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**THE CRANMER ABACUS:  
ITS USE IN TEACHING MATHEMATICS  
TO STUDENTS WITH VISUAL IMPAIRMENTS**

by

**Scott Isami Sakamoto**

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**A Dissertation Submitted to the Faculty of the**

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**In Partial Fulfillment of the Requirements  
For the Degree of**

**DOCTOR OF PHILOSOPHY**

**In the Graduate College**

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As members of the Final Examination Committee, we certify that we have read the dissertation prepared by Scott Isami Sakamoto

entitled THE CRANMER ABACUS: ITS USE IN TEACHING MATHEMATICS TO  
STUDENTS WITH VISUAL IMPAIRMENTS

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A handwritten signature in black ink, appearing to be 'S. J. ...', written over a horizontal line.

## TABLE OF CONTENTS

LIST OF FIGURES.....	6
LIST OF TABLES .....	7
ABSTRACT .....	8
<b>1 PRELIMINARIES .....</b>	<b>10</b>
1.1 Introduction .....	10
1.2 Purpose .....	12
1.3 Literature Review .....	17
1.3.1 History .....	17
1.3.2 The Cranmer Abacus .....	22
1.3.3 Alternatives .....	23
1.3.4 Research .....	26
1.4 Methodology .....	31
1.4.1 Methodology Introduction .....	31
1.4.2 The Survey .....	31
1.4.3 The Study .....	34
1.4.4 Notes on the Project .....	36
<b>2 THE SURVEY .....</b>	<b>37</b>
2.1 Training and Experience .....	37
2.2 The Use of Manuals .....	40
2.3 The Use of the IEP .....	42
2.4 Question 5 .....	43
2.5 Question 6 .....	45
2.6 Question 7 .....	50
2.7 Question 8 .....	55
<b>3 VIDEOTAPES AND ASSESSMENTS .....</b>	<b>61</b>
3.1 Student A and Teacher A .....	62
3.2 Student B and Teacher B .....	66
3.3 Student C and Teacher C .....	70
3.4 Student D1, Student D2, and Teacher D .....	73
3.5 Discussion .....	77

**TABLE OF CONTENTS- *Continued***

<b>4 SUMMARY AND CONCLUSIONS .....</b>	<b>82</b>
<b>4.1 General Discussion .....</b>	<b>82</b>
<b>4.2 Conclusions .....</b>	<b>90</b>
<b>APPENDIX A-- HUMAN/ANIMAL SUBJECTS APPROVAL .....</b>	<b>94</b>
<b>APPENDIX B- THE SURVEY .....</b>	<b>95</b>
<b>APPENDIX C- COVER LETTER .....</b>	<b>96</b>
<b>APPENDIX D- ASSESSMENT TOOL .....</b>	<b>97</b>
<b>APPENDIX E- VOLUNTEER SOLICITATION .....</b>	<b>104</b>
<b>APPENDIX F- PARENT PERMISSION .....</b>	<b>106</b>
<b>APPENDIX G- VIDEOTAPE PROTOCOL .....</b>	<b>107</b>
<b>REFERENCES .....</b>	<b>108</b>

**LIST OF FIGURES**

<b>FIGURE 1, The Cranmer abacus with the number 307 set .....</b>	<b>19</b>
<b>FIGURE 2, The soroban .....</b>	<b>19</b>
<b>FIGURE 3, The braillewriter .....</b>	<b>20</b>
<b>FIGURE 4, The numberaid.....</b>	<b>20</b>
<b>FIGURE 5, All peg settings for the Taylor slate.....</b>	<b>21</b>
<b>FIGURE 6, The cubarithm.....</b>	<b>21</b>

**LIST OF TABLES**

<b>TABLE 1, Experience .....</b>	<b>38</b>
<b>TABLE 2, Training.....</b>	<b>39</b>
<b>TABLE 3, College versus non-college training.....</b>	<b>39</b>
<b>TABLE 4, Likert scale questions.....</b>	<b>40</b>
<b>TABLE 5, Question 4 .....</b>	<b>41</b>
<b>TABLE 6, Question 5.....</b>	<b>44</b>
<b>TABLE 7, Question 6.....</b>	<b>46</b>
<b>TABLE 8, Question 7.....</b>	<b>51</b>
<b>TABLE 9, Question 8 .....</b>	<b>56</b>

## ABSTRACT

For more than a decade, little research has been done regarding the Cranmer abacus and its use. Attention to the Cranmer abacus and review of its potential as a learning aid has been minimal. Lack of recent abacus related study is addressed in this dissertation in two ways. First, some of the ideas previously examined from research conducted in the sixties, seventies, and early eighties is expanded on. Information is updated, and current facts and ideologies are summarized. Second, topics not mentioned in earlier studies are examined. Information pertaining to the Cranmer abacus was gathered in two ways.

1. A survey (Appendix B) was distributed to teachers in the United States who regularly teach mathematics with the abacus.
2. Part two was a semester long endeavor consisting of two components. A series of videotape sessions and student assessments (Appendix D) were analyzed.

The primary goal of this dissertation was to explore the present status of the Cranmer abacus' use in teaching mathematics to students with visual impairments in the United States.

The responses to the survey reveal that most teachers feel they are successfully teaching mathematics to visually impaired students with the abacus. The videotapes are further evidence that teachers are doing a good job, while not necessarily having an extensive mathematical background (teachers of the visually impaired are not required to take extra mathematics classes).

The four teachers who participated in the videotape study achieved varying degrees of success. Three cases resulted in nice progress throughout the semester. The fourth teacher had two students, one of which enjoyed little success. Generally speaking, all four teachers taught the abacus diligently. Also, these teachers' responses to the survey questions were common amongst the forty-five teachers who participated in the survey.

In reviewing both parts of this study, it is clear that teachers of the visually impaired are successfully incorporating the abacus into the curriculum for students with visual impairments. The only improvement that can be made is an increase in these teachers familiarity with mathematics education.

## 1 PRELIMINARIES

### 1.1 Introduction

In the early 1960's, T.V. Cranmer of the Kentucky School for the Blind created a tool called the Cranmer abacus (figure 1). He designed it for students with visual impairments to learn and practice mathematics. Actually, the design is an adaptation of the Japanese soroban. The soroban is one of many variations of the abacus. Chinese and Russian variations are discussed later. Relatives of the abacus date back thousands of years. The abacuses' evolution continued through medieval times and the late nineteenth century. Each country and each time period had its own procedures for doing arithmetic with the abacus (Moon, 1971). The form Cranmer chose to adapt was the soroban. The reasons for this choice will be discussed shortly.

The soroban is an arrangement of beads and rods used for centuries by Japanese merchants, businessmen, and many other members of society for mathematical purposes (Moon, 1971). There are thirteen columns of rods stabilized in a rectangular frame. Each rod contains five beads, where the top bead is separated from the rest by a dividing bar located near the top of the device. The four beads below the bar (unit counters) have a value of one, while the single bead above the bar (five counter) is worth five units. Each counter is said to be set if it is as close as possible to the dividing bar (unit counters all the way up and or five counter all the way down), and cleared if they are as far away from the bar as possible. Each rod represents a place value coinciding with written numbers. The rod to the farthest right is the units position, the next column to its left is the tens column, and so on. To illustrate, if a five bead and two

unit beads were set in the ones column while three unit counters were set in the hundreds (third from the right) column, you would read the number 307 (see figure 1). Operations such as addition, subtraction, multiplication, and division are simple manipulations on the soroban. These may be performed on whole numbers, fractions, and decimals. The Japanese have used the soroban for hundreds of years. Furthermore, in the nineteenth century they began experimenting (successfully) with its use in teaching mathematics to students with visual impairments (Moon, 1971). There were a few inherent problems, however, with its design causing difficulties in its use.

These concerns were addressed in Cranmer's adaptation, giving it immediate attention during research in the sixties and seventies. One of the problems was the instability of set counters. The beads moved too easily upon touch or shifting. Since the blind need to touch the abacus to read it, work became unintentionally erased or altered. As a solution, Cranmer placed a rubber matting behind the beads, resulting in a sturdier environment and more accurate calculations. Set beads could now withstand touch, shifting, and even dropping to the floor. Each bead always maintains contact with the matting, keeping it in place after being set or cleared.

The second and final alteration was to its size. The Cranmer abacus has standard, pocket-sized dimensions. While still having the calculating properties of the soroban, now the abacus was portable and stable. With these two design changes, the Cranmer abacus became a tool of great promise (Lewis, 1969).

For the next twenty years or so, the abacus received ample attention in research articles (discussed later). Mae Davidow in the sixties, Marian Lewis in the seventies,

and Arthur Steinbrenner in the early eighties are but a few of the people who did research involving the Cranmer abacus. In the mid-eighties, however, research on the abacus stopped. Only incidental references appear in articles regarding some other topic pertaining to the visually impaired. For example, Rapp, in 1992 reported the confusion the abacus causes some secondary school teachers, while the article dealt mainly with other topics (Rapp, 1992). But no research in the last ten to fifteen years focuses directly on the use of the Cranmer abacus.

### 1.2 Purpose

The lack of recent research is the motivating factor for the writing of this dissertation. It is difficult to know the current status of the Cranmer abacus. The surveys and videotapes illuminate the ideas teachers of the visually impaired have relating the using the abacus with students with visual impairments.

This paper is a simple exploration of the use of the Cranmer abacus. The thoughts of current teachers of the visually impaired (VI teachers) are shared in this report. The prominence of the abacus is also examined. There are two primary goals for this dissertation. The first is to inform VI teachers about the current use of the Cranmer abacus. The second is to initiate future research pertaining to the Cranmer abacus.

Many students have difficulties with mathematics (not just students with visual impairments). Imagine trying to learn mathematics without the luxury of sight. If the Cranmer abacus has the potential reported in previous literature (Lewis, 1969), every effort should be made to improve all aspects of teaching the abacus. Research could help students with visual impairments gain better mathematical experiences.

Mathematical restrictions need not be placed on students in early elementary grades. For this reason it is important for teachers to be well prepared in both content and pedagogy as they pertain to teaching with the abacus.

In order to analyze the information gathered with the surveys and videotapes and assessments, a few questions have been listed. These questions are the foundation for developing ideas in this report. Any theories or conclusions will be based upon the answers to the following questions.

*How much formal training do current teachers of the abacus have?*

Teacher training on the Cranmer abacus is mentioned often as an important research topic (Lewis 1969: Lewis 1970: Brothers, 1972: Steinbrenner and Becker, 1982). In 1982, Steinbrenner and Becker found teacher training in educational institutions to be incomplete and ineffective (Steinbrenner and Becker, 1982). Also, they noted that sixty-three percent of the teachers using the abacus responding to a survey conducted in 1982 were self- taught. Has training evolved or remained as Steinbrenner described? This is one question discussed in this dissertation.

*In which grade do teachers introduce the abacus to their students?*

Another question deals with the grade in which teachers choose to begin teaching the abacus to their students. Grade was chosen over age due to the variations in cognitive development among blind children (Shea and Bauer, 1994). One student of age six may have progressed successfully into the first grade, while another may still be struggling through kindergarten or preschool. It has been reported that many teachers

begin abacus instruction in early elementary grades (Lewis, 1970; Steinbrenner, Becker, and Kalina, 1980). In what grade today do teachers choose to introduce the abacus?

*What ideas do teachers have to motivate their students to learn the abacus?*

One idea that motivated students to learn to use the abacus was the abacus bee (Lewis, 1969). Similar to a spelling bee, the abacus bee is a contest where the contestants answer mathematics questions with the abacus. In 1967, the Tennessee School for the Blind began holding abacus bees. Eventually, the students gained the confidence to challenge a public school class to a mathematics contest. The abacus users performed as well as or better than the sighted contestants in speed and computation in all four basic operations (Lewis, 1969). The abacus bee appeared to be a good idea. What other ideas do teachers have to motivate their students?

These questions are raised from ideas discussed in previous research. A nation wide survey (Appendix B) was used to gather information on each question. With the changing of times, ideas, and people, however, new topics emerge. Topics yet to be treated in a research paper.

*Are IEP goals for abacus instruction appropriately set for the students they affect?*

*Do teachers and students consistently meet their IEP goals pertaining to the abacus?*

A very important part of the education of a student with visual impairments is an IEP (individualized education program). A team of educators and parents create IEPs. Collectively, they map out the goals students will work toward during the school year. This team is not the same for all students with visual impairments since not all of these students require the same services. The team could consist of the regular classroom

teacher, parents, and a vision teacher. These teams may include also O&M specialists (orientation and mobility is training for moving around in the environment), physical therapists for muscular or dexterity problems, or other specialists deemed necessary by doctors or vision teachers. The individual needs of students make individualized education programs important. No previous articles mention IEPs, leading to some questions about the implementation of IEPs. Two questions considered in this dissertation: Are IEP goals satisfied by students on a regular basis? Are the IEP goals usually appropriate for the child they affect?

*What skills do teachers require of their students before introducing the abacus?*

Another area of interest not mentioned in past research is exactly what skills teachers require of their students before introducing the abacus. Some articles report on grade of introduction, (Lewis, 1970: Steinbrenner and Becker, 1982), but none discusses the abilities students should have before beginning abacus work. Thus, the survey (Appendix A) asks respondents to note the skills they prefer their students to obtain prior to learning to use the abacus.

*Which manual, if any, do teachers prefer to use when teaching with the abacus?*

Similar to most other mathematics classes, there are books for the teachers and students to follow while learning to manipulate the abacus. There are Mae Davidow's *The Abacus Made Easy*, Rita Livingston's *Use of the Cranmer Abacus*, Nancy Jacquat Foster's *Detailed Instruction on the Use of the Cranmer Abacus*, and numerous other manuals. When most of the research pertaining to the abacus was published, the Cranmer abacus was still fairly new. Few manuals existed; when more became

available, none of the research reported on teacher preferences. Which manual, if any, do abacus teachers of today prefer and why?

*What information would current teachers of the abacus like to learn more about in a research article?*

With the lack of current research pertaining to the abacus, many questions may be unanswered. I have been able to list a number of ideas that I would like to learn more about. Other teachers of students with visual impairments, however, may have some concerns as well. With regards to issues addressed, what would teachers of the visually impaired (VI teachers) like to see in a research article?

The questions and ideas described above are thoroughly examined in this dissertation. Current statistics regarding these questions are unavailable due to the lack of recent research focusing on the abacus. The most recent topic to surface deals with the appropriateness of using the abacus for teaching mathematics to students with visual impairments. Some teachers of students with visual impairments question whether the abacus is the most effective tool when compared to the braillewriter or the talking calculator. This information is not available in published form, rather a few comments have appeared over the AER (Association for Education and Rehabilitation of the Blind and Visually Impaired) listserve. This is another topic that is considered in a later chapter.

