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SLOTS IN A PARALLEL PLATE WAVEGUIDE

by

Jeffrey Paul Quintenz

A Dissertation Submitted to the Faculty of the
DEPARTMENT OF ELECTRICAL ENGINEERING
In Partial Fulfillment of the Requirements
For the Degree of
DOCTOR OF PHILOSOPHY
In the Graduate College
THE UNIVERSITY OF ARIZONA

1975
I hereby recommend that this dissertation prepared under my direction by **Jeffrey Paul Quintenz** entitled **Slots in a Parallel Plate Waveguide** be accepted as fulfilling the dissertation requirement of the degree of **Doctor of Philosophy**

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**TABLE OF CONTENTS**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>v</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>x</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. SINGLE SLOT RADIATION</td>
<td>4</td>
</tr>
<tr>
<td>2.1 Derivation of Integral Equation</td>
<td>4</td>
</tr>
<tr>
<td>2.2 Application of Method of Moments</td>
<td>17</td>
</tr>
<tr>
<td>2.3 Low Frequency Analysis</td>
<td>29</td>
</tr>
<tr>
<td>2.4 High Frequency Analysis</td>
<td>34</td>
</tr>
<tr>
<td>2.5 Calculation of Aperture $\overline{H}$ Field and Power Radiated</td>
<td>38</td>
</tr>
<tr>
<td>2.6 Reflection, Transmission, and Radiation Coefficient Determination</td>
<td>42</td>
</tr>
<tr>
<td>2.7 Antenna Pattern Analysis</td>
<td>50</td>
</tr>
<tr>
<td>3. MULTIPLE SLOT RADIATION</td>
<td>58</td>
</tr>
<tr>
<td>3.1 Derivation of Integral Equation and Matrix Symmetries</td>
<td>58</td>
</tr>
<tr>
<td>3.2 Reflection, Transmission, and Radiation Coefficient Determination</td>
<td>65</td>
</tr>
<tr>
<td>3.3 Antenna Pattern Analysis</td>
<td>72</td>
</tr>
<tr>
<td>4. SINGLE AND MULTIPLE SLOT SHIELDING</td>
<td>82</td>
</tr>
<tr>
<td>4.1 Aperture Field Calculation</td>
<td>82</td>
</tr>
<tr>
<td>4.2 Power Coupling Coefficients</td>
<td>83</td>
</tr>
<tr>
<td>4.3 Low and High Frequency Analysis</td>
<td>93</td>
</tr>
<tr>
<td>4.4 Directional Coupler Analysis</td>
<td>94</td>
</tr>
<tr>
<td>5. TRANSIENT SHIELDING</td>
<td>98</td>
</tr>
<tr>
<td>6. CONCLUSIONS</td>
<td>135</td>
</tr>
<tr>
<td>APPENDIX A</td>
<td>137</td>
</tr>
<tr>
<td>LIST OF REFERENCES</td>
<td>138</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>Problem Geometry</td>
</tr>
<tr>
<td>2</td>
<td>Method of Imaging</td>
</tr>
<tr>
<td>3</td>
<td>Pulse Expansion of Aperture E Field</td>
</tr>
<tr>
<td>4</td>
<td>Triangle Expansions</td>
</tr>
<tr>
<td>5</td>
<td>Aperture Field for M = 10, a = 0.025\lambda, b = 0.1\lambda, \varepsilon_R = 0.4</td>
</tr>
<tr>
<td>6</td>
<td>Aperture Field for M = 20, a = 0.025\lambda, b = 0.1\lambda, \varepsilon_R = 0.4</td>
</tr>
<tr>
<td>7</td>
<td>Aperture Field for M = 30, a = 0.025\lambda, b = 0.1\lambda, \varepsilon_R = 0.4</td>
</tr>
<tr>
<td>8</td>
<td>Equivalent Dipole Images in Parallel Plates. Image Locations x=0, ± 2b, ± 4b</td>
</tr>
<tr>
<td>9</td>
<td>Low Frequency Approximation vs Moment Method for ak=10^{-5}, bk=10^{-2}</td>
</tr>
<tr>
<td>10</td>
<td>Low Frequency Approximation vs Moment Method for ak=10^{-6}, bk=10^{-3}</td>
</tr>
<tr>
<td>11</td>
<td>Geometrical Theory of Diffraction</td>
</tr>
<tr>
<td>12</td>
<td>Aperture H Field for a = 0.008\lambda, b = 0.063\lambda, E_o = 16</td>
</tr>
<tr>
<td>13</td>
<td>Aperture H Field for a = 0.125\lambda, b = 0.250\lambda, E_o = 4</td>
</tr>
<tr>
<td>14</td>
<td>Aperture H Field for a = 0.063\lambda, b = 0.16\lambda, E_o = 64</td>
</tr>
<tr>
<td>15</td>
<td>Time Average Power Through Slot</td>
</tr>
<tr>
<td>16</td>
<td>Reflection Coefficient</td>
</tr>
<tr>
<td>17</td>
<td>Transmission Coefficient</td>
</tr>
<tr>
<td>18</td>
<td>Radiation Coefficient</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>---------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>19</td>
<td>Transmission Coefficient Variation with Slot Width for Different Values of $k_b$</td>
</tr>
<tr>
<td>20</td>
<td>Reflection Coefficient Variation with Slot Width for Different Values of $k_b$</td>
</tr>
<tr>
<td>21</td>
<td>Antenna Pattern Geometry</td>
</tr>
<tr>
<td>22</td>
<td>Antenna Pattern for $a = .008\lambda$, $b = .250\lambda$</td>
</tr>
<tr>
<td>23</td>
<td>Antenna Pattern for $a = .016\lambda$, $b = .063\lambda$</td>
</tr>
<tr>
<td>24</td>
<td>Antenna Pattern for $a = 1.50\lambda$, $b = .250\lambda$</td>
</tr>
<tr>
<td>25</td>
<td>Multiple Slot Geometry</td>
</tr>
<tr>
<td>26</td>
<td>Field Pulse Expansion for Multiple Slots</td>
</tr>
<tr>
<td>27</td>
<td>Reflection Coefficient of Three Slots of Equal Width $2a$.</td>
</tr>
<tr>
<td>28</td>
<td>Transmission Coefficient of Three Slots of Equal Width</td>
</tr>
<tr>
<td>29</td>
<td>Radiation Coefficient of Three Slots of Equal Width</td>
</tr>
<tr>
<td>30</td>
<td>Total Time Average Power Through Slots of Equal Width ($a = .01\lambda$)</td>
</tr>
<tr>
<td>31</td>
<td>Power Out Slots vs. Slot Width When Slot Spacing is $d = 1\lambda$, Plate Spacing is $b = .20\lambda$</td>
</tr>
<tr>
<td>32</td>
<td>Array Geometry</td>
</tr>
<tr>
<td>33</td>
<td>Array Pattern for 2, $0.005\lambda$ Slots Separated by $d = 1\lambda$, Plate Spacing is $b = .20\lambda$</td>
</tr>
<tr>
<td>34</td>
<td>Antenna Pattern for 2, $0.005\lambda$ Slots Separated by $d = .50\lambda$, Plate Spacing is $b = .20\lambda$</td>
</tr>
<tr>
<td>Figures</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>35</td>
<td>Antenna Pattern for 2, .50λ Slots Separated by d = λ. Plate Spacing is b = .20λ</td>
</tr>
<tr>
<td>36</td>
<td>Mutual Coupling Effects of 3, .025λ Slots Spaced .25λ Apart With Plate Spacing of .2λ</td>
</tr>
<tr>
<td>37</td>
<td>Mutual Coupling Effects of 3, .075λ Slots Spaced .25λ with Plate Spacing of .2λ</td>
</tr>
<tr>
<td>38</td>
<td>Aperture E Field for a .025λ Slot with a .20λ Plate Spacing. Angle of Incidence is 45°</td>
</tr>
<tr>
<td>39</td>
<td>Aperture H Field for a .025λ Slot with a .20λ Plate Spacing. Angle of Incidence is 45°</td>
</tr>
<tr>
<td>40</td>
<td>Aperture E Field for 3 Slots. c₁ = -.50λ, c₂ = 0λ, c₃ = +.10λ, a₁ = .03λ, a₂ = .015λ, a₃ = .005λ, b = .20λ, θ = 60°</td>
</tr>
<tr>
<td>41</td>
<td>Aperture H Field for 3 Slots. c₁ = -.50λ, c₂ = 0λ, c₃ = +.10λ, a₁ = .03λ, a₂ = .015λ, a₃ = .005λ, b = .20λ, θ = 60°</td>
</tr>
<tr>
<td>42</td>
<td>Power Coupled into Each Mode for 0° Incidence, a = .05λ, b = 2.25λ</td>
</tr>
<tr>
<td>43</td>
<td>Power Coupled into Each Mode for 30° Incidence, a = .05λ, b = 2.25λ</td>
</tr>
<tr>
<td>44</td>
<td>Power Coupled into Each Mode for 60° Incidence, a = .05λ, b = 2.25λ</td>
</tr>
<tr>
<td>45</td>
<td>Power Coupled into Each Mode for 90° Incidence, a = .05λ, b = 2.25λ</td>
</tr>
<tr>
<td>46</td>
<td>High Frequency Approximation for a 5λ Slot with 10.1λ Plate Spacing. Angle of Incidence is 30°</td>
</tr>
<tr>
<td>47</td>
<td>Directional Coupler</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>48</td>
<td>$H_y$ Transfer Function at $x = -5cm$, $z = -1m$, $b' = 10cm$, $a = 1mm$, $\theta = 30^\circ$</td>
</tr>
<tr>
<td>49</td>
<td>$E_x$ Transfer Function at $x = -5cm$, $z = -1m$, $b' = 10cm$, $a = 1mm$, $\theta = 30^\circ$</td>
</tr>
<tr>
<td>50</td>
<td>$H_y$ Transfer Function at $x = -5cm$, $z = -1m$, $b' = 10cm$, $a = 1mm$, $\theta = 90^\circ$</td>
</tr>
<tr>
<td>51</td>
<td>$E_x$ Transfer Function at $x = -5cm$, $z = -1m$, $b' = 10cm$, $a = 1mm$, $\theta = 90^\circ$</td>
</tr>
<tr>
<td>52</td>
<td>$H_y$ Transfer Function at $x = -5cm$, $z = -1m$, $b' = 10cm$, $a = 1cm$, $\theta = 30^\circ$</td>
</tr>
<tr>
<td>53</td>
<td>$E_x$ Transfer Function at $x = -5cm$, $z = -1m$, $b' = 10cm$, $a = 1cm$, $\theta = 30^\circ$</td>
</tr>
<tr>
<td>54</td>
<td>$H$ Field Impulse Response of Single Slot for $30^\circ$ Incidence and 2mm Slot Width</td>
</tr>
<tr>
<td>55</td>
<td>$E$ Field Impulse Response of Single Slot for $30^\circ$ Incidence and 2mm Slot Width</td>
</tr>
<tr>
<td>56</td>
<td>$H$ Field Step Response of Single Slot for $30^\circ$ Incidence and 2mm Slot Width</td>
</tr>
<tr>
<td>57</td>
<td>$E$ Field Step Response of Single Slot for $30^\circ$ Incidence and 2mm Slot Width</td>
</tr>
<tr>
<td>58</td>
<td>$H$ Field Impulse Response of Single Slot for $90^\circ$ Incidence and 2mm Slot Width</td>
</tr>
<tr>
<td>59</td>
<td>$E$ Field Impulse Response of Single Slot for $90^\circ$ Incidence and 2mm Slot Width</td>
</tr>
<tr>
<td>60</td>
<td>$H$ Field Step Response of a Single Slot for $90^\circ$ Incidence and 2mm Slot Width</td>
</tr>
<tr>
<td>61</td>
<td>$E$ Field Step Response of Single Slot for $90^\circ$ Incidence and 2mm Slot Width</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>62</td>
<td>H Field Impulse Response of a Single Slot for 30° Incidence and 2 cm Slot Width</td>
</tr>
<tr>
<td>63</td>
<td>E Field Impulse Response for 30° Incidence and 2 cm Slot Width</td>
</tr>
<tr>
<td>64</td>
<td>H Field Step Response for 30° Incidence and 2 cm Slot Width</td>
</tr>
<tr>
<td>65</td>
<td>E Field Step Response for 30° Incidence and 2 cm Slot Width</td>
</tr>
<tr>
<td>66</td>
<td>H_y Time Response for .6m Slot with 6m Plate Spacing</td>
</tr>
<tr>
<td>67</td>
<td>H_y Time Response for 1.2m Slot with 12m Plate Spacing</td>
</tr>
<tr>
<td>68</td>
<td>H_y Time Response for .6m Slot with 12m Plate Spacing</td>
</tr>
<tr>
<td>69</td>
<td>H Field Transfer Function for 2, 12cm Slots with d = 1.5m, b = 1.2m, x = -.6m, z = -60m, θ = 0°</td>
</tr>
<tr>
<td>70</td>
<td>H Field Transfer Function for 2, 12cm Slots with d = 1.5m, b = 1.2m, x = -.6m, z = +60m, θ = 0°</td>
</tr>
<tr>
<td>71</td>
<td>H_y Time Response for 12cm Slot at x = -.6m, z = -60m</td>
</tr>
<tr>
<td>72</td>
<td>H_y Time Response for 12 cm Slot at x = -.6m, z = +60m</td>
</tr>
<tr>
<td>73</td>
<td>Time Response at z = -60m</td>
</tr>
<tr>
<td>74</td>
<td>Time Response at z = +60m</td>
</tr>
<tr>
<td>75</td>
<td>Result of Convolution of Double Exponential Pulse with 20 ns Width Rectangular Pulse</td>
</tr>
</tbody>
</table>
ABSTRACT

The problem of one or more infinite transverse slots in a parallel plate waveguide wall is considered. An integral equation is developed for the aperture electric field for the case of a single slot with sources in the guide. The integral equation is solved using the method of moments. The method used to solve the single slot case is then extended to treat two or more slots. Reflection, transmission, and coupling coefficients are obtained as well as total power coupled out of the guide and antenna patterns. Mutual coupling effects in an antenna array are discussed and the moment method solution is compared with simple array theory for multiple slots. The moment method is shown to fill a gap in previously presented results for medium slot widths.

