STARLIGHT EXCITATION OF PERMITTED LINES IN GASEOUS NEBULAE

by

Steven Aldridge Grandi

A Dissertation Submitted to the Faculty of the

DEPARTMENT OF ASTRONOMY

In Partial Fulfillment of the Requirements
For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

THE UNIVERSITY OF ARIZONA

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GRADUATE COLLEGE

I hereby recommend that this dissertation prepared under my direction by Steven A. Grandi

entitled Starlight Excitation of Permitted Lines in Gaseous Nebulae

be accepted as fulfilling the dissertation requirement of the degree of Doctor of Philosophy

John Gilbert — 12 NOV 1975

Dissertation Director — Date

After inspection of the final copy of the dissertation, the following members of the Final Examination Committee concur in its approval and recommend its acceptance:*

R. E. Williams — 10 Nov 1975

P. A. Strittmatter — 10 Nov 1975

J R P Angel — 10 Nov 1975

*This approval and acceptance is contingent on the candidate's adequate performance and defense of this dissertation at the final oral examination. The inclusion of this sheet bound into the library copy of the dissertation is evidence of satisfactory performance at the final examination.

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SIGNED: Steven A Eldridge Grandi

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ABSTRACT

The weak heavy element permitted lines observed in the spectra of gaseous nebula have, with only a few exceptions, been thought to be excited only by recombination. It is the purpose of this dissertation to investigate in detail (via model nebula calculations) the accuracy of this assumption for individual lines in the nebula spectra.

First, approximations and techniques of calculation are considered for the three possible excitation mechanisms: recombination, resonance fluorescence by the starlight continuum, and resonance fluorescence by other nebular emission lines.

Next, the permitted lines of OI as observed in gaseous nebula are discussed. Since OI $\lambda 8446$ was discovered in the spectrum of the Orion Nebula, there has been a continuing debate as to its excitation mechanism. I conclude that neither recombination nor resonance fluorescence by the Lyman lines can account for the strength of $\lambda 8446$ and the presence of the other observed OI lines. Basically, there is an insufficient amount of OII for recombination to be important and Lyman line fluorescence can be ruled out, because of the lack of Lyman line photons in the neutral regions of the nebula that contain OI. However, resonance fluorescence by starlight can successfully account for the strength of all the observed OI lines.

Thirdly, it is shown that varying combinations of recombination, resonance fluorescence by starlight and resonance fluorescence by other

nebula lines can successfully account for the observed strengths in the Orion Nebula of lines of the following ions: CII, NI, NII, NIII, OII, NeII, SiII, SiIII and SIII. A similar analysis is performed for the lines in the spectra of the planetary nebulae NGC7662 and NGC7027, and, with some exceptions, satisfactory agreement between the observed and predicted line strengths is found. The major exceptions are lines that arise from levels with more than one excited electron. Attempts to account for these lines by considering recombination from metastable states, dielectronic recombination, resonance fluorescence, and non LS permitted transitions following a normal recombination or photoexcitation have all proved unsuccessful.

Finally, observations of the far red spectra of the Orion Nebula, the planetary nebulae NGC3242, NGC6210, NGC2392, IC3568, IC4997, NGC7027 and NGC7662 and the reflection nebulae IC431 and NGC2068 are reported.

CHAPTER 1

INTRODUCTION

The strongest lines in the spectra of gaseous nebulae are the permitted recombination lines of hydrogen and helium and lines (usually forbidden) of heavier elements that arise from low-lying levels excited by collisions with electrons. The spectral lines that will be discussed in this dissertation, however, are the permitted lines of the elements heavier than helium. Such lines, although generally weak, are nonetheless astrophysically interesting -- a point I hope to make clear in this dissertation.

These heavy element-permitted lines (with the exception of those excited by the Bowen fluorescence mechanism [Bowen, 1935], such as OIII λ3133) were often offhandedly assumed to be entirely due to recombination (Aller and Menzel, 1945), but, as I will try to show in the succeeding chapters, resonance fluorescence, mainly by starlight, is as important an excitation mechanism. "Resonance fluorescence" is the term used to describe the mechanism of photoexcitation of an ion followed by spontaneous radiative de-excitation, usually by a different path through the ion's energy levels than the excitation.

Seaton (1968) first suggested that resonance fluorescence by starlight (the "case C" mechanism) is more important than recombination in the excitation of some weak heavy element permitted lines. Burgess and Seaton (1960) had calculated the ionic abundance of oxygen in

NGC7027 based on the assumption that the weak permitted lines of OIII, OIV and OV were excited by recombination. Seaton (and later Harrington 1969) concluded that the abundances calculated in this way were an order of magnitude too large as compared to the values expected from ionization calculations. Via a schematic calculation, Seaton showed that resonance fluorescence by starlight could remove this order of magnitude difference for the OIII ion. Leibowitz (1972) further justified Seaton's hypothesis by an explicit solution of the equations of statistical equilibrium and resonance line transfer, including resonance fluorescence by starlight, for the CIV ion. Leibowitz concluded that some lines of this ion are dominated by resonance fluorescence by starlight while others are dominated by recombination. Kaler (1972) used indirect arguments to show that recombination cannot account for the observed strengths of certain lines of CII and OIII in several gaseous nebulae and he concluded that some other excitation mechanism, presumably direct excitation by starlight, is required.

Further evidence that recombination is insufficient to explain all the heavy element-permitted lines in gaseous nebulae comes from the spectrum of OI in the Orion Nebula. Andrillat and Houziaux (1968a) demonstrated that the surprising strength of OI $\lambda 8446$ in the Orion Nebula cannot be due to recombination, and Morgan (1971) ruled out a mechanism involving HI Lyman-β similar to the Bowen fluorescence mechanism.

Therefore, I have undertaken to quantitatively analyze (albeit with numerous approximations) all of the heavy element-permitted lines observed in the Orion Nebula and in two planetary nebulae in an effort to determine the relevant excitation mechanisms for each line. In the following chapters I will first outline the theory of emission line excitation in gaseous nebulae and detail the approaches and approximations used in this investigation. The next chapter will be devoted to the problem of permitted lines of neutral oxygen in gaseous nebulae. Finally, the heavy element-permitted line spectra of the Orion Nebula and of two representative planetary nebulae will be discussed in the last two chapters.

CHAPTER 2

MECHANISMS FOR PERMITTED LINE EXCITATION IN NEBULAE

Under the low-density approximation used in nebular calculations, an ion is always assumed to be in its ground state except when it is temporarily in the process of cascade following a recombination or when it has been temporarily excited from the ground state to a higher energy state by a collision or by radiation. Such a non-L.T.E. situation must be treated by the equations of statistical equilibrium rather than by the Saha-Boltzman formulas. In this way, by equating the rates into and out of a particular level, the rates for radioactive downward transitions can be obtained. For the permitted lines to be considered here, the transition rate per second for a line from a particular level is merely the number of populations of this level per second (multiplied by the branching ratio if there is more than one downward line) since a permitted radiative transition will dominate all downward processes.

Recombination

First, I will consider excitation of a permitted line by recombination. The rate at which any level of the ion I is populated due to recombination from the ion I + 1 is:

(1) $$\alpha_i^{eff} N_e N_{I+1}$$

where N_{I+1} is the density of the ion I+1, N_e is the density of electrons, and α_i^{eff} is the effective recombination coefficient to the i^{th} level. Thus, the volume emission coefficient due to recombination for a line from this level is:

$$(2) \qquad j_{line}^{recomb} = N_e N_{I+1} \alpha_{level}^{eff} \frac{A_{line}}{\sum A} \frac{\nu_{line}}{4\pi}$$

where $\frac{A_{line}}{\sum A}$ is the branching ratio (including the multiplet fraction). In this discussion I will assume that the nebulae considered are optically thick in the light of all resonance lines, and so I will neglect the contribution to equation 2 from all such de-excitations, equating them to photoexcitations by the on-the-spot approximation.

The quantity α_i^{eff} is calculated not only from consideration of recombinations directly to the i^{th} level, but from cascades following recombinations to higher levels as well. Therefore, values of α_i^{eff} must be obtained from the solution to a set of statistical equilibrium equations (the capture-cascade equations) which entails the knowledge of the recombination coefficient to each level $(\alpha j)_{(j \geq i)}$ and the transition probabilities between these levels (see Burgess and Seaton, 1960; and Leibowitz, 1972).

The approximation to α_i^{eff} that will be used here is to use hydrogenic values for αn (αn includes recombination to the level n and all higher levels) for each principal quantum number n, as tabulated in Osterbrock (1974) and to divide these up according to statistical weights among the terms with the same principal quantum number.

Next, we multiply this quantity by the ratio of α_B (the recombination coefficient to all excited levels) for the ion in question (obtained from Aldrovandi and Pequignot, 1972) to α_B for H. The solution to the capture-cascade equation is crudely approximated by multiplying these values by a fraction (1-f) where f is the estimated probability that a recombination or cascade leading to the population of a level higher than n will cascade to a level below n. In all cases, f is assumed to be equal to 0.5. Transitions among terms with the same n are approximately taken into account by increasing the coefficients of lower lying terms.

As an example, consider the 3p $^2D°$ term of OII. The effective recombination coefficient can be expressed as follows:

$$\alpha^{eff}_{3p\ ^2D°} = \alpha_C(\mathrm{OII})\ \frac{1}{3}\ \frac{1}{2}\ \frac{\omega_{3p\ ^2D} + \omega_{3d\ ^2F°}}{\omega_{TOTAL}}\ \frac{\alpha_B\ (\mathrm{OII})}{\alpha_B\ (\mathrm{H})} \tag{3}$$

where α_C(OII) is the hydrogenic value for OII of the recombination coefficient to levels with principal quantum number of 3 and above, the factor of 1/3 represents the fact that 1/3 of the recombinations will go to doublet terms as opposed to quartet terms, and the factor of 1/2 approximately takes into account recombinations to higher levels that cascade to lower levels bypassing the n = 3 level. The second to last term in the equation represents the fraction of effective recombinations to doublet terms of principal quantum number 3 that go directly to 3p ^2D ($\omega_{3p^2D}/\omega_{total}$) and those recombinations that go to 3d $^2F°$ and cascade to 3p ^2D ($\omega_{3d^2F}/\omega_{total}$).

Table 1 lists the values of α_i^{eff} calculated in the course of this investigation. The values were determined for one of two electron temperatures: 7500°K for the Orion Nebula and 15000°K for the representative planetary nebulae. The accuracy of this technique of calculation is indicated by a comparison between values of α^{eff} calculated for some lines of OIII, OIV and OV by Burgess and Seaton (1960) and values calculated by the above method. The two techniques give values which agree to within a factor of 10.

Resonance Fluorescence

I now consider excitation of a permitted line by resonance fluorescence. The rate of population of a level i due to photoexcitation from the ground state by a resonance transition is $J_{\nu_i} B_i$ where J_{ν_i} is the mean intensity of radiation at the appropriate frequency and B_i is the Einstein B transition probability for the resonance line. In general, the population of any level due to resonance fluorescence is a sum of such terms (representing photoexcitations to higher levels), multiplied by branching ratios to the level in question. I will define $(JB)^{eff}$ to be analogous to α^{eff} so that $N_I(JB)_i^{eff}$ will be the rate of population of the i^{th} term of the I^{th} ion due to reconance fluorescence. In the analyses that follow, however, I will consider excitation of only a few important levels and assume that contributions from other levels will be negligible due to small branching ratios. Thus, the volume emission coefficient due to resonance fluorescence becomes:

$$(4) \qquad j_{line}^{fluor} = N_I \ (JB)_{line}^{eff} \ \left(\frac{A\ line}{\sum A}\right) \ \frac{\nu line}{4\pi}$$

Table 1. Computed recombination coefficients.

Ion	Term	Te (°K)	α^{eff} ($cm^3 sec^{-1}$)
CII	3d ^2D	7500	8.2×10^{-13}
CII	4s ^2S	7500	3.0×10^{-13}
CII	4f $^2F°$	7500	5.1×10^{-13}
CIII	3p $^3P°$	15000	3.0×10^{-12}
CIII	3d ^1D	15000	6.3×10^{-13}
CIII	5s ^3S	15000	3.3×10^{-13}
CIII	5g ^1G	15000	2.4×10^{-13}
CIII	5g ^3G	15000	7.5×10^{-13}
CIII	6h $^{3,1}H°$	15000	6.7×10^{-13}
CIV	3p $^2P°$	15000	5.6×10^{-12}
CIV	6h $^2H°$	15000	2.5×10^{-12}
CIV	7i ^2I	15000	1.9×10^{-12}
NI	3p $^4S°$	7500	9.8×10^{-15}
NI	3p $^4P°$	7500	2.1×10^{-14}
NI	3p $^4D°$	7500	3.0×10^{-14}
NII	3p ^3P	7500	3.4×10^{-13}
NII	3p ^3D	7500	5.5×10^{-13}
NII	3d $^1D°$	7500	7.4×10^{-14}
NII	3d $^3D°$	7500	2.2×10^{-13}
NII	4s $^3P°$	7500	2.2×10^{-13}

Table 1. Computed recombination coefficients--<u>continued</u>.

Ion	Term		Te (°K)	$\alpha^{eff}(cm^3 sec^{-1})$
NIII	3p	$^2P°$	7500	6.8×10^{-12}
NIII	3d	^2D	7500	4.4×10^{-12}
NIII	5f	$^2F°$	15000	8.0×10^{-13}
NIII	5g	^2G	15000	1.0×10^{-12}
NIV	3p	$^3P°$	15000	3.6×10^{-12}
NV	3p	$^2P°$	15000	1.5×10^{-11}
OI	3p	^3P	7500	2.9×10^{-14}
OII	3p	$^2D°$	7500	9.6×10^{-14}
OII	3p'	$^2D°$	7500	8.6×10^{-14}
OII	3p'	$^2F°$	7500	1.1×10^{-13}
OII	3p	$^4P°$	7500	3.6×10^{-13}
OII	3p	$^4D°$	7500	5.5×10^{-13}
OII	3d	^2F	7500	5.0×10^{-14}
OII	3d'	^2F	7500	5.0×10^{-14}
OII	3d'	^2G	7500	6.2×10^{-14}
OII	3d	^4P	7500	1.4×10^{-13}
OII	3d	^4D	7500	2.4×10^{-13}
OII	3d	^4F	7500	3.1×10^{-13}
OII	4f'	$^2H°$	7500	3.4×10^{-14}
OII	4f	$^4D°$	7500	9.8×10^{-14}
OII	4f	$^4G°$	7500	1.8×10^{-13}

Table 1. Computed recombination coefficients--continued.

Ion	Term	Te (°K)	$\alpha^{eff}(cm^3 sec^{-1})$
OIII	3d $^1F°$	15000	3.0×10^{-13}
OIII	3d $^3D°$	15000	6.6×10^{-13}
OIII	3d $^3F°$	15000	9.0×10^{-13}
OIV	3d ^2D	15000	7.9×10^{-12}
OV	3p $^1P°$	15000	3.7×10^{-12}
NeI	3p [5/2]	7500	3.6×10^{-14}
NeII	3d ^2D	7500	3.3×10^{-14}
NeII	3p $^4D°$	7500	4.4×10^{-13}
NeII	3p $^4P°$	7500	4.0×10^{-13}
SiII	4p $^2P°$	7500	6.2×10^{-13}
SiII	4d ^2D	7500	3.7×10^{-13}
SiII	5s ^2S	7500	1.5×10^{-13}
SiIII	4p $^1P°$	7500	6.6×10^{-13}
SiIII	4p $^3P°$	15000	5.2×10^{-13}
SiIV	4p $^2P°$	15000	4.2×10^{-12}
SiIV	6h ^2H	7500	3.3×10^{-12}
SII	4p $^4D°$	7500	3.8×10^{-13}

Table 1. Computed recombination coefficients--<u>continued</u>.

Ion	Term	Te (°K)	$\alpha^{eff}(cm^3 sec^{-1})$
SIII	$4p\ ^3S$	7500	7.3×10^{-13}
SIII	$4p\ ^3P$	7500	1.2×10^{-12}
SIII	$4p\ ^3D$	7500	1.7×10^{-12}

There are two sources of radiation that can cause resonance fluorescence: starlight from the central star or photons in another nebular emission line. First, I will consider starlight excitation. Instead of solving explicitly the equation of transfer for the resonance line, I have made the simplifying assumption that scattering may be ignored -- that once a degradable resonance line photon is absorbed, it will be degraded into cascade line photons in the same part of the nebula ("on-the-spot") where the initial resonance line photon was absorbed. Thus, the mean intensity of starlight at a radius r is:

$$(5) \qquad J\,(r) = H_{surface}\, R_*^2/r^2\, e^{-\tau}$$

where R_* is the radius of the central star, $H_{surface}$ is the surface Eddington flux, and τ is determined by the line and continuum opacities.

Besides continuous sources of opacity such as the ionization of H and He, radiation at the resonance line frequency is absorbed by the resonance line itself. The method I have adopted to take this into account is to divide the total line absorption coefficient $\frac{\pi e^2}{mc} f_{abs}$ by a frequency interval corresponding to ten Doppler widths and to treat the resulting cross-section at this frequency in the same way as the cross-sections due to H or He ionization. If nebulae were totally static, then approximately 2 Doppler widths would be the appropriate value for the extent of the absorption line, but observed emission lines are much wider (Münch and Taylor, 1974, give line widths of 40km/sec for OI λ8446 and [OI] λ6300 in the Orion Nebula, which, for λ6300, corresponds to 14 Doppler widths). If it is assumed that this line broadening is caused by microturbulence (as seems likely, at least, for the Orion Nebula from Munch, 1958, and Osterbrock, 1974), the broad lines imply that the absorption coefficient should be broader than 2 Doppler widths (the thermal width). Also, since the absorption coefficient applies outward from the central star to the point in question in the nebula, macroturbulent velocities are important in broadening the absorption coefficient.

Next, I consider resonance fluorescence by nebular emission lines. This process will be treated in the same way as the diffuse radiation from recombinations of H and He -- by use of the "on-the-spot" approximation. This approximation assumes that the mean intensity, due to an emission line, at any particular point in the nebula is representative only of the physical conditions at that point. In other words, the mean intensity due to a nebular emission line is (Osterbrock 1974):

$$J_\nu = \frac{j_\nu}{\Sigma a_i N_i} \qquad (6)$$

where j_ν is the continuum volume emission coefficient of the emission line and $\Sigma a_i N_i$ is the sum of the applicable absorption cross-sections. The value of j_ν is computed by dividing the total energy output in the nebular emission line by a frequency interval of 10 Doppler widths.

