A COMPARISON OF TWO METHODS OF MOTIVATION IN TEACHING FRACTIONS TO FOURTH AND SIXTH GRADE PUPILS

by

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I hereby recommend that this dissertation prepared under my direction by Robert Henry Richter

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ABSTRACT

This study sought to determine what differential in motivation occurs when verbal praise and monetary rewards are compared as distinct methods of motivational reinforcement while teaching fractions to boys and girls of the fourth and sixth grades.

The subjects of the research were 253 fourth and sixth grade pupils of Harelson Elementary School, Amphitheater School District 10, Tucson, Arizona. Gain scores were derived from two pretests and two posttests, based on one of each for part one of the study; addition and subtraction of fractions (four weeks) and part two of the study, multiplication and division of fractions (four weeks).

Four fourth grade classes and four sixth grade classes were pretested, taught addition and subtraction of fractions for four weeks, and then posttested. The teachers exchanged classes, pretested, taught multiplication and division of fractions for four weeks, and then posttested. All students were taught under both methods of motivational reinforcement, either in part I of the study or part II. Rewards were uniformly awarded under both methods for daily worksheets successfully completed.

Analysis of variance for the mixed design, two between- and one within-subjects variables with one factor

a repeated measure, was used in the study. A questionnaire was presented to the subjects to determine their attitudes toward the two methods.

The study produces the following results:

1. The gain scores, although consistently in favor of the monetary rewards, were not significantly different from those of verbal praise.
2. There was no significant difference in the gain scores of boys and girls under the two methods of motivation, for boys and girls at fourth grade level and for boys and girls at sixth grade level.
3. The differences in the gain scores of the fourth grade when compared to those of the sixth grade were significant. Fourth grade students performed as well under either method of motivation. Sixth grade students performed better under the monetary reward method than they did for verbal praise.
4. Almost half of all students indicated that they enjoyed the monetary reward better than verbal praise. More sixth grade than fourth grade students indicated enjoyment of monetary reward. More boys than girls indicated enjoyment of monetary reward.
5. All students indicated they worked harder for monetary reward than for verbal praise.
   a. There was little difference between grades as to how hard they felt they worked for either method.

b. Boys felt they worked harder for the monetary reward than did girls.

6. Both boys and girls in both grades felt that neither monetary reward nor verbal praise produced more work on a comparative basis.

7. Neither method of motivation appeared to cause undue pressure on the subjects. Students indicated they did not worry and were happy with both methods.
   a. Fourth grade subjects indicated a slightly greater tendency to worry than sixth grade subjects. Both groups appeared satisfied with either method, although there was a slight tendency to favor monetary reward.
   b. Girls indicated they felt equally satisfied with either method, while more boys tended to favor the monetary reward and showed less satisfaction with verbal praise.

8. More students indicated a preference for the monetary reward over the verbal praise.
   a. More sixth grade pupils indicated a preference for the monetary reward and less for the verbal praise than did the fourth grade.
   b. Girls indicated practically no preference for one method over the other. The largest percentage of girls indicated no difference. Boys clearly showed a preference for monetary reward.

Very few boys indicated a preference for verbal praise, while approximately one-third felt there would be no difference in preference.

CHAPTER I

THE PROBLEM, ASSUMPTIONS, LIMITATIONS, AND DEFINITIONS OF THE STUDY

The Problem

Introduction to the Problem

Throughout the world, and especially in the United States, there has been a great deal of work done to improve the mathematics curriculum. Attention has been given to the expansion and improvement of courses for elementary teachers in the area of mathematics, and more emphasis than ever before is being placed on the study of mathematics. Methods courses have increased understanding of the psychology of learning, which has helped to reduce fears and anxieties, for both the elementary teacher and the pupils (Glennon and Callahan, 1968, p. 54). In general, there has been significant development in mathematics instruction.

Work and research still remain to be done in terms of mathematical knowledges and understandings needed by people in modern day technological societies. If children are to reach for and achieve these knowledges and understandings, teachers must know what best motivates students toward these ultimate goals. Theories exist concerning motivation which apply to individuals in any activity, but

as Glennon and Callahan (1968) point out, little concrete research exists to substantiate these theories.

What motivates students to achieve or seek goals is often determined by the light in which the student perceives the task to be done. If the task produces pleasure to the student the probability is that he will successfully complete the task. The task, then, within itself provides a reward for the student. Often, when the task is tedious and produces no pleasure, teachers will substitute rewards which are external to the problem. Such a reward consists of approval by the teacher in terms of praise or a reward that satisfies a physical need of the student. The substitute reward acts as a sustaining drive to carry the student through the task in which the intrinsic reward is delayed. In order to perpetuate the learning activity of the student, the teacher continues to reinforce the expected responses by increasing or re-introducing the same or new rewards.

Several different approaches to extrinsic motivation have been featured currently in the literature. Two popular methods often cited are: (1) verbal praise reinforcement and (2) monetary reward reinforcement. In order to obtain clear-cut results it seemed advisable to test and compare these methods at two non-consecutive grade levels, fourth and sixth grades.

Statement of the Problem

It was the purpose of this study to seek an answer to the question: What differential in motivation occurs when verbal praise and monetary rewards are compared as distinct methods of motivational reinforcement while teaching fractions to boys and girls of the fourth and sixth grades in a departmentalized organization.

Significance of the Problem

Although mathematics teachers have been aware, consciously or unconsciously, of the effectiveness of various motivational techniques in their classrooms, little has been done to objectively compare these methods. There are undoubtedly many methods of motivational reinforcement. This study should contribute evidence relative to the efficacy of verbal praise reinforcement as compared with monetary reward reinforcement.

Woodworth and Schlosberg (1965) suggest, "Both ability and motivation are factors in performance and if either of them is entirely lacking, the performance does not occur. Ability is like a machine which cannot do its work unless power is applied" (p. 655). With this thought in mind it is anticipated that this research will produce evidence to aid teachers in making decisions pertaining to these two methods and their use in classroom situations. Furthermore, it may prove useful in assisting teachers to

better understand differentials which may exist between one motivational approach and another on a grade level basis.

Hypotheses to be Tested

The following hypotheses which ordered and provided direction to this study were tested:

1. There will be a difference in the mathematical achievement scores of the total population of the pupils of this study when the verbal praise method of reinforcement is used and compared to the use of the monetary reward method of reinforcement.
2. There will be a difference in the mathematical achievement scores of girls as compared to the mathematical achievement scores of boys with respect to (a) the use of the verbal praise method of reinforcement and (b) the use of the monetary reward method of reinforcement.
3. There will be a difference in the mathematical achievement scores of the fourth grade girls as compared to the mathematical achievement scores of fourth grade boys with respect to (a) the use of the monetary reward method of reinforcement and (b) the use of the verbal praise method of reinforcement.
4. There will be a difference in the mathematical achievement scores of sixth grade girls as compared to the mathematical achievement scores of sixth

grade boys with respect to (a) the use of the verbal praise method of reinforcement and (b) the use of the monetary reward method of reinforcement.

5. There will be a difference in the mathematical achievement scores of the fourth grade pupils as compared to the mathematical achievement scores of the sixth grade pupils when considered with respect to (a) the use of verbal praise method of reinforcement and (b) the use of the monetary reward method of reinforcement.
6. The students in the study will indicate a difference in the amount they enjoyed verbal praise in comparison to monetary reward when considered as (a) a total population (all students), (b) sixth grade to fourth grade, and (c) boys to girls.
7. The students in the study will indicate a difference in how hard they worked under verbal praise and monetary reward when considered as (a) a total population (all students), (b) sixth grade to fourth grade, and (c) boys to girls.
8. The students in the study will indicate a difference in the amount of work they performed under verbal praise and monetary reward when considered as (a) a total population (all students), (b) sixth grade to fourth grade, and (c) boys to girls.

9. The students in the study will indicate a difference in the pressure (positive or negative) felt under verbal praise and monetary reward when considered as (a) a total population (all students), (b) sixth grade to fourth grade, and (c) boys to girls.
10. The students in the study will indicate a preference for verbal praise method or monetary reward method when compared as (a) a total population (all students), (b) fourth grade to sixth grade, and (c) boys to girls.

Assumptions Underlying the Problem

The following assumptions were made in this study:

1. That the children in this study will respond naturally and in a representative manner.
2. That whereas all children are not able to achieve to the same degree, their performance will change in accordance with how they are motivated.
3. That motivation and achievement are related in teaching arithmetic to children.

Limitations of the Study

This study was limited in the following ways:

1. It was limited to the study of two incentive methods, verbal praise reinforcement and monetary reward reinforcement.

2. It examined only those boys and girls of Harelson Elementary School in the fourth and sixth grades.
3. These students were representative of middle and upper-middle class socioeconomic groups.
4. The groupings for this study were made without regard to age, intelligence, or ability.
5. It was limited to two four-week periods during the school year 1970-71.

Definitions of the Terms Used

For the purpose of this study the following terms, 1 through 11, are quoted from Good (1959):

1. Achievement is defined as "accomplishment or proficiency of performance in a given skill or body of knowledge" (p. 7).
2. Achievement, Pupil is defined as "the status of a pupil with respect to attained skills or knowledges as compared with other pupils or with the school's adopted standards" (p. 7).
3. Motive is defined "broadly considered, as any impulse, drive, attitude, whether conscious or not, that arouses, sustains, or regulates behavior" (p. 354).
4. Motivation is defined as

> . . . (2) the practical art of applying incentives and arousing interest for the purpose of causing a pupil to perform in a desired way; usually designates the act of choosing study materials of such a

> sort and presenting them in such a way that they appeal to the pupil's interests and cause him to attack the work at hand willingly and to complete it with sustained enthusiasm; also designates the use of various devices such as the offering of rewards or an appeal to the desire to excel (p. 354).

5. Motivation, External is defined as "environmental determinants of behavior, including a perceived goal object (such as food when hungry) and social incentives (such as a word of praise)" (p. 354).
6. Motivation, Extrinsic is defined as "the application of incentives that are external to a given activity to make work palatable and to facilitate performance, for example, offering a prize to the pupil who makes the highest score in a spelling test as an appeal to the extrinsic desire to excel" (p. 354).
7. Motivation, Group is defined as "aims or purposes that are held in common by the members of a group. Such as the desire of teammates to win a game" (p. 354).
8. Motivation, Internal is defined as "physiological drives and purposes that are located within the organism" (p. 354).
9. Motivation, Intrinsic is defined as the "determination of behavior that is resident within an activity and that sustains it, as with autonomous acts and interests" (p. 354).

10. Reinforcement is defined as the "strengthening of a conditioned response by reintroducing the original unconditioned stimulus; (2) Increase in the intensity or efficiency of a response to a stimulus brought about by the concurrent action of another stimulus" (p. 457).
11. Reward is defined as a "pleasant satisfying experience consequent upon a certain course of behavior and mediated by an external agent or by the self acting as agent in the hope of encouraging the repetition of the behavior" (p. 470).
12. Reward, Monetary is defined as a prize which can be spent or exchanged for goods or services which the pupil may value, and which will encourage him to continue the task at hand, to finish it willingly and see it as an enjoyable experience.
13. Reward, Verbal Praise is defined as words of appreciation used to approve of and encourage the pupil to continue the task at hand, to finish it willingly and see it as an enjoyable experience.

CHAPTER II

REVIEW OF RELATED LITERATURE

There has been a great deal of study in the area of motivation since the turn of the century. Certainly it is conceivable that man has always rewarded and punished children according to how well they achieved. But it is doubtful that much deliberate research has been done before the 1900's. The review of literature in this study will therefore limit itself to the modern day and only that literature which is relevant to the main concepts of motivation with which this research is concerned.

There have been some good reviews done on motivation during this time period. Most of these publications only touch on monetary reward and verbal praise. The first of these was done by Hurlock (1931); a second by Davis and Ballard (1932); one was done by Schmidt (1941); and one by Kennedy and Willcutt (1964).

Literature Concerning Praise

Kirby (1913) displayed individual charts to show students their performance in learning how to add in arithmetic at the third and fourth grade levels. He believed this would encourage the students to improve their

performance and scores. He claimed a 48% gain in the media scores of his students.

Praise was found to be more effective than reproof by Gilchrist (1916) in one of the earlier studies involving motivation.

Chapman and Feder (1917) indicated that there were definite differences between the gains made by groups that were offered small prizes or stars and a control group which received no such incentives. Using fifth grade students he showed that extrinsic rewards proved more effective.

Comparing encouragement to discouragement or repetition while working individually with college students, Gates and Rissland (1923) felt that encouragement was only a little better than discouragement or repetition.

A continuum of research was done by Hurlock. The first of these (1924) concluded that there was little difference between praise and reproof as motivational factors among older children. She also noted that praise seemed to motivate Negro children while verbal reproof seemed more apt to be effective with white children. Hurlock (1925a) again studied the effect of praise versus reproof when testing fourth and sixth grade students on an intelligence test. Either motivational method seemed to be more effective than just practice in itself. Hurlock (1925b) did a study over a longer period of time in which she worked with a praise group, a reproof group, and a group that was

completely ignored. Regardless of the age, sex, ability, or race, the praise group performed significantly better than the other three groups, with reproof second, the ignored group third.

In a study by Warden and Cohen (1931) the investigators felt that any reward or incentive which is a change from an established routine tended to be effective. By trying an incentive for a week at a time and then going to a new one, they determined that neither verbal praise nor reproof were as powerful as was commonly thought. They used promises of games, or a story, or a party, along with praise and reproof to establish their findings.

Davis and Ballard (1932) reviewed the literature and concluded that positive motivational methods were generally more effective than negative ones. Their findings were that, particularly among younger children, praise worked better than reproof.

Working with four groups of children, Chase (1932) studied the effectiveness of several different types of external motivating factors. In contrast with the findings presented by Hurlock, Chase determined that failure motivated to a higher degree than success, but that any motivation, whether positive or negative, is better than none at all.

Rivalry was studied as a motivating factor by Maller and Zubin (1932) and they found that there was no more gain

score by the rival group than for the group which was used as a control. Rivalry caused the students to try more problems on an intelligence test but it also brought about more mistakes.

Anderson and Smith (1933) followed up the Chase research, using the Chase study data. In this study they determined that success-reward was less effective than failure-punishment, and that failure repetition was a little better than success-repetition. Three years later, in 1936, Anderson analyzed the scores again from the motivation study which Chase had begun and, negating the findings by Chase, suggested that success was better in sustaining long range efficiency, and that a better motivational factor was obtained when using success-praise than when using failure-reproof.

Working with seventh and eighth grade groups' intelligence scores, Benton (1936) found no significant gain score differences between four incentives for motivation: praise, encouragement, promise of a prize, and having knowledge of the results before being retested.

Forlano and Axelrod (1937) working with fifth grade students, attempted to distinguish the difference in the gain scores of children with personality traits as a factor. They found that, especially among introverted students, blame was a greater motivator than either praise or indifference.

Schmidt (1941), in his review of the literature on motivational factors, praise and blame, expressed indetermination as to which is the better method. Research had failed to determine a clear-cut argument for one or the other.

In an attempt to identify performance of subjects at ages nine, twelve, fifteen, and eighteen, when subjected to reproof, Potter (1943) tested an arithmetic task and a motor task. He found that at age nine reproof hurt the arithmetic task while the motor task was bettered; at age twelve both task performances were less; and for both ages, fifteen and eighteen, reproof had little effect on the arithmetic or the motor task.

Thompson and Hunnicutt (1944) attempted to build upon the experiment of Forlano and Axelrod (1937) in the study of introverted and extroverted children. In working with fifth grade students with both introverts and extroverts included, they found praise and blame to be equally effective as a motivator; and both of these to be more effective than neither one. Contrary to Forlano and Axelrod, they found praise to be more effective for introverts and blame to be more effective for extroverts. The investigators felt that teachers have a tendency to praise or rebuff students as a group rather than single out a particular child, and that consequently, poor use of praise or

reproof could be harmful to both the personality and achievement of a student.

Negative and positive statements were found to be more effective than neutral statements according to Grace (1948) who worked with children of the sixth grade in which the students performed a simple learning task.

Using praise and reproof as an instrument of motivation with grades three through six while teaching reading, Silverman (1957) found neither to have much effect on the ability to read.

Sandstrom and Weinz (1958) studied effects of praise and reproof with an individually administered experimental task. The reproof group did better but this may have been because of the nature of the praise motivation, where the praise turned out to be more of an irritant than an encourager.

A study by Stevenson and Snyder (1960) revealed that when working with mentally retarded students, failure increased motivation if the subjects were given a level of expectation to aspire to or a comment on their performance being substandard. Failure lowers motivation when only a comment is made on the performance of the student.

According to Dollins, Angelino, and Mech (1960) the use of praise in a fourth grade classroom over a thirty-day period produced no remarkable effect on either the students'

personality or their ability to perform certain arithmetic tasks.

In working with kindergarten and first grade children, Terrell and Ware (1961) noted that an immediate reward achieved greater motivation than a delayed reward. They found the subjects learned more quickly when the reward was immediate.

Also working with kindergarten children, Hicks (1962) asserted that positive reinforcement was decidedly better than negative or no reinforcement at all. He spoke of achievement behavior and indicated that when a child had a history of supportive reinforcement, he was more apt to achieve than if he was accustomed to negative or no reinforcement at all.

Kennedy, Turner, and Lindner (1962) attempted to discriminate a difference between the effect of praise and blame as it relates to average and slightly above average students of the eleventh and twelfth grades. They found that whereas blame inhibited the performance of average or above average students, there was no difference between the blame or praise groups for the high intelligence students.

In a study of praise and reproof with respect to intelligence, Kennedy and Willcutt (1963) worked with fourth graders. They found that intelligence levels and reinforcement seemed to be unrelated. All intelligence levels

performed more effectively with praise than with either reproof or no incentive at all.

To study the effects of praise and reproof on educationally retarded students, Van De Riet (1964) tested ninety underachieving and non-achieving boys from grades four, five, and six. He concluded that praise was detrimental to these underachievers and reasoned that these boys felt they should fail. Praise then came as a threat to their security.

When Kennedy and Vega (1965) studied the effect of a white examiner in comparison to a Negro examiner using the same technique of praise and reproof on Negro students, the effects turned out to be the same for praise, but that reproof was a detriment when used by a white examiner and an implementor when used by a Negro examiner.

Allen (1966) sampled ninety-five kindergarten and ninety-five fourth and fifth grade boys to discover whether verbal support helps the child to remain with a task. She found that for younger children, verbal support reinforced the response while reproof deterred it. An older child will stay with a task longer with criticism than with supportive or no comments. When the task becomes more difficult, the comments become less effective.

In a study of generalized effects of praise in relation to the generalized effects of reproof, Anderson, White, and Wash (1966), when working with college students,

found that the effects of praise or reproof seemed to transfer from one class to another class in terms of performance levels.

It was found in a study by Rosenhan (1966) that with twenty-two Negro and white boys, the lower-class Negroes responded better to approval and worse with disapproval than did middle-class whites. He reasoned that for the lower socioeconomic class Negro whose punishment was usually physical, either verbal approval or disapproval would produce better results.

Review of Literature Concerning Praise

Over the years of testing and retesting it has been evident that there is no great agreement on the consequence of using verbal praise or reproof. Some researchers have found verbal praise better than verbal reproof, while others have reversed this conclusion. Still others feel that there is no difference between the two. The reviews seem to favor the verbal praise method for better results and effectiveness based on achievement. Certainly the later studies gave evidence that there are other factors involved. Age and grade have produced variations in the results. Socioeconomic and environmental background seem to affect the outcomes. Intelligence and ability grouping have certainly been considered variables of these studies. As the findings arrived at in the literature were reviewed there was always

present the feeling that there may be still other variables worth consideration. Perhaps novelty of method or teacher influence are variables that have not been considered. In any case, the need for further study is indicated.

Literature Concerning Material Rewards

The study of material rewards did not start simultaneously with the study of verbal praise. Before the thirties, few formal studies had been made in this area other than a few individual observations and some study with college students.

