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SOIL-STRUCTURE-ARCHING ANALYSIS

of BURIED FLEXIBLE STRUCTURES

by

Farrel Dwayne Nielson

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I hereby recommend that this dissertation prepared under my direction by F. Dwayne Nielson entitled Soil-Structure-Arching Analysis of Buried Flexible Structures be accepted as fulfilling the dissertation requirement of the degree of Doctor of Philosophy

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SIGNED: L. Dwayne Nielson

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F. Dwayne Nielson

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ABSTRACT

Recent observations on buried structures made in the laboratory have led to the conclusion that one is not justified in using the classical Marston theory indiscriminately for loads on underground pipe. In order to allow for pressure redistribution across the top of a buried flat-roofed structure, a different differential element must be assumed. The differential element chosen in this study was the shape of a circular arch for long structures, such as buried pipe, and a spherical dome for circular or square structures. After selection of the differential element, the problem of determining the location of the soil arch support in the soil mass for the differential element arises. There is no physical justification for assuming that the arch extends only across the prism of soil directly above the buried structure. The location of the soil arch support was assumed to be at the location of the maximum shear stress within the soil mass as determined by the theory of elasticity. If any movement or strain in the soil should occur, the majority of it should take place in the region of maximum shear stress. The theory

of elasticity is not used for determining the stress in the differential soil arch; it is used only to locate the region of maximum shear stress before any slippage of the soil grains occur. It is proposed that a movement or strain within the soil mass will cause the differential soil arch to form. A differential equation was written using the circular arch for two dimensional structures and a circular dome for three dimensional structures. In order to evaluate the stress at the soil arch support, model studies were used. When all of the necessary parameters, including a non-linear modulus of passive resistance for soil, were included, the resulting differential equation was not readily integrable; therefore, a numerical integration procedure was used to obtain solutions. Results were compared with model studies and with several field installations where adequate information was available.

INTRODUCTION

The Problem

The inadequacies of the methods of analysis of underground structures is very apparent when the actual deflection and ultimate load capacity of these structures is compared with the values predicted by existing theories. Moreover, more accurate design procedures for buried structures are becoming more important because of the expanding highway system and the resulting increased use of large drainage and underpass type structures. In addition the interest in blast and fallout shelters by the Office of Civil Defense has also created a need for an adequate design procedure. Several theories to determine loads on buried structures have been presented but they only partly explain the phenomena that have been observed. Design procedures used today are usually based on "experience tables," and very little basic engineering knowledge is applied.

Objectives

The objectives of this study are twofold: Firstly, to review the existing theories for transmitting forces through the soil to buried structures and point out the limitations of these existing theories: Secondly; to present a new theory based on arching; i.e., the soil above the buried structure is assumed to act as a series of arches, one acting on top of the other, and to apply the arching theory to buried circular pipes.

Scope of Problem

A differential equation governing arching is derived for two dimensional and three dimensional structures. Because of the shortage of adequate field data for comparison purposes, only the two dimensional equation for circular pipes will be evaluated and verified. Solution of the differential equation for other shapes of structures will not be attempted here.

CHAPTER I

HISTORICAL REVIEW

Marston Theory

One of the first studies made on the analysis of loads on underground conduits was undertaken by Dean Anston Marston (1913) of Iowa State University. Figure 1 shows a free-body diagram of the loading which Marston assumed in deriving the equation for loads on underground pipe.

The governing differential equation for ditch conduits can be derived by summing forces in the vertical direction on the differential element in Figure 1. The forces acting upward are

$$V + dV + 2ku' \frac{V}{B_d} dh.$$

The forces acting downward are

$$V + W B_d dh.$$

For equilibrium the sum of the vertical forces must equal zero or

$$V + dV + 2Ku' \frac{V}{B_d} dh = V + w B_d dh.$$

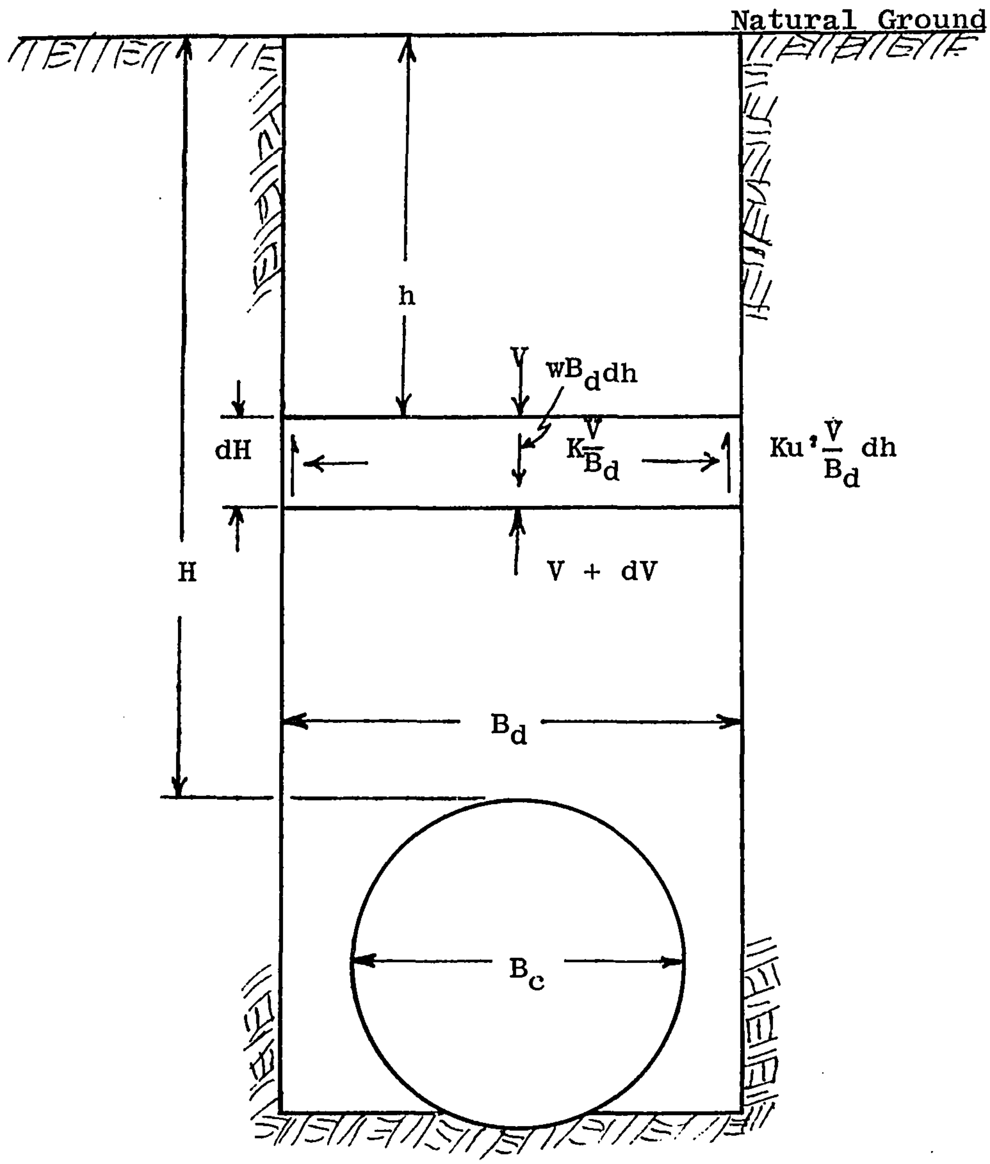

Figure 1 Classical Free Body Diagram of Loading Assumed on Buried Conduits

This is a linear differential equation; the solution is

$$V = W B_d^2 \frac{1-e^{-2Ku' \frac{h}{B_d}}}{2Ku'}$$

Where:

V = vertical pressure on any horizontal plane in backfill, in pounds per linear foot of ditch.

B_d= horizontal width of ditch at top of conduit in feet.

H = height of fill above top of conduit, in feet.

h = distance from ground surface down to any horizontal plane in backfill in feet.

u'= tan $\emptyset$ = coefficient of friction between fill material and sides of ditch.

K = ratio of active lateral unit pressure to vertical unit pressure.

e = base of natural logarithms.

The load transmitted to the pipe is

$$W_c = C_d W B_d^2$$

Where

W_c = load on conduit (lb/ft).

C_d = coefficient.

The forces transmitted to the underground conduit in installations other than the ditch condition depend on many factors, the major one being the relative movement of the soil with respect to the buried conduit or the settlement ratio as defined by Spangler (1960). Moreover, this relative movement depends upon the soil properties and the properties of the pipe. If the conduit is flexible enough to deflect more than the surrounding soil, it will not be required to carry the weight of the soil prism directly above it. Part of the load will be transmitted to the surrounding soil through arching action. But, if the conduit is relatively rigid with respect to the surrounding soil, the load on the conduit will be greater than the weight of the soil prism directly above it. As the soil moves downward around the conduit, arching action will transmit part of the weight of the surrounding soil onto the conduit.

Observations made in the laboratory on flexible membranes over flat-topped model structures yield results that the Marston theory cannot explain. Figure 2 shows a schematic diagram of the model used in the test. By placing a parapet wall above the top of the structure as shown, the pressure transmitted to the flexible membrane is greatly decreased (Engineering Research Laboratory 1964).

The Marston theory cannot explain pressure redistribution across the top of the membrane. Measured

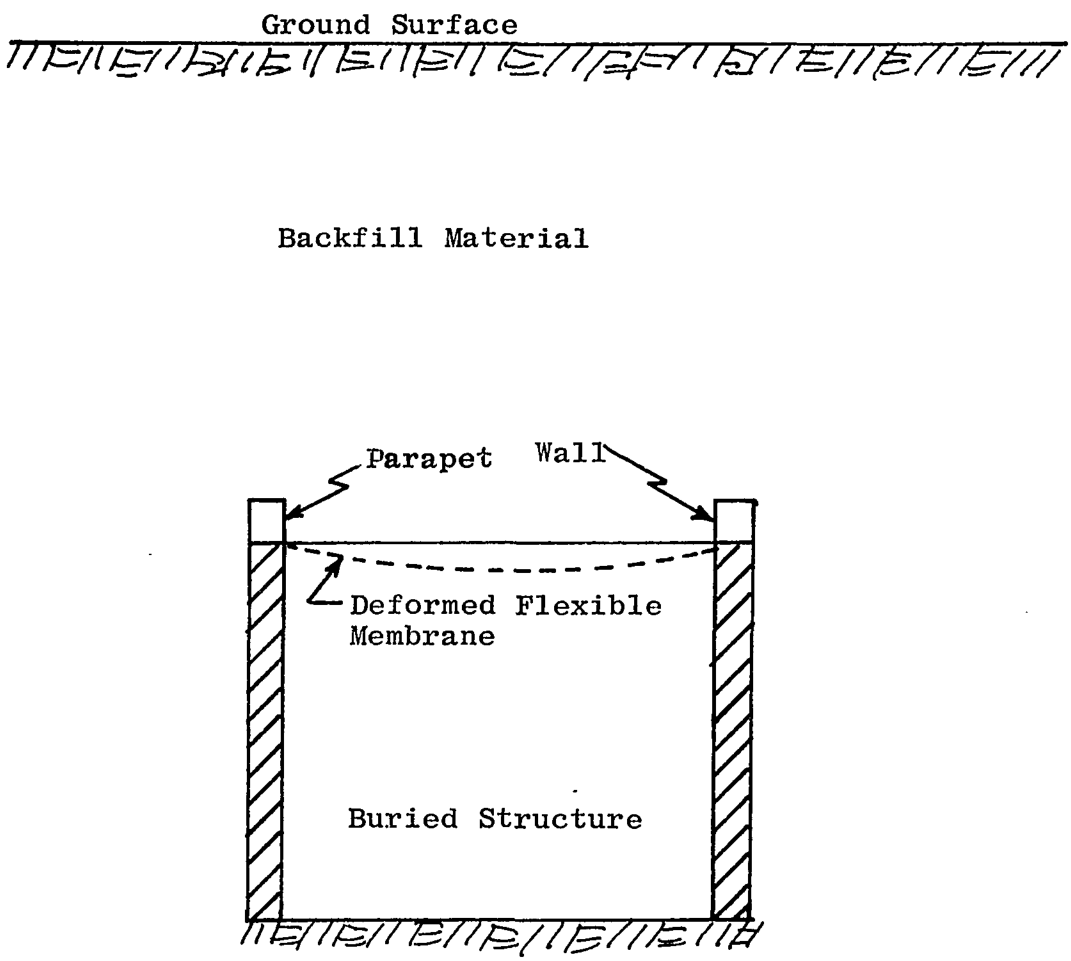

Figure 2 Schematic Diagram of Buried Flexible Membrane

results show the pressure at the edge of the membrane to be greater than the pressure at the center.

The Marston theory used a horizontal differential element. The element is assumed to act as a unit and retain its shape as it moves downward. However, laboratory studies indicate the element does not retain its shape but deflects as it moves downward. The soil at the center of the element moves more than the soil at the edge. This can be seen in Figures 12, 13, and 16 on Pages 33, 34, and 42.

Further, the differential element shown in Figure 1 is assumed to act as a beam having a uniformly distributed load with axial forces. If the flexural stresses in the beam or differential element are larger than the axial stress, a tension stress will occur in the bottom of the element.

It is generally assumed that soil cannot take tension, and if this assumption is applied here, it would invalidate the use of the horizontal differential element.

Because previous theory does not allow for pressure redistribution across the top of flat structures nor for pressure reduction caused by placing a parapet wall above the structure, a different shape of differential element has been selected.

Another disadvantage in the use of the Marston theory is the selection of the proper settlement ratio for installations other than the ditch conduit. It can be seen from tables presented by Spangler (1960) that the load transmitted to the buried structure depends greatly on the settlement ratio. At the present time the selection of the settlement ratio is based on experience with field installations.

Air Force Design Manual Theory

Newmark and Haltiwanger (1962) have presented a theory in the Air Force Design Manual (AFDM) to determine the pressure transmitted to an underground structure from a uniform loading on the ground surface. They use the same horizontal differential element as the Marston theory. They have not included the weight of the soil in the analysis.

Because the differential element is the same as that used by Marston, this theory is subject to the same limitations as the Marston theory with regard to the pressure redistribution and pressure reduction due to the parapet wall.

In order to eliminate the problem of obtaining a valid settlement ratio, they have given two different equations which include the deflection of the structure along with other variables. Usually it is necessary to know the pressures acting on a structure before the deflection can be obtained. The result should be a trial and error solution.

This theory is limited because of the assumed horizontal differential element and because it neglects the weight of the soil.

Elastic Theory

Attempts have been made to solve the problem through use of elastic theories.

One analysis, for circular pipe, has been presented by Burns and Richard (1964) and Burns (1965). The analysis assumes the soil to be an elastic medium and the pipe to be an elastic medium with a different stiffness. Equations are developed to give the stresses and displacement of the elastic media.

The original analysis by Burns and Richard does not include body forces, but only predicts the pressure acting on the buried structure due to a uniform pressure or surcharge load on the surface of the soil. Burns later included body forces in the analysis which was then only limited by the assumed linear elasticity. Normally, under deflections usually encountered with buried flexible pipe, soil does not exhibit linear elastic properties. An additional disadvantage is the restriction of the analysis to circular sections.

Another solution to determine arching in soil due to deflection of a rigid horizontal "trap door" has been presented by Chelapati (1964). He has assumed an elastic medium above a rigid base which contains a yielding rigid "trap door." As the rigid "trap door" is lowered, the pressure distribution across it is computed. The condition is imposed that the stresses in the elastic medium at the edge of the "trap door" do not go into tension. The assumption of a rigid "trap door" makes the pressure calculated at the center of the "trap door" higher than at the edges. Very few underground structures have covers that can be considered completely rigid. If the center of the span of the buried flat-topped structure deflects more than the edges, redistribution do to arching action will cause the pressure on the edges of the structure to be higher than at the center. Most engineering structures with flat roofs will deflect downward more at the center than at the edge because of beam or membrane action of the roof. Because of the difference in modes of deflection of the roof, limitations on the type of structure, and the assumed elasticity of soil, Chelapati's solution has limited application.

Shearing Theory

Truesdale and Vey (1964) have presented a solution based on vertical shearing elements. They assume a pressure distribution acting on the buried structure and calculate the deflection of the structure due to the assumed pressure. The transfer of shear from one element to another is determined, which is based on the computed deflection of the membrane and an assumed stress-strain diagram and strain distribution of the soil. A new pressure distribution is computed. The process is repeated until a compatible pressure and deflection have been reached. This theory adequately describes the pressure redistribution across the surface of a buried structure, but cannot explain the pressure reduction caused by the parapet wall. Another disadvantage of this theory is the necessity for assuming a stress-strain diagram and a strain distribution in the soil mass.

CHAPTER II

ANALYSIS OF PROBLEM

General Discussion

In order to allow for pressure redistribution across the top of the structure and pressure reduction due to the parapet wall, a different differential element must be used. There are several other types of elements that could be used. A shear element is shown in Figure 3. It would be difficult to explain the pressure reduction on the flexible membrane due to the parapet wall using this differential element.

In order for this element to describe the observed pressure reduction, the magnitude of the lateral forces would have to be increased greatly. Tests on the model shown in Figure 2 indicate that the lateral pressure would approximate the full passive pressure for one complete diameter above the structure. This is not consistent with general soil mechanics theory. The lateral pressure is usually less than the vertical pressure.

Using a differential element the shape of an arch, redistribution across the top of the structure and

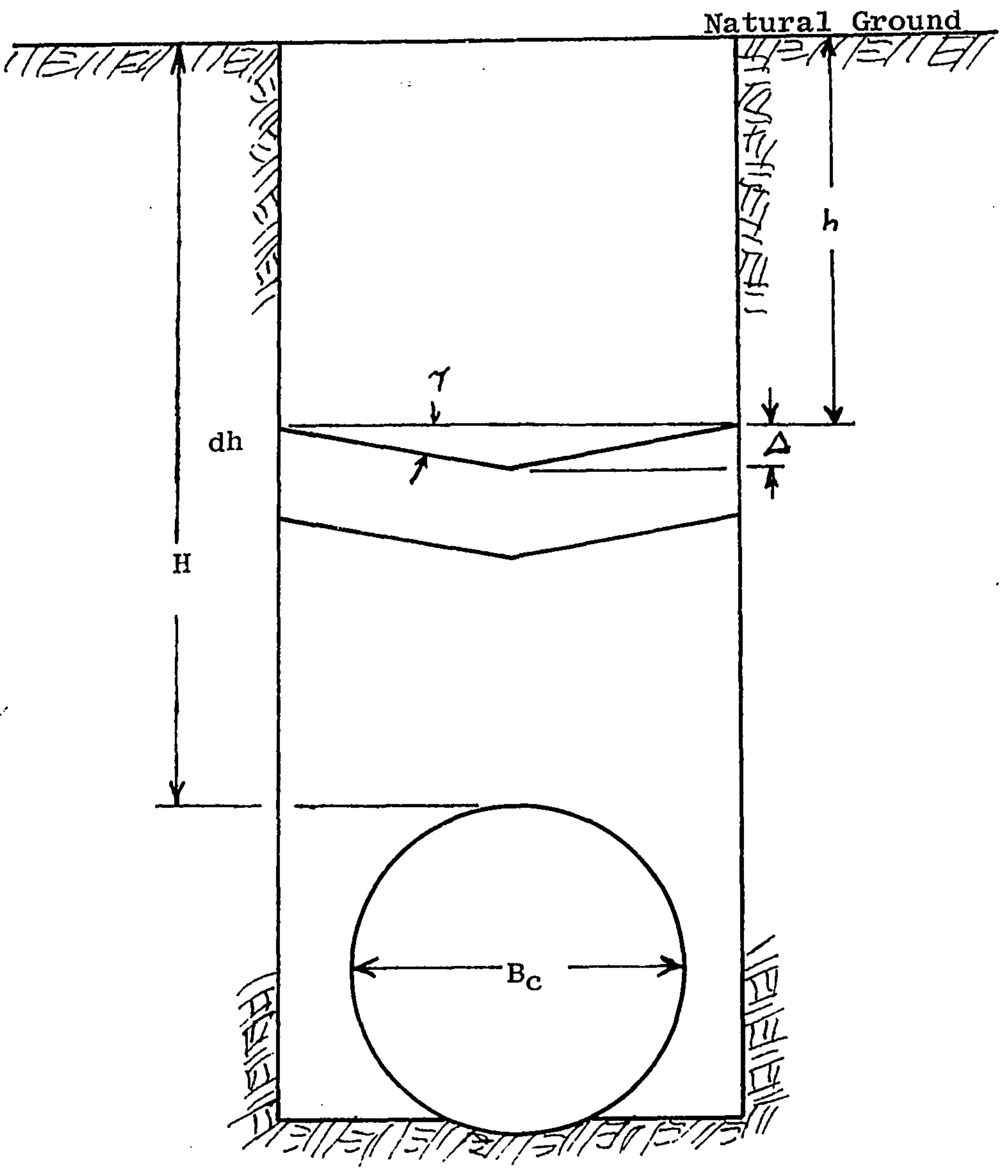

Figure 3 Free Body Diagram of Shear Element Above a Buried Conduit

pressure reduction caused by placing the parapet wall above the flexible membrane can be explained.

One arch is assumed to act on top of another. The pressure is transmitted away from the buried structure by the stress which acts as the differential arch support. As the support for the differential element gets closer to the structure, the stress gets larger. These stresses normally are slightly greater than the active pressure. Upon reaching the parapet wall, the stress will approach the "at rest" pressure. This will cause the arch to transmit its pressure to the parapet wall rather than to the roof of the structure. As the radius of the arch gets even smaller the assumed arch support will move out onto the roof of the structure. The arch support stress will be higher at the edge than at the center of the roof. The pressure on each differential arch will be lower than the one directly above it. This will cause the pressure at the support to be lower as it moves toward the center of the structure. The pressure on the roof is the arch support pressure.

If the loading on the differential arch is uniform vertical loading, as shown in Figure 4, an element with no tensile stresses will assume the shape of a parabola. No tensile stresses or moments will occur

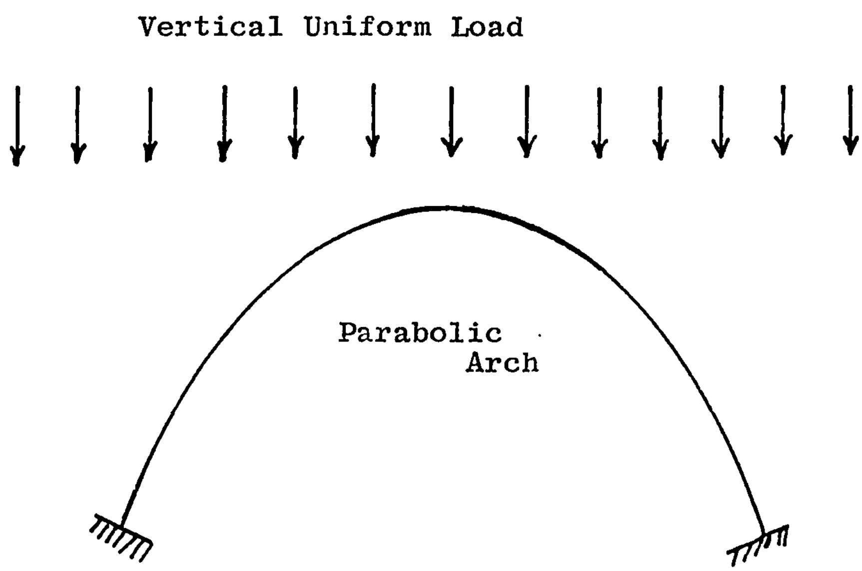

Figure 4 Compressive Arch Under Uniform Vertical Loading (no bending stresses in arch)

within the section. If the loading is a uniform radial loading, the shape of the arch is a circle as shown in Figure 5.

If one of the above loadings is assumed to act on the soil, a differential element can be selected so that no tensile stresses occur in it.

The vertical components from a uniform radial pressure distribution on a circular arch give the same vertical pressure distribution as the uniform vertical pressure on a parabolic arch. Therefore, when summing forces only in the vertical direction, the resulting vertical pressure distribution is the same. This is true only for a uniform vertical and a uniform radial loading.

The physical properties of the soil conduit system also enter into the location of the arch. In order to simplify the problem a circular arch is assumed. As more knowledge is obtained about stress redistribution characteristics of soil, a more rigorous analysis may be possible.