The third and final method of exciting a permitted line is collisional excitation by electrons. The electron temperature characteristic of HII regions and planetary nebulae is only sufficient to excite low lying states, which predominantly results in the emission of forbidden line photons. Since all the permitted lines presently observed in nebulae arise from levels far above the ground state, excitation by electron collision will not be further discussed in this dissertation.

The calculations of resonance fluorescence rates and of branching ratios require the knowledge of atomic transition probabilities. If possible, these probabilities were taken from the N.B.S. compilations (Wiese, Smith and Glennon 1966; Wiese, Smith and Miles 1969). Kelly (1964) lists many transition probabilities for ions of N and O, and these have been used if necessary. In other cases, the Coulomb approximation, as described in Bates and Damgaard (1949), was utilized to estimate transition probabilities. In all cases, unless specific transition probabilities have been calculated, LS coupling selection rules have been assumed. For example, transitions such as NII $2s\ 2p^3\ {}^3S^\circ - 2s^2\ 2p\ 3p\ {}^3P$ have been ignored because they violate the rule that only one electron may change state.

Model Nebulae

The goal of my calculations, based on the above premises, was line strengths for given permitted lines that could be compared with observations. The flux in a certain optically thin line (always expressed as a ratio to the flux in $H\beta$) was therefore calculated by integration through a model spherical nebula:

$$(7) \qquad F_{line}/F_{H\beta} = \int_0^R j_{line}\,(r)\ r^2\,dr \Big/ \int_0^R j_{H\beta}\,(r)\ r^2\,dr$$

The equations and techniques involved in the necessary task of constructing a model gaseous nebula are rather tedious and are well covered elsewhere (Osterbrock, 1974). Nevertheless, a discussion of the techniques that were actually used in the models built for this investigation is in order.

The basic ionization equations for H and He (in the "on-the-spot" approximation) are:

(8)
$$N_{HI} \left(\int_{\nu_1}^{\infty} \frac{4\pi J_\nu^*}{h\nu} a_{HI}(\nu)\, d\nu + N_e C_{HI} \right) + N_{HeII} N_e \left(y\, \alpha^{m=1}_{HeI} + p\, \alpha^{B}_{HeI} \right) + N_{HeIII} N_e \left(z\, \alpha^{B}_{HeII} + \alpha^{m=2}_{HeII} \right) = N_{HII} N_e \alpha^{B}_{HI}$$

$$N_{HeI} \left(\int_{\nu_2}^{\infty} \frac{4\pi J_\nu^*}{h\nu} a_{HeI}(\nu)\, d\nu + N_e C_{HeI} \right) + (1-y) N_{HeII} N_e \alpha^{m=1}_{HeI} = N_{HeII} N_e \alpha^{A}_{HeI}$$

$$N_{HeII} \left(\int_{\nu_3}^{\infty} \frac{4\pi J_\nu^*}{h\nu} a_{HeII}(\nu)\, d\nu + N_e C_{HeII} \right) = N_{HeIII} N_e \alpha^{B}_{HeII}$$

where J_ν^* is the mean intensity due to starlight at the frequency ν, N represents a density, ν_1, ν_2 and ν_3 are the ionization thresholds of HI, HeI and HeII respectively, $a(\nu)$ is the photoionization cross section, C is the collisional ionization coefficient, $\alpha^{m=1}$, α^{A}, $\alpha^{m=2}$ and α^{B} are the recombination coefficients to the first level only, to all levels, to the second level only and to all levels but the first respectively, y is the fraction of recombinations to the ground level of HeI that result in the production of a photon that ionizes a H atom, p is the fraction of recombinations to excited levels of HeI that result in the production of a photon that ionizes a H atom, and z is the fraction of recombinations to excited levels of HeII that result in a hydrogen ionizing photon. These three equations can be solved iteratively.

The determination of the ionization of the heavy elements is straightforward once the H and He ionization fractions and the electron density are found from equation 8. Diffuse radiation from the HI and HeI Lyman continuum, the HeII Balmer continuum, and HeI and HeII Lyman $-\alpha$ and 2 photon emission has been included (according to the "on-the-spot" approximation) in the photoionization flux integral for the heavy elements. Oxygen-hydrogen and nitrogen-hydrogen charge exchange have also been included in the calculations. Once the heavy element ionization structure is known, the electron cooling rates (due to lines excited by electron collision,recombination of H and He and free-free radiation from H and He) and heating rates (due to photoionization) can be determined.

The actual construction of the model proceeds in a series of shells (usually about 100) starting from the central star and working outward. The stellar radiation at each shell is determined from the absorption in the previous shells, and the electron temperature of the previous shell is used as the initial estimate for the next shell. Within each shell, the H and He ionization structure is solved, the heavy element ionization structure is determined and finally the electron heating and cooling rates are found. If the two rates are not close to being equal, the temperature is changed and the process is repeated until the rates are roughly the same. With my program, an entire model calculation took approximately 75 seconds of CDC6400 computer time.

Two particular nebular models were constructed: one which represents the Orion Nebula and one which represents a planetary nebula. The model for the Orion Nebula consists of a uniform spherical nebula with a density of 4000 H atoms/cm^3 surrounding an O7V star which is modeled by a 37,500 °K, log g = 4.0, non-L.T.E. stellar atmosphere (Mihalas, 1972). The central star is assumed to have a normal O star absorption line spectrum. The planetary nebula model -- roughly corresponding to NGC7662 -- consists of a uniform gas of 2000 H atoms/cm^3 between the radii of 0.029 and 0.072 pc surrounding a central star of 0.24 $R_\odot$ which is modeled by a 10^5 °K black body (Harrington, 1969). The chemical abundances of these two models are listed in Table 2. The abundances for the Orion Nebula model are the solar values from Withbroe (1971) while the abundances used in the planetary nebula model were taken from Harrington (1969).

In Table 3, a comparison of the observed and predicted line strengths for the Orion Nebula is made. The observational data are the fully corrected values from the list of Johnson (1968). In Table 4, a comparison is made between the planetary nebula model predictions and the observed line strengths taken from the compilation of Harrington

Table 2. Model chemical abundances.

	Orion	Planetary Nebula
He/H	$1. \times 10^{-1}$	1.6×10^{-1}
N/H	$2. \times 10^{-4}$	1.5×10^{-4}
Ne/H	$1. \times 10^{-4}$	5.5×10^{-5}
O/H	$6. \times 10^{-4}$	4.5×10^{-4}
C/H	$4. \times 10^{-4}$	$2. \times 10^{-4}$
S/H	1.6×10^{-5}	1.6×10^{-5}
Si/H	$4. \times 10^{-5}$	$4. \times 10^{-5}$

Table 3. Observed and predicted line strengths in the Orion Nebula.

Line		Observed Strength ($H\beta = 1$)	Computed Strength ($H\beta = 1$)
$H\alpha$		2.31	2.8
HeI	$\lambda 4471$	0.051	0.048
HeI	$\lambda 5876$	0.25	0.13
[NI]	$\lambda 5199$	0.007	0.026
[NII]	$\lambda 6584$	0.36	0.50
[OI]	$\lambda 6300$	0.046	0.024
[OII]	$\lambda 3727$	1.83	0.52
[OIII]	$\lambda 5007$	3.32	8.4
[NeIII]	$\lambda 3869$	0.27	0.33
[SII]	$\lambda 6730$	0.05	0.026
[SIII]	$\lambda 9532$	0.67	0.22

Table 4. Observed and predicted line strengths in NGC7662.

Line		Observed Strength ($H\beta = 1$)	Computed Strength ($H\beta = 1$)
HeI	λ4471	0.029	0.051
HeI	λ5876	0.068	0.14
HeII	λ4686	0.63	0.55
[NII]	λ6584	0.061	0.031
[OII]	λ3727	0.10	0.054
[OIII]	λ4363	0.17	0.075
[OIII]	λ5007	13.	9.0
[NeIII]	λ3869	0.80	0.34

(1969) for NGC7662. No pretense of detailed accuracy can be made for these two models. A better model for the Orion Nebula would include a density dropoff (Simpson, 1973) while density inhomogenities would improve the NGC7662 model (Harrington, 1969). Nevertheless, the computed models should be adequate to delineate the relative effects of the processes under discussion in this dissertation.

CHAPTER 3

OI PERMITTED LINES

An interesting result of the first investigations of the far-red spectrum of the Orion Nebula was the unexpected detection of the OI λ8446 3s $^3S°$-3p ^3P line. It is a fairly strong line and it shows large variations in strength over the face of the nebula (Danziger and Aaronson, 1974; Münch and Taylor, 1974). It was soon realized that the conventional explanation of recombination could not account for the strength of the line. λ8446 is not the only permitted OI line observed in the Orion Nebula: the work of Morgan (1971) and my own observations (see Appendix A) show several others.

The observed OI triplet transitions are shown on an energy level diagram in Figure 1 and the observed line strengths are listed in Table 5. All but the first two strengths come from my observations (see Appendix A). The components of all the transitions listed are so close together that they are unresolved at the dispersions utilized. The line at λ5958.5 appears as a blend with SiII λ5957.6, but a strength may be determined for this line by utilizing the theoretical LS-coupling multiplet ratios in the SiII 4p $^2P°$-5s ^2S multiplet and the observed strength of SiII λ5979.0 to subtract the λ5957.6 contribution from the blend.

The question of the excitation mechanism of λ8446 and the other OI lines seen in the Orion Nebula has been considered by several

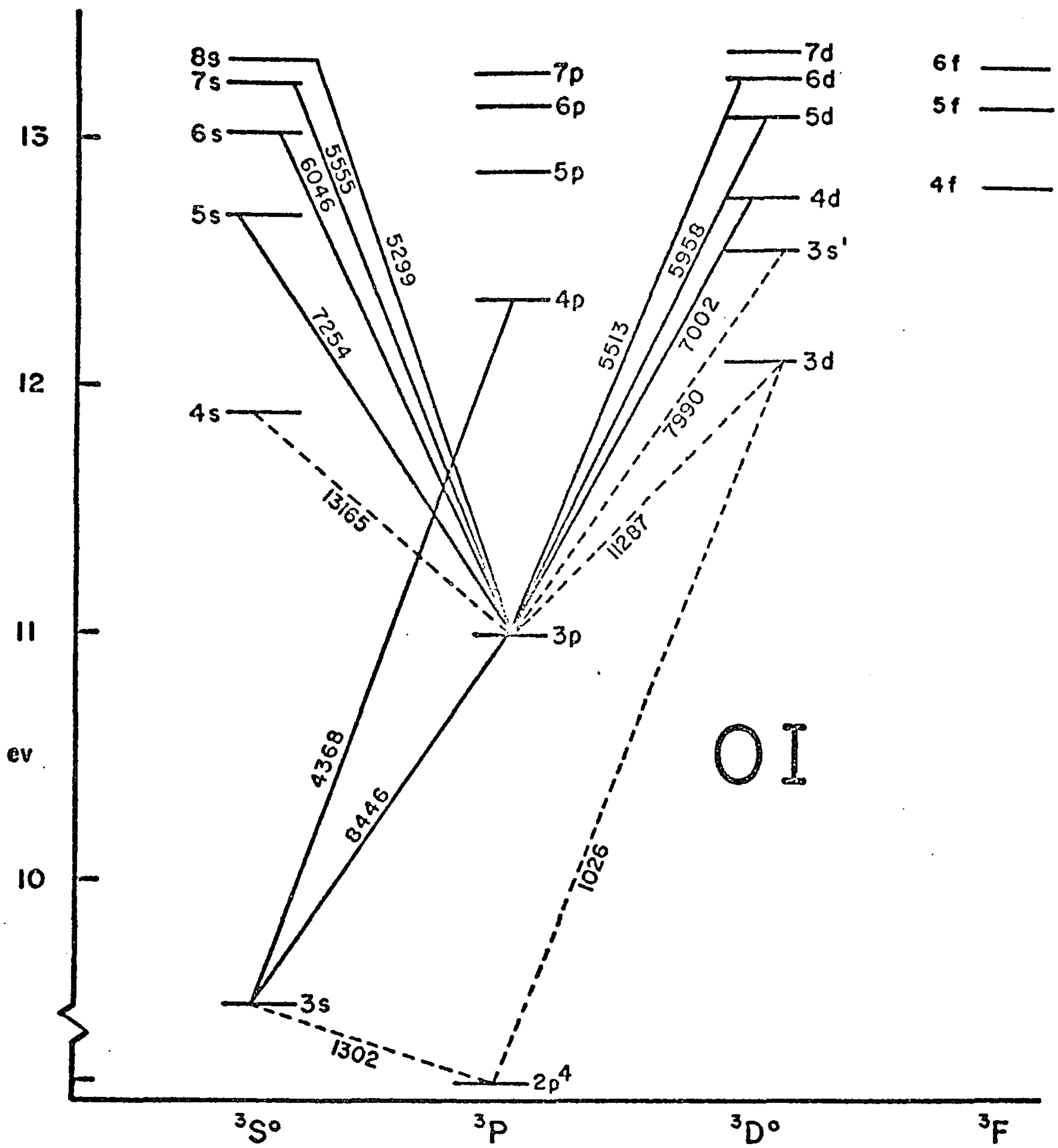


Figure 1. Triplet energy level diagram of OI showing important transitions.

Table 5. Observed strengths of OI lines.

Line (Å)	Transition	Observed Strength
4368.3	$3s\,^3S°-4p\,^3P$	0.001*
5299.0	$3p\,^3P-8s\,^3S°$	0.0006**
5513.7	$3p\,^3P-6d\,^3D°$	0.0009
5554.9	$3p\,^3P-7s\,^3S°$	0.0009
5958.5	$3p\,^3P-5d\,^3D°$	0.002
6046.4	$3p\,^3P-6s\,^3S°$	0.002
7002.2	$3p\,^3P-4d\,^3D°$	0.002
7254.4	$3p\,^3P-5s\,^3S°$	0.005
8446.4	$3s\,^3S°-3p\,^3P$	0.012

*From Kaler and Aller 1965.
**From Aller and Walker 1970.

authors beginning with Andrillat and Houziaux (1968a,-b). They compared the strength of λ8446 to the combined strength of the [OII] doublet at λ7319 and λ7330 and were thus able to exclude the possibility of recombination as the mechanism producing OI λ8446. Assuming an electron temperature of 9000 °K, an electron density of 4000 cm^{-3}, and a recombination coefficient derived under the assumption that all recombinations, except those to the ground state, pass through the 3p ^3P term, they derived a line ratio about 4 orders of magnitude below the observed value. A more qualitative reason for excluding recombination as an excitation mechanism for λ8446 in the Orion Nebula is the lack of quintet lines in the spectrum. If recombination was the source of excitation, we would expect that the quintet counterpart of λ8446, λ7774 (3s $^5S°$-3p ^5P), would be roughly 5/3 as strong as λ8446.

An alternative to recombination as an excitation mechanism is fluorescence of the 3d $^3D°$ term of OI by Lyman-β, which can cascade to 3p ^3P and generate the λ8446 lines. This mechanism has been discussed by Andrillat and Houziaux (1968a,-b), Morgan (1971), Münch and Taylor (1974) and Shields (1974). Bowen (1947) first pointed out the coincidence between Lyman-β at λ1025.72 and the strongest component of the OI resonance line $2p^4$ ^3P-3d $^3D°$ at λ1025.77. Swings (1955) discussed this mechanism in reference to aurorae and emission line stars. However, as Morgan (1971) pointed out (although disputed by Münch and Taylor, 1974), it appears that in the nebular environment where OI is found, not enough Lyman-β photons will escape degradation to appreciably excite $\lambda 8446$. Morgan went on to suggest that perhaps resonance fluorescence by starlight could account for the observed OI lines.

With this historical perspective, I will now rediscuss the three proposed excitation mechanisms for the OI lines in the Orion Nebula from the point of view developed in the last chapter. First, recombination can be easily discounted as a source of excitation for the observed OI lines by noting that the predicted ratio of $\lambda 8446$ to $H\beta$ due to recombination is 2.4×10^{-5} in the Orion Nebula model.

Excitation of the permitted OI lines by Lyman line fluorescence is an interesting problem, and it deserves a detailed analysis. First, ignoring the other OI lines, let us predict the line strength of $\lambda 8446$ under the assumption that it is excited only by Lyman β fluorescence. Using equation 6 (the "on-the-spot" approximation), the mean intensity of radiation due to Lyman β is:

$$(9) \qquad J_{Ly\beta} = \frac{j_{Ly\beta}}{a_{Ly\beta} N_{HI} + \underset{\lambda 1026}{a_{OI}} N_{OI}}$$

with

$$(10) \qquad j_{Ly\beta} = \frac{N_e N_{HII} \alpha^{eff}_{\substack{n=3 \\ H}} \dfrac{A_{Ly\beta}}{A_{Ly\beta} + A_{H\alpha}} h\nu_{Ly\beta} \dfrac{1}{4\pi}}{10 \, \Delta\nu_{\substack{Doppler \\ Ly\beta}}}$$

Using these formulas in the Orion Nebula model we obtain a predicted line ratio, $\lambda 8446/H\beta$, of 3.0×10^{-5}.

More physical insight can be obtained by ignoring the opacity due to OI in equation 9 and expressing the result, when equation 10 is inserted, as:

$$(11) \qquad J_{Ly\beta} \propto \frac{N_e N_{HII}}{N_{HI}}$$

Thus, the mean intensity of Lymanβ radiation will be large in the highly ionized regions of the nebula, but it will become much smaller where the nebula is less ionized.

Next, we note that due to change exchange (Williams, 1973), the number density ratios $N_{OII}{}^*/N_{OI}$ and N_{HII}/N_{HI} are approximately equal throughout the nebula (OII* here refers to all the ionized forms of oxygen). Equation 11 therefore becomes:

$$(12) \qquad J_{Ly\beta} \propto \frac{N_e N_{OII}{}^*}{N_{OI}}$$

The mean intensity of Lymanβ thus varies <u>inversely</u> with the OI fraction in the nebula. In other words, in the locations in the nebula where there is OI, there is little Lymanβ intensity, and vice-versa.

