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A KINEMATICAL STUDY OF FIELD  
SUPERGIANTS NEAR THE SUN

by

Philip Cooper Steffey

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## ABSTRACT

The galactic orbits of six field supergiants are traced into the recent past and compared with the motions of nearby OB aggregates to attempt location of the stars' birthplaces. Stellar model studies show that such stars evolve from early-type progenitors which form predominantly in groups, thus each supergiant considered here probably formed in a recognizable aggregate, having escaped due to acquisition of a higher than average velocity at the epoch of formation. Since the pre-nuclear phases of evolution appear much shorter in duration than subsequent stages of nuclear burnings, the onset of hydrogen burning occurs virtually simultaneously with ejection from the birthplace, i.e. at any later time the kinematical age is a close measure of the nuclear age of the star. Kinematical ages, where they can be determined by locating parent clusters of young field stars, are therefore useful as age indicators independent of evolutionary predictions.

The past space motions of the F-type supergiants  $\alpha$  Leporis and  $\alpha$  Carinae show that the former, and probably the

latter, originated in the extensive Scorpio-Centaurus association about  $2.1$  and  $2.7 \times 10^7$  years ago, consistent with both evolutionary and 'expansion' ages of the oldest B stars in the aggregate. The z-motion of  $\alpha$  Orionis (M2Iab) and rough evolutionary age reasoning suggest formation in the Orion association, but the 3-dimensional orbit fails to pass through the interior of the group at a plausible time in the past. This is believed due to some gross error in the motion of the association or of the star, rather than implying that the latter did not form in I Orion and is much older than expected for its great luminosity. The trajectory of  $\beta$  Ori, a B8Ia star now quite distant from the I Ori center though seemingly a member, does show an escape from the interior. About  $7 \times 10^6$  years ago Rigel was located 30-40 pc from the aggregate center and probably formed in that vicinity. The final low-velocity star treated is the classical Cepheid  $\zeta$  Gem, whose past motion indicates origin in one of the Canis Major B-star groups presently 800 pc distant  $2.5$  to  $3 \times 10^7$  years ago. The distances and peculiar motions of these aggregates are so uncertain that the exact parent group cannot be identified.

The high-velocity B supergiant  $\rho$  Leonis, now 500 pc from the galactic plane, is found to have left the disc not

over  $12 \times 10^6$  years ago, the most reliable orbital solution indicating ejection from the II Scorpii subgroup of Scorpius-Centaurus at 60 km/sec. Neither for this star nor any other considered has a significant discrepancy between kinematical and nuclear ages been discovered.

The accuracy of galactic orbits of young stars depends on (1) quality of data used to compute the present position and velocity; (2) the galactic distance scale, velocity of the local standard of rest, and force laws adopted; (3) accuracy of the numerical solution of the equations of motion. Locating the parent group of a field star involves comparing two orbits, minimizing the influence of (2). The greatest difficulties encountered here are in the net motions of the OB groups and the size of the star-forming areas in each, sources (1) and (3) above being of lesser importance.

# I

## Introduction

This dissertation is concerned with the galactic orbits of selected individual massive stars, the problems of determining and interpreting the orbits, and the information obtainable from such studies. This is not a study of general properties of galactic orbits, an area of rather intense interest in recent years (e.g. see Ollongren's (1962) thorough analysis). Although the results, which are of a preliminary nature, cannot alone be employed to attempt far-reaching general inferences, it will be argued that if the analysis discussed here were applied to a very large sample of stars for which all the necessary starting data existed, then some very important conclusions could be drawn regarding star formation, ejection and expansion mechanisms, and the meaning of stellar ages.

Ideally one would like to pose and solve the initial-value problem: Given the position and velocity of a star at the present time plus a knowledge of all forces acting on it, to find the motion of the star at all

previous times during which it has existed. In practice one finds that virtually none of the necessary conditions in the statement of the problem can be presently met with the accuracy that is desired.

For the stars which are considered here, one is not really given the position and velocity components, but only approximations to them. It is thus necessary to deal only with stars for which 1) the approximate positions and velocities are sufficiently accurate or 2) the uncertainties in these quantities do not materially alter the calculated results and interpretations. Such restrictions alone greatly limit the number of stars of all types for which meaningful trajectories can be computed, but the second half of the required information elicits further potential concern. Even if the most important force acting on a star throughout most of its lifetime is the net galactic gravitational field, basic uncertainties connected therewith must be faced. First, the gravitational force of the Galaxy acting on a star at any distance from the center and plane, respectively, is somewhat uncertain because neither the total mass, the mass distribution, nor the distance scale of the Galaxy are very well established. Second, the Galaxy is

presumably evolving dynamically, so it is not safe to assume that the present mass distribution has maintained itself throughout indefinite aeons of time, i.e. the gravitational potential is most probably time-dependent. Third, perturbations of stellar orbits due to critical encounters cannot be entirely ruled out. Although it has been convincingly established that star-star encounters are extremely rare (Chandrasekhar 1942), and interactions between stars and small interstellar clouds are only barely worrisome (Spitzer and Schwarzschild 1951), encounters of stars with large clouds are potentially serious. Spitzer and Schwarzschild (1953) have shown that encounters between a low velocity star and clouds of mass  $\sim 10^5 - 10^6 M_{\odot}$  may alter the star's velocity by an amount equal to its unperturbed value in about  $10^8$  years providing that a sufficient number of large clouds exist which remain always very near the galactic plane. These two conditions may not be strictly valid, yet it is imperative that one recognize the possibility that if they are even approximately correct, then individual stellar motions may be strictly determinate for only a few hundred million years in the past.

The effects of the latter two uncertainties can be avoided to a large extent by considering very young stars,

since they are likely to be still relatively close to their places of formation and except in the case of abnormally high-velocity stars the shortness of the orbital paths coupled with small curvature should allow simple numerical approximation to the true motion.

It is clear that by imposing certain restrictions on the objects studied and by adopting reasonable hypotheses, the problem of determining galactic stellar orbits can be meaningfully posed. The mathematical solution is, in principle, straightforward, but interpretation of the solution gives rise to further difficulties centered on the meaning of the existence of a star. The meaning of stellar ages is an essential piece of knowledge for understanding the motions of young stars. The most obvious measure is a star's evolutionary age; until quite recently this has been taken almost tacitly to mean the time spent by the star in various stages of thermonuclear evolution. Such a synonymous meaning is evidently an oversimplification, for stars develop from interstellar gas, passing through several interesting and dynamically significant evolutionary stages prior to firing hydrogen in their interiors. It is now possible to at least outline the bulk of the evolutionary history of a massive star,

i.e. a star of about 4 solar masses or more. In TABLE 1 this is summarized according to Spitzer (1962) (pre-main-sequence phases) and Hayashi, Hōshi, and Sugimoto (1962) (main-sequence and subsequent phases). The descriptions are necessarily short, so the reader should consult the articles quoted for details, particularly in reference to the pre-stellar phases, where theory suggests that massive stars are formed in groups. This is an important hypothesis of the present investigation, one strongly supported by statistics (Blaauw 1962).

In addition to a description of the phases of stellar evolution, TABLE 1 contains information on the fractional lifetime spent in each phase by stars of 15.6 and 4 solar masses, respectively, based on model computations summarized by Hayashi, Hōshi, and Sugimoto. Only for the Kelvin-Helmholtz contraction and the main-sequence phases are absolute lifetimes known for a wide range of mass. Since the fractional lifetime spent in the pre-main-sequence contraction stage is roughly independent of mass, absolute contraction times are not necessary at this point, however a reliable determination of main-sequence lifetimes is vital. TABLE 2 lists the adopted run of this quantity with mass according to some of the more recent theoretical

models. The mass-spectral type calibration is based on binary star data (Sahade 1960; Pecker and Schatzman 1959; Wood 1963; Harris, Strand, and Worley 1963) for Sp later than O9.5. For the O stars, fragmentary binary star data has been fit to observable parameters ( $M_V$ , B - V) predicted by the models quoted. It has been assumed that a star of  $65 M_{\odot}$  has spectral type O5 at zero age, an assumption born out by several lines of observational evidence according to Sahade's discussion. The run of  $M_V$  adopted here follows from the theoretical models and pertains to the zero-age configuration. To obtain these values, bolometric corrections thoroughly discussed by Wildey (1962) were employed, so in some cases there is disagreement with  $M_V$  given by the model authors.

The lucky circumstance that upper main-sequence stars spend 80 to 90 per cent of their nuclear lifetime in the hydrogen-burning stage allows one to estimate with fair accuracy the total nuclear age  $\tau_{\text{nuc}}$  providing the star in question is sufficiently separated from the observed main sequence in the color-magnitude diagram so that it has clearly evolved beyond the hydrogen-burning phase, that a good estimate of the present bolometric magnitude is possible, and the manner in which  $M_{\text{bol}}$  varies in time is known. It is the

TABLE 1

## The Evolutionary History of a Massive Star.

(Fractional times in various nuclear phases given for stars of 15.6 and 4  $M_{\odot}$  respectively. Energy lost to neutrinos neglected.)

Stage	Fraction of Total Nuclear Lifetime
1. Formation of interstellar cloud from uniform 'intercloud' medium.	-----
2. Collapse and fragmentation of cloud into cluster of 'proto-stars'; collapse of the individual proto-stars.	-----
3. Kelvin-Helmholtz contraction.	0.003, 0.004
4. Hydrogen-burning in core.	0.88, 0.80
5. Core contraction; hydrogen-burning in outer shell.	0.004, 0.042
6. Helium-burning in core; hydrogen-burning in outer shells.	0.065, 0.17
7. Core contraction; helium-burning and hydrogen-burning in outer shells.	0.003, 0.017
8. Carbon-burning in core; helium-burning in outer shell.	0.013, 0.013
9. Later phases, not well understood.	0.03:, 0.0

TABLE 2

Adopted Mass-Spectral Type Relation for Class V Stars and  
Main-Sequence Lifetimes for Chemical Composition  $X \sim 0.7$ ,  
 $Z \sim 0.02$ .

$M/M_{\odot}$	Mean Sp	$T_{ms}$ ( $10^6$ yrs)	$M_{vis}$	Model Source
62.7	O6	2	-5.7	Schwarzschild and Harm 1958
30	O9	3.1	-4.8	Stothers 1963
25	O9.5	4.2	-4.4	interpolated
20		5.8	-4.0	Heney, LeLevier, and Levee 1959
18	B0	6.6		interpolated
16	B0.5	7.8	-3.3	interpolated
14	B1	9.3		interpolated
12	B1.5	11.5		interpolated
11		13	-2.4	Heney et al. 1959
10	B2	14.5		interpolated
8.9		18	-1.8	Hoyle 1960
7.5	B3	23		interpolated
6	B4	40	-1.0	Heney et al. 1959
5	B5	70	-0.4	Polak 1962
3.9	B7	105	-0.1	Hoyle 1960

first condition which explains why evolved stars were chosen for this study.<sup>1</sup> Neither the absolute magnitudes nor intrinsic colors of individual O and B main-sequence stars are known to the accuracy needed to relate them to the zero-age locus in the color-magnitude diagram, hence to establish the precise time since they began generating energy by hydrogen fusion. But if one knows the bolometric magnitude of an evolved star and how much the luminosity has increased since zero (nuclear) age, it is possible to estimate the mass of the main-sequence progenitor by interpolating in initial models with the 'corrected' bolometric magnitude as argument. The mass found then yields the main-sequence lifetime, and the total nuclear age is but a few per cent greater.

The reason for choosing field supergiants poses no mystery, for it is seldom worthwhile - and actually quite difficult - to look for birthplaces of stars still located within their parent group.

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1. The choice of supergiants is not entirely arbitrary. Few nearby early-type giants are found in isolation, and class II and III stars of late spectral types are too old for this study.

## II

### Kinematical Ages

The only determinable stellar ages independent of evolutionary models are kinematical, or expansion ages. The kinematical age  $\tau_{\text{kin}}$  is defined as the time of flight of a star from its place of origin to its present location, therefore knowledge of this quantity implies that one knows where the star formed. The 'place of formation' is a rather loose term and means different things for different stars. For those discussed here it will be (hopefully) a known OB star group. For a high-galactic-latitude star such as the supergiant 89 Herculis it can be simply the plane of the Galaxy, since the time spent in traversing the thickness of the gas-dust stratum of the galactic disc, where the star probably formed, is small compared to the time necessary to rise to a distance  $z \sim 1$  kpc from the plane. For a group of somewhat older field stars, say classical Cepheids, the birthplace may have been in a spiral arm which has since been distorted beyond recognition by dynamical evolution. Kraft and Schmidt (1963) demonstrate rather convincingly that

only the more luminous Cepheids outline (fragments of) spiral arms; the intrinsically fainter (hence older) ones have moved too far from their place of formation owing to the cumulative effect of initial random velocities followed by separation due to differential galactic rotation.

Although the determination of kinematical stellar ages may be carried out without reference to theoretical evolutionary ages, it does not necessarily follow that the two are wholly unrelated. The relationship between  $\tau_{\text{kin}}$  and  $\tau_{\text{evol}}$  may depend in detail on the physical connection between the star-formation process and the ejection mechanism, but it very definitely depends on when, in the star's lifetime, the escape, or 'separation' velocity is imparted. The simplest situation to envisage is that the star acquires this separation velocity at the very time it begins to burn hydrogen in its core, for at any time thereafter when the star is observed the kinematical age is precisely a measure of the time since the star began thermonuclear evolution, i.e. since what is called zero age. Unfortunately this picture is a naive one; most of the ejection mechanisms which have been discussed in the literature cannot possibly operate on a genuine star, and it is the belief of this writer that most of the relatively low-velocity massive stars found in the

galactic field escaped their respective places of formation as a result of some impulse acting on them during an early pre-main-sequence phase. (There is no theoretical or thoroughly sound observational basis by which to distinguish low, or normal stellar velocities from high ones. The value  $V = 40$  km/sec relative to the Sun is adopted here as the dividing speed, in keeping with Blaauw's discussion (1961), but the distinction may well be artificial.) No attempt will be made here to discuss the details of ejection mechanisms, yet it is instructive to associate these mechanisms, albeit in a crude fashion, with the major evolutionary epochs in a star's history. TABLE 3 presents this material, where the nine stages of TABLE 1 have been condensed into four, an adequate subdivision. The major point of TABLE 3 is that quite different physical forces act on pre-stellar material than will affect the same material in a highly condensed state.

TABLE 3

Major Stages of Dynamical Importance in the Evolution of a  
Massive Star and Possibly Active Ejection Mechanisms.

Evolutionary Stage	Ejection or Expansion Mechanism	References
Undifferentiated pre-cluster gas- dust cloud.	Expanding HII region.	Oort 1954; Save- doff and Greene 1955.
	Supernova shell.....	Öpik 1953.
	Slow stellar mass e- jection.....	-----
	Turbulence.....	-----
Fragmentation and collapse of proto- stars.	Rocket acceleration....	Oort and Spitzer 1955.
Kelvin-Helmholtz contraction.  Stellar.	Dissolution of binary system.....	Blaauw 1961.
	Disruption of cluster by interstellar clouds. Gravitational instabil- ity of multiple system.	Spitzer 1958. Ambartsumian 1953, 1955.

The importance of when, in a star's history, it is ejected from its place of formation is illustrated concisely in FIGURE 1, which shows schematically the relationship between the nuclear age of a massive field star in the helium-burning phase and the kinematical age corresponding to three dynamical histories. If the star was ejected near the onset of hydrogen-burning,  $\tau_{\text{kin}} \sim \tau_{\text{nuc}}$ , but if ejection occurred before Kelvin-Helmholtz contraction began, the two may differ appreciably, i.e. the expansion interval (if any) and fragmentation-collapse period may have been significant compared to  $\tau_{\text{ms}}$ , which dominates the nuclear age. There is neither a satisfactory quantitative theory of expanding stellar groups nor adequate observations to admit estimation of the expansion interval (as it is meant here), which must be intimately related to the star-formation process itself. Once the collapse of a proto-star begins, however, some numbers may be written down.

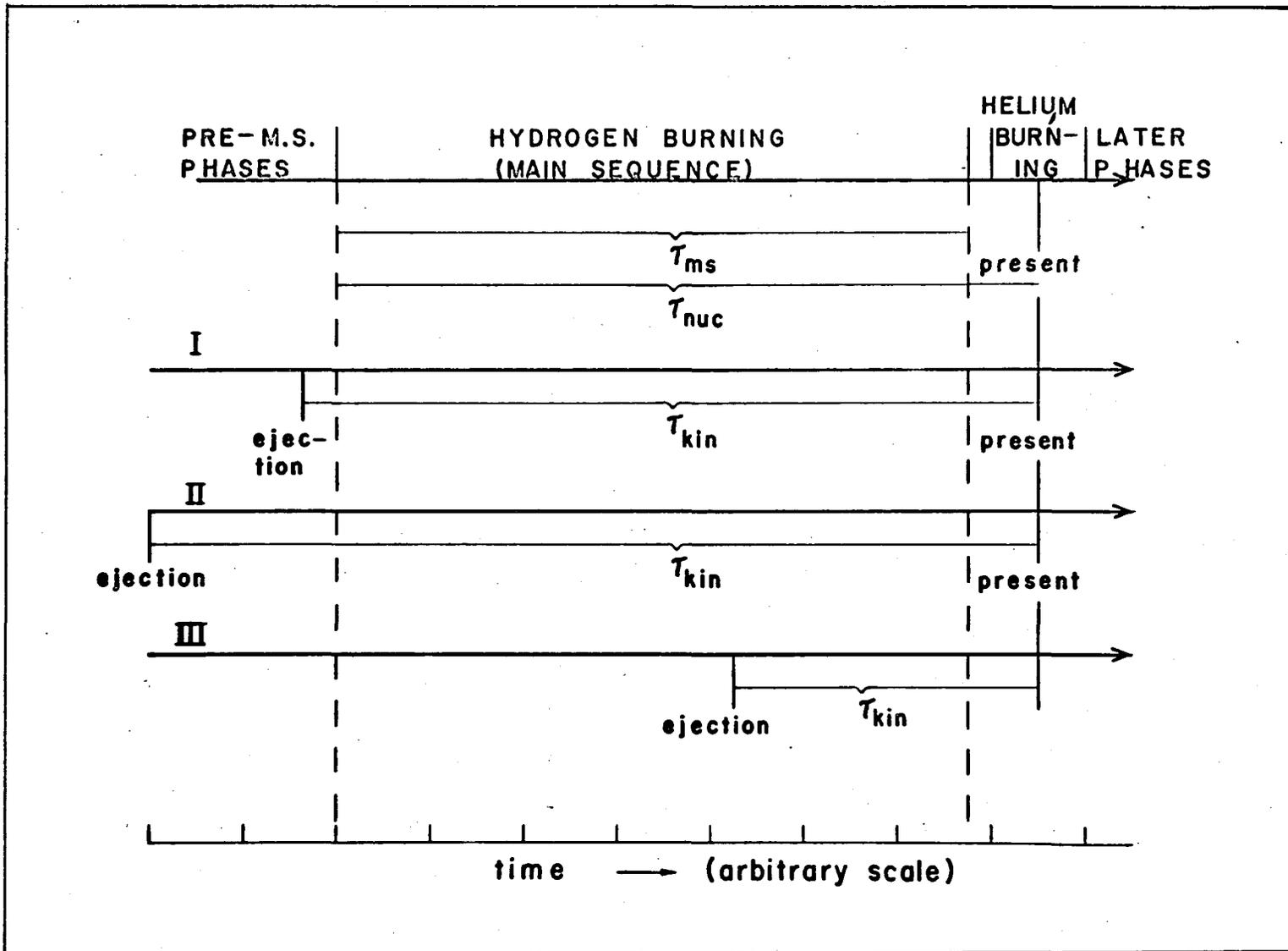
The contraction of a self-gravitating gas sphere of radius  $R_0$  and mass  $m$  with negligible internal pressure proceeds at a rate of order

$$\left( \frac{dR_0}{dt} \right)^2 = Gm/R_0, \quad (\text{II-1})$$

a well-known result. The time required for collapse to a

FIGURE 1. Parallel time lines illustrating the relation between the nuclear age of a helium-burning supergiant star and the kinematical age.

Three plausible dynamical histories are shown: Case I - ejection closely precedes the onset of hydrogen burning,  $\tau_{\text{nuc}} \sim \tau_{\text{kin}}$ . Case II - ejection significantly earlier than the onset of hydrogen burning,  $\tau_{\text{kin}} > \tau_{\text{nuc}}$ . Case III - ejection follows the onset of hydrogen burning,  $\tau_{\text{kin}} < \tau_{\text{nuc}}$ .



radius  $R \ll R_0$  is then approximately

$$t_{\text{ff}} = R_0^{3/2} / (Gm)^{1/2}. \quad (\text{II-2})$$

The importance of the free-fall time  $t_{\text{ff}}$  relative to the subsequent main-sequence lifetime depends mainly on the initial radius of the proto-star. Typical values  $R_0 = 0.1$  to 1 parsec appear appropriate for proto-OB stars, which form in groups of several dozen within a volume whose radius is 10 pc or less initially (Blaauw 1962). But for a given mass not all clouds will be unstable to collapse; the Virial Theorem distinguishes these from stable clouds by demanding that for a given mass and temperature the cloud will collapse only if its radius is less than a certain critical value. In the case of pure hydrogen spheres at  $T = 100^\circ\text{K}$  one derives from the stability condition (e.g. see Hoyle 1953) the critical radius

$$R = 5 \times 10^{-3} M/M_\odot \text{ pc}. \quad (\text{II-3})$$

This derivation ignores angular momentum, surface pressure, and other physics of great interest; however it is completely beyond the scope of this study to discuss such details.

Assuming a range of mass appropriate to O and early B stars, the critical radii and densities for free-fall collapse were computed, together with the radius at which the collapse ceases,  $R_f$ , owing to balancing of gravity by

internal pressure arising from much-increased opacity. Spitzer (1962) assigns a critical density for cessation of collapse of  $n_H \sim 10^{10} \text{ cm}^{-3}$ , which is employed in this calculation. The free-fall times are then computed from equation (II-2) for clouds initially just barely unstable, and these times are compared with the subsequent Kelvin-Helmholtz contraction intervals and main-sequence lifetimes for each mass. The results are given in TABLE 4, the Kelvin-Helmholtz contraction times according to the formula

$$\tau_{GC} = 6.2 \times 10^7 (M/M_{\odot})^2 (L/L_{\odot})^{-1} (R/R_{\odot})^{-1} \text{ years,}$$

which follows from the discussion by Herbig (1962a) of calculations by Henyey et al. (1955). The main-sequence lifetimes are carried over from TABLE 2 of this paper. Spitzer has pointed out that some of the physical factors neglected here may increase  $t_{ff}$  by up to 5 times its values in TABLE 4, thus the collapse interval for stars more massive than  $30 M_{\odot}$  might be appreciable. None of the stars considered in the present investigation is, however, quite this massive, so it appears that  $t_{ff}$  cannot contribute significantly to their evolutionary ages. Except for the extremely massive O stars, the case  $\tau_{kin} > 1.1\tau_{nuc}$  most likely would imply that ejection occurred during an evolutionary stage preceding the onset of gravitational collapse.

TABLE 4

Critical Physical Characteristics for Gravitational Collapse of Uniform Hydrogen Spheres at  $T=100^{\circ}\text{K}$  and Comparison of Free-Fall and Kelvin-Helmholtz Contraction Intervals with Main-Sequence Lifetimes for Various Masses.

$M/M_{\odot}$	$\rho >$ ( $\text{cm cm}^{-3}$ )	$R <$ (pc)	$R_f$ (pc)	$t_{\text{ff}}$ (	$T_{\text{GC}}$ $10^5$ yrs	$\tau_{\text{ms}}$ )
5	$5 \times 10^{-16}$	0.02	0.002	0.2	8.5	700
10	1	.05	.003	.4	2.1	145
20	$3 \times 10^{-17}$	.10	.003	.7	0.8	58
30	2	.15	.004	1.0	0.6	31
45	$7 \times 10^{-18}$	.2	.004	1.6	0.5	25:
60	4	.3	.005	2.	0.4	20:

There is some evidence for kinematical ages somewhat greater than nuclear ages among the rare high-galactic-latitude supergiants (Munch 1956; Bonsack and Greenstein 1956; Abt 1960), however more recent thought (Searle, Sargent, and Jugaku 1963) suggests these stars to be evolved 'runaway' stars, and in the discussion of  $\rho$  Leonis (section V) it will be seen that no significant discrepancy between  $\tau_{\text{kin}}$  and  $\tau_{\text{nuc}}$  exists. Case I of FIGURE 1 is therefore a valid hypothesis for even these unusual objects; for the much more common low-velocity supergiants it is probably equally good.