Responses to these questions may lead to some new ideas for future research. Ideas that can be inferred from the analysis of the data are noted as they occur. Also,

chapter four (the concluding chapter) reviews major themes that appear throughout this dissertation.

### 1.3 Literature Review

#### 1.3.1 History

A little over a 100 years ago, a counting board was discovered on the island of Salamis, near Athens. Now, it is on display at the National Museum of Athens. This white marble slab may date from as early as 400 B.C. (Moon, 1971). There is only speculation as to when the first counting board appeared. The Salamis board, however, may prove them to be at least 2000 years old.

Counting boards existed in medieval times and the Middle Ages. The ancient Romans actually had abaci made of metal plates and rods for beads to slide on (Moon1971). People in each era and location had their own version and rules for performing operations. They are all a part of the evolution of the abaci used mainly by visually impaired people in the United States today.

Three main configurations of the abacus appear during the last few centuries. The Russian abacus has ten beads per rod. Each bead has a value of one, hence there are no counters for fives, fifties, et cetera. On the other hand, the Chinese abacus does make accommodations for place values of five. It has seven beads on each rod; a bar separates the bottom five and the top two. The lower five are each worth one and the upper two have value of five each. Both have a drawback the Cranmer abacus avoids. Many numbers do not have unique settings on the Russian or Chinese abaci. For example, on the Russian abacus, the number 10 can be ten beads set in column one or

one bead set in column two. On the Chinese abacus, the number 10 can be represented by five ones and a five in column one, two fives in column one, or one unit counter set in column two. Since some numbers are not uniquely set on the Russian or Chinese abacus, the third configuration is the type we choose for modern day people with visual impairments or blindness.

The Japanese abacus or soroban (figure 2) allows unique representation of all numbers. For example, the number 10 can only be presented as one unit counter set in column two. In the early 1960's, an adapted form of the soroban (the Cranmer abacus) became the United States' primary computational abacus used by students with visual impairments (Lewis, 1970; Brothers, 1972; Steinbrenner and Becker, 1982).

Up until the nineteenth century, the abacus was a tool used by the general public (Moon 1971). It took thousands of years for someone to suggest its use in teaching the blind. For over one hundred years, the Japanese have used the soroban in teaching mathematics to the blind. In 1881, in Kyoto, Japan, the Kyoto asylum proclaimed the abacus disadvantageous for the blind (Becker and Kalina, 1975). The beads moved too easily, causing work to erase or change unexpectedly. The instability of the beads was a definite shortcoming of the soroban, but its use continued with success (Moon, 1971). The first documented discussion for its use in teaching mathematics to students with visual impairments in the western world came in 1920 by the American Association of Instructors of the Blind (Hattendorf, 1971). More time was required, however, before the abacus became a staple in teaching mathematics to the blind.

FIGURE 1, The Cranmer abacus with the number 307 set

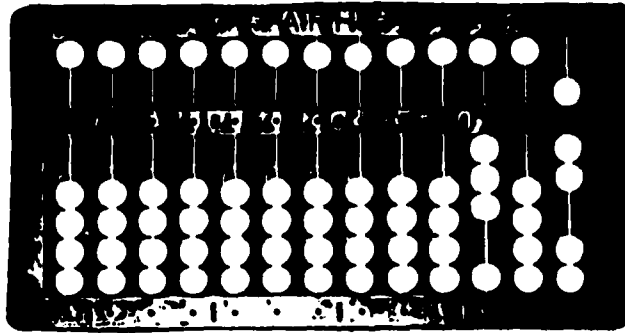


FIGURE 2, The soroban

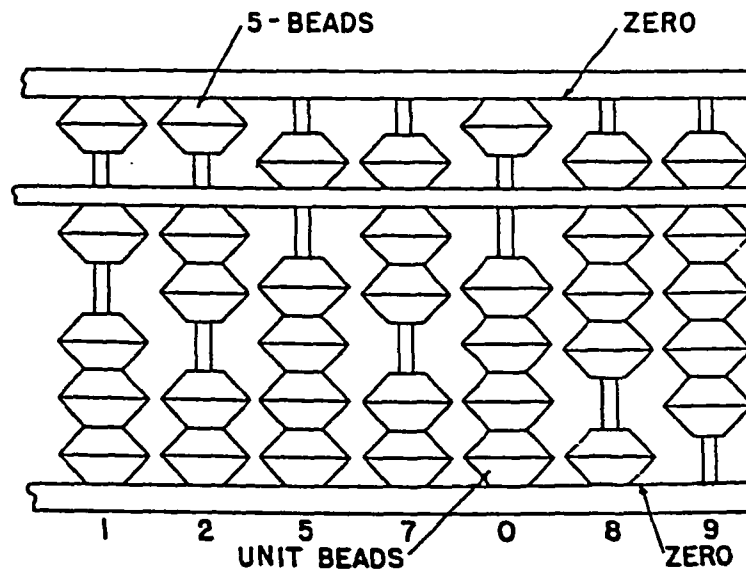


FIGURE 3, The braillewriter



FIGURE 4, The numberaid

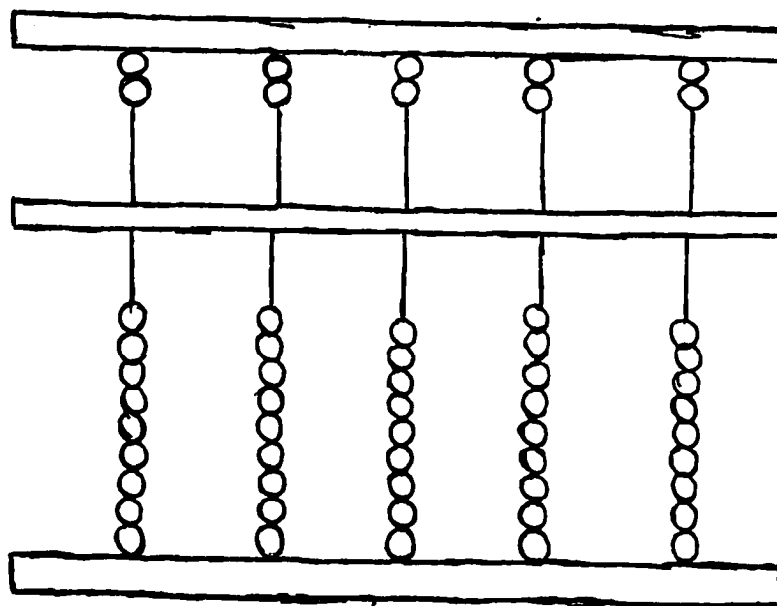


FIGURE 5, All peg settings for the Taylor slate (Hanninen, 1982)

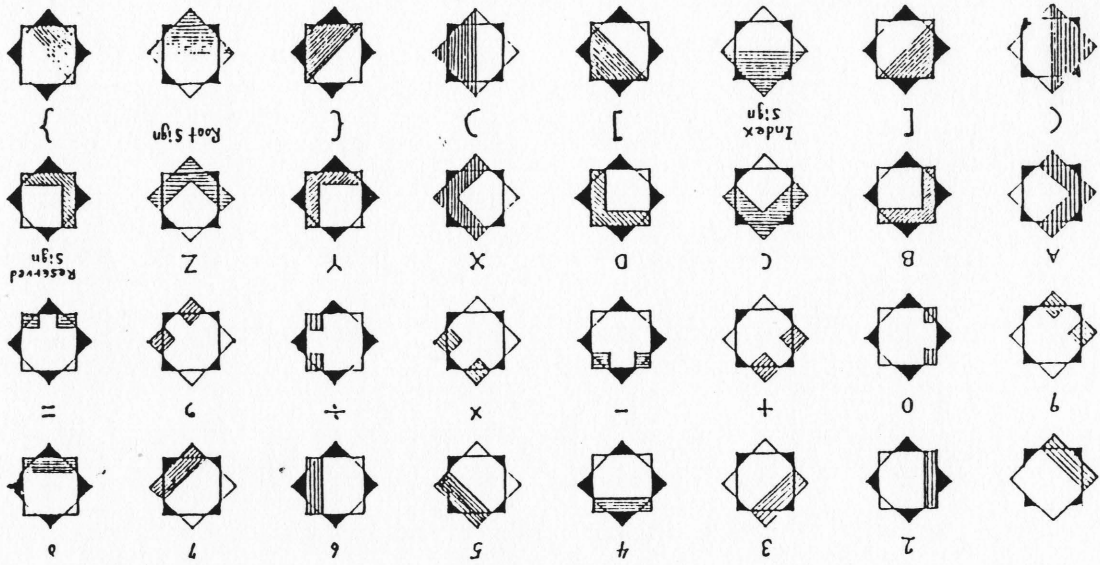
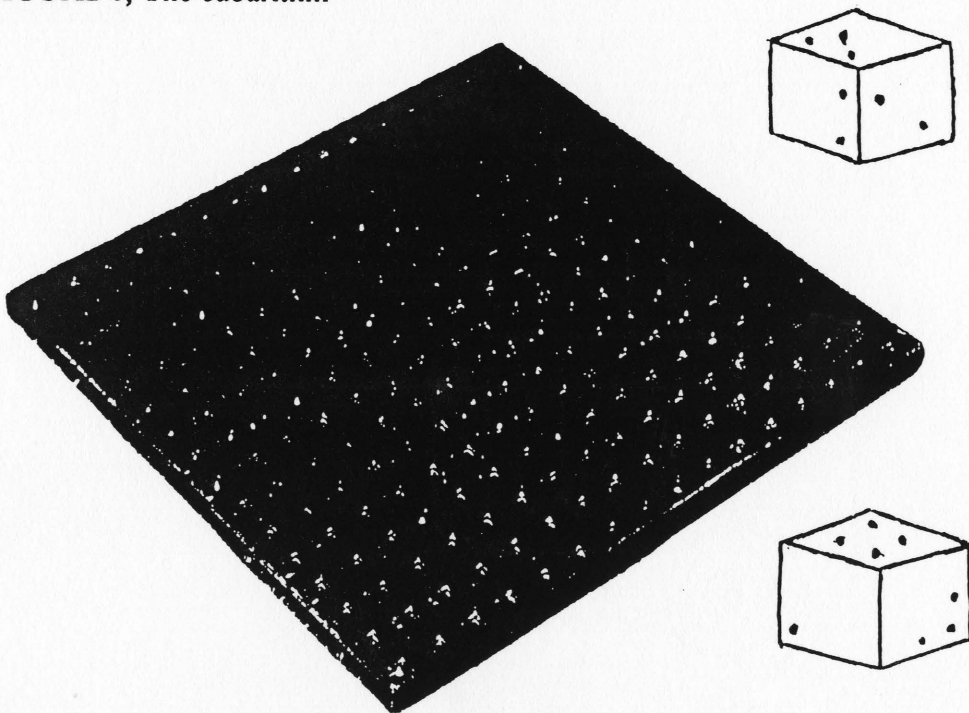


FIGURE 6, The cubarithm



### 1.3.2 The Cranmer Abacus

By 1956, T.V. Cranmer and his associates were brainstorming alterations to the soroban to increase its effectiveness in teaching blind people (Hass, 1965). A few years later the Cranmer abacus appeared. It solved many of the hindering problems without stopping the success of the Japanese. A rubber backing under the thirteen columns of white beads holds set beads firmly in place. This does not cause difficulty in moving the counters. Also, the dimensions are standard and pocket size for easy portability. With these two small adaptations, the Cranmer abacus gained immediate attention as a powerhouse in teaching mathematics to the blind. To quote Hattendorf, "The abacus is as important a basic skill to blind persons as is Braille." (Hattendorf, 1971)

In 1964, Fred Gissoni led the first Cranmer abacus workshop for teachers in Kentucky. Presenters included speakers from the U.S., the Danish school for the blind, and the University of Tokyo's school for the blind. Gissoni reported much enthusiasm for learning and teaching the abacus by the attendees (Gissoni, 1965). It was apparent that teacher training was crucial, which led to the workshop at the University of Kentucky the following summer of 1965 (Becker and Kalina, 1975). This need developed into workshops throughout the sixties and led to George Peabody University granting college credit for abacus training courses beginning in 1967 (Becker and Kalina, 1975). "The abacus shows great promise," (Lewis, 1969); therefore instructors must learn to use the Cranmer abacus. In fact, training remained a major issue of many of the research articles that followed (Lewis, 1969; Lewis, 1970; Brothers, 1972; Steinbrenner and Becker, 1982).

### 1.3.3 Alternatives

Before reviewing the research from the last thirty years, it is helpful for the reader to become familiar with some of the common tools of the past and present used by students with visual impairments to work mathematics. Perhaps the most prevalent tool used was the Perkins Braillewriter (figure 3). This is the manual-typing machine used for Braille. It parallels the manual typewriter in its functions, yet has only nine keys. The far left is the line up key, and the next three make up the left column of the six dot Braille cell. Next is the space bar, followed by three keys for the right column of the Braille cell and the backspace key. This is a basic necessity for typing Braille. Computing, however, becomes rather difficult. To line up a mathematics problem correctly, a series of backspace and linespace manipulations must occur. For example, the problem  $17+22$  requires Brailleing the first addend, linespacing down, and backspacing three spaces. This requires the user to type the plus sign and the second addend. More demanding questions, perhaps with two addends or more require even further manipulations. Though the Braillewriter is not the simplest computational tool, it is used widely in conjunction with other tools for recording purposes (Lewis, 1970).

Some other tools that have been used include numberaids (figure 4), Taylor slates (figure 5), and cubarithms (figure 6). The numberaid is similar to the Cranmer abacus. It consists of beads on rods that represent numbers, while the rods mimic place values. The beads stay in place due to their inner coating, not rubber matting like Cranmer's model. Only five rods make up a numberaid, however, making longer multiplication and division problems impossible. Of course the numberaid does have

some redeeming qualities. All the columns have different colored beads to help the low vision users, and all the moving pieces are larger and easier to discriminate.

Uniqueness is not available with the numberaid; there are nine unit counters and two five counters on each rod.

The Taylor slate is very different from the structure and use of the abacus. It is a metal slate with many rows and columns of octagonal holes. The user places one of two different pegs in the holes. One peg has a long bar on one end and two small squares on the other end. The second peg has two bars perpendicular to each other on one end and a triangle on the other end. The pegs are square, and there are sixteen ways to set each peg, with eight on either end giving thirty-two possible settings. The different settings represent different numbers, and the pegs are set side by side to create numbers of any length. The user may set numbers below or next to others and perform operations at will.

