BARTH, James Edmond, 1935-
ELECTROSTATIC CORRECTION OF THE SPHERICAL ABERRATION OF ELECTRON LENSES.

University of Arizona, Ph. D., 1967
Physics, electronics and electricity

ELECTROSTATIC CORRECTION OF THE SPHERICAL ABERRATION OF ELECTRON LENSES.

by

James Edmond Barth

A Dissertation Submitted to the Faculty of the

DEPARTMENT OF PHYSICS

In Partial Fulfillment of the Requirements
For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

THE UNIVERSITY OF ARIZONA

1967

THE UNIVERSITY OF ARIZONA

GRADUATE COLLEGE

I hereby recommend that this dissertation prepared under my direction by James Edmond Barth entitled ELECTROSTATIC CORRECTION OF THE SPHERICAL ABERRATION OF ELECTRON LENSES be accepted as fulfilling the dissertation requirement of the degree of DOCTOR OF PHILOSOPHY

Allen P. Wilska — April 14, 1967

Dissertation Director — Date

After inspection of the dissertation, the following members of the Final Examination Committee concur in its approval and recommend its acceptance:*

John W. Robson — April 14, 1967

Louis R. Marquet — April 14, 1967

Stephen H. Tollman — April 14, 1967

J. L. Lamont — April 14, 1967

B. R. Poole — April 25 - 1967

*This approval and acceptance is contingent on the candidate's adequate performance and defense of this dissertation at the final oral examination. The inclusion of this sheet bound into the library copy of the dissertation is evidence of satisfactory performance at the final examination.

STATEMENT BY AUTHOR

This dissertation has been submitted in partial fulfillment of requirements for an advanced degree at The University of Arizona and is deposited in the University Library to be made available to borrowers under rules of the Library.

Brief quotations from this dissertations are allowable without special permission, provided that accurate acknowledgment of source is made. Requests for permission for extended quotation from or reproduction of this manuscript in whole or in part may be granted by the head of the major department or the Dean of the Graduate College when in his judgment the proposed use of the material is in the interests of scholarship. In all other instances, however, permission must be obtained from the author.

SIGNED: James E. Barth

ACKNOWLEDGMENTS

This work was done in the electron optics group of Prof. Dr. Ir. J.B. Le Poole at the Technological University at Delft, The Netherlands. The author is grateful to Professor Le Poole and the members of his group, in particular Ir. J. Kramer, for their help in all aspects of the work.

TABLE OF CONTENTS

	Page
LIST OF ILLUSTRATIONS	v
INTRODUCTION	1
ACTION OF WEAK LENSES	3
Foil Lens	3
Gauze Lens	7
Magnetic Lens	12
USE OF FOIL AND GAUZE LENSES	18
CORRECTION OF A WEAK MAGNETIC LENS BY MEANS OF A GAUZE LENS	20
Correction of Spherical Aberration - 1	20
Correction of Spherical Aberration - 2	23
Errors Due to the Gauze Lens	28
Alignment Errors	28
Fifth Order Error	31
Chromatic Error	35
Errors Due to the Magnetic Lens	37
Fifth Order Error	37
Chromatic Error	39
Summary of Lens Errors	41
Experimental Test	45
Design of the Corrector Electrodes	45
Experimental System	45
Experimental Results	50
Possible Improvement of a Probe Lens	56
MULTIPLE GAUZE LENS	60
Compensation of the Gauze Hole Error	60
Spherical Aberration of the Gauze Hole Lenses	65
Residual Gauze Hole Error	66
Alignment Error	70
Summary	72
LIST OF REFERENCES	76

LIST OF ILLUSTRATIONS

Figure | Page

1. Design for Corrector Lens.......................... 46
2. Experimental System................................ 48
3. Test Lens and Corrector............................ 49
4. Image Profiles..................................... 53
5. Two Gauze Lens..................................... 61

ABSTRACT

The spherical aberration of electron lenses of which the refracting fields are axially symmetric, static, and space charge free is always positive. The spherical aberration of an electrostatic lens in which surface charges are introduced within the electron beam on a metal gauze or a conducting foil can be given an arbitrary value by adjusting the electrostatic field at the gauze or foil. The performance of gauze lenses is limited by the lens action of the gauze holes. If a gauze lens is given a zero spherical aberration and used to provide the principal lens action, the aberration figure due to the gauze holes of diameter Δ has a diameter in the image plane of about $(1+M)\Delta$ where M is the magnification of the lens. If a gauze lens is used to correct the spherical aberration of a normal magnetic lens which is used to provide the principal lens action, the aberration figure due to the gauze holes is a factor $2\Delta/h$ - where h is the radius of the lens aperture - times smaller than the minimum spherical aberration figure of the uncorrected lens. The other possible sources of error, such as misalignment, are discussed. Experimental results obtained with a gauze lens corrector are given. The use of two parallel gauzes as the central element of an electrostatic lens gives the possibility of making a gauze lens free of spherical aberration in which the error due to the gauze holes is reduced. Each pair of gauze holes forms a lens, but, by adjusting the intensity of the electric field between the gauzes, the strength of the hole lenses can be made zero. The design of such a lens is given together with estimates of the errors limiting the performance. The usefulness of foil lenses and correctors is limited by the scattering of electrons by the foil. Some experimental results are given for a foil corrector.

INTRODUCTION

Any axially symmetric electric and magnetic field is capable of forming a geometrically faithful electron image of an object in a plane normal to the axis of symmetry of the field, provided that the electron rays forming the image make a small enough angle with the axis, that the part of the object imaged covers an area small enough about the axis and that the electrons are monoenergetic. This ideal image, in which diffraction effects are also ignored, is called the Gaussian image. The geometrical aberrations are characterized by the deviations, in the Gaussian image plane, from the Gaussian image points. These deviations arise from the employment of imaging pencils of finite aperture and from the imaging of objects of finite extent. For axially symmetric systems the coordinates of the ray intersections in the image plane depend only on odd order terms in the object and aperture plane coordinates. The first order terms give the Gaussian approximation and the higher order terms the aberrations.

Terms of higher order than the third order aberrations are, for the small angles normally encountered in electron optics, negligible. The spherical aberration is the third order aberration which depends only on the ray coordinates in the aperture plane and thus does not vanish for an axial object point. The aberration figure is a circle about the Gaussian image point with a radius proportional to the cube of the aperture radius.
That is:

$$\rho = A\, r_a^3 \,.$$

The spherical aberration is the most important geometrical aberration as it cannot always be made negligible by reducing the aperture angle. In the case of lenses used to form a real image of an electron source, because the current into the image is reduced by reducing the aperture angle, the spherical aberration of the lens limits the current into a spot of given size. In the case of an electron microscope objective, the spherical aberration of the lens limits the resolution, because diffraction effects increase with reduced aperture angle.

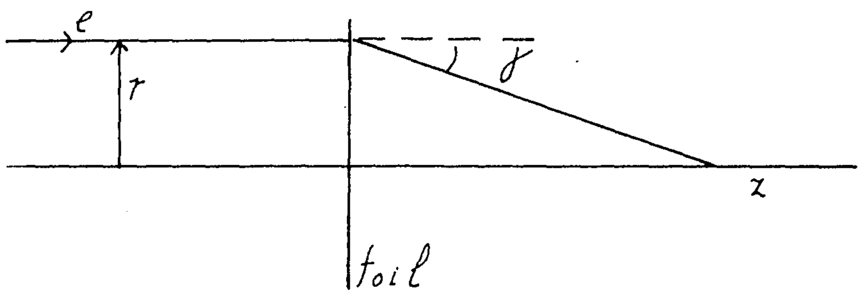
Under the conditions that the refracting fields are axially symmeyric, static, and space charge free, Scherzer (1936) has shown that the integrand of the integral expression for the spherical aberration coefficient A can be written as the sum of a number of squared terms. Thus A is always positive and can only be zero if each of the terms of the integrand is zero, which only happens if there is no lens. It was also Scherzer (1947) who showed that the removal of any one of these restrictions permits the construction of lenses of zero or negative spherical aberration. The use of combinations of non-axially symmetric lenses to achieve correction of axially symmetric lenses has received most of the experimental attention.

The present investigation concerns the experimental realization of spherical aberration correction by means of surface charges introduced within the electron beam on metal gauzes or electron permeable conducting foils.

ACTION OF WEAK LENSES

Action of a Foil Lens

Consider an axially symmetric electrostatic electron lens of which the central electrode is a conducting foil, transparent to electrons.



The lens is thin, that is, the extent of the refracting field is short compared to the distance of the lens from both the object and the image. The lens is weak, that is, the field strengths are such that an electron incident parallel to the axis will change neither its separation from the axis nor its axial velocity to an appreciable extent while passing through the lens. The electron trajectories are paraxial, that is, the angles of the electron rays with respect to the axis are small.

An electron of momentum p_z, passing through the lens parallel to the axis at a distance r from the axis, receives an impulse directed towards the axis

$$\Delta p = e\int_{-\infty}^{\infty} E_r(r,z)\,dt$$

where E_r is the radial component of the electric field. The lens is symmetric about the central plane, thus

$$\Delta p = 2e\int_{0}^{\infty} E_r\,dt$$

The axial velocity is constant, so that

$$dt = \frac{dz}{v_z}$$

gives

$$\Delta p = \frac{2e}{v_z}\int_0^\infty E_r\,dz$$

The electron is deflected through an angle

$$\gamma = \frac{\Delta p}{p_z} = \frac{2e}{m v_z^2}\int_0^\infty E_r\,dz$$

The energy of the electron can be written

$$\frac{1}{2} m v_z^2 = eU$$

where U is the energy in electron volts, giving

$$\gamma = \frac{1}{U}\int_0^\infty E_r\,dz$$

From Gauss' law, applied to a cylinder beginning just outside the foil,

$$2\pi r\int_0^\infty E_r\,dz = 2\pi\int_0^r E_z\,r\,dr$$

where $E_z = E_z(r,o)$ is the field in the z direction at the foil. Thus

$$\gamma = \frac{1}{U}\cdot\frac{1}{r}\int_0^r E_z\,r\,dr.$$

Expanding E_z in a power series in r,

$$E_z = E_{zo}(1 + a_2 r^2 + a_4 r^4 + \dots)$$

where, because of the axial symmetry of the lens, only even powers appear, gives

$$\gamma = \frac{E_{zo}}{U} \frac{1}{r} \int_0^r (r + a_2 r^3 + a_4 r^5 + \dots) dr$$

$$= \frac{E_{zo}}{U} \left(\frac{r}{2} + \frac{a_2}{4} r^3 + \frac{a_4}{6} r^5 + \dots \right)$$

The focal length f_e of the lens is given by

$$\frac{1}{f_e} = \frac{E_{zo}}{2U}$$

Thus a foil lens with a parabolic field distribution at the foil

$$E_z = E_{zo}\left(1 - \frac{r^2}{r_o^2}\right)$$

gives a deflection

$$\gamma = \frac{r}{f_e} - \frac{1}{2 f_e r_o^2} r^3$$

These results have been obtained in a weak lens approximation in which the electron velocity and the radius of the electron ray in the lens are taken to be constant and the differences between γ, $\sin \gamma$ and $\tan \gamma$, although of the third order, are ignored.

The application of the weak lens approximation to the third order terms in r is justified if the third order term $r^3/2f_e r_o^2$ is large compared to the third order terms in γ^3 resulting from the expansion of sin γ, tan γ, etc., that is, if

$$\gamma^3 \simeq \frac{r^3}{f_e^3} << \frac{r^3}{f_e r_o^2}$$

or

$$r_o << f_e$$

When a foil is used as a corrector for the spherical aberration of a normal lens it will be quite weak and the approximation rather good. If a foil lens is used to provide the principal lens action the approximation is not so good and there will be contributions which cannot be ignored. In either case the essential result is that the spherical aberration of a foil lens can be given a desired value by adjusting the field at the foil.

