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SERGEI IVANOVICH TANEEV'S DOCTRINE OF THE CANON:
A TRANSLATION AND COMMENTARY

by
Paul Richard Grove II

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A Dissertation Submitted to the Faculty of the
SCHOOL OF MUSIC AND DANCE
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For the Degree of
DOCTOR OF PHILOSOPHY
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THE UNIVERSITY OF ARIZONA

1999
As members of the Final Examination Committee, we certify that we have read the dissertation prepared by Paul Richard Grove II entitled Sergei Ivanovich Taneev's Doctrine of the Canon: A Translation and Commentary and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

Joel Timothy Kolosick

Edward W. Murphy

Kelland Thomas

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Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copy of the dissertation to the Graduate College.

I hereby certify that I have read this dissertation prepared under my direction and recommend that it be accepted as fulfilling the dissertation requirement.

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SIGNED: Paul Grove
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DEDICATION

This dissertation is dedicated to Sandi and Sierra.
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ABSTRACT

Sergei Ivanovich Taneev's Doctrine of the Canon (Moscow, 1929) is the complement of Convertible Counterpoint in the Strict Style (Moscow, 1909). Both works are unique in the history of music theory due to Taneev's application of algebra for the demonstration of general laws of vertical-and horizontal-shifting counterpoint in the strict style. Together they form the cornerstone of Russian, twentieth-century contrapuntal theory.

This dissertation provides a translation of Doctrine of the Canon into English, and commentary. The commentary offers a comparison of Taneev's method with those of his contemporaries, a synthesis of relevant information from Convertible Counterpoint in the Strict Style with information in Doctrine of the Canon, a discussion of political influences on the theories that developed from Doctrine of the Canon, and a summation of developments of Taneev's theories of imitative counterpoint found in the works of the Soviet music theorists Semyon Semyonovich Bogatyryov, Mark Kopytman, Evgeny Nikolaevich Korchinsky, Sergey Sergeevich Skrebkov, and Nikolay Andreevich Timofeev.
Preface

The purpose of this dissertation is to provide an English translation of Sergey Ivanovich Taneev's *Doctrine of the Canon* and to discuss how the theories contained in Taneev's text were developed by theorists in the Soviet era in Russia. The result of this study expands the quantity of information about the theories of Taneev that is available to English-speaking students and scholars of Russian music and Russian music theory--a quantity that is surprisingly small in comparison to the size of Taneev's influence on Russian music and Russian music theory.  

My interest in this project was inspired by Dr. Ellon Carpenter's seminal work, *The Theory of Music in Russia and the Soviet Union ca. 1650-1950.* Dr. Carpenter's text provided me with fundamental information about Taneev's life.

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and theories, knowledge of the importance of Taneev's theories to the science of music theory in Russia, and a detailed history of Russian music theory.

Part I of this dissertation consists of an English translation of Taneev's second work on counterpoint, *Doctrine of the Canon*. This text is the second and final part of Taneev's published writings on counterpoint, and its translation, along with G. Ackley Brower's translation of *Convertible Counterpoint in the Strict Style*, provides in the English language the complete foundation of contemporary Russian contrapuntal theory. The commentary that accompanies the translation provides clarification of theoretical concepts established in *Convertible Counterpoint in the Strict Style* that have been cited by Taneev in *Doctrine of the Canon*, indicates corrections made to the text, provides complete citations of musical and theoretical works cited by Taneev, and offers additional pertinent information.

Part II of this dissertation explains several developments of Taneev's theories of imitative counterpoint by music theorists in the Soviet era. For the most part, Part II deals with contrapuntal theory in Russia after 1950.

With regard to the transliteration of Russian words in bibliographical citations, I have relied on the Library of

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Congress transliteration system. This system is provided below.

Passages from Convertible Counterpoint in the Strict Style that are quoted in this dissertation have been translated by G. Ackley Brower. Unless noted, all other materials from Russian and German sources have been translated by the author of this dissertation. Several corrections to Taneev's text were made previously by Andreas Wehrmeyer in his German translation and are duly noted when they are used in this translation.4

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Library of Congress Transliteration System

A  B    V  G  L  Е    Ж  Е  К  И  И    К  Л  М  Н  О  П  Р  С  Т  У  Ф  Х  Ц  Ч  Ш  Ш  Ы  Ы  Ю  Я
a  b  v  g  d  e  ė  zh  z  i  i  k  l  m  n  o  p  r  s  t  u  f  kh  ts  ch  sh  shch  sh  e  i  u  ia
Introduction to Part I

Taneev's Stature and Influences

Sergey Ivanovich Taneev (1856-1915) is remembered in Russia as a remarkable composer, music theorist, pianist, and pedagogue. During his tenure as a professor at the Moscow Conservatory, Taneev influenced many of the most important musicians and music theorists in twentieth-century Russia, including Aleksandr Nikolaevich Scriabin (1871/72-1915), Sergey Vasilevich Rakhmaninov (1873-1943), Nikolay Karlovich Metner (1879/80-1951), Georgy Eduardovich Conus (1862-1933), and Boleslav Leopoldovich Yavorsky (1877-1942). As Carpenter points out, due to his "creative and scholarly methods [that] set an example for numerous young Russian theorists," Taneev is thought of as the founder of contemporary Russian musical science.  

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5 Taneev's influence in these areas is discussed in Liudmila Korabel'nikova, S. I. Taneev v moskovskoi konservatorii: iz istorii russkogo muzykal'nogo obrazovaniia [S. I. Taneev in the Moscow Conservatory: from the history of Russian musical education] (Moscow, 1974).

6 An extensive list of Taneev's students is provided in Grigorii Borisovich Bernardt, S. I. Taneev (Moscow, 1983), pp. 270-2.

Taneev's formal education in counterpoint began at the Moscow Conservatory with Nikolay Albertovich Gubert (1840-88). Following his graduation in 1875, he independently developed his mathematical theories of moveable counterpoint. His theories grew out of his approach to composition, which involved a systematic exploration of compositional potentialities of a given theme. In addition to facilitating the composition of several masterworks, his contrapuntal theories proved to be more succinct for pedagogy than the empirical methods of his contemporaries and inclusive in their explanation of vertical-shifting counterpoint and horizontal-shifting counterpoint.

Although he theorized about counterpoint in both the strict and free style, Taneev heavily emphasized strict-style

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3 How this approach was applied in Taneev's creative activities is discussed by Elena Viazkova, professor of counterpoint at the Gnessin Institute. See Elena Vasil'evna Viazkova, "O tvorcheskom protsesse S. I. Taneeva (po eckiznym materialam kantat)" [On the creative process of S. I. Taneev (according to the preliminary materials of cantatas), Protsessy muzykal'noogo tvorchestva [Processes of musical works] 130 (1993), pp. 90-111.

Tchaikovsky's commentary on and criticism of these exercises is printed in P. I. Chaikovskii, S. I. Taneev Pis'ma [P. I. Chaikovskii, S. I. Taneev Letters], ed. V. A. Zhdanov (Moscow, 1951).
counterpoint in his lectures at the Moscow Conservatory and in his published works. This emphasis resulted from Taneev's belief that strict-style counterpoint is the foundation of a historical approach to music theory. Taneev valued an historical approach—one in which counterpoint is studied before harmony—because it "compelled students to experience for themselves the entire historical process of the evolution of music, taught them to separate in art the essential from the secondary, [and] also to evaluate the strong, the fantastic, and the eternal in the past." Taneev also valued the technical facility that could be gained from composing within the limits of the strict style. As Carpenter points out, these views, which form the basis of Taneev's approach to theory pedagogy, were heavily influenced by the eminent critic Hermann Avgustovich Larosh (1845-1904).

9 Several of Taneev's views on counterpoint in the free style are found in letters to his student Aleksey Vladimirovich Stanchinsky (1888-1914). These letters, which deal with the composition of fugue, are published in S. I. Taneev. materials and documents [S. I. Taneev. materials and documents] (Moscow, 1952), pp. 241-9. Taneev's views on free-style counterpoint are also found in A. G. Mikhailenko. Fuga v teoreticheskom i tvorcheskom nasledii S. I. Taneeva: Stranitsa istorii russkoi polifonii [Fugue in the theoretical and artistic legacy of S. I. Taneev: a page of history of Russian polyphony] (Novosibirsk, 1992).


11 Carpenter, "The Theory of Music in Russia," 404. Larosh emphasized the importance of a historical approach to music theory and stated the technical benefits of composing in the strict style in his article "Mysli o muzykal'nom obrazovanii v Rossii" [Thoughts on musical education in Russia], Russkii Vestnik [Russian herald] 7 (1869). This article was later republished in G. A. Larosh, Sobranie muzykal'nokriticheskikh statei [A collection of musical-critical articles], vol. 1 (Moscow, 1913), pp. 214-45.
Taneev's use of mathematics to study the strict style was inspired by the writings of the rationalist Benedict (Baruch) Spinoza (1632-77), whose use of geometry in his philosophical theories inspired Taneev to use algebra to demonstrate his musical theories. Taneev summarized his view on the importance of mathematics to musical science with the following quotation by Leonardo da Vinci, which he placed at the beginning of Convertible Counterpoint in the Strict Style: "No field of human research can claim to be considered a true science unless it is mathematically verifiable."¹²

Despite his numerous pedagogical activities, Taneev published only one music theory book during his lifetime: Convertible Counterpoint in the Strict Style (1909). His second work on counterpoint, Doctrine of the Canon (1929), was compiled, edited, and published posthumously by the Russian musicologist Viktor Mikhaylovich Belyaev (1888-1968). These works form an interdependent collection of ideas. In his first book, Taneev sets forth the mathematical theories that he applies to imitative counterpoint in Doctrine of the Canon. These texts form only a portion of the works that

Taneev intended to publish on counterpoint. Unfortunately, other texts were not completed.\textsuperscript{13}

**Taneev's Contribution to the Study of Counterpoint**

Unlike the contrapuntal theories of his contemporaries, whose published works rely primarily on empirical observations and generally apply only to the most widely employed forms of double counterpoint, namely, at the octave, tenth, and twelfth, Taneev's theories of moveable counterpoint are distinguished by their use of mathematics, which makes possible concise descriptions of all facets of vertical- and horizontal-shifting counterpoint. The benefits of this contribution can be seen in a comparison of approaches to the study of double counterpoint at the tenth. An approach that is typical of those in the works of nineteenth-century theorists is found in Ernst Friedrich Richter's (1808-79) *Lehrbuch des einfachen und doppelten*

\textsuperscript{13} Titles of uncompleted works on counterpoint by Taneev are: *Tsvetistyi kontrapunkt strogoogo pis'ma* [Florid counterpoint in the strict style], *Imitatsiia v prostom kontrapunkte* [Imitation in simple counterpoint], *Obratimyi kontrapunkt* [Invertible counterpoint], *Fuga v strogom pis'me* [Fugue in the strict style], *Fuga v svobodnom pis'me* [Fugue in the free style], and *Kanonicheskie formy na osnove podvizhnogo kontrapunkta* [Canonic forms on the basis of moveable counterpoint]. Fedor Georgievich Arzamonov, "Zavety S. Taneeva" [Ordinances of S. Taneev], *Sovetskaia muzika* [Soviet music] 11 (1956): 31. Cited in Ellon D. Carpenter, *Contributions of Taneev, Catoire, Conus, Garbusov, Mazel, Tiulin.* Russian Theoretical Thought in Music, no. 10, ed. Gordon McQuere (Ann Arbor: UMI Research Press, 1983), p. 360.
Kontrapunkt (1872). I will compare his approach to Taneev’s approach.

Richter begins by discussing the treatment of tied dissonances. He states the following and supplies an example that is given here as Example Intro. 1: “The suspension by means of the fourth may occur in the lower part only; and that of the seventh in the upper.”

\[
\begin{array}{c|c}
\text{Bad} & \text{Inver. 10th below} \\
\hline
\text{Good} & \text{Inver. 10th below}
\end{array}
\]

Example Intro. 1. Richter’s demonstration of good and bad tied dissonances at double counterpoint at the tenth.

In contrast to this, Taneev shows these same rules in a concise numerical table: 0 1 2 3 4 5 6 7 8 9, where 0 = a unison, 1 = a second, 2 = a third, etc. The line below 3 indicates that this interval may be tied only in the lower voice, and the line above 6 indicates that this interval may be tied only in the upper voice in original combinations. In addition to these, in order to remain in the strict style,

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16 Taneev, *Convertible Counterpoint*, p. 318. Taneev describes his system for numeration as follows: “This new method...consists in taking the interval between two adjacent scale-degrees, i.e., a second, as the unit. The interval is then indicated by a figure showing the number of these units that it contains. The unison is indicated by 0, since in it this quantity is equal to zero. Therefore each interval is represented by a figure that is 1 less than its usual numerical designation: a third by 2, a fourth by 3, etc.” (Taneev, *Convertible Counterpoint*, § 1, p. 25.)
the second (1) may be tied only in the lower voice, and the
ninth (8) may be tied only in the upper voice.

With regard to successions of parallel consonant
intervals, Richter states:

"Inasmuch as the sixth becomes by inversion a
fifth, and the third and tenth become octave and unison,
similar motion, which generally consists of consecutive
thirds or sixths, must be treated with exceptional care,
because their character is so completely altered by
inversion." 17

In contrast to this, Taneev provides a mathematical
description that shows the variable and invariable properties
of all consonances at all vertical shifts.

He begins by positioning and labeling consonances in a
successive series, which is displayed in Example Intro. 2.

```
<table>
<thead>
<tr>
<th>Consonances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

Example Intro. 2: Taneev's successive series of consonances.
```

The negative signs that accompany the integers to the left of
the unison show that the upper voice has passed below the
lower voice. Taneev then separates all intervals into two
groups. Group one consists of intervals that appear in three
forms: perfect, augmented, and diminished. These are unisons
(labeled 0), fourths (3), fifths (4), and octaves (7). Group

17 Richter, Treatise on Counterpoint, p. 115.
two consists of all intervals that appear in four forms: major, minor, augmented, and diminished. These are seconds (1), thirds (2), sixths (5), and sevenths (6). Using this grouping, Taneev makes the following observations about the successive series of consonances (see Example Intro. 2):

Two consonances of the same type (i.e., both perfect or both imperfect) are separated from each other by a quantity contained in group one. Consonances of different groups (i.e., one perfect, the other imperfect) are separated from each other by an interval from group two. For example, consonance 2 (imperfect) is separated from consonance 5 (imperfect) by 3 (a member of group one); consonance -7 (perfect) is separated from -11 (perfect) by 4 (a member of group one). Conversely, consonance -5 (imperfect) is separated from -7 (perfect) by 2 (a member of group two); consonance -4 (perfect) from -9 (imperfect) by 5 (a member of group two), etc.\(^{18}\)

Taneev indicates that the only exception to this is between the two perfect intervals -4 and +4, whose distance is equal to 8, a member of group two \([8 - 7 (an\ octave) = 1]\).

With regard to double counterpoint at the tenth (i.e., counterpoint in which all original intervals are separated from their derivatives by 9, a member of group two), Taneev's theory tells us that imperfect consonances will become perfect and perfect consonances will become imperfect. As is evident from this comparison, Taneev's method provides concise, mathematically-accurate, inclusive principles of vertical-shifting counterpoint by using basic algebra.

\(^{18}\) Taneev, Convertible Counterpoint, § 15, pp. 30-1.
Conclusion

Taneev's life and work occurred at a significant point in the history of Russian musicology—a time when a distinctly Russian school of musical science was emerging. Taneev was at the forefront of this school and served as an inspiration to other significant Russian musical theorists, such as Conus and Yavorsky. Yavorsky expresses his appreciation for Convertible Counterpoint in the Strict Style as follows:

I rejoice and celebrate. Finally what I always considered as one of the very outstanding events in our musical life has been made general property. I attach such significance to your work, because in my opinion this is the first work that transfers musical art, that is, something unconscious, intangible for the human intellect, into the sphere of science. This is the first theoretical work based on precise laws, the logical application and development of which create a constructed edifice.\(^\text{19}\)

In Doctrine of the Canon, Taneev applies his mathematical principles to the composition of imitative counterpoint. Because Taneev's two contrapuntal texts were developed and written simultaneously, Doctrine of the Canon should be viewed as the complement of Convertible Counterpoint in the Strict Style. These two texts continue to be used by music students in Russia and would be

beneficial for music students in this country because they are applicable to the strict as well as the free style.

As is evident from the diversity of the works that were composed by Taneev—"the Russian Brahms"—and his students, such as Scriabin and Rakhmaninov, it is apparent that the true benefit of applying mathematical principles to the study of composition rests, not in an enforced mimicry of a given teacher of composition, but rather in the development of a disciplined and resourceful musical mind. This is perhaps Taneev's greatest legacy, the creation of a system for the study of counterpoint that through its thoroughness and rigor liberates the true creative vision and instincts of the composer.
Part I: The Translated Work

Sergei Ivanovich Taneev:  
*Doctrine of the Canon*
Abbreviations in the Translation and Musical Examples

G.D.-- A musical work from Henricus Glareanus, Dodecachordon, ed. Peter Bohn (Leipzig, 1888). The numbers that accompany this abbreviation are page references.

P.-- A musical work from Giovanni Pierluigi da Palestrina, Werke, ed. F. X. Haberl (Leipzig, 1862-1907). The Roman numerals that accompany this abbreviation are volume references; the Arabic numerals are page references.

C.C.-- A citation from Convertible Counterpoint in the Strict Style.

V.B.-- Victor Beliaev (information added by the editor)

P.G.-- Paul Grove (information added by the translator)
Forward

With the publication of Doctrine of the Canon, the second major work by S. I. Taneev that pertains to research on counterpoint and its application in artistic forms of polyphonic music has appeared. Both of these works (the first being, as is known to all, Convertible Counterpoint in the Strict Style) represent a necessary supplement to one another and may be examined as two parts of a single whole.\(^{22}\)

Doctrine of the Canon was completed by its author and partially rewritten on a typewriter for publication. The entire work consists of six separate notebooks that have been marked by the letters A, B, C, D, E, F, which sometimes contain numeration of the paragraphs that begin each time with a numeral, sometimes without this numeration but with an indication of the beginning of each paragraph.\(^{23}\) Following the model given in Convertible Counterpoint in the Strict Style, I adopted the system of the consecutive numeration of paragraphs for the entire work by S. I. Taneev and introduced uniformity to the titles of its separate sections. In the

\(^{20}\) This is the first appearance of the word "podvizhnoi" in the Russian text. Brower chose to translate this as "convertible." Based on the advice of a Russian language scholar, I have chosen to translate this word as "moveable." Henceforth, when podvizhnoi appears in the title of Taneev's first work on counterpoint it will be translated as convertible, in all other places it will be translated as moveable. The two words have virtually the same meaning and encompass such terms as double counterpoint and invertible counterpoint. (P.G.)

\(^{21}\) These notebooks are stored in Taneev's archive in the museum at Tchaikovsky's home in Klin, Russia. (P.G.)
majority of the aforementioned notebooks that originally comprised *Doctrine of the Canon* (some of these exist in several variants), the musical examples are absent. These had been written down by Taneev in separate musical notebooks that frequently contain musical examples that were written by him for other purposes, specifically for the aim of teaching. The greatest difficulty of my work has turned out to be the restoration of these musical examples to the appropriate places in the text. This work was made still more difficult because all of S. I. Taneev’s works in his archive turned out to be unorganized. In all, the entire matter of the preparation for printing of *Doctrine of the Canon* occupied about two years of persistent and laborious work.

Wishing to present the work of S. I. Taneev in the condition in which it came from the pen of its author, and at the same time seeing the necessity for some additions, I enclosed all of my own supplements in straight brackets so they would not become mixed in with the authentic text of S. I. Taneev. I believe that in such a condition *Doctrine of the Canon* will simultaneously have the appearance of both an authentic historical document and a guide that has been designed for practical use.

Both *Convertible Counterpoint in the Strict Style* and *Doctrine of the Canon* belong to a type of musical-scientific work that forms the foundation of an original theory of counterpoint, which has been constructed on the fundamentals
of mathematics. The greatest rationalist among all musical scholars and a musical theorist of genius, S. I. Taneev in these works removed from the study of counterpoint the remaining veils of mysticism that for centuries have obscured this subject of music theory and presented it in clear mathematical formulas. This circumstance makes the publication of *Doctrine of the Canon* completely relevant for our time. But our era requires not only rules and precise theoretical constructions, but also their practical utilization and popularization. With respect to this, for the time being nothing has been done, and the task of the coming years should be its adaptation to the matter of the practical teaching of music theory.

The *Doctrine of the Canon* received its general design in the time period between 1901 and 1903 (all investigative work by S. I. Taneev in the area of strict counterpoint embraces a period of not fewer than 25 years), but after this, up until 1912, it was subjected to significant alterations and was still not in the format in which it is now published.12 There is no doubt that if *Doctrine of the Canon* had been

published by its author it would have had some different final editing, but the whole would have differed from the present edition only in details, not in the general form of its presentation.

Both the preparation of *Doctrine of the Canon* for publication and the publication of the book itself turned out to be an exceptionally difficult matter. During its printing, a significant amount of proofreading was done by a student of S. I. Taneev, L. V. Nikolaev, who is currently a professor at the Leningrad State Conservatory, to whom I consider necessary to express my heartfelt gratitude. I also owe the same gratitude to S. S. Popov, who warmly supported me in my intention to prepare *Doctrine of the Canon* for publication.

*Viktor Beliaev*²¹

23.X.1929. Moscow

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²¹ Viktor Mikhailovich Beliaev (1888-1968) was a musicologist, folklorist, and pedagogue. He taught at both the Moscow and Leningrad Conservatories. As Beliaev states in the Forward, he was responsible for compiling this edition of the text. (P.G.)
Introduction
(The application of moveable counterpoint to canonic forms)

When teaching complex counterpoint parallel with a general course on strict counterpoint, one should not limit oneself to problems that pertain directly to this, but [one must] use any opportunity where what has been studied by the student in complex counterpoint may be applied.\textsuperscript{24} For example, if imitation in a chorale is studied, one should indicate how complex counterpoint can be applied in it. During the study of fugue, there is constantly an opportunity to use moveable counterpoint in the theme and countersubject, in episodes, strettos, etc. In addition to this, an area exists where complex counterpoint finds extensive application—in the teaching of canon. Exercises in canon should continually accompany the study of complex counterpoint. They have this importance because they contribute to a solid mastering of complex counterpoint and, in general, they develop an aptitude for contrapuntal thought. There is a very great difference between the contrapuntal technique of one who is in a position to write an assignment only in complex counterpoint and the technique of one who has familiarized himself with all possible applications of it and with

\textsuperscript{24} Taneiev describes complex counterpoint as "that kind of counterpoint in which an original combination of melodies yields one or more derivatives." (Serge Ivanovitch Taneiev, Convertible Counterpoint in the Strict Style, trans. G. Ackley Brower (Boston: Bruce Humphries, 1962), p. 19.)
confidence and ease, almost instinctively, uses it as an obedient tool in the most diverse opportunities. These exercises provide a possibility for the student to become convinced that what he has studied has practical application, they will stimulate his interest by diversifying his works, and they make accessible such tasks that for him were previously unsolvable.

Each of the existing forms of imitation can serve as a foundation for canon. In the doctrine of the canon set forth here, all exceptional forms of imitation have been removed: augmentation, diminution, inversion, cancrizan, etc.—those forms to which with some degree of justification the name tricks, "Kunsteleien," may be applied. Just as in a course on simple counterpoint, where these exceptional forms of imitation are studied only as an addition to its simple form, also in the study of canon it will be natural in the beginning to study those of its forms in which the simplest form of imitation serves as the foundation. It would be incorrect to maintain that among the enumerated exceptional forms there are not encountered those that would be useful for the development of contrapuntal technique. But, in the first place, we have set as the goal to trace a link between forms of complex counterpoint and canonic forms, on account

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25 Ludwig Bussler describes "Kunsteleien der Imitation" in Der Strenge Satz (Berlin, 1877), § 27, pp. 100-2. His discussion deals primarily with imitation in retrograde. Within the Russian tradition, exceptional forms of imitation were explored by Evgeny Korchinsky. His contributions are discussed in Chapter 19 of this dissertation. (P.G.)
of this we may not apply to canons forms of complex counterpoint that we have not yet studied, for example, invertible counterpoint. In the second place, no matter how strange this appears, in existing textbooks it is possible to find far more thorough information regarding exceptional forms of canon. This information is often completely unnecessary and owes its existence more to fabrications by idle theorists than to forms that create the most essential part of the doctrine of the canon. In the present work, we are also not concerned with those canons that have more than one main guiding voice (Proposta), such as double canon, triple canon, etc. Although these canons are also written with the help of forms of complex counterpoint that we have studied, their inclusion in the present work would have stretched this book too far beyond its limits. These canons

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26 See C.C., p. 9. (V.B.) Relative to Brower's translation, this citation corresponds to pages 21-2 of the Introduction. At that point, Taneev describes "mirror counterpoint" as a form of "metamorphosed counterpoint" and states that "Since metamorphosed counterpoint does not enter into the plan of the present work it will not be considered further." However, in the Introduction to the original, unpublished manuscript of Convertible Counterpoint in the Strict Style, Taneev briefly discusses issues that pertain to invertible, mirror counterpoint. This discussion is reprinted in Liudmilla Korabel'nikova, Fedor Arzamanov, eds., S. I. Taneev. Iz nauchno-pedagogicheskogo naslediia [S. I. Taneev. From the scientific-pedagogical legacy] (Moscow, 1929), pp. 42-8.

Invertible counterpoint was explored using Taneev's methodology in Semen Semenovich Bogatyrev, Obratimyi kontrapunkt [Invertible counterpoint] (Moscow, 1960). (P.G.)
could be the subject of a separate investigation.\(^\text{27}\) We begin with two-voice canons and move to three- and four-voice canons. Canons that have a greater number of voices are mentioned [here] only in passing.\(^\text{28}\)

Thus, the delimited sphere includes canons that have been based, as we have already mentioned, on the simplest forms of imitation that were in earlier times called canon simplex per motum rectum—simple canon in direct motion. Here we encounter canonic forms that were continually used in contrapuntal compositions by masters of both the strict and free style. Apart from its significance for the development of compositional technique, knowledge of these forms contributes to an understanding of the technical side of compositions of an earlier time. Having set as a goal to research as completely and comprehensively as possible this sphere of canonic forms that we have singled out, we then no longer make a choice between these forms, and, although briefly, we set forth all cases that are relevant to this, not excluding even those whose significance as applied to composition is extremely insignificant (for example, three-

\(^{27}\) Double canon was studied by Bogatyryov in Dvoinoi kanon [Double canon] (Moscow, 1948), and by Sergei Sergeevich Skrebkov, Teoriia imitatsionnoi polifonii [A theory of imitative counterpoint] (Kiev, 1983), pp. 119-29. These theories are discussed in Chapters 17 and 20 of this dissertation. (P.G.)

\(^{28}\) Within the Russian tradition, composition of canons with more than four voices is discussed in Mark R. Kopytman, "Mnogogolosnyi kanon (kanon i sekventsiia)" [Multi-voice canon (canon and sequence)], Voprosy muzykoznaniia [Questions of musical knowledge] 3 (1960): 195-296; Sergei Sergeevich Skrebkov, Teoriia imitatsionnoi polifonii [A theory of imitative counterpoint] (Kiev, 1983). These works are discussed in Chapters 18 and 20 of this dissertation. (P.G.)
voice and four-voice infinite canons at various intervals). Contributing to a clear understanding of the theoretical side of the questions that engage us, these cases, in light of their little practical significance, may remain entirely without exercises.

In general, when applying this doctrine of the canon as material for practical application and a better mastery of forms of moveable counterpoint that one has studied, it is not necessary to strive to teach the student this course in its entirety. Given the amount of time that is set aside for a course on strict counterpoint, for example, at the Petersburg and Moscow Conservatories (one year), it would not be possible to fulfill such a task. It will be sufficient for the student to write exercises in the main cases of canonic imitations and to master their methods of composition and analysis. In so doing, invariably one should be guided by this rule: Practice only those forms of canon that require an application of forms of moveable counterpoint that have already been studied. The teacher is presented with a vast choice of exercises of all sorts of cases of the latter, beginning with the easiest forms of two-voice canon and ending with the most difficult forms of four-voice. Through the entire doctrine of the canon, a division of canonic forms into two orders has been made, depending on whether they require an application of vertical-shifting or horizontal-
shifting counterpoint independently or in combination. To locate appropriate exercises for passable forms of complex counterpoint that correspond to the strengths of the student is a very easy task for the teacher, thanks to the indicated division.

Exercises in canonic forms, studied as supplementary parallel with the study of complex counterpoint, will be completely sufficient in order to acquaint the student with methods of writing canons and will make further independent study of canonic forms accessible to him, which is highly desirable for the development of compositional technique. Just as completely accomplished virtuosos find an essential portion of their time to devote to technical exercises for the maintenance and refinement of their technique, young composers would also find it useful not to abandon their contrapuntal exercises when they have finished a course on counterpoint. Those advantages that knowing how to perfectly use counterpoint will give to him—elegance, smoothness and logical succession of separate voices, an ease in the treatment of musical thoughts, and speed when extracting possible combinations that are contained in these—all these advantages of a complete mastery over the musical material compensate in abundance for time spent in contrapuntal study.

With few exceptions, the rules that have been presented in the doctrine of the canon have a general significance: they pertain to both the free and strict style. But to be
guided by them in the study of canon in the free style is possible only after a preliminary study of moveable counterpoint in the free style. In order to use this manual for canon in the free style, one must find new examples for guidance since all of the models cited here belong to the strict style.

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(A study of moveable counterpoint in the free style that employs Taneev's theories is present in Iosif Iakovlevich Pustyl'nik, Podvizhnui kontrapunkt i svobodnoe pis'mo [Moveable counterpoint and the free style] (Leningrad, 1966). (P.G.)
First Part:

Two-Voice Canon
§ 1. We assume the student is acquainted with imitation. The first exercises in this form pertain to a course on simple counterpoint and begin immediately after counterpoint of various species has been studied (see texts by Fux, Bellermann, Cherubini, and Bussler). In these two-voice exercises, it is not necessary to be limited to imitation of only that part of the initial voice that extends from its entrance up to the point of entrance of the imitative voice. Imitation may continue still further, employing the counterpoint that the initial voice performs with regard to the imitative voice. By means of this, canonic imitation arises that is accessible to the student from his first steps in the study of imitative forms. So, in Bellermann's textbook, the second of the cited examples in imitation presents a sample of canonic imitation that turns

31 The texts that Taneev is probably referring to are as follows: Johann Joseph Fux, Gradus ad Parnassum (Vienna, 1725); Heinrich Bellermann, Der Contrapunkt oder Anleitung zur Stimmführung in der musikalischen Composition (Berlin, 1862); Luigi Cherubini, Cours de contrepoint et fugue (Paris, 1835); Ludwig Bussler, Der Strenge Satz (Berlin, 1877). Taneev translated Bussler's work into Russian: Ludwig Bussler, Strogii stil' [The strict style], trans. Sergei Ivanovich Taneev (Moscow, 1885). (P.G.)
into a free conclusion. Such two-voice imitation, which does not require the application of any sort of complex counterpoint, will serve us as the starting point for the doctrine of canon.

§ 2. Using old terminology, we will call the beginning voice of imitation the Proposta and will indicate it with the letter P; the imitating voice—the Risposta—will be indicated with the letter R. The distance at which the entrance of the Risposta occurs with regard to the Proposta (a measure, two measures, a half measure, etc.) we will call the distance of entrance and schematically depict it as

\[
P \quad R
\]

and the interval at which the Risposta enters with regard to the Proposta we will call the interval of entrance (C.C.§ 67). At the same interval of entrance, there can be a distinction in the direction of entrance, for example, an entrance at the upper or lower fifth, at the upper or lower octave, etc. We will designate the direction of entrance with an arrow, placing it before the number of the interval of entrance. Thus, the expression \( \downarrow 1 \) means an entrance of

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33 At § 67 of *Convertible Counterpoint in the Strict Style*, Taneev establishes the possibility of measuring the interval of entrance by using the initial notes of two voices that do not enter simultaneously. (Taneev, *Convertible Counterpoint*, § 67, p. 48.) (P.G.)
the Risposta at an upper second, and \( \mathcal{V}_1 \), at a lower second; \( \mathcal{V}_2 \), at an upper third, \( \mathcal{V}_2 \), at a lower third; etc.\(^3\) At an entrance at the prime, the direction of entrance can be distinguished in precisely the same way (\( \mathcal{V}_0 \) and \( \mathcal{V}_0 \)), depending on whether the Risposta is the upper or lower voice. If a distinction in direction is not needed in the indication, then the arrow before the number will be placed horizontally: \( \rightarrow 4 \) (an entrance at the fifth), \( \rightarrow 9 \) (an entrance at the dezime), without a distinction in direction.

The interval and direction of entrance are indications by which two-voice canons are divided. Thus, canons are distinguished: at the prime (\( \rightarrow 0 \)), at the upper or lower second (\( \mathcal{V}_1, \mathcal{V}_1 \)), at the upper or lower third (\( \mathcal{V}_2, \mathcal{V}_2 \)), etc.

§ 3. We will cite a number of examples of two-voice canonic imitation. The sign * has been set at the place where a voice ceases to take part in imitation and shifts to free counterpoint.

\[^3\] For information about Taneev's system for numeration, see the sixteenth footnote of Introduction to Part I in this dissertation. (P.G.)
§ 4. In the following example, there is no pause before the entrance of the Risposta, and the Risposta forms a direct continuation of the preceding melody.

35 Josquin des Pres, "Pleni sunt" from Missa: Pange Lingua, mm. 32-41. (P.G.)
36 Giovanni Pierluigi da Palestrina, "Gloria" from Missa: O Regem Coeli, mm. 32-6. (P.G.)
§ 5. In this example:

both the Proposta and Risposta have not been separated by pauses from the preceding motion of the voice.

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37 Giovanni Pierluigi da Palestrina, "Credo" from Missa: Vitute magna, mm. 186-92. (P.G.)

38 Giovanni Pierluigi da Palestrina, "Gloria" from Missa: De Beata Virgine, mm. 40-6. (P.G.)
§ 6. In this example:

No. 5.\textsuperscript{19}

P.XI, 46/47.

\begin{center}
\includegraphics[width=\textwidth]{example.png}
\end{center}

the Risposta, which also has not been separated by a pause, has its first note lengthened: it is a whole note instead of a half note. This alteration of the length of the first note should not lead to error regarding the real distance of entrance. For a determination of the latter, the length of the initial note must be represented accurately. Consequently, in the example cited above, the distance of entrance of the Risposta is equal to a half measure. The following example presents the opposite situation—a shortening of the length of the first note of the Risposta.

\textsuperscript{19} Giovanni Pierluigi da Palestrina, "Credo" from Missa: Sine nomine, mm. 67-72. (P.G.)
No. 6.\textsuperscript{40}

Joannes Vannius

(G.D.266)

\begin{music}
\begin{staff}
\begin{ruler}
\end{ruler}
\begin{clef}
\begin{ruler}
\end{ruler}
\end{clef}
\end{staff}
\end{music}

§ 7. In the third measure of the following example, it is necessary to note the rare case of the appearance of a diminished fifth on the first strong beat.

No. 7.\textsuperscript{41}

P.XIV, 8.

\begin{music}
\begin{staff}
\begin{ruler}
\end{ruler}
\begin{clef}
\begin{ruler}
\end{ruler}
\end{clef}
\end{staff}
\end{music}

§ 8. In the previous examples, the distance of entrance of the Risposta was equal to either a measure or to a half

\textsuperscript{40} Joannes Vannius, "Attendite popule meus." mm. 146-53. (P.G.)

\textsuperscript{41} Giovanni Pierluigi da Palestrina, "Sanctus" from Missa: Aeterna Christi munera, mm. 1-4. (P.G.)
measure. In the next example, it is equal to a quarter of a measure.

No. 8.\textsuperscript{42}

P.XIV,28.

\begin{center}
\begin{tikzpicture}
\draw[thick] (0,0) -- (5,0);
\draw[thick] (0,-1) -- (5,-1);
\draw[thick] (0,0) -- (0,-1);
\draw[thick] (0,-1/2) -- (0,-1);
\draw[thick] (0,-1/2) -- (2,-1/2);
\draw[thick] (0,-1) -- (2,-1);
\draw[thick] (2,-1/2) -- (2,-1);
\draw[thick] (2,-1/2) -- (4,-1/2);
\draw[thick] (2,-1) -- (4,-1);
\end{tikzpicture}
\end{center}

Such close canonic imitation was called at an earlier time "fuga ad minimam." The name fugue in the fifteenth and sixteenth centuries indicated what we now call canon.\textsuperscript{43} "Minima" is the name of the note corresponding to our half note.

§ 9. The distance of entrance in the following two examples exceeds the previous. In the first, it is equal to a measure and a half; in the second, to two measures.

\textsuperscript{42} Giovanni Pierluigi da Palestrina, "Sanctus" from Missa: Jam Christus astra ascenderat, mm. 49-52. (P.G.)

\textsuperscript{43} The musicologist Imogene Horsley concurs and adds: "It was not until the eighteenth century that 'canon' was used universally to mean what it does today—the strict and continuous imitation of a leading part by one or more following parts, usually at fixed intervals of time and pitch." (Imogene Horsley, Fugue: History and Practice (New York, 1966), p. 6.) (P.G.)
No. 9.  
Joannes Mouton
(G.D.255)

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No. 10.  
Joannes Mouton
(G.D.256)

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44 Joannes Mouton, "Domine salvum fac Regem," mm. 61-6. In this example, instead of the labels "R" and "R" for the two voices, which are present in the original text, "P" and "R" have been added in the translation. (P.G.)

45 Joannes Mouton, "Domine salvum fac Regem," mm. 1-8. The arrows in the sixth measure indicate the presence of a hidden fifth—a fifth approached in the same direction by both voices. (P.G.)
In the sixth measure of the last example, we note the hidden fifths, which are quite often encountered in two-voice counterpoint by Netherland composers.

§ 10. The previous examples presented imitation at the fourth and fifth and did not include a crossing of the voices. But in imitations with a smaller interval of entrance, crossing becomes a common occurrence. To try to avoid it, for example, in imitation at the prime, would mean the abandonment of any freedom of voice leading. More or less prolonged imitation at the prime without crossing would have as a necessary result the movement of voices in a single direction. So, for example, if the Proposta is the upper voice, then, in order to remain the upper voice at the entrance of the Risposta at the prime, the Proposta will have to shift to a note above its beginning. When the Risposta approaches this note, then the Proposta will have to again move upwards, etc. For example:

No. 11.

If the Proposta had been the lower voice, then to avoid crossing both voices would have moved downwards in the same way. For this reason, crossing of voices in imitation at
close intervals should not only [not] be prohibited, but more likely prescribed. The next example—the beginning of a canon at the prime—contains numerous crossings of voices.

No. 12.\textsuperscript{46}
de Orto
(G.D.279)

§ 11. Further examples do not require explanations.

\textsuperscript{46} Marbriand de Orto, "Agnus Dei II" from Missa: \textit{J'ay pris amour}, mm. 1-16. (P.G.)
No. 13.  

P.XII, 32.

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No. 14.  

P.XII, 35.

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47 Giovanni Pierluigi da Palestrina, "Gloria" from Missa: Primi Toni, mm. 98-107. (P.G.)

48 Giovanni Pierluigi da Palestrina, "Credo" from Missa: Primi Toni, mm. 26-31. (P.G.)
No. 15.\textsuperscript{49}  
P.XIII, 17.

\begin{center}
\includegraphics{music1}
\end{center}

No. 16.\textsuperscript{50}  
P.XIII, 31.

\begin{center}
\includegraphics{music2}
\end{center}

\textsuperscript{49} Giovanni Pierluigi da Palestrina. "Gloria" from Missa: Secunda [\textit{Primi Toni}], mm. 20-3. (P.G.)

\textsuperscript{50} Giovanni Pierluigi da Palestrina. "Gloria" from Missa: Tertia [\textit{Jesu Nostra Redemptio}], mm. 1-6. (P.G.)
No. 17.\textsuperscript{51}

P.XII, 54.

Giovanni Pierluigi da Palestrina, "Credo" from Missa: Brevis, mm. 1-7. (P.G.)

No. 18.\textsuperscript{52}

P.XIV, 24.

Giovanni Pierluigi da Palestrina, "Credo" from Missa: Jam Christus astra ascenderat, mm. 123-9. (P.G.)
§ 12. Imitation may be a component part in a composition for a greater number of voices--two voices proceeding in canon, the others not in imitation. We will conclude this collection of examples with a short two-voice canonic imitation at the prime, which forms a part of the four-voice "Misere" from the mass De nostra Domina by Josquin. We cite this small selection in its entirety.
§ 13. In all of the examples that have been cited, the Proposta, and after it even the Risposta, does not return to its beginning and does not repeat again. They either transfer to nonimitative counterpoint, most often forming a cadence, or by turn they fall silent, as in Example 15. Such canoncic imitation carries the name finite, in contrast to infinite canonic imitation, where the Proposta and the

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54 Josquin des Pres. "Credo" from Missa: De nostra Domina, mm. 118-134. (P.G.)
Risposta return to their beginning and may be repeated an arbitrary number of times, as is evident in the following example of an infinite canon on a theme by Palestrina.\textsuperscript{55}

No. 21.\textsuperscript{56}

P.XIV,77.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{canon_example.png}
\caption{Example of an infinite canon on a theme by Palestrina.}
\end{figure}

§ 14. The proper term canon has been accepted to be applied to imitation only in that instance when it forms the most independent, complete composition, even if it is of a small size. From this viewpoint, only canonic imitation that ends in a cadence can be called a finite canon. However, in the study of canonic forms, this distinction is not essential. The cadence that concludes the canon presents a feature of secondary importance. It is written in simple counterpoint and turns out to have no influence on the methods of composing the canon itself. In a canon, canonic imitation is the most important part, and whether it will

\textsuperscript{55} Taneev was not alone in his use of infinite canons as a pedagogical tool. Ludwig Bussler also published examples of such canons in Der Strenge Satz, pp. 200-2. (P.G.)

\textsuperscript{56} Giovanni Pierluigi da Palestrina, "Credo" from Missa: Nigra sum, mm. 30-32. (P.G.)
conclude with a cadence or its voices do not cadence and take part in the overall course of the composition is immaterial. Having applied the usual viewpoint, which requires a canon to be a complete whole, we would narrow to an extreme the field of examples by which we may be guided.

You will find relatively few complete canons in the music of the masters of the strict style. Requiring this completeness, we would have to turn from real musical literature to stillborn inventions by theorists who wrote canons often of very dubious musical merit. On the contrary, not applying these requirements and being content with canonic imitations, even if they form a part of another composition and appear only periodically, we have at our disposal a large supply of models that have been included in the compositions of the great masters of counterpoint. Only then is it made clear how significant is the role of both two-voice and multi-voice canon in their compositions, with what ease they used canonic forms, how far from exhausting them, as will be confirmed. Canonic imitation and its employment are the foundation of moveable counterpoint, the mutual connection with which will be made clear in this work; they form an inalienable characteristic of the technique of contrapuntal writing, and without a clear understanding of them many facets of this technique will remain incomprehensible.
§ 15. The method of writing two-voice finite canons consists of the following. Having written a section of the Proposta up to the point of entrance of the Risposta, and having transferred this section to the latter, one continues the Proposta in counterpoint to the Risposta, and this counterpoint will again be shifted to the Risposta, continuing in this way until the concluding cadence. We will designate the order of sections that form the Proposta with the letters A, B, C, etc., and the order of the same sections in the Risposta with the letters A/, B/, C/, etc. Since the length of each section is equal to the distance between the beginning of the Proposta [and the beginning of the Risposta,] then a canon can be depicted schematically as:

(P) A B C D E etc.
(R) A/ B/ C/ D/ E/ etc.

or as follows if the Risposta enters from above:

(R) A/ B/ C/ etc.
(P) A B C etc.

Consequently, the way to write a canon is the same. A is written and is transferred to the Risposta with a shift by the required interval, forming section A/. Then counterpoint is written to this section (the combination A/+B), B is
shifted to the Risposta, and the combination B/+C is written, etc.\(^57\)

No. 22.\(^58\)
Josquin
(Publ. VI, 80)

§ 16. In such a canon, each combination of sections appears only once (A.+B, B/+C, C.+D, etc.), and as a result of this, between the combinations of sections the relationship of the original to the derivative is absent. For this reason, finite two-voice canons do not require the application of vertical-shifting counterpoint. And this occurs not as a result of the accidental circumstance that

\(^{57}\) The lack of control over the musical content of the Proposta in this sectional approach caused Evgeny Korchinsky to create his melodic theory of canon. Korchinsky's developments are discussed in Chapter 19 of this dissertation. (P.G.)

