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BASIC PSYCHOPHYSICAL FUNCTIONS IN YOUNG CHILDREN

by

John Edwin Kelley

A Dissertation Submitted to the Faculty of the

DEPARTMENT OF PSYCHOLOGY

In Partial Fulfillment of the Requirements
For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

THE UNIVERSITY OF ARIZONA

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THE UNIVERSITY OF ARIZONA

GRADUATE COLLEGE

I hereby recommend that this dissertation prepared under my direction by JOHN EDWIN KELLEY

entitled BASIC PSYCHOPHYSICAL FUNCTIONS IN YOUNG CHILDREN

be accepted as fulfilling the dissertation requirement of the degree of DOCTOR OF PHILOSOPHY

Ted L. Rosenthal — 6-28-74

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Lewis Heil	6-24-74
[signature]	6-24-74
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ABSTRACT

In two separate experiments, paired stimuli differing by a small constant deviation were sampled from 13 equidistant points along height and weight dimensions. Using a paired comparison procedure, children from preschool, first and third grade age levels judged the five pairs of lines with either small, medium, or large dimensional values and, in like manner, five pairs of weights. Within an over-all pattern of correct relational judgments for both experiments, errors for pairs significantly increased in a decelerating, monotonic relationship with increasing magnitude of stimulus dimensions. This curvilinear function held for all three age levels in the line experiment even though preschoolers made significantly more errors than the older children. The same general function could neither be verified nor rejected for preschoolers in the weight experiment, however, since their judgments approached random for all values of the dimensional range. Over-all error judgments for each dimension were found to approximate hyperbolic functions with asymptotes of perfect chance responding indicating good fit with the classic Weber and Fechner equations. Additional evidence was found suggesting that contrast effects can occur when the identical comparison is judged in restricted ranges of smaller- versus larger-valued comparisons.

INTRODUCTION

In previous research, Rosenthal, Kelley and White (in press) obtained unambiguous data documenting relational judgments in young children. Using brightness, texture, facial happiness, height, and weight as dimensions in five experiments, children (age 3.4 to 5.3 years) consistently shifted their judgments of intermediate-valued stimuli away from extreme-valued anchors. These relational shifts appeared virtually independent of children's ages, the provision or omission of overt visual memory props, and the sequential orders of both stimulus and anchor presentation. Additional evidence of relational judgment was found within the narrower range of the test stimuli themselves.

The observation of consistent anchoring shifts across diverse stimulus dimensions seemed to contradict Piaget's (1968) theoretical statement that young children cannot think relationally. Similarly, these results seemed inconsistent with O'Reilly and Steger's (1970) finding that five-year-old children did not shift their judgments of weights relative to a novel frame of reference. It therefore seemed germane to determine if the judgments of young children could be shown to approximate the classic Weber or Fechner equations. Judgmental relativity would be hard to deny if a psychophysical task not only produced evidence that young children could render difficult stimulus judgments in the

correct relational direction at a better than chance level, but also produced evidence that the patterning of their errors conformed to well-known perceptual effects in adults. The applicability of fundamental psychophysical functions to children follows directly from social judgment theory [Helson 1964, Rosenthal et al. (in press), Sherif and Hovland 1961] and social learning theory (Bandura 1969, 1971; Rosenthal and Zimmerman 1972).

There were two major pitfalls to avoid in planning the present research. First, response formats easy for adults or older children may assume subtle extra-task knowledge on the part of subjects which young children generally do not possess. The present research used simple, direct response procedures to facilitate the children's comprehension of their task, a point emphasized elsewhere [Rosenthal et al. (in press), Rosenthal and White 1972]. Second, typical psychophysical experiments require a great number of repetitive stimulus judgments, an unrealistic requirement for young children with limited attention spans. In the present research children were required to spend no more than 20 minutes rendering their judgments.

Two parallel experiments, using height and weight as dimensions, were conducted in an attempt to show that ease of discriminating a constant difference would be a negative function of the magnitude of paired stimuli, using children from three age levels. Each experiment involved a carefully graded set of 13 equidistant stimulus dyads. The stimuli of each dyad differed by a constant amount and were judged in a paired comparison procedure. All children participated in both experiments, but

compared only 5 of the 13 stimulus dyads for either dimension. Within each separate dimension, a child received the five dyads having only small, medium, or large magnitudes. Since the 13 dyads per dimension were divided into three range groups of five dyads each, one dyad was common to both the small and medium range groups and one was common to both the medium and large range groups.

The following specific hypotheses were advanced for dimensions.

1. The judgments of the small, medium, and large range groups will depart from random discrimination in the correct direction, i.e., there will be more correct than incorrect responses for each group.
2. Range groups given larger-valued dyads will make more errors than those given smaller-valued dyads.
3. Within range groups, generally, there will be more errors on larger-valued dyads than on smaller-valued dyads. This difference should be significant at least for the most extreme dyads within the small range group, and possibly for the medium or both the medium and large range groups as well. Although errors are expected to increase monotonically with increasing pair magnitude, the magnitude of successive error increments for the 13 unique dyads should decline asymptotically with a limit of chance performance. Hence, the error difference between extreme pairs should be greatest for the small range group and least for the large range group.

4. The same reasoning implies that the slope of error scores plotted across range groups will be greatest at the portion of the scale representing pairs of small magnitude and least at the portion representing pairs of large magnitude.
5. This reasoning furthermore suggests that, within range groups, any departure from linearity will occur such that larger-valued pairs have lesser slopes than smaller-valued pairs.
6. A contrast effect will occur for the two replicated comparisons, i.e., the identical discrimination should yield fewer errors when it occurs at the bottom of a larger value spectrum than as the topmost value of a smaller magnitude context.
7. As suggested by hypotheses 2 through 5, the observed function was expected to be monotonic and decelerating with an asymptote of chance responding for larger-valued dyads. Stimuli were chosen (see Methods section) so that dyads with the smallest magnitudes would elicit some errors. In effect, each separate dimension was intentionally truncated so that there was no anticipation of positive curve acceleration with an asymptote of perfect responding for the smallest dyads. Hence, it seemed reasonable to suppose that the data obtained for the sampled dimensional range would approximate a portion of one or several mathematical functions with a smooth, decelerating asymptotic gradient. The hyperbolic family of curves having the form $Y = aX^b + c$ were selected as the simplest functions consistent with the general assumptions and predictions of this research.

Therefore, it was predicted a priori that an analysis of departure of the 13 comparison pairs from a hyperbolic curve estimated from the empirical data would yield no difference from the overall error curve (or, if necessary, separate curves within range groups). This test was expected to be an approximate indication of goodness of fit with the general assumptions of the Weber and Fechner laws.

8. If intergroup variability differs, variability will tend to be greater for larger-valued pairs and less for smaller-valued pairs. Heteroscedasticity might be expected as an inherent property of the perceptual effects giving rise to the Weber-Fechner functions.

9. No age or sex effects were predicted. Obtained age and sex differences were anticipated to occur independent of the general pattern of relational judgments hypothesized above. For example, hypotheses 1 through 8 were predicted to hold within each individual age level despite differential numbers of errors across age levels.

METHOD

Height and weight were selected from the five stimulus dimensions studied by Rosenthal et al. (in press) for two experiments to be run concurrently with all subjects participating in both. These dimensions can be easily studied through simple but precisely quantified stimulus operations.

Subjects

Children from an early childhood education center in a university setting and a nearby grammar school in a middle-class, suburban section of northern Utah participated as subjects. From each of three age levels, 18 boys and 18 girls were randomly selected: preschool children ranged from 2.3 to 5.8 years old (mean = 3.8 years); first graders ranged from 6.2 to 7.7 years old (mean = 6.7 years); and third graders ranged from 8.0 to 9.5 years (mean = 8.5 years).