### III

#### A Brief Survey of Field Supergiants Near the Sun

It is well known that a number of late-type supergiants lying within a few hundred parsecs of the Sun are not obvious members of OB associations or clusters, yet current evolutionary theory suggests that these stars originated as early-type main-sequence objects, most of which form in groups (Blaauw 1962). In TABLES 5A and 5B are listed respective samples of Ia-ab and Ib field stars nearer than 700 parsecs. The adopted intrinsic colors are due to Johnson (1964), Johnson and Borgman (1963), Arp (1958), and Fernie (1963). In some cases the indicated reddening is so small that none has been included. The writer thanks Dr. Harold Johnson for making available unpublished photometric data for many of these stars. The absolute magnitude sources are the following. PL: Cepheid period-luminosity relation (Kraft and Schmidt 1963; Kraft 1963); HK: Equivalent widths of the CaII H and K emission cores (Wilson and Bappu 1957) re-calibrated to fit more recent  $M_v$ 's of the  $\eta$  and  $\chi$  Persei M-type supergiants (Wildey 1962), the resulting magnitudes

TABLE 5A

Ia, Iab Supergiants Brighter than  $V = + 5.0$  and Nearer than 700 Parsecs.

HR	Name	MK	V	B - V
1713	$\beta$ Ori	B8Ia	0.10	-0.03
1903	$\epsilon$ Ori	B0Ia	1.70	-0.19
2004	$\kappa$ Ori	B0.5Ia	2.06	-0.18
2061	$\alpha$ Ori	M2Iab	0.69v	+1.84v
2693	$\delta$ CMa	F8Ia	1.84	+0.66
6134	$\alpha$ Sco A	M1Ia-Ib	0.88v	+1.84
7924	$\alpha$ Cyg	A2Ia	1.26	+0.09

HR	$V_o$	$M_v$	d(pc)	Source of $M_v$	Cluster Member ?
1713	0.10	-6.9	260	hX	*
1903	1.55	-6.5	410	B	*
2004	2.00	-5.6	330	B	*
2061	-0.60v	-5.8	110	e	
2693	1.51	-7.6	665	C1	*
6134	-0.04v	-5.8	150	e(C1)	*
7924	0.99	-7.4	475	hX	*

TABLE 5B

Ib Supergiants Brighter than  $V = + 5.0$  and Nearer than 700 Parsecs.

HR	Name	MK	V	B - V
424	$\alpha$ UMi	F8Ib	2.01v	+0.60
429	$\gamma$ Phe	K5Ib	3.44	+1.56
834	$\eta$ Per	K3Ib	3.79	+1.70
1017	$\alpha$ Per	K5Ib	1.80	+0.48
1203	$\zeta$ Per	B1Ib	2.85	+0.11
1303	$\mu$ Per	G0Ib	4.20	+0.96
1845	119 Tau	M2Ib	4.35	+2.07
1865	$\alpha$ Lep	F0Ib	2.58	+0.22
1948-49	$\zeta$ Ori	O9.5Ib	1.78	-0.21
2326	$\alpha$ Car	F0Ib-II	-0.73	+0.15
2473	$\epsilon$ Gem	G8Ib	2.97	+1.41
2650	$\zeta$ Gem	F7-G3Ib	3.9v	+0.7
3188	$\zeta$ Mon	G2Ib	4.34	+0.99
3634	$\lambda$ Vel	K5Ib	2.24	+1.64
3699	$\iota$ Car	F0Ib	2.25	+0.17
4050	$\eta$ Car	K5Ib	3.41v	+1.55
4133	$\rho$ Leo	B1Ib	3.85	-0.12
6461	$\beta$ Ara	K3Ib	2.90	+1.45
6536	$\beta$ Dra	G2Ib	2.78	+0.99
6553	$\theta$ Sco	F0Ib	1.86	+0.39
7747	$\alpha^1$ Cap	G3Ib	4.26	+1.08
7796	$\gamma$ Cyg	F8Ib	2.23	+0.67
8079	$\xi$ Cyg	K5Ib	3.70	+1.65
8232	$\beta$ Aqr	G0Ib	2.85	+0.84
8279	$\eta$ Cep	B2Ib	4.74	+0.30
8308	$\epsilon$ Peg	K2Ib	2.38	+1.53
8313	$\eta$ Peg	G5Ib	4.31	+1.18
8414	$\alpha$ Aqr	G2Ib	2.92	+0.98
8465	$\zeta$ Cep	K1Ib	3.31	+1.55
8571	$\delta$ Cep	F5-G2Ib	3.96v	+0.66v

TABLE 5B--Continued

HR	$V_0$	$M_V$	d(pc)	Source of $M_V$	Cluster Member ?
424	1.86v	-3.0	100	PL	
429	3.44	-4.7	425	e(HK)	
834	2.65	-4.4	255	HK	
1017	1.44	-4.6	160	C1	*
1203	1.95	-5.3	280	hX	*
1303	3.51	-4.3	365	HK	
1845	3.18	-4.7	375	HK	
1865	2.37	-4.7	260	e	
1948-49	1.48	-6.2	350	B	*
2326	-0.73	-3.8	40	-----	
2473	1.62	-4.7	185	HK	
2650	-----	-4.2	330	PL	
3188	3.95	-4.6	515	HK	
3634	2.09	-4.7	230	e(HK)	
3699	2.19	-4.7	240	e	
4050	3.41v	-4.7	420	e(HK)	
4133	3.67	-5.3	620	hX	
6461	2.51	-4.4	240	e(HK)	
6536	2.39	-4.7	260	e	
6553	1.14	-4.8	150	e	?
7747	3.72	-4.6	460	e(HK)	
7796	1.87	-4.6	195	e	
8079	3.52	-4.7	440	e(HK)	?
8232	2.52	-4.7	280	HK	
8279	3.33	-5.4	560	hX	?
8308	1.42	-4.3	140	HK	
8313	3.62	-3.7	290	HK	
8414	2.56	-5.1	340	HK	
8465	2.05	-4.6	215	HK	
8571	-----	-3.1	260	PL	

being about  $0^m.3$  fainter than those given by Wilson and Bappu; hX: Calibration based on the h and  $\chi$  Persei association supergiants (Wildey 1962); B: Values based on the strength of H $\gamma$  calibrated by Bappu et al. (1962); Cl:  $M_V$  based on mean distance modulus of cluster to which the star in question belongs ( $\alpha$  Per according to Mitchell (1960),  $\delta$  CMa after Schmidt-Kaler (1961)); e: Estimate, the basis given in parentheses if definite. The distance of  $\alpha$  Carinae is discussed later in this dissertation.

TABLES 5A and 5B reveal a striking difference between the very luminous and less luminous supergiants. Of the 30 Ib stars in the latter table, only 3 are definite cluster members, two of which are luminous enough to really belong in TABLE 5A. By contrast, nearly every star in TABLE 5A belongs to a group. Apparently the very young Ia-Iab stars tend to occur in groups, while the Ib objects tend to be found in isolation. This is by no means a startling discovery. The investigation by Schmidt-Kaler (1961) of F0-K7 supergiants in galactic clusters and associations revealed that over half of these stars are extremely luminous; a few Ib stars are definite or probable cluster members, and class II stars are virtually absent from these groups. Of the hundreds of Pop I Cepheids known, systematic searches for

cluster members have to date been meagerly rewarded. These results are consistent with the idea that the supergiants have evolved from OB-type stars which formed in unstable associations, and with the prediction made in section II that a correlation exists between the kinematical and nuclear ages of these stars. It can hardly be denied that a real age effect is operative here: The very young Ia-Iab stars do not live long enough for a significant number to escape their parent groups whereas the older Ib supergiants have ample lifetimes in which to do so. Typical evolutionary lifetimes of the stars in TABLE 5B are 1 to  $5 \times 10^7$  years, suggesting that (at least the outer haloes of) their parent aggregates were relatively ephemeral.

Expansion of OB associations is an observed phenomenon which cannot be refuted (Spitzer 1963; Blaauw 1962), average expansion velocities around 10 km/sec being encountered. A star of 30 solar masses spends less than 5 million years in the hydrogen-burning phase; moving at 10 km/sec relative to the center of mass of its parent association, this object would cover only about 50 pc by the time it leaves the main sequence, roughly the radius of usual OB groups (Blaauw 1962). A star of 5 solar masses

on the other hand, exists about  $10^8$  years, giving it ample time to move over 1000 pc away from its place of formation by the time it becomes a supergiant. If the kinematical and nuclear ages of association stars do not markedly differ, this interplay between the two explains why so few O and early B main-sequence stars are found outside groups and why several associations are surrounded by a halo of supergiants, for example the  $\eta$  and  $\chi$  Persei aggregate. The expansion of associations, whatever its physical cause, is clearly capable of injecting low-velocity massive stars into the galactic field. It is now generally accepted that unique evolutionary ages cannot be assigned to all clusters and associations (Herbig 1962b), so the presence of compact clusters of early-type stars both with (e.g. Orion Nebula cluster) and without (e.g. NGC 2287) extensive haloes is no more remarkable from the astrophysical standpoint than from the dynamical. As FIGURE 16 shows, a typical first-generation expanding OB group is expected to disperse into the surrounding field, possibly leaving behind a much smaller 'nucleus-cluster' of second-generation stars. (This diagram will be discussed further in section V.)

An elementary calculation demonstrates that expanding associations are expected to dissolve into the background

of field stars in a time comparable to the main-sequence lifetime of an early B-type star, later to develop into a Ib supergiant. Imagine a spherical stellar group of uniform star density endowed somehow with a constant expansion speed, the stars having condensed out of the pre-association cloud a negligibly short time after the expansion began. Let  $R_i$  and  $R_f$  denote the initial and final radii of the group, the latter value applying to that epoch when the expansion has reduced the mean star density to the density of field O and B stars. One can adopt  $\rho_f = 10^{-4} M_{\odot}/\text{pc}^3$  as the final space density, after Oort (1958). Then if  $v_{\text{exp}}$  and  $\Delta t_D$  are the expansion speed and the time for  $\rho$  to decrease to  $\rho_f$ , one easily obtains

$$R_f = (3M/4\pi\rho_f)^{1/3} = R_i + v_{\text{exp}}\Delta t_D, \quad (\text{III-1})$$

where  $M$  is understood to mean the mass in the form of stars, the effect of gas and dust on the expansion being neglected. It follows that the dissolution time is

$$\Delta t_D = 1/v_{\text{exp}} [(3M/4\pi\rho_f)^{1/3} - R_i]. \quad (\text{III-2})$$

Thus if one assigns an initial radius, mass (in stars), and expansion speed, the dissolution time can be found on the basis of this simple model. Referring once more to Blaauw's (1962) compilation of data on nearby OB associations, values of  $M$  from 1 to  $5 \times 10^3 M_{\odot}$  and initial radii  $\sim 0$  to 20 pc

appear representative. In TABLE 6 computations of final radii and dissolution times for expansion speeds of 5 and 10 km/sec are summarized. Although the precise values of  $\Delta t_D$  should not be stressed owing to the crude model to which they refer, it is safe to conclude that associations expanding at moderate speeds will lose their identity in a few tens of millions of years. The oldest stellar evolutionary ages in Blaauw's list are about 30 million years, and there is a good correlation between the maximum evolutionary age and the linear extent of the groups considered.

The dispersal of associations poses a serious limitation on the methods of this investigation. Since some massive stars may acquire slightly higher than average 'expansion' velocity during formation in an unstable group and therefore become isolated from the latter in the course of a few times  $10^7$  years, the eventual dissolution of the parent group will render it impossible to locate the birthplace of the field star if this occurs in a time shorter than the star's kinematical age. Any attempt to locate the parent group of a star suspected (from evolutionary considerations) of being older than about 40 million years is hopelessly futile by the methods to be applied here.

TABLE 6

Dissolution Times for Spherical Associations of Uniform Density Expanding at Constant Speed. (Time Interval for Density Decrease to  $10^{-4}M_{\odot}/\text{pc}^3$ .)

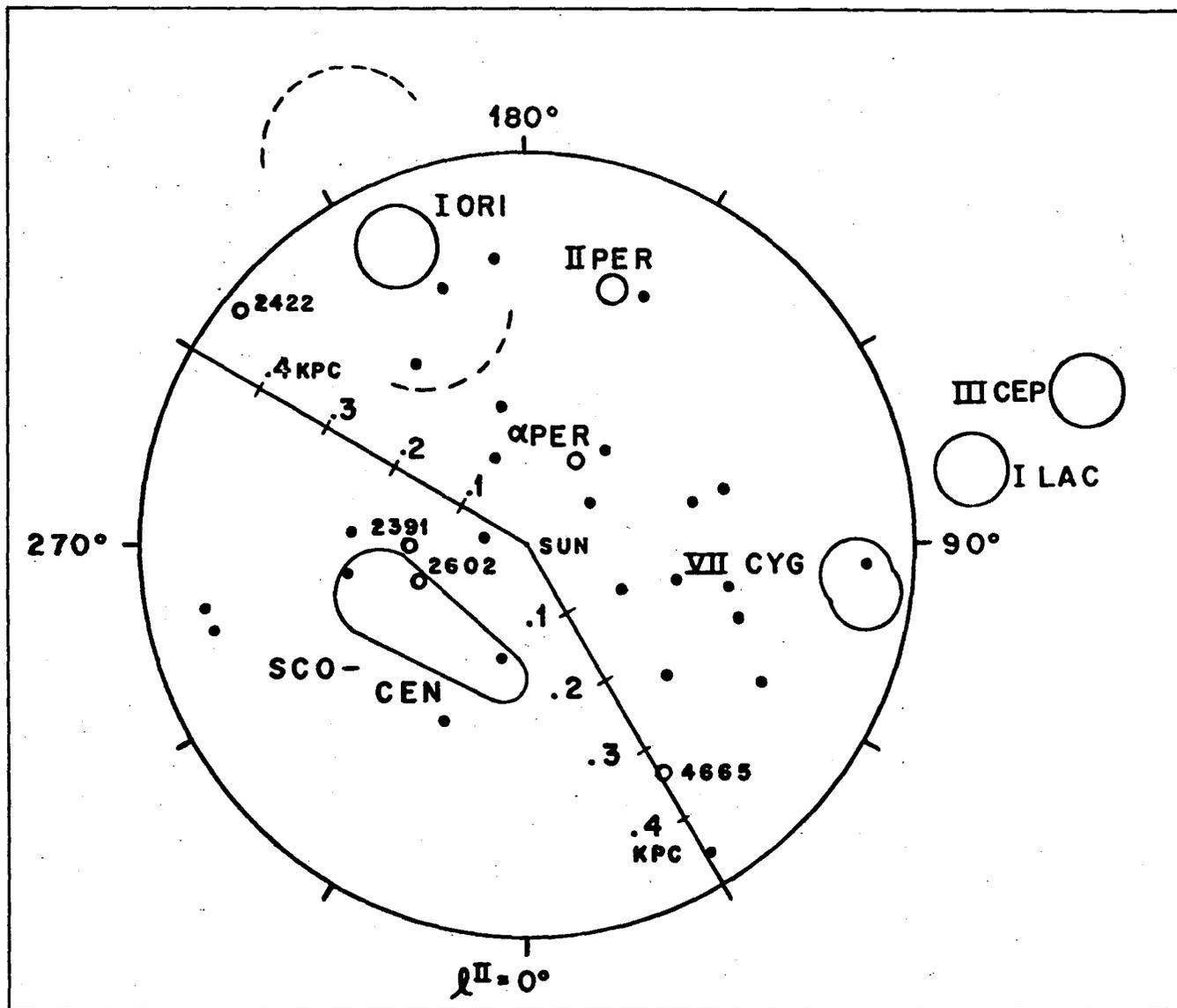
Initial Radius $R_i$ (pc)	Expansion Speed $v_{\text{exp}}$ (km/sec)	Mass (Stars) ( $M_{\odot}$ )	Final Radius $R_f$ (pc)	Dissolution Time $\Delta t_D$ ( $10^6$ yrs)
1.....	5.0	1000	140	26
		5000	230	45
	10.	1000	140	13
		5000	230	22
10.....	5.0	1000	140	24
		5000	230	43
	10.	1000	140	12
		5000	230	21
20.....	5.0	5000	230	41
		5000	230	20

The parent associations would likely have vanished, leaving at best small central clusters, which would have to be considered as the birthplaces. If the list by Johnson, Hoag, Iriarte, Mitchell, and Hallam (1961) is an indication, young clusters are quite numerous, so even if sufficiently accurate data were available for computing the peculiar space motions of each, the problem of associating a field star's orbit with one of them would be staggering.

In FIGURE 2 the non-cluster supergiants of TABLES 5A and 5B nearer than 500 parsecs have been plotted in projection on the galactic plane, where their positions may be compared with the major concentrations of O-B5 stars. Data for the groups are taken from Blaauw (1956), Kopylov (1958), Johnson et al. (1961), Alter, Ruprecht, and Vanýsek (1958), and Whiteoak (1962). The real extent of the Orion Aggregate (see section V) is indicated approximately by dashed arcs. Notice the clustering of Ib stars at  $d \sim 200-300$  pc in the direction  $l^{II} \sim 90^\circ$ , which may mark roughly a vanished OB association. (The nearby Cygnus OB aggregates are very extended and sparsely populated, according to Kopylov.) As will be mentioned in section V, a number of Ib-II stars may be connected with the Scorpio-Centaurus association, a few being shown in FIGURE 2.

This concludes the preliminary discussion. In the following section the mathematical determination of galactic orbits of young stars is treated together with sources of error and generalities concerning interpretation of the solutions. Six supergiants,  $\alpha$  Carinae,  $\alpha$  Leporis,  $\alpha$  Orionis,  $\beta$  Orionis,  $\rho$  Leonis, and  $\zeta$  Geminorum, have been chosen for specific orbital studies. It must be emphasized that these stars were not chosen because their space motions have easy interpretations. Each presents peculiar difficulties - despite recognition of the simplifying restrictions discussed in the Introduction - the totality of which can be expected to yield valuable insight into procedures possible for much larger numbers of stars.

FIGURE 2. The positions of field supergiants (TABLES 5A, 5B) nearer than 500 parsecs projected on the galactic plane together with the 7 nearest OB associations and 4 young galactic clusters: NGC 2422, IC 2391, 2602, and 4665.



## IV

### Calculation of the Galactic Orbits

#### A. The Observational Data

The data required to compute the velocity of any star relative to the Sun are the celestial coordinates, radial velocity, proper motion, and distance. TABLE 7 lists these data for the six stars to be examined in detail here. The equatorial coordinates are taken from the General Catalogue (Boss et al. 1937) and have been converted to galactic coordinates by means of the new Lund Tables (1961).<sup>2</sup> The radial velocities of  $\beta$  Ori,  $\alpha$  Lep,  $\alpha$  Car, and  $\rho$  Leo are those given by Wilson (1953); for  $\alpha$  Ori the average photospheric velocity adopted by Weymann (1962) is used, and the radial velocity of  $\zeta$  Gem is that listed in TABLE 1 of the paper by Kraft and Schmidt (1963). The proper motions, except for  $\zeta$  Gem, are taken from the catalogue by Morgan (1952) based on the N30 system; the

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2. Throughout this paper galactic coordinates in System II are used.

proper motion of the Cepheid is from the work of Blaauw and Morgan (1954), also in the N30 system (see, however, section IV C). The adopted distances are those given in TABLES 5A and 5B. For Canopus,  $M_V$  is quite well established because the trigonometric parallax is significant. Greenstein (1942) states: "A good Cape trigonometric parallax gives the absolute magnitude of  $\alpha$  Car as  $-3.8 \pm 0.3$ ". The Yale parallax  $0''.018 \pm .005$  (Jenkins 1952), combined with photometric data (Johnson 1957), yields  $M_V = -3.5$ , the value listed by de Vaucouleurs (1957). The spectroscopic absolute magnitude, determined by the method of Searle, Sargent, and Jugaku (1963), using Greenstein's atmospheric parameters, is  $-3.8 \pm 0.5$ . Utilizing this brightness, a distance of 40 pc is obtained.

TABLE 7

## Basic Data for Computing Space Velocities of 6 Supergiants

Star	MK	$\alpha$ -----1950-----	$\delta$	$l$ II
$\beta$ Ori	B8Ia	05 <sup>h</sup> 12 <sup>m</sup> .1	-08°16'	209°2
$\alpha$ Lep	FOIb	05 30.5	-17 51	221.1
$\alpha$ Ori	M2Iab	05 52.5	+07 24	199.3
$\alpha$ Car	FOIb-II	06 22.8	-52 40	261.2
$\zeta$ Gem	F7-G3Ib	07 01.1	+20 39	195.8
$\rho$ Leo	B1Ib	10 30.2	+09 34	234.9

Star	$b$ II	$v_r$ (km/sec)	$\mu_\alpha \cos \delta$ ---.001"/yr---	$\mu_\delta$	$d$ (pc)
$\beta$ Ori	-25°3	+20.7	+ 1.5	- 1.0	270
$\alpha$ Lep	-25.1	+24.7	+ 2.9	- 1.0	260
$\alpha$ Ori	-09.0	+20.6	+24	+ 9.0	120
$\alpha$ Car	-25.3	+20.5	+20.9	+17	40
$\zeta$ Gem	+11.9	+ 6.8	- 6.4	- 2.1	330
$\rho$ Leo	+52.8	+42.0	- 0.5	- 7.0	620

### B. Computation of the Space Velocity

The rectangular velocity components of the stars in TABLE 7 relative to the Sun ( $u$ ,  $v$ ,  $w$ ) were computed using a scheme described by Przybylski (1962). The quantities

$$A = (v_r \cos \delta - T_\delta \sin \delta) \cos \alpha' + T_\alpha \sin \alpha',$$

$$B = -(v_r \cos \delta - T_\delta \sin \delta) \sin \alpha' + T_\alpha \cos \alpha',$$

$$C = v_r \sin \delta + T_\delta \cos \delta,$$

are first computed, where  $T_\alpha = 4.74 \mu_\alpha \cos \delta d$ ,  $T_\delta = 4.74 \mu_\delta d$ , and  $\alpha' = \alpha - \alpha_c$ ,  $\alpha_c$  being the right ascension of the galactic center. The heliocentric velocity components are then

$$u = 0.8753A - 0.4835C,$$

$$v = 0.4113A + 0.5258B + 0.7446C,$$

$$w = 0.2542A - 0.8506B + 0.4602C.$$

Next, adding the components of solar motion (Kraft and Schmidt 1963)

$$u_\odot = +8 \text{ km/sec toward } l^{\text{II}} = 0^\circ, b^{\text{II}} = 0^\circ,$$

$$v_\odot = +13 \text{ " " " } l^{\text{II}} = 90^\circ, b^{\text{II}} = 0^\circ,$$

$$w_\odot = +6 \text{ " " " } b^{\text{II}} = 90^\circ,$$

one obtains the velocity components ( $u'$ ,  $v'$ ,  $w'$ ) with respect to the local standard of rest near the Sun. (It is immaterial in this study whether the above solar motion or the 'standard solar motion' is used, since the same transform

is applied to all objects considered.) The final transformation to cylindrical galactocentric components ( $\Pi, \Theta, Z$ ) is made by assuming that the local standard of rest executes circular motion about the galactic center with velocity  $\Theta(o)$ . A value  $\Theta(o) = 216$  km/sec has been adopted, corresponding to the Oort parameters  $A = +19.5$  and  $B = -6.9$  km/sec kpc and a distance  $R(o) = 8.2$  kpc of the Sun from the center of the Galaxy. Insofar as these assumptions may introduce errors in the results of the forthcoming sections, they will be discussed later.

TABLE 8 lists the computed space velocity in the heliocentric and galactocentric reference systems for the supergiants in TABLE 7, plus geometrical information needed to determine the galactic orbits. The quantity  $\theta - \theta_{\odot}$  is the angular difference between the direction of the galactocentric radius vector to the star and that to the Sun at present, positive for  $180^{\circ} < l^{II} < 360^{\circ}$ .

TABLE 8

Space Velocities and Geometrical Information for Six  
Supergiants.

