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AN EXAMINATION OF NON-SYMMETRICAL DISTRIBUTIONS OF RETURNS: THE CASE OF CONVERTIBLE BONDS

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Alan William Frankle

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GRADUATE COLLEGE

I hereby recommend that this dissertation prepared under my direction by Alan William Frankle

entitled An Examination of Non-Symmetrical Distributions of Returns: The Case of Convertible Bonds

be accepted as fulfilling the dissertation requirement of the degree of Doctor of Philosophy

Clark A Hawkins — March 11, 1974

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*This approval and acceptance is contingent on the candidate's adequate performance and defense of this dissertation at the final oral examination. The inclusion of this sheet bound into the library copy of the dissertation is evidence of satisfactory performance at the final examination.

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SIGNED: Alan W. Frankle

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My entire dissertation committee is owed a great debt as they were always available to offer comments and guidance. Special thanks belongs to Clark A. Hawkins, my dissertation chairman who offered encouragement and ideas, especially in the organization of the study. Many of his comments will be remembered long after this work is completed. Jean Weber was a constant source of constructive criticism in the areas concerning method of analyses and their interpretations. Her promptness and detail are appreciated and the dissertation benefitted greatly as a result of her efforts. John Emery provided insight into many of the theoretical areas on which the study came in contact. Also, he constantly attempted to make me state ideas as clearly as possible. John Buehler is thanked for his willingness to be a reader.

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ABSTRACT

This work develops an approach to evaluating convertible bonds and convertible bond premiums. The theoretical basis of the investigation is formed around the framework of the capital asset pricing model. This theory provides the background necessary to derive a truncated type bivariate normal distribution function which enables the study to relate convertible bond investments to the market portfolio. It is hypothesized that investors make convertible bond investment decisions using the product-moments generated by this bivariate distribution as proxies for risk and return. Such investment decisions determine the price of convertible bonds in the market, as well as their premiums. The premium on a convertible bond is defined as the difference between the market price and the greater of the straight bond value or conversion value. Empirical investigations follow, determining the variation of convertible bond premiums and descriptive characteristics of convertible bond prices.

Multiple regression is well suited for the analysis of variation in convertible bond premiums. It is hypothesized that premium value is a function of the product-moments generated from this study's model, and the income difference between the convertible bond coupon and the

dividends paid on the shares of common stock stated in the conversion ratio.

Data are collected for sixty-six convertible bonds for the time period January, 1968 to December, 1972. Monthly observations were gathered for convertible bond prices, straight bond values, and conversion values. For each convertible bond at each point in time, the data serve as input for the model's product-moment generator as well as providing information to calculate the premium and the income difference.

The analysis first examines the data cross-sectionally for each time period. This investigation stratifies the sample into two groups. The first group contains all observations where the conversion value is greater than the straight bond value; the second, all observations where the straight bond value is greater than the conversion value. In addition to the cross-sectional analysis, the data are pooled and relationships are examined by use of a multiple regression equation on the total data. The results appear to support the product-moment approach for explaining convertible bond premiums; however, the model performs better in cases where the conversion value is greater than the straight bond value.

Capital asset pricing theory postulates that the pricing of any asset depends upon the risk-return relationships of that asset with the total portfolio of assets. The results of this work support the theory embodied in the initial hypothesis that the described independent variables are reasonable surrogates for the risk-return relationships that exist between convertible bonds and the market portfolio.

CHAPTER 1

INTRODUCTION

Convertible bonds and their attractiveness to investors are the concerns of this study. These securities are unique in that they possess both equity and debt characteristics. That is, an investor may purchase a convertible bond and partake in a firm's equity growth while retaining the security of a debt instrument. Currently, convertible debentures have received much attention as a possible double hedge against inflation and deflation. According to Diller (1973),

> To supply this putative hedge convertibles do not merely combine stocks and bonds in a package whose yield is the average of the component yields; such packages can be assembled by investors, themselves, or purchased from packages, like investment companies. Rather, convertibles purport to offer the better of both worlds--in an inflation, at least the equity value, and in a deflation, at least the debt value (p. 1).

The primary question therefore is the value of such a security. Various models have been presented to explain the valuation of these securities. Until recently, however, the attempts did not include capital market equilibrium. The purpose of this study is to expand existing convertible bond theory to include capital market theory and to use this model to predict premiums for convertible bonds.

Statement of the Problem

Since convertible debentures have extential benefits of both debt and equity concurrently, cost of ownership is greater than for either the debt or equity equivalent held separately. The cost beyond the debt or equity equivalent is the premium paid for a convertible bond. The premium is defined as the difference between the market price of the convertible bond and the greater of the common stock conversion value or straight debt value.

The problem examined in this study is twofold: (1) which characteristics of the convertible bond influence the size of the premium; and (2) how does the addition of the premium to the theoretical mean value of the convertible bond alter the distribution of actual convertible bond prices from the theoretically derived distribution in this study.

By examining the relationship of each convertible bond with the market portfolio of common stocks, this study attempts to improve existing theory. Using as variables the product moments of the joint distribution of convertible bond and market portfolio log price relatives, the model attempts to explain the variation of premiums across the bond sample. Past models have been naive, using as variables the ratio of the common stock conversion value to the straight bond value, and this ratio squared.

The second point of inspection is comparing the actual log price relative distributions of each firm's common stock, convertible bond, and straight bond over the period of study. This investigation examines the benefits and liabilities of owning each security by analyzing their log price relative distributions.

Organization

This study is separated into five chapters and appendices. The five chapters and their content are as follows: Chapter 1 describes the area of study and states the purpose of the research. Chapter 2 reviews the significant literature in areas directly related to this paper. Chapter 3 derives the theoretical model and describes the necessary assumptions for its implementation. Chapter 4 presents the methodology used for the empirical tests which relate the actual data to the theoretical model. Chapter 5 summarizes the results of the study and discusses conclusions and implications for further research. The appendices include derivations, data samples, and regression results which are too extensive to include in the main body of the study.

CHAPTER 2

REVIEW OF THE LITERATURE

Research in the area of convertible bonds has progressed from a discounted returns model to complex valuation models to the integration of convertible debentures with capital market equilibrium. Since the outcome of owning a convertible bond is uncertain, the researcher is led to hypothesize the distribution of expected returns of the investment as well as the investor's utility function. Also, all securities involve the equilibrium of the capital market, for which necessary assumptions must be postulated.

This chapter reviews endeavors in the areas which directly relate to convertible bonds. The scope of the review will include writings on convertible bonds, utility and stock prices distribution theory, and capital market theory.

Utility Theory

Decisions arise when one encounters two or more alternative courses of action in any given situation. Rarely are the outcomes of the alternatives known with certainty. The classical economic model of investment assumes certainty has been extended to deal with situations in which outcomes are not known with certainty. Two models of decision making which discard certainty are (1) decision making with objective probabilities, or risk conditions and (2) decision making with subjective probabilities, or uncertainty conditions. Many scholars think it important to distinguish between risk and uncertainty. An example frequently used to present a risk situation is the lottery ticket, where the number of tickets and the prize list are known. In this instance the objective probability distribution and relative frequency of outcomes are known. Uncertainty, in contrast, may be observed in an experiment that is unique, where there exists no repetitive trials and results. Such situations result in persons developing subjective probability distributions for the event. Investment decisions are of this type, while lottery situations have objective probability distributions of possible outcomes.

However, with the acceptance of Baysian statistics, the differences which distinguish risk and uncertainty are not due to mathematical characteristics. Therefore, in this paper the terms risk and uncertainty will be interchangeable.

If the use of subjective probability distributions is accepted, an objective criterion to evaluate the rational man's subjective risk preferences must be ascertained. Utility theory provides many functions which may be used to approximate the behavior of a rational man under certain conditions, but none are either completely satisfactory or general. A major assumption upon which these theories are built defines rational behavior as a man attempting to maximize the expected utility of his decision. Expected utility is defined as the sum of the utilities of each component of an uncertain situation weighted by the probability of that component being the actual outcome. The difference between maximizing expected utility and maximizing expected value was first discussed by Bernoulli (1738, translated 1954) in his famous Saint Petersburg Parodox. Bernoulli's solution is that gamblers should not always evaluate bets on the mathematical expectations of winnings, but should evaluate them on the mathematical expectations of the utilities of winnings. The crux of Bernoulli's hypothesis is "since there is no reason to assume that of two persons encountering identical risks, either should have his desires more closely fulfilled, the risks encountered by each must be deemed equal in value." Thus, if two persons encounter the same bet, one may accept it while the other may reject it thereby signifying that the value of the bet is not determined by the mathematical expected value alone but by the expected utility the bet will yield. Bernoulli established a definite function for his utility theory concluding that the utility which results from a small increase in wealth usually is proportional to the quantity of goods previously possessed. This may be shown as:

$$dU = k \frac{dx}{x} \tag{2.1}$$

where dU is the increment in utility resulting from an increment dx of wealth and k is a constant. Hence, total utility is a logarithmic function of wealth,

$$U = k \log \frac{x}{c} \tag{2.2}$$

where c is the amount of wealth necessary for existence.

Stigler (1950a, 1950b) points out that Bernoulli's logarithmic function is intuitively appealing as it parallels the Weber-Fechner hypothesis that a just noticeable increment to any stimulus is proportional to the stimulus.

Early portfolio theorists such as Markowitz (1952) and Tobin (1958) assume the investor is willing to base his decision on a utility function of two parameters which correspond with the first two moments of the subjective probability distribution of expected returns. It is only under this assumption that the evaluation of uncertain outcomes may focus on the first two moments of the probability distribution. One such utility function is a quadratic utility function which is designated:

$$U = a + br - cr^2 \tag{2.3}$$

This quadratic utility function has been criticized for having a limited range of applicability (a to b) as shown in Figure 2.1.

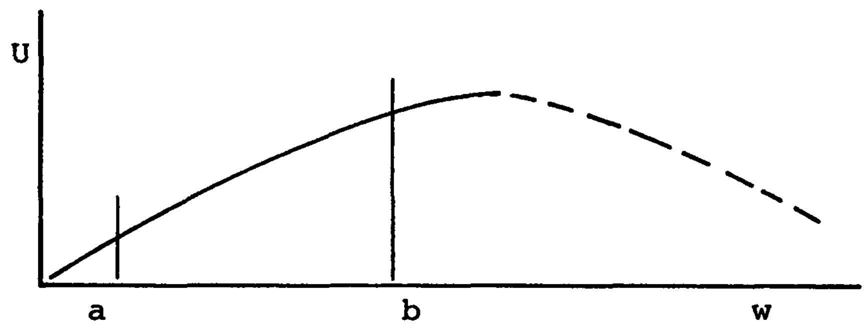

Figure 2.1. Quadratic Utility Curve

Another criticism of the quadratic utility curve is the implication of increasing absolute risk aversion as wealth increases. Also, the quadratic utility function signifies that the corresponding probability distribution of uncertain outcomes are adequately described by the first two moments. Such an assumption may not be realistic, although in some cases the model predicts well.

Investments which are represented by probability distributions possessing upper moments cannot in most cases be evaluated by a utility function that is quadratic. Richter (1960) states that when the invested maximizes expected utility and his portfolio preferences can be described in the first n portfolio income moments, then his utility is an nth degree polynomial in income.

Thus, when one seeks to maximize expected utility as an investment standard the order of moments may be restricted in two ways. The restriction may be upon the investors' utility function or upon the investment's probability distribution of expected return. For example, probability distributions such as the Poisson expresses all upper moments in terms of the first moment, mean return. It is not consistent, however, to state that an investor facing such a distribution of expected returns will seek to maximize the mean return.

Polynomials present yet another problem when representing a utility function. According to Arrow (1965), the risk-adverse individual should have the following fundamental characteristics:

1. $U'(y) > 0$, i.e., marginal utility of wealth is positive.
2. $U''(y) < 0$, i.e., marginal utility of wealth decreases with an increase of wealth.
3. $d[-U''(y)/U'(y)]/dy \leq 0$, i.e., marginal absolute risk-aversion should, if anything, decrease with an increase in wealth.
4. $d[-yU''(y)/U'(y)]/dy \geq 0$, i.e., marginal relative risk-aversion should if anything, increase with an increase in wealth.

Utility functions which are expressed in the polynomial form cannot satisfy these requirements concurrently, since at some point absolute utility begins to fall. There are, however, utility functions such as the negative exponential function $U(y) = (1-e^{-xy})$ and the family of constant elasticity utility functions $[1/(1-a)]\ y^{1-a}$, $(a>0)$, and $U(y) = \log y$ which satisfy Arrow's four conditions almost completely (Tsiang, 1972). Although these functions are not polynomials they can be transformed to fit into Richter's analysis. As long as the functions are continuous and have derivatives they can be expanded by a Taylor series. Taylor's series is a converging series of the following form:

$$U(y) = U(\bar{y} + h) = U(\bar{y}) + U'(\bar{y})h + U''(\bar{y})\frac{h^2}{2!} + U'''(\bar{y})\frac{h^3}{3!} + \ldots + U^{(n-1)}(\bar{y})\frac{h^{n-1}}{(n-1)!} + Rn \tag{2.4}$$

where Rn is the remainder and is equal to $U^{(n)}(y + \Sigma h)\ h^n/(n!)$, $(0 < \Sigma < 1)$. Thus, the utility function once expanded by the Taylor series becomes a polynomial in h. Expected utility then may be written as:

$$E[U(y)] = \int_{-\infty}^{\infty} U(\bar{y} + h)\ f(h)\ dh \tag{2.5a}$$

$$= U(\bar{y}) + U''(\bar{y}) \frac{\bar{m}_2}{2!} + U'''(\bar{y}) \frac{\bar{m}_3}{3!}$$

$$+ \ldots + U^{(n-1)}(\bar{y}) \frac{\bar{m}_{n-1}}{(n-1)!}$$

$$+ \frac{1}{N!} E[U^{(n)}(\bar{y} + \Sigma h)h^n], \quad (0 < \Sigma < 1) \tag{2.5b}$$

where f(h) is the density function of h, the deviation of y from $\bar{y}$, and $\bar{m}_2$, $\bar{m}_3$, ..., $\bar{m}_{n-1}$ are the second, third, and higher central moments of the distribution. Since the Tayl Taylor series is a converging series, the remainder will approach zero and an approximation of the function is the first (n-1) central moments. The number of moments needed to adequately describe the investor's utility function centers on speed of the series convergence which determines how many moments are necessary to approximate the utility function. Conventional portfolio theory has assumed the first two moments to be adequate. This paper explores a situation where a minimum of the first three moments are needed to adequately describe rational behavior.

To be consistent in comparing utility functions of investors with expected probability distributions of expected income from investments, the probability distributions of expected income must be established. To derive the probability distribution of expected returns for a convertible debenture, it is necessary to examine the corresponding stock's probability distribution of expected returns.

Distribution Theory

Many theories of common stock prices have been postulated, most of which have attempted to answer two questions. The questions are: (1) Does the current price "fully reflect" all available information, and (2) if successive price changes or returns are assumed to be identically distributed, which distribution is utilized.

The first to attempt to answer these questions was Bachelier (1967) in 1900, who suggested that future stock prices were independent of past price movements. Using arithmetic differences of stock prices, Bachelier assumed equal dollar gains and losses in the price of the stock. Thus at any time the expected value of profit from speculation of a price change in the security is zero. Therefore, Bachelier has assumed that current prices "reflect" all available information and since he assumed equal arithmetic gains and losses the expected distribution of stock prices is normal. Osborne (1959), apparently independent of Bachelier, introduced the measurement technique of geometric differences in stock prices. The procedure asserts an equal probability of a rise or fall in the logs of common stock prices. Osborne's hypothesis states that common stock prices are distributed log normally, and may be restated as the changes in the logs of prices being normally distributed. Osborne agreed with Bachelier on the question of current stock prices reflecting all available information, but regarding the type of distribution to which future prices conform, Osborne favored the log normal distribution. The log normal distribution is intuitively appealing as the limits of the stock prices are 0 and $+\infty$. Limits for the normal distribution when applied to common stock prices are $-\infty$ and $+\infty$.

Empirically the log normal distribution has received support from Kendall (1953, p. 24) and Moore (1964, pp. 116-123). Kendall's research was based upon nineteen indices of British industrial share prices as well as spot prices for New York cotton and Chicago wheat. By using successive arithmetic first differences Kendall found the serial correlations of the series to be weak and the distribution of successive price changes to be approximately normal. Moore, using the Standard and Poor Index and thirty randomly selected common stocks from the New York Stock Exchange found the changes in the logs of common stock prices also exhibit little serial correlation and appeared to be distributed approximately normal.

Many models have been derived to study whether the current price "reflects" all available information. Models such as the martingale, submartingale, and the weak, semi-strong, and strong form of random walk have been developed

(Fama, 1970). This paper will assume the weak form of random walk since the empirical work will only deal with historical prices rather than including other variables for informational content.

The question of distribution type has also provoked much interest in the literature. Other distributions which have been suggested and supported by empirical tests are the family of stable Paretian distributions. Mandelbrot (1963) and Fama (1963) introduced these distributions which are more peaked and whose variance is infinite. The normal distribution is a special case of this distribution family in which the variance is finite. These distributions are said to better describe common stock prices as the fatter tails which characterize the distributions accommodate the observed leptokurtic nature of changes in common stock prices.

Praetz (1972) suggests that Osborne's work can be modified to include the "temperature" of the share market which represents the market's degree of activity. This extension produces a scaled t-distribution which for Praetz's sample provides a better fit than either the stable Paretian distributions or Gaussian distributions. The empirical work is based upon weekly observations of seventeen share-price indices from the Sydney Stock Exchange for the period 1958-66.

This study views the advantages and disadvantages of the preceding mentioned distributions and has chosen to assume log normal distribution of common stock prices. The rationale behind this choice relies on the following facts:

1. Log normal theory has been supported both theoretically (Osborne, 1959; Aitchison and Brown, 1963) and empirically (Kendall, 1953; Moore, 1964).
2. The stable Paretian and scaled t-distribution are relatively new and have not been well defined in a generalized form.
3. There does not seem to be an applicable utility function that will satisfy Arrows' four conditions and when applied with a stable Paretian distribution of returns produce an expected utility which is positive and finite.
4. There is no generally accepted method of rejecting any of the mentioned distributions other than a best fit criterion, which taken by itself is not fully resolved.

To summarize, the log normal distribution will be assumed because: it is well defined theoretically, it is consistent with some empirical evidence, and it accommodates the most well defined utility functions such as the negative exponential and the constant elasticity functions. The theory developed in the review of utility and distribution theory will be combined with the definitions of convertible bonds to examine the existing models of convertible bond valuation.

Capital Market Theory

The concept of capital market equilibrium is relatively new. Markowitz (1952) introduced a normative approach which developed a model for combining securities with less than perfect correlation in a way which produces an efficient set of the portfolios in terms of two parameters, expected return and standard deviation of return. The investor then selects a portfolio from the efficient frontier which maximizes his expected utility. The necessary assumptions for the Markowitz model are:

1. All investors exhibit diminishing marginal utility of wealth and maximize one-period expected utility.
2. Investor risk estimates are proportional to the variability of the expected returns.
3. Investors are willing to base their decisions solely on the two parameters, expected return and risk measured by variability of expected return.
4. For any given level of risk, investors prefer higher returns to lower returns, or conversely, for any given level of rate of return, investors prefer less risk over more risk.

Tobin (1958) extended Markowitz' work toward a more testable positive theory. By separating risk-free assets and risky assets Tobin hypothesized that investors may adjust the expected returns and risk of a security by weighing it in combination with a risk-free asset. This concept was expounded upon by Sharpe (1964) and others to develop a capital market equilibrium model. In order to develop such a model, two more major assumptions were applied:

1. There is a common pure rate of interest, and all investors are able to borrow and lend on equal terms.

2. There exists homogeneity of investor expectations.

With the added assumptions "a capital market line" is constructed in the space E_R, σ_R, from the risk-free rate to the market portfolio on the efficient frontier. All investors would buy the market portfolio and then borrow or lend at the risk-free rate until they reach a point of expected return and risk which maximizes their expected utility.

Capital market theory does not conform nicely to empirical work. According to Douglas (1969) and Miller and Scholes (1972), the empirical work questions the validity and practicality of the theoretical model. This non-compatibility is the sequelant of two possible imperfections:

(1) error in the basic test equation, and (2) error in the measurement techniques for relevant risks and returns. With these shortcomings in mind many researchers attempt to apply capital market theory to various securities and investments.

Convertible Bond Theory

Convertible bonds have been the source of interest for many academicians and practitioners. Early works described the institutional characteristics of the security while later works explored various valuation techniques. Currently research has been directed toward explaining the premium associated with convertible bonds.

Description and Background

A convertible bond is a bond which may be converted at the option of the holder into common stock of the same corporation. The ratio of exchange of common stock for the convertible bond is stated in one of two ways: (1) a conversion price per share, or (2) a conversion ratio of shares per bond. Conversion privilege may vary over time to allow the issuer to replace each bond with less shares of stock as the time to maturity of the bond decreases. This increase in conversion price or decrease in conversion ratio may force the holder to convert or to suffer a decrease in the value of the security.

