QUANTUM INDUCTION AND HIGGS MASS

by

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When I am able to recognize a thing among others, without being able to say in what its differences or characteristics consist, the knowledge is confused .... It is when I am able to explain the peculiarities which a thing has, that the knowledge is called distinct.

Leibniz (Discourse on Metaphysics, xxiv)

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ABSTRACT

With our newly proposed dynamical Higgs mechanism and Quantum Induction programme, Higgs mass is predicted at $M_H \approx 190\,GeV$ by using our modified renormalization group equations. The same procedure also explains the top quark mass correctly.

CHAPTER 1

INTRODUCTION

The search for Higgs particles has become the holy grail of the current generation of particle accelerators. In the Minimal Standard Electroweak Model, the Higgs boson serves to generate the masses of fermions and gauge bosons. The process by which the particles get mass is called the 'Higgs mechanism'. In section 2.1., we begin with a brief review of local gauge invariance and spontaneous symmetry breaking which constitute the essential part of the Higgs mechanism. Then the simplest version of the Higgs mechanism for electroweak symmetry breaking, namely, the Minimal Standard Model with a single Higgs boson is discussed in section 2.2.. Here it should be noted that within the Standard Model, the mass of the Higgs particle is simply an input parameter of the theory. In other words, there is no way to calculate the Higgs mass without finding experimental information about the Higgs spectrum itself. However, in chapter three, we present a newly proposed Quantum Induction Programme, in which the Higgs mass is no longer a free parameter. Moreover, though the Standard Model is extremely successful in describing the experimental data, its Higgs sector has not been verified experimentally. There are still many theoretical objections to the simplest version of the Higgs mechanism described in section 2.2.. In chapter four, we propose an alternative which suggests that the electroweak symmetry is broken dynamically. With the results obtained in chapter three and four, we predict the Higgs mass by using our modified renormalization group equations appeared in chapter five. The same procedure also explains the top quark mass correctly.

CHAPTER 2

THEORETICAL OVERVIEW OF HIGGS PHYSICS IN THE MINIMAL STANDARD MODEL

One would expect people to remember the past and to image the future.
But in fact, when discoursing or writing about history,
they imagine it in terms of their own experience,
and when trying to gauge the future
they cite supposed analogies from the past:
till, by a double process of repetition,
they imagine the past and remember the future.
Lewis Namier [1]

2.1 Abelian Higgs Model

2.1.1 Abelian gauge field theory

One of the most important conceptual revolutions of the structure of theoretical physics in this century is the "*symmetry dictates interactions*" principle [1] which played an essential role in giving rise to various field theories. In other words, symmetry could be used as a guiding principle for formulating interactions in fundamental physics. For instance, Abelian (non-Abelian) gauge symmetry gives rise to Abelian (non-Abelian) gauge field theories which provide a universal framework for describing the four currently known fundamental physical interactions. [2] First

[1]L. Namier, " Symmetry and Repetition," *Conflicts*, (London and Basingstoke, 1942).

[2]It is generally believed that the Weinberg-Salam model and quantum chromodynamics, which are both based on non-Abelian gauge theories, represent the correct descriptions of the electroweak

of all, let us see how the Abelian gauge symmetry gives rise to electromagnetism. Consider the Lagrangian density [3] for a (*classical*) free Dirac field $\psi(x)$ of mass m

$$\mathcal{L}_o = \overline{\psi}(x)(i\partial\!\!\!/ - m)\psi(x) \tag{2.1}$$

with $\partial\!\!\!/ := \gamma^\mu \frac{\partial}{\partial x^\mu}$,[4] and the adjoint field $\overline{\psi} := \psi^\dagger \gamma^0$. Dirac matrices γ^μ are defined so that $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\eta_{\mu\nu}$ where the diagonal spacetime metric $\eta_{\mu\nu}$ has elements $\eta_{00} = 1$, $\eta_{11} = \eta_{22} = \eta_{33} = -1$. Also, $\gamma_5 := i\gamma^0\gamma^1\gamma^2\gamma^3$. Clearly (2.1) is invariant under the global U(1) transformation

$$\begin{aligned} \psi(x) &\longrightarrow e^{-iq\theta}\psi(x) , \\ \overline{\psi}(x) &\longrightarrow \overline{\psi}(x)e^{iq\theta} , \end{aligned} \tag{2.2}$$

where θ is an arbitrary real constant, independent of the spacetime coordinates x, and q is some coupling constant.

According to Noether's theorem [9], there is a conserved four current density J^μ associated with this invariance. That is

$$J^\mu(x) = q\overline{\psi}(x)\gamma^\mu\psi(x) \tag{2.3}$$

and strong interactions respectively. Gravitation is also a non-Abelian gauge theory, but in a sense which has not yet been totally clarified. Detailed discussions of the historical roots and limits of gauge theory can be found in refs. [2, 3, 4].

[3]For introductory treatment of gauge field theory, see, for instance, [5, 6, 7].

[4]We use the notations and conventions of Bjorken and Drell [8]. e.g. $c = \hbar = 1, kx = k^\mu x_\mu = k^0x^0 - \mathbf{kx}, \Box = \partial^2 = \partial_0^2 - \nabla^2$. Three-vectors will be denoted by bold-face letters. As usual, the Greek indices are spacetime indices going from 0 to 3.

with

$$\partial_\mu J^\mu(x) = 0 \ . \tag{2.4}$$

As the electric charge is defined by

$$\begin{aligned} Q(t) &:= \int J^0(x)\, d^3x \\ &\overset{2.3}{=} \int q\psi^\dagger(x)\psi(x)\, d^3x \ , \end{aligned} \tag{2.5}$$

it follows from the current conservation (2.4) that

$$\begin{aligned} \frac{dQ(t)}{dt} &= \int \partial_0 J^0(x)\, d^3x \\ &= -\int_R \partial_i J^i(x)\, d^3x \qquad (\ i \ = \ 1,2,3\ )\ . \end{aligned} \tag{2.6}$$

By the Gauss theorem, (2.6) becomes

$$\frac{dQ(t)}{dt} = -\int_{\partial R} \vec{\mathbf{J}} \cdot \mathbf{d}\vec{\mathbf{S}} \ , \tag{2.7}$$

and the surface integral at the right hand side vanishes under the assumption that the fields $\psi(x)$, and hence the current density, approach zero sufficiently quickly at infinity. Thus the conservation of electric charge in electromagnetism reflects the fact that the free Lagrangian (2.1) shows global gauge invariance under U(1).