The act of rewarding with money did not turn out to be directly proportional with learning according to Thorndike and Forlano (1933). They concluded that the learning rate was increased as the money reward was increased only up to the point where the tension or excitement would interfere with learning and then the reward would become a detriment.

In learning a sensori-motor task, Abel (1936) felt that when students understood the goal and results to be achieved, learning was brought about more readily. It was still more effective when verbal praise was added to that understanding, and more effective still if material rewards were offered beyond the first two.

In a study relevant to the present study Klugman (1944) examined intelligence test results of seventy-two

white and Negro children of grades two through seven. He found that for the tests there were no differences in their reliability for either verbal praise or money reward. For the Negro children, however, he found that the money reward produced better results on the intelligence test than when verbal praise was used.

Douvan (1956), in a series of tasks, reinforced with a reward of ten dollars as opposed to a gain of personal satisfaction, while working with lower and middle-class students. Middle-class subjects performed as well when they were told they achieved as when they were offered a sum of money. Lower-class subjects performed less well in the absence of the monetary reward as compared to the personal satisfaction. The investigator felt that the values and attitudes that the subject had when he started the tasks determined his "pattern of achievement."

In another related study Terrell and Kennedy (1957) investigated the effect of verbal praise, reproof, immediate candy reward, delayed candy reward, and a token reward. They presented a discriminatory task to eighty four- and five-year-old children. The outcome, in order, was that the candy group learned more quickly than the others, then the praise group, the token reward group, the reproof group, and last the control group.

Terrell (1958), in a follow-up study to the one in the previous paragraph, worked with a different

socioeconomic group. Whereas he and Kennedy had worked with a rural group of children, he now experimented with a group of middle-class environment children. The investigator used four incentive methods to study two types of delayed incentives. The children were given, in accordance to their grouping, an immediate candy reward, a reward manifested by transferring a bean from one jar to another which could later be traded for candy, a promise of a candy reward, and for the control group a light which would flash when a correct response was made. There was no appreciable difference between any of the groups which was in direct conflict with the previous research. He felt strongly that the difference must be related to the difference in the two socioeconomic groups used in the study. To test this he performed a follow-up research with another rural group. The results led him to reason that the reward presented may be related to the socioeconomic background of the subject; if subjects were used to having their promises fulfilled, they would learn more quickly under this method of reinforcement than subjects whose promises had not been fulfilled.

Terrell, Durkin, and Wiesley (1959) tested five- and six-year-old groups, and ten- and eleven-year-old groups to determine the comparative effectiveness of material reward versus non-material reward. The one group was rewarded, non-materially, by a light flash for correct responses,

while the other group received a piece of candy for their efforts. These lower-class subjects responded more favorably for the candy than they did for the non-material reward. The investigators decided that lower-class children respond to the materialistic things to which they can relate, and that the value of learning as a symbol of prestige has little motivation for them.

Working with three different reward methods, Miller and Estes (1961) attempted to discover differences in performance of seventy-two boys between the ages of nine to eleven. They gave a one cent reward, a fifty cent reward, and a knowledge of their results as motivation reinforcement. Oddly, both monetary reward groups did less well than the group which received knowledge of the results. The fifty cent reward may very well have interfered with the learning process as Thorndike and Forlano (1933) suggest, but it is doubtful that the one cent reward would have caused much excitement.

In a study which attempted to differentiate results between verbal motivation and prize motivation, Smock (1962) identified slow-learning, average, and bright children. He gave a test under normal conditions to 1,160 children whose ages ranged from nine to twelve. In the second part of his investigation he told one-half of these children to do as well as they could, while the other half were offered a variety of little prizes for doing well. The

average and high intelligence subjects did better under the prize reinforcement, but it was noted that as age increased so did the scores in general, under verbal motivation. Differences due to sex, socioeconomic, and intelligence rankings were not remarkable.

Hypothesizing that the difference in intelligence test taking was due to motivation (or the lack of it), Tiber and Kennedy (1964) used 480 second and third grade children of middle-class and lower-class, both white and Negro, to determine score differences. The investigators used four motivation methods: verbal praise, verbal reproof, candy reward, and a control method (nothing). There turned out to be no significant difference between any of the subgroups, and the hypothesis was rejected.

Storm (1965) attempted to determine differences in the use of material and non-material rewards with two separate ethnic groups of New Zealand: Thirty native lower-class (Maori) and thirty middle-class children of European descent (Pakeha). There seemed to be no difference between the groups of ten- and eleven-year-olds, while the Maori children of the five- and six-year-old group responded better for the material reward when they performed the manipulatory speed task.

In four studies concerned with the resulting performance of subjects as rewards were administered, Toppen (1966) examined the following aspects: (1) the magnitude of

the reward, (2) the timing, and (3) the contingency upon the performance. He observed that humans respond much like animals when they are offered rewards. As the value of the prize is enlarged, the response performance goes up. The effort expended grows with the incidence of reward.

Hollander (1968) studied four incentives: verbal praise, verbal reproof, a candy reward, and no reinforcement whatsoever. These were examined with respect to speed and accuracy displayed by fifth and sixth grade students in performing a simple arithmetic task. Her study produced the following conclusions: (1) fifth and sixth graders worked faster for candy; (2) girls worked faster than boys; (3) fifth and sixth graders worked more accurately for praise; (4) verbal reproof method produced less work and less accuracy than any of the other methods; (5) girls attempted more problems than boys; (6) girls completed more problems than boys; (6) girls completed more problems correctly than did boys in the candy reward group; (7) sixth graders responded to praise more favorably than fifth graders; and (8) in the candy reward group, more fifth graders attempted more problems than the sixth graders.

In the June, 1970, issue of the Phi Delta Kappan magazine, Elam reports that RLC (Rapid Learning Center), a private business organization, is now operating in Texarkana, Arkansas, under a five year, five million dollar dropout prevention project supported by the United States Office of

Education under Title VIII (dropout prevention) of the Elementary and Secondary Education Act. The RCL managers and assistant managers (the name teacher is not used) not only try to make learning fun and games, but give material rewards, such as prizes of transistor radios, television sets, and green stamps. When students complete lessons correctly they receive ten green stamps. Iowa Tests of Educational Achievement is used to measure grade-level advancement. If grade-level advancement is achieved, the student receives a transistor radio. The student who makes the best advancement in grade-level receives a television set. One winner advanced 5.1 grade levels in mathematics and 3.2 grade levels in reading, during a three-month period (this would seem to be in agreement with Toppen's study). Many other forms of reward are used by RCL such as games, puzzles, and popular magazines. It is interesting to note that although favorable claims have been made for RLC methods, there was a serious flaw in their research design. The article states that there was a failure to properly match the control and the treatment groups of the project.

Review of Literature Concerning Material Rewards

The studies reviewed seem to indicate that the recognition of student accomplishment, in some form, is a necessary item if pupils today are going to achieve to their capabilities. Generally speaking, the material rewards seem

to be more effective than non-material rewards. The values which the student carries to the task, as determined by his socioeconomic background seem to be important in terms of how well he will react or achieve under the different methods of motivation. Lower-class children seem to react well to material rewards, while middle income groups and more mature individuals seem to react well towards intrinsic rewards. There is an indication that a definite relation exists between low socioeconomic class groups and low achievement motivation groups.

It is still evident that the research has not answered all of the necessary questions concerning motivation. Most of the research has been done with test scores from learning that had already been done. Posttests immediately followed pretests which would mean if a subject's best effort were recorded on the first test, he could not present a better score on the next test even though he might be motivated to do so. Other studies worked with motor performance and skill acts of learning rather than the acquisition of knowledges and understandings. The present study will attempt to eliminate some of these weaknesses by first pretesting, then teaching, and then posttesting. It will be by groups rather than individuals, and in accordance with how much is learned under each motivational method rather than how the students test under each method. Attention will also be given to the students' perceptions of how

he feels regarding the material reward versus the verbal praise.

CHAPTER III

ORGANIZATION AND DESIGN OF THE STUDY

Population for the Study

This study was conducted at Harelson Elementary School. The entire fourth and sixth grades participated as subjects for the two method approaches to instruction. Harelson School is a member of the Amphitheater School District, Number 10, Tucson, Arizona.

The Amphitheater Schools serve some 7,500 children through seven elementary schools, two junior high schools, a special education department, and two high schools. The district is located in the north of Tucson and in Pima County. Seven of the eleven schools in the district are located within the City of Tucson. Harelson is located out of the City to the north.

The education of its children has long been the prime concern of this district. Although the area encompassed is great, the population is only approximately 45,000. Despite a relatively low per pupil ability to support ($5,385 assessed valuation per pupil), the expenditure for the 1970-71 school year (exclusive of Title ESEA funds) was approximately $674.45 per pupil.

Because of the proximity of The University of Arizona, excellent teachers and teaching standards have been maintained. The district has constant contact and cooperation with the College of Education and has worked together with the University on a program of inservice training for student teachers.

Harelson Elementary School is the northernmost elementary school in the Amphitheater District. Located at 826 West Chapala Drive, it serves a community which is almost totally residential. Eighty-two per cent of the homes are single family units and ninety per cent of the homes are owned by the inhabitants. The median price range for houses in this area is $32,307.

The children of Harelson come from families which average 3.37 persons per household. The area is quite stable with sixty per cent of the population having lived there for more than five years. Sixty-seven per cent of the household heads are college graduates with an average yearly income of $17,222. The population is dispersed over a large area and ninety per cent of the pupils are bused to school a distance of from one-half mile to fifteen miles. Due to a continued growth in this particular area, Harelson found itself in an overcrowded situation during the school year 1970-71. In order to insure quality education, the overflow of students were bused to another school where classrooms

and teachers were available. This also insured a relatively stable population for this study at Harelson.

The organizational plan for Harelson is based on the self-contained classroom for grades one through three with some homogeneous grouping in the communicative arts area. In grades four through six there is a turn towards departmentalization, with teachers teaching in two areas of competence. At this level there is a definite regrouping of students in the communicative arts. The philosophy of Harelson is one of continuous growth with an abundance of individual testing.

Selection of the Sample

Since the purpose of this study was to determine if there would be a difference in the motivation methods, verbal praise and monetary reward, as measured by gain scores of the subjects, it seemed essential to obtain a group in which the socioeconomic background of the subjects would not be too varied. If the backgrounds were too varied it would cause concern as to whether the obtained results were due to the methods used or to the differences in socioeconomic background. In a self-contained classroom the overlapping of subject matter is more likely to occur than in a departmentalized classroom in which only one subject at a time is emphasized. Also, in a self-contained classroom more teachers and more classes would be involved in a study

than if just mathematics-oriented teachers and classes were used. It was for these reasons that the investigator felt a departmentalized class arrangement was essential to insure that all students would receive the same treatment. Further criteria for the selection of a sample group demanded a fairly stable population; one which would not change too much during the approximately ten-week period required for the plan of the experiment.

Harelson Elementary School was selected for this study since the criteria for the sample were met. The entire fourth grade and sixth grade mathematics classes were used in the study. In all, 255 students were part of the research. When the data were collected only fifty-six fourth grade boys, fifty-six fourth grade girls, fifty-six sixth grade boys, and fifty-six sixth grade girls were randomly chosen so that equal groups could be utilized as required for the statistical design of this study.

Measures and Procedures Used in the Study

There were four investigator-made tests constructed especially for this study. They were reviewed and approved by three experts in the field of education and mathematics. All tests were twenty-six item tests in which the correct answer had to be found by the pupil. Each question of the tests was a standard type question as found in modern mathematics books. The questions were of a normal type

usage which left little doubt as to reliability or validity. The tests, labeled A through D (as shown in Appendix B), were written in parallel form. Test A was written to judge previous knowledge of addition and subtraction of fractions of arithmetic. Test B was to measure the amount of gain the student had achieved after twenty days of instruction. Test C was a pretest to measure previous knowledge of multiplication and division of fractions of arithmetic, and Test D was given as a posttest to measure the amount of gain in achievement after twenty days of teaching this topic.

All tests were administered and hand corrected by the four teachers involved in the study and reviewed by the investigator to insure uniformity in the scoring process.

The study was conducted in two distinct parts: Part I and Part II. Four fourth grade mathematics classes and four sixth grade mathematics classes of comparable size were arranged in random distribution. Four teachers of mathematics worked with the eight classes and were pre-instructed as to their roles in the study.

Part I:

1. All classes were given Test A, the pretest on addition and subtraction of fractions of arithmetic.
2. Two teachers, each teaching one fourth and one sixth grade class, taught using the monetary method of reinforcement with their fourth grade classes, and verbal praise method with their sixth grade classes.

The other two teachers, each teaching one fourth grade class and one sixth grade class, taught using verbal praise with their fourth grade classes, and monetary reward with their sixth grade classes.

3. All four teachers taught addition and subtraction of fractions for twenty days. Each teacher gave a minimum uniform prearranged presentation, and passed out a work sheet to be completed for the next day (Appendix C). The pupils were rewarded for the completion of these work sheets in accordance with the reward group they were part of at the time. Only these assignment sheets were rewarded by the teachers.
4. After teaching addition and subtraction of fractions for twenty days, Test B, the posttest on addition and subtraction of fractions, was given to measure achievement gain.

Part II:

1. All students were given Test C, the pretest over multiplication and division of fractions of arithmetic.
2. The first two teachers of Part I traded classes with the other two teachers and reversed the order of Part I. The first two teachers now taught using verbal praise on their fourth grade classes and the

monetary reward method with their sixth grade classes. The other two teachers taught using the monetary reward method on their fourth grade classes and the verbal praise method on their sixth grade classes (see Appendix A).

3. All four teachers now taught multiplication and division of fractions of arithmetic for twenty days. Again, each teacher uniformly gave a daily pre-arranged presentation and passed out work sheets (Appendix D). Again, only these assignment sheets were rewarded by the teachers.
4. After teaching multiplication and division of fractions for twenty days, Test D, the posttest on multiplication and division of fractions, was given to measure achievement gain.

Gain scores were figured by subtracting Test B from Test A, and Test D from Test C. Since there were a few negative gain scores, ten was added to all scores so that the data would be in a usable form. The gain scores were used in the statistical analysis.

The Verbal Praise Group

The verbal praise group was given encouraging words of praise on papers they completed successfully. There were no grades given but the papers which were executed in a noteworthy manner were rewarded with phrases and comments

such as: "Excellent work, I'm proud of you"; "Tremendous work, keep it up." For less distinguished work, comments like: "This is very good work, I'll bet you can do still better"; or "Work like this is very pleasing," were used. In no case was there any comment less than "good" given on any of the papers returned. If the work was poor comments were simply withheld or only words of encouragement were used. Teachers were encouraged to keep the classroom attitude pleasant and encouraging. After the initial presentation the teachers only helped the students when they were asked.

The Monetary Reward Group

The monetary reward group were all presented with bank books from the Catalina Savings and Loan, Tucson, Arizona, which sponsored the money for the research. This group was told that they could earn five cents for each assignment successfully completed. This was gauged by the individual teacher in view of the capabilities of the student. If the teacher felt that the student gave the work sheet a sincere effort, the student was given the reward even if the paper was not a hundred per cent correct. With twenty assignments, each pupil could earn a dollar total and upon presentation to the bank could either open a savings account or withdraw his money upon completion of the twenty days. Again no grades were given.

Statistical Design of the Study

To test the hypothesis for the independent variables concerning methods, sex, and grade level, an analysis of variance for a mixed design, two between- and one within-subjects variables were used. The gain score factor was the dependent variable and the one-factor repeated measurements design was used to strengthen the analysis.

According to Downey and Heath (1965) two basic assumptions must be met when using analysis of variance: (1) that subjects within the various subgroups be selected by random sampling from normally distributed populations, and (2) that the variance of the subgroups be homogeneous.

A table of random numbers was used to select the subjects from each subgroup that was considered to be representative of normally distributed populations. Myers (1966) feels strongly that the use of the repeated measures design greatly strengthens the analysis of the data. He states, "For the first time, the levels of our variables, S (subjects), may be considered a random sample from a population of levels" (p. 153). He further infers that it is also possible that the scores will be correlated when they are obtained from the same people. The same number of subjects was tested in each subgroup to assure the second assumption, homogeneity of variance (Winer, 1962).

When it was necessary to compare subgroups of the study, the Tukey test for finding significant differences

between pairs of means was used as indicated in Glass and Stanley (1970, p. 383).

A questionnaire (Appendix F) was used in this study to determine attitudes of the pupils with regard to the two methods of motivation. This questionnaire was presented after the conclusion of the experiment. Percentages were noted and Chi Square tests were used when applicable.

CHAPTER IV

PRESENTATION AND ANALYSIS OF THE DATA

This study was conducted in its entirety with the fourth and sixth grades of Harelson Elementary School, Amphitheater Elementary School District Number Ten, Tucson, Arizona. The purpose of the study was to investigate the comparative achievements of the students when they were taught the same materials under different methods of motivation.

The Original Grouping of the Pupils

Teachers and classes in which the pupils learned were designated by numerical subscripts in accordance with where they were originally placed. Teachers are T_1, T_2, T_3, and T_4. Classes are designated $C_1$4th, $C_2$4th, $C_3$4th, $C_4$4th, $C_1$6th, $C_2$6th, $C_3$6th, and $C_4$6th in accordance to which teacher they began with and which grade they were in. Table I exhibits how the classes were set up for Part I of the study. The letters M. R. signify the monetary reward method of motivation and the letters V. P. signify the verbal praise method of motivation.

Table II presents the distribution of the subjects in a fashion pertinent to Hypothesis Two of the study. For the question regarding the comparison of the two methods,

Table I. Distribution of Subjects in Part I of the Study by Teacher, Method, Class, and Sex

Teacher	Method	Class	Boys	Girls	Total
T_1	M. R.	C_1 4th	17	16	33
T_1	V. P.	C_1 6th	19	14	33
T_3	M. R.	C_3 4th	18	14	32
T_3	V. P.	C_3 6th	16	17	33
	Total M. R.	4th	35	30	65
	Total V. P.	6th	35	31	66
T_2	V. P.	C_2 4th	15	11	26
T_2	M. R.	C_2 6th	16	13	29
T_4	V. P.	C_4 4th	17	17	34
T_4	M. R.	C_4 6th	14	19	33
	Total M. R.	6th	30	32	62
	Total V. P.	4th	32	28	60

Table II. Distribution of Subjects in a Manner Relevant to the Hypotheses

	M. R.		V. P.		
	Girls	Boys	Girls	Boys	Total
4th	30	35	28	32	125
6th	32	30	31	35	128
Total	62	65	59	67	253

for Part I of the study, the table indicates 127 subjects under the monetary reward method compared to 126 subjects under the verbal praise method. Because it was later necessary to arrange for the same number of subjects in each cell under the statistical design indicated for this research, some of these subjects were later randomly discarded. However, since the repeated factor design was used all 253 subjects were exposed to both methods, doubling the possible number of subjects to 506. For the hypotheses concerning a comparison of the two grade levels under the two methods of motivation, the far right column in Table II indicates 125 fourth graders and 128 sixth graders. By adding both columns under girls and then both columns under boys the totals are: Girls, 121 and Boys, 132. Again, these numbers were later reduced to equalize the cells for this statistical design, but upon the completion of Part II of the study the number of subjects is doubled because of the repeated factor design (Appendix F).

Comparison of Experimental Groups

Comparison of Fourth Grade Boys to Fourth Grade Girls

There were 58 girls and 67 boys listed in the fourth grade. Test B subtracted from Test A and Test D subtracted from Test C produced two gain scores for each subject tested. Table III gives a partial view of the results.

Table III. Fourth Grade: Sex and Method Difference

		Monetary Reward		Verbal Praise		
	No.	Sum of Gains	Average Gain	Sum of Gains	Average Gain	Difference of Ave. Gains
Girls	58	527	9.086	536	9.241	-.155
Boys	67	692	10.328	603	9.000	1.328

Comparison of Sixth Grade Boys to Sixth Grade Girls

On the sixth grade level the results on the same basis are listed in Table IV.