The reciprocal problem applicable to shielding is considered next by formulating an integral equation for the aperture electric field when the sources are exterior to the waveguide. The interior fields are found by an integration of the aperture field using Green's theorem. Multiple slot shielding is considered and directional coupling effects examined. By repeating the above process at several frequencies, a transfer function is obtained for the shielding structure. This transfer function is used to produce transient responses in the guide by multiplication of the frequency spectrum of the desired input pulse by the transfer function and then performance of an inverse Fourier transform.
CHAPTER 1

INTRODUCTION

In this paper the electromagnetic boundary value problem consisting of an arbitrary number of infinite transverse slots in a parallel plate waveguide wall will be presented. Only the TM (to $z$) polarization is considered. R. F. Millar (1959) has considered the TE (to $z$) polarization for the case of a single slot when the incident field is the dominant TE mode in the parallel plate region. Millar also considers the TM$z$ polarization for a single slot when the incident field is the TEM mode. His results, however, are restricted to large slot widths in comparison to the incident field wavelength. Millar treats plate spacings where only the TEM mode can propagate but his analysis includes the effects of the evanescent modes. His method of approach consists of solving simultaneous integral equations asymptotically. Another approach to solving the single slot problem is given by Simmons (1957). He uses the variational principle to solve the problem of an incident TEM mode; his results, however, apply only to small apertures.

One purpose of this paper is to bridge the gap between the above two methods for medium slot widths and to allow plate spacings which permit higher order mode propagation. Another objective of this
work is to add several slots to the analysis. A final goal is to obtain the transient shielding properties of one or more slots in a parallel plate waveguide wall.

In this paper, the moment method is used to solve the integral equation for the aperture electric field. This integral equation is obtained by an application of Green's theorem and a subsequent matching of tangential $\mathbf{E}$ and $\mathbf{H}$ through the aperture. A rectangular pulse expansion of the aperture field is used in the moment method and proves to be satisfactory over a wide range of slot parameters. The edge effects of the slot are used as a priori knowledge of the aperture field for small slot sizes to obtain a quasi-static solution. Once the aperture field is known, all other quantities of interest are calculated using Green's theorem.

The sources considered here are both line and sheet magnetic currents located in the guide or magnetic line currents or incident plane waves outside the guide. The $\mathbf{T}_z$ polarization is of interest since it is the polarization for which maximum energy penetrates the slot, whether the case be that of an antenna (incident energy from within the guide) or the shielding problem (incident energy from outside the guide).

Chapter 2 considers the case where the incident field is a propagating mode inside the guide. This incident mode encounters a single transverse slot causing reflected and transmitted modes and radiated energy. Results are presented for the reflection, transmission,
and radiation (coupling) coefficients as well as the resultant antenna pattern for various slot widths and plate spacings.

Chapter 3 extends the results of Chapter 2 to include more than one slot in the radiation problem. Reflection and transmission coefficients are again presented and the antenna pattern, now for an antenna array, is discussed. The antenna patterns obtained are compared with simple array theory and the effects of mutual coupling by one array element on another are presented.

Chapters 4 and 5 are concerned with the shielding problem of single and multiple slot geometries. In Chapter 4 the steady state problem of a monochromatic field incident from the guide exterior is considered. The amount of power coupled into the guide modes is calculated for various plane wave incidence angles, slot widths and plate spacings. In Chapter 5, the guide transfer functions are obtained by calculating the fields interior to the waveguide for various frequencies. Finally transient results are obtained from these transfer functions for various input pulses using Fourier transform techniques.
CHAPTER 2

SINGLE SLOT RADIATION

In this chapter the problem of a single slot in the wall of a parallel plate waveguide will be considered. First the integral equation for the aperture field will be derived and then different methods of solving this equation will be discussed. Approximate solutions will be considered for the cases of low frequency (small aperture) and high frequency (large aperture) incident fields.

2.1 Derivation of Integral Equation

The problem geometry is shown in Figure 1. In this figure the walls of the parallel plate waveguide extend to infinity in both the positive and negative $y$ directions. $F_y(x_0, z_0)$ is an electric or magnetic line source, also infinite in the $y$ dimension, with no variation in $y$. The geometry is conveniently divided into two regions, one interior to the waveguide (Region I) and one exterior to the waveguide (Region II). The constitutive parameters of the two regions are $(\varepsilon_1, \mu_1)$ and $(\varepsilon_2, \mu_2)$ respectively.

By virtue of the $y$ independence of the geometry and the fact that the source is not a function of $y$, the problem is two-dimensional and hence scalar. The fields can thus be separated into two mode sets, one transverse magnetic to $z$ ($TM_z$) and the other transverse electric...
Figure 1. Problem Geometry
to \( z \) (TE\(_2\)). This separation is accomplished as follows. Let the Fourier transform pair \( F(\omega) \leftrightarrow f(t) \) be defined by

\[
F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt
\]

\[
f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega
\]

Maxwell's equations in the frequency domain then become

\[
\nabla \times \overrightarrow{E}(x,z) = -i\omega \mu \overrightarrow{H}(x,z) - \overrightarrow{M}(x,z)
\]

\[
\nabla \times \overrightarrow{H}(x,z) = \overrightarrow{J}(x,z) + i\omega \varepsilon \overrightarrow{E}(x,z)
\]

\[
\nabla \cdot \overrightarrow{E}(x,z) = \rho/\varepsilon, \quad \nabla \cdot \overrightarrow{H}(x,z) = 0
\]

where use has been made of the constitutive properties \( \overrightarrow{D} = \mu \overrightarrow{H} \) and \( \overrightarrow{B} = \varepsilon \overrightarrow{E} \). In (2) \( \overrightarrow{J} \) is the electric current density and \( \overrightarrow{M} \) is the magnetic current density. These are the only sources for this problem. By expanding the curl equations into their vector components, the following equation sets result.

\[
\begin{align*}
-\frac{\partial E_y}{\partial z} & = i\omega \mu H_x - M_x \\
\frac{\partial E_y}{\partial x} & = -i\omega \mu H_z - M_z \\
\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} & = J_y + i\omega E_y \\
-\frac{\partial H_y}{\partial z} & = J_x + i\omega E_x \\
\frac{\partial H_y}{\partial x} & = J_z + i\omega E_z \]
\end{align*}
\]

\[
\begin{align*}
\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} & = -i\omega H_y - M_y
\end{align*}
\]
It is evident that these two sets are completely decoupled. The first set contains the field components \((E_y, H_x, H_z)\) and the source components \((J_y, M_x, M_z)\). The second set contains the complementary components \((E_x, E_z, H_y)\) and \((J_x, J_z, M_y)\). Hence the two sets can be solved independently. The source is specified as either \(J_y\) or \(M_y\) with all other source components equal to zero. Using this result, the usual wave equations can be formed for each set by performing the prescribed derivatives on the first two equations and substituting the results into the third equation in each set. The results are

\[
\begin{align*}
(\nabla_{xz}^2 + k^2) E_y &= i\omega \mu J_y \\
H_x &= \frac{1}{i\omega\mu} \frac{\partial E_y}{\partial z} \\
H_z &= -\frac{1}{i\omega\mu} \frac{\partial E_y}{\partial x} \\
(\nabla_{xz}^2 + k^2) H_y &= i\omega\varepsilon M_y \\
E_x &= -\frac{1}{i\omega\varepsilon} \frac{\partial H_y}{\partial z} \\
E_z &= \frac{1}{i\omega\varepsilon} \frac{\partial H_y}{\partial x}
\end{align*}
\]

Equation set (5) forms the mode set transverse electric to \(z\) (TE\(_z\)) while (6) contains the transverse magnetic to \(z\) (TM\(_z\)) modes with the transverse electromagnetic mode (TEM) a special case. Since the TEM mode is the dominant mode in a parallel plate waveguide, only the solution of (6) will be considered.
The method of solution involves first solving the Green function equation for each of the two regions. With the Green function determined, Green's theorem is applied to each region to determine the field in each region in terms of the sources and the tangential components of the boundary fields, in this case with assumed perfectly conducting plates this reduces to only the aperture fields. Continuity of tangential $\mathbf{E}$ and $\mathbf{H}$ is then applied in the aperture to obtain the integral equation for the aperture fields. Once the aperture fields are determined, the fields everywhere follow from Green's theorem.

It should be realized that depending upon whether one begins solving for the Green function by using a spectral expansion in $x$ or $z$ followed by a closed form solution in $z$ or $x$ respectively, one will obtain two different forms of the desired Green function. Since this holds true in both region I and region II, there are four possible combinations of the Green functions in the final derivation of the integral equation. The steps present here will include only those which lead to the integral equation which will be solved in this paper. The other possible integral equations will be summarized later.

In region I the Green function equation is

$$(\nabla_{xz}^2 + k_1^2)g_1(x,z) = -\delta(x-x')\delta(z-z')$$

for

$$\frac{\partial g_1(x,z)}{\partial x} \bigg|_{x=0,b} = 0$$

and

$$g_1(x,z) \to 0 \text{ as } |z| \to \infty$$
The solution of (7) is obtained by first considering the spectral expansion of \( g(x,z) \) in the \( x \) dimension. Let

\[
g_1(x,z) = a_0(z) + \sum_{n=1}^{\infty} a_n(z) \cos(n\pi x) \tag{8}\]

Substituting this form of \( g_1(x,z) \) into (7) gives

\[
(\nabla_{xz}^2 + k_1^2) \left[ a_0(z) + \sum_{n=1}^{\infty} a_n(z) \cos(n\pi x) \right] = \delta(x-x')\delta(z-z') \tag{9}\]

or

\[
\sum_{n=1}^{\infty} \left\{ \left(-\frac{n\pi}{b}\right)^2 + k_1^2 \right\} a_n(z) + \frac{d^2a_n(z)}{dz^2} \cos(n\pi x) = \frac{1}{b} \delta(z-z') \tag{10}\]

Using the orthogonality relation for the cosines

\[
\int_0^b \cos \frac{n\pi x}{b} \cos \frac{m\pi x}{b} \, dx = \frac{b}{\epsilon_n} \delta_{nm} \tag{11}\]

where \( \epsilon_n \) is Neumann's number

\[
\epsilon_n = \begin{cases} 
1 & n = 0 \\
2 & n \neq 0 
\end{cases}
\]

(10) reduces to

\[
\frac{d^2a_n(z)}{dz^2} + k_n^2a_n(z) = -\delta(z-z')\epsilon_n \frac{\cos \frac{n\pi x}{b}}{b} \tag{12}\]

where

\[
k_n^2 = k_1^2 - \left(\frac{n\pi}{b}\right)^2
\]

Now with the substitution

\[
A_n(z) = \frac{b}{\epsilon_n} \frac{a_n(z)}{\cos \frac{n\pi x}{b}} \tag{13}\]
(12) becomes

\[ \frac{d^2 A_n}{dz^2} + k_n^2 A_n = -\delta(z-z') \]  \hspace{1cm} (14)

which has solutions

\[
A_n = \begin{cases} 
Be^{ik_n z} + Ce^{-ik_n z} & (z>z') \\
De^{ik_n z} + Ee^{-ik_n z} & (z<z')
\end{cases}
\]  \hspace{1cm} (15)

To specify uniquely the branch in the complex plane, \( k_n \) is chosen to have the following properties:

\[ \text{Re} (k_n) > 0 \quad \text{Im} (k_n) \leq 0 \]

Then for \( A_n \) to be well behaved as \( |z| \to \infty \) it must be that \( B = E = 0 \) in (15). By the usual Green function construction method, continuity of \( A_n(z) \) and the proper jump of \( A_n' \) at \( z = z' \) implies

\[
Ce^{-ik_n z'} = De^{ik_n z'}
\]

\[
-ik_n Ce = -ik_n De
\]

