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OPTICAL REFERENCE PROFILOMETRY

by

Stephan Richard Clark

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2000
As members of the Final Examination Committee, we certify that we have read the dissertation prepared by Stephan Clark entitled Optical Reference Profilometry and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

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I hereby certify that I have read this dissertation prepared under my direction and recommend that it be accepted as fulfilling the dissertation requirement.

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ABSTRACT

The Optical Reference Profilometer is a new coordinate measurement machine (CMM) configuration that utilizes a special optical referencing frame to provide a highly stable and highly accurate surface measurement. This new referencing frame provides several mechanical advantages that make it possible to use lower precision mechanical components while still maintaining a high measurement accuracy.

The Optical Reference Profilometer also provides a reduced measurement sensitivity to thermal variations of the system. With the addition a Super-Invar metering rod network, this CMM system is essentially thermally insensitive to temperature changes on the order of 1°C. This special feature makes the Optical Reference Profilometer functional at a high measurement accuracy level in an open lab environment.

For the current dissertation work, two profilometer designs were built: a prototype and a second-generation system. A discussion of both systems will be given where the advantages of the optical reference frame design will be shown. This dissertation will end with a discussion of the overall system performance and plans for future work that would increase the overall system accuracy.
The optical reference profilometer has proven to be a viable testing device. It provides a high accuracy surface measurement, 100nm peak-to-valley and 15nm rms for surface slopes up to 20 degrees, with simple mechanical structures while maintaining the versatility to measure a variety of surface shapes.
CHAPTER 1 INTRODUCTION

1.1 PROJECT OVERVIEW

The Optical Reference Profilometer was developed as part of a multi-year DARPA-funded program, which began in 1996. The project goal is to investigate the design, fabrication and testing issues related to the implementation of conformal surfaces in missile domes and aircraft windows (Shannon, 1999). The program is a consortium effort. The University of Arizona as well as several industrial and academic partners (Raytheon Systems Company, Boeing Aircraft Company, Rochester Photonics Corp., University of Rochester, Sinclair Optics, ORA, Morton CVD) worked together to develop new technologies that would meet the program goals.

The drive for the program is to improve current (optical-seeker) missile domes as well as aircraft windows used with optical systems. Different window shapes are needed to improve speed, distance of travel and survivability. The optical domes/windows needed are to conform firstly to aerodynamic and mechanical constraints as well as possible radar cross section issues. After the basic surface shape that meets these requirements is found, it will be optimized to achieve the best optical performance while still maintaining the base shape requirements. (The name conformal optics is applied to these type of optical surfaces.) In the past, the design process was reversed. The optics were designed first and the mechanical system was then molded around it. This design
methodology resulted in aerodynamic performance constraints that were undesirable, because aerodynamically favorable shapes are not necessarily optically favorable.

In order to give a better understanding of the changes that are required, two examples will be given—one for the missile dome and one for the aircraft window. Conventional optical-seeker missile domes are hemispherical in shape. The main reason for this shape is related to optical testing issues. A spherical shape is better optically than it is aerodynamic. The fineness ratio needs to be increased to make the missile more aerodynamic.

The fineness ratio is the ratio of the length of the dome to the diameter of the dome (L/D). The goal for this program was to take the hemispherical dome of fineness of \( \frac{1}{2} \) and stretch it out to fineness of 1 or 1.5. The resulting conformal shape would be more elliptical in nature and will have less aerodynamic drag. Figure 1.2 shows a comparison of the performance for the conventional missile dome and the conformal missile dome.
Current aircraft window designs can also be improved upon. Current window designs are either spherical or consist of a combination of faceted flat windows. In order to improve upon these current designs, a more toric shape should be used. This shape would enable the window to sit flush with the fuselage of the aircraft or on the wing. Making the window shape less spherical adds versatility to the overall system design—ultimately allowing the window to be placed on the aircraft in a more optimal location to get the biggest field of view possible with better targeting, tracking and damage assessment capabilities. Figure 1.3 shows a comparison of possible conformal shapes to conventional shapes for the aircraft window.
1.2 PROJECT TASK

Testing of conformal surfaces was and is a real challenge (Shannon, 1999). These surfaces are highly aspheric and in some cases, are non-rotationally symmetric. Rotationally symmetric aspheric surfaces are often tested using some type of interferometric null configuration. Interferometry is a differential measurement; i.e. the output is the difference between a reference wavefront and the test wavefront. In practice one would like the reference wavefront to match the wavefront from the desired test part shape. For example, if a spherical surface is being tested, the reference wavefront should be spherical. This means that if the test part is the desired spherical shape, then the output of the interferometer is a null, i.e. one bright fringe or one dark fringe. If the test part produces a non-spherical wavefront shape then the output of the interferometer is not a null. To correct for this situation, some type of null corrector can be placed in the interferometer. These nulling elements introduce the opposite amount of aberration into
the interferometer to cancel or null the wavefront that is either transmitted or reflected from the test optic. Meaning that if the test part is the desired shape, then the output of the interferometer is again a null situation. (Interferometric Null Testing is described in more detail in *Optical Shop Testing* by Daniel Malacara (1992)).

Null tests are generally expensive, alignment is difficult and the null elements must be validated. Test result interpretation can also be problematic. In the presence of test part errors, the rays do not follow the same design path through the optical test system, as in the initial design, and induced aberrations may result.

An alternative to this approach is stylus profilometry. This alternative will be discussed in greater detail later in this dissertation.

The testing program for conformal optics required an instrument capable of measuring highly aspheric surfaces that are relatively flat or have low curvature. Examples might be conformal windows or aspheric corrector elements used to correct the aberrations within an optical system. These parts could be either rotationally symmetric or non-rotationally symmetric. The surface measurement accuracy dictated by the program was around 150nm peak-to-valley (pv). This is the maximum allowed deviation of the measured surface from the actual surface. Based on this requirement, a goal of 100nm pv accuracy was established for this instrument. The final instrument was
required to work in an open-lab environment with a test part size capacity of 50.8mm x 178mm x 178mm, with a maximum measurable surface slope of 40 degrees.

For this portion of the conformal optics testing program, it was decided that a vertical probe Coordinate Measurement Machine (CMM) was needed. Other portions of the conformal optics program would address optical tests (transmitted wavefront tests). A CMM was felt to be a baseline testing procedure that would provide the versatility needed to test both steep aspheric and non-rotationally symmetric surfaces at the required accuracy level.

At the outset of the conformal optics program, a CMM was not available among the consortium members. It was felt that this type of testing technology was necessary for the success of the program, so a CMM was designed and built that would meet the program testing goals.

In the following chapters a discussion of an Optical Reference Profilometer (a type of Coordinate Measurement Machine) will be given. During the project two systems were developed: a prototype and a second generation Optical Reference Profilometer. Mechanical layouts, data correction algorithms, analysis software and system operation results will be given for both instruments. This dissertation will conclude with a discussion of the project goals and future work areas.
CHAPTER 2 OPTICAL REFERENCE PROFILOMETRY

2.1 COORDINATE MEASUREMENT MACHINES (CMM)

CMMs provide surface data in point map form. The CMM uses a probe/stylus that is brought in contact with the test surface. The relative displacement of the probe from a reference position is then measured. This displacement data can be taken over the whole surface, or along a single profile. The point sampling locations are not dependent on the surface type being tested, which is the feature that makes the CMM so versatile and attractive for this situation. In general, this surface point map is used to determine surface figure information. If a fine enough (or continuous) sampling is used, some machines can give surface roughness measurements (Bennett, 1981). In our program, the CMM was used for surface figure information only.

2.1.1 CONVENTIONAL COORDINATE MEASUREMENT MACHINES (ACCURACY AND LIMITS)

A wide variety of CMMs are available in the market today, each of varying accuracy and test part size capacity. CMMs have measurement accuracies ranging from around 2-3μm down to as low as a few nanometers (Shiozawa, 1998). One of the principle limitations on vertical probe or three-axis CMMs is part slope. There are five-axis machines available to remove this part slope limitation, however these systems have additional mechanical issues that must be dealt with. The focus of this dissertation is on
three-axis systems and no attempt will be made to discuss or compare these with the five-axis systems available.

For the highest accuracy CMMs (peak to valley measurement departures of a few nanometers), the maximum slope tends to be around 40-45 degrees. Beyond this surface slope 'limit' the accuracy goes down. There are several possible reasons for this 'limit'. The first is that a vertical probe is essentially a cantilever and is subject to lateral deflection. In section 4.2, the deflection effects of a stylus probe are discussed. In that section it is shown that as the surface slope increases, the probe deflection increases due to the increased transverse force for a given stylus resting weight.

The second possible reason for this 'limit' is that in general the high precision CMMs use air-bearings for the stylus. Air bearings tend to keep the stylus slide centered in the air sleeve, but the air-bearing slide can move in the transverse direction given a sufficient force. This allows the stylus to slide down the part surface in an unpredictable fashion as the contact slope becomes too great. Stylus sliding has two effects. It introduces a cosine error in the height measurement due to the stylus tilting in its sleeve, as well as introduces a positioning error, because it is actually contacting the test surface at a different location to that measured by the system. It should be noted that this effect is dependent on the actual air bearing used, therefore the region over which acceptable repeatability is achieved can be improved with a better stylus assembly design.
2.1.2 CMM DESIGN ISSUES

Like any high precision machine, a great deal of care must be taken to optimize the system accuracy. There are several factors that must be addressed to maintain a high accuracy instrument: geometric errors—squareness and parallelism, thermal expansion and thermal induced distortions, kinematic errors—motion errors between moving parts, static and dynamic errors—deflections and vibration and probe/tool interactions (Elshennawy, 1988).

Because these instruments have been around for many years, many people have investigated the above mentioned design issues. Several papers and books, as early as 1929 (Bosch), have been written on the construction, calibration and error reduction of CMMs. The National Institute of Standards and Technology has studied CMM design and performance for several years (Swyt, 1993). Other papers include those written by the following: Elshennawy (1990), Ni (1992) and Mast (1995).

The goal for this project was to build a CMM and to investigate several new design concepts to add to existing high accuracy CMM measurement technology. Obviously, the right starting point before beginning to improve upon existing technology is to review what others have done in that area. In the review process, common design features and operational methods were sought. These common features reveal operational
requirements and proven design techniques that provide the highest accuracy available today.

After reviewing the layouts for several high accuracy CMMs, a few common design features/philosophies emerged (instrument size, air-bearing slides and stages and temperature control), which will be discussed briefly here. To help illustrate these features a comparison of two commercial CMMs will be made. The first system was built by the Nikon Corporation (1-6-3 Nishi-Ohi, Shinagawa, Tokyo, 140 Japan). The second was built by the Anorad Corporation (Hauppauge, NY 11788).

Size is the first design feature investigated. Nikon Corporation's high accuracy CMM is called the Super Master. It has a vertical measurement range of 4" with horizontal range around 16". The overall system size is around 8'x8'x8'. The size is due to a large oil damping base and a large granite bridge assembly (Shiozawa, 1998). Figure 2.1 gives the system layout.
Anorad Corporation also makes a high accuracy CMM. The Anorad CMM will only give a 2-dimensional or linear profile. The part being tested must be moved manually for each new surface profile. Like the Nikon machine, the Anorad CMM has a granite bridge assembly. However, the Anorad system has a much larger horizontal measurement range of about 60" (Mast, 1995). Figure 2.2 shows the Anorad system.

Although the reasoning for using such a large system frame is not explicitly mentioned by either company, it is possible to speculate on the reason. The most obvious reason is rigidity. These massive structures are stiff and will not deform unless acted upon by a
Figure 2.2, Anorad Profilometer (All dimensions are in inches)

large force. Rigidity is essential to maintain a stable referencing structure during a measurement. This stability keeps the system accuracy and repeatability high.

The second common design feature used in these instruments is air-bearing slides and stages. Both the Nikon and Anorad instruments use air bearings for the stylus and the motion stages. The Nikon instrument moves the part on an air-bearing stage and uses a double air-bearing stylus assembly for height measurements. The Anorad instrument uses an air-bearing slide to move an air-bearing stylus assembly over the stationary part. Air
bearings can be made very flat (surface flatness can be less than 0.0001", or 2.5μm) and therefore make good distance-referencing structures. Air bearings are also very smooth, thus reducing motion induced measurement errors.

The final commonality between these instruments is temperature control. The Nikon instrument is held to less than +/- 0.02 °C from the base start temperature. The Anorad system does not mention an explicit temperature control mechanism but states that during a measurement scan temperatures change by less than 0.07 ° F. Controlling the temperature eliminates measurement errors induced by thermal expansion. Controlling the temperature also makes it possible to use common building materials instead of more expensive low-expansion materials.

These common features of size, air bearing slides and stages and temperature control provide a stable machine base that is very rigid, accurate and thermally stable. These design features are used in the Nikon machine to obtain measurement accuracy on the order of 19nm peak-to-valley with a repeatability of 2nm, and in the Anorad machine to obtain an accuracy of 254nm peak-to-valley and a repeatability of 15nm rms. Given the proven performance of these two machines, any future techniques should use the aforementioned design techniques as a basis.
2.2 NEW APPROACH

Because the Conformal Optics Program is a technology development program, the CMM design developed at the University of Arizona was to take a new approach to an old problem, while still keeping with proven design philosophies of rigidity, repeatable motion and temperature insensitivity. The new CMM design was to be effectively smaller (scaled appropriately to the part size capacity), to remove the need for temperature control and to change the entire bridge/air-bearing stage distance-referencing method to a new optical referencing frame technique that allows lower precision mechanical components to be used while still maintaining a high accuracy measurement.

2.2.1 Optical Reference Profilometer (CMM)

Instead of starting the design of this CMM as a mechanical system, this program started with the concept that the CMM is an opto–mechanical instrument. So, the principles of optical measurements were used in its design.

The conventional CMM design uses a differential measurement between a reference and the test part. Interferometry is also a differential measurement. Interferometry compares a wavefront from a reference optic to that of the wavefront from a test part. The difference between the two wavefronts is related back to the surface of the test part. The Fizeau Interferometer can be used as an example, see figure 2.3.
The Fizeau interferometer has essentially three components: the reference, the test part and a laser source. The laser source is the distance-measuring mechanism. The operation of the Fizeau Interferometer is very simple. A collimated laser beam is directed through the interferometer where a portion of the beam is reflected from each optical surface. Not every surface reflection is used in the interferometer. The non-used surfaces are angled so that the reflections from those surfaces are lost and don’t return through the system to the detector. The only important surface reflections come from the reference surface and the test part. When these two beams combine, they form the interference pattern seen on the detector.
Although the Fizeau interferometer is very simple, it provides very accurate surface measurements. Typical accuracies for interferometric surface measurements can be close to \( \lambda/100 \) at the nominal HeNe testing wavelength of 632nm.

The reason for this comparison is to identify operational features that could be implemented in a mechanical device with a similar accuracy. Looking back at the figure of the Fizeau interferometer, we see that this interferometer is essentially just a cavity made up of the reference surface and the test part surface. The fringe pattern observed at the output is a function of the separation of the reference mirror and the test part. If the mirror separation changes during a measurement, then the pattern will shift and the interferogram will be different. If this cavity is stable during a measurement, then the interference pattern observed can be used to obtain high accuracy surface measurements of the test optic. This single feature of the Fizeau Interferometer can be very simply implemented in a mechanical surface measurement while maintaining a high accuracy.

This is the operational idea for the Optical Reference Profilometer. The Optical Reference Profilometer is a 3-dimensional Coordinate Measurement Machine. It makes a differential measurement relative to an optical reference. The optical reference, a \( \lambda/20 \) mirror, is mounted face down on the top of a referencing frame. The test part is placed underneath the reference on a base plate. The separation of the reference mirror from the
test part is maintained by four Invar metering rods, see figure 2.4. This set up provides a small stable referencing structure.

Figure 2.4, Optical Reference Frame for the Optical Reference Profilometer

Invar is a very low expansion metal with an expansion coefficient, at room temperature as low as $1.24 \times 10^{-6}/°C$. The Invar metering rods maintain a constant separation between the reference and the test part and thus make up the stable measurement cavity.

The distance measuring mechanism for our profilometer is a Distance Measuring Interferometer (DMI). Three axes of the DMI are used to monitor $x$, $y$ and yaw while a fourth axis is used to monitor stylus probe displacement from the optical reference.
The DMI is a laser-based heterodyne technique for measuring distances. The measurement resolution (in vacuum) for the system is 1.24nm or 2.48nm, depending on the interferometer configuration used, single pass gives 2.48nm and a double pass gives 1.24nm. The actual measurement accuracy of the system is reduced slightly from this level by air turbulence and polarization leakage in the interferometers used for each axis of the measurement.

2.2.2 Operational Procedure of the Optical Reference Profilometer

Now that the individual components of the profilometer have been introduced, the system as a whole can be discussed. Because a contact probe can only measure one point at a time, some type of scanning motion is required to obtain a complete surface map. A crossed-roller-bearing stage is used to provide the scanning motion for the Optical Reference Profilometer. The reference frame mentioned earlier is kinematically mounted on top of the stage and moves as a unit during a scan. The surface sag of the test part is measured relative to the probe of the air-bearing stylus assembly and the z-axis reference flat. This assembly is held on a bridge that crosses through the reference frame. Moving the stage moves the test part underneath the stationary stylus assembly.

The sampling method is the set-and-sample technique on a circular or rectangular grid. At a sample location the stylus is lowered, a sag value is measured, the probe is raised, the stage is moved to the next position and the stylus is re-lowered for the next
point. This is used instead of the alternative method of drag-and-sample (the author's terminology for common sampling techniques), which simply pulls the probe along the surface as the stage is moved.

![Diagram](image)

Figure 2.5, Basic Optical Reference Profilometer Setup

2.2.3 Design Benefits

At this point, only the benefits of the overall operating procedure will be mentioned. Individual system performance discussions of the prototype and the second-generation system will be given for each system in later chapters.
The first benefit to this system is its insensitivity to pure z translations of the cross-roller-bearing stage. The optical reference frame/cavity provides a stable structure from which to make a distance measurement. The distance measured will always be the relative distance between the reference mirror and the test part. Z-axis motions of the stage cannot change the separation between the part and the reference flat, which is maintained by the metering rods; therefore these motion errors are never seen in a measurement. Thus, the optical reference frame removes the need for an ultra-precision air bearing stage to provide the scanning operation.

The second benefit of this system relates to the bridge design. The bridge is made of aluminum. Because an air bearing is used to make the differential distance measurement, the stylus can float on the part (i.e. the stylus can move freely in the air-bearing sleeve). This means that the bridge can also make pure z translation without disturbing the measurement. The large rigid bridge structure common to most high-accuracy CMMs is no longer required. In addition, this reference structure reduces the temperature sensitivity of the instrument, permitting it to operate in an open lab environment without additional temperature control.

Removing as many error sources as possible in a system is an important design consideration. The new reference frame approach loosens the constraints on the stage motion flatness. It also allows for a simpler and smaller bridge assembly to be used.
Because of these design features and benefits, the overall complexity of the design can be reduced. This in turn will potentially reduce cost and make the system smaller, while still maintaining a high-accuracy system.
3.1 INTRODUCTION

This chapter will discuss the individual components that were needed to construct the prototype system. Because this is a prototype, it is for proof of concept and is not expected to work at the project goal accuracy level, although it will be designed to be close to that level.

3.2 REFERENCE FRAME

The reference frame (See figure 3.1) consists of a top plate and base plate separated by four metering rods. The base plate and metering rods were made of Invar-36. Invar-36 has a elastic modulus of $14.7 \times 10^{10}$ N/m$^2$, a density of $8.05$ g/cm$^3$ and an expansion coefficient of $1.24 \times 10^{-6}$/C (Yoder, 1993). Invar was chosen due to its strength and low thermal expansion. The top plate was made of aluminum instead of Invar to keep costs down. The $\lambda/20$ reference mirror (at $\lambda = 632$nm) was mounted kinematically on the top plate by three balls and v-grooves. The top and base plates of the reference frame were 0.5 inches thick.

The Mechanical Engineering group at the Optical Sciences Center did a finite element analysis on the system. This type of analysis can provide information about deflection magnitudes and vibration modes with corresponding excitation frequencies.
The fundamental frequency for the prototype system was found to be a torsion mode of approximately 76.5Hz. The head of the Mechanical Engineering group is Dr. Ralph Richard. He stated that anything above about 50Hz is acceptable, (Richards, Private Communication) any vibrations will dampen quicker with the higher oscillation frequency.

Figure 3.1, Prototype Reference Frame with Stage Mirrors

The finite element analysis was done to verify rigidity, however, a different analysis had to be done to determine thermal expansion effects for the optical reference frame. Thermal expansions will affect the length of the metering rods as well as the lateral dimension of the base plate. The base plate holds the stage mirrors used to monitor the lateral position of the part during a measurement scan. If the base plate expands, the x or
y measurements for any sample point will show this change. Since the stylus probe did not move in a lateral direction when the base plate expanded, the height value taken will correspond to a different part location than that measured by the system. The real position for the height value measured is equal to the measured position minus the expansion of the base plate.

If the position error is uncorrected, the magnitude of the height error for the measured point is equal to the difference in the height from the correct part position to where the stylus is located on the part that the expansion change would suggest. Because these changes are typically small, simple geometry will give the corresponding height error.

\[ \text{HeightError} = \text{Slope} \times \Delta \text{Expansion} \quad \text{(Eq 3.1)} \]

The metering rod expansion effects are different from the base plate expansion in that movement of the metering rods translates directly into a z-height measurement error.

For the sake of estimating the thermal effects on the system, a typical temperature change was taken at about 0.2°C. If the core temperature of the Invar base plate were to change by 0.2°C, the corresponding positional error introduced in a measurement would be 58nm. If the metering rod temperature were to change by 0.2°C, the separation between the base plate and the reference flat would change by 94nm.
These values show the temperature sensitivity of the optical reference frame. They are to be considered as ballpark estimates, because this simplified model used to estimate the thermal sensitivity does not take thermal conductivity and part size/thickness into account.

3.3 TRANSLATION STAGE

To provide the x and y scanning motion for the system, a translation stage was used. The stage is a New England Affiliated Technologies model XYL-15-15 crossed-roller-bearing stage. It has a platform size of 15" by 15" and a load capacity of 800lbs. The motion specifications are as follows:

| Flatness and Straightness | Travel*0.0001" |
| Pitch and Yaw             | 20 - (travel*5) arc sec. |

It is important to have a smooth scanning motion, but because the system is insensitive to pure z-motion errors a simple crossed-roller bearing stage can be used instead of a more precise air-bearing stage. This change was desirable because of the substantial cost increase for an air-bearing translation stage. To help maintain a high accuracy measurement, the lateral position of the stage is monitored by the use of a Distance Measuring Interferometer (DMI) and λ/10 stage mirrors. The position resolution of the DMI is on the order of 2nm.
The main issue with this type of stage is the pitch and yaw specification. Tilting of the stage during a measurement scan causes a cosine error in the z-axis measurement as well as an x, y position error in the probe contact point. Given that the tilts are so small, the cosine errors can be ignored, however, the position error cannot be.

The positional error is surface slope dependent. The positional error is essentially the same as in the x, y expansion of the base plate. The stylus touches in a location other than that measured by the x and y interferometers. The magnitude is again given by the difference in heights on the surface from the measured coordinates to the actual contact coordinates.

It was found later in the project, that the stage used actually has a parabolic track motion (see chapter 4), so most of the error introduced by the pitch and yaw of the stage is a systematic power error. This could account for some of the radius of curvature fitting error found in the prototype system.

3.4 STYLUS ASSEMBLY

An air-bearing stylus was used to measure the separation between the reference mirror and the test part. The air bearing was a custom part made by Nelson Air of Milford, New Hampshire. Figure 3.2 shows a drawing of the stylus.
The stylus air bearing works on a differential pressure raise and lower mechanism. On the top of the stylus sleeve is an air cavity. Two air-lines come into this cavity from points A and B in the figure, and push the stylus up. If the pressure in these two lines is lowered, the stylus will fall. The lowering motion is a damped free fall, i.e. the rate at which the air can flow out of the cavity is restricted. The actual falling rate is around 0.35"/sec. Because the stylus is in a damped free fall, it can bounce slightly off the test surface. The magnitude of the bounce is obviously related to the lowering rate. Because of the fast lowering speed for this stylus, test samples were limited to hard glasses.
The figure of the stylus assembly also shows an air path down the center of the shaft of the air bearing. This shaft is for the laser. To remove the effects of the air-bearing slide expansion, a hollow shaft was made through the slide and a corner cube placed on the bottom. This allows the laser beam to be referenced directly from the corner cube, which is closer to the contact probe and thus the test part, instead of on top of the aluminum shaft.

3.4.1 Stylus Repeatability

One of the major issues that had to be addressed about the stylus was placement repeatability. Stylus repeatability is the distribution of sag values measured for the same point on a surface over and over again. Figure 3.3 is a plot of 100 samples of the same point. The air bearing had placement repeatability on the order of 100nm peak to valley.

Figure 3.3, Stylus Repeatability Plot
When the air bearing was assembled, spacers were used to maintain a slight displacement between the slide and the sleeve housing. This spacing makes it possible for the slide to move relative to the sleeve housing when air is forced into this spacing. The smoothness of travel and placement repeatability are a function of how well the air bearing surfaces are made and how large the spacing is. The system was a bit too loose, so the repeatability was low (Nelson Air, 1998, Private Communication).

3.4.2 Distance Measuring Setup

In order to monitor every degree of freedom of the stylus assembly and the translation stage, 12 axes of the Distance Measuring Interferometer (DMI) would have to be used. Six degrees of freedom for each rigid body: x, y, z translations as well as rotations about each of the three axes.

Commercially available DMI systems only support up to 6 axes per machine. So, in order to reduce costs, it was decided to start with 4 axes: x, y, yaw and z-axes. Figure 3.4 shows a block diagram of the placement of the different axes.
Distance Measuring Interferometers use cube interferometers, which are just different combinations of polarization beamsplitter cubes and waveplates. In the prototype Optical Reference Profilometer, two different configurations were used. Figure 3.5 shows both configurations.
The High Stability Plane Mirror Interferometer, on the left in the figure, was used for the x, y and yaw measurements. It was chosen because of its thermal insensitivity and ability to be used with plane mirrors. This configuration is called a double pass interferometer, because the measurement beam reflects twice from the stage mirror before being combined with the reference beam.

