac{\partial F}{\partial \sigma_{11}}, \frac{\partial F}{\partial \sigma_{22}}, \frac{\partial F}{\partial \sigma_{33}}, \frac{\partial F}{\partial \sigma_{12}}, \frac{\partial F}{\partial \sigma_{23}}, \frac{\partial F}{\partial \sigma_{31}} \right]^T \]  
(3.100)
\[
\frac{\partial F}{\partial \sigma_{ij}} = \frac{\partial F(J_1, J_{2D}, J_{3D})}{\partial \sigma_{ij}} = \frac{\partial F}{\partial J_1} \frac{\partial J_1}{\partial \sigma_{ij}} + \frac{\partial F}{\partial J_{2D}} \frac{\partial J_{2D}}{\partial \sigma_{ij}} + \frac{\partial F}{\partial J_{3D}} \frac{\partial J_{3D}}{\partial \sigma_{ij}}
\]

(3.101)

\[
\left\{ \frac{\partial F}{\partial \sigma} \right\} = \frac{\partial F}{\partial J_1} \left\{ \frac{\partial J_1}{\partial \sigma} \right\} + \frac{\partial F}{\partial J_{2D}} \left\{ \frac{\partial J_{2D}}{\partial \sigma} \right\} + \frac{\partial F}{\partial J_{3D}} \left\{ \frac{\partial J_{3D}}{\partial \sigma} \right\}
\]

(3.102)

\[F = J_{2D} - F_b F_s = 0\]

(3.103)

\[F_b = -\alpha J_1^n + \gamma J_1^2\]

(3.104)

\[F_s = (1 - \beta S_r)^m\]

(3.105)

\[S_r = \frac{\sqrt{27}}{2} J_{3D} \bar{J}_{2D}^2\]

(3.106)

\[k = \frac{\sqrt{27}}{2}\]

(3.107)

\[S_r = k \bar{J}_{3D} \bar{J}_{2D}^{\frac{3}{2}}\]

(3.108)

\[\frac{\partial F}{\partial J_1} = -F_s \frac{\partial F_b}{\partial J_1}\]

(3.109)

\[\frac{\partial F_b}{\partial J_1} = -n\alpha J_1^{n-1} + 2\gamma J_1\]

(3.110)

\[\frac{\partial F}{\partial J_{2D}} = 1 - F_b \frac{\partial F_s}{\partial J_{2D}}\]

(3.111)

\[\frac{\partial F_s}{\partial J_{2D}} = \frac{\partial}{\partial J_{2D}} (1 - \beta S_r)^m\]

\[= m(1 - \beta S_r)^{m-1} \left(-\beta \frac{\partial S_r}{\partial J_{2D}}\right)\]

(3.112)
\[
\frac{\partial S_r}{\partial J_{2D}} = -\frac{3}{2} k J_{3D} J_{2D}^{-\frac{3}{2}} \tag{3.113}
\]

\[
\frac{\partial F}{\partial J_{3D}} = -F_b \frac{\partial F_s}{\partial J_{3D}} \tag{3.114}
\]

\[
\frac{\partial F_r}{\partial J_{3D}} = m(1 - \beta S_r)^{m-1}(-\beta \frac{\partial S_r}{\partial J_{3D}}) \tag{3.115}
\]

\[
\frac{\partial S_r}{\partial J_{3D}} = k J_{2D} \tag{3.116}
\]

\[
\frac{\partial \bar{J}_1}{\partial J_1} = \frac{\partial}{\partial \bar{J}_1} \left( \frac{J_1 + J_{1*}}{p_0} \right) = \frac{1}{p_0} \tag{3.117}
\]

\[
\frac{\partial \bar{J}_{2D}}{\partial J_{2D}} = \frac{\partial}{\partial \bar{J}_{2D}} \left( \frac{J_{2D}}{p_0^2} \right) = \frac{1}{p_0^2} \tag{3.118}
\]

\[
\frac{\partial \bar{J}_{3D}}{\partial J_{3D}} = \frac{\partial}{\partial \bar{J}_{3D}} \left( \frac{J_{3D}}{p_0^2} \right) = \frac{1}{p_0^2} \tag{3.119}
\]

\[
J_1 = \sigma_{ii} \tag{3.120}
\]

\[
= \sigma_{11} + \sigma_{22} + \sigma_{33} \tag{3.121}
\]

\[
\frac{\partial J_1}{\partial \sigma_i} = \delta_{ij} \tag{3.121}
\]

\[
\left\{ \frac{\partial J_1}{\partial \sigma} \right\} = [1, 1, 1, 0, 0, 0]^T \tag{3.122}
\]

\[
\frac{\partial J_{2D}}{\partial s_{kl}} = \frac{\partial}{\partial s_{kl}} \left( \frac{1}{2} s_{ij} s_{ji} \right) = \frac{1}{2} \left( \frac{\partial s_{ji}}{\partial s_{kl}} + s_{ji} \frac{\partial s_{ij}}{\partial s_{kl}} \right) \tag{3.123}
\]

\[
= \frac{1}{2} \left( s_{ik} \delta_{jk} + s_{ji} \delta_{ik} \delta_{jl} \right) \tag{3.123}
\]

\[
= \frac{1}{2} \left( s_{ik} + s_{lk} \right) = s_{kl} \tag{3.123}
\]
\[ s_{ij} = \sigma_{ij} - \frac{J_1}{3} \delta_{ij} \]  

(3.124)

\[ \frac{\partial J_{2d}}{\partial \sigma_{kl}} = \frac{\partial J_{2D}}{\partial s_{ij}} \frac{\partial s_{ij}}{\partial \sigma_{kl}} \]

\[ = s_{ij} \frac{\partial}{\partial \sigma_{kl}} (\sigma_{ij} - \frac{1}{3} \sigma_{nn} \delta_{ij}) \]  

(3.125)

\[ = s_{kl} \]

\[ \left\{ \frac{\partial J_{2D}}{\partial \sigma} \right\} = \left[ \frac{\partial J_{2D}}{\partial \sigma_{11}}, \frac{\partial J_{2D}}{\partial \sigma_{11}}, \frac{\partial J_{2D}}{\partial \sigma_{22}}, \frac{\partial J_{2D}}{\partial \sigma_{22}}, \frac{\partial J_{2D}}{\partial \sigma_{33}}, \frac{\partial J_{2D}}{\partial \sigma_{33}}, \frac{\partial J_{2D}}{\partial \sigma_{12}}, \frac{\partial J_{2D}}{\partial \sigma_{23}}, \frac{\partial J_{2D}}{\partial \sigma_{31}} \right]^T \]  

(3.126)

\[ = [s_{11}, s_{22}, s_{33}, 2s_{12}, 2s_{23}, 2s_{31}]^T \]

Notice the factor of 2 in the matrix form. This is introduced in the conversion of tensor notation to matrix notation.

\[ J_{3D} = \frac{1}{3} s_{ij} s_{jk} s_{ki} \]  

(3.127)

\[ \frac{\partial J_{3D}}{\partial s_{lm}} = \frac{1}{3} \frac{\partial}{\partial s_{lm}} (s_{ij} s_{jk} s_{ki}) \]

\[ = \frac{1}{3} \left( s_{ij} s_{jk} \frac{\partial s_{ki}}{\partial s_{lm}} + s_{ij} \frac{\partial s_{jk}}{\partial s_{lm}} s_{ki} + \frac{\partial s_{ij}}{\partial s_{lm}} s_{jk} s_{ki} \right) \]  

(3.128)

\[ \frac{\partial J_{3D}}{\partial \sigma_{ij}} = \frac{\partial J_{3D}}{\partial s_{lm}} \frac{\partial s_{lm}}{\partial \sigma_{ij}} \]

\[ = s_{ik} s_{km} \frac{\partial}{\partial \sigma_{ij}} (\sigma_{lm} - \frac{1}{3} \delta_{lm} \sigma_{nn}) \]  

(3.129)

\[ = s_{ik} s_{km} - \frac{2}{3} J_{2D} \delta_{ij} \]

\[ \left\{ \frac{\partial J_{3D}}{\partial \sigma} \right\} = \left[ \frac{\partial J_{3D}}{\partial \sigma_{11}}, \frac{\partial J_{3D}}{\partial \sigma_{11}}, \frac{\partial J_{3D}}{\partial \sigma_{22}}, \frac{\partial J_{3D}}{\partial \sigma_{22}}, \frac{\partial J_{3D}}{\partial \sigma_{33}}, \frac{\partial J_{3D}}{\partial \sigma_{33}}, \frac{\partial J_{3D}}{\partial \sigma_{12}}, \frac{\partial J_{3D}}{\partial \sigma_{23}}, \frac{\partial J_{3D}}{\partial \sigma_{31}} \right]^T \]

\[ = \left\{ \begin{array}{l} s_{1k} s_{k1} - \frac{2}{3} J_{2D} \\ s_{2k} s_{k2} - \frac{2}{3} J_{2D} \\ s_{3k} s_{k3} - \frac{2}{3} J_{2D} \\ 2s_{1k} s_{k2} \\ 2s_{2k} s_{k3} \\ 2s_{3k} s_{k1} \end{array} \right\} \]  

(3.130)
Notice the factor of 2 in the matrix form. This is introduced in the conversion from tensor notation to matrix notation.

### 3.7.4 Drift Correction in Matrix Notation

#### 3.7.4.1 Method One

Eq. 3.63 is

\[ c = -\frac{F}{\left\{ \frac{\partial F}{\partial \sigma} \right\}^T \left\{ \frac{\partial F}{\partial \sigma} \right\}} \]  

(3.131)

Then Eq. 3.64 is given by

\[ \{\sigma'\} = \{\sigma\} - \frac{F}{\left\{ \frac{\partial F}{\partial \sigma} \right\}^T \left\{ \frac{\partial F}{\partial \sigma} \right\}} \left\{ \frac{\partial F}{\partial \sigma} \right\} \]  

(3.132)

#### 3.7.4.2 Method Two

Eq. 3.75 is

\[ c = \frac{F}{\left\{ \frac{\partial F}{\partial \sigma} \right\}^T \left[ C^e \right] \left\{ \frac{\partial Q}{\partial \sigma} \right\} - \frac{\partial F}{\partial \xi} \frac{\partial Q}{\partial \sigma} \} \]  

(3.133)

Then Eq. 3.76 is given by

\[ \{\Delta \sigma\} = -c[C^e] \left\{ \frac{\partial Q}{\partial \sigma} \right\} \]  

(3.134)

Then Eq. 3.76 is given by

\[ \{\sigma'\} = \{\sigma\} - \frac{F}{\left\{ \frac{\partial F}{\partial \sigma} \right\}^T \left[ C^e \right] \left\{ \frac{\partial Q}{\partial \sigma} \right\} - \frac{\partial F}{\partial \xi} \frac{\partial Q}{\partial \sigma} \} \left\{ \frac{\partial Q}{\partial \sigma} \right\} \]  

(3.135)

\[ \Delta \xi = c \left\| \frac{\partial Q}{\partial \sigma} \right\| \]  

(3.136)

and Eq. 3.77 as

\[ \xi' = \xi + \frac{F \left\| \frac{\partial Q}{\partial \sigma} \right\|}{\left\{ \frac{\partial F}{\partial \sigma} \right\}^T \left[ C^e \right] \left\{ \frac{\partial Q}{\partial \sigma} \right\} - \frac{\partial F}{\partial \xi} \frac{\partial Q}{\partial \sigma} \} \]  

(3.137)
3.7.5 Calculation of Intact Plastic Strain

Given strain to stress transfer matrix \([C^{ep}]\), and the total strain \(\{d\varepsilon\}\), the stress can be obtained as

\[
\{d\sigma\} = [C^{ep}]\{d\varepsilon\}
\]  \(3.138\)

The stress to strain transfer matrix \([D^e]\) can be obtained by

\[
[D^e] = [C^e]^{-1}
\]  \(3.139\)

and the elastic strain by

\[
\{d\varepsilon^e\} = [D^e]\{d\sigma\}
\]  \(3.140\)

and the plastic strain by

\[
\{d\varepsilon^p\} = \{d\varepsilon\} - \{d\varepsilon^e\}
\]  \(3.141\)

The final expression for the plastic portion of the given total strain is

\[
\{d\varepsilon^p\} = ([I] - [D^e][C^{ep}]) \{d\varepsilon\}
\]  \(3.142\)

3.7.6 \(\xi_0\), Initial Value of \(\xi\) Used in DSC Model

From the plastic yield function Eq. (2.1), at the given initial state of stress \(J_1, J_{2D}\) and \(J_{3D}\), and the determined material parameters \(\beta, \gamma\) and \(m\), the initial value of \(\alpha = \alpha_0\) can be obtained by

\[
\alpha_0 = \frac{1}{J_1^n} \left[ \gamma J_1^2 - \frac{\tilde{J}_{2D}}{(1 - \beta S_r)^m} \right]
\]  \(3.143\)

Since

\[
\alpha = \frac{a_1}{\xi_1^n}
\]
then the initial value of $\xi = \xi_0$ can be obtained by

$$\xi_0 = \left( \frac{a_1}{\alpha_0} \right)^{\frac{1}{\eta_1}} = \left[ \frac{a J_i}{\gamma J_i^2 - \frac{j_{2g}}{(1 - \beta S_{cr})^m}} \right]^{\frac{1}{\eta_1}}$$  \hspace{1cm} (3.144)

given that material parameters $a_1, \eta_1$ have been determined. In the case of hydrostatic initial stress, $\xi_0$ is given by

$$\xi_0 = \left( \frac{a_1 J_i^{n-2}}{\gamma} \right)^{\frac{1}{\eta_1}}$$  \hspace{1cm} (3.145)
Chapter 4
DETERMINATION OF MATERIAL PARAMETERS

In the DSC with the $\delta_0$ HiSS model to represent the RI state, there are thirteen material parameters to be determined from laboratory test data. Those parameters are listed and summarized in Table 4.1.

4.1 Parameters of Elasticity $K, G$

Two elastic parameters, the bulk modulus $K$ and the shear modulus $G$ can be determined from laboratory test data.

In a set of laboratory tests for finding the bulk modulus of elasticity $K$, there should be at least one conventional triaxial compression (CTC) tests. Fig. 4.1 shows the schematic plot of the test data used to determine $K$. By measuring the slope of the plot, the bulk modulus of elasticity $K$ can be calculated as

$$3K = \tan \phi_1$$

where $\phi_1$ is the slope shown in Fig. 4.1.

To determine the shear modulus, there must be at least shear test (simple shear SS test or triaxial compression TC test). Fig. 4.2 shows the sketch of the plot for the test data. By measuring the angle $\phi_2$ in Fig. 4.2, the shear modulus can be calculated as

$$2G = \tan \phi_2$$
### Table 4.1. Material Parameters To Be Determined

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameter</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$K$</td>
<td>the bulk modulus of elasticity</td>
</tr>
<tr>
<td>2</td>
<td>$G$</td>
<td>the shear of elasticity</td>
</tr>
<tr>
<td>3</td>
<td>$e_0^s$</td>
<td>material void ratio at fully adjusted state.</td>
</tr>
<tr>
<td>4</td>
<td>$\lambda$</td>
<td>slope of plot $e^s$ vs. In $\frac{J}{J_{1s}}$</td>
</tr>
<tr>
<td>5</td>
<td>$J_{1s}$</td>
<td>material bonding of (tensile) strength</td>
</tr>
<tr>
<td>6</td>
<td>$a_1$</td>
<td>constant in hardening function $\alpha$</td>
</tr>
<tr>
<td>7</td>
<td>$\eta_1$</td>
<td>constant in hardening function $\alpha$</td>
</tr>
<tr>
<td>8</td>
<td>$\beta$</td>
<td>response parameter associated with the shape in $\pi$ plane</td>
</tr>
<tr>
<td>9</td>
<td>$\gamma$</td>
<td>response parameter associated with the ultimate behavior</td>
</tr>
<tr>
<td>10</td>
<td>$\bar{\eta}$</td>
<td>constant</td>
</tr>
<tr>
<td>11</td>
<td>$n$</td>
<td>phase change parameter</td>
</tr>
<tr>
<td>12</td>
<td>$A$</td>
<td>Disturbance parameter</td>
</tr>
<tr>
<td>13</td>
<td>$Z$</td>
<td>Disturbance parameter</td>
</tr>
<tr>
<td>14</td>
<td>$D_u$</td>
<td>Ultimate disturbance ($=1$)</td>
</tr>
</tbody>
</table>
Figure 4.1. Plot For Determination of $K$
Figure 4.2. Plot For Finding Shear Modulus
4.2 Determination of $E$ and $\nu$

Usually Young's Modulus $E$ can be determined from CTC and RTE tests.

$$E = \frac{\Delta q}{\Delta \epsilon_1} \quad (4.3)$$

where

$$q = \sigma_1 - \sigma_3 \quad (4.4)$$

At least one CTC test is required.