The cubarithm uses a board with sixteen rows and columns of square slots for the placement of cubes. These cubes have Braille numbers on each side, but this does not limit users to six numbers. Since the Braille numbers only use the bottom two rows in a six-dot cell, the cubes can be turned so each side may be used for as many as four different numbers. The cubes are set in rows and place value is inferred, as in paper and pencil computation. Numbers are set below or next to any others to begin computations.

These alternatives of the past may still be used; the future, however, is technology. Today a common alternative is the talking calculator. The talking calculator simply gives voice output to depressed numbers, answers, operations, et

cetera. Almost any type of calculator can be made accessible to people with visual impairments through speech output. The most basic model, as well as the graphing calculators used in many mathematics classes, can be adapted for speech output. In fact, some researchers have conducted studies concluding that the talking calculator is the superior choice in teaching mathematics to students with visual impairments (Champion, 1976). In 1976, Champion showed that a group of nine blind students, from third to eighth grade, improved computational and concept scores twenty to thirty percent on the Stanford Achievement Test (pre-test A and post-test B) (Champion, 1976). These students received an hour of instruction using the talking calculator, followed by three weeks of practice. There was no control group, however, with which to compare the progress of the blind students. Also, reported was that the concept scores improved less than computational scores (Champion, 1976). Since the concept scores improved less, it is possible that other tools for mathematical concept development are indicated. The calculator is a definite asset for the mathematics student, sighted or not. However, just as it is not a replacement for paper and pencil computation for the average student (NCTM, 1989), it does not replace computational aids for the students with visual impairments (Steinbrenner, Becker, and Kalina, 1980). Though recently over the AER listserve there has been some controversy about the use of the abacus, it remains the choice of many teachers of the visually impaired.

The use of manipulatives in teaching mathematics to the visually impaired is an important method (Lewis, 1970). Among the numerous aids used by people with visual

impairments to do mathematics, the Cranmer abacus has been the most commonly utilized mathematical tool for students with visual impairments (Brothers, 1972; Steinbrenner, Becker, and Kalina, 1980).

#### 1.3.4 Research

In 1959, seventy-five percent of visually impaired students achieved below grade level in mathematics. They averaged fifteen points below normal on the SAT (Stanford Achievement Test) (Nolan and Ashcroft, 1959). Since 1959, many special educators have taken on the challenge of improving methods of teaching mathematics to students with visual impairments. Even today, mathematics requires more attention from teachers than other subjects (Rapp, 1992). T.V. Cranmer is one person who tried to improve methods used in teaching mathematics to students with visual impairments. After a few years of mulling over this problem, he created the Cranmer abacus; a tool with great promise (Lewis, 1969), whose ancestor, the soroban, was already proven successful in teaching the blind by the Japanese (Takashi, 1954; Moon, 1971).

In 1964, a group of fifth and sixth graders showed some of the earliest signs that the Cranmer abacus could be effective. They improved both accuracy and speed in mathematics after practicing on this exciting new tool (Becker and Kalina, 1975). This led a number of professional educators of the visually impaired to begin publishing research about the Cranmer abacus.

One of the earliest was Marian Lewis in 1969. The “abacus bee” was one of the main focuses of her 1969 research article (Lewis, 1969). In 1967, the Tennessee School for the Blind began regular instruction on the Cranmer abacus. Soon after, the classes

began competing among themselves and against other classes in the school. Later in the year, one class had the confidence to challenge a local public school to a mathematical contest. The abacus users of the Tennessee School for the Blind defeated the public schools calculator users in both speed and accuracy categories (Lewis, 1969). In fact, throughout history abacus users have outperformed or performed as well as non-abacus users in all four basic operations (Takashi, 1954; Moon, 1971). Victories such as those attained by the students of the Tennessee School for the Blind implanted the Cranmer abacus as a tool of great promise in teaching students with visual impairments number and place value concepts (Lewis, 1969).

By 1970, the Cranmer abacus was the second most common aid used by teachers of the blind in residential schools for mathematical purposes. Eighty-seven percent of schools polled in a study by Marian Lewis reported its use (Lewis, 1970). The braillewriter was used by ninety percent of the responding schools, though it was not necessarily the most widely used calculating aid. The braillewriter was used as a computational tool by some students, but many used it for recording purposes only (writing mathematics problems, writing answers, etc.). Mental calculation and the Taylor slate came in third and fourth respectively (Lewis, 1970). Interestingly, only thirty-five percent of the resource room instructors reported that they used the abacus. Resource rooms are separate, self-contained classes where students with disabilities may spend part of their school day working. The teachers in these rooms are not regular classroom teachers. Also, as regards resource rooms and related data, the abacus seemed to be used only in certain geographically concentrated areas. One area may have

had numerous resource room teachers using the abacus, while other areas had none at all. Fifty percent of residential and twenty-three percent of the resource rooms (those that used the abacus) introduced it by the end of third grade (Lewis, 1970). Though almost ninety percent of the residential schools responding to Lewis' survey used the abacus in one way or another (at the time of her article), its use still had not gained its expected popularity. Lewis blamed this on a lack of teacher training, a topic that both of her early articles emphasize. There was a need for workshops and college training courses (Lewis, 1970) that many of the articles that followed corroborate (Brothers, 1972; Steinbrenner and Becker, 1982).

Roy Brothers, in 1972, reported on a research study he performed in October of 1970 (Brothers, 1972). He questioned whether or not teachers were prepared to use tools such as the numberaid or calculaid, and made specific reference to the abacus. Brothers solicited twelve schools to participate in his study of students with visual impairments in third, fourth, sixth, and eighth grade. The primary, intermediate I, intermediate II, and advanced level of the SAT were administered by regular school personnel to the third, fourth, sixth, and eighth graders respectively. The abacus was used by about forty-five percent of the students in his study. Two interesting pieces of information resulted from Brothers' 1970 study. One is the fact that since the Nolan and Ashcroft study in 1959, Braille users' achievement in mathematics had dropped from fifteen percent below the norm to twenty-seven percent below the normal. The second item was that the highest achieving group among the eighth grade groups was the

abacus or abacus plus other device group (this study categorized students by the device they used), in which the students actually had the lowest mean IQ of all eighth grade groups. The abacus or abacus and other device group achieved .85 points (based on an 8.2 as an average score) higher than the next best group (mental calculation users), and their mean IQ was 7.4 points lower than the next lowest group (mental calculation as well) (Brothers, 1972).

A few articles were published in the early eighties, after it was determined the abacus was the most widely used mathematical aid for students with visual impairments. In 1975, Becker and Kalina drew this conclusion from circumstantial evidence (Becker and Kalina, 1975). They reported that C. Y. Nolan, educational research coordinator at the American Printing House for the Blind, (in a personal communication) stated that over 50,000 abaci had been sold by 1974 (Becker and Kalina, 1975). Unfortunately, no numbers were available in 1974 for them to confirm that the abacus was the most widely used mathematical aid for the blind (Becker and Kalina, 1975). However, the first of two articles written by Arthur Steinbrenner and his colleagues gave some statistics supporting Becker and Kalina's claim (Steinbrenner, Becker, and Kalina, 1980). Thirty residential schools responded (out of fifty) to a survey focusing on the Cranmer abacus. Fifty percent reported the abacus as the predominant tool used for mathematics by students with visual impairments, while only ten percent used the abacus in absolutely no way. The braillewriter was second in primary use at forty percent, and the calculator was utilized in some manner by eighty-four percent (second to ninety-percent for the abacus) of the students (Steinbrenner,

Becker, and Kalina, 1980). By this time, the tools such as Taylor slates, calculaids, and cubarithms mostly disappeared from the reports. The main devices involved were abaci, braillewriters, calculators, and mental calculation.

Steinbrenner also examined time periods when teachers introduced the abacus and teacher training in his 1980 article. Previous reports stated that three to five years was a good age to begin students learning mathematics skills (Lewis, 1970), and here in the United States, thirty-six percent of the residential schools that participated in Steinbrenner's survey began the abacus in early elementary grades (Steinbrenner Becker, and Kalina, 1980). Unfortunately, formal training on the abacus for teachers of the visually impaired was not yet predominant. Only sixty-three percent of teachers participating in Steinbrenner and Becker's survey claimed to be self-taught.

Two years later, Steinbrenner and Becker devoted an entire study to teacher education (Steinbrenner and Becker, 1982). They appeared to be reinforcing the points made in their 1980 article. Again the warning was made against reliance on the calculator for mathematics, and teacher training fell victim to further scrutiny. Although all of the twenty-five schools responding to their questionnaire stated abacus training occurred some time in their program, still the training was incomplete and ineffective (Steinbrenner and Becker, 1982). People were listening to earlier criticisms about teacher education, but more attentiveness was necessary for improvement to the process.

From 1982 to the present, few articles focused on or mentioned the use of the abacus. Until 1982, the topic was well covered, and its use and training processes

seemed to be following previously set guidelines. However, no one followed up on the facts in these papers. Perhaps people are having difficulties understanding the use and effectiveness of the abacus. Maybe the advances in technology have lessened the interest in tools such as the abacus. Never the less, this lack of recent research inspires the study described in section four.

## 1.4 Methodology

### 1.4.1 Methodology Introduction

The research methodology for this dissertation consisted of two parts. Part one was a nationwide survey. Teachers of the visually impaired from around the United States were given a survey dealing with several aspects of teaching students with visual impairments on the Cranmer abacus. Part two was a more in depth, small group study. This study is detailed in section 4.3, immediately following the discussion of the survey representing part one of this dissertation.

### 1.4.2 The Survey

The first part of this study grew out of a curiosity pertaining to current information regarding the abacus and its use. Knowledge of current trends in abacus teaching was necessary for a complete report. Also, reports of these ideologies would give VI teachers a resource when trying to determine popular methods for implementing the abacus into the curriculum for students with visual impairments. A foundation of knowledge was required to initiate the study described in section 4.3. In order to gather some current information, a nationwide survey was distributed to VI teachers (teachers of the visually impaired). The results of this survey gave new data on topics not

discussed in previous research articles. Also, comparisons were made between some study results of the past to today's students.

The one-page survey (Appendix B) was developed for distribution throughout the United States. Limiting the survey to one page was an attempt to create a greater response to the questions and to minimize the inconvenience for participating teachers. Apparently this worked. Although two sets of surveys had to be distributed, forty-five out of 111 surveys were received. In the fall of 1997, only twenty-seven surveys were received. The rest arrived after distributing ten more packets of surveys in the fall of 1998. The brevity of the survey also limited the number of questions appearing on the document.

Once the survey was prepared, a procedure was devised to distribute it to teachers in different parts of the country. Approximately twenty-five people (fifteen people in the fall of 1997 and ten people in the fall of 1998) were identified around the country that could help in the distribution of the forms. Each person received a packet of four surveys (except one person who asked for two copies and made her own copies to give to teachers in her area) along with a cover letter (Appendix C). This letter contained instructions on distributing the surveys, descriptions of the teachers who should receive them, and several methods for getting questions answered (phone, addresses, email, etc.). The chosen people may or may not be current teachers of the visually impaired, but were in the position to contact numerous VI teachers to request participation. To insure that the people contacted had the ability to find potential

participants, their names were chosen from a directory of services for visually impaired persons.

This process is similar to older research articles using surveys as a means of gathering information. Marian Lewis contacted residential schools from a list of educational institutions serving the blind created by the American Printing House for the Blind (Lewis, 1970). Arthur Steinbrenner recruited educational institutions that trained teachers for instruction on the abacus (Steinbrenner, Becker, and Kalina, 1980). For this dissertation, names were taken from the AFB (American Foundation for the Blind) Directory of Services for Blind and Visually Impaired Persons in the United States and Canada (1997). Names were drawn in such a way that teachers from many different geographic areas of the United States were involved. With each packet, four self-addressed, stamped envelopes were included for easy return by participants to minimize their expense.

Completed surveys were stored until all had arrived. Once the deadline for the surveys passed, an attempt was made to contact each person who received a packet of surveys. Since the actual number of surveys distributed may differ from the number sent out (four were in each packet and distributed depending on the number of teachers that use the abacus), It was important to know how many were distributed and to whom. After I talked to as many of the people who received packets as possible (some did not return calls or get in touch with me at my request), I estimate the number of teachers who received the survey as 111. Forty-five surveys were returned and three of these forty-five were not usable for this study. They were returned blank. The three people

who returned the unusable surveys reported that they did not use the abacus. Finally, each survey was carefully examined and the responses recorded for analysis in part one of this dissertation.

#### 1.4.3 The Study

In part two of this research, observations were made of teachers and students working together on the Cranmer abacus. Four teachers and their students were carefully observed and findings are discussed in detail in chapters three and four of this dissertation. Observations revealed the methods each teacher used in his or her lessons. Each student was visually impaired with no other reported disabilities. Each of the students was reviewed in two ways. First, at three different times during the semester, the students and their respective teachers were videotaped. At the beginning, middle, and end of a school semester, each participating teacher was contacted to set up an appointment. With one of the teachers, there were some difficulties arranging appointments. Thus, one set of videotapes began a few weeks into the fall semester and ended a few weeks into the spring semester. At every session an entire teaching activity was videotaped. In order to base all observations on the same criteria, a protocol (Appendix G) was designed for analyzing the videotapes. Data on student and teacher actions in these sessions were noted for use in this dissertation.

Second, at three separate times (simultaneous with the time of the tapings), the students completed an assessment (Appendix D) of their abacus skills. Three different assessment tools were used, one per visit. The number and types of problems, however, remained the same on each assessment. Only the numbers used in each problem varied.

Students involved in the study were doing different levels of mathematics. Before the assessment was written IEP goals and teacher comments pertaining to each student were considered. This allowed for an assessment that examined all topics that each student was working on during this semester. Each student took the same test, and all completed different sections of the assessment. The students were not compared to each other. Instead, the assessments were to gauge their progress throughout the semester. Combining the assessments with student knowledge noticed on the videotapes gave a good approximation of how the students were progressing. These two procedures embody the information gathering techniques for part two of this dissertation.

A letter (Appendix E) was distributed to recruit participants among VI teachers in the Tucson Unified School District and neighboring areas. Once volunteers were identified, each one was contacted to begin the study. The parents or guardians of students identified (by the volunteer teachers) as abacus users were contacted. A permission form (Appendix F), detailing the requested involvement of their child was sent home for signature confirming their permission. These papers will be kept on file. Finally, after everyone was comfortable with the procedures (parents, teachers, and students), the research began. The teachers were contacted about a week before each session to set up appointments for observations and assessments. No special requests were made of the teachers. Each taught as if it was a normal day. No interventions or curricular changes were required.

#### 1.4.4 Notes on the project

The results of both parts of the research were interesting. In some aspects, the results from the videotapes supported the results from the survey. In other ways, the results from the videotapes did not support those from the surveys. Chapter four discusses both parts of the research simultaneously. The next two chapters focus on the survey results and the videotapes respectively.

