TRANSISTOR DISTRIBUTED AMPLIFIERS

by

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Chapter 1

INTRODUCTION

The conventional techniques of wide-band amplifier designs have been explored thoroughly in recent years by Wheeler[1] and others. They have shown that there is an upper limit to the gain-bandwidth product associated with a given vacuum tube or transistor type, regardless of the complexity of the interstage coupling networks. The limit is governed primarily by shunt capacities, associated with the respective active device, across which a voltage must be developed. This places a definite limit on the bandwidth obtainable by cascading single stage amplifiers, for if the desired overall bandwidth is greater than the gain-bandwidth product of the individual stages, each stage attenuates instead of amplifying.

The vacuum tube distributed amplifier [2,3,4] can be used to obtain amplification over bandwidths in excess of the gain-bandwidth

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[1]Wheeler, H. A., "Wide-band Amplifiers for Television", Proc. IRE, Vol. 27, July, 1939, pp. 429-438.

[2]Ginzton, E. L., Hewlett, W. R., Jasberg, J. H., Noe, J.D., "Distributed Amplification", Proc. IRE, Vol. 36, August, 1948, pp. 956-969.

[3]Horton, W. H., Jasberg, J. H., Noe, J. D., "Distributed Amplifiers: Practical Considerations and Experimental Results", Proc. IRE, Vol. 38, July, 1950, pp. 748-753.

[4]Rogers, P. H., "Large Signal Analysis of Distributed Amplifiers", Tech. Report No. 52, Electronics Defense Group, University of Michigan, July, 1955.

product of its vacuum tube amplifying sections. The individual sections are connected together in such a manner that their input and output capacities form part of an artificial transmission line. Each section amplifies the voltage wave propagating down the input line (see Figure 1.1). The sections are arranged so that the amplified voltages are added in phase on the output transmission line. Since the voltages are combined by addition, the total amplification is equal to the sum of the section gains rather than the product. Amplification is then possible even though the gains of the individual sections are less than unity.

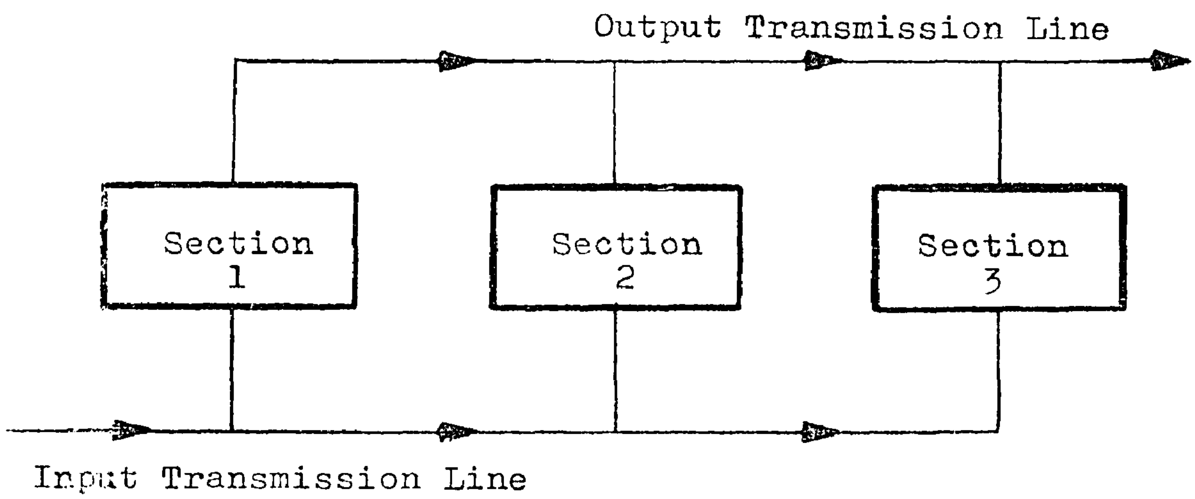

Figure 1.1 Distributed amplifier with three amplifying sections.

1.1 Statement of the Problem and Method of Approach

The purpose of this dissertation is to develop the required theory for the successful use of transistors in distributed amplification. The problem is separated into two parts. The first part develops a satisfactory transistor amplifying section. The input and output impedances are controlled in order to approximate reactive behavior and not cause attenuation in the transmission lines. In addition, it is possible to exchange voltage gain for bandwidth in order to obtain bandwidths in the order of magnitude of the gain-bandwidth product of the transistor.

The second part develops the transmission lines of the distributed amplifier. The input and output transmission lines contain the reactive input and output impedances of the amplifying sections as actual circuit elements. Two types of lines are discussed which propagate signal frequencies with minimum attenuation. The first line is developed on the classical image impedance basis, and the second through the application of modern network synthesis to the specific problem at hand.

Experimental results for distributed amplifiers constructed to illustrate both types of transmission lines are presented and discussed.

Chapter 2

DEVELOPMENT OF A SUITABLE WIDE-BAND TRANSISTOR AMPLIFYING SECTION

2.1 Introduction

The amplifying sections of a distributed amplifier should be extremely wide-band transistor video amplifiers, presumably with gains in the neighborhood of unity, for if the midfrequency gains were much greater than unity, cascading sections would give a greater overall gain. In addition, the input and output impedances must approach reactive behavior in the pass-band when compared to the impedance level of the respective transmission line; otherwise severe attenuation would result. The common emitter amplifier with emitter degeneration becomes the most logical configuration. The development in the following sections of this chapter determines an emitter impedance which causes the input impedance to approximate a pure capacity, and at the same time exchanges voltage gain for bandwidth in such a manner that the gain-bandwidth product remains approximately constant.

2.2 Development

The equivalent circuit used here for common emitter wide-band design is a modified form of the Johnson-Giacolleto hybrid-pi circuit[1] shown in Figure 2.1. The basic method of analysis utilized for the development is one first used by Bruun[2]. One difficulty in the analysis of wide-band transistor amplifiers occurs because of the non-unilateral nature of the transistor caused by the collector barrier capacitance C_c. Bruun's technique allows the effect of C_c to be approximated by a Miller capacity, greatly reducing the problem.

The following equations may be written from inspection of Figure 2.2:

$$V' = i_1 \left(\frac{1}{Y_{11}} + Z_e \right) + Y_{21} Z_e V$$

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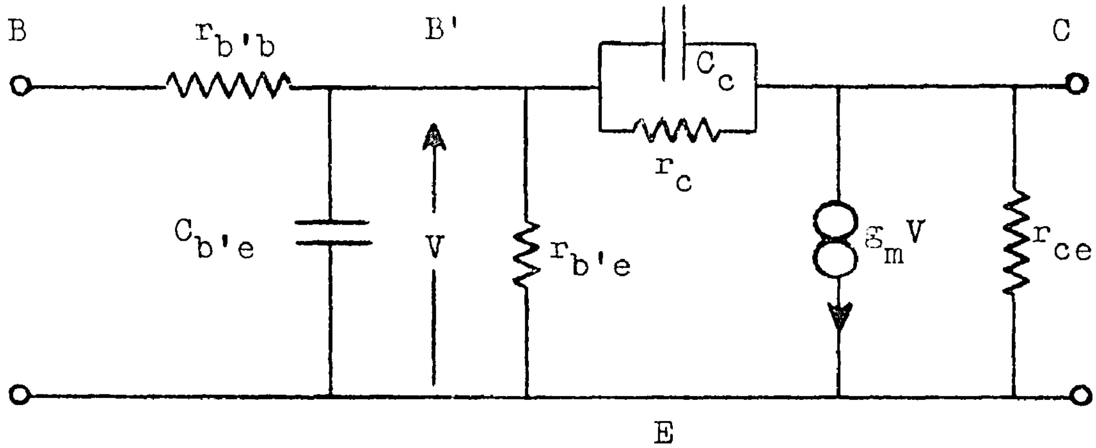
[1] Giacolleto, L. J., "Study of PNP Alloy Junction Transistors from DC Through Medium Frequencies", RCA Review, Vol. 14, No. 4, 1954, pp. 506-562.

[2] Bruun, Georg, "Common-emitter Transistor Video Amplifiers", Proc. IRE, Vol. 44, No. 11, Nov., 1956, pp. 1561-1572.



$r_{b'b}$ = base spreading resistance

$C_{b'e}$ = diffusion capacity plus barrier capacity
= $I_e(ma)/27\,\omega_t$

$r_{b'e}$ = emitter diode forward resistance

$$= \frac{27\ B_o}{I_e(ma)} = \frac{B_o}{g_m}$$

C_c = collector diode capacity, proportional to $\dfrac{1}{\sqrt{V_{c\ b}}}$

r_c = reverse diode resistance (large and usually neglectible)

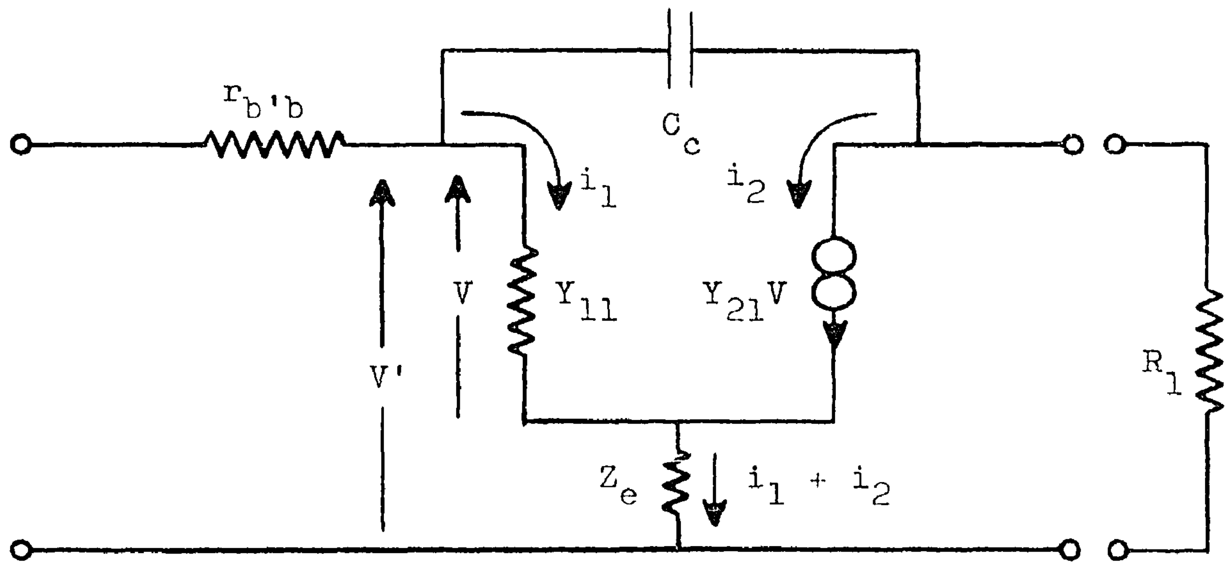
g_m = intrinsic transconductance = reciprocal of emitter diffusion resistance = $I_e(ma)/27$

B_o = low frequency short circuit current gain in the common emitter configuration

r_{ce} = space charge widening resistance (large and usually neglectible)

ω_t = frequency at which $|B(j\omega)| = 1$. For alloyed type transistors $\omega_t = \omega_a/1.22$. For other types ω_t is generally specified.

Figure 2.1 Johnson-Giacoletto hybrid-pi equivalent circuit for common emitter configuration. This circuit holds for all frequencies for which the transistor is capable of power gain.



$$Y_{11} = sC_{b'e} + \frac{g_m}{B_o} = s\frac{g_m}{\omega_t} + \frac{g_m}{B_o} = g_m\left(\frac{1}{B_o} + \frac{s}{\omega_t}\right)$$

$$Y_{21} = g_m$$

Figure 2.2 Equivalent circuit used to analyze emitter degeneration.

But

$$V = i_1 \left(\frac{1}{Y_{11}} \right)$$

So

$$V' = i_1 \left[\frac{1 + Z_e \ (Y_{11} + Y_{21})}{Y_{11}} \right] \qquad (2.1)$$

Also

$$i_2 = Y_{21} V = \frac{Y_{21}}{Y_{11}} \ i_1 = \frac{Y_{21} V'}{1 + Z_e \ (Y_{11} + Y_{21})} \qquad (2.2)$$

Equations 2.1 and 2.2 may be interpreted in the equivalent circuit shown in Figure 2.3.

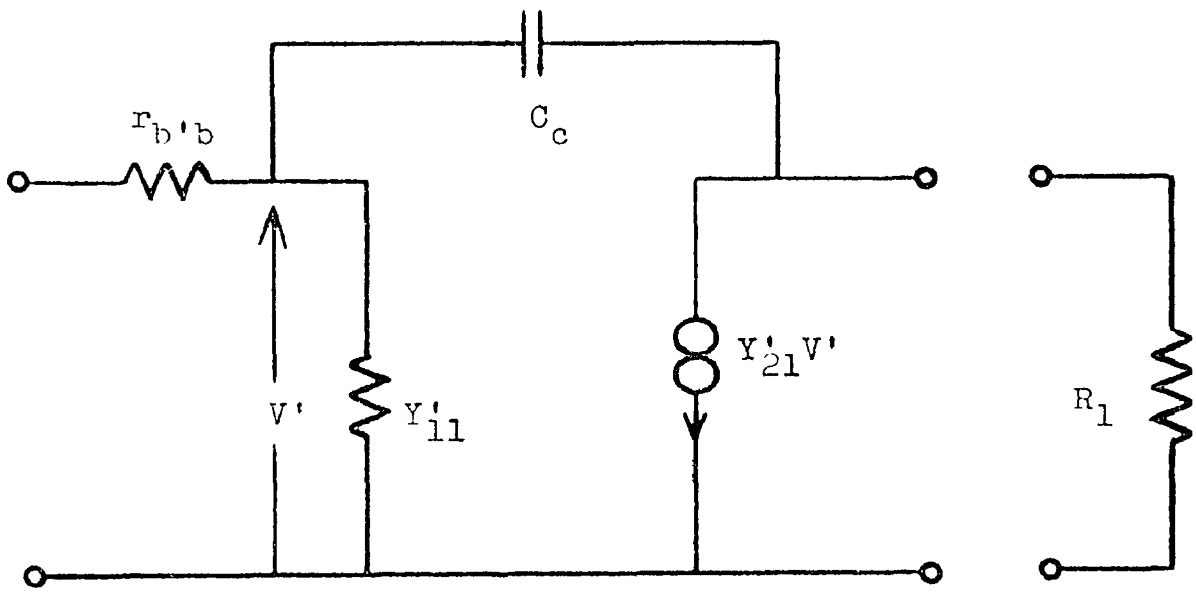
The expression

$$Y'_{21} = \frac{Y_{21}}{1 + Z_e \ (Y_{11} + Y_{21})}$$

is easily interpreted if it is expressed in terms of the relations in Figure 2.2.

$$Y_{21} + Y_{11} = g_m \left[1 + \frac{1}{B_o} + \frac{s}{\omega_t} \right]$$

$$\doteq g_m \left[1 + \frac{s}{\omega_t} \right] \quad \text{if } B_o \gg 1$$



where $$Y'_{11} = \frac{Y_{11}}{1 + Z_e(Y_{11} + Y_{21})}$$

$$Y'_{21} = \frac{Y_{21}}{1 + Z_e(Y_{11} + Y_{21})}$$

Figure 2.3 Emitter degeneration absorbed into equivalent circuit.

If

$$Z_e = \frac{R_e}{1 + \frac{s}{\omega_1}}$$

then

$$Y_{21}' = \frac{g_m}{1 + g_m R_e \dfrac{1 + \frac{s}{\omega_t}}{1 + \frac{s}{\omega_1}}}$$

If, in addition $\omega_1 = \omega_t$,

then

$$Y_{21}' = \frac{g_m}{1 + g_m R_e} = \frac{1/r_m}{1 + \frac{R_e}{r_m}} = \frac{1}{r_m + R_e}$$

If the definition

$$\frac{1}{r_m + R_e} = G_m$$

is made,

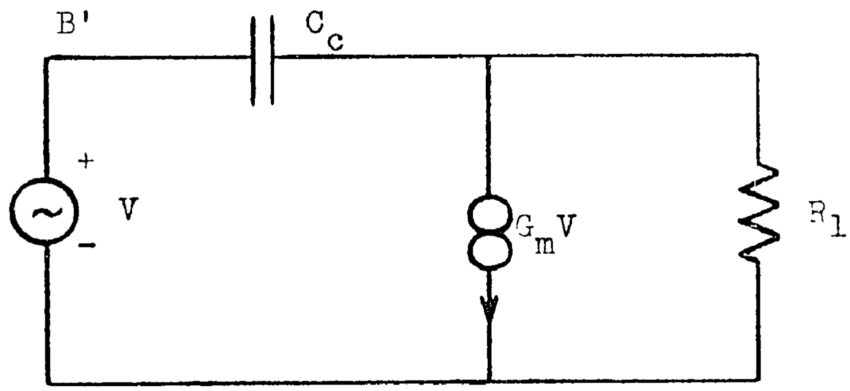
then

$$Y_{21}' = G_m \tag{2.3}$$

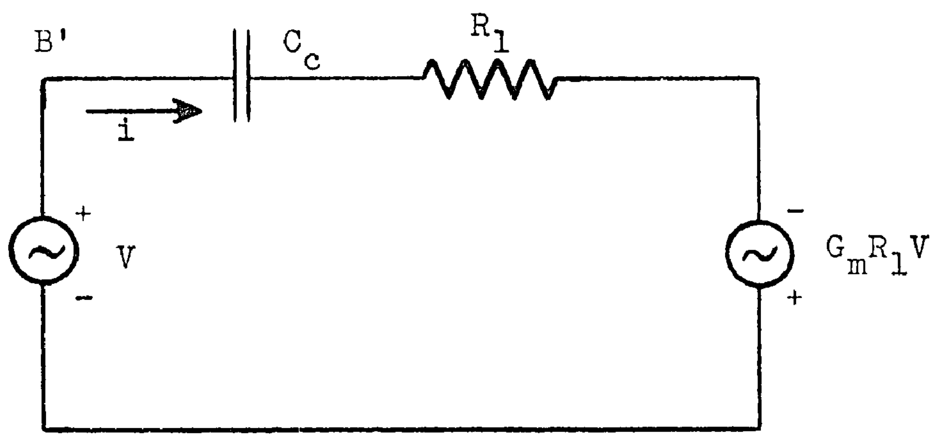
represents the effective transconductance of the composite transistor.