\(^{58}\) (Publ. VI, 80) refers to a collection of selected works published by Franz Commer. Volume VI is a publication of older practical and theoretical musical works issued by the Society for Musicology, Berlin 1877. Cited by Andreas Wehrmeyer in Sergej Taneev, *Die Lehre vom Kanon*, trans. Andreas Wehrmeyer (Berlin: Ernst Kuhn, 1994), p. 15. (P.G.)
Jv=0 is used in the given example, since this will be repeatedly encountered in subsequent canons. Here the absence of complex counterpoint is due to the very form of the canon—a characteristic setting finite two-voice canons apart into a special class, completely belonging to the sphere of simple counterpoint.\(^5\)

§ 17. By confirming that two-voice finite canons do not require the application of vertical-shifting counterpoint, we do not want to say that they do not allow its application. Two-voice canonic imitation, as a special type of two-voice counterpoint, may be subjected to the conditions of any complex counterpoint. As a result of this, there appears the possibility to obtain from one imitation a derivative

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\(^5\) Taneev's first description of Jv, the index of vertical-shifting counterpoint, occurs at § 27 of Convertible Counterpoint in the Strict Style. At this point, Taneev states: "The algebraic sum \(v\) (the letter \(v\) for 'vertical' (plural \(vv\)) is equal to the size of the vertical shift of a voice in the derivative combination) of two voices contrapuntally united is termed the index of vertical-shifting counterpoint, and is indicated by \(Jv\) (plural \(JJv\)), \(J\) standing for 'index' and \(v\) for 'vertical shift.' In distinction to the sign \(v\), referring to the individual voice (§ 21), the sign \(Jv\) can refer only to the combination of two voices." (Taneiev, Convertible Counterpoint, § 27, p. 37.)

The mathematical equations that reveal values of \(JJv\) are presented in Chapters 2 and 3 of this dissertation. For the present, it is sufficient to note that when Taneev writes about "Jv=0" and an "absence of complex counterpoint" he is stating that the interval of entrance between the two voices in the original combination is equal to the interval of entrance between the two voices in the derivative. If these values are not equal, then a vertical shift has occurred and complex counterpoint is present.

In the Russian text, Taneev uses \(Iv\) instead of \(Jv\) as an abbreviation for "Index Verticalis." In his translation, Brower substituted \(Jv\) for \(Iv\) in order to avoid confusion with Voice I. Brower's substitution is followed in this translation. This substitution is noted in Ellon D. Carpenter, "The Theory of Music in Russia and the Soviet Union, ca. 1650-1950" (Ph.D. diss., University of Pennsylvania, 1988), p. 447. (P.G.)
combination—a requirement placed on it from without and not arising from the very nature of its form. The insufficient distinction of these two concepts—to require and to allow—probably gave cause to Marx to mistakenly claim that two-voice canon must be written in double counterpoint at the octave (Kompositionslehre). Meanwhile, with the exception of two-voice infinite canon at the octave, no sort of two-voice canon must be written in double counterpoint at the octave, though any two-voice canon may be written in any complex counterpoint.

§ 18. The two-voice finite canon quite often plays the roll of a cantus firmus in multi-voice works. This is the case in Palestrina’s mass Repleatur os meum (Volume XII of the complete collection of his works, page 105), which contains several five-voice selections with two-voice canons at the intervals 7, 6, 5, 4, 3, 2, 1. The two-voice canon in the first Agnus Dei of this mass was written in augmentation, and in the second Agnus Dei, a three-voice canon performs the roll of a cantus firmus. When forming a part of a multi-voice composition, the two-voice canon is not obliged to be written in strict two-voice counterpoint: in this, on a

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60 The quotation by Marx that expresses this view is as follows: "All of these types of canon [i.e., two-voice canons at various intervals] have in common the following, the line of the upper voice will become the lower voice and the lower voice will become the upper voice, that is, both voices shall become inverted. They must be composed following the rules of counterpoint at the octave." Adolph Bernhard Marx, Die Lehre von der musikalischen Komposition (Leipzig, 1856), 2:447-8. Cited in Andreas Wehrmeyer, Sergej Taneev, Die Lehre vom Kanon, p. 16-7. (P.G.)
general basis, hidden successions and the free use of the fourth may be permitted. For example:

No. 23.

These two voices have been extracted from the five-voice "Kyrie" of the aforementioned mass. The fourth, which is encountered in the fifth measure, enters freely due to the supplementary note of the bass voice. In other masses by Palestrina, a canon in the accompaniment of free voices is quite often met in the closing section of the mass: the Agnus Dei. (See the Missa: Brevis in Volume XII, the Missa: Tertia in Volume XIII, and others.)

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61 Giovanni Pierluigi da Palestrina, "Kyrie" from Missa: Repleatur os meum, mm. 7-13. (P.G.)
Chapter 2

The Infinite Canon (general concepts)

§ 19. In the infinite canon, after the entrance of the Risposta, the Proposta returns to the beginning and repeats once again. Imitating it, the Risposta also repeats. In musical examples, we will separate these repetitions from the preceding with the sign //, and repetitions of the Proposta and Risposta will be indicated //P and //R. An example of an infinite canon:

No. 24. 62

See P.XIV,2.

Beginning from the point of entrance of //R, we have a repetition of a previous combination of voices. Therefore, this example could have been written in short by having enclosed the second and third measures in repeat signs. And since the theme does not begin with a full measure, we will

62 This passage does not occur in the portions of the Kyrie and Gloria from Missa: Aeterna Christi munera, which appear on page two of the fourteenth volume of the Breitkopf edition of Palestrina’s complete works. (P.G.)
add the missing half note in brackets on the strong beat at the return of the Risposta, at the beginning of the second measure.

No. 25.

§ 20. Each two-voice infinite canon can begin with either the upper or the lower voice, in other words, its Proposta can become the Risposta and vice versa. We will begin the previous canon with the lower voice:

No. 26.

Each two-voice infinite canon, consequently, is a canon at both the upper and lower interval, depending on whether we will begin it with the lower or upper voice.
§ 21. The previous canon can be presented schematically in both of its aspects:

Schema of distances and intervals of entrance

It is not necessary to write out \( \text{//R} \) since, beginning from the point of entrance of \( \text{//R} \), a previous combination of voices repeats (§ 19). In the schema that has been cited, it is possible to see the intervals of entrance and a likeness of both distances. Dashes, which are located on the first line of the staff, indicate the latter.

The distance between consecutive dashes will be considered equal to a single section. The length of this section does not have significance for the form of the canon. The following schema has a more general character. From it, the latter of the enumerated signs that characterize a given canon is visible: an equality of distances of entrance.

Schema of distances of entrance

We pause on these schematic depictions of the form of canon because with their help, particularly with regard to
multi-voice canons, they simplify to an extreme the search for rules according to which a canon must be written.

§ 22. Depending on a similarity or difference between both distances of entrance, infinite two-voice canons are divided into two orders. Canons where both distances are alike belong to the first order and, as we will see later, are written according to the rules of vertical-shifting counterpoint. Canons in which these entrances are different belong to the second order and require the application of horizontal-shifting counterpoint. All aforementioned infinite canons belong to the first order. The following canon belongs to the second order:

No. 27.

The distance between the Proposta and the Risposta is equal to a whole note, between the Risposta and //P it is three times greater. A schematic depiction of this canon is as follows:

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63 In order to correspond to the sentence that follows, the term "vertical-shifting" has been underlined in the translation. (P.G.)
a) Schema of distances and intervals of entrance

\[
\begin{array}{cc}
    & P \quad R \quad /P \\
\end{array}
\]

b) Schema of distances of entrances

\[
\begin{array}{cc}
    & P \quad R \quad /P \\
\end{array}
\]

On the basis of what was said at § 20, this canon can begin with the lower voice:

No. 28.

\[
\begin{array}{cc}
    & P \quad R \\
\end{array}
\]

and then its schema will be as follows:

a)

\[
\begin{array}{cc}
    & P \quad R \quad /P \\
\end{array}
\]

b)

\[
\begin{array}{cc}
    & P \quad R \quad /P \\
\end{array}
\]
§ 23. The infinite canon of both the first and second order allows the following subdivisions: a) The Proposta repeats on the same degrees—the infinite canon in the proper sense, to which all of the previous examples belong; b) The Proposta, and after it even the Risposta, at each repetition is shifted a certain number of degrees up or down. As a result, a canonic sequence is formed. For example:

a) Canonic sequence of the first order:

No. 29.\(^64\)

P.XII,32.

\[ \text{etc.} \]

b) Canonic sequence of the second order:

No. 30.\(^65\)

P.XIV,22.

\[ \text{etc.} \]

\(^{64}\) Giovanni Pierluigi da Palestrina, "Gloria" from Missa: Primi Toni, mm. 77-9. (P.G.)

\(^{65}\) Giovanni Pierluigi da Palestrina, "Credo" from Missa: Jam Christus astra ascenderat, mm. 57-9. (P.G.)
In the last example, the Proposta and Risposta return to the beginning after two and one half measures. Therefore, in the repetition, both voices have been moved from one part of the measure to another, and only at its appearance a third time, after five measures, does the melody occur at the same part of the measure as at the beginning.

We shift now to an exposition of rules according to which the aforementioned aspects of two-voice canon are written.
The infinite two-voice canon of the first order with a
Proposta that repeats on the same degrees (the proper
infinite canon of the first order)

The application of vertical-shifting counterpoint

§ 24. Since the distance between the Risposta and //P in
an infinite canon of the first order is the same as the
distance between the Proposta and Risposta, the number of
sections of such a canon is equal to the number of voices, as
is apparent from the following schema of sections:

\[
\begin{array}{c}
\text{(R)} \\
\text{(P)} \\
\end{array} \quad \begin{array}{c}
\text{A/ B/} \\
\text{B A} \\
\end{array}
\]

What has been said pertains even to multi-voice infinite
canons of the first order, i.e., to canons in which all
distances of entrance are identical. Thus, as we will see
subsequently, three-voice infinite canon of the first order
consists of three sections, the four-voice, of four, etc.

§ 25. It is apparent from the previous schema that in
the canon two simultaneous combinations of sections A and B
are encountered:

\[
\begin{array}{c}
\text{A/} \\
\text{B} \\
\end{array} \quad \text{and} \quad \begin{array}{c}
\text{B/} \\
\text{A} \\
\end{array}
\]
At the appearance in canons of such repetitions of sections that coincide among themselves and that are denoted by similar letters, one should examine the first of these combinations as original and the repetition as its derivative, and if there are several repetitions, a Jv for each will be located. In so doing, the Proposta must definitely form a part of each original two-voice combination, since of all the voices of the canon it is the only one that is composed, the other voices, i.e., the Risposta and P, are only its repetition. Therefore, all conditions according to which a canon must be written can be established only with regard to a combination in which the Proposta definitely participates. Any combination of voices or sections of which not one is composed anew and both reproduce that which was composed earlier should be examined as a derivative combination. In order to write a canon correctly, it will be necessary to locate an original for this derivative, to which the appropriate conditions of moveable counterpoint will be applied.

We will now return to the aforementioned schema. Thus, we will use the first combination of sections A and B as the original (A\textsubscript{1}+B) and their second combination (A+B\textsubscript{1}) as the derivative. We will call the interval of entrance the letter m and we will seek the Jv.

As is known (C.C.$\S$ 27), Jv is equal to the algebraic sum of the shift of two voices (in the given situation, to two
Having made a comparison between the original and derivative combinations with regard to the shift of both of their sections, we will see that section $A_1$, which was above in the original combination, has been shifted downwards in the derivative by $m$, i.e., it was shifted by $-m$. In exactly the same way, section $B$ was also shifted by $-m$.\(^6\)

\[
\begin{array}{c}
A_1 \quad (-m) \\
\downarrow \\
B_1 \\
\downarrow \\
A \\
\end{array}
\]

Having added these two values, we obtain a general formula for two-voice infinite canons of the first group, namely, $Jv = -2m$. This is the $Jv$ whose conditions the original combination, $A_1 + B$, must satisfy in order for the canon to turn out correctly.

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\(^6\) This section of *Convertible Counterpoint in the Strict Style* is quoted in its entirety at the fifty-ninth footnote in Chapter I. (P.G.)

\(^7\) Information regarding positive and negative values is provided in *Convertible Counterpoint in the Strict Style* at § 10. Taneiev summarizes this information with the following chart. In this chart, ascending motion is considered positive and descending motion negative for Voice I, and ascending motion is considered negative and descending motion positive for Voice II.

\[
\begin{array}{c|c}
+ & - \\
\hline
I & II \\
- & + \\
\end{array}
\]

With regard to the information in § 25 of *Doctrine of the Canon*, Section A is considered Voice I and Section B is considered Voice II. (Serge Ivanovitch Taneiev, *Convertible Counterpoint in the Strict Style*, trans. G. Ackley Brower (Boston: Bruce Humphries, 1962), § 10, p. 28.) (P.G.)
§ 26. On the basis of the formula $Jv = -2m$, for canon at the octave ($m=7$), $Jv = -14$ is required (double counterpoint at the octave); for canon at the ninth ($m=8$), $Jv = -16$ is required (double counterpoint at the dezime); for canon at the dezime ($m=9$), $Jv = -18$ is required (double counterpoint at the duodezime); etc. The three enumerated aspects of infinite canon—at the octave, ninth, and dezime—thus require the application of the most widely-used $JJv$: $-7$, $-9$, and $-11$, i.e., double counterpoint at the octave, dezime, and duodezime. These indices make these canons fit as an exercise at the very beginning of study of vertical-shifting counterpoint. The composition of these canons is facilitated when the limit of greatest distance between the voices is not

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68 Taneev lists the conditions for these $JJv$ as follows:

$Jv = -7$: $0 \ 1 \ 2 \ \frac{3}{2} \ 4^* \ 5 \ 6 \ 7$

$Jv = -9$: $0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9$

$Jv = -11$: $0 \ 1 \ 2 \ \frac{3}{2} \ 4^* \ 5 \ \frac{6}{2} \ 7 \ 8 \ 9 \ 10 \ 11$

These lists concisely describe the conditions that must be satisfied in an original two-voice combination in order for it to form correct counterpoint in a vertically-shifted derivative. A line above an integer indicates that a note that is tied at a given interval in an upper voice over a note in the lower voice is permitted in the original combination because the union will remain contrapuntally correct in the derivative. A line below an integer indicates that a note that is tied at a given interval in a lower voice below a note in the upper voice is permitted in the original combination because the union will remain contrapuntally correct in the derivative.

With regard to the "$x$" positioned to the right of the tie sign, Taneev states: "If a note with the sign $-$ progresses to a dissonance at the point where a resolution would normally occur, this dissonance must be treated as passing or auxiliary and takes the supplementary sign $x$." As for "$p$" he states: "When a derivative consonance is perfect it will be indicated by a small $p$." (Taneev, Convertible Counterpoint, §§ 134, 140, pp. 90, 92.) (P.G.)
7>, 9>, and 11>, but at an interval an octave greater: 14>, 16>, and 18>. As a consequence, in the canon (7) at § 13, and in the canon (7) at § 19 the voices could have move away from each other unimpeded at the dezime. 69

69 Taneev classifies the interaction of contrapuntal lines in vertical-shifting counterpoint as follows: direct shift, inverse shift, and mixed shift (C.C.§ 23). In the direct shift, Voice I, the upper voice, and Voice II, the lower voice, do not cross, i.e., the notes of Voice I remain above the notes of Voice II in the derivative combination. In the inverse shift, the melodies switch position in the derivative combination, i.e., the notes of Voice II are positioned above the notes of Voice I. This is often referred to as double counterpoint. In this instance, once the voices have crossed in a derivative combination, all of the notes of Voice II must remain above the notes of Voice I in order to be called an inverse shift. A mixed shift permits a crossing of voices, i.e., it is partially a direct shift and partially an indirect shift.

As a means of preventing a crossing of voices in a direct shift, Taneev introduces the concept of a "limiting interval for approaching voices of the original combination" (designated by the sign <) (C.C.§ 38). This sign is used only with the direct shift and indicates the limit of proximity of voices in an original combination that will insure that the voices will not cross in the derivative. This concept is demonstrated at Example 2.1.

Original Intervals

\[ Jv = -2 \]

\[ \begin{align*}
\text{\textless} & & \text{\textless} & & \text{\textless} & & \text{\textless} \\
0 & & 1 & & 2 & & 3 & & \text{etc.}
\end{align*} \]

Example 2.1. Demonstration of the limiting interval for approaching voices at \( Jv = -2, 2\text{<} \).

In the example cited above where \( Jv = -2 \), a direct shift is maintained in the derivative combination as long as the voices in the original combination do not approach closer than a third (2<). The "limiting interval for receding voices of the original combination" (designated by the sign >) applies to the indirect shift and indicates how far voices may recede from one another in an original combination in order that they will continue not to cross in the derivative (C.C.§ 39). This concept is demonstrated at Example 2.2.
§ 27. We will cite examples of canons that require the application of the aforementioned JJv.

No. 31.

Example 2.2. Demonstration of the limiting interval for receding voices at JJv=7, 7>.

With regard to Taneiev's statement at § 26 of Doctrine of the Canon, the benefits that arise by having limiting intervals of 14>, 16>, and 18>, instead of 7>, 9>, and 11>, can now be appreciated: the range of the voices may be expanded while simpler JJv continue to be employed. (Taneiev, Convertible Counterpoint, §§ 23, 38-9, pp. 34-5, 38-9.) (P.G.)
§ 28. We will cite three examples of canons that require the application of a less widely-used JJv.
This Jv was seldomly used due to its limited range. However, the octave equivalent, Jv=-11, is far more common. (P.G.)
§ 29. The problem could be in reverse: locate the interval of entrance according to a given \( Jv \). Since \( Jv = -2m \) (§ 25), then in order to obtain the interval of entrance it will be necessary to divide the given \( Jv \) by 2. It is understood that this is suitable only for even numbers. If the \( Jv \) is an odd number, then one should add 7 to its absolute value (C.C. § 47) and, having obtained an even number, divide by 2. \(^7\) We will assume that it will be necessary to locate an interval of entrance for a canon that

\(^7\) With regard to the relationship of \( JJv \) separated by an octave, Taneiev states the following at § 47 of Convertible Counterpoint in the Strict Style: "It is obvious that anything conforming to the conditions of simple counterpoint (i.e., a combination at \( Jv = 0 \)) would also be correct for \( Jv = 7 \), so this index does not require special rules. Similarly, to separate a derivative combination at \( Jv = 1 \) gives a derivative at \( Jv = 8 \).

A combination at \( Jv = 2 \) will serve equally well for \( Jv = 9 \), one at \( Jv = 3 \) for \( Jv = 10 \); the same relation holds between \( Jv = 4 \) and \( Jv = 11 \), etc. It is therefore unnecessary to formulate rules for positive indices equal to compound intervals; those equal to the corresponding simple intervals can be used instead." (Taneiev, Convertible Counterpoint, § 47, p. 41.) (P.G.)
requires \( Jv = -13 \). Having added 7 to 13, we will obtain 20.
Half of this number is the unknown interval: \( \text{m}=10 \), i.e., the
Risposta must enter at the undezime. What has been stated
here pertains to \( JJv \) that have an inverse shift, which we, up
to now, have applied to infinite canons.

§ 30. It does not matter which of the two voices will
begin the infinite canon of the first order. If one of the
aforementioned canons were to begin with the other voice,
i.e., the Risposta is taken as the Proposta and vice versa,
then the combination of sections A and B, which earlier was
derivative, will turn into an original, and the former
original will turn into a derivative. For example, [canon
No. 35 (Example No. 37) will begin here with the lower
voice]:

No. 37.

Since all canons that have been examined here have been
written at a \( Jv \) with an inverse shift, then, having begun the
canon with the other voice, i.e., having made a substitution
of an original for a derivative, we will cause no sort of change to the \( Jv \) (C.C. § 44).\(^\text{72}\)

§ 31. If \( m=0 \) will be taken, i.e., the Risposta will enter at the prime, then \( Jv \) also will equal zero, i.e., it is necessary to write the original combination in simple counterpoint and it will be identical to the derivative. For example:

No. 38.\(^\text{73}\)
Josquin (G.D.204)

\[
\begin{array}{c}
\text{No. 38.}\quad \text{Josquin (G.D.204)} \\
\end{array}
\]

\[
\begin{array}{c}
\text{Josquin des Pres. "Agnus Dei II" from Missa: Mater Patris, mm. 1-6. (P.G.)}
\end{array}
\]

\(^{72}\) Taneiev states the following at § 44 of Convertible Counterpoint in the Strict Style: "It is possible to regard every recurrence of a two-voice combination on the same degrees or its removal to other degrees as a shift at \( Jv=0 \), and therefore simple counterpoint can be understood as a special case of the vertical shift." (Taneiev, Convertible Counterpoint, § 44, p. 40.) (P.G.)

\(^{73}\) Josquin des Pres, "Agnus Dei II" from Missa: Mater Patris, mm. 1-6. (P.G.)
In both examples, the combination $A+B$ literally reproduces the combination $A\mathbb{V}+B$ on the same degrees. We will note the difference between the application of simple counterpoint in two-voice finite canons (§ 16) and the application of it in infinite canons examined here. There the canon is written in simple counterpoint because the very form of the finite canon does not require the application of complex counterpoint. Here the application of the latter is required by the form of the infinite canon, and simple counterpoint is only a particular type of complex on account of $Jv=0$. In the study of multi-voice canons, it will be repeatedly necessary for us to resort to $Jv=0$, which unites vertical-shifting counterpoint to simple and thereby facilitates the composition of canon.

§ 32. From canons at the prime, we will proceed to other canons with a direct shift. Here, as in previous

---

74 Antoine Brumel, "Benedictus," mm. 16-19. (P.G.)
canons, we will distribute the Proposta and Risposta on a relative pitch that conforms to their voices, placing on the upper line that which is comparatively higher. It is already known that a negative $Jv$ provides a direct shift in the instance when intervals smaller than the absolute value of the index are absent in the original combination. This absolute value even serves as the limiting interval of approach for the voices (C.C.§ 38). We will take $m=1$, i.e., a canon at the second. According to the formula at § 25, $Jv=-2$ in this canon. Thus, in order to obtain a direct shift, the third is the limit of approach for the voices.

For example:

No. 40.

---

75 Taneiev states the following at § 38 of Convertible Counterpoint in the Strict Style: "A successive series of original intervals for a negative $Jv$, in order to yield the direct shift, must start with an interval equal to the absolute value of $Jv$. Therefore, at the direct shift, an interval equal to the absolute value of the index is the limiting interval for approaching voices of the original combination; it is indicated by the sign $<$." For further clarification, see the eighth footnote of this chapter. (Taneiev, Convertible Counterpoint, § 38, p. 38.) (P.G.)
If the original and derivative combination are written out separately, having distributed melodies to voices that conform to their relative pitch, then it will become apparent that the original combination must be written at $Jv=-2$.\(^{76}\)

This is true also for canons written at other intervals with $JJv$ that have a direct shift. When $m=2$ (canon at the third), $Jv=-4$ is required; at $m=3$ (canon at the fourth), $Jv=-6$. Canons at larger intervals are awkward to write at a $Jv$ with a direct shift since this would require the voices to move too far away from one another. For example, in a canon at the fifth ($Jv=-8$), it would not be possible to bring voices nearer than a ninth. At odd $JJv=-1$, -3, and -5, it is not possible to write canons with a direct shift.

§ 33. If a canon that has been written at a negative $Jv$ with a direct shift will begin with the other voice, having taken the Proposta as the Risposta and vice versa, then in the original combination (which was previously derivative)

\(^{76}\) In the second example, the letter $v$ means vertical and refers to the vertical shift of a voice (for further clarification, see the fifty-ninth footnote in Chapter 1). In the schema, instead of "I $v=1$" and "II $v=1$", which are present in the original text, "I $v=-1$" and "II $v=-1$" have been added in the translation. These corrections are cited by Wehmeyer in Sergej Taneev, *Die Lehre vom Kanon*, trans. Andreas Wehmeyer (Berlin: Ernst Kuhn, 1994), p. 195. (P.G.)
negative intervals will result.\footnote{77} We will begin the previous
canon with the upper voice:

No. 41.

One should consider these negative intervals of the
original combination as positive and write the latter
according to the rules of a Jv whose sign has been changed to
the opposite. Having applied this rule to the previous
canon, and having considered the negative intervals as
positive, we must then write an original combination at a
positive Jv, namely, at Jv=2, as is apparent from the
following example:

\footnote{77 Taneiev considers these intervals as negative because "the
lowest tone belongs to the upper voice and the highest tone to the lower
voice." Thus, intervals that occur during voice crossings are measured
with negative intervals. (Taneiev, Convertible Counterpoint, § 4, p.
26.) (P.G.)}
Thus, in order to write an original combination in canon according to the rules of a positive Jv, one should write negative intervals in this combination, considering them as positive. By means of this, a possibility to use infinite canons as material for exercises, not only at negative, but also at positive, even indices is revealed.

§ 34. The section on infinite canons that has been examined has significance primarily for pedagogy. At the very beginning of study of vertical-shifting counterpoint, it is possible to begin exercises in infinite canon, applying to them the knowledge that has been acquired in this counterpoint. At the same time, these exercises present a natural passage to the application of moveable counterpoint in three-voice canons, which is a more difficult task. But the significance of infinite canons is exhausted by pedagogy. Their application in composition is very slight. The very repetition in succession of the same melody by the same voice imparts a sense of monotony to this canon, and only in the case of a rare exception is it desirable in a musical work. For example, in the composition of a fugue, the composer tries not to place the theme twice in succession in the same voice on the same degree, but where this is unavoidable, usually one repetition is separated from another by a prolonged episode of variable length. In the infinite canon, there is no place for an episode and, in general, means to disrupt the characteristic monotony of this canon are absent.
The canonic sequence, to which we now turn, has a comparably greater application in composition.
Chapter 3

The infinite two-voice canon of the first order with a Proposta that shifts to other degrees in an ascending or descending order (canonic sequence of the first order)

§ 35. We will examine the first example of canonic sequence of the first order that was cited at § 23.

No. 42.

In this canon, the intervals of entrance are as follows:

The original has been indicated by the letter m, and the derivative, by the letter n. The direction of these intervals is contrasting: m is upwards and n is downwards. The third interval of entrance has not been noted by a letter since it is identical to the first. R enters at this interval and, from the point of its entrance, it repeats on a degree below a combination of sections that earlier was at
the beginning \((A/\!+\!B)\). As a result of the sameness of this combination to an original, it is not necessary to examine it as a separate derivative. Two initial combinations of sections remain \((A/\!+\!B\) and \(A\!+\!B/\)\), which are with regard to one another an original and derivative. The \(Jv\) can be found in the previous way, having taken the algebraic sum of the shift of sections \(A/\) and \(B\). The difference in their movement compared to a canon where the Proposta is not shifted to other degrees is that in the latter the size of both intervals of entrance is identical. As a result of this, the algebraic sum of the shift of both sections equals \(-2m\). In the canonic sequence, the intervals of entrance are different. While section \(B\) shifts by \(-m\), section \(A/\) shifts by \(-n\), which produces the result \(Jv=\!-m\!-\!n\), as is apparent from the following schema. In this schema, the shift of section \(A/\) upwards has been indicated in the derivative combination by \(A\), which has been positioned above:

\[
Jv = -m-n
\]

(R)

\[
\begin{array}{c}
\text{(R)}
\end{array}
\]

(P)

\[
\begin{array}{c}
\text{(P)}
\end{array}
\]
In this schema, the Proposta shifts in a direction identical to the direction of the first interval of entrance. The formula cited above will remain unchanged even in the situation where the shift of the Proposta has occurred in a contrasting direction to \( m \), namely:

\[
\begin{align*}
A & \quad B \\
A' & \quad B' \\
\vdots & \quad \vdots
\end{align*}
\]

On the basis of this formula, \( Jv = -3 - 4 = -7 \) for the canon that has been cited in this section, i.e., the original combination must be written in double counterpoint at the octave.

§ 36. The formula \( Jv = -m - n \) is common to all infinite two-voice canons of the first order where the intervals of entrance have a contrasting direction. The previous formula for the infinite canon, where the Proposta repeats on the same degrees (\( Jv = -2m \)), is only a particular case of the formula \( Jv = -m - n \), where \( n = m \), and therefore the infinite canon in the proper sense may be examined as a particular type of canonic sequence. We will examine what sort of change would be introduced to the formula \( Jv = -m - n \) if the direction of both intervals of entrance was identical. In the previous schema, if section \( A' \) were to shift in the same direction as section
B, i.e., upwards, then this shift of A as an upper voice must be indicated by a plus instead of a minus and then, consequently, the algebraic sum of the shift is $J_v = -m+n$.

§ 37. Thus, when both intervals of entrance are in a contrasting direction, the formula for the index is the following:

a) $J_v = -m-n$,

and when their directions are identical it is the following:

b) $J_v = -m+n$.

The combination of these formulas provides a general formula for all canonic sequences:

$J_v = -m + (+/-n)$.

§ 38. Canonic sequences that have been written according to formula b) are encountered relatively rarely. This formula can be applied in that situation when intervals of entrance are not large and the voices do not move away from each other at large intervals; otherwise, the Proposta and the Risposta will stand too far from their repetitions, and their melodies acquire a range that is too large, which hinders their repetition on other degrees. In the example:
the intervals of entrance are as follows:

their directions are identical, consequently, \( Jv = -1 + 3 = 2 \).

Despite that these intervals are not large and the voices have been arranged fairly close together (not more than an octave in the original combination); nevertheless, both the range of the melody and the interval (a fifth) at which it shifts in each individual voice makes this melody awkward for shifting. For example, at the first repetition, the upper voice (\( / / R \)) already goes too high and causes this example to be unfit as a sequence. In general, canonic sequences at \( Jv = -m + n \), in light of the aforementioned awkwardness, are
rarely applied. The crisscrossing of voices that is encountered in this also contributes to their rarity: crisscrossing that occurs due to the comparatively higher melody at each repetition being positioned invariably above, this makes these sequences more monotonous than canonic sequences that have been written according to the formula $Jv=-m-n$, i.e., sequences in which the intervals of entrance are different among themselves in direction.

§ 39. Of the number of sequences with an identical direction of intervals of entrance, sequences at $Jv=0$ will be encountered more often than others. This index occurs in those situations when intervals of entrance with an identical direction are equal to each other. For example:

No. 44.

![Musical notation image]

The intervals of entrance in this example are as follows:

![Musical notation image]
$Jv = -3 + 3 = 0$. Thus, the derivative combination $(A + B/)$ is identical to the original $(A/ + B)$:

\[
\begin{align*}
\text{No. 45.} & \\
\end{align*}
\]

The intervals of entrance:

\[
\begin{align*}
Jv = 0. \text{ Using the method that has been indicated at § 37 for the search for } Jv, \text{ we obtain the following schemata of the two previous canons:}
\end{align*}
\]

---

78 Taneev is probably referring to Johann Phillipp Kirnberger, Die Kunst des reinen Satzes, 5th ed. (Berlin, 1777), 2:177-81. This same theme is present in the work that he cites in Serge Ivanovitch Taneiev, Convertible Counterpoint in the Strict Style, trans. G. Ackley Brower (Boston: Bruce Humphries, 1962), p. 297) (P.G.)
where the interval that has been positioned to the left of P forms a unison with the Risposta, which indicates that Jv=0.

§ 40. With the help of the two formulas that have been cited, it is possible to write canonic sequences at any Jv, not being limited, as in previous canons, to only even indices. In so doing, the first interval of entrance can be of an arbitrary quantity and direction.\(^79\) By means of this interval, its direction, and the given index, the size and direction of the second interval are determined. We will assume that Jv=-7 and a sixth (m=5) as the first interval of entrance have been given. On the basis of the formula Jv=-m+(+/-n), the value that will be necessary to add to -5 in order to obtain Jv=-7 is the second interval of entrance. This quantity is -2, i.e., a third, and the minus indicates that its direction is in contrast to the first interval of entrance:

\[^79\] One should exclude those cases when, at a negative Jv with an inverse shift, the first interval of entrance is equal to half of the Jv. This provides an infinite canon with P on the same degree as the Proposta. (Note by Taneev)
By means of this, we locate a schema of entrances for any Jv.

Here are examples of entrances:

a) at Jv=-9

![Diagram of entrances at Jv=-9]

b) at Jv=-11

![Diagram of entrances at Jv=-11]

A more complete list of entrances has been cited below in the section on three-voice canons.

§ 41. We will indicate another more obvious method of determination of Jv that consists of the following. To the left of //P is set aside an interval equal to the first interval of entrance but in a contrasting direction. For example:
The first interval of entrance (5) has an upwards direction; the interval that we have set aside has a direction that is downwards from P. The note that we have obtained (indicated •), standing from P at a distance that is identical to the distance between the Proposta and Risposta, rhythmically coincides with the note of entrance of the Risposta. The interval formed by these two notes (9 in the given example) is always equal to the sought-after Jv. The direction at which the note • is situated relative to the note of entrance of the Risposta with which it rhythmically coincides can either be identical to the first interval of entrance or in contrast to it. In the first case, Jv is positive, in the second, negative. In the example that has been cited, these directions are contrasting:

and Jv=-9. In the following example, these directions are identical and Jv is positive:
We will cite several additional cases:

\[
\begin{align*}
J_v &= -11 \quad P \quad R \quad P \\
J_v &= -7 \quad R \quad P \\
J_v &= 3 \quad R \quad P \\
J_v &= +10 \quad R \quad P \\
J_v &= -3 \quad P \quad R \quad P 
\end{align*}
\]

If the note of entrance of the Proposta turns out to be between the dot and the note of entrance of the Risposta, i.e., within the interval that has been formed by them, then there is an inverse shift; if it is outside of this interval, then there is a direct shift. When \( J_v = 0 \), then the note that has been positioned to the left of \( \uparrow \) \( P \) will form a unison with the note of entrance of the Risposta.

\[
\begin{align*}
J_v &= 0 
\end{align*}
\]

The method [of determination of \( J_v \)] that has been indicated is also fit for the infinite canon with a \( P \) that returns to the same degrees, [since this canon] can be examined as a particular type of canonic sequence (§ 36). For example:
As we will see below, by this method we can also determine the point of entrance of an imaginary voice in a canon of the second order. It will also be applied in multi-voice canons.

§ 42. The majority of examples cited below have been taken in part from works by Palestrina and in part from works by composers from the Netherlands that have been cited in Glarean's *Dodecachordon*. Several examples have been borrowed in their entirety, in others, only the theme has been taken.

No. 46.\(^{80}\)

P.XIV, 22.

This example at Jv = -9, which does not contain tied dissonances, allows doublings: I \(d = -9\), II \(d = -9\), II \(d = -2\).\(^{91}\)

Canonic imitations in which direct motion is absent quite

---

\(^{80}\) Giovanni Pierluigi da Palestrina. "Credo" from Missa: Jam Christus astra ascendereat, mm. 57-8. (P.G.)

\(^{81}\) In the original text, "d" is missing from "II \(d = -2\)". This correction is cited in Sergej Taneev, *Die Lehre vom Kanon*, trans. Andreas Wehrmeyer (Berlin: Ernst Kuhn, 1994), p. 195.

The use of the small letter "d" indicates that a voice may be doubled. Taneev discusses doubling in the Introduction to *Convertible Counterpoint in the Strict Style*. At that point, he separates complex counterpoint into three categories based upon methods for obtaining derivative combinations: 1) the shifting of voices, 2) duplications in imperfect consonances, and 3) transmutations (see Taneev, *Convertible Counterpoint*, p. 20). I \(d = -9\) indicates that Voice I may be doubled at a tenth below, II \(d = -9\) indicates that Voice II may be doubled at a tenth above, and II \(d = -2\) indicates that Voice II may be doubled at a third above. (P.G.)
often appear with doublings. The following example also has been written at \( Jv = -9 \):

No. 47.\(^{82}\)

P.XIV,97.

Besides a shift at \( Jv = -9 \), its original combination also permits a shift at \( Jv = -11 \) and \( Jv = -7 \). In such cases, a new sequence can almost always be obtained by applying various \( J^v \). For example:

No. 48.

No. 49.

\(^{82}\) Giovanni Pierluigi da Palestrina, "Kyrie" from Missa: Sicut liliwm inter spinas, mm. 65-7. (P.G.)
These sequences do not contain direct motion and the doublings that have been indicated are allowed in them.

§ 43. We stated that original combinations in canonic sequences that have several $\text{JJv}$ almost always allow the application of these $\text{JJv}$; thus, they provide a possibility to obtain new sequences on the same theme. Indeed, sometimes obstacles to this are met at the point of entrance of $/\text{P}$. For example, this can be seen in the first canonic sequence of the previous section ($\text{Jv}=-9$). Its original combination may also be shifted at $\text{Jv}=-11$.

\[ \begin{array}{c}
R^A & B^A \\
\text{R} & \text{P} & \text{A} & \text{B} \\
\text{P} & \text{R} & \text{A} & \text{B} \\
\end{array} \]

But $\text{Jv}=-11$ requires the following entrances:

\[ \begin{array}{c}
\text{Jv}=-11 \\
\text{R} & \text{P} & \text{A} & \text{B} \\
\text{P} & \text{R} & \text{A} & \text{B} \\
\end{array} \]

Whereas $/\text{P}$ may not enter from this note:

\[ \begin{array}{c}
\text{Jv}=-11 \\
\text{R} & \text{P} & \text{A} & \text{B} \\
\text{P} & \text{R} & \text{A} & \text{B} \\
\end{array} \]

because parallel fifths with the Risposta would occur:

\[ \begin{array}{c}
\text{Not allowed} \\
\text{Not allowed} \\
\text{Not allowed} \\
\end{array} \]
But this circumstance depends on the course of a given melody and presents a relatively rare phenomenon. If at the point of entrance of \( P \) forbidden successions with the previous consonance do not occur, then there are no obstacles to obtain a sequence at the new \( Jv \).

§ 44. From the following example:

No. 50.\(^{83}\)

P.XIV, 63.

Several canonic sequences can be extracted. Besides \( Jv=-9 \), the original combination of this sequence allows shifts at \( JJv=-7, -11, -4 \). At \( Jv=-7 \), the following sequence results:

\(^{83}\) Giovanni Pierluigi da Palestrina, "Benedictus" from Missa: Iste confessor, mm. 15-16. At Example No. 50., a "+" has been added between "I d=-9 + II d=-9". This addition is cited by Wehrmeyer in Die Lehre vom Kanon, p. 195. (P.G.)
The shift at $Jv=-11$ provides a leap of a sixth:

No. 52.

Since this sixth is minor in both voices, then this example is also adequate. The shift at $Jv=-4$ provides the following sequence:

No. 53.

Besides this possibility, there is also the following combination. The original combination simultaneously allows a shift at $Jv=-9$ and a doubling of the lower voice at a lower third: II d=2:
If the doubling of the lower voice is accepted as a voice of the original combination, and as its derivative the shift that has been indicated at \( Jv = -9 \) is taken, then as a result a shift at \( Jv = -13 \) is obtained.\(^{84}\)

The sequence at this \( Jv \) and at the given first interval of entrance (\( m=6 \)) requires the following entrances:

Hence, a new canonic sequence results:

No. 54.

\(^{84}\) In the passage that refers to the given example, all statements in the original text that say that the combination without doubling is at \( Jv = -11 \) have been changed in the translation to say that \( Jv = -9 \). (P.G.)
The number of possible combinations could be increased significantly if the first of the sequences on this theme were to begin with the lower voice instead of the upper, i.e., the Risposta will be placed in the position of the Proposta (§ 20), and to the original combination that is obtained, the same methods of constructing a sequence will be applied once more.

§ 45. In order to show several different combinations that may be extracted from a single theme, even if insignificant, we have intentionally chosen a theme that is so short that the original combination of the sequence consists of only two half notes. But in spite of its brevity and apparent musical insignificance, this theme has a great resemblance in harmony and beauty to a theme of one of the remarkable imitations—the canonic sequence from Ave verum by Mozart. This imitation by Mozart has been written in the free style. It contains an entrance of a seventh by leap on a weak beat and a tritone in the course of the melody at the transition to //P. Nevertheless, due to the melodic construction of its theme, the doubling of both voices in thirds, and the entrance of the second pair of voices on a dissonance, this imitation by Mozart carries the undoubtful imprint of a previous epoch of the strict style, which

85 Wolfgang Amadeus Mozart, Ave verum corpus for mixed choir, strings, and organ Kv. 618, Series 1, Werkgruppe 3, no. 18 (Kassel, 1963), mm. 30-4. (P.G.)
sharply distinguishes it from works of later music and imparts to this imitation inexplicable elegance.

No. 55.

These traces of the epoch of the strict style, so tangible in the technique of Mozart, after him begin to be smoothed down and finally they disappear. The former subtlety of contrapuntal writing relinquishes its place to a more harsh and coarse technique. To return to the perfect writing of Mozart, having combined it with all the improvements of contemporary harmony, to acquire the lost sensitivity to beauty and subtlety of voice leading, this is the ideal to which the contemporary composer should strive, the approach to which can be assisted by the study of the strict style.

§ 46. The infinite canon that has been cited at § 19 has been written at $Jv=-14$. But its original combination also allows a shift at $Jv=-11$. Since the first interval of
entrance is the octave, the second must be a fifth that will be placed in the opposite direction:

\[ \text{P R P} \]

Hence, the following canonic sequence results:

No. 56.\textsuperscript{86}

P.XIV,2.

\[ Jv = -11 \]

§ 47. At § 35, the canonic sequence has been written at Jv=-7, but, at the same time, the original combination may be shifted at Jv=-3. In so doing, the Jv of the entrance must be as follows:

\[ Jv = 3 \]

\[ \text{etc.} \]

---

\textsuperscript{86} This passage does not occur in the portions of the Kyrie and Gloria from Missa: Aeterna Christi munera that appear on page two of the fourteenth volume of the Breitkopf edition of Palestrina's complete works. (P.G.) (P.G.)
Hence, a sequence:

No. 57.\textsuperscript{87}  
P.XII,31.

\begin{music}
\begin{align*}
\text{\textit{etc.}}
\end{align*}
\end{music}

Since the original combination allows a shift at \( Jv=-7 \) and \( Jv=-3 \), then, having taken the latter as the original combination and the shift at \( Jv=-7 \) as its derivative, we obtain a new shift at \( Jv=-4 \). Hence, we obtain this sequence:

No. 58.\textsuperscript{88}  
P.XII,31.

\begin{music}
\begin{align*}
\text{\textit{etc.}}
\end{align*}
\end{music}

From the infinite canon by Brumel at \( Jv=0 \), which was cited at § 31, a canonic sequence can also be made at \( Jv=-4 \):

\textsuperscript{87} Giovanni Pierluigi da Palestrina, "Gloria" from Missa: Primi Toni, mm. 75-7. (P.G.)  
\textsuperscript{88} Ibid. (P.G.)
§ 48. From rarely-used Jjv we cited an example at Jv=-13 (§ 44). The following examples of canonic sequences have been written at other Jjv that are rarely encountered: -8, -10, and -12.

No. 60.

No. 61.

89 Antoine Brumel, "Benedictus," mm. 16-18. (P.G.)
§ 49. The canonic sequence can be a part of an infinite canon of a large size as a component part. For example:

No. 63.\(^{90}\)

Kirnberger

In this canon, two canonic sequences are maintained—one ascending, the other descending.

§ 50. In contrast to an infinite canon with a Proposta that repeats on the same degrees, canonic sequences have a

\(^{90}\) See the first note of this chapter. (P.G.)
very broad application in composition. On the basis of this, we cited so many examples of these sequences. Canonic sequences are continually encountered in compositions in the strict style, particularly in works by composers from the Netherlands. In exactly the same way, they play a large roll in counterpoint in the free style. Canonic sequences are material for episodes in fugue, and, in general, they have a wide application in a thematic work (in so-called division of a theme). The free style introduces a modulatory element into such sequences, converting sequences within a mode to sequences that modulate. A particular type of such a sequence is the circle canon (Zirkel-Canon), which modulates according to the circle of fifths. But it is precisely this mandatory modulation that excludes the circle canon from the number of forms that are studied in a course on the strict style.
Chapter 4

Infinite two-voice canon of the second order (the distances between entrances are diverse) with a Proposta that repeats on the same degrees (the proper infinite canon of the second order)

The application of horizontal-shifting counterpoint

§ 51. We will attempt to write an infinite canon of the second order, i.e., a canon in which the distances of entrances are diverse:

The beginning of this canon is written like any finite canon. Section A of the Proposta is shifted to the Risposta, to which the Proposta counterpoints section B. For its part, section B is shifted to the Risposta to which the Proposta counterpoints section C, etc. For the present, in the subsequent combination, a section of the Risposta is rewritten from a previous section of the Proposta and to it a new section of the Proposta is joined. Up to this point, the canon does not present any difference from the usual finite canon and does not require the application of any complex counterpoint. But just as we approach a combination of two sections of which neither is composed anew (in the given
canon this is the combination //A+E//, the combination that directly precedes this can not be written according to the example of the others. If section E were to be in simple counterpoint to section D, then E would not agree with //A. For example:

```
\begin{center}
\begin{tabular}{c c}
\textbf{Not allowed} & \textbf{Not allowed} \\
\end{tabular}
\end{center}
\begin{center}
\begin{tabular}{c}
\textbf{E} \\
\end{tabular}
\end{center}
```

That is why section E, which directly precedes //A, can not be written in the same way as the sections of the Proposta that precede it. From the point of entrance of //P, the combination //A+E// results. Neither one of these sections is composed, they both are rewritten from sections that were composed earlier, which is why the combination of these sections must be considered as derivative, for which an original combination must be located (§ 25). But the latter is absent. In the number of preceding combinations, not one is formed precisely from these sections. The original that is absent must be restored with the help of an imaginary combination.\footnote{Prior to this discussion of an imaginary combination, Taneev introduces the concept of a "mnimyi golos" [imaginary voice] in Serge Ivanovitch Taneiev, Convertible Counterpoint in the Strict Style, trans. G. Ackley Brower (Boston: Bruce Humphries, 1962), § 322, p. 209. As will be seen in Doctrine of the Canon, Taneev uses the imaginary combination in order to compose horizontal-shifting counterpoint. (P.G.)} In the make-up of the imaginary combination, which substitutes as an original, without fail the Proposta.
must enter (also on the basis of § 25). In order for such a combination to result, section A... must enter in the imaginary voice at precisely the same place where section E begins in the Proposta. The latter also will simultaneously be in counterpoint to both section A... of the imaginary voice and to section D of the Risposta.

\[ \text{(R)} \quad \text{A/ B/ C/ D/ E/ } //\text{A} \]
\[ \text{(P)} \quad \text{A B C D E } //\text{A } //\text{B} \]

(Imaginary combination) A...