Now let us generalize the transformation (2.2) to the *local* one where θ is a function of x. In other words, the parameter θ can be different at every space-time point. Under such a transformation, the fields transform as

$$\begin{aligned} \psi(x) &\longrightarrow e^{-iq\theta(x)}\psi(x) \ , \\ \overline{\psi}(x) &\longrightarrow \overline{\psi}(x)e^{iq\theta(x)} \ , \end{aligned} \tag{2.8}$$

whereas the free Lagrangian density (2.1) is not invariant under such local gauge transformation (2.8). However, if we *insist* that a global invariance hold locally, we must introduce a vector field $A_\mu(x)$, called the gauge field, and replace ∂_μ by the covariant derivative (*the minimal substitution*)

$$\mathcal{D}_\mu := \partial_\mu + iqA_\mu \ . \tag{2.9}$$

Then we get instead

$$\mathcal{L}(x) = \overline{\psi}(x)(i\not{\mathcal{D}} - m)\psi(x) \ , \tag{2.10}$$

which is invariant under (2.8), provided that at the same time we transform

$$A_\mu \longrightarrow A_\mu + \partial_\mu\theta(x) \ . \tag{2.11}$$

Next, consider the Lagrangian density for the vector fields A_μ. Since a mass term $A_\mu A^\mu$ spoils local gauge invariance, the vector field A_μ must be massless and thus the simplest gauge invariant Lagrangian term for A_μ (with a conventional normalization) is

$$\mathcal{L}_A = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{2.12}$$

with the field strength tensor

$$F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu \ . \tag{2.13}$$

Combining (2.10) and (2.12), we are led to the Lagrangian density of *electrodynamics*

$$\mathcal{L}_{EM} = \overline{\psi}(i\partial\!\!\!/ - m)\psi \ - \ q\overline{\psi}\gamma^\mu \psi A_\mu \ - \ \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \ , \tag{2.14}$$

which describes Dirac fields (electrons and positrons) interacting with Maxwell fields (photons). Thus the requirement of local gauge invariance gives rise to electrodynamics. In summary, if we demand that our theory is invariant under the local gauge transformations, a massless gauge field must be introduced and hence the nature of the interaction term is specified. In other words, the interaction is dictated by the insistence on (local) gauge invariance — gauge principle.

The idea of gauge invariance goes back to the work of Herman Weyl in 1919 [11] who, inspired by Einstein's sucessful formulation of general relativity in 1916 with the idea of coordinate-transformation invariance, attempted to deduce electromagnetism from a symmetry principle. [5] Few physicists took up Weyl's idea until

[5] It is important to note here that the original idea formulated in Weyl's 1919 paper was

1954 [18], Yang and Mills [20] generalized Weyl's gauge formulation of electromagnetism to the group SU(2) which started the modern non-Abelian gauge theory on which the QCD and Standard Electroweak Model are based. However, there were still two major problems. As seen from our previous discussion, in order to preserve local gauge invariance, the gauge fields must have zero mass. Photon and gluons are massless, but the weak interaction quanta W^+, W^- ($m_W = 80.4$ GeV), Z^0 ($m_Z = 91.2$ GeV) [21] certainly are not. We shall see in section (2.1.2) that there is a way around this, by introducing scalar fields with the added concept of a spontaneous symmetry-breaking mechanism. The other problem was the renormalization. In the early seventies, 't Hooft [22] and others proved that non-Abelian gauge theories are renormalizable [6] (provided they do not contain anomalies) and hence calculable in the same sense as QED.

2.1.2 Spontaneous symmetry breaking: the Higgs mechanism

For the gauge fields to acquire masses, the vacuum symmetry must be broken by scalar fields.[7] Here, we take the Abelian Higgs Model (local U(1) symmetry) as an generic example to illustrate the ideas of spontaneous symmetry breaking and Higgs mechanism.[8]

Consider a complex scalar (classical) field

$$\phi(x) := \rho(x)e^{i\eta(x)} , \tag{2.15}$$

actually scale invariance. It was abandoned soon after its proposal, since it would lead to conflict with known physical facts [12, 13]. However, with the development of Quantum mechanics, Weyl [14] and others [15, 16] realized that Weyl's original transformation should be reinterpreted as a change in the phase of a wavefunction and not a change of scale. Thus the idea of gauge invariance (i.e. phase invariance) was literally formulated. For a full account of this period of gauge theory, see [17, 4].

[6] For renormalization techniques, see, for instance, [23].

[7] The fields are required to be scalar in order to prevent spontaneous breaking of the Lorentz invariance with which our observed particle physics vacuum possesses.

[8] For a simple introduction and references for spontaneous symmetry breaking and Higgs mechanism, refs. [24, 25, 26] may be consulted.

interacting with a $U(1)$ gauge field $A_\mu(x)$, which is introduced via the gauge principle of section 2.1.1. Thus, with the introduction of the Goldstone potential

$$V(\phi) := \frac{m_\phi^2}{2}|\phi|^2 + \frac{\lambda}{4!}(|\phi|^2)^2 \quad (\lambda > 0, m_\phi^2 < 0) \quad , \tag{2.16}$$

the Lagrangian density becomes

$$\mathcal{L} = \frac{1}{2}(\mathcal{D}_\mu\phi)^*(\mathcal{D}^\mu\phi) \; - \; V(\phi) \; - \; \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \; , \tag{2.17}$$

where $\mathcal{D}_\mu$ is the $U(1)$ covariant derivative

$$\mathcal{D}_\mu := \partial_\mu + iqA_\mu \; , \tag{2.18}$$

and $F_{\mu\nu}$ is the gauge invariant field tensor

$$F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu \; . \tag{2.19}$$

It is clear on inspection that this Lagrangian density (2.17) possesses $U(1)$ global gauge symmetry

$$\phi(x) \longrightarrow e^{-iq\theta}\phi(x) \tag{2.20}$$

as well as $U(1)$ local gauge symmetry

$$\begin{aligned} \phi(x) \longrightarrow \quad \phi'(x) &= e^{-iq\theta(x)}\phi(x) \ , \\ A_\mu(x) \longrightarrow \quad A'_\mu(x) &= A_\mu(x) + \partial_\mu\theta(x) \ . \end{aligned} \tag{2.21}$$

Note that for $m_\phi^2 > 0$, the Lagrangian density (2.17) just describes the ordinary electrodynamics of a massive charged scalar particle (apart from the $|\phi|^4$ self interaction term) and the potential (2.16) $V(\phi)$ preserves the symmetry of the Lagrangian because it has a minimum at $|\phi| = 0$ that is unique. However, in the case $m_\phi^2 < 0$, $|\phi| = 0$ is not the ground state. [9] Here, we must understand that quantum field theory is really a perturbation calculation in which the fields are treated as quantum fluctuations around the ground state. Thus, we need to determine the true ground state first. As

$$\frac{dV}{d|\phi|} = m_\phi^2|\phi| + \frac{\lambda}{6}|\phi|^3 = 0 \tag{2.22}$$

and

$$\frac{d^2V}{d(|\phi|)^2}\Big|_{|\phi|=\sqrt{\frac{-6m_\phi^2}{\lambda}}} = -2m_\phi^2 > 0 \ , \tag{2.23}$$

hence, for $m_\phi^2 < 0$, the potential has minima at

[9]Up to this point the fields we have considered are classical. But in order to introduce the Higgs mechanism, in the rest of this section, the fields are interpreted as *quantum* ones.

$$|\phi| = \sqrt{\frac{-6m_\phi^2}{\lambda}} := v \tag{2.24}$$

corresponding to a whole ring of radius v in the complex ϕ plane at each of whose points $V(|\phi|)$ is at its minimum value. In other words, the field ϕ causes degenerate ground states and thus the symmetry is broken —- spontaneous [10] symmetry breaking.

As the physical fields, which are excitations above the vaccum, are realized by performing perturbation about the ground state, so we select a particular ground state which, without loss of generality, we choose to be (on quantization)

$$\langle 0|\phi(x)|0\rangle \ = \ v\ , \tag{2.25}$$

i.e.

$$\begin{aligned} \langle 0|\rho|0\rangle &= v\ , \\ \langle 0|\eta|0\rangle &= 0\ , \end{aligned} \tag{2.26}$$

and then introduce two hermitian fields $h(x)$ and $\xi(x)$, which represent the quantum fluctuations around the vacuum. [11] Thus in terms of $h(x), \xi(x)$, the original

[10]Note that $V(\phi)$ actually is potential *energy density.* Therefore the total energy barrier is infinite after $V(\phi)$ is multiplied by the infinite spatial volume in which the quantum field theory is defined. Hence quantum tunneling is impossible and it explains why spontaneous symmetry breaking can happen in quantum field theory but not in quantum mechanics.