Convertible bonds are issued for a variety of reasons:

1. Interest rates on convertible bonds are generally less than interest rates on the corresponding straight bond of the same quality.
2. Convertible bond issuers may desire to use equity financing but feel the common stock market is temporarily depressed. By issuing a convertible bond with a conversion price above the market price, the issuer may be able to obtain equity financing indirectly. This procedure has the advantage of reducing dilution for the current stockholders.
3. Convertible bond financing is popular with growth companies, especially where mergers are involved.

The investor who buys a convertible bond has been described as one who wants the secure return of a bond but who also wishes to speculate in the common stock of a corporation. This investor purchases the right to speculate by accepting an interest rate on the convertible bond which is less than the corresponding interest rate of a bond of similar quality. The convertible bond market price represents the greater of the conversion value or the straight debt value plus a premium. Factors underlying the premium value are discussed in the Weil, Segall, and Green (1968) article under review of evaluation models.

Convertible bonds have also been described as either an issue of common stock with a put option or as an issue of debt with a warrant attached. This approach will be explained further in the review of evaluation models.

Evaluation Models for Convertible Bonds

Brigham (1966). This model evaluates a convertible bond by an analysis including an annuity component which is the interest paid per period over the life of the bond, and a final value M". Brigham states that in the early years of the straight bond's life, the annuity feature dominates the total value, while in years closer to maturity or redemption, the final value M" dominates. His equation is

$$B_t = \sum_{k=1}^{T-t} \frac{I}{(1+i)^k} + \frac{M''}{(1+i)^{T-t}} \tag{2.6}$$

where: B_t = convertible bonds value as straight debt at time t.

T = original time to maturity.

k = time subscript from year 1 to year T.

i = market rate of interest or equivalent risk pure debt issue.

I = dollars of interest paid each year.

M'' = bonds redemption value at maturity, $1,000 par value.

If the convertibility feature is added to the equation, it is modified as:

$$M = \sum_{t=1}^{N} \frac{I}{(1+k)^t} + \frac{TV}{(1+k)^t} \tag{2.7}$$

where: M = price paid for the bond.

I = dollars of interest received per year.

TV = terminal value of the bond; call price if surrendered on call; maturity value if redeemed; conversion value if converted; or market price if sold.

N = number of years bond was held.

k = internal rate of return.

It may be noted that the price paid for the bond is now dependent on the price of the common stock as well as the terminal value of the bond. Brigham expresses the above model in a graphic manner by using as his axes dollar value on the ordinate and time on the abcissa (Figure 2.2).

Figure 2.2 illustrates that the premium paid is based on the difference between the market value of the convertible bond and the greatest of the values of the straight bond or the conversion price of common stock. At point X the two values are equal. One point which is not shown in Table 2.2 is the effect of interest rate changes on the market price of the straight bond. Any changes in the interest rate paid on debt in the convertible bond risk class will result in a shift in the floor value of the bond, the floor value being the value of the straight debt.

Brigham gives three reasons for the value of the premiums on convertible bonds to approach zero: (1) call feature, (2) diminishing loss protection, and (3) current

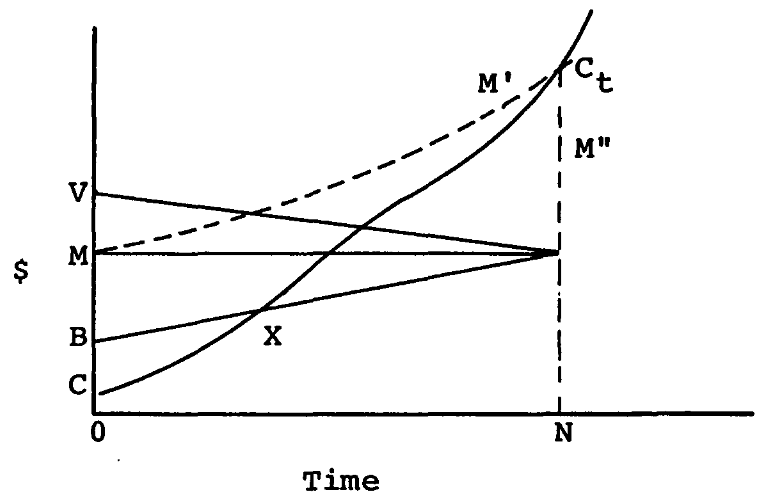

Figure 2.2. Brigham's Graphical Model

where: B M" = value of the straight bond over time,
M M" = par value over time,
V M" = called value of bond over time,
C C_t = value of the common stock over time,
M M' = market value of the convertible bond over time.

yield of the common stock becomes greater than the current yield of the bond.

Baumol, Malkiel, and Quandt (1966). This model values convertible bonds in two parts. The first part represents the security itself, either stock or bond, which may be emblematic of the discounted future cash inflows plus the expected value of the security at the end of the horizon period. The second source of value possessed by a convertible security is obtained from the insurance of a floor value if the security in the first part is a stock, or the value of a call option if the first part is valued as a bond.

Baumol, Malkiel, and Quandt assume that the aggregate investor's subjective probability distribution of common stock prices is predictable from past distributions of common stock prices. Their preliminary model consists of the following factors:

1. C--represents the value of the convertible.
2. B--represents the convertible bond equivalent value (straight bond value).
3. S--represents the number of shares of common stock into which the convertible can be exchanged.
4. P(t)--the price of a common stock share at date t so that p(t)S represents the convertibles' stock equivalent value (its stock conversion value).
5. i(t)--a price relative of a share at date t and left to be the date on which the converrible is being evaluated.

Therefore by definition:

$$P(t) = i(t)\ P\ (t_o) \text{ for any date } t \qquad (2.8)$$

If the convertible security is equal to or greater than its conversion value in common stock plus its insurance value, then:

$$C \geq P(t_o)\ S + V \qquad (2.9)$$

The value V refers to any possibility of the value of the bond B being greater than its conversion value into common stock P(t)S. So for any i(t) the value of this V will be B-i(t)P (t_o)S with the probability of $f(i,t_o)$ di(t). Hence, if we sum all probabilities keeping the value of B a constant $\bar{B}$, the result is the following integral:

$$V = \int_{0}^{\bar{B}/PS} f(i,t_o)[B - i(t)\ P(t_o)S]\ di(t) \qquad (2.10)$$

The limits of integration being i(t) = 0,i(t) = $\bar{B}$/PS. The lower limit is set at zero since the value of common stock cannot fall below zero. The upper limit of i(t) is set so that B = i(t) $P(t_o)$S which sets the bond value equal to the common stock conversion value. Substitute the integral for V [Equation (2.10)] in Equation (2.9). The result is:

$$C \geq Cs = P(t_o)\ S + \int_{0}^{\bar{B}/PS} f(i,t_o)\ [B - i(t)P(t_o)S]\ di(t) \qquad (2.11)$$

Since the option part of the expression in reality is summed over the range of (0,∞), an analogous relationship is found based on the flow value of the bond and the part of the option distribution in which the value of P(t)S exceeds B.

$$C \geq C_b = \bar{B} + \int_{B/PS}^{\infty} f(i,t_o)\ [i(t)p(t_o)S-\bar{B}]\ di(t) \qquad (2.12)$$

Poensgen (1966). The Poensgen model is developed similarly to the Baumol, Mailkiel, Quandt model but in more explicit terms. The model is developed from the basis of a convertible being a combination of a straight debt instrument and an option on common stock. The option is valued as follows for a given straight debt value (y):

$$\int_{y}^{\infty} x \; h(x/y) \; dx \qquad (2.13)$$

where: x = all values of x greater than y.

h(x/y) = the probability of a certain x value given a set value of straight debt y.

Time, t, for the above expression is held constant thereby not giving a completely accurate approach when valuating the option over all values of time.

The value of the option [Equation (2.13)] is added to the value of straight bond times the probability that the value of the stock (x) will be less than the value of the straight bond (y) [Equation (2.14)].

$$y \int_{0}^{y} h(x/y) \; dx \qquad (2.14)$$

The resulting expression is:

$$y \int_{0}^{y} h(x/y) \; dx + \int_{y}^{\infty} x \; h(x/y) \; dx \qquad (2.15)$$

However, there is the problem of changes in interest rate leading to changes in bond yield and consequently bond value. Hence, the above must be integrated for all values of y g(y) from 0 to ∞. Therefore, the expected value of the bond E(p) is:

$$E(p) = \int_0^\infty [y \int_0^y h(x/y)\, dx + \int_y^\infty x\, h(x/y)\, dx]\, g(y)\, dy \qquad (2.16)$$

Poensgen then substitutes h(x,y) for h(x/y)g(y) to obtain

$$E(p) = \int_0^\infty [y \int_0^y h(x,y)\, dx + \int_y^\infty xh(x,y)\, dx]\, dy \qquad (2.17a)$$

and by regrouping the integral obtains:

$$E(p) = \int_0^\infty \underbrace{[\int_0^\infty xh(x,y)\, dx}_{1} + \underbrace{\int_0^y (y-x)\, h(x,y)\, dx]}_{2}\, dy \qquad (2.17b)$$

where: 1 = expected stock value.

2 = floor value guarantee.

Rearranging again under the assumption that for all values of y the sum of the probabilities for each x sum to one:

$$\int_0^\infty h(x/y)\, dx = 1 \qquad (2.18)$$

and substituting into Equation A, Poensgen obtains:

$$E(p) = \underbrace{\int_0^y y\, g(y)\, dy}_{3} + \underbrace{\int_0^\infty \int_y^\infty (x-y) h(x,y)\, dxdy}_{4} \qquad (2.19)$$

where: 3 = expected straight debt value.

4 = expected value of conversion option.

Evaluation of the above integrals to arrive at a certain expected value for a convertible bond price does not provide complete information to the investor. Also, knowledge of the shape of the convertible bond price probability distribution will influence the investor's choice of investments.

Poensgen uses a log normal distribution of stock prices. He assumes stock prices follow a stochastic process in a multiplicative manner. This results in changes in the logs of stock prices then being distributed normally. The distribution of untransformed stock price changes is log normal with asymtotes of 0 and $+\infty$. The convertible bond distribution as shown by Poensgen (Figure 2.3) is truncated so every value less than a (the floor value of the bond) accumulates its frequency at point $\underline{a}$, thus we obtain a highly skewed distribution.

The δ segment is defined as an infinitesimally narrow mass which extends vertically to infinity. The mass is representative of the portion of the probability

Figure 2.3. Convertible Bond Price Change Distribution

distribution of returns of the common stock which falls to the left of the floor value a. Due to this δ accumulation, the third moment of the distribution (index of skewness) is non-zero. Poensgen then takes the first moment around the truncation point (rather than the origin) and obtains the mean, labeled EPW. The higher moments are then taken around the mean EPW by using a Taylor expansion. This approach does not measure the moments of a convertible bond rate of return but only the moments of the option portion around its mean.

Weil, Segall, and Green (1968). The article defines the theoretical premium on convertible bonds as being the difference between the present values of the cash stream expected to be received by convertible bond holders and the cash stream expected to accrue to the corresponding common stock. The objective of the Weil et al. (WSG hereafter) study is to find the degree to which observed premiums can be explained from the observable data. They list the following factors as determinants of the premium:

1. Transaction costs.
2. Differences in income streams of the convertible bond and the corresponding common stock.
3. Differences in financing costs of the convertible bond and corresponding common stock.
4. Anti-dilution clauses.
5. Convertible bond price floor.
6. Volatility of the price of the corresponding common stock.
7. Duration.

Only transaction costs, income differences and price floor are used, the other variables are omitted due to either empirical or theoretical inconsistencies.

The general form of the equation WSG use, is:

$$\frac{P}{B} = f(F) + \beta\frac{Yd}{B} + \gamma\frac{T}{B} + \delta\frac{1}{B} \qquad (2.20)$$

where: P = calculated premium.

B = bond price.

F = floor variable; the difference between the bond price and the straight debt value of the bond as reported in the Moody's Bond Survey.

Yd = difference in income streams; bond income less stock income; the study uses current income differences.

T = transaction cost difference; cost to buy the stock into which the bond is convertible less the cost to buy the bond including all transfer taxes.

The results of the authors' tests do not strongly support their interpretation of the theory of the premium on convertible bonds. WSG state that their main finding is the floor value having negligible importance in explaining the premium.

West and Largay (1972). In their comment on the WSG article, the authors make some clarifying remarks on the study in question. They find two major faults in the article: (1) the definition of premiums, and (2) the systematic omission of bonds with a conversion value less than $1000, although greater than the straight bond value. WSG define the floor variable as being equal to the difference between the market value of the convertible bond and its straight debt value. Usually, the floor variable is defined as the difference in the conversion value and the straight bond value. West and Largay point out that by misdefining the floor variable WSG regress the premium upon itself. This spawns results contrary to existing theory. Using the sample of WSG, West and Largay find a significant relationship between the premium and floor variable when the floor variable is defined correctly.

By restricting the sample to bonds which have a conversion value above $1000, WSG omit many cases in which their definition of the floor variable makes little sense. In these cases the conversion value and the straight bond value are nearly equal; hence, the premium and the floor value are nearly equal.

This comment is a needed clarification of the WSG work and returns support to conventional theory.

Walter and Que (1973). Walter and Que present a convertible bond valuation model which is compatible with the market model. In this model they examine convertible debentures in light of their relation to the market portfolio.

One problem which they identify is the asymmetry in Poensgen's equations [see Equations (2.17b) and (2.19)]. Since the integrals which represent the value of convertible bonds may be grouped in various ways, Walter and Que ascertain that the regression studies should depend on whether the conversion value exceeds or falls short of the straight bond value.

The authors also explain that the conversion privilege of convertible securities is different from the straight option or warrant. The conversion privilege possesses two important features which are not evident in warrants or straight options. The characteristics are: (1)

the possibility of involuntary termination, and (2) the value of the security and the conversion privilege associated with it depends on the value of the underlying common stock and also on the floor value plus a premium.

The market model presented by the authors relates the return of both the ith security and the market. The regression equation of this relationship is usually expressed:

$$R_{it} = \hat{\alpha}_i + \hat{\beta}_i \tilde{R}_{mt} + \varepsilon_{it} \qquad (2.21)$$

when ε_{it} is the residual error with assumed properties, $E(\varepsilon i) = 0$ and $E(\varepsilon_i, \tilde{R}_m) = 0$.

The authors regressed the price relatives of the convertible bonds and the price relatives of the underlying common stocks with the monthly price relatives of Standard and Poor's 500 index. The resulting beta values are greater for the common stock in 12 of the 13 cases over the ten year period of 1960-1969. This result precedes the authors' presumption that the floor value of a convertible bond operates by its effect upon systematic risk. This beneficial influence of the bond floor upon the systematic risk of the convertible debenture diminishes as the conversion value of the security approaches and exceeds the bond floor value.

The simulation model presented by the authors compensates for the drawbacks of the conventional and market model. The model is able to accommodate three cases: (1) involuntary termination due to conversion; (2) involuntary termination due to a sinking fund call; and (3) no calls or terminations. In addition, it accounts for differing risk characteristics and also transaction costs.

The preceding studies have developed convertible bond theory to its present state. By extending these efforts the model in Chapter 3 was derived.

CHAPTER 3

VALUATION OF CONVERTIBLE BONDS: A BIVARIATE MODEL

The model developed here is similar to that of Walter and Que (1973) in that it combines the convertible bond model of Poensgen (1966) with capital market theory. Where the Walter and Que article establishes beta coefficients empirically as risk surrogates for convertible bonds, this study establishes the theoretical background of convertible bond beta coefficients and questions beta's adequacy as a risk surrogate.

Poensgen's definition of the convertible bond distribution of expected prices is:

$$E_{(p)} = \int_0^{\infty} \left[\int_0^{\infty} xh(x,y)dx + \int_0^{y} (y - x)h(x,y)dx\right]dy \qquad (2.17b)$$

Assuming that the bond value y is not a stochastic variable, the integral outside the brackets is dropped, obtaining:

$$E_{(p)} = \int_0^{\infty} xh(x)dx + \int_0^{y} (y - x)h(x)dx \qquad (3.1a)$$

$$= \int_y^{\infty} xh(x)dx + \int_0^{y} yh(x)dx \qquad (3.1b)$$

Where h(x) is the log normal density function. Figure 2.2 represents the above equation.

As stated by Poensgen, this distribution is neither a truncated log normal nor a censored one. Aitchison and Brown (1963, pp. 87-99) define the truncated distribution as

$$P[X \leq x] = 0 \qquad \text{for } (x \leq y) \tag{3.2a}$$

$$P[X \leq x] = \frac{\Lambda(x|\mu,\sigma^2) - \Lambda(y|\mu,\sigma^2)}{1 - \Lambda(y|\mu,\sigma^2)} \qquad \text{for } (x > y) \tag{3.2b}$$

where y denotes the truncation point and $\Lambda(x|\mu,\sigma^2)$ is the cumulative lognormal distribution of x with mean,

$$\alpha = e^{\mu + 1/2\ \sigma^2} \tag{3.3}$$

and variance,

$$\beta^2 = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1). \tag{3.4}$$

Truncation of a distribution is defined as cutting off a portion of the distribution and redistributing it over the remaining portion of the distribution according to its corresponding density.

The definition of the censored lognormal distribution with y as the point of censorship is:

$$P[X \leq x] = \Lambda(x|\mu,\sigma^2) \qquad \text{for } (x \geq y) \tag{3.5a}$$

$$P[X \leq y] = \Lambda(y|\mu,\sigma^2) \qquad \text{for } (x \leq y) \tag{3.5b}$$

The density function is only known for the portion of the distribution above the point of censorship. Nothing is known of the density function below y except that its cumulative probability is

$$\Lambda(y|\mu,\sigma^2).$$

The distribution described by a convertible bond is not a case of censorship since it is known that the cumulative probability below y is massed at point y, hence the shape of the distribution is known and moments may be generated from the distribution. Also, the convertible bond distribution of expected returns is not a truncated distribution as truncated portion is not redistributed over the remaining part of the distribution.

Poensgen calls the convertible bond distribution a "truncated distribution of class 2." It is defined as

$$P[X = y] = \Lambda(y|\mu,\sigma^2) \qquad \text{for } (x = y) \tag{3.6a}$$

$$P[X \leq x] = \Lambda(x|\mu,\sigma^2) \qquad \text{for } (x > y) \tag{3.6b}$$

$$P[X < y] = 0 \qquad \text{for } (x < y) \tag{3.6c}$$

and is depicted in Figure 3.1.

To summarize, the convertible bond distribution is not truly a truncated distribution, as defined by statisticians, since the truncated portion is not redistributed over the remainder of the distribution with the corresponding density. The distribution is also not a case of censorship since the portion of the distribution below the truncation

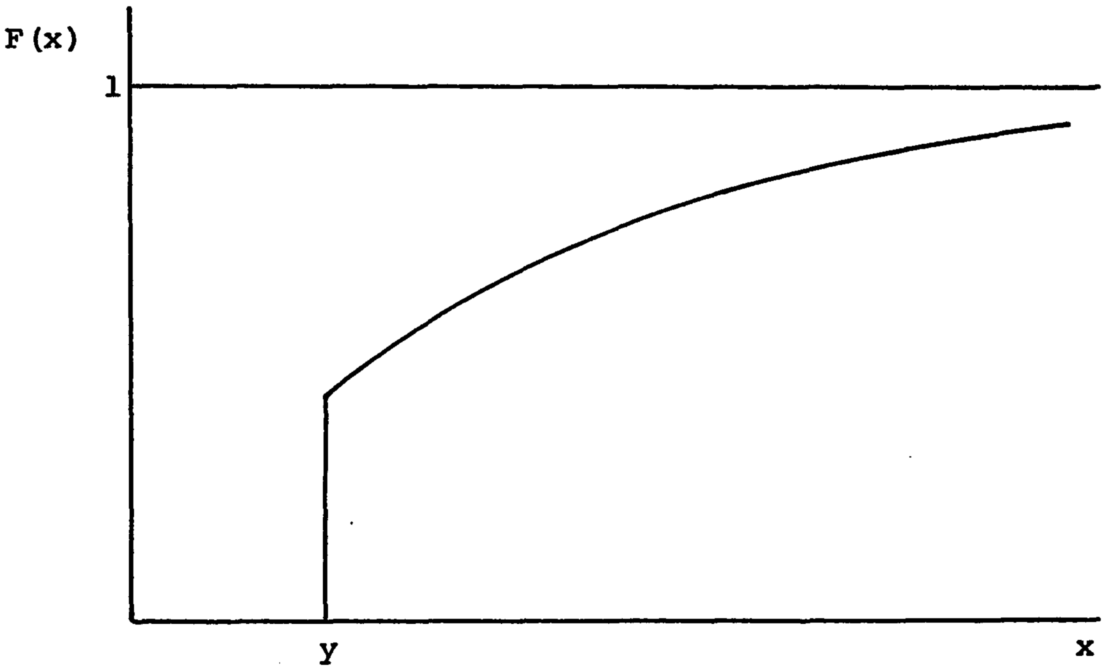

Figure 3.1. Cumulative Truncated Distribution Function of Class 2

point y is defined as being massed at y rather than being undefined as is the case of true censorship.

