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I hereby recommend that this dissertation prepared under my direction by DON BRADLEY PANTON

entitled A WRITER'S NONLINEAR VALUATION MODEL FOR THE CALL OPTION

be accepted as fulfilling the dissertation requirement of the degree of DOCTOR OF PHILOSOPHY

James E. West — Dissertation Director — Date 4/17/72

After inspection of the final copy of the dissertation, the following members of the Final Examination Committee concur in its approval and recommend its acceptance:*

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*This approval and acceptance is contingent on the candidate's adequate performance and defense of this dissertation at the final oral examination. The inclusion of this sheet bound into the library copy of the dissertation is evidence of satisfactory performance at the final examination.

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SIGNED: Don B. Panton

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ABSTRACT

The call option is a relatively little understood financial device which derives its value to the buyer from the possibility of providing an opportunity to purchase another asset below market price at some time in the future. Value to the call seller, or writer, is in the form of immediate compensation for the risk he bears and return on the capital he has committed. Determining a market-clearing price *a priori* for that value in terms of today's dollars, however, is a difficult task. Indeed, since the call is bought and sold by men with differing attitudes toward desirable and undesirable features in ventures of chance, the problem is not likely to be solved in other than a statistical sense.

The objective of this study is the derivation and testing of an intrinsically nonlinear pricing model for the call option under certain reasonable assumptions. That objective is developed in the following manner:

First, option market institutional constraints and the call writer's reasons for requiring compensation are discussed.

Second, the distribution of possible cash outflows as viewed by the call writer is related to option length, stock price

volatility, and stock market price at the time of the call's original issuance.

Third, a call writer's model relating the summation of a risk position and foregone interest on covering funds to immediate compensation is developed. Assumptions essential to the development of the model are discussed. Unknown parameters are left to be estimated with a curve-fitting technique.

Fourth, minimization of the sum of squared residuals is defined as the optimization criterion for the valuation equation, with $\underline{\theta}^*$ being the attending set of estimated parameters.

Fifth, a comparison of sums of squared residuals for the seller's model and a competitive model is defined as the relative measure of equation merit.

Sixth, iterative optimum seeking techniques are applied to the two models. Intervals of uncertainty for parameter spaces are reduced via golden sectioning, with successive estimates of $\underline{\theta}$ converging on $\underline{\theta}^*$.

Results

The error sum of squares for the competitive model was found to be 3.76 times that of the seller's model developed in this study.

Also, the magnitude of the mean error was larger for the competitive model for forty-seven out of the fifty stocks examined.

CHAPTER 1

INTRODUCTION

The main purpose of this study is to analyze the relationship between the price of a call option on common stock and an identifiable set of explanatory variables--ultimately leading to an operational model for the pricing of calls given the values of those variables.

When offering a valuation model one must identify not only those variables which are of concern but also the nature of the relation which links them. The statistical theory which has been developed to aid in identifying significant variables is considerable, e.g., design of experiments, analysis of variance, and regression analysis. Often the analysis is simplified by assuming the relation is linear, at least in the region of interest. However, when it is possible to postulate or derive a model from economic theory, the equation is rarely linear in the estimated parameters.

This paper will attempt to extend existent option theory by incorporating the pure rate of interest and dividends in an explicit, nonlinear call pricing model. Option valuation will be approached through the eyes of the writer, not the option buyer as in previous studies. Only two parameters, the market discount rate for call options on a given stock, and the expected variance of price relative logarithms, will be estimated using empirical data.

Characteristics of the Call Option

An "American" call is a contract that entitles the holder, at his option, to buy some number (usually 100) shares of a given common stock at some specified exercise price, minus any dividends paid since the time of issue, at any time during the life of the call. The striking price of a call indicates how many dollars must be paid on exercising the option for each share of the given common stock. The striking price is usually equal to the market price of the common stock at the time of issue.

If any cash dividends are declared and the stock goes ex-dividend during the life of the call, the stated striking price is reduced by the amount of the dividends. If the stock is sold ex-rights during the period, the striking price is reduced by the amount of the first sale of the rights on the day the stock sells ex-rights. If there is a stock dividend or split, the call holder receives additional shares upon exercising the call.

The option period or duration presently varies from twenty-one days to one year plus ten days. Contracts of six months and ten days are now the most popular maturity (possibly because this maturity allows the holder to take advantage of the long-term capital gain provisions of income tax laws if he sells the call in preference to exercising it).

As seen by the holder, the value of a call derives from the possibility of having the privilege to buy the associated common stock at a price below the market price at some time in the future.

The remaining life of the call is one factor influencing this value. When a call is about to expire its market price will approach the <u>exercise value</u>[1] (X-Y+D), if X>(Y-D) where X is the price of the stock, Y is the fixed striking price, and D is the amount of any dividends paid since the original issue of the call. An expiring call is of no value if X<(Y-D). In this case, the privilege of converting the call into common stock at the <u>effective striking price</u> of (Y-D) is worth nothing.

If a call is not about to immediately expire, its value will not simply be the positive difference between the current market price and the effective striking price; rather it will depend upon the probability distributions of future stock prices and the utility functions of the buyers and sellers of call options. To determine the expected value of a call analytically from the call buyer's viewpoint one must know before the fact just what action the buyer would take, given specific values for the option before expiration. That is, is there any exercise value or selling price which would induce the option holder to divest (exercise or sell the option outright) early? If a possibility of early exercise exists, *ex ante* distributions of stock prices and dividends to be paid must be postulated for all points in time before the option expires. Since options are negotiable, the option holder must also estimate the market value of the option if there is a possibility that the resale value may be higher than the exercise value.

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1. Some writers have misleadingly termed this quantity "theoretical value."

The option writer, or seller, is also interested in the option holder's set of alternatives. If there exists a possibility of early exercise the seller must estimate the distributions of unfavorable outcomes to himself at all points in time before the expiration date. If, on the other hand, any option holder who divests early does so by selling the option outright, the writer need only estimate the stock price distribution at one point in time, the expiration date.

In general, the term "option premium" refers to that dollar amount paid by the buyer and received by the seller in exchange for the option contract. Because of transactions costs, however, the amount paid by the buyer is greater than the amount received by the seller. In this study the buyer's premium refers to the total amount paid for the call option. The seller's premium refers to the amount received by the call writer. The buyer is willing to accept a certain and immediate cash outflow in return for a cash inflow which is uncertain both in timing and nominal amount. Conversely, the call writer, or seller, accepts a certain and immediate inflow in return for a commitment of an uncertain outflow. The maximum possible loss to the buyer of the call is limited to the buyer's premium, and the maximum possible gain to the seller is limited to the seller's premium. The difference between the two premiums is the dealer's spread which consists of, in the case of a put and call dealer:

1. An endorsing fee to the seller's broker

2. A discount to the buyer's broker[2]

3. New York State transfer tax[3]

4. Gross profit to the put and call house.

Organization

This study is organized as follows:

Chapter 1 states the purpose of this study and describes the characteristics of an "American" call option.

Chapter 2 reviews some previous works relating to call option valuation.

Chapter 3 presents the functional form of the call pricing relation. Assumptions essential to the development of the model are discussed.

Chapter 4 describes the iterative procedure used to estimate parameters for the functional equation developed in Chapter 3.

Chapter 5 specifies the method by which the call pricing model is evaluated.

Chapter 6 summarizes the results of this study and suggests some directions for further research.

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2. Endorsing fees and discounts paid by Lombard Street, Incorporated vary between \$6.25 and \$25.00. The majority of brokerage houses still charge \$6.25.

3. This tax is tied to the value of the optioned stock, not the option price. The structure is presently:

Stock Price	Tax/Share Optioned
$\$5 > S$	1.25 cents
$\$10 > S \geq \$5$	2.5 cents
$\$20 > S \geq \$10$	3.75 cents
$S \geq \$20$	5 cents

CHAPTER 2

REVIEW OF THE LITERATURE

Most of the recent studies of option pricing have taken the form of linear regression equations involving parameters that are estimated from actual market data. Since the venture's final outcome is uncertain, the investigator often attacks the problem by postulating a stable distribution of stock price changes and a stable investor utility function covering the range of possible outcomes. Dividends are often assumed to be zero to avoid the problem of variable striking prices.

This chapter reviews works having a direct bearing on call valuation. Included are studies on warrants, options, stock price distribution theory, and capital asset equilibrium theory.

Options

Charles Castelli

As early as 1877 Charles Castelli qualitatively related option premiums to future stock price distributions as estimated through previous price movements.

> The call is the option by which a person has the right to buy or not, at his choice, a certain quantity of stocks or shares at a determined price and at a fixed future date, by the payment of an agreed premium.
>
> The premium which is payable when the option expires, fluctuates according to the variations of the stock to be contracted: if the fluctuations are violent and

> numerous, and if, its future course liable to a great rise or great fall, then the premium asked is very heavy; if however, the stock has evenly kept its quotation, options can be negotiated for a very trifling premium. (Castelli 1877, pp. 7-9)

Castelli's logic certainly seems reasonable; however, without a precise, quantifiable definition for his explanatory variable "future course liable to a great rise or great fall," the statement is difficult to evaluate. No method was proposed by which comparative volatility could be measured.

The options described by Castelli were bought and sold on Lombard Street[1] in 19th century London; however, options are much the same today. The main differences being that the premium is now paid on a one day settlement basis and the buyer of an "American" type call may exercise his option at any time prior to the expiration date.

A. James Boness

In 1962 A. James Boness offered an operational call pricing model having rigorous theoretical backing. Essentially, Boness theorized that call market prices were equal to the present value of expected future cash flows as seen by the call buyer.

The following assumptions were essential to the development of the model:

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1. Hence the name of the corporation supplying much of the data in this study - Lombard Street, Incorporated.

1. Investors in Puts and Calls are indifferent to risk; nevertheless, the expected value of an option may be discounted at a rate higher than the pure interest rate.
2. Stock price changes are distributed log normally.
3. Call premiums are not affected by the magnitude of dividends on the underlying stock.
4. The utility functions of call buyers are such that early exercise is never desirable.

After deriving his theoretical non-linear model (T), as seen through the eyes of the call buyer, Boness then used the model as an independent variable in a linear regression equation of the form

$$\frac{\hat{C}}{X_o} = a + b\,\frac{(T)}{(X_o)} \qquad 2\text{-}1$$

The existence of fixed transaction costs and biases in demand and supply provided justification for Boness not using the pricing relation

$$\frac{\hat{C}}{X_o} = \frac{T}{X_o} \qquad 2\text{-}2$$

His rationale for the intercept (a) was that a scarcity of option writers and an oversupply of buyers would give positive rents to both writers and dealers. This supply demand bias was also seen in the reduction of (b), the regression coefficient, to a value less than unity.

For both explanatory equations (premiums to buyers, premiums to sellers), Boness empirically derived three parameters:

1. An intercept to adjust for transactions costs
2. A beta coefficient to adjust for biases in supply and demand, and
3. The rate at which the expected values of all options (on all stocks) are discounted.

The final sample regression functions offered by Boness were:

$$\frac{\hat{C}}{X_o} = 2.355\% + .78998 \left(\frac{T}{X_o} \right) \qquad R^2 = .792 \qquad 2\text{-}3$$

for options bought from dealers, and

$$\frac{\hat{C}}{X_o} = 3.263\% + .58702 \left(\frac{T}{X_o} \right) \qquad R^2 = .559 \qquad 2\text{-}4$$

for options sold to dealers.

