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WADE, STEVEN HOWARD

THE IMPLICATIONS OF DECREASING BLOCK PRICING FOR  
INDIVIDUAL DEMAND FUNCTIONS: AN EMPIRICAL APPROACH

*The University of Arizona*

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THE IMPLICATIONS OF DECREASING BLOCK PRICING  
FOR INDIVIDUAL DEMAND FUNCTIONS:  
AN EMPIRICAL APPROACH

by

Steven Howard Wade

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A Dissertation Submitted to the Faculty of the  
DEPARTMENT OF ECONOMICS  
In Partial Fulfillment of the Requirements  
For the Degree of  
DOCTOR OF PHILOSOPHY  
In the Graduate College  
THE UNIVERSITY OF ARIZONA

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THE UNIVERSITY OF ARIZONA  
GRADUATE COLLEGE

As members of the Final Examination Committee, we certify that we have read  
the dissertation prepared by Steven Howard Wade

entitled The Implications of Decreasing Block Pricing for Individual  
Demand Functions: An Empirical Approach

and recommend that it be accepted as fulfilling the dissertation requirement  
for the Degree of Doctor of Philosophy.

<u>James M. Walker</u>	<u>4/11/80</u> Date
<u>Ronald L. Ogawa</u>	<u>4/11/80</u> Date
<u>Foster D. Taylor</u>	<u>4-11-80</u> Date
_____	_____ Date
_____	_____ Date

Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copy of the dissertation to the Graduate College.

I hereby certify that I have read this dissertation prepared under my direction and recommend that it be accepted as fulfilling the dissertation requirement.

<u>Foster D. Taylor</u> Dissertation Director	<u>4-11-80</u> Date
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*Steven M. Wade*

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## ABSTRACT

Decreasing block pricing refers to the practice of selling a product at successively lower marginal prices as the amount purchased in any one time period increases. In more familiar terms, this practice can be thought of as any quantity discount scheme as long as marginal price does not vary continuously with quantity. Decreasing block pricing results in a faceted, non-convex budget set, and under standard assumptions concerning consumer preferences, yields several nonstandard theoretical implications. The central goal of this paper is to formulate an estimation technique which is consistent with these implications.

When the budget set is not convex, the uniqueness of consumer equilibrium is no longer guaranteed. It also follows that discontinuities in demand occur whenever consumer equilibrium shifts from one facet of the budget constraint to another. Prior empirical studies have not made use of demand functions consistent with these results. In Chapter 2, a utility-maximizing algorithm was developed to determine consumer equilibrium given the declining block pricing schedule and income for a Cobb-Douglas utility function. In developing this algorithm, it was made clear that the proper approach for estimating individual demand was through the use of a block-dependent independent variable.

The coefficient of this block-dependent independent variable provided an estimate of a utility function parameter which completely specified the Cobb-Douglas form. Incorporating this utility function estimate into the utility-maximization algorithm made it possible to

obtain estimates of consumption given changes in any or all of the rate schedule components.

While the use of a block-dependent independent variable is the theoretically correct method for estimating demand, it poses an inescapable problem of errors-in-variables. A Monte Carlo study was performed in Chapter 2 to investigate, among other things, the seriousness of the errors-in-variables bias. The results were quite encouraging. When using data incorporating extremely large error variances, amazingly precise estimates were obtained. Another encouraging Monte Carlo result was when comparing samples not containing a discontinuity with those with one, it was found that the latter produced estimates with statistically significant superiority.

Chapter 3 generalized the estimation technique of the previous chapter to allow the estimation of demand using cross-sectional data. The data base recorded monthly electricity consumption for households from a number of cities whose utilities had decreasing block rates. Seven of these cities were selected for analysis. The data also included various demographic characteristics and electric appliance stock information. The generalization was accomplished by assuming that all households had a Stone-Geary utility function. Also, the utility function parameter representing the minimum required quantity of electricity was assumed to depend linearly on the household's appliance stock and demographic characteristics. This allowed demand to vary across households on the basis of this parameter and income.

The results of applying this regression technique to the cross-sectional data were then compared with results from a conventional,

non-theoretically based demand specification. The data were used in pooled and individual month form with the former yielding much better statistical results. The Stone-Geary form provided a greater number of significant coefficients for price and income variables than the conventional version. The predominant failure of the conventional version was that the coefficient of marginal price was rarely significant and when significant, frequently of the wrong sign. For the same samples, the Stone-Geary results were quite acceptable except for the regressions involving one of the cities. Thus, it was demonstrated that a method consistent with the theoretical implications of decreasing block pricing is easily applied to cross-sectional data and produces better results than conventional techniques.

## CHAPTER 1

### INTRODUCTION

Decreasing block pricing refers to any pricing scheme for which the marginal price declines as the quantity purchased in any one time period increases. While the majority of commodities are single-priced, that is, price is invariant with the quantity consumed, several examples of decreasing block pricing can be enumerated. One of the most important and widespread uses of decreasing block pricing occurs in household electricity rate schedules. Also, natural gas for household consumption is frequently priced in this way. Quantity discounts are another form of decreasing block pricing. One often notices this practice in the supermarket where one can purchase the "regular-sized" package or the larger "economy-sized" package. These pricing policies may or may not be justified on the basis of reduced cost for supplying the larger amounts per period. In any event, decreasing block pricing has been shown to have important implications for consumer behavior at the theoretical level.

The fact that decreasing block pricing poses some additional theoretical problems not encountered when dealing with single-priced commodities has long been recognized in the literature. However, a complete and rigorous theoretical treatment of individual demand functions has appeared only recently. Empirical estimations of individual demand functions consistent with these theoretical implications are

nonexistent. As the title of this study suggests, the main goal of this research is to outline appropriate econometric methods consistent with these implications.

The earliest discussion of the use of decreasing block pricing in connection with the estimation of demand functions appears in an article by Houthakker (1951). In this article he criticizes the use of ex post average price in empirical studies of the demand for electricity. He notes that since average price depends inversely on quantity demanded, it is not an independent variable in the economic or statistical sense. This is a classic example of the simultaneity problem. In this case, an explanatory variable, average price, is correlated with the error term which produces biased and inconsistent coefficient estimates.

The empirical work contained in Houthakker's study uses marginal price to account for the effects of the rate schedule on consumption. The consumers in his sample faced a two-part rate schedule that consisted of a fixed monthly charge and a marginal price that applied to all kilowatt-hours purchased. The use of marginal price as a predictor did not pose the simultaneity problem that average price did because marginal price was invariant with quantity consumed. If the rate schedule had had more than one block, marginal price would have varied with quantity, and a simultaneity problem similar to the one discussed in connection with the use of average price would have occurred.

Houthakker's model excluded the fixed charge component of the rate schedule. He dismissed this component, arguing that it would have only an insignificant income effect on consumption. Since the exclusion

of a variable that properly belongs in a model always produces biased regression coefficients, a better approach would have been to include the fixed charge as part of his model.

Another paper which provides some insight into the nature of individual demand functions under decreasing block pricing is that of Buchanan (1952-53). One of his conclusions is (p. 199), "If the buyer exerts any control over price, no demand curve can be derived since quantity demanded depends on the whole market offer, not on price alone." The fact that a demand curve cannot be derived is due to the assumption that the decline of marginal price can be represented by a continuous function. That is, for any small increment in the quantity purchased, marginal price will decrease. In this case, it is not possible to vary the price of any particular unit without affecting the entire rate schedule. This is so because if such a price was changed, all other prices constant, then the decline of marginal price could not be represented by a continuous function. Thus a demand curve cannot be derived.

Under decreasing block pricing the decline of marginal price is not restricted to forms represented by a continuous function. For this reason, it is possible to derive a demand function. This is because with decreasing block pricing, it is possible to hold all but one of the block prices constant. Demand is then the relationship between quantity demanded and the block price that has been allowed to vary. Buchanan's statement that "quantity demanded depends on the whole market offer," remains valid as will be demonstrated in Chapter 2.

It is also interesting to note that the budget set under Buchanan's assumptions is nonconvex as it is for decreasing block pricing. This nonconvexity will be shown to be the cause of the potential discontinuity and multi-valuedness of demand. He (p. 203) anticipates this result in the following comment, "If an opportunity curve were of precisely the same degree of convexity as the indifference curve the behavior of the buyer would be completely erratic." By this statement he means that the actual quantity demanded would be subject to fluctuations determined by some random process. That is, ceteris paribus, consumption can vary from period to period since consumer equilibrium occurs at an infinite number of points. In the present case, only a finite number of equilibria can occur for any one schedule of rates.

Another result that has some bearing on the present discussion is given in Gabor (1955-56). The primary focus of this article concerns the efficiency of two-part tariffs in extracting a consumer's surplus. Gabor also dispels the once commonly held notion that a reduction in a block price at a quantity greater than the consumer's current equilibrium quantity could have no effect on the equilibrium position. Gabor (pp. 33-34) concludes that this type of behavior is rational only for a consumer who is ignorant of the existence of the reduced price or of the quantity at which this reduction becomes available. As in the Gabor article, it is assumed in this analysis that the consumer possesses complete knowledge of the entire rate schedule.

In each of the above cases, the results reported were more or less incidental to the main arguments. A more direct discussion of the problems posed by decreasing block pricing is found in Taylor (1975).

Taylor concludes (p. 77) that due to the piecewise linearity of the budget constraint, "the equilibrium of the consumer cannot be derived, as is conventionally the case, using the differential calculus. Mathematical programming must be used instead." In Chapter 2, such a method is utilized to generate consumer equilibria for alternative block structures.

Taylor also points out that the demand function will possess discontinuities at prices where consumer equilibrium "switches from one facet of the budget constraint to another." In addition, the demand curve will be multi-valued whenever the budget constraint has multiple tangencies with the same indifference curve. These results will be illustrated and discussed further in Chapter 2.

It was not until the work of Blattenberger (1977) that an exhaustive, rigorous study of the implications of decreasing block pricing for individual consumer demand functions was completed. Under the standard assumptions about consumer preferences, she proved (p. 86) that the number of discontinuities in an individual's demand function can never exceed the number of marginal blocks in the rate schedule minus one. She showed that for changes in price or income which cause quantity demanded to switch from one block to another, a discontinuity will always occur. Also for changes in price or income which do not result in a block change, demand is continuous as in the usual case. The implications of these results for the estimation of individual consumer demand functions will be expanded in subsequent chapters. Other results from her discussion of individual demand will be cited as they become necessary.

Taylor (1975) discusses one final issue relevant to the present case. This concerns the absence of empirical studies utilizing theoretically plausible demand functions. Two considerations must be addressed. First of all, theoretically plausible demand functions must satisfy the Slutsky symmetry conditions. Blattenberger (1977, pp. 91-104) has shown that for differentiable demand functions, the Slutsky restrictions hold "almost everywhere" under decreasing block pricing. Since the current discussion concerns the estimation of individual demand functions, and since complete sets of demand functions are estimated at the aggregate level, no further discussion of the Slutsky conditions is needed.

The second issue is that theoretically plausible demand functions must be consistent with utility maximization. This requires that an empirical estimation of individual demand must allow for the aforementioned potential discontinuity and multi-valuedness. Devising such an estimation is the central goal of this study.

As will be shown in Chapter 2, a theoretically consistent estimation technique can be devised through the use of an independent variable whose value depends upon the block in which consumption is observed. The computation of this variable is determined by maximizing the utility of a consumer whose consumption is assumed to fall alternatively in each of the blocks in the price schedule. This process also yields a block-dependent method for specifying demand.

To clarify the implementation of this procedure, consider the following experiment. A consumer is confronted with a decreasing block pricing schedule for which one of the price components has been

systematically varied holding all other determinants of demand constant. As price is varied, observations on equilibrium quantities are generated. With this information, the block-dependent independent variable is calculated. This variable measures both the income and price effects inherent in the theoretically derived demand models. The details of its computation are fully described in Chapter 2. Regression analysis is then used to estimate a utility function parameter which is substituted into the block-dependent demand function. Since Blattenberger has shown that demand within a block is continuous, this specification is consistent with her results.

Points of discontinuity in the demand function can be estimated by using an algorithm which returns a unique quantity which maximizes a utility function subject to the budget constraint implied by the decreasing block pricing schedule. By using the estimate of the utility function parameter to completely specify the utility function along with the series of prices used to generate the original data, the discontinuity points will be estimated to occur when the utility maximizing quantity changes blocks.

A simultaneity problem similar to the one that occurs when using average price as a predictor is present when employing the above methodology. The value of the independent variable is a function of the observed quantity and vice versa. Although average price continuously varies with quantity, the independent variable currently can only assume a finite number of values. This number is equal to the number of marginal blocks in the rate schedule.

Data generated through the systematic variation of one of the block prices is not available at this time. Monte Carlo data will be generated using the utility-maximization algorithm mentioned above. The results from applying the theoretically consistent estimation technique to these data will be used to address several issues. One of the most important is whether the simultaneous equations bias poses any severe estimation problems. Another issue concerns the relative efficiency of the estimates for samples containing a discontinuity as opposed to those without one.

Another consideration of Chapter 2 involves the use of the Tobit estimator. Tobit is needed to deal with a truncation bias that also occurs when implementing this estimation technique. Tobit is a consistent estimator in the statistical sense, and the results of applying Tobit to the Monte Carlo data will be contrasted with the OLS results.

The Monte Carlo data simulated the ideal data structure for employing a theoretically consistent estimation strategy. In recent years, various research groups and electric utilities have collected data bases recording individual household electricity consumption along with numerous appliance stock and demographic data. The variation in these data sets is across households with little or no variation in the rate schedule. Chapter 3 will generalize the method proposed in the previous chapter to allow the use of a theoretically consistent estimation technique on this type of data.

Since there is essentially no price variation, estimates of demand from utility function parameters must be obtained from other sources of variation. In a cross-sectional data base, considerable

income variation is normally present. Appliance stock data and demographic characteristics will also be incorporated. A conventional or ad hoc estimation will be employed on the same samples as the theoretically correct one. This will serve as a basis for making comparisons and evaluating results.

Finally, a concluding chapter will summarize the major points of the preceding chapters. Suggestions for further research and improvements in the area of data collection will also be included.

## CHAPTER 2

### THE THEORY AND ESTIMATION OF INDIVIDUAL DEMAND

In this chapter the theory and estimation of individual demand functions under decreasing block pricing will be investigated. As mentioned in the introductory chapter, Blattenberger (1977) has provided an in-depth analysis of the theory, so the section on theory will be short and draw heavily from Blattenberger's work. Given a specification of a utility function, a decreasing block rate schedule, and income, the estimation methodology is derived from and hinges upon an algorithm which will find the quantity or quantities of the good which maximize the consumer's utility. Such an algorithm will be introduced after the summary of the theory. Finally, this algorithm will be used to generate data for a Monte Carlo study of the proposed estimation technique.

As was mentioned in the introductory chapter, data appropriate for the estimation of a single individual's utility function using time-series observations are nonexistent. The Monte Carlo study will simulate the use of data of this nature. In recent years, research groups and utilities have collected cross-section, time-series data bases. A variant of the estimation technique of this chapter will be applied to one such data base in Chapter 3.

The results of the Monte Carlo study will be useful in judging the validity of the Chapter 3 estimations. This is due to the fact that the estimation problems are essentially the same for both methods. The

Monte Carlo study will shed light upon some of the estimation problems that would be encountered when and if time-series data on individual consumers become available.

### Consumer Theory for the Two-Block Case

Before discussing the estimation of individual demand functions it is necessary to review the theoretical background developed by Blattenberger (1977). The basic complication posed by decreasing block pricing is that the budget constraint is piecewise linear. Thus, if the consumer's utility function is continuous, an equilibrium will exist, but it is not necessarily unique. This follows from the fact that the budget set will remain compact under decreasing block pricing, although it is no longer convex. Existence follows since any continuous function will attain at least one maximum (and one minimum) over a compact domain. However, uniqueness is not guaranteed because convexity of the budget set is required. Also, the derivation of consumer equilibrium is complicated since it is not possible to specify a constraint with continuous and unique first partial derivatives for the usual Lagrangean function.

To begin this discussion, it is convenient to consider the least complicated decreasing block pricing structure. This is the case where the rate schedule consists of a fixed charge and two marginal block prices. Call these  $z_1$ ,  $z_2$ , and  $z_3$ , respectively. Note that decreasing block pricing requires that  $z_2$  be strictly greater than  $z_3$ . The theoretical results are easily generalized when the pricing structure consists of an arbitrary number of marginal blocks.

Let  $q_1$  denote the quantity of the commodity priced according to a decreasing block structure. In the present discussion, this will be referred to as the quantity of electricity measured in kilowatt-hours. If the data are generated in a short enough time interval, relative prices may be assumed constant. If so, we may define  $q_2$  as the Hicksian composite commodity without loss of generality. This is a necessary assumption since any further detailing of  $q_2$  would present formidable data collection problems.

The consumer's electric bill will be determined as follows:

$$\begin{aligned} & \$z_1 && \text{for } q_1 \leq KWH_1 \\ (1) \quad & z_2 \text{ ¢/kwh} && \text{for } KWH_1 < q_1 \leq KWH_2 \\ & z_3 \text{ ¢/kwh} && \text{for } q_1 > KWH_2. \end{aligned}$$

An algebraic expression for the budget constraint is given in Taylor (1975, p. 76) as:

$$(2) \quad g(z_2, q_1, KWH_1, KWH_2) + h(z_3, q_1, KWH_2) + p_2 q_2 = x - z_1,$$

with  $x$  denoting the consumer's income, and

$$(3) \quad g = \begin{cases} 0 & \text{for } q_1 \leq KWH_1 \\ z_2 (q_1 - KWH_1) & \text{for } KWH_1 < q_1 \leq KWH_2 \\ z_2 (KWH_2 - KWH_1) & \text{for } q_1 > KWH_2 \end{cases}$$

$$(4) \quad h = \begin{cases} 0 & \text{for } q_1 \leq KWH_2 \\ z_3(q_1 - KWH_2) & \text{for } q_1 > KWH_2. \end{cases}$$

Graphically, this budget constraint takes the form as given in Figure 1.

For generality, the only restriction placed on the consumer's utility function is that the indifference curves are differentiable and convex. All utility functions used in empirical studies obey this restriction. Therefore, the following results will hold regardless of the specific form of the utility function.

An illustration of some of the theoretical results is given in Figures 2 through 4. Figure 2 shows the occurrence of multiple equilibria. That is, the budget constraint is tangent to the same indifference curve at two locations. Note that the above smoothness conditions for the indifference curves implies that consumer equilibrium can never occur at  $KWH_2$ . For future reference, denote the prices at which multiple equilibria occur as  $z_1^*$ ,  $z_2^*$ , and  $z_3^*$ .