Table IV. Sixth Grade: Sex and Method Differences

		Monetary Reward		Verbal Praise		
	No.	Sum of Gains	Average Gain	Sum of Gains	Average Gain	Difference of Ave. Gains
Girls	63	519	8.238	395	6.270	1.968
Boys	65	522	8.031	435	6.692	1.339

The difference of the average gains for the monetary reward method and the verbal praise method seems to indicate a slight preference for the V. P. method among the fourth grade girls. In all the other cases the preference seems to

be for the M. R. method. When the two grades are put together for a picture of total girl population with respect to the total boy population, Table V shows both boys and girls performed better under the M. R. method, with the boys' gain scores somewhat better on the average than the girls' gain scores.

Table V. Total, Girls and Boys, Methods Differences

		Monetary Reward		Verbal Praise		
	No.	Sum of Gains	Average Gain	Sum of Gains	Average Gain	Difference of Ave. Gains
Girls	121	1046	8.662	931	7.756	.906
Boys	133	1214	9.180	1038	7.846	1.334

It is of interest to note that whereas there is no appreciable change in the Difference of Average Gains column for the boys, both fourth and sixth grade, the girls at the fourth grade level were -.155 and then went to 1.968 at the sixth grade level. This figure surpasses that of the sixth grade boys which was 1.339.

Comparison of the Fourth Grade to the Sixth Grade

When the total fourth grade is compared to the sixth grade, Table VI was relevant to Hypothesis Five. It will be

Table VI. Total, Fourth and Sixth Grades Methods Differences

		Monetary Reward		Verbal Praise		
	No.	Sum of Gains	Average Gain	Sum of Gains	Average Gain	Difference of Ave. Gains
Fourth	125	1219	9.752	1139	9.112	.640
Sixth	128	1041	8.133	830	6.484	1.649

noted that in this table the difference of average gains of the sixth grade seems to be considerably greater than the fourth grade.

In relation to the question posed in Hypothesis One the preceding figures in Tables III to VI seem to indicate a better general performance under the monetary reward than under the verbal praise method of reinforcement. Only in one specific instance (fourth grade girls) did the verbal praise scores average better, and that amount seemed negligible.

Presentation of the Problem for Analysis

Grouping the Subjects to Meet the Statistical Design

The statistical design of this study is an Analysis of Variance for the mixed design, two between- and one within-subjects variables with one factor a repeated measure. This design requires a randomly distributed population with

the same number of subjects in each combination (Myers, 1966, p. 183). Since the least number of subjects in any subgroup is shown to be the fourth grade girls under verbal praise in Table I, subjects from the other subgroups were randomly eliminated until each subgroup contained exactly 28 subjects. The data then were listed in Table VII in the following manner. A_1 and A_2 represent the fourth and sixth grades respectively. B_1 and B_2 represent the boys and girls respectively. The methods C_1 and C_2 represent verbal praise and monetary reward methods in the same order. The numbers are the sums of the gain scores for each group with totals at the end of each row and at the bottom of each column. $\overline{X}$ represents the average gain score plus ten.

Table VIII is a summary of the analysis of variance for the mixed design, two between- and one within-subjects variables with one factor a repeated measure.

Between Scores

The F for the first main effect (Grade Level) was 81.271 and significant at the .001 level.

The F value presented for the second main effect (Sex of the Subject), .3854, was not significant.

For the interaction effect between grade level and sex of the subject the F value was 6.7999 which represents a significant difference ($p < .01$) between means. Van Dalen (1962, pp. 396-398) suggests further interpretation of the

Table VII. Sums of Gain Scores and Mean Scores

	No.	C_1 (Verbal Praise)	C_2 (Monetary Reward)	
A_1 (4th):				
B_1 (boys)	28	530	609	1139
	28	529	433	962
	56	1059	1042	2101
		$\bar{X}$ = 18.910	$\bar{X}$ = 18.607	$\bar{X}$ = 18.758
B_2 (girls)	28	554	589	1143
	28	525	474	999
	56	1079	1073	2142
		$\bar{X}$ = 19.267	$\bar{X}$ = 18.982	$\bar{X}$ = 19.125
		2138	2105	4243
		$\bar{X}$ = 19.089	$\bar{X}$ = 18.794	$\bar{X}$ = 18.941
A_2 (6th):				
B_1 (boys)	28	468	565	1033
	28	485	455	940
	56	953	1020	1973
		$\bar{X}$ = 17.017	$\bar{X}$ = 18.214	$\bar{X}$ = 17.616
B_2 (girls)	28	436	531	967
	28	444	496	940
	56	880	1027	1907
		$\bar{X}$ = 15.714	$\bar{X}$ = 18.339	$\bar{X}$ = 17.026
		1833	2047	3880
		$\bar{X}$ = 16.366	$\bar{X}$ = 18.276	$\bar{X}$ = 17.321
				8123

Sub-totals for BC Cells:

	C_1	C_2		
B_1	2012	2062	4074	$\bar{X}$ = 18.187
B_2	1959	2090	4049	$\bar{X}$ = 18.075
	3971	4152	8123	
	$\bar{X}$ = 17.727	$\bar{X}$ = 18.535		

Table VIII. Analysis of Variance for the Data on Methods, Sex, and Grade

SV	df	SS	MS	F	
Total	447	9722.23			
Between S	223	1350.23			
A	1	294.12	294.12	81.271	P < .001
B	1	1.395	1.395	.3854	not significant
AB	1	24.609	24.609	6.7999	P < .01
S/AB	220	796.07	3.619		
Within S	224	8372.00			
C	1	73.127	73.127	2.0257	.10 < P < .20
AC	1	136.188	136.188	3.7727	.05 < P < .10
BC	1	14.646	14.646	.4057	not significant
ABC	1	206.458	206.458	5.7193	P < .025
SC/AB	220	7941.581	36.098		

interaction. Table IX provides the mean scores for each group and the differences between them. Figure 1 plots the fourth and sixth grades on the abscissa and the two mean scores for both boys and girls produce a line segment for each sex.

Table IX points out that although both boys and girls show less gain score in the sixth grade than is shown in the fourth grade, the girls start with more gain score in the fourth grade but have less gain score in the sixth grade. This would lead to a possible conclusion that somewhere between the fourth and sixth grades both would have the same gain scores, if there were any continuity. Figure 1 shows the line segment connecting the two mean scores of the girls to be of a greater incline than that of the boys, indicating that the change in mean scores was greater for the girls than for the boys from fourth grade to sixth grade.

Within Scores

The within subjects F score for the methods groups showed a difference at a significance level of between .10 and .20 ($.10 < p < .20$). Although the difference between the two methods exists in favor of the monetary rewards method, the data do not support acceptance of Hypothesis One at this level.

The F value of the interaction effect (Grade Level and Method) was p between .05 and .10 ($.05 < p < .10$).

Table IX. Interaction Means and Differences (Grade and Sex)

	4th	6th	Difference
Boys	8.8	7.6	1.2
Girls	9.1	7.0	2.1
Difference	-.3	.6	

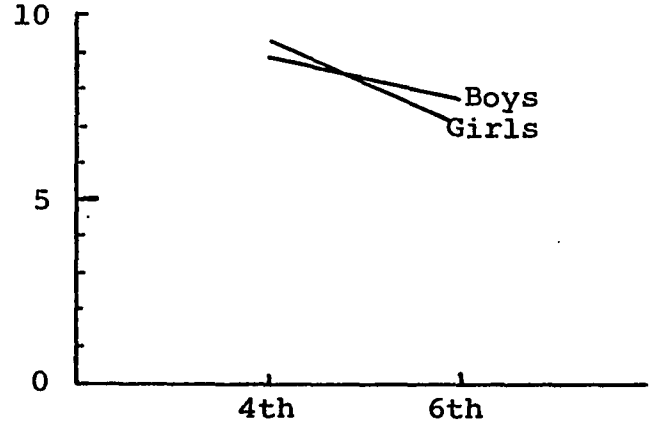


Figure 1. Graph of Interaction Means (Grade and Sex)

Table X and Figure 2 are shown for further interpretation of the data.

In Table X and Figure 2 both boys and girls show a slight increase from verbal praise to monetary reward, with the girls making the greater adjustment. A close scrutiny of Table VIII would point out that if just the fourth grade boys and girls were graphed as in Figure 2, the line segments would be almost parallel to the rewards axis, whereas for the sixth grade boys and girls the slope of the line segments would be much greater with that of the girls being the greatest. This is demonstrated in Figures 3 and 4.

When all three figures are considered (Figures 2-4) it clearly delineates the fact that the fourth graders are significantly less concerned about method than the sixth graders. Using the Tukey test for significance of differences between the means, the test showed the differences to be non-significant for the fourth grade. Whereas for the sixth graders the significance level was $p < .05$.

The F value for the interaction effect between grade and method was significant ($p < .06$). When the information is shown in both a table and a graph as indicated by Van Dalen (1962) there are some data worth noting. One interesting aspect to consider is the difference that exists between the two means on the verbal praise ordinate. The monetary reward ordinate is narrowed down to a point of little difference between the two means. Fourth grade again

Table X. Interaction Means and Differences (Method and Sex)

	V.P.	M.R.	Difference
Boys	7.96	8.42	-.46
Girls	7.49	8.66	-1.17
Difference	.47	-.24	

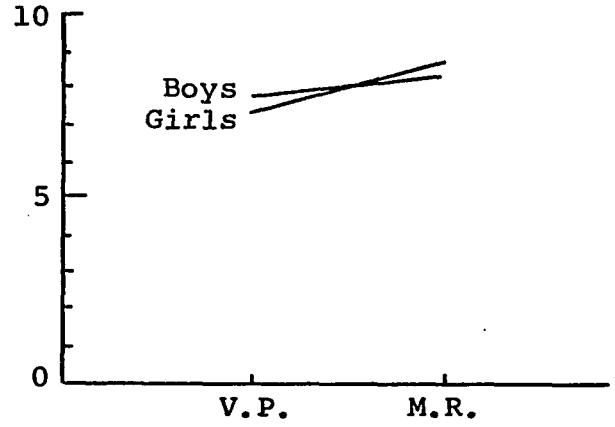


Figure 2. Graph of Interaction Means (Method and Sex)

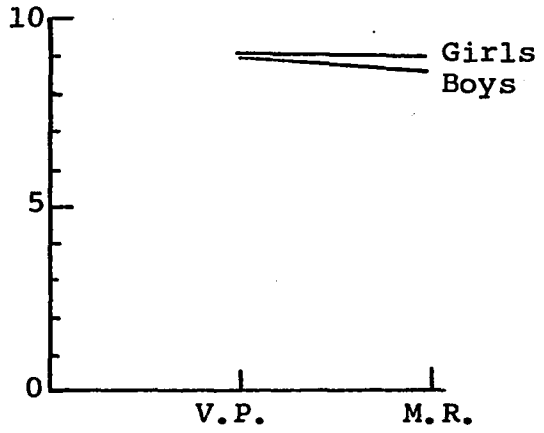


Figure 3. Fourth Grade Graph of Interaction Means (Method and Sex)

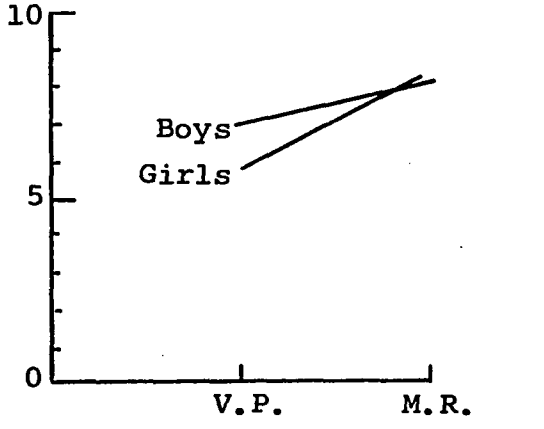


Figure 4. Sixth Grade Graph of Interaction Means (Method and Sex)

exhibits the lack of concern for method while the sixth grade distinctly displays a leaning toward the monetary reward method of motivation (Tukey Test, $p < .05$) (Table XI and Figure 5).

For the interaction of Grade, Sex, and Method the F value was 5.7193 and significant at the .025 level. Myers (1966) states: "A first-order interaction is significant when the differences among simple effects of one variable change significantly over the levels of the second variable. Similarly, a second order interaction effect is significant when the simple interaction effects of two variables change as a function of the level of the third variable" (pp. 396-398). The AB interaction is present since the matrices are

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{bmatrix} 8.8 & 7.6 \\ 9.1 & 7.0 \end{bmatrix} \end{array}$$

which produces $(8.8 - 9.1) - (7.6 - 7.0) = -.9$. If all AB interaction effects were absent, the result would be zero. For the matrices of the AC and BC interaction effects using the mean scores, the results are

Table XI. Interaction Means and Differences (Grade and Method)

	V.P.	M.R.	Difference
4th	9.1	8.8	.3
6th	6.4	8.3	-1.9
Difference	2.7	.5	

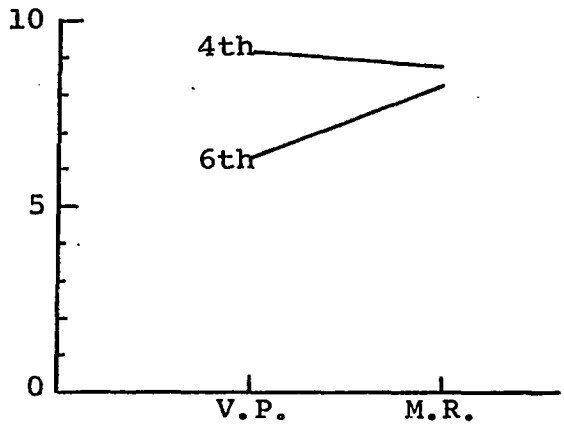


Figure 5. Graph of Interaction Means (Grade and Method)

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{bmatrix} 8.4 & 8.0 \\ 3.9 & 7.5 \end{bmatrix} \end{array} \quad \text{and} \quad \begin{array}{c} \\ B_1 \\ B_2 \end{array} \begin{array}{c} \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{bmatrix} 9.1 & 8.8 \\ 6.4 & 8.3 \end{bmatrix} \end{array}$$

$$(8.4 - 3.9) - (8.0 - 7.5) = 4.0$$

and

$$(9.1 - 6.4) - (8.8 - 8.3) = 2.2.$$

With the given definition the ABC interaction does contribute to the variability among cell means and is significant.

The Questionnaire Presented to Subjects on Methods

As an instrument of data collecting this study used a questionnaire to measure attitudes and judgments of the individual subjects insofar as their feelings for the two methods were concerned.

The first question was: Which class did you enjoy most. The one where you earned nickels or the other? This question had three possible selections from which the subjects could choose one. The results of this question are tabulated in Table XII in both percentages and Chi Square scores and level of significance.

The second question of the questionnaire was designed as an attempt to determine under which method the subjects felt they actually did more work. The question

Table XII. Question #1: Enjoyment of Methods

Which class did you enjoy the most. The one where you earned nickels or the other?

	Number	Nickels	Other	No Difference
		Fourth Grade Boys and Girls		
Boys	66	48.5%	18.2%	34.9%
Girls	53	32.1%	28.3%	39.6%
	$\chi^2 = 9.7047$	$p < .01$		
		Sixth Grade Boys and Girls		
Boys	60	58.3%	28.3%	13.3%
Girls	54	46.3%	26.0%	27.7%
	$\chi^2 = 23.0110$	$p < .001$		
		All Girls and Boys		
Boys	126	53.1%	24.2%	25.0%
Girls	107	39.3%	27.1%	33.6%
	$\chi^2 = 24.7048$	$p < .001$		
		Fourth Grade and Sixth Grade		
4th	119	41.2%	22.7%	37.0%
6th	114	53.5%	25.0%	21.1%
	$\chi^2 = 26.4627$	$p < .001$		

was: In which class did you work harder. The one where you earned nickels or the other? The results of this question are tabulated in Table XIII in both percentages and Chi Square scores and level of significance.

Among the fourth graders the boys felt that they worked harder for the nickels than they did for the verbal praise. About the same number of boys and girls felt that the method made little difference. The girls were relatively equally divided with a majority feeling that there was no difference between the two methods. The Chi Square was 19.6706 with a $p < .001$.

In the sixth grade there was an increase among the girls who felt they worked harder for the monetary reward. The ones who felt that the method made little difference in how hard they worked numbered about the same and it remained consistent for this question for all groups. The Chi Square for the sixth grade was 17.0334 with a $p < .001$.

Overall the boys felt they worked harder for the nickel rewards than did the girls. The Chi Square here was 36.1841 significant to the level $p < .001$.

The percentages for both grades were fairly even with most feeling that the monetary reward caused them to work harder or that it made little difference. Few felt that the verbal praise method made them work harder. The Chi Square was 30.9889, again highly significant at the level $p < .001$.

Table XIII. Question #2: Which Method Produced Harder Work?

In which class did you work harder. The one where you earned nickels or the other?

	Number	Nickels	Other	No Difference
		Fourth Grade Boys and Girls		
Girls	53	30.2%	24.5%	39.6%
Boys	66	51.5%	9.1%	39.4%
	$\chi^2 = 19.6706$	$p < .001$		
		Sixth Grade Boys and Girls		
Girls	54	40.7%	18.6%	40.7%
Boys	60	48.3%	14.8%	38.3%
	$\chi^2 = 17.0334$	$p < .001$		
		All Girls and Boys		
Girls	107	35.5%	21.5%	40.2%
Boys	128	49.2%	13.1%	38.3%
	$\chi^2 = 36.1841$	$p < .001$		
		Fourth Grade and Sixth Grade		
4th	119	42.8%	16.8%	40.3%
6th	114	44.7%	15.8%	39.5%
	$\chi^2 = 30.9889$	$p < .001$		

Questionnaire question number 3 is: In which class do you think you worked more? This question was designed to determine quantity rather than degree of effort as was the case in question number 2. In all groups the percentages were fairly equal. The Chi Squares were all below the level of significance. Questions number 2 and 3 seem to indicate that whereas the subjects worked with more intensity than before, they did not feel that they produced more work than before under either method (Table XIV).

Questions number 4 and 5 were both designed to discover if there was any attitudinal value produced by the methods, either negative or positive. Question number 4 was: Did you worry in either class? Based on the percentages, there seemed to be little worry under either method, but the boys seemed to worry less than the girls under monetary reward while the girls seemed to worry less than the boys under verbal praise method. In both cases sixth graders worried less under either method than did the fourth graders (Table XV).

Question number 5 asks the question: Were you happier in either class? Here the majority of the subjects seemed content and happy under either method with the percentages in favor of the monetary reward method. At the fourth grade level the boys seemed considerably happier with the nickel rewards, whereas at the sixth grade level the girls seemed happier with the monetary reward method. In

Table XIV. Question #3: Which Method Produces More Work?

	Number	Nickels	Other	No Difference
		Fourth Grade Boys and Girls		
Girls	53	34.0%	34.0%	32.1%
Boys	66	40.1%	30.3%	28.8%
	$\chi^2 = 2.1841$	N.S.		
		Sixth Grade Boys and Girls		
Girls	54	35.2%	25.9%	38.9%
Boys	60	36.7%	33.3%	30.0%
	$\chi^2 = 1.8433$	N.S.		
		All Girls and Boys		
Girls	107	34.6%	29.9%	35.5%
Boys	126	38.3%	31.3%	28.9%
	$\chi^2 = 2.4354$	N.S.		
		Fourth Grade and Sixth Grade		
4th	119	37.8%	31.9%	30.3%
6th	114	35.9%	29.8%	34.2%
	$\chi^2 = 1.8071$	N.S.		

Table XV. Question #4: On Worry About Methods

Did you worry in either class?