These two simultaneous equations are then solved and the results substituted into (15) to yield

\[
A_n(z) = \begin{cases} 
\frac{-ik_n(z-z')}{2ik_n} & (z>z') \\
\frac{-ik_n(z'-z)}{2ik_n} & (z<z')
\end{cases}
\]

or

\[
a_n(z) = \frac{e_n}{b} \cos \frac{n\pi x'}{b} \frac{e^{-ik_n|z-z'|}}{2ik_n}
\]
and finally

$$g_1(x,z;x',z') = \sum_{n=0}^{\infty} \frac{\epsilon_n}{b} \cos \frac{n\pi x'}{b} \cos \frac{n\pi x}{b} e^{\frac{-ik_n|z-z'|}{b}}$$  \hspace{1cm} (17)$$

In region II the Green function equation is

$$(\nabla^2_{x,z} + k_2^2)g_2(x,z) = -\delta(x-x')\delta(z-z')$$  \hspace{1cm} (18)$$

for

$$\frac{\partial g(x,z)}{\partial x} \bigg|_{x=0} = 0$$

and $g(x,z)$ is well behaved as $|z| \to \infty$, $x \to \infty$. This equation is most easily solved by the method of images. The magnetic line source can be imaged in the perfectly conducting plane as shown in Figure 2. The resulting Green function is just the sum of two free space Green functions in two dimensions, viz:

$$g_2(x,z;x',z') = \frac{1}{4\pi} H_0^{(2)}(k_2 \sqrt{(x-x')^2 + (z-z')^2})$$

$$+ \frac{1}{4\pi} H_0^{(2)}(k_2 \sqrt{(x+x')^2 + (z-z')^2})$$

\hspace{1cm} (19)$$

Green's theorem can be stated as follows

$$\int \frac{\phi\nabla^2 \psi - \psi \nabla^2 \phi}{\nabla \phi} \, dv = \oint_{\partial V} \frac{\partial \psi}{\partial n} - \frac{\partial \phi}{\partial n} \, ds$$  \hspace{1cm} (20)$$

where $\hat{n}$ is the outward directed normal to the surface enclosing the volume $V$. Applying this theorem to the Green function and $H_y$ field component in region I gives
Figure 2. Method of Imaging
The forcing function is a magnetic line source located at \( x = x_0 \) and \( z = z_0 \)

\[
f(x,z) = i\omega \mu_0 \delta(x-x_0)\delta(z-z_0)
\]

Then

\[
\int_{V_1} g_1 f(x,z) dz = i\omega \mu_0 \sum_{n=0}^{\infty} \frac{\cos \frac{n\pi x'}{b} \cos \frac{n\pi z_0}{b}}{2i k_1} \left| z_0 - z' \right|
\]

This forcing function, a line source, gives rise to all \( \text{TM}_z \) modes in the guide. It is more convenient to consider one mode at a time and this is easily accomplished by selecting the appropriate source distribution such that only the desired mode is excited. For example, a uniform sheet of magnetic current excites only the TEM mode, viz:

\[
f(x,z) = i\omega \mu_0 \delta(z-z_0)
\]

then

\[
\int_{V_1} g_1 f(x,z) dz = i\omega \mu_0 \frac{e^{-i k_1 \left| z_0 - z' \right|}}{2ik_1}
\]

which is the desired TEM mode. The value of the above integration is thus seen to be just the incident field and will be designated \( \text{inc} \) \( H_y(x',z') \). The boundary conditions on \( g(x,z;x',z') \) and \( H_y(x,z) \) can now be used to evaluate the surface integral in Green's theorem.

\[
\int_{V_1} (g_1 \nabla^2 H_y - H_y \nabla^2 g_1) dv_1 = \int_{V_1} g_1 f(x,z) dv_1 + H_y (x',z')
\]

\[
= \oint_{S_1} (g_1 \frac{\partial H_y}{\partial n} - H_y \frac{\partial g_1}{\partial n}) ds_1
\]
\[
\begin{align*}
\mathcal{G} \left[ g_1 \frac{\partial H_y}{\partial n^1} - H_y \frac{\partial g_1}{\partial n} \right] ds_1 &= \int_{-a}^{a} g_1(o,z;x',z') \frac{\partial H_y}{\partial x} (o,z) dz \\
\text{but} \quad \frac{\partial H_y}{\partial x} (o,z) &= i\omega \epsilon_1 E_z (o,z) \\
\text{then} \quad i\omega \epsilon_1 \int_{-a}^{a} g_1(o,z;x',z') E_z (o,z) dz &= H_y (x',z') + H_y (x,z) \\
\text{or changing variables} \quad H_y (x,z) &= i\omega \epsilon_1 \int_{-a}^{a} g_1(x,z;o,z') E_z (o,z') dz' - H_y (x,z) \\
\text{Following a similar development for region II} \quad H_y \ (x,z) &= -i\omega \epsilon_2 \int_{-a}^{a} g_2(x,z;o,z') E_z (o,z') dz' \\
\text{where the minus sign in front of the integral results from the fact that the surface normal is now directed in the } -z \text{ direction. The integral equation for the aperture field is obtained by equating the tangential } E \text{ and } H \text{ components in the aperture. Tangential } E \text{ is forced to be continuous in the aperture by defining an aperture field} \\
E_z (o,z') &= E_a (z') \\
\text{Now equating } H_y^1 \text{ and } H_y^2 \text{ in the aperture yields} & \\
i\omega \epsilon_1 \int_{-a}^{a} g_1(o,z;o,z') E_a (z') dz' - H_y (o,z) = & \\
& = -i\omega \epsilon_2 \int_{-a}^{a} g_2(o,z;o,z') E_a (z') dz' 
\end{align*}
\]
Or upon substituting (17) and (19) into (24) the integral equation becomes

\[
\begin{align*}
H_y^{\text{inc}} (o, z) &= \int_{-a}^{a} E_A(z') \sum_{n=0}^{\infty} \epsilon_n \frac{e^{-i k_n|z-z'|}}{2i k_n} \, dz' \\
&+ \frac{\omega \epsilon_2}{2} \int_{-a}^{a} E_A(z') H_O^{(2)} (k_2 |z-z'|) \, dz'
\end{align*}
\]

(25)

The above integral equation is the one chosen to be solved in this paper. Other possible integral equations for the aperture fields are summarized below.

Region I: Spectral expansion in \( x \), closed form in \( z \).
Region II: Spectral expansion in \( z \), closed form in \( x \).

\[
\begin{align*}
H_y^{\text{inc}} (o, z) &= \int_{-a}^{a} E_A(z') \sum_{n=0}^{\infty} \epsilon_n \frac{e^{-i k_n|z-z'|}}{2i k_n} \, dz' \\
&- \frac{i \omega \epsilon_2}{2\pi} \int_{-a}^{a} E_A(z') \int_{-\infty}^{\infty} \frac{i \alpha (z-z')}{ik_x} \, d\alpha \, dz'
\end{align*}
\]

where

\[
k_x^2 = k_2^2 - \alpha^2 \\
k_n^2 = k_1^2 - \frac{(m\pi)^2}{b} \\
\text{Im} \ (k_x) < 0 \\
\text{Im} \ (k_n) < 0
\]

Region I: Spectral expansion in \( z \), closed form in \( x \).
Region II: Spectral expansion in \( x \), closed form in \( z \).

\[
\begin{align*}
H_y^{\text{inc}} (o, z) &= -\frac{i \omega \epsilon_1}{2\pi} \int_{-a}^{a} E_A(z') \int_{-\infty}^{\infty} \frac{\cos k_x(x+b) e^{-i \beta (z-z')}}{k_x \sin k_x b} \, d\beta \, dz' \\
&- \frac{2i \omega \epsilon_2}{\pi} \int_{-a}^{a} E_A(z') \int_{0}^{\infty} \frac{e^{-i k_z |z-z'|}}{2ik_z} \, dv \, dz'
\end{align*}
\]
where

\[ k_z^2 = k_2^2 - v^2 \quad k_x^2 = k_1^2 - \beta^2 \]

\[ \text{Im}(k_z) < 0 \quad \text{Im}(k_x) < 0 \]

Region I: Spectral expansion in \( z \), closed form in \( x \).

Region II: Spectral expansion in \( z \), closed form in \( x \).

\[
\text{inc} \quad H_\gamma (o, z) = \frac{-i\omega_1}{2\pi} \int_{-a}^{a} E_A(z') \int_{-\infty}^{\infty} \frac{\cos k_x b e^{i\beta(z-z')}}{k_x(1)\sin k_x(1)b} \, db dz'
\]

\[
- \frac{i\omega_2}{2\pi} \int_{-a}^{a} E_A(z') \int_{-\infty}^{\infty} e^{i\alpha(z-z')} \, db dz'
\]

where

\[
k_x^{(1)} = k_1^2 - \beta^2 \quad k_x^{(2)} = k_2^2 - \alpha^2
\]

\[ \text{Im}(k_x^{(1)}) < 0 \quad \text{Im}(k_x^{(2)}) < 0 \]

Although these equations look different in form, it should be realized that they all have the same solution for \( E_A(z) \). The equations can all be shown to be equivalent by performing the proper integrations and special function identifications.

An in depth analysis of which of the above integral equations is best solved for different parameters such as plate spacing, slot width, or field point location has not been made. Such an analysis would include a study of integral and sum convergences as the parameters were varied. The integral equation chosen has proven to be quite satisfactory over all values of parameters of interest here.
2.2 Application of Method of Moments

The method of moments can now be applied to solve (25). The method is briefly described here. A more thorough treatment may be found in Harrington (1968).

Consider the linear inhomogeneous equation

\[ Lu = f \]

where \( L \) is a linear operator, \( u \) is the unknown solution and \( f \) is a forcing function. If \( u \) is assumed to be representable as an expansion in some known functions, \( u_m \), with unknown amplitudes \( A_m \)

\[ u = \sum A_m u_m \]

the linearity property of \( L \) can be used to interchange \( L \) and the summation, viz:

\[ Lu = L \sum A_m u_m = \sum A_m Lu_m = f \]

(if the summation is infinite the convergence of the resulting series must be considered as well as the linearity of \( L \) to interchange \( L \) and the summation). Forming an inner product with a known weighting function \( w_i \), reduces the equation to a matrix equation

\[ \langle w_i, \Sigma A_m Lu_m \rangle = \langle w_i, f \rangle \]

\[ \Sigma A_m \langle w_i, Lu_m \rangle = \langle w_i, f \rangle \]

\[ \Sigma A_m l_{1m} = f_1 \]

which can be solved by inverting the \( l_{1m} \) matrix.

\[ A_m = [l_{1m}]^{-1} [f_1] \]
Solution by the method of moments has several available forms. One may choose various functions as expansion functions of either the subdomain or entire domain type (Mitra 1973). There is also freedom as to what functions one uses in the weighting process. The choices made depend upon the ease with which the resulting integrals are performed and also the convergence properties of the resulting approximate solution. Any solution to a linear equation by the method of moments should consider the possible expansion and weighting functions and determine which combination is best in some sense. Various expansion functions have been considered for the integral equation in (25) and their results compared.

The first expansion functions considered were simple rectangular pulses as shown in Figure 3. The aperture $E$ field is then representable as

$$E_A(z) = \sum_{m=1}^{M} A_m p_m(z)$$

(26)

where

$$p_m(z) = \begin{cases} 1 & \beta_m \leq z \leq \alpha_m \\ 0 & \text{otherwise} \end{cases}$$

and

$$\beta_m = \left[ \frac{2(m-1)}{M} - 1 \right] a$$

$$\alpha_m = \left[ \frac{2m}{M} - 1 \right] a$$

Because of the integrals involved, delta function weighting is used throughout this paper. The $w_1$ functions are then

$$w_1 = \delta(z-z_1)$$
Figure 3. Pulse Expansion of Aperture E Field
Substituting the pulse expansion into (25) and then point matching with the weighting functions results in the following form of the matrix equation.

\[ f_1(z_1) = \sum_{l=1}^{M} A_m L_m \]  

where

\[ f_1(z_1) = \frac{\text{inc}}{i\omega_1} k_1 H_y(0, z_1) \]  

and

\[ L_{lm} = k_1 \sum_{n=0}^{\infty} \frac{\epsilon_n e^{-i k_n |z_1-z'|}}{2ibk_n} \]  

\[ + k_1 \varepsilon_2 \int \frac{C_m}{2i\omega_1} H_0^2(k_2 |z_1-z'|)dz' \]  

It is of importance to know whether the summation and integration in (29) may be interchanged. To verify the validity of this interchange two cases must be considered. The first case considered is when \( z_1 \notin [\beta_m, \alpha_m] \). For this case the series is absolutely convergent over the entire interval, and hence uniformly convergent, as proven by the ratio test for series (Goodman 1963), viz:

\[ \lim_{n \to \infty} \frac{\beta_{n+1}}{\beta_n} = \lim_{n \to \infty} \left| \frac{e^{-n+1} |z_1-z'|}{e^{-n} |z_1-z'|} \right| < 1 \]

Since the above limit is less than 1, the series is absolutely convergent. The second case is when \( z_1 \in [\beta_m, \alpha_m] \) as is the case in the self term (1=m). The series diverges when \( z_1 = z' \). That this is true can be seen as follows. For large \( n \) the series terms are of the form
\[ a_n = \frac{-dn}{n} \]

where

\[ d = \frac{\pi}{b} |z_1 - z'| \]

The ratio test applied here shows that the series converges if \( d > 0 \).