## 2 THE SURVEY

This section deals entirely with the responses given on the surveys. Almost every individual response is discussed. All trends and common responses are emphasized as well.

### 2.1 Training and Experience

In 1982, Steinbrenner and Becker reported that sixty-three percent of teachers in their study were self-taught on the abacus. Training and experience were included on this survey to make comparisons to data given fifteen years earlier. Table 1 exhibits the years of experience accumulated by teachers participating in this survey. The numbers range from three to twenty-seven years, while seven people abstained from responding. This may have been a result of spacing on the survey. There was less space on the fall, 1998 survey set for the training and experience request (the survey was retyped for the fall 1998 set), and there was a much higher rate of abstention. Five (27.8%) abstained on the 1998 set, while three (11.1%) abstained on the fall, 1997 set. Specifically asking where and how the participants learned to use the abacus could have resulted in more reasonable responses. Also, the training and experience question should have been given more space. On the survey, the request was placed one line before the first numbered question. Some teachers may not have noticed it or they may not have felt it was important because it was not a numbered question.

Table 1 reveals a wide range of experience for this survey (mean = 13.8 years and standard deviation = 7.5). Considering training and experience together, however, could have led to more interesting ideas about the status of training programs. Table 2

**TABLE 1, Experience**

Participating teachers' years of experience.

Years	Number of responses in the given range
0-4	5
5-9	6
10-14	7
15-19	5
20-24	7
25-29	3
Abstain	8

One teacher responded with the mathematical ideas he or she teaches on the abacus. This misunderstanding was not considered in this table.

gives the different methods by which teachers learned the abacus. Unfortunately, the high rate of abstention, twenty-two (52.3%), does not allow for any viable conclusions. Eleven out of nineteen (57.9%) teachers were trained to use the abacus in college. This percentage is higher than the thirty-seven percent reported in the Steinbrenner and Becker article (1982). However, with all of the abstain answers, the number of responses is too small for significant conclusions. A comparison of the number of teachers who were college trained before 1982 (time of the Steinbrenner and Becker article) with the number of teachers who were college trained after 1982 would reveal any increase in the number of teachers learning the abacus in their teacher training programs. Unfortunately, only a few of the teachers gave both years and training on the survey.

**TABLE 2, Training**

How teachers learned the abacus.

Method	Number of responses
College	11
In-service or workshop	5
Learned because of own visual impairment	3
Abstain	22

One teacher responded with incorrect information and was not considered. He or she gave information concerning the skills he or she teachers. Training and experience were not mentioned.

Table 3 gives the years of experience for both college trained and non-college trained (on the abacus) VI teachers. Seven out of twelve (58.3%) of the college trained teachers began working after 1982, while forty percent of the non-college trained teachers started their careers after 1982. This would imply an increase in abacus training at the college level. However, the small number of responses returned make this conclusion statistically insignificant.

**TABLE 3, College versus non-college training**

Years of experience for college trained and non-college trained teachers.

Training	Years of experience
College	3, 3, 3, 4, 9, 10, 13, 16, 20, 22, 24, 26
Non-college	10, 11, 20, 21, 27

## 2.2 The Use of Manuals

Questions one and four deal with the use of manuals while teaching the abacus. Since the Cranmer abacus surfaced in the early 1960's, a number of manuals have been written to help teachers learn and teach it (Livingston, Davidow, and Foster are a few who have written these manuals). Question one asks teachers if they use manuals and question four asks which manual they prefer. Table 4 gives information on question one,

**TABLE 4, Likert scale questions**

- Questions 1-3 refer to the use of manuals and IEP goals.
1. When teaching the abacus, do you follow the methods of a manual?
  2. Do you feel that you consistently meet the IEP goals pertaining to the abacus for your students?
  3. Do you feel that the IEP goals pertaining to the abacus set for your students are appropriate?

Question #	1 = always	2 = usually	3 = sometimes	4 = rarely	5 = never
1	17	15	8	1	1
2	20	15	5	1	0
3	18	17	4	1	0

Note that questions two and three did not apply to one teacher. Also, another teacher responded to question three by reporting that he/she usually does not set abacus goals.

while table 5 lists the preferred manuals of teachers.

The average score for question one was 1.9 and the standard deviation was .96. This means that in general, teachers do use manuals as part of their abacus instruction. Only one teacher never uses a manual. This shows that most teachers feel an abacus

manual is important for themselves and their students. In all mathematics courses, textbooks or manuals of some kind are used. It appears that manuals are important to understand the abacus for learners with visual impairments and their teachers. The results to question one relate directly to question four. It would be helpful to know what manual(s) teachers prefer while teaching the abacus.

It is evident from table 5 that the Davidow manual is the most popular (eleven out of forty-two). The Livingston, Willoughby, and Foster manuals are grouped behind

**TABLE 5, Question 4**

Question 4.

What manual(s) do you use when teaching the abacus?

Number of responses	Manual
11	"Abacus Made Easy", Davidow
5	"Detailed Instruction in the Use of the Cranmer Abacus", Foster
4	"The Use of the Cranmer Abacus", Livingston
5	"The Paper Compatible Abacus", Willoughby
2	"Cranmer Abacus", Green and White
1	Suella McCrimmon dissertation
1	Curriculum for Teaching Visually Impaired Abacus Section
3	University created materials
4	Makes own materials
5	Abstain

Note that some teachers responded with more than one answer. Thus, adding the numbers will exceed the number of surveys received. Also, one teacher uses photocopies from an unidentified source.

the Davidow manual. The table clearly shows that a number of teachers use materials of their own, or materials specific to their school or district. Once an understanding of how

to use the abacus is reached, it is not difficult to write a manual. Actually, any of these manuals can probably be used effectively by VI teachers.

### 2.3 The Use of the IEP

The IEP is another part of the education of students with visual impairments that has not received attention in research concerning the Cranmer abacus. Individualized education programs are created for all students with visual impairments. They permeate all aspects of these students' education. The IEP contains goals for the students and teachers to work on together for the following year. Goals related to the abacus are among those set by members of the IEP team. Since one of my curiosities about the abacus concerns the abilities of teachers to teach the abacus effectively, a couple of questions about IEPs were included in the survey.

Table 4 contains information on two questions that pertain to IEPs. Question 2 asks if teachers believe that they regularly meet the IEP goals pertaining to the abacus that are set for their students. Question 3 concerns the appropriateness of the goals that are set (see the survey in appendix C).

For question 2, the mean was 1.7 and the standard deviation was .79. These statistics, however, are not necessary to draw the conclusion that most teachers believe that they consistently meet the abacus instruction goals set for their students. Thirty-five out of 41 teachers gave an answer of two or one. Thus, both teachers and students appear to be working well with the abacus.

For question 3 the same inferences can be drawn. The mean and standard deviation are 1.7 and .876 respectively. Also, 35 out of 40 respondents answered one or

two for this question. Not only are goals consistently met, they are perceived to be appropriate for the students they regard as well. These results clearly show that teachers believe that they are progressing and succeeding when teaching the abacus to students with visual impairments.

This conclusion is, however, a little confusing. The responses to questions two and three imply that the abacus is a useful tool in teaching mathematics to students with visual impairments. However, some teachers question the effectiveness of the abacus. The positive results attained by many teachers who participated in the survey prove the potential benefits of the Cranmer abacus.

#### 2.4 Question 5

This section discusses the grade in which teacher prefer to introduce the Cranmer abacus. Steinbrenner, Becker and Kalina (1980), and Lewis (1970) have examined this question in their research. Both works suggest that abacus instruction commences in the early elementary grades. Table 6 gives some insight to current numbers relating to this question. There is still a preference for grades earlier than third, with a few exceptions. Only seven of the thirty people who gave numerical answers responded third or fourth grade.

Also, there is a wide range of answers. Some begin in preschool, while others wait until fourth grade. There does not appear to be a consensus about the grade in which the abacus should be introduced. In fact, there were some answers that did not specify a particular grade. Three people said they introduce the abacus as soon as possible. The students should start as soon as they are ready to effectively use the

abacus. Due to the developmental differences in cognitive abilities (Shea and Bauer, 1994) noted earlier, the decision is made on an individual basis.

Another response that did not specify a grade of introduction is discussed in the previous paragraph. Two people stated that the introduction of the abacus depends on the student. Anyone who works with students with visual impairments realizes that each student must be dealt with on an individual basis. This response suggests that the introduction of the abacus depends on the student's comfort and readiness to work with

TABLE 6, Question 5

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Question 5.

In what grade do you usually begin instructing on the abacus?

Grade	Number of responses
Preschool	2
Kindergarten	4
Kindergarten or first	5
First	8
First or second	3 (one of these said two or earlier)
Second	1
Third	6
Fourth	1
First through third	1
As early as possible	2
Depends on the student	3
Abstain	6

---

Two teachers do not teach the abacus and one of the teachers only teach in high school so they did not respond.

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the abacus. Unfortunately, The question asked for a grade and did not provide for this type of response. The possibility that teachers who participated in the survey are not

bound by their answers is good. They may have felt obligated to give a numerical response.

## 2.5 Question 6

Question six is meant to supplement the information given in question five. This is the first time prerequisite abilities have been studied, thus there is no comparison to be made with past reports. The question follows the idea that grade should not determine when the abacus should be introduced. Instead, it concerns the abilities of the students and how their abilities influence the introduction of the abacus.

Table 7 reports every answer given by the forty-two usable surveys. Notice that there are more than forty-two responses. Many teachers reported a number of abilities that they require of their students before introducing the abacus. There were four answers that were common among the surveys.

Nineteen out of forty-two (45.2%) people included basic addition and subtraction facts on their list. Counting was included on fourteen (33.3%) of the lists, while one to one correspondence appeared on twelve (28.6%) surveys. Finally, ten (23.8%) VI teachers required place value knowledge before beginning activities with the abacus. Each of these skills is very important in learning to use the abacus effectively. In fact, each of these abilities can be practiced and experienced with the abacus as well.

The four most popular responses to question six are important for new teachers to learn about. They give some insight into when to introduce the abacus to students

with visual impairments. Actually, each ability mentioned in table 7 is worthy of consideration. These abilities are examined in the next few paragraphs.

**TABLE 7, Question 6**

**Question 6.**

What mathematical abilities do you require your students to have before beginning instruction on the abacus?

<b>Ability</b>	<b>Number of responses</b>
One to one correspondence	12
Counting	14
Basic addition and subtraction facts	19
Place value	10
None	2
Previous manipulative work	1
Trading (i.e. five pennies for one nickel)	2
Basic skills	2
Independently set numbers	1
All skills up to carrying and borrowing	1
Problem solving abilities	1
Greater than / less than	1
Spatial concepts	1
Motor skills	1

The following are other statements made in response to this question.

- Introduces it in kindergarten when they really have not learned any math skills.
- We use the abacus as we learn new topics. When the class starts adding, we start addition.
- The students learn skills as they use the abacus.
- The students must know their numbers and how to set them before I begin instructing.
- Adding and subtracting three digit numbers by regrouping, beginning multiplication tables, and setting five digit numbers.

There were a number of other abilities appearing less frequently in the lists of responses to question six. Trading, greater than / less than, and all skills up to carrying and borrowing are a few of the skills required by some teachers before starting their

students with the abacus. Trading (similar to the idea of trading five pennies for a nickel) on the abacus is a necessity. When you cannot set five unit beads, you set one bead worth five. And when one column is full, you trade for one bead to the left. Incorporating the abacus into trading activities can definitely help the student's understanding.

Carrying and borrowing skills are also very important. They are a part of addition and subtraction and can even be explored with the abacus. In the problem  $16 + 9$ , the one must be carried after adding the nine and the six. On the abacus, we could simply set one bead to the left and the carrying function is complete. In fact, carrying results in the "secrets" of addition. These secrets are rules of addition that students and teachers can memorize. For example, there are two secrets for adding three. In the problem  $4 + 3$ , once the four beads are set in the ones column there is no room to set three more beads. Therefore we set a five counter and clear two unit beads. Thus one of the secrets for adding three is "set five, clear two." This is the same as adding five and subtracting two (which is the same as adding three). In the problem  $8 + 3$ , once the eight is set in the ones column there is no room to set three more beads. The secret here is "set one left and clear seven." This is the same as adding ten and subtracting seven. There are two secrets of addition for adding numbers up to four and there is only one secret for adding numbers greater than four. When adding a number like six, there is no way to set a five counter and clear unit counters. Borrowing can be explored in a similar manner.

Greater than / less than skills are simply the ability to determine the largest of a set of numbers, amounts, etc. One can look at the number of beads that is set on an abacus and do comparisons. Also, numbers can be set on each side of the abacus and the student can decide which is the larger number. Many activities on the abacus can reinforce and or require greater than / less than skills.

The responses to question six also included some skills that are experienced early in life, before school. Motor skills, previous manipulative work, spatial concepts, and problem solving abilities represent skills that VI teachers believe help students when they learn to manipulate the abacus. Children show signs of motor skills and spatial concepts at very early ages. Problem solving abilities and experience with manipulatives also come early. For example, a three-year old building a fort out of blocks is demonstrating all four skills. Hopefully, most students have experiences with these abilities before formal schooling begins. They are definitely necessary for successful use of the abacus.

There are only two answers from the table that have not been addressed. One teacher required the student to independently set numbers on the abacus before teaching the abacus. This is a little confusing. Doesn't a teacher have to begin instruction on the abacus before the students can learn to set numbers on it? The problem is in knowing when instruction actually begins. The final answer from the table to be discussed was included in two teacher's lists. These teachers answered "none" to question six. They engage the student as soon as possible, and the student may become comfortable with

the abacus early on. Also, as was discussed above, the students get access to one more tool that can illustrate many of the concepts that children need to learn.

Note that one of the answers to question six was not included in any discussion. The answer “basic skills” is ambiguous and gives no insight. There is a list of other interesting responses immediately following the table. These are worthy of attention.

The first one states that the abacus is introduced in kindergarten when the students have not learned any mathematics. The prompt introduction of the abacus is a good thing, but children do have mathematical experiences before kindergarten. A teacher should know this fact, though it is overlooked by teachers without (and some with) a mathematics background. The next two responses are very interesting as well. These teachers introduce new ideas on the abacus as the regular class learns mathematics i.e., when the class learns addition the VI teacher begins addition on the abacus. Students with visual impairments should do the same work as their peers. It is a great idea to keep students working at the same pace as the rest of the class (as long as the student has the cognitive ability to keep up).

The fourth statement is similar to one from the table. The teacher requires the student to know how to use the abacus before instruction begins. The final statement is very difficult to believe. I will let the reader make his or her own judgement about the impossible requirements of one of the responding teachers. I hope this teacher misinterpreted the question.