It turns out that over the whole (model) nebula, about 100 times more Lymanβ photons are generated per second that stellar photons capable of exciting the OI lines. However, the vast majority of these Lymanβ photons exist in the inner parts of the nebula where $(\frac{N_{HII}}{N_{HI}})\ N_e$ is large. Since these Lymanβ photons cannot travel into the outer regions of the nebula without being degraded to Hα by scatterings with neutral hydrogen, the Lymanβ density in the regions where OI exists will reflect the low value of $(\frac{N_{HII}}{N_{HI}}) N_e$ present there. Consequently, since the Lyman β photons are confined to the interior while the OI is confined to the exterior, Lyman β fluorescence is not important.

Some proposals have been made to circumvent this difficulty. Münch and Taylor (1974) propose that inside dense condensations straddling the transition region of the nebula, Lyman β photons and OI ions will coexist in sufficient quantity to generate the observed λ8446 flux. Diffusion of Lyman β photons in frequency space (Adams, 1975; Bahcall, 1966) might permit a large number of Lyman β photons to escape in the line wings into the neutral region of the nebula where they could conceivably excite some OI atoms. Shields (1974) has proposed that dense filaments, in the process of recombination and condensation, could contain a mixture of neutral and ionized hydrogen. Thus, there could be a large optical depth in Hα which would result in a large Lyman β flux due to Hα conversion. While the lack of an overly steep Balmer decrement in the Orion Nebula seems to rule out the last suggestion, all of these proposals fail to explain the presence of the other OI lines besides λ8446.

Morgan (1971) points out that in addition to the coincidence between OI $2p^4\ {}^3P-3d\ {}^3D°$ and Lyman β, there is a coincidence between Lyman ϵ at λ937.80 and a component of the OI resonance multiplet to 7s ${}^3S°$ at λ937.84. Also, Lyman ζ at λ930.75 corresponds to a component of the resonance multiplet to 7d ${}^3D$ at λ930.90. However, cascades from these terms cannot explain the strengths of all the OI lines. For example, the resonance lines to 5s $^3S°$, which excite λ7254, are located at λλ976.45, 977.96, 978.62 which are far from the Lyman γ line at λ972.54. In another case, let us assume that a cascade from 7s $^3S°$ through 6p ^3P to 6s $^3S°$ accounts for the strength of the λ6046 line. We find that the observed λ6046/Hβ line ratio of 0.002 implies a λ5555/Hβ line ratio of 0.32 compared with an observed ratio of 0.0009.

In Table 6, the OI/OI λ8446 line ratios predicted by a calculation of Lyman line fluorescence are listed. The calculation included fluorescence of the 3d ^3D, 7s ^3S and 7d ^3D levels of OI by Lyman β, Lyman ϵ , and Lyman ζ respectively. These line ratios are expected to be essentially model independent, since they depend only on the relative strengths of these Lyman lines inside the nebula, which are given by the solution to the hydrogen capture-cascade equations. As can be seen, the observed line ratios disagree widely with the observed values.

In contrast with Lyman line fluorescence, fluorescence by starlight can adequately account for not only λ8446 but the other observed OI lines in the Orion Nebula as well. In Table 7, I list the line strengths (as ratios to Hβ) of the OI lines as predicted from the resonance fluorescence by starlight mechanism and compare them with the observed strengths. The agreement is quite reasonable. The calculation included fluorescence of the 4s $^3S°$, 5s $^3S°$, 6s $^3S°$, 7s $^3S°$, 8s $^3S°$, 4d ^3D, 5d ^3D and 6d ^3D terms of OI.

Excitation of the 3d ^3D term was not considered because all the photons capable of exciting this resonance line will have been absorbed

Table 6. Line ratios of OI lines.

Line Ratio	Lyman Fluorescence	Starlight Fluorescence	Observed
4368/8446	2.1×10^{-3}	2×10^{-1}	8×10^{-2}
7254/8446	3.4×10^{-4}	1×10^{-1}	4×10^{-1}
6046/8446	1.7×10^{-4}	6×10^{-2}	2×10^{-1}
5555/8446	2.3×10^{-2}	2×10^{-2}	8×10^{-2}
7002/8446	2.4×10^{-5}	4×10^{-1}	2×10^{-1}
5958/8446	2.0×10^{-6}	6×10^{-2}	2×10^{-1}
5513/8446	1.4×10^{-8}	3×10^{-3}	8×10^{-2}

Table 7. Starlight excitation of OI lines.

Line	Transition	Predicted Strength	Observed Strength
λ1302	$2p^4\ ^3P - 3s\ ^3S°$	0.10*	- -
3692**	$3s\ ^3S° - 3p\ ^3P$	0.004	- -
4368	$3s\ ^3S° - 4p\ ^3P$	0.003	0.001
5299	$3p\ ^3P - 8s\ ^3S°$	0.0001	0.0006
5514	$3p\ ^3P - 6d\ ^3D°$	0.00005	0.0009
5555	$3p\ ^3P - 7s\ ^3S°$	0.0003	0.0009
5958	$3p\ ^3P - 5d\ ^3D°$	0.0003	0.002
6046	$3p\ ^3P - 6s\ ^3S°$	0.001	0.002
7002	$3p\ ^3P - 4d\ ^3D°$	0.006	0.002
7254	$3p\ ^3P - 5s\ ^3S°$	0.002	0.005
8446	$3s\ ^3S° - 3p\ ^3P$	0.016	0.012
11287	$3p\ ^3P - 3d\ ^3D°$	0.0003	- -
13165	$3p\ ^3P - 4s\ ^3S°$	0.005	- -

*Does not include possible contribution from collisional excitation by electrons.

**Blended with H16.

by neutral H as described above. Therefore, λ11287 (3p ^{3}P-3d ^{3}D°) should be much weaker than λ13150 (3p ^{3}P-4s ^{3}S°) which can be excited by starlight fluorescence. (The predicted λ13150 line strength listed in Table 7 corresponds to a λ13150/Paschen β flux ratio of 0.03, based on the recombination line calculations of Brocklehurst, 1971.) If Lyman β fluorescence were the dominant excitation mechanism of λ8446, λ13150 should be weaker than λ11287. The predicted λ11287 strength shown in Table 7 was derived from cascades via the 4p ^{3}P and 5p ^{3}P terms following starlight excitation of the 4d ^{3}D°, 5d ^{3}D°, 5s ^{3}S° and 6s ^{3}S terms.

The resonance line to the 3s ^{3}D° term of OI (with components λλ988.78, 990.21, 990.80, 988.66, 990.13 and 988.58) corresponds to the lowest lying NIII resonance line at λλ991.58, 989.79 and 991.51 (2s^2 2p ^{2}P°-2s 2p^2 ^{2}D) which should be quite strong in absorption in O star atmospheres (Smith, 1970). Therefore, OI λ7990 (3p ^{3}P-3s ^{3}D°) should be much weaker than lines such as λ7002. In fact, the λ7990 multiplet has not been observed. (The predicted strengths for the multiplet are 5.6×10^{-5}, 4×10^{-5} and 2.4×10^{-5} for the λλ7995, 7987 and 7982 components respectively if 3s ^{3}D° is excited from cascades from 5p ^{3}P only.)

Table 6 contains the OI/OI λ8446 line ratios predicted by direct starlight excitation. These ratios should be almost entirely model independent, since the ratios depend mostly on the spectrum of the exciting star. The predicted ratios agree reasonably well with the observed values; this further confirms the conclusion that the OI lines seen in the spectrum of the Orion Nebula are excited by resonance fluorescence by starlight.

One might expect that if starlight excitation is dominant in exciting the permitted lines of OI, the same mechanism should be just as dominant in exciting the lines of HI since the ionization potential and nebular ionization structures for these two ions are so similar. So why are the strong HI lines always assumed to be excited by recombination rather than by fluorescence? For one thing, the Lyman lines will be in absorption in the stellar atmosphere, thus cutting down the available flux. Also, while OII is not present in the nebular model except close to the transition region, HII is present throughout the nebula. Therefore, recombination is relatively unimportant for the excitation of the OI lines, but the opposite is true for HI. Other investigators have discussed the effect of starlight resonance fluorescence on the hydrogen recombination spectrum (the "Case-C" situation) and concluded that it has little effect (Van Blerkom, 1969; Seaton, 1969).

Danziger and Aaronson (1974) remark on the large spatial variation of $\lambda 8446$ in the Orion Nebula, and they note that it increases in strength dramatically on the bright arc near θ^2 Ori. However, the spatial variations are different from those of the forbidden lines or recombination lines. One might expect such variations in starlight excited lines in the actual Orion Nebula near another source of starlight excitation, and since the mechanisms involved are distinct, the mode of spatial variation should be different from that seen in recombination or collisionally excited lines.

Münch and Taylor (1974) present two very interesting image tube plates of the Orion Nebula, one taken in the light of [OI] $\lambda 6300$ and the other in the light of OI $\lambda 8446$. The $\lambda 8446$ image appears as a collection of filaments superimposed on a diffuse background. The $\lambda 6300$ image shows no such filaments. I believe that these $\lambda 8446$ filaments can be easily understood in terms of direct starlight excitation of this line. Consider a condensation immersed in the lower-density nebular gas. As we go from the outside of this condensation toward the center, the ionization will drop until all the Lyman continuum radiation has been absorbed and all the hydrogen and oxygen atoms are neutral. However, this inner part of the condensation will still receive radiation longer than the Lyman limit that will excite $\lambda 8446$. On the other hand, [OI] $\lambda 6300$ requires not only the presence of OI but also the presence of electrons to collisionally excite the forbidden line. Therefore, the radiation of $\lambda 8446$ will peak in the center of a condensation while the $\lambda 6300$ radiation will be confined to the edge of such a condensation and will be weaker because there is less OI in the transition region of the condensation than in the center. Consequently, the $\lambda 8446$ image should show clearly the high-density centers of the condensations while the $\lambda 6300$ image should not.

It is interesting to note that a plate taken in [OI] $\lambda 6300$ of the planetary nebula NGC6853 (Capriotti, Cromwell and Williams, 1971) shows quite a filamentary structure. On the basis of the present discussion, I would predict that even more filaments would be seen in a plate taken in OI λ8446.

Lines of OI also appear in the spectra of planetary nebula. OI λ8446 has been reported in NGC7027 (Kolotilov and Noskova, 1974) and in IC418, NGC6572 and IC4997 (Andrillat and Andrillat, 1961; Andrillat and Houziaux, 1968a; Andrillat, Baronne and Houziaux, 1975). λ8446 was not by Andrillat and his co-workers in 13 other planetaries. Recent far-red spectra (see Appendix A) show the presence of a weak λ8446 line in NGC6210 but not in IC3568, NGC2392 or NGC3242. A fair summary of this set of observational data is that when [OI] λ6300 is observed at any significant strength, OI λ8446 is also seen. When the relative sensitivities of the spectrographic systems at the wavelengths of λ6300 and λ8446 are considered, it seems likely that the presence or lack of OI λ8446 in the spectrum of a planetary nebula reflects the abundance of OI rather than some difference in the efficiency of the excitation mechanism.

That the excitation mechanism for OI λ8446 in these planetaries is predominantly starlight excitation rather than Lyman-line fluorescence is shown by the presence of OI λ4368 ($3s\ ^3S°$-$4p\ ^3P$) in NGC7027 (Aller, Bowen and Wilson, 1963), NGC6572 and IC4997 (Aller and Kaler, 1964a) and IC418 (Aller and Kaler, 1964b). In several cases, the λ4368 line is identified as CII $3d'\ ^4P^o_{5/2} - 4f'\ ^4D_{3/2}$, an unlikely possibility since theoretically stronger members of this multiplet do not appear. Also, a recently obtained far-red spectrum of IC4997, which has a very strong OI λ8446 line (Andrillat et al., 1975), shows the presence of OI λ7254 ($3p\ ^3P$-$5s\ ^3S°$). In contrast to this, in NGC3242, which shows no λ8446, and in NGC6210, where λ8446 is weak, λ4368 is not found.

In addition to HII regions and planetary nebulae, it is conceivable that OI lines could be excited by resonance fluorescence in reflection nebulae. There should be sufficient OI in such nebulae, and the radiation from the central star should be rich in the energy range necessary for resonance fluorescence of this atom. One possible problem is that the dust which is a major component of reflection nebulae could absorb enough UV radiation to prevent significant excitation. Observational efforts to detect OI λ8446 in reflection nebulae have so far been unsuccessful.

The permitted lines of OI are present in several other types of astronomical objects and they present some interesting problems. For example, the "nebulous star" LkHα-101 (the source of illumination for NGC1579) shows a very strong OI λ8446 line (Herbig, 1971), on the order of 1/8 the strength of Hα. The continuum fluorescence diagnostic lines λ7002.2 and λ7254.4 are present in LkHα-101 (although not identified as such in Herbig, 1971), but the λ7254/λ8446 line ratio is less than 0.02 in this object as compared to 0.4 in the Orion Nebula. Also λ7773, the quintet counterpart of λ8446, is seen in LkHα-101, at a strength of approximately 0.07 of λ8446. Therefore, in LkHα-101, there appears to be evidence for recombination exciting the λ7773 line, starlight fluorescence exciting the λ7254 and λ7002 lines (λ7254 is about five times stronger than what would be expected from recombination) and Lyman β fluorescence pumping the strong λ8446 line. Observations of the OI 1.1287μ and 1.3165μ lines in LkHα-101 would help to untangle the processes involved.

To summarize this chapter, the permitted lines of OI as seen in the Orion Nebula can be adequately explained by the mechanism of resonance fluorescence by starlight. Alternative explanations, recombination and Lyman line fluorescence, can be ruled out. Continuum fluorescence also seems to be the dominant excitation mechanism of the OI lines in planetary nebulae, but the situation in other types of objects (exemplified by LkHα-101) is more complicated.

CHAPTER 4

OTHER HEAVY ELEMENT PERMITTED LINES IN THE ORION NEBULA

In this chapter, the permitted line spectrum of the Orion Nebula will be investigated in some detail in order to determine the principal excitation mechanism for each observed line. Table 8 lists the permitted lines of the heavy elements as observed in the Orion Nebula (excepting OI). Only the theoretically strongest observed member of each multiplet is listed. The observed line strengths are from the compilation by Johnson (1968), (the line strengths are the "fully corrected" values) with additions from the author (Appendix A) and Aller and Walker (1970). The "predicted" and the "predicted recombination" line strengths and the "dominant mechanism" are calculated by the methods of Chapter 2 and will be discussed in more detail below.

If it is assumed that the recombination calculations of Brocklehurst (1971, 1972) for HI and HeI are correct, then the photographically determined strengths of the weak lines observed in the Orion Nebula are overestimated. Such an effect was noted by Miller (1971) in the analyses of NGC7027 and NGC7662. To correct this systematic error, all the line strengths obtained from the Johnson (1968) list have been divided by a factor that ranges from 1.5 for lines of strength 1×10^{-4} ($H\beta = 1$) to 1 for lines of strength 1×10^{-2} and stronger.

CII

Following the order of Table 8, I will first discuss CII. In order to illuminate the briefer discussions to follow, the analysis of CII will be described in some detail. The energy levels of CII are shown in Figure 2, along with the observed multiplets. Resonance fluorescence by starlight can pump the ^2S and ^2D terms, so excitations of the 4s ^2S, 3d ^2D, 5d ^2D and 6d ^2D terms were considered. Thus, the volume emission coefficients for the three observed lines become:

$$j\ \text{line} = \left(N_{CIII}\, N_e\, \alpha^{eff}_{line} + N_{CII}\, (JB)^{eff}\right) \frac{A\ \text{line}}{\Sigma A}\ \frac{h\nu\ \text{line}}{4\pi} \tag{13}$$

The values of α^{eff} for λ4267 (4f $^2F°$), λ7236 (3d ^2D) and λ3920 (4s ^2S) are listed in Table 1. The values for $(JB)^{eff}$ for each line can be found from:

$$(JB)^{eff}_{\lambda 4267} = J_{\lambda 560}\, B_{\lambda 560}\, \frac{A\lambda 10504}{\Sigma A} + J_{\lambda 543}\, B_{\lambda 543}\, \frac{A\lambda 6662}{\Sigma A} \tag{14}$$

$$(JB)^{eff}_{\lambda 7236} = J_{\lambda 687}\, B_{\lambda 687}$$

$$(JB)^{eff}_{\lambda 3920} = J_{\lambda 636}\, B_{\lambda 636}$$

Where λ560, λ543, λ687 and λ636 refer to the resonance lines 2p $^2P°$-5d ^2D, 2p $^2P°$-6d ^2D, 2p $^2P°$-3d ^2D and 2p $^2P°$-4s ^2S respectively. A λ10504/ΣA and Aλ6622/ΣA are the branching ratios from 5d ^2D to 4f $^2F°$ and 6d ^2D to 4f $^2F°$. The values for the Einstein B probabilities are 3.3×10^8, 13×10^8, 30×10^8 and 1.1×10^8 for λ560, λ543, λ687 and λ636 respectively.

Table 8. Permitted line strengths in the Orion Nebula.

Ion	λ(Å)	Transition	Observed Strength	Predicted Strength	Dominant Mechanism	Predicted Recombination Strength
CII	3921	$3p\ ^2P^o_{3/2} - 4s\ ^2S_{1/2}$	2.6×10^{-3}	1.4×10^{-3}	starlight	4.0×10^{-4}
CII	4267	$3d\ ^2D - 4f\ ^2F^o$	3.3×10^{-3}	1.1×10^{-3}	recombination	9.7×10^{-4}
CII	7236	$3p\ ^3P^o_{3/2} - 3d\ ^2D_{5/2}$	5×10^{-3}	1.2×10^{-3}	starlight-recombination	5.5×10^{-4}
NI	7468	$3s\ ^4P_{5/2} - 3p\ ^4S^o_{3/2}$	2×10^{-3}	6.2×10^{-3}	starlight	8.9×10^{-7}
NI	8216	$3s\ ^4P_{5/2} - 3p\ ^4P^o_{5/2}$	3×10^{-3}	5.3×10^{-3}	starlight	1.2×10^{-6}
NII	3838	$3p\ ^3P^o_2 - 4s\ ^3P_2$	1.3×10^{-3}	9.7×10^{-4}	line	2.0×10^{-4}
NII	4447	$3p\ ^1P_1 - 3d\ ^1D^o_2$	- -	4.1×10^{-4}	recombination	4.1×10^{-4}
NII	4630	$3s\ ^3P^o_2 - 3p\ ^3P_2$	6×10^{-4}	1.6×10^{-3}	line/recombination	7.8×10^{-4}
NII	5680	$3s\ ^3P^o_2 - 3p\ ^3D_3$	2×10^{-3}	2.1×10^{-3}	line/recombination	1.1×10^{-3}
NII	5942	$3p\ ^3P^o_2 - 3p\ ^3D^o_3$	6×10^{-4}	5.6×10^{-4}	recombination/starlight	4.1×10^{-4}
NIII	4097	$3s\ ^2S_{1/2} - 3p\ ^2P^o_{3/2}$	5×10^{-4}	4.4×10^{-4}	starlight/Bowen	1.8×10^{-4}
NIII	4641	$3p\ ^2P^o_{3/2} - 3d\ ^2D_{5/2}$	6×10^{-4}	3.6×10^{-4}	starlight/Bowen	1.4×10^{-4}

Table 8. Permitted line strengths in the Orion Nebula--Continued.