Stimulus Materials

For each separate experiment, 13 dyads of precisely quantified stimuli were constructed. The smaller and larger stimuli within each dyad differed by a small deviation which was constant for all pairs. The absolute values of the 13 pairs spanned a wide range of stimulus magnitudes with equal distances between adjacent pairs. The five smallest stimulus dyads (labeled I through V) belonged to the small range

group. The five largest dyads (labeled IX through XIII) belonged to the large range group. The five middlemost dyads (labeled V through IX) belonged to the medium range group. Hence, dyad V was common to both the small and medium range groups, and dyad IX was common to both the medium and large range groups.

The actual stimulus values for each dimension were selected arbitrarily with one major constraint. The values were chosen in informal piloting with children and adults so that subjects seemed to respond randomly to the pairs with the largest magnitudes and at a better than chance but not perfect level for the pairs with the smallest magnitudes. Thus, no attempt was made to formally document Weber- or Fechner-like effects until impressionistic data suggested that the sampled range was suitable for such a demonstration. It would have been fruitless, for instance, to sample only a restricted range of dimensional values which would elicit only perfect or only random judgments.

Height Experiment

The heights of the 13 pairs of lines are presented in Table 1. All 26 line stimuli were strips of 0.6 cm wide black embossing tape placed individually on pieces of white posterboard 32.9 cm high and 11.0 cm wide. Each strip was centered vertically on its posterboard background 1.0 cm from the bottom. As can be seen in Table 1, the shorter members of the 13 dyads ranged from extremes of 1.0 cm to 15.4 cm. The taller members of the 13 dyads ranged from extremes of 1.2 cm to 15.6 cm. A constant difference of 1.2 cm was maintained between the 13 successive shorter dyad members as well as between the 13 successive taller dyad

Table 1. Stimulus Values for Both Dimensions.

Stimulus Dyad	Height Dyads		Weight Dyads	
	Shorter Members	Taller Members	Lighter Members	Heavier Members
I	1.0 cm	1.2 cm	40.0 gm	60.0 gm
II	2.2 cm	2.4 cm	130.0 gm	150.0 gm
III	3.4 cm	3.6 cm	220.0 gm	240.0 gm
IV	4.6 cm	4.8 cm	310.0 gm	330.0 gm
V	5.8 cm	6.0 cm	400.0 gm	420.0 gm
VI	7.0 cm	7.2 cm	490.0 gm	510.0 gm
VII	8.2 cm	8.4 cm	580.0 gm	600.0 gm
VIII	9.4 cm	9.6 cm	670.0 gm	690.0 gm
IX	10.6 cm	10.8 cm	760.0 gm	780.0 gm
X	11.8 cm	12.0 cm	850.0 gm	870.0 gm
XI	13.0 cm	13.2 cm	940.0 gm	960.0 gm
XII	14.2 cm	14.4 cm	1030.0 gm	1050.0 gm
XIII	15.4 cm	15.6 cm	1120.0 gm	1140.0 gm

members. Thus, a fixed 0.2 cm deviation was sustained between the smaller and taller members of each dyad. The heights of the posterboard backgrounds were intentionally more than twice as great as the tallest stimulus line so that the children could not readily use the distance from the tops of the lines to the tops of the posterboard as a cue in making their judgments. With the stimuli so constructed, it was always theoretically easier to judge on the basis of the lines themselves.

Weight Experiment

The magnitudes of the 13 pairs of weights are presented in Table 1. All 26 weight stimuli were constructed of small orange juice cans 5.4 cm in diameter and 9.8 cm in height containing varying amounts of lead. The cans were painted flat black and the tops were covered with taut pieces of black cloth sealed down with black friction tape to make them homogeneous in appearance. As can be seen in Table 1, the lighter members of the 13 dyads ranged from extremes of 40.0 gm to 1120.0 gm. The heavier members of the 13 dyads ranged from extremes of 60.0 gm to 1140.0 gm. A constant difference of 90.0 gm was maintained between the 13 successive lighter dyad members as well as between the 13 successive heavier dyad members. Thus, a fixed 20.0 gm deviation was sustained between the lighter and heavier members of each dyad.

Procedure

Children were taken individually to a test room, seated at a small table across from the male experimenter, and given a brief introduction as follows:

Okay (child's name), we're going to play some games together. When we're all done playing, I have some candy for you as a treat.

The instructions for the line experiment proceeded as follows:

I'm going to show you two lines. One line will always be tall and one line will always be short. I want you to look at both lines and tell me which one is the tall one. Let's begin.

The experimenter began presenting pairs of line stimuli to the subject holding them upright on the table spaced 5.0 cm apart and continued:

Look at these lines. One is tall and one is short. Which one is the tall one?

Look at these lines. One is tall and one is short. Which is the tall one?

Look at these lines. Which is the tall one?

The experimenter proceeded saying either, "Which is the tall one?" or "Which is the tall line?" for the remaining 27 line judgments. If a subject's response was ever unclear, the experimenter asked the child to point to the "tall" line.

The instructions for the weight experiment proceeded as follows:

Which hand do you use to eat and write with? (obtains answer) Good! I'm going to show you two cans. One can will always be heavy and one can will always be light. I want you to use this hand (points to preferred hand) and lift each can one at a time and tell me which is the heavy one. Let's begin.

The experimenter began placing pairs of weight stimuli on the table in front of the subject several centimeters apart and continued:

Lift each can. One is heavy and one is light. Which one is the heavy one?

Lift each can. One is heavy and one is light. Which one is the heavy one?

Lift each can. Which is the heavy can?

The experimenter continued with "Which can is the heavy one?" and "Which is the heavy can?" for the remaining 27 weight judgments. If a subject's response was ever unclear, the experimenter asked the child either to point to or to hand him the "heavy" can.

At the conclusion of both experiments, the experimenter thanked the child and let him choose two pieces of wrapped candy. The average time required per subject was roughly 12 minutes. No child ever required as long as 20 minutes.

Experimental Variations

For each experiment considered separately, every child judged exactly five of the possible 13 stimulus dyads. Children assigned to the small range group for a given dimension judged the five pairs of smallest magnitude (i.e., dyads I through V as listed in Table 1); children assigned to the large range group judged the five pairs of largest magnitude (i.e., IX through XIII); and children assigned to the medium range group judged the five dyads of middlemost value (i.e., V through IX). Each child judged his allotted pairs six times apiece

for a total of 30 judgments in all. Stimulus presentation within any pair was balanced so as not to favor the left or right sides of any child across his six repetitions of a given stimulus pair. For example, a child receiving dyad III of the line dimension viewed the 3.4 cm line three times to the left and three times to the right of the 3.6 cm line. The order of pair presentation and the sequence of left-right placement of stimulus members within pairs were individually randomized for each child without further constraint.

Although the three range groups of five ordered pairs per stimulus dimension differed in their absolute magnitudes (except for two replications), the three sets were congruent in terms of their relative values, i.e., every pair held a precise ordinal position with respect to the remaining four. In this sense, the following five triads of comparison pairs were held to be parallel: (1) I, V, IX; (2) II, VI, X; (3) III, VII, XI: (4) IV, VIII, XII; and (5) V, IX, XIII. This within-subjects ordinal position variate was factorially crossed with the range group variate in the main statistical analyses.

From each age level, two boys and two girls were randomly assigned to each of nine treatment dyads created by factorially combining the three range groups of line stimuli (short, medium, and tall) and the three range groups of weight stimuli (light, middle, and heavy). Within each dyad so created, one boy and one girl from each age level (randomly chosen) judged the line stimuli before judging the weight stimuli. The remaining children judged the weight stimuli prior to the

line stimuli. The 18 treatment conditions so described served to control for irrelevant order and range effects between dimensions.

Scoring and Analyses

Each incorrect judgment was scored as 1.0 and each correct judgment as 0.0. Summing across individual children's six repeated judgments of particular stimulus pairs, raw scores ranging from 0.0 to 6.0 were obtained. The scores from the height and weight dimensions were separately analyzed by the same set of statistical procedures.