The effect of C_c on the input circuit may be approximated by reflecting it as a Miller admittance. The admittance seen at B' in Figure 2.4 b is

$$Y = 1/V = sC_c \left(1 + G_m R_1 \right) \frac{1}{sC_c R_1 + 1}$$



a. Circuit for determination of Miller admittance.



b. Circuit of part a with current generator replaced by Thevenin equivalent.

Figure 2.4 Calculation of Miller admittance.

which reduces to

$$Y = sC_c \; (1 + G_m R_1)$$

for $\omega \ll 1/R_1 C_c$. Hence, the Miller admittance is a pure capacitance, $C_m = C_c \; (1 + G_m R_1)$, in parallel with Y_{11}' as depicted in Figure 2.5.

Y_{11}' is readily obtained from Figures 2.2 and 2.3

$$Y_{11}' = \frac{Y_{11}}{1 + Z_e \; (Y_{11} + Y_{21})}$$

$$= \frac{g_m \; (\frac{1}{B_o} + \frac{s}{\omega_t})}{1 + g_m R_e \; \dfrac{1 + \frac{s}{\omega_t}}{1 + \frac{s}{\omega_1}}}$$

However, if $\omega_1 = \omega_t$ then

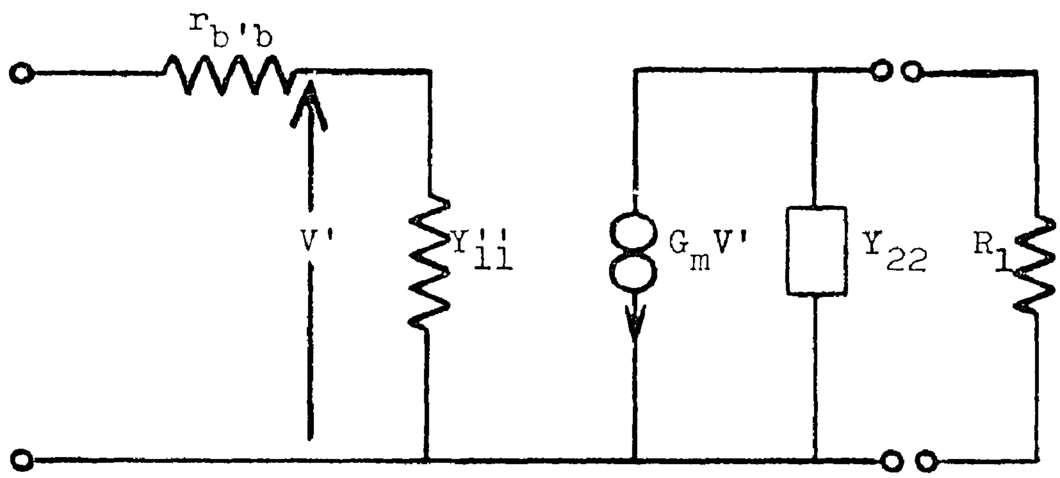
$$Y_{11}' = \frac{g_m}{1 + g_m R_e} \left(\frac{1}{B_o} + \frac{s}{\omega_t} \right)$$

which reduces to

$$Y_{11}' = G_m \left(\frac{1}{B_o} + \frac{s}{\omega_t} \right)$$

because $G_m = g_m/(1 + g_m R_e)$. Hence, Y_{11}' is a resistance $R_{11}' = B_o/G_m$ in parallel with a capacitance $C_{11}' = G_m/\omega_t$. C_{11}' and the Miller capacitance C_m are in parallel and may be added to obtain the total capacity C from the internal base of the transistor to ground, i.e.,

$$C = C_{11}' + C_m = G_m/\omega_t + C_c \; (1 + G_m R_1)$$



$$Y''_{11} = Y'_{11} + sC_c(1 + G_m R_1) \quad \text{for } \omega \ll \frac{1}{R_1 C_c}$$

where Y_{22} = output admittance

Figure 2.5 Emitter degeneration and Miller effect, because of C_c, absorbed into equivalent circuit.

It is useful to express this equation in a slightly different form:

$$C = F \frac{G_m}{\omega_t}$$

where

$$F = \frac{C_{11}' + C_m}{C_{11}'} = 1 + \omega_t C_c \ (R_1 + R_m).$$

From this equation it is possible to see immediately how much effect the Miller capacity has on the circuit when numerical values are used.

The admittance Y_{11}'' in Figure 2.5 is thus

$$Y_{11}'' = \frac{G_m}{B_o} + sC \tag{2.4}$$

The circuit used for determining the output admittance Y_{22} is that depicted in Figure 2.6. It has been assumed that the Miller capacity C_{11}', $r_{b'b}$, and R_{11}' all have negligible effect on the base line impedance. The first assumption requires $F \doteq 1.0$.

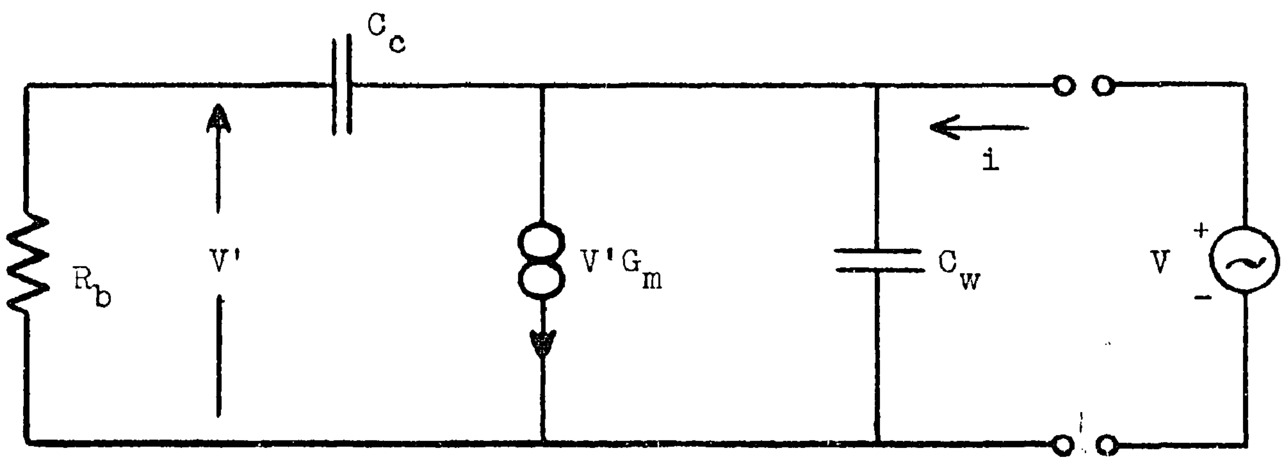


Figure 2.6 Circuit for determining the output admittance Y_{22}. R_b approximates the base line impedance and C_w the interstage wiring capacity.

The second and third assumptions are discussed in the next section and in Chapter 3.

From Figure 2.6

$$i = V'G_m + V \frac{1}{\frac{1}{sC_c} + R_b} + VsCw$$

$$= V \left(G_m \frac{R_b}{\frac{1}{sC_c} + R_b} + \frac{1}{\frac{1}{sC_c} + R_b} + sC_w \right)$$

Hence

$$i/V = \left(\frac{G_m R_b + 1}{sR_b C_c + 1} \right) sC_c + sC_w$$

If $\omega << \frac{1}{R_b C_c}$, then

$$Y_{22} \doteq sC_c \; (1 + G_m R_b) + sC_w \qquad (2.5)$$

Thus, the output admittance consists of a "reverse" Miller capacity in parallel with the parasitic wiring capacity of the interstage coupling network. For available high frequency transistors the wiring capacity is by far the larger of the two. Hence, even if the assumptions used in calculating the "reverse" Miller capacity are rather crude, usable results are obtained.

In view of equations 2.3, 2.4, and 2.5 the general equivalent circuit in Figure 2.5 reduces to the lumped element circuit shown in Figure 2.7.

The voltage gain is

$$V_o/V = (V_o/V')\ (V'/V) \left[\frac{G_m}{sC_o + G_1}\right] \frac{\dfrac{1}{sC + 1/B_oR_m}}{r'_{bb} + \dfrac{1}{sC + 1/B_oR_m}}$$

If $B_o \gg r'_{bb}/R_m$, then

$$V_o/V = \frac{A_r}{(s/\omega_o + 1)} \cdot \frac{1}{(s/\omega_2 + 1)}$$

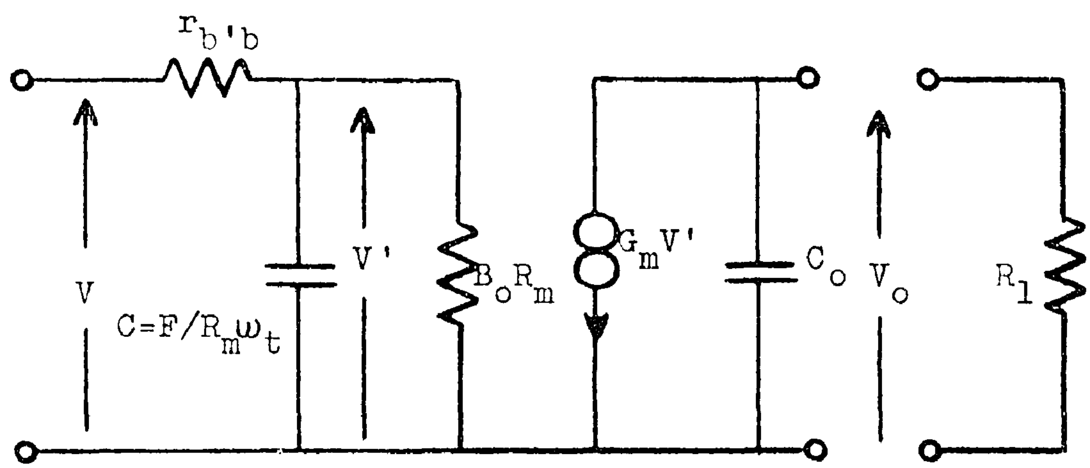
where:

$$A_r = G_mR_1$$

$$\omega_o = \omega_t \left(\frac{R_m}{r'_{bb}F} \right) = \frac{1}{r'_{bb}\ C}$$

$$\omega_2 = \frac{1}{R_1C_o}$$

Notice that ω_2 is the break frequency contributed by the output circuit. For moderate bandwidths, ω_2 is large enought that its effect is negligible. The voltage gain bandwidth is then equal to



$C_o = C_c(1 + G_mR_b) + C_w$

$R_m = 1/G_m = R_e + r_m$ = sum of external emitter resistance and emitter diffusion resistance

ω_t = frequency at which $|B(j\omega)| = 1$

$F = 1 + \omega_t C_c(r_m + R_e + R_1)$

R_b = base line impedance

Figure 2.7 Equivalent circuit of the composite transistor, with an emitter impedance of

$$Z_e = \frac{R_e}{1 + s/\omega_t}$$

the break frequency of the input circuit ω_o. The gain bandwidth product becomes

$$A_r\omega_o = \frac{R_1\omega_t}{r'_b{}_b F}$$

If the Miller capacity constitutes only a small portion of the capacity C shown in Figure 2.7, then F does not vary significantly with R_e; the gain-bandwidth product is essentially independent of R_e. Hence, gain may be exchanged for bandwidth by increasing R_e until ω_o approaches ω_2. Since the overall bandwidth would never exceed ω_2, there would be no apparent advantage obtained by sacrificing further gain to increase ω_o beyond ω_2.

2.3 Vacuum Tube Analogy

The previous section illustrated a method for increasing the impedance level of the input circuit of a transistor in its common emitter configuration without reducing its gain-bandwidth product. In the following section the circuit of Figure 2.7 is put in a form analogous to a grounded cathode vacuum tube circuit. Once this is accomplished, transistor distributed amplifiers can be designed from an exact analogy with vacuum tube distributed amplifiers.

The expression for the input admittance of the circuit shown in Figure 2.8 can be shown to be

$$Y_{in} = (1/R_{11}') \frac{1 + \omega^2/\omega_o\ \omega_b + j\omega\ (\ 1/\omega_b - 1/\omega_o\ )}{1\ (\ +\ (\omega/\omega_o)^2}$$

$$\doteq (1/R_{11}') \frac{(1 + \omega^2/\omega_o\ \omega_b) + j\omega/\omega_b}{1 + (\omega/\omega_o)^2}$$

since $R_{11}' = B_o R_m \gg r_b'\ _b$, and consequently $\omega_o \gg \omega_b$, where $\omega_b = 1/R_{11}'C$ and $\omega_o = 1/r_b'\ _b\ C$.

For $(\omega/\omega_o)^2 \ll 1.0$, the expression for Y_{in} becomes

$$Y_{in} \doteq (1/R_{11}') \left[(1 + \omega^2/\omega_o\ \omega_b) + j\ (\omega/\omega_o)\right]$$

which is shown schematically in Figure 2.9.

The resistance R_{11}' is large enough to be neglected in comparison to the impedance level of the interstage if the transistor is to be used as a wide-band amplifier. R' is analogous to the shunt resistance in vacuum tubes which decreases with the

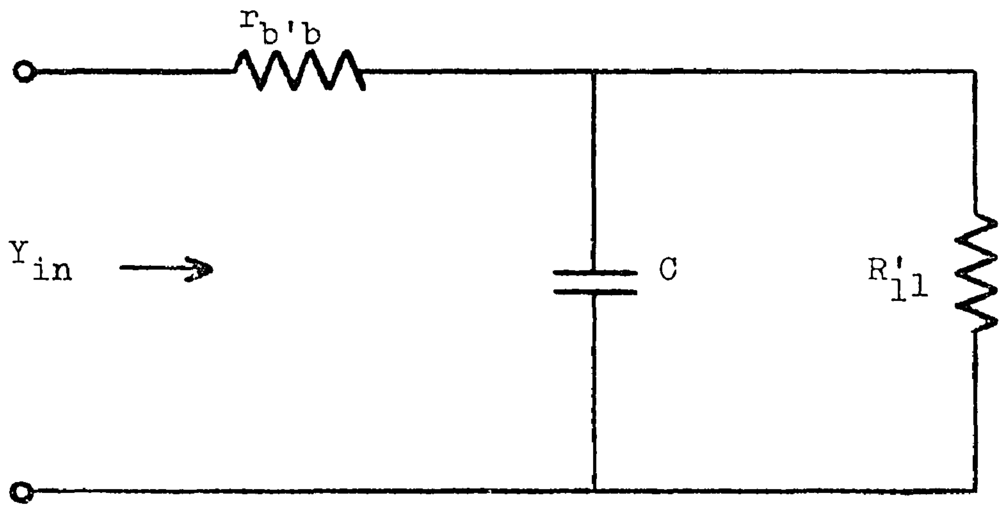
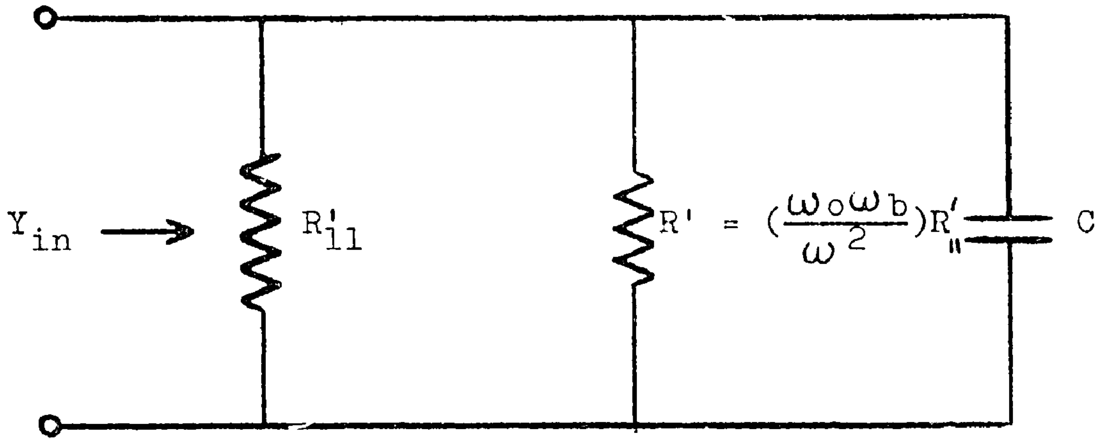


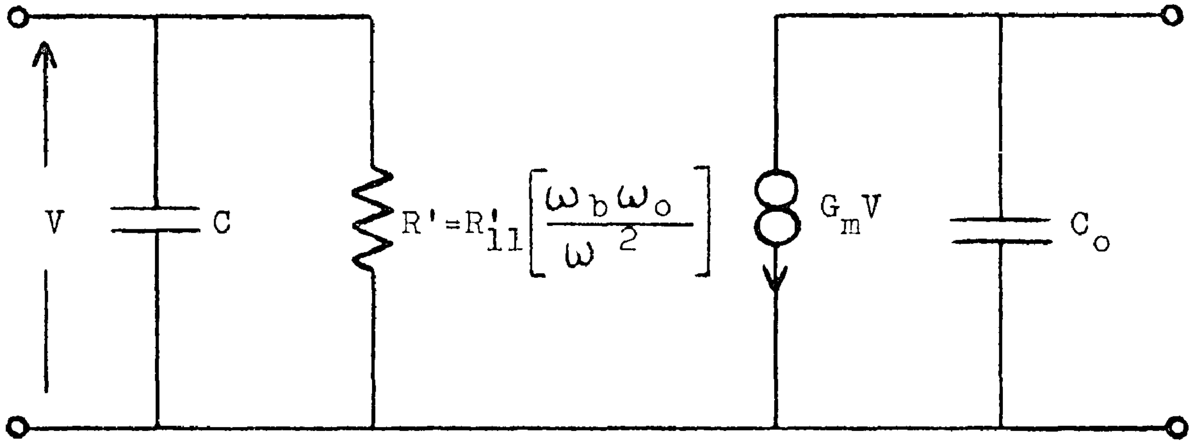
Figure 2.8 Input circuit of unilateralized composite transistor of Figure 2.7.