	Nickels			Other		
	No.	Yes	No	No.	Yes	No
			Fourth Grade Boys and Girls			
Boys	63	28.6%	71.4%	54	35.2%	64.8%
Girls	51	39.2%	60.8%	44	29.5%	70.5%
			Sixth Grade Boys and Girls			
Boys	56	25.0%	75.0%	53	34.0%	66.0%
Girls	52	30.8%	69.2%	49	20.4%	79.6%
			All Girls and Boys			
Boys	119	26.9%	73.1%	107	34.6%	65.4%
Girls	103	35.0%	65.1%	93	24.7%	75.3%
			Fourth Grade and Sixth Grade			
4th	114	33.3%	66.7%	98	32.7%	67.4%
6th	108	27.8%	72.2%	102	27.5%	72.6%

all cases the monetary reward method seemed to produce the most happiness (Table XVI).

The last question of the questionnaire was: Would you rather work in a class where you earned money or in the other type of class? This question was asked to determine a preference by the subjects. At the fourth grade level the boys preferred the nickels over the verbal praise but the greatest percentage felt it made little difference. The girls seemed to prefer verbal praise while 39.6% felt it made little difference. At the sixth grade level the boys were 60% in favor of the monetary reward method. The girls now favored the money at a percentage of 31.5% while 50% felt it made no difference which method was used. Overall the boys seemed to prefer the money reward while the girls demonstrated a feeling of no difference for most, while the remainder was comparatively even for each of the methods. At the sixth grade level 46.5% of all subjects preferred a money reward compared to 31.1% for the fourth graders. Forty-two per cent of the fourth graders felt it made no difference which method was used while only 36.0% of the sixth graders were unconcerned about money. All Chi Squares are highly significant with $p < .01$ (Table XVII).

In most cases the percentages reflect an attitude on the part of the subjects which seems to be manifested in the statistical data of the study.

Table XVI. Question #5: On Happiness in Both Methods

Were you happier in either class?

	Nickels			Other		
	No.	Yes	No	No.	Yes	No
			Fourth Grade Boys and Girls			
Girls	46	67.4%	32.6%	41	65.9%	34.2%
Boys	60	73.3%	26.7%	52	51.9%	48.1%
			Sixth Grade Boys and Girls			
Girls	52	76.9%	23.1%	50	66.0%	34.0%
Boys	57	68.2%	31.8%	50	60.0%	40.0%
			All Girls and Boys			
Girls	98	72.5%	27.6%	91	65.9%	34.1%
Boys	117	70.9%	29.1%	104	47.1%	52.9%
			Fourth Grade and Sixth Grade			
4th	106	70.8%	29.3%	93	58.1%	41.9%
6th	109	72.5%	27.5%	100	63.0%	37.0%

Table XVII. Question #6: About Method Preference

Would you rather work in a class where you earned money or in the other type of class?

	Number	Nickels	Other	No Difference
		Fourth Grade Boys and Girls		
Boys	66	37.9%	18.2%	43.9%
Girls	53	22.6%	37.7%	39.6%
	$\chi^2 = 9.9304$	$p < .01$		
		Sixth Grade Boys and Girls		
Boys	60	60.0%	16.7%	23.3%
Girls	54	31.5%	18.5%	50.0%
	$\chi^2 = 17.3602$	$p < .001$		
		All Girls and Boys		
Boys	126	47.7%	17.2%	33.6%
Girls	107	27.1%	28.0%	44.9%
	$\chi^2 = 24.5464$	$p < .001$		
		Fourth Grade and Sixth Grade		
4th	119	31.1%	26.9%	42.0%
6th	114	46.5%	17.5%	36.0%
	$\chi^2 = 19.0325$	$p < .001$		

CHAPTER V

SUMMARY, CONCLUSIONS, RECOMMENDATIONS, AND OBSERVATIONS

Summary

It was the purpose of this study to determine if there would be a differential in motivation when verbal praise and monetary rewards were employed as distinct methods of motivational reinforcements when teaching fractions to boys and girls of the fourth and sixth grades.

To answer this question, and to order and provide direction to this study, the following hypotheses were tested:

1. There will be a difference in the mathematical achievement scores of the total population of the pupils of this study when the verbal praise method of reinforcement is used and compared to the use of the monetary reward method of reinforcement.
2. There will be a difference in the mathematical achievement scores of girls as compared to the mathematical achievement scores of boys with respect to (a) the use of the verbal praise method of reinforcement and (b) the use of the monetary reward method of reinforcement.

3. There will be a difference in the mathematical achievement scores of the fourth grade girls as compared to the mathematical achievement scores of the fourth grade boys with respect to (a) the use of the monetary reward method of reinforcement and (b) the use of the verbal praise method of reinforcement.
4. There will be a difference in the mathematical achievement scores of sixth grade girls as compared to the mathematical achievement scores of sixth grade boys with respect to (a) the use of the verbal praise method of reinforcement and (b) the use of the monetary reward method of reinforcement.
5. There will be a difference in the mathematical achievement scores of the fourth grade pupils as compared to the mathematical achievement scores of the sixth grade pupils when considered with respect to (a) the use of the verbal praise method of reinforcement and (b) the use of the monetary reward method of reinforcement.
6. The students in the study will indicate a difference in the amount they enjoyed verbal praise in comparison to monetary reward when considered as (a) a total population (all students), (b) sixth grade to fourth grade, and (c) boys to girls.

7. The students in the study will indicate a difference in how hard they worked under verbal praise and monetary reward when considered as (a) a total population (all students), (b) sixth grade to fourth grade, and (c) boys to girls.
8. The students in the study will indicate a difference in the amount of work they performed under verbal praise and monetary reward when considered as (a) a total population (all students), (b) sixth grade to fourth grade, and (c) boys to girls.
9. The students in the study will indicate a difference in the pressure (positive or negative) felt under verbal praise and monetary reward when considered as (a) a total population (all students), (b) sixth grade to fourth grade, and (c) boys to girls.
10. The students in the study will indicate a preference for the verbal praise method or the monetary reward method when compared as (a) a total population (all students), (b) fourth grade to sixth grade, and (c) boys to girls.

It was assumed in this study that (1) the children would respond naturally and in a representative manner; (2) whereas all children are not able to achieve to the same degree, their performance would change in accordance with

how they are motivated; and (3) motivation and achievement are relative in teaching arithmetic to pupils.

This study accepted the definition of motivation as

> . . . (2) the practical art of applying incentives and arousing interest for the purpose of causing a pupil to perform in a desired way; usually designates the act of choosing study materials of such a sort and presenting them in such a way that they appeal to the pupil's interest and cause him to attack the work at hand willingly and to complete it with sustained enthusiasm; also designates the use of various devices such as the offering of rewards or an appeal to the desire to excel (Good, 1959, p. 354).

Verbal praise reward was defined as words of appreciation spoken or written to approve of and encourage the pupil to continue the task at hand, to finish it willingly, and to see it as an enjoyable experience.

Monetary reward was defined as a prize which can be spent or exchanged for goods or services which the pupil may value.

The study was conducted with the entire fourth and sixth grades at Harelson Elementary School, a member of the Amphitheater School District Number 10, Tucson, Arizona. There were four fourth grade classes and four sixth grade classes taught by four teachers whose major area was mathematics.

The study was done in two parts. In Part I the presentation was addition and subtraction of fractions, presented to all students in the same manner with only the

rewards methods differing. The same pretest, Test A, was administered to all subjects, with each teacher teaching a fourth grade and a sixth grade arithmetic class with verbal praise for one class and monetary reward for the other. Addition and subtraction was taught for a period of 20 days after which a posttest was administered. By subtracting the pretest from the posttest score, a gain score was arrived at in order to measure achievement.

Part II presented multiplication and division of fractions for a period of 20 days. Pretest C was given at the beginning of the presentation and posttest D was given to obtain a gain score as it was done in Part I.

To minimize teacher influence each teacher exchanged their classes with each other so that each had a new group and were presenting the rewards methods in a different order.

Daily work sheets were passed out and the children received their rewards based on their perserverance and attempt at correctly completing the work sheets.

The monetary rewards groups were given bank books into which they could deposit the nickel they would earn for successfully completing their assignment. If they did not do their work the nickel earnings were withheld. At the end of the presentation they had the option of cashing in their earnings or opening an account with the bank.

The verbal praise was administered either orally or written on the completed assignment sheets. The praise was always administered graciously and like the monetary reward was simply withheld whenever an assignment was not successfully completed. In no case were negative comments made.

All tests A through D were manually scored by both the teachers and the investigators. Daily work sheets were scored by the teachers in constant consultation with the investigator.

To test the hypotheses for the independent variables concerning methods, sex differences, and grade level differences, an analysis of variance for a mixed design, two between- and one within-subjects variables was used. The gain score factor was the dependent variable and the one-factor repeated measurements design was used to strengthen the analysis. In accordance with the demands of this statistical design as set forth by Myers (1966, p. 153) the groups were equalized to 28 each by randomly eliminating some of the subjects' scores.

A questionnaire was given to the subjects to determine their attitudes about the two methods and to support the data obtained from the study. Percentages were noted and Chi Square tests were used when they were applicable.

Conclusions

The findings listed below are based on the gain scores derived from the difference of Test A and Test B, and Test C and Test D, as they were administered under the two reinforcement methods, verbal praise and monetary rewards, to fourth and sixth grade students of Harelson Elementary School. Also listed are the findings from the questionnaire as they relate to the hypotheses:

1. There was a difference in the gain scores produced by the methods, monetary rewards when compared to verbal praise. The gain scores were consistently in favor of the monetary reward method over the verbal praise method. However, the differences in gain scores were not statistically significant with p between .10 and .20 ($.10 < p < .20$) and therefore from this study the first hypothesis cannot be accepted as true.
2. The difference between the gain scores for the boys and girls with respect to the two methods of motivation was not significant and the second hypothesis of this study cannot be accepted as true.
3. There was no supportive evidence statistically that the third hypothesis of the study was true. The third hypothesis states that there will be a difference between the mathematical achievement scores of the fourth grade girls as compared to the

mathematical achievement scores of the fourth grade boys with respect to (a) the use of the monetary reward method of reinforcement, and (b) the use of the verbal praise method of reinforcement. Using the Tukey test for comparing the mean scores, the differences of the girls and boys at fourth grade level was not significant for either verbal praise or monetary reward methods.

4. For the fourth hypothesis of the study: there will be a difference in the mathematical achievement scores of the sixth grade girls as compared to the mathematical achievement scores of the sixth grade boys with respect to (a) the use of the verbal praise method of reinforcement, and (b) the use of the monetary reward method of reinforcement, the differences were not significant when compared by the Tukey test. Although there was a distinct difference under verbal praise with the girls' mean scores below that of the boys the difference could not be considered statistically significant.
5. The fifth hypothesis was that there will be a difference in the mathematical achievement scores of the fourth grade pupils as compared to the sixth grade pupils when considered with respect to (a) the use of the verbal praise method of reinforcement, and (b) the use of the monetary reward method of

reinforcement. The interaction level for grade level and method was significant with $p < .06$. In view of the fact that there was little difference in the effects of the two methods of motivation on the fourth grade, $p < .06$ for the total population of the study would indicate that the sixth grade has a great deal of influence on this statistic. In order to substantiate this the Tukey test was used to discern the mean difference for (a) fourth grade comparison under the two methods and (b) sixth grade comparison under the two methods. The fourth grade test indicated no significant difference under either method, but the sixth grade test showed a significant difference between the two methods of $p < .05$ for the boys and a significant difference between the two methods of $p < .01$ for the girls. Monetary reward achieved a greater motivation for sixth grade students than did the verbal praise method of reinforcement. Sixth grade students were better motivated by the monetary reward method of reinforcement than they were by the verbal praise method of reinforcement.

6. a. Findings for the sixth hypothesis indicated that 47.4% of all students enjoyed the monetary reward method while 25.9% preferred the verbal praise method. Of the total number of students,

29.1% indicated no difference between the two methods. Using the Chi Square test the significance level of this finding was $p < .001$.

b. More sixth grade students indicated that they enjoyed monetary reward than did the fourth grade students. Whereas both classes felt that monetary reward was more enjoyable than verbal praise, sixth grade showed 53.5% as compared to 41.2% for the fourth grade. More fourth grade students (37.0%) than sixth grade students indicated that, for them, there was no difference in enjoyment of the two methods. The Chi Square test showed a significance level of $p < .001$.

c. Boys at both class levels showed that they enjoyed the monetary reward more so than the girls. However, it is interesting to note that as the age (grade level) increases, the interest in the monetary reward increases proportionally. The Chi Square test showed this item to be significant to $p < .001$.

7. a. When considering the total population of the study, 42.9% felt that they worked harder for the monetary reward than they did for verbal praise. Only 16.8% felt they worked harder for verbal praise and 39.6% felt there was no

difference in how hard they worked. Using the Chi Square test, the level of significance was $p < .001$.

b. There seemed to be little difference between fourth graders and sixth graders in how hard they felt they worked under the two methods of motivation. Both classes felt they worked harder for the monetary reward (42.0% and 44.7%) and only approximately 16% of both classes felt they worked harder for verbal praise. Approximately 40% of both groups felt they did not work any harder for the one method than they did for the other method ($p < .001$).

c. Boys (49.2%) felt they worked harder than the girls (35.5%) felt they worked for the monetary reward method. More girls (40.2%) felt there was no difference in effort expended than did the boys (38.3%). Comparably low percentages of both boys and girls felt they worked harder for verbal praise (13.1% boys and 21.5% girls) ($p < .001$).

8. a. The total population of pupils in the study indicated a near equal distribution among the three choices for the question as to whether they felt they worked more for one method than the other, or no difference at all. The Chi

Square test showed no significance level. The conclusion reached on this point is that there was no significant difference in the amount of work produced under either of the two methods, verbal praise or monetary reward. Hypothesis 8a was therefore rejected.

b. The questionnaire indicated no significant difference between boys and girls as to the amount of work they felt they produced under either method, verbal praise and monetary reward. Hypothesis 8b was rejected.

c. Fourth grade students indicated no significant difference from sixth grade in the amount of work they felt they produced under either method, verbal praise and monetary reward. Hypothesis 8c was rejected.

9. a. The total population of this study indicated that neither method of motivation, verbal praise or monetary reward, caused any worry (pressure). On an average, less than 30% of all subjects worried in either motivational category while an average of approximately 70% indicated they were happier in the monetary rewards group, and an average of approximately 65% showed they were happy in the verbal praise group. Neither method seemed to perturb either

group remarkably while percentages indicate that both methods were acceptable to the pupils.

b. In the comparison of the fourth grade to the sixth grade as to the amount of pressure felt, fourth graders showed a tendency to worry a little more than the sixth graders in both methods. As to which method they felt happier in, both groups indicated a higher percentage happy in the monetary reward method (70.8% and 72.5%) than in the verbal praise method (58.1% and 63.0%). However, neither group, fourth grade nor sixth indicate a great deal of adversity to either method. The tendency of the fourth grade seems to be towards more concern under either method. It raises the question as to whether or not this may be the nature of the fourth grade children rather than the effect of either method.

c. When happiness with the methods is considered comparing boys to girls, the boys and girls both seem to be happy under monetary reward to an almost equal degree (girls 72.5% and boys 70.9%). When verbal praise is considered the girls show they like verbal praise almost as much as they do the monetary reward method while the boys dropped back to 47.5%. This would seem to

indicate that boys are more conscious of the method of reward than are the girls. This point was borne out by the question of worry under the two methods. Boys worried more under verbal praise and less under monetary reward while girls worried more under monetary reward and less under verbal praise. Overall, boys seemed to prefer the monetary rewards more definitely than did the girls. Girls did not consider the motivational factor as critical as did the boys.

10. a. For the total population of this study there was an approximate average of 38.1% that favored working on in a monetary reward type class as opposed to approximately a 22.4% in favor of continued work in a verbal praise type class. Approximately 39.1% felt that it made no difference between the two methods or that they had no preference between the two methods.

    b. In contrast to the total population, whereas, the fourth grade percentage was down to 31.1% in preference for the monetary reward, the sixth grade jumped to a 46.5% preference for the monetary reward. While 42.0% of the fourth grade indicated there was no difference in which they would prefer, only 26.9% liked the verbal praise better than they liked monetary reward.

On the sixth grade level only 17.5% favored verbal praise while 36.0% felt they could not differentiate. This result clearly indicated that sixth graders exhibited a sharp difference in attitude toward the monetary reward type of motivation as compared to the fourth graders. The Chi Square test indicated a significance level of $p < .001$ on this item.

c. In reference to the question of method preference with regard to boys and girls, the boys clearly indicated their preference for the monetary reward method. Girls were almost equally divided between the two methods, with the biggest percentage (44.9%) indicating no preference between the two methods. Boys, on the other hand, had the biggest percentage (47.7%) in favor of monetary reward. Only 17.2% favored the verbal praise method, while 33.6% indicated no difference. The Chi Square test shows a $p < .001$.

Recommendations for Further Study

In view of the results and findings of this study certain recommendations would seem to be in order. The design of this study for investigation of further problems of a like nature is recommended. The questionnaire as it was used in this study should be considered an integral

part of the design. While it is important to find out what happens to the performance of students, it is probably even more important to be aware of what is happening to them as human beings. Their likes and dislikes should be an important aspect of the study.

The use of a questionnaire with question responses using a semantic differential scale might offer information regarding pupils' attitudes which would be of a more continuous nature. It would lend some insight as to the degree of like or dislike for a particular method that the students might have.

This study indicates that although there seems to be little difference in the achievement of the fourth grade under either method of motivation, the sixth grade clearly indicates a difference. It would be of interest to research for an answer as to why this grade level difference exists. Certainly it would be helpful to evaluate what differentials would exist for other types of rewards.

It is also suggested that this type of study could be successfully used in other fields of study such as reading, social studies, communicative arts, etc.

This study used a population which was fairly homogeneous with respect to socioeconomic background. It would be of interest to see if there would be different results with other socioeconomic groups, or between two or more socioeconomic groups. Also, it would be enlightening

to compare groups in accordance with their learning abilities (ability grouping).

There seems to be a turning point at which age group levels will respond differently to these methods of motivation. It would seem vital to further investigate reactions at other grade levels.

Further research with other strategies of motivating seem imperative, and it is hoped that other investigators would continue to fill in the gaps that exist in this area of study.

Observations

While this study can probably never be considered entirely conclusive, certain indications seemed to be clear-cut. Motivational method is important to learning.

At the fourth grade level, the subjects of this study performed equally as well under both methods of motivation. Even though both classes were being offered the same new material (addition and subtraction, and multiplication and division of fractions), the greatest average gains were made by fourth grade pupils (Table VII). This was due in part to the fact that the pretest scores by the sixth grade pupils were generally higher than those of the fourth grade pupils on both pretests. The most noticeable difference was on the pretest for the addition and subtraction of fractions. On the pretest for multiplication and division

of fractions, the two grades were closer together insofar as test scores were concerned.

Average gain scores for the fourth grade boys and girls were consistent under both methods indicating that boys and girls of the fourth grade achieved to the same extent under both methods. The average gain scores for sixth grade subjects were comparable with those of the fourth grade gain scores under monetary reward but distinctly below for verbal praise.

In view of the preceding information and the indications of the questions answered by the subjects in the study questionnaire, it is believed that the students of the fourth grade came to the subject intrinsically motivated. This is consistent with the thinking of previous studies in that there is a pattern of achievement motivation characteristic of each individual.

By the time the subjects have become sixth grade students they have lost some of this achievement motivation. How this comes about is not ascertained by this study although previous studies have hinted that motivational factors can become boring and should be changed. Perhaps the students have tired of learning for learning sake. The verbal praise did not serve as enough of a motivator to bring the achievement motivation of the sixth graders back up to what it was in the fourth grade, but the monetary reward did bring the effort back to the fourth grade level

(which may or may not be a maximum achievement motivational level).

Mecklenburger and Wilson (1971), in a survey of motivational techniques programs sponsored by both school districts and government grants, observes that many of these programs are using rewards to substitute for missing achievement motivation in the students' learning make-up. They state that these programs are combining any and all methods of motivation to get the student back to the point where he considers learning to be rewarding. The article intimates that it is foolish to believe that the student is intrinsically motivated, but that through extrinsic motivation perhaps he will come to like learning as a rewarding experience and so continue learning for the sake of gaining knowledge alone.