Action of a Gauze Lens

Consider the situation in which the foil is replaced by a gauze which has small holes of diameter Δ. For a single hole of radius R in a conducting sheet, where the fields approach the uniform fields $-E_+$ on the left and E_+ on the right, the potential on the axis of the hole is [see, for example, Zworykin (1945) page 385]

$$\phi(z) = -\frac{2RE_+}{\pi}\left(\frac{z}{R}\tan^{-1}\frac{z}{R} + 1\right)$$

For values of $z/R > 1$ this can be expanded as

$$\phi = -E_+ z - \frac{2RE_+}{3\pi}\left(\frac{R}{z}\right)^2 + \frac{2RE_+}{5\pi}\left(\frac{R}{z}\right)^4 \ldots$$

The axial field is then

$$E_z = -\frac{d\phi}{dz}$$

$$= E_+ - \frac{4E_+}{3\pi}\left(\frac{R}{z}\right)^3 + \cdots$$

and the disturbance in the field decreases as $(\Delta/z)^3$ for values of $z/\Delta > 1$. The contribution to the deflection, in the case of a foil, for the part of the path $-z$ to z, for small z, is

$$\delta(r,z) = \frac{1}{2U}\int_{-z}^{z} E_r\, dz$$

$$= \frac{1}{U}\int_{0}^{z} E_r\, dz$$

$$\simeq \frac{1}{U}\int_0^z \frac{\partial E_z}{\partial r}\, z\, dz$$

as the lens is symmetric about z equal zero and Curl $\underline{E}$ is zero. For a parabolic field

$$E_z = E_{z0}\left(1 - \frac{r^2}{r_0^2}\right)$$

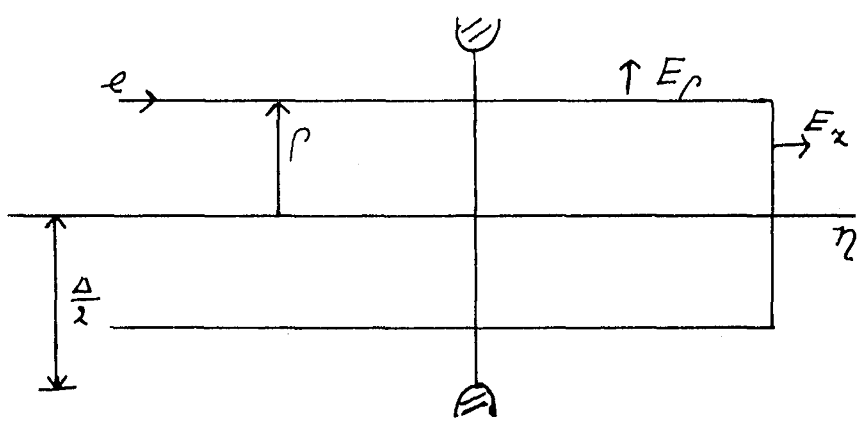
at the foil, this becomes

$$\gamma(r,z) = \frac{-1}{U}\int_0^z 2E_{z0}\frac{r}{r_0^2}\, z\, dz = -2\left(\frac{z}{r_0}\right)^2 \frac{E_{z0}\, r}{2U}$$

$$= -2\left(\frac{z}{r_0}\right)^2 \frac{r}{f_e}$$

Thus if $\Delta \ll r_0$ the action of the gauze can be considered to be that of a foil plus the disturbance due to the gauze holes.

Consider an electron passing through a hole centered at $r=r_c$ at a distance ρ from the axis of the hole.



The deflection is

$$\delta_\rho = \frac{1}{U}\int_0^{\bar{\eta}} E_\rho \, d\eta$$

where $\bar{\eta}$ is chosen to be:

Large enough compared to Δ so that the contribution to the lens action of the hole of the region $\eta > \bar{\eta}$ can be neglected.

Small enough compared to the dimensions of the lens so that the field in the plane $\eta = \bar{\eta}$ can be taken equal to the value for $z = 0$ in the case of a foil.

The change in E_z with r across a hole of diameter Δ is of the order

$$\Delta E_z \simeq \frac{\partial E_z}{\partial r}\,\frac{\Delta}{2} \simeq -E_{z0}\,\frac{r_c \Delta}{r_0^{2}}$$

Since the maximum deflection due to the lens action of a hole is proportional to $E_z\Delta$ the effect of the change in E_z across the hole will be of second order in Δ and can be neglected. Thus the field in the plane $\eta = \bar{\eta}$ is taken to be uniform and equal to the value for $z = 0$ in the case of the foil, that is

$$E_z(r, \bar{\eta}) \simeq E_z(r_c, 0)$$

From Gauss' law

$$2\pi \int_0^{\rho} E_z \rho \, d\rho = -2\pi\rho \int_0^{\bar{\eta}} E_\rho \, d\eta$$

Thus

$$\delta_\rho = \frac{-1}{U}\,\frac{1}{\rho}\int_0^{\rho} E_z \rho \, d\rho = \frac{-1}{U}\,E_z(r_c)\,\frac{\rho}{2}$$

$$= \frac{-E_{z0}}{2U}\left(1 - \frac{r_c^{2}}{r_0^{2}}\right)\rho$$

$$= -\frac{1}{f_e}\left(1 - \frac{r_c^2}{r_o^2}\right)\rho$$

The maximum value of ρ is $\Delta/2$, so the maximum deflection due to the lens action of a hole at $r=r_c$ is

$$\gamma_\Delta = \frac{-1}{2 f_e r_o^2}\left(r_o^2 - r_c^2\right)\Delta$$

The principal action of the lens is that of a foil lens, that is

$$\gamma_e = \frac{1}{f_e} r - \frac{1}{2 f_e r_o^2} r^3$$

If the field at the gauze is uniform, that is, if $1/r_o^2$ is zero, then the lens is free of spherical aberration. The maximum deflection due to the lens action of a hole is

$$\gamma_\Delta = \frac{-1}{2 f_e} \Delta$$

This gives an aberration figure in the focus of diameter

$$\delta_\Delta = 2 I / \gamma_\Delta /$$

$$= \frac{I}{f_e} \Delta$$

where I is the image distance.

The result that a uniform field at the gauze gives a lens free of spherical aberration is a weak lens approximation which ignores the existence of third order terms other than that due to the field. Thus to make the net third order coefficient zero will actually require a small negative contribution from the field. This will be considered, for the case of the multiple gauze lens, on page 66.

Action of a Magnetic Lens

In order to find the lens action of a thin, weak, axially symmetric magnetic lens, the following approximations to the equations of motion in polar coordinates are used.

$$\dot{v}_r = -\eta\, v_\theta B_z + \frac{1}{r} v_\theta^2$$

$$\dot{v}_\theta = -\eta\,(v_z B_r - v_r B_z) - \frac{2}{r} v_r v_\theta \simeq -\eta\, v_z B_r$$

$$\dot{v}_z = \eta\, v_\theta B_r \simeq 0$$

Here η is e/m. The quantities v_z and r are taken to be constant in the lens. The vector potential A(r,z), which because of the symmetry of the field has only one component, is expanded in terms of the axial field.

$$A(r,z) = \frac{r}{2} B_z - \frac{r^3 B_z''}{16} + \frac{r^5 B_z^{iv}}{384} - \dots$$

Integrating the equation for $\dot{v}_\theta$ gives

$$v_\theta = -\eta\, v_z \int_{-\infty}^{t} B_r\, dt = -\eta \int_{-\infty}^{z} B_r\, dz$$

$$= \eta \int_{-\infty}^{z} \frac{\partial A}{\partial z}\, dz = \eta\, A(r,z)$$

as

$$\dot{v}_z = 0 \, , \qquad B_r = - \frac{\partial A}{\partial z} \, .$$

and as A(r,z) has the form

$$A(r, z) = \sum D(r)\, C(z)$$

The deflection given an electron is

$$\delta(r) = \frac{-\Delta v_r}{v_z} = \frac{-1}{v_z} \int_{-\infty}^{\infty} \dot{v}_r \, dt$$

$$= \frac{-1}{v_z^2} \int_{-\infty}^{\infty} \dot{v}_r \, dz$$

$$= \frac{1}{v_z^2} \int_{-\infty}^{\infty} \left(\eta v_\theta B_z - \frac{v_\theta^2}{r} \right) dz$$

$$= \frac{\eta^2}{v_z^2} \int_{-\infty}^{\infty} \left(\frac{A}{r} \frac{\partial (rA)}{\partial r} - \frac{A^2}{r} \right) dz$$

$$= \frac{\eta}{2U} \int_{-\infty}^{\infty} A \frac{\partial A}{\partial r} \, dz$$

as

$$\dot{v}_z = 0 \, , \qquad B_z = \frac{1}{r} \frac{\partial (rA)}{\partial r}$$

and

$$eU = \frac{1}{2} m v_z^2$$

With

$$A\frac{\partial A}{\partial r} = \frac{r}{4}B_z^2 - \frac{r^3}{8}B_z B_z'' + \frac{r^5}{256}\left(3(B_z'')^2 + 2B_z B_z^{\prime v}\right) - \cdots$$

and making use of the integrals

$$\int_{-\infty}^{\infty} B_z B_z'' dz = -\int_{-\infty}^{\infty} (B_z')^2 dz$$

$$\int_{-\infty}^{\infty} B_z B_z^{\prime v} dz = \int_{-\infty}^{\infty} (B_z'')^2 dz$$

γ is evaluated through terms in the fifth power in r.

Let

$$\gamma = \frac{1}{f}r + \frac{C_3}{f^4}r^3 + \frac{C_5}{f^6}r^5$$

Then the coefficients are given by the integrals

$$\frac{1}{f} = \frac{\eta}{8U}\int_{-\infty}^{\infty} B_z^2 dz$$

$$\frac{C_3}{f^4} = \frac{\eta}{16U}\int_{-\infty}^{\infty} (B_z')^2 dz$$

$$\frac{C_5}{f^6} = \frac{5\eta}{512U}\int_{-\infty}^{\infty} (B_z'')^2 dz$$

The radius of the aberration figure in the Gaussian focus is:

$$\rho = I\Delta\gamma = \frac{I}{f}f\Delta\gamma$$

$$= (1+M)\, f \Delta \gamma$$

$$= (1+M)\left(\frac{C_s r^3}{f^3} + \frac{C_5 r^5}{f^5}\right)$$

where I is the image distance and M the magnification of the lens.

For a weak lens the shape of the magnetic field does not depend on the field strength, but only on the geometry of the lens. Thus the coefficients C_s/f^3 and C_5/f^5 are constants for weak lenses.

According to Van Ments and Le Poole (1947) for iron clad lenses of pole separation S to bore diameter D ratio of about unity the axial field is well represented by the function

$$B_z = B_{z0}\, e^{-\frac{z^2}{p^2}}$$

For a field of this form

$$\frac{1}{f} = \frac{\eta B_{z0}^2}{8U}\sqrt{\frac{\pi}{2}}\; p$$

$$\frac{C_s}{f^3} = \frac{1}{2p^2}$$

$$\frac{C_5}{f^5} = \frac{15}{64}\,\frac{1}{p^4} \simeq \frac{1}{4p^4} = \left(\frac{C_s}{f^3}\right)^2$$

Van Ments and Le Poole also found that in terms of the lens geometry the relation

$$\frac{C_s}{f^3} \simeq \frac{2,4}{S^2+D^2}$$

gives a rather good fit to values calculated from experimentaly measured fields for weak lenses over a wide range of the gap to bore ratio S/D.

The possibility of compensating for the third order aberration makes interesting an estimate of the fifth order error. This has been done in the weak lens approximation. The contributions to the aberrations due to the inclination of the electron trajectories and the change in axial velocity, which have been ignored, are smaller by a factor of the order p/f than the contribution from the field distribution. Archard (1960) has made a numerical calculation of the complete fifth order coefficient C_5/d for the bell-shaped field

$$B = \frac{B_0}{1+\left(\frac{z}{d}\right)^2}$$

for values of the excitation parameter

$$k^2 = \frac{\eta}{8U} B_0^2 d^2$$

between 0.2 and 2.0. For this field shape, the weak lens integrals give

$$\frac{f}{d} = \frac{0.64}{k^2}, \quad \frac{C_s}{f^3} d^2 = 0.25, \quad \frac{C_5}{f^5} d^4 = 0.12$$

The complete integrals give

$$\frac{f}{d} = 3.81 \quad \frac{C_s}{f^3} d^2 = 0.27 \quad \frac{C_5}{f^5} d^4 = 0.13.$$

for $k^2 = 0.2$ and

$$\frac{f}{d} = 1.03 \qquad \frac{C_s}{f^3}\,d^2 = 0.34 \qquad \frac{C_5}{f^4}\,d^4 = 0.08$$

for $k^2 = 2.0$.

USE OF FOIL AND GAUZE LENSES

As has been shown, a foil lens can be given an arbitrary spherical aberration. Thus a foil lens free of spherical aberration can be made, or a foil lens of negative spherical aberration can be combined with a lens of the normal type, having positive spherical aberration, to give a lens system free from spherical aberration. The problem of making a conducting foil thin enough so that the scattering of electrons by the foil is negligible, while maintaining sufficient mechanical strength, has prevented the use of such lenses. A foil lens may be applicable to the correction of electron microscope objectives where relatively high energies and only a very small aperture are required. This has been considered theoretically by Scherzer (1949).

The difficulty of using foils leads to the consideration of the use of lenses in which the foil is replaced by a gauze. As has been shown, a gauze lens acts like a foil, except that the holes in the gauze, as they act as lenses, introduce a new error. In the case of a spherical aberration free gauze lens the diameter of the aberration figure in the Gaussian image plane due to the lens action of the holes is

$$\delta_{\Delta} = \frac{I}{f_e} \Delta$$

where I is the image distance, f_e the focal length of the lens, and Δ is the diameter of the gauze holes. The problem of making holes small enough to make this error negligible, while maintaining sufficient transparency and mechanical strength, has restricted the use of such lenses.

However, if a gauze lens of negative spherical aberration is used to correct the positive spherical aberration of a normal lens the size of the holes imposes a much less severe restriction on the performance of the composite lens. Scherzer and Steigerwald (1959) have obtained a patent based on this idea. Rus (1965) has made calculations and obtained some preliminary experimental results which demonstrated the feasibility of this approach to the correction of spherical aberration. As will be shown, if a normal lens is corrected for spherical aberration by a gauze lens the aberration figure due to the gauze holes has a diameter

$$\delta_{\Delta} = \frac{2\Delta}{h}\,\delta_{s}$$

where h is the radius of the lens aperture and δ_s is the minimum spherical aberration figure of the uncorrected lens. Thus a substantial reduction in the aberration figure can be achieved. This is demonstrated experimentally.

The use of more than one gauze gives a second possibility for the reduction of the error due to the gauze holes. For example, two parallel gauzes can be used as the central element in constructing a spherical aberration free lens. Each pair of gauze holes forms a lens, but, by adjusting the intensity of the electric field between the gauze, the strength of the hole lenses can be made zero. The spherical aberration of the gauze hole lenses will be the limiting error. The aberration figure has, at unit magnification, a diameter

$$\delta_{s\Delta} = \frac{1}{9}\left(\frac{\Delta}{d}\right)\Delta$$

where d is the separation of the gauzes. This is a substantial reduction over the aberration figure of a single gauze lens. This type of lens is also considered.