$\omega_b = 1/R'_{11}C \qquad \omega_o = 1/r_{b'b}C$

Figure 2.9 The input circuit of the composite transistor of Figure 2.7 for $(\omega/\omega_o)^2 \ll 1$.

square of frequency because of transit time, cathode lead inductance and other high frequency effects. Hence, the composite transistor behaves like a vacuum tube when used as a wide-band amplifier. The final equivalent circuit is shown in Figure 2.10.



$$C = F/R_m \omega_t$$

$$\omega_o = 1/r_{b'b}C$$

$$\omega_b = 1/R'_{11}C = \omega_t/FB_o$$

$$R_m = 1/G_m = r_m + R_e$$

$$F = 1 + \omega_t C_c(R_m + R_1)$$

Figure 2.10 Final equivalent circuit for composite transistor in the frequency range $(\omega/\omega_o)^2 \ll 1$ and for interstage impedance levels on the input in the range $|Z| \ll B_o R_m$. Other assumptions are $B_o \gg 1$, $B_o \gg r_{b'b}/R_e$, $\omega \ll \frac{1}{R_1 C_c}$, $\omega \ll \frac{1}{R_b C_c}$, and $F \doteq 1$.

Chapter 3

CONSTANT K DISTRIBUTED AMPLIFIER

3.1 Development for Flat Frequency Response

The last chapter illustrated that emitter degeneration can be used to force a transistor to behave electrically like a vacuum tube.

Techniques for designing vacuum tube distributed amplifiers are well known.[1,2] The following sections review this material adapting it as required for the design of transistor distributed amplifiers.

The gain expression for a general distributed amplifier is shown in Appendix A to be

$$A_t(s) = A(s)\; x(s)^{n-1} \quad \frac{1 - k^n}{1 - k}$$

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[1] Horton, W. H., Jasberg, J. H., and Noe, J. D., "Distributed Amplifiers: Practical Considerations and Experimental Results," Proc. IRE, Vol. 38, July, 1950, pp. 748-753.

[2] Rogers, P. H., "Large Signal Analysis of Distributed Amplifiers," Tech. Report No. 52, Electronics Defense Group, University of Michigan, July, 1955.

where: $A(s)$ = amplifying section voltage gain.

$x(s)$ = voltage transfer function per section of collector line.

k = ratio of voltage transfer function per section of base line to the like function for the collector line.

n = number of amplifying sections.

The following statements can be made for a constant K amplifier using the composite transistor equivalent circuit of Figure 2.7:

(1) The collector line is essentially lossless and consequently $\left| x(j\omega) \right| = 1$, in the passband.

(2) The attenuation factor per section of the base line is $a = -\ln k$ nepers.

(3) The attenuation factor of the half section input matching network is $-\frac{1}{2} \ln k$. Consequently,

$$\left| A_t(j\omega) \right| = \left| A(j\omega) \right| e^{-a/2} \frac{1 - e^{-na}}{1 - e^{-a}}$$

$$= \left| A(j\omega) \right| e^{-\frac{na}{2}} \frac{\sinh(na/2)}{\sinh(a/2)} \tag{3.1}$$

The gain of an individual amplifying section is

$$\left| A(j\omega) \right| = G_m \frac{R_{oc}/2}{\sqrt{1 - (\frac{\omega}{\omega_c})^2}} \tag{3.2}$$

where

$$R_{oc} = \sqrt{L/C} = \text{characteristic impedance of the collector line (see Figure 3.1).}$$

$$\omega_c = 2/\sqrt{LC} = \text{cutoff frequency of the collector line.}$$

The attenuation factor per section of the base line must be determined. The constant K filter section of the base line appears as shown in Figure 3.1, with the exception that a resistance must be added in parallel with each capacitance to represent the input admittance of the transistor. If it were desirable to consider the attenuation because of coil loss, a resistance would be added in series with the coil. The attenuation factor per base line section may be written[3]

$$a = (R/2L + G/2C)\,\frac{d\phi}{d\omega} + 1/2(R/2L + G/2C)^2\,\frac{d^2\phi}{d\,\omega^2} + \ldots..$$

if $(R/2\omega L - G/2\,\omega C) \ll 1.0.$

In the pass-band of the multisection line, the attenuation factor per section is fairly small.

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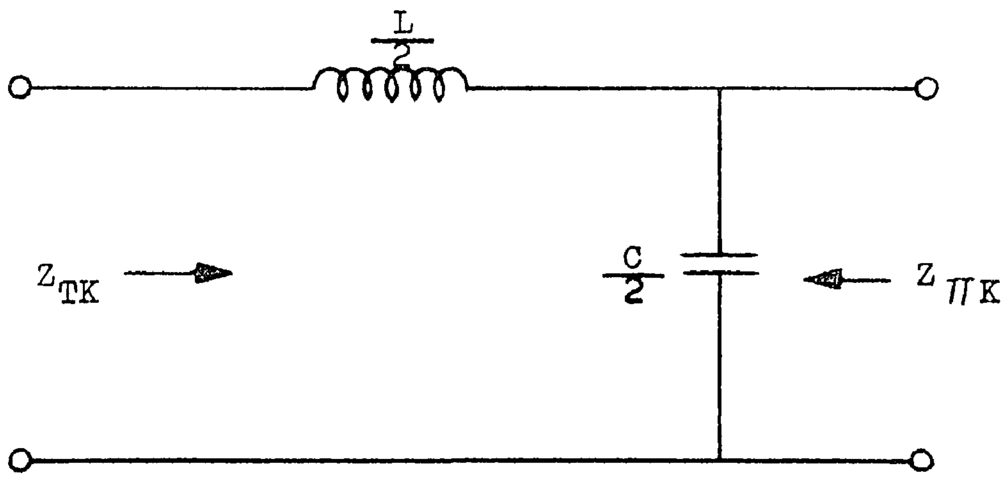
[3]Guillemin, E. A., Communication Networks, Vol. 2, Wiley, New York, New York, 1935, pp. 447.



where $Z_{TK} = R_o\sqrt{1 - \left(\frac{\omega}{\omega_c}\right)^2}$

$$Z_{\pi K} = \frac{R_o}{\sqrt{1 - \left(\frac{\omega}{\omega_c}\right)^2}}$$

$$\omega_c = \frac{2}{\sqrt{LC}} = 2\,\frac{R_o}{L} = 2/R_oC$$

$$\left.\begin{aligned} a &= 0 \\ \phi &= 2\sin^{-1}\left(\frac{\omega}{\omega_c}\right) \end{aligned}\right\} \quad 0 \leq \omega \leq \omega_c$$

$$\left.\begin{aligned} a &= 2\cosh^{-1}\left(\frac{\omega}{\omega_c}\right) \\ \phi &= \pi \end{aligned}\right\} \quad \omega_c \leq \omega \leq \infty$$

Figure 3.1 Low-pass constant K half-section. The phase and attenuation characteristics are for a full-section.

It is possible under these conditions to obtain a satisfactory approximation to the attenuation factor by neglecting all terms in the series higher than the first and using the phase function of a lossless line. Hence,

$$a = (G/2C)\,\frac{\partial \phi}{\partial \omega} = \frac{(1/r'_{bb})\,(\omega/\omega_o)^2}{2C}\,\frac{\partial}{\partial \omega}\left[2\,\tan^{-1}\frac{(\omega/\omega_c)}{\sqrt{1-(\omega/\omega_c)^2}}\right]$$

$$= (1/2)\,(\omega_c/\omega_o)\,\frac{2(\omega/\omega_c)^2}{\sqrt{1-(\omega/\omega_c)^2}} = a_o\,\frac{(\omega/\omega_c)^2}{\sqrt{1-(\omega/\omega_c)^2}} \qquad (3.3)$$

where

$$a_o = \frac{\omega_c}{\omega_o} = \frac{2r'_{bb}C}{R_{ob}C} = \frac{2r'_{bb}}{R_{ob}}$$

Coil losses have been neglected since they would be small.

Using the results of equations 3.1, 3.2 and 3.3, normalized curves for various values of $na_o/2$ may be plotted as a function of frequency.[4] The curves are shown in Figure 3.2.

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[4] Horton, W. H., "Distributed Amplifiers: Practical Considerations and Experimental Results," Proc. IRE., Vol. 38, July, 1950, pp. 748-753.

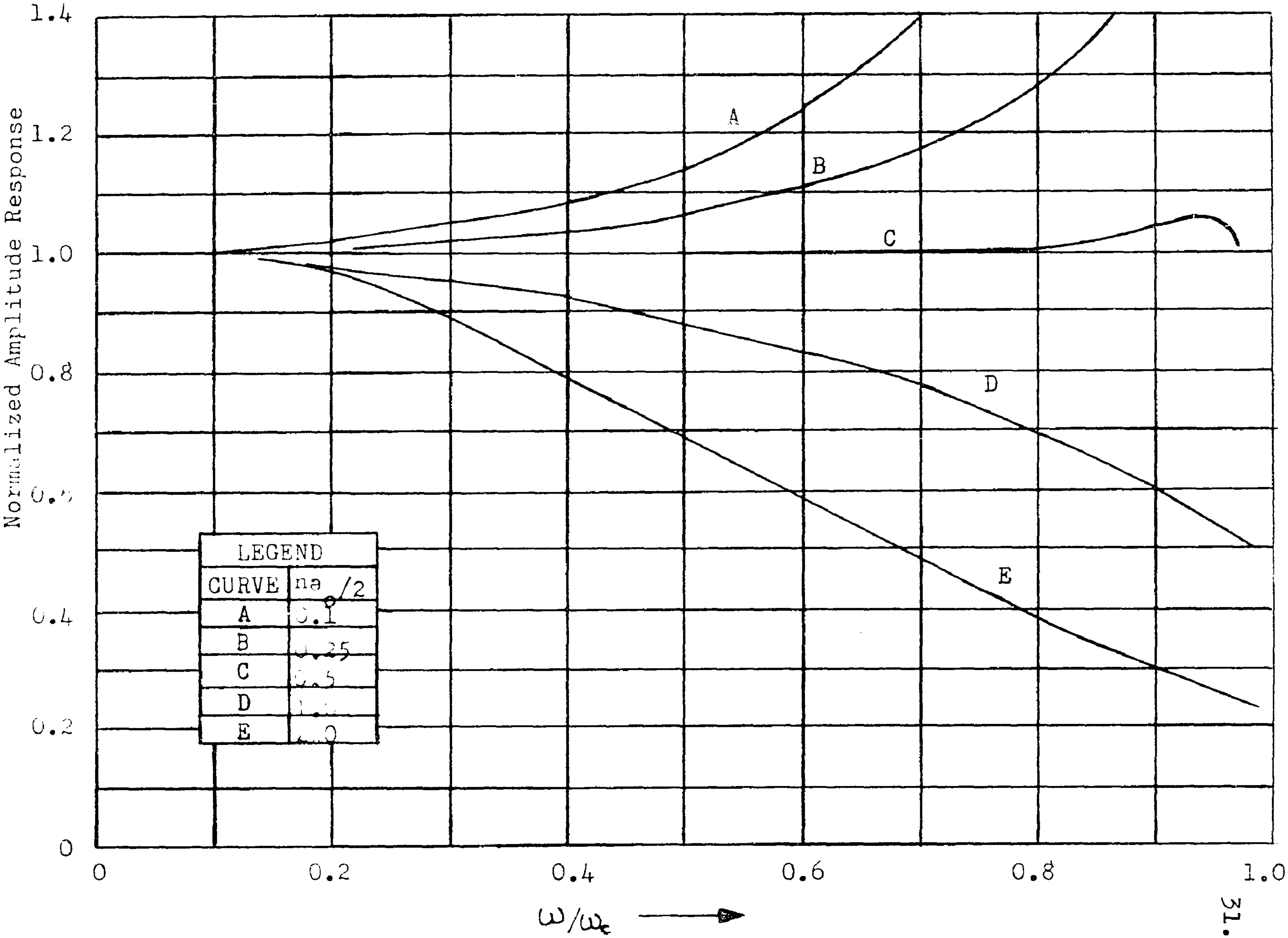

Figure 3.2 Normalized curves for various values of $na_o/2$ plotted as a function of frequency.

3.2 Figure of Merit

The figure of merit of the constant K distributed amplifier, F_K, is defined as the product of the available power gain and the bandwidth cubed, i.e., $F_K = Gf_\Delta^3$. An expression for F_K is derived in the following paragraphs.

The available power gain of the constant K amplifier is

$$G = \frac{P_{out}}{P_{in}} = A_t^2 \ (0) \ \frac{R_{ob}}{R_{oc}}$$

For $\omega = 0$, the base line has no attenuation. Consequently, in view of equation 1 in Appendix A,

$$A_t(0) = nA(0) = n \ G_m \ R_{oc}/2$$

So

$$G = \frac{n^2 \ R_{oc} \ R_{ob}}{4 \ R_m^2} \tag{3.4}$$

Let $na_o/2 = P$ and from Figure 3.2 pick the desired curve, A, B, C, D, or E which fixes P. From the graph also determine b , the value of (ω/ω_c) where the chosen curve is .707. Then

$$P = na_o/2 = (n/2) \ (\omega_c/\omega_o) = (n/2) \ \left(\frac{\omega_\Delta}{b\omega_o} \right) \text{ or } \omega_o = \frac{n \ \omega_\Delta}{2bP}$$

From Figure 2.7, $C = (1/r'_{b\ b}\ \omega_o) = F/R_m\ \omega_t$; solving for R_m yields

$$R_m = \frac{Fr'_{b\ b}\ \omega_o}{\omega_t}$$

Substituting into equation 3.4 results in

$$G = R_{oc} R_{ob} \left(\frac{b\ P\ \omega_t}{\omega_\Delta\ r'_{b\ b}\ F} \right)^2$$

Now $\dfrac{R_{ob}}{r'_{b\ b}} = \dfrac{2}{a_o} = n/P$ and $R_{oc} = (2/\omega_c C_o) = (2b/\omega_\Delta C_O)$; so

$$G = \frac{2Pb^3 n}{F^2\ \omega_\Delta^3} \left[\frac{\omega_t^2}{r'_{b\ b}\ C_o} \right]$$

If the definition $F_m = \dfrac{f^2{}_t}{r'_{b\ b}\ C_c}$ is made, then the desired result is $F_K = \dfrac{Pb^3\ n\ F_m}{\pi\ F^2}$

For curve C of Figure 3.2, $P = \frac{1}{2}$ and $b = 1$. A good approximation is generally $F = 2$. Under these conditions

$$F_K = \frac{n}{8\pi} \quad F_m$$

F_m is a figure of merit of the transistor which is valuable for comparing different transistors for use in this distributed amplifier configuration.

3.3 Design Procedure and Numerical Example

The theory underlying the use of transistors in constant K distributed amplifiers was developed in the preceding sections. The results are presented in a detailed design procedure and numerical example in the present section.

