INFORMATION TO USERS

This dissertation was produced from a microfilm copy of the original document. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the original submitted.

The following explanation of techniques is provided to help you understand markings or patterns which may appear on this reproduction.

1. The sign or “target” for pages apparently lacking from the document photographed is “Missing Page(s)”. If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting thru an image and duplicating adjacent pages to insure you complete continuity.

2. When an image on the film is obliterated with a large round black mark, it is an indication that the photographer suspected that the copy may have moved during exposure and thus cause a blurred image. You will find a good image of the page in the adjacent frame.

3. When a map, drawing or chart, etc., was part of the material being photographed the photographer followed a definite method in “sectioning” the material. It is customary to begin photoing at the upper left hand corner of a large sheet and to continue photoing from left to right in equal sections with a small overlap. If necessary, sectioning is continued again — beginning below the first row and continuing on until complete.

4. The majority of users indicate that the textual content is of greatest value, however, a somewhat higher quality reproduction could be made from “photographs” if essential to the understanding of the dissertation. Silver prints of “photographs” may be ordered at additional charge by writing the Order Department, giving the catalog number, title, author and specific pages you wish reproduced.

University Microfilms
300 North Zeeb Road
Ann Arbor, Michigan 48106
A Xerox Education Company
YOUNG, Donald Edward, 1941-
TIME DOMAIN AND FREQUENCY DOMAIN SCATTERING
FROM A CONDUCTING CYLINDER IN A PARALLEL
PLATE WAVEGUIDE.

The University of Arizona, Ph.D., 1972
Engineering, electrical

University Microfilms, A XEROX Company, Ann Arbor, Michigan
TIME DOMAIN AND FREQUENCY DOMAIN SCATTERING
FROM A CONDUCTING CYLINDER IN A
PARALLEL PLATE WAVEGUIDE

by

Donald Edward Young

A Dissertation Submitted to the Faculty of the
DEPARTMENT OF ELECTRICAL ENGINEERING
In Partial Fulfillment of the Requirements
For the Degree of
DOCTOR OF PHILOSOPHY
In the Graduate College
THE UNIVERSITY OF ARIZONA

1972
I hereby recommend that this dissertation prepared under my direction by Donald Edward Young entitled TIME DOMAIN AND FREQUENCY DOMAIN SCATTERING FROM A CONDUCTING CYLINDER IN A PARALLEL PLATE WAVEGUIDE be accepted as fulfilling the dissertation requirement of the degree of Doctor of Philosophy.

Dissertation Director  
Date

After inspection of the final copy of the dissertation, the following members of the Final Examination Committee concur in its approval and recommend its acceptance:

This approval and acceptance is contingent on the candidate's adequate performance and defense of this dissertation at the final oral examination. The inclusion of this sheet bound into the library copy of the dissertation is evidence of satisfactory performance at the final examination.
STATEMENT BY AUTHOR

This dissertation has been submitted in partial fulfillment of requirements for an advanced degree at The University of Arizona and is deposited in the University Library to be made available to borrowers under the rules of the Library.

Brief quotations from this dissertation are allowable without special permission, provided that accurate acknowledgment of source is made. Requests for permission for extended quotation from or reproduction of this manuscript in whole or in part may be granted by the Head of the Electrical Engineering Department or the Dean of the Graduate College when in his judgment the proposed use of the material is in the interests of scholarship. In all other instances, however, permission must be obtained from the author.

SIGNED: Donald E. Young
ACKNOWLEDGMENTS

The author is pleased to have had for his advisor Dr. Donald G. Dudley and wishes to thank him for his assistance in selecting the topic and scope of this dissertation.

This work was supported by funds from the U. S. Army SAFEGUARD Communications Agency (SAFCA) under contracts DAEA 18-70-C-0132 and DAEA 18-70-C-0283. The author wishes to thank the personnel of SAFCA and the MITRE Corporation, especially Dr. John M. Hamm, for providing the experimental data.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>ix</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>x</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. FREQUENCY DOMAIN SOLUTION</td>
<td>3</td>
</tr>
<tr>
<td>2.1 Integral Formulation of the Magnetic Field Intensity</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Frequency Domain Green's Function</td>
<td>6</td>
</tr>
<tr>
<td>2.3 Evaluation of the Integral Equation by Moment Methods</td>
<td>8</td>
</tr>
<tr>
<td>3. TIME DOMAIN SOLUTION</td>
<td>16</td>
</tr>
<tr>
<td>3.1 Integral Formulation of the Magnetic Field Intensity</td>
<td>18</td>
</tr>
<tr>
<td>3.2 Time Domain Green's Function</td>
<td>20</td>
</tr>
<tr>
<td>3.3 Method of Solution</td>
<td>21</td>
</tr>
<tr>
<td>4. DISCUSSION OF THE INTEGRAL EQUATION SOLUTIONS</td>
<td>26</td>
</tr>
<tr>
<td>4.1 Frequency Domain</td>
<td>26</td>
</tr>
<tr>
<td>4.2 Time Domain</td>
<td>27</td>
</tr>
<tr>
<td>4.3 Frequency Domain Versus Time Domain</td>
<td>38</td>
</tr>
<tr>
<td>5. ANALYSIS OF THE PARALLEL PLATE SIMULATOR</td>
<td>40</td>
</tr>
<tr>
<td>5.1 Frequency Domain Analysis</td>
<td>40</td>
</tr>
<tr>
<td>5.2 Time Domain Analysis</td>
<td>56</td>
</tr>
<tr>
<td>5.3 Comparison With Experimental Results</td>
<td>62</td>
</tr>
<tr>
<td>6. CONCLUSIONS</td>
<td>75</td>
</tr>
<tr>
<td>APPENDIX A: FREQUENCY DOMAIN SINGULARITY</td>
<td>77</td>
</tr>
<tr>
<td>APPENDIX B: TIME DOMAIN SINGULARITY</td>
<td>80</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS (Continued)

APPENDIX C: COMPUTER PROGRAM LISTINGS ...................... 85

Frequency Domain Program ...................................... 85
Time Domain Program ............................................ 89

REFERENCES ....................................................... 93
### LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Coordinate system of the parallel plate structure</td>
<td>4</td>
</tr>
<tr>
<td>2-2</td>
<td>Pole locations in the $\zeta$-plane</td>
<td>9</td>
</tr>
<tr>
<td>2-3</td>
<td>Subsection index</td>
<td>13</td>
</tr>
<tr>
<td>3-1</td>
<td>Coordinate system for the time domain solution</td>
<td>17</td>
</tr>
<tr>
<td>4-1</td>
<td>Square cylinder in free space</td>
<td>29</td>
</tr>
<tr>
<td>4-2</td>
<td>Impulse response of square cylinder, positions 1 through 4</td>
<td>30</td>
</tr>
<tr>
<td>4-3</td>
<td>Impulse response of square cylinder, positions 5 through 8</td>
<td>31</td>
</tr>
<tr>
<td>4-4</td>
<td>Impulse response of square cylinder, positions 9 through 12</td>
<td>32</td>
</tr>
<tr>
<td>4-5</td>
<td>Impulse response of square cylinder, positions 13 through 16</td>
<td>33</td>
</tr>
<tr>
<td>4-6</td>
<td>Impulse response of square cylinder, positions 17 through 20</td>
<td>34</td>
</tr>
<tr>
<td>4-7</td>
<td>Square cylinder current density (Comparison with Mei and Van Bladel)</td>
<td>37</td>
</tr>
<tr>
<td>4-8</td>
<td>Square cylinder current density (Comparison with Andreasen)</td>
<td>37</td>
</tr>
<tr>
<td>5-1</td>
<td>Building in a vertical parallel plate simulator</td>
<td>41</td>
</tr>
<tr>
<td>5-2</td>
<td>Bottom-corner, $kw=0.3$</td>
<td>42</td>
</tr>
<tr>
<td>5-3</td>
<td>Top-center, $kw=0.3$</td>
<td>44</td>
</tr>
<tr>
<td>5-4</td>
<td>Bottom-corner, $w=\lambda/8$</td>
<td>45</td>
</tr>
</tbody>
</table>
# LIST OF ILLUSTRATIONS (Continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-5. Top-center, $w=\lambda/8$.</td>
<td>46</td>
</tr>
<tr>
<td>5-6. Bottom-corner, $w=\lambda/4$.</td>
<td>47</td>
</tr>
<tr>
<td>5-7. Top-center, $w=\lambda/4$.</td>
<td>48</td>
</tr>
<tr>
<td>5-8. Bottom-corner, $w=3\lambda/8$.</td>
<td>49</td>
</tr>
<tr>
<td>5-9. Top-center, $w=3\lambda/8$.</td>
<td>50</td>
</tr>
<tr>
<td>5-10. Bottom-corner, $w=\lambda/2$.</td>
<td>51</td>
</tr>
<tr>
<td>5-11. Top-center, $w=\lambda/2$.</td>
<td>52</td>
</tr>
<tr>
<td>5-12. Current density at bottom-corner of a typical building.</td>
<td>54</td>
</tr>
<tr>
<td>5-13. Current density at top-center of a typical building.</td>
<td>55</td>
</tr>
<tr>
<td>5-14. Current density at bottom-corner of a typical building with input pulse.</td>
<td>57</td>
</tr>
<tr>
<td>5-15. Approximate impulse response at the bottom-corner of a typical building.</td>
<td>59</td>
</tr>
<tr>
<td>5-16. Current density at top-center of a typical building with input pulse.</td>
<td>60</td>
</tr>
<tr>
<td>5-17. Illustration of delay time paths.</td>
<td>61</td>
</tr>
<tr>
<td>5-18. Theoretical and experimental magnetic field intensities at the top-center of a mock-up building.</td>
<td>64</td>
</tr>
<tr>
<td>5-19. Horizontal array over a five foot high mock-up building.</td>
<td>66</td>
</tr>
<tr>
<td>5-20. Theoretical results for a five foot high mock-up building, with and without parallel plates.</td>
<td>67</td>
</tr>
<tr>
<td>5-21. Theoretical and experimental results for a five foot high mock-up building.</td>
<td>68</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>5-22.</td>
<td>Horizontal array over a 14 foot high mock-up building.</td>
</tr>
<tr>
<td>5-23.</td>
<td>Theoretical results for a 14 foot high mock-up building with and without parallel plates, input and positions 1 through 3.</td>
</tr>
<tr>
<td>5-24.</td>
<td>Theoretical results for a 14 foot high mock-up building with and without parallel plates, positions 4 through 7.</td>
</tr>
<tr>
<td>5-25.</td>
<td>Theoretical and experimental results for a 14 foot high mock-up building, positions 1 through 4.</td>
</tr>
<tr>
<td>5-26.</td>
<td>Theoretical and experimental results for a 14 foot high mock-up building, positions 5 through 7.</td>
</tr>
<tr>
<td>B-1.</td>
<td>Integration over the singularity.</td>
</tr>
<tr>
<td>Table</td>
<td>Page</td>
</tr>
<tr>
<td>---------------</td>
<td>------</td>
</tr>
<tr>
<td>5-1. Delay times</td>
<td>63</td>
</tr>
</tbody>
</table>
ABSTRACT

The problem of a perfectly conducting rectangular cylinder in a parallel plate waveguide excited by a TEM wave is presented. Frequency and time domain solutions for the surface current density over the cylinder are obtained by use of the method of moments to solve an integral equation. Singularities in the equations are removed analytically. The frequency domain solution leads to a set of simultaneous equations for the expansion function coefficients. The time domain solution is a progressive set of linear equations in which each unknown coefficient is computed in terms of previously computed coefficients. The first time increment is determined by the input signal and the initial conditions.