Figure 3 illustrates the occurrence of a discontinuity in the demand function. Initially, set  $z_3$  at some value greater than  $z_3^*$  while  $z_1$  and  $z_2$  remain fixed at  $z_1^*$  and  $z_2^*$ , respectively. Note that for any such value of  $z_3$  consumer equilibrium will occur at the quantity represented by point A. As one systematically lowers  $z_3$  in constant increments, the increment for which  $z_3$  first becomes less than  $z_3^*$  will cause a jump in equilibrium from point A to point B'. At this value of  $z_3$  a discontinuity in demand occurs. Of course, if  $z_3$  should ever equal  $z_3^*$  (a very low probability event) then equilibrium will occur at either A or B'. Blattenberger (1977, p. 86) provides a rigorous proof (for a

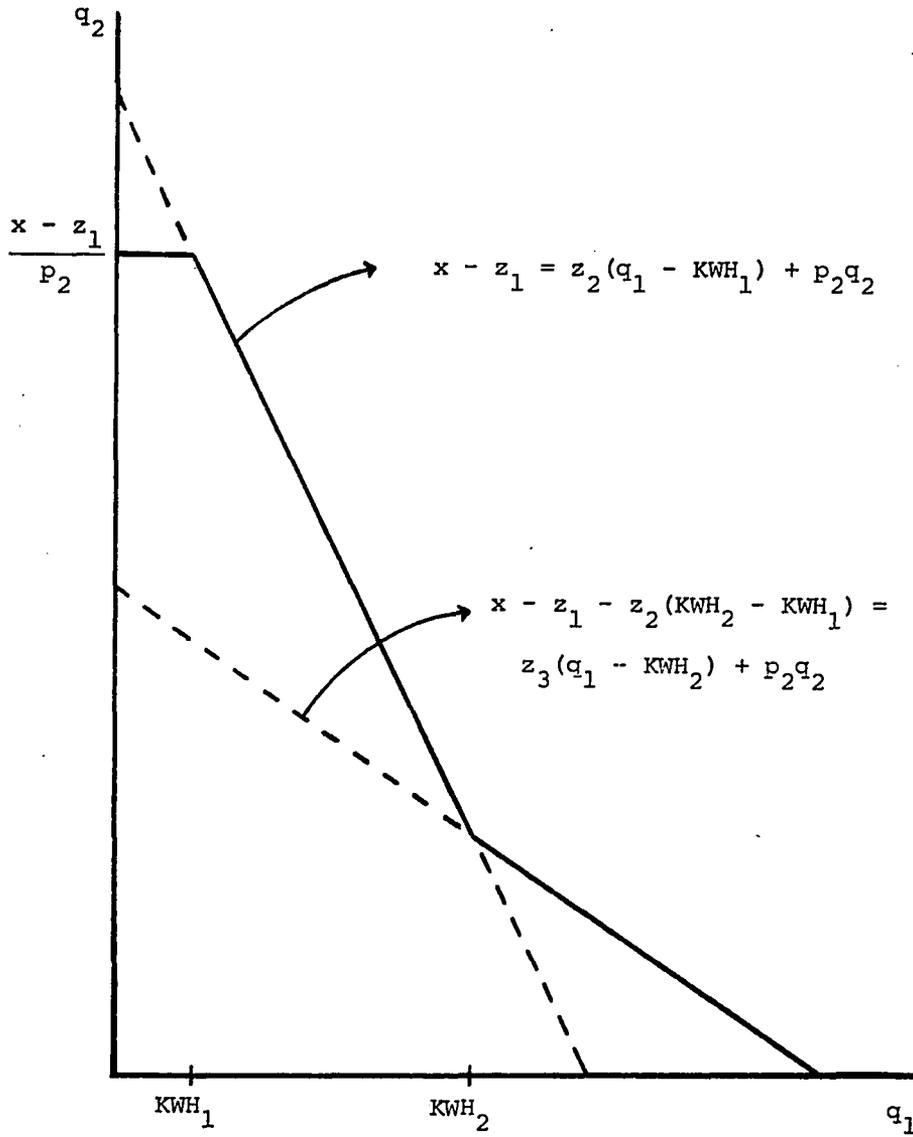


Figure 1. The Budget Constraint.

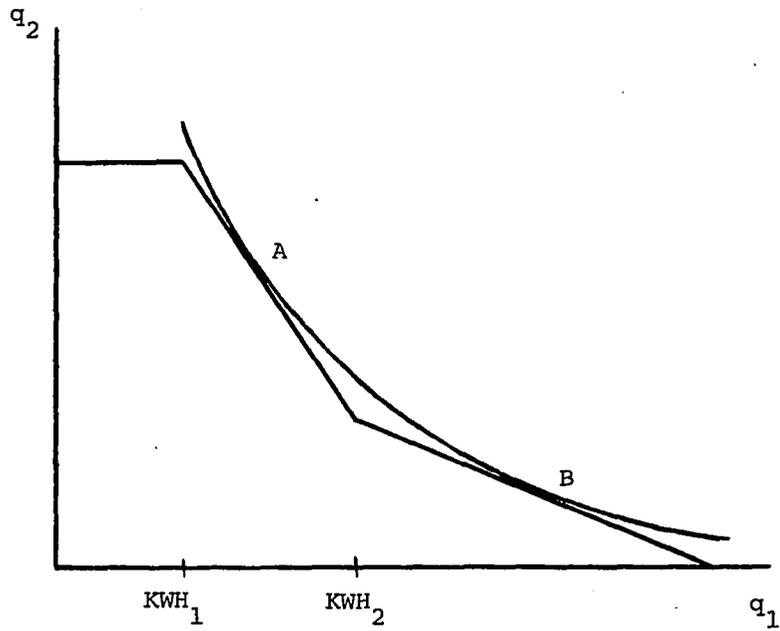


Figure 2. Multiple Equilibria.

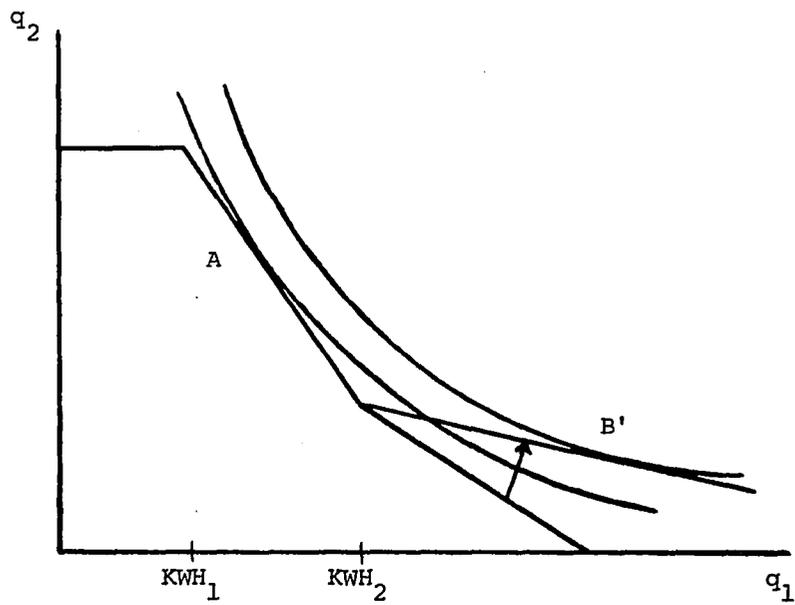


Figure 3. Change in  $z_3$  Resulting in Block Change.

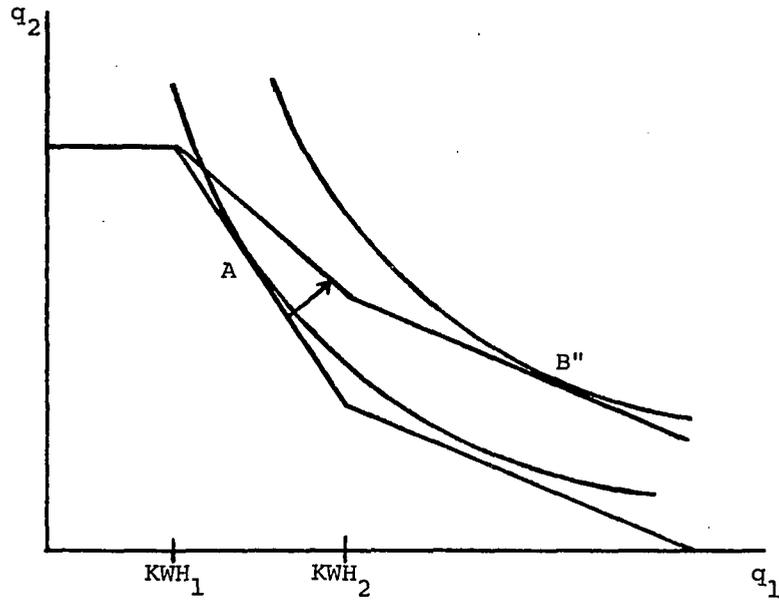


Figure 4. Change in  $z_2$  Resulting in Block Change.

more general case with  $m$  blocks) that demand will be continuous for the two subsets of data defined by  $z_3 > z_3^*$  and  $0 < z_3 < z_3^*$ .

A similar situation is depicted in Figure 4. However, this time assume  $z_1 = z_1^*$  and  $z_3 = z_3^*$ . Assume that initially  $z_2$  is greater than  $z_2^*$  (as usual  $A'$  will lie to the left of  $A$ ). Again incrementally lower  $z_2$  with the increment size chosen small enough so as to trace out several equilibria in the  $z_2$  block. When finally  $z_2$  becomes lower than  $z_2^*$ , equilibrium will again switch to the  $z_3$  block. Once more, it has been proven in Blattenberger (1977, p. 77) that the demand functions tracing out these two subsets are continuous.

The prior discussion illustrates the concept of demand as developed in Blattenberger (1977). It has been demonstrated that the maximum number of discontinuities in a consumer's demand function is equal to the number of marginal blocks minus one. These discontinuities always occur when consumer equilibrium switches from one marginal block to another. However, within any particular block, demand is continuous. This suggests the following estimation techniques (ignoring, for the moment, any complications posed by the error term). Separate the data into two subgroups as above, the first group containing all equilibria in the  $z_2$  block, the second group containing all equilibria in the  $z_3$  block. Since these two subgroups are observations generated by two continuous functions, simply estimate separate regressions for each subgroup. Equivalently, the appropriate set of dummy variables would also be sufficient to allow for the discontinuity. A method for estimating the parameters of the underlying utility function will be

discussed below, but first it is necessary to discuss the methodology for determining consumer equilibrium.

The Mathematical Programming  
of Consumer Equilibrium

As was mentioned earlier, the fact that the budget constraint is nonconvex poses some slight complications to the standard theoretical exercise of determining consumer equilibrium. To illustrate the method that will be used, recall the budget constraint of Figure 1. The equation of the portion of the constraint in the  $z_2$  block can be written using equations (2), (3), and (4) as:

$$(5) \quad x - z_1 = z_2(q_1 - KWH_1) + p_2q_2.$$

Similarly, the equation of the portion of the budget constraint in the  $z_3$  block can be written:

$$(6) \quad x - z_1 = z_2(KWH_2 - KWH_1) + z_3(q_1 - KWH_2) + p_2q_2.$$

Assume that the utility function has the following form,  $U = q_1^\alpha q_2^{1-\alpha}$ , where  $\alpha$  is greater than zero and less than one. This functional form obeys the necessary smoothness conditions for the utility function as outlined above. One can derive two quantities by maximizing this utility function subject to the budget constraints given by equations (5) and (6). They are, respectively:

$$(7) \quad q_1^0 = \alpha[x - z_1 + z_2 KWH_1]/z_2$$

$$(8) \quad q_1' = \alpha [x - z_1 - z_2(KWH_2 - KWH_1) + z_3 KWH_2] / z_3.$$

The superscripts on the quantities denote equilibria derived from the budget constraint of the  $z_2$  and  $z_3$  blocks, respectively. The two quantities are then checked for feasibility. That is, if  $q_1^0$  is greater than  $KWH_2$  then this value is set to zero since it is derived from (7) and is valid only for the  $z_2$  block. Similarly, if the value of  $q_1'$  is less than  $KWH_2$  then  $q_1'$  is inadmissible and set to zero. Note that at least one of the quantities must be admissible since the consumer always achieves equilibrium under DBP. If only one of the quantities is acceptable, then this value is returned by the subroutine as the unique maximum  $q_1^*$ . Finally, if both are feasible, then their respective levels of utility are calculated and the optimum is determined by choosing the level of consumption with the highest utility. If two quantities yield the same utility, one is randomly chosen as the maximum. For further details refer to the flowchart of Figure 5.

Finally, it can be seen that equations (7) and (8) include price terms for all the blocks up to and including the marginal block. For example, in equation (8), consumption occurs in the second ( $z_3$ ) block, and the term  $\alpha[-z_1 - z_2(KWH_2 - KWH_1) + z_3 \cdot KWH_2] / z_3$  represents the income effects due to consumption from prior blocks. Thus a change in  $z_2$  when consumption is in the  $z_3$  block causes a decrease in the quantity of electricity due to the deduction of  $z_2(KWH_2 - KWH_1)$  from income,  $x$ . Also note that the price  $z_3$  enters equation (8) in the usual inverse manner for accounting for changes in the marginal price. Thus, the problem of how to include price and income effects properly in demand

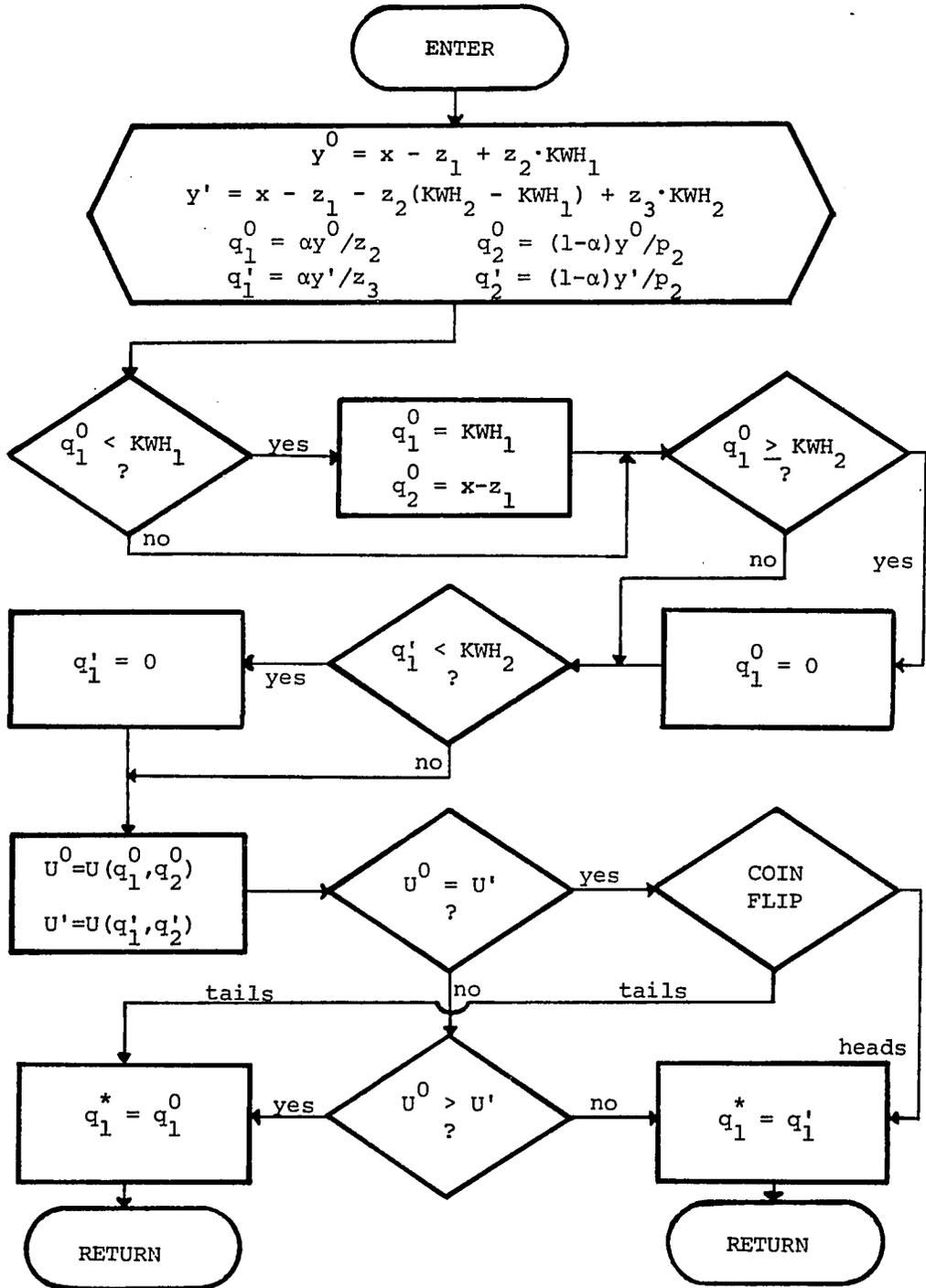


Figure 5. Flowchart for Determining Consumer Equilibrium for the Two-Block Case.

equations under decreasing block pricing has been solved through the use of the algorithm for determining the appropriate consumer equilibrium. However, since these price and income effects vary according to block, the specification of demand becomes block-dependent. Thus, it is not possible to write a single demand equation valid for all consumers as is the case when dealing with single-priced commodities.

When deriving these block-dependent demand equations caused by decreasing block pricing, the following generalization will simplify the work. For any particular marginal block, the demand equation for that block can be written in the same form as when the budget constraint is linear, except that price is replaced by marginal price and income is replaced by income adjusted by the effects of prior block consumption. This income effect can be computed as the difference between what the consumer's electric bill would have been if all electricity were purchased at the marginal price and the consumer's actual bill. This is the same adjustment that has been advocated for use in the empirical studies on aggregate demand in Taylor, Blattenberger, and Verleger (1977, pp. 2-11).

#### The Estimation of Utility Function Parameters

In this section a method for estimating individual demand functions under decreasing block rate schedules in a manner wholly consistent with the previously discussed theoretical implications will be investigated. This method must allow for the potentially discontinuous nature of demand when decreasing block pricing is employed. In the absence of an experimental data base constructed to facilitate the

estimation of demand in this case, Monte Carlo data will be used to test the robustness of this method. As will be seen shortly, statistical considerations will lead to the derivation of three additional methods, all variations of the original method, for estimating demand. Comparisons among these four methods will then be based on their relative performances when applied to the Monte Carlo data.

Returning to a discussion of the Cobb-Douglas form, an estimate of the parameter  $\alpha$  can be obtained directly from equations (7) and (8). Recall that these equations represent the functional form of demand in the  $z_2$  and  $z_3$  marginal blocks, respectively. Momentarily ignoring the statistical considerations posed by the error term, the following procedure is sufficient to deal with the theoretical implications for individual demand under decreasing block pricing. This technique involves making the computation of the appropriate independent variable contingent upon the block in which consumption is observed. For example if consumption falls in the  $z_2$  block, compute the independent variable as  $[x - z_1 + z_2 \cdot \text{KWH}_1]/z_2$ . Likewise, if consumption falls in the  $z_3$  block, the independent variable becomes  $[x - z_1 - z_2(\text{KWH}_2 - \text{KWH}_1) + z_3 \cdot \text{KWH}_2]/z_3$ . If there were more than two marginal blocks, the computation would involve additional terms in the income effect and changing the marginal price in the denominator.

The OLS coefficient of the regression of the quantity of electricity consumed on the above independent variable will provide an estimate of  $\alpha$ . This estimate can then be inserted into equations (7) and (8) to give an estimate of the segments of the demand function in the  $z_2$  and  $z_3$  blocks. An estimate of  $\alpha$  in equations (7) and (8) will

not alone allow the determination of the point of discontinuity in this estimated demand function. Using the estimate of  $\alpha$  in the algorithm of Figure 5, the price at which the discontinuity occurs in the estimated demand function can be iteratively derived.

Also note that once an estimate of  $\alpha$  is obtained, say from an experiment in which  $z_2$  was varied, an estimate of the demand function with  $z_3$  varying for any values of  $z_1$  and  $z_2$  can also be determined. This estimate of demand would come from equations (7) and (8) and the utility-maximization algorithm to give an estimate of the discontinuity point.

Another issue involves the treatment of changes in price or income. When the demand function is discontinuous, these are best handled in a manner that differs from the usual practice of using elasticities. The reason for this is that the concept of elasticity is inappropriate for price or income changes that result in a block change for consumer equilibrium or, in other words, a discontinuity. Since it would not be readily apparent whether or not any given price change would bridge the discontinuity price, any use of elasticities would be quite dubious. Indeed, the application of elasticities even when demand is continuous always involves an arbitrary choice of the base price and quantity. For instance, should one use a point elasticity or an arc elasticity formula in computing the effects of a price change? Rather than resort to such arbitrary choices, information about the functional form of demand embodied in the  $\alpha$ -estimate to provide estimates of the effects of price or income changes should be used. In fact, estimates of changes in more than just marginal price can be computed using the

utility-maximization algorithm. Substituting the new price (or prices) into this algorithm would generate an estimate of the new level of consumption. Again, the use of this algorithm permits the computation of results fully consistent with the theoretical implications discussed above. Such a method is very flexible because it can produce quantity-change estimates for any simultaneous variation in any of the prices, income or other rate schedule parameters.