From any test which provides a plot of volumetric strain vs. axial strain, Poisson's ratio can be obtained by the following steps:

$$\Delta \epsilon_v = \Delta \epsilon_1 + \Delta \epsilon_2 + \Delta \epsilon_3 \quad (4.5)$$

Since $\Delta \epsilon_2 = \Delta \epsilon_3$ in the CTC test,

$$\nu = \frac{1}{2} \left( \frac{\Delta \epsilon_v}{1 - \Delta \epsilon_1} \right) \quad (4.6)$$

4.3 Ultimate Behavior Parameters $\gamma, \beta, J_{1s}$

When the ultimate state is reached, the hardening function becomes zero, that is, $\alpha = 0$. Therefore, those two ultimate behavior parameters, $\beta$ and $\gamma$, can be determined as follows:

First, let $\alpha = 0$ and substitute the $\alpha$ into the yield function, Eq. (2.1). So the yield function becomes as

$$F = J_{2D} - \gamma J_1^2 (1 - \beta S_r)^m = 0 \quad (4.7)$$

Rearranging Eq. (4.7) leads to

$$\left[ \frac{J_{2D}}{(J_1^2)} \right]^{\frac{1}{2}} \gamma^{-\frac{1}{m}} + S_r \beta = 1 \quad \text{at ultimate state} \quad (4.8)$$
Note that Eq. (4.8) is linear with respect to two unknowns, $\gamma^{-\frac{1}{m}}$ and $\beta$. To solve the equation for $\gamma$ and $\beta$, at least two sets of test data are required. If more are available, the least-square method can be used to obtain optimal values for $\gamma$ and $\beta$.

At least two tests under different stress paths are required to find $\beta$ and $\gamma$, one in compression stress path and the other in extension stress path. Also, $\beta$ should be in the range of (see description in Section 5.3.1)

$$0 \leq \beta \leq 0.755923 \quad (4.9)$$

The ultimate state is asymptotic state and in reality it can not be reached. Therefore the value of the ultimate stress state can be set to a percentage higher than that of the stress peak point. In this implementation, the percentage is 10%.

In case that $\beta$ value is outside the range value in Eq. (4.9), it is set to the bounding value of 0.755.

For the bonding of tensile strength $J_{1s}$ as shown in Fig. 4.3, its value can be found approximately as

$$J_{1s} = \frac{\bar{\varepsilon}}{\sqrt{\gamma}} \quad (4.10)$$

as shown in Fig. 4.3.

### 4.4 Phase Change Parameter $n$

To determine the phase change parameter $n$, two particular states the material undergoes are needed – transition state and ultimate state.

The transition state is a state that denotes the stress state when the volume begins to dilative response material is reached under certain stress condition. At the transition state, the derivative of yield function $F$ respect to $J_1$ becomes zero, that is

$$\frac{\partial F}{\partial J_1} \bigg|_{\text{at transition state}} = 0 \quad (4.11)$$
FIGURE 4.3. Plot for determining $J_{1s}$. 
By using Eq. (2.1), Eq. (4.11) can be expanded as
\[ \frac{\partial F}{\partial J_i} = \frac{\partial F}{\partial J_i} \frac{\partial J_i}{\partial J} = \left[ -\alpha n J_i^{n-1} + 2\gamma J_i \right] (1 - \beta S_v) \frac{\partial J_i}{\partial J} = 0 \] (4.12)
where
\[ \frac{\partial J_i}{\partial J} = \frac{\partial}{\partial J} \left( \frac{J_i + J_{12}}{p_0} \right) = \frac{1}{p_0} \] (4.13)
Considering that the factor \((1 - \beta S_v)^m \neq 0\), Eq. (4.12) reduces as
\[ -\alpha n J_i^{n-1} + 2\gamma J_i = 0 \quad \text{(at transition state)} \] (4.14)
Rearranging Eq. (4.14) gives
\[ J_i = \left( \frac{2\gamma}{\alpha n} \right)^{\frac{1}{n-2}} \quad \text{(at transition state)} \] (4.15)
Also, rearranging Eq. (2.1) by making the yield function \(F = 0\), leads to
\[ \frac{J_{2D}}{J_i^2} = (-\alpha J_i^{n-2} + \gamma) F_s \] (4.16)
Now substituting Eq. (4.15) into Eq. (4.16) the following is obtained
\[ \frac{J_{2D}}{J_i^2} = \left( 1 - \frac{2}{n} \right) \gamma F_s \quad \text{(at transition state)} \] (4.17)
At the ultimate state the hardening function \(\alpha\) becomes zero, that is \(\alpha = 0\). Thus Eq. (2.1) becomes
\[ J_{2D} - \gamma J_i^2 F_s = 0 \quad \text{(at ultimate state)} \] (4.18)
Rearranging Eq. (4.18) gives
\[ \frac{J_{2D}}{J_i^2} = \gamma F_s \quad \text{(at ultimate state)} \] (4.19)
Now divide Eq. (4.17) by Eq. (4.19) and the following equation can be obtained
\[ \frac{J_{2D}}{J_i^2} \bigg|_{\text{at transition state}} = \left( 1 - \frac{2}{n} \right) \frac{F_s}{J_i^2} \bigg|_{\text{at ultimate state}} \] (4.20)
Now define

\[ f_1 = \left. \frac{J_{PP}}{J_f} \right|_{\text{at transition state}} \]

\[ f_2 = \left. \frac{F_s}{F_s} \right|_{\text{at ultimate state}} = 1 \]

Then Eq. (4.21) becomes

\[ f_1 = \left( 1 - \frac{2}{n} \right) f_2 = \left( 1 - \frac{2}{n} \right) \]

By further rearrangement \( n \) can be expressed as

\[ n = \frac{2}{1 - f_1} \]  

(4.24)

In determining \( n \) value by using Eq. (4.24), the test data at the transition state needs to be identified. The transition state is reached when \( \frac{\partial F}{\partial J_1} = 0 \). According to the theory of plasticity, the increment of plastic volumetric strain, \( d\varepsilon_v \), and the associated yield function \( F \) have the following relation

\[ d\varepsilon_v = 3\lambda \frac{\partial F}{\partial J_1} \]

(4.25)

Therefore, the transition state can be identified alternatively by finding state at which \( d\varepsilon_v = 0 \).

### 4.5 Hardening Parameters \( a_1, \eta_1 \)

The hardening function \( \alpha \) is defined in Eq. (3.1) and rewritten here as

\[ \alpha = \frac{a_1}{\xi^m} \]
Now material parameters $a$ and $\eta_1$ need to be determined from the test data. Taking logarithm on the both sides of the above equation leads to

$$\ln a_1 - \ln \eta_1 \ln \xi = \ln \alpha$$

(4.26)

Eq. (4.26) is often linear with respect of two unknowns $\ln \alpha$ and $\eta_1$. By now, all other parameters in Eq. (2.1), the yield function $F$, are determined and therefore the values of $\alpha$ can be calculated from $F = 0$. Thus the values of hardening function can be expresses as

$$\alpha = \frac{\gamma \dot{J}_1^2 - \dot{J}_2 D (1 - \beta S_r)^{-m}}{\dot{J}_1^n}$$

(4.27)

Note that at least two sets of tests are required in order to get two sets of data corresponding to the ultimate state, since these two sets of data are enough to solve Eq. (4.26). It is recommended that greater number of test data be collected and therefore the

4.5.1 $\xi_0$, Initial value of $\xi$ under Initial Stress Used in Finding $a_1$ and $\eta_1$

The following describes the procedure to calculate $\xi_0$, the initial value of $\xi$ to be used for finding $a_1$ and $\eta_1$ using Eq. (4.26). Assume that in HC test the unloading path will be parallel to the tangential line (observed in most cases), as shown in Fig. 4.4.

$$\Delta \varepsilon_\sigma^e = \Delta \varepsilon_\sigma - \Delta \varepsilon_\varepsilon^e$$

(4.28)

$$\frac{\Delta p}{\Delta \varepsilon_\varepsilon^e} = p' (\varepsilon_\sigma + \Delta \varepsilon_\sigma)$$

(4.29)

where $p'$ is derivative of function $p$, that is

$$p' = \frac{d}{d \varepsilon_\sigma} p(\varepsilon_\sigma)$$

(4.30)
Figure 4.4. Calculation of $\xi$ Based on HC Test
Rearranging the above equations leads

\[ \Delta e^p_v = \Delta e_v - \frac{\Delta p}{p'(\epsilon_v + \Delta \epsilon_v)} \] (4.31)

\[ \xi_0 = \int \sqrt{de^p_{ij}de^p_{ij}} \]
\[ = \sum \sqrt{(\Delta \epsilon^p_1)^2 + (\Delta \epsilon^p_2)^2 + (\Delta \epsilon^p_3)^2} \]
\[ = \sum \sqrt{3} |\Delta \epsilon^p_v| \]
\[ = \frac{1}{\sqrt{3}} \sum |\Delta \epsilon^p_v| \] (4.32)

Since \( a, \eta \) are parameters of the hardening function, it makes more sense to choose data points between one the hardening behavior obviously appears and one the peak stress value appears. Otherwise it is likely in most cases that an ill-conditioned or even singular matrix is encountered during data fitting process.

### 4.6 Critical State Parameters \( \epsilon^c_0, \lambda, \bar{m} \)

Relation Parameters \( \epsilon^c_0 \) and \( \lambda \) is defined as

\[ \epsilon^c = \epsilon^c_0 - \lambda \ln \frac{J_1}{3p_s} \] (4.33)

and can be determined by measuring the intercept and the slope respectively as showed in Fig. 4.5.

Parameter \( \bar{m} \) can be found by plotting the \( \sqrt{J_{2D}} \) vs. \( J_1 \) of values at critical state and the finding the slope of the best fit line passing through these points, as showed in Fig. 4.6.

If more sets of data are available, data fitting can be used to obtain optimal results.
FIGURE 4.5. Plot of $e^c$ vs. $\ln \frac{J_1}{3P_a}$ to Determine $\lambda$ and $e_0^c$. 
Figure 4.6. Plot of $J_1 - \sqrt{J_{2D}}$ to determine $m$
4.7 Disturbance Parameters $A, Z$

The disturbance parameters $A, Z$ appear in Eq. (3.32). For simplification let $D_u = 1$, Eq. (3.32) becomes as

$$D = 1 - \exp(-A\xi_D^2)$$ \hfill (4.34)

Rearranging Eq. (4.34) and taking logarithm of the both sides leads to

$$\ln[-\ln(1 - D)] = \ln A + Z\ln \xi_D$$ \hfill (4.35)

From the test data $D$ can be calculated as (Desai, 1995)

$$D = \frac{\sqrt{J_{1D}} - \sqrt{J_{2D}}}{\sqrt{J_{2D}} - \sqrt{J_{2D}}}$$ \hfill (4.36)

where $a, c, i$ denote the average, critical, intact, respectively. $\xi_D$ can be found from Eq. (3.5). Fig. 4.7 is the schematic in which values of parameters $A$ and $Z$ can be found.

4.8 Conventional Procedure of Parameter Determination

The conventional procedure to find the parameter values usually first find a value for a certain parameter from individual test by the formula described in the previous section of this chapter, then determine the value for that certain parameter by averaging all the values from each tests. In the following, $k$ denotes the total number of tests.

For the elastic parameters,

$$G = \frac{1}{k} \sum_{i=1}^{i=k} G_i$$ \hfill (4.37)

$$K = \frac{1}{k} \sum_{i=1}^{i=k} K_i$$ \hfill (4.38)
Figure 4.7. Plot for determining $A$ and $Z$
\[ \nu = \frac{1}{k} \sum_{i=1}^{i=k} \nu_i \]  

Ultimate behavior parameters:

\[ J_{1s} = \frac{1}{k} \sum_{i=1}^{i=k} (J_{1s})_i \]  

From Eq. (4.8), the following steps can be derived for data fitting to find value of \( \beta \) and \( \gamma \): The following gives the data fitting steps.

\[ [x] \{A\} = \{y\} \]  

where

\[ [x] = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ \vdots & \vdots \\ x_{i1} & x_{i2} \\ \vdots & \vdots \\ x_{k1} & x_{k2} \end{bmatrix}_{k \times 2} = \begin{bmatrix} S_{r1} (\frac{J_{1s}}{T})_{1}^{\frac{1}{m}} \\ S_{r2} (\frac{J_{1s}}{T})_{2}^{\frac{1}{m}} \\ \vdots \\ S_{ri} (\frac{J_{1s}}{T})_{i}^{\frac{1}{m}} \\ \vdots \\ S_{rn} (\frac{J_{1s}}{T})_{n}^{\frac{1}{m}} \end{bmatrix} \]  

and

\[ \{A\} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}_{2 \times 1} = \begin{bmatrix} \beta \\ \gamma^{-\frac{1}{m}} \end{bmatrix}_{2 \times 1} \]  

\[ \{y\} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{k \times 1} \]  

After \( A \) is solved by using the least square method, \( \beta \) and \( \gamma \) can obtained as

\[ \beta = A_1 \]
\[
\gamma = \frac{1}{A_i^n}
\]

Phase change parameter:

\[
n = \frac{1}{k} \sum_{i=1}^{i=k} n_i
\]

Hardening parameter \( a_1 \) and \( \eta_1 \):

\[
[x] \{ A \} = \{ y \}
\]

where

\[
[x] = \begin{pmatrix}
x_{11} & x_{12} \\
x_{21} & x_{22} \\
\vdots & \vdots \\
x_{i1} & x_{i2} \\
\vdots & \vdots \\
x_{k1} & x_{k2}
\end{pmatrix}_{k \times 2} = \begin{pmatrix}
1 & -\ln \xi_1 \\
1 & -\ln \xi_2 \\
\vdots & \vdots \\
1 & -\ln \xi_i \\
\vdots & \vdots \\
1 & -\ln \xi_k
\end{pmatrix}
\]

and

\[
\{ A \} = \begin{pmatrix}
A_1 \\
A_2
\end{pmatrix}_{2 \times 1} = \begin{pmatrix}
\ln a_1 \\
\eta_1
\end{pmatrix}_{2 \times 1}
\]

\[
\{ y \} = \begin{pmatrix}
\ln \alpha_1 \\
\ln \alpha_2 \\
\vdots \\
\ln \alpha_k
\end{pmatrix}_{k \times 1}
\]

After \( A \) is solved by using the least square method, \( a_1 \) and \( \eta_1 \) can obtained as

\[
a_1 = \exp(A_1)
\]

\[
\eta_1 = A_2
\]

Critical state parameters \( e_c^c \) and \( \lambda \): From Eq. (4.33),

\[
[x] \{ A \} = \{ y \}
\]
where

\[
\begin{bmatrix}
X_{11} & X_{12} \\
X_{21} & X_{22} \\
\vdots & \vdots \\
X_{i1} & X_{i2} \\
\vdots & \vdots \\
X_{k1} & X_{k2}
\end{bmatrix}
= \begin{bmatrix}
1 & -\ln\left(\frac{\xi}{3\nu}\right)_1 \\
1 & -\ln\left(\frac{\xi}{3\nu}\right)_2 \\
\vdots & \vdots \\
1 & -\ln\left(\frac{\xi}{3\nu}\right)_i \\
\vdots & \vdots \\
1 & -\ln\left(\frac{\xi}{3\nu}\right)_k
\end{bmatrix}
\]

(4.55)

and

\[
\{A\} = \begin{bmatrix}
A_1 \\
A_2
\end{bmatrix}_{2\times 1}
= \begin{bmatrix}
e_0 \\
\lambda
\end{bmatrix}_{2\times 1}
\]

(4.56)

\[
\{y\} = \begin{bmatrix}
(e^c)_1 \\
(e^c)_2 \\
\vdots \\
(e^c)_k
\end{bmatrix}_{k\times 1}
\]

(4.57)

After \(A\) is solved by using the least square method, \(e_0\) and \(\lambda\) can obtained as

\[
e_0 = A_1
\]

(4.58)

\[
\lambda = A_2
\]

(4.59)

\(\bar{m}\) can be found by

\[
\bar{m} = \frac{1}{k} \sum_{i=1}^{i=k} \bar{m}_i
\]

(4.60)

Disturbance parameters \(A\) and \(Z\): From Eq. (4.35), the following steps can be derived as

\[
[x]\{A\} = \{y\}
\]

(4.61)

where

\[
\begin{bmatrix}
x_{11} & x_{12} \\
x_{21} & x_{22} \\
\vdots & \vdots \\
x_{i1} & x_{i2} \\
\vdots & \vdots \\
x_{k1} & x_{k2}
\end{bmatrix}_{k\times 2}
= \begin{bmatrix}
1 & -\ln(\xi_D)_1 \\
1 & -\ln(\xi_D)_2 \\
\vdots & \vdots \\
1 & -\ln(\xi_D)_i \\
\vdots & \vdots \\
1 & -\ln(\xi_D)_k
\end{bmatrix}_{k\times 2}
\]

(4.62)
and

\[
\{A\} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}_{2 \times 1} = \begin{bmatrix} \ln A \\ Z \end{bmatrix}_{2 \times 1} \tag{4.63}
\]

\[
\{y\} = \begin{bmatrix} \ln(-\ln(1 - D_1)) \\ \ln(-\ln(1 - D_2)) \\ \vdots \\ \ln(-\ln(1 - D_k)) \end{bmatrix}_{k \times 1} \tag{4.64}
\]

After \( A \) is solved by using the least square method, \( A \) and \( Z \) can obtained as

\[
A = \exp(A_1) \tag{4.65}
\]

\[
Z = A_2 \tag{4.66}
\]
Chapter 5

Optimization in DSC Constitutive Modeling

In practice, any load-deformation problem requires constitutive models for the stress-strain behavior of all the materials involved. Stress is related to strain or vice versa of an engineering material through constitutive modeling. Sophisticated constitutive models have evolved over the last two decades or so. The Disturbed State Concept (DSC) is such a model which has been applied to a wide range of materials, interfaces and joints. Calibrating material parameters for the model is a non-trivial task (Traditional method of parameter calibration often rely on simple averaged procedures) to include effects of various factors on the material behavior. Even a sophisticated model calibrated using such procedures may not simulate the mechanical behavior of that material satisfactorily. In order to solve this problem, optimization is utilized to improve the accuracy of model simulations.