Star	Velocity Relative to Sun (km/sec)			Galactocentric Velocity (km/sec)		
	u	v	w	$\Pi$	$\Theta$	Z
$\beta$ Ori	-15.8	-11.1	- 7.8	+ 7.8	218	- 1.8
$\alpha$ Lep	-16.3	-17.2	- 7.8	+ 8.3	212	- 1.8
$\alpha$ Ori	-20.5	- 9.7	+11.0	+12.5	220	+17.0
$\alpha$ Car	- 5.4	-19.9	- 4.6	- 2.6	210	+ 1.4
$\zeta$ Gem	- 8.8	- 1.2	- 8.8	+ 0.8	228	- 2.8
$\rho$ Leo	- 6.9	-38.4	+25.5	- 1.1	191	+31.5

Star	Positional Data		
	R (kpc)	$\theta - \theta_{\odot}$ (degrees)	z (pc)
$\beta$ Ori	8.44	+0.88	-114
$\alpha$ Lep	8.40	1.17	-110
$\alpha$ Ori	8.31	0.28	- 19
$\alpha$ Car	8.20	0.28	- 18
$\zeta$ Gem	8.52	0.60	+ 68
$\rho$ Leo	8.42	2.09	+495

### C. Numerical Solution of the Equations of Motion

In a cylindrical polar coordinate system, the equations of motion for a star subject to an axially-symmetric gravitational potential  $\Phi(R,z)$  are (e.g. see Elvius 1961, p.51ff.):

$$d\Pi/dt = K_R + \varphi^2/R, \quad (\text{IV-1})$$

$$R\varphi = h = \text{a constant}, \quad (\text{IV-2})$$

$$dz/dt = K_z, \quad (\text{IV-3})$$

where  $\Pi = dR/dt$ ,  $\varphi = R d\theta/dt$ ,  $Z = dz/dt$  are the velocity components, and  $K_R = \frac{\partial \Phi}{\partial R}$ ,  $K_z = \frac{\partial \Phi}{\partial z}$  are the central and perpendicular force components, respectively. For the stars considered here,  $z \ll R$  and  $\Delta R/R \ll 1$ , hence the dependences of  $K_R$  on  $z$  and of  $K_z$  on  $R$  are sufficiently weak that the  $z$ -motion may be considered de-coupled from the motion parallel to the galactic plane. (An exception is  $\rho$  Leonis, for which the variation of  $K_R$  and  $K_z$  with both  $R$  and  $z$  is barely significant. See APPENDIX B.) Equations (IV-1) and (IV-2) therefore completely describe the motion 'in' the plane while (IV-3) describes the motion perpendicular to the plane. The HI principal plane ( $b^{\text{II}} = 0^\circ$ ) is taken as the plane of mass symmetry (hence  $K_z$ -symmetry) of the Galaxy, and orbital inclinations are assumed negligible in all cases.

The differential equations plus the equations defining  $\Pi$ ,  $\vartheta$ ,  $Z$  were replaced by simple forward-difference equations. Expanding these and including equation (IV-2), one obtains the complete difference system

$$\begin{aligned} R_{n+1} &= R_n + \Pi_n \Delta t, \\ \Pi_{n+1} &= \Pi_n + (\vartheta^2/R + K_R)_n \Delta t, \\ \vartheta_{n+1} &= h/R_{n+1}, \\ \theta_{n+1} &= \theta_n + (\vartheta/R)_n \Delta t, \\ z_{n+1} &= z_n + Z_n \Delta t, \\ Z_{n+1} &= Z_n + (K_Z)_n \Delta t, \end{aligned} \tag{IV-4} \text{ through (IV-9)}$$

where  $K_R = K_R(R_n)$ ,  $K_Z = K_Z(z_n)$ ,  $t = n\Delta t$ ;  $n = 0, 1, 2, 3$ , etc. ((IV-10) through (IV-12)). Together with appropriate starting conditions, the above equations constitute a routine initial-value problem. The 'initial' values here,  $R_0, \Pi_0, \vartheta_0, \theta_0, z_0, Z_0, K_R(R_0), K_Z(z_0)$ , are the present ones, thus the angular momentum constant is  $h = R_0 \vartheta_0$  and  $\Delta t$  is negative. The force component  $K_R$  is read from tables prepared from Schmidt's (1956) final model for the galactic mass distribution and so happens to be a linear function of  $R$ . For the required run of  $K_Z$  with  $z$  the values determined by Oort (1960), based on the  $z$ -distribution of K giants (Hill 1960), have been employed. These values are preferable to earlier determinations (e.g. Schmidt 1956), as the former correspond

to a total mass density in the galactic plane more in agreement with observational statistics. The reader is referred to the monograph by Elvius (1961) for a discussion of this point. From basic values of the Oort  $K_2$  a much more detailed table was prepared by graphical interpolation.

Equations (IV-4) through (IV-7) were solved on a desk calculator with time steps  $\Delta t = 3.169 \times 10^6$ ,  $1.584 \times 10^6$ ,  $1.000 \times 10^6$ , and  $3.169 \times 10^5$  years, the longer intervals being used for the motion projected on the galactic plane whereas the z-motion typically involved the smallest step lengths. The number of integration steps taken varied from 5 to about 50, depending on the star. The galactic orbits of the program stars will be discussed individually in later sections.

#### D. Sources of Error

There are essentially three classes of errors which may conspicuously affect a galactic orbit computed by the methods used here. They are (a) uncertainties in the observational data from which the space velocity is computed; (b) possible erroneous assumptions concerning the kinematical characteristics of the local standard of rest and the mass distribution of the Galaxy, i.e. the model adopted;

(c) errors is the numerical approximation to the differential equations of motion and the resulting solutions.

In TABLE 9A, the probable errors in the distance, radial velocity, and proper motion of each star are given, the sources of each being described. The range in the galactocentric velocity components resulting from these uncertainties have been computed, generally by fixing one quantity (e.g.  $d$ ) and varying the others ( $v_r$ ,  $\mu$ ), then interchanging them. Uncertainty in the proper motion is the most serious of the probable errors, except for  $\alpha$  Ori, whose distance is quite indefinite. In the case of  $\rho$  Leo, an uncertainty in  $v_r$  equal to 10 times the measurement error (Wilson 1953) has been assumed to demonstrate that the resulting range in galactocentric velocity is still moderate. Since a star travelling at  $x$  km/sec moves about  $x$  parsecs per  $10^6$  yrs, probable errors in  $d$ ,  $v_r$ , and  $\mu$  evidently do not yield a very significant uncertainty in the motions of young stars. Of the objects in TABLE 9A, only the  $z$ -motion of  $\alpha$  Orionis appears indefinite owing to such errors.

Systematic errors in the observational data are not so easily brushed aside, especially differences between proper motions in the various systems. Most of the space velocity and galactic orbit computations done for this study

TABLE 9A

Probable Errors in Adopted Observational Data and in the Resulting Galactocentric Velocity Components.

Star	Probable Errors in				Range in		
	d (pc)	$\mu_{\alpha} \cos \delta$ (.001"/yr)	$\mu_{\delta}$	$v_r$ (km/sec)	II	$\vartheta$ (km/sec)	Z
$\beta$ Ori	40	0.6	0.6	0.5	+ 7.3	217.4	- 1.2
					8.2	219.3	- 2.4
$\alpha$ Lep	50	0.8	0.8	0.5	6.3	211.0	- 0.7
					8.9	213.7	- 2.6
$\alpha$ Ori	-20	0.5	0.5	0.5	12.5	218.1	+17.0
	+70				13.3	219.8	25.3
$\alpha$ Car	16	1.2	1.5	0.5	negligible		
$\zeta$ Gem	33	0.6	0.6	0.9	negligible		
$\rho$ Leo	80	0.7	0.7	5.	- 1.5	191.1	+28.8
					+ 2.0	196.1	36.8

Star	Sources of Probable Errors; Comments
$\beta$ Ori	d: Based on range in $M_V$ of 5 B5-B9Ia stars in h and $\chi$ Persei (Wildey 1962). $v_r$ : Wilson (1953). $\mu$ : <u>General Catalogue</u> (Boss et al. 1937).
$\alpha$ Lep	d: Estimated as the usual 20 per cent uncertainty in spectroscopic absolute magnitudes. $v_r$ : Wilson (1953). $\mu$ : GC.
$\alpha$ Ori	d: Based on scatter in $M_V$ of 15 M-type supergiants in h and $\chi$ Persei (Wildey 1962). $v_r$ : Wilson (1953); the radial velocity varies by about 10 km/sec (Weymann 1962). $\mu$ : GC.
$\alpha$ Car	d: Based on p.e. in trigonometric parallax, 0".005. $v_r$ : Wilson (1953). $\mu$ : GC; the N30 p.e. is three times this large.
$\zeta$ Gem	d: Kraft and Schmidt (1963). $v_r$ : Compromise between Wilson (1953) and Kraft and Schmidt values for Cepheids. $\mu$ : Blaauw and Morgan (1954).
$\rho$ Leo	d: Based on range in $M_V$ of 7 B0.5-B2Ib stars in h and $\chi$ Persei (Wildey 1962). $v_r$ : 10 times Wilson p.e. assumed. $\mu$ : GC.

were completed early in 1963, prior to publication of the Fourth Fundamental Catalogue (Fricke, Kopff, et al. 1963).

TABLE 9B compares  $\mu$  for the 6 supergiants of concern here in the N30 and FK4 systems; corresponding galactocentric velocity components, keeping  $d$  and  $v_r$  adopted previously, are also given. For  $\rho$  Leonis a large discrepancy in  $\mu$  exists, thus the velocity vectors differ greatly. The discrepancy in the proper motion of  $\alpha$  Leporis is also significant, however for the four remaining stars it is small enough that solutions based on the previously adopted data will be adhered to by and large.

TABLE 9B

Comparison of Proper Motions in the N30 and FK4 Systems  
and Corresponding Galactocentric Velocity Components.

Star	System	$\mu_{\alpha} \cos \delta$ (.001"/yr)	$\mu_{\delta}$	$\Pi$	$\varphi$ ----- (km/sec) -----	Z
$\beta$ Ori	N30	+ 1.5	- 1.0	+ 7.8	218	- 1.8
	FK4	- 3.6	- 1.8	+ 6.4	221	- 7.5
$\alpha$ Lep	"	+ 2.9	- 1.0	+ 8.3	212	- 1.8
		- 5.3	+ 1.4	+ 9.8	219	- 9.4
$\alpha$ Ori	"	+24.0	+ 9.0	+12.5	220	+17.0
		+26.0	+ 9.7	+12.6	220	+18.2
$\alpha$ Car	"	+20.9	+17.0	- 2.6	210	+ 1.4
		+28.7	+22.2	- 1.9	209	+ 2.9
$\zeta$ Gem	"	- 6.4	- 2.1	+ 0.8	228	- 2.8
		- 7.7	- 3.2	+ 1.1	228	- 5.3
$\rho$ Leo	"	- 0.5	- 7.0	- 1.1	191	+31.5
		- 8.8	- 6.1	+20.4	188	+20.4

The effect of an error in the assumptions that (1) the local standard of rest executes circular motion about the galactic center and (2) the distance scale and Oort parameters are those given in the previous sub-section on the orbital calculations will be briefly considered. It is presently believed that  $R(o) \sim 10$  kpc,  $A \sim +14$  and  $B \sim -9$  km/sec kpc (Takase 1962; Rubin and Burley 1963; Kraft and Schmidt 1963), and that  $\Pi(o)$  may not be negligibly small. None of these quantities noticeably affects the z-motion of stars relatively close to the plane of the Galaxy and at large R from its center. As for the motion in the plane, the velocity components  $\Pi$  and  $\Theta$  will be altered by the same absolute amount for all objects considered, e.g. for any star and its suspected parent cluster. If  $\Pi(o) \neq 0$ , the major axes of both orbits would shift in direction with respect to the Sun's galactocentric radius vector. The critical parameters, which may affect the time scale (kinematical ages), are the orbital eccentricities and the force component  $K_R$ . A moderate  $\Pi(o)$  will not alter the eccentricities much, because it will be offset by the increase in  $\Theta$  corresponding to the kinematical parameters given immediately above, and the increase in the distance scale. The central force, in the plane-motion approximation (quite good for all the stars

studied here), is the same within a few per cent regardless of whether the parameters  $A$ ,  $B$ ,  $R(o)$  adopted or those suggested above are used. At the Sun's distance  $R(o)$ , for example,  $K_{8.2}(\text{old}) = 1.85 \times 10^{-8}$  and  $K_{10}(\text{new}) = 1.77 \times 10^{-8}$   $\text{cm sec}^{-2}$ . This means that an object moving in either force field will experience nearly identical acceleration.

Solutions for the motion of  $\alpha$  Orionis have been obtained using the new  $A$ ,  $B$ , and  $R(o)$ , and they confirm the above expectations that the assumptions made concerning the motion of the local standard of rest and adoption of the Schmidt model are not critical. At worst one might anticipate changes in the time scale by one or two million years for kinematical ages around  $2 \times 10^7$  years, which uncertainties are no larger than those expected to arise through errors in the starting data.

Impressive statements are made in texts on numerical analysis (e.g. Kunz 1957) that the difference scheme employed in this study to approximate the galactic orbital solutions is quite inaccurate. Actually the truncation error introduced is surprisingly moderate. Equations (IV-1), (IV-3), and the equations defining the velocity components  $\Pi$ ,  $\Theta$ ,  $Z$  may each be written in the form

$$dy/dt = f(t,y). \quad (\text{IV-13})$$

Upon replacing  $dy/dt$  by the difference quotient  $\Delta y/\Delta t$ , one truncates a term  $1/2 (\Delta t)^2 d^2y/dt^2 + O(\Delta t)^3$ , the truncation error consisting mostly of the first term, usually denoted  $\epsilon$ , which represents the difference between the true value of  $y(t)$  and an approximation obtained by solving the difference analogue of (IV-13).

In the equations relevant here, one does not know the analytical form of the functions  $f$ , but by entering the orbital solutions themselves and extracting representative values of the dependent variables ( $y=R, \Pi, \theta, z$ , and  $Z$ , respectively), the derivatives  $d^2y/dt^2$  can be evaluated. Performing these operations, one finds the average truncation error in each dependent variable shown in TABLE 9C. These errors are for a single time step  $\Delta t$ ; the error in subsequent steps is not the same, but rather is amplified by a factor approximately

$$A_i = \left\{ 1 + \frac{\Delta t}{\epsilon(y)} [f(t, y+\epsilon) - f(t, y)] \right\}^{n-i}.$$

The error in the  $n$ -th step is then

$$\epsilon_n = \sum_{i=1}^n (\Delta t)^2 A_i d^2 y_i / dt^2. \quad (\text{IV-14})$$

The factors  $A_i$  and accumulated error in the tenth integration step  $\epsilon_{10}$  have been estimated from the representative kinematical quantities in the solutions, the latter being

included in TABLE 9C. The error in  $\phi$  is due solely to  $\epsilon(R)$  through the angular momentum integral.

TABLE 9C indicates that the truncation error accumulating after a few integration steps is quite small, amounting to less than the errors propagated through uncertainties in the observational data going into the starting position and velocity. With time steps of  $10^{14}$  seconds, the errors become noticeable after  $n = 10$  steps, particularly in the angle  $\theta$ , and would render any solution carried beyond about  $n = 15$  ( $5 \times 10^7$  years) quite inaccurate. The use of smaller time steps has limited advantage, for although according to equation (IV-14),  $\epsilon_n \propto (\Delta t)^2$ , the greater number of steps needed to reach a given epoch allows the amplification factor  $A$  to increase sharply. With  $\Delta t = 10^{13}$  sec, the truncation error at any time earlier than 10 million years ago is probably no better than 1/10 the corresponding error for  $10^{14}$  sec. This reduction is, however, not insignificant, as the table indicates.

Round-off error in the solutions amounts to approximately the truncation error per single step, but does not accumulate in the same manner. Tabular interpolation to find  $K_R$  and  $K_Z$  introduces random errors in  $\Pi$  and  $Z$  of 0.01 km/sec. Neither source of error is thus cause for concern.

TABLE 9C

Truncation Errors per Time Step and Approximate Accumulated Errors in the Finite-Difference Approximation to the Differential Equations of Motion.

Variable; units of error	$\epsilon_1(\Delta t)$ error in first step			$\epsilon_{10}(\Delta t)$ accumulated error	
	$10^{14}$ sec	$5 \times 10^{13}$ sec	$10^{13}$ sec	$10^{14}$ sec	$10^{13}$ sec
	R (pc)	0.3	0.08	----	6.
$\dot{R}$ (km/sec)	0.05	0.012	----	1.	----
$\dot{\theta}$ (km/sec)	0.009	0.002	----	0.02	----
$\theta$ (degrees)	0.02	0.005	----	0.3	----
(pc @ $R=R(0)$ )	2.7	0.7	----	40.	----
z (pc)	3.0	0.75	0.03	65.	0.65
Z (km/sec)	0.1	0.025	0.001	10.	0.1

It is clear that errors in the data needed to compute the present position and velocity of stars younger than 30 million years far outweigh other uncertainties. For greater ages, the finite-difference approximation to the equations of motion used becomes increasingly unreliable, forcing one to employ a more accurate method. The differential nature of a search for places of origin, however, allows marginal data and uncertainties in the adopted galactic model which would be inadmissible if exact absolute motions were desired.

### Locating the Places of Origin

Locating the parent group of any star is an uncertain and sometimes very complex task, if possible at all. In recent years considerable interest has arisen over the places of formation of the OB 'runaway' stars, two different methods having been employed for re-tracing the motion of these objects:

- (A) Mapping the proper motion backward in time on the celestial sphere and seeking a coincidence with the position of some association. The curvature of the projected stellar motion as well as the peculiar motion of the parent group may be considered. The most famous application was the study of AE Aurigae and  $\mu$  Columbae by Blaauw and Morgan (1954). This method alone yields dubious results, since it demonstrates (in successful cases) only a 2-dimensional coincidence, so a further consideration, e.g. the radial velocities, is needed before the place of origin of a star can be convincingly located.

(B) Calculation of the stellar motion projected onto the galactic plane and investigating points of intersection with orbits of known OB associations, whose motion about the center of the Galaxy is assumed circular. This method has been applied successfully by van Albada (1961) to locate the group from which the B3V high-velocity star 72 Columbae was ejected. The z-motion is not a crucial factor in the result for this star. It is obvious that the proper-motion method (A) would be difficult to apply to 72 Col, which at present lies an enormous distance away from its place of formation (I Scorpii) on the celestial sphere.

The method of locating places of origin applied in this dissertation is a refinement of van Albada's method, i.e. the 3-dimensional motion of each star is considered in detail and where possible the peculiar motions of the OB aggregates are taken into account. The variety of galactic motions of both young stars and stellar groups demands this approach, and furthermore the motion perpendicular to the galactic plane, so far ignored in studies of the parent associations of massive stars, is an important 'probe' in searching for these groups. Approximate evolutionary age

considerations will be employed as an elimination process to narrow the occasionally sizable number of prospective parent associations, but ultimately a reasonable agreement between the kinematical age and evolutionary age (in the sense of section II) must result for the star in question.

As FIGURE 2 shows, there are only 5 OB associations within a distance of 500 pc from the Sun, namely the  $\alpha$  Persei group, II Persei, I Orionis, the Scorpio-Centaurus association and a loose group in Cygnus. There are in addition a few young galactic clusters (Hogg 1959), but these will only be of interest in connection with the star  $\zeta$  Geminorum. Approximate galactic orbits of the centers of the five associations mentioned have been determined by the method outlined in section IV; these motions will be discussed together with the stars which may have formed in the groups.

#### A. Alpha Leporis, Canopus, and Scorpio-Centaurus

TABLES 10A and 10B summarize the galactic orbits of the F-type supergiants  $\alpha$  Leporis and  $\alpha$  Carinae. In FIGURE 3 the positions and velocity vectors (projected on the galactic plane) of these stars are shown with respect to the Sun and the Scorpio-Centaurus association. This diagram, together with the z-motion and rough age considerations suggest

strongly that  $\alpha$  Lep and  $\alpha$  Car formed in Scorpio-Centaurus, and even very crude orbital solutions concur with this belief.

The classical study of motions of the Scorpio-Centaurus stars was carried out by Blaauw (1946), who demonstrated a tendency among them toward movement in a common direction. Owing mainly to this study, it came to be accepted that the Sco-Cen stars constitute a moving group, like the Hyades, and in a more recent investigation Bertiau (1958) determined the convergent point of the velocity vectors, the 'stream' velocity, and the so-called k-term, which indicates an expansion of the association superimposed on the mean group motion.

TABLE 10A

ALPHA LEPORIS  
 0=260 PC, GCRV VR, N30 P.M., SCHMIDT KR, OORT KZ

TIME (MEGAYRS)	R (KPC)	DELTH (DEGREES)	ZDIST (KPC)	PI (KM/SEC)	THETA (KM/SC)	Z (KM/SC)
.0000	8.4000	1.1699	-.1100	8.3000	212.0000	-1.8000
-1.5639	8.3866	3.4873	-.1062	8.4201	212.3381	-2.9135
-3.1279	8.3730	5.8121	-.1006	8.5262	212.6820	-3.9882
-4.6919	8.3593	8.1445	-.0934	8.6184	213.0310	-5.0061
-6.2559	8.3454	10.4846	-.0847	8.6970	213.3847	-5.9493
-7.8199	8.3315	12.8325	-.0745	8.7617	213.7424	-6.8003
-9.3839	8.3174	15.1883	-.0630	8.8124	214.1038	-7.5428
-10.9478	8.3033	17.5521	-.0504	8.8484	214.4681	-8.1614
-12.5118	8.2891	19.9240	-.0369	8.8691	214.8349	-8.6426
-14.0758	8.2749	22.3040	-.0228	8.8737	215.2035	-8.9749
-15.6398	8.2607	24.6922	-.0083	8.8611	215.5732	-9.1496
-17.2038	8.2466	27.0886	.0063	8.8303	215.9432	-9.1608
-18.7678	8.2325	29.4933	.0208	8.7800	216.3129	-9.0059
-20.3317	8.2185	31.9061	.0350	8.7092	216.6812	-8.6858
-21.8957	8.2046	34.3272	.0485	8.6170	217.0473	-8.2047
-23.4597	8.1909	36.7564	.0612	8.5026	217.4103	-7.5703
-25.0237	8.1774	39.1937	.0727	8.3656	217.7692	-6.7932
-26.5877	8.1642	41.6390	.0829	8.2057	218.1229	-5.8871
-28.1517	8.1512	44.0921	.0915	8.0231	218.4704	-4.8680

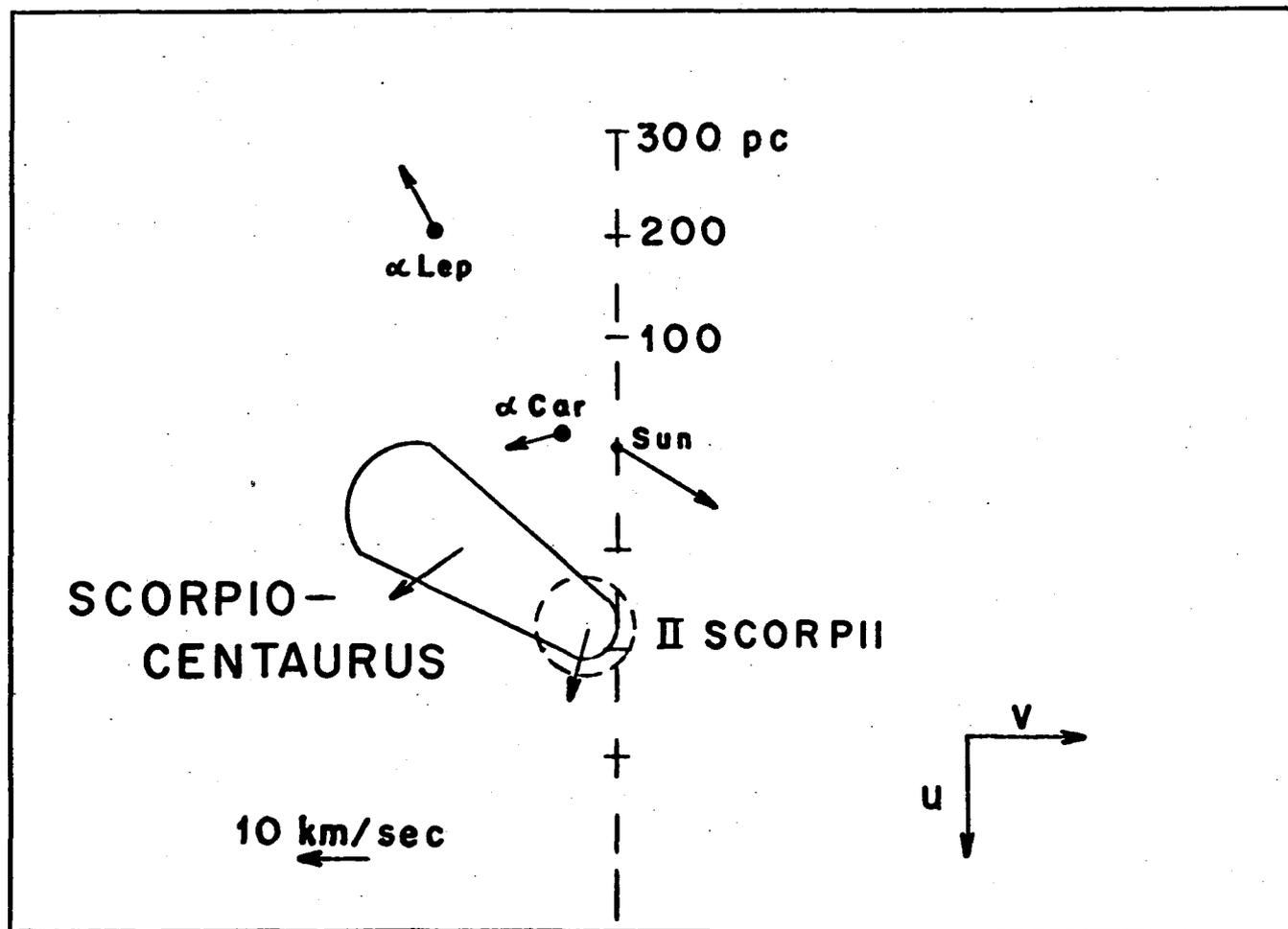
TABLE 10 B

ALPHA CARINAE  
 0=40 PC, GCRV VR, N30 P.M., SCHMIDT KR, OORT KZ

TIME (MEGAYRS)	R (KPC)	DELTH (DEGREES)	ZDIST (KPC)	PI (KM/SEC)	THETA (KM/SC)	Z (KM/SC)
.0000	8.2000	.2799	-.0170	-2.8000	209.6000	1.4000
-3.1279	8.2066	4.9624	-.0207	-1.5411	209.4306	.9604
-6.2559	8.2098	9.6391	-.0230	-.4713	209.3484	.4515
-9.3839	8.2096	14.3142	-.0236	.6016	209.3538	-.0898
-12.5118	8.2060	18.9915	-.0225	1.6702	209.4466	-.6252
-15.6398	8.1989	23.6748	-.0197	2.7267	209.6264	-1.1162
-18.7678	8.1885	28.3682	-.0154	3.7632	209.8924	-1.5269
-21.8957	8.1749	33.0753	-.0100	4.7717	210.2432	-1.8267
-25.0237	8.1580	37.8001	-.0038	5.7439	210.6771	-1.9919
-28.1517	8.1381	42.5462	.0025	6.6712	211.1917	-2.0081
-31.2796	8.1154	47.3172	.0087	7.5447	211.7840	-1.8708
-34.4076	8.0899	52.1167	.0143	8.3557	212.4505	-1.5871
-37.5356	8.0620	56.9480	.0188	9.0996	213.1864	-1.1753

FIGURE 3. Positions of  $\alpha$  Leporis and  $\alpha$  Carinae projected on the galactic plane relative to the Scorpio-Centaurus association and the Sun.