The two solutions are compared after inversion of the frequency domain solution by means of a numerical Fourier transform program. Numerical results are also checked by comparing them with published frequency domain results for a rectangular cylinder in free space and with experimental data from a parallel plate simulator.

The integral equations are used to evaluate the degree of distortion introduced when a parallel plate waveguide is used to simulate free space scattering conditions.
The effect on the surface current density as the plate spacing is increased is illustrated in both the frequency and time domains. For the frequency domain, the magnitude of the surface current density oscillates about the free space value with decreasing amplitude as the plate spacing is increased. At frequencies which are approximately midway between adjacent plate resonant frequencies, the surface current nearly equals the free space value. In the time domain, the effect of the parallel plates is to add to the incident electromagnetic pulse a damped oscillation which is a combination of the natural cylinder resonance and the plate resonances. The numerical results aid in the selection of an optimum plate spacing for experimental measurements.
CHAPTER 1

INTRODUCTION

Experimental measurements of induced currents on a scattering body or of radar cross sections are sometimes limited by physical dimensions or power requirements because of the need for an incident plane wave. This is especially true for low frequencies when frequency scaling of the obstacle is not possible. One method [1] that is used to overcome these difficulties is to use a parallel plate waveguide. The scattering object is placed inside of the parallel plate waveguide which is excited so as to produce an incident transverse electromagnetic wave (TEM).

The field incident upon the scattering object is identical to that of a plane wave in free space. However, the scattered field produced by the object in the waveguide is different than the scattered field produced by the object in free space because of multiple reflections from the parallel plates or, equivalently, mutual coupling between the object and its images. The degree of distortion caused by the parallel plate simulator is the subject of this dissertation.

For certain applications, both time and frequency characteristics of the parallel plate simulator are of
interest. In the past, most analyses of obstacles in waveguides have been done in the frequency domain. However, direct time domain solutions for objects in free space are now appearing in the literature [2]. Time domain solutions are of interest for wideband systems such as the response of a communication building to an electromagnetic pulse (EMP).

In this dissertation, an integral equation for the magnetic field intensity induced upon a scattering body is solved in both the frequency and time domains. The derivations and solutions of the frequency and time domain integral equations appear in Chapters 2 and 3, respectively. Discussion of the two solutions is included in Chapter 4.

The scattering object in this analysis of the parallel plate simulator is assumed to be perfectly conducting. Also, the scattering object, the source, and the parallel plates are of infinite extent in one dimension. Although the techniques used in this dissertation are applicable to an arbitrarily shaped cylinder, results are presented for only a rectangular cylinder. These results, along with some experimental results, appear in Chapter 5. Final comments and conclusions are given in Chapter 6.
CHAPTER 2

FREQUENCY DOMAIN SOLUTION

The coordinate system used to analyze the vertical parallel plate structure is shown in figure 2-1. The scattering object is a rectangular cylinder of width w and height h. The source of the TEM wave is a delta function voltage source of the form

\[ \tilde{M} = \mathcal{Q} V_0(\omega) \delta(z - z_0) \]  \hspace{1cm} (2-1)

In the frequency domain, Maxwell's equations are;

\[ \nabla \times \tilde{E} = -i\omega \mu \tilde{H} - \tilde{M} \]  \hspace{1cm} (2-2)

\[ \nabla \times \tilde{H} = i\omega \varepsilon \tilde{E} \]  \hspace{1cm} (2-3)

The boundary conditions are that the tangential \( \tilde{E} \) field be zero on the ground, parallel plates, and the sides of the rectangular cylinder. Since there is no variation in the y direction and the voltage source has only a y component, there exists only a y component of the magnetic field intensity, \( \tilde{H} \). Therefore, the letter \( H \) without the arrow or subscript is used to indicate the y component of the vector \( \tilde{H} \).
Figure 2-1. Coordinate system of the parallel plate structure.
The wave equation for $H$ obtained from equations (2-2) and (2-3) is

$$
\nabla^2 H + k^2 H = i\omega e M_y .
$$

(2-4)

The boundary conditions for equation (2-4) are obtained from setting the tangential $\vec{E}$ fields to zero which implies that

$$
\frac{\partial H}{\partial n} = 0
$$

(2-5)

on all of the perfectly conducting surfaces.

2.1 Integral Formulation of the Magnetic Field Intensity

The solution of equation (2-4) is simplified by first finding a Green's function, $G$, which satisfies

$$
\nabla^2 G(r) + k^2 G(r) = -\delta(x - x')\delta(z - z') .
$$

(2-6)

Multiplying equation (2-4) by $G$, equation (2-6) by $H$, subtracting the two results and integrating over the free space volume $V$,

$$
\int_V [G(r,r') \nabla^2 H(r') - H(r') \nabla^2 G(r,r')] \, dV' =
$$

(2-7)

$$
i\omega e \int_V G(r,r') M(r') \, dV' + \int_V H(r') \delta(x-x')\delta(z-z') \, dV' .
$$
Evaluating the trivial integration on the right hand side of equation (2-7) and using Green's theorem [3] on the left hand side,

\[ \int_S [G(r,r') \frac{\partial H(r')}{\partial n'} - H(r') \frac{\partial G(r,r')}{\partial n'}] dS' = \]

\[ i\omega e \int_V G(r,r') M(r') dV' + H(r) . \]

But on the surface S, bounding the free space volume V, \( \frac{\partial H(r')}{\partial n'} = 0 \). Also, if the boundary conditions on G are chosen such that \( \frac{\partial G}{\partial n'} = 0 \) on the plates and the ground, equation (2-8) reduces to

\[ H(r) = -i\omega e \int_V G(r,r') M(r') dV' - \int_{S_c} H(r') \frac{\partial G(r,r')}{\partial n'} dS' \quad (2-9) \]

where \( S_c \) indicates the surface of the rectangular cylinder. Therefore, once G is obtained, equation (2-9) is a well defined integral equation for H. The first term on the right hand side of equation (2-9) is designated \( H^{\text{inc}} \) for the incident field and the second term designated \( H^{\text{scat}} \) for the scattered field.

2.2 Frequency Domain Green's Function

Equation (2-6) can be simplified by taking the Fourier cosine transform with respect to the z variable. The result is
\[
\frac{\partial^2 G(x, \xi; x', z')}{\partial x^2} + \gamma^2 G(x, \xi; x', z') = -\delta(x-x') \sqrt{\frac{2}{\pi}} \cos \xi z' 
\]

where

\[
\gamma = \sqrt{k^2 - \xi^2} .
\]

Equation (2-10) is a simple one dimensional Green's function with boundary conditions, \( \frac{\partial G}{\partial x} = 0 \) at \( x = 0 \) and \( x = a \). By direct construction the solution is

\[
G(x, \xi; x', z') = \frac{\sqrt{2/\pi} \cos \xi z'}{-\gamma \sin \gamma a} [\cos \gamma x \cos \gamma (x'-a)U(x'-x) + \cos \gamma x' \cos \gamma (x-a)U(x-x')]
\]

where \( U(x-x') \) is the unit step function, being unity if the argument is positive and zero if the argument is negative.

The Green's function is given by the inverse Fourier cosine transform of equation (2-11), i.e.,

\[
G(x, \xi; x', z') =
\]

\[
\frac{2}{\pi} \int_0^\infty \cos \gamma x \cos \gamma (x'-a)U(x'-x) + \cos \gamma x' \cos \gamma (x-a)U(x-x') \frac{\cos \xi z' \cos \xi d\xi}{-\gamma \sin \gamma a}
\]

Since equation (2-12) is even in \( \gamma \), no branch cuts exist and the integral can be evaluated by contour integration by taking the sum of the residues. The poles are located
at
\[ \gamma_n = \frac{n\pi}{a}, \quad n = 0,1,2, \ldots \] (2-13)
or
\[ \zeta_n = \sqrt{k^2 - \left(\frac{n\pi}{a}\right)^2} \] (2-14)
as shown in figure 2-2.

The poles along the real axis are the propagating modes, while the poles along the imaginary axis are the cut-off modes.

Evaluating the residues,
\[ G(x,z;x',z') = \frac{-i}{2ka} \left[ e^{-ik(z+z')} + e^{-ik|z-z'|} \right] \] (2-15)
\[ \sum_{\gamma=1}^{\infty} \frac{\cos\frac{\pi x}{a}}{\Gamma_n} \frac{\cos\frac{\pi x'}{a}}{\Gamma_n} \left[ e^{-i\Gamma_n (z+z')} + e^{-i\Gamma_n |z-z'|} \right] \]
where
\[ \Gamma_n = \sqrt{k^2 - \left(\frac{n\pi}{a}\right)^2} . \]

2.3 Evaluation of the Integral Equation by Moment Methods

The first step in evaluating the integral equation is to compute the integral for \( H_{inc} \). The points of observation are on the cylinder, and hence, \( z < z' \) and

\[ H_{inc} = -i\omega \int_0^a \int_0^a V_0 \delta(z'-z_0)G(x,z;x',z') \, dz' \, dx' . \] (2-16)
Figure 2-2. Pole locations in the ζ-plane.
The z integration is trivial and for the x integration, the cosine terms of G [see equation (2-15)] integrate out to zero so that only the TEM mode is excited, as desired. Thus,

\[ H_{\text{inc}} = \frac{-\omega e}{k} V_0 e^{-ikz_0} \cos kz . \quad (2-17) \]

Equation (2-9) can be solved by moment methods [4] as follows. The unknown, H, is expanded in terms of a set of functions, \( f_i \), with constant coefficients, \( b_i \),

\[ H = \sum_{i=1}^{N} b_i f_i . \quad (2-18) \]

Substituting equation (2-18) into equation (2-9),

\[ \sum_{i=1}^{N} b_i [f_i(x,z) + \int_{S'} f_i(x',z') \frac{\partial G(x,z;x',z')}{\partial n'} \,ds'] = H_{\text{inc}}(x,z). \quad (2-19) \]

Next a set of testing functions, \( w_j \), is defined and the inner products (integral over the surface) of both sides of equation (2-19) with each \( w_j \) are set equal. This forms a set of linear equations of the form
\[ \sum_{i=1}^{N} b_i \int_{S} \left[ f_i(r) + \int_{S'} f_i(r') \frac{\partial G(r,r')}{\partial n'} \right] w_j(r) \, ds = \int_{S} H_{\text{inc}}(r) w_j(r) \, ds \quad j = 1, 2, \ldots, N . \]  

(2-20)

Equation (2-20) can be put in matrix form as 

\[
\begin{bmatrix}
  m_{11} & m_{12} & \cdots & m_{1N} \\
  m_{21} & m_{22} & \cdots & m_{2N} \\
  \cdots & \cdots & \cdots & \cdots \\
  m_{N1} & m_{N2} & \cdots & m_{NN}
\end{bmatrix}
\begin{bmatrix}
  b_1 \\
  b_2 \\
  \vdots \\
  b_N
\end{bmatrix}
= 
\begin{bmatrix}
  c_1 \\
  c_2 \\
  \vdots \\
  c_N
\end{bmatrix}
\]  

(2-21)

where

\[ m_{ji} = \int_{S} \left[ f_i(r) + \int_{S'} f_i(r') \frac{\partial G(r,r')}{\partial n'} \right] w_j(r) \, ds \]

and

\[ c_j = \int_{S} H_{\text{inc}}(r) w_j(r) \, ds . \]

The unknown coefficients, \( b_i \), are found by solving the above set of simultaneous equations. The approximate expression for \( H \) is then given by equation (2-18).