That is, the buyer's call premium, represented as a percentage of the optioned stock's value, had an intercept of 2.355% and a regression coefficient for the theoretical model of .78998. The seller's call premium also had a non-zero intercept and a regression coefficient less than one.

The first sample included 51 options purchased from dealers; the second sample included 38 options sold to dealers. Selection criteria for the samples were:

1. The option was on one of sixteen stocks having highest option volume during the period (February, 1958 - August, 1960).
2. Among all options on a given stock, only non-overlapping durations were eligible.

3. Extremely long or extremely short durations had preference over moderate durations. That is, 12 month and 2 month options were preferred to 6 month options.

4. Puts were preferred to calls. Boness used actual put premiums plus conversion charges as proxies for call premiums. The model assumed that the put premium was neither higher nor lower than a call having the same terms, minus conversion[2] charges.

In extreme cases these final equations yield unacceptable results. For example, a call option having market striking price and due to expire the next instant could be purchased for 2.355% of the stock's value. That same option could be sold to a dealer for 3.263% of the stock's value.

In view of the necessity of adding parameters a and b to the valuation model (whatever the rationale) one must conclude that the basic model (T) was biased.

An overall view of the study suggests that the theoretical model itself:

$$\tilde{M} = T + \tilde{\varepsilon} \qquad \text{2-5}$$

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2. An option house converts a put into a call by simultaneously buying a put and one hundred shares of the underlying stock, then selling a call of the same duration. The buyer's premium for the newlyborn call is equal to the summation of: the seller's premium for the put, prepaid interest on the covering funds for the life of the option, two floor brokerages, transfer taxes, and any differential between the put striking price and the stock market price when the stock is purchased by the conversion house. This differential adds to the call buyer's premium when the stock has risen, and reduces the call buyer's premium when the stock price has fallen.

1. Indicated too high a premium for long periods and/or high volatility.

2. Indicated too low a premium for short periods and/or low volatility.

Boness did not consider the effects of interest rates or stock dividends in his basic non-linear model. However, he did test the significance of dividends in the linear regression

$$\frac{\hat{C}}{X_o} = a + b_1 \frac{(T)}{(X_o)} + b_2 d \qquad 2\text{-}6$$

His conclusion was that dividends have no significant effect on option pricing. Boness did not consider the time location of dividends within the call period nor did he offer a theory as to why the effect of dividends in the valuation equation should be linear.

Burton G. Malkiel and Richard E. Quandt

Malkiel and Quandt (1969, pp. 28-31) worked with a cross sectional linear model incorporating as independent variables: stock price volatility, number of common shares outstanding, stock turnover ratio, and expectations on stock growth as obtained from a sample of security analysts. Although *a priori* rationale for the signs of zero order correlations is given, no attempt is made to justify a linear relationship. Coefficients of determination varied from .635 to .794. In several cases, regression coefficients of variables included in the final equation were not significant.

The general level of Malkiel and Quandt's R^2 is in agreement with Boness' results for a linear model.[3]

Apparently, linear models are capable of explaining much of the short-term option price variance; however, Malkiel and Quandt's results suggest the regression coefficients are not stable. In the case of the three regression equations (1964, 1965, 1966) coefficients for four of the five variables considered had high values at least 1.5 times the low value for the three years. One variable had a coefficient of .005 in 1964, and .022 in 1966.

Warrants

Although similar to call options of the same duration, stock purchase warrants differ in at least two aspects:

1. Warrants do not have fixed striking prices adjusted for dividends; instead, the warrant exercise price is often a series of time step functions.
2. Warrants represent a potential source of dilution. When warrants are exercised, the corporation simply sells from its supply of authorized but unissued stock, diluting outstanding stock. Essentially, the corporation is always covered; however, no actual cash is tied up in the covering operation. In contrast, the individual covered option

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3. In both models the dependent variable was call price expressed as a percentage of the stock price.

writer must forego the interest on funds used to cover the outstanding call. In the case of a writer of covered options these funds amount to the purchase price of the underlying stock; for the writer of naked options, foregone interest is on margin requirements. Exchange regulations require a minimum maintenance deposit of 30 percent of the optioned stock's market value. As a practical matter, however, the majority of endorsers require higher margin requirements than exchange minimums. Indeed, some firms will not endorse any call unless the writer owns the stock. (Securities and Exchange Commission 1961, p. 59.)

Case M. Sprenkle

Sprenkle (1961), using the assumption of a log normal price change distribution, derived a predictive model with warrant price equal to the expected value plus a premium the investor is willing to pay for the leverage of the warrant.

His explicit valuation equation depended upon those parameters estimated from actual market data. These parameters were: expected standard deviation (σ) of changes in stock price logs, a leverage premiums (P_e) and a measure (K) of the relation between the present price of the stock and the mean expected stock price. Although Sprenkle cited difficulty in obtaining stable estimates of all three parameters, he states that "the empirical results show first that the theory explains warrant prices with hardly any error." However, he did not offer an R^2 for his warrant valuation model itself after all parameters had been estimated.

Andrew H.Y. Chen

Chen (1969, 1970) used a dynamic programming model to investigate warrant pricing. Essentially the method consists of a multi-period decision problem in which the warrant owner:

1. Decides to exercise if expected returns during the next period do not satisfy minimum requirements, or
2. To continue to hold the warrant for one more period, at the end of which he faces the same choices.

Warrant life is divided into a series of periods--each period having a critical value which, if attained by the stock, results in the warrant being exercised immediately. Having decided upon a course of action for each possible value of the underlying stock at each period end, and also upon a probability density function for each period, Chen used successive backward optimization to find the warrant's value at t_o. The critical values (those values above which the warrant should be exercised and below which the warrant should be held one more period) described by Chen demonstrate the difficulty in assuming any one specific option holder owner will hold the option till just before it expires.[4] Since warrants might logically be exercised before maturity or the next striking price change, they would not necessarily be worth more in the resale market than their pure exercise value. Chen did not consider the alternative use of a warrant as a hedging device against a short position for the time left till the next change in exercise price.

Paul Samuelson and Robert C. Merton

Samuelson and Merton's model (1969) examined warrant prices in a world that included other capital assets. To arrive at a final explicit equation, however, one must postulate specific utility functions and the *ex ante* stock price distribution. The authors' assumption of a single utility curve so that "the behavior of a group of investors can be treated as if it resulted from the deliberation of a single mind" (p. 23) ensures the absence of a secondary market when the stock price is above the critical value determined by the holder. Given any specific utility function, Samuelson and Merton demonstrate that the extra value of the American option (exercisable any time before the contract expires)

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4. In a recent dissertation J.R. McGuigan (1971) compared computer simulated results for three timing strategies in the option market:

a. Exercise the option on the expiration date.
b. Exercise the option whenever an exercise signal is indicated by by a trading rule known as the filter technique.
c. Exercise the option whenever the dynamic programming "critical value" is attained.

McGuigan found that the buy-and-hold rule yields a higher average return than the dynamic programming model and the filter technique.

over the European option (exercisable only immediately before expiration) lies in the ability to readjust one's portfolio in light of changing market conditions. Only through re-balancing the portfolio can the investor assure the utility adjusted "effective probability" mean of every asset in his portfolio will be identical.

Stock Price Distribution Theory

In 1900, Louis Bachelier (translated 1964) proposed that the arithmetic differences of stock prices were independent of the order of past price movements. This hypothesis has been the basis for most of the theoretical analysis of options and warrants. One main point of later disagreement with Bachelier's analysis centered around the fact that the "arithmetic" Brownian motion leads to an equally probable rise or fall of any given dollar amount. Therefore, as the distribution's variance increases, there is an increasing probability of a negative stock price. Osborne (1959) later suggested the "geometric" Brownian motion which led to the log normal distribution of stock prices. This latter distribution met the *a priori* requirements of asymptotes at $X = 0$ and $X = \infty$. Under the "geometric" Brownian motion hypothesis there is an equal probability of a given rise or fall in the log of a stock's price.

Moore and Kendall have provided empirical evidence in support of the Gaussian hypothesis, Moore (1962, pp. 116-123) graphed weekly first differences of log price relations for eight New York Stock

Exchange stocks on probability paper. He concluded the fit was close enough to support the hypothesis of approximate normality. Kendall (1953, p. 24) observed that weekly price changes in British industrial common stocks seem to be approximately normally distributed.

Mandelbrot (1963) and Fama (1963) challenged the log normal Gaussian assumption with empirical evidence suggesting the true distribution has more events in the tails and center than predicted by the Gaussian density function. Based on those results Mandelbrot introduced the class of distribution called stable paretian. Except for the Gaussian distribution which has finite variance, the stable Paretian distribution in general has an infinite second moment, precluding the use of any standard statistical method which assumes finite variance.

Brada, Ernst, and Van Tassel (1966, pp. 334-335) pointed out that in the usual form of the Gaussian assumption price changes (or the change in logs of consecutive prices) are distributed normally over time, not numbers of transactions. After observing the price changes of 10 stocks over 102 trading days they found that "the distribution obtained by differencing across transactions did not have an excessive number of extreme events, although the center classes continued to be overcrowded." Brada et al. concluded that the differencing should be done across transactions and not across a unit of time--which may include any number of transactions. In order to jump from the "transaction domain" to the "time domain" one must assume that all future time intervals of equal length contain similar numbers of transactions.

So far there is no general agreement as to what specific distribution is empirically justified. The works of Moore, Kendall, and Brada et al. support the log normal distribution; whereas, the works of Mandelbrot and Fama support a non-normal stable paretion distribution.

This study will proceed under the assumption that the changes in consecutive price logarithms are distributed Gaussian (or that prices themselves are distributed log normally). The reasons for this choice are:

1. As demonstrated by Osborne (1959) and Aitchison and Brown (1963, also see Appendix A) the log normal distribution has strong theoretical backing, its derivation requiring relatively few assumptions.
2. No generally accepted method of rejecting the hypothesis of either the log normal or the generalized Pareto-Levy distribution has been generally accepted.

The final rationale, then, for using the log normal distribution is: it is theoretically strong, its density function is well defined, and it is consistent with some empirical evidence.

Capital Asset Equilibrium Theory

In general terms, an equilibrium pricing model contains provisions for the simultaneous consideration of all desirable and undesirable features of available allocations of capital. Theoretically, for a complete and rigorous solution to the valuation of one specific asset, the entire economic system must be considered. Therefore, the investigator who would offer an operational equilibrium system must claim knowledge of the decision-making processes used by all market participants for all states of nature.

A less ambitious, but more pragmatic, approach is to ignore (however reluctantly) the mutual dependence of some economic variables. This partial equilibrium approach assumes permission to consider certain variables in the relation as being largely of a determining nature rather than a determined one.

Utility Theory

In analyzing the St. Petersburg paradox, Daniel Bernoulli (1738, translated 1954) established that expected value along is not acceptable as a valuation model. Since then other authors have demonstrated that the two classes of variables which affect valuation are probability of wealth states and utility of wealth states. However, the application of this concept to security valuation presents extreme difficulties. Universal agreement exists on neither the underlying probability density function for security prices nor on the most appropriate model for "Everyman's" utility function. Given these two functions, an explicit valuation is possible. For example, in his dissertation dealing with mutual funds, Farrar (1967) showed that by expanding the investor's utility function about the mean in a Taylor's series, utility for a specific risky project could be expressed in terms of density function moments and utility function derivatives. That is,

$$E[U(x)] = U(\mu) + U'(\mu)\ E(x - \mu) + \frac{U''(\mu)}{2!}\ E\ (X - \mu)^2 + \frac{U'''(\mu)}{3!}\ E\ (x - \mu)^3 + \ldots \qquad 2\text{-}7$$

The expected utility of an uncertain investment is shown to equal the utility of the expected outcome only if all whole terms on the right side of the equation after the first are equal to zero. Either the utility derivative or the density function moment must be zero in each added term.

