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IMAGE QUALITY CRITERIA FOR ABERRATED SYSTEMS

by

Melvyn Howard Kreitzer

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THE UNIVERSITY OF ARIZONA

GRADUATE COLLEGE

I hereby recommend that this dissertation prepared under my direction by Melvyn Howard Kreitzer

entitled Image Quality Criteria for Aberrated Systems

be accepted as fulfilling the dissertation requirement for the degree of Doctor of Philosophy

Roland V. Shack — 27 AUG 76

Dissertation Director — Date

As members of the Final Examination Committee, we certify that we have read this dissertation and agree that it may be presented for final defense.

Robert R. Shannon — 27 August 76

James C. Wyant — 30 August 76

Final approval and acceptance of this dissertation is contingent on the candidate's adequate performance and defense thereof at the final oral examination.

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ABSTRACT

Image quality criteria that have quadratic aberrational dependence find wide application in the design and analysis of optical systems. Those members of this class that have direct physical significance are of special interest, particularly for use during the final aberration-balancing stages of the design process.

In this work a continuum is established between the physically significant extremes of the mean-squared spot size and the variance of the wavefront aberration. This is achieved by obtaining an analytical second-order expansion of the optical transfer function (OTF), including most of the wave aberrations through eighth order. This expression is then multiplied by a weighting function that approximates the transfer function of a typical detector and the product is integrated over spatial frequency and azimuthal angle. The continuum is a smooth function of a single parameter characterizing the width of the weighting function. The second-order OTF expansion and the continuum obtained from it are both expressed in orthonormalized form so that a detailed study can be made of the behavior of the separable balancing and weighting coefficients.

Results comparing the use of all these criteria are established by exact OTF calculations via autocorrelation of the pupil function. It is shown that the variance of the wavefront aberration is consistently the most successful aberration-balancing criterion within the

class being studied, regardless of the level of aberration. It is emphasized that this result applies only to the balancing coefficients associated with the variance criterion. The mean-squared spot size is shown to have limited applicability in terms of aberration-balancing, being suitable only when the OTF is high. This is because the use of this criterion is equivalent to a consideration of only the curvature, at the origin, of the OTF. It is also indicated that the choice of aberration-balancing criterion is important mainly at lower relative modulation (below about 0.6) where convenient analytical treatments are not available. Under these circumstances, the variance criterion offers the closest systematic approach to the optimal balancing of aberrations. At higher relative modulation, differences between the use of the variance and the OTF-based criteria are very small.

CHAPTER 1

INTRODUCTION

There is a class of optical image quality criteria characterized by having quadratic functional dependence on the wave aberration coefficients. This functional form is generally typical of the merit function in automatic optical design. There are certain members of this class that have direct physical significance, lending usefulness particularly to the final stages of automatic design. The mean-squared spot size and the variance of the wavefront aberration are generally regarded as being useful criteria in the extremes of systems having large and small amounts of aberration. A second-order expansion of the optical transfer function (OTF), known as the Hopkins criterion, occupies an intermediate position.

The objectives of this study were to develop a logical continuum between the spot-size and variance criteria and to explore the resultant gamut in terms of extent of validity and applicability to optical design. In this latter context, the problem of optimal balancing of aberrations is of particular interest.

The approach is firstly to develop an analytical version of the second-order OTF expansion up to, and including, most of the eighth-order wave aberration coefficients. It is intuitively obvious that this expression will have a close relationship to the spot-size criterion (at sufficiently low frequencies) and to the variance criterion (for sufficiently small aberrations). Once a suitable continuum is established, the process of orthonormalization is used to eliminate cross-terms in the aberration coefficients. This orthonormal form greatly facilitates the subsequent analysis and presentation of results.

Background

Two well-known optical image quality criteria date from the nineteenth century. The Rayleigh quarter wavelength rule (Born and Wolf, 1975, p. 468) imposes a requirement on the maximum deformation of the wavefront, relative to the exit pupil reference sphere. This concept is useful primarily for the establishment of tolerances on individual aberrations. It is less useful in the general case of mixed aberrations, where the shape of the wavefront and not just the maximum deformation requires consideration.

The Strehl Ratio (Born and Wolf 1975, pp. 461-2) compares the irradiance at the diffraction focus (point of maximum irradiance) of an aberrated point spread function relative to that for the same system without aberrations. It is physically meaningful only for small aberrations, under which circumstances the diffraction focus is unique. It is well-known experimentally that the primary effect of individual aberrations and defocus, in amounts of up to about a quarter of a wavelength, is a drain of energy away from the central maximum of the diffraction image. Quantitative calculations of this sort were first made in 1879 by Rayleigh; Richter, in 1925, was the first to calculate aberration-balancing relationships, for small amounts of aberration, using the

Strehl Ratio as a criterion (Born and Wolf, 1975, pp. 467-8). A significant development by Maréchal (1947) relates a physically meaningful criterion, viz., the concept of the Strehl Ratio for small aberrations, to the variance of the wavefront aberration. The variance of the wavefront aberration, is, in turn, easily related to the classical wave aberration coefficients. Alternatively, if the aberration function is expanded in terms of Zernike polynomials, the corresponding coefficients can be directly related to the Strehl Ratio in an analogous manner. The restriction to small aberrations still applies. Maréchal developed extensive tolerancing and balancing results, based on the requirement of the Strehl Ratio exceeding 0.8. The usefulness of the Maréchal relationship lies in the ability to consider the significance of an arbitrary combination of aberrations, subject to a prescribed value of the Strehl Ratio. The Maréchal tolerancing results correspond approximately, in terms of maximum allowable aberration, to the Rayleigh limit for individual aberrations.

Use of the Optical Transfer Function (Duffieux and Lansraux, 1945) is extensive in modern optical image evaluation. Its use in design is limited primarily to post facto analysis, since the mathematical form is generally complicated. The formal relationship to image quality is in terms of the Fourier Transform of the point spread function, under conditions of linearity and spatial stationarity. In terms of the scope of this investigation, useful approximations to the exact OTF are of interest. Steel (1953) obtained an expansion of the OTF to second order in the wave aberration coefficients. The resultant form is accurate for sufficiently small aberrations at all spatial frequencies; for larger aberrations accuracy is retained at progressively lower frequencies. This form of criterion was extensively developed by Hopkins (1957a). His results are formulated in a manner analogous to the Maréchal relationships. The relative modulation, as a function of the normalized spatial frequency, is defined as the ratio of the value of the OTF of an aberrated system to that of the same system without aberrations. This is related to the variance of the wave aberration difference function, which occurs in the argument of the autocorrelation formulation of the OTF. The requirement, both for accuracy and for satisfactory image quality, is that the relative modulation should exceed 0.8. The Hopkins criterion was used to establish aberration-tolerancing and balancing data, as functions of spatial frequency. The aberration-balancing information is potentially of the greater significance in optical design. Subsequently it was shown by King (1969) that the Hopkins criterion is reliable for relative modulations exceeding 0.6-0.7, depending on the type of aberration.

Another often-used approximation to the exact OTF is an expansion to second-order in spatial frequency (e.g., Sayanagi, 1963). Linfoot (1959) has shown that this form of approximation is equivalent to the concept of the mean-squared spot size or the radius of gyration of the point spread function. This is normally used in the context of the geometrical point spread function, but Linfoot has indicated that an extension to the diffraction image is valid, provided that the increase in the moment of inertia of the diffraction image is considered. This is to avoid the problem of infinite second moments occurring, for example, in the simple case of an aberration-free diffraction point spread function. In the historical context, criteria such as the mean-squared spot size and the minimum circle of confusion preceded the development of the OTF, since these were intuitive concepts in an era when diffraction-based criteria were either not available or were extremely tedious to evaluate.

A geometric-optical analysis of image formation is considered appropriate for systems having large aberrations because, under these circumstances, the point spread function predicted by geometrical optics, is similar, in general form, to the actual point image. More specifically, Hopkins (1955) and Miyamoto (1961) showed that geometric-optical analyses become increasingly accurate at lower spatial frequencies, for systems having aberrations exceeding about 2λ. Thus a strongly detector-limited optical system, having large aberrations, may reasonably be expected to be described in terms of optical performance by an image quality criterion such as the mean-squared spot size. In the other extreme, the variance of the wavefront aberration is a valid criterion for systems having small aberrations, where, in addition, good performance at all spatial frequencies up to the optical cut-off is of interest.

The development of the modern computer introduced the concept of the so-called merit function into optical design. In its most fundamental form, this function is expressed as a weighted sum of squared image defect terms. This general functional form is also characteristic of the three criteria discussed thus far, viz., the mean-squared spot size, the variance of the wavefront aberration and the Hopkins criterion. This suggests, as shown by Hopkins (1966), that these criteria can be directly incorporated as merit functions in automatic optical design. The need for the merit function to have physical significance is important mainly during the final stages of a design procedure (Glatzel and Wilson, 1967). This stage would include the optimal balancing of the residual higher-order aberrations by the adjustment of lower-order aberrations and by the location of the focal surface. This problem of the so-called aberration-balancing merit function has been discussed by many authors (e.g., Hopkins, 1966; King, 1970; Offner, 1970; and Itoh, 1971). Even if a merit function is not used, as in the design programs of Glatzel, the use of these criteria provides appropriate target-values for the aberration-balancing stage.

Although use of the spot-size and variance criteria is physically justifiable in the respective extremes discussed earlier, it is relevant to point out that this segregation is not rigorously adopted. Thus Linfoot (1955, p. 31) has used the spot-size criterion for the optimization of a Schmidt system having small aberrations. Unvala (1967) and Meiron (1968) have indicated that the variance criterion may be used beyond the limit of well-corrected systems; this is justified on the basis of its general geometrical significance in terms of the shape of the wavefront. This extended-range use has also been discussed by Tatian (1974). The quantitative extent to which these procedures are justified has not been discussed. Indeed, a comparative evaluation of all these criteria, in terms of the relative extent to which the OTF can be affected, has not been made. This leads to the statement of the problem to be investigated, as discussed in the next section.

Statement of the Problem

The fundamental task in this dissertation was to establish a continuum between the extremes of the mean-squared spot size and the variance of the wavefront aberration. The approach was to be along the lines of the Hopkins criterion, but the mathematical procedure was similar to that in the work by Steel, except that the classical expansion of the wavefront aberration function was adopted. The work by Hopkins discusses individual aberration types and includes the fourth-order classical wavefront aberrations, together with sixth-order spherical aberration and linear coma. Hopkins used a rectangular coordinate system in the integration over the sheared pupil; this necessitated the use of tabulated integrals for the subsequent presentation of the results. The use of the appropriate polar coordinates permits an analytical result which is potentially more conducive to the establishment of the mathematical trends stated earlier. The task was also to include all the wavefront aberrations through sixth order, together with eighth-order spherical aberration. An eighth-order oblique spherical aberration term was subsequently added.

The Hopkins criterion does not, in itself, form a logical continuum between the spot-size and variance criteria. However a possible process is suggested by the relationship of the spot-size criterion to the low-frequency OTF on the one hand, and by the relationship of the variance criterion to the normalized volume under the OTF, on the other hand. A reasonable continuum might be a progressive integration of the Hopkins criterion over increasingly larger ranges of spatial frequency. A logical reason for the choice of spatial frequency range might be the characteristics of a potential detector.

Once the appropriate criterion has been obtained, an important procedure is that of orthonormalizing the resultant quadratic form in order to eliminate cross-terms in the aberration coefficients. This procedure greatly simplifies the presentation of results, since the weighting and balancing coefficients are immediately identifiable. The orthonormalized form is both elegant and practical, since it is directly suitable for use as a merit function in automatic design.

Outline of the Remaining Chapters

The Hopkins criterion is developed in Chapter 2. The orthonormalization procedure is described and used to display the variation, with spatial frequency, of the coefficients in the Hopkins criterion.

In Chapter 3, the continuum between the spot-size and variance criteria is obtained. The coefficients are again plotted as a function of a parameter characterizing the properties of a hypothetical detector. The transitions to the two limits are shown and discussed. Chapter 4 is devoted to a discussion of aberration-balancing. This is done in the context of the so-called aberration-balancing merit function. It is shown that the use of the variance criterion in this connection is widely justifiable and that the Hopkins criterion confers only slight advantages at high relative modulation. At low values of the transfer function, the spot-size criterion yields the poorest results.

In Chapter 5, the extended-range use of the variance criterion as a merit function is discussed. Although this usage is often adopted, it is important that justification be made in terms of the extent to which the OTF can be optimized.

A summary of the results and the significant conclusions are given in Chapter 6.

CHAPTER 2

THE SECOND-ORDER EXPANSION OF THE OPTICAL TRANSFER FUNCTION

Mathematical Development

The following work is performed under the assumptions of incoherent irradiance, a circular exit pupil and the usual simplifying conditions of scalar diffraction theory, as applied to problems of image formation. Under these conditions, an unaberrated wavefront will yield a classical Fraunhofer diffraction pattern centered on the Gaussian image point; aberrations are regarded essentially as perturbations from a reference sphere centered on the Gaussian image point. These assumptions and simplifications are reasonable in the study of photographic lenses of moderate aperture and field coverage.

The two-dimensional OTF is the Fourier Transform of the point spread function. It is usually more convenient to consider a number of one-dimensional transfer functions, obtained as cross-sections of the two dimensional OTF, at various azimuthal angles. As is well-known, these one-dimensional OTFs are identical to the Fourier Transforms of the corresponding spread functions. Any desired section of the two-dimensional OTF can therefore be obtained by considering the imaging of an infinite line object, rotated to the appropriate azimuthal angle. It is necessary to formulate the aberration function so that this rotation, which is effectively a rotation of the coordinate systems throughout the optical system, is taken into account.

The Optical Transfer Function $O(\omega,\theta)$ can also be expressed as the autocorrelation of the pupil function $P(x,y)$ (e.g., Hopkins, 1957a)

$$O(\omega,\theta) = \frac{1}{\pi} \iint_{-\infty}^{\infty} P(x,y)\ P^*\ (x-2\omega,y)\ dxdy \qquad (2.1)$$

where (x,y) are the reduced pupil coordinates, ω is the normalized spatial frequency and θ is the azimuthal angle of the frequency component.

The coordinate system and the various parameters in the above expression are shown in Fig. 2.1. The pupil function $P(x,y)$ is related to the aberration function $W(x,y)$

$$P(x,y) = e^{\frac{2\pi i}{\lambda} W(x,y)} \qquad \text{within the pupil}$$

$$= 0 \qquad \text{outside the pupil} \qquad (2.2)$$

Since the calculation is to be performed with respect to the coordinate system (x,y) and the aberration function is normally expressed in the unrotated system it is necessary to re-express the aberration function. This is most simply done if polar coordinates are used; the aberration function is then formulated in terms of the rotational invariants

$$H^2\ ,\ \rho^2\ ,\ H\rho\cos(\phi-\theta)$$

where the (ρ,ϕ) system corresponds to the (x,y) rectangular coordinates; H is the normalized field parameter.

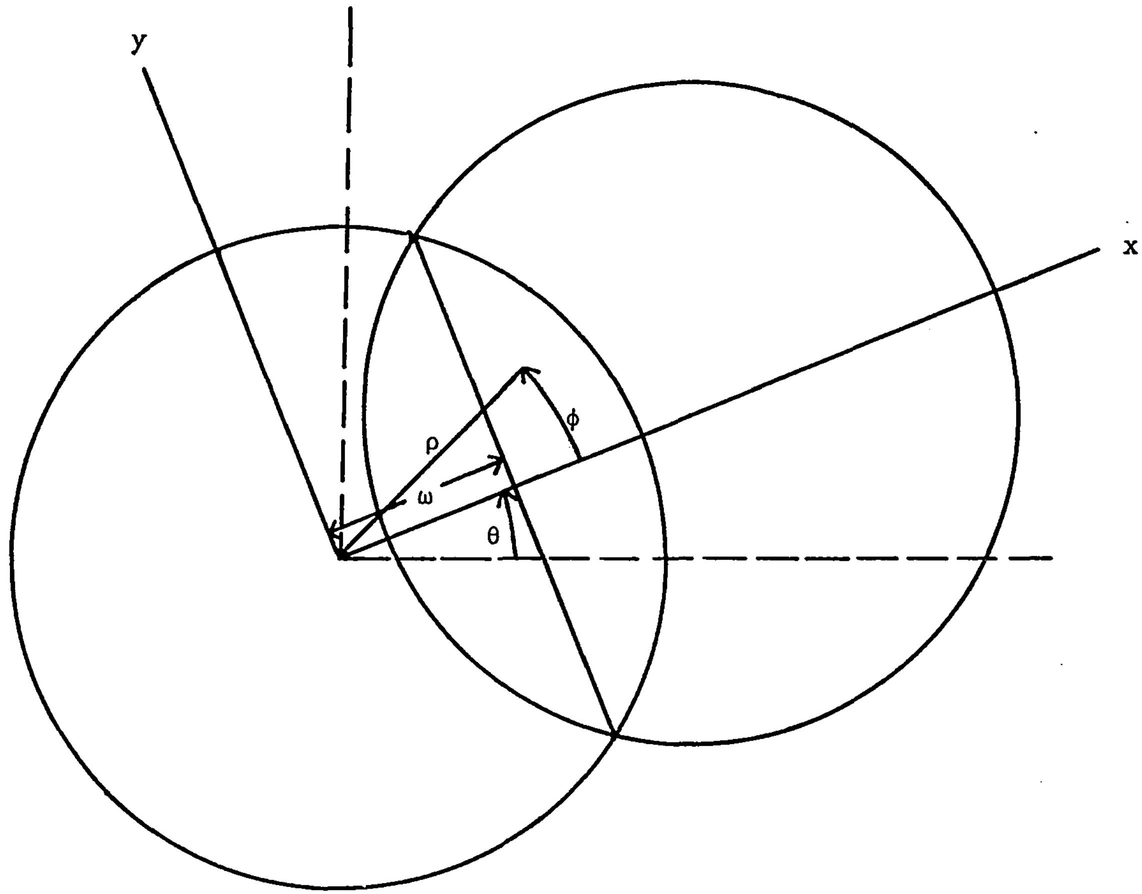

Fig. 2.1. The Coordinate System and Parameters Involved in the Calculation of the OTF via Autocorrelation of the Pupil Function.

The second-order expansion of the real part of $O(w,\theta)$ is given by

$$S(\omega,\theta) = \frac{1}{\pi}\left[\int\int \rho\, d\rho\, d\phi - \frac{2\pi^2}{\lambda^2}\int\int (W-W')^2\, \rho\, d\rho\, d\phi\right] \tag{2.3}$$

where the region of integration is the area common to both pupils (cf. Fig. 2.1) and W and W' are W(x,y) and W(x-2w,y), re-expressed in the (ρ,ϕ) coordinates.

In terms of the classical wave aberrations the expansion used for this work is

$$\begin{aligned}\frac{W}{\lambda} = {} & B\cos(\phi-\theta) + D\rho^2 + A\rho^2\cos^2(\phi-\theta) + C_1\rho^3\cos(\phi-\theta) \\ & + EC\rho^3\cos^3(\phi-\theta) + S_1\rho^4 + OB_1\rho^4\cos^2(\phi-\theta) + C_2\rho^5\cos(\phi-\theta) \\ & + S_2\rho^6 + OB_2\rho^6\cos^2(\phi-\theta) + S_3\rho^8\end{aligned} \tag{2.4}$$

where

$$B = Hw_{111} + H^3w_{311} + H^5w_{511} + H^7w_{711} + \ldots$$

$$D = w_{020} + H^2w_{220} + H^4w_{420} + H^6w_{620} + \ldots$$

$$A = H^2w_{222} + H^4w_{422} + H^6w_{622} + \ldots$$

$$C_1 = Hw_{131} + H^3w_{331} + H^5w_{531} + \ldots$$

$$EC = H^3w_{333} + H^5w_{533} + \ldots$$

$$S_1 = w_{040} + H^2w_{240} + H^4w_{440} + \ldots$$

$$OB_1 = H^2w_{242} + H^4w_{442} + \ldots$$

$$C_2 = Hw_{151} + H^3w_{351} + \ldots$$

$$S_2 = w_{060} + H^2w_{260} + \ldots$$

$$OB_2 = H^2w_{262} + \ldots$$

$$S_3 = w_{080} + \ldots$$

where the w_{ijk} are the classical wave aberration coefficients.

W' is obtained by substituting $\rho\cos\phi$ - 2w for $\rho\cos\phi$ in the expression for W, taking care to express ρ^2 as $\rho^2\cos^2\phi + \rho^2\sin^2\phi$, etc.

The resulting expression for W-W' is given in Appendix A. The calculation of S(w,θ) can now be done analytically once the limits of integration have been established. It can be seen in Fig. 2.1 that, over either half of the area of integration, the limits of integration for ρ are from $\frac{\omega}{\cos\phi}$ to 1, while ϕ ranges from -a to +a, where a = arc cos ω.

Some additional simplifications can be made. The calculation of S(w,θ) involves the integration of $(W-W')^2$. This will include even-even, even-odd and odd-odd aberration product terms. The even-odd products will cancel over the halves of the area of integration whereas the other products will reinforce.

The final expression for the second-order expansion of the OTF is therefore

$$S(\omega,\theta) = \frac{2}{\pi}\left[\int_{-a}^{a}\int_{\frac{\omega}{\cos\phi}}^{1} \rho\, d\rho\, d\phi - \frac{2\pi^2}{\lambda^2}\int_{-a}^{a}\int_{\frac{\omega}{\cos\phi}}^{1} (W-W')^2\, \rho\, d\rho\, d\phi\right]$$

$$= \frac{2}{\pi}(a - \omega\sin a) - \frac{4\pi}{\lambda^2}\int_{-a}^{a}\int_{\frac{\omega}{\cos\phi}}^{1} (W-W')^2\, \rho\, d\rho\, d\phi \qquad (2.5)$$

where $a = \text{arc}\cos\omega$

and $\sin a = (1-\omega^2)^{\frac{1}{2}}$

The evaluation of S(ω,θ) is lengthy but straightforward; the result in given in Appendix B. This second-order expansion of the real part of the OTF forms the basis for most of this work. However, a corresponding result for the imaginary part of the OTF is also useful and may be obtained as follows:

The exact OTF is given by

$$O(\omega,\theta) = \frac{1}{\pi} \iint e^{i\,K} \rho\, d\rho\, d\phi$$

where $K = \frac{2\pi}{\lambda}(W-W')$ and the integration is over the overlapping pupil region.

The imaginary part is given approximately by

$$I(\omega,\theta) \approx \frac{4}{\lambda} \int_{-a}^{a} \int_{\frac{\omega}{\cos\phi}}^{1} (W-W')\, \rho\, d\rho\, d\phi \qquad (2.6)$$

where (W-W') is given in Appendix A and the integral is evaluated in Appendix C.

The modulus and phase of the OTF may be obtained from the expressions for the real and imaginary parts in the usual manner.

The second-order expressions for the modulus, phase, real and imaginary parts of the OTF have been programmed. The DIFFMTF program, listed in Appendix D, permits any of these quantities to be calculated as functions of the wavefront aberration, spatial frequency and azimuthal angle.

In the remainder of this work the real part of the OTF, as opposed to the modulus, will be used. There is, of course, no difference as far as the spherical and astigmatic aberration types are concerned. Use of the azimuth-averaged real part will provide the link between the variance and spot-size extremes mentioned in Chapter 1.

In addition, the real part of the OTF retains aberration terms involving the B coefficient (lateral shift and distortion). This is desirable both for completeness and for any potential work in which accurate knowledge of the optimum lateral shift is required. The modulus of the OTF is, of course, independent of the lateral shift term; by the Fourier Transform relationship between the OTF and the intensity point spread function a lateral shift in the image space affects only the phase of the OTF. In addition, as mentioned above, the second-order expansion of the real part of the OTF will be averaged over azimuth for the work that follows.

Orthonormalization of a Quadratic Form

The second-order expansion of the (azimuth-averaged) OTF is a quadratic function of the wave aberration coefficients. It is highly desirable to orthonormalize this expression in order to eliminate cross-terms between the aberration coefficients. The resultant form will be a sum of squared terms each of which is a linear expression in the aberration coefficients. Minimization of this form is obtained by simply having each linear term equal zero. This yields information concerning the optimal balancing of aberrations and the choice of focal surface.

The process of orthonormalization is most simply discussed in terms of the familiar completion of the square. A general quadratic form is first expressed symmetrically:

$$
\begin{array}{l}
a_{11}A_1A_1 + a_{12}A_1A_2 + a_{13}A_1A_3 + \cdots + a_{1n}A_1A_n \\
+ a_{21}A_2A_1 + a_{22}A_2A_2 + \cdots + a_{2n}A_2A_n \\
\vdots \\
+ a_{n1}A_nA_1 + \cdots + a_{nn}A_nA_n
\end{array}
$$

Forming $\alpha_{1j} = \frac{a_{1j}}{a_{11}}$; $j = 1 \rightarrow n$.

The quadratic form can now be written as

$$
\begin{array}{l}
a_{11}(A_1 + \alpha_{12}A_2 + \cdots \alpha_{1n}A_n)^2 \\
+ b_{21}A_2A_1 + b_{22}A_2A_2 + \cdots \\
\vdots \\
+ b_{n1}A_nA_1 + \cdots + b_{nn}A_nA_n
\end{array}
$$

where $b_{kj} = a_{kj} - \frac{a_{k1}a_{1j}}{a_{11}}$; $j = 1 \rightarrow n$, $k = 2 \rightarrow n$

Repeating the process

Form $\beta_{2j} = \frac{b_{2j}}{b_{22}}$; $j = 2 \rightarrow n$

and $c_{\ell j} = b_{\ell j} - \frac{b_{\ell 2}\, b_{2j}}{b_{22}}$; $j = 1 \rightarrow n$, $\ell = 3 \rightarrow n$

This yields the second squared term and the process is repeated in an obvious manner. This procedure is easily programmed and the necessary steps are included in the TFORTHO program, listed in Appendix D. This program is used to obtain the coefficients for the orthonormalized form of the second-order expansion of the real part of the azimuth-averaged OTF, as a function of spatial frequency.

The General Orthonormalized Criterion

The image quality criteria having quadratic aberrational dependence that are discussed in this work can all be expressed in the following general orthonormalized form.

$$
\begin{aligned}
N\{w_{020} &+ (aw_{040}+bw_{060}+cw_{080}) + [(w_{220}+\tfrac{1}{2}w_{222}) + a(w_{240}+\tfrac{1}{2}w_{242}) \\
&+ b(w_{260}+ w_{262})]H^2 + [(w_{420}+\tfrac{1}{2}w_{422}) + a(w_{440}+\tfrac{1}{2}w_{442})]H^4 \\
&\qquad + (w_{620}+\tfrac{1}{2}w_{622})H^6\}^2 \\
&+ \alpha\{(w_{040}+ dw_{060}+ew_{080})+ [(w_{240}+\tfrac{1}{2}w_{242}) + d(w_{260}+\tfrac{1}{2}w_{262})]H^2 \\
&\qquad + (w_{440}+\tfrac{1}{2}w_{442})H^4\}^2 \\
&+ \beta[(w_{060}+fw_{080}) + (w_{260}+\tfrac{1}{2}w_{262})H^2]^2 + \gamma[w_{080}]^2 \\
&+ \delta[(w_{222}+gw_{242}+hw_{262})H^2 + (w_{422}+gw_{442})H^4 + W_{622}H^6]^2 \\
&+ \varepsilon[(w_{242}+iw_{262})H^2 + W_{442}H^4]^2 + \psi[w_{262}H^2]^2 \\
&+ \kappa\{w_{111}H + (jw_{131}+kw_{151})H + [w_{311} + j(w_{331}+3/4w_{333}) \\
&\qquad + kw_{351}]H^3 \\
&\qquad + [w_{511} + j(w_{531}+3/4w_{533})]H^5 + w_{711}H^7\}^2 \\
&+ \mu\{(w_{131}+\ell w_{151})H +[(w_{331}+3/4w_{333}) + \ell w_{351}]H^3 \\
&\qquad + (w_{531}+3/4w_{533})H^5\}^2 \\
&+ \eta[w_{151}H+w_{351}H^3]^2 + \tau[w_{333}H^3+w_{533}H^5]^2 ,
\end{aligned}
\qquad (2.7)
$$

where the w_{ijk} are the wave aberration coefficients and H is the normalized field parameter. The values of the weighting (α-τ) and balancing (a-ℓ) coefficients depend on the particular criterion. Results for the second-order expansion of the real part of the azimuth-averaged OTF are shown in Figs. 2.2(a-c) and 2.3, for N = 1. It is of interest to note

Fig. 2.2. Balancing Coefficients--Second-order OTF.

Figs. a-c illustrate the behavior of the coefficients corresponding to the orthonormalized second-order expansion of the real part of the azimuth-averaged OTF.

The corresponding values for the variance criterion are indicated by the horizontal bars on each curve. The values for the spot-size criterion are the zero-frequency limits.

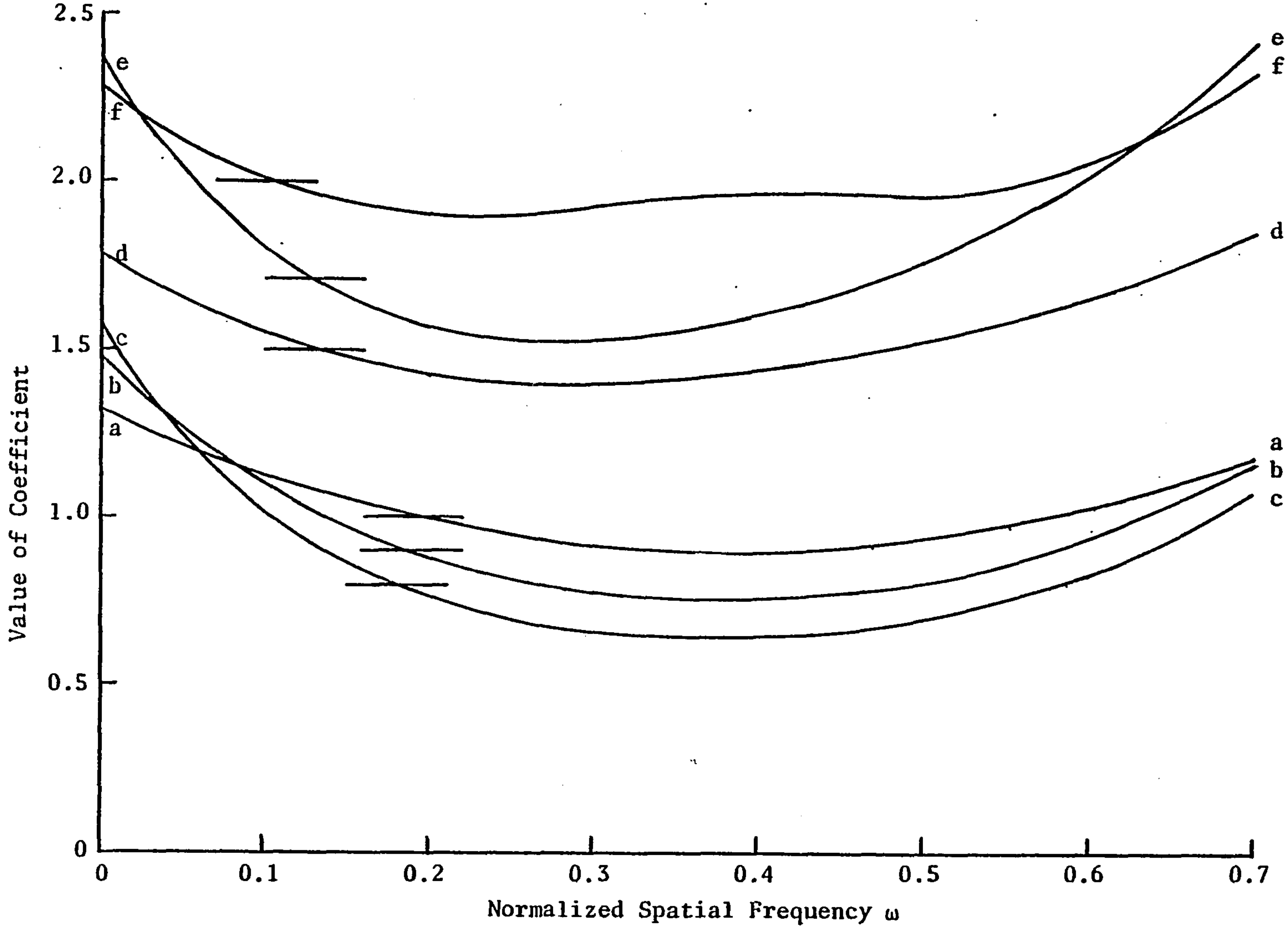

Fig. 2.2. Balancing Coefficients--Second-order OTF.