[11]Note that, for these new variables, the vacuum state makes $\langle 0|h|0\rangle \ = \ 0$ and $\langle 0|\xi|0\rangle \ = \ 0$.

field is given by

$$\phi(x) = (v + h(x))e^{i\frac{\xi(x)}{v}} . \tag{2.27}$$

Substituting (2.27) into (2.17) gives the Lagrangian density, expressed in the fields $h(x)$ and $\xi(x)$ as follows:

$$\begin{aligned} \mathcal{L}' \;=\; & \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + m_\phi^2 h^2 + (\partial^\mu \xi)(\partial_\mu \xi) \\ & + qvA_\mu(\partial^\mu \xi) + \frac{1}{2}q^2v^2A_\mu A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & + (h, \xi \; interactions) . \end{aligned} \tag{2.28}$$

By inspection, the gauge field A_μ has a mass $m_A = qv$ as implied by the term $\frac{1}{2}q^2v^2A_\mu A^\mu$. Similarly, the scalar field h carries a mass $m_h = \sqrt{-2m_\phi^2}$ by virtue of the term $m_\phi^2 h^2$ whereas the scalar field ξ, whose quantum is known as Goldstone boson,[12] is massless. However, since no such massless scalar boson has been observed, it must be eliminated from the theory. This is a typical case of spontaneous symmetry breaking – the Lagrangian is invariant under a symmetry operation, but the vacuum is not.[13] Note that the original Lagrangian $\mathcal{L}$ (2.17) is invariant under (2.20), but the reformulated Lagrangian $\mathcal{L}'$ (2.28) is not; the symmetry has been "broken". To put it the other way around, the overall symmetry of the system is "hidden" by our arbitrary selection of one of the degenerate minima. Here, I need

[12] In fact, this is a simple example of the Goldstone theorem [27, 28] which states that the spontaneous breaking of a continuous global symmetry inevitably generates one or more massless scalar bosons.

[13] A familiar example is the ferromagnet, the Hamiltonian for which is rotationally invariant but the group state of the theory consists of atoms whose spins are all aligned in the same direction, so breaking the invariance.

to emphasize that both Lagrangian densities $\mathcal{L}$ and $\mathcal{L}'$ describe exactly the same physical system. However, the original $\mathcal{L}$ would not lead to a useful perturbation series because the expansion is about the unstable point $|\phi| = 0$; whereas the reformulated one $\mathcal{L}'$ leads to a meaningful perturbation series. The difference in the interpretation arises when perturbations around the minimum are considered.

It is instructive to count the degrees of freedom (dof). Originally we had a complex scalar field (2 dof) and a transverse field for a massless photon (2 dof) for a total of 4 dof. After the spontaneous symmetry breaking, we have a massive photon (3 dof) [14] and two real scalar fields h, ξ (2 dof). Thus we seem to have gained one extra degree of freedom. A way out of the dilemma is to exploit the local gauge freedom of ϕ (2.21), i.e. $\phi(x) \longrightarrow e^{-iq\theta(x)}\phi(x)$. We can choose a particular gauge, for example, the unitary gauge reached by [15]

$$\theta(x) = \frac{\xi}{qv} \tag{2.29}$$

so that the field ξ does not appear in $\mathcal{L}'$.

Substituting (2.29) into (2.21) with (2.27) gives

$$\begin{aligned} \phi(x) \longrightarrow \quad \phi'(x) \quad &= v + h(x) \ , \\ A_\mu(x) \longrightarrow \quad A'_\mu(x) \quad &= A_\mu(x) + \frac{1}{qv}\partial_\mu \xi(x) \ . \end{aligned} \tag{2.30}$$

Now, due to gauge invariance, we may substitute these expressions into (2.17).

[14] A massless gauge field has two tranverse dof, while a massive one has an additional longitudinal dof.

[15] Detailed discussions about the choices for different gauges can be found in, e.g., [7] **Ch. 9**.

Then the Lagrangian density becomes

$$\begin{aligned}\mathcal{L}'' &= \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + m_\phi^2 h^2 - \frac{\lambda}{6} v h^3 - \frac{\lambda}{4!} h^4 \\ &\quad - \frac{1}{2} q A'_\mu (2v + h) + \frac{1}{2} q^2 v^2 A'_\mu A'^\mu - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} \ . \end{aligned} \tag{2.31}$$

Dropping the primes on A_μ and $F_{\mu\nu}$, we find the Lagrangian density in the form

$$\begin{aligned}\mathcal{L}'' &= \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + m_\phi^2 h^2 - \frac{\lambda}{6} v h^3 - \frac{\lambda}{4!} h^4 \\ &\quad - \frac{1}{2} q A_\mu (2v + h) + \frac{1}{2} q^2 v^2 A_\mu A^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \ . \end{aligned} \tag{2.32}$$

The field ξ disappears from the theory and the gauge field A_μ becomes massive. The Lagrangian now correctly represents the desired mass spectrum: it describes two interacting massive particles, a vector gauge boson A_μ, with mass $m_A = qv$ and a massive scalar h with mass $m_h = \sqrt{-2m_\phi^2} = \sqrt{\frac{\lambda}{3}}v$ which is called a Higgs particle. Moreover, the total number (four) of degree of freedom is unaltered. In fact, from (2.30) $A_\mu(x) \longrightarrow A'_\mu = A_\mu + \frac{1}{qv}\partial_\mu \xi(x)$, we see that the massless Goldstone boson is "swallowed" by the gauge field and reappears effectively as the longitudinal component of the gauge boson, giving it a mass. In other words, the extra degree of freedom actually corresponds to the gauge freedom and we have just "gauged away" the unwanted degree of freedom. After Higgs, this mechanism is known as the (Abelian) Higgs mechanism [29, 30, 31, 32, 33], built on the union of local gauge invariance and spontaneous symmetry breaking.

2.2 Higgs Boson in the Minimal Standard Electroweak Model

In the preceding section we have shown how the (Abelian) Higgs mechanism generates mass for the gauge field in an (Abelian) Higgs model. Here, we generalize the above argument to the $SU(2)_L \otimes U(1)_Y$ symmetry of the electroweak interactions with emphasis on the aspects relevant for Higgs physics. [16] In the following, we shall see how the Higgs mechanism provides mass for the W^+, W^- and Z^0 gauge bosons. [17]

The Standard Electroweak Model is an $SU(2)_L \otimes U(1)_Y$ gauge theory [37, 38, 39], containing four gauge fields: $W^i_\mu (i = 1, 2, 3)$ for SU(2) and B_μ for U(1) with a kinetic energy term

$$\mathcal{L}_{\mathcal{G}} = -\frac{1}{4} W^i_{\mu\nu} W_i^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} , \tag{2.33}$$

where the non-Abelian field strength tensors are defined as

$$W^i_{\mu\nu} := \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon^{ijk} W^j_\mu W^k_\nu \tag{2.34}$$

and the Abelian field strength tensor is given by

$$B_{\mu\nu} := \partial_\mu B_\nu - \partial_\nu B_\mu . \tag{2.35}$$

Just as in the Abelian Higgs model of the last section, in order to implement the

[16] Note that we follow the line of the last section. Only after introducing the spontaneous symmetry breaking, the fields are intrepreted as quantum ones.

[17] Details about the generating of fermions mass and other aspects of Standard Electroweak Model can be found in refs. [34, 35]. A comprehensive collection of the original papers on the Standard Model are collected in ref. [36].