CORRECTION OF A WEAK MAGNETIC LENS BY MEANS OF A GAUZE LENS

Correctiôn of Spherical Aberration - 1

A magnetic lens gives a deflection

$$\gamma_m = \frac{r}{f_m} + \frac{C_s}{f_m^4} r^3$$

and a foil or gauze lens can be made to give the deflection

$$\gamma_e = \frac{r}{f_e} - \frac{1}{2r_o^2} \frac{r^3}{f_e}$$

For $1/f_e << 1/f_m$, if the lenses are superimposed, with coincident principal planes, the net deflection is

$$\gamma = \frac{r}{f_m} + \left(\frac{C_s}{f_m^4} - \frac{1}{2r_o^2 f_e}\right) r^3$$

Thus if

$$\frac{C_s}{f_m^4} - \frac{1}{2r_o^2 f_e} = 0$$

the spherical aberration of the combined lens is zero. This requires

$$\frac{1}{2r_o^2 f_e} = \frac{C_s}{f_m^4}$$

That is

$$\frac{E_{zo}}{r_o^2} = \frac{4U}{f_m} \frac{C_s}{f_m^3}$$

which determines one parameter in the parabolic field distribution

$$E_z = E_{zo}\left(1 - \frac{r^2}{r_o^2}\right)$$

In the case of the gauze lens a new aberration, due to the lens action of the holes, is introduced. A hole centered at $r=r_c$, gives a maximum deflection of magnitude

$$\delta_\Delta(r_c) = \frac{-1}{2 f_e r_o^2}\left(r_o^2 - r_c^2\right)\Delta$$

This gives an aberration figure in the focus of diameter

$$\delta_\Delta(r_c) = 2 f_m \,/\, \delta_\Delta \,/$$

$$= \frac{f_m}{f_e r_o^2}\,/\,r_o^2 - r_c^2\,/\,\Delta$$

The coefficient $f_m/f_e r_o^2$ is fixed by the requirement that the net spherical aberration be zero. Let the maximum value of r_c be h. The quantity $r_o^2 - r_c^2$ has the value r_o^2 at r_c equal to zero, decreases to zero at r_c equal to r_o and increases to $r_o^2 - h^2$ at r_c equal to h. Increasing r_o from zero increases the value at r_c equal to zero and decreases it at r_c equal to h, thus the minimum of the maximum value of this quantity occurs when

$$\delta_\Delta(o) = \delta_\Delta(h)$$

That is, when

$$r_o^2 = h^2 - r_o^2$$

Thus the condition

$$r_0^2 = \frac{h^2}{2}$$

which fixes the second parameter in the parabolic field distribution, minimizes the aberration due to the lens action of the gauze holes.

The diameter of the aberration figure is then

$$\delta_\Delta = \frac{f_m}{f_e r_0^2} \cdot \frac{h^2 \Delta}{2} = \frac{f_m}{f_e} \Delta$$

But, as has been shown, in order to make the net spherical aberration zero

$$\frac{f_m}{f_e} = \frac{C_s}{f_m^3} \cdot 2 r_0^2 = \frac{C_s}{f_m^3} h^2$$

Thus

$$\delta_\Delta = \frac{C_s}{f_m^3} h^2 \Delta$$

The minimum spherical aberration figure for the magnetic lens has a diameter

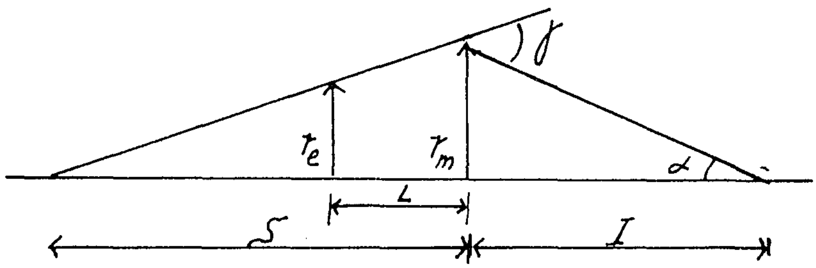
$$\delta_s = \frac{1}{2} \frac{C_s}{f_m^3} h^3$$

Thus the minimum aberration figure of the corrected lens is

$$\delta_\Delta = \frac{2\Delta}{h} \delta_s$$

Correction of Spherical Aberration - 2

Consider the case in which the electron source is at a finite distance S from the lens and the plane of the corrector is a distance L from the plane of the magnetic lens.



Here

$$\frac{1}{I} + \frac{1}{S} \simeq \frac{1}{f_m} \qquad \text{as } \frac{1}{f_e} << \frac{1}{f_m}$$

$$\frac{r_m}{r_e} = \frac{S}{S-L} = k$$

The deflections due to the magnetic lens and the corrector lens are

$$\gamma_m = \frac{r}{f_m} + \frac{C_s}{f_m^4} r^3$$

and

$$\gamma_e = \frac{r}{f_e} - \frac{1}{2 f_e r_o^2} r^3$$

The radius ρ of the error at the focus is

$$\rho = (I+L)\frac{r_e^3}{2 f_e r_o^2} - I\frac{C_s}{f_m^4} r_m^3 - \frac{I}{f_m}\frac{L r_e^3}{2 f_e r_o^2}$$

Let ρ = C.

Then

$$\left(1+\frac{L}{I}-\frac{L}{f_m}\right)\frac{1}{2f_e r_o^2}=\frac{C_s}{f_m^4}\frac{r_m^3}{r_e^3}$$

as

$$\left(1+\frac{L}{I}-\frac{L}{f_m}\right)=1-\frac{L}{S}=\frac{r_e}{r_m}$$

This gives

$$\frac{1}{2f_e r_o^2}=\frac{k^4}{f_m}\frac{C_s}{f_m^3}$$

so that

$$\frac{E_{ro}}{r_o^2}=k^4\frac{4U}{f_m}\frac{C_s}{f_m^3}$$

The error in the deflection due to the gauze holes is

$$\gamma_\Delta(r_e)=\frac{-1}{f_e}\left(1-\frac{r_e^2}{r_o^2}\right)\frac{\Delta}{2}$$

The maximum magnitude of this is minimized if

$$\frac{h_e^2}{r_o^2}=2$$

where h_e is the maximum value of r_e. This gives

$$\gamma_\Delta=\frac{\Delta}{2f_e}$$

The diameter δ_Δ of the error in the focus duc to this error in the deflection is

$$\delta_\Delta=2\left(I+L-\frac{L}{f_m}I\right)\gamma_\Delta=2\frac{I}{k}\gamma_\Delta$$

$$= 2\frac{I}{k}\frac{\Delta}{2f_e} = \frac{I}{k}\left(k^4\frac{h_e^2}{f_m}\frac{C_s}{f_m^3}\right)\Delta$$

$$= k\left(\frac{1}{2}\frac{I}{f_m}\frac{C_s}{f_m^3}h_m^3\right)\frac{2\Delta}{h_m}$$

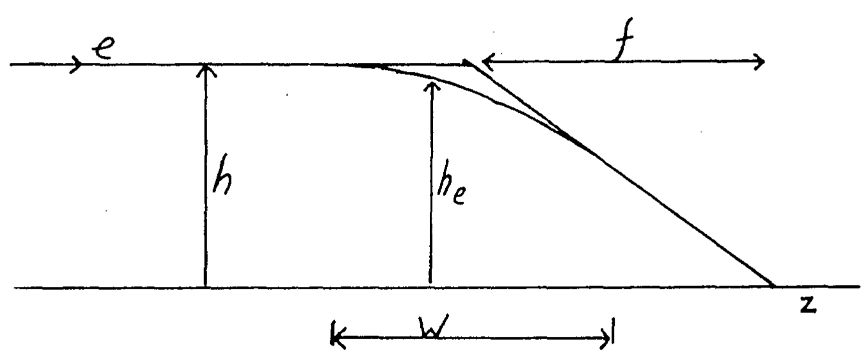
But the minimum spherical aberration figure of the magnetic lens has a diameter

$$\delta_s = \frac{1}{2}\frac{I}{f_m}\frac{C_s}{f_m^3}h_m^3$$

Thus

$$\delta_\Delta = k\frac{2\Delta}{h_m}\delta_s$$

In the case that the corrector is within the deflecting field of the magnetic lens it is not sufficient to take the radial coordinate of the electron to be that given by the straight line approximation to the electron trajectory unless the length of the magnetic lens is quite short compared to its focal length.



Consider an electron incident parallel to the axis of a magnetic lens of length w at a height h. Approximating the trajectory within the lens by

$$r_e(z) \simeq h\left(1 - \frac{1}{2} a^2 z^2\right)$$

and requiring

$$-r_e'(w) = h a^2 w = \frac{h}{f}$$

gives

$$r_e(z) \simeq h\left(1 - \frac{1}{2}\frac{z^2}{wf}\right)$$

If the corrector is at the center of the magnetic lens then the radial coordinate at the corrector is

$$h_e = h\left(1 - \frac{1}{8}\frac{w}{f}\right)$$

For the net spherical aberration to be zero, as

$$\gamma_e = \frac{1}{f_e} h_e - \frac{1}{2 f_e r_o^2} h_e^3$$

and

$$\gamma_m = \frac{1}{f} h + \frac{C_s h^3}{f^4}$$

$$\frac{1}{2 f_e r_o^2} h_e^3 = \frac{C_s}{f^4} h^3$$

That is

$$\frac{E_{z0}}{r_0^{\,2}} = \left(\frac{h}{h_e}\right)^3 \frac{4\mathcal{U}}{f} \frac{C_1}{f^3}$$

$$= \left(1 + \frac{1}{8}\frac{W}{f}\right)^3 \frac{4\mathcal{U}}{f} \frac{C_1}{f^3}$$

$$= k_w^{\,3} \; \frac{4\mathcal{U}}{f} \; \frac{C_1}{f^3}$$

The gauze hole error

$$\delta_\Delta = \frac{I}{f_e} \Delta$$

becomes

$$\delta_\Delta = \frac{I}{f} \frac{C_1}{f^3} \left(\frac{h}{h_e}\right)^3 h_e^{\,2} \Delta$$

$$= \left(\frac{h}{h_e}\right) \frac{I}{f} \frac{C_1}{f^3} h^2 \Delta$$

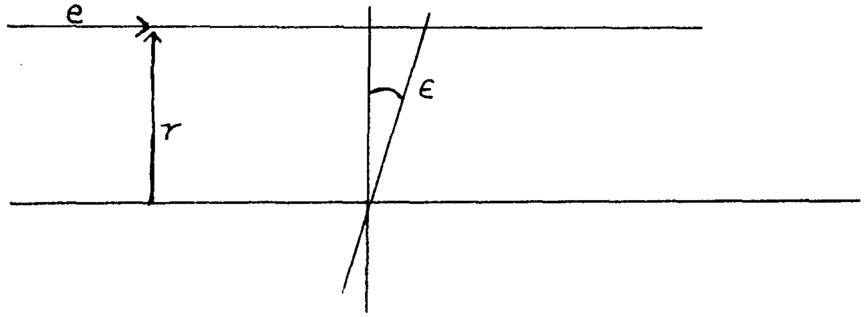
$$= k_w \frac{I}{f} \frac{C_1}{f^3} h^2 \Delta$$

Errors Due to the Gauze Lens

Alignment Errors

If the axis of the corrector does not coincide with that of the magnetic lens, errors will be introduced into the image.

Suppose that the corrector axis is tilted with respect to the axis of the magnetic lens. Consider electron rays in the plane of the two axes.



An electron, incident at a radius r parallel to the magnetic lens axis, is at a radius with respect to the corrector axis given by

$$r_e = \frac{r}{\cos \epsilon}$$

and thus is given a deflection by the corrector

$$\gamma_e = \frac{r_e}{f_e} - \frac{1}{f_e h^2} r_e^3$$

Thus there is an error in the deflection, taking

$$\frac{1}{\cos \epsilon} \simeq 1 + \frac{\epsilon^2}{2}.$$

given by

$$\Delta \gamma = \gamma^{\epsilon}_{(r)} - \gamma^{\epsilon = 0}_{(r)}$$

$$= \frac{r}{f_e}\frac{\epsilon^2}{2} - \frac{r^3}{f_e h^2}\frac{3\epsilon^2}{2}$$

$$= \left(1 - 3\frac{r^2}{h^2}\right)\frac{r}{f_e}\frac{\epsilon^2}{2}$$

This has its maximum value at r equal to h, where

$$\Delta\gamma = -\frac{h}{f_e}\epsilon^2$$

The width of the error in the focus is

$$\delta_\epsilon = 2 f_m |\Delta\gamma|$$

$$= 2\frac{f_m}{f_e} h \epsilon^2$$

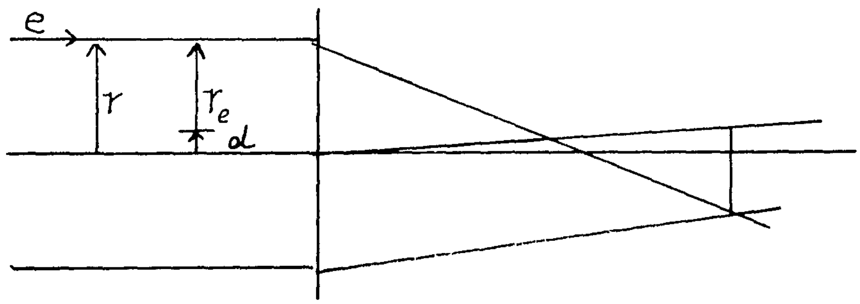
In terms of the gauze error,

$$\delta_\Delta = \frac{f_m}{f_e}\Delta$$

this is

$$\frac{\delta_\epsilon}{\delta_\Delta} = 2\frac{h}{\Delta}\epsilon^2$$

Suppose that the corrector axis is displaced parallel to the magnetic lens axis. Consider electron rays in the plane of the displacement.