The combination A...+E must be identical to the combination //A+E//, because the relationship between the points of entrance of voices in the original (imaginary) combination must be identical to the relationship between the points of entrance of voices in the derivative combination (of the Risposta and Proposta). This correlation between points of entrances is graphically presented in the following schema of the previous canon (see § 21):

```
Derivative combination

Imaginary (original) combination
```

The continuation of the imaginary voice proceeds up to the point of entrance of the //Proposta.
§ 52. The method that has been indicated for the formation of a schema of entrances at § 41 we will apply also in the given situation. In order to determine the point and interval of entrance of the imaginary voice, it will be sufficient to measure in the opposite direction from \( P \) a distance and interval of entrance that is equal to the distance and interval of entrance of the Risposta. In the given example, the Risposta enters a measure later than the Proposta and at a fifth above. The imaginary voice will enter a measure earlier than \( P \) and at a fifth below. If the first distance of entrance is greater than the second, then the imaginary voice will enter earlier than the Risposta. The length of the imaginary voice extends from its beginning up to the point of entrance of \( P \). This distance is equal to the initial distance of entrance (i.e., to the distance between the entrances of the Proposta and the Risposta).\(^2\)

\[ \text{\includegraphics[width=0.5\textwidth]{diagram.png}} \]

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\(^2\) Nikolay Andreevich Timofeev notes that if the entrances of such a canon are placed on graph paper, then correlations of combinations can be examined using elementary geometry, i.e., parallel lines will display original and derivative combinations. (N. A. Timofeev, *Prevrashchaemost' prostykh kanonov strogoogo pis'ma* [The transformation of simple canons in the strict style] (Moscow, 1981), p. 34.) Timofeev's theories are discussed in Chapter 21 of this dissertation. (P.G.)
§ 53. By knowing the point and interval of entrance of the imaginary voice and its duration, we can finish the canon that was begun earlier:

No. 64.

![Imaginary combination](image)

§ 54. In this canon, the first distance of entrance is smaller than the second. If the canon will begin with the upper voice (§ 20), then the schema will be as follows:

No. 65.

![Imaginary combination](image)

In this exchange of places by the Proposta and Risposta, the task is essentially not changed at all. The first five measures of the previous example (up to the entrance of //P) repeat on other degrees the same five measures of the canon in its previous aspect. The only difference is that in the second canon the roll of the Risposta of the first canon is
performed by an imaginary voice, and the roll of the imaginary voice of the first canon is performed by the Risposta in the second. Also, in the first four measures of the second canon, the imaginary combination is written in exactly the same way as the real combination of the first canon. The entire difficulty of both canons is concentrated in the fifth measure, where it is necessary for the Proposta to be in counterpoint simultaneously to two voices. The counterpoint in both situations is completely identical. In general, the difficulty in the infinite canon of the second order is proportional to the length of that place in the canon where it is necessary for the Proposta to be in counterpoint simultaneously with regard to an imaginary voice and with regard to a Risposta. In the infinite canon, the length of this place is always equal to the smaller of the distances of entrance in a given canon. Infinite canons of the second order (which are encountered in the musical literature primarily as a canonic sequences) usually begin with very close imitation, which facilitates to an extreme the composition of such a canon. In exercises, additional facilitation can be achieved if the point of entrance of //P is left to be written with discretion. If at a given place counterpoint to two voices presents difficulty, then the entrance of the imaginary voice (in a canon where the second distance of entrance is greater than the first, then the
entrance of the Risposta) is postponed until a more convenient point.

§ 55. In the composition of such canons, it is not necessary to determine beforehand the distance that must be between the Risposta and //P. Whatever the length of this distance, the imaginary voice always precedes //P at the same distance, equal to the distance between the Proposta and the Risposta, and it will always stand apart from the Proposta at an interval equal to the second interval of entrance. When beginning to write a canon, it is necessary to write out the imaginary voice separately (the length of it, as is known, extends to the point of entrance of //P, i.e., it is equal to the distance between the Proposta and the Risposta). If at the point where the imaginary voice is presumed to enter there are encountered any sort of difficulties for the Proposta to counterpoint simultaneously to both the imaginary voice and the Risposta, then the entrance of the former may be postponed and a more suitable point for it may be chosen. It goes without saying that once the imaginary voice enters the entrance of //P may not be postponed arbitrarily, and //P will enter at a definite distance from the imaginary voice. Therefore, in schemata that are posed to the student, it is not at all practical to indicate the distance between the

---

93 The adjective "first" in the original text "...and it will always stand apart from the Proposta at an interval equal to the first interval of entrance" has been changed to "second" in the translation. (P.G.)
Risposta and //P. For example, the schema at § 51 could be given in the following aspect:

\[
\begin{array}{c}
\text{p} & \text{n} \\
\text{---} & \text{---} \\
\text{-----} & \text{-----}
\end{array}
\]

§ 56. Infinite canons of the second order with a Proposta that repeats on the same degrees are as infrequently applied in compositions as analogous canons of the first order. We will cite a modest number of examples that have been, as before, either borrowed as a whole from the indicated sources or have been written on a theme borrowed from there.

a)

No. 66.\(^{94}\)

P.XI. 89.

\[\text{\begin{music}
\end{music}}\]

\(^{94}\) Giovanni Pierluigi da Palestrina, "Sanctus" from Missa: Aspice Domine, mm. 41-4. (P.G.)
A schema of this canon:

or in its more general aspect:

A section is equal to one half of a measure of the canon, as a result of the distance between the entrances of the Risposta and //P being equal to two and one half measures. The following two canons have been written according to a single schema:

or
It is understood that between adjacent notes of the same schema the rhythmic distances are identical.

b)

No. 67.\textsuperscript{95}

P.XI,149.

\begin{music}
\begin{musicexample}
\end{musicexample}
\end{music}

No. 68.\textsuperscript{96}

P.XIV,22.

\begin{music}
\begin{musicexample}
\end{musicexample}
\end{music}

A schema of canon No. 69:

\begin{music}
\begin{musicexample}
\end{musicexample}
\end{music}

No. 69.

\begin{music}
\begin{musicexample}
\end{musicexample}
\end{music}

\textsuperscript{95} Giovanni Pierluigi da Palestrina, "Benedictus" from Missa: Papae Marcelli, mm. 29-31. (P.G.)

\textsuperscript{96} Giovanni Pierluigi da Palestrina, "Credo" from Missa: Jam Christus astra ascenderat, mm. 57-9. (P.G.)
Chapter 5

The infinite two-voice canon of the second order with a Proposta that shifts to other degrees (the canonic sequence of the second order)

§ 57. The entire difference between the schema of this canon and the schema of the previous comes down to this: the degree from which //P enters is different than the one from which the Proposta enters. In all other respects, this schema is formed in entirely the same way as the schema of the previous canon. The determination of the point and interval of entrance of the imaginary voice (§ 52 and the following), the procedure of its composition, and conditions of its difficulty or ease, all apply to the canon where the Proposta remains in place as to the canon where it is shifted to other degrees. The form of the first of these canons can be examined as a particular type of the form of the second because, included in the number of possible shifts of the Proposta, there is also a shift at an interval equal to zero, i.e., the prime.

§ 58. We proceed to examples and their schemata. The following canonic sequence has been written according to the schema:
The following canon, with the exception of a large distance between the entrances of the Risposta and P, has been constructed according to a schema that is in all other ways similar to the previous:

b) No. 71.  
N. Craen  
(G.D.287)

[An additional canon:]

---

37 Giovanni Pierluigi da Palestrina, "Credo" from Missa: Jam Christus astra ascenderat, mm. 57-9. (P.G.)
38 In the original text, Taneev cites this work as being by Josquin. The correction is cited by Wehrmeyer in Sergej Taneev, Die Lehre vom Kanon, trans. Andreas Wehrmeyer (Berlin: Ernst Kuhn, 1994), p. 195. The work is Nicolaus Craen, "Ecce video coelos apertos," mm. 97-9. (P.G.)
c) No. 72.\textsuperscript{99}

Josquin

(G.D.387)

In [all] previous examples, the Proposta at its repetition is shifted upwards by a degree. In the two examples that follow, it is shifted downwards by a degree.

d) No. 73.\textsuperscript{100}

S. Dietrich

(G.D.288)

e) No. 74.\textsuperscript{101}

P.XVII, 94.

\textsuperscript{99} Josquin des Pres, "Planxit autem David," mm. 24-7. (P.G.)

\textsuperscript{100} Sixtus Dietrich, "Servus Tuus," mm. 26-8. (P.G.)

\textsuperscript{101} Giovanni Pierluigi da Palestrina, "Gloria" from Missa: Dum complerentur, mm. 121-2. (P.G.)
A canon with a Proposta that is shifted by a third:

f) No. 75.\(^{102}\)

G. Meyer

(G.D.373)

A canon with a Proposta that is shifted by a fourth:

g) No. 76.\(^{103}\)

L. Senfl

(G.D.297)

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\(^{102}\) At the last note of the third measure, instead of the half note g₁, which is present in the original text, a quarter note g₁ has been added in the translation. This correction is cited by Wehrmeyer in Taneev, *Die Lehre vom Kanon*, p. 195. This work is Gregorius Meyer, "Kyrie" from Missa: *De nostra Domina*, mm. 48-50. (P.G.)

\(^{103}\) Ludwig Senfl, "Deus in adjutorium meum," mm. 64-6. (P.G.)
§ 59. The canonic sequence of the second order
(including here even the canon with a Proposta that repeats on the same degrees) can, similar to a canon of the first order, permit modifications that will consist of //P entering from another degree. Due to this, the shift in the sequence is changed. These modifications can be done in that instance when the imaginary combination (P+R...) allows a direct shift at some other Jv. Then, by the interval that corresponds to this Jv, the entrance of //P can be moved. We will take as an example the imaginary combination from the canonic sequence a) § 58 (No. 70). This combination allows a shift at Jv=3:

\[ \text{Imaginary voice} \]

i.e., the imaginary voice could be shifted like this:

\[ \text{Imaginary voice} \]

Hence, the //Proposta may also be shifted at +3 and the canonic sequence will be in the following aspect:
§ 60. The imaginary combination in the canonic sequence b) § 58 (No. 71) allows a shift at the same Jv.

A shift of //P by the same quantity will provide as a result a new variant of this sequence:

No. 78.

§ 61. The imaginary combination in Example c) § 58 (No. 72) provides a shift at Jv=-2 and Jv=2:
Hence, there are the following modifications to this canonic sequence:

No. 79.

\[ Jv = -2 \]

\[ \text{etc.} \]

No. 80.

\[ Jv = 2 \]

\[ \text{etc.} \]

§ 62. The imaginary combination in canonic imitation d) § 58 (No. 73) permits a shift at \( Jv = -2 \):

\[ Jv = -2 \]

\[ \text{etc.} \]

Hence, there is the following variant of this sequence:

No. 81.

\[ \text{etc.} \]
§ 63. The imaginary combination in Example e) § 58 (No. 74) allows a shift at \( Jv=-1 \) and \( Jv=-3 \).

\[
\begin{align*}
Jv &= -1 \\
Jv &= -3
\end{align*}
\]

Hence, the following two sequences:

No. 82.

\[
\begin{align*}
\text{\ldots}
\end{align*}
\]

No. 83.

\[
\begin{align*}
\text{\ldots}
\end{align*}
\]

§ 64. In Example f) § 58 (No. 75), the imaginary combination permits a shift at \( Jv=1 \):

\[
\begin{align*}
Jv &= 1
\end{align*}
\]

this provides the following sequence:
§ 65. In the canonic sequence g) § 58 (No. 76), the imaginary combination provides a shift at $Jv=-3$:

the following infinite canon results because //P, by being shifted by $-3$, is a literal repetition of the Proposta on the same degrees.

No. 85.

This possibility of converting a canonic sequence into an infinite canon confirms the correctness of the view that the infinite canon is a particular type of canonic sequence.
§ 66. If a canonic sequence can be transformed into an infinite canon, then an infinite canon can be changed into a canonic sequence by the same way in reverse. It is apparent that each shift of //P turns an infinite canon into a sequence. Following the numerous examples that have been cited here, such an alteration of the canon will not represent any difficulty. We will limit ourselves to a single example. The infinite canon b) § 56 (No. 67) allows a shift at Jv=3 in the imaginary combination. As a result of this, it turns into the following canonic sequence:

No. 86.104

\[
\begin{array}{c}
\text{etc.}
\end{array}
\]

Jv = 3

§ 67. Those modifications to which we subjected infinite canons and canonic sequences dealt with intervals of entrance that are changed as a result of a shift of //P. We will mention in closing modifications that pertain to the distance between the entrances of the Risposta and //P. The canonic sequence can sometimes be compressed in a horizontal direction. If in the sequence the second interval of

104 "v=3" has been replaced by "Jv=3". This correction is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 195. (P.G.)
entrance is greater than the first and the imaginary voice can shift one section to the left, then //P can be shifted at the same distance and in the same direction. As a result of this, its repetitions will be located at a smaller space from one another. For example, in the canonic sequence c) § 58 (No. 72) the imaginary voice

\[ \text{entrada is greater than the first and the imaginary voice can shift one section to the left, then //P can be shifted at the same distance and in the same direction. As a result of this, its repetitions will be located at a smaller space from one another. For example, in the canonic sequence c) § 58 (No. 72) the imaginary voice} \]

\[ \text{can move a single section to the left, namely:} \]

\[ \text{By means of such a displacement of //P, a possibility to obtain the sequence in a new aspect is given:} \]

No. 87.

\[ \text{By means of such a displacement of //P, a possibility to obtain the sequence in a new aspect is given:} \]

\[ \text{For its part, this shift of //P will permit new combinations by means of a vertical shift. In the previous example, the} \]

\[ \text{For its part, this shift of //P will permit new combinations by means of a vertical shift. In the previous example, the} \]
imaginary voice can be shifted at \( Jv=2 \), this provides the following sequence:

No. 88.

\[
\text{\includegraphics{music.png}}\text{ etc.}
\]

§ 68. With this we will conclude the section on two-voice canons. In this section, we encountered a division of canons into two orders which will be studied even in the remaining parts of the doctrine of the canon. The methods for composing canonic imitations that we encountered in this section will also be applied in multi-voice canons to which we turn.
Second Part:

Multi-Voice Canon
§ 69. In the previous section, the doctrine of the two-voice canon was set forth in its full scope. It included finite and infinite canons. In spite of the little practical significance of the infinite canon in the proper sense, we expounded on it in detail, being guided mainly by pedagogical considerations, namely, by the following. In the application of moveable counterpoint to canons, it will be necessary to begin with the simplest two-voice exercises, and only after having mastered them, to shift to multi-voice canons. But the finite two-voice canon does not require the application of moveable counterpoint. Hence, it will be necessary to begin exercises with two-voice infinite canons, even though the latter only in the form of a canonic sequence has a direct relationship to composition. These considerations are inapplicable to the multi-voice canon. Multi-voice canons, beginning with finite three-voice, require moveable counterpoint in its most diverse applications. At the same time, they also have a very great significance for composition, forming an inalienable attribute of contrapuntal
technique of previous masters of the strict and free style. Whereas contrapuntal compositions of the strict and free style have been filled with finite canonic imitations, and we can in abundance cull from them examples of all sorts of cases of finite canons, multi-voice infinite canons (with the exception of canonic sequences) appear only as a rare exception in the musical literature and more likely belong to the category of professional works, which formerly at one time were in fashion with guild musicians, who practiced on them their musical wit, and with them maintained a quite often undeserved reputation for their erudition. We find models of finite canons, as well as canonic sequences, in the greatest works of earlier music, and we should look mainly in textbooks for examples of infinite canons with a Proposta that returns to its previous place. This exceptional position occupied by finite multi-voice canons, due to their significance simultaneously for both the aim of composition and for the aim of teaching, served as a reason for us to separate them into a single section and to expound on them as fully as possible. Three- and four-voice infinite canons also have been grouped into a single section, but they have been set forth more briefly. We dwell on them comparatively little in light of their far less significance as applied to composition, since only the group of canonic sequences can be an exception to this.
§ 70. The division of canons into two orders, which we encountered for the first time in the doctrine of the infinite two-voice canon, passes through the entire doctrine of multi-voice canons. A canon with identical distances of entrance belongs to the first order and requires the application of vertical-shifting counterpoint. With different distances of entrance, it belongs to the second order and is written according to the rules of horizontal-shifting counterpoint. In the four-voice canon, we will encounter mixed cases in which there is a simultaneous presence in the canon of both similar and different distances of entrance that require the simultaneous application of moveable counterpoint in both aspects. These cases we will also ascribe to the second order, the characteristic sign of which is, consequently, the presence of horizontal-shifting counterpoint.

§ 71. Thus, the distances of entrance have great importance when determining the order to which a canon belongs and, consequently, also the method of its composition. Similarity and difference between these distances we will express with letters, in order for it to be immediately clear in which order a given canon should be placed. The first distance of entrance (between the Proposta and the Risposta) we will indicate with the letter a. Of the distances that follow, those that are equal among themselves will be indicated by an identical letter, unequal will be
indicated by different letters in a successive order. On the basis of this, the three-voice finite canon will be expressed by one of two formulas: aa—a canon of the first order with equal distances, which requires the application of vertical-shifting counterpoint; and ab—a canon of the second order with unequal distances, which requires the application of horizontal-shifting counterpoint. Cases of four-voice canon are more numerous: aaa, aba, aab, abb, abc. Several of them have their own subdivisions about which we will speak in detail below.

The three-voice finite canon of the first order

The distances of entrance are identical (aa)

The application of vertical-shifting counterpoint

§ 72. As is already known (§ 22), all canons belong to the first order, which require the application of vertical-shifting counterpoint, in which the distances of entrance are identical, and are indicated by the expression aa. In order to be in a position to write such a canon not by groping ones way, but with complete awareness, it will be necessary first of all to give oneself an account of which two-voice combinations are originals, which are derivatives, and how many derivatives each original has. After this, it will be necessary to locate those JJv that determine the conditions according to which the given canon must be written. As we
will see below, for the three-voice canon only one \( J_v \) is required. This \( J_v \) may equal zero, as a result of this, in reality the canon will be written according to the rules of simple counterpoint. The degree of difficulty of each canon can be determined beforehand. It depends on the degree of difficulty of the indices and on their number.

**Original and derivative combinations in the three-voice finite canon**

§ 73. The three-voice canon has two Rispostas, the indicated entrances in order are \( R_1 \) and \( R_2 \). When indicating sections of each of the voices with letters, we will place near the letter the number of the Risposta to which a given section belongs. Therefore, sections of the first Risposta will be indicated by the letters \( A_1, B_1, C_1 \), etc.; sections of the second Risposta will be indicated by the letters \( A_2, B_2, C_2 \), etc. Letters that are not accompanied by numbers will belong to sections of the Proposta. We will examine the following schema of a three-voice canon of the first order:

\[
\begin{align*}
(R2) & \quad A_2 \quad B_2 \quad C_2 \quad D_2 \quad E_2 \\
(R1) & \quad A_1 \quad B_1 \quad C_1 \quad D_1 \quad E_1 \\
(P) & \quad A \quad B \quad C \quad D \quad E
\end{align*}
\]
The canon begins with section A performed by the Proposta. At the entrance of R1, a simultaneous combination of sections A and B (namely, A1+B) results. With the entrance of R2, a combination of three sections results: A, B, and C (A2+B1+C). It is obvious that the combination of these two sections, which initially appeared as A1+B, will repeat at each entrance of a new Risposta. If this were a four-voice canon, then at the entrance of R3 a combination of sections A3, B2, C1, and D would occur; thus, the combination A+B would appear a third time.

The first appearance of a combination of two sections (A1+B) where the Proposta is a member should be examined as original, and each repetition of these same sections by the Rispostas, as its derivative. In a four-voice canon, there would be two derivatives of A1+B, in a five-voice canon, three, etc. In general, each combination of Rispostas among themselves is a derivative combination. The number of derivatives of a given original combination in a canon of the first order is equal to the number of Rispostas in this canon that still have not entered.

§ 74. We will take the beginning of a three-voice canon with the following entrances:
No. 89.

Derivative at Jv = 2
The combination of sections A1+B, having appeared for the first time at the entrance of R1, will appear immediately between both Rispostas at the entrance of R2. Comparing these two combinations among themselves, we see that the second with regard to the first is derivative at Jv=2:

No. 90.\textsuperscript{105}

\begin{center}
\begin{music}
\guitar
dc\n\end{music}
\end{center}

Derivative (Measures 7-10)

The original combination is P+R1 and its derivative is R1+R2. The Proposta participates only in the original combination, R2, only in the derivative. As for that which precedes R1, it participates in both the original and derivative. We will note that the intervals by which the sections shift, the algebraic sum of which provides Jv=8-6=2, are the two intervals of entrance that have been cited above.

\textsuperscript{105} In the derivative, instead of "(Measures 7-8)", which is present in the original text, "(Measures 7-10)" has been added in the translation, and instead of "IIv=6", the correction "IIv=-6" has been added. The second correction is noted in Sergej Taneev, \textit{Die Lehre vom Kanon}, trans. Andreas Wehrmeyer (Berlin: Ernst Kuhn, 1994), p. 195. (P.G.)
What has been said with regard to A1+B applies also to the combination of sections in the same voices that follow after it: B1+C, which is repeated in the two Rispostas in the same order, at the same length, and at the same \( Jv=2 \):

No. 91.

Derivative at \( Jv = 2 \)

No. 92.

Original (Measures 7-9)

\[ \text{etc.} \]

\( Jv = 2 \) Derivative (Measures 10-12)

\[ \text{etc.} \]

\[ ^{106} \text{In order to maintain an accurate correlation to Example No. 89, the designation "A" for measures 1-3 of the top line of Example 91 has been changed to "A2". Also, the incorrect designation "A1" for measures 4-6 of the second line has been changed to "C1". These corrections are noted by Wehrmeyer in Taneev, } \textit{Die Lehre vom Kanon,} \textit{p. 195. (P.G.)} \]
Clearly, for as long as canonic imitation will last, all subsequent combinations of sections of the Proposta and Risposta will shift one after the other in the same order and at the same Jv. Therefore, we can say that P+R1 forms an original combination and R1+R2 forms its derivative, which is expressed schematically as:

\[
\begin{array}{c}
\text{Derivative} \\
\hline
P \hspace{1cm} R_1 \hspace{1cm} R_2 \\
\text{Original}
\end{array}
\]

This schema is the same for the two-voice infinite canon of the first order with the only difference being that the position of //P is occupied here by R2.

§ 75. At its entrance, R2 forms with the Proposta a combination of sections A2+C. Since R2 is the last Risposta in the canon to have entered, section A will not be met again, and, consequently, even the combination of A with C also will not be repeated. Obviously, further combinations of the Proposta and R2, such as B2+D, C2+E, etc., will not repeat. The combination P+R2 can not be derivative since the Proposta participates in it (§ 25). But it also may not be called an original since it does not have a derivative. Hence, it follows that the combination of the Proposta with the last Risposta in the canon is written in simple counterpoint. (On the basis of this, the application of moveable counterpoint in two-voice finite canon was also not necessary.) This rule does not apply to infinite canons
where //P, which follows after the last Risposta and is not a newly composed voice, is examined as a Risposta. (From the preceding, it also follows that in finite canons of the first order each combination of a Proposta with each Risposta, with the exception of the last in the canon, is an original combination. The number of original combinations in a finite canon is equal to the number of voices in the canon minus two.)

§ 76. For the original P+R1, it will be necessary to determine the JV. The size of the JV depends on the intervals of entrance and was made clear in the study of two-voice infinite canon (namely, the canonic sequence). The interval of entrance between the Proposta and R1 (the original combination) is original and is indicated by the letter m, and the interval of entrance between R1 and R2, the derivative combination, is indicated by the letter n. According to these two intervals, we determine the JV on the basis of the formula JV=−m+(+/−n). According to this formula, to the derivative interval (+/−n) is added the original, which has been taken with a negative sign. In the normal correlation of the notes of a melody to the position of a voice (§ 32), the original interval of entrance is positive. Consequently, it must be taken with a minus sign and added to the derivative interval. The latter can either be positive or negative. It is positive in the instance when it has the same direction as the original interval of
entrance, and it is negative when their direction is contrasting. As a reminder, the previous formula may be supplemented in this way:

\[ J_v = -m + \frac{n}{\ell} \]

Applying this formula to the previous canon, which has intervals of entrance in a single direction, we obtain \( J_v = -6 + 8 = 2 \).

The procedure for writing a canon

§ 77. The previous canon is written in the following way. The initial section A is composed (see the example) and is shifted by the indicated interval to R1. R1 is in counterpoint to the Proposta at \( J_v = 2 \). The combination \((A1+B)\) \( J_v = 2 \) occurs. Both of these sections are shifted by the indicated intervals to both Rispostas, forming a derivative combination, \( A2+B1 \). Section C of the Proposta is in counterpoint to these two sections while observing the following two conditions: a) The Proposta forms correct three-voice counterpoint with the sections of both Rispostas; b) Simultaneously, it forms a combination with R1 at the given \( J_v \), namely, \((B1+C)\) \( J_v = 2 \). It is necessary not to forget that with regard to R2 the Proposta is written in simple
counterpoint. The final two sections, Bl+C, are again shifted to both Rispostas and, again in observance of the same conditions, the Proposta counterpoints section D to them, etc., until canonic imitation turns into free counterpoint. In this same manner is written any three-voice finite canon of the first order at any index.

§ 78. The Jv of the previous canon had a direct shift. Now we will cite an example of a canon at a Jv that provides an inverse shift, namely, Jv=-11. This Jv is encountered extremely often in canonic imitations by masters of the strict style, more often than any other not excluding even Jv=-7, i.e., double counterpoint at the octave. In general, Jv=-11, i.e., double counterpoint at the duodezime, is connected in the most intimate way with the technique of the contrapuntal style. It is obligated to this style in part because of the proximity of its rules to the rules of simple counterpoint, which makes its use extremely easy, and mainly because of the conveniences that it presents for imitation. Canonic imitations at the most common intervals: the fifth and the octave, require this index. This latter consideration gives an advantage to double counterpoint at the duodezime over double counterpoint at the octave and explains the frequency of its application. If theorists would not overlook the link that exists between moveable counterpoint and canon and were to be aware of those situations where the application of at least double
counterpoint requires canonic imitation, then the usual statement, that of all double counterpoint the most widespread and therefore primarily deserving of study is double counterpoint at the octave, would not be expressed so categorically. On a plane with the latter would be established double counterpoint at the duodezime, which with regard to canonic imitations unquestionably must occupy the first position among all types of double counterpoint.

The canon to which we turn is the "Benedictus" from Palestrina's mass Gia fu chi m'hebbe cara. The following intervals of entrance are in this canon:

```
\begin{array}{c}
  4\quad 7\quad 11
\end{array}
```
No. 93.  

We will write out the beginning of combinations P+R1 and R1+R2, which provides a possibility to more graphically present the application of double counterpoint at the duodezime:  

---

107 Giovanni Pierluigi da Palestrina, "Benedictus" from Missa: Gia fu chi m'hebbe carra, mm. 1-26. (P.G.)

108 The integer "1" is added to the combination "P+R". This correction is noted by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 195. (P.G.)
§ 79. According to the same given information, we will determine the size of several other $Jv$. 

Here we locate $Jv$ according to intervals of entrance. It is possible to proceed in reverse—to locate intervals of entrance according to the index. How this is done was indicated in the section on two-voice canon (§ 40). The first interval of entrance can be of an arbitrary size and
direction. Having added the Jv to this, we obtain a second interval of entrance \((n=m+Jv)\). The plus or minus of the second interval indicates whether this interval has a direction identical to or different from the first.

§ 80. For example, at the entrance of the Risposta at the upper second \(^\text{V}1\) at \(Jv=-9\), at which interval does \(R2\) enter relative to \(R1\)?

Answer: at a ninth lower.

\[
\begin{array}{c}
\text{m:1} \\
Jv=-9 \\
\text{Jv=-1} + 8 = -9
\end{array}
\]

since \(1-9=-8\). At which interval would this entrance occur if \(R1\) entered relative to the Proposta at the lower fifth \(^\text{V}4\)？

Answer: at the upper sixth \(^\text{V}5\), since \(4-9=-5\).

\[
\begin{array}{c}
\text{m:4} \\
Jv=-4 + 5 = -9
\end{array}
\]

§ 81. We will apply to the examples at § 79 the method of determining \(Jv\) that was indicated at § 41, and we will set aside from \(R2\) to the left an interval equal to \(m\), but in the opposite direction.
In order to solve the opposite task by this method, i.e., to locate an interval of entrance according to a given Jv, it will be necessary to enlist the following method.

First of all, an interval that is equal to a given Jv will be positioned either above or below a note that corresponds to the entrance of Rl. For example:

Then, at an arbitrary interval to the left of Rl will be set aside a note that corresponds to the entrance of the Proposta. But the direction of this can not be arbitrary, it depends on whether the Jv that we have chosen is positive or negative. If it is positive, then this direction is identical to the direction of the interval that corresponds to the Jv; if the Jv is negative, then the direction of the interval is in contrast to it. Thus, we obtain the initial interval of entrance (see the places in the following examples that have been marked (a)). Then, from the note that has been indicated •, an interval equal in size and direction to the first interval of entrance remains to be set
aside to the right (see the places that have been indicated by (b)).

\[
\begin{align*}
Jv &= -11 \\
Jv &= -4 \\
Jv &= -9 \\
Jv &= +5 \\
\end{align*}
\]

etc.

It will be necessary to completely master these uncomplicated methods before proceeding further.

**The classification of three-voice finite canons of the first order**

§ 82. If the first interval of entrance at a given Jv is taken at an arbitrary size, and the second interval is determined by the first interval and the index, then according to the index and the first interval of entrance it is possible to classify three-voice finite canons of the first order. Having arranged the indices according to the order \( Jv = 0, 1, -1, 2, -2, \ldots \), and having enumerated for

---

109 The information in this section is incorrect. In order to obtain a positive Jv, it is necessary to position the Proposta and the dot in contrasting directions from R1, and in order to receive a negative Jv, it is necessary to place the entrance of the Proposta and the dot in the same direction from R1. Also, in the second group of schemata at § 81, the correct designation "Jv=-4" has replaced "Jv=+4" in the second example, and in the third group of schemata, instead of "Jv=-11, Jv=+4, Jv=-9, Jv=-5", the corrections "Jv=-11, Jv=-4, Jv=-9, Jv=+5" have been added. These corrections are cited by Wehrmeyer in Taneev, *Die Lehre vom Kanon*, p. 195. (P.G.)
each of them a successive row of intervals at which the voices of a two-voice canon can enter, which correspond to the intervals of entrance of a three-voice canon, we acquire a clear representation of the domain of the three-voice canon in all of its completeness. If the number of possible entrances of the two-voice canon will be denoted by the letter \((r)\) and the number of possible indices by the letter \((s)\), then \((r) \times (s)\) will equal the number of three-voice canons that have been differentiated according to intervals of entrance. Each canon acquires a definite position among the other canons that are similar to it, and the rules according to which the canon must be written are determined by its index. The degree of difficulty of the canon is also determined by the properties of the index that belong to it.

§ 83. The following table presents this classification according to indices and to the first interval of entrance.
Table of three-voice finite canons of the first order
Explanations for the table of three-voice canons of the first order (aa)

1. JJv have been positioned according to the order JJv=1, -1, 2, -2, etc. The indices JJv=-4, -5, and -6 have been cited twice: JJv=-4<, -5>, -5<, -5>, etc.

2. At each JJv a row of entrances for the three-voice canon that require this JJv have been written out. These entrances have been positioned according to the first interval of entrance (m) in the successive order m=0, m=1, m=2, etc.

3. The entrances at each JJv have been written out on two lines. On the upper line, the first interval of entrance has been taken in an ascending order, on the lower, in a descending order.

4. For negative JJv with a direct shift (from -1 up to -6< inclusive), the first m is equal to the index. For example, at JJv=-1, it is equal to 1, at JJv=-2, to 2, etc.

5. If three notes of entrance that belong to an index with a direct shift are to be taken in a reverse order, from the last note to the first, then the entrances that have been obtained by this method require the application of a JJv of

---

110 Instead of "(from -1 up to -6> inclusively)" the correction "(from -1 up to -6< inclusively)" has been added. (P.G.)
the same size but with the opposite sign. For example, for the entrances:

\[ Jv=3 \]

is required, and for the same entrances that have been taken in the reverse order:

\[ Jv=-3 \]

is necessary. For the entrances:

\[ Jv=2 \]

is necessary, and for the entrances:

\[ Jv=-2, \text{ etc.} \]

In the table, the indices 1 and -1, 2 and -2, etc., have been positioned on a single line in pairs, and entrances of each negative Jv, when taken in a reverse order, present entrances of a positive index of the same quantity, and vice versa.

6. If such a change in the order of entrances will be performed with regard to a Jv with an inverse shift, then once again the entrances that are obtained will require the
same index without any change. For example, for the entrances:

and

the same $Jv=-11$ is necessary; for the entrances:

and

$Jv=-9$ is identically required. In the table, this symmetry is apparent for each $JJv$ with an inverse shift. All entrances that belong to each index have a center from which on both sides entrances have been positioned symmetrically. For even $JJv$, in the center occurs a group of three entrances in which both intervals of entrance are equal, and the size of each is equal to half the size of the index.
Mutually corresponding groups of entrances are located on both sides of the center, they have been designated by identical letters.

For odd $JJv$, this center occurs in the space between two groups of entrances. For example:
Chapter 7

Three-voice finite canon of the first order

(a continuation)

§ 84. According to the intervals of entrance, we learned how to exactly determine a Jv, the rules according to which each three-voice canon must be written. But the rules of two-voice vertical-shifting counterpoint, as must already be known, undergo several changes when this counterpoint forms a part of multi-voice combinations. These changes are analogous to those that enter into two-voice simple counterpoint when there is an increase in the number of voices. We will be limited here to a short enumeration of these rules. The changes are reduced to the following:

a) With regard to consonances -- There is great freedom for hidden successions. Direct motion to a perfect consonance, which is not permitted in two-voice counterpoint, is quite often encountered in multi-voice. The absence of exact rules with regard to hidden successions in simple counterpoint hinders their establishment in complex.

b) With regard to dissonances -- The changes that are caused in two-voice counterpoint by an increase in the number of voices concern two intervals—fourths and ninths. The fourth, as was mentioned in Convertible Counterpoint in the Strict Style (§ 255 and the following), placed between an upper and middle voices (or between two middle voices) is
considered consonant and on this basis is not required to submit to limitations prescribed to it by the rules of two-voice counterpoint (3 instead of $\frac{3}{2}$).\textsuperscript{111} If its lower tone is located in the bottom voice, then it continues to be considered dissonant and obeys the previous rules: $\frac{3}{2}$. In this case, it goes without saying that nothing can prevent retention of limitations by this interval ($\frac{3}{2}$). Hence, with regard to the fourth, there are two ways of writing multi-voice combinations: 1) strictly adhering to the rules for its use in two-voice counterpoint ($\frac{3}{2}$) in whichever voices it is encountered; and 2) freeing it within the indicated limits from limitations. We will encounter both of these methods of writing in the composition of canons. Canons that have been written according to the first of these ways (i.e., with a strict use of the fourth: $\frac{3}{2}$) acquire a property that canons written according to the second way lack: the first is capable of transformation. At the same time, as the fourth acquires greater freedom in multi-voice combinations, the other interval—the ninth—acquires a new limitation compared to two-voice counterpoint (see C.C.§ 252 and the following).

\textsuperscript{111} The use of dashes above and below an integer in order to indicate a tie is discussed by Taneiev at §§ 88-94 in Convertible Counterpoint in the Strict Style, and at the seventh footnote of Chapter 2 in this dissertation. The line above integer 3 indicates that the interval of a fourth may be tied in an upper voice over a lower voice in an original combination. The line beneath the integer 3 indicates that the interval of a fourth may be tied in a lower voice beneath an upper voice in an original combination. Brackets around these lines indicate that a tie is forbidden in these situations. (Serge Ivanovitch Taneiev, Convertible Counterpoint in the Strict Style, trans. G. Ackley Brower (Boston: Bruce Humphries, 1962), §§ 88-94, pp. 66-9.) (P.G.)
This limitation consists of an allowance of a resolution of the ninth to an octave (\(\tilde{9}\)) only when the lower note is located in the bass: dissonant ninths (\(\tilde{9}\)) with regard to a middle voice are not allowed. Consequently, the conditions for its use when it is dissonant with regard to a middle voice will be expressed as \(\tilde{8}\) and \(\tilde{8}\). As we will see subsequently, for canons that are subject to transformation, the dissonant ninth above (\(\tilde{8}\)) is not at all allowed.

With these cited cases, the difference between the rules of simple counterpoint in two voices and multi-voice counterpoint is exhausted. In the latter, if any two voices are singled out, then the difference between the combination of these set-apart voices and a combination that has been written in two-voice counterpoint will be reduced to this: in the first place, in the former hidden successions can be encountered; and in the second place, the fourth is used freely. In all other respects, the combination of two voices that has been extracted from a multi-voice will in no other way be distinguished from simple two-voice counterpoint.

What has been said pertains to those combinations in which the bottom voice does not participate. The combination of the bottom voice with each of the other voices provides a correct two-voice combination with regard to the use of the fourth.

We will note that there is only a single case where the voices, forming correct two-voice counterpoint with each
other, never the less provide an incorrect three-voice combination: this is in disturbance of the rule that forbids doubling of the dissonant note.\textsuperscript{112}

\textbf{Exercises in three-voice finite canons of the first order}

\section*{§ 85. Exercises in three-voice canons of the first order are more practically assigned in the following sequence:}

1) The voices enter in succession of pitch--consequently, the direction of intervals of entrance are identical ($J_v$ with a direct shift): a) the intervals of entrance are of an identical quantity ($J_v=0$); and b) the intervals of entrance are different (the remaining $J_{Jv}$ with a direct shift).

2) The voices do not enter in succession of pitch--the direction of the intervals of entrance are diverse, i.e., $J_{Jv}$ with an inverse shift.

\section*{§ 86. We begin with a canon at $J_v=0$, the simplest of the canons of the first order. This $J_v$ results each time we

\begin{footnotesize}
\begin{footnotes}
\footnote{\textsuperscript{112} This rule is discussed by Taneev at § 266 in Convertible Counterpoint in the \textit{Strict Style}. At that point, Taneev states: "As is already known, the dissonant note in a suspension should not be doubled. Therefore all progressions are incorrect in which the bass, used as a suspended fourth (1 or 10) is double by another voice." (Taneiev, Convertible Counterpoint, § 266, p. 171.) (P.G.)}
\end{footnotes}
\end{footnotesize}
take intervals of entrance in a single direction that are equal in size (see the table at § 79).

\[
\begin{align*}
\text{Interval 1} & \quad \text{Interval 2} & \quad \text{Interval 3} & \quad \text{Interval 4} \\
\text{Example 1} & \quad \text{Example 2} & \quad \text{Example 3} & \quad \text{Example 4}
\end{align*}
\]

Taking into consideration what was said with regard to the conditions of multi-voice combinations (§ 84), we must take account of the following rules for the composition of canon at \( Jv=0 \):

**Rule one.** With regard to the canon that begins with an upper voice. In the canon at \( Jv=0 \), since any interval of the original combination provides the same interval in the derivative, then in a canon that begins with an upper voice free use of the fourth (3) is prohibited. During a shift of it to the bottom and middle voice, such a fourth would provide an inadmissible combination of voices:

\[
\begin{align*}
\text{Example 1} & \quad \text{Example 2} & \quad \text{Example 3} & \quad \text{Example 4}
\end{align*}
\]
The third measure of this canon, which presents entirely correct three-voice counterpoint, includes a fourth that in the following measure turns out to be incorrectly employed. Therefore, in such a canon, the fourth preserves the limitations imposed on it by the rules of two-voice counterpoint (3).

Rule two. With regard to a canon that begins with the bottom voice. In the canon at \( Jv=0 \) that begins with the bottom voice, tied upper ninths (\( \frac{8}{9} \)) are prohibited between the Proposta and \( R_1 \). When transferred to \( R_1 \) and \( R_2 \) (the upper and middle voices), such a ninth provides an incorrect combination:

\[
\begin{align*}
\text{Proposta} & : \quad \text{R}_1 \\
\text{R}_1 & : \quad \text{R}_2 \\
\end{align*}
\]

That which has been cited here as incorrect could be avoided by a crossing of voices (in the previous example, the Proposta could be the note above \( R_1 \)), but crossings in such cases quite often hinder the subsequent course of the canon and, in general, can cause significant inconveniences.

§ 87. We cite a canon by Palestrina at \( Jv=0 \).
a)

No. 95.\textsuperscript{113}

P.XI, 67.

\[ jv = 0 \]

\[ \text{Giovanni Pierluigi da Palestrina, "Benedictus" from Missa: Ad fugam, mm. 1-21. (P.G.)} \]
In this example, the voices enter each from the previous at an interval of a fifth and not once do they cross. At entrances at intervals of a smaller size, crossing in canonic imitation often becomes unavoidable. In the following example, it is repeatedly encountered:

b)

No. 96.\textsuperscript{114}

P.XVII.134/135.

\textsuperscript{114} Giovanni Pierluigi da Palestrina, "Sanctus" from Missa: Sacerdotes Domini, mm. 34-61. (P.G.)
§ 88. Before exercises will begin in canons with other JrV, it will be necessary to take into consideration the following additional rules for the conditions of these indices, which have been caused by a difference in the conditions for the use of fourths and ninths in multi-voice counterpoint compared to conditions for their use in two-voice.

Additional Rules

§ 89. Additional rules:

a) With regard to the fourth --

Rule one for an original combination in which the bottom voice does not enter.

At a given JrV, if the fourth provides a consonance in the derivative combination, then it may be liberated from its limitations ($\frac{3}{3}$ instead of $\frac{3}{2}$). If this consonance is imperfect, then a parallel succession of fourths may be allowed. By means of this, parallel motion enters JrV of the second group.\footnote{At § 11 in Convertible Counterpoint in the Strict Style, Taneev distinguishes between group one (0, 3, 4, 7), i.e., perfect intervals, and group two (1, 2, 5, 6), i.e., imperfect consonances and dissonances. In this case, if a succession of fourths is present in the upper voices of an original combination, they can become a succession of thirds or sixths at JrV from the second group. (Taneeiev, Convertible Counterpoint, § 11, pp. 28-9.) For additional information about groups see the Introduction to Parts I and II of this dissertation. (P.G.)} In all other cases, with the exception of those mentioned here, the fourth preserves its limitations.
(3). This requirement, that the bottom voice not enter the combination P+R1 in which a fourth is encountered, is caused by the necessity for the bottom voice to form a bass with regard to the fourth and thereby transform it into a consonance. If additional voices besides the voices that form the canon take part in the composition, then the roll of the bottom voice, which turns the fourth into a consonance, can be fulfilled by one of them. In the latter case, it does not matter which place according to pitch the voices of the original combination occupy among the other voices of the canon. The fourth, having support from the additional voice, in any case can be encountered without preparation between the others. In the following example:

No. 97.

\[ Jv = \text{Possible} \]

R2, at its entrance in the third measure, turns the freely employed fourth between the Proposta and R1 into a consonance, which in the derivative combination (R1+R2), at the beginning of the fourth measure, results in a consonance of the fifth. In this example:
we have an entire row of fourths that are consonant owing to the additional bottom voice. These parallelisms are possible because at $Jv=6$ an imperfect consonance—the tenth—results from the original fourth ($3+6=9$), a row of which we have in the derivative combination, $R1+R2$. We will note the unusual distance between the entrances of voices of this example, equal to five half notes. We will enumerate the $JJv$ where the fourth provides a derivative consonance. With bold type we have indicated those that have this imperfect consonance. The latter all belong to the second group of indices: $Jv=6, 4, 2, 1, -1, -3, -5, 7, -8, -10, -12, -14$.