Once the shape of the arch is selected, the problem of locating the arch supports arises. Unless the conduit is in a trench with relatively rigid sides, there is no physical basis for assuming that the arch

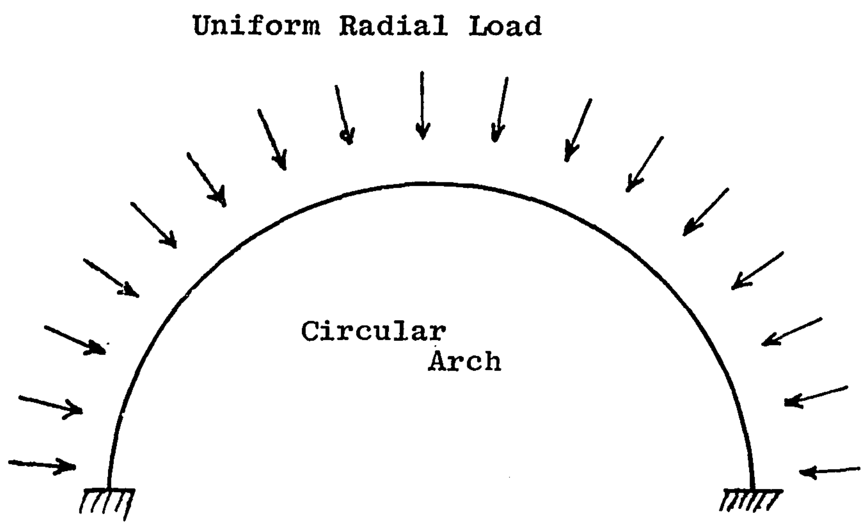

Figure 5 Compressive Arch Under Uniform Radial Loading (no bending stresses in arch)

extends through only the soil prism directly above the pipe. If the vertical prism cannot be assumed, other means must be used for locating the soil arch supports.

Location of Soil Arch Supports with Pipe in Trench with Vertical Sides

If the conduit is buried in a trench with relatively rigid vertical walls, the support of the arch can be assumed to be on the sides of the trench as shown in Figure 6.

The arch acts across the trench so that it subtends an angle of 90° at its center of curvature. The side of the trench, where most of the movement of the soil is assumed to take place, is the plane of maximum shear stress. Mohr's circle shows the major and minor principal stresses are at an angle of 45° to the maximum shear stress. As a result the line of action or the direction of the principal stress in the soil arch is at an angle of 45° with the vertical side. Therefore, the soil arch must intersect the side of the trench at 45°. This assumption may seem incompatible at the level of the differential soil arch directly above the buried conduit. At this level the radius of the soil arch is different from the radius of the conduit. This may be resolved by assuming that the

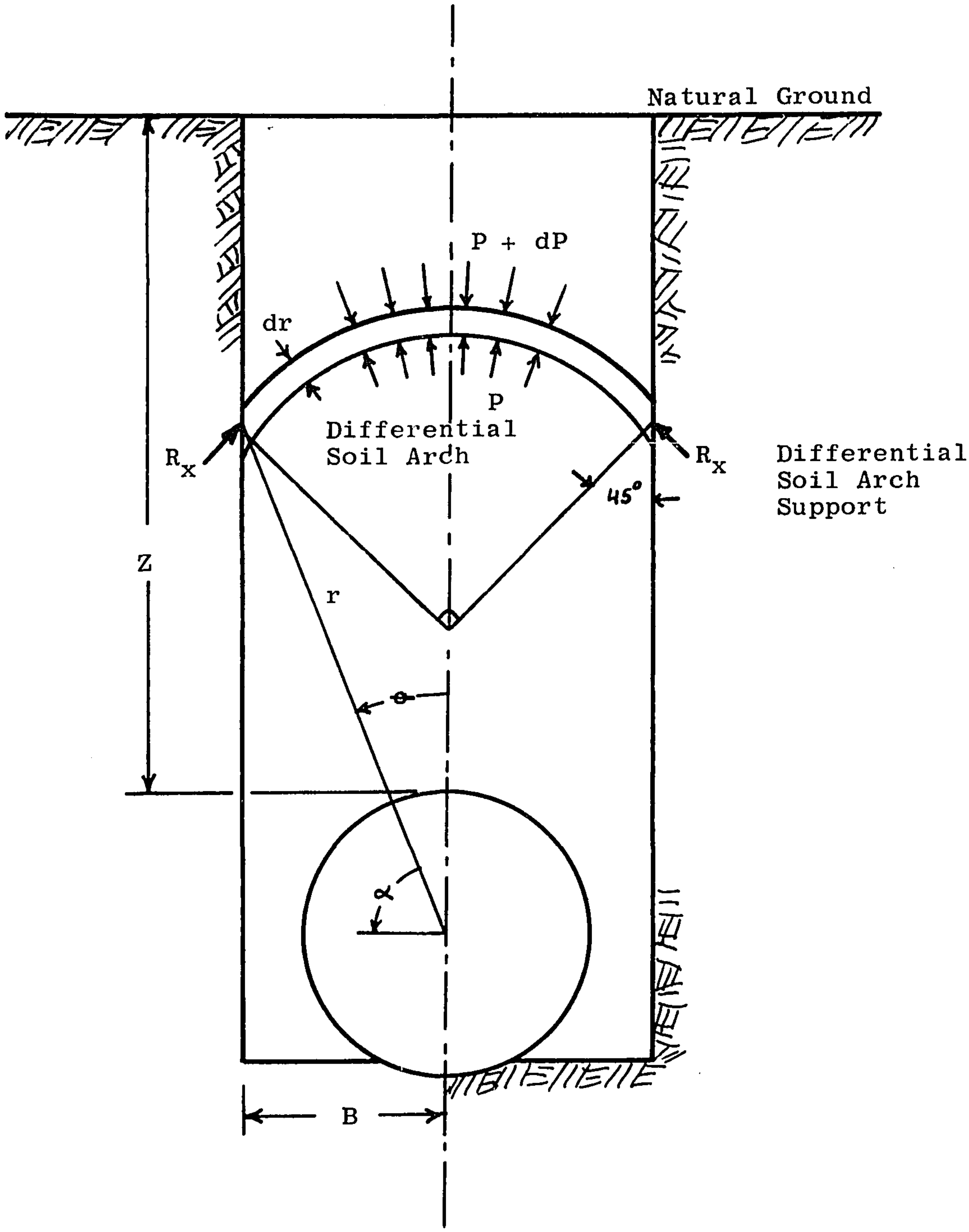

Figure 6 Free Body Diagram of Differential Soil Arch in Trench with Relatively Rigid Vertical Walls

soil, between the lowest possible differential soil arch and the conduit, simply transmits the pressure to the conduit in compressive forces.

The relationship between r and θ can be shown to be

$$\theta = \sin^{-1} \frac{B}{r} \tag{1}$$

The value of r and θ fix the location of the differential soil arch.

Location of Soil Arch Supports with Pipe in Trench with Sloping Sides

If the conduit is buried in a trench with relatively rigid sloping walls, as shown in Figure 7, the supports can be assumed to be at the side of the trench as before. The angle θ can be determined as follows. From the law of sines the angle c can be determined as

$$c = \sin^{-1} \left(\frac{B \sin(180-\gamma)}{r}\right) \tag{2}$$

By knowing that the sum of the interior angles in triangle ODE equals 180° the angle α can be found as

$$\alpha=\gamma - \sin^{-1} \left(\frac{B \sin(180^{\circ}-\gamma)}{r}\right) \tag{3}$$

Knowing the angle α the angle θ can be determined as

$$\theta = 90^{\circ} - \alpha \tag{4}$$

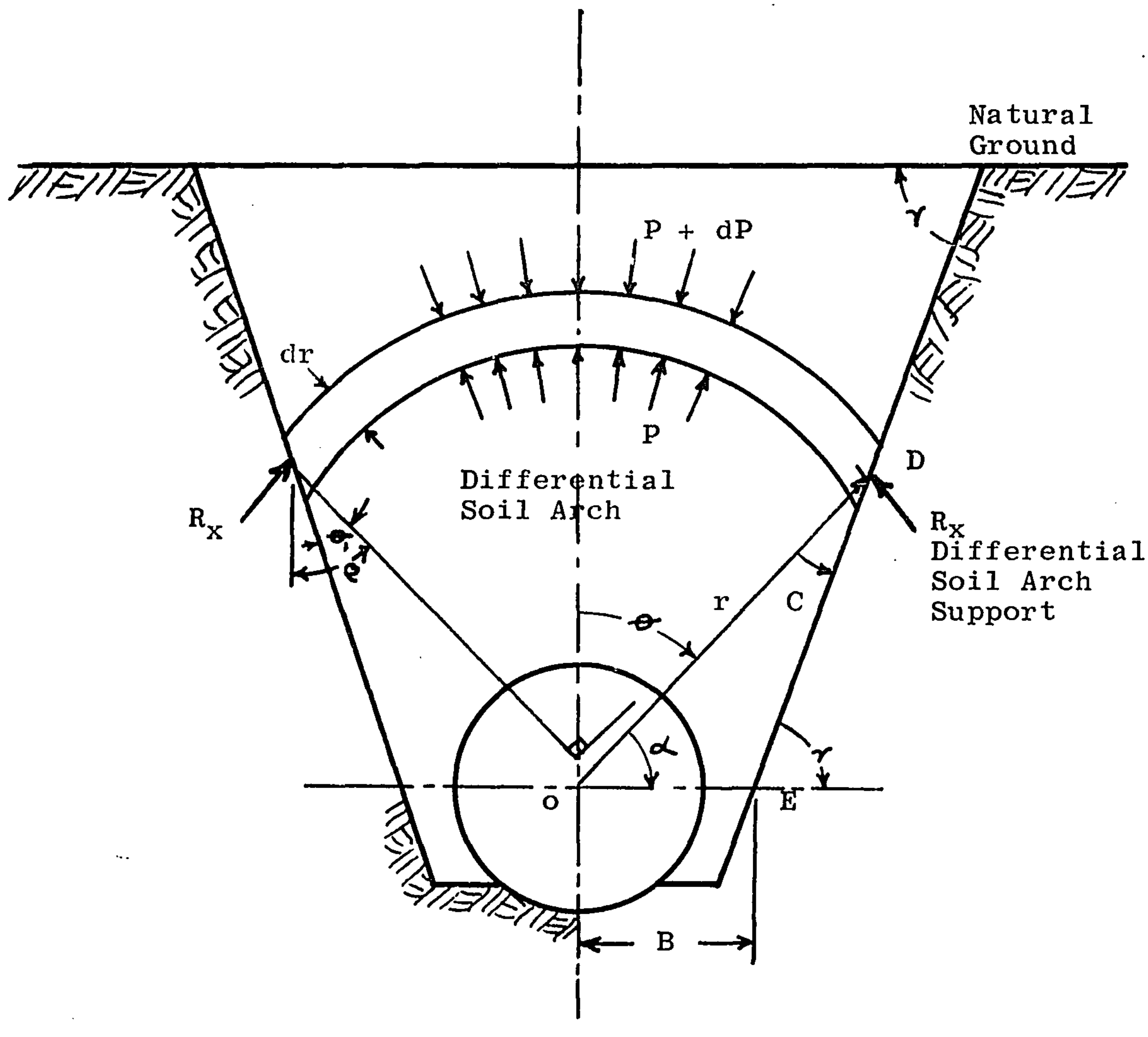

Figure 7 Free Body Diagram of Differential Soil Arch in Trench with Relatively Rigid Sloping Sides

The values of θ, γ, and r completely define the location of the soil arch support. The angle β in Figure 7 is assumed to be 45° regardless of the angle γ. When γ equals 90° the problem reduces to the vertical trench condition. When γ equals 45° the angle θ_1 is assumed to be 0° and hence β equals 45°.

<u>Location of Soil Arch Supports in an Earth Fill</u>

If the conduit is placed in an earth fill so that no well-defined shear planes are available, other means must be used for determining the location of the soil arch supports. If one assumes that most of the strain or movement within the soil mass above the conduit occurs between regions of maximum shear stress and that the movement of the soil between the regions of maximum shear stress is downward, a solution can be found. According to the theory of elasticity, two regions of maximum shear stress form, one on each side of a hole, in an infinite plate. The exact location will be shown later in this section. The maximum shear stress in a soil conduit system is assumed to act in approximately the same location.

The pressure on the flexible conduit causes it to deflect downward. The soil directly above the conduit will follow, causing the soil arch to form. The ends of the arch, or arch supports, are assumed to be at the region where shear stress was maximum before any movement occurred. As the soil moves down it acts as a wedge, and the more the movement the higher the stress in the soil arch. Consequently, more force is transmitted away from the pipe into the surrounding fill and the total load the pipe is required to carry is reduced.

As an example to illustrate the point, consider a funnel (Figure 8) which has been filled with sand. The sides of the funnel correspond to the region of the initial maximum shearing stress. Therefore, the sides of the funnel are also the region where the soil arch supports are assumed to form. The sand within the funnel moves downward until a compressive arch forms from one side of the funnel to the other. The sand is prevented from running out the bottom of the funnel but no pressure is applied on it from below. The surface of the sand will take an extremely large amount of pressure, after the soil arch has formed, before the sand can be pushed through the funnel. This is basically the same phenomenon that is assumed to occur over a buried pipe if the pipe is relatively flexible with respect to the soil.

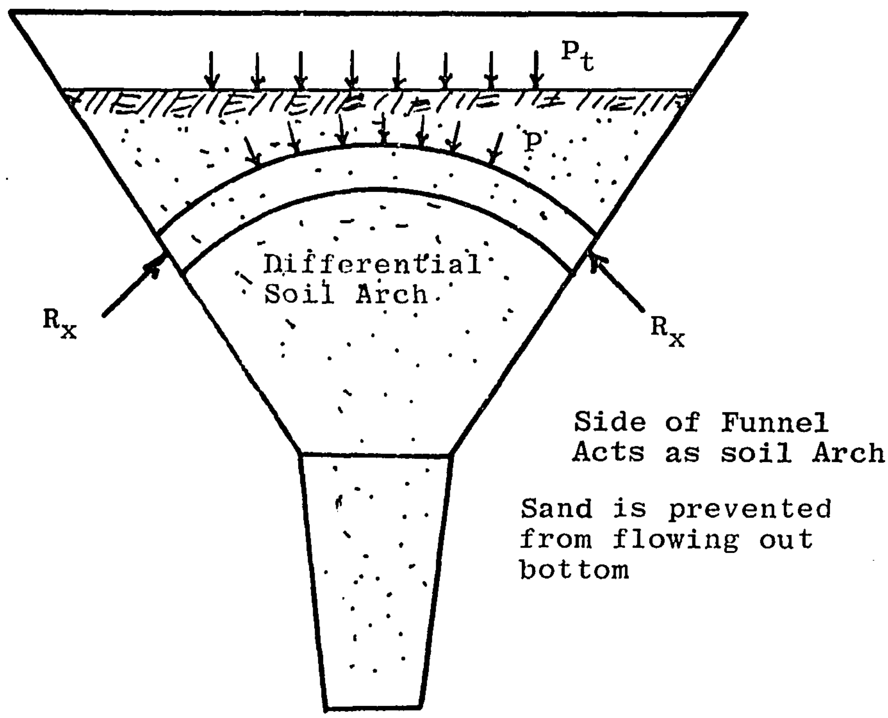

Figure 8 Funnel Analogy to Illustrate the Concept of the Formation of the Soil Arch

The region of maximum shearing stress can be determined from the theory of elasticity. However, it should be emphasized that once the shear stress in the soil causes the grains to move, there is a redistribution of stress and the theory of elasticity no longer applies. As soon as the soil grains move, the soil arch forms. Therefore, the theory of elasticity can be used to locate the supports resulting from incipient motion of the soil arch.

It is possible to take into account the stiffening effect on the conduit in locating the region of maximum shearing stress. Such an analysis has been presented by Burns and Richard (1964), using the theory of elasticity. The present knowledge of stress-strain and pressure redistribution characteristics of soil does not justify the added complication. For the sake of simplicity, the conduit stiffness will be assumed to be negligible.

Timoshenko and Goodier (1951) have given the equations for the stress at a point in an infinite plate with a circular hole as shown in Figure 9. Watkins and Nielson (1962) have taken these equations and determined the region of maximum shear stress. A brief summary of the derivation is given below.

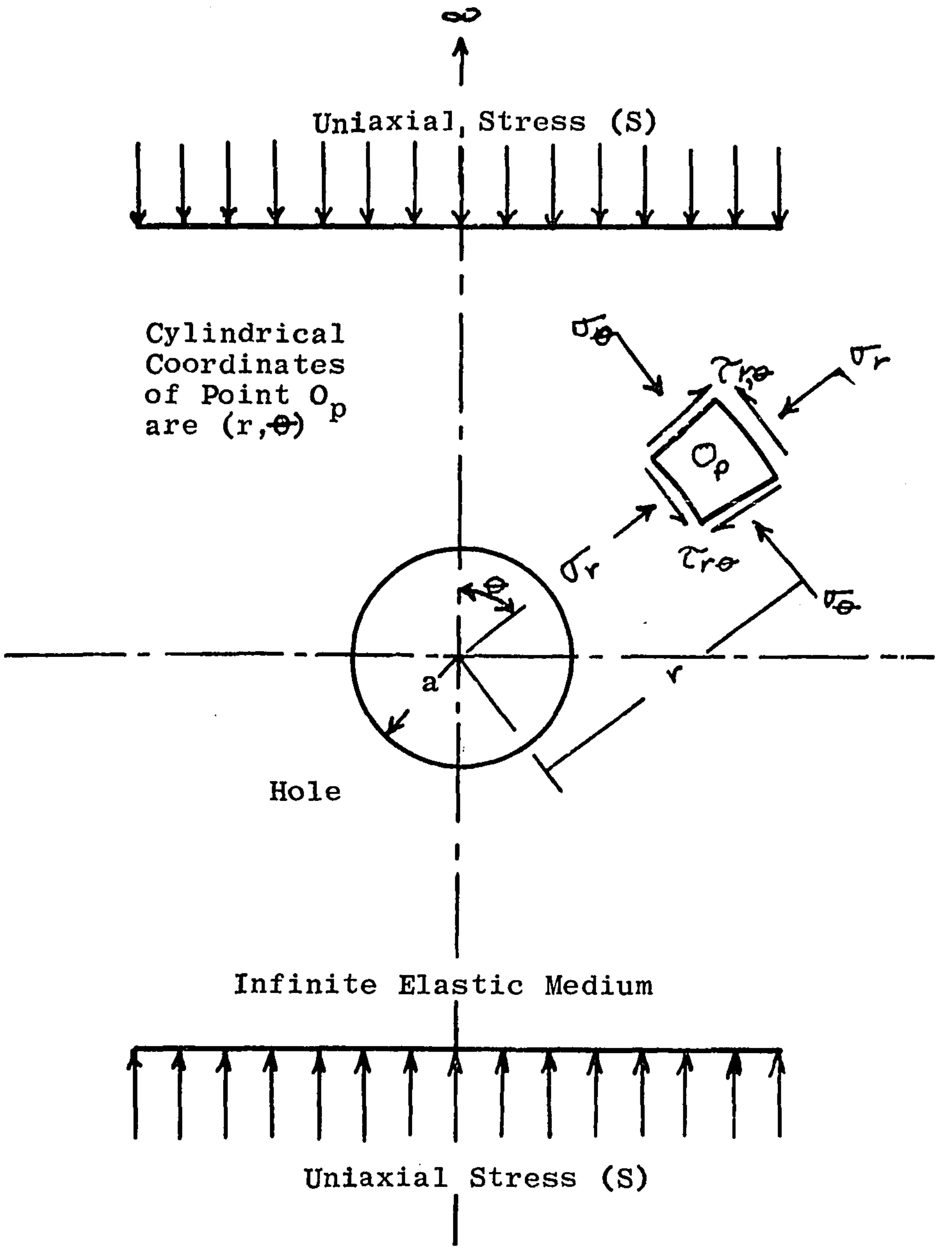

Figure 9 Stresses Acting at Point O_p near the Hole in an Infinite Elastic Medium with a Uniformly Distributed Uniaxial Load at Right Angles to the Axis of the Hole

The equations for the stresses at point (r,θ) are

$$\sigma_r = \frac{S}{2}(1-a^2/r^2) + \frac{S}{2}(1+3a^4/r^4-4a^2/r^2)\cos 2\theta \quad (5)$$

$$\sigma_\theta = \frac{S}{2}(1+a^2/r^2) - \frac{S}{2}(1+3a^4/r^4)\cos 2\theta \quad (6)$$

$$\tau_{(r,\theta)} = -\frac{S}{2}(1-3a^4/r^4+2a^2/r^2)\sin 2\theta \quad (7)$$

The various terms are shown in Figure 9. To reduce the length of the equations somewhat let

$$G = (1-a^2/r^2) \quad (8)$$

$$B = (1+3a^4/r^4-4a^2/r^2) \quad (9)$$

$$H = (1+a^2/r^2) \quad (10)$$

$$D = (1+3a^4/r^4) \quad (11)$$

and

$$E = (1-3a^4/r^4+2a^2/r^2) \quad (12)$$

Substituting Equations 8 through 12 into Equations 5, 6, and 7 yields the following simplified equations:

$$\sigma_r = \frac{S}{2}(G+B\cos 2\theta) \quad (13)$$

$$\sigma_\theta = \frac{S}{2}(H-D\cos 2\theta) \quad (14)$$

$$\tau_{(r,\theta)} = -\frac{S}{2}E\sin 2\theta \quad (15)$$

Once the values for the Equations 13, 14, and 15, are obtained the maximum shear stress can be found from a Mohr's circle analysis of the stress of a point. Figure 10 shows Mohr's circle for the stresses at a point.

The maximum shear stress is found from Figure 10 in terms of the other stresses as

$$\tau_{max} = \sqrt{\left(\frac{\sigma_r+\sigma_\theta}{2}\right)^2 + \tau^2_{(r,\theta)}} \tag{16}$$

The location of the maximum shear stress can be found by substituting Equations 13, 14 and 15 into Equation 16, differentiating with respect to θ, and equating to zero.

$$\frac{\partial \tau_{max}}{d\theta} = 0 \tag{17}$$

The angle θ at which the maximum shear stress occurs for any given value of the radius of the soil arch (r) is found to be

$$\cos 2\theta = \frac{TU}{4E^2\mu^2} \tag{18}$$

where T=G-H (from Equation 8 and 10)

U=3+D (from Equation 9 and 11)

If the values for T, U, and E are substituted into Equation 18 it becomes

$$\cos 2\theta = \frac{3a^4-2a^2r^2-r^4}{4(-3a^2r^2+2r^4)} \tag{19}$$

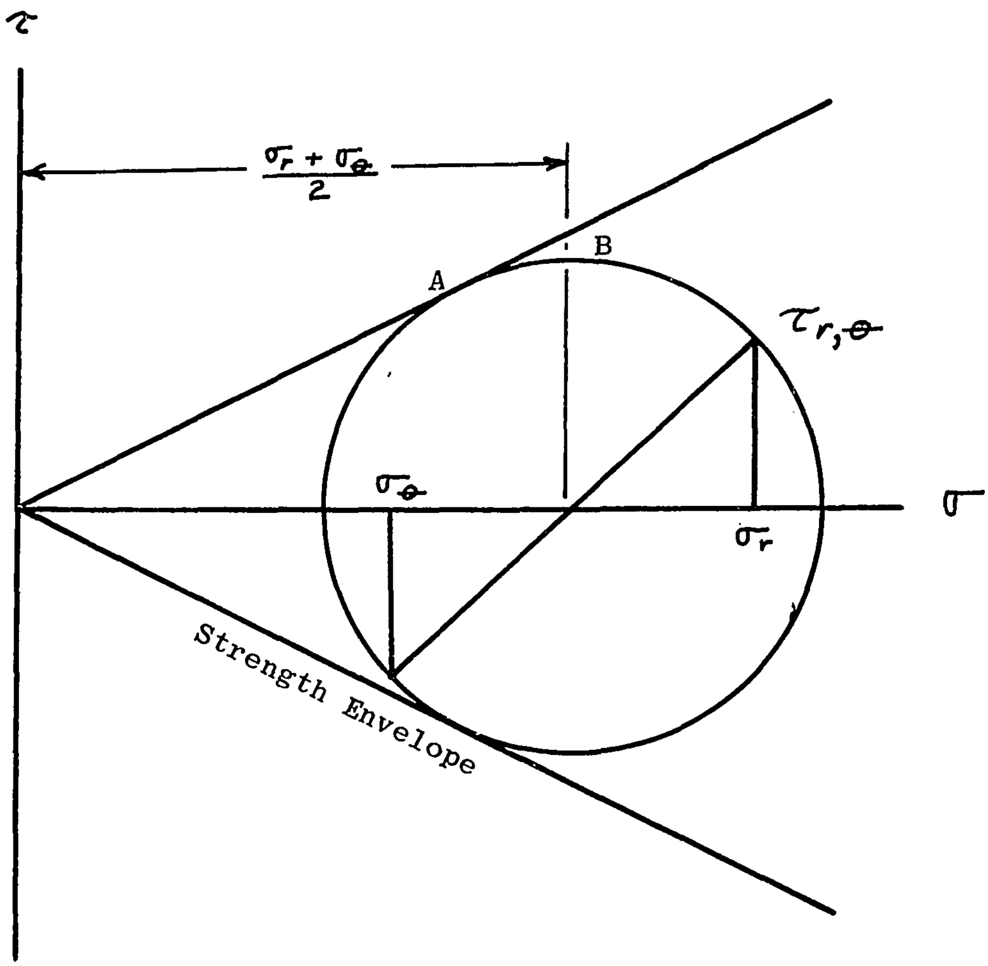

Figure 10 Mohr's Circle for Stress to Determine Maximum Shear Stress in Soil

This equation defines the point, a distance (r) from the center of the pipe, at which maximum shear stress occurs for any given value of pipe radius (a). The dotted line in Figure 11 shows the location of the plane of maximum shear stress as determined by Equation 19.