#### The Monte Carlo Study

A total of 366 regressions will be utilized in the Monte Carlo study. The following comments will apply to all of these regressions. First of all, the Cobb-Douglas utility function will be used without loss of generality. This is because all of the theoretical implications discussed above are valid when the indifference curves are differentiable and convex as they are for Cobb-Douglas. Cobb-Douglas has the additional advantage of being fully specified by the single parameter,  $\alpha$ . Other utility functions may require several parameters for specification, and this would lead to an underidentification of the parameters as will be seen more clearly when the estimation method is introduced. At the same time, the implications for individual demand (and even the general shape of demand) would remain the same as under the Cobb-Douglas model.

Secondly, the budget constraints over which utility will be maximized will be similar for all of the regressions. In all cases, income will be set to 300 dollars per month and the rate schedule will have a fixed charge of two dollars and two marginal blocks. For a given value of  $\alpha$ , two sets of consumer equilibria will be generated using the

Fortran subroutine based on the flowchart of Figure 5. These two sets correspond to those resulting from price variations similar to those illustrated in Figures 3 and 4. Recall that in Figure 3, the price in the second marginal block,  $z_3$ , was systematically lowered until consumer equilibrium shifted from the  $z_2$  to the  $z_3$  block. In the following cases when  $z_3$  varies, its range of variation will be from 4¢ per kwh to .755¢ in 60 equal decrements. At the same time  $z_2$  will be held constant at 4¢. Likewise, when  $z_2$  varies as in Figure 4,  $z_3$  will be 2¢ per kwh while  $z_2$  varies from 7.9¢ to 2¢, again in 60 equal decrements.  $KWH_1$  will always be 20 kwh per month, while  $KWH_2$  will take the value of either 400, 500 or 600 kwh per month.

Finally, normally distributed, zero-mean error terms will be added to each of the two sets of consumer equilibria. For each value of  $\alpha$  and  $KWH_2$ , seven different sets of error terms of increasing variance will be added to provide data for seven regressions.

To summarize the Monte Carlo studies which follow, the procedure will be to choose values for  $\alpha$  and  $KWH_2$ . For these values, two demand functions will be derived, one with  $z_3$  varying, the other with  $z_2$  varying. To these two demand functions the seven sets of error terms will be added. For two sets of  $\alpha$  and  $KWH_2$ , the number of observations will be 35 instead of 60. Table 1 shows the combinations of  $\alpha$  and  $KWH_2$  along with the number of observations per sample. Note also that two of the data sets were designed so that no discontinuity occurred to serve as a type of control group in the study.

Table 1. Organization of the Monte Carlo Data.

$\alpha$	$KWH_2$	Number of Observations	Occurrence of a Discontinuity ?
.03	400	60	Yes
.03	600	60	No
.04	400	60	Yes
.05	400	60	Yes
.05	500	60	Yes
.05	600	60	Yes
.06	500	60	Yes
.06	600	35	No
.07	600	35	Yes

To give an example of the appearance of a discontinuous demand function, refer to Figures 6 and 7. In these figures,  $\alpha$  was set to .05 and  $KWH_2$  to 500. Figure 6 plots the consumer equilibria resulting from a variation in  $z_3$  as was illustrated in indifference curve space in Figure 3. Figure 7 corresponds to the price variation represented in Figure 4.

Note that in Figure 6 there is a vertical segment to the demand function in the  $z_2$  block. The reason for this can be seen in Figure 3 where very small initial changes in  $z_3$  leave consumer equilibrium unchanged and in the  $z_2$  block. Figure 7 has no similar vertical segment because when consumer equilibrium is in the  $z_2$  block, a change in  $z_2$  results in both an income and substitution effect. Also when consumer equilibrium finally switches to the  $z_3$  block, further changes in  $z_2$  will still produce an income effect. It should be mentioned that in both of these figures, the appearance of the curves may not be smooth due to the discrete nature of the computer printer.

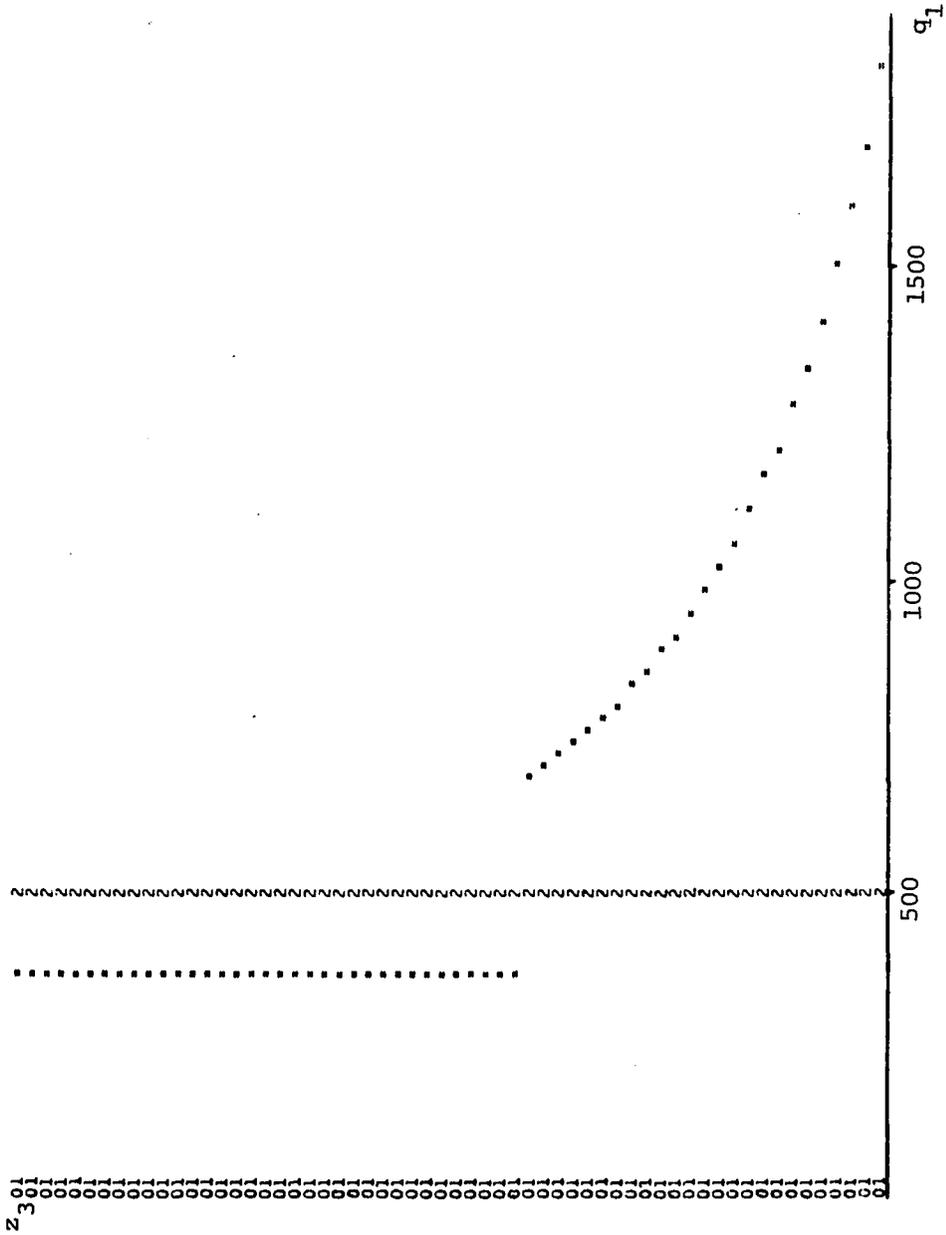


Figure 6. Demand when  $z_3$  varies,  $\alpha = .05$ ,  $KWH_2 = 500$ .

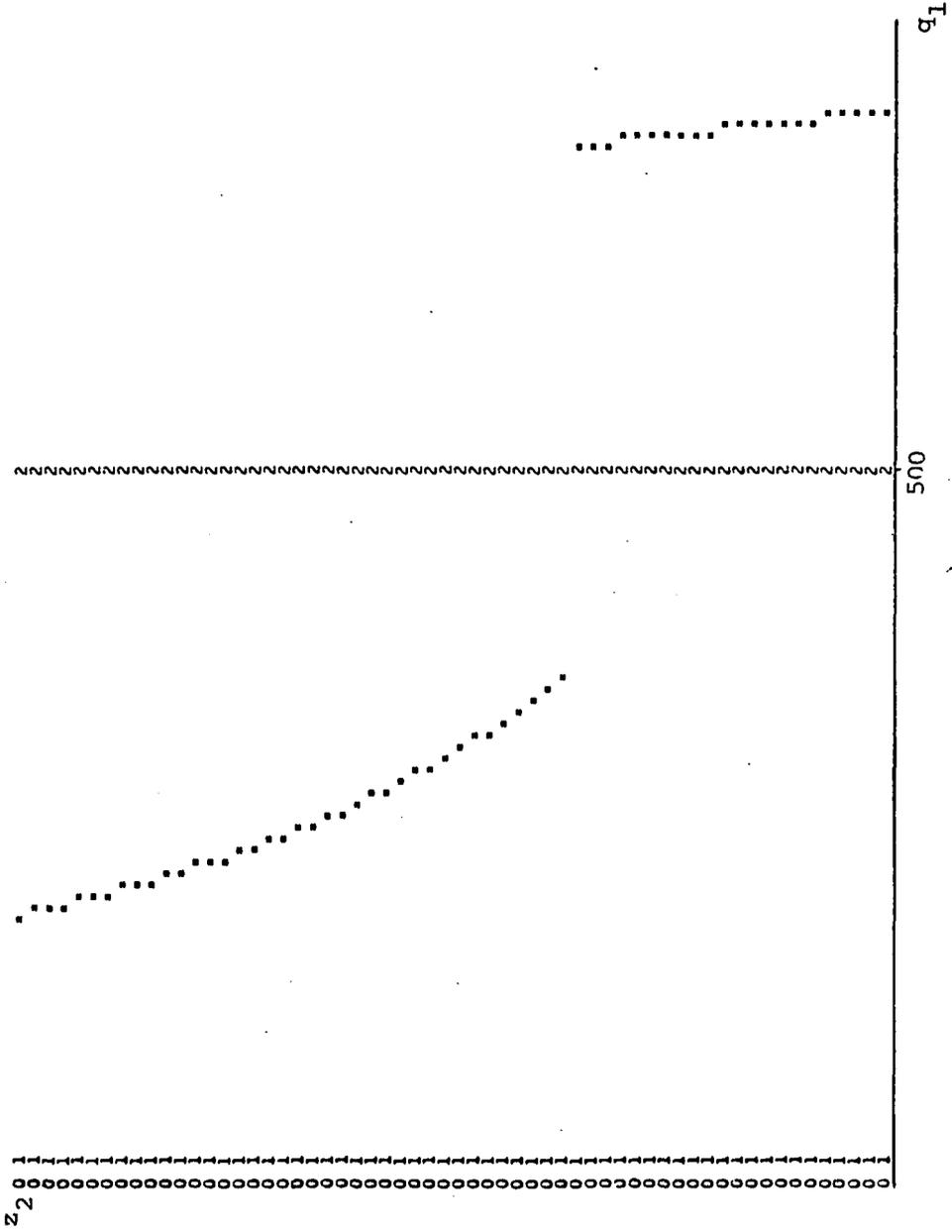


Figure 7. Demand When  $z_2$  Varies,  $\alpha = .05$ ,  $KWH_2 = 500$ .

An estimate of  $\alpha$  is obtained using OLS and the appropriately computed independent variable. The results of this methodology are illustrated in Figures 8 and 9. After adding error terms with variances of 22500, this technique is applied to the data from Figures 6 and 7. The dotted points represent the OLS fitted values, and the asterisks represent the data points with errors included. An equal sign indicates that the fitted and actual values were too close to be plotted separately. For both figures, .05 was the true value of  $\alpha$ . A comparison of the estimates given in Figures 8 and 9 with the underlying true values points out that reasonable estimates were obtained in both cases. The  $z_3$  regression of Figure 8 produced an  $\alpha$ -estimate of .0488, a 3.2% error. The results for the  $z_2$  regression were excellent with the estimated  $\alpha$ -value deviating from its true value by only .4%. Considering the amount of scattering caused by the error terms, the results of this estimation technique were quite good.

Eight of the actual or observed consumption points have been encircled in each of the Figures 8 and 9. These observations were highlighted to point out a statistical problem encountered when the computation of the independent variable is contingent upon the block in which the dependent variable is observed. The error term may displace observed consumption to a block different from the one in which the initial consumer equilibrium occurred. This was the case for the encircled points in Figures 8 and 9. In such a situation, application of the above technique will cause a miscalculation of the independent variable. As a result, an errors-in-variables problem exists which will cause bias in the OLS estimate of the parameter  $\alpha$ .

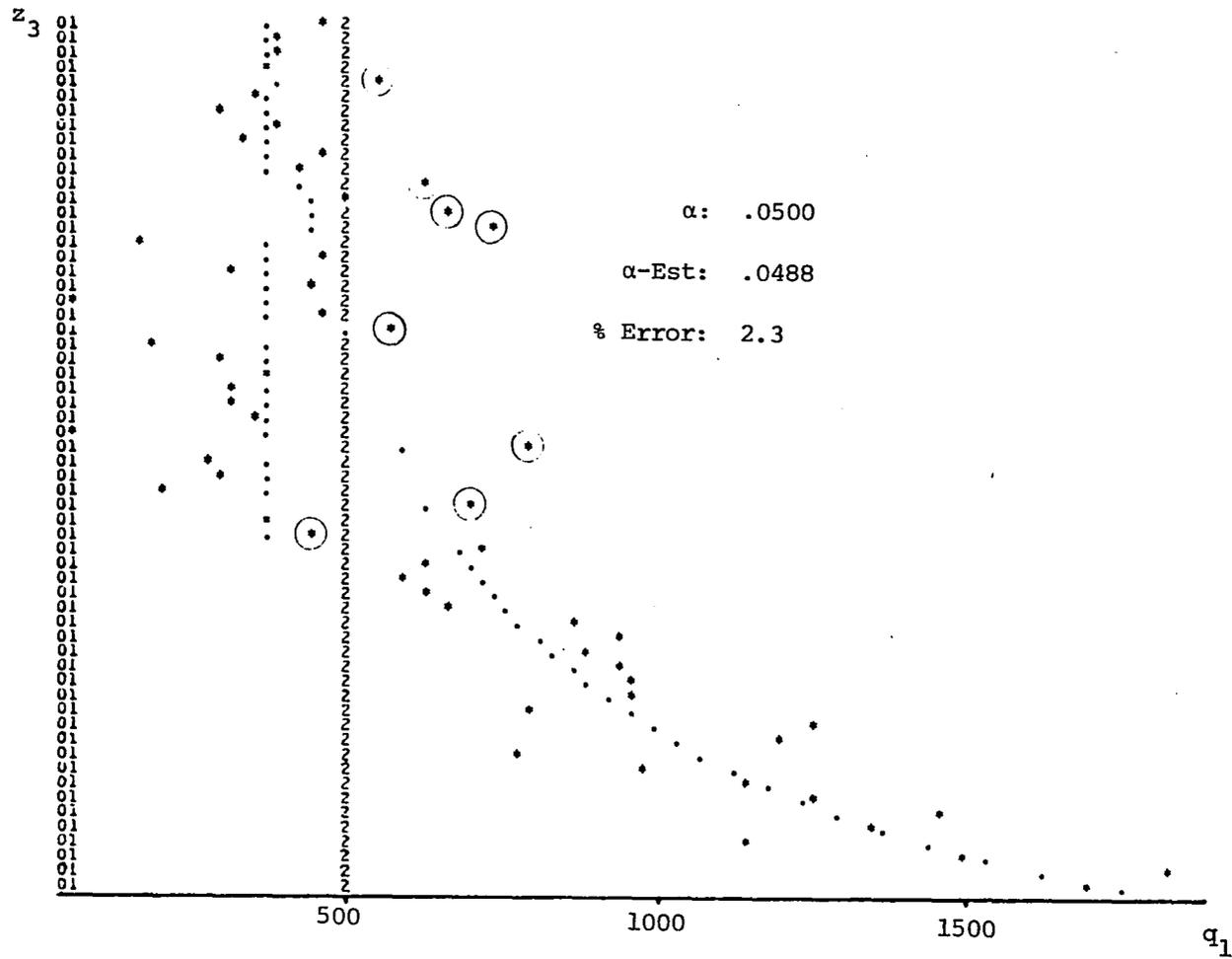


Figure 8. Error Variance Equals 22500,  $z_3$  Varies.

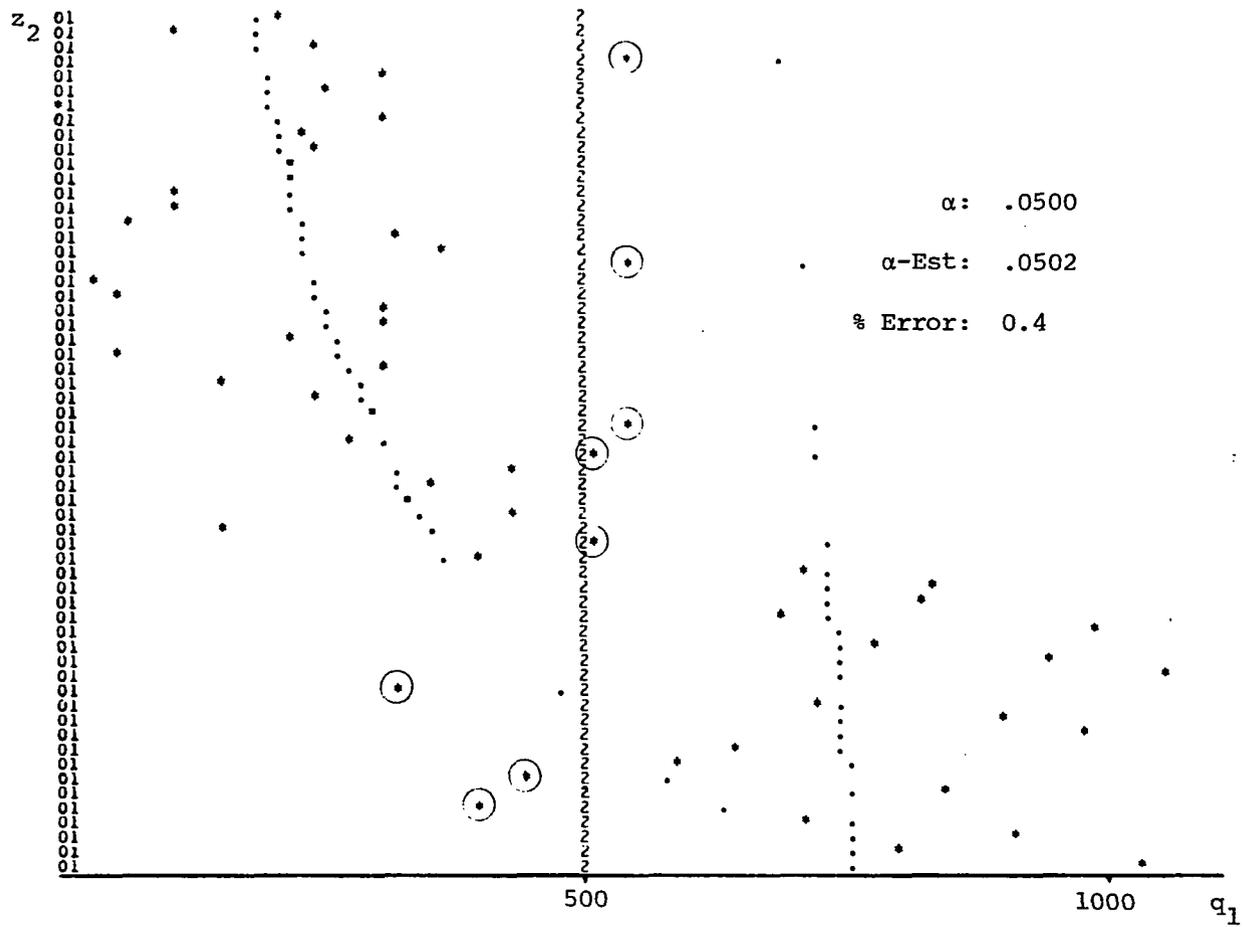


Figure 9. Error Variance Equals 22500,  $z_2$  Varies.