Rule two for an original combination where the bottom voice is absent from the derivative. (This condition is fulfilled in the three-voice canon when the Proposta is the

---

116 Giovanni Pierluigi da Palestrina, "Gloria" from Missa: Sine nomine, mm. 120-5. (P.G.)
bottom voice.) At a given \( Jv \), if the derivative fourth (positive or negative) results from a consonance, then this consonance may be liberated from the limitations that it has received from the fourth and may be freely employed. If this consonance is imperfect, then even parallel successions of it may be allowed. By means of this, parallel motion enters \( JJv \) of the second group. Absence of the bottom voice from the number of voices of the derivative combination must be understood here in the same sense as in the previous rule.\(^{117}\) A bottom voice of the canon is necessary in order to convert the fourth into a consonance. If there is an additional voice that will do this, then the aforementioned requirement is unnecessary. First of all, we will cite a series of examples at \( Jv=-7 \). At this \( Jv \), the fifth is liberated from the limitations that it has acquired from the derivative fourth (\( \frac{3}{4} \)) and is freely employed because the latter loses the property of a dissonance.

\(^{117}\) The word "original," which is present in the original text, has been replaced by "derivative" in the translation. (P.G.)
No. 99.\textsuperscript{118}  

P.XV, 15.

\begin{center}
\includegraphics[width=0.7\textwidth]{diagram1}
\end{center}

No. 100.\textsuperscript{119}  

P.XVIII, 55.

\begin{center}
\includegraphics[width=0.7\textwidth]{diagram2}
\end{center}

\textsuperscript{118} Giovanni Pierluigi da Palestrina, "Benedictus" from Missa: Dies sanctificatus, mm. 1-9. (P.G.)  
\textsuperscript{119} Giovanni Pierluigi da Palestrina, "Credo" from Missa: Vestiva i colli, mm. 200-2. (P.G.)
No. 101.\textsuperscript{120}

P.IV, 61.

\[ J_v = -7 \]

\[ \text{etc.} \]

No. 102.\textsuperscript{121}

P.II, 31.

\[ J_v = -7 \]

\[ \text{etc.} \]

\textsuperscript{120} Giovanni Pierluigi da Palestrina, Motet: \textit{Pulchra es amica mea}, mm. 1-5. Instead of "P.II, 61", which is present in the original text, "P.IV, 61" has been added in the translation. This correction is cited by Wehrmeyer in Sergej Taneev, \textit{Die Lehre vom Kanon}, trans. Andreas Wehrmeyer (Berlin: Ernst Kuhn, 1994), p. 196. (P.G.)

\textsuperscript{121} Giovanni Pierluigi da Palestrina, Motet: \textit{Derelinquat impius}, mm. 44-50.
We cite cases of the application of this same rule in canonic imitations at other JJv. These indices are: $Jv=1$, where the derivative fourth results from an imperfect consonance—the third ($2+1=3$); $Jv=-2$, where the fourth results also from an imperfect consonance—the sixth ($5-2=3$); $Jv=-5$, where the fourth (negative) results from the third ($2-5=-3$); and, finally, $Jv=-8$, where a derivative, negative fourth results from a sixth ($5-8=-3$).

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122 Giovanni Pierluigi da Palestrina, Motet: *Introductit me Rex in cellam*, mm. 16-21. Instead of "P.II,34", which is present in the original text, "P.IV,34" has been added in the translation. This correction is cited by Wehrmeyer in Taneev, *Die Lehre vom Kanon*, p. 196. (P.G.)
No. 104.  
P.XVII, 52.

Giovanni Pierluigi da Palestrina, "Credo" from Missa:  O admirabile commercium, mm. 127-30. (P.G.)

No. 105.  
P.II, 112/113.

Giovanni Pierluigi da Palestrina, Motet:  Benedicta tu (secunda pars), mm. 14-20. (P.G.)
No. 106.\textsuperscript{125}

P.II, 50.

\begin{align*}
Jv = -2
\end{align*}

No. 107.\textsuperscript{126}

P.XV, 23.

\begin{align*}
Jv = -5
\end{align*}

\textsuperscript{125} Giovanni Pierluigi da Palestrina, Motet: Exi Cito, mm. 15-18. (P.G.)

\textsuperscript{126} Giovanni Pierluigi da Palestrina, "Kyrie" from Missa: In te Domine speravi, mm. 28-32. Instead of "P.XV,22", which is present in the original text, the correction of "P.XV,23" has been added in the translation. Also, instead of "Jv=5\textsuperscript{>}", which accompanies the schema at the top of Example No. 107 in the original text, the correction "Jv=-5\textsuperscript{>}" has been added in the translation. These corrections are cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 196. (P.G.)
We will enumerate $J_{Jv}$ at which a derivative fourth results from a consonance, having indicated in bold print those where the consonance is imperfect (the latter all belong to the second group): $J_{Jv} = 6, 5, 3, 1, -1, -2, -3 -4<, -5>, -6<, -7, -8, -10, -12, -14$.

Rule three pertains to the combination of the Proposta with the last Risposta in the canon. As we know, a combination of the Proposta with the last Risposta in the canon is written in simple counterpoint (§ 75). If neither one of these voices is the bottom, then the fourth can be liberated from the limitations that are imposed on it by two-voice counterpoint.

---

127 Giovanni Pierluigi da Palestrina, "Credo" from Missa: Quem dicunt homines, mm. 193-5. (P.G.)
No. 109.  

P.XII, 118.

\[ Jv = -7 \]

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No. 110.  

P.XI, 12.

\[ R, Jv = -7 \]

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128 Giovanni Pierluigi da Palestrina, "Credo" from Missa: Repleatur os meum laude, mm. 43-5. (P.G.)

129 Giovanni Pierluigi da Palestrina, "Credo" from Missa: De Beata Virgine, mm. 180-3. The note ci, which precedes the entrance of R2 in the original text, has been omitted in the translation. This correction is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 196. (P.G.)
The aim of the enumerated additional rules is to facilitate the writing of canon by liberating the fourth from its limitations. Therefore, these rules are not mandatory in the sense that their non-observance will entail any kind of error, but will only complicate the composition of canon. The rule of the ninth, to which we now turn, is the opposite. It always must be observed otherwise inadmissible successions will result.

b) The rule of the ninth for an original combination in a three-voice canon that begins with the bottom voice—the bottom voice does not enter the derivative. Since both Rispostas enter the derivative of a three-voice canon, then a canon whose derivative does not include the bottom voice, without fail, begins with the bottom voice, i.e., the Proposta is the bottom voice. According to the rules of simple counterpoint, the ninth may not be dissonant above with regard to a middle voice. Therefore, the interval that produces a ninth in the derivative combination should not be dissonant above at a Jv with a direct shift and below at a Jv with an inverse shift. Having indicated this interval with the letter m, we will have $\overline{m}$ for a Jv with a direct shift and $\overline{m}$ for a Jv with an inverse shift. The sign of the interval on the inverse side will remain without change in that aspect in which it appears in the given. For example, at Jv=2 the derivative nine results from $\overline{2^*}$ [i.e., it may be used only in the form of a passing or auxiliary note]. At Jv=-11, the
interval that produces a derivative ninth (negative) is the fourth (3), the signs of which must be altered as such: $\overline{3}$.

§ 90. The additional rules for the fourth and ninth that have been set forth here have a general significance and pertain to all canons of the first order regardless of the number of voices that participate in them. They even include the rules that were set forth at § 86 for a canon at $Jv=0$. Even though the latter has been cited separately, this was done solely with the aim of not hampering the student by a sudden exposition of rules that concern all $JJv$ when, for the time being, additional rules only for $Jv=0$ were necessary. The cited rules for the fourth and ninth pertain to the domain of the strict style. All remaining rules that we will need to encounter in the doctrine of the canon—for example, rules that determine $Jv$ for a canon, rules that concern the entrance of an imaginary voice, etc., these have nothing to do with differences between the strict and free style; therefore, they are as identically fit for one as for the other.

If we shift from exercises in the strict style to exercises in the free style, then, conforming with the new requirements, it would be necessary to change the conditions of all $JJv$. As for rules that determine the application of these $JJv$ in this or the other case, which establish a link between entrances of voices in a canon and the indices that
are required by these entrances, all of these rules would remain without change even in a shift to the free style.

The rules of the fourth that liberate the fourth from limitations in the original combination and the consonance that produces a fourth in the derivative significantly facilitate composition of canon at difficult JJv by supplying a new interval that is free from limitations, thereby augmenting the number of fixed consonances. But these facilitators hinder the extraction from the canon to which they are applied of a new combination of voices, which would occur with the help of transformation.

§ 91. After the rules that concern fourths and ninths have been studied, and after having assignments of canons at \( JJv=0 \), one should shift to exercises in canons that require the application of other JJv. Models of such canons are located in the appendix. Of these indices, particularly at the beginning, one should choose the easiest for exercises. In a shift from two-voice canons to three-voice, the difficulty of composing canons grows significantly, and there will be no basis for it to be increased by the choice of awkward JJv. If the student acquires sufficient technique in the writing of canons at relatively easy indices, such as \( JJv=\pm 2, \pm 3, \pm 4, -7, -9, -11 \), then such a result may be

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130 Taneev describes "fixed" intervals at § 96 of Convertible Counterpoint in the Strict Style. A consonant interval in an original combination that remains consonant in a derivative is regarded by Taneev as fixed. (Taneiev, Convertible Counterpoint, § 96, p. 70.) (P.G.)
recognized as being entirely sufficient. As we will see later, composers of the strict style primarily used the easiest of the indices, applying the more difficult only in short canonic imitations. One should shift to canons that require more difficult JJv no earlier than when skill has been acquired in the application of the easiest of them to canonic imitation. Only on this condition can exercises in canons with difficult JJv be of use for the development of contrapuntal technique. Premature study of them can reflect poorly in the voice leading and, in general, in the aesthetic aspect of the works of the student. If the student wishes to test his skills on such tasks that earlier composers only touched upon, then even in the midst of the most usual and simple JJv, such as JJv=-7, -9, -11, he will locate entrances which, as can be stated with almost complete certainty, were not at all attempted. This other group of entrances can supply extensive material for exercises in unusual cases of canonic imitation, even if they require the most accessible indices. If the student wishes to test his skills in the application of more difficult indices, such as JJv=±1, ±6, -8, -10, -12, -13, then he will locate examples of these instances in the appendix.\footnote{This appendix was not located in the papers of S. I. Taneiev. (V.B.)} It is necessary to keep in mind that the application of additional rules that pertain to the fourth are permitted at a majority of these JJv, thereby
facilitating their use and counterbalancing the difficulties that arise during a shift from two-voice exercises to multi-
voice. It is not necessary to write such canons with overly large dimensions, and [one should] alternate these exercises with exercises in canons with simpler indices.

**Three-voice canonic imitation by composers of the epoch of the strict style**

§ 92. From the numerous examples that have been cited here, it is apparent that composers of the epoch of the strict style often used three-voice canonic imitation. The following entrances have predominant significance in these imitations (see the table at § 83).

\[ J_\text{v} = -11 \]

\[ \text{\begin{align*}
\text{\textbullet} & \quad \text{\textbullet} \\
\text{\textbullet} & \quad \text{\textbullet} \\
\text{\textbullet} & \quad \text{\textbullet}
\end{align*}} \]

The preference given to these entrances is fully understandable. Imitations at the fifth and octave correspond to the range of the voices and permit the melody to shift from voice to voice, preserving the relative position of its tones and semitones. A great majority of compositions in the strict style begin with imitation at these intervals, and in subsequent compositional styles, they are met primarily before all others. It is completely
natural that the conveniences presented by these intervals of entrance compelled them to be given preference even in canonic imitation. The reason why this and not a different succession of these degrees was chosen for entrances in canonic imitation of the first order is also completely understandable. The groups of entrances that have been enumerated here present the distinctive feature that they require one of the most accessible indices, namely, $Jv=-\frac{11}{2}$. This circumstance even is decisive in the choice of the indicated succession of entrances.

§ 93. The fact is interesting that composers of that epoch nearly each time, as if they had to, used the aforementioned degrees for entrances, but in a different order. For example:

\[ \begin{align*}
  Jv &= -1 \\
  Jv &= 1 \\
  Jv &= -10
\end{align*} \]

they systematically avoid a use that requires more difficult indices by these entrances, such as $JJv=\pm 1$ and $-10$, using for this aim the following methods:

1) Canonic imitation entirely ceases at the entrance of R2, as a result of this, it is not necessary to write the combination $P+R1$ in complex counterpoint. For example:
2) Imitation does not cease, but at its entrance, R2 moves by a single degree upwards or downwards in order to obtain counterpoint at $Jv=0$.

The following example with unchanged Rispostas would require $Jv=1$. But due to the shift of $R1$ by a degree downwards, $Jv=0$ results at the entrance of $R2$.

---

132 In the text that precedes Example No. 111, instead of "...as a result of this, it is not necessary to write the combination $P+R$ in complex counterpoint", which is present in the original text, "...as a result of this, it is not necessary to write the combination $P+R1$ in complex counterpoint" has been added in the translation. This correction is cited by Wehrmeyer in Taneev, *Die Lehre vom Kanon*, p. 196. Example No. 111 is from Giovanni Pierluigi da Palestrina, "Sanctus" from Missa: *O Regem Coeli*, mm. 1-8. (P.G.)
In the following example, the same $J_v$ results instead of $J_v=-1$ because of the shift of $R_1$ again by a degree downwards at a point that directly precedes the entrance of $R_2$.

No. 113.

\[ \text{Diagram of music notation} \]

In this example:

No. 114.\footnote{133}

P.XIV, 91.

\[ \text{Diagram of music notation} \]

\footnote{133} Giovanni Pierluigi da Palestrina, "Agnus Dei I" from Missa: \textit{Nigra sum}, mm. 1-6. Instead of "P.XIV,111", which is present in the original text, the correction "P.XV,91" has been added in the translation. This correction is cited by Wehrmeyer in Taneev, \textit{Die Lehre vom Kanon}, p. 196. (P.G.)
at the point of entrance of R2, changes in R1 still have not occurred, and R2 still enters at \( Jv = 1 \). But beginning with its following note, R1 shifts by a degree upwards and \( Jv = 0 \) results. In general, this method can be used in a substitution of an easy \( Jv \) for a difficult one, even if the easy \( Jv \) is not equal to 0. In the following example, we encounter such a substitution of indices. Instead of \( Jv = -12 \), which is required by these entrances,

\[
\text{\includegraphics[width=0.5\textwidth]{image.png}}
\]

as a result of the shift of R1 by a degree downwards, \( Jv = -11 \) results.

No. 115.\(^{134}\)
P.XI.28.

\[
\begin{align*}
\text{\includegraphics[width=0.5\textwidth]{image.png}}
\end{align*}
\]

\( Jv = -11 \) instead of \( Jv = -12 \)

\(^{134}\) Giovanni Pierluigi da Palestrina, "Credo" from Missa: Inviolata, mm. 23-27. (P.G.)
Subsequent composers also used these methods, as is apparent from the following place in the Mass in B Minor by Bach ("Et exspecto"): 

No. 116.\textsuperscript{135} 

For clarity, both aspects, which are the basic combination and a combination with a shifted interval, we cite separately: 

From a comparison of these combinations, it is apparent that correct imitation would here require $Jv=-15$: 

\textsuperscript{135} Johann Sebastian Bach, "Et exspecto" from the Mass in B Minor, BWV 232, vol. 6, ed. Bach-Gesellschaft (Leipzig: Breitkopf and Hartel, 1851-1926), mm. 18-24. (P.G.)
At the same time, the shift of the bass by a second upwards requires the simpler index of $J_v = -14$:

3) A canon of the first order is replaced by a canon of the second order as a result of the entrance of $R_2$ at a distance that is not equal to the first distance. $R_2$ either delays its entrance, waiting for a more accessible moment, for example:

No. 117.\textsuperscript{136}
P.XII,59.

or $R_2$ enters earlier than it would have to enter in a canon of the first order, for example:

\textsuperscript{136} Giovanni Pierluigi da Palestrina, *Sanctus* from Missa: Brevis, mm. 1-5. (P.G.)
How to write such canons will be discussed in the next chapter.

4) At equal distances of entrance, R1 imitates the theme of the melody exactly, but the length of its notes changes in order that they can adapt to the entrance of R2. In the following example, part of the Proposta (a) is imitated by the Risposta in augmentation (b), and, in this aspect, it is in counterpoint to R2.

\[\text{Giovanni Pierluigi da Palestrina, "Benedictus" from Missa: Repleatur os meum laude, mm. 39-42. Instead of "P. XII,129", which is present in the original text, the correction "P.XII,130" has been added in the translation. This correction is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 196. (P.G.)}\]
Here we enter the domain of modifications of the theme that can be diversified infinitely, and it is neither necessary nor possible to enumerate and classify all of them. As an interesting sample of such modifications, we will cite the following example:

No. 120.\textsuperscript{139}

P.XII, 129/130.

\footnotesize
\begin{verbatim}
(Jv = -11)
\end{verbatim}
At equal distances of entrance, this imitation would require $Jv = -10$.

The entrance of $R_2$ a half measure later than it would for a canon of the first order removes the necessity of applying this $Jv$. At the same time, the modifications to the theme of $R_1$ are such that the same part of the melody is in counterpoint to the first note of $R_2$ and to the first note of $R_1$, forming a combination at $Jv = -11$.

§ 94. The examples from the works of Palestrina that have been cited provide us with an understanding of which methods composers of the sixteenth century used to facilitate composition at difficult indices. With regard to this, the question suggests itself: Did these composers know the correlation that exists between intervals of entrance and the complex counterpoint that these intervals require? With regard to whether Palestrina's contemporaries knew that the aforementioned entrances at the fifth and octave require the application of double counterpoint at the duodezime, about this there can be no doubt. By that time, the use of these entrances in canonic imitations must have inevitably revealed the link between these entrances and double counterpoint at the duodezime, the rules of which were generally known and set forth in contemporary music texts of those times. The
connection of double counterpoint at the duodezime with the aforementioned order of entrances could have been made clear to the composer even by a different means. Each time there was a change in the order of entrances, he must have invariably encountered obstacles of awkwardness that accompany the application of more difficult and little-known JJv, and he must have observed in them that double counterpoint at the duodezime with an altered succession of entrances is less applicable. Another matter to be solved is the question of whether composers of the strict style knew all of the entrances at which precisely one and not another type of double counterpoint is required, and whether they could with regard to each order of entrances locate the index--this is expressed using our terminology--that corresponds to these entrances. To these questions it is apparently necessary to answer negatively. At least in compositions of the strict style it is difficult to locate confirmation of a contradiction to this opinion. In the same way, in theoretical treatises an explanation of these questions is absent. If we regard as a fact that musicians of an earlier time, similar to medieval architects, held in secret methods of their craft, then this opinion can pertain only to those methods that they used in compositions, and we speak here namely about the traces of which are imperceptible in their works.
§ 95. The section on three-voice canonic imitation which has been examined here has great importance for compositions in the contrapuntal style. This is confirmed by the abundance of examples of the application of this imitation that present themselves to us in contrapuntal compositions of the strict and free style. But in vain we would seek information that pertains to this section in theoretical works of earlier times and of our time. These canons are mentioned only in passing. The very separation of canons with equal distances of entrance from those with unequal, which substantially distinguishes one from the other according to the methods of their composition, is absent in textbooks. Examples of both orders are quite often introduced haphazardly in a jumble, and instructions about how to write them are not provided, if the advice given in many textbooks is not considered as instruction--alter and improve that which has been written in canon until the time when you come across counterpoint that will be suitable for all voices. The analysis of canons, in the majority of cases, comes down to a description of at which interval the voices enter. Regarding the participation of moveable-counterpoint in canons, almost nothing is said. An exception with regard to this is the textbook by Bussler.\footnote{As stated previously, Taneev translated Bussler's Der Strenge Satz into Russian. (See the first footnote of Chapter 1.) (P.G.)}
In this textbook, canons are divided as such: those where "double counterpoint" is applied and those that are written with the assistance of simple counterpoint. But such designations are very incomplete. In the first place, it is not known to which category canons where the Jv is a direct shift will be added, and also to where all canons with unequal distances of entrance will be placed. But it is necessary to give credit to Bussler in that he, more clearly than all others, perceived the link between complex counterpoint and canon and gave several rules of instruction, for example, regarding the application of double counterpoint in two-voice infinite canons.

§ 96. The insufficiency of theoretical information about this section of canonic imitation is all the more perceptible because this contrapuntal form could have great application even in contemporary music, which, in general, displays a striving towards contrapuntal writing. All of the forms of canonic imitation that the old masters used could even now find an application in compositions that are filled with new musical content. But the contemporary composer, who has been armed with all of the improvements of the latest techniques: chromaticism, modulations, an abundance of independent dissonant combinations, etc., is not limited by those bounds in which earlier composers maneuvered. Thus, in canonic imitation, the old masters particularly valued those entrances which, in the diatonicism of melodies of those
times, provided a possibility for melodies to be imitated exactly. For the contemporary composer, such considerations have far less significance. Using accidentals, modulations, etc., he will achieve exact imitation at any desired entrances. There exists an established opinion that the Netherland and Italian composers of the strict style exhausted all cases of canon. In refutation of this opinion, it is possible to indicate a whole area of canonic imitations that apparently still have not been attempted. These cases come down to two categories: a) Unusual entrances at generally used JJV; and b) Entrances that require the application of rarely encountered JJV. Cases of the first category are fully accessible to the student and with benefit may be applied in his works. Cases of the second category are more fit for the fully experienced and mature contrapuntalist. Both of these open for the contemporary composer many new combinations that so far have not been attempted. But even limiting oneself to those canonic forms that were used by masters of the strict style, and having freely mastered their use, the contemporary composer has already introduced something new into his compositions, since much of that which was known and able to be done by earlier masters has been completely forgotten by our contemporaries.
Chapter 8

The three-voice finite canon of the second order

The distances of entrance are different (a[b])\textsuperscript{141}

The application of horizontal-shifting counterpoint

§ 97. In the section on three-voice canon that was just covered, the distances of entrance were identical. As a result of this, between R1 and R2 combinations of sections occurred that were directly before in the Proposta and R1. By comparing the combinations of sections to one another and examining one as original and the other as derivative, we locate the Jv whose conditions the given canon will obey. But when R2 enters at a different distance from R1 than R1 entered from the Proposta, there occurs combinations of sections between both Rispostas that previously did not coincide.

§ 98. As an example we will take the following schema:

\[
\begin{align*}
(R2) & & A2 & B2 & C2 \\
(R1) & & A1 & B1 & C1 & D1 & E1 \\
(P) & & A & B & C & D & E
\end{align*}
\]

\textsuperscript{141} "(a)", which is present in the original text, appears to be an error. The correction "(ab)" has been added in the translation. This correction has been added in order to be consistent with Chapter 6, "Explanatic for the table of the three-voice canons of the first order (aa)". (P.G.)
At the entrance of R2, the combinations of sections A2+C1, B2+D1, etc., occur. Each of these combinations belong to both Rispostas, and on this basis they should be examined as derivative (§ 73). But the derivative does not have an original that corresponds to itself, and the latter must be established by means of an imaginary combination. Since this imaginary combination substitutes as an original, the Proposta must form a part of it. The imaginary combination, consequently, will be formed by the voices of the principle and the imaginary (P+R...), and its derivative by the voices R1+R2. The imaginary combination must be similar to the combination R1+R2 with regard to distance, direction, and interval of entrance. For the determination of the point and interval of entrance of the imaginary voice, one should measure out in the reverse direction from R2 a distance and interval of entrance equal to the distance and interval of entrance of R1 (see § 52). The preceding schema, with an addition of sections of the imaginary voice, will obtain the following aspect:

(R...)   A...B...C...
(R2)     A2  B2  C2
(R1)     A1  B1  C1  D1  E1
(P)       A   B   C   D   E
The point of entrance of R... is determined by the distance of entrance between the Proposta and R1, having been taken from R2 in the reverse direction, i.e., to the left, namely, R1 enters a section later than the Proposta, and R... enters a section earlier than R2.

By means of this, in the imaginary combination there occur the same coincidences of sections that are in the combination of the Rispostas among themselves: A...+C, B...+D, etc.

We will assume that intervals of entrance in this canon are as follows:

i.e., R1 enters one section later than the Proposta and at a fourth above, R... must enter one section earlier than R2 and (as we saw in the previous schema) at a fourth below.
If the first distance of entrance (between the Proposta and R1) is greater than the second (between R1 and R2), then the imaginary voice will enter earlier than R1. For example:

```
P \overrightarrow{R_4} \overrightarrow{R_5} \overleftarrow{R_{...}}
```

Thus, the schema of the finite three-voice canon of the second order is formed in the same way as the schema of the two-voice infinite canon of the same order, with the only difference being that R2 takes the place of //P in the three-voice canon.

We examined the combinations P+R... and R1+R2. The combination P+R2 remains to be examined. Since R2 is the last Risposta in the canon, its combination with the principle voice is written in simple counterpoint, and to this combination is applied the third of the additional rules that deal with the use of the fourth (§ 89).

§ 99. We cite a canon by Palestrina that has been written according to the preceding schema and which is presented as a separate, complete work.
No. 121.142

P.XI, 65.

Giovanni Pierluigi da Palestrina. "Sanctus" from Missa: Ad fugam, mm. 19-38. The citation "P.XI, 65", which is missing from the original text, has been added in the translation. This correction is cited by Wehrmeyer in Sergej Taneev, Die Lehre vom Kanon, trans. Andreas Wehrmeyer (Berlin: Ernst Kuhn, 1994), p. 196. (P.G.)
§ 100. The procedure for writing this canon is as follows. Up to the entrance of the third voice—whether this voice will be imaginary or real—the canon is written as a typical two-voice canon. (Regarding the possibility for the free use of the fourth in the imaginary combination, this is discussed below at § 102.) The part of the Proposta that has been written is rewritten at the required interval in all of the remaining voices, including here even the imaginary voice (see the second schema of sections at § 98). Then the Proposta is gradually composed one section at a time. Each section of the Proposta that has been composed is transferred to all voices. In so doing, the Proposta coincides with all of the voices of a given vertical column in order to form correct three-voice counterpoint with the real voices and, separately from them, correct two-voice counterpoint with the imaginary voice.

We will write out the imaginary combination for the canon of the second order cited above (§ 99).
§ 101. The rule for the use of the fourth and ninth in the imaginary combination: If both Rispostas are in the middle and upper voice—or stated another way, if the canon begins with the bottom voice—then in the imaginary combination a free use of the fourth is allowed; the ninth that is dissonant above (8) is not allowed. In all other cases, the imaginary combination is written according to the rules of two-voice simple counterpoint, to which hidden successions may be introduced to the degree that they are permissible in multi-voice counterpoint (§ 84). In the following example, the imaginary combination begins with an
unprepared fourth, which will be transferred to real voices where it turns into a consonance.

No. 123.

In this example, the first note of R2 has been reduced by half of its length. The change in the first note of the imaginary voice corresponds to this. A free use of the fourth in the imaginary combination is also encountered in the following example:
We will note the following peculiarity of this example. At the beginning of the seventh measure, R1, by shifting to free counterpoint, ceases to imitate the Proposta. But this part of the melody, which is independent of the Proposta, is imitated by R2 at the beginning of the ninth measure. Thus, beginning from the seventh measure, a two-voice canon is made from a three-voice, and the former R1 becomes its Proposta. In R1 we indicated this change as (=P), i.e., from this point R1 serves as the Proposta for a two-voice canon. Its Risposta is the former R2, which has been indicated as (=R1).

143 Giovanni Pierluigi da Palestrina, "Benedictus" from Missa: Repleatur os meum laude, mm. 1-9. The tie between the two half notes in measures 7-8 in R1 has been restored in the translation. This correction is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 196. (P.G.)
§ 102. One should not mix the aforementioned cases of
the free use of the fourth together with the so-called
c consonant fourth with regard to a tied note of the lower
voice (see Bellermann, Der Contrapunct, p. 220), which enters
as a syncopation as if preparing itself. For example:

Such a use of the fourth is always permissible in the
imaginary combination, even if one of the Rispostas is the
bottom voice. For example:

---

144 Heinrich Bellermann, Der Contrapunkt, 4th ed. (Berlin, 1901),
p. 220. In the first schema, the indications C.F. and * have been added
in order to correspond to the second schema. These additions are cited
by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 196. (P.G.)
§ 103. Regarding the length of the imaginary combination, it will be necessary to note the following. The imaginary combination is a future combination of two Rispostas. In the combination of the latter (R1+R2), if we note the place where one of them ceases its imitation, then, at the corresponding place in the imaginary combination, the imaginary voice must be interrupted. If the Proposta in the imaginary combination shifts to free counterpoint earlier than this point, then from this point the imaginary combination becomes superfluous.

145 Joannes Mouton, "Miseremini me," mm. 3-11. (P.G.)
§ 104. In multi-voice compositions in the strict style, quite often three voices form canonic imitation while the remaining serve as supplementary to them. We cite the beginning of a canon that has been extracted from a six-voice Agnus Dei by Palestrina. In the fifth measure, the fourth that is encountered turns into a consonance by means of a supplementary voice.
Giovanni Pierluigi da Palestrina, "Agnus Dei II" from Missa: Primi Toni, mm. 1-9. The bass clef in the lowest voice has been replaced by a baritone clef. This correction is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 196. (P.G.)
§ 105. Completing the doctrine of the three-voice finite canon, we will mention the significance that this canon has for exercises in moveable counterpoint. The application of the latter in three-voice canon is in forms far more diverse than the application of it in two-voice infinite canon. The necessity for the Proposta to be in counterpoint to R1 while counterpointing to an imaginary voice; furthermore, to form correct three-voice counterpoint together with both Rispostas, is a requirement that is absent from two-voice canon. This new requirement, which has been introduced by three-voice canons, while complicating the task, greatly assists the development of facility and flexibility in the skillful use of moveable counterpoint. We have not yet spoken about the advantage that three-voice finite canon has over two-voice infinite canon as applied to composition. Examining three-voice canon only from a pedagogical viewpoint, as material for exercises in two-voice moveable counterpoint, one should give it preference even over four-voice. While four-voice canon must continually deal with forms that require the application of three-voice moveable counterpoint, three-voice canon, in its entirety, only requires the application of two-voice moveable counterpoint. Therefore, in a textbook, for the study of the latter, three-voice finite canon should occupy the first place among all others.
II. Four-voice finite canon

Chapter 9

Four-voice finite canon of the first order

The distances of entrance are identical (aaa)

The application of vertical-shifting counterpoint

§ 106. We will examine a schematic depiction [of a four-voice finite canon of the first order:][1]

\[
\begin{array}{cccccc}
(R3) & A3 & B3 & C3 & D3 & E3 & F3 \\
(R2) & A2 & B2 & C2 & D2 & E2 & F2 \\
(R1) & A1 & B1 & C1 & D1 & E1 & F1 \\
(P) & A & B & C & D & E & F \\
\end{array}
\]

It is known (§ 73) that in the finite canon of the first order each combination of the Proposta with each of the Rispostas, with the exception of the last, is an original combination. It is also known that the number of original combinations in each finite canon is equal to the number of voices of the canon minus two (§ 75). Thus, in the four-voice finite canon of the first order, two combinations are original: P+R1 and P+R2. Each original combination has as many derivatives as there are Rispostas that enter after the

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[1] On the top line of this schema, instead of "A", which is present in the original text, the correction "A3" has been added in the translation. This correction is cited by Wehrmeyer in Sergej Taneev, *Die Lehre vom Kanon*, trans. Andreas Wehrmeyer (Berlin: Ernst Kuhn, 1994), p. 196. (P.G.)
one that forms part of the original. In the four-voice canon, since there are three Rispostas, the original combination P+R1 has two derivatives: R1+R2 and R2+R3, and the original P+R2 has one derivative, R1+R3, as is apparent from the following schema:

![Diagram of canon schema]

We will verify what has been said in [the aforementioned schema, having divided it into sections that correspond to the first original and its derivatives, as is apparent below:

**(R3)**

\[
\begin{array}{ccccccc}
A3 & B3 & C3 & D3 & E3 & F3 \\
\end{array}
\]

**(R2)**

\[
\begin{array}{ccccccc}
A2 & B2 & C2 & D2 & E2 & F2 \\
\end{array}
\]

**(R1)**

\[
\begin{array}{ccccccc}
A1 & B1 & C1 & D1 & E1 & F1 \\
\end{array}
\]

**(P)**

\[
\begin{array}{ccccccc}
A & B & C & D & E & F \\
\end{array}
\]

Here we see that at the entrance of R1 a combination of sections A1+B results. These same sections coincide twice more between themselves in the further course of the canon—at each entrance of a new Risposta, namely, A2+B1 and A3+B2.
In the combination P+R we took sections A and B, but even those following after these, which coincide in the voices as B1+C, C1+D, etc., for their part repeat twice: in the combination R1+R2 and in the combination R2+R3, shifting in the same order. Thus, P+R1 is an original combination and R1+R2 and R2+R3 are its derivatives.

We turn to the following original: P+R2. [According to the schema that has been cited below:]

\[
\begin{array}{ccccccc}
\text{(R3)} & \text{A3} & \text{B3} & \text{C3} & \text{D3} & \text{E3} & \text{F3} \\
\text{(R2)} & \text{A2} & \text{B2} & \text{C2} & \text{D2} & \text{E2} & \text{F2} \\
\text{(R1)} & \text{A1} & \text{B1} & \text{C1} & \text{D1} & \text{E1} & \text{F1} \\
\text{(P)} & \text{A} & \text{B} & \text{C} & \text{D} & \text{E} & \text{F}
\end{array}
\]

coinciding sections at the entrance of R2: A2+C, coincide only one time further, at the entrance of the last Risposta in the aspect of A3+C1, i.e., at R3 and R1. The subsequent coincidence of sections of P+R2 repeats in exactly the same way, it has been shifted in the same order at R1 and R3. Thus, the latter combination is derivative with regard to the original P+R2. This original does not have another derivative since R3, forming a part of its derivative, is the last Risposta in the canon (§ 75). This original is distinguished from the combinations that have been examined up to now because the voices that form it do not enter directly one after the other, but across one voice (across
R1). Such an entrance of voices is also in its derivative combination.

We have examined two original combinations: P+R1 and P+R2, together with their derivatives. As for the combination of the Proposta with the last Risposta in the canon (P+R3), this combination, which does not have a combination similar to itself in the subsequent course of the canon, can neither be original nor derivative and is written in simple counterpoint. (Regarding the use of the fourth in this combination, see § 89 above, rule three.)

§ 107. The original P+R1, which has two derivatives, is written at a dual Jv that we will indicate as Jv'. The first of the two indices of the double Jv serves for the first derivative (R1+R2); the second, for the second derivative (R2+R3). [In order to present this more graphically, we will take the following schema of entrances of a four-voice finite canon:

In this schema, we see the first original combination (P+R1) with its two derivatives. Jv' of the first derivative:
which in this case still can be indicated as \(Jv'\), unlike the \(Jv\) of the second derivative, is equal here to \(-m+n\) (§ 36), i.e., \(-4+4=0\).

\(Jv'\) of the second derivative, which can be indicated as \(Jv'^2\), unlike \(Jv'\), as is apparent from the following schema:

\[
\begin{array}{c}
\alpha \\
\alpha'
\end{array}
\]

equals \(-m-n\), i.e., \(-4-3=-7\). Thus, \(Jv'\) (i.e., the double \(Jv\) of the first original combination) will be \(Jv'=0, -7\).

As for the \(Jv\) of the second original combination \((P+R2)\), which we will indicate as \(Jv''\), in accordance with the following schema:

\[
\begin{array}{c}
\alpha \\
\alpha''
\end{array}
\]

it will equal \(-8+1=-7\), i.e., it will equal the value of \(Jv'^2\).

From this we can observe that \(Jv'\) always will be the same for a three-voice canon that begins with the same degrees. [We will cite here an entire schema of entrances of a four-voice finite canon that up until now we have cited only in parts (separately for each derivative combination):
and we will also provide a summary of all indices that these entrances require:

\[ J' = -4 + 4 = 0 \quad \text{or} \quad J' = 0, -7 \]
\[ J'' = -4 - 3 = -7 \]
\[ J'' = -8 + 1 = -7 \]

§ 108. We will cite two more similar schemata:

The indices of schema a) will be as follows:

\[ J' = 0, -9 \]
\[ J'' = -9 \]

The indices of schema b) are as follows:\(^{148}\)

\[ J' = -10, -2 \quad (J' = -3 - 7 = -10, J' = -3 + 1 = -2) \]
\[ J'' = 2 \quad (J'' = -4 + 6 = +2). \]

As is apparent from the schemata of entrances that have been examined, between \( J' \) (which pertains to the second derivative: \( R_2 + R_3 \)) and \( J'' \) (\( R_1 + R_3 \)) there exists a continual correlation that is reflected by both of these indices always being of the same quantity. In so doing, their signs are either identical (i.e., both indices are positive or

---

\(^{148}\) Following Wehrmeyer's translation, parenthesis have been added to "\( J' = -3 - 7 = -10, J' = -3 + 1 = -2 \)". This addition is cited in Taneev, Die Lehre vom Kanon, p. 196. (P.G.)
negative) or different (one Jv is positive while the other is negative).

Both of these indices are completely identical (according to both quantity and sign) in that case when, of the first three voices of the canon, the Proposta is an outside voice.

Both of the indices are identical according to quantity and different according to sign (i.e., one of them is positive and the other negative) when, of the first three voices of the canon, the Proposta is a middle voice.

§ 109. [In order for what has been said to be still more graphic, we will cite the following example of the beginning of a four-voice finite canon that has been written according to the schema:]

No. 127.149

P.XX, 110.

---

149 Giovanni Pierluigi da Palestrina. "Gloria" from Missa: Alma Redemptoris, mm. 21-4. (P.G.)
[The first original of this canon has been written at a double $Jv'=-11$, 0, as is apparent from a comparison of this original with both of its derivatives:]

No. 128.

$Jv = -11$, 0

$Jv = 0$

$Jv = 0$

[The second original of this canon has been written at $Jv''=0$, as is apparent from the following example, which contains both the original and derivative combination:]^{150}

No. 129.

$Jv = 0$

$Jv = 0$

^{150} "Jv=-0", which is present in the original text, has been changed to "Jv=0" in the translation. (P.G.)
The classification of four-voice finite canons of the first order

§ 110. Each three-voice canon can serve as a beginning for many four-voice canons. The three-voice canon is determined entirely by the first interval of entrance (with an indication of its direction) and by the index (§ 40).151 By means of these givens, the second interval of entrance is determined. For a canon of four voices, it will also be necessary to add to these givens [a definition of a third interval of entrance, i.e., to locate] Jv'2, which pertains to the second derivative [of the first original combination], (R2+R3), in other words, for a canon of four voices, a double index must be indicated (Jv'). This is determined by both the size and direction of the third interval of entrance and by the index of Jv", which is situated in a continual correlation with Jv'2, as was indicated above (§ 108). Speaking theoretically, the entrances of each three-voice canon can serve as the beginning of as many four-voice canons as there are JJv. Retaining without change the initial entrances of any three-voice canon, we can in turn take one index after the other as Jv'2. With a change in the quantity of Jv'2, the third interval of entrance will also change; thus, we will obtain a list of all four-voice canons that

151 Instead of "(§ 76)", which is present in the original text, "(§ 40)" has been added in the translation. (P.G.)
begin with the same entrances of the first three voices. If the number of all possible entrances in two-voice canon will be designated by the letter \( r \) and the number of widely-used JJv by the letter \( s \), then the expression \( r \times s \) will indicate the number of possible three-voice canons, and \( r \times (s^2) \) will indicate the number of four-voice canons. Considering that \( r = 14 \) and \( s = 20 \) approximately, the number of three-voice canons with different entrances is equal to \( r \times s = 280 \), and \( r \times (s^2) = 5600 \) equals the number of canons with four voices. This number must be reduced to significantly more modest dimensions since a large number of canons that require the application of difficult JJv must be added to the unsolvable.

The abundance of possible entrances for canons compelled theoreticians to think that it was not possible to establish definite rules for the writing of canons, but from the content of this work it is clear that the number of combinations does not serve as an obstacle to classification of all of them by exact, definite rules.

As a pattern for classification of the four-voice canon, we will cite two examples:

1) canon at an upper third at \( Jv^1 = 2 \), and
2) canon at an upper fifth at \( Jv^1 = -11 \).
Preserving the two initial intervals of entrance without change, as $Jv'_{2}$ we take all $JJv$ one after the other, as a result of this, the third interval of entrance changes each time.

1) canon at the upper third ($Jv'_{2}=2$):

\[
\begin{align*}
&Jv'=2.0 \quad Jv'=2.1 \quad Jv'=2.2 \quad Jv'=2.3 \quad Jv'=2.4 \\
&Jv'=2.0 \quad Jv'=2.1 \quad Jv'=2.2 \quad Jv'=2.3 \quad Jv'=2.4
\end{align*}
\]

2) canon at the upper fifth ($Jv'_{2}=-11$):

\[
\begin{align*}
&Jv'=-11.0 \quad Jv'=-11.1 \quad Jv'=-11.2 \quad Jv'=-11.3 \quad Jv'=-11.4 \\
&Jv'=-11.0 \quad Jv'=-11.1 \quad Jv'=-11.2 \quad Jv'=-11.3 \quad Jv'=-11.4
\end{align*}
\]

Both of these tables of entrances are incomplete since they may easily be completed by the student. They have been formed according to the same principle that formed the "Table of Three-Voice Canons" (§ 83), for $JJv'_{2}$ they go in both an ascending (the first line) and descending order (the second line of the example). Each of the entrances of "The Table of

\[\text{footnote}{152} \text{ A negative sign has been added to } JJv'_{2} \text{ of Examples 2-5 on the second line. (P.G.)}\]
Three-Voice Canons" can be positioned as the basis for a row of four-voice canons (forward and in reverse).

As for additional rules [for the use of the fourth and ninth in the four-voice canon,] they are the same as those for the three-voice canon (§§ 89-91).

Exercises in four-voice finite canons of the first order

§ 111. [In four-voice finite canons of the first order, as also in three-voice canons,] one should begin exercises with canons where all JJv=0. Then [it will be necessary] to proceed to those canons where Jv'2=0 and, consequently, Jv''=0. Of these canons, one should give preference to those where Jv'1 belongs to the category of the most simple: -7, -9, -11. Following these exercises, it will be possible to proceed to those canons where Jv'2 does not equal 0, choosing for this index the simplest cases.\textsuperscript{153} Canons that are encountered more often than others in the musical literature, besides canons at the prime or octave, in the majority of cases have been written at Jv'=-11, 0. Quite often canons at Jv'=-11, -7 or Jv'=-11, 7 are also encountered. The presence of the latter indices (Jv=-7, Jv=7) causes a shift of R3 to a different octave.

\textsuperscript{153} In this sentence, "Jv'1", which is present in the original text, has been replaced by "Jv'2". (P.G.)
Chapter 10

The four-voice finite canon of the second order

All or several distances of entrance are different. The application of horizontal-shifting counterpoint alone or in combination with vertical-shifting

§ 112. In the four-voice canon, if even one of the distances between the entrances is different from the others, then such a canon will be added to canons of the second order. By indicating similar distances between entrances with identical letters and dissimilar with different letters, we will obtain the following aspects of four-voice canons of the second order: aba, aab, abb, abc:

With the exception of the first case, aba, in all other cases subdivisions are encountered that rely on whether the sum of two neighboring distances is equal to the third or whether they are unequal. In order to indicate that two distances are equal to the third, we will connect the appropriate letters with a bracket below. For example, a\textsubscript{ab} indicates that in the given canon the first two distances equal the
third; \textit{abb} indicates that the second and third distances are equal to one another and, at the same time, are equal to the first. If a bracket is absent, it is assumed that in the canon there are not two distances whose sum is equal to the third distance. Thus, canons aab and abb each have two subdivisions: aab and \textit{aab} then abb and \textit{abb}. In the form abc, there are three possible subdivisions: abc, \textit{abc}, and \textit{abc}. The aforementioned Example b), aab, is essentially \textit{aab}, and Example c) is \textit{abb}. Example abc is an example in which all of the distances of entrance are different. Examples \textit{abc} and \textit{abc} we provide below:

\begin{center}
\includegraphics[width=0.8\textwidth]{example.png}
\end{center}

§ 113. In the instance of equality of \{two neighboring distances of entrance to the third,\} the position of the imaginary combination provides the Jv. [We will test this in the aforementioned examples. We already know (§ 73) that the number of derivative combinations in each finite canon is equal to the number of paired combinations of all Rispostas in the canon.] In the four-voice canon, there are three such combinations: \textit{R1+R2}, \textit{R2+R3}, and \textit{R1+R3}. Each derivative must have its own original, which can either be a real combination
(P+R) or imaginary (P+R...), and in both of these original combinations the Proposta must participate. In order to locate an original combination for each of the derivatives, it will be necessary to set aside to the right of the Proposta a distance equal to the distance between the voices of the derivative; thus, we will locate the point of entrance for the voice that forms with the Proposta an original combination with regard to a given derivative. If this point coincides with the entrance of one of the Rispostas, then the Proposta together with this Risposta is an original real combination, and for this combination, it will be necessary to locate a Jv. If this point occurs at a place where there is no entrance of either of the Rispostas, then at this place an imaginary voice must enter. Its combination with the Proposta forms the sought-after original combination which, consequently, will be imaginary.