In this chapter the optimization procedure with respect to DSC model is discussed, including proposal of objective functions for the optimization and a brief review of mathematical optimization strategies. Then the results of model verification are presented, to verify the effectiveness of the optimized model. The verification is carried out through using data obtained from laboratory tests, and simulations of the triaxial cylindrical tests and shaking table test for a saturated soil-structure system.

5.1 Objective Function In Optimization

Optimization basically involves two steps, constructing of objective function, and carrying out the calculation with an optimization strategy.

An objective function is a scalar measure of goodness or satisfaction of a particular solution to a constitutive model. In this proposal, the basic concept for constructing
an objective function is to define a function expressing the total error between the experimental observations and model prediction, and, then to minimize the objective function, that is to say, minimize the error. The total error can be expressed by summing the discrete residuals \((r)\) at each of the experimental points used in the determination of the material parameters:

\[
 r(x) = \frac{1}{x_1 - x_2} \int_{x_1}^{x_2} \left[ \frac{\hat{f}(x, \bar{X})}{f(x)} - 1 \right]^2 dx
\]  

(5.1)

where \(\hat{f}(x, \bar{X})\) denotes model predictions, \(f(x)\) is experimental observation, and \(x_1, x_2\) are end points of sampling data. Ideally, \(r\) should become zero. The problem is that a model prediction is always an approximation to the stress-strain response. Efforts must be made to get the best or optimized approximation. This means that the objective function should follow:

\[ r(x) \longrightarrow min. \]  

(5.2)

5.2 Proposed Objective Functions and the Constraints

A good constitutive model should be able to account for the behavior of a material under significant factors that influence the behavior. Therefore, the objective function should be chosen as such that it will reflect as many factors such as stress paths and density as possible.

In practice, the conventional (cylindrical) triaxial device is often used for the stress-strain behavior of geologic materials. In this apparatus two of the major principal stresses are always equal, that is, \(\sigma_2 = \sigma_3\). Therefore, all possible stress paths followed in cylindrical triaxial device will fall on a plane in the stress space, called triaxial plane, shown in Figure 5.1.

Figure Fig. 5.2 shows a projection of the space diagonal on the triaxial plane. There are nine basic stress paths as follows:
FIGURE 5.1. Representation of Triaxial Plane for Stress Path
Figure 5.2. Stress Paths in the Triaxial Plane
1. CTC – Conventional Triaxial Compression
2. CTE – Conventional Triaxial Extension
3. HC – Hydrostatic Impression
4. RTC – Reduced Triaxial Compression
5. RTE – Reduced Triaxial Extension
6. PL – Proportional Loading
7. TC – Triaxial Compression
8. TE – Triaxial Extension
9. SS – Simple Shear

The objective function is proposed here consisting of weighted basic objective functions for various stress paths. The general form of this functions is given by

\[
    r = \sum_i w_i r_i \quad (5.3)
\]

\[
    \sum w_i = 1 \quad (5.4)
\]

where \( r_i \) is a basic objective function for a certain stress path and \( w_i \) is the weight for the corresponding stress path. The weight can be decided based on the engineering judgment. In practice, for example, when a problem is about a dam stability, the extension behavior of the material may count more, and therefore properties of the extension stress path should weigh more than that of other stress paths. In contrast, in a problem of footing on top of soil, the properties of compression stress path should weigh more.
5.3 Constraints

Constraints in the optimization of DSC model are discussed in the following sections.

5.3.1 Constraint of $\beta$

The constraint ranges for $\beta$ is defined as such that the values of $\beta$ maintain the yield locus convex. The yield function defined in Eq. (2.1) can be expressed as

$$J_{2D} = C_1 \left(1 - \frac{\sqrt{27}}{2} \beta \frac{J_{3D}}{J_{2D}^{3/2}}\right)^{-1/2}$$  \hspace{1cm} (5.5)

where $C_1$ is given as

$$C_1 = -\alpha(J_1 + J_{1s})^n + \gamma(J_1 + J_{1s})^2$$ \hspace{1cm} (5.6)

Also note that $J_{3D} = S_1 S_2 S_3$ where $S_i$ is the principal deviatoric stress. In the $\pi$ plane of the principal stress space (See Figure 5.3), the following equations hold:

$$r^2 = 2J_{2D}$$
$$y_i = S_i$$
$$\theta_i = \frac{y_i}{r}$$

where $y_i$ is $y$ value in Cartesian system, correspondingly; $r$ is radius of polar coordinate system and $\theta_i$ is Lode angle accordingly. The Eq. (5.5) can be further expressed as

$$r = \sqrt{2C_1} (1 - \beta \sin 3\theta)^{-1/4}$$ \hspace{1cm} (5.7)

Now $r$ is the radius of the yield locus in the $\pi$ plane. From plasticity theory, the locus must be convex. That is, mathematically, $\beta$ is such that it makes the curvature of the locus no less than zero. The curvature is expressed as

$$\chi = \frac{2r'^2 - rr'' + r^2}{(r^2 + r^2)^{3/2}}$$ \hspace{1cm} (5.8)

Numerically solving Eq. 5.8 gives

$$0 \leq \beta < 0.755923$$ \hspace{1cm} (5.9)
Figure 5.3. Pi Plane
5.3.2 Constraint of $\gamma$

To maintain the yield locus convex, the hardening function $\alpha$ needs to conform to the condition

$$\alpha \geq 0 \quad (5.10)$$

and $\alpha$ can be derived as

$$\alpha = \frac{\gamma J_1^n - J_2D(1 - \beta S_r)^{-m}}{J_1^n} \quad (5.11)$$

Rearranging the above equation gives

$$\gamma J_1^n - J_2D(1 - \beta S_r)^{-m} \geq 0 \quad (5.12)$$

$$\gamma \geq \frac{J_2D}{J_1^n}(1 - \beta S_r)^{-m} \quad (5.13)$$

$$\gamma \geq \frac{J_2D}{J_1^n(1 - \beta S_r)^m} \quad (5.14)$$

So Eq. (5.14) is the constraint condition for parameter $\gamma$.

5.3.3 Constraint of $n$

The condition for the constraint value of parameter $n$ is such that its value shall ensure the plastic yield locus remain convex. For a certain value of parameter $\beta = 0$, the relation of $\sqrt{J_2D} - J_1$ can be derived from the yield function function (Eq. (2.1)) as

$$\sqrt{J_2D} = (-\alpha J_1^n + \gamma J_1^2)^{1/2} \quad (5.15)$$

The curvature for this projection curve is expressed as

$$\chi = \frac{\frac{\partial^2 \sqrt{J_2D}}{\partial J_1^2}}{\left[1 + \left(\frac{\partial \sqrt{J_2D}}{\partial J_1}\right)^2\right]^{3/2}} \quad (5.16)$$
To maintain the yield locus convex, the curvature $\chi$ must not be less than 0, that is, $\chi \geq 0$. Solving Eq. (5.16) numerically, the value of parameter $n$ is:

$$n = 2.00017318 > 2$$  \hspace{1cm} (5.17)

So Eq. (5.17) is the constraint condition for parameter $n$.

### 5.3.4 Constraints of $G$ and $K$ or $E$ and $\nu$

For these parameters, the constraints are

$$G > 0$$  \hspace{1cm} (5.18)

$$K > 0$$  \hspace{1cm} (5.19)

or

$$E > 0$$  \hspace{1cm} (5.20)

$$0 < \nu < 0.5$$  \hspace{1cm} (5.21)

### 5.4 Selected Numerical Method of Optimization

In this study, quasi-Newton method is utilized as the numerical strategy. To help understand this method, first, it is necessary to introduce briefly the Newton's method.

#### 5.4.1 Newton's Method

A function can be expressed in a quadratic form by obtaining a truncated Taylor series expansion of $f(x)$ about $x^{(k)}$, as illustrated in Fig. 5.4, which can be written as (Fletcher, 1987)

$$f\left(x^{(k)} + \delta\right) \approx q^{(k)}(\delta) = f^{(k)} + g^{(k)T} \delta + \frac{1}{2} \delta^T G^{(k)} \delta$$  \hspace{1cm} (5.22)
Figure 5.4. Schematic Plot of Taylor Expansion of a Function
where $\mathbf{x}$ is a vector

$$\mathbf{x} = [x_1, x_2, \ldots, x_i, \ldots, x_n]^T$$

and $q^{(k)}$ is the resulting quadratic and

$$\delta = \mathbf{x} - \mathbf{x}^{(k)}$$

and $\mathbf{g}(\mathbf{x})$ is a vector of first partial derivatives or gradient vector given by

$$\mathbf{g}(\mathbf{x}) = \nabla f(\mathbf{x})$$

and $\mathbf{G}(\mathbf{x})$ is a matrix of second partial derivatives or Hessian matrix given by

$$\mathbf{G}(\mathbf{x}) = \nabla^2 f(\mathbf{x})$$

Then the iterate $\mathbf{x}^{(k+1)}$ in Newton’s method is simply taken to be $\mathbf{x}^{(k)} + \delta^{(k)}$, where the correction $\delta^{(k)}$ minimizes $q^{(k)}(\delta)$. The method requires zero, first and second derivatives of $f$ to be available at any point, so that the coefficients $f^{(k)}, \mathbf{g}^{(k)}$, and $\mathbf{G}^{(k)}$ which define $q^{(k)}(\delta)$ are available. Also since $q^{(k)}(\delta)$ only has a unique minimizer if $\mathbf{G}^{(k)}$ is positive definite, the method is only well defined in these circumstances. In this case $\delta^{(k)}$ is defined by the condition that $\nabla q^{(k)}(\delta^{(k)}) = 0$, so the $k$th iteration of Newton’s method can be written as

1. solve $\mathbf{G}^{(k)} \delta = -\mathbf{g}^{(k)}$ for $\delta = \delta^{(k)}$
2. set $\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \delta^{(k)}$

### 5.4.2 Quasi-Newton Method

The main disadvantage of Newton’s method, even when modified to ensure global convergence, is that the user must supply formulae from which the second derivative matrix $\mathbf{G}$ can be evaluated. This is often a major disincentives to its use.
The above disadvantages are avoided in the much more important class of quasi-
Newton methods, the introduction of which greatly increases the range of problems
which can be solved. This type of method is like Newton's method with line search,
except that \( G^{(k)} \) is approximated by a symmetric positive definite matrix \( H^{(k)} \), (where
\( H \) is an approximation to \( G^{-1} \) ) which is corrected or updated from iteration to
iteration. Thus the \( k \)th iteration has the basic structure

1. set \( s^{(k)} = -H^{(k)}g^{(k)} \)

2. line search along \( s^{(k)} \) giving \( x^{(k+1)} = x^{(k)} + \alpha^{(k)}s^{(k)} \)

3. update \( H^{(k)} \) giving \( H^{(k+1)} \)

where \( \alpha \) is the line search step length which is a scalar quantity and discussed in
Section 5.4.3. The initial matrix \( H^{(1)} \) can be an any positive definite matrix, although
in the absence of any better estimate, the choice \( H^{(1)} = I \) is often made, where \( I \) is
an identity matrix defined as

\[
I = \begin{pmatrix}
1 & 0 & 0 & \ldots & 0 \\
0 & 1 & 0 & \ldots & 0 \\
\vdots \\
0 & 0 & 0 & \ldots & 1
\end{pmatrix}
\]  

(5.27)

Much of the interest lies in the updating formulae which enable \( H^{(k+1)} \) to be
calculated from \( H^{(k)} \). Here, the BFGS (was suggested by Broyden (1970), Fletcher
(1970), Goldfarb (1970), and Shanno (1970), so after four authors' name initials)
formula is used, given by (Fletcher, 1987)

\[
H^{(k+1)}_{BFGS} = H^{(k)} + \left( 1 + \frac{\gamma^T H^{(k)} \gamma}{\delta^T \gamma} \right) \frac{\delta \delta^T}{\delta^T \gamma} - \left( \frac{\delta \gamma^T H^{(k)} + H^{(k)} \gamma \delta^T}{\delta^T \gamma} \right)
\]

(5.28)

where

\[
\gamma^{(k)} = g^{(k+1)} - g^{(k)}
\]  

(5.29)
5.4.3 Line Search Strategy

Numerous strategies for carrying out the line search have been proposed, and a good choice is important since it can have a considerable effect on the performance of the algorithm in which it is embedded. The one illustrated in Fig. 5.6 is efficient and stable (Fletcher, 1987) and, therefore, is chosen as the line search strategy. The following equations are the descent and stable conditions used in the line search (to calculate $\alpha$, the step length) (Fletcher, 1987):

\[
\begin{align*}
    f^{(k)} - f^{(k+1)} & \geq -\rho g^{(k)}T \delta^{(k)} \\
    g^{(k+1)T} s^{(k)} & \geq \sigma g^{(k)} s^{(k)}
\end{align*}
\]

(5.30)

(5.31)

where $\rho$ is a fixed scalar quantity in range of $(0, \frac{1}{2})$, and $\sigma$ is a fixed scalar quantity in a range of $(\rho, 1)$. Fig. 5.5 is a schematic illustration for the conditions.

The following equations are to be used for updating $\alpha$ used in the line search:

\[
\alpha - \bar{\alpha} = 0.5 \frac{\bar{\alpha} - \bar{\alpha}_1}{\left(1 + \frac{f_k - f_{k-1}}{(\bar{\alpha} - \bar{\alpha}_1)f_k'}\right)}
\]

(5.32)

\[
\hat{\alpha} - \bar{\alpha} = \frac{(\bar{\alpha} - \bar{\alpha}_1)f'}{(f_{k-1} - f_k')}
\]

(5.33)

5.5 Implementation of Optimization Procedure

The optimization should be carried out in following two major steps.

5.5.1 Constrained Objective Function

The objective function is defined as the error residual $r$ of the model back-prediction as given by

\[
    r = \frac{1}{n} \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ \frac{\hat{q}_j(x)}{q_j} - 1 \right]^2
\]

(5.34)
FIGURE 5.5. Descent and Stable Conditions
Evaluate

\(\bar{a}_0 = 0, \bar{a}_0 = a_{\text{large number}} \) (e.g. 1.0e+30) \(\bar{a}_0 > 0\) is given

\(f' := f_i, f'_i := \bar{g}_i, y_i < 0\) (':=' means to assign value)

\[ f = f(x_0 + a_i s_i) \]

Does Eq. 5.30 hold?

Yes

\[ g := g(x^{(i)} + a_i s^{(i)}) \]

\[ f' := gT_s^{(i)} \]

No

Calculate \(\hat{a}\) using Eq. 5.32

\[ \bar{a}_1 := \bar{a} \]

\[ a := \hat{a} \]

Does Eq. 5.31 hold?