Analyses of Slope

The predicted curvilinear monotonic relationship between magnitude of pair values and number of judgment errors was tested in a 3(range group) x 5(ordinal position) x 3(age level) x 2(sex) factorial design. Post-hoc trend analyses or Scheffé tests were used to determine the locus of significant effects. In this analysis, significant positive slopes for range group and ordinal position were indicants of the proper monotonic relationship. A significant interaction between these variates such that slope was less for pairs in range groups having larger stimulus values was indicative of the proper curvilinear relationship. Significant quadric trend components indicating decelerating slope for either or both of these variates were further evidence supporting the predicted curvilinearity. Conceptually relevant age and sex effects appeared as interactions with either range group or ordinal position or both. The absence of such interactions indicated similar patterns of

relational responding independent of any main effects for the age or sex variates.

Contextual Analyses

Error differences for the two replicated comparisons per dimension were separately analyzed in a 2(contextual magnitude) x 3(age level) x 2(sex) factorial design. Again, age and sex were of conceptual interest only if they interacted with contextual magnitude.

Goodness-of-Fit Tests

Raw error scores were converted into scores indicating the total proportion of correct responding for each of the 13 comparison pairs per dimension. A hyperbolic function of the form $Y = aX^b + c$ with the comparison dyad (arbitrarily valued 1.0 to 13.0 with increasing dyad magnitude), Y the proportion of correct relational responding, and c the theoretical asymptote of 50.0 percent correct responding was obtained for each dimension. Parameters a and b were derived from the data by determining the reduction line of best fit for $\log(Y - c)/\log X$ (Lewis 1960). An F test of goodness-of-fit for any function having both independent and related samples was used to determine if the 15 samples from the entire dimensional range (including the two replicated comparisons) departed significantly from the obtained hyperbolic equation. The relevant equation for the F ratio was developed from Lewis' (1960) derivations of F tests for independent samples and related samples considered separately. The complete derivation of this formula is presented in Appendix A.

Tests for Homoscedasticity

For each separate dimension, variances based upon the average number of errors each child made on his five dyads were computed for the nine age level x range group combinations. The following two sets of comparisons were performed using Cochran's test for homogeneity of variance (Myers 1971):

1. Within each separate age level, the variances for the small, medium, and large range groups were compared (three tests per dimension).
2. Within each separate range group, the variances for preschoolers, first graders, and third graders were compared (three tests per dimension).

In addition, these variances were inspected to determine if there were any consistent trends which did not attain statistical significance.

RESULTS

Theoretically, if judgments were purely random, the expected number of errors for any treatment combination would be 3.00. The obtained means for all major treatment combinations in both experiments were less than 3.00 indicating response patterns consistently better than chance. All significance levels reported in this paper were based on two-tailed probability estimates. Significance levels for all post-hoc trend analyses were set at a familywise error rate of .05 through the use of Dunn's procedure for dividing the critical region of significance into equal portions (Kirk 1968). Similarly, significance levels for all Scheffé tests employed a .05 error rate per family of comparisons.

Slope

Height Dyads

Figure 1 presents the mean number of errors for each range group with preschooler means and composite first and third grader means separately displayed. Analysis of variance revealed an unpredicted age level main effect ($F = 39.46$; $df = 2/90$; $p < .001$). Scheffé tests indicated that preschoolers made significantly more errors than either first or third graders (both p's $< .001$), but that the two older groups did not differ. Since age level did not interact with any variate, all other effects occurred in parallel for the three age groups as hypothesized.

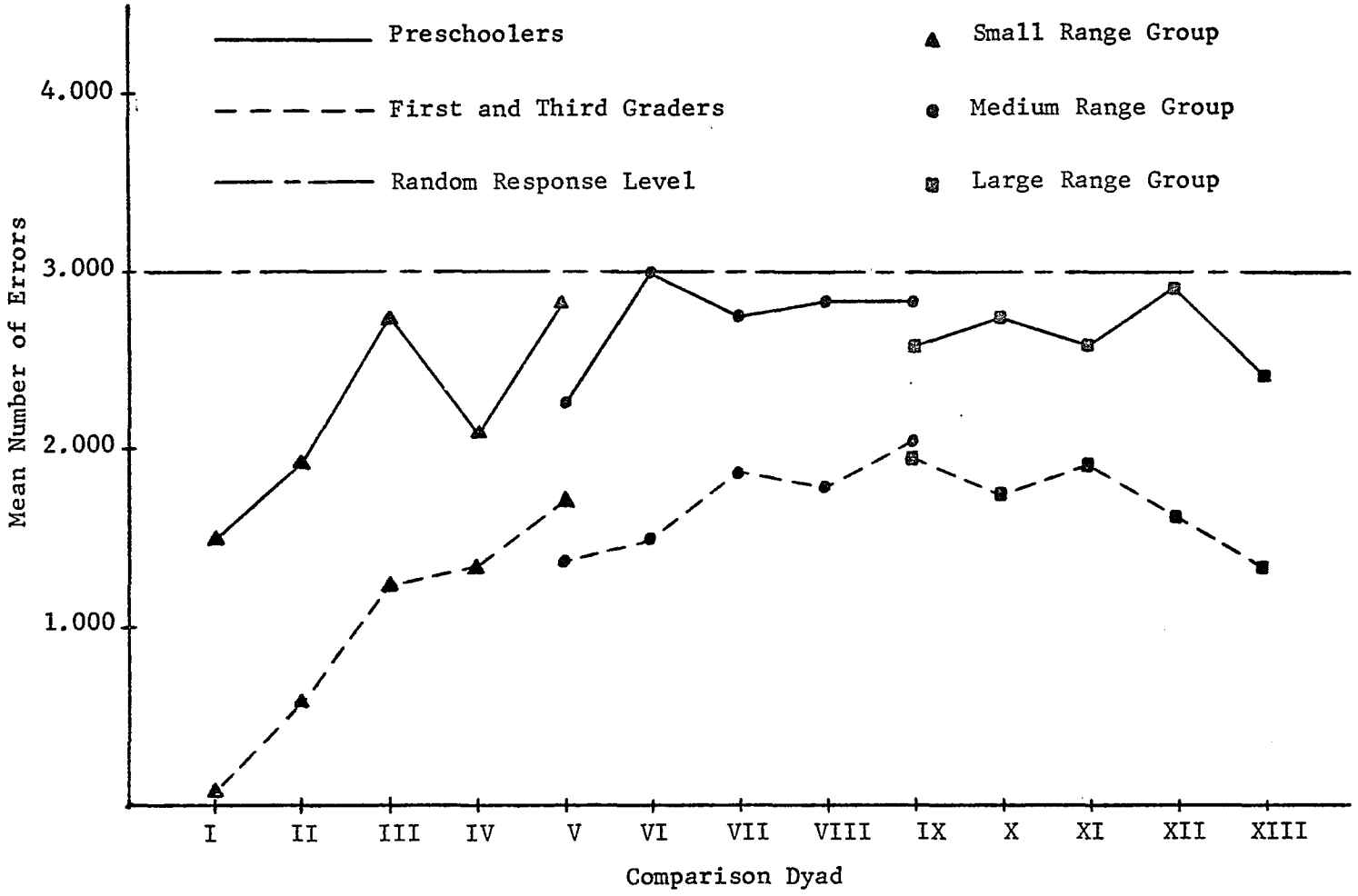

Figure 1. Error Curves for Height.

Table 2 presents the mean number of errors for each range group. Inspection reveals the expected increase in errors from the small to the medium range, but also an unanticipated decline one-twentieth the previous amount from the medium to the large range group. Analysis of variance disclosed a significant main effect for these obtained differences ($F = 14.07$; $df = 2/90$; $p < .001$). Post-hoc trend tests confirmed the presence of the predicted positive linear relationship between errors and range magnitude ($F = 19.98$; $df = 1/90$; $p < .005$) as well as the hypothesized decrease in slope from the small to the large group ($F = 8.16$; $df = 1/90$; $p < .025$).

Table 2. Mean Errors for Range Groups.

Dimension	Small Range	Medium Range	Large Range
Height	1.400	2.055	2.022
Weight	2.183	2.555	2.600

Table 3 presents the mean number of errors for ordinal position of dyads within range groups. Inspection reveals positive increments from the first to the third positions with a subsequent flattening of the curve from the third through the fifth positions. Analysis of variance disclosed a significant main effect for these observed differences ($F = 7.08$; $df = 4/360$; $p < .001$). Post-hoc trend analyses confirmed that the predicted positive linear relationship existed between number

Table 3. Mean Errors for Ordinal Position of Dyads.