Velocity vectors with respect to the local standard of rest are shown, the mean group velocity of Scorpio-Centaurus according to Blaauw (1946, 1956), the remaining velocities as discussed in the text. The dashed line lies in the direction of the galactic center (down) and anti-center (up).



If the work of Blaauw and Bertiau had turned out to be entirely tenable, it would have made the present problem much easier to handle than has been the case. However, Petrie (1962) has criticized the treatment of Scorpio-Centaurus as a moving group (particularly to establish distances), and his discussion of the internal motions raises anew the question: Just how has the association moved as a whole in the past? In the present author's opinion astrophysical information must supplement purely mechanical arguments to answer this, such as the following:

- (1) Virtually all the interstellar material in Scorpio-Centaurus is found in or near the II Scorpii subgroup, the northernmost area of the association. There is strong evidence that star formation occurs only in areas where high concentrations of interstellar matter are present (Spitzer 1962), so one suspects that stars have not formed recently in the lower association, and possibly never did.
- (2) There exists a definite age gradient among the association stars, the older ones lying mostly in the intermediate and lower areas. FIGURE 4 outlines the group boundaries and shows the various subgroup regions, following largely Bertiau's terminology. The spectral

types of member stars in the lists of Bertiau (entire aggregate) and Hardie and Crawford (1961, II Scorpii) together with any adopted mass-spectral type calibration indicate a higher average mass per star in II Scorpii than the other regions. Herbig (1962a) notes the presence of H $\alpha$ -emission stars in the Scorpio-Ophiuchus dark clouds, which are evidently related to the upper association and believed to be very young.

- (3) Blaauw (1962) gives a total mass for the association of  $5800 M_{\odot}$ , a large fraction of which apparently occupies the II Scorpii area. Using the mass-spectral type calibration of TABLE 2 for member stars, a mass totalling less than 15 per cent of Blaauw's value is found. Presumably the remainder is interstellar gas and dust plus unseen stars, all of which are preferentially concentrated in the upper association. The center of mass of Scorpio-Centaurus thus lies in or near this area and so the net group motion is most likely weighted heavily by that of II Scorpii.

FIGURE 4. Subgroups of the Scorpio-Centaurus association in reference to the galactic coordinate system.

All boundaries are idealized and no definite members in the Carina-Vela region are known. The dots show approximate positions of Ib-II stars which may have formed in the association. Alpha Scorpii, in the II Scorpii subgroup, is the only definite supergiant member.

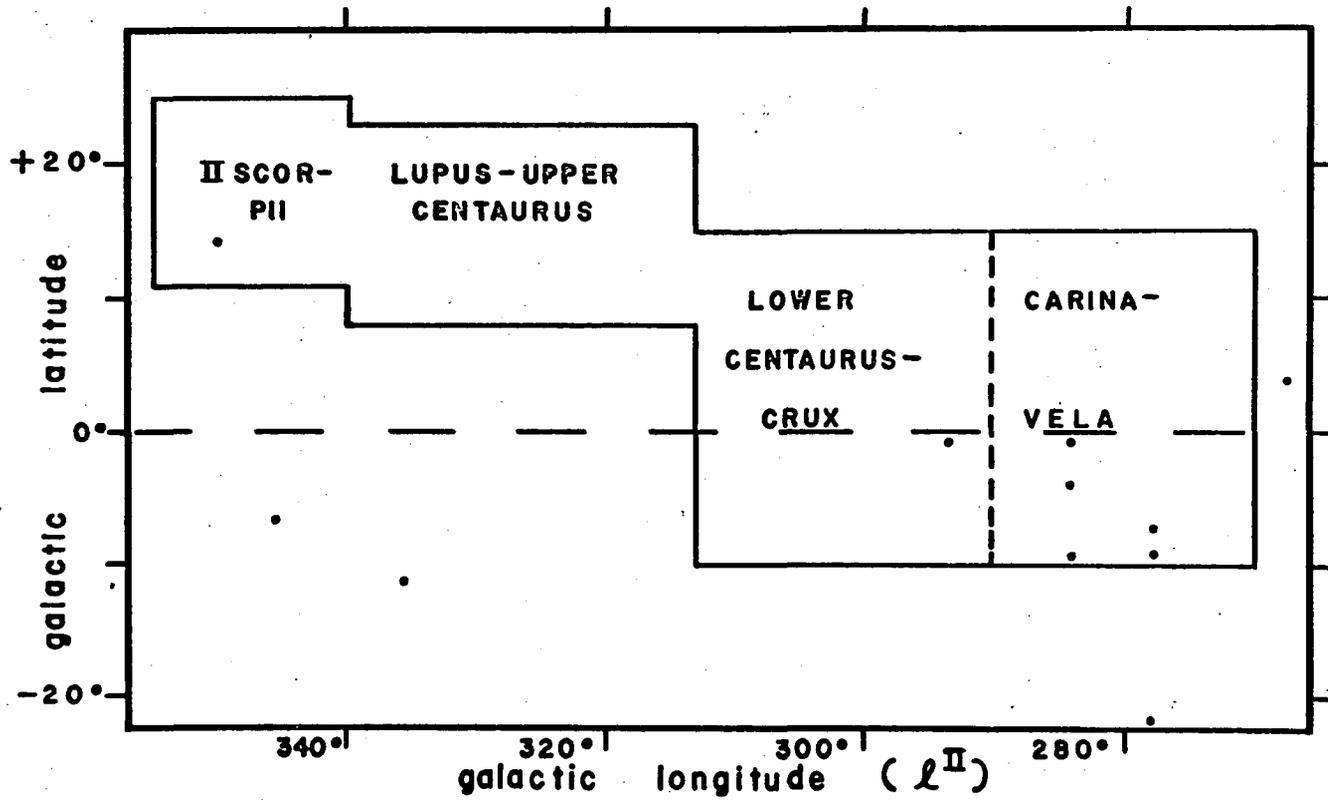


TABLE 11

Mean Space Motion Data for Four Scorpio-Centaurus Stellar Subgroups.

No.	Sub-group	Range in galactic long.	$\bar{l}^{\text{II}}$
1	II Scorpii	341° - 358°	351°
2	Upper Centaurus-Lupus	322 - 337	330
3	Intermediate	307 - 322	310
4	Lower Centaurus-Crux	290 - 307	300

No.	$\bar{d}$ (pc)	$\bar{S}'_0$ (km/sec)	Mean direction	Number of (1) Stars
1	170	12	341°	18
2	170	12.8	319	6
3	150	9.5	301	6
4	150	9.8	293	15

1. Includes only stars within 200 pc used to compute mean space velocities.

TABLE 11--Continued

No.	$\bar{u}'_0$ ---(km/sec)---	$\bar{v}'_0$ ---(km/sec)---	$\bar{u}'(3)$ ----(km/sec)----	$\bar{v}'(3)$ ----(km/sec)----
1	-----	-----	+10	- 3.0
2	+9.7	-8.4	+ 7.7	-10.4
3	+4.9	-8.1	+ 2.9	-10.1
4	+3.8	-9.0	+ 1.8	-11.0
mean (2)	+6.9	-9.5	+ 4.9	-11.5

No.	$\bar{z}$ (pc)	$\bar{R}$ (kpc)	$\theta - \theta_{\odot}$	$\Pi$ --(km/sec)--	$\varphi$
1	+60	8.03	+0 <sup>o</sup> .18	-10	213.5
2	+40	8.05	0 .60	- 7.7	206.1
3	+25	-----	-----	- 2.9	206.4
4	+10	8.13	0 .90	- 1.8	205.5
mean (2)	---	-----	-----	- 4.9	205

2. Blaauw (1946).

3. The measured mean velocity components were corrected for the difference between the standard solar motion and that adopted here, viz.

$$u' = u'_0 - 2 \text{ km/sec,}$$

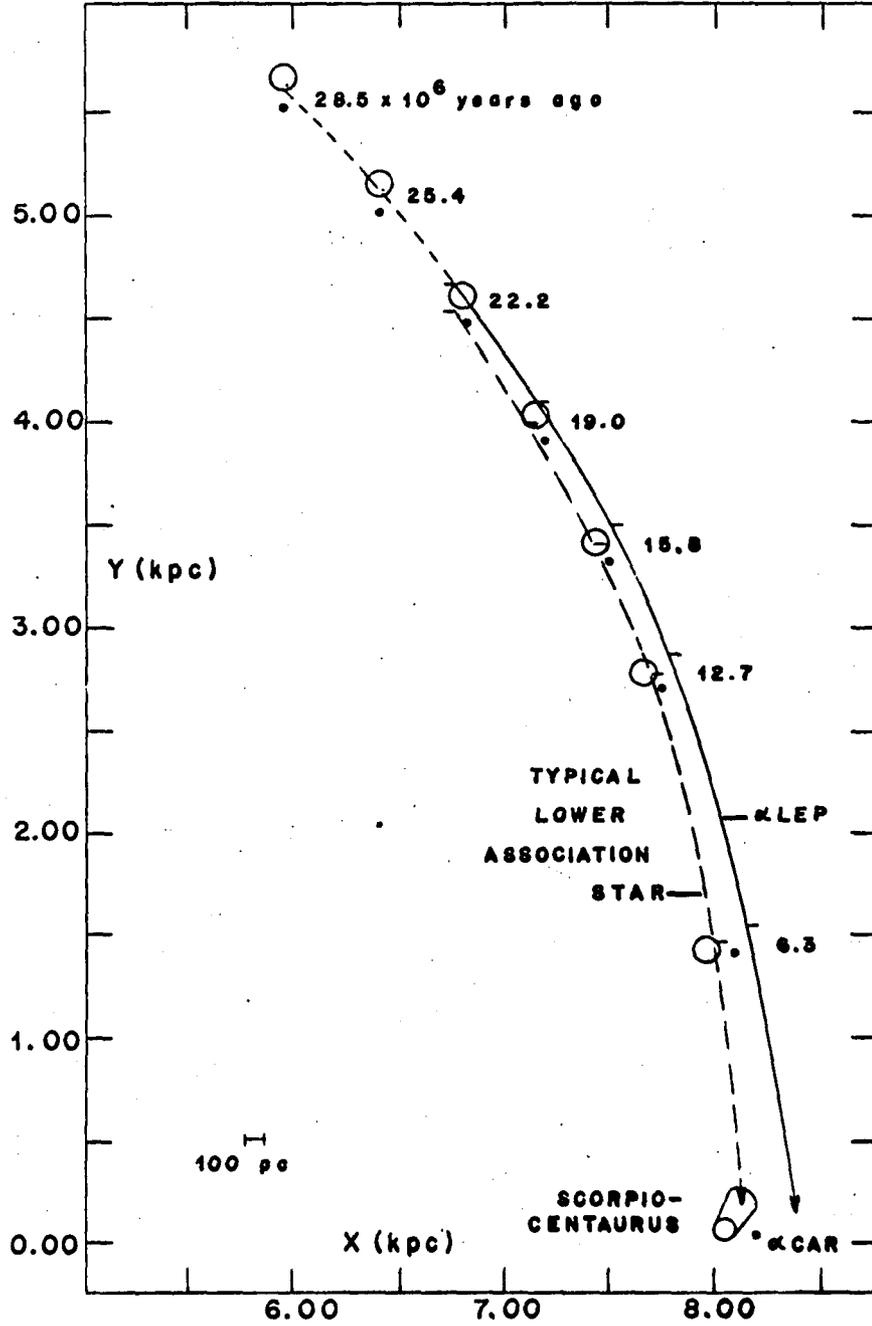
$$v' = v'_0 - 2 \text{ km/sec.}$$

With the above in mind, it has been assumed that the physically meaningful velocity centroid is not the average velocity of all association stars, but rather that of the II Scorpii members, which should reflect the motion of the massive gas-dust complex from which they, and somewhat earlier, the lower association stars formed. Petrie's investigation, like Bertiau's, shows clear evidence of an expansion, indicating a genetic relationship between the association stars. Ignoring the minor effect of differential galactic rotation, his Figure 3 may be used to estimate the mean velocity of subgroups of Scorpio-Centaurus, the results being listed in TABLE 11 together with similar data for II Scorpii derived from radial velocities and proper motions according to Bertiau. The divergence of the mean velocity vectors is striking; the II Scorpii stars, on the average, are moving almost toward the galactic center (relative to the local standard of rest) while the Lower Centaurus-Crux stars are moving nearly anti-parallel to the direction of galactic rotation. The speed of the 4 subgroups varies only slightly, thus taking  $\bar{S} = 12$  km/sec, convergence is found 200 pc distant about 17 million years ago, in good agreement with Bertiau's expansion age  $20 \pm 5$  million years.

Orbital solutions have been carried out for the II Scorpii subgroup and for the average of the Lower Centaurus-Crux stars. These motions, together with those of  $\alpha$  Lep and  $\alpha$  Car, are plotted in projection on the galactic plane in FIGURE 5. Notice first that the 'typical lower association star' did lie in the II Scorpii region about 16 million years ago, confirming the notion that (at least some) lower association stars formed here. A star of spectral type B2 has a main-sequence lifetime around 16 million years; the lower Sco-Cen stars average B3-B4, being understandably still main-sequence objects following the hypothesis that they formed very shortly before acquiring separation velocity. Between 19 and 22 million years ago the position of  $\alpha$  Leporis coincided with that of the upper association, suggesting its formation there, the star having acquired a separation velocity 15 km/sec relative to the place of formation, only 3 km/sec higher than the outermost B-type members of the group. There is a weaker indication that Canopus also formed in II Scorpii a few million years earlier; the lack of a definite spatial coincidence does not rule out this possibility since the size of the original star-forming region cannot be reliably established and moreover the uncertainties in the orbital solution are significant near 30 million years ago.

FIGURE 5. Galactic orbits of  $\alpha$  Lep and  $\alpha$  Car projected on the plane.

Circles: Successive positions of the II Scorpil region of Scorpio-Centaurus (the dashed circle of Fig. 3). Dots: Positions of Canopus. Tick marks on the paths of  $\alpha$  Lep and a typical lower association B-type star show positions at the times indicated. In the lower right corner the present positions of the 2 F supergiants are shown together with the entire Scorpio-Centaurus group.

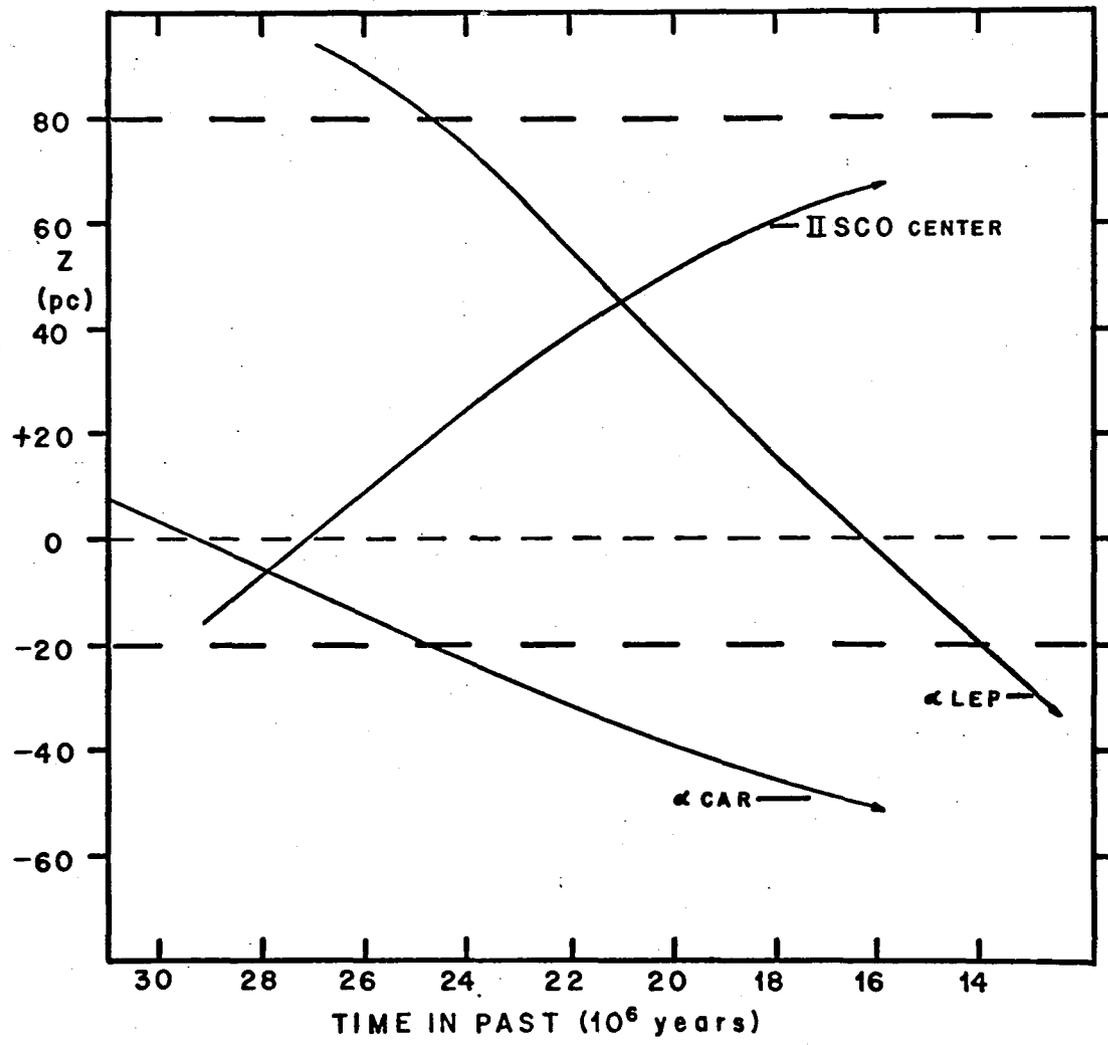


The motions of both F supergiants perpendicular to the galactic plane agree very well with formation in II Scorpii at the same epochs indicated by FIGURE 5, as shown in FIGURE 6. If  $\alpha$  Carinae did form in Scorpio-Centaurus, it is clearly one of the oldest such objects. Alpha Leporis and the lower association B stars formed more recently, and presently the II Scorpii area is occupied by a still younger generation.

Are the kinematical ages of the F supergiants found here consistent with independent evidence? TABLE 12 summarizes available age data for such stars and for the Scorpio-Centaurus group. FIGURE 7 illustrates the astrophysical relationship of  $\alpha$  Lep and  $\alpha$  Car to the association; from this the stars' evolutionary ages have been deduced as follows. Assume these stars are now in the early helium-burning stage b-c. The models indicate a change in luminosity  $\Delta M_{\text{bol}} = 1.0$  mag. since the onset of hydrogen burning. The  $M_V$ 's from TABLE 5B and Arp's (1958) bolometric corrections yield present bolometric magnitudes  $M_{\text{bol}} = -5.0$  and  $-4.1$  for  $\alpha$  Lep and  $\alpha$  Car, respectively, therefore the initial values were  $-4.0$  and  $-3.1$ . Interpolation in the main-sequence models yields masses  $8$  and  $6.5 M_{\odot}$  for the progenitors, whose ZAMS spectral types were B2.5 and B4 and whose main-sequence lifetimes were around 20 and 30 million years. This agreement with

FIGURE 6. The z-motions of  $\alpha$  Lep and  $\alpha$  Car between 30 and 14 million years ago.

The motion of the II Scorpii center is also shown and heavy dashed lines show the approximate present z-limits of the Scorpio-Centaurus association.



the kinematical ages indicated by the space motion is more than satisfactory. One further bit of evidence for the age of F supergiants follows indirectly from Abt's (1958) study of line-broadening mechanisms in Ib-star atmospheres. Abt found that the observed rotational velocities may be reasonably well explained if these stars evolved from progenitors of spectral type B2.5 on the average and have rotated differentially, i.e. with angular momentum conserved separately in shells, since evolving off the main sequence. Though the results are statistical in nature, it happens that  $\alpha$  Leporis was one of the stars investigated. The case for its formation in Scorpio-Centaurus is thus sound on all counts, while that for Canopus, though less convincing, is still plausible. The motions of these supergiants depend on present velocities derived from N30 proper motions. TABLE 9B indicates that the velocity of  $\alpha$  Lep based on the FK4 proper motion differs somewhat, and in fact an origin in an intermediate Sco-Cen area about 19 million years ago is indicated if the adopted motion of the association is maintained. Since no radical departure from the above conclusions arises, it is hardly worthwhile to pursue this detail further.



Alpha Leporis and Canopus may not be the only supergiants related to the Scorpio-Centaurus group. A coarse survey of the lists of de Vaucouleurs (1957) and MacRae (1964) provides a number of tempting objects, some of which were seen in FIGURE 2. TABLE 13 contains some relevant data for these stars; their apparent positions relative to the Sco-Cen subgroups are seen in FIGURE 4 and their positions in the color-magnitude diagram are plotted in FIGURE 7. No kinematical calculations have yet been carried out for these objects, so it is premature to propose a dynamical connection with the association. Future orbital determinations will probably indicate, however, that some of them formed in the group.

It has been tacitly assumed that the only significant force acting on the stars in question since they formed was the net galactic gravitational field. The separation velocities found (15 and 16 km/sec for  $\alpha$  Lep and  $\alpha$  Car, respectively) are encouraging in this respect; they probably resulted from random internal motions of a few km/sec superimposed on a steady expansion at 10-12 km/sec. At present the internal random velocities are about 1 km/sec. In the past, when there were few stars and much dust and gas, these motions may have been more violent, so if their distribution

TABLE 13

Luminosity Class Ib, II Stars Possibly Related to the Scorpio-Centaurus Association.