Sometimes not only the amount of the final payoff is unknown, but the timing as well. The utility curve, then, must be applicable to the time the cash flow is to be realized.[5]

When an individual's utility function is known with certainty (in terms of present day dollars) and the density function postulated, an explicit valuation model can be derived. However,

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5. State preference theory suggests this utility function may possibly vary with additional factors describing the state of the world (e.g., health, marital status, age).

there is no *a priori* rationale which specifies the "correct" utility function.

Capital Market Theory

In 1952 Markowitz proposed a portfolio selection model which demonstrated that by combining securities having less than perfect correlation, an "efficient" set of portfolios (i.e., those portfolios which maximize expected return for a given level of return standard deviation) could be determined. From this set an investor selects the portfolio that maximizes his expected utility. Explicit assumptions are:

1. Each investor chooses the investments in his portfolio on the basis of only expected portfolio return and portfolio return standard deviation.
2. Each investor prefers more expected return and will refuse to incur increased standard deviation unless compensated by sufficient increased expected return.
3. Distributions of common stock returns have finite standard deviations.

Application of the theory requires estimation of all expected returns, standard deviations, and covariances for all available securities.

Sharpe (1964, 1967) extended the Markowitz work to a general theory of capital market equilibrium by applying two additional assumptions:

1. All investors may lend and borrow without limit at the pure rate of interest, and
2. All investors are willing to use identical estimates of security expected returns, standard deviations, and covariances.

The result was a "capital market line" in portfolio ($E - \sigma$) space. A major implication of Sharpe's analysis was that all efficient portfolios differ only in the proportion of wealth invested in a unique "Market Portfolio" and the proportion borrowed or lent at the pure rate of interest.

The most serious limitations of capital market theory have to do with the inability of the model to forecast. Evidence supplied by Lintner (1965), Douglas (1969), and Miller and Scholes (1972) indicates that the mean-variance model does not provide a complete description of the structure of security returns.

State-Preference Theory

Arrow (1964), Debreu (1959), Hirshleifer (1965, 1966), and Jacob (1970) have described an axiomatic approach to market equilibrium. Actual market securities are viewed as being composed of contingent claims to income for specified states of the world. A "state" here refers to a complete world-environment for the investor. Individuals are allowed to differ with respect to the utility of a given payoff in a given state, and with respect to the likelihood of a given state occuring. Consider the payoff matrix below:

	E_1		E_j		E_N
a_1	R_{11}	$\cdots$	R_{1j}	$\cdots$	R_{1N}
	$\vdots$		$\vdots$		$\vdots$
a_i	R_{i1}	$\cdots$	R_{ij}	$\cdots$	R_{iN}
	$\vdots$		$\vdots$		$\vdots$
a_M	R_{M1}	$\cdots$	R_{Mj}	$\cdots$	R_{MN}

Fig. 1. Payoffs in Possible States of the World for All Available Actions

Across the top of the table is an exhaustive set E of mutually exclusive, equally probable states for one period in the future. The column to the left is comprised of the set A of actions (securities) presently available to the investor. If the investor chooses portfolio $a_i \varepsilon A$ and state $E_j \varepsilon E$ occurs, his dollar payoff will be R_{ij}.

A general equilibrium pricing model is possible when given:

1. Agreement upon payoff elements.
2. Each market participant's wealth available for investment.
3. The decision criteria used by market participants. Three possible approaches are:
   a. Allocate funds such that the sum of possible outcomes is the greatest, i.e., $\underset{a_i \varepsilon A}{\text{Max}} \sum_{j=1}^{N} R_{ij}$

b. Allocate funds such that the highest payoff results should the most unfavorable state occur, i.e.,

$$\max_{a_i \varepsilon A} \; \min_{E_j \varepsilon E} \; R_{ij}$$

c. Allocate funds such that the most favorable payoff results should the most favorable state occur, i.e.,

$$\max_{a_i \varepsilon A} \; \max_{E_j \varepsilon E} \; R_{ij}$$

The analysis to this point assumes complete ignorance as to which state E_1, E_2, ...E_N will occur. In this situation it is difficult to deny that all states are equally probable.

If probabilities are assigned to each of the states E_j and the decision maker uses expected utility as his maximizing objective, he must determine the portfolio that leads to: $\max_{a_i \varepsilon A} \sum_{j=1}^{N} P(E_j)U_j(R_{ij})$;

the consideration being that the utility-of-wealth function for any investor may be related to the state that obtains.

Depending upon the shape of the investor's utility curve, individual portfolios may be priced to yield rates higher than, lower than, or equal to the riskless rate. Whereas a riskless portfolio promising to pay \$(1+r*) in one period would be priced at \$1 today, the price of a risky portfolio having different outcomes for each state is entirely dependent upon the summation of prices for the payoffs in each state, i.e.,

$$\text{Price } a_i = \sum_{j=1}^{N} \frac{P(E_j)\,R_{ij}}{1+r_j} \qquad 2\text{-}8$$

Just as the investor's utility function is state dependent, so is his required rate of return.

If, however, it is assumed that investor future utility is independent of the state that occurs, the present price of a portfolio may be written

$$\text{Price } a_i = \frac{1}{1+r} \sum_{j=1}^{N} P(E_j)\ R_{ij} \qquad 2\text{-}9$$

This latter equation is a subset of the more general state-preference theory and is the basis for the model developed later in this study. The implication is that a dollar will yield the same utility in a state of sickness or health, war or peace, or old age as opposed to youth.

Summary

Call premiums have been related previously to:

1. Uncertain parameters (future dividends, future stock prices, future stock transactions, future volume).
2. Certain parameters (option durations, unadjusted striking price, the underlying stock price at the time of the call writing, and ratios based on accounting data).

The relationship has been described qualitatively and statistically in linear regressions.

Qualitative statements provide insight into the direction of a change in call premiums; however, no information is given regarding the relative size of the change.

Linear regressions go a step further by estimating the relative weight to be given each independent variable. The regression form is usually additive in regressors and the error term.

Non-linear theories have taken the form of discounted expected values, the expected values being derived from assumed unit time or individual trial density functions, and the discount rate estimated from empirical data.

Individual investigators have suggested that option (or warrant) prices are best described by:

1. A buyer's view of expected value discounted by a constant rate determined by actual market data. (Boness-options)
2. A buyer's view of expected value plus a leverage premium. (Sprenkle-warrants)
3. A linear combination of option regressors (length, striking price, *ex post* volatility of stock prices, and ratios obtained from corporate annual reports). (Malkiel and Quandt-options)
4. A buyer's view of utility-adjusted value where the investor utility function and stock price distribution are left unspecified. (Samuelson and Merton-warrants)

Only Boness and Malkiel and Quandt offered "goodness-of-fit" measures for their respective call pricing models. Boness' non-linear model (when used as an independent variable in a linear regression) produced an adjusted R^2 of .792. The simple linear model used by Boness for comparison produced an adjusted R^2 of .722. Malkiel and Quandt's linear models had R^2s ranging from .635 to .794; however, not all regression coefficients were significant. Also, the values of the regression coefficients were not stable for the three years examined.

The call pricing model to be developed in the following chapter draws most heavily on the works of Osborne and Boness. Osborne's theory that the logarithms of consecutive stock prices are distributed Gaussian was chosen for use in this study because the density function has been well-defined and the theory is consistent with some empirical evidence. Although the question of the correct form for the stock price distribution is not without controversy, most of the previous works on warrant/call valuation (Sprenkle, 1961; Boness, 1962; Samuelson, 1965; Chen, 1969) are based on the log-normal distribution.

Of the call pricing models presently available in the literature, only that of Boness is rigorous and based on an assumed underlying stock price distribution. However, the Boness model offers little insight into the valuation process as seen by the writer. As a result, some additional variables of concern to the writer (and, therefore, to the pricing relation) are not included in his valuation equation.

CHAPTER 3

FORMULATION OF THE CALL VALUATION MODEL

Theoretically, perfect information would enable a model builder to construct an abstract relation which could represent the true underlying state of the world. To arrive at prices for available assets he would require specific descriptions of the utility functions for all market participants and the nature of their expectations for future price movements.

Conceptually, it would be possible to interview all market participants and thus obtain some Von Neumann-Morgenstern measure of their utility functions. For the purposes of the present study, however, this approach is not feasible. Therefore, the model presented in this chapter is accompanied by a few simple, but reasonable, assumptions.

The Writer's Alternatives

By choosing to write a naked call or a covered call, the writer decides upon the type of risk which is the least unpleasant to bear. In writing a covered call he bears the risk of falling stock prices--his potential loss being limited to the stock price at the time of the call minus the seller's premium. In writing a naked call he bears the same potential loss as one who sells short--unlimited loss. Of these two alternatives, the vast majority of writers choose the former position--that of writing covered calls.[1]

The "safety-first" criterion may explain this behavior. Roy (1952), Telser (1955-6), and Kataoka (1963) have suggested that investors behave so as to maximize expected return subject to a disaster constraint. For those investors the possibility of a very large, if not unlimited, loss which accompanies a naked position is unacceptable.

The Writer's Estimate of Ultimate Exercise Timing

With an "American" type call the holder has the right to exercise his option at any time before 3:15 p.m. New York time on the final day of the option contract. The option is the buyer's option; the writer has no alternative but to respond at the whim of the buyer. The question under consideration here is whether, in a frictionless market, the option would ever be exercised early. If the answer is "no", the privilege of being able to exercise early is worthless and the "American" type call option is no more valuable than the "European" type call option.

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1. Public Policy Aspects of a Futures-Type Market in Option on Securities, prepared for the Chicago Board of Trade by Robert Nathan Associates, Nov. 1969, p. 13, reports that "about 90% of today's call writers are covered." Report on Put and Call Options, Securities and Exchange Commission, August 1961, p. 7, reports that "writers were long in the stock in about 80% of the calls written."

At any point in time the call owner must decide whether to hold the call or to divest. Obviously he will not consider exercising if the stock price (X) has fallen below the adjusted striking price $(Xo - D_t)$. If at some time before the call expiration date the stock price is above the adjusted striking price and he wishes to divest himself of all downside risk, he may do so in two ways:

1. Exercise the call and sell the called stock.
2. Sell the call outright.

With the first method the speculator realizes a net cash inflow of $X - (Xo - D_t)$, and all rights to further gains during the remainder of the option period are lost.

If at least one opportunity exists by which divesting option holders can realize ultimate profits greater than $X - (Xo - D_t)$ with no attending chance of loss, the call will not be exercised at that time.

Consider a converter[2] who exposes himself to no chance of loss. The converter would initially pay $X - (X_o - D_t)$ for a call "in the money", the same amount as the call holder could get by exercising. At the same time he would short the stock--freeing up X. His net cash inflow would be $(X_o - D_t)$, and he has the *possibility*

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2. This converter could be any brokerage house having free use of stock left in the "street name" or an individual owning the stock. In either case the owner could demand and get a percentage of the profits ensuing from the use of his stock.

of gains on the stock-call combination should the stock price fall below $(X_o - D_t)$. The amount $(X_o - D_t)$ could be invested at the riskless rate of interest for the remaining life of the call.

To entice call holders to sell their options rather than exercise, the conversion house offers to split in some ratio any subsequent profits after the sale, plus a portion of the interest on funds freed by the short sale.