No conclusion can be drawn from the ratio test if \( d = 0 \). An application of Canchy’s integral test confirms the divergence of this series when \( d = 0 \). As detailed by Titchmarsh (1939), if

\[ \int_{\beta_m}^{\beta_m} \left( \sum a_n(z') \right) \, dz' = \sum_{\beta_m}^{\beta_m} a_n(z') \, dz' \]

for all values of \( c \) less than \( z_1 \), (as is the case here since the series is uniformly convergent over the interval \( [\beta_m, c] \) for \( c < z_1 \) then

\[ \int_{\beta_m}^{\beta_m} \left( \sum a_n(z') \right) \, dz' = \sum_{\beta_m}^{\beta_m} a_n(z') \, dz' \]

provided that

\[ \sum_{\beta_m}^{\beta_m} \left| a_n(z') \right| \, dz' \]

is convergent. Now, for large \( n \)

\[ \int_{\beta_m}^{\beta_m} |a_n(z')| \, dz' = \frac{1}{2} \left( \frac{b}{n\pi} \right)^2 \left[ 1 - e^{-\frac{n\pi}{b}(z_1 - \beta_m)} \right] \]

and

\[ \sum_{\beta_m}^{\alpha_m} |a_n(z')| \, dz' \]

is seen to converge by an application of Canchy’s integral test. A similar case can be made for the integral over \( [z_1, \alpha_m] \). The interchange of summation and integration in (29) is therefore valid.

It is desired to make the evaluation of the matrix elements as efficient as possible especially when considering large apertures or
many slots. By interchanging the summation and integration in (29) and making a variable change in the Hankel function integral a new representation of the matrix element is obtained, viz:

If \( l \neq m \)

\[
L_{lm} = S(|m-l|+1) - S(|m-l|) + P(|m-l|+1) - P(|m-l|)
\]

If \( l = m \)

\[
L_{lm} = -2S(1/2) + 2S(1) + 2P(1)
\]

where

\[
P(m) = \frac{k_2 \Delta(m-\frac{1}{2})}{2ik_2 \varepsilon_1} \int \frac{H_0^{(2)}(x)}{x} dx
\]

\[
S(m) = k_1 \sum_{n=0}^{\infty} \frac{\varepsilon_n}{2bk_n^2} e^{-ik_n\Delta(m-\frac{1}{2})}
\]

\[
\Delta = \alpha_m^{-\beta_m} = \frac{2a}{M}
\]

From the above it is evident that the matrix elements depend only upon the magnitude of the difference between \( l \) and \( m \). This fact points out a symmetry property which greatly enhances the matrix filling and inversion. A matrix whose elements depend only upon \(|m-l|\) is called a symmetric Toeplitz matrix (Zohar 1969). It has the general form shown below

\[
\begin{array}{cccccc}
  a & b & c & d & e & \\
  b & a & b & c & d & \\
  c & b & a & b & c & \\
  d & c & b & a & b & \\
  e & d & c & b & a & \\
  \cdot & \cdot & \cdot & \cdot & \cdot & \\
  \cdot & \cdot & \cdot & \cdot & \cdot & \\
  \cdot & \cdot & \cdot & \cdot & \cdot & \\
\end{array}
\]
It is immediately obvious that only $M$ elements of the $M \times M$ matrix need be calculated and stored. Preis (1972) has written an algorithm which enables the calculation of the inverse of an $M \times M$ Toeplitz matrix. This algorithm is very efficient compared to the regular Gauss-Jordan matrix reduction which might be used for a general matrix in that it requires only $\frac{M(M+1)}{2}$ storage locations compared with $M^2$ for the Gauss-Jordan and involves only $\sqrt{M^2}$ multiplications compared with $\sqrt{M^3}$ for the Gauss-Jordan method.

Other expansion functions which have been tested consist of the triangle functions

$$u(z) = \sum_{m=1}^{M} A_m T(z)$$

and the triangle functions with half triangles at the aperture ends. Both of these expansion functions are shown in Figure 4. A comparison of these triangle functions with the pulse expansion is shown in Figure 5 for the case of $M = 10$, Figure 6 for the case of $M = 20$, and Figure 7 for the case of $M = 30$. For each of these figures a TEM mode is incident from the $-z$ direction in the guide. The aperture $E$ field is shown for each of these cases. It is evident from these figures that while the triangle and pulse functions converge to the same solution, the pulse functions converge more rapidly. In this context an expansion is said to have converged, if any further increase in the number of expansion terms considered does not appreciably change the aperture field.
(a) Full Triangles

(b) Full Triangles Plus Half Triangles

Figure 4. Triangle Expansions
The half triangle case adds a dip to the aperture field near the slot edge. This dip has occurred before in results obtained at the University of Mississippi (Tsai and Beren 1974). The dip is a product of the half triangle expansion and not a true representation of the field at that point. This anomaly rules out the use of the half triangle expansion in this problem. Because of the more rapid convergence of the pulse expansion and the fact that the matrix involved in the pulse expansion is more easily calculated, this expansion is used exclusively in all that follows.

The representation of the field as shown in Figures 5-7 displays an edge singularity (Meixner 1954) at the aperture ends. With the a priori knowledge that the field has this edge singularity, it is possible to enhance the convergence of the pulse expansion by adding two entire domain expansion functions which have the proper singularity at the aperture ends (Prettie 1974). The field then has the expansion

$$E_A(z) = \sum_{m=1}^{M} A_m p_m + \frac{A_{m+1}}{\sqrt{(a+z)k_1}} + \frac{A_{m+2}}{\sqrt{(a-z)k_1}}$$

Using this expression in the integral equation leads to the solutions labeled singular in Figures 5-7. While the convergence of the solution is enhanced with this type expansion, the matrix loses its Toeplitz form and the inherent trade off of increased computational time verses faster convergence favors the pulse expansion without the singular terms added. In the quasi-static approximation, the singular terms have more meaning.
5. Aperture Field for $M = 10$, $a = 0.025\lambda$, $b = 0.1\lambda$, $\varepsilon_R = 0.4$
Figure 6. Aperture Field for $M = 20$, $a = 0.025\lambda$, $b = 0.1\lambda$, $\varepsilon_R = 0.4$.
Figure 7. Aperture Field for $M = 30$, $a = 0.025\lambda$, $b = 0.1\lambda$, $\epsilon_R = 0.4$. 
2.3 Low Frequency Analysis

A low frequency approximation for the aperture field is of great use in calculating the field for small slots ($2a \ll \lambda$). The low frequency approximation is usually much more efficient to calculate than a direct application of the moment method and the accuracy of the approximation can be quite good in the appropriate frequency range. The low frequency solution, also called the quasi-static solution, can be found from several different methods.

Bethe (1944) first considered the problem of coupling by small apertures. His method is to evaluate equivalent electric and magnetic dipoles which replace the aperture fields in the quasi-static limit. Taylor (1973) has used this method to investigate the coupling of a small circular or ellipsoidal aperture in a parallel plate structure. In his paper Taylor points out that the infinite parallel plates image the dipoles as shown in Figure 8. Another method which can be used to obtain the low frequency solution is a power series solution in $\omega$ as discussed by Fano, Chu, and Adler (1960). The low frequency solution is obtained from the first one or two terms in the power series solution.

Perhaps the most obvious method to obtain the low frequency solution and the one used here, is to take the low frequency limit of the exact integral equation and solve for the low frequency form of the aperture field. For small $\omega$ (25) becomes
Figure 8. Equivalent Dipole Images in Parallel Plates. Image Locations $x=0, \pm 2b, \pm 4b, \ldots$. 
inc \( k_1 H_y (o,z) = \frac{1}{2ib} \int_{-a}^{a} E_A(z') e^{\frac{-ik_1|z-z'|}{b}} dz' + k_1 \sum_{n=1}^{\infty} E_n(z') e^{\frac{-n\pi|z-z'|}{b}} \)

\[ \text{(30).} \]

\[
+ \frac{\varepsilon_{2k_1}}{2i\varepsilon_1} \int_{-a}^{a} E_A(z') dz' - \frac{\varepsilon_{2k_1}}{\pi\varepsilon_1} \int_{-a}^{a} E_A(z') \log (\frac{\delta_{k_2}|z-z'|}{2}) dz'
\]

where \( \delta = 1.728 \)

Now the aperture field may be obtained by solving (30) with the method of moments as before, however a further simplification is possible.

By virtue of the fact that the edge condition is a low frequency phenomena and that this condition dominates the field structure for slots small when compared with a wave-length, the form of the aperture field can be assumed

\[ E_A(z') = \frac{A}{\sqrt{(a+z')k_1}} + \frac{B}{\sqrt{(a-z')k_1}} \]

and the unknowns A and B evaluated by matching at two points in the aperture (say the aperture edges). When the above assumed form of the aperture field is inserted into (30) and matched at the two points, a matrix equation for A and B is obtained, viz:

\[
\begin{bmatrix}
H_{11} & H_{12} \\
H_{12} & H_{11}
\end{bmatrix}
\begin{bmatrix}
A \\
B
\end{bmatrix}
= G
\]

where

\[
H_{11} = \frac{1}{ibk_1} \sqrt{\frac{n^2}{2}} \left[ c(\sqrt{\frac{4ak_1}{\pi}}) - is(\sqrt{\frac{4ak_1}{\pi}}) \right] + bk_1 \sum_{n=1}^{\infty} \text{erf}(\sqrt{\frac{2an\pi}{b}})
\]

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} dt
\]
+ \varepsilon_2 \frac{\sqrt{2ak_1}}{ie_1} - 2\varepsilon_2 \frac{\sqrt{2ak_1}}{\pi e_1} \log \frac{ak_2}{2} \quad [12]

H_{12} = e^{-2ak_1} \frac{\sqrt{\pi}}{2} \left[ C(\sqrt{\frac{4ak_1}{\pi}}) - iS(\sqrt{\frac{4ak_1}{\pi}}) \right] + \frac{2}{\pi} \frac{bk_1}{\sqrt{\pi}} 1 \frac{\Phi d (2an\pi)}{b} \frac{ik_2}{n\sqrt{n}}

+ \varepsilon_2 \frac{\sqrt{2ak_1}}{ie_1} - 2\varepsilon_2 \frac{\sqrt{2ak_1}}{\pi e_2} \left[ \ln(4ak_2) - 2 \right]

and

\begin{align*}
G_1 &= \frac{k_1}{i\omega e_1} H_y^{inc}(o,-a) \\
G_2 &= \frac{k_1}{i\omega e_1} H_y^{inc}(o,a)
\end{align*}

The special functions \( c(x), s(x), \text{erf}(x), \) and \( d(x) \) are defined and evaluated in Appendix A. This equation is easily solved as

\[
A \left[ \begin{array}{c}
H_{11} & -H_{12} \\
-1 & 0
\end{array} \right] \left[ \begin{array}{c}
G_1 \\
G_2
\end{array} \right] = \left[ \begin{array}{c}
1 \\
2
\end{array} \right]
\]

The results using the above quasi-static method are compared with the exact solution using moment method in Figure 9 for the case of a TEM mode incident. The method is seen to be accurate for low frequency cases. If the frequency is allowed to become even lower, the phase variation of the aperture field becomes negligible and \( A = B \) in the above derivation. There are three criteria which must be satisfied to obtain good results with this assumed aperture field in the low
Figure 9. Low Frequency Approximation vs Moment Method for $a_k=10^{-5}$, $b_k=10^{-2}$
frequency limit. One obvious condition is that the frequency be such that the slot size is small enough in wavelengths that the edge effects dominate the aperture field \((ak < 10^{-4})\). This criterion follows from the fact that the assumed aperture field is obtained from Meixner's edge condition for the slotted plane where no second plane is considered. A second criteria is that the plate spacing be large enough that the slot appears as a slotted plane not appreciably affected by the presence of the second plane \((bk > 10ak)\). The final criteria is that the plate spacing be small enough at the frequency of interest to allow the asymptotic form of the equation to be used \((bk < 10^{-3})\). Also the integral equation can be approximated even further to arrive at the relation

\[ A = \frac{H_0}{\varepsilon_1} \frac{bk_1}{2\sqrt{2ak_1}} \]

where \(H_0\) is the amplitude of the incident TEM mode \(\overline{H}\) field. Figure 10 displays the resulting \(E\) field in the aperture if this approximation is made compared with the moment method \(E\) field. Again it is seen that the approximate field agrees quite well with the more exact representation.