(a) Spherical Aberrations

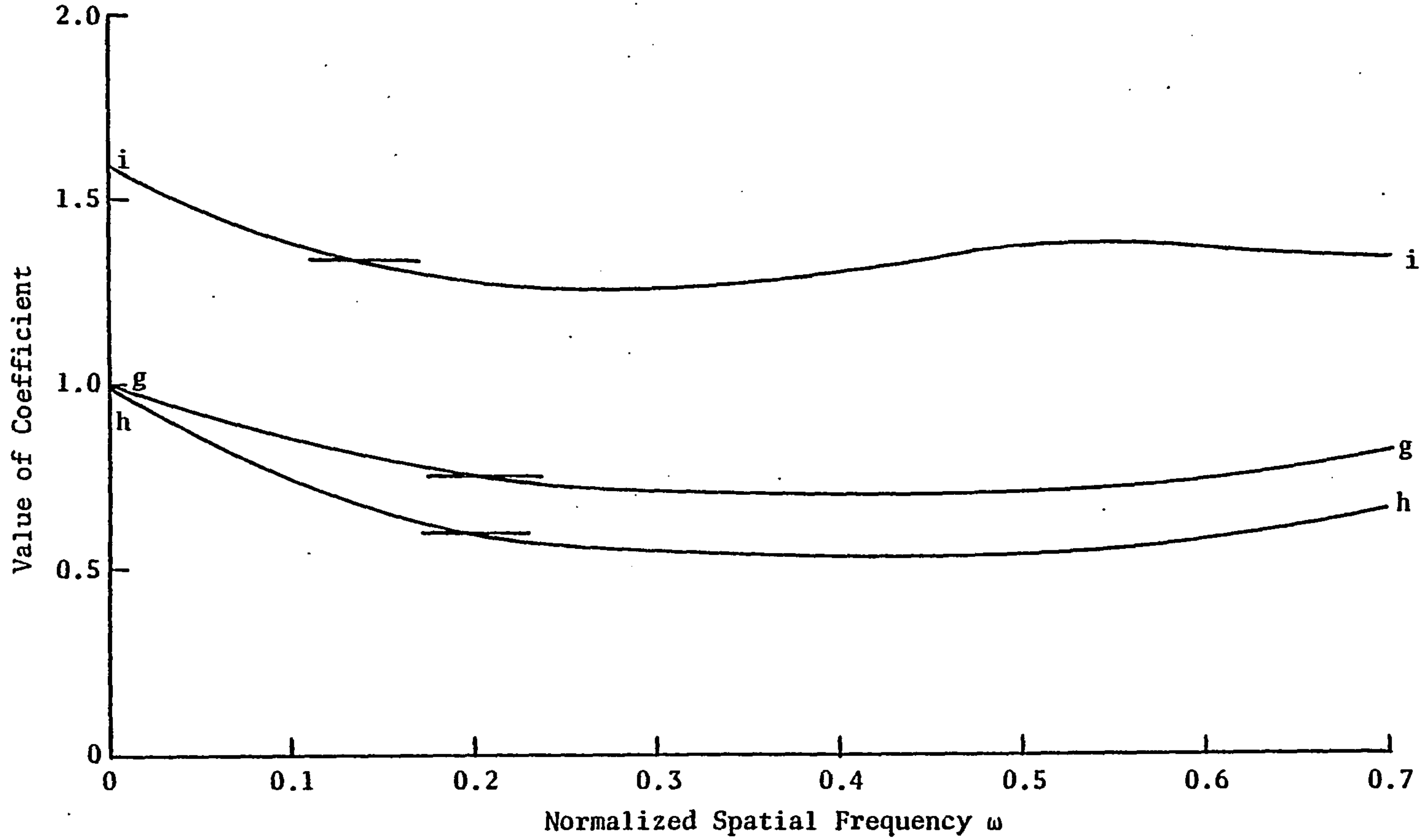

Fig. 2.2. Continued.

(b) Astigmatic Aberrations.

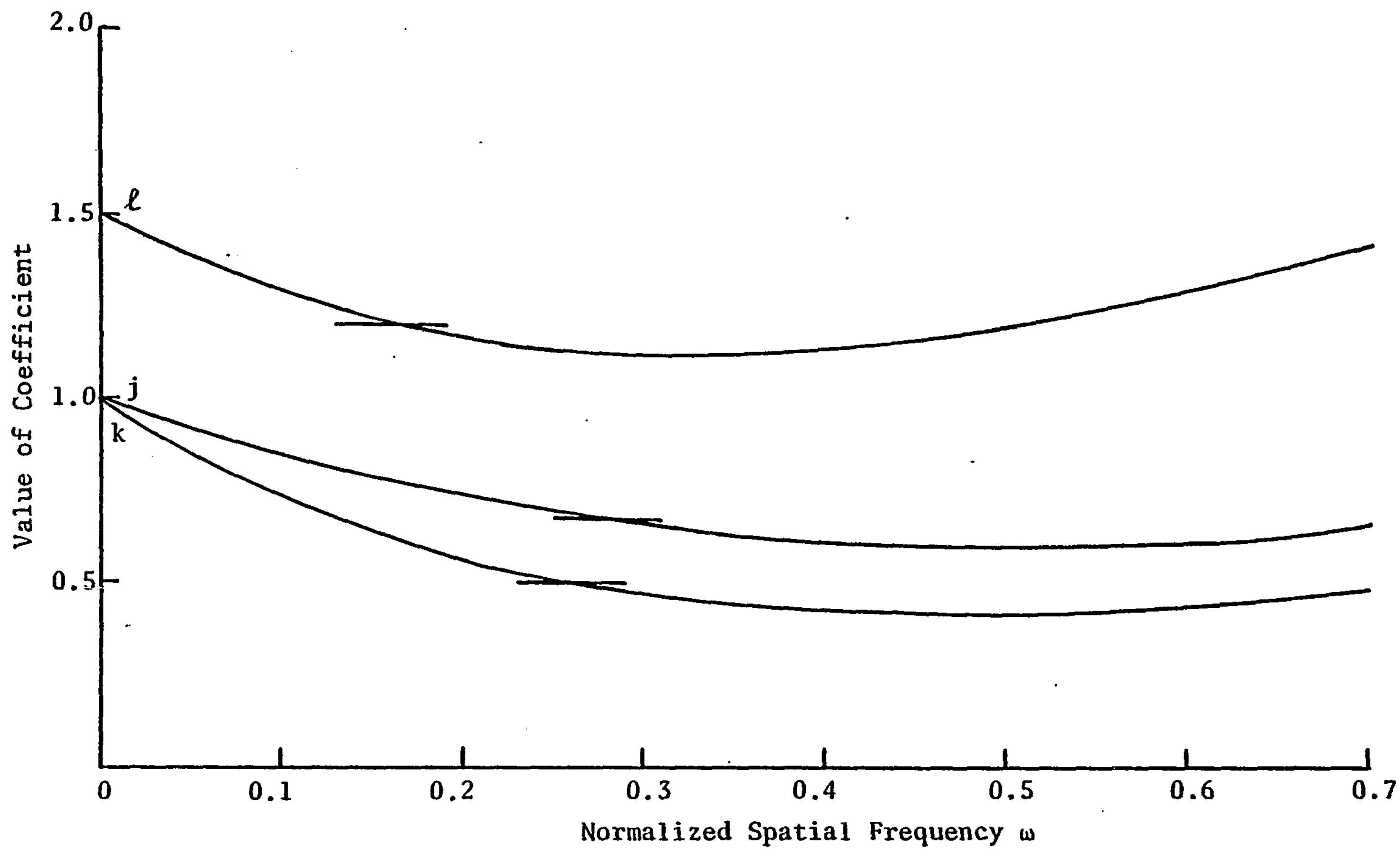

Fig. 2.2. Continued.

(c) Comatic Aberrations.

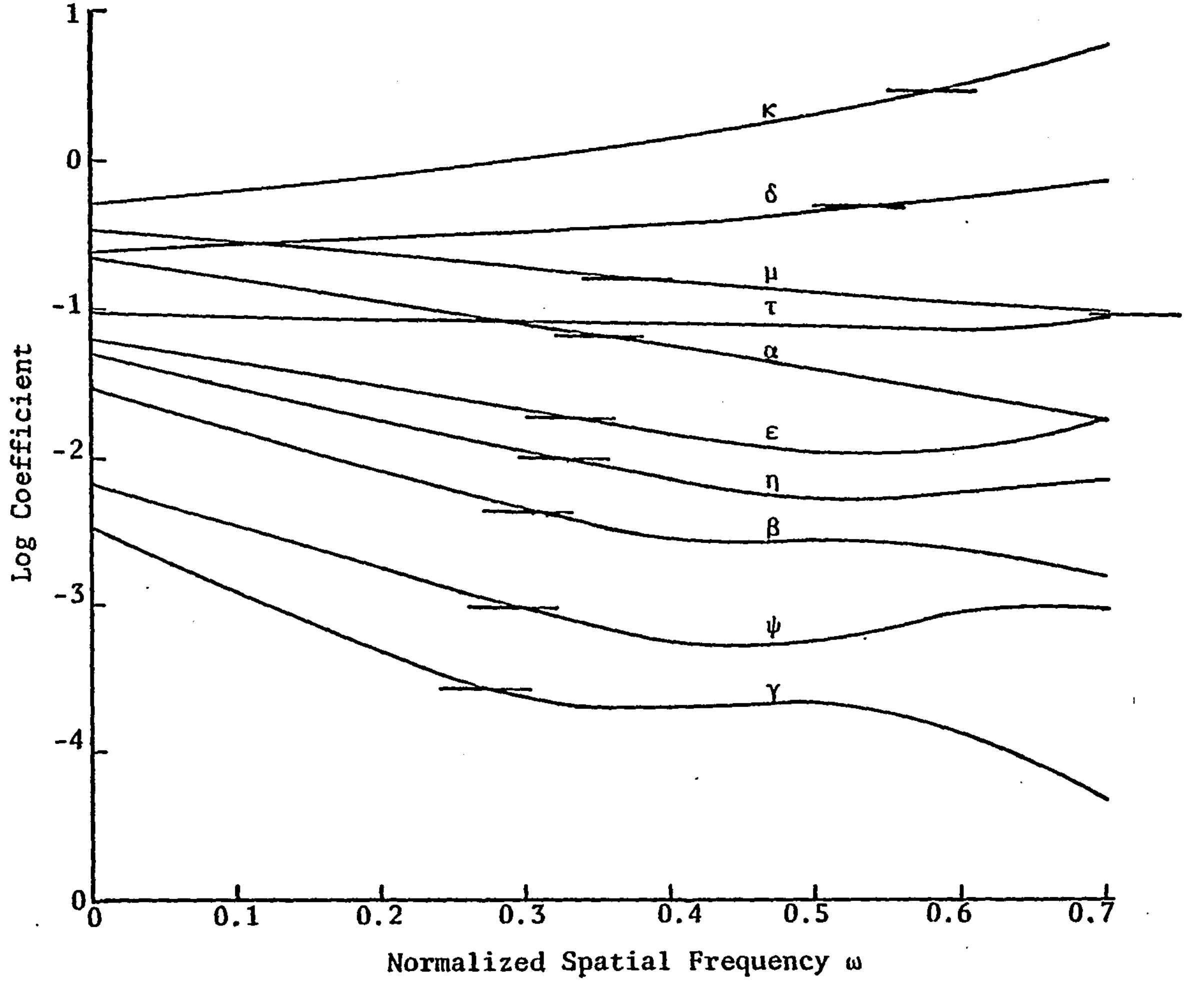

Fig. 2.3. Weighting Coefficients--Second-order OTF.

α, β, γ --Spherical Aberrations,
δ, ε, ψ --Astigmatic Aberrations,
κ,μ,η,τ --Comatic Aberrations.

that aberrations having the same aperture dependence, but with differing azimuthal nature, always have the same constant relationship; e.g., w_{020} and w_{222}, w_{131} and w_{333}, w_{242} and w_{040}, etc. Clearly, any desired field point or field averaging can be considered. Wiese (1974) has indicated how the primary chromatic aberrations can also be included.

CHAPTER 3

THE CONTINUUM BETWEEN THE MEAN-SQUARED SPOT SIZE AND THE VARIANCE OF THE WAVEFRONT ABERRATION

The mean-squared spot size and variance of the wavefront aberration criteria may both be obtained from the OTF-based criterion developed in Chapter 2. The algebraic procedure is best illustrated by a simple example. The second-order expansion of the real part of the OTF for the D and S_1 aberration only is (cf. Appendix B)

$$\begin{aligned}
&\frac{2}{\pi}(a - \omega\sin a) - 64\pi\omega^2\frac{D^2}{\lambda^2}\left[a\left(\frac{1}{4} + \omega^2\right) - \omega\sin a\left(\frac{13}{12} + \frac{1}{6}\omega^2\right)\right] \\
&- 256\pi\omega^2\frac{DS_1}{\lambda^2}\left[a\left(\frac{1}{6} + 2\omega^2 + 2\omega^4\right) - \omega\sin a\left(\frac{7}{6} + \frac{25}{9}\omega^2 + \frac{2}{9}\omega^4\right)\right] \\
&- 256\pi\omega^2\frac{S_1^2}{\lambda^2}\left[a\left(\frac{1}{8} + \frac{17}{6}\omega^2 + 8\omega^4 + 4\omega^6\right)\right. \\
&\qquad\left. - \omega\sin a\left(\frac{29}{24} + \frac{139}{20}\omega^2 + \frac{292}{45}\omega^4 + \frac{14}{45}\omega^6\right)\right]
\end{aligned} \tag{3.1}$$

If this expression is expanded in powers of ω the lowest order terms are

$$1 - \frac{4}{\pi}\omega - \frac{8\pi^2\omega^2}{\lambda^2}\left(D^2 + \frac{8}{3}DS_1 + 2S_1^2\right) \tag{3.2}$$

and it is seen that the coefficient of the term in ω^2 is, to within a numerical constant, the expression for the mean-squared spot size. If (3.1) is integrated over spatial frequency and azimuth, i.e., over the volume under the two-dimensional OTF, we obtain

$$\frac{\pi}{4} - \frac{\pi^3}{12\lambda^2}\left(D^2 + 2DS_1 + \frac{16}{15}S_1{}^2\right) \tag{3.3}$$

Normalizing by $\frac{\pi}{4}$ we obtain

$$1 - \left(\frac{2\pi}{\lambda}\right)^2\left[\frac{1}{12}\left(D^2 + 2DS_1 + \frac{16}{15}S_1{}^2\right)\right] \tag{3.4}$$

This is the familiar formulation of the Strehl Ratio and the expression in the square brackets is the variance of the wavefront aberration.

The example given suggests the procedure whereby a logical continuum can be achieved between the extremes of the spot-size and variance criteria. The second-order OTF expansion is multiplied by a weighting function approximating the transfer function of a hypothetical detector. The produce is then integrated over the spatial frequency and azimuth. As the width of the weighting function tends to infinity, i.e., as the function tends to a constant value, the result of the integration should tend smoothly towards the variance criterion. Similarly, as the function becomes very narrow, the relative weights between the aberration terms should approach those for the spot-size criterion.

This procedure has been carried out for the entire OTF expression in orthonormalized form. The weighting function used was

$$F = \frac{1}{1 + (qS)^2} \tag{3.5}$$

where q is a parameter characterizing the width of the spread function of the detector and S is the spatial frequency. This expression can be re-expressed in terms of the normalized spatial frequency ω

$$F = \frac{1}{1 + (R\omega)^2} \tag{3.6}$$

where $R = \frac{q}{\lambda(\text{f-number})}$

The upper limit of integration is the value of ω for which F falls to O.I, or the optical cut-off, whichever occurs first (i.e., depending on the value of R). The truncation of F is necessary for the effective isolation of the appropriate frequency region of the OTF expression.

The results of this work yield an orthonormalized criterion that is again of the generalized form Eq. (2.7) given in Chapter 2, except that the coefficients are now a function of the parameter R. The computational work is described in the listing of the TFIORTH program, given in Appendix D. The variation of the coefficients as a function of R is shown in Figs. 3.1(a-c) and 3.2 (again for N = 1) and the smooth transition from the spot size to the variance limits is evident. The coefficient values for the spot size and variance limits are given in Appendix E.

Fig. 3.1. Balancing Coefficients--Integrated Second-order OTF.

These illustrations show the behavior of the coefficients corresponding to the orthonormalized image quality criterion that forms a continuum between the spot-size and variance criteria. This criterion is a second-order expansion of the real part of the OTF that is multiplied by a weighting function of the form $1/1 + (R\omega)^2$ and integrated over azimuthal angle and spatial frequency. The upper frequency limit of integration is

$$\omega_{max} = \frac{3}{R} \; ; \quad \frac{3}{R} \leq 1.0$$

$$= 1.0; \quad \frac{3}{R} > 1.0$$

(a) Spherical Aberrations

(b) Astigmatic Aberrations

(c) Comatic Aberrations

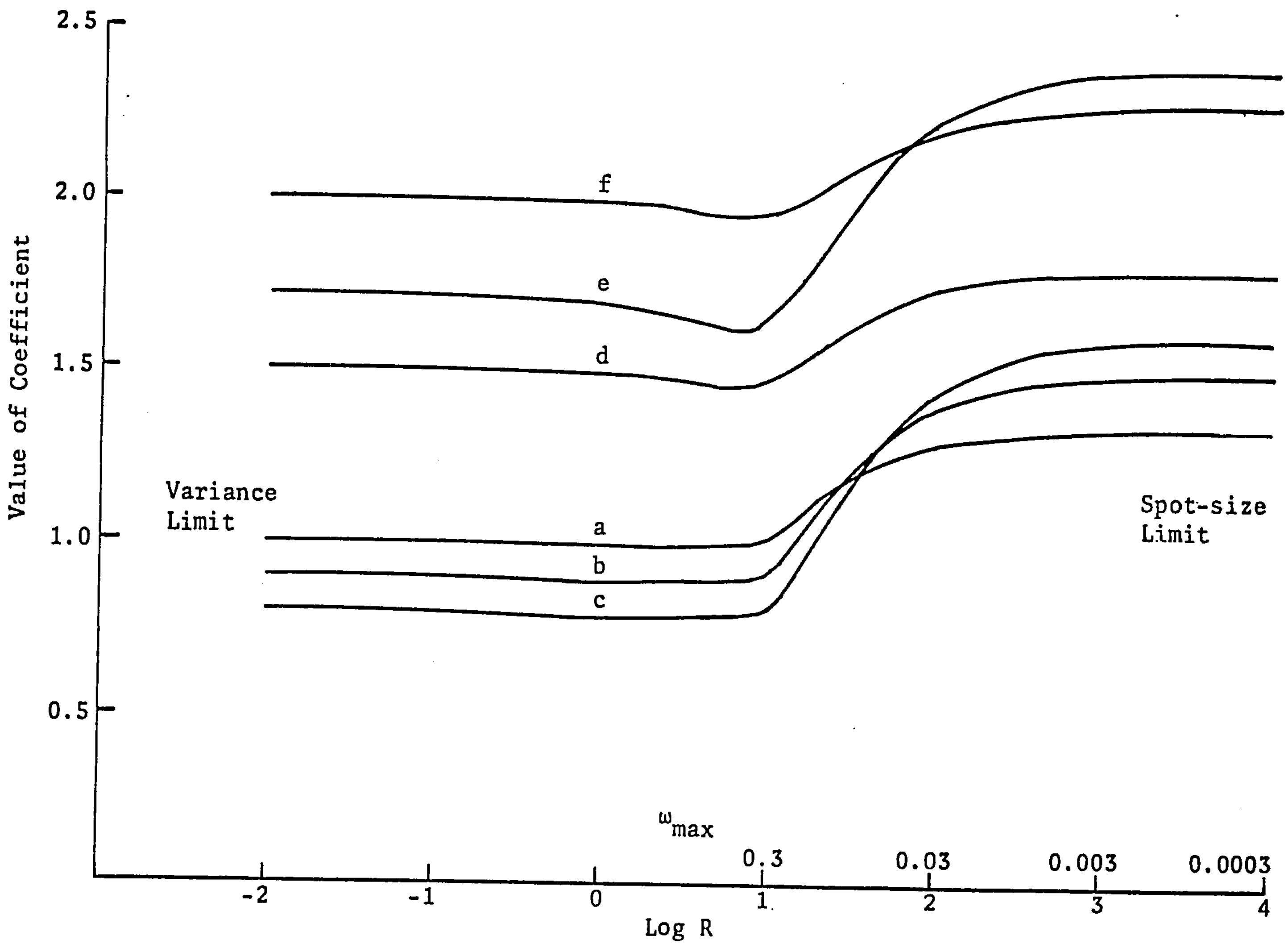

Fig. 3.1. Balancing Coefficients--Integrated Second-order OTF.

(a) Spherical Aberrations.

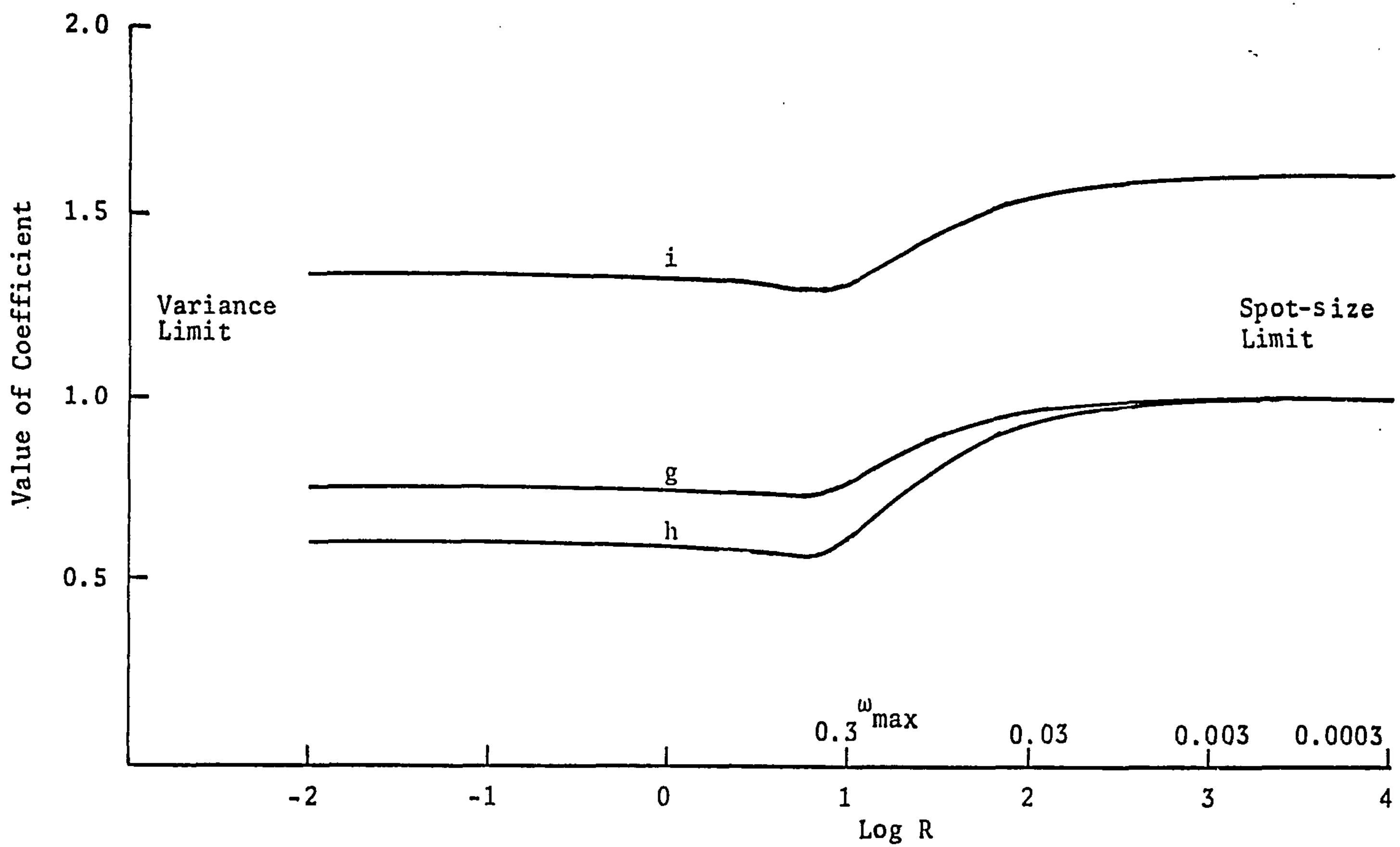

Fig. 3.1. Continued.

(b) Astigmatic Aberrations.

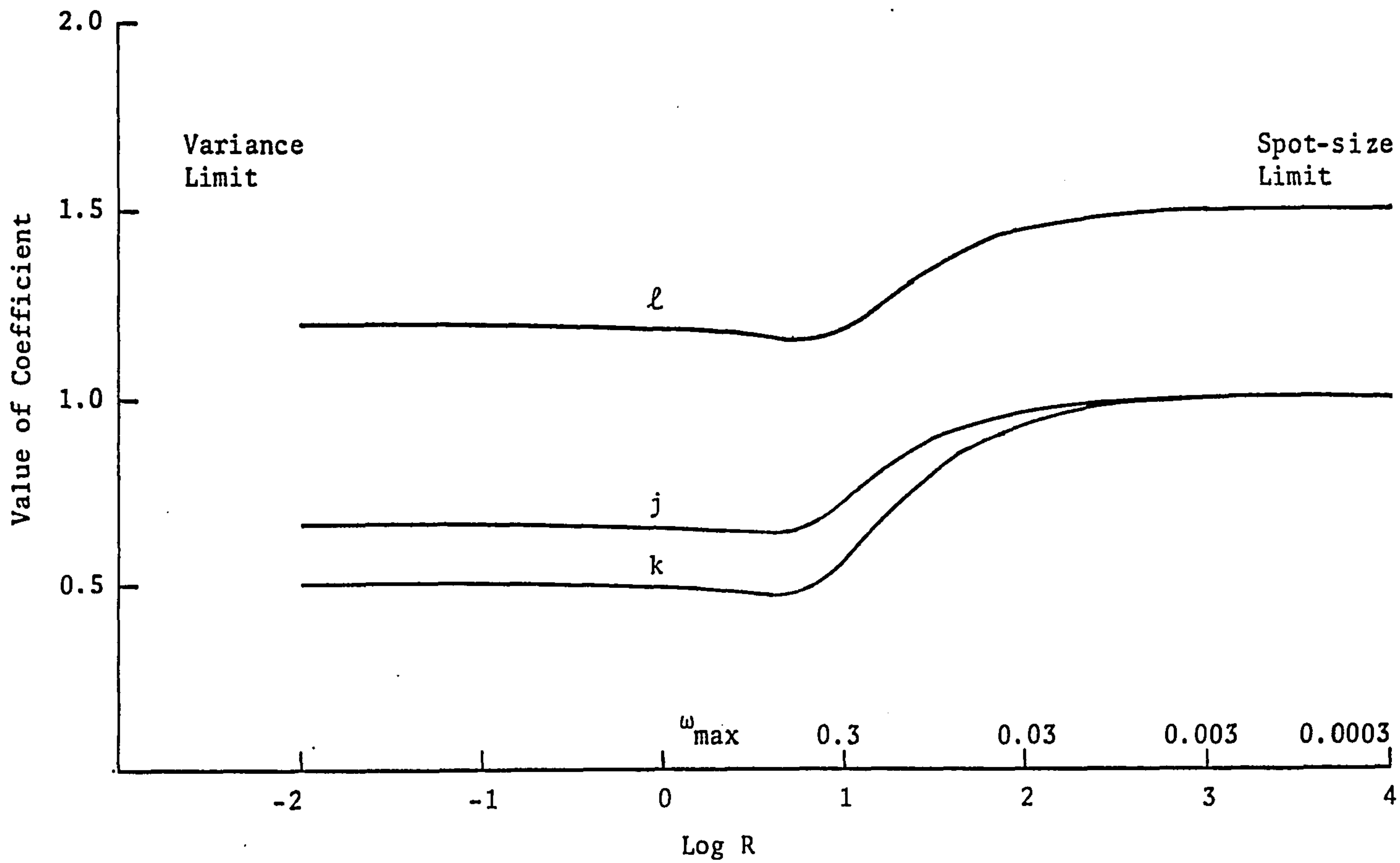

Fig. 3.1. Continued.

(c) Comatic Aberrations.

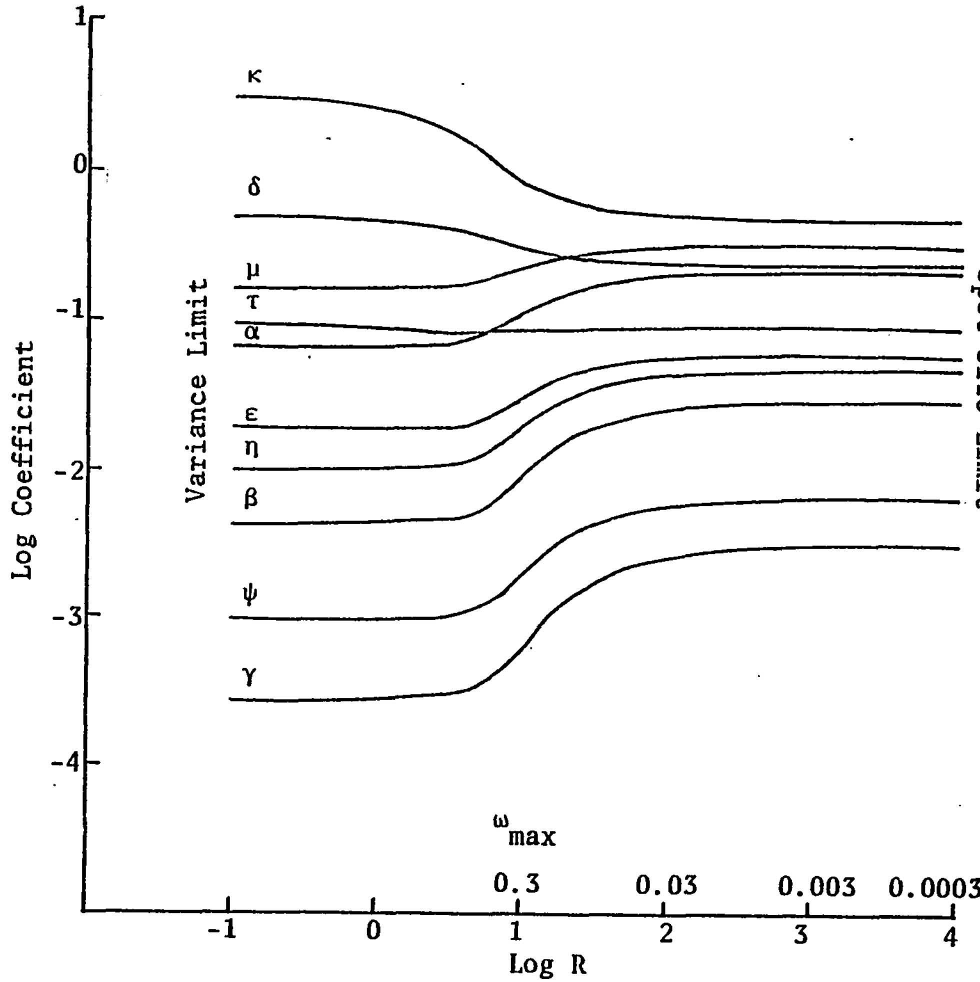

Fig. 3.2. Weighting Coefficients--Integrated second-order OTF.