The above discussion suggests that any remaining life of the call option is of some value. The analysis is valid assuming:

1. There exists a converter having access to the underlying stock.
2. There is zero (or negligible) probability of subsequent dividends during the remaining life of the call *summing to more than the present adjusted striking price.*
3. A short sale depresses the market stock price no more than a long sale.

Note that in the case of market entry by the converter, the set of investments available to the public is larger than if the call holder had exercised his call outright. Admittedly, the above analysis does not consider any shifts which might occur in the capital market line as a result of hedging operations.

In a frictionless market reflecting perfect competition conditions, arbitrage operations prevent the "American" call option from being exercised early.[3] The plan under study by the Chicago Board of Trade promises to offer less friction and reduce the number of calls exercised early.

Robert R. Nathan Associates state in their report (1969, p. 34) to the Chicago Board of Trade,

> The options resale, or secondary, market envisaged by the Chicago Board . . . should also reduce risk for both the holder and writer, as both should be able to change their position at any time when they believe that they can increase their gain or curtail a loss. The holder should be able to realize some value for the unexpired portion of an option he sells.

The Covered Writer's Valuation of the Call Option

Kruizenga (1967a, p. 389) demonstrated that a long position in stock combined with a short position in the associated call yields the same risk as a short position in a put with no cover.[4] However, the two strategies are not exactly identical. In the first strategy, more of the writer's cash is tied up during the option life. Also, any dividends paid before the option is exercised represent a cash flow to the call writer. For the naked put writer, dividend payments

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3. See Boness (1962) and Chen (1969) for different approaches to the consideration of early conversion.

4. Kruizenga's notation for this is: $\begin{vmatrix} 1 \\ -1 \end{vmatrix} + \begin{vmatrix} -1 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ -1 \end{vmatrix}$

represent no cash flow until the put is exercised. There are many other combinations of options and stock which result in identical final outcomes; however, as shown in the example above, the intermediate cash flows may be quite different. Any serious option valuation model must recognize the amounts and timing of intermediate as well as "end-of-game" cash flows. The model developed in this section attempts to value the relevant cash flows as seen by the covered call writer.

The owner of a share of common stock has claim at any time T in the future to the share price plus any reinvested dividends which are paid at time t before T. The expected value of this claim at T is:

$$E[\tilde{X}_T + \tilde{d}_T] = \int_0^\infty \tilde{X}_T \; f(\tilde{X}_T) \; d\tilde{x}_T + \bar{d}_t \; e^{rd(T-t)} \qquad 3\text{-}1$$

where $\tilde{X}_T$ is a random variate corresponding to the market stock price at time T, $\tilde{d}_T$ is a random variate corresponding to the value of dividends received at time t and reinvested until time T. This expected value may also be partitioned as follows:

$$\begin{aligned} E[\tilde{X}_T + \tilde{d}_T] = & \int_{X_o - d}^\infty [\tilde{X}_T - (X_o - \bar{d})] \; f(\tilde{X}_T) \; d\tilde{x}_T \\ & - \int_0^{X_o - \bar{d}} [(X_o - \bar{d} - \tilde{X}_T] \; f(\tilde{X}_T) \; d\tilde{x}_T \\ & + (X_o - \bar{d}) \int_0^\infty f(\tilde{X}_T) \; d\tilde{x}_T + \bar{d} \; e^{rd\;(T-t)} \end{aligned} \qquad 3\text{-}2$$

The terms on the right hand side of this equation represent, respectively:

1. A long position in a European-type call expiring at T.
2. A short position in a European-type put expiring at T.
3. A long position in a Treasury bill having face value $(X_o - \overline{d})$, maturing at T.
4. The expected value of dividends payable at t, reinvested at rate r_d for the time period from t to T.

As a result, the expected value of a given stock's price at any time in the future is always equal to the expected value of these four terms. Therefore, the present price of the stock is equal to the sum of the discounted expected values of the positions noted for a share owner who

1. writes a call against his stock
2. plans to remain covered throughout the call period, and
3. is certain the call will not be exercised until the end of the option period, if at all.

In equation form this may be written:

$$X = e^{-r_c T} E[\tilde{c}_\tau] - e^{-r_p T} E[\tilde{p}_\tau] + e^{-\rho T} E[\text{T-Bill}] + \overline{d}\, e^{-r_d t} \qquad 3\text{-}3$$

Transposing, the call premium demanded by the writer equals:

1. the price of the underlying common stock, plus
2. the price of a put on the same stock, having identical striking price and length as the call, minus
3. a Treasury bill of face value $(X_o - \overline{d})$, maturing when the call expires, minus

4. the present value of expected dividends, to be paid at time t before the call expires.

$$C = X_o + [P] - (X_o - \bar{d})\, e^{-\rho T} - \bar{d}\, e^{-r_d t} \qquad 3\text{-}4$$

If more than one dividend payment is expected during the option period, the valuation equation may be written:

$$C = X_o + [P] - (X_o - \bar{D})\, e^{-\rho T} - \bar{d}_1\, e^{-r_d t_1} - \bar{d}_2 e^{-r_d t_2} - d_3\, e^{-r_d t_3} - d_4\, e^{-r_d t_4} \qquad 3\text{-}5$$

where $\bar{D}$ represents Σd_i.

Assuming that dividends are discounted and reinvested at the pure rate of interest,

$$C = X_o(1 - e^{-\rho T}) + [P] + [\bar{D}\, e^{-\rho T} - d_1\, e^{-\rho t_1} - \bar{d}_2\, e^{-\rho t_2} - \bar{d}_3\, e^{-\rho t_3} - \bar{d}_4\, e^{-\rho t_4}] \qquad 3\text{-}6$$

where

C = the call premium to the writer

X_o = the stock price at the time of the call's original issue

ρ = the pure rate of interest

[P] = the price of a European-type put at the time of the call's original issue

T = the length of the call period in years

τ = the length of the call period in weeks[5]

d_1, $\bar{d}_2$, $\bar{d}_3$, $\bar{d}_4$, = the expected values of dividends to be paid at t_1, t_2, t_3, t_4 respectively before the option expires. With quarterly dividends the maximum number of dividends (for a one year option) is four.

$\bar{D}$ = The sum of those dividends for which the stock goes ex dividend during the option period. This is equal to the sum of dividends paid unless the stock has gone ex dividend but the dividend has not been paid when the option expires. The two figures become equal when the time between the ex dividend date and the payment goes to zero.

The value of the equivalent European put in Equation 3-4 is, of course, dependent upon the *ex ante* probability distribution of stock prices, and the writer's utility function.

The discounted expected value is:

$$[P] = e^{-r_p T} E[\tilde{p}_\tau] = e^{-rT} \int_0^{X_0-\bar{D}} (X_0-\bar{D}-\tilde{X})\ f(\tilde{X})\ dx \qquad 3\text{-}7$$

For reasons detailed earlier (and in Appendix A), the log of stock returns $Y = \ln [X_\tau/X_0]$ is assumed to be distributed Gaussian, with zero mean and variance σ^2, where τ stands for the number of

5. Theoretically, since τ represents statistical trials or observations, it can only take on integer values. Only the volume-time assumption permits the transformation from statistical observations to continuous time (see Appendix A).

basic time units (which in this study is one week). That is,

$$\phi(Y)dy = \exp\{-Y^2/2\sigma^2\tau\} \frac{dy}{\sqrt{2\pi\sigma^2\tau}} \quad \text{3-8}$$

The probability distribution of X_τ itself is:

$$\phi(X_\tau)\ dx = \phi\ [Y = \ln\ (X_\tau/X_o)]\ (dy/dx)\ dx \quad \text{3-9}$$

$$= \exp\ \{-[\ln\ (X_\tau/X_o)]^2\ /2\sigma^2\tau\}\ \frac{dx}{X_\tau\sqrt{2\pi\sigma^2\tau}}$$

Under these conditions equation 3-7 simplifies to:

$$[P] = e^{-rT}[\int_0^{X_o-\bar{d}} \frac{X_o-\bar{D}-X}{X\sqrt{2\pi\sigma^2\tau}} \exp \frac{-[(\ln X-\ln X_o)/\sigma\sqrt{\tau}]^2}{2}\ dx] \quad \text{3-10}$$

Letting $\lambda = (\ln X-\ln X_o)/\sigma\sqrt{\tau};\ d\lambda = \dfrac{dx}{X\sigma\sqrt{\tau}}$

$$dx = X\sigma\sqrt{\tau}d\lambda;\ \lambda\sigma\sqrt{\tau} = \ln X - \ln X_o$$

$$\ln X = \lambda\sigma\sqrt{\tau} + \ln X_o;\ X = e^{\lambda\sigma\sqrt{\tau}}\cdot X_o$$

$$[P] = e^{-rT}[\frac{X_o-\bar{D}}{\sqrt{2\pi}} \int_{-\infty}^{A} \exp\ [-\frac{\lambda^2}{2}]\ d\lambda \quad \text{3-11}$$

$$- \frac{X_o}{\sqrt{2\pi}}\ \exp\ [\frac{\sigma^2\tau}{2}] \int_{-\infty}^{A} \exp\{\lambda\sigma\sqrt{\tau} - \frac{\lambda^2}{2} - \frac{\sigma^2\tau}{2}\}\ d\lambda$$

where $A = \dfrac{\ln\ (X_o-\bar{D}) - \ln X_o}{\sigma\sqrt{\tau}}$

Letting $u = \lambda - \sigma\sqrt{\tau}$; $du = d\lambda$

$$P = e^{-rT} \{ \frac{X_o - \bar{D}}{\sqrt{2\pi}} \int_{-\infty}^{A} \exp [- \frac{\lambda^2}{2}] \; d\lambda \qquad 3\text{-}12$$

$$- \frac{X_o \exp [\frac{\sigma^2 \tau}{2}]}{\sqrt{2\pi}} \int_{-\infty}^{B} \exp [- \frac{u^2}{2}] \; du \}$$

$$\text{where } B = \frac{\ln(X_o - \bar{D}) - \ln X_o - \sigma^2 \tau}{\sigma\sqrt{\tau}} = A - \sigma\sqrt{\tau}$$

Note that both integrals are in the form of normalized cumulative density functions.

Representing the area under the normal curve from $-\infty$ to A as $N(A;\ 0,1)$ and combining the value of the equivalent Put with Equation 3-6, the theoretical price of a 100 share call is:

$$\hat{C} = 100\{X_o(1-e^{-\rho T}) + e^{-rT}[(X_o - \bar{D})N(A;\ 0,1) \qquad 3\text{-}13$$

$$- X_o \exp (\frac{\sigma^2 \tau}{2}) \; N(B;\ 0,1)] + \bar{D} \; e^{-\rho T}$$

$$- \sum_{i=1}^{N} \bar{d}_i \; e^{-\rho \, t_i}\}$$

Essentially, this equation states that covered call writers stand ready to sell call options at a price determined by the present stock market price, the pure rate of interest, the option length, expected dividends, expected variance of successive stock price logarithms, and a risk discount rate. Even as the variance of the logarithms of successive stock prices approaches zero, the covered writer requires compensation for foregone interest on committed funds.

An application of equation 3-13 is given below.

Consider a six month - 10 day call on Leasco Data Products Corporation Common Stock.