The bias induced by the errors-in-variables problem is probably not important in Figures 8 and 9. However, the problem is greatly magnified when errors with larger variance are used. When a variance of 122,500 is added, 13 observations were shifted for the  $z_3$  regression, 16 for the  $z_2$  regression. With approximately one-fourth of the observations switching blocks, one must consider alternate schemes that will tend to reduce the bias caused by this phenomenon.

In actual empirical work, there are a large number of determinants of electricity consumption which cannot be modeled. The error term can be viewed as arising from such determinants. Examples include changes in consumption caused by weather (such as staying home and watching television instead of playing tennis because the wind is strong), mistakes (accidentally leaving an electric oven on overnight), unexpected visits by friends or relatives, changes in tastes, and so on. In such work, the investigator would not know a priori the extent and importance of such factors. Thus, as in the Monte Carlo cases with large variances, methods for reducing bias caused by block-switching would be useful.

Although there is no theoretical method for quantifying the direction or magnitude of this bias, eliminating all observations that lie within a certain distance of the block demarcation quantity,  $KWH_2$ , may mitigate the problem. Before the error term is added, the consumer will never be observed to consume  $KWH_2$  units of electricity. In fact, the consumer will never consume electricity within a certain neighborhood of  $KWH_2$ . Thus, after the error term is included, observations close to or equal to  $KWH_2$  are more likely to have contributed to

errors-in-variables bias than are observations near zero or much in excess of  $KWH_2$ . The elimination of such points will be referred to as trimming and will be applied to the Monte Carlo data for comparison with the original or untrimmed estimation technique.

There is another statistical problem which must be addressed in the Monte Carlo study. This problem arises from the truncation of the error term caused by the impossibility of obtaining negative consumption values. This is an example of a limited dependent variable. In this situation, the standard assumptions concerning the error term will be violated. As a result, OLS estimates will be biased. This problem will be present when using either real world data or Monte Carlo data.

When constructing the Monte Carlo data, negative values for consumption can occur if the error term is large enough and negative. Merely excluding these negative observations produces the truncation bias. To illustrate, consider the following regression model with  $T$  observations:

$$(9) \quad y_t = \beta'x_t + u_t \quad t = 1, \dots, T.$$

Let  $x_t$  be scalar rather than a vector since in the case of Cobb-Douglas there is only one independent variable. In vector notation, this model becomes:

$$(10) \quad Y = \beta'X + U$$

where  $Y$ ,  $X$  and  $U$  are  $T \times 1$  vectors. Assuming  $U$  is distributed  $N(0, \sigma^2)$ , the OLS estimate of  $\beta$  is unbiased since

$$\begin{aligned}
 E(\hat{\beta}) &= \beta + E((X'X)^{-1}X'U) \\
 (11) \quad &= \beta + (X'X)^{-1}X'E(U) \quad (\text{since } X \text{ is fixed}) \\
 &= \beta. \quad (\text{since } E(U) = 0)
 \end{aligned}$$

Excluding negative or zero values of  $y_t$  produces bias in two ways. First, if observations on  $y_t$  are permitted only when  $u_t > -\beta'x_t$ ,  $E(U)$  will no longer be zero. Second,  $U$  and  $X$  will no longer be independent since an observation will tend to be omitted when  $u_t$  is large and negative. Thus,  $E((X'X)^{-1}X'U) \neq 0$  implying  $\hat{\beta}$  is biased.

Under such conditions, the Tobit estimator proposed by Tobin (1958) will provide a consistent estimate of  $\beta$ . The model for which the Tobit estimator was derived is as follows:

$$(12) \quad y_t = \begin{cases} \beta'x_t + u_t & \text{if } u_t > -\beta'x_t \\ 0 & \text{if } u_t \leq -\beta'x_t. \end{cases}$$

By setting the negative observations equal to zero rather than excluding them, Tobit estimates can be obtained for the Monte Carlo data. This estimator is derived by maximizing a likelihood function which explicitly includes the zero observations. It is a standard result that maximum likelihood estimators are consistent.

Fair (1977) describes a very efficient iterative method for obtaining the Tobit estimator. In the case where there is only one independent variable, these computations are particularly simple. To obtain the maximum computational efficiency, I developed a Fortran subroutine that produces Tobit estimates for one independent variable.

Another considerable savings in computer time was realized by having the program automatically compute Tobit estimates whenever appropriate.

For each of the data sets described in Table 1, estimates of  $\alpha$  will be produced in the following four ways. First,  $\alpha$  will be estimated using the original technique which computes the independent variable contingent upon the observed consumption value. Second, the same method will be used to obtain an estimate of  $\alpha$  from a trimmed sample. In this sample, 10 observations will be excluded, the five closest to  $KWH_2$  on each side of  $KWH_2$ . Since both of these procedures require the elimination of negative or zero consumption values, the number of observations in the sample will generally decrease as the variance of the error term increases. This can be seen in any of the tables contained in Appendix A.

Since omitting those observations with negative or zero consumption values results in truncation bias, an estimation procedure which can account for this fact is required. Using Tobit analysis, two additional values of  $\alpha$  are obtained, one from the original and one from the trimmed sample. When any observations are eliminated, these two estimates are automatically produced. In all cases, the number of observations for the Tobit results will equal the number in the original sample before any were excluded.

To begin the discussion of the Monte Carlo study, it is useful to make a few general observations about the regressions summarized in Appendix A. These tables contain OLS estimates of  $\alpha$ , the absolute value of the percent error, the number of observations in the sample, the Tobit estimate of  $\alpha$  and the absolute value of its percent error.

T-statistics were not reported for OLS since the smallest for any regression was 11.8. In general, the t-statistics declined as variance increased. Asymptotic t-statistics were also computed for the Tobit estimates. These are also omitted for the same reason. These asymptotic t-statistics showed the same decline as above. However, they were smaller than the t-statistics in almost every case. The smallest asymptotic t-statistics found was 7.4.

Upon inspection of the tables in Appendix A, the following comments can be made. First, in nearly all of the OLS regressions, the ones involving  $z_3$  have smaller percent errors than those involving  $z_2$ . A similar result holds for the majority of the Tobit estimates. The most plausible explanation for this can be seen by referring to Figures 8 and 9. The results for the regressions represented in these figures are an exception to this tendency as can be seen in Table A8. With  $z_3$  varying, consumption ranges from approximately 375 to more than 1500 kwh per month. Consumption ranges from around 175 to something less than 1000 kwh per month for the  $z_2$  regression. Also, the points in Figure 9 show a greater scattering than those of Figure 8. This is because the magnitude of the error relative to average consumption is smaller for the  $z_3$  regression than for the  $z_2$  regression. Under such circumstances, better results for the  $z_3$  regressions are expected on average as the evidence in Appendix A suggests.

Another observation is that the OLS and Tobit estimates for the  $z_2$  samples (both the original and trimmed ones) tend to be larger than their true values. A possible explanation for the OLS estimates is that as observations that become negative are omitted, the mean of the error

term for the remaining observations becomes positive. This would impart a positive bias in the estimate of  $\alpha$ . Tobit should mitigate this bias since under the conditions fulfilled by the experimental design, it will be a consistent estimator. This seems to be the case for the  $z_2$  Tobit results since their percent errors are generally smaller.

No such pattern was observed for the  $z_3$  OLS regressions. These estimates were less than the true value approximately 60% of the time and greater than the true value about 35% of the time. Such a split does not indicate any severe problems with bias. The Tobit results were not encouraging since Tobit nearly always underestimated the true value of the parameters. In every case, the Tobit estimate is smaller in absolute value than the corresponding OLS regression coefficient. Tobit appears to be compensating for bias that is introduced by eliminating the observations which become negative upon the addition of a random error term.

Another finding is that the estimates for the trimmed samples usually have larger percentage errors than do those for the original samples. Since trimming was used to mitigate the errors-in-variables bias produced by error terms which cause observations to switch blocks, an improvement in the results was expected. Unfortunately, this was not the case.

It is also important to point out that less precise estimates were found for those samples with no discontinuity. Since there is no guarantee that observations will be found in both blocks when a discontinuity is not present, trimming was not applied. When the sample actually contained a discontinuity, both the OLS and Tobit estimates

were much better on average. One possible explanation for this is since average quantities for the samples with no discontinuity are slightly smaller than those for the  $z_2$  regressions, higher percentage errors would generally be expected. A similar rationale explained the relatively poor performance of the  $z_2$  results compared with the  $z_3$  results. However, this explanation is not entirely sufficient. A comparison of Tables A1 and A3 shows that even when average quantities are roughly equal, the samples with no discontinuity still have much higher percentage errors. For example, mean consumption for the  $z_3$  sample containing a discontinuity is 282 kwh, while the corresponding mean for the sample without one is 224. The average percentage error for the OLS estimates for samples with no discontinuity is 19.7 compared with 2.0 for the samples which contained a discontinuity. Similarly, mean consumption for the  $z_2$  samples containing a discontinuity is 207 compared with 203 for the no-discontinuity samples. Even though mean consumption is roughly equal, the mean OLS percentage error for the no-discontinuity samples was 25.1 compared with 15.3 for those containing one. Although somewhat less pronounced, similar differences in mean percentage errors were found for the Tobit estimates for these samples. Thus, it appears that the existence of a discontinuity actually improves the efficiency of the estimates.

The data presented in Table 2 will be used to highlight the differences in mean percentage errors for the Monte Carlo results summarized in Appendix A. For both OLS and Tobit, means have been computed for all of the samples with  $z_2$  and  $z_3$  varying. Also, the  $z_2$  and  $z_3$  percentage errors were combined and reported as the means for the  $z_2+z_3$

Table 2. Statistical Summary of the Monte Carlo Results.

Sample	Mean % Error	Variance of % error	Correlation (% error, $\sigma^2$ )	No. of Obs.
OLS:				
$z_3$ original	2.1	7.1	.313	49
$z_3$ no discon.	12.2	316.0	.696	14
$z_3$ trimmed	2.6	13.1	.266	49
$z_2$ original	6.9	71.9	.704	49
$z_2$ no discon.	14.9	287.0	.542	14
$z_2$ trimmed	10.0	109.0	.790	49
$z_2+z_3$ original	4.5	45.0	.504	98
$z_2+z_3$ no discon.	13.6	292.0	.618	28
$z_2+z_3$ trimmed	6.3	74.3	.532	98
Tobit:				
$z_3$ original	4.4	15.4	.00404	27
$z_3$ no discon.	11.3	73.6	.162	9
$z_3$ trimmed	5.5	25.0	-.0998	27
$z_2$ original	6.3	35.5	-.0844	34
$z_2$ no discon.	10.7	68.6	.150	11
$z_2$ trimmed	9.1	52.9	-.167	34
$z_2+z_3$ original	5.5	27.0	-.0318	61
$z_2+z_3$ no discon.	10.9	67.2	.132	20
$z_2+z_3$ trimmed	7.5	43.3	-.103	61

samples. Finally, correlations between the percentage errors in the estimates of  $\alpha$  and the variance of the error term,  $\sigma^2$ , have been included in this table.

Data in Table 2 show that all of the correlations between the error variance and the absolute value of the percentage error for the OLS estimate are positive, with most greater than one-half. This is not surprising since estimates are expected to get progressively worse as the error variance is increased. What is surprising, however, is that similar results are in no way evidenced for the Tobit estimates. None of the correlations are greater than .2 whereas none were less than .266 for the OLS results. Ostle (1972) outlines an analysis of variance procedure for testing the significance of the difference between two sample correlations. Applying this test to corresponding correlations from the OLS and Tobit samples revealed that the following four differences were significant at the 99.9% level: the  $z_2$  original, the  $z_2$  trimmed, the  $z_2+z_3$  trimmed. No other differences were significant at even the 90% level. In general, it appears that Tobit destroys any correlation between the error term and the percent error. This implies that Tobit performs poorly relative to OLS for smaller variances, and better for larger ones. That is, Tobit is a more even performer.

Table 3 summarizes the results of applying the t-test for the difference between two means between all possible relevant comparisons between the means reported in Table 2. Only those differences significant at a 90% level or better have been reported. Also, the relevant means from Table 2 have been included in parentheses for clarity. To handle these results in order, first note that Tobit does a

Table 3. Significant Differences between Monte Carlo Mean Percentage Errors.

Means Compared	Significance Level
OLS $z_3$ Original (2.1)--Tobit $z_3$ Original (4.4)	.05
OLS $z_3$ Trimmed (2.6)--Tobit $z_3$ Trimmed (5.5)	.05
Tobit $z_2$ Original (6.3)--Tobit $z_2$ Trimmed (9.1)	.10
Tobit $z_2+z_3$ Original (5.5)--Tobit $z_2+z_3$ Trimmed (7.5)	.10
OLS $z_2$ Original (6.9)--OLS $z_3$ Original (2.1)	.01
OLS $z_2$ Trimmed (10.0)--OLS $z_3$ Trimmed (2.6)	.01
Tobit $z_2$ Trimmed (9.1)--Tobit $z_3$ Trimmed (5.5)	.05
OLS $z_3$ Original (2.1)--OLS $z_3$ No Discontinuity (12.2)	.10
OLS $z_2+z_3$ Original (4.5)--OLS $z_2+z_3$ No Discontinuity (13.6)	.01
Tobit $z_3$ Original (4.4)--Tobit $z_3$ No Discontinuity (11.3)	.05
Tobit $z_2+z_3$ Original (5.5)--Tobit $z_2+z_3$ No Discontinuity (10.9)	.05
OLS $z_3$ No Discontinuity (12.2)--OLS $z_3$ Trimmed (2.6)	.10
OLS $z_2+z_3$ No Discontinuity (13.6)--OLS $z_2+z_3$ Trimmed (6.3)	.05
Tobit $z_3$ No Discontinuity (11.3)--Tobit $z_3$ Trimmed (5.5)	.10
Tobit $z_2+z_3$ No Discontinuity (10.9)--Tobit $z_3$ Trimmed (7.5)	.10

significantly worse job than OLS when working with the original and trimmed methods for variation in  $z_3$ . Second, the original method does better than the trimmed method for the  $z_2$  and  $z_2+z_3$  Tobit results. A third comparison involves the initial postulate that the  $z_2$  results were worse than those for  $z_3$ . This is confirmed at high levels of significance for the OLS original method, OLS trimmed method and Tobit trimmed method.

The remaining comparisons in Table 3 concern those between the samples with no discontinuity and the original and trimmed methods for samples with discontinuities. In general, the samples with no discontinuity had significantly higher mean percent errors than the  $z_3$  samples and the combined  $z_2+z_3$  samples. As was previously noted, some of the differences observed in the means should be expected since overall the mean of the no-discontinuity samples was slightly less than the mean of those samples with  $z_2$  varying containing a discontinuity. This does not entirely explain the increase in the mean percent errors for the samples without discontinuities. Most likely, this improvement can be attributed to the fact that samples containing a discontinuity had greater variation in the independent variable defined in equations (7) and (8). Since samples with a discontinuity have observations in both marginal blocks, greater variation in the independent variable will occur than for samples all of whose observations were generated in only one block.

In conclusion, this chapter has succeeded in developing an estimation methodology fully consistent with the implications of decreasing block pricing. In the course of its development, it was found that an unavoidable errors-in-variables bias was present. Monte Carlo data

were used as a vehicle for investigating the seriousness of this bias. When generating the Monte Carlo data, a truncation problem occurred which would also bias the estimates. The Tobit estimator provided consistent parameter estimates and mitigated the effects of the truncation bias. A procedure for trimming the original samples was proposed in an attempt to reduce the errors-in-variables bias. The results of the Monte Carlo study were encouraging since good results were obtained even with the addition of error terms with quite large variances. It was also found that the trimming procedure generally failed to improve the estimates. Even though trimming failed, the errors-in-variables bias did not preclude the obtaining of good parameter estimates. In fact, samples containing a discontinuity yielded much better results than those not containing one. This was the most encouraging of all the results. At the outset, it seemed quite possible that the existence of a discontinuity and the resulting error-in-variables bias might seriously hamper the ability of this technique to obtain precise parameter estimates, thereby reducing the usefulness of this methodology.

## CHAPTER 3

### A MODIFIED METHOD FOR CROSS-SECTIONAL DATA

In the previous chapter, a method was developed for estimating the potentially discontinuous demand functions that occur when a utility maximizing consumer purchases a commodity whose price is determined by a decreasing block pricing schedule. This method is most appropriate for data experimentally generated through the variation of one of the block prices while all other components of the rate schedule remain fixed. Such an experimental data base has never been collected, thus it was necessary to make use of Monte Carlo data. While the results of applying this method to the Monte Carlo data were encouraging, it is still desirable to apply it to actual consumption data.

Presently, the analysis of Chapter 2 will be modified to allow the use of cross-sectional data in the estimation of individual demand functions. This modified approach will be applied to these data and then compared with a more conventional estimation procedure.

To estimate individual demand functions from cross-sectional data, the assumption that all consumers possess the same type of utility function is required. Variations in the specific functional form may be taken into account by allowing the numerical value of one or more of the parameters of the utility function to depend on certain household characteristics. In this manner, a distinct utility function is estimated for each household. From this information, an estimate of the

household's demand for electricity can be obtained by the method detailed in Chapter 2. This procedure selects the optimum consumption by maximizing utility subject to each segment of the budget constraint.

Rather than estimate demand directly, the use of a utility function to derive the demand function indirectly provides at least two distinct advantages over conventional methods. First, a demand function derived from a utility function is by definition fully consistent with utility theory. Therefore, estimates of demand so derived allow for the potential discontinuity and multi-valuedness previously described. Second, once a consumer's utility function has been estimated, this consumer's response to changes in any or all of the elements of the decreasing block rate schedule can also be estimated by generalizing the algorithm underlying Figure 5 to the appropriate utility functions. The estimation of price and income elasticities from the conventional or ad hoc approach is especially limited when dealing with commodities employing decreasing block pricing. In this case, if a change in price or income causes a household's consumption to shift to a different block, a discontinuity in demand occurs, and the use of an elasticity becomes meaningless.

#### Estimating the Stone-Geary Utility Function

To demonstrate the usefulness of this approach, estimates of demand derived from the Stone-Geary utility function will be made using cross-sectional data. Each household in the sample is assumed to possess a utility function of the form:

$$(13) \quad U = \beta_1 \log(q_1 - \alpha_1) + \beta_2 \log(q_2 - \alpha_2),$$

where the subscripts pertaining to the household have been omitted for simplicity of notation. As in Chapter 2,  $q_1$  is the quantity of electricity consumed and  $q_2$  is the quantity of all other goods or income. The parameters of the utility function will be subject to the usual restrictions:

$$(14) \quad 0 < \beta_1 < 1, \beta_1 + \beta_2 = 1 \text{ and } q_i - \alpha_i > 0 \quad i = 1, 2.$$

The parameters  $\alpha_1$  and  $\alpha_2$  can be interpreted as minimum required quantities of the two commodities, electricity and all other goods. Under such an interpretation, these parameters would be required to be non-negative. Brown and Deaton (1972, p. 1195) point out that this interpretation can be overly restrictive. As long as the quantities  $q_i - \alpha_i$  are greater than zero, all of the strictures of neoclassical utility theory are met.

With  $\alpha_1$  and  $\alpha_2$  being minimum required quantities, the concept of noncommitted income can be introduced as  $x - p_1\alpha_1 - p_2\alpha_2$ . The parameters  $\beta_1$  and  $\beta_2$  are then interpreted as the marginal propensity to consume the respective goods out of the noncommitted portion of income. The restriction that these parameters sum to one guarantees that total expenditure on all goods equals income.

If one or more of the utility function parameters is allowed to vary across households, a distinct utility function will be estimated for each household. In the empirical work that follows, only the parameter  $\alpha_1$  will be allowed to vary. The reasons for this assumption will be discussed below.

