FAIRBANK, Jr., Benjamin Ayer, 1942-
EXPERIMENTS ON THE TEMPORAL ASPECTS OF NUMBER PERCEPTION.

University of Arizona, Ph.D., 1969
Psychology, experimental

EXPERIMENTS ON THE TEMPORAL ASPECTS OF NUMBER PERCEPTION

by

Benjamin Ayer Fairbank, Jr.

A Dissertation Submitted to the Faculty of the

DEPARTMENT OF PSYCHOLOGY

In Partial Fulfillment of the Requirements
For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

THE UNIVERSITY OF ARIZONA

1 9 6 9

THE UNIVERSITY OF ARIZONA

GRADUATE COLLEGE

I hereby recommend that this dissertation prepared under my direction by Benjamin Ayer Fairbank, Jr. entitled Experiments on the Temporal Aspects of Number Perception be accepted as fulfilling the dissertation requirement of the degree of Doctor of Philosophy

Neil R. Bartlett — Dissertation Director

11/1/68 — Date

After inspection of the final copy of the dissertation, the following members of the Final Examination Committee concur in its approval and recommend its acceptance:*

Clinton L. Trafton	Nov 1 68
Neil R. Bartlett	11/1/68
Wm. L. McCauley	Nov. 1, 1968
Jack E. Capehart	Oct 31 1968
Mac E. Hadley	Oct. 31, 1969

*This approval and acceptance is contingent on the candidate's adequate performance and defense of this dissertation at the final oral examination. The inclusion of this sheet bound into the library copy of the dissertation is evidence of satisfactory performance at the final examination.

STATEMENT BY AUTHOR

This dissertation has been submitted in partial fulfillment of requirements for an advanced degree at The University of Arizona and is deposited in the University Library to be made available to borrowers under rules of the Library.

Brief quotations from this dissertation are allowable without special permission, provided that accurate acknowledgment of source is made. Requests for permission for extended quotation from or reproduction of this manuscript in whole or in part may be granted by the head of the major department or the Dean of the Graduate College when in his judgment the proposed use of the material is in the interests of scholarship. In all other instances, however, permission must be obtained from the author.

SIGNED: Benjamin Ayer Fairbank, Jr.

PREFACE

This dissertation, having been prepared primarily for the Department of Psychology, contains material written in accordance with the stylistic conventions peculiar to the field of Psychology. For readers not familiar with those conventions the following notes may be useful.

The single letter S designates any one of the subjects who takes part in an experiment. The experimenter is designated by the letter E. Thus the sentence "E instructed Ss to react as quickly as possible," is read "(The) experimenter instructed (the) subjects to react as quickly as possible."

The statistical procedures used in this paper are described fully in most intermediate texts on general statistics. The reader unfamiliar with statistical methodology may refer to such books for further details of the methods of data analysis.

The unit of time measurement employed in six of the experiments reported here is the millisecond, abbreviated ms. A millisecond is one-thousandth of a second; electronic measuring apparatus has made measurement of durations to the nearest millisecond both accurate and convenient.

ACKNOWLEDGMENTS

Professor Neil R. Bartlett, the author's Dissertation Director, has, at every step of this project, provided encouragement, stimulation, and insight. For four years he has been the author's research director and throughout those years his invariably constructive and patient suggestions have been of the greatest help to the author. Professor Jack E. Capehart, the author's advisor, has provided helpful comments and suggestions in many areas, but particularly in those of statistical methodology, research design, and adaptation level theory. Professor Clinton L. Trafton, who replaced Dr. Robert W. Lansing as a committee member when the latter left for a sabbatical year, has been especially helpful in editing early drafts of this dissertation. His perceptive comments have helped to eliminate ambiguities and to clarify some of the more complex sections, most notably those on model construction.

Professors William J. McCauley, Peter E. Pickens, and Mac E. Hadley of the Zoology Department have, as members of the Minor Committee, all given their time generously to help the author complete his program.

Miss Dianne M. Timbers of the Department of Sociology, through her aid in computer programming and in other areas, has helped to make this dissertation a reality.

In addition, numerous individuals from several university departments have assisted the author over the last four years. Among those whose help is very much appreciated are Mrs. Rufulyn Andrew, Mr. Louis Pfautsch, Mr. Frank Bonneville, Mrs. Joanna McComb, and the personnel of the University of Arizona Computer Center.

While carrying out most of the research reported on here, the author was supported by a National Aeronautics and Space Administration Predoctoral Traineeship. Much of the equipment used was purchased with funds made available by the National Science Foundation through Grant GB 3955 to The University of Arizona.

Finally, the author expresses with love his gratitude to his parents, Mr. and Mrs. Benjamin A. Fairbank of Cambridge, Massachusetts, who have spent twenty-seven years helping to make this dissertation possible.

TABLE OF CONTENTS

TABLE OF CONTENTS--Continued

TABLE OF CONTENTS--Continued

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS--Continued

LIST OF ILLUSTRATIONS--Continued

LIST OF TABLES

ABSTRACT

In 1967 Moyer and Landauer reported that the time required to select the larger of two digits varies negatively with the difference between the digits. This phenomenon is investigated further. In a series of experiments several hypothetical explanations of this effect are proposed and examined. Moyer and Landauer's suggested explanation (numbers may be processed in a way analogous to the processing of stimuli differing in physical magnitude) is generally supported, although it is modified in several details.

A similar phenomenon is found when one substitutes either months of the year or letters of the alphabet for single digit numbers, suggesting a generalized phenomenon peculiar to serial lists. In a related experiment, however, the process of judging letter pairs is shown to be different in at least one major respect from the process of judging number pairs.

It is also found that the larger of two digits is more quickly selected than the smaller. The relevance of adaptation level theory to this finding is discussed. In an experiment designed to further test the relevance of adaptation level theory to numerical-perceptual phenomena, evidence is found suggesting that human subjects may

process numbers in a way which involves a power function transformation of the given stimulus number.

An earlier study which resulted in a report that certain digits are read aloud more quickly than other digits is not found replicable. Two other experiments also fail to find differences in the speeds with which the various digits are perceived.

In arriving at an equation for calculating the time required to select the larger of two digits the following factors must be considered (in approximate order of decreasing importance): the difference between the digits, the size of the smaller digit, the size of the larger digit, the fractional exponent, if any, to which a number is effectively raised when it is processed, and the irreducible minimum decision time.

INTRODUCTION

In 1967 Moyer and Landauer first reported an effect not previously described in the psychological literature. Briefly, they found that when one has to choose the larger (in numerical value) of two digits the time required for the decision varies negatively with the magnitude of the difference between the digits. This effect will be referred to as the Moyer-Landauer effect, or, more simply, the MLE. Figure 1 is a reproduction of Moyer and Landauer's original illustration of the effect.

As Moyer and Landauer point out, there are at least three somewhat different processes by which one could make a decision regarding which of two digits is the larger. First, one could have memorized the larger digit of all possible digit pairs and the decision process would then consist simply of identifying the given pair and selecting the larger digit from memory. Or, second, one might employ some kind of counting procedure, counting either from one digit to the other, or to one of the digits from one or nine. This counting could presumably go on at an unconscious level, so the subject would not be aware of any such process. Finally, there is the possibility that each of the digits is represented internally in a form which is in some way analogous to its magnitude. The decision process

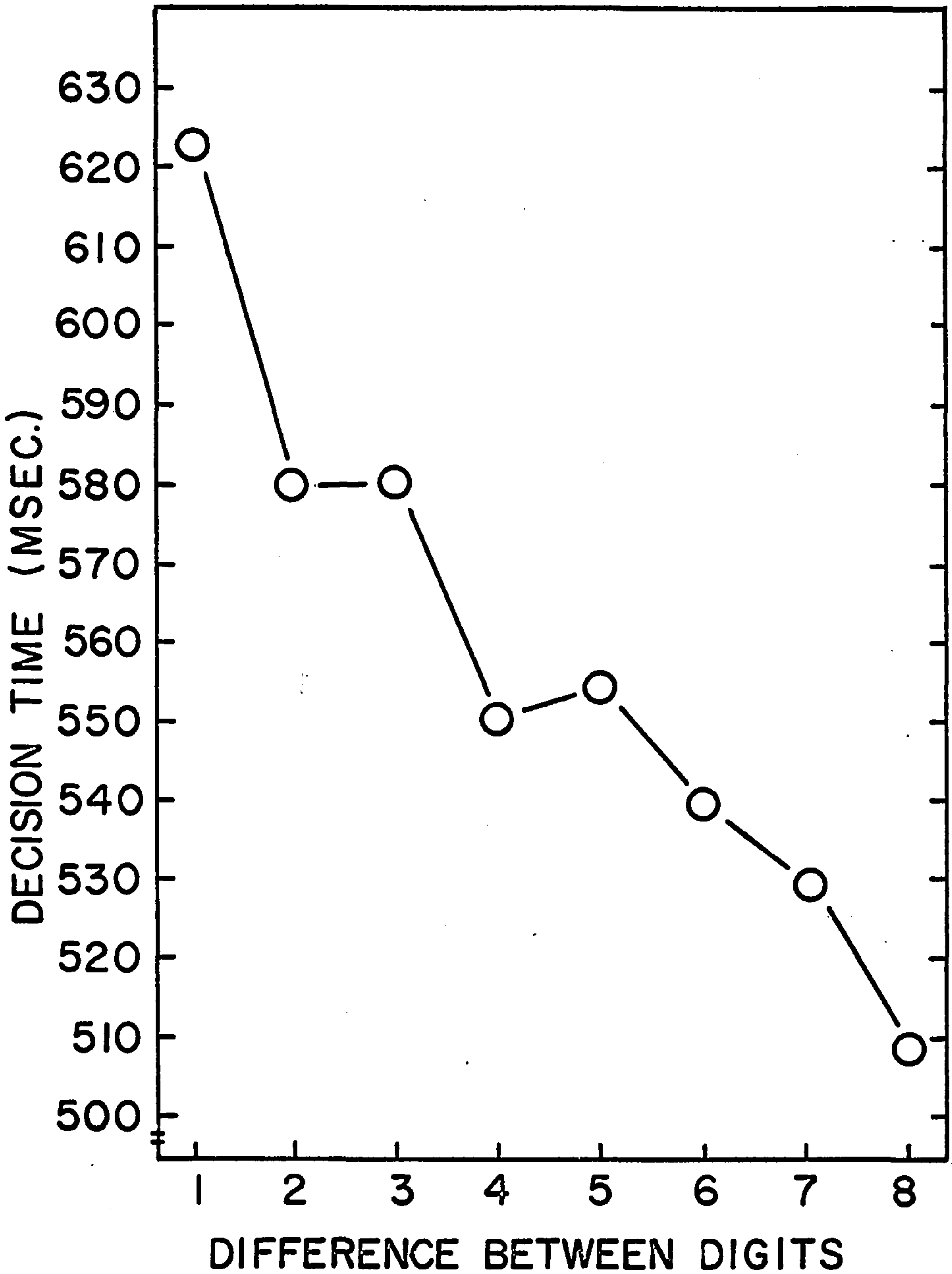

Fig. 1. Decision time as a function of the difference between digits as found by Moyer and Landauer (1967).

for identifying the larger digit would consist of comparing the internal magnitudes and selecting the larger.

As Moyer and Landauer point out, fortunately each of these three basic processes leads to one or more specific predictions regarding decision times.

If the decision process is based upon retrieval of the larger digit from memory then there is no reason to expect that the MLE would be observed. All digit pairs should be associated with substantially the same decision times, assuming that all pairs have been equally well learned during the course of a person's experience with numbers.

Various counting models yield at least two quite different predictions concerning decision times. The fundamental technique involved in deriving predictions from such models entails making up a table of all possible digit pairs, and determining for each pair exactly how many "counting steps" would be required to select the larger digit, given the constraints specified in the model being considered. By then grouping together all pairs which have the same difference between the larger and the smaller digits (e.g., 4-1, 7-4, 2-5, etc., being those pairs with a difference of three) one can find the mean number of counting steps involved in selecting the larger of two digits with a given difference. By doing this for all pairs of digits after the pairs have been grouped according to "difference," one can construct a table or a graph showing expected decision time in counting steps as a function of difference between digits. Decision time is assumed to be proportional to the number of such counting steps required in judging a pair. Moreover, all counting steps are presumed to be of equal duration.

Such predictions have been made for four counting type models and are illustrated below. The four counting models discussed here will be referred to throughout this paper as counting-model one, counting-model two, etc., lest they be confused with other models discussed.

Assume initially that for all counting models the subjects know in advance the range of digits to be included in the pairs, and although the assumption is not essential for generalized counting models, assume additionally that in the cases under discussion here the range of digits included in the pairs is from one to nine.

For counting-model one assume that the subject (S), when presented with two digits, selects one of the digits at random, and then counts up from the selected digit until either he reaches the other digit or he reaches nine. If he reaches the other digit first, then the other digit is larger, while if he reaches nine then the digit he initially selected is the larger. The decision times predicted by such a model are shown as a function of numerical difference (larger digit minus smaller digit) in Figure 2. In

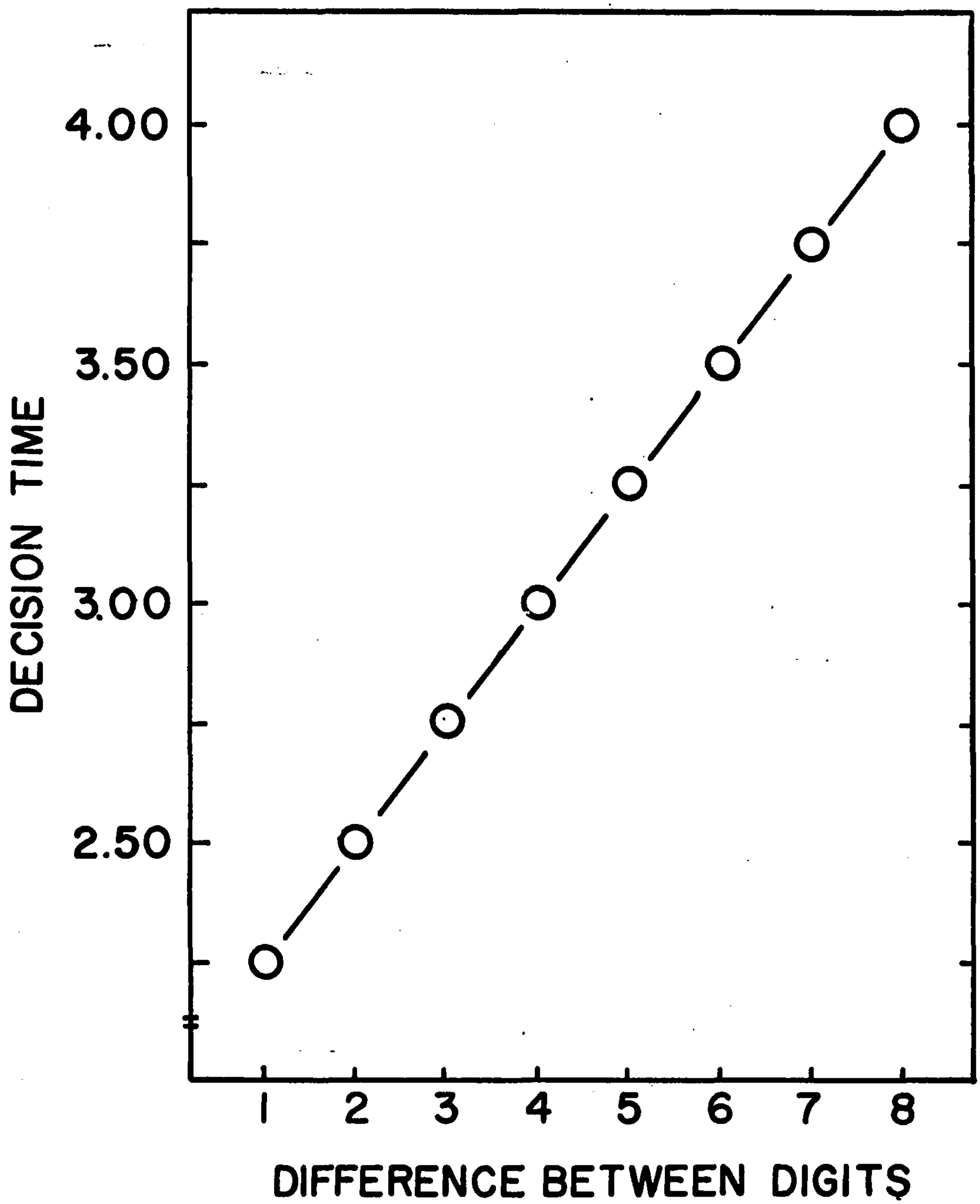

Fig. 2. Decision time in arbitrary "counting steps" as a function of the difference between digits as predicted by counting-model one.

Figures 2 through 5 the units on the ordinate are counting steps.

Counting-model two is similar to the first in that a subject chooses a digit at random, but in this case he counts down and if he reaches the other digit then the digit with which he started is assumed to be the larger. If he reaches one, the digit which he did not select is assumed larger. The predictions arising from this model are shown in Figure 3.

Since neither Figure 2 nor 3 is similar in shape to Figure 1, we may conclude that neither counting-model one nor counting-model two is adequate to describe the data.

A model somewhat similar to those described, counting-model three, is one in which S is assumed to start counting at one, and continues counting until he reaches one of the numbers; then he assumes that the other number is the larger. The predictions of this model are shown in Figure 4. A parallel model (counting-model four) exists in which one counts down from nine until one of the numbers is reached and then that number is assumed to be the larger. Figure 5 shows the predictions of such a model.

Figures 4 and 5 bear general resemblance to Figure 1, thus suggesting that either counting-model three or four might be a suitable description of the way in which a S arrives at a decision. This resemblance alone does not establish the validity of either counting model; other

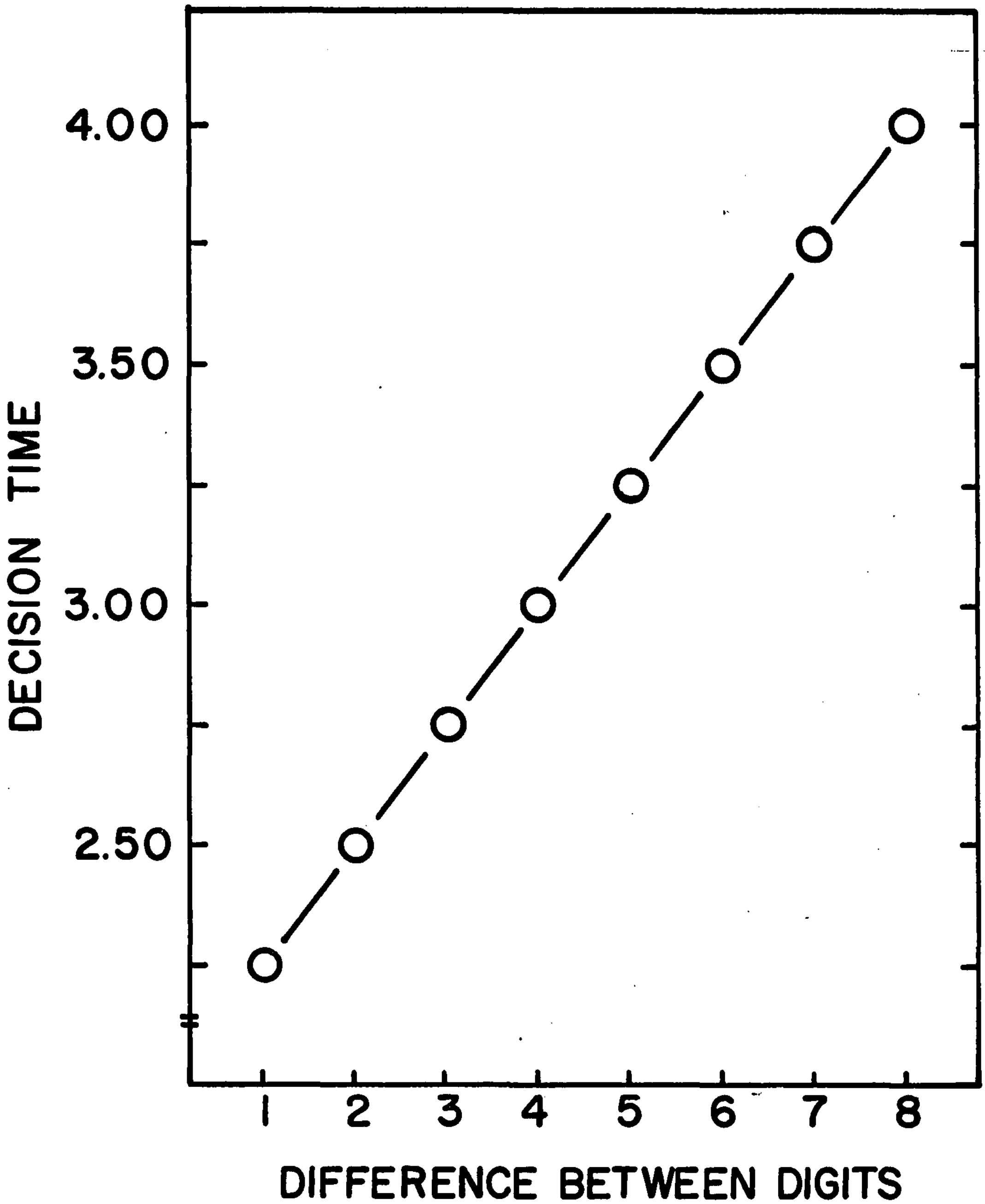

Fig. 3. Decision time in arbitrary "counting steps" as a function of the difference between digits as predicted by counting-model two.