Yes

No

Calculate \(\hat{a}\) using Eq. 5.33

\[ \bar{a}_2 := \bar{a} \]

\[ f_i := f \]

\[ f'_i := f' \]

\[ \bar{a} := \hat{a} \]

terminate with \(a_i := \bar{a}\)

**Figure 5.6.** Flow Chart for Numerical Line Search Strategy
where \( \hat{q}_j \) is the model predicted stress value in the DSC model at the \( j \)th data point, while \( q_j \) is the observed stress value. Here \( w_i \) is the weight of the \( i \)th stress path, which is a user controlled value based on the engineering judgment and \( \sum w_i = 1 \), for example, when a set of tests includes one CTC test and one RTE test, \( w_1 = 0.6, w_2 = 0.4 \) for the CTC and RTE tests, respectively; \( m \) the total number of various stress paths; \( n \) the total number of data sampling points; And where \( x \) is a vector of the material parameters defined as

\[
x = [K, G, n, \beta, \gamma, a_1, \eta_1, \bar{m}, \lambda, \varepsilon^0, A, Z]^T
\]

or

\[
x = [E, \nu, n, \beta, \gamma, a_1, \eta_1, \bar{m}, \lambda, \varepsilon^0, A, Z]^T
\]

Here the parameters \( J_1 \) (binding of tensile strength), \( D_u \) (ultimate disturbance) are assumed constant and not considered in the optimization.

Thus the problem is to find a solution \( x \) to

\[
\text{minimize } r(x)
\]

subject to:

\[
0 \leq \beta \leq \beta_{\text{max}}
\]

\[
\gamma \geq \frac{\bar{J}_D}{\bar{J}_1^2 (1 - \beta S_r)^m}
\]

\[
n > 2
\]

\[
E > 0 \quad \text{or} \quad G > 0
\]

\[
0 < \nu < 0.5 \quad \text{or} \quad K > 0
\]
\[ a_1 > 0 \quad (5.42) \]
\[ \eta_1 > 0 \quad (5.43) \]

The gradient of the error residual can be obtained by calculating partial derivatives of Eq. (5.34) and given as

\[
g = \nabla r(x) = \frac{2}{n} \sum_{i=1}^{n} w_i \sum_{j=1}^{n} \frac{1}{q_j} \nabla q_j(x) \left[ \frac{\hat{q}_j(x)}{q_j} - 1 \right] \quad (5.44)
\]

5.5.2 Transformation of the Constrained Problem to the Unconstrained Problem

To solve the equations for optimization-with-constraints like Eq. (5.36), there is a need to make transformations from a constrained minimization problem to a form that is more readily solved. There are a number of ways to directly or indirectly account for simple bounds. These include the use of a constrained optimization procedure (in the numerical method), the use of barrier and penalty functions, Lagrange multipliers, variable transformations or elimination (Fletcher, 1987). After a review of available literature, it was concluded that variable transformation would provide the most straightforward solution. One of the variable transformations is trigonometric variable transformation which transforms the kind of problem of minimize \( f(x) \) subject to \( l_i \leq x_i \leq u_i \) where \( u_i \) and \( l_i \) are the upperbound and lowerbound for the \( i \)th variable, respectively. The trigonometric variable transformation gives the following transformation as

\[ x_i = l_i + (u_i - l_i) \sin^2(y_i) \quad (5.45) \]

or

\[ y_i = \sin^{-1} \sqrt{\frac{x_i - l_i}{u_i - l_i}} \quad (5.46) \]
Thus the problem is to minimize \( f(x(y)) = f(y) \), where the components of \( y \) can now vary in the domain of \(-\infty \leq y_i \leq +\infty\). The above transformation was found to work extremely well in the application, as described by Gill and Murray (1981) and DeNatale et al. (1983). Another kind of the problem is to minimize \( f(x) \) subject to \( x_i \geq l_i \). To make a transformation of this kind constraints to a form ready to be solved in range of \((-\infty, +\infty)\), the following transformation is proposed

\[
x_i = l_i + (y_i)^2
\]  

or

\[
y_i = \sqrt{x_i - l_i}
\]

Both kinds of transformations indeed has been found to work very well and no problem has been detected in DSCOPT, the computer program developed based on the optimization procedure discussed here.

The following are the actual numerical procedures in implementing transformation from a constrained problem to a unconstrained problem:

\[
\beta = \beta_{\text{max}} \sin^2(\beta^*)
\]  

\[
\gamma = \gamma_{\text{min}} + (\gamma^*)^2
\]  

\[
n = 2 + (n^*)^2
\]  

\[
E = (E^*)^2
\]  

\[
\nu = 0.5 \sin^2(\nu^*)
\]  

\[
a_1 = (a_1^*)^2
\]
Thus the problem of the unconstrained optimization is to find a solution to

\[
\text{minimize } r(x^*)
\]

where \( x^* \) is

\[
x^* = [E^*, \nu^*, n^*, \beta^*, \gamma^*, \alpha^*, \eta_1^*, \bar{m}^*, \lambda^*, e_0^*, A^*, Z^*]^T
\]

with the initial values \( x_0^* \) as

\[
\beta_0^* = \sin^{-1} \sqrt{\frac{\beta_0}{\beta_{\text{max}}}}
\]

\[
\gamma_0^* = \sqrt{\gamma_0 - \gamma_{\text{min}}}
\]

\[
n_0^* = \sqrt{n_0 - 2}
\]

\[
E_0^* = \sqrt{E_0}
\]
\[ \nu_0^* = \sin^{-1} \sqrt{2 \nu_0} \tag{5.67} \]

\[ (a_1)_0^* = \sqrt{(a_1)_0} \tag{5.68} \]

\[ (\eta_1)_0^* = \sqrt{(\eta_1)_0} \tag{5.69} \]

\[ \bar{m}_0^* = \sqrt{\bar{m}_0} \tag{5.70} \]

\[ \lambda_0^* = \sqrt{\lambda_0} \tag{5.71} \]

\[ (e_0^5)_0^* = \sqrt{(e_0^5)_0} \tag{5.72} \]

\[ A_0^* = \sqrt{A_0} \tag{5.73} \]

\[ Z_0^* = \sqrt{Z_0} \tag{5.74} \]

Note that the subscript 0 denotes the initial value and all the initial values here are to be found from the conventional procedure of parameter determination described in the previous chapters.

### 5.5.3 Iteration Steps in Numerical Optimization Implementation

The major iterations in the computer program are as follows:

1. Calculate \( x_0^* \), the initial values of the transformed model parameters, using Equations 5.49 to 5.74.

2. Initialize the parameter vector variable \( x^* \leftarrow x_0^* \)

3. Calculate the error residual \( r^{(k)} = r(x^{*(k)}) \) from Eq. (5.34)
4. check the convergence condition:

\[
\left| \frac{r^{(k)}}{r^{(k+1)}} - 1 \right| < \epsilon
\]  

(5.75)

(where \( \epsilon = 1.0 \times 10^{-12} \) could be a good choice) if Eq. (5.75) is true, stop; otherwise go to Step 5

5. Calculate Hessian matrix \( H_{BFGS} \) from Eq. (5.28).

6. Calculate the gradient of the error residual, \( g \) from Eq. (5.44).

7. Determine the line search direction \( s^{(k)} = -H^{(k)}g^{(k)} \).

8. Calculate line search step length \( \bar{\alpha} \) as described in Section 5.4.3.

9. Calculate the parameter vector increment \( \Delta x^* = \bar{\alpha}s \)

10. Update the parameter vector \( x^{*(k+1)} = x^{*(k)} + \Delta x^* \)

11. Go back to Step 3

5.5.4 Variable Barrier Method of Constrained Optimization

As an experimental investigation, the variable barrier method of solving constrained optimization problem is also implemented in DSCOPT. In this method, instead of transforming the constraint variable into \((-\infty, +\infty)\) domain by using method described in Section 5.5.2, the numerical implementation just simply reset the constrained variable to the lowerbound or the upperbound value, whenever it exceeds the range. Thus the iteration steps are:

1. Get \( x_0 \), the initial values of the model parameters, which is determined by methods described in Chapter 4.

2. Initialize the parameter vector variable \( x \leftarrow x_0 \)
3. Calculate the error residual $r^{(k)} = r(x^{(k)})$ from Eq. (5.34)

4. Check the converging condition: if the equation below is true,

$$\left| \frac{r^{(k)}}{r^{(k+1)}} - 1 \right| < \varepsilon$$  \hspace{1cm} (5.76)

if Eq. (5.76) true, stop; otherwise go to Step 5.

5. Calculate Hessian matrix $H_{BFGS}$ from Eq. (5.28).

6. Calculate the gradient of the error residual, $g$ from Eq. (5.44).

7. Determine the line search direction $s^{(k)} = -H^{(k)}g^{(k)}$.

8. Calculate line search step length $\alpha$, as described in Section 5.4.3.

9. Calculate the parameter vector increment $\Delta x = \alpha s$

10. Update the parameter vector $x^{(k+1)} = x^{(k)} + \Delta x$

11. Check the constrained variables:

   if $x_i > (x_i)_{max}$ then set $x_i \leftarrow (x_i)_{max}$ if $x_i < (x_i)_{min}$ then set $x_i \leftarrow (x_i)_{min}$

12. Go back to Step 3

5.5.5 Comments

In implementation of DSCOPT, the variable barrier method has been found to experience convergence problem most of times and therefore, the variable transformation method is used.

The numerical procedure of the optimization is described in Fig. 5.7.
input test data
find parameters using conventional procedure
transforming constrained problem into unconstrained

constraint conditions
DSC model
quasi-Newton Method
line search procedure

optimized model parameters
FEM analysis for soil-structure interaction

FIGURE 5.7. Main Optimization Procedure Implementation
5.6 Application and Verification of the DSC Model Optimization

To verify the effectiveness of the DSC model optimization, the present study investigate the following cases:

1. Backprediction of Laboratory tests of sands performed by Armaleh (1990)
2. Prediction of independent laboratory test not used in the parameter determination.
3. Finite element analysis of the laboratory test
4. Simulation a shaking table test for a saturated sand using finite element analysis with the DSC model optimization

All results will give the comparison between the optimized and averaged results. Here the averaged results are meant the result found from the conventional procedure. The optimization is carried out by using DSCOPT, the computer program implemented based on the procedure discussed in this chapter. The description of DSCOPT will be given in Chapter A. Note that all HC tests are used for determining initial values and not used in optimization process of minimizing error residuals.

5.6.1 Stress-Strain Backprediction of LB Sand Tests

This section depicts the results of Leighton Buzzard (LB) Sand Test conducted by Armaleh (1990). The test summary is given in Table 5.1. The sand was obtained from Leighton Buzzard, England. It is a subrounded, closed graded (U.S. Sieve 20 - 30) sand. It has a specific gravity $G_s$ of 2.66, a maximum void ratio of 0.81, and a minimum void ratio of 0.53. All tests were conducted on dry specimens as consolidated drained tests.
### Table 5.1. Laboratory Tests of Leighton Buzzard Sand

<table>
<thead>
<tr>
<th>Stress Path</th>
<th>Density ($D_r$)</th>
<th>Confining Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>95%</td>
<td>65%</td>
</tr>
<tr>
<td>CTC (cylindrical)</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>RTE (cylindrical)</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HC (cylindrical)</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>RTC (cubical)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TC (cubical)</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TE (cubical)</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Armaleh (From 1990)
- * denotes the test performed.

- Confining Pressure:
  - low = 89.6 kPa.
  - med = 275.6 kPa.
  - high = 826.8 kPa.

- The void ratio: $\varepsilon_{\text{max}} = 0.81$, $\varepsilon_{\text{min}} = 0.53$.

Accordingly,
- $D_r = 95\% \Rightarrow \varepsilon_0 = 0.544$
- $D_r = 65\% \Rightarrow \varepsilon_0 = 0.628$
- $D_r = 10\% \Rightarrow \varepsilon_0 = 0.782$
5.6.2 Nonindependent Prediction of Tests

This section presents the backprediction of nonindependent tests of the LB sands. Here, a nonindependent prediction is such that the test data is used to find the parameter values and also is predicted or simulated by the DSC model.

Table 5.2 shows 5 groups of tests for relative density $D_r = 95\%$ to be used for parameter determination and stress-strain response backprediction.

Table 5.3 shows parameter values calculated by using both the conventional and optimized procedure for Group 1 test data. It also shows the value percentage change due to the optimization. Fig. 5.8 to Fig. 5.11 shows the stress-strain backprediction where it can be seen that the optimized procedure yields a better simulation agreement with the observed response. In Table 5.4, it is also shown that the percentage change of the error residual of the backprediction, as a result of the optimization, quantitatively indicates the better simulation results by using the optimized DSC model.

Table 5.5 shows parameter values calculated by using both the conventional and optimized procedure for Group 2 test data. It also shows the value percentage change due to the optimization. Fig. 5.12 to Fig. 5.15 shows the stress-strain backprediction where it can be seen that the optimized procedure yields a better simulation agreement with the observed response. In Table 5.6, it is also shown that the percentage change of the error residual of the backprediction, as a result of the optimization, quantitatively indicates the better simulation results by using the optimized DSC model.

Table 5.7 shows parameter values calculated by using both the conventional and optimized procedure for Group 3 test data. It also shows the value percentage change due to the optimization. Fig. 5.16 to Fig. 5.19 shows the stress-strain backprediction where it can be seen that the optimized procedure yields a better simulation agreement with the observed response. In Table 5.8, it is also shown that the percentage
<table>
<thead>
<tr>
<th>No.</th>
<th>Stress Path</th>
<th>Confining Stress (kPa)</th>
<th>Group of Data Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>HC</td>
<td>N/A</td>
<td>x     x     x     x     x</td>
</tr>
<tr>
<td>2</td>
<td>CTC</td>
<td>89.6</td>
<td>x     x     x     x     x</td>
</tr>
<tr>
<td>3</td>
<td>CTC</td>
<td>275.6</td>
<td>x     x     x     x     x</td>
</tr>
<tr>
<td>4</td>
<td>CTC</td>
<td>826.8</td>
<td>x     x     x     x     x</td>
</tr>
<tr>
<td>5</td>
<td>RTE</td>
<td>89.6</td>
<td>x     x     x     x     x</td>
</tr>
<tr>
<td>6</td>
<td>TC</td>
<td>275.6</td>
<td>x     x     x     x     x</td>
</tr>
<tr>
<td>7</td>
<td>TE</td>
<td>89.6</td>
<td>x     x     x     x     x</td>
</tr>
</tbody>
</table>

**TABLE 5.2. Various Combinations of Test Data ($D_r = 95\%$)**
change of the error residual of the backprediction, as a result of the optimization, quantitatively indicates the better simulation results by using the optimized DSC model.

Table 5.9 shows parameter values calculated by using both the conventional and optimized procedure for Group 4 test data. It also shows the value percentage change due to the optimization. Fig. 5.20 to Fig. 5.23 shows the stress-strain backprediction where it can be seen that the optimized procedure yields a better simulation agreement with the observed response. In Table 5.10, it is also shown that the percentage change of the error residual of the backprediction, as a result of the optimization, quantitatively indicates the better simulation results by using the optimized DSC model.