The Cat's Eye Custom Interferometer, found on the left in the figure, was used for the z-axis measurements. This configuration was used because it is insensitive to tilt alignment errors in the interferometer beam relative to the reference flat. This configuration is called a single pass interferometer, because it only reflects once from the reference mirror. (The bottom corner cube shown for this configuration is actually mounted directly on top of the stylus contact probe, not seen in the figure.)

To get a better feeling for how these cubes work, the High Stability Plane Mirror Configuration will be discussed in detail. To begin the discussion, a more in depth explanation of the laser source used with DMI system is required than that given in chapter 2.

The laser source used with the DMI produces two orthogonal beams of different frequency. This frequency splitting can be achieved by placing a magnetic field on the laser cavity (Zeeman Split) or by passing the beam through an opto-acoustic modulator. The Hewlett Packard 5527A DMI design uses the Zeeman splitting (HP manual) and the
Zygo ZMI-1000 DMI system uses the opto-acoustic modulator (Zygo manual). The Zeeman split laser has a 2MHz frequency difference and the opto-acoustic modulated laser has a frequency splitting of 20MHz. This large splitting ultimately allows for a better measurement resolution. Due to this increase in measurement resolution, the ZYGO system was chosen for use with the Optical Reference Profilometer.

If the combined beam, i.e. both polarizations, passes through a polarization beam splitter cube, then one beam will be reflected while the other will be transmitted. This means that one frequency can act as the reference beam in the interferometer and the other will be the measurement beam, as the two beams interrogate different mirrors following different optical paths. In the High Stability Plane Mirror configuration one polarization will follow the path A and the other path B as shown in figure 3.6.

Path A

Path B

Figure 3.6, Paths of the Different Polarizations
The quarter waveplates change the polarization of the beam to allow it to either transmit or reflect at each pass. The two output beams overlap and the combined beam is collected through a fiber optic connector. This combined beam is passed through a polarizer at 45° to the horizontal polarization axis allowing interference. When two different optical frequencies interfere, a beat signal results. This is an amplitude modulation of the combined wavefield that oscillates at a frequency equal to one half the difference in the two individual frequencies. This frequency is much slower than regular optical frequencies and is therefore detectable by conventional electronics. (For a discussion on beats the reader is reference to an optics text written by Hecht (1990).)

This beat frequency is used to measure displacement information in the cube interferometers. If one or both of the two mirrors in the above interferometer is displaced, a Doppler shift is introduced into one or both of the beams. This changes the measured beat frequency. This Doppler shifted beat frequency is then compared to a reference beam frequency that is maintained in the DMI head with an up/down counter.

There are two channels in an up/down counter, one for the internal reference beat frequency and the second for the measurement beat frequency. The measurement frequency is the beat frequency caused by the two legs of the cube interferometer. The reference beat frequency is output directly from the laser head of the DMI.
This counter looks for changes in one beat frequency to another. The up-down counters will only register a value if the number of zero-crossings in a cycle is different for the two counters. As the measurement axis of the interferometer moves, there is a Doppler shift in the frequency of that leg. This shift will register in a difference in the measured beat frequency and thus a count will be measured.

Direction information is encoded in the Doppler shift either as a plus shift for path length reduction or minus shift for an increase in the path length. The actual sign of the shift will also be dependent on which frequency is the reference leg of the cube interferometer.

To convert the counts given by the DMI board into a physical displacement, the following equation must be used, where \( n \) is the current index of refraction of the air, \( N \) is the number of counts measured and the other terms are scale factors for the cube interferometer configuration used, 256 is used for single pass and 512 for double.

\[
D = \frac{N \lambda}{256 \times n}
\]  
(Eq 3.2)

3.5 BRIDGE

For the Optical Reference Profilometer system, the stylus assembly must be held in a stationary position between the reference flat/mirror and the test part. The desired
positioning setup for the stylus was required to pass through the reference frame and still allow the frame to move on top of the translation stage. This requires the use of a bridge. Figure 3.7 shows a CAD drawing of the final bridge design. The bridge was made of aluminum (for reasons mentioned in chapter 2). The center beam, along with the side supports, were made of ¼ inch thin wall rectangular pipe.

Using thin wall rectangular pipe made the bridge strong but lightweight to keep the resonant frequency high and deflection low. A finite element analysis of the bridge was not done; instead the performance of the bridge design was estimated by the use of the following equations.

The self-weight deflection of the bridge was calculated by the following formula (Griffel, 1966).
\[
\frac{-5 \cdot W \cdot L^3}{384 \cdot E \cdot I} \quad (Eq. 3.3)
\]

\( W \) = weight at center of bridge  
\( L \) = total length of the bridge  
\( E \) = elastic modulus of bridge material  
\( I \) = inertial moment of bridge cross-section

For the bridge design chosen, the deflection magnitude was 1.24 microns. Because the bridge was holding a mass at its center (the stylus assembly), it will have an additional deflection besides its self-weight deflection.

The following two formulas will calculate the deflection due to the mass and the resultant resonant frequency for the bridge (Griffel, 1966).

\[
\frac{-1 \cdot W \cdot L^3}{48 \cdot E \cdot I} \quad (Eq. 3.4)
\]

\[
\omega^2 = \frac{g}{z_{max}} \quad (Eq. 3.5)
\]

\( W \) = weight at center of bridge  
\( L \) = total length of the bridge  
\( E \) = elastic modulus of bridge material  
\( I \) = inertial moment of bridge cross-section  
\( g \) = gravity  
\( \omega \) = resonant frequency of bridge

The additional deflection was much less at about 100nm. The fundamental frequency of this combination was then 1575Hz. This is well above the 50Hz limit suggested by Dr. Richards.
In addition to vibration issues, one must be cautious of thermal effects. A temperature gradient of 0.2 °C from one side of the bridge to the other will cause a tilt in the stylus of approximately 0.5 arc seconds. If this temperature gradient occurred during a measurement cycle, a complicated error form would be introduced into the measured surface map. Because a 0.5 arc second tilt of the stylus does not introduce significant errors in the measured surfaces of low slope parts, this tilt was ignored in our analysis of the prototype system. However, a more detailed discussion of stylus tilt errors is given in chapter 3.

3.6 MIRROR MOUNTS

The Optical Reference Profilometer uses two different reference optics. Tenth wave stage mirrors ($\frac{\lambda}{10}$ at 632nm) are used to reference the horizontal position of the part under test, and a $\frac{\lambda}{20}$ reference mirror is used to reference the z-heights or sag values.

The stage mirrors were connected to the tilt and rotation mounts by Urethane adhesive. Three balls were placed between the mirror and the mirror platform. The adhesive connected the balls to the mirrors and the balls to the mirror platform. Urethane adhesive was used due to its zero shrinkage and elastic nature that would minimize the stress due to mounting. Induced deformation due to the adhesive was not investigated.
The mirror substrate was quite thick so this effect was ignored. Figure 3.8 shows a CAD drawing of the connection.

![Figure 3.8, Stage Mirror Mount](image)

The top or z-axis reference mirror was mounted by ball and v-groove connections (three-point mount). The balls were bonded to the mirror face by a Urethane adhesive. V-grooves are placed in the top mounting plate of the optical referencing frame such that the grooves have their centers pointing toward the center of the top plate. This kinematic connection makes it possible for the top plate to move relative to the reference mirror without inducing stress. Figure 3.9 shows a CAD drawing of this mount.
Although the reference flat/mirror is rated at $\lambda/20$ surface flatness, this number can change depending on the mounting configuration. In Yoder’s book entitled *Opto-Mechanical System Design* (Yoder, 1993), a formula is given to calculate the self-weight deflection for a simply supported mirror on a ring contact.

$$\Delta Y = \frac{-3W \cdot (m-1) \cdot (5m+1) \cdot a^2}{16 \cdot \pi \cdot E \cdot m^2 \cdot t^3}$$  (Eq. 3.6)

- $a$ = plate diameter = 10 inches
- $t$ = plate thickness = 3 inches
- $W$ = total load (weight) = 9.768kg
- $m$ = reciprocal of Poisson’s ratio = 1.0/0.24
- $E$ = elastic modulus = $9.06 \times 10^{10}$N/m$^2$

According to this equation, the self-weight deflection of the Zerodur mirror used would be 4nm. Although the mount used with this system is a three-point mount and not a ring contact, this formula still gives an order of magnitude for the deflection. The result suggests that the self-weight deflection of the reference mirror will be negligible.
Although the deflection is of negligible magnitude, it is interesting to note that the deflection pattern for a three-point mount will be a trefoil pattern, which is a three-lobe pattern that sags in between the contact points. Below is a picture of a trefoil pattern taken from page 442 of Yoder's book on Opto-Mechanical System Design (Yoder, 1993).

Figure 3.10, Trefoil Deflection Pattern (contour levels shown)
3.7 SYSTEM LAYOUT

The final mechanical description to be given in this chapter is the system as a whole. Figure 3.11 shows a photograph of the assembled system. It is on a standard optical table 4' wide and 18'' thick.

![Prototype System Layout](image)

Figure 3.11, Prototype System Layout

Also visible in this picture are two towers at 90 degrees to each other (one to the left and one behind the profilometer). These towers hold the cube interferometers and beam...
directing optics for the system. The reference frame also shows side supports. These supports were ultimately removed because they were deemed unnecessary for this system. (With the addition of these supports the fundamental vibration frequency went up to well above 400Hz, clearly unnecessary for the system since the vibrations of the reference frame are allowed to settle after every move; a benefit of the Set-and-Sample data collection method.)
CHAPTER 4 CMM ERRORS AND THEIR CORRECTION

4.1 INTRODUCTION

In the following chapter data correction algorithms will be discussed. Due to the interaction between the test part and the machine alignment/motion, systematic and non-systematic measurement errors can result. These errors must be removed from the raw data in order to produce high accuracy measurements. The first correction presented will be the stylus Ball Tip—Stylus Tilt.

4.2 BALL TIP—STYLUS TILT CORRECTION

All vertical-probe CMMs have an inherent ball tip error. Figure 4.1 shows that due to the finite size of the stylus probe ball, the probe ball will not touch the part along the stylus axis, but rather at some point on the side of the ball where the test part slope and the probe ball slope match.

Figure 4.1, Finite Ball Tip Error
Because the ball does not touch the test part along the stylus axis there will be an error in the measured height. In the above example, the measured height will be too small by a value equal to the distance $p$. As can be seen in the figure, this value varies with part slope. For rotationally symmetric curved surfaces this distance variation translates into a power error. For flats this has no effect and for irregularly shaped surfaces this error term cannot be given a specific form: it will add a profile that is a function of the ball size, surface feature frequency (Al-Jumaily, 1987) and surface slope.

There are a couple of possible correction techniques for this error. One technique involves least squares fitting of the measured surface map to a Taylor Polynomial and then removing those terms that are known to be generated from this error (Mast, 1995). These error terms are derived analytically, but are only for low slope parts. An approximation is made that the spherical surface can be represented by the first two terms in the Taylor Series for a sphere.

$$Z = a \cdot \rho^2 + b \cdot \rho^4 \quad (Eq. 4.1)$$

The resulting error term caused by the finite size of the probe ball is $c\rho^2$. The main limitation for this type of correction is that it limits the slope of the surface being tested to one that can be represented well by the first two terms in the series expansion.
Another possible correction technique involves a surface-construction using Surface Kriging. The technique uses a vector approach to find the ball center for the contact probe and interpolates the original surface (Mayer, 1997).

The Surface Kriging approach seems more sensible than the direct approximation to low slope surfaces. However, neither the Surface Kriging nor the least squares fitting method is general enough to cover both high slope surfaces as well as stylus tilt error correction.

Stylus tilt is a misalignment of the stylus relative to the x and y measurement plane of the system. (The stylus is not perpendicular to that plane.) This misalignment of the stylus causes a systematic error in the measured data for the surface.

It turns out that there are two parts to the stylus shaft tilt error. The first is that the stylus does not contact the surface at the assumed x, y measurement location (even ignoring the finite probe ball size). The second is that there is a cosine error in the z-height measurement at the actual contact location. This combination of errors leads to a skewed coordinate system. The error therefore cannot be corrected by a simple rotation of the part to the stylus shaft tilt angle. See figure 4.2.
As the figure 4.2 shows, the probe ball tip error will also change with stylus shaft tilt angle. This coupled effect mandates that the correction method treat the two errors in a combined fashion. In both of the aforementioned techniques, assumptions are made about the stylus perpendicularly to the reference surface. The Kriging approach assumes that the stylus shaft is exactly perpendicular to the reference. In the least squares fitting approach, the stylus can be tilted slightly. However, the error forms generated are applicable only for low slope surfaces. In reality the stylus shaft is never going to be exactly perpendicular to the reference surface. The determination for correcting this error is dependent on the magnitude of the stylus tilt, the test part surface slope as well as the desired measurement accuracy for the part under test.

Given the project goal of testing 40-degree surface slopes with 100nm accuracy or better, some type of data correction was necessary. A correction method similar to that
proposed by Mast (1995) was investigated first, however, when this method was extended to a 3-dimensional map on higher slope parts, the number of error terms needed to describe the error accurately grew so large that the correction method was no longer practical. Therefore a new approach was developed that was feasible to implement, that coupled the correction of the probe ball tip error and the stylus shaft tilt and was applicable to steeply sloped surfaces.

The correction method developed is iterative in nature. An initial estimate is made about the surface; this estimate is then used to make the first correction to the raw data. The corrected data is then used to find a better estimate and so on until the sequence converges to a solution. In order to insure that convergence occurs, the original error data is always used in the subsequent data corrections. This insures that the final surface determination will be the one that is consistent with the original error data set. A detailed example of the iteration process follows. For this example a sphere is used. While a sphere might be considered as a special case, no assumptions of sphericity are made in the algorithm. The ball tip error results solely from the fact that the test surface is not flat and is not dependent on sphericity. Thus making the correction technique valid over any type of asphere. Figure 4.3 shows a two dimensional example of the correction method.
The iteration method used is actually three-dimensional but for the purposes of this discussion two dimensions are sufficient.

Figure 4.3 shows that if the correction parameters \( d, \delta, \theta \) are known, that the effect of the Ball Tip—Stylus Tilt error can be exactly compensated. In the figure the value \( L' \) is the measured height along the stylus axis for the point \( x_0, z_0 \) (probe ball center). To obtain the corrected height value for the point \( x_0', z_0' \) the following equations are used.

\[
\begin{align*}
x' &= x_0 + r \cdot \cos \theta_x \quad (Eq. \ 4.2) \\
z' &= z_0 + r \cdot \cos \theta_z \quad (Eq. \ 4.3) \\
d &= \sqrt{(z_p - z')^2 + (x_p - x')^2} \quad (Eq. \ 4.4)
\end{align*}
\]
$$\Delta = f(x_p) - f(x_0')$$  \hspace{1cm} (Eq. 4.5)

$$L_o = (L' - r - d) \cdot |\cos \theta_z| + \Delta$$  \hspace{1cm} (Eq. 4.6)

To extend these equations to three dimensions simply add the y dependence into the equation for d and 
\Delta.

$$d = \sqrt{(zp - z')^2 + (xp - x')^2 + (yp - y')^2}$$  \hspace{1cm} (Eq. 4.7)

$$\Delta = f(x_p, y_p) - f(x_0', y_0')$$  \hspace{1cm} (Eq. 4.8)

To determine the correction values of d and \Delta, the probe contact point x_c,z_c and stylus tilt angle \theta must be determined. The contact point x_c,z_c is found by a damped-
least-squares iteration. The stylus axis/part intersection point x_p,z_p is found by ray tracing. For the moment assume that \theta is known. A calibration process that will be discussed in chapter 5 can determine the actual angle.

In order for these techniques to work, an analytical description of the surface must be known. The surface description can be given by one equation for the entire surface or by locally defined patches that collectively describe the surface to the desired accuracy level.
As stated earlier, the initial estimate of the surface is just the measured surface map. So to begin the iteration, the measured surface map is fit to a tenth-order Taylor polynomial either over the entire surface or to a second or third-order polynomial on a patch-wise basis. This surface will be used to find the first set of correction values of d and delta. Because least-squares iterations and ray tracing equations for aspheric surfaces are well known, they will not be discussed in detail here. However, the technique used to find the initial guess for the contact point iteration will be discussed briefly. For a more detailed discuss of least squares fitting and ray tracing the reader is referred to "The Art and Science of Optical Design" by Robert Shannon, 1997 (Shannon, 1997).

To start the process, a geometrical estimate is made for where the contact point will be. This estimate is determined by knowledge of the stylus tilt angle, the measured slope for the point being corrected and the measured height for that point. The probe ball center can be approximated when the probe is in contact with the test part by using the length of the measured value L' at the point that is being corrected and subtracting or adding the value of the probe tip radius. The local part slope as well as the stylus tilt direction relative to the local part slope is taken into account to determine the sign of the probe radius used. This compensated z-height value for the probe ball center is then used with the stylus tilt angle to find the x-shift of the ball center.

The next adjustment depends on the local surface slope. The slope at the point being corrected is determined by differentiating the Taylor Polynomial used to describe the
measured surface. This slope value is then found on the probe ball relative to its center. The position offset relative to the ball center, be it positive or negative, is added to the probe ball center value previously determined. This combined value is used to find the first estimate at the contact point. This estimate can be narrowed if necessary by looking at the part slope again at this laterally shifted contact position and then matching that slope to a new point on the probe ball. This adjusted contact position shift is then used again in the contact point iteration. This whole process continues till the iteration converges.

The stylus axis–surface intersection point is found using a simple binomial iteration. The initial estimates for this iteration do not have to be anywhere near as close to the actual solution as the contact point iteration. The solution space is much more linear. The actual estimate (high and low values used for the binomial iteration) is just the physical measurement boundaries of the stylus assembly. These values can be adjusted by hand if needed.

Because this first set of correction values is generated from an estimate of the real part surface, they cannot generate the correct answer immediately. However, they are close enough to the real surface to generate a better estimate than the raw measurement data. This improved estimate is used to find another set of correction values that are even closer to the real values. The original data set is corrected with these estimates until the surface generated is the surface that generated the original measurement values.
There is one important feature of the iteration that needs to be mentioned again. The
original $L'$ values given by the raw data set must be used in each correction phase of the
iteration in order for the iteration to converge to the right answer. In general the iteration
must go through 2-4 cycles to find the right answer. Figures 4.4 and 4.5 show the
convergence process for an actual data set. These figures show the surface residual
departure from the known surface for a $R=52.135\,\text{mm}$ sphere with a diameter of 43\,mm.

![Figure 4.4, First Iteration (330\,nm PV, 72\,nm rms)](image1)
![Figure 4.5, Second Iteration (237\,nm pv, 47\,nm rms)](image2)

After the second iteration on this surface, the base radius of curvature matched the
interferometrically determined radius by less than 0.03\%. The residual surface structure
seen in figure 4.5 is due to a noisy data set. The interferometric measurement of the same
surface showed a peak-to-valley departure from a sphere at 32\,nm.
The only real limitation of this correction method deals with using a least-squares-fitted surface as the actual surface representation. This type of surface representation is not a perfect replica of the raw data or even of the actual surface. The sampling rate of the surface determines what frequencies on the surface are visible in any one measurement. The measured surface map may also suffer from large noise spikes in the raw data set caused by vibration, dust, or other random errors. These also can distort the fitted surface relative to the actual part surface.

It can happen that the fitting function form is unable to match the generated data set accurately. Data fitting is actually a smoothing process and can remove structure from the raw data. Thus the fitted surface can have surface structure that was not present in the raw data nor the actual surface. In order to get a better fit between the raw data and the fitting function, a different function can be used instead of the Taylor Polynomial or the Taylor Polynomial can be used in a local, patch-wise surface description. However, the patch-wise surface fit is more noise susceptible.

4.3 STYLUS PROBE CONTACT AREA/DEFORMATION AND DEFLECTION

If any two surfaces come in contact with each other, the two surfaces will deform and there will be a finite region over which contact occurs. In Young’s book (1989) of
formulas for stress and strain, a formula is given to calculate the deformation ($\alpha$) and the contact area ($2a$) of the two surfaces.

\[
a = \sqrt{\frac{3F \left[ \frac{1-\sigma_1^2}{E_1} + \frac{1-\sigma_2^2}{E_2} \right]}{\left[ \frac{1}{d_1} + \frac{1}{d_2} \right]}}
\]  
(Eq. 4.9)

$\sigma_1$ = Poisons ratio for material one  
$\sigma_2$ = Poisons ratio for material two  
$F$ = load of surface one on surface two.  
$D_1$ = diameter of sphere one  
$D_2$ = diameter of sphere two

\[
\alpha = 0.53 \times (V_1 + V_2)^2 \times \left( \frac{F^2}{D} \right)^{1/3}
\]  
(Eq. 4.10)

$V_i = (1-\sigma_i)/E_i$  
$E_i$ = elastic modulus of material i  
$F$ = load of surface one on surface two  
$D_1$ = diameter of sphere one  
$D_2$ = diameter of sphere two

As an example, if a 1mm diameter sapphire ball with a one gram weight is resting on a flat plate of BK7 glass, the contact area is 7.54\mu m, with a surface deformation of 28nm.

The net deformation is material dependent but in this case most materials will be close to the hardness of BK7. The deformation is a constant term over a measurement on
any one given surface. It therefore simply adds a DC value to all measurement points and can be ignored.

If a softer material is tested with the system, such as metal or crystalline, then care must be taken to lower the resting weight so as not to permanently damage the surface. This is practical system limitation for any contact probe measurement device.

The final aspect to mention about the stylus is probe deflection. The stylus probe has a small diameter shaft from which a ruby ball is connected. The ball is what actually contacts the surface. Because the surfaces under test can be curved, a bending moment can be introduced by contacting the surface at an angle other than the surface normal. This effect is depicted in figure 4.6.

![Figure 4.6, Stylus Deflection on a Curved Surface](image-url)
The magnitude of the deflection is a function of the probe resting weight, the local surface slope of the part under test, the probe shaft diameter and length and of course, the probe shaft material. The following equation can be used to calculate the deflection (Young, 1989 p.100).

\[ \delta = \frac{F \cdot L^3}{3 \cdot E \cdot I} \]  
\( (Eq. \ 4.11) \)

\( \theta \) = angle between stylus axis and surface normal  
\( F = \) (resting weight*\( \sin \theta \))  
\( L = \) probe shaft length  
\( I = \) shaft inertial moment  
\( E = \) elastic modulus of the probe shaft

For example, if a carbide-steel stem probe is used with a length of 15mm, a stem diameter of 1mm and a lateral force of 0.0049N, the deflection magnitude is on the order of 8 microns. Shortening the stem length and/or increasing the stem diameter can reduce this value.

The above equation is slightly misleading, since the force \( F \) is not constant across the surface. In the equation, the force \( F \) is assumed to be acting in a direction perpendicular to the stylus shaft. If the surface normal is plotted against the z-axis or probe direction, the resultant force vector in the direction normal to the z-axis, will vary as the surface normal changes. This deflection variation has been ignored in the prototype system analysis, because simply lowering the resting weight of the stylus can reduce the overall effect.
4.4 MEASUREMENT PLANE TO MOTION PLANE TRANSFORMATION

The next error that will be discussed deals with a skewed x and y position measurement. This error is caused by a misalignment between the stage motion plane and the plane defined by the stage mirrors on the reference frame base plate. Figure 4.7 shows a picture of the problem.

![Figure 4.7, Motion Plane to Measurement Plane Error](image)

This error can be corrected by a simple 3-D rotation. The measurement plane defined by the stage mirrors must be rotated to match the stage motion plane or part plane because the part is moving on the stage. The following equations will perform the rotation.
\[ xp = l_1 \cdot x_{\text{not}} + m_1 \cdot y_{\text{not}} + n_1 \cdot d \]
\[ yp = l_2 \cdot x_{\text{not}} + m_2 \cdot y_{\text{not}} + n_2 \cdot d \]

(Eq. 4.12a,b)

In the above equations \( d = 0 \). The values \( x_{\text{not}} \) and \( y_{\text{not}} \) are the measured \( x \) and \( y \) values for the test part, \( l_1, m_1 \) and \( n_1 \) are the direction cosines from the original \( x, y \) and \( z \)-axes to the new \( x \)-axis and \( l_2, m_2 \) and \( n_2 \) are the direction cosines from the original \( x, y \) and \( z \)-axes to the new \( y \)-axis.

4.5 STAGE MOTION ERRORS

4.5.1 Yaw Errors

Translation stages have several different motion errors. In the prototype and second-generation Optical Reference Profilometers only one motion error is corrected—yaw. This is a rotation of the stage platform in the \( x,y \) plane. It is caused by imperfections in the roller bearing track. This error will rotate the part under the stylus assembly resulting in a measurement positioning error (i.e. the \( x \) and \( y \) positions measured are not the correct \( x \) and \( y \) locations for the actual contact point on the surface). The magnitude and direction of this rotation can vary from point to point along the measurement path.
If the rotation angle for each data point is known, this error can be corrected. (Figure 3.4 in chapter 3 showed the interferometer layout for monitoring the yaw motion of the stage.) In order to determine the rotation angle, both the x and yaw interferometers must be used. The difference in the distance measured by these interferometers divided by the separation between them gives the tangent of the angle of the stage platform (or the yaw rotation angle). Using simple geometry, the correct x and y contact positions on the part can be determined from the rotated/measured x and y positions. The following equations correct each data point for the yaw rotation.

\[
\theta = \tan^{-1}\left(\frac{x_2 - x_1}{d}\right) \quad (Eq. 4.13)
\]

\[
y_o' = y_{cen} \cdot \cos \theta \quad (Eq. 4.14)
\]
\[ xo' = \left[ \frac{x_2 - x_1}{d} \cdot \text{dis} + x_1 \right] \cdot \cos(\tan^{-1}\left(\frac{x_2 - x_1}{d}\right)) \]

(Eq. 4.15)

\[ \text{dis} = \text{the separation between the x1 axis location and xcen} \]
\[ d = \text{the separation between the x1 and x2 axes} \]

4.5.2 Stage Tilts

Although the system is insensitive to pure z motion errors of the stage, it is not insensitive to tilts. The prototype system does not correct for stage motion tilt, because the prototype profilometer does not have the capability to monitor this error. Although this error is not corrected, its effect will be discussed briefly here.

It turns out that the stage actually has a parabolic travel path. The measured radius of curvature in the x direction is 545.742mm and slightly different from this for the y direction. The radius was determined by using an autocollimator. As the stage moves along the parabolic trajectory, the tilt of the base plate was measured. This measured slope value can then be related to the radius of curvature of the stage motion by using the parabolic surface equation.