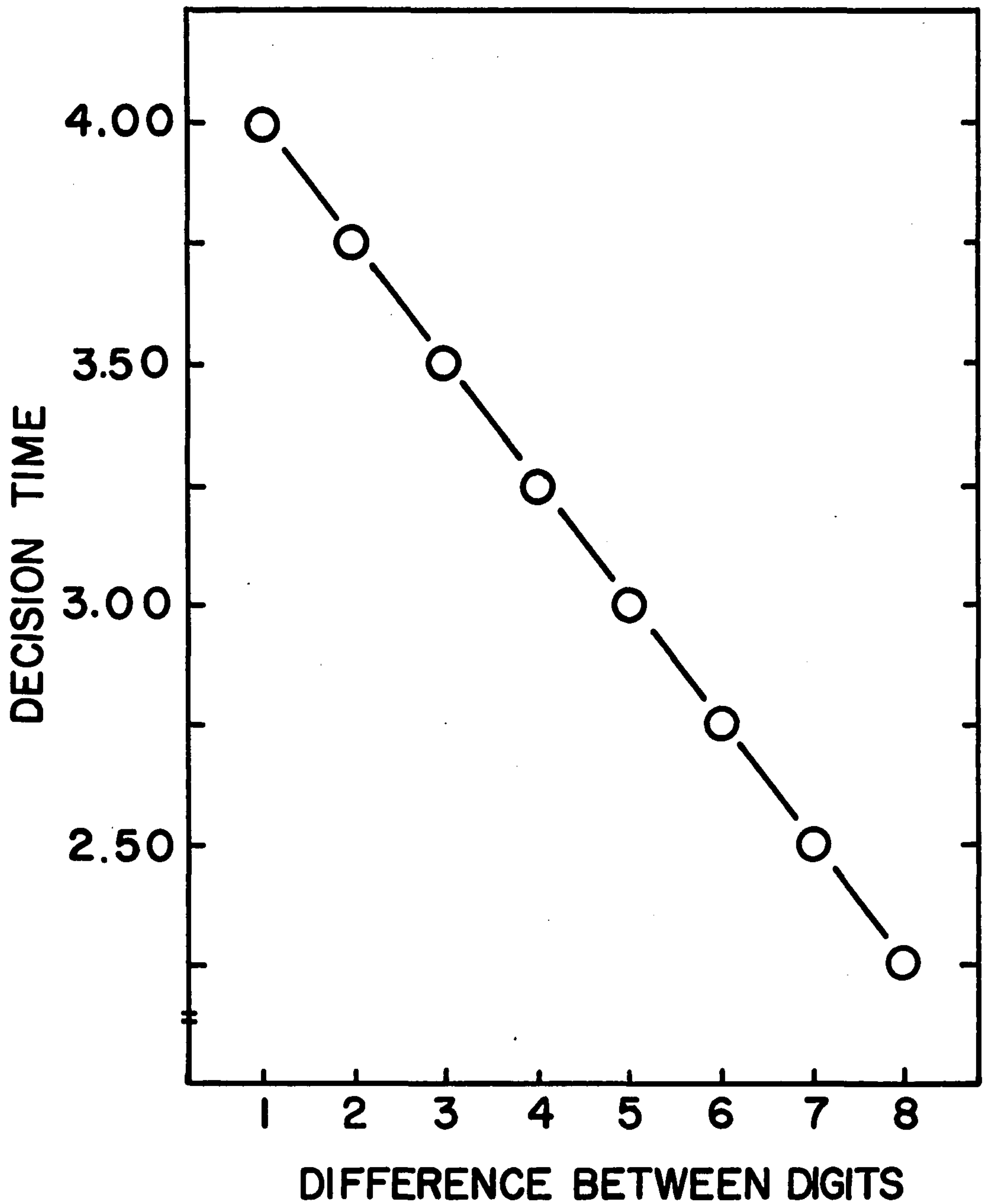

Fig. 4. Decision time in arbitrary "counting steps" as a function of the difference between digits as predicted by counting-model three.

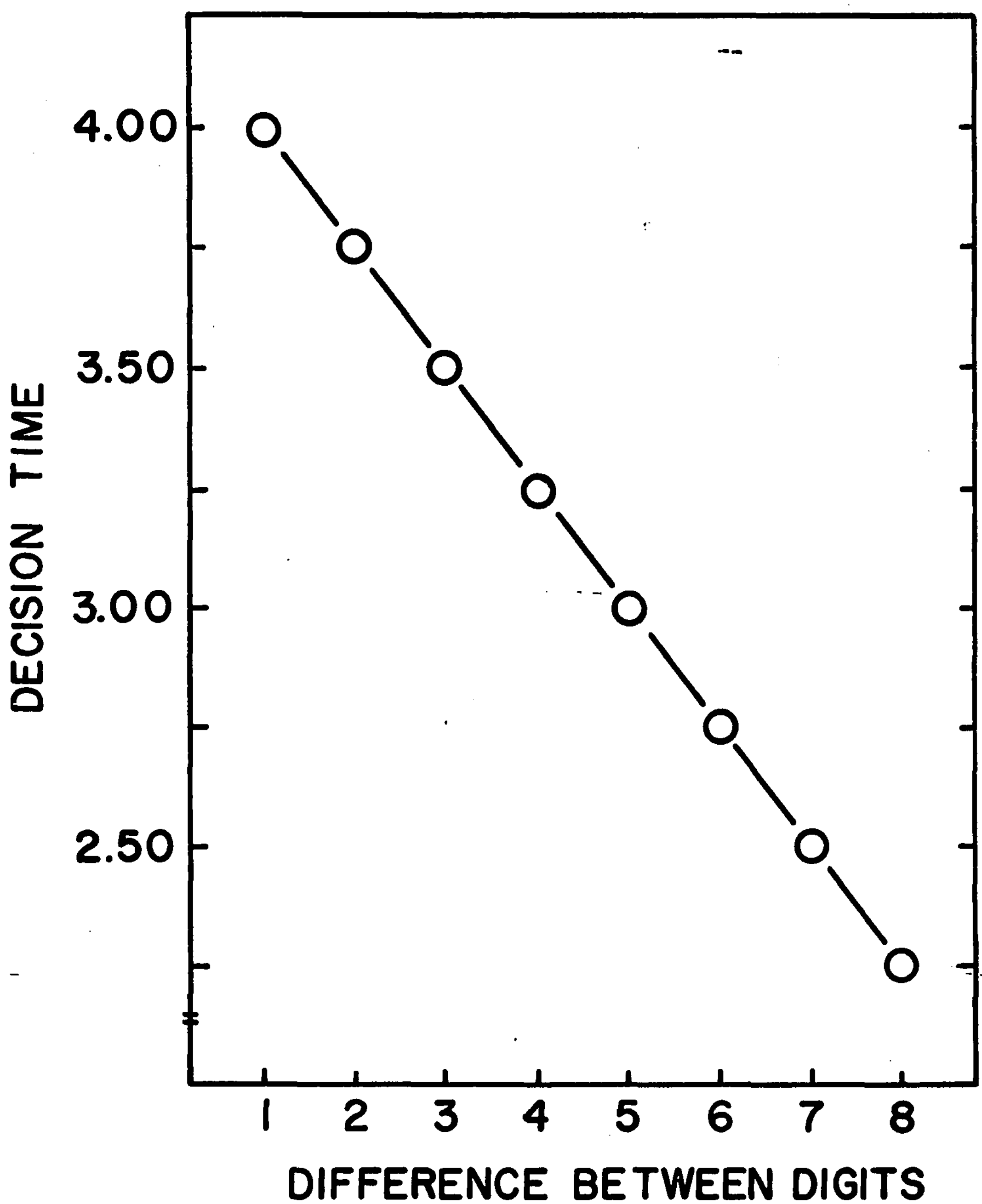

Fig. 5. Decision time in arbitrary "counting steps" as a function of the difference between digits as predicted by counting-model four.

predictions, also derived from these models, should be tested. If counting-model three is tenable, then all pairs of digits containing the digit one should have the same (and the minimum) decision time. If counting-model four is appropriate, then all pairs of digits containing the digit nine should have the same (and again the minimum) decision time. More completely, the decision times for all pairs under counting-model three should vary directly with the magnitude of the smaller digit, while if counting-model four is correct the decision times should vary negatively with the size of the larger digit. These predictions are very easy to test, given the proper data, but Moyer and Landauer (1967) did not publish the required data, so a replication of their study is necessary.

Before turning to Moyer and Landauer's third theoretical explanation of the MLE, consideration of a recent article by North, Grant, and Fleming (1967) is in order. They reported that when a S is reading numbers aloud from a display panel, and the interval between stimulus onset and response initiation is measured by a timer stopped by a voice-activated relay (voice key), the digits 1, 2, 6, and 9 are responded to more quickly than are the digits 3, 4, 5, 7, and 8. North et al. did not attempt an explanation of their finding, but noted that it was significant at the 0.01 level or better. The same effect was observed with numbers written out as words, although it was not significant at as high a level. Thus apparently more than strictly peripheral processes were functioning (i.e., the physical configuration of the stimulus is apparently not a determining variable). Since "6" and "7" are reported read with different latencies, the initial consonant is not the only relevant variable.

Making some assumptions prompted by this finding, a model can be constructed which again shows expected decision time as a function of the difference between digits. First, assume that the differences shown by North et al. are differences involved in the perception of the digits, rather than ones associated with the articulation of the names of the numerals or with the characteristics of the voice key. Then assume that the larger digits of all number pairs are stored in memory and that once the numbers have been perceived by S̲ the decision times are equally long. This is essentially the retrieval-from-memory model rejected by Moyer and Landauer (1967), but they implicitly assumed that all digits were perceived equally quickly. If, instead, it is assumed that 1, 2, 6, and 9 are perceived more quickly than the other digits, then the model leads to predictions quite different from those of equal decision times for all digit pairs. The model requires a number of further assumptions. Assume first that a digit pair which contains no "fast digit" (1, 2, 6, or 9) will be reacted to (i.e., the larger digit selected) more slowly than one which contains one "fast digit," and a pair which contains two "fast digits" will be reacted to more quickly than one which contains only one "fast digit." Further, assume that decision times associated with all pairs of digits with an equal difference between digits is a linear function of the mean number of "fast digits" per pair. Thus for example, consider the pairs with a difference between the digits of seven, i.e., the pairs 2-9, 9-2, 1-8, and 8-1. These four pairs have six out of eight "fast digits," thus in constructing the model a value of 1.5 "fast digits" per pair is assigned to those pairs with a numerical difference of seven.

Assuming that the decision time is a linearly decreasing function of the number of "fast digits" in a pair, a graph of expected decision time (or number of "fast digits") as a function of the difference between the digits can be prepared. Figure 6 shows the predicted relation. This graph is so similar to Figure 1 that the model warrants further consideration. The findings of North et al. (1967) must first be replicated. Further predictions based on their findings can also be tested. For example, the pairs 1-2, 1-6, 1-9, 2-6, 2-9, and 6-9 (and the reverse pairs) all contain two "fast digits," and they are the only pairs which do. Thus if the model derived from North et al. is tenable, then there should be no significant difference between the decision times to all of these pairs. Moreover,

Fig. 6. Mean number of "fast digits" per pair as a function of difference between digits.

According to the model, speed of decision varies directly with the number of "fast digits," hence the ordinate also represents predicted decision time, although appropriate units are not indicated.

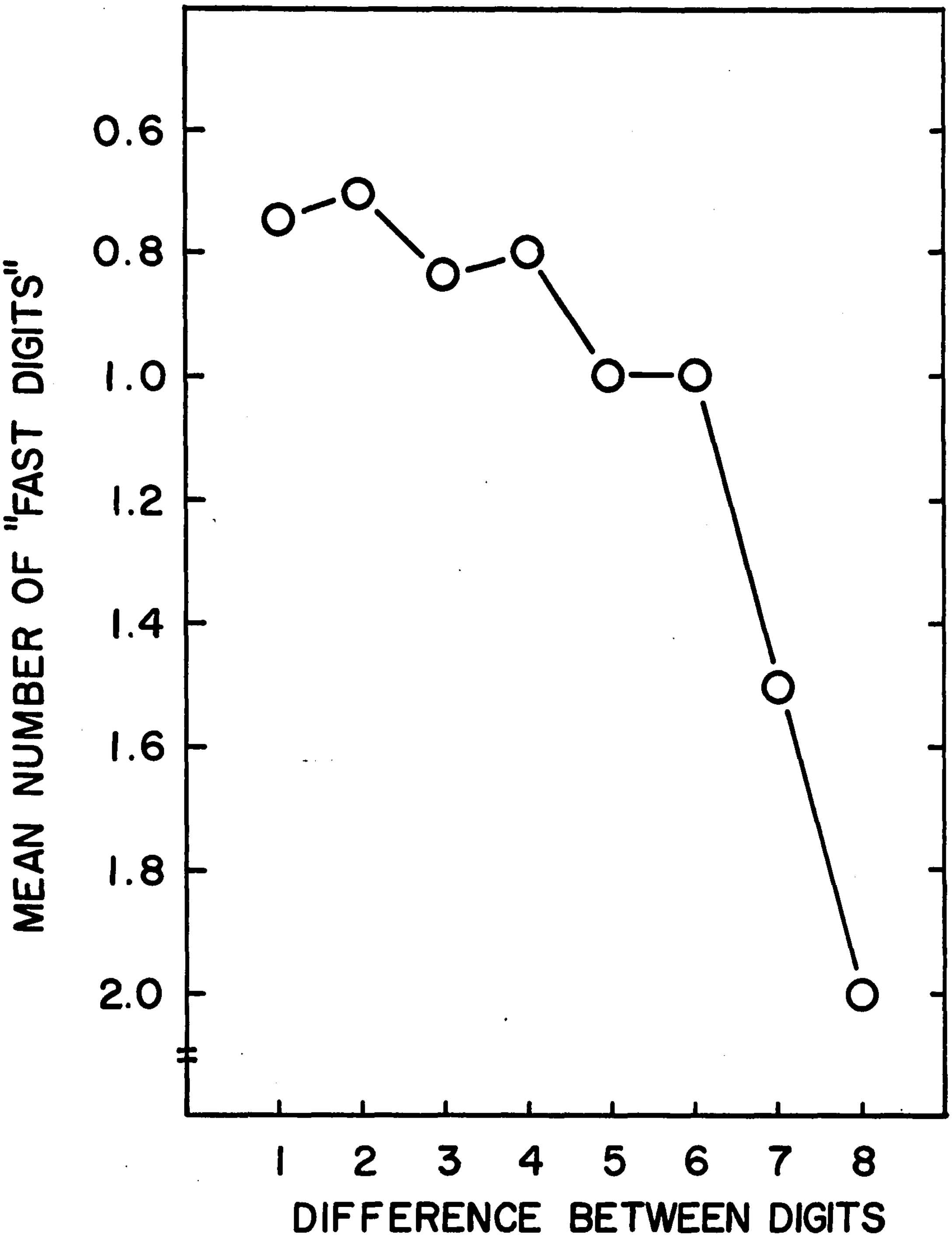

Fig. 6. Mean number of "fast digits" per pair as a function of difference between digits.

none of the other pairs should be significantly faster than any of these pairs. This is probably the most discriminating test of the model. Moyer and Landauer (1967), unfortunately, do not present their data in a way which makes such a test possible.

Moyer and Landauer's third postulated explanation for the MLE (after memory retrieval and counting procedures), the one upon which they settled as being most probably correct, is backed by a rather long history of psychophysical investigation. The phenomenon underlying the explanation is that when a subject is choosing one of two non-numerical stimuli on the basis of a difference on a single stimulus continuum, then the time required for him to make the decision is greater when the stimuli are closer together on the continuum than it is when they are well separated. Moyer and Landauer's theoretical explanation entails assuming that the central nervous system processes numbers in a way that is very similar to the way in which it processes physical magnitudes.

In 1906 Henmon described (as part of a test for color blindness) a choice reaction time effect similar to the one described above. Her subjects were required to sort a number of cards on the basis of color, and she found that the color-blind subjects were slower than the normal subjects, even if just as accurate. In 1927 a paper by Lemmon described an experiment somewhat similar to that of

Moyer and Landauer, except that printed numbers were not used; the Ss had to subitize (detect without actually counting) the number of stimulus lights presented in a stimulus display. Moreover, there was a constant difference of one between the two stimuli, i.e., there were one and two, two and three, three and four, and four and five stimulus lights presented in pairs, but never one and three, one and four, etc. S had to select the stimulus display which had the larger number of lights on it. This experiment is relevant to the MLE only if one is interested in seeing if the decision times are constant for a difference of one or if the decision times in the MLE change as the magnitude of the numbers involved changes. Again the need arises for data which are not available in the Moyer and Landauer paper. Lemmon found that the greater the number of lights in a given stimulus display, the greater the decision time, given a constant difference (between the left and right displays) of a single stimulus light.

Woodworth and Schlosberg (1954) devote a paragraph to the discussion of the disjunctive reaction time (RT), and they also discuss the experiments of Henmon (1906) and those of Lemmon (1927). They summarize the findings in the field by saying "The more alike the stimuli, the longer the disjunctive RT." They support this assertion with data from experiments on lengths of lines, frequency of tones, and colors differing in hue.

Festinger (1943) in a publication based on his doctoral dissertation reported that when Ss are judging lengths of lines in a situation in which only two are presented, the judgments of which the Ss are least sure take the greatest amount of time. This is similar to a finding reported in Woodworth and Schlosberg (1954).

A. T. Welford (1960) of the Psychological Laboratory in Cambridge (England) has devoted much study to information processing mechanisms in human observers and in a 1960 paper reviews a number of studies bearing on the MLE. After reviewing several studies which all generally bear out the generalization of Woodworth and Schlosberg (1954), Welford synthesizes several findings to develop a theory applicable to choice reaction time data.

The relevance of such studies to the MLE is of course questionable. The very phenomenon under discussion does not seem to be a judgmental one in the sense of judging which of two lines is the longer. Instead there is involved a process of a more symbolic nature. An example will perhaps make this difference clear. If one is judging the relative lengths of lines, then after the judgment one might still be a bit uncertain of one's accuracy. With numbers, this would not be the case. One does not "judge" four to be greater than two, one simply "knows" that this is the case. There is no uncertainty. Nevertheless, it is the very real nature of the MLE which forces upon us the relevance of the disjunctive or choice reaction time studies. The adequacy or inadequacy of alternative models as examined below should determine whether or not it is necessary to assign the judgment of numerical inequalities to the same class of judgments as those of physical inequalities.

If an effect such as the MLE is present for numbers, then one might look for it with other stimuli, such as letters of the alphabet, months of the year, or any other ordered list. If results are obtained which are parallel to those of Moyer and Landauer (1967), then this evidence would be suggestive of there being a judgmental process common to a wide variety of inequality-type situations.

There is, of course, the chance that the MLE is an artifact of the methodology by which it was first detected. Moyer and Landauer in their study used typewritten cards displayed behind a half-silvered mirror. A light would illuminate the cards making them suddenly visible from the front side of the mirror where the S observed. S would then respond with either his right or his left hand according to the side on which the larger digit appeared.