Assume given:

f_Δ = desired overall 3 db bandwidth = 250 mc

G = desired power gain = 10

P = $na_o/2$ = parameter determined from Figure 3.2 to give the desired frequency response = 1.0

b = f_Δ/f_c = 0.8 = value of (ω/ω_c) where chosen curve D is equal to 0.707

$r'_{b\,b}$ = 50 ohms

C_c = 0.5 $\mu\mu$f

f_t = 250 mc

r_m = $1/g_m$ = 5.4 ohms

C_w = 1.0 $\mu\mu$f

The transistor parameters are typical of the 2N502.

(a) Determine the transistor figure of merit F_m.

$$F_m = \frac{f_t^2}{r'_{b\,b}\,C_o} = 833 \times 10^{24}$$

where C_o = 1.5 $\mu\mu$f has been approximated by realizing that C_o is equal to the sum of C_w and something less than twice C_c. This will be checked later.

(b) Determine the required number of transistors n from the relation

$$n = \frac{\pi F^2 G f_\Delta^3}{P b^3 F_m}$$

From experience the Miller factor is chosen $F = 2.0$. This will also be checked later.

$$n = 4.6$$

$= 5$ when rounded off to the next highest integer.

(c) Compute $f_o = (n/2bP)f_\Delta = 782$ mc.

(d) Compute $R_m = r'_{b\,b}F\,(f_o/f_t) = 312$ ohms

(e) $R_e = R_m - r_m = 306$ ohms

$C_e = 1/\omega_t R_e = 2.12\ \mu\mu f$

(f) Compute $R_{oc} = \dfrac{2}{\omega_c\,C_o} = \dfrac{2b}{\omega_\Delta\,C_o} = 680$ ohms

$R_{ob} = r'_{b\,b}\,(n/P) = 250$ ohms

(g) $L_c = \dfrac{2R_{oc}}{\omega_c} = \dfrac{2bR_{oc}}{\omega_\Delta} = 0.70\ \mu h$

$L_b = L_c(R_{oc}/R_{ob}) = 0.257\ \mu h$

The constant K transmission lines should be terminated in the standard m-derived half-sections.[5]

(h) Calculate $C_o = C_c\,(1 + G_m R_{ob})$ plus wiring capacity and $F = 1 + \omega_t C_c (R_m + R_1)$ and compare it with the value assumed in (a). If there is a significant difference, the design procedure should be repeated using the new value.

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[5] Reference Data for Radio Engineers, Fourth Ed., International Telephone and Telgraph Co., 1956, pp. 166.

The determination of R_1 is difficult, however, since the load impedance seen by each transistor in a distributed amplifier is a function of the particular section in question, the number of sections, the line attenuation and frequency.[6] Hence, an average value must be selected. Experimentally, it has been found that usable results are obtained if the selection $R_1 = R_{oc}$ is made. Better results could probably be obtained if some attempt were made to take the position of the transistor on the line into account, however, the complexity of the problem increases rapidly. Hence,

$$F = 1 + \omega_t C_c (R_m + R_{oc}) = 1.8$$

and $C_o = C_c(1 + G\, R_b) + C_w = 1.7\ \mu\mu f$

which are in fair agreement with F = 2.0 and C = 1.5 $\mu\mu$f which were assumed in part (a) and (b).

---

[6] Rogers, P. H., "Large Signal Analysis of Distributed Amplifiers," Tech. Report No. 52, Electronics Defense Group, University of Michigan, July, 1955, pp. 58-63

3.4 Experimental Results

The 5 section constant K distributed amplifier designed in section 3.3 was constructed; the schematic diagram appears in Figure 3.3. Experimentally, it was found to have a 3 db bandwidth of 235 mc and a power gain of 10.7 db. The saturated power output was 61 milliwatts. The frequency response is shown in Figure 3.4.

It was found necessary to maintain a fairly constant collector supply voltage. The Miller capacity was quite large, and, since it varies with the square root of the collector to base voltage, the input transmission line was sensitive to changes in the supply voltage.

Variations in the Miller capacity from section to section were corrected by slight adjustment of the emitter capacitances C_e.

Temperature and transistor variations did not produce severe effects because of the emitter feedback.

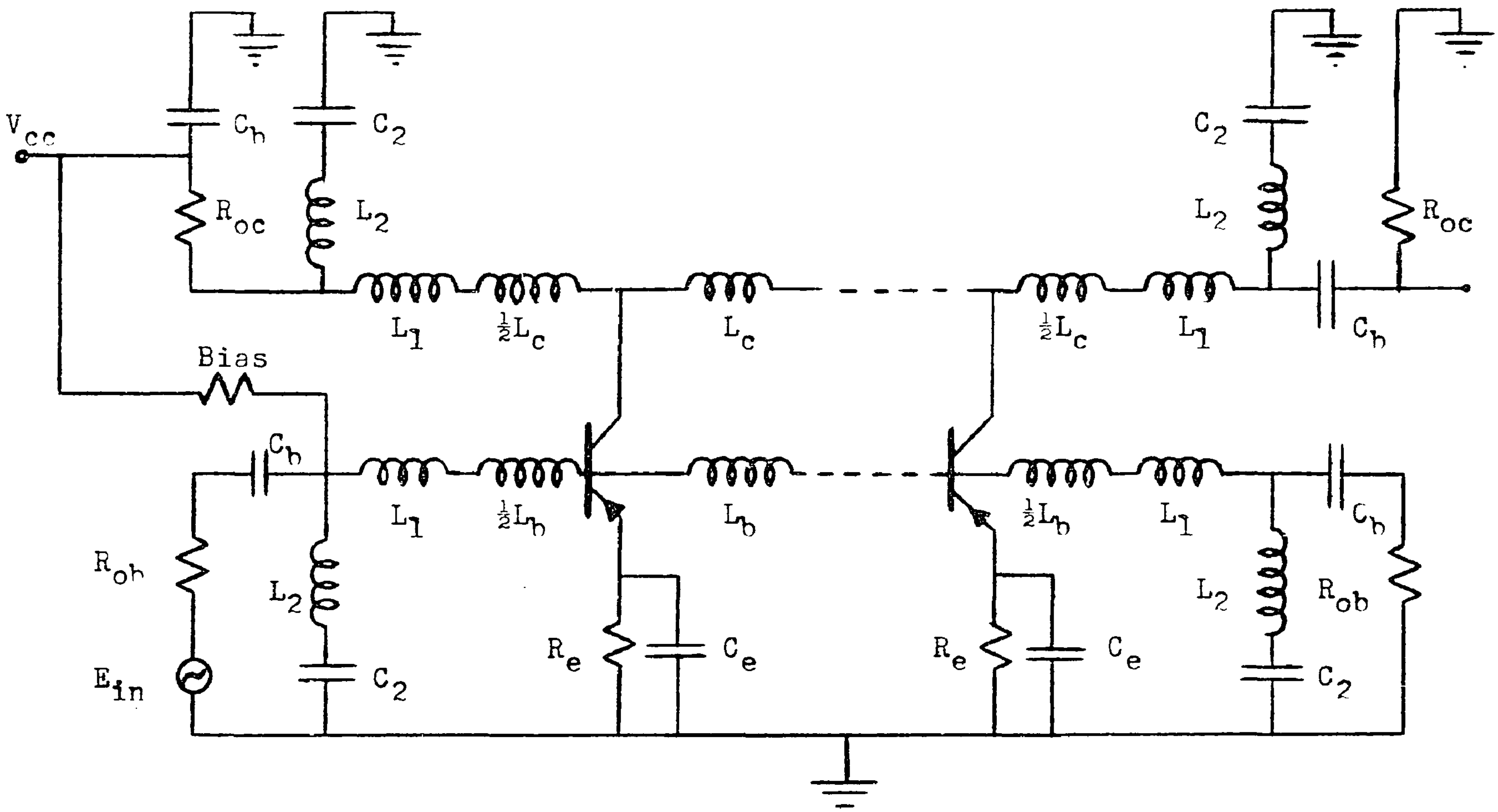

Figure 3.3 Schematic of constant K amplifier.

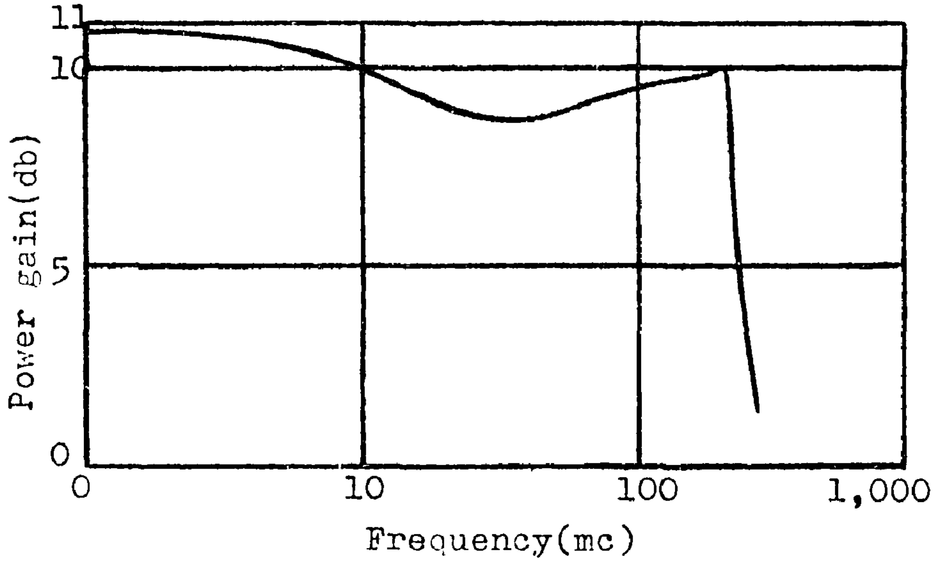

Figure 3.4 Experimental frequency response of 5 section constant K distributed amplifier designed in section 3.3.

Chapter 4

CONSTANT RESISTANCE AMPLIFIERS

4.1 Introduction

A wide-band transistor amplifying section suitable for distributed amplifier application was developed in Chapter 2. The input circuit contained resistances which were not desirable elements of a supposedly lossless transmission line. It was shown in Chapter 3 that the resulting attenuation could be partially compensated for by the rising gain function of the amplifying section. However, it would seem that better results might be obtainable if the existence of the resistances were recognized, and a transmission line developed which contained them as an integral part of the line. The techniques of modern network synthesis have been used to develop interstage and filter networks which, for a given complexity, provide a smaller absolute deviation from the ideal filter characteristic than the classical constant K and m-derived networks. Hence, there seems to be no reason for considering the constant K amplifier developed in Chapter 3 as ultimate in any sense of the word.

4.2 General Constant Resistance Amplifier

The amplifying section developed in Chapter 2 is shown in Figure 4.1. The input circuit consists of a series resistance and capacitance while the output circuit consists of a capacitance in parallel with a current generator. It is desired to synthesize transmission lines for a distributed amplifier which contains these elements.

In order to reduce the complexity of the problem, the following assumptions are made:

(1) The transmission lines are to be composed of ladder networks.

(2) The phase shift per section of the base line must equal that of the collector line for a particular amplifying section.

(3) The driving point impedance of the base line looking to the right from each amplifying section must be independent of the number of sections.

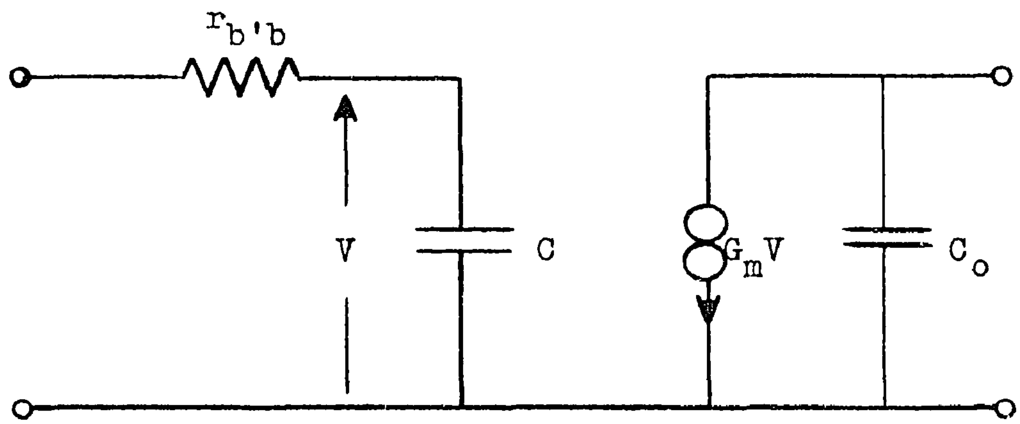

Figure 4.1 Wide-band transistor amplifying section developed in Chapter 2. The resistance B_oR_m, in parallel with C, shown in Figure 2.7, has been neglected because it is very large compared to the impedance level of any wide-band interstage.

(4) The driving point impedance of the collector line looking in both directions from each amplifying section must be independent of the number of sections.

The problem at this point is reduced to determining a satisfactory iterative line impedance. Unfortunately, virtually nothing has appeared in the literature on artificial transmission lines containing resistances as building block elements. A successful approach, however, is developed in the following sections by choosing the iterative line impedance equal to a constant resistance.

4.2.1 Base Line

The input circuit of a transistor amplifying section may be normalized in frequency and impedance levels so that $r'_{b\,b} = 1$ ohm and $\omega_o = 1$ radian/second, as shown in Figure 4.2. If the resulting RC combination is used to terminate a three terminal lossless network, the complete network becomes a lossless network terminated in a one ohm resistance. If the circuit is limited to being minimum susceptive, it has a complement (the input impedance of the two circuits connected in parallel is a constant resistance).

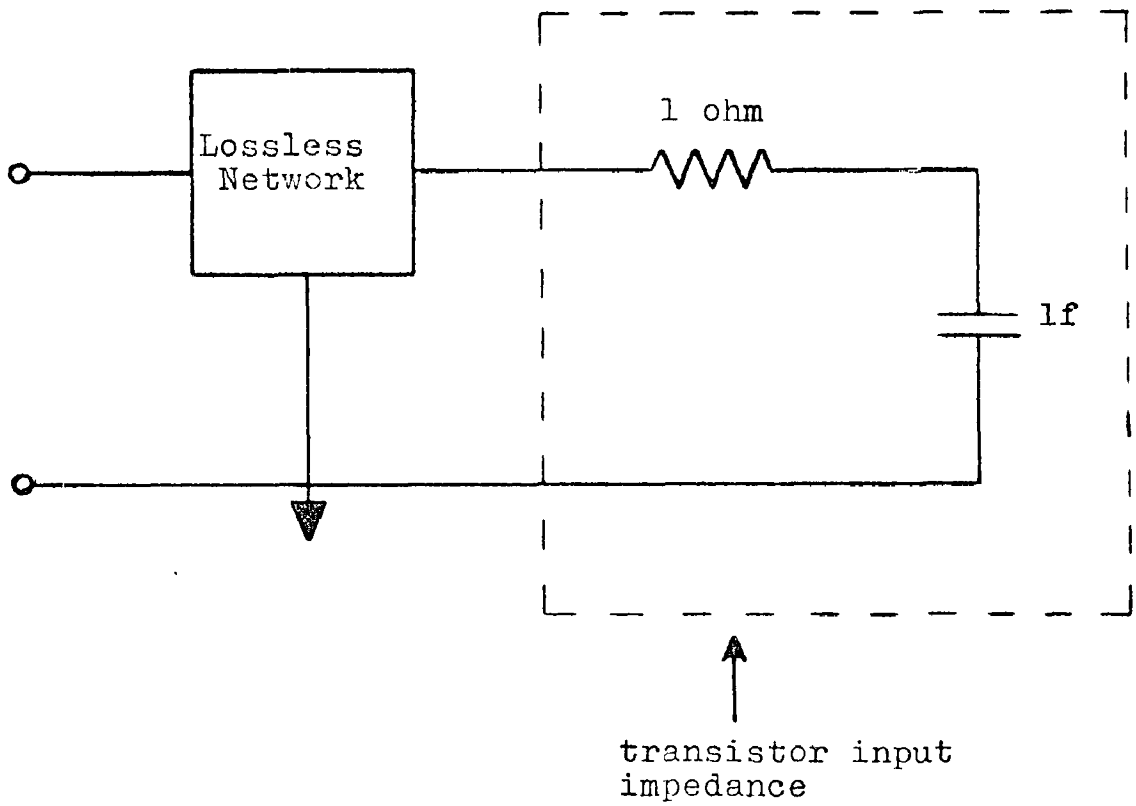

Figure 4.2 A lossless three terminal network terminated in the input impedance of the composite transistor depicted in Figure 4.1. The impedance and frequency levels have been normalized to unity.

The second circuit may also be synthesized as a lossless network terminated in a resistance, as shown in Figure 4.3. If $R_1 = R_2$, a base line may be generated by using the constant resistance input of section number 1 to terminate section number 2. The constant resistance input of section number 2 can then be used to terminate section number 3 and so on. The transfer function per line section would be equal to the transfer admittance of the network N_c to within a multiplicative constant. The conditions under which $R_1 = R_2$ are determined in the following paragraphs.[1]

Assume that the complementary network N_c is a low-pass prototype. If band-passing becomes desirable, this may be done later. Let $Y(s)$ represent the input admittance of N, and $Y_c(s)$ represent the input admittance of N_c. If $Y_c(s)$ is a lossless ladder terminated in a resistance, it must have either a pole or zero at infinity. Since it must be minimum susceptive, it must have a zero there. Hence, if

$$Y_c(s) = \frac{a_o + a_1 s + \ldots\ldots a_{k-1} s^{k-1}}{1 + b_1 s + \ldots\ldots a_k s^k}$$

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[1]Balabanian, N., <u>Network Synthesis</u>, Prentice Hall, Inc., New York, New York, 1958, pp. 244-246.