[We will turn now to examples. Example a), aba, has three derivatives for which it will be necessary to locate originals. These derivatives are R1+R2, R2+R3, and R1+R3. The original for the derivative R1+R2 will be P+R..., which is apparent from the schema cited below, where to the right of the Proposta has been set aside an interval equal to the interval between the voices of the derivative and at the same distance:
This Risposta is imaginary and its combination with the Proposta provides the imaginary combination \(P+R\ldots\), the \(Jv\) of which will be 0. Proceeding to the derivative \(R_2+R_3\), we find that its original will be \(P+R_1\) and \(Jv=0\), as is apparent from the following schema:

where the interval that has been positioned to the right of the Proposta coincides not only in distance, but also in size with \(R_1\). The derivative \(R_1+R_3\) has as its original the combination \(P+R_2\) also at \(Jv=0\), as is apparent from the following schema:

The interval that has been positioned to the right of the Proposta coincides with \(R_2\). The lettered schema of this canon, similar to those schemata that we employed for four-voice finite canons of the first order, is as follows:

\[
\begin{align*}
(R_1) & \quad A_1 & B_1 & C_1 & D_1 & E_1 & F_1 \\
(R_3) & \quad A_3 & B_3 & C_3 & D_3 & E_3 & F_3 \\
(P) & \quad A & B & C & D & E & F \\
(R_2) & \quad A_2 & B_2 & C_2 & D_2 & E_2 & F_2 \\
(R\ldots) & \quad A\ldotsB\ldotsC\ldotsD\ldotsE\ldotsF\ldots
\end{align*}
\]
from which it is apparent that the derivative combination A₂+C₁ does not have its own real original, which must be restored to it by the imaginary combination A...+C.

Example b), aabb, has the following originals for its derivatives:

1) P+R₁, Jᵥ=−11

2) P+R₂, Jᵥ=−7

3) P+R..., Jᵥ=0

In the first and second of the schemata that have been cited above, the intervals that have been positioned to the right of the Proposta, coinciding with the entrances of R₁ and R₂, are original real combinations at the JJV that have been

---

154 Instead of "1) P+R₁, Jᵥ=11", which is present in the original text, "1) P+R₁, Jᵥ=−11" has been added in the translation. (P.G.)
indicated above, and only the third schema provides an imaginary combination. We will note the affiliation of the entrances of this example with the group of entrances where two distances are equal to the third (§ 112).

Example c), \(\text{abb}\), has the following originals for its derivatives, if they are examined separately:

1) \(P+R\ldots, Jv=0\):

\[
\begin{array}{c}
\text{P} \\
\text{R}...
\end{array}
\]

2) \(P+R\ldots, Jv=0\):

\[
\begin{array}{c}
\text{P} \\
\text{R}...
\end{array}
\]

3) \(P+Rl, Jv=-7\):

\[
\begin{array}{c}
\text{P} \\
\text{R}...
\end{array}
\]

In the third of the aforementioned schemata, the interval that has been set aside to the right of the Proposta coincides with \(Rl\) according to distance, and the Rispostas of the first two schemata coincide with each other, due to this,
they must be replaced by a single Risposta that has been written at \( Jv=-11 \). Thus, this canon has only two original combinations: one imaginary at \( Jv=-11 \) and the other real at \( Jv=-7 \).

Example d), abc, consists entirely of imaginary original combinations. If each original is examined separately, then there will be three:

1) \( P+R... \), \( Jv=0 \):

![Diagram](image1)

2) \( P+R... \), \( Jv=0 \):

![Diagram](image2)

3) \( P+R... \), \( Jv=0 \):

![Diagram](image3)

Examples e) and f) belong to a category of canons in which the sum of two entrances is equal to the third, due to this, one of the intervals that has been set aside to the right of the Proposta coincides with a real voice.
Example e), abc, has three original entrances:

1) P+R..., Jv=0:

\[\text{Diagram}\]

2) P+R2, Jv=-7:

\[\text{Diagram}\]

3) P+R..., Jv=-0:

\[\text{Diagram}\]

The Jv of the second original is equal to -7 because the interval that has been set aside in the schema of this original to the right of the Proposta rhythmically coincides with R2.

Example f), abc, also has three original entrances:

1) P+R..., Jv=0:

\[\text{Diagram}\]
2) $P + R \ldots, Jv = 0$:

3) $P + R_l, Jv = -7$:

In the third schema of the canon that was just cited, we again locate a real original combination because the interval that has been set aside to the right of the Proposta rhythmically coincides with $R_l$.

§ 114. The row of examples cited above (with different distances between entrances) should assist the student in the search for both the original entrances for all derivatives of each four-voice finite canon of the second order and for their indices. From this we see that each imaginary combination must be with regard to its derivative, above all, a combination at $Jv = 0$, i.e., it will precisely reproduce this derivative. But cases exist, [as we have seen], when the imaginary combination has two derivatives (i.e., two combinations with distances between entrances of voices that are identical to the imaginary combination). Then the imaginary combination must be written in vertical-shifting
counterpoint and yields to the conditions of the \( J_v \) that is required by its second derivative. As is already known, the number of imaginary combinations and \( J_{Jv} \) (it is all the same whether the \( J_v \) belongs to a real or to an imaginary combination) in all is equal to the number of derivatives, in the four-voice canon, consequently, it is equal to three. The easiest canons in this section [are those] in which there is one imaginary combination and all \( J_{Jv} \) are equal to 0. These canons should be the primary material for exercises. Compared to three-voice canons of the second order, they present the next level of difficulty—the Proposta, forming a part of a two-voice combination, is at the same time a member of a four-voice real combination. An increase in the number of imaginary combinations, as well as the replacement of \( J_{Jv} \) that equal 0 by others, even if they are the simplest \( J_{Jv} \), significantly expand the difficulty of writing this canon.

§ 115. [We will cite several examples of four-voice finite canons of the second order, providing, as we did in the chapter on four-voice finite canons of the first order, only the beginning of the canon and omitting the conclusions, which are written in simple counterpoint.

Canon aba:
No. 130.\textsuperscript{155}

Ludwig Senfl

(G.D.294)

has been written according to the schema:

and has three original combinations: \( P+R\ldots \), \( P+R_1 \), and \( P+R_2 \), the JJv of which are equal to 0. Example No. 131 provides the first of these original combinations with its derivative:

No. 131.

\begin{itemize}
  \item Original (Measures 1-4)
  \item Derivative (Measures 1-5)
\end{itemize}

\textsuperscript{155} Ludwig Senfl, "Deus in adjutorium," mm. 56-62. (P.G.)
Like the first original, Examples No. 132 and 133 provide the second and third original combinations of this canon with their derivatives at $Jv=0$.

No. 132.

In Example No. 130, the voices of the canon entered in succession of pitch, beginning from the bottom. In Example No. 134, which also has been written according to the formula aba, the entrances of the voices have been divided.
No. 134, which also has been written according to the formula aba, the entrances of the voices have been divided.

No. 134.\textsuperscript{155}

P.X. 160.

A schema of the entrances of this canon:

\begin{center}
\includegraphics[width=0.8\textwidth]{schema.png}
\end{center}

The original combinations of this canon are $P+R\ldots$, $P+R_1$, and $P+R_2$. For all of the original combinations the $JJ_v$ are equal to 0, as is apparent in the following excerpts from this canon of all three originals with their derivatives:

No. 135.

\begin{center}
\includegraphics[width=0.8\textwidth]{excerpts.png}
\end{center}

\textsuperscript{155} Giovanni Pierluigi da Palestrina, “Gloria” from Missa: Sine nomine. mm. 110-14. (P.G.)
The following canon, which has been written according to the same formula, aba:
No. 138.\textsuperscript{157}

Josquin

(G.D.320)

has as its basis a different schema of entrances and distances:

Its originals (at $jv=0$) are as follows: $P+R_\ldots$, $P+R_1$, and $P+R_2$, as is apparent in Examples No. 139, 140, and 141 where they have been given together with their derivatives:

\textsuperscript{157} Josquin des Pres, "Ave Maria," mm. 64-73. (P.G.)
No. 139.

\[ P + R \]

\[ Jv = 0 \]

Original

Derivative

No. 140.

\[ P + R_1 \]

\[ Jv = 0 \]

Original

Derivative

No. 141.

\[ P + R_2 \]

\[ Jv = 0 \]

Original

Derivative
The following canon has been written according to the formula \( abb \):

No. 142.\(^{158}\)

Heinrich Isaac

(G.D.310)

Its schema:

It has only two originals: \( P+R1... \) and \( P+R2... \) (see § 113, Example c)).\(^{159}\) The index of the first original, which has two derivatives, is -11; the index of the second original is

\(^{158}\) Heinrich Isaac, "Anima mea," mm. 30-8. (P.G.)

\(^{159}\) Instead of "(see § 113, Example b))", which is present in the original text, "(see § 113, Example c))" has been added in the translation. (P.G.)
0, as is apparent from Examples No. 143 and 144, which have been extracted from the canon:

No. 143.

\[
\begin{align*}
\text{Original} & : J_0 = 0 \\
\text{First derivative} & : J_1 + R_1 = 0 \\
\text{Second derivative} & : J_2 + R_2 = 11
\end{align*}
\]

No. 144.

\[
\begin{align*}
\text{Original} & : J_0 = 0 \\
\text{Derivative} & : J_1 = 0
\end{align*}
\]

The last canon that we cite has been written according to the formula abc:
The schema of this canon:

Two of its original entrances are imaginary, one is real—P+R2. The imaginary combinations, according to the general rule, have $J_\nu=0$, as for the real combination, its index equals -14. The three examples below (No. 146, 147, and 148) provide us with extractions from the canon of all three of its original combinations, each with its own derivative.

---

160 Giovanni Pierluigi da Palestrina, "Sanctus" from Missa: Iste Confessor, mm. 1-7. (P.G.)
No. 146.

\[ P + R_1 \quad \text{Original} \]

\[ R_1 \quad \text{Derivative} \]

\[ J_V = 0 \]

No. 147.

\[ P + R_2 \quad \text{Original} \]

\[ R_2 \quad \text{Derivative} \]

\[ J_V = -14 \]

No. 148.

\[ P + R_3 \quad \text{Original} \]

\[ R_3 \quad \text{Derivative} \]

\[ J_V = 0 \]
In Example No. 148, in both the original and derivative combinations, we encounter a succession of two fourths—one is augmented and one is perfect. The possibility for an appearance of these fourths in the imaginary combination has been caused by the circumstance that its derivative combination, while being a real combination, is simultaneously a combination of middle voices that allows a free use of the fourth with a supporting bass (see the rule of the fourth § 89).

**Comment:** In the search for original combinations of a four-voice finite canon, it is possible to offer a second way. In the following schema, which has been borrowed from § 115 (the schema of entrances of the canon at No. 142):

![Schema 1](image1)

the initial intervals of the first entrances and their directions are as follows: ^7, ^4. Initially, we will measure the first of these intervals in the opposite direction from R2, i.e., to the left and upwards. We obtain:

![Schema 2](image2)
Then we add to this the two initial intervals, which have been set aside from R3 in the same reverse order:

![Diagram of initial intervals]

after this, we obtain a full schema of original entrances of this canon:

First original combination $Jv=0$

Second original combination $Jv=-11$

Third original combination $Jv=0$

which can be abbreviated in the following way:

First and second original combinations $Jv=0$, $-11$

Third original combination $Jv=0$
and in which the first original combination will be the Proposta plus the interval that has been measured to the left of R2, the second original combination will be the Proposta plus the second interval that has been measured to the left of R3, and the third original combination will be the Proposta plus the first interval that has been measured to the left of R3. The following three combinations in succession will be the derivatives for these three originals: R1+R2, R2+R3, and R1+R3.

We will take another schema from this same paragraph (the schema of entrances of the canon at No. 130):

\[
\begin{array}{c}
\hline
\hline
\hline
\hline
\hline
\end{array}
\]

In this schema, both intervals that have been set aside to the left and downwards from R3 coincide with real entrances, and only the first interval that has been set aside downwards from R2 creates an imaginary entrance.\(^{161}\)

\(^{161}\) The original text: "In this schema, both intervals that have been set aside to the left and downwards from R2 coincide with real entrances, and only the first interval that has been set aside downwards from R3 creates an imaginary entrance" has been changed to: "In this schema, both intervals that have been set aside to the left and downwards from R3 coincide with real entrances, and only the first interval that has been set aside downwards from R2 creates an imaginary entrance". This correction is cited by Wehrmeyer in Sergej Taneev, \textit{Die Lehre vom Kanon}, trans. Andreas Wehrmeyer (Berlin: Ernst Kuhn, 1994), p. 196. (P.G.)
Thus, the following three combinations will be the original combinations of this schema:

First original combination \( Jv=0 \)

Second original combination \( Jv=0 \)

Third original combination \( Jv=0 \)

of which the first is imaginary.
Section C

Three and Four-Voice Infinite Canon of the First and Second Order

I. The three-voice infinite canon

Chapter 11

The multi-voice infinite canon (general concepts)

§ 116. In the infinite canon, the Proposta, and after it all Rispostas that participate in the canon, returns to the beginning and repeats. These repetitions of voices will be indicated as before: //P, //R1, //R2, etc. The entrances that precede //P will be called primary; the entrances beginning with //P we will call secondary. The intervals between the entrances of the Proposta, R1, R2, and //P we will call primary intervals of entrance. Thus, the number of primary intervals in the infinite canon is equal to the number of voices: three for the three-voice canon, four for the four-voice.

§ 117. Infinite multi-voice canons, on a level with all other canons, divide into two orders. The canon of the first order has distances of entrance that are identical, considering in this number even the distance between the last Risposta and //P, and is written according to the rules of vertical-shifting counterpoint. The number of sections in this canon is equal to the number of voices. The canon of
the second order has all or several distances of entrance that are different and is written according to the rules of horizontal-shifting counterpoint alone or in combination with vertical-shifting.

§ 118. The canon of both the first and second order has, similar to two-voice infinite canon, the following subdivisions: a) The Proposta and all of the Rispostas after it literally repeat on the same degrees—the infinite canon in the proper sense; and b) The Proposta and all subsequent Rispostas at their repetitions shift to other degrees in a rising or falling order—the canonic sequence.

§ 119. First and foremost, any infinite canon yields to the conditions to which the finite canon with the same primary entrances is subject to (Jv for the canon of the first order, the imaginary voice for the canon of the second order). But in the infinite canon, to these conditions are added new ones that are introduced by the group of secondary entrances.

§ 120. It was stated above (§ 25) that any combination of voices of which not one is composed, but both repeat that which was composed earlier, should be examined as a derivative combination. On this basis, in the infinite canon, to the voices that reproduce that which was composed earlier, it will be necessary to add the voices that enter a second time: //P, //R1, etc., (as we did in the two-voice infinite canon with regard to //P). These voices are on a
level with primary Rispostas, and they form a part of
derivative combinations and expand their number. Hence, the
sum of those conditions grows that pertains to the
composition of the infinite canon compared to the finite
canon that has been written for the same number of voices.

The infinite three-voice canon of the first order with
secondary entrances on the same degrees (the proper infinite
canon of the first order)

The application of vertical-shifting counterpoint

§ 121. In the three-voice infinite canon of the first
order, the number of sections is equal to three (§ 24). The
way in which a combination of one section occurs with another
is apparent in the following schema:

(R2)   \[
\begin{array}{ccc}
A_2 & B_2 & C_2 \\
\end{array}
\]
(R1)   \[
\begin{array}{ccc}
A_1 & B_1 & C_1 \\
\text{//A_1} \\
\end{array}
\]
(P)  \[
\begin{array}{ccc}
A & B & C \\
\text{//A} \\
\text{//B} \\
\end{array}
\]

Sections of voices that enter a second time have been
indicated by the sign //,. The combination //A_1+//B, which
literally repeats the combination A_1+B, has been made
apparent by dotted lines. In the canonic sequence, we will
note that this combination is shifted to other degrees
compared to A_1+B, but the correlation between rhythmically
coinciding sections is preserved identically in both this and the other case.

§ 122. We will determine the number and reciprocal relationship of original and derivative combinations according to the following schema:

The repetition of the last Proposta in the canon (\(//R2\)) is not necessary to include in the schema since, at the point of its entrance, the canon is an exact repetition of that which was earlier than the point of entrance of R2 (with a shift to other degrees in a canonic sequence). We will begin with a determination of derivative combinations. It is known that a combination of each Risposta with each of the others is a derivative combination. But in the infinite canon, one should also add to the Rispostas the voices that enter a second time since they, similar to the primary Rispostas, are voices that are not newly composed and only imitate the Proposta. In the aforementioned schema, from the number of repeated voices we excluded the repetition of the last
Risposta in the canon (i.e., //R2), the other voices: R1, R2, //P, and //R1, are members of derivative combinations.

§ 123. In order to obtain a complete list of derivative combinations, it will be necessary to combine among themselves each of these voices with each of the others. We will begin with combinations of neighboring voices in pairs. For these derivatives, the combination P+R1 will be original. Having excluded the combination //P+//R1 since it literally repeats the original P+R1 (the combination //P+//R1 has been indicated by a dotted line), we will obtain two derivatives, namely, R1+R2 and R2+//P.¹⁶²

\[
\begin{array}{c}
\text{Derivative} \\
P \quad R_1 \quad R_2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\
\alpha \quad \alpha' \quad \alpha'' \\
\end{array}
\]

Original

§ 124. Proceeding to combinations of voices that are not consecutive, we will also obtain two derivative combinations: R1+//P and R2+//R1, for which the combination P+R2 is original. Thus, with regard to the infinite canon, the rule is invalid, according to which the combination of the Proposta with the last Risposta in the canon must be

¹⁶² Instead of "Having excluded the combination //P+//R...", which is present in the original text, the correction "Having excluded the combinations //P+//R1..." has been added in the translation. (P.G.)
written in simple counterpoint since in the finite canon this combination may not be in the number of originals.\textsuperscript{163}

\[
\begin{array}{c}
\text{Derivative} \\
\alpha \\
\beta' \\
P \\
\beta \\
R_1 \\
R_2 \\
\beta \\
\alpha \\
\beta'' \\
\end{array}
\]

Second Original

Further combinations--bridging over two voices--are not possible since it would be necessary to combine not two separate voices, but repetitions of one and the same. Thus, we have two original combinations that each have two derivatives, namely:

Originals: \hspace{1cm} Their derivatives:

1) P+R1 \hspace{1cm} R1+R2
   \hspace{1cm} R1+/P
   \hspace{1cm} R2+/P

2) P+R2 \hspace{1cm} R1+/P
   \hspace{1cm} R2+/R1

\textsuperscript{163} In the schema that follows this text, instead of the indication "P" for the second entrance of the Proposta, which is present in the original text, "//P" has been added in the translation. Also in this schema, instead of "R3", the correction "//R1" has been added. These corrections are cited by Wehrmeyer in Sergej Taneev, Die Lehre vom Kanon, trans. Andreas Wehrmeyer (Berlin: Ernst Kuhn, 1994), p. 196. (P.G.)
§ 125. We will note that in general the number of original combinations in the infinite canon of the first order is equal to the number of its Rispostas. Also equal to this number is the maximum number of derivative combinations that each original has and, consequently, the maximum number of its JJv.

§ 126. It should not be overlooked that all conditions to which the canon yields pertain only to original combinations, i.e., to combinations in which the Proposta and one of the primary Rispostas participate, and, consequently, the effect of these conditions ceases at the point of entrance of //P.

§ 127. The JJv for each original combination is deduced by one of two known ways. The first way, which is in order to locate a JJv according to the schema that has been cited above, the original, which has been taken with a negative sign, is added to the derivative interval of entrance, does not require explanations. The second way, which consists of setting aside the intervals of entrance on the opposite side and in the opposite direction according to pitch (in a schema this is expressed by notes), is in several instances more simple and practical. In order to apply this method, it is necessary to adhere to the following fundamentals that apply to all cases of three-voice infinite canon and canonic

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164 The word "derivative", which is present in the original text, has been replaced by "original" in the translation. (P.G.)
sequences. The primary intervals of entrance are measured out to the left of real entrances in the opposite direction according to pitch (in order, beginning with the first interval). The measuring out of intervals begins from R2, from which the first primary interval is set aside in the indicated way, from //P, two intervals are set aside, from //R1, three. In any canon, for as many voices that have been written, the measuring out of intervals begins with the entrance of R2 and ends on the secondary entrance of the penultimate Risposta in the canon.

§ 128. We will assume that the primary intervals of entrance and their directions are as follows: ↑4 ↓10. At first we will measure the first of these intervals to the left and downwards from R2. We obtain:

Then we add to this two primary intervals that have been set aside from //P:

and, having added the intervals that have been set aside from //R1:
we obtain a full schema. Such a schema, which here required three musical staffs for its gradual formation, can

[Employing the method that we applied in the search for Jv at § 113, we can set aside to the right of the Proposta the intervals of all entrances that form derivative combinations. By knowing that there are four derivatives in the three-voice infinite canon: R1+R2, R2+//P, R1+//P, and R2+//R1, we can set aside in order all intervals that are formed by the entrances of these voices to the right of the Proposta and obtain:

- $R_1 + R_2$
- $R_2 + //P$
- $R_1 + //P$
- $R_2 + //R_1$

By combining the results that have been obtained into a single schema we receive:

i.e., the same that we have at § 128. (V.B.)

In this passage, the following corrections have been made: instead of "Employing the method that we applied in the search for Jv at § 133...", which is present at the beginning of this passage in the original text, the correction "Employing the method that we applied in the search for Jv at § 113..." has been added in the translation; instead of "By knowing that there are four derivatives in the two-voice infinite canon", the correction "By knowing that there are four derivatives in the three-voice infinite canon" has been added in the translation; and instead of "R1+//R1", which accompanies the fourth schema in the original text, the correction "R2+//R1" has been added in the translation. Corrections one and three are cited by Wehrmeyer in Taneev, *Die Lehre vom Kanon*, p. 196. (P.G.)
be formed in three stages on one musical staff, on which the measured-out intervals are gradually layered. According to this schema, we will seek indices. We consider the Proposta and Risposta of this schema.

![Musical Staff Diagram]

It is already known that the interval between the note of entrance of R1 and each dark note that is situated on a single vertical line with it is equal to an index for the original combination P+R1. Therefore, this combination has two JJv—one equal to 2, the other equal to 14. The first of these JJv is positive (since the directions from the Proposta to R1 and from the Proposta to the note that is positioned a third higher than R1 are identical), and the second JJv is negative since the directions from the Proposta to R1 and from the Proposta to the note that is positioned two octaves lower than R1 are contrasting. Then we take P+R2 from this schema—the second original—and by the same reasoning (having taken into consideration the directions of the intervals between the Proposta and R2 and the Proposta and each of the two dark notes that are situated on a single vertical line with R2) we arrive at the conclusion that P+R2 also has two JJv, namely, Jv=-2 and Jv=-16.
In several respects, this method is more practical than the first, which is awkward because the schema, on which each original and each derivative combination are noted by brackets, is overly marked.

The task that we encounter [in three-voice infinite canons] is extremely difficult [and is, in any case, more difficult than that which we encountered in four-voice finite canons of the first order. This is because] here we enter the region of three-voice vertical-shifting counterpoint on account of the demand that has been made on the Proposta—to form a vertical-shifting combination with each of the two Rispostas. In this situation, this requirement is inevitable. With the exception of the canon where all of the voices enter at the prime and therefore all JJv=0, any other three-voice infinite canon of the first order requires, without fail, two original combinations. But if the number of originals can not be decreased, then, in order to compensate for this, the number of derivatives can be decreased, and to this point one should turn particular attention in the shift to practical exercises. It is possible to choose such entrances at which each original combination will have only one Jv, and this Jv will be the
same for both original combinations. Such cases with the simplest JJv should serve as the main material for exercises.

§ 129. For a reduction of the number of derivative combinations one should make several of the JJv equal to zero, which, as is known, is the result of a similarity in size and direction of the intervals of entrance. Of the three primary intervals of entrance, if two are identical in size and direction, then both original combinations will have the same Jv. This Jv is negative and is equal to the size of an identical interval that has been multiplied by the number of voices in the canon. Having indicated this interval with the letter m, for the three-voice canon we will obtain:

\[ Jv = -3m \]

Thus,

if \( m = 1 \), then \( Jv = -3 \)

" \( m = 2 \), " \( Jv = -6 \)

" \( m = 3 \), " \( Jv = -9 \) (double counterpoint at the dezime)

" \( m = 4 \), " \( Jv = -12 \)

" \( m = 5 \), " \( Jv = -15 \)

" \( m = 6 \), " \( Jv = -18 \) (double counterpoint at the duodezime)

" \( m = 7 \), " \( Jv = -21 \) (double counterpoint at the octave)
§ 130. We will cite examples of such entrances:

\[ m=1. \ J_v=-3 \]
\[ m=2. \ J_v=-6 \]
\[ m=3. \ J_v=-9 \]
\[ m=4. \ J_v=-12 \]

In the first two examples, those of the derivative combinations whose order of entrances is similar to either an original or to a previous derivative have been indicated with a dotted line, which is why they do not have significance for the determination of \( J_v \).

Beginning with Example c), indications by a dotted line are absent, and the entrance of \(/ /R_1\) has also been omitted, as a result of this, the combination in which it forms a part, \( R_2 + / /R_1\), repeats a previous derivative.

§ 131. From these schemata, it is clear that as a result of the return of the Proposta to its initial degree, the interval that is distinguished from the others by its size should be distinguished from the others also by its

---

166 In the first example at Example b), instead of "P, [], R1, //R2, //R1", which is present in the original text, "P, R1, R2, //P, //R1" has been added in the translation. In the first example at Example c), instead of "-5" between R2 and //P, the correction "-6" has been added. These corrections are cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 196. (P.G.)
direction, i.e., it will be a negative interval. Since we are required to add an original interval with the opposite sign, i.e., also negative, to this interval in order to obtain a \( Jv \), then it is apparent that positive \( JJv \) are inapplicable in such a canon. It is also clear that the size of the negative interval, being equal to the two other positives, is equal to their sum. Based on this and the formula that was cited above (which pertains to cases where two intervals are equal), \( Jv = -3m \). By means of such considerations, it is not difficult to arrive at the conclusion that \( Jv = -4m \) for the four-voice infinite canon of the first order that has secondary entrances on the same degrees and a similarity of size and direction of three of the intervals of entrance. Regarding two-voice infinite canon, we already know that \( Jv = -2m \). Thus, in a similar infinite canon of the first order, if one of the primary intervals of entrance has been distinguished from the others (we do not forget that to the primary intervals belongs also the interval between //P and the Risposta that precedes it) and the others are similar between themselves according to size and direction, then all of the original combinations have a single \( Jv \), this \( Jv \) is negative and is equal to the smaller interval, which has been multiplied by the number of voices in the canon.

This \( Jv \) pertains to a combination of the Proposta with each of the Rispostas. The three-voice infinite canon that
has the aforementioned entrances should, consequently, be written according to the following conditions: \((P+R_1) Jv=-3m, (P+R_2) Jv=-3m\).

§ 132. In so doing, it is not necessary for similar intervals to consecutively follow one after the other. In whichever order the intervals of entrance of such a three-voice canon are arranged, if two of them are similar in size and direction, then the \(Jv\) will always equal the smaller interval multiplied by three, it will always be negative, and it will always pertain to both original combinations \((P+R_1\) and \(P+R_2\):

\[
\begin{align*}
\text{a)} & \quad -3 \quad -9 \\
\text{b)} & \quad -3 \quad -9 \\
\text{c)} & \quad -9 \quad -9
\end{align*}
\]

For all of these entrances, \((P+R_1) Jv=-9\) and \((P+R_2) Jv=-9\).

The canon that has been written according to these conditions may begin with each of the voices (see the two-voice canon, § 20).

§ 133. The infinite canon of the first order where all entrances are \(-0\) and all \(JJv=0\) is not difficult to write. Methods for its composition can be found in any textbook on canon. The following method is the most practical. The Proposta is written up to the point of entrance of \(R_1\). Then, from the first musical line, the melody is transferred to a second line and is continued in counterpoint to that which has been written on the first line. Arriving at the end of
this, the melody is transferred to a third line, forming three-voice simple counterpoint with the two previous voices. If it is a four-voice canon, then a fourth line is written in the same way. At the end of the last line, the melody should approach the beginning of the first line, for example:

No. 149.

Taken successively, these three sections form the Proposta. At the beginning of the second section, R1 enters with the same melody, at the beginning of the third, R2.

No. 150.
From the point of entrance of the last Risposta in the canon, the same three-voice combination repeats, the sections of which occur in turn in the voices. By means of this method, it is not difficult to write a canon at \( \rightarrow 0 \) for a large number of voices.

§ 134. In the following canon by Glinka, which has been borrowed from his hand-written exercises housed in the Public Library, the voices enter at the octave. On account of this, the Proposta must be in double counterpoint at the octave with each of the Rispostas \( (m=7, Jv=-21) \).

![Diagram of the canon by Glinka]

Regarding this, every similar canon, which has two intervals of entrance that are identical in size and direction, can begin with any of the voices, as was already mentioned at § 20.
No. 151.167

Glinka

When there is an instrumental accompaniment or, in general, when there are supplemental voices, the composition of canon at the octave is made significantly easier. The fifth and fourth may be used freely, and, of all requirements of double counterpoint at the octave, only two remain: Avoid successions of parallel fourths and avoid ninths that are dissonant above. Due to these conditions, canon at the octave presents as little difficulty as canon at the prime. In such a canon, there is quite often encountered an alternation of entrances at the octave with entrances at the prime. The ease of composition of such a canon contributed to its prevalence. An entire literature of canons at the prime and octave exists, from children's ditties to serious musical works inclusively. This form of canon carries the

167 The library mentioned in this passage is the Saltikov-Shedrin Library in St. Petersburg. This exercise is found in Mikhail Glinka, Uchebnie raboty [Student works], vol. 17 of Polnoe sobranie sochinenie [The complete collection of his works], prepared by N. Zagornie (Moscow, 1969), p. 111. This information is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 141. (P.G.)
name Rota in the Italian language, in English, Round.\textsuperscript{168} The close entrances of voices that continually return to the beginning can easily impart a shade of comedy to this canon and make it mainly suitable for texts of a humorous character. In serious compositions, the Proposta has a more or less significant length and is quite often equal to an entire period (the canon in \textit{Fidelio}, \textit{Ruslan}, etc.).\textsuperscript{169}

§ 135. From the canon at the prime and octave, we proceed to canon at other intervals. The following canon requires double counterpoint at the dezime: $m=3$, $Jv=-9$.

No. 152.

A canon that has been written at a $Jv$ from the second group, owing to the absence of direct motion, quite often allows doubling of imperfect consonances, which imparts to it

\textsuperscript{168} The word "Rund" in the original text has been changed to "Round" in the translation. This correction is cited by Wehrmeyer in Taneev, \textit{Die Lehre vom Kanon}, p. 197. (P.G.)

greater harmonic fullness. The canon cited above is awkward for such doublings because of the tritone that occurs in the melody. For example, when doubling the Proposta at the upper third or tenth, we receive a tritone in the second measure,

![Musical notation]

when doubling R1 at the lower tenth, although they have been separated by a pause, there is a tritone between the last note of the melody and the first note of its repetition. It is not necessary to explain that in the free style such tritones in the melody may be encountered unimpeded.

In the cited example, if the last note of the Proposta is shortened to half its length (having taken « instead of «), then the unprepared sixth, which enters on the second half of the third measure, will be eliminated from the combination P+R1. Owing to this alteration, the combination P+R1 will satisfy conditions not only at Jv=−9, but also at Jv=−11. In its present aspect, the combination P+R2 also will satisfy conditions at Jv=−11. We will use this means subsequently in order to obtain a canonic sequence from this canon.

§ 136. The following canon is also in double counterpoint at the dezime.
No. 153.

\[ Jv = -9 \]

\[ \text{\includegraphics[width=0.4\textwidth]{no153.png}} \]

It sounds empty in three voices, but with doublings it acquires harmonic fullness and euphony.

No. 154.

\[ \text{\includegraphics[width=0.4\textwidth]{no154.png}} \]

This canon can also be executed in four voices (if one of the outer voices is removed from the previous example).

§ 137. We cite one additional canon also at \( Jv = -9 \).

This canon also has two intervals of entrance that are similar in size and direction, but it is distinguished from the previous canons because the entrances of the voices are not in a succession of pitch, namely:

\[ \text{\includegraphics[width=0.2\textwidth]{no137.png}} \]
But we already know that such an order of entrances does not turn out to have an influence on the value of the $J_v$, which, even for such a canon, equals the negative value of the smallest interval multiplied by the number of voices in the canon.

No. 155.

§ 138. The following two canons have been written in counterpoint at the duodezime: $m=6$, $J_v=-18$. In each of these canons, the distance of entrance is equal to two measures.

No. 156.
§ 139. In conclusion, we will cite a canon that requires a less common \( Jv \), namely, \( Jv = -12 \).

No. 158.\(^\text{170}\)

This canon can be executed in four voices by either doubling the Proposta at the lower third:

\(^{170}\) Instead of the designation "R", which accompanies the second Risposta in this example in the original text, the designation "R2" has been added in the translation. Cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 197. (P.G.)
No. 159.\textsuperscript{171}

or R2 at the upper third. In order to avoid the tritone, one should change the key signature.

No. 160.

The Proposta of this canon is not only in counterpoint to each of the Rispostas at Jv=-12, but also forms with each of them a combination in double counterpoint at the octave, which subsequently will provide us with a possibility to obtain a sequence from this canon.

The aforementioned doublings of the Proposta and R2 may not be done simultaneously since they would be hindered in the fifth measure by an incorrect resolution of the seventh: 6.

\textsuperscript{171} Instead of the designations "R" and "R", which accompany the first and second Rispostas in this example in the original text, the designations "R1" and "R2" have been added in the translation. This correction is cited by Wehrmeyer in Taneev, \textit{Die Lehre vom Kanon}, p. 197. (P.G.)
If we will take their doublings instead of the Proposta and R2, then we will obtain the beginning of canonic imitation with two identical intervals of entrance \((m=6)\).

However, even here the return of the Proposta to the same degrees at (a) is hindered by the seventh that provides an incorrect resolution at (b): 6.

Thus, we can not obtain the infinite canon in the proper sense at these entrances. But as we will see below, here there is a possibility for a return of the Proposta with a shift to other degrees, i.e., a canonic sequence is possible. For the time being, we will note that the Proposta is situated with two other voices in double counterpoint at the dezime: \((P+R1) Jv=-9\) and \((P+R2) Jv=-9\).

§ 140. Infinite canons of the first order, similar to those that have been cited above (with the exception of canons at the prime and octave), one would not succeed in encountering in textbooks on counterpoint. Some theorists
disclaim the very possibility of their existence, for example, Hauff, who maintains that such canons are possible only at the prime or octave (he says about this "nur ebenfalls möglich," from which it is apparent that he even considers two-voice infinite canons of the first order at other intervals as not possible).¹⁷² But if the possibility of the existence of such canons is not subject to doubt, then their significance is limited only to pedagogy. They can serve as excellent exercises in three-voice vertical-shifting counterpoint. In composition, they are on a level with two-voice infinite canons in the proper sense because it is unlikely their application will be located. Unavoidable monotony of a short Proposta prevents this. This monotony occurs due to the return of each voice to the same phrase, repeated on the same degrees. In an increase in the length of the Proposta, this monotony, although it ceases to be sensed, instead significantly expands the difficulty that accompanies the writing of this canon, which makes its employment awkward. Moreover, an attainment of an exactness of imitation by each of the voices remains imperceptible to the listener since the length of the Proposta, due to the complex relations in which the voices enter with one another, is not possible to follow in exact imitation.

¹⁷² Johann Hauff, Die Theorie der Tonsetzkunst, vol. 4 of Der Canon (Frankfurt, 1872), p. 6. (P.G.)
Chapter 12

The infinite three-voice canon of the first order with secondary entrances on other degrees (the canonic sequence of the first order)

§ 141. For the three-voice canonic sequence, we will also give preference to those entrances where the number of derivative combinations is two and, consequently, both original combinations have the same Jv. Similar to two-voice sequences, where for each given interval of entrance we could write a sequence at any Jv, depending on whether the Proposta at its return must be shifted by a certain number of degrees either upwards or downwards, in the three-voice canonic sequences examined here—with two intervals of entrance equal in size and direction (indicated by the letter m)—we can also use any JJv with a negative sign for the given degrees P, R1, and R2, which will also determine the interval of entrance of /P. For example, when m=2, for the application of Jv=-7, it is necessary to shift the Proposta by a degree
in the opposite direction of \( m \); for the application of \( Jv=-8 \), by two degrees; for the application of \( Jv=-9 \), by three, etc.

If \( m=3 \), for the application of \( Jv=-7 \) it is necessary for the Proposta to shift by two degrees in the same direction as \( m \), etc. We will take \( m=3 \):

Of these examples, the one that has \( Jv=-9 \) shows that the canon with a Proposta that repeats on the same degree is only a particular type of canonic sequence.
§ 142. JJv that have a direct shift, which are not possible in a canon with secondary entrances on the same degrees, are possible in canonic sequences, but they are extremely awkward, causing unavoidable shifts of secondary entrances by excessively large intervals.

§ 143. We cite examples of canons with an application of double counterpoint at the octave, dezime, and duodezime.

At JJv=−14

No. 161.

No. 162.

Canon at JJv=−16.
The following canons require \( Jv=-11 \).

No. 164.\(^1\)

P.XII, 52.

\(^1\) Instead of "\( Jv=-14 \)", which is present in the original text, the correction "\( Jv=-16 \)" has been added in the translation. This correction is cited by Wehrmeyer in Sergej Taneev, *Die Lehre vom Kanon*, trans. Andreas Wehrmeyer (Berlin: Ernst Kuhn, 1994), p. 197. (P.G.)

\(^{1\text{a}}\) Giovanni Pierluigi da Palestrina, "Gloria" from Missa: *Brevis*, mm. 25-8. The schemata of the voices for Examples 164 and 165 have been joined with their corresponding musical examples. This correction is cited by Wehrmeyer in Taneev, *Die Lehre vom Kanon*, p. 197. (P.G.)
No. 165.

Giovanni Pierluigi da Palestrina, "Gloria" from Missa: Vitute magna, mm. 34-37. (P.G.)
§ 144. In the infinite canon of the first order, quite often the Proposta is in counterpoint with regard to each Risposta at a Jv that is required by the given canon while simultaneously forming with each of them a combination at some other Jv, which has arisen by chance and is not required by the given canon. In examples that have been examined above, we noted such cases of a concurrence of two indices. The second Jv provides a possibility to create a canonic sequence from the infinite canon by shifting the Proposta in such a way that entrances result that correspond to the second index. We have already encountered similar instances

176 Giovanni Pierluigi da Palestrina, "Credo" from Missa: Jam Christus astra ascenderat, mm. 99-102. Two superfluous notes, which are present in the schema in the original text, have been removed from the schema in the translation. This correction is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 197. (P.G.)
in the presentation of the doctrine of the two-voice infinite canon (§ 42).

§ 145. We will do an experiment with several of the examples of the infinite canon cited above. When citing the infinite canon No. 152 (§ 135), which has been written at \( Jv = -9 \), we mentioned that if the last note of the Proposta is shortened to half of its length, then it turns out that the Proposta forms a combination with each of the Rispostas not only at \( Jv = -9 \), but also at \( Jv = -11 \).\(^{177}\) Having preserved the first two intervals of entrance without change, we will alter the third in accordance with the new \( Jv \), and we will obtain a canonic sequence with voices that are lowered by a third. This sequence must have the following entrances:

No. 169.

\[ \text{\begin{center} \includegraphics[width=\textwidth]{image.png} \end{center}} \]

\(^{177}\) Instead of "When citing the infinite canon No. 153 (§ 136)", which is present in the original text, the correction "When citing the infinite canon No. 152 (§ 135)" has been added in the translation. This correction is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 197. (P.G.)
§ 146. We proceed to the canon cited above at $Jv=-12$, No. 158 (§ 139). As was previously stated, the Proposta of this canon is also situated in double counterpoint at the octave with the two other voices. This provides a possibility to obtain a canonic sequence with a shift of the voices by a third downwards with the entrances:

\[ Jv = -14 \]

in which R1 permits a doubling at the upper sixth:

No. 170.

§ 147. We noted above (§ 139) that in canonic imitation of the same theme at $m=6$, the Proposta forms double counterpoint at the dezime with the two other voices. Hence, it is possible to obtain a canonic sequence that ascends by a third with the following entrances:
In the example cited below, the order of entrances has been changed: the sequence begins with the middle voice. The upper voice in this sequence allows doubling at the lower third.

No. 171.

\[ Jv = -16 \]  

§ 148. In order to obtain a succession of entrances at a given \( Jv \), it is necessary to divide the absolute value of the \( Jv \) into two parts—either equal or unequal. Of these two parts, we consider one as the interval that is met twice in the schema and the other part as the remaining interval. If one part is twice as great as the other, then \( //P \) enters on the same degree as the Proposta and an infinite canon in the proper sense results. For any other relationship of these parts, a canonic sequence results. The interval by which the Proposta is shifted is determined in the following way: To the sum of the similar intervals, which is taken with a minus
sign, is added the value of the dissimilar interval taken with a positive sign. The absolute value of the algebraic sum that is obtained indicates by which interval the Proposta is shifted. The direction of this shift is determined by the sign of the sum: If the sign is positive, then the direction of the shift of the Proposta is in a contrasting direction to the similar intervals; if the sign is negative, then this direction is the same as the direction of the similar intervals.

I. The Inverse Shift

a) The smaller part of the index \(< Jv/3\)

\[ Jv = -9 \]

Having divided the Jv into two equal or unequal parts, we take one part twice in a single direction and the other a single time in the opposite direction:
The smaller part < \( \frac{Jv}{3} \): the Proposta shifts in the direction of the dissimilar interval.

b) One part = \( \frac{Jv}{3} \), the other = \( 2 \frac{Jv}{3} \).

The smaller part = \( \frac{Jv}{3} \): the Proposta does not shift.

c) The smaller part > \( \frac{Jv}{3} \).

The smaller part > \( \frac{Jv}{3} \): the Proposta shifts in the direction of the similar intervals.\(^{178}\)

The sum of the two identical intervals and the dissimilar are compared, the direction of the shift of the Proposta is determined by the direction of the greater of the two.

\(^{178}\) At § 148 Example a), instead of "The smaller part < \( \frac{Jv}{3} \): the Proposta shifts in the direction of the similar intervals", which is present in the original text, "The smaller part < \( \frac{Jv}{3} \): the Proposta shifts in the direction of the dissimilar interval" has been added in the translation; at Example c), instead of "The smaller part > \( \frac{Jv}{3} \): the Proposta shifts in the direction of the dissimilar interval", "The smaller part > \( \frac{Jv}{3} \): the Proposta shifts in the direction of the similar intervals" has been added in the translation. (P.G.)
II. The Direct Shift

It is necessary to divide the \( Jv \) (either positive or negative) into two parts, of which one should have the sign + and the other the sign -. One of these parts is taken as an interval of entrance a single time, the other, twice; therefore, the direction of all three intervals is identical. That which has the sign (-) must be the primary interval of entrance; the order of the remaining is irrelevant. (Thus, the intervals of entrance, once noted, can be distributed in six combinations.)

\[
Jv=-1=-2+1
\]

179 Mark Kopytman shows that the formula \( Jv=-1=-2+1 \) can not be applied to the second example of the first line because it does not take into account the change of sign of the \( Jv \) that occurs with regard to the second derivative of \( P+R2 \). Also, he shows that the formula \( Jv=1=-1+2 \) can not be applied to the second example of the third line because it does not take into account the change of sign of the \( Jv \) that occurs with regard to the second derivative of \( P+R2 \).

\[
\begin{align*}
P+R1 & \quad Jv=-1,0 \quad \mathbf{Jv=I} \\
P+R2 & \quad Jv=0,+1 \\
P+R1 & \quad Jv=+1,0 \\
P+R2 & \quad Jv=0,-1
\end{align*}
\]

(Mark R. Kopytman, "Mnogogolosnyi kanon (kanon i sekventsiia)" [Multi-voice canon (canon and sequence)], Voprosy muzikoznaniia [Questions of musical research] 3 (1960): 259.) (F.G.)
The interval between the Proposta and //P is equal to the sum of all three intervals of entrance. Its direction is the same as the direction of each of them.
Chapter 13

The infinite three-voice canon of the second order (all or several distances of entrance are different) The application of horizontal-shifting counterpoint alone or in combination with vertical-shifting

§ 149. In the infinite canon of the second order, all or several of the distances of entrance are different. This canon is written according to the rules of horizontal-shifting counterpoint alone or in combination with vertical-shifting (§ 22), and, similar to the canon of the first order, it may have a Proposta that returns on other degrees (the canonic sequence). The infinite canon of the second order can have primary entrances that are identical to entrances of the finite canon of either the first or second order. In the first instance, the distance that is distinguished from the others will be the distance between R2 and //P. Those conditions to which the finite canon with the given entrances would yield (whether they are expressed by a Jv or an imaginary voice) remain in effect even in the infinite. But to these conditions new ones are added, imposed by a return of the voices to the beginning.