By careful examination of the data from a number of replications taken on the same Ss it should be possible to determine whether or not the effect decreases with time and practice. There is some indication that it would do so; Mowbray and Rhoades (1959) found that choice reaction times initially vary with the number of stimuli and alternatives, but with increased practice a S is able to handle the incoming information more economically and the choice reaction time becomes a constant, independent of the number of alternatives. The experiment used to elucidate this point was not exactly parallel to the Moyer and Landauer situation, but was close enough so that an investigation would be worthwhile.

In a current article on choice reaction time E. E. Smith (1968) makes the distinction between experiments in which there is a one-to-one mapping between stimuli and responses, and experiments in which the mapping changes from trial to trial. Unfortunately his review does not consider the latter class of experiments, hence the applicability of his review to the MLE is limited. But it does suggest a further experiment of the MLE. A typical MLE experiment contains all possible pairs of digits, and S indicates the side on which the larger digit has appeared. Thus the mapping between stimulus and response is not one-to-one for two reasons. First, the response to seven of the digits, two through eight, may be either as a larger or as a smaller digit, and second, even with the digits one and nine the response may be right or left, depending on which side of the pair the digit appears on. It would be worthwhile to investigate the magnitude of the MLE when there is a one-to-one mapping between stimulus and response.

For example, two stimulus lists might be presented, one consisting of ones and nines, and the other consisting of eights and nines, and if the MLE is present regardless of the one-to-one mapping then there should be a considerable difference in the times for the completion of two lists of equal length. Such an experiment would be ideally suited to an experimental technique such as "crossing out," to be discussed in the methods section of the appropriate experiment.

Smith (1968), in discussing the aspects of the choice reaction time includes a paragraph quite relevant to the MLE:

> The task is structured so that error rates are reasonably low (approximately 1-10 per cent), and no comparison between two simultaneously presented stimuli is required. This is in contrast to perceptual judgment experiments, which frequently measure response latency. Until more is known about the relation between choices made among alternatives represented only in memory (CRT tasks) and choices made among alternatives present in the environment (perceptual judgment tasks), it will be difficult to specify the relation between the processes involved in CRT and perceptual judgment experiments (p. 78).

The MLE is an excellent tool with which to investigate the "relation between choices made among alternatives represented only in memory and choices made among alternatives present in the environment."

In a personal communication one investigator has suggested the relevance of adaptation level theory to the

MLE. If a subject is judging whether a given stimulus is heavy or light, small or large, hot or cold, etc., his judgment time or decision time is longer the closer the presented test stimulus is to the adaptation level. Since the AL is roughly equivalent to the weighted geometric mean of all stimuli, then the heavier stimuli (larger, louder, etc.) are identified more quickly than the lighter ones. (This is because the geometric mean is closer to the smaller items on a continuum.) We may look for the same effect in the MLE by giving subjects the same series of stimuli four times in an ABBA order and having him select the larger digit in two cases and the smaller digit in two. An experiment of the "crossing out" type would be appropriate in this case. If the AL conception is relevant then the decision time to the series when *S* had to pick the larger numeral should be shorter. Since the individual times would not be as interesting as the overall averages for larger choices as compared to smaller choices, the "crossing out" method would seem to be ideally suited to this experiment.

The present series of experiments was designed to test the various alternative hypothetical explanations of the MLE, to investigate related phenomena such as the North et al. (1967) findings, and to extend the findings of Moyer and Landauer (1967), if possible, to stimuli other than numbers. The description of each experiment is preceded by a brief introduction explaining the rationale of the experiment.

EXPERIMENTS

Summary of Experiments

The Experiments section of this dissertation consists of the full descriptions of eleven experiments. In order that the reader may follow more clearly the course of the experiments and better understand their relevance to the MLE, this brief outline of the Experiments section is included.

Experiment One is simply a replication of the work of Moyer and Landauer (1967) using very similar methodology. The following experiment is identical, except that instead of numbers being used as stimuli, letters of the alphabet are used. Experiment Three investigates the nature of the MLE when one uses months of the year instead of numbers as stimuli. In this experiment the methodology is shifted from the electronic timing of individual responses to a paper and pencil technique. Experiment Four is another replication of Moyer and Landauer's work, but in this case an attempt is made to compile a large number of data from highly practiced subjects. In addition, the effects of choosing the smaller instead of the larger digit are investigated. The experiments following this one are all on related but essentially different topics.

Experiment Five is the first of three experiments designed to evaluate the relevance and significance of the North et al. (1967) work. This experiment is an attempt to replicate the work of North et al., and as it is unsuccessful in so doing, the following two experiments are included in order to find out more about the relative latencies with which the several digits are perceived. Experiment Six requires simply that the Ss recognize given digits, while Experiment Seven determines the time required for judgments of equality of digits as a function of the digits used.

Experiment Eight makes possible a comparison of decision times for selecting the larger digit with decision times for selecting the smaller digit. Since this experiment is successful in supplying evidence that numbers might be handled internally in a manner similar to the handling of sensory magnitudes, Experiment Nine investigates that possibility more fully by having Ss estimate but not compute the midpoint between various numbers.

Since the substitution of letters for numbers did not change the essential findings in going from Experiment One to Experiment Two, Experiment Ten is essentially a replication of Experiment Eight employing letters instead of numbers as stimuli. The purpose of this experiment is to determine how far the parallels hold up between phenomena involving numbers and those involving letters.

Finally, Experiment Eleven investigates the effect of a one-to-one mapping between stimulus and response by examining differences between the decision times to the pairs 8-9 and 1-9.

Experiment One: A Replication of the Work of Moyer and Landauer

Introduction

In order to verify the reality of the MLE a replication study was carried out, differing only in details from the original Moyer-Landauer experiment. An additional reason for this replication was the need for data to test the hypothetical explanatory models advanced in the introductory section of this paper. These models are based on counting procedures, and on the implications of the North et al. (1967) paper. The Moyer and Landauer data as published do not permit the tests.

Method

Stimuli were presented on an in-line readout 1.5 meters in front of a seated S; lighted numerals two cm. high and eight cm. apart were presented on a dark background in a dimly lit room. S selected the larger of the two digits and indicated his choice by pressing either the left or right one of two microswitch (BZ2RW80) keys with the forefinger of his right hand. The Ss were instructed to press the key that was on the same side as the larger digit. Because of a metal partition between the keys, it was impossible to press both keys simultaneously with the same finger.

In this and all other experiments Ss were university students, some volunteer, and some paid.

When E had selected the stimulus pair to be presented, he would so indicate by turning on a small "ready light" in S's isolation room. When S felt ready he would press a foot switch which turned out the "ready light" and initiated a two-second foreperiod. At the end of the foreperiod the two numbers came on and stayed on for one second. The time between the onset of the numbers and S's response was measured by a Hewlett-Packard 522B counter. Ten Ss judged all of the 72 non equal digit pairs from 1-2 to 9-8 three times each, making a total of 216 judgments for each S. After each 72 judgments S took a five minute rest. S was instructed to make his decisions as quickly as possible, but to be accurate above all. The order in which the digit pairs were presented to the Ss was random.

Results

The data may be presented in a variety of ways. Figure 7 shows the overall mean decision time as a function of the differences between the presented digits (lowest curve). For comparison, the original Moyer-Landauer data are also shown. (The uppermost curve illustrates the

Fig. 7. Decision time as a function of the difference between digits or between letters.

Upper curve presents results from Experiment Two, the middle curve presents Moyer and Landauer's (1967) data, and the lower curve presents the results of Experiment One.

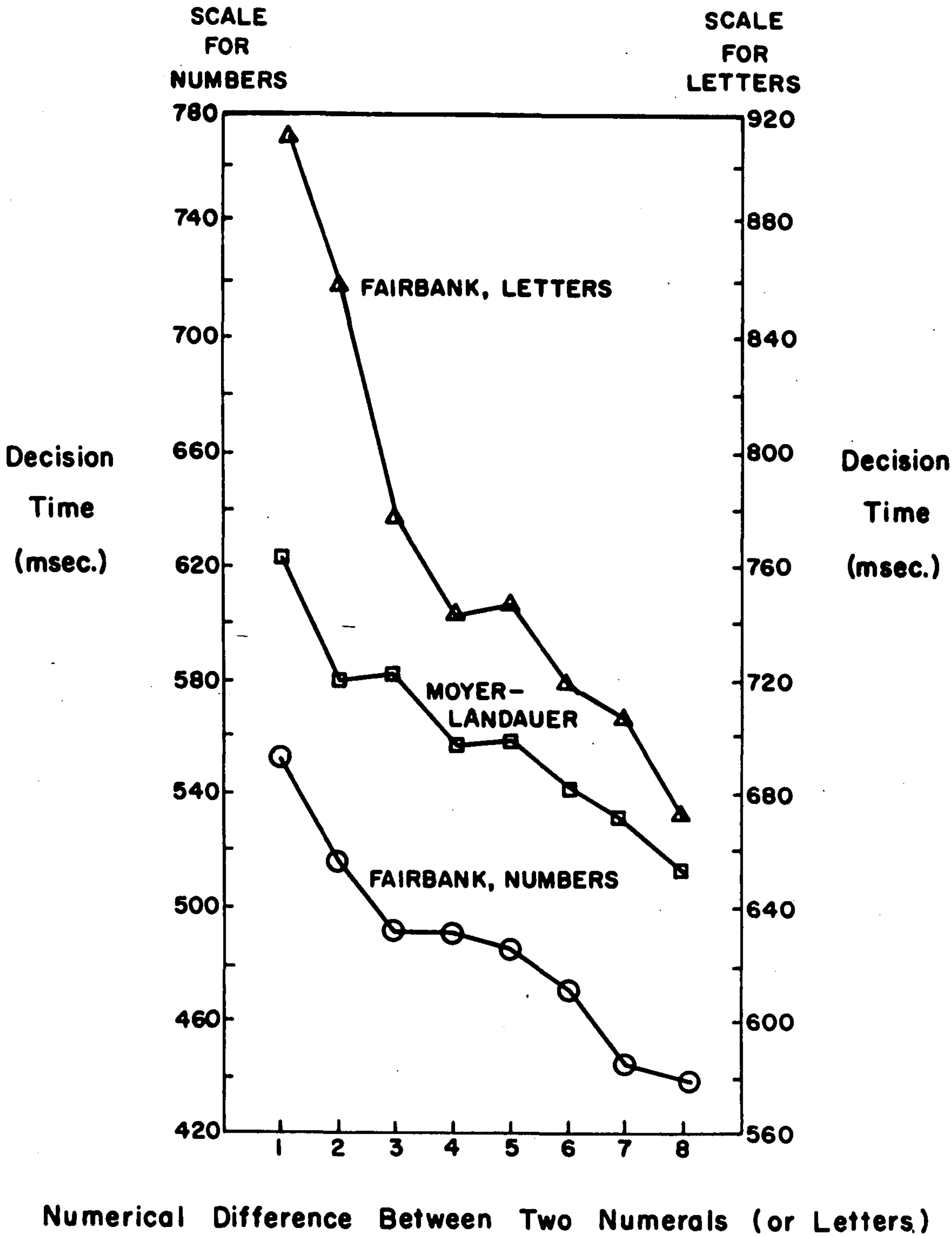

Fig. 7. Decision time as a function of the difference between digits or between letters.

findings of Experiment Two, discussed below.) Table 1 shows the mean decision times for all "values of the differences between the elements of the pairs" (hereafter called "deltas") for all Ss; in addition, at the bottom, the overall means as plotted in Figure 7 appear. In this table, as throughout this paper, times are given in milliseconds. Table 2 presents the correlation coefficients (r) between deltas and decision times for each of the ten Ss. The equation for the straight line which best fits each S's data is also given. The overall r and the overall best fitting equation (this equation and the preceding ones were fitted by the method of least squares) appear as well. Using a one-tailed test for the significance of correlation, all of these correlations are significantly different from zero beyond the 0.005 level. This particular test is not completely applicable, however, since it is difficult to meet the assumption of random sampling. To be conservative the significance level might be doubled to the 0.01 level.

In order to test counting-models three and four (see Introduction) which predicted that decision times would be, respectively, an increasing linear function of the magnitude of the smaller digit or a decreasing linear function of the magnitude of the larger digit in the pairs, Figure 8 was prepared. Figure 8 shows the mean decision times as a function of the size of the smaller digit and of

Table 1

Mean Decision Times (in ms.) for Each Delta and for Each S Data from Experiment One

Subject	Difference between digits							
	1	2	3	4	5	6	7	8
1	628	594	504	521	510	524	487	430
2	610	550	525	533	554	516	500	485
3	577	531	521	509	491	489	467	446
4	458	450	435	444	448	416	420	400
5	624	560	541	498	503	483	482	470
6	490	467	454	474	454	448	416	401
7	499	468	431	435	460	438	423	411
8	591	574	553	553	546	480	484	491
9	539	507	515	488	472	454	465	449
10	498	449	437	433	424	423	396	382
Means	551	515	491	489	486	467	454	437

Note.--Means may not exactly equal column means because they were computed from column data before columns were rounded to nearest ms.

Table 2

Correlation Coefficients and Regression Equations for all Ss and Pooled Data of Experiment One

Subject	Correlation coefficient	Regression equation
1	-.42	DT = -24.1(delta) + 631
2	-.29	DT = -14.4(delta) + 598
3	-.39	DT = -17.0(delta) + 579
4	-.21	DT = -6.2(delta) + 463
5	-.39	DT = -23.9(delta) + 622
6	-.21	DT = -9.3(delta) + 494
7	-.25	DT = -10.7(delta) + 492
8	-.31	DT = -16.6(delta) + 608
9	-.33	DT = -13.7(delta) + 546
10	-.35	DT = -14.4(delta) + 494
All data pooled	-.29	DT = -15.0(delta) + 553

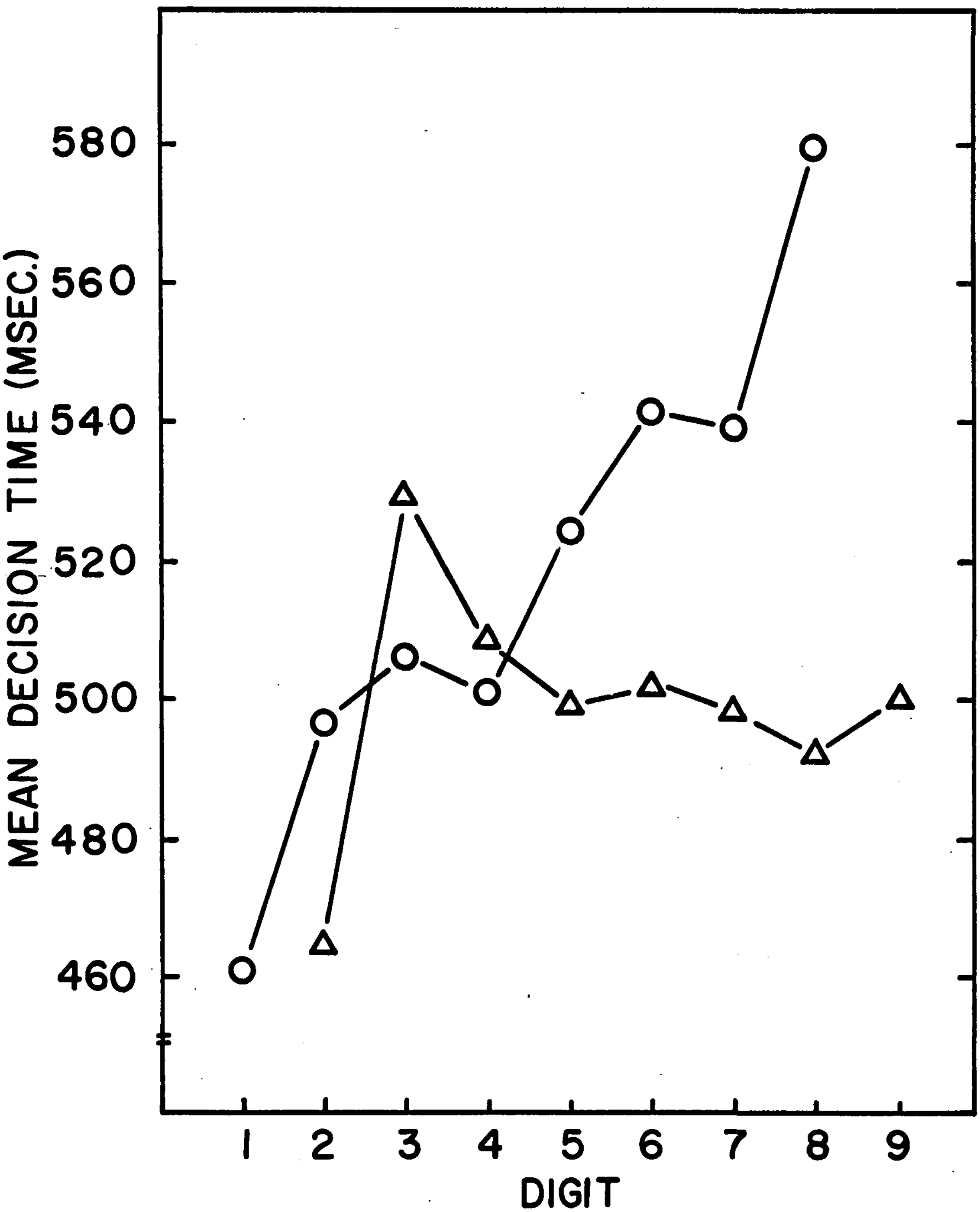

Fig. 8. Decision time as a function of the larger digit (triangles) and of the smaller digit (circles) in a pair.

the size of the larger digit. The values for the points on the upper curve were computed by simply taking the mean of all decision times for those pairs which had a common smaller digit. The other curve was prepared analogously. The prediction of counting-model three is born out strikingly--decision times increase approximately linearly with the magnitude of the smaller digit in a pair. Unfortunately, there is a contaminating factor in this particular method of data presentation. The mean difference between the digits (or the mean delta) of all pairs having any common lowest digit is a linear function of the magnitude of the smaller digit. For examples, the mean delta for the pairs containing _1_ is 4.5, the mean delta for those pairs containing _2_ as a smaller digit is 4.0, that for the pairs containing _3_ as a smaller digit is 3.5, that for those with _4_ the smaller is 3.0, and so on down to the pair containing _8_ as the smaller digit, where the delta is 1.0. Thus the MLE itself would tend to produce a slope such as the one observed in the upper curve of Figure 8. Exactly parallel circumstances hold for the line showing decision time as a function of the size of the larger digit. The mean deltas vary directly as a function of the size of the larger digit, hence since decision time varies negatively with the size of the delta, it should vary negatively with the size of the larger digit.

To recapitulate the status of these two counting-models, counting-models three and four explain or "predict" the MLE on the basis of certain "counting" operations; further predictions made on the basis of these models are supported in the case of counting-model three, but not counting-model four. The test used on the models, however, contains its own biasing factor which should influence the test toward acceptance of the models. If the models are completely valid, then there are two reasons why the test should support them; first, the mean DT as a function of the smaller (or larger) digit would yield a predictable slope; and second, given the fact that the MLE does exist, the slopes should be affected by the mean delta of each group of pairs with identical smaller (or larger) elements. This last should be true even if the models are not valid. Counting-model four, which predicts a linearly decreasing decision time as a function of the magnitude of the larger digit, may be rejected, since the slope of the appropriate line in Figure 8 is obviously very nearly zero, after a brief initial wavering.

The agreement with prediction seen in the upper curve of Figure 8 must not be overemphasized, because of the contaminating factor discussed above. Thus further predictions based on counting-model three must be tested. The model requires that S starts counting up from one and continues until he reaches either digit. He then selects the other digit as being larger. If this counting process is a valid representation of the real processes involved, then all pairs of numbers with the same smaller digit should have the same mean decision time, or, stated differently, a graph of a family of seven curves showing decision times as a function of the size of the larger digit, given that the smaller digit of all pairs represented in a single curve is the same, should show a family of seven zero-slope curves. Figure 9 is such a graph, showing the mean decision times when the smaller member of a pair was held constant. The slopes of these seven lines are so obviously different from zero that no statistical test is required. Counting-model three is not tenable.

Discussion

Two main points emerge from this experiment; first, the MLE is real and replicable, and second, none of the proposed counting-models is adequate to explain the data. The adequacy of the model based on the finding of North et al. (1967) is the basis for discussion in a later section; at this point one adequate model of the MLE is that advanced by Moyer and Landauer (1967): that the digits are encoded somehow as analog magnitudes and judgments are based on comparisons of these magnitudes.