( $m_V$  is either  $HP_V$  given by A. de Vaucouleurs (1957) or V by Johnson (1957) or MacRae (1964). Other data from de Vaucouleurs, Keenan (1963), and TABLE 5B.)

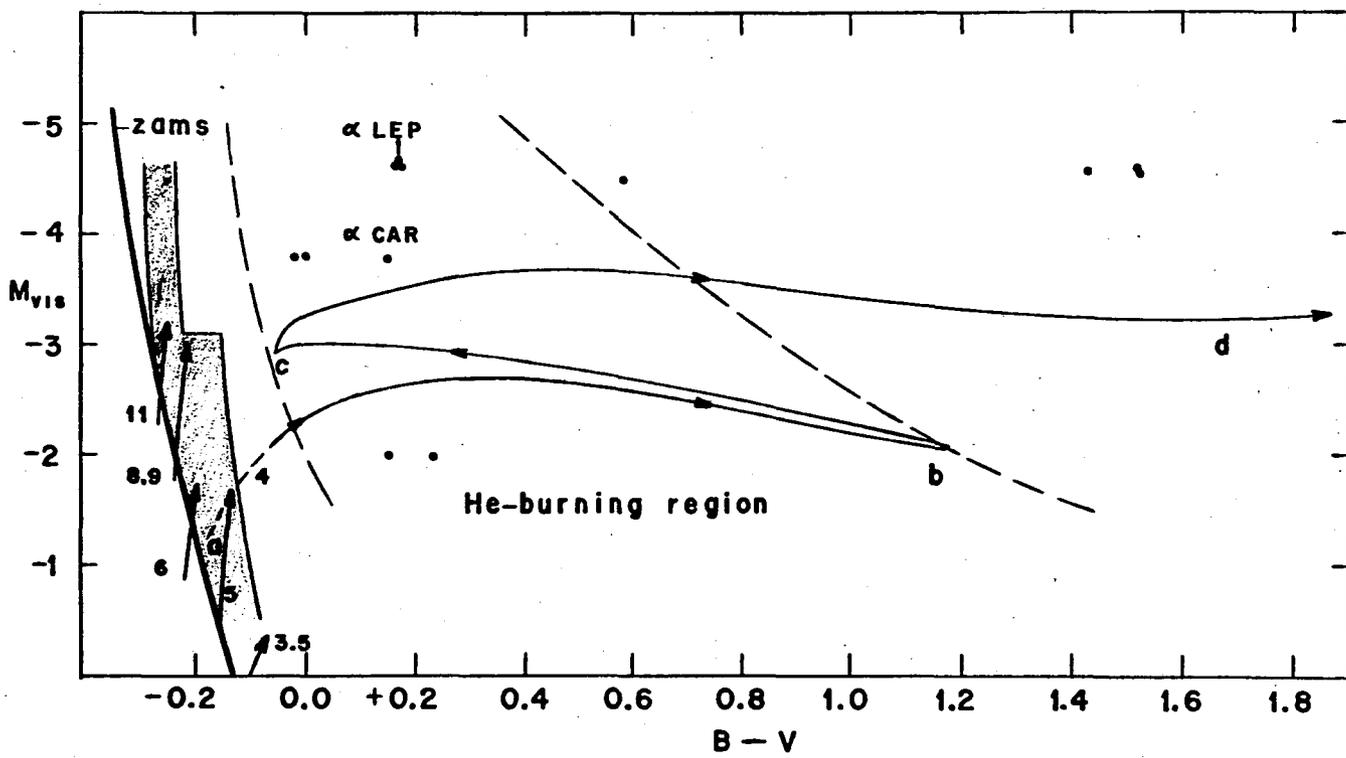
HR	Name	MK	$m_V$	$M_V$	d(pc)
2803	$\delta$ Vol	F8I-II	3.94	----	335
	c Car	B8II	3.88	-3.4	275
3634	$\lambda$ Vel	K5Ib	2.24	-4.7	230
3699	$\iota$ Car	FOIb	2.25	-4.7	240
3890	$\upsilon$ Car A	A9II	2.95	-2.0	100
4050	$\eta$ Car	K5Ib	3.41v	-4.7	420
	s Car	FOI-II	4.00	----	160
4467	$\lambda$ Cen	B9II	3.15	-3.4	200
6461	$\beta$ Ara	K3Ib	2.90	-4.4	240
6553	$\theta$ Sco	FOIb	1.86	-4.8	150

FIGURE 7. The color-magnitude diagram of certain and probable members of Scorpio-Centaurus between galactic longitudes 270 and 340 degrees, according to Bertiau (1958) and A. de Vaucouleurs (1957), together with some evolved stars (luminosity classes Ib, Ib-II, II) which may be genetically related.

The shaded region contains approximately 80 B-type stars. Stars of the II Scorpii subgroup are not included, as these appear to be distinctly younger.

The evolutionary tracks for stars of 3.5, 6, and 11 solar masses, according to Henyey, LeLevier, and Levee (1959) are shown, plus tracks for 5 and 8.9 solar masses (Polak 1962; Hoyle 1960) during the hydrogen-burning phase. Advanced evolutionary stages of a star of 4 solar masses according to Hayashi, Hoshi, and Sugimoto (1962) are also shown, the letters on this track denoting the following:

- a----b: gravitational core contraction subsequent to hydrogen exhaustion;
- b: onset of helium burning;
- c----d: core contraction during helium-exhaustion phase;
- d: onset of carbon burning.



resembled a Maxwellian distribution of speeds, the probability of speeds as high as 5 km/sec (i.e. total velocities  $V_{\text{exp}} + V_{\text{rand}} = 15$  km/sec) is a few per cent. There are about 100 known and provisional members of Scorpio-Centaurus, thus a few stars in addition to  $\alpha$  Lep and  $\alpha$  Car may have attained total speeds sufficient for them to escape, a second reason for thinking that some of the supergiants in TABLE 13 may have formed in this association.

The influence of the gravitational attraction of the association on the computed orbits can be shown negligible unless the initial mass was 10 times greater than the 5800 solar masses presently contained and all this material was compressed in a volume of the order 10 parsecs in radius, within or near which  $\alpha$  Lep and Canopus formed. These are improbable circumstances. It is not possible to assess the influence of internal dynamical encounters since there is no information available on the space distribution of member stars during the early stages of the association's history. Evidently the simple analysis presented is satisfactory and these complications have not affected the motions of the two F supergiants.

### B. Alpha, Beta Orionis, and the Orion Aggregate

Estimation of the present distance of  $\alpha$  Orionis is necessarily based on indirect means, the trigonometric parallax  $0''.005 \pm .004$  (Jenkins 1952) being hardly useful. The well-known correlation between luminosity and the width of the emission core in the calcium H and K lines is one such method and direct comparison with M supergiants whose distance has been independently established is another. Wilson and Bappu (1957) give  $M_V = -5.7$  for Betelgeuse, based on extrapolation of the current calibration  $M_V = f(W_O)$ , and later confirmed when three red supergiants in  $\eta$  and  $\chi$  Persei were measured to extend the calibration (Wilson 1959). The photometric data and spectra of  $\alpha$  Ori and the M supergiants in the Perseus aggregate are so similar that it is reasonable to assume physical identity, wherein, for example, the mean absolute magnitude of the ten brightest Perseus stars,  $M_V = -5.3$  (Willey 1962) would apply to Betelgeuse.

Recent photometric measurements obtained by Johnson and his associates (Johnson 1964) using infra-red techniques (Johnson 1962; Johnson and Mitchell 1963; Low and Johnson 1964) indicate that  $\alpha$  Orionis is heavily reddened, thus complicating the luminosity estimation. The visual absorption

of 1.3 magnitudes found yields distances corresponding to  $M_V = -5.3, -5.7,$  and  $-6.9$  of 90, 110, and 190 pc, respectively. The indicated reddening law,  $A_V = 7E_{B-V}$ , is similar to that obeyed by the obscuring matter in parts of the Orion association (Johnson and Borgman 1963). The latter extreme luminosity cannot be entirely ruled out, for YZ Persei, an M2Iab star in the  $\eta$  and  $\chi$  Persei aggregate, has  $M_V = -6.7$  according to Wildey. On the other hand, absolute magnitudes as (relatively) faint as  $-4$  are found among the Perseus stars, but such a value for Betelgeuse would correspond to a distance much less than 100 pc and it would appear strange that a significant trigonometric parallax does not exist. The value  $d = 120$  pc adopted for this study yields  $M_V$  close to the luminosity given by the Wilson-Bappu effect; however, before the anomalous reddening of  $\alpha$  Ori became known to this writer orbital computations were carried out using  $d = 190$  pc (Weymann 1962), and it is not obvious at this point that these latter solutions should be ruled out.

Having established that Betelgeuse is a very luminous star, it is appropriate to next examine evidence for a genetic relation between it and the early-type supergiants of I Orionis. Ten evolved O- and B-type stars are known to be members of the Orion association, most being found in the

extensive list by Sharpless (1952). TABLE 14 contains relevant data for 9 of these objects ( $\pi^5$  Ori, B2III, is omitted). Absolute magnitudes are based on the calibration by Bappu et al. (1962), who used the strength of  $H\gamma$  as a luminosity criterion, and the intrinsic colors are from these authors or Johnson and Borgman. Bolometric corrections are due to Arp (1958) or Schmalberger (1960), found by entering graphs prepared by Wildey which employ  $(B - V)_0$  as the argument. The color-magnitude diagram of these stars is plotted in FIGURE 8 together with hydrogen-burning evolutionary tracks for a range of masses plus the post-main-sequence track for a star of  $15.6 M_{\odot}$  (Hayashi and Cameron 1962). From this diagram and the results of the models the initial bolometric magnitude has been estimated for each of the stars in TABLE 14 as described in the previous subsection for  $\alpha$  Lep and  $\alpha$  Car. The masses of the main-sequence progenitors follow by interpolation in ZAMS models and the main-sequence lifetimes are found by entering TABLE 2. According to FIGURE 8, only  $\beta$  Ori has (probably) reached the helium-burning evolutionary phase, thus the main-sequence lifetimes in TABLE 14 must be close to  $\tau_{\text{nuc}}$  for the other stars and not much different for Rigel. The most luminous of these stars have ages in agreement with the figure  $4 \times 10^6$  years frequently quoted for the

association (Sharpless 1962), but the three B giants have ages more like that given by Blaauw (1962),  $15 \times 10^6$  years, as the maximum stellar ages in the group.

Next, one may compare  $\alpha$  Orionis with other M-type supergiants believed to be cluster members. This comparison is shown in TABLE 15, where for  $\alpha$  Ori and  $\alpha$  Sco the data in the first five columns is due to Johnson (1964), that for the  $\eta$  and  $\chi$  Persei stars having been taken from Wildey, and information for BU and TV Gem is according to Crawford, Limber, Mendoza, Schulte, Steinman, and Swihart (1955). A later paper by Hardie, Seyfert, and Gullledge (1960) questions the reality of the I Geminorum association, so the results for the last two stars should be viewed with caution. The estimated age of I Orionis follows from TABLE 14 and the above discussion. The ages of the II Scorpii stars were found by plotting photometric data of Hardie and Crawford (1961) in a color-magnitude diagram, noting that the brightest stars turn off the zero-age main sequence at  $M_V \sim -3$ . There are apparently very young stars connected with this group, viz. the O9.5 'runaway' star  $\zeta$  Oph (Blaauw 1959) and pre-main-sequence objects, but no ages greater than about 11 million years are apparent. In the  $\eta$  and  $\chi$  Persei association there exist three branches off the main sequence: If the M

supergiants are associated with the uppermost branch, their age is about  $6 \times 10^6$  years, but if they have evolved across the H-R diagram from the intermediate branch, an age 17 million years is assigned (Wildey 1962). This latter value may be criticized, however, as it is based on a stellar model ( $15.6 M_{\odot}$ ) having an uncommon initial chemical composition for a Pop I star. With the composition of TABLE 2, such a star has a main-sequence lifetime of 8 or 9 million years.

Inspection of TABLES 14 and 15 reveals that the M supergiants possess ages in the range  $6 - 11 \times 10^6$  years, compared to  $3 - 11 \times 10^6$  years for the I Orionis OB giants and supergiants. Alpha Orionis, evidently similar to the other M stars in TABLE 15, therefore has an age comparable to the typical luminous I Ori star. Further evidence that M supergiants are massive stars comes from the eclipsing binary VV Cephei, M2Ia-Iab+B (Sahade 1960). Pecker and Schatzman (1959) give  $47 M_{\odot}$  for the M star, McLaughlin (1961) states  $M = 24 M_{\odot}$  if the two components have equal mass, and Peery (1962) finds  $84 M_{\odot}$  if the orbital inclination is  $90^{\circ}$ . All determinations yield a radius at least  $1000 R_{\odot}$ .

By comparison, evidence for moderate masses (with corresponding ages, say, over  $3 \times 10^7$  years) of the M supergiants is feeble, being based almost solely on the spread in

FIGURE 8. The positions of several I Orionis members in luminosity classes I-III compared with  $\alpha$  Ori in the color-magnitude diagram.

The evolutionary tracks of stars of 11, 20, and 30 solar masses in the hydrogen-burning phase are due to Henyey et al. (1959). The track of a star of  $15.6 M_{\odot}$  based on models by Hayashi and Cameron (1962) shows the evolution in the following stages:

- a---b: gravitational core contraction subsequent to hydrogen exhaustion;
- b---c: helium burning in core plus hydrogen burning in outer shell;
- c---d: core contraction during helium exhaustion;
- d: onset of carbon burning in core.

The initial composition of this star is  $X = 0.90$ ,  $Y = 0.08$ . The dashed track a---b' shows the post-hydrogen-exhaustion core contraction phase for a star of  $10.1 M_{\odot}$  having  $X = 0.61$ ,  $Y = 0.37$  initially (Hayashi and Cameron 1962).

The zero-age main sequence in this diagram and in FIG. 7 is based on values given by Blaauw (1963) for  $M_V > -3$  and is extended to brighter luminosities using photometric data by Walker (1961) for the extremely young cluster NGC 6611. The helium-burning zone in the color-magnitude diagram follows data according to Hayashi, Hōshi, and Sugimoto (1962), referring here to the region where the star spends most of its time in this stage.

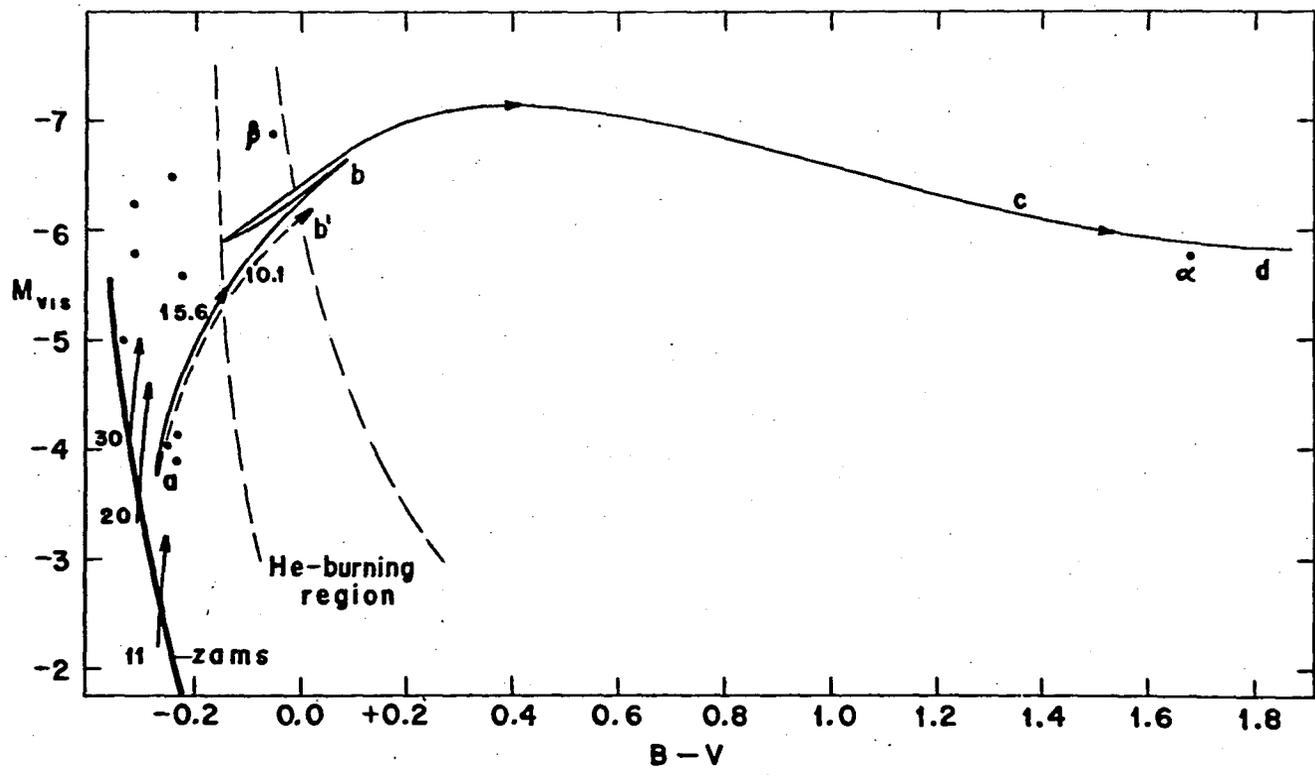


TABLE 14

Summary of the Evolution of I Orionis Stars of Luminosity  
Classes I - III

Star	MK	$M_V$	$(B - V)_0$	B.C.
$\pi^4$ Ori	B2III	-3.9	-0.23	-2.6
$\beta$ Ori	B8Ia	-6.9	-0.05	-0.8
$\delta$ Ori	O9.5II	-5.8	-0.31	-2.9
$\iota$ Ori	O9III	-5.0	-0.33	-3.0
$\epsilon$ Ori	B0Ia	-6.5	-0.24	-2.6
$\zeta$ Ori	O9.5Ib	-6.2	-0.31	-2.9
HD37756	B3III	-4.2	-0.23	-2.6
$\kappa$ Ori	B0.5Ia	-5.6	-0.20	-2.2
55 Ori	B2III:	-4.0	-0.25	-2.6

Star	$M_{bol}$	$M_{bol,i}$	$M/M_\odot$	$\tau_{ms}(10^6 \text{ yrs})$
$\pi^4$ Ori	-6.5	-5.5	12	11
$\beta$ Ori	-7.7	-7.0	20	5.5
$\delta$ Ori	-8.7	-8.2	30	3.0
$\iota$ Ori	-8.0	-7.9	30	3.5
$\epsilon$ Ori	-9.1	-8.7	40	2.8
$\zeta$ Ori	-9.2	-8.7	40	2.8
HD37756	-6.6	-5.5	12	11
$\kappa$ Ori	-7.8	-7.2	22	5.0
55 Ori	-6.6	-5.7	13	10

TABLE 15

 $\alpha$  Orionis and M Supergiants in OB Associations Compared

Star	MK	$M_V$	$(B - V)_O$	B.C.
$\alpha$ Ori	M2Iab	-5.8v	+1.68	-1.4
$\alpha$ Sco	M1Ia-Ib	-5.8v	1.68	-1.4
HD13136	M2Ib	-5.2	1.91	-2.0
AD Per	M2.5Iab	-5.4v	1.82	-1.7
HD14404	M1Ib	-5.7	1.80	-1.6
SU Per	M3.5Iab	-5.2v	1.84	-1.8
RS Per	M4.5Iab	-5.2v	1.83	-1.7
+56 <sup>o</sup> 595	M0.5Iab	-5.1	1.79	-1.6
HD14580	M0Iab	-5.5	1.70	-1.3
HD14826	M3Iab	-5.6	1.67	-1.3
+58 <sup>o</sup> 445	(cM1)	-5.1	1.98	-2.2
YZ Per	M2Iab	-6.7v	2.19	-3.6:
BU Gem	M1Ia	-6.1v	----	-1.6:
TV Gem	M1Iab	-5.3v	----	-1.6:

Star	$M_{bol}$	Parent Group	Group Age (10 <sup>6</sup> yrs)
$\alpha$ Ori	- 7.2	I Ori ?	3 - 11
$\alpha$ Sco	- 7.2	II Sco	5 - 11
HD13136	- 7.2	I Per	6 - 17:
AD Per	- 7.1	"	"
HD14404	- 7.3	"	"
SU Per	- 7.0	"	"
RS Per	- 6.9	"	"
+56 <sup>o</sup> 595	- 6.7	"	"
HD14580	- 6.8	"	"
HD14826	- 6.9	"	"
+58 <sup>o</sup> 445	- 7.4	"	"
YZ Per	-10.3:	"	"
BU Gem	- 7.7:	I Gem	6:
TV Gem	- 6.9:	"	"

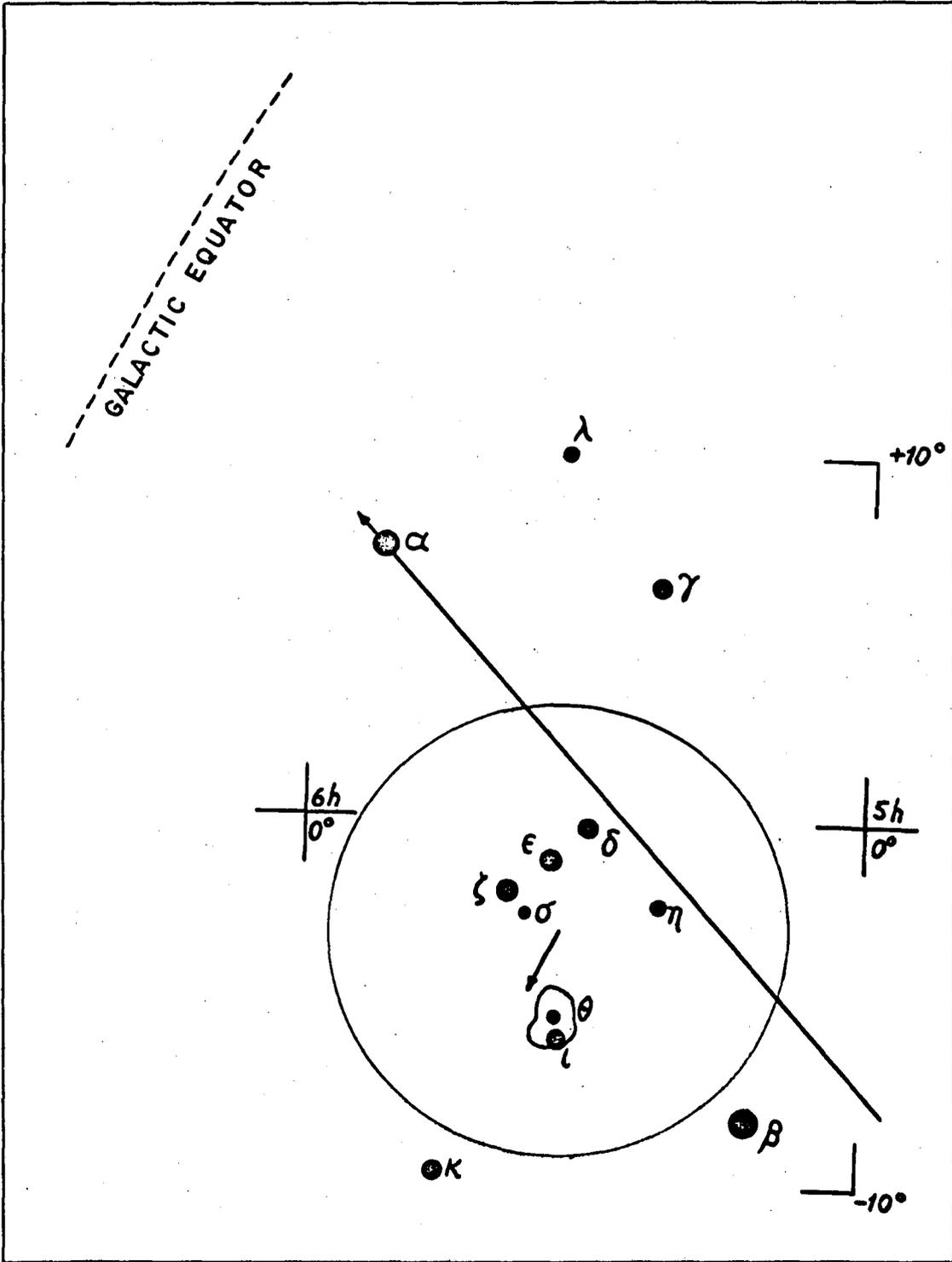
$M_V$  of the red supergiants in  $\eta$  and  $\chi$  Persei and in the main-sequence lifetime variation in the same association, which ranges up to perhaps 40 or 50 million years. It is not impossible that the least luminous M stars in the group did evolve from the lowermost main-sequence turnoff, but the majority almost certainly have come from much higher up.