2.4 High Frequency Analysis

Approximate solutions in the high frequency range can usually be found by one of two methods. The first method involves an application of Keller's theory of diffraction (Yee, Felsen, and Keller 1968)
Figure 10. Low Frequency Approximation vs Moment Method for $a_k=10^{-6}$ $b_k=10^{-3}$
where the incident field in the guide can be thought of as being composed of bouncing rays (see Figure 11) which encounter the edges of the aperture and diffract (Rudduck and Tsai, 1969). (See Figure 11b) The total field outside the guide is then composed of the diffracted waves whose magnitudes are determined by Keller's diffraction coefficients. This method is valid to obtain far field values for the radiation pattern and near field results for aperture admittance and reflection coefficients. Better results can be obtained by considering higher order diffraction terms as shown in Figure 11c.

The second method which can be used to obtain the high frequency asymptotic fields is the Kirchhoff approximation (Johnson 1965). In the Kirchhoff approximation the assumptions made are that the field components in the aperture are the same components which would exist at the aperture location were there no conductors present. In other words the assumed aperture field is the incident field. Unfortunately the Kirchhoff approximation breaks down when the angle of incidence of the incident field is along the conductor as in the case presented here. The reason for the breakdown is that the incident field in the aperture has no tangential component of electric field. Hence the Kirchhoff approximation can not be used to obtain high frequency results for the problem of the waveguide radiating. The Kirchhoff approximation can, however, be used for the case of the shielding problem as will be shown in Chapter 4.
(a) Incident field as bouncing rays

(b) Diffracted rays

(c) Higher order diffracted rays

Figure 11. Geometrical Theory of Diffraction
2.5 Calculation of Aperture $\overline{H}$ Field and Power Radiated

Once the aperture field has been obtained, all other quantities of interest can be calculated. Some of the obvious quantities of interest are the aperture $\overline{H}$ field, the total power coupled out the slot, the reflection, transmission, and radiation coefficients of the slot, and the antenna pattern created by the slot.

The aperture $\overline{H}$ field can be calculated from the aperture $\overline{E}$ field by a direct application of the equation for the $\overline{H}$ field in region II, viz:

$$H_y(o,z) = -\frac{\omega \varepsilon_2}{2} \int_{-a}^{a} E_A(z') H_0 \left(k_2 |z-z'| \right) dz'$$

For the case where the aperture field has been expanded in pulses as in (26) the above relation becomes

$$H_y(o,z_1) = -\frac{\omega \varepsilon_2}{2} \sum_{m=1}^{M} A_m H_0 \left(k_2 |z_1-z'| \right) dz'$$

Figures 12-14 show the aperture $\overline{H}$ field for various parameters. With the aperture $\overline{H}$ Field known the total power out the slot can be derived. The time average power flowing through a surface $S_a$ is

$$<P> = \frac{1}{2} \text{Re} \int (\overline{E} \times \overline{H}^*) \cdot \hat{n} \, ds$$

Where $\hat{n}$ is the normal to the surface. The total time average power through the slot is then

$$<P> = \frac{1}{2} \text{Re} \int_{-a}^{a} E_A(z) H_y^*(o,z) \, dz$$

or in the case of a pulse expansion

$$<P> = -k_1 a \text{Re} \sum_{m} A_m B_m^*$$
Figure 12. Aperture $H$ Field for $a = .008\lambda$, $b = .063\lambda$, $E_o = 16$
Figure 13. Aperture $H$ Field for $a = 0.125\lambda$, $b = 0.250\lambda$, $E_O = 4$
Figure 14. Aperture $H$ Field for $a = .063\lambda$, $b = .016\lambda$, $E_0 = 64$
where

\[
H_y(0,z) = \begin{cases} 
B_1 & \beta_1 < z < \alpha_1 \\
0 & -a < z < \beta_1 \text{ or } \alpha_1 < z < a
\end{cases}
\]

defines \( B_m \). The time average power through a slot versus the slot width is plotted in Figure 15 for various plate spacings when a TEM mode of unit \( E \) field magnitude is incident down the guide. An examination of this figure presents some interesting results. The total power out the slot is seen to increase as the slot width is increased. This is expected since a wider slot would be expected to permit more energy to couple out of the guide. A wider plate spacing is seen to allow more power to couple out of the guide for wider slots and less for narrower slots than a smaller plate spacing permits.

2.6 Reflection, Transmission, and Radiation Coefficient Determination

The power reflection coefficient of the slot is defined as the ratio of the power reflected back down the guide to the power of the incident field. Likewise the power transmission coefficient is defined as the forward power in the guide beyond the slot divided by the power in the incident field. The time average power passing through a cross section of the guide can be obtained using the expression for the total field in the guide

\[
H_y(x,z) = i\omega \epsilon_j \int_a^\infty \sum_{n=0}^\infty \frac{\epsilon_n}{\epsilon_m} \cos \frac{n\pi x}{b} \frac{-i\kappa_n}{\zeta} e^{-i\kappa_n|z-z'|} \int_{-a}^{b} E_A(z')dz' - H_y(x,z)
\]
Figure 15. Time Average Power Through Slot
and the relation defining the $E$ field in the guide

$$E_x(x,z) = \frac{1}{j\omega\varepsilon_0} \frac{\partial H_y}{\partial z}(x,z)$$

in the relation for the power (31). These equations can be applied for $|z| \gg 1$ to evaluate the real power flow since only the propagating modes contribute. When a pulse expansion is used, the total time average power in the reverse direction for the $n^{th}$ propagating mode can be written

$$<P>_{\text{refl}} = \frac{\varepsilon_0}{4\varepsilon_1 k_1} \left( 1-\cos(k_n\Delta) \right) \sum_{m=1}^{M} A_m e^{-ik_n\alpha_m} \sum_{j=1}^{M} A_j^* e^{ik_n\alpha_j}$$

and in the forward direction

$$<P>_{\text{trans}} = \frac{\varepsilon_0}{4\varepsilon_1 k_1} \left( 1-\cos(k_n\Delta) \right) \sum_{m=1}^{M} A_m e^{ik_n\alpha_m} \sum_{j=1}^{M} A_j^* e^{-ik_n\alpha_j} + <P>_{\text{inc}}$$

where $\Delta = \alpha_m - \beta_m$. Hence, the power reflection coefficient is

$$R = \frac{<P>_{\text{refl}}}{<P>_{\text{inc}}}$$

and the power transmission coefficient is

$$T = \frac{<P>_{\text{trans}}}{<P>_{\text{inc}}}$$

It is convenient to define a radiation coefficient to account for the remainder of the energy. For example the total output power of the system must be equal to the total input power. Then the input power must equal the sum of the reflected power, the transmitted power, and the radiated power, viz:

$$<P>_{\text{inc}} = <P>_{\text{refl}} + <P>_{\text{trans}} + <P>_{\text{radiated}}$$
Then a radiation coefficient $S$ can be defined

$$S = \frac{\langle P \rangle_{\text{radiated}}}{\langle P \rangle_{\text{inc}}}$$

note also that

$$R + T + S = 1$$

Figures 16-18 display the voltage coefficients, which are the square roots of the power coefficients, for various slot and plate parameters when a TEM mode is incident in the guide. Figure 16 shows the reflection coefficient. The reflection coefficient increases as the slot width increases and as the plate spacing decreases. This is to be expected since a wider slot or the same sized slot in a narrower plate spacing presents a greater discontinuity to a wave in the guide. Figure 17 shows the transmission coefficient. It is seen to be the converse of the reflection coefficient as expected. If more energy is reflected back down the guide, less is expected to go forward beyond the slot. The radiation coefficient is shown in Figure 18.

Millar (1959) has plotted his asymptotic results for large slot width against the measured results of Simmons (1957) for the reflection and transmission coefficients. His plots are reproduced here with the moment method solutions added. In Figure 19 the transmission coefficient is shown for three different values of plate spacing $k_b$. The moment method solution agrees extremely well with the measured values of Simmons. The variations between the moment method solutions and Millar's results are appreciable at smaller values of slot width $k_a$ but as $k_a$ increases this difference decreases. This is
Figure 16. Reflection Coefficient
Figure 17. Transmission Coefficient
Figure 18. Radiation Coefficient
Figure 19. Transmission Coefficient Variation with Slot Width for Different Values of $k_b$. (Open Circles and Crosses are measured values scaled from the Report of Simmons for $k_b = 1.57, 1.88$ Respectively)
to be expected since Millar's results become better as slot width is increased. The fact that Millar's asymptotic results approach the moment method solution as the slot width is increased and that the moment method solution matches the experimental results for small slot widths is an indication that the moment method can be used to bridge the gap between the range of validity of Millar's results \((ka \geq 10)\) and Simmons results \((ka \leq 2)\). Figure 20 shows the reflection coefficient for two values of \(kb\).

2.7 Antenna Pattern Analysis

The last information of interest is the antenna pattern which the slot produces. Figure 21 displays the geometry of the aperture and defines the quantities of interest in the following derivation. In the upper half-space

\[ H_y(x,z) = \frac{\omega e_2}{2} \int_{-a}^{a} H_0^{(2)}(k_2 \sqrt{x^2 + (z-z')^2}) E_A(z') dz' \]

In the far field region where the antenna pattern is to be calculated, the large argument approximation for the Hankel function may be substituted in the above expression.

\[ H_0^{(2)}(x) \sim \frac{2}{\sqrt{\pi x}} e^{-i(x-\pi)} \text{ for large } x \]

Then

\[ H_0^{(2)}(k_2 \sqrt{x^2 + (z-z')^2}) \sim \frac{2}{\sqrt{\pi k_2}} e^{-ik_2|\vec{r} - \vec{r}'|} e^{ik} \]
Figure 20. Reflection Coefficient Variation with Slot Width for Different Values of $k_b$. (Open Circles are Measured Values Scaled From the Report of Simmons for $k_a = 1.57$)
Figure 21. Antenna Pattern Geometry
For $\rho \gg a$

$$\left| \rho - \rho' \right| \sim \rho - z' \cos \theta$$

Then

$$H_y(x,z) \sim C \int_{-a}^{a} e^{ik_2z' \cos \theta} E_A(z')dz'$$

where $C$ is a constant which is not of importance because the pattern will be normalized to unit peak value. For a pulse expansion the above expression can be shown to be

$$H_y(x,z) \sim C \sum_{m=1}^{M} \left[ \frac{ik_2 \cos \theta \alpha_m}{e^{-ik_2 \cos \theta}} \right]$$

or

$$H_y(x,z) \sim C \sum_{m=1}^{M} A_m (\alpha_m - \beta_m) k_1, \theta = 90^\circ$$

The antenna patterns are given for various slot widths and plate spacings in Figures 22-24. The incident field in the guide is assumed to be the TEM mode. The pattern shown in Figure 22 is typical of the narrower slot. The fact that the pattern is nearly uniform can be explained by realizing that, in the radiation problem, the aperture field can be replaced by an equivalent directed magnetic current. If this magnetic current is imaged in the infinite planes, the equivalent problem, at least as far as the upper half-space is concerned, is that of a magnetic line source radiating in free space. This current source would be expected to radiate uniformly if it were filamentary as a narrow slot would approximate. The pattern shown in Figure 24.
Figure 22. Antenna Pattern for \( a = 0.008\lambda \), 
\( b = 0.250\lambda \)
Figure 23. Antenna Pattern for $a = .016\lambda$, $b = .063\lambda$
Figure 24. Antenna Pattern for $a = 1.50\lambda$, $b = .250\lambda$
deviates from the uniform one because the slot is wider allowing some phase variation across it and the equivalent current source is no longer filamentary. In the next chapter the results given here are extended for the case of more than one slot.
CHAPTER 3

MULTIPLE SLOT RADIATION

3.1 Derivation of Integral Equation and Matrix Symmetries

The derivation of the integral equations for the aperture fields for the case of more than one slot in a parallel plate waveguide wall follows closely the derivation presented in the last chapter for the case of one slot. This is to be expected since the case of several slots can be treated as several coupled singled slots (in which case the formulation results in a set of coupled integral equations) or as one large slot with strips of metal interposed (in which case only one integral equation is obtained). It can be shown that the two treatments above result in identically the same integral equation for the aperture field. This statement is made plausible by considering the following. If the $N$ slot problem were treated as $N$ slots each with its own aperture field $E_1$, then after a suitable expansion of the unknown fields and a matching of these fields at the required number of points, $N$ coupled equations of the form

$$
\int_{-a}^{a} E_1(z') K(z_1|z') dz' + \cdots + \int_{-a}^{a} E_N(z') K(z_1|z') dz' = f(z_1)
$$

= $f(z_1)$
would be obtained. If, however, these slots were considered as simply portions of one large slot with unknown aperture field $E_A(z')$, after expansion and matching, $N$ equations of the form

$$\int_{-a_l}^{a_N} E_A(z') K(z_1 | z') \, dz' = f(z_1)$$

would result. That these two forms are identical is seen when the equivalence

$$E_A(z') = E_1(z') + E_2(z') + \ldots + E_N(z')$$

is made.