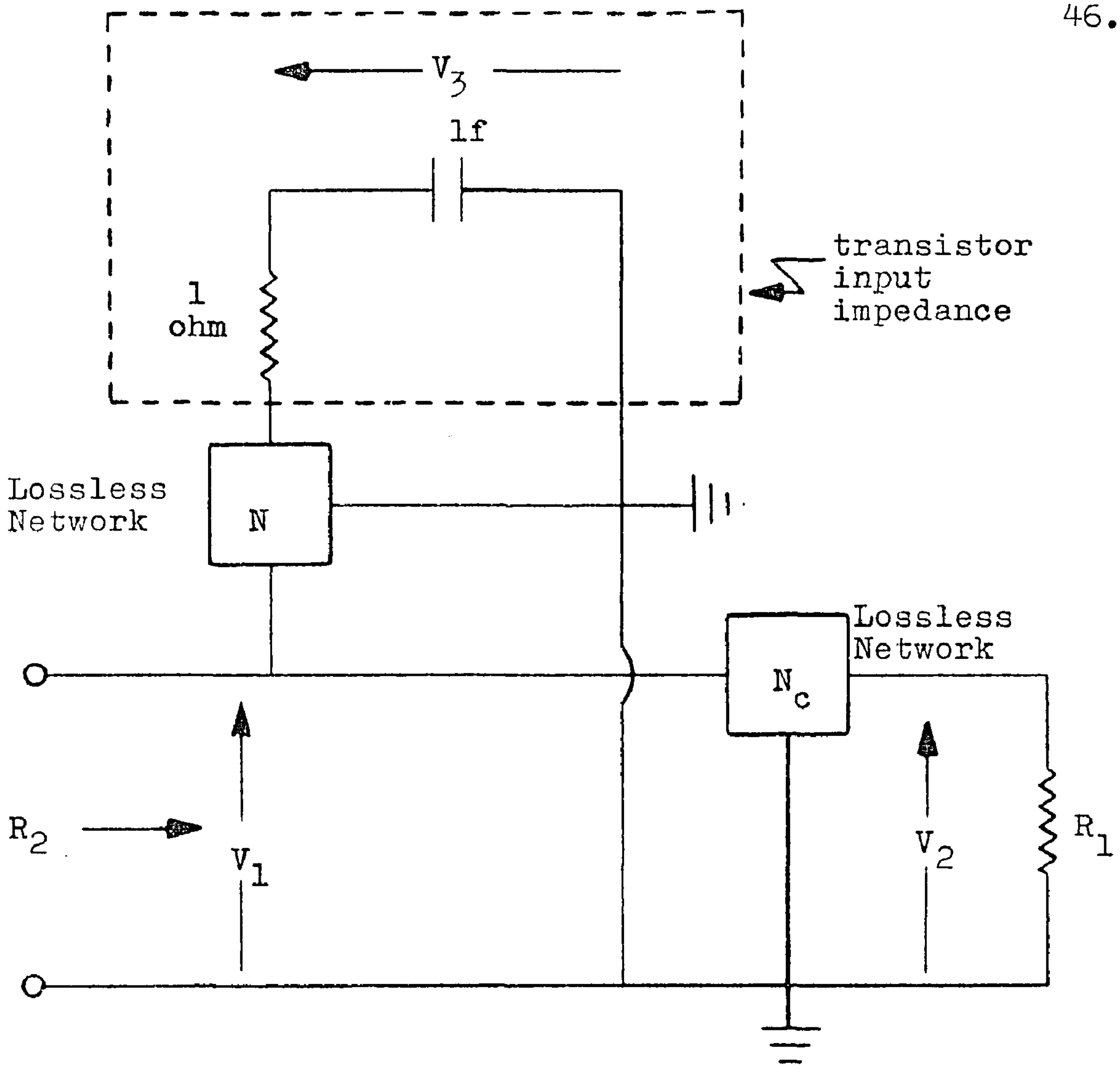


Figure 4.3 The network N, representing the transistor input impedance and lossless interstage, has been complemented by the network N_c.

and, if G_2 is the maximum value of the real part of $Y_c(j\omega)$, then

$$Y(s) = G_2 - Y_c(s)$$

$$= \frac{(G_2-a_o)+(G_2b_1-a_1)s+\ldots(G_2b_{k-1}-a_{k-1})s^{k-1}+G_2b_ks^k}{1+b_1s+\ldots\ldots b_ks^k}$$

is a positive real function.

In view of the desired low-pass characteristic of the section of transmission line, the attenuation at the origin must be zero. This requires that N be a high-pass filter with a transmission zero at the origin and consequently, $G_2 = a_o$. The constant, a_o, is the zero frequency value of Y_c, and, since the imaginary part is zero at s = 0, it is also the zero frequency value of the real part of $Y_c(j\omega)$. Remembering that G_2 is the maximum value of the real part of $Y_c(j\omega)$, it follows that the maximum value of $\left|Y_{12c}(j\omega)\right|^2$ and the maximum value of the real part of $Y_c(j\omega)$ both occur at the origin.

The infinite frequency value of Y(s) can be seen from above to be G_2. However, from the circuit of Figure 4.3, it can be seen that if $Y_{12}(s)$ is to have its maximum gain constant, the infinite frequency value of the admittance of the high-pass network N must be one mho. Likewise, the zero frequency value of the admittance of the low-pass network must be G_1. Hence,

$$G_2 = G_1 = a_o = 1$$

and consequently, the constant resistance input of one line section can be used to terminate another section for the generation of a base line.

The relation between the voltage transfer function per section of base line and the voltage transfer function from the line to the internal base of the transistor follows directly. $\left|Y_{12}(j\omega)\right|$ represents the transfer admittance of the network N. Thus, the transfer function from the line to the transistor is $\left|T(j\omega)\right| = 1/\omega \left|Y_{12}(j\omega)\right|$, which may be expressed in terms of the transfer function per line section, $Y_{12c}(j\omega)$:

$$\left|T(j\omega)\right|^2 = 1/\omega^2 \left|Y_{12}(j\omega)\right|^2 = 1/\omega^2 \left[1 - \left|Y_{12c}(j\omega)\right|^2\right]$$

It is important to observe that once the line section transfer function has been chosen, the transfer function to the transistor is precisely determined by the above equation.

4.2.2 Collector Line

If maximum gain is to be realized, the phase shift between two amplifying sections on the output line must be equal to that between the corresponding two sections on the input line. This implies that for an exact solution the transfer functions must also be equal. Hence, all restrictions on the base line affect the generation of the collector line.

The following paragraphs determine further limitations placed upon the transmission lines by the collector line requirements mentioned in section 4.2.

Minimum susceptive and minimum reactive networks may both be complemented to generate transmission lines. Minimum susceptive networks are connected in parallel, while minimum reactive networks are connected in series. Either technique, however, will yield satisfactory transmission lines from a theoretical point of view.

4.2.2.1 Collector Line A

The first collector line is generated by paralleling minimum susceptive networks, as shown in Figure 4.4. The following assumptions are made:

(1) The networks N_c are lossless.

(2) $Y + Y_c = 1$ (N and N_c form complementary networks when each is terminated in a one ohm resistance).

The transfer and driving point admittance of N_c are fixed by the corresponding quantities on the base line, which in turn fix Y. Hence, N may be synthesized exactly as it is in the base line. The transfer impedance from the current generator of the transistor to the line follows directly from Thevenin's theorem.

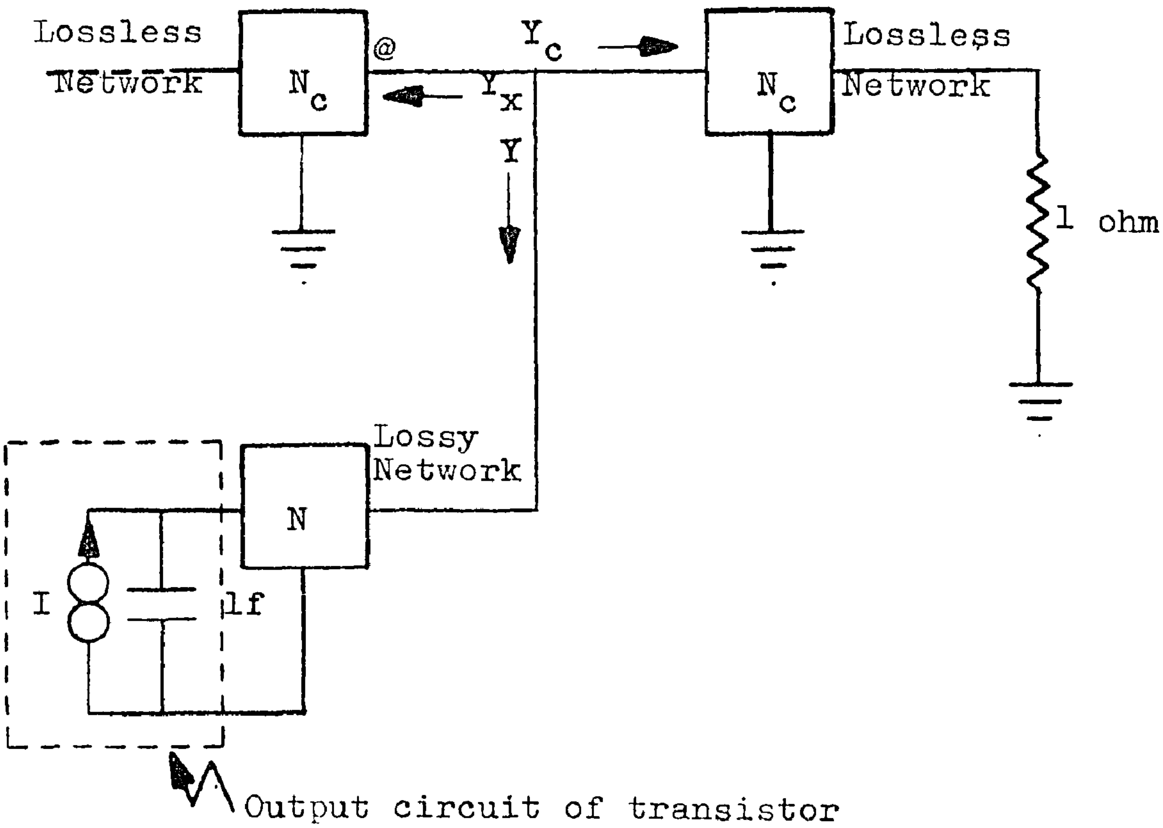


Figure 4.4 Proposed collector line A.

The circuit in Figure 4.4 is broken at a and redrawn in Figure 4.5. The Thevenin equivalent generator impedance, as seen from terminals 2-2', is a resistance of one ohm. The Thevenin equivalent generator is $V_2 = Z_{12}I$. However, because of reciprocity and the one ohm driving point impedance seen at terminals 2-2', $Z_{12} = Z_{21} = V_1/V_2 = T(j\omega)$. Observe that $T(j\omega)$ was defined in section 4.2.1 as the voltage transfer function from the base line to the transistor.

The Thevenin equivalent, with the reverse transmission line reconnected, is shown in Figure 4.6. It is obvious that the transfer impedance from the transistor current generator to the transmission line is

$$Z_{12t} = \frac{I}{1 + Y_x}$$

In section 4.2 it was assumed that the driving point impedance of the collector line seen in both directions from each amplifying section was independent of the number of sections. This requires that

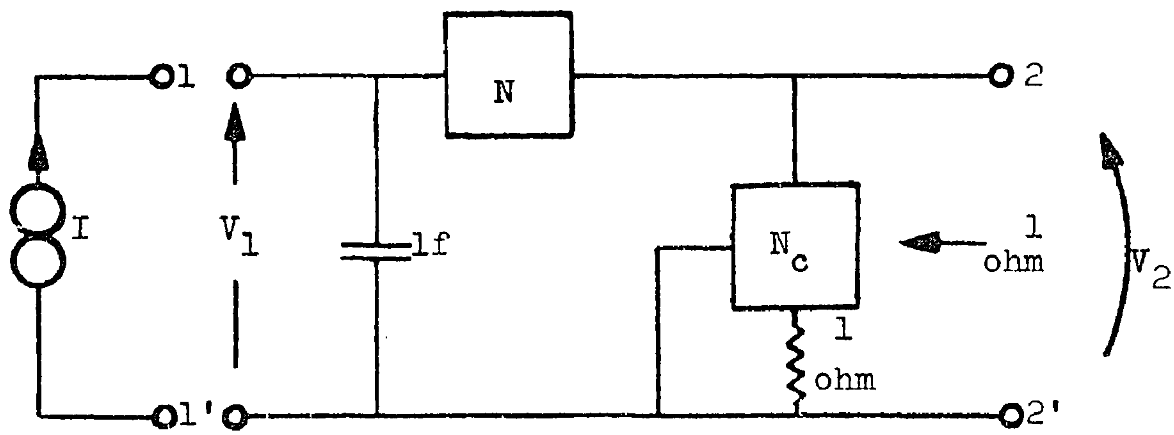


Figure 4.5 Circuit of Figure 4.4 broken at @ and redrawn. Terminals 2-2' correspond to point @.

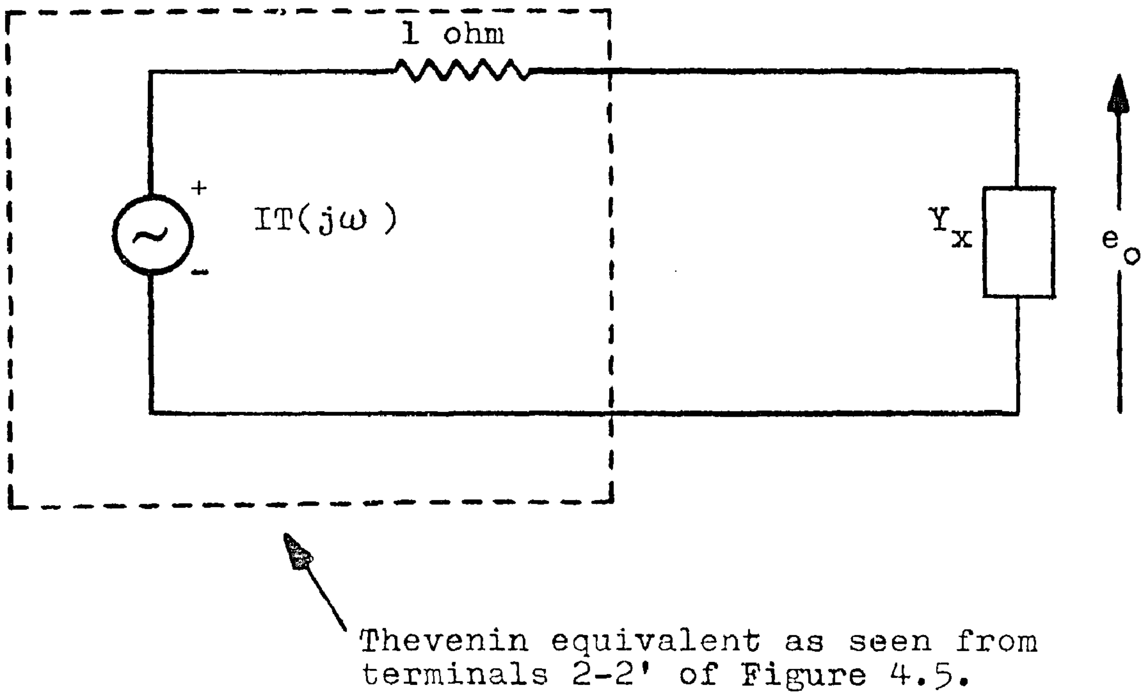


Figure 4.6 Circuit used for the determination of the transfer impedance from the transistor current generator to the collector line.

the Y_x's for all amplifying sections be equal. It is shown below that they are equal if N_c is a symmetrical network. Consider the last few line sections on the reverse end as shown in Figure 4.7. Let N_{c1} be terminated on the left by a one ohm resistance. Since $Y_1 + Y_{c2} = 1$, N_{c1} is terminated on the right side by a one ohm resistance also. If N_{c1} is a symmetrical network, it presents the same driving point impedance from either terminal pair since the opposite terminal pair is terminated in a one ohm resistance. Hence,

$$Y_{x1} = Y_{c1} = Y_{c2} \text{ and, so } Y_{x1} + Y_1 = 1 \text{ and}$$

$$Y_{x2} + Y_2 = 1 \text{ and so on.}$$

The result is that all Y_x's are equal to Y_c. The overall gain expression for the amplifier is seen from equation 1 in the appendix to be

$$\left|A_t(j\omega)\right| = n/\omega^2 \quad \frac{\left|Y_{12c}(j\omega)\right|^n \left[1 - \left|Y_{12c}(j\omega)\right|^2\right]}{\left|1 + Y_c(j\omega)\right|}$$