The deflection due to the corrector lens is

$$\gamma_e = \frac{r_e}{f_e} - \frac{1}{f_e h^2} r_e^3$$

Here,

$$r_e = r - d$$

where r is the radial coordinate of the electron ray with respect to the magnetic axis. Thus there is an error in the deflection

$$\Delta \gamma = \gamma^{d}_{(r)} - \gamma^{d=0}_{(r)}$$

$$= \frac{-d}{f_e} + \frac{3r^2 d}{f_e h^2}$$

from which,

$$\Delta \gamma(0) = -\frac{1}{f_e} d$$

$$\Delta \gamma(h) = \frac{2}{f_e} d$$

Thus the width of the error in the focus is

$$\delta_d = 3 \frac{f_m}{f_e} d$$

In terms of the diameter of the gauze error, this is

$$\frac{\delta_d}{\delta_\Delta} = 3 \frac{d}{\Delta}$$

Fifth Order Error

Consider the next term in the expansion of the field at the gauze.

Let

$$E_z = E_{z_0}\left(1 - (1+\zeta)\frac{r^2}{r_0^2} + \zeta \frac{r^4}{r_0^4}\right)$$

Then, as

$$\gamma_e = \frac{1}{rU} \int_0^r E_z \, r \, dr$$

this gives

$$\gamma_e = \frac{1}{f_e}\left(r - \frac{(1+\zeta)}{2 r_0^2} r^3 + \frac{\zeta}{3 r_0^4} r^5\right)$$

Combining this with

$$\gamma_m = \frac{1}{f_m}\left(r + \frac{C_3}{f_m^3} r^3\right)$$

the net aberration is

$$\Delta\gamma = \frac{C_s}{f_m^4}\left(1 - \frac{(1+j)\,f_m^4}{2 f_e\, r_o^2\, C_s}\right) r^3 + \frac{j}{3 f_e\, r_o^4}\, r^5$$

Consider that j << 1, let

$$|E_z(0)| = |E_z(h)|$$

in order to minimize δ_Δ. This gives

$$r_o^2 = \frac{h^2}{2}(1-j)$$

Introduce the parameter x, which is small for small j,

$$\frac{f_m^4}{2 f_e\, r_o^2 C_s} = 1 + x$$

and ignore terms in j^2 and jx. Then

$$\Delta\gamma = \left(-(j+x)\,r^3 + \frac{4j}{3h^2}\,r^5\right)\frac{C_s}{f_m^4}$$

If (j+x) is zero the error is in the fifth order only. The maximum diameter of the aberration figure in the Gaussian focus is

$$\delta_5 = 2 f_m |\Delta\gamma(h)| = \frac{8}{3}\frac{C_s}{f_m^3}\, h^3 |j|$$

However, this does not represent the minimum aberration figure.

The diameter of the combined error in the Gaussian focus is the absolute value of

$$\delta_{3,5}(r) = 2 f_m\, \Delta\gamma(r)$$

$$= \left[\frac{4}{3}\left(\frac{r}{h}\right)^5 - \left(1+\frac{x}{j}\right)\left(\frac{r}{h}\right)^3\right]\frac{2C_s h^3}{f_m^3}/j/$$

Let

$$\frac{r}{h} = X, \quad \frac{3}{4}\left(1+\frac{x}{j}\right) = q$$

Then

$$\delta_{3,5}(X) = \left(X^5 - qX^3\right)\delta_5$$

Let X_0 be the value of X satisfying

$$\frac{\partial\ \delta_{3,5}}{\partial X} = 0$$

That is, let

$$X_0 = \sqrt{\frac{3}{5}q}$$

Thus, as a function of the adjustable parameter X_0

$$\delta_{3,5}(X) = \left(X^5 - \frac{5}{3}X_0^2 X^3\right)\delta_5$$

Increasing X_0 increases $/\delta_{3,5}(X_0)/$ but decreases $/\delta_{3,5}(1)/$ so that the best value of X_0 is that for which

$$/\delta_{3,5}(X_0)/ = /\delta_{3,5}(1)/ \equiv \delta_{3,5}$$

That is, it satisfies

$$2X_0^5 + 5X_0^2 - 3 = 0$$

The approximate solution is

$$X_0 \simeq \frac{7.02}{\sqrt{2}}$$

Thus

$$\delta_{3,5} = \frac{2}{3} X_0^{\ 5} \simeq \frac{7}{8} \ \delta_5$$

$$= \frac{2}{3} \ \frac{C_s}{2 f_m^{\ 3}} \ h^3 / \gamma / = \frac{2}{3} \ \delta_s / \gamma /$$

where δ_s is the minimum diameter of the third order spherical aberration error. In terms of the gauze error this becomes, since

$$\frac{\delta_s}{\delta_\Delta} = \frac{h}{2\Delta}$$

$$\delta_{3,5} = \frac{h}{2\Delta} \ \frac{2\delta_\Delta}{3} / \gamma /$$

The relative error in the field at the maximum radius h is

$$\frac{\Delta E_z(h)}{E_{z_0}} = \frac{/ E_z^{\gamma}(h) - E_z^{\gamma=0}(h) /}{E_{z0}} = 2 / \gamma /$$

So that in terms of the field error at h

$$\frac{\delta_{3,5}}{\delta_\Delta} = \frac{h}{2\Delta} \cdot \frac{1}{3} \ \frac{\Delta E(h)}{E_{z0}}$$

Chromatic Error

The focal length of the combined lens is given by

$$\frac{1}{f} = \frac{1}{f_m} + \frac{1}{f_e}$$

The variation in this focal length due to variation in the strength of the corrector is given by

$$\frac{\Delta f}{f} = \frac{f_m}{f_e}\,\frac{\Delta f_e}{f_e}$$

The dependence of the strength of the corrector on the voltage v applied to the electrodes and the electron energy U is

$$\frac{1}{f_e} = \frac{k v}{U}$$

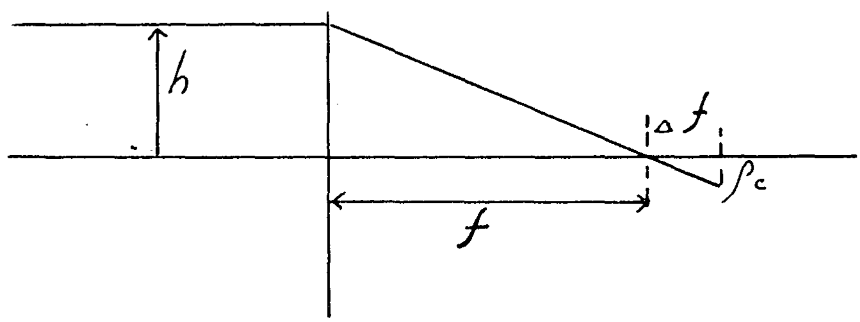
where k is a geometrical constant of the lens. Thus

$$-\frac{\Delta f_e}{f_e} = \left(\frac{\Delta v}{v} - \frac{\Delta U}{U}\right)$$

Since f_m/f_e is one, the contribution of the $\Delta U/U$ term is dominated by the chromatic aberration of the magnetic lens. So, as a function of $\Delta v/v$

$$-\frac{\Delta f}{f} = \frac{f_m}{f_e}\,\frac{\Delta v}{v}$$

As can be seen from the following sketch,



the diameter of the aberration figure due to a variation in the focal length is

$$\delta_c = 2\rho_c = 2\frac{|\Delta f|\,h}{f}$$

Thus here

$$\delta_c = 2\frac{f_m}{f_e}\;\frac{\Delta\nu}{\nu}\,h$$

As the diameter of the gauze aberration is

$$\delta_\Delta = \frac{f_m}{f_e}\;\Delta$$

then

$$\frac{\delta_c}{\delta_\Delta} = \frac{2h}{\Delta}\;\frac{\Delta\nu}{\nu}$$

Errors Due to the Magnetic Lens

Fifth Order Error

Consider the expansion of the deflection in the magnetic lens to the next higher order in the radius of the electron ray. Let

$$\gamma_m = \frac{1}{f_m} r + \frac{C_3}{f_m^4} r^3 + \frac{C_5}{f_m^6} r^5$$

The diameter of the aberration figure due to a fifth order error only is

$$\delta_5 = 2 f_m \, \Delta \gamma_5(h) = 2 \frac{C_5}{f_m^5} h^5$$

But, as has been shown for the case of the fifth order error in the corrector lens, when a fifth order error is combined with the appropriate amount of third order error, the minimum aberration figure has a diameter

$$\delta_{3,5} = \frac{1}{8} \delta_5$$

$$= \frac{1}{4} \frac{C_5}{f_m^5} h^5$$

For a weak lens the factor C_5/f^5 is a constant, depending only on the field geometry. For an axial field of the form

$$B_z = B_{z0} \, e^{-\frac{z^2}{\rho^2}}$$

which is a good approximation to the field of a magnetic lens of the form used as a test lens, C_5/f^5 is given by

$$\frac{C_5}{f_m^5} \simeq \left(\frac{C_1}{f_m^3}\right)^2$$

Thus

$$\delta_{3,5} \simeq \frac{7}{4}\left(\frac{C_1}{f_m^3}\right)^2 h^5$$

In terms of the gauze error,

$$\delta_\Delta = \frac{C_1}{f^3} h^2 \Delta$$

This becomes

$$\frac{\delta_{3,5}}{\delta_\Delta} = \frac{7}{4} \frac{C_1}{f^3} \frac{h^3}{\Delta}$$

Chromatic Error

The diameter of the aberration figure due to a variation in the focal length is

$$\delta_c = \frac{2/\Delta f/h}{f}$$

The focal length of a magnetic lens is dependent on the energy of the electrons and the lens current. For a weak lens,

$$f = k\frac{U}{A^2}$$

where U is the electron energy, A is the lens current and k is a geometrical constant of the lens. Thus

$$\frac{\Delta f}{f} = \frac{\Delta U}{U} - 2\frac{\Delta A}{A}$$

$$\equiv \epsilon_u - 2\epsilon_A$$

As the variations in the energy and lens current are mostly independent, the diameter of the aberrations due to these variations can be taken to be

$$\delta_u = 2\epsilon_u h$$

and

$$\delta_A = 4\epsilon_A h$$

In terms of the gauze error these are

$$\frac{\delta_u}{\delta_\Delta} = \frac{2}{h\Delta}\left(\frac{C_1}{f^3}\right)^{-7} \epsilon_u$$

and

$$\frac{\delta_A}{\delta_\Delta} = \frac{4}{h\Delta}\left(\frac{C_1}{f^3}\right)^{-7} \epsilon_A$$

Summary of Lens Errors

The errors of the corrector and the magnetic lens that have been considered are listed below together with estimates of their magnitude for the experimental system. A corrector with a gauze electrode with holes about 30 micron in diameter and a magnetic lens with a spherical aberration coefficient C_s/f^3 of about 10^{-3} mm^{-2} were used. The magnification was small, that is, I/f is about one. Values are given for $h \simeq 1mm$ and $h \simeq 2mm$, and in that order. The errors are calculated as fractions of the gauze hole error

$$\delta_\Delta = \frac{C_s}{f^3} h^2 \Delta$$

The errors of the corrector are:

The alignment error due to the tilt, by an angle ϵ , of the corrector axis with respect to the magnetic axis.

$$\frac{\delta_\epsilon}{\delta_\Delta} = \frac{2h\epsilon^2}{\Delta}$$

$$= 0.7 \quad , \quad 0.2$$

for $\epsilon \simeq 1/30$.

The alignment error due to the parallel displacement, by a distance d, of the corrector axis with respect to the magnetic axis.

$$\frac{\delta_d}{\delta_\Delta} = \frac{3d}{\Delta}$$

$$\simeq 0.7$$

for $d \simeq 1$ micron.

The error due to the fifth order error, of magnitude ΔE at h, in the field.

$$\frac{\delta_{35}}{\delta_\Delta} = \frac{h}{6\Delta} \frac{\Delta E(h)}{E_{x0}}$$

$$\simeq 0.1 \quad , 0.2$$

for $\Delta E/E \simeq 0.02$.
The chromatic error due to a fractional fluctuation of amplitude ϵ_V in the voltage supplied to the corrector.

$$\frac{\delta_c}{\delta_\Delta} = \frac{2h}{\Delta} \epsilon_V$$

$$= 0.7 \quad , 0.7$$

for $\epsilon_V \simeq 0.001$.