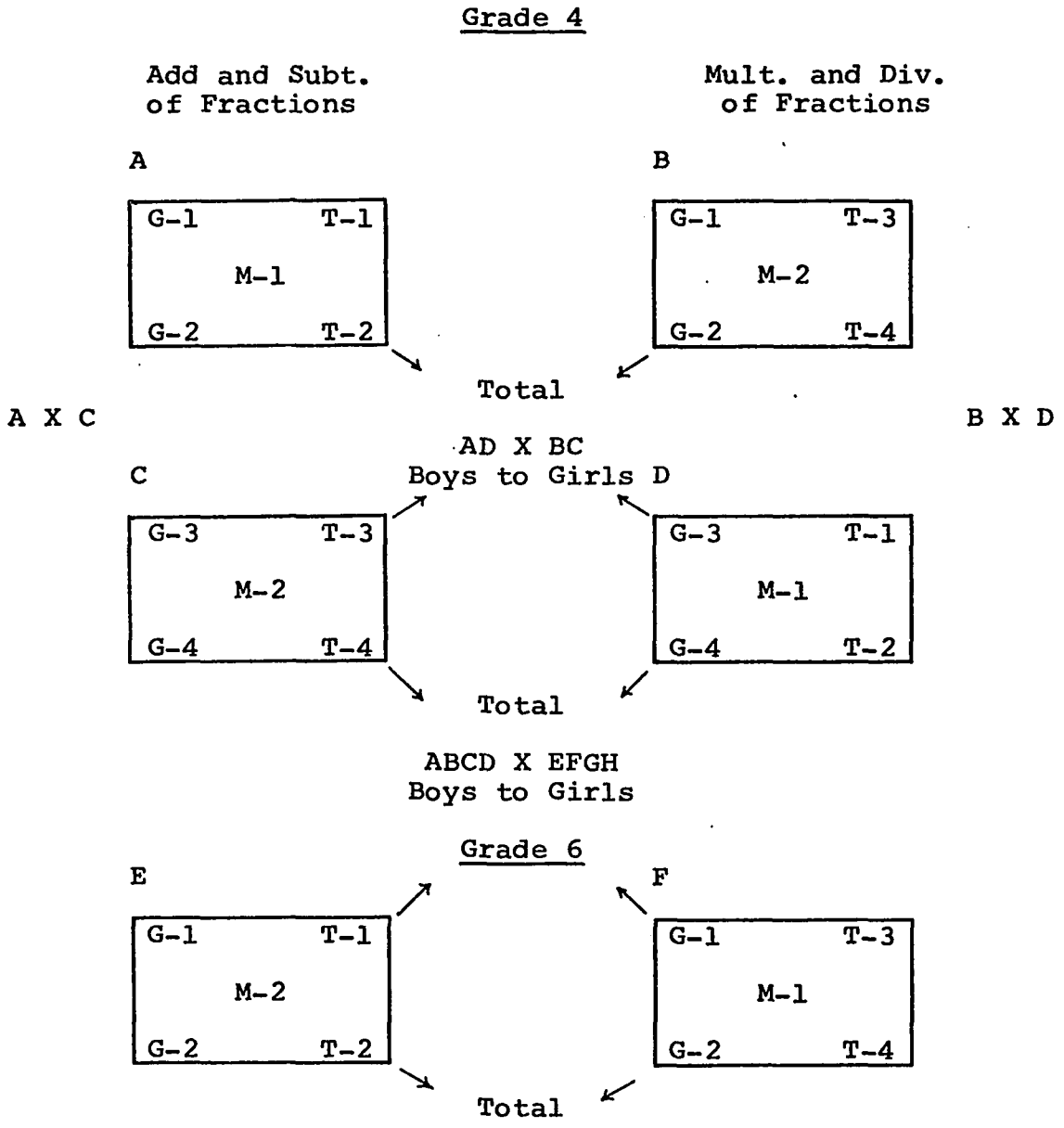
Further observations of this study are that students not only like the monetary reward at the higher grade level, but will perform better for it. Boys will achieve no better than girls for either method of reward, but girls do seem to be more unconcerned about the reward method than the boys. Boys indicated at both levels that they preferred the monetary reward method to the verbal praise method while girls at both grade levels did not have as distinctive a preference.

Generally speaking, it appears that learning habits change as the students progress from grade level to grade

level. It is possible that as the students move to higher grade levels they become disenchanted with learning for learning sake, and may be brought back to achievement by a strong enough motivational reinforcement. Attitudes towards verbal praise and monetary rewards methods differ between boys and girls and grade level to grade level. Generally boys prefer monetary reward methods to verbal praise methods, and sixth grade students prefer monetary reward methods to verbal praise methods. Generally, fourth graders and girls are less critical as to which method of motivation is being employed.

APPENDIX A

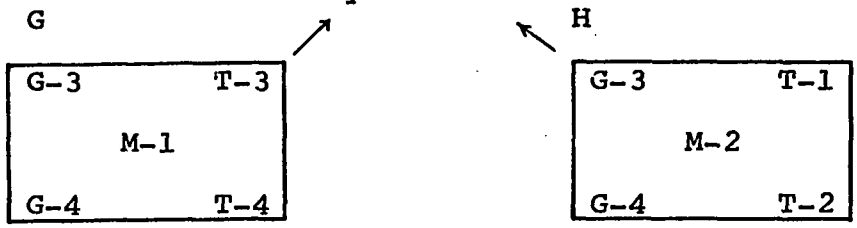
PROCEDURES SCHEMATIC



E X G

EH X GF
Boys to Girls

F X H



Groups (classes) . . . G-1, G-2, G-3, G-4

Teachers T-1, T-2, T-3, T-4

Verbal praise method . . . M-1

Monetary reward method . . M-2

APPENDIX B

TESTS A, B, C, AND D, FOR COLLECTING GAIN SCORES

Test A

Boy Girl
Name_____
Section_____

Answers

_____ 1. What fraction of this bar is shaded?

_____ 2. $1 = \frac{?}{4}$

_____ 3. $13(\frac{1}{13}) = \frac{?}{13}$

_____ 4. $3(\frac{1}{5}) = \frac{3}{?}$

_____ 5. $5(\frac{1}{6}) = \frac{?}{6}$

_____ 6. $\frac{2 \times ?}{3 \times ?} = \frac{10}{5}$

_____ 7. $\frac{3 \times ?}{7 \times ?} = \frac{6}{14}$

_____ 8. $\frac{1}{2} = \frac{?}{6}$

_____ 9. $\frac{2}{3} = \frac{?}{9}$

_____10. $\frac{3 \div 3}{9 \div 3} = ?$

_____11. $4(\frac{1}{10}) = \frac{?}{5}$

_____12. $\frac{7}{14} = \frac{?}{2}$

_____13. $4(\frac{1}{7}) + 2(\frac{1}{7}) = ?(\frac{1}{7})$

_____14. $5(\frac{1}{11}) - 3(\frac{1}{11}) = ?(\frac{1}{11})$

_____15. $\frac{4}{9} + \frac{3}{9} = ?$

_____16. $\frac{4}{10} + \frac{3}{10} = ?$

_____17. $\frac{6}{7} - \frac{2}{7} = ?$

_____18. $\frac{1}{4} + \frac{1}{2} = ?$

_____19. $\frac{2}{5} + \frac{1}{3} = ?$

_____20. $\frac{1}{2} - \frac{2}{5} = ?$

_____21. $\frac{2}{7} + \frac{3}{8} = ?$

_____22. $\frac{5}{12} - \frac{1}{18} = ?$

_____23. $\frac{5}{18} + \frac{4}{21} = ?$

_____24. The least common denominator of $\frac{5}{8}$ and $\frac{7}{12}$ is ___?___.

_____25. At lunch time Joe ate $\frac{1}{8}$ of a pie. At dinner time he ate another $\frac{1}{5}$ of the pie. How much of the pie did he eat in all?

_____26. Sue has $\frac{3}{5}$ of a dollar. Joan has $\frac{3}{4}$ of a dollar. What fraction of a dollar more does Joan have than Sue?

Test B

Answers

Boy Girl
Name_____
Section_____

_____ 1. What fraction of the circles have an X inside.

_____ 2. $1 = \frac{?}{7}$

_____ 3. $15(\frac{1}{15}) = \frac{?}{15}$

_____ 4. $\frac{6}{6} = ?$

_____ 5. $\frac{\triangle}{\triangle} = ?$

_____ 6. $3(\frac{1}{5}) = \frac{?}{5}$

_____ 7. $4(\frac{1}{7}) = \frac{4}{?}$

_____ 8. $\frac{3 \times ?}{4 \times ?} = \frac{9}{12}$

_____ 9. $\frac{1}{3} = \frac{?}{6}$

_____10. $\frac{3}{4} = \frac{?}{12}$

_____11. $6(\frac{1}{10}) = ?(\frac{1}{5})$

_____12. $\frac{6 \div ?}{9 \div ?} = \frac{2}{3}$

_____13. $\frac{8}{12} = \frac{?}{3}$

_____14. $\frac{3}{54} = \frac{1}{?}$

_____15. $2(\frac{1}{9}) + 3(\frac{1}{9}) = ?(\frac{1}{9})$

_____16. $\frac{3}{7} + \frac{2}{7} = ?$

_____17. $7(\frac{1}{13}) - 3(\frac{1}{13}) = ?(\frac{1}{13})$

_____18. $\frac{7}{9} - \frac{2}{9} = ?$

_____19. $\frac{1}{3} + \frac{1}{2} = ?$

_____20. $\frac{1}{6} + \frac{2}{3} = ?$

_____21. $\frac{1}{3} - \frac{2}{7} = ?$

_____22. $\frac{5}{14} - \frac{2}{21} = ?$

_____23. $\frac{7}{15} + \frac{3}{25} = ?$

_____24. Eddie and Joe are working together. Eddie has $\frac{4}{9}$ of the work finished, Joe has $\frac{3}{8}$ of the work finished. What fraction of the work is done?

_____25. The least common denominator of $\frac{1}{12}$ and $\frac{2}{15}$ is ?

_____26. Bob has $\frac{7}{20}$ of a dollar. Jim has $\frac{6}{25}$ of a dollar. What fraction of a dollar does Bob have more than Jim?

Test C

Answers

Boy Girl
Name_____
Section_____

_____ 1. Dividing by or multiplying by ___?___ gives back the same number?

_____ 2. $\frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = ? \times \frac{1}{7}$

_____ 3. $\frac{2}{11} + \frac{2}{11} + \frac{2}{11} = ? \times \frac{1}{11}$

_____ 4. We don't divide by ___?___

_____ 5. $(1 \times 7) \div ? = 1$

_____ 6. $(4 \times ?) \div 6 = 4$

_____ 7. $\frac{8}{11} \times ? = 8$

_____ 8. $\frac{9}{?} \times 10 = 9$

_____ 9. $\frac{4}{5} \times ? = 1$

_____ 10. $? \times \frac{7}{8} = 1$

_____ 11. $\frac{2}{3} \times \frac{4}{5} = ?$

_____ 12. $\frac{3}{7} \times ? = \frac{6}{21}$

_____ 13. $\frac{3}{4} \times \frac{4}{7} = ?$

_____ 14. $\frac{4}{5} \times \frac{15}{16} = ?$

_____ 15. $\frac{3}{4} \div 4 = ?$

_____ 16. $\frac{1}{4} \div \frac{2}{3} = ?$

_____ 17. $\frac{3}{25} \div \frac{4}{5} = ?$

_____ 18. $\frac{7}{9} \div \frac{28}{27} = ?$

_____ 19. $\frac{2}{3} \div \frac{4}{7} = \frac{2}{3} \times ?$

_____ 20. $(\frac{1}{3} + \frac{2}{5}) \times \frac{15}{16} = ?$

_____ 21. $(\frac{3}{4} \times ?) - 3 = 0$

_____ 22. $\frac{(4 \times ?) + 4}{2} = 6$

_____ 23. The reciprocal of $\frac{1}{3}$ is ___?___.

_____24. Joe said to Mike, "I have $\frac{3}{5}$ of a pie. I will give you $\frac{1}{2}$ of it." How much pie did Mike get?

_____25. Bill said, "If you divide the fraction I'm thinking of by $\frac{2}{3}$ you will get 1. What is my fraction?"

_____26. John grew $\frac{3}{4}$ of an inch over the last four months. How much did he average each of the four months?

Test D

Boy Girl
Name_____
Section_____

Answers

_____ 1. Dividing by 4 is the same as multiplying by ?

_____ 2. $\frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = ? \times \frac{1}{9}$

_____ 3. $\frac{3}{13} + \frac{3}{13} + \frac{3}{13} = ? \times \frac{1}{13}$

_____ 4. What number is never used as a denominator of a fraction?

_____ 5. $(1 \times 9) \div ? = 1$

_____ 6. $(6 \times ?) \div 5 = 6$

_____ 6. $\frac{5}{13} \times ? = 5$

_____ 8. $\frac{8}{?} \times 11 = 8$

_____ 9. $\frac{4}{7} \times ? = 1$

_____ 10. $? \times \frac{5}{9} = 1$

_____ 11. $\frac{3}{4} \times \frac{2}{5} = ?$

_____ 12. $\frac{4}{5} \times ? = \frac{8}{15}$

_____ 13. $\frac{4}{11} \times \frac{11}{13} = ?$

_____ 14. $\frac{5}{7} \times \frac{21}{25} = ?$

_____ 15. $\frac{4}{5} \div 5 = ?$

_____ 16. $\frac{1}{7} \div \frac{3}{4} = ?$

_____ 17. $\frac{1}{7} \div \frac{5}{7} = ?$

_____ 18. $\frac{9}{7} \div \frac{81}{49} = ?$

_____ 19. $\frac{3}{5} \div \frac{2}{9} = \frac{3}{5} \times ?$

_____ 20. $(\frac{2}{7} + \frac{3}{5}) \times \frac{35}{35} = ?$

_____ 21. $(\frac{5}{6} \times ?) - 3 = 2$

_____ 22. $\frac{(4 \times ?) + 4}{2} = 3$

_____ 23. The reciprocal of $\frac{2}{5}$ is ?

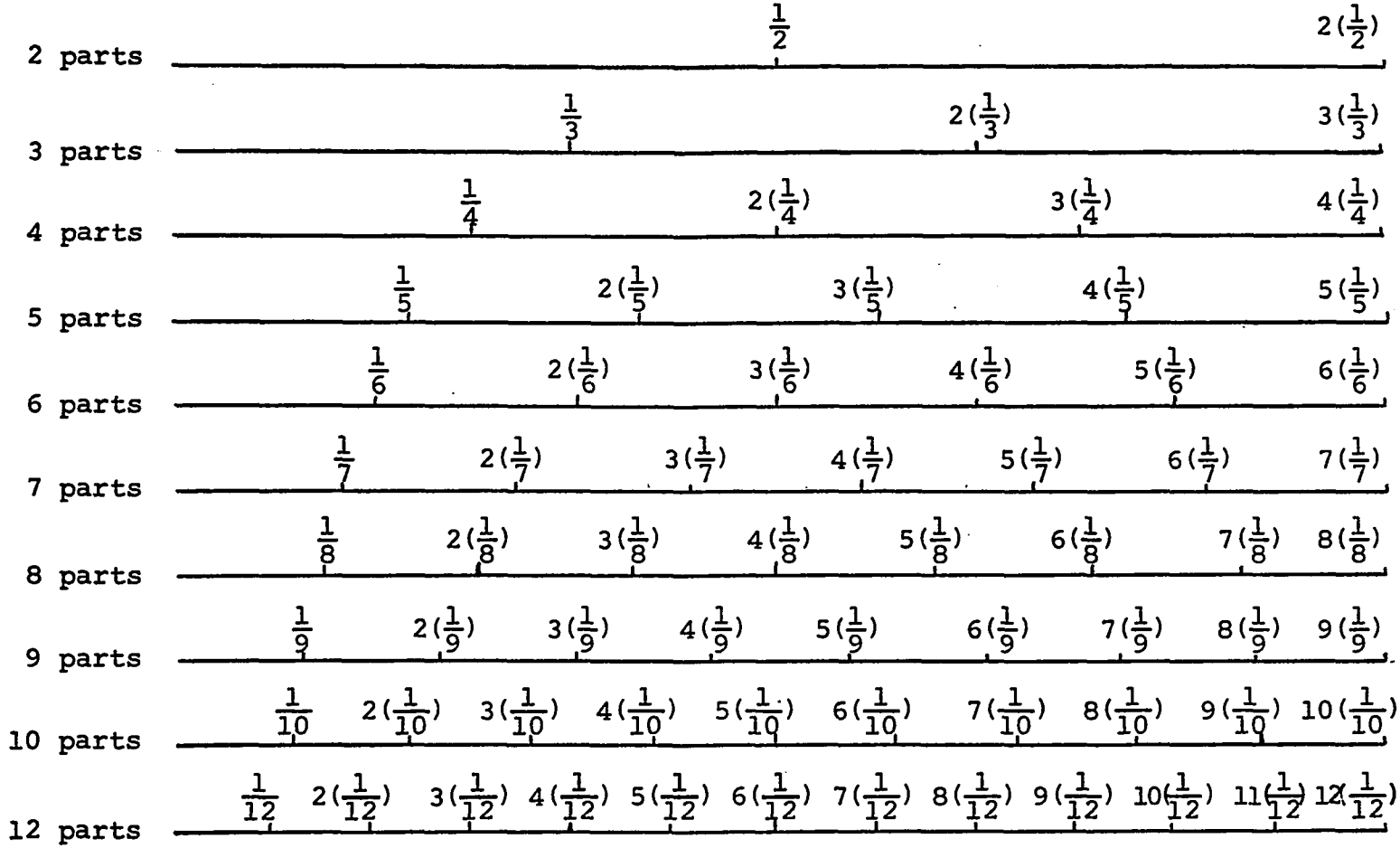
_____24. Jeff said to Ken, "I have $\frac{4}{7}$ of a cake. I will give you $\frac{2}{5}$ of it." How much cake will Ken get?

_____25. Bob said, "If you divide $\frac{3}{4}$ by $\frac{7}{8}$ you will get the number I am thinking of." What number was Bob thinking of?

_____26. John said to his five friends, "There were 50 pieces of candy in this box. I have $\frac{3}{5}$ of a box left. Each of you may have $\frac{1}{6}$ of what is left." How many candies does each one get?