For ease of computation, the expansion functions, \( f_i \), and testing functions, \( w_j \), are selected as pulse functions. That is, the functions are constant over a
subsection of the rectangular cylinder surface and are zero outside of the subsection.

Thus,

\[ m_{ji} = \int_{S_j} \left[ \delta_{ij} + \int_{S_i} \frac{\partial G(r, r')}{\partial n'} \, dr' \right] \, dr \quad (2-22) \]

and

\[ c_j = -\frac{V_0}{n} e^{-ikz_0} \int_{S_j} \cos kz \, dr \]

where \( \delta_{ij} = \begin{cases} 1, & i=j \\ 0, & \text{otherwise} \end{cases}, \) and \( n = \frac{1}{\varepsilon}. \)

The subsections are illustrated in figure 2-3 with the numbering starting at the bottom of the cylinder and going up to subsection \( P, \) the highest subsection on the vertical wall, and then proceeding along the top surface to \( x = 0. \)

Evaluating the \( c_j \) elements,

\[ c_j = \begin{cases} -\frac{V_0 e^{-ikz_0}}{\eta k} [\sin kz_{j+1} - \sin kz_j] & j = 1, 2, \ldots, P \\ \frac{V_0}{\eta} e^{-ikz_0} \cos kh (x_{j+1} - x_j) & j = P+1, P+2, \ldots, N \end{cases} \quad (2-23) \]
Figure 2-3. Subsection index.
When both \( i \) and \( j \) are less than or equal to \( P \),

\[
m_{ji} = \int_{z_j}^{z_{j+1}} \left[ \delta_{ij} - \int_{z_i}^{z_{i+1}} \frac{\partial G(x,x')}{\partial x'} \, dz' \right] \, dz . \tag{2-24}
\]

When \( i \leq P \) and \( j > P \),

\[
m_{ji} = -\int_{x_j}^{x_{j+1}} \int_{z_i}^{z_{i+1}} \frac{\partial G(x,x')}{\partial x'} \, dz' \, dx . \tag{2-25}
\]

When \( i > P \) and \( j \leq P \),

\[
m_{ji} = -\int_{z_j}^{z_{j+1}} \int_{x_i}^{x_{i+1}} \frac{\partial G(x,x')}{\partial z'} \, dx' \, dz . \tag{2-26}
\]

When \( i > P \) and \( j > P \),

\[
m_{ji} = \int_{x_j}^{x_{j+1}} \left[ \delta_{ij} - \int_{x_i}^{x_{i+1}} \frac{\partial G(x,x')}{\partial z'} \, dx' \right] \, dx . \tag{2-27}
\]

When \( i \) and \( j \) are equal \( \frac{\partial G}{\partial n} \) has a singularity which can be removed analytically as shown in Appendix A.

The expressions for \( m_{ji} \) which are not derived in Appendix A are obtained by straightforward term by term differentiation and integration. For \( i \) and \( j \) less than or equal to \( P \) but not equal to each other, equation (2-24) becomes
\[ m_{ji} = \frac{i\pi}{2a^2} \sum_{n=1}^{\infty} \frac{n}{\Gamma_n^3} \sin^{2n\pi w} \frac{n\pi x_{j+1}}{a} - \sin^{2n\pi x_j} \frac{n\pi x_j}{a} [1 - e^{-i\Gamma_n \Delta z}] \]

(2-28)

\[ + e^{-i\Gamma_n (|z_j - z_i| - \Delta z)} \]

where \( \Delta z = z_{i+1} - z_i = z_{j+1} - z_j \).

For \( i < P \) and \( j > P \),

\[ m_{ji} = \frac{1}{a} \sum_{n=1}^{\infty} \frac{1}{\Gamma_n^2} \sin^{n\pi w} \frac{n\pi x_{j+1}}{a} - \sin^{n\pi x_j} \frac{n\pi x_j}{a} [1 - e^{-i\Gamma_n \Delta z}] \]

(2-29)

\[ \cdot [e^{-i\Gamma_n (h - z_{i+1})} + e^{-i\Gamma_n (h + z_i)}] \]

For \( i > P \) and \( j < P \),

\[ m_{ji} = \frac{\Delta x}{ka} e^{-ikh} (\sin k z_{j+1} - \sin k z_j) - \frac{1}{i\pi} \sum_{n=1}^{\infty} \frac{1}{n\Gamma_n} \cos^{n\pi w} \frac{n\pi x_{i+1}}{a} - \sin^{n\pi x_i} \frac{n\pi x_i}{a} [1 - e^{-i\Gamma_n \Delta z}] \]

(2-30)

\[ \cdot [e^{-i\Gamma_n (h - z_{j+1})} + e^{-i\Gamma_n (h + z_j)}] \]

Since all of the matrix elements are defined, the expansion coefficients, \( b_i \), can be found by solving the set of linear equations in equation (2-21). Equation (2-21) can be solved on a digital computer by any suitable algorithm such as a Gauss-Jordan reduction.
CHAPTER 3

TIME DOMAIN SOLUTION

The coordinate system of the parallel plate structure without the conducting ground is shown in figure 3-1. The source at \( z = z_0 \) is a magnetic current of the form

\[
\dot{m}(r, t) = \mathcal{Q} v(t) U(t) \delta(z - z_0) .
\]  

(3-1)

In the time domain, Maxwell's equations are

\[
\nabla \times \mathbf{E}(r, t) = -\mu \frac{\partial \mathbf{H}(r, t)}{\partial t} - \dot{m}(r, t)
\]

(3-2)

\[
\nabla \times \mathbf{H}(r, t) = \varepsilon \frac{\partial \mathbf{E}(r, t)}{\partial t} .
\]

The partial differential equation for the \( y \) component of the magnetic field intensity is obtained from Maxwell's equations and is

\[
\nabla^2 h(r, t) - \mu \epsilon \frac{\partial^2 h(r, t)}{\partial t^2} = \epsilon \frac{\partial m(r, t)}{\partial t}
\]

(3-3)

with initial conditions and boundary conditions;

\[
h(r, 0) = 0 \quad \text{for all } r
\]

\[
\frac{\partial h(r, 0)}{\partial t} = 0 \quad \text{for all } r
\]
Figure 3-1. Coordinate system for the time domain solution.
and
\[ \frac{\partial h(r,t)}{\partial n} = 0 \text{ on conducting surfaces.} \]

### 3.1 Integral Formulation of the Magnetic Field Intensity

An integral equation for the magnetic field intensity on the cylinder is obtained by introducing a Green's function, \( g(r,r',t,t') \), satisfying the partial differential equation

\[ \nabla^2 g(r,r',t,t') - \mu \varepsilon \frac{\partial^2 g(r,r',t,t')}{\partial t'^2} = -\delta(r-r')\delta(t-t') \quad (3-4) \]

\( g \equiv 0 \text{ for } t < t'. \)

Multiplying equation (3-3) by \( g \), subtracting equation (3-4) multiplied by \( h \), and integrating over space and time gives

\[ \int_0^T \int V \left[ g(\nabla^2 h - \mu \varepsilon \frac{\partial^2 h}{\partial t'^2}) - h(\nabla^2 g - \mu \varepsilon \frac{\partial^2 g}{\partial t'^2}) \right] dv' dt' = \]

\[ \int_0^T \int V \left[ \varepsilon \frac{\partial m}{\partial t'} dv' dt' + h(r,t), \ T > t'. \right. \quad (3-5) \]
Using the time domain Green's theorem [5],

\[
\int_V (g \frac{\partial h}{\partial t} - h \frac{\partial g}{\partial t}) \, dv' + \int_V (-g \frac{\partial h}{\partial t} + h \frac{\partial g}{\partial t}) \, dv' + \int_S \int_0^T (g \frac{\partial h}{\partial n} - h \frac{\partial g}{\partial n}) \, dr \, dt' = \varepsilon \int_V \int_0^T g \frac{\partial m}{\partial t} \, dv \, dt' + h(r,t). \tag{3-6}
\]

The first integral above on the left vanishes due to the initial conditions on \( h \), the second vanishes due to the initial conditions on \( g \), and the first term of the third integral vanishes due to the boundary conditions on \( h \). Thus,

\[
h(r,t) = -\varepsilon \int_0^T \int_V g(r,r',t,t') \frac{\partial m(r',t')}{\partial t'} \, dv' \, dt' - \int_0^T \int_S h(r',t') \frac{\partial g(r,r',t,t')}{\partial n'} \, dr' \, dt'. \tag{3-7}
\]

Note that equation (3-7) can also be obtained by taking the inverse Fourier transform of equation (2-9), the corresponding frequency domain equation.

Up to this point, no mention of the spatial boundary conditions on \( g \) has been made. The surface \( S \) presently refers to the entire surface bounding the interior volume \( V \). By choosing \( \frac{\partial g}{\partial n} = 0 \) on \( x = 0 \) and \( x = a \), the surface \( S \) refers to the surface of the cylinder only.
3.2 Time Domain Green's Function

One form of the time domain Green's function is obtained by taking the inverse Fourier transform of the frequency domain Green's function, equation (2-15), i.e. [6],

\[ g(r,r',t,t') = \frac{c}{2a} U(ct-ct'-|z-z'|) + \frac{c}{a} \sum_{n=1}^{\infty} \cos \frac{n\pi x}{a} \cos \frac{n\pi x'}{a} \]

\[ \cdot J_0 \left( \frac{n\pi}{a} \sqrt{(ct-ct')^2 - |z-z'|^2} \right) U(ct-ct'-|z-z'|). \]

The above form is not suitable for numerical computations since for large \( n \), each term only decreases by \( 1/\sqrt{n} \).

An alternate form of the Green's function is obtained by using images. The result is [7],

\[ g(r,r',t,t') = \frac{c}{2\pi} \sum_{i=-\infty}^{\infty} \frac{U(ct-ct'-R_i)}{[(ct-ct')^2 - R_i^2]^{1/2}} \]

where \( R_i^2 = (x-x_i)^2 + (z-z_i)^2 \)

\[ x_i' = \begin{cases} x' + ia & i = 0, \pm 2, \pm 4, \cdots \\ -x' + (i+1)a & i = \pm 1, \pm 3, \cdots \end{cases} \]

The above representation is suitable for short time periods since it is a finite sum (due to the unit step function) of closed form expressions.