α, β, γ --Spherical Aberrations
δ, ε, ψ --Astigmatic Aberrations
κ, μ, η, τ --Comatic Aberrations

CHAPTER 4

ABERRATION-BALANCING AND OPTIMIZATION OF THE OPTICAL TRANSFER FUNCTION

As mentioned in Chapter 1, the class of image quality criteria developed in this work is of particular significance for use as a merit function during the final aberration-balancing stage of an optical design procedure. In practice, consideration of the characteristics of the entire imaging process would determine the spatial frequency range for which the optical system has to be optimized. This would, in turn, determine the values of the coefficients for the orthonormalized criterion. The resultant criterion and the balance of aberrations will be optimal provided that the relative modulation of the optical system is sufficiently high.

It is of considerable practical interest to examine the results of different aberrational balances over a wide range of relative modulations. A systematic approach is to compare the various balances corresponding to the spot size, the variance of the wavefront aberration and the OTF-based criteria. The basis of comparison will be the extent to which the exact OTF can be optimized. The exact OTF has been calculated via autocorrelation of the pupil function, using the Hopkins algorithm (Hopkins, 1957b). This procedure is described in the AUTCORR program, listed in Appendix D.

In the examples that follow, the balancing relationships are obtained by setting the appropriate orthogonal components of the particular criterion to zero. Results at frequencies below 0.1 are of the greater interest since it can be seen from Figs. 2.2(a-c), pages 19-21, that the balancing coefficients for the variance and OTF-based criteria are similar in value at higher frequencies. Moreover, good correction is required to give appreciable OTF values at the higher frequencies and hence the variance criterion becomes appropriate anyway.

Figure 4.1(a-e) illustrates the effect of the choice of criterion for aberration-balancing. There are some consistently demonstrable results. Differences at high OTF values are small. At lower OTF values, use of the spot size criterion becomes inappropriate, even for large aberrations and at low frequencies [cf. Fig. 4.1(e)].

Use of the OTF-based criterion is advantageous mainly at relative modulations in the range 0.65-0.85 and at frequencies such that the numerical balance is appreciably different from those for the other criteria [cf. Fig. 4.1(a-c)]. A typical result is shown in Fig. 4.1(a) at a frequency of 0.08. In general, the choice of balancing criterion is seen to be important mainly at lower OTF levels.

These results are again illustrated for data obtained for an actual lens. This is a 150mm Double-Gauss (Stavroudis and Sutton, 1965), analyzed at f/3.5 and a 14½-degree half-field. The performance is adequately described by the wave aberration coefficients through eighth order; these were obtained using the PROXI program (G. W. Hopkins, 1976).

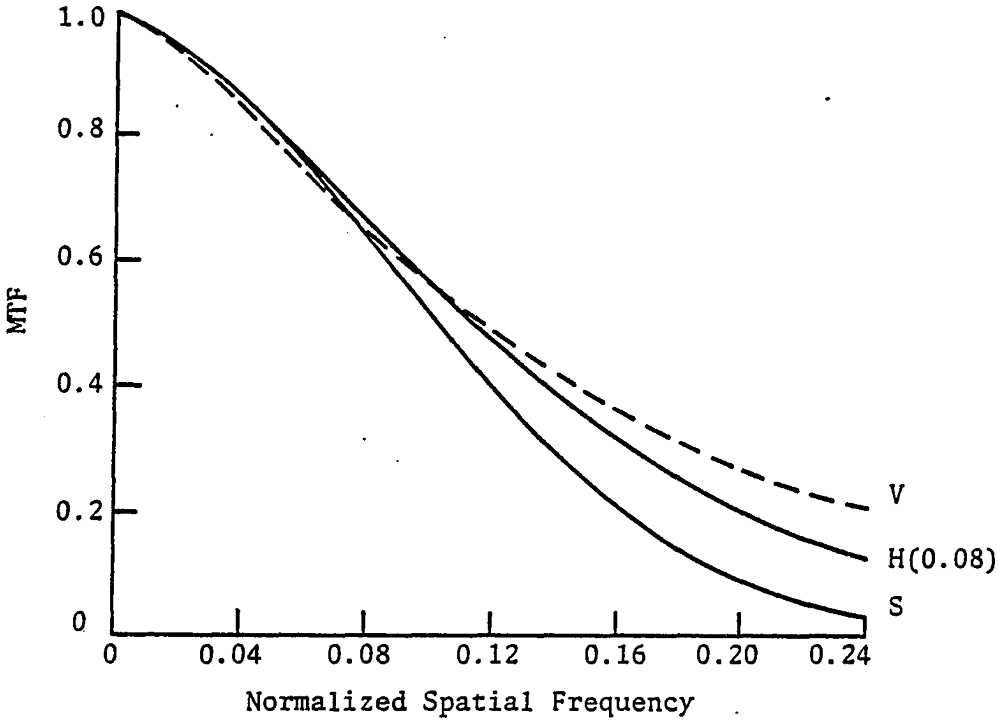

Fig. 4.1. Transfer Functions for Aberrational Balances using Different Criteria.

These illustrations show the effect of the choice of criterion for aberration-balancing. In each case, the MTF has been calculated by autocorrelation of the pupil function. The various lower-order balances are distinguished as follows:

V --minimization of the variance of the wavefront aberration

S --minimization of the mean-squared spot size

H(x) --optimization, at a frequency of x, of the second-order expansion of the OTF.

(a) $S_1 = 2\lambda$.
V -- D = -2λ
H(0.08) -- D = -2.34λ
S -- D = $-8/3\lambda$

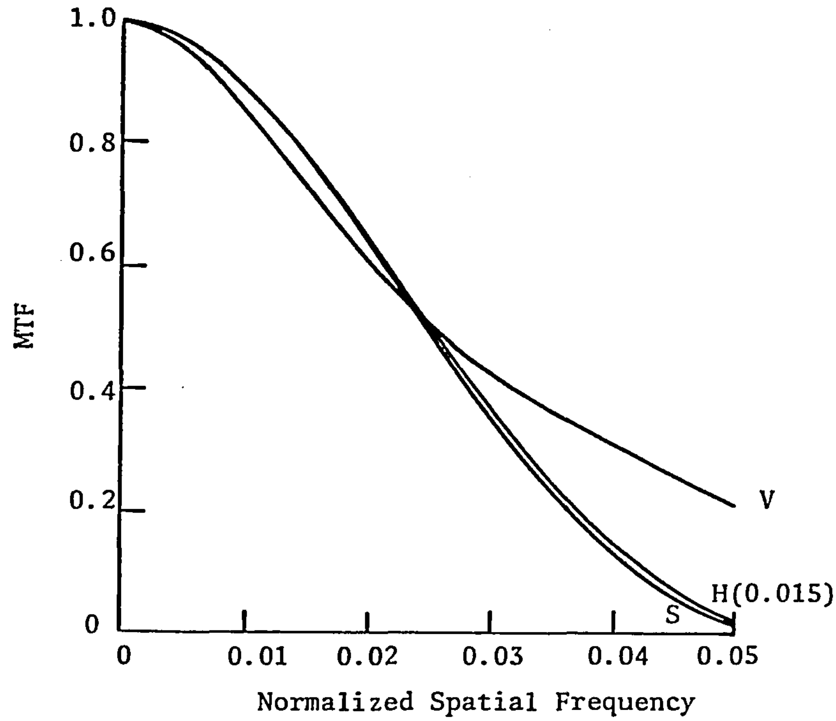

Fig. 4.1. Continued, Transfer Functions.

(b) $S_1 = 8\lambda$.
V -- D = -8λ
H(0.015) -- D = -10.4λ
S -- D = $-32/3\lambda$

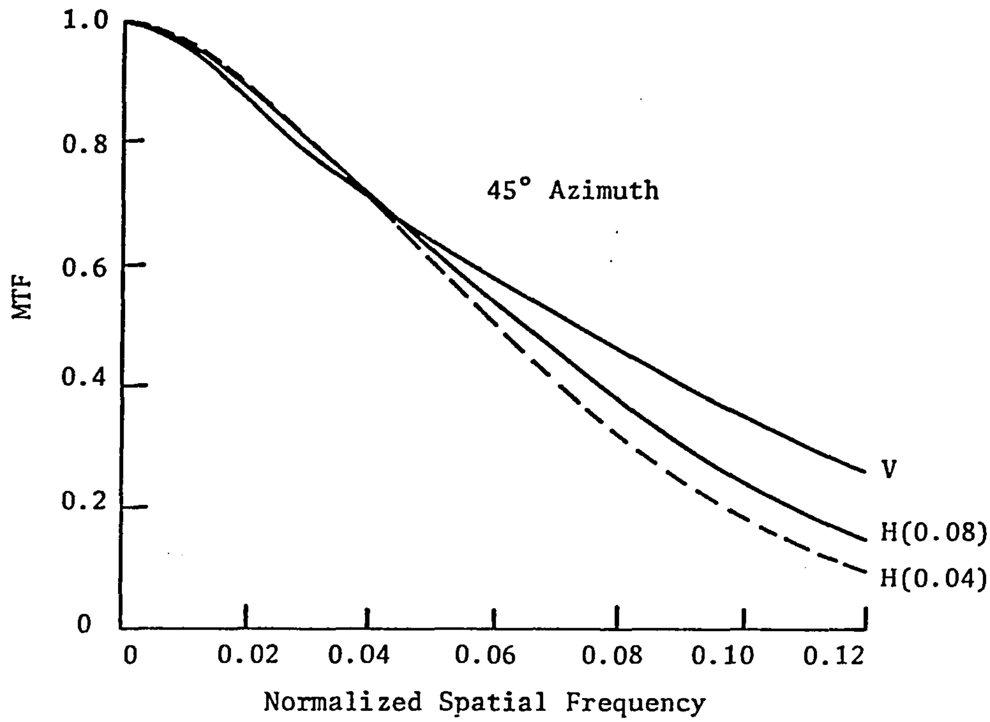

Fig. 4.1. Continued, Transfer Functions.

(c) $C_2 = 8\lambda$.

V	$--C_1$	=	-9.6λ
H(0.08)	$--C_1$	=	-10.59λ
H(0.04)	$--C_1$	=	-11.24λ

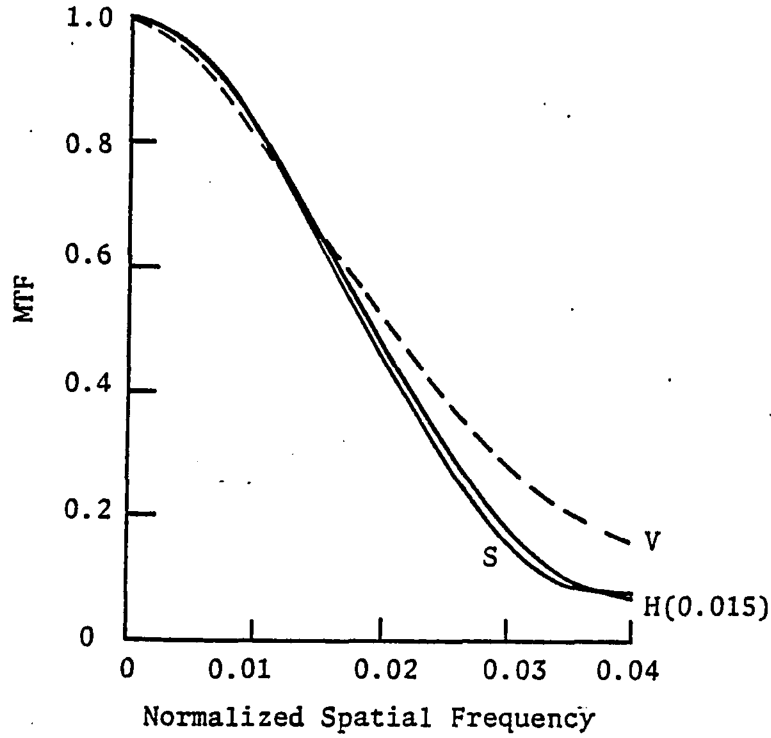

Fig. 4.1. Continued, Transfer Functions.

(d) $OB_2 = 21\lambda$; $S_3 = 21\lambda$;
$C_2 = 12\lambda$; $EC = 12\lambda$

Curve:	V:	H(0.015):	S:
S_2	-52.5λ	-57.34λ	-58.5λ
S_1	41λ	50.65λ	52.8λ
D	-10.2λ	-14.90λ	-15.9λ
OB_1	-28λ	-32.77λ	-33.6λ
A	8.4λ	11.98λ	12.6λ
B	3.6λ	5.70λ	6λ
C_1	-23.4λ	-26.55λ	-27λ

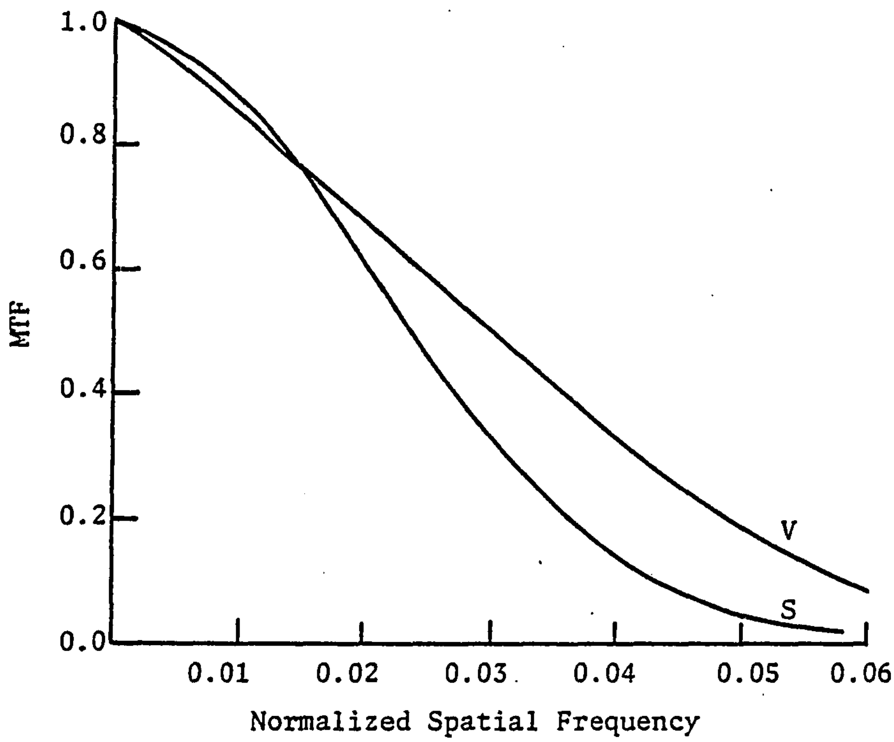

Fig. 4.1. Continued, Transfer Functions.

(e) $S_3 = 70\lambda$.
V -- $S_2 = -140\lambda$; $S_1 = 90\lambda$; $D = -20\lambda$
S -- $S_2 = -160\lambda$; $S_1 = 120\lambda$; $D = -32\lambda$

This example illustrates that the spot-size criterion can be inappropriate even for large aberrations and at low frequencies.

Figure 4.2 shows OTF curves corresponding to different balances using only the fourth-order wave aberrations and the choice of focal surface as variables. All the data are for the edge of the field. Chromatic aberrations were not included in this analysis.

Optimal Aberration-balancing at Lower Relative Modulation

It can be seen from the examples given that the variance criterion yields the best results at lower relative modulation, regardless of the level of aberration. This indicates that the true optimal aberrational balance moves towards the variance values as the relative modulation decreases. There are, however, no convenient analytical procedures for obtaining the precise values corresponding to the true optimal aberrational balances at lower relative modulation. The author has investigated an extension of the OTF expansion to fourth order and has found that very little additional insight is provided; the expansion has poor convergence. It is easily shown empirically (see, for example, Born and Wolf, 1975, p. 488) that, at lower relative modulation, the true optimal balance differs from that obtained using any of the second-order criteria. All that can be inferred is that, under these circumstances, use of the variance criterion yields the closest systematic approach to the true optimum.

It is also easily shown that the aberrational balance that is optimal (obtained empirically) at frequencies for which the relative modulation is low will yield a corresponding deterioration in the response at lower frequencies where the relative modulation is high.

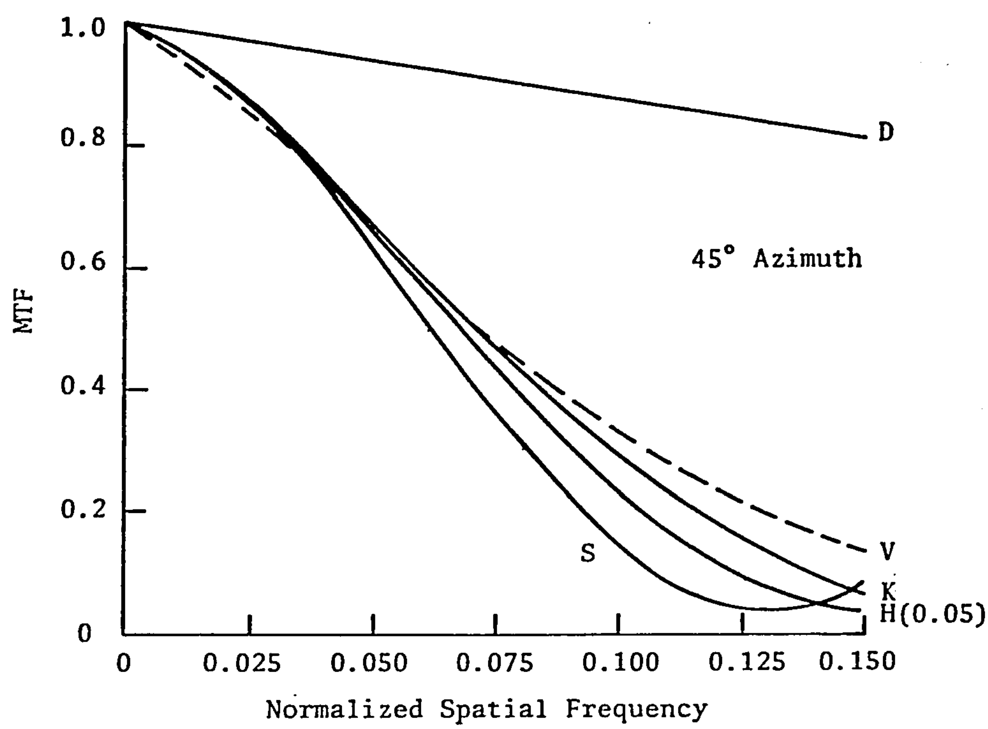

Fig. 4.2. Transfer Functions for a Double-Gauss Lens.

$OB_2 = 0.0\lambda$; $OB_1 = 5.66\lambda$; $S_3 = -0.35\lambda$;
$S_2 = -2.97\lambda$; $EC = -0.73\lambda$; $C_2 = +0.31\lambda$

The above data were obtained for a 150mm Double-Gauss lens evaluated at f/3.5 and at a half-field of 14½°. The lower-order balances are as follows:

	S1	D	A	C1	B
V	2.20λ	0.02λ	-4.25λ	0.18λ	0.09λ
K	2.46λ	0.04λ	-4.83λ	0.16λ	0.11λ
H(0.05)	2.82λ	-0.11λ	-5.22λ	0.13λ	0.13λ
S	3.35λ	-0.40λ	-5.66λ	0.09λ	0.15λ

The curve labeled K illustrates the use of the integrated version of the OTF-based criterion, described in Chapter 3. The weighting function used falls to a value of 0.1 at a frequency of 0.150 (i.e., R=20). The curve labeled D shows the diffraction-limited performance for a lens of the same specifications. V and S refer to the variance and spot-size criteria.

Under these circumstances the aberrational balance provided by the use of the variance criterion appears to yield the most satisfactory systematically obtainable compromise between optimum performance at both high and low relative modulation. This compromise is in the sense that significant improvement at lower relative modulation can be obtained with only a small deterioration at higher relative modulation. This conflict between good performance at both low and high relative modulation (i.e., referred to different regions of the same OTF curve) means that the aberration-balancing problem cannot be adequately approached by considering only the properties of the optical system.

CHAPTER 5

THE EXTENDED-RANGE USE OF THE VARIANCE OF THE WAVEFRONT ABERRATION

This chapter is devoted to a discussion involving the extent to which the variance of the wavefront aberration can be used as a general-purpose merit function in automatic optical design.

This merit function is invariably a sum of squared image defect terms. As shown in this work this type of function can adequately describe optical image quality, provided, essentially, that the relative modulation is sufficiently high (above 0.6-0.7). Otherwise, it is necessary to use a more complicated function such as the actual OTF. This has not been done although there is no fundamental problem. The computational complexity has apparently deterred most attempts in this direction and, instead, a great deal of work has been devoted to optimization techniques involving the sum of squares type of merit function.

The need for a physically significant merit function, whether of sum of squares form or otherwise, is of importance primarily for the aberration-balancing stage. It has already been indicated that the variance criterion has extensive applicability in this connection. It is to this extent that the variance can serve as a general-purpose merit function. It is, however, important to note that the value of the variance does not necessarily correlate with image quality for other than well-corrected systems. This is best illustrated by an example, shown in Fig. 5.1. In this example, each OTF curve corresponds to a value of the variance that is five times that for which the Maréchal results start becoming inaccurate, i.e., an rms variance of 1/3λ. It can be seen that large aberrations of different types cannot be reliably compared using the variance as a measure of image quality. The weighting, as opposed to the balancing, coefficients in the orthonormalized expression for the variance do not have applicability beyond the domain of well-corrected systems. However, it is clear that the variance will correlate with image quality if a given system progresses from a certain state of correction to another differing primarily in the amount and not the type of aberration.

Empirically, it appears that the variance is generally applicable as an aberration-balancing merit function for situations where the relative modulation exceeds about 0.5, although this value depends on the type of aberration. This conclusion results from some trial and error attempts to find the optimal aberration balances for fourth-order spherical aberration plus defocus, and for third- and fifth-order coma. This is worthy of further investigation, but the process is clearly a tedious one.

These conclusions about the appropriateness of the orthogonality, as opposed to the normalization, of the variance criterion are a justification for the expansion of the wavefront aberration in terms of the Zernike circle polynomials. This expansion is clearly not limited to situations where the aberrations are small.

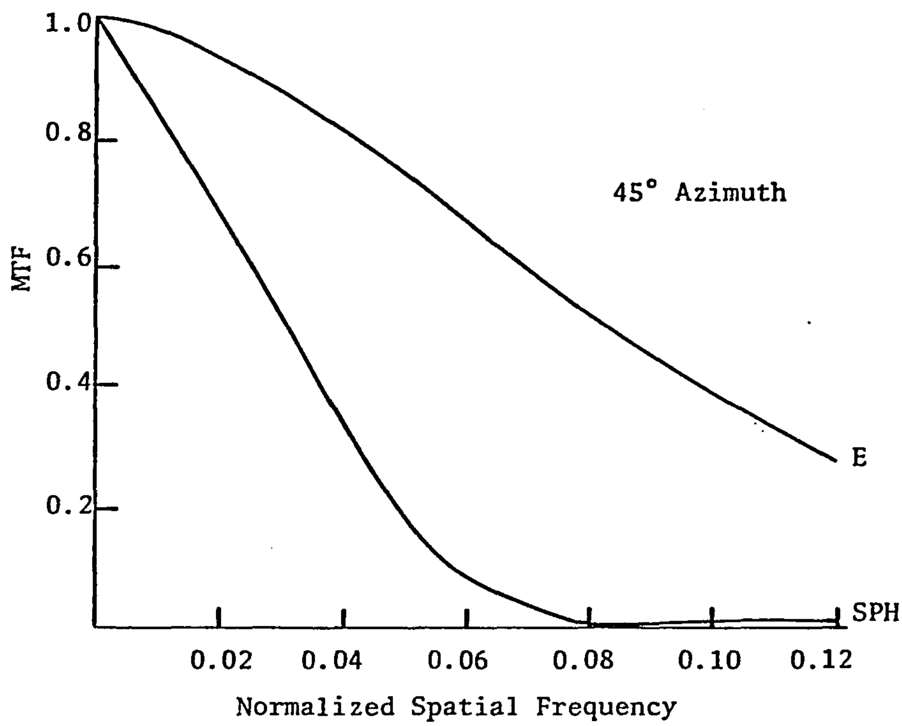

Fig. 5.1. Transfer Functions Corresponding to the Same Variance Value.

Both curves represent the same rms variance of the wavefront aberration = $1/3\lambda$.

E ——— EC = 3.77λ; C = -2.83λ
(The 45° azimuth does not differ significantly from an azimuthal average)

SPH ——— $S_3 = 70\lambda$; $S_2 = -140\lambda$; $S_1 = 90\lambda$; $D = -20\lambda$
In each case, the lower-order balance is such as to minimize the variance of the wavefront aberration.

CHAPTER 6

SUMMARY AND CONCLUSIONS

The original goal of this work was to develop a continuum of physically significant optical image quality criteria, usable for any amount of aberration, and subject to the functional form that has quadratic aberrational dependence. While it was anticipated that this would yield a useful tool for the analysis and comparison of optical systems, it was expected that the main use would be in terms of a merit function suitable for automatic optical design. Specifically, the work would provide the required aberration-balancing information necessary for the final optimizational stages of a design procedure. All this has been achieved but the significant conclusions that have emerged are, in the author's opinion, more important than the routine attainment of the initial objectives.

Firstly, it has been shown that the use of criteria based on a second-order expansion of the OTF is of somewhat limited value in optical design. This type of criterion is theoretically accurate only at high relative modulation and, under these circumstances, the choice of criterion for aberration-balancing is not critical. The maximum advantages obtainable do not appear to significantly exceed 0.03 OTF units as compared with aberration-balancing using the variance of the wavefront aberration as a criterion. These small advantages can be of significance for situations where it is desirable to maximize the irradiance at a point on a detector; in these situations, any additional energy, however, small, can be of importance.

Secondly, it has been shown that use of the variance of the wavefront aberration as an aberration-balancing criterion is widely justifiable beyond the usual limitation of small aberrations. Indeed, this offers the only systematic approach, known to the author, towards the problem of optimal aberration-balancing at lower relative modulation. It appears that, as the relative modulation decreases, the optimal balance of aberration moves from the value for the OTF-based criterion, at any frequency, towards the value for the variance criterion and then proceeds beyond this value, in the same directional sense. This last point requires further investigation. The significance of these results lies not only in an establishment of the extent to which the "sum of squares" type of merit function can be reliably employed. There is also an indication that a more complicated merit function, such as the OTF itself, may not be necessary for most situations in automatic optical design. This is an important avenue for further research.

Thirdly, it has been shown that the mean-squared spot-size criterion should be used with caution. It is a reliable criterion only when the OTF is high, typically above 0.7-0.8, depending on the type of aberration. At lower OTF values, even for large aberrations and at low frequencies, the spot-size criterion becomes a progressively poorer aberration-balancing criterion. This might have been anticipated, although it does not seem entirely obvious, from the limited relationship of this criterion to the curvature of the OTF at the origin.

Finally, as indicated at the start of this chapter, it is anticipated that the orthonormalized criteria developed in this work will have some intrinsic value in the analysis and understanding of optical systems, as, for example, in the use of field-focus displays. In addition, the explicit aberrational dependence permits a systematic investigation, at least for higher relative modulation, of the comparative effects that different aberrations have on the diffraction-based OTF.