Poensgen then presents a moment generating function for his truncated class 2 distribution:

$$V_j = y^j \int_0^y \frac{1}{x\sqrt{2\pi}\sigma} e^{(-1/2)\left(\frac{\ln x - \mu}{\sigma}\right)^2} dx$$

$$+ e^{j\mu + 1/2\, j^2\sigma^2} \int_y^\infty \frac{1}{x\sqrt{2\pi}\sigma} e^{-1/2\left(\frac{\ln x - \mu - j\sigma^2}{\sigma}\right)^2} dx \qquad (3.7)$$

He then transforms it to a normal distribution by substituting:

$$\lambda = \frac{\ln x - \mu}{\sigma}, \quad \frac{d\lambda}{dx} = \frac{1}{x\sigma} \tag{3.8a}$$

$$\lambda = \frac{\ln x - \mu - j\sigma^2}{\sigma}, \quad \frac{d\lambda}{dx} = \frac{1}{x\sigma} \tag{3.8b}$$

obtaining:

$$V_j = \int_{-\infty}^{\frac{\ln y - \mu}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{(-1/2)\lambda^2}\, d\lambda + \int_{\frac{\ln y - \mu - j^2}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{(-1/2)\lambda^2}\, d\lambda \tag{3.9}$$

therefore:

$$V_j = y^j \int_{-\infty}^{\frac{\ln y - \mu}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{(-1/2)\lambda^2}\, d\lambda + e^{j\mu + 1/2\, j^2\sigma^2} \int_{\frac{\ln y - \mu - j\sigma^2}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{(-1/2)\lambda^2}\, d\lambda \tag{3.10}$$

Thus the moments may be calculated by substituting values for j and evaluating the probabilities under the integrals from the cumulative normal probability tables. The only variables that must be known are the mean and standard deviation for the expected price distribution of the underlying common stock and the point of truncation. The point of truncation is the value of the straight bond in terms of the price of the common stock and the conversion ratio.

The model in this study parallels Poensgen's model to a point. The log-normal distribution of common stock prices is transformed to the normal distribution of logs of price relatives using the current prices (p_o) as the base price and (p_t) the price at period end. Since random walk theory states that the best estimation of a future common stock price is its current price, actual future prices will be distributed around the current price. Thus if p_o equals p_t the price relative is one. If the prices are assumed to be distributed log-normally then the log relatives are distributed normally with a mean of zero. Dividing through the parameters, mean and standard deviation, by the standard deviation creates a standardized distribution, N(0,1). Letting i(t) represent price relatives, the distribution is illustrated in Figure 3.2 where the standardized truncation point a is obtained by dividing the log of the straight bond value minus the log of the current conversion value by the standard deviation of the logs of consecutive period-end common stock prices adjusted for the life and conversion ratio of the particular convertible bond. In this instance call possibilities are ignored.

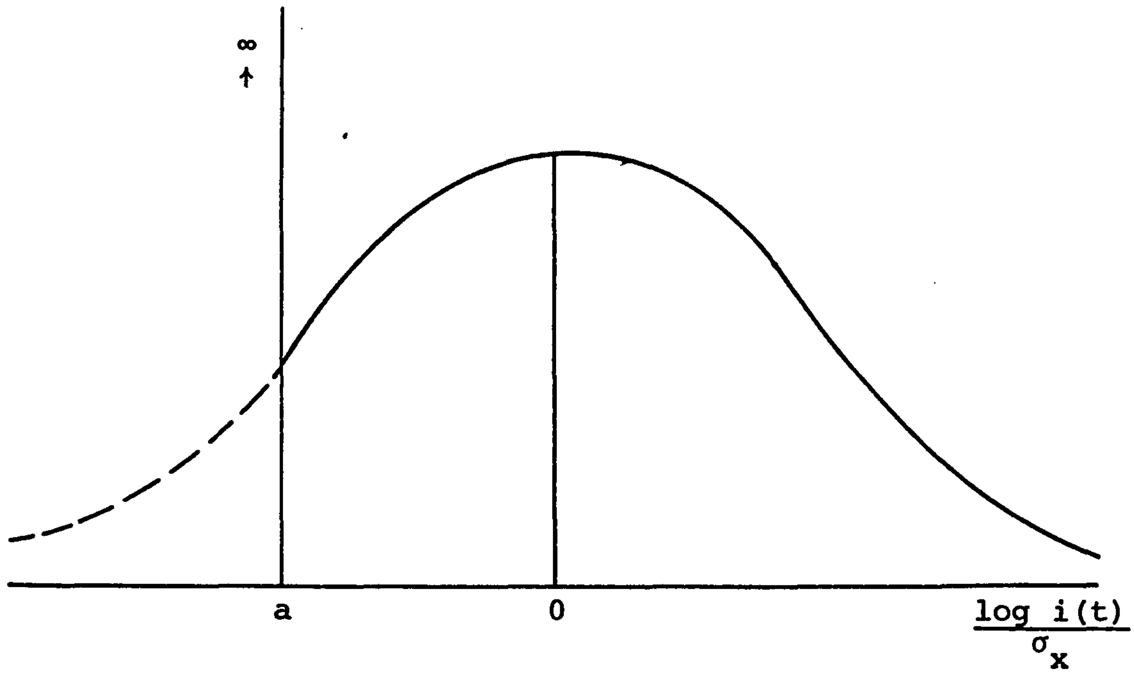

Figure 3.2. Standardized Distribution of the Logs of Convertible Bond Price Relatives

Assuming the straight bond value to be non-stochastic the distribution in Figure 3.2 may be expressed:

$$E_{\ln(i(t))_{CB}} = a \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}\sigma_x} e^{(-1/2)\left(\frac{x-\mu_x}{\sigma_x}\right)^2} dx$$

$$+ \int_{a}^{+\infty} x \frac{1}{\sqrt{2\pi}\sigma_x} e^{(-1/2)\left(\frac{x-\mu_x}{\sigma_x}\right)^2} dx \qquad (3.11)$$

where: $E_{\ln(i(t))_{CB}}$ = expected value of the logs of convertible bond price relatives.

a = straight debt value expressed in logs of conversion value price relatives.

x = possible logs of price relatives of the corresponding common stock conversion value.

μ_x = mean of the logs of conversion value price relatives.

σ_x = standard deviation of the logs of conversion value price relatives.

The purpose of this model is to describe the resulting holding period returns of a portfolio consisting of a convertible bond and the market portfolio, thus combining a truncated distribution of class two with a normal distribution. Where Poensgen used variance and skewness as measures of risk, this model separates systematic and unsystematic risk and theoretically evaluates the skewness factors. This task is accomplished by developing a model which utilizes the available theory of bivariate normal distributions. Convertible bonds are defined as in Equation (3.9). The market portfolio distribution of log relatives is defined as:

$$E_{\ln(i(t))_M} = \int_{-\infty}^{\infty} y \frac{1}{\sqrt{2\pi}\sigma_y} e^{-1/2\left(\frac{y-\mu_y}{\sigma_y}\right)} dy \qquad (3.12)$$

where: $E_{\ln(i(t))_M}$ = the expected value of the logs of the market portfolio price relatives.

y = possible logs of the market portfolio price relatives.

μ_y = mean of the logs of the market portfolio price relatives.

σ_y = standard deviation of the logs of the market portfolio price relatives.

Combining the convertible bond and market portfolio distribution of log relatives forms a truncated bivariate distribution which is expressed:

$$E_{\ln(i(t))_{CB,M}} = \int_{-\infty}^{a}\int_{-\infty}^{\infty} ay \frac{1}{2\pi\sigma_x\sigma_y(1-\rho^2)^{1/2}} \exp\Big[-\frac{1}{2(1-\rho^2)}$$

$$\Big[\Big(\frac{x-\mu_x}{\sigma_x}\Big)^2 + 2\rho\Big(\frac{x-\mu_x}{\sigma_x}\Big)\Big(\frac{y-\mu_y}{\sigma_y}\Big) + \Big(\frac{y-\mu_y}{\sigma_y}\Big)^2\Big]\,dydx$$

$$+ \int_{a}^{\infty}\int_{-\infty}^{\infty} xy \frac{1}{2\pi\sigma_x\sigma_y(1-\rho^2)^{1/2}} \exp\Big[-\frac{1}{2(1-\rho^2)}$$

$$\Big[\Big(\frac{x-\mu_x}{\sigma_x}\Big)^2 + 2\rho\Big(\frac{x-\mu_x}{\sigma_x}\Big)\Big(\frac{y-\mu_y}{\sigma_y}\Big) + \Big(\frac{y-\mu_y}{\sigma_y}\Big)^2\Big]\,dydx \tag{3.13}$$

The above equation is in general form and there is no weighting of the assets. ρ is the correlation coefficient for the underlying common stock and market portfolio price relatives. The product-moment generating function derived in Appendix B generates covariance and coskewness product-moments for the two asset model consisting of a convertible bond and the market portfolio of common stocks about the origin. Thus the product-moment generation establishes theory for calculation of beta coefficients for convertible bonds in terms of the mean and standard deviation of the underlying common stock distribution and the straight bond value (point of truncation). The generation of the product-moments also gives theoretical insight into the third moment and its relationships with the market index although its interpretation is not yet clear. The product moment generating function is:

$$E(x^r, y^s) = \exp[t_1 a + t_2\mu_y + 1/2(t_2^2\sigma_y^2)]\ \Phi\left(\frac{a-\mu_x-\rho\sigma_y\sigma_x t_2}{\sigma_x}\right)$$

$$+ \exp[t_1\mu_x + t_2\mu_y + 1/2(t_1^2\sigma_x^2 + 2\rho t_1 t_2\sigma_x\sigma_y + t_2^2\sigma_y^2]$$

$$[1-\Phi\left(\frac{a-\mu_x-t_1\sigma_x^2-t_2\rho\sigma_y\sigma_x}{\sigma_x}\right)] \quad . \qquad (3.14)$$

This means μ_x, μ_y are both zero since the means of the logs of the price relatives of both the underlying common stock and the market portfolio are assumed to be zero. To evaluate the moments generated $t_1 = t_2 = 0$, thus both functions of Φ are reduced to

$$\Phi\left(\frac{a - \mu_x}{\sigma_x}\right) = \Phi\left(\frac{a}{\sigma_x}\right) \qquad (3.15)$$

which will henceforth be denoted as Φ. The product-moments for the first three parameters are:

$$\mu_{10} = a\Phi + \sigma_x\phi \tag{3.16}$$

$$\mu_{01} = 0 \tag{3.17}$$

$$\mu_{20} = a^2\Phi + \sigma_x^2(1 - \Phi) - \sigma_x^2\phi' \tag{3.18}$$

$$\mu_{02} = \sigma_y^2 \tag{3.19}$$

$$\mu_{11} = \rho\sigma_x\sigma_y(1 - \Phi) - a\rho\sigma_y\phi - \rho\sigma_x\sigma_y\phi' \tag{3.20}$$

$$\mu_{30} = a^3\Phi + 3\sigma_x^3\phi + \sigma_x^3\phi'' \tag{3.21}$$

$$\mu_{21} = 3\rho\sigma_x^2\sigma_y\phi + \rho\sigma_x^2\sigma_y\phi'' - a^2\rho\sigma_y\phi \tag{3.22}$$

$$\mu_{12} = a\sigma_y^2\Phi + \sigma_x\sigma_y^2\phi + 2\rho^2\sigma_x\sigma_y^2\phi + a\rho^2\sigma_y^2\phi' + \rho^2\sigma_x^2\sigma_y\phi'' \tag{3.23}$$

$$\mu_{03} = 0 \tag{3.24}$$

The symbol Φ represents the cumulative normal distribution from $-\infty$ to point $\underline{a}$ and Φ is the ordinate normal value at point $\underline{a}$ and the first derivative of the cumulative normal function. Further derivatives will be denoted by primes such as Φ' for the second derivative of the cumulative normal function. The value for Φ and its derivatives are given in tables such as those in Burington and May (1953).

The mean of the convertible bond distribution of expected price relatives given by moment μ_{10}, is

$$\text{mean}_{CB} = a\Phi + \sigma_x\phi \tag{3.25}$$

and is always greater than or equal to the expected mean of the greater of the common stock conversion value's or the straight bond value's probability distribution of price relatives.

In order to be consistent with past research the product-moments should be around the mean of the distribution. These product-moments are derived in Appendix B. The variance of the expected price relative distribution of a convertible bond is based upon moment μ_{20}, which is the raw second moment about the origin $(E(x^2))$. The variance is the second moment about the mean E(x) and is written $E(x^2) - (E(x))^2$. In terms of μ_{10} and μ_{20} the variance is written:

$$\text{variance} = \sigma_x^2(1 - \Phi) + a^2\Phi - \sigma_x^2\phi' - (a\Phi + \sigma_x\phi)^2 \tag{3.26a}$$

$$= \sigma_x^2(1 - \Phi) + a^2\Phi - \sigma_x^2\phi' - a^2\Phi^2 - 2a\sigma_x\Phi\phi - \sigma_x^2\phi^2 \tag{3.26b}$$

The covariance term as produced by moment μ_{11} is shown as

$$\text{covariance}_{CB,M} = \rho\sigma_x\sigma_y(1 - \Phi) - a\rho\sigma_y\phi - \rho\sigma_x\sigma_y\phi' \tag{3.27a}$$

For all truncation values of a,

$$a\rho\sigma_y\phi = -\rho\sigma_x\sigma_y\phi' \; \underline{1/} \tag{3.27b}$$

thus for all values the terms cancel reducing the covariance to

$$\text{covariance}_{CB,M} = \rho\sigma_x\sigma_y(1 - \Phi). \tag{3.27c}$$

Therefore, from the covariance equation it is obvious that both the covariance and the market and the beta coefficients are smaller for the convertible bonds than for their corresponding common stocks.

The third moment about the mean may be written

$$\mu'_{30} = \mu_{30} - 3\mu_{10}\mu_{20} + 2\mu_{10}^3 \tag{3.28a}$$

Expanding,

$$\mu'_{30} = a^3\Phi + 3\sigma_x^3\phi + \sigma_x^3\phi'' - 3(a^3\Phi^2 + a\sigma_x^2(\Phi - \Phi^2) - a\sigma_x^2\Phi\phi' + a^2\sigma_x\Phi\phi + \sigma_x^3(\phi - \Phi\phi) - \sigma_x^3\phi\phi') + 2(a^3\Phi^3 + 3a^2\sigma_x\Phi^2\phi + 3a\sigma_x^2\Phi\phi^2 + \sigma_x^3\phi^3) \tag{3.28b}$$

$\underline{1/}$

$$\phi(t) = \left(\frac{1}{\sqrt{2\pi}}\right) e^{\frac{-t^2}{2}} \qquad \phi'(t) = \left(\frac{1}{\sqrt{2\pi}}\right)\left(-te^{\frac{-t^2}{2}}\right)$$

Substituting $\left(\frac{a}{\sigma_x}\right)$ for t

$$a\rho\sigma_y\left(\frac{1}{\sqrt{2\pi}}\right) e^{\frac{-\left(\frac{a}{\sigma_x}\right)^2}{2}} = \frac{a}{\sigma_x}\rho\sigma_x\sigma_y\left(\frac{1}{\sqrt{2\pi}}\right) e^{\frac{-\left(\frac{a}{\sigma_x}\right)^2}{2}}$$

This value indicates the skewness for the convertible bond distribution of expected price relatives.

The two co-skewness product moments about the mean would be symmetric for the bivariate normal case. However, the unusual characteristics of this model result in the following product-moments:

$$\mu'_{21} = \mu_{21} - 2\mu_{11}\mu_{10} \tag{3.29a}$$

$$\mu'_{12} = \mu_{12} - \sigma_y^2\mu_{10} \tag{3.30a}$$

Upon expansion:

$$\mu'_{21} = 3\rho\sigma_x^2\sigma_y\phi + \rho\sigma_x^2\sigma_y\phi'' - a^2\rho\sigma_y\phi - 2\rho\sigma_x\sigma_y(a\Phi + \sigma_x\phi)(1 - \Phi) \tag{3.29b}$$

$$\mu'_{12} = 2\rho^2\sigma_x\sigma_y^2\phi + a\rho^2\sigma_y^2\phi' + \rho^2\sigma_x^2\sigma_y\phi'' \tag{3.30b}$$

These values indicate the co-skewness values of the convertible log price relatives with those of the market.

To summarize the effect of the point of truncation μ on the mean, variance, covariance with the market, skewness and coskewness with the market, three special cases are examined below. These cases present an overview of the truncation effect from the points minus infinity to plus infinity.

Case 1: $a \to -\infty$

The point of truncation in case 1 is infinitely to the left of the mean of the log relative distribution of the underlying common stock. Therefore, there is no effect from truncating at this point and the log relative distributions of expected prices for the convertible bond and the corresponding common stock are identical. This result is easily shown by evaluating the derived product moments for the convertible bonds when Φ, ϕ, ϕ', ϕ'' are all equal to zero. The results of this evaluation are the product-moments of a normal distribution.

$$\text{mean}_{CB} = 0 \tag{3.31a}$$

$$\text{variance}_{CB} = \sigma_x^2 \tag{3.31b}$$

$$\text{covariance}_{CB,M} = \rho\sigma_x\sigma_y \tag{3.31c}$$

$$\text{skewness}_{CB} = 0 \tag{3.31d}$$

$$\text{coskewness}_{CB,CB,M} = 0 \tag{3.31e}$$

$$\text{coskewness}_{CB,M,M} = 0 \tag{3.31f}$$

Case 2: $a = \mu_x = 0$

The truncation point in case 2 is located at the mean of the log relative distribution of the underlying common stock. This situation exists when the convertible bond's straight bond value is equal to its common stock conversion value. At this truncation point the following values are found in Burington and May (1953, pp. 267-275).

$$\Phi = .5$$

$$\phi = .3989$$

$$\phi' = 0$$

$$\phi'' = -.3989$$

Evaluation of the product moments produce:

$$\text{mean}_{CB} = .3989\sigma_x \tag{3.32a}$$

$$\text{variance}_{CB} = .3409\sigma_x^2 \tag{3.32b}$$

$$\text{covariance}_{CB,M} = .5\rho\sigma_x\sigma_y \tag{3.32c}$$

$$\text{skewness}_{CB} = .3276\sigma_x^3 \tag{3.32d}$$

$$\text{coskewness}_{CB,CB,M} = .3989\rho\sigma_x^2\sigma_y \tag{3.32e}$$

$$\text{coskewness}_{CB,M,M} = .7978\rho^2\sigma_x\sigma_y^2 - .3989\rho^2\sigma_x^2\sigma_y \tag{3.32f}$$

<u>Case 3: $a \to +\infty$</u>

Case 3 is a situation where the distribution of the convertible bond's expected log relatives are only dependent upon the straight bond value. The corresponding common stock has no effect upon the expected log relatives of the convertible bond. The truncation point is infinitely to the right of the common stock's distribution of log price relatives. Although the mean of such a convertible bond is infinitely to the right of the conversion value base, the mean is then valued solely as the mean of the straight bond.

A constraint of this model is the straight bond value being defined as a non-stochastic variable. However, upon lifting this restriction the straight bond value becomes a stochastic variable with normal tendencies and a mean of zero. Hence, evaluating at this point of truncation using the tabular values:

$$\Phi = 1$$

$$\phi = 0$$

$$\phi' = 0$$

$$\phi'' = 0$$

The product moments produced are:

$$\text{mean}_{CB} = 0 \tag{3.33a}$$

$$\text{variance}_{CB} = 0 \tag{3.33b}$$

$$\text{covariance}_{CB,M} = 0 \tag{3.33c}$$

$$\text{skewness}_{CB} = 0 \tag{3.33d}$$

$$\text{coskewness}_{CB,CB,M} = 0 \tag{3.33e}$$

$$\text{coskewness}_{CB,M,M} = 0 \tag{3.33f}$$

To summarize, the truncated bivariate model establishes return, variance, covariance with the market, and skewness product moments for convertible bonds. Table 3.1 is a summary of product moments evaluated at various truncation points in terms of the variance of the common stock, the truncation point, and the correlation coefficient of the underlying common stock and the market.

Table 3.1. Summary of Product Moments

Product Moment	$a = -\infty$	$a = \mu_x = 0$	$a = +\infty$
μ_{10}	0	$.3989\sigma_x$	0
μ_{20}	σ_x^2	$.3409\sigma_x^2$	0
μ_{11}	$\rho\sigma_x\sigma_y$	$.5\rho\sigma_x\sigma_y$	0
μ_{30}	0	$.3276\sigma_x^3$	0
μ_{21}	0	$.3989\rho\sigma_x^2\sigma_y$	0
μ_{12}	0	$.7978\rho^2\sigma_x\sigma_y^2$	0
		$-.3989\rho^2\sigma_x^2\sigma_y$	

CHAPTER 4

METHODOLOGY

Chapter 3 develops concepts and theory upon which this chapter's empirical inveatigations are based. The derived bivariate model combines the log relative distribution of prices for the convertible bond with the market index of common stocks. Chapter 4's purpose is to investigate the actual distributions of the convertible bond as well as those distributions of the underlying common stock and straight bond in order to examine the consistency of the model with actual data. Also, the premium is analyzed using the product-moments generated by the bivariate distribution to explain its variations. This model's results will be compared with the results obtained from other models that have been subjected to empirical validation. The basis of comparison is the model of Weil et al. (1968) discussed in Chapter 2.