With the exception of the area immediately adjacent to the pipe, Equation 19 has essentially the same shape as a hyperbola. Equation 20, which is the equation of a hyperbola, can be substituted for Equation 19 with very little error.

$$\cos 2\theta = a^2/r^2 \tag{20}$$

The solid line in Figure 11 is a plot of Equation 20. The accuracy of the analysis does not justify the added complication of using Equation 19.

The validity of the above analysis as applied to soils can be evaluated by a critical study of analyses that have been made to determine the movement of soil around a buried pipe. One such study was made by Watkins (1957), who placed lead shot in a grid pattern in the soil mass around a model pipe. As the model was loaded, the movement of the lead shot was followed by taking a series of x-ray pictures. Figures 12, 13, and 14 show the results of the x-ray study.

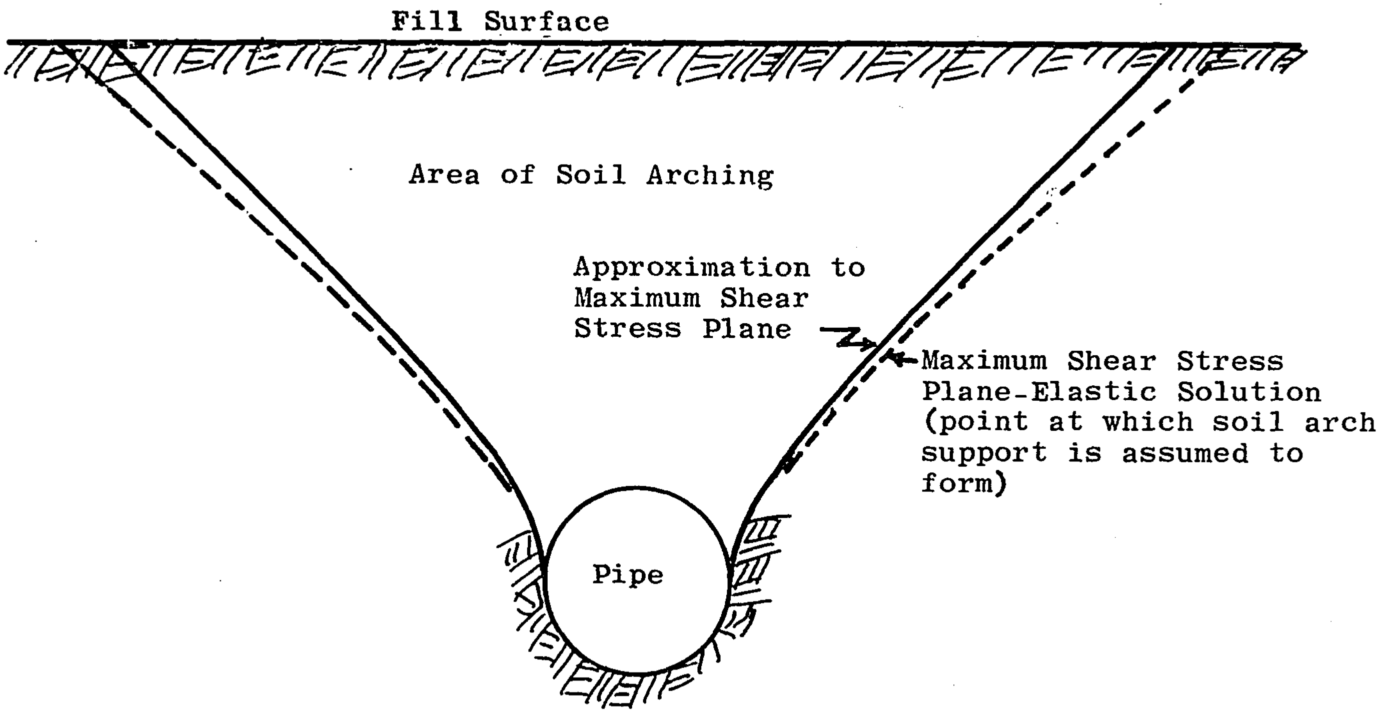

Figure 11 Comparison of Elastic Solution and the Assumed Appromimation for Location of the Region of Maximum Shear Stress which is Assumed as the Location of the Soil Arch Support

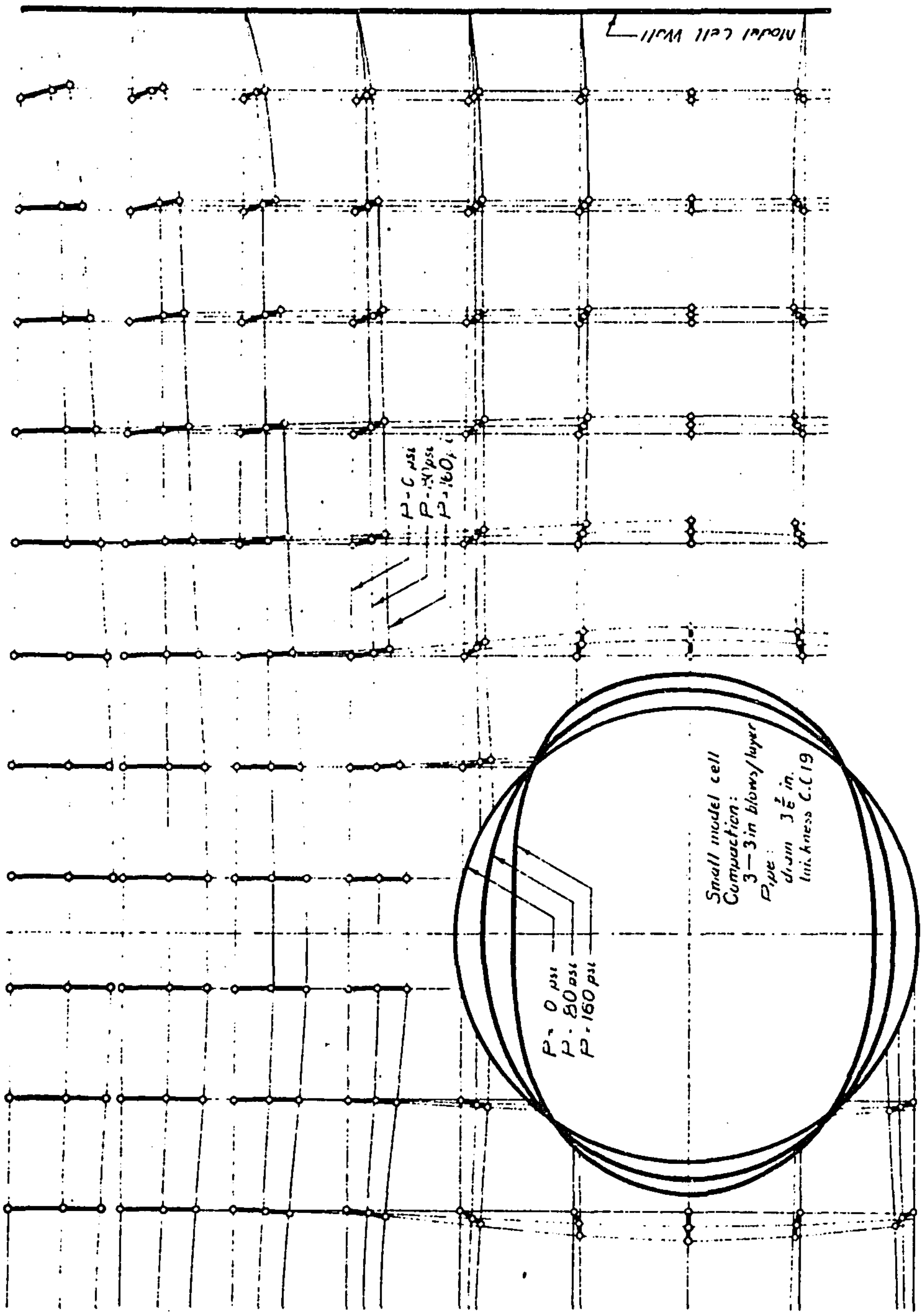

Figure 12 **Superimposed Position of Lead Shot in Loess from X-Ray Photographs at Various Pressures (after Watkins)**

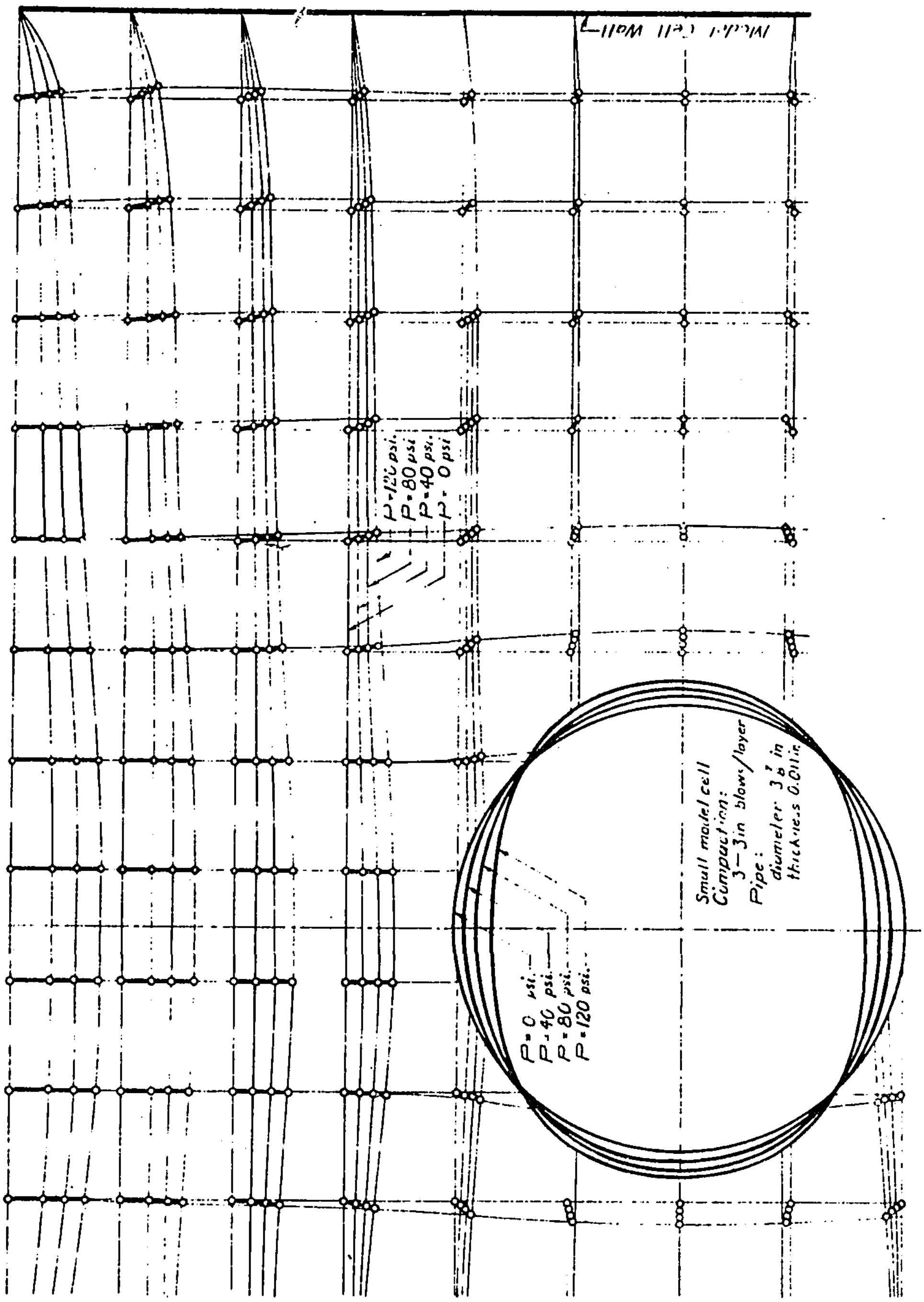

Figure 13 Superimposed Position of Lead Shot in Loess from X-Ray Photographs at Various Pressures (after Watkins)

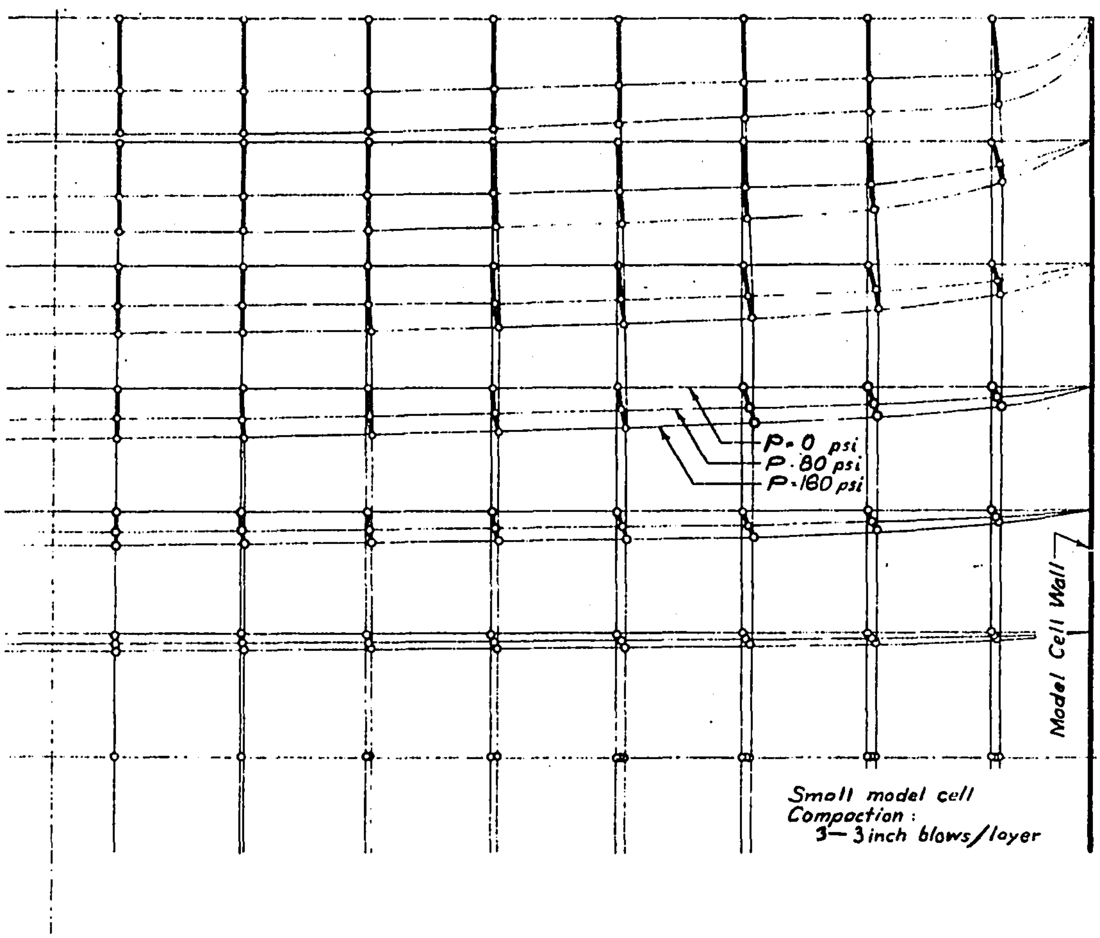

Figure 14 Superimposed Positions of Lead Shot From X-Ray Photographs for Loess with no Model Pipe at Various Pressures (after Watkins)

In order to determine the stress pattern induced by the addition of the pipe in the soil, the displacement of the soil without the pipe (Figure 14) was subtracted from the displacement of the soil with the pipe in place (Figure 12). These results are shown in Figure 15. The relative magnitude and direction of the displacements are shown by arrows which represent the movement of the soil due only to the influence of the pipe. The direction of the arrows indicates the direction of major principal stress in the soil medium. The minor principal stress is at an angle of 90° to the major principal stress. If a line is drawn through the soil mass perpendicular to the major principal stress at all points, as shown in Figure 15, it will trace out the line of action of a differential soil arch that has no shear or bending stress within its section.

The differential element traced out is somewhat flatter than the circular arch. It is not known what effect the boundaries of the cell had on the displacement pattern obtained. In order to get some idea of the magnitude of the influence of the cell wall, an analysis of the same problem was made using the theory of elasticity. The displacements of an infinite elastic plate with a stiffening ring were calculated using the equations presented

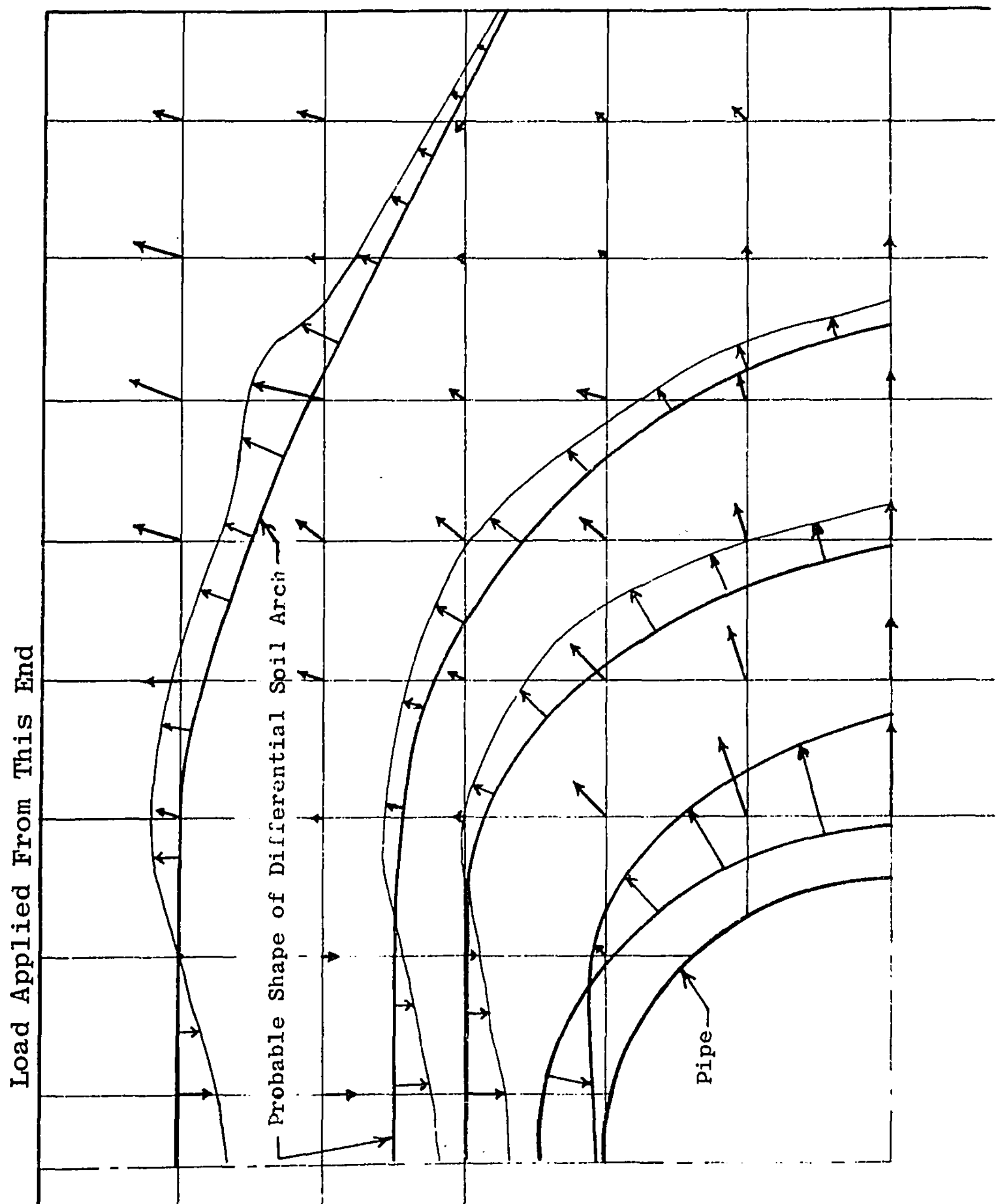

Figure 15 Movement of Lead Shot Due Only to Influence of Pipe as Determined From X-Ray Study-Determined by Subtracting Movement in Figure 14 From Movement in Figure 12

by Burns and Richard (1964). The displacements were calculated for the plate with the ring, and again without any hole at all. The displacements of the plate with no hole were then subtracted from the displacements of the plate with the ring. The results should represent the displacement of the medium due only to the influence of the ring. These results are presented in Figure 15a. The displacement pattern is approximately the same as that obtained from the x-ray analysis with the exception that the line following the direction of the minor principal stress is somewhat higher than a circle. The differential element which would be obtained from this analysis would be more the shape of a parabola. There is one difference between the displacements obtained in the soil by the x-ray analysis and those obtained from the theory of elasticity. The movement of the soil directly above the pipe is larger in the soil than in the elastic medium. As a result of the movement in the soil mass, the arch in the soil would be somewhat flatter or it would approach the configuration of a circle. It seems justifiable to assume that the shape of the differential arch in the soil mass is a circle, without adding any appreciable error to the solution.

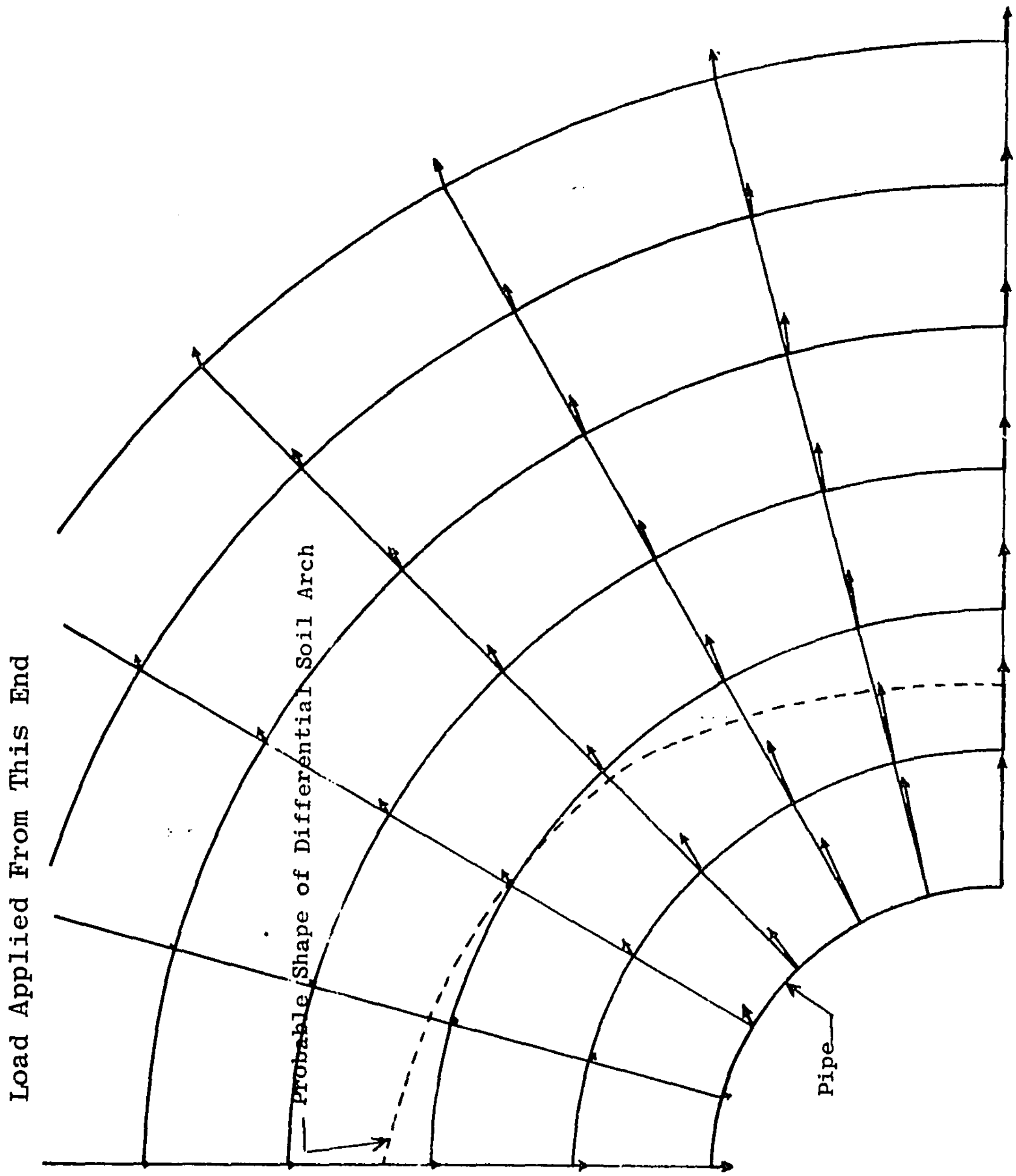

Figure 15a Displacements in Elastic Plate as Determined From Theory of Elasticity-Obtained by Subtracting Displacements in Plate With No Hole From Displacements in Plate With Pipe

Figure 15 was obtained by subtracting the displacement of the lead shot at a pressure of 160 psi in Figure 14 from the displacement of the lead shot at the same pressure in Figure 12. Displacements of the shot at 80 psi were in the same direction as at 160 psi with one exception. The upper limit of downward displacement of soil directly above the pipe was closer to the pipe at 80 psi than at 160 psi.