Question six resulted in a lot of information concerning VI teachers and their knowledge of mathematics education. The ability of these teachers to teach the abacus

is not in question. The responses to the Likert scale questions imply that teachers believe they are successful when teaching mathematics on the abacus. However, there are some issues with mathematics that may deserve some attention. The responses to question seven give some interesting insight to some of the previously mentioned ideas.

## 2.6 Question 7

Question 7 asked teachers to share any good ideas that they believe may be useful for other teachers of students with visual impairments to explore. Table 8 depicts the pool of answers given to question seven. The numbers in parentheses represent the number of teachers who included that response on their survey. The bottom note reveals that twenty-one teachers had no methods to share (50%). Sixteen people abstained, while five answered “none” to the question. If these teachers have no good ideas to motivate their students to use the abacus, it may be helpful for them to hear how other teachers motivate their students.

Regardless of the large number of teachers without suggestions for fellow teachers to explore, there were responses deserving discussion. Table 8 is a list of suggestions appearing in the responses to question seven. Some are good ideas, while others are confusing or contradictory. For example, the first two suggestions are exact opposites. One teacher suggests teaching from right to left, while another prefers teaching left to right. The terms “right to left” and “left to right” refer to the order in which to add digits in a problem such as  $12 + 12$ . Adding the twos first and then adding the ones represents going right to left. Left to right starts with the ones. Arguments can be made for both cases. Right to left operations parallel paper and pencil procedures.

Though some blind students will never use paper and pencil, they will use braillewriters. It is much more efficient to go right to left on the braillewriter. Also, some students who use the abacus do use handwriting. There are advantages to teaching

**Table 8, Question 7**

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**Question 7.**

Do you use any methods that you feel may be useful for other teachers to explore? Explain.

- Go left to right.
- Go right to left.
- Follow the ideas of a manual.
- Make manual explanations more clear since they are very confusing.
- Help other adults in the school learn the abacus so they can help if you are not around.
- Get the entire class (regular class) to experience the abacus.
- (2) Work through problems out loud.
- (2) Make the abacus fun.
- Use the abacus in everyday life.
- I am not fond of the abacus and use calculators once they know the process.
- Practice skills a lot.
- Follow along with the mathematics that the regular class is doing.
- Begin instruction in a one to one setting.
- Many kids recycle, so sell the abacus as a paper saver.
- Use the secrets.
- (2) Use counting rather than memorizing secrets.
- Relate fives and tens to nickels and dimes.
- (2) Use the paper compatible method rather than Cranmer's method.
- Use two abaci for division and decimals.

Sixteen people abstained from answering, and five people answered that they had no ideas to share.

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left to right as well. As the addition process begins on the left and moves to the right, the numbers being added are becoming much smaller. In adding 1483, once the 1000 is

added, there is a much smaller number to deal with. This could make it easier for the children to transfer skills from adding two digit numbers to adding three digit numbers and so on. Both methods have supporting reasons, preference being primary.

Certain teachers may find left to right easier, while others find left to right more difficult. It is important to remember that students do not always agree with their teachers, so it is imperative that teachers of the visually impaired know both methods. In this way, the one that is used can be based on the student's abilities and success. If a student started with right to left instruction and was struggling, it is feasible to try left to right instruction. Of course there are no guarantees of success.

Another pair of conflicting suggestions pertains to the manuals. One teacher believes the manuals should be followed (this teacher uses the Green and White manual). They are clear and easy to understand. Another teacher wishes the manuals were made clearer. They are confusing and hard to follow. This teacher makes his or her own materials for teaching the abacus. These sentiments will vary from teacher to teacher. However there is not enough information given to determine which manuals are the easiest to follow. A separate research project that asks teachers to review several manuals could help determine the easiest manual to follow. If there are teachers having trouble with the manuals, however, this will affect their ability to teach the abacus.

Some teachers disagree on the different methods that can be used with the abacus. One teacher recommends the "secrets," one recommends counting, and another prefers the paper compatible method. The "secrets" are the rules children use (described earlier) to add or subtract. The counting method would be simply adding one bead at a

time. When there is no room left, we set one in the column to the left and clear the column we are working in. In the problem  $18 + 3$ , we start by setting the eighteen. Then set one to have nineteen. There is no room to set another so set one left and clear the ones column. The number twenty is now set. Finally, set one more, which makes a total of three settings. The answer (21) is now set correctly. The paper compatible method is meant to illustrate the processes that occur in regular paper and pencil computation. For example, in adding 14 to 8, you set the 14 on the far right and the 8 on the far left. Next, use the fact that 8 plus 4 is 12 and clear the ones column on the left and set a 2. Also, set one bead in the tens column to illustrate carrying the one. The answer 22 is now set. For a more detailed description see Doris Willoughby's *Handbook for Itinerant and Resource Teachers of Blind and Visually Impaired Students* (1989).

Since all of these methods appear in the suggestions, all must have qualities making them successful. It is important to be mindful of the student's abilities. Make sure to use the method best suited for the individual student.

The ideas discussed above are matters of teacher preference. Manuals and order are important topics for VI teachers to learn about. However, any manual can probably be used effectively. Also, students can succeed using left to right or right to left methods. Some of the next suggestions from question seven could be useful in developing student's understanding of the abacus.

One idea is to teach other adults in the school to use the abacus so they may help the students in case the teacher of the visually impaired is not present. In a school for the blind, adults with abacus knowledge should be plentiful. However, in a public

school they may be scarce. Itinerant teachers travel from school to school. They are not always accessible to answer questions. If the regular classroom teacher knew how to use the abacus, students would not be limited in opportunities to complete work.

Another idea that may be helpful is including the entire class in abacus activities. It is very important to students with visual impairments that they fit in with other children. Social problems can greatly affect academic abilities for everyone. Furthermore, the activities can help the entire class improve their mathematical knowledge.

Some of the other ideas might be useful and should be followed. Practicing skills and relating the abacus to everyday life are important concepts. In fact, all mathematics students use these methods. If the student is in a public school class, one teacher suggests following along with the mathematics that the rest of the class is learning. The students can incorporate the ideas they learn in class with the lessons they receive on the abacus. Another teacher suggests making the abacus fun. Although this statement is vague, it is a good idea, if the instructor knows how to achieve "fun."

A couple of the suggestions could help in making the abacus fun. One suggests if the students like to recycle, the instructor can show how the abacus would save paper. Another teacher says relating the abacus to money can help clarify the use of the abacus. Determining what is fun for the student and incorporating it into abacus lessons can be helpful in motivating students to learn the abacus. If a student is competitive, create a contest in which the student keeps score. If the student likes basketball,

incorporate basketball into some of the questions you ask. Incorporating the student's interests can make the abacus more interesting.

Some of the other suggestions are preferences and should be used according to the students' abilities. One to one sessions (which are probably necessary), working out loud, and using two abaci may be very helpful for any student. There is one more response that which requires attention.

One teacher (who is not fond of the abacus) lets the kids use calculators once they learn to use the abacus. Unfortunately, if students do not practice what they have learned, many of the ideas will be forgotten. All students must practice their skills to keep and build upon them (Willoughby, 1990). An immediate transition to calculators would make any efforts or success with the abacus in vain.

There were several participants who had no methods to share with their colleagues. About half of the respondents had some ideas. This makes me curious as to how these teachers actually instruct on the abacus. Also, these teachers may use very effective techniques, but did not report them for some reason. The success reported in the responses to questions two and three imply that many teachers use some kind of effective methods to teach with the abacus.

## 2.7 Question 8

The last question on the survey has a simple purpose. Although I have many questions about the abacus and how it is used to teach mathematics to students with visual impairments, other teachers' concerns should also be addressed. Table 9 lists the ideas that teachers of the visually impaired would like to learn about. Some of the

suggestions are good ideas for future research. Others are as interesting but can be answered by simple inquiries. Each of the responses to question eight is discussed in the Table 9, Question 8

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**Question 8.**

In reading an article dealing with teaching and learning the abacus, what questions and ideas would you like to see addressed?

- (5) Compare methods of teaching.
- Which is better, Left to right or right to left?
- Why is it used in state testing (i.e. psycho-educational) instead of calculators?
- What states require non-calculator testing in their programs?
- (3) Fractions.
- What are others teachers doing with the abacus?
- How to get all kids to use it since it is such a good tool.
- (3) An article explaining how important the abacus is.
- (6) Is it really worth it to teach the abacus?
- (3) Is the abacus still relevant, and what does it teach?
- Abacus for Deaf-blind.
- (2) Abacus for multi-disabled.
- Abacus in Junior high school.
- Abacus in preschool.
- Abacus for adults.
- I have not had a student use the abacus in a practical manner. Do other teachers encounter the same situation?
- When and why to teach it.
- Applications.
- (2) How far to go with the abacus before transferring to the calculator?
- How to help the child that cannot learn math facts?
- Teacher training programs.
- (2) Use in the regular class.
- What to do with kids who move everything or refuse to touch anything?
- What to do with kids who can only memorize and do not know concepts?
- Who uses the abacus?
- Specific procedures that teachers use to teach the abacus.
- A supplemental workbook for extra practice.

Six people abstained from answering.

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next few paragraphs.

Investigating various strategies that teachers use to teach the abacus is an excellent idea for research. Many teachers have no or few ideas for incorporating the abacus into their students' curriculum (see question 7). A report on the specific methods teachers use to teach the abacus would give all teachers a pool of ideas for future reference. In fact, a comparison of these methods (which five teachers suggested), could give teachers an idea of current trends in teaching the abacus, understanding that some methods may be better for one student than another. Knowing more than one method for teaching the abacus will help teachers reach a larger number of students. The abacus is a very important tool. Some teachers who participated in the survey reported an interest in reading an article that explains the importance of the abacus.

There are numerous attributes making the Cranmer abacus important for students with visual impairments. The abacus has manipulative qualities that often help foster mathematics understanding (Baroody, 1989 & Sowell, 1989). Also, the abacus is an appropriate alternative for students who cannot use paper and pencil for calculating. The abacus builds understanding and procedural knowledge. Some teachers, however, argue that braillewriters or talking calculators are more efficient than the abacus. Unfortunately, these two aids lack some of the qualities that the abacus does not.

Using braillewriters is similar to using manual typewriters for mathematics. Mathematics problems require a lot of back spacing and line spacing. Talking calculators are far more procedural than conceptual. Letting students with visual impairments rely on calculators would be comparable to allowing sighted students to

work exclusively with calculators. These two tools have credibility and should be incorporated into mathematics programs. The strength of the abacus supplements the weakness of the others. The abacus is much easier to learn and quicker to use compared to the braillewriter. Also, conceptual as well as procedural knowledge can be attained from using the abacus. The abacus is an important tool for students with visual impairments (Hattendorf, 1971 & Lewis, 1970), though not all teachers of the visually impaired share this theory.

Some teachers question whether or not the abacus is a useful tool. The concerns of these instructors deserve consideration. Questions such as, "Is it worthwhile to teach the abacus?" and "Is the abacus still relevant, and what does it teach?" are valid. Some teachers have difficulties with the abacus (Rapp, 1994). These difficulties can easily lead to a lack of confidence in the abacus. One teacher reported that he/she has never had a student use the abacus in a practical manner. It is difficult to find worth in a tool that has yet to show any value (in this individual case). The next suggestion is related to the difficulties encountered by some teachers of the visually impaired.

The teachers who are having difficulties with the abacus may not be prepared to instruct with the abacus. In fact, the successful teachers may have the same preparatory problems. One teacher requested that teacher preparation programs be investigated in research. Past research has addressed whether or not abacus training is included in college programs (Steinbrenner, Becker, and Kalina, 1982). One of the next topics that should be examined concerns what actually happens in teacher preparation programs (pertaining to abacus training).

Another point of interest is the notion that many teachers requested information about teaching the abacus to certain populations. The potential value the abacus holds for people of different ages and abilities would be helpful to study. The visually impaired population is diversified. Applications of the abacus would also be nice to read about. Making mathematics real to students is important (NCTM, 1989). Lastly, a study focusing on the use of the abacus in the regular classroom was suggested by one of the respondents. The TIMSS (Third International Mathematics and Science Study) (Stigler and Hiebert, 1997) has found American students to be lagging behind countries such as Japan in mathematics. All students in Japan are taught the abacus. Perhaps there is a connection.

Some of the statements appearing in table 9 were not discussed in the previous few paragraphs. These requests are valid, but research may not be necessary to respond to them. One teacher asks if left to right or right to left is better. This comparison is made in the discussion of question seven. Other teachers wonder about state requirements for standardized testing. Simply calling the testing offices should answer these questions. When students cannot learn mathematics facts or can only learn procedures and not concepts, it is tough for teachers to deal with the frustration. Some students take longer than others, however, and the abacus can help with both of these problems. Also, a report on methods that teachers use to teach the abacus (discussed earlier) could give teachers some ideas for managing these problems. These methods might also help with the child that is tactile defensive or unable to use his or her hands appropriately.

How far to go with children before transitioning them to a calculator is determined on an individual basis. However, it is not necessary to abandon one tool for the other. Calculators, abaci, and other aids can be utilized simultaneously.

There are so many questions that need to be answered. No one has given the abacus more than secondary standing in recent research. Many questions have accumulated and few have been answered. The results of this survey are just the beginning. A few facts were uncovered, but many more questions were raised by the responses to the survey. The direction of future research will be discussed in chapter four. An analysis of the videotapes and assessments (part two of this dissertation) will be discussed in the next chapter.

### 3 VIDEOTAPES AND ASSESSMENTS

At the end of the fall semester of 1997, letters of recruitment (Appendix D) were distributed to teachers of the visually impaired in the Tucson, Arizona area. Three teachers agreed to participate in the series of videotapes and assessments. Therefore, three students and their teachers (one short of the four that was desired) were studied in the spring of 1998. A fourth volunteer was located at the end of the summer and the fourth student-teacher situation was examined in the fall of 1998.

In order to maintain anonymity, no names will be used in this dissertation. The teachers will be referred to as teachers A, B, C, and D. The students of these teachers will be referred to as students A, B, C, and D respectively. In one of the situations studied, there were two student involved. The teacher had two visually impaired students in the same class and she taught them the abacus simultaneously. I will let the letter D represent this situation. In this case the students will be students D1 and D2. The use of these names will become clear as this chapter progresses.

In order to be consistent while examining the videotapes, a protocol (Appendix F) for studying the videotapes was developed. The topics described in the protocol are the foundation for each videotape analysis. Not all topics on the protocol can be examined through the videotapes alone. Therefore, the videotapes and assessments are discussed simultaneously.