Ion	λ(Å)	Transition	Observed Strength	Predicted Strength	Dominant Mechanism	Predicted Recombination Strength
OII	3882	3p $^4D^o_{7/2}$ - 3d $^4D_{7/2}$	- -	3.8×10^{-4}	recombination	3.8×10^{-4}
OII	4072	3p $^4D^o_{5/2}$ - 3d $^4F_{7/2}$	1.1×10^{-3}	1.4×10^{-3}	recombination	1.4×10^{-3}
OII	4089	3d $^4F_{9/2}$ - 4f $^4G^o_{11/2}$	3×10^{-4}	5.5×10^{-4}	recombination	5.5×10^{-4}
OII	4119	3p $^4P^o_{5/2}$ - 3d $^4D_{7/2}$	6×10^{-4}	1.0×10^{-3}	recombination	1.0×10^{-3}
OII	4153	3p $^4P^o_{3/2}$ - 3d $^4P_{5/2}$	7×10^{-3}	2.4×10^{-4}	recombination	2.0×10^{-4}
OII	4190	3p' $^2F^o_{7/2}$- 3d' $^2G_{9/2}$	3×10^{-4}	4.3×10^{-4}	recombination	4.3×10^{-4}
OII	4254	3d' $^2G_{9/2}$- 4f' $^2H^o_{11/2}$	6×10^{-4}	3.1×10^{-4}	recombination	3.1×10^{-4}
OII	4304	3d $^4P_{5/2}$ - 4f $^4D^o_{7/2}$	3×10^{-4}	5.5×10^{-4}	recombination	5.5×10^{-4}
OII	4349	3s $^4P_{5/2}$ - 3p $^4P^o_{5/2}$	6×10^{-4}	2.2×10^{-3}	recombination	2.2×10^{-3}
OII	4351	3s' $^2D_{5/2}$- 3p' $^2D^o_{5/2}$	1×10^{-4}	8.2×10^{-4}	recombination	8.2×10^{-4}
OII	4415	3s $^2P_{3/2}$ - 3p $^2D^o_{5/2}$	5×10^{-4}	9.1×10^{-4}	recombination	9.1×10^{-4}
OII	4591	3s' $^2D_{5/2}$- 3p' $^2F^o_{7/2}$	3×10^{-4}	1.0×10^{-3}	recombination	1.0×10^{-3}
OII	4649	3s $^4P_{5/2}$ - 3p $^4D^o_{7/2}$	2.0×10^{-3}	3.6×10^{-3}	recombination	3.6×10^{-3}
OII	4699	3p' $^2D^o_{5/2}$- 3d' $^2F_{7/2}$	3×10^{-4}	2.9×10^{-4}	recombination	2.9×10^{-4}
OII	4705	3p $^2D^o_{5/2}$ - 3d $^2F_{7/2}$	3×10^{-4}	4.3×10^{-4}	recombination	4.3×10^{-4}

Table 8. Permitted line strengths in the Orion Nebula--Continued.

Ion	λ(Å)	Transition	Observed Strength	Predicted Strength	Dominant Mechanism	Predicted Recombination Strength
NeII	3335	3s $^4P_{5/2}$ - 3p $^4D^o_{7/2}$	2×10^{-3}	5.7×10^{-4}	recombination	5.7×10^{-4}
NeII	3694	3s $^4P_{5/2}$ - 3p $^4P^o_{5/2}$	1.0×10^{-3}	5.7×10^{-4}	recombination	5.7×10^{-4}
NeII	3830	3p $^2P^o_{3/2}$ - 3d $^2D_{5/2}$	5×10^{-4}	1.0×10^{-4}	starlight	1.9×10^{-5}
SiII	3856	3p $^2D_{5/2}$ - 4p $^2P^o_{3/2}$	3.8×10^{-3}	3.1×10^{-3}	starlight	2.9×10^{-5}
SiII	5056	4p $^2P^o_{3/2}$ - 4d $^2D_{5/2}$	2×10^{-3}	4.6×10^{-3}	starlight	4.8×10^{-5}
SiII	5979	4p $^2P^o_{3/2}$ - 4s $^2S_{1/2}$	5×10^{-3}	3.8×10^{-3}	starlight	1.7×10^{-5}
SiII	6347	4s $^2S_{1/2}$ - 4p $^2P^o_{3/2}$	4×10^{-3}	5.3×10^{-3}	starlight	4.9×10^{-5}
SiIII	5740	4s $^1S_0$ - 4p $^1P^o_1$	4.0×10^{-4}	1.2×10^{-4}	starlight/recombination	5.6×10^{-5}
SIII	3325	3d $^3P^o_2$ - 4p $^3P_2$	2×10^{-3}	1.0×10^{-4}	starlight	3.0×10^{-5}
SIII	3709	3d $^3P^o_1$ - 4p $^3D_2$	- -	2.4×10^{-6}	recombination	1.4×10^{-6}
SIII	3718	4s $^3P^o_2$ - 4p $^3S_1$	2.1×10^{-3}	1.8×10^{-4}	starlight/recombination	1.1×10^{-4}
SIII	3838	4s $^3P^o_1$ - 4p $^3P_1$	9×10^{-4}	9.1×10^{-5}	starlight	2.7×10^{-5}
SIII	3929	3d $^3D^o_3$ - 4p $^3P_2$	5×10^{-4}	2.0×10^{-4}	starlight	6.3×10^{-5}
SIII	4285	4s $^3P^o_1$ - 4p $^3D_2$	3×10^{-4}	2.2×10^{-4}	recombination	1.5×10^{-4}

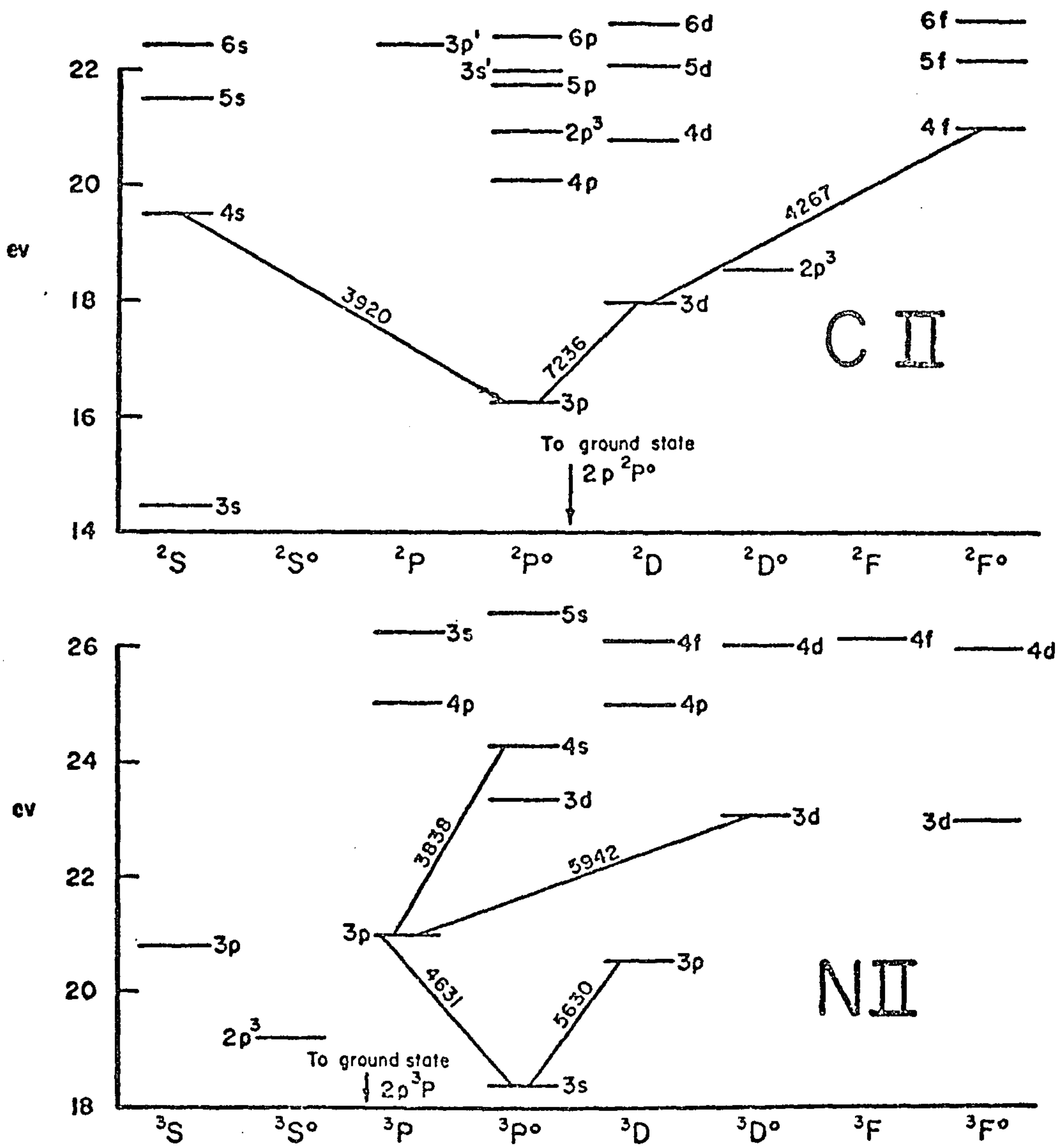


Figure 2. Energy level diagrams of CII and NII showing lines observed in the spectrum of the Orion Nebula.

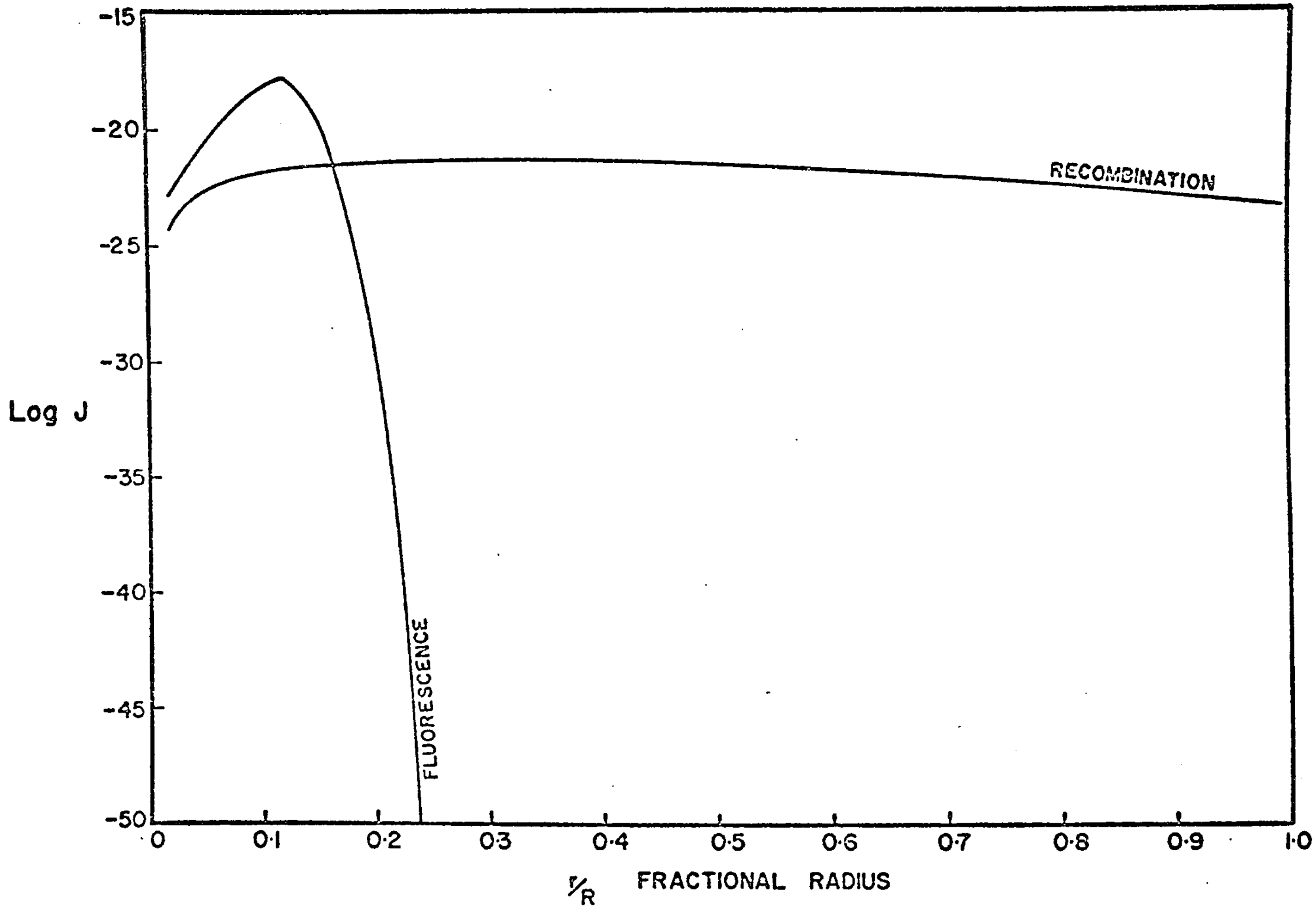
The continuum absorption coefficient due to a CII resonance line is:

$$(15) \qquad a = \frac{\frac{\pi e^2}{mc} f_{line}}{10 \frac{1}{\lambda\, line} \sqrt{\frac{2kTe}{m_{CII}}}}$$

where m_{CII} is the mass of the CII ion, and Te is the electron temperature. This coefficient, when combined with the absorption coefficient due to the ionization of H, provides the value for r in equation 5. At the wavelengths of the resonance lines under consideration here, the diffuse radiation from the ionization of H is negligible, so only starlight was considered as the source of continuum radiation.

The values of the volume emission coefficient due to recombination and due to resonance fluorescence by starlight for the λ7236 line of CII (3d ^2D - 3p $^2P°$) are shown in Figure 3. Resonance fluorescence dominates over recombination in the inner region of the nebula, but it soon drops precipitously as the starlight photons in the resonance line are absorbed. Nevertheless, resonance fluorescence contributes as much as recombination in the total integrated line strength. The emission coefficient for recombination, reflecting the abundance of CIII, varies slowly across the nebula.

The result of the inclusion of resonance fluorescence is to bring the predicted and observed CII line strengths into much better agreement (Table 8) as compared to the recombination-only prediction.



VOLUME EMISSION COEFFICIENTS (J) FOR CII λ 7236

Figure 3. Volume emission coefficient of CII λ7236 due to starlight fluorescence and recombination in the model Orion Nebula.

The spectrum of CII illustrates a useful qualitative point: lines which arise from terms which have large L values tend to be excited largely by recombination. For example, CII λ4267 (3d ^{2}D - 4f ^{2}F°) and OII λ4089 (3d ^{4}F - 4f ^{4}G°) have upper terms which cannot be reached by permitted resonance lines. Therefore, resonance fluorescence can only populate these terms via cascades from other, higher terms. The larger the value of L, the lower the probability of excitation of these high-L terms by fluorescence. (The ground state of all ions discussed in this paper are either S or P terms.) Also, the high-L terms have higher statistical weights than terms with lower values of L so the population of the high-L terms via recombination is enhanced. On the other hand, lines which originate from terms that are connected to the ground state by resonance lines are quite likely to be excited by fluorescence.

Kaler (1972) has shown that among nebulae with central stars of different temperatures, the ratio of the strengths of CII λ3921 to λ4267 decreases with temperature. Kaler assumed that λ3921 is excited largely by starlight excitation so that its strength represents the abundance of CII in the nebula. On the other hand, λ4267, which is excited almost entirely by recombination, represents the CIII abundance. As the temperature of the central star of a nebula is increased the ionization will go up, and the amount of CII will go down while the amount of CIII will roughly stay the same. Thus λ3920 will decrease in strength relative to λ4267. In the Orion Nebula, I find that resonance fluorescence is 2.5 times more effective than recombination in exciting λ3921 while recombination dominates by an order of magnitude the excitation of λ4267. Therefore, Kaler's suggestion is confirmed for the Orion Nebula.

NI

Moving down Table 8, I will now consider NI. This ion, like OI, is confined to the transition regions of the nebula. Also, again like OI, some of the NI resonance lines have wavelengths longward of the Lyman limit, so starlight photons at the energies of these resonance lines can penetrate into the transition region. I consider starlight excitation of the 3d ^4P and 4s ^4P terms of NI (see Figure 4), and as can be seen from Table 8, resonance fluorescence by starlight readily accounts for the observed line strengths and easily dominates over recombination. The predicted line strength ratio for the unobserved line λ8680 (3s $^4P_{5/2}$ - 3p $^4D^o_{5/2}$) is 3.9×10^{-3}. The presence of these NI permitted lines in the Orion Nebula was not discovered until I made a specific search for them (see Appendix A) based on the above prediction.