The direction of displacement of soil is interpreted to be the direction of the major principal stress in the soil mass. Since the direction of soil displacement is essentially constant for 80 psi and 160 psi, the direction of the principal stress along the plane defined by Equation 20 is assumed to be constant and independent of the applied pressure at any given point. If the direction of the major principal stress is constant at a point, the direction of the minor principal stress must be constant also. The direction of the minor principal stress on this plane corresponds to the direction that the assumed soil arch intersects this plane.

The displacement of the soil shown in the upper right hand corner of Figure 15 is steeper than would be expected assuming a circular arch. It is believed that the influence of the cell wall is responsible for causing the increase in slope in this area.

The results of one other study by Watkins and Nielson (1962) is presented in Figure 16. It shows only the difference in vertical displacements between the soil mass with the pipe in place and the same soil mass under the same loading condition without the model pipe.

There is one major difference between this study and the x-ray study. The pipe in this study was bored into place. The soil was compacted without the pipe then a hole was bored slightly larger than the diameter of the pipe and the pipe was placed in the hole. This will explain the difference in soil displacements between the vertical plane at the side of the pipe and the plane defined by Equation 20. As the pipe was loaded it could exert only limited, if any, horizontal pressure because the hole was larger than the pipe. Therefore the horizontal pressure component exerted by the pipe on the soil mass was missing.

The center section of soil directly above the pipe moves down more than the rest of the soil mass. The soil will move down until the vertical force builds up. As the vertical pressure increases, it will cause the soil to act as a wedge causing the soil arch to form. The arch supports lie on the plane defined by Equation 20.

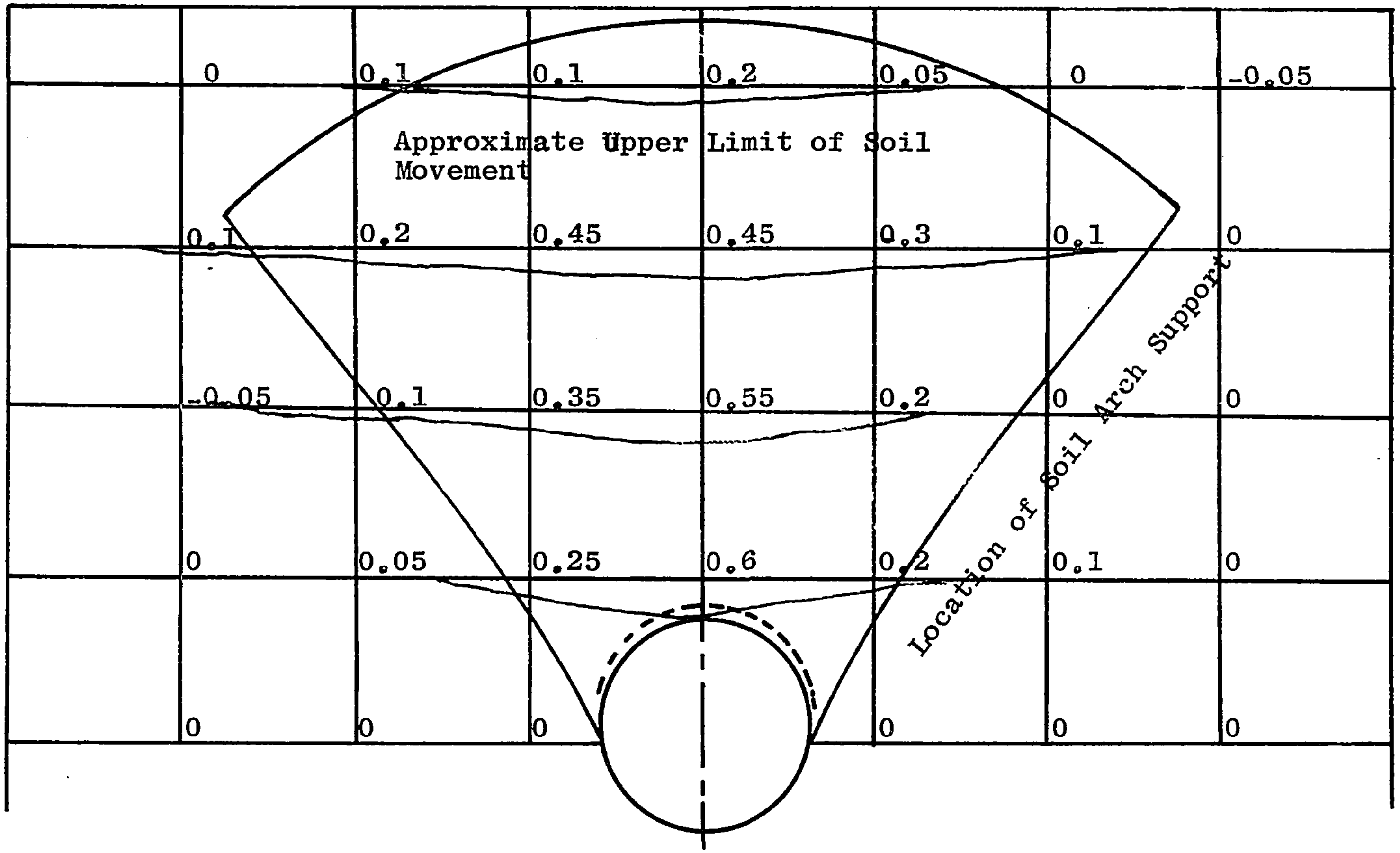

Figure 16 Vertical Soil Deformation at Various Elevations above a Model Conduit (each number is the difference in soil movement with and without the conduit in place)

The study shows that the vertical displacements of the soil due to the influence of the pipe occurs within the area bounded by Equation 20.

A study made by Terzaghi and Richart (1952) tends to point to the same stress distribution in rock about a tunnel that is assumed to act around the culvert. They point out that the stresses around an arch are lower for an arch than for any other shape of tunnel investigated. If the stresses in the surrounding rock are lower in the arch than in the other shapes, the arch would be the configuration that a natural cavity would tend to develop. They indicate that in natural limestone cavities, arched roofs with spans of up to 600 feet have been encountered.

The roof configuration that develops during stoping in mines is also an arch shape. Moreover, they also state that during collapse of a mine roof the collapse structure moves outward in a radial manner.

CHAPTER III

OPERATIONAL AND CONCEPTUAL DEFINITIONS

Modulus of Passive Resistance of Soil

At this point, a discussion of the modulus of passive resistance (hereafter called the soil modulus), as defined first by Spangler (1942) and later modified by Watkins and Spangler (1957), is in order.

In the original derivation of the equation of equilibrium used to determine the deflections of flexible conduits Spangler (1942) assumed that the soil modulus was similar to the modulus of subgrade reaction used in highway work. The units of the original soil modulus (e) were lb/in^3. Later Watkins and Spangler (1958) pointed out that the soil modulus (e) was not constant but depended upon the radius of the pipe. The true soil modulus was er=E'. However, it was still assumed to be linear.

Later Watkins and Nielson (1962) described the nonlinearity of the soil modulus, which was observed in model studies made in the laboratory. Figure 17 shows the apparatus used in the model tests. They also constructed an instrument called the modpares device for measuring the soil modulus. Figure 18 shows a schematic

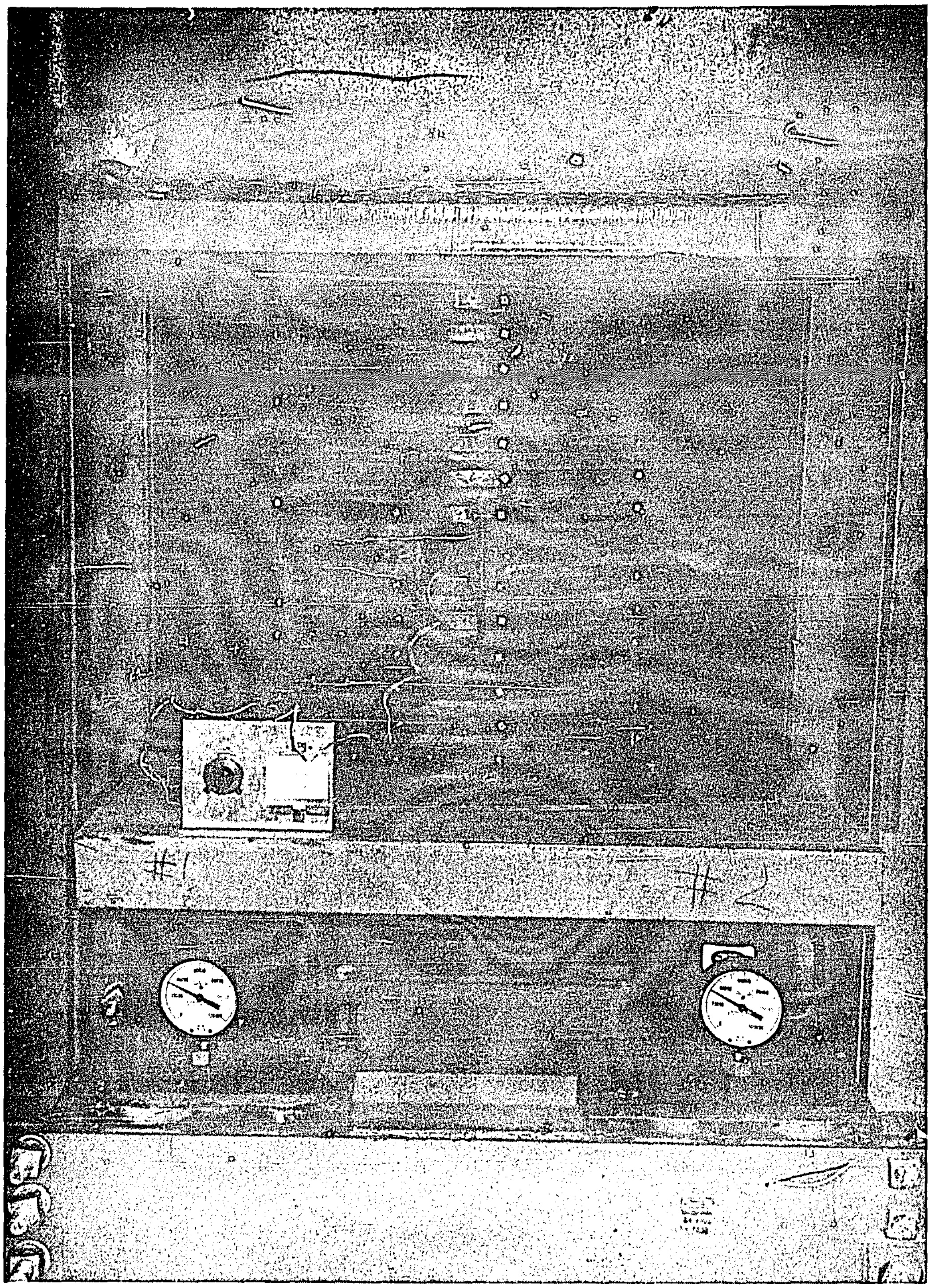

Figure 17 Testing Frame and Cell Used in Model Study by Watkins and Nielson

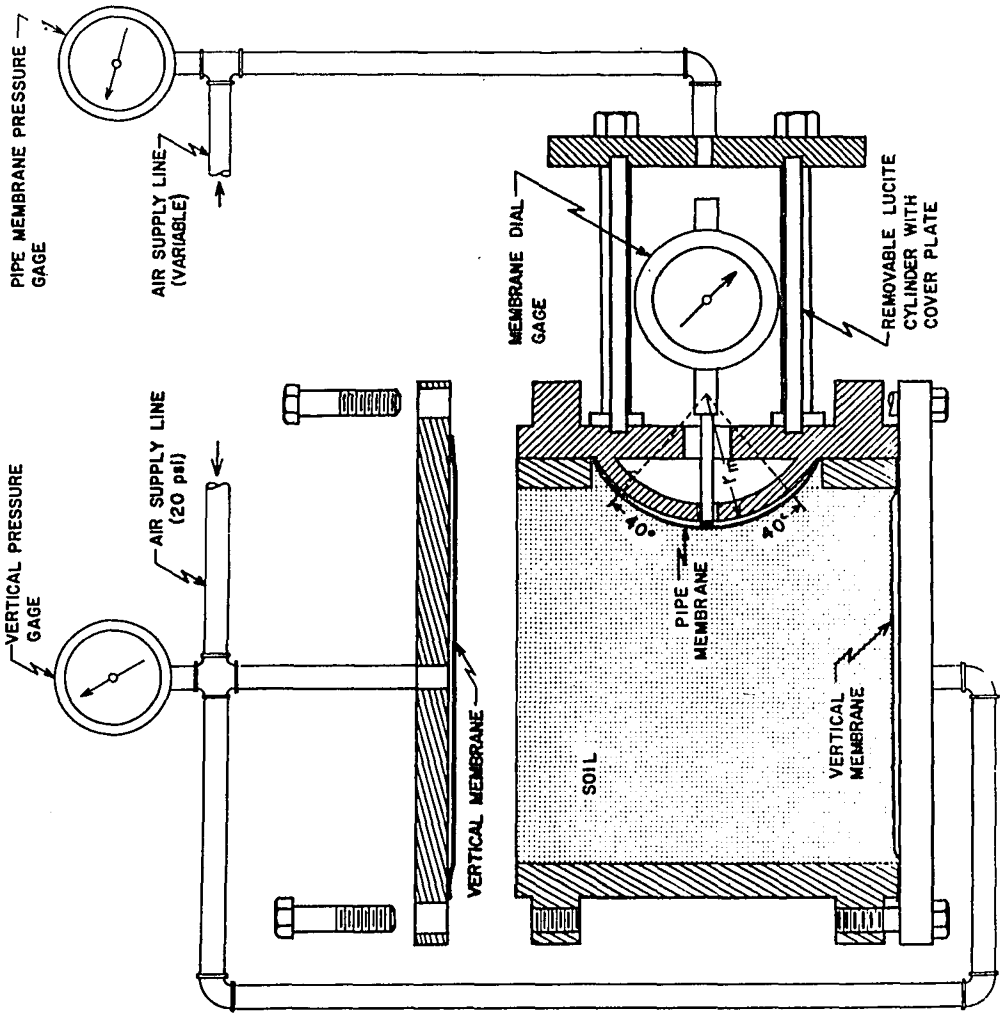

Figure 18 Schematic Diagram of Modpares Device

diagram of the apparatus. As the pressure is applied to the rubber membrane around the circular pipe the resulting membrane deflection is measured. The soil modulus is defined as

$$E' = 2h/(dX/D) \tag{21}$$

where h = soil pressure at the side of the pipe or membrane

dX= horizontal deflection of pipe membrane

D = diameter of the pipe or membrane.

A typical plot from the modpares device is shown in Figure 19. For example of curves of different soil types see paper by Watkins and Nielson (1962).

Both the model tests shown in Figure 20 and the curves from the modpares device (Figure 19) plainly show the non-linearity of the soil modulus. According to Spangler's deflection theory the curves should be straight if the modulus was linear. The vertical pressure on the modpares device was adjusted until the soil modulus at 5 percent deflection was the same as that obtained when calculated from Spangler's equation (Equation 22) and the model studies (Figure 23).

$$dX = \frac{KW_c r^3}{EI+.061E'r^3} \tag{22}$$

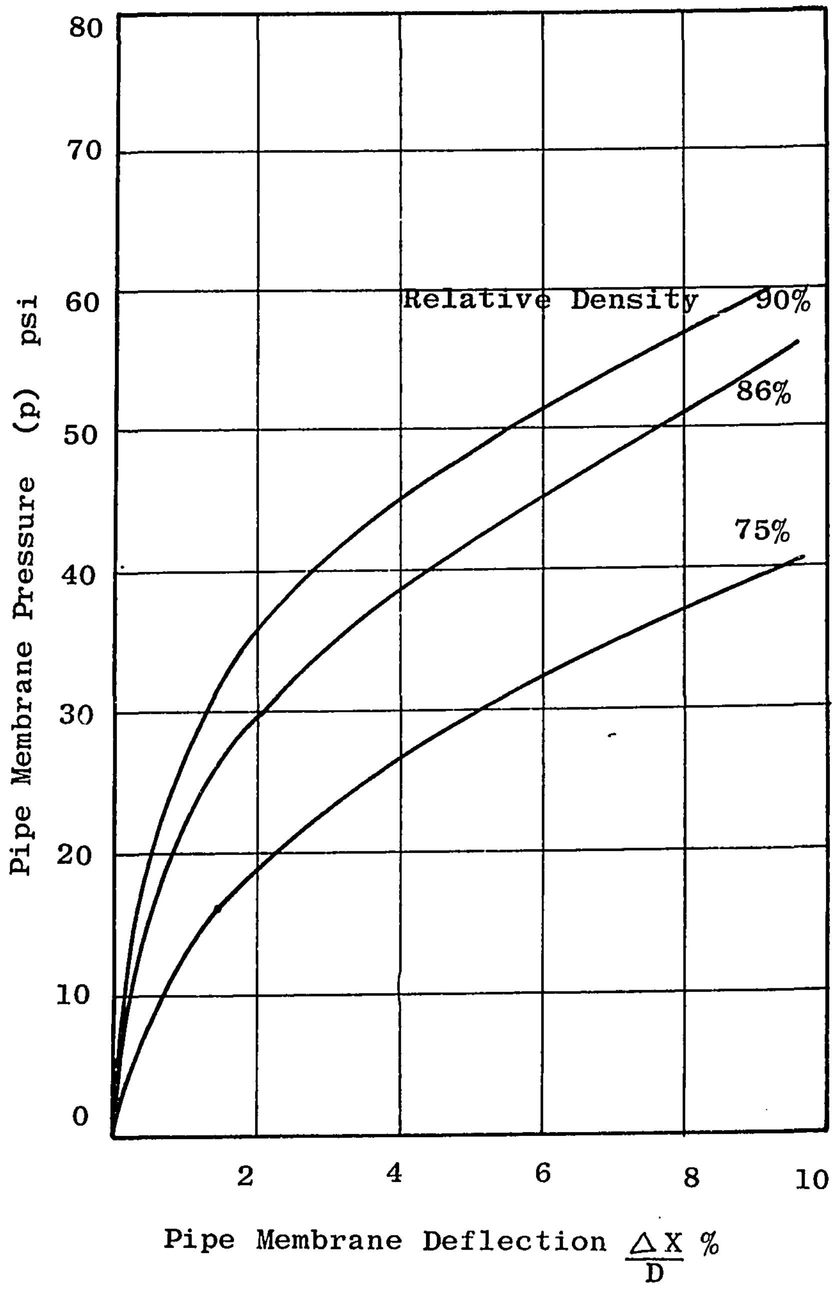

Figure 19 Typical Plots of Modpares Data for Cohesive Soil

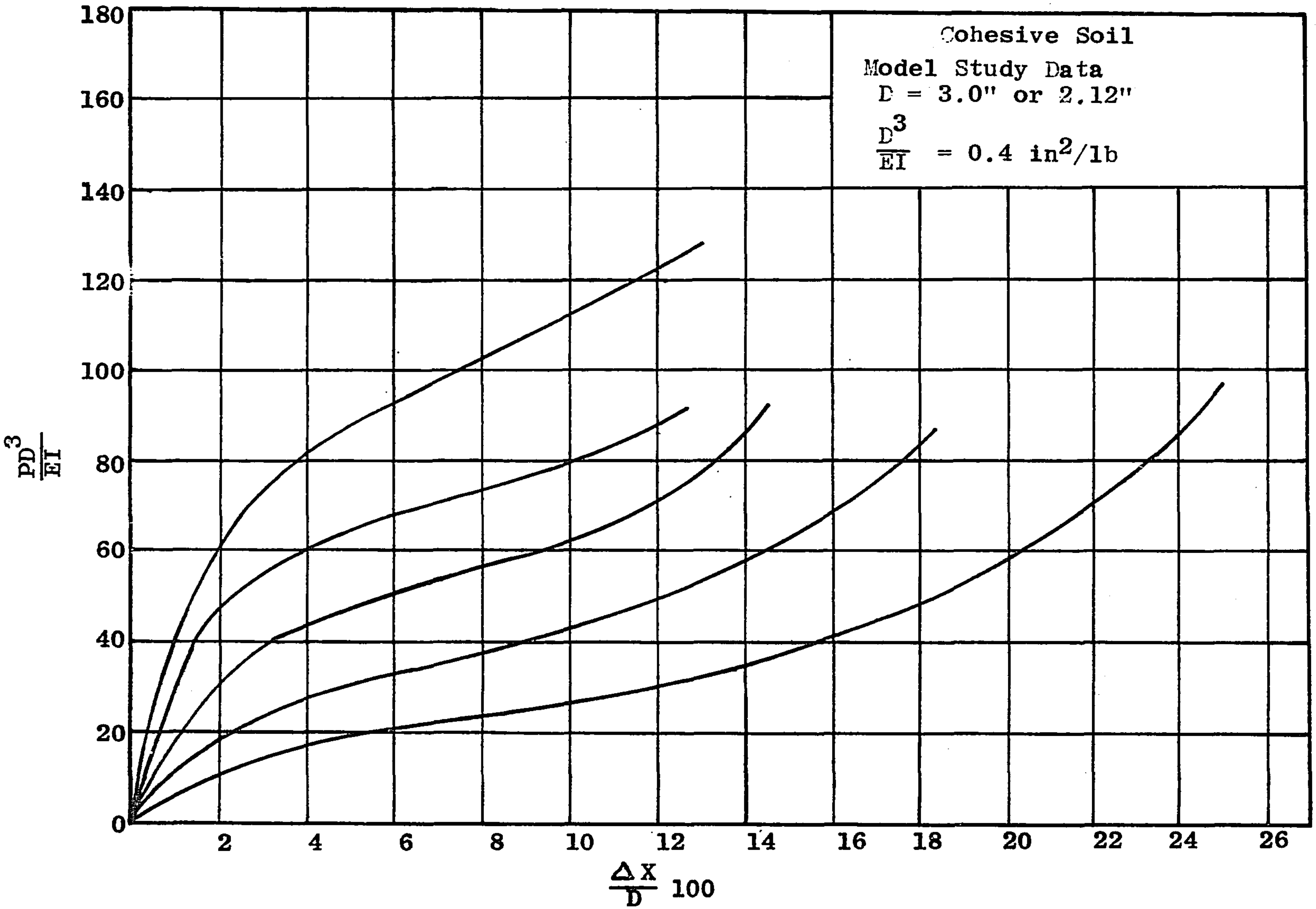

Figure 20 Load Deflection Curves for Cohesive Soil Determined by Model Study

where

dX = deflection of the horizontal diameter of the pipe

KW = bedding constant (see Spangler (1960) for values)

W_c = load in pounds per foot on the pipe

r = radius of the pipe

E = modulus of elasticity of the pipe

I = moment of inertia of the pipe wall

E' = modulus of passive resistance of the soil.

If the curve obtained from the modpares device is used directly in the analysis for deflection that is described later, it will predict deflections that are approximately 30 to 50 percent high for small deflections of the order of 2 or 3 percent of the pipe diameter. This limits the use of the modpares device in predicting the deflection directly from the modpares curve. However, this does not prevent the modpares device from being used to predict the deflection and loads on underground pipe. A series of soil modulus curves can be calculated from the model data and in design problems the proper curve can be selected with the modpares device.

For design purposes, only one curve for sand and one curve for clay soils need be calculated from the model data. Once the form of the curve is obtained, the other curves can be approximated by multiplying the pressure (h) in Equation 21 by the ratio of the pressure obtained from the modpares device at a deflection of 5 percent of the diameter to the pressure obtained from the model data at this same percentage. This correction is generally valid at deflections less than five percent; however, at large deflections there are some deviations. This does not limit its use, because very few installations are designed for deflections greater than 5% of the diameter.

The load deflection curves from the model test have somewhat the same general shape as the constrained or confined modulus of elasticity of soils. This may allow the soil modulus to be determined from a confined compression test. The soil modulus would have to be determined from the theory of elasticity using the constrained modulus of elasticity. Watkins (1960) has made studies to determine the soil modulus from a confined compression test. This area, however, needs further investigation and is not included in the scope of this research.