The overall correlation between decision time and delta obtained in this experiment is lower than the one

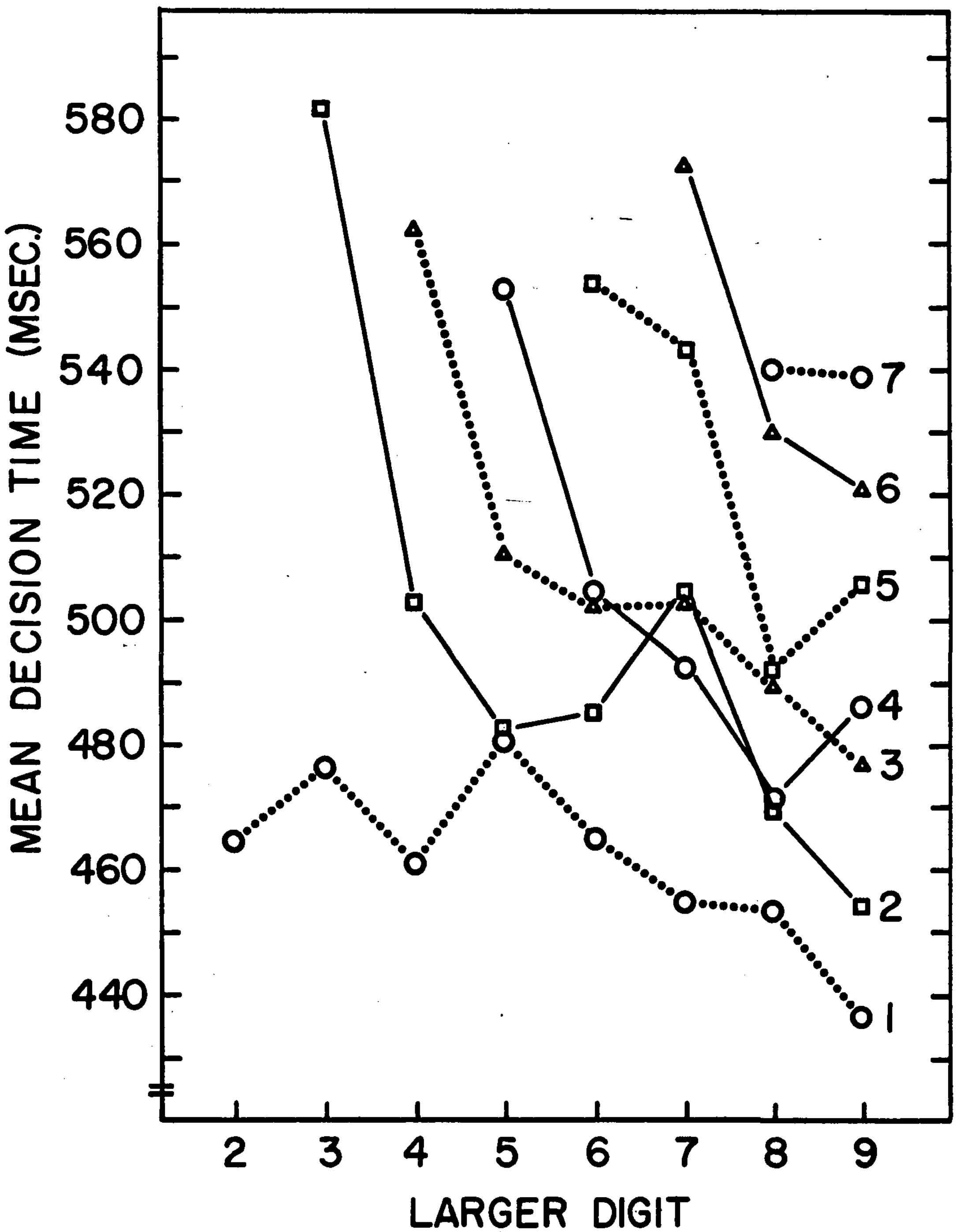

Fig. 9. Mean decision time per pair as a function of the larger digit; one curve shown for each of the smaller digits.

reported by Moyer and Landauer (-0.29 instead of -0.63); possible reasons for this are discussed in the Conclusions section below.

Experiment Two: The Moyer-Landauer Effect Using Letters

Introduction

In order to establish the generality of the MLE with non-numerical stimuli, letters instead of numbers were used in the Moyer-Landauer situation.

Method

The equipment and the method used for this study were exactly identical to those of the first experiment, except that instead of the numbers one through nine appearing as stimuli, the letters A through I appeared. Thus nine letters took the place of the nine digits. *S*s were instructed to press the key on the same side as the letter which was further into the alphabet, or nearer Z.

Results

For the sake of clarity, the same terminology will be used in the discussion of this experiment as was used in that of Experiment One, i.e., a pair with a delta of four will indicate a pair such as C-G in which three letters intervene between the members of the pair.

The uppermost curve of Figure 7 shows the summarized data from this experiment. The MLE is clearly apparent. Table 3 gives correlation coefficients (all significant beyond 0.005) and least squares regression lines for the ten Ss and for all of the data taken together. There are differences between the results obtained using numbers as stimuli and those obtained using letters, but these differences are ones of degree, not kind. The correlation coefficients are somewhat higher with letters, the slopes considerably steeper, and the overall times are slower. Note that the two different scales of Figure 7 make comparison of number and letter data difficult in this illustration.

In both this and the previous experiment a practice effect was noticeable: the Ss received the 216 pairs of stimuli in groups of 72; the overall means for the successive groups of 72 pairs decreased over the three administrations. Table 4 shows the extent of the decreases with the administrations for Experiments One and Two, averaged over all subjects.

Discussion

The MLE is clearly not restricted to numbers. The data gathered using letters are as convincing as those gathered using numbers as stimuli, and so raise a difficult problem. Perhaps the occurrence of the MLE with letters is

Table 3

Correlation Coefficients and Regression Equations for all <u>S</u>s and Pooled Data of Experiment Two

Subject	Correlation coefficient	Regression equation
1	-.27	DT = -28.8(delta) + 941
2	-.30	DT = -20.1(delta) + 755
3	-.43	DT = -60.9(delta) + 1093
4	-.34	DT = -44.2(delta) + 979
5	-.42	DT = -39.7(delta) + 978
6	-.47	DT = -48.4(delta) + 969
7	-.36	DT = -39.3(delta) + 913
8	-.27	DT = -32.1(delta) + 928
9	-.44	DT = -45.6(delta) + 1009
10	-.22	DT = -17.7(delta) + 746
All data pooled	-.33	DT = -37.7(delta) + 931

Table 4

Decision Times Averaged Over All Ss and All Stimuli for the Three Consecutive Administrations of the Stimulus Lists

Administration	Data from Experiment One	Data from Experiment Two
First	524	855
Second	495	800
Third	489	761

a manifestation of the same processes which produce the MLE with numbers. However, a tenable explanation of the MLE is the encoding of numbers as analog magnitudes. The existence of a MLE with letters requires us to consider the possibility of representing letters internally as analog magnitudes. This sounds unlikely on first consideration, and as Experiment Ten shows, the parallel between letter and number phenomena is not complete, hence conclusions regarding the numerical MLE may not in some cases be applied to the alphabetic MLE.

Experiment Three: The Moyer-Landauer Effect and Months of the Year

Introduction

In order to look for an effect similar to the MLE, but involving months of the year, a new study was undertaken. A further purpose of performing this study was to employ a methodology substantially different from the original Moyer-Landauer methodology while looking for evidence of a similar effect, thus obtaining evidence that the MLE is not an artifact of methodology.

Method

Once again, the term "delta" will be used with the same meaning as in Experiment One, e.g., the month pair June-August has a delta of two.

Six lists of 264 pairs of months were prepared. There are 132 different month-pairs, excluding pairs involving identical months, but including all other pairs. Each of the six lists prepared included each one of these 132 pairs once. The presence of these 132 pairs was intended simply as filler or camouflage for the "test" pairs included in the lists. (If the lists had not included the filler pairs, then one list would, for example, have consisted entirely of the pairs December-January and January-December.) The other 132 pairs in a given list were the so-called test pairs. The test pairs for a given list all had the same delta. Thus all of the test pairs on a given list (half of the total number of pairs) were month-pairs whose elements were separated by the same number of months. The deltas were the same given only the important assumption that one always counts from January to December; so that, for example, the pair November-February has a delta of nine, not three, as when one counts through the end of the calendar year. Of the six complete lists there was one each with deltas of one, three, five, seven, nine, and eleven months. Pairs appeared in AB and BA order with equal frequency. A complete randomization of the order of test and filler pairs was made concurrently with a complete randomization of the side of the pair on which appeared the month nearer December.

Each of these six lists was then split into two halves and typed on two duplicator master sheets with the months abbreviated to their first three letters. The result was six pairs of sheets, each being very nearly half filler and half test, with random orders of test and filler appearing on all sheets. Ten test booklets were then prepared, each consisting of one of each of the twelve pages. The first six pages of a booklet were made up of one each of the pages from the first halves of the lists, while the second half contained the second halves of the same lists in exactly the opposite order, to counterbalance for possible order effects. The ten booklets all used different orders of presentation of the lists to eliminate any systematic effects due to the presentation of the lists in a particular order. In addition to the ten test booklets, ten identical sheets were prepared, made up of all 132 nonidentical pairs in random order.

The tests were administered to ten Ss in a group setting. The experimenter (E) instructed the Ss to cross out with pen or pencil as quickly as possible whichever one of the two months in each pair was nearer to December, or further into the year. The single sheets were distributed first to give the Ss a chance to practice, and to ask any questions if the instructions were not completely clear. The booklets were then distributed, and the ten Ss were given one minute of working time for each of the ten pages, with one minute rests between the pages.

The number of pairs completed for each delta was the dependent variable. It was obtained by adding together the number of items crossed out on each of the two pages with the same delta for all six pairs of pages and for all ten Ss.

Results

Each of the ten Ss provided six scores, each one corresponding to a delta; the correlation coefficient (r) between delta and number completed was +0.65 (n = 60); this is significant beyond the .005 level, but again the sample is not truly random, hence the significance level must be set somewhat lower than the tabled value of 0.005. The positive r is consistent with MLE phenomena since a higher number of pairs completed indicates a shorter time per pair as delta increases. In other words, the positive correlation in this instance is evidence of the same general class of phenomenon that led to the negative correlations in Experiments One and Two, since pairs completed per unit of time was the dependent variable, not time per completed pair as in the earlier experiments. Figure 10 shows the number of month pairs completed as a function of delta.

In order to determine whether or not the differences in the number of pairs completed in the six groups were

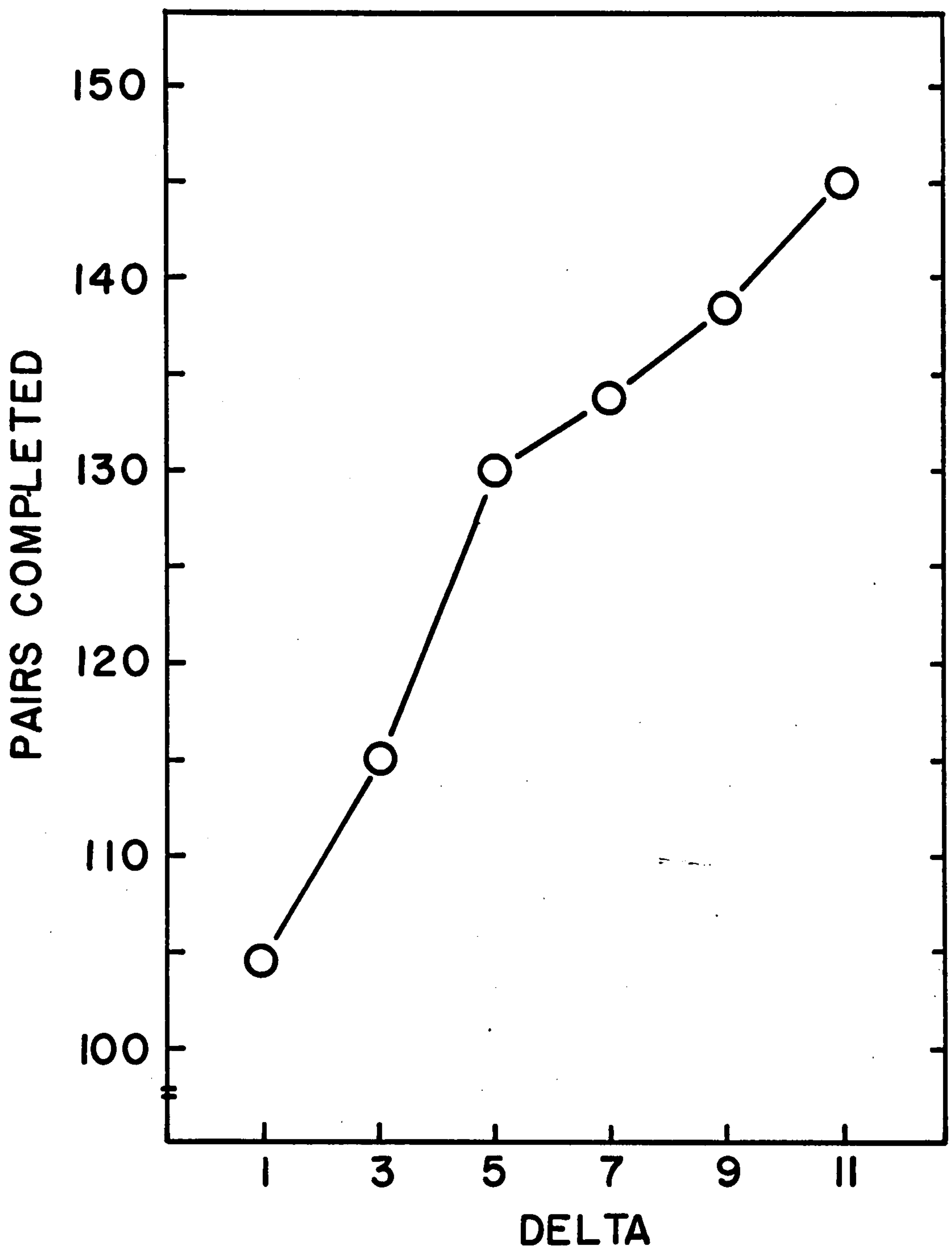

Fig. 10. Mean number of month pairs completed as a function of delta.

significant, a subjects by conditions analysis of variance was performed on the data. Since this particular test will be used several times in the following experiments, a brief explanation of it is given here. In using this test one is assuming that either there are no significant differences between the various subjects, or that if there are such differences one is not interested in them. The analysis essentially discounts all variance in the data which arises from subject differences and tests only for the effects of the treatment. As such it is a very powerful design and is the design most highly favored when the administration of one treatment to a group of Ss will not affect their performance in the other treatments. In this case the analysis showed that the differences between the groups (each group corresponding to one of the deltas) was significant well beyond the 0.001 level. F with five and nine degrees of freedom equaled 66.65, or, written in standard notation, $F_{5,9} = 66.65$.

Discussion

The results, both in direction and in significance, are clearly consistent with a generalized concept of the MLE. It appears that the MLE may be observed whenever one is selecting one item of a pair when the pairs are drawn from a serial list whose order is known by the S(s), and the rule governing the correct selection is that the selected item be nearer one end of the list. This hypothesis is based on a small number of lists, three, but there is apparently no major difference in the nature of the phenomenon in the cases tested.

Experiment Four: The Moyer-Landauer Effect with Well Practiced Subjects

Introduction

This experiment yielded the greatest number of data of any of the experiments described in this dissertation; these data may be examined as functions of several variables, and although no statistical hypotheses are tested in this experiment, the data are summarized in several ways for descriptive clarity.

Essentially this experiment is a replication of Experiment One, but instead of ten Ss being run once each to yield 2160 data, two Ss were run twice a day for ten days to yield 8640 data. From this mass of data it was hoped that the answers to a number of theoretical questions would begin to appear. The first question to be considered was whether or not the magnitude of the MLE (i.e., the slope of the best fitting line when decision time is given as a function of delta) decreases with practice. The second question concerns the shape of the Moyer-Landauer curve as the Ss become more and more practiced. If the curve becomes smoother then it should be possible to estimate the shape of the "true" curve, that curve which would be found with an indefinitely large number of data. The third problem investigated concerns whether or not there is a difference in decision speed when one selects the smaller instead of the larger digit. Until this time all Moyer-Landauer type experiments have had the Ss select the larger digit. The fourth question was "How does the correlation coefficient vary with practice?," and the fifth question was simply concerned with how long it took before Ss' mean decision times per session reached an asymptote, indicating that the S was fully practiced.

The variability of simple reaction time decreases with practice; if the variability of decision time decreases similarly then one would expect an increase in the correlation between delta and decision time. As each S, through practice, reduces his own variability, the total variance in the data should decline. However, the variance which is in reality due to the deltas should not decline, and hence should account for a relatively larger proportion of the total variance. The correlation coefficient measures indirectly the amount of variance accounted for by the independent variable, thus if S's variance declines while variance due to the effects of the deltas does not, the correlation coefficients should increase with practice.

Method

One female (MA) and one male (DH) subject were tested for twenty sessions each, two sessions per day. The procedure for each session was the same as that described for Experiment One. The equipment was the same as that used in the first experiment. Each session consisted of the taking of 216 decision times in groups of 72 with five minutes rest between groups. Each group of 72 stimulus pairs contained all of the possible non-identical pairs once each, and the order in which the pairs appeared was randomized anew for each administration of the 72 pairs. In one session each day _S_ selected the larger digit, and in the other session, the smaller digit. If _S_ started with smaller digits in the morning and larger in the afternoon, then the reverse order was used the next day, and on subsequent days the orders alternated.

Results

There are so many data from this experiment that resort is best made to graphs for presentations. Figure 11 is in the familiar graphical form for the MLE; decision time as a function of delta. This graph, although showing the results for only two _Ss_, is based on four times as many data as is any of the curves in Figure 7. The relationship appears to be curvilinear; the best fitting straight line (as found by minimizing the squares of the deviations of

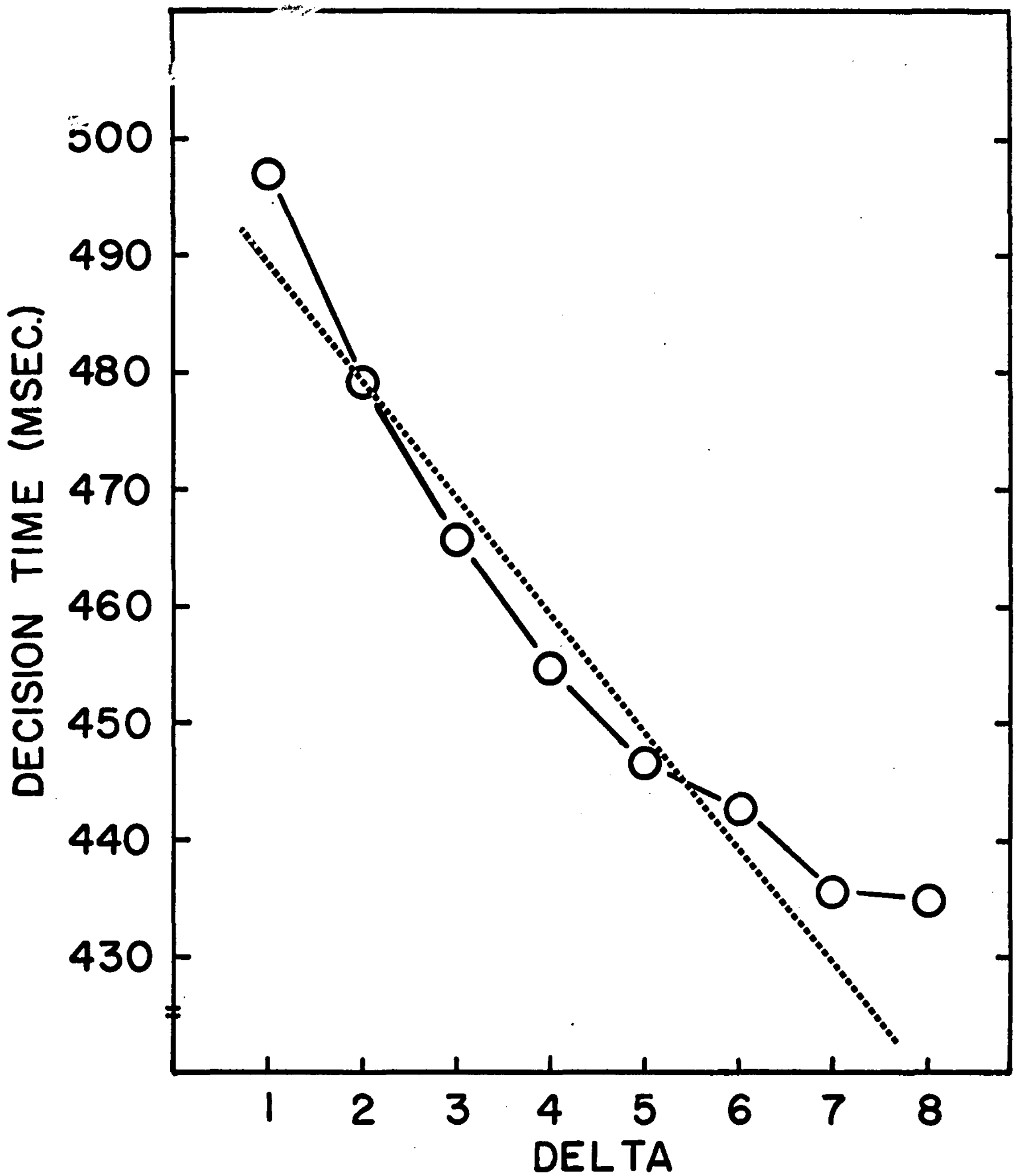

Fig. 11. Decision time as a function of delta, with least-squares line (dotted) shown also.