§ 150. The infinite canon of the second order in which only horizontal-shifting counterpoint is applied, i.e., a canon written with the help of imaginary combinations and not having a single Jv, can begin with entrances that belong to
the canon of the first order, but, in such a case, the Jv of
the latter must be equal to 0.

§ 151. To form a schema that indicates the points of
entrance of the imaginary voices of the canon of the second
order that does not have a Jv or where Jv=0 is done most
easily with the help of the second way that has been
indicated above, i.e., by positioning the primary intervals
of entrance in the opposite direction to the left of R2, //P,
and //R1 (§ 115, Comment).

§ 152. Having placed the required interval to the left
of R2, we discover the conditions to which the finite canon
with the same entrances would also have to yield. Opposite
to this, the intervals that have been positioned to the left
of //P and //R1 indicate the conditions that are especially
characteristic of infinite canon.

§ 153. This method has an advantage in that the exact
distance between R2 and //P is not defined by it, so when
writing the canon the entire group of entrances of //P through
//R2, together with the imaginary voices that precede them,
which have been measured out to the left of them, can in the
case of any sort of awkwardness that is present in the
counterpoint of an imaginary Risposta be shifted ahead to a
point at which the Proposta can more freely be in
counterpoint to them.
The infinite three-voice canon of the second order with secondary entrances on the same degrees (the proper infinite canon of the second order)

§ 154. We cite an example of an infinite canon at $\to 0$ by Palestrina.

No. 172.\textsuperscript{180}

P.XXX, 3.

The primary entrances of this canon are identical to the entrances of a finite canon of the first order.

\textsuperscript{180} Giovanni Pierluigi da Palestrina, Sacred Song: "Illumina oculos meos," mm. 1-9. (P.G.)
§ 155. To write a similar canon is not difficult. Owing to the imitation at \( \rightarrow 0 \), there will be no Jv whatsoever. The imaginary voices repeat a former real combination, i.e., a contrapuntally correct combination, and to position a Proposta in counterpoint to them does not present particular difficulty. All of the difficulty of this canon is concentrated at the point that directly precedes the entrance of //P, where it is necessary for the Proposta to be in counterpoint to several imaginary voices at once. In a canon, the length of such a place is equal to the distance between the entrances of the Proposta and the last Risposta in the canon (in the three-voice canon this is R2). The closer the voices enter at the beginning of the canon, the shorter the group of imaginary voices that precede //P and the fewer difficulties it will be necessary to overcome. We do not forget that once an imaginary voice has entered it continues up to the point of entrance of //P, stopping at its first note.

§156. We will form a schema of canon No. 173.
It begins with the following entrances:

If this were a finite canon, then its conditions would be expressed as such:

In order to add to this the conditions of the infinite canon, we will set aside the primary intervals of entrance to the left of //P and //R1, preserving the same distance between them.

---

181 Giovanni Pierluigi da Palestrina. "Sanctus" from Missa: Ad fugam, mm. 19-27. (P.G.)
Having combined both parts of the schema, we will obtain an aggregate of the conditions that the Proposta of this canon must satisfy. In so doing, it is possible to avoid determining the distance between these two parts of the schema.\textsuperscript{182}

Of the three imaginary voices in the second half of the schema, voices a) and b) are situated in the same relationship to one another as are the voices of the real combination P+Rl. These two imaginary voices, which are located among themselves in a contrapuntal combination, are more conveniently written in the completed task on a single musical line, and voice c) is separate from them. By means of this, we will obtain an exact reproduction of the entrances of two primary real voices and the imaginary that follows them. It would be possible to write out voices a) and c) on a separate line:

\begin{center}
\includegraphics[width=0.5\textwidth]{image.png}
\end{center}

\begin{itemize}
\item $\text{R}$
\item $\text{(a)}$
\end{itemize}

\textsuperscript{182} Instead of "R", which designates the first Risposta in the schema in the original text, "Rl" has been added in the translation. This correction is cited by Wehrmeyer in Sergej Taneev, \textit{Die Lehre vom Kanon}, trans. Andreas Wehrmeyer (Berlin: Ernst Kuhn, 1994), p. 197. (P.G.)
which reproduces another real combination, R1+R2.

§ 157. We will pause on the previous schema. The imaginary voice between R1 and R2, which also belongs to the finite canon with the same entrances, will remain and no changes are introduced to the schema of the infinite canon. If the point of entrance of //P were to be postponed, then even the other imaginary voices that directly precede it would be preserved in the previous order and in the previous relationship with it. It is also possible to use this circumstance in the composition of canon. If at the entrance of this group of imaginary voices there turned out to be any sort of awkwardness for the Proposta to be in counterpoint to them, then the entrance of all of these voices could be delayed and for their entrance a more suitable point could be chosen. If //P is brought nearer to R2, then it can occur that any one of the imaginary voices that directly precede //P can coincide with one of the voices of the first half of the schema (the primary group).\textsuperscript{183} In this case, the voice with whose entrance the imaginary has rhythmically coincided (irregardless of whether this voice is real or imaginary) acquires a Jv, which will replace a former imaginary voice, for example:

\textsuperscript{183} Instead of "If //P is brought nearer to R1...", which is present in the original text, the correction "If //P is brought nearer to R2..." has been added in the translation. This correction is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 197. (P.G.)
Here the canon has become smaller by a single imaginary voice, but on the other hand the combination $P+R_2$ has acquired $J_v=-14$, namely:

§ 158. We will bring $P$ closer to $R_2$ by an additional section:

Here the imaginary voice of the primary group has acquired $J_v=-11$ because an imaginary voice of the group of secondary entrances coincided with it. In such a coincidence of an imaginary voice with an imaginary, we will enclose in brackets the one of which belongs to the group of secondary entrances, as is apparent in the given example.

§ 159. In the schema, when two imaginary voices coincide among themselves, each of them can be the voice with which the Proposta should enter in combination, provided that
this combination will fulfill the requirement of the Jv. Therefore, the first imaginary combination of the cited schema:

\[ Jv = -11 \]

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succession of parallel fourths, etc., will be encountered between them. Returning to the example that has just been cited, we see that both of the coinciding voices form with the imaginary that directly precedes //P a combination that is similar to one of the real combinations, namely:

Combination a) is similar to $R_2+//P$, combination b) is similar to $R_1+R_2$. But since a lower voice of the canon is part of the first of the real combinations ($R_2+//P$), we will give preference to combination a) and, consequently, we will leave the imaginary voice that belongs to the group of primary entrances in the schema. As a result of this, the schema will acquire the following final aspect:

§ 160. If we shift //P closer to $R_2$ by one additional section, then we will obtain the following schema:

or in a different way
Such a canon would be very difficult to write because $J_v = -10$ and, in particular, $J_v = -8$ are encountered in it.

§ 161. We drew $P_2$ closer to $R_2$ each time by a distance equal to a single section, and, by means of this, we obtained coincidences of imaginary voices of the group of secondary entrances with real voices of the primary group or with an imaginary that also belongs to this group. However, it would be possible to entirely avoid these combinations when drawing $P_2$ closer to $R_2$. One would have only to move $P_2$ towards $R_2$ by a distance equal to part of a section or by a fraction of a section, for example:

![Diagram 1]

hence:

![Diagram 2]

§ 162. Thus, we have examined schemata of three-voice infinite canons where $JJ_v$ are absent, as well as those where $JJ_v$ are present along with imaginary combinations. With
regard to these latter schemata, it will be necessary to note the following:

1) Each original combination of the three-voice canon of the second order—whether this combination is real or imaginary—can not have more than a single Jv.

2) In the three-voice canon of the second order, if there is a voice—real or imaginary—that forms a combination with the Proposta that has a Jv, then, at the same rhythmic distance at which this voice entered from the Proposta, there is situated to the left of //P an entrance of a voice (also real or imaginary) whose combination with the Proposta has the same Jv. The instance when there is only one combination in the canon that has a Jv does not contradict this. Then the voice with which the Proposta combines at the given Jv stands at an equal distance from //P, it is located exactly in the middle between the Proposta and //P.

From what has been said, it follows that:

3) If a three-voice canon of the second order has two JJv, then both JJv are identical.

4) If such a canon has three JJv, then two of them are similar and the third belongs to a combination of the Proposta with a voice that enters in the middle between the Proposta and //P, at an equal distance from each of them.

These propositions, which only pertain to three-voice canons of the second order, can serve as verification of the schemata that have been compiled.
§ 163. We cite several additional examples of canon of the second order that have indices as follows:

With a single $J_v=-14$:

No. 174.\textsuperscript{184}

L. Senfl

(G.D.297)

With $J_v=-16$ on the same theme but shifted to other degrees:

No. 175.\textsuperscript{185}

L. Senfl

(G.D.297)

\textsuperscript{184} Ludwig Senfl, "Deus in adjutorium meum," mm. 64-6. (P.G.)

\textsuperscript{185} Ibid. (P.G.)
With two identical indices at $jv=-11$, of which one belongs to the combination of the Proposta with an imaginary voice:

No. 176.\textsuperscript{186}

P.XX, 110.

This canon also begins at the first original as a canon of the first order.

\textsuperscript{186} Giovanni Pierluigi da Palestrina, "Gloria" from Missa: Alma Redemptoris, mm. 21-3. (P.G.)
In the last example below, a canonic sequence has been made with a Proposta that is shifted by a fourth downwards.

The following example:

which has been written in 5/2 time, is not used in the strict style.

---

187 Giovanni Pierluigi da Palestrina, "Gloria" from Missa: Tertia [O Magnum Mysterium], mm. 48-50. The bass clef, which is present in the lower voice in the original text, has been replaced by an alto clef in the translation. This correction is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 197. (P.G.)
If similar canonic imitation were encountered in the works of earlier composers, then it would have been fit into 4/2 meter and the return of the Proposta would occur each time at different parts of the measure. Cases where the length of the repeating Proposta can not fit into a whole number of measures are quite often in works by composers of the epoch of the strict style, and examples of such cases are cited by Ambros (Geschichte der Musik). Similar cases are also met at a later time. For example, we will cite a fugue for piano in F Major by Handel, where a sequence has been placed in measures of 4/4, the motive of which equals 5/4.

§ 164. With this we will finish with infinite three-voice canons where the Proposta returns to previous degrees, and we will proceed to examples of canonic sequences of the same order.

---

August W. Ambros, ed., Geschichte der Musik (Breslau, 1868). (P.G.)

Taneev is not specific about which fugue he means. Handel's complete works contain two possibilities: Fugue in F Major from the Second Piano Suite, ser. 4, vol. 1; Fugue in F Major, ser. 4, vol. 17, in the Hallishce Handel-Ausgabe, eds. Max Schneider and Rudolph Steglich (Kassel, 1955).
Chapter 14

Infinite three-voice canon of the second order with secondary entrances on other degrees (the canonic sequence of the second order)

§ 165. We will cite examples of the three-voice canonic sequence of the second order. Following the explanations that have been provided above, these examples, as well as their schemata, will be entirely clear. We will begin with a sequence without JJv and with a single imaginary combination, the Proposta of which has been borrowed from Example No. 173.

No. 180.\textsuperscript{190}

P.XI,65.

\[\text{\footnotesize{\textsuperscript{190} Giovanni Pierluigi da Palestrina, "Sanctus" from Missa: Ad Fugam, mm. 19-23. A tenor clef has been replaced by a bass clef on the second line of this example. This correction is cited by Wehrmeyer in Sergei Taneev, Die Lehre vom Kanon, trans. Andreas Wehrmeyer (Berlin: Ernst Kuhn, 1994), p. 197. (P.G.)}}\]
§ 166. The subsequent examples have [various] JJv.


No. 181.\textsuperscript{191}

P.X.60.

\begin{center}
\includegraphics[width=0.5\textwidth]{example181}
\end{center}

No. 182.\textsuperscript{192}

G. Meyer

(G.D.373)

\begin{center}
\includegraphics[width=0.5\textwidth]{example182}
\end{center}

\textsuperscript{191} Giovanni Pierluigi da Palestrina, "Gloria" from Missa: Vitute magna, mm. 55-56. (P.G.)

\textsuperscript{192} Gregorius Meyer, "Kyrie" from Missa: De nostra Domina, mm. 48-50. (P.G.)
An example with two identical JJv. The following example has both JJv=-14. Its Proposta is the same as the Proposta in the canon at No. 178.193

No. 183.

Finally, we will cite examples with three JJv, of which two JJv are identical.

2JJv=-11, Jv=-7.

---

193 Instead of "Its Proposta is the same as the Proposta in the canon at No. 25", which is present in the original text, the correction "Its Proposta is the same as the Proposta in the canon at No. 178" has been added in the translation. This correction is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 198. (P.G.)
No. 184. 194

P.XII, 53.

194 Giovanni Pierluigi da Palestrina, "Gloria" from Missa: Brevis, mm. 56-8. (P.G.)
We will conclude the section on the three-voice infinite canon with these examples, and we will shift to the infinite four-voice canon.

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195 Giovanni Pierluigi da Palestrina, "Gloria" from Missa: Vitute magna, mm. 34-7. The natural sign over b1 in the sixth measure of this example, which is missing in the original text, has been added in the translation. This correction is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 198. (P.G.)
II. Four-voice infinite canon

Chapter 15

Infinite four-voice canon of the first order with Propostas that repeat on the same degrees

§ 167. The primary entrances of the four-voice infinite canon are P, R1, R2, and R3; the secondary entrances are P, R1, R2, and R3. Like the others, these canons are divided into two orders, depending on whether all of the distances between entrances are rhythmically identical (a canon of the first order, which requires the application of vertical-shifting counterpoint) or all or several of them are different among themselves (a canon of the second order, which requires the application of horizontal-shifting counterpoint alone or in combination with vertical-shifting). Regarding the division of the infinite canon into either being in the proper sense or a canonic sequence, this was discussed earlier (§ 23).

The number of sections of an infinite canon of the first order is equal to the number of voices in the canon (§ 24); consequently, [the number of sections of a four-voice infinite canon is equal] to four.

196 Instead of "The primary entrances of the four-voice infinite canon are P1, R1, R2, and R3", which is present in the original text, "The primary entrances of the four-voice infinite canon are P, R1, R2, and R3" has been added in the translation. This correction is cited by Wehrmeyer in Sergej Taneev, Die Lehre vom Kanon, trans. Andreas Wehrmeyer (Berlin: Ernst Kuhn, 1994), p. 198. (P.G.)
§ 168. The reciprocal relationship of original and derivative combinations [in the four-voice infinite canon] is apparent in the following schema:

\[
\begin{array}{cccccccc}
I & j & j & i & I & \mu^{m} & \mu^{1} & \mu^{n} \\
\alpha & \alpha & \alpha & \beta & \beta & \beta^{n} & \beta^{1} & \beta^{m} \\
\alpha & \alpha & \alpha & \alpha & \alpha & \alpha^{m} & \alpha^{1} & \alpha^{n} \\
P & R_{1} & R_{2} & R_{3} & R_{1} & R_{2} & R_{1} & R_{2} \\
\beta & \beta & \beta & \beta & \beta & \beta & \beta & \beta \\
\mu & \mu & \mu & \mu & \mu & \mu & \mu & \mu \\
\end{array}
\]

The secondary entrance of the last Risposta in the canon (/R3) has not been included in the schema since at its entrance all four voices carry out that which was earlier at the entrance of R3. In the infinite canon in the proper sense, this repetition occurs on previous degrees; in the canonic sequence, it is shifted by an interval either upwards or downwards.

§ 169. The number of original combinations is equal to the number of Rispostas in the canon; consequently, here it is equal to three (P+R1, P+R2, P+R3).\textsuperscript{197} The number of derivative combinations of each original is also equal to this number, understanding that several of the latter can

\textsuperscript{197} Taneev's reference to § 23 in this sentence has been omitted. (P.G.)
have \( Jv = 0 \). When there is an absence of indices that are equal to zero, the number of derivative combinations of each original is also equal to three. The maximum number of \( JJv \) of such a canon is, consequently, equal to 9, included in this number are indices that are equal to one another. In reality, the number of indices is reduced, in the first place by \( Jv = 0 \), and in the second place by an equality of indices that belong to the same original.

§ 170. The schema cited above graphically depicts the number of original and derivative combinations and their mutual dependence. Nevertheless, in light of the complexity and cumbersomeness that hampers the formation of such a schema, one should prefer the schema that has been formed by the other method, which has been indicated in the Comment at § 115. As is already known, this method consists of primary intervals of entrance being measured out to the left and in a reverse direction initially from the points of entrance of each primary Risposta, and, as was indicated above, according to these marked-off degrees, \( JJv \) are found. By means of this, we obtain a group of conditions that is identically required by both a finite and infinite canon with the same entrances. But for the latter, to this primary group of conditions others must be added—the conditions that occur from the measuring out of primary intervals of entrance from the point of each secondary entrance, excluding the entrance of the last repeated Risposta (//R3), i.e., from //P, //R1, and
//R2. JJv that have been obtained from the setting aside of primary intervals of entrance to the left of points of entrance of the aforementioned repeated voices present conditions that belong exclusively to the infinite canon, and together with the primary group of conditions they form an aggregate of rules for a given infinite canon (and canonic sequence).

§ 171. By this means, we will form a schema of the following canon, for example:

Initially, we will deduce a group of rules that is shared by a finite canon that has the same entrances. To accomplish this aim, we momentarily disregard all secondary entrances and measure out primary intervals from R2, then from R3 (from each entrance are set aside as many primary intervals of entrance as there are Rispostas that have entered before).
Thus, we will obtain the first group of conditions:

\[(P+R_1) \ Jv=3,-6\]

\[(P+R_2) \ Jv=-6\]

Now to these conditions we will gradually add intervals of entrance to the second group of entrances. At first we will add intervals that have been set aside from \(//P:\)

![Diagram of intervals](image1)

to this we will add intervals that have been set aside from \(//R_1:\)

![Diagram of intervals](image2)

and, finally, intervals that have been set aside from the penultimate Risposta (from \(//R_2\)).

![Diagram of intervals](image3)
With regard to the first note that has been set aside to the left of //R2 and that has been enclosed in brackets [•], we will indicate that this note does not indicate any sort of Jv and is necessary only in order to serve as a step to additional measured-out primary intervals. In general, one should keep in mind that JJv can only occur in combinations of the Proposta with entrances of primary Rispostas and not repetitions, and if a set-apart note coincides with //P or with secondary entrances that follow after it, then it will not indicate any sort of Jv. Now we have a full list of conditions for the infinite canon with the aforementioned entrances (how JJv are found according to such a schema was indicated above). Here is the list of conditions:

\[ Jv=3, -6, -13 \]
\[ Jv=-6, -16, -22 \]
\[ Jv=-7, -13, -16 \]

In this example, Jv=0 is absent, as are equivalent JJv that would pertain to a single original, and that is why it includes the maximum number of derivatives, i.e., three for

---

198 It is not necessary to consider the dots that follow after //P to the right as voices in the canon because they repeat relationships that occured between corresponding primary entrances. (Note Taneev)
each of the three originals, in accordance with the number of primary Rispostas in the canon.

All that has been said pertains identically to canonic sequences.

§ 172. From the examined schema, it is apparent how great the difficulty is for composing a four-voice infinite canon of the first order. Due to the complexity and intricacy of the mutual relationship of the voices and to an abundance of original and derivative combinations, this canon surpasses all aspects of canon that have been examined so far. Even if we use the methods indicated here, and have an opportunity to theoretically explain the correlations of voices in each individual case, and an opportunity to explain the complex conditions that the canon obeys; nevertheless, to write a canon that satisfies all of these conditions can turn out to be extremely difficult and sometimes entirely impossible. It will be important to protect the student from an unproductive waste of time as a result of insurmountable assignments and to choose those cases that present the least difficulty so that he can focus his attention primarily on these tasks. As examples, we choose those cases of the infinite canon (and of the canonic sequence) whose composition does not present insurmountable difficulty and whose schemata can serve as independent studies for the student.
§ 173. When presenting the doctrine of the three-voice infinite canon of the first order, we indicated that even if the number of original combinations in the infinite canon cannot be diminished, it will be possible on the other hand to limit the number of derivative combinations and, consequently, also the number of JJv by making several of the JJv equal to zero. Jv=0 is the result of similarities in size and direction of intervals of entrance—of the interval of an original and the interval of a derivative. Based on this, as the main material for exercises in three-voice infinite canon of the first order, we recommended placing all primary entrances in a single direction and at a single interval from each other. By means of this method, we reduced the number of JJv to a minimum—namely, to a single Jv for each original combination with an identical value for all originals.

§ 174. Now we will examine the results that this method will provide when applied to four-voice infinite canon of the first order. It is known that when there is a similarity of size and direction of primary intervals of entrance the value of the Jv that pertains to each original combination is equal in the four-voice canon to Jv=−4m, where m is understood to be the size of each of the primary intervals of entrance that are equal to one another. We saw also that the order of entrances of the canon may change and, therefore, it can
begin with any of the entrances, and the remaining follow after the first without a change in their order, for example:

\[
\begin{array}{ccc}
\text{m=0} & Jv=0 \\
m=1 & Jv=-4 \text{ (double counterpoint at the fifth)} \\
m=2 & Jv=-8 \\
m=3 & Jv=-12 \\
m=4 & Jv=-16 \text{ (double counterpoint at the dezime)} \\
m=5 & Jv=-20 \\
m=6 & Jv=-24 \\
m=7 & Jv=-28 \text{ (double counterpoint at the octave)}
\end{array}
\]

In so doing, any of these voices can serve as the Proposta. In such an alteration, the canon maintains all at \( Jv=-4m \), which always pertains to each of the three combinations of the Proposta with each of the primary Rispostas and always stipulates an inverse shift.\(^{199}\) On the basis of the formula \( Jv=-4m \), we obtain the following list of indices:

\(^{199}\) Instead of "...which always pertains to each of the three combinations of the Risposta with each of the primary Rispostas", which is present in the original text, "...which always pertains to each of the three combinations of the Proposta with each of the primary Rispostas" has been added in the translation. This correction is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 198. (P.G.)
From this table, we see that if the entrance \( \rightarrow 0 \) is not considered, at which complex counterpoint is absent, and the entrance \( \rightarrow 7 \), even though it requires double counterpoint at the octave \( (Jv=-28) \), but is unfit as a result of the large distance—three octave—between the outer voices, the following entrances will remain that require comparatively less difficult JJv, namely, \( m=1 \), which requires \( Jv=-4> \); and \( m=4 \), which requires \( Jv=-16 \) (i.e., double counterpoint at the tenth).

As for \( Jv=-4> \), even though this is not a difficult \( Jv \), the range in which the voices must move is too small \( (4>) \), and that is why such a canon allows a Proposta of only the most insignificant range. Such a canon will be cited below.

Thus, only canon at the fifth—\( m=4, Jv=-16 \)—remains accessible.

But to add this canon to the category of those that are easily written is not possible since the absence of direct motion between the Proposta and each of the Rispostas is a condition so inconvenient that either it is necessary to interrupt the voices with pauses (as was done in the canon with these entrances cited below) or introduce hidden progressions, which can be badly answered in the voice leading.

For the type of four-voice infinite canon examined here (with a Proposta that returns to the same degrees), we will
note that it is not necessary to consider the possibility of writing the canon at $Jv=-11$, i.e., double counterpoint at the duodezime, the most suitable $Jv$ due to an abundance of fixed consonances and to the presence of direct motion for any sort of contrapuntal combination.\footnote{For a definition of "fixed" intervals see the one-hundred and thirtieth footnote in Chapter 7. (P.G.)} We would only obtain this double counterpoint if $m=8$. When $m=8$, $Jv=-32$, i.e., we would have double counterpoint at the duodezime. But obviously to write a canon with these entrances that does not exceed the allowable limits of the voices is not possible.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{music_example}
\caption{Example of double counterpoint at the duodezime.}
\end{figure}

§ 175. From what has been said, to what degree there is a paucity of resources that exist for the examined aspects of four-voice infinite canon is apparent. The main reason for this is that the most accessible distributions of intervals for the three-voice canon—identical intervals according to size and direction—leads to those $JJv$ in four-voice infinite canon that are the most awkward and difficult. The following example can confirm what has been said. The four-voice canon at $m=2$, $Jv=-8$, i.e., where the simultaneous application of double counterpoint at the ninth to three Rispostas is
required, is perhaps the most difficult of all that exist. At these conditions, to write a four-voice canon in the strict style can be considered an unfulfillable task. We will imagine that it is necessary for us to write not a four-voice, but a seven-voice canon with the same primary entrances at the third \((m=2)\). It would seem that the impossibility of writing a canon in four-voices would make an attempt at writing a seven-voice canon with these same entrances imprudent. Whereas this is not so. In seven voices, this canon must have \(Jv=-7 \times m=-14\), i.e., in all it requires double counterpoint at the octave—the easiest of all that exist. Moreover, if we wish to write an eight-voice canon, even then our task would not be impossible because only double counterpoint at the dezime—\(Jv=-8 \times m=-16\)—would be necessary.\(^{201}\)

Returning to the four-voice infinite canon of the first order, we will note that the difficulties that accompany its composition must have had as their consequence a great number of failed attempts at its composition. It is unquestionably necessary to add to this circumstance that even though

\(^{201}\) Instead of "In seven voices, this canon must have \(Jv=-7, m=-14\)" and "...even then our task would not be impossible because only double counterpoint at the dezime—\(Jv=-8, m=-16\)—would be necessary", which are present in the original text, "In seven voices, this canon must have \(Jv=-7 \times m=-14\)" and "...even then our task would not be impossible because only double counterpoint at the dezime—\(Jv=-8 \times m=-16\)—would be necessary" have been added in the translation. This correction is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 198. (P.G.)
theorists composed canons in great numbers, so few infinite
102 canons of the first order are encountered.

§ 176. We proceed to exercises in infinite canon with a
Proposta that returns to the initial degree. The easiest and
most often encountered case of all such canons is canon at
the prime. But as we know, this canon does not require
applications of any complex counterpoint, and that is why we
will not cite four-voice examples of this canon. They were
sufficiently discussed in the section on three-voice infinite
canon, and models of this canon can be encountered in great
numbers in existing texts.

§ 177. The canon in double counterpoint at the octave
presents the next level of difficulty. Usually in such a
canon, entrances at the octave alternate with entrances at
the prime (counting from the Proposta). Such an alternation
provides an advantage to this canon compared to canon where
all entrances are at 0. The entrances that alternate between
0 and 7 allow mixed voices to take part in the canon that are
awkward with regard to the range of the voices at entrances
exclusively at \( \rightarrow 0 \). At the same time, such an alternation of
entrances liberates the Proposta from the necessity of being
written in counterpoint to each of the voices at \( Jv=-7 \), since

\[ \text{Instead of "...so few infinite canons of the second order"}
\] which is present in the original text, "...so few infinite canons of the
first order" has been added in the translation. This correction is
cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 198. (P.G.)
the Proposta begins in simple counterpoint with voices that enter on the same pitch with it. In Example No. 186:

No. 186.\textsuperscript{203}

P.III,123.

\begin{center}
\includegraphics[width=\textwidth]{example.png}
\end{center}

it is apparent that unprepared fourths can be freely placed between P+R1 (a) only if another pair of voices simultaneously contains the bottom voice, which transforms this fourth into a consonance. As soon as this fourth shifts to R2+R3, then the bottom voice will turn out to be in the first pair of voices (c), and the previous arrangement of intervals will be repeated.\textsuperscript{204}

\footnotesize
\textsuperscript{203} Giovanni Pierluigi da Palestrina, Motet: \textit{Judica me Deus, et discerne}, mm. 74-7. Instead of "P.III,133", which is present in the original text, "P.III,123" has been added in the translation. This correction is cited by Wehrmeyer in Taneev, \textit{Die Lehre vom Kanon}, p. 198. (P.G.)

\textsuperscript{204} Instead of "As soon as this fourth shifts to R1+R2...", which is present in the original text, "As soon as this fourth shifts to R2+R3..." has been added in the translation. This correction is cited by Wehrmeyer in Taneev, \textit{Die Lehre vom Kanon}, p. 198. (P.G.)
§ 178. In the following example, also by Palestrina:

No. 187.\textsuperscript{205}

P.XII,75.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{example.png}
\end{figure}

the incorrectness of the unprepared fourth is eliminated by
the sustained note in the bass voice. In contemporary music,
an excellent example of an infinite canon at the octave and
prime (also on a sustained note in the bass) is present in
the canon in Ruslan: "Kakoe chudnoe mgnovenie."\textsuperscript{206}

§ 179. Before canon at the prime and octave will be
abandoned, we will note that besides the sustained note in
the bass there is an additional way that permits the
unhindered use of the unprepared fourth and fifth and reduces

\textsuperscript{205} Giovanni Pierluigi da Palestrina, "Kyrie" from Missa: L'homme arme, mm. 10-13. In this example, the designation "Rl" for the first Risposta has replaced "R", which is present in the original text. This correction is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 198. (P.G.)

\textsuperscript{206} Mikhail Ivanovich Glinka, "Kakoe chudnoe mgnovenie" [What a beautiful moment], from Ruslan i Liudmila, in the complete works of Glinka, vols. 14-15 (Moscow, 1955- ). (P.G.)
the limitations of double counterpoint at the octave to a single requirement: Avoid parallel fourths and ninths that are dissonant above. This way consists of writing a canon over a repeating bass that is equal to the length of a section, i.e., to the rhythmic distance between neighboring entrances. This canon is written in absolutely the same way as the canon at 0, the only difference is that each line of the Proposta is written simultaneously in counterpoint to that which has been written on the preceding line and to the bass voice. Such a canon (if, as was said, two fourths in succession and ninths are avoided) can have entrances at both 0 and 7.

§ 180. We proceed to canon at other more difficult intervals, and we will recall what has already been stated about this (§ 174). In the first case we consider \( m = 1 \), \( j\nu = -4 \).
In this example, the melody of the Proposta is limited to three notes. In spite of this melodic simplicity, in the fourth measure it was necessary to resort to a pause since it turned out to be impossible to locate for the Proposta a

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207 In the second example, the soprano clef on the second line, which is present in the original text, has been replaced by an alto clef in the translation. This correction is cited by Wehrmeyer in Taneev, *Die Lehre vom Kanon*, p. 198. To R3 of the fourth example, a whole rest has been added in the seventh measure, and to the Proposta of the fifth example, a whole rest has been added in the fourth measure. (P.G.)
continuation of the melody that would be within the conditions of \( Jv=-4 \) with regard to each Risposta.\(^{208}\)

§ 181. In this canon, the combination P+R allows a very large number of shifts at a variety of JJv, with the exception of a shift that provides \( \bar{6} \) at a point of entrance of a Risposta from above and a shift that provides \( \bar{1} \) at a point of entrance of a Risposta from below. Here are these diverse combinations:

Due to this possibility for numerous shifts and also to the pause, which occupies the length of a whole section and removes the combination P+R3 and, consequently, also its derivatives, it is possible to take advantage of this

\(^{208}\) Instead of "...in the third measure it was necessary to resort to a pause", which is present in the original text, "...in the fourth measure it was necessary to resort to a pause" has been added in the translation. (P.G.)
Proposta in infinite canons with extremely diverse entrances, as [is apparent in Example No. 188].

We will derive JJv for any of these canons, for example, for the fourth. For the original combination P+R3, we will note that it is not necessary to indicate a Jv due to the pause that removes this combination. In multi-voice canons, if JJv are to be deduced by the first of the methods that have been indicated here, then it will be more convenient to do this on a separate musical line for each original, namely:

Thus, we can deduce JJv for all canons that have this Proposta. But this richness of combinations has been acquired at the cost of melodic poverty of the theme. If we attempt to continue or to augment the sections of this canon,

\[ R_1 \quad R_2 \quad R_3 \quad R_4 \]

\[ Jv=-4+3=-1 \]
\[ Jv=-4-11=-15 \]

\[ Jv=-7+7=0 \]
\[ Jv=-7-7=-14 \]

\[ Jv=-11-4=-15 \]
\[ Jv=-11+3=-14 \]

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209 Instead of "... and removes the combination P+R2", which is present in the original text, "... and removes the combination P+R3" has been added in the translation. This correction is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 198. (P.G.)
it would immediately turn out that we would be entangled from all sides by numerous constraints, which are inseparably linked to this type of canon. For such a short and meager musical theme we will note that it was certainly not necessary for the composition of each of the aforementioned canons to make calculations of all JJv, examples of which we cited. Here it would be possible to locate diverse entrances more quickly by simply trying them out rather than by deriving complex calculations that would be, in the given situation, an unnecessary complication, but which are difficult to avoid in cases of greater complexity.

§ 182. In the following example:

No. 189.\textsuperscript{110}

\begin{center}
\includegraphics[width=0.5\textwidth]{example.png}
\end{center}

because \( Jv = -9 \), significant difficulties present a restriction regarding direct motion. In the fourth measure, the Proposta

\textsuperscript{110}Instead of the designation "R" for the third Risposta, "R3" has been added in the translation. This correction is cited by Wehrmeyer in Taneev, \textit{Die Lehre vom Kanon}, p. 198. (P.G.)
has a pause equal to a whole measure, in this instance, it is equal to a whole section as a result of the impossibility to continue counterpoint of the Proposta at the required conditions. Due to the defects of this example, it is necessary to place empty consonances on the strong part of each measure. Several large harmonic half notes make doubling of the Proposta at the upper tenth possible due to the absence of direct motion between the voices.

§ 183. Example No. 190:

presents a more complex case of entrances, as can be seen in the supplemental schemata. This example requires no further explanations.
§ 184. [The schema of the canon at the duodezime that has been cited at § 174:

\[
\begin{align*}
&J_v = -4 - 7 = -11 \\
&J_v = -4 - 3 = -7 \\
&J_v = 4 + 7 = 3
\end{align*}
\]

which is awkward with regard to the tesituras of the voices that participate in this canon, can have practical application by reducing several of the intervals of entrance by an octave, as is apparent in the following schema:²¹¹

\[
\begin{align*}
&J_v = -4 - 7 = -11 \\
&J_v = -4 - 3 = -7 \\
&J_v = 4 + 7 = 3
\end{align*}
\]

according to which the following canon has been written:

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²¹¹ Instead of the starting note "F" in the Proposta, "A" has been added in the translation. This correction is cited by Wehrmeyer in Taneev, *Die Lehre vom Kanon*, p. 198. (P.G.)
§ 185. On this we will finish the four-voice infinite canon of the first order with a Proposta that returns to the same degrees. This canon and, in general, similar infinite canons, almost never has application in composition (with the exception of canons at 0 and 7). The great difficulties that are involved in its composition make this canon of little use for pedagogical aims, and exercises in them are almost unnecessary. Canonic sequences of the first order, to which we now turn, present a completely opposite phenomenon.

Canonic sequence of the first order

§ 186. The canonic sequence is written with far fewer difficulties than the infinite canon in the proper sense because we can arbitrarily select the last interval between R3+/P. The application [of canonic sequences] in composition is extremely significant. We encounter them in music by composers of the strict style and in music by composers of
later epochs. They were used by Bach, Handel, and Mozart. If they are a rare phenomenon in music by contemporary composers, this must not be attributed to a lack of appropriateness for contemporary musical style, but more likely to a general collapse in the technique of complex counterpoint. As an exception in this regard, we will cite Schumann (the sequences in the last movement of the piano quartet) and Glazunov (the Seventh Symphony).

§ 187. Similar to three-voice canonic sequences, where the simplest cases are those entrances in which two of the intervals are identical in size and direction and the third has a contrasting direction to them, here the easiest cases are those where three intervals of entrance are similar in size and direction and the fourth has a contrasting direction to them. As is known, in these sequences there is only one $J_v=-m-n$, where one of these letters is considered the size of the interval similar to the others, and the other interval has the contrasting direction. At this single $J_v$, the Proposta should be in counterpoint to each of the Rispostas.

Initially, we will cite the simplest cases of entrances that require $J_v=-7$ or $J_v=-14$.

\[\text{---}^{212}\text{---}\]

`212` Robert Schumann, Klavierquartett in Es-Dur, op. 47, Finale, ser. 5, vol. 1 of Schumann's complete works, ed. Clara Schumann (Leipzig, 1879); Alexsander Glazunov, Symphony no. 7 in F-Dur, op. 77 (Moscow, 1949). (P.G.)
These sequences require so-called quadruple counterpoint at the octave, i.e., counterpoint of the Proposta to each of the Rispostas at \( Jv = -7 \) or at \( Jv = -14 \). In spite of the relative ease of such a task, this sequence is almost unfit for the strict style. With the first repetition on other degrees in each of the aforementioned series of entrances, entrances occur on each of the degrees of a major scale. Consequently, even if the Proposta does not have a range that exceeds the fourth, in one of its shifts to other degrees, it will turn out to be within the range of an augmented fourth. However, this circumstance, despite being a hindrance for such sequences in the strict style, does not hinder their composition in the free style, and the student may try his skills on similar relatively easy sequences, using augmented and diminished intervals, chromaticism, etc. As for the strict style, for canonic sequences, and also for any sort of imitation in general, it is necessary to mainly use such entrances where the melody is shifted by either an octave or by the most accessible interval after the octave—the fifth. As is known, in such cases a dominant roll is acquired by
§ 188. In Example No. 192, as in examples of the infinite canon that were cited earlier, the original P+R3 and its derivative combinations are absent as a result of a pause that is equal to a whole section.

No. 192.

\[ \text{P.X.134.} \]

§ 189. In Example No. 193, the Proposta also has a pause, but it is equal to three fourths of a measure, in the given situation, [to three fourths] of a section; therefore, the end of the Proposta overlaps the first note of R3.

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\[ ^{213} \text{Giovanni Pierluigi da Palestrina, "Agnus Dei I" from Missa: Ad coenam Agni providi, mm. 20-2. (P.G.)} \]
§ 190. With regard to Example No. 194, it will be necessary to note that its three lower voices were cited above as an example of three-voice canonic sequence of the second order.  

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214 "c1" and "g" have been added to the Proposta in the third complete measure. "b flat" has been added to //P in the seventh complete measure. These corrections are cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 198. (P.G.)

215 The three-voice canonic sequence referred to in this passage is at § 166, Example No. 184. (P.G.)
§ 191. Example No. 195 could be continued further in the free style, and a modulatory sequence could be made from it.

No. 195.\textsuperscript{216}

P.XVII.94.

\textsuperscript{216}Giovanni Pierluigi da Palestrina, "Gloria" from Missa: Dum complerentur, mm. 121-3. (P.G.)
§ 192 In Example No. 196, in addition to the entrances at the fifth and octave, there is also an entrance at the third (R1), which makes this schema more complex and causes the participation of several JJv: (P+R1) JJv=5, -16; (P+R2) JJv=5, -11, -16. But these JJv do not present significant difficulties.

No. 196.²¹⁷

P.XIII,98.
§ 193. In conclusion, we introduce an example of a sequence that has been built on the Proposta from Example No. 188:

No. 197.
Chapter 16

Infinite four-voice canon and the canonic sequence of the second order with secondary entrances on other degrees

§ 194. Following the explanations that were provided earlier, it is sufficient to limit ourselves to a citation of examples of the infinite canon and canonic sequence of the second order. An infinite canon of the second order (in the proper sense) is incomparably easier to write than a canon of the first order. Due to this, it is possible to explain why in textbooks on counterpoint almost all four-voice infinite canons that are encountered belong to the second order.

§ 195. We will cite examples of the infinite canon of the second order.
The primary entrances of this canon belong to the first order and require $Jv = -11$ for $P+R_1$. In the given situation, as we already know, this same $Jv$ pertains to the combination of the Proposta with the voice that will be set aside to the left of $//P$ at the same distance that $R_1$ will stand to the

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218 Giovanni Pierluigi da Palestrina, "Credo" from Missa: Sanctorum merits, mm. 147-54. (P.G.)
right of the Proposta that enters the first of the combinations that has been mentioned. When writing such an example, we will recall that we can shift the entrances of the imaginary voices of the second group of entrances to a point where the Proposta is conveniently in counterpoint to them—this circumstance significantly facilitates the composition of canon of the second order in comparison to canon of the first order.

We will note that three of the imaginary voices present entrances that occur in the same order as the first three primary entrances, i.e., they form a contrapuntally correct combination that facilitates their counterpoint with the Proposta.

§ 196. In Example No. 199, to JV=-11, which belongs to the group of primary entrances, we add JV=-2 for P+R3, which owes its origin to the group of secondary entrances.

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Instead of "...this same JV pertains to the combination of the Proposta with the voice that will be set aside to the left of P", which is present in the original text, the correction "...this same JV pertains to the combination of the Proposta with the voice that will be set aside to the left of /P" has been added in the translation. This correction is cited by Wehrmeyer in Sergej Taneev, *Die Lehre vom Kanon*, trans. Andreas Wehrmeyer (Berlin: Ernst Kuhn, 1994), p. 198. (P.G.)

Instead of "In Example No. 42...", which is present in the original text, the correction "In Example No. 199..." has been added in the translation. This correction is cited by Wehrmeyer in Taneev, *Die Lehre vom Kanon*, p. 198. (P.G.)
In conclusion, we cite an example of a canonic sequence of the second order:

221 Giovanni Pierluigi da Palestrina, "Gloria" from Missa: Alma Redemptoris, mm. 81-2. An alto clef has been added in place of a bass clef for the lower voice. This correction is cited by Wehrmeyer in Taneev, Die Lehre vom Kanon, p. 198. (P.G.)
In this example, double counterpoint is applied at the duodezime in the form of Jv=-25, which is rarely encountered. Here there is only one imaginary voice. The remaining imaginary voices coincided with other real voices and with this imaginary.

222 Josquin des Pres, "Planxit autem David," mm. 24-7. (P.G.)
Part II: Commentary
Introduction to Part II

Taneev's Theories in Russia during the Soviet Era and in the Present

As Carpenter points out, in contrast to the theories of other prominent, prerevolutionary musical theorists, Taneev's approach to moveable counterpoint and canon continued to be "utilized essentially as Taneev conceived it" during the Soviet era.223 His theories and several of the developments of his theories by Soviet music theorists remained to a large degree free of Communist propaganda because they conformed to two fundamental Party objectives for the composition and study of music: Taneev's theories deal with a conservative type of tonality, and the developments of Taneev's theories by his followers participate in a tradition established by an eminent Russian music theorist.

With regard to the first point, in order to promote socialist political ideology, the Soviet authorities favored music that was accessible to all members of society. Because Taneev consistently expressed conservative views about tonality, and because his contrapuntal theories deal with the modal language of strict-style counterpoint, his theories were acceptable to the Communist Party.

With regard to the second point, the Party's emphasis on an unbroken link with those aspects of Russian culture that could be viewed as traditional or classical protected several of the developments of Taneev's theories from revision. That Taneev was seen by the Soviets as a founder of such a tradition is confirmed in the words of Boris Asafiev, the foremost pioneer of Soviet musicology. Asafiev called Convertible Counterpoint in the Strict Style "a scientific treatise which is not inferior to the works of theorists of the Renaissance and the Age of Enlightenment."

Furthermore, Taneev's contrapuntal theories remained unaltered because they were practical for training composers and theory students.

During the Soviet era, Taneev's theories proceeded in two distinct directions. According to Aleksandr Rovenko, a Russian pedagogue and music theorist, these directions were pedagogy and development. With regard to the application of Taneev's theories to the instruction of counterpoint,

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several pedagogical texts were published. These texts vary in level of sophistication and purpose.

With regard to a scientific development of the theories presented in *Doctrine of the Canon*, there were several notable contributions: developments by Semyon Semyonovich Bogatyryov (1890-1960), Mark Kopytman (b. 1929), Evgeny Nikolaevich Korchinsky (1904-65), Sergey Sergeevich Skrebkov (1905-67), and Nikolay Andreevich Timofeev (1906-78). Their theories received various degrees of practical use and development, and each will be discussed in Part II of this dissertation.