The tilting of the stage actually causes two errors. The first is a positioning error and the second is a cosine error. The cosine error term introduced by the tilt is
inconsequential since the angles are so small. However, the positioning error is dependent on the slope and the lateral extent of the part under test.

For a spherically symmetric curved part, this parabolic stage motion will introduce a power error. As an example, if a spherical surface with a maximum surface slope of 20 degrees and lateral part diameter of 43mm is tested, the measurement data will have approximately a 1.5 micron power error.

The parabolic stage motion also affects non-rotationally symmetric surfaces. A non-rotationally symmetric part will suffer from a systematic skew of the x and y position coordinates that are dependent of the part being tested.

4.6 NON-ORTHOGONAL STAGE MIRRORS

Another systematic measurement error that can occur in CMM measurements is a position skew of the measured x and y coordinates due to non-orthogonal stage mirrors. The effect of this alignment error is to introduce an astigmatism like term into the measured surface map. Figure 4.9 shows a simulated error term for a mirror misalignment of 10 arc seconds. The part used for this simulation was a sphere with a max surface slope of 41.8 degrees. If the misalignment were 1 arc second, then the peak to valley error would be reduced by a factor of 10. To generate the plot the following relation is substituted in for the normal x variable in the standard sphere equation.
The magnitude of the error is dependent on part slope as well as on the actual angular misalignment magnitude. This term was not considered for correction in the prototype system because part slopes for the prototype were limited to about 10 degrees and the alignment of the stage mirrors was better than 5 arc seconds. The second-generation
system discussed later in the dissertation had a better alignment and measured surface up to 40 degrees without correction.

4.7 TEMPERATURE COMPENSATION

4.7.1 Metal Expansion

In order to create a high accuracy measurement machine, all axes of the CMM must be thermally stable. To correct for possible thermal expansion induced measurement errors, a series of thermistors were used to monitor the temperature changes of the metal mounting surface for the system components. The temperature in the room where the prototype system was used was relatively constant with typical metal surface changes of about less than 0.2 °C. These changes were over a relatively short time frame of about 20-45 minutes. Although this correction was made in the data set, it was shown to have little affect on the measurement. (A more detailed discussion of this correction will be given in Chapter 6 Prototype Performance.)

4.7.2 Index of Refraction Variations

Because Distance Measuring Interferometers use laser light, the measurements are susceptible to index of refraction changes in the air. The index of refraction of air is a function of temperature, pressure, humidity and gas composition. As the index of refraction of the air changes through which the light propagates, the wavelength of the
light changes and will thus introduce a path length change. This path length change can be compensated if an active index of refraction measurement is made. The index of refraction can be determined from Edlen’s Equation (HP 5527A Users Manual) if the air temperature, pressure and relative humidity are known.

\[
n = 1.0 + (3.8369 \times 10^{-7} \cdot P) \cdot \frac{1.0 + P \cdot (0.817 - 0.0133 \cdot T) \cdot 10^{-6}}{1.0 + 0.003661 \cdot T} - 5.607943 \times 10^{-8} \cdot f
\]

\[
f = \frac{RH}{100} \cdot (4.07859739 + 0.44301857 \cdot T + 0.00232093 \cdot T^2 + 0.00045785 \cdot T^3)
\]

\hspace{10cm} (Eq. 4.17)

- \( RH \) = relative humidity
- \( T \) = air temperature
- \( P \) = air pressure

Active index of refraction compensation is vital not only for accurate motion measurements but also for deadpath correction. Deadpath is defined to be the difference in the optical path length of the reference and the measurement arms for the zero or start positions in the interferometers used (HP 5527A user manual). This is shown pictorially in the following figure where D is the deadpath distance and L is the actual motion displacement.
Figure 4.10, Deadpath Definition

This definition can actually be refined to be the unequal air path lengths in the arms of the interferometers in their zero or starting positions, if the glass path changes are either ignored or will be compensated for by a temperature compensation technique. (For the prototype Optical Reference Profilometer the glass path changes were ignored.)

The effect of the deadpath distance is to shift the defined zero position of the interferometer if a change in the air index of refraction occurs. For example, if \( L \), the physical displacement of the measurement leg of the interferometer, were equal to zero and the index of refraction in the path length \( D \) were to change, the DMI would measure a motion when no motion occurred, only the path length changed due to an index change in \( D \). The DMI measures distance in counts (see section 3.4.2). These counts are the sum of the real motion of the object whose position is being monitored and any shifts in the zero position of the interferometer.
To give a better picture of the effect of this error an example follows. If the deadpath of one axis of an interferometer is 10.96mm and the temperature, pressure and humidity of the air in that axis change by 1 °C, 0.25mm Hg and 10% respectively. The deadpath error is 28.4nm for the uncorrected situation. Obviously as the deadpath distance increases the magnitude of this error will also increase.

This position error can be corrected, however. If the deadpath distance $D$ is known and the index of refraction of the path length $D + L$ is actively monitored, then the additional motion created by the shift in the interferometer zero position can be removed.

To correct for the deadpath error, the deadpath distance in terms of counts at the initial index of refraction of the air is calculated. At each new measurement point the deadpath distance is recalculated in terms of counts, at the new index of refraction. These two count values are subtracted and then the result is removed from the measured counts in the output buffer of the DMI axis card. This removes the added distance due to the zero position shift. The remaining counts are used with the current index of refraction to find the physical displacement. The above discussion can be summarized in the following equation.

$$N_L = N_M + N_{NOT} - N_P \quad (Eq. 4.18)$$
$N_L =$ actual displacement in counts
$N_M =$ measurement displacement in counts
$N_NOT =$ initial deadpath in counts
$N_P =$ current deadpath at new index in counts.

The most obvious solution to this deadpath error is to make the path lengths of the measurement leg and reference leg of the interferometer equal. This means $D = 0$. For situations where this configuration is not possible a stable measurement environment is required to reduce the magnitude of this error. It is also beneficial to actively monitor the air index of refraction to obtain the highest accuracy possible. For the prototype profilometer system, the zero deadpath configuration was not possible, so an active monitoring system was implemented.

4.8 CUBE INTERFEROMETER POLARIZATION LEAKAGE

There are several error sources for cube interferometers. The errors typically come from polarization leakage. The laser used in DMI has two frequencies of orthogonal polarization. These frequencies must remain separate in polarization in order to avoid measurement errors. The reason for this can be understood by analyzing what happens when two light beams, in the same plane, combine with different amplitudes and phases. It can be shown that the resultant beam will have a new amplitude and phase dependent on the relative amplitudes and phase of the two individual beams.
For example, if a small portion of frequency 1 were to leak into the polarization plane of frequency 2, it would follow the same optical path as frequency 2. The remaining portion of frequency 1 would have a different path length because it followed a different optical path. After the individual beams pass through the polarizer at the detector, the different beams will be in the same plane (refer back to chapter 2 for DMI operational basics). This means that the small portion of frequency 1, that leaked into frequency 2's path, will combine with the other portion of that frequency and will make a new beam of the same frequency but with different amplitude and phase. This phase will vary with optical path length changes for frequency 2 because a portion of frequency 1 follows the same path. The resultant output beam will therefore vary in phase because of the leakage. This polarization error is cyclic in nature. The magnitude for this error can be found by calculating the new phase value $\chi$ while one beam varies in phase from 0 to $2\pi$ radians.

$$\tan \chi = \frac{A_1 \sin \theta_1 + A_2 \sin \theta_2}{A_1 \cos \theta_1 + A_2 \cos \theta_2} \quad (Eq. \ 4.19)$$

The above equation can be used to calculate the new phase $\chi$ upon the addition of two waves with differing amplitude and phase. In the equation $A_1$ and $\theta_1$ are the amplitude and phase of wave one and $A_2$ and $\theta_2$ are the amplitude and phase of wave 2.
Polarization leakage can occur when the coatings on the polarizing beamsplitter cubes are not perfect, when the DMI laser axes are tilted with respect to polarization axes of the polarization beamsplitter cubes or when the orientation of the waveplates used in the cube interferometers is wrong so that the output is not exactly linear but rather slightly elliptical. Figure 4.11 shows a plot of a real measurement of this error for one of our cube interferometers.

Number of Sample Points

Figure 4.11, Polarization Error Plot (error given in mm)
If the noise is ignored, the cyclic polarization error can be approximated at about 10nm peak to valley for this cube.

Comparing the measured motion of a precision stage with the known displacement performs the measurement for this polarization error. Several points are taken as the stage is moved by 158nm. The difference in the actual position and the measured position for each point is then plotted to obtain the polarization error plot shown above.

The best corrections for this error are to use high quality polarization components and to align the waveplates and polarization beamsplitter cubes precisely. Typically the error is in the noise for the system so an active correction is not necessary nor feasible.

4.9 ERROR BUDGET

The error budget presented here is considered a worst-case estimate (i.e. the individual error magnitudes will be no greater than these values). The error budget presented here shows the main error sources for a measurement excluding the Ball Tip--Stylus Tilt error. It should be noted that when a least square fitted surface is used instead of the raw data, that the measurement accuracy can be improved from this worst-case magnitude estimate. Some values given in the error budget are dependent on the surface slope of the part being tested as well as the time per scan. For these errors a maximum
surface slope of 15 degrees and had typical scan time under 45 minutes is used to evaluate their respective magnitudes.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stylus Repeatability (with deadpath correction)</td>
<td>100nm</td>
</tr>
<tr>
<td>Reference Flatness</td>
<td>30nm</td>
</tr>
<tr>
<td>Δx position errors (λ/10 stage mirrors)</td>
<td>30nm</td>
</tr>
<tr>
<td>Stage mirror non-orthogonality (1'-10'')</td>
<td>10-100nm</td>
</tr>
<tr>
<td>Polarization periodic error</td>
<td>37nm</td>
</tr>
<tr>
<td>Metal expansion x, y and z total</td>
<td>40nm</td>
</tr>
<tr>
<td>Random Stage tilts (30”)</td>
<td>10nm</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td><strong>257-347nm (Linear Sum)</strong></td>
</tr>
<tr>
<td><strong>RSS Totals</strong></td>
<td><strong>122-158nm (Root Sum Square)</strong></td>
</tr>
</tbody>
</table>
CHAPTER 5 ALIGNMENT AND CALIBRATION

5.1 ALIGNMENT

Alignment can be a difficult process. Misalignment of critical components results in systematic errors in the measured surface map. In general, the alignments presented here are intended to reduce the need for data correction algorithms. But because most of the systematic errors induced by alignment errors can be compensated for, the alignment tolerances were relaxed to easily achievable tolerances.

5.1.1 Stage Mirror Alignment (Perpendicularity)

The first alignment necessary for the system was the stage mirrors. These mirrors form the reference for the x and y system axes. As such, they must be aligned perpendicular to one another. The stage mirror alignment made use of an autocollimator and a precision mirror polygon. The faces of the polygon were certified by the National Institute of Standards and Technology to within 1 arc second of the angular measure given. For the purposes of this alignment, only the 90-degree faces were used.

The alignment setup was quite simple. The reference polygon was placed on the base plate of the reference frame. It was roughly aligned so one of the faces of the polygon was within 10 arc minutes of perpendicular with one of the stage axes. This was done with a reflection from a parallel plate attached to the side of the stage. The sides of the
stage were assumed to be parallel enough to the stage motion to use as the reference. The data eventually taken with the prototype showed this as a correct assumption. The autocollimator was then adjusted relative to the polygon so that the projection reticle of the autocollimator returned as close to the zero, zero angular position of the collimator as can be resolved (about 0.2 arc seconds). The stage mirror was then placed in the path of the autocollimator. It was adjusted until the projection reticle returned again to the original zero, zero angular position. Once the stage mirror was in place, the autocollimator was moved 90 degrees to align with the second face of the reference polygon. The same steps involved for alignment of mirror one are repeated till the second mirror is as close to the reference as possible. Figure 5.1 shows the alignment setup.

Figure 5.1, Stage Mirror Alignment Setup
The overall estimate of the alignment accuracy in the prototype was about 5 arc seconds. The reason for this was the mount used was not easily aligned. When the individual mirrors were tightened down into position, they would rotate out of alignment. The end result was to compensate for the rotation in the tightening process. This made it difficult to be very precise.

One method of alignment verification was to examine the surface residual measurement of a known optical surface. Because the form of the error introduced by having non-orthogonal stage mirrors is known (astigmatism-like), the error can be identified in the surface residual, and an estimate made of the actual alignment accuracy. The surface measurements taken with the prototype system did not show a definite contribution from this error. However, this error could have been compensated for in the measurement plane to stage/part motion plane transformation because the form of the two errors is similar.

Because the individual error terms are found with the use of a damped least squares optimization routine on known optical surfaces, the exact contribution of the errors cannot be known. A solution determined by this method minimizes the difference between the known surface and the measured surface by any combination of the errors that will give a minimum. The same problem exists in lens design optimization routines. The algorithms will find local solution space minimums but are never guaranteed to find the exact solution or even the best solution.
5.1.2 Stylus Assembly Alignment

The stylus assembly should be aligned so that its motion axis is perpendicular to the reference flat. In practice, the alignment will never be perfect. However, the Ball Tip—Stylus Tilt correction mentioned in chapter 4 can compensate for tilt errors as large as a few arc minutes. Because of the correction possibility, the stylus for the prototype was simply mounted on the aluminum bridge without any additional alignments. The mechanical tolerances used when constructing the bridge were assumed close enough to be correctable.

Aligning with making the stylus motion axis perpendicular to the reference flat, the laser beam used to measure the z-heights must also be made perpendicular to this surface. This alignment can be done visually by watching the measurement beam displacement relative to the reference beam as the stylus shaft was moved. The displacement occurs due to the offset introduced by the corner cube at the bottom of the shaft. If a tilt exists between the stylus air-bearing shaft and the reference flat, as the shaft translates downward the beam will move further out on the corner cube face. Because of this shift the output beam will also shift. This results in a beam separation between the reference and the measurement that is a function of the vertical position of the stylus shaft. Figure 5.2 illustrates this point.
In the actual alignment process, the reference beam and the measurement beam were seen to move slightly relative to each other; this introduced a very small cosine error in the z-height measurements but was too small to be considered.

As part of the stylus assembly alignment, adjustments had to be made to the cube interferometer to match the angle and placement of the air bearing to the reference flat. The most difficult part of that adjustment dealt with the focusing lens. The interferometer is in a Cat's Eye configuration to return a collimated beam from the reference flat.

To focus the lens, the interferometer was placed on a moveable platform. The relative size of the measurement beam to the reference beam was the only measure of focus used. Because the moveable platform used for focusing could tilt, the cube interferometer had to be realigned each adjustment of the platform. Therefore the alignment for the z-axis
interferometer had to be coupled to the stylus assembly to keep everything aligned to each other. (See figure 3.5 for interferometer configuration details.)

Because the Cat's Eye interferometer configuration is a single pass interferometer, a focusing lens must be used to keep the measurement and reference beams of the cube interferometer collimated to each other. If the lens were not present, the two beams would be tilted with respect to each other. The presence of tilt fringes on the plane of the detector causes phase fluctuations across the detector’s active area. The detector will only see the average of these fluctuations. So, as the measurement leg of the interferometer moves, the relative phase across the detector will change but the average will remain constant and thus no displacement is measured. Because of this problem, it is essential that the two beams remain collinear so the DMI can measure displacement information.

5.1.3 Cube Interferometer Alignment

The x and y interferometers suffer from a tilt error when the input beam is not perfectly matched with the stage mirror surface normal. In the Hewlett Packard DMI manual there is a plot for the OPL error induced by this tilt as a function of stage mirror displacement. For the prototype system the x1, x2 and y interferometers were aligned to less than 2 arc minutes of tilt relative to the stage mirrors. The chart given in the Hewlett
Packard manual shows that for this tilt error and for a maximum x or y travel of 3.2 inches, the maximum OPL error was less than 30nm.

The stage used for the prototype system has a parabolic motion path. Because of the parabolic stage motion, the input tilt angle of the measurement beam will change as function of measurement position. This tilt error was not considered in the overall error budget for the prototype CMM. The reason for this was that the motion error was not detected until after the prototype system was disassembled. This error was not corrected in the second-generation system either due to the cost increase necessary to implement an active correction technique. However, this error will be discussed briefly again in connection with the Second-Generation system in chapter 7.

5.2 CALIBRATION

Calibration can be one of the most difficult parts of precision machine development. The goal in the calibration process is to remove any systematic errors introduced by component misalignment. Calibration is required to remove the Ball Tip—Stylus Tilt error as well as the Measurement Plane to Motion Plane Transform error (the measurement plane to stage motion/part plane skew). A software package was written to aid in the process of finding the stylus tilt angles and plane rotation angles necessary to correct for these errors. A copy of the software used is found in the appendix.
The process begins with the measurement of a known sphere. The surface figure on this reference surface is better than $\lambda/10$ (pv), and the radius of curvature of the sphere is known from interferometric measurements. For the purposes of this discussion, the surface is effectively a perfect sphere of known radius.

Because the surface that is being tested is of known figure and shape, the surface residual after the known part description is removed must be the combined contribution of the plane skewing and the Ball Tip—Stylus Tilt errors. The other error terms mentioned in chapter 3 will have been already removed from the raw data.

The calibration routine begins with the assumption that there is no stylus tilt error and no measurement plane to part plane transformation is necessary. The algorithms in the software will then remove the base power error due to the finite probe ball size, leaving the linear and cubic-like terms from the stylus tilt and a saddle-shaped structure due to the skew in the x and y positional coordinates.

The calibration routine then tries to remove the stylus tilt error. This is done by manually adjusting the value of the stylus tilt angle until as much of the cubic term is removed as possible.

The stylus tilt error also has a linear term. There are actually two linear terms that can exist in the measured surface map. The first is due to the stylus tilt and the second is due
to a wedge between the reference flat and the base plate. This is why the “cubic” term is used to determine if the right stylus tilt angle was used.

After the stylus tilt error is minimized, the plane rotation angles are changed by hand until the ‘saddle’ term is removed. Once both the base ‘saddle’ and ‘cubic’ terms are removed, final adjustments are made by trying slightly different combinations of these angles until a minimum in the residual surface map is observed.

There is one difficulty that is involved in this process, and that is that the calibration tilt and rotation angles are not guaranteed to be the actual tilt and rotation angles. The technique only minimizes the surface residual. This means that the remaining surface residual is a combination of the real surface map and a fitting error cause by using the wrong combination of tilt and rotation angles. The only way to really separate each error would be to find a calibration measurement that isolates one term from the other.

5.2.1 Software Calibration Procedure

The following discussion will be of the particulars of how the calibration software works, using the previous explanation as an overview of the whole process. This software package has been written to lead the user through the alignment process.
The first step in the calibration process is to correct the raw data for the Ball Tip—Stylus Tilt error; the algorithm mentioned in chapter 4 does this. There are two features that need to be mentioned about the raw data before the next step in the calibration process can be explained.

Because the test part is not centered in the machine, the vertex or low point on the surface map will not be at the measured coordinate (0,0). This also means that the measured surface map will not be symmetric about the vertex.

The second feature is that the part is not leveled to be parallel to the reference flat before a measurement is taken. This results in a rotation of the actual test surface about a point that is unknown.

In order to remove an ideal surface map of the reference optic, the orientation and point sampling of the measurement data must be defined. Because the reference/calibration optic is a sphere, any tilting or translation in its positioning result simple in a newly defined sphere vertex. Therefore, to make the calibration process easier, the raw data set is rotated so that the measured scan is symmetric about some new vertex point and then re-sampled on a uniform grid. This technique makes positioning of the reference optic surface map relative to the measured map much easier.
To make the measurement scan symmetric, a plane is fit to the raw data. This plane defines the new vertex surface normal. Once this new vertex normal direction is defined, the entire raw data set is rotated so that this surface normal direction is aligned with the current z-axis. This rotation also makes the sampled surface symmetric, i.e. the vertex of the data is in the middle of the scan. The rotated data is then re-sampled to a uniform grid. The re-sampling is done by fitting the rotated data to a Taylor polynomial and then evaluating the function at a predefined point grid.

After the data has been rotated and sampled on a uniform grid, the ideal reference surface is subtracted from the result. The sampling of the reference surface is now exactly the same as the measurement data, i.e. the sample points have the exact same relative space from the part vertex. This ensures that the sampled points are correlated to the same points on the sphere.

After the ideal data has been subtracted, the residual tilt is removed. The residual surface error, after this first set of corrections have been made, is stored for comparison with each new guess for the stylus tilt and measurement plane to part plane rotation angles.

The residual surface map is checked for ‘cubic’ residuals. The next trial uses the adjusted angle values based on the visual residual of this term. The entire calibration
process is repeated with the new estimate of the stylus tilt angles until the residual cubic term is gone.

The next step is to go back and start rotating the measurement plane to remove the x and y position skew. The 'saddle' like term is the mark of this skew. The rotation angles are changed by hand until they start to correct for this saddle. When the surface map residual is close to the known residual, both stylus tilt and plane rotation angles are moved until the minimum in surface figure departure from the ideal part is observed. This entire process can take a couple of hours for the trained observer or a few days for a beginner.
CHAPTER 6. PROTOTYPE PERFORMANCE ANALYSIS

6.1 ANALYSIS

Before the performance of the prototype can be discussed, an explanation of how the data was analyzed must be given. The analysis package used for this CMM uses a damped least squares optimization to determine the optimal placement of the measured surface such that when the actual surface is subtracted the surface residual is a minimum. One of the nicest features about this analysis method lies in the part alignment. Because the actual orientation of the test part relative to the system axes can be found with the analysis routine, no additional alignment processes or equipment are required.

The analysis routine begins by using the stylus tilt and plane transformation angles found in the calibration routine discussed in chapter 5 to correct the raw data for the Plane Rotation/Transformation and the Ball Tip—Stylus Tilt errors. This corrected data set is then used in the iteration to match the ideal surface.

In any least squares optimization, there is a set of target values the iteration is trying to match. In this process the corrected raw data now represents what is being matched with the placement of the ideal surface. This ideal surface is shifted in the all directions x, y and z and rotated to match the corrected measurement data at the same sampled points given in the measurement data set.
For example, if the part being measured is aligned perpendicular to the reference flat, but was not sampled symmetrically about its vertex, than the measured 0,0 point would be simply displaced. In equational form this could be represent by the following.

\[ F(x) = g(x + xo) \]  
\text{Eq. 6.1) }

For this example, the analysis routine would shift the ideal surface \( F(x) \) until the value of the shift were \( xo \). This results in the minimum value of the target function, (i.e. the minimum difference between the measured data and the ideal surface).

If the measured surface is rotated relative to the reference flat, then the situation is a little more complicated. In these situations the ideal surface is rotated by the following equations (Spiegel, 1999).

\[
\begin{align*}
x_{pr} &= l_1 \cdot x_{val} + l_2 \cdot y_{val} + l_3 \cdot z_{val} \\
y_{pr} &= m_1 \cdot x_{val} + m_2 \cdot y_{val} + m_3 \cdot z_{val} \\
z_{pr} &= n_1 \cdot x_{val} + n_2 \cdot y_{val} + n_3 \cdot z_{val}
\end{align*}
\]  
\text{(Eq. 6.2)}

In the above equations the values of \( l_i, m_i \) and \( n_i \) are the directional cosines from the old \( x, y \) and \( z \) axes relative to the new \( x, y \) and \( z \) axes of the rotated coordinate system. \( l_1 \) is from the old \( x \)-axis to the rotated \( x \) axis, \( l_2 \) is from the old \( x \) axis to the rotated \( y \)-axis and \( l_3 \) is from the old \( x \) axis to the new \( z \)-axis—the others follow in a similar fashion. The values of \( x_{val}, y_{val} \) and \( z_{val} \) are the original data in the un-rotated coordinate system and \( x_{pr}, y_{pr} \) and \( z_{pr} \) are the rotated coordinates.
In order to perform this rotation, the values of the directional cosines must be known. Not any set of directional cosines will work. There are certain requirements on the relative values of these cosines. The rotated coordinate system axes must remain orthogonal to each other and the sum of the squares of each set of vectors that define one axis must be equal to 1. To satisfy these requirements the following equations must be solved (Spiegel, 1999).

\[ m_1^2 + n_1^2 + l_1^2 = 1 \]
\[ m_2^2 + n_2^2 + l_2^2 = 1 \]
\[ m_3^2 + n_3^2 + l_3^2 = 1 \]
\[ l_1 * l_2 + n_1 * n_2 + m_1 * m_2 = 0 \]
\[ l_2 * l_3 + n_2 * n_3 + m_2 * m_3 = 0 \]
\[ l_3 * l_1 + n_3 * n_1 + m_3 * m_1 = 0 \]

(Eq. 6.3)

From the above equations, it can be seen that there are 9 variables and only 6 equations, so an exact solution is not available. It therefore becomes necessary to make assumptions about the directional cosines. The analysis algorithm uses \( l_3, m_3 \) and \( m_1 \) as the variables and solves the above equations to find the remainder of the cosine values. The derivation of the solution of these equations is long and would provide little insight into the problem, so it will not be included here.
As with any iteration, initial estimates must be made for each variable to begin the solution finding process. At the beginning of the optimization, the values for each of the rotation and shift variables are set to zero. The optimization routine will change these values until the solution is found. The routine also has the ability to change the step size taken in the damped least squares iteration. As the difference between the ideal surface and the measured surface becomes smaller, larger step sizes are allowed in finding the solution. This variability allows for a faster convergence. For more details on the least squares optimization routine see Shannon (1997).

The iteration takes several minutes to find the solution on a 200MHz Pentium computer. The main limit on the speed for this correction lies in the point by point data set rotation. The time interval for each iterative cyclic is therefore dependent on the number of sample data points taken.

Although this analysis method works well, there are a couple of possible drawbacks to this approach. Because the correction algorithms and rotations are point by point, the entire process takes several minutes. Second, if the part being measured has a cubic term in it, then it can be removed in the analysis process. If two spheres are subtracted, where one is shifted slightly to one side relative to the other, the residual surface map would have a linear term and higher odd order terms that depend on the slope of the two spheres. Thus if surface is measured that has any of these odd order terms, the shift and rotate algorithm in the analysis software would simply shift the reference surface relative
the measured sphere till these terms disappear. It is essentially impossible to remove this
effect unless fiducials are used to mark the plane of the measured surface and its vertex
i.e. 0,0 location. The use of fidusials was never implemented on this system. Although
this would make the entire analysis process much easier, not all parts are going to have
this type of data registration available.

6.2 REFERENCE OPTICS MEASURED

Now that the analysis routine has been discussed the results of prototype system can
be given. Several reference optics were measured to estimate system accuracy and
repeatability.

