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REAL-TIME WAVEFRONT CORRECTION THROUGH BRAGG DIFFRACTION OF LIGHT BY SOUND WAVES

by

Virendra Nath Mahajan

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I hereby recommend that this dissertation prepared under my direction by Virendra Nath Mahajan

entitled Real-Time Wavefront Correction Through Bragg Diffraction of Light by Sound Waves

be accepted as fulfilling the dissertation requirement of the degree of Doctor of Philosophy

Jack D. Gaskill — Dissertation Director

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After inspection of the final copy of the dissertation, the following members of the Final Examination Committee concur in its approval and recommend its acceptance:*

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SIGNED: Virendra Nath Mahajan

To my family

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ABSTRACT

In the work reported here the index variation, or wave, produced by a sound wave traveling in a liquid is obtained from the Lorentz-Lorenz equation. In a solid, it is obtained by considering the photoelastic effect on the solid's index ellipsoid. The index waves produced by two orthogonal sound waves are also obtained.

It is shown that a sound wave of low frequency and narrow width behaves as an optical phase grating (Raman-Nath diffraction). When light is incident on such a sound wave, it is diffracted into various orders such that the corresponding positive and negative orders have the same irradiance. The frequency of the light in a given order is shifted from that of the incident light by order number times the acoustic frequency.

If the frequency and width of the sound wave are increased sufficiently, all diffraction effects generally disappear due to the destructive interference of the diffracted waves generated at different points of an acoustic wavefront. However, when light is incident at an angle approximately equal to half the ratio of the optical and acoustic wavelengths, it is diffracted efficiently into one order only, the order corresponding to approximate reflection of the incident light from the acoustic wavefronts (Bragg diffraction). A transition length (sound beamwidth) separating the regions of Raman-Nath and Bragg diffraction is defined, and expressions for the diffracted waves in these two cases are obtained.

Diffraction of light by a sound wave is also considered in terms of a photon-phonon interaction. The frequency and wave vector shifts are explained in terms of energy and momentum conservation, respectively.

A partial wave analysis of diffraction of light by a sound wave, based on the optical wave equation for the acousto-optic medium is given. The Raman-Nath equation is derived and its solutions are obtained in the Raman-Nath and Bragg regions of diffraction. A similar analysis of diffraction of light by two orthogonal sound waves is also given. It is shown that the diffraction effects produced by two such waves, when present simultaneously, are identical to the ones produced when they are present successively.

Successive Bragg diffraction of light by two sound waves of the same frequency but different wavelengths propagating in approximately parallel directions is described. It is shown that the direction of propagation and phase of the diffracted-diffracted wave can be controlled by varying the acoustic frequency and the relative phase of the two sound waves, respectively. Since the frequency shifts produced by the two sound waves are of equal magnitude but opposite signs, they cancel each other with the consequence that the d-d wave is coherent with the incident light. It is proposed that if a wavefront distorted by atmospheric turbulence is divided into subwavefronts, each subwavefront being planar, it can be corrected in real time if it is allowed to pass through an array of Bragg cells. Each cell carries two pairs of orthogonal sound waves to correct for tilts and phase errors of a corresponding subwavefront.

Some experiments on the direction and phase of a Bragg-diffracted light beam are described. The dependence of direction on the acoustic frequency has been demonstrated by observing the movement of the diffracted beam on a distant screen. The phase shifts have been observed in terms of the displacement of fringes in the interference pattern formed by two light beams whose relative phase was changed by varying the relative phase of the acoustic waves.

Doppler interpretation of the frequency shifts, acoustic beam steering by a phased array transducer, and diffraction of light by a standing sound wave are considered in the appendices.

CHAPTER 1

INTRODUCTION

In recent years various techniques have been developed to process a distorted image in order to improve its quality (National Academy of Sciences, 1966), and the search for new methods goes on. These techniques are generally called post-processing techniques because the image is processed after it has been recorded. Although the image gets distorted for various reasons such as object motion, optical-system aberrations and imperfections, film grain noise, etc., the most fundamental distortion of the images of airborne or space objects obtained with a ground-based telescope is caused by the atmospheric turbulence. Due to its random nature the atmospheric turbulence distorts a wavefront emanating from the object by varying amounts as a function of position on the wavefront, and distortion at any position varies independently of the distortions at other positions as a function of time. For a small-aperture telescope, the distortion at any time may be approximated by a tilt in the wavefront. However, for a large-aperture telescope, various sections of a wavefront are tilted by varying amounts. The net result is a distorted image even if the exposure time is small. Large-aperture telescopes reduce the diffraction effects and thus improve resolution, but this advantage may be completely lost because of the effects of the atmospheric turbulence.

If the distorted wavefronts could be corrected before reaching the telescope, i.e., in real time, the image obtained would be considerably improved. The phenomenon of Bragg diffraction of light by sound waves is proposed to achieve this task (Mahajan and Gaskill, 1974a).

It is well known that when light is diffracted by a grating, the ℓth-order diffracted beam, assuming normal incidence, appears at an angle θ_ℓ given by

$$d \sin\theta_\ell = \ell\lambda, \tag{1.1}$$

where d is the grating period and λ is the optical wavelength. Thus by varying the grating period we can vary the angle at which a diffracted beam appears. This gives us an idea as to how we might control the direction of propagation of a wavefront.

While we know well the directional dependence of a diffracted beam on the grating period, we generally do not appreciate that the relative phase of the light in a given order depends upon the position of the grating. If we could vary the position of the grating, the phase of light in any diffraction order would also vary. Thus in order to control the direction and phase of a diffracted beam, we must be able to vary the period and position of the grating. The period and position of a grating can be conveniently varied if this grating is generated in some medium by a sound wave traveling in it.

Sound waves are generated by attaching a piezoelectric transducer to the medium. The wavelength, which is equivalent to the grating period, and phase of the sound wave are varied by varying the frequency and phase of the electrical signal applied to the transducer, respectively. If the medium itself is piezoelectric, no external transducer is necessary, and a sound wave can be generated by applying the electrical signal directly to the medium.

Sound waves produce periodic variations in the refractive index of the medium in which they are traveling. A light beam traveling through the acoustic medium sees the index variations as a moving phase grating and is therefore diffracted into various orders. The diffraction is called Raman-Nath diffraction. Because of the moving nature of the grating, the frequency of light in a given order is shifted from that of the incident light by the order number times the acoustic frequency. By varying the acoustic frequency and phase, the direction and phase of a diffracted beam can be varied.

Now, we would like to diffract as much light as possible into a given order, an order whose beam direction and phase we are interested in controlling. This can be done by generating a sound wave of high frequency ($\gtrsim$ 50 MHz). When the acoustic frequency is high, the incident light is diffracted efficiently into the first order only provided it is incident at an angle approximately equal to the corresponding Bragg angle. The diffracted light appears in a direction corresponding to near reflection of the incident light from the acoustic wavefronts. The light is said to be Bragg diffracted in analogy with selective diffraction of x rays in crystals. The angle between the incident (or the undiffracted) and the diffracted light beams is equal to the ratio of the optical and acoustic wavelengths, or twice the Bragg angle.

A transparent medium carrying a sound wave of high frequency is referred to as a Bragg cell.

If a wavefront distorted by atmospheric turbulence is divided into small subwavefronts, each subwavefront being planar, the direction and phase of the corresponding diffracted beams can be controlled as the directions and phases of the subwavefronts change if the wavefront is incident on an array of Bragg cells. Each subwavefront is diffracted by a corresponding Bragg cell and the combination of the Bragg diffracted subwaves forms the corrected wavefront.

There is one difficulty, however. Since the subwavefronts are tilted by different amounts by the atmospheric turbulence from their true orientations, their corrections require sound waves of correspondingly different frequencies, which in turn produce diffracted waves whose frequencies are slightly different from each other. This as such may not be so much of a problem since the difference in their frequencies would be very small compared to the frequency bandwidth of the incident light. However, if the electrical signals applied to generate the sound waves are statistically independent, i.e., if the random fluctuations of the signals are statistically independent of each other, the diffracted subwaves will be incoherent with each other and consequently will not form a meaningful wavefront. Thus what is required is an electrical signal genrator with multichannel output, each channel capable of giving a signal of variable frequency such that any random fluctuation in it is the same as in the signals of all the other channels.

The problem of a multichannel output signal generator can be eliminated if each diffracted subwave is diffracted by another sound wave of the same frequency as the first in such a way that the diffracted beam produced by the second sound wave has the same frequency as the incident light. Thus if one sound wave produces a diffracted wave of frequency upshifted from that of the incident light, the second sound wave produces a diffracted wave of downshifted frequency. The relative phase of the two sound waves determines the phase of the diffracted-diffracted, of d-d, light wave. In order that the direction of the d-d wave depend on the frequency of the two sound waves, it is necessary that the Bragg angles corresponding to them be different. For the same Bragg angles, since they are very small in practice, the direction of the d-d beam is independent of the acoustic frequency and parallel to that of the incident light.

Since the Bragg angle depends on the acoustic wavelength, it can have different values for a given acoustic frequency if the acoustic velocities are different. Choosing either two different media in which the acoustic velocities for a given frequency are different, or two different acoustic modes of different velocities for a given frequency in the same medium, we can generate two sound waves of the same frequency but different wavelengths.

Successive Bragg diffraction of a subwavefront by two sound waves of the same frequency (but different wavelengths) not only eliminates the coherence problem but also reduces significantly the phase shifts produced in the d-d waves corresponding to the subwavefronts that are not incident at the Bragg angle since the phase shifts produced by the two sound waves tend to cancel.

If to each of the sound waves is added an orthogonally propagating sound wave, the two additional sound waves having the same frequency (although not necessarily the same as that of the existing two) but different wavelengths, a subwavefront can be corrected for an arbitrary tilt and a planar phase by varying the frequencies and relative phases of the sound waves. The system of four sound waves forms a subwavefront correcting element. A distorted wavefront divided into planar subwavefronts can be corrected if it is intercepted by an array of these elements, each element correcting a corresponding subwavefront.

Assuming that the atmosphere forms a linear shift invariant system with respect to the light propagating through it over a subwavefront area, the wavefronts emanating from different object points are corrected simultaneously by the elemental array. Moreover, since the distortions produced by the atmospheric turbulence do not change for a period of a few milliseconds and the electrical signals (generating the sound waves) can be changed in a few microseconds, the wavefronts can be corrected in real time. The distortions in a wavefront are determined by the methods used in optical testing, such as a Hartmann test, and corrections are made by adjusting the frequencies and phases of the electrical signals by corresponding amounts.

In the following chapters we find the index waves produced by sound waves and describe diffraction of light by them from three different points of views: (1) the sound wave as a phase grating (Raman and Nath, 1935a, 1935b, 1936a) and constructive and destructive interference of diffracted waves generated at various points of an acoustic wavefront (Adler, 1967) or by two acoustic wavefronts, (2) diffraction in terms of photon-phonon interactions (Damon, Maloney, and McMahon, 1970), and (3) diffraction on the basis of the optical wave equation (Raman and Nath, 1936b, 1936c; Klein and Cook, 1967, Mahajan and Gaskill, 1974b; Mahajan, 1974a). Diffraction by two orthogonal sound waves is described next (Mahajan and Gaskill, 1974c). This is followed by a discussion on the real-time wavefront correction and experiments on the direction and phase of a Bragg diffracted light beam. Finally, some recommendations for future research in the area of real-time wavefront correction are made.

Doppler interpretation of the frequency shifts, acoustic beam steering, and diffraction of light by a standing sound wave are considered in the appendices.

CHAPTER 2

SOUND WAVES AND THE CORRESPONDING INDEX WAVES

Sound waves traveling in a medium produce variations in its refractive index. In a liquid, the index variations produced can be obtained from the Lorentz-Lorenz equation. In a solid they are obtained by considering the photoelastic effect on the index ellipsoid of the solid. Although only longitudinal sound waves are considered here, the index variations produced by transverse waves traveling in solids can be obtained in a similar manner. Since the index variation is periodic like the sound wave producing it, it is referred to as the corresponding index wave.

2.1. Sound Waves in Liquids

Sound waves traveling in a medium produce periodic variations in its refractive index. In a liquid, sound waves can be generated by placing a piezoelectric transducer in it. However, only a longitudinal sound wave can propagate in it, and the corresponding index wave produced is generally independent of the optical polarization or wave vector. For an exception refer to the work of Riley and Klein (1969).

Consider a plane sound wave of frequency Ω and wave number K traveling along the x axis in a liquid of refractive index n and density ρ. The frequency and the wave number are related to each other by the acoustic velocity V according to $\Omega = KV$. If $u(x,t)$ is the particle displacement, the wave motion may be described by

$$u(x,t) = u \cos(\Omega t - Kx + \Phi), \tag{2.1}$$

where u and Φ are the amplitude and phase constant of the sound wave, respectively. The strain produced by the sound wave is given by

$$\varepsilon(x,t) = \frac{\partial u}{\partial x} = Ku \sin(\Omega t - Kx + \Phi). \tag{2.2}$$

The relation between the index variation and the strain is obtained from the Lorentz-Lorenz equation (Born and Wolf, 1970)

$$\frac{n^2 - 1}{n^2 + 2} = \frac{4\pi}{3} \alpha N, \tag{2.3}$$

where α and N are the mean polarizability and the number of molecules per unit volume of the liquid, respectively. Differentiating Eq. (2.3) we get the index variation produced by a sound wave,

$$\Delta n(x,t) = - \frac{(n^2 - 1)(n^2 + 2)}{6n} \varepsilon(x,t), \tag{2.4}$$

where we have substituted $\varepsilon(x,t)$ for $\Delta N(x,t)/N$. Substituting Eq. (2.2) into Eq. (2.4), we obtain the index wave produced in the liquid

$$\Delta n(x,t) = \Delta n \sin(\Omega t - Kx + \Phi), \tag{2.5a}$$

where

$$\Delta n = - \frac{(n^2 - 1)(n^2 + 2)}{6n} Ku. \tag{2.5b}$$

Since the index variation is proportional to the density variation, the refractive index is higher in the regions of compression and lower in the regions of rarefaction. This is illustrated in Fig. 2.1.

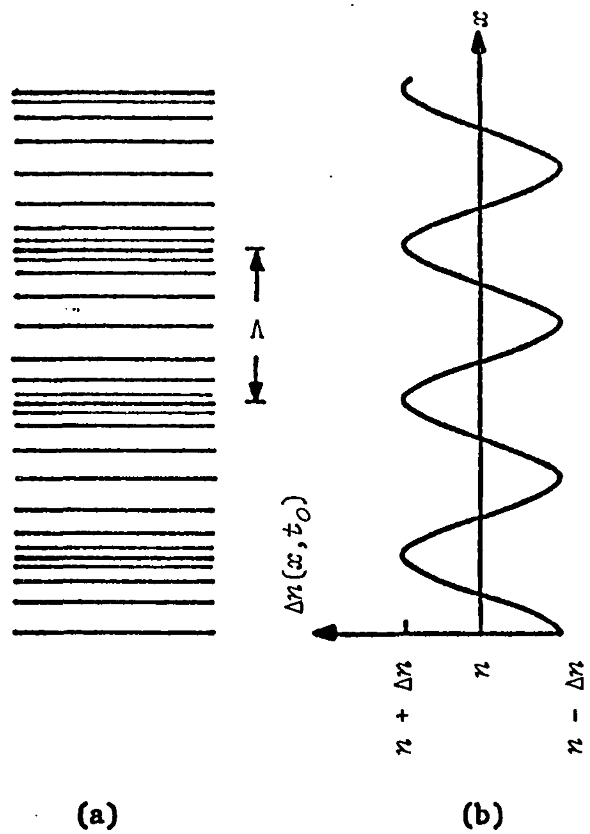


Fig. 2.1. Low-Frequency Sound Wave in a Transparent Liquid of Refractive Index n.

(a) Alternating regions of compression and rarefaction that travel at the sound velocity V. (b) Instantaneous spatial variation of the refractive index produced by the sound wave at some time t_o.

The time-averaged acoustic energy density (energy per unit volume) is given by $\frac{1}{2}\rho V^2\varepsilon^2$, where $\varepsilon = Ku$ is the amplitude of strain. The acoustic irradiance (power per unit area) is V times the energy density. The relation between the acoustic power P_{ac} and strain is therefore given by

$$P_{ac} = \tfrac{1}{2} HL\rho V^3 \varepsilon^2, \tag{2.6}$$

where H and L are the height and the width of the sound wave, respectively. The quantity HL is the area of an acoustic wavefront and is approximately equal to the area of the transducer.

2.2. Sound Waves in Solids

In a solid, the index variations produced by a sound wave depend on the optical and acoustic polarizations and wave vectors. In an anisotropic (crystalline) solid, for any given acoustic wave vector $\vec{K}$, there are three independent but mutually perpendicular particle-displacement vectors. These displacement vectors, whose directions define the acoustic polarizations, represent three sound waves with different frequencies and velocities of propagation. For certain directions of $\vec{K}$, one of the displacement vectors is parallel to it, in which case, the other two lie in a plane perpendicular to it. The wave with displacement vector parallel to $\vec{K}$ is called a pure longitudinal wave. The remaining two waves, with displacement vectors perpendicular to $\vec{K}$, are called pure transverse (shear) waves.

In an isotropic solid (e.g., glass), for any direction of $\vec{K}$, one of the three waves is purely longitudinal, and the other two are purely transverse. The two transverse waves propagate with the same velocity, which is always smaller than the velocity of the longitudinal wave.

Sound waves in a solid are generally generated by attaching a piezoelectric transducer to it. Transducers can be made to generate longitudinal or transverse sound waves (Mason, 1960). If the solid itself is piezoelectric (e.g., crystalline lithium niobate), no external transducer is necessary, and sound waves can be generated by applying an electrical signal directly to the solid.

2.3. Photoelastic Effect in Isotropic Solids

Index variations produced by a sound wave traveling in a solid are determined from Fresnel's index ellipsoid, which is also called Fletcher's indicatrix. A sound wave produces periodic distortions in the index ellipsoid by means of the photoelastic effect.

To illustrate, we consider, for simplicity, a sound wave traveling in an isotropic solid of refractive index n. In the absence of a sound wave, the index ellipsoid is a sphere described by

$$B^0(x^2 + y^2 + z^2) = 1, \tag{2.7}$$

where $B^0 = 1/n^2$. The sound wave modifies the index ellipsoid to

$$B_1x^2 + B_2y^2 + B_3z^2 + 2B_4yz + 2B_5zx + 2B_6xy = 1, \tag{2.8}$$

where the coefficients B_m are the relative dielectric impermeabilities given by (Nye, 1960; Nelson and Lax, 1970, 1971)

$$B_m = B_m{}^0 + p_{mn}\varepsilon_n; \quad m,n = 1, 2, . . ., 6. \tag{2.9}$$

In Eq. (2.9) p_{mn} are the (strain-optic) photoelastic constants of the medium and ε_n are the strains produced by the sound wave. It is evident from Eq. (2.7) that

$$B_m^0 = B^0 \text{ for } m = 1, 2, 3$$
$$= 0 \text{ for } m = 4, 5, 6. \tag{2.10}$$

The photoelastic matrix for an isotropic solid is given by

$$p_{mn} = \begin{bmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(p_{11}-p_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(p_{11}-p_{12}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(p_{11}-p_{12}) \end{bmatrix}. \tag{2.11}$$

Note that there are only two independent photoelastic constants.

The refractive index for any light wave is obtained from the elliptical section of the index ellipsoid formed by a central plane perpendicular to the optical wave vector. The elliptical section is called the optical index ellipse and its semi-major and semi-minor axes give the refractive indices for light with electric vectors (strictly speaking the electric displacement vectors) vibrating parallel to these axes, respectively.

2.4. Longitudinal Sound Wave in an Isotropic Solid

If the sound wave under consideration is a longitudinal wave traveling along the x axis, the particle-displacement vector describing the wave motion can be written

$$\vec{u} = [u \cos(\Omega t - Kx + \Phi), 0, 0] \tag{2.12}$$

where u, Ω, K, and Φ, as before, are the amplitude, frequency, wave number, and phase constant of the sound wave, respectively. The corresponding strains are given by

$$\varepsilon_1 = Ku \sin(\Omega t - Kx + \Phi); \; \varepsilon_n = 0, \; n \neq 1. \tag{2.13}$$

Substituting Eqs. (2.10), (2.11), and (2.13) into Eq. (2.9), we obtain

$$B_1 = \frac{1}{n^2} + p_{11}\varepsilon_1, \quad B_2 = \frac{1}{n^2} + p_{12}\varepsilon_1, \quad B_3 = \frac{1}{n^2} + p_{12}\varepsilon_1,$$

$$B_4 = B_5 = B_6 = 0. \tag{2.14}$$

Therefore Eq. (8) for the deformed ellipsoid becomes

$$\left(\frac{1}{n^2} + p_{11}\varepsilon_1\right)x^2 + \left(\frac{1}{n^2} + p_{12}\varepsilon_1\right)y^2 + \left(\frac{1}{n^2} + p_{12}\varepsilon_1\right)z^2 = 1. \tag{2.15}$$

It is evident from Eqs. (2.7) and (2.15) that a longitudinal sound wave does not produce any rotation of the principal axes of the index ellipsoid but only variations of their lengths. This is illustrated in Fig. 2.2.

Although in the following chapters, we shall consider light waves with wave vectors making small angles with the z axis, these angles are so small ($\lesssim 10^{-2}$ rad) that, for the purpose of finding the index waves, we can assume that the light is traveling along the z axis. The corresponding index ellipse is then obtained from Eq. (2.15) by putting $z = 0$:

$$\left(\frac{1}{n^2} + p_{11}\varepsilon_1\right)x^2 + \left(\frac{1}{n^2} + p_{12}\varepsilon_1\right)y^2 = 1. \tag{2.16}$$

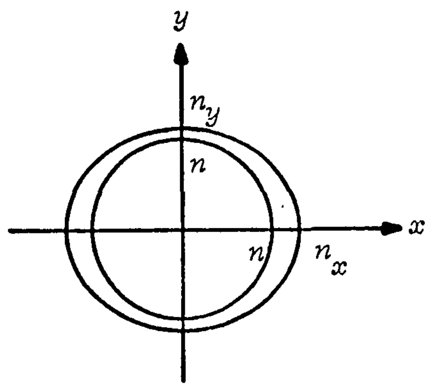


Fig. 2.2. Index Ellipse for Light Propagating Along the z Axis in an Isotropic Solid.

The circle represents the index ellipse in the absence of a sound wave. The refractive index n is independent of the optical polarization. A longitudinal sound wave propagating along the x axis distorts the circle into a periodically fluctuating ellipse and the solid becomes nonisotropic. The refractive indices for optical polarizations parallel to the x and y axes are given by n_x and n_y, respectively.

Accordingly, the refractive indices for optical polarizations parallel to the x and y axes are given by

$$\frac{1}{n_x{}^2} = \frac{1}{n^2} + p_{11}\varepsilon_1 \tag{2.17}$$

and

$$\frac{1}{n_y{}^2} = \frac{1}{n^2} + p_{12}\varepsilon_1 , \tag{2.18}$$

respectively. The corresponding index waves are therefore given by

$$[\Delta n(x,t)]_x = (\Delta n)_x \sin(\Omega t - Kx + \Phi) , \tag{2.19a}$$

where

$$(\Delta n)_x = -\tfrac{1}{2}n^3 K u p_{11} \tag{2.19b}$$

and

$$[\Delta n(x,t)]_y = (\Delta n)_y \sin(\Omega t - Kx + \Phi), \tag{2.20a}$$

where

$$(\Delta n)_y = -\tfrac{1}{2}n^3 K u p_{12}. \tag{2.20b}$$

The subscripts in these equations refer to the direction of optical polarization.