X_o = \$25

ρ = 6% per year

T = .53 years

τ = 27 weeks

r = 12%

$\hat{\sigma}$ = .09

} See Chapter 4 for estimation procedure

$\bar{d}_1, \bar{d}_2,$ = 55¢ The last dividend was a quarterly dividend of 55¢; the next dividends are due two months and five months from now. $t_1 = .167$ $t_2 = .416$

$\bar{D} = \bar{d}_1 + \bar{d}_2$ = \$1.10

$$A = \frac{\ln (23.90) - \ln 25}{(.09)\ 27} = -.0198$$

$$B = -.0198 - .09\sqrt{27} = -.4878$$

$N(A; 0,1) = .4922$

$N(B; 0,1) = .3135$

$$\hat{C} = 100\{\$25(.0314) + .938[\$23.90(.4922) - \$25(1.115)\ (.3135)] + \$1.10(.97) - \$.55(.999 + .991)\}$$

$$= 100\{\$.785 + .938[\$11.76 - 8.68] + (\$1.07) - (\$1.09)\}$$

$$= 100\{3.08 - .02\} = \$306$$

In this example, interest charges ($78.50) on the initial covering funds amount to more than 25% of the total seller's premium. The buyer could expect to pay an additional dealer's spread of approximately $55 (S.E.C., 1961, p. 8).

In summary, the model states that the covered call writer has two reasons for requiring compensation: foregone interest on covering funds and possible loss of wealth in the future. Whether the assumptions accompanying the development of the model are good enough for the purposes at hand can only be determined ultimately by seeing if the final equation fits empirical evidence to a greater degree than the best competitive model.

CHAPTER 4

ESTIMATION OF PARAMETERS

The goal of the previous chapter was to develop a call valuation model under the assumption that all explanatory terms were readily available. The objective of the current chapter is to present a method by which those parameters known with less than certainty may be estimated, utilizing empirical data.

The pricing model developed in chapter 3 may be written:

$$\hat{C}_i = f(X_{i1}, X_{i2}, \ldots, X_{ij}; \theta_1, \theta_2, \ldots, \theta_k) \qquad 4\text{-}1$$

where the regressors ($\underline{X}$) are well defined and can be observed by option buyer and seller alike at the time of the option sale (see Appendix B), and the parameters ($\underline{\theta}$) are estimated on the basis of empirical observations. Regressors $\underline{X}$ and parameters $\underline{\theta}$ are defined as follows:

Regressors:

$x_1 = x_o$ = the current market price of the underlying stock

= the unadjusted striking price of the call option.

$x_2 = \rho$ = the risk-free rate at the time of the option sale.

$x_3 = T =$ option life expressed in years.

$x_4 = \tau =$ the option life expressed in weeks.

$x_5 = \bar{D} =$ the sum of dividends expected to be paid during option life (*ex post* data used as proxy).

$x_6, x_8 \ldots =$ amount of individual dividends expected to be paid during option life.

$x_7, x_9 \ldots =$ length of time in years till dividend payments $x_6, x_8 \ldots$

Note that if dividends are paid quarterly there would be a maximum of four dividend payments in an option life of one year. The time values x_7, x_9, x_{11}, x_{13} would be .25 years apart.

Parameters (Θ):

$\Theta_1 = r =$ the rate determined by the seller market for discounting the expected value of the equivalent put.

$\Theta_2 = \sigma =$ the seller market's estimation of the standard deviation of changes in the logarithms of week's end prices. This estimated value is expected to be in effect over the life of the option.

Optimization Criterion

For the standard linear model under classical assumptions, parameter estimates are calculated using minimization of the sum of the squared residuals as the objective criterion. Applying the

Gauss-Markov theorem (see Theil 1971, p. 119), these estimators can be shown to be best-linear-unbiased estimators (BLUE) of the true (unknown) parameters. For the nonlinear regression function the least squares criterion does not, in general, insure these desirable traits given any finite sample size. However, if the disturbances are assumed to be normally distributed, the least squares and maximum likelihood estimators correspond, yielding asymptotically efficient and consistent parameter estimators for large samples.

The error or "discrepancy" function to be minimized may be defined as

$$S(\underline{\Theta}) = \sum_{i=1}^{N} \{c_i - f(\underline{X}_i, \underline{\Theta})\}^2 \tag{4-2}$$

$\underline{\Theta}^*$ will denote that value of $\underline{\Theta}$ which minimizes $S(\underline{\Theta})$. Since c_i and $\underline{X}_i$ are fixed observations, the sum of squares is a function of $\underline{\Theta}$. The criterion for determining $\underline{\Theta}^*$ shall be to select that matrix $\hat{\underline{\Theta}}$ which minimizes $S(\underline{\Theta})$.

In an attempt to find unique values for $\underline{\Theta}^*$, which minimize the sum of squared residuals, one may take partial derivatives of equation 4-2 and equate to zero.

$$\text{(a)} \quad \sum_{i=1}^{N} \{c_i - f(\underline{X}_i , \underline{\Theta})\} \frac{\partial f(\underline{X}_i, \underline{\Theta})}{\partial \Theta_1} = 0 \tag{4-3a}$$

(b) $$\sum_{i=1}^{N} \{c_i - f(\underline{X}_i, \underline{\Theta})\} \frac{\partial f(\underline{X}_i, \underline{\Theta})}{\partial \Theta_2} = 0 \qquad \text{4-3b}$$

When $f(\underline{X}_i, \underline{\Theta})$ is linear in $\underline{\Theta}$ the partial derivatives are functions of $\underline{X}_i$ only and not of $\underline{\Theta}$. Thus in the linear case the normal equations are a set of k linear equations having k unknowns, and these are solvable if the matrix of coefficients is non-singular. However, in the non-linear case in point, the normal equations 4-3 have no explicit solution and must be solved by an iterative method.

Iterative Method Used

Iterative optimization techniques require an initial point Θ_o to be specified, and proceed by generating a sequence of estimates $\hat{\underline{\Theta}}$ which represent improved approximations to the solution. Several techniques are available - the trade-off usually being between simplicity and rapid convergence. The optimization method used in this study was a direct univariate search utilizing golden sections. By repeated evaluation of $S(\Theta)$ the minimum can be located to any required precision; the efficiency of the search process depends upon the algorithm used to choose points for function evaluation (see Appendix C).

The optimum sampling number (N) for estimating parameters of a stable distribution, using a consistent estimator, is equal to the population of price changes since the stock came on the market.

However, parameters of the generating process may have been altered by changing conditions (e.g., creation of Federal Reserve control over margin requirements, wage-price-dividend controls). The σ in the model is that value believed by the seller market to be in effect over the life of the call option. In order to get an initial estimate of that expected value, weekly stock price changes from January 1970 to January 1971 (excluding ex dividend weeks) were taken from "Barrons."

$$\hat{\sigma}_o^2 = \sum_{t=1}^{N} \frac{[\ln x_{t+1} - \ln x_t]^2}{N-1} \qquad 4\text{-}4$$

With an initial estimate for Θ_2 derived from weekly price changes during 1970, the feasible region for Θ_1 is systematically reduced via the Golden Section technique. (See Appendix C.)

The objective function $S(\Theta)$ is first minimized on Θ_1, then on Θ_2, then Θ_1 again, etc. The process is illustrated below:

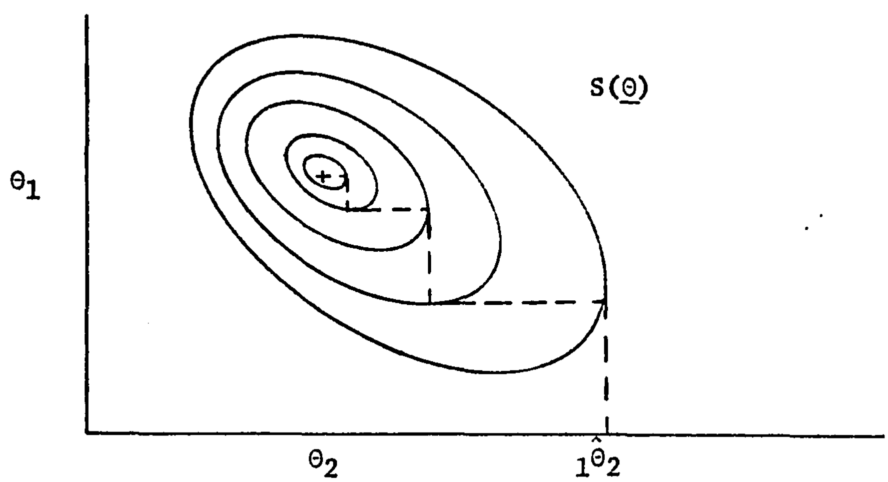

Fig. 2. Multivariable Direct Search for the Optimum of a Unimodal Function

Step 1: An initial estimate for Θ_2 is introduced $({}_1\hat{\Theta}_2)$.

Step 2: The function $S(\Theta)$ is evaluated at $S(-.5,\hat{\Theta}_2)$ and $S(-.25,\hat{\Theta}_2)$. If $S(-.5,\hat{\Theta}_2) > S(-.25,\hat{\Theta}_2)$, the function is evaluated at $S(.25,\hat{\Theta}_2)$, $S(.5,\hat{\Theta}_2)$, etc., until a test point on Θ_1 yields a larger $S(\underline{\Theta})$ than the previous test. The Θ_1 interval containing the minimum $S(\underline{\Theta})$ for a given $\hat{\Theta}_2$ is then sectioned and tested until the "probabilistic interval of uncertainty" [Wilde 1964, p. 184] for Θ_1 is less than a specified δ (in this case the δ was .0001). The best estimate of Θ_1 is saved and used in the process of minimizing $S(\underline{\Theta})$ over Θ_2.

Step 3: The function is evaluated at points $({}_1\hat{\Theta}_1,0)$ and $({}_1\hat{\Theta}_1,.01)$. If $S({}_1\hat{\Theta}_1,0) > S({}_1\hat{\Theta}_1,.01)$, the function is then evaluated at $S({}_1\hat{\Theta}_1,.02)$, $S({}_1\hat{\Theta}_1,.04)$, $S({}_1\hat{\Theta}_1,.08)$, $S({}_1\hat{\Theta}_1,.16)$, etc. until a test point on Θ_2 yields a larger $S(\underline{\Theta})$ than the previous test point. The Θ_2 interval containing the minimum $S(\underline{\Theta})$ for a given Θ_1 is then systematically sectioned and tested until the probabilistic region of uncertainty for Θ_2 is less than .0001.

Step 4: The program continues to alternately minimize $S(\Theta)$ over Θ_1 and Θ_2, using the most recent best estimate for one parameter while searching to find the new best estimate for the second parameter.[1] The entire process is halted when the change in Θ_1 over successive minimization cycles is less than .001.

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1. Response surfaces for three representative stocks were depicted graphically from several viewing angles. All computer-generated displays indicated the surfaces were unimodal and without ridges.

The following information is then printed for each of the fifty stocks included in the sample.

Θ_1^* = estimated value for r

Θ_2^* = estimated value for σ

$S(\Theta^*)$ = error sum of squares $= |C_i - \hat{C}_i|' \, |C_i - \hat{C}_i|$

$$\text{Mean error} = \frac{1}{N} \sum_{i=1}^{N} (\hat{C}_i - C_i)$$

A complete list is provided in Appendix D.

Summary

Given the call valuation model developed in Chapter 3, this chapter presented the method by which parameters were estimated. The optimization criterion used in the iterative process was the minimization of the sum of squared residuals. With the estimated parameters held constant at those optimum values, terms relating to the model's goodness-of-fit were defined.

CHAPTER 5

EVALUATING THE MODEL

Having developed the basic model $\hat{C} = f(\underline{X},\underline{\Theta})$ in Chapter 3, and a method for estimating $\underline{\Theta}$ in Chapter 4, the objective of this chapter is to present the method used to evaluate the equation's merit.