Using the image representation of the Green's function, equation (3-7) becomes
\[ h(r,t) = h^{\text{inc}}(r,t) - \int \int h(r',t') \frac{3}{\partial n'} \left[ -\frac{c}{2\pi} \right] dr'dt' \]

(3-10)

\[ \sum_{i=-\infty}^{\infty} \frac{U(ct-ct'-R_i)}{[(ct-ct')^2 - R_i^2]^{1/2}} ] dr'dt' \]

where

\[ h^{\text{inc}}(r,t) = -\varepsilon \int \int g(r,r',t,t') \frac{\partial m(r',t')}{\partial t'} \text{dv'dt'.} \]

By letting \( r \) approach \( r_s \), a point on the surface, an integral equation for \( h(r_s,t) \) is obtained. The limiting process introduces a singularity into the integrand which can be removed as shown in Appendix B. From Appendix B,

\[ h(r_s,t) = 2h^{\text{inc}}(r_s,t) - \frac{c}{\pi} \int \sum_{i=-\infty}^{\infty} \frac{\hat{n} \cdot \hat{a}}{R_i} \int_{S-\sigma}^{t-R_i/c} \frac{\partial h(r',t')}{\partial t'} \frac{(t-t')}{[(ct-ct')^2 - R_i^2]^{1/2}} \text{dt'dr'} \]

(3-11)

where \( \sigma \) is an arbitrarily small region about \( r_s \).

After integration by parts, the single image case \((i=0)\) in the above equation is equal to Bennett and Weeks' [2] equation (4) for the TE polarization which is derived from the magnetic vector potential.

### 3.3 Method of Solution

Equation (3-11) is solved by the method of moments. The expansion functions for \( h \) are chosen to be a product of
space and time functions of the following type. The surface $S$ is subdivided into $J$ subsections with $r_j$ being the center of subsection $j$. A pulse function defined by

$$P_j(r) = \begin{cases} 1 & \text{if } |r-r_j| < \frac{\Delta r_j}{2} \\ 0 & \text{otherwise} \end{cases}$$

where $\Delta r_j$ is the length of subsection $j$, is used for the space expansion. A triangle function defined by

$$T_n(t) = \begin{cases} 1 - \frac{|t-t_n|}{\Delta t} & \text{if } |t-t_n| < \Delta t \\ 0 & \text{otherwise} \end{cases}$$

is used for the time expansion. That is,

$$h(r_s,t) = \sum_{j=1}^{J} \sum_{n=1}^{N} a_{jn} P_j(r) T_n(t).$$

Substituting the above expansion into equation (3-11) and performing the time differentiation in the integrand,

$$\sum_{j=1}^{J} \sum_{n=1}^{N} a_{jn} P_j(r) T_n(t) = 2h^{inc}(r_s,t) - \frac{c}{\Delta t} \sum_{i=-\infty}^{\infty} \frac{\hat{n} \cdot \hat{u}}{R_i}$$

$$\cdot \int_{R_i}^{R_i/c} \frac{(t-t') dt' dr'}{[(ct-ct')^2 - R_i^2]^{1/2}} \sum_{j=1}^{J} \sum_{n=1}^{N} a_{jn} P_j(r')$$

$$\cdot [U(t'-t_{n-1}) - 2U(t'-t_n) + U(t'-t_{n+1})] .$$
The testing functions are chosen to be

\[ w_{km} = \delta(r_k) \delta(t_m) \quad (3-16) \]

where \( r_k \) is the center of subsection \( k \) and \( t_m = m\Delta t \).

Multiplying equation (3-16) by a given testing function and integrating over space and time gives

\[
a_{km} = 2h^{inc}(r_k, t_m) - \frac{c}{\Delta t} \sum_{j=1}^{J} \sum_{n=1}^{N} a_{jn} \int_{S_j} \int_{i=-\infty}^{\infty} \frac{\hat{n} \cdot \hat{u}}{R_i} \int_{0}^{t_m-R_i/c} (t-t') \, dt' \, dr' \frac{(t_m-t') \, dt' \, dr'}{[(ct_{m}-ct')^2 - R_i^2]^{1/2}} [U(t'-t_{n-1})-2U(t'-t_{n})+U(t'-t_{n+1})] .
\]

The time integration is performed analytically to give

\[
a_{km} = 2h^{inc}(r_k, t_m) - \frac{1}{\pi c \Delta t} \sum_{j=1}^{J} \sum_{n=1}^{N} a_{jn} \int_{S_j} \int_{i=-\infty}^{\infty} \frac{\hat{n} \cdot \hat{u}}{R_i} \left[ \sqrt{(ct_{m}-ct_{n-1})^2 - R_i^2} U(ct_{m}-ct_{n-1}-R_i) ight. \\
- \left. 2\sqrt{(ct_{m}-ct_n)^2 - R_i^2} U(ct_{m}-ct_n-R_i) + \sqrt{(ct_{m}-ct_{n+1})^2 - R_i^2} U(ct_{m}-ct_{n+1}-R_i) \right] \, dr' ,
\]

or by re-indexing,

\[
a_{km} = 2h^{inc}(r_k, t_m) - \frac{1}{\pi c \Delta t} \sum_{j=1}^{J} \sum_{n=1}^{N} [a_{j,n-1} - 2a_{jn} + a_{j,n+1}] .
\]
\[
\int \sum_{i=-\infty}^{\infty} \frac{\hat{n} \cdot \hat{u}}{R_i} \sqrt{(ct_m - ct_n)^2 - R_i^2} U(ct_m - ct_n - R_i) \, dr' \quad (3-19)
\]

where \( a_{j,0} = a_{j,-1} = 0 \).

Note that the largest value of \( N \) that gives a non-zero term is

\[
N \leq \frac{ct_m - R_i}{c \Delta t} = m - \frac{R_i}{c \Delta t} . \quad (3-20)
\]

Since \( R_i = 0 \) is excluded (\( R_i = 0 \) occurs in the integration over \( \sigma \)), the maximum value of \( N \) is \( m-1 \). This maximum value of \( N \) occurs only if \( \frac{R_i}{c \Delta t} \leq 1 \).

Therefore, if the distance between subsections is greater than \( c \Delta t \), the maximum value of \( N \) occurs only for the \( k \)th subsection; i.e., equation (3-19) can be written as

\[
a_{km} = 2h \text{inc}(r_k, t_m) - \frac{1}{\pi c \Delta t} \sum_{j=1}^{m} \sum_{n=1}^{m-2} [a_{j,n-1} - 2a_{jn} + a_{j,n+1}]
\]

\[
\int \sum_{i=-\infty}^{\infty} \frac{\hat{n} \cdot \hat{u}}{R_i} \sqrt{(ct_m - ct_n)^2 - R_i^2} U(ct_m - ct_n - R_i) \, dr' \quad (3-21)
\]

\[
- \frac{1}{\pi c \Delta t} [a_{k,m-2} - 2a_{k,m-1} + a_{k,m}] \int \sum_{i=-\infty}^{\infty} \frac{\hat{n} \cdot \hat{u}}{R_i} \sqrt{(c \Delta t)^2 - R_i^2} U(c \Delta t - R_i) \, dr' .
\]
The above equation is easily solved for $a_{km}$ in terms of coefficients $a_{jn}$ with $n$ less than or equal to $m-1$. Hence, matrix inversion is not needed if one starts by first computing $a_{k,1}$ for all $k$. 
CHAPTER 4

DISCUSSION OF THE INTEGRAL EQUATION SOLUTIONS

The frequency and time domain integral equations are solved in Chapters 2 and 3, respectively. Both are useful in analyzing obstacle scattering in a parallel plate waveguide. In principle, only one solution is needed since one can be obtained from the other by means of the Fourier transform. Depending upon the nature of the analysis, one solution may be more advantageous than the other. This chapter discusses some of the properties of the two solutions and gives some criteria for selecting one solution over the other.

4.1 Frequency Domain

The frequency domain integral equation is solved by moment methods in Chapter 2. The solution uses pulse functions for both the expansion and testing functions. The above choices lead to a matrix equation which is easily solved on a digital computer. The matrix is not excessively large and the matrix elements are not difficult to compute. In particular, for the choice of the pulse functions the number of subsections required to give a reasonable
representation of the surface current is of the order of six to ten subsections per wavelength.

The matrix elements as given in equations (2-28) to (2-30) and (A-8) and (A-13) are all exact for the chosen testing and expansion pulse functions and are easily programmed for the computer. All expressions converge at a rate equal to or greater than $1/n^2$. For most of the elements, there is an exponential factor of the form $e^{-n\pi z/a}$ for large $n$ which gives rapid convergence for $a/\lambda$ ratios of the order of ten or less. The few terms which do not contain the exponential factor can be computed by simply taking more terms or by improving the convergence. The convergence is improved by adding and subtracting a known series which has the same asymptotic behavior such as [8]

$$\sum_{i=2}^{\infty} \frac{\sin nx}{n^2 - 1} = \frac{\sin x}{4} - \sin x \ln(2\sin \frac{x}{2}). \quad (4-1)$$

By so doing, the resulting series converges at a rate of $1/n^4$.

4.2 Time Domain

In the time domain solution, the time increments should be small enough so as to give a reasonable representation of the input signal. The space increments are then chosen so that the condition $\Delta r \geq c\Delta t$ is met. This allows for a progressional time solution without matrix inversion.
The upper bound on $\Delta r$ is chosen so as to give a good representation of the current on the surface. In general, $\Delta r = c\Delta t$.

Any signal, $v(t)$, can be used as the input signal for the time domain integral equation. However, an approximation to the delta function, such as a narrow Gaussian pulse,

$$v(t) = \exp\left(-\frac{(t-t_0)^2}{T^2}\right)$$

(4-2)

has certain advantages. The first obvious advantage is that once the approximate delta function response is obtained, the response for any $v(t)$ is obtained by simple convolution. The Gaussian pulse is also useful in interpreting the physical scattering properties of the scatterer as illustrated in the following example of a square cylinder in free space.

The cylinder configuration, shown in figure 4-1, is obtained by rotating figure 3-1 ninety degrees and imaging the cylinder about the plate at $x = 0$. The half width of the cylinder is three meters. The parameters of the input Gaussian signal are $T=3$ nanoseconds and $t_0=8$ nanoseconds. For $|t-t_0|$ greater than eight nanoseconds, the Gaussian pulse is truncated to zero.

The impulse response at the different points on the cylinder are displayed in figures 4-2 through 4-6. At position one, for the first ten nanoseconds, the cylinder looks infinitely flat and, hence, the current is twice the
Figure 4-1. Square cylinder in free space.
Figure 4-2. Impulse response of square cylinder, positions 1 through 4.
Figure 4-3. Impulse response of square cylinder, positions 5 through 8.
Figure 4-4. Impulse response of square cylinder, positions 9 through 12.
Figure 4-5. Impulse response of square cylinder, positions 13 through 16.
Figure 4-6. Impulse response of square cylinder, positions 17 through 20.
incident Gaussian pulse. After ten nanoseconds, the fields scattered from the two leading corners, A and C, begin to arrive. The signals scattered from each corner produce the negative pulse beginning at about 13 nanoseconds in figure 4-2a. In figures 4-2b,c,d and figure 4-3a, the two pulses scattered from the corners are seen separately due to the different travel times to the observation points. For points along the top surface of the cylinder, the main response is that of the passing incident field. The fields scattered by the corners on the shadow side of the cylinder give rise to the fields seen on the back side of the cylinder. At the center point of the back side, as shown in figure 4-6d, the two pulses coalesce into one pulse. A creeping wave traveling around the cylinder is heavily attenuated and is not observed.

A partial check on the accuracy of the time domain solution can be made by observing the size of the current density at position twenty. There should be no field before the time it takes a signal, traveling at the speed of light, to arrive at position twenty by way of along the top of the cylinder and then down the backside (30ns.). However, from equation (3-11) the incident field is nonzero after the time it takes a signal to travel the width of the cylinder (20ns.) and must be cancelled by the scattered
field. For this particular example, the numerical error is less than $10^{-3}$ below the input signal.