The errors of the magnetic lens considered are:
The error due to the fifth order spherical aberration error of the lens.

$$\frac{\delta_{3,5}}{\delta_\Delta} = \frac{1}{4} \frac{C_s}{f^3} \frac{h^3}{\Delta}$$

$$= 0.0 \quad , 0.7$$

The chromatic error due to a fractional fluctuation of amplitude ϵ_u in the energy of the electrons.

$$\frac{\delta_u}{\delta_\Delta} = \frac{2}{h\Delta} \left(\frac{C_s}{f^3}\right)^{-1} \epsilon_u$$

$$= 0.3 \quad , 0.2$$

for $\epsilon_u \simeq 5.10^{-6}$
The chromatic error due to the fractional fluctuation of amplitude ϵ_A in the lens current.

$$\frac{\delta_A}{\delta_\Delta} = \frac{4}{h\Delta} \left(\frac{C_s}{f^3}\right)^{-1} \epsilon_A$$

$$= 0.3 \quad , 0.2$$

for $\epsilon_A \simeq 2.5.10^{-6}$.

Considering that the total error, δ_T can be taken to be

$$\frac{\delta_T}{\delta_\Delta} = \sqrt{1 + \sum_i \left(\frac{\delta_i}{\delta_\Delta}\right)^2}$$

gives

$$\frac{\delta_T}{\delta_\Delta} \simeq 1.1$$

for both values of h.

One important source of errors that has not yet been discussed is the astigmatism of the lenses due to the departure of physical electrodes from ideal rotational symmetry. Thus there are errors in the deflection dependent on the azimuthal angle θ as well as the radius of incidence r of the electron rays. These errors can be corrected for by introducing weak multipole lenses of variable strength and orientation. Normally, if the first order astigmatism is corrected, the higher order astigmatism is negligible compared to the spherical aberration. If the spherical aberration is corrected, then it may be necessary to correct the second order astigmatism.

Another source of errors which is important practically is the disturbances due to the charging of contaminated surfaces. The surfaces which limit the electron beam are the most critical. That is, the lens aperture and, in the case of the corrector, the edges of the gauze holes.

These errors can be reduced by maintaining clean surfaces, which requires a good vacuum in the region of the lenses.

Experimental Test

Design of the Corrector Electrodes

Electrode configurations which give the required field distributions at the gauze,

$$\frac{E_x}{E_{xo}} = 1 - \frac{r^2}{r_o^2}$$

were determined with the aid of a resistance network analogue. The network corresponds to one of the half planes of a system of rotational symmetry. The network has 20 meshes in the radial direction and 40 meshes in the axial direction. The accuracy of the field determination is about one percent.

The best configuration found is shown in figure 1. The shape of the field is not very sensitive to small changes in the configuration. The deviation of the field at the central plane from parabolic form is less than about two percent over the range $0 \leq r \leq h$ for h less than 9 meshes where h is the aperture radius and where, in order to minimize the gauze hole error, r_o has been set equal to $h/\sqrt{2}$. The parameter r_o is determined by the ratio of the ring electrode voltage to the gauze electrode voltage V_R/V_G. The parameter ϵ (r_o), where $E_{zo}/r_o^2 = \epsilon(r_o)V_R$, varies only slowly with r_o. See figure 1.

Experimental System

The experimental electron optical system, which consists of an electron microscope split above the objective lens to admit the corrected lens, is shown in figure 2. The condensor lenses are used to form a highly demagnified image of the gun crossover. The corrected lens forms an image of this source in the object plane of the microscope objective. The Gaussian size of the image can be made small compared to the aberration figure. The microscope was normally used at a magnification of 10^4. The 4 micron wide bars of a

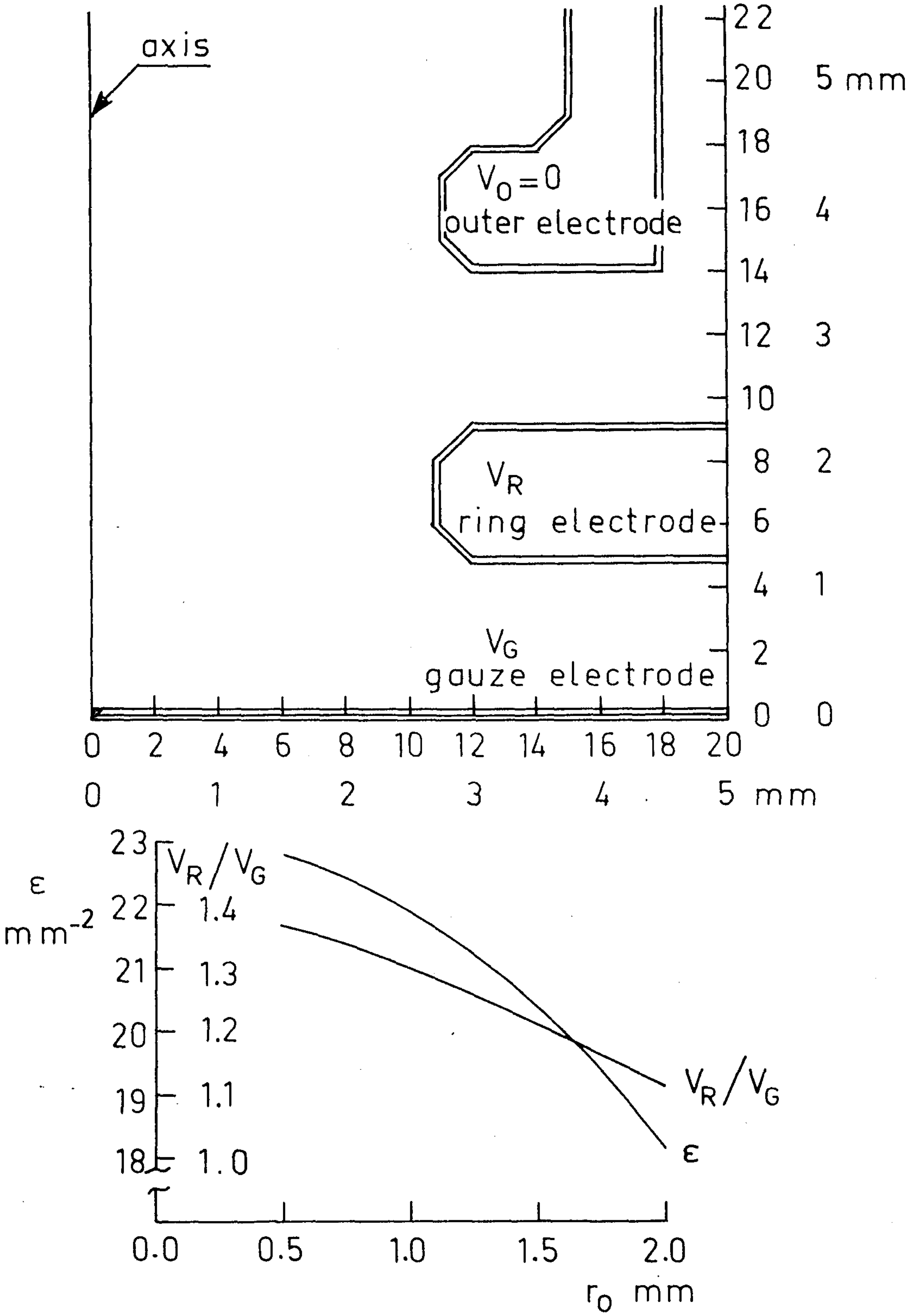


Fig.1 Design for corrector lens

gauze were used as a standard. An electron energy of 70 kV was normally used.

The test lens and corrector are shown in figure 3. The test lens has a bore of 3 cm and a pole separation of 4 cm. The upper pole piece is centerable and the corrector can be centered on the magnetic lens. First and second order astigmatism correctors are built into the lens. The corrector holder was made of vinidur plastic, the electrodes of brass and the gauze of copper. The gauze was annealed in vacuum which made it possible to achieve a high degree of flatness when it was placed in the corrector. The test lens and corrector electrode holder are those designed and used by Rus (1965) in a preliminary investigation.

The spherical aberration coefficient C_s/f^3 for the test lens was determined by measuring the fractional change in lens current needed to defocus from the Gaussian plane to the plane of minimum cross section.
This gave

$$\frac{C_s}{f^3} \simeq 1.2 \cdot 10^{-3} \ mm^{-2}$$

The size of the image was normally determined simply by observing the image on a screen with reference markings of one millimeter separation which corresponds to 0.1 micron in the image. In order to obtain information on the intensity distribution in the image, the profile across a slit was determined. The beam was driven across a thin slit at the screen position by means of one pair of the deflection coils below the condensor lenses and the electrons passing through the slit were detected by means of a scintillator, light pipe and photomultiplier and the signal displayed on a storage oscilloscope. The magnified image was driven at 50 cps with an amplitude of 20 mm at the screen. As the image diameter at the screen was only about one millimeter the velocity across the slit was taken to be constant. Referred to the object plane of the microscope, this was 0.63 mm/s. The