In keeping with the notation of the previous chapter, define the rate schedule consisting of  $n-1$  marginal blocks as follows:

$z_1$  = the fixed customer charge for  $q_1 \leq KWH_1$ ,

$z_2$  = the marginal price for  $KWH_1 < q_1 \leq KWH_2$ ,

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$z_n$  = the  $n-1$ <sup>th</sup> marginal price for  $q_1 > KWH_{n-1}$ .

Finally, let

$p_2$  = the price of  $q_2$ ,

$x$  = household income,

$D_k$  = the household's  $k$ <sup>th</sup> demographic characteristic,

$S_k$  = the household's  $k$ <sup>th</sup> element of the electric appliance stock.

Since the slope of the budget constraint and the income effect due to intramarginal block consumption vary according to the block in which consumption falls, the specification of the demand equation must also be block-dependent. While this block-dependency presents certain statistical difficulties, it is the theoretically correct approach for estimating individual demand under decreasing block pricing. The budget constraint for a household whose consumption falls in the  $j$ <sup>th</sup> block is as follows:

$$(15) \quad x = z_1 + z_2(KWH_2 - KWH_1) + \dots + z_{j-1}(KWH_{j-1} - KWH_{j-2}) \\ + z_j(q_1 - KWH_{j-1}) + p_2q_2.$$

Maximizing utility subject to this budget constraint gives the demand equation for a household with consumption in the  $j^{\text{th}}$  block as:

$$q_1 = \alpha_1(1-\beta_1) + \beta_1[x - z_1 - z_2(\text{KWH}_2 - \text{KWH}_1) - \dots] \\ (16) \\ z_{j-1}(\text{KWH}_{j-1} - \text{KWH}_{j-2}) + z_j \cdot \text{KWH}_{j-1} / z_j - \beta_1 \alpha_2 \cdot p_2 / z_j.$$

For future reference, denote income minus the bracketed terms in equation (16) as the fixed charge. This fixed charge results in the income effect of intramarginal block consumption.

Only the minimum required quantity of electricity will be allowed to vary across households. The parameter  $\alpha_2$  will not be allowed to vary because there are only two variables available in the data base upon which it might depend, the number of residents and income. Taken together, these two variables may be good predictors of the minimum required quantity of all other goods. However, this would exclude the use of income as a separate independent variable since multicollinearity would be introduced. A comparison of the last two terms on the RHS of (16) points this out. Under the assumptions that  $\alpha_2 = a_2 x$  and  $p_2 = 1$ , the last term would be rewritten as  $\beta_1 a_2 \cdot x / z_j$ . This would be highly correlated with the previous term since the only difference between the two is a rather small income effect adjustment. Alternatively, the number of residents could be assumed to be proportional to the minimum required expenditure on other goods. However, without the inclusion of income, this relationship would be expected to be rather weak. A better procedure for allowing  $\alpha_2$  to vary across households would be to make a more direct approximation. This could be achieved through the use of

survey data on such items as the monthly mortgage or rent payment, minimum expenditures on food, transportation and so forth.

The final potential candidate for variation across households is  $\beta_1$ . Permitting  $\beta_1$  to vary introduces considerable estimation complications, primarily because it enters all three terms on the RHS of equation (16). If  $\beta_1$  is a linear function of some set of variables, estimates of it could be obtained directly from the second term on the RHS of (16) and indirectly from the first term. Thus, this parameter is overidentified. This overidentification could be solved in principle by using the appropriate constraint on the coefficients. However, the implementation of such a scheme would be quite formidable and will not be considered in this analysis.

As a result of these difficulties,  $\alpha_1$  remains as the only utility function parameter which can be varied feasibly across households. In order to account for the effects of the appliance stock and demographic data on electricity consumption in a theoretically correct manner, it is essential to allow at least one of the parameters to vary. While it would be interesting to compare the results when allowing  $\beta_1$  to vary with the results when  $\alpha_1$  varies, an illustration of this methodology can be based on the variation of  $\alpha_1$ .

It will be assumed that  $\alpha_1$  is a linear function of the stock of electric appliances and certain demographic characteristics. It can thus be written as:

$$(17) \quad \alpha_1 = \sum_{k=1}^{n_1} a_k S_k + \sum_{l=n_1+1}^{n_2} a_l D_l$$

where  $n_1$  is the number of appliances and  $n_2$  minus  $n_1$  is the number of demographic characteristics. Replacing  $\alpha_1$  in equation (16) by its value in (17) yields the following estimating form for the demand function:

$$(18) \quad q_1 = \sum_{k=1}^{n_1} A_0^k S_k + \sum_{l=n_1+1}^{n_2} A_0^l D_l + A_1 [\text{ADJINC}] / z_j + A_2 [1/z_j] + \varepsilon.$$

The relationship between the utility function parameters and those of the estimating equation is as follows:

ADJINC = income minus the fixed charge due to intramarginal block consumption from equation (16).

$$(19) \quad \begin{aligned} A_0^k &= a_k (1 - \beta_1), \\ A_0^l &= a_l (1 - \beta_1), \\ A_1 &= \beta_1, \\ A_2 &= -\beta_1 \alpha_2, \\ \varepsilon &= \text{a random error term.} \end{aligned}$$

The estimation procedure involves computing the value of ADJINC for each household and employing the marginal price as indicated in equation (18). This approach presents a statistical difficulty in that both ADJINC and the marginal price,  $z_j$ , depend upon  $q_1$  which, in turn, is correlated with the error term. Therefore, an errors-in-variables situation analogous to the one discussed in Chapter 2 occurs in (18). To deal with this problem, it would be possible to devise a trimming procedure similar to the one used in Chapter 2. This procedure failed to improve the results in the Monte Carlo study and will not be used on the cross-sectional data. Thus, it has been decided to proceed with the

estimation of (18), even though some problems with errors-in-variables are present.

Assuming that the error term enters additively in the demand function of equation (18) rather than in the utility function allows the use of OLS as the efficient estimation technique. If a normally distributed error with zero mean and covariance matrix,  $\sigma^2 I$ , enters additively in the parameter  $\alpha_1$  in equation (17), then the covariance matrix of  $\epsilon$  in (18) is  $(1-\beta_1)^2 \sigma^2 I$ . The error could also enter additively in (18) in which case the covariance matrix is assumed to be  $\sigma^2 I$  as above. Thus, the efficient estimation technique in either case is OLS.

One final problem in the estimation of equation (18) is that it is quite possible for the estimated coefficient of the independent variable ADJINC to be negative. This did occur in some of the preliminary regressions. Such a result implies that electricity is an inferior good and that the demand curve is positively sloped. Neither of these conclusions is theoretically acceptable. To avoid this problem, the estimate of this coefficient can be constrained to be between zero and one. Johnston (1972, pp. 221-227) has outlined a procedure for constraining the estimated coefficient. This method was successfully applied to the preliminary data, but it was unnecessary for the final form of the regression.

Before proceeding with a discussion of the particulars of the data base, a few comments about the form of the ad hoc equations are in order. All ad hoc equations will be estimated as linear functions and will contain a constant term. Among the variables to be included in the equations are the marginal price of electricity, the fixed charge and

income. This fixed charge is computed as the difference between the actual electric bill and what the bill would have been if all electricity were purchased at the marginal price. The fixed charge should not be confused with the customer charge,  $z_1$ . Under decreasing block pricing, the income effect on consumption of the fixed charge is always negative. Thus, the sign of its coefficient is expected to be negative. The inclusion of marginal price and the fixed charge as independent variables results in a model with errors-in-variables. In view of the Monte Carlo results presented in Chapter 2, this problem is not judged to be serious. Finally, for comparison purposes, all electric appliance stock variables and demographic characteristics will be the same as for the corresponding estimations of the Stone-Geary utility function.

#### The Data Base

In applying the above methodology, data from the Midwest Research Institute (MRI) Survey of 1976 will be used. This data base includes observations on the electricity consumption for 1,985 households in 16 cities for roughly a one-year period. This survey also includes a rather detailed breakdown of the households' electric appliance stock, monthly fuel adjustments, and demographic information such as income, the number of persons in the household, a measure of dwelling size and so forth. Information on rate schedules was obtained from the Federal Power Commission's National Electric Rate Book.

From the original 16 cities, seven were selected for inclusion in the sample on the basis of the structure of their rate schedules. For example, cities in which nearly all of the households would be

expected to have consumption in the same block were excluded. Also, decreasing block prices were not in effect for some cities. In principle, the methodology described in the previous section could be used to estimate residential electricity demand for the nine excluded cities. Since the objective of this research is to measure the impacts of declining block rates on residential electricity consumption, data for these cities were omitted from the analysis.

For the selected cities, some monthly data contained in the survey were also omitted from the data base. The reason for these exclusions is basically the same reason certain cities were entirely excluded. Rate schedules tended to have more blocks in the winter rather than the summer months, leading to greater variation in the marginal price and the fixed charge in the winter period. When those months with limited data variation are excluded, the resulting sample includes roughly 2,000 monthly observations on approximately 600 households. For some cities only one month was chosen, while for others, between three and 10 months were included. Stone-Geary and ad hoc regressions will be computed for each city and each month for a total of 23 separate data groupings. Next, pooled equations will be estimated for each city using all available months for any city with more than one included month. This pooling yields five additional data groupings. Finally, one winter month will be chosen from each city to include in a pooled estimation across different rate schedules. This sample contains 562 observations for the indicated months for the following cities: Des Moines, November; St. Louis, November; Owensboro, November; and New Orleans, February. Since this final pooling produces the greatest

variation in both marginal price and the fixed charge, relatively good results are anticipated.

Before the MRI data could be analyzed, some adjustments were necessary. Within each city significant variation in both the period covered by the billing cycle and the number of days in the billing cycle was readily apparent. Data on heating and cooling degree-days, which were not included in the MRI data base, had to be calculated independently. Since the dates covered by a particular bill were always included in the MRI survey, it was possible to determine both the number of days and the number of heating or cooling degree-days corresponding to each monthly observation. For the computation of degree-days, National Weather Bureau data on the mean daily temperatures for each of the cities were used. Heating degree-days were defined as the sum over the billing period of the difference between 65° and the mean temperature for each day. Cooling degree-days were similarly computed also using 65° as the base temperature. To account for the difference in the length of the billing period, electricity consumption and heating and cooling degree-days were normalized to correspond to 30-day months. With these adjustments, comparisons across households and even cities is made possible.

Not all demographic and appliance stock data reported in the MRI survey were included in the regressions. Based on preliminary regressions several variables were eliminated in the final models, most notably the measure of dwelling size. In all the models including both dwelling size and electric appliance stock variables, some apparent multicollinearity resulted in statistically insignificant parameter

estimates for dwelling size. Furthermore, dwelling size was not measured consistently across households. Two-thirds of the households reported the number of rooms as a measure of dwelling size with the remaining one-third reporting the number of square feet. Thus, if dwelling size were included, the sample had to be divided according to which measure of size was given. This doubled the number of regressions and diminished degrees of freedom for each data grouping.

Another undesirable feature of the MRI data base is that while a rather detailed description of the household's appliance stock is given, information on watt capacity is not included. As a result, aggregation of related appliances is not possible. Since the number of degrees of freedom in the data base, even with the exclusion, is quite large, including separate variables for each appliance is not prohibited. Still, some degree of aggregation of the appliance stock data would be desirable.

Finally, one additional statistical difficulty is presented because in the MRI survey, annual income is measured by categories only. While this type of income measurement is commonly used in the construction of electric data base, it greatly limits the usefulness of such information. The estimating procedure of this analysis requires a dollar measure of income rather than information on income class. Under decreasing block pricing, income must be adjusted by the fixed charge of equation (16) before it can be entered into the demand function in the theoretically correct manner. To make this adjustment, each household is assigned an income equal to the geometric mean of its income class. From this figure, monthly income is computed and then adjusted

by the fixed charge. The fixed charge ranges from as little as a few dollars to a maximum of fifty dollars per month. At the same time, the use of the geometric mean resulted in measurement errors in monthly income as large as several hundred dollars. While this measurement error is an undesirable consequence of this procedure, it cannot be avoided, given the data limitations.

### The Empirical Results

To provide some basis for judging the results of this modified technique, two models were estimated from each of the data groupings. An ad hoc demand function and a demand function derived from the Stone-Geary utility function are displayed in pairs in Appendices B and C. The ad hoc regressions are assumed to be linear functions of income, the fixed charge, the marginal price of electricity and the various appliance stock and demographic variables. The estimate of the demand function derived from the Stone-Geary utility function is obtained by performing the regression indicated by equation (18).

Demand functions derived from the Stone-Geary utility function do not include a constant term while the ad hoc models do. This means that comparisons between the corrected correlation coefficients of the two methods cannot be made since the computation of  $R^2$  differs when an equation does not contain a constant term. In equations which include a constant term, the computation of  $R^2$  is equivalent to computing the square of the simple correlation,  $r^2$ , between the actual and predicted values of the dependent variable. For comparison purposes,  $r^2$  was computed for both the ad hoc and Stone-Geary regressions.

Estimation of equation (18) yields the utility function parameters,  $\alpha_1$ ,  $\alpha_2$  and  $\beta_1$  which can then be substituted into the adaptation of the utility-maximization algorithm to the Stone-Geary utility function. This yields estimates of the utility maximizing levels of electricity consumption for each household. These are then correlated with the actual level of electricity consumption. In Appendices B and C, the square of this correlation is reported in the line simply entitled correlation. No corresponding estimate is available for the ad hoc regressions since they are not derived from utility maximization. Finally, the own-price elasticity of electricity demand is computed for each household, and the average over all households is reported for comparison with the point of means elasticity of the ad hoc regression. These elasticities have been provided for comparison purposes only. In practice, the utility-maximization algorithm is used to compute the effects of price changes since this method is consistent with the aforementioned theoretical results.

In what follows, the variables entering the equations will be discussed along with other general observations. However, their order of discussion will not necessarily follow their order as presented in the summary tables. Also, these variables have been pre-selected on the basis of the preliminary regressions and a discussion of any excluded variables is omitted.

One of the most important findings of this study is that much better statistical results are obtained from the pooled samples. This is true for pooling across months within a city as well as for pooling across cities. A summary of the significant t-statistics for the

regression coefficients from the tables in Appendices B and C is presented in Table 4 and supports this fact. Two-tailed tests were used whenever the sign of a variable could not be predicted. They were also used for HDD since the majority of its significant coefficients had signs opposite of what would be expected. The percentage of significant coefficients for both the pooled and non-pooled regressions shown in Table 4 emphasizes two main points. First, the pooled regressions tended to have a larger percentage of significant coefficients for all variables. For example, the income variable from the ad hoc regressions was significant at the 90 percent level or better in 65 percent of the non-pooled regressions and in 83 percent of the pooled regressions. Second, the pooled percentages are skewed toward higher significance levels.

The fact that pooling produces better results is not surprising. Pooling across months within a city gives more variation in marginal price, the fixed charge and heating or cooling degree-days. Pooling across cities provides even greater variation in the above predictors and additionally results in increased variation in appliance stocks and demographic characteristics. Also, a larger number of degrees of freedom is available when pooling.

Another general observation is that the squared correlations ( $r^2$ ) between the actual and predicted consumption are always larger for the ad hoc regressions than for the Stone-Geary ones. The presence of two additional predictors in the ad hoc equations, the fixed charge and a constant term, may partially explain this fact. In most cases these variables had t-statistics in excess of unity, and this difference alone

Table 4. Percentage of Significant Variables from the Non-Pooled and Pooled Regressions.

Independent Variables	Non-Pooled						Pooled					
	Ad Hoc			Stone-Geary			Ad Hoc			Stone-Geary		
Significance Level*	99%	95%	90%	99%	95%	90%	99%	95%	90%	99%	95%	90%
Income	22	39	4	--	--	--	50	0	33	--	--	--
ADJINC/z <sub>j</sub>	--	--	--	57	21	4	--	--	--	100	0	0
Marg. Price	17	4	0	--	--	--	17	0	0	--	--	--
# Residents	43	30	17	57	21	4	100	0	0	100	0	0
Insulation	0	4	0	4	0	0	0	0	0	0	0	0
# Refrig.	13	30	17	4	13	35	50	17	33	50	17	33
Dishwasher	4	13	9	13	18	17	50	17	0	83	17	0
Elec. Dryer	9	9	9	9	13	17	17	17	33	50	17	0
CDD	0	33	17	17	17	0	100	0	0	100	0	0
Central Air	83	17	0	83	17	0	100	0	0	100	0	0
Fixed Chg.	39	0	9	--	--	-	83	0	17	--	--	--
Significance Level**	98%	95%	90%	98%	95%	90%	98%	95%	90%	98%	95%	90%
(Marg. P) <sup>-1</sup>	--	--	--	57	0	0	--	--	--	50	17	0
Energy Sav.	4	4	13	4	9	9	33	17	0	17	0	50
HDD	0	6	12	29	24	6	0	17	0	33	0	0
Constant	26	0	4	--	--	--	33	17	0	--	--	--

\*Using a one-tailed test.

\*\*Using a two-tailed test.

would help to explain the larger  $r^2$  values. Both specifications yielded correlations (either  $\bar{R}^2$  or  $r^2$ ) which were quite good for cross-sectional data bases.

A major focus of this analysis is the identification of the income and price effects due to declining block rates. In the Stone-Geary models, income and price effects are incorporated in the one independent variable,  $ADJINC/z_j$ . This variable was significant in 82 percent of the non-pooled cases and 100 percent of the pooled ones. Since utility theory dictated the entry of the income variables in this manner, this is a notable result. It was also surprising to find that its coefficient,  $\beta_1$ , was positive and less than one even when not statistically significant. This is especially important since theory restricts it to this range. Thus, the constrained least squares routine developed on the basis of the preliminary results was never employed.

In the ad hoc regressions income and price responses are measured by three separate variables, income, the fixed charge, and marginal price. As previously mentioned, income was generally shown to be a significant determinant of electricity consumption in both the pooled and non-pooled regressions. The fixed charge was significant 48 percent of the time for the non-pooled runs and 100 percent for the pooled ones. This result is somewhat surprising considering that income was measured in such broad categories. This type of measurement caused errors which could have easily overshadowed its income effect. Moreover, its coefficient was negative as expected in the majority of cases.

The coefficient of marginal price in the ad hoc regressions was rarely significant and frequently estimated to be positive when

significant. For non-pooled regressions, the only variation in marginal price was caused by consumers purchasing electricity in different blocks. Under decreasing block pricing, the relationship between marginal price and quantity consumed is negative by definition. That is, as quantity increases, marginal price must decrease. This poses a simultaneity problem since the rate schedule is fixed and demand varies from one household to the next. This problem is similar to the one caused by using average price as a predictor. However, a subtle difference exists because average price varies continuously with quantity whereas marginal price does so in discrete intervals. The resultant bias for the price coefficient should be in the negative direction. Thus, it is perplexing to find the coefficient for marginal price frequently positive and significant.

The results for St. Louis presented an exception to this pattern since the coefficient of marginal price was negative and significant at better than the 99 percent level for all regressions. It was hoped that some clue concerning the frequent positive signs would be provided by this exception. Unfortunately, an inspection of the rate schedules, excluding fuel adjustments, given in Table 5 suggested no obvious explanation. In fact, the rate schedule for St. Louis is quite similar to the one for Des Moines whose marginal price coefficient was always positive whenever it was significant.

A related matter concerns the coefficient of the inverse of marginal price, an independent variable in the demand equations derived from the Stone-Geary utility function. It should be mentioned at the outset that the simultaneity problem of the ad hoc specification is not

Table 5. Rate Schedules.