Data are from Experiment Four, averaged over all days and both subjects.

the points from the line) is drawn over the data, but it clearly does not depict the data adequately.

Figure 12 presents the overall daily means for both Ss. This figure indicates the magnitude of the effect of practice. DH is consistently faster than MA, but after the fourth day no major practice effects are visible for either S. The first four days, then, were ones in which the Ss were steadily improving their speed. Figure 13 shows the correlation coefficient values for the two Ss as a function of the session. Unlike the absolute speed of decision, shown in Figure 12, the correlations did not change systematically with practice. The correlations are generally lower and more variable for MA than for DH. At no time did either S, even when well practiced, come close to the correlation coefficient of -0.63 reported by Moyer and Landauer (1967). Possible reasons for this will be considered in the Conclusions section. The overall correlations were -0.15 for MA and -0.30 for DH.

Judging from the slopes of the least squares regression lines presented in Figure 14, there is a practice effect, at least for DH. However, after the fourth day (eighth session) the improvement seems to cease. MA showed no detectable systematic change over the ten days. Mowbray and Rhoades (1959) found decision time (or, as they call it, choice reaction time) could, with practice, be reduced to a minimum which did not vary with the stimulus presented.

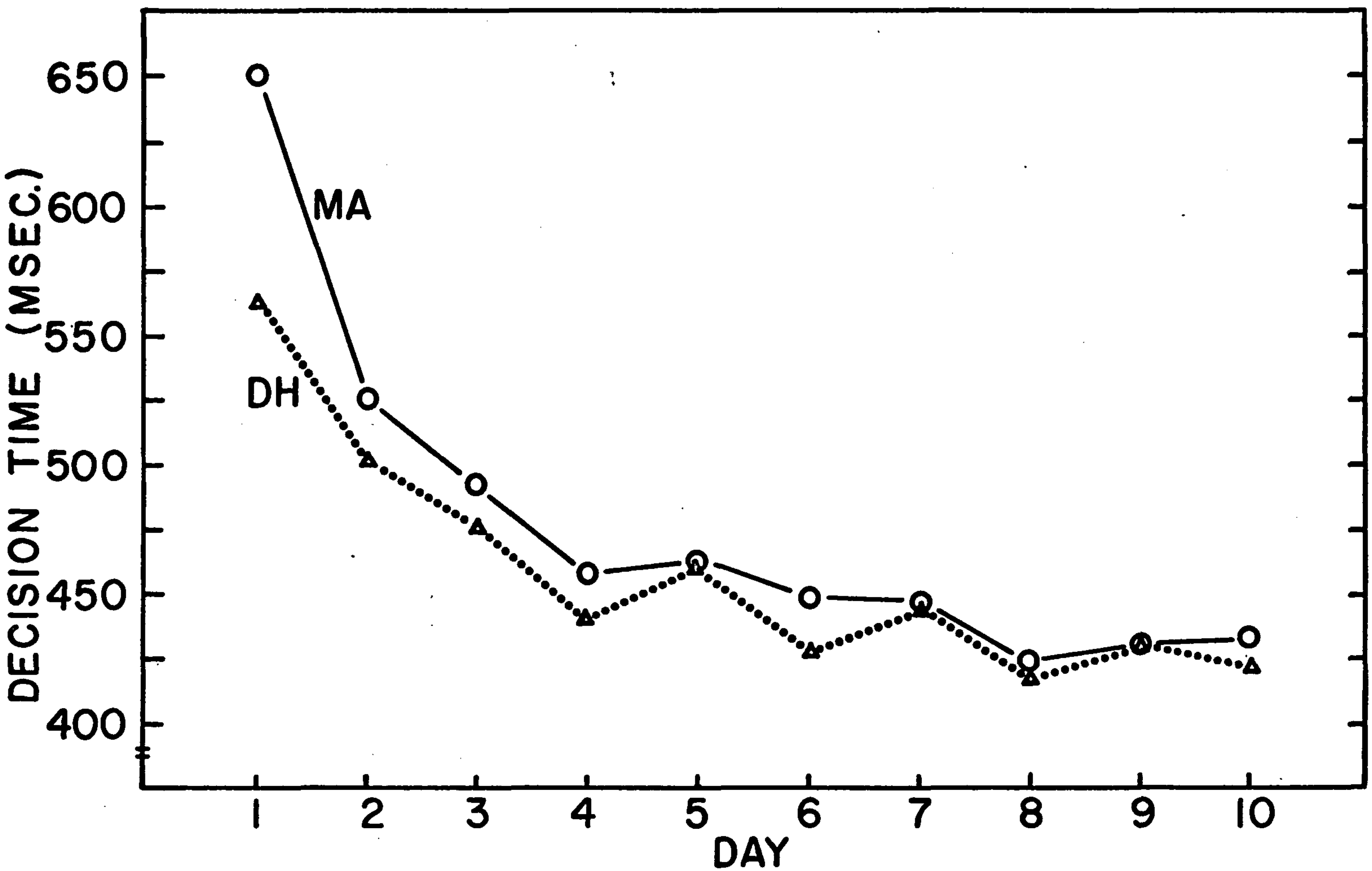

Fig. 12. Overall mean decision time as a function of day of testing for two subjects.

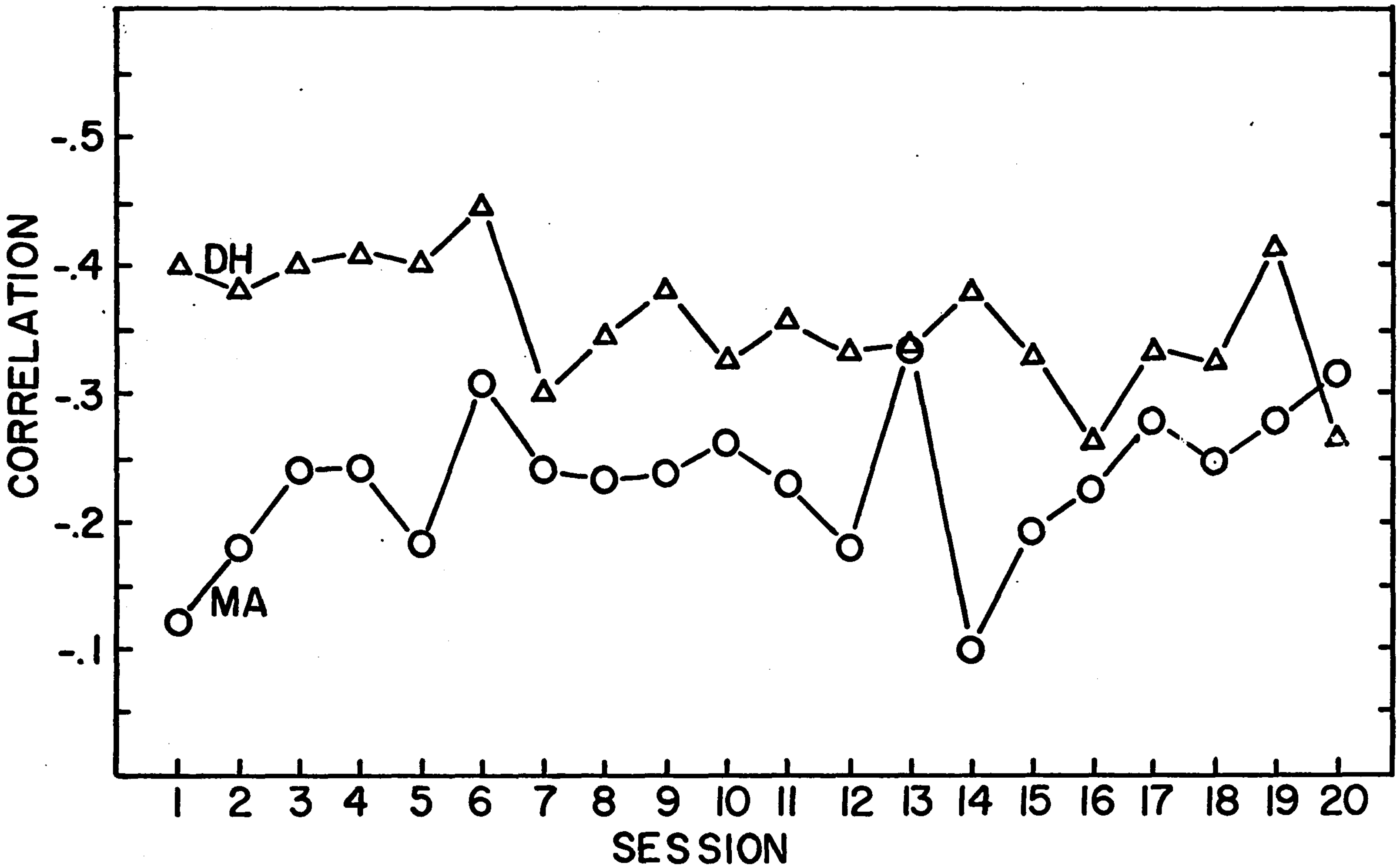

Fig. 13. Correlation between delta and decision time as a function of of session for two subjects.

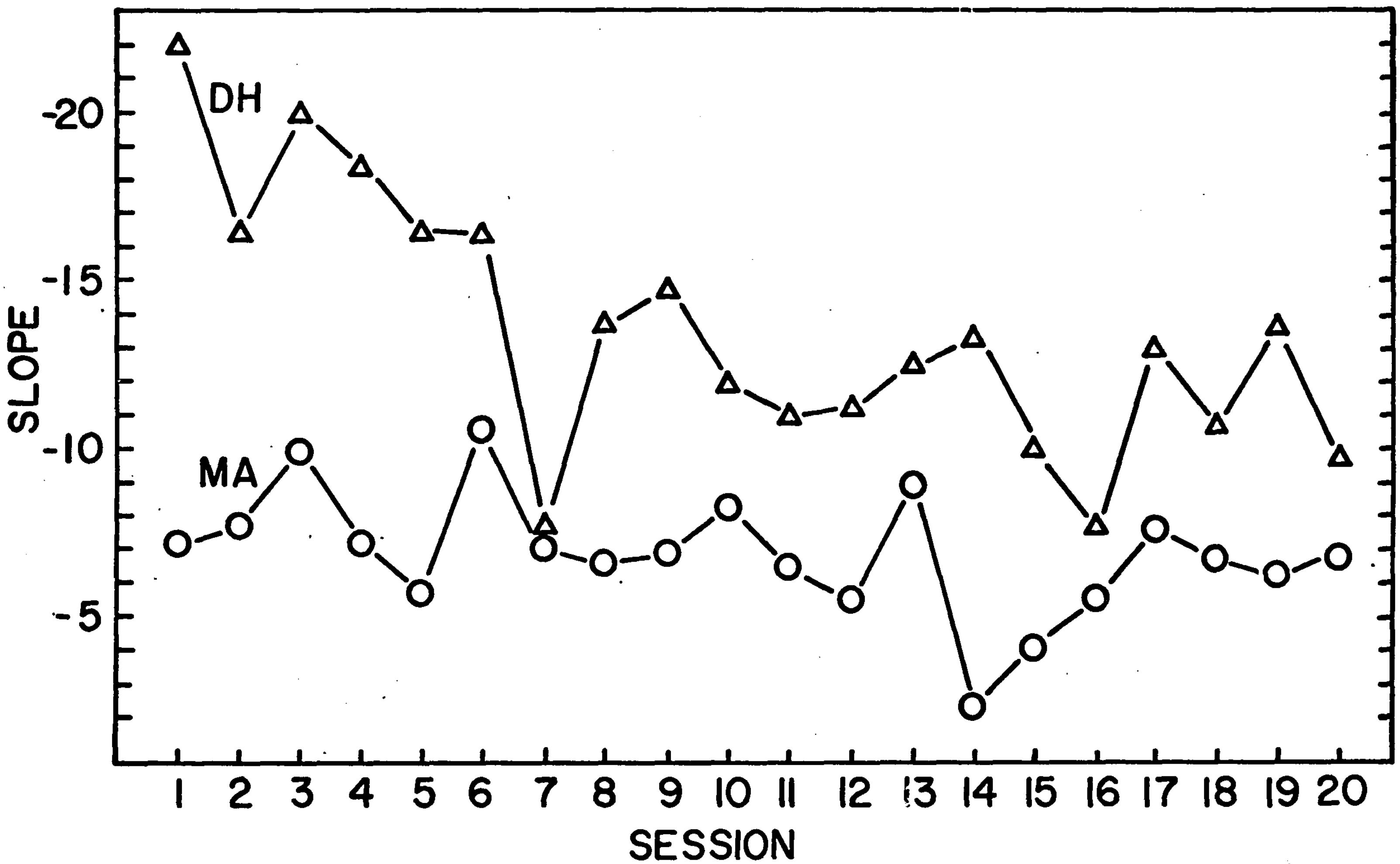

Fig. 14. Slope of least-squares lines as a function of session for two subjects.

However, they used a less complex stimulus population; the twenty sessions in which subjects MA and DH ran are probably not enough to exclude definitely the possibility that practice effects would eventually reduce the slopes to zero, as the Mowbray and Rhoades article leads one to expect. Figure 14 shows the slopes as a function of the session, which means that the graphed points include the effects of an alternation between morning and afternoon session slopes, and of a double alternation between choosing the higher and choosing the lower digit, after the first session. In order to present a function whose only independent variable is the day of running, Figure 15 was prepared. Unlike Figure 14, each point represents the average of the slopes of two regression lines, one morning and one afternoon, one for choosing the larger digit, and one for choosing the smaller digit. If there is a long-term trend toward a reduction of slope with practice, the twenty sessions of this experiment were insufficient to uncover evidence for that reduction.

Figure 16 presents the data for the two Ss as a function of the day of running, with two points computed for each day, one for mean of the decision times associated with choosing the larger digit, and the other for the mean of the decision times associated with choosing the smaller digit. DH is apparently faster when choosing the larger digit, while MA shows no consistent difference. The

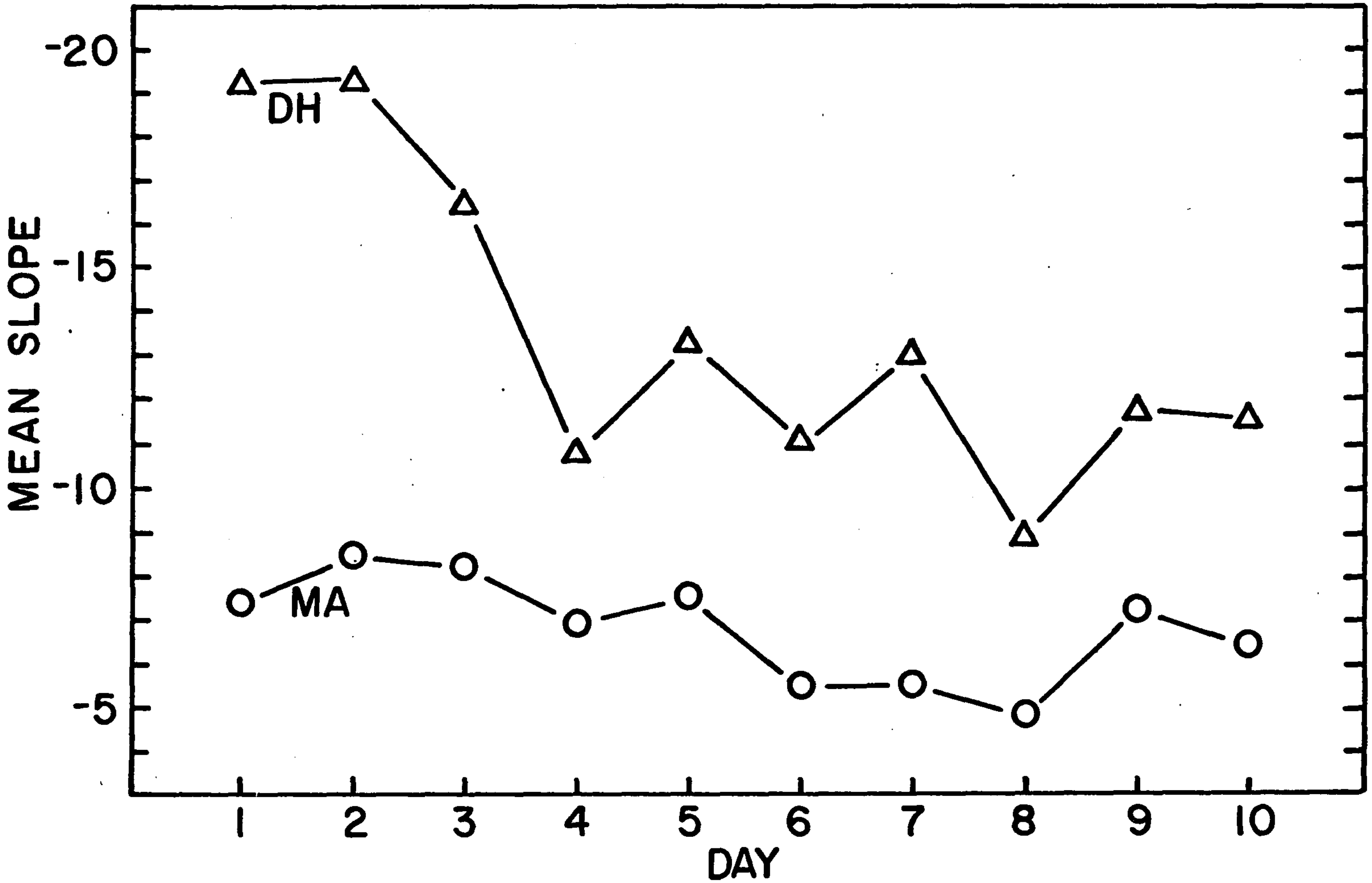

Fig. 15. Mean slopes of least-squares lines from A.M. and P.M. sessions as a function of day of testing for two subjects.