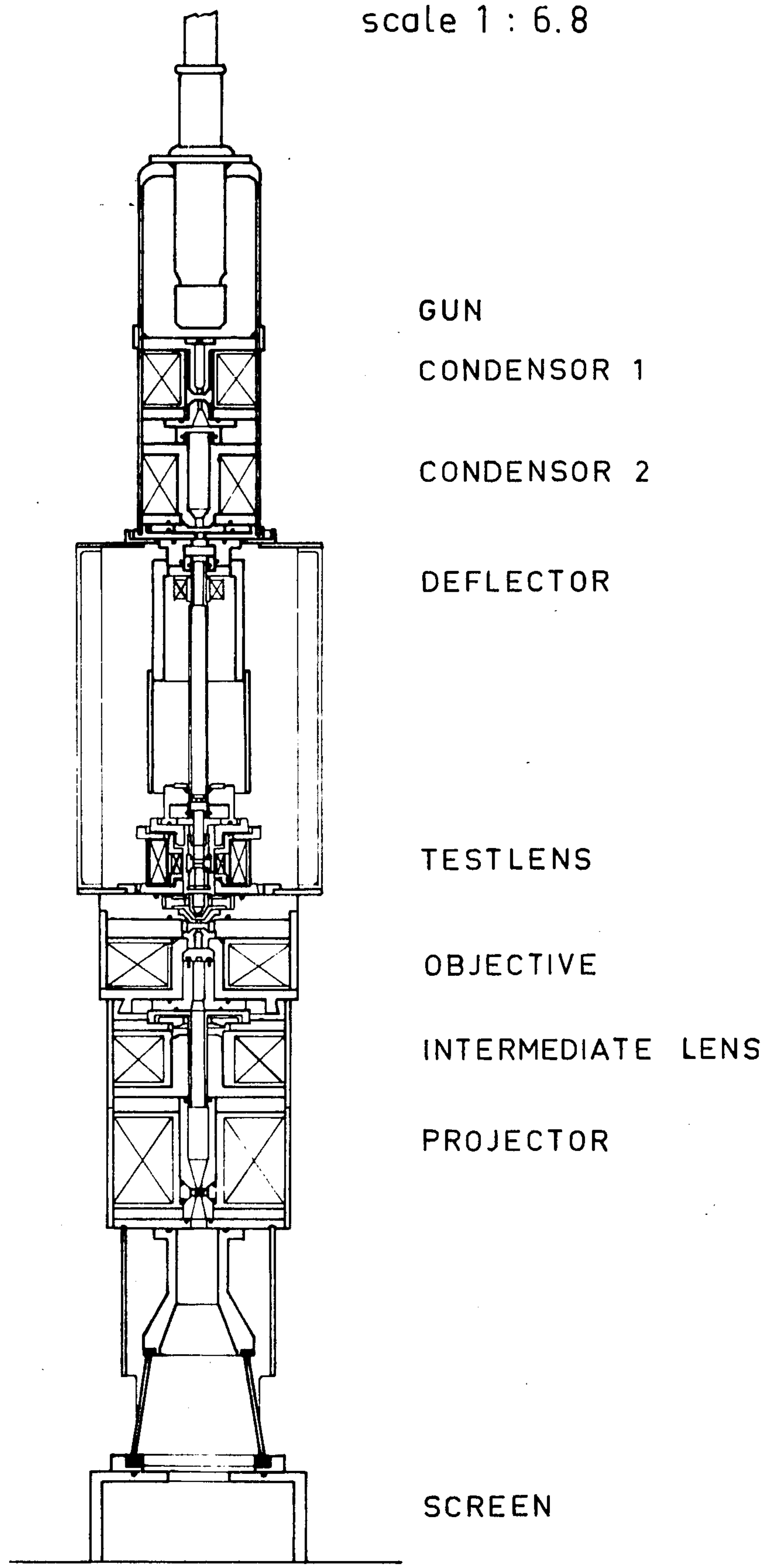


Fig. 2 Experimetal system

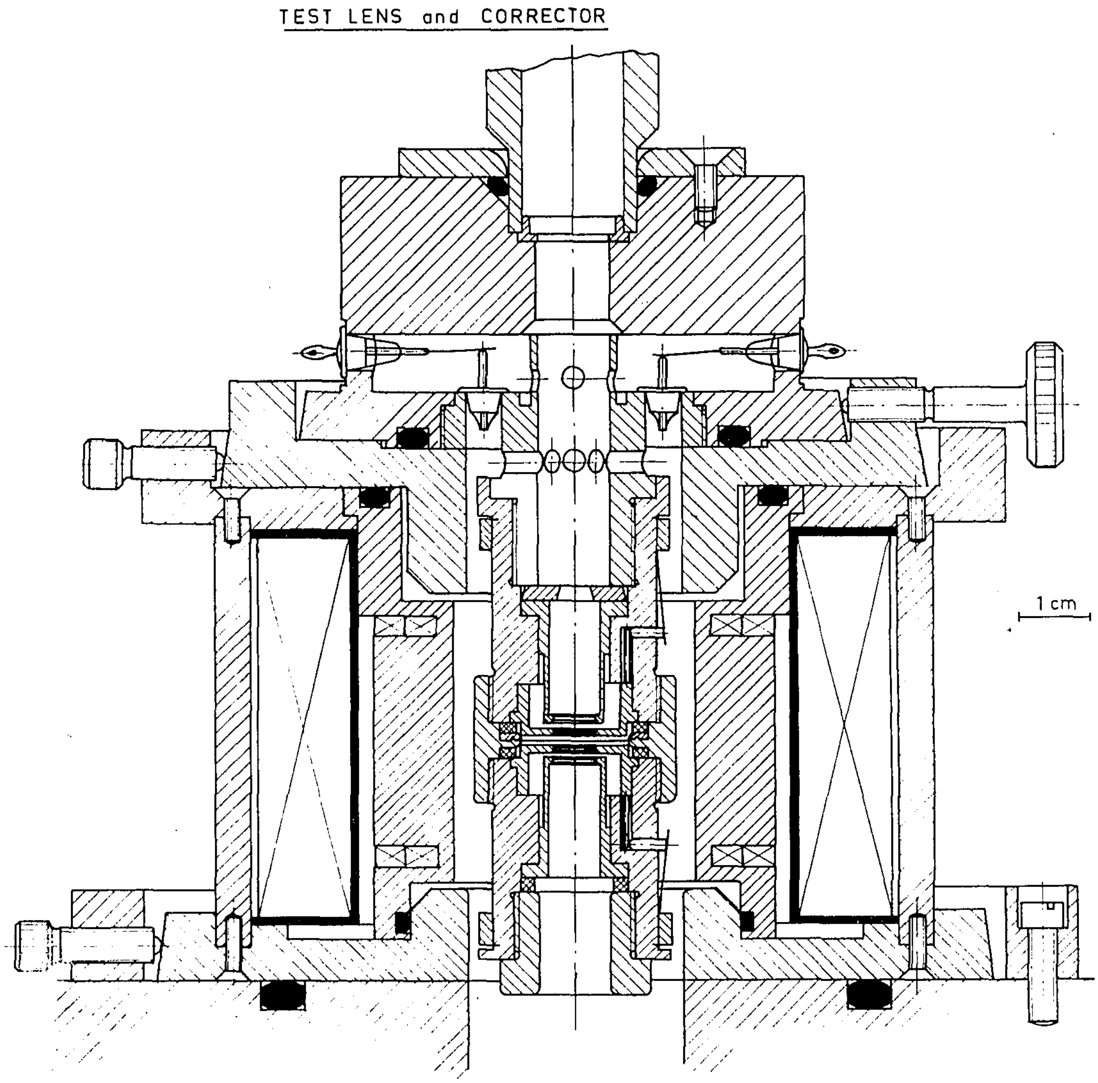


Fig. 3 Test lens and corrector

oscilloscope was driven at 0.02 s/m, thus the magnification at the oscilloscope screen was 0.013 micron/mm. The width of the slit was about 50 micron which is small compared to the image sizes.

Experimental Results

The diameter of the minimum spherical aberration figure is given by

$$\delta_s = \frac{1}{2} \frac{I}{f} \frac{C_s}{f^3} h^3 .$$

The object distance S was 32 cm, the image distance I was 6 cm and the physical aperture of diameter a was a distance L equal 2 cm above the center of the lenses. Thus

$$\frac{I}{f} = 1 + \frac{I}{S} = 1.19$$

$$h = \frac{S}{S-L} \frac{a}{2} = 0.53\ a$$

For the test lens

$$\frac{C_s}{f^3} = 1.2 \cdot 10^{-3}\ mm^{-2}$$

Thus

$$\delta_s = 0.11\ a^3\ micron$$

where a is in millimeters.

If the spherical aberration of the magnetic lens is corrected by means of a gauze lens, placed at the center of the magnetic lens, the diameter of the gauze hole aberration figure is

$$\delta_\Delta = k_w \frac{I}{f} \frac{C_s}{f^3} h^2 \Delta$$

where Δ is the diameter of the gauze holes. The gauze used was a square mesh copper gauze with holes 25 microns square. Verster (1963) has studied the lens action of the holes of square mesh gauzes by means of a ray tracer. The maximum deflection is perpendicular to the bars and is about 20 percent greater than in the round hole case. Thus the 25 micron square holes are about equivalent to 30 micron round holes. Thus Δ is taken to be 30 micron. The parameter k_w which arises because the corrector is at the center of the magnetic lens is given by

$$k_w = \left(1 + \frac{1}{8} \frac{W}{f}\right)$$

where w is the distance over which the lens action of the magnetic lens can be considered to take place. Taking w to be equal to the pole piece separation gives

$$k_w = \left(1 + \frac{1}{8} \frac{4}{6}\right) = 1.1$$

Thus

$$\delta_\Delta = 0.013\ a^2 \text{ micron}$$

where a is in millimeters.

The spherical aberration of the system test lens plus corrector will be zero if

$$\frac{E_{zo}}{r_o^2} = k_w^3 \frac{4U}{I} \frac{I}{f} \frac{C_s}{f^3}$$

For an electron energy U of 70 kV

$$\frac{E_{zo}}{r_o^2} = 8.5 \quad V/mm^3$$

The maximum diameter of the gauze hole aberration figure is minimized if

$$r_o = \frac{h_e}{\sqrt{2}} = \frac{1}{k_w} \frac{h}{\sqrt{2}}$$

In terms of the physical aperture of diameter a this is

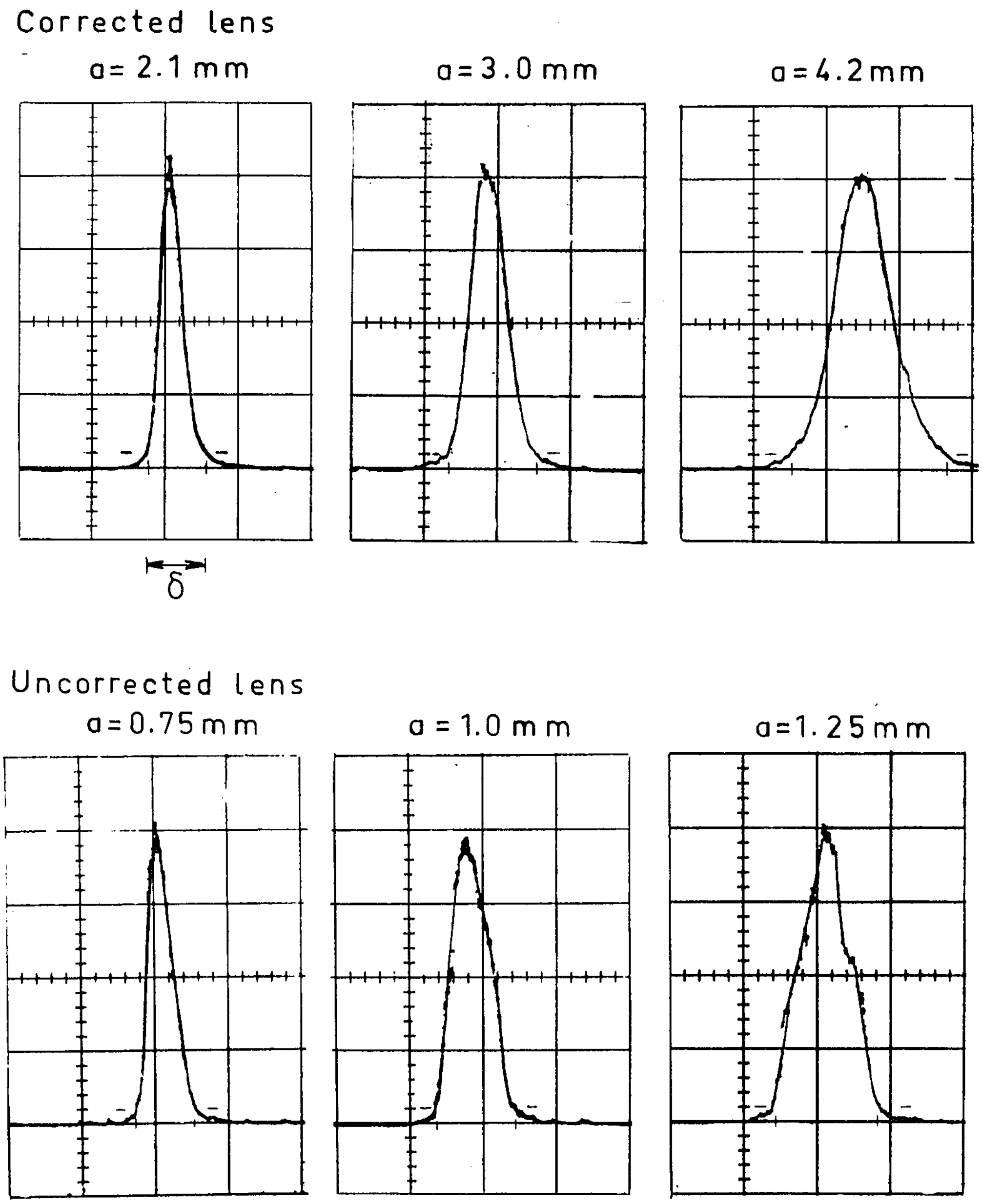
$$r_o = 0.34 \; a$$

The corrector electrode voltages which correspond to these values of the parameters E_{zo}/r_o^2 and r_o can be determined from the curves of figure 1 where V_R/V_G and $\epsilon(r_o)$ are given as a function of r_o. V_R is given by

$$V_R = \frac{E_{zo}/r_o^2}{\epsilon(r_o)}$$

Apertures 2.1, 3.0, and 4.2 mm in diameter were used.

V_R, V_R/V_G, the centering of the corrector and the first and second order astigmators were adjusted to minimize the diameter of the aberration figure. The profiles of the images are shown in figure 4 . The profiles of images for the uncorrected lens, where the corrector has been replaced, by a tube of 7.5 mm inside diameter, for apertures 0.75, 1.00,



Scale is 0.013 μ/mm referred to the test lens image plane

Fig.4 Image profiles

and 1.25 mm in diameter are also shown in figure 4 . The diameter of the images, δ , is taken to be the width of the profile at 1/20 of the maximum. For a Gaussian distribution this would include 99 percent of the current into the image.

Tabulating the results, for the uncorrected lens

a (mm)	$\delta_s (\mu)$	$\delta (\mu)$
0.75	0.05	0.10
1.00	0.11	0.14
1.25	0.21	0.19

and for the corrected lens

a (mm)	$\delta_s (\mu)$	$\delta_\Delta (\mu)$	$\delta (\mu)$
2.1	1.0	0.06	0.10
3.0	2.9	0.12	0.15
4.2	8.0	0.23	0.26

where a is the diameter of the physical aperture, δ_s the calculated diameter of the minimum spherical aberration figure, δ_Δ the calculated diameter of the minimum gauze hole aberration figure and δ the measured diameter of the minimum aberration figure. As the vacuum in the microscope was not very good, the image size at small apertures was limited by contamination. The calculated V_R, and experimental V_R^*, voltages were

a (mm)	V_R(v)	V_R^*(v)	V_R/V_G	V_R^*/V_G^*
2.1	380	385	1.33	1.33
3.0	390	400	1.29	1.27
4.2	415	430	1.22	1.19

For the two larger apertures the reproducibility of V_R^* was about $\pm$ 5 volts and for the 2.1 mm aperture about $\pm$ 30 volts.

The reproducibility of V_R^*/V_G^* was about $\pm$ 0.03 for the larger apertures. For the 2.1 mm aperture the ratio was set equal to the calculated value.

A few trials were made with foil electrodes. Aluminum oxide windows about 6 mm in diameter and 300 A thick were made in aluminum foil electrodes by anodic oxidation. Carbon was evaporated into the electrodes to make them conducting which gave a total of about 1000 A. An uncorrected image 8 micron in diameter could be reduced by the foil corrector to an image about 0.2 micron in diameter which had sharp edges and contained about 1/3 of the incident electrons. The remaining electrons were scattered over large enough angles so that they did not come through the microscope.

Possible Improvement of a Probe Lens

Consider the case of a lens used to form a real image of an electron source. The current into the image of an electron source of brightness B is

$$J = \frac{\pi^2 i^2}{4}\left(\frac{h}{I}\right)^2 B$$

where h/I is the angular aperture of the lens and i is the Gaussian diameter of the image. The total diameter of the image can be taken to be

$$d = \sqrt{i^2 + \delta^2}$$

where δ is the diameter of the disc of confusion due to the aberrations of the lens. Normally, the limiting aberration is the spherical aberration, of which the minimum disc of confusion is

$$\delta_s = \frac{m}{2}\,\frac{C_s}{f^3}\,h^3$$

where

$$m = \frac{I}{f} = 1 + M$$

M being the magnification of the lens.