The geometry of this problem is shown in Figure 25. The overall aperture field will be expanded in pulses and delta function testing will be used in the same manner as for the single slot in Chapter 2. This field expansion is demonstrated in Figure 26. The matrix equation thus obtained is given below

$$f_1(z_1) = \sum_{m=1}^{M} A_m \, \Sigma_{lm}$$

where

$$f_1(z_1) = k_1 H_y(o,z_1)$$

$$\Sigma_{lm} = k_1 \omega \int_{-\infty}^{\infty} -ik_n \left| z_1 - z' \right| dz' + k_1 \epsilon_2 \int_{2ibk_n}^{2i\epsilon_1} \frac{a_m}{\beta_m} (2) \, dz'$$

where the unknown field is given as

$$E_A(z) = \sum_{m=1}^{M} A_m P_m(z)$$
Figure 25. Multiple Slot Geometry
Figure 26. Field Pulse Expansion for Multiple Slots
where again

\[ p_m(z) = \begin{cases} 
1 & \beta_m < z < \alpha_m \\
0 & \text{otherwise}
\end{cases} \]

It is now obvious that the above equation is identically equation (29) where only \( \alpha_m \) and \( \beta_m \) have different values.

As was pointed out in the preceding chapter, an equal width pulse expansion of the aperture field in a single slot leads to a matrix which is Toeplitz and symmetric. Unfortunately this is not the case when more than one slot is considered. The overall matrix is no longer Toeplitz since the distance between pulse centers within one slot remains the same, an integral number of pulse widths, whereas the distance between two pulses in different slots is not an integral number of pulse widths. Therefore a different matrix inversion routine must be employed to solve the matrix equation. Certain symmetries are present, however, and it is worthwhile to consider them as special cases.

The most general case which can be considered is that of an arbitrary slot spacing and slot width. It is desired that the expansion pulse width within each slot be the same while the pulse widths from slot to slot may be different. In order to make the computer program more tractable in this case it is also desired that each slot be divided into the same number of pulses. If this is the case, the \( l_{mn} \) matrix has the form, (here for example \( M=9, N=3 \))
The boxed submatrices above are symmetric Toeplitz and represent the self coupling within each aperture. The symmetric Toeplitz submatrices enable the calculation of $M^2(1-L) + M$ instead of $M^2$ terms in the matrix fill. The overall matrix is, however, not symmetric and unless an inversion routine is written to make use of the Toeplitz submatrices by some sort of partitioning, a general matrix inversion routine must be used.

The next more specific case considered is that of arbitrary slot spacing but each slot width the same. Then when the pulse widths are chosen to be the same the $l_{mn}$ matrix has the form

\[
\begin{bmatrix}
11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 \\
12 & 11 & 12 & 24 & 25 & 26 & 27 & 28 & 29 \\
13 & 12 & 11 & 34 & 35 & 36 & 37 & 38 & 39 \\
41 & 42 & 43 & 44 & 45 & 46 & 47 & 48 & 49 \\
51 & 52 & 53 & 45 & 44 & 45 & 57 & 58 & 59 \\
61 & 62 & 63 & 46 & 45 & 44 & 67 & 68 & 69 \\
71 & 72 & 73 & 74 & 75 & 76 & 77 & 78 & 79 \\
81 & 82 & 83 & 84 & 85 & 86 & 78 & 77 & 78 \\
91 & 92 & 93 & 94 & 95 & 96 & 79 & 78 & 77 \\
\end{bmatrix}
\]
Again the boxed submatrices are symmetric Toeplitz. Now, however, the overall matrix is also symmetric and there are other internal relations between matrix elements. Altogether the matrix fill requires the calculation of \( \frac{M + (2N - 1) (M^2 - N)}{2} \) rather than \( M^2 \) elements. A symmetric matrix inversion routine can be used to decrease the time and storage required to calculate the inverse.

A final example of matrix symmetries for special cases is obtained when the slot widths and spacings are all the same. Then the matrix has the form
Again the boxed submatrices are symmetric Toeplitz and the overall matrix is symmetric. Now there are added relations between other matrix elements which requires only $\frac{2M}{N} (\frac{M}{N} - 1) + 1$ terms be calculated instead of the $M^2$ terms generally required to fill an $M \times M$ matrix. Computer codes have been written for all of the above cases.

3.2 Reflection, Transmission, and Radiation Coefficient Determination

The power coefficients of interest are calculated in the same way for the multiple slot case as described for the single slot case in Chapter 2. The coefficients are computed for the entire array rather than the individual slots. Thus the reflection coefficient is the ratio of that power which is reflected back down the guide by the entire slot array (rather than at some intermediate point between two successive slots) to the incident power. Similarly the transmission
coefficient is the ratio of the power which flows beyond the last slot to the incident power. Figures 27-29 show the power coefficients as defined above for various slot configurations when a TEM mode is incident in the guide.

The total time average power out each slot is shown for various cases in Figures 30 and 31. In Figure 30, the slot spacings are considered to be the same, \( d \), and \( d \) is varied while the slot widths are all the same, \( 2a \), and held constant. It is interesting to note that there are distinct crossovers at which the power out each slot is the same. These crossovers occur at integer multiples of \( \frac{\lambda}{2} \) in spacing. The reason for the existence of these crossovers can be seen by considering the path differences from one slot to another. When the slots are separated by an integer multiple of \( \frac{\lambda}{2} \), the amplitudes of the scattered fields at any slot position from all other slots are equal (assuming the slots are small and scatter identically) and the relative phases of these scattered fields are all the same. The fields are then necessarily of the same amplitude in each slot with only phase shifts allowed. Figure 31 shows the power out each slot if all the slots have the same constant spacing, \( d \), and the same width, \( 2a \), while \( a \) is varied.

Implementation of the asymptotic results for low and high frequencies is basically the same as for the single slot. It is obvious, however, that the GTD approximation becomes much more complicated to implement because of the added number of diffractions.
Figure 27. Reflection Coefficient of Three Slots of Equal Width 2a
Figure 28. Transmission Coefficient of Three Slots of Equal Width
Figure 29. Radiation Coefficient of Three Slots of Equal Width
Figure 30. Total Time Average Power Through Slots of Equal Width ($a = 0.01\lambda$)
Figure 31. Power Out Slots vs. Slot Width When Slot Spacing is $d = 1\lambda$. Plate Spacing is $b = .20\lambda$. 

- Slot 1
- Slot 2
- Slot 3
3.3 Antenna Pattern Analysis

The antenna pattern of the multiple slot array is perhaps of most interest since this configuration approximates a slotted array. It is interesting to note that use of the moment method to solve for the aperture field facilitates comparison of the effects of mutual coupling of the array elements on the overall antenna pattern. These results will be demonstrated below.

It should be possible to predict the array pattern using basic array theory. It is a well known result (Ramo, Whinnery, and Van Duzer 1965) of array theory that the field radiated by a linear equally spaced array of $N$ current elements is

$$E \propto \sum_{n=0}^{N-1} J_n \cos \theta$$

where the pattern is calculated in the plane perpendicular to the element axes, and $\theta$ and $d$ are defined in Figure 32. Here $J_n$, the current in the $n^{th}$ radiator, can be written $I_n e^{i\alpha_n}$ where $I_n$ is the amplitude and $\alpha_n$ the phase of the $n^{th}$ radiator. The slotted array formed by multiple slots in the parallel plate waveguide wall approximates the above linear array if the slots are narrow ($2a<\lambda$) and equally spaced. Then $I_n$ corresponds to the amplitude of the current (field) in the $n^{th}$ slot and $\alpha_n$ is the phase of the current (field) in that slot. The phase variation for narrow slots arises from the phase difference of the exciting field in the waveguide at each slot. In fact, for narrow slots spaced $d$ apart,

$$\alpha_n = -\beta nd$$
Figure 32. Array Geometry
where $\alpha_0$ is assumed to be 0 and $\beta$ is the propagation constant of the $m$th mode in the guide ($\beta = k_1 \sqrt{1 - \left(\frac{m\pi}{dk_1}\right)^2}$). Then if the field strengths in each slot are approximately the same the pattern produced by this incident mode can be written:

$$\sum_{n=0}^{N-1} ik_2 d \cos \theta - i \beta nd$$

or

$$\sum_{n=0}^{N-1} i k_d (\cos \theta - \beta)$$

As an example of the above, consider the case where $k_2 = k_1$ and the incident mode is the TEM mode. Then

$$N-1 \frac{ik_1 nd (\cos \theta - 1)}{e}$$

The summation above can be evaluated since it is a geometric progression. Then

$$E = \frac{ikd(\cos \theta - 1)N}{e^{ikd(\cos \theta - 1)}}$$

The pattern then has maxima at

$$\cos \theta = 1 - \frac{2\pi l}{kd}, \quad l = 0, 1, 2, \ldots$$

If the spacing is chosen to be $\lambda$ then the pattern has principle maxima at $\theta = 0^\circ, 90^\circ$ and $180^\circ$. This case is shown for the slotted waveguide in Figure 33. If the spacing is $\frac{\lambda}{2}$ then the pattern has principle maxima at $\theta = 0^\circ$ and $180^\circ$. This example is shown in Figure 34 for the
Figure 33. Array Pattern for 2, .005λ Slots Separated by \( d = 1\lambda \). Plate Spacing is \( b = .20\lambda \)
Figure 34. Antenna Pattern for 2, .005λ Slots Separated by 
\( d = .50\lambda \). Plate Spacing is \( b = .20\lambda \)
slotted guide. In Figures 33 and 34 the solid line represents the actual pattern while the dashed line represents the pattern predicted by simple array theory.

If the slots are wide enough to have appreciable phase variation across their width, then the above simple analysis no longer pertains. Figure 35 shows the pattern for wide slots (2a=λ/2) spaced λ apart. When compared with Figure 33 the failure of the simple analysis to hold is evident. The reason for this breakdown is the fact that the element phase is no longer simply defined by the phase progression of the incident field but also contains phasing effects of the slot itself.

Mutual coupling effects of one slot on another are very easily investigated using the moment method. The matrix obtained when solving for the aperture fields contains submatricies which define the effects of sources in one slot on field points within that same slot and also contains submatricies which define the effects of sources in one slot on field points within other slots. Since the mutual coupling effects are a result of the submatrix components relating one slot to another, these effects can be neglected by setting the appropriate matrix components to zero. It is common practice in designing arrays to neglect mutual coupling effects as a first order approximation. The kinds of errors which can result from this neglect of mutual coupling are displayed in Figures 36 and 37. The effects of mutual coupling are shown for two different cases. Figure 36 considers the case of .2λ
Figure 35. Antenna Pattern for 2,50λ Slots Separated by d = λ. Plate Spacing is b = .20λ.
Figure 36. Mutual Coupling Effects of 3, .025λ Slots Spaced .25λ Apart With Plate Spacing of .2λ
Figure 37. Mutual Coupling Effects of 3, \(0.075\lambda\) Slots Spaced \(0.25\lambda\) with Plate Spacing of \(0.2\lambda\)
plate spacing with three slots spaced \(0.25\lambda\) apart. The slot widths are all \(0.025\lambda\). In Figure 37 the above parameters remain the same except the slot sizes which now become \(0.075\lambda\). In these figures the solid line represents the antenna pattern calculated with all mutual coupling effects accounted for and the dashed line represents the antenna pattern calculated neglecting mutual coupling.

The single and multiple slot transmitting antenna problems have been considered above. The opposite problem, that of considering the shielding effectiveness of the slotted structure will be considered next.
CHAPTER 4

SINGLE AND MULTIPLE SLOT SHIELDING

The problem of calculating the shielding effectiveness of the slotted structure when a plane wave is incident from outside the guide can also be thought of as a calculation of the power coupled into the guide by the antenna when used as a receiver. The objective is to calculate the power coupled into each waveguide mode in both the forward and reverse directions for various angles of incidence of a monochromatic plane wave. The calculation of field values in the guide resulting from a field incident from outside follows a very similar development. In the next chapter the methods developed here will be used to calculate the frequency transfer function of the structure and the time response for various inputs will be analyzed.

4.1 Aperture Field Calculation

The integral equation which results for the case where the source is a plane wave incident from outside the guide is derived from an application of Green's theorem. In region I the field is

\[
H_y(x, z) = \text{i} \omega \varepsilon_0 \int_{-a}^{a} E_A(z') \sum_{n=0}^{\infty} \frac{\varepsilon_n \cos \frac{n \pi x'}{b} \cos \frac{n \pi x}{b}}{2i kn} e^{-i k_n |z-z'|} \, dz'
\]

(1)

and in region II \(H_y\) is

82
The aperture $E$ fields can be found by the moment method with again pulse expansions and point matching. The $H$ field in the guide is obtained by an integration over the aperture field. Figures 38 and 39 show some values of aperture $E$ and $H$ fields for various parameters for single slot cases. Figure 40 and 41 show these fields for multiple slot cases. In Figures 40 and 41 the geometry is as follows. Slot 1 is at $c_1 = -0.5\lambda$ and has width $2a_1 = 0.06\lambda$. Slot 2 is located at $c_2 = 0$ and has width $2a_2 = 0.03\lambda$. Slot 3 is located at $c_3 = +0.1\lambda$ and has width $2a_3 = 0.01\lambda$. The plate spacing in these figures is $b = 0.2\lambda$.