The velocity of propagation down the backside is approximately 0.8 times the speed of light which is the same as observed by Bennett [9] for similar objects.

The frequency domain response of the cylinder is obtained by numerically taking the Fourier transform of the time response and dividing by the transform of the Gaussian pulse, i.e.,

$$H(\omega) = \frac{e^{i\omega t_0} e^{(\omega T)^2}}{\sqrt{\pi} T} \int_{-\infty}^{\infty} h(t) e^{-i\omega t} \, dt. \quad (4-3)$$

The frequency domain results obtained from equation (4-3) are compared with the results of Mei and Van Bladel [10] in figure 4-7.

Figure 4-7 is a duplication of Mei and Van Bladel's figure 15 with the Fourier transformed time domain results superimposed. For $kw = 0.3$ and $kw = 1$ the agreement is excellent. At $kw = 5$ there is slight disagreement, however, this was also noted by Andreasen [11]. The Fourier transformed results for $kw = 5$ are compared with Andreasen's results in figure 4-8 and much better agreement is obtained. For $kw = 5$ the time domain solution uses approximately six subsections per wavelength. Since the number of subsections was fixed, at $kw = 10$ there are only about three
Figure 4-7. Square cylinder current density (Comparison with Mei and Van Bladel).

—— Mei and Van Bladel

kw=

• kw=0.3

• kw=1

• kw=5

Figure 4-8. Square cylinder current density (Comparison with Andreasen).

—— Andreasen

kw=

• kw=5

• kw=10
subsections per wavelength and the accuracy greatly deteriorates as shown in figure 4-8.

4.3 Frequency Domain Versus Time Domain

The use of the frequency domain or the time domain integral equations depends upon the nature of the desired output. For example, if one only wants to know the fields at a given frequency for a particular physical configuration, then the frequency domain solution is the best choice. Thus, for meaningful comparisons, it is assumed that one is interested in either a frequency spectrum or a time history or both.

First suppose that one is interested in a frequency domain spectrum. In order to obtain the spectrum from the frequency domain integral equation, one simply solves the integral equation at each desired point in the spectrum. If the time domain integral equation is used, one first finds the approximate impulse response by means of a suitable input signal such as the Gaussian pulse. The frequency spectrum is then found by repeated use of a numerical Fourier transform program. If the time domain impulse response is such that it decays in a short period of time, then it is advantageous to solve the time domain integral equation and use the Fourier transform program repeatedly since, in general, less computation time is required. If, on the other hand, the time domain impulse response does
not decay rapidly (such as due to multiple reflections) then a complete time domain impulse response, as found via the time domain integral equation, may be impractical.

Next, suppose that one is interested in a time domain response. If one is interested in only a short period of time, or if the impulse response decays rapidly, then the time domain integral equation is the most direct approach. But, if one is interested in a long period of time and the impulse response decays slowly, then the approximate frequency domain impulse response, coupled with the inverse Fourier transform, is more advantageous. Thus, the "break even" point between the time and frequency domain solutions depends upon the relative speeds of the two computer programs and the number of time and frequency points desired.

Computer program listings of the frequency and time domain integral equations are in Appendix C.
CHAPTER 5

ANALYSIS OF THE PARALLEL PLATE SIMULATOR

The purpose of this chapter is to determine the amount of distortion introduced by the parallel plates. Most of the results are for a rectangular cylinder which has the dimensions of a typical communications building. For the time domain analysis, the input signal is a model of an electromagnetic pulse (EMP) generated by a nuclear blast. Comparisons with experimental results are made for several mock-up buildings.

5.1 Frequency Domain Analysis

The simulator configuration is shown in figure 5-1 for the case of vertical incidence upon a building on a perfectly conducting ground. Because of symmetry, the fields are unchanged if a plate is inserted in the $x = 0$ plane. Thus, the fields for the configuration of figure 5-1 are identical with those of figure 2-1.

The results that follow are for a typical building, 3.91 meters high and 7.46 meters wide. Figure 5-2 is for a low frequency ($kw = 0.3$) and displays the current density at the bottom-corner of the building as a function of plate
Figure 5-1. Building in a vertical parallel plate simulator.
Figure 5-2. Bottom-corner, kw=0.3.
spacing. Since the frequency is very low, the building looks like a small perturbation in a shorted waveguide. Thus, the current density is nearly twice the incident field for all plate spacings. The same holds true for all points on the building as indicated in figure 5-3 for the current density at the top-center of the building. The asymptotic value of infinite plate spacing was determined via the time domain integral equation and the Fourier transform. All other points where found by use of the frequency domain integral equation.

Figures 5-4 and 5-5 display the current density for a higher frequency \( w = \lambda/8 \) at the bottom-corner and top-center of the building. At a plate spacing of \( a = 15 \) meters, the first plate resonance occurs. At much larger plate spacings \( (a = 100 \) to 110 meters), the current density begins to approach the asymptotic value. The same type of variation is also seen in figures 5-6 and 5-7 for \( w = \lambda/4 \) and in figures 5-8 and 5-9 for \( w = 3\lambda/8 \). In each case, the plate resonances decrease in magnitude as the plate spacing increases and after ten wavelengths, as in figures 5-8 and 5-9 for \( a = 100 \), the current density nearly equals the asymptotic value. At \( w = \lambda/2 \), however, the current density approximately equals the asymptotic value for almost all plate spacings as shown in figures 5-10 and 5-11. The same type of behavior also occurs at \( w = \lambda \).
Figure 5-3. Top-center, kw=0.3.
Figure 5-4. Bottom-corner, $w=\lambda/8$. 
Figure 5-5. Top-center, \( w = \lambda / 8 \).
Figure 5-6. Bottom-corner, $w=\lambda/4$. 

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure56}
\caption{Bottom-corner, $w=\lambda/4$.}
\end{figure}
Figure 5-7. Top-center, \( w = \lambda / 4 \).
Figure 5-8. Bottom-corner, w=3λ/8.
Figure 5-9. Top-center, \( w=3\lambda/8 \).
Figure 5-10. Bottom-corner, w=λ/2.
Figure 5-11. Top-center, \(w=\lambda/2\).
Figures 5-2 through 5-11 indicate that for a given frequency, one can find a plate spacing which gives the same current density as the free space condition. In general, such a plate spacing is midway between two adjacent plate resonances. An electromagnetic pulse, however, has a spectrum of frequencies, so that it is also of interest to view the frequency domain impulse response over a wide frequency range for various plate spacings.

Figure 5-12 shows the frequency domain impulse response from zero to sixty megahertz for plate spacings of \( A = 10.4m, 13.4m, 16.7m, 20m, 30m, \) and \( \infty \). The effects of the plate resonances stand out clearly and a trend can be seen, as the plate spacing increases, for the spectrum to approach the free space values of figure 5-12f. At the smaller plate spacings, the building resonance at 16 megahertz is enhanced.

Figure 5-13 is similar to figure 5-12 except that it is for the current density at the top-center of the building. The comments are the same as for figure 5-12 except that the building resonances are at 20 and 48 megahertz. The data for figures 5-12a,b,c,d,e and 5-13a,b,c,d,e were computed from the frequency domain integral equation and figures 5-12f and 5-13f were prepared by data from the time domain integral equation.
Figure 5-12. Current density at bottom-corner of a typical building.
Figure 5-13. Current density at top-center of a typical building.
5.2 Time Domain Analysis

The assumed electromagnetic pulse has a very fast rise time and a slow decay time and is modeled by an expression of the form
\[ v(t) = e^{-\alpha t} - e^{-\beta t}, \quad t > 0 \] (5-1)
with corresponding Fourier transform,
\[ V(\omega) = \frac{1}{\alpha + i\omega} - \frac{1}{\beta + i\omega}. \] (5-2)

In the following figures, \( \alpha \) and \( \beta \) are such that the rise time is twenty nanoseconds and the decay time (1/e) is 350 nanoseconds.

Figure 5-14 displays the time domain response for an input EMP of the form of equation (5-1) at the same plate spacings of figure 5-12. Twice the incident EMP is included for comparison. Figures 5-14a,b,c,d, and e where obtained by multiplying equation (5-2) times the frequency domain impulse response data of figure 5-12 and numerically taking the inverse Fourier transform. Figure 5-14f was prepared by data from the time domain integral equation.

In order to apply figure 3-1 to the case of a conducting ground, an additional source is added at \( z = -z_0 \). The system is then symmetrical about \( z = 0 \) and equivalent to figure 2-1. For each plate spacing, a frequency of approximately 16 megahertz, the building resonance, is added to the input signal, decreasing in magnitude as the plate spacing is increased.
Figure 5-14. Current density at bottom-corner of a typical building with input pulse.
Data for figures 5-14d and e where also computed from the time domain integral equation using a Gaussian pulse as input and convolving the results with equation (5-1). The difference between the data obtained via the time domain and frequency domain equations is of the order of $10^{-3}$. The approximate impulse response for the cases corresponding to figures 5-14d,e, and f are shown in figure 5-15. Note that for an infinite plate spacing the impulse response decays rapidly, while for the finite plate spacings the multiple reflections cause a slow decay.

The time domain response at the top-center of the building is shown in figure 5-16. Again, figures 5-16a,b, c,d, and e where obtained via the data of figure 5-13 and use of the inverse Fourier transform. Figure 5-16f was obtained via the time domain integral equation. In each of the figures, the pulse reflected from the ground can be seen to begin at 38.5 nanoseconds, the time it takes the pulse to travel twice the height of the building and half of the width.

In figures 5-14 and 5-16 a certain portion of each curve is independent of the plate spacing. At the bottom-corner of the building, the first effects of the parallel plates are due to the field scattered off of the corner C and reflected off of the plate at the point Q as shown in figure 5-17. The time delay, $t_1$, between the arrival of
Figure 5-15. Approximate impulse response at the bottom-corner of a typical building.
(a) $A = 10.4$ meters  
(b) $A = 13.4$ meters  
(c) $A = 16.7$ meters  
(d) $A = 20$ meters  
(e) $A = 30$ meters  
(f) $A = \infty$ meters

Figure 5-16. Current density at top-center of a typical building with input pulse.
Figure 5-17. Illustration of delay time paths.
the incident pulse at point B and the arrival of the field along the path CQB is

\[ t_1 = \frac{1}{c} \sqrt{h^2 + (A-W)^2} - h] . \quad (5-3) \]

The time delay, \( t_2 \), at position T is the time it takes the field, scattered off of the corner C, to reflect off of the plate at point P and arrive at position T, i.e.,

\[ t_2 = \frac{1}{c}[A - w] . \quad (5-4) \]

The delay times for the plate spacings presented in figures 5-14 and 5-16 are tabulated in table 5-1.

5.3 Comparison With Experimental Results

For the vertical array, a direct comparison between theory and experiment is made for a mock-up building. The mock-up building is 1.2 meters high and 6.6 meters wide with parallel plates 13.4 meters apart. The theoretical input is of the form of equation (5-1). The results are shown in figure 5-18. The theoretical curve was obtained via the frequency domain integral equation and the inverse Fourier transform and, hence, shows a very steep rise at \( t = 0 \). The experimental curve does not have this steep of a rise because of the high frequency limitations and time delays of the instrumentation. Except for the instrumentation errors, the two results compare favorably.
Table 5-1. Delay times.