Description of Data

Various data are needed to implement the empirical comparisons; the data used in this study are monthly observations for the time period from January 1968 to December 1972 inclusively, a total of sixty observations.

Sixty-six convertible bonds were studied; these are the only bonds found to meet the following criteria:

1. The firm issuing the bond had its common stock listed on the New York Stock Exchange during all of the sample period.
2. None of the bonds were called during the study.
3. The rating remained the same throughout the study period and was classified by Standard and Poor's as either A, BBB, or BB.
4. The value of the convertible bond was not greatly influenced by merger activity of the firm.

Sources of data for the study are Standard and Poor's Convertible Bond Statistical Analysis and the stock price tape developed by the Center for Research in Security Prices, University of Chicago (CRSP). Standard and Poor's Convertible Bond Statistical Analysis supplied various monthly observations for the specified period. The items collected from this source are:

1. Convertible bond coupon.
2. Dividend income for the number of shares of common stock per bond.
3. Convertible bond price.
4. Conversion value in common stock shares.
5. Investment worth of the straight bond.
6. Maturity data of the issue.

In some cases investment worth was not calculated by Standard and Poor's. Whenever this calculation was not available these observations were estimated using another bond of like rating and maturity.

Price data from the CRSP File was used to calculate the standard deviations of the log relatives for the underlying common stocks as well as the correlation coefficients for the log price relatives of the common stocks and Fisher's Combination Index (Fisher, 1966, pp. 196-197).

The collected data were then used in the following empirical comparisons.

Comparison 1: Descriptive Characteristics

For each of the sixty-six convertible bonds in the study, the mean, variance, skewness, and beta coefficients are calculated for the convertible bond itself and the corresponding common stock and straight bond. The relative values of each set, convertible bond, common stock, and straight bond are tabulated and compared with the derived model for consistency. The premium addition is discussed as well as its effects on the pricing of convertible bonds.

The calculation of these parameters are:

$$\text{mean} = \bar{X} = \frac{\Sigma_{i=1}^{n} X_i}{N} \tag{4.1}$$

$$\text{variance} = S^2 = \Sigma_{i=1}^{n} \frac{(X_i - \overline{X})^2}{N} \tag{4.2}$$

$$\text{skewness} = \frac{\Sigma_{i=1}^{n} \left(\frac{X_i - \overline{X}}{S}\right)^3}{N} \tag{4.3}$$

Beta = the regression coefficient in the linear regression equation

$$r_i = a_i + \beta_i r_m + \varepsilon \tag{4.4}$$

The derivations in Chapter 3 suggest that various relationships exist between the common stock, straight bond, and convertible bond for any given firm. The parameters defined above are used to describe and measure these relationships. Mathematical derivations in the preceding chapter are the basis for *a priori* expectations which are reflected in the following hypotheses:

1. Ho_1. The bond floor increases the expected value of the log relatives of the convertible bond over the expected value of the common stock by truncating the lower tail of the log relative distribution.
2. Ho_2. The convertible bond is less risky than the common stock as measured by a total risk concept (variance) since the range of variation os its log relatives is truncated by introduction of the floor variable.

3. Ho_3. The truncation converts the symmetrical normal distribution into an asymmetric distribution which is highly skewed to the right. Therefore distributions of log price relatives for convertible bonds will possess a positive skewness which is greater than that of the corresponding common stock.
4. Ho_4. The common stock follows the improvement of the market index according to its beta coefficient. Since the convertible bond has a reduced range of variation it is not able to follow the common stock precisely. Thus the beta coefficient of the convertible bond with the market index is expected to be less than the beta coefficient of the common stock and the market index.

Consistent relationships are expected between the straight bond value and the convertible bond value since an assumption of the model holds the straight bond value constant. If it were to vary, however, parameters such as the expected value, variance, and beta coefficient for the convertible bond would be expected to fall between their values for the common stock and for the straight bond. Also, the skewness value of the convertible bond will continue to be greater than the skewness factor of the underlying common stock, since conventional theory associates little if any positive skewness with the log price relative distribution of straight bonds.

Another effect which must be analyzed with respect to the empirical results and the stated hypotheses is the effect of the addition of a premium. The hypotheses are made on the basis of theoretical distributions for which the effect of a premium is absent. Data used in this study are actual prices which include the effect of a premium. The effect of a premium is expected to increase the variance term and decrease the expected value and skewness terms associated with convertible bonds. This result is due to the mean price of the convertible bond increasing upon addition of a premium to the theoretically defined mean value.

Comparison 2: Cross-Section Regression Model

Convertible bonds usually sell at a premium above their straight bond or stock conversion prices. Using the derived model to estimate variables, this portion of the study tests which characteristics of convertible bonds determine the premium associated with a certain bond at any given time. Past models (Weil et al., 1968; Walter and Que, 1973) have used as independent variables the difference of the conversion value and the straight bond value, differences in the income streams of the dividends for common stock and the interest for convertible bonds, historical beta coefficients of the convertible bond with the market index and the differences in transaction costs of convertible bonds and common stock. This study has incorporated the predicted product-moments of the distribution of convertible bond log relatives along with the difference in dividend and coupon income. The product-moments more accurately describe the effects on the premium of the difference between the conversion value and straight bond value. The standard deviation of the log relatives of the firm's corresponding common stock is used in calculating the product-moments. In past studies standard deviation of the firm's common stock has been dismissed as unimportant. In this study it is an important variable although it is not used directly as an independent variable.

Past research has confined the analysis of convertible bond premiums to the special case which occurs when conversion value is greater than straight bond value. The model presented in this study is a general model which is able to analyze premiums regardless of the relationship between conversion value and straight bond value. For the purpose of comparing the product-moment model with the model of Weil et al. (1968) the sample in this study is also stratified into two groups depending on the relationship between conversion value and straight bond value.

Multiple regression analysis is used to explain the variations in convertible bond premiums. The regression equation contains seven independent variables and has the following form:

$$P = a_o + b_1\mu_{10} + b_2\mu_{20} + b_3\mu_{11} + b_4\mu_{30} + b_5\mu_{21} + b_6\mu_{12} + b_7\ \mathrm{DFlow} + \varepsilon \qquad (4.5)$$

where: P = Premium on convertible bonds.

$\mu_{10}, \ldots, \mu_{12}$ = Product moments derived from the expected distribution of convertible bond log relatives.

DFlow = Difference in the income streams of coupons and dividends.

This regression equation is used for the basis of three different regressions. The results of each will be compared to the revised Weil et al. (1968) equations.[1]

The Weil et al. equations are divided into two cases, the first being when the conversion value is greater than the straight bond value. This equation is:

$$P = \alpha + \beta_1 \left(\frac{CV}{SBV}\right) + \beta_2 \left(\frac{CV}{SBV}\right)^2 + \beta_3\ \mathrm{DFlow} + \varepsilon \qquad (4.6a)$$

In the cases where the straight bond value is greater than the conversion value the following equation is used:

$$P = \alpha + \beta_1 \left(\frac{SBV}{CV}\right) + \beta_2 \left(\frac{SBV}{CV}\right)^2 + \beta_3\ \mathrm{DFlow} + \varepsilon \qquad (4.6b)$$

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1. The original regression equation was revised in a comment by West and Largay (1972) which restates the calculation of the floor variable.

where: P = Premium on convertible bonds.

CV = Common stock conversion value.

SBV = Straight bond value.

DFlow = Difference in the income streams of coupons and dividends.

Cross-section analyses of both the product-moment model and the Weil et al. model are done using multiple regression. There are sixty cross-sectional periods consisting of sixty-six convertible bonds. This analysis is similar to past empirical studies and expands the Weil et al. study by using a larger data base. Cross-section analysis enables comparisons between bonds with different features such as coupons, maturities, and underlying common stock log price relative distributions.

In order to compare the regression model developed in this study to that of Weil et al. (1968) as revised by West and Largay (1972), a regression is run on the total data base. Since this data base is a combination of time series and cross-section observations, a pooling technique is utilized. The technique employed (Kmenta, 1971, pp. 508-512) subjects the observations to two transformations. The first removes first order auto-regression which may be present in the time series model and the second removes heteroskedasticity in the cross-sectional model. The first transformation finds the correlation coefficient of the residuals of an ordinary least squares regression on the data base. The correlation coefficient $\hat{\rho}_i$ is calculated:

$$\hat{\rho}_i = \frac{\Sigma\ e_{it}\ e_{i,\ t-1}}{\Sigma\ e^2_{i,\ t-1}} \tag{4.7}$$

The dependent, independent, and error terms are calculated as follows:

$$Y^*_{it} = Y_{it} - \hat{\rho}_i\ Y_{i,t-1}, \tag{4.8}$$

$$X^*_{it,k} = X_{it,k} - \hat{\rho}_i X_{i,t-1,k} \quad (k = 1, 2, \ldots, k), \tag{4.9}$$

$$u^*_{it} = \varepsilon_{it} - \hat{\rho}_i\ \varepsilon_{i,t-1}, \tag{4.10}$$

$t = 2, 3, \ldots, T,$

$i = 1, 2, \ldots, N.$

The original N x T matrix is converted into a N x (T-1) matrix since one observation per time period is deleted in calculating Y^*_{it}, X^*_{it}, and u^*_{it}. The transformed variables are free of first order auto-regression but still may be heteroskedastic.

Another transformation is necessary in order to meet the requirements of homoskedasticity. This is accomplished by dividing the independent, dependent, and residual values of an ordinary least squares regression on the nonauto-regressive transformed variables by an estimated variance S_{ui}:

$$Y^{**}_{it} = \frac{Y^{*}_{i}}{S_{ui}}, \tag{4.11}$$

$$X^{**}_{it,k} = \frac{X_{it,k}}{S_{ui}} \qquad (k = 1, 2, \ldots, k), \tag{4.12}$$

$$U^{**}_{it} = \frac{U^{*}_{it}}{S_{ui}}, \tag{4.13}$$

$t = 2, 3, \ldots, T,$

$i = 1, 2, \ldots, N.$

The final transformed regression can be estimated by the ordinary least squares method, its residuals free of first order auto-regression and heteroskedasticity for N(T-1) pooled observations.

These transformations were performed on the Weil et al. (1968) model and on the product-moment model developed in this study.

CHAPTER 5

EMPIRICAL RESULTS AND CONCLUSIONS

The objectives of this chapter are to discuss and analyze the empirical comparisons described in Chapter 4 in order to draw logical conclusions concerning the validity of the model derived in Chapter 3. Results from each empirical comparison are reported and analyzed in separate sections. All comparisons are then summarized, conclusions are drawn, and areas of further research are indicated.

Distribution Characteristics of Convertible Bonds and Associated Prices

The first empirical work undertaken in this study utilizes time series price data for convertible bonds, their corresponding common stock conversion values, their straight bond values, and Fischer's Combination Index. The log relatives of each are calculated and the distributional characteristics are evaluated. Also, the beta coefficients of each log relative series with the log relative values of Fischer's Combination Index are calculated. The full set of results is given in Appendix E. Table 5.1 summarizes the relative order of each set (convertible bond, conversion value, and straight bond value) in terms of greatest to smallest value. That is, highest value for each

Table 5.1. Summary of Distributional Characteristics

	Mean			Variance			Skewness			Beta		
Series	1	2	3	1	2	3	1	2	3	1	2	3
Convertible Bond	8	53	5	2	50	14	13	21	32	13	49	4
Conversion Value	17	15	34	51	13	2	13	34	19	53	13	0
Straight Bond	46	10	10	3	10	53	40	11	15	0	4	62

characteristic is ranked one while the lowest value is ranked three. Equal values are given equal rankings, either ones or twos.

During the sample period the highest log relative mean values were consistently observed in the straight bond series, the convertible bond series had the next highest mean values, followed by the means for the series of common stock conversion values. Clearly these rankings are highly dependent upon the period of study. It appears, however, that the convertible bond mean is a hedge between the mean returns of the equity and straight debt. This result conforms to general a priori expectations. These expectations are not concerned with the ranking of returns between debt and equity over a stated period of time, but with the use of convertible bonds as a hedge between the returns of debt and equity.

The calculations of variances show the log relatives of common stock conversion values to have the largest variances followed by the log relatives of the convertible bond and straight bond series. Conventional theory and *a priori* predictions support this result as equity securities are more volatile than straight or convertible bonds. The variance calculations are not as sensitive to the period of study as the calculations for the mean and skewness parameters as shown by the results in Table 5.1. The convertible bond again appears to be a hedge, in this case a hedge against the risk associated with the corresponding common stock.

Skewness characteristics of each series do not conform with *a priori* expectations based on existing theory. Perhaps the time period has influenced this result. The straight bond values are shown to be most positively skewed, while the convertible bond values have the least positive skewness. This result conflicts with the distribution of convertible bonds derived in the present study and other studies. The truncation effect of the distribution insures the convertible bond of a higher positive skewness than the corresponding common stock. There are two possible explanations which are not necessarily mutually exclusive. The first, period of study, has already been mentioned. Over time one would expect the log relative distributions of common stock prices to be more positively skewed than those of straight bond prices. This point albeit true does not completely solve the problem since it is generally accepted that the convertible bond is more positively skewed than the corresponding common stock. The second possible explanation which specifically addresses this point is the effect of the premium addition. A derived theoretical distribution, such as Poensgen's model, is a valuation without the addition of a premium. The addition of the premium reduces the skewness effect as well as the mean return of the convertible bond by moving the price of the convertible above the larger of the conversion or straight bond value.

The results of calculating beta coefficients are interesting. The highest beta coefficients are between the log relatives of the common stock conversion values and the index. These beta coefficients are followed in order of value by those of the convertible bond and the straight bond series. As in the Walter and Que (1973) study, the beta coefficients of the common stock conversion values are greater than those of the convertible bond series in 53 out of 66 cases. The exceptions are shown in Table 5.2. From the values presented it is apparent that the exceptions are all cases where the beta values are nearly equal. The complete set of beta values are shown in Appendix D. The

Table 5.2. Beta Coefficient Exceptions

Bond	Rating	Coup	MPT	Convertible Bond Beta	Conversion Value Beta
Alcoa	BBB	5-1/4	1991	.73558	.67547
Bulova	BB	4-1/2	1984	1.71486	1.70948
Celanese	BB	4	1990	.43663	.23652
Commonwealth Oil	BB	4-1/4	1992	.41826	.35251
General Tele. Corp.	BBB	4-1/2	1977	1.15124	1.11628
Grants Co.	BBB	4	1990	.98402	.94944
Hawaiian Elect.	BBB	4-1/8	1982	.43147	.31455
Interstate Dept. Store	BB	4-5/8	1981	2.08749	1.84680
Keystone Steel & Wire	BB	4-1/2	1981	.41273	.40175
Northern Ind. Pub. Ser.	A	4	1976	.72904	.72346
Rohr Ind.	BB	5-1/4	1986	.82153	.60358
Sola Basic Ind.	BB	4-1/2	1992	.77142	.76080
United Aircraft	BB	4-1/2	1988	1.13858	1.07768

beta values for the straight bond series are quite small in most cases between zero and .2.

Cross-Section Results for Observations Where Conversion Value is Greater Than Straight Bond Value

The results of regression equations for each monthly data period are calculated for this study's model and the revised Weil et al. model. The resulting unadjusted R^2's are greater for the product-moment model in 59 out of the 60[2] cross-section regression equations. The average R^2's of each model for the 60 regression equations are:

Product-moment model average R^2 = .7195

Weil et al. model average R^2 = .6192

Thus, on the average, this study's model appears to explain 10 per cent more of the premium's variation than the comparative model.

Comparing standard errors of each model shows the product-moment model to have a lower standard error of the

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2. The unadjusted R^2's are reported to be larger in 59 out of 60 cases for the product-moment model. When the R^2's are corrected for degrees of freedom the product-moment model is superior in 56 of 60 cases. The adjusted figures are consistent with the observed standard errors of estimate. The adjusted R^2 is calculated in the following way:

$$\text{adjusted } R^2 = 1 - \frac{n - 1}{n - k}(1 - R^2)$$

where: n = number of observations.
k = total number of variables.

estimate in 56 out of 60 cases. The average of the standard errors for the 60 regression equations are 4.38 for the product-moment model and 4.89 for the Weil et al. model. The lower standard error for this study's model indicates a slightly superior predictive value when forecasting convertible bond premiums.

Both regression models have F-values that are highly significant. The product-moment model has significant F-values at the .01 level in 58 out of 60 cases. The remaining two cases have F-values which fall below the .05 significance level. The regression equations for the Weil et al. model also show 58 out of 60 cases with F-values significant at the .01 level. However, one of the two remaining cases has an F-value which is significant at the .05 level. There is little difference in the significance properties of the two models. The complete regression results of both models are in Appendix F.

The signs and significance of the regression variables are summarized in Table 5.3.

The cross-section regressions for the above case yield both expected and unexpected results. The variable for difference in income flow always has a positive coefficient and in 55 out of 60 cases the associated t-value is at the .01 level. The product-moment variables are not as consistently significant as the income difference variable

Table 5.3. Summary of Regression Variables Using Observations Where Conversion Value Exceeds Straight Bond Value

	DFlow	μ_{10}	μ_{20}	μ_{11}	μ_{30}	μ_{21}	μ_{12}
Positive Signs	60	47	24	44	38	50	50
Significance .01	55	10	0	3	2	2	7
Significance .05	0	6	4	4	5	10	14
Negative Signs	0	10	36	15	16	10	9
Significance .01	0	0	0	0	0	1	0
Significance .05	0	0	6	2	1	0	0

but generally provide enough significance to support their relationships with the convertible bond premium.

The mean of the convertible bonds distribution of log link relatives appears to be directly related to the premium. In 16 cases this variable's coefficients have significant t-values at the .05 level or above. This result is expected since the premium should increase as the mean of the convertible bond increases above the mean of the base distribution. The common stock distribution is the base for these observations.

The variance (μ_{20}) results are mixed; that is, no concrete relationship is evident from the results of the 60 cross-section regression equations. The signs of the coefficients are split, 24 positive and 36 negative. Of the positive coefficients 4 have t-values significant at the .05 level; six of the negative coefficients are significant at the .05 level.

The covariance variable μ_{11} is in general directly related to the premium. The coefficient is positive in 44 cases and negative in 15 cases. Seven of the positive and two of the negative coefficients have t-values significant at the .05 level or above. It appears from these results that investors prefer convertible bonds with a high expected association with the market portfolio of common stocks.

All of the skewness variables μ_{30}, μ_{21}, and μ_{12} appear to be directly related to the premium. The most significant variable is the co-skewness product-moment μ_{12}. This variable has a positive coefficient in 50 cases and a negative coefficient in 9 cases. The positive coefficients are significant at the .01 level in seven cases and an additional 14 cases are significant at the .05 level. The other co-skewness product-moment, μ_{21}, is positive in 50 cases and negative 10 times. Twelve positive t-values are significant at the .05 level or above while only one negative t-value is significant above the .05 level. The skewness factor μ_{30} is nearly as influential having 38 positive and 16 negative coefficients. Significant t-values above the .05 level are found for 7 of the positive coefficients and one of the negative coefficients.

In summary, the results from the product-moment model in the case where conversion value exceeds straight bond value support the theoretical model derived in Chapter 3.

Analyzing the regression variables, it is interesting to note the lack of any distributional characteristics which are consistently inversely related to the convertible bond premium. This is contrary to the usual risk return context in which securities are evaluated. The usual surrogate for rish is either the variance of the security or its systematic variation with the market portfolio. In this case variance is inconclusive in its relationship with the premium and the systematic variation surrogate increases with the premium's value. Such a result leads to the questions of what, if any, risk proxy is needed for convertible bonds.

The first and third product-moment characteristics show a positive relationship with the premium as was expected <u>a priori</u>. Thus all of the first six product-moments appear to be positively associated with the convertible bond premium.

Cross-Section Results for Observations Where Straight Bond Value is Greater Than Conversion Value

This is the most difficult case for which to explain the premium attached to the convertible bond. Past researchers have ignored it by stating that the problem is asymmetric.

The regression equation for this study's model explains more of the premium's variation than the Weil et al. (1968) model in all of the 60[3] cross-section time periods in which regressions were run. The average R^2's for each model are:

Product-moment model average R^2 = .756

Weil et al. average R^2 = .539

Standard errors of estimate are calculated for each cross-section regression equation; in 47 out of 60 cases this study's model has a lower standard error than the Weil et al. model. Over the sixty time periods, the average standard errors of estimate for each model are:

Product-moment model average s_e = 3.55

Weil et. average s_e = 4.37

Therefore, on the average, the product moment model seems to have greater predictive value than the Weil et al. model.

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3. See footnote 2.

Results of significance tests are similar for both models; each model has 26 cross-section regressions with significant F-values at the .01 level. At the .05 level 13 additional product-moment regressions have significant F-values while the Weil et al. model has 15 cross-section regressions with significant F-values. Neither model defines a regression equation with consistent significance over the 60 cross-section periods. This is not surprising since this particular situation occurs when equity prices are depressed and many outside forces may have significant effects on a convertible security. At such times investors are continually reevaluating the risk-return features of their portfolio. During times of very high or very low equity prices it is possible that investors are more influenced by the inflow of new information. Information which would not normally have any great effect on the market may have a magnified effect during these periods leading to investors over reacting for short periods of time.