APPENDIX A

CALCULATION OF THE WAVE ABERRATION DIFFERENCE FUNCTION

The expression W-W', the wave aberration difference function, occurs in the autocorrelation formulation of the OTF. In terms of the wave aberration coefficients used in this work, the following result is obtained.

$$
\begin{aligned}
W-W' = \lambda\omega \Big\{ & 4D\left[(\rho\cos\phi - \omega)\right] + 8S_1\left[(\rho\cos\phi - \omega)\,(\rho^2 - 2\rho\omega\cos\phi + 2\omega^2)\right] \\
& + 4S_2\left[(3\rho^5\cos\phi - 12\rho^4\omega\cos\phi - 3\rho^4\omega + 16\rho^3\omega^2\cos^3\phi + 24\rho^3\omega^2\cos\phi \right. \\
& \qquad \left. - 48\rho^2\omega^3\cos^2\phi - 12\rho^2\omega^3 + 48\rho\omega^4\cos\phi - 16\omega^5)\right] \\
& + 16S_3\left[(\rho^7\cos\phi - 6\rho^6\omega\cos^2\phi - \rho^6\omega + 16\rho^5\omega^2\cos^3\phi + 12\rho^5\omega^2\cos\phi \right. \\
& \qquad - 16\rho^4\omega^3\cos^4\phi - 48\rho^4\omega^3\cos^2\phi - 6\rho^4\omega^3 + 64\rho^3\omega^4\cos^3\phi \\
& \qquad \left. + 48\rho^3\omega^4\cos\phi - 96\rho^2\omega^5\cos^2\phi - 16\rho^2\omega^5 + 64\rho\omega^6\cos\phi - 16\omega^7)\right] \\
& + 4A\left[\cos\theta\,(\rho\cos(\phi-\theta) - \omega\cos\theta)\right] \\
& + 40B_1\left[(2\rho^3\cos^3\phi - 6\rho^2\omega\cos^2\phi + 8\rho\omega^2\cos\phi - 4\omega^3)\cos^2\theta \right. \\
& \qquad + \rho^2\sin^2\phi\,(\rho\cos\phi - \omega) \\
& \qquad \left. + \rho\sin\phi\sin\theta\cos\theta\,(3\rho^2\cos^2\phi + \rho^2\sin^2\phi - 6\rho\omega\cos\phi + 4\omega^2)\right]
\end{aligned}
$$

$$
\begin{aligned}
+ 40B_2 \Bigg[& \cos^2\theta \Big(2\rho^5\cos^3\phi + \rho^5\cos\phi - 4\rho^4\omega\cos^4\phi - 10\rho^4\omega\cos^2\phi - \rho^4\omega \\
& + 24\rho^3\omega^2\cos^3\phi + 16\rho^3\omega^2\cos\phi - 52\rho^2\omega^3\cos^2\phi - 8\rho^2\omega^3 \\
& \quad + 48\rho\omega^4\cos\phi - 16\omega^5\Big) \\
& + 2\sin^2\theta \Big(- \rho^5\cos^3\phi + \rho^5\cos\phi + 2\rho^4\omega\cos^4\phi - \rho^4\omega\cos^2\phi \\
& \qquad - \rho^4\omega - 4\rho^3\omega^2\cos^3\phi + 4\rho^3\omega^2\cos\phi \\
& \qquad \quad + 2\rho^2\omega^3\cos^2\phi - 2\rho^2\omega^3\Big) \\
& + \sin\theta\cos\theta \Big(4\rho^5\cos^2\phi\sin\phi + \rho^5\sin\phi - 8\rho^4\omega\cos^3\phi\sin\phi \\
& \qquad - 12\rho^4\omega\cos\phi\sin\phi + 32\rho^3\omega^2\cos^2\phi\sin\phi \\
& \qquad + 8\rho^3\omega^2\sin\phi - 40\rho^2\omega^3\cos\phi\sin\phi + 16\rho\omega^4\sin\phi\Big) \Bigg] \\
+ 2B & \left[\cos\theta\right] \\
+ 2C_1 \Big[& 2\rho^2\cos\phi\cos(\phi-\theta) + \rho^2\cos\theta - 6\rho\omega\cos\phi\cos\theta - 2\rho\omega\sin\phi\sin\theta \\
& + 4\omega^2\cos\theta \Big] \\
+ 2C_2 \Big[& 4\rho^4\cos^2\phi\cos\theta + 4\rho^4\cos\phi\sin\phi\sin\theta + \rho^4\cos\theta - 8\rho^3\omega\cos^3\phi\cos\theta \\
& - 12\rho^3\omega\cos\phi\cos\theta - 8\rho^3\omega\cos^2\phi\sin\phi\sin\theta - 4\rho^3\omega\sin\phi\sin\theta \\
& + 32\rho^2\omega^2\cos^2\phi\cos\theta + 16\rho^2\omega^2\cos\phi\sin\phi\sin\theta + 8\rho^2\omega^2\cos\theta \\
& - 40\rho\omega^3\cos\phi\cos\theta - 8\rho\omega^3\sin\phi\sin\theta + 16\omega^4\cos\theta \Big] \\
+ 2EC\cos\theta \Big[& 3\rho^2\cos^2\phi\cos^2\theta + 3\rho^2\sin^2\phi\sin^2\theta + 6\rho^2\sin\phi\cos\phi\sin\theta\cos\theta \\
& - 6\rho\omega\cos\phi\cos^2\theta - 6\rho\omega\sin\phi\sin\theta\cos\theta + 4\omega^2\cos^2\theta \Big] \Bigg\}
\end{aligned}
$$

APPENDIX B

THE EXPANSION OF THE OTF TO SECOND ORDER IN THE WAVE ABERRATION COEFFICIENTS

The second-order expansion of the real part of the OTF is given by

$$S(\omega,\theta) = \frac{2}{\pi}(a - \omega\sin a) - \frac{4\pi}{\lambda^2}\left[\int_{-a}^{a}\int_{\frac{\omega}{\cos\phi}}^{1} (W-W')^2\,\rho\,d\rho\,d\phi\right]$$

where $a = \text{arc}\cos\omega$

$\sin a = (1-\omega^2)^{\frac{1}{2}}$

and $(W-W')$ is given in Appendix A.

The integration can be done analytically and the result is

$$S(\omega,\theta) = \frac{2}{\pi}(a-\omega\sin a)$$

$$- 64\pi\omega^2 D^2\left\{a\left(\frac{1}{4}+\omega^2\right) - \omega\sin a\left(\frac{13}{12}+\frac{1}{6}\omega^2\right)\right\}$$

$$- 256\pi\omega^2 DS_1\left\{a\left(\frac{1}{6}+2\omega^2+2\omega^4\right) - \omega\sin a\left(\frac{7}{6}+\frac{25}{9}\omega^2+\frac{2}{9}\omega^4\right)\right\}$$

$$- 384\pi\omega^2 DS_2\left\{a\left(\frac{1}{8}+3\omega^2+10\omega^4+\frac{16}{3}\omega^6\right) - \omega\sin a\left(\frac{29}{24}+\frac{1477}{180}\omega^2+\frac{389}{45}\omega^4+\frac{2}{5}\omega^6\right)\right\}$$

$$- 512\pi\omega^2 DS_3\left\{a\left(\frac{1}{10}+4\omega^2+28\omega^4+48\omega^6+16\omega^8\right) - \omega\sin a\left(\frac{37}{30}+\frac{247}{15}\omega^2+\frac{3644}{75}\omega^4+\frac{724}{25}\omega^6+\frac{64}{75}\omega^8\right)\right\}$$

$$- 128\pi\omega^2 DA\cos^2\theta\left\{a\left(\frac{1}{4}+\omega^2\right) - \omega\sin a\left(\frac{13}{12}+\frac{1}{6}\omega^2\right)\right\}$$

$$- 128\pi\omega^2\ DOB_1 \left\{ \begin{array}{l} \cos^2\theta \left[a\left(\frac{1}{4} + \frac{7}{2}\omega^2 + 4\omega^4\right) - \omega \sin a \left(\frac{113}{60} + \frac{27}{5}\omega^2 + \frac{7}{15}\omega^4\right)\right] \\ + \left[a\left(\frac{1}{24} + \frac{1}{4}\omega^2\right) - \omega \sin a \left(\frac{9}{40} + \frac{7}{90}\omega^2 - \frac{1}{90}\omega^4\right)\right] \end{array} \right\}$$

$$- 128\pi\omega^2 DOB_2 \left\{ \begin{array}{l} \cos^2\theta \left[a\left(\frac{5}{16} + \frac{49}{6}\omega^2 + 29\omega^4 + 16\omega^6\right) \right. \\ \left. - \omega \sin a \left(\frac{757}{240} + \frac{2803}{120}\omega^2 + \frac{2317}{90}\omega^4 + \frac{11}{9}\omega^6\right)\right] \\ + 2\sin^2\theta \left[a\left(\frac{1}{32} + \frac{5}{12}\omega^2 + \frac{1}{2}\omega^4\right) \right. \\ \left. - \omega \sin a \left(\frac{113}{480} + \frac{151}{240}\omega^2 + \frac{17}{180}\omega^4 - \frac{1}{90}\omega^6\right)\right] \end{array} \right\}$$

$$- 256\pi\omega^2 S_1{}^2 \left\{ \begin{array}{l} a\left(\frac{1}{8} + \frac{17}{6}\omega^2 + 8\omega^4 + 4\omega^6\right) \\ - \omega \sin a \left(\frac{29}{24} + \frac{139}{20}\omega^2 + \frac{292}{45}\omega^4 + \frac{14}{45}\omega^6\right) \end{array} \right\}$$

$$- 768\pi\omega^2 S_1 S_2 \left\{ \begin{array}{l} a\left(\frac{1}{10} + \frac{15}{4}\omega^2 + 22\omega^4 + \frac{100}{3}\omega^6 + \frac{32}{3}\omega^8\right) \\ - \omega \sin a \left(\frac{37}{30} + \frac{2497}{180}\omega^2 + \frac{15679}{450}\omega^4 + \frac{8692}{450}\omega^6 + \frac{264}{450}\omega^8\right) \end{array} \right\}$$

$$- 1024\pi\omega^2 S_1 S_3 \left\{ \begin{array}{l} a\left(\frac{1}{12} + \frac{47}{10}\omega^2 + 48\omega^4 + \frac{448}{3}\omega^6 + 144\omega^8 + 32\omega^{10}\right) \\ - \omega \sin a \left(\frac{5}{4} + \frac{118}{5}\omega^2 + \frac{815}{7}\omega^4 + \frac{90556}{525}\omega^6 + \frac{99304}{1575}\omega^8 \right. \\ \left. \qquad + \frac{2048}{1575}\omega^{10}\right) \end{array} \right\}$$

$$- 256\pi\omega^2 S_1 A \cos^2\theta \left\{ a\left(\frac{1}{6} + 2\omega^2 + 2\omega^4\right) - \omega \sin a \left(\frac{7}{6} + \frac{25}{9}\omega^2 + \frac{2}{9}\omega^4\right)\right\}$$

$$
- 256\pi\omega^2 S_1OB_1 \left\{ \begin{aligned} &\cos^2\theta \left[\begin{aligned} &a\left(\frac{3}{16} + \frac{29}{6}\,\omega^2 + 15\omega^4 + 8\omega^6\right) \\ &- \omega\sin a\left(\frac{467}{240} + \frac{1517}{120}\,\omega^2 + \frac{1151}{90}\,\omega^4 + \frac{29}{45}\,\omega^6\right) \end{aligned} \right] \\ &+ \left[\begin{aligned} &a\left(\frac{1}{32} + \frac{5}{12}\,\omega^2 + \frac{1}{2}\,\omega^4\right) \\ &- \omega\sin a\left(\frac{113}{480} + \frac{151}{240}\,\omega^2 + \frac{17}{180}\,\omega^4 - \frac{1}{90}\,\omega^6\right) \end{aligned} \right] \end{aligned} \right.
$$

$$
- 256\pi\omega^2 S_1OB_2 \left\{ \begin{aligned} &\cos^2\theta \left[\begin{aligned} &a\left(\frac{1}{4} + \frac{81}{8}\,\omega^2 + \frac{188}{3}\,\omega^4 + 98\omega^6 + 32\omega^8\right) \\ &- \omega\sin a\left(\frac{193}{60} + \frac{32603}{840}\,\omega^2 + \frac{2843}{28}\,\omega^4 + \frac{18173}{315}\,\omega^6 \right. \\ &\qquad \left. + \frac{562}{315}\,\omega^8\right) \end{aligned} \right] \\ &+ 2\sin^2\theta \left[\begin{aligned} &a\left(\frac{1}{40} + \frac{9}{16}\,\omega^2 + \frac{5}{3}\,\omega^4 + \omega^6\right) \\ &- \omega\sin a\left(\frac{29}{120} + \frac{157}{112}\,\omega^2 + \frac{6281}{4200}\,\omega^4 + \frac{401}{3150}\,\omega^6 \right. \\ &\qquad \left. - \frac{19}{1575}\,\omega^8\right) \end{aligned} \right] \end{aligned} \right.
$$

$$
- 512\pi\omega^2 S_2{}^2 \left\{ \begin{aligned} &a\left(\frac{3}{32} + \frac{207}{40}\,\omega^2 + \frac{199}{4}\,\omega^4 + 150\omega^6 + 144\omega^8 + 32\omega^{10}\right) \\ &- \omega\sin a\left(\frac{45}{32} + \frac{401}{16}\,\omega^2 + \frac{16479}{140}\,\omega^4 + \frac{60371}{350}\,\omega^6 + \frac{11034}{175}\,\omega^8 \right. \\ &\qquad \left. + \frac{228}{175}\,\omega^{10}\right) \end{aligned} \right\}
$$

$$- 512\pi\omega^2 S_2 S_3 \left\{ \begin{aligned} & a\left(\frac{3}{14} + \frac{33}{2}\omega^2 + \frac{1208}{5}\omega^4 + 1224\omega^6 + 2352\omega^8 + 1568\omega^{10} + 256\omega^{12}\right) \\ & - \omega\sin a\left(\frac{53}{14} + \frac{21089}{210}\omega^2 + \frac{161962}{210}\omega^4 + \frac{105184}{49}\omega^6 + \frac{2558392}{1225}\omega^8 + \frac{661216}{1225}\omega^{10} + \frac{9792}{1225}\omega^{12}\right) \end{aligned} \right\}$$

$$-384\pi\omega^2 S_2 A\cos^2\theta \left\{ \begin{aligned} & a\left(\frac{1}{8} + 3\omega^2 + 10\omega^4 + \frac{16}{3}\omega^6\right) \\ & - \omega\sin a\left(\frac{29}{24} + \frac{1477}{180}\omega^2 + \frac{389}{45}\omega^4 + \frac{2}{5}\omega^6\right) \end{aligned} \right\}$$

$$- 128\pi\omega^2 S_2 O B_1 \left\{ \begin{aligned} & \cos^2\theta \left[\begin{aligned} & a\left(\frac{9}{20} + 19\omega^2 + 120\omega^4 + 192\omega^6 + 64\omega^8\right) \\ & - \omega\sin a\left(\frac{119}{20} + \frac{5161}{70}\omega^2 + \frac{103576}{525}\omega^4 + \frac{60308}{525}\omega^6 + \frac{632}{175}\omega^8\right) \end{aligned} \right] \\ & + \left[\begin{aligned} & a\left(\frac{3}{40} + \frac{7}{4}\omega^2 + 6\omega^4 + 4\omega^6\right) \\ & - \omega\sin a\left(\frac{29}{40} + \frac{499}{105}\omega^2 + \frac{2059}{350}\omega^4 + \frac{268}{525}\omega^6 - \frac{8}{175}\omega^8\right) \end{aligned} \right] \end{aligned} \right\}$$

$$-128\pi\omega^2 S_2 O B_2 \left\{ \cos^2\theta \left[a\left(\frac{5}{8} + \frac{371}{10}\omega^2 + \frac{747}{2}\omega^4 + 1160\omega^6 + 1136\omega^8 + 256\omega^{10}\right) - \omega\sin a\left(\frac{391}{40} + \frac{77731}{420}\omega^2 + \frac{63111}{70}\omega^4 + \frac{33833}{25}\omega^6 + \frac{264044}{525}\omega^8 + \frac{5528}{525}\omega^{10}\right)\right] + 2\sin^2\theta \left[a\left(\frac{1}{16} + \frac{43}{20}\omega^2 + \frac{49}{4}\omega^4 + 20\omega^6 + 8\omega^8\right) - \omega\sin a\left(\frac{59}{80} + \frac{6479}{840}\omega^2 + \frac{561}{28}\omega^4 + \frac{4653}{350}\omega^6 + \frac{386}{525}\omega^8 - \frac{4}{75}\omega^{10}\right)\right]\right\}$$

$$-1024\pi\omega^2 S_3{}^2 \left\{ a\left(\frac{1}{16} + \frac{89}{14}\omega^2 + \frac{388}{3}\omega^4 + \frac{4898}{5}\omega^6 + 3136\omega^8 + \frac{12544}{3}\omega^{10} + 2048\omega^{12} + 256\omega^{14}\right) - \omega\sin a\left(\frac{61}{48} + \frac{38753}{840}\omega^2 + \frac{36229}{70}\omega^4 + \frac{439367}{189}\omega^6 + \frac{28426988}{6615}\omega^8 + \frac{32771008}{11025}\omega^{10} + \frac{18883552}{33075}\omega^{12} + \frac{209024}{33075}\omega^{14}\right)\right\}$$

$$-512\pi\omega^2 S_3 A\cos^2\theta \left\{ a\left(\frac{1}{10} + 4\omega^2 + 28\omega^4 + 48\omega^6 + 16\omega^8\right) - \omega\sin a\left(\frac{37}{30} + \frac{247}{15}\omega^2 + \frac{3644}{75}\omega^4 + \frac{724}{25}\omega^6 + \frac{64}{75}\omega^8\right)\right\}.$$

$$- 512\pi\omega^2 S_3 0 B_1 \left\{ \cos^2\theta \left[a\left(\frac{1}{8} + \frac{79}{10}\omega^2 + 86\omega^4 + 280\omega^6 + 280\omega^8 + 64\omega^{10}\right) - \omega\sin a\left(\frac{241}{120} + \frac{17351}{420}\omega^2 + \frac{22562}{105}\omega^4 + \frac{24886}{75}\omega^6 + \frac{65816}{525}\omega^8 + \frac{464}{175}\omega^{10}\right) \right] + \left[a\left(\frac{1}{48} + \frac{3}{4}\omega^2 + 5\omega^4 + \frac{28}{3}\omega^6 + 4\omega^8\right) - \omega\sin a\left(\frac{59}{240} + \frac{2473}{840}\omega^2 + \frac{3776}{420}\omega^4 + \frac{691}{105}\omega^6 + \frac{116}{315}\omega^8 - \frac{8}{315}\omega^{10}\right) \right] \right\}$$

$$- 512\pi\omega^2 S_3 0 B_2 \left\{ \cos^2\theta \left[a\left(\frac{5}{28} + \frac{59}{4}\omega^2 + \frac{2251}{10}\omega^4 + 1172\omega^6 + 2296\omega^8 + 1552\omega^{10} + 256\omega^{12}\right) - \omega\sin a\left(\frac{1381}{420} + \frac{38699}{420}\omega^2 + \frac{138238}{189}\omega^4 + \frac{13778227}{6615}\omega^6 + \frac{22707008}{11025}\omega^8 + \frac{17816072}{33075}\omega^{10} + \frac{266464}{33075}\omega^{12}\right) \right] + 2\sin^2\theta \left[a\left(\frac{1}{56} + \frac{7}{8}\omega^2 + \frac{33}{4}\omega^4 + 26\omega^6 + 28\omega^8 + 8\omega^{10}\right) - \omega\sin a\left(\frac{209}{840} + \frac{497}{120}\omega^2 + \frac{5377}{270}\omega^4 + \frac{421613}{13230}\omega^6 + \frac{31852}{2205}\omega^8 + \frac{3676}{6615}\omega^{10} - \frac{208}{6615}\omega^{12}\right) \right] \right\}$$

$$
- 64\pi\omega^2 A^2 \cos^2\theta \left\{ \begin{array}{l} \sin^2\theta \left[\frac{1}{4} a - \omega \sin a \left(\frac{5}{12} - \frac{1}{6} \omega^2 \right) \right] \\ + \cos^2\theta \left[a \left(\frac{1}{4} + \omega^2 \right) - \omega \sin a \left(\frac{13}{12} + \frac{1}{6} \omega^2 \right) \right] \end{array} \right\}
$$

$$
- 128\pi\omega^2 AOB_1 \left\{ \begin{array}{l} \cos^4\theta \left[a \left(\frac{1}{4} + \frac{7}{2} \omega^2 + 4\omega^4 \right) - \omega \sin a \left(\frac{113}{60} + \frac{27}{5} \omega^2 + \frac{7}{15} \omega^4 \right) \right] \\ + \cos^2\theta \left[a \left(\frac{1}{24} + \frac{1}{4} \omega^2 \right) - \omega \sin a \left(\frac{9}{40} + \frac{7}{90} \omega^2 - \frac{1}{90} \omega^4 \right) \right] \\ + \sin^2\theta \cos^2\theta \left[a \left(\frac{1}{4} + \omega^2 \right) - \omega \sin a \left(\frac{19}{20} + \frac{13}{30} \omega^2 - \frac{2}{15} \omega^4 \right) \right] \end{array} \right\}
$$

$$
- 128\pi\omega^2 AOB_2 \left\{ \begin{array}{l} \cos^4\theta \left[\begin{array}{l} a \left(\frac{5}{16} + \frac{49}{6} \omega^2 + 29\omega^4 + 16\omega^6 \right) \\ - \omega \sin a \left(\frac{757}{240} + \frac{2803}{120} \omega^2 + \frac{2317}{90} \omega^4 + \frac{11}{9} \omega^6 \right) \end{array} \right] \\ + \sin^2\theta \cos^2\theta \left[\begin{array}{l} a \left(\frac{5}{16} + \frac{7}{2} \omega^2 + 5\omega^4 \right) \\ - \omega \sin a \left(\frac{469}{240} + \frac{707}{120} \omega^2 + \frac{7}{6} \omega^4 - \frac{1}{5} \omega^6 \right) \end{array} \right] \end{array} \right\}
$$

$$- 64\pi\omega^2 OB_1{}^2 \left\{ \begin{aligned} &\cos^4\theta \left[a(5\omega^2 + 24\omega^4 + 16\omega^6) - \omega\sin a \left(\frac{46}{35} + \frac{125}{7}\omega^2 + \frac{852}{35}\omega^4 + \frac{52}{35}\omega^6 \right) \right] \\ &+ \cos^2\theta \left[a\left(\frac{3}{8} + \frac{14}{3}\omega^2 + 6\omega^4 \right) - \omega\sin a \left(\frac{433}{168} + \frac{3119}{420}\omega^2 + \frac{389}{315}\omega^4 - \frac{62}{315}\omega^6 \right) \right] \\ &+ \left[a\left(\frac{1}{64} + \frac{1}{8}\omega^2 \right) - \omega\sin a \left(\frac{221}{2240} + \frac{37}{672}\omega^2 - \frac{13}{840}\omega^4 + \frac{1}{420}\omega^6 \right) \right] \end{aligned} \right\}$$

$$- 128\pi\omega^2 OB_1 OB_2 \left\{ \begin{aligned} &\cos^4\theta \left[\begin{aligned} &a\left(\frac{9}{20} + \frac{151}{8}\omega^2 + \frac{719}{6}\omega^4 + 192\omega^6 + 64\omega^8 \right) \\ &- \omega\sin a \left(\frac{2483}{420} + \frac{61717}{840}\omega^2 + \frac{414349}{2100}\omega^4 + \frac{180884}{1575}\omega^6 + \frac{814}{225}\omega^8 \right) \end{aligned} \right] \\ &+ \sin^2\theta\cos^2\theta \left[\begin{aligned} &a\left(\frac{19}{40} + \frac{43}{4}\omega^2 + \frac{109}{3}\omega^4 + 24\omega^6 \right) \\ &- \omega\sin a \left(\frac{3713}{840} + \frac{1018}{35}\omega^2 + \frac{12317}{350}\omega^4 + \frac{5042}{1575}\omega^6 - \frac{536}{1575}\omega^8 \right) \end{aligned} \right] \\ &+ 2\sin^4\theta \left[\begin{aligned} &a\left(\frac{1}{80} + \frac{3}{16}\omega^2 + \frac{1}{4}\omega^4 \right) \\ &- \omega\sin a \left(\frac{57}{560} + \frac{503}{1680}\omega^2 + \frac{253}{4200}\omega^4 - \frac{1}{75}\omega^6 + \frac{1}{525}\omega^8 \right) \end{aligned} \right] \end{aligned} \right\}$$

$$
\begin{aligned}
- 64\pi\omega^2 OB_2{}^2 \Bigg\{ & \cos^4\theta \Bigg[a\left(\frac{13}{24} + \frac{1351}{40}\omega^2 + 352\omega^4 + 1122\omega^6 + 1120\omega^8 + 256\omega^{10}\right) \\
& - \omega \sin a \left(\frac{7321}{840} + \frac{37177}{216}\omega^2 + \frac{3267347}{3780}\omega^4 + \frac{2090101}{1575}\omega^6 + \frac{2369194}{4725}\omega^8 + \frac{7184}{675}\omega^{10}\right)\Bigg] \\
& + 4\sin^4\theta \Bigg[a\left(\frac{1}{96} + \frac{39}{160}\omega^2 + \frac{3}{4}\omega^4 + \frac{1}{2}\omega^6\right) \\
& - \omega \sin a \left(\frac{349}{3360} + \frac{18659}{30240}\omega^2 + \frac{10823}{15120}\omega^4 + \frac{499}{6300}\omega^6 - \frac{127}{9450}\omega^8 + \frac{8}{4725}\omega^{10}\right)\Bigg] \\
& + \sin^2\theta\cos^2\theta \Bigg[a\left(\frac{7}{12} + \frac{423}{20}\omega^2 + 131\omega^4 + 228\omega^6 + 96\omega^8\right) \\
& - \omega \sin a \left(\frac{583}{84} + \frac{100861}{1260}\omega^2 + \frac{140923}{630}\omega^4 + \frac{27584}{175}\omega^6 + \frac{4796}{525}\omega^8 - \frac{368}{525}\omega^{10}\right)\Bigg]\Bigg\}
\end{aligned}
$$

$$- 16\pi\omega^2 B^2 \cos^2\theta \left\{ a - \omega \sin a \right\}$$

$$- 64\pi\omega^2 BC_1 \cos^2\theta \left\{ a\left(\frac{1}{2} + 2\omega^2\right) - \omega \sin a \left(\frac{11}{6} + \frac{2}{3}\omega^2\right) \right\}$$

$$- 32\pi\omega^2 BC_2 \cos^2\theta \left\{ a(1 + 12\omega^2 + 16\omega^4) - \omega \sin a \left(\frac{19}{3} + 20\omega^2 + \frac{8}{3}\omega^4\right) \right\}$$

$$
\begin{aligned}
- 32\pi\omega^2 BEC \Bigg\{ & \cos^4\theta \left[a\left(\frac{3}{4} + 4\omega^2\right) - \omega \sin a \left(\frac{13}{4} + \frac{3}{2}\omega^2\right)\right] \\
& + \sin^2\theta\cos^2\theta \left[\frac{3}{4} a - \omega \sin a \left(\frac{5}{4} - \frac{1}{2}\omega^2\right)\right] \Bigg\}
\end{aligned}
$$

$$- 64\pi\omega^2 C_1^2 \left\{ \begin{aligned} &\cos^2\theta \left[a\left(\frac{3}{8} + \frac{17}{4}\omega^2 + 4\omega^4\right) - \omega\sin a\left(\frac{101}{40} + \frac{83}{15}\omega^2 + \frac{17}{30}\omega^4\right)\right] \\ &+ \sin^2\theta \left[a\left(\frac{1}{24} + \frac{1}{4}\omega^2\right) - \omega\sin a\left(\frac{9}{40} + \frac{7}{90}\omega^2 - \frac{1}{90}\omega^4\right)\right] \end{aligned} \right\}$$

$$- 32\pi\omega^2 C_1 C_2 \left\{ \begin{aligned} &\cos^2\theta \left[\begin{aligned} &a\left(\frac{7}{4} + \frac{122}{3}\omega^2 + 124\omega^4 + 64\omega^6\right) \\ &- \omega\sin a\left(\frac{1007}{60} + \frac{3149}{30}\omega^2 + \frac{4646}{45}\omega^4 + \frac{244}{45}\omega^6\right) \end{aligned} \right] \\ &+ \sin^2\theta \left[\begin{aligned} &a\left(\frac{1}{4} + \frac{10}{3}\omega^2 + 4\omega^4\right) \\ &- \omega\sin a\left(\frac{113}{60} + \frac{151}{30}\omega^2 + \frac{34}{45}\omega^4 - \frac{4}{45}\omega^6\right) \end{aligned} \right] \end{aligned} \right\}$$

$$- 32\pi\omega^2 C_1 E C \left\{ \begin{aligned} &\cos^4\theta \left[a\left(\frac{5}{4} + 16\omega^2 + 16\omega^4\right) - \omega\sin a\left(\frac{183}{20} + \frac{217}{10}\omega^2 + \frac{12}{5}\omega^4\right)\right] \\ &+ \sin^2\theta\cos^2\theta \left[a\left(\frac{5}{4} + 6\omega^2\right) - \omega\sin a\left(\frac{111}{20} + \frac{67}{30}\omega^2 - \frac{8}{15}\omega^4\right)\right] \end{aligned} \right\}$$

$$- 16\pi\omega^2 C_2^2 \left\{ \begin{aligned} &\cos^2\theta \left[\begin{aligned} &a\left(\frac{11}{5} + 85\omega^2 + \frac{1520}{3}\omega^4 + 784\omega^6 + 256\omega^8\right) \\ &- \omega\sin a\left(\frac{413}{15} + \frac{33359}{105}\omega^2 + \frac{426506}{525}\omega^4 + \frac{727144}{1575}\omega^6 + \frac{22928}{1575}\omega^8\right) \end{aligned} \right] \\ &+ \sin^2\theta \left[\begin{aligned} &a\left(\frac{2}{5} + 9\omega^2 + \frac{80}{3}\omega^4 + 16\omega^6\right) \\ &- \omega\sin a\left(\frac{58}{15} + \frac{157}{7}\omega^2 + \frac{12562}{525}\omega^4 + \frac{3208}{1575}\omega^6 - \frac{304}{1575}\omega^8\right) \end{aligned} \right] \end{aligned} \right\}$$

$$
- 32\pi\omega^2 C_2 EC \left\{ \begin{aligned} &\cos^4\theta \left[a\left(\frac{3}{2} + 38\omega^2 + 120\omega^4 + 64\omega^6\right) - \omega\sin a\left(\frac{153}{10} + \frac{301}{3}\omega^2 + \frac{1534}{15}\omega^4 + \frac{28}{5}\omega^6\right)\right] \\ &+ \sin^2\theta\cos^2\theta \left[a\left(\frac{3}{2} + 18\omega^2 + 24\omega^4\right) - \omega\sin a\left(\frac{101}{10} + 29\omega^2 + \frac{26}{5}\omega^4 - \frac{4}{5}\omega^6\right)\right] \end{aligned} \right\}
$$

$$
- 16\pi\omega^2 EC^2\cos^2\theta \left\{ \begin{aligned} &\cos^4\theta \left[a\left(\frac{9}{8} + 15\omega^2 + 16\omega^4\right) - \omega\sin a\left(\frac{339}{40} + \frac{421}{20}\omega^2 + \frac{13}{5}\omega^4\right)\right] \\ &+ \sin^2\theta\cos^2\theta \left[a\left(\frac{9}{4} + 15\omega^2\right) - \omega\sin a\left(\frac{243}{20} + \frac{67}{10}\omega^2 - \frac{8}{5}\omega^4\right)\right] \\ &+ \sin^4\theta \left[\frac{9}{8} a - \omega\sin a\left(\frac{99}{40} - \frac{39}{20}\omega^2 + \frac{3}{5}\omega^4\right)\right] \end{aligned} \right\}
$$

Some of these terms are plotted, as functions of spatial frequency ω, in Figs. B.1(a-h).