Lambe (1960) has performed field tests which show the non-linearity of the soil modulus. He cut two circular plates with different diameters out of the side of a buried pipe. Figure 21 shows the pressure deflection

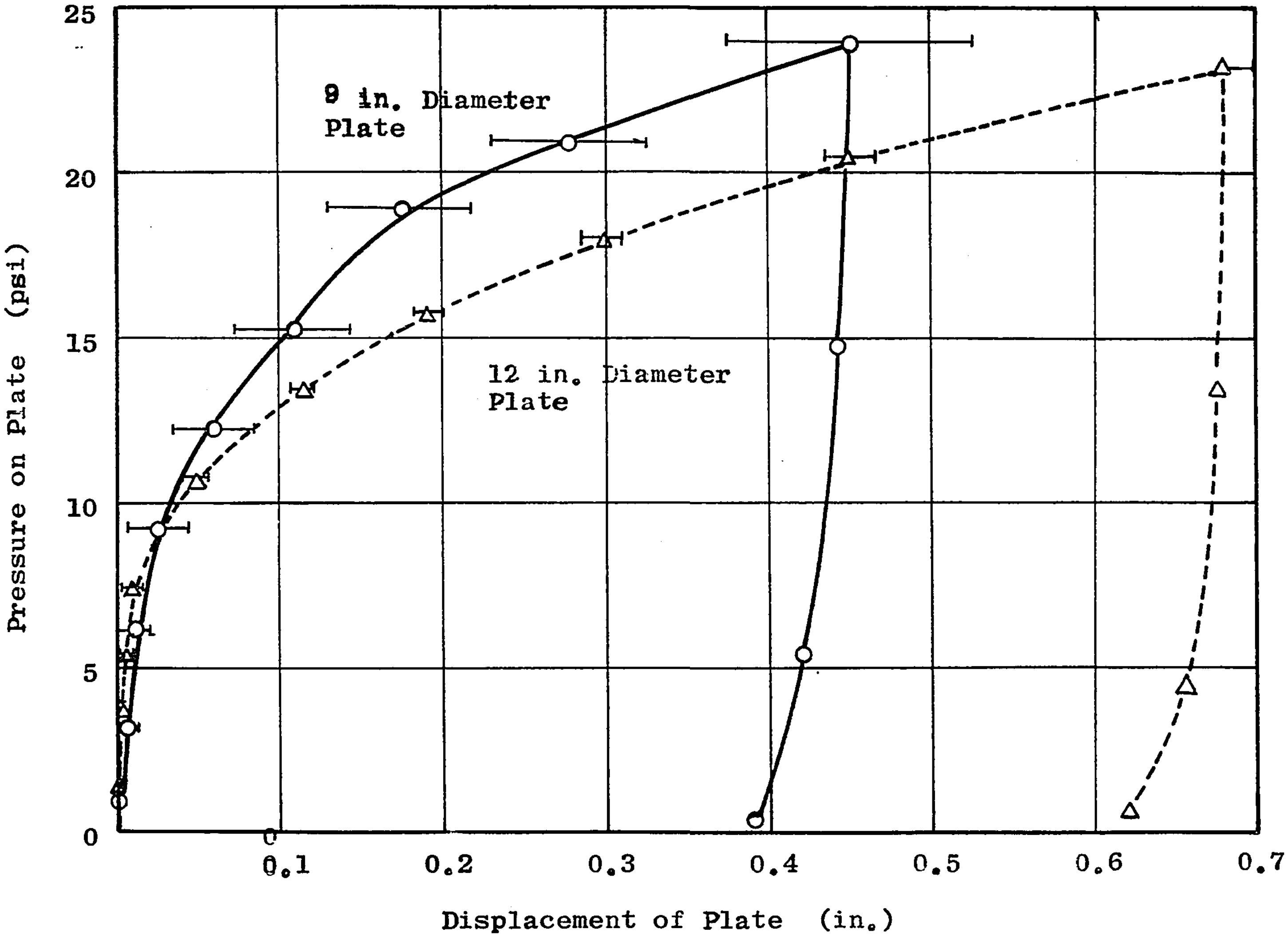

Figure 21 Pressure-Deflection Curves to Determine Soil Modulus
(after T. W. Lambe)

curves obtained as the plates were jacked into the soil. There are two contributing factors causing the smaller plate to have a higher pressure than the larger for a given deflection. The first is due to the perimeter shear stresses. The ratio of perimeter to area is larger for the smaller plate than the larger plate. The bearing pressure on the plate is due to the perimeter shear and bearing capacity of the soil. This concept was introduced by Housel (Capper and Cassie 1953). The second contributing factor is the larger stress bulb in the soil under the larger plate. The larger stress bulb will have more soil under strain. Deflection is proportional to strain multiplied by the length, therefore, the deeper stress bulb should have more settlement.

One would expect the shape of the curve obtained from Lambe's study for the soil modulus to be similar in shape to the curve obtained from the plate-bearing test because of the similarity of the two tests. The curves for most plate-bearing tests are similar to Figure 21. This presents another possible way of identifying the soil modulus to be used in predicting the deflection of a flexible conduit.

There is one very basic reason why the shape of the curves obtained from the plate bearing test and the modpares device differ from the shape of the curve obtained from the model test. Neither the modpares curve nor the plate bearing curve show the reverse curvature that the model test shows. The plate-bearing test has a lateral or confining pressure proportional to the vertical stress. The modpares test has a biaxial stress but the vertical stress is held constant at 20 psi. On the other hand, the model test has a biaxial stress field in which the confining stress increases non-linearly due to the increased height of fill above the pipe plus the deflection of the pipe into the side fill material.

The model test can be analyzed by an analogous change in the vertical pressure in the modpares device. As the pressure causing the pipe membrane to deflect is increased the vertical pressure could also be increased. As the vertical pressure is then increased it causes the soil to be more resistant to pipe membrane deflection resulting from the pipe membrane pressure. When the vertical pressure becomes sufficiently large a reverse curvature in the pipe membrane pressure vs. deflection curve will occur and has been measured. The main difficulty in using the modpares device as a model pipe is selecting the proper ratio between the pipe membrane pressure and the vertical pressure. Almost any shape of curve can be obtained simply by varying the pressure ratio. This difficulty was overcome by setting the vertical pressure at 20 psi and using the modpares device only as a classification device to select the proper soil modulus curve which was obtained from model data. With the exception of the model tests and the constrained modulus, each of the curves plot as a straight line on log-log or semi-log paper. This would make the equation for the line of parabolic or exponential form. However, to make the computer program used later more versatile a polynomial equation is used.

Several other forms of equation have been suggested for the non-linear stress-strain diagram. None of them, however, will allow a reverse curvature such as the constrained modulus exhibits.

Loads on Underground Conduits

The loads transmitted through the soil to an underground conduit can be determined using the concept of the soil arch. Figure 22 shows a free body diagram of the assumed loading. The differential equation can be obtained by summing the vertical forces on the differential element. A circular arch is assumed because of simplicity. Also the exact direction of the principal stresses in the soil mass are not known. The validity of the circular arch

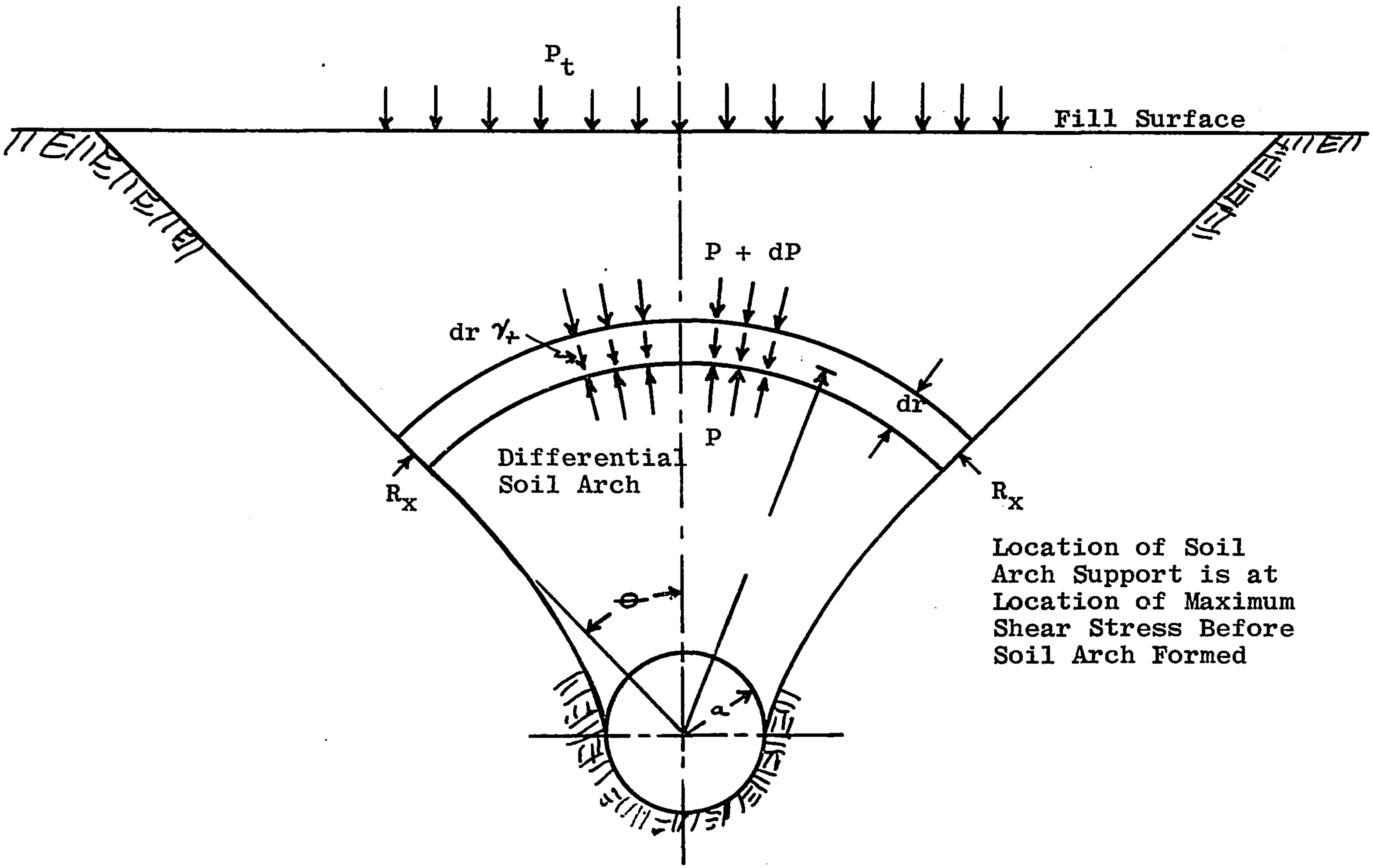

Figure 22 Free-Body Diagram for Determining Loads on Buried Conduits by an Arching Analysis

may be questioned at the pipe wall, because of the radial pressure distribution necessary for this type of arch. But, if one takes the loading on a circular pipe as assumed by Spangler (1942) and finds the radial components, it can be seen that the assumption of radial pressure across the top is not greatly in error. For this reason and for simplicity, the radial distribution will be used in the analysis presented in this study.

By summing the vertical forces on the differential arch in Figure 22 the differential equation governing the problem can be developed. The vertical component of the force acting down on top of the differential arch

$$2 \int_0^{\theta} (P+dP) \cos\theta d\theta = 2(P+dp)r\sin\theta$$

(For notation see Figure 22).

Likewise, the vertical component of the force acting on the bottom of the differential arch is

$$2 \int_0^{\theta} P\cos\theta d\theta = 2pr\sin\theta.$$

The above two equations, which were obtained for a radial pressure distribution, show that the vertical components of the force are the same as those obtained from a uniform vertical loading on a differential element. The distance between arch supports is $2r\sin\theta$. The right hand side of the above equations can be obtained by multiplying $2r\sin\theta$ by the appropriate uniform vertical pressure. The total weight of the soil in the differential circular arch is

$$2\ \gamma(dr)\int_0^{\theta} r\,d\theta\ (1) = 2r\gamma\theta(dr).$$

If the circular arch is used, an equivalent radial pressure (P') should be applied to the arch to include the influence of the weight of soil. The equivalent pressure would be equal to the weight of soil in the element divided by the distance between supports or

$$P' = 2\gamma\ \theta_{max}\ r\ dr/2r\sin\theta_{max} = \frac{\gamma\theta_{max}dr}{\sin\theta_{max}}$$

Where θ_{max} = the value of θ at the soil arch support.

The vertical component of equivalent load would be equal to the weight of soil in the differential element.

The vertical component of the force acting on the support of the differential element is

$$R_x\ dr\ (1)\ \sin\theta$$

where

R_x = stress in the line of action of arch at the soil arch support.

By taking the summation of the forces on the element equal to zero, the differential equation can be derived.

$$2(P\text{-}dp)r\sin\theta - 2pr\sin\theta + 2r\theta rdr - 2R_x dr\sin\theta = 0$$

or

$$dP = \frac{R_x \sin\theta dr - r\theta\gamma dr}{r\sin\theta} \tag{23}$$

where

P = pressure acting on the differential arch

R_x= stress in line of action of arch at the arch support

r = radius of circular arch

θ = angle at center of pipe from the vertical to the arch support

γ = unit weight of soil.

It may be argued that the weight of the soil should be assumed to act radially on the arch to be compatible with the assumption of no bending moments in arch. If this assumption is made the vertical component of the force due to the weight of the soil is

$$2\ \gamma(dr) \int_0^\theta r\ \cos\theta d\theta = 2\gamma\ r(dr)\sin\theta.$$

If the above equation is used, the differential equation can be obtained as before.

$$2(P+dP)\ r\ \sin\theta - 2pr\sin\theta + 2\gamma\ r\sin\theta\ dr - 2\ R_x(dr)\sin\theta = 0$$

$$dP = \frac{R_x - \gamma rdr}{r} \tag{23a}$$

These equations cannot be integrated unless the value of R_x and θ are determined in terms of the radius r. The relationship between θ and r has been determined in a previous section. It can be substituted into Equation 23. The equation used depends on the construction practice and the general environment in which the pipe is placed. If the pipe is placed in a vertical trench, the differential soil arch would extend only across the trench and Equation 1 would be used, but if it is in an earth fill, Equation 19 or 20 would be used. Equilibrium conditions are satisfied by Equation 23. Compatibility of the soil deformation and pipe deformation will be discussed in the following section. The relationship between the force R_x at the end of the soil arch and the radius r will be developed in the following section.

Stresses at the Differential Soil Arch Support

The value of the support pressure R_x at the differential soil arch support is difficult to determine. The first approach assumed the stress was simply the radial pressure in the soil at that point multiplied by the active pressure coefficient (K_a). In this approach, when the friction angle of the soil is reduced, the theory using the active pressure predicted more attenuation of pressure. Laboratory measurements (Terzaghi, (1943)) indicate that it should be just the opposite. Therefore it was necessary to use some other means of determining the pressure at the soil arch support.

If the radial pressure in the soil is multiplied by the passive pressure coefficient (K_p) and the product used to evaluate the support pressure, the attenuation increases with an increase in friction angle of the soil. This, however, is obviously inconsistent with the physical characteristics of the system in which active soil condition exist.

In order to obtain some type of a relationship for R_x, the following tabulated variables are considered to be important in the determination of the stress at the soil arch support. The variables considered are as follows:

Variable	Symbol
Stress at Soil Arch Support	R_x
Soil Modulus Horizontal Deflection of Pipe	E'
One-half Length of Soil Arch	L
Radius of Pipe	a
Pressure on Pipe	$p = W_c/2a$
Pipe Wall Stiffness/in of length	EI

The modulus of elasticity of the soil is not included in the above list of variables because it is not independent of the soil modulus (E'). Also, the radius of the pipe, pressure on the pipe, pipe wall stiffness, and the horizonal deflection of the pipe are not all independent variables. It is assumed that these variables enter into the determination of R_x only through the deflection of the pipe. Spangler's deflection equation (Equation 22) relates these variables to one another as follows:

$$\Delta x = \frac{KW_c r^3}{EI + .061\ E'r^3}$$

By letting $\Delta\chi$ represent the influence of these variables, the number of primary independent quantities is reduced to four. It is assumed that these variables can be combined in the following form to determine the equation for the stress on the differential soil arch support.

$$R_x = \frac{\Delta}{L} E' \times C \qquad (24)$$

Where C = Constant.

This approximation assumes a hyperbolic distribution of stress in the soil mass. The length of the arch (L) can be expressed in terms of the radius of the soil arch (r) and the angle (θ). The angle θ is approximately 45° throughout the soil mass, L is assumed to equal 0.785 x r.

When the substitution is made:

$$R_x = \frac{\Delta}{0.785r} E'C$$

When r becomes large, the value of R_x approaches zero, indicating that the arching takes place immediately in the vicinity of the pipe. If the pipe is infinitely rigid, the value of Δ will be zero, and the value of R_x will be zero; allowing no arching action to occur. If the pipe is very flexible, all of the stress must be transmitted into the surrounding soil by arching action. The deflection in an element, similar in shape to the arch shown in Figure 5 can be shown to be

$$y = \int_0^r \frac{P}{E} dy = \frac{P}{E} r$$

The deflection must also equal the value of Δ in Equation 24, or

$$\frac{R_x r}{E} = \frac{KW_c r^3}{EI + .061Er^3}$$

If the pipe is very flexible, the stiffness (EI) can be neglected and:

$$\frac{R_x r}{E} = \frac{KW_c}{0.061 E'}$$

Substituting in $\sigma/\frac{\Delta x}{D}$ for E' and $2\sigma r$ for W_c and reducing yields:

$$R_x = \frac{2K\Delta E}{0.061\ r}$$

Spangler (1960) gives values of K ranging from 0.083 to 0.110, if K = 0.110

$$R_x = 3.62\ \frac{E\Delta}{R}$$

For Poissons ratio equal to one fourth, E' can be shown to be 1.8E. Substituting in the value of E' yields:

$$R_x = \frac{2E'\Delta}{r}$$

Thus the constant in Equation 24 is approximately two.

To compare with field data the value of R_x used in the analysis was

$$R_x = \frac{2E'\Delta}{0.785r} \qquad (25)$$

Where

R_x = compressive pressure on the soil arch support perpendicular to the line of action in the arch.

Δ = deflection of soil arch support at the pipe boundary.

r = distance from center of pipe to the arch as shown in Figure 22.

E' = modulus of passive resistance of soil.

The value of the stress at the soil arch support depends on the soil modulus (E') and the deflection of the soil arch support. But the soil modulus also depends on the applied stress and the deflection of the soil arch support, which in turn depend on the stress level and on the soil modulus. In order to make some allowance for the interaction of the above-mentioned variables, Equation 25 was used to determine the stress at the soil arch support.

As an additional basis for justifying Equation 25, one should consider the soil modulus (E') in Equation 22, which is used to predict the deflection of an underground conduit as derived by Spangler. In the original derivation of the equation the value $E' r^3$ in the denominator was $e r^4$, where e had the dimensions of lb/in^3. It is analogous to the modulus of subgrade reaction (K) used in highway engineering. Watkins and Spangler (1957) pointed out that the value of the soil modulus depends on the radius of the pipe and that er=E' was the correct constant. A value with units similar to e is the modulus with the proper dimensions to be used in Equation 25. Therefore, the value of E' must be divided by some dimension of length. The radius of one-half the differential element seems to fit best.

Equation 25 can be substituted into Equation 23 for the value of R_x, and such Equations as 4, 19, or 20 can be substituted for the value of θ. The resulting equations become difficult to integrate. Each type of trench condition would require a separate integration. In each case the equation for θ would have to be substituted into Equation 23.

At this point it is easier to turn to the computer and a numerical integration procedure than it is to integrate each individual problem as it arises. Appendix A contains a computer solution for Equation 23, for a circular conduit.

The value of Δ in Equation 25 is unknown. It is therefore impossible to solve the equation because there are two unknowns and only one equation. Another equation must be obtained in order to meet compatibility requirements. Equation 22 with a modification for the non-linearity of the soil modulus will be used.

With a relationship for Δ, it is possible to solve the equation for the pressure at the top of the conduit. It is still a trial-and-error procedure because of the non-linear soil modulus.

CHAPTER IV

VERIFICATION OF LOAD THEORY

In order to verify any theory it is necessary to resort to laboratory or field studies. Model analysis in the laboratory is one of the most rewarding methods. Figure 23 shows a comparison between the computer solution and model tests obtained by Watkins and Nielson (1962) for a clay soil. These curves are the ones used to evaluate the functional relationship in Equation 25. Figure 24 shows a comparison between the computer solution and the model tests for sandy soils.

There are several field studies reported in the literature. One study was made by Kaiser Aluminum Corporation (1963). Figure 25 shows a comparison between their measured values and the values obtained from the computer solution. The value for the soil modulus in this particular soil was measured by Watkins in the modpares device. Kaiser Aluminum supplied him with the soil samples. Figure 26 shows the curve from the modpares device used in the analysis.

The measured curve in Figure 25 does not agree very well with the computer solution. One variable which