The motivation for this lengthy discussion lies in the forthcoming interpretation of the motion of  $\alpha$  Orionis. Briefly, if the star is younger than 20 million years, the Orion association is the only plausible birthplace, whereas a greater age would suggest formation near the galactic plane (30 to 40 million years ago). The latter alternative is very difficult to defend.

Apparent proximity of Betelgeuse to the Orion group as well as its proper motion hint at formation in the association, as indicated by FIGURE 9. The first criterion is, unfortunately, spurious, while the latter is only partially valid. The proof (or disproof) of a dynamical relationship between the supergiant and I Ori requires knowledge of the past position and motion of both.

FIGURE 9. The constellation Orion, showing the apparent relationship of  $\alpha$  Orionis to the brightest I Orionis stars.

The proper motion of  $\alpha$  Ori corrected for solar parallactic motion following Smart (1960) and the charts by Pearce and Hill (1931) is shown, neglecting curvature and projection effects. The large circle outlines the hydrogen cloud in the Orion Aggregate according to Menon (1958), its motion due to galactic rotation being indicated by a short arrow.



The galactic orbit of Betelgeuse, computed from the data of TABLE 7, is presented in TABLE 16, and TABLE 17 compares this motion with that based on a present distance of 190 pc. The use of the FK4 proper motion instead of the N30 would lead to negligibly differing space motion. As for the motion of the Orion association, all the data going into its determination (except the apparent position in the sky) are highly uncertain. Various techniques yield a mean distance of the association stars, or of the Orion Nebula cluster, differing by more than 100 pc, and the discovery of different absorption-to-reddening ratios within the area (Johnson and Borgman 1963) does not improve the situation. It is not certain that the Nebula region coincides with the center of the larger stellar association, though it is quite possibly the nucleus of the enormous HI gas cloud existing there (Menon 1958). Radial velocities of stars in the group and the various gaseous components vary considerably in published investigations, and are difficult to interpret because of the bizarre internal motions (expansion of the Orion Nebula, the Nebula cluster stars, the HI cloud, and possibly rotation of the stellar association (Parenago 1953, 1954, 1959; Strand 1958; Menon 1958)). Proper motions of the association stars are so few and so discordant on absolute systems that they must

TABLE 16

## The Galactic Orbit of Alpha Orionis

(The star probably did not exist at times below the line.)

time in past $10^6$ yrs	R kpc	$\theta - \theta_0$ degrees	$\Pi$ km/sec	$\Theta$ km/sec	z pc	Z km/sec
0.00	8.310	+0.28	+12.50	219.8	- 18.7	+17.03
0.95	8.298	1.75	12.26	220.1	- 35.2	16.83
1.90	8.286	3.23	12.00	220.4	- 51.4	16.52
2.85	8.274	4.72	11.74	220.8	- 67.4	16.09
3.80	8.263	6.20	11.46	221.1	- 82.8	15.53
4.75	8.252	7.69	11.18	221.4	- 97.7	14.87
5.70	8.241	9.19	10.88	221.6	-112	14.10
6.66	8.230	10.69	10.58	221.9	-125	13.23
7.61	8.220	12.19	10.27	222.2	-138	12.30
8.56	8.209	13.70	9.95	222.5	-150	11.30
9.51	8.200	15.21	9.62	222.8	-160	10.24
10.46	8.191	16.72	9.28	223.0	-170	9.14
11.41	8.181	18.24	8.93	223.3	-178	7.99
12.36	8.172	19.77	8.57	223.5	-186	6.81
13.31	8.164	21.29	8.20	223.7	-192	5.61
14.26	8.156	22.82	7.82	224.0	-197	4.38
15.21	8.148	24.35	7.43	224.2	-201	3.13
16.16	8.141	25.88	7.04	224.4	-204	1.87
17.11	8.135	27.42	6.64	224.5	-205	0.61
-----						
18.06	8.128	28.95	6.24	224.7	-205	- 0.7
19.01	8.122	30.49	5.83	224.9	-204	- 1.9
22.2	8.10	35.6	4.4	225	-197	- 4.4
25.4	8.09	40.8	3.0	226	-175	- 8.4
28.5	8.08	46.0	1.5	226	-141	-12
31.7	8.07	51.2	- 0.1	226	- 97	-15
34.8	8.07	56.4	- 1.6	226	- 45	-17
38.0	8.08	61.6	- 3.2	226	- 2	-17
41.2	8.09	66.8	- 4.7	226	+ 41	-17
44.4	8.10	72.0	- 6.3	225	+ 92	-15

TABLE 17

Comparison of the Space Motion of Alpha Orionis for Assumed Present Distances (a) 120 and (b) 190 Parsecs, Since 16 Million Years Ago.

time in past ( $10^6$ yrs)-----	R(a) (kpc)---	R(b) (kpc)---	$\Delta\theta$ (a) --(degrees)--	$\Delta\theta$ (b) --(degrees)--	z(a) -----	z(b) -----
0.0	8.31	8.38	0.28	0.43	- 18.7	- 29.7
3.17	8.27	8.34	5.21	5.27	- 72.6	-112
6.34	8.23	8.30	10.19	10.15	-121	-192
9.51	8.20	8.26	15.21	15.09	-160	-262
12.7	8.17	8.22	20.28	20.07	-188	-320
15.8	8.14	8.19	25.37	25.09	-203	-362

be considered virtually unknown. At the distance of the Orion association the proper motion, however, indicates a tangential velocity quite small compared to the radial velocity.

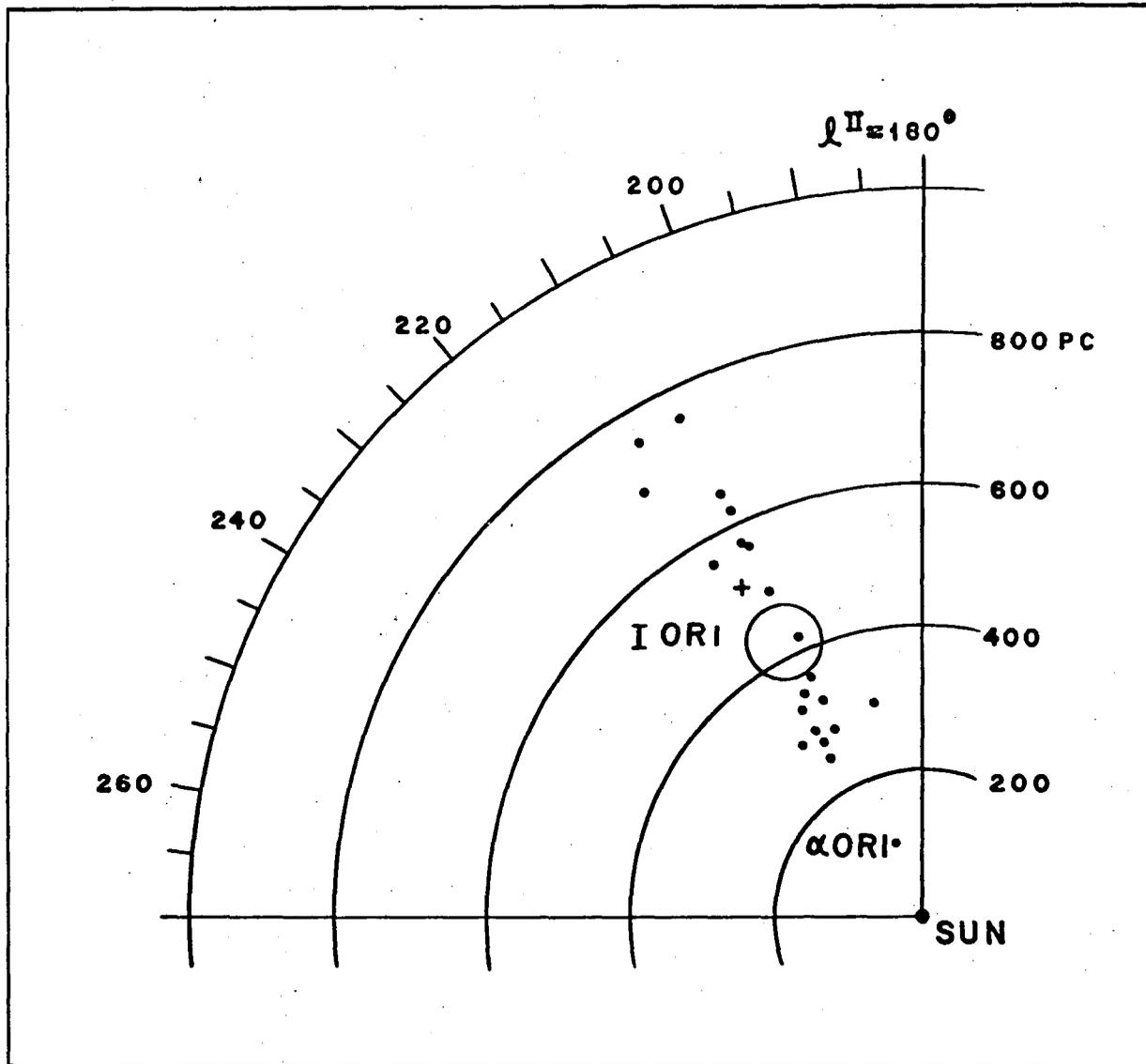
In addition to the above uncertainties, the spatial extent of I Orionis is not well known. A linear diameter  $D \sim 100$  pc usually is assigned (Blaauw 1956), but individual absolute magnitude estimates suggest  $D$  is 3 or 4 times greater. Bappu et al. (1962) find a mean distance modulus for 23 of the brighter I Ori stars equal to 8.1, but the scatter is huge, implying distances ranging from 250 to 750 pc! A plot of these stars in projection on the galactic plane (FIGURE 10) demonstrates that the idea of I Ori as a fairly compact, spherical aggregate is naive. The highly elongated distribution presented may be the consequence of a selection effect in galactic longitude, and to be sure, some revision of individual distance moduli can be expected, yet the great extent of the Orion Aggregate must be accepted as quite possibly real. This is not at all a unique phenomenon; Blaauw has repeatedly emphasized the existence of a complex of physically and kinematically different subgroups within the Scorpio-Centaurus association (Blaauw 1959, 1962, 1964), some aspects of which were described in the previous subsection.

The 'classical' photometric distance of the I Orionis center is about 420 pc (Johnson et al. 1961); this value plus an alternative 370 pc - chosen to allow for a slight reduction due to possibly widespread heavy reddening - will be adopted henceforth. FIGURE 10 implies an extension of the association to as near as 300 pc; until a convincing demonstration is given that the outlying Orion stars have achieved their present location through an overall expansion of the aggregate, the likelihood that star formation has occurred outside the accepted boundary must be admitted.

The radial velocity is the most important factor determining the present galactocentric velocity of the Orion association. A range of  $v_r$  from +28 km/sec (Markowitz 1949: Orion Nebula cluster) to +35 km/sec (Struve and Titus 1944: mean of  $\theta^1$  Ori A, C, D and  $\theta^2$  Ori A) is assumed to apply to the I Orionis center. There is no other approach possible, for to adopt some mean velocity averaged over the larger aggregate would risk selection of those stars nearer than the group center, which may possess peculiar motions of considerable magnitude relative to the center of mass.

The proper motion of  $\theta^1$  Ori A is given in the General Catalogue (Boss et al. 1937), but no Trapezium proper motions are known in the FK4 system. A few of the outer supergiants

FIGURE 10. Distribution of 20 luminous I Orionis stars projected on the plane of the Galaxy, relative to the usually accepted association configuration (open circle) and the position of the Orion Nebula cluster (cross) according to Strand (1958).



have published values in all systems, the tendency being for GC and N30 values around  $+0''.001-.003$  and FK4 values of the same magnitude but with negative sign. These do not, however, refer to the association center, and since 'internal' motions may be responsible for a large part of the proper motions of these stars, it would not be wise to apply means over them to the association center. Numerous velocity computations have been made, using  $v_r$ ,  $\mu$ , and  $d$  in the range suggested by the preceding discussion; these reveal that the  $\Pi$  component of the association center is dependent mainly on  $v_r$ ,  $\Theta$  on  $\mu_\delta$ , and  $Z$  on  $\mu_\alpha \cos \delta$ . The most serious influence of the proper motion is on the velocity perpendicular to the galactic plane, but because the group diameter is at least 100 pc - comparable to its present distance from the plane - it is a fairly good approximation to ignore the  $z$ -motion. This is tantamount to assuming a small positive value of  $\mu_\alpha \cos \delta$ , and for the sake of consistency the proper motion of  $\theta^1$  Ori A on the GC system has been adopted.

TABLE 18 gives several sets of data for the center of I Ori which have been employed to compute the past space motion. The trajectory of  $\alpha$  Orionis perpendicular to the galactic plane is shown in FIGURE 11 relative to the Orion group, and the motions of star and association projected on

TABLE 18

## Alternative Data for the I Orionis Center

$$\alpha = 5^{\text{h}}33^{\text{m}}.0 \quad \ell^{\text{II}} = 209^{\circ}.0 \quad (1950)$$

$$\delta = -05^{\circ}25' \quad b^{\text{II}} = -19^{\circ}.3$$

Set No.	d (pc)	$v_r$ (km/sec)	$\mu_{\alpha} \cos \delta$ --- (.001"/yr) ---	$\mu_{\delta}$
1A	370	+28.3	+3.0	+3.0
B	370	+34.6	+3.0	+3.0
2A	420	+28.3	+3.0	+3.0
B	420	+34.6	+3.0	+3.0

Set No.	R (kpc)	$\Delta\theta$ (degrees)	z (pc)	$\Pi$ ----- (km/sec) -----	$\Theta$	Z
1A	8.54	+1.10	-130	+18.5	217.4	+3.3
B	8.54	+1.10	-130	+23.8	214.7	-0.5
2A	8.57	+1.36	-140	+18.9	217.5	-4.1
B	8.57	+1.36	-140	+24.2	214.8	-7.9

FIGURE 11. The motions of  $\alpha$  and  $\beta$  Orionis perpendicular to the galactic plane since 12 million years ago, assuming the stars existed at these times.

The solution for Betelgeuse is based on a present distance of 120 parsecs. The Orion association, shown for comparison, is assumed fixed in  $z$  during the interval in question.

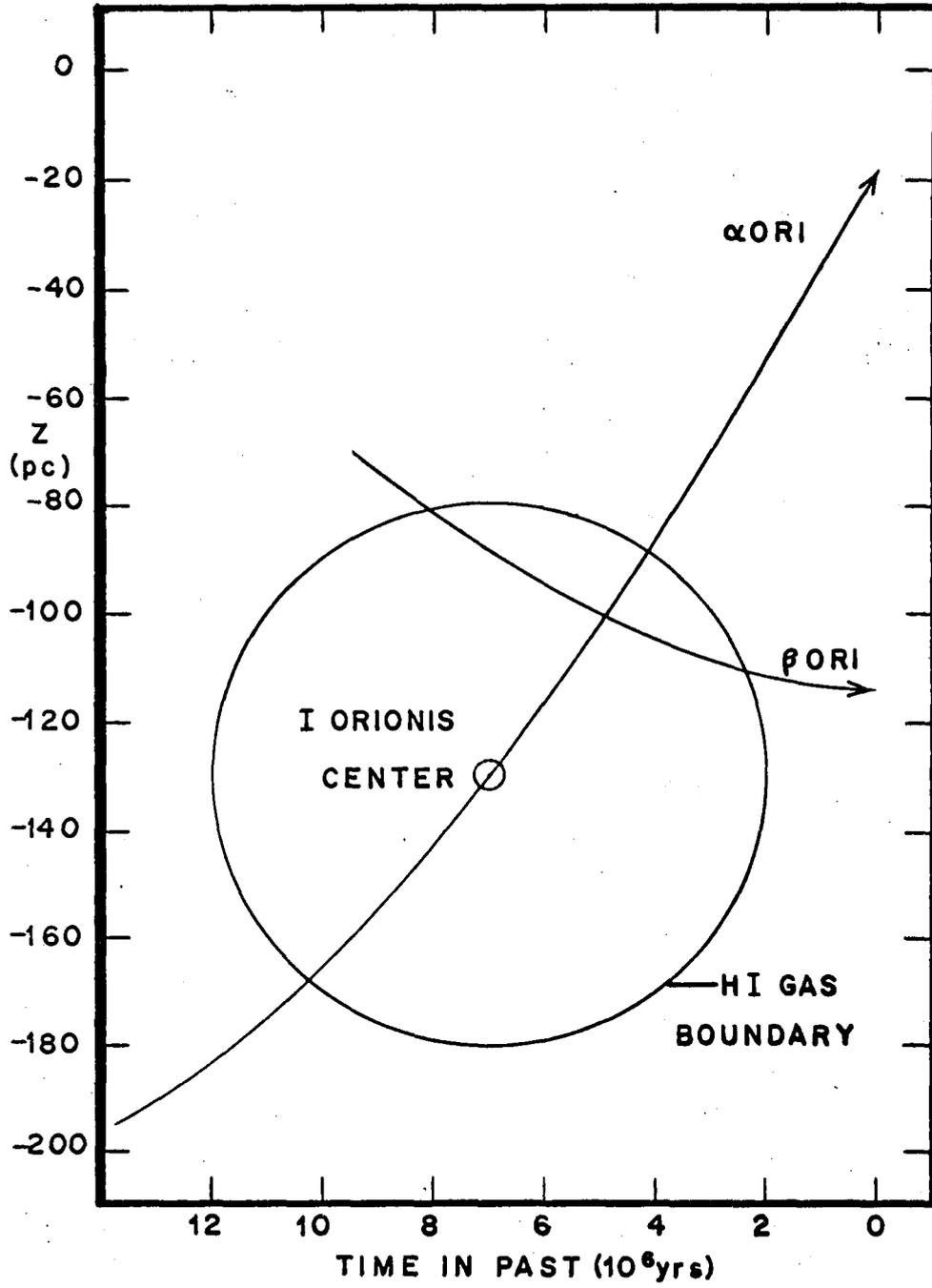
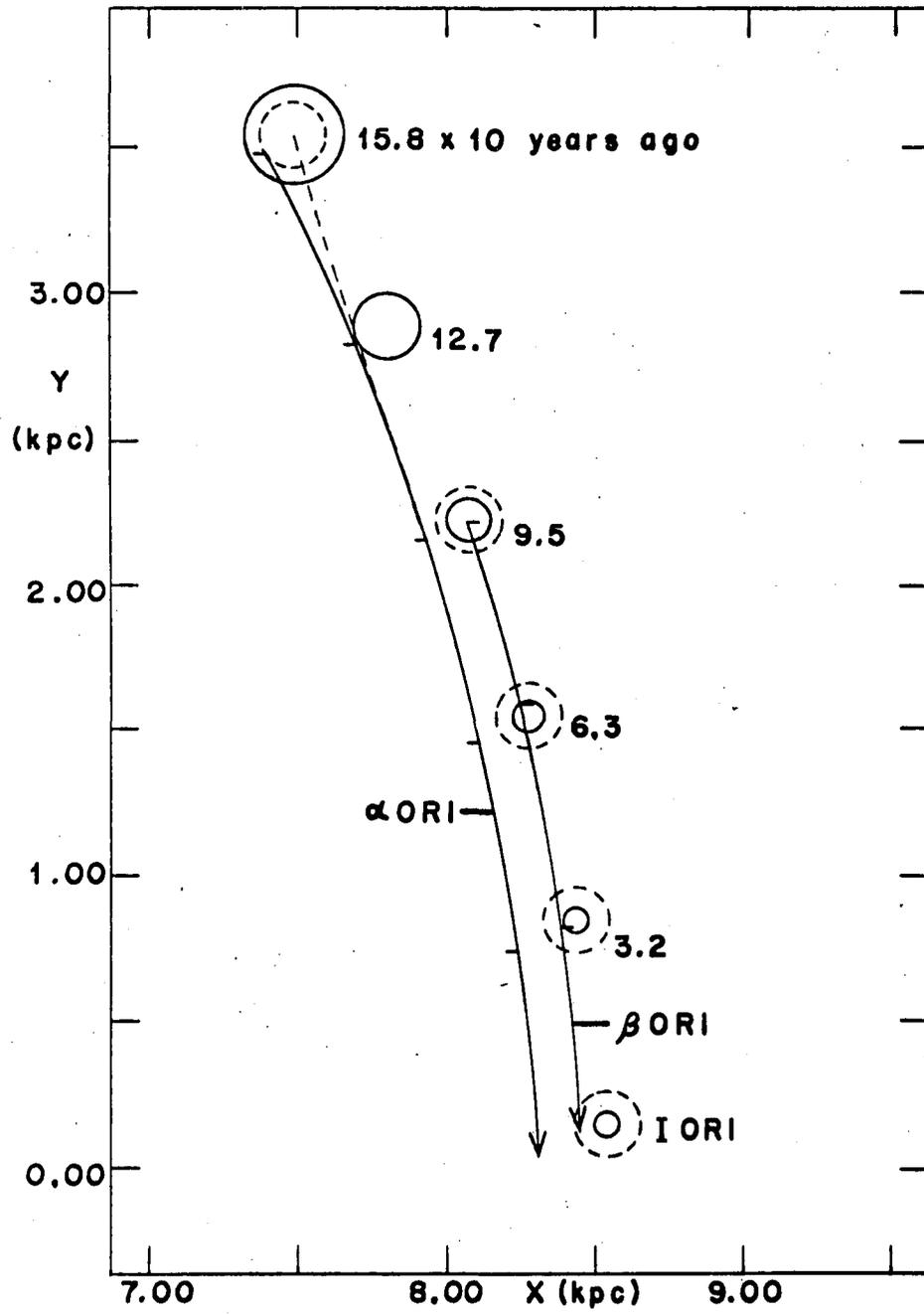


FIGURE 12. The motion of  $\alpha$  and  $\beta$  Orionis parallel to the galactic plane, relative to I Orionis.

The stellar orbits are based on data given in section IV, that of I Ori following from data set 1A, TABLE 18. Tick marks on the stellar tracks show positions at the times labelled. The dashed circles show the idealized association boundary including outer stars (see FIG. 2 and FIG. 10), while the small solid circle at the present location marks the hydrogen cloud boundary. Solid circles of decreasing size illustrate a hypothetical change in the star formation area of the aggregate, and the dashed portion of the trajectory of Betelgeuse between 15.8 and 9.5 million years ago is discussed in the text.



the plane are illustrated in FIGURE 12, the orbit of I Ori being based on data set 1A. Although the z-motion of  $\alpha$  Ori hints at formation in the association, FIGURE 12 indicates that no real spatial coincidence has occurred during the past 16 million years (at least). If all the data entering the respective orbital solutions is correct - and one really believes that Betelgeuse formed in I Orionis - then some rather unusual circumstance is required to explain the failure of the trajectories to intersect in the past. A number of such hypotheses have been incorporated into FIGURE 12 to this end. No single explanation of this sort can account for all of the large discrepancy in R between star and association, yet there may be some truth in all of them, e.g. the idea that the star-formation area in Orion was larger than the quoted present association diameter. A strong retardation of the early motion of  $\alpha$  Ori, either due to the gravitational field of the aggregate or to some gas-dynamical effect (such as the decelerated expansion mechanism of Öpik (1953)) operative on the pre-stellar material from which the star later formed, seems ruled out. In the first case, a mass in excess of  $10^6 M_{\odot}$  contained in a volume not over 30 pc in radius would have had to act on Betelgeuse. It appears that I Orionis encompasses at most one tenth this much material

(Menon 1958; Parenago 1959). A decelerated expansion also seems unlikely, for the 'expansion interval' required must be at least 5 million years, and it is doubtful if an expanding shell can really continue to accrete matter efficiently from the surrounding medium over this length of time, when its dimensions increase enormously. The initial trajectory shown by the dashed curve in FIGURE 12 is based on an actual decelerated expansion model, but the conditions required for this solution are so restrictive (e.g. formation of  $\alpha$  Ori a few million years after the expansion began) that the reality of such motion is highly dubious.