### 4.2 Power Coupling Coefficients

The quantity of most interest as far as shielding effectiveness is concerned is the total power which couples into the guide. This quantity is calculated in the same way as was given in Chapter 2 to calculate time average powers of forward and reflected waves in the guide. By integrating Poyntings vector over the slots, the total time
Figure 38. Aperture E Field for a .025λ Slot with a .20λ Plate Spacing. Angle of Incidence is 45°
Figure 39. Aperture H Field for a .025λ Slot with a .20λ Plate Spacing. Angle of Incidence is 45°
Figure 40. Aperture E Field for 3 Slots. $c_1 = -0.5\lambda$, $c_2 = 0\lambda$, $c_3 = +1.0\lambda$, $a_1 = 0.03\lambda$, $a_2 = 0.015\lambda$, $a_3 = 0.005\lambda$, $b = 0.20\lambda$, $\theta = 60^\circ$. 

$E_A(z)$ (volts/m)
Figure 41. Aperture H Field for 3 Slots. \( c_1 = -0.5\lambda, c_2 = 0\lambda, c_3 = 0.1\lambda, a_1 = 0.03\lambda, a_2 = 0.015\lambda, a_3 = 0.005\lambda, b = 0.2\lambda, \theta = 60^\circ \).
average power which couples into the guide can be calculated. An 
integration of Poyntings vector for each propagating mode over appro-
ipriate surfaces of constant $z$ within the guide determines the total 
time average power in each of these modes. Figures 42-45 display these 
power values for various angles of incidence of the incoming plane 
wave. The plate spacing is $b = 2.25\lambda$ and the slot width is $2a = .1\lambda$
in each of these figures. The total power is the total power coupled 
into the guide normalized to a unit amplitude ($E_0 = 1$) incident plane 
wave. The power in each propagating mode is expressed in % of this 
total power. All power is accounted for by noting that the sum of 
powers in each mode add up to 100%. In Figures 42-45 the forward di-
rection is considered to be the $+z$ direction and the reverse direction 
is the $-z$ direction. Since the plate spacing is 2.25$\lambda$, not only the 
TEM mode propagates but also the first $4$ TM modes. The power is seen 
in each of these figures to distribute among the propagating modes 
with more power going into the highest order TM modes and decreasing 
amounts going into TM modes of decreasing order. The amount of power 
which is coupled into each mode depends upon the source distribution 
in the guide (Dudley and Quintenz 1972). A uniform distribution in $x$
would put all power in the TEM mode. As the source distribution be-
comes more nonuniform, more energy is coupled into higher order modes. 
Since the excitation of the fields interior to the guide is accom-
plished through the boundary condition in the slot which occurs at one 
point in $x$ and is hence highly nonuniform in $x$, it is not surprising
Total Power $9.89 \times 10^{-4}$ watts

Forward Power

Reverse Power

Figure 42. Power Coupled into Each Mode for $0^\circ$ Incidence, $a = 0.05\lambda$, $b = 2.25\lambda$
Figure 43. Power Coupled into Each Mode for 30° Incidence,
\[ a = 0.05\lambda, \quad b = 2.25\lambda \]
Total Power $1.02 \times 10^{-3}$

Forward Power

Reverse Power

Figure 44. Power Coupled into Each Mode for $60^\circ$ Incidence, $a = 0.05\lambda$, $b = 2.25\lambda$
Figure 45. Power Coupled into Each Mode for 90° Incidence, 
\[ a = 0.05\lambda, \quad b = 2.25\lambda \]
that the power couples dramatically into the higher order TM modes. As the angle of incidence increases from 0° to 90° the difference between the power in the forward and backward modes decreases until at 90° they become the same. This is to be expected since at 90° the problem becomes symmetric in \( z \).

### 4.3 Low and High Frequency Analysis

The low frequency asymptotic results are handled in the same way for the shielding problem as they were for the antenna problem of Chapters 2 and 3. The high frequency GTD asymptotic method could also be applied to the shielding problem as described for the antenna problem. As was eluded to in Chapter 2, the Kirchhoff approximation has application to the shielding problem as demonstrated now.

The Kirchhoff approximation assumes that the aperture field is the field which would exist were there no upper plate. The incident field is

\[
\bar{E}_{\text{inc}} = (-\cos \theta \hat{a}_x + \sin \theta \hat{a}_z) e^{ik_2(x \sin \theta + z \cos \theta)}
\]

\[
\bar{H}_{\text{inc}} = \frac{\sqrt{\varepsilon_2}}{\mu_2} e^{ik_2(x \sin \theta + z \cos \theta)} \hat{a}_y
\]

Hence assume

\[
E_z(z)\big|_{x=0} = E_A(z) = \sin \theta e^{ik_2 z \cos \theta}
\]

\[
H_y(z)\big|_{x=0} = \frac{\sqrt{\varepsilon_2}}{\mu_2} e^{ik_2 z \cos \theta}
\]
Then the field in the guide is approximately

\[ H_y(x,z) \propto i\omega e_1 \int_{-a}^{a} \sum_{n=0}^{\infty} \frac{\varepsilon_n}{b} \cos \frac{n\pi x}{b} e^{\frac{-ik_n|z-z'|}{2}} \sin \theta e^{\frac{ik_2 z' \cos \theta}{2ik_n}} \frac{dz'}{2} \]

or after some manipulation

\[ H_y(x,z) \propto i\omega e_1 \sin \theta \sum_{n=0}^{\infty} \frac{\varepsilon_n}{k_n} \cos \frac{n\pi x}{b} \frac{2}{i(k_n^2-k_2^2 \cos^2 \theta)} \left[ -ik_2 z \cos \theta e^{\frac{-ik_2 a \cos \theta -ik_n^2}{e^{\frac{-ik_n a}{e}}}} \right] \]

The aperture fields as calculated using the moment method are compared with the fields predicted by the Kirchhoff approximation for a single slot in Figure 46. As can be seen from these figures the Kirchhoff approximation gives very good results for an aperture of this size. The approximate results become better as the frequency is increased.

4.4 Directional Coupler Analysis

A final example to be presented here of coupling energy from outside the waveguide into waveguide modes resembles directional coupler effects. It is well known that if a series of slots are placed appropriately in a wall common to two waveguides a directional coupler can be made. By placing the slots one quarter wavelength apart as shown in Figure 47 the energy from wave guide I which is coupled into wave guide II travels in a preferred direction. A simple explanation of this result is found by considering phase differences of waves traveling different paths. At point 1 the wave which travels path a arrives with some phase, say \( \phi_0 \). The wave which travels path b arrives with
Figure 46. High Frequency Approximation for a 5λ Slot with 10.1λ Plate Spacing. Angle of Incidence is 30°
Figure 47. Directional Coupler
phase $\phi_0 + \pi$ or $180^\circ$ out of phase with the other wave causing cancelation of energy in this direction. At point 2 on the other hand both paths result in the same phase and the two waves add in phase.

The above situation can be simulated by spacing small slots $\lambda$ apart and allowing a plane wave to be incident along $\theta = 0^\circ$. This case has been considered for two different plate spacings when the slot widths are maintained at $.02\lambda$. In the first case $b = .2\lambda$ and the forward ($+z$ traveling) TEM mode contains 7.7 percent of the total coupled power as opposed to the backward ($-z$ traveling) TEM mode's 92.3 percent. In the second case $b = .75\lambda$ allowing the $TM_1$ mode to propagate also. Here the forward and backward TEM modes contained 1.6 percent and 23.7 percent of the total coupled power respectively while the forward and backward $TM_1$ modes respectively contained 10.0 percent and 64.7 percent. Smaller plate spacings and slot widths improve this directivity.

By solving the slot problem over a band of frequencies, the transfer function can be obtained. These results are presented in the next chapter.
CHAPTER 5

TRANSIENT SHIELDING

The transient shielding characteristics of the slotted parallel plate waveguide are obtained using Fourier transform techniques. The steps involved are calculation of the system transfer function $H(\omega)$ and multiplication by a desired input function frequency spectrum $F(\omega)$ to obtain the frequency spectrum of the desired response $G(\omega)$, viz.

$$H(\omega) \cdot F(\omega) = G(\omega)$$

The transient response $g(t)$ is then the inverse Fourier transform of $G(\omega)$, viz:

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{G(\omega) \cdot e^{i\omega t}}{i\omega} \, d\omega$$

The system function is determined from the monochromatic shielding problem considered in Chapter 4. The number of frequency points required to describe the system function depends upon the plate spacing, slot widths and locations, and field point locations. Since the system function is to be multiplied by an input spectrum it is the product of these two functions which must be adequately described to give a valid inverse transform. It is possible for the system function to vary slowly over a frequency range where the input spectrum is varying rapidly. Since the system function is usually more costly to evaluate, it is worthwhile to use a polynomial approximation to the system function.
at intermediate values of frequency which are needed to detail the
input spectrum. This is the case in this problem where low frequencies
are concerned. The transfer function varies very little in the low
frequency limit whereas some of the input pulses used here have large
variations over the same range.

The fields in the guide are found using the following expres-
sions once the aperture $\mathbf{E}$ field has been calculated.

$$(1) \quad H_y(x,z) = i\omega_1 \int_{-a}^{a} \sum_{n=0}^{\infty} \frac{e_n}{b} \cos \left( \frac{\pi n x}{b} \right) e^{-ik_n |z-z'|} E_A(z') dz'$$

$$E_x = -\frac{1}{i\omega_1} \frac{\partial H_y}{\partial z}$$

$$E_z = +\frac{1}{i\omega_1} \frac{\partial H_y}{\partial x}$$

In the case of a single slot where a pulse expansion has been
used to approximate the aperture field as in (26) the field components
take on the following convenient form if $z < -a, 0 < x < b$.

$$H_y(x,z) = \sqrt{\frac{E_1}{\mu_1}} \left[ \psi_0 - \sum_{n=1}^{\infty} \psi_n \right]$$

$$E_x(x,z) = -\psi_0 + \sum_{n=1}^{\infty} \psi_n \frac{k_n}{k_1}$$

$$E_z(x,z) = \frac{\pi}{ibk_1} \sum_{n=1}^{\infty} n \psi_n \tan \left( \frac{\pi n x}{b} \right)$$

where

$$\psi_0 = i(\alpha_m - \beta_m) k_1 \frac{ik_1 z}{2bk_1} \sum_{m=1}^{M} A_m e^{-ik_1 \alpha_m}$$
\[
\psi_n = \frac{\imath^n \cos \left(\frac{\pi nx}{b} \right) e^{i k_n z} (1 - e^{-ik_n (\alpha_m - \beta_m)})}{\sum_{m=1}^{M} A_m e^{-ik_n \alpha_m}} \left( \frac{k_n^2}{k_1} \right)^{\frac{b}{2k_1}}
\]

Similar results hold for \(-a \leq z \leq a\) and \(z > a\).

In the low frequency limit these field components approach

(again for \(z < -a\))

\[
H_y (x,z) \propto 2 \sqrt{\frac{\varepsilon_2}{\mu_2}} e^{ik_1 z}
\]

\[
E_x (x,z) \propto -2 \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\mu_1 \mu_2 \varepsilon_1}} e^{ik_1 z}
\]

\[
E_z (x,z) \propto 0
\]

The low frequency limits of the field components demonstrate that the \(H_y\) field component goes to twice the amplitude of the incident field as does the \(E_x\) field component. These components comprise a TEM mode traveling in the negative \(z\) direction. There is no longitudinal component of field at low frequencies (\(E_z = 0\)).

A few transfer functions for the \(H_y\) and \(E_x\) field components are presented next. In order to eliminate unnecessary phase variations in the transfer functions, each has been multiplied by a phase factor \(e^{ik_1 \sqrt{x^2 + z^2}}\). This multiplication in the frequency domain results in a linear shift in the time domain corresponding to the time required for an excitation to travel by the most direct route from the slot center to the field point. By removing this time delay, which amounts to shifting the time origin to the instant at which an excitation arrives
at the field point, the unnecessary phase variation is eliminated and fewer frequency points are required to detail the transfer functions.

Figures 48 and 49 display the transfer functions for the $H_y$ and $E_x$ field components at $x = -5 \text{cm}, z = -1 \text{m}$ when the plates are separated by $b = 10 \text{cm}$ and the slot width is $2\text{mm}$. The incident field is a unit amplitude $\left( E_0 = \frac{1}{m} \right)$ plane wave with incidence angle $\theta = 30^\circ$. Figures 50 and 51 are for the same case except $\theta$ is now $90^\circ$. Figures 52 and 53 are for the case $x = -5 \text{ cm}, z = -1 \text{ m}, b = 10 \text{ cm}$, and the slot width is $2 \text{ cm}$. In these figures the incidence angle is $30^\circ$. Figures 48-53 represent transfer functions for relatively small slot widths and plate spacings. The plate spacing is such that only the TEM mode propagates for all frequencies that are of interest for the input pulses considered. As a result of the fact that only the TEM mode propagates, the magnitude of the $H_y$ transfer function is less than the magnitude of the $E_x$ transfer function by a factor equal to the characteristic impedance of the guide medium. The transfer functions fall off at higher frequencies as $1/\omega$ in accordance with the TEM mode fall off of a parallel plate waveguide. These transfer functions can now be multiplied by the frequency spectra of desired input pulses to obtain transient results.