The variables signs and significance are summarized in Table 5.4.

The regression results in this case are not as conclusive as the case summarized in Table 5.3. The independent variable income flow has negative coefficients in seven out of fifty-eight regressions. However, eleven positive coefficients have t-values significant at the

Table 5.4. Summary of Regression Variables Using Observations Where Straight Bond Values Exceed Conversion Values

	DFlow	μ_{10}	μ_{20}	μ_{11}	μ_{30}	μ_{21}	μ_{12}
Positive Signs	51	13	37	31	35	28	25
Significance .01	11	0	1	0	2	0	1
Significance .05	11	0	6	3	2	3	3
Negative Signs	7	41	19	25	20	28	30
Significance .01	0	6	0	1	6	1	2
Significance .05	0	12	3	3	3	3	3

.01 level and an additional eleven positive coefficients have t-values significant at the .05 level. There are no negative coefficients with t-values that are significant at or above the .05 level.

Variables described by the product-moment model are more abstract in this particular case. The relationships are simplified when one realizes the maximum premium theoretically occurs when the conversion value is equal to the straight bond value. The mean of the convertible bond distribution of log relatives μ_{10} has a negative coefficient in 41 out of 54 cases. All coefficients that are significant are negative. Six have t-values which are significant at the .01 level while another 12 are significant at the .05

level. This result is consistent with theory although it may appear contrary. The product-moment model is based on the distribution of the underlying common stock as shown in Figure 3.2. In cases where the straight bond value is greater than the current conversion value, the mean product-moment is defined as:

$$\mu_{10} = a\Phi + \sigma_x\phi \qquad (3.16)$$

is biased. The premium in the present case is related only to the second term in Equation (3.16). As the difference between the straight bond value and the conversion value becomes larger, this value decreases.

The variance μ_{20} results indicate a positive relationship with the premium in 37 out of 56 cases. Of these 37 cases one coefficient has a t-value significant at the .01 level and another 6 have t-values significant at the .05 level. Coefficients that are negative have t-values significant at the .05 level in 3 cases. The investor appears to prefer convertible bonds with higher variance. This is reasonable since the convertible bonds in these observations are trading at low value supported by the straight bond values. Hence at low price levels investors prefer convertibles with high variation since the downward possibilities are small as a result of the floor value.

The covariance variable μ_{11} is not consistently related with the premium. The coefficients have 31 positive signs and 25 negative signs. The positive coefficients have significant t-values at the .05 level for 3 cross-section regression time periods. The negative coefficients have 1 time period in which the t-value is significant at the .01 level and another 3 time periods where the t-values are significant at the .05 level. Thus covariance or beta coefficient does not appear to be a relevant proxy for risk in analyzing these premium observations.

The skewness variable μ_{30} is positively related to the premium in 35 out of 55 cases. However, of the 35 cases only 4 have significant t-values at or above the .05 level. The negative coefficients have 6 t-values significant at the .01 level and another 3 t-values significant at the .05 level. The two co-skewness variables μ_{21} and μ_{12} have mixed coefficients. Variable μ_{21} has 28 positive and 28 negative coefficients. Three of the positive coefficients have t-values significant at the .05 level while 4 negative coefficients have significant t-values at the .05 level or above. Variable μ_{12} has 25 coefficients with positive signs and 30 with negative signs. The t-values are significant for 4 positive and 5 negative coefficients at the .05 level or above. Neither of the co-skewness terms appears to give consistent results as an explanatory variable.

Summarizing, it appears that the regression results for these cross-section observations are not conclusive. The asymmetry problem does seem to exist. The model presented in this study may be able to better accommodate the problem than can the Weil et al. model. By using the product-moments the regression model is more apt to reflect the changing relationships which exist when the straight bond value increases above the value of the common stock conversion. The variables used in the Weil et al. study are designed for that specific case and are not as general as the variables defined in this study. One point which may help eliminate the asymmetry problem is to let the base distribution vary between the underlying common stock and straight bond distributions. This change would eliminate all a values in the derivations in Chapter 3. A complication of this extension is to change the straight bond value to a stochastic variable. The derivations would then have to accommodate two stochastic variables rather than one. This would result in a more general and accurate model.

An observation from the regression results is the lack of explanatory value in the market associated product-moments. Possibly this occurred because the market for common stocks were depressed and investors are emphasizing other factors related to the value of convertible bonds. During such times characteristics of the common stock distribution may be swamped by expectations in the term structure of interest rates.

Pooling of Data

The results of the regression for pooled time series and cross-sectional data are reported for this study's model and the revised Weil et al. (1968) model.

Using regression Equation (4.5) the following results are obtained for this study's model.

$$P = .473 - \underset{(1.76)^{*}}{1.363\mu_{10}} - \underset{(2.75)^{**}}{10.804\mu_{20}} + \underset{(27.55)^{***}}{1729.739\mu_{11}} - \underset{(14.87)^{***}}{51.312\mu_{30}}$$

$$+ \underset{(24.06)^{***}}{1468.725\mu_{21}} + \underset{(6.44)^{***}}{536.343\mu_{12}} + \underset{(36.28)^{***}}{.226\ \text{DFlow}}$$

$R^2 = .53507$

$F = 638.8894^{***}$

$D\text{-}W = 1.05034$

$s_e = .92969$

*** = Significant at the .0005 level

** = Significant at the .005 level

* = Significant at the .2 level

For the WSG regression, Equations (4.6a) and (4.6b), the results are as follows:

$$P = .391 + \underset{(20.13)^{***}}{3.865\ \text{floor}} + \underset{(22.51)^{***}}{1.401\ \text{FLSQ}} + \underset{(34.42)^{***}}{.119\ \text{DFlow}}$$

R^2 = .3245

F = 622.95294***

D-W = .94246

s_e = .9551

*** = Significant at the .0005 level

This study's pooled regression model explains 53.5 per cent of the variation in the dependent variable, the convertible bond premium. WSG's pooled regression model explains only 32.5 per cent of the variation in the dependent variable. Both regression equations have very high F-values partially due to the large number of total observations (3894). The standard error of the estimate of this study's model is (.92969), slightly smaller than that of the WSG model (.95511). Both Durbin-Watson statistics show high positive autocorrelation, although both equations have been transformed to eliminate first order autocorrelation. Therefore, the relationship between the residuals must be more involved than simple correlation between residuals of successive time periods. It would be difficult and very time consuming to pursue these relations further, even though it would improve the rigor of the tests.

From a statistical point of view it is not proper to compare the two models directly, since they have different numbers of independent variables. In order to obtain more comparable results, the pooled regression for this study's model uses stepwise regression. The following results are found after the introduction of the third independent variable:

$$P = .525 + 1485.619\mu_{11} + 941.212\mu_{21} + .211\ DFlow$$

$$(27.86)^{***} \quad (40.64)^{***} \quad (38.78)^{***}$$

$R^2 = .50516$

$F = 1323.7341^{***}$

$S_e = .95863$

*** = Significant at the .0005 level

This regression equation explains 50.5 per cent of the variation in the premium. The F value is very high and is significant at the .0005 level for the entire equation. Each independent variable also has a significant t-value at the .0005 level. The standard error of the above regression equation is (.95863), which is slightly larger than the standard error of this study's regression equation with seven independent variables.

In summary, the regression model presented in this section results in a higher R^2 for both the seven variable and three variable interpretations than the corresponding statistics for the Weil et al. model. Although the statistical results seem to favor the model developed in this study, the strongest argument for the model is its theoretical development. WSG introduce a squared term for the floor variable to account for non-linearity, which increases the R^2. The product moment model theoretically justifies relationships of non-linearity rather than including an ad hoc variable.

In both models the income flow difference is an important variable having the highest t-value. The coefficients are positive for both models as predicted *a priori* and are consistent with current theory. Income flow difference is defined as the difference in the yearly income of the convertible bond coupon and the yearly income from the dividend of one share of common stock times the conversion ratio. Thus the premium increases as the income flow difference increases. Income flow difference explains 28.77 per cent of the variation of the premium in this regression model and 19.99 per cent in the WSG model.

The use of the theoretically derived product moments in the regression models appears to produce a better fit than the naive variables, floor value, and floor value squared used by WSG. All product moments except μ_{10} and μ_{20} are highly significant as indicated by their t-values. The independent variable μ_{10} is significant at the .2 level. Its coefficient does not have the expected positive sign, which would signify a direct relationship between the mean and the premium. It is possible that the bias mentioned in the cross-section regression for observations where straight bond value exceeds conversion value also influences μ_{10} in the pooled case.

The variance of a convertible bond μ_{20} has a negative coefficient indicating that investors react negatively to increased variance in convertible bonds. Therefore the lower the variance the higher the premium the investors are willing to pay.

An interesting result is that the coefficient of the covariance term is positively related to the premium. This term is the basis for the convertible bond's beta coefficient with the market. Thus two measurements of risk used in analyzing common stocks are contrarily related to convertible bond premiums. The convertible bond investor pays a higher premium for both lower variance and higher comovement with the market. This result may have four possible explanations:

1. The data base used to calculate the standard deviation and correlation with the market of the price log relatives of each common stock price may have produced estimates with high standard errors. This could influence the resulting covariance term to some degree. However, the covariance term is highly significant and it is doubtful that the data could be the source of error to such an influential degree.

2. The risk measurement for convertible bonds may not be consistent with the two parameter capital market equilibrium model. The upper moments may be influential in the assessment of total equilibrium risk.
3. It is also possible that convertible bond holders are investors who are willing to accept a lower average return for the small probability of possible large gains. The investor undergoes a large probability of a small loss (the difference in a straight bond yield and a convertible bond yield in the same risk class) as opposed to a small probability of a large gain (the chance of the common increasing significantly in value).
4. Convertible bond investors may view risk differently depending upon the relationship between conversion value and straight bond value. As shown in the cross-section analysis of these special situations, the variance and covariance variables change signs and show inconsistencies in their significance.

The coefficient of the third moment about the mean μ_{30} is negative. This result is contrary to the generally held belief that investors place a positive value upon positive skewness. One possible explanation is the addition of the two co-skewness product moments μ_{21} and μ_{12}. Both of these terms have positive coefficients that are highly significant. Thus, investors may evaluate skewness on the basis of its relationship with the market portfolio rather than the skewness associated with the convertible bond as a single security.

Final Conclusions and Implications for Further Research

In summary, it is appropriate to view this work in terms of its initial objectives. To recapitulate, two primary objectives were stated:

1. To ascertain characteristics of convertible bonds which influence the size of the premium paid by investors.
2. How does the addition of a premium to the theoretical value of a convertible bond alter the distribution of actual convertible bond prices from the theoretically derived distribution in this study.

Thus the success of the study should be evaluated in terms of these objectives.

A normative model is developed to ascertain proxies for the risk-return relationship for valuing convertible bonds. This model is an extension of the capital asset pricing model and existing convertible bond valuation models. The model's product-moment generating function allows the risk-return analysis to include upper product-moments of the bivariate distribution. This enables an evaluation of convertible bonds with respect to the market portfolio by including the product-moments, mean, variance, covariance, skewness, and two coskewness factors.

Multiple regression is used to apply the derived normative model. The result is a positive model that serves as a predictive tool and outlines the approach of multiple regression and its application to determining convertible bond premiums. Also, descriptive statistics are defined for the distributions of log relatives for actual convertible bond, straight bond, and conversion values.

The results of the multiple regression equations appear to support the product-moment approach for explaining convertible bond premiums. The model performs better in cases where the conversion value is greater than the straight bond value.

Two important features are shown as the result of the descriptive characteristics for the actual distribution of convertible bond log price relatives. Both the skewness and the beta coefficient factors for convertible bonds are less than those characteristics for the underlying common stock. This result has previously been shown for beta coefficients in studies which employ small samples; however, the relationship of the skewness factors is an unexpected result.

Further research is needed in at least three areas. The introduction of varying interest rates is a necessary step toward understanding the behavior and characteristics of convertible bonds. Also simulation models, such as Walter and Que (1973), are needed to incorporate the more sophisticated models such as the one presented in this paper. The results of this study indicate that other variables may be important when evaluating convertible bonds and their premiums. Factors which may be of value and have been brought out by this study are the activity of both the capital and the money markets. These variables were not incorporated in the product-moment model but their importance has been indicated indirectly in the empirical results and conclusions.

APPENDIX A

DERIVATION OF A BIVARIATE NORMAL MOMENT GENERATING FUNCTION WHEN ONE VARIABLE IS TRUNCATED AND ALL THE TRUNCATED MASS IS CENTERED AT TRUNCATION POINT a

The bivariate distribution moment generating function is:

$$m(t_1,t_2) = \int_a^\infty \int_{-\infty}^\infty e^{xt_1 + yt_2} f(x,y)\, dydx$$

$$+ \int_{-\infty}^a \int_{-\infty}^\infty d^{at_1 + yt_2} f(x,y)\, dydx$$

Substituting the normal bivariate density function for f(x,y) the moment generating function becomes:

$$m(t_1,t_2) = \int_a^\infty \int_{-\infty}^\infty e^{t_1x + t_2y} \frac{1}{2\pi\sigma_x\sigma_y\sqrt{(1-\rho^2)}}$$

$$\exp\left\{- 1/2 \left[\frac{(x-\mu_x)^2}{\sigma_x^2} - \frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y}\right.\right.$$

$$\left.\left.+ \frac{(y-\mu_y)^2}{\sigma_y^2}\right]\right\} dydx + \int_{-\infty}^a \int_{-\infty}^\infty e^{t_1a + t_2y} \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}$$

$$\exp \left\{ - 1/2 \left[\frac{(x - \mu_x)^2}{\sigma_x^2} - \frac{2\rho (x - \mu_x)(y - \mu_y)}{\sigma_x \sigma_y} \right.\right.$$

$$\left.\left. + \frac{(y - \mu_y)^2}{\sigma_y^2} \right] \right\} dydx$$

Separating the variables

$$\int_a^\infty e^{t_1 x} \frac{1}{\sqrt{2\pi}\sigma_x} e^{-1/2} \frac{(x - \mu_x)^2}{\sigma_x} \left\{ \int_{-\infty}^\infty e^{t_2 y} \frac{1}{\sqrt{2\pi\sigma_y^2(1 - \rho^2)}} \right.$$

$$\left. \exp \left[- \frac{1}{2\sigma_y^2(1 - \rho^2)} \left(y - \mu_y - \frac{\rho\sigma_y (x - \mu_x)}{\sigma_x} \right)^2 \right] dy \right\} dx$$

$$+ \int_{-\infty}^a e^{t_1 a} \frac{1}{\sqrt{2\pi}\sigma_x} e^{-1/2} \left(\frac{x - \mu_x}{\sigma_x} \right)^2 \left\{ \int_{-\infty}^\infty e^{t_2 y} \frac{1}{\sqrt{2\pi\sigma_y^2(1 - \rho^2)}} \right.$$

$$\left. \exp \left[- \frac{1}{2\sigma_y^2(1 - \rho^2)} \left(y - \mu_y - \frac{\rho\sigma_y (x - \mu_x)}{\sigma_x} \right)^2 \right] dy \right\} dx$$

Completing the squares and integrating with respect to y, the function becomes

$$\int_a^\infty e^{t_1 x} \frac{1}{\sqrt{2\pi}\sigma_x} e^{-1/2} \left(\frac{x - \mu_x}{\sigma_x} \right)^2 \exp \left\{ t_2 \mu_y + \frac{t_2 \rho \sigma_y}{\sigma_x} (x - \mu_x) \right.$$

$$\left. + 1/2 t_2^2 \sigma_y^2 (1 - \rho^2) \right\} dx$$

$$+ \int_{-\infty}^{a} e^{t_1 a} \frac{1}{\sqrt{2\pi}\sigma_x} e^{-1/2\left(\frac{x - \mu_x}{\sigma_x}\right)^2} \exp\left\{t_2\mu_y\right.$$

$$\left. + \frac{t_2\rho\sigma_y}{\sigma_x} (x - \mu_x) + 1/2t_2^2\sigma_y^2 \ (1 - \rho^2)\right\} dx$$

Rewriting with all constants outside the integral one obtains:

$$\left\{\exp\left[t_2\mu_y \ \rho\frac{\sigma_y}{\sigma_x} t_2\mu_x + \frac{t_2^2\sigma_y^2(1 - \rho^2)}{2}\right]\right\}$$

$$\left\{\int_{a}^{\infty} e^{t_1 x + t_2\rho \frac{\sigma_y}{\sigma_x} x} \frac{1}{\sqrt{2\pi}\sigma_x} e^{-1/2} \left(\frac{x - \mu_x}{\sigma_x}\right)^2 dx\right.$$

$$\left. + e^{t_1 a} \int_{-\infty}^{a} e^{t_2\rho\frac{\sigma_y}{\sigma_x} x} \frac{1}{\sqrt{2\pi}\sigma_x} e^{-1/2} \left(\frac{x - \mu_x}{\sigma_x}\right)^2 dx\right\}$$

Completing the squares:

$$\left\{\exp\left[t_2\mu_y - \rho t_2\mu_x \frac{\sigma_y}{\sigma_x} + \frac{t_2\sigma_y^2(1 - \rho^2)}{2}\right]\right\}$$

$$\left\{\int_{a}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left[- \frac{1}{2\sigma_x^2}\left[(x - \mu_x - \rho\sigma_x\sigma_y t_2 - t_1\sigma_x^2)^2\right.\right.\right.$$

$$\left.\left. - \mu_x^2 - (\mu_x + t_1\sigma_x^2 + t_2\rho\sigma_y\sigma_x)^2\right]\right] dx$$

$$+ e^{t_1 a} \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left[- \frac{1}{2\sigma_x^2}\left[(x - \mu_x - \rho\sigma_x\sigma_y t_2)^2\right.\right.$$

$$\left.\left. - \mu_x^2 - (\mu_x^2 + \rho\sigma_y\sigma_x t_2)^2\right]\right] dx \Bigg\}$$

Re-arranging and summing terms:

$$\left\{\exp\left[t_1\mu_x + t_2\mu_y + 1/2(t_1^2\sigma_x^2 + 2\rho t_1 t_2\sigma_x\sigma_y + t_2\sigma_y^2)\right]\right\}$$

$$\left\{\int_a^{\infty} \frac{1}{\sqrt{2\pi}\sigma_x} e^{-1/2\left(\frac{x - \mu_x - t_1\sigma_x^2 - t_2\rho\sigma_y\sigma_x}{\sigma_x}\right)^2} dx\right\}$$

$$+ \left\{\exp\left[t_1 a + t_2\mu_y + 1/2(t_2^2\sigma_y^2)\right]\right\}$$

$$\left\{\int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}\sigma_x} e^{-1/2\left(\frac{x - \mu_x - \rho\sigma_y\sigma_x t_2}{\sigma_x}\right)^2} dx\right\}$$

Integrating each term with respect to x:

$$\left\{\exp\left[t_1\mu_x + t_2\mu_y + 1/2(t_1^2\sigma_x^2 + 2\rho t_1 t_2\sigma_x\sigma_y + t_2\sigma_y^2\right]\right\}$$

$$\left\{1 - \Phi\left[\frac{a - \mu_x - t_1\sigma_x^2 - t_2\rho\sigma_y\sigma_x}{\sigma_x}\right]\right.$$

$$+ \left\{\exp\left[t_1 a + t_2\mu_y + 1/2(t_2^2\sigma_y^2)\right]\right\}$$

$$\left\{\Phi\left[\frac{a - \mu_x - \rho\sigma_y\sigma_x t_2}{\sigma_x}\right]\right\}$$

This is the product-moment generation for a bivariate distribution of a normal distribution and a truncated distribution of class 2, as described by Poensgen.

APPENDIX B

DERIVATION OF MOMENTS ABOUT THE MEAN

The product-moment generator derived in Appendix A is for moments about the origin. In order to conform with existing theory they must be transformed to product-moments about the mean. These derivations result in the needed product-moments.