Higgs mechanism, we postulate the existence of a background field permeating all space, called the Higgs field, coupling to the gauge fields. In the present case, we require to break the $SU(2)_L \otimes U(1)_Y$ symmetry spontaneously in a manner that leaves the photon massless while the W and Z bosons become massive. This can be achieved with a weak isospin $SU(2)$ doublet of complex scalar fields

$$\begin{aligned} \phi &:= \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} \\ &:= \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \end{aligned} \tag{2.36}$$

where the electric charges, positive for $\phi^{(+)}$ and neutral for $\phi^{(0)}$, follow if to the Higgs doublet one assigns the weak hypercharge $Y = 1$. [18] Again the scalar (classical) potential that will be used to break the symmetry spontaneously is [19]

$$V(\phi) := \frac{m_\phi^2}{2}|\phi^\dagger\phi| + \frac{\lambda}{4!}|\phi^\dagger\phi|^2 \tag{2.37}$$

in which m_ϕ and λ are constants with $m_\phi^2 < 0$ and $\lambda > 0$.

After introducing the interaction between the scalar fields ϕ and the gauge boson fields via the gauge principle discussed in section 2.1.1., we arrive at the

[18] With the help of the Gell-Mann - Nishijima relation of the weak interactions $\hat{Q} = \hat{T}_3 + \frac{\hat{Y}}{2}$ [37] where $\hat{T}_3$ is the third component of the weak isospin. Note that this relation is an additional condition and not included in the basic principle of $SU(2)_L \otimes U(1)_Y$ gauge invariance.

[19] In the simplest version of the Higgs mechanism, there is no deeper theoretical explanation about the choice of potential $V(\phi)$. It can't be derived from more fundamental principles. Thus the form of $V(\phi)$ has a somewhat arbitrary feature though it can be shown to be the most general form which can be renormalized and is $SU(2)$ gauge invariant. However, we shall see in chapter four that it is not the case under our newly proposed dynamical Higgs mechanism in which the potential (2.37) can be recovered with a deeper physical meaning. Moreover, it should be noted that what we discuss in this chapter is just the classical picture of spontaneous symmetry breaking. The potential (2.37) is actually the tree approximation to the effective potential V_{eff} [40, 41].

$SU(2)_L \otimes U(1)_Y$ invariant Lagrangian density for these scalar fields

$$\mathcal{L}_{\phi G} = \frac{1}{2}(\mathcal{D}_\mu \phi)^\dagger (\mathcal{D}^\mu \phi) - \frac{m_\phi^2}{2}|\phi^\dagger \phi| - \frac{\lambda}{4!}|\phi^\dagger \phi|^2 , \tag{2.38}$$

where the covariant derivative is

$$\mathcal{D}_\mu := \partial_\mu - ig\frac{\hat{\tau}_i}{2}W_\mu^i - i\frac{g'}{2}\hat{Y}B_\mu . \tag{2.39}$$

Here g, $\frac{g'}{2}$ are the coupling constants associated with the weak isospin group $SU(2)$ and weak hypercharge $U(1)$ respectively, while $\frac{\hat{\tau}_i}{2}$ are the generators of the $SU(2)$ algebra with

$$\hat{\tau}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \hat{\tau}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \hat{\tau}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \tag{2.40}$$

and $U(1)$ has the single generator

$$\hat{Y} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} . \tag{2.41}$$

It is straightforward, if tedious, to show that $\mathcal{L}_{\phi G}$ (2.38) is manifestly invariant under $SU(2)_L \otimes U(1)_Y$ transformations

$$\phi(x) \longrightarrow \phi'(x) = \hat{U}_2\hat{U}_1\phi(x) , \tag{2.42}$$

$$\frac{\hat{\tau}_i}{2}W_\mu^i \longrightarrow \frac{\hat{\tau}_i}{2}W_\mu'^i = \hat{U}_2\frac{\hat{\tau}_i}{2}W_\mu^i\hat{U}_2^\dagger - \frac{i}{g}(\partial_\mu\hat{U}_2)\hat{U}_2^\dagger , \tag{2.43}$$

$$B_\mu \longrightarrow B'_\mu = B_\mu - \partial_\mu\theta(x) , \tag{2.44}$$

where

$$
\begin{aligned}
\hat{U}_1 &:= e^{-i\frac{g'}{2}\theta(x)\hat{Y}} , \\
\hat{U}_2 &:= e^{-ig\theta^i(x)\frac{\hat{\tau}_i}{2}} ,
\end{aligned}
\tag{2.45}
$$

and the functions $\theta(x)$, $\theta^i(x)$ are independent.

As usual, the vacuum state in the case $m_\phi^2 < 0$, $\lambda > 0$ is not at $|\phi|^2 = 0$ and the scalar field has a non-zero vacuum expectation value. Proceeding as in the (Abelian) Higgs model, by demanding $\frac{dV}{d|\phi|} = 0$, we get for the classical potential $V(\phi)$ a minimum at

$$
|\phi|^2 = (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = -\frac{6m_\phi^2}{\lambda} . \tag{2.46}
$$

Without loss of generality, we can choose the vacuum to have

$$
\begin{aligned}
\phi_1 &= \phi_2 = \phi_4 = 0 , \\
\phi_3^2 &= -\frac{6m_\phi^2}{\lambda} \\
&:= v^2 ,
\end{aligned}
\tag{2.47}
$$

which was originally proposed by Weinberg [38].

Hence, the vacuum expectation value of ϕ is chosen as

$$
\phi_0 := \langle 0|\phi|0\rangle
$$

$$= \begin{pmatrix} 0 \\ v \end{pmatrix} , \tag{2.48}$$

thereby spontaneously breaking the $SU(2)_L \otimes U(1)_Y$ symmetry and thus generating mass for gauge bosons. However, the $U(1)_{em}$ symmetry stays unbroken and the photon remains massless. This can be easily seen as follows. Out of the four $SU(2)_L \otimes U(1)_Y$ generators, namely $\frac{\hat{\tau}_i}{2}$ and $\hat{Y}$, we can form a new generator

$$\begin{aligned} \hat{Q} &:= \frac{\hat{\tau}_3 + \hat{Y}}{2} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \end{aligned} \tag{2.49}$$

which is a linear combination of $\frac{\hat{\tau}_3}{2}$ and $\hat{Y}$. As

$$\hat{Q}\phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} = 0 , \tag{2.50}$$

any $\theta(x)$ leaves

$$\phi_0 \xrightarrow{2.42} e^{i\theta(x)\hat{Q}} \phi_0 = \phi_0 . \tag{2.51}$$

Moreover, according to the weak Gell-Mann - Nishijima relation, $\hat{Q}$ actually represents the charge operator. In short, the vacuum is invariant under the local $U(1)_{em}$ symmetry. Hence the corresponding gauge field (photon field), being a linear combination of W_μ^3 (belonging to $\frac{\hat{\tau}_3}{2}$) and B_μ (belonging to $\hat{Y}$), [20] acquires no mass.

[20]We shall see later how the fields W_μ^3 and B_μ are mixed to form the photon field explicitly.