2.5. Two Orthogonal Longitudinal Sound Waves in an Isotropic Solid

Next we consider two orthogonal longitudinal sound waves, one traveling along the x axis and the other along the y axis, and find the index waves produced by them. The particle-displacement vector in this case can be written

$$\vec{u} = [u_1 \cos(\Omega_1 t - K_1 x + \Phi_1),\ \ u_2 \cos(\Omega_2 t - K_2 y + \Phi_2),\ 0]. \tag{2.21}$$

In Eq. (2.21), the quantities with subscript 1 refer to the sound wave traveling along the x axis. Similarly, quantities with subscript 2 refer to the sound wave traveling along the y axis. The strains produced by the sound waves are given by

$$\begin{aligned} \varepsilon_1 &= K_1 u_1 \sin(\Omega_1 t - K_1 x + \Phi_1), \\ \varepsilon_2 &= K_2 u_2 \sin(\Omega_2 t - K_2 y + \Phi_2), \\ \varepsilon_3 &= \varepsilon_4 = \varepsilon_5 = \varepsilon_6 = 0. \end{aligned} \tag{2.22}$$

Following the same procedure as for one longitudinal sound wave, the equation for the deformed index ellipsoid is found to be

$$\left(\frac{1}{n^2} + p_{11}\varepsilon_1 + p_{12}\varepsilon_2\right)x^2 + \left(\frac{1}{n^2} + p_{12}\varepsilon_1 + p_{11}\varepsilon_2\right)y^2$$

$$+ \left(\frac{1}{n^2} + p_{12}\varepsilon_1 + p_{12}\varepsilon_2\right)z^2 = 1. \tag{2.23}$$

The index ellipse for light with the wave vector along the z axis is therefore given by

$$\left(\frac{1}{n^2} + p_{11}\varepsilon_1 + p_{12}\varepsilon_2\right)x^2 + \left(\frac{1}{n^2} + p_{12}\varepsilon_1 + p_{11}\varepsilon_2\right)y^2 = 1. \tag{2.24}$$

The refractive indices for optical polarizations parallel to the x and y axes are given by

$$\frac{1}{{n_x}^2} = \frac{1}{n^2} + p_{11}\varepsilon_1 + p_{12}\varepsilon_2 \tag{2.25}$$

and

$$\frac{1}{{n_y}^2} = \frac{1}{n^2} + p_{12}\varepsilon_1 + p_{11}\varepsilon_2, \tag{2.26}$$

respectively. The corresponding index waves can be written in the form

$$[\Delta n(x,y,t)]_x = [(\Delta n)_1]_x \sin(\Omega_1 t - K_1 x + \Phi_1)$$

$$+ [(\Delta n)_2]_x \cdot \sin(\Omega_2 t - K_2 y + \Phi_2), \tag{2.27a}$$

where

$$[(\Delta n)_1]_x = -\tfrac{1}{2}n^3 K_1 u_1 p_{11} \tag{2.27b}$$

and

$$[(\Delta n)_2]_x = -\tfrac{1}{2}n^3 K_2 u_2 p_{12}; \tag{2.27c}$$

and

$$[\Delta n(x,y,t)]_y = [(\Delta n)_1]_y \sin(\Omega_1 t - K_1 x + \Phi_1)$$

$$+ [(\Delta n)_2]_y \sin(\Omega_2 t - K_2 y + \Phi_2), \tag{2.28a}$$

where

$$[(\Delta n)_1]_y = -\tfrac{1}{2}n^3 K_1 u_1 p_{12} \tag{2.28b}$$

and

$$[(\Delta n)_2]_y = -\tfrac{1}{2}n^3 K_2 u_2 p_{11}. \tag{2.28c}$$

CHAPTER 3

RAMAN-NATH AND BRAGG DIFFRACTION

A sound wave causes diffraction of light by producing periodic variations in the refractive index of the medium in which it is traveling. When its frequency is low and its width is narrow, it acts like a moving phase grating and the diffraction produced by it is called Raman-Nath diffraction (Klein and Cook, 1967). If its frequency and width are sufficiently increased, all diffraction effects disappear unless the light is incident at an angle approximately equal to the Bragg angle (Bhagavantam and Rao, 1948). When light is incident at the Bragg angle, the diffracted wave appears to be reflected from the acoustic wavefronts. This selective diffraction is called Bragg diffraction in analogy with x-ray diffraction in crystals. A transition length (sound beamwidth) separating the regions of Raman-Nath and Bragg diffraction is defined. The diffracted waves in the two cases are determined.

3.1. Raman-Nath Diffraction

As indicated in Fig. 3.1, consider a plane wave of light of frequency ω, wave vector $\vec{k} = (-k \sin\theta, 0, k \cos\theta)$, and amplitude U traveling in a transparent medium of refractive index n. Let a sound wave of frequency Ω and wave number K, traveling along the x axis with a velocity V, be generated in the medium. The refractive index of the medium

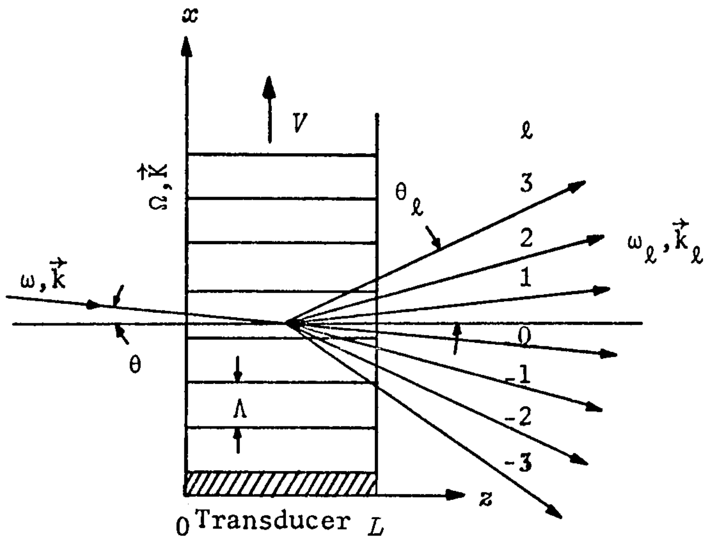


Fig. 3.1. Diffraction of Light by a Sound Wave.

A sound wave of frequency Ω and wave vector $\vec{K}$ propagating along the x axis in a medium of refractive index n diffracts light of frequency ω and wave vector $\vec{k}$ into waves of frequency $\omega_\ell = \omega + \ell\Omega$ and wave vectors $\vec{k}_\ell = \vec{k} + \ell\vec{K}$. The angle θ_ℓ is given by $\tan\theta_\ell = -\tan\theta + \ell K/k \cos\theta$.

in the region of the sound wave can be written (from Chapter 2)

$$\begin{aligned} n(x,t) &= n + \Delta n(x,t) \\ &= n + \Delta n \sin(\Omega t - Kx), \end{aligned} \tag{3.1}$$

where $\Delta n(x,t)$ represents an index wave of amplitude Δn produced by the sound wave. The light field incident on the sound wave is given by

$$E_i = \tfrac{1}{2}U \exp i(\omega t - \vec{k} \cdot \vec{r}) + \text{c.c.}, \tag{3.2}$$

where c.c. denotes the complex conjugate. Raman-Nath diffraction occurs for low acoustic frequencies, in which case the sound wave acts like a moving phase grating, and the light emerges from the sound beam phase modulated. The emergent light field can therefore be written

$$E_e = \tfrac{1}{2}U \exp i[(\omega t - \vec{k} \cdot \vec{r}) + \Delta\phi(x,t)] + \text{c.c.}, \tag{3.3}$$

where $\Delta\phi(x,t)$ is the phase modulation produced by the sound wave.

The phase modulation can be obtained by defining a parameter s along the path of the light beam, as shown in Fig. 3.2.

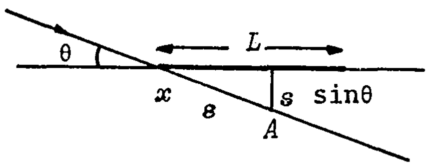


Fig. 3.2. Index Variation as a Function of Parameter s Along the Path of the Incident Light Beam.

$xA = s$, and $\Delta n(x,s,t) = \Delta n \sin(\Omega t - Kx + Ks \sin\theta)$. The angle of incidence θ is exaggerated for clarity.

The index variation as a function of s, observed by light entering at a point x, is given by

$$\Delta n(x,s,t) = \Delta n \sin(\Omega t - Kx + Ks \sin\theta). \tag{3.4}$$

The phase modulation of the emerging light field is therefore given by

$$\Delta\phi(x,t) = \frac{k}{n} \int_0^{L/\cos\theta} \Delta n(x,s,t)ds, \tag{3.5}$$

where L is the width of the sound beam. Substituting Eq. (3.4) into

Eq. (3.5) and performing the integration, we obtain

$$\Delta\phi(x,t) \;=\; \Delta\phi \,\sin(\Omega t - Kx + \tfrac{1}{2}KL\,\tan\theta), \tag{3.6}$$

where

$$\Delta\phi \;=\; \frac{\Delta n}{n}\,\frac{kL}{\cos\theta}\,\mathrm{sinc}\left(\frac{L}{\Lambda}\tan\theta\right). \tag{3.7}$$

In Eq. (3.7) $\Lambda = 2\pi/K$ is the acoustic wavelength and $\mathrm{sinc}x = \sin(\pi x)/\pi x$. Substituting Eq. (3.6) into Eq. (3.3), we obtain

$$E_e \;=\; \tfrac{1}{2}U\,\exp i[(\omega t - \vec{k}\cdot\vec{r}) + \Delta\phi\,\sin(\Omega t - Kx + \tfrac{1}{2}KL\,\tan\theta)] + \text{c.c.}, \tag{3.8}$$

which shows that a plane wavefront incident on a sinusoidal sound wave emerges as a sinusoidal wavefront from it. As shown below, the emerging wavefront represents an infinite number of light waves of different frequencies propagating in different directions.

Using the identity (Watson, 1966, p. 22)

$$\exp(ia\,\sin b) \;=\; \sum_{\ell=-\infty}^{\infty} J_\ell(a)\,\exp(i\ell b), \tag{3.9}$$

where J_ℓ is the ℓth-order Bessel function of the first kind, we can write Eq. (3.8) as

$$E_e \;=\; \tfrac{1}{2}U \sum_{\ell=-\infty}^{\infty} J_\ell(\Delta\phi)\,\exp i(\omega_\ell t - \vec{k}_\ell\cdot\vec{r} + \tfrac{1}{2}\ell KL\,\tan\theta) + \text{c.c.}, \tag{3.10}$$

where

$$\omega_\ell \;=\; \omega + \ell\Omega \tag{3.11}$$

and

$$\vec{k}_\ell = (\ell K - k \sin\theta, 0, k \cos\theta). \tag{3.12}$$

Equations (3.10) to (3.12) show that a low frequency sound wave diffracts the incident light into various orders ℓ. The frequency and wave vector of the ℓth-order diffracted wave are ω_ℓ and $\vec{k}_\ell$, respectively. The shift in the frequency of a diffracted wave from that of the incident light can be interpreted as a Doppler shift due to the motion of the acoustic wavefronts (Appendix A). Equations (3.11) and (3.12) represent conservation of energy and momentum, respectively, of a photon-phonon interaction in which ℓ phonons are absorbed or emitted depending on whether ℓ is positive or negative (Chapter 4). The ℓth-order wave, according to Eq. (3.12) appears at an angle θ_ℓ where

$$\tan\theta_\ell = \frac{\ell K}{k \cos\theta} - \tan\theta. \tag{3.13a}$$

Or, for small angles

$$\theta_\ell = \frac{\ell K}{k} - \theta. \tag{3.13b}$$

If the direction of sound propagation is reversed, the light in each positive order is replaced by the light in the corresponding negative order and vice versa. Note that like the wave vectors $\vec{k}_\ell$, the angles are also measured inside the medium. The corresponding angles outside the medium can be determined by using Snell's law.

The amplitude and time-averaged irradiance of the ℓth-order wave are given by

$$U_\ell = UJ_\ell(\Delta\phi) \tag{3.14}$$

and

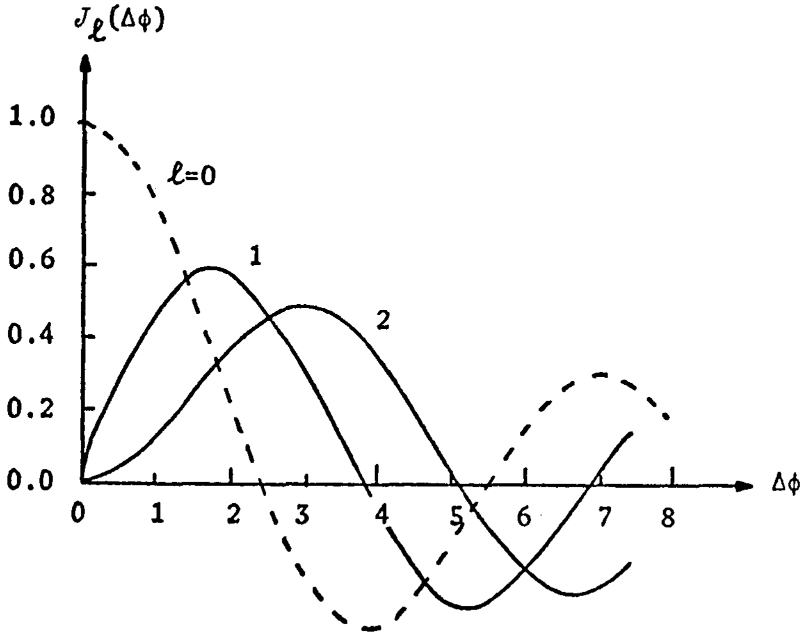
$$I_\ell = \tfrac{1}{2}|U_\ell|^2 = IJ_\ell^{\,2}(\Delta\phi), \tag{3.15}$$

respectively. In Eq. (3.15), $I = \frac{1}{2}U^2$ is the time-averaged irradiance of the incident light. Since $J_{-\ell} = (-1)^\ell J_\ell$, $J_{-\ell}^{\,2} = J_\ell^{\,2}$, i.e., the irradiances of the corresponding positive- and negative-order waves are equal for all angles of incidence. The positive and negative ℓth-order waves disappear simultaneously when $\Delta\phi$ is equal to any root of the ℓth-order Bessel function. The variation of amplitude and irradiance of the diffracted waves as a function of $\Delta\phi$ for the first few orders is shown in Fig. 3.3. Note that when $\Delta\phi = 2.4$ rad, the zero-order wave disappears, i.e., all the incident light is diffracted. For small values of $\Delta\phi$, maximum diffraction occurs for normal incidence (θ=0) since $\Delta\phi$ is maximum in that case.

Since $J_\ell(0) = 0$ except when $\ell = 0$ in which case it is 1, diffraction effects disappear whenever $\Delta\phi = 0$. This happens, according to Eq. (3.7) when the angle of incidence $\theta = \theta_m$ where

$$\tan\theta_m = m\Lambda/L, \quad m = \pm 1, \pm 2, \ldots . \tag{3.16}$$

At these angles of incidence the light enters and leaves the sound wave at points differing in acoustic phase by $2\pi m$. As shown in Fig. 3.4, $\Delta\phi$ is maximum when light is incident normally and decreases as the angle of incidence increases. It approaches zero when $\tan\theta = \Lambda/L$, and consequently no amount of light is diffracted. As the angle of incidence is



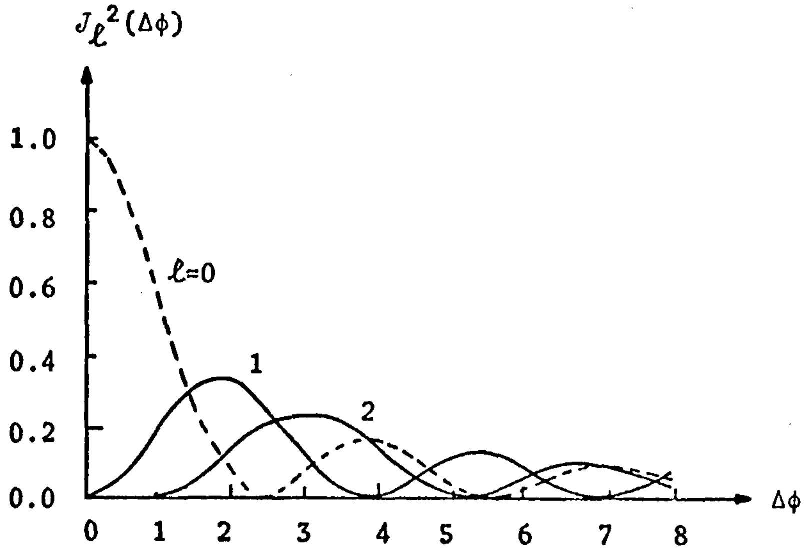


Fig. 3.3. Raman-Nath Diffraction. J_ℓ and $J_\ell{}^2$ are the Relative Amplitude and Irradiance of the ℓth-Order Diffracted Wave.

Since $J_\ell{}^2 = J_{-\ell}{}^2$, the irradiances of the corresponding positive- and negative-order waves are equal. $\Delta\phi$ is the amplitude of phase modulation produced by a sound beam of width L.

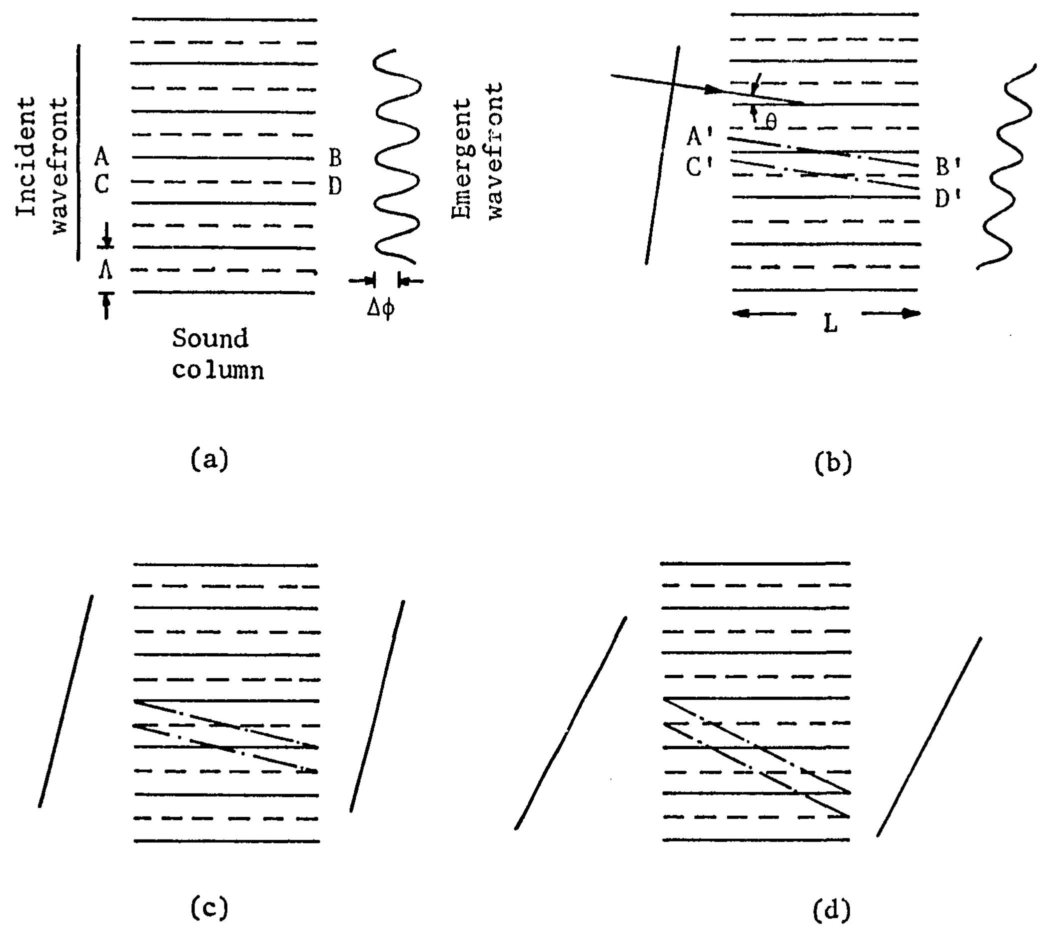


Fig. 3.4. Raman-Nath Diffraction as a Function of Angle of Incidence θ.

(a) Diffraction effects are maximum when $\theta = 0$. (b) Diffraction effects decrease as θ is increased since the amplitude of phase modulation $\Delta\phi$ decreases. Note that the optical path difference between paths A'B' and C'D' is less than that between AB and CD since the optical path along A'B' is less than that along AB and the optical path along C'D' is greater than that along CD. (c) and (d) Diffraction effects disappear when $\tan\theta = m\Lambda/L$ where $m = \pm 1, \pm 2, \pm 3$, etc. In (c) and (d), m = 1 and 2, respectively.

further increased, $\Delta\phi$ increases to a smaller maximum and then decreases to zero when $\tan\theta = 2\Lambda/L$, and so on.

3.2. The Transition Length

Although low frequency sound waves cause maximum diffraction of normally incident light, the diffraction effects disappear if the acoustic frequency is sufficiently increased. When the acoustic frequency is high so that the width of the sound wave is much larger than its wavelength, the diffracted waves disappear due to destructive interference (Adler, 1967).

Consider, for example, the positive-first-order wave. For normal incidence, according to Eq. (3.13), it appears in a direction making an angle of approximately λ/Λ with the direction of incident light, where λ is the optical wavelength in the medium. Contributions to this order generated at different points along an acoustic wavefront do not add in phase. When the width of the sound wave $L \simeq 2\Lambda^2/\lambda$, these contributions cancel each other completely since, as shown in Fig. 3.5, the path difference (OA - OB) between contributions from the extreme ends of the acoustic wavefront is λ. The same holds for the negative-first-order wave. The higher-order waves, owing to their appearance at higher angles, disappear for even smaller values of L. For example, the second-order waves disappear when $L \simeq \Lambda^2/2\lambda$. Thus when $L \geq 2\Lambda^2/\lambda$, all the light goes into the zero order. The value $2\Lambda^2/\lambda$ of L is called the transition length:

$$L_t = 2\Lambda^2/\lambda. \qquad (3.17)$$

Note that the higher the acoustic frequency the smaller the transition length.

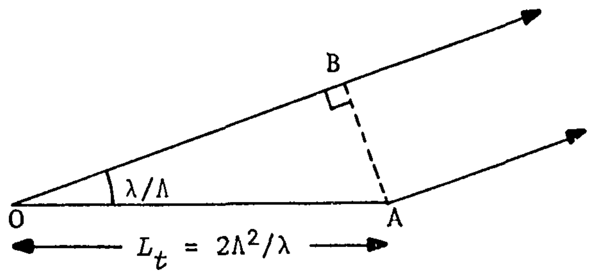


Fig. 3.5. The Transition Length L_t.

For normal incidence (θ=0), contributions to the positive first order (ℓ= 1) generated at different points along an acoustic wavefront OA interfere destructively since the path difference OA - OB = λ.

3.3. The Bragg Equation

It is clear that if contributions along an acoustic wavefront interfere constructively, some light will be diffracted. This happens for the positive-first-order wave if the light is incident at an angle $\lambda/2\Lambda$ as shown in Fig. 3.6a. The positive-first-order waves, still appearing at an angle λ/Λ with respect to the incident light, now appear to be reflected from the acoustic wavefronts as if they were partially reflecting mirrors. No matter how long we make L, all points on an acoustic wavefront contribute to the positive-first-order wave in phase. This cumulative interaction occurs for the positive-first-order wave only; the negative-first-order and all higher-order waves still disappear owing to destructive interference. When $\theta \approx \lambda/2\Lambda$, or more precisely $\sin\theta = \lambda/2\Lambda$, not only the contributions from a given acoustic wavefront interfere constructively, but as shown in Fig. 3.6b, contributions from two acoustic wavefronts also interfere constructively since their path difference is equal to λ:

$$2\Lambda \sin\theta = \lambda. \tag{3.18}$$

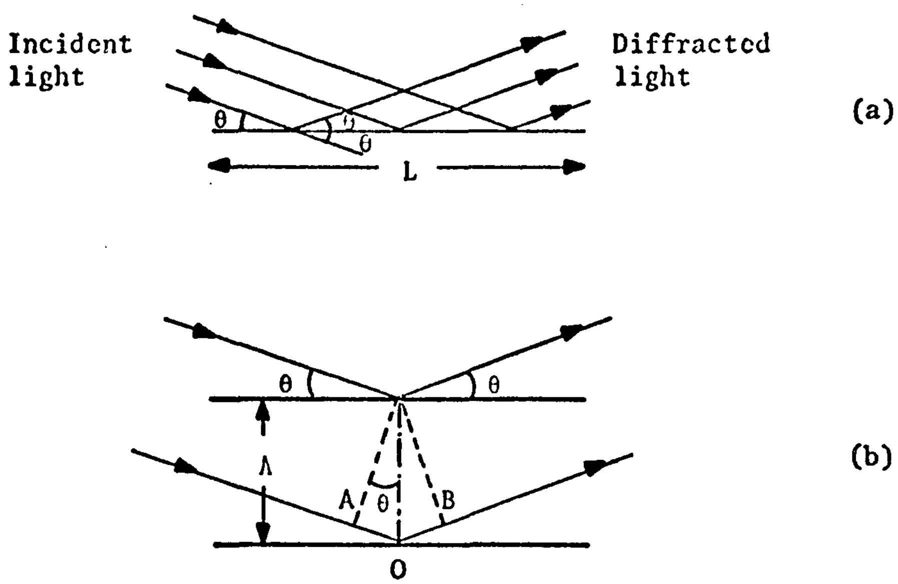


Fig. 3.6. Bragg Diffraction.