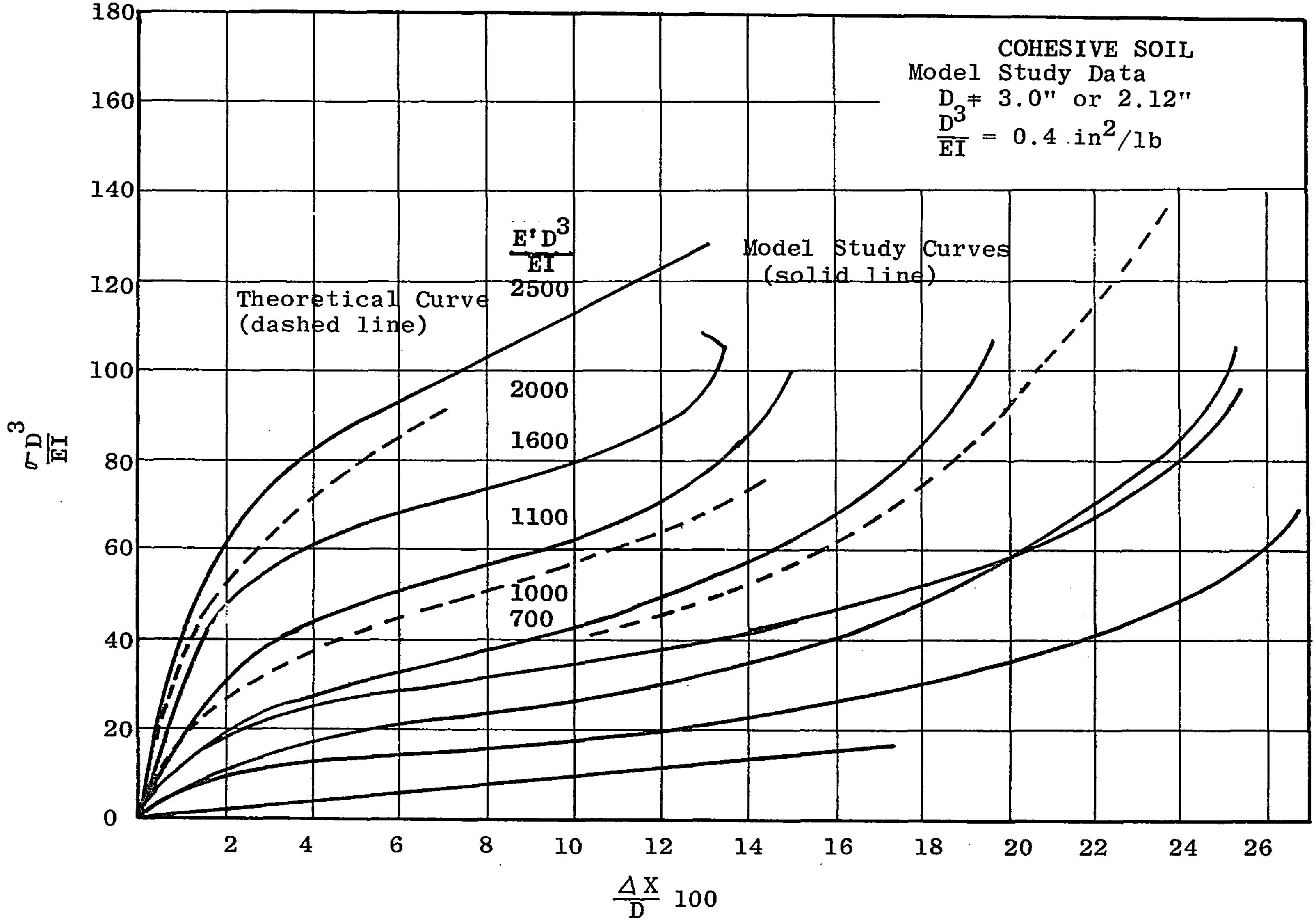

Figure 23 Comparison of Model Study Curves and Theoretical Curves

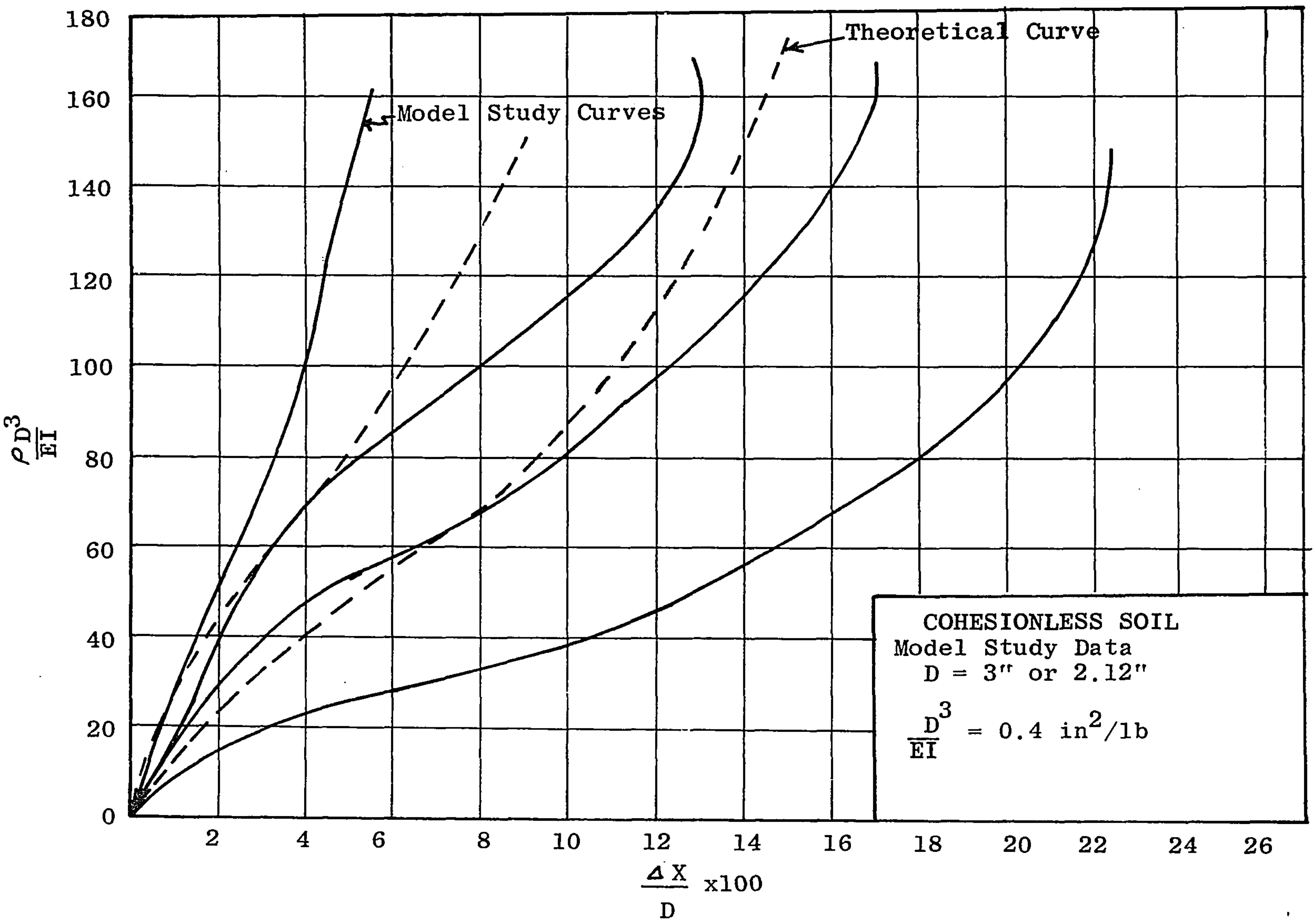

Figure 24 Comparison of Model Study Curves and Theorefical Curves

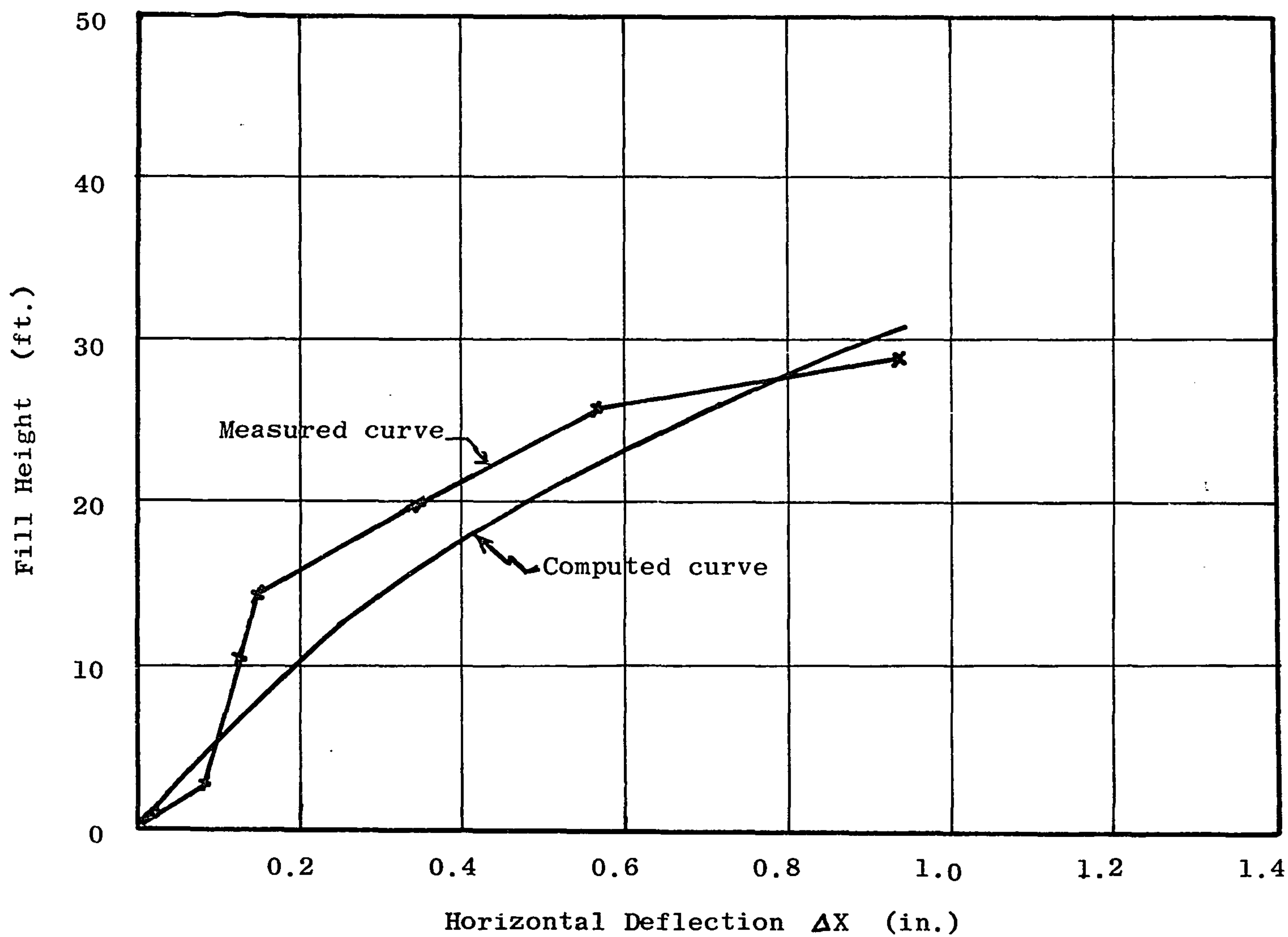

Figure 25 Comparison of Deflection of 78" pipe Measured by Kaiser Aluminum Corp. and Values Computed by Arching Theory

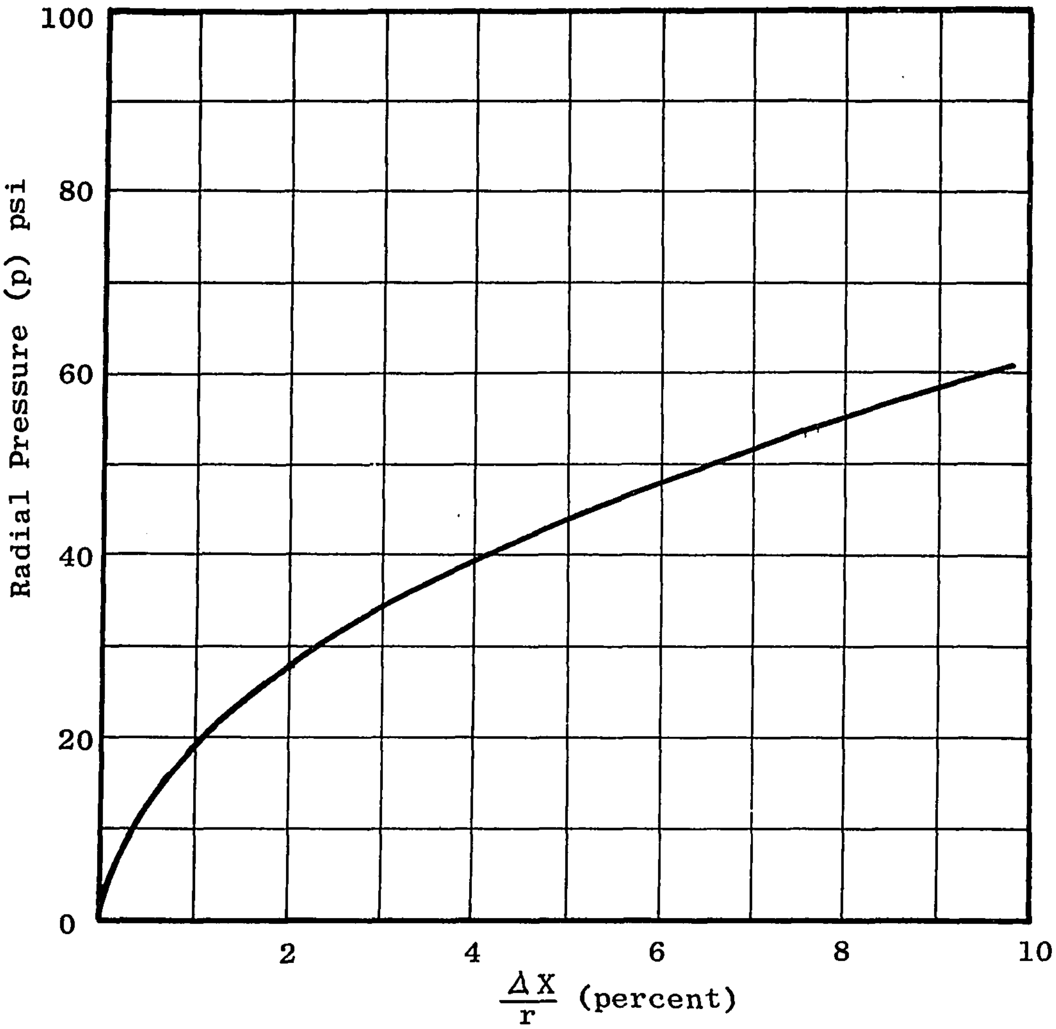

Figure 26 Modpares curve for Kaiser Aluminum Study
Curve was Made By Watkins

may be responsible for the difference is a deflection study involving live loads which was made with about 2.4 feet of cover above the pipe. The live load would introduce an increase in the effective preload (compaction) pressure. The effects of the live load would not be overcome until the pressure due to the fill height was greater than the live pressure. No indication was given as to how the live load was placed on the fill at the beginning of the test. One would expect some deflection of the pipe during loading at such low fill heights; however, none was recorded.

In order to determine the approximate height of fill necessary to overcome the increased compaction caused by the live load, the equivalent height of fill can be determined. The deflection curves would not be expected to converge until after this effective height had been reached.

The effective height can be determined as follows:

$$He = \frac{P}{v_t}$$

where

P = preload pressure

v_t = unit weight of soil

He = effective height of fill.

The H20 live load used in the test is 4 tons on the front axle and 16 tons on the rear axle. This would make 16,000 pounds per set of dual wheels. The depth of fill to the top of the pipe during the live load test was 2.4 feet.

If the assumption is made that the stress is distributed downward on a 1 to 2 ratio (1 horizontal to 2 vertical) the pressure at the level of the top of the pipe due to the live loading can be found to be approximately 1400 lb/ft^2. This gives an effective height (He) of 11.3 feet due to the live load. Added to the 2.4 feet of existing fill this makes an equivalent height of fill of 13.7 feet. The curve in Figure 25 has a sharp break at about 14 feet. After the break the load deflection curve follows almost parallel to the calculated curve almost to the maximum load.

The Kaiser Aluminum pipe was placed in a fill which was compacted to an average density of about 90 percent of Proctor density or 123 lb/cu. feet. The fill height was approximately 30 feet.

The pipe had a diameter of 78 inches with a vertical elongation before placing of 6.6 percent. The pipe was made of aluminum with a modulus of elasticity of 10.2x10^6 psi, a moment of inertia of 0.083 in^4/in, and a section modulus of 0.064 in^3/in.

Koepf (1962) has also presented the results of a field study. Two aluminum pipes were used in the study. One was 48 inches in diameter and the other was 60 inches. Koepf gave the moment of inertia as 0.0332 x t ($in.^4/in.$) (2-2/3 in. x 1/2 in. pattern) where t is the thickness in inches. The modulus of elasticity (E) was given as 10×10^6 psi.

The data for the soil modulus was taken from Table 3 in Koepf's report, corresponding to the 48 and 60 inch diameter pipes. Figure 27 shows the data plotted to give the numerical value of the soil modulus. These values were obtained by measuring the pressure at the side of the pipe as it deflected into the soil and then plotting a curve similar to the modpares curve. The curves were not plotted by Koepf in his report. The deflection was computed using Koepf's data. Figures 28 and 29 show a comparison of the measured and computed deflection for each pipe.

The agreement between these curves and the predicted curve is sufficiently close to justify the approach and the assumption made.

In addition to comparing the arching theory with various field tests, it was compared with the theory of elasticity. In order to make this comparison it was

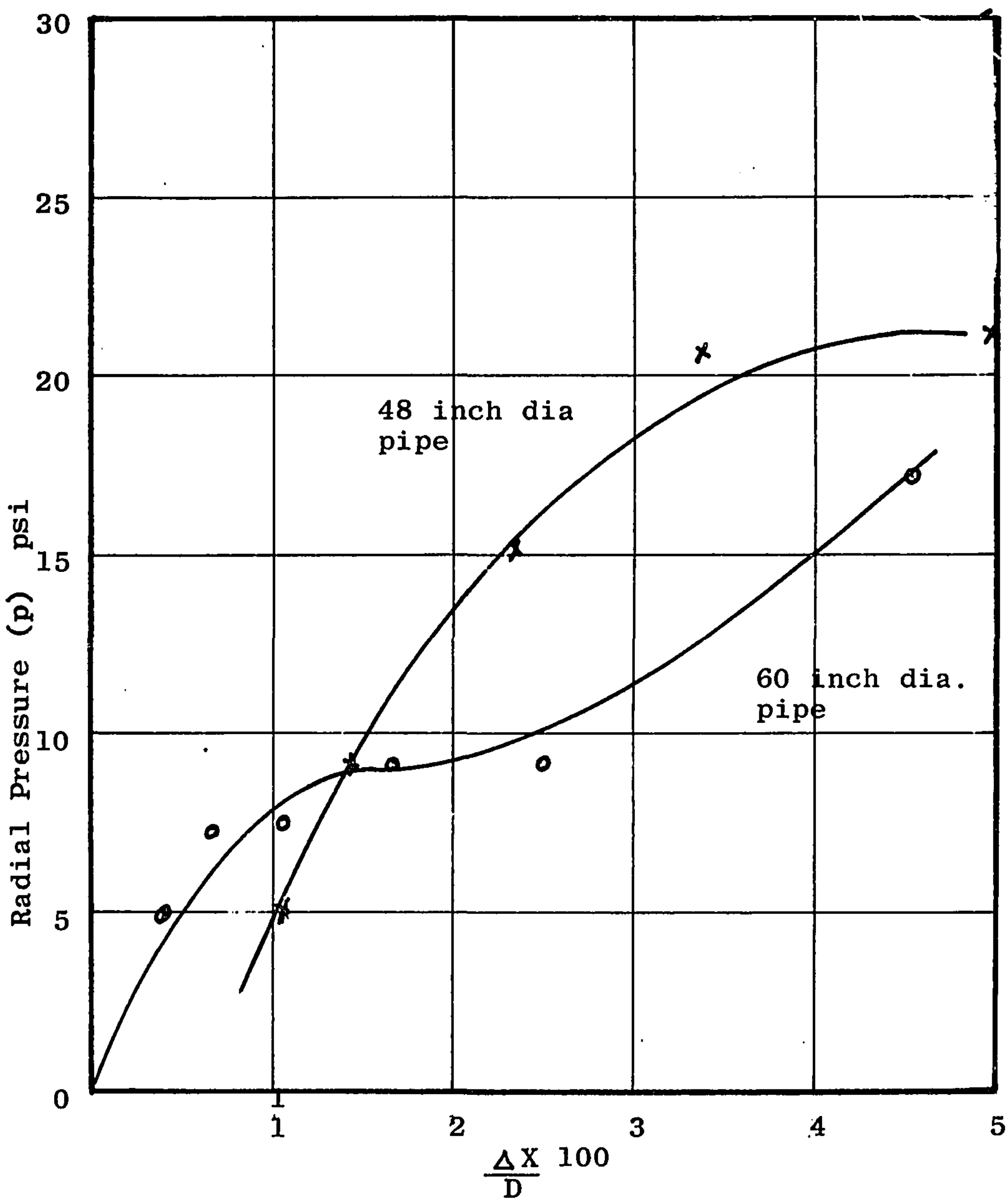

Figure 27 Soil Data for Koepf's Study which was Used to Compare with the Theoretical Analysis

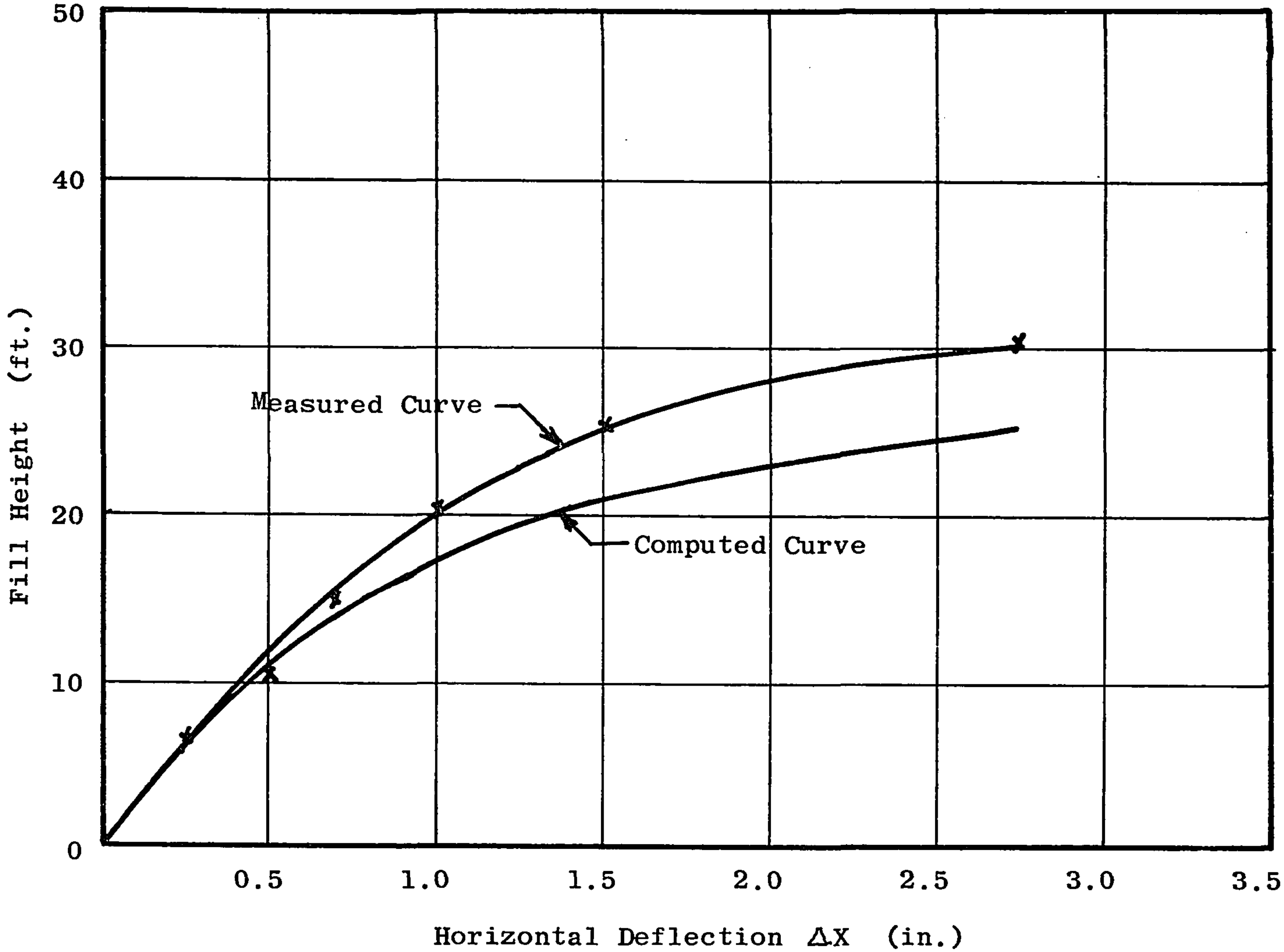

Figure 28 Comparison of Deflection of 60" Pipe Measured by Koepf and Values Computed by Arching Theory

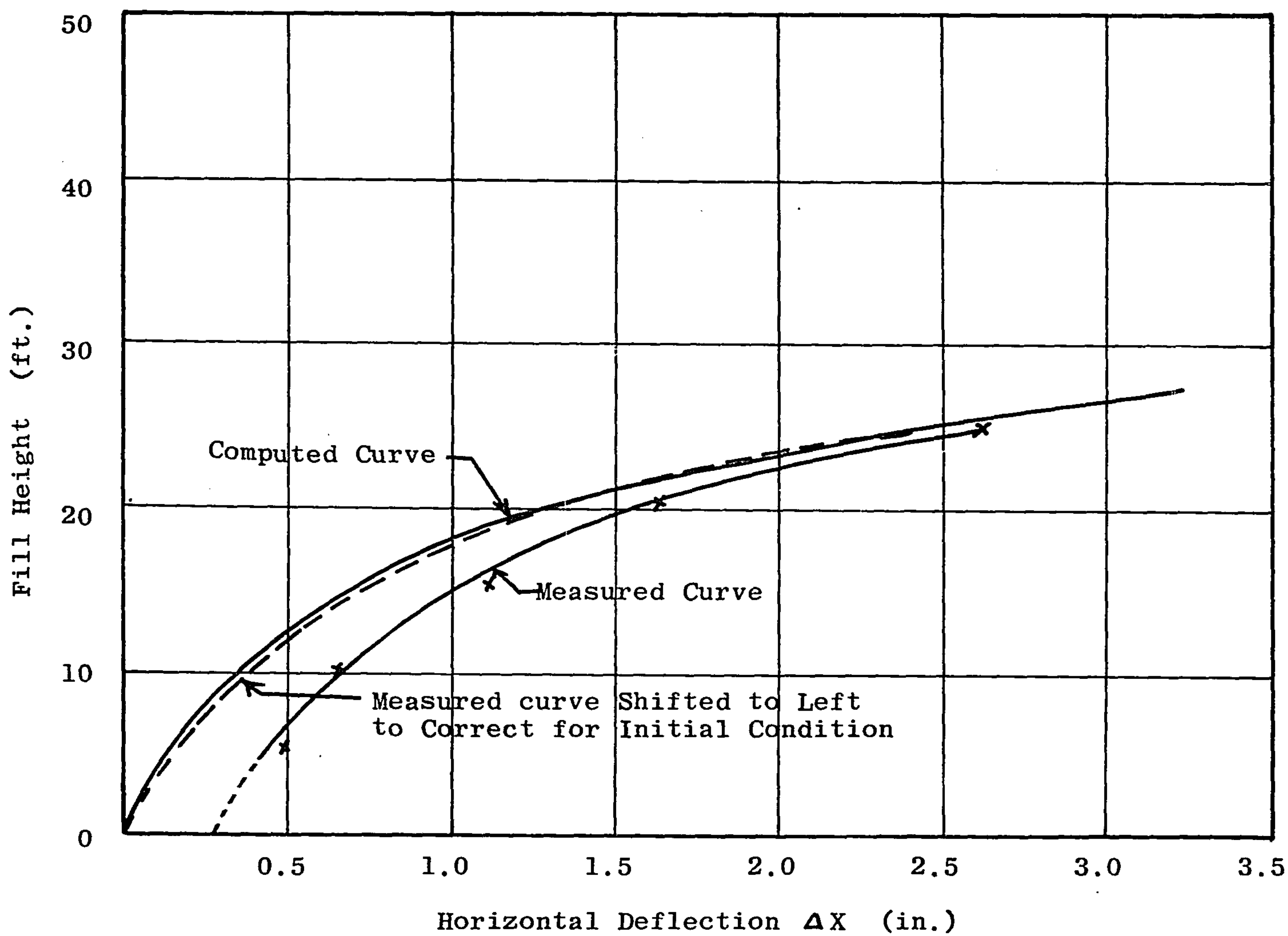

Figure 29 Comparison of Deflection of 48" Pipe Measured by Koepf and Values Computed by Arching Theory

necessary to determine a relationship between the modulus of passive resistance used in the arching theory with the modulus of elasticity and Poissons ratio used in the elastic theory. The equations for the deflection and pressure surrounding a conduit embedded in an elastic medium, as given by Burns and Richard (1964), were used to determine the relationship between the two variables.