Now, in order to investigate the particle spectrum, let us expand $\phi(x)$ about our chosen vacuum (2.48) $\phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}$. In terms of η_1, η_2, η_3, $\mathcal{H}$ which describe the quantum fluctuations around the vacuum,[21] $\phi(x)$ can be parametrized as

$$\phi(x) = \begin{pmatrix} \eta_2(x) + i\eta_1(x) \\ v + \mathcal{H}(x) - i\eta_3(x) \end{pmatrix} , \tag{2.52}$$

or in terms of its 'modulus' and 'phase' [26]

$$\phi(x) = e^{i\tau_i \frac{\eta^i}{v}} \begin{pmatrix} 0 \\ v + \mathcal{H}(x) \end{pmatrix} . \tag{2.53}$$

Proceeding in a way similar to the Abelian case, the next step is to 'gauge' away the unphysical $\vec{\eta}(x)$ field. Let us choose a gauge, say,

$$\begin{aligned} \theta(x) &:= 0 , \\ \theta^i(x) &:= \frac{2\eta^i}{gv} . \end{aligned} \tag{2.54}$$

Then inserting these into (2.42) with (2.53), we obtain

$$\phi(x) \longrightarrow \phi^{'}(x) = \begin{pmatrix} 0 \\ v + \mathcal{H}(x) \end{pmatrix} , \tag{2.55}$$

[21] Hence, η_1, η_2, η_3, $\mathcal{H}$ have zero vacuum expectation values.

i.e. only the real field $\mathcal{H}(x)$ is left.

Similarly, inserting (2.54) into (2.43)(2.44), we have

$$
\begin{aligned}
\frac{\hat{\tau}_i}{2}W_\mu^i \longrightarrow \quad \tfrac{\hat{\tau}_i}{2}W_\mu^{\prime i} &= \hat{U}_2\frac{\hat{\tau}_i}{2}W_\mu^i\hat{U}_2^\dagger - \frac{i}{g}(\partial_\mu\hat{U}_2)\hat{U}_2^\dagger \,, \\
B_\mu \longrightarrow \quad B_\mu^{\prime} &= B_\mu \,,
\end{aligned}
\tag{2.56}
$$

where

$$
\hat{U}_2 = e^{-i\eta^i\frac{\hat{\tau}_i}{2}} \,. \tag{2.57}
$$

Now, due to gauge invariance, we may substitute (2.55) (2.56) into (2.38), writing $\mathcal{D}_\mu$ as a 2×2 matrix and using the explicit forms (2.40) of the Pauli matrices. Dropping the primes on W_μ^i, we arrive at a final Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\phi G} &= \frac{1}{2}\partial_\mu\mathcal{H}\partial^\mu\mathcal{H} + m_\phi^2\mathcal{H}^2 - \frac{\lambda v}{6}\mathcal{H}^3 - \frac{\lambda}{4!}\mathcal{H}^4 \\
&\quad - \frac{g^2}{8}(v+\mathcal{H})^2(W_\mu^1 - iW_\mu^2)(W^{1\mu} + iW^{2\mu}) \\
&\quad - \frac{1}{8}(v+\mathcal{H})^2(gW_\mu^3 - g'B_\mu)^2 + 0(g'W_\mu^3 + gB_\mu)^2 \\
&= \frac{1}{2}\partial_\mu\mathcal{H}\partial^\mu\mathcal{H} + m_\phi^2\mathcal{H}^2 - \frac{\lambda v}{6}\mathcal{H}^3 - \frac{\lambda}{4!}\mathcal{H}^4 \\
&\quad - \frac{g^2}{4}(v+\mathcal{H})^2W_\mu^+W^{-\mu} - \frac{g^2+g'^2}{8}(v+\mathcal{H})^2Z_\mu Z^\mu + 0A_\mu A^\mu
\end{aligned}
\tag{2.58}
$$

Here we have defined the charged Bose fields [22]

[22]Note that in some literature the notations $W_\mu^\dagger := W_\mu^+$, $W_\mu := W_\mu^-$ are also used.

$$\begin{aligned} W_\mu^+ &:= \frac{1}{\sqrt{2}}(W_\mu^1 - iW_\mu^2) \ , \\ W_\mu^- &:= \frac{1}{\sqrt{2}}(W_\mu^1 + iW_\mu^2) \ ; \end{aligned} \tag{2.59}$$

and the fields W_μ^3 and B_μ are mixed to form the neutral Bose fields [23]

$$\begin{aligned} Z_\mu &:= \frac{1}{\sqrt{g^2+g'^2}}(gW_\mu^3 - g'B_\mu) \ , \\ A_\mu &:= \frac{1}{\sqrt{g^2+g'^2}}(g'W_\mu^3 + gB_\mu) \ . \end{aligned} \tag{2.60}$$

By inspection,

$$m_W = \frac{1}{2}gv \ , \tag{2.61}$$

$$m_Z = \frac{1}{2}\sqrt{g^2+g'^2}v \ , \tag{2.62}$$

$$m_\gamma = 0 \ , \tag{2.63}$$

$$m_{\mathcal{H}} = \sqrt{-2m_\phi^2} = \sqrt{\frac{\lambda}{3}}v \ . \tag{2.64}$$

Hence, we are left with four gauge bosons having the desired properties and one neutral scalar Higgs particle. It is again instructive to perform some 'bookkeeping' on the number of degrees of freedom. Originally we had

[23]Equation (2.60) can also be written in terms of the Weinberg angle θ, where $g sin\theta = g' cos\theta$.

$$\begin{array}{rl} 4 & \text{(complex scalar field } \phi) \\ 6 & \text{(3 massless SU(2) } W_\mu^i \text{ fields)} \\ +\quad 2 & \text{(massless } U(1) \text{ gauge field } B_\mu) \\ \hline 12\ . & \end{array}$$

After the spontaneous symmetry breaking we have

$$\begin{array}{rl} 1 & \text{(physical real scalar field } \mathcal{H}) \\ 9 & \text{(massive } W \text{ and } Z \text{ fields)} \\ +\quad 2 & \text{(massless photon field } A_\mu) \\ \hline 12\ . & \end{array}$$

Here, the scalar degrees of freedom have been turned into the longitudinal polarizations of the massive W and Z gauge bosons.

Furthermore, the parameter v has the value $v = 246$ GeV which is determined by the weak interaction data. The argumentation is as follows. We know that in terms of the V-A form of the weak current, the matrix element of the μ decay ($\mu \longrightarrow e\overline{\nu}_e\nu_\mu$) can be written, with the Fermi coupling constant G_F,

$$\mathcal{M}(\mu \longrightarrow e\overline{\nu}_e\nu_\mu) = \frac{G_F}{\sqrt{2}}[\overline{\nu}_\mu\gamma^\sigma(1-\gamma_5)\mu][\bar{e}\gamma_\sigma(1-\gamma_5)\nu_e]\ , \tag{2.65}$$

while under the vector boson picture of weak interactions

$$\mathcal{M}(\mu \longrightarrow e\overline{\nu}_e\nu_\mu) = [\frac{g}{\sqrt{2}}\overline{\nu}_\mu\gamma^\sigma\frac{1-\gamma_5}{2}\mu][\frac{1}{m_W^2-q^2}][\frac{g}{\sqrt{2}}\bar{e}\gamma_\sigma\frac{1-\gamma_5}{2}\nu_e]\ . \tag{2.66}$$

Since q^2 is of the order m_μ^2, it can be neglected in comparison with m_W^2 and we make the identification

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} \ . \tag{2.67}$$

Thus, (2.61) and the measured value of G_F from muon decay give

$$\begin{aligned} v^2 &= \left(\sqrt{2}G_F\right)^{-1} \\ &= (246 \ \ GeV)^2 \ . \end{aligned} \tag{2.68}$$

In the preceeding sections we have briefly discussed the Higgs mechanism. Here, some remarks need to be made.