Given: Product-moments about the origin

$U_{10} = E(\tilde{x})$

$U_{20} = E(\tilde{x}^2)$

$U_{11} = E(\tilde{x}\tilde{y})$

$U_{30} = E(\tilde{x}^3)$

$U_{21} = E(\tilde{x}^2\tilde{y})$

$U_{12} = E(\tilde{x}\tilde{y}^2)$

Find: Product-moments about the mean

$U'_{10} = 0$

$U'_{20} = E[(\tilde{x} - \overline{x})^2]$

$U'_{11} = E[(\tilde{x}-\overline{x})(\tilde{y}-\overline{y})]$

$U'_{30} = E[(\tilde{x}-\overline{x})^3]$

$$U'_{21} = E[(\tilde{x}-\bar{x})^2(\tilde{y}-\bar{y})]$$

$$U'_{12} = E[(\tilde{x}-\bar{x})(\tilde{y}-\bar{y})^2]$$

$$U'_{20} = E[(\tilde{x}^2 - 2\tilde{x}\bar{x} + \bar{x}^2)]$$

$$= E(\tilde{x}^2) - 2E(\tilde{x})\bar{x} + \bar{x}^2$$

$$= E(\tilde{x}^2) - E(\tilde{x})^2 \qquad \text{since } E(\tilde{x}) = \bar{x}$$

$$U'_{20} = U_{20} - U_{10}^2$$

$$U'_{11} = E[(\tilde{x}-\bar{x})(\tilde{y}-\bar{y}]$$

$$= E(\tilde{x}\tilde{y} - \bar{x}\tilde{y} - \tilde{x}\bar{y} + \bar{x}\bar{y})$$

$$= E(\tilde{x}\tilde{y}) \qquad \text{since } E(\tilde{y}) = \bar{y} = 0$$

$$U'_{11} = U_{11}$$

$$U'_{30} = E[(\tilde{x}-\bar{x})^3]$$

$$= E[(\tilde{x}^3 - 3\tilde{x}^2\bar{x} + 3\tilde{x}\bar{x}^2 - \bar{x}^3)]$$

$$= E(\tilde{x}^3) - 3E(\tilde{x}^2)\ E(\tilde{x}) + 2E(\tilde{x})^3$$

$$\text{since } E(\tilde{x}) = \bar{x}$$

$$U'_{30} = U_{30} - 3U_{20}U_{10} + 2U_{10}^3$$

$$U'_{21} = E[(\tilde{x}-\bar{x})^2(\tilde{y}-\bar{y})]$$

$$= E[(\tilde{x}^2\tilde{y} - 2\tilde{x}\tilde{y}\bar{x} + \bar{x}^2\tilde{y} - \tilde{x}^2\bar{y} - 2\tilde{x}\bar{x}\bar{y} + \bar{x}^2\bar{y})]$$

$$= E(\tilde{x}^2\tilde{y}) - 2E(\tilde{x}\tilde{y})\ E(\tilde{x}) \qquad \text{since } E(\tilde{y}) = \bar{y} = 0$$

$$U'_{21} = U_{21} - 2U_{11}U_{10}$$

$$U'_{12} = E[(\tilde{y}-\bar{y})^2(\tilde{x}-\bar{x})]$$

$$= E[(\tilde{y}^2\tilde{x} - 2\tilde{y}\bar{y}\tilde{x} + \bar{y}^2\tilde{x} - \tilde{y}^2\bar{x} - 2\tilde{y}\bar{y}\bar{x} + \bar{y}^2\bar{x})]$$

$$= E(\tilde{y}^2\tilde{x}) - E(\tilde{y}^2)\ E(\tilde{x}) \qquad \text{since } E(\tilde{y}) = \bar{y} = 0$$

$$U'_{12} = U_{12} - U_{02}U_{10}$$

APPENDIX C

CONVERTIBLE BOND SAMPLE

Listing of convertible bond issues, ratings, coupons, and maturities for all bonds used in this study. The number is used in following Appendices for identification.

Issue	Rating	Coupon	Maturity
1. Air Reduction	BBB	3-7/8	1987
2. Allied Stores	BBB	4-1/2	1981
3. Allied Stores	BBB	4-1/2	1992
4. Alcoa	BBB	5-1/4	1991
5. Amerace Esna	BBB	5	1992
6. American Airlines	BB	4-1/4	1992
7. American Mach. & Found.	BB	4-1/4	1981
8. Armstrong Rubber	BB	4-1/2	1987
9. Ashland Oil	BBB	4-3/4	1993
10. Baltimore Gas & Electric	A	4-1/4	1974
11. Baxter Labs.	BBB	4	1987
12. Black & Decker	BBB	4	1992
13. Bobbie Brooks	BB	5-1/4	1981
14. Bulova Watch	BB	4-1/2	1984
15. Burlington Industries	BBB	5	1991
16. Celanese Corp.	BB	4	1990
17. Central Hudson Gas & Electric	BBB	4-3/8	1981
18. Cessna Aircraft	BB	3-7/8	1992
19. Chock Full of Nuts	BB	4-1/2	1981
20. Cluett Peabody	BB	4-1/2	1984
21. Commercial Solvents	BB	4-1/2	1991
22. Commonwealth Oil Ref.	BB	4-1/4	1992
23. Control Data	BB	3-3/4	1989
24. Copperweld Steel	BB	5	1979
25. E. G. & G.	BB	3-1/2	1987
26. FMC	BBB	3-1/8	1981
27. FMC	BBB	4-1/4	1992
28. Foremost Dairies	BB	5-1/2	1980
29. General Instruments	BB	4-1/4	1985
30. General Instruments	BB	5	1992

	Issue	Rating	Coupon	Maturity
31.	General Telephone Corp.	BBB	4-1/2	1977
32.	General Telephone & Electric	BB	4	1990
33.	General Telephone & Electric	BB	5	1992
34.	Grants (W.T.) Co.	BBB	4	1990
35.	Grumman Air	BB	4-1/4	1992
36.	Hawaiian Electric	BBB	4-1/8	1982
37.	Intl. Minerals	BB	4	1991
38.	Interstate Bakeries	BB	5-1/4	1990
39.	Interstate Dept. Stores	BB	4-5/8	1981
40.	Interstate Dept. Stores	BB	4	1992
41.	Keystone Steel & Wire	BB	4-1/2	1981
42.	Macy's	BBB	4-1/4	1990
43.	Macy's	BBB	5	1992
44.	McGraw Hill	A	3-7/8	1992
45.	National Distillers	BB	4-1/2	1992
46.	Northern Indiana Pub. Sv.	A	4	1976
47.	Northern Indiana Pub. Sv.	A	4-1/4	1992
48.	Northrop Corp.	BB	4-3/4	1987
49.	Owens-Illinois	BBB	4-1/2	1992
50.	Parker-Hannifin	BB	4	1992
51.	RCA	A	4-1/2	1992
52.	Reeves Brothers	BB	4	1991
53.	Revere Copper	BB	5-1/2	1992
54.	Reynolds Metal	BB	4-1/2	1991
55.	Rohr Ind.	BB	5-1/4	1986
56.	Skil Corp.	BB	5	1992
57.	Sola Basic Ind.	BB	4-1/2	1992
58.	Southern Calif. Edison	A	3-1/8	1980
59.	Stauffer Chemical	BBB	4-1/2	1991
60.	Stokely-Van Camp	BB	4-1/4	1982
61.	Trane Co.	A	4	1992
62.	Trans World Air	BB	4	1992
63.	United Aircraft	BB	4-1/2	1988
64.	United Aircraft	BB	5-3/8	1991
65.	United Aircraft	BB	4-1/2	1992
66.	United Merch. & Manuf.	BB	4	1990

APPENDIX D

STATISTICS FOR THE ASSOCIATED COMMON STOCKS

Standard deviations of common stock prices and their correlation coefficients with the market are calculated from the CRSP data file of monthly stock prices. The index used to calculate correlation coefficients is the Fisher Combination Index.

Common Stock	Standard Deviation	Correlation Coefficient
Air Reduction	.10523	.55103
Allied Stores	.67407	.83406
Alcoa	1.01381	.38954
Amerace Esna	.84137	.28138
American Airlines	.18080	.32062
American Machine & Found.	.13200	.52362
Armstrong Rubber	.71113	.63888
Ashland Oil	1.10790	.45158
Baltimore Gas & Electric	1.00000	.14417
Baxter Labs	.79653	.46299
Black & Decker	.84770	.59304
Bobbie Brooks	.84140	.27466
Bulova Watch	.10449	.58933
Burlington Industries	1.17297	.58194
Celenese Corp.	.90191	.76101
Central Hudson Gas & Electric	1.00000	.30883
Cessna Aircraft	.81121	.49385
Chock Full of Nuts	.12017	.23208
Cluett Peabody	.63493	.52103
Collins and Ackman	.81687	.02153
Commercial Solvents	.79261	.58442
Commonwealth Oil Ref.	.29952	.48504
Control Data	.94792	.31387
Copperweld Steel	.81105	.63296
E. G. & G.	.76686	.79530
FMC	1.02141	.58420
Foremost Dairies	1.00000	.55465

Common Stock	Standard Deviation	Correlation Coefficient
General Instruments	.77550	.53475
General Telephone Corp.	1.38366	.31115
Grants (W.T.) Co.	.83969	.51471
Grumman Air	.82247	.44354
Hawaiian Electric	1.00000	.58665
Intl. Minerals	.88793	.60284
Interstate Bakery	1.00000	.55644
Interstate Dept. Stores	1.14278	.69293
Keystone Steel & Wire	.05322	.39495
Macy's	.80441	.65089
McGraw Hill	.93225	.47821
National Distillers	1.07142	.48332
Northern Indiana Public Ser.	1.00000	.48124
Northrup Corp.	.91525	.25172
Owens Illinois	1.00506	.54657
Parker Hannifen	.09248	.61894
RCA	1.28996	.66991
Reeves Brothers	.71414	.47945
Revere Copper	.74788	.72533
Reynolds Metal	1.05043	.62548
Rohr Ind.	.80605	.40661
Skil Corp.	.84212	.44639
Solv Basic Ind.	.15037	.31628
Southern Cal Edison	1.00000	.50843
Stauffer Chemical	.94981	.43370
Stokley Van Camp	.90214	.67982
Trane Co.	.93358	.38151
Trans World Air	.11786	.64525
United Air Craft	.96246	.65341
United Merch. & Manuf.	1.10997	.58776

APPENDIX E

DISTRIBUTION CHARACTERISTICS OF CONVERTIBLE BOND AND ASSOCIATED PRICES

Tabular presentation of the characteristics of the series of log relatives for convertible bond values, conversion values, and straight bond values. The sample period is January, 1968 to December, 1972. The first four moments and the beta coefficients are reported. Set number corresponds with the bond number in Appendix C.

The securities are designated as follows:

CB = Convertible bond,

CS = Common stock,

SB = Straight bond.

Set	Security	Mean	Variance	Skewness	Kurtosis
1	CB	-.007	.009	-.842	12.131
	CS	-.011	.027	-.162	8.508
	SB	.000	.005	-.380	7.379
2	CB	.000	.014	.056	2.483
	CS	.000	.021	.157	5.364
	SB	.001	.003	.490	6.055
3	CB	-.001	.004	.379	.238
	CS	-.000	.011	-.038	1.009
	SB	-.000	.001	-.143	.006
4	CB	-.004	.002	.078	.444
	CS	-.006	.012	.529	2.901
	SB	-.001	.000	-.805	2.099

Set	Security	Mean	Variance	Skewness	Kurtosis
5	CB	-.003	.004	.300	1.312
	CS	-.004	.008	.044	.133
	SB	-.001	.001	-.201	4.552
6	CB	-.001	.006	.165	1.051
	CS	-.003	.018	-.693	.601
	SB	-.003	.004	-.327	8.937
7	CB	.005	.002	.060	-.208
	CS	.016	.011	.043	1.884
	SB	.001	.006	-.204	17.486
8	CB	-.003	.004	-.834	2.036
	CS	-.004	.006	.140	.022
	SB	-.001	.000	.077	.498
9	CB	-.003	.003	-.323	-.084
	CS	-.004	.010	-.409	1.673
	SB	.001	.004	-.375	21.207
10	CB	.002	.003	.264	.267
	CS	.002	.003	.066	-.106
	SB	.001	.000	.491	-.061
11	CB	.013	.004	-.578	-.001
	CS	.016	.004	-.554	.010
	SB	-.001	.001	-.613	2.696
12	CB	.016	.004	.106	-.345
	CS	.018	.004	-.224	.361
	SB	.000	.005	-.525	6.390
13	CB	-.002	.002	.099	.233
	CS	-.007	.015	.147	-.650
	SB	-.002	.001	3.283	20.017
14	CB	-.003	.012	-.433	.259
	CS	-.002	.013	-.441	.174
	SB	-.000	.001	3.125	16.982
15	CB	-.002	.002	-.202	1.043
	CS	-.002	.007	.016	.829
	SB	-.000	.001	-.957	8.576
16	CB	-.005	.003	-.233	.005
	CS	-.008	.007	-.409	.996
	SB	-.001	.001	2.211	10.638

Set	Security	Mean	Variance	Skewness	Kurtosis
17	CB	-.001	.001	-.748	1.954
	CS	-.001	.003	.661	1.983
	SB	-.000	.004	-.281	17.645
18	CB	.006	.005	-1.095	3.141
	CS	.009	.011	-.542	1.404
	SB	-.002	.010	.037	6.610
19	CB	-.005	.001	-.035	-.848
	CS	-.017	.013	.533	-.066
	SB	.001	.002	3.727	22.335
20	CB	-.004	.012	-.829	3.193
	CS	-.004	.010	-1.441	4.010
	SB	.000	.043	.113	25.257
21	CB	-.004	.002	-.094	.083
	CS	-.011	.017	.272	.395
	SB	-.001	.001	3.624	20.572
22	CB	-.008	.003	-.019	-.850
	CS	-.013	.025	-.271	6.920
	SB	-.002	.001	-.425	1.069
23	CB	-.011	.011	-.340	-.295
	CS	-.014	.020	-.370	.564
	SB	-.001	.002	1.174	4.731
24	CB	.002	.002	.505	1.268
	CS	.004	.008	.387	.906
	SB	.000	.001	4.579	28.245
25	CB	-.011	.006	.315	.168
	CS	-.018	.018	.547	.013
	SB	.000	.001	3.159	17.020
26	CB	-.006	.007	.205	-.462
	CS	-.006	.008	.122	-.175
	SB	.001	.024	-.357	25.370
27	CB	-.000	.004	2.178	9.222
	CS	.002	.013	1.521	5.461
	SB	-.001	.002	.358	11.374
28	CB	-.004	.006	-.269	-.304
	CS	-.004	.006	-.703	1.129
	SB	.001	.001	4.957	31.031

Set	Security	Mean	Variance	Skewness	Kurtosis
29	CB	-.010	.008	-.160	-.220
	CS	-.012	.017	.015	-.570
	SB	-.001	.002	3.626	23.786
30	CB	-.007	.004	-.166	-.357
	CS	-.013	.016	.018	-.477
	SB	-.003	.005	.435	10.304
31	CB	-.005	.006	-.222	.423
	CS	-.005	.006	-.120	.317
	SB	-.001	.002	-.025	.949
32	CB	-.004	.004	-.003	.486
	CS	-.005	.006	-.253	.514
	SB	-.002	.002	2.473	13.192
33	CB	-.004	.002	-.075	-.069
	CS	-.005	.006	.120	.253
	SB	.001	.003	1.696	7.531
34	CB	.005	.023	-2.061	15.490
	CS	.005	.035	-1.902	18.016
	SB	.001	.005	-.325	20.047
35	CB	-.011	.004	.389	-.020
	CS	-.019	.010	.384	.124
	SB	-.004	.001	-.012	.181
36	CB	.003	.003	.497	.548
	CS	.004	.004	.520	-.005
	SB	-.000	.001	1.088	3.585
37	CB	-.001	.003	-.195	-.137
	CS	-.001	.015	-.053	-1.107
	SB	-.004	.002	1.577	12.240
38	CB	-.005	.005	1.520	4.322
	CS	-.007	.007	.857	2.066
	SB	-.001	.003	1.619	10.850
39	CB	-.025	.015	-.217	-.735
	CS	-.029	.014	-.237	-.809
	SB	-.001	.001	3.571	19.704
40	CB	-.012	.015	-.023	11.806
	CS	-.028	.015	.027	-.367
	SB	-.002	.002	2.899	18.942

Set	Security	Mean	Variance	Skewness	Kurtosis
41	CB	-.001	.002	-.815	1.399
	CS	-.006	.006	-.129	.983
	SB	.000	.001	3.730	19.989
42	CB	.004	.006	-.251	-.080
	CS	.004	.009	.138	-.155
	SB	.000	.000	.057	.105
43	CB	.001	.006	-.294	-.007
	CS	.004	.008	-.193	-.545
	SB	-.001	.000	-.280	.505
44	CB	-.008	.003	-.314	1.978
	CS	-.020	.011	.358	1.151
	SB	-.001	.004	.640	9.084
45	CB	-.003	.002	.380	.341
	CS	-.002	.004	.485	-.286
	SB	-.001	.001	3.350	18.818
46	CB	-.001	.008	.322	.262
	CS	-.001	.008	.329	.111
	SB	.001	.002	-1.059	14.270
47	CB	-.002	.008	-.044	10.151
	CS	-.001	.030	.199	6.234
	SB	-.000	.001	-.098	.174
48	CB	-.019	.027	-1.771	13.074
	CS	-.027	.060	-1.317	10.146
	SB	-.003	.002	.325	5.320
49	CB	-.003	.004	-.679	2.123
	CS	-.005	.006	-.062	-.530
	SB	-.000	.001	-.014	.867
50	CB	-.003	.003	-.947	1.249
	CS	-.007	.010	-.391	1.405
	SB	-.007	.003	-.143	11.567
51	CB	-.005	.003	.255	.019
	CS	-.006	.008	.160	-.198
	SB	-.001	.001	-.798	5.058
52	CB	-.002	.005	.356	.364
	CS	-.001	.010	.458	.431
	SB	-.002	.010	.208	12.682

Set	Security	Mean	Variance	Skewness	Kurtosis
53	CB	-.007	.003	-.155	2.414
	CS	-.019	.011	-1.073	4.404
	SB	-.003	.004	.039	10.227
54	CB	-.008	.003	.554	-.066
	CS	-.017	.009	.266	-.495
	SB	-.001	.001	3.415	18.905
55	CB	-.007	.007	-1.020	2.785
	CS	-.009	.009	.362	.152
	SB	-.001	.001	3.826	22.274
56	CB	.003	.007	-1.461	4.148
	CS	.005	.020	-.678	2.255
	SB	-.002	.003	2.824	20.847
57	CB	.003	.005	.167	-.285
	CS	-.003	.010	.179	-.173
	SB	-.002	.004	2.659	23.528
58	CB	-.001	.002	-.041	1.342
	CS	-.003	.006	.192	-.416
	SB	.001	.001	.593	2.175
59	CB	-.002	.003	-.449	.993
	CS	-.000	.007	.030	-.355
	SB	-.001	.001	-.274	.722
60	CB	-.015	.006	-.322	1.934
	CS	-.017	.013	-.248	.221
	SB	-.001	.002	3.341	19.561
61	CB	.001	.030	-.010	23.401
	CS	.004	.004	-.821	.453
	SB	-.001	.000	-.111	.359
62	CB	-.002	.009	1.130	4.684
	CS	.002	.024	-.111	.864
	SB	-.001	.003	1.695	7.220
63	CB	-.009	.008	-.066	-.922
	CS	-.009	.022	-.174	7.616
	SB	-.002	.006	.285	20.470
64	CB	-.007	.002	-.085	-.146
	CS	-.009	.015	-.338	1.275
	SB	-.003	.001	-.185	4.191

Set	Security	Mean	Variance	Skewness	Kurtosis
65	CB	-.007	.002	-.163	.319
	CS	-.009	.016	-.139	2.653
	SB	-.003	.002	-.291	5.626
66	CB	-.003	.005	-.394	.659
	CS	-.003	.006	-.099	-.095
	SB	-.001	.001	2.311	10.712

Straight Bond, Convertible Bond, and Common Stock Beta Coefficients

	SB	CB	CS
01	-.00811	.29846	.45987
02	.05412	1.12276	1.32725
03	.12333	.57124	1.20740
04	.11425	.73558	.67547
05	.09210	.51210	1.10928
06	-.45108	.87051	1.37787
07	.15332	.16853	.92517
08	.08205	.26107	.98206
09	.22520	.54839	.89951
10	.12564	.51585	.55004
11	.03200	.87883	.99068
12	.18140	.56488	.70871
13	.01817	.39956	1.65513
14	.06316	1.71486	1.70948
15	.02986	.22783	.61543
16	.07648	.43663	.23652
17	.02950	.19333	.55328
18	.46156	.45270	.99378
19	.96455	.21264	1.04493
20	.52660	.62750	.83369
21	.07005	.48078	.67454
22	.13077	.41826	.35251
23	.05722	1.47527	1.82746
24	.04756	.41136	.54605
25	.05872	.86296	1.99864
26	.63997	.83675	.91910
27	.16911	.64173	1.03106
28	.04313	.62886	.66632
29	.07426	.90736	1.59248
30	-.01294	.63521	1.53100
31	.19648	1.15124	1.11628
32	-.03631	.84432	1.23388
33	.20720	.73752	1.28731

	SB	CB	CS
34	.00457	.98402	.94944
35	.97865	.68336	1.11663
36	.18518	.43147	.31455
37	.12031	.41281	.93133
38	.11054	.56905	.71313
39	.06216	2.08749	1.84680
40	.05061	1.13485	1.83140
41	.06019	.41273	.40175
42	.14868	1.08576	1.28438
43	.10943	.52107	1.27972
44	.25217	.08339	1.02592
45	.04295	.27945	.45719
46	.16758	.72904	.72346
47	.14839	.11076	.73101
48	.04187	.49671	1.75145
49	.01091	.76212	1.06752
50	.02913	.26276	1.06902
51	.11309	.82182	1.43988
52	.61283	.69444	1.38273
53	.07288	.33668	.76626
54	.05813	.52157	.98909
55	.03645	.82153	.60358
56	.00896	.55125	1.76555
57	.05958	.77142	.76080
58	.20452	.28251	.38816
59	.08886	.47216	.62545
60	.12080	.65561	1.16428
61	.11307	-.54910	.33421
62	.00999	.77645	1.35263
63	.02085	1.13858	1.07768
64	.07900	.62157	1.42281
65	.09493	.55664	1.39646
66	.07844	.52036	.71035

APPENDIX F

REGRESSION RESULTS FOR CROSS-SECTION ANALYSIS WHEN CONVERSION VALUE IS GREATER THAN STRAIGHT BOND VALUE

Summary statistics for the sixty-cross-section regressions for cases where the conversion value in common stock exceeds the straight bond value. The identifying numbers conform with monthly time periods from January, 1968 to December, 1972.