(a) When light is incident at an angle $\theta \approx \lambda/2\Lambda$, the positive-first-order light appears to be reflected from an acoustic wavefront and the contributions generated at different points are in phase. (b) Contributions from two acoustic wavefronts are in phase. The path difference AO + OB = $2\Lambda \sin\theta = \lambda$. Angle θ is called the Bragg angle, later denoted by Θ.

Equation (3.18) is called the Bragg equation (or condition). The angle θ given by this equation is called the Bragg angle, later denoted by Θ. Diffraction of light by sound waves of high frequency is called Bragg diffraction in analogy with selective diffraction of x rays in crystals.

3.4. Bragg Diffraction

To determine the irradiance of the Bragg diffracted light, let the phase modulation produced by an element of length dL be

$$d\phi(x,t) = d\phi \sin(\Omega t - Kx), \tag{3.19}$$

where

$$d\phi = k\frac{\Delta n}{n}\frac{dL}{\cos\Theta}. \tag{3.20}$$

The complex amplitude $U_0 \exp i(\omega t - \vec{k}\cdot\vec{r})$ of the light incident at the Bragg angle (zero-order wave) is modified on passing through this element to

$$\begin{aligned} &U_0 \exp i[\omega t - \vec{k}\cdot\vec{r} + d\phi \sin(\Omega t - Kx)] \\ &\simeq U_0[1 + i d\phi \sin(\Omega t - Kx)] \exp i(\omega t - \vec{k}\cdot\vec{r}) \\ &= U_0 \exp i(\omega t - \vec{k}\cdot\vec{r}) + \tfrac{1}{2}U_0 d\phi \exp i[(\omega + \Omega)t - \vec{k}\cdot\vec{r} - Kx] \\ &\quad -\tfrac{1}{2}U_0 d\phi \exp i[(\omega - \Omega)t - \vec{k}\cdot\vec{r} + Kx]. \end{aligned} \tag{3.21}$$

In writing Eq. (3.21), we have used the fact that $d\phi$ is small. The last two terms in this equation represent the positive- and negative-first-order waves that diverge from the zero-order wave (represented by the first term) owing to their wave vector component $\pm K$ in the x direction, respectively. As explained above, only the positive-first-order wave grows; the negative-first-order wave is eliminated by destructive interference. The amplitude of the positive-first-order wave is given by

$$dU_1 = \tfrac{1}{2}U_0 d\phi. \tag{3.22}$$

Its rate of change with distance L can be written

$$\frac{dU_1}{dL} = \tfrac{1}{2}U_0 \frac{d\phi}{dL}. \tag{3.23}$$

The conservation of energy requires that the sum of the irradiances of the diffracted and undiffracted waves be equal to the irradiance of the incident wave. Therefore,

$$U^2 = U_0{}^2 + U_1{}^2. \tag{3.24}$$

Differentiating Eq. (3.24) and using Eq. (3.23), we obtain

$$\frac{dU_0}{dL} = -\tfrac{1}{2}U_1 \frac{d\phi}{dL}. \tag{3.25}$$

Solving Eqs. (3.23) and (3.25) subject to the boundary conditions

$$U_0(0) = U \tag{3.26a}$$

and

$$U_1(0) = 0, \tag{3.26b}$$

we obtain the amplitudes of the diffracted and undiffracted waves emerging from the sound beam

$$U_1 = U \sin\left(\tfrac{1}{2} \frac{\Delta n}{n} \frac{kL}{\cos\Theta} \right) \tag{3.27}$$

and

$$U_0 = U \cos\left(\tfrac{1}{2} \frac{\Delta n}{n} \frac{kL}{\cos\Theta} \right), \tag{3.28}$$

respectively. The corresponding time-averaged irradiances are given by

$$I_1 = I \sin^2\left(\tfrac{1}{2} \frac{\Delta n}{n} \frac{kL}{\cos\Theta} \right) \tag{3.29}$$

and

$$I_0 = I \cos^2\left(\tfrac{1}{2} \frac{\Delta n}{n} \frac{kL}{\cos\Theta} \right). \tag{3.30}$$

As shown in Fig. 3.7, whenever $kL\Delta n/n \cos\Theta$ is an odd integral multiple of π, all the light incident at the Bragg angle is diffracted. The frequency of the diffracted light is $\omega + \Omega$. If the direction of sound propagation is reversed, the diffracted light appears in the same direction as before but its frequency becomes $\omega - \Omega$.

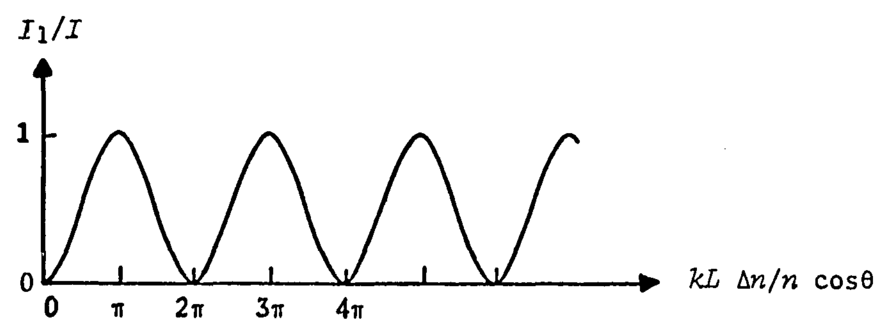


Fig. 3.7. Relative Irradiance of Bragg-Diffracted Light as a Function of the Amplitude of Index Variation.

For large L and high Ω we have shown that unless the light is incident at the Bragg angle, it will not be diffracted. This would be strictly true if there were only one plane sound wave causing diffraction. However, any wave limited in extent can be represented as a sum of plane waves of different amplitudes traveling in different directions. This is the phenomenon of diffraction of a wave by an aperture. As a consequence the Bragg condition is relaxed, and some light is diffracted even when it is incident at an angle slightly different from the Bragg angle. The diffracted light may be considered as resulting from partially constructive interference of diffracted waves generated at various points of an acoustic wavefront. For a uniform sound beam of width L, the relative amplitude of the plane wave component traveling at an angle $\Delta\theta$ with respect to the x axis (Fig. 3.8) is given by $\mathrm{sinc}[(L/\Lambda)\sin\Delta\theta]$, or approximately, $\mathrm{sinc}(L\Delta\theta/\Lambda)$. Hence when light is incident at an angle different from the Bragg angle by $\Delta\theta$, the amplitude of the diffracted wave becomes

$$U_1 = U \sin\left[\tfrac{1}{2} \frac{\Delta n}{n} \frac{kL}{\cos\Theta} \,\mathrm{sinc}\, \frac{L\Delta\theta}{\Lambda} \right]. \tag{3.31}$$

For low acoustic powers, i.e., small Δn, the amount of diffracted light is reduced from its value for Bragg incidence according to

$$\frac{I_1(\Delta\theta)}{I_1(0)} = \mathrm{sinc}^2(L\Delta\theta/\Lambda). \tag{3.32}$$

Variation of $I_1(\Delta\theta)$ as a function of $\Delta\theta$ is shown in Fig. 3.9. Note that when

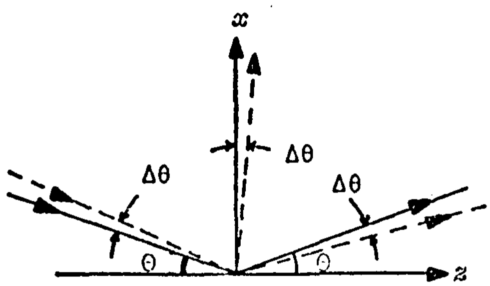


Fig. 3.8. When Light is Incident at an Angle $\Theta + \Delta\theta$, Bragg Diffraction is Caused by the Sound Wave Component Traveling in a Direction Making an Angle $\Delta\theta$ with the x Axis.

For a uniform sound beam of small amplitude, the irradiance of diffracted light is reduced by a factor of $\mathrm{sinc}^2(L\Delta\theta/\Lambda)$. The angle between the undiffracted and diffracted light beams is always 2Θ.

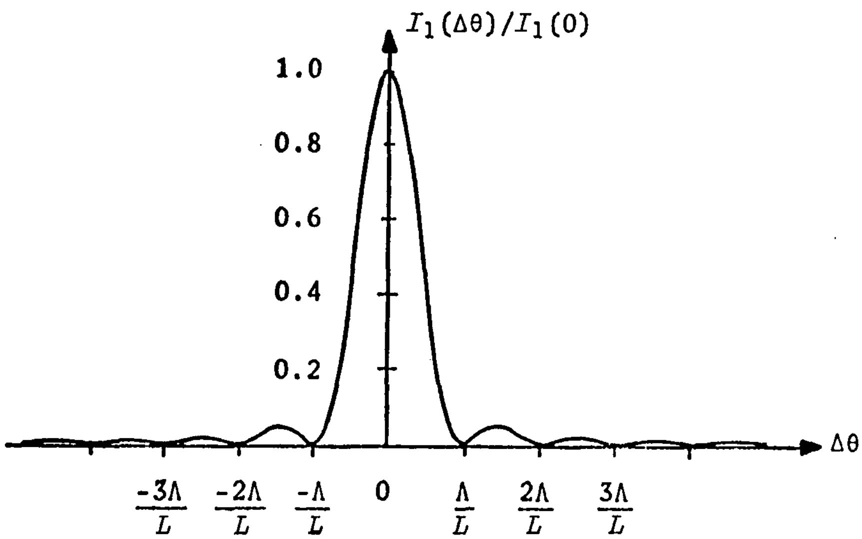


Fig. 3.9. Relative Irradiance of Bragg-Diffracted Light as a Function of Deviation $\Delta\theta$ in the Angle of Incidence from the Bragg Angle; $\Delta\theta = \theta - \Theta$.

$$\Delta\theta = m\Lambda/L, \quad m = \pm 1, \pm 2, \ldots . \tag{3.33}$$

the diffracted wave disappears. It can be shown that for these values of $\Delta\theta$, the contributions to diffracted waves from different parts of an acoustic wavefront interfere destructively.

The angle between the diffracted and undiffracted light beams is always equal to twice the Bragg angle. Thus when the angle of incidence is increased by $\Delta\theta$, the angle at which the diffracted beam appears is decreased by the same amount, as shown in Fig. 3.8. The direction of the diffracted beam can be held fixed at angle Θ (Fig. 3.10) if the

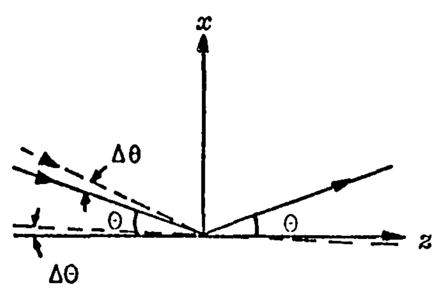


Fig. 3.10. When the Angle of Incidence Changes by $\Delta\theta$, the Direction of the Diffracted Beam is Held Fixed by Changing the Bragg Angle by $\Delta\Theta = \Delta\theta/2$ Through a Change in the Acoustic Frequency by $\Delta\Omega = (\Omega\Delta\Theta)(1/\tan\Theta)$.

Diffraction is caused by the sound wave component whose wavefronts make an angle of $\Theta + \Delta\Theta$ with the incident light. Irradiance of the diffracted beam is reduced by a factor of $\mathrm{sinc}^2(L\Delta\theta/2\Lambda)$ compared to when $\Delta\theta = 0$. Note that the angle between the diffracted and undiffracted beams is twice the new Bragg angle, $2(\Theta + \Delta\Theta) = 2\Theta + \Delta\theta$.

acoustic frequency is changed from Ω to $\Omega + \Delta\Omega$ where $\Delta\Omega = kV\Delta\theta \cos\Theta$, so that the Bragg angle changes by $\Delta\theta/2$. Its irradiance can attain its maximum value if the sound beam is rotated (about the y axis) by $\Delta\theta/2$. Moreover, if the phase of the sound wave is changed by Φ, a phase of this amount is also introduced in the diffracted wave. This can be seen by inserting a phase constant Φ in the index wave. The dependence of direction and phase of the Bragg diffracted wave on the frequency and phase of the sound wave forms the basis of real-time wavefront correction to be described later in detail.

CHAPTER 4

PARTICLE PICTURE OF RAMAN-NATH AND BRAGG DIFFRACTION

Diffraction of light by sound waves is considered in terms of photon-phonon interactions (Damon et al., 1970). Depending on whether one or more phonons are involved in an interaction, diffraction is called Bragg or Raman-Nath diffraction, respectively. A transition length is defined above which multiple-phonon interaction does not occur. Momentum conservation leads to vector diagrams that are useful in determining the direction of the diffracted waves.

4.1. Photon-Phonon Interaction and the Bragg Equation

In the particle picture, a light beam with propagation vector $\vec{k}$ and frequency ω is considered to consist of a stream of particles (photons) with momentum $\hbar\vec{k}$ and energy $\hbar\omega$, where $\hbar = h/2\pi$ and h is the Planck's constant. A sound beam, with propagation vector $\vec{K}$ and frequency Ω, likewise, can be thought of as made up of particles (phonons) with momentum $\hbar\vec{K}$ and energy $\hbar\Omega$.

A three-particle interaction in which a phonon is absorbed is illustrated in Fig. 4.1. The direction of $\vec{K}$ is reversed when a phonon is generated. To distinguish between incident and scattered photons, the parameters of the latter are denoted with a prime. As in the previous chapter, the vectors, $\vec{k}$, $\vec{K}$, and $\vec{k}'$ are assumed to lie in the zx plane. The absorption or emission of a phonon is described by the

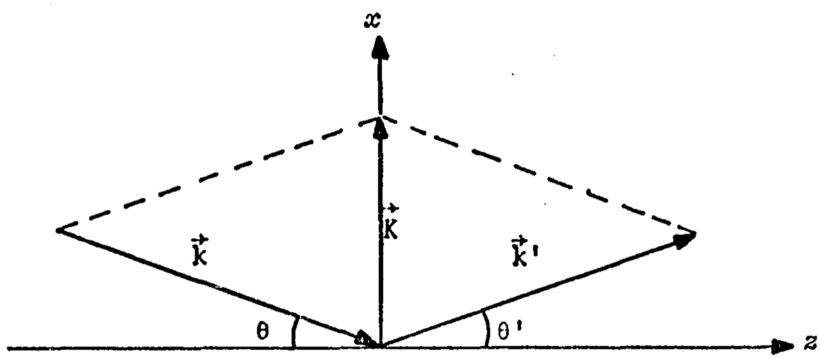


Fig. 4.1. Vector Diagram for Photon-Phonon Interaction in Which a Phonon is Annihilated. Direction of $\vec{K}$ is Reversed When a Phonon is Created.

conservation of energy and momentum:

$$\begin{aligned} \omega + \Omega &= \omega' \\ \vec{k} + \vec{K} &= \vec{k}' \end{aligned} \tag{4.1}$$

and

$$\begin{aligned} \omega &= \Omega + \omega' \\ \vec{k} &= \vec{K} + \vec{k}'. \end{aligned} \tag{4.2}$$

Equations (4.1) correspond to phonon absorption and Eqs. (4.2) to phonon emission. Since $\Omega << \omega$, we may assume $\omega \simeq \omega'$, and therefore $k \simeq k'$ for isotropic materials. Regardless of whether a phonon is absorbed or emitted, the conservation of momentum can be written in terms of its components as

$$k \cos\theta = k' \cos\theta' \tag{4.3a}$$

and

$$-k \sin\theta + K = k' \sin\theta'. \tag{4.3b}$$

Since $k = k'$, the first of these equations yields $\theta = \theta'$. The second equation, therefore, gives

$$\sin\theta = \frac{K}{2k} = \frac{\lambda}{2\Lambda} . \tag{4.4}$$

This is the Bragg equation (3.18) obtained earlier in which λ and Λ are the optical and acoustic wavelengths, respectively. Note that it limits the sound wavelength, which may be scattered to a value greater than $\lambda/2$. The case $\Lambda = \lambda/2$ corresponds to back scattering, i.e., $\vec{k}' = -\vec{k}$. The angle θ given by Eq. (4.4) is called the Bragg angle and is denoted by Θ.

4.2. Angular Width of the Wave Vectors and the Transition Length

So far, we have treated the diffraction of light by sound as a sum of separate three-particle interactions. Let us now consider the effects of finite widths of light and sound beams. It was pointed out in Chapter 3 that a wave restricted in size spreads due to diffraction. The angular spread for a uniformly illuminated rectangular aperture can be characterized by the angle from the normal at which the first zero of the far-field diffraction pattern occurs. Thus we can write the angular width of light and sound beams of width W and L as

$$\delta = 2\,\frac{\lambda}{W} \tag{4.5}$$

and

$$\Delta_s = 2\,\frac{\Lambda}{L} , \tag{4.6}$$

respectively. The subscript s denotes sound and is used to avoid confusion with the variation Δ in n, θ, Ω, etc. Because of the spread in vectors $\vec{k}$ and $\vec{K}$, the three-particle Bragg condition is relaxed and becomes an angular distribution of three-particle interactions, thereby permitting scattering to occur at various angles.

The angular spread in K-vector (if sufficient) can also cause multiple (i.e., successive) interactions. The conservation of momentum for a two-stage interaction is shown in Fig. 4.2. The energy conservation enters into this figure by the requirement that

$$k = k' = k''$$

and

$$\theta = \theta' = \theta'' = \theta''' = \Theta. \tag{4.7}$$

We note from the figure that the conservation of energy and momentum can occur in a two-step process only if there are two phonons with their

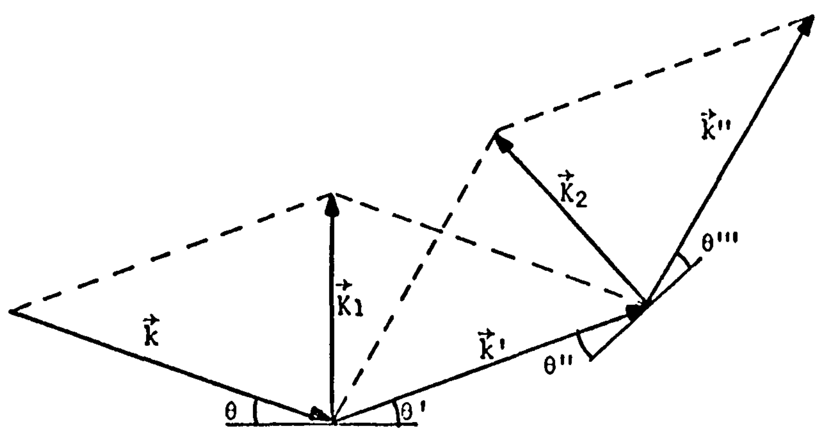


Fig. 4.2. Vector Diagram for Successive Scattering of a Photon $\vec{k}$ by Two Phonons $\vec{K}_1$ and $\vec{K}_2$.

K-vectors properly aligned. Moreover, the smaller the Bragg angle, the smaller the spread in K-vector required for multiple scattering. In other words, multiple scattering is possible at low acoustic frequencies (since Θ is then small), and energy and momentum conservation cannot both occur if multiple scattering takes place at sufficiently high acoustic frequencies. In this sense multiple scattering becomes a forbidden process at high acoustic frequencies.

Consequently, we can distinguish the region for which multiple scattering is highly probable (Raman-Nath region) from the one for which it is relatively improbable (Bragg region). Multiple scattering will readily occur if the angular half width of K-vector is large compared to the Bragg angle and will not readily occur when the reverse is true. The limit between the two regions can be expressed by the equation

$$\tfrac{1}{2}\Delta_s = \Theta. \tag{4.8}$$

Or, using $\Theta \simeq \sin\Theta = \lambda/2\Lambda$ and $\Delta_s = 2\Lambda/L$, we get

$$L = 2\Lambda^2/\lambda. \tag{4.9}$$

This is the transition length L_t defined in Eq. (3.17).

4.3. Bragg Diffraction and the Angular Width of the Wave Vectors

In the remaining part of this chapter, we shall assume high acoustic frequencies so that the scattering is always in the Bragg region. For $\Omega \ll \omega$, we assume $\omega = \omega'$ and therefore $k = k'$ and make the elementary but useful construction shown in Fig. 4.3, which is drawn in momentum space (Gordon, 1966). The locus of the scattering interaction in this space is a circle of radius k. The scattering (or deflection) angle is 2Θ. If we assume that neither the optical nor the acoustic beam has any angular width (i.e., if they are both plane waves), only a phonon of precisely the correct $\vec{K}$ will scatter $\vec{k}$. If the direction of $\vec{k}$ or $\vec{K}$ is changed, thereby changing the angle of incidence Θ, the vector sum $\vec{k} + \vec{K}$ no longer falls on the circle and, therefore, the intensity of the scattered (diffracted) light goes to zero.

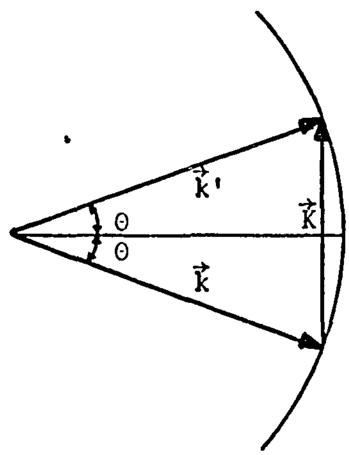


Fig. 4.3. Vector Diagram for Bragg Interaction of Plane Monochromatic Optical and Acoustic Waves.

If the direction of $\vec{k}$ or $\vec{K}$ is changed, thereby changing the angle of incidence Θ, the vector sum $\vec{k} + \vec{K}$ no longer falls on the circle, and the intensity of the scattered wave becomes zero.

To appreciate the effect of nonzero diffraction spread, first consider the case in which the angular width of the acoustic beam is larger than that of the optical beam, i.e., $\Delta_s > \delta$. The sound wave vector K has a well defined magnitude but an angular width of Δ_s. If δ is zero, only one K-vector (which is properly aligned with respect to the incident k-vector) can contribute to the scattered light. For a finite δ, all the K-vectors in the angular range δ about the vertical scatter light. Consequently, the angular width of the scattered light is also δ as illustrated in Fig. 4.4. Since its wavelength is also the same as that of the incident light ($k' = k$), we see that the coherence of the light beam, both spatial and temporal, is preserved. Note that only a portion of the acoustic energy is useful in scattering.

We now consider the other case, i.e., the one in which the diffraction spread of the optical beam is larger than that of the acoustic beam ($\delta > \Delta_s$). Figure 4.5 illustrates that in this case only a portion of the incident light is scattered, and the angular width of the scattered light is equal to Δ_s, which is smaller than that of the incident light. Thus the angular width of the scattered light always corresponds to the smaller of the two angles δ and Δ_s.

Finally, we consider the effect of variation in acoustic frequency. In what follows we shall neglect the diffraction spread δ. If the direction of the incident light changes, that of the scattered light can be held fixed by an appropriate change in the acoustic frequency as shown in Fig. 4.6. When the angle of incidence increases from Θ to Θ + $\Delta\theta$, the direction of the scattered light remains unchanged if the

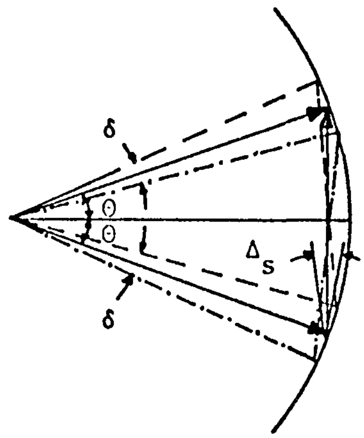


Fig. 4.4. Vector Diagram for Bragg Diffraction of an Optical Beam of Angular Width δ by an Acoustic Beam of Angular Width $\Delta_s > \delta$.

All the k-vectors in the angular range δ are diffracted by the corresponding K-vectors in the range δ. The remaining K-vectors do not take part in the acousto-optic interaction, and therefore only a portion of the acoustic energy is useful in scattering. Note that the angular width of the diffracted-beam is the same as that of the incident beam.