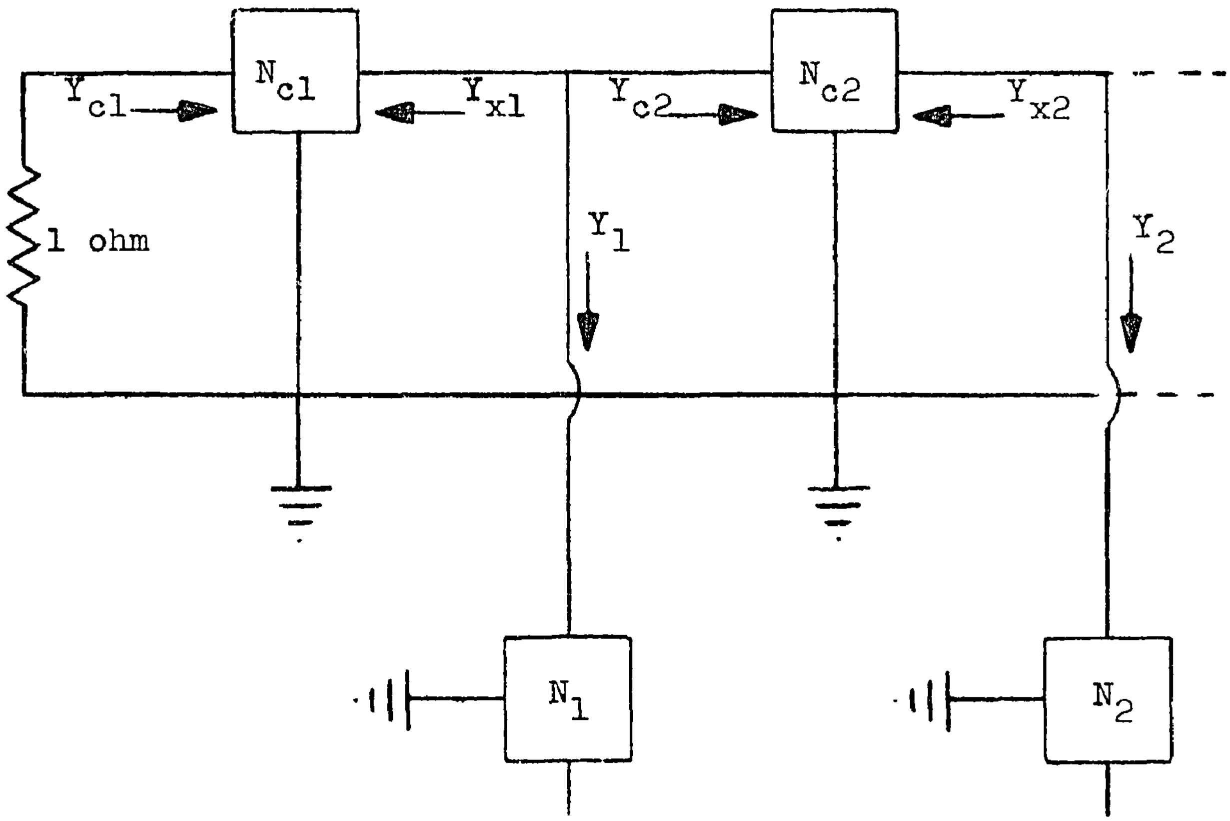


Figure 4.7 Reverse end of the collector line. N_c networks are identical. Subscripts are for designation purposes only. Thus, $Y_{c1} = Y_{c2}$ and $Y_1 = Y_2$.

Notice that $Y_{12}(j\omega)$ is raised to the nth power rather than the n-1 power because of the low-pass section between the last transistor and the terminating resistor of the collector line.

4.2.2.2 Collector Line B

The second collector line is generated by synthesizing the duals of the networks N and N_c in the base line and connecting them in series as shown in Figure 4.8. It follows that Z' and Z_c' are minimum reactive and complementary, and that the transfer impedance of the line section is equal to the transfer admittance of the base line section.

The transmission line is generated by using the driving point impedance of the last section to terminate the preceding section. Since the network N_c' is a minimum reactive low-pass filter, its first element must be a shunt capacitance. This capacitance can represent the shunt capacity of the transistor output section.

The transfer impedance from the transistor current generator to the collector line is easily derived by considering the last few

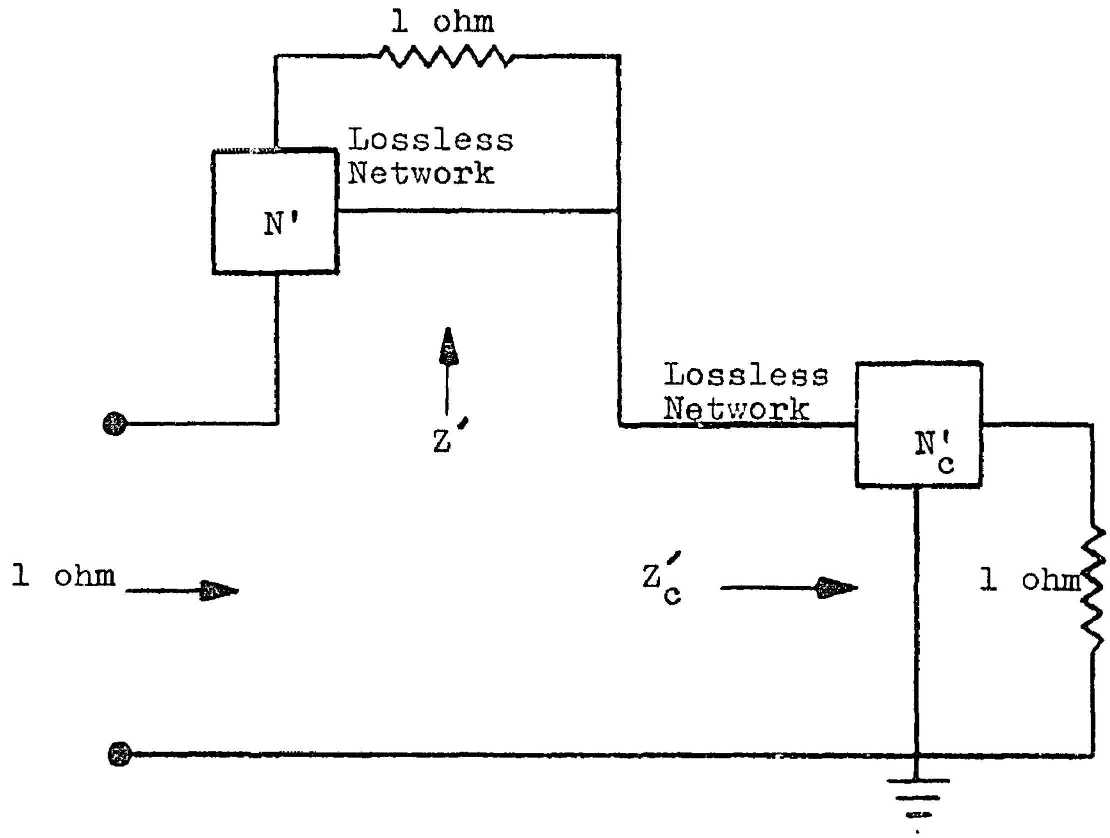


Figure 4.8 The duals of networks N and N_c of the base line connected in series.

sections on the terminating end depicted in Figure 4.9. The transfer impedance is equal to the driving point impedance seen by the current generators. Consider the left hand generator: To the left it sees one ohm, and to the right it sees the impedance Z_c'. Now, consider the second current generator: It sees Z_c' to the right, but what it sees to the left is not obvious at first. Again restrict N_c' to be symmetrical. Hence, Z_c' is seen looking to the left from a. The current generator sees $Z_c' + Z' = 1$ looking to the right. Thus, the driving point impedance seen by the right-hand current generator is likewise one ohm in parallel with Z_c'. Following this line of reasoning it can be shown that the impedance seen by all of the current generators is

$$Z_1 = \frac{Z_c'}{1 + Z_c'}$$

Because of the impedance normalization and the fact that Z_c' of the collector line is Y_c of base line, then

$$Z_1 = \frac{Y_c}{1 + Y_c}$$

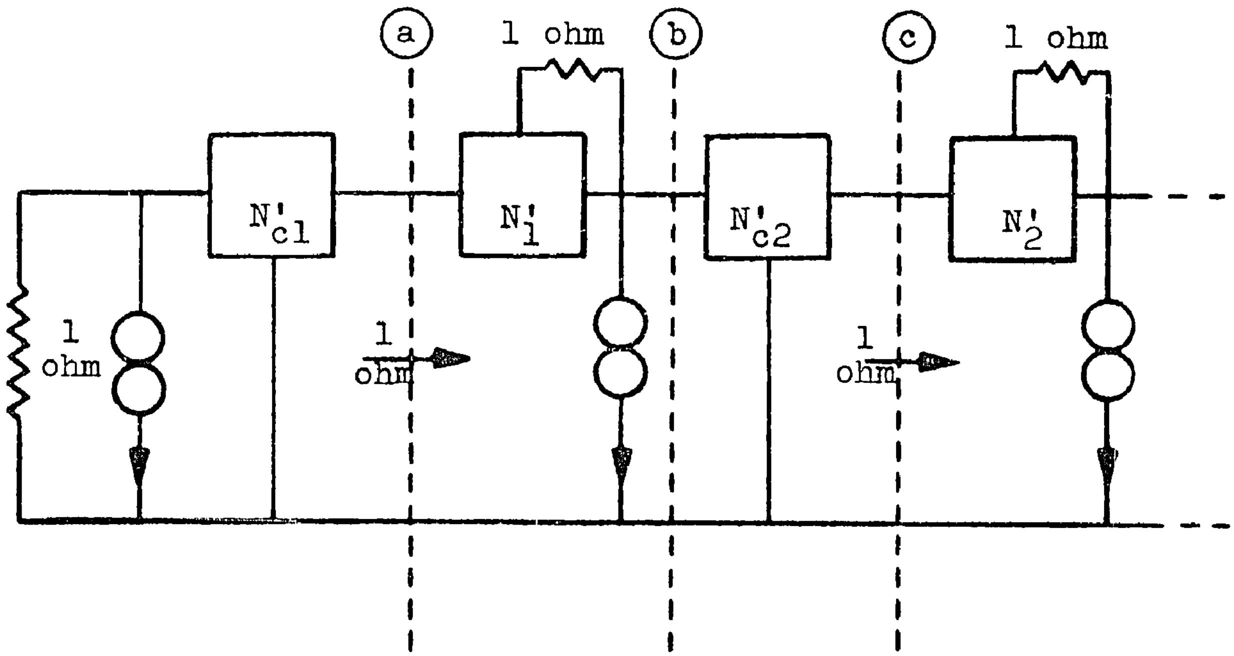


Figure 4.9 The terminating end of the transmission line. All N' networks are identical, as are the N'_c networks.

The overall gain expression is

$$\left|A_t(j\omega)\right| = n/\omega \left|Y_{12c}(j\omega)\right|^{n-1} \sqrt{1 - \left|Y_{12c}(j\omega)\right|^2} \left|\frac{Y_c(j\omega)}{1 + Y_c(j\omega)}\right|$$

4.2.3 Practical Transmission Lines

The previous sections of this chapter discussed limitations placed upon the transmission line section networks because of their constant resistance nature and because of the configuration of a distributed amplifier. The present section discusses other limitations placed upon the networks because of their desired low-pass characteristics. It is shown that, other than the single-pole case, low-pass maximally flat and equal ripple functions are not practical response functions for the transmission lines.

4.2.3.1 Maximally Flat Transmission Lines

The input signal traverses n-1 transmission line sections and one amplifying section before reaching the output. It follows that the attenuation characteristics of the line sections are significantly more important than those of the amplifying sections. The maximally flat function would seem to be a very desirable line transfer function.

Consider a general maximally flat transfer function

$$\left|Y_{12c}(j\omega)\right|^2 = \frac{1 + a_2\omega^2 + \ldots a_{2m}\,\omega^{2m}}{1 + a_2\omega^2 + \ldots a_{2m}\omega^{2m} + b_{2n}\omega^{2n}}$$

where $m < n$.

To be realizable as a LC network terminated in a one ohm resistance, the numerator must contain double order j-axis zeros only. The transfer function from the line to the internal base of the transistor is

$$\left| T(j\omega) \right|^2 = 1/\omega^2 \left| Y_{12}(j\omega) \right|^2 = 1/\omega^2 \left[1 - \left| Y_{12c}(j\omega) \right|^2 \right]$$

$$= 1/\omega^2 \left[\frac{b_{2n}\,\omega^{2n}}{1 + a_2\omega^2 + \ldots a_{2m}\omega^{2m} + b_{2n}\omega^{2n}} \right]$$

Since $T(j\omega)$ is to be a low-pass function, then $n = 1$ and $m = 0$. Consequently,

$$\left| T(j\omega) \right|^2 = \frac{1}{1 + b_2\omega^2}$$

and

$$\left| Y_{12c}(j\omega) \right|^2 = \frac{1}{1 + b_2\omega^2}$$

which are both single-pole Butterworth functions. It is easily shown that the Y and Y_c of the above functions are symmetrical, and hence, suitable for distributed amplifier application.

The maximal flat "concept" did not prove to be very rewarding. The degree of maximal flatness is measured by the number of derivatives of $1 - \left|Y_{12c}(j\omega)\right|^2$ which vanish at the origin. This number, n, is precisely the number of zeros of $Y_{12}(s)$ at the origin. Yet since T(s) is to be a low-pass function, $Y_{12}(s)$ is limited to one zero there. Consequently, the single-pole Butterworth is the best that can be obtained.

4.2.3.2 Equal Ripple Transmission Lines

The equal ripple or Tschebyscheff functions do not concentrate their approximating ability at the origin as does a maximally flat function. Instead they distribute the error evenly or uniformly through the approximation band. As a consequence, their gain-bandwidth products are superior to that of a Butterworth or maximally flat function for a given capacity. Figure 4.10 gives great insight into the T(s) resulting from an equal ripple three pole function.

The high-pass complement $Y_{12}(s)$ has simple zeros at the origin and at ω_3. Since $T(s) = 1/sY_{12}(s)$, it is seen that the zero of $Y_{12}(s)$ at the origin is cancelled by the pole of T(s) at the origin. Hence, the voltage transfer function from the line to the internal base of the transistor T(s) is a band-stop function. The attenuation is infinite at ω_3, but frequencies above and below are transmitted. The pass-band extending from the origin to the vicinity of ω_3 represents the desired pass-band of the amplifying section.

Equal ripple functions greater than three pole functions

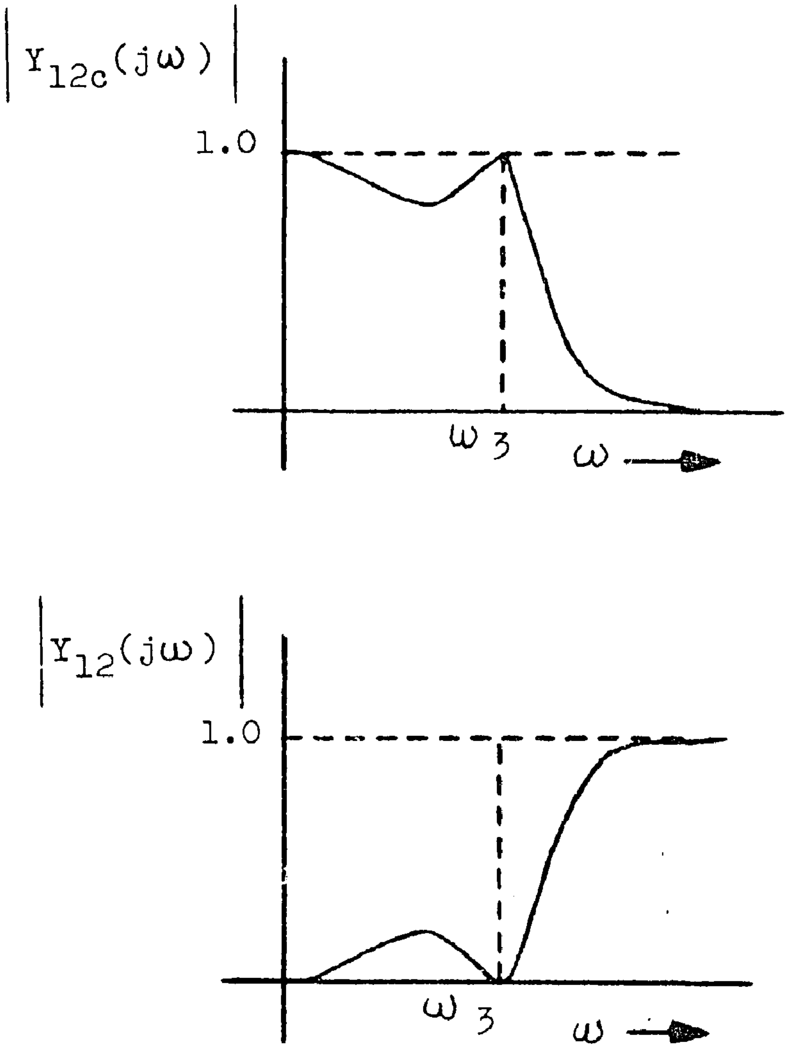


Figure 4.10 Three pole low-pass equal ripple filter and complement.

would be undesirable because each ripple introduces an additional point of infinite attenuation in $T(s)$.