Table 5.11 shows parameter values calculated by using both the conventional and optimized procedure for Group 5 test data. It also shows the value percentage change due to the optimization. Fig. 5.24 to Fig. 5.27 shows the stress-strain backprediction where it can be seen that the optimized procedure yields a better simulation agreement with the observed response. In Table 5.12, it is also shown that the percentage change of the error residual of the backprediction, as a result of the optimization, quantitatively indicates the better simulation results by using the optimized DSC model.
Table 5.3. Parameter Values Determined from Data Group 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>101450 kPa</td>
<td>104507 kPa</td>
<td>+3.01</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.41</td>
<td>0.32584</td>
<td>-20.53</td>
</tr>
<tr>
<td>$m$</td>
<td>-0.5</td>
<td>-0.5</td>
<td>0</td>
</tr>
<tr>
<td>$n$</td>
<td>2.537465</td>
<td>2.40913</td>
<td>-5.06</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.7</td>
<td>0.60156</td>
<td>-14.06</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.08102</td>
<td>0.0894954</td>
<td>+10.46</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.00296</td>
<td>3.00302E-4</td>
<td>-89.85</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>0.2849</td>
<td>0.298106</td>
<td>+4.64</td>
</tr>
<tr>
<td>$J_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{m}$</td>
<td>0.00929</td>
<td>9.31397E-3</td>
<td>+0.26</td>
</tr>
<tr>
<td>$e_0$</td>
<td>0.544</td>
<td>0.544</td>
<td>0</td>
</tr>
<tr>
<td>$e_0^c$</td>
<td>0.683211</td>
<td>0.752341</td>
<td>+10.12</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.0317</td>
<td>0.2940957</td>
<td>+827.75</td>
</tr>
<tr>
<td>$A$</td>
<td>0.4579</td>
<td>0.84379</td>
<td>+84.27</td>
</tr>
<tr>
<td>$Z$</td>
<td>0.3201</td>
<td>0.499684</td>
<td>+56.10</td>
</tr>
</tbody>
</table>
Table 5.4. Error Residual Change Due to Optimization ($D_r = 90\%$, Group 1)

<table>
<thead>
<tr>
<th>Test</th>
<th>Simulation Response</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC ($\sigma_0 = 89.6kPa$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.188100</td>
<td>0.065981</td>
<td>-64.92</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>0.586827</td>
<td>0.350201</td>
<td>-40.32</td>
</tr>
<tr>
<td>CTC ($\sigma_0 = 275.6kPa$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.215176</td>
<td>0.181337</td>
<td>-15.73</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>0.724846</td>
<td>0.463418</td>
<td>-36.07</td>
</tr>
</tbody>
</table>
FIGURE 5.8. Conventional and Optimized Stress Backprediction of CTC Test ($D_r = 95\%, \sigma_0 = 89.6\, \text{kPa}$)
Figure 5.9. Conventional and Optimized Strain Backprediction of CTC Test ($D_e = 95\%, \sigma_0 = 89.6$ kPa)
FIGURE 5.10. Conventional and Optimized Stress Backprediction of CTC Test ($D_r = 95\%, \sigma_0 = 275.6\text{ kPa}$)
Figure 5.11. Conventional and Optimized Strain Backprediction of CTC Test ($D_r = 95\%, \sigma_0 = 275.6\text{kPa}$)
### Table 5.5. Parameter Values Determined from Data Group 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>136903.6 kPa</td>
<td>117318.6 kPa</td>
<td>-14.31</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.395</td>
<td>0.4313507</td>
<td>+9.20</td>
</tr>
<tr>
<td>$m$</td>
<td>-0.5</td>
<td>-0.5</td>
<td>0</td>
</tr>
<tr>
<td>$n$</td>
<td>2.35515</td>
<td>2.4009107</td>
<td>+1.94</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.7</td>
<td>0.544112</td>
<td>-22.27</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0662344</td>
<td>0.0819761</td>
<td>+23.77</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.0021461</td>
<td>2.0024E-3</td>
<td>-6.70</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>0.253249</td>
<td>0.311191</td>
<td>+22.88</td>
</tr>
<tr>
<td>$J_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\tilde{m}$</td>
<td>0.1288136</td>
<td>0.110721</td>
<td>-14.05</td>
</tr>
<tr>
<td>$c_0$</td>
<td>0.544</td>
<td>0.544</td>
<td>0</td>
</tr>
<tr>
<td>$c_0^2$</td>
<td>0.692136</td>
<td>0.402381</td>
<td>-41.86</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.0454751</td>
<td>0.1392104</td>
<td>+206.12</td>
</tr>
<tr>
<td>$A$</td>
<td>1.098959</td>
<td>1.27563</td>
<td>+16.08</td>
</tr>
<tr>
<td>$Z$</td>
<td>0.5205417</td>
<td>0.659812</td>
<td>+26.75</td>
</tr>
</tbody>
</table>
Table 5.6. Error Residual Change Due to Optimization ($D_r = 90\%$, Group 2)

<table>
<thead>
<tr>
<th>Test</th>
<th>Simulation Response</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC ($\sigma_0 = 89.6\text{kPa}$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.166968</td>
<td>0.065981</td>
<td>-60.48</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>0.692613</td>
<td>0.350201</td>
<td>-49.44</td>
</tr>
<tr>
<td>CTC ($\sigma_0 = 826.8\text{kPa}$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.159886</td>
<td>0.038606</td>
<td>-75.85</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>1.000786</td>
<td>0.851287</td>
<td>-14.94</td>
</tr>
</tbody>
</table>
FIGURE 5.12. Conventional and Optimized Backprediction of CTC Test \((D_e = 95\%, \sigma_0 = 89.6\text{kPa})\)
FIGURE 5.13. Conventional and Optimized Strain Backprediction of CTC Test ($D_r = 95\%$, $\sigma_0 = 89.6\text{kPa}$)
Figure 5.14. Conventional and Optimized Stress Backprediction of CTC Test ($D_r = 95\%, \sigma_0 = 826.8\text{kPa}$)
Figure 5.15. Conventional and Optimized Strain Backprediction of CTC Test ($D_r = 95\%, \sigma_0 = 826.8\text{kPa}$)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>31185.45 kPa</td>
<td>35176.9 kPa</td>
<td>+12.80</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.4</td>
<td>0.3736</td>
<td>-6.60</td>
</tr>
<tr>
<td>$m$</td>
<td>-0.5</td>
<td>-0.5</td>
<td>0</td>
</tr>
<tr>
<td>$n$</td>
<td>2.7</td>
<td>2.48745</td>
<td>-7.87</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.7</td>
<td>0.755</td>
<td>+7.86</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.368903</td>
<td>0.424063</td>
<td>+14.95</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.0794703</td>
<td>0.0690234</td>
<td>-13.15</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>0.080068</td>
<td>0.06012</td>
<td>-24.91</td>
</tr>
<tr>
<td>$J_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{m}$</td>
<td>0.273942</td>
<td>0.199052</td>
<td>-27.34</td>
</tr>
<tr>
<td>$e_0$</td>
<td>0.544</td>
<td>0.544</td>
<td>0</td>
</tr>
<tr>
<td>$e_0^*$</td>
<td>0.592089</td>
<td>0.400987</td>
<td>-32.28</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.105828</td>
<td>0.209711</td>
<td>+98.16</td>
</tr>
<tr>
<td>$A$</td>
<td>3.67549</td>
<td>3.07893</td>
<td>-16.23</td>
</tr>
<tr>
<td>$Z$</td>
<td>0.236983</td>
<td>0.199761</td>
<td>-15.71</td>
</tr>
</tbody>
</table>
Table 5.8. Error Residual Change Due to Optimization ($D_r = 90\%$, Group 3)

<table>
<thead>
<tr>
<th>Test</th>
<th>Simulation Response</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC ($\sigma_0 = 89.6\text{kPa}$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.086922</td>
<td>0.065981</td>
<td>-24.09</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>1.121994</td>
<td>0.350201</td>
<td>-68.79</td>
</tr>
<tr>
<td>RTE ($\sigma_0 = 89.6\text{kPa}$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.568781</td>
<td>0.188989</td>
<td>-66.77</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>0.715170</td>
<td>0.422243</td>
<td>-40.96</td>
</tr>
</tbody>
</table>
Figure 5.16. Conventional and Optimized Stress Backprediction of CTC Test \( (D_r = 95\%, \sigma_0 = 89.6\, \text{kPa}) \)
Figure 5.17. Conventional and Optimized Strain Backprediction of CTC Test ($D_r = 95\%$, $\sigma_0 = 89.6$ kPa)
Figure 5.18. Conventional and Optimized Stress Backprediction of RTE Test ($D_r = 95\%$, $\sigma_0 = 89.6\text{kPa}$)
FIGURE 5.19. Conventional and Optimized Strain Backprediction of RTE Test ($D_r = 95\%, \sigma_0 = 89.6\text{kPa}$)
Table 5.9. Parameter Values Determined from Data Group 4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Conventional Value</th>
<th>Optimized Value</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>61267.9 kPa</td>
<td>61268.8 kPa</td>
<td>+0.24</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.4</td>
<td>0.3471</td>
<td>-13.22</td>
</tr>
<tr>
<td>$m$</td>
<td>-0.5</td>
<td>-0.5</td>
<td>0</td>
</tr>
<tr>
<td>$n$</td>
<td>2.7</td>
<td>2.6812</td>
<td>-0.70</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.7</td>
<td>0.724</td>
<td>+3.43</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0875272</td>
<td>0.0802723</td>
<td>-8.29</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.00383195</td>
<td>0.00283195</td>
<td>-26.10</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>0.2594</td>
<td>0.305194</td>
<td>+17.65</td>
</tr>
<tr>
<td>$J_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{m}$</td>
<td>0.128624</td>
<td>0.286237</td>
<td>+122.54</td>
</tr>
<tr>
<td>$e_0$</td>
<td>0.544</td>
<td>0.544</td>
<td>0</td>
</tr>
<tr>
<td>$e_0^c$</td>
<td>0.730089</td>
<td>0.120524</td>
<td>-83.49</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.102535</td>
<td>0.0664085</td>
<td>-35.23</td>
</tr>
<tr>
<td>$A$</td>
<td>3.17788</td>
<td>17.8969</td>
<td>+462.57</td>
</tr>
<tr>
<td>$Z$</td>
<td>1.33896</td>
<td>1.32403</td>
<td>-1.12</td>
</tr>
</tbody>
</table>
Table 5.10. Error Residual Change Due to Optimization ($D_r = 90\%$, Group 4)

<table>
<thead>
<tr>
<th>Test</th>
<th>Simulation Response</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC ($\sigma_0 = 89.6$ kPa)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.121148</td>
<td>0.065981</td>
<td>-45.54</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>0.573248</td>
<td>0.350201</td>
<td>-38.91</td>
</tr>
<tr>
<td>TC ($\sigma_0 = 275.6$ kPa)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.278506</td>
<td>0.226898</td>
<td>-18.53</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>1.468561</td>
<td>0.538697</td>
<td>-63.35</td>
</tr>
</tbody>
</table>
**Figure 5.20.** Conventional and Optimized Stress Backprediction of CTC Test ($D_r = 95\%, \sigma_0 = 89.6\text{kPa}$)
FIGURE 5.21. Conventional and Optimized Strain Backprediction of CTC Test ($D_r = 95\%, \sigma_0 = 89.6\text{kPa}$)
Figure 5.22. Conventional and Optimized Stress Backprediction of TC Test ($D_r = 95\%, \sigma_0 = 275.6kPa$)
Figure 5.23. Conventional and Optimized Strain Backprediction of TC Test ($D_r = 95\%, \sigma_0 = 275.6\text{kPa}$)
Table 5.13 shows groups of tests for relative density $D_r = 10\%$ to be used for parameter determination and stress-strain response backprediction.

Table 5.15 shows parameter values calculated by using both the conventional and optimized procedure for Group 6 test data ($D_r = 10\%$). It also shows the value percentage change due to the optimization. Fig. 5.28 to Fig. 5.35 shows the stress-strain backprediction where it can be seen that the optimized procedure yields a better simulation agreement with the observed response.

In Table 5.4, it is also shown that the percentage change of the error residual of the backprediction, as a result of the optimization, quantitatively indicates the better simulation results by using the optimized DSC model.
Table 5.11. Parameter Values Determined from Data Group 5

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>40567.9 kPa</td>
<td>44194.6 kPa</td>
<td>+8.94</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.4</td>
<td>0.35904</td>
<td>-10.24</td>
</tr>
<tr>
<td>$m$</td>
<td>-0.5</td>
<td>-0.5</td>
<td>0</td>
</tr>
<tr>
<td>$n$</td>
<td>3</td>
<td>2.89034</td>
<td>-3.66</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.75</td>
<td>0.75213</td>
<td>+0.28</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0827565</td>
<td>0.07904</td>
<td>-4.49</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.00128265</td>
<td>0.001282</td>
<td>-0.05</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>0.307546</td>
<td>0.31001</td>
<td>+0.80</td>
</tr>
<tr>
<td>$J_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>0.102821</td>
<td>0.110981</td>
<td>+7.94</td>
</tr>
<tr>
<td>$e_0$</td>
<td>0.544</td>
<td>0.544</td>
<td>0</td>
</tr>
<tr>
<td>$e_0^*$</td>
<td>0.783012</td>
<td>0.679201</td>
<td>-13.26</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.182442</td>
<td>0.2956</td>
<td>+62.02</td>
</tr>
<tr>
<td>$A$</td>
<td>0.965391</td>
<td>0.889101</td>
<td>-7.90</td>
</tr>
<tr>
<td>$Z$</td>
<td>0.29839</td>
<td>0.27092</td>
<td>-9.21</td>
</tr>
</tbody>
</table>
### Table 5.12. Error Residual Change Due to Optimization ($D_r = 90\%$, Group 5)

<table>
<thead>
<tr>
<th>Test</th>
<th>Simulation Response</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC ($\sigma_0 = 89.6\text{kPa}$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.065981</td>
<td>0.056479</td>
<td>-16.82</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>0.584242</td>
<td>0.350201</td>
<td>-40.06</td>
</tr>
<tr>
<td>TE ($\sigma_0 = 275.6\text{kPa}$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.302183</td>
<td>0.223382</td>
<td>-26.08</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>4.725935</td>
<td>0.988664</td>
<td>-378.01</td>
</tr>
</tbody>
</table>
Figure 5.24. Conventional and Optimized Stress Backprediction of CTC Test ($D_r = 95\%, \sigma_0 = 89.6\text{kPa}$)
Figure 5.25. Conventional and Optimized Strain Backprediction of CTC Test ($D_r = 95\%, \sigma_0 = 89.6\text{kPa}$)
FIGURE 5.26. Conventional and Optimized Stress Backprediction of TE Test ($D_r = 95\%, \sigma_0 = 275.6\,kPa$)
Figure 5.27. Conventional and Optimized Strain Backprediction of TE Test ($D_r = 95\%, \sigma_0 = 275.6\text{kPa}$)
Table 5.13. Various Combinations of Test Data ($D_r = 10\%$)

<table>
<thead>
<tr>
<th>No.</th>
<th>Stress Path</th>
<th>Confining Stress (kPa)</th>
<th>Group of Data Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>HC</td>
<td>N/A</td>
<td>x     x    x</td>
</tr>
<tr>
<td>2</td>
<td>CTC</td>
<td>89.6</td>
<td>x</td>
</tr>
<tr>
<td>3</td>
<td>CTC</td>
<td>275.6</td>
<td>x</td>
</tr>
<tr>
<td>4</td>
<td>CTC</td>
<td>826.8</td>
<td>x</td>
</tr>
<tr>
<td>5</td>
<td>RTE</td>
<td>89.6</td>
<td>x</td>
</tr>
</tbody>
</table>
Table 5.14. Error Residual Change Due to Optimization ($D_r = 10\%$)

<table>
<thead>
<tr>
<th>Test</th>
<th>Simulation Response</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC ($\sigma_0 = 89.6kPa$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.209563</td>
<td>0.150474</td>
<td>-39.27</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>1.101648</td>
<td>0.819540</td>
<td>-34.42</td>
</tr>
<tr>
<td>CTC ($\sigma_0 = 275.6kPa$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.271977</td>
<td>0.210964</td>
<td>-22.43</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>1.095929</td>
<td>0.621755</td>
<td>-43.27</td>
</tr>
<tr>
<td>CTC ($\sigma_0 = 826.8kPa$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.176116</td>
<td>0.062711</td>
<td>-64.39</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>1.583151</td>
<td>0.207948</td>
<td>-86.86</td>
</tr>
<tr>
<td>RTE ($\sigma_0 = 826.8kPa$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.172218</td>
<td>0.024830</td>
<td>-85.58</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>0.904684</td>
<td>0.419520</td>
<td>-53.63</td>
</tr>
</tbody>
</table>
### Table 5.15. Parameters Determined from Data Group 6

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>35456.1 kPa</td>
<td>42830.5 kPa</td>
<td>+20.80</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.4</td>
<td>0.379</td>
<td>-5.25</td>
</tr>
<tr>
<td>$m$</td>
<td>-0.5</td>
<td>-0.5</td>
<td>0</td>
</tr>
<tr>
<td>$n$</td>
<td>2.6</td>
<td>2.758</td>
<td>+6.08</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.7</td>
<td>0.751456</td>
<td>+7.35</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.049215225</td>
<td>0.05315372</td>
<td>+8.00</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.00122739</td>
<td>0.00102946</td>
<td>-16.13</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>0.304157</td>
<td>0.3375602</td>
<td>10.98</td>
</tr>
<tr>
<td>$J_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{m}$</td>
<td>0.268212</td>
<td>0.2459766</td>
<td>-8.29</td>
</tr>
<tr>
<td>$e_0$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$e_0^c$</td>
<td>0.0327233</td>
<td>0.02002876</td>
<td>-38.79</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.0194252</td>
<td>0.0234572</td>
<td>+20.76</td>
</tr>
<tr>
<td>$A$</td>
<td>1.53464</td>
<td>1.300956</td>
<td>-15.23</td>
</tr>
<tr>
<td>$Z$</td>
<td>0.0800357</td>
<td>0.0689714</td>
<td>-13.82</td>
</tr>
</tbody>
</table>
Figure 5.28. Conventional and Optimized Stress Backprediction of CTC Test ($D_r = 10\%, \sigma_0 = 89.6\text{kPa}$)
Figure 5.29. Conventional and Optimized Strain Backprediction of CTC Test \((D_r = 10\%, \sigma_0 = 89.6\text{kPa})\)
Figure 5.30. Conventional and Optimized Stress Backprediction of CTC Test ($D_r = 10\%, \sigma_0 = 826.8\text{kPa}$)
**Figure 5.31.** Conventional and Optimized Strain Backprediction of CTC Test ($D_r = 10\%, \sigma_0 = 826.8\text{kPa}$)
**Figure 5.32.** Conventional and Optimized Stress Backprediction of CTC Test ($D_r = 10\%, \sigma_0 = 275.6\text{kPa}$)
Figure 5.33. Conventional and Optimized Strain Backprediction of CTC Test ($D_r = 10\%, \sigma_0 = 275.6\text{kPa}$)
5.6.3 Independent Prediction of Tests

This section presents the independent prediction of the LB tests. Here, an independent prediction means that the simulation done by the model is performed for the test data that has been excluded in finding the parameter values.