City	KWH	¢/KWH	¢/KWH	¢/KWH	Customer Charge
Des Moines		Mar., Apr.		Oct., Nov.	None
	0 < q ≤ 50	6.5		6.8	
	50 < q ≤ 200	4.2		4.5	
	200 < q ≤ 400	3.6		4.15	
	400 < q ≤ 1000	2.9		3.45	
	q > 1000	2.6		3.15	
Minneapolis			Oct.		\$2.50
	0 < q ≤ 500		3.53		
	500 < q ≤ 1000		3.09		
	q > 1000		1.90		
New Orleans			All Months		None
	0 < q ≤ 20			7.25	
	20 < q ≤ 50			2.7	
	50 < q ≤ 120			2.5	
	120 < q ≤ 500			2.11	
	q > 500			1.67	
Owensboro		Mar., Apr.		Nov.	None
	0 < q ≤ 20	5.5		6.08	
	20 < q ≤ 70	4.18		4.62	
	70 < q ≤ 150	2.2		2.43	
	150 < q ≤ 600	1.98		2.19	
	q > 600	1.375		1.52	
St. Louis		Mar., Apr., Nov.			None
	0 < q ≤ 100			4.29	
	100 < q ≤ 250			3.42	
	250 < q ≤ 600			3.04	
	600 < q ≤ 1000			2.76	
	q > 1000			2.22	
Tucson			Nov.		None
	0 < q < 100		6.95		
	100 < q < 600		5.1386		
	600 < q < 1000		3.8138		
	q > 1000		2.7586		
Philadelphia			Sept.		None
	0 < q < 500		4.34		
	q > 500		3.70		

present in the Stone-Geary formulation since the rate schedule is explicitly utilized in the derivation of demand. Referring to the equations in (19) for converting the estimating parameters to the structural parameters, it is seen that the coefficient for the inverse of marginal price,  $A_2$ , will have the opposite sign of the product of  $\beta_1$  and  $\alpha_2$ . Since  $\beta_1$  was always found to be positive, this coefficient will have the opposite of the sign of  $\alpha_2$ .

The interpretation of  $\alpha_2$  as the minimum required expenditure on all other goods leads to the expectation that  $\alpha_2$  will be positive. This was not the case for the majority of the regressions. While this is not an appealing result, at least it poses no problems for maximizing the estimated utility function since taking logarithms of negative numbers will never arise. On the other hand, for cities for which  $\alpha_2$  was positive, problems with the sign of  $q_2 - \alpha_2$  sometimes occurred. This was a major factor in the case of New Orleans where the utility-maximization algorithm often failed due to  $q_2 - \alpha_2$  become negative. There were a large number of low income households in the New Orleans sample so a separation of the sample by income level was made. This separation provided minimal gains at best, and the value of  $q_2 - \alpha_2$  remained negative for many households in both the low and high income groups.

In cases where  $q_2 - \alpha_2$  was negative for more than a few households, the correlation between the dependent variable and the values predicted from the utility maximization algorithm was placed in parentheses in Appendices B and C. These correlations were placed in parentheses because they are misleadingly high. This is because when the maximization algorithm failed, the regression value of  $\hat{q}_1$  was used in

place of a utility maximizing value. When the maximization algorithm places the household in the same block in which its consumption actually occurred, then the value from the algorithm coincides with the value of  $\hat{q}_1$  from equation (18). These values differ whenever the algorithm places the household in a block different from the one in which actual consumption occurred. The process of replacing the regression value,  $\hat{q}_1$ , with the value found by the algorithm necessarily reduces the correlation between actual and estimated consumption since least squares maximizes the correlation. Since this replacement process was not employed for a large portion of the New Orleans sample, the computed correlation will be high when compared with the values for cities for which the algorithm never failed.

The reason for computing these correlations was to provide insight into the usefulness of the utility-maximization algorithm for predicting consumption given changes in the rate schedule. The average across 17 non-pooled regressions of the ratio of correlation to  $r^2$  equaled 72 percent. This figure excludes the six New Orleans regressions for which the algorithm failed. Similarly, the average for the pooled regressions was 82 percent. This time both New Orleans aggregations were excluded.

These averages are encouraging in spite of some relatively low correlations. The lowest was .174 for St. Louis in November. As will be brought up later, improvements in the data base would lead to better overall results along with these correlations. This implies that this methodology could provide useful information to utility planners and regulatory commissions concerning the effects of modifying rate schedules.

It was also found that large positive values for  $\alpha_2$  could cause the estimate of the household's elasticity of demand to become positive. Excluding New Orleans, the own-price elasticity estimates were all negative. The average elasticities, excluding New Orleans, were -1.3 for the non-pooled and -.42 for the pooled. New Orleans tended to have large positive values for  $\alpha_2$  which caused the elasticity estimate to be positive. It is easily seen why this is the case since from equation (16) one can compute the own-price elasticity approximately as:

$$(20) \quad \epsilon = -\beta_1 (\text{ADJINC} - \alpha_2) / z_j q_1,$$

where  $z_j$  is the marginal price.

Elasticities at the point of means were calculated for the ad hoc specifications. The results were clearly inferior to those obtained from the Stone-Geary equations. While New Orleans proved to be a nemesis for the Stone-Geary forms, no such pattern was observed for the ad hoc equations. The results were very inconsistent and produced positive elasticities in almost half of the cases. This came as no great surprise since the ad hoc specification is not in any way consistent with the restrictions of homogeneity in income and prices. Nor does this specification in any way take into consideration the potentially discontinuous nature of demand.

Thus it has been shown that when  $\alpha_2$  is positive and large, the pathological result of a positive own-price elasticity is possible. On the other hand, when  $\alpha_2$  is negative, its interpretation as the minimum required expenditure on all other goods is no longer valid. In view of these problems, it would be quite desirable to derive an estimate of

this parameter from survey data directly. Both of these problems would be eliminated if such an estimate could be made. However, data more detailed than those available in the MRI survey would be required. Information of this type would be beneficial for other formulations of demand also.

The number of residents was significant more often than any other variable. In the pooled equations, the number of residents was significant and its coefficient positive at the 99 percent level in all cases. For the non-pooled equations, it was significant in 90 percent of the ad hoc regressions and 82 percent of the Stone-Geary ones. It is very reasonable to expect the number of residents to be positively correlated with the consumption of electricity. This is because there are a number of electricity uses which are proportional to the number of people in the household. Examples of such uses include electricity used for cooking, washing and drying clothes, washing dishes and probably even lighting and television.

Another result that is not as one would expect was the effect of heating degree-days (HDD) on electricity consumption. One would expect the consumption of electricity to increase as the number of HDD increases. There are at least two reasons for expecting this. First, although none of the households heated directly with electricity, electricity is almost always used for the circulation of the heated air or water. Second, an indirect effect might occur if people tend to remain in the home more often on cold days since this would tend to increase other uses of electricity. The results show that neither of these effects is strong. HDD was rarely significant in the ad hoc regressions.

In the Stone-Geary regressions, the results were quite surprising. The coefficient for HDD was significant and negative in 59 percent of the non-pooled equations and 33 percent of the pooled ones. There is no readily apparent hypothesis that would lead one to expect this result. It was reassuring to find that when pooling across months that the coefficient for HDD was positive and significant two of four times in the Stone-Geary equations.

Cooling degree-days provided results more in line with what one would expect. Its coefficient was always positive in sign when significant. It proved significant in two of six individual month equations. In the pooled equation for New Orleans summer months, the coefficient had a t-statistic of roughly 10 for both specifications of demand. Much of the effect of CDD can probably be attributed to air conditioning which is very electricity intensive when compared with non-electric home heating. Also, the dummy variable for central air conditioning was positive and significant in all of the summer regressions. It is possible that the use of this variable along with CDD overshadowed the effect of the CDD variable in the individual month equations. Note that in principle, both CDD and air conditioning should be included together since the air conditioner will have to work harder to achieve the same inside temperature as the number of CDD increases.

The variable energy savers was significant roughly 20 percent of the time for both specifications using the data in non-pooled form. When pooling these percentages increased to 50 percent for the ad hoc form and 67 percent for the Stone-Geary equations. The category called energy savers consisted of a group of small appliances whose use could

potentially save electricity. Examples of these appliances are electric fans which could substitute for air-conditioning and cooking appliances such as a microwave oven or a slow electric cooker which are energy efficient when compared with conventional cooking methods. These appliances could also increase consumption if they are used in conjunction with or more intensively than their substitute appliances. Two-tailed t-tests were used since the sign of its coefficient could be either positive or negative.

In conclusion, it was found that both demand specifications showed improvements in terms of the percentage of significant coefficients when using pooled data as opposed to the non-pooled data. Also the t-statistics were in general much higher when pooling. These statements hold for pooling across months or cities. The rationale for this result is that pooling produces greater variation in the data. It also greatly increases the degrees of freedom in the sample. Further work in the area of estimating theoretically consistent demand functions should, therefore, focus on the use of pooled data.

There are several areas where improvements in the data could be made. More accurate measurements of income would undoubtedly improve the results. Since the MRI survey measured income only in wide ranges, the actual value of income could only be approximated. Using an approximate value for income as an independent variable is a classic example of errors-in-variables which biases the estimates of the utility function parameters. It is especially important to measure income accurately when estimating the demand for a commodity employing decreasing block

pricing since income must be adjusted by some relatively small income effects.

Another improvement would be to measure appliances in terms of watt capacity as opposed to merely stating how many of what kind of appliance. By measuring them in watts, appliances of similar nature could be grouped into a smaller number of aggregated appliance stocks. Grouping was not done with the MRI survey data except for the variable energy savers. Grouping of appliances would have two desirable consequences. One would be to increase the number of degrees of freedom of the regression. The other would be a reduction in any multicollinearity which happened to be present among the disaggregated appliances.

Finally, in the context of using the Stone-Geary utility function, additional survey information on the minimum required expenditure on commodities such as food, transportation, housing, clothing and certain other commodities would allow a direct estimate of the minimum required expenditure on all other goods,  $\alpha_2$ . This would provide greater flexibility in the estimation process by automatically allowing another utility function parameter to vary across households. Also, it would no longer be necessary to estimate this parameter in the regression, thereby reducing the number of independent variables by one. This would also eliminate the problems that were experienced with the sign and magnitude of  $\alpha_2$ . These problems were the determining factor in the failure of the utility-maximization algorithm in the case of New Orleans.

With these improvements, the application of the methodology developed in this chapter could be very useful to utilities and their regulatory commissions. By estimating demand through the use of utility

function parameters, estimates of effects due to rate schedule changes can be made from data containing little or no variation in the rate schedule. As has been pointed out, this method for determining the impacts of rate schedule changes is conceptually superior to the use of elasticities.

Finally, this chapter demonstrated that an estimation technique consistent with the theoretical implications of decreasing block pricing is easily applied to cross-sectional data. The empirical results have shown that this method is more powerful than the ad hoc specification in determining the price and income effects of decreasing block pricing. The most serious failure of the ad hoc version was its inability to measure the price effects. Marginal price was rarely significant, and when it was, its coefficient had the wrong sign in nearly half of the regressions. For these same samples, the Stone-Geary formulation produced acceptable results for all regressions except those involving New Orleans data. The results found in this chapter show promise, and hopefully, they will stimulate further research in this area of demand analysis.

## CHAPTER 4

### CONCLUSIONS

It is a well-known fact that decreasing block pricing has important implications for individual demand functions. At the same time, empirical procedures designed to deal with these implications have been largely ignored. Empirical estimations to date have attempted to measure price responses through the use of ex post average price, marginal price, and marginal price along with the income effects of prior block consumption. Since these demand functions have not been based on the utility maximizing behavior of the consumer, the resulting specifications are not consistent with the usual theoretical restrictions. Furthermore, the additional and very important restrictions posed by decreasing block pricing have generally been overlooked.

In Chapter 2, standard utility theory was used to derive a model of electricity demand which explicitly accounted for the impacts of a declining block rate schedule. These demand equations were derived by maximizing a Cobb-Douglas utility function subject to a budget constraint with the price of electricity determined by a rate schedule with two marginal blocks. Demand models derived from other utility functions can also be developed in the same general fashion. Generalizing the methodology to deal with rate schedules having more than two marginal blocks is easily accomplished. Such a generalization was made for the Stone-Geary utility function used for the empirical work in Chapter 3.

Inherent in this methodology is one potentially serious statistical problem. The proper approach for estimating individual demand under decreasing block pricing involves the use of a block-dependent independent variable. This block-dependency introduces the problem of errors-in-variables bias. The extent of this bias is one of the main issues addressed in the analysis.

Data appropriate for estimating these theoretically determined models could be generated by the following conceptual experiment. An individual consumer would be presented with a rate schedule for which one of the marginal prices was systematically varied. For each price, the corresponding level of consumption would be recorded. To construct such a data base, Monte Carlo techniques were employed. This methodology used the utility-maximization algorithm previously described and a random number generator.

One important advantage of using Monte Carlo data is that the extent of the bias problem previously mentioned can be directly assessed. Surprising accuracy in the utility function parameter estimates was achieved even when large error variances were introduced.

Another noteworthy Monte Carlo result was that the mean percentage error of the utility function parameter estimates was smaller for samples which contained a discontinuity than for those that did not. Initially, the expectation was that the presence of a discontinuity would reduce the accuracy of the results. In the case where a discontinuity is present, greater variation in the block-dependent independent variable exists. Apparently, this increased variation leads to more efficient parameter estimates.

If real world data appropriate for the application of the methodology of Chapter 2 become available, it would afford an opportunity to test for these highly specific theoretical restrictions. While this alone would be a worthwhile endeavor, a more practically oriented procedure must be based on a cross-section of consumers. This would allow the estimation of the effects of rate schedule changes on the total residential demand for electricity of a given utility. Such information would be useful for rate cases and capital planning.

To make the transition from estimating demand for one consumer over time to estimating demand for a cross-section of consumers, it is necessary to introduce further assumptions. In Chapter 3, this type of analysis is undertaken. Demand models estimated in this chapter are derived from the Stone-Geary utility function. The parameter representing the minimum required quantity of electricity was permitted to vary from household to household. This provided an appealing way to incorporate appliance stock and demographic variables into the utility function. That is, this utility function parameter was assumed to be a linear function of the household's appliance stock and demographic characteristics. Controlling for these influences of demand should result in more precise estimates of price responses.

The MRI data base was used to test these theories. The results were encouraging despite the following shortcomings of the data base. First, income was measured by categories producing measurement error in the calculation of the adjusted income variable. Direct measurement of income would correct this situation. Second, data on appliance stocks did not include information concerning watt capacities. By measuring

their wattages, meaningful groupings of appliances would be made possible. This would reduce any multicollinearity that might be present among the disaggregated appliances. Finally, additional information on expenditures on housing, food, transportation and other key commodities would allow the direct estimation of the minimum required expenditure on other goods. This last item has been mentioned with the Stone-Geary utility function in mind. Since minimum required expenditure on other goods had to be estimated in the regression, this would no longer be necessary. Problems with the New Orleans samples stemmed from the large positive values that occurred in the estimation of this parameter. These problems would be eliminated by its direct measurement. Furthermore, by obtaining an estimate directly from the survey information, this parameter would automatically vary across households. This would also provide greater flexibility to the estimation process since two parameters would vary instead of one.

The results of estimating demand derived from the Stone-Geary utility function were compared with results of an ad hoc, non-theoretically based regression model. Both models produced better results when using data pooled across months or cities. This was to be expected since greater variation in many of the independent variables occurs with pooling. On the basis of detecting the price and income effects of decreasing block pricing, the Stone-Geary model was judged superior. The predominant failure of the ad hoc version was its inability to measure the price effects. Marginal price was rarely significant, and when it was, its coefficient had a positive sign in roughly half of the cases. At the same time, the Stone-Geary results

were quite acceptable for all regressions except those involving New Orleans data.

The use of block-dependent independent variables for estimating individual demand can be applied either to observations on one consumer or cross-sectional observations. This approach results in estimates fully consistent with the theoretical implications of decreasing block pricing. In either application, an errors-in-variables bias is inherent. On the basis of the Monte Carlo study in Chapter 2, this statistical problem did not appear to pose any significant barriers to the estimation process. The empirical results from Chapter 3 also support this conclusion. Further work in this area would be illuminating in terms of the application of utility theory to micro-consumption data. It would also provide insight into the residential demand for electricity since decreasing block pricing is a widespread phenomenon in the electric utility industry.

## APPENDIX A

### MONTE CARLO RESULTS

Appendix A contains a summary of the Monte Carlo study undertaken as a part of Chapter 2. The organization of the information contained in this appendix follows the outline that was presented in Table 1. For each combination of  $\alpha$  and  $KWH_2$  listed in Table 1, two sets of observations were generated, one with  $z_3$  varying, the other with  $z_2$  varying. For both sets of observations, two procedures, OLS and Tobit, were employed. The results of these two procedures are summarized in the tables entitled "Original Method." For samples which contained a discontinuity (see Table 1 for a list of these), a trimming procedure which eliminated 10 observations was applied to the two original sets of observations with  $z_3$  and  $z_2$  varying, respectively. The tables entitled "Trimmed Method" parallel those of the original method except the trimmed samples were used. In all of the following tables,  $N$  represents the number of observations in a sample, and means are reported for the percent errors in the estimates of  $\alpha$ .

Table A1. Original Method,  $\alpha = .03$ ,  $KWH_2 = 400$ .

Error Variance (100's)	OLS			TOBIT (N=60)	
	$\alpha$ -Est.	%-Error	N	$\alpha$ -Est.	%-Error
$z_3$ varies:					
25	.0303	1.0	60	--	--
100	.0292	2.7	59	.0291	3.0
225	.0295	1.7	56	.0279	4.1
400	.0320	6.6	49	.0294	2.0
625	.0302	.5	46	.0270	10.0
900	.0300	.0	43	.0259	13.6
1225	.0304	<u>1.4</u>	42	.0257	<u>14.5</u>
		2.0			7.9
$z_2$ varies:					
25	.0308	2.8	60	--	--
100	.0312	4.1	57	.0306	2.1
225	.0315	5.1	52	.0276	2.8
400	.0323	7.8	49	.0290	3.4
625	.0357	18.9	49	.0313	4.2
900	.0390	30.1	47	.0340	13.3
1225	.0415	<u>38.3</u>	49	.0372	<u>23.9</u>
		15.3			8.3

Table A2. Trimmed Method,  $\alpha = .03$ ,  $KWH_2 = 400$ .

Error Variance (100's)	OLS			TOBIT (N=50)	
	$\alpha$ -Est.	%-Error	N	$\alpha$ -Est.	%-Error
$z_3$ varies:					
25	.0296	1.3	50	--	--
100	.0285	5.1	49	.0283	5.8
225	.0286	4.8	46	.0278	7.5
400	.0316	5.3	39	.0286	4.7
625	.0289	3.6	36	.0255	15.0
900	.0298	.6	33	.0249	17.1
1225	.0303	<u>1.0</u>	32	.0245	<u>18.4</u>
		3.1			11.4
$z_2$ varies:					
25	.0300	.1	50	--	--
100	.0302	.6	47	.0291	3.1
225	.0309	2.9	42	.0275	8.2
400	.0325	8.4	39	.0274	8.6
625	.0352	17.4	39	.0288	4.0
900	.0400	33.3	37	.0333	10.9
1225	.0440	<u>46.7</u>	39	.0378	<u>26.1</u>
		15.6			10.2

Table A3. No Discontinuity,  $\alpha = .03$ ,  $KWH_2 = 600$ .

Error Variance (100's)	OLS			TOBIT (N=60)	
	$\alpha$ -Est.	%-Error	N	$\alpha$ -Est.	%-Error
$z_3$ varies:					
25	.0300	.1	60	--	--
100	.0291	2.9	59	.0287	4.3
225	.0334	11.3	55	.0300	.0
400	.0302	.7	47	.0247	17.8
625	.0359	19.5	45	.0246	17.9
900	.0438	46.0	41	.0230	23.6
1225	.0472	<u>57.2</u>	40	.0239	<u>20.4</u>
		19.7			14.0
$z_2$ varies:					
25	.0314	4.6	60	--	--
100	.0320	6.6	57	.0313	4.3
225	.0351	16.9	52	.0311	3.7
400	.0353	17.6	49	.0305	1.8
625	.0393	30.9	49	.0322	7.4
900	.0451	50.5	46	.0357	19.1
1225	.0446	<u>48.9</u>	49	.0389	<u>29.2</u>
		25.1			11.0

Table A4. Original Method,  $\alpha = .04$ ,  $KWH_2 = 400$ .