(I). The one that we have discussed is the simplest version of the Higgs mechanism for electroweak symmetry breaking. However, this has not been verified experimentally and the physics that underlies electroweak symmetry breaking is still not certain. Indeed, several alternatives have been suggested [24]. For instance, some have considered that the electroweak symmetry may be broken dynamically by the technicolor approach [44, 45] which postulates the existence of technicolor gauge interactions among a set of technifermions. In contrast, in chapter four, we also propose a model with dynamical electroweak symmetric breaking mechanism, but the role played by fermions is different. Another popular alternative to the minimal Standard Model with a single Higgs boson is to make the Standard Model supersymmetric [46, 47, 48]. Indeed many other attempts including the effort to construct an electroweak model without Higgs mechanism [49, 50] are still under investigation.

[24] For a general overview about the Higgs physics, [42, 43] can be consulted.

(II). There is no prediction for the Higgs boson mass which is a free parameter within the electroweak theory, though some theoretical constraints on Higgs mass can be derived. For example, the condition for perturbative unitarity [51, 52] requires $m_{\mathcal{H}} < 870\,\mathrm{GeV}$ [25]. However, the mass of the Higgs particle is no longer a free parameter under our newly proposed Quantum Induction Programme which we are going to discuss in the next chapter.

[25]This simply means that the perturbation theory is not valid and the electroweak sector of the theory is strongly interacting for heavier masses.

CHAPTER 3

OUTLINE OF THE QUANTUM INDUCTION PROGRAMME

Everything should be made as simple as possible, but not simpler.
A. Einstein

From the previous discussion, we saw that the Higgs mass depends on λ (2.64) which is a parameter characteristic of the scalar potential. However, within the Standard Model, there is no way to calculate or determine λ without finding experimental information about the Higgs spectrum itself. It is due to the fact that the conventional local quantum field theory has intrinsic ultraviolet divergencies which are supposed to be circumventable by the renormalization procedure [53], and thus requiring a number of renormalization constants which cannot be determined by the theory and are regarded as free parameters to be fixed by experiment. However, in the following, we shall see that the proposed Quantum Induction Programme [54, 55] provides a way for avoiding ultraviolet divergencies and replacing them by some consistency conditions. Hence we have fewer adjustable parameters and thus λ can be estimated.

Since Quantum Induction is a vast subject, an introductory presentation necessarily involves choices of emphasis and of omission. Other aspects of Quantum Induction, for instance, the unitarity preserving finiteness [56] and the access to gravity [57, 58, 59, 60] won't be discussed here.

In the following, first of all, we shall discuss basic postulates of our proposed Quantum Induction in section 3.1. Then various implied properties of Bose fields in Quantum Induction will be fully explained in section 3.2. In contrast with the usual theory where quantum field equations are never mentioned, they are induced by our postulates in Quantum Induction appeared in section 3.3. Finally, in section 3.4, we shall derive two important results from our postulates, namely, the non-canonical Bose propagators and bare vertex functions. Hence our couplings approach bare constants at asymptotic energies on which our modified renormalization group equations appeared in Chapter five are based.

3.1 Basic Postulates

At the most fundamental level, we adopt the "Wightman axioms" for quantum field theory [61] except the asymptotic completeness and positive definiteness axioms. Here, we do not intend to present these axioms in detail, however, there is an important point that should be mentioned.

It has been realized since the classic analysis of field measurements for the electromagnetic field in quantum electrodynamics [62, 63] that it is physically meaningless to ask for the value of a quantized field $\phi(x)$ at a given space-time point because it cannot be a proper observable. In fact, it appears evident that such a measurement at a point would necessitate infinite energy. This suggests that the quantized field $\phi(x)$ can only be interpreted as a field to be smeared, i.e. to every test function $f(x)$ on Minkowski space there corresponds an (unbounded) operator $\phi[f]$ acting on the Hilbert space $\mathcal{H}$, the space of states, in a manner indicated by

$$\phi[f] = \int d^4x\, f(x)\phi(x) \qquad \in \mathcal{A}\,. \tag{3.1}$$

Here $f(x) \in \mathcal{D}$ where $\mathcal{D}$ is the space of those infinitely differentiable complex-valued functions which are of compact support, and $\mathcal{A}$ denotes the algebra of smeared fields . In fact, all serious attempts [64, 65] to attribute a rigorous mathematical meaning to $\phi(x)$ have led to the same conclusion — only when the quantized field $\phi(x)$ is treated as an *operator valued distribution* over Minkowski space, [1] it is well-defined.

First of all, let us come to the postulates of our Quantum Induction Programme. For brevity's sake, we take the "interacting" Dirac equation [2]

$$\{i\partial\!\!\!/ - \mathcal{B}(x)\}\Psi(x) = 0 \tag{3.2}$$

and the equal-time anticommutation relations

$$\begin{aligned} [\Psi(x), \Psi^\dagger(0)]_+ \delta(x^0) &= \delta(x) \qquad (3.3) \\ [\Psi(x), \Psi^T(0)]_+ \delta(x^0) &= 0 \qquad (3.4) \end{aligned}$$

as our basic postulates. In the formulae above, Ψ is a spinor of 96 components which describes the 24 leptons and quarks [3], while $\Psi^\dagger$ and Ψ^T denote its hermitian conjugate and transpose respectively. Of course, a unit matrix of the appropriate dimension is *implied* on the right-hand side of (3.3) and thus its left-hand side

[1]Note that this can be viewed as the mathematical expression of the physical idea of modifying the exact point model because only the space-time averages of a quantum field $\phi(x)$ over test functions are considered as proper observables.

[2]Note that the conventional form of free Dirac's equation is $(i\partial\!\!\!/ - m)\psi = 0$ which describes a free fermion ψ with mass m. However, when one regards all matter as made out of leptons and quarks (i.e. fermions), with Bose fields $\mathcal{B}(x)$ mediating their forces, it is most natural to start from (3.2). In fact, Dirac adopted a similar equation in his early work [66]. In fact, if one approximates Bose fields $\mathcal{B}(x)$ by its vacuum expectation value, the "interacting" Dirac equation will be reduced to the free Dirac's equation. Moreover, it should be pointed out here that (3.2) does not hold in the usual theory for *interacting* Bose fields $\mathcal{B}(x)$ [68] since coincident canonical quantum field products are divergent. More will be discussed at footnote 5.

[3]In the current view, the spectrum of the Standard Model for the fermions consists of 6 leptons and 6 'flavors' of quarks, which come in three 'colors'.

should be understood as

$$[\Psi(x), \Psi^\dagger(0)]_+^{\zeta\eta} := \Psi^\zeta(x)\Psi^{\dagger\eta}(0) + \Psi^{\dagger\eta}(0)\Psi^\zeta(x) , \tag{3.5}$$

where ζ, η are spinor indices. Several remarks need to be made here first.

(i). Actually, the postulates (3.2) through (3.4) can be derived from Feynman's path integral with the basic action

$$\mathsf{S}[\Psi] = \int dx\, \overline{\Psi}(x)(i\partial\!\!\!/ - \mathcal{B}(x))\Psi(x) , \tag{3.6}$$

which is regarded as our primary postulate (Appendix A). However, for pedagogic purposes, we start from (3.2) through (3.4).

(ii). As we know, free fields can be quantized canonically. For interacting fields, the infinite renormalization [67] is invoked and so the *renormalized* fields violate the canonical equal-time relations [68, 69, 70] [4]. However, if one postulates that *only* Dirac's field Ψ is quantized canonically, then the infinite renormalization is prevented and thus the canonical quantization by (3.3) and (3.4) can be demanded to hold for *both* free and interacting Dirac fields. Moreover, (3.2) through (3.4) will provide so strong consistency conditions that a (*non-canonical*) quantization of Bose fields $\mathcal{B}$ is implied. More will be discussed in the following sections.