Dimension	Ordinal Position				
	1	2	3	4	5
Height	1.453	1.703	2.018	1.925	2.027
Weight	2.231	2.287	2.398	2.481	2.833

of errors and the ordinal position of stimulus dyads (F = 18.60; df = 1/90; $p < .005$), but indicated that the anticipated decrease in slope with increasing pair value reached only borderline significance (F = 5.44; df = 1/90; $.05 < p < .10$).

Examination of Figure 1 reveals that slope within range groups decreased for successive range groups. Analysis of variance disclosed a significant range group x ordinal position interaction (F = 5.51; df = 8/360; $p < .001$) confirming that, as hypothesized, slope dissipated with increasing range magnitude. Post-hoc trend analyses disclosed a significant linear range group x linear ordinal position interaction (F = 33.84; df = 1/90; $p < .001$). Hence, change in linear slope across adjacent range groups appeared to be constant. It should be noted that, contrary to expectation, dyad XIII elicited considerably fewer errors than the other four large range pairs which differed little in total errors. A t test between this pair and dyad IX (which tied for the greatest number of errors in the large range group) was non-significant (t = 1.60; df = 35; $p > .10$). Thus, this error decrement probably reflected chance variation.

Analysis of variance also revealed an unpredicted sex main effect ($F = 4.08$; $df = 1/90$; $p < .05$) indicating that boys made significantly fewer errors (mean = 1.711) than girls (mean = 1.940). Since sex did not interact with any variate, all other effects were parallel for male and female children as hypothesized.

Weight Dyads

Figure 2 presents the mean number of errors for each range group with preschooler means and composite first and third grader means separately displayed. Analysis of variance disclosed an unpredicted age level main effect ($F = 19.55$; $df = 2/90$; $p < .001$). Scheffé tests indicated that preschoolers made significantly more errors than either first or third graders (both p's $< .005$), but that the two older groups did not differ. Contrary to hypothesis, age level interacted with range group indicating dissimilar response curves across the different range magnitudes (see below). The absence of further age level interactions indicated that, as expected, remaining effects occurred in parallel for the three groups of children.

Table 2 presents the mean number of errors for each range group. Inspection reveals an increase in errors for successive range groups as well as a decrease in slope with increased range magnitude. Analysis of variance disclosed a significant main effect for these obtained differences ($F = 6.79$; $df = 2/90$; $p < .005$). Post-hoc tests confirmed the presence of the predicted positive linear relationship between errors and range magnitude ($F = 11.27$; $df = 1/90$; $p < .01$), but disclosed that the observed curvature was non-significant, although it was in the

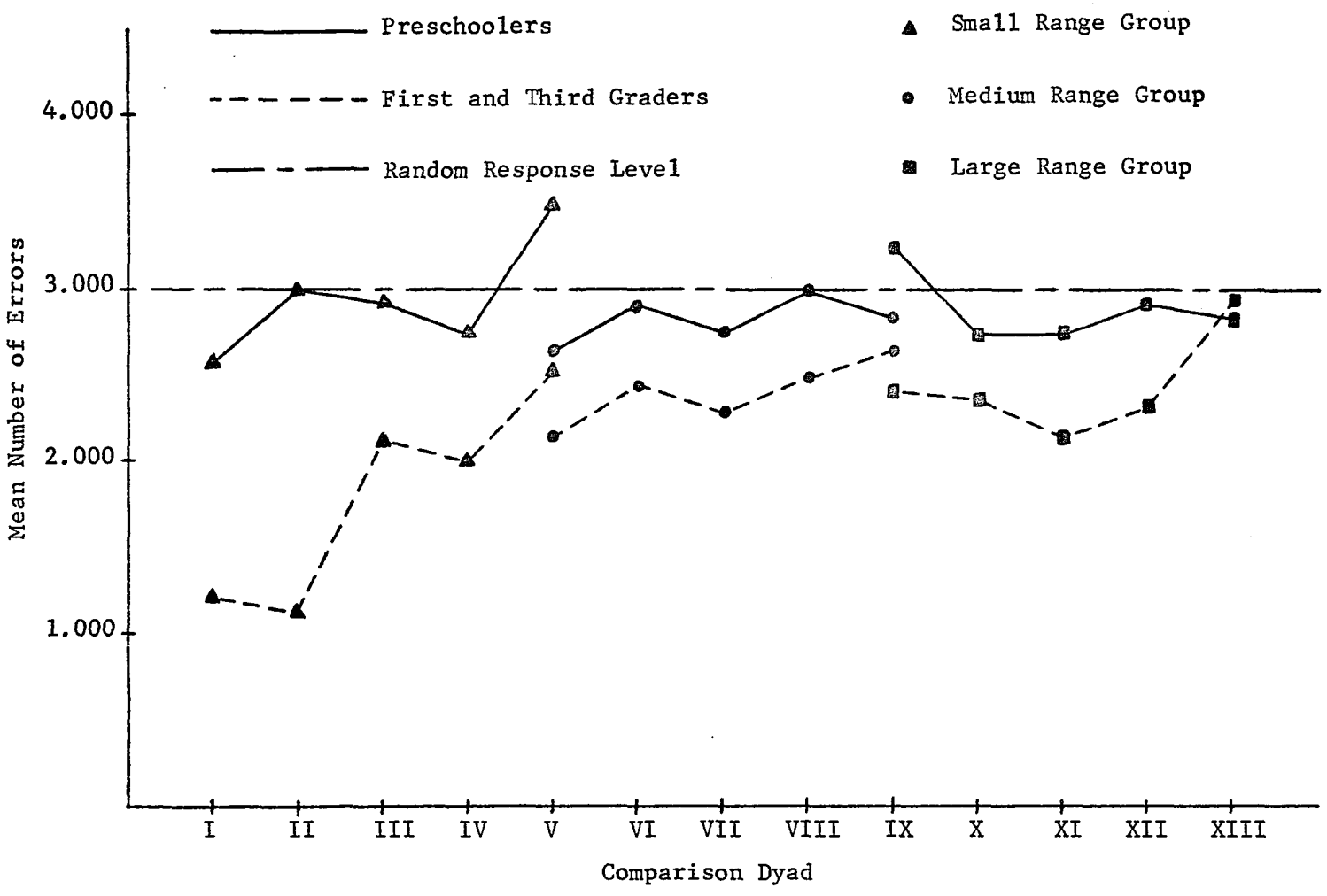

Figure 2. Error Curves for Weight.

expected direction. Table 4 presents the same data partitioned into age levels for each range group. Examination reveals that, except for a slight decrease in errors from the medium to the large range group for first graders, errors for the two older groups of children conformed to general expectations. In contrast, however, errors for the preschoolers were essentially equal for all range magnitudes and only negligibly better than chance. Analysis of variance disclosed a significant range group x age level interaction ($F = 3.37$; $df = 4/90$; $p < .025$) confirming differences in slope for the several age levels. Scheffé tests comparing linear trend across range groups indicated that preschoolers differed significantly from third graders ($p < .025$), but that first graders did not differ from the other two age levels. No differences in quadratic curvature were obtained.

Table 4. Error Means by Range Group and Age Level for Weight.

Age Level	Range Group		
	Small	Medium	Large
Preschool	2.950	2.833	2.900
First Grade	1.883	2.500	2.283
Third Grade	1.716	2.333	2.616

Table 3 presents the mean number of errors for ordinal position of dyads within range groups. Inspection reveals positive increments for successive pairs, but no evidence of any quadratic curvature. Analysis of variance disclosed a significant main effect for these observed differences ($F = 5.00$; $df = 4/360$; $p < .001$). Post-hoc trend analyses confirmed the presence of the predicted linear relationship between errors and pair magnitude ($F = 16.99$; $df = 1/90$; $p < .005$) as well as the absence of the anticipated slope deceleration.