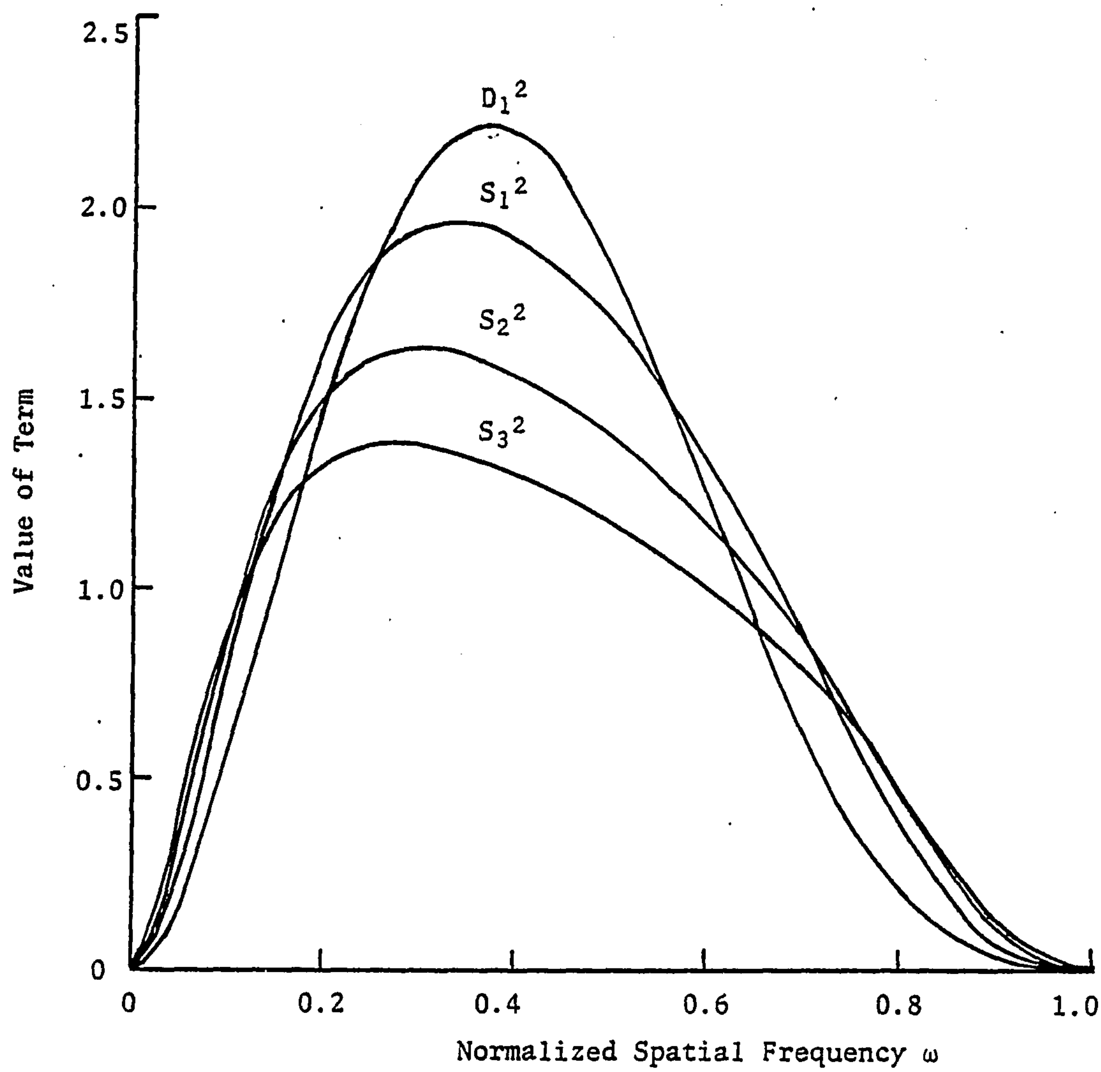

Fig. B.1. Second-order OTF Expansion.

(a) Spherical aberration terms.

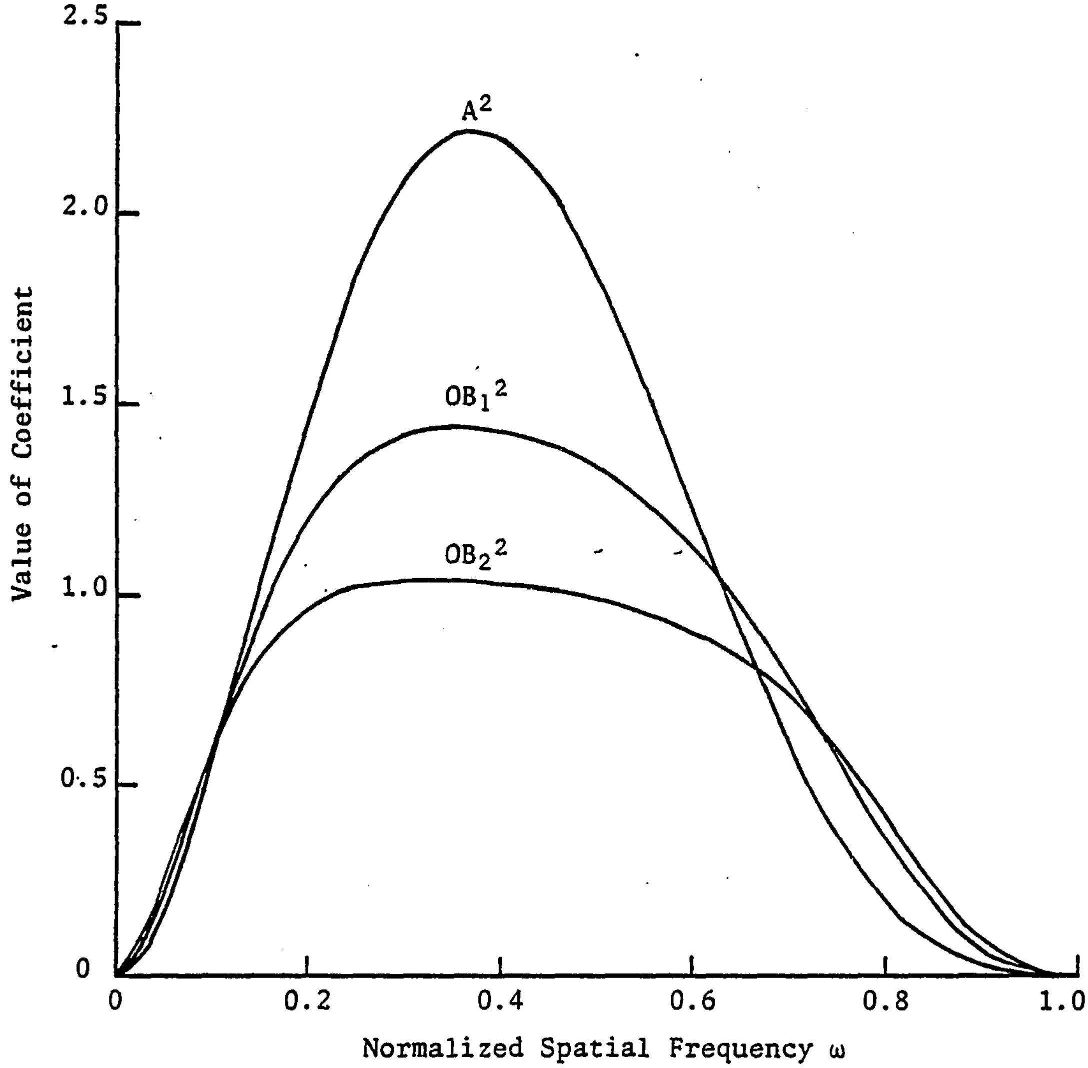

Fig. B.1. Continued.

(b) Astigmatic aberration terms, $\theta = 0^{\circ}$

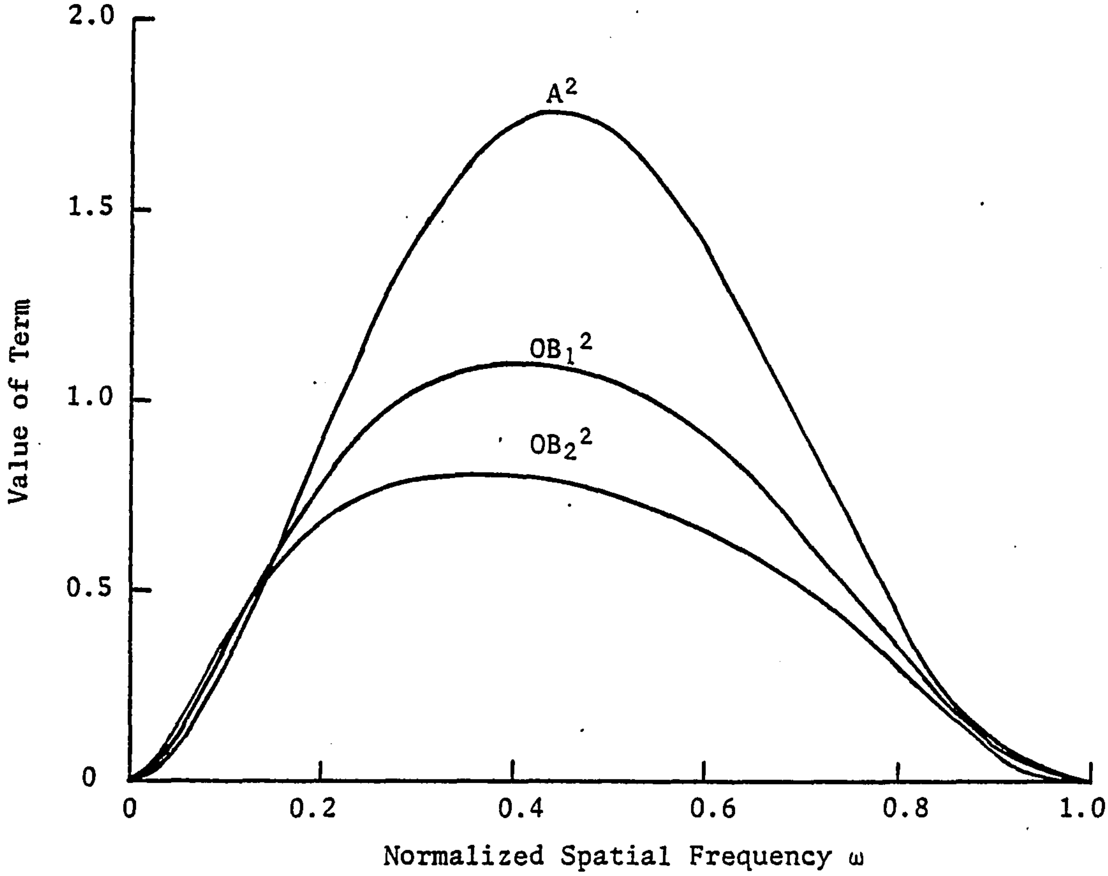

Fig. B.1. Continued.

(c) Astigmatic aberration terms, $\theta = 45^{\circ}$

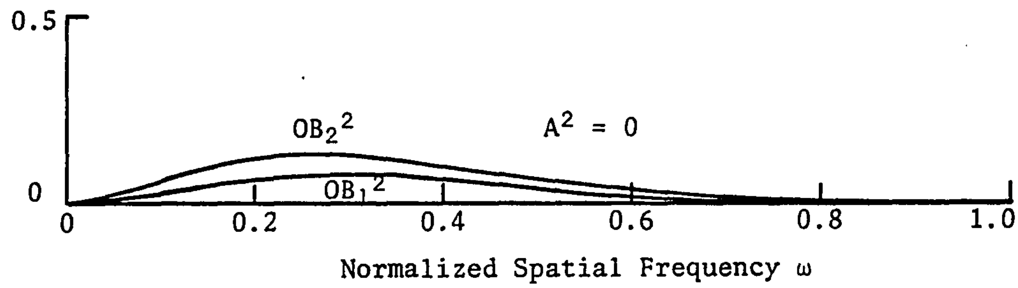

Fig. B.1. Continued, Second-order OTF Expansion.

(d) Astigmatic aberration terms, $\theta = 90°$

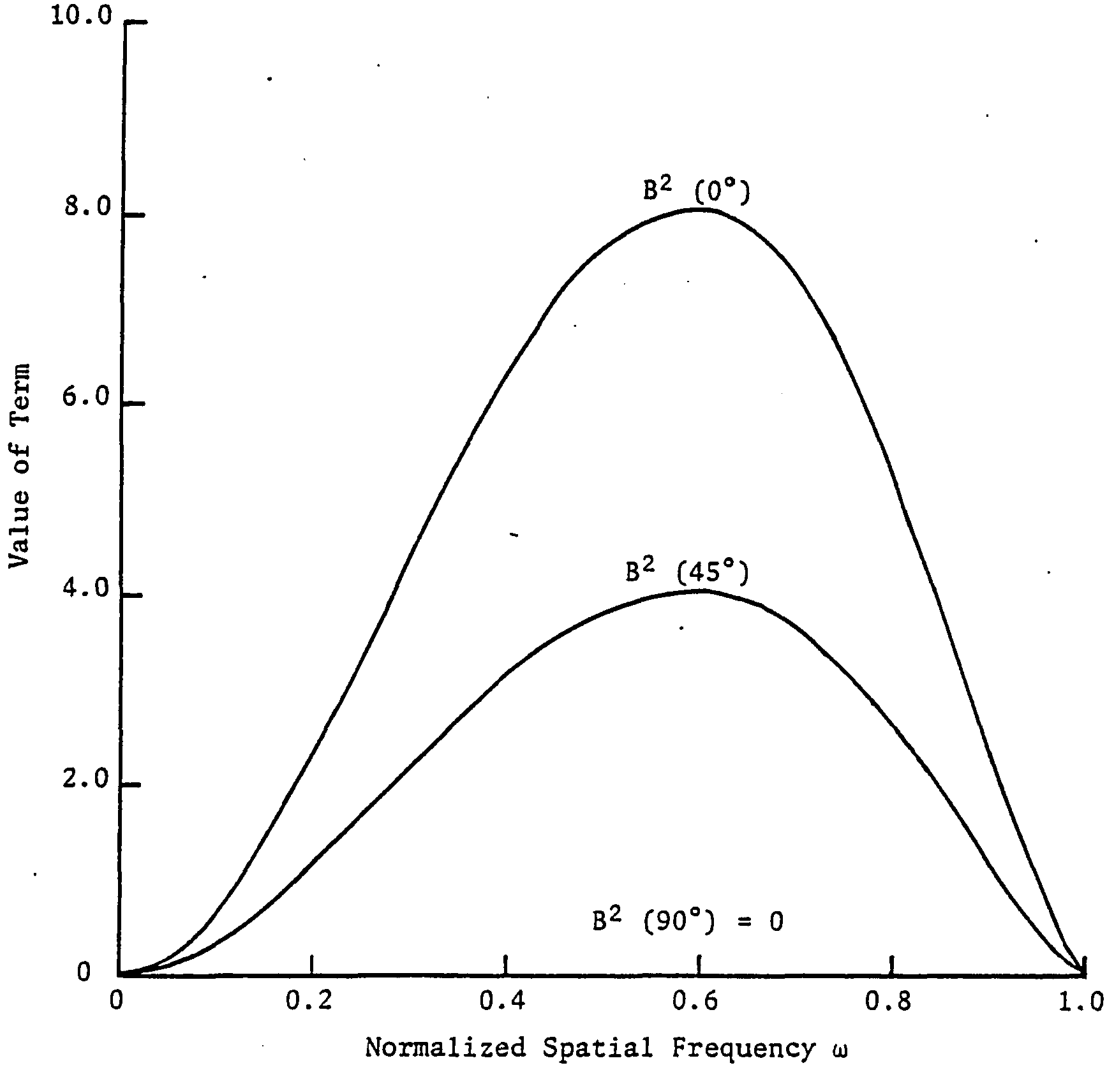

Fig. B.1. Continued.

(e) B^2 term

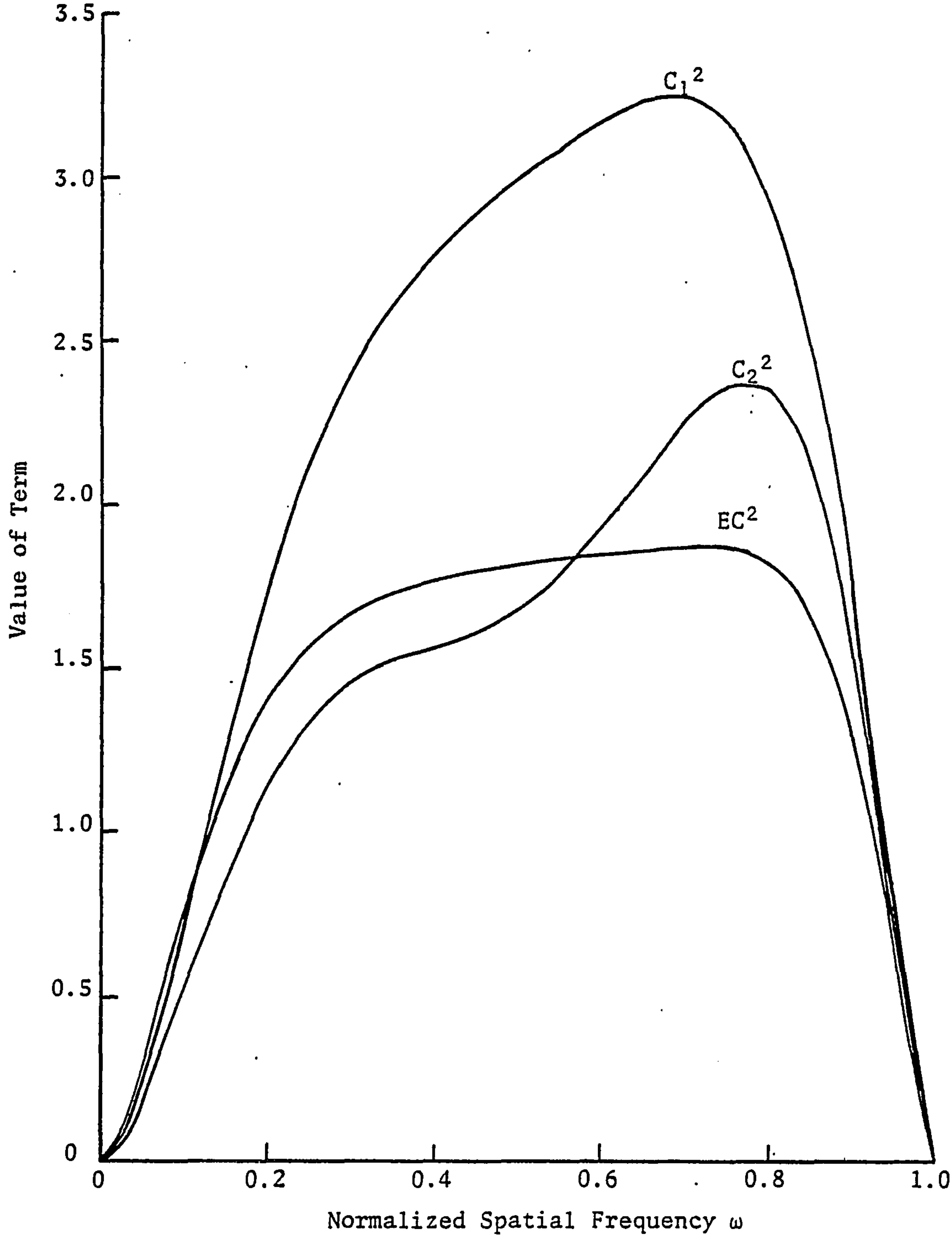

Fig. B.1. Continued.

(f) Comatic aberration terms, $\theta = 0^{\circ}$

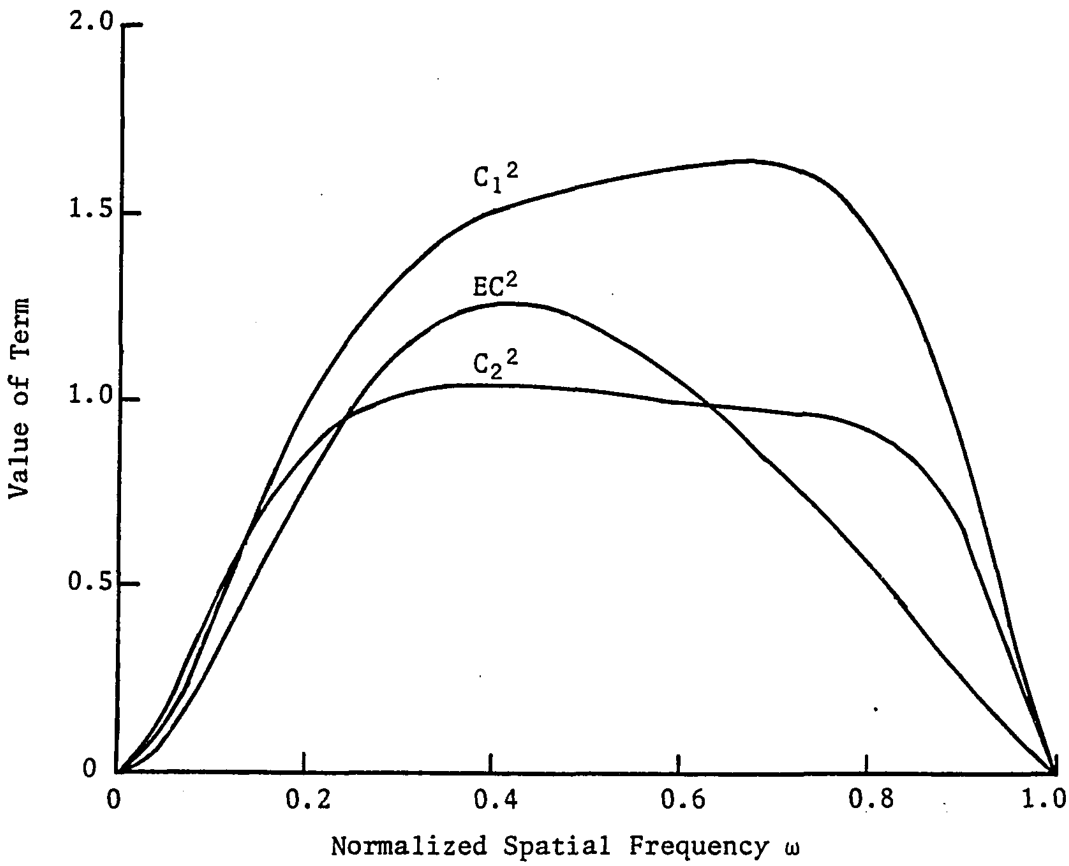

Fig. B.1. Continued.

(g) Comatic aberration terms, $\theta = 45^\circ$

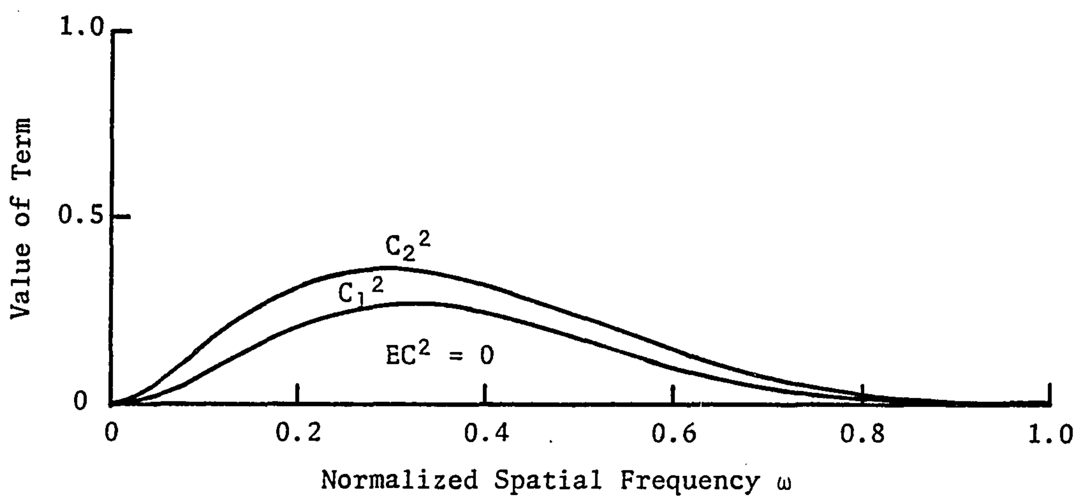

Fig. B.1. Continued, Second-order OTF Expansion.

(h) Comatic aberration terms, $\theta = 90^o$

APPENDIX C

THE IMAGINARY PART OF THE OTF

Calculation of the imaginary part of the OTF requires the evaluation of the following expression

$$\int_{-a}^{a}\int_{\frac{\omega}{\cos\phi}}^{1} (W-W')\;\rho\, d\rho\, d\phi$$

where W-W' is the wave aberration difference function. The result is

$$2\omega\lambda\cos\theta \left\{ \begin{aligned} & B\left[a - \omega\sin a\right] \\ & + C_1\left[a(1 + 4\omega^2) - \omega\sin a\left(\frac{11}{3} + \frac{4}{3}\,\omega^2\right)\right] \\ & + C_2\left[a(1 + 12\omega^2 + 16\omega^4) - \omega\sin a\left(\frac{19}{3} + 20\omega^2 + \frac{8}{3}\,\omega^4\right)\right] \\ & + EC\left[\begin{aligned} & \cos^2\theta\left(a\left(\frac{3}{4} + 4\omega^2 - \omega\sin a\left(\frac{13}{4} + \frac{3}{2}\,\omega^2\right)\right)\right) \\ & + \sin^2\theta\left(\frac{3}{4}\,a - \omega\sin a\left(\frac{5}{4} - \frac{1}{2}\,\omega^2\right)\right) \end{aligned}\right] \end{aligned} \right\}$$

The above result is for half the overlapping pupil area. The result for the other half is the same as the above for the odd aberrations and opposite in sign for the even aberrations.

APPENDIX D

COMPUTER PROGRAMS

This appendix contains listings of the following programs:

DIFFMTF-----second-order expansion of the OTF. The modulus, phase, real and imaginary parts of the OTF are calculated as functions of the wave aberration coefficients, the normalized spatial frequency and the azimuthal angle.

TFORTHO-----the second-order expansion of the real part of the OTF is orthogonalized, averaged over azimuthal angle and displayed as a function of spatial frequency.

TFIORTH-----the second-order expansion of the real part of the OTF is multiplied by a Lorentzian weighting function of the form $\frac{1}{1+(R\omega)^2}$. The result is then integrated over spatial frequency and azimuthal angle; the upper integration limit for the spatial frequency is either the frequency at which the Lorentzian has fallen to 0.I or the cutoff frequency. The expression is then orthogonalized and the output is displayed as a function of the parameter R.