Fig. 5.36 to Fig. 5.39 shows the stress-strain prediction of independent tests, where it can be seen that, like in the backprediction of nonindependent tests, the optimized procedure also yields a better simulation agreement with the observed response. In Table 5.16, it is also shown that the percentage change of the error residual of the prediction, as a result of the optimization, quantitatively indicates the better simulation results by using the optimized DSC model.

5.6.4 Verification Using Parameter Values from $D_r = 95\%$ Sand to Predict Response for $D_r = 10\%$ Sand, and Vice Verse

To further verify the optimized procedure, this section presents the prediction for tests of $D_r = 10\%$ sand using parameter values from tests of $D_r = 95\%$ sand, and vice verse.

By using parameter values founded from one CTC ($\sigma_0 = 89.6\,\text{kPa}$) test and one RTE ($\sigma_0 = 89.6\,\text{kPa}$) test of $D_r = 95\%$ sand as shown in Table 5.7 to predict responses of one CTC ($\sigma_0 = 89.6\,\text{kPa}$) test and one RTE ($\sigma_0 = 826.8\,\text{kPa}$) test of $D_r = 10\%$ sand, the prediction results are shown in Fig. 5.40 to Fig. 5.43 and the error residual results are shown in Table 5.17. From those figures, it is observed that the optimization procedure yields closer agreement of the predicted response with the observed. Table 5.17 shows that the residual change is -20.36% (stress response), decreasing from $r = 0.124764$ to $r = 0.099362$, and -66.13%, decreasing from $r = 4.345876$ to $r = 1.472093$.

By using parameter values founded tests of $D_r = 10\%$ sand as shown in Table 5.15 to predict responses of one CTC test and one RTE test of $D_r = 95\%$ sand.
Figure 5.34. Conventional and Optimized Stress Backprediction of RTE Test ($D_r = 10\%, \sigma_0 = 826.8\text{kPa}$)
Figure 5.35. Conventional and Optimized Strain Backprediction of RTE Test ($D_r = 10\%, \sigma_0 = 826.8\text{kPa}$)
Figure 5.36. Conventional and Optimized Stress Prediction of Independent CTC Test ($D_r = 95\%$, $\sigma_0 = 275.6$ kPa)
FIGURE 5.37. Conventional and Optimized Strain Prediction of Independent CTC Test ($D_r = 95\%, \sigma_0 = 275.6\text{kPa}$)
Optimized Backprediction

Conventional Backprediction

Observed

Axial Strain

\(\sigma_1 - \sigma_3\) (kPa)

Figure 5.38. Conventional and Optimized Stress Prediction of Independent TE Test \((D_r = 95\%, \sigma_0 = 275.6\text{kPa})\)
Figure 5.39. Conventional and Optimized Strain Prediction of Independent TE Test ($D_r = 95\%, \sigma_0 = 275.6\text{kPa}$)
Table 5.16. Independent Prediction: Error Residual Change Due to Optimization ($D_e = 90\%$)

<table>
<thead>
<tr>
<th>Test</th>
<th>Simulation Response</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE ($\sigma_0 = 89.6\text{kPa}$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.230119</td>
<td>0.092185</td>
<td>-59.94</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>0.692441</td>
<td>0.400124</td>
<td>-42.21</td>
</tr>
<tr>
<td>CTC ($\sigma_0 = 275.6\text{kPa}$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.300074</td>
<td>0.259912</td>
<td>-13.38</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>0.792585</td>
<td>0.538762</td>
<td>-32.03</td>
</tr>
</tbody>
</table>
Figure 5.40. Stress Prediction for CTC test of $D_r = 10\%$ ($\sigma_0 = 89.6\text{kPa}$) Sand Using Parameter Values From $D_r = 95\%$ Sand
Figure 5.41. Strain Prediction for CTC test of $D_r = 10\%$ ($\sigma_0 = 89.6\text{kPa}$) Sand Using Parameter Values From $D_r = 95\%$ Sand
Figure 5.42. Stress Prediction for RTE test of $D_r = 10\%$ ($\sigma_0 = 826.8\text{kPa}$) Sand Using Parameter Values From $D_r = 95\%$ Sand
Figure 5.43. Strain Prediction for RTE test of $D_r = 10\%$ ($\sigma_0 = 826.8$ kPa) Sand Using Parameter Values From $D_r = 95\%$ Sand
Table 5.17. Error Residual Change Due to Optimization ($D_r = 90\%$, Group 3)

<table>
<thead>
<tr>
<th>Test</th>
<th>Simulation Response</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC ($\sigma_0 = 89.6\text{kPa}$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.124764</td>
<td>0.099362</td>
<td>-20.36</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>4.345876</td>
<td>1.472093</td>
<td>-66.13</td>
</tr>
<tr>
<td>RTE ($\sigma_0 = 89.6\text{kPa}$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.397849</td>
<td>0.216362</td>
<td>-45.62</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>0.643217</td>
<td>0.398651</td>
<td>-38.02</td>
</tr>
</tbody>
</table>
the prediction results are shown in Fig. 5.44 to Fig. 5.47 and the error residual results are shown in Table 5.18. From those figures, it is observed that the optimization procedure yields closer agreement of the predicted response with the observed. Table 5.18 shows that the residual change is -35.06% (stress response), decreasing from \( r = 0.207521 \) to \( r = 0.134761 \), and -25.11% (strain response), decreasing from \( r = 0.476311 \) to \( r = 2.254714 \).

5.6.5 Effect of Weight for Different Stress Paths

The optimization procedure allows for varying weight \( w_i \) for different stress paths. Values of weight for the stress paths can be decided by users based on their engineering judgment. In practice, for example, when a problem is about a dam or slope stability, the extension behavior of the material may count more, and therefore properties of the extension stress path should weigh more than that of other stress paths. In contrast, in a problem of footing on top of soil, the properties of compression stress path should weigh more. This section demonstrates the weight effect on the values the optimized \( \text{nSC} \) model and the stress-strain backprediction under different values of the weight. As examples, Test Group 3 (that is, one CTC test and one RTE test) is used to calibrate the \( \text{nSC} \) model with different weight values.

First, the values of weights are set as

\[
\begin{align*}
   w_1 &= w_{\text{CTC}} = 0.5 \\
   w_2 &= w_{\text{RTE}} = 0.5
\end{align*}
\]

and the parameter values are calculated using the optimized procedure and the back-prediction is performed. Then the weight values are set as

\[
\begin{align*}
   w_1 &= w_{\text{CTC}} = 0.65 \\
   w_2 &= w_{\text{RTE}} = 0.35
\end{align*}
\]

and also the parameter values are obtained and backprediction is performed. Table 5.19 shows the result of this weight combination effect on the parameter values. Fig. 5.48, Fig. 5.49, Fig. 5.50, Fig. 5.51 show the effect on the response backprediction.
Figure 5.44. Stress Prediction for CTC test of \( D_r = 10\% \) Sand Using Parameter Values From \( D_r = 95\% \) Sand
Figure 5.45. Strain Prediction for CTC test of $D_r = 10\%$ Sand Using Parameter Values From $D_r = 95\%$ Sand
Figure 5.46. Stress Prediction for RTE test of $D_r = 10\%$ Sand Using Parameter Values From $D_r = 95\%$ Sand.
Figure 5.47. Strain Prediction for RTE test of $D_r = 10\%$ Sand Using Parameter Values From $D_r = 95\%$ Sand
Table 5.18. Error Residual Change Due to Optimization ($D_r = 90\%$, Group 3)

<table>
<thead>
<tr>
<th>Test</th>
<th>Simulation Response</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC ($\sigma_0 = 89.6$kPa)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.207521</td>
<td>0.134761</td>
<td>-35.06</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>0.476311</td>
<td>0.356713</td>
<td>-25.11</td>
</tr>
<tr>
<td>RTE ($\sigma_0 = 89.6$kPa)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.145174</td>
<td>0.102562</td>
<td>-29.35</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>2.674139</td>
<td>2.254714</td>
<td>-15.68</td>
</tr>
</tbody>
</table>
from which it is observed that the higher weight for the stress path yields relatively the closer agreement of model simulation for that stress path. This observation can be verified by the error residual change due to the weight change, as shown in Table 5.20. In the this case that the weights are changed form \( \{w_1 = w_{CTC} = 0.5, \ w_2 = w_{RTE} = 0.5\} \) to \( \{w_1 = w_{CTC} = 0.65, \ w_2 = w_{RTE} = 0.35\} \), the error residual percent change for CTC test is -23.80\% (stress response), from \( r = 0.065981 \) to \( r=0.050277 \), and -16.68\% (strain response), error residual from \( r = 0.0350201 \) to \( r = 0.291748 \) (see Fig. 5.48, Fig. 5.49 and Table 5.19), While for RTE test the error residual change is +116.97\% (stress response) from \( r = 0.188989 \) to \( r = 0.410043 \), and +11.27\% from \( r = 0.422243 \) to \( r = 0.469819 \), (see Fig. 5.50, Fig. 5.51 and Table 5.19).

As another case showing the weight effect on the model optimization, the tests are the same but the values of the weight are set as opposite to the above example. that is

\[
\begin{align*}
w_1 &= w_{CTC} = 0.35 \quad w_2 = w_{RTE} = 0.65
\end{align*}
\]

The effect results are shown in Table 5.21 Fig. 5.52, Fig. 5.53, Fig. 5.54, Fig. 5.55 Table 5.22. and the same conclusion can be arrived. In this second case that the weights are changed form \( \{w_1 = w_{CTC} =0.5, \ w_2 = w_{RTE} = 0.5\} \) to \( \{w_1 = w_{CTC} = 0.35, \ w_2 = w_{RTE} = 0.65\} \), the error residual percent change for CTC test is +21.88\% (stress response), from \( r = 0.065981 \) to \( r=0.08417 \), and +27.93\% (strain response), error residual from \( r = 0.0350201 \) to \( r = 0.448001 \) (see Fig. 5.52, Fig. 5.53 and Table 5.21), While for RTE test the error residual change is -22.33\% (stress response) from \( r = 0.188989 \) to \( r = 0.1.46793 \), and -12.31\% from \( r = 0.422243 \) to \( r = 0.370265 \), (see Fig. 5.54, Fig. 5.55 and Table 5.21).

The results of the weight effect show that the error residual for the simulation with relatively higher weight for the stress path is lower than that with relatively lower weight value, and vice verse.
Table 5.19. Effect of Weight on Parameter Values of Test Group 3 ($w_{CTC} = 0.65$, $w_{RTE} = 0.35$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$w_{CTC} = w_{RTE} = 0.5$</th>
<th>$w_{CTC} = 0.65$, $w_{RTE} = 0.35$</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>35176.9 kPa</td>
<td>37334.2 kPa</td>
<td>+6.13</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.3736</td>
<td>0.3549</td>
<td>-5.01</td>
</tr>
<tr>
<td>$m$</td>
<td>-0.5</td>
<td>-0.5</td>
<td>0</td>
</tr>
<tr>
<td>$n$</td>
<td>2.48745</td>
<td>2.56391</td>
<td>+3.07</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.755</td>
<td>0.6013</td>
<td>-20.36</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.424063</td>
<td>0.498171</td>
<td>+17.48</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.0690234</td>
<td>0.0649979</td>
<td>-5.83</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>0.06012</td>
<td>0.05537</td>
<td>-7.90</td>
</tr>
<tr>
<td>$J_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{m}$</td>
<td>0.199052</td>
<td>0.190444</td>
<td>-4.32</td>
</tr>
<tr>
<td>$e_0$</td>
<td>0.544</td>
<td>0.544</td>
<td>0</td>
</tr>
<tr>
<td>$e_0^c$</td>
<td>0.400987</td>
<td>0.431145</td>
<td>+7.52</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.209711</td>
<td>0.347845</td>
<td>+65.87</td>
</tr>
<tr>
<td>$A$</td>
<td>3.07893</td>
<td>2.84561</td>
<td>-7.58</td>
</tr>
<tr>
<td>$Z$</td>
<td>0.199761</td>
<td>0.198043</td>
<td>-0.86</td>
</tr>
</tbody>
</table>
Figure 5.48. Effect of Weight on Stress Backprediction of CTC Test ($D_r = 95\%, \sigma_0 = 89.6\text{kPa}$)

\[ (w_1 = w_{CTC} = 0.65, w_2 = w_{RTE} = 0.35) \]
Figure 5.49. Effect of Weight on Strain Backprediction of CTC Test ($D_r = 95\%, \sigma_0 = 89.6\text{kPa}$)

(w_1 = w_{CTC} = 0.65, w_2 = w_{RTF} = 0.35)
(\(w_1 = w_{CTC} = 0.65, w_2 = w_{RTE} = 0.35\))

**Figure 5.50.** Effect of Weight on Stress Backprediction of RTE Test \((D_r = 95\%, \sigma_0 = 89.6\, \text{kPa})\)
(w₁ = w_{CTC} = 0.65, w₂ = w_{RTE} = 0.35)

Figure 5.51. Effect of Weight on Strain Backprediction of RTE Test (D_r = 95%, σ₀ = 89.6kPa)
Table 5.20. Effect of Weight on Error Residual ($D_r = 90\%$, Test Group 3, $w_{CTC} = 0.65$, $w_{RTE} = 0.35$)

<table>
<thead>
<tr>
<th>Test</th>
<th>Simulation Response</th>
<th>$w_{CTC} = w_{RTE} = 0.5$</th>
<th>$w_{CTC} = 0.65$, $w_{RTE} = 0.35$</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC ($\sigma_0 = 89.6\text{kPa}$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.065981</td>
<td>0.050277</td>
<td>-23.80</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>0.350201</td>
<td>0.291784</td>
<td>-16.68</td>
</tr>
<tr>
<td>RTE ($\sigma_0 = 89.6\text{kPa}$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.188989</td>
<td>0.410043</td>
<td>+116.97</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>0.422243</td>
<td>0.469819</td>
<td>+11.27</td>
</tr>
</tbody>
</table>
Table 5.21. Effect of Weight on Parameter Values of Test Group 3 ($w_{CTC} = 0.35$, $w_{RTE} = 0.65$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$w_{CTC} = w_{RTE} = 0.5$</th>
<th>$w_{CTC} = 0.35, w_{RTE} = 0.65$</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>35176.9 kPa</td>
<td>36714.7 kPa</td>
<td>+4.37</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.3736</td>
<td>0.3802</td>
<td>+1.77</td>
</tr>
<tr>
<td>$m$</td>
<td>-0.5</td>
<td>-0.5</td>
<td>0</td>
</tr>
<tr>
<td>$n$</td>
<td>2.48745</td>
<td>2.40071</td>
<td>-3.49</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.755</td>
<td>0.7551</td>
<td>+0.01</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.424063</td>
<td>0.419147</td>
<td>-1.16</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.0690234</td>
<td>0.0657892</td>
<td>-4.69</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>0.06012</td>
<td>0.05927</td>
<td>-1.41</td>
</tr>
<tr>
<td>$\dot{J}_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>0.199052</td>
<td>0.254588</td>
<td>+27.90</td>
</tr>
<tr>
<td>$e_0$</td>
<td>0.544</td>
<td>0.544</td>
<td>0</td>
</tr>
<tr>
<td>$e_0^c$</td>
<td>0.400987</td>
<td>0.423476</td>
<td>+5.61</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.209711</td>
<td>0.189097</td>
<td>-9.83</td>
</tr>
<tr>
<td>$A$</td>
<td>3.07893</td>
<td>3.08342</td>
<td>+0.15</td>
</tr>
<tr>
<td>$Z$</td>
<td>0.199761</td>
<td>0.202367</td>
<td>+1.30</td>
</tr>
</tbody>
</table>
Figure 5.52. Effect of Weight on Stress Backprediction of CTC Test ($D_r = 95\%$, $\sigma_0 = 89.6\text{kPa}$)

($w_1 = w_{CTC} = 0.35$, $w_2 = w_{RTF} = 0.65$)
Figure 5.53. Effect of Weight on Strain Backprediction of CTC Test ($D_r = 95\%, \sigma_0 = 89.6\text{kPa}$)

$\left( w_1 = w_{CTC} = 0.35, w_2 = w_{RTE} = 0.65 \right)$
Figure 5.54. Effect of Weight on Stress Backprediction of RTE Test ($D_r = 95\%$, $\sigma_0 = 89.6\text{kPa}$)

$(w_1 = w_{CTC} = 0.35, w_2 = w_{RTE} = 0.65)$
$w_1 = w_{CTC} = 0.35, \ w_2 = w_{RTE} = 0.65$

Figure 5.55. Effect of Weight on Strain Backprediction of RTE Test ($D_r = 95\%, \sigma_0 = 89.6kPa$)
Table 5.22. Effect of Weight on Error Residual ($D_r = 90\%$, Test Group 3, $w_{CTC} = 0.35$, $w_{RTE} = 0.65$)

<table>
<thead>
<tr>
<th>Test</th>
<th>Simulation Response</th>
<th>$w_{CTC} = w_{RTE} = 0.5$</th>
<th>$w_{CTC} = 0.35$, $w_{RTE} = 0.65$</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC ($\sigma_0 = 89.6$ kPa)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.065981</td>
<td>0.080417</td>
<td>+21.88</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>0.350201</td>
<td>0.448001</td>
<td>+27.93</td>
</tr>
<tr>
<td>RTE ($\sigma_0 = 89.6$ kPa)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.188989</td>
<td>0.146793</td>
<td>-22.33</td>
</tr>
<tr>
<td></td>
<td>Volumetric Strain</td>
<td>0.422243</td>
<td>0.370265</td>
<td>-12.31</td>
</tr>
</tbody>
</table>
5.6.6 Verification Using Finite Element Analysis of Simulating CTC Test

To verify the effectiveness of the model optimization, some finite element analysis were carried out as described in the following.