226 A chronological list of significant pedagogical works that employ Taneev's theories of moveable counterpoint are: A. V. Abutkov, *Slozhnyi kontrapunkt strogoi pis'ma* [Complex counterpoint in the strict style] vol. 2 of *Rukovodstvo k izucheniiu kontrapunkta, kanona i fugi* [Guide to the study of counterpoint, canon and fugue] (Moscow, 1913); Viktor Mikhailovich Beliaev, *Kratkoe izlozhenie ucheniia o kontrapunkte i ucheniia o muzikal'nykh formakh* [A brief exposition of the doctrine of counterpoint and the doctrine of musical forms] (Moscow, 1915); Georgii Eduardovich Konius, *Kurs kontrapunkta strogoi pis'ma v ladakh* [A course on counterpoint in the strict style in modes] (Moscow, 1930); Sergey Sergeevich Skrebkov, *Uchebnik polifonii* [Textbook of polyphony] (Moscow, 1951); S. Pavliuchenko, *Rukovodstvo k prakticheskomu izucheniiu osnov inventsionnoi polifonii* [Guide to the practical study of the foundations of inventive polyphony] (Moscow, 1953); -----, *Prakticheskoe rukovodstvo po kontrapunktu strogoi pis'ma* [Practical guide to counterpoint in the strict style] (Leningrad, 1963); Genrikh Il'ich Litinskii, *Obrazovanie imitatsii strogoi pis'ma* [The formation of imitation in the strict style] (Moscow, 1971); Aleksandr Sergeevich Stepanov and Aleksandr Georgievich Chugaev, *Polifonia* [Polyphony] (Moscow, 1972); M. I. Roitershtein, *Prakticheskaiia polifonii* [Practical polyphony] (Moscow, 1972); I. Pustyl'nik, *Prakticheskie rukovodstvo k napisaniu kanona* [A practical guide to the writing of canon] (Leningrad, 1975); Aleksandr Georgievich Chugaev, *Nekotorye voprosy prepodavaniia polifonii v muzikal'nom uchilishche* [Several issues of the teaching of polyphony in a musical vocational school] (Moscow, 1976); S. Grigor'ev and Teodor Miuller, *Uchebnik polifonii* [Textbook of polyphony], 4th ed. (Moscow, 1985); Viktor Pavlovich Fraenov, *Uchebnik polifonii* [Textbook of polyphony] (Moscow, 1987); Teodor Miuller, *Polifonia* [Polyphony] (Moscow, 1989).
As for the status of Taneev's theories of moveable counterpoint and canon in the present, they form the basis of current Russian pedagogical practice in counterpoint. They are employed at the Russian conservatories as well as at the musical vocational schools, and they continue to be developed by theorists such as Yury Igorevich Neklyudov, Aleksandr Ivanovich Rovenko, and Kirilina Yuzhak.

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Chapter 17
Bogatyryov: Double Canon

Introduction

Semyon Semyonovich Bogatyryov (1890-1960) was a highly respected music theorist and pedagogue in Soviet Russia.\textsuperscript{228} He taught at the Moscow Conservatory from 1943 to 1960, and he wrote two published texts on music theory, both of which contribute to the development of Taneev’s theories.\textsuperscript{229} In addition to his work as a theorist, he was responsible for the restoration of Tchaikovsky’s Symphony in E-flat.

Of all the theorists who developed Taneev’s theories of imitative counterpoint, Bogatyryov, in \textit{Dvoinoi kanon} [Double canon], follows the model of \textit{Doctrine of the Canon} the closest. He does this by applying Taneev’s system of classification to double canon, i.e., he studies finite double canon of the first order, finite double canon of the second order, infinite double canon of the first order, and

\textsuperscript{228} On March 7, 1995, during an interview with Viktor Pavlovich Frayonov, a music theorist and former student of Bogatyryov, in addition to commenting about his respect for Bogatyryov’s skill as a teacher, Frayonov spoke about his admiration of Bogatyryov’s courage. As a demonstration of this courage, he cited Bogatyryov’s teaching of Schoenberg’s music in 1953 in Stalinist Russia.

infinite double canon of the second order. The primary contribution of his theory of double canon is that it offers insight for avoiding difficult indices and for limiting the number of indices in vertical-shifting counterpoint in finite and infinite double canons of the first order.

Finite Double Canon of the First Order

In order to establish simple JJv for a four-voice double canon, Bogatyryov applies Taneev’s formula $Jv = -m + (+/\,-n)$ as follows. The value of $m$, the interval of entrance of an original combination, is assigned to the interval between the entrances of the two Propostas; $n$, the interval of entrance of a derivative combination, is assigned to the interval between the entrances of the two Rispostas. Similar to Taneev’s theories, $m$ is always equal to a positive value. $n$ is equal to a positive value when the entrance of R2 is below R1, and it is negative when the entrance of R2 is above R1. In Dvoinoi kanon, Bogatyryov provides entrances of original and derivative combinations with simple JJv. These are cited at Example 17.1.
Example 17.1. Entrances of original and derivative combinations in finite double canon of the first order with simple JJv.

\[ Jv = -m + (+/-n): \]

1) \[ Jv = -18 + 4 = -14 \]
2) \[ Jv = -18 + 2 = -16 \]
3) \[ Jv = -4 - 3 = -7 \]
4) \[ Jv = -7 - 7 = -14 \]
5) \[ Jv = -9 - 9 = -18 \]

Bogatyryov also provides simple indices for five- and six-voice double canons of this same type. For five-voice double canon he initially determines original and derivative combinations and then establishes correlations between the two that provide simple JJv. Following Taneev’s premise that a Proposta must enter all original combinations (Doctrine of the Canon, § 25), Bogatyryov establishes the following three original combinations and their derivatives for a double canon in which a three-voice canon is positioned above a two-voice canon.

In the sectionalized portrayal of this canon, the integer 1 indicates that a section belongs to the first canon; the integer 2 indicates that a section belongs to the second canon. The letters a and b are used to differentiate between the two Rispostas of the three-voice canon.
Sectionalized Portrayal of a Five-Voice Canon

\[
\begin{array}{ccccccc}
P1 & A1 & B1 & C1 & D1 & E1 & F1 \\
R1 & Aa1 & B1 & Ca1 & Da1 & Ea1 & Fa1 \\
Rb1 & Ab1 & Bb1 &Cb1 & Db1 & Eb1 & Fb1 \\
P2 & A2 & B2 & C2 & D2 & E2 & F2 \\
R2 & A2 & B2 & C2 & D2 & E2 & F2 \\
\end{array}
\]

Original Combinations

1. \( P1+P2 \)
2. \( P1+R1 \)
3. \( P2+R1 \)

Derivative Combinations

1. \( R1+R2 \), i.e., \( Aa1+A2 \)
2. \( R1+Rb1 \), i.e., \( Ba1+Ab1 \)
3. \( R2+Rb1 \), i.e., \( B2+Ab1 \)

In order to have dealings with only one \( Jv \) when composing this canon, Bogatyryov sets one of the three \( JJv \) equal to 0. This relationship occurs in three intervalic equivalencies: 1) \( P1+P2 = R1+R2 \), 2) \( P1+R1 = R1+Rb1 \), 3) \( P2+R1 = R2+Rb1 \). In each of these, the interval of an original combination is equal to the interval of its
derivative thereby causing one \( J_v \) to equal zero and the other two to equal one another. Several results of these correlations are displayed at Example 17.2.

Example 17.2. Entrances for original and derivative combinations in five-voice double canon.

1) \( n-(m)=J_v \)  2) \( n-(m)=J_v \)  3) \( n-(m)=J_v \)  4) \( n-(m)=J_v \)

\[
\begin{align*}
P_1+P_2 &= 5-16=-11 & 5-12=-7 & 9-2=7 & 9-9=0 \\
P_1+R_1 &= -4-7=-11 & -3-4=-7 & 3-3=0 & 5-5=0 \\
P_2+R_1 &= 9-9=0 & 8-8=0 & 12-5=7 & 14-14=0
\end{align*}
\]

Bogatyryov divides six-voice finite canons of the first order into two subdivisions: 1) \((P_1+R_1+R_b)+(P_2+R_a+R_b)\),

and 2) \((P_1+R_1)+(P_2+R_a+R_b+R_c)\) or \((P_1+R_1+R_b+R_c)+(P_2+R_2)\).

When this canon is written according to the first
subdivision, there are five original combinations and six derivatives. The originals are as follows:

**Sectionalized Portrayal of the Canon**

<table>
<thead>
<tr>
<th>P1</th>
<th>A1</th>
<th>B1</th>
<th>C1</th>
<th>D1</th>
<th>E1</th>
<th>F1</th>
<th>G1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ra1</td>
<td>Aa1</td>
<td>Ba1</td>
<td>Ca1</td>
<td>Da1</td>
<td>Ea1</td>
<td>Fa1</td>
<td>Ga1</td>
</tr>
<tr>
<td>Rb1</td>
<td>Ab1</td>
<td>Bb1</td>
<td>Cb1</td>
<td>Db1</td>
<td>Eb1</td>
<td>Fb1</td>
<td>Gb1</td>
</tr>
<tr>
<td>P2</td>
<td>A2</td>
<td>B2</td>
<td>C2</td>
<td>D2</td>
<td>E2</td>
<td>F2</td>
<td>G2</td>
</tr>
<tr>
<td>Ra2</td>
<td>Aa2</td>
<td>Ba2</td>
<td>Ca2</td>
<td>Da2</td>
<td>Ea2</td>
<td>Fa2</td>
<td>Ga2</td>
</tr>
<tr>
<td>Rb2</td>
<td>Ab2</td>
<td>Bb2</td>
<td>Cb2</td>
<td>Db2</td>
<td>Eb2</td>
<td>Fb2</td>
<td>Gb2</td>
</tr>
</tbody>
</table>

**Original Combinations**

1. P1+Ra1
2. P1+P2
3. P1+Ra2
4. P2+Ra1
5. P2+Ra2
Derivative Combinations

1. Ral+Rbl, i.e., Bal+Ab1
2a. Ral+Ra2, i.e., Aal+Aa2  
2b. Rbl+Rb2, i.e., Abl+Ab2
3. Ral+Rb2, i.e., Bal+Ab2
4. Rbl+Ra2, i.e., Abl+Ba2
5. Ra2+Rb2, i.e., Ba2+Ab2

Bogatyryov determines two arrangements of voices that produce the fewest JJv for this canon. These arrangements are those where the intervals of entrance within each of the two three-voice canons are identical, and those where the interval of an original of one of the two three-voice canons is equal to the interval of a derivative of the other canon. Bogatyryov demonstrates the first of these arrangements at Example 17.3.

Example 17.3. Entrances of a six-voice double canon with simple JJv. The intervals of entrance within each of the two three-voice canons are identical.
There are two \( J^v \) that belong to the original combination \( P_1+P_2 \) because it has two derivatives, namely, \( R_1+R_2 \) and \( R_1+R_2 \). Based upon the arrangements of the entrances at Example 17.3, Bogatyryov makes the following observations about this canon: Excluding the second \( J^v \) for the original combination \( P_1+P_2 \), when \( J^v \) is not equal to 0 for all combinations, this double canon requires the application of a single \( J^v \) for three of the original combinations (the two remaining \( J^v \) are equal to 0). The second \( J^v \) of the combination \( P_1+P_2 \) is made easier when entrances are chosen that create \( J^v=-7 \) for this original with regard to its first derivative.\(^{230} \) Furthermore, the absolute value of the \( J^v \) for this arrangement, excluding the second \( J^v \) of the original combination \( P_1+P_2 \), is equal to the sum of the intervals of

\begin{align*}
\begin{array}{cccccc}
1) & 2) & 3) & 4) & 5) \\
\text{P}_1+\text{Ra}_1 &= 0 & 0 & 0 & 0 & 0 \\
\text{P}_1+\text{P}_2 &= 0,0 & -2,-4 & -7,-14 & -11,-22 & 2,4 \\
\text{P}_1+\text{Ra}_2 &= 0 & -2 & -7 & -11 & 2 \\
\text{P}_2+\text{Ra}_1 &= 0 & -2 & -7 & -11 & 2 \\
\text{P}_2+\text{Ra}_2 &= 0 & 0 & 0 & 0 & 0
\end{array}
\end{align*}

\(^{230}\) This is true because the second \( J^v \) for this original combination is equal to twice the value of the first. With regard to the relationship of \( J^v \) separated by an octave, Taneev states the following at § 47 of Convertible Counterpoint in the Strict Style: "It is obvious that anything conforming to the conditions of simple counterpoint (i.e., a combination at \( J^v=0 \)) would also be correct for \( J^v=7 \), so this index does not require special rules. (Serge Ivanovitch Taniev, Convertible Counterpoint in the Strict Style, trans. G. Ackley Brower (Boston: Bruce Humphries, 1962) § 47, p. 41.)
entrance of the combinations \[(P_1+R_{al})+(P_2+R_{a2})=Jv\] when the entrances of one of the canons occur in a contrasting direction to the entrances of the other, and the \(Jv\) is equal to the difference between these intervals of entrance \[(P_1+R_{al})-(P_2+R_{a2})=Jv\] when all entrances occur in the same direction.

As stated above, the second arrangement of this six-voice canon that produces simple \(Jv\) occurs when an interval of an original combination of one of the three-voice canons is equal to an interval of a derivative combination of the other canon. Bogatyryov demonstrates this arrangement by establishing the relationships \(P_1+R_{al} = R_{a2}+R_{bl}\) and \(P_2+R_{a2} = R_{al}+R_{bl}\) between the following entrances at Example 17.4.

Example 17.4. Entrances that demonstrate the second simplified arrangement of a six-voice canon. An interval from an original combination of one of the three-voice canons is equal to an interval in a derivative of the other canon.
In the second example, $Jv$ is equal to both $+7$ and $-7$. In this case, because these $Jv$ are separated by two octaves, their coexistence does not increase the level of difficulty when writing this canon. However, if these dissimilar $Jv$ were equal to any other values, new difficulties would arise. In order to avoid new limitations, Bogatyryov prescribes that the interval between the derivative entrances $Ra1+Ra2$ should be smaller than the interval between the original entrances $P1+P2$. In such a case, the sign of the $Jv$ will be the same for each original combination.

The second subdivision of a six-voice double canon consists of the following original and derivative combinations:

<table>
<thead>
<tr>
<th></th>
<th>1)</th>
<th>2)</th>
<th>3)</th>
<th>4)</th>
<th>5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P1+Ra1$</td>
<td>0</td>
<td>0</td>
<td>-7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$P1+P2 = -7,-7$</td>
<td>7,7</td>
<td>7,7</td>
<td>-11,-11</td>
<td>-9,-9</td>
<td></td>
</tr>
<tr>
<td>$P1+Ra2$</td>
<td>0</td>
<td>0</td>
<td>-7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$P2+Ra1=-7$</td>
<td>7</td>
<td>0</td>
<td>-11</td>
<td>-9</td>
<td></td>
</tr>
<tr>
<td>$P2+Ra2=-7$</td>
<td>-7</td>
<td>0</td>
<td>-11</td>
<td>-9</td>
<td></td>
</tr>
</tbody>
</table>
Sectionalized Portrayal of the Canon

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>A1</th>
<th>B1</th>
<th>C1</th>
<th>D1</th>
<th>E1</th>
<th>F1</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>A1</td>
<td>B1</td>
<td>C1</td>
<td>D1</td>
<td>E1</td>
<td>F1</td>
<td></td>
</tr>
<tr>
<td>P2</td>
<td>A2</td>
<td>B2</td>
<td>C2</td>
<td>D2</td>
<td>E2</td>
<td>F2</td>
<td></td>
</tr>
<tr>
<td>Ra2</td>
<td>Aa2</td>
<td>Ba2</td>
<td>Ca2</td>
<td>Da2</td>
<td>Ea2</td>
<td>Fa2</td>
<td></td>
</tr>
<tr>
<td>Rb2</td>
<td>Ab2</td>
<td>Bb2</td>
<td>Cb2</td>
<td>Db2</td>
<td>Eb2</td>
<td>Fb2</td>
<td></td>
</tr>
<tr>
<td>Rc2</td>
<td>Ac2</td>
<td>Bc2</td>
<td>Cc2</td>
<td>Dc2</td>
<td>Ec2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Original Combinations

1. P1+P2
2. P1+Ra2
3. P1+Rb2
4. P2+Ra2
5. P2+Rb2

Derivative Combinations

1. R1+Ra2, i.e., A1+Aa2
2. R1+Rb2, i.e., B1+Ab2
3. R1+Rc2, i.e., C1+Ac2
4a. Ra2+Rb2, i.e., Ba2+Ab2  4b. Rb2+Rc2, i.e., Bb2+Ac2
5. Ra2+Rc2, i.e., Ca2+Ac2
JJv are decreased and simplified by means of the same arrangements that were applied in the other subdivision of the six-voice canon: 1) Jv=0 in the four-voice canon, and 2) the interval of entrance in the original combination P+R1 is equal to the interval of entrance of a derivative combination in the four-voice canon. Examples in which the Jv is simplified by the use of Jv=0 in the four-voice canon are present at Example 17.5.

Example 17.5. The entrances of a four-voice and two-voice canon in combination. Jv=0 in the four-voice canon.

<table>
<thead>
<tr>
<th></th>
<th>1)</th>
<th>2)</th>
<th>3)</th>
<th>4)</th>
<th>5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1+P2=</td>
<td>-7</td>
<td>-5</td>
<td>-2</td>
<td>-9</td>
<td>5</td>
</tr>
<tr>
<td>P1+Ra2=</td>
<td>-7</td>
<td>-5</td>
<td>-2</td>
<td>-9</td>
<td>5</td>
</tr>
<tr>
<td>P1+Rb2=</td>
<td>-7</td>
<td>-5</td>
<td>-2</td>
<td>-9</td>
<td>5</td>
</tr>
<tr>
<td>P2+Ra2=</td>
<td>0,0</td>
<td>0,0</td>
<td>0,0</td>
<td>0,0</td>
<td>2,4</td>
</tr>
<tr>
<td>P2+Rb2=</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
In each of these examples, Jv in the four-voice canon is equal to 0, and all of the original combinations of the double canon that are not composed at Jv=0 are subject to the same Jv.

At Example 17.6, Bogatyryov displays the second method of simplification, i.e., the interval of entrance of the two-voice canon is situated in a derivative combination in the four-voice canon.

Example 17.6. The entrances of a four-voice and two-voice canon in combination. The interval of entrance of the two-voice canon is situated in a derivative combination in the four-voice canon.

<table>
<thead>
<tr>
<th></th>
<th>1)</th>
<th>2)</th>
<th>3)</th>
<th>4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P2+Ra2 = 0, -11</td>
<td>-7, 0</td>
<td>-11, 0</td>
<td>-7, 0</td>
<td></td>
</tr>
<tr>
<td>P2+Rb2= -11</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>P1+P2= -11</td>
<td>-7</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>P1+Ra2= -11</td>
<td>0</td>
<td>-11</td>
<td>-7</td>
<td></td>
</tr>
<tr>
<td>P1+Rb2= 0</td>
<td>-7</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
Infinite Double Canon of the First Order

Bogatyryov classifies this type of double canon by the arrangement of the entrances of the Propostas and Rispostas. His results are as follows: The entrances of a canon can occur in a layered arrangement [nasloenie], an intersecting arrangement [perekreshchivanie], or an encircled arrangement [okruzhenie]. Each is displayed below with original combinations and formulas for JJv at Figures 17.1, 17.2, and 17.3.\(^{231}\) The value of m1 pertains to shifts of the Proposta and Risposta in the first canon; the value of m2 pertains to shifts of the Proposta and Risposta in the second canon.

\[
\begin{align*}
I & \quad P1 \quad A1 \\
II & \quad R1 \quad A1 \\
III & \quad P2 \quad A2 \\
IV & \quad R2 \quad A2 \\
\end{align*}
\]

\[
\begin{align*}
I + II & \quad (P1+R1) \quad Jv=-2m1 \\
I + III & \quad (P1+P2) \quad Jv=-m1+m2 \\
I + IV & \quad (P1+R2) \quad Jv=-m1-m2 \\
II + III & \quad (R1+P2) \quad Jv=m1+m2 \\
II + IV & \quad (R1+R2) \quad Jv=m1-m2 \\
III + IV & \quad (P2+R2) \quad Jv=-2m2 \\
\end{align*}
\]

Figure 17.1. The layered arrangement of entrances and formulas for JJv.

\(^{231}\) The combination R1+R2 is given in the list of original combinations in order to provide the second Jv for the original combination P1+P2.
I + II (P1+R2) Jv = -m1+m2
I + III (P1+R1) Jv = -2m1
I + IV (P1+P2) Jv = -m1-m2
II + III (R2+R1) Jv = -m2-m1
II + IV (R2+P2) Jv = -2m2
III + IV (R1+P2) Jv = m1-m2

Figure 17.2. The intersecting arrangement of entrances and formulas for \( Jv \).

I + II (P1+R2) Jv = -m1+m2
I + III (P1+R1) Jv = -m1-m2
I + IV (P1+P2) Jv = -2m1
II + III (R2+P2) Jv = -2m2
II + IV (R2+R1) Jv = -m2-m1
III + IV (P2+R1) Jv = m2-m1

Figure 17.3. The encircled arrangement of entrances and formulas for \( Jv \).

If duplicate combinations are excluded, there are six possible \( Jv \) for the layered arrangement, five for the
intersecting, and four for the encircled. Bogatyryov indicates that the layered and intersecting arrangements are simplified when \( m_1 = m_2 \). This produces the following results when applied to the arrangements in the previous examples:

Layered:

\[
\begin{align*}
I + II \ (P_1 + R_1) & \ J_v = -2m \\
I + III \ (P_1 + P_2) & \ J_v = 0 \\
I + IV \ (P_1 + R_2) & \ J_v = -2m \\
II + III \ (R_1 + P_2) & \ J_v = 2m \\
II + IV \ (R_1 + R_2) & \ J_v = 0 \\
III + IV \ (P_2 + R_2) & \ J_v = -2m
\end{align*}
\]

Intersecting:

\[
\begin{align*}
I + II \ (P_1 + R_1) & \ J_v = 0 \\
I + III \ (P_1 + P_2) & \ J_v = -2m \\
I + IV \ (P_1 + R_2) & \ J_v = -2m \\
II + III \ (R_1 + P_2) & \ J_v = -2m \\
II + IV \ (R_1 + R_2) & \ J_v = -2m \\
III + IV \ (P_2 + R_2) & \ J_v = 0
\end{align*}
\]

In the layered arrangement, two \( J_Jv \) are present. They are equal in value; however, they have contrasting signs. All \( J_Jv \) are the same in the intersecting arrangement.

As for the encircled arrangement, Bogatyryov recommends that the interval separating \( m_1 \) from \( m_2 \) be one or more
octaves. This is apparent in Example 17.4 where m2=14 and m1=7.\textsuperscript{232}

<table>
<thead>
<tr>
<th>I</th>
<th>R2</th>
<th>1+II</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>P1</td>
<td>2</td>
</tr>
<tr>
<td>III</td>
<td>R1</td>
<td>3</td>
</tr>
<tr>
<td>IV</td>
<td>P2</td>
<td>4</td>
</tr>
</tbody>
</table>

\[ A2 \rightarrow B2 \]
\[ A1 \rightarrow B1 \]
\[ A1 \rightarrow B1 \]
\[ B2 \rightarrow A2 \]

I+II \ (R2+P1) \ Jv=-m2+m1=-7
I+III \ (R2+R1) \ " \ "
IV+I \ (P2+R2) \ Jv=-2m2=-28
II+III \ (P1+R1) \ Jv=-2m1=-14
IV+II \ (P2+P1) \ Jv=-m2-m1=-21
IV+III \ (P2+R1) \ " \ "

Figure 17.4. An encircled arrangement where m1=7 and m2=14.

In this instance there is one Jv: Jv=-7.

As for the infinite double canon of the first order where the Propostas repeat on different degrees, i.e., a double canonic sequence, Bogatyryov accounts for the shifts of entrance by substituting a derivative value, n, for the intervals between the entrances of the Rispostas and the

---

\textsuperscript{232} The formulas for JJv are altered slightly from those at Figure 17.3 because the entrances are arranged in a different order. However, both examples contain an encircled arrangement because a Proposta and Risposta of one canon encircles the Proposta and Risposta of the other. At Figure 17.3, the encircling canon is the first canon. At Figure 17.4, the encircling canon is the second canon.
repetitions of the Propostas. This is demonstrated at Figures 17.5, 17.6, and 17.7.

\[
\begin{align*}
\text{I} + \text{II} (P1+R1) & \quad Jv=-m_1-n_1 \\ 
\text{I} + \text{III} (P1+P2) & \quad Jv=-m_1+m_2 \\ 
\text{I} + \text{IV} (P1+R2) & \quad Jv=-m_1-n_2 \\ 
\text{II} + \text{III} (R1+P2) & \quad Jv=n_1+m_2 \\ 
\text{II} + \text{IV} (R1+R2) & \quad Jv=n_1-n_2 \\ 
\text{III} + \text{IV} (P2+R2) & \quad Jv=-m_2-n_2
\end{align*}
\]

Figure 17.5. Arrangement and formulas for the layered canonic sequence.

\[
\begin{align*}
\text{I} + \text{II} (P1+R2) & \quad Jv=-m_1-n_2 \\ 
\text{I} + \text{III} (P1+R1) & \quad Jv=-m_1-n_1 \\ 
\text{I} + \text{IV} (P1+P2) & \quad Jv=-m_1+m_2 \\ 
\text{II} + \text{III} (R2+R1) & \quad Jv=n_2-n_1 \\ 
\text{II} + \text{IV} (R2+P2) & \quad Jv=-n_2-m_1 \\ 
\text{III} + \text{IV} (R1+P2) & \quad Jv=-n_1-m_2
\end{align*}
\]

Figure 17.6. Arrangement and formulas for the intersecting canonic sequence.
In order to simplify and have all $J_{Jv}=0$, 7, or -7 for layered and intersecting canonic sequences, Bogatyryov recommends that the sum of the original and derivative intervals in one of the two-voice sequences be equal to the sum of the original and derivative intervals of the other two-voice sequence. Furthermore, he recommends that a combination of all intervals in the canon be equal to a dividend that is divisible by 7. A result of this is given at Example 17.7.
Example 17.7. A layered canonic sequence with less complex $J_{JV}$.

For the encircled arrangement, there are again only four originals that will produce $J_{JV}$ because $-m_1+m_2=n_2-n_1$ and $-m_1-n_2=-m_2-n_1$ (see Figure 17.7). According to Bogatyryov, the easiest arrangement of entrances occurs when the sums of the original and derivative intervals of each canon ($m_1+n_1$ and $m_2+n_2$) differ from one another by 14 and are equal to a number that is divisible by 7. For example, for the entrances given at Example 17.8, when $m_1=11$, $n_1=10$, $m_2=4$, $n_2=3$, the following $J_{JV}$ are required:
Example 17.8. An encircled canonic sequence with less complex JJv.

For a double canon in which the direction of the interval of entrance of P1+R1 is different than the direction of the interval of entrance of P2+R2, Bogatyryov offers the following simplifications. For the encircled and intersecting canonic sequences, the canons are simplified when \((m_1+n_1)-(m_2+n_2)=14\) or \(-14\). This is apparent at Figure 17.8 where a schema of entrances and formulas for JJv are presented for an encircled canon.
Figure 17.8. Arrangement and formulas for the encircled canonic sequence where the direction of Pl+Rl is not equal to the direction of P2+R2.

This arrangement produces a simple Jv when ml=11, n1=10, m2=3, n2=4; ml=2, n1=5, m2=12, n2=9; etc.

The formula (ml+n1)-(m2+n2)=14 or -14 also is useful for simplifying the intersecting double canon where the direction of Pl+Rl is different than the direction of P2+R2. In addition to this formula, the composition of this canon is simplified when ml+n1=m2+n2. The simplest case of this occurs when ml=n2 and m2=n1. This is apparent at Figure 17.9, when ml=13, n1=12, m2=12, n2=13.
Figure 17.9. Arrangement and formulas for the intersecting canonic sequence where the direction of $P_1+R_1$ is not equal to the direction of $P_2+R_2$.

In this instance, $J\nu=0$ and $-25$.

For the layered double canon where the direction of $P_1+R_1$ does not equal the direction of $P_2+R_2$, the simplest arrangements are those where $m_1+n_1=7$ and $m_2+n_2=21$, or where $m_1+n_1=14$ and $m_2+n_2=28$. This can be seen at Figure 17.10 when $m_1=4$, $m_2=10$, $n_1=3$, $n_2=11$. 
Figure 17.10. Arrangement and formulas for the layered canonic sequence where the direction of Pl+Rl is not equal to the direction of P2+R2.

I + II (P1+R1) Jv=-ml-n1
I + III (P1+R2) Jv=-ml+n2
I + IV (P1+P2) Jv=-ml-m2
II + III (R1+R2) Jv=n1+n2
II + IV (R1+P2) Jv=n1-m2
III + IV (R2+P2) Jv=-n2-m2
Conclusion

Of all the Russian theorists who developed Taneev’s theories of imitative counterpoint, Bogatyryov, using Taneev’s classification system throughout his theory, adheres the closest to the model of Doctrine of the Canon. The primary contribution of his theory is that it shows relationships of intervals of entrance in double canon of the first order that provide fewer and less complex JJv. His theory of double canon of the second order has been omitted here because it applies the same procedure used by Taneev in Doctrine of the Canon, i.e., the employment of imaginary combinations for the composition of real ones.
Chapter 18
Kopytman: Multi-Voice Canon (canon and sequence)

Introduction

Mark Kopytman (b. 1929) is a Ukrainian-born composer and music theorist. He received a medical degree in 1952 and a music degree from the Lwow Academy of Music in 1955. He continued his studies at the Moscow Conservatory where he received a doctorate degree in 1958. In 1972 he emigrated to Israel.

Kopytman's theory of multi-voice canon is found in his dissertation "Mnogogolosnyi kanon (kanon i sekventsiia)" [Multi-voice canon (canon and sequence)] and in a published article with the same title. The theories contained in these texts expand Taneev's theories of imitative counterpoint by employing an innovative procedure for composing and analyzing canons and canonic sequences with four or more voices.

Kopytman's procedure, which he named "metodika tsepochki" [method of the chain], examines the interaction of groups of three consecutive entrances in a multi-voice canon.

---

Except for the first Jv of a canon (i.e., the Jv of the original combination P+R1 with regard to its first derivative, R1+R2), which is found in the usual way, and which Kopytman refers to as a "real" Jv, the JJv of the other groups of three consecutive entrances are considered "corrective" JJv. By means of these corrective JJv, Kopytman shows how it is possible to reveal all JJv for a multi-voice finite or infinite canon of the first order. During analysis, Kopytman's procedure simplifies the task of locating JJv for a multi-voice canon by using consecutive entrances, which permits the analyst to avoid having to skip over entrances in order to discover JJv for original combinations such as P+R2, P+R3, etc. During composition, as we will see, this approach allows the composer to limit the number and complexity of JJv in a multi-voice canon.

The Method of the Chain Employed in Analysis

Kopytman demonstrates his procedure by using the following excerpt from the "Kyrie" from Mozart's Requiem.\(^\text{234}\)

This excerpt is provided at Example 18.1.

Example 18.1. Excerpt from the "Kyrie" of Mozart's Requiem.

Example 18.2. The intervals of entrance in Mozart's "Kyrie."

In Example 18.2, the letter M labels the intervals of entrance. The positive or negative value of each M is determined by its direction in relation to M1, which is always considered positive. Based upon this, the entrances in Example 18.2 are as follows: M1=+4, M2=+4, M3=+4, and M4=-10.

Initially, Kopytman discovers the "real" Jv for the first three voices by using Taneev's formula Jv=-m+(+/-n). Applying Kopytman's modifications, which involve substituting M1 for m and M2 for n, the formula appears as follows: Jv1=-(M1)+(+/-M2). Based upon this, Jv1=-(4)+(4)=0. As is apparent in Figure 18.1, Kopytman places this value to the left and right of a vertical line. Henceforth, all values placed to the left are "corrective" JJv (indicated KJJv), and all values placed to the right are "real" JJv.

0 0

Figure 18.1. Jv for the first three consecutive entrances in Mozart's "Kyrie."
The formula is applied to the next three consecutive voices. However, the obtained value, $KJv2=-(M2)+(-/+/M3)=-(4)+(4)=0$, is not a real $Jv$; rather, it is a corrective $Jv$. Due to this, it is placed to the left of the vertical line, as is apparent at Figure 18.2.

![Figure 18.2. Jv and KJv2 for Mozart's "Kyrie."](image)

In order to obtain the final $KJv$ of this canon, the formula is applied to the last three entrances:

$KJv3=-(M3)+(-/+/M4)=-(4)+(-10)=-14$. Similar to the other $KJJv$, this value is positioned to the left of the vertical line, as is apparent at Figure 18.3.

![Figure 18.3. Jv, KJv2, and KJv3 for Mozart's "Kyrie."](image)
Using these values, Kopytman determines all real \(J_{Jv}\) for this five-voice canon. The \(J_{Jv}\) of the first original combination, \(P+R_1\), whose derivatives are \(R_1+R_2\), \(R_2+R_3\), and \(R_3+R_4\), are placed on the first horizontal row to the right of the vertical line. They are discovered as follows: To the initial \(Jv\), which is positioned in the first place to the right of the vertical line, is added the value of \(K_{Jv_2}\): \(K_{Jv_2}(0)+Jv(0)=0\). This \(Jv\) is caused by the derivative \(R_2+R_3\) and is positioned to the right of the first \(Jv\). The \(Jv\) caused by the derivative \(R_3+R_4\) is discovered by adding the value of \(K_{Jv_3}\) to the second \(Jv\) of this horizontal row: \(K_{Jv_3}(-14)+Jv_2(0)=-14\). These \(J_{Jv}\) are displayed at Figure 18.4.

\[
\begin{array}{cccc}
0 & 0 & 0 & -14 \\
K_{Jv_2} & 0 & & \\
K_{Jv_3} & -14 & & \\
\end{array}
\]

Figure 18.4. \(J_{Jv}\) for the original combination \(P+R_1\) positioned on the top horizontal row.

In order to discover the remaining \(J_{Jv}\), Kopytman relies on the same observation that Taneev made in *Doctrine of the Canon*: \(J_{Jv}\) for the first original combination (\(P+R_1\)) are equal to \(J_{Jv}\) of other original combinations, i.e., \(P+R_2\), \(P+R_3\), etc.\(^{235}\) Therefore, Kopytman transfers \(J_{Jv}\) for the

\(^{235}\) Sergei Ivanovich Taneev, *Uchenie o kanone* [Doctrine of the canon] (Moscow, 1929), §§ 107-8, pp. 114-16.
original combination P+R1 to corresponding positions where they apply to other original combinations. These transferences are shown at Figure 18.5.

<table>
<thead>
<tr>
<th>Original</th>
</tr>
</thead>
<tbody>
<tr>
<td>P+R1</td>
</tr>
<tr>
<td>P+R2</td>
</tr>
<tr>
<td>P+R3</td>
</tr>
</tbody>
</table>

Figure 18.5. The transference of JJv of the original combination P+R1 to other originals.

The Jv for the original combination P+R2 with regard to its second derivative is discovered by adding KJv2 and KJv3 to the first Jv of the second horizontal row:
KJv3(-14) + KJv2(0) + the first Jv of the second horizontal row (0) = the second Jv for the original combination P+R2: (-14). Figure 18.6 displays all JJv and KJJv for Mozart's "Kyrie."

<table>
<thead>
<tr>
<th>Original</th>
</tr>
</thead>
<tbody>
<tr>
<td>P+R1</td>
</tr>
<tr>
<td>P+R2</td>
</tr>
<tr>
<td>P+R3</td>
</tr>
</tbody>
</table>

Figure 18.6. A complete list of all JJv and KJJv for the entrances of Mozart's "Kyrie."
In this instance, these transferences of JJv were successful only because all Rispostas were positioned above the entrance of the Proposta. In general, if the entrances of the Rispostas are positioned around the entrance of the Proposta, the values of the JJv for original combinations P+R2, P+R3, etc., will correspond to the values of the JJv of the original combination P+R1 by number; however, they will not correspond according to sign.\textsuperscript{236} This has significant ramifications when Jv is equal to a value other than the octave.

The method of the chain provides an expedient alternative to the methods of analysis discussed in \textit{Doctrine of the Canon} because it allows the analyst to consider only neighboring entrances in a canon of the first order.

The Composition of Multi-Voice Finite Canons

Kopytman simplifies the process of composing multi-voice finite canons by establishing relationships between real and corrective JJv that will produce a single Jv for the entire canon. These relationships are given in the chart at Figure 18.7.

\textsuperscript{236} Ibid., § 108, p. 116.
Figure 18.7. Relationships of JJv and KJJv that will produce a single Jv in a multi-voice canon.

When this chart is used in conjunction with the formula (+/-n)=Jv+m, i.e., Taneev's formula Jv=-m+(+/−n) positioned in order to obtain the value of a derivative interval of entrance, the points of entrance in a multi-voice canon at a single Jv can be discovered. Kopytman demonstrates this in a four-voice canon by selecting Jv1=0, KJv2=Jv from the chart (the second horizontal line) to which he applies Jv=-14. Having arbitrarily selected an interval of entrance for the original combination P+R1, he discovers the point of entrance of R2 by using the formula M2=Jv+(M1), i.e., (+/-n)=Jv+m.

When the first entrance M1=6 and Jv1=0 are introduced to this formula, M2=0+(+6)=+6 is the result. This is the
interval of the derivative combination $R_1+R_2$ and is displayed at Example 18.3.

Example 18.3. A partial display of the entrances of a four-voice canon that has a single $Jv$.

M3 is discovered by using the same formula; however, in order to study the following three consecutive entrances the value of $KJv2$ is substituted for $Jv$ and $M2$ is substituted for $M1$: $M3=KJv2+(+/-M2)$, i.e., $M3=(-14)+(+6)=-8$. When this entrance is added to the previous two, an aggregate results of all entrances in this canon. This is presented at Example 18.4.

Example 18.4. A complete display of the entrances of a four-voice canon that has a single $Jv$. 
In the same manner, the chart at Figure 18.7 can be used to locate the entrances of five-voice and six-voice canons that have a single Jv.

In order to avoid having a Proposta surrounded by Rispostas, thereby causing JJv with contrasting signs, Kopytman introduces the concept of a "minimal'nyi interval imitatsii" [minimal interval of imitation]. This interval indicates the interval of entrance between the Proposta and the first Risposta that causes all Rispostas, with the possible exception of the last, which is composed in simple counterpoint, to be positioned either above or below the Proposta.

The minimal interval of entrance can be discovered using Kopytman's charts as follows: A row number that is indicated in the far right vertical column at Figure 18.7 is located on the top horizontal line of the chart at Figure 18.8. The vertical columns of Figure 18.8 provide minimal intervals of entrance for various JJv. For example, in order to compose a five-voice canon with Jv1=Jv, KJv2=0, and KJv3=-Jv where Jv= -11 (i.e., Jv, 0, -Jv, the ninth horizontal line in Figure 18.7), the minimal interval of entrance will be 8 because the ninth horizontal line of Figure 18.7 refers the composer to the third vertical column of Figure 18.8 where for Jv=-11 the minimal interval of entrance is listed as 8. 8 indicates that the interval of entrance between the Proposta and R1
must be at least a ninth in order to avoid JJv with contrasting signs when composing this canon.

<table>
<thead>
<tr>
<th>Row</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jv</td>
<td>-7</td>
<td>-9</td>
<td>-11</td>
<td>-14</td>
<td>-16</td>
<td>-18</td>
<td>-21</td>
<td></td>
</tr>
<tr>
<td>M&gt;</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Jv</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Figure 18.8. Kopytman's chart for the determination of the minimal interval of entrance in a multi-voice canon.

Kopytman, like Taneev, also addresses the issue of analyzing and composing canonic sequences of the first order. In analysis, Kopytman indicates that the method of the chain can be applied to the primary entrances of multi-voice canonic sequences in the same manner as it is applied to multi-voice finite canons. Kopytman locates JJv that involve secondary entrances by using JJv correlations and values for KJJv.

For example, the entrances at Example 18.5 provide JJv for a five-voice finite canon where the secondary entrance //P
is considered the fifth voice. JJv and KJJv for this canon are displayed at Figure 18.9. When it enters, //R1 is a secondary entrance and is a member of derivative combinations in the canonic sequence.

Example. 18.5. Entrances of a five-voice finite canon.

Original

\[
\begin{align*}
P+R1 &\quad -3 & -3 & 0 & -24 \\
P+R2 \ (KJv2=+3) &\quad 0 & -21 \\
P+R3 \ (KJv3=-24) &\quad -24 
\end{align*}
\]

Figure 18.9. JJv and KJJv for the entrances given at Example 18.5.

All of these values can be determined by using the same methods that were employed to determine JJv for Mozart's "Kyrie." JJv that pertain to secondary Rispostas are located as follows: KJv3 is placed in the last position of the third
horizontal row. To this is added KJv2 which provides the value of the second Jv of the third horizontal row. The remaining Jv, which pertains to the third derivative of P+R2, is discovered by extending the correlations of Jv that were established above for the five-voice finite canon. At Figure 18.10, the circles attached by lines indicate corresponding Jv. The complete list of Jv and KJv for this canonic sequence is given at Figure 18.11.

![Diagram](image)

Figure 18.10. Corresponding Jv of a four-voice canonic sequence.

<table>
<thead>
<tr>
<th>Original</th>
<th>P+R1</th>
<th>-3</th>
<th>-3</th>
<th>0</th>
<th>-24</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P+R2 (KJv2=+3)</td>
<td>0</td>
<td>-21</td>
<td>-21</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P+R3 (KJv3=-24)</td>
<td>-24</td>
<td>-21</td>
<td>-24</td>
<td></td>
</tr>
</tbody>
</table>

Figure 18.11. A complete list of Jv and KJv for the four-voice canonic sequence that employs the entrances given at Example 18.5.
A similar procedure yields JJv and KJJv for a five-voice canonic sequence. The entrances of such a canon are given as a six-voice finite canon in which the secondary entrance \( P \) serves as the sixth voice. These entrances are displayed at Example 18.6.

![Example 18.6. Entrances of a six-voice finite canon.](image)

Several of the JJv for these entrances are discovered using the method of the chain and the correlations of JJv given at Figure 18.12. The Jv for original combination P+R2 with regard to its third derivative--R3+/\( P \)--is located as follows: the sum of KJv3+KJv4 is added to the second Jv of the second horizontal row. Figure 18.12 is used in order to discover the second derivative of the original combination P+R3.
Figure 18.12. Corresponding JJ\nu of a five-voice canonic sequence.

JJ\nu and KJJ\nu are provided at Figure 18.13 for the first six entrances of this canonic sequence.

Original

\[
\begin{align*}
P+R1 & \quad -14 & & -14 & & 0 & & -14 & & -21 \\
P+R2 (KJ\nu2=+14) & \quad 0 & & 0 & & -21 \\
P+R3 (KJ\nu3=-14) & \quad -14 & & -21 \\
P+R4 (KJ\nu4= -7) & \quad -21
\end{align*}
\]

Figure 18.13. JJ\nu and KJJ\nu for the entrances given at Example 18.6.

The additional JJ\nu are discovered as follows (for clarity, see Figures 18.12, 18.13, 18.14): KJ\nu4 is placed in the last position of the fourth horizontal row. To this is added KJ\nu3 in order to find the third Jv of this row. To the third Jv is added KJ\nu2 in order to find the second Jv.
Except for the third Jv of the third horizontal row, which is discovered by adding KJv3 and KJv2 to the final Jv of that row, the other JJv are discovered by using the correlations at Figure 18.12. A complete list of all JJv and KJJv for this canon is displayed at Figure 18.14.

<table>
<thead>
<tr>
<th>Original</th>
</tr>
</thead>
<tbody>
<tr>
<td>P+R1</td>
</tr>
<tr>
<td>P+R2 (KJv2=+14)</td>
</tr>
<tr>
<td>P+R3 (KJv3=-14)</td>
</tr>
<tr>
<td>P+R4 (KJv4=-7)</td>
</tr>
</tbody>
</table>

Figure 18.14. A complete list of JJv and KJJv for the five-voice canonic sequence that employs the entrances given at Example 18.6.

For the entrances of the six-voice canonic sequence presented at Example 18.7, Kopytman proceeds similarly, i.e., he discovers JJv and KJJv for a seven-voice finite canon where /P is considered the seventh voice.

Example 18.7. Entrances of a seven-voice finite canon.
JJv of the second horizontal row are discovered by adding the sum of KJv2+KJv3 to the first Jv of this row in order to find the second Jv, the third Jv is discovered by adding the sum of KJv3+KJv4 to the second Jv, and the fourth Jv is discovered by adding the sum of KJv4+KJv5 to the third Jv (for clarity, see Figure 18.16). Based upon an extension of the correlations of JJv at Figure 18.12, which is presented as Figure 18.15, these values are transferred to the third and fourth horizontal rows.

Figure 18.15. Corresponding JJv of a six-voice canonic sequence.