6.2.1 λ/5 Mirror

It makes sense from discussions made in chapter 4 that flats are easier to test than
curved surfaces. Therefore, the first measurement made with the Optical Reference
Profilometer was on a flat. The measurement grid was square, and the measurement area
11cm x 11cm. For the contour plots shown, residual tilt is removed. Also, for the
profilometer surface measurement, the contour plot is interpolated to appear more
continuous. The interpolation routine uses a linear extrapolation between data points.
Although the two contour plots really look nothing alike, useful information about the system can be drawn from this result. This first plot shows how large the noise level in the prototype is. For flat surfaces, the noise level is around 200nm. This is not within the desired program limit but is close enough to be correctable in the second-generation system.

In a later section of this chapter, a discussion of the possible noise sources will be given along with possible correction mechanisms.
6.2.2 $\lambda/20$ Sphere (Radius of Curvature = 753.19mm)

The next class of surfaces tested with the prototype Optical Reference Profilometer were spheres. Low slope spheres were measured first. The reference sphere was a $\lambda/20$ with a radius of curvature of 753.19mm. The maximum slope on this part was 1 degree. The measurement area was on a square grid of 2cm x 2cm.

![Image of sphere measurement]

<table>
<thead>
<tr>
<th>Profilometer Measurement</th>
<th>Interferometric Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_v = 0.244\mu m$</td>
<td>$P_v = 0.037\mu m$</td>
</tr>
<tr>
<td>$RMS = 0.037\mu m$</td>
<td>$RMS = 0.005\mu m$</td>
</tr>
</tbody>
</table>

Figure 6.2, R=753.19mm Sphere
The plots shown have residual power and tilt removed. The profilometer measurement set is again interpolated to appear more continuous.

Because all sag/height values for the test part are measured relative to a flat, the data can be used to make a direct radius of curvature measurement. For this surface the best-fit radius was less than 0.03\% different from that found by an interferometer.

Although the radius of curvature fit was extremely close, the measured surface residual differed significantly. The measured surface does seem to have some of the same type surface structure, however the magnitudes are so much different that it is difficult to believe that the structure wasn’t just noise induced.

It should be noted that the measurement surfaces presented for these comparisons are not the raw data but rather a least squares fitted surface, the most likely surface in a least squares sense (Frieden, 1991), of the raw data to a Taylor Polynomial unless otherwise noted.

6.2.3 \( \lambda/10 \) Sphere (Radius of Curvature = 52.135mm)

The final surface tested with the prototype system was a higher slope sphere. This part had a maximum surface slope of 10 degrees. The measurement area was again square with an area of 2cm x 2cm.
The plots shown have residual power and tilt removed. The profilometer measurement set is again interpolated to appear more continuous. The radius of curvature fit was again quite close to the interferometrically measured value with a difference of about 0.08%. However, the measured surface structure is again too large to represent the actual surface structure accurately.
The prototype system was mainly built for concept verification and to be used as a learning tool before endeavoring to make the final instrument. Although, the prototype did not work at the desired accuracy level, it does provide an important conclusion. The main conclusion drawn from the test results presented in section 6.2 was that the optical reference frame idea works. The measured surface residuals observed are too small to have been caused by either the z-axis motion errors of the translation stage or the bridge.

Had the operational concept of the Optical Reference Profilometer not been valid, the measurement data would have had several microns of surface error. For the smallest scan area of 1 inch the stage is rated at a 0.0001" (2.5μm) motion flatness. Under typical operating conditions, with a temperature change of 0.2°C, the bridge and stylus assembly could have moved up to 5μm in the z-axis direction. (The temperature changes were not always linear, so the net temperature induced motion of the bridge could introduce structure different from that of a tilt error.) Because neither of these terms existed in the measurement set, it appears that our system is insensitive to both of these motion errors.
6.3.1 Interferometer Configuration Limitation

Although the concept of the optical referencing frame worked, the prototype system had two major problems. The first was that the scan area was too small. This limitation was due to the Cat's Eye interferometer configuration used for the z-axis measurement.

The Cat's Eye interferometer focuses the laser beam down to a 7 micron spot. This spot is adjusted to be directly on the mirrored surface of the z-axis reference flat. As the part is scanned, the optical reference frame mentioned in chapter 2 and 3 moves across the interferometer focus. If this focus spot comes across a small particle of dust on the reference mirror during the scanning motion, then the measurement signal for the DMI is scattered and the signal is lost. This causes an error in the measurement values taken after that point. Although the data after the error point can be corrected, the correction is very difficult to do well, so typically the entire measurement of the surface must be taken again. The ultimate effect of this problem was to limit the possible scan area to a region of about 1 inch in the center of the reference mirror, which was the largest region that could be kept clean.
6.3.2 System Noise (Systematic and Non-Systematic)

The other problem with the prototype system was the measured surface residuals were too high. This residual surface noise was caused by a combination of several factors: stylus slippage, translation stage tilt, air turbulence, x and y positional errors and polarization leakage errors from the cube interferometers. Because the air turbulence seemed like the easiest to correct it was investigated first.

6.3.2.1 Air Turbulence

The DMI is sensitive to index of refraction changes in the air, therefore it is possible to have large measurement errors when the temperature, pressure or the humidity of the air changes. To reduce the magnitude of this error an active index of refraction measurement is made. Any changes in the index are corrected in the measurement data, but the correction is only valid in a long-term sense. The temperature, pressure and humidity are not measured at the exact same time that the distance is measured. This lag is due to software instrument control limitations. Therefore there will be fluctuations in the measurement data due to this sampling lag as well as from detector response lag due to short turbulent pulses.

In order to get a better perspective on the magnitude of these fluctuations, the z-axis measurement values for the system were tested when the stylus air was off. The air from
the stylus air bearing is the largest source of turbulence for the system. The beam path for the laser passes right down the middle of the air bearing stylus shaft. So, turbulent air fluctuations from the air bearing have a direct impact on the z-axis measurement.

A series of tests were done to monitor the z-axis values when the stylus was at rest on a flat surface with the air off. The average of the tests was taken to be the estimate on ambient air turbulence. This value was then compared to the average when the stylus air bearing was turned back on (the stylus was still resting on a flat). The peak-to-valley average for the air off was 32.5nm with rms of 8.7nm. When the stylus air was on, the peak to valley measurement was 48.2nm with rms of 12.7nm. This showed that the air bearing was contributing only about 20nm to the overall z-axis air turbulence noise.

The x and y axes were also monitored for turbulent fluctuations, but the lead screw on the stage would expand and settle giving jumps in the measurements thereby masking any large-scale fluctuations from being detectable. It was assumed therefore, that the ambient air turbulence was on the same order as the z-axis with the air bearing turned off. Because these interferometers are only a few inches apart, this is a reasonable assumption.

The tests showed that in order to reduce the z-axis measurement noise some type of air shield was needed to block the air flow from coming into the beam path of the cube.
interferometer. It also suggests that some type of cover might help reduce the ambient air-flow for the room in which the instrument operates.

6.3.2.2 Thermal Expansion

The next noise/error source investigated was the x and y positional errors caused by thermal expansion. The first attempt to qualify the possible magnitude of this error was a using a theoretical model. In this simplified model, the metal surfaces of the instrument had immediate responses to any temperature changes that occur during a measurement. From a simple expansion relation, the positional variations in the cube interferometer locations can then be estimated.

\[ \Delta L = L \alpha \delta t \]

In the above expression \( L \) is the length of the object at temperature \( T_1 \), \( \alpha \) is the thermal expansion coefficient, \( \delta t \) is the change in temperature of the object in question and \( \Delta L \) is the change in length of the object due to the temperature change.

Considering a temperature change of 0.2°C, the estimated position error in the x and y directions was on the order of 250nm.

To find the influence of this error experimentally, the peak to valley residual was monitored for when the x and y positions were corrected for expansions and when they
were left alone. The resulting difference was not more than a few nanometers with little variation in the measured surface structure. There are two possible reasons for this result. The first is that the metal didn’t expand as much as the surface temperature sensors stated. And the second, they did actually move by the values determined, but the effect was masked by some other error not investigated at that time. (In the second generation system a vibration was found that caused the x and y positions read out to fluctuate by about 200-250nm. The suspected cause was the air bearing. Forcing air through a small cavity can cause the stylus assembly to vibrate and thus anything else on the optical table. This error would have been seen in prototype as well and could have masked some the expansion error.) It is the opinion of the author of this dissertation, that the metal did not really move and the vibrations found merely added position measurement noise.

6.3.2.3 Reference Optic Flatness

Because the stage mirror surface used to reference the x and y positions of the test part are not perfectly flat, they introduce position measurement errors. However, the x and y position errors caused by the \( \lambda/10 \) stage mirror surface flatness are potentially correctable with some type of line reversal technique (Evans, 1999). A line reversal calibration technique uses a series of special surface measurements related through some functional mathematical form to solve for the unknown surface. Evans stated that this type of correction technique is only helpful in certain situations, where the motion error, or other type error that is being corrected, is large as compared to the noise of the
measurement system. It is difficult to say if the prototype Optical Reference Profilometer has the right noise ratio, but because the right equipment was not available to perform this correction, it was never tried. The z-axis noise caused by the $\lambda/20$ reference flat could also potentially be reduced by the same technique.

6.3.2.4 Stylus Motion/Placement

Stylus motion errors were another large error source, potentially the worst source of error in the system. It was found that the stylus motion of the prototype system was too fast. Because the lowering mechanism of the stylus is a damped free fall, the probe may not always set down on the part slowly. If the probe comes down too fast, it will flex when it contacts the surface. This introduces a variable measurement error that is not correctable after the data is taken. This may also cause the air-bearing slide to settle in a new position in the sleeve and thus generate an inconsistent stylus placement. When the stylus comes down too fast, it can also bounce off the surface. This can damage the test surface as well as the test probe. Because stylus repeatability is so important, several upgrades were made to correct the aforementioned problems. The discussion of these upgrades is left for chapter 6—Second-Generation Optical Reference Profilometer.
6.3.2.5 Polarization Leakage and Stage Motion Tilt

The remainder of the noise was generated by polarization leakage and stage motion tilts. In order to correct the polarization errors, higher quality polarization optics coupled with better alignment must be used. This upgrade was too expansive in relation to the gain in performance to make it feasible. The random stage tilts are correctable by either a better translation stage or adding additional cube interferometers to monitor the tilt actively.

6.4 PERFORMANCE SUMMARY

Before concluding this section, two points need to be mentioned. The first is that the stage mirror alignment did not seem to contribute enough to be seen as an error term on its own. It must be assumed that the error was on the order of 1-4 arc seconds or that the Plane Transformation mentioned earlier removed the error.

The second thing was the above analysis used the least square fitted surface and not the raw data. This helps to reduce the effects of uniformly and symmetrically distributed noise sources, because on average these errors give the right answer. A least squares fitted surface reproduces the most likely surface in the presence of noise. This means that the fitted surface will represent more of the overall systematic structure found in the raw data and will ignore most of the uniform high frequency fluctuations. For the prototype
system, the three main sources for the systematic errors were the stylus motion errors, the stage tilts and the reference optic surface flatness. All the others added mainly to the high frequency fluctuations.

Despite the high noise level in the prototype profilometer system, the results obtained show that the concept of the Optical Reference Frame works. The performance was not at the desired accuracy level, however good results were obtained for surfaces up to 10 degrees. The limitations found in the prototype were addressed before the second-generation system was built, thus insuring a better instrument on the second attempt.
CHAPTER 7 SECOND-GENERATION OPTICAL REFERENCE PROFILOMETER

7.1 INTRODUCTION

The following section will discuss the upgrades made to the prototype system. The discussion will be on the mechanical design and performance expectations. A new error budget will be given where systematic errors that can be removed will not be included. The purpose for the upgrades was to improve overall system performance and to ease the system alignment.

7.2 STYLUS ASSEMBLY

One of the main changes in the prototype system was the stylus assembly. The prototype configuration had a limited scan area and the repeatability was too low. The final system was required to have at least a six-inch diameter scan area and was desired to have 7.5 inches (limit imposed by stage mirror sizes). The limited scan area was caused solely by the Cat’s Eye interferometer configuration used with the prototype system. It was therefore necessary to use a new configuration. The new interferometer configuration had to be tilt insensitive and easier to align. It also had to have a reduced overall airpath length, due to DMI deadpath and index of refraction fluctuation errors. A reduce airpath length makes the air index of refraction compensation work better (the possible fluctuations along a beam are more uniform the shorter the beam path, i.e. the entire beam path has the same variation). The new interferometer design is shown in figure 7.1.
The only difficulty with this configuration is the large path length difference in the measurement beam to the reference beam. The reference beam in this interferometer passes right through the polarization beam splitter (PBS) into the corner cube and then returns back out the PBS. The measurement beam, however, must reflect from both mirrors twice, as well as pass through the corner cube before coming back out the PBS. These mirror absorption losses, combined with the Fresnel glass surface reflections from each pass through the PBS reduced the fringe contrast to 50 percent, which is close to the bottom of the modulation threshold for the DMI.
This new interferometer configuration necessitated the change of the air bearing design. The slide used in the prototype was not able to pass both beams of the new interferometer configuration.

It was also desirable to redesign the air bearing stylus because the placement repeatability of the prototype air bearing was also too low. The new air bearing design was to have a better slide flatness and smoother descent. In order to insure that the next stylus was better than the first, a set of performance specifications was outlined. The following is a list of those specifications.

1. Slide surface flatness < 0.0001"  
2. Slide resting weight < 1 gram  
3. Slide lowering speed < 0.2"/sec  
4. Air bearing travel limit 2"  
5. Air bearing size: vertical 5"  
   Cross section 2" x 2"  
6. Temperature insensitive (temp delta allowed 1° C –10nm sag variation)  
7. Slide raise/lower mechanism pneumatic controlled.

To keep the stylus motion as smooth as possible, a digital pressure regulator was used to control the decent. The regulator was computer controlled and had a 0.1psi step resolution. Because the pressure regulator was computer controlled, the stylus resting weight could also be adjusted to a minimum value of about 1 gram.
A series of experiments were performed by the manufacturer of the air bearing (Nelson Air, New Hampshire) to find the optimal damped free fall profile for the stylus. It was found that a small curvature on the upper cavity walls gave the right damped free fall rates. Nelson Air also advised that a slightly wider slide would make it easier to grind to slide surface flatness requirement. The final slide was a thin wall aluminum shaft. The thin walls help to control heating variations in the slide as it is ground thus helping to maintain a good surface flatness. Figure 7.2 shows the final design chosen for the second-generation stylus. The shaded sections in the figure make up the movable slide portion of the air bearing.

Figure 7.2, Second-Generation Stylus Design
The second-generation design is seen to have a shaft running down the middle of the air-bearing slide. This shaft is made of Zerodur, a very low-expansion glass. The top of the shaft is a mirror to be used with the new interferometer configuration. The ends of the shaft were made parallel to each other to within 10 arc seconds. The bottom walls of the air-bearing slide were made perpendicular to the slide walls to within 10 arc seconds. These tolerances limited the potential cosine errors caused by tilts. They also made the alignment of the stylus easier. The entire stylus assembly would now require only one adjustment to make it perpendicular to the top reference flat. The second-generation stylus also implements an air shield to keep the extra airflow coming from the air bearing out of the interferometer beam path.

7.3 REFERENCE FRAME

The changes in the reference frame were mainly in overall dimensions and construction materials. The height of the second-generation system was made shorter. This reduced possible air path lengths and made the system more rigid. The metering rods were changed from Invar-36 to Super-Invar, a lower expansion version of Invar-36.

The second dimensional change was on the base plate. The worry was that the plate was too thin and could vibrate with stage motion. Since the reference frame is placed kinematically on the stage top, the center of the base plate is unsupported and could have
some kind of dynamic flexing induced by the stage scanning motion. This would in turn move the test part up and down resulting in noisy data.

In the prototype system, the base plate was a $\frac{1}{2}$ inch thick plate of Invar-36. In the second-generation system the base plate was made of Super-Invar with a thickness of 1 inch. To keep the reference frame from bowing at the top with different metal expansion rates, the top plate mirror mount was also made of Super-Invar.

Shortening the optical reference frame and using lower expansion metering rods reduced the thermal expansion sensitivity of the system. The overall effect was to reduce the possible delta-z at a 1° C temperature change to under 80 nm. (This expansion value seems quite high. However, it must be noted that the metering rods are 1.5 inches thick and with an extremely low thermal conductivity the measurement room environment would have to experience a very large temperature swing before the core temperature of the metering rods were to change by 1° C.)

The final changes made to the reference frame were for alignibility. To reduce the magnitude of the linear term introduced by a wedge in the reference frame, adjustment screws were added to the top plate of the optical reference frame. The adjustment screws were built with hemi-spherical balls on the bottom, thus allowing the top plate of the reference frame to rest in v-grooves (the number of metering rods was reduced to three to make this connection work). There were two options with this mounting configuration.
The top plate could either rest kinematically on top of the three metering rods or be bolted down to them through a hole placed in the adjustment screws, see figure 7.3.

![Figure 7.3, Top Flat Adjustment Screw](image)

Ultimately the top mirror mounting plate was bolted down to the metering rods, thus increasing the rigidity of the entire optical reference frame.

7.4 STAGE MIRROR MOUNTS

The stage mirror mounts were redone mainly due to difficulty in alignment. The v-groove sections at either end of the old mirror mount were not mechanically connected. This meant that they could easily rotate relative to one another, thus adding to the difficulty of making the two stage mirrors orthogonal to each other.
This problem was fixed by using a single mounting base instead of two separate sections. To help with the orthogonality adjustment, a 4-80 adjustment screw and spring combination was placed on the side of the reference frame base plate and mirror mount.

The final adjustment made to the stage mirror mounting system was connecting the mirrors to the mounting plates. Instead of placing balls in between the mounting plate and the mirrors and using an adhesive to connect them, v-grooves were made in the mounting plate and hemi-spherical balls were epoxied to the underside of the stage mirrors. This allowed the mirror to rest in the grooves and not distort or vibrate. Eventually a small amount of urethane adhesive was added to this connection to ensure a good contact between the balls and the mounting plate (it was noticed that the balls were not setting perfectly in the v-grooves and could therefore move—a ball placement error).

7.5 METERING ROD NETWORK

The idea of the metering rod network was to maintain a thermally stable positioning system for any surface measurement. For any part measurement, the temperature of the system can change. These temperature changes cause thermal expansions of the metal infrastructure for the system. These expansions cause measurement errors.

In order to reduce these effects, the temperature of the system must either be controlled or the system must be insensitive to these temperature fluctuations. The
second-generation system uses low thermal expansion materials combined in a distance referencing network to reduce the system's overall temperature sensitivity. This structural network is referred to as the metering rod network. Figure 7.4 shows a block diagram for this structure.

Figure 7.4, Metering Rod Network Concept Design

The x, y and yaw interferometers were discussed in an earlier chapter. However, the x_center and y_center interferometers are new. If the stylus assembly were to move laterally during a measurement scan, the measured x and y positions for a given height value would be in error.
The measured x and y values for a given part measurement are relative to the machine center. When a measurement scan is taken, the stylus assembly center is at some position relative to that location. If the stylus assembly remains in that same lateral position during the measurement scan, the measured height values for the part under test correspond directly to the measured x and y positions. If the stylus moves during the scan, the measured height values correspond to a shifted x and y location different from those measured by the x and y cube interferometers. If the displacement of the stylus assembly as a function of each scan point is known, the correct lateral position for a given height value can be determined. This correction allows for greater positioning accuracy and more accurate surface measurements.

The metering rod network references the x, y, yaw and x_center and y_center cube interferometer locations with the stylus center position or machine center. This network is constructed with Super-Invar rods/plates. Because of the very low expansion of this material, the x, y, yaw and x_center and y_center cube interferometer locations are stable to within 30nm even with a 1°C temperature change in the metal (theoretical calculation).

In order to implement this metering rod network, several structural changes had to be made. The first of these was the stage would have to rest on a plate that would allow the metering rods to pass underneath. The metering rods are attached to the center of this plate by a ball-swivel connection, see Figure 7.5.
This type of joint allows rotations of the two base rods but constrains the radius of the rotation. This was necessary because the whole Optical Reference Profilometer was to be mounted on a standard optical table, which was made of stainless steel. The expansion coefficients for stainless and Invar are extremely different and therefore temperature gradients could cause shifts in the base rod locations.

If the rotations are constrained to a small distance, the cosine error caused by the rotation is negligible. The constraining action for the second-generation system was done by a flexure connection to the optical table. The flexure connection will also help to reduce thermal-expansion-induced distortions in the rod network. In the system, each connection of the rod network with a non-Invar piece was done by a flexure. Figure 7.6 shows one of these connections.
The flexure was designed to hold a vertical load but to twist or pivot with a torque at the flexure connection.
The remainder of the metering rod network can be seen in figure 7.8. The network is the shaded portion in the figure.

Although the metering rod network is quite simple, it provides a high degree of temperature insensitivity. As mentioned earlier, the referencing network will maintain positioning accuracy to around 30nm for a 1°C temperature change. This high degree of temperature insensitivity adds the positioning stability necessary for machine operation in an open lab environment.
7.6 ERROR BUDGET

This error budget assumes that expansion errors can either be calibrated out or that the metering rod network corrects them to within the state tolerance. The reference frame metering rod expansion is ignored due to the thickness of the rods. Although the rods can move 80nm for a 1°C metal temperature change, typical runs will only see a portion of this value, because the air temperature typically changes by less than 0.2 °C. For runs about an hour with air temperature changes of 0.2°C, the metal temperature is assumed to move by a quarter to a tenth of this value.

By simply looking at the worst case error budget estimate, it would seem that given the upgrades that were made, the accuracy goal of about 100nm over a 6” diameter at up to 40 degree surface slopes does not seem possible. Therefore two error budgets are worked out. The first was to determine the region over which the worst case estimate would reach the project accuracy goals. The second was a worst case estimate over the desired operation range. Error budget 1 will be over the center 3” of the measurement region, and be valid for surface slopes of up to 10 degrees. Budget 2 is for a 6” diameter with up to 40-degree surface slopes.
### Error Budget #1

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Ref. Mirror Flatness (cen. 3&quot;)</td>
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</tr>
<tr>
<td>X, Y Position Errors (mirror flatness &amp; turbulence)</td>
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</tr>
<tr>
<td>Stylus Repeatability</td>
<td>45nm</td>
</tr>
<tr>
<td>Cyclic Polarization Error</td>
<td>10nm</td>
</tr>
<tr>
<td>Random Stage Tilts</td>
<td>10nm</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td><strong>115nm (Linear Sum)</strong></td>
</tr>
<tr>
<td><strong>RSS Totals</strong></td>
<td><strong>61nm (Root Sum Square)</strong></td>
</tr>
</tbody>
</table>

### Error Budget #2

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref. Mirror Flatness (cen. 6&quot;)</td>
<td>30nm</td>
</tr>
<tr>
<td>X, Y Position Errors (mirror flatness &amp; turbulence)</td>
<td>90nm</td>
</tr>
<tr>
<td>Stylus Repeatability</td>
<td>245nm</td>
</tr>
<tr>
<td>Cyclic Polarization Error</td>
<td>10nm</td>
</tr>
<tr>
<td>Random Stage Tilts</td>
<td>10nm</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td><strong>385nm (Linear Sum)</strong></td>
</tr>
<tr>
<td><strong>RSS Totals</strong></td>
<td><strong>263nm (Root Sum Square)</strong></td>
</tr>
</tbody>
</table>

The stylus repeatability value for Error Budget #2 was based on after the fact data on the repeatability of the new stylus at larger surface slopes.

It should also be noted that these numbers are peak to valley magnitude estimates. Since the analysis routine uses the least squares fitted surface, the measured surface
residual will most likely be much less than the linear sum worst case estimates predicted, as will be shown in the chapter 9. A better estimate would be the RSS total or a rms sum.

7.7 CONCLUSION

The redo of the prototype system was mainly to improve the stylus assembly and increase the measurement range of the system. Along with these changes, a more thermally stable measurement structure was implemented. In the second-generation system design process more attention was given to details, especially in the area of kinematic mounting.

7.8 SYSTEM PICTURES

The following section shows pictures of the second-generation Optical Reference Profilometer. There will be two pictures. One picture will show an overview of the whole system, and the second will give a close-up view of the referencing frame and new stylus assembly.
The picture shows the two towers that house the system interferometers. The one in front and the second on the left.
Figure 7.10, Optical Reference Frame Close-Up
CHAPTER 8 SECOND-GENERATION OPTICAL REFERENCE PROFILOMETER ALIGNMENT

8.1 INTRODUCTION

This section will contain a supplementary explanation of the alignment process of the second-generation system. The main alignments such as cube interferometers and stage mirror alignments are carried out in an identical fashion to that described in chapter 5.

8.2 STAGE MOTION—OPTICAL REFERENCE FRAME ALIGNMENT

The first step in the alignment process for the second-generation system was to make the base plate of the optical reference frame parallel to the stage motion plane. This alignment reduces cosine error effects in the sag measurements of a surface. Because the motion path of the stage was more parabolic in nature than flat, there was a limit on how well this alignment could be done. The parabolic motion path is caused by a flexing of the translation stage as the Optical Reference Frame mass is redistributed to the outer ends of the stage slideways. The measurement of this deflection was done with the use of a simple runout gauge. The value of the probe displacement was monitored as the stage is moved under the runout gauge. Figure 8.2 shows the stage deflection for both the x and y directions. Each plot shows a total of 20 sample points across a 7 inch scan distance.
Figure 8.1, Stage Deflection

Figure 8.2, X and Y Stage Motion Paths (Sag given in Microns)

Figure 8.2 shows that the two slideways of the x-y translation stage have differing bending moments. This astigmatic stage bending ultimately resulted in an additional
astigmatism term being added to the surface measurements of the spherical reference optics. For non-spherical surfaces this will generate systematic errors of differing magnitude in the orthogonal directions. If the bending moments were the same, a power error would have been introduced.

8.3 OPTICAL REFERENCE FRAME AND STYLUS ASSEMBLY ALIGNMENT

The first step in the alignment of the optical reference frame to the stylus assembly was to define the orientation of the surface normal of the optical reference frame base plate. This was done by use of a plane parallel plate (plate wedge < 10 arc seconds). The parallel plate was placed on the ground surface of the base plate (0.001" surface flatness). After the parallel plate was placed on the base plate of the reference frame, an autocollimator along with a high accuracy pentaprism (10 arc second output beam orthogonality) were used to locate the surface normal. See figure 8.3.
The autocollimator serves as the alignment reference source. This means any additional optics placed in between the base plate and z-axis reference flat will be aligned relative to the surface normal defined by the autocollimator.