Inspection of Figure 2 reveals that slope within range groups decreased for successive range groups. Analysis of variance disclosed a significant range group x ordinal position interaction ($F = 2.12$; $df = 8/360$; $p < .05$) confirming that, as hypothesized, slope dissipated with increasing range magnitude. Post-hoc tests disclosed a linear range group x linear ordinal position interaction ($F = 8.46$; $df = 1/90$; $p < .05$). Hence, change in linear slope across adjacent range groups appeared to be constant. Although close inspection of Figure 2 reveals a shallower positive gradient for preschoolers than for the older children in the small range group, analysis of variance did not disclose a significant range group x ordinal position x age level interaction to confirm any important departure from the predicted parallel slopes. Nevertheless, the preschooler data cannot realistically be interpreted as partially supporting the experimental hypotheses, since all five error means for the small range dyads were so close to the chance value of 3.00.

Error means for boys and girls did not differ. The absence of any sex interaction effects confirmed the hypothesis of parallel effects for male and female children.

Contextual Differences

Height Experiment

Table 5 presents the mean number of errors for the replicated comparison dyads V and IX in their contexts of smaller and larger stimulus values (i.e., the small versus the medium or the medium versus the large range groups). Inspection reveals that each separate dyad elicited more errors when it was judged in a context of smaller magnitude pairs than when it was judged in a context of larger magnitude pairs. Analysis of variance disclosed borderline significance for the difference for dyad V ($F = 3.41$; $df = 1/60$; $.05 < p < .10$), but also that the difference for dyad IX did not approach significance. Although no significant contextual effects emerged, the observed differences were in the predicted direction.

Table 5. Error Means for Replicated Comparison Dyads.

Dimension	Dyad	Smaller Context	Larger Context
Height	V	2.083	1.666
	IX	2.305	2.138
Weight	V	1.716	1.400
	IX	1.633	1.616

Several age level and sex effects were obtained in the above analyses. Since these findings merely corroborated effects obtained in the slope analyses, they were not of special conceptual import, but are reported here for the interested reader. For dyad V, analysis of variance revealed a significant age level main effect ($F = 11.67$; $df = 2/60$; $p < .001$) and a borderline sex main effect ($F = 3.41$; $df = 1/60$; $.05 < p < .10$). For dyad IX, analysis of variance disclosed a significant age level main effect ($F = 3.62$; $df = 2/60$; $p < .05$), a significant sex main effect ($F = 5.70$; $df = 1/60$; $p < .025$), and a borderline age level x sex interaction ($F = 2.62$; $df = 2/60$; $.05 < p < .10$).

Weight Experiment

Table 5 presents the mean number of errors for dyads V and IX in their contexts of smaller and larger stimulus values. Inspection reveals that each separate dyad elicited more errors when it was judged in a context of smaller magnitude pairs than when it was judged in a context of larger magnitude pairs. Analysis of variance disclosed borderline significance for the obtained difference for dyad V ($F = 3.46$; $df = 1/60$; $.05 < p < .10$) and, for dyad IX, a borderline contextual magnitude x age level effect ($F = 3.06$; $df = 2/60$; $.05 < p < .10$) indicating that first graders tended to score in the predicted direction whereas preschoolers and third graders tended to score in the opposite direction. Again, although no significant contextual effects emerged, over-all differences were in the predicted direction.

One age level main effect was found in the above analyses supporting an earlier finding in the slope analyses. Analysis of

variance for dyad V revealed a significant age level main effect ($F = 3.29$; $df = 2/60$; $p < .05$).

Goodness of Fit

Height Dimension

The hyperbolic equation $Y = .369\ X^{-.385} + .500$ was estimated from the data by solving for the reduction line of best fit of $\log(Y - .500)/\log X$. The Pearson product moment correlation coefficient (r) for the reduction line was -.847. Hence, linear correlation appeared to account for about 71 percent of the obtained variance of the 13 means from this line.

Figure 3 presents a plot of the 15 dyad samples for the combined age levels (including the two replicated comparisons) against the obtained hyperbolic function. An F test of goodness of fit of the 15 samples to the estimated equation (see Appendix A) did not quite attain rejection level ($F = 1.73$; $df = 13/420$; $p < .05$). It should be noted that, although contextual differences were apparently small, contextual influences in the present analysis would have tended to inflate the numerator of the F ratio, hence, making rejection of the null hypothesis more likely.

Weight Dimension

The hyperbolic equation $Y = .254\ X^{-.660} + .500$ was estimated from the data by solving for the reduction line of best fit of $\log(Y - .500)/\log X$. The Pearson product moment correlation for the line of best fit was -.724. Hence, approximately 52 percent of the

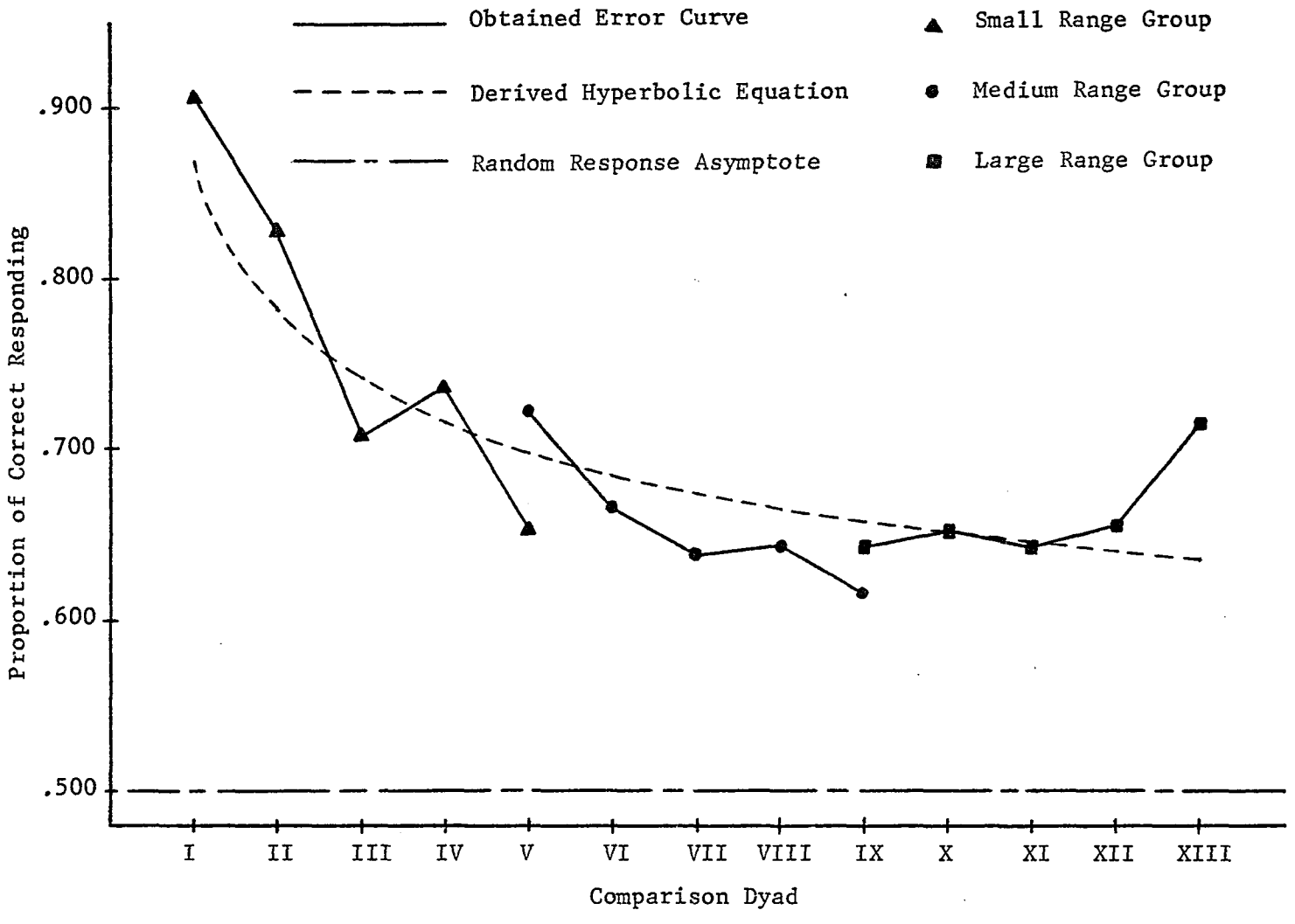

Figure 3. Departure from a Hyperbolic Curve for Height.

variance of the 13 means appeared to be accounted for by linear correlation.