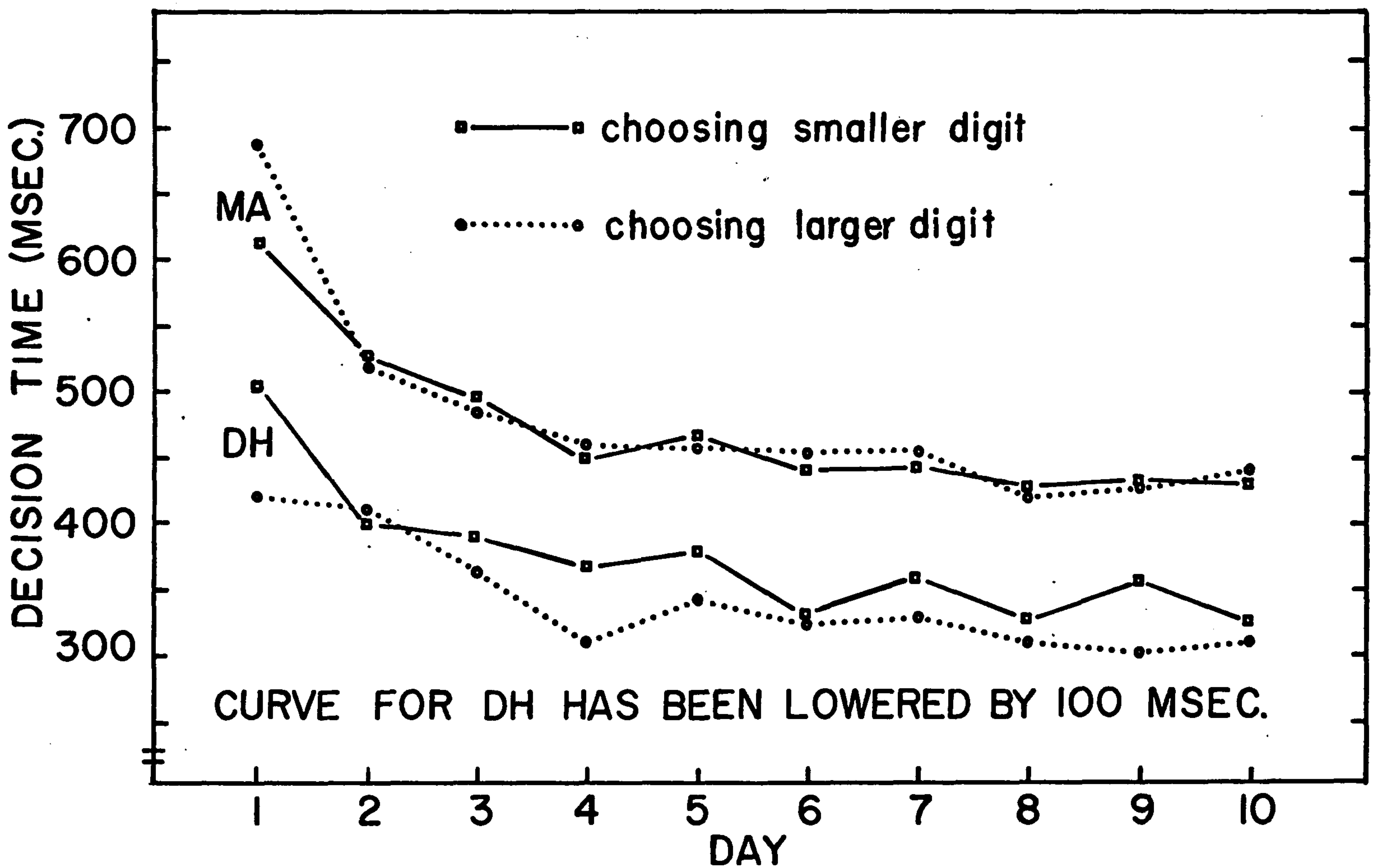

Fig. 16. Decision time for choosing larger digit compared with that for choosing smaller digit for two subjects as a function of day.

inconclusiveness of this aspect of the experiment prompts further investigation; see Experiment Eight.

For reference, Table 5 gives the mean decision times for each stimulus pair. Each pair was presented to each S six times per session (thrice in AB and thrice in BA order) for twenty sessions. Thus each DT of Table 5 represents 240 averaged data, grouped according to delta.

Discussion

This experiment has yielded many data but no firm conclusions. The most interesting discovery based on this experiment is the shape of the curve in Figure 11. The least squares straight line obviously does not provide a satisfactory approximation of the data. Moyer and Landauer (1967) suggest that the best fitting equation would be of the nature DT = K log (larger/delta), but they do not provide any suggested K for their own data. If that equation were suitable for describing the data, then a graph of decision time as a function of log (larger/delta) should be linear. Figure 17 is such a graph, made up of 36 points, one for each pair of digits, averaging together the overall mean DT for each pair and its reverse (e.g., 4-2 and 2-4). Although the points are well spread out, particularly at the upper end of the abscissa, the function looks as if it would be approximated by a straight line far better than might the function of Figure 11. Figure 18

Table 5

Overall Mean DTs for Both Ss, All Sessions, for All Stimulus Pairs, Arranged in Order of Increasing Delta

Stimulus pair	Mean DT for pair (and the reverse pair)	Delta
1 2	444 ms.	1
2 3	472	1
3 4	479	1
4 5	472	1
5 6	521	1
6 7	537	1
7 8	528	1
8 9	524	1
1 3	438	2
2 4	464	2
3 5	462	2
4 6	466	2
5 7	497	2
6 8	527	2
7 9	504	2
1 4	434	3
2 5	438	3
3 6	457	3
4 7	474	3
5 8	498	3
6 9	495	3
1 5	435	4
2 6	441	4
3 7	456	4
4 8	469	4
5 9	475	4
1 6	432	5
2 7	449	5
3 8	449	5
4 9	458	5
1 7	434	6
2 8	441	6
3 9	453	6
1 8	432	7
2 9	440	7
1 9	435	8

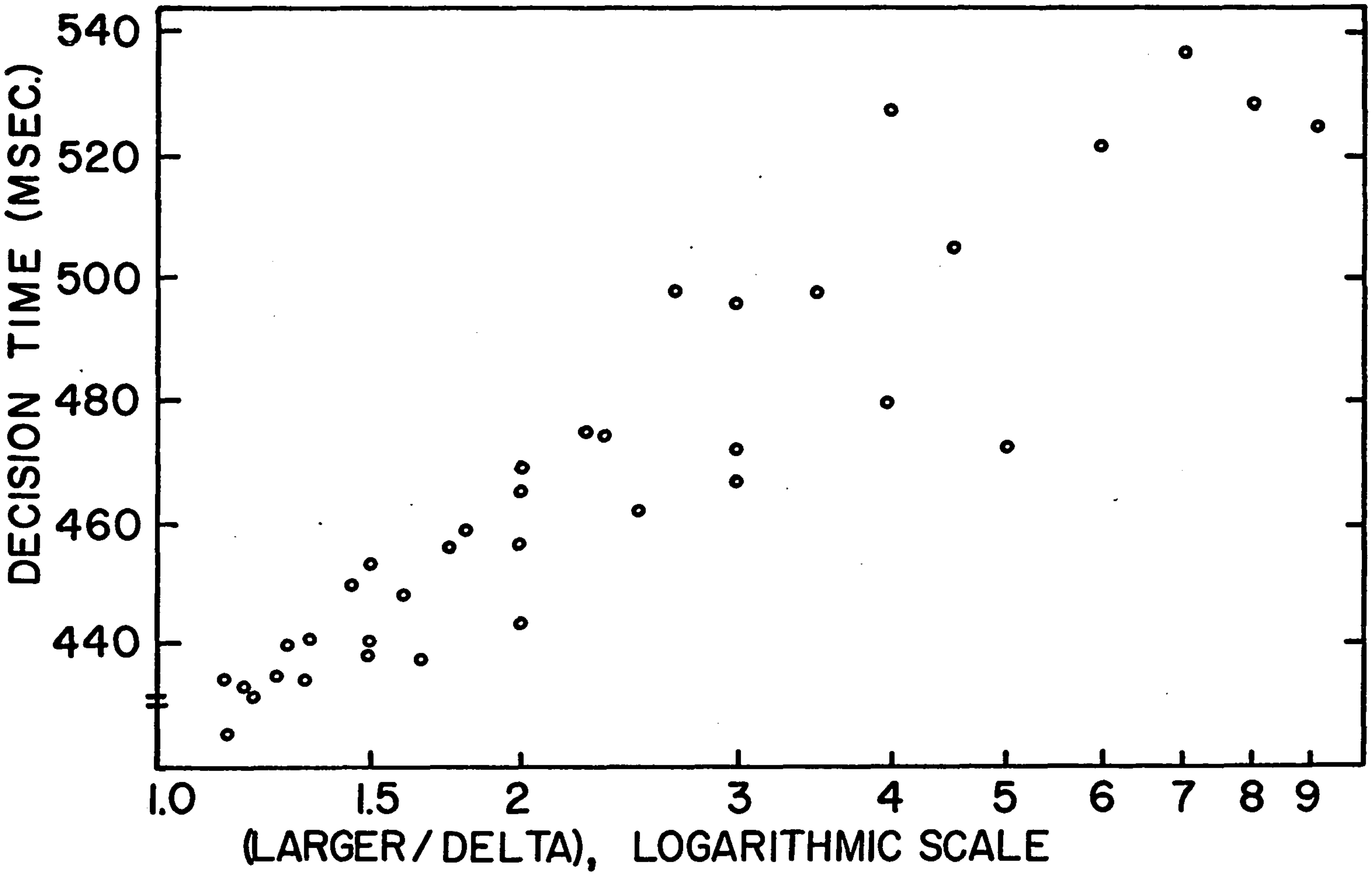

Fig. 17. Decision time as a function of the larger digit divided by delta; abscissa is a logarithmic scale.

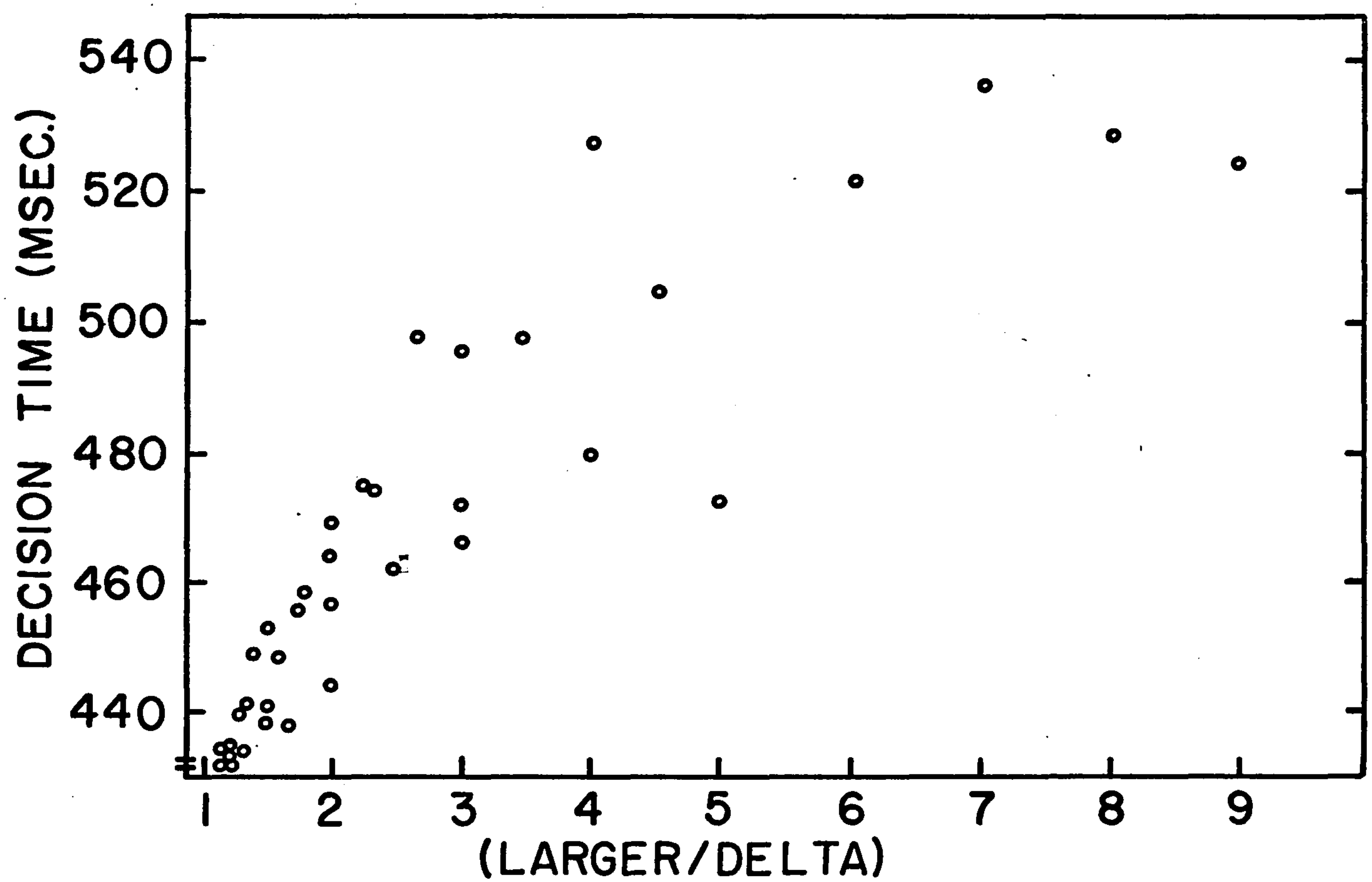

Fig. 18. Decision time as a function of the larger digit divided by delta; abscissa is a linear scale.

shows the same data as Figure 17, but in linear coordinates to give an intuitive idea of the logarithmic nature of the function. Obviously a straight line fits these points less well than in the case of the points of Figure 17. This transformation of the data suggested by Moyer and Landauer (1967) may or may not be a valid one (see Welford [1960] for a full discussion of the applicability of such a function to other choice reaction time situations involving stimuli varying along various physical continua), but since it prevents us from considering decision time as a function of delta, it is difficult to compare the fits obtained with such a transformation and obtained with ~~the~~ data graphed as in Figure 11. The curve in Figure 11 is so smooth that a resort to a log (larger/delta) transformation seems unnecessary. The curve might be described by a number of functions (logarithmic, hyperbolic, exponential), but in the absence of a suitable *a priori* theoretical framework it is difficult to select the right one. If the numbers are judged in a manner analogous to the way in which physical quantities are judged, as Moyer and Landauer suggest, then it is tempting to suppose that they might be encoded in the same way; in other words that the analog magnitude is proportional to the logarithm, or perhaps some power, of the stimulus value. These possibilities are the subject for further discussion in the Conclusions section.

Experiment Five: An Attempted Replication of the North, Grant, and Fleming Experiment

Introduction

This experiment and the two following were designed to evaluate the applicability of the North et al. (1967) findings to the MLE. As pointed out in the Introduction, North et al. found that the latencies involved in reading aloud the digits one, two, six, and nine were significantly lower than those required for the readings of the other digits. From these relationships one can derive a "prediction" of the MLE. Thus it is important to verify the North et al. findings; doing so is the aim of this experiment.

Method

Using the center element of the readout described in Experiment One, ten *S*s were presented with approximately 200 random digits, not including zero, at the rate of one digit every 1.5 seconds. The digits stayed on for one second, and *S* read the digit aloud as quickly as possible into a microphone. As the digit came on, a Hewlett-Packard 522B counter began measuring the interval. A microphone connected with a Grason Stadler voice relay stopped the counter when *S* named the digit. *S*s worked in the same dimly lit room as used in Experiment One, and sat approximately 1.5 meters from the readout.

Results

Each S read each digit a minimum of twenty times. The data resulting from an averaging of the twenty readings per digit were combined over all ten Ss and are presented in Figure 19. The results of North et al. are also given for comparison purposes.

Discussion

The wide differences between the two curves in Figure 19 make it obvious that the results of North et al. were not reproduced with the above described instrumentation and methodology.

Some characteristic of North et al.'s stimuli might have accounted for their results; for example the digits which were pronounced faster might have been larger, clearer, brighter, of greater contrast, or otherwise different from the other digits. Since, however, they obtained essentially the same results with the names of the digits written out in letters, their results are more likely to be due to an artifact connected with the use of the voice key relay. Such relays usually have a number of sensitive adjustments; differences between their settings and those on the equipment used in the present experiment might account for the differences in obtained results. The likelihood of North et al.'s having found a significant

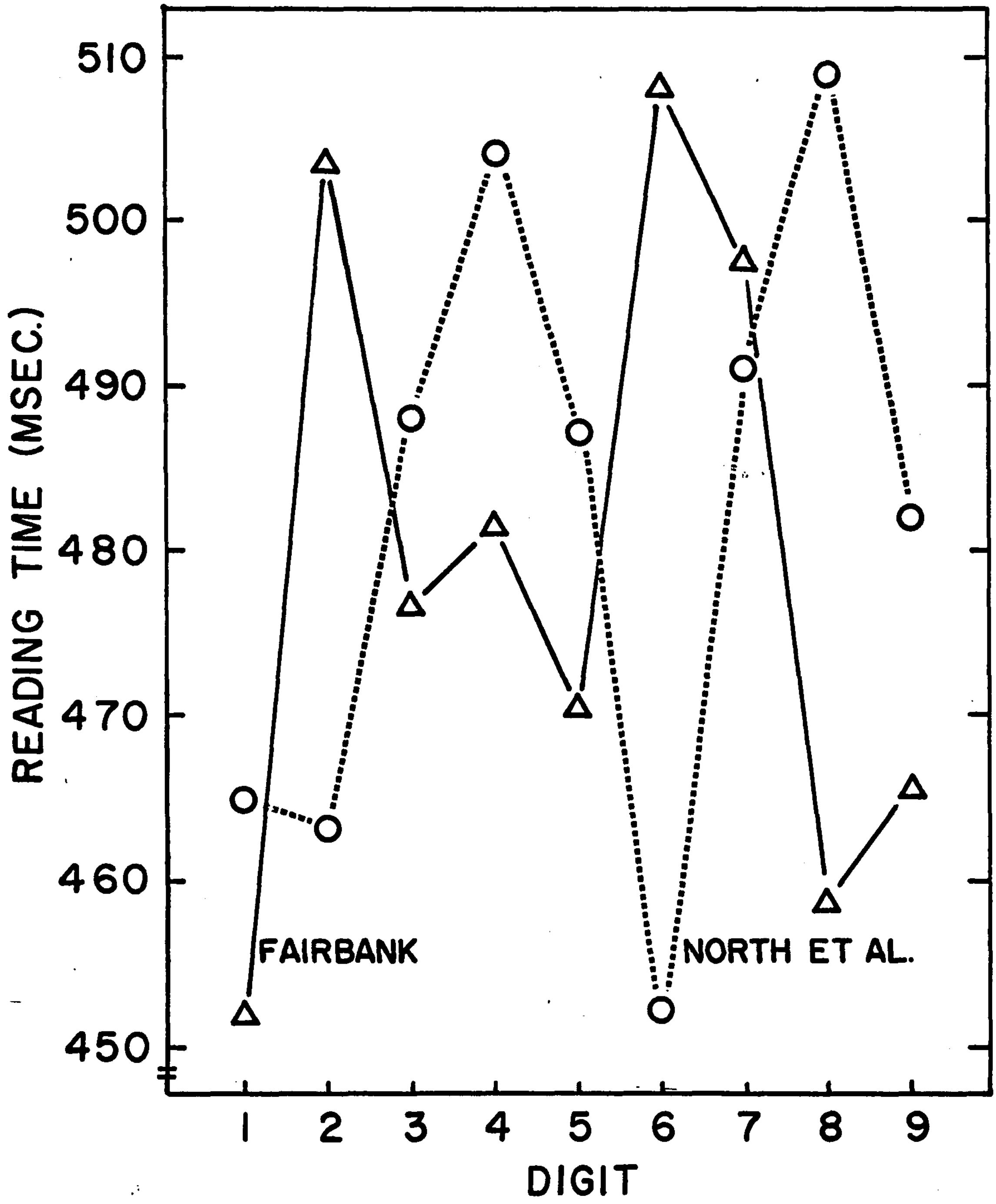

Fig. 19. Time required for first sound in reading the nine digits.

Data from North, Grant, and Fleming (1967) and from Experiment Five.

difference when one did not in fact exist is small, since they obtained similar results with written out numbers.

Experiment Six: The Speed of Digit Recognition Without Vocalization

Introduction

In order to determine whether the various digits are recognized equally quickly even when it is not necessary to pronounce the digits' names, the following experiment was performed.

Method

The stimulating apparatus and rate of stimulus presentation were identical to those used in Experiment Five. Instead of speaking the digit names, however, S pressed a microswitch whenever a particular digit for which he had been told to look appeared. While attending the display of random digits, each of the ten Ss looked for each of the nine non-zero digits in a different random order. The random order method was employed to minimize the chance of fatigue effects or loss of vigilance influencing the results. S attended the parade of digits until ten of the digits for which he had been looking had appeared, then E instructed him to look for another digit. S was instructed to press the key as quickly as possible after the onset of the digit for which he was looking. A

Hewlett-Packard 522B counter measured the time between digit onset and button pressing.

Results

The data, averaged over all ten subjects, are illustrated by Figure 20. A subjects by conditions analysis of variance (similar in principal to the one described in Experiment Three) showed no significant differences between the speeds with which one perceives and identifies the various digits ($F_{8,72} = 0.929$).