The Modulus of passive resistance is defined as $E' = h/\Delta x/D$ where h is equal to the pressure at the side of the pipe on the horizontal diameter and $\Delta x/D$ is the percent change in the horizontal diameter at pressure h. When the equations from the theory of elasticity for h and Δx were substituted into the expression for E' it presented a rather formidable expression to try to reduce. The computer was used to solve the equation with input data varying over a wide range. Poisson's ratio varied from 0.1 to 0.4. The radius of the pipe varied from 20 inches to 180 inches and the modulus of elasticity varied from 200 psi to 2000 psi. As can be seen in Figure 30 the modulus of passive resistance is almost directly proportional to the constrained modulus of elasticity. The solid line in Figure 30 shows that the modulus of passive resistance is approximately 1.5 times the constrained modulus. This ratio was used in comparing

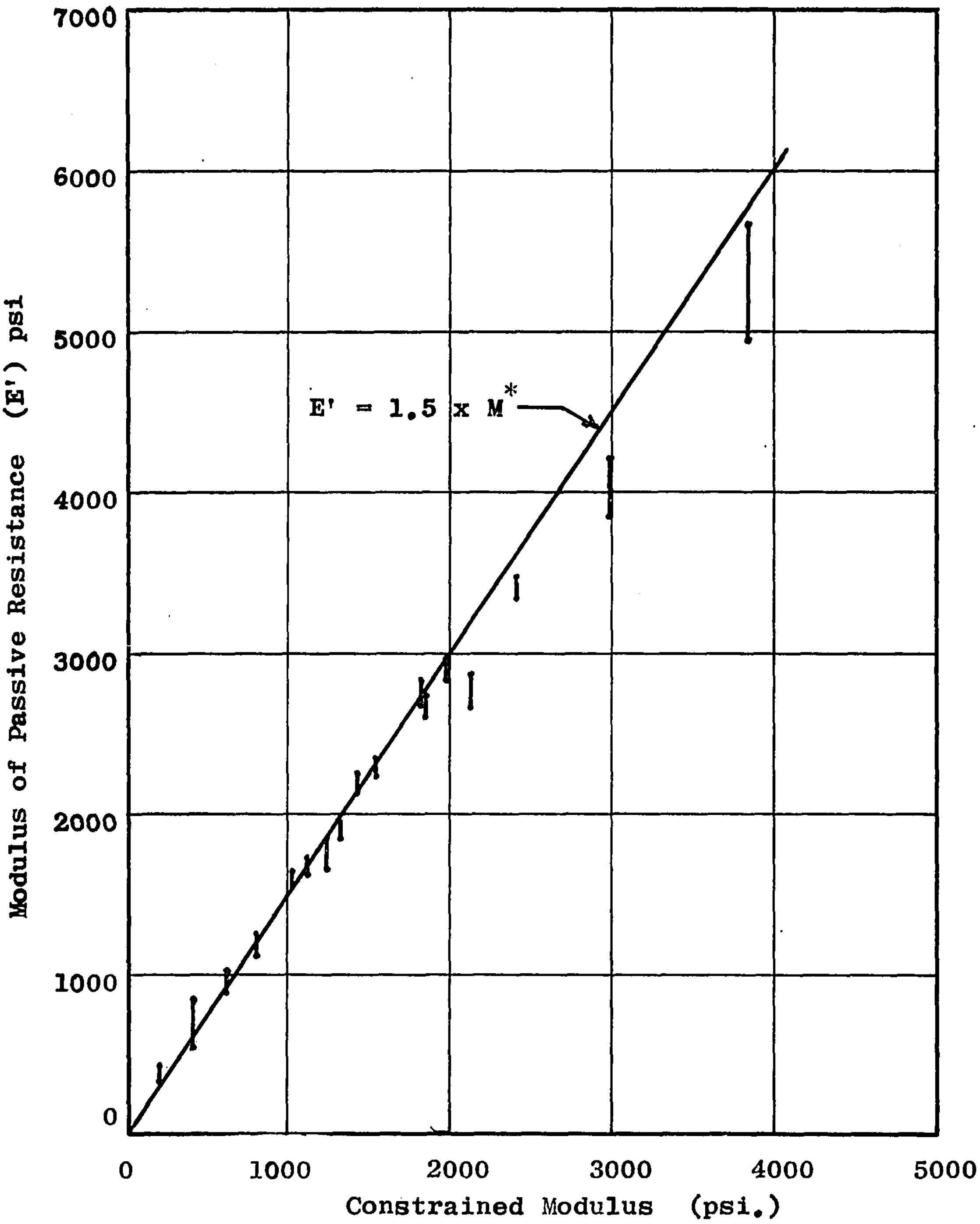

Figure 30 Comparison of Modulus of Passive Resistance and Constrained Modulus of Elasticity as Determined from the Theory of Elasticity

the two theories.

Figure 31 shows a comparison of the deflections calculated from the two theories. The agreement is reasonably good for fill heights below about 60 feet. The deflection predicted by the arching theory is on the conservative side if the deflection computed from the theory of elasticity is assumed to be correct.

The pressure distribution through the soil mass above the pipe as determined by the arching theory is somewhat higher than that determined by the theory of elasticity, (Figure 32); however, the pressures acting on the pipe as computed from each theory are very close to one another. For example, at a depth of 100 feet of fill the pressure computed from the arching theory is about 79 psi while it is 76 psi computed from the theory of elasticity. Figure 33 shows a comparison of the pressures acting on the pipe versus the height of fill. The difference between these curves is probably due to the difference in stress distribution assumed by Spangler in the derivation of the deflection equation (Equation 22) and the stress distribution surrounding the conduit in the elastic medium.

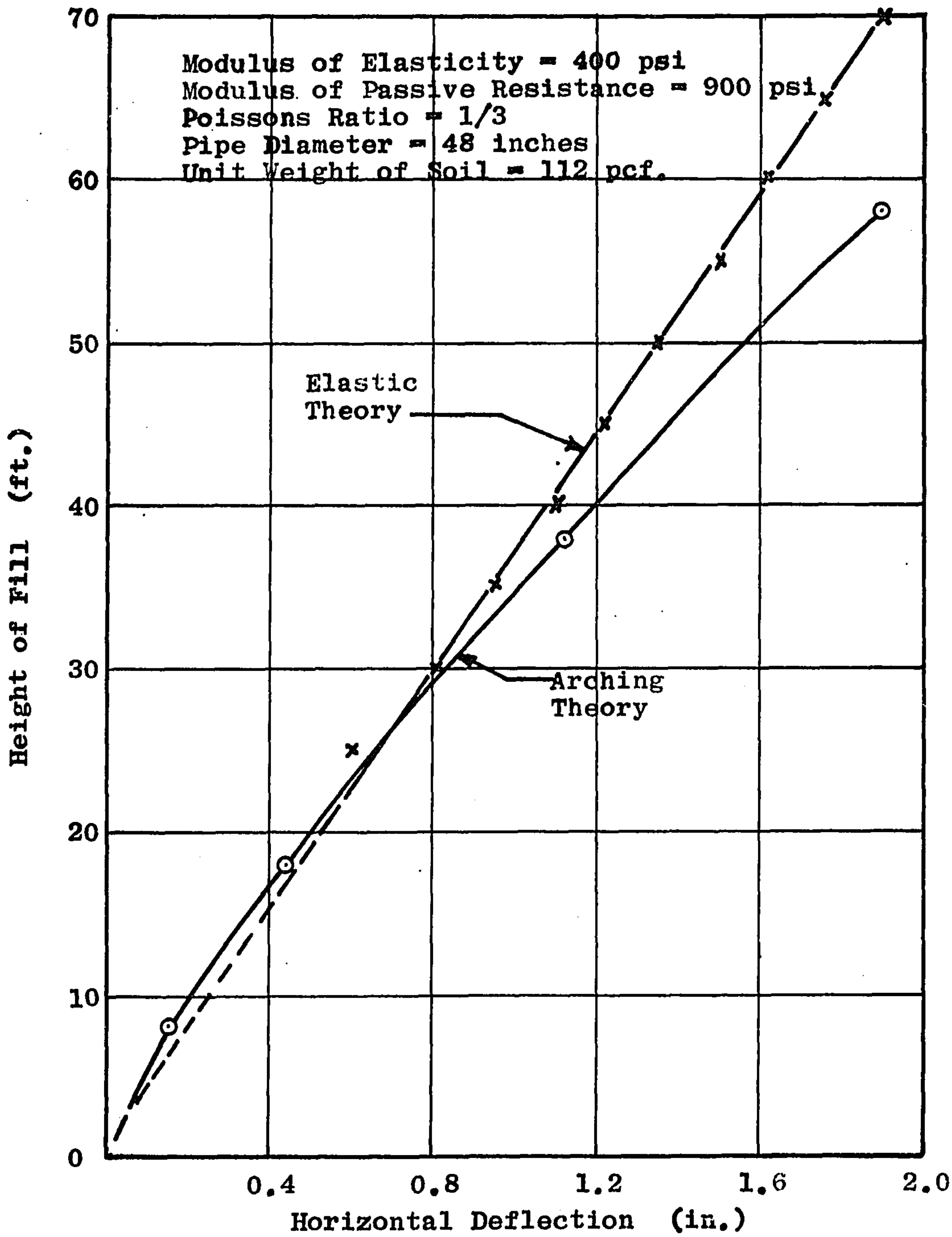

Figure 31 Comparison of Deflections Calculated from Elastic Theory and Arching Theory

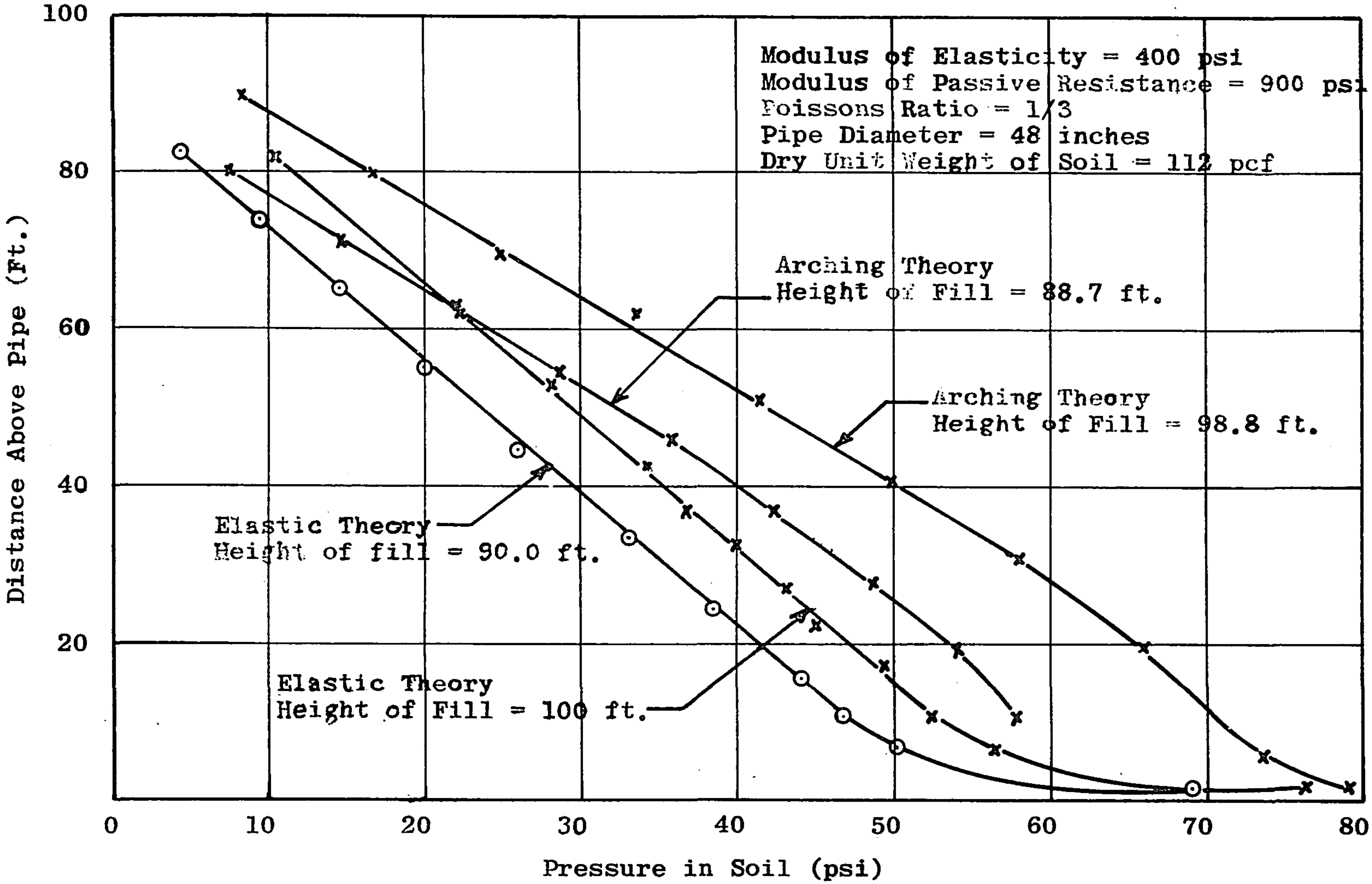

Figure 32 Comparison between pressure computed in fill from elastic theory and arching theory

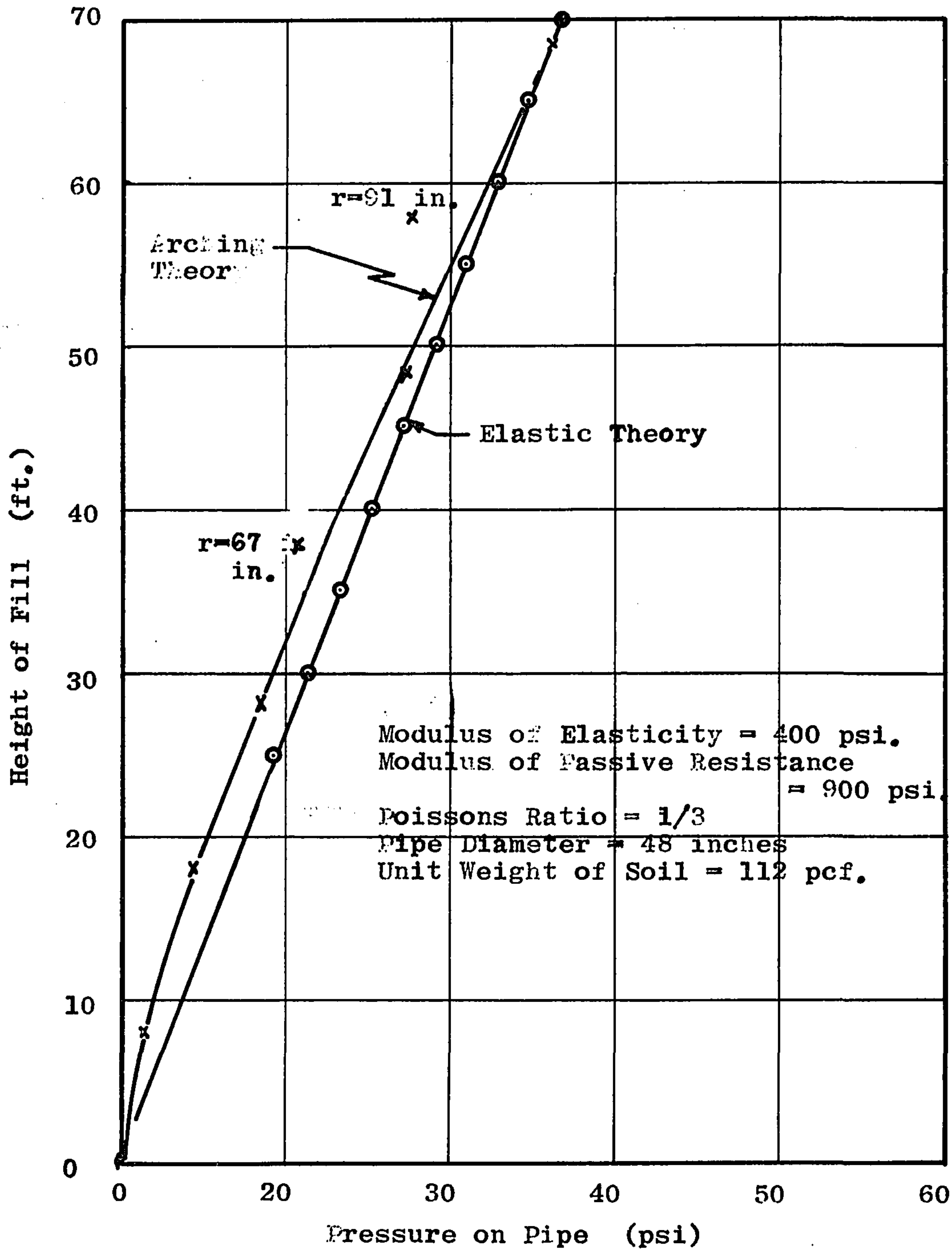

Figure 33 Comparison of Pressure Acting on Top of Pipe From Arching Theory and Elastic Theory

The agreement between the two theories is sufficiently close to justify the use of the arching theory in a non-linear medium such as soil.

CHAPTER V

APPLICATION OF THEORY TO NON-CIRCULAR STRUCTURES

Pipe Arches

The theory is applicable to pipe arches with a few modifications. One of the major problems is the determination of an equation, similar to Equation 22, to meet the compatibility requirements. One would also need to assume that the center of the pipe arch would correspond to the center of curvature of the upper section.

The loading distribution of the pipe arch may be assumed to be a function of the radius of curvature as shown in Figure 34. The force on the sides is a function of the displacement Δx at these points, but the displacement is a function of the force; therefore, a relation between these two variables must be determined. Such a relationship is the modulus of passive resistance with the appropriate correction for non-linearity. The problem is most easily solved by numerical methods. The solution of this problem is not included in the scope of this research.

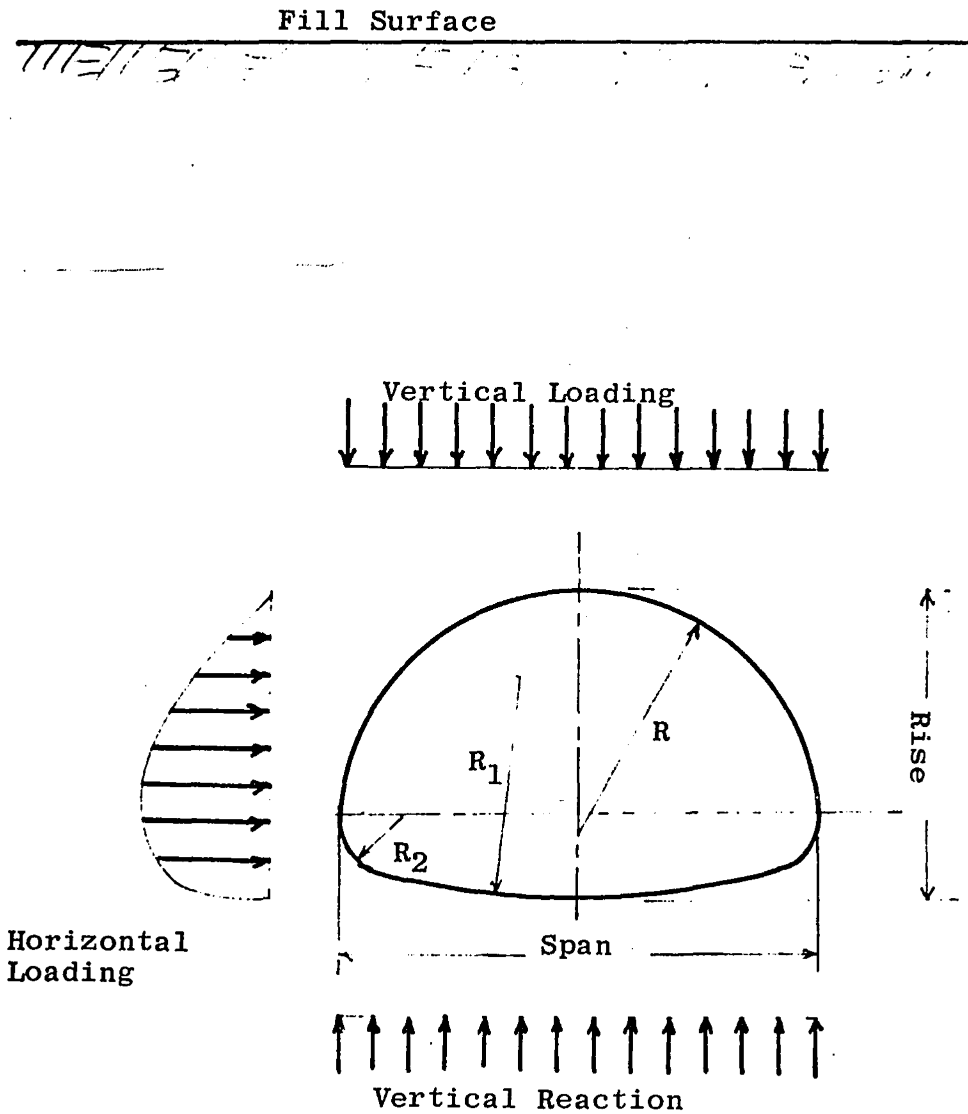

Figure 34 Assumed Loading Distribution Which Can be used on Pipe Arches (No attempt is Made to Solve the Problem)

Long Flexible Sections

The theory is also applicable to long flexible sections if the additional equation for compatibility can be found. Part of the soil mass directly above the structure must be assumed to be part of the structure. It can be assumed that the center of curvature of the soil arch is located at the top center of the rectangular section as shown in Figure 35. The load on the structure must include the weight of the soil mass assumed to be part of the structure as well as the pressure computed in the analysis. Any arching below the point where the soil arch touches the edge of the structure would only cause a redistribution of stress along the top. This redistribution could be evaluated if the value of the stress at the soil arch support could be determined.

As an approximation, the stress at the soil arch support that is touching the structure may be approximated by the at-rest soil pressure. This would cause greater stress at the edge of the structure than at the center. This is in agreement with laboratory observations.

No attempt will be made here to give the equation needed for compatibility. The type of structure would dictate the type of compatibility equation needed. After

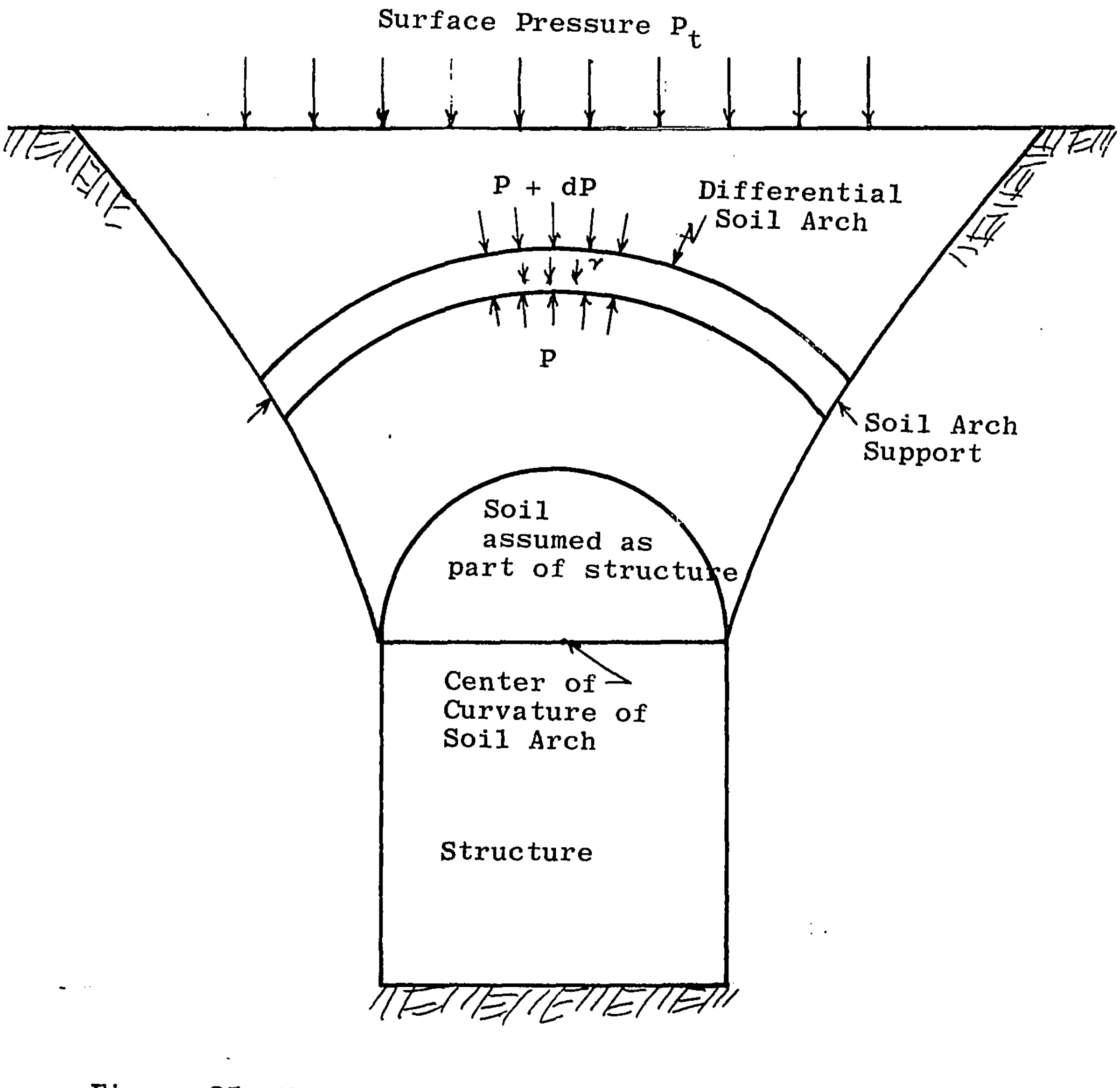

Figure 35 Free Body Diagram of Assumed Loading on Buried Rectangular Structures

the equation for compatibility has been obtained, it could be substituted into the computer program for solution of the problem.