Figure 4 presents a plot of the 15 dyad samples for the combined age levels against the obtained hyperbolic function. An F test goodness of fit did not approach rejection level ($F = 1.32$; $df = 13/420$; $p > .20$). Again, contextual influences would have tended to inflate the numerator of the F ratio enhancing the likelihood of rejecting the null hypothesis.

Homoscedasticity

Height Stimuli

Table 6 presents the variances for the nine age level x range group combinations for the height experiment. Five of the six separate tests for homogeneity of variance were non-significant. Within the small range group, Cochran's test attained statistical significance ($C = .6106$; $df = 11$; $p < .05$). Inspection of Table 6 reveals that judgmental variability declined with increasing age level for this portion of the stimulus dimension. The same pattern also occurred within the medium range, but variability was least for preschoolers within the large range group. Inspection further discloses that variability increased consistently with increasing range magnitude with one exception, i.e., variability for preschoolers in the large range group was very low relative to variability for most other age level x range group combinations. However, none of these trends approached significance.

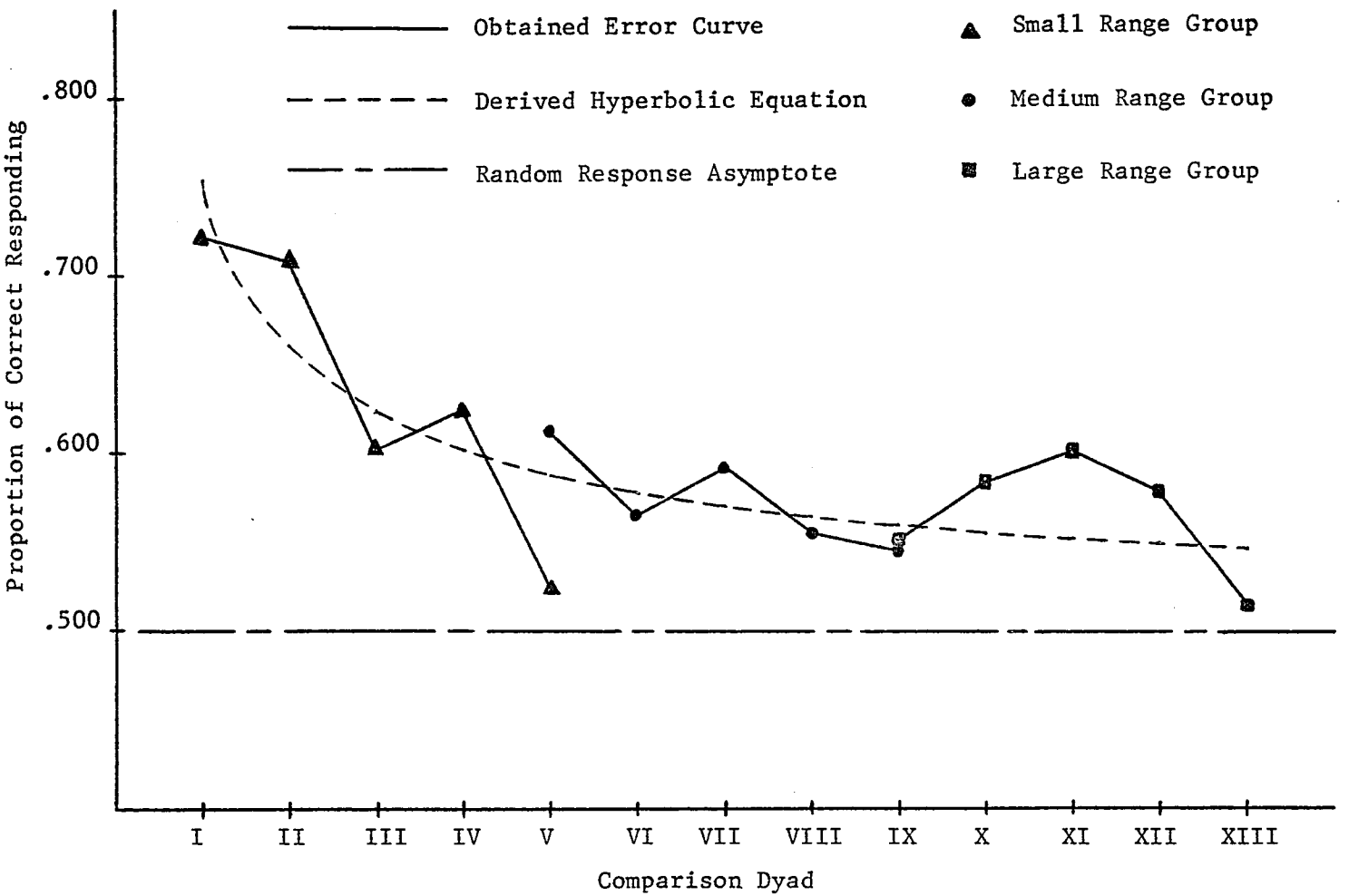

Figure 4. Departure from a Hyperbolic Curve for Weight.

Table 6. Variances by Range Group and Age Level for Height.

Age Level	Range Group		
	Small	Medium	Large
Preschool	0.672	0.890	0.175
First Grade	0.308	0.410	0.558
Third Grade	0.119	0.257	0.422

Weight Stimuli

Table 7 presents the variances for the nine age level x range group combinations for the weight experiment. All six of the six separate tests for homogeneity of variance were non-significant. Inspection of Table 7 reveals that no consistent relationships appeared between age level and judgmental variability within range groups. Inspection further discloses that no consistent trends emerged between range magnitude and variability within age levels.

Summary

The major findings supporting the experimental hypotheses were as follows:

1. Over-all, children consistently rendered both height and weight judgments in the correct relational direction.
2. Over-all, for both dimensions, the same constant stimulus deviation elicited more errors for larger magnitude pairs than for smaller magnitude pairs. These differences were observed both between range groups and within range groups.

Table 7. Variances by Range Group and Age Level for Weight.

Age Level	Range Group		
	Small	Medium	Large
Preschool	0.277	0.275	0.090
First Grade	0.163	0.483	0.214
Third Grade	0.512	0.198	0.272

3. Over-all, for both experiments, error slopes declined with increasing stimulus magnitude. These trends also were evident both between and within range groups.
4. In the line experiment, despite unpredicted age level and sex main effects, all groups exhibited parallel effects across the different stimulus magnitudes. The unpredicted main effects themselves did not contradict any hypotheses.
5. For both height and weight, over-all curves of the proportion of correct responding conformed well to hyperbolic curves consistent with the general assumptions and predictions of the Weber and Fechner functions.
6. The paucity of evidence for heteroscedasticity did not contradict any experimental hypotheses. For the height experiment, both the trend suggesting a negative relationship between age level and variability as well as the trend hinting at a positive relationship between range magnitude and variability were consistent with all predictions.

The following discrepant effects were also found:

1. For the weight dimension, preschoolers rendered judgments almost at random, although they still rendered slightly more correct than incorrect judgments. The unpredicted age level main effect itself did not contradict any experimental hypotheses.
2. A corollary to the previous item, preschoolers did not exhibit the predicted monotonic, decelerating relationship between errors and stimulus magnitude for the weight experiment.
3. None of the predicted contrast effects for the two replicated comparisons per experiment reached significance. Still, all four differences were in the expected direction.