AUTCORR-----this calculates the exact OTF by autocorrelation of the pupil function, using the Hopkins algorithm. The output displays the modulus, phase, real and imaginary parts of the OTF, as functions of the wave aberration coefficients, the normalized spatial frequency and the azimuthal angle.

```
      PROGRAM DIFFMTF(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
COMPUTATION OF APPROXIMATE DIFFRACTION-BASED OTF
C     THIS PROGRAM COMPUTES THE DIFFRACTION-BASED OTF TO SECOND ORDER
C     IN THE WAVE-ABERRATION COEFFICIENTS
 5    READ(5,10)D,S1,S2,S3,C1,C2,EC,A,OB1,OB2,B
 10   FORMAT(11(F5.2))
 11   IF(D.EQ.999.)90,15
 15   READ(5,20)THETA
      IF(THETA.EQ.999.)5,16
 16   RTHETA=THETA*PI/180.
 17   U=(COS(RTHETA))**2.
 18   V=(SIN(RTHETA))**2.
 20   FORMAT(F3.0)
 21   WRITE(6,22)
 22   FORMAT(1H1,4X,*THETA*,8X,*D*,8X,*S1*,8X,*S2*,8X,*S3*,8X,*OB1*,8X,
     2*OB2*,8X,*C1*,8X,*C2*,8X,*EC*,8X,*A*,8X,*B*)
 23   WRITE(6,24)THETA,D,S1,S2,S3,OB1,OB2,C1,C2,EC,A,B
 24   FORMAT(5X,F4.0,7X,F6.2,3X,F6.2,3X,F6.2,8(4X,F6.2))
 30   READ(5,35)W
 35   FORMAT(F5.3)
 36   IF(W.EQ.2.)15,28
 28   WRITE(6,29)
 29   FORMAT(///,4X,*W*,6X,*DIFF*,6X,*DD*,7X,*S1S1*,7X,*2DS1*,7X,*S2S2*,
     27X,*2S1S2*,7X,*2DS2*,10X,*AA*,7X,*2AD*,7X,*2AS1*,7X,*2AS2*,5X,*C1C
     31*)
 40   PI=3.14159265358979
 41   Y=ACOS(W)
 42   Z=W*(SQRT(1.-(W*W)))
 43   X=PI*W*W
 44   T=Y-Z
      C=W*W
      E=W**6.
      F=W**8.
      G=W**10.
      H=W**12.
      P=W**14.
      Q=W**4.
      S3S3=1024.*S3*S3*X*((Y*((1./16.)+(C*89./14.)+(Q*388./3.)+(E*4898./
     25.)+(F*3136.)+(G*12544./3.)+(H*2048.)+(P*256.)))-(Z*((61./48.)+(C*
     338753./840.)+(Q*36229./70.)+(E*439367./189.)+(F*28426988./6615.)+(
     4G*32771008./11025.)+(H*18883552./33075.)+(P*209024./33075.))))
      S2S3=512.*S2*S3*X*((Y*((3./14.)+(C*33./2.)+(Q*1208./5.)+(E*1224.)+
     2(F*2352.)+(G*1568.)+(H*256.)))-(Z*((53./14.)+(C*21089./210.)+(Q*16
     31962./210.)+(E*105184./49.)+(F*2558392./1225.)+(G*661216./1225.)+(
     4H*9792./1225.))))
      S1S3=1024.*S1*S3*X*((Y*((1./12.)+(C*47./10.)+(Q*48.)+(E*448./3.)+(
     2F*144.)+(G*32.)))-(Z*((5./4.)+(C*118./5.)+(Q*815./7.)+(E*90556./52
     35.)+(F*99304./1575.)+(G*2048./1575.))))
      DS3=512.*D*S3*X*((Y*((1./10.)+(C*4.)+(Q*28.)+(E*48.)+(F*16.)))-(Z*
     2((37./30.)+(C*247./15.)+(Q*3644./75.)+(E*724./25.)+(F*64./75.))))
      AS3=512.*A*S3*U*X*((Y*((1./10.)+(C*4.)+(Q*28.)+(E*48.)+(F*16.)))-
     2(Z*((37./30.)+(C*247./15.)+(Q*3644./75.)+(E*724./25.)+(F*64./75.)
     3)))
```

```
 O81O81A=(Y*((C*5.)+(Q*24.)+(E*16.)))-(Z*((46./35.)+(C*125./7.)+(Q*
2852./35.)+(E*52./35.)))
 O81O81B=(Y*((3./8.)+(C*14./3.)+(Q*6.)))-(Z*((433./168.)+(C*3119./4
220.)+(Q*389./315.)-(E*62./315.)))
 O81O81C=(Y*((1./64.)+(C/8.)))-(Z*((221./2240.)+(C*37./672.)-(Q*13.
2/840.)+(E/420.)))
 O81O81=((U*U*O81O81A)+(U*O81O81B)+O81O81C)*64.*X*O81*O81
 O81S2A=(Y*((9./20.)+(C*19.)+(Q*120.)+(E*192.)+(F*64.)))-(Z*((119./
220.)+(C*5161./70.)+(Q*103576./525.)+(E*60308./525.)+(F*632./175.))
3)
 O81S2B=(Y*((3./40.)+(C*7./4.)+(Q*6.)+(E*4.)))-(Z*((29./40.)+(C*499
2./105.)+(Q*2059./350.)+(E*268./525.)-(F*8./175.)))
 O81S2=((U*O81S2A)+O81S2B)*128.*X*S2*O81
 O81S1A=(Y*((3./16.)+(C*29./6.)+(Q*15.)+(E*8.)))-(Z*((467./240.)+(C
2*1517./120.)+(Q*1151./90.)+(E*29./45.)))
 O81S1B=(Y*((1./32.)+(C*5./12.)+(Q/2.)))-(Z*((113./480.)+(C*151./24
20.)+(Q*17./180.)-(E/90.)))
 O81S1=((O81S1A*U)+O81S1B)*256.*X*O81*S1
 O81DA=(Y*((1./4.)+(C*7./2.)+(Q*4.)))-(Z*((113./60.)+(C*27./5.)+(Q*
27./15.)))
 O81DB=(Y*((1./24.)+(C/4.)))-(Z*((9./40.)+(C*7./90.)-(Q/90.)))
 O81D=((U*O81DA)+(O81DB))*128.*X*D*O81
 O81AA=(Y*((1./4.)+(C)))-(Z*((19./20.)+(C*13./30.)-(Q*2./15.)))
 O81A=128.*X*O81*A*U*((O81DA*U)+O81DB+(V*O81AA))
 O81S3A=(Y*((1./8.)+(C*79./10.)+(Q*86.)+(E*280.)+(F*280.)+(G*64.)))
2-(Z*((241./120.)+(C*17351./420.)+(Q*22562./105.)+(E*24886./75.)+(F
3*65816./525.)+(G*464./175.)))
 O81S3B=(Y*((1./48.)+(C*3./4.)+(Q*5.)+(E*28./3.)+(F*4.)))-(Z*((59./
2240.)+(C*2473./840.)+(Q*3776./420.)+(E*691./105.)+(F*116./315.)-(G
3*8./315.)))
 O81S3=512.*X*O81*S3*((U*O81S3A)+O81S3B)
 O82O82A=(Y*((13./24.)+(C*1351./40.)+(Q*352.)+(E*1122.)+(F*1120.)+(
2G*256.)))-(Z*((7321./840.)+(C*37177./216.)+(Q*3267347./3780.)+(E*2
3090101./1575.)+(F*2369194./4725.)+(G*7184./675.)))
 O82O82B=(Y*((1./96.)+(C*39./160.)+(Q*3./4.)+(E/2.)))-(Z*((349./336
20.)+(C*18659./30240.)+(Q*10823./15120.)+(E*499./6300.)-(F*127./945
30.)+(G*8./4725.)))
 O82O82C=(Y*((7./12.)+(C*423./20.)+(Q*131.)+(E*228.)+(F*96.)))-(Z*(
2(583./84.)+(C*100861./1260.)+(Q*140923./630.)+(E*27584./175.)+(F*4
3796./525.)-(G*368./525.)))
 O82O82=64.*X*O82*O82*((O82O82A*U*U)+(O82O82B*4.*V*V)+(O82O82C*U*V)
2)
 O82S3A=(Y*((5./28.)+(C*59./4.)+(Q*2251./10.)+(E*1172.)+(F*2296.)+(
2G*1552.)+(H*256.)))-(Z*((1381./420.)+(C*38699./420.)+(Q*138238./18
39.)+(E*13778227./6615.)+(F*22707008./11025.)+(G*17816072./33075.)+
4(H*266464./33075.)))
 O82S3B=(Y*((1./56.)+(C*7./8.)+(Q*33./4.)+(E*26.)+(F*28.)+(G*8.)))-
2(Z*((209./840.)+(C*497./120.)+(Q*5377./270.)+(E*421613./13230.)+(F
3*31852./2205.)+(G*3676./6615.)-(H*208./6615.)))
 O82S3=512.*X*O82*S3*((U*O82S3A)+(2.*V*O82S3B))
 O82O81A=(Y*((9./20.)+(C*151./8.)+(Q*719./6.)+(E*192.)+(F*64.)))-(Z
2*((2483./420.)+(C*61717./840.)+(Q*414349./2100.)+(E*180884./1575.)
3+(F*814./225.)))
```

```
      OB2OB1B=(Y*((19./40.)+(C*43./4.)+(Q*109./3.)+(E*24.)))-(Z*((3713./
     2840.)+(C*1018./35.)+(Q*12317./350.)+(E*5042./1575.)-(F*536./1575.)
     3))
      OB2OB1C=(Y*((1./80.)+(C*3./16.)+(Q/4.)))-(Z*((57./560.)+(C*503./16
     280.)+(Q*253./4200.)-(E/75.)+(F/525.)))
      OB2OB1=128.*X*OB2*OB1*((U*U*OB2OB1A)+(U*V*OB2OB1B)+(2.*V*V*OB2OB1C
     2))
      OB2S2A=(Y*((5./8.)+(C*371./10.)+(Q*747./2.)+(E*1160.)+(F*1136.)+(G
     2*256.)))-(Z*((391./40.)+(C*77731./420.)+(Q*63111./70.)+(E*33833./2
     35.)+(F*264044./525.)+(G*5528./525.)))
      OB2S2B=(Y*((1./16.)+(C*43./20.)+(Q*49./4.)+(E*20.)+(F*8.)))-(Z*((5
     29./80.)+(C*6479./840.)+(Q*561./28.)+(E*4653./350.)+(F*386./525.)-(
     3G*4./75.)))
      OB2S2=128.*X*OB2*S2*((U*OB2S2A)+(2.*V*OB2S2B))
      OB2DA=(Y*((5./16.)+(C*49./6.)+(Q*29.)+(E*16.)))-(Z*((757./240.)+(2
     2803.*C/120.)+(Q*2317./90.)+(E*11./9.)))
      OB2DB=(Y*((1./32.)+(C*5./12.)+(Q/2.)))-(Z*((113./480.)+(C*151./24
     20.)+(Q*17./180.)-(E/90.)))
      OB2D=128.*X*OB2*D*((U*OB2DA)+(2.*V*OB2DB))
      OB2AA=(Y*((5./16.)+(C*7./2.)+(Q*5.)))-(Z*((469./240.)+(C*707./120.
     2)+(Q*7./6.)-(E/5.)))
      OB2A=128.*X*OB2*A*U*((U*OB2DA)+(V*OB2AA))
      OB2S1A=(Y*((1./4.)+(C*81./8.)+(Q*188./3.)+(E*98.)+(F*32.)))-(Z*((1
     293./60.)+(C*32603./840.)+(Q*2843./28.)+(E*18173./315.)+(F*562./315
     3.)))
      OB2S1B=(Y*((1./40.)+(C*9./16.)+(Q*5./3.)+(E)))-(Z*((29./120.)+(C*1
     257./112.)+(Q*6281./4200.)+(E*401./3150.)-(F*19./1575.)))
45    DIFM=2.0*(ACOS(W)-(W*SQRT(1.0-(W**2.0))))/PI
      OB2S1=256.*X*OB2*S1*((OB2S1A*U)+(OB2S1B*V*2.))
      AAA=V*((Y/4.)-(Z*((5./12.)-(W*W/6.))))
      AAB=U*((Y*((1./4.)+(W*W)))-(Z*((13./12.)+(W*W/6.))))
131   AA=U*64.*X*A*A*(AAA+AAB)
136   AD=128.*X*A*D*AAB
140   AS1=256.*A*S1*X*U*((Y*((1./6.)+(W*W*2.)+((W**4.)*2.)))-(Z*((7./6.)
     2+(W*W*25./9.)+((W**4.)*2./9.))))
145   AS2=384.*A*S2*X*U*((Y*((1./8.)+(W*W*3.)+((W**4.)*10.)+((W**6.)*16.
     2/3.)))-(Z*((29./24.)+(W*W*1477./180.)+((W**4.)*389./45.)+((W**6.)*
     332./5.))))
      DM=64.*X*D*D*((Y*((1./4.)+(W*W)))-(Z
     2*((13./12.)+((1./6.)*W*W))))
      S1M=256.*S1*S1*W*W*PI*((Y*((1./8.)+(17.*W*W/6.)+((W**4.)*8.)
     2+((W**6.)*4.)))-(Z*((29./24.)+(139.*W*W/20.)+(
     3292.*(W**4.)/45.)+(14.*(W**6.)/45.))))
      DS1M2=256.*D*S1*W*W*PI*((Y*((1./6.)+(2.*W*W)+((W**4.)*2.)))-
     2(Z*((7./6.)+(25.*W*W/9.)+((W**4.)*2./9.))))
      S2S2=4.*X*S2*S2*((Y*((12.)+(6624./10.*W*W)+(6368.*(W**4
     2.))+(19200.*(W**6.))+(18432.*(W**8.))+(4096.*(W**10.))))-((Z)*
     3((180.)+(3208.*W*W)+(527328./35.*(W**4.))+(7727488./350.
     4*(W**6.))+(2824704./350.*(W**8.))+(58368./350.*(W**10.)))))
      S1S2=768.*S1*S2*X*((Y*((1./10.)+(W*W*15./4.)+((W**4.)*2
     22.)+((W**6.)*100./3.)+((W**8.)*32./3.)))-((Z)*((37.
     3/30.)+(W*W*2497./180.)+((W**4.)*15679./450.)+((W**6.)*8692./450.)+
     4((W**8.)*264./450.))))
```

```
      DS2=384.*D*S2*X*((Y*((1./8.)+(W*W*3.)+((W**4.)*10.)+((W*
     2*6.)*16./3.)))-((Z)*((29./24.)+(W*W*1477./180.)+((W
     3**4.)*389./45.)+((W**6.)*2./5.))))
      C1C1A=64.*X*((Y*((3./8.)+(W*W*17./4.)+((W**4.)*4.)))-(Z*
     2((101./40.)+(W*W*83./15.)+((W**4.)*17./30.))))
      C1C1B=64.*X*((Y*((1./24.)+(W*W*1./4.)))-(Z*((9./40.)+(W*W*7./90.)-
     2((W**4.)/90.))))
      C1C1C=4.*W*((Y*((1./2.)+(2.*W*W)))-(Z*((11./6.)+(W*W*2./3.))))
60    C1C1=C1*C1*((C1C1A*U)+(C1C1B*V)-(C1C1C*C1C1C*4.*PI*U/T))
      C2C2A=U*((Y*((11./5.)+(85.*W*W)+((W**4.)*1520./3.)+((W**6.)*784.)
     2+((W**8.)*256.)))-(Z*((413./15.)+(33359.*W*W/105.)+((W**4.)*426506
     3./525.)+((W**6.)*727144./1575.)+((W**8.)*22928./1575.))))
      C2C2B=V*((Y*((2./5.)+(9.*W*W)+((W**4.)*80./3.)+((W**6.)*16.)))-(Z*
     2((58./15.)+(157.*W*W/7.)+((W**4.)*12562./525.)+((W**6.)*3208./1575
     3.)-((W**8.)*304./1575.))))
      C2C2C=Y*(1.+(12.*W*W)+((W**4.)*16.))-(Z*((19./3.)+(W*W*20.)+((W**4
     2.)*8./3.)))
100   C2C2=C2*C2*16.*X*(C2C2A+C2C2B-(C2C2C*C2C2C*U/T))
      C1C2A=U*((Y*((7./4.)+(122.*W*W/3.)+((W**4.)*124.)+((W**6.)*64.)))-
     2(Z*((1007./60.)+(3149.*W*W/30.)+((W**4.)*4646./45.)+((W**6.)*244./
     345.))))
      C1C2B=V*((Y*((1./4.)+(W*W*10./3.)+((W**4.)*4.)))-(Z*((113./60.)+(1
     251.*W*W/30.)+((W**4.)*34./45.)-((W**6.)*4./45.))))
      C1C2C=U*((Y*((1./2.)+(W*W*2.)))-(Z*((11./6.)+(W*W*2./3.))))*C2C2C
110   C1C2=32.*X*(C1C2A+C1C2B-(C1C2C*2./T))*C1*C2
      ECECA1=((Y*((9./8.)+(15.*W*W)+((W**4.)*16.)))-(Z*((339./40.)+(4
     221.*W*W/20.)+((W**4.)*13./5.))))
      ECECA=ECECA1*U*U
      ECECB=U*V*((Y*((9./4.)+(15.*W*W)))-(Z*((243./20.)+(W*W*67./10.)-((
     2W**4.)*8./5.))))
      ECECC=V*V*((Y*9./8.)-(Z*((99./40.)-(W*W*39./20.)+((W**4.)*3./5.)))
     2)
      ECECD=(U*((Y*((3./4.)+(W*W*4.)))-(Z*((13./4.)+(W*W*3./2.)))))+(V*(
     2(Y*3./4.)-(Z*((5./4.)-(W*W/2.)))))
115   ECEC=16.*EC*EC*X*U*(ECECA+ECECB+ECECC-(ECECD*ECECD/T))
      ECC1A=U*((Y*((5./4.)+(W*W*16.)+((W**4.)*16.)))-(Z*((183./20.)+(W*W
     2*217./10.)+((W**4.)*12./5.))))
      ECC1B=V*((Y*((5./4.)+(W*W*6.)))-(Z*((111./20.)+(W*W*67./30.)-((W**
     24.)*8./15.))))
      ECC1C=C1C1C*ECECD
120   ECC1=EC*C1*U*((32.*X*(ECC1A+ECC1B))-(16.*PI*W*ECC1C/T))
      ECC2A=U*((Y*((3./2.)+(W*W*38.)+((W**4.)*120.)+((W**6.)*64.)))-(Z*(
     2(153./10.)+(W*W*301./3.)+((W**4.)*1534./15.)+((W**6.)*28./5.))))
      ECC2B=V*((Y*((3./2.)+(W*W*18.)+((W**4.)*24.)))-(Z*((101./10.)+(W*W
     2*29.)+((W**4.)*26./5.)-((W**6.)*4./5.))))
      ECC2C=C2C2C*ECECD
125   ECC2=32.*X*EC*C2*U*(ECC2A+ECC2B-(ECC2C/T))
      PC1=C1C1C*C1*2.*PI*COS(RTHETA)/T
      PC2=C2C2C*C2*4.*PI*W*COS(RTHETA)/T
      PEC=ECECD*EC*4.*PI*W*COS(RTHETA)/T
      PB=B*4.*PI*W*COS(RTHETA)
      PTF=PC1+PC2+PEC+PB
65    XMTF=DIFM-DM-S1M-DS1M2-C1C1-S2S2-S1S2-DS2-AA-AS1-AS2-C1C2-C2C2-
```

```
     2ECC1-ECC2-ECEC-DS3-S1S3-S2S3-S3S3-AS3-C81D-081A-081S2-081S3-081081
     3-081S1-082D-082A-082S1-082S2-082S3-082081-082082-AD
      AUX=COS(RTHETA)*((2.*W*B*T)+(C1*C1C1C)+(2.*W*C2*C2C2C)+(2.*W*ECECD
     2*EC))
      RE=XMTF-(4.*PI*AUX*AUX/T)
      XIM=4.*AUX
76    WRITE(6,77)W,DIFM,DM,S1M,DS1M2,S2S2,S1S2,DS2,AA,AD,AS1,AS2,C1C1
77    FORMAT(2X,F6.3,F9.5,F10.5,F10.5,6(X,F10.5),3(F10.5))
79    WRITE(6,80)
80    FORMAT(/,2X,*2C1C2*,7X,*C2C2*,7X,*2ECC1*,4X,*2ECC2*,6X,*ECEC*,6X,*
     22DS3*,7X,*2S1S3*,5X,*2S2S3*,8X,*S3S3*,5X,*2AS3*,5X,*2081D*,5X,*208
     31A*,5X,*2081S1*)
81    WRITE(6,82)C1C2,C2C2,ECC1,ECC2,ECEC,DS3,S1S3,S2S3,S3S3,AS3,081D,08
     21A,081S1
82    FORMAT(3(F10.5),X,F10.5,X,F10.5,F10.5,X,7(F10.5))
83    WRITE(6,84)
84    FORMAT(/,2X,*2081S2*,7X,*2081S3*,7X,*081081*,7X,*2082D*,7X,*2082A*
     2,7X,*2082S1*,7X,*2082S2*,7X,*2082S3*,7X,*2082081*,6X,*082082*,6X,
     3*MTF*)
85    WRITE(6,86)081S2,081S3,081081,082D,082A,082S1,082S2,082S3,082081,0
     282082,XMTF
86    FORMAT(F10.5,X,F10.5,3(3X,F10.5),2(2X,F10.5),3(3X,F10.5),F10.5)
87    WRITE(6,88)
88    FORMAT(/,2X,*PC1*,7X,*PC2*,9X,*PEC*,8X,*PB*,9X,*RE*,9X,*IM*,70X,
     2*PTF*)
150   WRITE(6,160)PC1,PC2,PEC,PB,RE,XIM,PTF
160   FORMAT(2(F10.5),4(X,F10.5),59X,F10.5)
      GO TO 30
90    STOP
      END
```

```
      PROGRAM TFORTHO(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C     THIS PROGRAM CALCULATES THE REAL PART OF THE OTF TO SECOND ORDER
C     IN THE WAVE ABERRATION COEFFICIENTS.THE RESULTING QUADRATIC FORM
C     IS INTEGRATED OVER AZIMUTH AND ORTHOGONALIZED.THE OUTPUT DISPLAYS
C     THE ORTHOGONAL COEFFICIENTS AS A FUNCTION OF SPATIAL FREQUENCY
      DIMENSION EV(7,7),ALPHA(7,7),BV(7,7),BETA(7,7),CV(7,7),GAMMA(7,7),
     2DV(7,7),DELTA(7,7),FV(7,7),EP(7,7),GV(7,7),PHI(7,7),HV(7,7),RHO(7,
     37),OD(4,4),OALPHA(4,4),OBV(4,4),OBETA(4,4),OCV(4,4),OGAMMA(4,
     44),ODV(4,4),ODELTA(4,4)
 30   READ(5,35)W
 35   FORMAT(F5.3)
 36   IF(W.EQ.2.0)90,40
 40   PI=3.14159265358979
 41   Y=ACOS(W)
 42   Z=W*(SQRT(1.-(W*W)))
 43   X=PI*W*W
 44   T=Y-Z
      C=W*W
      E=W**6.
      F=W**8.
      G=W**10.
      H=W**12.
      P=W**14.
      Q=W**4.
      V=1.0
      U=1.0
      D=1.0
      A=1.0
      S1=1.0
      S2=1.0
      S3=1.0
      OB1=1.0
      OB2=1.0
      B=1.0
      C1=1.0
      C2=1.0
      EC=1.0
      DS1M2=256.*D*S1*W*W*PI*((Y*((1./6.)+(2.*W*W)+((W**4.)*2.)))-
     2(Z*((7./6.)+(25.*W*W/9.)+((W**4.)*2./9.))))
      S1M=256.*S1*S1*W*W*PI*((Y*((1./8.)+(17.*W*W/6.)+((W**4.)*8.)
     2+((W**6.)*4.)))-(Z*((29./24.)+(139.*W*W/20.)+(
     3292.*(W**4.)/45.)+(14.*(W**6.)/45.))))
      DM=64.*X*D*D*((Y*((1./4.)+(W*W)))-(Z
     2*((13./12.)+((1./6.)*W*W))))
      AAA=V*((Y/4.)-(Z*((5./12.)-(W*W/6.))))
      AAB=U*((Y*((1./4.)+(W*W)))-(Z*((13./12.)+(W*W/6.))))
 131  AA=U*64.*X*(AAA+AAB)
 136  AD=128.*X*AAB
 140  AS1=256.*A*S1*X*U*((Y*((1./6.)+(W*W*2.)+((W**4.)*2.)))-(Z*((7./6.)
     2+(W*W*25./9.)+((W**4.)*2./9.))))
 145  AS2=384.*A*S2*X*U*((Y*((1./8.)+(W*W*3.)+((W**4.)*10.)+((W**6.)*16.
     2/3.)))-(Z*((29./24.)+(W*W*1477./180.)+((W**4.)*389./45.)+((W**6.)*
     32./5.))))
      S3S3=1024.*S3*S3*X*((Y*((1./16.)+(C*89./14.)+(Q*388./3.)+(E*4898./
     25.)+(F*3136.)+(G*12544./3.)+(H*2048.)+(P*256.)))-(Z*((61./48.)+(C*
     338753./840.)+(Q*36229./70.)+(E*439367./189.)+(F*28426988./6615.)+(
     4G*32771008./11025.)+(H*18883552./33075.)+(P*209024./33075.))))
```

```
 S2S3=512.*S2*S3*X*((Y*((3./14.)+(C*33./2.)+(Q*1208./5.)+(E*1224.)+
2(F*2352.)+(G*1568.)+(H*256.)))-(Z*((53./14.)+(C*21089./210.)+(Q*16
31962./210.)+(E*105184./49.)+(F*2558392./1225.)+(G*661216./1225.)+(
4H*9792./1225.))))
 S1S3=1024.*S1*S3*X*((Y*((1./12.)+(C*47./10.)+(Q*46.)+(E*448./3.)+(
2F*144.)+(G*32.)))-(Z*((5./4.)+(C*118./5.)+(Q*815./7.)+(E*90556./52
35.)+(F*99304./1575.)+(G*2048./1575.))))
 DS3=512.*D*S3*X*((Y*((1./10.)+(C*4.)+(Q*28.)+(E*48.)+(F*16.)))-(Z*
2((37./30.)+(C*247./15.)+(Q*3644./75.)+(E*724./25.)+(F*64./75.))))
 AS3=512.*A*S3*U*X*((Y*((1./10.)+(C*4.)+(Q*28.)+(E*48.)+(F*16.)))-
2(Z*((37./30.)+(C*247./15.)+(Q*3644./75.)+(E*724./25.)+(F*64./75.)
3)))
 OB1OB1A=(Y*((C*5.)+(Q*24.)+(E*16.)))-(Z*((46./35.)+(C*125./7.)+(Q*
2852./35.)+(E*52./35.)))
 OB1OB1B=(Y*((3./8.)+(C*14./3.)+(Q*6.)))-(Z*((433./168.)+(C*3119./4
220.)+(Q*389./315.)-(E*62./315.)))
 OB1OB1C=(Y*((1./64.)+(C/8.)))-(Z*((221./2240.)+(C*37./672.)-(Q*13.
2/840.)+(E/420.)))
 OB1OB1=((U*U*OB1OB1A)+(U*OB1OB1B)+OB1OB1C)*64.*X*OB1*OB1
 OB1S2A=(Y*((9./20.)+(C*19.)+(Q*120.)+(E*192.)+(F*64.)))-(Z*((119./
220.)+(C*5161./70.)+(Q*103576./525.)+(E*60308./525.)+(F*632./175.))
3)
 OB1S2B=(Y*((3./40.)+(C*7./4.)+(Q*6.)+(E*4.)))-(Z*((29./40.)+(C*499
2./105.)+(Q*2059./350.)+(E*268./525.)-(F*8./175.)))
 OB1S2=((U*OB1S2A)+OB1S2B)*128.*X*S2*OB1
 OB1S1A=(Y*((3./16.)+(C*29./6.)+(Q*15.)+(E*8.)))-(Z*((467./240.)+(C
2*1517./120.)+(Q*1151./90.)+(E*29./45.)))
 OB1S1B=(Y*((1./32.)+(C*5./12.)+(Q/2.)))-(Z*((113./480.)+(C*151./24
20.)+(Q*17./180.)-(E/90.)))
 OB1S1=((OB1S1A*U)+OB1S1B)*256.*X*OB1*S1
 OB1DA=(Y*((1./4.)+(C*7./2.)+(Q*4.)))-(Z*((113./60.)+(C*27./5.)+(Q*
27./15.)))
 OB1DB=(Y*((1./24.)+(C/4.)))-(Z*((9./40.)+(C*7./90.)-(Q/90.)))
 OB1D=((U*OB1DA)+(OB1DB))*128.*X*D*OB1
 OB1AA=(Y*((1./4.)+(C)))-(Z*((19./20.)+(C*13./30.)-(Q*2./15.)))
 OB1A=128.*X*OB1*A*U*((OB1DA*U)+OB1DB+(V*OB1AA))
 OB1S3A=(Y*((1./8.)+(C*79./10.)+(Q*96.)+(E*280.)+(F*280.)+(G*64.)))
2-(Z*((241./120.)+(C*17351./420.)+(Q*22562./105.)+(E*24886./75.)+(F
3*65816./525.)+(G*464./175.)))
 OB1S3B=(Y*((1./48.)+(C*3./4.)+(Q*5.)+(E*28./3.)+(F*4.)))-(Z*((59./
2240.)+(C*2473./840.)+(Q*3776./420.)+(E*691./105.)+(F*116./315.)-(G
3*8./315.)))
 OB1S3=512.*X*OB1*S3*((U*OB1S3A)+OB1S3B)
 OB2OB2A=(Y*((13./24.)+(C*1351./40.)+(Q*352.)+(E*1122.)+(F*1120.)+(
2G*256.)))-(Z*((7321./840.)+(C*37177./216.)+(Q*3267347./3780.)+(E*2
3090101./1575.)+(F*2369194./4725.)+(G*7184./675.)))
 OB2OB2B=(Y*((1./96.)+(C*39./160.)+(Q*3./4.)+(E/2.)))-(Z*((349./336
20.)+(C*18659./30240.)+(Q*10823./15120.)+(E*499./6300.)-(F*127./945
30.)+(G*8./4725.)))
 OB2OB2C=(Y*((7./12.)+(C*423./20.)+(Q*131.)+(E*228.)+(F*96.)))-(Z*(
2(583./84.)+(C*100861./1260.)+(Q*140923./630.)+(E*27584./175.)+(F*4
3796./525.)-(G*368./525.)))
 OB2OB2=64.*X*OB2*OB2*((OB2OB2A*U*U)+(OB2OB2B*4.*V*V)+(OB2OB2C*U*V)
2)
 OB2S3A=(Y*((5./28.)+(C*59./4.)+(Q*2251./10.)+(E*1172.)+(F*2296.)+(
2G*1552.)+(H*256.)))-(Z*((1381./420.)+(C*38699./420.)+(Q*138238./18
39.)+(E*13778227./6615.)+(F*22707008./11025.)+(G*17816072./33075.)+
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```
      4(H*266464./33075.)))
       OB2S3B=(Y*((1./56.)+(C*7./8.)+(Q*33./4.)+(E*26.)+(F*28.)+(G*8.)))-
      2(Z*((209./840.)+(C*497./120.)+(Q*5377./270.)+(E*421613./13230.)+(F
      3*31852./2205.)+(G*3676./6615.)-(H*208./6615.)))
       OB2S3=512.*X*OB2*S3*((U*OB2S3A)+(2.*V*OB2S3B))
       OB2OB1A=(Y*((9./20.)+(C*151./8.)+(Q*719./6.)+(E*192.)+(F*64.)))-(Z
      2*((2483./420.)+(C*61717./840.)+(Q*414349./2100.)+(E*180884./1575.)
      3+(F*814./225.)))
       OB2OB1B=(Y*((19./40.)+(C*43./4.)+(Q*109./3.)+(E*24.)))-(Z*((3713./
      2840.)+(C*1018./35.)+(Q*12317./350.)+(E*5042./1575.)-(F*536./1575.)
      3))
       OB2OB1C=(Y*((1./80.)+(C*3./16.)+(Q/4.)))-(Z*((57./560.)+(C*503./16
      280.)+(Q*253./4200.)-(E/75.)+(F/525.)))
       OB2OB1=128.*X*OB2*OB1*((U*U*OB2OB1A)+(U*V*OB2OB1B)+(2.*V*V*OB2OB1C
      2))
       OB2S2A=(Y*((5./8.)+(C*371./10.)+(Q*747./2.)+(E*1160.)+(F*1136.)+(G
      2*256.)))-(Z*((391./40.)+(C*77731./420.)+(Q*63111./70.)+(E*33833./2
      35.)+(F*264044./525.)+(G*5528./525.)))
       OB2S2B=(Y*((1./16.)+(C*43./20.)+(Q*49./4.)+(E*20.)+(F*8.)))-(Z*((5
      29./80.)+(C*6479./840.)+(Q*561./28.)+(E*4653./350.)+(F*386./525.)-(
      3G*4./75.)))
       OB2S2=128.*X*OB2*S2*((U*OB2S2A)+(2.*V*OB2S2B))
       OB2DA=(Y*((5./16.)+(C*49./6.)+(Q*29.)+(E*16.)))-(Z*((757./240.)+(2
      2803.*C/120.)+(Q*2317./90.)+(E*11./9.)))
       OB2DB=(Y*((1./32.)+(C*5./12.)+(Q/2.)))-(Z*((113./480.)+(C*151./24
      20.)+(Q*17./180.)-(E/90.)))
       OB2D=128.*X*OB2*D*((U*OB2DA)+(2.*V*OB2DB))
       OB2AA=(Y*((5./16.)+(C*7./2.)+(Q*5.)))-(Z*((469./240.)+(C*707./120.
      2)+(Q*7./6.)-(E/5.)))
       OB2A=128.*X*OB2*A*U*((U*OB2DA)+(V*OB2AA))
       OB2S1A=(Y*((1./4.)+(C*81./8.)+(Q*188./3.)+(E*98.)+(F*32.)))-(Z*((1
      293./60.)+(C*32603./840.)+(Q*2843./28.)+(E*18173./315.)+(F*562./315
      3.)))
       OB2S1B=(Y*((1./40.)+(C*9./16.)+(Q*5./3.)+(E)))-(Z*((29./120.)+(C*1
      257./112.)+(Q*6281./4200.)+(E*401./3150.)-(F*19./1575.)))
       OB2S1=256.*X*OB2*S1*((OB2S1A*U)+(OB2S1B*V*2.))
       S2S2=4.*X*S2*S2*((Y*((12.)+(6624./10.*W*W)+(6368.*(W**4
      2.))+(19200.*(W**6.))+(18432.*(W**8.))+(4096.*(W**10.))))-((Z)*
      3((180.)+(3208.*W*W)+(527328./35.*(W**4.))+(7727488./350.
      4*(W**6.))+(2824704./350.*(W**8.))+(58368./350.*(W**10.)))))
       S1S2=768.*S1*S2*X*((Y*((1./10.)+(W*W*15./4.)+((W**4.)*2
      22.)+((W**6.)*100./3.)+((W**8.)*32./3.)))-((Z)*((37.
      3/30.)+(W*W*2497./180.)+((W**4.)*15679./450.)+((W**6.)*8692./450.)+
      4((W**8.)*264./450.))))
       DS2=384.*D*S2*X*((Y*((1./8.)+(W*W*3.)+((W**4.)*10.)+((W*
      2*6.)*16./3.)))-((Z)*((29./24.)+(W*W*1477./180.)+((W
      3**4.)*389./45.)+((W**6.)*2./5.))))
       C1C1A=64.*X*((Y*((3./8.)+(W*W*17./4.)+((W**4.)*4.)))-(Z*
      2((101./40.)+(W*W*83./15.)+((W**4.)*17./30.))))
       C1C1B=64.*X*((Y*((1./24.)+(W*W*1./4.)))-(Z*((9./40.)+(W*W*7./90.)-
      2((W**4.)/90.))))
       C1C1C=4.*W*((Y*((1./2.)+(2.*W*W)))-(Z*((11./6.)+(W*W*2./3.))))
60     C1C1=C1*C1*((C1C1A*U)+(C1C1B*V)-(C1C1C*C1C1C*4.*PI*U/T))
       C2C2A=U*((Y*((11./5.)+(85.*W*W)+((W**4.)*1520./3.)+((W**6.)*784.)
      2+((W**8.)*256.)))-(Z*((413./15.)+(33359.*W*W/105.)+((W**4.)*426506
      3./525.)+((W**6.)*727144./1575.)+((W**8.)*22928./1575.))))
       C2C2B=V*((Y*((2./5.)+(9.*W*W)+((W**4.)*80./3.)+((W**6.)*16.)))-(Z*
```

```
     2((58./15.)+(157.*W*W/7.)+((W**4.)*12562./525.)+((W**6.)*3208./1575
     3.)-((W**8.)*304./1575.))))
      C2C2C=Y*(1.+(12.*W*W)+((W**4.)*16.))-(Z*((19./3.)+(W*W*20.)+((W**4
     2.)*8./3.)))
      C1C2A=U*((Y*((7./4.)+(122.*W*W/3.)+((W**4.)*124.)+((W**6.)*64.)))-
     2(Z*((1007./60.)+(3149.*W*W/30.)+((W**4.)*4646./45.)+((W**6.)*244./
     345.))))
      C1C2B=V*((Y*((1./4.)+(W*W*10./3.)+((W**4.)*4.)))-(Z*((113./60.)+(1
     251.*W*W/30.)+((W**4.)*34./45.)-((W**6.)*4./45.))))
      C1C2C=U*((Y*((1./2.)+(W*W*2.)))-(Z*((11./6.)+(W*W*2./3.))))*C2C2C
      ECECA1=((Y*((9./8.)+(15.*W*W)+((W**4.)*16.)))-(Z*((339./40.)+(4
     221.*W*W/20.)+((W**4.)*13./5.))))
      ECECA=ECECA1*U*U
      ECECB=U*V*((Y*((9./4.)+(15.*W*W)))-(Z*((243./20.)+(W*W*67./10.)-((
     2W**4.)*8./5.))))
      ECECC=V*V*((Y*9./8.)-(Z*((99./40.)-(W*W*39./20.)+((W**4.)*3./5.)))
     2)
      ECECD=(U*((Y*((3./4.)+(W*W*4.)))-(Z*((13./4.)+(W*W*3./2.)))))+(V*(
     2(Y*3./4.)-(Z*((5./4.)-(W*W/2.)))))
115   ECEC=16.*EC*EC*X*U*(ECECA+ECECB+ECECC-(ECECD*ECECD/T))
      ECC1A=U*((Y*((5./4.)+(W*W*16.)+((W**4.)*16.)))-(Z*((183./20.)+(W*W
     2*217./10.)+((W**4.)*12./5.))))
      ECC1B=V*((Y*((5./4.)+(W*W*6.)))-(Z*((111./20.)+(W*W*67./30.)-((W**
     24.)*8./15.))))
      ECC1C=C1C1C*ECECD
      ECC2A=U*((Y*((3./2.)+(W*W*38.)+((W**4.)*120.)+((W**6.)*64.)))-(Z*(
     2(153./10.)+(W*W*301./3.)+((W**4.)*1534./15.)+((W**6.)*28./5.))))
      ECC2B=V*((Y*((3./2.)+(W*W*18.)+((W**4.)*24.)))-(Z*((101./10.)+(W*W
     2*29.)+((W**4.)*26./5.)-((W**6.)*4./5.))))
      ECC2C=C2C2C*ECECD
      EV(1,1)=DM
      EV(1,2)=DS1M2/2.
      EV(1,3)=DS2/2.
      EV(1,4)=DS3/2.
      EV(1,5)=AD/4.
      EV(1,6)=((DB1DA/2.)+DB1DB)*64.*X
      EV(1,7)=DB2D/4.
      EV(2,2)=S1M
      EV(2,3)=S1S2/2.
      EV(2,4)=S1S3/2.
      EV(2,5)=AS1/4.
      EV(2,6)=((DB1S1A/2.)+DB1S1B)*128.*X
      EV(2,7)=DB2S1/4.
      EV(3,3)=S2S2
      EV(3,4)=S2S3/2.
      EV(3,5)=AS2/4.
      EV(3,6)=((DB1S2A/2.)+DB1S2B)*64.*X
      EV(3,7)=DB2S2/4.
      EV(4,4)=S3S3
      EV(4,5)=AS3/4.
      EV(4,6)=((DB1S3A/2.)+DB1S3B)*256.*X
      EV(4,7)=DB2S3/4.
      EV(5,5)=((AAA/8.)+(AAB*3./8.))*64.*X
      EV(5,6)=((DB1DA*3./8.)+(DB1DB/2.)+(DB1AA/8.))*64.*X
      EV(5,7)=((DB2DA*3./8.)+(DB2AA/8.))*64.*X
      EV(6,6)=((DB1DB1A*3./8.)+(DB1DB1B/2.)+DB1DB1C)*64.*X
      EV(6,7)=((DB2DB1A*3./8.)+(DB2DB1B/8.)+(DB2DB1C*3./4.))*64.*X
```

```
      EV(7,7)=((O82O82A*3./8.)+(O82O828*3./2.)+(O82O82C*1./8.))*64.*X
      EV(2,1)=EV(1,2)
      EV(3,1)=EV(1,3)
      EV(3,2)=EV(2,3)
      EV(4,1)=EV(1,4)
      EV(4,2)=EV(2,4)
      EV(4,3)=EV(3,4)
      EV(5,1)=EV(1,5)
      EV(5,2)=EV(2,5)
      EV(5,3)=EV(3,5)
      EV(5,4)=EV(4,5)
      EV(6,1)=EV(1,6)
      EV(6,2)=EV(2,6)
      EV(6,3)=EV(3,6)
      EV(6,4)=EV(4,6)
      EV(6,5)=EV(5,6)
      EV(7,1)=EV(1,7)
      EV(7,2)=EV(2,7)
      EV(7,3)=EV(3,7)
      EV(7,4)=EV(4,7)
      EV(7,5)=EV(5,7)
      EV(7,6)=EV(6,7)
      DO 5 J=1,7
5     ALPHA(1,J)=EV(1,J)/EV(1,1)
      DO 10 K=2,7
      DO 10 J=1,7
10    BV(K,J)=EV(K,J)-(EV(K,1)*EV(1,J)/EV(1,1))
      DO 16 J=1,7
16    BETA(2,J)=BV(2,J)/BV(2,2)
      DO 20 K=3,7
      DO 20 J=1,7
20    CV(K,J)=BV(K,J)-(BV(K,2)*BV(2,J)/BV(2,2))
      DO 25 J=1,7
25    GAMMA(3,J)=CV(3,J)/CV(3,3)
      DO 31 K=4,7
      DO 31 J=1,7
31    DV(K,J)=CV(K,J)-(CV(K,3)*CV(3,J)/CV(3,3))
      DO 37 J=1,7
37    DELTA(4,J)=DV(4,J)/DV(4,4)
      DO 39 K=5,7
      DO 39 J=1,7
39    FV(K,J)=DV(K,J)-(DV(K,4)*DV(4,J)/DV(4,4))
      DO 45 J=1,7
45    EP(5,J)=FV(5,J)/FV(5,5)
      DO 50 K=6,7
      DO 50 J=1,7
50    GV(K,J)=FV(K,J)-(FV(K,5)*FV(5,J)/FV(5,5))
      DO 55 J=1,7
55    PHI(6,J)=GV(6,J)/GV(6,6)
      DO 61 J=1,7
61    HV(7,J)=GV(7,J)-(GV(7,6)*GV(6,J)/GV(6,6))
      DO 65 J=1,7
65    RHO(7,J)=HV(7,J)/HV(7,7)
      OO(1,1)=8.*X*T
      OO(1,2)=16.*X*((Y*((1./2.)+(2.*C)))-(Z*((11./6.)+(2./3.*C))))
      OO(1,3)=8.*X*((Y*((1.)+(12.*C)+(16.*Q)))-(Z*((19./3.)+(20.*C)+(8./
     23.*Q))))
```

```
      OD(1,4)=8.*X*(((((Y*((3./4.)+(4.*C)))-(Z*((13./4.)+(3./2.*C))))*3./
     28.)+(((Y*((3./4.)))-(Z*((5./4.)-(1./2.*C))))*1./8.))*2.
      OD(2,2)=(C1C1A+C1C1B)/2.
      OD(2,3)=6.*X*(C1C2A+C1C2B)
      OD(2,4)=16.*X*((ECC1A*3./8.)+(ECC1B/8.))
      OD(3,3)=8.*X*(C2C2A+C2C2B)
      OD(3,4)=16.*X*((ECC2A*3./8.)+(ECC2B/8.))
      OD(4,4)=16.*X*((ECECA1*5./16.)+(ECECB/16.)+(ECECC/16.))
      OD(3,1)=OD(1,3)
      OD(3,2)=OD(2,3)
      OD(2,1)=OD(1,2)
      OD(4,1)=OD(1,4)
      OD(4,2)=OD(2,4)
      OD(4,3)=OD(3,4)
      DO 110 J=1,4
  110 OALPHA(1,J)=OD(1,J)/OD(1,1)
      DO 120 K=2,4
      DO 120 J=1,4
  120 OBV(K,J)=OD(K,J)-(OD(K,1)*OD(1,J)/OD(1,1))
      DO 125 J=1,4
  125 OBETA(2,J)=OBV(2,J)/OBV(2,2)
      DO 130 K=3,4
      DO 130 J=1,4
  130 OCV(K,J)=OBV(K,J)-(OBV(K,2)*OBV(2,J)/OBV(2,2))
      DO 135 J=1,4
  135 OGAMMA(3,J)=OCV(3,J)/OCV(3,3)
      DO 141 J=1,4
  141 ODV(4,J)=OCV(4,J)-(OCV(4,3)*OCV(3,J)/OCV(3,3))
      DO 146 J=1,4
  146 ODELTA(4,J)=ODV(4,J)/ODV(4,4)
      WRITE(6,400) W
  400 FORMAT(1H1,*W=*,F6.3)
      WRITE(6,405)
  405 FORMAT(//,10X,*EVEN ABERRATIONS*)
      WRITE(6,500) (EV(1,J),J=1,7)
  500 FORMAT(//,7(E13.6,3X))
      WRITE(6,600) (EV(2,J),J=1,7)
  600 FORMAT(7(E13.6,3X))
      WRITE(6,700) (EV(3,J),J=1,7)
  700 FORMAT(7(E13.6,3X))
      WRITE(6,800) (EV(4,J),J=1,7)
  800 FORMAT(7(E13.6,3X))
      WRITE(6,850) (EV(5,J),J=1,7)
  850 FORMAT(7(E13.6,3X))
      WRITE(6,900) (EV(6,J),J=1,7)
  900 FORMAT(7(E13.6,3X))
      WRITE(6,950) (EV(7,J),J=1,7)
  950 FORMAT(7(E13.6,3X))
      WRITE(6,70)
   70 FORMAT(///,2X,*ORTHOGONAL COEFFICIENTS*)