Rejection Criterion

The regression equation previously derived in Chapter 3 may be written:

$$\hat{C}_i = f(\underline{X_i},\underline{\Theta}) \qquad 5\text{-}1$$

Under the classical regression assumption that the ε are independent and $N(0,\sigma^2)$, maximizing the likelihood function

$$L(\Theta) = (2\pi\sigma^2)^{-n/2} \prod_{i=1}^{n} \exp\{-[\Sigma\{c_i - f(\underline{X_i},\underline{\Theta})\}^2/2\sigma^2]\} \qquad 5\text{-}2$$

is equivalent to minimizing the error sum of squares

$$S(\Theta) = \sum_{i=1}^{n} \{c_i - f(\underline{X_i},\underline{\Theta})\}^2 \qquad 5\text{-}3$$

As shown in the previous chapter the resulting normal equations in general have no explicit solutions when $f(\underline{X_i},\underline{\Theta})$ is non-linear in $\underline{\Theta}$.

To permit application of the well developed linear techniques for parameter estimation in the present study would require transformation of the basic equation. Also, the error term in the transformed equation must meet the $N(0,\sigma^2)$ assumptions. In any case, the estimated parameters would result in a minimum sum of squared residuals for the transformed equation, not the valuation equation which is the object of our search. As a practical matter, the equation developed in this study is not transformable into a linear relation even if the error term were assumed to be multiplicative. Therefore, an iterative method was required to estimate the parameters $(\underline{\Theta})$ -- $\underline{\Theta}^*$ being that set of estimated parameters resulting in the minimum sum of squared residuals.

The writer's nonlinear model was then compared to a competitive pricing model. Parameter estimates were determined using the same iterative procedure for both models. Having specified the optimum parameter estimates, sums of squared residuals were calculated and compared. The sample regression function having the largest error sum of squares was rejected.

Comparison Model

Having been developed from the buyer's viewpoint the Boness equation takes the form:

$$\frac{\hat{C}}{X_o} = \beta_o + \beta_1 \{P[x<(\frac{\beta_2}{s} + \frac{s}{2})t^{1/2}] - e^{-\beta_2 t} P[x<(\frac{\beta_2}{s} - \frac{s}{2})t^{1/2}]\} \qquad 5\text{-}4$$

where x = the variable of the normal probability integral,

X_o = market stock price, call striking price

t = time till option maturity in weeks

s = estimate of the standard deviation of the weekly distribution of changes in stock price logarithms

β_0, β_1, β_2 = empirically determined parameters

In order to facilitate a direct comparison, the Boness model was altered in two ways:

1. Both sides of the equation were multiplied by X_o in order to express $\hat{C}$ as a dollar figure. Note that the equation provides for three estimated parameters:

   A. An adjustment for fixed transactions costs (β_o).

   B. An adjustment for biases in supply and demand (β_1).

   C. The estimated discount rate applied by call buyers to the expected value of future cash flows (β_2).

   These parameters are independent of the stock on which the call is written. That is, identical parameter estimates are used to explain call prices on Natomas and U.S. Steel.

2. The technique for determining the three parameters discussed above is the same as that employed in Chapter 4 of this paper. That is, an iterative method was used to determine those parameter values minimizing the sum of squared residuals for the equation $C = \hat{C} + \tilde{\varepsilon}$. Boness, on the other hand, found the parameter values by minimizing the sum of squared residuals for the equation

$$\frac{C_i}{X_{oi}} = \beta_o + \beta_1 \frac{T_i}{X_{oi}} + \varepsilon_i \qquad 5\text{-}5$$

The two procedures do not, in general, result in identical parameter estimates.

Results

As may be seen in Appendix D, the error sum of squares for the competitive model was found to be 3.76 times that of the seller's model developed in this study. Also, the absolute value of the mean error was larger for the competitive model in forty-seven out of the fifty stocks examined. On the basis of the first statement, the seller's model was judged to be superior. Primary causal factors for the superior fit of the writer's model are felt to be:

1. Inclusion of foregone interest in the valuation equation. Even though the writer's risk of loss may approach zero for calls on extremely stable securities, the covered call writer requires compensation for foregone interest on the covering funds.

2. Allowance of the discount rate (r) to take on different values, depending upon the risk class of the underlying stock. Only those calls on a given stock are assumed to be in the same risk class.

3. Recognition within the valuation equation that expected future volatility may be different from any *ex post* measure over a specified time period.

The Boness equation requires three parameter estimates. Since all calls are assumed to be in the same risk class, the final sample regression function may be used to predict prices for calls on all stocks. The writer's pricing model presented in this study requires two parameter estimates--each parameter estimate being dependent on the underlying stock upon which the call is written. Calls on different stocks are not assumed to be in the same risk class.

Regarding the exercise timing of the "American" call, Boness assumed the original buyer's utility function to be such that early exercise would never be desirable. The present study demonstrates that, in a frictionless market, an "American" call has a market value which is greater than its current exercise value at all points in time over the call's stated option length. Therefore, any holder wishing to divest would choose to sell the call in the marketplace rather than exercise the option. As a result, no call would be exercised early. In the normative writer's pricing model no constraints were placed upon the buyer's utility function; however, the writer was assumed to act as if his maximizing objective were subject to a "safety-first" constraint--thus preferring a covered position to a naked position throughout the life of the call.

CHAPTER 6

CONCLUSION

Based upon the comparison of minimum sums of squared residuals, the seller's model has been shown to fit empirical data to a greater extent than does the competitive Boness model. Primary reasons for the superior fit are felt to be:

1. The writer's consideration of foregone interest on covering funds was included in the pricing model.
2. The model has the capability of pricing calls being in different risk classes.
3. The model recognizes that expected volatility may be different from any *ex post* measure of volatility.

Notably absent from the results are parameter confidence intervals, significance tests, and R^2 values--such measures are most often found in the domain of linear regression analysis. Desiring to take advantage of the well-developed tools that linear regression offers, an experimenter may be tempted to force his model into the form of a linear relation. Often a transform operator is applied to convert a nonlinear relation into a more workable linear relation. In such cases the original relation is termed intrinsically linear. The error term in the original equation is assumed to take on that form which produces in the transformed equation a serially uncorrelated error term having constant variance and zero expected value. After the transformation, best-linear-unbiased parameter estimates may be calculated using the least squares criterion. These estimates are not, in general, identical to those estimates resulting in a minimum sum of squared residuals when applying the untransformed data to the original equation.

If the regression equation is left in its original nonlinear form (because the experimenter is unwilling to make the accompanying assumptions, or because the original equation is intrinsically nonlinear) no confidence intervals are possible without postulating the functional relation of error dispersion to all regressors. To specify that relation may be stretching the model-maker's license when several regressors must be considered.

In summary, when faced with the problem of estimating parameters in an intrinsically linear equation, one may choose:

1. To transform the equation, being ready to accept whatever must be assumed in order to apply the techniques of linear regression, or
2. To leave the equation in its original form, recognizing one's inability to make rigorous, quantitative statements about model significance and confidence intervals.

When the form of the relation is intrinsically nonlinear (cannot be transformed), the experimenter has no choice but to apply the optimization technique to the original equation.

Suggestions for Further Research

The major obstacle to options research has been that of obtaining large representative samples of empirical data. This problem should be alleviated when the Chicago Board Options Exchange begins active trading in early 1973. The following areas should receive attention at that time:

Value of Segments of the Total Stock Price Distribution

The plan proposed by the Chicago Board Options Exchange calls for several striking prices in $5 intervals for options on stocks selling at $50 and below, and $10 intervals for options on stocks selling for more than $50. By comparing premiums on calls having different striking prices, one may determine the market prices of various portions of the stock price appreciation distribution. For example, consider three calls having the same expiration date and striking prices of $50, $55, and $60. These calls (100 shares) are presently selling for $290, $250, and $220 respectively. Therefore, that portion of the price appreciation distribution (for the given maturity) from $50 to $55 is worth 40¢ per share to call buyers; the portion from $55 to $60 is worth 30¢ per share.

Volume Determinants

Because not all sales of new options are included in the figures provided by the Put and Call Brokers and Dealers Association, no reliable information regarding option volume is available. Data from the Chicago Board Options Exchange should enable investigators to determine the relation of option volume to recent stock volume and price movements.

Measure of Market Imperfections

An organized options exchange will enable investors to purchase not only whole shares of equity capital stock, but individual components of the shares as well. By comparing total costs of the option combinations to the price of the whole security, friction costs may be estimated.

The model presented in this study has been developed under the assumption that the price-making mechanism operates as if no friction costs were present. Although this assumption is reasonable for the high-volume calls written on the fifty stocks examined, the absence of an active secondary market may reduce the effectiveness of the model in explaining premiums on low-volume calls. The Chicago Board Options Exchange promises to reduce friction costs significantly, add depth to the options market, and provide transaction information to the public. A specialist operating on the floor of the exchange will be required to maintain a bid-asked spread of no more than $50 per option contract (100 shares). Since bid and asked quotes will also be accepted from the investing public, the spread may often be smaller. Not only will the Board increase the efficiency of the option market by providing information and reducing friction, but will assist research as well.

APPENDIX A

DERIVATION OF THE LOG NORMAL DISTRIBUTION

Suppose that a given stock initially has price X_o and that after j trades the price is X_j, reaching its final value X_τ after τ steps. At the jth step the change in the stock price is a random proportion of the value already attained; thus[1]

$$X_j - X_{j-1} = \varepsilon_j X_{j-1} \qquad \text{A-1}$$

where the set $\{\varepsilon_j\}$ is mutually independent, has zero mean and finite variance, and is also independent of the set $\{X_j\}$.

Rewriting A-1,

$$\frac{X_j - X_{j-1}}{X_{j-1}} = \varepsilon_j \qquad \text{A-2}$$

Summing over τ steps,

$$\sum_{j=1}^{\tau} \frac{X_j - X_{j-1}}{X_{j-1}} = \sum_{j=1}^{\tau} \varepsilon_j \qquad \text{A-3}$$

If each is small,

$$\sum_{j=1}^{\tau} \frac{X_j - X_{j-1}}{X_{j-1}} \simeq \int_{X_o}^{X_\tau} \frac{dx}{X} = \ln X_\tau - \ln X_o \qquad \text{A-4}$$

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1. The content of this appendix is based upon the exposition of J. Aitchison and J.A.C. Brown, The Log Normal Distribution. Cambridge: Cambridge University Press, 1963.

yielding

$$\ln X_\tau - \ln X_o = \varepsilon_1 + \varepsilon_2 + \ldots + \varepsilon_\tau \qquad \text{A-5}$$

By the central limit theorem $\ln [X_\tau/X_o]$ is distributed normally with mean μ and variance σ^2.

$$\mu = E[\ln X_\tau - \ln X_o] = E[\varepsilon_1 + \varepsilon_2 + \ldots + \varepsilon_\tau] = 0 \qquad \text{A-6}$$

$$\sigma = \sqrt{E(\varepsilon_1 + \varepsilon_2 + \ldots \varepsilon_\tau)^2 - [E(\varepsilon_1 + \varepsilon_2 + \ldots + \varepsilon_\tau)]^2} \qquad \text{A-7}$$

$$= \sqrt{\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_\tau^2}$$

Assuming that each change in the log of prices $[\ln(X_j/X_{j-1}) = \ln X_j - \ln X_{j-1}]$ has the same dispersion σ_j,

$$\sigma = \sqrt{\tau\sigma_j^2} = \sigma_j\sqrt{\tau} \qquad \text{A-8}$$

That is, the dispersion in the probability distribution of an individual stock's price increases as the square root of the number of transactions.