NII

Five multiplets of NII are observed in the Orion Nebula (Fig. 2, Table 8) and once again recombination appears insufficient to account for the observed triplet line strengths. Resonance fluorescence by starlight was included in the calculations for the 4s $^3P^o$, 3d $^3P^o$ and 3d $^3D^o$ terms, but it was found that resonance fluorescence of the 4s $^3P^o$ term by the recombination line HeI $1s^2\ ^1S$ - 1s 8p $^1P^o$ was dominant over starlight excitation for the multiplets at λ3838, λ4630

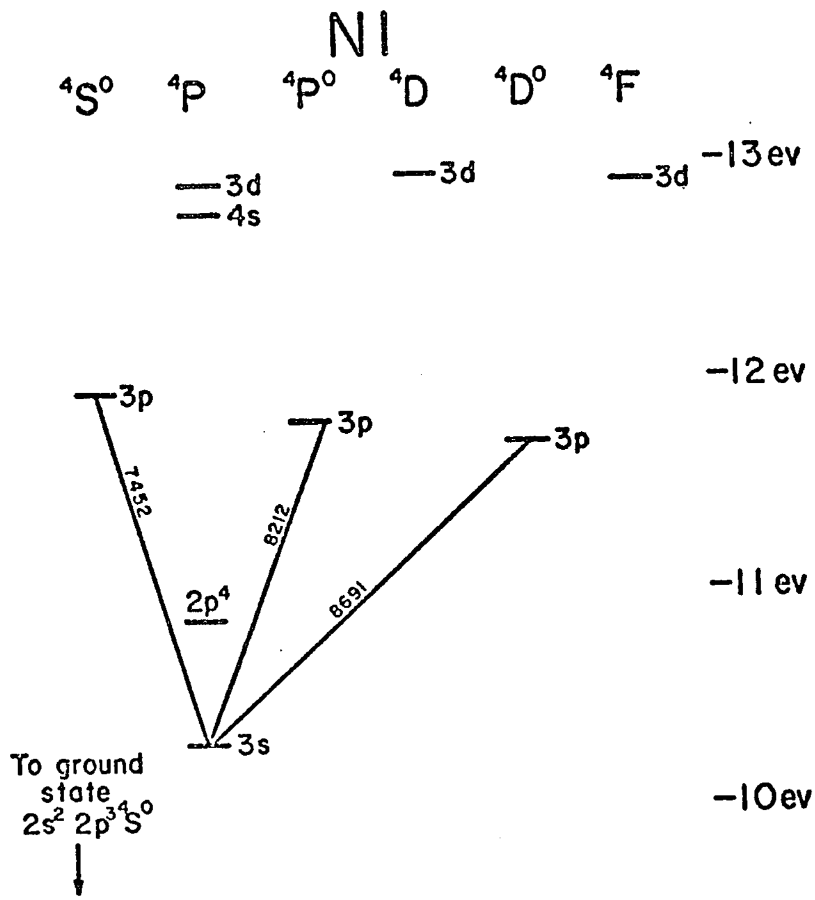


Figure 4. Energy level diagram of NI showing important transitions.

and λ5680. The strongest component of the NII resonance multiplet $2p^2$ ^3P - 2p 4s $^3P°$ is at λ508.697 while the wavelength of the HeI line is λ508.643 (Kelly 1968). Integrated over the nebula, HeI λ508.7 is nine times more efficient in populating 4s $^3P°$ than starlight.

NII λ5942 cannot be excited by HeI λ508.7, but recombination and starlight excitation seems to adequately account for the observed line strength. The singlet line λ4447 appears weakly in the Orion Nebula (although no strength is given) and the predicted recombination strength seems to be quite adequate.

NIII

Two doublet multiplets of NIII are seen in the spectrum of the Orion Nebula (Table 8, Figure 5). Recombination is incapable of producing the observed line strengths since the Orion Nebula model shows that there is an insufficient amount of NIV. Both the observed multiplets at λ4097 and λ4641 are capable of being excited by resonance fluorescence of the 3d ^2D term, however. Starlight excitation adds little to the contribution from recombination because the NIII resonance lines are present in absorption in the central O star's atmosphere.

The wavelength of the components of the resonance line to the 3d ^2D term of NIII, λ374.204 and λ374.441, are virtually coincident with the OIII resonance line to the 3s $^3P°$ term, which has components at λ374.005, λ374.075, λ374.165, λ374.331 and λ374.436 (Kelly 1968). It is just this coincidence which causes the λ4097 and λ4634 multiplets

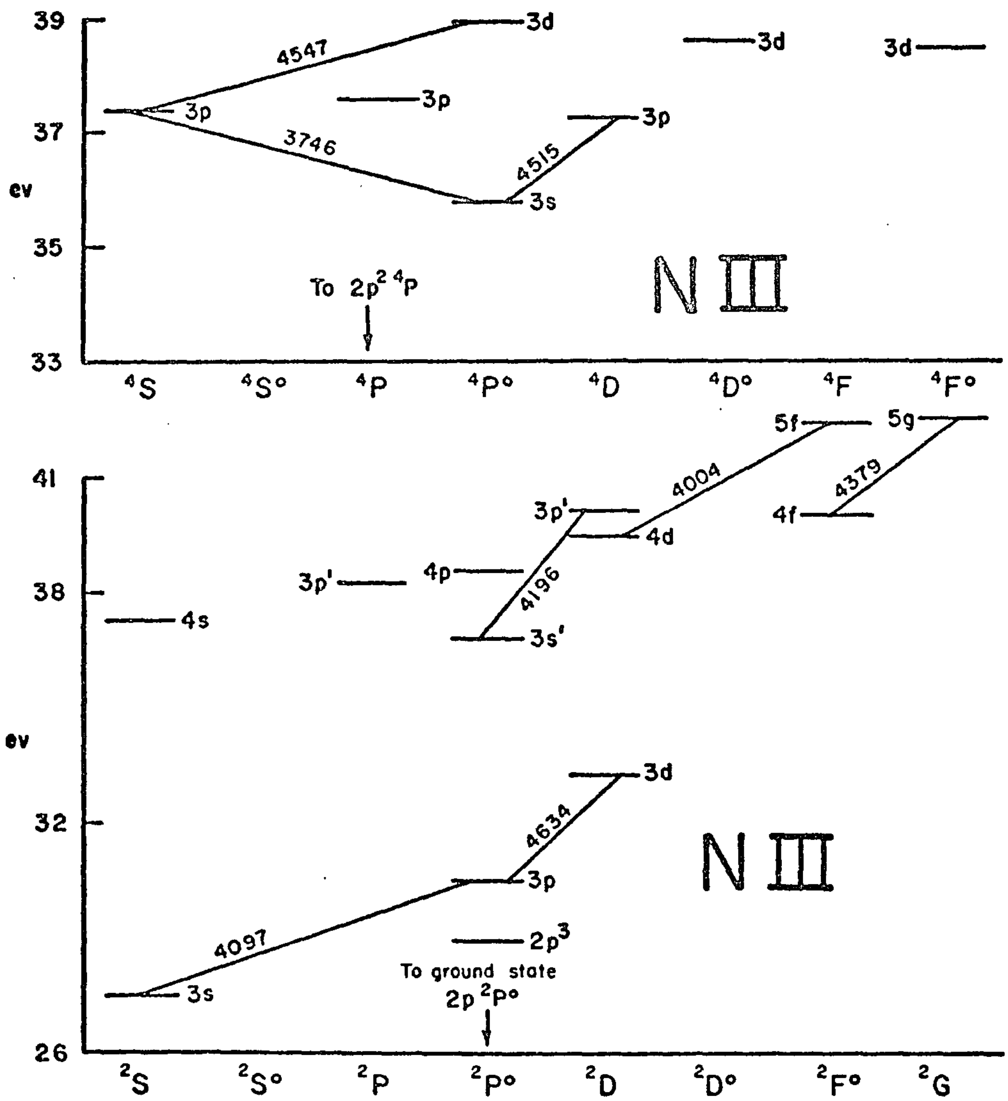


Figure 5. Energy level diagram of NIII showing lines observed in the spectra of the Orion Nebula and planetary nebulae.

of NIII to be so strong in planetary nebulae due to the Bowen fluorescence mechanism (Bowen, 1935; Weymann and Williams, 1969).

The OIII λ374 line is the raie ultime -- it connects the ground state of OIII with the first excited state that can be reached by a permitted transition. Therefore, because the Orion Nebula will be optically thick to the resonance lines of OIII, each excitation of an OIII ion will eventually result in the production of an λ374 photon. Also, λ374 photons will not be effectively absorbed by OIII ions -- an absorption will be immediately followed by an emission. In the Orion Nebula the level of ionization is too low for the existence of HeIII or OIV in any quantity, so neither HeII Lyman-α photons nor recombinations from OIV could excite the OIII in the Orion Nebula. An alternative is starlight excitation of the OIII present in the nebula. Even though stellar absorption lines will cut down the excitation of OIII due to any individual resonance line, there are approximately 10 OIII resonance lines that are capable of being excited by starlight -- and each one will result in the emission of a λ374 photon. In the calculations that resulted in the numbers listed in Table 8, I utilized λ304 (OIII $2p^2\ {}^3P - 3d\ {}^3P°$) as a representative OIII resonance line and further assumed that the contribution of the 10 resonance lines cancelled the diminution of the stellar flux due to OIII absorption lines in the stellar atmosphere. As can be seen from Table 8, the predicted line strengths of the NIII lines excited by OIII λ374 are in fair agreement with the observed line strengths.

OII

Lines of OII are numerous in the spectrum of the Orion Nebula and as is apparent from the presence of many lines that are quite unlikely to be excited by resonance fluorescence, recombination appears to be the only plausible excitation mechanism (see Figure 6). This is confirmed by the calculations whose results appear in Table 8. Starlight resonance fluorescence is of largely negligible importance, because OII is not abundant in the nebula except near the transition region where H and He are becoming neutral -- consequently the opacity in the OII resonance lines due to HI and HeI is quite large. As can be seen in Table 8, starlight excitation of the OII $2p^3\ ^4S^o - 3d\ ^4P$ line at λ430 contributes only 20% as much as recombination to the λ4153 line.

NeII

Two quartet multiplets and one doublet multiplet of NeII are observed in the Orion Nebula (Table 8, Figure 7). The two quartet lines cannot be excited by LS permitted lines from the ground state $2p^5\ ^2P^o$ (nor could the upper levels of the 2 observed quartets be excited by any electric dipole line from the ground term since their parities are the same). Therefore, recombination must be the excitation mechanism for the quartet multiplets. The excitation of the doublet multiplet at λ3830 is dominated by starlight excitation of the $3d\ ^2D$ term as shown in Table 8.

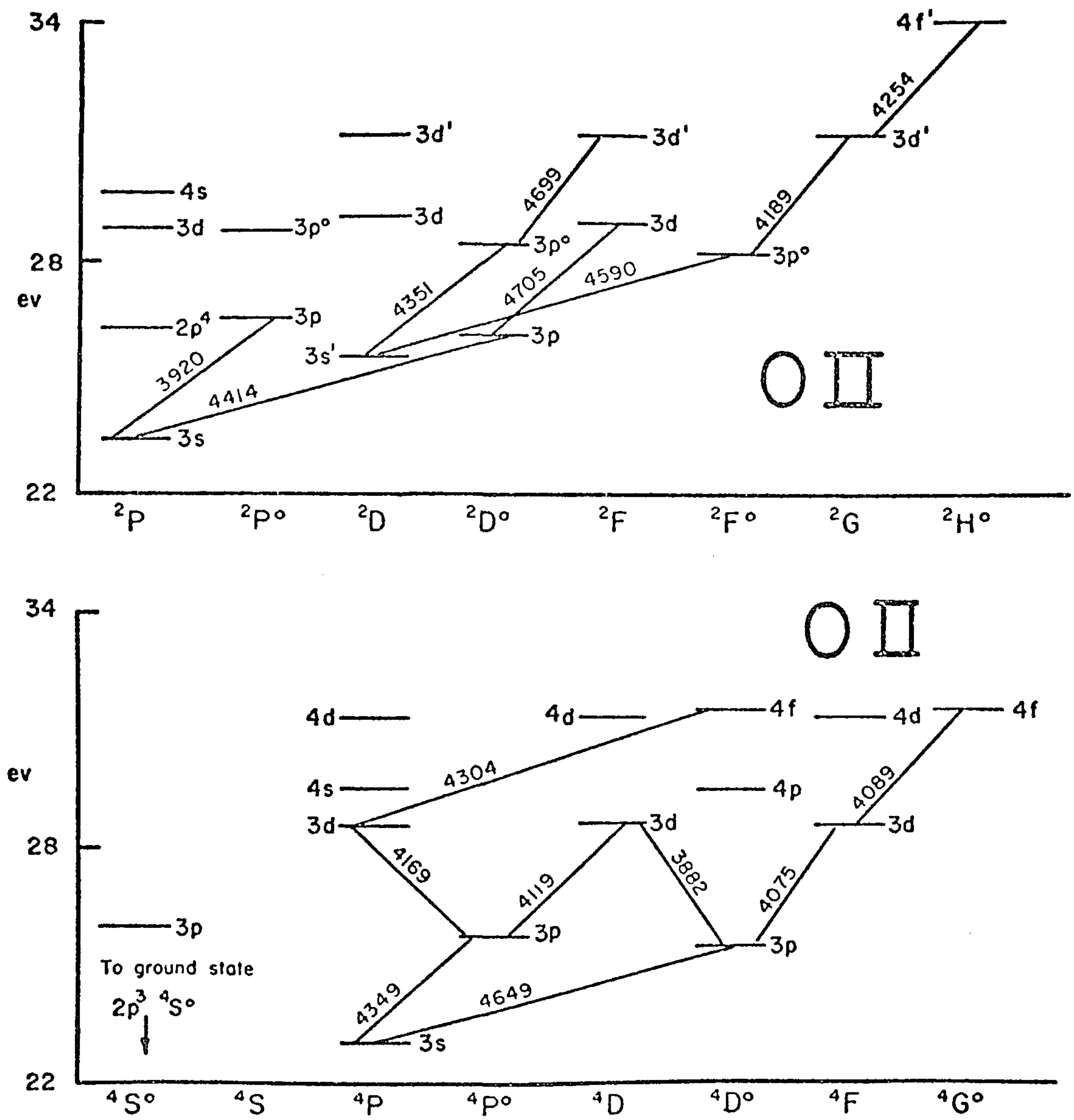


Figure 6. Energy level diagram of OII showing lines observed in the spectrum of the Orion Nebula.

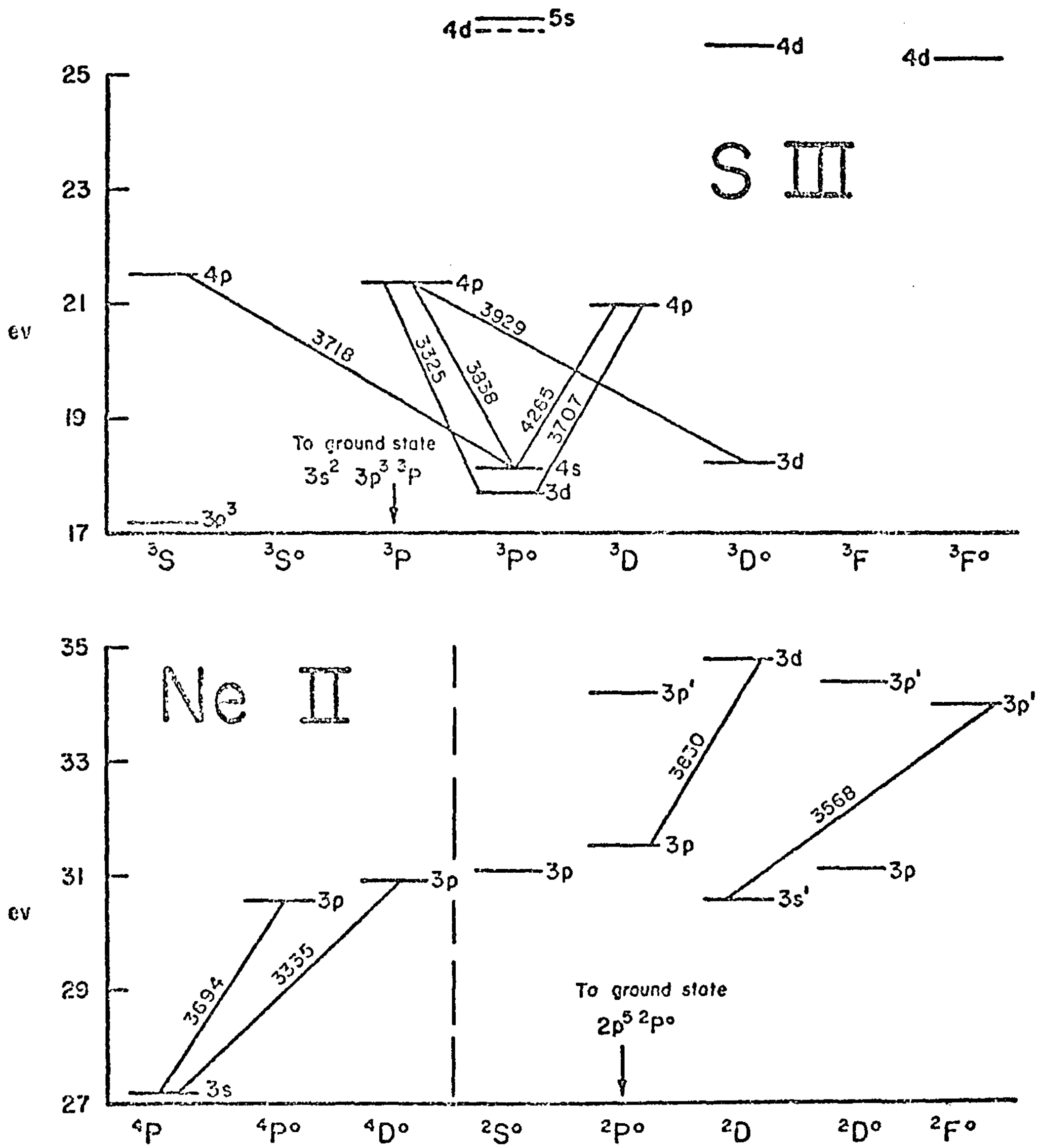


Figure 7. Energy level diagram of SIII and NeII showing lines observed in the spectrum of the Orion Nebula.

SiII

The presence of SiII in the Orion Nebula is, like OI and NI, confined to the HII-HI transition region of the nebula. Also, like OI and NI, some resonance lines of SiII have wavelengths (see Figure 8) longward of the Lyman limit, so the starlight is not absorbed in the ionization of H atoms. According to the calculations summarized in Table 8, starlight excitation of the 5s ^2S and 4d ^2D terms dominates over recombination by roughly two orders of magnitude in producing the observed lines and easily accounts for the observed strengths. Resonance fluorescence of the 6s ^2S and 5d ^2D terms could result in emission of lines at λ3339 and λ3210 respectively; but since the resonance lines to these terms have wavelengths shorter than λ912, it is not surprising that λ3339 and λ3210 are not observed in the Orion Nebula.

SiIII

One weak SiIII line is present in the spectrum of the Orion Nebula (Figure 8, Table 8). The contribution to the line from starlight resonance fluorescence of the 4p $^1P°$ level roughly equals the contribution from recombination (the calculation assumed that there was no absorption by stellar SiIII lines).