      WRITE(6,75) EV(1,1),(ALPHA(1,J),J=1,7)
   75 FORMAT (/,E13.6,8X,7(F11.6,4X))
      WRITE(6,80) BV(2,2),(BETA(2,J),J=1,7)
   80 FORMAT (F13.6,8X,7(F11.6,4X))
      WRITE(6,85) CV(3,3),(GAMMA(3,J),J=1,7)
   85 FORMAT (F13.6,8X,7(F11.6,4X))
      WRITE(6,91) DV(4,4),(DELTA(4,J),J=1,7)
```

```
91    FORMAT (F13.6,8X,7(F11.6,4X))
      WRITE(6,95) FV(5,5),(EP(5,J),J=1,7)
95    FORMAT (F13.6.8X,7(F11.6,4X))
      WRITE(6,100) GV(6,6),(PHI(6,J),J=1,7)
100   FORMAT (E13.6,8X,7(F11.6,4X))
      WRITE(6,105) HV(7,7),(RHO(7,J),J=1,7)
105   FORMAT (E13.6,8X,7(F11.6,4X))
      WRITE(6,410)
410   FORMAT(/////,10X,*ODD ABERRATIONS*)
      WRITE(6,415) (OD(1,J),J=1,4)
415   FORMAT(//,4(E13.6,3X))
      WRITE (6,420) (OD(2,J),J=1,4)
420   FORMAT(4(E13.6,3X))
      WRITE (6,425) (OD(3,J),J=1,4)
425   FORMAT(4(E13.6,3X))
      WRITE (6,430) (OD(4,J),J=1,4)
430   FORMAT(4(E13.6,3X))
      WRITE (6,435)
435   FORMAT(///,2X,*ORTHOGONAL COEFFICIENTS*)
      WRITE (6,440) OD(1,1),(OALPHA(1,J),J=1,4)
440   FORMAT(/,E13.6,8X,4(F11.6,4X))
      WRITE (6,445) OBV(2,2),(OBETA(2,J),J=1,4)
445   FORMAT(E13.6,8X,4(F11.6,4X))
      WRITE (6,450) OCV(3,3),(OGAMMA(3,J),J=1,4)
450   FORMAT(E13.6,8X,4(F11.6,4X))
      WRITE (6,455) ODV(4,4),(ODELTA(4,J),J=1,4)
455   FORMAT(E13.6,8X,4(F11.6,4X))
      WRITE (6,461)
461   FORMAT(//,20X,*ORDER OF ABERRATIONS*)
      WRITE (6,462)
462   FORMAT(/,10X,*EVEN*,5X,*D*,5X,*S1*,4X,*S2*,4X,*S3*,4X,*A*,5X,*OB1*
     2,3X,*OB2*)
      WRITE (6,463)
463   FORMAT(/,10X,*ODD*,6X,*B*,5X,*C1*,4X,*C2*,4X,*EC*)
      WRITE (6,464)
464   FORMAT(//,2X,*FIRST COLUMN OF ORTHOGONAL COEFFICIENTS NORMALISED B
     2Y EV(1,1) AND PRINTED IN ORDER FROM THE FIRST EVEN TO THE LAST ODD
     3 COEFFICIENT*)
      EVN=EV(1,1)/EV(1,1)
      BVN=BV(2,2)/EV(1,1)
      CVN=CV(3,3)/EV(1,1)
      DVN=DV(4,4)/EV(1,1)
      FVN=FV(5,5)/EV(1,1)
      GVN=GV(6.6)/EV(1,1)
      HVN=HV(7,7)/EV(1,1)
      ODN=OD(1,1)/EV(1,1)
      OBVN=OBV(2,2)/EV(1,1)
      OCVN=OCV(3,3)/EV(1,1)
      ODVN=ODV(4,4)/EV(1,1)
      WRITE (6,466) EVN,BVN,CVN,DVN,FVN,GVN,HVN,ODN,OBVN,OCVN,ODVN
466   FORMAT(/,2X,11(E11.4))
      GO TO 30
90    STOP
      END
```

```
      PROGRAM TFIORTH(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C      THIS PROGRAM COMPUTES THE DIFFRACTION-BASED REAL PART OF THE OTF TO
C      SECOND ORDER IN THE WAVE-ABERRATION COEFFICIENTS.THE RESULTING
C      QUADRATIC FORM IS THEN INTEGRATED OVER AZIMUTH AND SPATIAL FREQUENCY
C     AFTER MULTIPLICATION BY A LORENTZIAN WEIGHTING FUNCTION
C      WHICH APPROXIMATES THE MTF OF A DETECTOR.THE UPPER LIMIT OF THE SPATIAL
C      FREQUENCY VARIABLE IS TAKEN TO BE EITHER THE FREQUENCY AT WHICH THE
C      LORENTZIAN FALLS TO 0.1 OR TO THE OPTICAL CUT-OFF,WHICHEVER OCCURS FIRST.
C      FINALLY,THE QUADRATIC FORM IS ORTHOGONALIZED.THE RESULTS ARE
C      DISPLAYED AS A FUNCTION OF THE PARAMETER R,CHARACTERIZING
C      THE WIDTH OF THE LORENTZIAN.
      DIMENSION EV(7,7),ALPHA(7,7),BV(7,7),BETA(7,7),CV(7,7),GAMMA(7,7),
     2DV(7,7),DELTA(7,7),FV(7,7),EP(7,7),GV(7,7),PHI(7,7),HV(7,7),RHO(7,
     37),OD(4,4),OALPHA(4,4),OBV(4,4),OBETA(4,4),OCV(4,4),OGAMMA(4,
     44),ODV(4,4),ODELTA(4,4)
 3    READ(5,4) R
 4    FORMAT(F8.2)
      DO 998 K=1,7
      DO 998 J=1,7
 998  EV(J,K)=0.0
      DO 999 K=1,4
      DO 999 J=1,4
 999  OD(J,K)=0.0
      W=0.0
      IF(R.EQ.99.0)90,7
 7    IF(R-3.0) 8,8,9
 8    WM=1.0
      GO TO 12
 9    WM=3./R
 12   PT=0.0
 15   W=W+(WM/100.0)
      WMD=WM/100.
 40   PI=3.14159265358979
 41   Y=ACOS(W)
 42   Z=W*(SQRT(1.-(W*W)))
 43   X=PI*W*W
 44   T=Y-Z
      C=W*W
      E=W**6.
      F=W**8.
      G=W**10.
      H=W**12.
      P=W**14.
      Q=W**4.
      V=1.0
      U=1.0
      D=1.0
      A=1.0
      S1=1.0
      S2=1.0
      S3=1.0
      OB1=1.0
      OB2=1.0
      B=1.0
      C1=1.0
      C2=1.0
      EC=1.0
```

```
      DS1M2=256.*D*S1*W*W*PI*((Y*((1./6.)+(2.*W*W)+((W**4.)*2.)))-
     2(Z*((7./6.)+(25.*W*W/9.)+((W**4.)*2./9.))))
      S1M=256.*S1*S1*W*W*PI*((Y*((1./8.)+(17.*W*W/6.)+((W**4.)*8.)
     2+((W**6.)*4.)))-(Z*((29./24.)+(139.*W*W/20.)+(
     3292.*(W**4.)/45.)+(14.*(W**6.)/45.))))
      DM=64.*X*D*D*((Y*((1./4.)+(W*W)))-(Z
     2*((13./12.)+((1./6.)*W*W))))
      AAA=V*((Y/4.)-(Z*((5./12.)-(W*W/6.))))
      AAB=U*((Y*((1./4.)+(W*W)))-(Z*((13./12.)+(W*W/6.))))
131   AA=U*64.*X*(AAA+AAB)
136   AD=128.*X*AAB
140   AS1=256.*A*S1*X*U*((Y*((1./6.)+(W*W*2.)+((W**4.)*2.)))-(Z*((7./6.)
     2+(W*W*25./9.)+((W**4.)*2./9.))))
145   AS2=384.*A*S2*X*U*((Y*((1./8.)+(W*W*3.)+((W**4.)*10.)+((W**6.)*16.
     2/3.)))-(Z*((29./24.)+(W*W*1477./180.)+((W**4.)*389./45.)+((W**6.)*
     332./5.))))
      S3S3=1024.*S3*S3*X*((Y*((1./16.)+(C*89./14.)+(Q*388./3.)+(E*4898./
     25.)+(F*3136.)+(G*12544./3.)+(H*2048.)+(P*256.)))-(Z*((61./48.)+(C*
     3338753./840.)+(Q*36229./70.)+(E*439367./189.)+(F*28426988./6615.)+(
     4G*32771008./11025.)+(H*18883552./33075.)+(P*209024./33075.))))
      S2S3=512.*S2*S3*X*((Y*((3./14.)+(C*33./2.)+(Q*1208./5.)+(E*1224.)+
     2(F*2352.)+(G*1568.)+(H*256.)))-(Z*((53./14.)+(C*21089./210.)+(Q*16
     31962./210.)+(E*105184./49.)+(F*2558392./1225.)+(G*661216./1225.)+(
     4H*9792./1225.))))
      S1S3=1024.*S1*S3*X*((Y*((1./12.)+(C*47./10.)+(Q*48.)+(E*448./3.)+(
     2F*144.)+(G*32.)))-(Z*((5./4.)+(C*118./5.)+(Q*815./7.)+(E*90556./52
     35.)+(F*99304./1575.)+(G*2048./1575.))))
      DS3=512.*D*S3*X*((Y*((1./10.)+(C*4.)+(Q*28.)+(E*48.)+(F*16.)))-(Z*
     2((37./30.)+(C*247./15.)+(Q*3644./75.)+(E*724./25.)+(F*64./75.))))
      AS3=512.*A*S3*U*X*((Y*((1./10.)+(C*4.)+(Q*28.)+(E*48.)+(F*16.)))-
     2(Z*((37./30.)+(C*247./15.)+(Q*3644./75.)+(E*724./25.)+(F*64./75.)
     3)))
      OB1OB1A=(Y*((C*5.)+(Q*24.)+(E*16.)))-(Z*((46./35.)+(C*125./7.)+(Q*
     2852./35.)+(E*52./35.)))
      OB1OB1B=(Y*((3./8.)+(C*14./3.)+(Q*6.)))-(Z*((433./168.)+(C*3119./4
     220.)+(Q*389./315.)-(E*62./315.)))
      OB1OB1C=(Y*((1./64.)+(C/8.)))-(Z*((221./2240.)+(C*37./672.)-(Q*13.
     2/840.)+(E/420.)))
      OB1OB1=((U*U*OB1OB1A)+(U*OB1OB1B)+OB1OB1C)*64.*X*OB1*OB1
      OB1S2A=(Y*((9./20.)+(C*19.)+(Q*120.)+(E*192.)+(F*64.)))-(Z*((119./
     220.)+(C*5161./70.)+(Q*103576./525.)+(E*60308./525.)+(F*632./175.))
     3)
      OB1S2B=(Y*((3./40.)+(C*7./4.)+(Q*6.)+(E*4.)))-(Z*((29./40.)+(C*499
     2./105.)+(Q*2059./350.)+(E*268./525.)-(F*8./175.)))
      OB1S2=((U*OB1S2A)+OB1S2B)*128.*X*S2*OB1
      OB1S1A=(Y*((3./16.)+(C*29./6.)+(Q*15.)+(E*8.)))-(Z*((467./240.)+(C
     2*1517./120.)+(Q*1151./90.)+(E*29./45.)))
      OB1S1B=(Y*((1./32.)+(C*5./12.)+(Q/2.)))-(Z*((113./480.)+(C*151./24
     20.)+(Q*17./180.)-(E/90.)))
      OB1S1=((OB1S1A*U)+OB1S1B)*256.*X*OB1*S1
      OB1DA=(Y*((1./4.)+(C*7./2.)+(Q*4.)))-(Z*((113./60.)+(C*27./5.)+(Q*
     227./15.)))
      OB1DB=(Y*((1./24.)+(C/4.)))-(Z*((9./40.)+(C*7./90.)-(Q/90.)))
      OB1D=((U*OB1DA)+(OB1DB))*128.*X*D*OB1
      OB1AA=(Y*((1./4.)+(C)))-(Z*((19./20.)+(C*13./30.)-(Q*2./15.)))
      OB1A=128.*X*OB1*A*U*((OB1DA*U)+OB1DB+(V*OB1AA))
      OB1S3A=(Y*((1./8.)+(C*79./10.)+(Q*86.)+(E*280.)+(F*280.)+(G*64.)))
```

```
2-(Z*((241./120.)+(C*17351./420.)+(Q*22562./105.)+(E*24886./75.)+(F
3*65816./525.)+(G*464./175.)))
 OB1S3B=(Y*((1./48.)+(C*3./4.)+(Q*5.)+(E*28./3.)+(F*4.)))-(Z*((59./
2240.)+(C*2473./840.)+(Q*3776./420.)+(E*691./105.)+(F*116./315.)-(G
3*8./315.)))
 OB1S3=512.*X*OB1*S3*((U*OB1S3A)+OB1S3B)
 OB2OB2A=(Y*((13./24.)+(C*1351./40.)+(Q*352.)+(E*1122.)+(F*1120.)+(
2G*256.)))-(Z*((7321./840.)+(C*37177./216.)+(Q*3267347./3730.)+(E*2
3090101./1575.)+(F*2369194./4725.)+(G*7184./675.)))
 OB2OB2B=(Y*((1./96.)+(C*39./160.)+(Q*3./4.)+(E/2.)))-(Z*((349./336
20.)+(C*18659./30240.)+(Q*10823./15120.)+(E*499./6300.)-(F*127./945
30.)+(G*8./4725.)))
 OB2OB2C=(Y*((7./12.)+(C*423./20.)+(Q*131.)+(E*228.)+(F*96.)))-(Z*(
2(583./84.)+(C*100861./1260.)+(Q*140923./630.)+(E*27584./175.)+(F*4
3796./525.)-(G*368./525.)))
 OB2OB2=64.*X*OB2*OB2*((OB2OB2A*U*U)+(OB2OB2B*4.*V*V)+(OB2OB2C*U*V)
2)
 OB2S3A=(Y*((5./28.)+(C*59./4.)+(Q*2251./10.)+(E*1172.)+(F*2296.)+(
2G*1552.)+(H*256.)))-(Z*((1381./420.)+(C*38599./420.)+(Q*138238./18
39.)+(E*13778227./6615.)+(F*22707008./11025.)+(G*17816072./33075.)+
4(H*266464./33075.)))
 OB2S3B=(Y*((1./56.)+(C*7./8.)+(Q*33./4.)+(E*26.)+(F*28.)+(G*8.)))-
2(Z*((209./840.)+(C*497./120.)+(Q*5377./270.)+(E*421613./13230.)+(F
3*31852./2205.)+(G*3676./6615.)-(H*208./6615.)))
 OB2S3=512.*X*OB2*S3*((U*OB2S3A)+(2.*V*OB2S3B))
 OB2OB1A=(Y*((9./20.)+(C*151./8.)+(Q*719./6.)+(E*192.)+(F*64.)))-(Z
2*((2483./420.)+(C*61717./840.)+(Q*414349./2100.)+(E*180884./1575.)
3+(F*814./225.)))
 OB2OB1B=(Y*((19./40.)+(C*43./4.)+(Q*109./3.)+(E*24.)))-(Z*((3713./
2840.)+(C*1018./35.)+(Q*12317./350.)+(E*5042./1575.)-(F*536./1575.)
3))
 OB2OB1C=(Y*((1./80.)+(C*3./16.)+(Q/4.)))-(Z*((57./560.)+(C*503./16
280.)+(Q*253./4200.)-(E/75.)+(F/525.)))
 OB2OB1=128.*X*OB2*OB1*((U*U*OB2OB1A)+(U*V*OB2OB1B)+(2.*V*V*OB2OB1C
2))
 OB2S2A=(Y*((5./8.)+(C*371./10.)+(Q*747./2.)+(E*1160.)+(F*1136.)+(G
2*256.)))-(Z*((391./40.)+(C*77731./420.)+(Q*63111./70.)+(E*33833./2
35.)+(F*264044./525.)+(G*5528./525.)))
 OB2S2B=(Y*((1./16.)+(C*43./20.)+(Q*49./4.)+(E*20.)+(F*8.)))-(Z*((5
29./80.)+(C*6479./840.)+(Q*561./28.)+(E*4653./350.)+(F*386./525.)-(
3G*4./75.)))
 OB2S2=128.*X*OB2*S2*((U*OB2S2A)+(2.*V*OB2S2B))
 OB2OA=(Y*((5./16.)+(C*49./6.)+(Q*29.)+(E*16.)))-(Z*((757./240.)+(2
2803.*C/120.)+(Q*2317./90.)+(E*11./9.)))
 OB2OB=(Y*((1./32.)+(C*5./12.)+(Q/2.)))-(Z*((113./480.)+(C*151./24
20.)+(Q*17./180.)-(E/90.)))
 OB2O=128.*X*OB2*O*((U*OB2OA)+(2.*V*OB2OB))
 OB2AA=(Y*((5./16.)+(C*7./2.)+(Q*5.)))-(Z*((469./240.)+(C*707./120.
2)+(Q*7./6.)-(E/5.)))
 OB2A=128.*X*OB2*A*U*((U*OB2OA)+(V*OB2AA))
 OB2S1A=(Y*((1./4.)+(C*81./8.)+(Q*188./3.)+(E*98.)+(F*32.)))-(Z*((1
293./60.)+(C*32603./840.)+(Q*2843./28.)+(E*19173./315.)+(F*562./315
3.)))
 OB2S1B=(Y*((1./40.)+(C*9./16.)+(Q*5./3.)+(E)))-(Z*((29./120.)+(C*1
257./112.)+(Q*6281./4200.)+(E*401./3150.)-(F*19./1575.)))
 OB2S1=256.*X*OB2*S1*((OB2S1A*U)+(OB2S1B*V*2.))
 S2S2=4.*X*S2*S2*((Y*((12.)+(6624./10.*W*W)+(6368.*(W**4
```

```
     2.))+(19200.*(W**6.))+(18432.*(W**8.))+(4096.*(W**10.))))-((Z)*
     3((180.)+(3208.*W*W)+(527328./35.*(W**4.))+(7727488./350.
     4*(W**6.))+(2824704./350.*(W**8.))+(58368./350.*(W**10.)))))
      S1S2=768.*S1*S2*X*((Y*((1./10.)+(W*W*15./4.)+((W**4.)*2
     22.)+((W**6.)*100./3.)+((W**8.)*32./3.)))-((Z)*((37.
     3/30.)+(W*W*2497./180.)+((W**4.)*15679./450.)+((W**6.)*8692./450.)+
     4((W**8.)*264./450.))))
      DS2=384.*D*S2*X*((Y*((1./8.)+(W*W*3.)+((W**4.)*10.)+((W*
     2*6.)*16./3.)))-((Z)*((29./24.)+(W*W*1477./180.)+((W
     3**4.)*389./45.)+((W**6.)*2./5.))))
      C1C1A=64.*X*((Y*((3./8.)+(W*W*17./4.)+((W**4.)*4.)))-(Z*
     2((101./40.)+(W*W*83./15.)+((W**4.)*17./30.))))
      C1C1B=64.*X*((Y*((1./24.)+(W*W*1./4.)))-(Z*((9./40.)+(W*W*7./90.)-
     2((W**4.)/90.))))
      C1C1C=4.*W*((Y*((1./2.)+(2.*W*W)))-(Z*((11./6.)+(W*W*2./3.))))
60    C1C1=C1*C1*((C1C1A*U)+(C1C1B*V)-(C1C1C*C1C1C*4.*PI*U/T))
      C2C2A=U*((Y*((11./5.)+(95.*W*W)+((W**4.)*1520./3.)+((W**6.)*784.)
     2+((W**8.)*256.)))-(Z*((413./15.)+(33359.*W*W/105.)+((W**4.)*426506
     3./525.)+((W**6.)*727144./1575.)+((W**8.)*22928./1575.))))
      C2C2B=V*((Y*((2./5.)+(9.*W*W)+((W**4.)*80./3.)+((W**6.)*16.)))-(Z*
     2((56./15.)+(157.*W*W/7.)+((W**4.)*12562./525.)+((W**6.)*3208./1575
     3.)-((W**8.)*304./1575.))))
      C2C2C=Y*(1.+(12.*W*W)+((W**4.)*16.))-(Z*((19./3.)+(W*W*20.)+((W**4
     2.)*8./3.)))
      C1C2A=U*((Y*((7./4.)+(122.*W*W/3.)+((W**4.)*124.)+((W**6.)*64.)))-
     2(Z*((1007./60.)+(3149.*W*W/30.)+((W**4.)*4646./45.)+((W**6.)*244./
     345.))))
      C1C2B=V*((Y*((1./4.)+(W*W*10./3.)+((W**4.)*4.)))-(Z*((113./60.)+(1
     251.*W*W/30.)+((W**4.)*34./45.)-((W**6.)*4./45.))))
      C1C2C=U*((Y*((1./2.)+(W*W*2.)))-(Z*((11./6.)+(W*W*2./3.))))*C2C2C
      ECECA1=((Y*((9./8.)+(15.*W*W)+((W**4.)*16.)))-(Z*((339./40.)+(4
     221.*W*W/20.)+((W**4.)*13./5.))))
      ECECA=ECECA1*U*U
      ECECB=U*V*((Y*((9./4.)+(15.*W*W)))-(Z*((243./20.)+(W*W*67./10.)-((
     2W**4.)*8./5.))))
      ECECC=V*V*((Y*9./8.)-(Z*((99./40.)-(W*W*39./20.)+((W**4.)*3./5.)))
     2)
      ECECD=(U*((Y*((3./4.)+(W*W*4.)))-(Z*((13./4.)+(W*W*3./2.)))))+(V*(
     2(Y*3./4.)-(Z*((5./4.)-(W*W/2.)))))
115   ECEC=16.*EC*EC*X*U*(ECECA+ECECB+ECECC-(ECECD*ECECD/T))
      ECC1A=U*((Y*((5./4.)+(W*W*16.)+((W**4.)*16.)))-(Z*((183./20.)+(W*W
     2*217./10.)+((W**4.)*12./5.))))
      ECC1B=V*((Y*((5./4.)+(W*W*6.)))-(Z*((111./20.)+(W*W*67./30.)-((W**
     24.)*8./15.))))
      ECC1C=C1C1C*ECECD
      ECC2A=U*((Y*((3./2.)+(W*W*38.)+((W**4.)*120.)+((W**6.)*64.)))-(Z*(
     2(153./10.)+(W*W*301./3.)+((W**4.)*1534./15.)+((W**6.)*28./5.))))
      ECC2B=V*((Y*((3./2.)+(W*W*18.)+((W**4.)*24.)))-(Z*((101./10.)+(W*W
     2*29.)+((W**4.)*26./5.)-((W**6.)*4./5.))))
      ECC2C=C2C2C*ECECD
      WF=1./(1.+(R*W*W*R))
      IF(W-(WM-WMD)) 11,11,14
11    DMI=DM*WF*W
      DS1M2I=DS1M2*WF*W/2.
      DS2I=DS2*WF*W*WMD/2.
      DS3I=DS3*WF*WMD*W/2.
      ADI=AD*WF*WMD*W/4.
```

```
 OB1DI=((OB1DA/2.)+OB1DB)*64.*X*W*WF*WMD
 OB2DI=OB2D*WF*W*WMD/4.
 S1MI=S1M*WF*W*WMD
 S1S2I=S1S2*WF*WMD*W/2.
 S1S3I=S1S3*W*WMD*WF/2.
 AS1I=AS1*WF*W*WMD/4.
 OB1S1I=((OB1S1A/2.)+OB1S1B)*128.*X*WF*W*WMD
 OB2S1I=OB2S1*WF*W*WMD/4.
 S2S2I=S2S2*WF*W*WMD
 S2S3I=S2S3*WF*W*WMD/2.
 AS2I=AS2*WF*W*WMD/4.
 OB1S2I=((OB1S2A/2.)+OB1S2B)*64.*X*WF*W*WMD
 OB2S2I=OB2S2*W*WF*WMD/4.
 S3S3I=S3S3*W*WF*WMD
 AS3I=AS3*W*WF*WMD/4.
 OB1S3I=((OB1S3A/2.)+OB1S3B)*256.*X*W*WF*WMD
 OB2S3I=OB2S3*WF*W*WMD/4.
 AAI=((AAA/8.)+(AAB*3./8.))*64.*X*W*WF*WMD
 OB1AI=((OB1DA*3./8.)+(OB1DB/2.)+(OB1AA/8.))*64.*X*WMD*WF*W
 OB2AI=((OB2DA*3./8.)+(OB2AA/8.))*64.*X*WF*W*WMD
 OB1OB1I=((OB1OB1A*3./8.)+(OB1OB1B/2.)+OB1OB1C)*64.*X*WF*W*WMD
 OB2OB1I=((OB2OB1A*3./8.)+(OB2OB1B/8.)+(OB2OB1C*3./4.))*64.*X*WF*W*
2WMD
 OB2OB2I=((OB2OB2A*3./8.)+(OB2OB2B*3./2.)+(OB2OB2C*1./8.))*64.*X*WM
2D*W*WF
 ECECI=16.*X*W*WF*WMD*((ECECA1*5./16.)+(ECECB/16.)+(ECECC/16.))
 C2ECI=16.*X*W*WF*WMD*((ECC2A*3./8.)+(ECC2B/8.))
 C2C2I=8.*X*W*WF*WMD*(C2C2A+C2C2B)
 C1ECI=16.*X*W*WF*WMD*((ECC1A*3./8.)+(ECC1B/8.))
 C1C2I=8.*X*W*WF*WMD*(C1C2A+C1C2B)
 C1C1I=(C1C1A+C1C1B)*W*WF*WMD/2.
 BECI=8.*X*W*WMD*WF*((((Y*((3./4.)+(4.*C)))-(Z*((13./4.)+(3./2.*C))
2))*3./8.)+(((Y*((3./4.)))-(Z*((5./4.)-(1./2.*C))))*1./8.))*2.
 BC2I=8.*X*WF*W*WMD*((Y*((1.)+(12.*C)+(16.*Q)))-(Z*((19./3.)+(20.*C
2)+(8./3.*Q))))
 BC1I=16.*X*W*WF*WMD*((Y*((1./2.)+(2.*C)))-(Z*((11./6.)+(2.*C/3.)))
2)
 BBI=8.*X*T*W*WF*WMD
 PT=PT+(8.*PI*T*WMD*WF*W)
 OD(4,4)=OD(4,4)+ECECI
 OD(3,4)=OD(3,4)+C2ECI
 OD(3,3)=OD(3,3)+C2C2I
 OD(2,4)=OD(2,4)+C1ECI
 OD(2,3)=OD(2,3)+C1C2I
 OD(2,2)=OD(2,2)+C1C1I
 OD(1,4)=OD(1,4)+BECI
 OD(1,3)=OD(1,3)+BC2I
 OD(1,2)=OD(1,2)+BC1I
 OD(1,1)=OD(1,1)+BBI
 EV(7,7)=EV(7,7)+OB2OB2I
 EV(6,7)=EV(6,7)+OB2OB1I
 EV(6,6)=EV(6,6)+OB1OB1I
 EV(5,7)=EV(5,7)+OB2AI
 EV(5,6)=EV(5,6)+OB1AI
 EV(5,5)=EV(5,5)+AAI
 EV(4,7)=EV(4,7)+OB2S3I
 EV(4,6)=EV(4,6)+OB1S3I
```

```
      EV(4,5)=EV(4,5)+AS3I
      EV(4,4)=EV(4,4)+S3S3I
      EV(3,7)=EV(3,7)+OB2S2I
      EV(3,6)=EV(3,6)+OB1S2I
      EV(3,5)=EV(3,5)+AS2I
      EV(3,4)=EV(3,4)+S2S3I
      EV(3,3)=EV(3,3)+S2S2I
      EV(2,7)=EV(2,7)+OB2S1I
      EV(2,6)=EV(2,6)+OB1S1I
      EV(2,5)=EV(2,5)+AS1I
      EV(2,4)=EV(2,4)+S1S3I
      EV(2,3)=EV(2,3)+S1S2I
      EV(2,2)=EV(2,2)+S1MI
      EV(1,7)=EV(1,7)+OB2DI
      EV(1,6)=EV(1,6)+OB1DI
      EV(1,5)=EV(1,5)+ADI
      EV(1,4)=EV(1,4)+DS3I
      EV(1,3)=EV(1,3)+DS2I
      EV(1,2)=EV(1,2)+(DS1M2I*WMD)
      EV(1,1)=EV(1,1)+(DMI*WMD)
      GO TO 15
14    EV(1,1)=(EV(1,1)+(DMI*WMD/2.))/PT
      EV(1,2)=(EV(1,2)+(DS1M2I*WMD/2.))/PT
      EV(1,3)=(EV(1,3)+(DS2I/2.))/PT
      EV(1,4)=(EV(1,4)+(DS3I/2.))/PT
      OD(4,4)=(OD(4,4)+(ECECI/2.))/PT
      OD(3,4)=(OD(3,4)+(C2ECI/2.))/PT
      OD(3,3)=(OD(3,3)+(C2C2I/2.))/PT
      OD(2,4)=(OD(2,4)+(C1ECI/2.))/PT
      OD(2,3)=(OD(2,3)+(C1C2I/2.))/PT
      OD(2,2)=(OD(2,2)+(C1C1I/2.))/PT
      OD(1,4)=(OD(1,4)+(BECI/2.))/PT
      OD(1,3)=(OD(1,3)+(BC2I/2.))/PT
      OD(1,2)=(OD(1,2)+(BC1I/2.))/PT
      OD(1,1)=(OD(1,1)+(BBI/2.))/PT
      EV(7,7)=(EV(7,7)+(OB2OB2I/2.))/PT
      EV(6,7)=(EV(6,7)+(OB2OB1I/2.))/PT
      EV(6,6)=(EV(6,6)+(OB1OB1I/2.))/PT
      EV(5,7)=(EV(5,7)+(OB2AI/2.))/PT
      EV(5,6)=(EV(5,6)+(OB1AI/2.))/PT
      EV(5,5)=(EV(5,5)+(AAI/2.))/PT
      EV(4,7)=(EV(4,7)+(OB2S3I/2.))/PT
      EV(4,6)=(EV(4,6)+(OB1S3I/2.))/PT
      EV(4,5)=(EV(4,5)+(AS3I/2.))/PT
      EV(4,4)=(EV(4,4)+(S3S3I/2.))/PT
      EV(3,7)=(EV(3,7)+(OB2S2I/2.))/PT
      EV(3,6)=(EV(3,6)+(OB1S2I/2.))/PT
      EV(3,5)=(EV(3,5)+(AS2I/2.))/PT
      EV(3,4)=(EV(3,4)+(S2S3I/2.))/PT
      EV(3,3)=(EV(3,3)+(S2S2I/2.))/PT
      EV(2,7)=(EV(2,7)+(OB2S1I/2.))/PT
      EV(2,6)=(EV(2,6)+(OB1S1I/2.))/PT
      EV(2,5)=(EV(2,5)+(AS1I/2.))/PT
      EV(2,4)=(EV(2,4)+(S1S3I/2.))/PT
      EV(2,3)=(EV(2,3)+(S1S2I/2.))/PT
      EV(2,2)=(EV(2,2)+(S1MI/2.))/PT
      EV(1,7)=(EV(1,7)+(OB2DI/2.))/PT
```

```
      EV(1,6)=(EV(1,6)+(OB1DI/2.))/PT
      EV(1,5)=(EV(1,5)+(ADI/2.))/PT
      EV(2,1)=EV(1,2)
      EV(3,1)=EV(1,3)
      EV(3,2)=EV(2,3)
      EV(4,1)=EV(1,4)
      EV(4,2)=EV(2,4)
      EV(4,3)=EV(3,4)
      EV(5,1)=EV(1,5)
      EV(5,2)=EV(2,5)
      EV(5,3)=EV(3,5)
      EV(5,4)=EV(4,5)
      EV(6,1)=EV(1,6)
      EV(6,2)=EV(2,6)
      EV(6,3)=EV(3,6)
      EV(6,4)=EV(4,6)
      EV(6,5)=EV(5,6)
      EV(7,1)=EV(1,7)
      EV(7,2)=EV(2,7)
      EV(7,3)=EV(3,7)
      EV(7,4)=EV(4,7)
      EV(7,5)=EV(5,7)
      EV(7,6)=EV(6,7)
      DO 5 J=1,7
5     ALPHA(1,J)=EV(1,J)/EV(1,1)
      DO 10 K=2,7
      DO 10 J=1,7
10    BV(K,J)=EV(K,J)-(EV(K,1)*EV(1,J)/EV(1,1))
      DO 16 J=1,7
16    BETA(2,J)=BV(2,J)/BV(2,2)
      DO 20 K=3,7
      DO 20 J=1,7
20    CV(K,J)=BV(K,J)-(BV(K,2)*BV(2,J)/BV(2,2))
      DO 25 J=1,7
25    GAMMA(3,J)=CV(3,J)/CV(3,3)
      DO 31 K=4,7
      DO 31 J=1,7
31    DV(K,J)=CV(K,J)-(CV(K,3)*CV(3,J)/CV(3,3))
      DO 37 J=1,7
37    DELTA(4,J)=DV(4,J)/DV(4,4)
      DO 39 K=5,7
      DO 39 J=1,7
39    FV(K,J)=DV(K,J)-(DV(K,4)*DV(4,J)/DV(4,4))
      DO 45 J=1,7
45    EP(5,J)=FV(5,J)/FV(5,5)
      DO 50 K=6,7
      DO 50 J=1,7
50    GV(K,J)=FV(K,J)-(FV(K,5)*FV(5,J)/FV(5,5))
      DO 55 J=1,7
55    PHI(6,J)=GV(6,J)/GV(6,6)
      DO 61 J=1,7
61    HV(7,J)=GV(7,J)-(GV(7,6)*GV(6,J)/GV(6,6))
      DO 65 J=1,7
65    RHO(7,J)=HV(7,J)/HV(7,7)
      OD(3,1)=OD(1,3)
      OD(3,2)=OD(2,3)
      OD(2,1)=OD(1,2)
```

```
      OD(4,1)=OD(1,4)
      OD(4,2)=OD(2,4)
      OD(4,3)=OD(3,4)
      DO 110 J=1,4
110   OALPHA(1,J)=OD(1,J)/OD(1,1)
      DO 120 K=2,4
      DO 120 J=1,4
120   OBV(K,J)=OD(K,J)-(OD(K,1)*OD(1,J)/OD(1,1))
      DO 125 J=1,4
125   OBETA(2,J)=OBV(2,J)/OBV(2,2)
      DO 130 K=3,4
      DO 130 J=1,4
130   OCV(K,J)=OBV(K,J)-(OBV(K,2)*OBV(2,J)/OBV(2,2))
      DO 135 J=1,4
135   OGAMMA(3,J)=OCV(3,J)/OCV(3,3)
      DO 141 J=1,4
141   ODV(4,J)=OCV(4,J)-(OCV(4,3)*OCV(3,J)/OCV(3,3))
      DO 146 J=1,4
146   ODELTA(4,J)=ODV(4,J)/ODV(4,4)
      WRITE(6,400) R
400   FORMAT(1H1,*R=*,E10.3)
      WRITE(6,405)
405   FORMAT(//,10X,*EVEN ABERRATIONS*)
      WRITE(6,500) (EV(1,J),J=1,7)
500   FORMAT(//,7(E13.6,3X))
      WRITE(6,600) (EV(2,J),J=1,7)
600   FORMAT(7(E13.6,3X))
      WRITE(6,700) (EV(3,J),J=1,7)
700   FORMAT(7(E13.6,3X))
      WRITE(6,800) (EV(4,J),J=1,7)
800   FORMAT(7(E13.6,3X))
      WRITE(6,850) (EV(5,J),J=1,7)
850   FORMAT(7(E13.6,3X))
      WRITE(6,900) (EV(6,J),J=1,7)
900   FORMAT(7(E13.6,3X))
      WRITE(6,950) (EV(7,J),J=1,7)
950   FORMAT(7(E13.6,3X))
      WRITE(6,70)
70    FORMAT(///,2X,*ORTHOGONAL COEFFICIENTS*)
      WRITE(6,75) EV(1,1),(ALPHA(1,J),J=1,7)
75    FORMAT (/,E13.6,8X,7(F11.6,4X))
      WRITE(6,80) BV(2,2),(BETA(2,J),J=1,7)
80    FORMAT (E13.6,8X,7(F11.6,4X))
      WRITE(6,85) CV(3,3),(GAMMA(3,J),J=1,7)
85    FORMAT (E13.6,8X,7(F11.6,4X))
      WRITE(6,91) DV(4,4),(DELTA(4,J),J=1,7)
91    FORMAT (E13.6,8X,7(F11.6,4X))
      WRITE(6,95) FV(5,5),(EP(5,J),J=1,7)
95    FORMAT (E13.6,8X,7(F11.6,4X))
      WRITE(6,100) GV(6,6),(PHI(6,J),J=1,7)
100   FORMAT (E13.6,8X,7(F11.6,4X))
      WRITE(6,105) HV(7,7),(RHO(7,J),J=1,7)
105   FORMAT (E13.6,8X,7(F11.6,4X))
      WRITE(6,410)
410   FORMAT(/////,10X,*ODD ABERRATIONS*)
      WRITE(6,415) (OD(1,J),J=1,4)
415   FORMAT(//,4(E13.6,3X))
```

```
      WRITE (6,420) (OD(2,J),J=1,4)
420   FORMAT(4(F13.6,3X))
      WRITE (6,425) (OD(3,J),J=1,4)
425   FORMAT(4(F13.6,3X))
      WRITE (6,430) (OD(4,J),J=1,4)
430   FORMAT(4(E13.6,3X))
      WRITE (5,435)
435   FORMAT(///,2X,*ORTHOGONAL COEFFICIENTS*)
      WRITE (6,440) OD(1,1),(OALPHA(1,J),J=1,4)
440   FORMAT(/,E13.6,8X,4(F11.6,4X))
      WRITE (6,445) OBV(2,2),(OBETA(2,J),J=1,4)
445   FORMAT(E13.6,8X,4(F11.6,4X))
      WRITE (6,450) OCV(3,3),(OGAMMA(3,J),J=1,4)
450   FORMAT(E13.6,8X,4(F11.6,4X))
      WRITE (6,455) ODV(4,4),(ODELTA(4,J),J=1,4)
455   FORMAT(E13.6,8X,4(F11.6,4X))
      WRITE (6,461)
461   FORMAT(//,20X,*ORDER OF ABERRATIONS*)
      WRITE (6,462)
462   FORMAT(/,10X,*EVEN*,5X,*D*,5X,*S1*,4X,*S2*,4X,*S3*,4X,*A*,5X,*OB1*
     2,3X,*OB2*)
      WRITE (6,463)
463   FORMAT(/,10X,*ODD*,6X,*B*,5X,*C1*,4X,*C2*,4X,*EC*)
      WRITE (6,464)
464   FORMAT(//,2X,*FIRST COLUMN OF ORTHOGONAL COEFFICIENTS NORMALISED B
     2Y EV(1,1) AND PRINTED IN ORDER FROM THE FIRST EVEN TO THE LAST ODD
     3 COEFFICIENT*)
      EVN=EV(1,1)/EV(1,1)
      BVN=BV(2,2)/EV(1,1)
      CVN=CV(3,3)/EV(1,1)
      DVN=DV(4,4)/EV(1,1)
      FVN=FV(5,5)/EV(1,1)
      GVN=GV(6,6)/EV(1,1)
      HVN=HV(7,7)/EV(1,1)
      OON=OD(1,1)/EV(1,1)
      OBVN=OBV(2,2)/EV(1,1)
      OCVN=OCV(3,3)/EV(1,1)
      ODVN=ODV(4,4)/EV(1,1)
      WRITE (6,466) EVN,BVN,CVN,DVN,FVN,GVN,HVN,OON,OBVN,OCVN,ODVN
466   FORMAT(/,2X,11(E11.4))
      GO TO 3
90    STOP
      END
```

```
      PROGRAM AUTCORR(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C      THIS PROGRAM CALCULATES THE OPTICAL TRANSFER FUNCTION BY AUTO-
C      CORRELATION OF THE PUPIL FUNCTION.IT USES THE H.H.HOPKINS
C      ALGORITHM WITH E=0.02 FOR THE NUMERICAL INTEGRATION.
 2    READ(5,3) D,S1,S2,S3,A,OB1,OB2,B,C1,C2,EC
 3    FORMAT(11(F7.3))
      READ(5,300) P
 300  FORMAT(F3.0)
      PI=3.1415926535898
      IF(D.EQ.999.)90,6
 6    READ(5,7) THETA
 7    FORMAT(F5.0)
      IF(THETA.EQ.999.)2,19
 19   RTH=THETA*PI/180.
      WRITE(6,200)
 200  FORMAT(1H1,3X,*D*,9X,*S1*,8X,*S2*,8X,*S3*,8X,*A*,9X,*OB1*,7X,*OB2*
     2,7X,*B*,9X,*C1*,8X,*C2*,8X,*EC*)
      WRITE(6,205) D,S1,S2,S3,A,OB1,OB2,B,C1,C2,EC
 205  FORMAT(9(F10.3),2(F10.3))
      U=COS(RTH)
      V=SIN(RTH)
      IF(THETA.EQ.0.0)9,8
 8    IF(THETA.EQ.90.0)9,12
 9    L=2
      K=25
      GO TO 21
 12   K=50
      L=1
 21   WRITE(6,210) THETA
 210  FORMAT(//,3X,*AZIMUTHAL ANGLE=*,F5.0)
 13   READ(5,5) W
 5    FORMAT(F6.3)
      OXISUM=0.0
      OXSUM=0.0
      X=-0.98
      IF(W.EQ.2.)6,29
 29   DO 25 I=1,50
      Y=-0.98
      DO 20 J=1,K
      XA=ABS(X)
      CX=((XA-W)*(XA-W))+(Y*Y)
      CY=((XA+W)*(XA+W))+(Y*Y)
 18   IF(CX.LT.1.)10,24
 10   IF(CY.LT.1.)15,24
 15   FD=4.*X*D
      FXD=4.*D
      FS1=2.*S1*((4.*X*((X*X)+(Y*Y)))+(4.*W*W*X))
      FXS1=8.*S1*((3.*X*X)+(Y*Y)+(W*W))
      FYS1=16.*S1*X*Y
      FS2=2.*S2*((6.*(X**5))+(20.*X*X*X*W*W)+(6.*X*(W**4))+(6.*X*(Y**4))
     2+(12.*X*X*X*Y*Y)+(12.*X*Y*Y*W*W))
      FXS2=2.*S2*((30.*(X**4))+(60.*X*X*W*W)+(6.*(W**4))+(6.*(Y**4))+(36
     2.*X*X*Y*Y)+(12.*Y*Y*W*W))
      FYS2=48.*S2*X*Y*((X*X)+(Y*Y)+(W*W))
      FS3=16.*S3*((X**7.)+(7.*W*W*(X**5.))+(7.*W*((X*W)**3.))+(X*(W**6.)
     2)+(3.*Y*Y*(X**5.))+(10.*X*((X*Y*W)**2.))+(3.*X*Y*Y*(W**4.))+(3.*Y*
     3((X*Y)**3.))+(3.*W*W*X*(Y**4.))+(X*(Y**6.)))
```

```
      FXS3=16.*S3*((7.*(X**6.))+(35.*W*W*(X**4.))+(21.*X*X*(W**4.))+(W**
     26.)+(15.*Y*Y*(X**4.))+(30.*((X*Y*W)**2.))+(3.*Y*Y*(W**4.))+(9.*X*X
     3*(Y**4.))+(3.*W*W*(Y**4.))+(Y**6.))
      FYS3=16.*S3*((6.*Y*(X**5.))+(20.*Y*X*X*X*W*W)+(6.*X*Y*(W**4.))+(12
     2.*((X*Y)**3.))+(12.*X*W*W*(Y**3.))+(6.*X*(Y**5.)))
      FA=((4.*X*U*U)+(4.*Y*U*V))*A
      FXA=4.*U*U*A
      FYA=4.*U*V*A
      F081=2.*081*((U*U*((4.*X*X*X)+(4.*X*W*W)))+(2.*U*V*((3.*X*X*Y)+(Y*
     2*3.)+(Y*W*W)))+(2.*X*Y*Y))
      FX081=2.*091*((U*U*((12.*X*X)+(4.*W*W)))+(12.*X*Y*U*V)+(2.*Y*Y))
      FY081=2.*081*((2.*U*V*((3.*X*X)+(3.*Y*Y)+(W*W)))+(4.*X*Y))
      FB=2.*U*B
      FC1=2.*C1*((U*((3.*X*X)+(W*W)+(Y*Y)))+(2.*Y*X*V))
      FXC1=2.*C1*((6.*U*X)+(2.*Y*V))
      FYC1=2.*C1*((2.*U*Y)+(2.*X*V))
      FC2=((2.*U*((5.*(X**4))+(10.*X*X*W*W)+(W**4)+(6.*X*X*Y*Y)+(2.*Y*Y*
     2W*W)+(Y**4)))+(8.*X*Y*V*((X*X)+(Y*Y)+(W*W))))*C2
      FXC2=(2.*C2*U*((20.*X*X*X)+(20.*X*W*W)+(12.*X*Y*Y)))+(8.*C2*V*((3.
     2*X*X)+(W*W)+(Y*Y))*Y)
      FYC2=(2.*C2*U*((12.*X*X*Y)+(4.*Y*W*W)+(4.*Y*Y*Y)))+(8.*X*V*C2*((
     2X*X)+(W*W)+(3.*Y*Y)))
      FEC=2.*EC*((U*U*U*((3.*X*X)+(W*W)))+(6.*X*Y*V*U*U)+(3.*Y*Y*U*V*V))
      FXEC=12.*EC*U*U*((X*U)+(Y*V))
      FYEC=12.*EC*U*V*((X*U)+(Y*V))
      F082=4.*082*U*U*((3.*(X**5.))+(10.*X*X*X*W*W)+(3.*X*(W**4.))+(4.*X
     2*X*X*Y*Y)+(4.*X*Y*Y*W*W)+(X*(Y**4.)))+(4.*Y*U*V*082*((5.*(X**4.))+
     3(10.*X*X*W*W)+(W**4.)+(6.*X*X*Y*Y)+(2.*Y*Y*W*W)+(Y**4.)))+(8.*X*Y*
     4Y*V*V*082*((X*X)+(Y*Y)+(W*W)))
      FX082=4.*032*((U*U*((15.*(X**4.))+(30.*X*X*W*W)+(3.*(W**4.))+(12.*
     2X*X*Y*Y)+(4.*Y*Y*W*W)+(Y**4.)))+(Y*U*V*((20.*X*X*X)+(20.*X*W*W)+(1
     32.*X*Y*Y)))+(2.*Y*Y*V*V*((3.*X*X)+(W*W)+(Y*Y))))
      FY082=4.*082*((U*U*((8.*X*X*X*Y)+(8.*X*Y*W*W)+(4.*X*Y*Y*Y)))+(U*V*
     2((5.*(X**4.))+(10.*X*X*W*W)+(W**4.)+(18.*X*X*Y*Y)+(6.*W*W*Y*Y)+(5.
     3*(Y**4.))))+(2.*X*V*V*((2.*Y*X*X)+(4.*(Y**3.))+(2.*Y*W*W))))
      FX=2.*W*PI*0.01*(FX0+FXS1+FXS2+FXS3+FXA+FX081+FX082+FXC1+FXC2+FXEC
     2)*2.
      FY=2.*W*PI*0.02*(FYS1+FYS2+FYS3+FYA+FY081+FY082+FYC1+FYC2+FYEC)
      F=2.*PI*W*(F0+FS1+FS2+FS3+FA+F081+F082+FB+FC1+FC2+FEC)
      SINKFX=SIN(FX)/FX
      SINKFY=SIN(FY)/FY
      IF(FX.EQ.0.0)35,36
35    SINKFX=1.
36    IF(FY.EQ.0.0)37,38
37    SINKFY=1.
38    CONTINUE
      O=SINKFX*SINKFY*COS(F)*0.0016/PI
      OXSUM=OXSUM+O
      IF(P.EQ.0.0) GO TO 24
      OI=SINKFX*SINKFY*SIN(F)*0.0016/PI
      OXISUM=OXISUM+OI
24    CONTINUE
      C=J
      E=I
20    Y=-0.98+(C*0.04)
25    X=-0.98+(E*0.04)
      OSUM=OXSUM*L
```

```
      OISUM=OXISUM*L
      XMOD=SQRT((OSUM*OSUM)+(OISUM*OISUM))
      PHASE=ATAN(OISUM/OSUM)
      WRITE(6,215)
215   FORMAT(/,2X,*SPATIAL FREQUENCY*,4X,*RE(OTF)*,14X,*IM(OTF)*,14X,
     2*MTF*,18X,*PHASE*)
      WRITE(6,220) W,OSUM,OISUM,XMOD,PHASE
220   FORMAT(7X,F6.3,5X,F10.4,11X,F10.4,10X,F10.4,12X,F10.4)
      GO TO 13
90    STOP
      END
```