Error Variance (100's)	OLS			TOBIT (N=60)	
	$\alpha$ -Est.	%-Error	N	$\alpha$ -Est.	%-Error
$z_3$ varies:					
25	.0401	.2	60	--	--
100	.0392	2.1	60	--	--
225	.0393	1.8	58	.0391	2.4
400	.0412	3.1	57	.0408	1.9
625	.0400	.1	54	.0390	2.5
900	.0396	.9	52	.0382	4.6
1225	.0402	<u>.4</u>	48	.0376	<u>6.0</u>
		1.2			3.5
$z_2$ varies:					
25	.0411	2.7	60	--	--
100	.0418	4.4	59	.0417	4.2
225	.0411	2.8	57	.0406	1.6
400	.0410	2.4	54	.0399	3.7
625	.0425	6.2	52	.0408	2.1
900	.0484	21.0	51	.0460	15.1
1225	.0495	<u>23.8</u>	53	.0476	<u>18.9</u>
		9.0			7.6

Table A5. Trimmed Method,  $\alpha = .04$ ,  $KWH_2 = 400$ .

Error Variance (100's)	OLS			TOBIT (N=50)	
	$\alpha$ -Est.	%-Error	N	$\alpha$ -Est.	%-Error
$z_3$ varies:					
25	.0401	.1	50	--	--
100	.0392	2.0	50	--	--
225	.0393	1.8	48	.0390	2.4
400	.0416	3.9	47	.0411	2.6
625	.0397	.8	44	.0386	3.6
900	.0395	1.3	42	.0379	5.3
1225	.0408	<u>1.9</u>	38	.0378	<u>5.5</u>
		1.7			3.9
$z_2$ varies:					
25	.0419	4.9	50	--	--
100	.0430	7.4	49	.0427	6.9
225	.0425	6.3	47	.0418	4.6
400	.0428	7.0	44	.0414	3.5
625	.0445	11.3	42	.0423	5.8
900	.0516	29.0	41	.0484	21.0
1225	.0534	<u>33.5</u>	43	.0509	<u>27.2</u>
		14.2			11.5

Table A6. Original Method,  $\alpha = .05$ ,  $KWH_2 = 400$ .

Error Variance (100's)	OLS			TOBIT (N=60)	
	$\alpha$ -Est.	%-Error	N	$\alpha$ -Est.	%-Error
$z_3$ varies:					
25	.0501	.2	60	--	--
100	.0494	1.1	60	--	--
225	.0494	1.2	60	--	--
400	.0506	1.2	59	.0505	1.0
625	.0494	1.1	58	.0492	1.6
900	.0495	1.0	58	.0492	1.6
1225	.0502	<u>.4</u>	56	.0494	<u>1.3</u>
		.9			1.4
$z_2$ varies:					
25	.0508	1.6	60	--	--
100	.0506	1.2	60	--	--
225	.0515	3.0	60	--	--
400	.0514	2.7	55	.0507	1.3
625	.0518	3.5	57	.0514	2.8
900	.0551	10.1	53	.0538	7.5
1225	.0571	<u>14.1</u>	56	.0564	<u>12.8</u>
		5.2			6.1

Table A7. Trimmed Method,  $\alpha = .05$ ,  $KWH_2 = 400$ .

Error Variance (100's)	OLS			TOBIT (N=50)	
	$\alpha$ -Est.	%-Error	N	$\alpha$ -Est.	%-Error
$z_3$ varies:					
25	.0501	.2	50	--	--
100	.0494	1.3	50	--	--
225	.0496	.8	50	--	--
400	.0510	2.0	49	.0509	1.8
625	.0499	.1	48	.0496	.8
900	.0500	.1	48	.0496	.8
1225	.0501	<u>.2</u>	46	.0492	<u>1.6</u>
		.7			1.3
$z_2$ varies:					
25	.0513	2.6	50	--	--
100	.0520	4.0	50	--	--
225	.0542	8.4	50	--	--
400	.0535	7.0	45	.0527	5.4
625	.0541	8.3	47	.0537	7.4
900	.0588	17.5	43	.0572	14.4
1225	.0607	<u>21.4</u>	46	.0599	<u>19.7</u>
		9.9			11.7

Table A8. Original Method,  $\alpha = .05$ ,  $KWH_2 = 500$ .

Error Variance (100's)	OLS			TOBIT (N=60)	
	$\alpha$ -Est.	%-Error	N	$\alpha$ -Est.	%-Error
$z_3$ varies:					
25	.0501	.1	60	--	--
100	.0489	2.3	60	--	--
225	.0488	2.3	60	--	--
400	.0501	.3	58	.0498	.4
625	.0490	2.0	56	.0483	3.5
900	.0489	2.3	56	.0481	3.9
1225	.0494	<u>1.3</u>	53	.0477	<u>4.5</u>
		1.5			3.1
$z_2$ varies:					
25	.0511	2.2	60	--	--
100	.0510	2.0	60	--	--
225	.0502	.4	59	.0500	.0
400	.0526	5.2	54	.0511	2.3
625	.0510	2.0	55	.0499	.3
900	.0592	18.4	51	.0563	12.5
1225	.0593	<u>18.6</u>	53	.0570	<u>14.0</u>
		7.0			5.8

Table A9. Trimmed Method,  $\alpha = .05$ ,  $KWH_2 = 500$ .

Error Variance (100's)	OLS			TOBIT (N=50)	
	$\alpha$ -Est.	%-Error	N	$\alpha$ -Est.	%-Error
$z_3$ varies:					
25	.0501	.1	50	--	--
100	.0490	1.9	50	--	--
225	.0484	3.2	50	--	--
400	.0499	.3	48	.0495	1.0
625	.0485	2.9	46	.0477	4.5
900	.0487	2.5	46	.0479	4.3
1225	.0489	<u>2.2</u>	43	.0471	<u>5.8</u>
		1.9			3.9
$z_2$ varies:					
25	.0519	3.9	50	--	--
100	.0523	4.5	50	--	--
225	.0535	7.0	49	.0532	6.5
400	.0552	10.4	44	.0532	6.4
625	.0537	7.4	45	.0522	4.5
900	.0630	26.0	41	.0591	18.2
1225	.0640	<u>27.9</u>	43	.0608	<u>21.7</u>
		12.4			11.5

Table A10. Original Method,  $\alpha = .05$ ,  $KWH_2 = 600$ .

Error Variance (100's)	OLS			TOBIT (N=60)	
	$\alpha$ -Est.	%-Error	N	$\alpha$ -Est.	%-Error
$z_3$ varies:					
25	.0501	.1	60	--	--
100	.0489	2.2	60	--	--
225	.0476	4.8	60	--	--
400	.0493	1.5	57	.0485	2.9
625	.0482	3.6	53	.0463	7.3
900	.0469	6.2	54	.0454	9.2
1225	.0476	<u>4.7</u>	50	.0447	<u>10.6</u>
		3.3			7.5
$z_2$ varies:					
25	.0504	.8	60	--	--
100	.0502	.5	60	--	--
225	.0501	.3	59	.0499	.2
400	.0522	4.4	54	.0501	.3
625	.0514	2.9	55	.0499	.3
900	.0563	12.6	51	.0530	6.1
1225	.0590	<u>17.9</u>	52	.0555	<u>11.1</u>
		5.6			3.6

Table All. Trimmed Method,  $\alpha = .05$ ,  $KWH_2 = 600$ .

Error Variance (100's)	OLS			TOBIT (N=50)	
	$\alpha$ -Est.	%-Error	N	$\alpha$ -Est.	%-Error
$z_3$ varies:					
25	.0500	.0	50	--	--
100	.0487	2.6	50	--	--
225	.0470	6.0	50	--	--
400	.0487	2.5	47	.0479	4.1
625	.0472	5.5	43	.0452	9.7
900	.0469	6.2	44	.0451	9.8
1225	.0483	<u>8.5</u>	40	.0448	<u>10.4</u>
		3.8			8.5
$z_2$ varies:					
25	.0505	.9	50	--	--
100	.0506	1.2	50	--	--
225	.0501	.2	49	.0496	.7
400	.0537	7.4	44	.0503	.6
625	.0520	3.9	45	.0497	.7
900	.0579	15.8	41	.0533	6.7
1225	.0603	<u>20.7</u>	42	.0555	<u>11.1</u>
		7.2			4.0

Table A12. Original Method,  $\alpha = .06$ ,  $KWH_2 = 500$ .

Error Variance (100's)	OLS			TOBIT (N=60)	
	$\alpha$ -Est.	%-Error	N	$\alpha$ -Est.	%-Error
$z_3$ varies:					
25	.0599	.1	60	--	--
100	.0593	1.2	60	--	--
225	.0592	1.4	60	--	--
400	.0601	.1	59	.0600	.1
625	.0590	1.7	58	.0586	2.3
900	.0591	1.5	59	.0590	1.7
1225	.0590	<u>1.6</u>	56	.0581	<u>3.2</u>
		1.1			1.8
$z_2$ varies:					
25	.0610	1.5	60	--	--
100	.0608	1.3	60	--	--
225	.0600	.0	60	--	--
400	.0600	.1	55	.0591	1.4
625	.0621	3.5	56	.0614	2.3
900	.0648	8.0	53	.0632	5.4
1225	.0673	<u>12.2</u>	56	.0665	<u>10.8</u>
		3.8			5.0

Table A13. Trimmed Method,  $\alpha = .06$ ,  $KWH_2 = 500$ .

Error Variance (100's)	OLS			TOBIT (N=50)	
	$\alpha$ -Est.	%-Error	N	$\alpha$ -Est.	%-Error
$z_3$ varies:					
25	.0599	.1	50	--	--
100	.0592	1.3	50	--	--
225	.0592	1.4	50	--	--
400	.0606	.9	49	.0604	.7
625	.0594	1.1	48	.0590	1.7
900	.0597	.5	49	.0595	.8
1225	.0600	<u>.0</u>	46	.0589	<u>1.8</u>
		.8			1.3
$z_2$ varies:					
25	.0614	2.3	50	--	--
100	.0622	3.7	50	--	--
225	.0629	4.8	50	--	--
400	.0624	3.9	45	.0613	2.1
625	.0649	8.2	46	.0641	6.8
900	.0687	14.4	43	.0667	11.2
1225	.0709	<u>18.2</u>	46	.0699	<u>16.5</u>
		7.9			9.2

Table A14. No Discontinuity,  $\alpha = .06$ ,  $KWH_2 = 600$ .

Error Variance (100's)	OLS			TOBIT (N=35)	
	$\alpha$ -Est.	%-Error	N	$\alpha$ -Est.	%-Error
$z_3$ varies:					
25	.0615	2.4	35	--	--
100	.0609	1.5	35	--	--
225	.0586	2.3	35	--	--
400	.0567	5.5	35	--	--
625	.0601	.2	33	.0569	5.1
900	.0646	7.6	34	.0629	4.8
1225	.0685	<u>14.1</u>	33	.0646	<u>7.6</u>
		4.8			5.8
$z_2$ varies:					
25	.0601	.1	35	--	--
100	.0605	.8	35	--	--
225	.0576	4.0	34	.0559	6.9
400	.0681	13.5	30	.0570	4.9
625	.0573	4.5	30	.0510	15.0
900	.0642	7.0	27	.0529	11.8
1225	.0615	<u>2.5</u>	29	.0555	<u>12.5</u>
		4.6			10.2

Table A15. Original Method,  $\alpha = .07$ ,  $KWH_2 = 600$ .

Error Variance (100's)	OLS			TOBIT (N=35)	
	$\alpha$ -Est.	%-Error	N	$\alpha$ -Est.	%-Error
$z_3$ varies:					
25	.0701	.1	35	--	--
100	.0696	.5	35	--	--
225	.0707	1.0	35	--	--
400	.0705	.6	35	--	--
625	.0722	3.1	35	--	--
900	.0790	12.9	35	--	--
1225	.0787	<u>12.4</u>	35	--	--
		4.4			
$z_2$ varies:					
25	.0702	.3	35	--	--
100	.0710	1.5	35	--	--
225	.0673	3.9	35	--	--
400	.0672	4.0	32	.0649	7.2
625	.0675	3.6	31	.0649	7.3
900	.0689	1.5	28	.0637	9.0
1225	.0707	<u>1.0</u>	31	.0680	<u>2.8</u>
		2.3			6.6

Table A16. Trimmed Method,  $\alpha = .07$ ,  $KWH_2 = 600$ .

Error Variance (100's)	OLS			TOBIT (N=25)	
	$\alpha$ -Est.	%-Error	N	$\alpha$ -Est.	%-Error
$z_3$ varies:					
25	.0691	1.3	25	--	--
100	.0688	1.6	25	--	--
225	.0706	.9	25	--	--
400	.0695	.8	25	--	--
625	.0736	5.1	25	--	--
900	.0816	16.5	25	--	--
1225	.0833	<u>19.0</u>	25	--	--
		6.5			
$z_2$ varies:					
25	.0697	.4	25	--	--
100	.0703	.4	25	--	--
225	.0708	1.1	25	--	--
400	.0700	.1	22	.0659	5.9
625	.0716	2.4	21	.0668	4.5
900	.0743	6.1	18	.0651	7.0
1225	.0761	<u>8.7</u>	21	.0715	<u>2.1</u>
		2.7			4.9

## APPENDIX B

### REGRESSION RESULTS FOR INDIVIDUAL MONTH EQUATIONS

Appendix B contains the results of 23 individual month regressions discussed in conjunction with Chapter 3. That is, for each of the cities, a separate regression was performed for each month in its sample. Three cities, Tucson, Minneapolis and Philadelphia, had samples consisting of only one month. St. Louis and Owensboro had three month samples. Des Moines' sample included four months while New Orleans had 10 months in its sample.

For each month, two regressions were performed. These are reported as pairs for comparison purposes. The first regression of a pair is the ad hoc formulation while the second is the Stone-Geary specification. T-statistics are given in parentheses. Note that each method employs several variables not used by the other. The ad hoc equations contain the variables income, marginal price, the fixed charge effect and a constant term, none of which appear in the Stone-Geary specification. On the other hand, the Stone-Geary regressions include ADJINC divided by marginal price, the inverse of marginal price and an additional statistic called correlation not included for the ad hoc results.

Independent Variable	Tucson November		Minneapolis October	
Income	.054 (1.75)	--	.045 (1.92)	--
ADJINC/z <sub>j</sub>	--	.0041 (2.58)	--	98.E-5 (1.38)
Marginal Price	39.E3 (1.83)	--	-32.E3 (-1.96)	--
(Marginal Price) <sup>-1</sup>	--	20.8 (4.63)	--	12.7 (3.83)
# Residents	16.2 (1.32)	13.8 (.97)	41.4 (3.70)	48.5 (3.88)
Insulation	-23.3 (-.87)	-67.0 (-2.51)	33.7 (.99)	-1.66 (-.04)
# Refrig.	56.5 (1.70)	31.1 (.85)	-9.55 (-.43)	-3.6 (-.14)
Dishwasher	49.7 (1.13)	38.2 (.75)	-11.5 (-.36)	24.7 (.67)
Electric Dryer	42.4 (.89)	79.7 (1.48)	93.9 (2.72)	85.7 (2.12)
Energy Savers	9.93 (.82)	-6.75 (-.51)	11.1 (1.19)	6.23 (.57)
# TV's	20.5 (.57)	27.6 (.67)	10.2 (.60)	.148 (.01)
HDD	.038 (1.05)	-.68 (-2.34)	.024 (.21)	-.24 (-1.98)
CDD	--	--	--	--
Central Air Cond.	--	--	--	--
Fixed Charge	-92.1 (-3.05)	--	32.4 (1.07)	--
Constant	-1890. (-1.80)	--	922. (1.63)	--
R <sup>2</sup>	.793	.932	.600	.953
r <sup>2</sup>	.842	.780	.651	.530
Correlation	--	.365	--	.261
# Observations	47	47	87	87
Price Elasticity	4.0	-1.6	-1.4	-1.1

Independent Variable	Des Moines March		Des Moines April	
Income	.037 (1.82)	--	.043 (2.25)	--
ADJINC/z <sub>j</sub>	--	.0029 (3.67)	--	.0027 (3.56)
Marginal Price	1.2E3 (1.06)	--	23.E3 (2.58)	--
(Marginal Price) <sup>-1</sup>	--	12.7 (4.62)	--	14.5 (4.53)
# Residents	36.7 (4.39)	35.9 (3.43)	43.9 (5.97)	51.6 (5.68)
Insulation	23.9 (1.21)	35.6 (1.46)	7.28 (.39)	26.4 (1.12)
# Refrig.	11.7 (.67)	5.72 (.26)	29.1 (1.77)	32.3 (1.56)
Dishwasher	55.6 (2.01)	59.6 (1.73)	39.5 (1.57)	73.3 (2.35)
Electric Dryer	69.3 (2.85)	90.7 (3.04)	44.6 (1.89)	62.7 (2.13)
Energy Savers	8.25 (1.14)	2.99 (.34)	2.89 (.41)	-7.19 (-.83)
# TV's	-1.79 (-.11)	6.16 (.32)	-1.91 (-.14)	-4.35 (-.25)
HDD	-.231 (-2.05)	-.539 (-4.18)	-.439 (-1.77)	-1.17 (-4.95)
CDD	--	--	--	--
Central Air Cond.	--	--	--	--
Fixed Charge	-107. (-3.62)	--	-135. (-5.90)	--
Constant	-570. (-1.08)	--	-1060. (-2.58)	--
R <sup>-2</sup>	.798	.956	.811	.950
r <sup>2</sup>	.828	.731	.833	.728
Correlation	--	.532	--	.588
# Observations	75	75	97	97
Price Elasticity	.08	-1.1	1.6	-1.2

Independent Variable	Des Moines October		Des Moines November	
Income	.021 (.94)	--	.022 (.88)	--
ADJINC/z <sub>j</sub>	--	.00058 (.40)	--	.0019 (1.50)
Marginal Price	38.E3 (4.24)	--	7640. (.94)	--
(Marginal Price) <sup>-1</sup>	--	5.16 (.86)	--	25.3 (2.92)
# Residents	28.4 (2.85)	55.1 (3.49)	19.7 (1.75)	45.8 (3.00)
Insulation	-4.67 (-.20)	8.96 (.24)	-10.0 (-.37)	-4.95 (-.02)
# Refrig.	8.76 (.43)	58.2 (1.86)	-16.5 (-.72)	-57.9 (-1.83)
Dishwasher	39.6 (1.30)	119. (2.46)	44.3 (1.22)	69.7 (1.36)
Electric Dryer	47.8 (1.67)	82.1 (1.80)	19.7 (.58)	46.8 (.98)
Energy Savers	14.1 (1.74)	11.3 (.87)	9.10 (1.01)	12.3 (.97)
# TV's	7.04 (.40)	-30.6 (-1.12)	61.9 (3.15)	74.0 (2.68)
HDD	.084 (.90)	.132 (.99)	.266 (1.67)	-.317 (-2.02)
CDD	--	--	--	--
Central Air Cond.	--	--	--	--
Fixed Charge	-209. (-9.27)	--	-130. (-5.94)	--
Constant	-2060. (-4.79)	--	-712. (-1.64)	--
R <sup>-2</sup>	.757	.897	.762	.927
r <sup>2</sup>	.783	.431	.794	.587
Correlation	--	.393	--	.431
# Observations	106	106	83	83
Price Elasticity	2.5	-.30	.48	-1.4