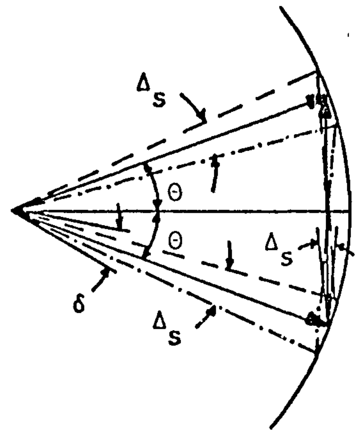


Fig. 4.5. Vector Diagram for Bragg Diffraction When $\delta > \Delta_s$.

Only the k-vectors in the angular range Δ_s are diffracted, and therefore a portion of the incident light takes part in scattering. The angular width of the scattered light is equal to Δ_s and, therefore, smaller than that of the incident light.

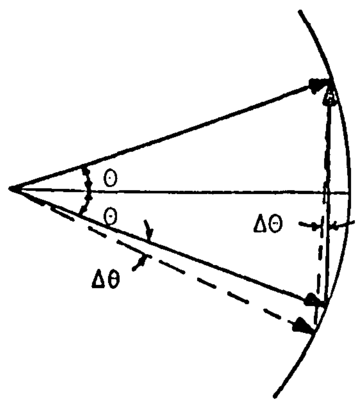


Fig. 4.6. Vector Diagram for Bragg Diffraction.

When the angle of incidence changes by $\Delta\theta$, the diffracted beam is held fixed by changing the Bragg angle by $\Delta\Theta = \Delta\theta/2$ through a change in the acoustic frequency by $\Delta\Omega = \Omega\Delta\Theta/\tan\Theta$. Intensity of the diffracted beam becomes maximum if the sound column is also rotated by $\Delta\Theta$. Diffraction spread δ of the optical beam is assumed to be zero.

acoustic frequency is increased by

$$\Delta\Omega = \frac{\Omega}{2\tan\Theta}\Delta\theta. \tag{4.10}$$

An increase in the frequency by $\Delta\Omega$ changes the Bragg angle from Θ to $\Theta + \Delta\Theta$ where $\Delta\Theta = \Delta\theta/2$. The irradiance of the scattered light for a uniform sound beam of width L will be less compared to when $\Delta\theta = 0$ by a factor of $\mathrm{sinc}^2(L\Delta\Theta/\Lambda)$ for low acoustic powers. It can be increased to its maximum value if the sound column is rotated by an angle $\Delta\Theta$, thus satisfying the Bragg condition at the new frequency.

If, however, the direction of the incident light is fixed, a change in the acoustic frequency can be detected by a corresponding change in the direction (and intensity) of the scattered light. We see from Fig. 4.7 that as the frequency changes from Ω to $\Omega + \Delta\Omega$, the direction of the scattered light changes by $2\Delta\Theta$ (where $\Delta\Theta$ is the change in the Bragg angle corresponding to a change $\Delta\Omega$ in the frequency), i.e., the scattering angle changes from 2Θ to $2(\Theta + \Delta\Theta)$.

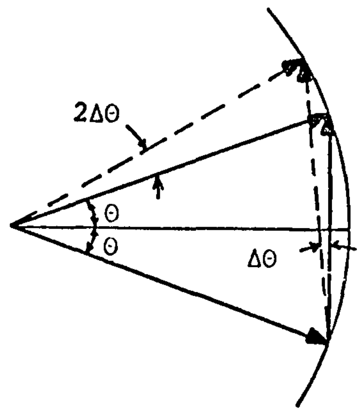


Fig. 4.7. Vector Diagram for Bragg Diffraction Showing the Effect of a Change in Acoustic Frequency.

A change of $\Delta\Omega$ in the acoustic frequency can be detected by measuring the change $2\Delta\Theta$ in the direction of the diffracted beam where $\Delta\Theta = (\Delta\Omega/\Omega) \tan\Theta$ is the change in Bragg angle corresponding to a change $\Delta\Omega$ in the frequency. Note that the direction of incident light is held fixed. Diffraction spread δ of the optical beam is assumed to be zero.

CHAPTER 5

PARTIAL WAVE ANALYSIS OF DIFFRACTION OF LIGHT BY SOUND WAVES

Recently, diffraction of light by sound waves according to the vector wave equation was described (Mahajan and Gaskill 1974b; Mahajan, 1974a). It was shown that the scalar wave equation describes the diffraction effects adequately except for large Bragg angles. Since we shall consider small angles, the scalar wave equation will be assumed to be adequate. The polarization of the incident light may be assumed to be parallel to one of the principal axes of the index ellipse in the presence of a sound wave. An arbitrary polarization can be resolved into two components along the principal axes, and the diffraction of each component can be considered separately. The net amplitude of each diffracted wave can be obtained by finding the vector sum of the two diffracted component amplitudes.

From the scalar wave equation, the Raman-Nath equation for the amplitudes of the diffracted waves is derived. Analytic solutions of this equation are obtained in the Raman-Nath and Bragg regions of diffraction. Results of Chapter 3 are obtained as special cases.

5.1. Optical Wave Equation and the Partial Waves

Referring to Fig. 2.3, we consider a plane wave of light of amplitude U, frequency ω, and wave vector $\mathrm{k} = (-k \sin\theta, 0, k \cos\theta)$ traveling in a transparent medium of refractive index n. When a sound wave of frequency Ω, wave number K, and phase constant Φ traveling along the x axis is generated in the medium, its refractive index in the region of the sound wave can be written (from Chapter 2)

$$n(x,t) = n + \Delta n \sin(\Omega t - Kx + \Phi), \tag{5.1}$$

where Δn is the amplitude of the index wave produced by the sound wave. The light field $E(\vec{r},t)$ at time t and at a point $\vec{r}$ inside the sound beam satisfies the optical wave equation, which for a nonconducting and nonmagnetic medium, can be written

$$\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 D}{\partial t^2}, \tag{5.2}$$

where c is the speed of light in vacuum and $D = En^2(x,t)$ is the electric displacement. Substituting for $n(x,t)$ from Eq. (5.1) and neglecting the term containing $(\Delta n)^2$, since $\Delta n << 1$, Eq. (5.2) can be written in the form

$$\nabla^2 E = \frac{1}{v^2}\frac{\partial^2}{\partial t^2} E[1 + 2\frac{\Delta n}{n}\sin(\Omega t - Kx + \Phi)], \tag{5.3}$$

where $v = c/n$ is the speed of light in the medium.

Confining the sound wave to the region $0 \leq z \leq L$, the light field incident on it is given by

$$E_i(\vec{r},t) = \tfrac{1}{2}U \exp i(\omega t - \vec{k}\cdot\vec{r}) + \text{c.c.}, \qquad z \leq 0 \tag{5.4}$$

where c.c. denotes the complex conjugate. Since the index variation produced by the sound wave is a periodic function of x and t, its effect on the incident light will also be periodic in these variables. Hence the light field in the region of the sound wave may be epxressed as

$$E(\vec{r},t) = \tfrac{1}{2}A(x,z,t)\ \exp i(\omega t - \vec{k}\cdot\vec{r}) + \text{c.c.},\quad z \geq 0 \tag{5.5}$$

where $A(x,z,t)$ is a periodic function of x and t, with the same periods as the refractive index $n(x,t)$. Since A is a periodic function, it may be expanded in a complex Fourier series

$$A(x,z,t) = \sum_{\ell=-\infty}^{\infty} U_\ell(z)\ \exp i\ell(\Omega t - Kx). \tag{5.6}$$

Substituting Eq. (5.6) into Eq. (5.5), we obtain

$$E(\vec{r},t) = \tfrac{1}{2}\sum_{\ell=-\infty}^{\infty} U_\ell(z)\ \exp i(\omega_\ell t - \vec{k}_\ell\cdot\vec{r}),\quad z \geq 0 \tag{5.7}$$

where

$$\omega_\ell = \omega + \ell\Omega \tag{5.8}$$

and

$$\vec{k}_\ell = \vec{k} + \ell\vec{K} = (-k\sin\theta + \ell K,\ 0,\ k\cos\theta). \tag{5.9}$$

It is evident that the acoustic wave vector $\vec{K} = (K, 0, 0)$.

Equation (5.7) represents a partial wave expansion of the light field inside the sound beam. Each partial wave, except the zero order, represents a diffracted plane wave and originates from the emission or absorption of one or more phonons by the incident light. The amplitude, frequency, and wave vector of the ℓth-order diffracted wave are given by $U_\ell(z)$, ω_ℓ, and $\vec{k}_\ell$, respectively. Equations (5.8) and (5.9) represent the conservation of energy and momentum in a photon-phonon interaction. As shown in Appendix A, the difference $\ell\Omega$ between the frequency of the ℓth-order wave and that of the incident light can also be interpreted as a Doppler shift due to the motion of the acoustic wavefronts. The amplitudes $U_\ell(z)$ obey the initial conditions

$$\begin{aligned} U_\ell(0) &= U \text{ for } \ell = 0 \\ &= 0 \text{ otherwise.} \end{aligned} \tag{5.10}$$

Note that for $z = 0$, Eq. (5.7) reduces to Eq. (5.4). The time-averaged irradiance of the diffracted waves emerging from the sound beam is given by

$$I_\ell = \tfrac{1}{2}|U_\ell(L)|^2, \tag{5.11}$$

where L is the width of the sound beam along the z axis.

5.2. The Raman-Nath Equation

The Raman-Nath equation for the amplitudes of the diffracted waves is obtained by substituting Eq. (5.7) into Eq. (5.3). The substitution leads to the equation

$$\tfrac{1}{2}\sum_{\ell=-\infty}^{\infty}\left(\frac{d^2U_\ell}{dz^2} - k_\ell{}^2U_\ell - 2ik\cos\theta\,\frac{dU_\ell}{dz}\right)\exp i(\omega_\ell t - \vec{k}_\ell \cdot \vec{r}) + \text{c.c.}$$

$$= -\tfrac{1}{2}\frac{1}{v^2}\sum_{\ell=-\infty}^{\infty}\omega_\ell{}^2[U_\ell - i\,\frac{\Delta n}{n}\,\{U_{\ell-1}\exp(i\Phi) - U_{\ell+1}\exp(-i\Phi)\}]$$

$$\exp i(\omega_\ell t - \vec{k}_\ell \cdot \vec{r}) + \text{c.c.} \tag{5.12}$$

Comparing the coefficients of $\exp i(\omega_\ell t - \vec{k}_\ell \cdot \vec{r})$, we obtain

$$\frac{d^2U_\ell}{dz^2} - k_\ell{}^2U_\ell - 2ik\cos\theta\,\frac{dU_\ell}{dz}$$

$$= -(\omega_\ell/v)^2[U_\ell - i\,\frac{\Delta n}{n}\,\{U_{\ell-1}\exp(i\Phi) - U_{\ell+1}\exp(-i\Phi)\}]. \tag{5.13}$$

If $U_\ell(z)$ is a slowly varying function of z, so that change in it is small in an optical wavelength, its second derivative with respect to z may be neglected compared to other terms in Eq. (5.13). Moreover, since $\Omega << \omega$, $\omega_\ell \simeq \omega$ and we may write $\omega_\ell/v = k$. Thus Eq. (5.13), after some rearrangement of terms, reduces to

$$\frac{dU_\ell}{dz} + \frac{k\Delta n}{2n\cos\theta}\,[U_{\ell-1}\exp(i\Phi) - U_{\ell+1}\exp(-i\Phi)]$$

$$= \frac{i}{2k\cos\theta}\,(k_\ell{}^2 - k^2)U_\ell. \tag{5.14}$$

Substituting for $k_\ell{}^2$ from Eq. (5.9), Eq. (5.14) becomes

$$\frac{dU_\ell}{dz} + \frac{k\Delta n}{2n\cos\theta}\,[U_{\ell-1}\exp(i\Phi) - U_{\ell+1}\exp(-i\Phi)]$$

$$= i\,\frac{\ell K^2}{2k\cos\theta}\,\left(\ell - \frac{2k}{K}\sin\theta\right)U_\ell. \tag{5.15}$$

This equation, called the Raman-Nath equation, is a set of coupled difference differential equations relating the amplitudes of the diffracted waves. Analytic solutions of it can be obtained in the

Raman-Nath and Bragg regions of diffraction, for then it reduces to simpler equations.

According to coupled mode theory, the partial waves in Eq. (5.7) are the normal modes and the coupling coefficient, which is proportional to $k\Delta n/2n \cos\theta$, couples only the adjacent modes. For a given coupling coefficient, the energy transfer between two modes depends on their degree of synchronization. Two modes with the same coefficient of the right-hand side of Eq. (5.15) are synchronous, and energy transfers from one to the other most efficiently. For low acoustic frequencies, the adjacent modes are nearly synchronous, and the energy transfers from the zero-order mode to the first, first to the second, second to the third, etc. This is the case of Raman-Nath diffraction. For high frequencies, however, energy can transfer efficiently only to the first-order modes provided $\sin\theta \simeq \pm K/2k$. This corresponds to Bragg diffraction. Note that although the zero- and the ℓth-order modes are synchronous for $\sin\theta = \ell K/2k$, they are adjacently coupled only for $\ell = \pm 1$.

5.3. Raman-Nath Diffraction

For sound waves of low frequency, the right-hand side of Eq. (5.15) is usually neglected. However, since ℓ in practice is generally less than or equal to 10, and unless θ is approximately zero, the term containing K may be retained. Thus neglecting the term containing K^2, Eq. (5.15) reduces to

$$\frac{dU_\ell}{dz} + \frac{k\Delta n}{2n \cos\theta} [U_{\ell-1} \exp(i\Phi) - U_{\ell+1} \exp(-i\Phi)] = -i\ell K \tan\theta U_\ell . \tag{5.16}$$

Using the identity for Bessel functions of the first kind (Watson, 1966, p. 17)

$$J_{\ell}(z) = \frac{z}{2\ell}\,[J_{\ell-1}(z) + J_{\ell+1}(z)] \tag{5.17}$$

and

$$2\,\frac{dJ_{\ell}(z)}{dz} = J_{\ell-1}(z) - J_{\ell+1}(z), \tag{5.18}$$

the solution of Eq. (5.16), subject to the boundary conditions, Eq. (5.10), is found to be

$$U_{\ell}(z) = UJ_{\ell}\left[\frac{\Delta n}{n}\,\frac{kz}{\cos\theta}\,\mathrm{sinc}\left(\frac{z}{\Lambda}\tan\theta\right)\right] \exp i\ell(\pi + \Phi - \tfrac{1}{2}Kz\tan\theta). \tag{5.19}$$

The time-averaged irradiance of the diffracted waves emerging from the sound beam, according to Eq. (5.11), are given by

$$I_{\ell} = IJ_{\ell}^{\,2}(\Delta\phi), \tag{5.20}$$

where $I = \frac{1}{2}U^2$ is the time-averaged irradiance of the incident light and

$$\Delta\phi = \frac{\Delta n}{n}\,\frac{kL}{\cos\theta}\,\mathrm{sinc}\left(\frac{L}{\Lambda}\tan\theta\right). \tag{5.21}$$

Note that Eq. (5.20) is the same as Eq. (3.15), which was obtained on the assumption that low frequency sound waves act like a moving phase grating.

5.4. Bragg Diffraction

Sound waves of high frequency diffract light only if it is incident at an angle approximately equal to the Bragg angle. Thus when

$\sin\theta \simeq K/2k$, light is diffracted efficiently into the positive-first-order only and Eq. (5.15) reduces to two equations:

$$\frac{dU_0}{dz} - \xi U_1 \exp(-i\Phi) = 0 \tag{5.22}$$

and

$$\frac{dU_1}{dz} + \xi U_0 \exp(i\Phi) = 2i\psi U_1, \tag{5.23}$$

where the first equation corresponds to $\ell = 0$ and the second to $\ell = 1$, and

$$\xi = \frac{\Delta n}{n} \frac{k}{2\cos\theta} \tag{5.24}$$

and

$$\psi = \frac{K^2}{4k\cos\theta} \left(1 - \frac{2k}{K}\sin\theta\right). \tag{5.25}$$

Solving Eqs. (5.22) and (5.23), subject to the boundary conditions

$$U_0(0) = U \tag{5.26a}$$

and

$$U_1(0) = 0, \tag{5.26b}$$

we obtain the amplitudes of the diffracted and undiffracted waves

$$U_1(z) = U(\xi/\zeta)\sin(\zeta z) \exp i(\pi + \Phi + \psi z) \tag{5.27}$$

(5.27)

and

$$U_0(z) = U[\cos(\zeta z) - i(\psi/\zeta)\sin(\zeta z)] \exp(i\psi z), \tag{5.28}$$

respectively, where

$$\zeta = (\xi^2 + \psi^2)^{\frac{1}{2}}. \tag{5.29}$$

The corresponding time-averaged irradiances of the waves emerging from the sound beam are given by

$$I_1 = I(\xi/\zeta)^2 \sin^2(\zeta L) \tag{5.30}$$

and

$$I_0 = I - I_1. \tag{5.31}$$

Figure 5.1 shows the variation of I_1 as a function of ψL for various values of ξL. The curves become slightly sharper and the secondary maxima higher as ξL increases, especially when ξL approaches $3\pi/4$. Note that according to Eq. (5.9) the direction of the Bragg diffracted wave depends on the acoustic frequency. Moreover, its phase, according to Eq. (5.27), depends on the acoustic phase.

For Bragg incidence, $\psi = 0$, and therefore Eqs. (5.30) and (5.31) become

$$I_1 = I \sin^2(\xi L) = I \sin^2\left(\frac{1}{2}\,\frac{\Delta n}{n}\,\frac{kL}{\cos\Theta}\right) \tag{5.32}$$

and

$$I_0 = I \cos^2(\xi L) = I \cos^2\left(\frac{1}{2}\,\frac{\Delta n}{n}\,\frac{kL}{\cos\Theta}\right), \tag{5.33}$$

respectively. In Eqs. (5.32) and (5.33), Θ is the Bragg angle given by $\sin\Theta = K/2k$. These equations are the same as Eqs. (3.29) and (3.30), respectively. Since $\cos\Theta \simeq 1$, all the light is diffracted when L is equal to $2n/\Delta n$ optical wavelengths. Since $\Delta n \sim 10^{-5}$, the transfer of

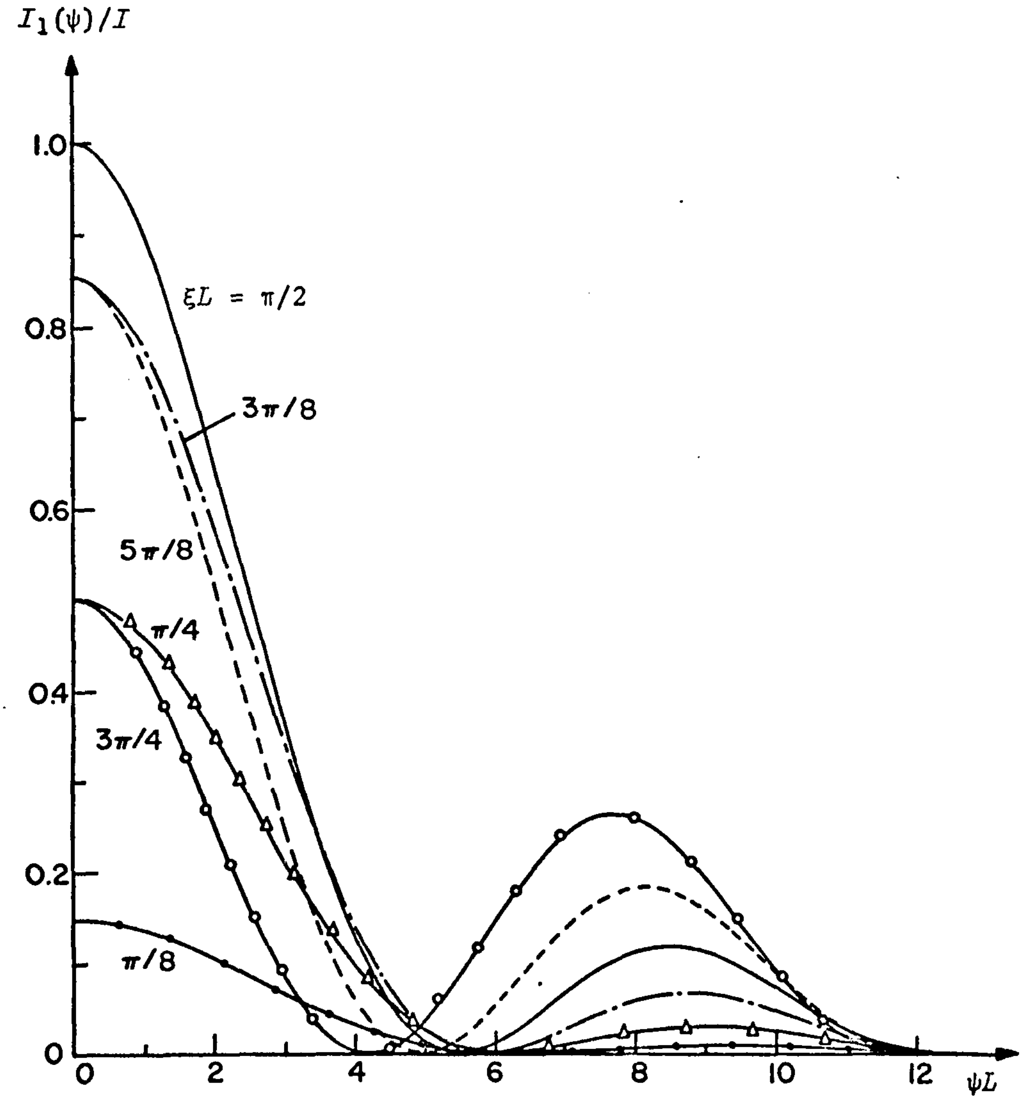


Fig. 5.1. Relative Irradiance of Bragg Diffracted Light as a Function of Deviation from the Bragg Condition for Various Values of the Parameter ξL.

energy from the zero order to the first as a function of z is extremely slow as was assumed in neglecting the second derivative of $U_\ell(z)$ with respect to z. This is also true for Raman-Nath diffraction in which case the zero-order beam is depleted even more slowly (see Fig. 3.5).

For small ξ, i.e., for low acoustic power, Eq. (5.30) can be written

$$I_1 = I(\xi/\psi)^2 \sin^2(\psi L). \tag{5.34}$$

When $\theta = \Theta + \Delta\theta$, where $\Delta\theta$ is small

$$\psi \simeq -\pi\Delta\theta/\Lambda \tag{5.35}$$

so that

$$\frac{I_1(\Delta\theta)}{I_{1(0)}} = \mathrm{sinc}^2(L\Delta\theta/\Lambda). \tag{5.36}$$

Note that Eq. (5.36) is the same as Eq. (3.32). Thus for low acoustic powers, the irradiance of the Bragg diffracted light measured as a function of the angle of incidence determines the angular distribution of the acoustic energy (Cohen and Gordon, 1965).

From the Bragg condition $\sin\Theta = K/2k$, we note that a deviation $\Delta\theta$ in the angle of incidence, $\Delta\Lambda$ in the acoustic wavelength, or Δk in the optical wave number from the corresponding Bragg values, where

$$\frac{k\cos\Theta}{K}\Delta\theta = \frac{k\sin\Theta}{2\pi}\Delta\Lambda = \frac{\sin\Theta}{K}\Delta k, \tag{5.37}$$

produce the same change in the irradiance of the Bragg diffracted light.

5.5. Diffraction in the Transition Region

We have obtained analytic solutions of the Raman-Nath equation (5.15) for low and high acoustic frequencies. For sound waves of intermediate frequencies, i.e., for diffraction in the transition region, simple, analytic solutions of this equation cannot, in general, be obtained. However, it can be solved numerically as has been done by Klein and Cook (1967). Their results clearly show how transition from the Raman-Nath to Bragg diffraction takes place as a function of a parameter

$$Q = K^2 L/k. \tag{5.38}$$

For instance, they show that for $kL(\Delta n/n) \leq 3$, the diffraction effects disappear for normal incidence. For oblique incidence, the irradiance of the zero-order wave is symmetric about $\theta = 0$, but the irradiances of the first-order waves are symmetric about the Bragg angle. In general, for $kL(\Delta n/n) \leq 2\pi$, the regions of diffraction are divided as follows:

$$Q \leq 0.5 \qquad \text{Raman-Nath Diffraction} \tag{5.39}$$

$$Q \geq 10. \qquad \text{Bragg Diffraction} \tag{5.40}$$

The intermediate values of Q belong to the transition region. It is interesting to note that the condition $Q \geq 10$ implies a transition length of

$$L_t = 10\,\frac{\Lambda^2}{2\pi\lambda}\,, \tag{5.41}$$

which is approximately equal to the one given by Eq. (3.17).