The following symbols are used:

B = Partial regression coefficients,

t = t-value for partial regression coefficients,

df = degrees of freedom in regression equation,

* = significant at the .05 level,

** = significant at the .01 level.

Period df	Statistic	DFlow	μ_{10}	μ_{20}	μ_{11}	μ_{30}	μ_{21}	μ_{12}	F-Value	R^2
T01	B	.262	-14.930	-131.766	1254.397	-50.904	469.363	1859.643	25.539**	.743
7,52	t	6,595**	.462	2.083*	1.603	1.214	.857	1.119		
T02	B	.271	48.625	-72.266	669.386	49.625	-232.054	2131.837	21.879**	.769
7,46	t	5.374**	1.292	.995	.679	1.081	.308	1.064		
T03	B	.235	48.349	-107.411	220.659	-13.039	-221.393	1471.110	22.800**	.780
7,45	t	4.463*	1.414	1.563	.222	.272	.319	.772		
T04	B	.198	17.447	-120.391	1076.558	-32.773	335.449	2669.641	26.433**	.804
7,45	t	4,882**	.618	2,119*	1.309	.853	.592	1.643		
T05	B	.174	24,537	-51.334	911.217	15.822	515.423	2431.934	23.256**	.765
7,50	t	4,624**	.-83	.892	1.135	.435	.850	1.469		
T06	B	.158	20.307	-46.119	976.471	28.851	590.912	3394.674	32.012**	.821
7,49	t	4.507**	,718	,879	1.343	.877	1.124	2.313*		
T07	B	,200	52.255	-36.792	1031,096	42,065	271.679	2916.694	27.050**	.788
7,51	t	4,906**	1,742*	,647	1,263	1.117	.447	1.819*		
T08	B	,180	48,461	-75.447	509,949	37,481	-129.682	2223.137	29.165**	.806
7,49	t	3.852**	1.703*	1.270	.567	.878	.205	1.216		
T09	B	.177	19.762	-62.468	1166.838		526.252	3276.752	24.908**	.769
6,45	t	4.725**	.977	1.216	1.602		.975	2.018*		
T10	B	.210	56.784	-6.326	208.377	34.737	312.928	2060.618	21.367**	.753
7,49	t	4.903**	1.810*	.100	.261	.900	.508	1.240		
T11	B	.247	38.978	-19.223	1267.142	32.094	591.905	3913.792	31.430**	.818
7,49	t	5.831**	1,215	.322	1.649*	.731	1.148	2.560		
T12	B	,172	91,593	69.454	1092.265	113.274	326.527	4042.688	29.901**	.772
7,52	t	4,311**	2,698**	1,200	1,374	2,669**	,644	2.580**		
T13	B	,195	23,627	-82,528	605,276	36.029	-98.282	1553.038	18.118**	.717
7,50	t	3,930**	,640	1,307	,636	,768	.155	,822		

Period df	Statistic	DFlow	μ_{10}	μ_{20}	μ_{11}	μ_{30}	μ_{21}	μ_{12}	F-Value	R^2
T14	B	.248	39.201	6.791	1681.055	7.270	1096.804	4928.821	20.417**	.737
7,51	t	5.972**	1.228	.122	2.040*	.189	2.042*	3.004**		
T15	B	.197	38.122	-3.610	1512.562	43.010	624.450	4145.155	17.308**	.712
7,49	t	4,222**	1.062	.055	1.719*	1.005	.961	2.357**		
T16	B	.230	17.084	-12.945	2020.912	-5.566	1405.484	4966.326	24.014**	.771
7,50	t	5.600**	.503	.221	2.692**	.141	2.440**	3.677**		
T17	B	.226	62.845	95.485	2042.383	70.071	1021.976	6296.396	15.636**	.691
7,49	t	5.529**	1.516	1.471	2.586**	1.640	1.750*	3.590**		
T18	B	.161	3.350	-52.402	1774.027		853.550	3726.827	20.719**	.717
6,49	t	4.299**	.164	1.158	2.367**		1.667*	2.286*		
T19	B	.116	30.910	-33.947	915.942	50.293	198.043	2732.391	14.403**	.677
7,48	t	2,435**	.834	.499	.905	1.029	.243	1.344		
T20	B	.171	-43.851	-128.780	1335.735	-67.890	966.979	2316.406	10.521**	.642
7,41	t	3.257**	1.082	1.628	1.243	1.255	1.036	.964		
T21	B	.125	5.681	-79.040	671.587		298.532	1573.228	19.933**	.727
6,45	t	3.575**	.308	1.614	1.016		.565	.928		
T22	B	.116	57.384	-60.912	-285.240	39.606	-528.083	-185.923	17.691**	.751
7,41	t	3.258**	1.625	.999	.363	1.086	.849	.105		
T23	B	.145	66.199	40.355	497.109	45.294	589.123	2344.402	16.871**	.724
7,45	t	3,697**	2,394**	.760	.627	1.348	1.019	1.470		
T24	B	.103	4,947	-78,666	1230.782	-10.964	563.125	1854.616	18.433**	.746
7,44	t	2.643**	,195	1.420	1.551	,324	.825	1.095		
T25	B	,104	-20.287	-106.907	102.145	-26.454	158.968		9.799**	.582
6,42	t	2,610**	,567	2.038*	,187	.656	.348			
T26	B	.128	-6,388	-113,072	-826,119	20,861	-485,421	-3079.541	9.794**	.662
7,35	t	2,614*	.141	1,406	,853	.397	.605	1.551		

Period df	Statistic	DFlow	μ_{10}	μ_{20}	μ_{11}	μ_{30}	μ_{21}	μ_{14}	F-Value	R^2
T27	B	.142	20.989	-38.743	146.275	4.847	313.941	593.552	21.559**	.786
7,41	t	4.944**	.931	.872	.239	.179	.606	.485		
T28	B	.166	71.574	68.982	-117.061	73.462	192.636	1834.202	22.213**	.795
7,40	t	5.805**	2.848**	1.477	.172	2.343*	.358	1.389		
T29	B	.138	-15.231	-18.123	2218.585	-8.178	927.055	4825.816	6.699**	.602
7,31	t	2.807**	.400	.210	1.568	.155	1.110	1.434		
T30	B	.053	-56.903	-209.606	-454.009	-62.623	-437.528	-1927.153	3.570**	.463
7,29	t	.894	1.114	1.694*	.303	1.043	.387	.617		
T31	B	.063		-113.885	-2051.949		-491.840	-2642.436	7.006**	.556
5,28	t	1.160		1.674*	1.478		.547	.956		
T32	B	.190	-38.604	-81.198	-80.975	-86.689	1152.305	-416.719	6.908**	.617
7,30	t	3.678**	.791	.865	.061	1.314	1.407	.164		
T33	B	.044	66.993	89.131	2161.919	49.942	890.055	6202.813	1.588	.246
7,34	t	.483	.947	.596	.927	.568	.628	1.330		
T34	B	.154	-19.400	-95.142	278.420	-54.080	738.789	1550.426	18.391**	.772
7,38	t	4.164**	.812	1.597	.315	1.709*	1.246	.853		
T35	B	.146	22.689	-54.263	677.655	-30.010	549.442	2052.778	6.528**	.596
7,31	t	2.722**	.425	.566	.492	.470	.648	.737		
T36	B	.127	44.690	50.046	282.303		1233.824	2662.260	9.416**	.638
6,32	t	2.931**	1.896	.832	.256		1.761*	1.210		
T37	B	.182		26.125	709.219	-27.458	2019.730	3700.396	24.821**	.784
6,41	t	4.824**		.687	.576	1.127	3.288**	2.036*		
T38	B	.173		-41.959	556.647	-12.010	1111.533	2942.622	31.580**	.815
6,43	t	5.098**		1.157	.696	.609	2.059*	1.877*		
T39	B	.138	-33.387	-139.210	-519.770	-18.862	223.630	-294.370	35.294**	.858
7,41	t	4.911**	1.088	2.200*	.642	.697	.428	.167		

Period df	Statistic	DFlow	μ_{10}	μ_{20}	μ_{11}	μ_{30}	μ_{21}	μ_{14}	F-Value	R^2
T40	B	.151	27.459	12.037	337.151	39.517	885.862	3179.893	29.816**	.826
7,44	t	4.723**	1.015	.219	.434	1.459	1.666*	2.154*		
T41	B	.159	4.504	-29.877	544.527		954.374	2850.060	30.852**	.808
6,44	t	5.259**	.900	.649	.774		1.869*	2.007*		
T42	B	.121	21.202	25.272	375.721	23.476	1034.939	2503.530	17.610**	.750
7,41	t	3.562**	.775	.467	.480	.756	1.804*	1.797*		
T43	B	.092	44.992	19.446	467.224	14.787	924.304	3203.949	16.651**	.754
7,38	t	2.623**	1.434	.274	.707	.443	1.422	1.805*		
T44	B	.099	51.981	34.389	184.533	41.232	687.317	2501.805	17.407**	.782
7,34	t	3.118**	1.503	.406	.173	1.217	.893	1.055		
T45	B	.111	8.585	-2.451	631.636	-5.068	1186.318	2698.766	20.405**	.803
7,35	t	3.912**	.330	.035	.718	.195	1.750	1.432		
T46	B	.145	38.501	75.234	146.425	27.154	1334.214	3480.894	15.001**	.761
7,33	t	3.904**	1.465	1.037	.118	.740	1.683*	1.463		
T47	B	.074	141.746	122.951	-1116.108	94.338	494.109	2595.956	5.997**	.646
7,23	t	1.515	1.986*	1.049	.597	1.304	.391	.741		
T48	B	.124	108.205	178.499	1043.512	49.350	1863.646	7114.847	15.136**	.822
7,23	t	3.610**	3.675**	2.444*	1.071	1.294	2.452*	3.154**		
T49	B	.124	59.539	63.901	-563.930	21.099	863.223	3008.899	16.269**	.803
7,28	t	3.997**	2.126*	.944	.613	.601	1.276	1.469		
T50	B	.100	99.719	90.520	-1350.247	73.024	239.069	1608.152	18.834**	.800
7,33	t	3.233**	3.952**	1.416	1.493	2.380*	.333	.801		
T51	B	.129	77.223	72.522	210.306	79.348	357.240	3631.548	16.619**	.769
7,35	t	4.004**	2.704**	1.098	.241	2.200*	.548	2.005*		
T52	B	.148	36.463	51.141	105.942	26.701	966.588	3674.594	9.290**	.684
7,30	t	4.538**	1.168	.777	.122	.707	1.385	2.162*		

Period df	Statistic	DFlow	μ_{10}	μ_{20}	μ_{11}	μ_{30}	μ_{21}	μ_{12}	F-Value	R^2
T53	B	.134	84.943	127.713	426.026	78.887	890.446	4325.283	10.451**	.696
7,32	t	4.102**	2.784**	1.749*	2.064*	2.076*	1.227	2.345*		
T54	B	.122	30.664	26.647		30.664	710.378	3376.484	12.375**	.692
6,33	t	3.800**	1.418	.488		.962	1.375	3.336**		
T55	B	.039	-40.083	-23.231	1801.519	18.538	1214.001	3030.830	.754	.164
7,27	t	.402	.404	.108	.601	.134	.463	.519		
T56	B	.151	89.360	126.202	-706.400	81.736	817.751	2554.196	13.464**	.777
7,27	t	4.518**	2.993**	1.939*	.666	1.897*	.935	1.238		
T57	B	.137	68.402	98.138	-448.941	36.171	1049.061	2925.020	13.313**	.775
7,27	t	5.321**	2.344**	1.807*	.568	1.189	1.542	2.046*		
T58	B	.214	65.442	-118.203	-2972.895	164.157	-3566.809	-3735.614	20.334**	.851
7,25	t	6.048**	1.946*	1.377	2.423*	4.358**	3.471**	1.605		
T59	B	.084	79.815	14.733	-2037.773	56.276	-427.093	-980.070	14.584**	.803
7,25	t	2.908**	2.523**	.207	2.335*	1.594	.391	.541		
T60	B	.126	29.027	-69.921	-1917.972	10.848	-523.398	-674.764	7.268**	.629
7,30	t	2.790**	.720	.746	1.458	.241	.559	.245		

Weil, Segall, Green Cross-Sectional Regression: Conversion Value Greater Than Straight Bond Value

Period df	Statistic	DFlow	FLSQ	Floor	F-Value	R^2
T01	B	.23	3.33	-21.34	28.82**	.61122
3,56	t	5.041	4.163	5.192		
T02	B	.26	4.59	-29.67	30.68**	.64799
3,50	t	4.504	5.530	6.353		
T03	B	.21	4.87	-30.75	27.85**	.63034
3,49	t	3.620	5.181	5.994		
T04	B	.18	3.56	-25.40	29.55**	.64401
3,49	t	3.733	6.034	7.056		
T05	B	.19	2.55	-19.83	32.20**	.64146
3,54	t	4.313	5.930	7.185		
T06	B	.19	2.62	-21.59	52.55**	.74839
3,53	t	4.831	7.939	9.638		
T07	B	.20	2.93	-22.25	35.96**	.66231
3,55	t	4.243	5.980	7.417		
T08	B	.18	4.63	-31.06	40.46**	.69607
3,53	t	3.641	6.173	7.746		
T09	B	.16	3.09	-22.74	32.26**	.66387
3,48	t	3.691	6.717	7.708		
T10	B	.20	2.86	-21.21	33.40**	.65402
3,53	t	4.408	5.608	11.783		
T11	B	.19	2.75	-22.14	43.19**	.70971
3,53	t	4.164	6.707	7.823		
T12	B	.17	2.40	-19.59	31.70**	.62936
3,56	t	3.606	5.581	6.779		
T13	B	.18	3.14	-23.44	29.45**	.62069
3,54	t	3.527	5.509	6.716		
T14	B	.17	2.96	-22.02	38.28**	.67618
3,55	t	3.902	6.041	7.340		
T15	B	.16	2.78	-21.42	29.53**	.62570
3,53	t	3.357	5.451	6.452		

Period df	Statistic	DFlow	FLSQ	Floor	F-Value	R^2
T16	B	.17	2.62	-20.74	35.17**	.66146
3,54	t	3.864	6.390	7.487		
T17	B	.19	1.86	-15.39	35.88**	.67010
3,53	t	5.115	6.000	6.964		
T18	B	.12	2.49	-19.28	26.59**	.60535
3,52	t	2.800	5.791	6.886		
T19	B	.11	2.77	-20.86	25.16**	.59211
3,52	t	2.443	5.540	6.643		
T20	B	.13	3.25	-21.88	18.73**	.55530
3,45	t	2.728	4.167	5.234		
T21	B	.11	2.14	-17.05	26.46**	.62318
3,48	t	2.781	5.487	6.875		
T22	B	.13	2.64	-18.21	23.39**	.60926
3,45	t	3.469	4.714	5.818		
T23	B	.14	3.17	-20.72	28.02**	.63174
3,49	t	3.750	5.197	6.260		
T24	B	.11	4.11	-25.12	29.07**	.64504
3,48	t	2.711	5.269	6.441		
T25	B	.10	3.09	-18.97	15.85**	.51382
3,45	t	2.624	4.120	4.940		
T26	B	.15	3.59	-22.16	16.54**	.55988
3,39	t	3.096	2.679	3.473		
T27	B	.14	3.30	-19.93	32.00**	.67739
3,45	t	4.246	4.783	5.744		
T28	B	.15	4.02	-22.74	42.02**	.74126
7,44	t	4.934	3.558	4.476		
T29	B	.15	2.34	-14.60	14.03**	.54594
3,35	t	3.282	.818	1.312		
T30	B	.09	4.37	-22.13	6.71**	.37885
3,33	t	1.658	.995	1.361		
T31	B	.10	4.17	-24.20	9.06**	.47535
3,30	t	1.904	.871	1.395		

Period df	Statistic	DFlow	FLSQ	Floor	F-Value	R^2
T32	B	.18	3.38	-18.43	13.21**	.53829
3,34	t	3.797	.988	1.412		
T33	B	.07	5.96	-28.76	3.41*	.21224
3,38	t	.858	1.345	1.653		
T34	B	.15	6.50	-32.74	32.65**	.69992
3,42	t	4.127	3.916	4.858		
T35	B	.16	5.67	-28.79	13.05**	.52800
3,35	t	3.300	2.232	2.779		
T36	B	.13	4.72	-24.84	13.89**	.54345
3,35	t	3.006	1.950	2.532		
T37	B	.14	6.63	-36.23	36.39**	.71271
3,44	t	3.425	3.923	5.053		
T38	B	.16	5.98	-34.21	48.19**	.75863
3,46	t	4.708	3.858	5.129		
T39	B	.17	5.21	-30.80	51.55**	.77460
3,45	t	5.472	4.530	5.992		
T40	B	.17	4.12	-26.81	59.98**	.78941
3,48	t	5.506	5.086	6.839		
T41	B	.17	3.72	-24.41	47.07**	.75029
3,47	t	5.458	5.471	7.014		
T42	B	.15	2.85	-19.78	38.20**	.71803
3,45	t	4.998	3.750	5.205		
T43	B	.11	4.33	-25.95	27.33**	.66127
3,42	t	3.282	4.977	6.063		
T44	B	.13	3.21	-20.98	29.40**	.69892
3,38	t	4.463	3.776	4.879		
T45	B	.14	3.03	-19.38	25.57**	.66299
3,39	t	4.540	4.391	5.210		
T46	B	.17	2.68	-19.37	29.74**	.70689
3,37	t	5.058	2.735	4.078		
T47	B	.10	3.36	-20.90	7.26**	.44642
3,27	t	2.223	2.087	2.662		

Period df	Statistic	DFlow	FLSQ	Floor	F-Value	R^2
T48	B	.10	5.37	-28.66	11.59**	.56286
3,27	t	2.594	3.216	3.899		
T49	B	.11	3.59	-21.29	17.54**	.62182
3,32	t	3.432	3.590	4.520		
T50	B	.11	3.12	-20.69	20.16**	.62040
3,37	t	3.321	3.089	4.275		
T51	B	.15	3.95	-25.29	29.17**	.69175
3,39	t	4.811	5.000	6.244		
T52	B	.14	2.71	-18.07	18.47**	.61293
3,34	t	4.627	3.188	4.144		
T53	B	.14	2.33	-16.31	14.54**	.54784
3,36	t	4.197	2.987	3.820		
T54	B	.05	2.02	-14.85	1.52	.11554
3,36	t	.607	1.080	1.447		
T55	B	.15	2.99	-20.74	15.81**	.59720
3,31	t	4.429	3.560	4.285		
T56	B	.19	2.87	-20.52	23.07**	.69064
3,31	t	5.595	3.932	4.728		
T57	B	.15	1.95	-13.71	19.39**	.65236
3,31	t	5.583	2.910	3.637		
T58	B	.25	3.78	-24.89	25.93**	.72847
3,29	t	6.197	4.154	4.824		
T59	B	.12	2.33	-16.98	18.09**	.65171
3,29	t	3.867	3.585	4.422		
T60	B	.14	2.29	-16.21	11.99**	.51400
3,34	t	3.098	2.516	3.094		

Summary of Standard Error of the Estimate Comparisons for Conversion Value Greater Than Straight Bond Value

Period	Product Moment Model Standard Error	WSG Model Standard Error
T01	4.54	5.39
T02	5.27	6.26
T03	5.01	6.24
T04	4.34	5.62
T05	4.45	5.28
T06	4.07	4.64
T07	4.68	5.67
T08	4.89	5.89
T09	4.30	4.98
T10	4.69	5.34
T11	4.39	5.32
T12	4.50	5.54
T13	5.22	5.82
T14	4.67	4.99
T15	4.94	5.42
T16	4.45	5.18
T17	4.42	4.39
T18	4.39	5.04
T19	4.93	5.32
T20	5.17	5.50
T21	3.92	4.46
T22	3.66	4.38
T23	4.07	4.51
T24	4.08	4.62
T25	4.21	4.38
T26	4.93	5.33
T27	3.12	3.66
T28	3.18	3.41
T29	4.54	4.56
T30	5.05	5.09
T31	5.00	5.25
T32	4.74	4.88
T33	8.49	8.21
T34	3.66	3.99
T35	5.38	5.47
T36	4.71	5.03
T37	4.39	4.38
T38	3.97	4.89
T39	3.39	4.07
T40	3.85	4.05
T41	3.77	4.16
T42	3.91	3.97
T43	3.72	4.16

Period	Product Moment Model Standard Error	WSG Model Standard Error
T44	3.46	3.84
T45	3.13	3.87
T46	4.04	4.23
T47	4.55	5.26
T48	3.13	4.53
T49	3.05	3.95
T50	3.31	4.30
T51	3.75	4.10
T52	3.78	3.86
T53	3.80	4.35
T54	3.65	10.38
T55	11.39	4.30
T56	3.87	4.26
T57	2.91	3.37
T58	3.92	4.91
T59	3.17	3.92
T60	4.89	5.26

APPENDIX G

REGRESSION RESULTS FOR CROSS-SECTION ANALYSIS WHEN STRAIGHT BOND VALUE IS GREATER THAN CONVERSION VALUE

Summary statistics for the sixty cross-section regressions for cases where the conversion value in common stock is less than the straight bond value. The identifying numbers conform with monthly time periods from January, 1968 to December, 1972.