After the base plate surface normal was defined using the front surface reflection of the parallel plate, the bridge and stylus assemblies were placed in the system. The bridge was fitted with three adjustment screws to align the stylus to the base plate. After the stylus was adjusted to the autocollimator-defined axis, the bridge was bolted down.
The final step in the alignment process of the reference frame was to align the z-axis reference flat to the base plate. This was done with the use of two autocollimators. See figure 8.5

Figure 8.4, Bridge and Stylus Assembly Alignment

Figure 8.5, Z-axis Reference Flat Alignment
For this part of the alignment the penta prism was replaced with a right angle prism (30 arc second output beam orthogonality). The right angle prism was able to reference both the reference frame base plate and a 45° plane relative to that plate. The 45° surface is used to reflect the autocollimator output up to the z-axis reference flat. To keep the autocollimator beam aligned with the base plate surface normal, the right angle prism was placed on the mirror face of the stylus. The mirrored surface was made parallel to the base plate in the previous bridge and stylus alignments.

Figure 8.5 shows two autocollimators. Both collimators were necessary to keep the alignment as accurate as possible. When the right angle prism is placed on the stylus mirror, autocollimator A is adjusted to match the return. This redefines the direction of the base plate surface normal. Once autocollimator A is repositioned, the right angle prism is removed. Autocollimator B is then adjusted to accept the output beam of autocollimator A. Because the right angle prism must be rotated 180° to use the back surface, the 45° surface relative to the base plate, for the reflection to the z-axis mirror, a reference point must be used to insure that the rotation is correct. The second autocollimator acts as this alignment reference for the right angle prism.

In order to have the reflected autocollimator beam as close to 90° to the stylus mirror surface normal as possible, the rotation of the right angle prism must be very close to 180°. If the angle is less than this value, the reflected beam will lie in a different plane
defined by the autocollimator output beam and the surface normal of the right angle prism.

To get the correct rotation, the right angle prism is placed in the output beam of autocollimator B. The front face of the prism is then adjusted to return the output beam of autocollimator B to match the previous input from autocollimator A. This insures that the back surface of the right angle prism is now aligned so the reflected beam of autocollimator A will be in the stylus mirror surface normal direction.

The accuracy of these alignments was determined from the calibration process mentioned in chapter 5. From this process, it was determined that the base plate and z-axis reference mirror were aligned to less than 30 arc seconds parallelism and the stylus tilt alignment was within a few arc minutes of perpendicular to the z-axis reference mirror.
CHAPTER 9 SECOND-GENERATION PERFORMANCE SUMMARY

9.1 INTRODUCTION

In this section a summary of the results obtained for the second-generation system are given. Issues of accuracy and repeatability are also addressed. The measured surface data presented here was analyzed using the same methods discussed in chapter 6.

9.2 6" FLAT

The second-generation system does not suffer from the same scan area limitation as the prototype, so a full 6" flat could be tested. The measurement area was on a 20x20 grid. This size sampling grid takes approximately 1.5 hours.

Interferometric Measurement
Pv = 0.128µm
RMS = 0.018µm

Profilometer Measurement
Pv = 0.131µm
RMS = 0.022µm

Figure 9.1, 6" Flat
The measurement plot shown from the profilometer is actually interpolated between sample points. The interpolation algorithm uses a linear extrapolation between the sampled data points. Both surface plots shown have tilt removed, so the contour plots represent surface figure departure from a flat.

The profilometer measurement also had a systematic astigmatism error removed on the order of 100nm peak-to-valley. The source of the astigmatism is from an asymmetry in the translation stage used (see chapter 8). Other than this systematic error, the residual surface map measured by the profilometer does not have any data correction. Ball Tip—Stylus Tilt error mentioned in earlier chapters does not affect flat surface measurements. Thus the plot shows the raw data deviation from the best fit plane.

Although the measured surface shows some of the same basic features found in the actual surface, the noise level of the system is still too great to obtain a more exact measurement of the actual surface. It should be noted that with a finer sampling grid that the higher frequency fluctuations found in the profilometer measurement could be smoothed out giving a better representation of the surface. This follows from the statistical relation that the standard deviation of a measurement set is reduced by the square root of the number of samples (Freiden, 1991, p. 235).
9.3 4" COMA PLATE

The next surface tested was a flat with a coma term polished into the surface. The magnitude of the coma term was about 3 waves at 632nm. The contour plot shown is on a 20x20 grid, over a 4 inch scan area.

![Contour Plot](image)

**Interferometric Measurement**
- $P_v = 1.916\mu m$
- RMS = $0.318\mu m$

**Profilometer Measurement**
- $P_v = 1.940\mu m$
- RMS = $0.331\mu m$

Figure 9.2, 4" Coma Plate

The measured surface form is replicated very well. The profilometer measurement plot is again interpolated using a linear extrapolation between data points. (This will be true for all measured data set shown in this chapter.) A tilt term is again removed from each of these plots. Because the slope of the comatic surface is so low, a Ball Tip—Stylus Tilt error correction was not necessary.
For these two plots, the same part orientation is used to see how closely the profilometer can replicate the interferometer measurement. When the two measurements are subtract, i.e. the interferometric from the profilometer measurement, the residual map shows a 200nm peak-to-valley difference. This is primarily due to a high surface roughness on the coma plate.

9.4 SPHERE R=753.19mm

The next surface tested was a λ/20 sphere. The measurement grid was again 20x20. The measurement area for this surface was 43mm with a maximum measured slope of 1 degree.

Interferometric Measurement
Pv = 0.037μm
RMS = 0.005μm

Profilometer Measurement
Pv = 0.065μm
RMS = 0.012μm

Figure 9.3, Sphere, R=753.19mm
In both plots shown above, tilt and residual power are removed. The profilometer measurement shown also has the Ball Tip—Stylus Tilt error corrected. The measurements presented here are not at the same orientation. However, the general structure shows that they are close.

Because the Optical Reference Profilometer measures height differences directly from a flat, the measured data can be used to make a direct radius of curvature measurement. The measured radius of curvature for this sphere differed by less than 0.03% from the interferometrically determined radius. Note that on a high precision specification for an optical element is on the order 0.1% for radius of curvature tolerances (Shannon, 1997). The Optical Reference Profilometer measurement is obviously well below that.

9.5 SPHERE, R=52.135mm

The next surface measured was a higher sloped sphere. It had a radius of curvature of 52.135mm. The maximum surface slope of this part was 20 degrees, and the scan area was 43mm on a 20x20 grid.
Both plots shown have residual power and tilt removed. In addition to the normal Ball Tip—Stylus Tilt correction for curved surfaces, this surface displayed an astigmatism term that had to be removed. The source of this astigmatism was the asymmetry found in the stage track motion (chapter 8). To verify that this was indeed the source of the error, the test part was rotated three times, in 120° increments, and re-measured. The residual surface departure from the ideal sphere was analyzed to verify the orientation of the astigmatism. In all cases, the astigmatism orientation and magnitude remained constant.
Comparing the residual surface structure for these two measurements hints at the random nature of the measurement noise found in this system, because the measured structure for the two systems is quite different. Although the residual surface structure difference was higher than in the earlier case, the radius of curvature fit differed by less than 0.03% from the interferometrically determined radius of curvature.

9.6 SPHERE, R=33.78mm

The final surface tested was a sphere with a radius of curvature of 33.78mm. The scan area was 39mm with a maximum surface slope of 35 degrees. The measurement grid was again 20x20.

<table>
<thead>
<tr>
<th>Interferometric Measurement</th>
<th>Profilometer Measurement</th>
</tr>
</thead>
</table>
| P
v = 0.064µm                | P
v = 0.243µm              |
| RMS = 0.010µm              | RMS = 0.027µm            |

Figure 9.5, Sphere, R=33.78mm
Figure 9.5 shows that the system measurement accuracy is reducing quickly as a function of surface slope. However, the radius of curvature measurement still within 0.03% from the interferometrically determined radius. For the plots shown, residual power and tilt are removed. For the profilometer measurement, the Ball Tip—Stylus Tilt error is corrected and the systematic astigmatism term caused by the stage motion asymmetry is removed. The part orientations for this measurement are not the same, although the measured surface structure pattern might suggest otherwise.

9.7 MEASUREMENT REPEATABILITY AND ACCURACY

To determine the repeatability for the second-generation system, a comparison was made of two measurements of the same surface. The measurements were taken several hours apart. The two surfaces used in this comparison were the R=52.135mm and R=33.78mm spheres. The R=52.135mm sphere is shown first. For both sets of plots shown, the orientation of the test part remains constant. So, any residual surface, as measured from the ideal sphere, that is observed is solely due to the repeatability of the instrument. The plots that follow have systematic error structure removed as well as tilt and residual power.
These plots show a similar magnitude for the measured surface residual, however the individual surface features are quite different. If the two surfaces are subtracted, the magnitude of the difference is on the order of 100nm peak-to-valley and 15nm rms. This is mainly due to the large data peaks seen in the second measurement. Even if several more measurements are taken of this surface, the measured peak-to-valley surface error does not deviate by more that a few nanometers from the measurement value shown above, however, as figure 9.6 shows, the measured surface structure observed will differ significantly.
Due to the significant variations or randomness in the measured surface structure for this part slope, the system accuracy must be estimated at the same order as the repeatability. So, for a 20 degree surface slope, the system repeatability and accuracy are on the order of 100nm peak-to-valley and 15nm rms. This estimate ignores the 0.03% power error.

It should be noted that in all of the surface measurements shown, there is a systematic astigmatism error that has been removed (see chapter 8). For these two surfaces of 20 degree surface slope, the magnitude of the astigmatism is about 220nm (pv).

Measurement #1
Pv = 0.251μm
RMS = 0.031μm

Measurement #2
Pv = 0.243μm
RMS = 0.027μm

Figure 9.7, R=33.78mm Measurement Comparison
The R=33.78mm surface measurements show a similar pattern to that found for the R=52.135mm surface, where the peak to valley measurements were very close but the surface features were significantly different. The repeatability is therefore measured at 250nm peak-to-valley and 31nm rms. Following suit with the previous measurements, if the actual R=33.78mm surface map is subtracted from these measurement the residual is on the order of 250nm peak-to-valley. Thus, for a 35 degree surface slope, the measurement accuracy is given a 250nm peak-to-valley and 30nm rms. This estimate ignores the 0.03% power error.

As mentioned earlier, the systematic astigmatism term is present for all surfaces and for this slope was around 400nm peak to valley.

The accuracy and repeatability for the R=753.19mm sphere follow in a similar manner where the peak to valley measurements are close but the physical structure changes. The accuracy for flats and low slope surfaces is given at 75nm peak to valley and about 12nm rms. This estimate ignores the 0.03% power error.

9.8 TEMPERATURE STABILITY

The previous sets of measurements were taken in a normal lab environment. The typical temperature fluctuations were between 0.2-0.8°C. Although the goal of the project was to maintain a high accuracy measurement level with standard temperature
fluctuations, it was decided to just see what would happen with larger temperature fluctuations. The results were quite surprising.

The first test was performed while measuring the R=52.135mm sphere. The measurement duration was the same as for all previous measurements at about one hour and twenty minutes.

During the measurement scan, the temperature in the room was changed by approximately 3 °C. This was accomplished by placing a space heater next to one side of the system. After the measurement, the data was analyzed and a large systematic error on the order of 1.2μm was found. The form of the error was similar to astigmatism.

For the next test, the heat source position was rotated to another side of the instrument. The R=52.135mm sphere was measured again with a very similar result to that of the first. The only exception was the magnitude of the astigmatism-like term was now 2.29μm. The environmental conditions for the two tests were a little different though. For the first test, the x-axis temperature changed by 1°C during the measurement and the y-axis temperature changed by 3°C during the measurement. In the second test, the x-axis temperature changed by 3°C during the measurement and the y-axis changed by 2.5°C.
It is reasonable for the magnitude of the error to increase with an increase in temperature. However, the form and magnitude of the error are hard to accept. It was seen in another experiment that the vibration amplitude of the x and y axes were different. If an impulse is given to the optical table, upon which the profilometer rests, the vibration magnitude of the x-axis is on the order of 1\mu m, while the y-axis remains relatively constant at 300nm. This seems to confirm that there is an asymmetry between the two axes of the profilometer. Although this may not have been the cause of the error structure seen, it confirms that there are asymmetries in the system that could cause an asymmetric measurement residual.

The observed measurement asymmetry can also be cause by temperature gradients caused by the location of the heat source relative to the system in the experiment. Possible methods for changing the system temperature in more uniform fashion are currently under investigation.

A few simple tests were performed on the system to locate the source of the asymmetry, such as tightening screws and checking that all parts were aligned properly, however none of these could be accepted as the source of the problem. The most likely source is a temperature gradient.
9.9 STYLUS PROBE POSITION MONITORING

Before concluding this section, one other feature about all of the aforementioned results must be explained. For all of the measurement result reported so far, the x and y stylus position interferometers were not used.

When the initial measurements were made with this system, the x and y position of the stylus were monitored and corrected. The correction values were those measured by the x and y center interferometers. During these initial measurements very large unrepeatable systematic structure on the order of 400nm (pv) was seen in the surface measurements. It was felt that the position shifts of the stylus assembly were false. Meaning the displacements measured were really not due to the stylus moving but rather to flexing or motion of the metering rod network or some type of settling in the reference ring that holds the corner cubes used with the center/stylus position interferometers.

To verify this assumption that the position shifts of the stylus were wrong, the x and y position axes were disconnected. Several measurement sets of the R=52.135mm sphere were then taken with the assumption that the stylus assembly did not move. In reality the stylus has to move slightly, however the motions are assumed to be small enough not to make a contribution over 10nm (theoretical value).
The reason for this assumption was because the stylus assembly was placed directly in the middle of the bridge. With a uniform thermal expansion of the bridge, symmetry would suggest that the stylus assembly will not move relative to that location. With gradients however, the position of the stylus could change. But with the bridge bolted down at both ends, the most likely effect of the gradient would be to simply induce stress into the bridge structure.

The series of measurements of the R=52.135mm surface were taken over several days and the results compared for repeatability. The results showed that the removal of the x and y center position axes gave repeatable surface measurements. The repeatability was on the order of 100nm peak to valley and 15nm rms. Although this was an indirect measurement and does not prove that the displacements measured were from the metering network flexing or from some type of settling, it does show however, that the measurements of the x and y positions of the stylus were in error. Therefore to obtain the highest accuracy possible the x and y center interferometers were not used. At the current time, the reason for the possible flexing of the metering rod structure has been traced to a design flaw in two of the flexure connections with the interferometer towers. This connection flaw will cause the top x any stylus/center position interferometers to move when the towers, holding the lateral positioning cube interferometers, expand vertically. This explanation is consistent with observed measurement results.
CHAPTER 10 DISCUSSION/CONCLUSIONS

10.1 INTRODUCTION

The purpose of this section is to discuss the results of the system on a more general level with regards to program goals and possible future improvements as well as to give a semi-detailed discussion of the performance limiting elements of the second-generation system.

10.2 PROGRAM GOALS

The goals for this program were to be able to measure up to 40-45 degree surface slopes with a surface residual measurement accuracy of around 100nm (pv). This accuracy was to be maintained over a part size of between 6" and 7.5". At the outset of the program, it was also envisioned that the Optical Reference Profilometer would be effectively smaller than conventional high accuracy CMMs (scaled according to measurable part size). It would potentially cost less for that same high accuracy and be functional in an open laboratory environment. In conclusion of the project, it would seem that the decision on whether these goals were met is not an easy one. Some are clearly a yes, while some are clearly no, while others are maybe. Because some of the project goals were so general, no clear-cut answer is available.
The first set of performance criteria, of measuring 40-45 degree surface slopes at an accuracy of around 100nm peak to valley, was not achieved. The maximum surface slope for which the measurement accuracy was within the desired tolerance was 20 degrees. For higher surface slopes, the accuracy measurement went down to 250nm peak to valley.

The new system was able to measure a full 6" part at the desired accuracy level. However, this ignores systematic astigmatism and power errors found in the second-generation profilometer system. The maximum measurable part size had to be limited to 6", due to an oversight in the mechanical design of the optical reference frame. In a future upgrade this problem will be easy to fix.

Both the 6" measurement range and the 20 degree surface slope measurements were accomplished in an open lab environment, where the maximum temperature variation was approximately 0.8°C. If the temperature variation were more than that, the measurement accuracy becomes limited by large systematic errors. As mentioned in chapter 8, the source of these errors is unknown.

The attainment of the next set of program goals, of size and cost reduction, does not have a clear answer. In order to make a fair comparison of the Optical Reference Profilometer relative to conventional high accuracy CMMs, a chart was made that compares several CMM and/or profiling systems on the basis of measurement range,
resolution, accuracy, size, operational temperature range and cost. The instruments listed are thought to be in the high accuracy range or close enough to be considered for this discussion.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Scan Size</th>
<th>Resolution/Accuracy</th>
<th>Operating Temperature range</th>
<th>Cost (dollars)</th>
<th>System Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Umpire CM6</td>
<td>13.8''x13.8''x13.8''</td>
<td>Res = 1μm, Acc = 3 + L/250 μm</td>
<td></td>
<td>68,000 retail</td>
<td>40''x40''x40''</td>
</tr>
<tr>
<td>Zeiss U-series</td>
<td>16''x20''x16''</td>
<td>Res = 0.2μm, Acc = 0.8 + L/600 μm</td>
<td>68°C +/- 0.2°C</td>
<td>300,000+ retail</td>
<td>6'x8'x11'</td>
</tr>
<tr>
<td>Taylor-Talyscan 150</td>
<td>4''x4''x0.5''</td>
<td>Res = 13nm, Acc = 0.5 μm</td>
<td></td>
<td>120,000 retail</td>
<td>2'x2'x3'</td>
</tr>
<tr>
<td>Nikon Super Master</td>
<td>16''x16''x4''</td>
<td>Res = 2nm, Acc = 20nm (at 45 degree surface slope)</td>
<td>+/- 0.02°C</td>
<td>8'x8'x8'</td>
<td></td>
</tr>
<tr>
<td>Anorad Custom Profilometer</td>
<td>60''x 7'' (?)</td>
<td>Res = 15nm (rms), Acc = 254nm (at 8.6 degree surface slope)</td>
<td>+/- 0.07°C</td>
<td>50''x40''x144''</td>
<td></td>
</tr>
<tr>
<td>Optical Reference Profilometer</td>
<td>6''x6''x2''</td>
<td>Res = 2nm, Acc = 65nm (at 20 degree surface slope)</td>
<td>+/- 0.40°C</td>
<td>75,000 parts only</td>
<td>2'x2'x4'</td>
</tr>
</tbody>
</table>

Figure 10.1, CMM and Profiling Instrument Comparison

This chart clearly shows that the Optical Reference Profilometer has a similar performance to any of the other high accuracy systems. A key feature when evaluating this area is to note the operational temperature range for which the stated accuracy level is valid. Most of the high accuracy CMMs have a very limited temperature range of less
than +/- 0.1°C. The Optical Reference Profilometer does have a much larger operational temperature range but does not have the same accuracy level. This trade off leads to cost.

The system costs listed do not include the temperature control system needed to maintain the stated accuracy level. It is unfortunate that no cost information was available for the highest accuracy system listed. The Nikon machine is undoubtedly the best system when measurement range and accuracy are considered. However, the cost of controlling the system temperature to +/- 0.02°C must be enormous. The elimination of temperature control definitely adds to the utility of the Optical Reference Profilometer.

The final comparison criteria of these systems is size. There are two systems that are of comparable size to the Optical Reference Profilometer: the Taylor-Hobson Talyscan and the Umpire CM6. The Umpire system has a large measurement range, but has a very low resolution and accuracy. The Talyscan has a high resolution but has a very small height measurement range of 10mm. If the other CMM systems are scaled down to the same measurement range as the Optical Reference Profilometer, they are still slightly larger, however, it is too close to say that the Optical Reference Profilometer is truly a smaller system.

If there is one clear conclusion that can be drawn from this discussion, it is that there is no definitive answer whether the Optical Reference Profilometer is a better overall instrument. But it should also be clear, that the Optical Reference Profilometer is a
comparable system in all regards. The one real advantage of the Optical Reference Profilometer is that it is functional at a high accuracy level without the need for a temperature control system. This feature alone drives the utility of this system to the top of the list when comparing to the other systems.

It must be noted that the concept of the Optical Reference Profilometer is scaleable. Although the Optical Reference Profilometer does not have any fundamental restrictions on it size, there may be practical limitations that would hamper its utility after a given point is reached, such considerations might be cost, optic mounting or equipment availability for that particular system size.

10.3 SYSTEM ACCURACY LIMITATIONS

The Optical Reference Profilometer is a system in progress, i.e. there is always something that can be improved upon. In order to identify those areas for improvement, it is important to find the limiting factors for the system performance. One of the largest error sources found in the second-generation profilometer was the stylus repeatability at large surface slopes. The error sources discussed below are for system operation limited to a 1°C temperature change.

It was found experimentally that the average value for the stylus repeatability was between 200-250nm peak-to-valley and 62.6nm rms for surface slopes above 20 degrees.
It must be noted that this is an average and the maximum repeatability error was almost double this number. The reason for this lack of repeatability was thought to be due to the high damped free fall rate of the air bearing stylus. The variability of the free fall rate changes how the stylus slides on the test surface, thus introducing a non-repeatable placement. Although this rate was lowered from the prototype design, it was still too fast to maintain a high repeatability.

Because of the type of dampen used to adjust the stylus free fall rate is so variable, i.e. the air turbulence inside the air bearing lift cavity provides the motion damping, the free fall rate could not be controlled to a highly repeatable motion. The fall rate was also found to be sensitive to the relative humidity level and temperature of the room air. Thus fluctuations in the ambient humidity levels and air temperature would cause fall rate fluctuations. These fluctuations, combined with the variability of the dampening motion, caused the stylus to contact the test surface an unrepeatable fashion.

For flat surfaces, these fluctuations didn’t seem to affect the measurement. However, the magnitude of the placement repeatability error increased with increasing surface slope, thus making a lower repeatability for high slope surfaces and ultimately causing the measurement accuracy goal of 100nm peak to valley to be valid only for surface slopes under 20 degrees.
The second largest error source found in the second-generation profilometer system comes from vibration. If the x or y-axes of the profilometer are sampled over a short time period, small amplitude fluctuations were found. The magnitude of these fluctuations is on the order of 150-250nm. Several experiments were performed to ascertain the source, however these were unsuccessful and the vibration source is still unknown. It is possible that these fluctuations are causing part of the (high slope) stylus repeatability problem, however since the vibration source could not be found this hypothesis could not be proved.

The next few error sources are lesser in magnitude than those previously discussed. The first of these is a random stage tilt. As discussed early in chapter 8, the translation stage has a parabolic motion path. This path is not perfectly parabolic and therefore has minor tilt fluctuations that will cause non-systematic positioning errors. The magnitude of these tilt variations is small. Although it has never been fully quantified as a deviation from the systematic parabolic tilt, the magnitude of these minor tilt variations must be much less than the overall stage rating of less than 50 arc seconds.

The next low magnitude error sources are air turbulence and polarization leakage. The combined peak to valley magnitude of these errors is on the order of 40nm. These actually represent a fundamental peak to valley measurement accuracy on the raw data. If the least squares surface is used, the combined rms value should be a better estimate of this fundamental measurement limit. The current rms values for these errors are
approximately 5nm and 0nm respectively. The polarization error is cyclic and has an average value of zero.

The final error source is due to reference optic surface flatness. Obviously no surface can be perfectly flat. The stage mirrors used with this system have a 63nm peak to valley error and the z-axis reference flat used has a 32nm peak to valley error. At the current noise level of the profilometer, these surface flatness errors are non-correctable. Meaning any type of line reversal or calibration method would be correcting to noise. If the noise level for the system could be reduced to around 10nm peak to valley positioning error, these type corrections would be useful. At a noise level of 10nm peak-to-valley, a three-flat test might become a viable calibration technique (Malacara, 1992 p 42, 580). However, at the current noise level of 45nm for the z-axis and 200-250nm for the x and y axes, it does not make sense.

10.4 FUTURE WORK

The first area for improvement would be with the stylus. As mentioned earlier in this chapter, the stylus motion was found to be unrepeatable at high surface slopes. In order to improve upon this value, a motion-controlled stage could be added that would be used along with the air-bearing stylus. This combination would lessen the overall motion
range needed on the air bearing as well as change the motion mechanism to a more repeatable and softer placement method.

There are several different stylus assembly designs used in other CMMs. Most seem to use something similar to the assembly previously mentioned. The most notable configuration is used on the Super Master from Nikon. The Nikon system uses a double air bearing stylus assembly. The first air bearing is attached to a positioning actuator to provide the base height movement to measure the surface. The second air bearing is attached to the first and serves as the actual contact probe holder. The stylus in the second bearing is attached to a spring. The spring is used to reduce the stylus weight. Although this system is more complicated than most, it was successfully used to a repeatability of 2nm (Shiozawa, 1998).

The next improvement, after a stylus assembly upgrade, would be to find and remove the vibration source from the system. Although a series of tests were performed to find possible vibrational sources in the system, none of these possibilities could be definitively linked to this problem. It is possible that a more in depth mechanical analysis of the profilometer would reveal the source.
After the stylus repeatability and vibration are corrected, a stage tilt correction could be added. This would allow the systematic power and astigmatism error terms caused by the stage asymmetry to be corrected. The idea of this correction is similar to the Ball Tip--Stylus Tilt correction. Where now the tilt angle of the stylus would now be a variable that is adjusted to compensate for the parabolic stage motion tilt as well as the non-systematic tilt. This variable would be defined on a point by point basis. The actual tilt angles could be determined by an additional cube interferometer axis or by using an autocollimator and some type of look up table.

The comparison of the stage tilt to the Ball Tip--Stylus Tilt error is made because the measurement effect of the stage tilting is identical to the stylus tilt problem. Figure 10.2 shows an exaggerated view of the optical reference frame when tilted with respect to the nominal start position. This figure shows that when the stage is tilted, the effect is the same as tilting the stylus relative to the nominal reference frame start position.
Figure 10.2, Stage Tilt Model

If a stage tilt correction routine were to be implemented, the effect of this correction would mainly be seen with higher slope surfaces. There, even the very small non-systematic tilts can cause noticeable errors.