CHAPTER 6

DIFFRACTION OF LIGHT BY TWO ORTHOGONAL SOUND WAVES

Diffraction of light by two orthogonal sound waves is described assuming, as in Chapter 5, the scalar wave equation to adequately do so (Mahajan and Gaskill, 1974c). The Raman-Nath equation for the amplitudes of the diffracted waves is derived and shown to separate into two Raman-Nath equations, one for each of the two sound waves. Thus it is shown that diffraction by two orthogonal sound waves present simultaneously in a medium is equivalent to diffraction by the two sound waves present successively. Analytic solutions of the Raman-Nath equation are obtained in the Raman-Nath and Bragg regions of diffraction.

6.1. Optical Wave Equation and the Partial Waves

Consider a plane wave of light of amplitude U, frequency ω, and wave number k propagating in a medium of refractive index n with direction cosines $(\alpha = \cos\bar{\alpha},\ \beta = \cos\bar{\beta},\ \gamma = \cos\bar{\gamma})$ as indicated in Fig. 6.1. In Chapter 5, we had $\alpha = -\sin\theta$, $\beta = 0$, and $\gamma = \cos\theta$. When two orthogonal sound waves, one of frequency Ω_1, wave number K_1, and phase constant Φ_1 traveling the x axis, and the other of frequency Ω_2, wave number K_2, and phase constant Φ_2 traveling along the y axis are generated, the refractive index of the medium in the region of sound waves can be written (from Chapter 2)

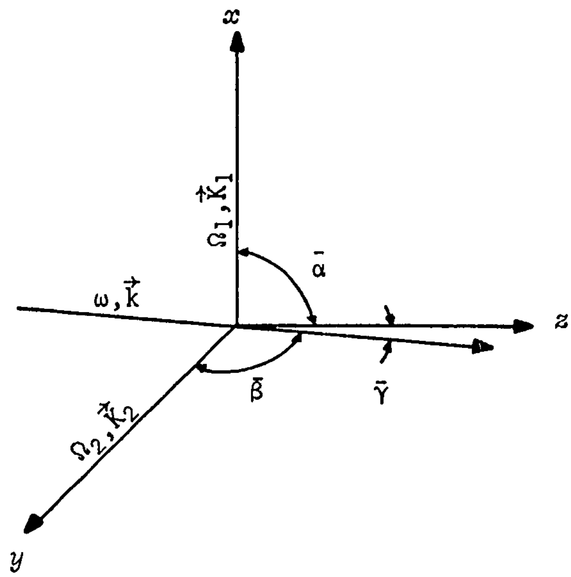


Fig. 6.1. Diffraction of Light by Two Orthogonal Sound Waves, One Traveling Along the x Axis and the Other Along the y Axis.

$$n(x,y,t) = n + (\Delta n)_1 \sin(\Omega_1 t - K_1 x + \Phi_1) + (\Delta n)_2 \sin(\Omega_2 t - K_2 y + \Phi_2), \tag{6.1}$$

where $(\Delta n)_1$ and $(\Delta n)_2$ are the amplitudes of the corresponding index waves produced by the sound waves. Let L_1 and L_2 be the widths of the two sound waves measured along the z axis. The sound waves diffract light when it is incident close to the z axis.

The light field inside the region of sound waves satisfies the optical wave equation, which for a nonconducting and nonmagnetic medium can be written

$$\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 D}{\partial t^2}, \tag{6.2}$$

where c is the speed of light in vacuum and $D = En^2(x,y,t)$ is the electric displacement. Substituting for $n(x,y,t)$ from Eq. (6.1) and neglecting the terms containing squares or products of $(\Delta n)_1$ and $(\Delta n)_2$, Eq. (6.2) can be written

$$\nabla^2 E = \frac{1}{v^2}\frac{\partial^2}{\partial t^2} E\{1 + \frac{2}{n}[(\Delta n)_1 \sin(\Omega_1 t - K_1 x + \Phi_1) + (\Delta n)_2 \sin(\Omega_2 t - K_2 y + \Phi_2)]\} \tag{6.3}$$

where $v = c/n$ is the speed of light in the medium. The light field incident on the sound waves is given by

$$E_i(\vec{r},t) = \tfrac{1}{2}U \exp i(\omega t - \vec{k}\cdot\vec{r}) + \text{c.c.}, \quad z \le 0 \tag{6.4}$$

where c.c. denotes the complex conjugate. Since the index variation produced by the sound waves is periodic in x, y, and t, its effect on the incident light will also be periodic in these variables. Hence the light field in the region of the sound waves may be expressed as

$$E(\vec{r},t) = \tfrac{1}{2}A(x,y,z,t) \exp i(\omega t - \vec{k}\cdot\vec{r}) + \text{c.c.}, \quad z \ge 0 \tag{6.5}$$

where $A(x,y,z,t)$ is a periodic function of x, y, and t with the same periods as the refractive index $n(x,y,t)$. Since A is a periodic function, it may be expanded in a complex Fourier series

$$A(x,y,z,t) = \tfrac{1}{2}\sum_{\ell=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} U_{\ell m}(z) \exp i[\ell(\Omega_1 t - K_1 x) + m(\Omega_2 t - K_2 y)] + \text{c.c.} \tag{6.6}$$

Note that if one of the two sound waves is absent, Eq. (6.6) reduces to an equation similar to Eq. (5.6) for the case of a single sound wave. Substituting Eq. (6.6) into Eq. (6.5), we obtain

$$E(\vec{r},t) = \tfrac{1}{2} \sum_{\ell=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} U_{\ell m}(z) \exp i(\omega_{\ell m} t - \vec{k}_{\ell m} \cdot \vec{r}) + \text{c.c.}, \quad (6.7)$$

where

$$\omega_{\ell m} = \omega + \ell\Omega_1 + m\Omega_2 \quad (6.8)$$

and

$$\vec{k}_{\ell m} = \vec{k} + \ell\vec{K}_1 + m\vec{K}_2 = (k\alpha + \ell K_1,\ k\beta + mK_2,\ k\gamma). \quad (6.9)$$

It is evident that the acoustic wave vector $\vec{K}_1 = (K_1,\ 0,\ 0)$ and $\vec{K}_2 = (0,\ K_2,\ 0)$.

Equation (6.7) represents a partial wave expansion of the light field in the region of sound waves. Each partial wave, except the (0,0) order, originates from the emission or absorption of one or more phonons by the incident light. The amplitude, frequency, and wavevector of the (ℓ,m)th-order diffracted wave are given by $U_{\ell m}(z)$, $\omega_{\ell m}$, and $\vec{k}_{\ell m}$, respectively. Equations (6.8) and (6.9) represent conservation of energy and momentum in a photon-phonon interaction, respectively. As indicated in Appendix A, the difference $(\ell\Omega_1 + m\Omega_2)$ between the frequency of the (ℓ,m)th-order wave and that of the incident light represents a Doppler shift due to the motion of the acoustic wavefronts. The amplitudes $U_{\ell m}(z)$ obey the initial conditions

$$\begin{aligned} U_{\ell m}(0) &= U \text{ for } \ell = 0 = m \\ &= 0 \text{ otherwise.} \end{aligned} \quad (6.10)$$

6.2. The Raman-Nath Equation

The Raman-Nath equation for the amplitudes of the diffracted waves is obtained by substituting Eq. (6.7) into Eq. (6.3). The substitution leads to the equation

$$\frac{1}{2}\sum_{\ell=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\left(\frac{d^2U_{\ell m}}{dz^2} - k^2{}_{\ell m}U_{\ell m} - 2i\gamma k\frac{dU_{\ell m}}{dz}\right)$$

$$\cdot \exp i(\omega_{\ell m} - \vec{k}_{\ell m}\cdot\vec{r}) + \text{c.c.}$$

$$= -\frac{1}{2}\frac{1}{v^2}\sum_{\ell=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\omega_{\ell m}{}^2\Big\{U_{\ell m} - i\frac{(\Delta n)_1}{n}$$

$$\cdot\ [U_{\ell-1,m}\exp(i\Phi_1) - U_{\ell+1,m}\exp(-i\Phi_1)]$$

$$-i\frac{(\Delta n)_2}{n}[U_{\ell,m-1}\exp(i\Phi_2) - U_{\ell,m+1}\exp(-i\Phi_2)]\Big\}$$

$$\cdot \exp i(\omega_{\ell m}t - \vec{k}_{\ell m}\cdot\vec{r}) + \text{c.c.} \tag{6.11}$$

Comparing the coefficients of $\exp i(\omega_{\ell m}t - \vec{k}_{\ell m}\cdot\vec{r})$, we obtain

$$\frac{d^2U_{\ell m}}{dz^2} - k^2{}_{\ell m}U_{\ell m} - 2i\gamma k\frac{dU_{\ell m}}{dz}$$

$$= -\left(\frac{\omega_{\ell m}}{v}\right)^2\Big\{U_{\ell m} - i\frac{(\Delta n)_1}{n}[U_{\ell-1,m}\exp(i\Phi_1) - U_{\ell+1,m}\exp(-i\Phi_1)]$$

$$-i\frac{(\Delta n)_2}{n}[U_{\ell,m-1}\exp(i\Phi_2) - U_{\ell,m+1}\exp(-i\Phi_2)]\Big\}. \tag{6.12}$$

If $U_{\ell m}(z)$ is a slowly varying function of z, so that the change in it is small in an optical wavelength, its second derivative with respect to z may be neglected compared to other terms in Eq. (6.12). Moreover, since Ω_1 and Ω_2 are much lower than ω, $\omega_{\ell m} \simeq \omega$ and we may write $\omega_{\ell m}/v = k$. Thus Eq. (6.12), after some rearrangement of terms, reduces to

$$\frac{dU_{\ell m}}{dz} + \frac{k(\Delta n)_1}{2\gamma n}\left[U_{\ell-1,m}\exp(i\Phi_1) - U_{\ell+1,m}\exp(-i\Phi_1)\right]$$

$$+ \frac{k(\Delta n)_2}{2\gamma n}\left[U_{\ell,m-1}\exp(i\Phi_2) - U_{\ell,m+1}\exp(-i\Phi_2)\right]$$

$$= \frac{i}{2\gamma k}\left(k_{\ell m}{}^2 - k^2\right)U_{\ell m}. \tag{6.13}$$

Substituting for $k_{\ell m}{}^2$ from Eq. (6.9), Eq. (6.13) becomes

$$\frac{dU_{\ell m}}{dz} + \frac{k(\Delta n)_1}{2\gamma n}\left[U_{\ell-1,m}\exp(i\Phi_1) - U_{\ell+1,m}\exp(-i\Phi_1)\right]$$

$$+ \frac{k(\Delta n)_2}{2\gamma n}\left[U_{\ell,m-1}\exp(i\Phi_2) - U_{\ell,m+1}\exp(-i\Phi_2)\right]$$

$$= i\left[\frac{\ell K_1{}^2}{2\gamma k}\left(\ell + \frac{2\alpha k}{K_1}\right) + \frac{mK_2{}^2}{2\gamma k}\left(m + \frac{2\beta k}{K_2}\right)\right]U_{\ell m} \tag{6.14}$$

This is the Raman-Nath equation for the diffracted amplitudes. If we substitute

$$U_{\ell m}(z) = V_\ell(z)W_m(z) \tag{6.15}$$

into Eq. (6.14) and divide throughout by $U_{\ell m}$, we obtain

$$\frac{1}{V_\ell}\left\{\frac{dV_\ell}{dz} + \frac{k(\Delta n)_1}{2\gamma n}\left[V_{\ell-1}\exp(i\Phi_1) - V_{\ell+1}\exp(-i\Phi_1)\right]\right\}$$

$$- i\,\frac{\ell K_1{}^2}{2\gamma k}\left(\ell + \frac{2\alpha k}{K_1}\right)$$

$$= -\frac{1}{W_m}\left\{\frac{dW_m}{dz} + \frac{k(\Delta n)_2}{2\gamma n}\left[W_{m-1}\exp(i\Phi_2) - W_{m+1}(-i\Phi_2)\right]\right\}$$

$$+ i\,\frac{mK_2{}^2}{2\gamma k}\left(m + \frac{2\beta k}{K_2}\right). \tag{6.16}$$

Since the two sound waves are independent of each other, each side of Eq. (6.16) must equal zero. Thus the Raman-Nath equation (6.14) separates into two Raman-Nath equations, one for V_ℓ and the other for W_m:

$$\frac{dV_\ell}{dz} + \frac{k(\Delta n)_1}{2\gamma n}\left[V_{\ell-1}\exp(i\Phi_1) - V_{\ell+1}\exp(-i\Phi_1)\right]$$

$$= i\,\frac{\ell K_1{}^2}{2\gamma k}\left(\ell + \frac{2\alpha k}{K_1}\right)V_\ell \tag{6.17}$$

and

$$\frac{dW_m}{dz} + \frac{k(\Delta n)_2}{2\gamma n}\left[W_{m-1}\exp(i\Phi_2) - W_{m+1}\exp(-i\Phi_2)\right]$$

$$= i\,\frac{mK_2{}^2}{2\gamma k}\left(m + \frac{2\beta k}{K_2}\right)W_m. \tag{6.18}$$

From Eqs. (6.17) and (6.18), it is clear that light diffraction by two simultaneously present orthogonal sound waves is equivalent to that due to the two waves present successively. Thus, whether the two sound waves travel in the same portion of the medium (one acousto-optic cell) or in its different portions (two cascaded acousto-optic cells), the net diffraction effect is the same. It is assumed, of course, that in the latter case all the light diffracted by one sound wave is incident on the other. In any application of this effect, the choice between simultaneous and successive diffraction will be made by practical considerations and the properties of the medium, etc. Equation (6.15) may be interpreted as follows. The incident light is diffracted by the sound wave traveling along the x axis giving waves of amplitude $V_\ell(z)$, frequency $\omega_\ell = \omega + \ell\Omega_1$, and wave vector $\vec{k}_\ell = \vec{k} + \ell\vec{K}_1$. Each of these diffracted waves is then diffracted by the sound wave traveling along the y axis. For example, the ℓth diffracted wave is diffracted producing waves of amplitude $V_\ell(z)W_m(z)$, frequency $\omega_{\ell m} = \omega_\ell + m\Omega_2$, and wave vector $\vec{k}_{\ell m} = \vec{k}_\ell + m\vec{K}_2$.

Equations (6.17) and (6.18) may now be solved subject to the boundary conditions

$$\begin{aligned} V_\ell(0) &= U \text{ for } \ell = 0 \\ &= 0 \text{ otherwise} \end{aligned} \tag{6.19}$$

and

$$\begin{aligned} W_m(0) &= 1 \text{ for } m = 0 \\ &= 0 \text{ otherwise,} \end{aligned} \tag{6.20}$$

respectively. Because of their similarity with Eq. (5.15), their solutions can be obtained following the solutions of this equation. The time-averaged irradiance of the (ℓ,m)th-order diffracted wave emerging from the sound beams is given by

$$\begin{aligned} I_{\ell m} &= \tfrac{1}{2}|V_\ell(L_1)W_m(L_2)|^2 \\ &= \tfrac{1}{2}|V_\ell(L_1)|^2|W_m(L_2)|^2. \end{aligned} \tag{6.21}$$

6.3. Raman-Nath Diffraction

In the region of Raman-Nath diffraction, i.e., for low acoustic frequencies, the term containing $K_1{}^2$ in Eq. (6.17) may be neglected. The solution of the resulting equation subject to the boundary conditions, Eq. (6.19), is given by

$$V_\ell(z) = UJ_\ell\left[\frac{(\Delta n)_1}{n}\,\frac{kz}{\gamma}\,\mathrm{sinc}\left(\frac{\alpha z}{\gamma\Lambda_1}\right)\right]\exp i\ell(\pi + \Phi_1 + \tfrac{1}{2}K_1\alpha z/\gamma). \tag{6.22}$$

Similarly, neglecting the term containing $K_2{}^2$ in Eq. (6.18), the solution of the resulting equation subject to the boundary conditions, Eq. (6.20), is given by

$$W_m(z) = J_m\left[\frac{(\Delta n)_2}{n}\,\frac{kz}{\gamma}\,\mathrm{sinc}\left(\frac{\beta z}{\gamma\Lambda_2}\right)\right]\exp im(\pi + \Phi_2 + \tfrac{1}{2}K_2\beta z/\gamma). \tag{6.23}$$

The time-averaged irradiance of the (ℓ,m)th-order diffractive wave according to Eq. (6.21) is given by

$$\begin{aligned} I_{\ell m} &= IJ_\ell{}^2\left[\frac{(\Delta n)_1}{n}\,\frac{kL_1}{\gamma}\,\mathrm{sinc}\left(\frac{\alpha L_1}{\gamma\Lambda_1}\right)\right] \\ &\quad\cdot J_m{}^2\left[\frac{(\Delta n)_2}{n}\,\frac{kL_2}{\gamma}\,\mathrm{sinc}\left(\frac{\beta L_2}{\gamma\Lambda_2}\right)\right] \end{aligned} \tag{6.24}$$

where $I = \frac{1}{2}U^2$ is the time-averaged irradiance of the incident light. Maximum diffraction occurs for normal incidence, i.e., for (α=0, β=0, γ=1), in which case Eq. (6.24) becomes

$$I_{\ell m} = IJ_{\ell}^{2}\left[\frac{(\Delta n)_1}{n}kL_1\right]J_{m}^{2}\left[\frac{(\Delta n)_2}{n}kL_2\right]. \tag{6.25}$$

6.4. Bragg Diffraction

In the region of Bragg diffraction, i.e., for high acoustic frequencies, a sound wave diffracts the incident light only if it is incident near the Bragg angle. Thus if α is negative and approximately equal to $K_1/2k$, the incident light is diffracted into the ℓ = 1 order by the sound wave traveling along the x axis. If however, $\alpha \approx K_1/2k$, then this sound wave diffracts the incident light into the ℓ = -1 order. Assuming that $\alpha \approx -K_1/2k$, we can neglect all V_ℓ in Eq. (6.17) except V_0 and V_1. Thus Eq. (6.17) reduces to the following two equations:

$$\frac{dV_0}{dz} - \xi_1 V_1 \exp(-i\Phi_1) = 0 \tag{6.26}$$

and

$$\frac{dV_1}{dz} + \xi_1 V_0 \exp(i\Phi_2) = 2i\psi_1 V_1, \tag{6.27}$$

where the first equation corresponds to ℓ = 0 and the second to ℓ = 1, and

$$\xi_1 = \frac{k(\Delta n)_1}{2\gamma n} \tag{6.28}$$

and

$$\psi_1 = \frac{K_1^2}{4\gamma k}\left(1 + \frac{2k\alpha}{K_1}\right). \tag{6.29}$$

Solving Eqs. (6.26) and (6.27) subject to the boundary conditions

$$V_0(0) \;=\; U \tag{6.30a}$$

and

$$V_1(0) \;=\; 0, \tag{6.30b}$$

we obtain

$$V_0(z) \;=\; U[\cos(\zeta_1 z) - i(\psi_1/\zeta_1)\,\sin(\zeta_1 z)]\,\exp(i\psi_1 z) \tag{6.31}$$

and

$$V_1(z) \;=\; U(\xi_1/\zeta_1)\,\sin(\zeta_1 z)\,\exp i(\pi + \Phi_1 + \psi_1 z), \tag{6.32}$$

where

$$\zeta_1 \;=\; (\xi_1{}^2 + \psi_1{}^2)^{\frac{1}{2}}. \tag{6.33}$$

Similarly, if $\beta \simeq -K_2/2k$, Eq. (6.18) can be solved to give

$$W_0(z) \;=\; [\cos(\zeta_2 z) - i(\psi_2/\zeta_2)\,\sin(\zeta_2 z)]\,\exp(i\psi_2 z) \tag{6.34}$$

and

$$W_1(z) \;=\; (\xi_2/\zeta_2)\,\sin(\zeta_2 z)\,\exp i(\pi + \Phi_2 + \psi_2 z), \tag{6.35}$$

where

$$\xi_2 \;=\; \frac{k(\Delta n)_2}{2\gamma n}, \tag{6.36}$$

$$\psi_2 \;=\; \frac{K_2{}^2}{4\gamma k}\left(1 + \frac{2k\beta}{K_2}\right), \tag{6.37}$$

and

$$\zeta_2 \;=\; (\xi_2{}^2 + \psi_2{}^2)^{\frac{1}{2}}. \tag{6.38}$$

The time-averaged irradiance of the four waves emerging from the sound beams are, according to Eq. (6.21), given by

$$I_{00} = I[\cos^2(\zeta_1 L_1) + (\psi_1/\zeta_1)^2 \sin^2(\zeta_1 L_1)] \cdot [\cos^2(\zeta_2 L_2) + (\psi_2/\zeta_2)^2 \sin^2(\zeta_2 L_2) \ , \tag{6.39a}$$

$$I_{01} = I[\cos^2(\zeta_1 L_1) + (\psi_1/\zeta_1)^2 \sin^2(\zeta_1 L_1)] \cdot [(\xi_2/\zeta_2)^2 \sin^2(\zeta_2 L_2)], \tag{6.39b}$$

$$I_{10} = I[(\xi_1/\zeta_1)^2 \sin^2(\zeta_1 L_1)] \cdot [\cos^2(\zeta_2 L_2) + (\psi_2/\zeta_2)^2 \sin^2(\zeta_2 L_2)], \tag{6.39c}$$

and

$$I_{11} = I[(\xi_1/\zeta_1)^2 \sin^2(\zeta_1 L_1)][(\xi_2/\zeta_2)^2 \sin^2(\zeta_2 L_2)]. \tag{6.39d}$$

For Bragg incidence, i.e., for $\alpha = -K_1/2k$ and $\beta = -K_2/2k$, the quantities ψ_1 and ψ_2 become zero. Therefore, Eqs. (6.39) reduce to

$$I_{00} = I \cos^2(\xi_1 L_1) \cos^2(\xi_2 L_2), \tag{6.40a}$$

$$I_{01} = I \cos^2(\xi_1 L_1) \sin^2(\xi_2 L_2), \tag{6.40b}$$

$$I_{10} = I \sin^2(\xi_1 L_1) \cos^2(\xi_2 L_2), \tag{6.40c}$$

and

$$I_{11} = I \sin^2(\xi_1 L_1) \sin^2(\xi_2 L_2), \tag{6.40d}$$

respectively. From Eqs. (6.40), it is clear that the incident light is divided equally among the four diffracted waves if $\xi_1 L_1 = \xi_2 L_2 = \pi/4$, i.e., if $(\Delta n)_1 L_1 = (\Delta n)_2 L_2 = \pi\gamma n/2k$. From Eq. (6.9) we note that while the (0,0)-order wave proceeds in the same direction as the incident light, the (0,1)-order wave appears to be reflected from the zx plane, the (1,0)-order wave appears to be reflected from the yz plane, and the (1,1)-order wave appears to be reflected from both the zx and yz planes.

The frequency of these waves is given by Eq. (6.8) by appropriately substituting for ℓ and m.

From Eqs. (6.40) we note that if $\xi_1 L_1 = \xi_2 L_2 = \pi/2$, then $I_{11} = I$, i.e., all the incident light is diffracted into the (1,1)-order wave. For small deviations from Bragg incidence, a reasonably large fraction of the incident light will be diffracted into this wave. According to Eq. (6.9), the direction of propagation of this wave depends on the acoustic wave numbers. Moreover, according to Eqs. (6.32) and (6.35), its phase depends on the acoustic phases. Thus by varying the acoustic frequencies and phases, the direction and phase of the (1,1)-order wave can be controlled. This result will be considered for real-time correction of wavefronts distorted by atmospheric turbulence in Chapter 7.

6.5. Mixed Raman-Nath and Bragg Diffraction

Mixed Raman-Nath and Bragg diffraction can be observed if one of the two orthogonal sound waves causes diffraction in the Raman-Nath region and the other in the Bragg region. Specifically, let the sound waves traveling along the x and y axes cause diffraction in the Raman-Nath and Bragg regions, respectively. Then if the light is incident in a direction such that $\alpha = 0$ and $\beta = \pm K_2/2k$, mixed Raman-Nath and Bragg effects will be observed. If, however, the light is incident normally to both sound waves, i.e., if $\gamma = 1$, Bragg effects will disappear and diffraction due to one wave only will be observed. Similarly, Raman-Nath effects will disappear whenever $\alpha = 2\gamma q/K_1 L_1$, where q is a nonzero integer.