Independent Variable	St. Louis March		St. Louis April	
Income	.031 (1.90)	--	.0088 (.47)	--
ADJINC/z <sub>j</sub>	--	.0020 (2.38)	--	.0024 (2.88)
Marginal Price	-76.E3 (-5.18)	--	-28.E3 (-3.73)	--
(Marginal Price) <sup>-1</sup>	--	11.8 (3.52)	--	12.9 (4.28)
# Residents	20.2 (3.04)	35.4 (3.39)	27.1 (3.42)	31.3 (2.64)
Insulation	-3.27 (-.21)	-4.20 (-.17)	16.1 (.90)	10.5 (.39)
# Refrig.	17.8 (1.32)	16.3 (.78)	35.5 (2.43)	12.2 (.56)
Dishwasher	17.8 (.91)	47.5 (1.56)	50.3 (2.49)	45.0 (1.49)
Electric Dryer	17.8 (.94)	42.8 (1.42)	31.1 (.15)	18.2 (.59)
Energy Savers	-2.16 (-.35)	-4.28 (-.44)	-3.73 (-.56)	-4.11 (-.41)
# TV's	-5.21 (-.42)	20.0 (1.03)	7.21 (.55)	9.16 (.46)
HDD	-.032 (-.27)	-.388 (-2.19)	.080 (.37)	-.899 (-3.11)
CDD	--	--	--	--
Central Air Cond.	--	--	--	--
Fixed Charge	.267 (.01)	--	-68.9 (-4.93)	--
Constant	2860. (5.18)	--	1030. (3.71)	--
R <sup>2</sup>	.781	.939	.762	.934
r <sup>2</sup>	.803	.518	.785	.526
Correlation	--	.322	--	.290
# Observations	114	114	112	112
Price Elasticity	-4.4	-.92	-1.7	-1.1

Independent Variable	St. Louis November		Owensboro March	
Income	.0026 (.14)	--	-.00030 (-.01)	--
ADJINC/z <sub>j</sub>	--	.0015 (2.24)	--	.00051 (1.41)
Marginal Price	-85.E3 (-4.87)	--	-60.E5 (-2.85)	--
(Marginal Price) <sup>-1</sup>	--	42.8 (10.6)	--	7.33 (4.93)
# Residents	3.16 (.41)	18.9 (2.00)	34.1 (4.62)	33.2 (4.29)
Insulation	14.8 (.89)	-2.54 (-.12)	60.7 (2.11)	21.8 (.81)
# Refrig.	29.0 (1.92)	35.0 (1.75)	55.2 (2.53)	59.6 (2.58)
Dishwasher	44.8 (2.23)	98.0 (3.89)	35.8 (1.11)	34.2 (1.01)
Electric Dryer	13.0 (.64)	60.2 (2.32)	34.5 (1.24)	42.2 (1.42)
Energy Savers	-2.05 (-.32)	-13.3 (-1.62)	-9.20 (-1.06)	-14.8 (-1.68)
# TV's	24.3 (2.06)	16.8 (1.09)	-2.56 (-.14)	-3.77 (-.20)
HDD	-.031 (-.18)	-1.21 (-9.74)	-.188 (-.74)	-.620 (-3.78)
CDD	--	--	--	--
Central Air Cond.	--	--	--	--
Fixed Charge	3.11 (.15)	--	9930. (2.84)	--
Constant	3020. (5.01)	--	14.E4 (2.86)	--
R <sup>-2</sup>	.857	.967	.717	.958
r <sup>2</sup>	.871	.772	.755	.711
Correlation	--	.174	--	.447
# Observations	115	115	83	83
Price Elasticity	-3.9	-2.8	-208.	-.94

Independent Variable	Owensboro April		Owensboro November	
Income	-.033 (-1.06)	--	.024 (.56)	--
ADJINC/z <sub>j</sub>	--	70.E-6 (.14)	--	.00059 (.55)
Marginal Price	-1260. (-.07)	--	-3550. (-.04)	--
(Marginal Price) <sup>-1</sup>	--	10.5 (5.67)	--	27.2 (4.70)
# Residents	55.9 (6.14)	63.6 (6.49)	44.0 (3.50)	46.7 (3.75)
Insulation	22.1 (.75)	17.5 (.54)	56.3 (1.41)	59.2 (1.52)
# Refrig.	5.82 (.21)	23.6 (.78)	62.0 (1.67)	61.6 (1.65)
Dishwasher	90.1 (2.13)	90.0 (2.00)	54.5 (.88)	71.3 (.12)
Electric Dryer	-5.38 (-.15)	-13.3 (-.34)	68.7 (1.38)	80.0 (1.62)
Energy Savers	-4.55 (-.44)	-4.07 (-.36)	-14.9 (-.99)	-16.4 (-1.09)
# TV's	-.72 (-.04)	-10.7 (-.46)	10.7 (.35)	5.02 (.16)
HDD	.145 (.21)	-1.98 (-4.03)	-.325 (-.64)	-1.03 (-4.61)
CDD	--	--	--	--
Central Air Cond.	--	--	--	--
Fixed Charge	-93.5 (-2.97)	--	-69.4 (-.50)	--
Constant	5.23 (.01)	--	285. (.10)	--
R <sup>-2</sup>	.717	.931	.623	.909
r <sup>2</sup>	.752	.695	.669	.660
Correlation	--	.368	--	.487
# Observations	87	87	91	91
Price Elasticity	-.05	-1.4	-.17	-2.3

Independent Variable	Philadelphia September		New Orleans January	
Income	.104 (3.43)	--	.179 (2.32)	--
ADJINC/z <sub>j</sub>	--	.0049 (3.29)	--	.0059 (2.43)
Marginal Price	-621. (-.15)	--	67.E3 (.82)	--
(Marginal Price) <sup>-1</sup>	--	12.6 (1.47)	--	-16.1 (-1.20)
# Residents	17.7 (1.41)	24.1 (1.82)	10.4 (.37)	35.9 (1.26)
Insulation	49.4 (1.80)	73.8 (2.60)	27.5 (.33)	6.55 (.07)
# Refrig.	12.1 (.34)	14.8 (.40)	24.6 (.29)	22.4 (.25)
Dishwasher	-10.4 (-.20)	19.0 (.35)	21.1 (.13)	246. (1.58)
Electric Dryer	46.9 (.96)	74.3 (1.43)	-224. (-1.57)	-116. (-.79)
Energy Savers	-10.2 (-.70)	-16.4 (-1.07)	2.55 (.08)	-2.34 (-.08)
# TV's	24.4 (.83)	25.0 (.79)	209. (3.40)	225. (3.39)
HDD	.183 (.77)	-.183 (-.91)	1.98 (1.39)	.706 (.67)
CDD	--	--	--	--
Central Air Cond.	--	--	--	--
Fixed Charge	-64.5 (-3.45)	--	-278. (-1.62)	--
Constant	-2.45 (-.01)	--	-3790. (-1.00)	--
R <sup>2</sup>	.630	.919	.659	.841
r <sup>2</sup>	.695	.632	.742	.677
Correlation	--	.564	--	(.677)
# Observations	64	64	46	46
Price Elasticity	-.06	-.90	4.2	.79

Independent Variable	New Orleans February		New Orleans March	
Income	.178 (2.71)	--	.220 (2.31)	--
ADJINC/z <sub>j</sub>	--	.0068 (2.74)	--	.0081 (2.43)
Marginal Price	-48.E3 (-.80)	--	-56.E3 (-.43)	--
(Marginal Price) <sup>-1</sup>	--	-3.87 (-.33)	--	-9.49 (-1.15)
# Residents	53.2 (2.26)	74.3 (3.00)	50.0 (1.64)	73.7 (2.36)
Insulation	47.4 (.85)	63.5 (1.06)	42.1 (.56)	71.0 (.92)
# Refrig.	83.0 (1.11)	120. (1.52)	47.4 (.49)	91.8 (.91)
Dishwasher	-80.1 (-.72)	76.9 (.68)	-19.3 (-.13)	149. (1.06)
Electric Dryer	-214. (-1.82)	-179. (-1.41)	-300. (-1.93)	-262. (-1.60)
Energy Savers	22.8 (.89)	19.4 (.73)	43.0 (1.26)	27.1 (.79)
# TV's	104. (2.37)	104. (2.18)	115. (1.96)	119. (1.94)
HDD	-1.82 (-.81)	-1.31 (-.77)	.314 (.10)	-.527 (-.38)
CDD	--	--	--	--
Central Air Cond.	--	--	--	--
Fixed Charge	-66.0 (-.50)	--	-76.1 (-.28)	--
Constant	1870. (.65)	--	1890. (.34)	--
R <sup>-2</sup>	.571	.810	.473	.756
r <sup>2</sup>	.635	.553	.553	.478
Correlation	--	(.549)	--	(.478)
# Observations	75	75	74	74
Price Elasticity	-2.9	-.07	-3.2	.19

Independent Variable	New Orleans October		New Orleans April	
Income	.049 (.89)	--	.231 (2.38)	--
ADJINC/z <sub>j</sub>	--	.0033 (1.82)	--	.0098 (2.77)
Marginal Price	-76.E4 (-1.12)	--	6340. (.02)	--
(Marginal Price) <sup>-1</sup>	--	.359 (.63)	--	9.33 (.57)
# Residents	43.7 (2.17)	59.7 (3.08)	3.61 (.11)	25.3 (.73)
Insulation	27.4 (.57)	59.7 (.30)	-80.0 (-.85)	-66.3 (-.67)
# Refrig.	86.7 (1.49)	99.2 (1.66)	133. (1.31)	135. (1.26)
Dishwasher	60.4 (.63)	222. (2.45)	-105. (-.61)	170. (1.02)
Electric Dryer	-121. (-1.25)	-53.4 (-.52)	-203. (-1.22)	-79.6 (-.45)
Energy Savers	31.6 (1.58)	35.5 (1.67)	121. (3.25)	122. (3.14)
# TV's	28.6 (.81)	19.4 (.51)	13.3 (.21)	-25.6 (-.38)
HDD	.339 (.50)	-.898 (-2.00)	--	--
CDD	--	--	.98 (.35)	-1.87 (-.94)
Central Air Cond.	--	--	480. (3.09)	453. (2.70)
Fixed Charge	1380. (1.01)	--	-253. (-.34)	--
Constant	30.E3 (1.11)	--	-1029. (-.06)	--
R <sup>-2</sup>	.634	.882	.573	.788
r <sup>2</sup>	.702	.633	.648	.569
Correlation	--	.623	--	.543
# Observations	60	60	69	69
Price Elasticity	-43.	-.41	.30	-.86

Independent Variable	New Orleans May		New Orleans June	
Income	.429 (3.69)	--	.413 (2.30)	--
ADJINC/z <sub>j</sub>	--	.015 (3.77)	--	.016 (3.54)
Marginal Price	-5938. (-.11)	--	-97.E3 (-1.09)	--
(Marginal Price) <sup>-1</sup>	--	-27.7 (-1.18)	--	3.17 (.12)
# Residents	65.8 (1.64)	75.5 (1.84)	129. (2.37)	123. (2.20)
Insulation	-23.6 (-.22)	-35.5 (-.33)	41.7 (.26)	-27.5 (-.17)
# Refrig.	262. (2.11)	234. (1.83)	429. (2.44)	288. (1.63)
Dishwasher	57.8 (.30)	185. (.97)	-33.6 (-.13)	154. (.57)
Electric Dryer	-186. (-.91)	-175. (-.83)	-408. (-1.36)	-352. (-1.13)
Energy Savers	72.9 (1.59)	60.5 (1.32)	143. (2.06)	168. (2.49)
# TV's	50.7 (.65)	53.9 (.67)	162. (1.49)	160. (1.42)
HDD	--	--	--	--
CDD	1.75 (.71)	1.65 (.86)	8.60 (1.33)	-1.28 (-.71)
Central Air Cond.	696. (3.77)	707. (3.74)	828. (3.34)	827. (3.18)
Fixed Charge	-136. (-1.01)	--	-195. (-.84)	--
Constant	-1060. (-.37)	--	-2029. (-1.00)	--
R <sup>-2</sup>	.629	.859	.648	.871
r <sup>2</sup>	.690	.657	.711	.669
Correlation	--	(.657)	--	.664
# Observations	74	74	68	68
Price Elasticity	-.16	.64	-1.7	-.43

Independent Variable	New Orleans July		New Orleans August	
Income	.596 (3.84)	--	.551 (3.17)	--
ADJINC/z <sub>j</sub>	--	.022 (4.12)	--	.015 (2.74)
Marginal Price	12.E4 (.88)	--	-25.E4 (-.92)	--
(Marginal Price) <sup>-1</sup>	--	135. (1.41)	--	-73.6 (-2.41)
# Residents	102. (1.89)	90.1 (1.62)	108. (2.00)	122. (2.11)
Insulation	-140. (-.88)	-147. (-.97)	-202. (-1.33)	-82.5 (-.52)
# Refrig.	384. (2.09)	281. (1.55)	380. (2.18)	255. (1.37)
Dishwasher	-40.9 (-.16)	18.8 (.07)	137. (.50)	225. (.77)
Electric Dryer	-257. (-.81)	-192. (-.58)	-123. (-.41)	-176. (-.54)
Energy Savers	103. (1.69)	127. (2.00)	108. (1.66)	93.2 (1.36)
# TV's	159. (1.43)	165. (1.44)	193. (1.75)	209. (1.75)
HDD	--	--	--	--
CDD	8.49 (.55)	-8.27 (-1.46)	12.2 (2.00)	4.45 (2.20)
Central Air Cond.	968. (3.92)	1000. (3.91)	823. (3.24)	839. (3.04)
Fixed Charge	-770. (-2.53)	--	49.7 (.90)	--
Constant	-11.E3 (-1.09)	--	2356. (.29)	--
$\bar{R}^2$	.600	.878	.608	.856
$r^2$	.658	.622	.674	.597
Correlation	--	.593	--	(.597)
# Observations	83	83	73	73
Price Elasticity	2.0	-3.6	-4.4	1.7

Independent Variable	New Orleans September	
Income	.220 (1.60)	--
ADJINC/ $z_j$	--	.0057 (1.55)
Marginal Price	16.E4 (.87)	--
(Marginal Price) <sup>-1</sup>	--	-38.4 (-3.70)
# Residents	141. (3.33)	141. (3.33)
Insulation	-33.4 (-.27)	5.07 (.04)
# Refrig.	212. (1.61)	197. (1.47)
Dishwasher	258. (1.21)	376. (1.81)
Electric Dryer	-390. (-1.74)	-383. (-1.65)
Energy Savers	104. (1.97)	81.9 (1.60)
# TV's	155. (1.86)	174. (2.04)
HDD	--	--
CDD	3.40 (1.98)	3.20 (4.13)
Central Air Cond.	323. (1.69)	350. (1.77)
Fixed Charge	-507. (-1.31)	--
Constant	-8010. (-1.07)	--
$\bar{R}^2$	.613	.853
$r^2$	.684	.647
Correlation	--	(.647)
# Observations	67	67
Price Elasticity	4.2	1.4

## APPENDIX C

### REGRESSION RESULTS FOR THE POOLED EQUATIONS

Appendix C contains the results of pooling across months for the four cities which had more than one month in their samples. Two poolings have been done for New Orleans, one for summer months and one for winter months. Pooling across months therefore yields five sets of data. Another data set was provided by pooling across cities. These results are discussed in Chapter 3. The organization of the tables in this appendix is exactly the same as the organization of Appendix B.

Independent Variable	Des Moines		St. Louis	
Income	.0148 (1.37)	--	.0165 (1.63)	--
ADJINC/z <sub>j</sub>	--	.0016 (2.83)	--	.0026 (5.21)
Marginal Price	20.E3 (5.96)	--	-37.E3 (-6.91)	--
(Marginal Price) <sup>-1</sup>	--	3.54 (2.06)	--	78.6 (4.73)
# Residents	34.3 (7.32)	56.9 (8.40)	20.0 (4.82)	35.1 (5.13)
Insulation	-.699 (-.06)	14.7 (.88)	11.7 (1.21)	-12.1 (-.76)
# Refrig.	13.2 (1.34)	19.4 (1.34)	32.0 (3.90)	17.8 (1.31)
Dishwasher	42.0 (2.75)	98.3 (4.38)	45.8 (4.06)	54.5 (2.92)
Electric Dryer	43.8 (3.14)	76.2 (3.69)	16.2 (1.43)	53.8 (2.86)
Energy Savers	10.1 (2.51)	35.8 (.60)	-3.42 (-.94)	-11.4 (-1.87)
# TV's	8.15 (.97)	6.40 (.51)	13.6 (1.92)	17.8 (1.51)
HDD	.046 (2.00)	.133 (5.39)	-.067 (-2.95)	.042 (1.41)
CDD	--	--	--	--
Central Air Cond.	--	--	--	--
Fixed Charge	-149. (-1.58)	--	-50.2 (-6.44)	--
Constant	-1070. (-7.01)	--	1410. (7.02)	--
R <sup>2</sup>	.772	.919	.813	.934
r <sup>2</sup>	.779	.521	.820	.562
Correlation	--	.473	--	.361
# Observations	361	361	341	341
Price Elasticity	1.3	-.32	-2.0	-.73

Independent Variable	Owensboro		New Orleans Summer	
Income	.0022 (.11)	--	.420 (7.56)	--
ADJINC/z <sub>j</sub>	--	.0011 (3.04)	--	.0149 (7.92)
Marginal Price	5990. (.73)	--	-13.E3 (-.87)	--
(Marginal Price) <sup>-1</sup>	--	1.31 (1.41)	--	-45.9 (-8.57)
# Residents	45.6 (8.03)	55.9 (8.25)	95.8 (4.90)	93.2 (4.59)
Insulation	46.7 (2.49)	20.9 (.93)	-33.9 (-.65)	-11.5 (-.21)
# Refrig.	40.7 (2.38)	70.2 (3.40)	272. (4.33)	191. (2.95)
Dishwasher	55.8 (2.11)	93.0 (2.94)	33.8 (.35)	165. (1.67)
Electric Dryer	35.3 (1.58)	48.1 (1.78)	-293. (-2.75)	-288. (-2.57)
Energy Savers	-9.35 (-1.39)	-13.8 (-1.72)	98.9 (4.37)	91.5 (3.85)
# TV's	4.14 (.30)	-10.4 (-.63)	130. (3.28)	118. (2.85)
HDD	-.066 (-.48)	.152 (3.94)	--	--
CDD	--	--	2.80 (9.92)	3.27 (11.5)
Central Air Cond.	--	--	689. (7.48)	710. (7.34)
Fixed Charge	-87.2 (-5.82)	--	-181. (-3.12)	--
Constant	-176. (-1.04)	--	-1440. (-2.13)	--
$\bar{R}^2$	.675	.906	.646	.855
$r^2$	.688	.549	.655	.618
Correlation	--	.484	--	(.618)
# Observations	261	261	434	434
Price Elasticity	.24	-.26	-.29	1.2

Independent Variable	New Orleans Winter		Across Cities Winter	
Income	.170 (5.22)	--	.059 (4.11)	--
ADJINC/z <sub>j</sub>	--	.0062 (4.90)	--	.0023 (4.63)
Marginal Price	2900. (.25)	--	-446. (-.36)	--
(Marginal Price) <sup>-1</sup>	--	-5.10 (-1.45)	--	2.99 (2.31)
# Residents	45.2 (3.64)	67.9 (5.26)	41.0 (7.21)	52.0 (8.53)
Insulation	30.2 (1.03)	47.9 (1.52)	21.2 (1.57)	26.4 (1.78)
# Refrig.	54.8 (1.42)	87.6 (2.13)	40.9 (2.81)	50.9 (3.26)
Dishwasher	-24.4 (-.41)	148. (2.53)	77.1 (3.81)	106. (4.81)
Electric Dryer	-196. (-3.11)	-140. (-2.07)	43.5 (2.23)	59.2 (2.76)
Energy Savers	29.5 (2.26)	23.6 (1.68)	-1.70 (-.30)	-3.72 (-.59)
# TV's	104. (4.30)	103. (3.96)	48.0 (4.28)	47.2 (3.90)
HDD	-.300 (-1.14)	-.593 (-2.32)	-.050 (-1.67)	-.056 (-1.73)
CDD	--	--	--	--
Central Air Cond.	--	--	--	--
Fixed Charge	-173. (-4.88)	--	-54.5 (-11.2)	--
Constant	-524. (-.97)	--	7.26 (.10)	--
R <sup>2</sup>	.583	.819	.540	.879
r <sup>2</sup>	.601	.528	.549	.441
Correlation	--	(.526)	--	.381
# Observations	255	255	562	562
Price Elasticity	.16	.03	-.03	-.35

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