<table>
<thead>
<tr>
<th>plate spacing A</th>
<th>delay time $t_1$</th>
<th>delay time $t_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.4 meters</td>
<td>3.4 nanoseconds</td>
<td>22.5 nanoseconds</td>
</tr>
<tr>
<td>13.4 &quot;</td>
<td>10.9 &quot;</td>
<td>32.5 &quot;</td>
</tr>
<tr>
<td>16.7 &quot;</td>
<td>20.7 &quot;</td>
<td>43.5 &quot;</td>
</tr>
<tr>
<td>20.0 &quot;</td>
<td>31.0 &quot;</td>
<td>54.5 &quot;</td>
</tr>
<tr>
<td>30.0 &quot;</td>
<td>63.2 &quot;</td>
<td>87.5 &quot;</td>
</tr>
</tbody>
</table>
Figure 5-18. Theoretical and experimental magnetic field intensities at the top-center of a mock-up building.

- Experimental
- Theoretical

\[ a = 6.7 \text{ m} \]
\[ w = 3.3 \text{ m} \]
\[ h = 1.2 \text{ m} \]
The horizontal array configuration (obtained by rotating figure 3-1 by ninety degrees) is shown in figure 5-19 for a mock-up building which is five feet high and ten feet wide with a plate spacing of 22 feet. Theoretical results for the cases of with and without the plates are given in figure 5-20. For the plate spacing of 22 feet, only a small ripple is added to the free space current.

Experimental results for the building configuration of figure 5-19 are given in figure 5-21 along with the corresponding theoretical results. The overall agreement is quite good considering that the theoretical model is two dimensional and that the experimental source excites other modes in addition to the desired incident TEM mode.

Additional horizontal array results are given for a building which is 14 feet high and ten feet wide as shown in figure 5-22. Theoretical results for the cases of with and without the parallel plates are given in figures 5-23 and 5-24. Figure 5-23a shows the incident magnetic current density. In figures 5-23b,c, and d the finite plate spacing causes a slight increase in the current density after the major peak occurs. In figure 5-24a, the top-center of the building, a small ripple is added to the free space current density by the parallel plate. On the shadow side (figures 5-24b,c, and d), the plate causes the current density to initially dip below the free space value.
Figure 5-19. Horizontal array over a five foot high mock-up building.
Figure 5-20. Theoretical results for a five foot high mock-up building, with and without parallel plates.
Figure 5-21. Theoretical and experimental results for a five foot high mock-up building.
Figure 5-22. Horizontal array over a 14 foot high mock-up building.
Figure 5-23. Theoretical results for a 14 foot high mock-up building with and without parallel plates, input and positions 1 through 3.
Figure 5-24. Theoretical results for a 14 foot high mock-up building with and without parallel plates, positions 4 through 7.
Experimental and theoretical results for the building shown in figure 5-22 are given in figures 5-25 and 5-26. Again, the comparison is quite good indicating that the two dimensional theoretical model is able to predict the major field effects caused by the parallel plate simulator.
Figure 5-25. Theoretical and experimental results for a 14 foot high mock-up building, positions 1 through 4.
Figure 5-26. Theoretical and experimental results for a 14 foot high mock-up building, positions 5 through 7.
CHAPTER 6

CONCLUSIONS

The frequency and time domain integral equations for the current density on the surface of a perfectly conducting cylinder in a parallel plate waveguide were solved by the method of moments. The resulting solutions are readily programmed for the computer. Depending upon the nature of a particular analysis problem, either the frequency or time domain solution may be more efficient in terms of data handling and computer time.

The time and frequency domain integral equations can be used to predict the degree of distortion introduced by a parallel plate simulator. The results shown in this dissertation are of the magnetic field intensity on the surface of the cylinder. However, other quantities, such as electric field intensity or cross section, can be derived from the surface current by use of Maxwell's equations and equations (2-9) and (3-10).

The derivations in this dissertation are for a two dimensional structure with TE to y polarization. The extension to TM to y polarization is straightforward and follows in analogous fashion. The physical size of the scatterer is limited by computer storage and running time, however,
for cylinders of more than ten wavelengths in circumference, geometrical optics should be satisfactory.

Future work would be to apply the techniques used in this dissertation to three dimensional scatterers and, perhaps, to parallel plates of finite extent. One could also apply these techniques to obstacles in other waveguide structures, such as rectangular waveguide.

The extension to three dimensional scatterers can be done in several ways. The simplest is to consider only thin wire objects. The thin wire can be solved by a scalar equation for the wire current in a manner similar to Taylor's [12] for a dipole in a parallel plate waveguide. For a general shape of scatterer, however, the surface current has two tangential vector components and a vector solution is required. The resulting equations for the surface current are three coupled scalar integral equations. Although more complicated than the two dimensional problem, the equations can be solved by the method of moments. For the time domain solution, there is some simplification in that the three dimensional Green's function is a delta function of space and time rather than the decaying wake response of two dimensions. Thus, the time and space integration is reduced to a space (or time) integration only.
APPENDIX A

FREQUENCY DOMAIN SINGULARITY

The singularity of equation (2-24) is removed as follows. From equation (2-12), as \( x \) approaches \( x' = w \) from the right,

\[
\frac{\partial G}{\partial x'} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin\gamma w \cos\gamma (x-a)}{\sin\gamma} \cos\zeta' \cos\zeta \, d\zeta . \tag{A-1}
\]

Using the identity

\[
2\sin A \cos B = \sin(A+B) + \sin(A-B) \tag{A-2}
\]

and letting \( x \) approach \( w \)

\[
\frac{\partial G}{\partial x'} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\sin\gamma(2w-a) + \sin\gamma a}{\sin\gamma} \cos\zeta \cos\zeta' \, d\zeta . \tag{A-3}
\]

But the second term is the Fourier cosine representation of the delta function so that

\[
\frac{\partial G}{\partial x'} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\sin\gamma(2w-a)}{\sin\gamma} \cos\zeta \cos\zeta' \, d\zeta + \frac{\delta(z-z')}{2} . \tag{A-4}
\]

Evaluating the above integral by residues

\[
\frac{\partial G}{\partial x'} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{n\pi}{a} \sin\left(\frac{2n\pi w}{a}\right) \left\{ e^{-i\Gamma_n(z+z')} + e^{-i\Gamma_n|z-z'|} \right\} + \frac{1}{2} \delta(z-z') . \tag{A-5}
\]
Thus, equation (2-24) becomes

\[
m_{ii} = \int_{z_i}^{z_{i+1}} \left[ \frac{1}{2} - \frac{i}{2} \right] \int_{z_i}^{z_{i+1}} \sum_{n=1}^{\infty} \frac{n}{a^2 \Gamma_n} \sin^{2n\pi w} a
\]

\[
\cdot [e^{-i\Gamma_n(z-z')} + e^{-i\Gamma_n|z-z'|} dz']dz,
\]

which by straightforward integration and use of the identity

\[
\sum_{n=1}^{\infty} \frac{n \sin n x}{n^2 - b^2} = \frac{\pi \sin b(\pi - x)}{2\sin \pi b}
\]

(A-7)

yields

\[
m_{ii} = \Delta z_i [1 + \frac{\sin k(a-2w)}{2\sin ka}] + \frac{\pi}{2ia^2} \sum_{n=1}^{\infty} \frac{n}{\Gamma_n^3} \sin^{2n\pi w} a
\]

\[
\cdot (1-e^{-i\Gamma_n \Delta z_i}) [2-(1-e^{-i\Gamma_n \Delta z_i}) e^{-i2\Gamma_n z_i}]
\]

(A-8)

where \( \Delta z_i = z_{i+1} - z_i \).

The same result can also be obtained by differentiating equation (2-15), performing the integrations, and then letting \( x \) approach \( w \).

The singularity of equation (2-27) is taken care of by taking the partial derivative of equation (2-15) term by term and letting \( z \) approach \( z' = h \), i.e.,

\[
\frac{\partial G}{\partial z'} = \frac{1}{2a} [e^{-i2kh} - 1] - \frac{1}{a} \sum_{n=1}^{\infty} \cos^{n\pi x/a} \cos^{n\pi x'/a} [e^{-i2\Gamma_n h} - 1].
\]

(A-9)
Rearranging terms and noting that the representation of $\delta(x-x')$ is given by

$$\delta(x-x') = \frac{1}{a} + \frac{2}{a} \sum_{n=1}^{\infty} \cos \frac{n\pi x}{a} \cos \frac{n\pi x'}{a},$$  \hspace{1cm} (A-10)$$
equation (A-9) becomes

$$\begin{aligned} \frac{\partial G}{\partial z'} &= \frac{1}{2} \delta(x-x') - \frac{e^{-i2kh}}{2a} - \frac{1}{a} \sum_{n=1}^{\infty} \cos \frac{n\pi x}{a} \cos \frac{n\pi x'}{a} e^{-i2\Gamma_n h}. \\
(A-11) \end{aligned}$$

Substituting the above equation into equation (2-27),

$$m_{ji} = \int_{x_{j+1}}^{x_j} \left[ \frac{i}{2} + \frac{e^{-i2kh}}{2a} + \frac{1}{a} \sum_{n=1}^{\infty} \cos \frac{n\pi x}{a} \cos \frac{n\pi x'}{a} e^{-i2\Gamma_n h} \right] dx. \hspace{1cm} (A-12)$$

which by straightforward integration gives

$$m_{ji} = \frac{1}{2} (x_j - x_{j+1}) \delta_{ij} + \frac{e^{-i2kh}}{2a} (x_i - x_{i+1})(x_j - x_{j+1})$$

$$+ \sum_{n=1}^{\infty} \frac{a}{(n\pi)^2} (\sin \frac{n\pi x_i}{a} - \sin \frac{n\pi x_{i+1}}{a})(\sin \frac{n\pi x_j}{a} - \sin \frac{n\pi x_{j+1}}{a}) e^{-i2\Gamma_n h}. \hspace{1cm} (A-13)$$
APPENDIX B

TIME DOMAIN SINGULARITY

Applying the chain rule of partial differentiation to the integrand of equation (3-10) and taking the limit as $r \to r_s$,

$$h(r_s, t) = h^{inc}(r_s, t) - \frac{c}{2\pi} \lim_{r \to r_s} \int_{s}^{T} \int_{0}^{s} h(r', t') \sum_{i=-\infty}^{\infty} \frac{\partial R_i}{\partial R_i} \frac{U(ct-ct'-R_i)}{[(ct-ct')^2 - R_i^2]^{1/2}} dr' dt'$$  \hspace{1cm} (B-1)

After performing the differentiation with respect to $R_i$,

$$h(r_s, t) = h^{inc}(r_s, t) - \frac{c}{2\pi} \lim_{r \to r_s} \int_{s}^{T} \sum_{i=-\infty}^{\infty} \frac{\partial R_i}{\partial R_i} \int_{0}^{\infty} h(r', t')$$

\hspace{1cm} \cdot \left[ \frac{-\delta(ct-ct'-R_i)}{[(ct-ct')^2 - R_i^2]^{1/2}} + \frac{R_i U(ct-ct'-R_i)}{[(ct-ct')^2 - R_i^2]^{3/2}} \right] dt' dr'$$  \hspace{1cm} (B-2)