CHAPTER 7

REAL-TIME CORRECTION OF OPTICAL WAVEFRONTS DISTORTED BY ATMOSPHERIC TURBULENCE

When an object in space is imaged by a ground-based telescope, some of the object detail is lost owing to distortion of the light waves by atmospheric turbulence. Since the turbulence is random in space as well as in time, an optical wavefront emanating from an object point is distorted, as it traverses the atmosphere, by different amounts as a function of position on the wavefront, and the distortions change independently as a function of time. If these distortions could be removed before the wavefronts reach the telescope, a considerably improved image would be obtained. It is proposed here that the phenomenon of Bragg diffraction of light by sound waves can be used to correct the distorted wavefront in real time if these wavefronts are allowed to pass through an array of Bragg cells before arriving at the telescope (Mahajan and Gaskill, 1974a).

7.1. Bragg Diffraction of Light by a Sound Wave

As in Chapter 5 and indicated in Fig. 7.1, we consider a plane wave of light of amplitude U, frequency ω, and wave vector $\vec{k} = (-k \sin\theta, 0, k \cos\theta)$ traveling in a transparent medium of refractive index n. When a sound wave of high frequency Ω, wave number K, phase constant Φ, and width L, traveling along the x axis with a velocity V, is generated

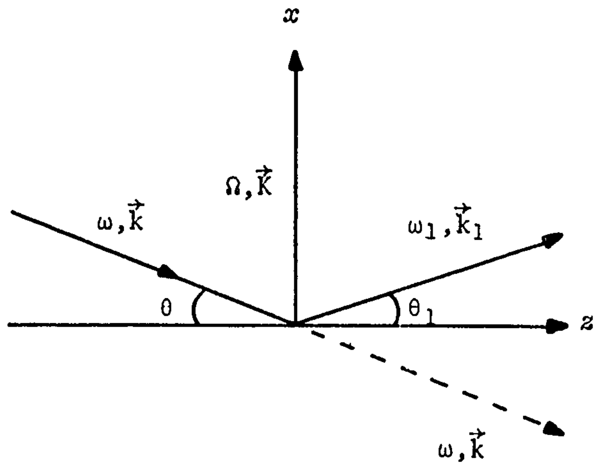


Fig. 7.1. Bragg Diffraction of Light of Frequency ω and Wave Vector $\vec{k} = (-k \sin\theta,\ 0,\ k \cos\theta)$ by a Sound Wave of Frequency Ω and Wave Vector $\vec{K} = (K,\ 0,\ 0)$.

The frequency and wave vector of the diffracted wave are given by $\omega_1 = \omega + \Omega$ and $\vec{k}_1 = \vec{k} + \vec{K}$, respectively.

in the medium, its refractive index in the region of the sound wave can be written

$$n(x,t) = n + \Delta n \sin(\Omega t - Kx + \Phi), \tag{7.1}$$

where Δn is the amplitude of the index wave produced by the sound wave. The light field incident on the sound wave is given by

$$E_i = \tfrac{1}{2} U \exp i(\omega t - \vec{k} \cdot \vec{r}) + \text{c.c.}, \tag{7.2}$$

where c.c. denotes the complex conjugate. If θ is approximately equal to the Bragg angle Θ, where

$$\sin\Theta = K/2k, \tag{7.3}$$

a part of the light is Bragg diffracted. The diffracted field, after emerging from the sound beam, is given by

$$E_1 = \tfrac{1}{2}U_1 \exp i(\omega_1 t - \vec{k}_1 \cdot \vec{r}) + \text{c.c.}, \tag{7.4}$$

where the amplitude, frequency, and wave vector of the diffracted wave are given by

$$U_1 = U(\xi/\zeta) \sin(\zeta L) \exp i(\pi + \Phi + \psi L), \tag{7.5}$$

$$\omega_1 = \omega + \Omega, \tag{7.6}$$

and

$$\vec{k}_1 = (K - k \sin\theta,\ 0,\ k \cos\theta), \tag{7.7}$$

respectively. In Eq. (7.5), the quantity ξ is a measure of the index change produced by the sound wave, and ψ is a measure of the deviation from the Bragg angle. These quantities are given by

$$\xi = \frac{\Delta n}{n} \frac{k}{2 \cos\theta} \tag{7.8}$$

and

$$\psi = \frac{K^2}{4k \cos\theta} \left[1 - \frac{2k}{K} \sin\theta\right], \tag{7.9}$$

respectively. The quantity ζ is given by

$$\zeta = (\xi^2 + \psi^2)^{\frac{1}{2}}. \tag{7.10}$$

The diffracted wave, according to Eq. (7.7), appears at an angle θ_1 where

$$\tan\theta_1 = \frac{K}{k \cos\theta} - \tan\theta. \tag{7.11}$$

The amplitude and therefore the irradiance of the diffracted wave are maximum when $\theta = \Theta$. Since the Bragg angle Θ is generally very small, the angle θ, which is approximately equal to Θ, is also very small. Accordingly, we may write $\cos\theta = 1$ and $\sin\theta = \theta$. Equations (7.8), (7.9), and (7.11) can then be written in the form

$$\xi = \pi \frac{\Delta n}{n\lambda}, \tag{7.12}$$

$$\psi = -\pi \frac{\Delta\theta}{\Lambda}, \tag{7.13}$$

and

$$\theta_1 = \Theta - \Delta\theta = \frac{\lambda}{2\Lambda} - \Delta\theta, \tag{7.14}$$

respectively, where λ and Λ are the optical and acoustic wavelengths and

$$\Delta\theta = \theta - \Theta. \tag{7.15}$$

Note that the diffracted wave appears at an angle smaller than the Bragg angle by the same amount as the angle of incidence is larger, which is consistent with the fact that the angle between the diffracted and undiffracted waves is twice the Bragg angle. The angles outside the medium are obtained by using Snell's law. Thus if the entrance and exit faces of the medium are perpendicular to the acoustic wavefronts, the angles outside the medium are larger than the corresponding angles inside by a factor of n, the refractive index of the medium. The frequency of the diffracted wave is upshifted from that of the incident light by the acoustic frequency.

If the light is incident from below the z axis, as shown in Fig. 7.2, the incident field can be written

$$E_i = \tfrac{1}{2}U \exp i(\omega t - \vec{k} \cdot \vec{r}) + \text{c.c.}, \tag{7.16}$$

where

$$\vec{k} = (k \sin\theta,\ 0,\ k \cos\theta). \tag{7.17}$$

Using

$$\vec{k}_\ell = (k \sin\theta + \ell K,\ 0,\ k \cos\theta), \tag{7.18}$$

it can be shown from Eq. (5.14) that the diffracted wave in this case corresponds to the $\ell = -1$ partial wave. Its field, after emerging from the sound beam, is accordingly given by

$$E_{-1} = \tfrac{1}{2}U_{-1} \exp i(\omega_{-1} t - \vec{k}_{-1} \cdot \vec{r}) + \text{c.c.}, \tag{7.19}$$

where

$$U_{-1} = U(\xi/\zeta) \sin(\zeta L) \exp i(-\Phi + \psi L), \tag{7.20}$$

$$\omega_{-1} = \omega - \Omega \tag{7.21}$$

and

$$\vec{k}_{-1} = (k \sin\theta - K,\ 0,\ k \cos\theta). \tag{7.22}$$

The diffracted wave appears at an angle θ_{-1}, where

$$\tan\theta_{-1} = \tan\theta - \frac{K}{k \cos\theta}, \tag{7.23}$$

or, for small angles

$$\theta_{-1} = \Delta\theta - \Theta = \Delta\theta - \frac{\lambda}{2\Lambda}. \tag{7.24}$$

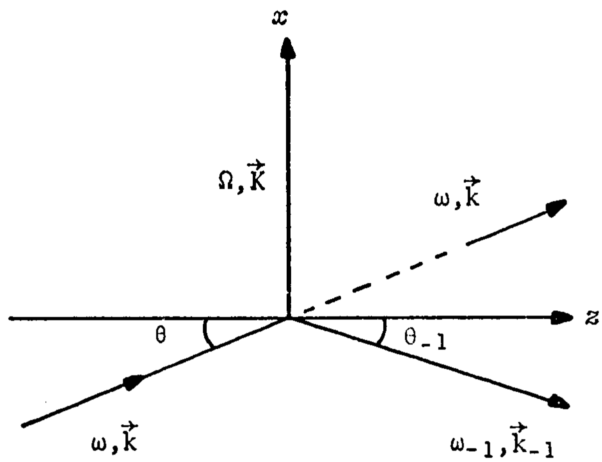


Fig. 7.2. Bragg Diffraction of Light of Frequency ω and Wave Vector $\vec{k} = (k \sin\theta,\ 0,\ k \cos\theta)$ by a Sound Wave of Frequency Ω and Wave Vector $\vec{K} = (K,\ 0,\ 0)$.

The frequency and wave vector of the Bragg diffracted wave are given by $\omega_{-1} = \omega - \Omega$, and $\vec{k}_{-1} = \vec{k} - \vec{K}$, respectively.

It is clear from Eq. (7.21) that the frequency of the diffracted wave is downshifted from that of the incident light by the acoustic frequency.

Thus, depending on whether the light is incident from above or below the z axis, keeping the direction of sound propagation fixed, the frequency of the Bragg diffracted wave is upshifted or downshifted from that of the incident light by the acoustic frequency, respectively. The diffracted wave appears in a direction corresponding to near reflection of the incident light from the acoustic wavefronts. For Bragg incidence, it corresponds to exact reflection.

7.2. Successive Bragg Diffraction of Light by Two Sound Waves of the Same Frequency but Different Wavelengths

Consider two sound waves of the same frequency Ω but different wavelengths Λ and Λ' present successively in a given medium or two different media. Since $\Omega = KV = 2\pi V/\Lambda$, two sound waves of the same frequency will have different wavelengths if their velocities of propagation are different. If they are traveling in a given medium (although different portions of it) they may be generated in such a way that their velocities of propagation (and therefore their wavelengths) are different. In an isotropic medium, for example, one of the sound waves may be longitudinal and the other transverse. If the sound waves travel in different media, their wavelengths will be different since their velocities of propagation will be different. Since the Bragg angle depends on the optical and acoustic wavelengths ($\Theta = \lambda/2\Lambda$), the two sound waves under consideration will have different Bragg angles for light of a given wavelength. We shall assume that the directions of propagation of the two sound waves make an angle equal to the difference between their Bragg angles. This has the effect that light diffracted by one sound wave at its Bragg angle is incident on the other sound wave at its Bragg angle, resulting in an optimum diffraction efficiency.

Let the index waves produced by the sound waves be given by

$$\Delta n(x,t) = \Delta n \sin(\Omega t - Kx + \Phi) \tag{7.25}$$

and

$$\Delta n'(x',t) = \Delta n' \sin(\Omega t - K'x' + \Phi'), \tag{7.26}$$

where Δn and $\Delta n'$ are the amplitudes of the index waves produced by sound waves of phase constants Φ and Φ', respectively. It should be clear that the angle between the x and x' axes, the directions of propagation of two sound waves, is equal to the difference between their Bragg angles. We shall assume that the sound wave of wave number K travels in a medium of refractive index n. Similarly, the sound wave of wave number K' travels in a medium of refractive index n'. Let the widths of the two sound waves be L and L', respectively.

When light is incident on the composite Bragg cell (Fig. 7.3), part of it is diffracted by the first sound wave and part of this diffracted wave is in turn diffracted by the second sound wave. The diffracted wave produced by diffraction of the incident light by both sound waves will be referred to as the diffracted-diffracted (d-d) wave. We shall describe the incident light and d-d wave in free space using the subscripts i and dd, respectively. Thus, for example, we shall denote the amplitude, wave number, wavelength, and angle of the incident light in free space by U_i, k_i, λ_i, and θ_i, respectively.

When light represented by field

$$\mathcal{E}_i = \tfrac{1}{2}U_i \exp i(\omega t - \vec{k}_i \cdot \vec{r}) + \text{c.c.}, \tag{7.27}$$

where

$$\vec{k}_i = (-k_i \sin\theta_i,\ 0,\ k_i \cos\theta_i) \tag{7.28}$$

is incident on the composite Bragg cell, the field of the d-d wave,

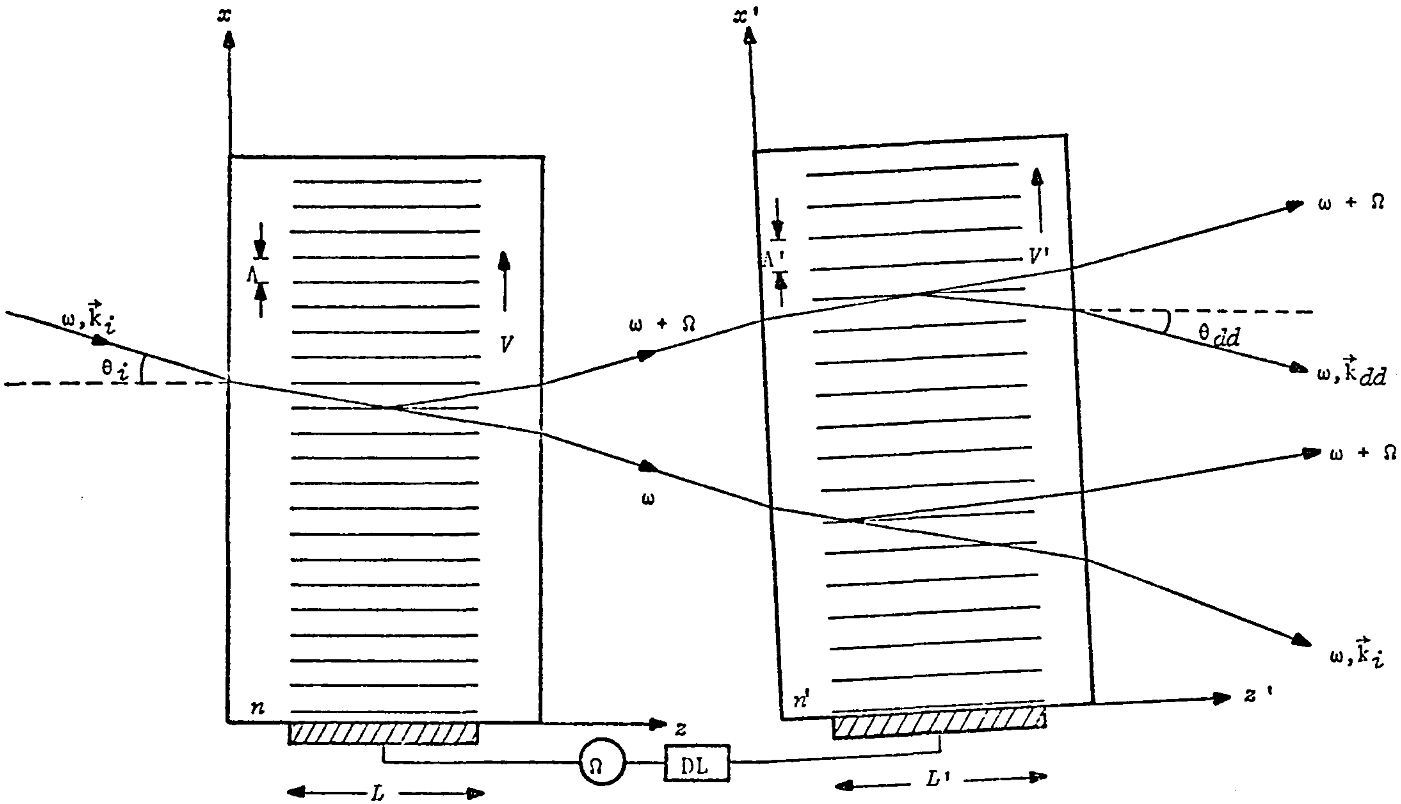


Fig. 7.3. Successive Bragg Diffraction of Light of Frequency ω and Wave Vector $\vec{k}_i = (-k_i \sin\theta_i, 0, k_i \cos\theta_i)$ by Two Sound Waves of the Same Frequency Ω but Different Wavelengths Λ and Λ'.

The sound waves have widths L and L' and propagate with velocities V and V' in media of refractive index n and n', respectively. The angle between their directions of propagation is equal to the difference in the Bragg angles corresponding to them. DL is a delay line with which the relative phase of the two sound waves can be varied. The wave vector of the d-d wave is $\vec{k}_{dd} = (-k_i \sin\theta_{dd}, 0, k_i \cos\theta_{dd})$. Its frequency and wavelength are the same as those of the incident light, respectively.

neglecting absorption and reflection, is given by

$$\mathcal{E}_{dd} = \tfrac{1}{2}U_{dd} \exp i(\omega t - \vec{k}_{dd} \cdot \vec{r} + \Phi - \Phi') + \text{c.c.}, \qquad (7.29)$$

where the amplitude U_{dd} and the wave vector $\vec{k}_{dd}$ are given by

$$U_{dd} = U_i[(\pi\Delta n/\lambda_i\zeta) \sin(\zeta L)][(\pi\Delta n'/\lambda_d\zeta') \sin(\zeta' L')]$$

$$\cdot \exp i\pi\left[1 - \Delta\theta_i\left(\frac{L}{n\Lambda} - \frac{L'}{n'\Lambda'}\right)\right] \qquad (7.30)$$

and

$$\vec{k}_{dd} = (-k_i \sin\theta_{dd}, 0, k_i \cos\theta_{dd}). \qquad (7.31)$$

In Eqs. (7.30) and (7.31)

$$\Delta\theta_i = \theta_i - \lambda_i/2\Lambda, \qquad (7.32)$$

$$\lambda_d = 2\pi c/(\omega + \Omega), \qquad (7.33)$$

$$\zeta = \pi[(\Delta n/\lambda_i)^2 + (\Delta\theta_i/n\Lambda)^2]^{\frac{1}{2}}, \qquad (7.34)$$

$$\zeta' = \pi[(\Delta n'/\lambda_d)^2 + (\Delta\theta_i/n'\Lambda')^2]^{\frac{1}{2}}, \qquad (7.35)$$

and

$$\theta_{dd} = \theta_i - \frac{\lambda_i}{\Lambda} + \frac{\lambda_d}{\Lambda'} . \qquad (7.36)$$

The quantity λ_d, given by Eq. (7.33), is the wavelength of the diffracted wave produced by the first sound wave. Since $\Omega << \omega$, we may write $\lambda_d = \lambda_i$ where $\lambda_i = 2\pi c/\omega$.

From Eq. (7.29) we note that the d-d wave has the same frequency, and therefore the same wavelength, as that of the incident light. It is therefore coherent with the incident light. Its phase depends on the relative phase $\Phi - \Phi'$ of the two sound waves. According to Eq. (7.36), it appears at an angle θ_{dd} that is different from the angle of incidence θ_i by twice the difference in the Bragg angles of the two sound waves. Since the Bragg angles can be varied by varying the acoustic frequency, the direction of propagation of the d-d wave can be held fixed if the direction of the incident light changes. For a change $d\theta_i$ in the angle of incidence θ_i, the change in acoustic frequency required to keep θ_{dd} fixed is, according to Eq. (7.36), given by

$$df = \frac{VV'}{\lambda_i(V' - V)} \, d\theta_i, \tag{7.37}$$

where $f = \Omega/2\pi$ (the acoustic frequency in hertz), V and V' are the velocities of propagation of the two sound waves, and we have written $\lambda_d = \lambda_i$. Thus by varying the frequency and relative phase of the two sound waves, the direction and phase of the d-d wave can be controlled. By generating two more sound waves of a given frequency, one propagating along the y axis in the first medium and the other along the y' axis in the second medium, the direction and phase of the wave diffracted by all four sound waves can be controlled for arbitrary tilts and phase changes of the incident light.

7.3. Real-Time Wavefront Correction

An optical wavefront emanating from a point on an object in space is distorted as it traverses the atmosphere. A ground-based telescope receiving the distorted wavefronts forms a distorted image of the object. If the wavefronts could be corrected before they reach the telescope, an improved image would be formed. The Bragg diffraction of light by sound waves can be used to achieve this task.

Let us divide a distorted wavefront into small sections such that each section, referred to as a subwavefront, is planar. Owing to the random nature of the atmospheric turbulence, the direction of propagation and relative phase of each subwavefront change independently of the changes in other subwavefronts. If a subwavefront is allowed to pass through a Bragg cell, the direction of propagation and the relative phase of the corresponding d-d wave can be held fixed, as the orientation and relative phase of the subwavefront change, by varying the frequencies and phases of the sound waves in the cell. Thus if a distorted wavefront is intercepted by an array of Bragg cells, the wavefront formed by the d-d waves will represent a corrected wavefront.

For an object of finite size, the angle of incidence θ_i at a Bragg cell has a finite range. From Eq. (7.30) we note that the d-d wave has a phase shift that depends on the angle of incidence. This phase shift given by

$$\pi\Delta\theta_i\left(\frac{L}{n\Lambda} - \frac{L'}{n'\Lambda'}\right)$$

can, however, be made very small. Let the angle subtended by the object on the telescope be approximately 10^{-4} rad. If the light from the center of the object is incident at the Bragg angle, the maximum value of $\Delta\theta_i$ is 5×10^{-5} rad. Letting $L = L' = 2.5$ cm, $n = n' = 2$, $V = 4 \times 10^5$ cm/sec, $V' = 5 \times 10^5$ cm/sec, and $f = 100$ MHz, we obtain a maximum phase shift of 10^{-2} rad. This phase shift is very small and may be neglected. From Eq. (7.37) we note that if the atmosphere produces a maximum tilt of a subwavefront of 5×10^{-5} rad, the corresponding change in acoustic frequency required to keep the d-d wave direction fixed is about 1.66 MHz for light of 600 nm wavelength. It may be added that the Bragg angles for the light and acoustic frequencies under consideration are less than 10^{-2} rad. For deviations of 10^{-4} rad from the Bragg angle, $\psi L = -\pi L(\Delta\theta_i/n\Lambda) < -0.1$ and, therefore, from Fig. 5.1 we note that the diffraction efficiency is very nearly equal to the corresponding Bragg value.

It should be clear from the above that if a distorted wavefront is divided into subwavefronts, each subwavefront being planar and 2.5 x 2.5 cm or less in size, it can be corrected if it is incident on an array of Bragg cells. The distortions in a wavefront are determined by the methods used in optical testing, such as the Hartmann test. The information on the distortions is supplied to the cell array, and corrections are made by varying the frequencies and phases of the various sound waves. The distortions produced by the atmospheric turbulence are assumed to be linear over a subwavefront area and constant in time over a period of a few milliseconds. Since the electrical signals applied to the transducers (generating the sound waves) can be changed in a few microseconds, the wavefronts can be corrected in real time. Moreover, for small subtense angles under consideration, we may assume that the atmosphere forms a linear shift invariant system with respect to the light propagating through it over a subwavefront area. Under this assumption, light waves emanating from different object points in different directions are distorted identically by the atmosphere, and therefore corrected simultaneously by the Bragg cell array.

CHAPTER 8

EXPERIMENTS ON THE DIRECTION AND PHASE OF A BRAGG DIFFRACTED LIGHT BEAM

The real-time wavefront correction of a distorted wavefront is based upon the direction and phase dependence of a Bragg diffracted light beam on the frequency and phase, respectively, of the sound wave causing the diffraction. The dependence of direction on the frequency can be observed by observing the movement of the diffracted beam as the acoustic frequency is varied. The phase shifts can be observed through the displacement of a fringe in an interference pattern formed by two beams whose relative phase depends on the acoustic phase. Experiments carried out along these lines are described beginning with a brief description of the Bragg cell.

8.1. The Bragg Cell

The Bragg cell used for the experiments was the Zenith D-70R Acousto-Optic Light Deflector. The cell consists of a block of dense glass in which a longitudinal sound wave of velocity 3.8×10^5 cm/sec is generated by a phased array of 10 PZT transducers. The cell works at a central frequency of 70 MHz. The Bragg angle, $\lambda_i/2\Lambda$, for He-Ne laser light at this frequency is 5.8×10^{-3} rad, or 1/3 degree. The cell was mounted on a device with which it could be adjusted for Bragg incidence.

An electrical signal from a Hewlett Packard 608D Signal Generator was amplified with a Zenith WPA-40 Broad Band RF Linear Power Amplifier and applied to the transducer. The phase of the electrical signal and therefore that of the sound wave was varied with a Computer Devices DV575-5 Delay Line. The delay line is capable of producing a time delay of up to 25 nsec. A time delay of 14.3 nsec produces a change of 2π in the phase of a sound wave of 70 MHz frequency.

The signal generator, power amplifier, delay line, and the cell are shown in Fig. 8.1.