The three videotape and assessment sessions were meant to gauge the student's progress throughout a full semester of instruction. The three sessions were scheduled to

occur at the beginning, middle, and end of the semester. In the three cases from the spring of 1998, the sessions occurred in the second week of January, the first week of March, and the first week of May respectively. Case D was videotaped and assessed in the second week of September (1998), first week of November, and first week of February in 1999. About eight to ten weeks elapsed between sessions (except between the last two sessions in case D). There were some difficulties making the last appointment for session three in case D. Conflicting schedules, early release days, and winter break delayed the third session a few weeks. The extra time, however, did not allow students D1 and D2 to progress more than the other cases. In fact, case D was far less successful relative to the other three cases. These three cases are analyzed first in this chapter.

### 3.1 Student A and Teacher A

Student A was a fifth grade student attending a public school in a district neighboring Tucson, Arizona. She is blind, so her use of vision with the abacus is not pertinent. Teacher A is an itinerant teacher who spends most of her work day with student A. Teacher A has been an itinerant teacher for three years, thus teacher A has less experience than most of the teachers who participated in the survey. However, teacher A's responses on the survey were similar to the most common answers reported in chapter two..

Teacher A often utilizes the methods of a manual when teaching the abacus. Also, she feels that the IEP goals for student A are appropriate and consistently met. Teacher A believes that the abacus should be introduced as early as possible. Also,

teacher A has no special methods for teaching the abacus that could be explored by other instructors. By referring back to the discussion of the survey in chapter two, it is clear that teacher A's philosophies are common amongst teachers of the visually impaired.

Upon viewing the videotapes for this teacher and student, I noticed that teacher A used several techniques during each session. Furthermore, teacher A's methods varied from session to session. What remained consistent throughout each session was duration. The sessions lasted twenty-two, twenty-four, and twenty minutes.

Each session was one on one. Student A was pulled out of class to work privately with teacher A. The room was always quiet, so both teacher and student could concentrate on the abacus. In fact, both seemed to have a great attitude towards the abacus. Teacher A appeared genuinely interested in the abacus, and she always explained its importance and relevance to class work. Student A had no trouble keeping her attention on the abacus. Student A followed instructions and enjoyed adequate success with the abacus. Both the assessment results and the frequency of correct responses present on the videotapes support this success. Generally speaking, both teacher and student created a positive atmosphere, which led to an exceptional learning environment.

During the first session, teacher and student worked solely on the abacus. The first few multiplication problems were completed out loud. The last few were finished silently by student A. There were small breaks between problems so they could talk about the problems. They discussed why the problems were important, and how they

pertain to students A's regular class work. Also these breaks gave student A the opportunity to rest. If every session followed this pattern, monotony may have resulted in a lack of interest in the abacus. However, Teacher A was able to succeed by varying instruction to benefit the student substantially.

In the second session, they were again working on problems. They did some out loud and some silently. This time student A was using the talking calculator to check answers. Also, the braillewriter was used to record answers. Not only was student A learning the abacus, but she was getting some valuable experience with other tools that will be very important for her continuing mathematics education.

The third session was a problem solving session. Student A simply worked on problems while the teacher did other work. Teacher A was available to answer questions if they arose. Once student A got working on the problems, she had little trouble completing her assignment. She worked well without a teacher to guide her. This session also incorporated the braillewriter and the talking calculator. Student A was proficient on all three apparatuses.

If all teachers could create this type of an environment, the abacus would be a much easier tool with which to work. Granted, it is not only the teacher who determines how smooth a teaching session progresses. Student attitude is very influential as well. However, the teacher helped immensely in making all three of these sessions pleasant and educational. Teacher A responded with excitement over correct answers, while being kind and helpful when student A made mistakes. Student and teacher got a lot of work done. This was evident in the videotapes and in the assessments.

Student A progressed from simple multiplication, to division, to decimals during the semester that I observed. In fact, she surpassed her IEP goals for the year. During each assessment, student A did very well. Her most common mistakes were with the secrets. At times, she would forget to clear beads, or forget to set certain beads. For example, in the problem  $274 - 187$ , she started by clearing one from the hundreds column (thus she moved left to right). She then cleared one left (from the hundreds column) and set two in the tens column for the subtraction of eight from seven. Finally, she should have cleared one left (from the tens column) and set three in the ones column for subtracting the seven. She missed this secret. She knew it, but just made a careless mistake. The only questions she had difficulty with were those she had yet to learn from the teacher. Student A was clearly learning the abacus with good success.

Although teacher A used different tools and methods during the sessions, they were mostly process oriented. None of the lessons incorporated conceptual activities or methods. Teacher A did explain how the mathematics they were doing pertained to the regular classroom. She did not (during any of the three videotaped sessions) mention, however, why the abacus works as it does. For example, the secret for adding eight is to set one bead left and clear two. This is similar to adding ten and subtracting two. The reason the secret works was not discussed. This exemplifies ideas that were missing from all sessions for all four teachers involved in this study. The procedural knowledge was taught, while the conceptual knowledge was not emphasized.

Another aspect of these lessons was the lack of motivating activity. Though the reasons necessitating things such as billions place values were explained, there were no

precursor activities to motivate the student to work. The student was already willing and able to work hard and efficiently with the abacus. The time constraints placed on pull out sessions might have forced teacher A to act according to student A's existing motivation.

It is not known if teacher A would have the same success with a student who was less cooperative.

This first case study gives some positive results. Although the methods used in these lessons followed the pattern of showing the procedure, doing some examples, and working on problems, they were effective in helping student A learn mathematics. This was probably the most successful case of the four cases studied. Student A had a good attitude and she progressed quicker than the other students in the study.

### 3.2 Student B and Teacher B

Student B was a ten-year old boy attending a residential school for the blind. He is an upper elementary student (this school does not put children in numbered grades, but student B would be in fourth or fifth according to the teacher) working on multiplication of numbers up to three digits. In contrast to student A, student B is a low vision student. This means he has usable vision. Furthermore, student B uses his vision more than touch to read and work the abacus.

Similar to teacher A, teacher B's survey responses were not out of the ordinary. He uses manuals often, and he feels the IEP goals for his students that pertain to the abacus are usually appropriate and consistently completed. The prerequisite abilities he

requires are similar to other teachers (basic addition and subtraction, counting, etc.).

Overall teacher B is representative of the average teacher responding to the survey.

Since this case was at a residential school, most of the teaching occurred within a classroom containing two or three other students. This resulted in a different atmosphere and teaching methodology. With the other students and an aide in the room, there was more background noise than there was in case A. However, the distractions were minimal. Student B had no difficulties keeping his attention on his work. There was no pressure in the class, and the teacher made it easy for students to do meaningful work.

Teacher B used fewer variations in teaching methods than teacher A. The first session was similar to the one from case A. Teacher and student worked together through some multiplication problems. Student B talked through the procedures so the teacher could help if mistakes were made. He never made student B feel stupid for making mistakes. When problems were completed correctly, teacher B was congratulatory. The entire first session continued in this manner.

The next two lessons were simply problem solving sessions. Student B used this time to practice skills he had learned a few days earlier. Teacher B said that he would show the process, work through problems together, and then let student B practice on his own. During this time, teacher B was helping other students and occasionally checking to see that student B was working well. If student B had questions, he would ask and the teacher would come over to help with the problems.

This was a much different method compared to teacher A. However, since the situations were different (case A was one on one while case B was in a classroom with other students), these methodology differences must be expected. Teacher B divided his time equally among all of the students. He just had to spend less one on one time with each student. Teacher B made class a nice place for the children to do their work. There were a few things he did differently from teacher A. These items might be good suggestions for teacher B's future lessons.

In all three sessions, only the abacus was used. Student B did record answers on paper, but he never used a calculator to check his work. Also, similar to teacher A, there were no motivational activities to precede the abacus. He used the same method that all teachers in this part of the study used. He used a method similar to the one used in case A. Lecture, examples, and problem solving embodied the learning activities for student B. Incorporating activities that develop concepts and motivate students to use the abacus may be a good idea for all teachers of the visually impaired to explore. It may be more important in a residential school. In public schools, students participate in the activities of the regular class. These activities can be useful in explaining concepts, which help foster understanding. In residential schools, there is no such luxury. Students do not attend other classes in which mathematics is discussed.

Teacher B did a fine job making sure student B's activities were worthwhile. The videotapes revealed that student B had become efficient with the abacus. The assessments verified this claim. He did well on the multiplication problems and was nearly perfect on the addition and subtraction questions. There were only a few

mistakes on secrets and multiplication facts. He was, however, able to correct himself when a mistake was pointed out. In fact, during the last assessment, student B had already started learning three-digit multiplication. His IEP goals stopped at three-digit multiplication. Student B progressed well throughout the semester.

Student B uses the secrets and teacher B explains why the secrets work. At one point on one of the videotapes student B had to add two but there was not room. Since student B had been working for a while, he forgot the secret. Teacher B explained how adding two is like adding five and subtracting three. Student B was familiar with this type of explanation. Many teachers will give no reason why the secrets work. In fact, some may not know (I make this judgement after taking a class that trained teachers on the abacus). It is nice to see a teacher explaining the concepts behind a procedure.

Generally speaking, teacher B was effective in instructing with the abacus. The lessons were twenty-two, twenty-six, and eighteen minutes long. He knew how long student B could concentrate on the abacus. In some public school classes sighted students are being taught mathematics so that they understand the concepts as well as the procedures. Teacher B touched on this when discussing the reasons the secrets work. There is no reason not to teach students to understand as well as perform mathematics. The students with visual impairments in public school classes can get this conceptual experience from the regular classroom teacher. Students in residential schools can explore the environment to get many of the same concepts.

### 3.3 Student C and Teacher C

Student C was in a lower elementary classroom in a residential school. There were two or three other students in the classroom while the lessons were taking place. In this classroom, the background noise was much more apparent than in the first two case studies. The teacher even had to attend to a student who was making a lot of noise, which was disturbing the lesson. Student C was able to keep her attention on her work. This was impressive.

Student C is a blind student who accesses the abacus through only touch. For the first two sessions there was another student learning the abacus with student C, but he went to a new class and was unable to participate in the entire study.

Teacher C's (just like teacher A and teacher B) responses to the survey were similar to those of other teachers. Although she reported that she does not use a manual while teaching the abacus, she explained that she does teach methods that would be found in a manual. After watching the videotapes, I believe she uses the methods of a manual, though she does not refer to one during a lesson. Teacher C's responses to the IEP and prerequisite ability questions were also similar to those of other teachers responding to the survey. Also, she had no suggestions that could be useful for other teachers to explore. Teacher C is representative of the VI teacher population discussed in chapter two.

The atmosphere of the class was a little more difficult than the previous two cases. This is understandable since teacher C deals with much younger students. Teacher C's demeanor is positive. She helps when mistakes are made, and she applauds

good work. This teacher, however, is more no-nonsense than the others. She is straightforward and to the point when teaching the abacus. In all three sessions, she simply drilled the procedures with continuous questions. Combining this method with the fact that the problems student C was doing (the most difficult was two-digit plus one-digit) were not complicated, resulted in short lessons. Sessions one, two, and three lasted eleven, seven, and five minutes respectively. The last session may have been shorter because the second student who was participating in the study transferred to a new class. The third session included only one student as opposed to the first two sessions, which included two students. The teacher commented that she was stopping when she felt the students could not concentrate anymore.

Although teacher C's methods were direct, student C did learn to use the abacus. According to the IEP, student C was to learn skills up to borrowing and carrying in subtraction and addition. At the time of the last session, they were starting to add two digit numbers. Student C was already successful at carrying when adding one digit numbers to two digit numbers. Also, student C was about to begin learning subtraction. She may have been a little behind, but there was still some time left in the semester.

In all three sessions, teacher C kept her full attention on student C (of course during the first two sessions there was another student at the table working with teacher and student C). She asked questions and moved on once they were answered correctly. During this time there was very little explanation of why the secrets work. On the third videotape it was clear she had student C working to memorize the secrets. At first this seemed a little stale, but she ended the lessons with helpful activities. At the end of tape

one, she asked her students how to add two numbers to get ten or five. For example, teacher C would ask, "What number do you add to seven to get ten?" The students would respond "three," and they would move to a new number. They quickly did about ten of these and the lesson was over.

This activity was a precursor to the next mathematics lesson. Teacher C could have put this as a suggestion for question seven on the survey. Another thing teacher C did that is a good idea was to incorporate the braillewriter in abacus lessons. Although the first two lessons only involved the abacus, the third included the braillewriter. Student C had to read (in Braille) the questions she was to answer with the abacus. She did very well, and worked rather quickly. Throughout all three sessions, student C was efficient with the abacus. In fact, when the other student was present at the first two sessions, student C was more accurate. Teacher C would alternate between the children when asking for answers. Student C had a little more success.

The success attained by student C was not as outstanding as the first two cases, but those two students were older and seemed well above average with their abacus skills. Teacher C was able to teach with the abacus, and she did it in a different style than the other two teachers. There were a few other differences besides her no-nonsense approach. Teacher C did not ask her students to talk through their solutions. She watched and made comments if she saw a mistake. Also, in asking student C a two-digit plus two-digit problem on the assessment, I noticed that she was working right to left. The first two teachers had their students work left to right. The atmosphere was harder

to deal with and the teaching style was unique for this study, yet, student C was learning to use the abacus.

Teacher C's success demonstrated that there is not one single correct method for teaching with the abacus. Teacher C's style greatly contrasted with the first two teachers. In fact, all three teachers used different methods for certain parts of their abacus lessons. All three teachers gained some level of success with their students. Also, it was apparent that none of the teachers was using methods that may be found in public school classrooms. There were no motivating activities, which might have been helpful. Also, the students did not have to figure out much for themselves. Admittedly, this could be difficult, but children with special needs require the best methods available. Maybe these are the best methods, or maybe they can be improved. Future research exploring the incorporation of motivational activities and conceptual understanding of mathematics is reasonable.

If all teachers enjoyed the success described in the previous three sections, there would not be so many questions about the use of the Cranmer abacus. The last study illustrates some of the difficulties a teacher can experience while teaching mathematics with the abacus.

#### 3.4 Student D1, Student D2, and Teacher D

This was the only study in which two students were present during all three videotape sessions. Both students are in a public school fourth grade class. Student D1 is a girl with low vision. She relies completely on her vision to read the abacus. Student D2 has some vision, but his is far worse than student D1. Therefore, student D2 uses

touch when reading the abacus. Although neither student appeared particularly excited to use the abacus, both were able to concentrate on their work for most of the sessions.

The room was quiet and there were no other students who might disrupt the lessons. Teacher D pulled the students out of the regular class for their abacus lessons. The atmosphere was adequate for learning, the teacher was easy to work with and the children respected her. The teacher's attitudes and techniques were similar to those from the previous three cases.