SIII

Six multiplets of SIII are observed in the Orion Nebula, but a satisfactory analysis is impeded by a lack of knowledge of the energy levels of SIII. Starlight excitation of all the observed lines via the 4d $^3D°$, 5s $^3P°$ and 4d $^3P°$ terms appears possible, but the calculations

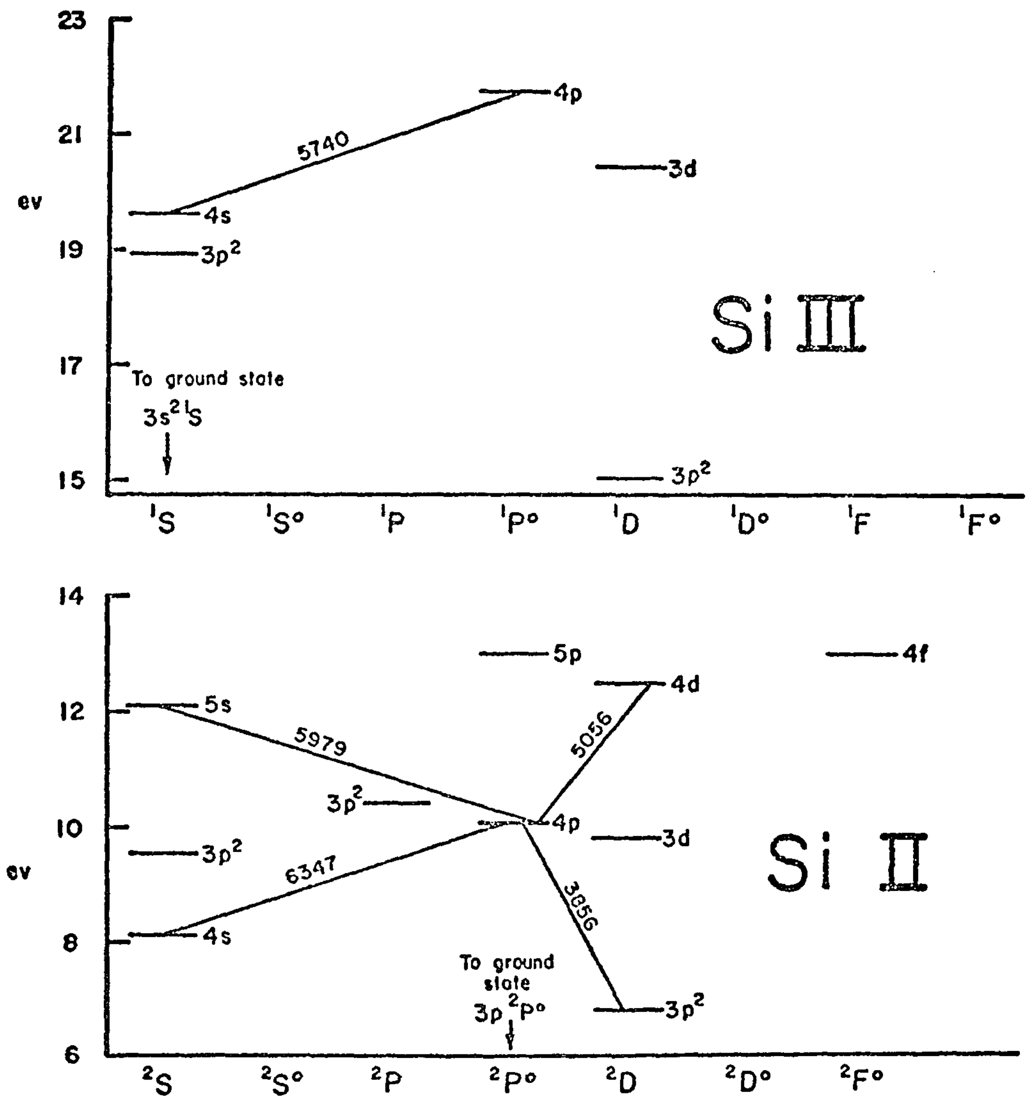


Figure 8. Energy level diagram of SiIII and SiII showing lines observed in the spectrum of the Orion Nebula.

are hindered by the unknown position of the 4d $^3P^o$ term (I have rather arbitrarily assumed a position as shown in Figure 7). The coincidence between the OII recombination line $2p^3\ ^2D^o$ - 3d ^2F and the SIII $3p^2\ ^3P$-4d $^3D^o$ resonance line at λ485 was also included in the calculations. As shown in Table 8, the results for the set of SIII lines is still somewhat below the observations.

Unobserved Lines

Before leaving the Orion Nebula, mention should be made of ions whose heavy element permitted lines do not appear. NeI and SII are similar to NI, OI and SiII in that they are confined to the transition region of the nebula; but unlike the others, the resonance lines of NeI and SII that could lead to the excitation of visible lines by fluorescence all have wavelengths shorter than the Lyman limit. Consequently, the resonance lines will be absorbed by the neutral H in the transition region, and appreciable excitation by starlight of lines of NeI and SII is impossible. Due to the relatively low abundance of NeII and SIII, recombination also appears incapable of appreciable excitation (as is also the case for NI, OI and SiII). For example, predicted recombination line strength ratios for SII λ5606 (3d ^4F - 4p $^4D^o$) and NeI λ6402 (3s[3/2]° - 3p[5/2]) to Hβ are 6.1×10^{-6} and 2.4×10^{-6} respectively.

Lines of CIII and OIII cannot be excited by recombination because CIV and OIV have quite low abundances in the Orion Nebula. Due to absorption lines in the stellar atmosphere, starlight resonance fluorescence should be rather ineffective in exciting lines of these two ions. For example, starlight excitation of the OIII $\lambda 3444$ (3p $^3P_2$ - 3d $^3P^\circ_2$) line (through $\lambda 304$) leads to a predicted line strength ratio to $H\beta$ of 3.5×10^{-6}. For the same reasons, only those NIII lines capable of being excited by the pseudo-Bowen fluorescence mechanism described above should be observed in the Orion Nebula. NeIII also falls into this class of ions, but it is the lack of stellar flux at the resonance line wavelengths ($\lambda 251$, $2p^4\ ^3P$ - $2p^3\ 3d\ ^3D^\circ$, is the lowest energy resonance line capable of exciting cascade lines) as well as stellar absorption lines that rules out excitation by starlight fluorescence.

SiIV has a fairly large abundance in the Orion Nebula, but calculations show that lines of this ion should not be easily observable. $\lambda 4089$ (4s ^2S - 4p $^2P^\circ$) which can be excited by resonance fluorescence by starlight of $\lambda 458$ (3s ^2S - 4p $^2P^\circ$) has a predicted line ratio to $H\beta$ of 3.9×10^{-5}, while $\lambda 4654$ (5g ^2G - 6h $^2H^\circ$) has a predicted line ratio due to recombination of 3.6×10^{-4}.

In summary, all the heavy element permitted lines seen in the spectrum of the Orion Nebula, excepting perhaps those of SIII, can be adequately accounted for by either recombination, resonance fluorescence by starlight, or resonance fluorescence by other nebular emission lines (see Table 8).

CHAPTER 5

HEAVY ELEMENT PERMITTED LINES IN PLANETARY NEBULAE

In this chapter, the heavy element permitted line spectra of planetary nebulae -- specifically NGC7027 and NGC7662 as representative examples -- will be discussed. As is the case with the Orion Nebula, no one process can account for all the lines of all the ions, and the discussion to follow will consider the processes applicable to each ion. A line list for NGC7027 was compiled from Aller, Bowen and Minkowski (1955), Aller, Bowen and Wilson (1963) and Kolotilov and Noskova (1974), and a list for NGC7662 was compiled from Aller, Kaler and Bowen (1966), Wyse (1942) and Aller and Walker (1970). Line strengths for the lines in NGC7027 were obtained from Kaler, Aller, Czyzak and Epps (in press). Reddening corrections to the observed line strengths from these sources were obtained from Kaler (1966) using the Whitford (1958) extinction curve.

Miller (1971) has shown that the intensities of the weak lines observed by Aller, Kaler and Bowen (1966) in NGC7662 are systematically too high. To correct this, the line strengths for NGC7662 have been divided by a factor that ranges from 1.5 for lines 1×10^{-4} as strong as $H\beta$ to 1.0 for lines 1×10^{-2} the strength of $H\beta$ and stronger.

SiII, CII, OII, NeII

Both NGC7027 and NGC7662 show SiII lines which, by all indications, must be excited by resonance fluorescence by starlight, as in the Orion Nebula. Since neither OI nor SiII exists in any quantity in the planetary nebula model (which is optically thin to the H continuum), no meaningful predictions can be made for the strengths of these lines, which presumably arise in the transition regions of dense clumps inside the nebula.

The lines of CII, OII and NeII which appear in the spectra of NGC7027 and NGC7662 are quite similar to those that appear in the Orion Nebula spectrum and they may be accounted for by the same excitation mechanisms as mentioned in Chapter 2 (see Table 9). The results listed in Table 9 for the CII lines $\lambda3921$ and $\lambda4267$ differ widely from the observed values. The planetary nebula model predicts that $\lambda3921$ should be much stronger than $\lambda4267$, which is the opposite of reality (as I discussed in the previous chapter, in reference to Kaler's work). However, this can be explained by noting that the planetary nebula model, for some reason, predicts a much higher abundance of CII than the model of Harrington (1969).

Lines from Doubly Excited Levels

Among the permitted lines in the spectra of planetary nebulae that do not appear in the Orion Nebula are several lines arising from terms with more than one excited electron. These lines are unexpected since such terms cannot be reached by LS permitted transitions from the ground state of that ion or from recombination and subsequent LS

Table 9. Strengths of CII, OII and NeII lines in NGC7662.

Ion	Line	Transition	Observed Strength ($H\beta = 1$)	Predicted Strength ($H\beta = 1$)	Dominent Mechanism
CII	3921	$3p\ ^2P^o_{3/2} - 4s\ ^2S_{1/2}$	4×10^{-4}	1.2×10^{-3}	Starlight
	4267	$3d\ ^2D - 4f\ ^2F^o$	7.1×10^{-3}	2.5×10^{-4}	Recombination
OII	4089	$3d\ ^4F_{9/2} - 4f\ ^4G_{11/2}$	9.0×10^{-4}	2.6×10^{-4}	Recombination
	4414	$3s\ ^2P_{3/2} - 3p\ ^2D_{5/2}$	1.0×10^{-3}	5.0×10^{-4}	Recombination
NeII	3777	$3s\ ^4P_{1/2} - 3p\ ^4P_{3/2}$	5.9×10^{-4}	7.0×10^{-5}	Recombination

permitted cascades from the ground state of the next highest ion. Table 10 lists the lines from multiply excited terms that have been seen in the spectra of NGC7027 and NGC7662 (only the theoretically strongest member that has been observed is listed for each multiplet). Three multiplets -- CIII λ4666, NIII λ4515 (see Figure 5) and OIV λ3386 -- have more than one member present and are therefore fairly certain identifications.

Consider the NIII λ4515/OIV λ3386 2s 2p 3s $^4P^\circ$ - 2s 2p 3p ^4D transition. The ground state of NIV/OV is $2s^2\ ^1S$, so normal recombination of an electron with a NIV/OV ion in its ground state cannot form a term such as NIII/OIV 2s 2p 3p ^4D. Nikitin (1959) has proposed that recombinations from the "metastable" 2s 2p ^3P term of NIV (principally excited by photoionization of a 2s electron from the $2s^2$ 2p ground state of NIII) can account for the observed strength of NIII λ4515. However, as I will show below, use of better atomic constants than were available to Nikitin in a fairly detailed calculation leads to the conclusion that this mechanism fails by several orders of magnitude. Furthermore, consideration of dielectronic recombination and resonance fluorescence or radiative recombination via transitions that are forbidden under LS coupling do not lead to any viable excitation mechanisms for NIII λ4515/OIV λ3386 or most of the other lines listed in Table 10.

In order to test Nikitin's proposal, I have constructed and solved the statistical equilibrium equations for the ions of N considering NI, NII, NIII, NV and NVI as one state ions (their ground

Table 10. Lines from doubly excited terms.

Ion	Line		Nebula
CIII	λ4326	2p 3s $^1P^o_1$ - 2p 3p $^1D0$	NGC7027
	4666	2p 3s $^3P^o_2$ - 2p 3p $^3P_2$	NGC7027
NIII	3746	2s 2p 3s $^4P^o_{1/2}$ - 2s 2p 3p $^4S_{3/2}$	NGC7027
	4196	2s 2p 3s $^2P^o_{1/2}$ - 2s 2p 3p $^2D_{3/2}$	NGC7027
	4515	2s 2p 3s $^4P^o_{5/2}$ - 2s 2p 3p $^4D_{7/2}$	NGC7027
	4547	2s 2p 3s $^4S_{3/2}$ - 2s 2p 3d $^4P^o_{5/2}$	NGC7027
	4196	2s 2p 3s $^2P^o_{1/2}$ - 2s 2p 3p $^2D_{3/2}$	NGC7662
	4535	2s 2p 3p $^4S_{3/2}$ - 2s 2p 3d $^4P^o_{3/2}$	NGC7662
OII	4146	2s $2p^3$ 3p $^6P_{7/2}$ - 2s $2p^3$ 3d $^6D^o_{9/2}$	NGC7662
OIII	3351	2s $2p^2$ 3s $^5P_3$ - 2s $2p^2$ 3p $^5P^o_3$	NGC7027
	3710	2s $2p^2$ 3s $^5P_1$ - 2s $2p^2$ 3p $^5D^o{}_0$	NGC7662
OIV	3349	2s 2p 3s $^2P^o_{3/2}$ - 2s 2p 3p $^2D_{5/2}$	NGC7027
	3386	2s 2p 3s $^4P^o_{5/2}$ - 2s 2p 3p $^4D_{7/2}$	NGC7027
	3737	2s 2p 3p $^4D_{7/2}$ - 2s 2p 3d $^4F^o_{9/2}$	NGC7027
	3386	2s 2p 3s $^4P^o_{5/2}$ - 2s 2p 3p $^4D_{7/2}$	NGC7662
	3737	2s 2p 3p $^4D_{7/2}$ - 2s 2p 3d $^4F_{9/2}$	NGC7662

state), and NIV as a 4 state ion consisting of $2s^2\ ^1S$ (the ground state) and $2s\ 2p\ ^3P^o_{0,1,2}$. All events resulting in the population of a triplet or singlet state of NIV were assumed to be immediately followed by a cascade to the $^3P^o$ or ^1S term, respectively. The transition probability for the dipole transition NIV $2s^2\ ^1S_0$ - $2s\ 2p\ ^3P^o_1$ (caused by the slight breakdown of LS coupling) was set at 471 sec^{-1} (Nussbaumer, 1972) while the magnetic quadrupole transition probability for $^1S_0$ - $^3P^o_2$ was assumed to be 6.6×10^{-3} sec^{-1} (Osterbrock, 1970; Garstang, 1967). The $2s^2\ ^1S_0$ - $2s\ 2p\ ^3P^o_0$ transition is completely forbidden for all single photon events, and I consequently assigned a zero value to this transition probability. Estimates of the probability for the electric quadrupole transition $2s\ 2p\ ^3P^o_2$ - $^3P^o_0$ and for the magnetic dipole transitions $^3P^o_2$ - $^3P^o_1$ and $^3P^o_1$ - $^3P^o_0$ were taken from the estimates of Nikitin (1959). The collision strengths for the electron collisional transitions $2s^2\ ^1S_0$ - $2s\ 2p\ ^3P^o_{0,1,2}$, $^3P^o_0$ - $^3P^o_2$, $^3P^o_0$ - $^3P^o_1$ and $^3P^o_1$- $^3P^o_2$ were taken from Osterbrock (1970) and Blaha (1968).

Photoionization cross sections for inner and outer shell electrons were taken from the compilations of Kurucz (1970) and MacAlpine (1971). Recombination coefficients were obtained from Aldrovandi and Pequignot (1972), and the recombination coefficient from NV $\rightarrow$ NIV was allocated among the 4 states of NIV according to their statistical weights. Ionizations which remove a 2s electron from NIII $2s^2$ 2p were similarly allocated among the 4 NIV states.

The results of this calculation as applied to the planetary nebula model are shown in Figure 9 wherein is plotted the fraction of nitrogen in the 4 NIV states as a function of position in the nebula.

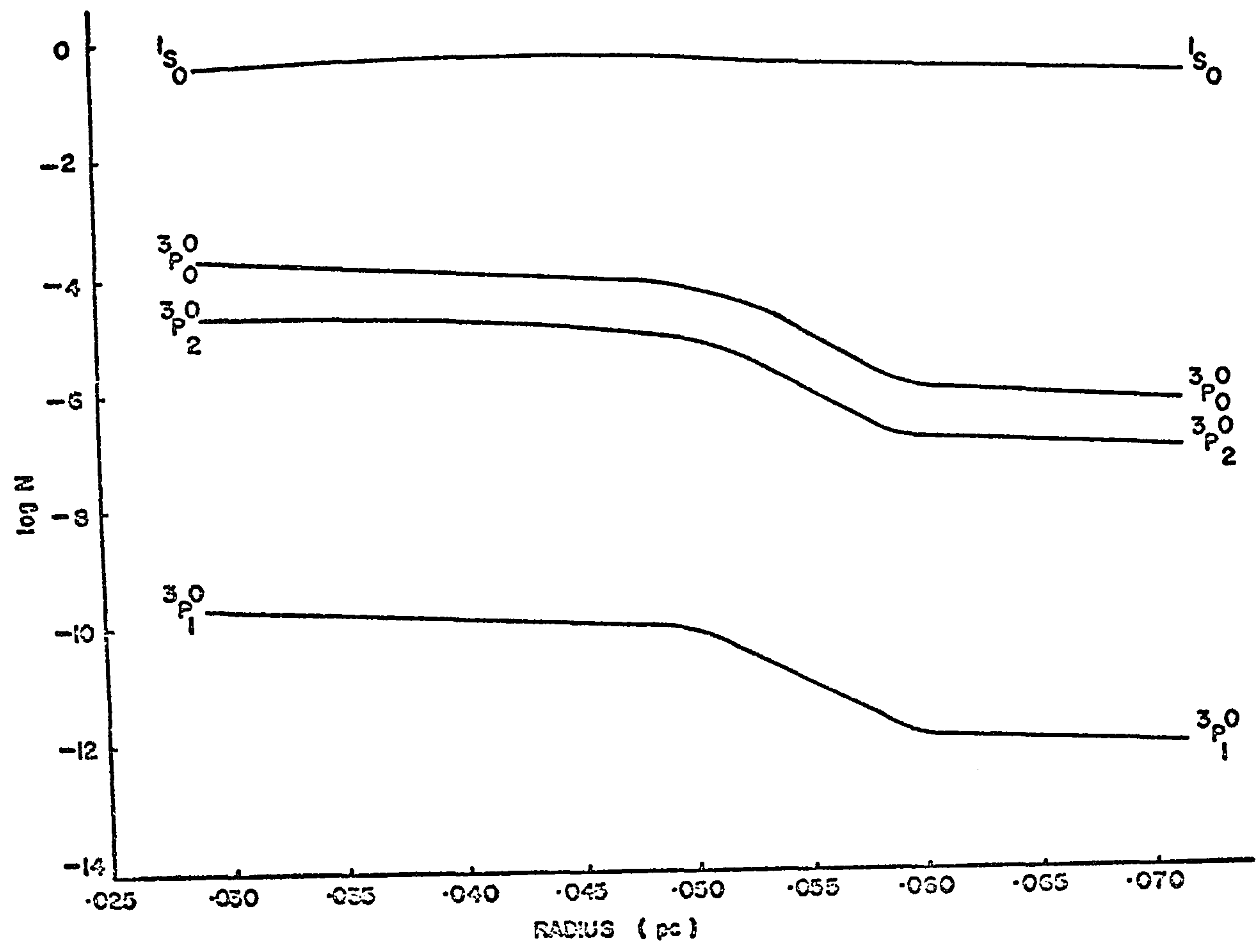


Figure 9. A plot of the calculation of the relative number of nitrogen atoms in the four lowest lying levels of NIV versus radius in the model planetary nebula.