The third Jv of the third horizontal row is discovered by adding the sum of KJv3+KJv4+KJv5 to the second Jv of this row. This completes the list of JJv for this seven-voice finite canon. These are displayed at Figure 18.16.
Figure 18.16. JJv and KJJv for the entrances given at Example 18.7.

The additional JJv are discovered in a similar manner to that which was used for the four-voice and five-voice canonic sequences presented above (for clarity, see Figure 18.17): KJv5 is placed in the last position of the fifth horizontal row. To this is added KJv4 in order to discover the fourth Jv of this row, to which is added KJv3 in order to discover the third Jv, to which is added KJv2 in order to discover the second Jv. The other JJv at Example 18.17 are discovered by using Figure 18.15.

<table>
<thead>
<tr>
<th>Original</th>
</tr>
</thead>
<tbody>
<tr>
<td>P+R1</td>
</tr>
<tr>
<td>P+R2 (KJv2= 0)</td>
</tr>
<tr>
<td>P+R3 (KJv3=+14)</td>
</tr>
<tr>
<td>P+R4 (KJv4=-14)</td>
</tr>
<tr>
<td>P+R5 (KJv5=-14)</td>
</tr>
</tbody>
</table>
The remaining JJv are discovered as follows (for clarity, see Figure 18.18): To the final Jv of the fourth horizontal row is added the sum of KJv4+KJv3, this provides the fourth Jv of this row, to which is added the sum of KJv3+KJv2 in order to discover the third Jv. Based on the information provided at Figure 18.15, the third Jv of this row can be transferred to the fourth position of the third horizontal row. This provides a complete list of JJv and KJJv for this six-voice canonic sequence. The results are displayed at Figure 18.18.
Figure 18.18. A complete list of JJv and KJJv for the six-voice canonic sequence that employs the entrances given at Example 18.7.

As we have seen, multi-voice finite canons are components of canonic sequences. Due to this, Kopytman culls the following arrangements from Figure 18.7 that provide an opportunity to compose canonic sequences at a single Jv.

<table>
<thead>
<tr>
<th>Correlation of JJv of component canons</th>
<th>P counterpoints to R, R, R, R, R</th>
<th>Row M&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jv, KJv, KJv, KJv, KJv,</td>
<td>Jv 0</td>
<td></td>
</tr>
<tr>
<td>Three</td>
<td>Jv 0</td>
<td>1</td>
</tr>
<tr>
<td>Four</td>
<td>Jv 0, Jv 0, Jv</td>
<td>3</td>
</tr>
<tr>
<td>Five</td>
<td>Jv 0, Jv, Jv</td>
<td>2</td>
</tr>
<tr>
<td>Five</td>
<td>Jv 0, Jv, Jv</td>
<td>1</td>
</tr>
<tr>
<td>Six-voice canon</td>
<td>Jv 0, Jv, Jv</td>
<td>5</td>
</tr>
<tr>
<td>Jv 0, Jv, Jv 0, Jv 0</td>
<td>Jv 0, Jv, Jv 0, Jv 0</td>
<td>6</td>
</tr>
<tr>
<td>Jv 0, Jv 0, Jv, Jv 0, Jv 0</td>
<td>Jv 0, Jv 0, Jv 0, Jv 0</td>
<td>3</td>
</tr>
<tr>
<td>Jv 0, Jv 0, Jv, Jv 0, Jv 0</td>
<td>Jv 0, Jv 0, Jv 0, Jv 0</td>
<td>1</td>
</tr>
<tr>
<td>Jv 0, Jv 0, Jv, Jv, Jv 0, Jv 0</td>
<td>Jv 0, Jv 0, Jv 0, Jv 0</td>
<td>7</td>
</tr>
</tbody>
</table>

Figure 18.19. Relationships of JJv and KJJv that will produce a single Jv for a multi-voice canonic sequence.
Similar to multi-voice finite canons, this chart is used with
the formula \((+/n)=Jv+m\) for composition.

Conclusion

Using his method of the chain, Kopytman extends Taneev's
*Doctrine of the Canon* by exploring the analysis and
composition of multi-voice canons and multi-voice canonic
sequences that contain more than four voices. In contrast to
Taneev's solution for the composition of canons at a single
Jv, which consists of the entrances of all voices except for
one at the same interval and in the same direction (see §
131, *Doctrine of the Canon*), Kopytman provides an exhaustive
list of relationships between JJv and KJv that will provide
canons and canonic sequences with diverse entrances at a
single Jv.
Chapter 19

Korchinsky: The Fundamentals of a Melodic Theory of Canon

Introduction

The theories of Evgeny Nikolaevich Korchinsky (1904-65) are considered by contemporary Russian theorists as a "genuine achievement of Soviet musical-theoretical thought." His theories have been applied in practical music pedagogy, and they were influential to the eminent Soviet music theorist Sergey Sergeevich Skrebkov (1905-67).

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238 Teodor Müller, a former professor of music theory at the Moscow Conservatory, discusses Korchinsky's theories in his counterpoint textbook Polifonia [Polyphony] (Moscow, 1989), pp. 104-6. In the textbook Prakticheskie osnovy stretno-imitatsionnoi polifonii [Practical fundamentals of stretto-imitative polyphony] (Moscow, 1986), pp. 106-18, Professor Aleksandr Rovenko of the Odessa Conservatory combines the theories of Korchinsky and Skrebkov in order to investigate both strict and free style counterpoint. An original aspect of Rovenko's text is that Korchinsky's theories are applied to complete motivic figures that contain diverse rhythmic characteristics.
Korchinsky's theory of imitative counterpoint is described primarily in his dissertation "K voprosu o teorii kanonicheskoi imitatsii (Osnovy melodicheskoi teorii kanona)" [To the question of the theory of canonic imitation (Fundamentals of a melodic theory of canon)] and in several contemporary Russian theorists.\(^{239}\)

An important stimulus for the completion [of this work] was also the acquaintance of the author with the theoretical work of the Soviet musicologist E. N. Korchinsky; to the question of the theory of canonic imitation (1960). This insightful work establishes for the first time, and precisely formulates, an objective dependence between the melody of a two-voice canon and the harmonic verticalities that result.

articles that evolved from this work. Despite the fact that the bulk of his publications occurred at the end of his lifetime and posthumously, his main thesis, which deals with the generative properties of the linear intervals of a Proposta, evolved over the course of thirty years. In addition to his work as a music theorist, he taught at music schools in Leningrad and Tomsk and at the Ural Conservatory. He was a teacher of the composer Edison Vasilevich Denisov (b. 1929).

Korchinsky's theories develop two issues that remained unexamined by Taneev in Doctrine of the Canon: the composition of "exceptional" forms of canon, such as canon in inversion, augmentation, and diminution, and the composition of a canon based on a cantus firmus, i.e., a canon in which the Proposta is not composed in sections. As Korchinsky indicates, the omission of a theory for the former in

Doctrine of the Canon does not mean that Taneev did not study or teach exceptional forms of counterpoint. Evidence to the contrary is found in Taneev's course outline and in the notes of his students.241

As for canon based on a complete cantus firmus, the methodology followed by Taneev in Doctrine of the Canon—that is, one which relies on a sectional construction of the Proposta—is not adequate. Taneev's approach is as follows:

The method of writing two-voice finite canons consists of the following. Having written a section of the Proposta up to the point of entrance of the Risposta, and having transferred this section to the latter, the Proposta continues in counterpoint to the Risposta and this counterpoint will again be shifted to the Risposta, continuing in this way until the concluding cadence.242

A sectional approach to the composition of canon was not unique to Taneev; rather, it was a standard pedagogical practice.243 Korchinsky faulted the proponents of this practice—namely, music theorists such as Marx, Cherubini, Bussler, and Bellermann—because "they did not attempt to

241 In his course outline for his counterpoint classes at the Moscow Conservatory in 1890, Taneev indicates that in the first year of contrapuntal studies he will teach imitation in augmentation, diminution, and invertible counterpoint. This citation and a list of other musical courses taught by Taneev at the Moscow Conservatory between 1880 and 1902 is cited in Liudmilla Korabel'nikov, Fedor Arzamanov, eds., S. I. Taneev. Iz nauchno-pedagogicheskogo naslediia [S. I. Taneev. From the scientific-pedagogical legacy] (Moscow, 1967), pp. 49-54. In this same work, musical examples from these classes and commentary are given by Taneev's student Sergei Vasilevich Evseev (1894-1956) (see pp. 121-49).
242 Sergei Ivanovich Taneev, Uchenie o kanone [Doctrine of the Canon] (Moscow, 1929), § 15, pp. 18-19.
243 A salient example of another theorist who may have provided a model of this approach for Taneev is Adolph Bernhard Marx, Die Lehre von der musikalischen Komposition (Leipzig, 1842), 2:423-426.
link the study of canon with the study of melody."  

Korchinsky illustrates how far removed this sectional, vertically-oriented approach is from actual compositional practice in the conclusion of his dissertation where he explores the compositional methodology of the Soviet polyphonist Anatoly Nikolaevich Aleksandrov (1888-1982), Taneev's former student.  

When asked by Korchinsky about his method for composing imitation, Aleksandrov replied that he relied on an understanding of the melody in order to judge whether or not it is fit for imitation. This view is contrary to the sectional approach generally applied in textbooks and helped to instigate Korchinsky's search for a theory that more precisely mirrored the creative thinking of composers.

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244 Korchinsky, "K voprosu o teorii kanonicheskoi imitatsii," p. 3.  
245 Aleksandrov was a professor of composition at the Moscow Conservatory. His biography is V. Kokushkin, A. Aleksandrov (Moscow, 1987).  
246 Korchinsky, "K voprosu o teorii kanonicheskoi imitatsii," p. 91. As Carl Phillip Emanuel Bach attests, Johann Sebastian Bach also relied on the nature of the melody in order to judge its imitative potential:

> When he listened to a rich and many-voiced fugue, he could soon say, after the first entries of the subjects, what contrapuntal devices it would be possible to apply, and which of them the composer by rights ought to apply, and on such occasions, when I was standing next to him, and he had voiced his surmises to me, he would joyfully nudge me when his expectations were fulfilled.

An additional source of inspiration for Korchinsky's melodic approach to imitation was his teacher Khristofor Stepanovich Kushnarev (1890-1960), with whom he studied at the Leningrad Conservatory. Kushnarev was generally critical of Western theorists of polyphony and Johann Fux (1660-1741) in particular because he believed their theories were abstract and did not take into account "the expressive qualities of the polyphonic methods" of the great composers of the Renaissance. Therefore, Kushnarev maintained as his primary pedagogical aim the creation of a new approach that, in the words of his colleague Yury Nikolaevich Tiulin (1893-1978), "relies on expressive musical language and authentic musical rules that in their simplest and most natural aspects can be observed in the works of the old masters of polyphony in the strict style."

Kushnarev believed the origin of this "expressive musical language" exists in the quality of the musical

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247 Kushnarev was appointed to the faculty of the Leningrad Conservatory in 1925 due to the influence of the Soviet musicologist Alexander Ossovsky. His appointment occurred at a time when Soviet musical-theoretical methods were becoming very distinct from those employed in the West. The bulk of Kushnarev's writings on polyphony are found in his archives. Several of these writing are published in Kh. S. Kushnarev, O polifonii [About polyphony], eds. Iu. N. Tiulin and I. Ia. Pustyl'nik (Moscow, 1971).

248 This assessment of Kushnarev's view was made by Yury Tiulin. It is found in the Preface to Kh. S. Kushnarev, O polifonii, p. 6.

subjects, not in vertical combinations that arise as a result of species counterpoint. Additional evidence of Kushnarev's emphasis on the cultivation of melodic skills in the study of counterpoint is summarized in the following quotation by Kushnarev's student Gaiane Moiseevna Chebotarian (b. 1918), which sets forth Kushnarev's criticism of Fux's system:

...the addition of contrapuntal voices in the strict style to a given cantus firmus of the choral type lies at the basis of the method of Fux. When doing this, it is necessary to fit into the unbelievably narrow limits of five "species."

Such a mechanistic method of adding voices impedes the development of melodic thinking of the student. In these tasks, melody is a secondary factor. It is entirely dependent on the cantus firmus, the notes of which hold it secure and do not allow it to develop.\(^\text{250}\)

Kushnarev's views influenced Korchinsky to develop a theory of imitation that emphasizes melody as the determinant of vertical unions that occur when a Proposta and Risposta are united in imitation. Korchinsky is successful in this endeavor to the extent that he mathematically demonstrates a connection between linear intervals of a Proposta and the subsequent verticalities that are formed in a contrapuntal union. His application of this insight to the study of "exceptional" forms of canon is a further development of Taneev's theories.

Korchinsky's Basic Formula

Korchinsky's basic formula is as follows: \( Q = N + M(d) \). Using the linear intervals of a given Proposta, this formula predicts the vertical intervals that will occur when the Proposta is imitated in canon by a Risposta. The value of \( Q \), i.e., the predicted verticality, is equal to the interval between notes of the Proposta (\( M \)), separated by a distance (\( d \)), added to the interval between the initial notes of the Proposta and Risposta (\( N \))—the interval of entrance. The value (\( d \)) is always equal to the rhythmic distance between the entrances of a Proposta and its Risposta. In Korchinsky's basic formula, the value of (\( d \)) remains constant and is used to determine all values of \( M \). In the canon at Example 19.1, \( Q \), \( M \), \( N \), and (\( d \)) are labeled.

Example 19.1. A canon of the "simple type" with indications of \( Q \), \( M \), \( N \), and (\( d \)).

In his dissertation, Korchinsky explains his basic formula by using canons of the "simple type," which he composes for the purpose of demonstration, and which only
contain a single rhythmic value. The positive or negative value of M is ascertained by using the precept established by Taneev in *Convertible Counterpoint in the Strict Style*:

Ascending linear motion is positive and descending motion is negative when the Proposta begins above the Risposta; ascending linear motion is negative and descending motion is positive when the Proposta begins below the Risposta. As for the value of Q (the predicted vertical interval), in a combination where the upper voice remains above the lower voice, the value of Q is positive. In a combination where the lower voice has crossed above the upper voice, then the value of Q is negative.\(^\text{251}\) In contrast to these changing values, the value of N (the interval of entrance) is always positive.

For the Proposta that follows at Example 19.2, the distance of entrance between the Proposta and Risposta will be equal to a whole note; therefore, (d) will also equal a whole note, causing the linear intervals of the Proposta that provide values for M to be separated from each other by a whole note. For this canon the value of N will be equal to 7 and the Risposta will enter below the Proposta. Based upon this information, the following equations at Example 19.2, which employ a constant value (N) and a value determined by intervals gleaned only from the Proposta (M), accurately

predict values for Q that will occur when the Risposta participates.\textsuperscript{252}

\[ N+M(d)=Q: \ 1) \ 7+4=11, \ 2) \ 7+(-2)=5, \ 3) \ 7+(-2)=5, \ 4) \ 7+5=12, \ 5) \ 7+(-2)=5 \]

Example 19.2. The Proposta of Korchinsky's canon of the "simple type" with values for M.

The equations given at Example 19.2 are verified by the canon at Example 19.3.

Example 19.3. Korchinsky's canon with a Proposta and Risposta.

In the discussion of his basic formula, Korchinsky offers an additional development of Taneev's work by

\textsuperscript{252} This canon of the "simple type" is by Korchinsky. Korchinskii, "K voprosu o teorii kanonicheskoi imitatsii," pp. 11-13.
indicating a possibility for using semitones in calculations for Q. He suggests the use of twelve half steps, as opposed to the seven degrees in Taneev's system, in order to more precisely determine the quality of intervals. By applying this numbering system in the formulas for the canon of the simple type at Example 19.3, the following revised results are obtained: 1) 12+7=19 (a perfect fifth), 2) 12+(-4)=8 (a minor sixth), 3) 12+(-3)=9 (a major sixth), 4) 12+8=20 (a minor sixth), 5) 12+(-3)=9 (a major sixth).

**Canons in Inversion, Augmentation, and Diminution**

Korchinsky expands his formula in order to predict verticalities for canons with an inverted Risposta, in augmentation, and in diminution. In the application of his theory to canons with an inverted Risposta, Korchinsky introduces the following formula that, when applied to the Proposta, will predict vertical intervals that will occur during a simultaneous statement of a Proposta and its inverted Risposta: Q=Qo+2J.

In this formula, Qo is equal to the same value that N is equal to in the formula Q=N+M(d)—that is, it is equal to the interval of entrance. J is equal to the interval between the

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initial note of the Proposta and the note whose Q value is in question. Because the Risposta is inverted, this value is multiplied by two. These values are labeled at Example 19.4.

Example 19.4. The beginning of a canon with an inverted Risposta. Q, Qo, and J are labeled.

Korchinsky demonstrates this formula using the following Proposta from Bach's *Well Tempered Clavier.*\(^{254}\) The equations that accompany Example 19.5 accurately predict the intervalic content of the canon when the inverted Risposta is added below at Qo=10. The results of these equations are verified by the union of the Proposta with the Risposta at Example 19.6.

\[
Qo + 2J = Q: \begin{align*}
1) & 10 + 2(0) = 10, \\
2) & 10 + 2(-1) = 8, \\
3) & 10 + 2(0) = 10, \\
4) & 10 + 2(1) = 12, \\
5) & 10 + 2(0) = 10, \\
6) & 10 + 2(3) = 16
\end{align*}
\]

Example 19.5. A Proposta by Johann Sebastian Bach from "Fugue No. VIII in D# Minor," The Well Tempered Clavier II, mm. 43-4.

Example 19.6. A Proposta and Risposta by Johann Sebastian Bach from "Fugue No. VIII in D# Minor," The Well Tempered Clavier II, mm. 43-4.

When this formula is applied to canons where the two voices do not enter simultaneously, it is modified as follows: $Q = N + 2J - M$. The value of $J$ once again indicates the interval between the initial note of the Proposta and the note of the Proposta whose $Q$ value is in question. Since the Proposta is not sounded at the same time as the Risposta, a value for $M$—the melodic interval between the note in question and the note that precedes it at the rhythmic distance of entrance—is added to the equation. These values are labeled at Example 19.7.
Example 19.7. Q, M, N, and (d) labeled in a canon with an inverted Risposta that does not enter simultaneously with the Proposta.

The verticalities predicted at Example 19.8 for a Proposta by Palestrina pertain to a situation where the Proposta is the upper voice, N=4, and the distance for determining M is equal to a whole note. For the sake of clarity, only combinations that occur on strong beats have been considered. The predictions made at Example 19.8 are verified when the Proposta is combined with the Risposta at Example 19.9.\(^{255}\)


Example 19.9. A Proposta and Risposta from Palestrina's "Magnificat," mm. 1-4.\(^{256}\)

In order to study canons with an augmented Risposta, Korchinsky modifies the basic formula by refining his definition of the value of \(d\) as follows: \(Q = N + M (\text{dm} = d + k - 1/k \times d1).\) In this equation, \(k\) is equal to the coefficient of augmentation, i.e., the alteration of the duration of the notes in the augmented Risposta in comparison with the Proposta. \(d\) is equal to the rhythmic distance between the entrances of the Proposta and the Risposta, and

\(^{256}\) Ibid.
(d1) is equal to the rhythmic distance between the note of the Proposta that coincides with the first note of the Risposta and the note of the Proposta whose Q value is in question. The value of (dm) is equal to the rhythmic distance between the two notes of the Proposta that will provide the value of M.

At Example 19.10, Korchinsky demonstrates this formula with a Proposta from Bach's Well Tempered Clavier.\(^{257}\)


In this canon, the Risposta will enter on the third beat of the first measure thereby causing (d) to be equal to four eighth notes. In order to determine the value of Q for the F# on the third beat of the third measure, values for (d1) and (k) must be found. (d1) is equal to sixteen eighth notes, and (k=2) because all rhythmic values of the Proposta are doubled in the Risposta—that is, eighth notes are

augmented to quarters, quarters are augmented to half notes, etc. Based on these values, \((dm)\) is equal to twelve eighth notes: \((dm = 4 + (2-1)/2 \times 16 = 4 + 1/2 \times 16 = 12)\). Because the note A\# is positioned twelve eighth notes to the left of F\#, and the Proposta in this instance is the lower voice, \(M=+2\) and \(Q=7+(+2)=9\). The complete excerpt of this canon is given below at Example 19.11.


For canons with a Risposta in diminution, Korchinsky uses the following formula: \(Q=N+M\) \([dm = d - (k-1) \times d1]\).

Korchinsky demonstrates this formula by using a bass line from the opening measures of "Contrapunctus VII" from Bach's The Art of the Fugue as a Proposta.\(^{258}\) This is given at

\(^{258}\) The Proposta at Example No. 19.12 is the bass line from Johann Sebastian Bach. "Contrapunctus VII," The Art of the Fugue, ed. Helmut Walcha (Frankfurt, 1967), mm. 5-10. The contrapuntal example at Example
Example 19.12. Korchinsky has transposed this melody and will combine it with a Risposta that begins above at N=3. Because the note values of the Risposta will be four times smaller than the note values of the Proposta, i.e., whole notes will become quarter notes, half notes will become eighth notes, etc., the coefficient of diminution (k) is equal to four.

In order to discover Q for the second quarter note of the sixth measure when the Risposta enters on the first beat of the fifth measure, the following values are introduced to the equation: (d) is equal to thirty-two eighth notes, and (dl) is equal to ten eighth notes. Thus,

\[ dm = 32 - (4-1) \times 10 = 32 - (3) \times 10 = 2 \text{ eighth notes} \].

By counting two eighth notes to the left of the second quarter note of the sixth measure, it is apparent that M=0; therefore, the following value of Q applies to this note: Q=N+M=Q=3+0=3. This is verified by the complete excerpt at Example 19.13.


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19.13. which uses this Proposta, is by Korchinsky, "K voprosu o teorii kanonicheskoi imitatsii," p. 89.

Conclusion

Korchinsky developed Taneev's theories of imitative counterpoint by introducing an alternative method to the sectional approach for composing canons that was advocated in pedagogical writings by Taneev and his contemporaries. Korchinsky's method employs a basic mathematical formula in order to determine vertical relationships that will occur in a two-voice canon. His formula is unique because it is based on the melodic course of the Proposta.

Korchinsky's emphasis on the linear, melodic aspects of the Proposta in contrapuntal combinations was inspired by his Soviet musical contemporaries who sought an approach to counterpoint that more closely resembled the authentic
thinking of composers. By placing primary emphasis on the Proposta he diverged from the approach employed by Taneev, which emphasizes the vertical relationship of a Proposta with a Risposta; however, by using mathematics in order to verify his theories he continued in Taneev’s theoretical lineage. Thus, Korchinsky’s work is a synthesis of traditional Russian contrapuntal theory with progressive trends of Soviet music-theory pedagogy.
Chapter 20

Skrebkov: A Theory of Imitative Polyphony

Introduction

Sergey Sergeevich Skrebkov (1905-67) was an eminent and prolific music theorist during the Soviet era whose influence is still present in Russia due to his numerous and significant published works. His writings are influential not only because of their high quality, but also because of the breadth of subjects covered by them, including musical analysis, harmony, acoustics, and polyphony. These texts were inspired by his teaching activities at the Moscow Conservatory which extended from 1932 until 1967.

While teaching at the conservatory, Skrebkov extended Taneev's theories of imitative counterpoint in the manuscript Teoriia imitatsionnoi polifonii [A theory of imitative polyphony], which was published posthumously. Skrebkov began this text prior to the second world war and completed it in 1962. In 1965, several fundamental concepts of this theory were published in Chapter 10 of the third edition of Uchebnik polifonii [Textbook of polyphony]. Even though Teoriia imitatsionnoi polifonii was not published until 1983, the

259 Skrebkov's texts on polyphony are Polifonicheskii analiz [Polyphonic analysis] (Moscow, 1940); Uchebnik polifonii [Textbook of polyphony] (Moscow, 1951); Polifonia i polifonicheskie formy [Polyphony and polyphonic forms] (Moscow, 1962); Teoriia imitatsionnoi polifonii [A theory of imitative counterpoint] (Kiev, 1983).
ideas therein have been influential to Russian theorists since their inception.\footnote{260}

In Skebkov's theory of imitative polyphony, Taneev's influence is evident because, like Taneev, Skrebkov uses mathematical abstractions for the purpose of demonstrating a wide range of compositional possibilities. Skrebkov encourages the acquisition of knowledge of a wide range of compositional recourses in order to "emancipate [the composer's] artistic imagination and to develop creativity."\footnote{261} Skrebkov's work extends the theories in Doctrine of the Canon by providing insight into the composition of canonic sequences with more than four voices, and double and triple canons.

Skrebkov does this by abandoning Taneev's position, stated initially by Heinrich Bellermann in Der Contrapunkt, that "multi-voice counterpoint [should be regarded] as the union of various combinations of two-voice counterpoint."\footnote{262} Skrebkov rejected this view because he believed that the number of two-voice combinations in multi-voice polyphony escalate to a point where they are no longer perceived as

\footnote{260 All information cited here about Teoriia imitatsionnoi polifonii is cited in Aleksandr Rovenko, "Taneevskie printsipy v sovremennoi teorii kontrapunkta i imitatsii" [Taneev's principles in contemporary theory of counterpoint and imitation] (dissertation for the degree of Doctor of Musicology, Odessa State Conservatory, 1988), pp. 91-2.}
\footnote{261 Skrebkov, Teoriia imitatsionnoi polifonii, p. 5.}
\footnote{262 Serge Ivanovitch Taneiev, Convertible Counterpoint in the Strict Style, trans. G. Ackley Brower (Boston: Bruce Humphries, 1962), § 262, p. 169. As the foundation for his view, Taneev cites Heinrich Bellermann, Der Contrapunkt, 4th ed. (Berlin, 1901), p. 200.}
distinct. As Skrebkov indicates, in a four-voice canon there are six two-voice combinations, in a five-voice canon, ten two-voice combinations, etc. Skrebkov maintained:

So great a quantity of individual two-voice complexes can not be grasped simultaneously by a single act of perception. Therefore, the musical ear of the composer, performer, and listener combines the polyphonic voices into harmonies and follows their motion partly as "harmonic positions."  

Skrebkov's perception of polyphony as harmony led to his observation that all canons in the strict style may be reduced to basic triads that can interact in a finite number of ways. In order to study and categorize this interaction, Skrebkov begins with a type of musical reduction that disregards non-harmonic tones and positions the harmonies that result on primary beats of a measure. These harmonies participate in a harmonic framework [karkas] of a given work.  

Skrebkov applies his theory to harmonic frameworks.

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263 Skrebkov, Teoriia imitatsionnoi polifonii, p. 7.
264 Skrebkov, Teoriia imitatsionnoi polifonii, p. 12. Other Russian theorists who have applied Skrebkov's method of reduction to polyphony are Aleksandr Rovenko and Yury Nekliudov. Rovenko uses harmonic frameworks in conjunction with Korchinsky's theories in his textbook Prakticheskie osnovy strettimoimitatsionnoi polifonii [Practical fundamentals of stretto-imitative polyphony] (Moscow, 1986).

In his dissertation, Yury Nekliudov applies this method to melodies in order to study archetypical melodic structures in the music of Johann Sebastian Bach. (Iurii Igorevich Nekliudov, "Interpretatsiia i razvitie teorii kontrapunkta taneevskoi traditsii" [Interpretation and development of the theory of counterpoint in the tradition of Taneev] (dissertation for the degree of Doctor of Musicology, Gnessin Institute, 1990), pp. 150-62.)
Skrebkov's Theory of Imitative Polyphony

Skrebkov demonstrates his theory using canonic sequences rather than finite canons because he believed that "the majority of finite canons can be theoretically presented as altered canonic sequences or altered infinite canon." He maintains that all canonic sequences are one of two types, namely, sequences in which the roots of consecutive triads shift at a constant interval [postoiannyi] and sequences where the roots of consecutive triads shift at variable intervals [peremenny]. Both of these types may be compared by assigning a constant root motion to sequences whose root motion is variable. Using this classification system, Skrebkov proceeds to show how the number of voices in a canonic sequence may be augmented, and he offers new insight for the study and composition of horizontal-shifting counterpoint and canons with more than one Proposta.

In order to accomplish these tasks, he considers each note of a contrapuntal line as being either a root (1), a third (3), or a fifth (5) within a corresponding harmony. When these harmonies are studied in progressions, he states that there are only three ways that the courses of the melodies that form the harmonies can be classified: 1) 5-3, 3-5, 3-1, 1-3—the "tertsovyi" type [the third]; 2) 5-5, 3-3,

\[265\] Skrebkov, Teoriia imitatsionnoi polifonii, p. 84.
1-1--the "oktavnyi" type [the octave]; and 3) 5-1, 1-5--the "kvintovyi" type [the fifth].

In this classification, the Arabic numerals refer to the quality of a note within a triad, i.e., Taneev's numbering system has been abandoned and 1 = the tonic of the triad, 3 = the third of the triad, and 5 = the fifth of the triad. Therefore, 5-3 in the first set of melodic progressions indicates that the fifth of a triad will proceed to the third of the triad that follows.

**Multi-Voice Combinations**

Using musical reduction and his classifications of melodic progressions, Skrebkov maintains that numerical combinations may be combined vertically to the extent that rules of the strict style are not broken. For example, a canonic sequence that uses the "tertsovyi" type of linear motion may consist of the following two arrangements that are displayed at Figure 20.1.

$$1-3-1-3-1-3 \quad 3-5-3-5-3-5$$

$$1-3-1-3-1-3 \quad \text{or} \quad 3-5-3-5-3-5$$

Figure 20.1. Numerical frameworks of a sequence of the "tertsovyi" type.
These arrangements may be combined vertically to form a four-voice combination. The harmonic framework of this combination and its numerical abstraction are displayed at Example 20.1.

Example 20.1. The harmonic framework and its numerical abstraction of the combination of numerical frameworks at Figure 20.1.

Skrebkov maintains that compositional recourses can be expanded further by applying the following root motions to the previous framework: The root remains constant from one triad to the next, the root ascends a second $2\uparrow$, the root descends a second $2\downarrow$; the root ascends a third $3\uparrow$, the root descends a third $3\downarrow$; the root ascends a fourth $4\uparrow$, the root descends a fourth $4\downarrow$.

At Example 20.2, the application of root motion by $2\uparrow$, $2\downarrow$ has been applied.
Example 20.2. The harmonic framework of Example 20.1 with the application of 2↑, 2↓ root motion.

Skrebkov indicates that a similar harmonic framework is present in the following passage from Johann Sebastian Bach's F-sharp Minor fugue from The Well-Tempered Clavier I (Example 20.3).\(^{266}\) At Example 20.3, the root motion ascends a fourth.


**Canonic Sequences with Variable Root Motion**

In the previous example, the numerical designations correspond to the true positions of notes within a harmony and the two-integer numerical series are imitated in canon. When the root motion of a sequence is variable, if the numerical abstraction of a harmonic framework correctly designates the positions of notes within a harmony, then the numerical series are not imitated in canon. This is demonstrated at Example 20.4 in Dmitry Shostakovich's Fugue in E Minor.²⁶⁷

²⁶⁷ Dmitri Shostakovich, "Fugue in E Minor," *24 preliudii i fugy dlia fortepiano* [24 preludes and fugues for fortepiano, op. 87], ed. K. Soroskin (Moscow, 1956), mm. 53-5.
Example 20.4. Dmitry Shostakovich, "Fugue in E Minor," from Twenty-four Preludes and Fugues for Fortepiano, op. 87, mm. 53-5.

At Example 20.4, the root motion is variable—that is, it proceeds up a third and down a fourth. The numbers 3-1 and 5-1 correctly reflect the positions of notes in the given harmonic intervals; however, their interaction does not reflect a canon, i.e., the designation 3-1 does not correspond to 5-1 in a similar proportion. A correlation between these numerical sets is obtained when the notes of the harmonic framework are assigned conditional designations [uslovnye oboznachenii] (see Example 20.5).

Example 20.5. The application of conditional designations to the harmonic framework of a canonic sequence from Shostakovich's "Fugue in E Minor."
Conditional numerical designations occur when a constant root motion is assigned to a sequence that possesses in reality a variable root motion (in Example 20.5, a repeated root motion on B has been assigned to the previous canon). Constant root motion is assigned in order to allow us to carry out a direct graphic comparison of these complex sequences with the more simple type...and, as a whole, to facilitate a theoretical awareness of their rules.\textsuperscript{268}

Often, several different numerical assignments are possible for a canonic sequence with variable root motion. Of all possible numerical assignments, the chosen one generally consists of integers with the simplest values. For the harmonic framework at Example 20.4, the numerical assignments given at Example 20.6 are preferable to those given at Example 20.5 because they are "simpler and more apparent than the previous."\textsuperscript{269} In order to produce this result, root motion of an ascending third has been assigned.

\textsuperscript{268} Skrebkov, Teoriiia imitatsionnoi polifonii, p. 23.
\textsuperscript{269} Ibid.
Example 20.6. The harmonic framework of Example 20.4 with an assigned root motion that ascends a third.

Horizontal-Shifting Counterpoint

With regard to horizontal-shifting counterpoint, Skrebkov applies horizontal shifts to the extent that the canon remains in the strict style. This approach is demonstrated at Figure 20.2.
Figure 20.2. The application of horizontal-shifting counterpoint to a numerical framework that contains a four-note sequence.

These horizontal shifts are permissible because no rules of strict-style counterpoint have been broken.

Double and Triple Canonic Sequences

In contrast to Example 20.1, where only melodic progressions of the tertsovyi type were combined to form multi-voice combinations, Skrebkov indicates that diverse types of melodic progressions may be combined in order to create double and triple canonic sequences. For example, a canonic sequence can be written using three types of melodic progression as long as the combination remains in the strict
style. A possible result of such a combination is given in the triple canon at Example 20.7.

\[
\begin{array}{cccccccc}
1 & 5 & 1 & 5 & 1 & 5 & 1 & 5 \\
1 & 5 & 1 & 5 & 1 & 5 & 1 & 5 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\
3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\
\end{array}
\begin{array}{cccccccc}
3 & 5 & 3 & 5 & 3 & 5 & 3 & 5 \\
3 & 5 & 3 & 5 & 3 & 5 & 3 & 5 \\
1 & 3 & 1 & 3 & 1 & 3 & 1 & 3 \\
1 & 3 & 1 & 3 & 1 & 3 & 1 & 3 \\
\end{array}
\]

Figure 20.3. Numerical framework of a triple canon.

Conclusion

Skrebkov's theory of imitative counterpoint extends Taneev's theories by supplying new insight for the composition of canonic sequences with more than four voices and for horizontal-shifting counterpoint. At the core of his theory is an emphasis on the harmonic characteristics of polyphony. This emphasis helped him to create a reduction system that allows the application of an innovative means of classifying, analyzing, and composing canons.
Chapter 21
Timofeev: The Transformation of Simple Canons in the Strict Style

Introduction

Nikolay Andreevich Timofeev (1905-78) was a noteworthy Russian composer and pedagogue. His only published work on music theory, Prevrashchaemost' prostyh kanonov strogogo pis'ma [The transformation of simple canons in the strict style] "is a further development of the classical work Doctrine of the Canon." However, Timofeev's approach differs from Taneev's approach because Timofeev examines imitative counterpoint with geometry as opposed to algebra. Timofeev's aim was to present a clearer means of displaying potential derivative combinations of a given original.

Timofeev's Theory

Timofeev's graphic presentations are premised upon Taneev's and Bellermann's belief that "multi-voice counterpoint [should be regarded] as the union of various

\[270\] Nikolai Andreevich Timofeev, Prevrashchaemost' prostyh kanonov strogogo pis'ma [The transformation of simple canons in the strict style] (Moscow, 1981). The quotation is Dmitry Shostakovich's assessment of Timofeev's book. (Ibid., p. 2)
combinations of two-voice counterpoint."\textsuperscript{271} Based upon this view, Timofeev introduces the concept of a "canonic twin" [kanon-bliznets], which is a derivative of a multi-voice canon and consist of a rearrangement of correct two-voice contrapuntal combinations. At Example 21.1, using entrances extracted from Taneev's "Table of Three-Voice Canons of the First Order." Timofeev displays pairings of original combinations with canonic twins.\textsuperscript{272}

Direct


\textsuperscript{272} Sergei Ivanovitch Taneev, Uchenie o kanone [Doctrine of the Canon] (Moscow, 1929), § 83, p. 74. Cited in Timofeev, Prevrashchaemost' prostykh kanonov strogogo pis'ma, p. 20.
Example 21.1. Entrances of three-voice canons with direct and indirect shifts displayed with their canonic twins.

For each of the paired three-voice combinations, the same two-voice unions are present; however, the order of the two-voice combinations has been changed in the derivative canonic twin.

Timofeev geometrically and musically displays a canonic twin at Example 21.2, Figure 21.1, and Example 21.3.
Example 21.2. Entrances of an original combination and its canonic twin.
Figure 21.1. A geometrical representation of the entrances presented at Example 21.2.

Original
Example 21.3. A canon and its canonic twin composed using the entrances at Example 21.2.\

In a comparison of the geometric depiction at Figure 21.1 with the musical examples at Example 21.3, it is apparent that point A corresponds to the entrance of the Proposta, and points B and C correspond to the entrances of the Rispostas in the original combination. Points B1 and C correspond to the entrances of the Rispostas in the canonic twin.

\[^{273}\text{Timofeev is the composer of these canons. They are present in Timofeev, }\text{Prevrashchaemost' pro stykh kanonov strogogo pis'ma, pp. 54-5.}\]
A horizontal reading of the geometric graph at Figure 21.1 provides information about the rhythmic positions of the entrances; a vertical reading provides information about the notes on which the voices enter. Timofeev employs a line segment between two points of entrance in order to show that the combination is contrapuntally correct. Therefore, in this canon, there are three correct two-voice combinations: K(A B), K(A C), and K(B C)—the letter K indicates a correct union. The dotted line segments K(A B1) and K(B1 C) indicate correct unions of voices in the canonic twin.

Timofeev's geometric representation of correct two-voice relationships in the canonic twin can be viewed as a unique approach to the display of an imaginary voice in a three-voice canon of the second order. Viewed in this way, segment K(A B1) of the canonic twin serves as an original imaginary combination for K(B C). This relationship causes K(A B) = K(B1 C); therefore, K(A B1) = K(B C) and K(A B) = K(B1 C), thereby forming a rectangle with two sets of parallel lines.

Throughout his theory, Timofeev uses parallel lines as a means of discovering canonic twins. For example, at Figure 21.2, A E F D is the original canon for A E B D because K(A F) = K(E B).
Figure 21.2. Geometric depiction of the original A E F D and its canonic twin A E B D.

The relationship of an original canon to its canonic twin is present because $K(A F)$ is both parallel and equal to $K(E B)$. Therefore, all two-voice combinations in A E F D are also present in A E B D. This rearrangement of two-voice combinations is apparent in the musical examples at Examples 21.4 and 21.5.²⁷⁴

²⁷⁴ Timofeev is the composer of these canons. They are present in Timofeev, *Prevrashchaemost’ prostykh kanonov strogogo pis’ma*, pp. 78-9, 81-2.
Example 21.4. A canon based on the graph $A \ E \ F \ D$ at Figure 21.2.
Example 21.5. A canon based on the graph A E B D at Figure 21.2.

In contrast to three-voice combinations, Timofeev shows that four-voice combinations may have several twins. For example, with regard to the canon at Example 21.4 (i.e., A E F D), if segment K(A H) is set parallel to segment K(E D), all two-voice combinations of the original A E F D are
maintained in the canonic twin A E B H. This is demonstrated at Figures 21.3 and 21.4.

Figure 21.3. Geometric depiction of the original canon A E F D in which segment K(A H) is parallel to segment K(E D).

Figure 21.4. Canonic twin A E B H.
Conclusion

Based upon the premise that multi-voice canons in the strict style consist of an aggregate of contrapuntally correct combinations of two voices, Timofeev shows how derivative canons (canonic twins) can be geometrically determined from originals by using parallel lines in graphs of entrances of voices in canon. A limitation of this approach is that it assumes that the difficult task of composing a Proposta that is contrapuntally correct with all Rispostas, real and imaginary, has been accomplished. An advantage is that it provides a systematic way for the composer to become graphically aware of a wide range of potential derivatives in a multi-voice canon.
Conclusion

Taneev wrote *Doctrine of the Canon* in order to make the study of imitative counterpoint more efficient. The result was a highly organized method which offers insight into all vertical and horizontal shifts in canon.

For present-day musicologists, composers, and music students, Taneev's theories continue to have relevance because they offer insight into twentieth-century compositional thought in Russia. However, the insight they offer is not into any single musical style, but rather into a pedagogical process that served as the rich soil from which a diverse variety of compositional branches sprang.

Despite the fact that Taneev developed his theories in order to make his own method of composition more practical, and the fact that he advocated the study of his theories in order to bring about a more restrained approach to composition by his students and contemporaries, there is a great difference between the compositional style of Taneev and the compositional styles of his students, as well as between the styles of his students themselves. Taneev's students, such as Gliere, Scriabin, and Rakhmaninov, each cultivated their own unique musical voice; however, a flowering of their personal creativity was greatly aided by an abundance of compositional technique. To a large degree, this technique resulted from their ability to fluently
maneuver within the limitations of the strict style, a skill that was developed using Taneev's system.275

This act of problem solving within a given set of limitations is at the center of Taneev's approach to musical composition and pedagogy. This same approach is echoed later in the philosophy for composition of Igor Stravinsky. Stravinsky writes:

My freedom will be so much greater and more meaningful the more narrowly I limit my field of action and the more I surround myself with obstacles. Whatever diminishes constraint diminishes strength. The more constraints one imposes, the more one frees oneself of the chains that shackle the spirit.276

This approach, which proposes a paradox consisting of freedom through constraint, remains relevant to our time because it interjects a necessary viewpoint into the discussion of contemporary approaches to musical pedagogy and composition.

As for the legacy of Taneev's theories during the Soviet era and in the present day, much valuable work has been done and continues to be done by Russian musical theorists. The allure that Taneev's work holds for musical scholars is the demand that it places on the theorist to move away from

275 Salient appearances of the influence of Taneev's methods in the music of these, as well as other twentieth-century Russian composers is documented in Vladimir Vasil'evich Protopopov, Istoriia polifonii v ee vazhneishikh iavleniakh [A history of polyphony and very important occurrences], (Moscow, 1962-65); -----, Istoriia polifonii [A history of polyphony], (Moscow, 1985-87).

speculative pursuits to that which is mathematically verifiable.

In this dissertation an English translation of *Doctrine of the Canon* has been set forth and a discussion of the development of theories from this text during the Soviet era has occurred. What remains to be done is the pedagogical application of theories of shifting counterpoint to various facets of free style counterpoint. This would have as a result the development of compositional recoursefullness with regard to contemporary approaches to tonality and would "introduce something new into his [the student's] compositions, since much of that which was known and able to be done by earlier masters has been completely forgotten by our contemporaries."\(^{277}\)

\(^{277}\) Sergei Ivanovich Taneev, *Uchenie o kanone* [Doctrine of the canon] (Moscow, 1929), § 96, p. 102.
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"Benedictus" from Missa: Ad fugam.

"Sanctus" from Missa: Ad fugam.

"Gloria" from Missa: Alma Redemptoris.

"Sanctus" from Missa: Aspice Domine.

Motet: Benedicta tu (secunda pars).

"Credo" from Missa: Brevis.

"Gloria" from Missa: Brevis.

"Sanctus" from Missa: Brevis.

"Credo" from Missa: De Beata Virgine.

Motet: Dereliquat impius.

"Benedictus" from Missa: Dies sanctificatus.

"Kyrie" from Missa: De Beata Virgine.
"Gloria" from Missa: Dum complerentur.

Motet: Exi Cito.

"Benedictus" from Missa: Gia fu chi m'hebbe carra.

Sacred Song: "Illumina oculos meos."

"Credo" from Missa: Inviolata.

"Kyrie" from Missa: In te Domine speravi.

Motet: Introduxit me Rex in cellam.

"Benedictus" from Missa: Iste confessor.

"Sanctus" from Missa: Iste Confessor.

"Credo" from Missa: Jam Christus astra ascenderat.

"Sanctus" from Missa: Jam Christus astra ascenderat.

Motet: Judica me deus, et discerne.

"Kyrie" from Missa: L'homme arme.

"Agnus Dei I" from Missa: Nigra sum.

"Credo" from Missa: Nigra sum.

"Credo" from Missa: O admirabile commericum.

"Benedictus" from Missa: O Regem Coelli.

"Gloria" from Missa: O Regem Coeli.

"Sanctus" from Missa: O Regem Coeli.

"Benedictus" from Missa: Papae Marcelli.

"Credo" from Missa: Primi Toni.

"Gloria" from Missa: Primi Toni.

Missa: Pulchra es amica mea.

"Credo" from Missa: Quem dicunt homines.

"Agnus Dei II" from Missa: Prima Toni.

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"Credo" from Missa: Repleatur os meum laude.
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"Credo" from Missa: Sanctorum merits.
"Gloria" from Missa: Secunda.
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"Credo" from Missa: Sine nomine.
"Gloria" from Missa: Sine nomine.
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