Discussion

The purpose of this experiment was to make sure that no digit had a recognition time which was significantly different from that of any other digit. While it appears (Figure 20) that it may take approximately 25 ms. longer to recognize the digit four than to recognize the other digits, the analysis of variance showed that the differences were far from significant. Thus the digits as presented are recognized equally quickly.

When the digits are ranked in order from most quickly recognized to least quickly recognized (even though the differences are not significant), and a similar ranking is made for the reading latencies of the preceding experiment, the rank-order correlation coefficient (rho) is an insignificant +0.06. Thus again it appears that no one number is recognized more quickly than any other number.

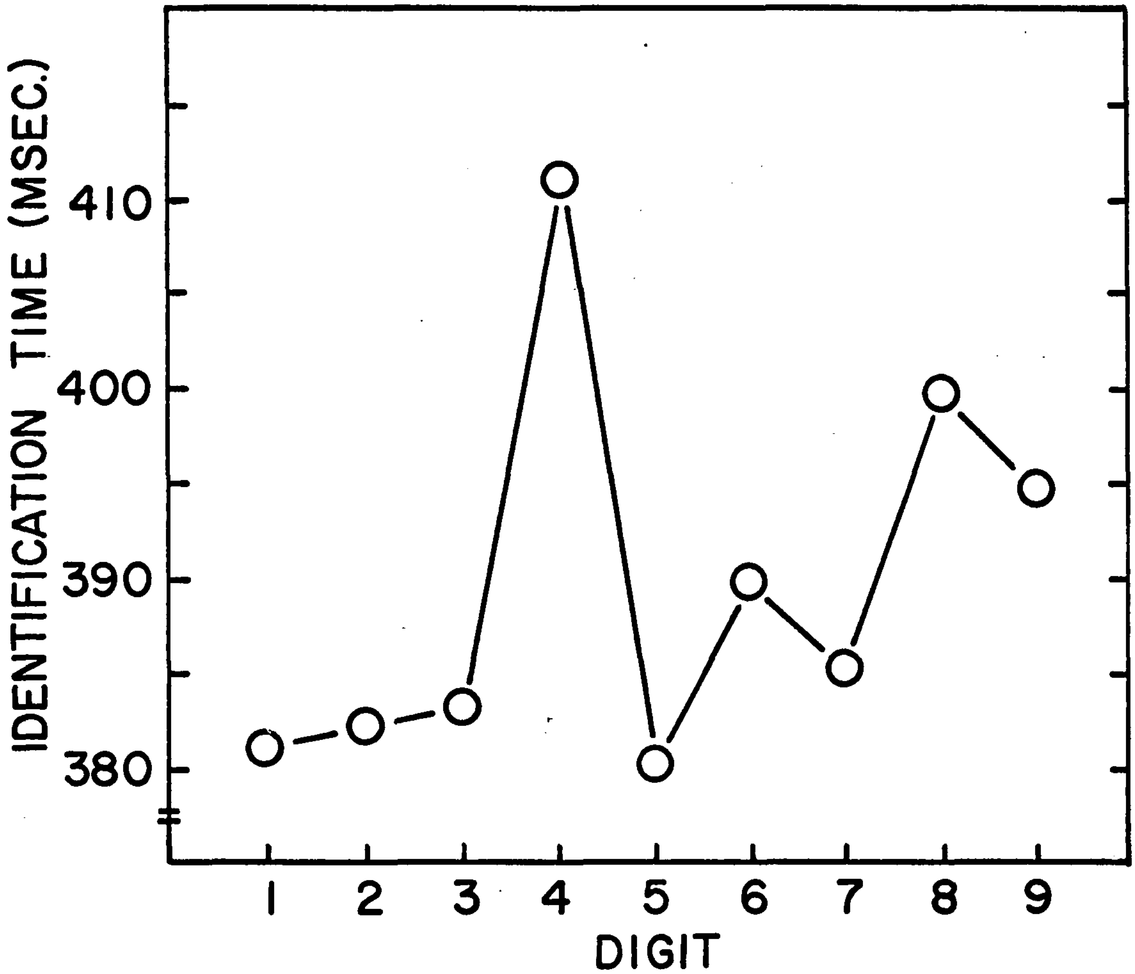

Fig. 20. Mean time required to identify the nine digits.

Experiment Seven: The Time Required to Judge Two Digits as Being the Same

Introduction

As a final check on the equality of speed with which Ss perceive the various digits, an experiment was performed in which Ss had simply to press a button if two presented digits were the same, and to take no action if they were different. Presumably such decisions can be made on the basis of visual information alone; recent work by Posner and Keele (1967) implies that in identifying two letters as identical the name of the letter need not be called up, the matching may be made on the basis of strictly visual information.

Method

Five Ss had the task of pressing a microswitch if two digits presented on the apparatus of Experiment One were the same. Each S judged 180 pairs of digits, 90 of which were identical pairs, each identical pair appearing ten times. The order in which the pairs appeared was random. A Hewlett-Packard 522B counter measured the time between the onset of the identical pairs and the pressing of the microswitch.

Results

Figure 21 presents the mean recognition times for identical pairs as a function of the digits making up the

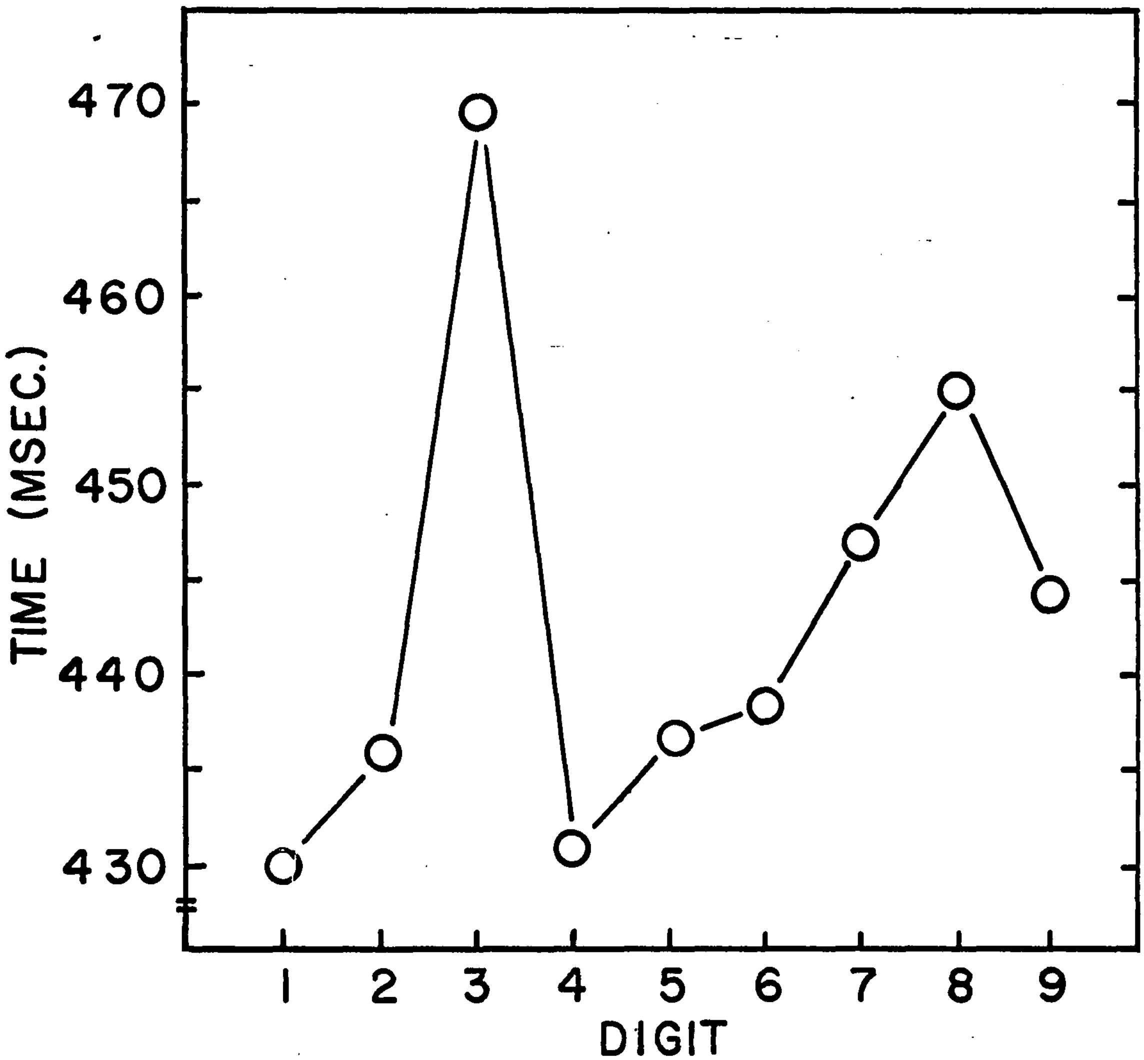

Fig. 21. Mean time required to recognize two digits as the same as a function of the digits presented.

pair. There was little variance between the Ss' overall means, but each individual S varied considerably over the nine digits. The only consistent difference for all nine Ss was the peak at three which shows clearly in Figure 21.

Discussion

The obvious peak at three in the graph was surprising, but its explanation is quite simple. Because of some peculiarity of the readout, the two digits "3" (one at each end of the display) were not as bright as the other eight digits. The readout was capable of presenting up to three digits in a horizontal line; in the experiments where two digits were presented, the end two spaces were used, and in the experiments where only one digit was presented, the center space was used. In the two preceding experiments the center space alone was used, this brightness anomaly did not influence the results of those experiments, since it was confined to the two end digits. The dimness of the two end "threes" was present only in this experiment and in the first experiment.

From Experiments Five, Six, and Seven it is evident that whatever the origin of the MLE, the speeds with which Ss perceive the individual digits involved is not a major factor, since there is no convincing evidence that the speeds are different.

As the final argument against the model derived from North et al. (1967) note that when the data from Experiment One are averaged over all _Ss_ and over all presentations of each different pair (counting the pair appearing in the order A-B as the same as the corresponding pair B-A) and are then grouped to place together all pairs which have a common number of "fast" digits (as discussed in the Introduction), then the mean decision times for the pairs with no fast digit is 516 ms., that for the pairs with one fast digit is 559 ms., and that for the pairs with two fast digits is 471 ms. A one way analysis of variance showed these differences to be insignificant ($F_{2,33} = 2.9$). Furthermore, the direction of the indicated but insignificant differences are not completely consistent with the predictions of the model derived from the North et al. paper.

Experiment Eight: Decision Time for Choosing the Larger of Two Digits Compared With That for Choosing the Smaller of Two Digits

Introduction

Experiment Four indicated that one _S_ was able to select the larger digits more quickly than he was able to select the smaller digits, while the other _S_ showed no such systematic difference. As mentioned in the Introduction, adaptation-level theory leads to a prediction that it should take longer to select the smaller digits than to select the larger digits. If numbers are indeed encoded as analog magnitudes then such an AL effect might well be observed. This experiment, by ignoring the decision times to individual pairs and concentrating on the difference between selecting large and selecting small, makes such differences maximally apparent.

Method

The full list of 72 possible non-identical pairs of digits was randomized eight times. Two complete random orders of these digits appeared on each of four spirit duplicating masters, yielding 144 pairs of digits on each master. Twenty-four copies of each master were made, and twenty-four booklets of four pages each were made up. Each booklet had the four pages arranged in a different order, thus all possible orders of pages were used. On each page, six columns of twenty-four pairs appeared, the digits of each pair were typed three spaces apart, adjacent pairs were separated by eight spaces horizontally and double spacing vertically. Ss had the task of crossing out the larger or the smaller digit, depending on instructions, of each pair as quickly as possible, and were timed for each page. Ss were given a one minute rest between pages. Half of the Ss crossed out the larger digits on the first page, the smaller digits on the next two pages, and the larger ones on the last page. The other half started by crossing out the smaller digits, then did two pages of larger digits, and finally did one page of smaller ones. Thus there was virtually no chance that the order of administration affected the outcome of the experiment. Twenty-four Ss took part, each working alone so that E could time the pages accurately. The dependent variable was the length of time required to complete each individual page. Timing was done with a Clebar brand stopwatch.

Results

The mean time per page for crossing out the larger digits was 89.8 seconds, while the mean for crossing out the smaller digits was 93.3 seconds per page. Seventeen of the twenty-four Ss were faster crossing out the larger digits, and one showed no difference. The times required for the pages with the same instructions (e.g., cross out the smaller digit) were combined to make a two column by twenty-four row data table which was suitable for an analysis of variance. A subjects by conditions analysis of variance (similar to that described in Experiment Three) was performed on these data and showed that the difference between selecting the larger and selecting the smaller digit was significant at the 0.0011 level ($F_{1,23} = 13.91$). The average time difference of 3.5 seconds between the pages may also be interpreted as a difference of 24 ms. per pair, i.e., on the average, Ss were able to cross out the larger element of a pair 24 ms. faster than they were able to cross out the smaller.

Discussion

The larger digits are more quickly selected beyond a doubt, thus suggesting that the adaptation level concept is relevant to the interpretation of the MLE. The results of this experiment alone do not, however, prove that adaptation level is relevant to the problem of selecting the larger or smaller of two digits. The reason that the AL falls at or near the geometric mean of a series of stimuli is thought to be because magnitudes (or other attributes of stimuli) are encoded in a way which makes the internal representation proportional to the logarithm of the stimulus intensity (weight, frequency, etc.). If this is also the case with the encoding of numbers, then the obtained results of this experiment would indeed be as observed. However, it is by no means certain that numbers are encoded as a function of the logarithm of their size, even though the existence of the MLE implies that numbers are encoded as some kind of analog magnitudes, at least when one must perform judgmental operations with those numbers. The following experiment was intended to investigate the possibility that numbers are encoded in a way that is related more closely to their logarithms than to the numbers themselves.

Experiment Nine: The Estimation of the Midpoint Between Two Given Numbers

Introduction

If numbers presented to Ss are coded in a way similar to the way in which AL theory postulates physical magnitudes to be coded (i.e., a logarithmic transformation) then we can predict that the perceived midpoint between two numbers will be nearer to the geometric mean than to the arithmetic mean of the given numbers. However, if college age Ss are asked to find the midpoint between two numbers, most of them know enough to attempt to find the arithmetic mean. There is some evidence that when it is manifestly impossible to compute the mean, Ss tend to underestimate it (Parducci et al., 1960), but unfortunately the report of the study in which this phenomenon was observed did not include a discussion of the suitability of the geometric mean as a possible substitute for the arithmetic mean. The present experiment was intended to determine whether the arithmetic mean or the geometric mean was the better predictor of the "perceived" midpoint between two two-digit numbers for a group of college students who were supposedly constrained from calculating the arithmetic mean.

Method

Thirty-three Ss took part in a group administration of this experiment. E distributed experimental sheets with twenty numbered spaces on them and the following instructions at the top:

> This is an experiment on guessing or estimating a middle number when one is given two unequal numbers. The experimenter will read off two numbers, each between ten and one hundred, and you are to write down the number you think is in the middle between them. DO NOT USE ARITHMETIC. Just use hunches, guesses, and estimates. You will have only about three seconds in which to make your choice and write it down, so you must work just as fast as you can. Just before presenting each pair, the experimenter will say STOP. If you are completely unable to produce any number at all that is between the two given numbers, then draw a line in the space, thus: 14) ——— . This should, however, never be necessary, since *any* number which falls between the given numbers is better than none.

After asking whether or not there were any questions (there were none), *E* read off the following list of number pairs, one pair every five seconds:

19-63
82-56
18-66
68-25
17-81
98-64
32-75
98-25
35-90
38-13
44-82
76-43
46-71
88-57
45-81
92-24
17-42
78-46
29-84
49-22

These pairs alternate between larger number first and smaller number first, and were selected from a random number table with the constraints that no number be less than 11, that the difference between the members of the pairs be at least 20 (to ensure separation of geometric and arithmetic means), and that there be no immediately obvious mean value for any pair. (E.g., had the pair 30 and 70 come up it would not have been included in the list.) Ss did not know the purpose of the experiment.

Results

The results from all thirty-three Ss were averaged for each pair (omitting data which did not fall between the presented stimulus numbers). Figure 22 presents the geometric means of the pairs, the arithmetic means, and the average "midpoint estimates" arranged arbitrarily as the geometric means increase. Although it appears that there is a systematic tendency to underestimate the real midpoint, the arithmetic mean still provides a better predictor of the estimated midpoints than does the geometric mean. The sum of the squared deviations from the geometric mean is 448.7, while that from the arithmetic mean is 225.6.

Discussion

This experiment does not support the contention that numbers are coded and dealt with in a manner

Fig. 22. Geometric means, arithmetic means, and midpoint estimates.

Data arbitrarily arranged with geometric mean increasing. Arithmetic means shown by upper curve, geometric means shown by lower curve. Dotted line represents midpoint estimates.

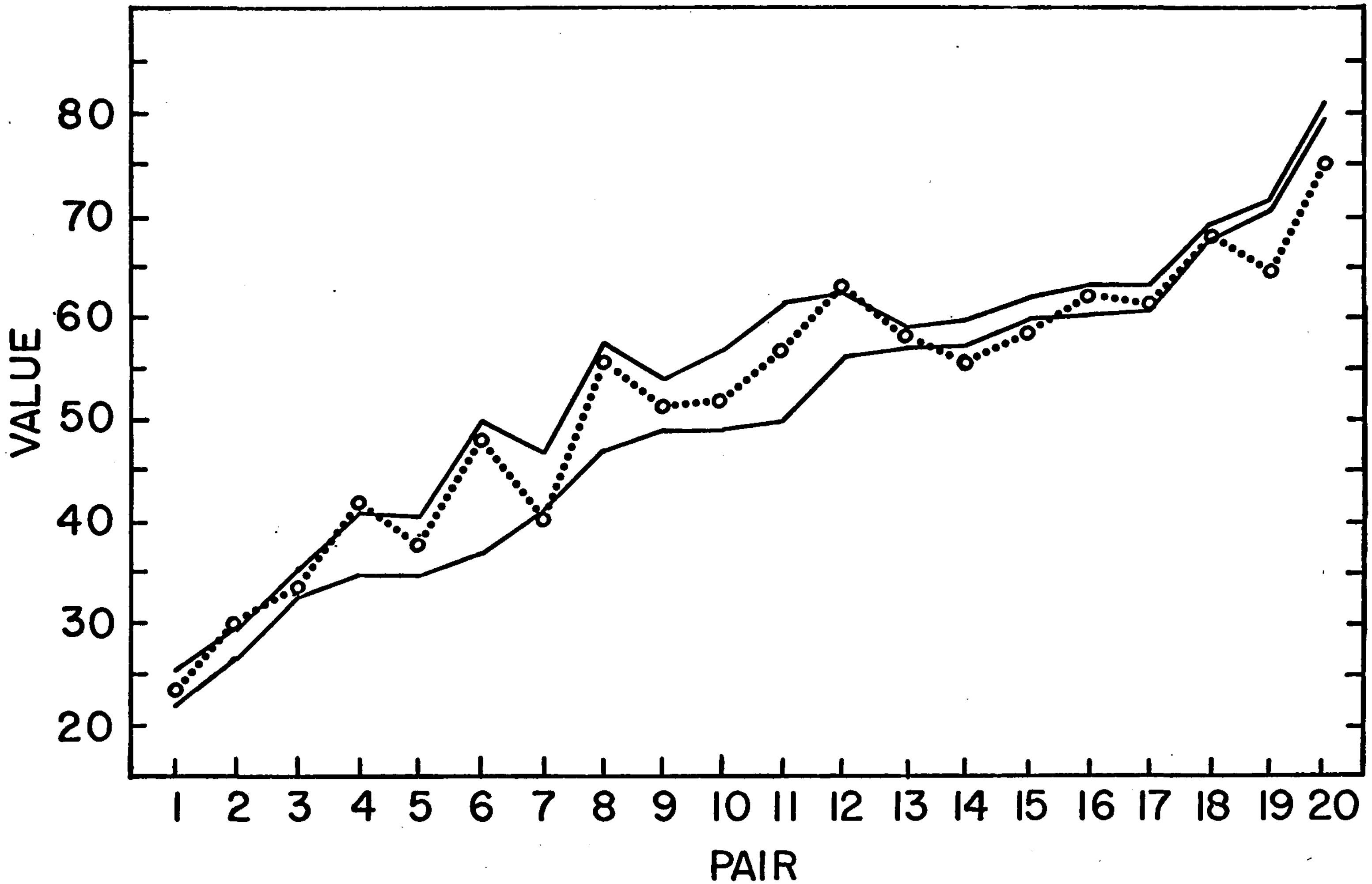

Fig. 22. Geometric means, arithmetic means, and midpoint estimates.

consistent with an internal logarithmic transformation. However, it does indicate that the mean is generally underestimated, as Parducci et al. (1960) found. While the data are not sufficiently conclusive to draw definite conclusions, it appears that especially when the arithmetic and geometric means differ widely the arithmetic mean provides the better predictor of the subjectively estimated midpoint. When the geometric mean is close to the arithmetic mean the tendency to underestimate the arithmetic mean naturally drops the estimate closer to the geometric mean. In short, this experiment has shown that numbers are probably not dealt with internally as if they were coded accurately in a linear fashion, but the assumption of a logarithmic coding is still less predictive than that of linear coding.