The input pulses chosen here are of the double exponential type.

$$f(t) = N \left( e^{-\alpha t} - e^{-\beta t} \right)$$  \hspace{2cm} (32)

In expression (32) $N$ is a normalization constant determined so that $f(t)$
Figure 48. $H_y$ Transfer Function at $x = -5 \text{ cm}$, $z = -1 \text{ m}$, $b = 10 \text{ cm}$, $a = 1 \text{ mm}$, $\theta = 30^\circ$
Figure 49. $E_x$ Transfer Function at $x = -5\text{cm}$, $z = -1\text{m}$, $b = 10\text{cm}$, $a = 1\text{mm}$, $\theta = 30^\circ$
Figure 50. $H_y$ Transfer Function at $x = -5\text{cm}$, $z = -1\text{m}$, $b = 10\text{cm}$, $a = 1\text{mm}$, $\theta = 90^\circ$
Figure 51. $E_x$ Transfer Function at $x = -5\text{cm}$, $z = -1\text{m}$, $b = 10\text{cm}$, $a = 1\text{mm}$, $\theta = 90^\circ$
Figure 52. $|H_y|$ Transfer Function at $x = -5\text{cm}$, $z = -1\text{m}$, $b = 10\text{cm}$, $a = 1\text{cm}$, $\theta = 30^\circ$
Figure 53. $E_x$ Transfer Function at $x = -5\text{cm}$, $z = -1\text{m}$, $b = 10\text{cm}$, $a = 1\text{cm}$, $\theta = 30^\circ$
reaches a peak of 1.0. The parameters $\alpha$ and $\beta$ can be varied to produce the desired rise and fall times of the incident pulse. A simple Fourier transform yields the spectrum of $f(t)$ as

$$F(\omega) = N \frac{(\alpha - i\omega)}{\alpha^2 + \omega^2} - \frac{(\beta - i\omega)}{\beta^2 + \omega^2}$$

Two different incident pulses are considered for each of the cases presented in Figures 48, 50, and 52. The first incident pulse resembles an impulse with rise time .2 ns and fall time .65 ns. For this pulse $\alpha = 4.5 \times 10^9$ and $\beta = 5.5 \times 10^9$. The second pulse resembles a step function with rise time 3 ns and fall time 1 ms. For this pulse $\alpha = 1 \times 10^3$ and $\beta = 8 \times 10^8$. These responses will be labeled impulse and step responses but it should be remembered that they are only approximations to these functions.

Figures 54-65 present the transient responses to each of these input pulses for the three cases previously mentioned. In each of these cases when compared with the incident field the response is seen to have a longer rise time. This result is a product of the input pulses high frequency content being reduced in relation to its low frequency content by the parallel plate transfer function. The impulse response peak value is less than the incident field peak value while the step response peak value is greater than that of the incident field. This is understood by considering the frequency spectra of the step and impulse responses and the transfer function of the guide. The step function spectrum has an appreciable low frequency content which is
Figure 54. H Field Impulse Response of Single Slot for 30° Incidence and 2mm Slot Width
Figure 55. E Field Impulse Response of Single Slot for 30° Incidence and 2mm Slot Width
Figure 56. H Field Step Response of Single Slot for 30° Incidence and 2mm Slot Width
Figure 57. E Field Step Response of Single Slot for 30° Incidence and 2mm Slot Width
Figure 58. H Field Impulse Response of Single Slot for 90° Incidence and 2mm Slot Width
Figure 59. E Field Impulse Response of Single Slot for 90° Incidence and 2mm Slot Width
Figure 60. \( \mathbf{H} \) Field Step Response of a Single Slot for 90° Incidence and 2mm Slot Width
Figure 61. E Field Step Response of Single Slot for 90° Incidence and 2mm Slot Width.
Figure 62. H Field Impulse Response of a Single Slot for 30° Incidence and 2 cm Slot Width
Figure 63. E Field Impulse Response for 30° Incidence and 2 cm Slot Width
Figure 6b. H Field Step Response for 30° Incidence and 2 cm Slot Width
Figure 65. E Field Step Response for 30° Incidence and 2 cm Slot Width
accentuated by the transfer function of the guide resulting in a large peak value of the time response. The impulse function spectrum, on the other hand, has an appreciable high frequency content which is reduced by the guide transfer function and the result is the reduced peak magnitude seen in the above figures.

Resonance effects are seen in the time responses when the guide plate spacing is made large enough that the resonant frequency occurs at a value where the input pulse spectrum has appreciable energy content. Three cases are considered here where this effect is shown. Figure 66 presents the transient response to a 10 ns rise, 350 ns fall pulse (α = 3x10^6, β = 10^8) for a 6 m plate spacing and a .6 m slot width. The field point is x = -3m, z = -30 m. The incidence angle is 30°. A large plate spacing as presented in this case is seen to add a ripple to the response. The ripple frequency is seen to have a period of approximately 20 ns. This period corresponds to the second resonance of the guide (50 mHz). The first resonance is not seen since the field point is x = -b and the first resonance corresponding to cos\(\frac{x}{b}\) is zero there (Dudley and Quintenz 1972).

Figure 67 presents the transient response for the case b = 12m, 2a = 1.2m, x = -6m, z = -60m, θ = 30°. Again the incident pulse has a 10 ns rise and a 350 ns fall. The dimensions in this case are twice those of Figure 66 and the resulting ripple frequency has the expected 40 ns period. The final single slot case is shown in Figure 68. Here the parameters are b = -12m, 2a = .6m, x = -6m, z = -60m, and θ = 30°.
Figure 66. $H_y$ Time Response for .6m Slot with 6m Plate Spacing
Figure 67. $H_y$ Time Response for 1.2m Slot with 12m Plate Spacing
The same incident field as above is again used. The cases displayed in Figures 67 and 68 are identical except for the slot widths. In Figure 68 a narrower slot width is considered. The narrower slot width allows less energy to penetrate. Less of the energy in the spectrum near the guide resonance penetrates and hence the ripple is not as pronounced. The rise time is lengthened with the narrowing of the slot.

A multiple slot case is considered next. In Figures 69 and 70, the transfer functions for two field points within a waveguide with two slots are presented. The plate spacing is 1.2 m and the slots are separated by 1.5 m. Both slots are 12 cm wide. The field points represented in Figures 69 and 70 are $x = -60$ cm, $z = -60$ m and $x = -60$ cm, $z = +60$ m respectively. These parameters were chosen so that the slot separation would be a quarter wavelength at a frequency of interest (50 MHz) to display the directional coupling effects discussed in Chapter 4. The incidence angle in this case is $0^\circ$.

Figure 69 and 70 display the effects of directional coupling in the transfer functions. The location considered in Figure 69 is in the direction of maximum coupling whereas the location in Figure 70 is in the direction of minimum coupling. At approximately 50 MHz (not exactly 50 MHz because of slot width effects) Figure 70 displays a dip in the transfer function. This is the expected quarter wave spacing effect. At 100 MHz the effective spacing is one half wavelength and no cancellation (dip) occurs. At 150 MHz the effect is repeated and continues at each odd multiple of 50 MHz. The first plate resonance (250 MHz) is
Figure 68. $h_y$ Time Response for .6m Slot with 12m Plate Spacing
Figure 69. H Field Transfer Function for 2, 12cm Slots with 
\( d = 1.5\text{m}, \ b = 1.2\text{m}, \ x = -0.6\text{m}, \ z = -60\text{m}, \ \theta = 0^\circ \)
Figure 70. $H$ Field Transfer Function for 2, 12cm Slots with $a = 1.5m$, $b = 1.2m$, $x = -0.6m$, $z = +60m$, $\theta = 0^\circ$. 
seen in both figures. The effect of the periodic cancellation caused by the directional coupling is seen to alter the transfer function for \( z > 0 \) such that, for frequencies less than 250 MHz, it resembles \( \frac{\sin x}{x} \). The Fourier transform of a rectangular pulse has the same \( \frac{\sin x}{x} \) behavior and this fact will be used to help explain the following transient results.

Figures 71 and 72 present transient results for the above two cases when a 10 ns rise 350 ns fall pulse is incident. These two responses have very little difference. This result is explained by the fact that the input pulse has the majority of its energy in the low frequencies (<50 MHz) where the two transfer functions have similar values. These responses should be compared with those shown in Figures 73 and 74 where a incident field with 1 ns rise and 10 ns fall is used. This incident pulse has appreciable energy in its spectrum beyond 50 MHz and the two responses for \( z = +60 m \) and \( z = -60 m \) are quite different. Figure 73 displays the response at \( z = -60 m \). The transfer function for this case is relatively smooth, without the dips found in the \( z = +60 m \) case. The transient response is as expected. The rise time is lengthened and the fall time increased due to the transfer function's attenuation of the high frequencies. In Figure 74 the effect of the dips in the transfer function for \( z = +60 m \) is very noticeable. When the similarity of this transfer function and the transform of a rectangular pulse is recognized, the time response can be understood by the convolution theorem. A product of two functions in the frequency domain
Figure 71. $H_y$ Time Response for 12cm Slot at $x = -0.6m$, $z = -60m$
Figure 72. $H_y$ Time Response for 12 cm Slot at $x = -6m$, $z = +60m$
Figure 73. Time Response at $z = -60m$
Figure 74. Time Response at $z = +60\text{m}$
results in a function whose inverse Fourier transform is the same time function which would result from the convolution of the original two functions of time, viz:

If \[ F(\omega) = G(\omega) H(\omega) \]
and
\[ f(t) \leftrightarrow F(\omega) \]
\[ g(t) \leftrightarrow G(\omega) \]
\[ h(t) \leftrightarrow H(\omega) \]

then
\[ f(t) = \int_{-\infty}^{\infty} g(y) h(t-y) dy \]

The time response for this case can then be approximated by the convolution of the incident time pulse with a rectangular pulse of width determined by the period of the \( \sin \frac{x}{X} \) nulls shown in Figure 70. A convolution with a rectangular pulse is known as smoothing (Papoulis 1962). Figure 75 shows this smoothing effect of the convolution of a double exponential pulse with a rectangular pulse. The rectangular pulse has width 20 ns. The resulting time function resembles the response in Figure 74. The effect of the nulls is thus a smoothing of the incident pulse. The ripples seen in this response are due to the 250 MHz plate resonance.
Figure 75. Result of Convolution of Double Exponential Pulse with 20 ns Width Rectangular Pulse
CHAPTER 6

CONCLUSIONS

The moment method has been used to solve the integral equation for the coupled slot problem. The low and high frequency asymptotic solutions have been shown to represent adequately the aperture field in their respective ranges of validity and have extended the frequency range over which aperture fields could be calculated. The moment method has been shown to bridge the gap of medium slot sizes between previously obtained results for narrow slots and wide slots.

In the single slot case the recognition of the existence of Toeplitz symmetry in the matrix to be inverted has greatly reduced the required matrix storage and inversion time thus increasing the slot size which could be economically considered. The computer code developed to solve for interior fields in the shielding problem has been demonstrated capable of calculating a transfer function that was then used to obtain transient results.

The transient results demonstrated that the shielding effectiveness of the slot is dependent upon the frequency content of the incident pulse. A wider slot permits more energy to couple into the guide. The shape of the penetrating pulse demonstrated that the energy content in the higher order modes added ripples of characteristic frequency to the time response.
By recognizing certain symmetries in the matrix of the multiple slot case, a number of slots were successfully treated to obtain an antenna array pattern. Mutual coupling effects have been presented. Including the mutual coupling effects between different slots in an array has been shown to increase the pattern nulls for the arrays considered. A directional coupling phenomena has been demonstrated in the steady state and this effect has explained coupling differences in the forward and backward traveling transient responses.
The four special functions used in the evaluation of the singular expansion terms are defined below.

**Fresnel Integrals**

\[
c(x) = \int_0^x \cos\left(\frac{\pi}{2} t^2\right) dt
\]

\[
s(x) = \int_0^x \sin\left(\frac{\pi}{2} t^2\right) dt
\]

**Dawson's Integral**

\[
d(x) = e^{-x^2} \int_0^x e^{t^2} dt
\]

**Error Function**

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt
\]

The Fresnel integrals are evaluated using the approximation of Boersma (1960) which has maximum error \(0.5 \times 10^{-9}\). Dawson's integral is evaluated using Hummer's expansion (1964). The error function is computed from the expansion given by Abramowitz and Stegun (1964).
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