The second term in the above equation can be integrated by parts to give
\[ h = h^{\text{inc}} - \frac{c}{2\pi} \lim_{r \to r_s} \int_S \sum_{i=-\infty}^{\infty} \frac{\delta R_i}{\delta n^i} \left\{ - \int_0^T h(r', t') \delta(ct-ct'-R_i) \, dt' \right\} \frac{(t-t') h(r', t') U(ct-ct'-R_i)}{R_i [(ct-ct')^2 - R_i^2]^{1/2}} \bigg|_0^T \]

\[ + \int_0^T \frac{(t-t')}{R_i [(ct-ct')^2 - R_i^2]^{1/2}} \left\{ \frac{\partial h(r', t')}{\partial t'} U(ct-ct'-R_i) - c h(r', t') \delta(ct-ct'-R_i) \right\} \, dt' \bigg|_0^T \]  

The delta function terms cancel and the limits at \( t' = 0 \) and \( T \) vanish due to the initial condition on \( h \) and the initial condition (the step function) on \( g \). Hence, the integral equation reduces to

\[ h = h^{\text{inc}} - \frac{c}{2\pi} \lim_{r \to r_s} \int_S \sum_{i=-\infty}^{\infty} \frac{\hat{n} \cdot \hat{u}}{R_i} \int_0^T \frac{\partial h(r', t')}{\partial t'} \, dt' \bigg|_0^T \frac{(t-t') \, dt' \, dr'}{[(ct-ct')^2 - R_i^2]^{1/2}} \]

where \( \hat{n} \) is the unit normal vector directed towards the surface and

\[ \hat{u} = (-1)^i \frac{(x-x_i') \hat{x} + (z-z') \hat{z}}{R_i} . \]

The above form has a singularity as \( r \) approaches the surface. The singularity can be removed analytically.
by separating the surface integration into two segments,

\( h = h^\text{inc} - \frac{c}{2\pi} \lim_{r \to r_s} \int_{\sigma} - \frac{c}{2\pi} \lim_{r \to r_s} \int_{\sigma} \)  

(B-5)

where \( \sigma \) is an arbitrarily small section of the surface covering the point \( r' = r_s \).

For the integral over \( \sigma \), the integrand remains finite except for the \( i = 0 \) term. Thus, for an arbitrarily small \( \sigma \), the \( \sigma \) integral reduces to

\[- \frac{c}{2\pi} \lim_{r \to r_s} \int_{\sigma} \left( \frac{\partial h(r', t')}{\partial t'} \right) \frac{(t-t') \ dt' \ dr'}{\left[ (ct-ct')^2 - R_0^2 \right]^{1/2}}.\]  

(B-6)

The \( \sigma \) integration is illustrated in figure B-1 with \( x \) being the axis of the path of integration and \( z \) being the axis normal to the surface.

Rewriting the \( \sigma \) integration in terms of the coordinates of figure B-1 gives

\[- \lim_{z \to 0} \frac{c}{2\pi} \int_{\sigma} \left( -\hat{z} \cdot \hat{n} \right) \frac{\partial h(x', t')}{\partial t'} \frac{(t-t') \ dt' \ dx'}{\left[ (ct-ct')^2 - R_0^2 \right]^{1/2}}.\]  

(B-7)

Considering the integral as a distribution, the limit may be taken inside the \( \sigma \) integral to yield
Figure B-1. Integration over the singularity.
\[ \frac{c}{2} \int \lim_{z \to 0} \frac{z}{\pi (x'^2 + z^2)} \int_0^{t-x'/c} \frac{\partial h(x',t')}{\partial t'} \frac{(t-t') \, dt' \, dx'}{[(ct-ct')^2 - x'^2]^{1/2}}. \]  

(B-8)

But the limit of the term in the brackets is a representation of \( \delta(x') \).

Using the multiplication property of delta functions, \( \delta(x)f(x) = \delta(x)f(0) \), the \( \sigma \) integral reduces to

\[ \frac{c}{2} \int_0^t \delta(x') \, dx' \int_0^t \frac{\partial h(x',t')}{\partial t'} \, dt', \]  

or simply \( \frac{h(x,t)}{2} \).

Thus, the integral equation reduces to

\[ h(r_s,t) = 2h(r_s,t) - \frac{c}{\pi} \int_{S-\sigma} \sum_{i=1}^{\infty} \frac{n \cdot \hat{u}}{R_i} \int_0^{t-R_i/c} \frac{\partial h(r',t')}{\partial t'} \]  

\[ \cdot \frac{(t-t') \, dt' \, dr'}{[(ct-ct')^2 - R_i^2]^{1/2}}. \]  

(B-10)
APPENDIX C

COMPUTER PROGRAM LISTINGS

The frequency and time domain computer program listings are presented in the following pages. Both programs are written in Fortran IV for use on a CDC 6400 computer.

Frequency Domain Program

PROGRAM CONDR(INPUT,OUTPUT,TAPE5=INPUT)
C DIMENSION Y(NPTS+1),Z(NBRK),C(NPTS),SY(NX,NMAX),
C E(2*NZ+1,NMAX),AM(NPTS,NPTS)
C MAXIMUM NX,18,NZ,18,NMAX,100
REAL K,K2
COMPLEX ONEM(100),E(37,100),AM(36,36),C(36),B(36)
COMPLEX CDUMY,SUM,TERM,PART,ALFA,BETA,TEMP
DIMENSION CW(100),G2(100),G(100),STWO(100),SW(100),
1 SY(18,100),Y(37),Z(19),AA(99),FREQ(99)
DATA PI/3.14159265358979/
1 READ 3002,W,H
3002 FORMAT(2F10.6)
IF(EOF,5) 1001,2
2 READ 3010,NX,NZ,NAA,NFREQ,NMAX
3010 FORMAT(5I5)
PRINT 3001
3001 FORMAT(1H1)
PRINT 3003,W,H,NX,NZ,NMAX
3003 FORMAT(9X2HW=,F10.6,2X2HH=,F10.6,2X3HNX=,I3,2X3HNZ=,
1I3,3X5HNMAX=,I5)
DO 3 I = 1,NAA
3 READ 3020,AA(I)
3020 FORMAT(F10.4)
DO 4 I = 1,NFREQ
4 READ 3030,FREQ(I)
3030 FORMAT(F10.3)
NPTS=NX+NZ $ NBRK=NZ+1 $ NZNZ=NBRK+NZ
Z(1) = 0.0 $ Z(NBRK) = H $ DELZ=H/FLOAT(NZ)
DO 6 I = 2,NZ
6 Z(I) = DELZ*FLOAT(I-1)
DELX=W/FLOAT(NX) $ Y(NPTS+1)=0.
DO 5 I = 1,NX
5 Y(NPTS+1-I) = DELX*FLOAT(I)
DO 1000 III=1,NAA
A=AA(III) $ FACT = W/A $ SD=A-W
DO 1000 JJJ=1,NFREQ
F = FREQ(JJJ)
PRINT 4006,F,A
4006 FORMAT(/20X,7HFREQ = ,E12.4,5X4HA = ,F9.4//)
XL=3.0E08/F $ ANN = A/XL $ WN=W/XL
K=2.09439510239319E-08*F $ SDN=SD/XL
PRINT 4019
4019 FORMAT(14X1HA,12X1HW,9X4HX(1),7X5HLAMDA)
PRINT 4020,A,W,SD,XL
PRINT 4020,ANN,WN,SDN,FACT
4020 FORMAT(10X,4E12.4)
DO 10 J=2,NBRK
10 C(J-1)= -(SIN(K*Z(J)) - SIN(K*Z(J-1)))/K
CC = -COS(K*H)*DELX
DO 12 J= NBRK,NPTS
12 C(J) = CC
PRINT 4030,C
4030 FORMAT(/2X,10E12.4)
C LOAD MATRIX AM
POA = PI/A $ K2 = K*K $ A2 = A*A
WPOA=W*POA $ MODE=NMAX+1
DO 13 N = 1,NMAX
AN=N $ G2(N)=K2-(AN*POA)**2 $ CW(N)=COS(AN*WPOA)
SW(N)=SIN(AN*WPOA) $ STWO(N)=2.*CW(N)*SW(N)
E(1,N)=(1.,0.,0.)
IF(N.GT.MODE) GO TO 54
IF(G2(N)) 51,8,52
8 PRINT 4001,F,A,N
4001 FORMAT(10X,*RESONANCE AT F= *,E12.4,4HA = ,F10.2,
1 4HN = ,I3)
GO TO 1000
51 MODE = N
54 G(N) = SQRT( -G2(N) )
DO 7 IX = 2,NZNZ
AIX = DELZ*FLOAT(IX-1)
7 E(IX,N) = EXP( -G(N)*AIX )
GO TO 19
52 G(N) = SQRT( G2(N) )
DO 14 IX = 2,NZNZ
AIX=DELZ*FLOAT(IX-1) $ DD=G(N)*AIX
14 E(IX,N) = CMPLX( COS(DD) , -SIN(DD) )
19 DO 15 I = 1,NX
15 SY(I,N)=SIN(AN*Y(NBRK+I)*POA)-SIN(AN*Y(NZ+I)*POA)
ONEM(N) = 1.0 - E(2,N)
13 CONTINUE
MODEM = MODE-1  $  DDD = 0.5*PI/A2
PPP=0.5*DELZ*(1.+SIN(K*(2.*SD-A))/SIN(K*A))
DO 30 I = 1,NZ
   ID = I+I-1  $  SUM = (0.0,0.0)
IF(MODE.EQ.1) GO TO 21
DO 9 N=1,MODEM
   AN=N  $  PART=CMPLX(0.0,-AN*STWO(N)/(G2(N)*G(N)))
   TERM= PART*ONEM(N)*(2.-ONEM(N)*E(ID,N))
9   SUM = SUM + TERM
21 DO 20 N=MODE,NMAX
   AN=N  $  DD=AN*STWO(N)/(G2(N)*G(N))
   TERM=DD*ONEM(N)*(2.-ONEM(N)*E(ID,N))
20 SUM = SUM + TERM
AM(I,I) = PPP + DDD*SUM
30 CONTINUE
DO 50 J=2,NZ
   I = 1
17   IDP = I + J - 1
   IDM = J - I  $  SUM = (0.,0.)
IF(MODE.EQ.1) GO TO 36
DO 35 N=1,MODEM
   AN=N  $  PART=CMPLX(0.0,-AN*STWO(N)/(G2(N)*G(N)))
   TERM=PART*(E(IDP,N)+E(IDM,N))*ONEM(N)**2
35 SUM = SUM + TERM
36 DO 40 N=MODE,NMAX
   AN=N  $  DD=AN*STWO(N)/(G2(N)*G(N))
   TERM=DD*ONEM(N)*(2.-ONEM(N)*E(ID,N))
40 SUM = SUM + TERM
AM(J,I) = -DDD*SUM
AM(I,J) = AM(J,I)  $  I = I + 1
IF(I.LT.J) GO TO 17
50 CONTINUE
RD = DELX/(A*K)
CDUMY = RD*CMPLX(COS(H*K),-SIN(H*K))
DO 90 J = 1,NZ
   DD = SIN(K*Z(J+1)) - SIN(K*Z(J))
   JDM = NBRK - J  $  JDP = NZ + J
   DO 90 I=NBRK,NPTS
   ID = I - NZ  $  SUM = (0.,0.)
IF(MODE.EQ.1) GO TO 71
DO 70 N=1,MODEM
   AN=N  $  PART=CMPLX(0.0,-SY(ID,N)*CW(N)/(AN*G(N)))
   TERM=ONEM(N)*(E(JDM,N)+E(JDP,N))
70 SUM = SUM + TERM*PART
71 DO 80 N=MODE,NMAX
   AN=N  $  TERM=SY(ID,N)*CW(N)/(AN*G(N))
80 SUM=SUM+TERM*ONEM(N)*(E(JDM,N)+E(JDP,N))
AM(J,I)=DD*CDUMY-SUM/PI
90 CONTINUE
DO 130 J=NBRK,NPTS
JD = J - NZ
DO 130 I=1,NZ
IDM=NBRK-I $ IDP=NZ+I $ SUM=(0.,0.)
DO 120 N=1,NMAX
RD = SW(N)*SY(JD,N)/G2(N)
120 SUM=SUM+RD*ONEM(N)*(E(IDM,N)+E(IDP,N))
130 AM(J,I) = SUM/A
RD = DELX*DELX/(2.*A) $ DD = 2.*K*H
PART = RD*CMPLX(COS(DD),-SIN(DD))
DD = A/(PI*PI) ' $ NBRKP = NBRK+1
DO 200 I=NBRKP,NPTS
ID = I - NZ $ J = NBRK
175 JD = J - NZ
SUM = (0.,0.)
DO 190 N=1,NMAX
AN=N $ RD = SY(ID,N)*SY(JD,N)/AN**2
190 SUM = SUM + RD*E(NZNZ,N)
AM(J,I) = PART + DD*SUM
AM(I,J) = AM(J,I)
J = J + 1
IF(I.EQ.J) GO TO 200
GO TO 175
200 CONTINUE
DO 220 I=NBRK,NPTS
ID = I - NZ $ SUM = (0.,0.)
DO 210 N=1,NMAX
AN=N $ RD = SY(ID,N)**2/(AN*AN)
210 SUM = SUM + RD*E(NZNZ,N)
220 AM(I,I) = 0.5*DELX + PART + DD*SUM
C AM MATRIX LOADED
C SOLUTION OF SYSTEM OF COMPLEX LINEAR EQUATIONS
NM1 = NPTS - 1
DO 690 KK=1,NM1
KKP1 = KK + 1 $ L = KK
DO 600 I=KKP1,NPTS
D=REAL(AM(I,KK)) $ P=AIMAG(AM(I,KK))
S=REAL(AM(L,KK)) $ T=AIMAG(AM(L,KK))
600 IF( (D*D+P*P).GT.(S*S+T*T) ) L = I
IF(L.EQ.KK) GO TO 620
DO 610 J = KK,NPTS
TEMP = AM(KK,J) $ AM(KK,J) = AM(L,J)
610 AM(L,J) = TEMP
TEMP=C(KK) $ C(KK)=C(L) $ C(L)=TEMP
620 DO 690 I = KKP1,NPTS
BETA = AM(I,KK)/AM(KK,KK)
DO 650 J = KKP1,NPTS
650 AM(I,J) = AM(I,J) - BETA*AM(KK,J)
690 C(I) = C(I) - BETA*C(KK)
B(NPTS) = C(NPTS)/AM(NPTS,NPTS) $ I = NM1
710 IP1 = I + 1
SUM = (0.,0.)
DO 700 J = IP1,NPTS
700 SUM = SUM + AM(I,J)*B(J)
B(I) = (C(I)-SUM)/AM(I,I) $ I = I - 1
IF(I.GE.1) GO TO 710
C END OF SOLUTION
PRINT 4050
4050 FORMAT(/10X1HN,4X6HREAL B,6X6HIMAG B,8X5HB-MAG,
1 7X5HB-ARG/)
DO 410 I = 1,NPTS
SUM=B(I) $ SUMM=CABS(SUM)
SUMA=ATAN2(AIMAG(SUM),REAL(SUM))
410 PRINT 4005,I,SUM,SUMM,SUMA
4005 FORMAT (10X,12,3E12.4,F12.4,F12.2)
1000 CONTINUE
GO TO 1
1001 CALL EXIT
END