The students would talk through problems with teacher D. She was helpful and kind when addressing mistakes and she was verbally excited when the students completed work correctly. Teacher D would ask the students to answer questions and guide them to the right answer. Sometimes the children would not talk through the procedures so teacher D would watch as the students manipulated the beads. There was no discussion of reasons as to why the abacus works as it does, and there were no motivating activities to generate interest in the abacus. I have written about these techniques three times before (when discussing the other three teachers). The similarities between case D and the others end here.

Teacher D uses techniques similar to those used by other teachers. Her students, however, were not having the same success as the other students. Student D2 did make progress during the semester. He started with adding and progressed to multiplication. He was expected to be proficient at addition, subtraction, and multiplication of three digit numbers within two months of the third videotape session. However, at the last assessment, he was still working on problems with one-digit multiplicands and

reviewing addition and subtraction. If he had been efficient with these processes at the time of the third videotape session, he would have been well on his way to completing his goals. But he could get very few problems correct independently. He would forget how to deal with adding when there was no room for directly setting numbers. For example, in the problem  $6 + 9$ , once he set the six he was unsure how to deal with the nine. He was progressing, but at a slower rate than the other students involved with the study.

Student D1, on the other hand, made very little progress during the four months of the study. In fact, she seemed to regress. During the first videotape session student D1 was having some success with setting numbers. She had yet to learn anything else. On the assessment she was asked to set 625 and she set 526. The numbers were right, but in the wrong order. During the second session, student D1 was doing some addition and even multiplication. But she could not do the work independently. She needed guidance on all problems. At the third visit, I did not require her to complete any assessment problems. Just watching her was sufficient to assess her abilities. She could not even get close to setting numbers correctly on the abacus. At one point, teacher D asked student D1 to set 300 and she set 100. She needed a lot of help to catch her mistake. Although, teacher D's techniques were not dissimilar from the other teachers, there was not as much success.

One suggestion I might have for teacher D would be to slow down a little. She spoke rather quickly and moved from topic to topic fast. During the last session, teacher D tried to have student D1 do some addition problems. She did this even though student

D1 had many problems with setting numbers. Teacher D did talk some about place value. But, she could have discussed it a little more, and given more time for information to sink in. A better understanding of place value could have helped student D1 with setting numbers on the abacus.

The two students were very close in abilities when the videotape sessions began. By the time of the last session, teacher D separated them. The two students were different in their levels of ability. Teacher D decided it would be more effective to teach them one on one. I also noticed the students (particularly student D2) were not using the secrets. Teacher D taught the counting method described in chapter two. She does use the secrets with some students, but she felt the secrets were not suitable for these two students. Both students have trouble memorizing rules. Teacher D clearly teaches according to the students' abilities. Her own preferences do not bias her decisions.

There were a number of factors that may have effected teacher D's success in teaching with the abacus. In fact, all teachers have room for improvement. If teacher D had been working with some of the other students, she may have had just as much success. Teacher D's methods and philosophies were similar to the other three teachers, but she was not achieving the same success with her students.

However, just like the other teachers, teacher D focused on procedures rather than conceptual knowledge. The students are shown a procedure and they practice them with teacher D. The homework that the regular class receives is where students D1 and D2 can practice their skills. There were no motivational activities to generate interest in the abacus. And there was no discussion about why the abacus works as it does or how

the abacus improves mathematical concept knowledge. The lessons were procedural and they lacked conceptual understanding.

All three sessions were run in this fashion. Also, teacher D occasionally incorporated other tools. The braillewriters were used during the second session (student D1 was writing with dark pens). And the talking calculator was used to check answers during the third videotaping. However, the teacher was the one using the calculator. The children never had an opportunity to check their own work. Of course student D1 had no answers to check.

Teacher D is representative of the four teachers participating in this study. Her style is similar, but her students did not achieve the same success. There are, however, many factors besides teaching methods that could have influenced this students success (intelligence, interest in mathematics, etc.). In fact (just like the other three teachers), teacher D is representative of the population of teachers who responded to the survey discussed in chapter two. Almost all of teacher D's responses to the survey were popular answers. The only difference was that she made the suggestion that the counting method may work better for some students. All four of these teachers have philosophies similar to the majority of teachers of the visually impaired.

### 3.5 Discussion

The protocol described in appendix F can be followed to answer some general questions pertaining to the use of the Cranmer abacus by the teachers and students discussed in sections 3.1 through 3.4. One question that many VI teachers would benefit from hearing the answers to is, "What do other teachers do when teaching the abacus?"

Descriptions of the methods observed on the videotapes could shed some light on this question.

All four teachers share a number of tactics. All were pleasant and showed very little frustration with their students. When their students' work was correct, the teachers responded with praise. When mistakes were made, the teachers were kind and helpful. In each case, the teachers used positive attitudes to enhance the learning environment. Lessons were halted when teachers felt the children were unable to remain focussed. Actually, many of the lessons were scheduled for a certain amount of time because the teachers understood their students' threshold. Although one teacher's lessons lasted half as long as the others, this was about the length of time her younger students could concentrate.

The actual format of all twelve videotape sessions was similar as well. Basically, each teacher explained a process, worked through some problems with the students, and let the students practice on their own. The extent to which each of these three items was used differed in every case. There were no motivating activities and very little discussion of conceptual knowledge. By considering some of the questions that have come about regarding the value of the Cranmer abacus, it is time to incorporate some new methods in teaching mathematics with the abacus to students with visual impairments. Most likely these teachers were trained to use the abacus in a similar fashion to how they teach with the abacus. It is difficult to fault the teachers for the methods they use. Actually, the styles these teachers used may be the most effective.

However, some further investigation into letting the students experience and develop ideas about mathematics and using the abacus could be very exciting.

There were a number of things that teachers did differently in this study. One teacher incorporated the braillewriter and the talking calculator in many of her lessons (teacher A). Though some of the others occasionally incorporated these tools, none of their students gained any valuable experience with them. Also, half of the students worked from left to right, while the other half worked from right to left. The best direction is still a matter of debate. Also, the secrets were taught by three of the teachers. The fourth one said she teaches them to some students, depending on the abilities of the students. However, the students she was teaching the counting method to were the lowest achieving of all students in this study. The reason teacher D was teaching the counting method was because she knew these children would have difficulties memorizing the secrets.

All of the students had trouble with the secrets. The trouble was usually minimal, but each student from cases A, B, and C made mistakes with the secrets. For this reason, I felt that some exploration of the secrets could be helpful. By helping the students to develop and understand the secrets and see how they work, they might have an easier time using them to solve mathematics problems.

There were also some patterns that teachers may find helpful to know about. Of course the residential school students did their work in the classroom. The public school students were pulled out for one on one instruction. I have seen very few situations in which itinerant teachers worked with their students on the abacus in the regular

classroom. Some teachers help their students follow along with regular classroom lessons and this is a good idea once the student has learned an operation on the abacus.

Two of the students who were studied used vision to access information from the abacus. When the condition is stable, this is great. The abacus has white beads on a dark background to accommodate low vision users. Unfortunately, one of the students from this study has an unstable condition, and it might be helpful to have her do some of the work tactually to prepare her in case her vision decreases. Although she will have the memory of the use of her vision, transitions would be smoother if tactual experience was increased. This student's teacher realizes this, but it is difficult to convince a young student to use touch when she is able to use her vision.

Each teacher tried to create a quiet environment so the students could concentrate. Although the lower elementary case (C) was more hectic than the others, all students were able to focus on their abacus work. The teachers created appropriate environments with their choice of location and their attitudes. They all appear to be effective teachers of the visually impaired. In the one on one situation, the teachers kept their attention on their students. When other students were present (the residential cases) the teachers were able to attend to all of the students. At times, the residential teachers needed to work one on one with particular students. In these cases, the aides would attend to the rest of the class. All students received optimum attention.

In chapter two, I questioned two things. The ability of teachers to teach the abacus and the training these teachers receive in their training programs. Since I feel the teachers in this study are good, I am not concerned with their ability to teach the abacus.

Only the training these teachers receive, particularly regarding mathematics education, can be greatly improved. Few of the teachers from the surveys or the videotapes exhibited mathematical backgrounds. Of course, teachers of students with visual impairments are not expected to be experts in mathematics education. However, increasing these teachers understanding of mathematics could be helpful in optimizing the use of the Cranmer abacus.

## 4 SUMMARY AND CONCLUSIONS

### 4.1 General Discussion

This section is devoted to discussing the questions listed in chapter one using data from the survey and or the videotape and assessment study.

#### *How much formal training do current teachers of the abacus have?*

Tables 1 and 2 list the responses given to the background question on the survey pertaining to experience and training. The primary goal of this question was to determine if there was an increase in the number of teachers who were trained on the abacus in college. Unfortunately, no conclusive information resulted from this question. Table 1 does show that teachers ranged from three to twenty-seven years of experience. Also, table 2 illustrates that eleven out of nineteen (57.9%) VI teachers were college trained. There were too many non-responses, however, to the training part of the request. The percentage is very close to the sixty-three percent reported in the Steinbrenner and Becker study (1982), but the numbers were not large enough to draw valid conclusions.

Table 3 combines experience with training. Since the Steinbrenner and Becker article came out in 1982, a comparison between the years of experience of college trained (on the abacus) teachers versus non-college trained (on the abacus) teachers could have solidified any arguments about the existence of abacus training in college. Unfortunately, only seventeen people gave both training and experience on the survey. The college trained teachers averaged 12.8 years of experience while the non-college-trained teachers averaged 15.8. Although the numbers were small, there are hints that

the occurrence of abacus training in college has increased. I believe this question should be asked again in future research pertaining to the abacus.

None of the other questions on the survey received such sparse responses. The training and experience request was not a numbered question. Participants may have overlooked its importance. Also, There was very little space allotted for this request. It was easy to miss. The question should have been worked specifically to ask for both how teachers were trained and how long they have been teaching. These are things to remember for future research projects.

*In which grade do teachers introduce the abacus to their students?*

The range of grades for this question was pre-school to fourth grade. However, most teachers responding to the survey prefer to introduce the abacus in early elementary grades (second grade or earlier). This is good information for teachers to know if they are unsure when to introduce the abacus. Knowing what other teachers do when teaching may help develop one's own philosophies. The wide range of responses could be of some concern. The age difference between children in pre-school and fourth grade could be six years or more. Most children with these age differences have different abilities. The reason behind the variations in responses to question five is unclear. Perhaps the variation in cognitive development in children with visual impairments (mentioned earlier) is a factor. Also, Each most teachers only have abacus students once every few years. They may not have had enough students to make a good estimate. Never the less, variations are to be expected, however, the range of responses

to question five is a little wide. No particular grade seemed to be the clear-cut choice for introducing the abacus (although some grades were more popular than others were).

*What skills do teachers require of their students before introducing the abacus?*

Table 7 lists numerous skills required by VI teachers before introducing the abacus. All of these were discussed in chapter two. The most common answers were basic addition and subtraction, counting, place value, and one to one correspondence. Other skills were not commonly required among teachers who participated in the survey. For new teachers or teachers who have had difficulties incorporating the abacus into their students' curriculum, this information could be helpful. Most of the teachers believe they are having success with the abacus (see questions two and three). A familiarity with the philosophies of successful teachers could help other teachers develop their own philosophies. By knowing the abilities required by successful teachers, one can develop ideas about what skills to work on before introducing the abacus.

I have only one suggestion to complement the popular answers to question six. All of these skills can be practiced with the abacus. Like Braille, the abacus takes time to master. By waiting until students have certain mathematical concepts mastered, the students are losing valuable practice time with manipulating the abacus. When a student is learning to add two-digit numbers, the challenges do not need to be compounded with the difficulties of learning to work the abacus.

For example, a student can learn to add using large manipulatives (fruit, sticks, etc.). The abacus can be used to reinforce the ideas of simple addition. Then when it is

time for longer addition problems, the student will have experience using the abacus.

Also, they will be familiar with the process of addition on the abacus. This would allow for more time to be spent on the addition and less time to be spent on learning to use the abacus.

Reading about colleagues' ideas might be a good way to improve your teaching abilities. However, some of the teachers seem to use a backward approach to teaching the abacus. They teach mathematical ideas before introducing these ideas on the abacus. Why not include the abacus among the other activities used to teach concepts such as counting and place value? The teachers participating in the survey believe they are achieving success with the abacus. The students in the videotapes support these beliefs. Of the five students participating in the videotapes, three made excellent progress, one made some progress, and only one made no progress. The student who made no progress, however, received a lot of practice with manipulating the abacus. When she is ready to learn to use the abacus efficiently, she will not have the difficulties incorporated in learning to work with new tools.

*What ideas do teachers have to motivate their students to learn the abacus?*

A number of tactics that VI teachers use to motivate their students to learn the abacus were discussed in chapter two. Some of the ideas were good, while others were contradictory or reflected individual preferences. Learning the techniques fellow teachers use can help people strengthen their teaching repertoire. Unfortunately, only half of the respondents had activities to share. Participating teachers wanted to learn about what other teachers do with the abacus (see results to question 8), suggesting that

some teachers are genuinely stumped when it comes to creative and effective ways to teach with the abacus.

It would have been nice to hear about more of the methods that teachers use when teaching with the abacus. Some of the participants who gave no response could learn a lot from other teachers. Also, some of the teachers who gave a non-response may not have realized that they have valuable ideas to share. They may have felt their methods to be standard or commonly used. It would have been great to know a lot more about what each of the participating teachers do with the abacus.

It was noted earlier that all four teachers in the videotapes taught on the abacus with examples and practice problems. In fact, they all used these methods effectively, and their students were learning to use the abacus. This shows either an inability or an unwillingness to use more creative methods in their teaching. Also, it could mean that these are the best methods these teachers have found. In response to question seven, one teacher reported that she did not know what other teachers did with the abacus. However, this teacher incorporated an excellent idea into her lessons.

This teacher used an activity that might be useful for other teachers to explore. Unfortunately, she did not explain this activity when responding to question seven. This teacher had not discussed secrets of addition, but she was preparing the children to learn them. At the end of one of the videotapes, she was having the children figure out two numbers that add to ten or five. In fact, Mae Davidow in "The Abacus Made Easy" calls these partners. For example, four and one are partners in making five. These ideas are the basics behind the secrets of addition. I thought this was a great idea. It prepared the

children for the secrets, even though the children did not realize it. Preliminary activities are effective in motivating and fostering understanding of mathematical topics.

Many teachers may have good ideas for teaching with the abacus; they just may not realize it. A more widespread study of what teachers do in the classroom (or one on one) may uncover some very exciting techniques for other VI teachers to explore. Teachers should have a variety of ideas for teaching any subject. Also, teachers should realize when they are using tactics that are effective and be able to describe them clearly to others. I believe these problems to be a result of training. The small amount of mathematics course work and practicum required of certified teachers of the visually impaired hinders their ability to create and detect reasonable methods for teaching with the abacus. However, this is merely conjecture. A more in depth investigation of the abacus-training teachers receive is warranted. Previous research only examined whether or not training occurred in college course work. A study of what happens within these training sessions could be enlightening.