APPENDIX C

ADDITION AND SUBTRACTION WORK SHEETS


2 parts
$\frac{1}{2}$ $2(\frac{1}{2})$
3 parts
$\frac{1}{3}$ $2(\frac{1}{3})$ $3(\frac{1}{3})$
4 parts
$\frac{1}{4}$ $2(\frac{1}{4})$ $3(\frac{1}{4})$ $4(\frac{1}{4})$
5 parts
$\frac{1}{5}$ $2(\frac{1}{5})$ $3(\frac{1}{5})$ $4(\frac{1}{5})$ $5(\frac{1}{5})$
6 parts
$\frac{1}{6}$ $2(\frac{1}{6})$ $3(\frac{1}{6})$ $4(\frac{1}{6})$ $5(\frac{1}{6})$ $6(\frac{1}{6})$
7 parts
$\frac{1}{7}$ $2(\frac{1}{7})$ $3(\frac{1}{7})$ $4(\frac{1}{7})$ $5(\frac{1}{7})$ $6(\frac{1}{7})$ $7(\frac{1}{7})$
8 parts
$\frac{1}{8}$ $2(\frac{1}{8})$ $3(\frac{1}{8})$ $4(\frac{1}{8})$ $5(\frac{1}{8})$ $6(\frac{1}{8})$ $7(\frac{1}{8})$ $8(\frac{1}{8})$
9 parts
$\frac{1}{9}$ $2(\frac{1}{9})$ $3(\frac{1}{9})$ $4(\frac{1}{9})$ $5(\frac{1}{9})$ $6(\frac{1}{9})$ $7(\frac{1}{9})$ $8(\frac{1}{9})$ $9(\frac{1}{9})$
10 parts
$\frac{1}{10}$ $2(\frac{1}{10})$ $3(\frac{1}{10})$ $4(\frac{1}{10})$ $5(\frac{1}{10})$ $6(\frac{1}{10})$ $7(\frac{1}{10})$ $8(\frac{1}{10})$ $9(\frac{1}{10})$ $10(\frac{1}{10})$
12 parts
$\frac{1}{12}$ $2(\frac{1}{12})$ $3(\frac{1}{12})$ $4(\frac{1}{12})$ $5(\frac{1}{12})$ $6(\frac{1}{12})$ $7(\frac{1}{12})$ $8(\frac{1}{12})$ $9(\frac{1}{12})$ $10(\frac{1}{12})$ $11(\frac{1}{12})$ $12(\frac{1}{12})$

#1 Name_____

1. Using the edge of a paper, find and list all the fractions that line up with $\frac{1}{2}$.

   $\frac{1}{2} = \square = \square = \square = \square = \square$

2. List all the fractions that line up with $\frac{1}{4}$

   $\square = \square = \square$

3. How many parts are $(\frac{1}{9})$s.
4. Which line segment has the most parts? _____
5. With paper and pencil mark off $\frac{1}{3}$ and the $2(\frac{1}{4})$s. The two together measure $\square$ $(\frac{1}{12})$s.
6. $2(\frac{1}{5})$s with 2 more $(\frac{1}{5})$s is $\square$ $(\frac{1}{5})$s?
7. $3(\frac{1}{12})$s and $3(\frac{1}{12})$s more are $\square$ $(\frac{1}{12})$s?

   a) How many $(\frac{1}{8})$s does $3(\frac{1}{12})$s and $3(\frac{1}{12})$s equal? $\square$

   b) How many $(\frac{1}{4})$s? $\square$

#2 Name_____

1. When a line segment is divided into:

   _____ a) Six parts, each segment (part) is called $(\frac{?}{?})$.

   _____ b) Four parts, each segment is called $(\frac{?}{?})$.

   _____ c) 12 parts, each segment is called $(\frac{?}{?})$.

2. What are all the names of parts that match up with $2(\frac{1}{3})$.

   a) _____ b) _____ c) _____

3. What are all the names of parts that match up with $1(\frac{1}{4})$.

   a) _____ b) _____

4. Can $9(\frac{1}{12})$ be expressed on another part line? _____

5. $6(\frac{1}{6}) = 1$ What is $8(\frac{1}{8})$ = _____

   $10(\frac{1}{10})$ = _____ $\triangle(\frac{1}{\triangle})$ = _____

6. Give two names for $2(\frac{1}{5}) + 3(\frac{1}{5})$

   a) _____ b) _____

7. Using paper measure and add:

   a) $1(\frac{1}{5}) + 1(\frac{1}{2})$ = _____ b) $1(\frac{1}{2}) + 1(\frac{1}{5})$ = _____

   c) $1(\frac{1}{4}) + 1(\frac{1}{2})$ = _____ d) $1(\frac{1}{2}) + 1(\frac{1}{4})$ = _____

   e) $1(\frac{1}{3}) + 3(\frac{1}{9})$ = _____ f) $3(\frac{1}{9}) + 1(\frac{1}{3})$ = _____

#3 Name_____

1. Using paper measure and add:

a) $1(\frac{1}{5}) + 2(\frac{1}{5}) =$ _____ b) $2(\frac{1}{5}) + 1(\frac{1}{5}) =$ _____

c) $1(\frac{1}{6}) + 2(\frac{1}{9}) =$ _____ d) $2(\frac{1}{9}) + 1(\frac{1}{6}) =$ _____

e) $1(\frac{1}{4}) + 9(\frac{1}{12}) =$ _____ f) $9(\frac{1}{12}) + 1(\frac{1}{4}) =$ _____

2. If you take away $1(\frac{1}{4})$ from $3(\frac{1}{4})$, what is left? _____

Another name? _____

3. $2(\frac{1}{5})$ from $4(\frac{1}{5}) =$ _____

4. $1(\frac{1}{2}) - 1(\frac{1}{4}) =$ _____ 5. $5(\frac{1}{8}) - 1(\frac{1}{4}) =$ _____

6. $9(\frac{1}{12}) - 1(\frac{1}{4}) =$ _____ 7. $7(\frac{1}{9}) - 2(\frac{1}{3}) =$ _____

Read all before doing 8-12

8. $1(\frac{1}{3}) + 1(\frac{1}{3}) = (1 + 1)(\frac{1}{3}) = 2(\frac{1}{3})$ _____

9. $2(\frac{1}{5}) + 1(\frac{1}{5}) = (2 + 1)(\frac{1}{5}) = 3(\frac{1}{5})$ _____

10. $3(\frac{1}{7}) + 2(\frac{1}{7}) = (3 + 2)(\frac{1}{7}) = 5(\frac{1}{7})$ _____

11. $3(\frac{1}{8}) + 4(\frac{1}{8}) = (3 + 4)(\frac{1}{8}) = 7(\frac{1}{8})$ _____

12. $5(\frac{1}{12}) + 2(\frac{1}{12}) = (5 + 2)(\frac{1}{12}) = 7(\frac{1}{12})$ _____

$4(\frac{1}{7})$ can be abbreviated to $\frac{4}{7}$
(shortened)

Abbreviate problems 8 through 12

When reading $\frac{5}{12}$, it is called five-twelfths and may be thought of as five of twelve parts of something. Read your book for other names of parts of numbers.

#4 Name_____

Some problems to do.

Example: Add $\frac{2}{5} + \frac{1}{5}$

$\frac{2}{5} + \frac{1}{5}$ means $2(\frac{1}{5}) + 1(\frac{1}{5})$, so $(2 + 1)(\frac{1}{5}) = 3(\frac{1}{5}) = \frac{3}{5}$

1. $\frac{4}{7} + \frac{2}{7} = \square(\frac{1}{7}) + \bigcirc(\frac{1}{7}) = (\_\_ + \_\_)(\frac{1}{7}) = \_\_(\frac{1}{7}) =$

2. $\frac{3}{8} + \frac{2}{8} =$ = = =

3. $\frac{2}{9} + \frac{3}{9} =$ = = =

4. $\frac{8}{12} + \frac{3}{12} =$ = = =

5. $\frac{3}{6} + \frac{2}{6} =$ = = =

- - - - - - -

$5(\frac{1}{5}) = \frac{5}{5}$ this means five of five parts. When you have all of the parts of something, you have the whole thing--or 1 whole thing. So $\frac{5}{5} = 1$. $1 = \frac{3}{3}$, $1 = \frac{4}{4}$, $1 = \frac{5}{5}$

$1 = \frac{\triangle}{\triangle}$

More problems

6. $\frac{2}{3} + \frac{1}{3} =$ _____

7. $\frac{3}{8} + \frac{5}{8} =$ _____

8. $\frac{9}{12} + \frac{3}{12} =$ _____

9. $\frac{3}{8} + \frac{4}{8} + \frac{1}{8} =$ _____

10. $1 = \frac{\quad}{7}$

11. $1 = \frac{8}{\quad}$

- - - - - - -

A number divided by itself is one!!

#5 Name_____

Example: On your multiplication table 5 x 1 = 5, 6 x 1 = 6, 7 x 1 = 7, 8 x 1 = 8, and so on. In fact △ x 1 = △. You give this meaning. _____

What is:

1. $\frac{1}{5} \times 1 =$ _____

2. $\frac{3}{7} \times 1 =$ _____

3. $\frac{5}{9} \times 1 =$ _____

4. $\frac{3}{10} \times 1 =$ _____

Yesterday we learned $\frac{\triangle}{\triangle} = 1$. Give 3 examples.

$\frac{\quad}{9} = 1$ $\frac{2}{\quad} = 1$ $\frac{\quad}{6} = 1$

Example: $\frac{1}{2} \times 1 = \frac{1}{2}$ and $1 = \frac{5}{5}$, put $\frac{5}{5}$ in place of 1 and we have

$$\frac{1}{2} \times \frac{5}{5} \text{ or } \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$

On your table is $\frac{1}{2}$ the same measure as $\frac{5}{10}$?

- - - - - - -

Example: $\frac{2}{3} \times 1 = \frac{2}{3}$ and $1 = \frac{3}{3}$

is $\frac{2}{3} \times \frac{3}{3}$ or $\frac{2 \times 3}{3 \times 3} = \frac{6}{9}$. Is $\frac{2}{3} = \frac{6}{9}$

5. $\frac{4 \times \square}{5 \times \square} = \frac{8}{10}$

6. $\frac{3 \times \square}{4 \times \square} = \frac{9}{12}$

7. $\frac{2 \times \square}{3 \times \square}$ $\frac{8}{12}$

8. $\frac{\square \times 6}{\square \times 6}$ $\frac{6}{12}$

#6 Name_____

Problems to do

1. $\frac{5 \times \square}{6 \times \square} = \frac{10}{12}$

2. $\frac{1 \times \square}{3 \times \square} = \frac{2}{6}$

3. $\frac{\square \times 4}{\square \times 4} = \frac{4}{12}$

4. $\frac{\square \times 2}{\square \times 2} = \frac{4}{8}$

5. $\frac{1 \times \square}{2 \times \square} = \frac{2}{4}$

6. $\frac{2 \times 2}{4 \times 2} =$ _____

7. $\frac{4 \times \square}{8 \times \square}$ $\frac{8}{16}$

8. $\frac{\square \times 3}{\square \times 3} = \frac{6}{9}$

Yes or No:

_____ 9. $\frac{1}{2} = \frac{4}{8}$

_____ 10. $\frac{1}{2} = \frac{6}{12}$

_____ 11. $\frac{1}{2} = \frac{2}{4}$

_____ 12. $\frac{1}{2} = \frac{3}{6}$

_____ 13. $\frac{1}{2} = \frac{2}{3}$

_____ 14. $\frac{1}{2} = \frac{5}{10}$

_____ 15. $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$ (check by measuring)

_____ 16. $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$

- - - - - - -

A fraction is a division problem. The top numeral is called the numerator. It tells how many parts. The bottom numeral is called the denominator. It tells in how many parts the whole is divided.

$\frac{2}{3}$ → numerator
→ denominator {2 of 3 parts}

#7 Name

To change 1 of 2 parts to another name with 12 parts, multiply $\frac{1}{2} \times 1$. But remember!!

$1 = \frac{\triangle}{\triangle}$ so $\frac{1}{2} \times 1 = \frac{1}{2} \times \frac{6}{6} = \frac{1 \times 6}{2 \times 6} = \frac{6}{12}$ OK?

Let's try it $\frac{1}{3} = \frac{?}{12}$, $\frac{1 \times ?}{3 \times ?} = \frac{?}{12}$, $3 \times 4 = 12$

so $\frac{1 \times 4}{3 \times 4} = \frac{4}{12}$

1. $\frac{1}{4} = \frac{?}{12}$, $\frac{1 \times \triangle}{4 \times \triangle} = \frac{\square}{12}$
2. $\frac{2}{3} = \frac{?}{12}$, $\frac{2 \times \triangle}{3 \times \triangle} = \frac{\square}{12}$
3. $\frac{2}{3} = \frac{\square}{6}$
4. $\frac{3}{4} = \frac{\square}{8}$
5. $\frac{1}{2} = \frac{\square}{10}$
6. $\frac{4}{5} = \frac{\square}{10}$
7. $\frac{2}{3} = \frac{\square}{9}$
8. $\frac{10}{12} = \frac{\triangle \times 5}{\triangle \times 6}$
9. $\frac{2}{7} = \frac{\square}{14}$
10. $\frac{2}{5} = \frac{\square}{15}$
11. $\frac{6}{8} = \frac{\triangle \times 3}{\triangle \times 4}$
12. $\frac{9}{12} = \frac{\triangle \times \square}{\triangle \times 4}$
13. $\frac{3}{5} = \frac{\square}{20}$
14. $\frac{4}{7} = \frac{\square}{21}$

#8 Name_____

More Problems

1. Change $\frac{1}{3}$ to $\frac{\square}{12}$ and $\frac{1}{4}$ to $\frac{\square}{12}$

   Jan wanted to add $\frac{1}{4}$ to $\frac{1}{3}$ but she had lost her "part lines." She thought if I change $\frac{1}{3}$ to twelfths and $\frac{1}{4}$ to twelfths I could add the twelfths.

   $\frac{1}{3} = \frac{\square}{12}$ and $\frac{1}{4} = \frac{\square}{12}$ How many twelfths?

2. Can you add $\frac{1}{2}$ to $\frac{1}{3}$? (Check your "part lines" if you can't)

3. $\frac{1}{2} + \frac{2}{3} = \frac{\square}{6} + \frac{\bigcirc}{6} = \frac{\quad}{6}$

4. $\frac{2}{5} + \frac{1}{3} =$

5. $\frac{1}{2} + \frac{1}{5} =$

6. $\frac{1}{6} + \frac{1}{2} =$

7. $\frac{3}{4} + \frac{1}{12} =$

8. $\frac{1}{4} + \frac{3}{8} =$

9. $\frac{2}{5} + \frac{1}{4} =$

10. $\frac{1}{7} + \frac{1}{3} =$

11. $\frac{3}{7} + \frac{1}{4} =$

12. $\frac{1}{6} + \frac{2}{5} =$

#9 Name_____

John saw Jan add $\frac{1}{3}$ and $\frac{1}{4}$ to get $\frac{7}{12}$. She said $\frac{1 \times 4}{3 \times 4} = \frac{4}{12}$ and $\frac{1 \times 3}{4 \times 3} = \frac{3}{12}$, so $\frac{4}{12} + \frac{3}{12}$ is $4(\frac{1}{12}) + 3(\frac{1}{12}) = (4 + 3)(\frac{1}{12})$ $= 7(\frac{1}{12}) = \frac{7}{12}$. He thought ----- and thought  Why not $\frac{1}{3} + \frac{1}{4} = \frac{4 \times 1 + 3 \times 1}{3 \times 4}$ then $\frac{4 + 3}{12} = \frac{7}{12}$. They tried this again with $\frac{2}{5} + \frac{1}{3}$. They said 5 x 3 = 15 so the denominator will be 15.

$$\frac{2 \times 3 + 5 \times 1}{3 \times 5} = \frac{6 + 5}{15} \text{ or } \frac{11}{15}$$

Can you do these?

1. $\frac{1}{7} + \frac{1}{4} =$ _____ = _____

2. $\frac{1}{3} + \frac{1}{2} =$ _____ = _____

3. $\frac{3}{8} + \frac{2}{5} =$ _____ = _____

4. $\frac{3}{5} + \frac{1}{4} =$ _____ = _____

#10 Name_____

Let's check ourselves. Can we do these problems?

1. $5(\frac{1}{7}) = \frac{\triangle}{\square}$

2. $\frac{2}{3} = \square(\frac{1}{3})$

3. $6(\frac{1}{6}) = \square$

4. $\frac{3}{4} = \frac{\square}{12}$

5. $\frac{2}{5} = \frac{\square}{10}$

6. $\frac{3}{7} = \frac{9}{\square}$

7. $\frac{9}{12} = \frac{\square}{4}$

8. $\frac{4}{8} = \frac{\square}{2}$

9. $\frac{21}{21} = \square$

10. $4(\frac{1}{9}) + 3(\frac{1}{9}) = \square(\frac{1}{9})$

11. $\frac{1}{6} + \frac{5}{6} = \square$

12. $\frac{2}{7} + \frac{3}{7} = \boxed{-}$

13. $5(\frac{1}{7}) - 2(\frac{1}{7}) = \square(\frac{1}{7})$

14. $\frac{5}{12} - \frac{4}{12} = \boxed{-}$

15. $\frac{1}{3} + \frac{1}{2} = \boxed{-}$

16. $\frac{2}{5} + \frac{1}{6} = \boxed{-}$

#11 Name_____

Mary said to John, "John, will your way of adding fractions work for subtraction?" John said, "Let's see."

$\frac{3}{4} - \frac{1}{3}$ would be $\frac{3 \times 3 - 4 \times 1}{4 \times 3} = \frac{9 - 4}{12} = \frac{5}{12}$

They checked this on the "parts lines." Did it check?

- - - - - - -

They then tried these problems.

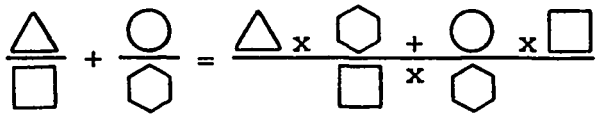
1. $\frac{4}{5} - \frac{1}{3} =$ _____ = _____

2. $\frac{5}{7} - \frac{1}{2} =$ _____ = _____

3. $\frac{3}{8} - \frac{1}{5} =$ _____ = _____

4. $\frac{2}{3} - \frac{2}{5} =$ _____ = _____

John said, "Here is a model for adding two fractions":



#12 Name_____

Problems to do: Find the $\frac{\text{sum}}{\text{(add)}}$ or $\frac{\text{difference}}{\text{(subtract)}}$

1. $\frac{7}{8} + \frac{1}{4} =$ _____

2. $\frac{3}{4} - \frac{1}{6} =$ _____

3. $\frac{5}{6} + \frac{3}{4} =$ _____

4. $\frac{3}{5} - \frac{1}{10} =$ _____

5. $\frac{3}{4} + \frac{1}{10} =$ _____

6. $\frac{2}{9} - \frac{1}{6} =$ _____

7. $\frac{7}{8} + \frac{3}{4} =$ _____

8. $\frac{1}{4} - \frac{1}{10} =$ _____

Joe said to Marty, "The answers I got to these problems can all be changed to fractions of less parts (denominators). Look at the first problem.

$$\frac{7 \times 4 \ + \ 8 \times 1}{8 \times 4} = \frac{28 + 8}{32} \text{ or } \frac{36}{32}$$

This means I have $\frac{32}{32}$ with $\frac{4}{32}$ more!"

(more tomorrow)

#13 Name_____

Marty said to Joe, "I have really been thinking about what you said about yesterday's problems and the only thing I can say is if you have more parts than you have in the whole line segment, you must have a whole segment with extra parts." Joe said, "You're right." We know:

$\frac{\triangle}{\triangle} = 1$, so $\frac{32}{32} = 1$, so $\frac{36}{32} = \frac{32}{32} + \frac{4}{32}$ or $1 + \frac{4}{32}$

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Can you go back over yesterday's problems and find answers like Marty and Joe did?

Marty said to Joe, "But look, Joe, I really meant the $\frac{4}{32}$ could be changed. Look!

$4 = 4 \times 1$ and $32 = 4 \times 8$ so

$\frac{4 \times 1}{4 \times 8} = \frac{1}{8}$!!!"

Can you do what Joe has done to other problems from yesterday's problems.

#14 Name_____

Alice said to Joe and Marty, "I've got some problems in addition and subtraction. Can you show me how you can find the sums or differences with smaller numbers?"

1. $\frac{1}{4} + \frac{3}{8} =$ _____

2. $\frac{5}{8} - \frac{1}{2} =$ _____

3. $\frac{5}{6} - \frac{1}{9} =$ _____

4. $\frac{5}{12} + \frac{1}{4} =$ _____

5. $\frac{7}{12} + \frac{1}{3} =$ _____

6. $\frac{5}{6} - \frac{1}{3} =$ _____

7. $\frac{7}{10} + \frac{1}{2} =$ _____

8. $\frac{1}{4} + \frac{5}{6} =$ _____

Alice said, "I am going to think about this while the teachers are away at their conference."