Basically, the transition probabilities for the $2s^2\ ^1S - 2s\ 2p\ ^3P°$ multiplet, while low, are high enough to prevent a large buildup of ions in the NIV $2s\ 2p\ ^3P°$ term which could recombine to form 2s 2p nl quartet terms of NIII, and excite λ4515. (The two orders of magnitude drop in the population of the $^3P°$ term between .05 and .06 pc is due to the transition from HeIII to HeII in the nebula, which has the consequence that starlight which could cause inner shell ionization of NIII is absorbed.) The prediction from this calculation for the NIII λ4515/Hβ line ratio is 1×10^{-7} as compared to the observed strength of 1.3×10^{-3} in NGC7027 (Aller, Bowen and Wilson, 1963 with reddening corrections from Kaler, 1966 and Whitford, 1958). Similar results can be expected for OIV λ3386.

For the same reason, recombinations from "metastable" states cannot account for the sextets of OII or the quintets of OIII. The transition probabilities for OIII $2s^2\ 2p^2\ ^3P - 2s\ 2p^3\ ^5S°$ and OIV $2s^2\ 2p\ ^2P° - 2s\ 2p^2\ ^4P$ are expected to be high enough (Edlen et al., 1969) to prevent significant populations in the OIII $2s\ 2p^3\ ^5S°$ or OIV $2s\ 2p^2\ ^4P$ levels.

At first glance, the process of dielectronic recombination appears to be a likely alternate mechanism for the production of ions in doubly excited states. Consider the following dielectronic process (Burgess, 1964):

$$(16) \qquad \text{NIV}\ (2s^2) + e^-(E, \ell + 1) \rightleftharpoons \text{NIII}\ (2s\ 2p\ nl)$$

which consists of a recombination to an autoionizing state of NIII.

If such a recombination were to be followed by a stabilizing transition of the form

$$(17) \qquad \text{NIII } (2s\ 2p\ nl) \rightarrow \text{NIII } (2s\ 2p\ n'l') + h\nu$$

where the energy of NIII (2s 2p n'l') is below the ionization threshold, terms such as 2s 2p 3p ^4D could be populated by subsequent cascades. Unfortunately, two facts seem to rule out such a population mechanism. First, since most dielectronic recombinations will occur through states with large values of n (Burgess, 1965), the probability for a "core" stabilizing transition such as

$$(18) \qquad \text{NIII } (2s\ 2p\ nl) \rightarrow \text{NIII } (2s^2\ nl) + h\nu$$

is much larger than for a transition such as (17), (Shore, 1969). Second, the electron temperature characteristic of planetary nebulae is too low to permit significant dielectronic recombinations through the NIV 2s 2p $^1P°$ or $^3P°$ terms (based on calculations using the techniques of Burgess, 1965). Therefore, dielectronic recombination cannot account for the presence of NIII λ4515/OIV λ3386 in planetary nebula. The same problems appear to rule out dielectronic recombination as an excitation mechanism for the doubly excited triplets of CIII, the sextets of OII and the quintets of OIII as well.

Another candidate excitation mechanism that is easy to rule out is resonance fluorescence. The transition NIII $2s^2$ 2p $^2P°$ - 2s 2p 3p ^4D, while forbidden under the LS selection rule against changes in S, is likely to have a spontaneous transition probability on the order of

10^1 to 10^2 sec^{-1}. This number is based on an extrapolation along the BI isoelectronic sequence of the transition probabilities of the $2s^2$ $2p\ ^2P°$ - $2s\ 2p^2\ ^4P$ line calculated in a full intermediate coupling treatment (including configuration interaction) by Shamey (1971). Calculations show, however, that an Einstein A value of roughly 10^8 sec^{-1} is required for starlight excitation of 2s 2p 3p ^4D to be able to account for the observed line strength of λ4515.

Yet another excitation mechanism for the doubly excited levels might be non-LS permitted transitions following a normal recombination or photoexcitation. Based on Shamey's results, it appears that multiply excited states with the same multiplicity as singly excited states are quite likely to have reasonably strong lines (due to configuration interaction) connecting them (see also Berry et al., 1970). Just such a line is CIII λ4156 2p 3p $^3D_2$ - 2s 5f $^3F^{\circ}_3$ which is observed in NGC7662 and NGC7027. Thus observed lines like CIII λ4666 2p 3s $^3P°$ - 2p 3p ^3P, or NIII λ4196 2s 2p 3s $^2P°$ - 2s 2p 3p ^2D are quite likely to be due to such LS coupling forbidden transitions populating the doubly excited state after a normal radiative recombination. However, transitions that involve a change in S have probabilities which are substantially lower (Shamey, 1971 quotes a factor of 10^{-5}) than comparable spin allowed transitions that connect doubly and singly excited terms. Therefore, the excitation mechanism for the quartet transitions of NIII and OIV as well as the sextets of OII and the quintets of OIII (as listed in Table 10) cannot be normal recombination followed by a multielectron transition.

Briefly summarizing the above discussion, the only lines in Table 10 that can be adequately explained are those that are of the same multiplicity as normal fluorescent or recombination lines. Recombination from "metastable" states cannot account for the other lines in Table 10, and dielectronic recombination, resonance fluorescence and S-changing transitions are all incapable of exciting the doubly excited levels as well.

Other Lines

The remaining lines to be discussed in the spectra of NGC7662 and NGC7027 are normal singly excited lines that do not appear in the spectrum of Orion and are listed in Tables 11 and 12 (lines that appear only in the spectrum of NGC7027 are listed in Table 12). Only the strongest (theoretically) unblended component of each multiplet is listed.

The ground state of the CIII ion is $2s^2\ ^1S$, so the three observed triplet lines in the spectrum of NGC7662 undoubtedly arise from recombination. The singlet line $\lambda4187$ has a ^1G term as its upper level, so it too is predominantly excited by recombination. No prediction can be made for $\lambda4156$ $3p\ ^3D_2 - 5f\ ^3F^o_3$ since no transition probability is available for this LS forbidden transition.

Three other CIII lines appear in the spectrum of NGC7027. The triplet lines $\lambda4517$ and $\lambda8196$ must be due to recombination, while the weakly observed $\lambda5696$ line can be excited by starlight excitation of the $4p\ ^1P^o$ term as well. Calculations show, however, that due to the

Table 11. Lines in the spectrum of NGC7662 and NGC7027

Ion	Line	Transition	Observed Strength in NGC7662	Observed Strength in NGC7027	Predicted Strength	Dominant Mechanism	Predicted Recombination Strength
CIII	λ4070	$4f\ ^3F^o_4 - 5g\ ^3G_5$	9.3×10^{-3}	5.4×10^{-3}	1.1×10^{-3}	recombination	1.1×10^{-3}
	4156	$3p'\ ^3D_2 - 5f\ ^3F^o_3$	*	4.1×10^{-4}	- -		
	4187	$4f\ ^1F^o_3 - 5g\ ^1G_4$	4.6×10^{-3}	2.9×10^{-3}	9.0×10^{-4}	recombination	9.0×10^{-4}
	4647	$3s\ ^3S_1 - 3p\ ^3P^o_2$	7.8×10^{-3}	4.4×10^{-3}	5.1×10^{-3}	recombination	5.1×10^{-3}
CIV	4659	$5g\ ^2G - 6h\ ^2H^o$	7.1×10^{-3}	4.9×10^{-3}	1.8×10^{-3}	recombination	1.8×10^{-3}
	5801	$3s\ ^2S_{1/2} - 3p\ ^2P^o_{3/2}$	4.4×10^{-3}	4.0×10^{-3}	3.4×10^{-3}	starlight/ recombination	2.1×10^{-3}
NIII	4004	$4d\ ^2D_{5/2} - 5f\ ^2F^o_{7/2}$	2.4×10^{-4}	not seen	2.4×10^{-4}	recombination	2.4×10^{-4}
	4097	$3s\ ^2S_{1/2} - 3p\ ^2P^o_{3/2}$	3.2×10^{-2}	2.0×10^{-2}	- -	Bowen	
	4379	$4f\ ^2F^o - 5g\ ^2G$	1.4×10^{-3}	1.1×10^{-3}	2.1×10^{-3}	recombination	2.1×10^{-3}
	4634	$3p\ ^2P_{1/2} - 3d\ ^2D^o_{3/2}$	3.0×10^{-2}	1.4×10^{-2}	- -	Bowen	
OIII	3133	$3p\ ^3S_1 - 3d\ ^3P^o_2$	1.05	0.84	- -	Bowen	
	3266	$3p\ ^3D_3 - 3d\ ^3F^o_4$	not seen	5.4×10^{-3}	2.4×10^{-3}	recombination	2.4×10^{-3}
	3341	$3s\ ^3P^o_2 - 3p\ ^3S_1$	0.17	0.13	- -	Bowen	

Table 11. Lines in the spectrum of NGC7662 and NGC7027--continued.

Ion	Line	Transition	Observed Strength in NGC7662	Observed Strength in NGC7027	Predicted Strength	Dominant Mechanism	Predicted Recombination Strength
OIII	λ3444	$3p\ ^3P_2 - 3d\ ^3P^o_2$	0.23	0.22	- -	Bowen	
	3732	$3p\ ^3P_2 - 3d\ ^3D^o_1$	3.4×10^{-3}**	1.5×10^{-3}**	1.1×10^{-3}	starlight	4.4×10^{-4}
	3760	$3s\ ^3P^o_2 - 3p\ ^3D_3$	6.9×10^{-2}	5.4×10^{-2}	- -	Bowen	
OIV	3412	$3p\ ^2P^o_{3/2} - 3d\ ^2D_{5/2}$	3.3×10^{-3}	1.1×10^{-2}	1.7×10^{-2}	recombination	1.4×10^{-2}
SiIV	4116	$4s\ ^2S_{1/2} - 4p\ ^2P^o_{1/2}$	1.6×10^{-4}	not seen	1.3×10^{-3}	starlight/ recombination	7.2×10^{-4}

*Blend.
**Blend with HeII Pickering 26.

Table 12. Lines in the spectrum of NGC7027 that are not in the spectrum of NGC7662.

Ion	Line	Transition	Observed Strength	Predicted Strength	Dominant Mechanism	Predicted Recombination Strength
CIII	λ4517	$4p\ ^3P^o_2 - 5s\ ^3S_1$	1.7×10^{-4}	3.0×10^{-4}	recombination	3.0×10^{-4}
CIII	5696	$3p\ ^1P^o_1 - 3d\ ^1D_2$	- -	1.4×10^{-5}	recombination	1.4×10^{-5}
CIII	8196	$5g\ ^{3,1}G - 6h\ ^{3,1}H^o$	4.0×10^{-3}	1.2×10^{-3}	recombination	1.2×10^{-3}
CIV	7726	$6h\ ^2H^o - 7i\ ^2I$	2.4×10^{-3}	8.2×10^{-4}	recombination	8.2×10^{-4}
NIV	3479	$3s\ ^3S_1 - 3p\ ^3P^o_2$	1.3×10^{-3}*	1.2×10^{-3}	recombination	1.2×10^{-3}
NV	4604	$3s\ ^2S_{1/2} - 3p\ ^2P^o_{3/2}$	2.1×10^{-4}	4.5×10^{-4}	starlight	7.7×10^{-5}
OIII	3047	$3s\ ^3P^o_2 - 3p\ ^2P_2$	0.15	- -	Bowen	
OIII	3962	$3p\ ^1D_2 - 3d\ ^1F^o_3$	6.3×10^{-4}	4.5×10^{-4}	recombination	4.5×10^{-4}
OV	5112	$3s\ ^1S_0 - 3p\ ^1P^o_1$	2.6×10^{-4}	5.4×10^{-6}	starlight ?	**

*Blend with HeI $2p\ ^3P^o - 15d\ ^3D$. 2/3 of observed strength assumed to be due to NIV.
**OVI was not included in the planetary nebula model.

low value of the branching ratio involved, resonance fluorescence by starlight is over an order of magnitude less effective than recombination in the excitation of λ5696.

Two multiplets of CIV are observed in NGC7662. λ5801 3s $^2S_{1/2}$ - 3p $^2P^o_{3/2}$ is excited in roughly equal amounts by recombination and by resonance fluorescence by starlight of λ312 2s ^2S - 2p $^2P^o$, while λ4659 is principally excited by recombination since its upper term, 6h ^2H, has such a large value of L. A question remains, however, as to why λ5801 is weaker than λ4659 in NGC7662 when both my results and the much more detailed calculations of Leibowitz (1972) give the opposite result. Another recombination dominated line of CIV, λ7726, is seen in NGC7027.

The λ4097 and λ4637 multiplets of NIII owe their rather large strengths in NGC7662 to the Bowen fluorescence mechanism. On the other hand, λ4004 and λ4379 (see Figure 5), which have upper terms of 5f ^2F and 5g ^2G respectively, should be predominantly excited by recombination.

The very strong OIII multiplets at λ3047, λ3133, λ3341, λ3444 and λ3760 are excited by the Bowen mechanism. λ3266, arising from a ^3F term, would seem to be dominated by recombination, while λ3732 is dominated by resonance fluorescence of λ306 $2p^2$ ^3P - 3d $^3D^o$ by starlight. λ3760 can also be excited by starlight excitation of 3d $^3D^o$, but calculations based on the observed strengths of the very strong Bowen mechanism lines show that the Bowen mechanism dominates the excitation of λ3760. A singlet line of OIII, λ3962, is seen in the spectrum of NGC7027. This line must be excited by recombination.

Kaler (1972) has shown that OIII permitted lines are present in planetary nebulae (primarily NGC6572 and NGC6543) that show no recombination lines of HeII. Since there is no HeIII present in these nebulae, the Bowen fluorescence mechanism cannot operate, and because the ionization potentials of HeII and OIII are quite similar, the lack of HeIII implies a lack of OIV which would rule out recombination as a source of the OIII permitted lines. Therefore, resonance fluorescence by starlight must be the operative mechanism. The OIII lines identified by Kaler may all be explained by this interpretation, with the possible exception of a singlet line which is identified in the faintest of the three nebulae considered.

The OIV $\lambda3412$ 3p $^2P°$ - 3d ^2D line can be excited by resonance fluorescence of the $\lambda238$ 2p $^2P°$ - 3d ^2D line by starlight. The calculations show that recombination is three times as effective as starlight excitation.

The observed SiIV $\lambda4116$ 4s $^2S_{1/2}$ - 4p $^2P°_{1/2}$ line can also be excited by resonance fluorescence by starlight. In this case, recombinations are equally as effective as starlight excitation of the $\lambda458$ 3s $^2S°$ - 4p ^2P resonance line.

Lines of four ions are found in NGC7027 that are not found in NGC7662 (Table 12). NIV $\lambda3479$ is a triplet line, while the ground state of NIV is ^1S. Therefore, $\lambda3479$ has to be excited by recombination. NV $\lambda4604$ can be excited by resonance fluorescence by starlight of $\lambda204$ 2s ^2S - 3p $^2P°$. The calculations show that this process dominates over recombination by a factor of 5.

OV λ5112 3s ^1S - 3p $^1P°$ seems an unlikely line to be observed since the case B branching ratio of this particular multiplet is only 0.0035 (due to very strong 2 electron transitions such as 2p^2 ^1D - 2s 3p $^1P°$), (Nussbaumer, 1972). The calculation listed in Table 12 is for resonance fluorescence by starlight of the λ172 2s^2 ^1S - 2s 3p $^1P°$ resonance transition. As is expected from the low branching ratio, the disagreement between theory and observation is quite large. Since this line is a singlet transition, confirmation of its identification by other members of the multiplet is not possible.

In concluding this discussion, it can be said that three excitation mechanisms, recombination, resonance fluorescence by starlight and resonance fluorescence by other nebular lines, can account for almost all of the heavy element permitted lines observed in the two planetary nebulae NGC7027 and NGC7662. The dominant excitation mechanism for each individual line is listed in Tables 11 and 12. However, an understanding of certain doubly excited lines in the spectra of planetary nebulae is still lacking. Also, misidentifications are suspected in a few instances where the predicted strength is much lower than the observed strength (e.g., OV λ5112).

Finally, it is necessary to point out the markedly different excitation of the planetary nebula NGC6778. The heavy element permitted lines in the spectrum of this nebula are quite strong (Czyzak and Aller, 1973) and, from the lines that are observed, it is likely that resonance fluorescence by starlight is quite dominant. For example, all the OII lines observed are quartet lines; while doublet lines (such as 4415 3s ^2P - 3p $^2D^o$ which is seen in NGC7027 and NGC 7662), which cannot be excited by permitted resonance transitions, are not. However, to quote Czyzak and Aller, 1973: "A difficulty arises in trying to understand how the faint central star can provide such a rich ultraviolet radiation field for the excitation of discrete lines" (p. 817).