The final correction that might be added to improve system accuracy deals with reference optic calibration. Because the reference surfaces are not perfectly flat, the variations in the surface flatness will introduce measurement errors. For the z-axis reference flat, these are direct height errors and for the x and y stage mirrors, these introduce lateral positioning errors. The most common approach to eliminate these errors
is to use a line reversal technique (Evans, 1999) to subtract out the systematic surface flatness error. This technique was explained earlier in chapter 6.

It is the feeling of the author of this dissertation, that if these corrections could be made, the accuracy of the current system could be improved to meet the surface slope requirement of up to 40-45 degrees.

10.5 SUMMARY

In summary, a few comments must be made to the overall performance and potential of the Optical Reference Profilometer. The optical reference frame measurement technique is very promising. The operational concept works very well and can be used to obtain high accuracy surface measurements. For surface slopes under 20 degrees, the measurement accuracy is approximately 100nm peak-to-valley and 15nm rms. For low slope surfaces on the order of 1 degree or less, the accuracy is approximately 65nm peak-to-valley and 12nm rms. The current profilometer design also provides very accurate radius of curvature measurements. The difference in the interferometrically measured radius of curvature and the radius of curvature determined from the profilometer data is less than 0.03% in all cases investigated, i.e. measured surface slopes of up to 35 degrees.
The data correction algorithms used with this system also add to its versatility. The system alignment tolerance can be relaxed to easily achievable values due to the calibration and error correction routines described in chapters 3 and 4. The analysis of the test part is also benefitted by the least squares optimization routine used to subtract the ideal surface from the measurement data. This routine provides high precision results without the need of additional test part alignment techniques.

Finally, the reduced temperature sensitivity for this system is a real bonus and makes it extremely versatile. Given that this system is functional at a high accuracy level in an open lab environment, the Optical Reference Profilometer provides a real advantage to conventional high accuracy CMM designs. Obviously, the current system performance can be improved upon and in time be made to work at an even higher accuracy level.
APPENDIX

; THE FOLLOWING SECTIONS OF CODE WILL HELP TO UNDERSTAND THE CALIBRATION
; AND BALL TIP—STYLUS TILT CORRECTION ALGORITHMS. THE CODE THAT FALLOWS IS
; A SUBSECTION OF THE ACTUAL CODE.

; this function performs a plane to plane rotation it is a cosine type error for the meas plane to the part plane

function trans, x_not, y_not, l3, m3, scan_number

n3 = 1.0*sqrt(1.0-l3^2-m3^2)

m1 = 0.000000D

n1 = abs(-2.0*l3*m3*m1 + sqrt(abs((2.0*l3^2+n3^2)*((m3*m1)^2-n3^2+n3^2*m1^2))/(2.0*(l3^2+n3^2)))); this should always be positive
if l3 lt 0.0 or l3 eq 0.0 then n1 = sqrt(abs(1.0-l1^2-m1^2))
if l3 gt 0.0 then n1 = -1.0*sqrt(abs(1.0-l1^2-m1^2))

Aprime = (n1*m3-m1*n3)/(l1*n3-n1*l3)
Bprime = ((m3+13*Aprime)/n3)^2
m2 = sqrt(abs(1.0/(1.0+Aprime^2+Bprime)))); this should always be positive
l2 = m2*Aprime
if m3 lt 0.0 or m3 eq 0.0 then n2 = sqrt(abs(1.0-m2^2-l2^2))
if m3 gt 0.0 then n2 = -1.0*sqrt(abs(1.0-m2^2-l2^2))

; direction cosines to adjust the stage plane to the interferometer plane
; l1 m1 n1 ; this is the x axis direction
; l2 m2 n2 ; this is the y axis direction
; l3 m3 n3 ; this is the z axis direction

d = dblarr(sqrt(scan_number), sqrt(scan_number))

xp = dblarr(sqrt(scan_number), sqrt(scan_number))

yp = dblarr(sqrt(scan_number), sqrt(scan_number))

; this rotates the measured z = 0 plane to the actual part z = 0 plane
xp = l1*x_not + m1*y_not + n1*d ; this generates the rotated x data
yp = l2*x_not + m2*y_not + n2*d ; this generates the rotated y data

Ro={fx:xp, fy:yp}; structure definition returning the rotated coordinates
return, Ro
end
:computes the sag values for the reference surface

function reference, x, y, mask

c = 1.0/52.090D;
sag = 1.0*c^((x)^2+(y)^2)/(1.0+(1.0-c^2*((x)^2+(y)^2)^.5))-.25.614975;
sag = sag*mask
return, sag
end

: this is the base iteration for finding the stylus tilt angles

function find_thetas, x, y, z, scannumber, mask, ax1, ay1

mask1 = intarr(sqrt(scannumber), sqrt(scannumber))
  check_radius = 0.0
  radial_dist = 0.0
  ac = (sqrt(scannumber)-1)/2.0
  bc = (sqrt(scannumber)-1)/2.0
  check_radius = ac-1.5

; mask array is set to 1 for x,y positions that lie within the desired circular region and zero outside that region
for i=0, sqrt(scannumber)-1 do begin
  for j=0, sqrt(scannumber)-1 do begin
    radial_dist = SQRT((ac-i)^2 + (bc-j)^2)
    if radial_dist lt check_radius or (radial_dist eq check_radius) then begin
      mask1[i,j] = 1
    endif else begin
      mask1[i,j] = 0
    endelse
  endfor
endfor

ref_x = dblarr(sqrt(scannumber), sqrt(scannumber))
ref_y = dblarr(sqrt(scannumber), sqrt(scannumber))

for j=0, sqrt(scannumber)-1 do begin
  for i=0, sqrt(scannumber)-1 do begin
    ref_x[i,j] = i*20.0/15.0-10.0;
    ref_y[i,j] = j*20.0/15.0-10.0;
  endfor
endfor
; initial angle for the stylus
pie = 3.14159265359 ; value used in the finite derivative
increment = 0.0001 D;

az1 = pie-acos(sqrt(1.0-cos(ax1)^2-cos(ay1)^2));

; choosing x and y angles fixes the z angle directional cosines

\[ \cos(\alpha_1)^2 + \cos(\beta_1)^2 + \cos(\gamma_1)^2 = 1 \]

angles1 = dblarr(3)

angles1(0) = ax1
angles1(1) = ay1
angles1(2) = az1

; these are the initial guesses for the plane transformation of the meas plane to part plane z=0
increment2 = 0.0001

l3_guess = -0.027144280D;
m3_guess = 0.027144280D;

l3_guess2 = l3_guess + increment2
m3_guess2 = m3_guess + increment2

; incremented values of the variables used to find the finite differences necessary to evaluate equations

angles2 = dblarr(3)
ad = ax1 + increment
ab = ay1 + increment

az2 = pie-acos(sqrt(1.0-cos(ax2)^2-cos(ay2)^2))

Alphav = dblarr(scannumber); these are the difference values between the reference and the data used in the least squares iteration

alpha_target1 = dblarr(sqrt(scannumber),sqrt(scannumber)); these are the reference surface sag values
alpha_data1 = dblarr(sqrt(scannumber),sqrt(scannumber)); these are the data values with the ax1 ay1 stylus angles
ro_data1 = dblarr(sqrt(scannumber),sqrt(scannumber)); this is the rotated data

plane_trans = trans(x,y,l3_guess,m3_guess,scannumber); this corrects for the meas. plane to part plane difference

x2 = plane_trans.fx
y2 = plane_trans.fy

alpha_data1 = correction_trial1(x2,y2,z,scannumber,mask,angles1); this evaluates the data at the ax1 ay1 stylus angles

print,'rotate data'

ro_data1 = rotate_surf_trial2(x2,y2,alpha_data1,scannumber,mask); this rotates the corrected data by the amounts given by the directional cosines

zs = ro_data1.fz; this is the corrected, rotated, z shifted data

alpha_target1 = reference(ref_x,ref_y,mask); the position routine (xyshifting) uses the definition of the reference surface as well

; generate the alpha1 values

l = 0; index variable

for j=0,sqrt(scannumber)-1 do begin
for i=0, sqrt(scannumber)-1 do begin
    Alpha_values(i) = (zs[i,j]-alpha_target1[i,j]);
    i = i + 1
endfor
endfor

; this section computes the peak to valley and rms numbers
zll = dblarr(sqrt(scannumber),sqrt(scannumber))
p = 0 ; index variable
for j=0, sqrt(scannumber)-1 do begin
    for i=0, sqrt(scannumber)-1 do begin
        zll[i,j] = Alpha_values(p)
p = p + 1
    endfor
endfor
j = steve_tilt5(zll, ref_x, ref_y, scannumber, mask1)

; generate the alpha1 values
l = 0 ; index variable
for j=0, sqrt(scannumber)-1 do begin
    for i=0, sqrt(scannumber)-1 do begin
        Alpha_values(l) = zll[i,j]
l = l + 1
    endfor
endfor

; because of edge fitting effects use a smaller mask to determine pv and rms.
gg = dblarr(3)
gg = find_min(zll, scannumber, mask1)
pv = 0.0D
ggl = dblarr(3)
ggl = find_max(zll, scannumber, mask1)
pv = ggl(0)-gg(0)
print,'pv = ',pv
window, 0
contour, mask1*zll, /fill, isotropic, levels = min(mask1*zll)+findgen(60)*(max(mask1*zll)-
min(mask1*zll))/60.0
window, 1
surface, mask1*zll

; put the difference values in the smaller mask1 into new array and find the rms value
p=0
for j=0, sqrt(scannumber)-1 do begin
    for i=0, sqrt(scannumber)-1 do begin
        if mask1[i,j] eq 1 then p=p+1
    endfor
endfor
aprime = dblarr(p)
p1 = 0
\[ w = 0 \]
for \( j = 0, \sqrt{\text{scannumber}} - 1 \) do begin
     for \( i = 0, \sqrt{\text{scannumber}} - 1 \) do begin
             if \( \text{mask}[i,j] \) eq 1 then begin
                 \( \text{aprime}[p1] = z11[i,j]; \alpha_{values}[w] \)
                 \( p1 = p1 + 1 \)
             endif
             \( w = w + 1 \)
     endfor
endfor

\[ \text{rms}_l = \text{MOMENT}(\text{aprime}) \]
\[ \text{RMS} = \sqrt{\text{rms}_l[1]} \]
print,'rms = ',rms
stop
print,ax1,ay1,i3\_guess,m3\_guess

; generate the derivative matrix, this is used in the least square iteration process
q = 0 ; index value
der\_matrix = dblarr(4,\text{scannumber});dblarr(4,\text{scannumber});this matrix contains the derivatives with respect to the three variables

\( f1 = zs; \) this is the corrected, rotated, z shifted data for the first set of variable values
\( f2x = \text{dblarr}(\sqrt{\text{scannumber}},\sqrt{\text{scannumber}}) \)

; this is the corrected sag values for ax changing while holding everything else constant
angles2(0) = ax2
angles2(1) = ay1
angles2(2) = \( \pi - \arccos(\sqrt{1.0 - \cos(ax2)^2 - \cos(ay1)^2}) \)
plane\_trans = trans(x,y,i3\_guess,m3\_guess,\text{scannumber}); this corrects for the meas. plane to part plane difference
\( x2 = \text{plane\_trans.fx} \)
\( y2 = \text{plane\_trans.fy} \)
\( f2x = \text{correction\_trial1}(x2,y2,z,\text{scannumber},\text{mask},\text{angles2}) \)
ro\_data = rotate\_surf\_trial2(x2,y2,f2x,\text{scannumber},\text{mask})
der\_ax1 = ro\_data.fz;+zshift1

\( f2y = \text{dblarr}(\sqrt{\text{scannumber}},\sqrt{\text{scannumber}}) \)
; this is the corrected sag values for ay changing while holding everything else constant
angles2(0) = ax1
angles2(1) = ay2
angles2(2) = \( \pi - \arccos(\sqrt{1.0 - \cos(ax1)^2 - \cos(ay2)^2}) \)
plane\_trans = trans(x,y,i3\_guess,m3\_guess,\text{scannumber}); this corrects for the meas. plane to part plane difference
\( x2 = \text{plane\_trans.fx} \)
\( y2 = \text{plane\_trans.fy} \)
\( f2y = \text{correction\_trial1}(x2,y2,z,\text{scannumber},\text{mask},\text{angles2}) \)
; this is the corrected sag values for ay changing while holding ax constant
ro\_data = rotate\_surf\_trial2(x2,y2,f2y,\text{scannumber},\text{mask})
der\_ay1 = ro\_data.fz;+zshift1

\( f2i3 = \text{dblarr}(\sqrt{\text{scannumber}},\sqrt{\text{scannumber}}) \)
this is the corrected sag values for l3 changing while holding everything else constant
angles2(0) = ax1
angles2(1) = ay1
angles2(2) = pi-acos(sqrt(1.0-cos(ax1)^2-cos(ay1)^2))
plane_trans = trans(x,y,l3_guess2,m3_guess,scannumber);this corrects for the meas. plane to part plane difference
x2 = plane_trans.fx
y2 = plane_trans.fy
f2l3 = correction_trial1(x2,y2,z,scannumber,mask,angles2);this is the corrected sag values for ay changing while holding ax constant
ro_data = rotate_surf_trial2(x2,y2,f2l3,scannumber,mask)
der_l3 = ro_data.fz;+zshift1
f2m3 = dblarr(sqrt(scannumber),sqrt(scannumber));this is the corrected sag values for l3 changing while holding everything else constant
angles2(0) = ax1
angles2(1) = ay1
angles2(2) = pi-acos(sqrt(1.0-cos(ax1)^2-cos(ay1)^2))
plane_trans = trans(x,y,l3_guess,m3_guess2,scannumber);this corrects for the meas. plane to part plane difference
x2 = plane_trans.fx
y2 = plane_trans.fy
f2m3 = correction_trial1(x2,y2,z,scannumber,mask,angles2);this is the corrected sag values for ay changing while holding ax constant
ro_data = rotate_surf_trial2(x2,y2,f2m3,scannumber,mask)
der_m3 = ro_data.fz;+zshift1
dx1 = der_ax1-fl
u = steve_tilt5(dx1,ref_x,ref_y,scannumber,mask)
dy1 = der_ay1-fl
u = steve_tilt5(dy1,ref_x,ref_y,scannumber,mask)
dl3 = der_l3-fl
u = steve_tilt5(dl3,ref_x,ref_y,scannumber,mask)
dm3 = der_m3-fl
u = steve_tilt5(dm3,ref_x,ref_y,scannumber,mask)
for t1=0,sqrt(scannumber)-1 do begin
for t=0,sqrt(scannumber)-1 do begin
    der_matrix(0,q) = (dx1[t,t1])/(increment)
    der_matrix(1,q) = (dy1[t,t1])/(increment)
    der_matrix(2,q) = (dl3[t,t1])/(increment2)
    der_matrix(3,q) = (dm3[t,t1])/(increment2)
    q = q + 1
endfor
endfor
;generate the variable vector for the three variables xshift,yshift,zshift
shift_vector = dblarr(4);dblarr(4)
GP = (invert(transpose(der_matrix))##der_matrix))##(transpose(der_matrix))

shift_vector = -GP##Alpha_values

shift_vector = shift_vector*0.25;0.005D
ax1 = ax1+shift_vector(0)
ay1 = ay1+shift_vector(1)
az1 = pie-acos(sqrt(1.0-cos(ax1)^2-cos(ay1)^2))
angles1(0) = ax1
angles1(1) = ay1
angles1(2) = az1

l3_guess = l3_guess + shift_vector(2)
m3_guess = m3_guess + shift_vector(3)

zshift1 = zshift1+shift_vector(2)

;increment = increment/10.0D
print,angles1,l3_guess,m3_guess

for tt=0.0 do begin
 ;incremented values of the variables used to find the finite differences necessary to evaluate equations
ax2 = ax1+increment
ay2 = ay1+increment
az2 = pie-acos(sqrt(1.0-cos(ax2)^2-cos(ay2)^2))
l3_guess2 = l3_guess + increment2
m3_guess2 = m3_guess + increment2

plane_trans = trans(x,y,l3_guess,m3_guess,scannumber);this corrects for the meas. plane to part plane difference
x2 = plane_trans.fx
y2 = plane_trans.fy

alpha_data1 = correction_trial1(x2,y2,z,scannumber,mask,angles1);this evaluates the data at the ax1 ay1 stylus angles
print,'rotate data'
ro_data1 = rotate_surf_trial2(x2,y2,alpha_data1,scannumber,mask);this rotates the corrected data by the amounts given by the directional cosines
zs = ro_data1.fz;this is the corrected,rotated,z shifted data

alpha_target1 = reference(ref_x,ref_y,mask);the position routine (xyshifting) uses the definition of the reference surface as well

;generate the alpha1 values
l = 0; index variable
for j=0,sqrt(scannumber)-1 do begin
 for i=0,sqrt(scannumber)-1 do begin
   Alpha_values(l) = (zs[i,j]-alpha_target1[i,j]):/\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\n
endfor
endfor

;this section computes the peak to valley and rms numbers
zl I = dblarr(sqrt(scannumber),sqrt(scannumber))
p = 0 ; index variable
for j=0,sqrt(scannumber)-1 do begin
    for i=0,sqrt(scannumber)-1 do begin
        zl I[i,j] = Alpha_values(p)
p = p + 1
    endfor
endfor
j = steve_tilt5(zl I ,ref_x,ref_y,scannumber,mask I)

;generate the alpha values
l = 0 ; index variable
for j=0,sqrt(scannumber)-1 do begin
    for i=0,sqrt(scannumber)-1 do begin
        Alpha_values(l) = zl I[i,j]
l = l + 1
    endfor
endfor

;because of edge fitting effects use a smaller mask to determine pv and rms.
gg = dblarr(3)
gg= find_min(zl 1,scannumber,mask I)
pv = 0.0D
gg I = dblarr(3)
gg I = find_max(zl 1,scannumber,mask I)
pv = gg I(0)-gg(0)
print,'pv =',pv

;put the difference values in the smaller mask I into new array and find the rms value
p=0
for j=0,sqrt(scannumber)-1 do begin
    for i=0,sqrt(scannumber)-1 do begin
        if mask I[i,j] eq 1 then p=p+1
    endfor
endfor
aprime =dblarr(p)
pl = 0
w = 0
for j=0,sqrt(scannumber)-1 do begin
    for i=0,sqrt(scannumber)-1 do begin
        if mask I[i,j] eq 1 then begin
            aprime[pl]=alpha_values[w]
            pl = pl+1
        endif
        w = w+1
    endfor
endfor
endfor

rmsl = MOMENT(aprime)
RMS = SQRT(rmsl[1])
print,'rms = ',rms
;stop
print,ax1,ay1,13_guess,m3_guess

;generate the derivative matrix, this is used in the least square iteration process
q = 0 ; index value
der_matrix = dblarr(4,scannumber);dblarr(4,scannumber);this matrix contains the derivatives with respect to the three variables

f1 = zs;this is the corrected,rotated,z shifted data for the first set of variable values
f2x = dblarr(sqrt(scannumber),sqrt(scannumber))

;this is the corrected sag values for ax changing while holding everything else constant
angles2(0) = ax2
angles2(1) = ay1
angles2(2) = pie-acos(sqrt(1.0-cos(ax2)^2-cos(ay1)^2))
plane_trans = trans(x,y,13_guess,m3_guess,scannumber);this corrects for the meas. plane to part plane difference
x2 = plane_trans.fx
y2 = plane_trans.fy
f2x = correction_triall(x2,y2,z,scannumber,mask,angles2)
ro_data = rotate_surf_trial2(x2,y2,f2x,scannumber,mask)
der_ax1 = ro_data.fz;+zshift1

defy = dblarr(sqrt(scannumber),sqrt(scannumber))
;this is the corrected sag values for ay changing while holding everything else constant
angles2(0) = ax1
angles2(1) = ay2
angles2(2) = pie-acos(sqrt(1.0-cos(ax1)^2-cos(ay2)^2))
plane_trans = trans(x,y,13_guess,m3_guess,scannumber);this corrects for the meas. plane to part plane difference
x2 = plane_trans.fx
y2 = plane_trans.fy
f2y = correction_triall(x2,y2,z,scannumber,mask,angles2);this is the corrected sag values for ay changing while holding ax constant
ro_data = rotate_surf_trial2(x2,y2,f2y,scannumber,mask)
der_ay1 = ro_data.fz;+zshift1

def13 = dblarr(sqrt(scannumber),sqrt(scannumber))
;this is the corrected sag values for l3 changing while holding everything else constant
angles2(0) = ax1
angles2(1) = ay1
angles2(2) = pie-acos(sqrt(1.0-cos(ax1)^2-cos(ay1)^2))
plane_trans = trans(x,y,13_guess2,m3_guess,scannumber);this corrects for the meas. plane to part plane difference
x2 = plane_trans.fx
y2 = plane_trans.fy
f2l3 = correction_triall(x2,y2,z,scannumber,mask,angles2);this is the corrected sag values for ay changing while holding ax constant
ro_data = rotate_surf_trial2(x2,y2,f2m3,scannumber,mask)
der_l3 = ro_data.fz;zshift1

f2m3 = dblarr(sqrt(scannumber),sqrt(scannumber))
;this is the correct sag values for l3 changing while holding everything else constant
angles2(0) = ax1
angles2(1) = ay1
angles2(2) = pie-acos(sqrt(1.0-cos(ax1)^2-cos(ay1)^2))
plane_trans = trans(x,y,l3_guess,m3_guess2,scannumber);this corrects for the meas. plane to part plane difference
x2 = plane_trans.fx
y2 = plane_trans.fy
f2m3 = correction_trial1(x2,y2,z,scannumber,mask,angles2);this is the corrected sag values for ay changing while holding ax constant
ro_data = rotate_surf_trial2(x2,y2,f2m3,scannumber,mask)
der_m3 = ro_data.fz;zshift1
dx1 = der_ax1-f1
u = steve_tilt5(dx1,ref_x,ref_y,scannumber,mask)
dy1 = der_ay1-f1
u = steve_tilt5(dy1,ref_x,ref_y,scannumber,mask)
dl3 = der_l3-f1
u = steve_tilt5(dl3,ref_x,ref_y,scannumber,mask)
dm3 = der_m3-f1
u = steve_tilt5(dm3,ref_x,ref_y,scannumber,mask)
for t1=0,sqrt(scannumber)-1 do begin
  for t=0,sqrt(scannumber)-1 do begin
    der_matrix(0,q) = (dx1[t,t1])/(increment)
    der_matrix(1,q) = (dy1[t,t1])/(increment)
    der_matrix(2,q) = (dl3[t,t1])/(increment2)
    der_matrix(3,q) = (dm3[t,t1])/(increment2)
    q = q + 1
  endfor
endfor
:generate the variable vector for the three variables xshift,yshift,zshift
shift_vector = dblarr(4);dblarr(4)
GP = (invert(transpose(der_matrix)#der_matrix))#(transpose(der_matrix))
shift_vector = -GP#Alpha_values
shift_vector = shift_vector*0.25;0.005D
ax1 = ax1+shift_vector(0)
ay1 = ay1+shift_vector(1)
az1 = pie-acos(sqrt(1.0-cos(ax1)^2-cos(ay1)^2))
angles1(0) = ax1
This will use an iterative method to find the stylus tilt angles.

```
pro calibration2

scannumber = 400;
OPEN_FILE
position = dblarr(3)
xpos = dblarr(scannumber)
ypos = dblarr(scannumber)
zpos = dblarr(scannumber)
;Read data from file and store in data arrays
index = 0
WHILE(index LT (scannumber)) DO BEGIN
  position = READ_DATA()
  xpos[index] = position[0]
  ypos[index] = position[1]
  zpos[index] = position[2]
  index = index + 1
ENDWHILE

FLUSH, 1
CLOSE, 1

;this puts the data into 2_D arrays
data = dblarr(sqrt(scannumber),sqrt(scannumber))
x_not = dblarr(sqrt(scannumber),sqrt(scannumber))
y_not = dblarr(sqrt(scannumber),sqrt(scannumber))
noise1 = dblarr(sqrt(scannumber),sqrt(scannumber))
p = 0

for j=0,sqrt(scannumber)-1 do begin
  for i=0,sqrt(scannumber)-1 do begin
    data[i,j] = zpos(p)
```
x_not[i,j] = xpos(p);
y_not[i,j] = ypos(p);

p=p+1
endfor
endfor

mask_on_off  = 'on'
mask = intarr(sqrt(scannumber),sqrt(scannumber))
   check_radius  = 0.0
   radial_dist  = 0.0
   ac = (sqrt(scanNumber)-1)/2.0
   bc = (sqrt(scanNumber)-1)/2.0
   check_radius = ac+0.5

;mask array is set to 1 for x,y positions that lie within the desired circular region and zero outside that region
for i=0,sqrt(scannumber)-1 do begin
   for j=0,sqrt(scannumber)-1 do begin
       radial_dist=SQRT((ac-i)^2 + (bc-j)^2)
       if(radial_dist lt check_radius) or (radial_dist eq check_radius) then begin
           mask[i,j] = 1
       endif else begin
           mask[i,j] = 0
       endelse
   endfor
endfor
if mask_on_off eq 'off' then mask = 1.0

ax = 1.5708597000D;
ay = 1.5718315000D;

stuff = 0.0D
stuff = find_thetas(x_not,y_not,data,scannumber,mask,ax,ay)
close,1
end
; THIS SECTION OF CODE IS FOR THE BALL TIP—STYLUS TILT CORRECTION

; this function will find the difference in the test piece and stylus ball slope in the x direction at the guess x,y,z_not

function diff_functsx, x,y,z_not,x_not,y_not,coeff

common share4
r = 0.5000000D

; since the z_not value generated by the x and y slopes is the same now I just chose to use the x slope eqs to find the z surface difference function value

A = double(0.0); x slope of the part being tested can be either positive or negative
B = double(0.0); just a simplification var used in the root equation
C1 = double(0.0); just a simplification var used in the root equation
D = double(0.0); just a simplification var used in the root equation
E = double(0.0); just a simplification var used in the root equation
difference = double(0.0); just a simplification var used in the root equation

A = derivative(coeff,'x',x,y); test piece slope in the x direction
B = (x-x_not)
C1 = (y-y_not)
D = cos(thetax)/cos(thetaz)
E = cos(thetay)/cos(thetaz)