APPENDIX E

COEFFICIENTS FOR THE ORTHONORMALIZED FORM OF THE SPOT-SIZE AND VARIANCE CRITERIA

The coefficient values for the spot-size and variance criteria are given below. The coefficients are labeled in accordance with Eq. 2.7

Coefficient	Spot-size Criterion	Variance Criterion
N	2	1/12
a	4/3	1
b	3/2	9/10
c	8/5	4/5
d	9/5	3/2
e	12/5	12/7
f	16/7	2
g	1	3/4
h	1	3/5
i	8/5	4/3
j	1	2/3
k	1	1/2
l	3/2	6/5
α	4/9	1/180
β	3/50	1/2800
γ	8/1225	1/44100
δ	1/2	1/24
ε	1/8	1/640
ψ	1/75	1/12600
κ	1	1/4

Coefficient	Spot-size Criterion	Variance Criterion
μ	2/3	1/72
η	1/10	1/1200
τ	3/16	1/128

LIST OF REFERENCES

Born, M., and E. Wolf, Principles of Optics, 5th Edition, Chap. IX., Pergamon Press, New York (1975).

Duffieux, P. M., and G. Lansraux, "Les facteurs de transmission et la lumière diffractée," Revue d'Optique 24, 65, 151, 215 (1945).

Glatzel, E., and R. Wilson, "Adaptive automatic correction," Lens Design with Large Computers, University of Rochester (1967).

Hopkins, G. W., Aberrational analysis of optical systems: A proximate ray trace approach. Ph. D. dissertation, The University of Arizona (1976).

Hopkins, H. H., "The frequency response of a defocused optical system," Proc. Royal Society, London, A 231, 91 (1955).

Hopkins, H. H., "The aberration permissible in optical systems," Proc. Phys. Soc. B 70, 449 (1957a).

Hopkins, H. H., "The numerical evaluation of the frequency response of optical systems," Proc. Phys. Soc. B 70, 1002 (1957b).

Hopkins, H. H., "The use of diffraction-based criteria of image quality in automatic optical design," Optica Acta 13, 343 (1966).

Itoh, Y., "Diffraction-based merit function as a quadratic form of aberration coefficients," J. Opt. Soc. Am. 61, 302 (1971).

King, W. B., "Correlation between the relative modulation and the magnitude of the variance of the wave-aberration difference function," J. Opt. Soc. Am. 59, 692 (1969).

King, W. B., "Use of the modulation transfer function (MTF) as an aberration-balancing merit function in automatic lens design," Optical Instruments and Techniques, ed. J. H. Dickson, Oriel Press, Newcastle-upon-Tyne (1970).

Linfoot, E. H., Recent Advances in Optics, Clarendon Press, Oxford (1955).

Linfoot, E. H., "Contrast transmission at low spatial frequencies," Optica Acta 6, 387 (1959).

Maréchal, A., "Étude des effets combinés de la diffraction et des aberrations géométriques sur l'image d'un point lumineux," Revue d'Optique 26, 257 (1947).

Meiron, J., "The use of merit functions based on wavefront aberrations in automatic lens design," Appl. Opt. 7, 667 (1968).

Miyamoto, K., "Wave optics and geometrical optics in optical design," V. I, Chap. 2, Progress in Optics, ed. E. Wolf, North-Holland Publishing Co., Amsterdam (1961).

Offner, A., "Extended range diffraction-based merit function for least squares type optimization," Optical Instruments and Techniques, ed. J. H. Dickson, Oriel Press, Newcastle-upon-Tyne (1970).

Sayanagi, K., "The role of optical transfer function in optical design techniques," Proceedings of the Conference on Optical Instruments and Techniques, London, ed. K. J. Habell, John Wiley and Sons, Inc., New York (1963).

Stavroudis, O. N., and L. E. Sutton, "Spot diagrams for the prediction of lens performance from design data," NBS Monograph 93, 46 (1965).

Steel, W. H., "Étude des effets combinés des aberrations et d'une obturation centrale de la pupille sur le contraste des images optiques," Revue d'Optique 32, 162 (1953).

Tatian, B., "Aberration balancing in rotationally symmetric lenses," J. Opt. Soc. Am. 64, 1083 (1974).

Unvala, H., "The orthonormalization of aberrations," Lens Design with Large Computers, University of Rochester (1967).

Wiese, G. E., Use of physically significant merit functions in automatic lens design. Master's thesis, The University of Arizona (1974).