DISCUSSION

Aggregately, the present results clearly demonstrate that young children exhibit perceptual effects approximating the Weber and Fechner effects found in adults. Evidence of the predicted monotonic, decelerating relationship between errors and dyad magnitude was obtained both at the comparatively gross level of measurement between range groups and at the much finer level of slope analysis within range groups. Two separate experiments with stimulus dimensions involving different sensory modalities yielded similar response curves, although one major age discrepancy occurred for the youngest children. Goodness of fit tests demonstrated that the over-all data from both experiments adequately approximated hyperbolic curves with random response as asymptotes despite any differences due to age level or sex. Hence, errors in the children's relational judgments seemed to reflect a reasonably smooth mathematical function congruent with the assumptions and predictions of the classic Weber and Fechner equations.

Preschoolers made significantly more errors than first and third graders for both the height and weight experiments. Although unpredicted, these effects did not in and of themselves contradict any of the experimental hypotheses. Clearly, on discrimination tasks similar to the present two, older children may outperform younger children for any one of a variety of theoretically plausible reasons. For instance, they

may have more training or experience with the relevant dimension, or they may have superior attentional skills requisite for tolerating the experimental procedures, or they even may be slightly more competent physiologically in detecting the relevant stimulus differences. Some of the preschoolers in the present sample were younger than had been originally anticipated, and these factors may have contributed in part to the poorer performance of this group. In any case, age level main effects were not of prime concern in the present investigation. The important conceptual issue was whether or not parallel perceptual effects would occur independent of any age level main effects.

Parallel perceptual effects for age level occurred in the line experiment, but preschoolers did not exhibit the hypothesized effects in the weight experiment. Several differing interpretations of this discrepancy are possible including the following:

1. Preschoolers can judge heights relationally, but not weights.
2. Weber-Fechner-like effects operate in preschoolers for some dimensions such as height, but not for others including weight.
3. Parallel perceptual effects do operate in preschoolers and first and third graders for both dimensions, but some aspects of the experimental procedures acted mainly to the detriment of the youngest children with crucial significance in the weight experiment.

Alternative 1 is difficult to entertain, since Rosenthal et al. (in press) obtained strong evidence of relational judgments with weights using preschoolers. In addition it hardly seems surprising that the preschoolers

performed so poorly in the present weight experiment, since the older children also made substantially more errors than in the height experiment. The difference between error means for the preschoolers and the older children was 57 percent greater in the height experiment than in the weight experiment. Alternative 2 is defensible even given previous evidence of judgmental relativity with weights. Confirmation of this possibility would require a lack of evidence for the hypothesized effects despite an over-all pattern of judgments consistently in the correct relational direction. Conversely, confirmation of alternative 3 would require the emergence of the predicted effects whenever judgmental relativity is obtained. Unfortunately, since preschoolers performed very close to chance, the present weight experiment was not an adequate test forcing choice in favor of 2 or 3. This issue might be resolved very simply by replicating the weight experiment with a larger constant deviation between members of each stimulus dyad. For instance, 30.0 gm differences should elicit fewer errors than did the present 20.0 gm deviations. The smaller stimulus members of each dyad could merely retain their current values in order to have the same span of the stimulus dimension sampled.

In accordance with expectation, a replicated dyad tended to elicit more errors when it held the maximum magnitude in a spectrum of smaller values than when it held the minimum magnitude in a range of greater values. No obtained differences were significant, but the directional consistency of the differences for the four replicated pairs suggests that small contextual effects may have been present. Of further

interest, the differences for the comparisons replicated in the medium and large range groups were smaller than the deviations for those repeated in the small and medium groups. This observation was consistent with theory, since contrast should be least between contexts which are closest in terms of absolute difficulty. In general, however, contextual effects were of negligible importance in the present experiments, even though they may have mildly inflated the critical ratios in the goodness of fit tests by contributing to the variance of the dyad sample means from the estimated hyperbolic equations.

The present experimental procedures may have inherently constrained variability so that heteroscedasticity was unlikely, although its absence did not contradict any hypotheses. Many children tended to heavily favor either the dyad member on their left or right when they were responding at a random level, or similarly, they tended to choose from both alternatives in large blocks of consecutive left or right judgments. Hence, since the placement of the smaller and larger dyad members was balanced left and right for each individual child, many children made exactly three errors on the most difficult dyads. There seemed to be fewer instances of random deviation from the error mode of three than one might have expected without these left-right response biases. The net effect may have been to constrain variability for higher magnitude dyads thereby preserving homoscedasticity across the total dimensional spectrum. Nevertheless, despite this inherent limitation, individual balancing of left-right dyad members was probably the safest way to protect against more damaging adverse effects due to response bias.

On methodological grounds, Rosenthal et al. (in press) questioned O'Reilly and Steger's (1970) finding that children below age six could not even respond relationally to the presence of an anchor. Similarly, on the basis of the consistent relational shifts in judgment they obtained, they also questioned Piaget's (1968) view that children are incapable of even elementary relativism of thought. The quantitative evidence now reported casts further doubt upon the methodological adequacy of the O'Reilly and Steger study as well as upon the soundness of Piaget's theoretical pronouncement. Not only did children respond relationally on stimulus dyads purposely made difficult, but their pattern of incorrect relational judgments was shown to approximate what would be expected on the basis of two classic psychophysical functions well-known in adults. It now seems still harder to deny that similar fundamental principles of adult human judgment extend to even young children.

APPENDIX A

DERIVATION OF F FOR INDEPENDENT REPLICATIONS OF RELATED SAMPLES

Lewis (1960), in his discussion of F tests of goodness of fit, derives F ratios appropriate for evaluating any complex function with either independent samples or related samples. In order to evaluate the over-all fit of the error values to the obtained hyperbolic functions in the present research, it was necessary to derive a similar formula for testing the over-all fit of three independent sets (range groups) of five related samples (five pairs within each range group). The derivation of the appropriate F ratio is presented below.

In an experimental investigation involving an independent variable X and a dependent variable Y, the data may be approximated by the equation

$$Y_{ikc} = f(X_{ik}) \qquad (1)$$

on the basis of theory or some curve-fitting procedure where Y_{ikc} represents the calculated or theoretical value of Y at different X_{ik} samples of X. In the ik th sample of X where the i th set of subjects is tested at each of k different values, n_{ik} values of Y will be obtained. Samples of X within the i th set will be statistically related, but samples across the i sets will be statistically independent. Some of the ik samples may occur at identical values of X, but at least k

different values will be sampled and no more than ik. The mean of the ik th sample may be defined by

$$M_{ik} = \frac{1}{n_{ik}} \Sigma_j Y_{ikj} \tag{2}$$

where Y_{ikj} is the symbol for single values of Y and Σ_j stands for summation over all of the Y values in that sample. The mean of the ij th subject will be given by

$$M_{ij} = \frac{1}{n_{ij}} \Sigma_k Y_{ikj} \tag{3}$$

where n_{ij} is the number of experimental samples within the i sets. A population value of the mean (Y_{ikc}) may be associated with each sample value of the mean (M_{ik}).

An expression for the total sum of squares of the deviations of the N values of Y_{ikj} from Y_{ikc} is simply

$$SS_t = \Sigma_k \Sigma_j (Y_{ikj} - Y_{ikc})^2. \tag{4}$$

Adding and subtracting M_{ik} inside parentheses yields

$$SS_t = \Sigma_k \Sigma_j (Y_{ikj} - M_{ik} + M_{ik} - Y_{ikc})^2$$

or

$$SS_t = \Sigma_k \Sigma_j [(Y_{ikj} - M_{ik}) + (M_{ik} - Y_{ikc})]^2 \tag{4a}$$

Expansion produces

$$SS_t = \Sigma_k\Sigma_j(Y_{ikj} - M_{ik})^2 + \Sigma_k\Sigma_j(M_{ik} - Y_{ikc})^2$$

$$+ 2\,\Sigma_k\Sigma_j(Y_{ikj} - M_{ik})(M_{ik} - Y_{ikc}) \qquad (4b)$$

It may easily be shown that the cross-products term vanishes leaving

$$SS_t = \Sigma_k\Sigma_j(Y_{ikj} - M_{ik})^2 + \Sigma_k\Sigma_j(M_{ik} - Y_{ikc})^2 \qquad (4c)$$

Since by definition $\Sigma_j = n_{ik}$, then

$$SS_t = \Sigma_k\Sigma_j(Y_{ikj} - M_{ik})^2 + n_{ik}\Sigma_k(M_{ik} - Y_{ikc})^2 \qquad (4d)$$

Division by the hypothetical variance σ_i^2 yields

$$\frac{SS_t}{\sigma_i^2} = \frac{\Sigma_k\Sigma_j(Y_{ikj} - M_{ik})^2}{\sigma_i^2} + \frac{n_{ik}\Sigma_k(M_{ik} - Y_{ikc})^2}{\sigma_i^2} \qquad (5)$$

The two terms on the right are mutually independent and each is distributed as χ^2. The numerator of the second term is simply the weighted sum of squares of the deviations of the sample means from their calculated or theoretical means.