In order to determine the aperture which gives the maximum current in an image of total diameter d_s the current is written as a function of the aberration δ_s. That is

$$J = k\left(d_s^2 - \delta_s^2\right)\delta_s^{2/3}$$

The solution to the maximizing equation

$$\frac{dJ}{d\,\delta_s} = 0$$

is

$$\delta_s = \frac{1}{2} d_s$$

Thus the radius of the optimum aperture for the total image size d_s is

$$h_s = \left(\frac{f^3}{m C_s}\right)^{1/3} d_s^{\,1/3}$$

The current into d_s is

$$J_s = \frac{3\pi^2}{16}\left(\frac{f^3}{m C_s}\right)^{2/3} \frac{d_s^{\,8/3}}{I^2} B$$

Suppose that the spherical aberration is corrected by means of a gauze lens and that the gauze hole error then determines the size of the aberration disc. The diameter of the aberration disc is

$$\delta_\Delta = m \frac{C_s}{f^3} h^2 \Delta$$

In this case the optimum value of δ_Δ is

$$\delta_\Delta = \frac{d_\Delta}{\sqrt{3}} \simeq \frac{d_\Delta}{2}$$

Thus,

$$h_\Delta = \left(\frac{f^3}{m C_s}\right)^{1/2} \left(\frac{d_\Delta}{2\Delta}\right)^{1/2}$$

is the optimum aperture radius. The current into d_Δ is

$$J_\Delta = \frac{3\pi^2}{16}\left(\frac{f^3}{mC_s}\right)\frac{d_\Delta^{\,3}}{2\Delta}\,\frac{T}{I^2}\,B$$

where T is the transmission of the gauze.
Thus, for a given image distance I and total image diameter d,

$$\frac{J_\Delta}{J_s} = T\left(\frac{f^3}{mC_s}\right)^{1/3}\frac{d^{1/3}}{2\,\Delta}$$

or, for a given current and image size

$$\frac{I_\Delta}{I_s} = \left[T\left(\frac{f^3}{mC_s}\right)^{1/3}\frac{d^{1/3}}{2\,\Delta}\right]^{1/2}$$

The ratio of the optimum aperture sizes is

$$\frac{h_\Delta}{h_s} = \left[\left(\frac{f^3}{mC_s}\right)^{1/3}\frac{d^{1/3}}{2\,\Delta}\right]^{1/2}$$

For a given current and image distance,

$$\frac{d_\Delta}{d_s} = \left[\frac{1}{T}\left(\frac{mC_s}{f^3}\right)^{1/3}\frac{2\Delta}{d_s^{1/3}}\right]^{1/3}$$

Here the ratio of the aperture sizes is

$$\frac{h_\Delta}{h_s} = \frac{1}{T^{1/6}}\left[\left(\frac{f^3}{mC_s}\right)^{1/3}\frac{d_s^{\,1/3}}{2\,\Delta}\right]^{1/3}$$

As a numerical example let

$$\frac{C_s}{f^3} = 10^{-3}\ mm^{-2}$$

$$m = 1 + M \simeq 1$$

$$d_s = 1 \text{ micron}$$

$$\Delta = 25 \text{ micron}$$

$$T = 0,5$$

Then

$$h_s = 1 \text{ mm}$$

and

$$\frac{J_\Delta}{J_s} = 10$$

$$\frac{I_\Delta}{I_s} = 3,2$$

with

$$h_\Delta = 4,5 \text{ mm}$$

and

$$d_\Delta = 0,5\ \mu$$

with

$$h_\Delta = 3.0 \text{ mm.}$$

MULTIPLE GAUZE LENS

Compensation of the Gauze Hole Error

Each hole of a gauze lens of strength $1/f_e$, of which the central element is a single gauze, acts as an aperture lens of strength $-1/f_e$. The lens action of the holes gives an error in the focus of diameter

$$\delta_\Delta = \frac{I}{f_e} \Delta$$

where I is the image distance and Δ is the diameter of the gauze holes. However, if more than one gauze is used, the lens action of the gauze holes is not fixed by the strength of the lens itself, but can be made zero by adjusting the fields between the gauzes.

Consider the lens sketched in figure 5, of which the central element is formed by two parallel gauzes. The electrodes are designed, with the aid of the resistance network described on page 45, to give a uniform field at the gauze. Thus the principal lens action is, within the weak lens approximation used so far, free of spherical aberration. The strength of the two halves of the lens are

$$\frac{1}{f_1} = \frac{E_1}{4U_{10}} \quad , \quad \frac{1}{f_2} = \frac{E_2}{4U_{20}}$$

where U_{10}, U_{20} are the potentials of the outer electrodes. In terms of the potentials and the lens geometry the fields are

$$E_1 = \frac{U_1 - U_{10}}{l} \quad , \quad E_2 = \frac{U_2 - U_{20}}{l} \quad ,$$

$$E_3 = \frac{U_2 - U_1}{d}$$

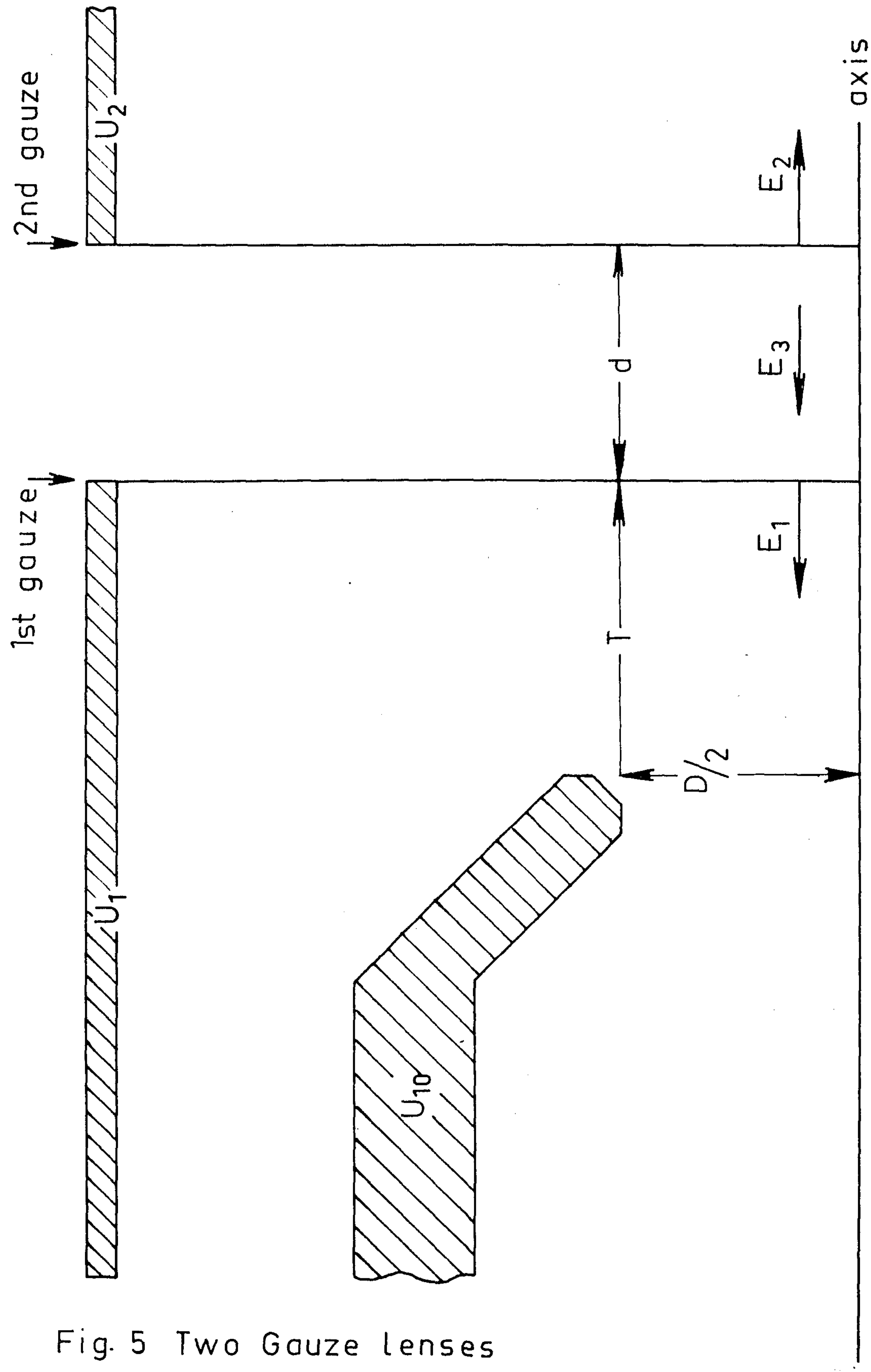
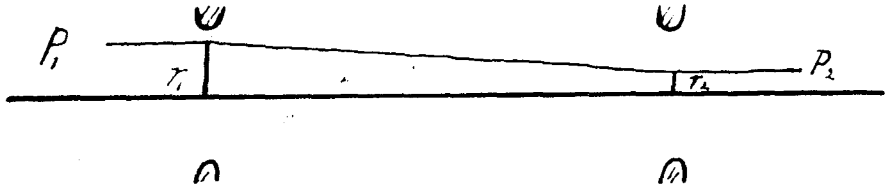


Fig. 5 Two Gauze lenses

If f_1 is equal to the object distance S, then the electrons will be incident perpendicular to the first gauze. The gauze holes act as aperture lenses of strength

$$\frac{1}{f_{\Delta 1}} = -\frac{(E_1 - E_3)}{4U_1} \quad , \quad \frac{1}{f_{\Delta 2}} = -\frac{(E_2 + E_3)}{4U_2}$$

If the net impulse directed toward the axis of a hole is zero, the electrons will emerge perpendicular to the second gauze.



It is required that

$$\Delta p_{r_1} + \Delta p_{r_2} = 0$$

$$p_1 \frac{r_1}{f_{\Delta_1}} + p_2 \frac{r_2}{f_{\Delta_2}} = 0$$

$$p_1 \frac{1}{f_{\Delta_1}} + \frac{r_2}{r_1} p_2 \frac{1}{f_{\Delta_2}} = 0$$

From

$$\dot{p} = eE_3 \quad , \quad p_1 = \sqrt{2meU_1} \quad , \quad p_2 = \sqrt{2meU_2}$$

$$\frac{r_2}{r_1} = 1 - p_1 \frac{p_2 - p_1}{p_2^2 - p_1^2} \frac{2d}{f_{\Delta_1}}$$

Thus

$$p_1 \frac{d}{f_{\Delta 1}} + p_2 \frac{d}{f_{\Delta 2}} - \frac{2 p_2 p_1}{p_1 + p_2} \frac{d^2}{f_{\Delta 1} f_{\Delta 2}} = 0$$

That is

$$\frac{(E_1 - E_3) d}{4\sqrt{U_1}} + \frac{(E_2 + E_3) d}{4\sqrt{U_2}} +$$

$$\frac{2\sqrt{U_1 U_2}}{\sqrt{U_1} + \sqrt{U_2}} \frac{(E_1 - E_3) d}{4 U_1} \frac{(E_2 + E_3) d}{4 U_2} = 0$$

But, from

$$E_1 = \frac{U_1 - U_{10}}{\ell} , \qquad E_2 = \frac{U_2 - U_{20}}{\ell}$$

$$\frac{1}{S} = \frac{E_1}{4 U_{10}} , \qquad \frac{1}{I} = \frac{E_2}{4 U_{20}}$$

the quantities

$$\frac{E_1 d}{4 U_1} = \frac{d}{S} \frac{1}{1 + \frac{4\ell}{S}} = g_1 ,$$

$$\frac{E_2 d}{4 U_2} = \frac{d}{I} \frac{1}{1 + \frac{4\ell}{I}} = g_2$$

are seen to be constants of the lens geometry. Let g_1 equal g_2, that is, let the lens have unit magnification. Then

$$g(\sqrt{U_2} + \sqrt{U_1}) + \frac{(U_2 - U_1)}{4} \left(\frac{1}{\sqrt{U_2}} - \frac{1}{\sqrt{U_1}} \right) +$$

$$\frac{2\sqrt{U_1 U_2}}{\sqrt{U_1}+\sqrt{U_2}}\left(g-\frac{U_2-U_1}{4U_1}\right)\left(g+\frac{U_2-U_1}{4U_2}\right)=0$$

With

$$r=\sqrt{U_2} \qquad , \quad s=\sqrt{U_1}$$

This can be written

$$(r-s)^2-\frac{8}{3}grs+\frac{4}{3}g(r-s)^2-\frac{16}{3}g^2\frac{r^2s^2}{(r+s)^2}=0$$

As g is small only the first two terms are kept. This gives

$$r^2-2\left(1+\frac{4g}{3}\right)rs+s^2=0$$

To the lowest order in g the required solution is

$$\frac{U_2}{U_1}=1+\sqrt{\frac{32}{3}g}$$

From

$$\frac{E_1 d}{4U_1}=\frac{U_1-U_{10}}{4\ell U_1}=g$$

$$\frac{E_2 d}{4U_2}=\frac{U_2-U_{20}}{4\ell U_2}=g$$

It follows that

$$\frac{U_{10}}{U_1}=\frac{U_{20}}{U_2}=\frac{1}{1+\frac{4\ell}{I}}$$

Thus the error of a spherical aberration free gauze lens due to the lens action of the gauze holes can be removed by using two gauzes, with the field between them adjusted to make the strength of the two-hole lenses zero. In doing this, a new error is introduced, which is due to the spherical aberration of the two-hole lenses.