Since Y_{12c} (alternately Z_{12c}) was given as a lossless network terminated in a one ohm resistance, the corresponding minimum susceptive (reactive) driving point admittance (impedance) is unique. If this unique driving point immittance, as determined by Bode's or Gewertz's method, is written

$$H(s) = \frac{m_1(s) + n_1(s)}{m_2(s) + n_2(s)}$$

where

$m_i(s)$ = even polynomial in s and

$n_i(s)$ = odd polynomial in s,

it can be shown that for a low pass network to be symmetrical, it is necessary that $m_1(s) = m_2(s)$.[1,2]

---

[1] This is **Case** A of Darlington's procedure. Case B is not applicable because if the numerator of z_{12}(alternately y_{12}) were odd, it would necessarily have a zero at the origin. This implies that Z_{12} (alternately Y_{12}) would also have a zero at the origin, which is contrary to the desired low-pass characteristics of the transmission line.

[2] Guillemin, E. A., Synthesis of Passive Networks, John Wiley and Sons, Inc., New York, New York, 1957, p. 371.

With the exception of the single-pole function, the equal ripple class of functions does not satisfy this condition, and hence, is not suitable for distributed amplifier application.

4.3 A Practical Constant Resistance Amplifier

The previous sections developed the general theory of the constant resistance amplifier. It was demonstrated that the line sections could be synthesized using single-pole Butterworth functions. The transfer admittance of the base line, on a normalized basis, is

$$Y_{12c}(s) = \frac{1}{s + 1}$$

The unique minimum susceptive driving point admittance which corresponds to this transfer admittance is

$$Y_c(s) = \frac{1}{s + 1}$$

and its complementary network is

$$Y(s) = \frac{s}{s + 1}$$

The generation of the base line is illustrated in Figure 4.11.

If the collector line is synthesized on an admittance basis, it is identical to the base line. The transfer impedance from the transistor current generator to the line is

$$Z_{12t} = \frac{T}{1 + Y_c} = \frac{\frac{1}{s+1}}{1 + \frac{1}{s+1}} = \frac{1}{s+2}$$

The overall voltage gain of the amplifier is then

$$A_t(s) = \frac{nA_r}{2} \left[\frac{1}{(s+1)^{n+1}\ (s/2 + 1)} \right] \qquad (4.1)$$

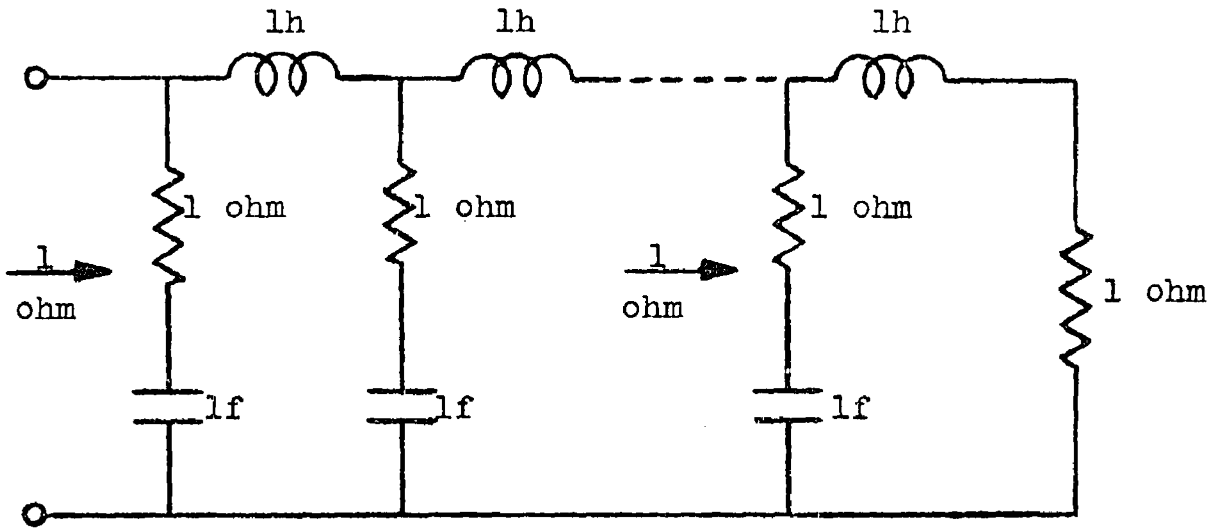
If the collector line is synthesized on an impedance basis the circuit of Figure 4.12 results. The transfer impedance from the transistor current generator to the line is

$$Z_{12t} = \frac{Z_c}{1 + Z_c} = \frac{\frac{1}{s+1}}{1 + \frac{1}{s+1}} = \frac{1}{s+2}$$

The resulting overall voltage gain of the amplifier is

$$A_t(s) = \frac{nA_r}{2} \quad \frac{1}{(s+1)^n\ (s/2 + 1)} \qquad (4.2)$$

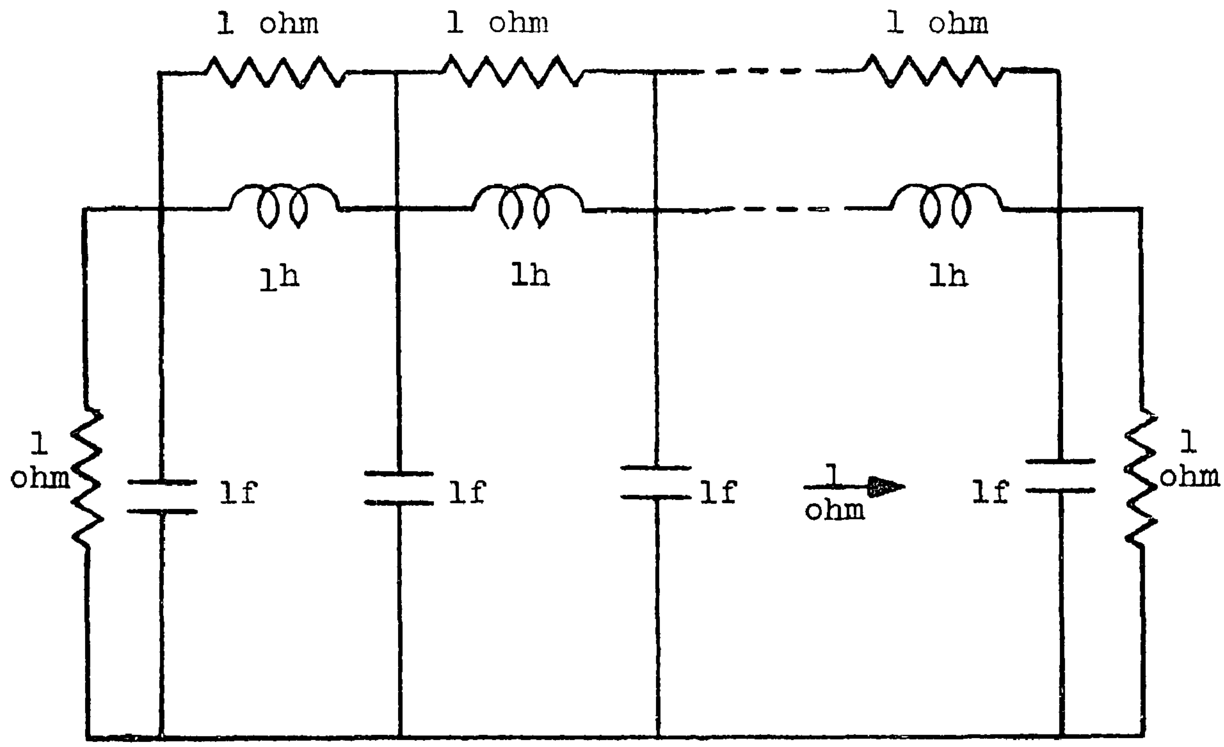
Notice that in equation 4.2 the factor $\frac{1}{s+1}$ is raised to the nth power, while in equation 4.1, it is raised to the n + 1 power.



$$Y_{12}(s) = \frac{1}{s + 1}$$ (voltage transfer function per section)

$$T(s) = \frac{1}{s + 1}$$ (voltage transfer function from line to transistor)

Figure 4.11 Constant resistance base line with a single pole Butterworth response per section. The shunt ladder arms represent the input impedance of the transistor amplifying sections.



$$Z_{12}(s) = \frac{1}{s+1}$$ (voltage transfer function per section)

$$Z_{12t}(s) = \frac{Z_c'}{1 + Z_c'} = \frac{1}{s+2}$$ (transfer impedance from transistor current generator to line)

Figure 4.12 Constant resistance collector line, generated on an impedance basis, with a single pole Butterworth response per section. The shunt capacitances represent the output capacity of the transistor amplifying sections.

Hence, equation 4.2 is the most desirable response and the collector line generated on the impedance basis is the better circuit.

4.3.1 <u>Figure of Merit</u>, F_R

In section 3.2 the figure of merit of the constant K amplifier, F_K, was defined as the product of the available power gain and the bandwidth cubed, i.e., $F_K = Gf_\Delta^3$. The figure of merit of the single-pole Butterworth constant resistance amplifier, F_R, is defined in exactly the same manner. In the following paragraphs an expression for F_R is derived and compared with F_K.

The available power gain of the constant resistance amplifier is

$$G = \frac{P_{out}}{P_{in}} = A_t(0)\,\frac{R_{ob}}{R_{oc}}$$

$$= n^2 A_r^2 \; \frac{r'_{b\,b}}{R_{oc}}$$

Now $A_r = G_m R_{oc}/2$ so

$$G = (n^2/4)\;(G_m^2\, r'_{b\,b}\, R_{oc})$$

Also, $G_m = \dfrac{\omega_t}{\omega_o r'_{b\,b} F}$ and $R_{oc} = \dfrac{1}{C_o\,\omega_o}$.

Consequently the available power gain becomes

$$G = \frac{n^2 \omega_t^2}{4\omega_o^3 r'_b {}_b C_o F^2}$$

If the factor $\frac{1}{s/2 + 1}$ in equation 4.2 is neglected, the overall bandwidth becomes

$$f_\Delta = \frac{f_o}{\sqrt{2^{1/n} - 1}}$$

However, for n > 3 this expression essentially reduces to[3]

$$f_\Delta \cong \frac{f_o}{1.2 \sqrt{n}}$$

Employing the definition for $F_R = Gf_\Delta^3$ yields the desired result:

$$F_R = \frac{\sqrt{n}}{13.8\pi F^2} F_m \qquad (4.1)$$

where

$$F_m = \frac{f_t^2}{r'_b {}_b C_o} \qquad (4.2)$$

Notice that F_R increases with the square root of n. In contrast, the figure of merit for the constant K amplifier, shown in section 3.2 to be

$$F_K = \frac{n}{2\pi F^2} F_m ,$$

varies directly as the number of stages n.

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[3] Martin, T. L. Jr., Electronic Circuits, Prentice-Hall, Inc., New York, New York, 1955, pp. 175-176.

Further, the multiplicative constant is larger for the constant K amplifier. However, the effect of the latter is partially cancelled by the fact that the Miller constant, F, is larger for the constant K amplifier because the collector load impedance increases rather than decreases with frequency.

As mentioned in section 3.3, F is the ratio of the total input capacity to the input capacity with the Miller effect neglected, and varies only slightly from transistor to transistor. Hence, F_m of equation 4.2 may be used as a figure of merit for comparing transistors for use in the constant resistance distributed amplifier as well as for the constant K amplifier.

4.3.2 Design Procedure and Numerical Example

The theory underlying the use of transistors in constant resistance distributed amplifiers was developed in the preceding sections. The results are presented in a detailed design procedure and numerical example in the present section.

Assume given:

$f_\triangle$ = desired overall 3 db bandwidth

= 150 mc

G = desired power gain

= 10

$r'_{b\,b}$ = 50 ohms

$C_c = 0.5\ \mu\mu f$

$f_t = 250$ mc

$r_m = 1/g_m = 5.4$ ohms

The transistor parameters are typical of the 2N502.

(a) Determine the transistor figure of merit F_m.

$$F_m = \frac{f_t^2}{r'_{b\,b}\, C_o}$$

where $C_o = 1.5\ \mu\mu f$ has been approximated by realizing that C_o is equal to the sum of the output interstage wiring capacity and something less than twice C_c. The exact value is calculated later as a check.

(b) Determine the required number of transistors n from the amplifier figure of merit.

$$n = \left[\frac{13.8\pi\ F^2 G f_\triangle^3}{F_m} \right]^2 = 6.9$$

Choose n = 7

From experience the Miller factor was chosen F = 1.2. This will also be checked later.

(c) Calculate $f_o = 1.2 \sqrt{n}\ \ f_\triangle = 476$ mc

(d) Calculate $R_m = \left(\frac{f_o}{f_t} \right) r'_{b\,b} F = 186.9$ ohms

(e) $R_e = R_m - r_m = 181.5$ ohms

$$C_e = \frac{1}{\omega_t R_e} = 3.52\ \mu\mu f$$

(f) $R_{oc} = \frac{1}{\omega_o C_o} = 223$ ohms

$$R_{ob} = r'_{b\,b} = 50 \text{ ohms}$$

(g) $L_c = \frac{R_{oc}}{\omega_o} = 0.0745\ \mu h$

$$L_g = \frac{r'_{b\,b}}{\omega_o} = L_c \frac{r'_{b\,b}}{R_{oc}} = 0.0167\ \mu h$$

(h) Calculate $F = 1 + \omega_t C_c (R_m + R_1)$ and $C_o = C_c(1 + G_m R_b) + C_w$, and compare with the values assumed in (a) and (b).

If there is a significant difference, the design procedure should be repeated using the new values as mentioned in section 3.3. The determination of R_1 is difficult since the load impedance seen by each transistor in a distributed amplifier is a function of the particular section in question, the number of sections, the line attenuation, and frequency. Hence, an average value is selected. Experimentally, it has been found that usable results are obtained if the selection $R_1 = R_{oc}/2$ is made. Hence,

$$F = 1 + \omega_t C_c \ (R_m + R_{oc}/2) = 1.234$$

and

$$C_o = C_c(1 + G_m r'_b \ _b) + C_w = 1.56 \ \mu\mu f$$

which are in fair agreement with $F = 1.2$ and $C_o = 1.5 \ \mu\mu f$ which were assumed in (a) and (b).

4.3.3 Experimental Results

An 8-section distributed amplifier utilizing constant resistance ladder networks for transmission lines was constructed and tested. The active elements were Philco 2N502 transistors. The schematic diagram appears in Figure 4.13. The design values were:

$$G = 10 \text{ db}$$

$$f_\Delta = 150 \text{ mc}$$

The measured values were:

$$G = 9.85 \text{ db}$$

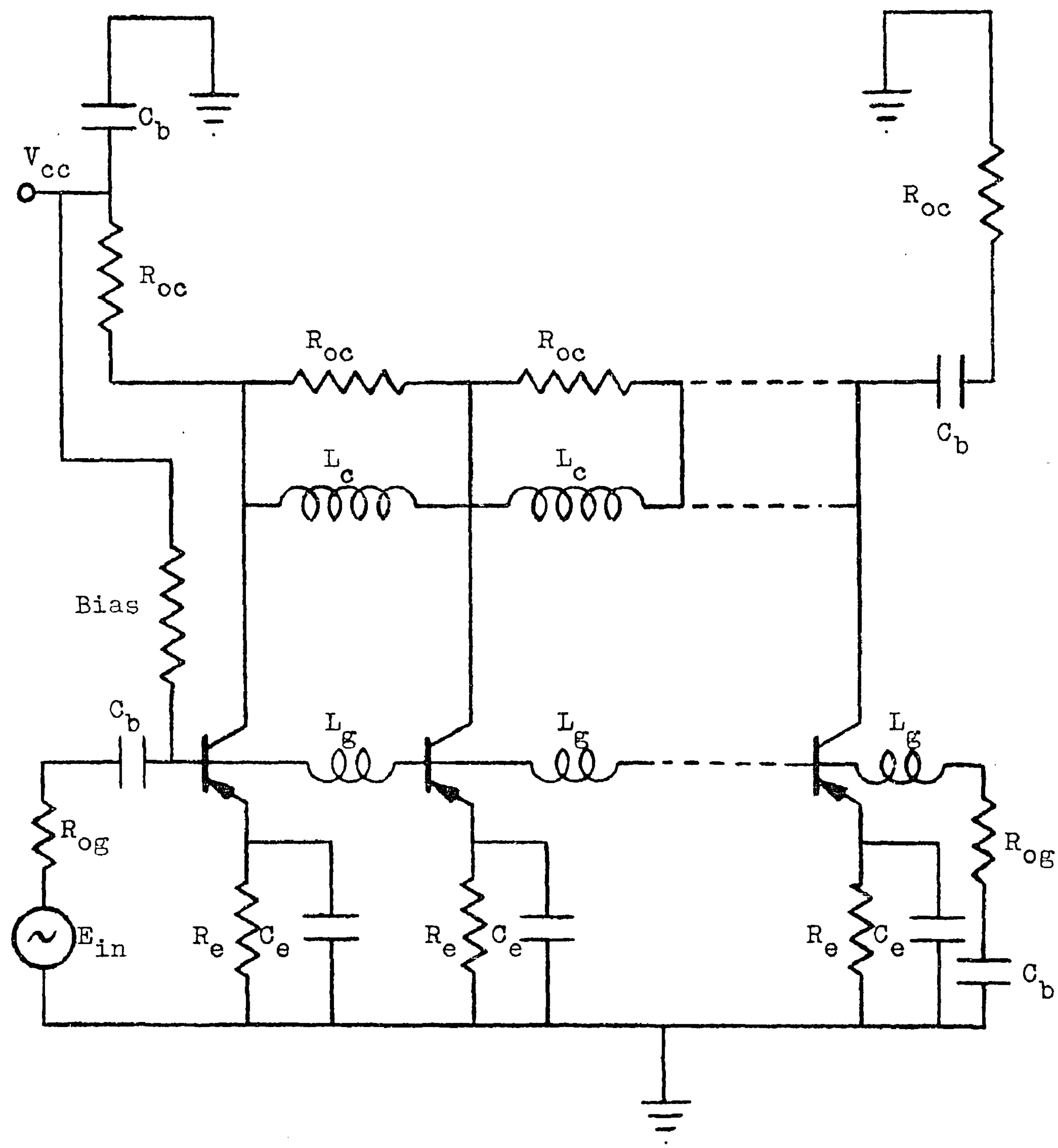

Figure 4.13 Schematic of constant resistance amplifier.

$$f_{\Delta} = 155 \text{ mc}$$

The saturated power output was 100 milliwatts. The frequency response is shown in Figure 4.14.