Experiment Ten: The Alphabetic MLE and the Choosing of the Letter Nearer A or Nearer Z

Introduction

Experiment Two established the existence of a MLE for letters, while Experiment Eight demonstrated that choosing the larger number was faster than choosing the smaller number. If the processes by which we select the correct letter are completely analogous to those used in selecting numbers, as Experiment Two suggests, then selecting the letter nearer Z would be faster than selecting the letter nearer A. This experiment tests that possibility.

Method

Twenty-four test booklets of four sheets each, arranged in the twenty-four different possible orders constituted the stimulus materials. Stimulus lists (one per page) were made up of 108 pairs of letters per page. Seventy-six of the pairs consisted of one letter from the first six letters of the alphabet and one from the last six, while the remaining pairs contained two letters from either the first six or last six letters. No identical pairs were used. The pairs were arranged in random orders, a different order for each page. Twenty Ss took part; they were divided into two groups of ten. As in Experiment Eight, half of the Ss worked in an AZZA order, and half worked in a ZAAZ order. The Ss had one minute in which to cross out the letters nearer A or Z as the case may have been, then a one minute rest, then another page, and so on through the four pages.

Results

On the sheets where the task was crossing out the letters nearer A, Ss completed a mean of 73 pairs. When working on the letter nearer Z, they completed an average of 66. This difference was subjected to test in a subjects by conditions analysis of variance, similar to the one described in Experiment Eight. The difference proved to be marginally significant, $F_{1,19} = 4.78$, which is significant only at the 0.05 level.

Discussion

The parallels between choosing letters and numbers break down in this situation. No obvious explanation is apparent, and the Ss themselves were not able to articulate why choosing the letter nearer A was faster. There was general agreement that it was easier, but no explanation of why. Most people are generally more familiar with the first part of the alphabet because of serial ordering of outlines, papers, etc., but this is far from being an explanation. In brief, this experiment shows merely that while there may be similarities between numerical and alphabetic MLEs, the underlying decision processes are not identical.

Experiment Eleven: Judgment of Numerical Inequalities on the Basis of Visual Information Alone

Introduction

As pointed out in the Introduction section, Smith (1968) in his review of choice reaction time literature makes the distinction between experiments in which there is a one-to-one mapping between stimulus and response and those in which there is not. In all of the experiments so far described in this dissertation there has not been such a one-to-one mapping. Instead, the correct response has depended upon a relation between the two stimuli. Experiment Eleven is included to investigate the effects of delta on the decision time when there is a one-to-one mapping between stimulus and response.

To demonstrate that the delta would not govern the decision time if one was able to choose the larger digit without actually making a quantitative comparison, the following small study was run. The idea was that given only the pairs 8-9 (and 9-8) or 9-1 (and 1-9) and told to cross out the larger number, the decisions would be equally fast, since there is in both cases a one-to-one mapping between stimulus and response, i.e., _9_ was always to be crossed out.

Method

Ten each of four sheets were prepared; two sheets had one hundred 1-9 and 9-1 pairs in random orders, while the other two sheets bore the pairs 8-9 and 9-8 in random orders. The sheets were assembled into packets of four; five packets in an ABBA order and five in a BAAB order. Ten _S_s were told simply to cross out the larger number of each pair. They were given fifteen seconds for each sheet and instructed to work as quickly as possible. Fifteen second rests were provided between successive sheets.

Scores were obtained by adding the number completed on the first sheet and last sheet, and on the middle two sheets.

Results

Nine subjects completed more of the items on the pages with the 9-1 pairs, while one subject showed no difference. This difference, as tested by a sign test, is significant beyond the 0.01 level. Because of this clearly significant sign test finding, an analysis of variance was not performed on these data. In the fifteen seconds allowed, an average of 2.5 more pairs was completed on the 1 9 sheets than on the 8 9 sheets.

Discussion

The rejection of the null hypothesis of no difference may mean that the difference between the values of the numbers in a group of pairs (i.e., the delta) affects the decision time even when only one pair is presented many times. One S summed up the feeling of many, however, when she said "You don't have to make a decision, you just have to look and see where they [the 9s] are." Without further testing it would be impossible to determine exactly what is causing the observed difference, but in this case it seems likely that the problem of visual discrimination may be playing a large part. The typewritten 8 is more similar to the typewritten 9 than is the typewritten 1, hence if the

_S_s are making a purely visual discrimination the results obtained would be the results expected.

CONCLUSIONS

The experiments described have covered a wide range. From an initial attempt merely to replicate the work of Moyer and Landauer to a consideration of how the human information processing system handles items drawn from serial lists and more particularly how it processes numbers, the experiments have not led to one grand conclusion, but rather have illuminated several related points.

One fundamental question is whether or not Moyer and Landauer's (1967) original experiment stands replicated. Moyer and Landauer reported a correlation coefficient of -0.63, which accounts for a rather high 40% of the variance in the decision times. In Experiment One of this report the correlation coefficient was only -0.30, which accounts for a mere 9% of the variance in decision times. "Why the difference?" is an appropriate question, given the assertion that Moyer and Landauer's experiment has been replicated. Taking the mean for all decision times for all subjects for each delta in Experiment One will yield one grand mean decision time for each delta, and nine pairs of numbers (deltas and decision times). The correlation coefficient for these nine data pairs is -0.97. In general, the more data one averages before computing a correlation coefficient, the higher the correlation will be, assuming that a linear relation is initially masked by high error variance. While Moyer and Landauer do not mention having averaged any of their data before computing the correlation, neither do they specify that the correlation coefficient was run an raw data. It therefore seems a reasonable assumption that they performed some preliminary averaging before computing the correlation coefficient. An averaging of, for example, the three decision times for each stimulus pair for each <u>S</u> would reduce the variance not due to deltas and raise the correlation coefficient somewhat.

The alternative assumptions, that Moyer and Landauer's subjects were inherently less variable, or that there was less variability in their raw data due to methodological differences are plausible but not wholly tenable. Generally speaking, the error variance of response latency measures (reaction times) increases as the latency increases, thus if one group of latency measures has a lower mean than a second group, the first group will usually have a smaller error variance as well. If this is true for decision times as it is for simple reaction times, then the data from Experiment One should have less error variance (and hence higher correlation coefficients between decision time and delta) than Moyer and Landauer's data (Figure 7 shows the overall means as being lower for Experiment One than for Moyer and Landauer's original experiment).

In short, Moyer and Landauer's correlation of -0.63 accounts for four times as much variance as this investigator has been able to account for with a correlation of -.30. The differences may be due either to greater experimental precision on the part of Moyer and Landauer, or to their having averaged out some of the variance in their data before computing the correlation coefficient. Given only their one report, it is impossible to say definitely which, if either, is the correct reason. However, since the correlation in Experiment One is clearly significant, the MLE is replicable.

The major unresolved theoretical question involving the MLE concerns the nature of the equation which best predicts decision time as a function of the two stimulus digits. The exact type of equation most suitable to describe the data illustrated in Figure 11 is indeterminate. If we assume that decision time (DT) varies inversely with delta, then an equation of the form DT = (K/delta) + C may be appropriate. This equation, if appropriate, would imply that a graph of decision time as a function of the reciprocal values of delta would be a straight line. However, such a graph showed the relationship to be nonlinear, although it was more nearly linear than a graph in delta and decision time (Figure 11).

Alternatively, assuming that as delta increases, DT decreases at a negatively accelerated rate, then an equation of the form DT = K(delta)^a + C might be appropriate. Actually, the constants for several forms of equations were derived to approximate the data illustrated in Figure 11, using the methods described by Lewis (1966, ch. 4). Of the functions tested, the one providing the best fit turned out to be the following:

$$DT = (98.2)e^{(-0.31)delta} + 426$$

The sums of the squares of the differences between the observed data and the values calculated from this empirical equation is 5.45, thus indicating that the equation provides an excellent fit of the data points. This empirical equation with its calculated constants regretably does not rest on any theoretical derivation. The curve of Figure 11 is, in a sense artifactual, since the decision times depend upon more than the deltas alone.

Figure 23 shows the mean decision times for all pairs with a delta of four, with the data taken from all sessions and both subjects of Experiment Four (Table 5). Since, as Figure 23 shows, decision time clearly depends on far more than the delta alone, Figure 11 represents an unjustifiable oversimplification of the situation. Any equation which describes the data must take into account at least the size of one of the two digits, and preferably that of both. Moyer and Landauer suggested the equation DT = K log (larger/delta) to explain their data, asserting

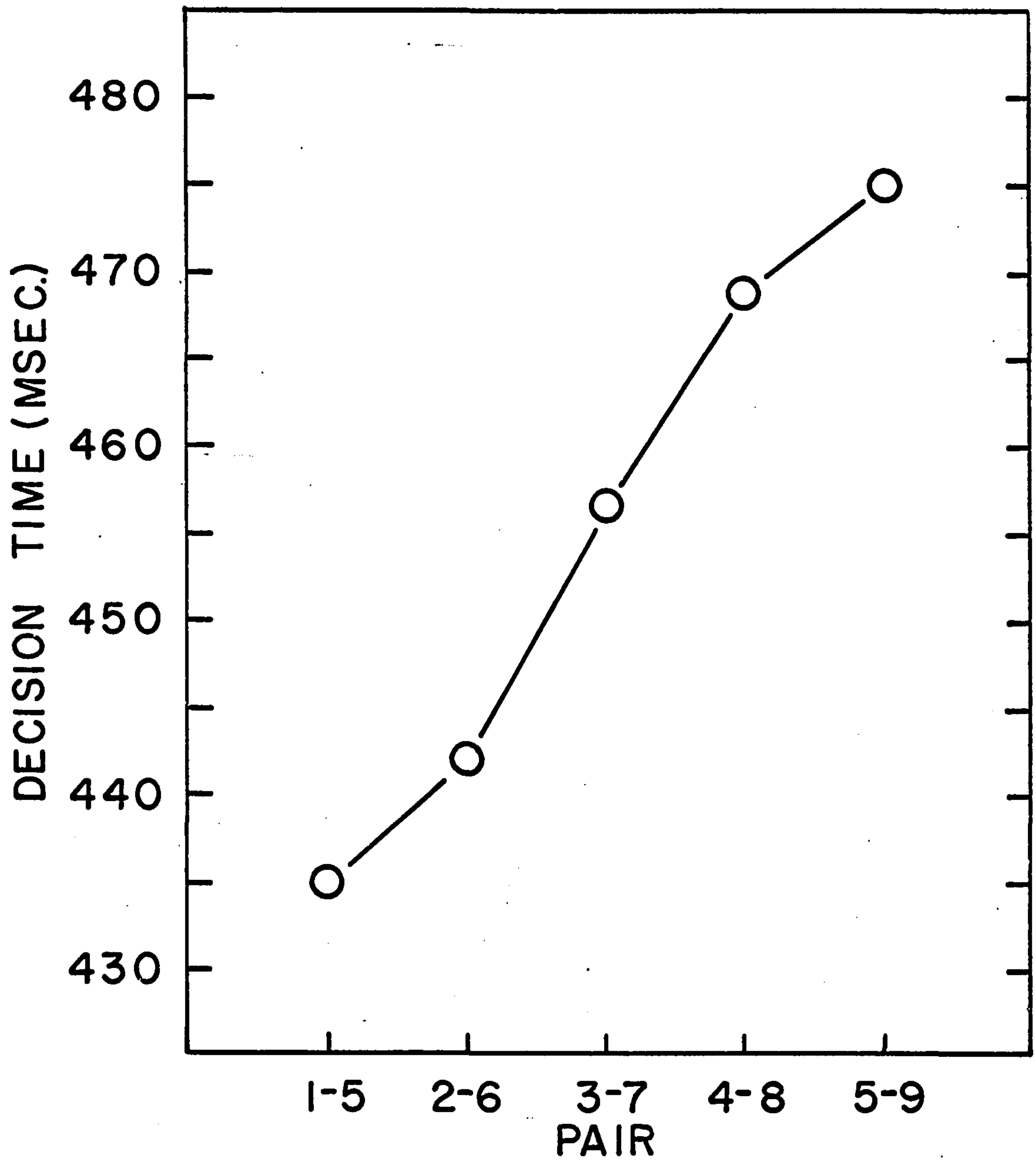

Fig. 23. Decision time as a function of the five pairs with a delta of four.

Data from Experiment Four.

that such a model yields a correlation of 0.75 for their data. But again, they do not specify whether or not the data had been averaged.

The equation which Moyer and Landauer suggest might well be the one most appropriate to describe the data, but Figure 7 illustrates that the effect of the larger digit on decision time is much less than the effect of the smaller digit. The data from Experiment Four were arranged into a graph similar to Figure 7, and showed similar trends except that the initial wavering of the curve relating DT to the size of the larger digit was eliminated, and that curve showed an overall positive slope, although a much smaller slope that of the curve relating DT to smaller digit size. A postulated equation such as Moyer and Landauer's which virtually neglects the contribution of the smaller digit in determining DT is probably less predictive than one which gives weight to such a term.

The curve which Moyer and Landauer propose does nonetheless provide a reasonably good approximation of the data. Since this curve is the same as that used to describe latencies in choice reaction time tasks where the stimuli are actually of different physical magnitudes, it is reasonable to infer, as Moyer and Landauer do, that numbers are represented internally in some kind of analog form. Given the present state of our knowledge it is not possible to say what form the internal representation of numbers takes.

Fechner (see, for example, Corso, 1963) advanced the notion that stimuli, in general, are represented internally on the basis of a logarithmic transformation of intensity (weight, luminance, pressure, etc.). Stevens (e.g., Stevens, 1961) has proposed a power law which postulates that the internal representation of a stimulus magnitude is a power function of the physical magnitude of the stimulus. If either of these views on the nature of stimulus magnitude transformations for internal representation is correct, then perhaps the equation which predicts decision time as a function of the digits in the presented pair will involve the logarithms or a power function of the stimuli presented. Whether or not such equations would prove more predictive than the Moyer-Landauer equation (DT = K log [larger/delta]) remains an area for further investigation.

If the internal coding of two-digit numbers is carried out in the same way as the coding of one-digit numbers (unfortunately there are no data to indicate whether or not this is the case), then data from Experiment Nine might give some indication of the magnitude of the exponent, if any, used in internal coding. One interesting model for predicting the "perceived midpoint" of the stimulus pairs given in Experiment Nine is as follows. First assume that the stimulus numbers in Experiment Nine are represented internally as analog variables of some kind whose magnitude is determined by the stimulus value raised to some exponent, call it P. Then assume that to find the "midpoint" number, S takes the true mean of the internal representation values, and finds the P^{th} root of that mean. Symbolically, let the perceived midpoint equal M, let the stimulus values equal X_1 and X_2, and the expression becomes

$$M = \left(\frac{X_1^P + X_2^P}{2} \right)^{1/P} \qquad \text{(equation 1)}$$

Since Experiment Nine gives actual values for X_1 and for X_2, as well as empirical values for M, the only unknown in equation 1 is P. Raising both sides of the equation to the power P, multiplying through by 2, and rearranging terms gives

$$X_1^P + X_2^P = 2M^P \qquad \text{(equation 2)}$$

The solution of such an equation for each of the twenty stimulus pairs and estimated midpoints in Experiment Nine would give twenty estimates of P; if these clustered about one value, the model postulating internal representation of numbers as powers of the stimulus numbers would be strengthened.

If one assumes a P value of 0.67 and calculates the midpoints on the basis of equation 1, then the resulting computed "midpoints" provide a better fit to the data of Experiment Nine than does either the geometric or the arithmetic mean. The sum of the squared deviations from predictions (i.e., $[\text{observed-predicted}]^2$) is, as reported in Experiment Nine, 448.7 for the geometric mean as a predictor, 225.6 for the arithmetic mean, but is only 183.3 for the use of equation 1 when $P = 0.67$. This value of P was selected only because it is one of the easier values with which to work (values of any number raised to the 0.67 power may be read directly off any slide rule having both K and A or B scales). While solutions of the twenty equations whose general form is given by equation 2 should yield a better value of P, the predictive power of equation 1 with $P = 0.67$ already makes very plausible the idea of coding numbers as analogs whose magnitude is a fractional power of the original number.

The most satisfactory mathematical description of the MLE will quite probably involve the stimulus numbers raised to a power. Whether or not this power will be the same as that which best fits the data from Experiment Nine remains to be seen.

To sum up all that we now know about the MLE and to try to postulate a general predictive equation which takes into account all relevant factors, we must consider the following:

1. A strong linear relationship exists between DT and the smaller digit (Figure 7).
2. DT varies to some extent with the magnitude larger digit (mentioned earlier in Conclusion).
3. There is an irreducible minimum decision time.
4. Decision time tends to vary inversely with the delta. (See discussion of best fitting curve earlier in this section.)
5. Numbers, in some situations, may be coded as analogs proportional to some power of the original number.

An equation which takes into account all of these observations is the following:

$$DT = \frac{A(\text{larger}) + B(\text{smaller})}{\text{larger}^Q - \text{smaller}^Q} + C \qquad \text{(equation 3)}$$

The constants A, B, C, and Q must all be determined empirically, and any of the three A, B, or C might equal zero, but not both A and B. It is also likely that B is larger than A, and that Q is less than one. Either A or B might be negative.

One task for further work in this area will be to find whether or not equation 3 provides a representation of the data which is better than the equation postulated by Moyer and Landauer.

REFERENCES

Corso, J. F. A theoretico-historical review of the threshold concept. Psychological Bulletin, 1963, 60, 356-370.

Festinger, L. Studies in decision: I. Decision-time, relative frequency of judgment and subjective confidence as related to physical stimulus difference. Journal of Experimental Psychology, 1943, 32, 291-306.

Henmon, V. A. C. The detection of color blindness. Archives of Philosophy, Psychology, and Scientific Methodology, 1906, No. 8, 341-344.

Lemmon, V. W. The relation of reaction time to measures of intelligence, memory, and learning. Archives of Psychology, 1927, 15(92).

Lewis, D. Quantitative methods in psychology. Iowa City: The University of Iowa Press, 1966.

Mowbray, G. H., and Rhoades, M. V. On the reduction of choice reaction times with practice. Quarterly Journal of Experimental Psychology, 1959, 11, 16-23.

Moyer, R. S., and Landauer, T. K. Time required for judgments of numerical inequality. Nature, 1967, 215, 1519-1520.

North, J. A., Grant, D. A., and Fleming, R. A. Choice reaction time to single digits, spelled numbers, "right" and "wrong" arithmetic problems and short sentences. Quarterly Journal of Experimental Psychology, 1967, 19, 73-77.

Parducci, A., Calfee, R. C., Marshall, L. M., and Davidson, L. P. Context effects in judgment: Adaptation level as a function of the mean, midpoint, and median of the stimuli. Journal of Experimental Psychology, 1960, 60, 65-77.

Posner, M. I., and Keele, S. W. Decay of visual information from a single letter. Science, 1967, 158, 137-139.

Smith, E. E. Choice reaction time: An analysis of the major theoretical positions. Psychological Bulletin, 1968, 69, 77-110.

Stevens, S. S. The psychophysics of sensory function. In W. Rosenblith (Ed.), Sensory communication. Cambridge: The M. I. T. Press, 1961. Pp. 1-34.

Welford, A. T. The measurement of sensory-motor performance: Survey and reappraisal of twelve years' progress. Ergonomics, 1960, 3, 189-230.

Woodworth, R. S., and Schlosberg, H. Experimental psychology. New York: Holt, Rinehart, and Winston, 1954.