Aballx = (B-z_not*D)/sqrt(r^2-(B-z_not*D)^2-(C1-z_not*E)^2)

difference = Aballx-A
return, difference
end

; this function will find the difference in the test piece and stylus ball slope in the y direction at the guess x,y,z_not

function diff_functsy, x,y,z_not,x_not,y_not,coeff

common share4
r = 0.5000000D
A = double(0.0); y slope of the part being tested can be either positive or negative
B = double(0.0); just a simplification var used in the root equation
C1 = double(0.0); just a simplification var used in the root equation
D = double(0.0); just a simplification var used in the root equation
E = double(0.0); just a simplification var used in the root equation

difference = double(0.0); just a simplification var used in the root equation

A = derivative(coeff,'y',x,y); test piece slope in the y direction
B = (x-x_not)
C1 = (y-y_not)
D = cos(thetax)/cos(thetaz)
E = cos(thetay)/cos(thetaz)

Abally = (C1-z_not*E)/sqrt(r^2-(B-z_not*D)^2-(C1-z_not*E)^2)

difference = Abally-A
return, difference
end

;=============
;this is the difference function for the stylus axis iteration
;=============

function diff_functa, a,xt,yt,coeff
common share4

x = xt + a*(cos(thetax)); this is the x location given a starting x location xt and a direction vector length a
y = yt + a*(cos(thetay)); this is the y location given a starting y location yt and a direction vector length a

value = double(0.0)

; this checks to see if the value of a will make the vector lie right on the test surface
value = a*(cos(thetaz))-sag_values(x,y,coeff)

return, value
end

;=============
;this function is the base iteration for the stylus axis intersection
;=============

function starta,x_not,y_not,coeff
common share4

;this section is a bisection iteration for the directional cosine method for finding a ray intersection point on a surface
;this will allow any surface intersection point to be found, just by altering the expression in the difference function
xl = double(0.0)
xr = double(0.0)
xs = double(0.0)
xl = 4.0; since the solution to the path length a must be between 0 and 100mm, typically between 15 and 30
\[ \text{xr} = 30.0 \]

; this is a bisection iteration for finding the vector length of the stylus vector (i.e. the length of the stylus vector such that as it travels down the stylus angles it will intersect the test surface--xs is actually the length of the vector not the x location of intersection. I'm using some old code to do this process

\[ \text{diff\_function1} = \text{double}(0.0) \]
\[ \text{diff\_function2} = \text{double}(0.0) \]
\[ \text{diff\_function3} = \text{double}(0.0) \]
\[ \text{diff\_function1} = \text{diff\_functa}(\text{xl}, \text{x\_not}, \text{y\_not}, \text{coeff}) \]
\[ \text{diff\_function2} = \text{diff\_functa}(\text{xr}, \text{x\_not}, \text{y\_not}, \text{coeff}) \]

\[ \text{while diff\_function1} \times \text{diff\_function2} > 0.0 \text{ do begin} \]
  \[ \text{print,'wrong initial guess interval'} \]
  \[ \text{print,'input initial guess xleft>'} \]
  \[ \text{stop} \]
  \[ \text{read,xl} \]
  \[ \text{print,'input initial guess xright>'} \]
  \[ \text{read,xr} \]
  \[ \text{diff\_function1} = \text{diff\_functa}(\text{xl}, \text{x\_not}, \text{y\_not}, \text{coeff}) \]
  \[ \text{diff\_function2} = \text{diff\_functa}(\text{xr}, \text{x\_not}, \text{y\_not}, \text{coeff}) \]
\[ \text{endwhile} \]

\[ \text{xs} = \text{double}(0.5) \times (\text{xl} + \text{xr}) \]
\[ \text{diff\_function3} = \text{diff\_functa}(\text{xs}, \text{x\_not}, \text{y\_not}, \text{coeff}) \]

\[ \text{interval1} = \text{double}(0.0) \]
\[ \text{interval2} = \text{double}(0.0) \]

\[ \text{for i =0, 35 do begin} \]
  \[ \text{interval1} = \text{diff\_function3} \times \text{diff\_function1} \]
  \[ \text{interval2} = \text{diff\_function3} \times \text{diff\_function2} \]
  \[ \text{if interval1} \text{ eq 0.0 or interval2} \text{ eq 0.0 then begin} \]
    \[ \text{if diff\_function1} \text{ eq 0.0 then xs} = \text{xl} \]
    \[ \text{if diff\_function2} \text{ eq 0.0 then xs} = \text{xr} \]
    \[ \text{if diff\_function3} \text{ eq 0.0 then xs} = \text{xs} \]
  \[ \text{endif else begin} \]
  \[ \text{if interval1} \text{ lt 0.0 then begin} \]
    \[ \text{xr} = \text{xs} \]
    \[ \text{diff\_function2} = \text{diff\_function3} \]
  \[ \text{endif else begin} \]
  \[ \text{xl} = \text{xs} \]
  \[ \text{diff\_function1} = \text{diff\_function3} \]
\[ \text{endelse} \]
xs = double(0.5)*(xr + xl)
diff_function3 = diff_functa(xs,x_not,y_not,coeff)

endelse
endfor

h = dblarr(3)

xp = x_not + xs*cos(thetax);this is the x coordinate of the intersection point, xs is just the length of the vector
yp = y_not + xs*cos(thetay);this is the y coordinate of the intersection point, xs is just the length of the vector
zp = xs*cos(thetaz);this is the z coordinate of the intersection point, xs is just the length of the vector
h(0) = xp
h(1) = yp
h(2) = zp

return, h

end

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;this function will find the difference between the z value of the ball and the test piece at the x,y
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
function diff_functz, x,y,z_not,x_not,y_not,coeff

common share4

r = 0.5000000D

B = double(0.0);just a simplification var used in the root equation
C1 = double(0.0);just a simplification var used in the root equation
D = double(0.0);just a simplification var used in the root equation
E = double(0.0);just a simplification var used in the root equation
z = double(0.0);test piece sag equation
zball = double(0.0);stylus ball sag equation

B = (x-x_not)
C1 = (y-y_not)
D = cos(thetax)/cos(thetaz)
E = cos(thetay)/cos(thetaz)

z = sag_values(x,y,coeff);
p = (r^2-(B-z_not*D)^2-(C1-z_not*E)^2)
if p lt 0.0 then stop
zball = z_not-(p)^.5

value = double(0.0)
value = z-zball; difference in the z heights of the ball and the surface given the x, y location and slope conditions

return, value
end

;===============
;this function will generate xslope values for a stylus ball centered at 0,0, to be used to help generate accurate initial guesses for the contact points
;===============
function ballx, x, y

r = 0.500000D
slope = x/(r^2-x^2-y^2)^.5

return, slope
end

;===============
;this function will generate yslope values for a stylus ball centered at 0,0, to be used to help generate accurate initial guesses for the contact points
;===============
function bally, x, y

r = 0.500000D
slope = y/(r^2-x^2-y^2)^.5

return, slope
end

;===============
;this function will find the location on the stylus ball x,y for a given part slope_x, slope_y. If abs(slope_x) gt 0.23 or abs(slope_y) gt 0.23 then stop
;this is done with a ball at the origin and the data is used to generate the guess for start_slope
;===============
function find_xy_loc, x1, y1, slope_x, slope_y

; if abs(slope_x) gt 0.23 or abs(slope_y) gt 0.23 then stop

increment = 0.0005D

x2 = x1+increment
y2 = y1+increment

ddx = (ballx(x2,y1)-ballx(x1,y1))/increment
ddy = (bally(x1,y2)-bally(x1,y1))/increment
ddx1 = (bally(x2,y1) - bally(x1,y1))/increment
ddy1 = (bally(x1,y2) - bally(x1,y1))/increment

der_matrix = dblarr(2,2); this matrix contains the finite differences needed to solve the equations

der_matrix(0,0) = ddx
der_matrix(0,1) = ddx1
der_matrix(1,0) = ddy
der_matrix(1,1) = ddy1

GP = dblarr(2,2); just an intermediate matrix to solve the equations
GP = invert(der_matrix)

Alpha = dblarr(2); this is the difference function values at x1,y1
Alpha(0) = ballx(x1,y1) - slopex
Alpha(1) = bally(x1,y1) - slopey

HP = dblarr(2); this contains the increments the guesses x,y must move
HP = -GP##Alpha

x1 = x1 + HP(0)
y1 = y1 + HP(1)
increment = increment/10.0D

for i=0,2 do begin

x2 = x1 + increment
y2 = y1 + increment

ddx = (ballx(x2,y1) - ballx(x1,y1))/increment
ddy = (ballx(x1,y2) - ballx(x1,y1))/increment
ddx1 = (bally(x2,y1) - bally(x1,y1))/increment
ddy1 = (bally(x1,y2) - bally(x1,y1))/increment

der_matrix = dblarr(2,2); this matrix contains the finite differences needed to solve the equations

der_matrix(0,0) = ddx
der_matrix(0,1) = ddx1
der_matrix(1,0) = ddy
der_matrix(1,1) = ddy1

GP = dblarr(2,2); just an intermediate matrix to solve the equations
GP = invert(der_matrix)

Alpha = dblarr(2); this is the difference function values at x1,y1
\[ \alpha(0) = ball(x_1, y_1) - \text{slopec} \]
\[ \alpha(1) = bally(x_1, y_1) - \text{slopec} \]

\[ HP = \text{dblarr}(2); \text{this contains the increments the the guesses } x,y \text{ must move} \]

\[ HP = -GP##\alpha \]

\[ x_1 = x_1 + HP(0) \]
\[ y_1 = y_1 + HP(1) \]
\[ \text{increment} = \text{increment}/10.0D \]

\[ \text{end for} \]

\[ h = \text{dblarr}(2) \]
\[ h(0) = x_1 \]
\[ h(1) = y_1 \]
\[ \text{return}, h \]
\[ \text{end} \]

\[ \text{function start_slope}, x\_\text{not}, y\_\text{not}, \text{coeff} \]

\[ \text{common share4} \]

\[ r = 0.5000000D \]

\[ \text{num} = \text{sag_values}(x\_\text{not}, y\_\text{not}, \text{coeff}); \]
\[ \text{increment}_\theta = \frac{\text{num}}{-\text{tan}(\theta_\text{ax})}; \text{this is used when the stylus is tilted down hill} \]
\[ \text{increment}_\theta_y = \frac{\text{num}}{-\text{tan}(\theta_\text{ay})} \]
\[ \text{increment}_\theta = \frac{\text{num}+1.0}{-\text{tan}(\theta_\text{ax})}; \text{this is use when the stylus is tilted up hill} \]
\[ \text{increment}_\theta_y = \frac{\text{num}+1.0}{-\text{tan}(\theta_\text{ay})} \]

\[ x = x\_\text{not} \]
\[ y = y\_\text{not} \]

; since the z_n not value generated by the x and y slopes is the same now I just chose to use the x slope eqs to find the z surface difference function value
\[ Ay = \text{double}(0.0); y \text{ slope of the part being tested} \]
\[ Ax = \text{double}(0.0); x \text{ slope of the part being tested can be either positive or negative} \]
\[ Ay = \text{derivative(coeff,'y',x,y);} \]
\[ Ax = \text{derivative(coeff,'x',x,y);} \]

;if abs(Ay) gt 0.23 or abs(Ax) gt 0.23 then stop

if abs(Ax) gt 1.0 or abs(Ax) eq 1.0 then locx = 0.35
if (abs(Ax) gt 0.6 or abs(Ax) eq 1.0) and abs(Ax) lt 1.0 then locx = 0.25
if (abs(Ax) gt 0.3 or abs(Ax) eq 0.6) and abs(Ax) lt 0.6 then locx = 0.15
if abs(Ax) lt 0.3 then locx = 0.1
if abs(Ay) gt 1.0 or abs(Ay) eq 1.0 then locy = 0.35
if (abs(Ay) gt 0.6 or abs(Ay) eq 1.0) and abs(Ay) lt 1.0 then locy = 0.25
if (abs(Ay) gt 0.3 or abs(Ay) eq 0.6) and abs(Ay) lt 0.6 then locy = 0.15
if abs(Ay) lt 0.3 then locy = 0.1

;this generates the location on a ball where the slope is the same as the part slope at the x_not,y_not
increment = 0.0005D
if Ax gt 0.0 then begin
  if Ay gt 0.0 then begin
    x3 = locx
    y3 = locy
  endif else begin
    x3 = locx
    y3 = -locy
  endelse
endif else begin
  if Ay gt 0.0 then begin
    x3 = -locx
    y3 = locy
  endif else begin
    x3 = -locx
    y3 = -locy
  endelse
endelse

d = dblarr(2); these are the values of x,y that have same slope on centered ball(at origin) at part slope
d = find_xy_loc(x3,y3,Ax,Ay); find_xy is the 2 variable iteration to find the x,y coordinates of the slope
condition just described

xp = abs(d(0))
yp = abs(d(1))

guesses = dblarr(3)

; this section will create a best guess for the contact point it depends on slope sign and stylus tilt angles
if Ax gt 0.0 or Ax eq 0.0 then begin
  if thetax lt 1.57079632D or thetax eq 1.57079632D then xs = x_not+increment_theta1+xp
  if thetax gt 1.57079633D or thetax eq 1.57079633D then xs = x_not+increment_theta+xp
  if thetax gt 1.57079632D and thetax lt 1.57079633 then xs = x_not+xp
endif
if Ax lt 0.0 then begin
  if thetax lt 1.57079632D or thetax eq 1.57079632D then xs = x_not+increment_theta-xp
  if thetax gt 1.57079633D or thetax eq 1.57079633D then xs = x_not+increment_theta-xp
  if thetax gt 1.57079632D and thetax lt 1.57079633 then xs = x_not-xp
endif
if Ay gt 0.0 or Ay eq 0.0 then begin
if thetay lt 1.57079632D or thetay eq 1.57079632D then ys = y_not+increment_thetay1+yp
if thetay gt 1.57079633D or thetay eq 1.57079633D then ys = y_not+increment_thetay1+yp
if thetay gt 1.57079632D and thetay lt 1.57079633 then ys = y_not+yp
ys = y_not+increment_thetay+yp
endif
if Ay lt 0.0 then begin
  if thetay lt 1.57079632D or thetay eq 1.57079632D then ys = y_not+increment_thetay*yp
  if thetay gt 1.57079633D or thetay eq 1.57079633D then ys = y_not+increment_thetay1-yp
  if thetay gt 1.57079632D and thetay lt 1.57079633 then ys = y_not-yp
  ys = y_not+increment_thetay-yp
endif
num1 = sag_values(xs,ys,coeff);
cen_sag_offset = r-(sqrt(r^2-d(0)^2-d(1)^2))
z0_guess = num1+r-cen_sag_offset
for hh=0,4 do begin
  Ay = derivative(coeff,y',xs,ys);
  Ax = derivative(coeff,x',xs,ys);
d =find_xy_loc(x3,y3,Ax,Ay)
  xp = abs(d(0))
  yp = abs(d(1))
;if abs(Ay) gt 0.30 or abs(Ax) gt 0.30 then stop

increment_theta = (z0_guess)/(-tan(thetax));this is used when the stylus is tilted down hill
increment_thetay = (z0_guess)/(-tan(thetay))
increment_thetay1 = (z0_guess)/(-tan(thetay));this is use when the stylus is tilted up hill
increment_thetay1 = (z0_guess)/(-tan(thetay))

;this section will create a best guess for the contact point it depends on slope sign and stylus tilt angles
if Ax gt 0.0 or Ax eq 0.0 then begin
  if thetax lt 1.57079632D or thetax eq 1.57079632D then xs = x_not+increment_thetay1+xp
  if thetax gt 1.57079633D or thetax eq 1.57079633D then xs = x_not+increment_thetay+xp
  if thetax gt 1.57079632D and thetax lt 1.57079633 then xs = x_not+xp
endif
if Ax lt 0.0 then begin
  if thetax lt 1.57079632D or thetax eq 1.57079632D then xs = x_not+increment_thetay-xp
  if thetax gt 1.57079633D or thetax eq 1.57079633D then xs = x_not+increment_thetay1-xp
  if thetax gt 1.57079632D and thetax lt 1.57079633 then xs = x_not-xp
  :xs = x_not+increment_thetay-xp
endif
if Ay gt 0.0 or Ay eq 0.0 then begin
  if thetay lt 1.57079632D or thetay eq 1.57079632D then ys = y_not+increment_thetay1+yp
  if thetay gt 1.57079633D or thetay eq 1.57079633D then ys = y_not+increment_thetay+yp
  if thetay gt 1.57079632D and thetay lt 1.57079633 then ys = y_not+yp
  :ys = y_not+increment_thetay+yp
endif
if Ay lt 0.0 then begin
  if thetay lt 1.57079632D or thetay eq 1.57079632D then ys = y_not+increment_thetay-yp
  if thetay gt 1.57079633D or thetay eq 1.57079633D then ys = y_not+increment_thetay1-yp
endif
if \theta_y > 1.57079632D and \theta_y \leq 1.57079633 then \( y_s = y_{not} - y_p \) 
\quad ; y_s = y_{not} + \text{increment}_\theta - y_p 
endif

num1 = sag values(xs, ys, coeff);

cen_sag_offset = r - (\sqrt{r^2 - d(0)^2 - d(1)^2})

z0_guess = num1 + r - cen_sag_offset 
endfor

guesses(0) = xs
\quad guesses(1) = ys
\quad guesses(2) = z0_guess

return guesses 
end

; =========
; this function is the base iteration for the stylus axis intersection
; =========
; function start, x_not, y_not, coeff

common share4
\quad xs = double(0.0); x guess
\quad ys = double(0.0); y guess
\quad xl = double(0.0)
\quad y1 = double(0.0)
\quad x2 = double(0.0)
\quad y2 = double(0.0)
\quad ddx = double(0.0); numerical slope x (finite differences) for the z heights difference eq.
\quad ddy = double(0.0); numerical slope y (finite differences) for the z heights difference eq.
\quad ddx \_l = double(0.0); numerical slope x (finite differences) for the x part slope difference eq.
\quad ddy \_l = double(0.0); numerical slope y (finite differences) for the x part slope difference eq.
\quad ddx \_2 = double(0.0); numerical slope x (finite differences) for the y part slope difference eq.
\quad ddy \_2 = double(0.0); numerical slope y (finite differences) for the y part slope difference eq.
\quad increment = double(0.0)
\quad increment = 0.001D
startvalues = dblarr(3)

startvalues = start_slope(x_not, y_not, coeff)
\quad xs = startvalues(0)
\quad ys = startvalues(1)
\quad zs = startvalues(2)
\quad xl = xs
\quad yl = ys
\quad zl = zs
\quad x2 = xl + \text{increment}
\quad y2 = yl + \text{increment}
\quad z2 = zl + \text{increment}
ddx = (diff_functz(x2,y1,z1,x_not,y_not,coeff)-diff_functz(x1,y1,z1,x_not,y_not,coeff))/increment  
ddy = (diff_functz(x1,y2,z1,x_not,y_not,coeff)-diff_functz(x1,y1,z1,x_not,y_not,coeff))/increment  
ddz = (diff_functz(x1,y1,z2,x_not,y_not,coeff)-diff_functz(x1,y1,z1,x_not,y_not,coeff))/increment  

ddx1 = (diff_functsx(x2,y1,z1,x_not,y_not,coeff)-diff_functsx(x1,y1,z1,x_not,y_not,coeff))/increment  
ddy1 = (diff_functsx(x1,y2,z1,x_not,y_not,coeff)-diff_functsx(x1,y1,z1,x_not,y_not,coeff))/increment  
ddd1 = (diff_functsx(x1,y1,z2,x_not,y_not,coeff)-diff_functsx(x1,y1,z1,x_not,y_not,coeff))/increment  

ddx2 = (diff_functsy(x2,y1,z1,x_not,y_not,coeff)-diff_functsy(x1,y1,z1,x_not,y_not,coeff))/increment  
ddy2 = (diff_functsy(x1,y2,z1,x_not,y_not,coeff)-diff_functsy(x1,y1,z1,x_not,y_not,coeff))/increment  
ddz2 = (diff_functsy(x1,y1,z2,x_not,y_not,coeff)-diff_functsy(x1,y1,z1,x_not,y_not,coeff))/increment  

der_matrix = dblarr(3,3); this matrix contains the finite differences needed to solve the equations  
der_matrix(0,0) = ddx  
der_matrix(0,1) = ddx1  
der_matrix(0,2) = ddx2  
der_matrix(1,0) = ddy  
der_matrix(1,1) = ddy1  
der_matrix(1,2) = ddy2  
der_matrix(2,0) = ddz  
der_matrix(2,1) = ddz1  
der_matrix(2,2) = ddz2  

GP = dblarr(3,3); just an intermediate matrix to solve the equations  
GP = (invert(transpose(der_matrix)#(der_matrix)))#(transpose(der_matrix))  

Alpha = dblarr(3); this is the difference function values at x1,y1  
Alpha(0) = diff_functz(x1,y1,z1,x_not,y_not,coeff)  
Alpha(1) = diff_functsx(x1,y1,z1,x_not,y_not,coeff)  
Alpha(2) = diff_functsy(x1,y1,z1,x_not,y_not,coeff)  

HP = dblarr(3); this contains the increments the the guesses x,y must move  
HP = -GP##Alpha  
if abs(HP(0)) gt 0.15 then begin  
  if hp(0) gt 0.0 then hp(0) = 0.055D  
  if hp(0) lt 0.0 then hp(0) = -0.055D  
endif  
if abs(HP(1)) gt 0.15 then begin  
  if hp(1) gt 0.0 then hp(1) = 0.055D  
  if hp(1) lt 0.0 then hp(1) = -0.055D  
endif  
if abs(HP(2)) gt 0.15 then begin  
  if hp(2) gt 0.0 then hp(2) = 0.055D  
endif
if hp(2) lt 0.0 then hp(2) = -0.055D
endif

xl = xl+HP(0)
yl = yl+HP(1)
zl = zl+HP(2)

zboundl = zs-0.15
zboundr = zs+0.15
if zl lt zboundl then zl = zs-0.05
if zl gt zboundr then zl = zs+0.05
:increment = increment/10.0D

i = 0
jj = 0
while (abs(total(Alpha)) gt 1.0e-009) or (Alpha(0) gt 1.0e-009 or Alpha(1) gt 1.0e-009 or Alpha(2) gt 1.0e-009) do begin

x2 = xl+increment
y2 = yl+increment
z2 = zl+increment

ddx = (diff_functz(x2,y2,zl,x_not,y_not,coeff)-
      diff_functz(x1,y1,zl,x_not,y_not,coeff))/increment
ddy = (diff_functz(x1,y2,zl,x_not,y_not,coeff)-
      diff_functz(x1,y1,zl,x_not,y_not,coeff))/increment
ddz = (diff_functz(x1,y2,z2,x_not,y_not,coeff)-
      diff_functz(x1,y1,zl,x_not,y_not,coeff))/increment

ddx1 = (diff_functsx(x2,y2,zl,x_not,y_not,coeff)-
        diff_functsx(x1,y2,zl,x_not,y_not,coeff))/increment
ddy1 = (diff_functsx(x1,y2,zl,x_not,y_not,coeff)-
        diff_functsx(x1,y1,zl,x_not,y_not,coeff))/increment
ddz1 = (diff_functsx(x1,y2,z2,x_not,y_not,coeff)-
        diff_functsx(x1,y1,zl,x_not,y_not,coeff))/increment

ddx2 = (diff_functsy(x2,y2,zl,x_not,y_not,coeff)-
        diff_functsy(x1,y2,zl,x_not,y_not,coeff))/increment
ddy2 = (diff_functsy(x1,y2,zl,x_not,y_not,coeff)-
        diff_functsy(x1,y1,zl,x_not,y_not,coeff))/increment
ddz2 = (diff_functsy(x1,y2,z2,x_not,y_not,coeff)-
        diff_functsy(x1,y1,zl,x_not,y_not,coeff))/increment

der_matrix = dblarr(3,3);this matrix contains the finite differences needed to solve the equations

der_matrix(0,0) = ddx
der_matrix(0,1) = ddx1
der_matrix(0,2) = ddx2
der_matrix(1,0) = ddy
der_matrix(1,1) = ddy1
der_matrix(1,2) = ddy2
der_matrix(2,0) = ddz
der_matrix(2,1) = ddz1
der_matrix(2,2) = ddz2

GP = dblarr(3,3); just an intermediate matrix to solve the equations
GP = (invert(transpose(der_matrix))##der_matrix))##(transpose(der_matrix))

Alpha = dblarr(3); this is the difference function values at x1,y1
Alpha(0) = diff_functz(x1,y1,z1,x_not,y_not,coeff)
Alpha(1) = diff_funcsx(x1,y1,z1,x_not,y_not,coeff)
Alpha(2) = diff_funcsy(x1,y1,z1,x_not,y_not,coeff)

HP = dblarr(3); this contains the increments the the guesses x,y must move
HP = -GP##Alpha
if abs(HP(0)) gt 0.15 then begin
    ; if i eq 0 then begin
        if hp(0) gt 0.0 then hp(0) = 0.025D
        if hp(0) lt 0.0 then hp(0) = -0.025D
    ; endif
    ; if i ne 0 then begin
        if hp(0) gt 0.0 then hp(0) = 0.015/double(i)
        if hp(0) lt 0.0 then hp(0) = -0.015/double(i)
    ; endif
endif

if abs(HP(1)) gt 0.15 then begin
    ; if i eq 0 then begin
        if hp(1) gt 0.0 then hp(1) = 0.025D
        if hp(1) lt 0.0 then hp(1) = -0.025D
    ; endif
    ; if i ne 0 then begin
        if hp(1) gt 0.0 then hp(1) = 0.015/double(i)
        if hp(1) lt 0.0 then hp(1) = -0.015/double(i)
    ; endif
endif

if abs(HP(2)) gt 0.15 then begin
    if hp(2) gt 0.0 then hp(2) = 0.025D
    if hp(2) lt 0.0 then hp(2) = -0.025D
endif

x1 = x1 + HP(0)
y1 = y1 + HP(1)
z1 = z1 + HP(2)
if z1 lt zboundl then z1 = zs-0.025
if z1 gt zboundr then z1 = zs+0.025
;print.alpha,x1,y1,z1
if increment gt 1.0e-009 and \( \text{abs}(\text{total}(HP)) \) lt 0.075 then increment = increment/10.0D

\[
\text{i} = \text{i} + 1
\]

if i gt 15 then begin
print,'flag',x1,y1,z1,alpha
p = dblarr(3)
p = narrow(x_not,y_not,coeff)
x1 = p(0)
y1 = p(1)
z1 = p(2)
i = 0
increment = 1.0e-010
jj = jj+1
if jj eq 2 then stop
endif
endwhile