*Are IEP goals appropriately set for the students they affect?*

*Do teachers and students consistently meet their IEP goals?*

There appears to be a lack of understanding among teachers of the visually impaired concerning the use of the abacus. Most teachers seem to know the procedures, but they are not sure of the best techniques for instructing with the abacus nor do they understand the mathematics involved. These potential problems, however, do not seem to hinder the success many of the teachers obtain with their students.

The responses to questions two and three imply that most teachers feel they are successful when teaching the abacus. Few participating teachers felt as though the IEP goals for their students were inappropriate or not successfully completed (see table 4). This success implies that the abacus can be a useful tool for many children with visual impairments.

The videotape and assessment sessions support these claims. Four out of five students in the study completed their IEP goals. In fact, two of them surpassed their goals for the semester. The student who made no progress had no IEP goals pertaining to the abacus. The teacher decided to try teaching her the abacus at the beginning of the semester when I was visiting. However, this student was introduced to concepts such as setting, adding, and subtracting numbers. For the most part, the students in the study progressed nicely through the semester. The success achieved by the teachers and students strengthens the argument for the potential benefits of the Cranmer abacus.

*What information would teachers of the abacus like to learn more about in a research article?*

In contrast to the implications of the success discussed in the previous section, some teachers question the effectiveness of the abacus. There were numerous responses to the question concerning the information teachers would like to know about the abacus (question 8 on the survey). Each of these statements was discussed in chapter two. One of the most prevalent inquiries targeted the usefulness of the abacus. Nine respondents suggested a study dealing with the relevance of the abacus in teaching mathematics today.

This is another good idea for future research. Two good arguments can be made for the incorporation of the abacus into the mathematics curriculum for students with visual impairments. The success described above is the first. The second involves broadening the experiences that students have with the abacus. Showing procedures and giving problems for practice is not enough to maximize understanding of the abacus and comprehension of mathematics. For example, the abacus is set up nicely for activities pertaining to place value. Furthermore, melding some of the tactics used by public school teachers into the curriculum for students with visual impairments could improve these students' abilities with the abacus and their mathematical understanding.

*Which manual(s), if any, do teachers prefer to use when teaching with the abacus?*

The manuals would be a good place to begin when trying to update teaching methods. The manuals are confusing to many VI teachers (Rapp, 1994). All teachers must have a good understanding of the abacus before they can teach with it. The manuals show the procedures and explain why they work; they provide examples and problems for practice. The most popular were listed in table 5.

These manuals usually focus on procedures and practice problems. There is not a lot of discussion of mathematical concepts nor are there any activities to motivate students to use the abacus. It would be helpful to supplement the information and exercises in the manuals with some activities that build concepts and understanding. Some teachers find the abacus easy to learn. Others have difficulties that must be addressed. Why not teach the abacus the same way we teach mathematics?

An assortment of activities that can be included in a teacher preparation program would be a good place to start in updating the training teachers receive with the abacus. Although the manuals should not be discarded, these activities could supplement the information in manuals. Also, a conglomeration of activities to be used with students would be an excellent resource for teachers. Discarding the existing manuals is not what I recommend. Supplementing the manuals with activities that motivate and foster understanding would help more teachers and students become comfortable and efficient with the abacus.

#### 4.2. Conclusions

Chapters two, three, and the first part of four examine many of the intricacies regarding teaching the abacus to students with visual impairments. Specific conclusions are drawn throughout these chapters. This section will be a summary of a few generalized topics. Also, I will restate some ideas that may be interesting topics for future research and list some possible ways to improve teacher education (pertaining to the abacus).

Earlier in this dissertation I discussed some concerns I had with teacher training programs and teachers' knowledge of the abacus. After analyzing the data from the surveys and the videotape and assessment study, these concerns remain but are lightened. Each teacher in the videotape study did a good job instructing with the abacus. Only one student had troubles, but her teacher used methods similar to those of the other teachers in the study. These teachers deserve applause for their ability to create positive learning environments and to teach the mathematics they know. The

teachers who participated in the survey deserve the same acknowledgment according to their responses to questions two and three. VI teachers are having success teaching the abacus to students with visual impairments.

This success is an excellent argument for the continued use of the Cranmer abacus. The training programs these teachers participated in deserve some credit as well. The concerns cited above deal mainly with how VI teachers are trained to use and teach the abacus. Being a good teacher requires much more than simply knowing the subject matter. The teachers participating in the videotapes show teaching skills that should be applied in all subjects. The training programs play a major role in helping teachers develop their teaching skills. Also, it is clear that it is possible to be successful when teaching with the abacus even if the teacher lacks a thorough understanding of mathematics.

There were several suggestions of ways to improve the preparation that VI teachers have pertaining to the abacus scattered throughout this paper. These suggestions were:

1. To do more than simply illustrate the use of the abacus in preparation programs.  
Give the teachers the knowledge of how and why the abacus works as it does.
2. To incorporate some activities that may be found in preparation courses for public school teachers. The activities used to build future elementary school teachers' knowledge can be adapted to build VI teachers' knowledge of the abacus.
3. To create a set of activities to supplement the information in current abacus manuals.

The people who train future VI teachers on the abacus could learn by watching how many elementary mathematics content courses are taught. This will help the teachers understand the abacus better, and in turn, give them a base of knowledge to develop activities for their students. Teaching students with visual impairments is different from teaching sighted students. But, this does not mean we should ignore current views on teaching mathematics.

There were also a few ideas for future studies that appeared during discussion of the surveys or the videotapes.

1. A specific study of how teachers learned to use the abacus and how long they have been teaching. Comparisons between the two can give solid information about the incorporation of abacus training in college programs.
2. A study comparing manuals. Determining the most understandable methods can lead to some improvements in the information found in current abacus manuals.
3. A study of what actually happens (pertaining to the abacus) in preparation programs for VI teachers.

Investigating the training VI teachers receive is the next logical step in improving the efficiency in using the abacus. These investigations would be helpful in improving what are generally good programs. Once we know what the majority of training programs do with their students, we can begin working to increase VI teachers' teaching abilities and understanding of the abacus. It is already clear that many VI teachers are not well versed in mathematics education. Broadening these teachers' abilities in instructing mathematics and the abacus could help students with visual impairments

better learn mathematics through the use of the Cranmer abacus. Continued study of the abacus is necessary to optimize the mathematics experience of students with visual impairments. Since there was such a long period between studies pertaining to the abacus, this dissertation is only a first step. Teachers are succeeding when they instruct with the abacus. However, improvements can be made. The success proves that the abacus can be effective. Further research will facilitate the realization that the abacus is a useful tool for students with visual impairments.

## APPENDIX A

## HUMAN/ANIMAL SUBJECTS APPROVAL

Human Subjects Committee



1622 E. Mable St  
 P.O. Box 215137  
 Tucson, Arizona 85721-5137  
 (520) 626-6721

8 May 1998

Scott Sakamoto, B.S.  
 c/o Steve Willoughby, Ph.D.  
 Department of Mathematics  
 Mathematics Building, Room 504  
 PO BOX 210089

RE: A STUDY OF THE USE OF THE CRANMER ABACUS IN TEACHING  
 MATHEMATICS TO STUDENTS WITH VISUAL IMPAIRMENTS

Dear Mr. Sakamoto:

We have received documents concerning your above cited project. It is our understanding that most of the data collection for this project has already been completed and that you were unaware IRB review should have taken place prior to enrollment. If this project had been submitted prior to study initiation, you would have been notified that regulations published by the U.S. Department of Health and Human Services [45 CFR Part 46.101(b) (2)] exempt this type of research from review by our Committee. Please submit any future proposals for review prior to initiation.

Thank you for informing us of your work. If you have any questions concerning the above, please contact this office.

Sincerely yours,

A handwritten signature in cursive script, appearing to read "W.F. Denny".

William F Denny, M.D.  
 Chairman  
 Human Subjects Committee

WFD:js  
 cc: Departmental/College Review Committee

## APPENDIX B

### THE SURVEY

This survey is for teachers of the visually impaired and blind who instruct regularly on the Cranmer abacus. Please only answer the questions pertaining to the situation where the student has no other reported disabilities besides visual. Please return your responses by November 14, 1997. Thank you.

Employer:

District:

Position Title:

Date:

Training and experience using the abacus (include years of teaching):

For items 1-3, use the scale 1=always, 2=usually, 3=sometimes, 4=rarely, 5=never

1. When teaching the abacus, do you follow the methods of a manual?

1      2      3      4      5

2. Do you feel that you consistently meet the IEP goals pertaining to the abacus for your students?

1      2      3      4      5

3. Do you feel that the IEP goals pertaining to the abacus set for your students are appropriate?

1      2      3      4      5

4. What manual(s) do you use in teaching the abacus?

5. In what grade do you usually begin instructing on the abacus?

6. What mathematical abilities do you require your students to have before beginning instruction on the abacus?

7. Do you use any methods that you feel may be useful for other teachers to explore? Explain.

8. In reading an article dealing with teaching and learning the abacus, what questions and ideas would you like to see addressed

## APPENDIX C

### COVER LETTER

For many years, the abacus has been an important tool used by visually impaired students to learn mathematics. I find that most of the research I read was written in the sixties and seventies, and I am curious about present applications and implications. I am a doctoral student in mathematics, and I am working on a study with Jane Erin at the University of Arizona dealing with the Cranmer abacus. Rather than addressing only my own thoughts in this study, I prefer to entertain the ideas of professionals around the country. I understand that you are in a position to help me distribute these short questionnaires to VI teachers who regularly instruct mathematics on the abacus. Please distribute one envelope (containing the one page survey) to each teacher you feel can help in my research. I would appreciate your cooperation in this survey greatly! On each form I have asked that they be returned by November 14, 1997. I have also sent self-addressed stamped envelopes in order to minimize any participants' expense.

If you or anyone else has any questions about the survey, don't hesitate to call or e-mail. Thank you for your time.

Phone: 520-881-0109 e-mail: [sakamoto@math.arizona.edu](mailto:sakamoto@math.arizona.edu)

Sincerely,

Scott Sakamoto

P.S. You may make copies of these surveys if you know of more teachers who would like to participate in this study. Thanks again.

**APPENDIX D****ASSESSMENT TOOL**

This is an example of the assessment that was used during part two of this project. Three versions will be made, one for each session. They will only differ by numbers used, length and content will remain the same.

Student name:

Date:

Setting numbers

1. 28

2. 640

3. 907

Addition

4. 7

+8

**APPENDIX D- *Continued*****5. 36****+15****6. 22****+36****7. 374****+170**

**APPENDIX D- *Continued*****Subtraction**

**8. 4**

**-2**

**9. 16**

**-3**

**10. 32**

**-18**

**11. 274**

**-187**

**APPENDIX D- *Continued*****Multiplication****12. 5****x3****13. 6****x24****14. 472****x 6**

**APPENDIX D- *Continued***

**15. 83**

**x56**

**Division**

**16. 14/7**

**17. 416/3**

**APPENDIX D- *Continued*****18. 336/16****19. 507/8****20. 762/12****Decimals****21. Set 4.8**

**APPENDIX D- *Continued*****22. Set 29.4396****23. 25.35****+ 2.9****24. 5.6****x 3.1****25. 49.7/7**

## APPENDIX E

### VOLUNTEER SOLICITATION

This letter was given to teachers that use the abacus with at least one student. Its purpose was to solicit volunteers for part two of the research project.

Dear teacher of the visually impaired,

As a doctoral student in mathematics at the University of Arizona, I believe the abacus is an important tool in educating students with visual impairments. Along with Jane Erin, I am conducting a semester-long study investigating some basic facts about teaching and learning the abacus. I am trying to find volunteers who have at least one student that they regularly instruct on the Cranmer abacus. The only work involved for you is a written statement of your philosophies about the abacus, teaching methods, and any other abacus related topic you feel important. In order to study the ways students learn the abacus, I need you to identify one student to whom you regularly teach the abacus. Information about instructional procedures will be gathered in the following manner.

1. Three times during the Spring semester of 1998 (once at the beginning, once in the middle, and once at the end) I will videotape one or two sessions of you teaching the abacus. None of these tapes will be distributed in any way, and I will request parent permission according to your school's policies.
2. I would like to assess each of your student's progress shortly after each video session. I will only use this information for my study.

**APPENDIX E- *Continued***

No names will ever be used in this project, nor is this an attempt to evaluate the teacher. I noticed that very little recent research pertains to the Cranmer abacus, and I would like to see how this tool is being utilized today.

For your participation, I will gladly give you a complete report of my findings.

Thanks for your time,

Scott Sakamoto

Note that the same contact information given on earlier appendices will be given at the bottom of this form. Also they will be delivered in self-addressed stamped envelopes.

**APPENDIX F****PARENT PERMISSION**

January 5, 1998

Dear Parent,

I am a doctoral student in Mathematics at the University of Arizona. I am studying the use of the Cranmer abacus in teaching mathematics to students with visual impairments. This letter is a request for permission that your child participate in my research. Your child's involvement includes the following.

1. Three videotape sessions (one in the beginning, middle, and end of the Spring school semester of 1998). These tape sessions will record both teacher and student in the learning environment.
2. Three assessments (also at the beginning, middle, and end of the semester). These assessments simply will monitor their progress made through the semester.

This study is by no means an attempt to evaluate your child. The information gathered is for my use only, and no names will ever be used. If you agree to the above, please sign the consent portion of this document below. Thank you for your time.

Sincerely,

Scott Sakamoto

Child name

Parent /Guardian name

Signature

Date

## APPENDIX G

### VIDEOTAPE PROTOCOL

The following is a list of items that were examined in the review of the videotapes. Of course these topics do not represent the only items to be discussed. However, they give a basis for extracting information from the videotapes.

#### 1. Teaching Method

To determine teaching method, the following items will be noted:

Do the students go right to left or vice versa?

Are the secrets used?

Do students talk through problems with their teachers or is much of the work silent?

Are there any activities used to motivate students to use the abacus?

Do teachers focus on the student or do they wander the room helping others at the same time?

How do the teachers respond to correct and incorrect answers?

2. Duration of lessons will be noted.
3. Do students use their vision when it is appropriate?
4. Can the students stay focussed on their tasks for an entire lesson?
5. What is the atmosphere like in the teaching environment? Is it loud or quiet? Is it friendly or strict? Etc.

Also, since I am videotaping only three sessions, it was easy to miss some techniques the teachers may use. Some informal questioning may occur at any of the sessions.

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