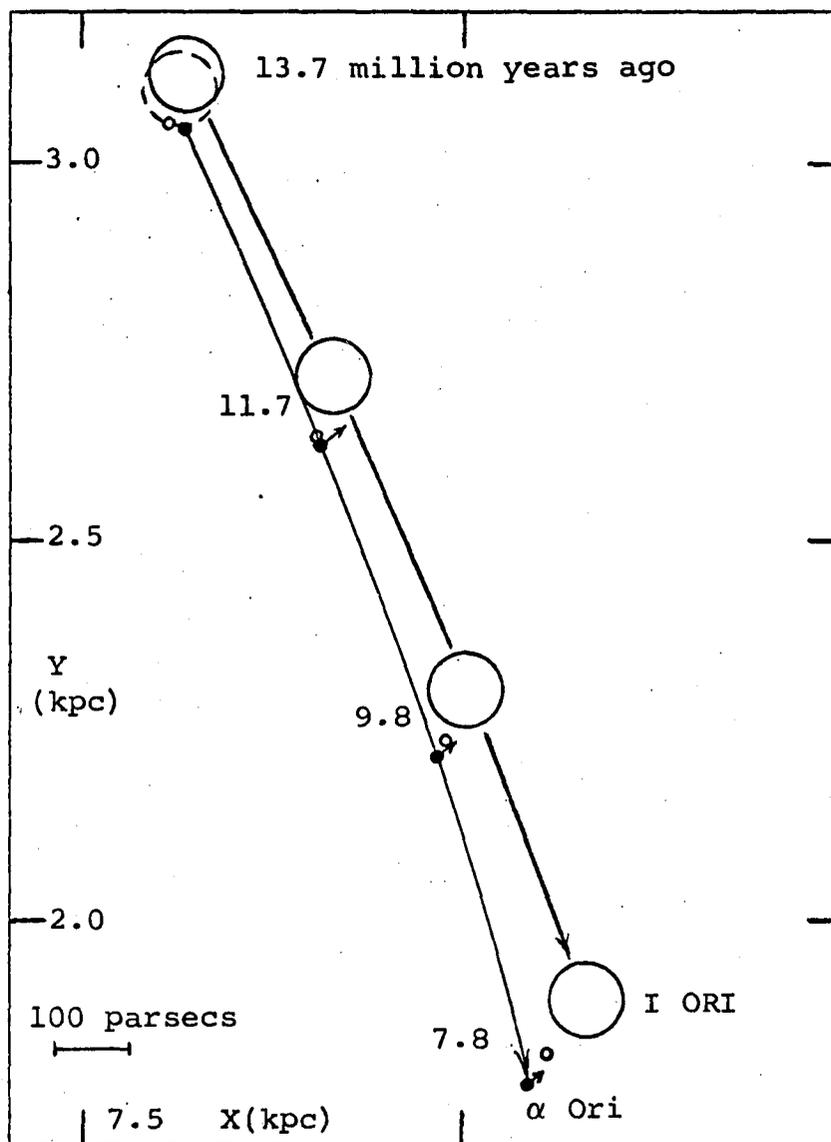
A somewhat more satisfactory way out of the dilemma is to admit that the observational data pertinent to the respective motions are probably incorrect. An increase in the present distance of Betelgeuse to around 200 pc is not unreasonable and would improve matters. The only other questionable datum for the star is the radial velocity; if for some strange reason the adopted  $v_r$  does not describe the line-of-sight motion of its center of mass, then further reduction in  $\Delta R$  between it and I Ori is possible. Inasmuch as the well-known asymmetrical line shifts in the spectrum of  $\alpha$  Ori have been traced to the superposition of lines arising in an expanding circumstellar envelope on the photosphere

components (Adams 1956; Weymann 1962), this notion has little foundation, yet a reduction in the center-of-mass radial velocity by only 3 or 4 km/sec would remove an additional large fraction of the positional difference between the supergiant and Orion association. The final improvement arises in adoption of a different motion of the I Ori center. Referring to TABLE 18, it is seen that data sets 1B and 2B produce  $\Pi \sim +24$  km/sec for the group, a significant increase over set 1A. Orbital computations have been made for the association using these data, projected plots of which are shown in FIGURE 13. The galactic orbit of Betelgeuse based on a present distance of 190 pc is shown for comparison.

A closer dynamical relationship of star to association is indicated by FIGURE 13 than by the earlier solutions. Without altering the size of I Ori in the past, proximity of  $\alpha$  Ori thereto occurred about 13 million years ago or earlier. Previous discussion, however, suggests that this interpretation is not entirely satisfactory, i.e. if the star is presently 190 pc distant, its luminosity is perhaps too great for an age over 10 million years. Furthermore, the motion of  $\alpha$  Ori perpendicular to the galactic plane based on  $d = 190$  pc leads to  $z \sim -300$  pc at  $13 \times 10^6$  years ago, quite a distance from I Ori even if the latter has remained strictly centered

FIGURE 13. The motions of  $\alpha$  Orionis and the Orion association parallel to the galactic plane between 13.7 and 7.8 million years ago, based on the modified present positions and velocities described in the text.

Small arrows show the displacement of Betelgeuse if its present radial velocity were reduced to 17 km/sec. The small open circle marks the motion of a point presently 300 pc distant in the direction of the association center. At  $t = 13.7 \times 10^6$  years ago, the association would be displaced to a position shown by the dashed circle if its present central distance is 370 (instead of 420) parsecs.



at  $z \sim 140$  pc throughout this interval. A more consistent picture is that the star did not form in the area now occupied by the Orion association, but outside this as a member of a more extended aggregate. This event may have happened 6 to 10 million years ago, judging from the 3-dimensional motion, and the astrophysical data would then agree.

It was hoped that the motions of several I Orionis supergiants could be determined in order to gain further insight into the problem of  $\alpha$  Ori and the large-scale kinematics of the entire aggregate. Unfortunately there was not time for such an ambitious undertaking, nor do the data - especially proper motions - measure up in required accuracy to justify this. Additional study was therefore restricted to the B8Ia star  $\beta$  Orionis, whose galactic orbit is given in TABLE 19 (numerically) and graphically in FIGURES 11 and 12, as the reader must have already noticed. This extremely luminous object seems to have formed in the interior of the Orion association, though possibly at some distance from the center. Formation 6 to  $7 \times 10^6$  years ago yields a satisfactory agreement with the nuclear age estimate (TABLE 14). This result hinges on an orbital solution obtained ultimately from the N30 proper motion, however the star's age is so small that the corresponding trajectory based on the FK4

TABLE 19

The Motion of Beta Orionis Since 10 Million Years Ago

time in past ( $10^6$ yrs)	R (kpc)	$\theta - \theta_{\odot}$ (degrees)	z (pc)	II ----- (km/sec) -----	$\vartheta$	Z
0.0	8.440	+ 0.880	-114	+7.78	218.4	-1.79
0.98	8.432	2.363	-112	7.1	218.6	-2.50
1.95	8.425	3.849	-109	7.23	218.8	-3.20
2.93	8.418	5.338	-106	6.95	218.9	-3.89
3.91	8.411	6.829	-102	6.67	219.1	-4.55
4.89	8.404	8.322	- 97	6.38	219.3	-5.19
5.86	8.398	9.818	- 91	6.08	219.4	-5.80
6.84	8.392	11.316	- 85	5.78	219.6	-6.38
7.82	8.386	12.816	- 79	5.48	219.8	-6.92
8.80	8.381	14.318	- 72	5.17	219.9	-7.41
9.77	8.376	15.822	- 64	4.86	220.0	-7.87

proper motion differs by less than 40 pc at the epoch of formation, hence no change is suggested. Such a benefit accrues from consideration of very young stars.

It is not possible, on the other hand, to interpret the motion of  $\alpha$  Ori with such abandon. Uncertainties in its motion and moreso in the motion of the Orion association preclude an unambiguous determination of the kinematical age. The z-motion suggests  $\tau_{\text{kin}} \sim 8$  million years with an uncertainty at least 25 per cent. The failure of the 3-dimensional orbit to intersect the center of I Ori at a comparable time in the past is probably deceptive, as star formation may have occurred in a region outside the accepted association boundary. Further knowledge of the spatial extent and dynamical evolution of the Orion Aggregate would be immeasurably helpful toward a more definite conclusion.

### C. The High-Galactic-Latitude Supergiant Rho Leonis

Rho Leonis, together with other evolved B stars at large galactic latitude, was discussed by Münch (1956), who called attention to an apparently large discrepancy between the time of flight from the galactic plane (where formation is supposed to occur) to the present location of these objects and their nuclear ages. If this discrepancy really exists, one is led to think either that formation did not happen near the plane, rather at moderate or large  $z$ -distances, or that these stars were ejected from the disc while still in some early pre-main-sequence phase of evolution. The discussion of section II centered on TABLE 4 has indicated that the sum of free-fall collapse and Kelvin-Helmholtz contraction times equals at most 10 per cent of the subsequent hydrogen-burning lifetime of massive stars, so any cases of  $\tau_{\text{kin}} \geq 1.1\tau_{\text{nuc}} \sim 1.2\tau_{\text{ms}}$  (for stars in the helium-burning stage) probably mean that the ejection velocity was imparted during an evolutionary phase preceding the onset of collapse to the main sequence, i.e. stage 1 or early stage 2 of TABLE 1. Formation of young stars outside the gas-dust stratum of the galactic disc is less plausible in the light of current ideas about physical conditions conducive to this process. The

work of Schmidt (1959) shows that the star formation rate is proportional to the second or third power of the interstellar gas density; thus, considering the strong concentration of this gas near the plane of the Galaxy, stellar condensations must occur very rarely, if ever, further than 100 to 200 pc from the plane. Practically every known OB association lies at  $|z| < 150$  pc (Schmidt 1957; Kopylov 1958).

A thorough re-examination of the high-galactic-latitude supergiants is clearly called for, using the three-dimensional galactic orbits (where they may be obtained with reasonable accuracy) together with careful estimates of the nuclear ages, before serious concern is generated over disagreements between the kinematical and evolutionary ages. There is already evidence that the discrepancy is small or non-existent for two high-latitude F supergiants, 89 Her and HD 161796 (Searle, Sargent, and Jugaku 1963). The remainder of this subsection will be devoted to the motion of  $\rho$  Leo, a B1Ib star of moderately high velocity situated about 500 pc 'north' of the galactic plane.

Solutions for the motion of  $\rho$  Leo have been found from the data of TABLE 8 and from corresponding velocity components based on the FK4 proper motion. The results, termed Solution 1 and Solution 2, are contained in TABLES

20 and 21. FIGURE 14 shows the motion perpendicular to the galactic plane since 18 million years ago according to Solution 1. It is seen that the star, if it existed then, left the disc of the Galaxy barely 10 million years ago, having passed through the plane 2 million years earlier. The nuclear age of  $\rho$  Leo may be estimated by assuming a mass equal to the B1Ib-II component of V448 Cyg (Sahade) and entering TABLE 2. The resulting main-sequence lifetime, about  $9 \times 10^6$  years (at least 10 per cent uncertain), when added to about 1 million years for the post-main-sequence evolution, strongly favors the idea that the star formed in the galactic disc. Solution 2 yields a slightly slower z-motion, departure from the disc approximately  $11 \times 10^6$  years ago being indicated.

The motion of  $\rho$  Leonis projected on the galactic plane differs enormously according as Solution 1 or 2 is accepted. Not only does the systematic difference in the proper motion (between the N30 and FK4 system values) produce a great change in the velocity component  $\Pi$ , but even the 'probable' error in  $\mu$  on either system leads to considerable uncertainty. Within the limits imposed by the latter uncertainty, Solution 1 suggests (weakly) that the supergiant was expelled from the area of the Orion association, while Solution 2 indicates formation in the young II Scorpii

TABLE 20

RHO LEONIS  
D=620 PC, GCRA VR, N30 P.M., SCHMIDT KR, OORT KZ

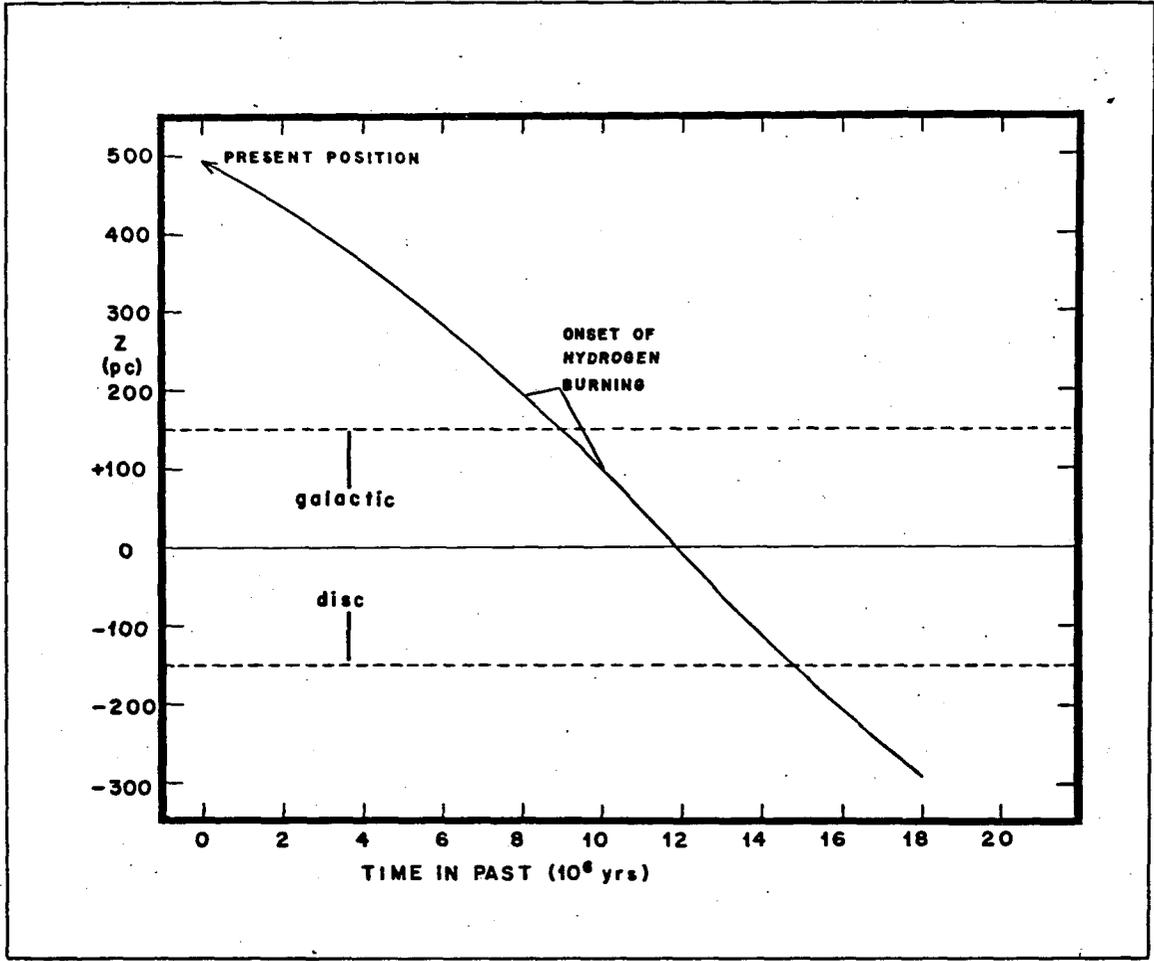
TIME (MEGAYRS)	R (KPC)	DELTA (DEGREES)	ZDIST (KPC)	PI (KM/SEC)	THETA (KM/SC)	Z (KM/SC)
.0000	8.4210	2.0879	.4940	.2100	193.6000	32.8000
-.9774	8.4204	3.4053	.4601	.9573	193.6133	34.9801
-1.9549	8.4190	4.7229	.4240	1.7294	193.6442	37.0885
-2.9324	8.4169	6.0410	.3859	2.5270	193.6931	39.1071
-3.9099	8.4140	7.3600	.3458	3.3500	193.7607	41.0141
-4.8874	8.4102	8.6800	.3039	4.1984	193.8476	42.7840
-5.8649	8.4056	10.0013	.2603	5.0709	193.9545	44.3883
-6.8424	8.4000	11.3242	.2152	5.9658	194.0818	45.7957
-7.8199	8.3936	12.6490	.1688	6.8803	194.2303	46.9742
-8.7974	8.3863	13.9759	.1214	7.8105	194.4004	47.8927
-9.7749	8.3780	15.3054	.0731	8.7516	194.5926	48.5238
-10.7523	8.3688	16.6376	.0244	9.6979	194.8071	48.8458
-11.7298	8.3586	17.9729	-.0244	10.6430	195.0441	48.8458
-12.7073	8.3475	19.3116	-.0731	11.5804	195.3038	48.5204
-13.6848	8.3354	20.6541	-.1213	12.5034	195.5860	47.8764
-14.6623	8.3225	22.0005	-.1687	13.4060	195.8904	46.9298
-15.6398	8.3086	23.3513	-.2151	14.2828	196.2169	45.7037
-16.6173	8.2939	24.7068	-.2601	15.1294	196.5649	44.2263
-17.5948	8.2784	26.0672	-.3034	15.9422	196.9339	42.5281

TABLE 21

RHO LEONIS  
D=620 PC, GCRV VR, FK4 P.M., SCHMIDT KR, OORT KZ

TIME (MEGAYRS)	R (KPC)	DELTH (DEGREES)	ZDIST (KPC)	PI (KM/SEC)	THETA (KM/SC)	Z (KM/SC)
.0000	8.4210	2.0879	.4940	20.3900	187.7800	20.4100
-.9774	8.4001	3.3687	.4724	21.3922	188.2469	22.6115
-1.9549	8.3782	4.6561	.4487	22.4019	188.7389	24.7892
-2.9324	8.3553	5.9503	.4229	23.4202	189.2565	26.9334
-3.9099	8.3313	7.2518	.3949	24.4479	189.8001	29.0318
-4.8874	8.3064	8.5610	.3648	25.4857	190.3706	31.0695
-5.8649	8.2803	9.8782	.3328	26.5341	190.9686	33.0284
-6.8424	8.2533	11.2039	.2988	27.5931	191.5948	34.8869
-7.8199	8.2252	12.5385	.2630	28.6623	192.2499	36.6197
-8.7974	8.1960	13.8825	.2256	29.7405	192.9349	38.1984
-9.7749	8.1657	15.2362	.1867	30.8260	193.6504	39.5913
-10.7523	8.1343	16.6002	.1465	31.9159	194.3972	40.7652
-11.7298	8.1018	17.9749	.1053	33.0065	195.1761	41.6866
-12.7073	8.0683	19.3609	.0632	34.0929	195.9877	42.3242
-13.6848	8.0337	20.7586	.0207	35.1693	196.8326	42.6507
-14.6623	7.9980	22.1686	-.0219	36.2292	197.7112	42.6456
-15.6398	7.9612	23.5915	-.0644	37.2654	198.6239	42.2969
-16.6173	7.9234	25.0277	-.1063	38.2705	199.5707	41.6020
-17.5948	7.8847	26.4779	-.1475	39.2371	200.5517	40.5677

FIGURE 14. The motion of  $\rho$  Leonis perpendicular to the galactic plane since 18 million years ago, according to Solution 1, TABLE 20.

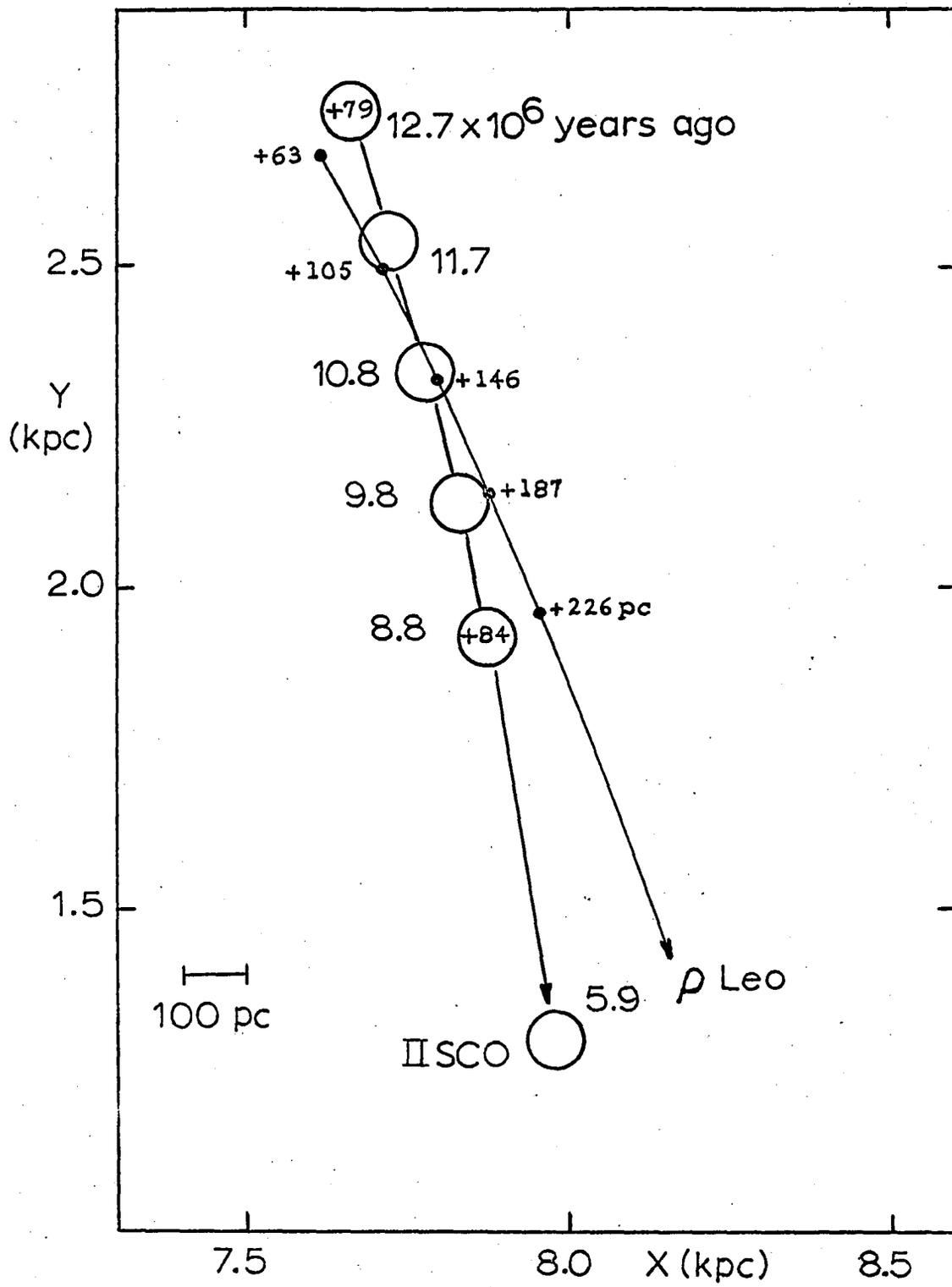


subgroup of the Scorpio-Centaurus association. FIGURE 15 demonstrates how consistent the latter interpretation is on kinematical grounds. It will be recalled furthermore that evolutionary age estimates earlier in this study gave an upper limit around 11 million years for II Scorpii, a value in pleasing agreement with the kinematical age of  $\rho$  Leo if it formed in that group. TABLE 22 presents a comparison of nuclear and kinematical ages of the star.

The conclusion that  $\rho$  Leo is an evolved 'runaway' star is inescapable - the ejection speed was at least 60 km/sec - but due to the highly uncertain motion parallel to the galactic plane, identification of the place of origin with II Scorpii is but provisional. Formation in the disc of the Galaxy is indicated regardless of which observational data the velocity is computed from. There is neither evidence for formation outside the disc, nor for a significant difference between the kinematical and evolutionary ages.

FIGURE 15. The trajectory of the B supergiant  $\rho$  Leonis in projection on the galactic plane (Solution 2, TABLE 21), compared with the motion of II Scorpii between 13 and 6  $\times 10^6$  years ago.

Signed numbers give the z-distances of  $\rho$  Leo and II Sco, respectively, unsigned numbers denoting the epochs at which the positions are shown.