**Time Domain Program**

PROGRAM TR(INPUT,OUTPUT,PUNCH,TAPE10)
C DIMENSION P AND PP NPTS*(NTIMES+1),G NPTS*NTIMES
DIMENSION P(2268),PP(2268),G(2250),XC(40),ZC(40),
1 RR(40),COEF(40),BBB(40),Q(8)
DATA BBB/1.,-1.,-1.,1.,1.,-1.,-1.,1.,1.,
1 1.,-1.,-1.,1.,1.,-1.,-1.,1.,1.,
2 -1.,1.,1.,-1.,1.,1.,-1.,1.,-1./
DATA/3.14159265358979/
MIN=0 $ C = 3.E+08
REWIND 10
READ 5000,Q
5000 FORMAT(8A10)
PRINT 5001,Q
5001 FORMAT(1H1/10X8A10)
PUNCH 5000,Q
READ 5002,W,H,A,NIMAGE
5002 FORMAT(3F10.4,I5)
READ 5003,DT,CAPT,TZ,NX,NZ,NTIMES
5003 FORMAT(F10.4,2E10.3,315)
PRINT 5004,DT,CAPT,TZ,NX,NZ,W,H,A
5004 FORMAT(1X3HDTE=,F7.3,5HCAPT=,E10.3,3HTZ=,E10.3,3HNX=,
1 I2,3HNZ=,I2,2HW=,F8.4,2HH=,F8.4,2HA=,F8.4)
PUNCH 5004,DT,CAPT,TZ,NX,NZ,W,H,A,
DT=DT*(1.E-09) $ DELTA=C*DT $ PID=PI*DELTA
DX=W/FLOAT(NX) $ DZ=(H+H)/FLOAT(NZ) $ NXPNZ=NX+NZ
NPTS=NXPNZ+NX $ NP1=NPTS+1 $ XC(NPTS)=XC(1)=0.5*DX
DO 1 I=2,NX
1 XC(NP1-I) = XC(I) = XC(I-1) + DX
NXP1 = NX + 1 $ NXP2 = NXP1 + 1
DO 2 I = NXP1,NXPNZ
2 XC(I) = W
DO 15 I = 1,NX
15 ZC(NP1-I) = H
ZC(I) = -H
ZC(NXP1) = -H + 0.5*DZ
DO 16 I = NXP2,NXPNZ
16 ZC(I) = ZC(I-1) + DZ
DO 17 I = 1,NPTS
17 PP(I) = XC(I)
AIM=0. $ L1=L+1+NPTS $ L2=NPTS+NPTS $ ASSIGN 130 TO IC
DO 150 IMAGE = 2,NIMAGE
AIM=AIM+A $ GO TO IC,(120,130)
120 DO 125 I = L1,L2
125 PP(I) = PP(I-NPTS) + AIM
ASSIGN 130 TO IC $ GO TO 140
130 DO 135 I = L1,L2
135 PP(I) = -PP(I-NPTS)
ASSIGN 120 TO IC
140 L1=L1+NPTS
150 L2=L2+NPTS
NPTSH = NPTS/2
DO 500 I = 1,NPTSH
IF(I-NX) 124,124,126
124 J1=NXP1 $ J2=NPTS $ IK=1
GO TO 131
126 J1=1 $ J2=NPTS $ IK=2
GO TO 131
131 DO 400 J = J1,J2
IF(J-NXPNZ) 222,222,221
221 ASSIGN 291 TO JM $ K1=1 $ NIM=J-NPTS $ ZZ=ZC(I)-ZC(J)
GO TO 226
222 IF(J-NX) 224,224,223
223 ASSIGN 292 TO JM $ K1=IK $ NIM=J-NPTS
IF(IK.EQ.2) NIM=J
GO TO 226
224 ASSIGN 291 TO JM $ NIM=J-NPTS $ K1=1 $ ZZ=ZC(J)-ZC(I)
226 ZZZZ=(ZC(I)-ZC(J))**2
DO 230 K=K1,NIMAGE
NIM=NIM+NPTS $ RR(K) = (XC(I)-PP(NIM))**2 + ZZZZ
GO TO JM,(291,292)
291 COEF(K) = ZZ*DX/RR(K)
GO TO 230
292 XX = (XC(I) - PP(NIM))*BBB(K)
COEF(K) = XX*DZ/RR(K)
230 CONTINUE
DN = 0. $ NNPTS = -NPTS
DO 400 N = 1, NTIMES
DN = DN + DELTA $ DN2 = DN**2 $ NNPTS = NNPTS + NPTS $ SUM = 0.
DO 250 K = K1, NIMAGE
ARG = DN2 - RR(K)
IF (ARG .LE. 0.) GO TO 250
SUM = SUM + COEF(K) * SQRT(ARG)
250 CONTINUE
400 G (J + NNPTS) = SUM
500 WRITE (10) G
C-----G LOADED -----
ENDFILE 10
REWIND 10
IMAX = NPTS * (NTIMES + 1)
DO 100 I = 1, IMAX
100 P(I) = PP(I) = 0.
DO 110 I = 1, NPTS
P(NPTS + I) = 2. * PINC (ZC(I), DT, H, TZ, CAPT)
110 PP(I) = P(NPTS + I)
PRINT 5010, DT
PRINT 5011, (P(L + NPTS), L = 1, NPTS)
PUNCH 5013, (P(L + NPTS), L = 1, NPTS)
T = DT $ NT = NPTS
DO 1000 IT = 2, NTIMES
NT = NT + NPTS $ T = T + DT
DO 900 I = 1, NPTSH
READ (10) G
IP = NP1 - I $ SUMI = SUMIP = 0.
IF (I - NX) 501, 501, 503
501 J1 = NXPL $ J2 = NPTS $ GO TO 505
503 J1 = 1 $ J2 = NPTS
505 DO 800 J = J1, J2
JP = NP1 - J $ MAX = IT - 1 $ LL = J - NPTS
LLP = JP - NPTS $ INDEX = J + NT
DO 800 ITAU = MIN, MAX
LL = LL + NPTS $ LLP = LLP + NPTS $ INDEX = INDEX - NPTS
SUMI = SUMI + PP(LL) * G(INDEX)
SUMIP = SUMIP + PP(LLP) * G(INDEX)
800 CONTINUE
NTI = NT + I $ NTIP = NT + IP
P(NTIP) = 2. * PINC (ZC(IP), T, H, TZ, CAPT) + SUMIP / PID
PP(NTI) = 2. * PINC (ZC(I), T, H, TZ, CAPT) + SUM / PID
1 + P(NTIP)
1 + P(NTI)
900 CONTINUE
REWIND 10
PRINT 5010, T
5010 FORMAT (/40X2HT=, 1PE11.3)
PRINT 5011, (P(L+NT),L=1,NPTS)
5011 FORMAT(10(1PE12.3))
PUNCH 5013, (P(L+NT),L=1,NPTS)
5013 FORMAT(8F10.6)
1000 CONTINUE
END

FUNCTION PINC(Z,T,H,TZ,CAPT)
 TAU = T-TZ+(Z-H)/3.0E+08
 IF(ABS(TAU).GE.TZ) GO TO 1
 PINC = EXP(-(TAU/CAPT)**2)
 RETURN
1 PINC = 0.
 RETURN
END
REFERENCES