So far the discussion of stock price distribution has been in terms of the number of trades. To move to the chronological time domain an assumption regarding the relationship between time and trading volume must be made. A reasonable assumption is that trades are spread fairly evenly over time. (See Osborne 1959, pp. 145-173.) Alternatively, one can by-pass the statistical derivation relating trades and the price distribution, and simply postulate that prices are log normally distributed over chronological time, trading occurrences in round lots, or a combination of both. The former approach seems more rigorous.

If volume is spread out evenly over time [i.e., τ = K• unit time] then the dispersion of a stock's price for a specific point in time in the future increases as the square root of the multiple of time units. In other words, if it is known that the variance of the logs of weekly prices is σ^2, the variance in the logs of prices from t_o to 25 weeks in the future is $5\sigma^2$.

APPENDIX B

SOURCES OF REGRESSION DATA

The hypothesis is that the sellers call premium can be explained by the equation:

$$C_i = 100 \{Xo[1-e^{-\rho T}] + \overline{D}e^{-\rho T} - \sum_{i=1}^{N} \overline{d}_i \, e^{-\rho t_i} \qquad \text{B-1}$$

$$+ e^{-rT} [(Xo - \overline{D}) \, N(A;0,1) - Xo$$

$$\exp \frac{\sigma^2 \tau}{2} \quad N(B;0,1)]\} + \tilde{\varepsilon}_i$$

where C_i = The seller's call premium

Xo = The price of the covering stock at t_o

ρ = The risk free interest rate

T = The duration of the call option in years

τ = The duration of the call option in weeks

r = The discount rate for the equivalent "European" short put held by the option writer

$\overline{d}_i$ = Dividends expected to be paid during the option period

t_i = The length of time in years till expected dividend payment d_i

$\bar{D}$ = Sum of dividends for which the underlying stock goes ex dividend during the option period

$\tilde{\varepsilon}_i$ = The error term

σ^2 = The variance of the logs of consecutive week-end stock prices

N(A;0,1), N(B;0,1) = Cumulative normal distributions from $-\infty$ to A or B, respectively

$$A = \frac{\ln (X_o - \bar{D}) - \ln X_o}{\sigma\sqrt{\tau}}$$

$$B = \frac{\ln (X_o - \bar{D}) - \ln X_o - \sigma^2\tau}{\sigma\sqrt{\tau}} = A - \sigma\sqrt{\tau}$$

In equation B-1 all but two terms (parameters r, σ) on the right hand side can be verified in

1. The Federal Reserve Bulletin
2. Moody's Dividend Record
3. Barrons
4. Lombard Street Records

A. Option Terms

All data defining option terms was provided by Lombard Street Incorporated, a New York put and call house, which has been in existence since 1968. Through their facilities, both the buying and selling sides of more than 4,000 put and call transactions were obtained. Basic data for each transaction included:

1. The symbol of the optioned stock
2. The Seller's premium (C_i)
3. The striking price (X_o)
4. The trade and expiration dates

Although there were more than 4,000 data points in all, total call options amounting to fifteen or more occurred in the cases of only fifty stocks. This result was expected in light of the S.E.C.'s earlier report that "almost half of the option volume was in 50 stocks" (S.E.C. 1961, p. 5).

The options on these fifty stocks were used as the basic sample for the valuation portion of this study. The total number of calls was 1132.

An individual call was excluded if its exercisable price did not fall within the range of actual prices quoted for the optioned stock on the day of the option transaction. If the striking price fell within the range of quoted prices, it was assumed that market price equaled striking price at the time of the transaction.

B. The Amount and Timing of Dividends ($\bar{d}_i$, $\bar{D}$, t_i)

The model incorporates an expected value for dividends to be paid during the option life. It is true, of course, that the dividend amount is not certain, nor for that matter is the dividend payment date. Given any specific functional relationship between the dividend amount and stock price, one could replace $\bar{d}$ with the integrated density function.

Lacking knowledge of any specific functional relationship, the expected dividend amount used in the model was equal to the most recent dividend paid; the next dividend payment (t_1) was assumed to be three months, six months, or a year from the most recent dividend payment date depending upon the interval between the last two consecutive dividend payments. Data was taken from Moody's Dividend Record.

C. The Riskless Rate of Return (ρ)

Adjusted Treasury Bill rates for three months, six months, nine months, or one year were used depending upon the option length. Data was taken from the Federal Reserve Bulletin.

APPENDIX C

REDUCTION OF PARAMETER SPACE

The direct search technique used in this study maximized an objective criterion by sequentially determining minimum values of $S(\underline{\Theta})$ in each dimension of $\underline{\Theta}$, utilizing all previously determined best estimates in all other dimensions of $\underline{\Theta}$. That is, while minimizing $S(\underline{\Theta})$ over Θ_1, the experimenter treats parameters Θ_2, Θ_3 . . . . Θ_N as known constants $\hat{\Theta}_2$, $\hat{\Theta}_3$ . . . $\hat{\Theta}_N$.

Under the assumption that $S(\underline{\Theta})$ is unimodal in N-space, the problem is to choose values of Θ_1, belonging to feasible region R, in such a manner as to systematically converge on Θ_1^* yielding the minimum S. Referring to Figure 3 below,

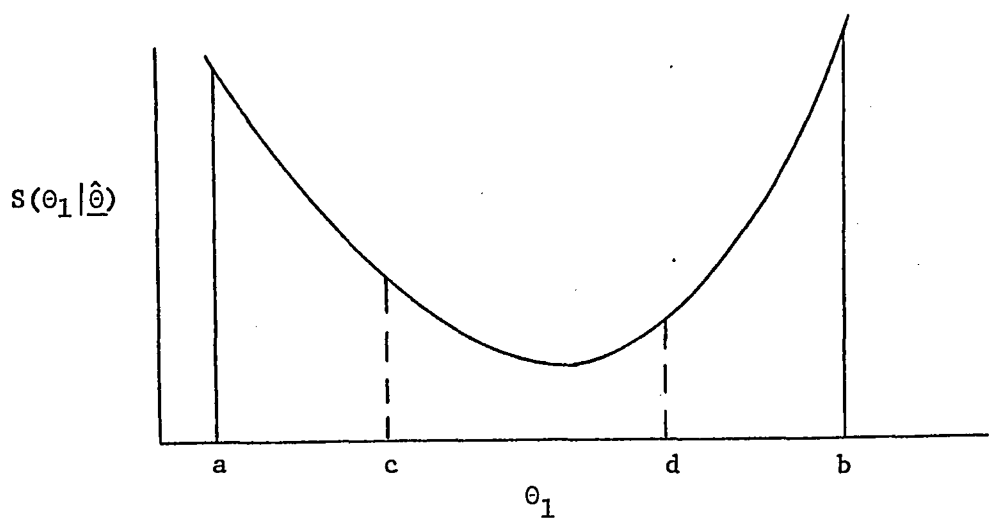


Fig. 3. The Discrepancy Function as Related to Θ_1, Given Estimates of All Other Parameters.

$S(\Theta_1|\underline{\Theta})$ represents the value of the function $S(\Theta_1)$ treating all other estimated parameters as known constants.

Having determined that Θ_1^* lies within the region defined by (a,b), the feasible region may be reduced by evaluating the function S at two interior points (see Kowalik and Osborne 1968, p. 10). If the function is evaluated at the interior point c, then the minimum of S can lie to either side of the point. To determine the interval within which Θ_1^* lies, another evaluation is necessary. Let $a < c < d < b$; then if $S(c) > S(d)$ the minimum lies in (c,b), otherwise in (a,d).

By repeated evaluation of S the minimum can be located to any required precision, the efficiency of attaining this precision depends upon the choice of interior points at which S is evaluated. One method of choosing interior points is by the "Golden Section" technique. This search procedure title is derived from the name given by ancient geometers to the division of an interval into two segments. The division is such that the ratio of the whole interval to the larger segment is the same as the ratio of the larger segment to the smaller segment. This ratio can be shown to be $(1 + \sqrt{5})/2$ (Box 1969, p. 12). Note that by positioning (in Figure 3) point c that distance from b such that $\frac{1}{\gamma}(ab) = ad = bc$ the feasible region is reduced to $\frac{1}{\gamma}$ times its former size with each new evaluation at specified interior points. Thus, in N function evaluations (N-1 iterations since the first iteration requires 2 function evaluations) search by Golden Section reduces the original interval by the factor $1/\gamma^{N-1}$. Because three of the four evaluations from the previous iteration may be used, each new iteration requires only one new evaluation of $S(\Theta)$.

The process may be illustrated as follows:

Initial conditions:

Feasible region = $x_2 - x_1$

Step 1: Evaluate at x_3 and x_4;

choose interval $x_4 - x_1$

Step 2: $x_4 - x_1 = \frac{1}{\gamma}(x_2 - x_1)$

Evaluate at x_5; choose interval

$x_3 - x_1$

Step 3: $x_3 - x_1 = \frac{1}{\gamma^2}(x_2 - x_1)$

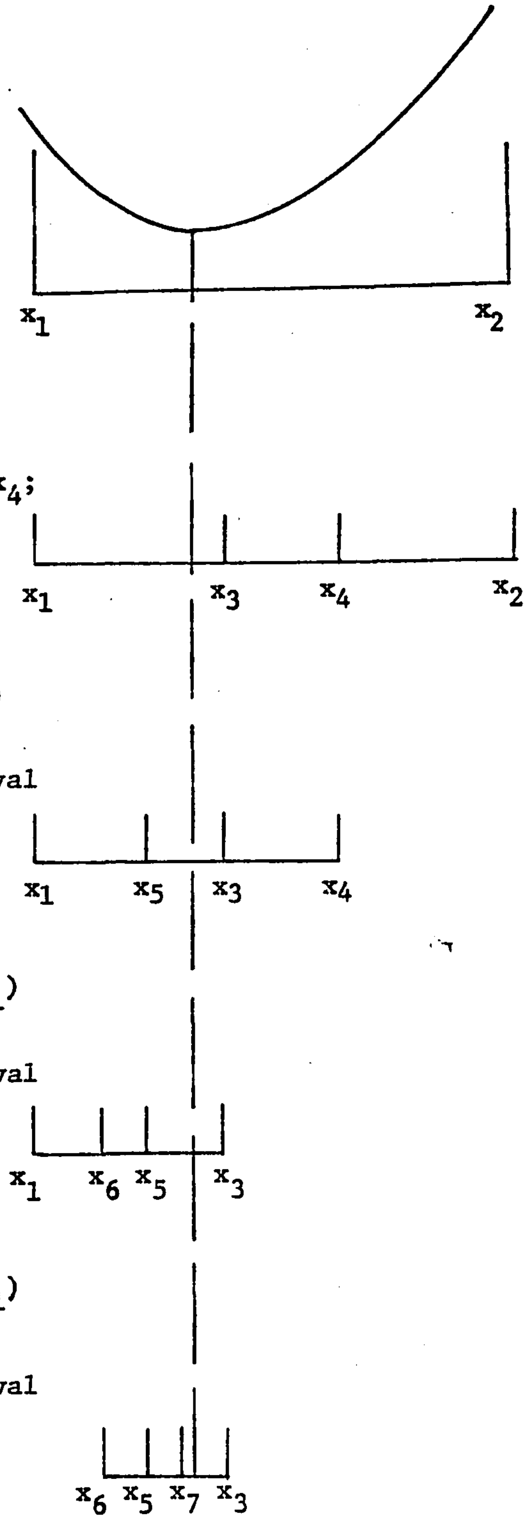
Evaluate at x_6; choose interval

$x_3 - x_6$

Step 4: $x_3 - x_6 = \frac{1}{\gamma^3}(x_2 - x_1)$

Evaluate at x_7; choose interval

$x_3 - x_5$



etc. . . . .