#### D. The Field Cepheid Zeta Geminorum

The discovery in recent years that classical Cepheid variables occur in galactic star clusters is certainly of major importance, yet at the time of this writing only 5 definite cases of cluster membership have been established, according to the review article by Kraft (1963). It would be fortunate if additional Cepheids presently lying in the field could be traced back to their parent clusters, but apparently this will not be possible (by the methods applied here) except for the most luminous such objects. The ages of the Cepheids seem to range from  $10^7$  to  $10^8$  years judging from their luminosity class (Ib-II for most) and cursory evolutionary considerations, but in section III it was concluded that places of formation of stars older than 4 or  $5 \times 10^7$  years cannot be convincingly located by the present analysis and interpretation, which age limitation forces exclusion of a large number of these variables. FIGURE 16 summarizes concisely the problems faced in attempting to determine where intermediate-age stars originated. If such a star is born in an expanding association as a first-generation member and escapes owing to a slightly higher than average initial velocity, its parent group will still be clearly visible for a relatively short time

thereafter, about  $2-3 \times 10^7$  years, viz. TABLE 6. During this interval, location of the place of formation of the released star (observed outside the association) will be aided not only by visibility of the group, but also by its size, i.e. even a fairly crude galactic orbit will probably lead back to some area of the association. After perhaps 30 million years the outer (first-generation) cluster stars will have spread themselves over an enormous volume of space, and may be lost among the surrounding field stars. The association may actually vanish (rendering location of the ejected star's origin impossible) or there may remain a compact 'nucleus-cluster' of stars which formed later than the original generation. Connection of the escaped star with the association in this case requires a very reliable orbit since even a slightly inaccurate one may miss the central cluster, i.e. the association center, by tens of parsecs. Furthermore, the stars in this remaining group will be somewhat younger than the field star in question, making the identification indefinite on evolutionary grounds. The Scorpio-Centaurus association and the F supergiants  $\alpha$  Carinae and  $\alpha$  Leporis seem to match this idealized model in an approximate manner. Zeta Geminorum may be a star with a similar kinematical history.

The age of  $\zeta$  Gem has been estimated by the method described for  $\alpha$  Lep and  $\alpha$  Car; TABLE 23 compares results with those for three cluster Cepheids of similar luminosity. Most of the information for the stars is taken from or based on Kraft's article (1963) and photometric data for the parent clusters of S Nor, DL Cas, and U Sgr are due to Landolt (1964) and Johnson et al. (1961). The ages of the clusters NGC 6087 and M25 are in good agreement with the estimates for the Cepheids, and for NGC 129 a slightly bluer main-sequence turnoff color would reduce the age considerably. It appears that the four stars in TABLE 23 have similar ages not differing greatly from those found earlier for the two non-variable F supergiants, hence location of the place of origin of  $\zeta$  Gem is not ruled out solely by time considerations.

The galactic orbit of  $\zeta$  Gem, summarized in TABLE 24, was computed in the usual fashion. Throughout its lifetime this star was evidently somewhat farther from the galactic center than at present, and one is faced with a lack of data on the motions of star clusters and associations in the area traversed. To even attempt tentative location of the parent star group of  $\zeta$  Gem, one must make some simplifying assumptions. An inspection of the motion during the past 25 million years leads to the suspicion that the birthplace lies

TABLE 23

A Comparison of Zeta Geminorum with Three Cluster Cepheids

Star	$\langle M_V \rangle$	$\langle (B - V)_O \rangle$	P	$\langle B.C. \rangle$	$\langle M_{bol} \rangle$	$M/M_\odot$	Age ( $10^6$ yrs)
$\zeta$ Gem	-4.2	+0.63	$10^d .2$	-0.03	-4.26	7.5	25
S Nor	-4.0	0.78	9.8	- .09	-4.05	7.1	28
U Sgr	-3.9	0.60	6.7	- .02	-3.94	6.9	30
DL Cas	-3.8	0.72	8.1	- .06	-3.90	6.8	32

Star	Parent Cluster	$(B - V)_T$	$(M/M_\odot)_T$	Cluster Age ( $10^6$ yrs)
S Nor	NGC 6087	-0.23	8	20
U Sgr	M 25	-0.2	7-8	25
DL Cas	NGC 129	-0.16	5-6	40-50

TABLE 24

The Galactic Orbit of Zeta Geminorum Since  $3 \times 10^7$  Years Ago

time in past ( $10^6$ yrs)	R (kpc)	$\theta - \theta_{\odot}$ (degrees)	$\Pi$ (km/sec)	$\varphi$	z (pc)	Z (km/sec)
0.0	8.520	+ 0.60	+ 0.8	228.3	+68	-2.8
3.17	8.518	5.58	- 2.6	228.4	77	-1.2
6.34	8.526	10.56	- 5.3	228.1	81	+0.7
9.51	8.543	15.53	- 7.9	227.7	79	2.7
12.7	8.569	20.48	-10.5	227.0	70	4.5
15.8	8.603	25.40	-13.0	226.1	55	6.0
19.0	8.645	30.28	-15.5	225.0	36	7.1
22.2	8.695	35.12	-17.8	223.7	13	7.7
23.8	8.72	37.5	-18.9	223.0	0	7.8
25.4	8.753	39.87	-20.0	222.2	-13	7.7
28.5	8.818	44.59	-22.1	220.6	-36	7.1
31.7	8.889	49.24	-24.1	218.8	-55	6.0

presently at  $d < 800$  pc,  $200^\circ < \ell^{II} < 245^\circ$ , probably in a group whose brightest main-sequence stars are of spectral type B3 to B6. Three good candidates among the young galactic clusters are NGC 2287 (M41), NGC 2301, and NGC 2422; there also exists in this region an extended association designated I Canis Major by Ruprecht, Alter, and Vanýsek (1958, 1962) and II Canis Major by Kopylov (1958), probably also synonymous with the  $o^1o^2$  Canis Majoris group discussed by Schmidt-Kaler (1961). Relevant geometrical and kinematical information for the three clusters and the association (two sets of data) is given in TABLE 25, taken from the authors mentioned above except Schmidt-Kaler. For a preliminary study of the motions of these groups, circular motion about the galactic center was assumed, the velocities having been obtained by entering the rotation curve  $\vartheta_C(R)$  derived by Schmidt (1956) from his final model for the galactic mass distribution. Using the angular velocity so determined ( $\omega$ ), the galactocentric angle  $\theta - \theta_\odot$  was found between the epochs 28.5 and  $22.2 \times 10^6$  years ago, these values being compared with corresponding ones for  $\zeta$  Gem in TABLE 26. The distances  $R$  of the four groups remain respectively fixed, of course, but  $R$  varies in the case of the Cepheid. NGC 2422 has been omitted from this table since its present  $R$  is less than that of the

TABLE 25

Positional and Kinematical Data for 4 Young Star Groups  
in Canis Major (Data Refers to Center of Each Aggregate)

Group	d (pc)	$l^{II}$ (degrees)	$b^{II}$	z (pc)	R (kpc)	$\theta - \theta_{\odot}$ (degrees)
I CMa	950 800	232 "	- 1 "	- 16 - 13	8.83 8.72	+4.87 4.13
NGC 2287	670	231.1	-10.2	-120	8.64	3.45
NGC 2301	790	212.5	- 0.3	- 4	8.89	2.73
NGC 2422	480	231.0	- 3.1	- 26	8.51	2.48

Group	$\omega$ (deg/10 <sup>6</sup> yrs)	$\theta_c$ (km/sec)	Comments
I CMa	1.378 1.407	207.5 209.3	diameter 70 pc = II CMa; diam. 200 pc
NGC 2287	1.433	210.8	earliest Sp B3
NGC 2301	1.365	207.1	earliest Sp B6
NGC 2422	1.463	212.4	earliest Sp B4

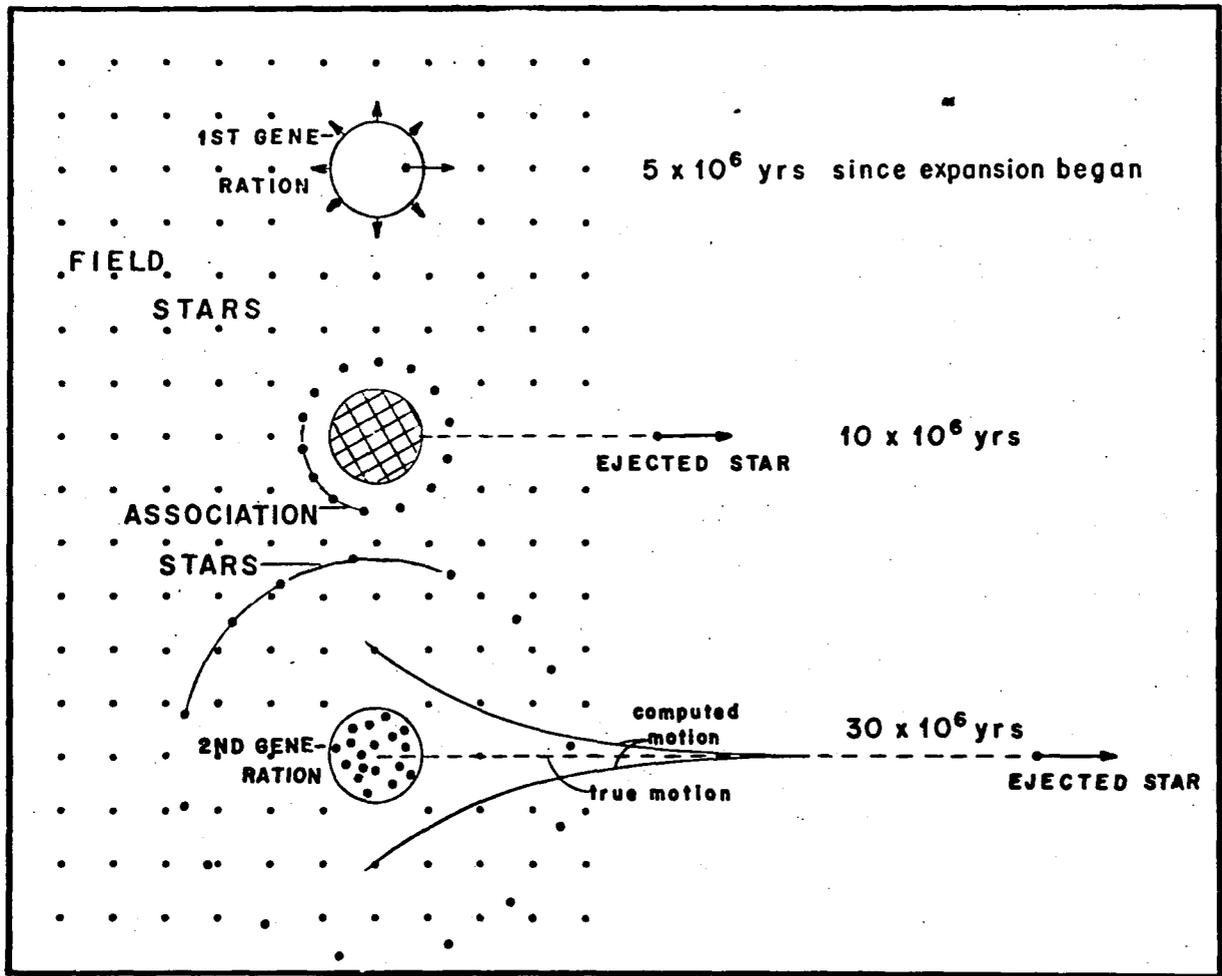
TABLE 26

Angular Motion of 3 Canis Major Aggregates Compared with the Motion of  $\zeta$  Gem

time in past ( $10^6$ yrs)	$(\theta - \theta_{\odot}$ in degrees)					$\zeta$ Gem	
	I CMa R = 8.72 kpc	I CMa R = 8.83 kpc	NGC 2287 R = 8.64 kpc	NGC 2301 R = 8.89 kpc	R (kpc)	$\theta - \theta_{\odot}$ (degrees)	
22.18	35.34	35.43	35.23	33.01	8.70	35.12	
23.76	37.57	37.61	37.50	35.17	8.72	37.5	
25.35	39.80	39.80	39.77	37.33	8.75	39.87	
28.52	44.26	44.27	44.31	41.66	8.82	44.59	

star, and unless NGC 2287 possesses a z-motion away from the plane of the Galaxy, it is too far away to have been the parent group. An examination of TABLE 26 reveals that although  $\zeta$  Gem was indeed in the area of the Canis Major aggregate 25 million years in the past, it does not seem to have formed in any of the smaller clusters. Rather than implying that the Cepheid could not have originated in one of these compact groups, the results demonstrate the need for consideration of the peculiar space motions of each, i.e. the circular motion approximation is likely insufficient. Schmidt-Kaler's discussion of the o CMa group, lying in this region, shows clearly the existence of non-circular motions, yet there remains confusion over distances, causing disagreement even in the designations of the OB aggregates there. Until a more definite picture of the clusters and real associations existing in Canis Major can be constructed and reliable data becomes available permitting calculation of their peculiar motions, no further conclusions can be ventured concerning the origin of  $\zeta$  Geminorum.

FIGURE 16. The dynamical history of an idealized expanding association and an ejected member star.



## VI

### Summary

The investigation reported here should be considered a feasibility study of (1) use of the galactic orbit of a star traced backward in time to seek its place of origin and (2) the derivation of the star's kinematical age where the parent group is identified. This has been applied to a small sample of young stars in post-hydrogen-burning evolutionary phases because careful consideration of the difficulties in the mechanical problem and in interpreting results is important, the analysis being much simpler for young stars than old ones. It has been hypothesized that the kinematical age does not differ significantly from the nuclear age, and results for 4 of the 6 stars treated support this view. For the remaining two ( $\alpha$  Ori and  $\zeta$  Gem), the kinematical analysis alone is inadequate, so  $\tau_{\text{kin}}$  and  $\tau_{\text{nuc}}$  are assumed equal to permit some definite conclusions to be drawn concerning the motions.

The most troublesome uncertainties arising in this study are in the net motions of nearby OB aggregates and the size of the star-forming region within these groups. Here

is clearly the area of knowledge requiring the most improvement. Although data required to compute present velocities and galactic positions of the rare, usually distant young stars are not highly accurate, critical examination of the effects of these uncertainties leads to quite reliable orbits by comparison with those of the associations. Inaccuracy in the solutions due to use of incorrect force laws exists, but the search for places of origin involves the differences between trajectories, so the newer force laws and distance scale will not yield much different results than those found using the Schmidt-Oort force field and old scale (see APPENDIX C). Errors due to the numerical approximation scheme used are not serious for the stars investigated here.

To conclude, the galactic orbits of young stars can be used to locate their respective birthplaces, and ages can thereby be determined without resorting to evolutionary arguments. This type of investigation should have a fruitful future, for although it cannot be hoped to locate the parent cluster of every star, each successful identification has intrinsic interest, and the firm conviction of the author is that the definite results, no matter how many or how few, will overshadow the inconclusive ones.

## APPENDIX A

TABLE 9C expresses quantitatively the mean accuracy with which the various kinematical variables ( $R$ ,  $\Pi$ ,  $z$ , etc.) can be computed by the numerical approximation to the galactic orbits employed in this research. A perhaps more meaningful demonstration of the reliability of the solutions so obtained is direct comparison with a published stellar orbit. Recently van Albada (1961) has determined the past space motion of the B3V 'runaway' star 72 Columbae parallel to the galactic plane using the energy and angular momentum integrals, the motion in the  $z$ -direction being relatively insignificant. Using his present velocity and positional data, the orbit of this star has been computed by the method of section IV. Comparison of the latter results ('S') with van Albada's ('vA') is summarized in TABLE 27A. His values of  $R$ ,  $\Pi$ , and  $\varphi$  were found by interpolating in Table 1 and Figure 1 of his paper; unfortunately it was not possible to obtain the angle  $\theta$  with sufficient accuracy. The author's solution was found using time steps of  $1.58 \times 10^6$  years.

The last three columns of TABLE 27A give the percentage differences ( $v_A - S$ ) in the values of  $R$ ,  $\Pi$ , and  $\varphi$  obtained by the two methods, algebraic signs being omitted. The differences are quite small until about 10 million years ago and are not seriously large even earlier. Interpolation errors of  $\pm 0.04$  kpc in  $R$  and  $\pm 2$  km/sec in the velocities from van Albada's solution probably account for most of the irregular variation in the differences at the first few time points in the table, but the systematic trend in the last three steps is due to accumulation of truncation error in the numerical solution. It is gratifying that the two solutions differ so little, for the motion of 72 Col is extremely different from the stars considered in the main body of the present study. The divergence of solutions beginning at  $t = 10$  million years ago happens because of the rapid change of  $R$ . For low-velocity stars, whose distance from the galactic center varies by 100 to 200 pc, such behavior of the approximate orbital solution is not expected until after many more time steps.

TABLE 27A

Comparison of Quantities Pertaining to the Galactic Orbit of 72 Columbae Projected on the Plane as Determined by van Albada (Using Energy and Angular Momentum Integrals) and by the Numerical Method of this Study

time in past ( $10^6$ yrs)	R (kpc)		$\Pi$ (km/sec)		$\varphi$ (km/sec)		Percentage Differences		
	vA	S	vA	S	vA	S	R	$\Pi$	$\varphi$
0.0	8.38	8.38	+182	+182	240	240	0.0	0.0	0.0
1.58	8.07	8.08	179	180	250	249	0.1	0.6	0.4
3.17	7.79	7.79	176	177	259	258	0.0	0.6	0.4
4.74	7.50	7.51	171	173	269	268	0.1	1.1	0.4
6.34	7.26	7.23	166	168	278	278	0.4	1.2	0.0
7.92	6.97	6.96	158	162	289	289	0.1	2.5	0.0
9.51	6.72	6.69	150	154	300	300	0.4	2.7	0.0
11.1	6.50	6.44	141	145	310	312	0.9	2.8	0.6
12.7	6.27	6.21	128	133	321	324	1.0	3.9	0.9

## APPENDIX B

Here the validity of the assumption that the motions of stars at small  $z$ -distances parallel and perpendicular to the galactic plane are practically independent is examined. The most extreme case in the present investigation is the star  $\rho$  Leonis, which lies at  $z \sim 500$  pc. TABLE 27B compares orbital solutions A, neglecting the  $z$ -variation of  $K_R$  and B, including the  $z$ -dependence of  $K_R$ . The starting data are taken from section IV of the main text, i.e. velocity components based on the N30 proper motion. In both cases  $K_z$  is assumed independent of  $R$ , which has varied by only 80 pc during the star's lifetime according to the data adopted. (The solution based on velocity components derived from the FK4 proper motion is, however, quite different.)

It is seen in TABLE 27B that the motion differs far too little to affect any conclusions made concerning the origin of the star. The discussion in section V C. favors the solution obtained ultimately from the FK4 proper motion, yet the comparison presented here is more applicable to the typical low-velocity star, whose maximum  $z$ -distance is about

half that of  $\rho$  Leo and whose R-distance varies but little.

The assumption for such stars that the motion perpendicular to the galactic plane may be considered de-coupled from that parallel to the plane is evidently sound.

TABLE 27B

The Space Motion of  $\rho$  Leonis: (A) Neglecting the z-Variation of  $K_R$ ;  
 (B) Including the z-Variation of  $K_R$

time in past ( $10^6$ yrs)	R (kpc)		$\theta - \theta_{\odot}$ (degrees)		z (pc)	Z (km/sec)
	A	B	A	B		
0.0	8.421	8.421	+2.09	+2.09	+494	+33
2.93	8.417	8.417	6.04	6.04	386	39
4.89	8.409	8.410	8.68	8.68	304	43
6.84	8.397	8.400	11.32	11.32	215	46
8.80	8.381	8.386	13.99	13.98	121	48
10.75	8.359	8.369	16.65	16.64	24	49
12.71	8.336	8.348	19.32	19.31	- 73	49

## APPENDIX C

Force Laws  $K_R(R)$  for the Plane of Symmetry of the Galaxy,  
Based on  $R(o) = 10$  kpc, Compared with the Schmidt Model.

At the time of writing, no complete three-dimensional model for the mass distribution of the Galaxy based on the new distance scale and Oort parameters is available. A few limited approximations to the entire run of  $K_R$  in the galactic plane are shown in FIGURE 17, where the ordinates are in units of  $100 \text{ km}^2 \text{ sec}^{-2} \text{ kpc}^{-1} = 3.24 \times 10^{-10} \text{ cm sec}^{-2}$  and the abscissae are in kpc. Curve (1) is due to Contopoulos and Stromgren (Contopoulos 1964):

$$K_R = -(73340/R^2 - 1581.8 + 3442.03R - 402.621R^2 + 12.9402R^3).$$

Curve (2) is Schmidt's (1956)  $K_R$  law transformed in  $R$  to the new distance scale. For points  $|R - R(o)| < 1$  kpc,  $|z| < 200$  pc, approximately, Poisson's equation plus the defining equations for  $A$  and  $B$  yield

$$\partial K_R / \partial R = -(A - B)^2 + 4A(A - B).$$

Integrating, one obtains

$$K_R = [-(A - B)^2 + 4A(A - B)]R - 4A(A - B)R(o).$$

FIGURE 17 gives solutions for  $A = +14.4$ ,  $B = -9.0$  (curve (3)) and  $A = +14.5$ ,  $B = -12.9 \text{ km sec}^{-1} \text{ kpc}^{-1}$  (curve (4)), combinations in the range of recent determinations, some of which were quoted in section IV D.

The force law of Contopoulos and Stromgren evidently predicts a stronger  $K_R$  than the Schmidt model at corresponding points whereas relation (3) is virtually identical with the latter in its region of validity. The most salient feature at any rate, as regards the present study, is that the slope of  $K_R$  is nearly the same for  $8 < R < 12 \text{ kpc}$  (new scale), or  $6.6 < R < 9.8 \text{ kpc}$  (old scale), regardless of which of the four force laws one believes. In this range (where all stars and groups treated have moved during their lifetimes), the slope of (1) averages  $835$  and (2)  $900 \text{ km}^2 \text{ sec}^{-2} \times \text{kpc}^{-2}$ . This difference implies a velocity difference (star minus origin) at the end of  $10^7$  years amounting to only  $0.7 \text{ km/sec}$  per  $100 \text{ pc}$  of average separation. Use of the Schmidt  $K_R$  thus does not give results concerning places of formation of nearby young low-velocity stars that differ much at all from conclusions to which the newer force laws would lead.

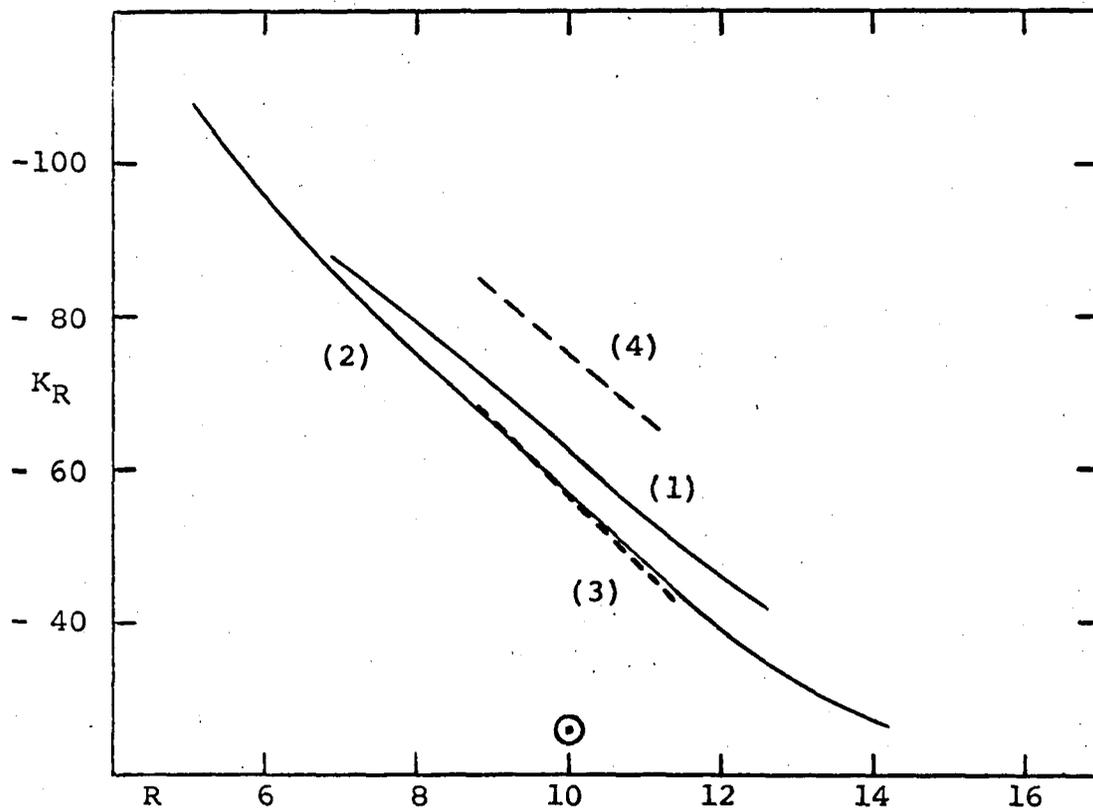


FIGURE 17. Galactic gravitational force laws  $K_R(R)$  in the plane based on  $R(0) = 10$  kiloparsecs.

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