5.6.6.1 Simulation with 40-Element Mesh

Fig. 5.56 shows the finite element mesh of 40 elements and 55 nodes, which is a quarter portion the test specimen of size 71 mm in diameter and 196 mm in height. The analysis were completed by using FEM computer program POROUSD developed by Desai and his co-workers. Fig. 5.57 shows the results of shear stress response of one of CTC test under a confining pressure 89.6 kPa. Fig. 5.58 shows the results of volumetric strain response of the same test.

5.6.6.2 Simulation with 160-Element Mesh

A 160-element mesh model as shown in Fig. 5.63 and Fig. 5.64 was used to simulate the same CTC test as did in the previous FEM simulation. The main difference is that in the 160-element model there are restraints on the top of the specimen. The results for stress-strain response for Elements 4, 64 and 156 are shown in Fig. 5.65, Fig. 5.66, Fig. 5.67, Fig. 5.68, Fig. 5.69, Fig. 5.70. The averaged results are shown in Fig. 5.71, Fig. 5.72.

The following is the procedure to calculate the average stress on the top elements:

\[
\bar{\sigma}_1 = \frac{1}{A} \sum_{i=1}^{8} (\sigma_1)_i A_i \tag{5.77}
\]

where \( A \) is the section area of the specimen given by

\[
A = \sum_{i=1}^{8} A_i = \pi R^2 \tag{5.78}
\]

\[
R = 0.0355 \text{ m} \quad \text{(radius of the specimen)} \tag{5.79}
\]

and \( A_i \) are the section area of the elements involved given by

\[
A_i = \pi [(r_2)_i^2 - (r_1)_i^2] \Delta r \tag{5.80}
\]
Figure 5.56. Finite Element Mesh of CTC Test Sample (Quarter Portion)
**Figure 5.57.** Finite Element Analysis Result of Shear Stress Response for CTC Test ($D_r = 95\%, \sigma_0 = 89.6\text{kPa}$), Element 2
Figure 5.58. Finite Element Analysis Result of Volumetric Strain Response for CTC Test ($D_r = 95\%, \sigma_0 = 89.6\text{ kPa}$), Element 2
Figure 5.59. Finite Element Analysis Result of Shear Stress Response for CTC Test \((D_r = 95\%, \sigma_0 = 89.6\text{kPa})\), Element 27
**Figure 5.60.** Finite Element Analysis Result of Volumetric Strain Response for CTC Test ($D_r = 95\%$, $\sigma_0 = 89.6\text{kPa}$), Element 27
FIGURE 5.61. Finite Element Analysis Result of Shear Stress Response for CTC Test ($D_r = 95\%, \sigma_0 = 89.6\text{kPa}$), Element 37
FIGURE 5.62. Finite Element Analysis Result of Volumetric Strain Response for CTC Test \( (D_r = 95\%, \sigma_0 = 89.6\, \text{kPa}), \) Element 37
### Table 5.23. Error Residual Change Due to Optimization in Finite Element Analysis

<table>
<thead>
<tr>
<th>Test</th>
<th>Simulation Response</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finite Element Analysis ($\sigma_0 = 89.6\text{kPa}$)</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.209734</td>
<td>0.097607</td>
<td>-53.46</td>
</tr>
</tbody>
</table>
and \( (r_1)_i \) and \( (r_2)_i \) are element radius of inner and outer edges, respectively; and

\[
\Delta r = \frac{R}{8} = \frac{0.0355}{8} = 0.044375 \quad (5.81)
\]

The average volumetric strain over the volume of the specimen is

\[
\bar{\varepsilon}_v = \frac{1}{V} \sum_{i=1}^{9} \varepsilon_v \Delta V_i \quad (5.82)
\]

where \( H = 0.098 \text{m} \), the height of the specimen; \( u_i \) are the vertical displacements of the top nodes.

\[
\varepsilon_v = \frac{1}{V} \sum_{i=1}^{160} (\varepsilon_v)_i \Delta V_i \quad (5.83)
\]

where \( V \) is the total volume of the specimen given by

\[
V = \sum_{i=1}^{160} V_i \quad (5.84)
\]

and \( V_i \) are the element volume; \( (\varepsilon_v)_i \) are element volumetric strain.
FIGURE 5.63. 160 Finite Element of CTC Test Sample (Quarter Portion) with Initial Stress Condition
FIGURE 5.64. 160 Finite Element of CTC Test Sample (Quarter Portion) with Displacement Load
Figure 5.65. FEM (160-element) result of shear stress response for CTC Test ($D_r = 95\%$, $\sigma_0 = 89.6\text{kPa}$), Element 4
Figure 5.66. FEM (160-element) Result of Volumetric Strain Response for CTC Test ($D_r = 95\%, \sigma_0 = 89.6\text{kPa}$), Element 4
Figure 5.67. FEM (160-element) Result of Shear Stress Response for CTC Test ($D_r = 95\%, \sigma_0 = 89.6\text{kPa}$), Element 64
Figure 5.68. FEM (160-element) Result of Volumetric Strain Response for CTC Test ($D_r = 95\%, \sigma_0 = 89.6\text{kPa}$), Element 64
Figure 5.69. FEM (160-element) Result of Shear Stress Response for CTC Test ($D_r = 95\%, \sigma_0 = 89.6\text{kPa}$), Element 156
Figure 5.70. FEM (160-element) Result of Volumetric Strain Response for CTC Test \(D_r = 95\%, \sigma_0 = 89.6\text{kPa}\), Element 156
Table 5.24. Error Residual Change Due to Optimization in FEM (160-element)

<table>
<thead>
<tr>
<th>Element No.</th>
<th>Simulation</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.200811</td>
<td>0.091212</td>
<td>-54.58</td>
</tr>
<tr>
<td>84</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.200795</td>
<td>0.089715</td>
<td>-55.32</td>
</tr>
<tr>
<td>156</td>
<td>$\sigma_1 - \sigma_3$</td>
<td>0.325731</td>
<td>0.173267</td>
<td>-46.81</td>
</tr>
</tbody>
</table>
Figure 5.71. FEM (160-element) Result of Averaged Stress Response for CTC Test ($D_r = 95\%, \sigma_0 = 89.6$ kPa), Top Elements: 153 through 160
Figure 5.72. FEM (160-element) Result of Averaged Strain Response for CTC Test ($D_r = 95\%, \sigma_0 = 89.6\text{kPa}$), All Elements
5.6.7 Average Changes of Error Residual for Various Test Cases Due to Optimization

To summarize the effectiveness of the optimized DSC model, Fig. 5.73 and Fig. 5.74 show the error residual for various test cases as categorized as

A CTC \((\sigma_0 = 89)\), CTC \((\sigma_0 = 275)\)

A CTC \((\sigma_0 = 89)\), CTC \((\sigma_0 = 826)\)

A CTC \((\sigma_0 = 89)\), RTE \((\sigma_0 = 89)\)

A CTC \((\sigma_0 = 89)\), TC \((\sigma_0 = 275)\)

A CTC \((\sigma_0 = 89)\), TE \((\sigma_0 = 275)\)

A CTC \((\sigma_0 = 89)\), CTC \((\sigma_0 = 275)\), CTC \((\sigma_0 = 826)\), RTE \((\sigma_0 = 826)\)

A TE \((\sigma_0 = 89)\), CTC \((\sigma_0 = 275)\)

A CTC \((\sigma_0 = 89)\), RTE \((\sigma_0 = 89)\)

A CTC \((\sigma_0 = 89)\), RTE \((\sigma_0 = 89)\)

It is observed from Fig. 5.73 and Fig. 5.74 that the average residual for the stress-strain simulation decreases more than 40%, which obviously indicates that the optimization procedure significantly improves the DSC model simulation.

5.6.8 Simulation of Shaking Table Test of Saturated Soil-Structure System

As a comparable example of the verification, a simulation of the Shaking table test is carried out by using finite element analysis with the conventional and the optimized procedure. This finite element simulation is to simulate the soil-structure interaction
Stress ($\sigma_1 - \sigma_3$) Error Residual Change

(Average Change: -40.79%)

From Conventional Procedure
From Optimization Procedure

FIGURE 5.73. Stress ($\sigma_1 - \sigma_3$) Error Residual for Various Test Cases
Figure 5.74. Volumetric Strain Error Residual for Various Test Cases
of the saturated sand under dynamic load, and to compare the simulation results with the laboratory result performed by Akiyoshi et al. (1996).

5.6.8.1 Testing Setup and Result Fig. 5.78 shows the setup of the shaking table test for saturated sand. The input motion used in the test was a sinusoidal wave with a frequency of $5H_z$ and a maximum acceleration of about $0.13g$. The measured result for the excess pore water pressure and the acceleration is shown in Fig. 5.84.

5.6.8.2 Finite Element Meshing In the finite element simulation, both structure (the container) and the soil are considered as a whole soil-structure system. The finite element mesh is shown in Fig. 5.79.

5.6.8.3 Soil Used in Simulation The testing data for soil properties used in the shaking table test are sufficient for determining the parameter values of the DSC model. Alternatively, Ottawa sand (specific gravity=2.64, maximum void ratio=0.77, minimum void ratio =0.46) is adopted in the simulation. The parameter values are determined based the test data of the Ottawa sand under cyclic stress paths of CTC and RTE, performed by Gyi (1996), as shown in Figs. 5.75, 5.76, and 5.77.

5.6.8.4 Model Parameters of Soil Two sets of the DSC model parameters are found through both the conventional and the optimized procedures. The values are shown in Table 5.25.

5.6.8.5 Result Comparison The finite element simulation results from the optimized procedure for the excess pore water pressure and the acceleration are shown in Fig. 5.82 and Fig. 5.83. The results from the conventional procedure for the excess pore water pressure and the acceleration are shown in Fig. 5.80 and Fig. 5.81. By

---

1The parameter values determine through the conventional procedure were determined by Mr. I. Park, who is a fellow student also studying the DSC modeling for his doctoral degree.

2Both the conventional and optimized simulations of the excess water pore pressure through the finite element analysis were carried out with active help from Mr. I. Park, who is a fellow student.
Figure 5.75. Cyclic Test of Ottawa Sand ($\sigma_0=69$ kPa)
FIGURE 5.76. Cyclic Test of Ottawa Sand ($\sigma_0=138 \text{ kPa}$)
Figure 5.77. Cyclic Test of Ottawa Sand ($\sigma_0=207$ kPa)
inspecting the excess pore water results in Fig. 5.80, Fig. 5.82, it can be observed that while the both the pore pressure and acceleration simulations give much similar results and agree well with the measured results, the simulation through the optimized model procedure for the excess pore water pressure gives closer agreement when input shaking time is in the range of 0 second to 2 second.

5.7 Remarks and Conclusions

An optimization procedure was presented in this chapter for optimizing the DSC model and computer numerical implementation of the optimization was discussed.

In all the cases presented in the previous section of data verification, it is generally observed that, in both the conventional and optimized procedures, overall simulation quality of shear stress responses are better than that of volumetric strain responses, or in other word, overall simulation quality of volumetric strain responses is not as good as that of shear stress responses. There are also three categories of simulation quality:

<table>
<thead>
<tr>
<th>Cases Presented in Figures</th>
<th>Conventional</th>
<th>Optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Very Good</td>
<td>Better</td>
</tr>
<tr>
<td>2</td>
<td>Good</td>
<td>Very Good</td>
</tr>
<tr>
<td>3</td>
<td>Not Good</td>
<td>Not Very Good</td>
</tr>
</tbody>
</table>

For the category 1, that is, cases showing good results form conventional procedure and better results from the optimized procedure, and for the category 2, that is, cases showing not good results from the conventional procedure and good results form the optimized procedure, all indicates that the DSC model is capable of simulating stress-strain behavior of the material under various stress paths.

For the category 3, that is, simulations from both the conventional and optimized procedures are Not Good, the lower quality contributes to the mixed independent of tests such as the model parameters were defined and optimized from sands of
FIGURE 5.78. Setup of Shaking Table Model Test

(From Akiyoshi et al. (1996))
Figure 5.79. Finite Element Mesh of Shaking Table Model
### TABLE 5.25. Parameters Values of Ottawa Sand

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Conventional</th>
<th>Optimized</th>
<th>Change(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>193000 kPa</td>
<td>248199 kPa</td>
<td>+28.60</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.38</td>
<td>0.379</td>
<td>-0.26</td>
</tr>
<tr>
<td>$m$</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.00</td>
</tr>
<tr>
<td>$n$</td>
<td>2.45</td>
<td>2.442</td>
<td>-0.33</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.713</td>
<td>1.7925</td>
<td>+4.64</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.845</td>
<td>0.82415</td>
<td>-2.47</td>
</tr>
<tr>
<td>$\eta_l$</td>
<td>0.0215</td>
<td>0.1748</td>
<td>-18.70</td>
</tr>
<tr>
<td>$J_1s$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\tilde{\eta}$</td>
<td>0.200</td>
<td>0.182</td>
<td>-9.00</td>
</tr>
<tr>
<td>$e_0$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$e_0^s$</td>
<td>0.593</td>
<td>0.5796</td>
<td>-2.26</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.019</td>
<td>0.0213</td>
<td>+12.11</td>
</tr>
<tr>
<td>$A$</td>
<td>4.22</td>
<td>4.1272</td>
<td>-2.20</td>
</tr>
<tr>
<td>$Z$</td>
<td>0.43</td>
<td>0.42538</td>
<td>-1.07</td>
</tr>
</tbody>
</table>
Figure 5.80. Conventional Procedure Result: Excess Pore Water Pressure Simulation by Finite Element Analysis
Figure 5.81. Conventional Procedure Result: Acceleration Simulation by Finite Element Analysis
Figure 5.82. Optimized Procedure Result: Excess Pore Water Pressure Simulation by Finite Element Analysis
Figure 5.83. Optimized Procedure Result: Acceleration Simulation by Finite Element Analysis
FIGURE 5.84. Excess Pore Water Pressure and Acceleration Measured at Depth of 300 mm

(From Akiyoshi et al. (1996))
$D_r = 10\%$ and were used to predict behavior of sand of $D_r = 10\%$, and vice versa. This is the most challenging and most difficult cases in the constitutive modeling. Nevertheless, the results from the optimized procedure were overall improved.

The optimized prediction of responses of stress and strain from the laboratory tests have shown closer agreement with the observed values than that of unoptimized or only averaged. Also, from the finite element analysis in the boundary value problem, the results obtained through the optimized model showed more much better agreement with the laboratory tests than that of unoptimized or only averaged. In addition, it is shown that with the proposed weighted objective function, the effect of weight on response simulation is significant and it can be used to emphasize the importance of certain stress path condition in practical problems, based on the engineering judgment. For example, when a footing problem is considered, the compression properties could affect more on the solution so greater weight can be assigned to CTC stress path. In contrast, when a dam stability is considered, the extension properties could affect more on the solution so greater weight can be assigned to RTE/TE stress path.
Chapter 6
SENSITIVITY OF DSC MODEL PARAMETERS

This chapter covers the sensitivity analysis of the DSC model parameters.

6.1 Test Data Preparation

The input testing data for DSCOPT, as shown in Table 6.1, was changed +20% and -20% for every single parameter while other were kept unchanged at one time. So there were 24 data sets plus the original unchanged data set. Then run DSCOPT with the 25 data sets, the following figures are obtained, as shown in Fig. 6.1, Fig. 6.2, and Fig. 6.3.