The amplifier was found to be fairly insensitive to voltage, temperature, and transistor variations; in this respect it was superior to the constant K amplifier.

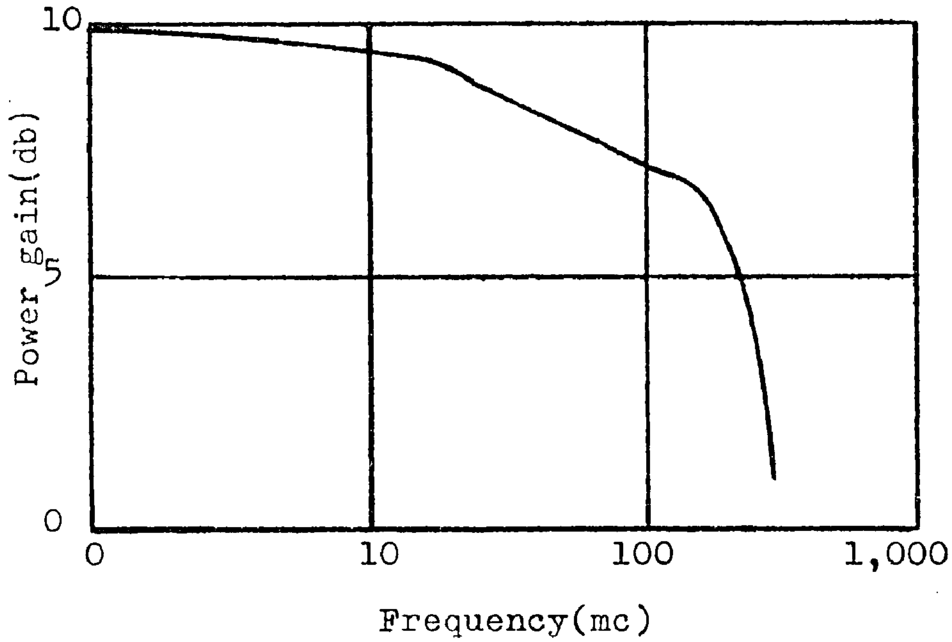

Figure 4.14. Experimental power gain versus frequency. Frequency response of constant resistance distributed amplifier designed for 150 mc bandwidth and 10 db gain. The amplifier uses eight 2N502 transistors with 50 and 270 ohms input and output impedance levels. The bandwidth obtained experimentally was 155 mc.

Chapter 5

CONCLUSIONS

5.1 Results of Research

1. A simple series feedback technique has been developed to make a transistor behave electrically like a vacuum tube when used in the common emitter configuration for wide-band application. The circuit allows voltage gain to be directly exchanged for bandwidth over a very large range.

2. A design procedure has been presented which allows wide-band constant K distributed amplifiers to be successfully designed using transistors, as discussed in the above paragraph, in place of vacuum tubes.

3. A new type distributed amplifier was developed and tested which used constant resistance ladder networks for transmission lines. The resistive components of the input impedance of the amplifying sections were used as actual building block elements of the transmission line.

5.2 Conclusions

1. For wide-band applications, emitter feedback can be used to make a transistor behave electrically like a vacuum tube. Hence, it is possible to obtain transistor circuits, in many

instances, for given vacuum tube circuits simply by replacing the vacuum tubes by their transistor counterparts. This dissertation applied the principle to distributed amplifiers; however, this is but one out of many possible applications.

2. Wide-band transistor constant K distributed amplifiers are easily designed using the technique mentioned in the paragraph above. Electrically they compete very well with their vacuum tube counterparts. They are relatively insensitive to temperature and transistor variation because of the emitter feedback. However, the frequency response varies noticeably if the collector supply voltage changes over a volt or two. The amplifiers have the small size, the small power requirements, and the excellent reliability characteristics of transistor circuits in general.

3. The constant resistance amplifier using single-pole Butterworth line sections differs electrically in several aspects from the constant K amplifier.

(a) The figure of merit is inferior, increasing only with the square root of n.

Constant Resistance Amplifier:

$$F_R = \frac{\sqrt{n}\, F_m}{13.8\pi\, F^2}$$

Constant K Amplifier:

$$F_K = \frac{nF_m}{2\pi\, F^2}$$

(b) The impedance levels of the lines are considerably lower, being much closer to cable impedance levels.

(c) The pulse response should be excellent because all poles of the gain function are on the "s" plane negative real axis.

(d) The amplifier is much less sensitive to voltage, temperature, and transistor variations. The Miller capacity can almost be neglected and the constant resistance networks can be terminated exactly.

All of these factors must be examined in view of the problem at hand before either circuit can be selected as optimum.

5.3 Suggestions for Further Research

During the course of this investigation several problems were encountered which seem worthy of further study.

1. The most important factor limiting the bandwidth of the constant K distributed amplifier discussed in this dissertation was Miller feedback, caused by the non-unilateral nature of the transistor. Considerable improvement would result if this effect could be eliminated. Amplifying sections using two transistors offer some promise. In particular, the common emitter-common base and common collector-common base configurations provide excellent unilateralization as well as

high input and output impedances. These circuits as well as others should be investigated.

2. A new class of filters should be developed for use in the constant resistance amplifier in order to obtain a greater and improved variety of response functions. The low-pass prototype should be minimum susceptive, symmetrical and synthesizable as a lossless network terminated in a one ohm resistance. In addition, the complementary high-pass function should have only one zero at the origin. A number of restrictions may be stated immediately:[1]

a. $Y_t(s)$ has neither a pole nor a zero at the origin, and has a zero at infinity. These conditions follow directly from the low-pass minimum susceptive restrictions.

b. The degree of the denominator must be odd. Otherwise, z_{11} and y_{22} have a private pole at infinity, which is contrary to the desired symmetry.

c. The z parameters have a noncompact pole at infinity; y parameters have a zero at infinity.

d. Coefficients of next to highest-power terms in numerator and denominator may or may not be equal. If they are equal, this implies that z_{11} and z_{22} have a semiprivate pole at infinity.

The above conditions either guarantee that the following restrictions are satisfied or result from them:

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[1] Balabanian, N., Network Synthesis, Prentice-Hall, Inc., New York, New York, 1958, pp. 234-235.

(1) The filter is low-pass and minimum susceptive.

(2) The filter is realized as a lossless network terminated in a resistance.

(3) The y parameters of the lossless network are compact.

Still to be determined are conditions which will guarantee:

(4) $y_{11} = y_{22}$ (requirements for a symmetrical network).

(5) The high-pass complement has only one zero at the origin.

If the collector line is to be synthesized on an impedance basis, the dual of the above discussion holds. Once conditions (4) and (5) are determined the approximation problem must be solved subject to these restrictions, and a suitable synthesis procedure selected or developed.

Appendix A

GENERAL DISTRIBUTED AMPLIFIER THEORY

A.1 Development of a General Gain Function Assuming a Unilateral Active Device

The general distributed amplifier consists of n amplifying sections connected in parallel between two transmission lines. The transmission lines muss have attenuation characteristics consistent with the desired overall response and at the same time must contain the terminal driving point impedances of the amplifying sections. If maximum gain is to be realized, the phase shift between two amplifying sections on the input line must be the same as the phase shift between the corresponding two sections on the output line. Generally, it is required that the amplifying sections be identical, and that the phase shift per section of transmission line be the same for all sections. These additional requirements are imposed to simplify the problem, since to date no one has succeeded in showing that better gain-bandwidth products can be obtained by using sections which are not identical.

The block diagram of the resulting distributed amplifier is shown in Figure A.1. If the transmission line networks are linear, and if the amplifying sections are linear and unilateral, then the law of superposition may be used to calculate the overall gain

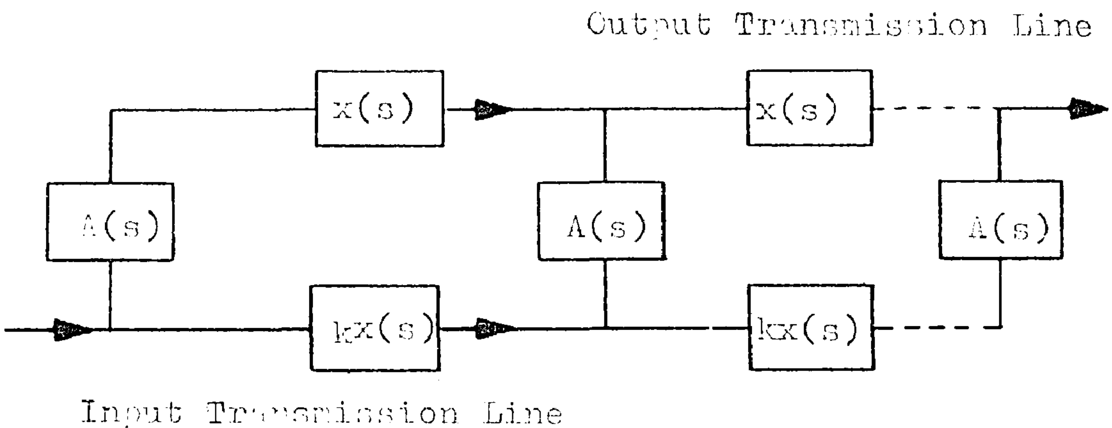

Figure A.1 General distributed amplifier.

x(s) = forward transfer function per section of collector line.

k = positive constant which is the ratio of the transfer function per section of collector line to transfer function per section of base line.

A(s) = transfer function of amplifying section.

expression.[1] It will be equal to the sum of the gains obtained by considering each amplifying section in turn, with the remaining sections replaced by their internal impedances.

$$A_t(s) = x(s)^{n-1} \left[A(s) + kA(s) + k^2 A(s) + \ldots . k^{n-1} A(s) \right]$$

$$= x(s)^{n-1} A(s) \left[1 + k + k^2 + \ldots . k^{n-1} \right]$$

$$= x(s)^{n-1} A(s) \sum_{i=0}^{n-1} k^i$$

Since $k < 1$, the infinite series obtained by allowing $n \rightarrow \infty$ would be absolutely convergent. Hence, the following manipulations are justified:

$$\sum_{i=0}^{n-1} k^i = \sum_{0}^{\infty} k^i - k^n \sum_{0}^{\infty} k^i$$

$$= \frac{1 - k^n}{1 - k}$$

The gain expression becomes

$$A_t(s) = A(s)\ x(s)^{n-1} \ \frac{1 - k^n}{1 - k}$$

---

[1] Ginzton, Hewlett, Jasberg, Noe, "Distributed Amplification", Proc. IRE, Vol. 36, August, 1958, pp. 956-969.

If $k \rightarrow 1$ then the partial sum sums to n and the gain expression reduces to

$$A_t(s) = nx(s)^{n-1} A(s) \tag{A.1}$$

Notice that the overall gain expression is proportional to the sum of the individual amplifying section gains. Hence, amplifier gains greater than unity are obtainable from aplifying sections which have gains less than unity.

A.2 Parasitic Capacitances and Ultimate Gain-bandwidth Product

Before attempting to obtain an actual configuration or design of a practical transistor distributed amplifier, it would be desirable to determine the ultimate results one might obtain under ideal conditions. In other words, what is the best amplifier obtainable if one were able to use ideal transistors coupled with an infinite number of ideal circuit elements for interstages

Define the "figure of merit" F_a of a distributed amplifier as the product of the maximum available power gain[2] G and the square of the 3 db bandwidth $f_\triangle$. The theoretical limitations on the maximum value of this figure of merit for an n section distributed amplifier will now be developed.

---

[2]The maximum available power gain is defined as the ratio of the power supplied to the load to the power supplied to the amplifier, under matched conditions.

The Giacoletto equivalent circuit for a transistor is shown in Chapter 2. For an ideal transistor[3], the base spreading resistance $r'_{b\,b}$, the collector barrier capacitance C_c, the emitter diode forward resistance $r'_{b\,e}$, the collector diode reverse resistance r_c, and the space charge widening resistance r_{ce} would all be negligible. The resulting equivalent circuit is shown in Figure A.2.

In view of equation A.1, the expression for the magnitude of the voltage gain would be

$$\left|A_t(j\omega)\right| = n \left|x(j\omega)\right|^{(n-1)} \left|A(j\omega)\right|$$
$$= n \left|A(j\omega)\right|$$

if $\left|x(j\omega)\right| = 1$, as it can for lossless networks such as the bridged -T. If the impedance levels of the input and output lines are equal, the power gain is simply the square of the voltage gain or

$$G = n^2 \left|A_t(j\omega)\right|^2$$

Wheeler[4] has shown that the maximum voltage gain-bandwidth product, for a circuit such as that of Figure A.2, regardless of the complexity of the coupling networks, is

$$A_r f_\Delta = \left[\frac{g_m}{\pi\sqrt{C_o C_1}}\right]$$

[3]For a discussion of a usable wide-band transistor equivalent circuit, see Chapter 2.

[4]Wheeler, H.A., "Wide-band Amplifiers for Television," <u>Proc. IRE</u>, Vol. 27, July, 1939, pp. 429.

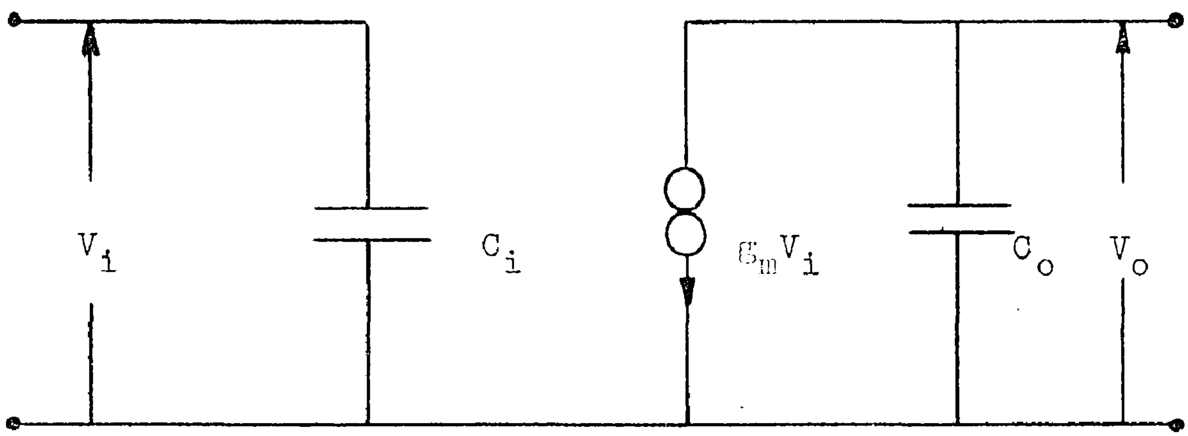

Figure A.2 Equivalent circuit of ideal transistor for determination of theoretical maximum figure of merit F_a .

It was assumed in the development of this equation that the output impedance level was transformed by a lossless transformation to be equal to the input impedance. Hence, substituting directly into the above equation yields

$$G = n^2 \left[\frac{g_m}{\pi f_\triangle \sqrt{C_o C_i}} \right]^2$$

or

$$F_a = f_\triangle^2 G = \left[\frac{n g_m}{\pi} \right]^2 \frac{1}{C_o C_i}$$

This figure of merit will allow one to determine that point at which the increasing circuit complexity and transistor perfection commence to overshadow the corresponding improvement in performance. The point of diminishing returns is an important one in the practical world.

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