APPENDIX A

FAR-RED OBSERVATIONS OF GASEOUS NEBULAE

In this appendix I will discuss the observations undertaken in support of the investigation reported on in this dissertation. First, observations of the Orion Nebula (obtained in a search for OI and NI permitted lines) will be discussed. Several spectra with exposure times of up to an hour were taken covering the wavelength range 6500-9100 Å at a dispersion of 94 Å mm^{-1} with an ITT image tube with a 40-mm S25 photocathode attached to the Cassegrain spectrograph of the Steward Observatory 90-inch (2.3 m) reflector. The slit was oriented east-west and was placed on the bright inner part of the nebula, about 15" south of the Trapezium stars. Spectra of the same region were also taken with a dispersion of 48 Å mm^{-1} covering the wavelength range from 5000 to 6300 Å with an RCA C33063 image tube.

Table 13 lists the lines identified from the far red spectra. Column (1) contains the measured wavelength, column (2) contains the wavelength of the identified line, column (3) lists the ion producing the line, and column (4) contains a crude estimate of the strength (on the one-hour exposure) for some of the lines (on a scale such that Hβ = 1). For wavelengths below 8250 Å, the strengths were obtained by calibrating the observed relative intensities (derived from a density tracing of a calibrated stepwedge) by a 1-hour exposure of the white dwarf EG54 which has been measured in absolute energy units by Oke

Table 13. Line identifications in the ITT Orion Nebula spectrum.

λobs (Å)	ID (Å)	Ion	Strength (Hβ = 1)	Corrected Strength	λobs	ID (Å)	Ion	Strength (Hβ = 1)
... .	6548.1	[NII]	*		8272.1.......	8271.9	P33	
... .	6562.8	Hα	*		8276.7.......	8276.3	P32	
... .	6583.4	[NII]	*		8281.4.......	8281.4	P31	
6677.8.........	6678.1	HeI	*		8287.5.......	8286.4	P30	
6716.3.........	6716.4	[SII]	0.05	0.03	8292.5.......	8292.3	P29	
6730.4.........	6730.8	[SII]	0.09	0.06	8299.1.......	8298.8	P28	
7002.6.........	7002.2	OI	0.003	0.002	8306.6.......	8306.1	P27	
7065.1.........	7065.3	HeI	*		8314.7.......	8314.3	P26	
7136.1.........	7135.8	[ArIII]	*		8323.8.......	8323.4	P25	
7155.1.........	7155.1	[FeII]	0.009	0.005	8334.5.......	8334.8	P24	
7231.2.........	7231.3	CII	0.01	0.005	8345.8.......	8345.6	P23	
7236.9.........	7236.4	CII	0.01	0.005	8359.2}	8359.0	P22	
7254.2.........	7254.4	OI	0.01	0.005	8361.2}	8361.7	HeI	
7281.6.........	7281.3	HeI	0.03	0.02	8374.9.......	8374.5	P21	
7320.3.........	7319.9	[OII]	*		8393.0.......	8392.8	P20	
7330.2.........	7330.2	[OII]	*		8413.7.......	8413.3	P19	
7378.6.........	7377.9	[NiII]	0.005	0.003	8438.3.......	8438.0	P18	
7412.1.........	7411.6	[NiII]	0.002	0.001	8447.2.......	8446.4	OI	0.012
7469.0.........	7468.3	NI	0.005	0.002	8467.7.......	8467.3	P17	
7751.3.........	7751.0	[ArIII]	*		8502.9.......	8502.5	P16	

Table 13. Line identifications in the ITT Orion Nebula spectrum--continued.

λobs (Å)	ID (Å)	Ion	Strength (Hβ = 1)	Corrected Strength	λobs	ID (Å)	Ion	Strength (Hβ = 1)
7816.3.........	7816.2	HeI	0.004	0.002	8545.8.......	8545.4	P15	
8046.5.........	8046.1	[ClIV]	0.003	0.001	8579.5**.....	8579.5	[ClII]	0.001
8185.1.........	8184.8	NI	0.004	0.002	8598.8.......	8598.4	P14	
8188.7.........	8188.0	NI	0.005	0.002	8617.1.......	8617.0	[FeII]	0.0007
8204.9.........			0.002	0.0009	8665.6.......	8665.0	P13	
8216.9.........	8216.3	NI	0.006	0.003	8751.0.......	8750.5	P12	
8223.7.........	8223.1	NI	0.008	0.003	8863.5.......	8862.8	P11	
8243.4.........	8242.3	NI	0.004	0.002	9015.7.......	9014.9	P10	
8268.0.........	8267.9	P34			9069.6.......	9069.0	[SIII]	

*Line is overexposed.
**Line appears to be a blend.

Table 14. Line identifications in the RCA Orion Nebula spectrum.

λobs (Å)	ID (Å)	Ion	Strength (Hβ = 1)	Corrected Strength
4959.1.....	4958.9	[OIII]	*	...
... .	5006.8	[OIII]	*	...
... .	5015.7	HeI	*	...
5047.1.....	5047.7	HeI	0.002	0.002
5056.2.....	5056.1	SiII	0.002	0.002
5158.5.....	5158.3	[FeVII]	0.002	0.002
5191.6.....	5191.4	[ArIII]	0.002	0.002
5198.6.....	5198.7	[NI]	0.006	0.006
5270.5.....	5270.3	[FeIII]	0.006	0.006
5513.4.....	5513.7	OI	0.001	0.0009
5517.5.....	5517.7	[ClIII]	0.007	0.006
5537.7.....	5537.6	[ClIII]	0.01	0.009
5554.6.....	5554,9	OI	0.001	0.0009
5576.9.....	5577.4	[OI]	0.005	0.004
5679.7.....	5679.6	NII	0.002	0.002
5754.7.....	5754.6	[NII]	0.02	0.02
5875.7.....	5875.6	HeI	*	...
5931.4.....	5931.8	NII	0.0005	0.0004
5941.1.....	5941.7	NII	0.0008	0.0006
5958.0.....	{5957.6	SiII}	0.003	0.002
	{5958.5	OI }		
5978.9.....	5979.0	SiII	0.003	0.002
6046.4.....	6046.4	OI	0.003	0.002
6087.3.....	...	...	0.002	0.002
6300.6.....	6300.3	[OI]	0.008	0.006
6312.4.....	6312.4	[SIII]	0.009	0.007

*Line is overexposed.

(1974). These strengths were normalized to Hβ via the line strength of the [SII] doublet at λ6716/6730 as given by Aller and Liller (1959). The line strengths were then fully corrected for the smoothed reddening of the Trapezium as listed in Mathis (1957) and discussed in Johnson (1968). Column (5) of Table 13 contains the line strengths corrected for reddening. For lines longward of 8250 Å, the strengths were compared with those of the neighboring Paschen lines which were in turn related to the strength of Hβ through the theoretical hydrogen recombination line strengths for $T_e = 10^4$K and $N_e = 10^4$ calculated by Brocklehurst (1971). Reddening corrections are, of course, unnecessary for these values.

Table 14 lists the lines identified on the RCA spectra. While the wavelength calibration of the ITT plate came from an He-Ar comparison source, the easily identified strong nebular lines were used to calibrate the RCA plate. The strength estimates contained in column (4) of Table 14 were obtained by normalizing the observed intensities of the stronger lines to the values listed in Aller and Walker (1970). These strengths were corrected for reddening as described above and the corrected values are listed in column (5).

The line identifications in both Tables 13 and 14 came from Johnson (1968), Morgan (1971), Aller and Walker (1970), and Danziger and Aaronson (1974). OH airglow lines identified by Chamberlain (1961) were excluded from consideration, as were unidentified lines which were also present on the spectrum of EG54.

Lines of [NiII] are here detected in the Orion Nebula for the first time. The two strongest members of the a^2D (the ground term)-a^2F multiplet at λ7377.9 and λ7411.5 (Garstang, 1958; Thackeray, 1953) are present. Although NiII has an ionization potential of only 18.2 eV and is therefore unlikely to be the dominant ionization stage of Ni, FeII has an even smaller ionization potential (16.2 eV) and many lines of [FeII] have been identified in the spectra of the Orion Nebula (Wyse, 1942). The two strongest members of the NII 3p ^3P-3d $^3D°$ multiplet at 5941.7 and 5931.8 Å are also identified for the first time in the Orion Nebula.

Permitted lines of NI are also seen for the first time. The five strongest members of the multiplet 3s ^4P - 3p $^4P°$ at 8184-8242 Å and the strongest member of the multiplet 3s ^4P - 3p $^4S°$ at 7468 Å are all identified.

Morgan (1971) identified the same OI lines that we report from our own spectra, and her line strengths agree reasonably well with those reported here, except for λ5513.7. Morgan lists this line as four times stronger than our strength estimate. There is a possibility of a very weak line at 7772.9 Å that might be OI λ7773.4 (the quintet counterpart of λ8446) which was identified by Morgan (1971), but this is highly uncertain.

Several far-red spectra of planetary nebulae were taken with the ITT image tube at the same dispersion and with the same wavelength coverage as the Orion Nebula spectrum discussed above. The purpose of these observations was mainly to search for OI λ8446 and the weaker

OI lines, but it was also found that spectra obtained showed weaker lines than the spectra presented by Andrillat and Andrillat (1961), Andrillat and Houziaux (1968a) and Andrillat et al. (1975). (However, the spectrum of NGC7027 discussed by Kolotilov and Noskova (1974) goes as deep as our spectrum of this object.) It is of some interest, therefore, to list the line identifications for these spectra. Generally, the far-red spectra of these planetary nebulae fit well with their visible spectra in terms of ion excitation. The lines seen in each nebula are listed in Tables 15 through 21 for NGC3242, NGC6210, NGC2392, IC3568, IC4997, NGC7027 and NGC7662 respectively.

Besides the wavelength and source ion for each line, an estimate of the strength of the line on the plate is given on a scale from 0 to 5 for all lines with wavelengths shorter than λ8300. A strength of 5 indicates a completely burned out line while a strength of 0 indicates a line whose existence is dubious. Crude strength estimates were made for lines intermixed with the Paschen series by taking a ratio between the line and adjacent Paschen lines (using an average HD curve) and then using recombination theory to estimate a line ratio to Hβ (see above). Strengths estimated in this way are also listed in Tables 15 through 21.

Uncalibrated scans of IC4997 obtained during an early run with the Steward Observatory S.I.T. photon counting spectrograph system (Gilbert, Angel and Grandi, 1974) confirm the line identifications listed in Table 18. Also, the λ7774 line of OI is seen on these scans, indicating that recombination is present in IC4997. However, since

Table 15. Line identification in NGC3242 (2 hour exposure). -- Bright double ring planetary.

λ	Ion	Strength	λ	Ion	Strength
6300.2	[OI]	2	7592.7	HeII	3
6548.1	[NII]	5	7751.0	[AIII]	4
6562.8	Hα	5	8045.6	[ClIV]	3
6583.4	[NII]	5	8196.5	CIII	1
6678.1	HeI	4	8236.8	HeII	3
6716.4	[SII]	1	8359.0	P22	
6730.8	[SII]	2	8374.5	P21	
6890.9	HeII	2	8381		
7005.6	[AV]	1	8392.4	P20	
7065.3	HeI	4	8413.3	P19	
7135.8	[AIII]	5	8438.0	P18	
7170.6	[AIV]	1	8467.3	P17	
7177.5	HeII	2	8502.5	P16	
7237.3	[AIV]	2	8545.4	P15	
7262.8	[AIV]	1	8598.4	P14	
7281.3	HeI	3	8665.0	P13	
7319.9	[OII]	3	8750.5	P12	
7330.2	[OII]	3	8862.8	P11	
7530.5	[ClIV]	2	9069.0	[SIII]	

Table 16. Line identification in NGC6210 (2-1/2 hour exposure).

λ	Ion	Strength	λ	Ion	Strength
6300.2	[OI]	4	8333.8	P24	
6312.1	[SIII]	4	8345.6	P23	
6363.8	[OI]	4	8359.0	P22	
6548.1	[NII]	5	8374.5	P21	
6562.8	Hα	5	8381		.004
6583.4	[NII]	5	8392.4	P20	
6678.1	HeI	4	8413.3	P19	
6716.4	[SII]	4	8438.0	P18	
6730.8	[SII]	4	8446.4	OI	.0008
7065.3	HeI	4	8467.3	P17	
7135.8	[AIII]	4	8502.5	P16	
7281.3	HeI	3	8545.4	P15	
7319.9	[OII]	4	8579.5	[ClII]	.003
7330.2	[OII]	4	8598.4	P14	
7530.5	[ClIV]	2	8665.0	P13	
7751.0	[AIII]	4	8750.5	P12	
8045.6	[ClIV]	3	8862.8	P11	
8314.3	P26		9014.9	P10	
8323.4	P25		9069.3	[SIII]	

Table 17. Line identification in NGC2392 (3 hour exposure). -- Fragmented double ring planetary - Eskimo Nebula.

λ	Ion	Strength	λ	Ion	Strength
6300.2	[OI]	1	7177.5	HeII	2
6312.1	[SIII]	2	7262.8	[AIV]	1
6363.0	[OI]	1	7281.3	HeI	2
6548.1	[NII]	4	7319.9	[OII]	3
6562.8	$H\alpha$	5	7330.2	[OII]	3
6583.4	[NII]	4	7530.5	[ClIV]	1
6678.1	HeI	3	7592.7	HeII	1
6716.4	[SII]	4	7751.0	[AIII]	3
6730.8	[SII]	4	8045.6	[ClIV]	1
6890.9	HeII	1	8236.8	HeII	2
7005.7	[AV]	1	8502.5	P16	
7065.3	HeI	3	8545.4	P15	
7135.8	[AIII]	4	8598.4	P14	
7154		1	8665.0	P13	
7170.6	[AIV]	1	8750.5	P12	

Table 18. Line identification in IC3568 (3-1/2 hour exposure). -- Symmetrical, low excitation planetary.

λ	Ion	Strength	λ	Ion	Strength
6312.1	[SIII]	1	8045.6	[ClIV]	1
6548.1	[NII]	2	8392.4	P20	
6562.8	Hα	5	8413.3	P19	
6583.4	[NII]	3	8438.0	P18	
6678.1	HeI	4	8467.3	P17	
7065.3	HeI	4	8502.5	P16	
7136.8	[AIII]	4	8545.4	P15	
7281.3	HeI	3	8598.4	P14	
7319.9	[OII]	3	8665.0	P13	
7330.2	[OII]	3	8750.5	P12	
7530.5	[ClIV]	1	8862.8	P11	
7751.0	[AIII]	3			

Table 19. Line identification in IC4997 (1-1/4 hour exposure). -- Bright, intermediate excitation, variable planetary.

λ	Ion	Strength	λ	Ion	Strength
6548.1	[NII]	4	8298.8	P28	
6562.8	Hα	5	8306.1	P27	
6583.4	[NII]	4	8314.3	P26	
6678.1	HeI	4	8323.4	P25	
6716.4	[SII]	2	8333.8	P24	
6730.8	[SII]	3	8345.6	P23	
7065.3	HeI	4	8359.0	P22	
7135.8	[AIII]	4	8374.5	P21	
7154		1	8392.4	P20	
7170.6	[AIV]	2	8413.3	P19	
7237.3	[AIV]	1	8438.0	P18	
7254.4	OI	1	8446.4	OI	.02
7262.8	[AIV]	1	8467.3	P17	
7281.3	HeI	3	8502.5	P16	
7319.9	[OII]	4	8545.4	P15	
7330.2	[OII]	4	8598.4	P14	
7751.0	[AIII]	4	8665.0	P13	
7816.2	HeI	1	8750.5	P12	
8242.3	NI	1	8862.8	P11	
8281.1	P31		9014.9	P10	
8286.4	P30		9059.3	[SIII]	
8292.3	P29				

Table 20. Line identification in NGC7027 (1-1/2 hour exposure). -- Bright planetary.

λ	Ion	Strength	λ	Ion	Strength
6548.1	[NII]	5	8286.4	P30	
6562.8	$H\alpha$	5	8292.3	P29	
6583.4	[NII]	5	8298.8	P28	
6678.1	HeI	4	8306.1	P27	
6716.4	[SII]	4	8314.3	P26	
6730.8	[SII]	4	8323.4	P25	
6890.9	HeII	2	8333.8	P24	
7005.7	[AV]	4	8345.6	P23	
7065.3	HeI	4	8359.0	P22	
7135.8	[AIII]	5	8374.5	P21	
7154		1	8392.4	P20	
7170.6	[AIV]	3	8413.3	P19	
7177.5	HeII	3	8438.0	P18	
7237.3	[AIV]	3	8446.4	OI	
7262.8	[AIV]	2	8467.3	P17	
7281.3	HeI	3	8502.5	P16	
7319.9	[OII]	5	8545.4	P15	
7330.2	[OII]	5	8579.5	[ClII]	
7530.5	[ClIV]	3	8598.4	P14	
7592.7	HeII	3	8665.0	P13	
7726.2	CIV	2	8727.4	[CI]	
7751.0	[AIII]	4	8750.5	P12	
8045.6	[ClIV]	3	8862.8	P11	
8196.6	CIII	2	9014.9	P10	
8236.8	HeII	3	9069.4	[SIII]	
8281.1	P31		9229.0	P9	

Table 21. Line identification in NGC7662 (3 hour exposure). -- Bright, double ringed planetary.

λ	Ion	Strength	λ	Ion	Strength
6548.1	[NII]	1	7319.9	[OII]	2
6562.8	$H\alpha$	5	7330.2	[OII]	2
6583.4	[NII]	2	7530.5	[ClIV]	2
6678.1	HeI	3	7592.7	HeII	2
6716.4	[SII]	1	7751.0	[AIII]	3
6730.8	[SII]	1	8045.6	[ClIV]	2
7065.3	HeI	3	8196.5	CIII	1
7135.8	[AIII]	3	8236.8	HeII	1

λ8446 is much stronger than λ7774, the conclusion that resonance fluorescence by starlight is dominant is still a valid one.

Two unidentified lines (λ7154 and λ8381) that are present in more than one of the planetary nebulae spectra deserve notice. The λ8381 line is contaminated by an OH night sky line which might mask its presence in other nebula.

Finally, ITT spectra of two reflection nebulae were obtained in a search for OI λ8446 emission. Unfortunately, 1 and 3 hour exposures of IC431 and a 2 hour exposure of NGC2068 (M78) show nothing but night sky lines.

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