8.2. Direction of a Bragg Diffracted Light Beam

The experimental arrangement for observing a Bragg diffracted light beam is shown in Fig. 8.2a and sketched in Fig. 8.2b. He-Ne laser light from a Spectra Physics 155 laser was incident on the Bragg cell and diffracted by a sound wave of 79 MHz frequency. The diffracted and undiffracted light beams were recorded on a Polaroid High Speed Film Type 57 placed about 140 cm away from the light entrance face of the cell. The incident beam was then slightly tilted with the mirror M, and the point of entry at the cell was kept unchanged by lowering the laser. The film was translated, and the diffracted and undiffracted beams were recorded again. Finally, the frequency of the electrical signal was changed so that the diffracted beam moved to the same position as before the incident beam was tilted. The film was translated, and the diffracted and undiffracted beams were once again recorded. The set of six spots thus obtained is shown as an enlargement in Fig.

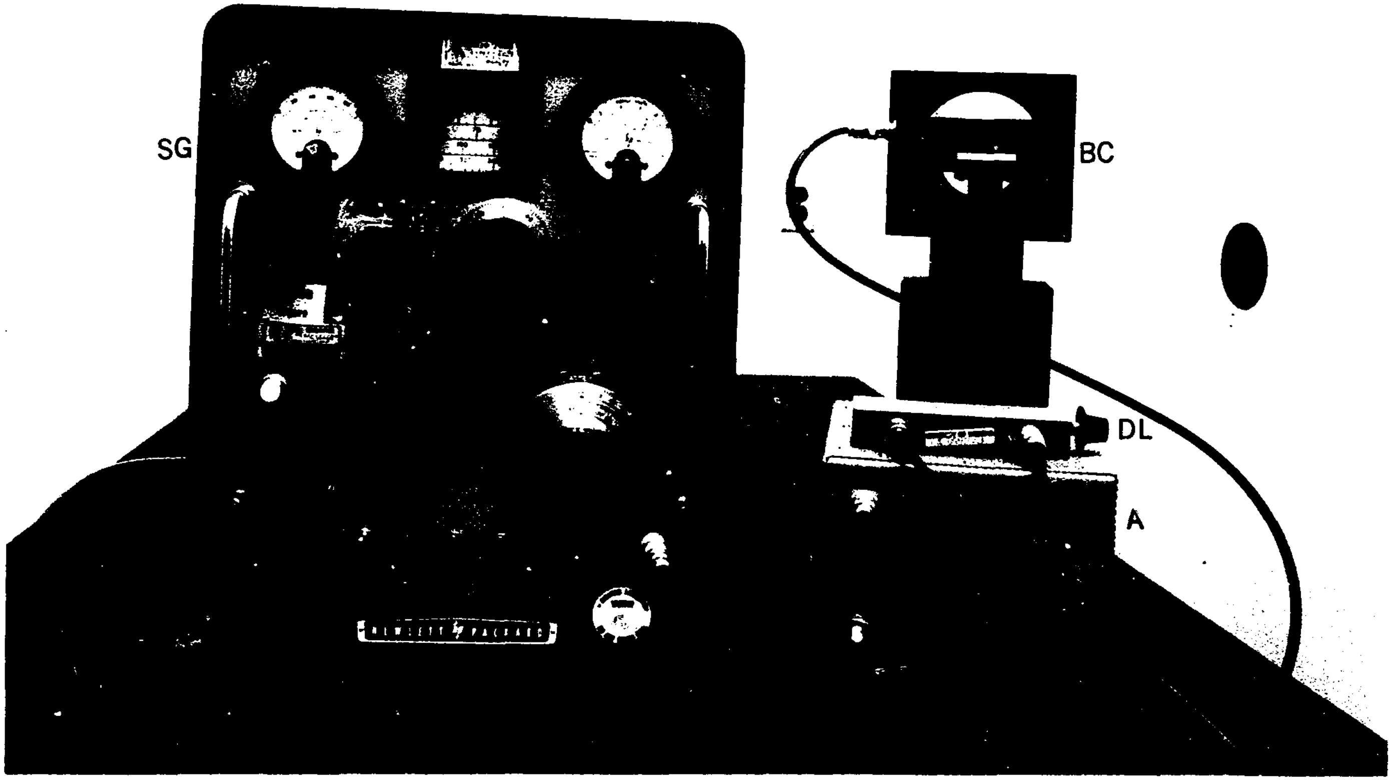


Fig. 8.1. Some of the Equipment Used for Experiments on the Direction and Phase of a Bragg Diffracted Light Beam.

SG (signal generator), A (amplifier), DL (delay line), BC (Bragg cell).

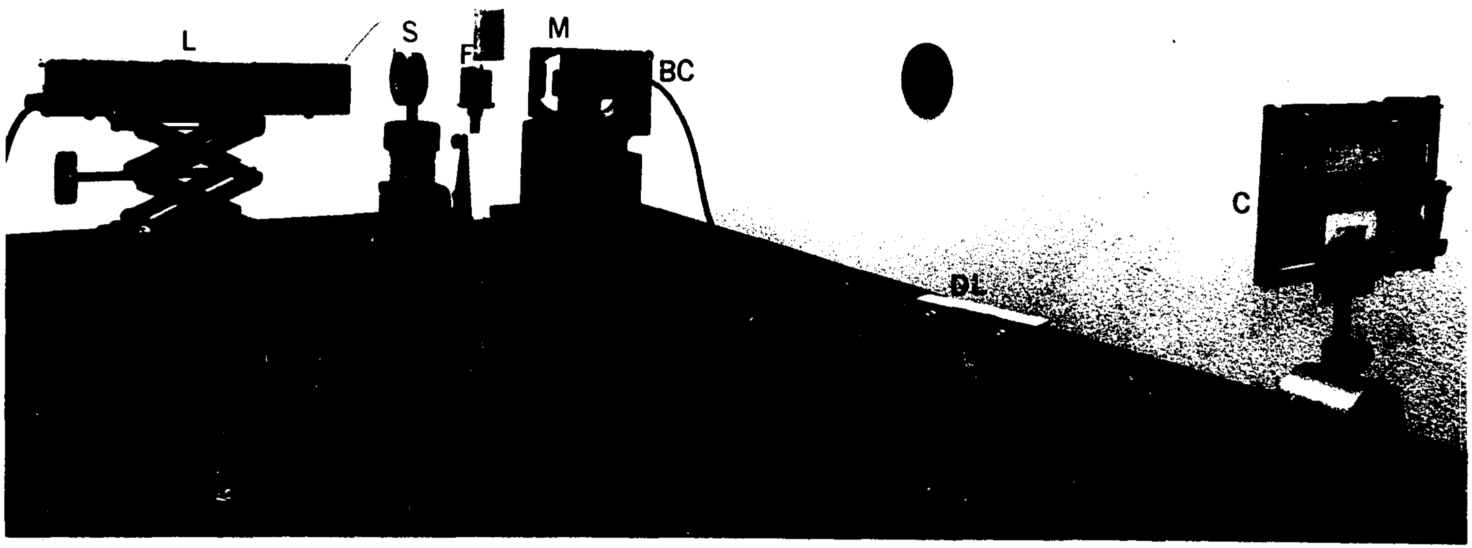


Fig. 8.2. Arrangement for Observing the Direction of a Bragg Diffracted Light Beam as a Function of the Acoustic Frequency.

(a) Experimental arrangement. L (laser), S (shutter), F (neutral density filter), M (mirror), BC (Bragg cell), C (camera), DL (delay line).

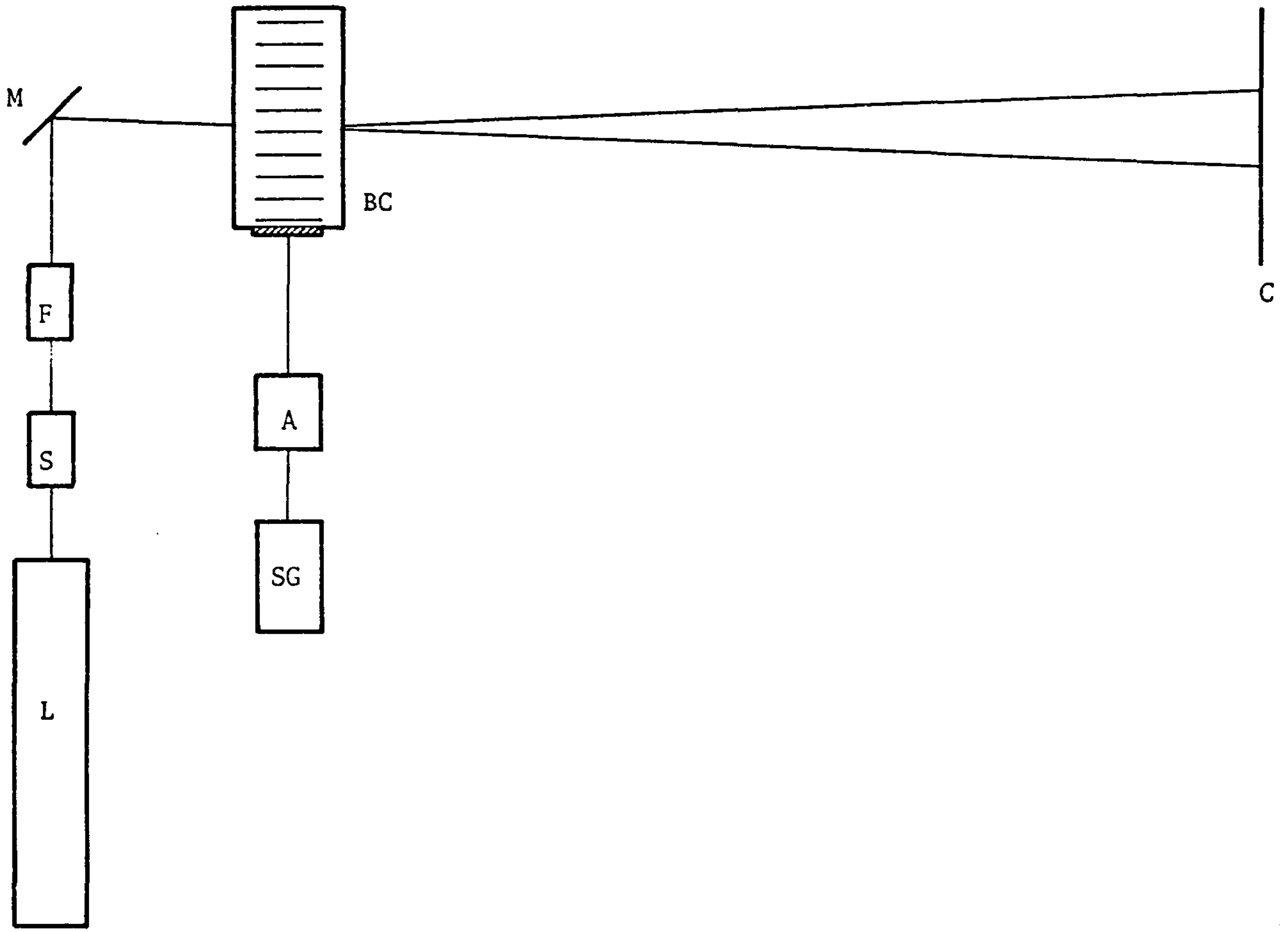


Fig. 8.2, continued.

(b) Schematic diagram. L (laser), S (shutter), F (neutral density filter), SG (signal generator), A (amplifier), BC (Bragg cell), C (camera), M (mirror).

8.3. This shows that the direction of the Bragg diffracted beam can be held fixed, as the direction of the incident beam changes, by varying the acoustic frequency.

The change in frequency required was 11 MHz, which within the experimental error is the same as expected from the relation

$$df = (V/\lambda_i)d\theta_i \tag{8.1}$$

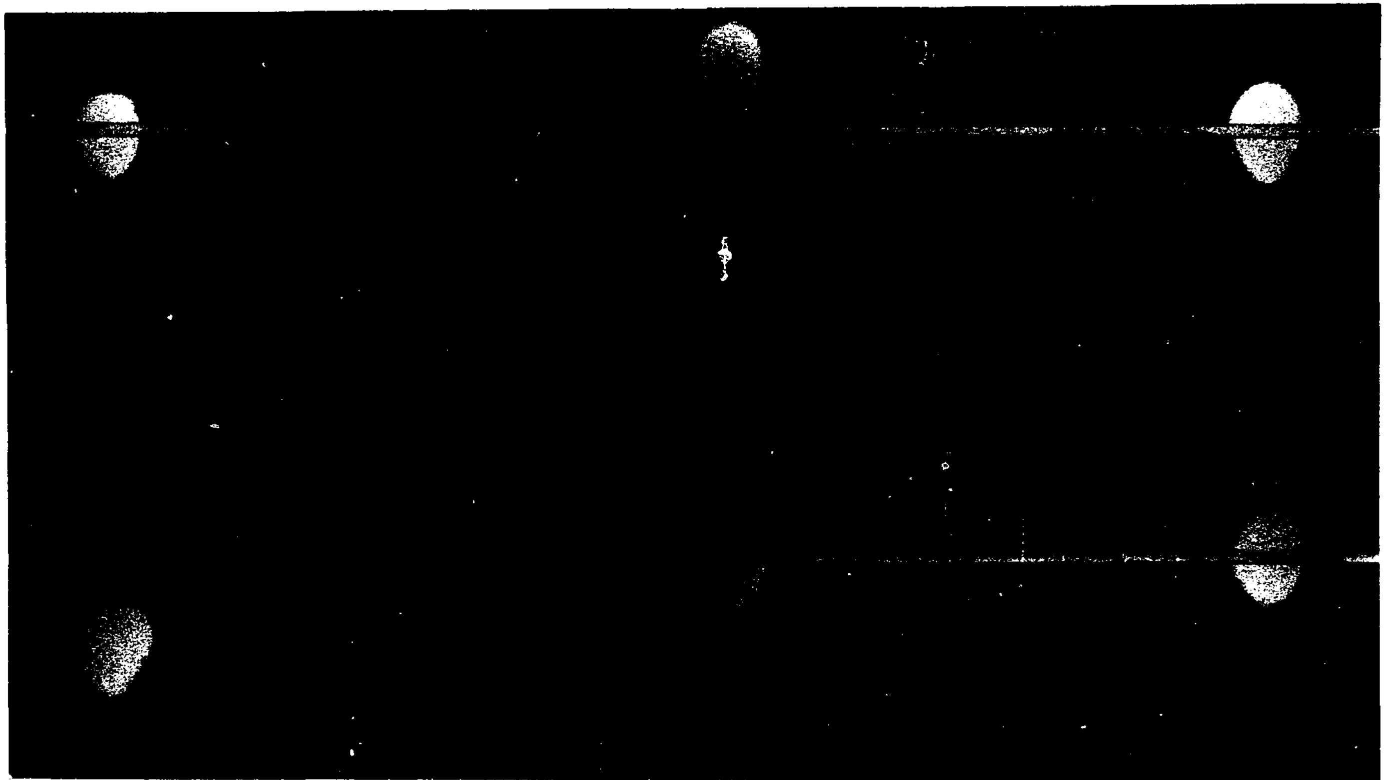
in which $V = 3.8 \times 10^5$ cm/sec, $\lambda_i = 633$ nm, and the tilt in the incident beam is $d\theta_i = 1.8 \times 10^{-3}$ rad. If a combination of two cells with sound waves of the same frequency but different wavelengths (or velocities) had been used, the change in frequency required to keep the direction of the d-d beam fixed would have been given by Eq. (7.37).

8.3. Phase of a Bragg Diffracted Light Beam

The dependence of the phase of a Bragg diffracted light wave on the acoustic phase was observed in several different ways.

(1) Two parallel beams derived from an He-Ne laser beam were incident on a Bragg cell at different points. The corresponding two diffracted beams were allowed to interfere, and an interference pattern was obtained. Since the two incident beams interacted with different parts of the cell sound wave, in general, the diffracted beams were shifted in phase by different amounts. Their relative phase, which is independent of the acoustic phase constant, was changed by varying the acoustic frequency according to

$$d\Phi = 2\pi(D/V)df, \tag{8.2}$$



(a) (b) (c)

Fig. 8.3. Effect of the Acoustic Frequency on the Direction of a Bragg Diffracted Light Beam.

(a) The diffracted and undiffracted beams for acoustic frequency of 79 MHz. (b) The two beams when the angle of incidence was slightly changed keeping the point of incidence at the Bragg cell unchanged. (c) The two beams when the acoustic frequency is 68 MHz. Note that the positions of the diffracted beams in (a) and (c) are at the same height indicating that they were parallel.

where $d\Phi$ is the change in the relative phase of the diffracted beams corresponding to a change df in the acoustic frequency and D is the separation of the incident beams. A fringe movement was observed, as expected, when the acoustic frequency was varied.

(2) In the second method two Bragg cells driven by electrical signals derived from the same signal generator were used. The relative phase of the electrical signals was changed by placing a delay line in one of them. An He-Ne laser beam was divided into two parts, and one part was incident on one cell and the other on the other cell. The cells were adjusted so that both produced diffracted beams of the same frequency. The two diffracted beams were allowed to interfere, and an interference pattern was obtained. A fringe movement was observed as the signal applied to one cell was delayed with the delay line.

(3) Finally, an He-Ne laser beam was successively diffracted by two similar Bragg cells. As in method 2, the cells were driven by electrical signals derived from the same signal generator. A delay line was placed in one of the two signals. The cells were adjusted so that the d-d wave produced by them had the same frequency as that of the incident light, or the undiffracted-undiffracted (u-u) wave. The d-d and the u-u waves were allowed to interfere, and an interference pattern was obtained. The experimental arrangement is shown in Fig. 8.4a and sketched in Fig. 8.4b. The interference pattern obtained is shown in Fig. 8.5. When the phase of one of the electrical signals was changed with the delay line, a corresponding shift in the fringes was observed. The lower part of Fig. 8.5a shows the fringe displacement

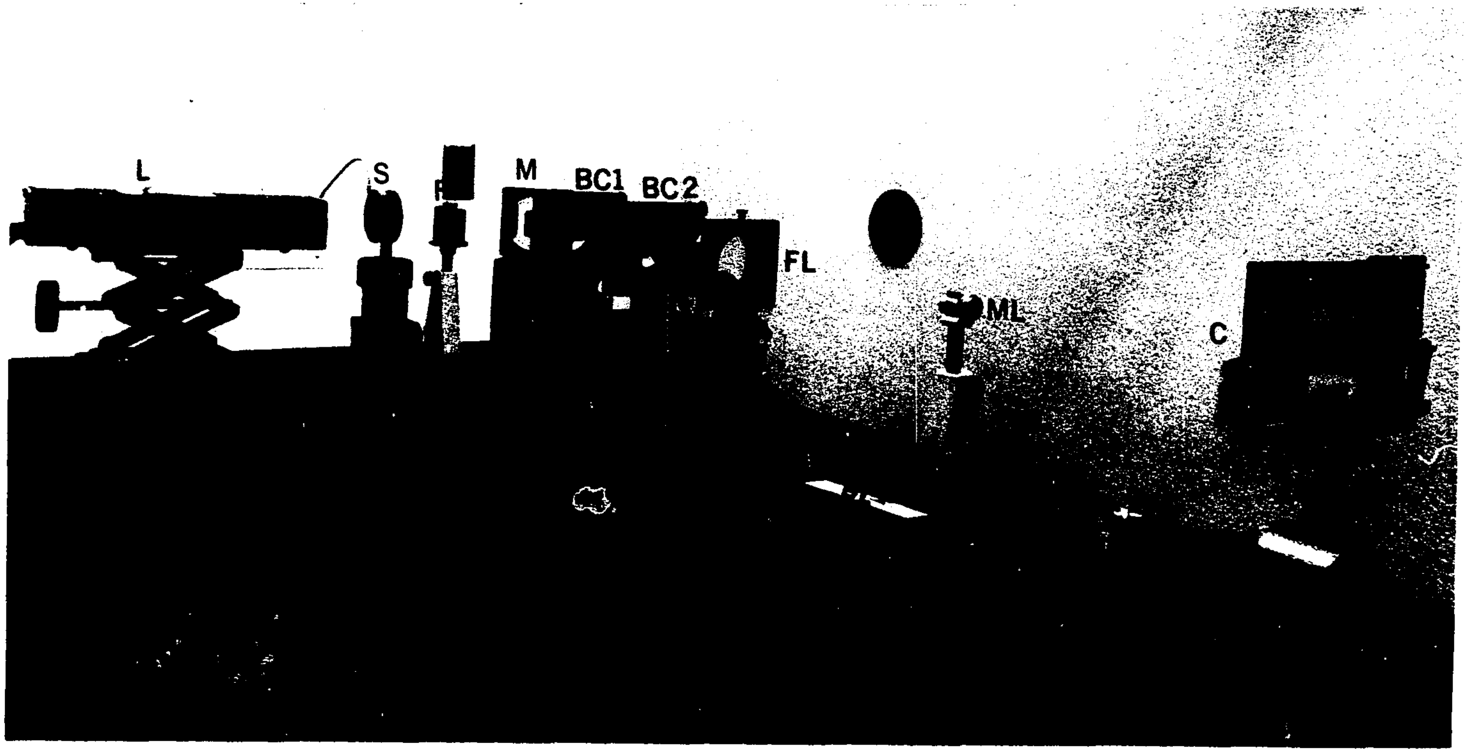


Fig. 8.4. Arrangement for Observing the Phase Dependence of a Bragg Diffracted Light Beam on the Phase of the Diffracting Sound Wave.

(a) Experimental arrangement. Light is successively diffracted by two Bragg cells BC1 and BC2, and the phase of the d-d wave is changed by varying the relative phase of the two sound waves with a delay line DL. L (laser), S (shutter), F (neutral density filter), M (mirror), FL (focusing lens), ML (magnifying lens), C (camera).

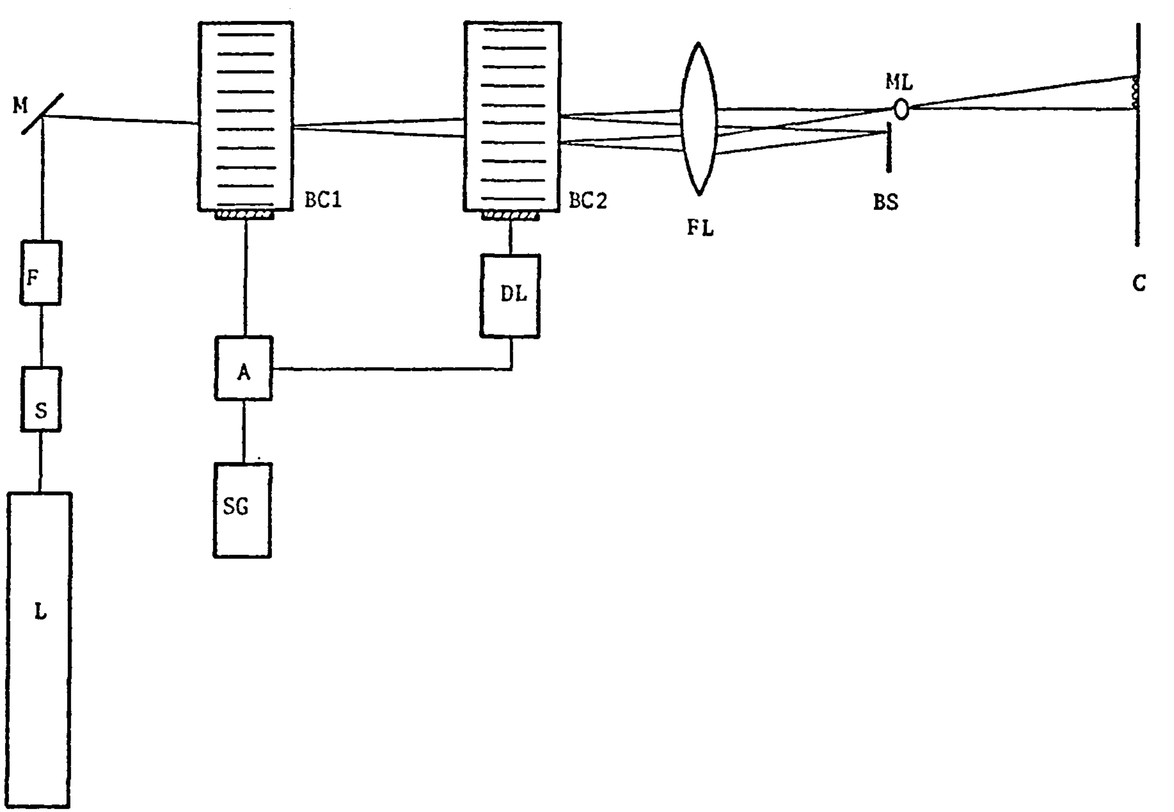


Fig. 8.4, continued.