#15 Name_____

This morning Alice caught Joe and Marty. She said, "While the teachers were away last Friday I was thinking.

$$\frac{5}{6} + \frac{3}{4} = \frac{4 \times 5 + 3 \times 6}{6 \times 4} = \frac{20 + 18}{24} = \frac{38}{24},$$

$$\frac{38}{24} = \frac{24}{24} + \frac{14}{24} = 1 + \frac{14}{24},\ 14 = 2 \times 7 \text{ and } 24 = 2 \times 12$$

$$\text{so } \frac{14}{24} = \frac{2 \times 7}{2 \times 12} = \frac{7}{12} \text{ and } \frac{5}{6} + \frac{3}{4} = 1 \text{ and } \frac{7}{12}$$

"We knew all that," Marty said. "I know but look," Alice went on excitedly.

$\frac{5}{6} = \frac{5}{2 \times 3}$ and $\frac{3}{4} = \frac{3}{2 \times 2}$ "If I just use the 2 in each denominator once I get 2 x 3 = 6 and 6 x 2 = 12." Marty was thoughtful but Joe said, "So?" "Look Joe," Alice continued; "Don't you see?"

$$\frac{5}{6} + \frac{3}{4} = \frac{5}{2 \times 3} + \frac{3}{2 \times 2} \text{ and } \frac{2 \times 5 + 3 \times 3}{(2 \times 3) \times 2} =$$

$$\frac{10 + 9}{12} \text{ or } \frac{19}{12} \text{ or } \frac{12}{12} + \frac{7}{12} \text{ or } 1 + \frac{7}{12}$$

"Wow!!!" said Marty.

#16 Name_____

Marty said to Alice, "I tried what you said on a problem and it worked!" "Look:

$\frac{1}{6} + \frac{4}{15} = \frac{1}{2 \times \textcircled{3}} + \frac{4}{\textcircled{3} \times 5}$, so $\frac{5 \times 1 + 4 \times 2}{(2 \times 3) \times 5} = \frac{5 + 8}{6 \times 5} = \frac{13}{30}$ which is the least number of parts." (The least common denominator) Let's try some.

1. $\frac{1}{3} + \frac{1}{6} = \frac{1}{\textcircled{3} \times 1} + \frac{1}{\textcircled{3} \times 2} = \frac{2 \times 1 + 1 \times 1}{(3 \times 1) \times 2} =$ _____ = _____

2. $\frac{3}{8} + \frac{1}{4} =$ _____ + _____ = _____ = _____ = _____

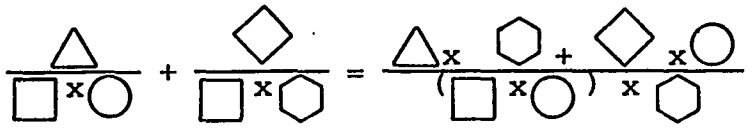
3. $\frac{3}{8} + \frac{5}{12} = \frac{\quad}{\textcircled{4} \times} + \frac{\quad}{\textcircled{4} \times} =$ _____ = _____ = _____

4. $\frac{1}{6} + \frac{2}{9} =$ _____ + _____ = _____ = _____ = _____

5. $\frac{5}{8} - \frac{1}{6} =$ _____ + _____ = _____ = _____ = _____

#17 Name_____

"Alice," Marty said, "I think I have a model for adding or subtracting the fractions we have been doing. This is it."



"I think you have it!" Alice responded gleefully. "Let's try it out."

1. $\frac{3}{10} + \frac{1}{4} =$ ——— + ——— = ——— = ——— = ———

2. $\frac{1}{8} - \frac{1}{12} =$ ——— − ——— = ——— = ——— = ———

3. $\frac{3}{8} + \frac{1}{6} =$ ——— + ——— = ——— = ——— = ———

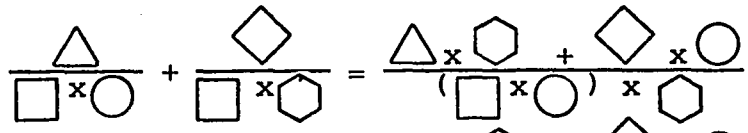
4. $\frac{7}{9} + \frac{1}{3} =$ ——— + ——— = ——— = ——— = □ + ———

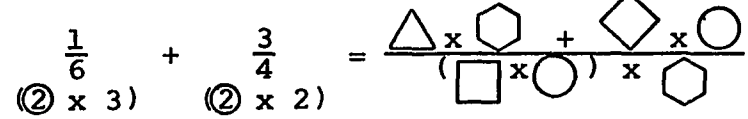
5. $\frac{3}{8} + \frac{5}{12} =$ ——— + ——— = ——— = ——— =

"You know," Alice said to Marty, "Whenever I see a whole number plus a fraction, I'm going to shorten it, like $1 + \frac{3}{7}$ is the same as $1\frac{3}{7}$." "Alright," said Marty.

#18 Name_____

Let's try some problems using Marty's model:



1. $\frac{1}{6} + \frac{3}{4} =$  = _____ = _____

   (② x 3) (② x 2)

2. $\frac{3}{8} + \frac{3}{10} =$ _____ = _____ = _____

3. $\frac{5}{8} - \frac{1}{6} =$ _____ = _____ = _____

4. $\frac{1}{7} + \frac{1}{14} =$ _____ = _____ = _____

5. $\frac{5}{6} - \frac{3}{8} =$ _____ = _____ = _____

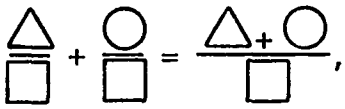
6. $\frac{3}{10} - \frac{1}{5} =$ _____ = _____ = _____

7. $\frac{3}{4} - \frac{1}{6} =$ _____ = _____ = _____

#19 Name_____

Today Kim said to Marty, "You only need 3 models for adding fractions. If the fraction looks like this:

1. $\frac{4}{7} + \frac{2}{7}$ you use this model

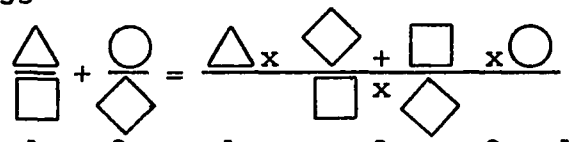


, so

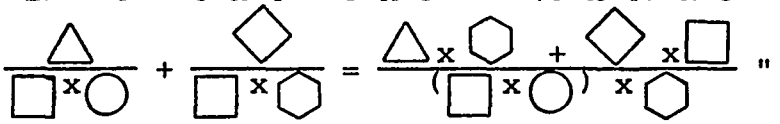
$\frac{4}{7} + \frac{2}{7} = \frac{4 + 2}{7}$ or $\frac{6}{7}$

2. Like this: $\frac{2}{5} + \frac{2}{7} = \frac{2 \times 7 + 5 \times 2}{5 \times 7}$ or $\frac{14 + 10}{35}$

or $\frac{24}{35}$ you use this model



3. For $\frac{1}{12} + \frac{3}{9} = \frac{1}{3 \times 4} + \frac{3}{3 \times 3} = \frac{3 \times 1 + 3 \times 4}{(3 \times 4) \times 3}$

or 

"

See if you can make up some problems from the models.

APPENDIX D

MULTIPLICATION AND DIVISION WORK SHEETS

#1 Name_____

1. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = ?(\frac{1}{8}) = \frac{\quad}{8}$

2. $\frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = ?(\frac{1}{12}) = \frac{\quad}{12}$

3. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} =$ ——

4. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} =$ ——

5. $\frac{2}{11} + \frac{2}{11} + \frac{2}{11} = \square(\frac{2}{11}) = \square \times 2(\frac{1}{11}) = \frac{\quad}{11}$

6. $\frac{2}{15} + \frac{2}{15} + \frac{2}{15} + \frac{2}{15} = \square(\frac{2}{15}) = \frac{\square}{15}$

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7. 7 x ☐ = 56

8. 6 x ☐ = 42

9. 9 x ☐ = 45

10. 6 x ☐ = 54

11. 5 x ☐ = 40

12. 7 x ☐ = 35

7 x ☐ = 21 is the same as saying 21 ÷ 7 = ☐

----so----

13. 54 ÷ 9 = ☐

14. 28 ÷ 7 = ☐

15. 64 ÷ 8 = ☐

16. 63 ÷ 9 = ☐

17. 20 ÷ 4 = ☐

18. 12 ÷ 3 = ☐

#2 Name_____

1. $5 \times 1 = \square$
2. $5 \div 1 = \square$
3. $\square \times 1 = 9$
4. $\square \div 1 = 9$
5. $7 \times \square = 7$
6. $7 \div \square = 7$

When you multiply a number by 1 or divide a number by 1 you get back the s... n.....

- - - - - - -

7. $5 \times 0 = 0$ is $0 \div 0 = 5$
8. $7 \times 0 = 0$ is $0 \div 0 = 7$
9. $8 \times 0 = 0$ is $0 \div 0 = 8$
10. $1 \times 0 = 0$ is $0 \div 0 = 1$

What can you say about dividing by zero? (Let's not divide by zero!!!)

11. $1 \times 8 = \square$
12. $\square \div 8 = 1$
13. $1 \times 4 = \square$
14. $\square \div 4 = 1$
15. $2 \times 7 = \square$
16. $\square \div 7 = 2$

#3 Name

1. $6 \times 9 = \square$
2. $\square \div 9 = 6$
3. $5 \times 8 = \square$
4. $\square \div 8 = 5$

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5. $(1 \times 7) \div \square = 1$
6. $(1 \times 4) \div \square = 1$
7. $(5 \times 6) \div \square = 5$
8. $(8 \times 9) \div \square = 8$
9. $4 \times (3 \div \square) = 4$
10. $5 \times (9 \div \square) = 5$
11. $9 \times (3 \div \square) = 9$
12. $3 \times \square \div 5 = 3$
13. $4 \times \square \div \square = 4$
14. $(4 \div 3) \times \square = 4$
15. $(18 \div 9) \times \square = 18$
16. $42 \div 6 \times \square = 42$
17. $117 \div 47 \times 47 = \square$
18. $1 \times 97 \div \square = 1$

Let's play a game called undoing what our friend does. Can you tell how the game is played?

#4 Name_____

A fraction is a division problem. Another way of saying $8 \div 4$ is $\frac{8}{4}$. The answer to a division problem is called the quotient. The quotient of $\frac{8}{4}$ is 2.

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Joe said, "I'm thinking of a fraction. If I multiply it by 8, I get 1. What is my fraction?"

- - - - - - -

Mary said to John, "I've found a model for the game we have been playing. Look!"

$\triangle \div \square \times \square = \triangle$ or $\frac{\triangle}{\square} \times \square = \triangle$

1. $\frac{6}{7} \times \boxed{?} = 6$

2. $\frac{2}{9} \times \square = 2$

3. $\frac{\square}{3} \times 3 = 2$

4. $\frac{9}{\square} \times 10 = 9$

- - - - - - -

Let's play the game of undoing what has been done. Make up 5 problems and challenge your friend to undo what you have done.

#5 Name_____

1. $\frac{8}{1} = \square$ 2. $\frac{7}{1} = \square$ 3. $\frac{3}{1} = \square$

4. $1 \times 8 = \square$ 5. $1 \times \square = 7$ 6. $\square \times 3 = 3$

- - - - - - - -

7. $\frac{6}{7} \times 1 = \square$ 8. $\frac{2}{7} \times 2 = \square$

9. $\frac{2}{9} \times 3 = \square$ 10. $\frac{4}{11} \times \frac{2}{1} = \square$

11. $\frac{2}{5} \times \frac{1}{1} = \square$ 12. $\frac{1}{9} \times \frac{4}{1} = \square$

13. $\frac{2}{7} \times \frac{1}{3} = \square$ 14. $\frac{2}{3} \times \frac{2}{5} = \square$

15. $\frac{4}{5} \times \frac{1}{2} = \square$ 16. $\frac{2}{9} \times \frac{3}{7} = \square$

17. $\frac{3}{7} \times \frac{2}{7} = \square$ 18. $\frac{2}{5} \times \frac{3}{7} = \square$

Extra for experts

$\frac{2}{3} \times \square + 1 = 3$

#6 Name_____

Can you make a model for multiplying two fractions?

_____ x _____ = _____

1. $\frac{7}{9} \times \frac{9}{7}$ means $7 \div 9 \times 9 \div 7$, $7 \div 9 \times 9 = \square$

   $\square \div 7 = 1$

2. $\frac{3}{4} \times \frac{4}{3} = \square$

3. $\frac{2}{7} \times \frac{7}{2} = \square$

4. $\frac{2}{3} \times \square = 1$

5. $\frac{3}{4} \times \square = 1$

6. $\frac{4}{5} \times \square = 1$

7. $\square \times \frac{4}{7} = 1$

8. $\frac{1}{3} \times \square = 1$

9. $\square \times \frac{7}{8} = 1$

10. $3 \times \square = 1$

11. $\square \times \frac{1}{4} = 1$

When two numbers (fractions) have a product of 1, these numbers are called <u>reciprocals</u>. What is the reciprocal "mate" of $\frac{3}{8}$?

#7 Name_____

Find the reciprocal mates of these.

1. $\frac{2}{5}$ $\square$

2. $\frac{7}{10}$ $\square$

3. $\frac{11}{15}$ $\square$

4. $\frac{\triangle}{\square}$ $\bigcirc$

- - - - - - -

More problems

5. $\frac{6}{7} \times \frac{1}{6} = \frac{6 \times 1}{7 \times 6} = \frac{6 \times 1}{6 \times 7} = \frac{6}{6} \times \frac{1}{7} = 1 \times \frac{1}{7} = \square$

6. $\frac{2}{3} \times \frac{3}{7} = \frac{2 \times 3}{3 \times 7} = \frac{2 \times 3}{7 \times 3} = \frac{2}{7} \times \square = \square$

7. $\frac{4}{9} \times \frac{1}{4} = \square$

8. $\frac{5}{8} \times \frac{8}{9} = \square$

9. $\frac{12}{13} \times \frac{2}{12} = \square$

10. $\frac{3}{7} \times \frac{7}{8} = \square$

11. $\frac{9}{10} \times \frac{1}{9} = \square$

12. $\frac{\triangle}{4} \times \frac{3}{\triangle} = \square$

Extra for experts

13. $\frac{2}{7} \times \square = \frac{6}{35}$

14. $\frac{\square}{7} \times \frac{5}{3} = \frac{5}{7}$

#8 Name_____

Warm ups

1. The reciprocal of $\frac{7}{8}$ is $\square$

2. The reciprocals of 8 is $\square$

3. $\frac{5}{7} \times \frac{2}{3} = \square$

4. $\frac{2}{9} \times \frac{9}{11} = \square$

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5. $\frac{4}{5} \times \frac{3}{8} = \frac{4 \times 3}{5 \times 8} = \frac{4 \times 3}{8 \times 5} = \frac{4}{8} \times \frac{3}{5} = \frac{1}{2} \times \frac{3}{5} = \square$

6. $\frac{3}{4} \times \frac{1}{9} = \frac{3 \times 1}{9 \times 4} = \frac{1\cancel{3} \times 1}{(\cancel{3} \times 3) \times 4} = \frac{1}{\square}$

Still shorter!

7. $\frac{1\cancel{6}}{\cancel{6}} \times \frac{1}{\cancel{15}_{3}} = \frac{1}{\square}$

8. $\frac{3}{7} \times \frac{1}{6} = \square$

9. $\frac{6}{7} \times \frac{7}{12} = \square$

10. $\frac{5}{6} \times \frac{2}{3} = \square$

Experts only

$\frac{7}{\square} \times \frac{2}{3} = \frac{7}{12}$

#9 Name_____

QUIZ

Let's multiply fractions.

1. $\frac{1}{4} \times \frac{1}{2} = \square$
2. $\frac{1}{2} \times \frac{3}{4} = \square$
3. $4 \times \frac{2}{9} = \square$
4. $\frac{3}{8} \times 8 = \square$
5. $\frac{3}{8} \times 2 = \square$
6. $2 \times \frac{5}{10} = \square$
7. $\frac{9}{10} \times \frac{1}{3} = \square$
8. $\frac{6}{7} \times \frac{7}{10} = \square$
9. $\frac{3}{8} \times \frac{4}{15} = \square$
10. $\frac{7}{8} \times \frac{2}{7} = \square$
11. $\frac{5}{9} \times \frac{3}{10} = \square$
12. $\frac{6}{14} \times \frac{7}{12} = \square$
13. $\frac{5}{9} \times \boxed{-} = \frac{1}{3}$
14. $\frac{3}{4} \times \boxed{-} = \frac{1}{2}$
15. $\frac{7}{9} \times \square = \frac{1}{3}$
16. $\frac{3}{10} \times \square = \frac{1}{5}$

Experts

$$\frac{\frac{2}{3} \times \square + 2}{3} + 1 = 2$$

#10 Name_____

More multiplication with fractions.

1. $\frac{\cancel{5}^{\textcircled{5}}}{\cancel{24}_{\textcircled{6}}} \times \frac{6}{\cancel{25}_{\textcircled{5}}} = \frac{1 \times 1}{4 \times 5} = \text{———}$

2. $\frac{12}{25} \times \frac{10}{21} = \frac{4 \times \textcircled{3}}{\textcircled{5} \times 5} \times \frac{\textcircled{5} \times 2}{7 \times \textcircled{3}} = \frac{4 \times 2}{5 \times 7} = \text{———}$

3. $\frac{21}{27} \times \frac{18}{28} = \text{———} \times \text{———} = \text{———} = \square$

4. $\frac{24}{35} \times \frac{28}{32} = \text{———} \times \text{———} = \text{———} = \square$

5. $\frac{30}{63} \times \frac{9}{10} = \text{———} \times \text{———} = \text{———} = \square$

6. $\frac{2}{3} \times \boxed{-} = 1$

7. $\frac{7}{8} \times \square = 7$

8. $\frac{4}{5} \times \boxed{-} = \frac{2}{5}$

9. $\frac{6}{15} \times \boxed{-} = \frac{1}{3}$

<u>For experts</u>:

$$\frac{\frac{2}{3} \times \square + 2}{2} - 2 = 0$$

#11 Name_____

1. $\frac{\triangle}{\square \times \bigcirc} \times \frac{\square}{\triangle \times 1} = \frac{1}{\square}$

2. $\frac{1}{3} \times \boxed{-} = \frac{2}{15}$

3. $\frac{3}{4} \times \boxed{-} = \frac{1}{2}$

4. $\frac{3}{5} \times \boxed{-} = \frac{3}{15}$

5. $\frac{4}{7} \times \square = 4$

When you ask the question

$\frac{2}{3} \times \boxed{-} = \frac{4}{9}$ you are really asking the question

$\boxed{-} = \frac{4}{9} \div \frac{2}{3}$. You may have found

$\frac{2}{3} \times \boxed{\frac{2}{3}} = \frac{4}{9}$ so $\frac{4}{9} \div \frac{2}{3} = \frac{2}{3}$.

If you took $\frac{4}{9}$ and multiplied by the <u>reciprocal</u>, of $\frac{2}{3}$, $\frac{3}{2}$ then

$\frac{4}{9} \times \frac{3}{2} = \frac{2}{3}$. $\frac{4}{9} \div \frac{2}{3}$ is the same as $\frac{4}{9} \times \frac{3}{2}$

6. $\frac{5}{8} \div \frac{2}{7} = \frac{5}{8} \times \frac{7}{2} = \frac{35}{16}$ or $\square + \frac{3}{16}$

#12 Name_____

Let's divide!