[4]Note that this violation under infinite renormalization is rarely mentioned in the usual theory, because neither canonical relations nor (quantum) field equations occur in the S-matrix. Nevertheless, it can easily be seen by using the fact that the generic renormalized field ϕ_r relates with the bare field ϕ_b through $\phi_r = \frac{1}{\sqrt{Z}}\phi_b$ with the renormalization constant $Z = 0$. Hence, the idea that the canonical commutation relations hold in any literal sense must be abandoned.

So, with the above mentioned postulates, *how are the ultraviolet divergencies prevented in Quantum Induction*? Here, it should be noted that the usual quantum field theory was regarded as a fundamental theory which means that all high-energy processes had to be taken into consideration. Hence, in order to take into account the high-energy effects on low-energy phenomena, the cutoff is taken to infinity which results in the ultraviolet divergencies. Or, equivalently, we may say that these divergencies originated from the usual attempt to make *all* basic fields canonical, so that the canonical quantum field products such as $\phi^a(x)\phi^b(y)$, $\phi^a_{,\mu}(x)\phi^b(y)$ are meaningless when they occur at the *same* point $x \equiv y$ [5] Here, let us take the usual interacting field equation $(\Box + m^2)\phi = \overline{\psi}(x)\psi(x)$ as an example. Actually, in usual theory, this quantum field equation is seldom used. Instead, its corresponding Lagrangian and S matrix are constructed. The perturbative calculations based on such S matrix always led to the appearance of numerous divergent integrals. By the traditional renormalization method, the resulting divergences are isolated, and are considered to represent infinite but unobservable renormalizations of appropriate masses, charges, and wave-functions. In this sense, the usual theory never touches the divergent coincident field products seriously. However, in Quantum Induction, this is not the case. They can be avoided in Quantum Induction in

[5]Here, dimensional analysis provides some useful insights. As we know, for canonical Bose field ϕ (in d dimensions), $[\phi] = L^{1-d/2}$ (L is length). Hence, the leading singular term for the short distance expansion of the canonical Bose field product $\phi(x+z)\phi(x-z)$ is z^{-2}. Note that it is simple to check that ϕ fulfills the equal-time commutation relations $[\partial_0\phi(x), \phi(0)]_-\delta(x^0) = -i\delta(x)$, $[\phi(x), \phi(0)]_-\delta(x^0) = [\partial_0\phi(x), \partial_0\phi(0)]_-\delta(x^0) = 0$ by using the fact that $[\delta(x)] = L^{-d}$ (because $\int dx\, \delta(x) = 1$) and $[\partial_\mu] = L^{-1}$. As for canonical Dirac field Ψ, $[\Psi] = L^{\frac{1-d}{2}}$, so the leading singular term for the short distance expansion of the canonical Dirac field product $\Psi(x+z)\Psi^\dagger(x-z)$ is $\not{z}^{-3}$. Similarly, it is simple to check that Ψ fulfills the equal-time anticommutation relations $[\Psi(x), \Psi^\dagger(0)]_+\delta(x^0) = \delta(x)$, $[\Psi(x), \Psi(0)]_+\delta(x^0) = [\Psi^\dagger(x), \Psi^\dagger(0)]_+\delta(x^0) = 0$. With the above expansions, it is trivial that the canonical 'coincident' field products are divergent. Note that the above dimensional analysis does not apply for classical fields since they are not satisfied the translational invariance.

the following ways.

(i). Postulate (3.6) indicates that point interactions of Dirac field Ψ with itself are excluded. Moreover, it will be seen that in Quantum Induction the Dirac field Ψ and its adjoint $\overline{\Psi}$ never couple at the same point since only the local limits of "$\Psi(x+z)\overline{\Psi}(x-z)$" (for instance with 'subtractions' done first) are used in all our calculations as appearing in (3.42) and (3.214).

(ii). In section 3.4, we shall see that in order to keep our postulates be possible, the coincident field products "$\mathcal{B}^P(x)\mathcal{B}^Q(x)$" are required to be finite and thus the Bose field $\mathcal{B}$ is regarded as a *non-canonical quantum* field. It is important to keep in mind that $\mathcal{B}$ is neither assumed as a non-quantized field with which the theory of heat kernels [71, 72] deals, nor as a prescribed non-quantized field which appears in external field theory [73, 74, 75]. .

(iii). The Dirac equation (3.2) taken as postulate demands

$$\int \mathcal{B}(x)\Psi(x)\,dx\,f(x) \;=\; -i\,\gamma^\mu \int \Psi(x)\,dx\,\partial_\mu\,f(x) \tag{3.7}$$

for every test function $f(x) \in \mathcal{D}$ while $\Psi(x) \in \mathcal{A}$ is treated as an operator valued distribution. Thus the 'coincident' field operator (matrix-contracted) "$\mathcal{B}(x)\Psi(x)$" [6] is also required to exist as an operator valued distribution in a space $\mathcal{A}'$ which is affiliated to $\mathcal{A}$.

[6] Here the product "$\mathcal{B}(x)\Psi(x)$" has the components "$\mathcal{B}^{\zeta\eta}(x)\Psi^\eta(x)$" where ζ, η are the spinor indices.

3.2 Bose Fields

In this section, we explore some implied properties of Bose fields $\mathcal{B}$ which are direct consequences of the basic postulates (3.2), (3.3) and (3.4).

3.2.1 Basic properties

(I). Since Ψ is a spinor of 4 components [7], so from our basic postulate (3.2), $\mathcal{B}$ is required to be a 4×4 matrix which is regarded as a general member of Dirac's Clifford algebra ($\mathcal{C}l_{\mathcal{D}}$) [8] [76]. Its generating elements γ^0, γ^1, γ^2, γ^3 are restricted by

$$\gamma^{(\mu}\gamma^{\nu)} = \eta^{\mu\nu} \ , \tag{3.8}$$

where

$$\gamma^{(\mu}\gamma^{\nu)} := \frac{1}{2}[\gamma^\mu, \gamma^\nu]_+ \ , \tag{3.9}$$

$$\eta := \mathrm{diag}(1, -1, -1, -1) \ . \tag{3.10}$$

Here, *no specific matrix representation* is adopted. However, each γ^μ is assumed unitary [9]

$$\gamma_\mu^{-1} = \gamma_\mu^\dagger \ . \tag{3.11}$$

Thus, with the introduction of the Dirac adjoint,

[7] Note that Ψ is a spinor of 4 components when only Clifford algebra is under consideration. If the $U(1) \times SU(2) \times SU(3)$ Lie algebra is further considered, as we shall see later, then Ψ will have 96 components. Moreover, when Ψ is treated as the quantum operator in the Hilbert space, it has infinite dimensions under field algebra.

[8] A set of matrices satisfying the commutation relations (3.8) is said to generate a Clifford algebra.

[9] Note that $\gamma_\mu = \eta_{\mu\nu}\gamma^\nu$.

$$\overline{\Gamma} := \gamma_0 \Gamma^\dagger \gamma_0 \qquad (\text{for every } \Gamma \in Cl_{\mathcal{D}}) \ , \tag{3.12}$$

we have

$$\gamma_\mu^\dagger = \gamma_\mu^{-1} = \gamma^\mu = \overline{\gamma}^\mu \quad \in Cl_{\mathcal{D}} \ . \tag{3.13}$$

Note that

$$\gamma^k = -\gamma^{k\dagger} \qquad (k = 1,2,3) \ , \tag{3.14}$$

i.e. γ^k is anti-hermitian. Moreover, with the introduction of

$$\gamma_5 := i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma_5^{-1} = \gamma_5^\dagger = -\overline{\gamma}_5 \quad \in Cl_{\mathcal{D}} \tag{3.15}$$

and

$$\sigma^{\mu\nu} := i\gamma^{[\mu}\gamma^{\nu]} := \frac{i}{2}[\gamma^\mu, \gamma^\nu]_- = \bar{\sigma}^{\mu\nu} = -\sigma^{\nu\mu} \ , \tag{3.16}$$

we can form the set

$$\Gamma^a = \{ \mathbf{1},\ i\gamma_5,\ \gamma^\mu,\ \gamma^\mu\gamma_5,\ \sigma^{\mu\nu} \} \ , \tag{3.17}$$

which constitutes a *basis* in $Cl_{\mathcal{D}}$, i.e. they are linearly independent. Before we show this, let us recall the following theorems first.