Three Dimensional Problems

The analysis up to this point has considered only two-dimensional problems i.e. structures that are long enough so that the influence of arching in the third dimension is small. No effects from the third dimension have been considered.

If the structure is very short, so that the width of the structure is equal to the length, the effects of arching at the end of the structure must also be considered. Instead of having a differential arch the differential element would have to be the shape of a dome or shell.

If the structure is constructed in rock or very well compacted soil, the supports of the shell would be on the wall or side of the excavation. This could present some serious problems when one tries to write the differential equation for the stresses at the shell support. Evaluation of the differential equation to determine stress in non-circular three-dimensional structures that are not shells of revolution is very difficult. It may be possible to use a funicular shell with a square base for a large majority of the structures. At the present time, however, the added effort does not seem justified. Therefore, for the sake of simplicity an "equivalent circular" shell or dome with an area equal to the area of the excavation for the structure can be used.

If the structure is placed in a uniform backfill material, the location of the soil arch supports can be determined from the theory of elasticity as before, if one assumes that the maximum shearing stress plane is rotated to form a surface of revolution. This is only an approximation to the location of the maximum shearing stress. However, three-dimensional problems involving the theory of elasticity become extremely complicated.

By summing the vertical forces on the differential shell in Figure 36, the differential equation can be formed as follows:

$$[\Sigma V=0]$$

$$(P+dP)\pi r^2\sin^2\theta+2\pi r^2(1-\cos\theta)\gamma_t dr=$$

$$P\pi r^2\sin^2\theta+R_x\pi 2r\sin^2\theta dr$$

or

$$dP = \frac{R_x 2\sin^2\theta dr-2r(1-\cos\theta)\gamma_t dr}{r\sin^2\theta} \qquad (26)$$

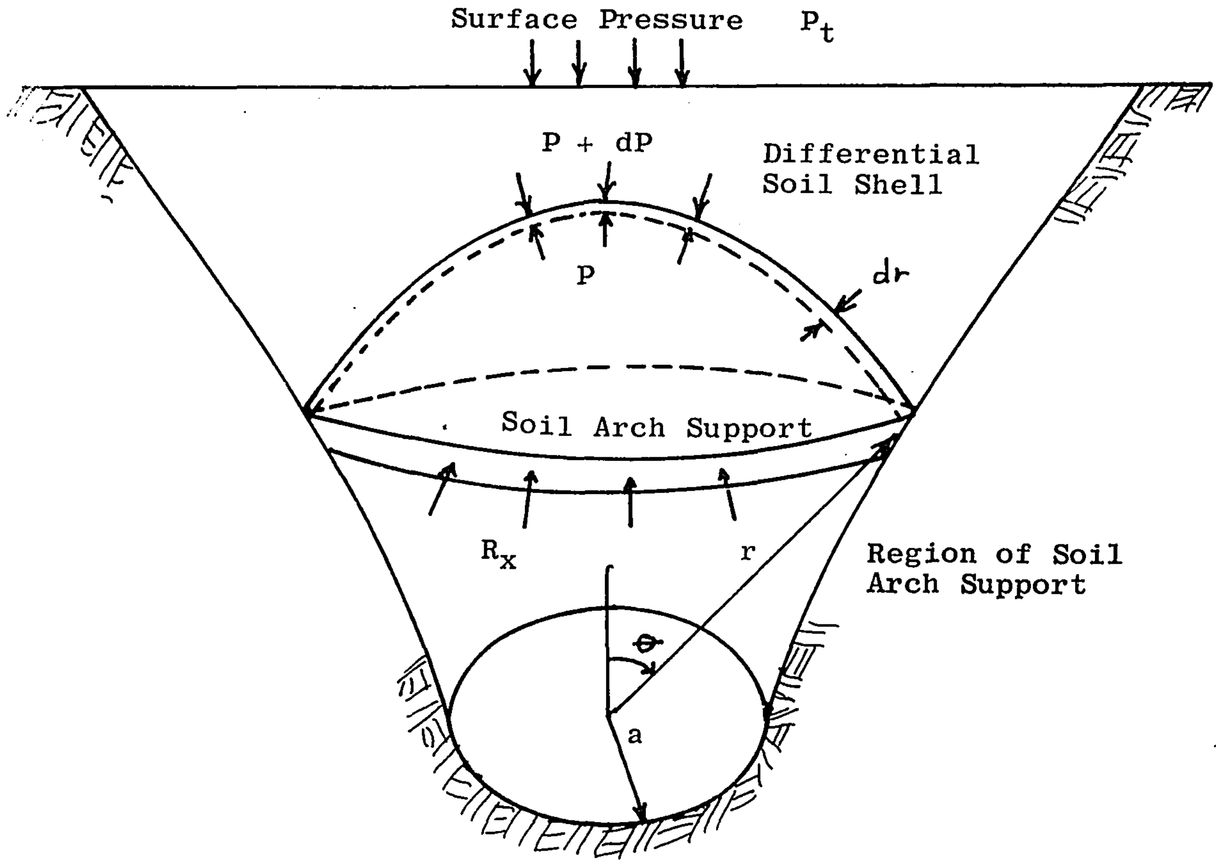

Figure 36 Schematic Diagram of Loading assumed on Three Dimensional Structures

Equation 26, like Equation 23, cannot be integrated until the relationship between R_x and r, and θ and r can be determined. The relationship between R_x and r (Equation 26) that was used in the two-dimensional analysis can be used again. However, a new compatibility relationship must be found to replace Spangler's equation for the flexible pipe. These equations will be discussed in the following section relating to the problem involved.

Loads on Buried Circular Membranes

With recent emphasis on the construction of underground blast shelters, the concept of a buried flexible membrane roof has been introduced. The load can also be predicted on such a structure. Equation 26 is applicable to the buried circular membrane. Equation 25 is still applicable for the value of R_x. A different relationship must be determined between the deflection of the membrane and the applied pressure. The equation relating pressure and deflection for a circular membrane roof is given by

$$P/(f/r_o) = 4\frac{1}{r_o/w + w/r_o} \tag{27}$$

where

P = uniform pressure on membrane in psi

f = line load in membrane in lb/in

r_o = radius of membrane in inches

w = deflection at center of membrane due to pressure.

An initial deflection can be assumed as before and substituted into Equation 27 to determine the pressure necessary to cause the given deflection of the circular membrane roof. The same deflection could be used to go into the soil modulus curve to determine its value to be used in Equation 25. Once the value of R_x is known, Equation 26 can be integrated.

The pressure distribution across the top of the membrane is not uniform. The pressure around the edge is much higher than the pressure at the center. Model analysis on 4-inch diameter membranes shows the pressure near the edge to be approximately 3 times the pressure at the center.

The higher pressures measured at the edge of the membrane support the concept of the soil arch. As the radius of the soil arch becomes slightly less than the radius of the membrane, the stress in the soil arch support approaches the value of the at-rest pressure at that point. At the next differential arch inward, the pressure on the membrane is reduced much more, due to the increased stress at the soil arch support. Therefore, the pressure on the soil arch support, which is the same pressure that the membrane experiences, is reduced as the arch support moves toward the center of the membrane. The assumption of uniform pressure is only an approximation, but is necessary to get a solution to the problem.

The deflection of the membrane downward has the same effect as the downward movement of the circular pipe in developing the differential arch. As soon as the deflection starts to take place the differential shell starts to form.

The value of the line load (f) for mild steel can be assumed to be 40,000 lb/in^2 x thickness of membrane if the yield point of the material is 40,000 psi. For aluminum or other materials that do not have a well defined yield point, the line load (f) will have to be determined by other means.

No attempt will be made to solve this problem. The methods used in the solution for the circular conduit with the necessary modifications could be used.

CHAPTER VI

SUMMARY AND CONCLUSIONS

Applications of the Soil Arch Concept

The concept of a differential soil arch has been used as the differential element in determining the load transmitted to underground structures. The differential equation was developed and, because of difficulties in integration, it was necessary to resort to numerical procedures.

The concept of the soil arch has been applied to circular pipe. Possible methods of application for flat top rectangular structures, pipe arch, and buried flexible membrances have been made.

The following conclusions are drawn:

1. The concept of a soil arch can be used to calculate the load transmitted to an underground structure. The results seem to indicate that there is good agreement with the few field installations that have been checked. However, more field studies need to be made to correlate laboratory data with field performance.

2. Due to the soil-structure interaction, the load transmitted to an underground structure depends on the deflection of the structure as well as the height of fill above the structure.

3. The modulus of passive resistance of the soil can be determined from model studies by calculation from Spangler's equation. Most other attempts to determine the modulus of passive resistance of soil have met with limited success. The modulus can be successfully determined by model studies on natural soils. There is a difference in the shape of the curve representing the modulus of passive resistance obtained for cohesive and cohesionless soils which is to be expected. For design purposes, however, two general classifications for the different soil can be used. One for cohesive soils, and the other for cohesionless soil. There will always be the question as to which to use for design in the intermediate soils. In such a soil the curve which gives the most severe condition should be used either from the standpoint of the load on the structure or the deflection of the structure.

4. Model tests need not be made for each installation. Using either the curve for cohesionless or cohesive soil,the modulus of passive resistance can be

determined by testing the material in the Modpares device. A correction can be applied to the model curve to get the proper modulus for use in design.

5. The modulus of passive resistance is non-linear rather than the constant value assumed by Spangler in the derivation of the equation to predict the deflection of underground flexible conduits. This non-linear modulus can be incorporated into the analysis by assuming that the modulus changes with deflection and incorporating a secant modulus into the solution. This makes it a trial-and-error solution which has been solved by use of the digital computer. The computer program is presented in the Appendix.

Recommendations

The following recommendations are made for future research.

1. Investigation of the effects of the fabric of cohesive materials on the modulus of passive resistance should be undertaken.

2. The modulus of passive resistance should be investigated to see if it could be determined from some of the more common tests that are already made on the soil. It is indicated that the modulus of passive

resistance is approximately proportional to the density of the soil. It may be possible to determine the modulus from the optimum density of the soil and a curve of dry density vs. compactive effort. The slope of the curve of log compactive effort vs. density may be an indication of the effect of additional compactive effort on the soil modulus.

3. It may also be possible to determine the modulus of passive resistance from some of the tests used in highway construction. It is recommended that a study using either the Hveem stabilometer or California Bearing Ratio be made for possible use in determining the modulus of passive resistance of the soil.

4. A study is recommended to determine the best configuration of the differential arch to be assumed in the analysis. This would allow a more exact solution to the problem.

5. A more detailed investigation should be made into the assumptions used in calculation of the stresses at the soil arch support (Equation 25).

APPENDIX

COMPUTER PROGRAM FOR SOLUTION OF PRESSURES AND DEFLECTIONS OF BURIED CIRCULAR PIPE

The computer solution is divided into three parts. Each part is written in Fortran II for an IBM 1620 computer. Part One takes the data to be used for determination of the modulus of passive resistance and fits a polynomial equation to it. Because of the range of the numbers involved the program first takes the square root of the pressure. It also multiplies the deflection term by one thousand and then takes the square root of it. The least-squares fit of a polynomial equation is applied to the modified data. The coefficients of the resulting equation are punched out to be used as input in part two or part three.

Part Two takes the data and coefficients from part one and determines a correction factor from data from the modpares device. It then applies the corrections to each pressure used in defining soil modulus curves to make it applicable to the given set of conditions. A least-squares fit is applied to the adjusted data coefficients of the corrected curve for the determination of E'.

Part Three will take the output from either Part One or Part Two. The program will calculate the pressure on the pipe, the horizontal deflection of the pipe, and some dimensionless variables which were used to compare the computed data with model data.

The computer program is started by assuming an initial deflection of the structure. It makes very little difference what value is assumed because the computer will adjust the assumed value in an iterative procedure until the correct one is obtained. A value of 6% of the diameter of the structure is used. The input into the program consists of the number of points used in the least-squares fit to define the soil modulus curve (M), the order of polynomial equation desired to describe the soil (N), the coefficients of the resulting equation C(I), the depth of cover above the structure (Z) in inches, the radius of the structure (A) in inches, the unit weight of the soil in pounds per cubic foot (GAM), the surface pressure (PT) in psi, the number of finite arches used in the numerical integration (IN), the pipe wall stiffness (EI) in psi x in^4/in or lb.-in. the pressure at five percent deflection of modpares device (P5M) in psi and STOPP which is a number which tells the computer that this is the last set of data. If the number is zero or negative the computer will look for more data.

The integration is started with the top differential soil arch. The pressure transferred away from the structure due to the differential soil arch supports is computed. The pressure on the next arch down is determined by subtracting the differential pressure from the pressure on top of the arch. Then the radius of the differential arch is compared with the radius of the pipe to determine if the element just used is at the level of the pipe. If the element is not yet down to the level of the pipe the computer goes to the next element and the same procedure is repeated until the level of the pipe is reached. Upon reaching the pipe level the computed pressure in the soil mass is compared with the pressure necessary to deflect the buried structure the same amount. If the two pressures are within three percent of each other, the pressure is assumed to be the pressure acting on the pipe. If there is more than three percent difference the assumed deflection is adjusted in the appropriate direction and the process is repeated. The same procedure is repeated until the two pressures are within the required three percent of each other. The three percent convergence limit was set to reduce the number of iterations necessary to reach the final answer. It would be unrealistic to assume that greater accuracy in the analysis is justified. The output from the computer consists of the depth of soil above the structure (ZZ) in feet at which the pressures and deflections of the structure are calculated, the pressure acting on the structure (P) in psi, the deflection of the structure (DEL) in inches, the soil modulus (X) in psi, the percent deflection of the structure (C2), a dimensionless number PD^3/EI to compare with Figures 23 and 24, and another dimensionless number $\gamma HD^3/EI$ which will allow a comparison of the pressure on the pipe with the weight of a prism of soil directly above the pipe.

PART 1

```
      DIMENSION AZ(17,6), RESUL(5,6), C(5), DATA(17,2), B(17,6)
      DIMENSION DAT(17,2)
C     M = NUMBER OF SETS OF DATA POINTS
C     N = ORDER OF POLYNOMIAL EQUATION TO BE USED
      READ 20,M , N
      PRINT20,M , N
   20 FORMAT (2I10)
      READ 21,((DAT(I,J), I = 1,M), J = 1,2)
      PRINT21,((DAT(I,J), I = 1,M), J = 1,2)
   21 FORMAT (8F10.2)
   15 N1 = N + 1
      N2 = N + 2
      DO210  I = 1,M
      DATA (I,1) = SQRTF ( DAT (I,1))
      DATA (I,2) = SQRTF ( DAT (I,2) * 1000.)
      AZ(I,N2)= DATA(I,1)
      DO210 J = 1,N1
  210 AZ(I,J)= DATA (I,2)**(J-1)
      DO212 K = 1,N1
      DO214 I = 1,M
      DO214 J = 1,N2
  214 B(I,J) =AZ(I,J) * DATA (I,2) **(K-1)
      DO212 J = 1,N2
      RESUL(K,J) = 0
      DO212 I=1,M
  212 RESUL (K,J) = RESUL (K,J) + B(I,J)
C     RESUL = SIMULTANEOUS EQUATIONS TO BE SOLVED
```

```
C     SOLUTION OF SIMULTANEOUS EQUATIONS BY ELIMINATION
C     FULL REDUCTION TO IDENTITY MATRIX
      DO 322 I = 1, N1
      BB= 1. / RESUL (I,I)
      DO 321 J = 1,N2
  321 RESUL (I,J) = RESUL (I,J) * BB
      DO 322 L = 1,N1
      IF (L-I) 323, 322, 323
  323 D = RESUL(L,I)
      DO 325 JJ = I,N2
  325 RESUL (L,JJ) = RESUL(L,JJ) - RESUL(I,JJ) * D
  322 CONTINUE
      DO330 I = 1, N1
  330 C(I) = RESUL(I,N2)
      DO 597 I = 1,N1
  597 PUNCH 500, C(I)
  500 FORMAT (5F15.3)
      END
        16          4
          -.116
          1.326
          -.149
           .009
          0.000
```

PART 2

```
      DIMENSION AZ(17,6), RESUL(5,6), C(5), DATA(17,2), B(17,6)
      DIMENSION DAT(17,2)
      READ 20, M, N
      PRINT 20, M, N
   20 FORMAT (2I10)
      N1 = N + 1
      N2 = N + 2
      READ 22,((DAT(I,J), I = 1,M), J = 1,2)
      PRINT22,((DAT(I,J), I = 1,M), J = 1,2)
   22 FORMAT (8F10.3)
      READ 21, (C(I), I = 1, N1)
      PRINT21, (C(I), I = 1, N1)
   21 FORMAT (F15.3)
   10 READ 23, P5M
      PRINT23, P5M
   23 FORMAT (F10.3)
    1 DEF = 0.05
      X1 = DEF * 1000.
      X = SQRTF (X1)
      YY = 0
      DO 410 I = 1, N1
  410 YY = YY + X**(I-1) * C(I)
      YY = YY **2
      X =(YY * 1000.) / X1
```

```
    IF (X) 460,461,461
460 X = 1000000.
461 COR = P5M / YY
    DO 16 I19 = 1,M
 16 DAT (I19,1) = DAT (I19,1) * COR
    DO 610 I = 1,M
    DATA(I,1) = SQRTF (DAT (I,1))
    DATA (I,2) = SQRTF ( DAT(I,2) * 1000.)
    AZ   (I,N2) = DATA (I,1)
    DO 610 J = 1,N1
610 AZ (I,J) = DATA (I,2)**(J-1)
    DO 612 K = 1,N1
    DO 614 I = 1,M
    DO 614 J = 1,N2
614 B(I,J) = AZ (I,J) * DATA (I,2) ** (K-1)
    DO 612 J = 1,N2
    RESUL (K,J) = 0
    DO 612 I = 1,M
612 RESUL (K,J) = RESUL (K,J) + B(I,J)
    DO 722 I = 1,N1
     BB= 1. / RESUL (I,I)
    DO 722 L = 1,N1
    IF (L-I) 723, 722, 723
723 D = RESUL (L,I)
    DO 725 JJ = 1,N2
725 RESUL (L,JJ) = RESUL(L,JJ) - RESUL(I,JJ) * D
```

```
722 CONTINUE
    DO 730 I = 1,N1
730 C(I) = RESUL (I, N2)
    DO 8  I = 1, N1
    PUNCH 19, C(I)
    PRINT 19, C(I)
 19 FORMAT (/F15.4)
  8 CONTINUE
    GO TO 10
    END
```

PART 3

```
C       3 560 DWAYNE NIELSON
        DIMENSION C(6)
  114   FORMAT (/ 6H ERROR, 3E10.3)
  400   READ 2, M, N
        PRINT2, M, N
    2   FORMAT (2I10)
        N1 = N+ 1
        DO 6 I = 1, N1
    6   READ 1, C(I)
        PRINT 1,(C(I), I = 1,N1)
    1   FORMAT (F20.3)
    8   FORMAT (8HCHECK =     , F10.1    )
        READ 3, Z, A, GAM, PT, IN, EI, P5M, STOPP
        PRINT3, Z, A, GAM, PT, IN, EI, P5M, STOPP
    3   FORMAT (4F10.1,I10, 3F10.1)
        COUNT = 0
        B = 12.
   29   PRINT 49
   49   FORMAT(/75H     DEPTH   PRESSURE     DEFL.    E@              DX/D
       1S-TERM    Z-TERM                        /)
        C1 = 0
        DEF = 0.05
        DELTA = 0.03 * A
```

```
      AA = A + 72.
      A3 = (Z-AA) / 10.
 115  FORMAT(/32HDID NOT REACH CONVERGENCE LIMIT      )
      K8 = AA
      KZ = Z + A3
      KA3 = A3
      DO 20  K2= K8, KZ, KA3
      IF (DELTA - .0001) 52,52,53
  52  DELTA = 0.01 * A
  53  Z = K2
      IF (C1 - 200.) 50, 50, 20
  50  CH 1 = 0
      CH 2 = 1.
      X1 = DEF * 1000.
      X = SQRTF (X1)
      YY = 0
      DO 810 I = 1,N1
 810  YY = YY + X ** (I-1) * C(I)
      YY = YY **2
      X = (YY * 1000.) / X1
      IF (X) 860,861,861
 860  X = 1000000.
 861   FN = IN
      XX = X
      DR = (Z-A) /FN
```

```
      CORR = 0.5 * DELTA
      CORR 2 = CORR
      PI  = 3.14159
      E1= 3.14159 / 2.
      G1=E1 / 2.
      ITER = 0
   14 R = Z
      IF (SENSE SWITCH 1) 952, 953
  952 PRINT 114, P, PSTRU, DELTA
  953 IF(SENSE SWITCH 2) 898, 899
  898 K2 = K2 + 1
      PAUSE
       GO TO 52
  899 IF( SENSE SWITCH 3) 895, 896
  895 ACCEPT 897, DELTA
  897 FORMAT(F10.3)
  896 ITER 2 = 0
      IF (DELTA) 693, 693, 694
  693 DELTA = ABSF (DELTA)
  694 CALL = DELTA / A
      X1 = CALL * 1000.
      X =  SQRTF (X1)
      YY= 0
      DO910 I = 1, N1
  910 YY=YY + X **(I-1) * C(I)
      YY=YY ** 2
```

```
    X = (YY * 1000.)/X1
    IF (X) 960,961,961
960 X = 1000000.
961 P = PT
 37 RX = X * 4. * DELTA / R  * .785
    ITER 2 = ITER 2 + 1
    IF (ITER 2 - 500) 42,42, 117
117 PRINT 115
    PRINT 118
118 FORMAT (/26H EXCEEDED  500 ITERATIONS     )
    GO TO 26
 42 IF (A/R) 999,999, 998
999 G = 3.14159 / 2.
    GO TO 43
998 CO2 = (A/R) ** 2
    G =1.571 - ATANF (SQRTF((1.- CO2) / (1. + CO2))  )
 43 DP  = (RX * SINF(G)*DR-GAM* 2.*R*G * DR/1728.) /(R*SINF(G))
    P = P - DP
    PRINT 888, P, R,
888 FORMAT (2E15.3)
    IF (P) 110, 110, 111
110 PS = DELTA * (EI+ 0.061 * X * (A**3))
    PSTRU = (PS / (0.083 * (A**4) *   2. )) * 2.
    P = (P + PSTRU)/2.
    DELTA = (.083 * P * 2. *(A**4))/(EI + .061 * X * (A**3) ) / 2.
    GO TO 14
```

```
111 R = R - DR
    IF (R-A) 36,36,37
 36 PS = DELTA * (EI+ 0.061 * X * (A**3))
    PSTRU = (PS / (0.083 * (A**4) *   2. )) * 2.
    CH 5 = P / PSTRU
    IF (CH5 - .94)110,26,60
 60 IF(CH 5 - 1.06 ) 26, 26, 110
 26 P =(P + PSTRU)/ 2.
    C1 = P * ((2. * A) **3) / EI
    C2 = DELTA /A
    C3 = XX * ((2. * A) **3) / EI
    DEL = DELTA * 2.
    ZZ = Z/12.
    C11 = Z * (GAM/1728.) * ((2.*A) **3) / EI
 32 PRINT 35,ZZ, P, DEL, X , C2, C1, C11
 35 FORMAT (  7E10.3)
    COUNT = COUNT + 1.
    IF ( COUNT - 50.) 20,20, 11
 11 PRINT 49
    COUNT = 0
 20 CONTINUE
    DCOEI = ( 2. * A) ** 3 / EI
    DIA = 2. * A
    PRINT 500, DIA, XX, C3
500 FORMAT(/ 5HDIA =, F5.1,3X, 4HE@ =, E10.3, 3X, 11HSOIL TERM =,F9.0)
    PRINT 501, DCOEI, EI
501 FORMAT (11HPIPE TERM =, F6.2, 3X, 4HEI =,E10.3)
```

```
   IF (STOPP) 400, 400,     71
 71 END
      16          4
        -.116
        1.326
        -.149
         .009
        0.000
1200.   24.     112.     0          10   53592. 50.
```

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