;print,xs-x 1 ,ys-y 1 ,zs-z 1
if (abs(\text{total}(\text{Alpha})) gt 1.0e-009) then stop

x_cen = double(0.0)
y_cen = double(0.0)
z_not = z1
x_cen = x_not+z_not*cos(thetax)/cos(thetaz)
y_cen = y_not+z_not*cos(thetay)/cos(thetaz)
numbers = dblarr(7)
numbers(0) = x1;x contact point
numbers(1) = y1;y contact point
numbers(2) = x_not;sample x
numbers(3) = y_not;sample y
numbers(4) = x_cen;x ball center
numbers(5) = y_cen;y ball center
numbers(6) = z_not;z ball center
num = sag_values(x_cen,y_cen,coeff);
return,numbers
end

:==================
;find the location of the ball centers
:==================

function find_ball_center,x_not,y_not,scannumber,coeff,mask

temp = dblarr(7)
contact_array = dblarr(sqrt(scannumber),sqrt(scannumber),7);the first element in any row is the z location of the ball center, the second is the x contact with surface
for \( j = 0, \sqrt{\text{scannumber}} - 1 \) do begin
    for \( i = 0, \sqrt{\text{scannumber}} - 1 \) do begin
        if \( \text{mask}[i,j] \) eq 1.0 then begin
            temp = start(x\_not(i,j), y\_not(i,j), \text{coeff}); start is a function that will do the base bisection iteration process
        endif else begin
            temp(0) = 0.0
            temp(1) = 0.0
            temp(2) = x\_not(i, j)
            temp(3) = y\_not(i, j)
            temp(4) = 0.0
            temp(5) = 0.0
            temp(6) = 0.0
        endelse
        contact_array(i, j, 0) = temp(0); axis intersection x
        contact_array(i, j, 1) = temp(1); axis intersection y
        contact_array(i, j, 2) = temp(2); sampling x
        contact_array(i, j, 3) = temp(3); sampling y
        contact_array(i, j, 4) = temp(4); x location of ball center
        contact_array(i, j, 5) = temp(5); y location of ball center
        contact_array(i, j, 6) = temp(6); z location of ball center
    endfor
endfor
; stop
return contact_array
end

; this function will find the intersection points of the stylus with the part surface

function find\_stylus\_axis, x\_not, y\_not, scannumber, coeff, mask

\text{temp} = \text{dblarr}(3)
\text{contact \_array} = \text{dblarr}(\sqrt{\text{scannumber}}, \sqrt{\text{scannumber}}, 3); this array will contain the x, y, z intersection points for the stylus

for \( j = 0, \sqrt{\text{scannumber}} - 1 \) do begin
    for \( i = 0, \sqrt{\text{scannumber}} - 1 \) do begin
        if \( \text{mask}[i,j] \) eq 1.0 then begin
            \text{temp} = \text{start}(x\_not(i,j), y\_not(i,j), \text{coeff}); start is a function that will do the base bisection iteration process
        endif else begin
            \text{temp}(0) = 0.0
            \text{temp}(1) = 0.0
            \text{temp}(2) = 0.0
        endelse
        \text{contact\_array}(i, j, 0) = \text{temp}(0); axis intersection x
        \text{contact\_array}(i, j, 1) = \text{temp}(1); axis intersection y
        \text{contact\_array}(i, j, 2) = \text{temp}(2); axis intersection z
function find_delta, x_not, y_not, scannumber, axis_intersection, coeff, mask

common share

contact_array = dblarr(sqrt(scannumber), sqrt(scannumber), 3); has ideal x location and the delta value there

for j = 0, sqrt(scannumber) - 1 do begin
  for i = 0, sqrt(scannumber) - 1 do begin
    if mask[i, j] eq 1.0 then begin
      contact_array[i, j, 0] = x_not(i, j); ideal x stylus axis location
      contact_array[i, j, 1] = y_not(i, j); ideal y stylus axis location
      contact_array[i, j, 2] = axis_intersection(i, j, 2) - sag_values(x_not(i, j), y_not(i, j), coeff); delta value at the ideal x location
    endif else begin
      contact_array[i, j, 0] = x_not(i, j); ideal x stylus axis location
      contact_array[i, j, 1] = y_not(i, j); ideal y stylus axis location
      contact_array[i, j, 2] = 0.0
    endelse
  endfor
endfor

return, contact_array
end

; this section will correct the errored data

function correct_data, scannumber, dd1, dd2, dd3, dd4, mask

common share

r = 0.500000D

data = dblarr(sqrt(scannumber), sqrt(scannumber), 3)

for j = 0, sqrt(scannumber) - 1 do begin
  for i = 0, sqrt(scannumber) - 1 do begin
    if mask[i, j] eq 1.0 then begin
      data[i, j, 0] = dd1(i, j, 2); x stylus axis location (sampled)
      data[i, j, 1] = dd1(i, j, 3); y stylus axis location (sampled)
    endif else begin
      data[i, j, 0] = dd1(i, j, 2); x stylus axis location (sampled)
      data[i, j, 1] = dd1(i, j, 3); y stylus axis location (sampled)
    endelse
  endfor
endfor

return, data
end
data(i,j,2) = (dd4(i,j)-r-(dd2(i,j,6)-r*cos(thetaz))^2 + (dd2(i,j,0)-dd1(i,j,4)-r*cos(thetax))^2 +
(dd2(i,j,1)-dd1(i,j,5)-r*cos(thetay))^2)^.5)*abs(cos(thetaz)) - dd3(i,j,2)
endif else begin
  data(i,j,0) = dd1(i,j,2); x stylus axis location (sampled)
  data(i,j,1) = dd1(i,j,3); y stylus axis location (sampled)
  data(i,j,2) = 0.0
endelse
endfor

return data
end

;================================================================================================
;this is the starting point in the program
;================================================================================================

function correct_trial2,x_not,y_not,data2,scannumber,mask,angles

common share4,thetax,thetay,thetaz,c

thetax = angles(0)
thetay = angles(1)
thetaz = angles(2)
c = 1.0/52.120; 1.0/753.0; 1.0/13.165D
ref_x = dblarr(sqrt(scannumber),sqrt(scannumber))
ref_y = dblarr(sqrt(scannumber),sqrt(scannumber))

for j=0,sqrt(scannumber)-1 do begin
  for i=0,sqrt(scannumber)-1 do begin
    ref_x[i,j] = i*43.0/19.0-21.5;
    ref_y[i,j] = j*43.0/19.0-21.5;
  endfor
endfor

sag = 1.0*c*((ref_x)^2+(ref_y)^2)/(1.0+(1.0-c^2*((ref_x)^2+(ref_y)^2))^5)-20.0;
data2 = data2-0.50000000D

mask1 = intarr(sqrt(scannumber),sqrt(scannumber))
  check_radius = 0.0
  radial_dist = 0.0
  ac = (sqrt(scannumber)-1)/2.0
  bc = (sqrt(scannumber)-1)/2.0
  check_radius = ac-1.5

;mask array is set to 1 for x,y positions that lie within the desired circular region and zero outside that region
  for i=0,sqrt(scannumber)-1 do begin
for $j = 0, \sqrt{\text{scannumber}} - 1$ do begin
    \[
    \text{radial\_dist} = \sqrt{((ac - i)^2 + (bc - j)^2)}
    \]
    if (radial\_dist < \text{check\_radius}) or (radial\_dist == \text{check\_radius}) then begin
        mask1[i,j] = 1
    endif else begin
        mask1[i,j] = 0
    endelse
endfor

: this section removes spikes
coeff = dblarr(93)
coeff = steve\_fitm(data2, x\_not, y\_not, scannumber, mask)
sur = sag\_values(x\_not, y\_not, coeff)

: for $qi = 0, \sqrt{\text{scannumber}} - 1$ do begin
    : for $qi = 0, \sqrt{\text{scannumber}} - 1$ do begin
        : if mask[qi,qj] == 1 then begin
            : if abs(sur[qi,qj]-data2[qi,qj]) > 0.00022 then data2[qi,qj]=sur[qi,qj]
            : endif
        : endif
    : endfor
    : endfor
: this changes the biggest spike that is left to the value of the sur array at that point
rrmax = dblarr(3)
rrmin = dblarr(3)
rrmax = find\_max1(sur-data2, scannumber, mask)
rrmin = find\_min1(sur-data2, scannumber, mask)
data2[rrmin(1),rrmin(2)] = data2[rrmin(1),rrmin(2)] + rrmin(0)
data2[rrmax(1),rrmax(2)] = data2[rrmax(1),rrmax(2)] + rrmax(0)
print, rrmax(0)-rrmin(0)

: refit after spikes removed
coeff = steve\_fitm(data2, x\_not, y\_not, scannumber, mask)
sur = sag\_values(x\_not, y\_not, coeff)

rrmax = find\_max1(sur-data2, scannumber, mask)
rrmin = find\_min1(sur-data2, scannumber, mask)
data2[rrmin(1),rrmin(2)] = data2[rrmin(1),rrmin(2)] + rrmin(0)
data2[rrmax(1),rrmax(2)] = data2[rrmax(1),rrmax(2)] + rrmax(0)
rrmax = find\_max1(sur-data2, scannumber, mask)
rrmin = find\_min1(sur-data2, scannumber, mask)
print, rrmax(0)-rrmin(0)

: stop
sur = sur+0.5000000002D
data2 = data2+0.5000000002D
dd1 =dblarr(sqrt(scannumber),sqrt(scannumber),7);ball center info
dd2 =dblarr(sqrt(scannumber),sqrt(scannumber),3);stylus axis intersection
dd3 =dblarr(sqrt(scannumber),sqrt(scannumber),3);delta value used in data correction
dd5 =dblarr(sqrt(scannumber),sqrt(scannumber),3);corrected data
dd1 = find_ball_center(x_not, y_not, scannumber, coeff, mask)
dd2 = find_stylus_axis(x_not, y_not, scannumber, coeff, mask)
dd3 = find_delta(x_not, y_not, scannumber, dd2, coeff, mask)
dd5 = correct_data(scannumber, dd1, dd2, dd3, sur, mask); sur in for data2

for i=0,1 do begin
    print, i
    coeff = steve_fitm(dd5(0:sqrt(scannumber)-1,0:sqrt(scannumber)-1,2), x_not, y_not, scannumber, mask)
surl = sag_values(x_not, y_not, coeff)
    :window,i
    :surface,mask*(surl-dd5(0:sqrt(scannumber)-1,0:sqrt(scannumber)-1,2))
    rrmmax = find_max1(surl-dd5(0:sqrt(scannumber)-1,0:sqrt(scannumber)-1,2), scannumber, mask)
    rrmmin = find_min1(surl-dd5(0:sqrt(scannumber)-1,0:sqrt(scannumber)-1,2), scannumber, mask)
    dd5[rrmmin(1), rrmmin(2), 2] = dd5[rrmmin(1), rrmmin(2), 2] + rrmmin(0)
    dd5[rrrmmax(1), rrmmax(2), 2] = dd5[rrrmmax(1), rrmmax(2), 2] + rrmmax(0)
    rrmmax = find_max1(surl-dd5(0:sqrt(scannumber)-1,0:sqrt(scannumber)-1,2), scannumber, mask)
    rrmmin = find_min1(surl-dd5(0:sqrt(scannumber)-1,0:sqrt(scannumber)-1,2), scannumber, mask)
    print, rrmmax(0)-rrmmin(0)
    coeff = steve_fitm(dd5(0:sqrt(scannumber)-1,0:sqrt(scannumber)-1,2), x_not, y_not, scannumber, mask)
    :surl = sag_values(x_not, y_not, coeff)
    dd1 = find_ball_center(x_not, y_not, scannumber, coeff, mask)
    dd2 = find_stylus_axis(x_not, y_not, scannumber, coeff, mask)
    dd3 = find_delta(x_not, y_not, scannumber, dd2, coeff, mask)
    dd5 = correct_data(scannumber, dd1, dd2, dd3, sur, mask); sur in for data2
endfor

return, dd5(0:sqrt(scannumber)-1,0:sqrt(scannumber)-1,2)

end
THE FOLLOWING SECTIONS OF CODE ARE FOR THE ANALYSIS ROUTINE. THEY SHIFT AND ROTATE AND IDEAL SURFACE TO MATCH THE MEASURED DATA IN A LEAST SQUARES SENSE.

; The code performs calculations for analyzing data
;
function reference2,x,y,xo,yo,zo,mask

  c = 1.0/52.118;
  sag = 1.0*c*((x+xo)^2+(y+y0)^2)/(1.0+(1.0-c^2*((x+xo)^2+(y+y0)^2)^.5)-14.479451 + zo;17.58747

  sag = sag*mask
  return,sag
end

function rot_shift2,x,y,z,scannumber,mask

; This is the base iteration for finding the stylus tilt angles
function rot_shift2,x,y,z,scannumber,mask

; This is the final data mask used to mask out raw data
mask1 = intarr(sqrt(scannumber),sqrt(scannumber))
  check_radius = 0.0
  radial_dist = 0.0
  ac = (sqrt(scannumber)-1)/2.0
  bc = (sqrt(scannumber)-1)/2.0
  check_radius = ac-1.5;analysis mask value

; Mask array is set to 1 for x,y positions that lie within the desired circular region and zero outside that region
for i=0,sqrt(scannumber)-1 do begin
  for j=0,sqrt(scannumber)-1 do begin
    radial_dist=SQR((ac-i)^2 + (bc-j)^2)
    if(radial_dist lt check_radius) or (radial_dist eq check_radius) then begin
      mask1[i,j] = 1
    endif else begin
      mask1[i,j] = 0
    endelse
  endfor
endfor

ref_x = dblarr(sqrt(scannumber),sqrt(scannumber))
ref_y = dblarr(sqrt(scannumber),sqrt(scannumber))
;this is the reference x and y positions used to subtract the ideal data from the raw data
;change these values to correspond to the data pattern taken.
for j=0,sqrt(scannumber)-1 do begin
  for i=0,sqrt(scannumber)-1 do begin
    ref_x[i,j] = i*43.0/34-21.5;
    ref_y[i,j] = j*43.0/34-21.5;
  endfor
endfor
increment = 0.0001
;initial directional cosines for the rotation plane, new z axis is perpendicular to this
icosines1 = dblarr(4)
l31 = 0.00000D; new z axis direction relative to the old x axis
m31 = 0.00000D; new z axis direction relative to the old y axis
n31 = sqrt(1.0-l31^2-m31^2); new z axis direction relative to the old z axis
m11 = 0.00000D; new x axis direction relative to the old y axis
icosines1(0) = l31
icosines1(1) = m31
icosines1(2) = n31
icosines1(3) = m11

;this is the shift of the dc term for the sphere
dd = dblarr(2)
dd = centering(z,x,y,scannumber)
offsetx = (max(x)-min(x))/2.0
offsety = (max(y)-min(y))/2.0
xshift1 = 0.00000D;
yshift1 = 0.000000D;
zshift1 = 0.000000D;

;incremented values of the variables used to find the finite differences necessary to evaluate equations
icosines2 = dblarr(4)
l32 = l31+increment
m32 = m31+increment
n32 = sqrt(1.0-l32^2-m32^2)
m12 = m11+increment
xshift2 = xshift1+increment
yshift2 = yshift1+increment
zshift2 = zshift1+increment

Alpha_values = dblarr(scannumber); these are the difference values between the reference and the data used in the least squares iteration
alpha_target1 = z; these are the reference surface sag values (the errored data is the ref and i'm moving the ideal sphere around)
alpha_data1 = dblarr(sqrt(scannumber),sqrt(scannumber)); these are the data values for the ideal sphere
ro_data = dblarr(sqrt(scannumber),sqrt(scannumber)); this is the rotated data
alpha_data1 = reference2(ref_x-xshift1,ref_y-yshift1,xshift1,yshift1,zshift1,mask); this moves the reference surface by xshift,yshift,zshift
print,'rotate data'
ro_data1 = rot_surf_1(x,y,alpha_data1,scannumber,mask,cosines1,ref_x-xshift1,ref_y-yshift1);this rotates
the corrected data by the amounts given by the directional cosines

zs = ro_data1.fz;this is the rotated and shifted data evaluated at the sampled points

;stop
;generate the alpha1 values
l = 0 ; index variable
for j=0,sqrt(scannumber)-1 do begin
   for i=0,sqrt(scannumber)-1 do begin
      Alpha_values(l) = zs[i,j]-alpha_target[i,j]
l = l + 1
   endfor
endfor
;this section computes the peak to valley and rms numbers
z11 = dblarr(sqrt(scannumber),sqrt(scannumber))
p = 0 ; index variable
for j=0,sqrt(scannumber)-1 do begin
   for i=0,sqrt(scannumber)-1 do begin
      z11[i,j] = Alpha_values(p)
p = p + 1
   endfor
endfor
;fff = dblarr(3)
fff = steve_tilt4(z11,ref_x,ref_y,scannumber,mask1)

gg = dblarr(3)
for j=0,sqrt(scannumber)-1 do begin
   for i=0,sqrt(scannumber)-1 do begin
      gg[i] = find_min1(z11,scannumber,mask1)
pv = 0D
   endfor
endfor
for j=0,sqrt(scannumber)-1 do begin
   for i=0,sqrt(scannumber)-1 do begin
      if mask1[i,j] eq 1 then p=p+1
   endfor
endfor
aprime = dblarr(p)
p1 = 0
w = 0
for j=0,sqrt(scannumber)-1 do begin
   for i=0,sqrt(scannumber)-1 do begin
      if mask1[i,j] eq 1 then begin
         aprime[p1]=z11[i,j];alpha_values[w]
p1 = p1+1
      endif
      w = w+1
   endfor
endfor
$\text{end for}$

$\text{end for}$

$\text{rms} = \text{MOMENT(p)}$ \(\text{aprime} \) \\
$\text{RMS} = \text{SQRT(rms(l))}$ \\
$\text{print, 'rnis = ', rms}$ \\
$\text{;contour, mask1*z11/fill/isotropic, levels = min(mask1*z11) + findgen(60)*(max(mask1*z11)-}$ \\
$\text{min(mask1*z11))/60.0}$

$\text{stop}$

$\text{;generate the derivative matrix, this is used in the least square iteration process}$ \\
$q = 0 ; \text{index value}$ \\
$\text{der_matrix = dblarr(6,scannumber);this matrix contains the derivatives with respect to the three variables}$ \\
$f1 = zs; \text{this is the },z\text{ shifted data for the first set of variable values}$

$\text{;this is the corrected sag values for l3 changing while holding everything else constant}$ \\
$\text{cosines2(0)} = l32$ \\
$\text{cosines2(1)} = m31$ \\
$\text{cosines2(2)} = \text{sqrt(1.0-l32^2-m31^2)}$ \\
$\text{cosines2(3)} = m11$ \\
$\text{alpha_data1 = reference2(ref_x-xshift1,ref_y-yshift1,xshift1,yshift1,zshift1,mask)}$ \\
$\text{ro_data = rot_surf_1(x,y,alpha_data1,scannumber,mask,cosines2,ref_x-xshift1,ref_y-yshift1)}$ \\
$\text{der_l3 = ro_data.fz}$

$\text{;this is the corrected sag values for m3 changing while holding everything else constant}$ \\
$\text{cosines2(0)} = l31$ \\
$\text{cosines2(1)} = m32$ \\
$\text{cosines2(2)} = \text{sqrt(1.0-l31^2-m32^2)}$ \\
$\text{cosines2(3)} = m11$ \\
$\text{alpha_data1 = reference2(ref_x-xshift1,ref_y-yshift1,xshift1,yshift1,zshift1,mask)}$ \\
$\text{ro_data = rot_surf_1(x,y,alpha_data1,scannumber,mask,cosines2,ref_x-xshift1,ref_y-yshift1)}$ \\
$\text{der_m3 = ro_data.fz}$

$\text{;this is the corrected sag values for m1 changing while holding everything else constant}$ \\
$\text{cosines2(0)} = l31$ \\
$\text{cosines2(1)} = m31$ \\
$\text{cosines2(2)} = \text{sqrt(1.0-l31^2-m31^2)}$ \\
$\text{cosines2(3)} = m12; \text{try to ignore the m11 variable for now}$ \\
$\text{alpha_data1 = reference2(ref_x-xshift1,ref_y-yshift1,xshift1,yshift1,zshift1,mask)}$ \\
$\text{ro_data = rot_surf_1(x,y,alpha_data1,scannumber,mask,cosines2,ref_x-xshift1,ref_y-yshift1)}$ \\
$\text{der_m1 = ro_data.fz}$

$\text{;this is the corrected sag values for zshift1 changing while holding everything else constant}$ \\
$\text{alpha_data1 = reference2(ref_x-xshift2,ref_y-yshift1,xshift2,yshift1,zshift1,mask)}$ \\
$\text{ro_data = rot_surf_1(x,y,alpha_data1,scannumber,mask,cosines1,ref_x-xshift2,ref_y-yshift1)}$ \\
$\text{der_xshift = ro_data.fz}$

$\text{;this is the corrected sag values for yshift1 changing while holding everything else constant}$ \\
$\text{alpha_data1 = reference2(ref_x-xshift1,ref_y-yshift2,xshift1,yshift2,zshift1,mask)}$ \\
$\text{ro_data = rot_surf_1(x,y,alpha_data1,scannumber,mask,cosines1,ref_x-xshift1,ref_y-yshift2)}$ \\
$\text{der_yshift = ro_data.fz}$
;this is the corrected sag values for zshift1 changing while holding everything else constant

\text{alpha_data} I = \text{reference2(ref_x-xshift1,ref_y-yshift1,xshift1,yshift1,zshift2,mask)}
\text{ro_data} = \text{rot_surf_1(x,y,alpha_data1,scannumber,mask,cosines1,ref_x-xshift1,ref_y-yshift1)}
\text{der_zshift} = \text{ro_data.fz}

dl3 = \text{der_l3-fl}
\text{u} = \text{steve_tilt4(dl3,x,y,scannumber,mask)}

dm3 = \text{der_m3-fl}
\text{u} = \text{steve_tilt4(dm3,x,y,scannumber,mask)}

dm1 = \text{der_m1-fl}
\text{u} = \text{steve_tilt4(dm1,x,y,scannumber,mask)}

dxs = \text{der_xshift-fl}
\text{u} = \text{steve_tilt4(dxs,x,y,scannumber,mask)}

dys = \text{der_yshift-fl}
\text{u} = \text{steve_tilt4(dys,x,y,scannumber,mask)}

dzs = \text{der_zshift-fl}
\text{u} = \text{steve_tilt4(dzs,x,y,scannumber,mask)}

\text{for} \ t = 0, \sqrt{\text{scannumber}} - 1 \ \text{do begin}
\text{for} \ t = 0, \sqrt{\text{scannumber}} - 1 \ \text{do begin}
\text{der_matrix}(0,q) = (dl3[t,t1])/(\text{increment})
\text{der_matrix}(1,q) = (dm3[t,t1])/(\text{increment})
\text{der_matrix}(2,q) = (dm1[t,t1])/(\text{increment})
\text{der_matrix}(3,q) = (dxs[t,t1])/(\text{increment})
\text{der_matrix}(4,q) = (dys[t,t1])/(\text{increment})
\text{der_matrix}(5,q) = (dzs[t,t1])/(\text{increment})
\text{q} = q + 1
\text{endfor}
\text{endfor}

;generate the variable vector for the three variables xshift,yshift,zshift
\text{shift\_vector} = \text{dblar(6)}
\text{GP} = (\text{invert(transpose(der\_matrix))##der\_matrix))##(transpose(der\_matrix))\n\text{shift\_vector} = -\text{GP##Alpha\_values}\n\text{shift\_vector} = \text{shift\_vector*0.25D}\n
l31 = l31+\text{shift\_vector}(0)
m31 = m31+\text{shift\_vector}(1)
n31 = \sqrt{\text{sqrt(l31^2-m31^2)}}
m11 = m11+\text{shift\_vector}(2)
\text{cosines1(0)} = l31
\text{cosines1(1)} = m31
cosines1(2) = n31
cosines1(3) = m11
xshift1 = xshift1+shift_vector(3)
yshift1 = yshift1+shift_vector(4)
zshift1 = zshift1+shift_vector(5)
if abs(l31) gt 1.0 then l31 = 0.000000000D
if abs(m31) gt 1.0 then m31 = 0.000000000D
if abs(m11) gt 1.0 then m11 = 0.000000000D
increment = increment/10.0
print,cosines1,xshift1,yshift1,zshift1

;---------------
;this procedure will rotate the data back to a flat position (this will essentially remove all tilt from the profile)
;---------------

function rot_surf_1,xsam,ysam,zval,scannumber,mask,dir_cosines,xval,yval

l3 = dir_cosines(0)
m3 = dir_cosines(1)
n3 = dir_cosines(2);this should always be positive
m1 = dir_cosines(3)

l1 = abs(-2.0*l3*m3*m1 + sqrt((2.0*l3*m3*m1)^2-4.0*(l3^2+n3^2)*((m3*m1)^2-n3^2+n3^2*m1^2)))/(2.0*(l3^2+n3^2));this should always be positive
if l3 lt 0.0 or l3 eq 0.0 then n1 = sqrt(1.0-l1^2-m1^2)
if l3 gt 0.0 then n1 = -1.0*sqrt(1.0-l1^2-m1^2)

Aprime = (n1*m3-m1*n3)/(l1*n3-n1*l3)
Bprime = ((m3+l3*Aprime)/n3)^2
m2 = sqrt(1.0/(1.0+Aprime^2+Bprime));this should always be positive
l2 = m2*Aprime
if m3 lt 0.0 or m3 eq 0.0 then n2 = sqrt(1.0-m2^2-l2^2)
if m3 gt 0.0 then n2 = -1.0*sqrt(1.0-m2^2-l2^2)

;xthis section rotates the data back to its original position
xpr = dblarr(sqrt(scannumber),sqrt(scannumber))
ypr = dblarr(sqrt(scannumber),sqrt(scannumber))
zpr = dblarr(sqrt(scannumber),sqrt(scannumber))

xpr = l1*xval + m1*yval + n1*zval ;this generates the rotated x data
ypr = l2*xval + m2*yval + n2*zval ;this generates the rotated y data
zpr = l3*xval + m3*yval + n3*zval ;this generates the rotated z
zpr = zpr*mask

d = dblarr(sqrt(scannumber),sqrt(scannumber))
d = regular_gridding3(xpr,ypr,zpr,scannumber,mask,xsam,ysam)

A={fx:xpr, fy:ypr, fz:d}; structure definition returning the rotated coordinates
return,A

cend
REFERENCES


REFERENCES - Continued