APPENDIX D

SUMMARY DATA FOR COMPARISON OF BUYER'S AND SELLER'S MODELS

Given functional forms for the competing equations, this section describes the quantitative measures used in determining which equation represents the better explanation of empirical data.

Employing the direct, iterative search procedure outlined in Chapter 4, optimum parameter estimates were determined for both sample regression functions. These estimates are listed in this appendix as Final Estimate of σ and Final Estimate of r for the seller's model and β_o^*, β_1^*, β_2^* for the buyer's model. The Initial Estimate of σ was used in the seller's model as an initial estimate of the writer's expectation for the standard deviation of consecutive stock price logarithms over the life of the call; the same value was used in the buyer's model as a fixed estimate of the log standard deviation.

Applying the sample regression functions to empirical data, Mear Errors and sums of squared residuals, $S_s(\Theta^*)$ and $S_b(\Theta^*)$, were calculated. The Number of Data Points used in estimating parameters for each writer's sample regression function is listed; the same data were used to estimate parameters for the buyer's model. The total overall sum of squared residuals was 1.853×10^6 for the seller's model and 6.979×10^6 for the buyer's model.

Table 1

Summary Data for Comparison of
Buyer's and Seller's Models

	Seller's Model (C_s)				Buyer's Model (C_b)			
Stock	Final Est. of σ	Final Est. of r	Mean Error	$S_s(\theta^*)$	Number of Data Points	Initial Est. of σ	Mean Error	$S_b(\theta^*)$
1. American Motors	.2050 E-1	-.2774 E0	-.2028 E1	.7818 E4	19	.0701	-.2202 E2	.1796 E5
2. American Telephone and Telegraph	.3228 E-2	-.3779 E0	.5651 E1	.3904 E5	16	.0799	.3078 E3	.1591 E7
3. Ampex Corporation	.2411 E-1	-.2111 E0	-.1429 E2	.4352 E5	16	.0832	-.4699 E1	.4418 E5
4. Beverly Enterprises	.5463 E-1	-.2503 E0	-.1196 E1	.6047 E4	23	.1124	-.4835 E2	.8457 E5
5. Boeing Company	.9874 E-2	-.3793 E0	.4727 E-1	.2144 E5	22	.0769	-.2263 E2	.6440 E5

Table 1, Continued. Summary Data

	Stock	Final Est. of σ	Final Est. of r	Mean Error	$S_s(\Theta^*)$	Number of Data Points	Initial Est. of σ	Mean Error	$S_b(\Theta^*)$
6.	California Computer Products	.4627 E-1	-.2152 E0	-.2861 E1	.2502 E5	27	.1020	-.6050 E2	.1845 E6
7.	Chrysler Corporation	.1942 E-1	-.2325 E0	-.1606 E1	.3840 E5	40	.0626	-.5463 E2	.1987 E6
8.	City Investing Co.	.4614 E-1	-.1089 E0	-.2486 E1	.2127 E5	26	.1390	.1627 E2	.4193 E5
9.	Clark Oil & Refining	.7439 E-1	.1074 E0	-.4771 E1	.1407 E5	16	.0736	.1376 E2	.1476 E5
10.	Computer Sciences Corp.	.9429 E-1	-.3621 E-01	-.7168 E0	.8789 E4	30	.1065	-.5049 E2	.9929 E5
11.	Data Processing Financial and General Corp.	.5657 E-1	-.1177 E0	-.1953 E2	.4329 E5	19	.1264	-.4674 E2	.5808 E5
12.	Data Products Corporation	.7496 E-1	.8158 E-2	-.9802 E1	.1816 E5	16	.1355	-.3781 E2	.3764 E5

Table 1, Continued. Summary Data

	Stock	Final Est. of σ	Final Est. of r	Mean Error	$S_s(\theta^*)$	Number of Data Points	Initial Est. of σ	Mean Error	$S_b(\theta^*)$
13.	Dixilyn Corporation	.7547 E-1	-.6811 E-1	-.1950 E1	.1982 E5	27	.0944	-.4255 E2	.8508 E5
14.	Eastern Air Lines	.1756 E-1	-.2876 E0	-.6203 E0	.6226 E4	16	.0695	-.2580 E2	.3481 E5
15.	Ecological Sciences	.4505 E-1	-.2207 E0	-.1000 E1	.5842 E4	14	.0860	-.2544 E2	.3042 E5
16.	Equity Funding Corporation	.7620 E-1	.2969 E-1	-.2800 E1	.2003 E5	24	.1026	-.1134 E2	.5016 E5
17.	Fairchild Camera Corp.	.4963 E-1	-.2235 E0	-.5641 E1	.3669 E5	19	.0967	-.6919 E2	.1803 E6
18.	Fleetwood Enterprises	.3533 E-1	-.2257 E0	-.5264 E1	.3756 E5	19	.0761	-.4773 E2	.1185 E6
19.	Flying Tiger Corporation	.7523 E-1	.4551 E-1	-.5721 E0	.1090 E6	52	.0748	.1785 E2	.2171 E6

Table 1, Continued. Summary Data

	Stock	Final Est. of σ	Final Est. of r	Mean Error	$S_s(\theta^*)$	Number of Data Points	Initial Est. of σ	Mean Error	$S_b(\theta^*)$
20.	Gulf Oil	.1861	-.2048	-.6770	.2553	25	.0440	.9992	.2716
	Corporation	E-1	E0	E0	E5			E2	E6
21.	International	.5925	-.1736	-.6546	.2640	28	.1423	-.4972	.1041
	Industries	E-1	E0	E1	E5			E2	E6
22.	Gulf &	.1688	-.2957	-.1424	.1125	15	.0752	.7859	.2093
	Western	E-1	E0	E1	E5			E-1	E5
23.	Kentucky	.8544	.5886	.7094	.3088	19	.0950	-.3655	.3161
	Fried Chicken	E-1	E-2	E0	E4			E2	E5
24.	Leasco Data	.8991	.1192	-.8743	.2382	17	.1723	-.2823	.2897
	Products	E-1	E0	E1	E5			E2	E5
25.	Litton	.7216	.4700	-.4054	.3355	21	.0737	.8262	.5289
	Industries	E-1	E-1	E1	E5			E0	E5
26.	Lockheed	.4914	-.1825	-.1094	.4492	27	.0908	-.4931	.9576
	Aircraft	E-1	E0	E2	E5			E2	E5

Table 1, Continued. Summary Data

	Stock	Final Est. of σ	Final Est. of r	Mean Error	$S_s(\Theta^*)$	Number of Data Points	Initial Est. of σ	Mean Error	$S_b(\Theta^*)$
27.	Marshall	.9095	.2901	-.9761	.2392	15	.1062	-.7360	.1133
	Industries	E-1	E-1	E1	E5			E2	E6
28.	McCulloch	.3815	-.1593	-.5948	.6745	42	.0771	.3386	.2968
	Oil Corp.	E-1	E0	E1	E5			E2	E6
29.	Memorex	.7494	-.0900	-.2491	.2737	15	.1101	-.1052	.5200
	Corporation	E-1	E-01	E1	E6			E2	E6
30.	Molybdenum	.6996	.7720	.3893	.2368	16	.0701	-.3817	.4400
	Corporation	E-1	E-2	E0	E5			E2	E5
31.	Natomas	.1255	.1543	-.5944	.1984	18	.1278	-.1361	.5409
	Company	E0	E0	E1	E6			E2	E6
32.	Northwest	.3634	-.1428	-.2598	.1625	19	.0738	.3029	.6974
	Airlines	E-1	E0	E1	E5			E2	E5
33.	Occidental	.1992	-.3147	-.7993	.3375	56	.3028	.1122	.1067
	Petroleum	E-1	E0	E0	E5			E2	E6

Table 1, Continued. Summary Data

	Stock	Final Est. of σ	Final Est. of r	Mean Error	$S_s(\Theta^*)$	Number of Data Points	Initial Est. of σ	Mean Error	$S_b(\Theta^*)$
34.	Phillips Petroleum	.1425 E-1	.2778 E0	-.3109 E1	.4751 E5	18	.0477	.4828 E2	.7901 E5
35.	Reynolds Metals	.9425 E-2	-.3063 E0	-.7230 E0	.3024 E5	16	.0544	.4322 E2	.7945 E5
36.	Saxon Industries	.2510 E-1	-.3017 E0	.6422 E1	.7944 E5	26	.1624	-.4220 E2	.2848 E6
37.	SCM Corporation	.6186 E-1	-.3334 E-1	-.1656 E0	.5965 E4	24	.0619	-.3943 E1	.7163 E4
38.	Solitron Devices	.6898 E-1	-.1337 E0	.1132 E1	.1145 E5	13	.1055	-.6567 E2	.7466 E5
39.	Sperry Rand Corporation	.1518 E-1	-.2381 E0	-.1122 E2	.5546 E5	15	.0611	.4147 E2	.9183 E5
40.	Superscope Incorporated	.6375 E-1	-.1114 E0	-.1060 E2	.3467 E5	16	.1082	-.4965 E2	.7299 E6

Table 1, Continued. Summary Data

	Stock	Final Est. of σ	Final Est. of r	Mean Error	$S_s(\Theta^*)$	Number of Data Points	Initial Est. of σ	Mean Error	$S_b(\Theta^*)$
41.	Syntex	.1504	-.2885	.5888	.3401	16	.0799	.3564	.1831
	Corporation	E-1	E0	E1	E5			E2	E6
42.	Teledyne	.3610	-.1780	-.2960	.2253	28	.0749	.8985	.4452
	Incorporated	E-1	E0	E1	E5			E0	E5
43.	Telex	.8917	.4194	-.2895	.1236	23	.2591	-.2704	.3246
	Corporation	E-1	E-1	E0	E5			E2	E5
44.	Texas Gulf	.2444	-.2609	-.9018	.2204	33	.0653	-.1304	.4220
	Sulphur Co.	E-1	E0	E0	E5			E2	E5
45.	Tool Research	.7198	.2189	-.3127	.1077	18	.0792	-.5021	.5743
	& Engineering	E-1	E-1	E1	E5			E2	E5
46.	Transamerica	.6963	.9645	-.3846	.1250	14	.0697	.2104	.1473
	Corporation	E-1	E-1	E1	E5			E2	E5
47.	Twentieth	.6564	-.1802	-.2706	.8515	16	.0899	-.1970	.1905
	Century Fox	E-1	E-1	E1	E4			E2	E5

Table 1, Continued. Summary Data

	Stock	Final Est. of σ	Final Est. of r	Mean Error	$S_s(\Theta^*)$	Number of Data Points	Initial Est. of σ	Mean Error	$S_b(\Theta^*)$
48.	Universal	.2288	-.2405	.6033	.8853	16	.0673	.2560	.2660
	Oil Products	E-1	E0	E0	E4			E2	E5
49.	University	.7080	-.1304	-.6453	.1275	48	.1255	-.5989	.3745
	Computing Co.	E-1	E0	E0	E6			E2	E6
50.	Vikoa	.1083	.2469	-.4235	.1427	18	.1083	-.4929	.4413
	Incorporated	E0	E-1	E0	E4			E2	E5
51.	All Stocks				.1853 E7	1132			.6979 E7

Buyer's model estimated parameters are: $\beta_o^* = .06605$

$\beta_1^* = .1400$

$\beta_2^* = 1.3980$

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