1. $\frac{5}{6} \div \frac{2}{3}$ means $\frac{5}{6} \times \frac{3}{2} = \square$

2. $\frac{3}{8} \div \frac{1}{2} = \frac{3}{8} \times \frac{2}{1} = \square$

3. $\frac{2}{5} \div \frac{4}{5} = \_\_\_\_ \times \_\_\_\_ = \square$

4. $\frac{1}{2} \div 4 = \_\_\_\_ \times \frac{1}{4} = \square$

5. $\frac{2}{7} \div 3 = \_\_\_\_ \times \_\_\_\_ = \square$

6. $\frac{7}{9} \div \frac{7}{3} = \_\_\_\_ \times \_\_\_\_ = \square$

7. $\frac{5}{7} \div \frac{2}{7} = \_\_\_\_ \times \_\_\_\_ = \square$

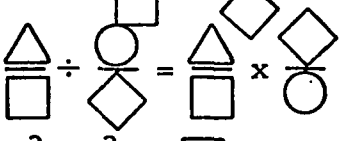
8. $\frac{6}{11} \div \frac{4}{11} = \_\_\_\_ \times \_\_\_\_ = \square$

Experts:

$$\frac{9 \times \square}{7} = 9$$

#13 Name_____

To divide  by  , multiply  by 



1. $\frac{3}{5} \div \frac{3}{8} = \square$
2. $\frac{3}{7} \div \frac{6}{5} = \square$
3. $\frac{5}{14} \div \frac{3}{7} = \square$
4. $\frac{3}{7} \div \frac{5}{14} = \square$
5. $\frac{2}{9} \div \frac{4}{3} = \square$
6. $\frac{4}{3} \div \frac{2}{9} = \square$
7. $\frac{7}{12} \div \frac{5}{9} = \square$
8. $\frac{5}{9} \div \frac{7}{12} = \square$
9. $\frac{3}{5} \div 1 = \square$
10. $1 \div \frac{3}{5} = \square$
11. $0 \div \frac{3}{5} = \square$
12. What about $\frac{3}{5} \div 0 =$?????
13. $\frac{2}{5} \div 4 = \square$
14. $4 \div \frac{2}{5} = \square$

Experts:

$\frac{2}{5} \div \square = \frac{1}{2}$

#14 Name_____

Let's have fun!

1. $15 \times (\frac{2}{5} + \frac{1}{3}) = 15 \times \frac{2}{5} + 15 \times \frac{1}{3} = 3 \times 2 + 5 = \square$

2. $9 \times (\frac{4}{9} + \frac{2}{3}) = 9 \times \frac{4}{9} + 9 \times \frac{2}{3} = \quad = \square$

3. $18 \times (\frac{5}{6} + \frac{4}{9}) = 18 \times \frac{5}{6} + 18 \times (\frac{4}{9}) = \quad = \square$

4. $\frac{20}{20} \times (\frac{3}{4} + \frac{2}{5}) = \frac{\overset{5}{\cancel{20}}}{20} \times \frac{3}{\cancel{4}} + \frac{\overset{4}{\cancel{20}}}{20} \times \frac{2}{\cancel{5}} = \frac{15}{20} + \frac{8}{20} = \frac{\square}{20}$

Do you remember $\frac{\triangle}{\triangle} = 1$?

5. $\frac{21}{21} \times (\frac{1}{3} + \frac{1}{7}) = \frac{21}{21} \times \frac{1}{3} + \frac{21}{21} \times \frac{1}{7} = \quad = \boxed{-}$

6. $\frac{24}{24} \times (\frac{3}{8} + \frac{1}{6}) = \quad = \square$

7. $\boxed{-} \times (\frac{2}{3} + \frac{1}{5}) = \quad = \frac{\square}{15}$

#15 Name_____

$\frac{\triangle}{\triangle} = 1$; $\triangle \times 1 = \triangle$; $\square \times (\triangle + \bigcirc) = \square \times \triangle + \square \times \bigcirc$

$1 \times (\frac{2}{5} + \frac{1}{3}) = \frac{15}{15} \times (\frac{2}{5} + \frac{1}{3}) = \frac{\cancel{15}^{3}}{15} \times \frac{2}{\cancel{3}} + \frac{\cancel{15}^{5}}{15} \times \frac{1}{\cancel{3}} =$

$\frac{3 \times 2}{15} + \frac{5 \times 1}{15} = \frac{6 + 5}{15} = \frac{11}{15}$

You try it!

1. $\frac{1}{4} + \frac{2}{3} = \boxed{-} \times (\frac{1}{4} + \frac{2}{3}) = \boxed{-} \times \frac{1}{4} + \boxed{-} \times \frac{2}{3} =$

____ + ____ = ____ = ____

2. $\frac{1}{7} + \frac{3}{8} =$

3. $\frac{1}{4} + \frac{3}{5} =$

4. $\frac{1}{6} + \frac{2}{5} =$

Experts:

$\frac{2 \times \boxed{-} + 3}{2} - 1 = 1$

#16 Name_____

Mark said, "I just got an answer of $\frac{25}{3}$ is that a number?"
"Sure Mark," said Ken. "For every 3 in 25 you have a whole number. Look!"

" $\frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{1}{3}$

or $8 \times 1 + \frac{1}{3}$ or $8 + \frac{1}{3}$ or $8\frac{1}{3}$ "

"That's too slow," said Luke. "Just divide 3 into 25. $3 \times 8 = 24$. So $\frac{25}{3} = 8$ with $\frac{1}{3}$ left over." "Try it!"

1. $\frac{15}{7} = \square$
2. $\frac{20}{3} = \square$
3. $\frac{17}{5} = \square$
4. $\frac{13}{2} = \square$
5. $\frac{19}{4} = \square$
6. $\frac{13}{4} = \square$
7. $\frac{38}{7} = \square$
8. $\frac{65}{9} = \square$

Experts:

$\square + \frac{2}{3} = \frac{14}{3}$

#17 Name_____

Tom said, "My Dad told me that when the numerator of a fraction is larger than the denominator, it's called an <u>improper fraction</u>. When you divide the de_____ into the n_____ you get a <u>mixed number</u>." "I can change a mixed number into an improper fraction," said Frank. "Look!"

"$4\frac{1}{2} = 4 + \frac{1}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{1}{2} = \frac{9}{2}$ "

Kevin said, "Why not just multiply 2 x 4 and add in a 1.

Like $\frac{2 \times 4 + 1}{2} = \frac{8 + 1}{2} = \frac{9}{2}$ "

"Here's a model," said Lenny.

$$\triangle + \frac{\bigcirc}{\square} = \frac{\triangle \times \square + \bigcirc}{\square}$$

Can you do these? (Change to mixed numbers)

1. $3\frac{2}{3} = \square$
2. $4\frac{1}{5} = \square$
3. $8\frac{4}{7} = \square$
4. $6\frac{1}{7} = \square$
5. $7\frac{3}{11} = \square$
6. $2\frac{9}{10} = \square$

#18 Name_____

Let's review and do fractions:

1. $\frac{6}{7} \times \square = 6$

2. $(4 \times \square) \div 6 = 4$

3. $\frac{7}{9} \times \square = 1$

4. $\square \times \frac{1}{3} = 1$

5. $\frac{2}{7} \times \frac{1}{3} = \square$

6. $\frac{2}{5} \times \frac{3}{11} = \square$

7. $\frac{3}{4} \times \frac{4}{7} = \square$

8. $\frac{4}{5} \times \frac{15}{16} = \square$

9. $\frac{2}{9} \div 3 = \square$

10. $\frac{1}{8} \div \frac{2}{3} = \square$

11. $\frac{3}{25} \div \frac{4}{5} = \square$

12. $\frac{3}{4} \times (\frac{1}{4} + \frac{1}{2}) = \square$

13. The reciprocal of $\frac{2}{3}$ is $\square$

14. The top numeral of a fraction is called the _____

15. The bottom numeral is called the _____

APPENDIX E

THE QUESTIONNAIRE PRESENTED TO SUBJECTS ON METHODS

Boy ☐ Girl ☐ Fourth Grade ☐ Sixth Grade ☐

1. Which class did you enjoy the most? The one where you earned nickels or the others?

   Nickels ☐ Other ☐ No Difference ☐

2. In which class did you work harder? The one where you earned nickels or the other?

   Nickels ☐ Other ☐ No Difference ☐

3. In which class do you think you learned more? The one where you earned nickels or the other?

   Nickels ☐ Other ☐ No Difference ☐

4. Did you worry in either class?

   Nickels___Yes ☐ No ☐ Other___Yes ☐ No ☐

5. Were you happier in either class?

   Nickels___Yes ☐ No ☐ Other___Yes ☐ No ☐

6. Would you always rather work in a class where you earned money or in the other type of class?

   Nickels ☐ Other ☐ No Difference ☐

APPENDIX F

RAW SCORE DATA

Group I

Fourth Grade

Boys

	Monetary--Teacher I			Verbal--Teacher II		
	Test A	Test B	Difference	Test C	Test D	Difference
1.	22	10	12	21	14	7
2.	23	9	14	22	13	9
3.	20	11	9	22	16	6
4.	25	13	12	21	7	14
5.	25	14	11	26	17	9
6.	21	10	11	20	14	6
7.	23	12	11	21	12	9
8.	22	14	8	18	16	2
9.	20	11	9	21	11	10
10.	23	14	9	21	17	4
11.	19	7	12	18	12	6
12.	22	12	10	21	14	7
13.	23	14	9	19	17	2
14.	22	12	10	21	11	10
15.	22	7	15	19	9	10
16.	24	8	16	20	14	6
17.	21	12	9	22	14	8

Group I

Fourth Grade

Girls

	Monetary--Teacher I			Verbal--Teacher II		
	Test A	Test B	Difference	Test C	Test D	Difference
1.	20	9	11	23	15	8
2.	24	8	16	22	12	10
3.	20	13	7	20	17	3
4.	20	7	13	21	11	10
5.	24	11	13	23	14	9
6.	20	8	12	24	9	15
7.	23	15	8	21	18	3
8.	19	12	7	22	17	5
9.	24	13	11	23	15	8
10.	26	19	7	25	14	11
11.	20	9	11	23	7	16
12.	20	9	11	21	14	7
13.	26	11	15	23	21	2
14.	23	9	14	19	8	11
15.	23	9	14	23	12	11
16.	26	12	14	18	11	7

Group II

Fourth Grade

Boys

	Monetary--Teacher III			Verbal--Teacher IV		
	Test A	Test B	Difference	Test C	Test D	Difference
1.	26	9	17	19	12	7
2.	22	12	10	22	7	15
3.	25	13	12	25	14	11
4.	26	11	15	17	11	6
5.	26	10	16	24	9	15
6.	26	13	13	23	16	7
7.	25	13	12	22	14	8
8.	26	9	17	21	7	14
9.	26	12	14	23	12	11
10.	26	16	10	25	8	17
11.	25	13	12	20	11	9
12.	23	10	13	21	8	13
13.	26	10	16	24	12	12
14.	26	21	5	22	12	10
15.	26	17	9	26	13	13
16.	26	25	1	26	19	7
17.	24	10	14	22	11	11
18.	25	12	13	21	13	8

Group II

Fourth Grade

Girls

	Monetary--Teacher III			Verbal--Teacher IV		
	Test A	Test B	Difference	Test C	Test D	Difference
1.	26	24	2	26	16	10
2.	23	10	13	20	13	7
3.	24	16	8	21	16	5
4.	21	19	3	24	5	19
5.	26	7	19	21	8	13
6.	26	14	12	25	12	13
7.	26	15	11	24	16	8
8.	25	8	17	22	4	18
9.	26	14	12	21	11	10
10.	26	13	13	21	8	13
11.	24	11	13	25	15	10
12.	26	17	9	26	17	9
13.	26	21	5	26	14	12
14.	25	13	12	20	11	9

Group III

Fourth Grade

Boys

	Verbal--Teacher II			Monetary--Teacher I		
	Test A	Test B	Difference	Test C	Test D	Difference
1.	25	15	10	25	18	7
2.	24	11	13	23	16	7
3.	26	22	4	24	22	2
4.	26	15	11	24	22	2
5.	26	23	3	24	21	3
6.	26	20	6	25	24	1
7.	23	8	15	20	9	11
8.	26	13	13	25	21	4
9.	25	19	6	25	22	3
10.	26	20	6	17	18	-1
11.	26	11	5	21	23	-2
12.	26	23	3	24	22	2
13.	26	22	4	26	25	1
14.	26	19	7	22	21	1
15.	26	21	5	23	23	0

Group III

Fourth Grade

Girls

	Verbal--Teacher II			Monetary--Teacher I		
	Test A	Test B	Difference	Test C	Test D	Difference
1.	24	12	12	24	13	11
2.	25	17	8	25	20	5
3.	26	25	1	25	16	9
4.	26	21	5	22	22	0
5.	25	18	7	23	23	0
6.	25	13	12	25	24	1
7.	26	16	10	22	20	2
8.	21	15	6	24	24	0
9.	26	18	8	24	17	7
10.	25	17	8	22	12	10
11.	25	19	6	23	21	2

Group IV

Fourth Grade

Boys

	Verbal--Teacher IV			Monetary--Teacher III		
	Test A	Test B	Difference	Test C	Test D	Difference
1.	25	11	14	23	10	13
2.	26	20	6	25	23	2
3.	25	11	14	21	15	6
4.	24	12	12	18	14	4
5.	23	12	11	22	16	6
6.	26	23	3	24	12	12
7.	20	18	2	19	9	10
8.	23	14	9	24	14	10
9.	25	15	10	25	25	0
10.	26	15	11	24	11	13
11.	23	8	15	23	17	6
12.	25	12	13	24	9	15
13.	25	13	12	20	13	7
14.	17	11	6	22	17	5
15.	24	10	14	22	10	12
16.	26	18	8	21	18	3
17.	20	7	13	21	10	11

Group IV

Fourth Grade

Girls

	Verbal--Teacher IV			Monetary--Teacher III		
	Test A	Test B	Difference	Test C	Test D	Difference
1.	24	13	11	17	10	7
2.	24	19	5	23	16	7
3.	23	14	9	17	10	7
4.	26	15	7	22	12	10
5.	22	12	10	23	13	10
6.	25	18	7	21	15	6
7.	20	11	9	21	6	15
8.	25	8	17	17	10	7
9.	24	16	8	22	13	9
10.	25	14	11	21	10	11
11.	24	14	10	24	23	1
12.	25	15	10	22	11	11
13.	22	12	10	18	14	4
14.	22	10	12	21	10	11
15.	26	13	13	25	18	7
16.	25	17	8	25	10	15
17.	24	20	4	21	12	9

Group V

Sixth Grade

Boys

	Verbal--Teacher III			Monetary--Teacher IV		
	Test A	Test B	Difference	Test C	Test D	Difference
1.	13	1	12	11	2	9
2.	16	8	8	24	17	7
3.	12	3	9	20	7	13
4.	11	2	9	20	8	12
5.	15	4	11	19	13	6
6.	17	7	10	16	16	0
7.	20	7	13	18	13	5
8.	10	4	6	21	7	14
9.	13	3	10	19	7	12
10.	16	7	9	21	11	10
11.	10	0	10	13	0	13
12.	16	9	7	24	10	14
13.	8	0	8	19	6	13
14.	6	2	4	18	5	13
15.	20	14	6	23	15	8
16.	9	1	8	17	2	15

Group V

Sixth Grade

Girls

	Verbal--Teacher III			Monetary--Teacher IV		
	Test A	Test B	Difference	Test C	Test D	Difference
1.	11	2	9	19	12	7
2.	20	3	17	19	10	9
3.	12	9	3	21	10	11
4.	14	2	12	15	11	4
5.	14	1	13	16	7	9
6.	16	5	11	23	11	6
7.	16	3	13	25	7	18
8.	14	6	8	20	9	11
9.	21	9	12	21	17	4
10.	7	4	3	18	9	9
11.	12	7	5	19	7	12
12.	14	5	9	20	6	14
13.	10	8	2	21	10	11
14.	11	4	7	18	7	11
15.	4	4	0	20	7	13
16.	10	5	5	19	13	6
17.	7	3	4	21	7	14

Group VI

Sixth Grade

Boys

	Monetary--Teacher II			Verbal--Teacher I		
	Test A	Test B	Difference	Test C	Test D	Difference
1.	15	9	7	24	8	16
2.	10	2	8	19	9	10
3.	8	2	6	17	3	14
4.	14	13	1	16	14	2
5.	3	2	1	16	3	13
6.	15	3	12	21	16	5
7.	9	4	5	19	9	10
8.	11	3	8	6	4	2
9.	10	3	7	18	13	5
10.	17	6	11	11	19	-8
11.	3	1	2	16	1	15
12.	10	4	6	9	2	7
13.	10	0	10	16	13	3
14.	15	0	15	15	9	6
15.	17	8	9	18	4	14
16.	3	0	3	21	2	19

Group VI

Sixth Grade

Girls

	Monetary--Teacher II			Verbal--Teacher I		
	Test A	Test B	Difference	Test C	Test D	Difference
1.	23	20	3	12	9	3
2.	12	0	12	15	14	1
3.	6	2	4	20	6	14
4.	11	1	10	21	11	10
5.	11	2	9	14	11	3
6.	14	4	10	13	10	3
7.	17	5	16	13	9	4
8.	10	6	4	19	7	12
9.	14	4	10	15	10	5
10.	16	12	4	20	7	13
11.	10	6	4	16	7	9
12.	10	1	9	15	14	1
13.	6	5	1	18	11	7
14.	19	4	15	13	21	-8
15.	12	7	5	24	14	10
16.	8	6	2	12	14	-2
17.	6	1	5	17	3	14
18.	9	2	7	17	8	9
19.	7	1	6	14	8	6

Group VII

Sixth Grade

Boys

	Monetary--Teacher IV			Verbal--Teacher III		
	Test A	Test B	Difference	Test C	Test D	Difference
1.	18	14	4	23	16	7
2.	21	15	6	24	18	6
3.	11	13	-2	20	16	4
4.	25	13	12	24	14	10
5.	13	10	3	22	21	1
6.	14	11	3	20	14	6
7.	23	15	8	22	20	2
8.	15	13	2	21	20	1
9.	16	13	3	20	19	1
10.	11	3	8	11	10	1
11.	26	22	4	24	22	2
12.	15	4	11	24	12	12
13.	25	16	9	20	21	-1
14.	11	9	2	24	10	14

Group VII

Sixth Grade

Girls

	Monetary--Teacher IV			Verbal--Teacher III		
	Test A	Test B	Difference	Test C	Test D	Difference
1.	18	14	4	22	13	9
2.	14	8	6	22	7	15
3.	18	11	7	20	11	9
4.	18	10	8	20	16	4
5.	10	2	8	18	15	3
6.	17	10	7	22	11	11
7.	22	11	11	24	14	10
8.	16	12	4	21	13	8
9.	19	12	7	22	16	6
10.	21	19	2	22	21	1
11.	25	10	15	15	13	2
12.	21	5	16	23	10	13
13.	15	7	8	19	9	10

Group VIII

Sixth Grade

Boys

	Verbal--Teacher I			Monetary--Teacher II		
	Test A	Test B	Difference	Test C	Test D	Difference
1.	3	3	0	10	7	3
2.	18	4	14	21	18	3
3.	9	4	5	20	7	13
4.	9	7	2	19	4	15
5.	11	5	6	12	3	9
6.	16	10	6	21	13	8
7.	10	4	6	11	5	6
8.	10	4	6	11	2	9
9.	11	4	7	16	14	2
10.	3	3	0	15	7	8
11.	13	7	6	21	7	14
12.	19	15	4	14	8	6
13.	13	5	8	20	7	13
14.	3	4	-1	15	6	9
15.	13	5	8	23	9	14
16.	9	4	5	16	4	12
17.	12	3	9	18	9	9
18.	22	5	17	21	9	12
19.	4	6	-2	16	7	9

Group VIII

Sixth Grade

Girls

	Verbal--Teacher I			Monetary--Teacher II		
	Test A	Test B	Difference	Test C	Test D	Difference
1.	10	5	5	18	4	14
2.	6	6	0	14	6	8
3.	10	6	4	11	9	2
4.	13	4	9	18	7	11
5.	12	19	-7	14	11	3
6.	9	5	4	13	6	7
7.	15	11	4	22	13	9
8.	12	6	6	20	9	11
9.	14	6	8	19	8	11
10.	6	9	-3	23	12	11
11.	13	5	8	16	14	2
12.	7	5	2	16	9	7
13.	8	4	4	16	10	6
14.	7	4	3	14	5	9

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