(b) Schematic diagram. Light is successively diffracted by two Bragg cells BC1 and BC2, and the phase of the d-d wave is changed by varying the relative phase of the two sound waves with a delay line DL. L (laser), S, (shutter), F (Neutral density filter), M (mirror), FL (focusing lens), ML (magnifying lens), C (camera), A (amplifier), SG (signal generator), BS (blocking screen, actually a part of the ML holder).

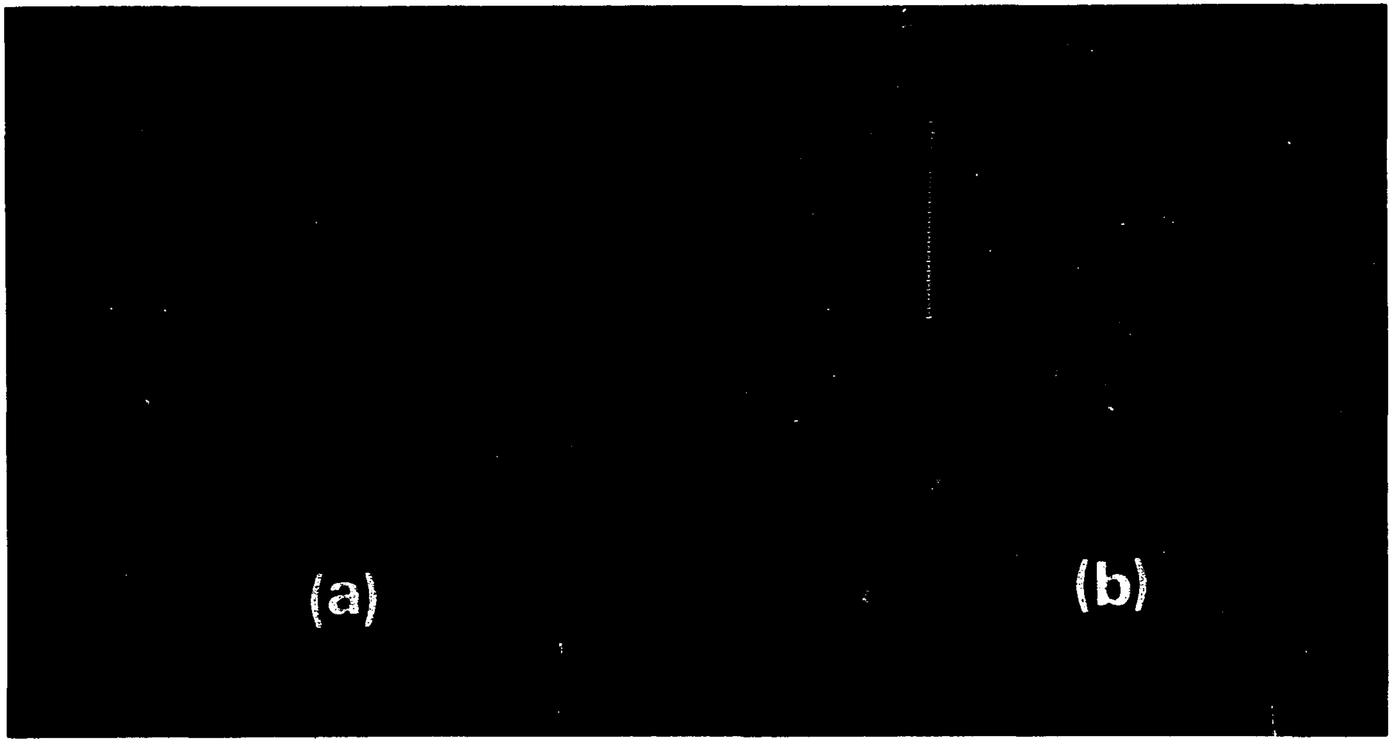


Fig. 8.5. Effect of the Acoustic Phase on the Phase of a Bragg Diffracted Light Beam.

(a) Fringe pattern obtained by the interference of the d-d and the u-u waves. The lower and upper parts indicate a fringe displacement when the relative phase of the two sound waves causing the diffraction was changed by about π.
(b) Fringe pattern obtained by the interference of the d-u and u-d waves.

corresponding to a phase change of about π. Figure 8.5b shows similar observations with the remaining two beams, the diffracted-undiffracted (d-u) and the undiffracted-diffracted (u-d) waves. The fringe pattern of this figure moved as a whole, due to the change in the direction of the beams, when the acoustic frequency was varied.

CHAPTER 9

CONCLUSIONS AND RECOMMENDATIONS

The theory of diffraction of light by a sound wave, two orthogonal sound waves, or two nearly parallel sound waves present successively, has been given in detail. It has been shown, in principle, that a wavefront distorted by atmospheric turbulence can be corrected in real time if it is allowed to pass through an array of Bragg cells, each cell carrying four sound waves.

It has been shown theoretically that the direction and phase of a d-d wave, produced by successive diffraction of a light beam by two nearly parallel sound waves of the same frequency but different wavelengths, can be controlled by varying the frequency and the relative phase of the two sound waves, respectively. For optimum interaction, the sound waves travel in directions making an angle with each other equal to the difference in the Bragg angles corresponding to them. The Doppler shifts in the frequency produced by them have the same magnitude, equal to the acoustic frequency, but opposite signs. Therefore, the d-d wave and the incident light have the same frequency and are, consequently coherent with each other.

The phase dependence of the d-d wave on the relative phase of the two successive sound waves has been demonstrated experimentally using two similar cel.s, i.e., two sound waves of the same frequency and the same wavelength. As the phase of the electrical signal generating one of the two sound waves was varied by a certain amount, a corresponding fringe displacement in the interference pattern formed by the interference of the d-d and u-u waves was observed. A corresponding fringe displacement was also observed in the pattern obtained by the interference of the d-u and the u-d waves. Moreover, this second pattern moved as a whole when the acoustic frequency was varied, demonstrating the directional dependence of a Bragg diffracted beam on the acoustic frequency. Such a dependence was also demonstrated using one cell only.

Although no difficulty is anticipated, it is recommended that similar experiments be done with two cells in which sound waves of the same frequency but different wavelengths can be generated. By generating orthogonal sound waves, a second dimension may also be added. It is suggested that a signal generator with two separate outputs, each having a slightly different frequency from that of the other but identical fluctuation, be developed to see if the diffracted waves produced by the sound waves generated by these two signals would interfere giving a time-independent fringe pattern. Eventually, experiments should be carried out to correct a distorted wavefront by an array of Bragg cells in real time.

APPENDIX A

DOPPLER INTERPRETATION OF THE FREQUENCY SHIFTS OF LIGHT DIFFRACTED BY SOUND WAVES

When light is diffracted by a sound wave, the frequencies of the diffracted waves are shifted from the frequency of the incident light by an integral multiple of the acoustic frequency. These frequency shifts are interpreted as a relativistic Doppler effect due to the motion of the acoustic wavefronts, which act as the source of the diffracted waves (Mahajan and Gaskill, 1974d).

While describing diffraction of light by sound waves, Raman and Nath showed that for a sinusoidally varying sound wave, the frequency of the ℓth-order diffracted wave differed from the frequency of the incident light (the zero-order wave) by ℓ times the acoustic frequency (Raman and Nath, 1936a). Without elaborating, they attributed this frequency shift to the Doppler effect.

Quantum mechanically, the frequency and the wave vector of the ℓth-order diffracted wave are given by

$$\omega_\ell = \omega + \ell\Omega, \tag{A.1}$$

$$\vec{k}_\ell = \vec{k} + \ell\vec{K}, \tag{A.2}$$

where ω is the frequency and $\vec{k}$ the wave vector of the incident light, and Ω and $\vec{K}$ are similar quantities for the sound wave. Equations (A.1) and (A.2) represent the conservation of energy and momentum of a photon-phonon interaction, respectively. The integer ℓ represents the number of phonons involved and is positive or negative depending upon whether they are absorbed or emitted.

The measured frequency shifts were found to be in agreement with Eq. (A.1) by Cummins et al. (1963). Using their results, Adrion (1970) showed that when light is Bragg diffracted, the observed frequency shift (equal to the acoustic frequency) did not agree with the expected Doppler shift. He thus concluded that the shift in the frequency could not be called a Doppler shift. Later Auth (1971) and Askne (1971), who also considered Bragg diffraction, pointed out the error made by Adrion, and showed that the measured frequency shift was indeed a Doppler shift. All three considered the diffracted beam to be produced by a reflection of the incident beam from a moving mirror (which represented an acoustic wavefront).

It is clear that a mirror produces only one diffracted beam for a given incident beam. While the moving mirror analogy may be appropriate in the Bragg region of diffraction, where only one order of diffraction is observed, it cannot hold in the Raman-Nath and the transition regions of diffraction where several orders of diffraction are simultaneously observed. It is evident that if the frequency shifts are truly a Doppler effect, they must be so irrespective of the region of diffraction. The region of diffraction dictates the irradiance of the various diffracted beams but not their frequency. Moreover, in the rest frame of the mirror, the incident and the reflected waves have the same wavelength. In the rest frame of the acoustic wavefronts, however, the situation is different; outside the region of the sound wave the diffracted waves have the same frequency and the same wavelength as the incident wave, while inside they have the same frequency but different wavelengths, according to an equation similar to Eq. (A.2). It should be emphasized that inside the sound column, where the diffraction takes place, the temporal periodicity of the column gives rise to the wavelength shifts. Thus for a diffracting column that is spatially periodic only (e.g., a holographic record of a sinusoidal fringe pattern), there are no frequency shifts but the various orders have different wavelengths according to Eq. (A.2).

We now show that the frequency shifts of light diffracted by a sinusoidally varying sound wave can be interpreted as Doppler shifts due to the motion of the acoustic wavefronts irrespective of the region of diffraction. The mathematics is somewhat similar to that of Askne (1971), but the interpretation is different. From Fig. A.1, we see that the light waves scattered from successive acoustic wavefronts interfere constructively when

$$(2\pi/\lambda)\Lambda \sin\theta + (2\pi/\lambda_\ell)\Lambda \sin\theta_\ell = 2\pi\ell, \qquad \text{(A.3)}$$

i.e., when the phase difference is an integral multiple of 2π. Here λ and λ_ℓ are the wavelengths of the incident light and the ℓth-order diffracted wave inside the medium, respectively, and Λ is the acoustic wavelength. Equation (A.3) represents the x component of Eq. (A.2) and can be written in terms of wave numbers as

$$k \sin\theta + k_\ell \sin\theta_\ell = \ell K. \qquad \text{(A.4)}$$

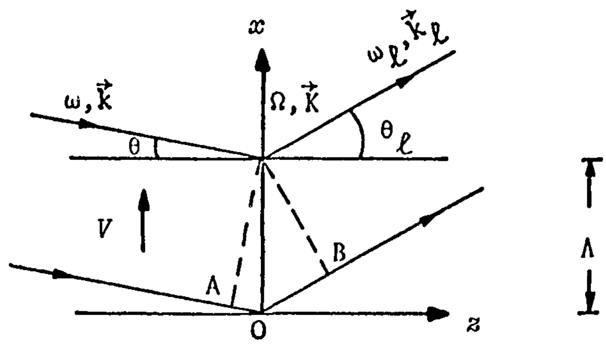


Fig. A.1. Scattering of Light from Moving Acoustic Wavefronts.

Light scattered from successive acoustic wavefronts interferes constructively when it makes an angle θ_ℓ with them where OA = Λ sinθ, OB = Λ sinθ_ℓ, and $k \cdot$ OA + $k_\ell \cdot$ OB = $2\pi\ell$.

According to Eq. (A.2) the z component of the k vectors of all the diffracted waves is the same. Because of the symmetry of the problem, it does not enter the condition for constructive interference, Eq. (A.3) or (A.4). Note that for a given angle of incidence θ, a diffracted beam appears at all angles θ_ℓ satisfying Eq. (A.3). The irradiance of these beams is obtained by solving the wave equation for the acousto-optic medium.

According to the special theory of relativity, when a light wave has a frequency ω and a wave vector $\vec{k}$ in one frame of reference, its frequency ω' in a second frame moving with a uniform velocity $\vec{V}$ relative to the first is given by

$$\omega' = \gamma(\omega - \vec{V} \cdot \vec{k}), \tag{A.5}$$

where $\gamma = (1 - V^2/v^2)^{-\frac{1}{2}}$ and v is the speed of light in the medium. Equation (A.5) is the relativistic Doppler formula relating the frequency ω measured by an observer to the frequency ω' of a source moving with a velocity $\vec{V}$ relative to the observer.

In the present problem the acoustic wavefronts that generate the various orders of diffracted light act as a source moving with a velocity $\vec{V}$ parallel to the x axis. Applying Eq. (A.5) to both the diffracted and the undiffracted (zero-order) waves, we obtain

$$\omega' = \gamma(\omega + Vk \sin\theta) \tag{A.6}$$

and

$$\omega_\ell' = \gamma(\omega_\ell - Vk_\ell \sin\theta_\ell), \tag{A.7}$$

respectively. But in the rest frame of the acoustic wavefronts, the frequency of each of the waves is the same,

$$\omega' = \omega_\ell'. \tag{A.8}$$

From Eqs. (A.6) through (A.8) we find that

$$\omega_\ell = \omega + V(k \sin\theta + k_\ell \sin\theta_\ell). \tag{A.9}$$

Then, using Eq. (A.4) and $\Omega = KV$, we get the desired result, Eq. (A.1):

$$\omega_\ell = \omega + \ell\Omega. \tag{A.10}$$

The above discussion can be extended to the diffraction of light by more than one sound wave by first considering the diffraction produced by a single sound wave, then finding the frequencies of the various diffracted waves, and finally determining the frequency shifts produced by a second sound wave acting on each of the diffracted waves, etc.

APPENDIX B

ACOUSTIC BEAM STEERING

Owing to the diffraction spread of the acoustic beam (assuming that the incident light is a plane wave), some light is diffracted even when it is incident at an angle slightly different from the Bragg angle. The range of the angle of incidence can be increased by using a narrow acoustic beam so that the acoustic energy is spread in a wide angular range. The scattering is evidently inefficient, but one could in principle supply the acoustic power needed to diffract a large fraction of the incident light. However, as the acoustic power is increased, its dissipation also increases. A large amount of power dissipation can set thermal gradients in the acoustic medium, which in turn can distort the light passing through it. The power requirements can be reduced if the transducer is designed to steer the radiated acoustic beam in such a way as to maintain the Bragg condition as the angle of incidence changes, i.e., if the acoustic wave direction can be changed as the angle of incidence changes so that the scattering is always at its optimum value.

To understand the theory of acoustic beam steering from a simple point of view, it is important to note that the angular distribution of acoustic power radiated from a transducer does not change (except for phase factors) as a function of distance from the transducer. Moreover, the amount of light diffracted at low acoustic powers (and consequently low diffraction efficiencies) does not depend on which part of the acoustic diffraction field the light traverses. Although the acousto-optic interaction usually takes place in the near acoustic field, analysis is convenient in the far field. Coquin, Griffin, and Anderson (1970) have shown that the numerical results obtained by solving Eq. (5.15) for the case of Δn and Φ dependent on z (multi-element phased array transducer described below) are only slightly different from those one would obtain from this type of analysis even when as much as 80% of the incident light is diffracted.

Acoustic beam steering can be achieved if the single-element transducer is replaced by a multi-element phased array transducer shown in Fig. B.1. The transducer elements are equally spaced by a distance S and are driven separately by equal amplitude signals whose phases are advanced, from left to right, by an amount Ψ per element. At a given time the wavefronts generated by the array form a staircase, which are equivalent in the far field of the array to planar wavefronts, making an angle θ_e with the transducer faces. Assuming that θ_e is small, one can write it in terms of Ψ as

$$\theta_e \simeq \tan\theta_e = \Psi/KS, \tag{B.1}$$

where K is the acoustic wave number.

If the incident light makes an angle θ with the plane of the array, the angle it makes with the effective acoustic planes is $\theta + \theta_e$. The incident light satisfies the Bragg condition if the angle $\theta + \theta_e$ is equal to the Bragg Θ. Accordingly we can define a beam steering error

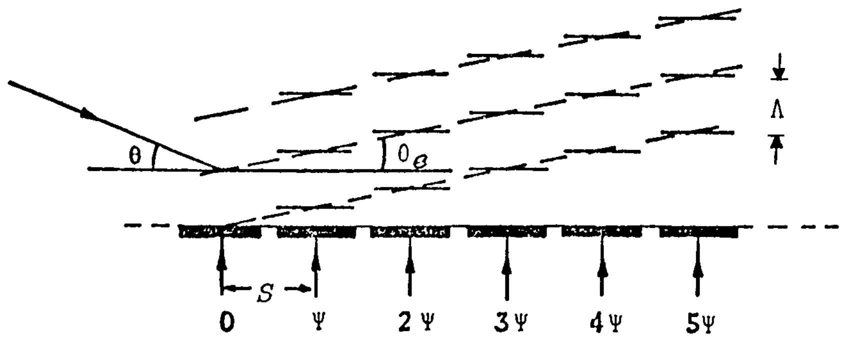


Fig. B.1. Multi-element Phased Array Transducer.

Solid lines indicate actual acoustic wavefronts and dashed lines indicate the effective acoustic wavefronts. Λ is the acoustic wavelength. When light is incident making an angle θ with the plane of the array, the effective angle of incidence is $\theta + \theta_e$. The angles shown are exaggerated for clarity.

θ_{se} as the difference between these two angles, i.e.,

$$\theta_{se} = (\theta + \theta_e) - \Theta = \theta + \theta_e - \frac{K}{2k}, \tag{B.2}$$

where k is the optical wave number and we have assumed that $\Theta \simeq \sin\Theta = K/2k$.

Suppose that initially $\theta_{se} = 0$ so that the Bragg condition is satisfied. If now θ changes, the direction of the diffracted light can be held fixed by changing the acoustic frequency by a proper amount. However, when the frequency is changed the Bragg condition is no more satisfied. It can be satisfied if the acoustic beam is steered by choosing Ψ properly so that the steering error is zero. Setting $\theta_{se} = 0$ in Eq. (B.2) and solving for the required perfect beam steering phase Ψ_p, we obtain

$$\Psi_p = KS\left(\frac{K}{2k} - \theta\right). \tag{B.3}$$

Note that Ψ_p is a quadratic function of the acoustic wave number or frequency. Coquin et al. (1970) have shown that a multi-element array steers the acoustic beam satisfactorily if the phase applied to each transducer is within $\pm\pi/4$ of the perfect beam steering phase at all frequencies. They find, for example, that with this type of steering the amount of light diffracted by a 10-element transducer is at the most 0.8 dB down from the case of perfect beam steering.

Some acoustic beam steering takes place even if Ψ is fixed. The factor π is an experimentally convenient choice for a fixed Ψ since it can be achieved by series interconnecting the elements (Gordon, 1966; Korpel, Adler, and Desmares, 1965). Such a transducer is called a first-order beam-steered transducer. Since a planar phased array emits radiation in two equal lobes inclined in opposite directions with respect to the array plane and only one of them can be used for Bragg diffraction, half of the acoustic power is wasted. This power can be utilized if the elements are placed on steps (like a blazed grating) because the energy in that case is radiated in a single lobe (Coquin et al., 1970; Korpel et al., 1966; Pinnow, 1971). The Bragg cell used for the experiments consists of ten elements placed in this manner.

APPENDIX C

DIFFRACTION OF LIGHT BY A STANDING SOUND WAVE

Recently diffraction of light by a standing sound wave, with emphasis on the frequency spectrum of a diffracted wave, has been studied (Mahajan and Gaskill, 1974e; Mahajan, 1974b). It has been shown that, whereas a traveling sound wave produces monochromatic diffracted waves, a standing sound wave produces diffracted waves such that light in any even (odd) order consists of waves whose frequencies are shifted from the frequency of the incident light by even (odd) multiples of the frequency of the component sound waves. These frequency shifts do not depend on the standing wave ratio or the region of diffraction. Moreover, the irradiance of light in any order is periodic in time with a period equal to half the period of the component sound waves. Below, we shall find the irradiances of the Raman-Nath and Bragg diffracted waves.

Consider a beam of light of frequency ω and wave vector $\vec{k} = (-k \sin\theta, 0, k \cos\theta)$ propagating in a nonconducting and nonmagnetic medium of refractive index n (see Fig. C.1). Let a general standing sound wave be generated in the medium producing index variations,

$$\Delta n(x,t) = \Delta n \sin(\Omega t - Kx) + a\Delta n \sin(\Omega t + Kx). \tag{C.1}$$

The first term on the right-hand side of Eq. (C.1) represents an index wave of peak amplitude Δn produced by a sound wave of frequency Ω and

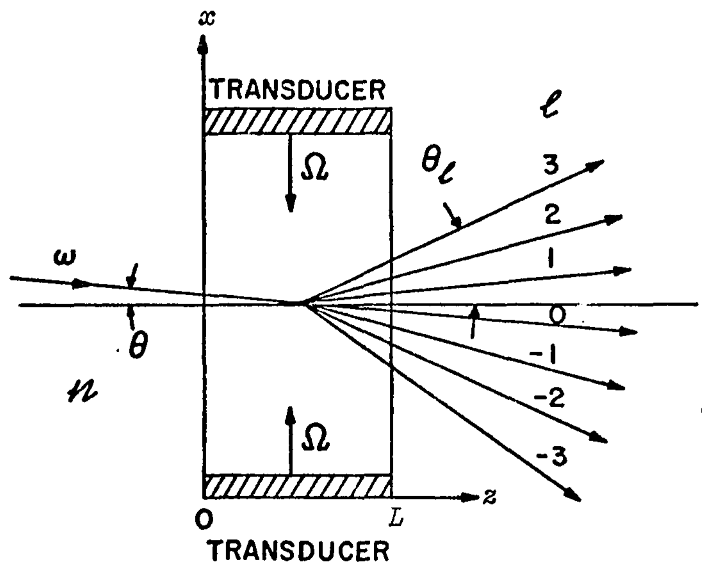


Fig. C.1. Diffraction of Light by a Standing Sound Wave.

wave number K traveling along the x axis. The second term represents an index wave of peak amplitude $a\Delta n$ produced by a sound wave traveling in the negative x direction. The sound waves are generated either by two separate transducers or one sound wave is generated by a transducer and the other is produced by a reflection of the first at the medium-air interface. The quantity a is called the standing wave ratio of the sound wave and in the latter case represents the ratio of the reflected and incident amplitudes.

If we separate the temporal and spatial dependence of the index variations, Eq. (C.1) can be written in the form

$$\Delta n(x,t) = \Delta n(t) \sin(\alpha - Kx), \tag{C.2}$$

where

$$\Delta n(t) = \Delta n(1 + a^2 - 2a \cos 2\Omega t)^{\frac{1}{2}} \tag{C.3}$$

and

$$\tan\alpha = \frac{1 + a}{1 - a} \tan\Omega t. \tag{C.4}$$

The refractive index of the medium in the region of the sound waves can be written

$$n(x,t) = n + \Delta n(t) \sin(\alpha - Kx). \tag{C.5}$$

Comparing Eq. (C.5) with Eq. (5.1) and proceeding as in Chapter 5, diffraction of light by a standing sound wave can be described. Thus, for example, the irradiance of the ℓth-order Raman-Nath diffracted wave emerging from the sound waves is given by

$$I_\ell(t) = IJ_\ell{}^2[\Delta\phi(t)], \tag{C.6}$$

where I is the time-averaged irradiance of the incident light and

$$\Delta\phi(t) = \frac{\Delta n(t)}{n} \frac{kL}{\cos\theta} \operatorname{sinc}\left(\frac{L}{\Lambda} \tan\theta\right). \tag{C.7}$$

As in Chapter 5, J_ℓ is the ℓth-order Bessel function of the first kind; L and Λ are the width and the wavelength of the sound waves. Note that the irradiance of a wave according to Eq. (C.6) is time dependent. It has been assumed in this equation that the faster variation at the optical frequency has been averaged out.

Similarly, it can be shown that the irradiance of the Bragg diffracted wave (assuming diffraction in the Bragg region and that θ is approximately equal to the Bragg angle) is given by

$$I_1(t) = I\frac{\xi^2(t)}{\xi^2(t) + \psi^2} \sin^2\{[\xi^2(t) + \psi^2]^{\frac{1}{2}}L\} \tag{C.8}$$

where

$$\xi(t) = \frac{\Delta n(t)}{n}\frac{k}{2\cos\theta} \tag{C.9}$$

and

$$\psi = \frac{K^2}{4k\cos\theta}\left(1 - \frac{2k}{K}\sin\theta\right). \tag{C.10}$$

The wave vector of a diffracted wave is given by Eq. (5.9).

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