6.2 Sensitivity Measurement - Cumulatively Relative Difference

\[ CumulativelyRelativeDifference = \sum \left| \frac{\bar{q} - q}{q} \right| \] (6.1)

where \( q \) is value of \( \sigma_1 - \sigma_3 \), \( \bar{q} \) denotes the value computed from the changed parameter value. Fig. 6.4 and Fig. 6.5 show the sensitivity according to the cumulatively relative difference. Fig. 6.6 shows the averaged cumulatively relative difference.

6.3 Remarks

By both inspecting the plots in Fig. 6.1, Fig. 6.2, and Fig. 6.3, and comparing the Cumulatively Relative Difference, it can be observed that phase change parameter \( n \) and ultimate parameter \( \gamma \) are the most sensitive. Care must be taken when to determine those very sensitive parameter values.
Table 6.1. Input Testing Data

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$E$</td>
<td>98.980 kPa</td>
</tr>
<tr>
<td>2</td>
<td>$\nu$</td>
<td>0.35</td>
</tr>
<tr>
<td>3</td>
<td>$n$</td>
<td>2.6</td>
</tr>
<tr>
<td>4</td>
<td>$\beta$</td>
<td>0.6</td>
</tr>
<tr>
<td>5</td>
<td>$\gamma$</td>
<td>0.0502571</td>
</tr>
<tr>
<td>6</td>
<td>$a_1$</td>
<td>0.000386232</td>
</tr>
<tr>
<td>7</td>
<td>$\eta_1$</td>
<td>0.7112</td>
</tr>
<tr>
<td>8</td>
<td>$\bar{m}$</td>
<td>0.079039</td>
</tr>
<tr>
<td>9</td>
<td>$e_0^c$</td>
<td>0.0883205</td>
</tr>
<tr>
<td>10</td>
<td>$\lambda$</td>
<td>0.049583</td>
</tr>
<tr>
<td>11</td>
<td>$A$</td>
<td>1.0153</td>
</tr>
<tr>
<td>12</td>
<td>$Z$</td>
<td>0.4284032</td>
</tr>
</tbody>
</table>

Table 6.2. Order of Parameter Sensitivity

<table>
<thead>
<tr>
<th>1</th>
<th>Parameter</th>
<th>Averaged Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$n$</td>
<td>34.31</td>
</tr>
<tr>
<td>3</td>
<td>$\gamma$</td>
<td>14.11</td>
</tr>
<tr>
<td>4</td>
<td>$\eta_1$</td>
<td>6.48</td>
</tr>
<tr>
<td>5</td>
<td>$\beta$</td>
<td>5.99</td>
</tr>
<tr>
<td>6</td>
<td>$a_1$</td>
<td>4.22</td>
</tr>
<tr>
<td>7</td>
<td>$A$</td>
<td>3.43</td>
</tr>
<tr>
<td>8</td>
<td>$Z$</td>
<td>3.13</td>
</tr>
<tr>
<td>9</td>
<td>$\bar{m}$</td>
<td>2.73</td>
</tr>
<tr>
<td>10</td>
<td>$e_0^c$</td>
<td>2.36</td>
</tr>
<tr>
<td>11</td>
<td>$\lambda$</td>
<td>2.35</td>
</tr>
<tr>
<td>12</td>
<td>$E$</td>
<td>1.28</td>
</tr>
<tr>
<td>13</td>
<td>$\nu$</td>
<td>0.05</td>
</tr>
</tbody>
</table>
FIGURE 6.1. Plots of Parameter Sensitivity Result
FIGURE 6.2. Plots of Parameter Sensitivity Result
FIGURE 6.3. Plots of Parameter Sensitivity Result
Cumulatively Relative Difference of CTC Test
Due to Parameter Value Changes

**Figure 6.4. Parameter Sensitivity of CTC Test**
Cumulatively Relative Difference of RTE Test Due to Parameter Value Changes

Figure 6.5. Parameter Sensitivity of RTE Test
Average of Cumulatively Relative Difference of CTC/RTE Test
Due to \( \pm 20\% \) of Parameter Value Changes

![Bar chart showing the order of DSC model parameter sensitivity.](image)

**Figure 6.6.** Order of DSC Model Parameter Sensitivity
Chapter 7

SUMMARY AND CONCLUSIONS OF DSC OPTIMIZATION AND ITS APPLICATION

In order to improve the accuracy of sophisticated DSC constitutive modeling, an optimization procedure is proposed for optimizing the model. As an application of the proposed procedure, a computer software application called DSCOPT has been developed and verified.

The code of the application used a Quasi-Newton optimization strategy to locate the set of material parameter values which minimizes the weighted error residual between the model predictions and the observed data obtained from the laboratory tests. Through application of finite element analysis to a number of real soils, this new procedure has found to be an efficient, reliable and practical means of accomplishing optimized model calibration.

7.1 Summary

1. An optimization objective function with input weighted factor for different stress paths was developed for the constitutive modeling of Disturbed State Concept (DSC). In dealing with optimization of the constrained problem of DSC modeling, the variable transformation method was used. Quasi-Newton method was used as the optimization strategy with the BFGS formula in the line search technique.

2. The optimization of DSC modeling was verified through simulation of various laboratory test data, including nonindependent and independent tests, and simulation using finite element analysis. The results have shown that the DSC model is capable of simulating satisfactorily responses of stress-strain of geo-materials
such as clay, sands, rock, and the interfaces. With the application of optimization, the DSC model can yield improved constitutive simulation in terms of enhanced agreement with testing observation, resulting lower error residual than the conventional or averaged procedure.

7.2 Contribution of This Study

1. Insights into how the optimization technique can be used to significantly improve the constitutive modeling, particularly modelings using the Disturbed State Concept (DSC).

2. Developed a general procedure to optimize the advanced constitutive model—Disturbed State Concept (DSC) Model so that the optimized DSC model will be capable of better predicting the stress-strain response of a material and boundary value problems with the finite element analysis. The verification has shown that the optimized procedure significantly decreases the error residual of the constitutive simulation of the DSC model.

3. Addressed the technical issue of force boundary condition corrections when drift correction is applied in calculating the plastic yield surface, especially for simulation of stress-strain response of conventional triaxial tests. The correction procedure solves a problem encountered when drift correction is applied to simulation of conventional triaxial tests and the drift correction does not converge to the force boundary values. This study has shown that the proposed boundary condition correction procedure indeed is effective to correct the boundary value drifting during the yield surface drift correction.

4. A computer program DSCOPT has been developed for the constitutive modeling. DSCOPT was implemented in C++, over 14,000 lines of source code. It is unique in that
- It is the first ever computer program capable of doing 3-D stress-strain analysis and prediction (the program computes in 3-D stress-strain space internally, and outputs 3-D stress and strain values);
- It is capable to process laboratory test data under all possible distinct stress paths;
- It uses data lexing and parsing techniques (discussed in Chapter A) in input data processing which greatly reduce the error and time consumed in the input data preparation;
- It provides a graphics plotting interface to external gnuplot software tools.

5. Parameter Sensitivity Analysis was carried out. From the analysis, it is observed that parameters $n, \gamma$ are the most sensitive and care should be taken to the values of those parameters
Appendix A

DSCOPT – THE COMPUTER PROGRAM IMPLEMENTED WITH DSC OPTIMIZATION

DSCOPT is a computer program implemented with the optimization procedure proposed in Chapter 5. It is a robust and handy tool for calibrating the Disturbed State Concept (DSC) model. DSCOPT has more than 14,000 lines of source code written in C. This chapter presents the basic descriptions of the usage and the implementation features.

A.1 Usage of DSCOPT

With various command line options, DSCOPT can be used to do the following things:

- To check the syntax of the input data file.
- List syntax check of the input data file.
- Display and print out plots of stress-strain based on the input data file.
- To calibrate HiSS model itself, that is, calculate the material parameters of HiSS model.
- To calibrate DSC model, that is, to calculate all the material parameters including parameters of both the relative intact state material and the disturbed state material.

A.2 Command Line Options

The following is DSCOPT command line options.
usage: dscopt [-ihlp] [filename]

h -- invoke this help info

b[ia] -- backpredict relation specified in the input file
   i -- relative intact state relation
   a -- average relation

c[123] -- Calculates constants only:
   1 -- calculate intact parameters
   2 -- calculate critical parameters
   3 -- calculate disturbance parameters

d[01] -- Drift correction method
   0 -- disable drift correction
   1 -- enable drift correction

i -- Implementation methods

l[adptuw] -- only listing input data without executing
   d -- data
   p -- loading paths
   t -- titles
   u -- user provided parameter values
   w -- weight of path for model parameters
   a -- all the above

m[n] -- make intact backprediction plot of n curves (default n=1)

p[f] -- plot input data
   f -- plot fitted input data

s[idi] -- State to be calculated
   i -- intact state
   d -- disturbed state

V -- show version number
A.3 Format of Input Data File

The format of input data file for DSCOPT is defined by a grammar. The advantage of using a grammar for the data file over a traditional Fortran style input data is that users only need to follow an easy remembered grammar instead of puzzles of counting bits in a data file. Most times users are frustrated by the confusion of Fortran style data file. From day one when the DSCOPT was implemented, this unique technique has been designed in mind, it can greatly reduce the time-consuming part of data preparation.

A.3.1 Grammar of Input Data Format

The format of input data file is defined by a grammar which is defined as follows:

\[
\text{input} : \text{one_input} \\
| \text{multiple_input}
\]

\[
\text{one_input} : \text{test_data_part} \\
| \text{loading_path_weight_part} \\
| \text{parameter_value_part}
\]

\[
\text{test_data_part} : \text{test} \{ \text{test_data_block} \}
\]

\[
\text{Loading_path_weight_part} : \text{path_wei} \{ \text{path_wei_list} \}
\]

\[
\text{parameter_value_part} : \{ \text{parameter_value_list} \}
\]

\[
\text{test_data_block} : \text{test_desc_list}, \text{data_content}
\]

test_desc_list : test_desc
    | test_desc_list, test_desc

    test_desc : title = string
        | init_strs = value ( strs_unit )
        | used_to_find_param = yes_or_no
        | back_predict = yes_or_no
        | loading_path = path_desc

    data_content_part : data_content ( resp_desc, resp_desc ) data_list

    path_desc : one_set_data_path
        | one_set_data_path

    one_set_data_path : HC

    two_set_data_path : CTC
        | CTE
        | RTC
        | TC
        | TE

    strs_unit : psi
        | kpsi
        | mpsi
        | pa
| kpa
| mpa

yes_or_no : yes
| no

resp_desc : vol_strn
 | axi_strn
 | vol_strn ( % )
 | axi_strn ( % )
 | axi_strs ( strs_unit )
 | conf_strs ( strs_unit )
 | J2_strs ( strs_unit )
 | sigma1_sigma3 ( strs_unit )

data_list : value value
 | data_list value value

value : number
 | - number
 | + number

number : integer
 | float

path_weight_part : param_path_wei { param_path_wei_list }
param_path_wei_list : param_path_wei
                   | param_path_wei_list param_path_wei

param_path_wei : param_desc :

path_wei_list : path_wei
               | path_wei_list path_wei

path_wei : path_desc = number
          | default = number

param_val_part : param_val { param_val_list }

param_val_list : param_val
                | param_val_list param_val

param_val : param_with_unit = value strs_unit
          | param_without_unit = value

param_with_unit : E
                | K
                | G

param_without_unit : nu
                   | lambda
                   | mbar
                   | gamma
                   | beta
A.3.2 Comments inside Input Data File

The grammar of DSCOPT input data takes a C style comment at anywhere inside a input data file, that is, all things between /* and */ will be treated as a comment which in fact will be ignored by DSCOPT.

A.4 Data Examples of Valid Input Data

Here some basic valid input data (according to the grammar) are showed in Data Example A.4.0.1, Data Example A.4.0.2 on Page 233, and Data Example A.4.0.3 on Page 234.

A.5 Program Modules of DSCOPT

The computer program of DSCOPT has basically following modules:

1. Data parsing and syntax checking
2. constitutive model calibrating
3. optimization computing
**Data Example A.4.0.1 Valid Input Data**

```plaintext
test{
  used_to_find_param = yes,
  back_predict = yes,
  loading_path = HC,
  data_content (vol_strn, conf_strs (psi))
  0.000000 0.000000
  0.006546 10.258844
  0.016497 60.220117
  0.018476 120.682189
}
```

4. Data output

5. Constitutive model computing

6. Constitutive response computing

7. Data plotting

8. Matrix and vector computing

The relation of these modules are outlined in Fig. A.1.
Data Example A.4.0.2 Valid Input Data

/*
Comment: figure 4.24a
*/
test {
title = "Fig. 4.24a DCTC Strs-Strn Resp (Dr=65%)",
init_strs = 13 (psi),
used_to_find_param = yes,
back_predict = yes,
loading_path = DCTC,
/* No. */ data.content (axi.strn, sigmal-sigma3 (psi))
/* 1 */ 0.000000 0.000000
/* 2 */ 0.000444 2.382564
/* 3 */ 0.000845 10.743970
/* 4 */ 0.002061 17.757466
/* 5 */ 0.004906 26.796816
/* 6 */ 0.009644 37.234344
/* 7 */ 0.022934 41.176491
/* 8 */ 0.028997 43.646975
/* 9 */ 0.058567 44.546897
/* 10 */ 0.096725 43.833754
/* 11 */ 0.145158 43.011959
/* 12 */ 0.216532 43.011754

/* No. */ data.content (axi.strn, vol_strn (%))
/* 1 */ 0.000000 0.000000
/* 2 */ 0.001941 0.080675
/* 3 */ 0.004461 0.166955
/* 4 */ 0.009326 -0.008786
/* 5 */ 0.021851 -0.582287
/* 6 */ 0.041758 -1.523226
/* 7 */ 0.048621 -2.199992
/* 8 */ 0.096849 -4.029505
/* 9 */ 0.128035 -4.962719
/* 10 */ 0.190059 -6.159372
/* Added */ 0.216632 -8.011754
}
### Data Example A.4.0.3 Valid Input Data

#### param_val{

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.7</td>
</tr>
<tr>
<td>( E )</td>
<td>12077.2 psi</td>
</tr>
<tr>
<td>( G )</td>
<td>4052.75 psi</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.06579845</td>
</tr>
<tr>
<td>( \psi )</td>
<td>0.000194817</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.638579</td>
</tr>
<tr>
<td>( J_{ls} )</td>
<td>0</td>
</tr>
<tr>
<td>( A )</td>
<td>1.71128</td>
</tr>
<tr>
<td>( Z )</td>
<td>0.457797</td>
</tr>
</tbody>
</table>

#### param_path_weight{

<table>
<thead>
<tr>
<th>Material</th>
<th>Value</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K )</td>
<td>HC=1</td>
<td>0.5</td>
</tr>
<tr>
<td>( \nu )</td>
<td>DCTC=1 DRTE</td>
<td>0.5</td>
</tr>
<tr>
<td>( E )</td>
<td>DCTC=1</td>
<td>0.1</td>
</tr>
<tr>
<td>( \psi )</td>
<td>DCTC=2</td>
<td>0.2</td>
</tr>
<tr>
<td>( A )</td>
<td>DCTC=5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

#### test {

<table>
<thead>
<tr>
<th>Data Content</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( vol_{strn}, conf_{strs} (psi) )</td>
<td>0.000000 0.000000</td>
</tr>
<tr>
<td>( axi_{strn}, sigmal-sigma3 (psi) )</td>
<td>0.000863 10.422619</td>
</tr>
<tr>
<td>( axi_{strn}, vol_{strn} )</td>
<td>0.000000 0.002561</td>
</tr>
</tbody>
</table>

Comment: figure 4.25a */

#### test {

<table>
<thead>
<tr>
<th>Data Content</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( vol_{strn}, conf_{strs} (psi) )</td>
<td>0.000000 0.000000</td>
</tr>
<tr>
<td>( axi_{strn}, sigmal-sigma3 (psi) )</td>
<td>0.134823 102.278301</td>
</tr>
<tr>
<td>( axi_{strn}, vol_{strn} )</td>
<td>0.205848 101.836002</td>
</tr>
</tbody>
</table>

Comment: figure 4.25b */

<table>
<thead>
<tr>
<th>Data Content</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( vol_{strn}, conf_{strs} (psi) )</td>
<td>0.000000 0.002561</td>
</tr>
<tr>
<td>( axi_{strn}, vol_{strn} )</td>
<td>0.0007735 0.000873</td>
</tr>
</tbody>
</table>
FIGURE A.1. Program Modules of DSCOPT
REFERENCES


Toth, J. C. and Desai (1994). Development of lunar ceramic composites, testing and constitutive modeling, including cemented sand. Technical report, Department of Civil Engineering and Engineering Mechanics, University of Arizona, Tucson, AZ, USA. Report to the National Science Foundation.