The first term of equation (4d) may be further broken down by adding and subtracting the quantity $(M_{ij} - M_{iY})$ obtaining

$$\Sigma_k\Sigma_j(Y_{ikj} - M_{ik})^2 = \Sigma_k\Sigma_j[(Y_{ikj} - M_{ik} - M_{ij} + M_{iY}) + (M_{ij} - M_{iY})]^2 \quad (6)$$

where M_{iY} is the mean of the $\underline{i}$ th set of subjects. Expansion yields

$$\begin{aligned}\Sigma_k\Sigma_j(Y_{ikj} - M_{ik})^2 = {} & \Sigma_k\Sigma_j(Y_{ikj} - M_{ik} - M_{ij} + M_{iY})^2 \\ & + \Sigma_k\Sigma_j(M_{ij} - M_{iY})^2 \\ & + 2\,\Sigma_k\Sigma_j(Y_{ikj} - M_{ik} - M_{ij} + M_{iY}) \\ & \cdot (M_{ij} - (M_{iY}) \end{aligned} \quad (6a)$$

Again, the cross-products term vanishes leaving

$$\begin{aligned}\Sigma_k\Sigma_j(Y_{ikj} - M_{ik})^2 = {} & \Sigma_k\Sigma_j(Y_{ikj} - M_{ik} - M_{ij} + M_{iY})^2 \\ & + \Sigma_k\Sigma_j(M_{ij} - M_{iY})^2 \end{aligned} \quad (6b)$$

Division by σ_i^2 yields

$$\frac{\Sigma_k\Sigma_j(Y_{ikj} - M_{ik})^2}{\sigma_i^2} = \frac{\Sigma_k\Sigma_j(Y_{ikj} - M_{ik} - M_{ij} + M_{iY})^2}{\sigma_i^2} + \frac{\Sigma_k\Sigma_j(M_{ij} - M_{iY})^2}{\sigma_i^2} \quad (7)$$

The two terms on the right are once more mutually independent and distributed as χ^2.

The numerator of the first term on the right in equation (6b) is the sum of squares for the interaction between level of X (i.e., X_{ik}) and individual subjects (I).

$$SS_{X \times I} = \Sigma_k \Sigma_j (Y_{ikj} - M_{ik} - M_{ij} + M_{iY})^2 \tag{8}$$

The numerator of the second term is the sum of squares of the deviations of the individual subject means from the general mean of the $\underline{i}$ set (M_{iY}), i.e., the between-individuals sum of squares.

$$SS_{bI} = \Sigma_k \Sigma_j (M_{ij} - M_{iY})^2 \tag{9}$$

Hence, use of the first term eliminates any real differences that are due to variations in the mean performance of the individual subjects and will provide an appropriate error term for testing the deviations of the sample means from the "curve of best fit."

The second term on the right from equation (5) gives the relation

$$\chi_{1i}^2 = \frac{n_{ik}\Sigma_k (M_{ik} - Y_{ikc})^2}{\sigma_i^2} \text{ with } r_{1i} = n_{ij} \tag{10}$$

Similarly, the first term on the right from equation (7) gives the relation

$$\chi_{2i}^2 = \frac{\Sigma_k \Sigma_j (Y_{ikj} - M_{ik} - M_{ij} + M_{iY})^2}{\sigma_i^2} \qquad (11)$$

$$\text{with } r_{2i} = (n_{ij} - i)(n_{ik} - i)$$

Utilizing the additive property of χ^2, summation over the n_i sets yields

$$\chi_1^2 = \Sigma_i \frac{n_{ik} \Sigma_k (M_{ik} - Y_{ikc})^2}{\sigma_i^2} \qquad (10a)$$

$$\text{with } r_1 = \Sigma_i (n_{ij}) = n_i (n_{ij})$$

and $$\chi_2^2 = \Sigma_i \frac{\Sigma_k \Sigma_j (Y_{ikj} - M_{ik} - M_{ij} + M_{iY})^2}{\sigma_i^2} \qquad (11a)$$

$$\text{with } r_2 = \Sigma_i (n_{ij} - 1)(n_{ik} - 1)$$

$$= n_i (n_{ij} - 1)(n_{ik} - 1)$$

Therefore, since

$$F = \frac{\chi_2^2 / r_2}{\chi_1^2 / r_1} = \frac{\chi_2^2}{r_2} \cdot \frac{r_1}{\chi_1^2} \qquad (12)$$

$$\text{with } \gamma_1 = r_2 - 1 \text{ and } \gamma_2 = r_1 - 1$$

equations (10a) and (11a) may be utilized to obtain the desired F ratio. Hence,

$$F = \frac{\Sigma_i[n_{ik}\Sigma_k(M_{ik} - Y_{ikc})^2/\sigma_i^2]}{n_i(n_{ij})} \cdot \frac{n_i(n_{ij} - 1)(n_{ik} - 1)}{\Sigma_i[\Sigma_k\Sigma_j(Y_{ikj} - M_{ik} - M_{ij} + M_{iY})^2/\sigma_i^2]} \tag{13}$$

$$\text{with } \gamma_1 = n_i(n_{ij}) \text{ and } \gamma_2 = n_i(n_{ij} - 1)(n_{ik} - 1)$$

If the $n_i(n_{ij})$ values of Y_{ikc} are obtained from a source other than the sample data, equation (13) is the proper formula for F. However, if C constants (parameters) are calculated from the empirical data, this equation assumes the general form

$$F = \frac{\Sigma_i[n_{ik}\Sigma_k(M_{ik} - Y_{ikc})^2/\sigma_i^2]}{n_i(n_{ij}) - C} \cdot \frac{n_i(n_{ij} - 1)(n_{ik} - 1)}{\Sigma_i[\Sigma_k\Sigma_j(Y_{ikj} - M_{ik} - M_{ij} + M_{iY})^2/\sigma_i^2]} \tag{13a}$$

$$\text{with } \gamma_1 = n_i(n_{ij}) - C \text{ and } \gamma_2 = n_i(n_{ij} - 1)(n_{ik} - 1)$$

All terms in the general equation can be directly evaluated from the sample data with the single exception of σ_i^2. Since its value is hypothetical and not deducible, it cannot be placed outside the summation signs without restrictions. Therefore, it is necessary to assume that

σ_i^2 is independent of X, i.e., $\sigma_i^2 = \sigma^2$. In fact, an implicit assumption of the entire derivation of equation (13a) is that the variance is the same in every cell. A further tacit assumption of the derivation is that the slope of the function $Y_{ikc} = f(X_{ik})$ holds for all individuals within sets. However, given that these two assumptions seem tenable, equation (13a) reduces to its final form

$$F = \frac{\Sigma_i n_{ik} \Sigma_k (M_{ik} - Y_{ikc})^2}{n_i(n_{ij}) - C} \cdot \frac{n_i(n_{ij} - 1)(n_{ik} - 1)}{\Sigma_i \Sigma_k \Sigma_j (Y_{ikj} - M_{ik} - M_{ij} + M_{iY})^2} \qquad (13b)$$

with $\gamma_1 = n_i(n_{ij}) - C$ and $\gamma_2 = n_i(n_{ij} - 1)(n_{ik} - 1)$

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