The following symbols are used:

B = Partial regression coefficients,

t = t value for partial regression coefficients,

df = degrees of freedom in regression equation,

* = significant at the .05 level,

** = significant at the .01 level.

Period df	Statistic	DFlow	μ_{10}	μ_{20}	μ_{11}	μ_{30}	μ_{21}	μ_{12}	F-Value	R^2
T01	B	.135		-595.279	-3062.994		-2210.108		139.438	.998
4,1	t	5.777		16.420*	7.555*		9.093*			
T02	B	.023	-90.143	-308.270	1408.849	-1291.787	-2621.719	1092.186	6.239*	.916
7,4	t	.138	1.642	.656	1.228	1.247	1.524	.757		
T03	B	.224	-94.546	486.685	-442.632			2429.644	5.409*	.794
5,7	t	2.668*	2.429*	1.731	.445			2.344*		
T04	B	-.023	-50.403	122.584	-690.137	-156.994	506.254	1558.152	4.436	.861
7,5	t	.187	1.323	.329	.640	.438	.464	1.263		
T05	B		14.910	-893.597	114.854	-1809.530	-1162.476	-2891.082	.886	.842
6,1	t		.045	.292	.014	.338	.490	.219		
T06	B	.036	-82.272	637.748	-588.323	1290.431	-1218.757	978.991	83.784	.998
7,1	t	1,010	7.041*	3.059	1.489	2.763	4.671	2.326		
T07	B	.416		2251.514	3636.216	8979.835		1374.073	11.724	.983
5,1	t	3.111		3.939	3.528	3.585		2.089		
T08	B	.127	-76.821	100.332	3212.393	-852.633	-1476.004	-576.227	1865.613*	1.000
7,1	t	20.622*	20.402*	2.453	26.327*	17.017*	16.086*	7.639*		
T09	B	.187	-95.749		-857.214	-915.845	-1855.575	673.054	5.910*	.835
6,7	t	1.731	3.601**		.550	2.135*	1.332	.731		
T10	B	.300	-72.556	905.894	3859.744	2573.117	-2342.088	-694.576	1165.595*	1.000
7,1	t	11.171*	8.763*	7.998*	8.124*	4.425	15.296*	2.781		
T11	B	.105	-40.406	562.915	1024.171	2023.413	-805.622	-525.032	19.350	.993
7,1	t	3,755	2,067	2.964	1.574	2.530	1.821	.950		
T12	B		209.698	-800.739		12378.681		-5839.660	1.377	.846
4,1	t		.565	.737		.670		.629		

Period df	Statistic	DFlow	μ_{10}	μ_{20}	μ_{11}	μ_{30}	μ_{21}	μ_{14}	F-Value	R^2
T13	B	.366	-217.435	927.328	-1054.672		-7779.939	4939.225	3.619	.956
6,1	t	2.973	1.692	.735	.401		2.343	1.076		
T14	B			-4320.457	-22816.341	-4777.812	-4237.726	7900.247	113.282	.998
5,1	t			3.885	4.293	3.237	2.471	4.131		
T15	B	.012	57.093		1323.710	1023.312	2701.163	-1693.036	.557	.625
6,2	t	.045	.634		.404	.402	1.095	.748		
T16	B	-.339	-106.941		6404.825	-11061.337	3788.529	396.812	9.860	.983
6,1	t	1.039	2.648		2.474	2.382	1.586	.434		
T17	B	-.125	-102.513	2418.452	3762.036	8115.139	-722.782	408.631	4.048	.966
7,1	t	2.034	2.370	2.828	4.081	1.806	.580	.535		
T18	B	.275	-59.873	1583.461	-4129.220	9798.547	-4076.049	475.821	38.674*	.993
7,2	t	8.869**	5.922*	10.767**	8.645**	10.508**	10.271**	2.176		
T19	B	.066	16.251	504.170	2244.114	1752.379	1150.054	-1048.748	1.479	.838
7,2	t	.518	.426	.828	.951	1.000	1.217	1.464		
T20	B	.173	-46.661	549.709	3453.686	625.827	530.235		2.041	.551
6,10	t	1.032	1.352	1.123	1.088	.718	.377			
T21	B	.307	-78.128	466.673		681.798	180.048	746.132	18.757**	.941
6,7	t	5.228**	3.451**	1.364		.520	.336	1.521		
T22	B	-.077	15.166	-1322.498	-4012.297	-3830.692	-956.944	-911.915	1.292	.501
7,9	t	.540	.205	1.414	1.028	.811	.346	.551		
T23	B	.188	43.981	704.369	5694.962	3595.567	3081.419	-2639.335	6.400*	.900
7,5	t	1.778	.744	.838	1.512	1.013	1.049	1.795		
T24	B	.105	-65.877	281.444	-2210.702	82.229	-285.333		4.858*	.806
6,7	t	1.274	3.043**	.696	1.237	.164	.237			

Period df	Statistic	DFlow	μ_{10}	μ_{20}	μ_{11}	μ_{30}	μ_{21}	μ_{12}	F-Value	R^2
T25	B	-.036	60.878	-64.070	1672.518	815.826	1314.050	-2388.382	2.175	.629
7,9	t	.270	.996	.134	.843	1.057	1.028	1.654		
T26	B	.232	-66.101	343.872	-211.070	129.504	-397.529	1078.510	2.537	.542
7,15	t	2.585*	1.898*	1.158	.171	.399	.473	1.113		
T27	B	.033	-29.802	-305.827	-269.715	-788.282	-457.836	103.976	3.782*	.746
7,9	t	.305	.767	1.155	.173	2.904**	.820	.141		
T28	B	-.166	-8.183	579.327	2882.860	783.993	461.800	-167.742	.711	.332
7,10	t	.596	.101	.912	.894	1.331	.283	.071		
T29	B	.497	-69.220	352.734	-1444.513	27.829		124.809	4.434**	.571
6,20	t	3.273**	1.941*	1.259	1.098	.163		.118		
T30	B	.194	-53.572	191.856	-351.334	-78.824	-478.625	380.895	2.284	.432
7,21	t	1.777*	1.600	.822	.207	.514	.687	.411		
T31	B	.173	-58.436	201.203	-1001.023	-156.769	-176.952	1021.079	9.072**	.726
7,24	t	3.483**	4.095**	1.965*	1.276	2.349*	.662	3.202**		
T32	B	.229	-34.245	187.399	-155.316	160.277	-346.979	493.336	3.364*	.541
7,20	t	2.662**	1.662	1.193	.158	1.367	.911	1.043		
T33	B	.226	-59.343	230.127	-611.326	195.471	-718.434	1433.862	4.120**	.643
7,16	t	2.781**	2.480*	1.204	.713	.617	1.289	2.474*		
T34	B	.241	-23.926	111.452	201.901	200.929	193.410	447.854	14.016**	.891
7,12	t	5.542**	.913	.451	.315	.517	.351	.860		
T35	B	.071	-15.449	123.537	-251.279	183.202	-28.070	375.521	3.423*	.558
7,19	t	.835	.750	.834	.375	1.742	.055	.729		
T36	B	.168	-46.701	209.091	-883.141	85.970	-314.230	836.749	7.180**	.726
7,19	t	2.265*	2.202*	.645	1.190	.941	.589	1.305		
T37	B	.320	-13.909	575.261	3476.749	470.916	2501.926	-595.831	9.249**	.866
7,10	t	3.915**	.395	2.072*	2.525*	2.267*	1.842*	.598		

Period df	Statistic	DPlow	μ_{10}	μ_{20}	μ_{11}	μ_{30}	μ_{21}	μ_{12}	F-Value	R^2
T38	B	.288	76.933	-125.461	2126.287	752.371	2030.999	-3151.593	2.780	.709
7,8	t	3.021**	1.564	.949	1.251	.539	.221	1.920*		
T39	B	.314			352.118	836.300	1142.936	-772.363	12.082**	.846
5,11	t	2.937**			.344	1.502	1.319	1.358		
T40	B	.260		-443.574	1185.363	-699.311	369.180	-268.355	8.741**	.882
6,7	t	2.730*		.901	.951	.612	.305	.277		
T41	B	.069	-153.949	1712.695	-3327.578	1661.502	2391.279	4847.578	10.196**	.911
7,7	t	.867	3.421**	2.877*	2.677*	1.520	1.115	2.466*		
T42	B	.062	81.796	-637.670	633.034	40.088	3022.809	-1986.879	5.412**	.808
7,11	t	.659	1.751	1.530	.462	.145	1.628	1.315		
T43	B	.010	31.404	-206.004		237.271	1266.398	-2505.061	10.266**	.826
6,13	t	.118	1.045	1.121		1.730	1.761	2.945**		
T44	B	-.067	-15.819	325.542	1247.716	315.284	1427.063	-1341.638	2.088	.477
7,16	t	.375	.266	.912	.514	.926	.927	.826		
T45	B	.111	-44.046	270.675	1155.173	58.840	230.377	-626.111	2.838*	.570
7,15	t	1.065	.770	.759	.687	.276	.224	.486		
T46	B	.064	-6.563	35.883	1318.660	268.395	-367.489	-1359.660	6.152**	.717
7,17	t	.832	.226	.152	1.230	1.487	.546	1.954		
T47	B	.072	-37.998	268.648	743.783	150.711	-167.116	-617.165	6.335**	.622
7,27	t	1.126	1.538	1.399	.504	1.242	.292	.670		
T48	B	.112	-31.424	203.573	76.828	111.422	-133.825	-748.803	9.210**	.705
7,27	t	2.119*	.617	1.249	.055	.967	.298	1.371		
T49	B	.101	-47.475	502.969	391.150	674.020	-181.302	-596.967	11.012**	.778
7,22	t	2.092*	1.886*	2.154*	.297	2.997**	.385	1.107		
T50	B	.051	-77.527	730.581	2357.554	662.735	321.093		7.672**	.719
6,18	t	.737	3.282**	2.499*	1.243	2.316*	.491			

Period df	Statistic	DFlow	μ_{10}	μ_{20}	μ_{11}	μ_{30}	μ_{21}	μ_{12}	F-Value	R^2
T51	B	.107	-50.125	-45.900	-2303.920		-1393.712		2.830*	.454
5,17	t	1,037	2.104*	.272	1.161		1.335			
T52	B	.059	-26.772	-289.842	-442.917	-897.229	-512.582	-252.818	3.780**	.570
7,20	t	.878	.846	1.082	2.500**	.274	.589	.390		
T53	B	.041	-29.464	-190.908	325.274	-626.843	19.814	-193.960	8.207**	.761
7,18	t	.618	1.279	1.060	.247	3.384**	.028	.432		
T54	B	.024	13.465	-449.348	-1490.937	-437.215	67.481	-469.486	4.596**	.641
7,18	t	.307	.510	2.077*	.978	2.764**	.110	.965		
T55	B	.111	-13.214	-90.911	-273.911	-158.467	342.799	-253.433	6.639**	.669
7,23	t	1.849*	.709	.713	.260	1.745*	.904	.885		
T56	B	.093		-121.564		-82.072	226.006	-491.205	7.584**	.603
5,25	t	1.722*		1.894*		1.297	.666	2.510**		
T57	B	.075		-150.280	575.057	-189.741	628.836	-410.752	11.725**	.746
6,24	t	.0004		.758	.701	2.268*	1.723*	2.071*		
T58	B	.061	20.776	-195.816	-541.325		489.570	-728.538	4.698**	.520
6,26	t	.902	.994	1.292	.534		1.157	1.816		
T59	B	.243	-30.478	86.629	880.634	-170.834	782.682	33.203	14.616**	.804
7,25	t	4.546**	1.878*	.857	1.1306	1.809*	2.423*	.128		
T60	B	.074	-30.755	70.610	1308.126	-207.883	560.903	146.498	6.433**	.692
7,20	t	1.1622	1.126	.388	1.204	1.3706	1.357	.328		

Weil, Segall, Green Cross-Sectional Regression: Straight Bond Value Greater Than Conversion Value

Period df	Statistic	DFlow	FLSQ	Floor	F-Value	R^2
T01	B	.80	166.63	-533.89	2.84	.73971
3,2	t	1.889	1.958	2.016		
T02	B	.21	-31.75	66.25	4.03	.60197
3,8	t	1.289	.868	.648		
T03	B	.13	-4.91	3.53	2.65	.46896
3,9	t	1.229	.358	.083		
T04	B	.05	1.41	-18.74	6.06*	.66882
3,9	t	.488	.115	.497		
T05	B	-.16	-12.23	26.74		
3,4	t	.802	.333	.236		
T06	B	.13	17.24	-66.47	3.05	.64669
3,5	t	1.149	.702	.967		
T07	B	.11	64.51	-178.80	.36	.26435
3,3	t	.509	.554	.593		
T08	B	.19	39.69	-128.52	2.51	.60063
3,5	t	1.090	1.277	1.425		
T09	B	.18	37.65	-123.85	6.41*	.68126
3,10	t	1.461	1.934	2.246		
T10	B	.20	53.09	-155.07	3.24	.66014
3,5	t	1.801	1.547	1.691		
T11	B	.16	45.09	-130.70	3.83	.69702
3,5	t	2.590	1.594	1.728		
T12	B	.03	20.59	-65.84	.89	.57256
3,2	t	.152	.129	.171		
T13	B	.21	142.30	-358.28	2.08	.60986
3,4	t	2.027	1.306	1.357		
T14	B	.01	-63.68	170.31	22.45*	.95735
3,3	t	.019	3.450	2.892		
T15	B	.14	-6.21	2.78	1.14	.40534
3,5	t	.953	.169	.027		

Period df	Statistic	DFlow	FLSQ	Floor	F-Value	R^2
T16	B	-.05	37.78	-117.20	2.46	.64811
3,4	t	.279	1.158	1.293		
T17	B	-.07	-23.54	68.24	.57	.25625
3,5	t	.783	1.126	1.092		
T18	B	.00072	15.80	-51.43	1.16	.36713
3,6	t	.007	.905	1.010		
T19	B	.09	-8.34	19.52	3.38	.62811
3,6	t	1.374	.969	.721		
T20	B	.18	3.91	-24.91	3.42*	.44105
3,13	t	1.373	.363	.697		
T21	B	.37	24.66	-92.71	12.76**	.79293
3,13	t	4.366	2.618	3.135		
T22	B	.01	1.47	-14.13	1.55	.26359
3,13	t	.094	.092	.287		
T23	B	.25	23.07	-82.37	8.60**	.74131
3,9	t	3.298	.952	1.291		
T24	B	.13	36.36	-123.40	10.02**	.75029
3,11	t	2.218	2.159	2.593		
T25	B	.20	13.31	-52.88	3.63*	.45598
3,13	t	2.241	.662	.913		
T26	B	.28	6.24	-34.42	4.63*	.42255
3,19	t	3.077	.804	1.445		
T27	B	.22	15.68	-59.34	4.46*	.50706
3,13	t	2.344	1.577	2.034		
T28	B	.07	-8.42	17.34	.89	.15991
3,14	t	.453	.723	.465		
T29	B	.49	4.03	-32.07	8.69**	.53120
3,23	t	3,750	.660	1.576		
T30	B	.22	4.17	-25.31	5.27**	.38756
3,25	t	2.551	.908	1.571		
T31	B	.21	6.71	-31.82	9.34**	.50007
3,28	t	3.972	2.252	2.938		

Period df	Statistic	DFlow	FLSQ	Floor	F-Value	R^2
T32	B	.21	1.28	-12.45	3.68*	.31517
3,24	t	2.271	.247	.669		
T33	B	.26	20.39	-68.88	3.81*	.36396
3,20	t	2.943	1.798	2.006		
T34	B	.31	.49	-12.05	13.29**	.71369
3,16	t	5.525	.047	.385		
T35	B	.16	-3.69	4.99	5.05**	.39721
3,23	t	2.168	1.073	.410		
T36	B	.22	-.90	-8.58	10.45**	.57673
3,23	t	2.934	.280	.720		
T37	B	.37	-13.51	25.69	9.38**	.66786
3,14	t	3.391	1.603	.932		
T38	B	.26	-51.10	129.58	6.75**	.62807
3,12	t	3.111	2.531	2.332		
T39	B	.51	67.92	-36.76	10.57**	.70933
3,13	t	5.090	.859	1.209		
T40	B	.43	-4.53	-8.80	6.11*	.64704
3,10	t	3.817	.083	.059		
T41	B	.22	108.23	-294.08	4.88*	.57111
3,11	t	2.288	1.807	1.991		
T42	B	.27	6.07	-31.47	4.65*	.51746
3,15	t	3.426	.128	.261		
T43	B	.17	3.23	-35.20	7.06**	.56949
3,16	t	1.894	.152	.600		
T44	B	.11	25.97	-96.27	4.15*	.38373
3,20	t	.784	1.497	1.860		
T45	B	.21	30.16	-104.93	4.04*	.38921
3,19	t	2.257	1.534	1.829		
T46	B	.16	9.39	-42.85	4.62*	.39772
3,21	t	1.835	.903	1.341		
T47	B	.13	7.07	34.81	6.98**	.40323
3,31	t	1.865	1.119	1.679		

Period df	Statistic	DFlow	FLSQ	Floor	F-Value	R^2
T48	B	.13	3.80	-24.42	12.29**	.54324
3,31	t	2.293	.765	1.484		
T49	B	.13	4.39	-26.30	8.67**	.50006
3,26	t	2.178	.543	1.053		
T50	B	.11	4.20	-29.24	6.65**	.48722
3,21	t	1.331	.426	.916		
T51	B	.15	9.78	-42.39	2.76	.30376
3,19	t	1.698	1.040	1.396		
T52	B	.15	17.97	-67.52	6.50**	.45892
3,24	t	2.184	1.832	2.245		
T53	B	.17	21.37	-82.28	12.45**	.62924
3,22	t	2.982	2.861	3.450		
T54	B	.13	6.50	-32.78	5.47**	.41635
3,22	t	1.797	1.134	1.663		
T55	B	.15	4.44	-27.04	11.36**	.56721
3,27	t	2.755	1.461	2.299		
T56	B	.12	.56	-10.58	10.56**	.53992
3,27	t	2.088	.281	1.236		
T57	B	.15	1.66	-15.98	14.28**	.61336
3,27	t	2.829	1.085	2.389		
T58	B	.09	-1.02	-3.43	8.11**	.45616
3,29	t	1.413	.580	.445		
T59	B	.25	1.97	-18.15	19.94**	.67350
3,29	t	4.371	1.713	3.241		
T60	B	.08	2.04	-16.04	7.71**	.49092
3,24	t	1.212	1.153	2.033		

Summary of the Standard Error of the Estimate Comparisons for Straight Bond Value Greater Than Conversion Value

Period	Product Moment Model Standard Error	WSG Model Standard Error
T01	.72	5.01
T02	3.82	5.88
T03	3.19	4.52
T04	3.53	4.07
T05	5.85	4.64
T06	.61	3.90
T07	1.49	5.73
T08	.15	4.95
T09	3.58	4.29
T10	.15	3.64
T11	.91	2.62
T12	2.99	3.53
T13	2.46	3.67
T14	1.21	3.44
T15	5.11	4.07
T16	1.66	3.83
T17	.99	2.06
T18	.68	3.66
T19	2.51	2.20
T20	5.99	5.86
T21	2.24	3.52
T22	5.83	5.89
T23	2.69	3.22
T24	3.33	3.16
T25	3.96	3.99
T26	4.19	4.18
T27	2.72	3.19
T28	6.69	6.35
T29	5.74	5.59
T30	5.58	5.31
T31	2.66	3.33
T32	3.52	3.92
T33	3.12	3.72
T34	1.93	2.71
T35	3.05	3.24
T36	3.22	3.64
T37	3.19	4.24
T38	3.96	3.66
T39	3.15	3.98
T40	3.76	5.45
T41	3.45	6.02
T42	4.12	5.43
T43	3.09	4.38

Period	Product Moment Model Standard Error	WSG Model Standard Error
T44	7.52	7.30
T45	5.07	5.36
T46	4.00	5.25
T47	4.26	5.00
T48	3.63	4.21
T49	3.28	4.53
T50	4.53	5.67
T51	5.66	6.05
T52	8.79	4.58
T53	3.53	3.98
T54	6.51	4.98
T55	3.59	3.79
T56	3.80	3.93
T57	3.54	4.12
T58	4.77	4.81
T59	3.43	4.11
T60	4.15	4.88
Ave.	3.55	4.37

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