Spherical Aberration of the Gauze Hole Lenses

As the field between the gauzes is much greater than the fields outside the latter can be ignored in calculating the spherical aberration of the two-hole lenses. The spherical aberration coefficient is given by the integral

$$\frac{C_s}{f^4} = \frac{5}{64}\frac{1}{U^2}\int_{-\infty}^{\infty}(E_z')^2\,dz$$

For the axial field of a hole of radius R, where the fields approach the uniform fields E- on the left and E+ on the right, E_z' is given by

$$E_z' = \frac{2}{\pi R}(E_+ - E_-)\frac{1}{\left[1+\left(\frac{z}{R}\right)^2\right]^2}$$

Thus for the two-hole lens

$$\frac{C_s}{f^4} = \frac{5}{64}\frac{4E_3^2}{\pi^2 R^2}\left(\frac{1}{U_1^2}+\frac{1}{U_2^2}\right)\int_{-\infty}^{\infty}\frac{dz}{\left[1+\left(\frac{z}{R}\right)^2\right]^4}$$

As

$$\int_{-\infty}^{\infty}\frac{dz}{\left[1+\left(\frac{z}{R}\right)^2\right]^4} = \frac{5\pi R}{12}$$

and

$$E_3^2\left(\frac{1}{U_1^2}+\frac{1}{U_2^2}\right) \simeq \frac{64}{3}\,\frac{1}{Id}$$

This gives

$$\frac{C_s}{f^4} = \frac{25}{9\pi}\,\frac{1}{RId}$$

The minimum aberration figure is

$$\delta_{s\Delta} = \frac{I}{2}\,\frac{C_s}{f^4}R^3 = \frac{25}{18\pi}\left(\frac{R}{d}\right)R$$

$$= \frac{25}{72\pi}\left(\frac{\Delta}{d}\right)\Delta \simeq \frac{1}{9}\left(\frac{\Delta}{d}\right)\Delta$$

Residual Gauze Hole Error

The result that a uniform field at the gauze gives a lens free of spherical aberration depends on the assumption that the lens is weak and, in particular, on the assumption that the axial velocity is constant. In finding the aberrations of the lens this is not adequate. Taking into account the variation of axial velocity, the deflection given an electron becomes

$$\gamma = \frac{e}{mV_{z0}}\int_{-\infty}^{\infty} E_r\,\frac{dz}{V_z(r,z)}$$

Making use of the expansion of the potential in terms of the axial potential

$$\Phi(r,z) = \phi(z) - \frac{r^2}{4}\phi''(z) + ----$$

gives

$$E_r = \frac{-\partial\phi}{\partial r} \simeq \frac{r}{2}\phi''$$

and

$$v_z(r,z) \simeq \sqrt{\frac{e\Phi}{2m}} \simeq v_{z0}\left(1 - \frac{r}{4}\frac{E_r}{U_{10}}\right)$$

Thus

$$\gamma = \frac{1}{2U_{10}}\int_{-\infty}^{\infty} E_r\left(1 + \frac{r}{4}\frac{E_r}{U_{10}}\right) dz$$

$$= \frac{1}{2U_{10}}\int_{-\infty}^{\infty} E_r\, dz + \frac{r}{8U_{10}^2}\int_{-\infty}^{\infty} E_r^2\, dz$$

In each half of the lens $E_r(r,z)$ is approximated by a function $E_r(r)$ over a region of width w and by zero elsewhere. Then,

$$\frac{1}{2U_{10}}\int_{-\infty}^{0} E_r\, dz = \frac{E_{z0}\, r}{4U_{10}} = \frac{E_r W}{2U_{10}}$$

Thus

$$E_r(r) = \frac{E_{z0}\, r}{2W}$$

The error in the deflection is

$$\Delta y_1 = \frac{r}{8U_{10}^2}\int_{-\infty}^{0} E_r^2 \, dx = \frac{r^3}{8U_{10}^2}\left(\frac{E_{x0}}{2W}\right)^2 W$$

$$= \frac{1}{2}\left(\frac{E_{x0}}{4U_{10}}\right)^2 \frac{1}{W} r^3 = \frac{1}{2}\,\frac{1}{I^2 W}\, r^3$$

Taking w to be the width at half height of E_r as measured on the resistance network gives, for the lens shown in figure 5

$$W \simeq \frac{D}{2}$$

where D is the diameter of the outer electrodes. Thus

$$\Delta y_D = \frac{2}{I^2 D}\, r^3$$

This can be corrected, as in the case of the use of a one gauze lens to correct a lens of the normal type, by causing the field at the gauzes to be

$$E_x = E_{x0}\left(1 - \frac{r^2}{r_0^2}\right)$$

This gives an additional deflection

$$\Delta y_{r_0} = \frac{-2}{I\, r_0^2}\, r^3$$

Thus by requiring

$$\Delta y = \Delta y_D + \Delta y_{r_0}$$

$$= \left(\frac{2}{I^2 D} - \frac{2}{I r_o^2} \right) r^3 = 0$$

which gives

$$\frac{1}{r_o^2} = \frac{1}{ID}$$

this error can be made zero. The deviation from uniformity introduced in the field,

$$\pm \frac{\Delta E_x}{E_{xo}} = \pm \frac{E_{xo} - E_x(h)}{2\, E_{xo}} = \pm \frac{h^2}{2\, r_o^2}$$

$$= \pm \frac{h^2}{2ID}$$

gives a deflection, due to the lens action thus given to the hole lenses, of maximum range

$$\Delta y_\Delta' = \pm \frac{\Delta E}{2 U_1} \frac{\Delta}{2} = \pm \frac{\Delta E}{E_{xo}} \frac{E_{xo}}{4 U_1} \Delta$$

$$\simeq \pm \frac{\Delta E}{E} \frac{\Delta}{I} = \pm \frac{h^2}{2ID} \frac{\Delta}{I}$$

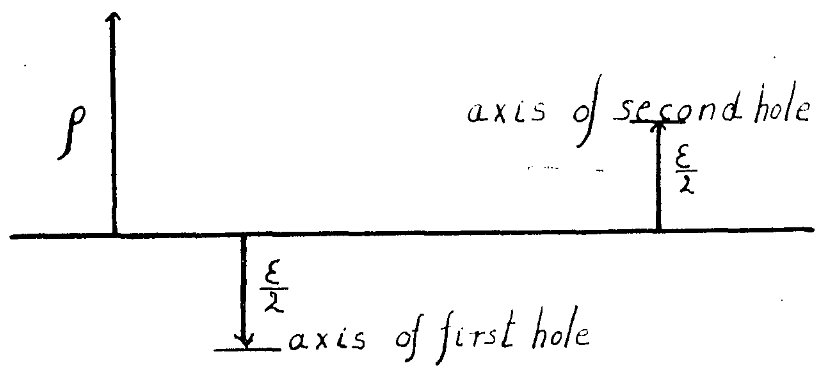
This gives an error in the focus of diameter

$$\delta_\Delta = 2I \left| \Delta y_\Delta' \right| = \frac{h^2}{ID} \Delta$$

This error arises, because, for the principal lens action to be spherical aberration free, the field at the gauzes must be slightly non-uniform, which gives a gauze hole lens action which cannot be made zero over the entire aperture by adjusting the uniform field between the two parallel gauzes. However, by shaping the gauzes, the field between them can be given the appropriate value over the entire aperture.

Alignment Error

If the pairs of holes in the two gauzes are not aligned this will give an error in the focus. A parallel displacement of one of the gauzes will, to the first order in the displacement, give only a displacement of the focus. If the misalignment is rotational, however, the direction of the displacement of the focus will vary around the lens and will thus give an error in the size of the focal spot. Suppose, that the holes are misaligned by a small angle ϵ and consider a pair of holes at a radius r from the lens axis:



The linear displacement is then

$$\varepsilon = r\epsilon$$

The deviation in the deflection in the plane of the displacement ε is

$$\Delta \gamma_{\epsilon} \simeq \frac{\rho + \frac{\varepsilon}{2}}{f_{\Delta 1}} + \frac{\rho - \frac{\varepsilon}{2}}{f_{\Delta 2}}$$

$$= \rho\left(\frac{1}{f_{\Delta 1}} + \frac{1}{f_{\Delta 2}}\right) + \frac{\varepsilon}{2}\left(\frac{1}{f_{\Delta 1}} - \frac{1}{f_{\Delta 2}}\right)$$

$$= \frac{\varepsilon}{f_{\Delta 1}}$$

as

$$\frac{\rho}{f_{\Delta 1}} + \frac{\rho}{f_{\Delta 2}} \simeq 0$$

As

$$\frac{1}{f_{\Delta 1}} \simeq \frac{E_3}{4U_1} = \frac{U_2 - U_1}{4dU_1} = \frac{1}{4d}\left(\frac{U_2}{U_1} - 1\right)$$

$$= \frac{1}{4d}\sqrt{\frac{32 g}{3}} \simeq \sqrt{\frac{2}{3}\,\frac{1}{Id}}$$

the deviation is

$$\Delta \gamma_\epsilon = \sqrt{\frac{2}{3}\,\frac{1}{Id}}\;\varepsilon$$

The diameter of the aberration figure is

$$\delta_\epsilon = 2\,I\,|\Delta\gamma| = 2\,I\sqrt{\frac{2}{3}\,\frac{1}{Id}}\;\varepsilon_{max}$$

$$= \sqrt{\frac{8}{3}\,\frac{I}{d}}\;h_\epsilon$$

Summary and Numerical Example

The use of more than one gauze to form the central element of a spherical aberration free electrostatic lens gives a reduction in the error due to the gauze holes. By adjusting the field between the gauzes the error due to the lens action of the pairs of gauze holes can be minimized. The appropriate potentials satisfy

$$\frac{U_2}{U_1} = 1 + \sqrt{\frac{32}{3} g}$$

where

$$g = \frac{d}{I} \frac{1}{1 + \frac{4\ell}{I}}$$

The potentials which give the required principal lens action satisfy

$$\frac{U_{10}}{U_1} = \frac{U_{20}}{U_2} = \frac{1}{1 + \frac{4\ell}{I}}$$

The lens is considered at unit magnification. The image distance is I, the radius of the aperture is h and Δ is the diameter of the gauze holes. D, T, d, ℓ and $1/r_o^2$ are parameters of the lens geometry. See figure 5

The fields at the gauzes are given by

$$E_1 = \frac{U_1 - U_{10}}{\ell}\left(1 - \frac{r^2}{r_o^2}\right)$$

and

$$E_2 = \frac{U_2 - U_{20}}{\ell}\left(1 - \frac{r^2}{r_o^2}\right)$$

For the spherical aberration of the lens to be zero it is required that

$$\frac{1}{r_o^2} = \frac{1}{\ell D}$$

The parameters ℓ and $1/r_o^2$ are functions of the lens geometry. Varying T and keeping the rest of the lens geometry fixed, measurements on the resistance network give the following values:

T/D	ℓ/D	D^2/r_o^2
0.62	0.78	0.0
0.69	0.86	0.1
0.75	0.94	0.2

The errors of the two gauze lens considered are: The error due to the spherical aberration of the two-hole lenses,

$$\delta_{s\Delta} = \frac{1}{9}\left(\frac{\Delta}{d}\right)\Delta$$

the residual gauze hole error due to the non-uniformity of the field at the gauzes,

$$\delta_{\Delta} = \frac{h^2}{\ell D}\,\Delta$$

and the error due to a rotational misalignment of the pairs of gauze holes,

$$\delta_{\epsilon} = \sqrt{\frac{8}{3}\,\frac{\ell}{d}}\; h\epsilon$$

Take as a numerical example a lens with the following values of the parameters:

$I = 100$ mm

$h = 4$ mm

$D = 10$ mm

$d = 5$ mm

$\Delta = 200$ micron

The requirement

$$\frac{1}{r_0^2} = \frac{1}{ID}$$

is satisfied if

$$T = 6{,}9 \text{ mm}$$

which gives

$$\ell = 8.6 \text{ mm}$$

Then the potential ratios are

$$\frac{U_{10}}{U_1} = \frac{U_{20}}{U_2} = \frac{1}{1 + \frac{4\ell}{I}} = 0{,}74$$

and

$$\frac{U_2}{U_1} = 1 + \sqrt{\frac{32}{3}\,\frac{d}{I}\,\frac{1}{1+\frac{4\ell}{I}}} = 1.6$$

The errors in the focus are

$$\delta_{J\Delta} = \frac{1}{9}\left(\frac{\Delta}{d}\right)\Delta \simeq 0.9\ \mu$$

$$\delta_{\Delta} = \frac{h^2}{ID} \simeq 3\mu$$

$$\delta_\epsilon = \sqrt{\frac{8}{3}\,\frac{I}{d}}\, h_\epsilon \simeq 7\, h_\epsilon$$

If δ_ϵ is to be less than $\delta_{s\Delta}$, h_ϵ must be less than 0,1 micron which is a very severe requirement. If this can be achieved then it becomes worth while to reduce δ_Δ by shaping the gauze. It should be possible to keep h_ϵ to within about 0,5 micron which would give a total error of about 5 micron.

A single gauze lens would require gauze holes of about 3 micron to achieve this. A normal lens of similar dimensions would have a spherical aberration error of about 100 micron.

LIST OF REFERENCES

Archard, G.D. (1960) Brit. J. Appl. Phys. 11, 521

Ments, M. v. and J.B. Le Poole (1947) Appl. Sci. Res. B1, 3

Rus, P.J. (1965) "Correctie van Sferische Aberratie met behulp van een Gaaslens" Afstudeerverslag - Technische Hogeschool Delft.

Scherzer, O. (1936) Z. Physik 101, 593

Scherzer, O. (1947) Optik 2, 114

Scherzer, O. (1949) J. Appl. Phys. 20, 20

Scherzer, O. and K.H. Steigerwald (1959) DBR patent nr. 1134769

Verster, J.L. (1963) "On the Use of Gauzes in Electron Optics" Proefschrift - Technische Hogeschool Delft, Philips Res. Repts. 18, 465.

Zworykin, V.L., G.A. Morton, E.G. Ramberg, J. Hillier and A.W. Vance (1945) "Electron Optics and the Electron Microscope", Wiley, New York.