Theorem 1 :

$$Tr(AB) = Tr(BA) \ , \tag{3.18}$$

it follows that

$$Tr(ABC) = Tr(CAB) = Tr(BCA) \; . \tag{3.19}$$

Theorem 2 : The trace of the product of an odd number of generating matrices γ^μ is zero.

Proof

$$\begin{aligned} Tr(\gamma^{\mu_{i_1}}\gamma^{\mu_{i_2}}\ldots\gamma^{\mu_{i_n}}) &\stackrel{3.15}{=} Tr(\gamma_5\gamma_5\gamma^{\mu_{i_1}}\gamma^{\mu_{i_2}}\ldots\gamma^{\mu_{i_n}}) \\ &\stackrel{3.8}{=} (-1)^n Tr(\gamma_5\gamma^{\mu_{i_1}}\gamma^{\mu_{i_2}}\ldots\gamma^{\mu_{i_n}}\gamma_5) \\ &\stackrel{3.18}{=} (-1)^n Tr(\gamma^{\mu_{i_1}}\gamma^{\mu_{i_2}}\ldots\gamma^{\mu_{i_n}}) \; . \end{aligned} \tag{3.20}$$

■

Theorem 3:

$$Tr(\gamma_5) = 0 \; . \tag{3.21}$$

Proof

$$\begin{aligned} Tr(\gamma_5) &\stackrel{3.15}{=} i\, Tr(\gamma^0\gamma^1\gamma^2\gamma^3) \\ &\stackrel{3.8}{=} -i\, Tr(\gamma^1\gamma^2\gamma^3\gamma^0) \\ &\stackrel{3.18}{=} -i\, Tr(\gamma^0\gamma^1\gamma^2\gamma^3) \; . \end{aligned} \tag{3.22}$$

■

With theorem 2, 3 and using (3.8), (3.15), it is quite straight forward to verify

Theorem 4:

$$Tr(\Gamma^a\Gamma^b) = 0 \qquad (\text{ if } a \neq b \text{ }) \; . \tag{3.23}$$

To show that those sixteen elements in (3.17) are linearly independent, let us assume that

$$\sum_a \mathcal{C}_a \Gamma^a = 0 \, . \tag{3.24}$$

Then multiply (3.24) by Γ^b and take the trace. By using theorem 4, we find that $\mathcal{C}_b = 0$. As b is arbitrary, this means that all coefficients are zero and thus these sixteen elements are linearly independent. Therefore, using (3.17) as the basis, Bose fields $\mathcal{B}(x)$ can be written as [10]

$$\begin{aligned} \mathcal{B}(x) &= \mathcal{S}^+(x) + i\gamma_5 \mathcal{S}^-(x) + \gamma^\mu \mathcal{V}_\mu^+(x) + \gamma^\mu \gamma_5 \mathcal{V}_\mu^-(x) + \sigma^{\mu\nu} \mathcal{T}_{\mu\nu}(x) \\ &= \mathcal{S}(x) + \gamma^\mu \mathcal{V}_\mu(x) + \sigma^{\mu\nu} \mathcal{T}_{\mu\nu}(x) \, , \end{aligned} \tag{3.25}$$

where

$$\mathcal{S}(x) := \mathcal{S}^+(x) + i\gamma_5 \mathcal{S}^-(x) \, , \tag{3.26}$$

$$\mathcal{V}_\mu(x) := \mathcal{V}_\mu^+(x) + \gamma_5 \mathcal{V}_\mu^-(x) \, . \tag{3.27}$$

Note that $\mathcal{S}^\pm(x)$, $\mathcal{V}_\mu^\pm(x)$ and $\mathcal{T}_{\mu\nu}(x)$ only act on the flavor and color components of $\Psi(x)$. Thus they do not contain any γ_μ matrices. Moreover, from the parity transformations of the 16 elements in (3.17), the fields $\mathcal{S}^+(x)$, $\mathcal{S}^-(x)$, $\mathcal{V}^+(x)$, $\mathcal{V}^-(x)$, $\mathcal{T}_{\mu\nu}(x)$ can be easily identified as scalar, pseudoscalar, vector, pseudovector and tensor fields respectively.

[10] However, in sectioin 3.2.3. we shall find $\mathcal{T}_{\mu\nu} = 0$. Thus, $\mathcal{B}(x)$ is actually not spanned by all 16 basis elements of Dirac's Clifford algebra, but only by 10 of them.

(II). In contrast to the usual theory where $\overline{\Psi} := \Psi^\dagger\gamma_0$ is assumed from the start and one lets Ψ and $\overline{\Psi}$ vary independently without any explanation (see, for instance, [77]), $\Psi(x)$ and $\overline{\Psi}(x)$ are treated as two different fields initially in the quantum induction programme, so their variations are independent. However, by demanding the consistency of Dirac's equation (A.49)

$$(i\partial\!\!\!/ - \mathcal{B})\Psi = 0 \tag{3.28}$$

and its adjoint (A.50)

$$\overline{\Psi}(i\overleftarrow{\partial\!\!\!/} + \mathcal{B}) = 0 \ , \tag{3.29}$$

we shall show

$$\overline{\Psi} \ = \ \Psi^\dagger\gamma_0 \ , \tag{3.30}$$

$$\overline{\mathcal{B}} \ := \ \gamma_0\mathcal{B}^\dagger\gamma_0 \ = \ \mathcal{B} \ . \tag{3.31}$$

In fact, by taking the hermitian conjugate of (3.29), we have

$$\begin{aligned}
& & (i\partial\!\!\!/ + \mathcal{B})^\dagger\overline{\Psi}^\dagger &= 0 \\
\Rightarrow & & (-i\partial\!\!\!/^\dagger + \mathcal{B}^\dagger)\overline{\Psi}^\dagger &= 0 \\
\Rightarrow & & -\gamma_0(-i\partial\!\!\!/^\dagger + \mathcal{B}^\dagger)\gamma_0\gamma_0\overline{\Psi}^\dagger &= 0 \\
\Rightarrow & & (-i\partial\!\!\!/ - \gamma_0\mathcal{B}^\dagger\gamma_0)\gamma_0\overline{\Psi}^\dagger &= 0 \ .
\end{aligned} \tag{3.32}$$

Comparing with Dirac's equation (3.28), we have

$$\begin{aligned} \Psi &= \gamma_0 \overline{\Psi}^\dagger \qquad\qquad (\,\text{or } \overline{\Psi} = \Psi^\dagger \gamma_0\,)\ , \\ \mathcal{B} &= \gamma_0 \mathcal{B}^\dagger \gamma_0 := \overline{\mathcal{B}}\ . \end{aligned} \tag{3.33}$$

(III). As a consequence of (3.31) with (3.25), we have
(i).

$$\mathcal{S} = \overline{\mathcal{S}}\ . \tag{3.34}$$

Using the fact that $\mathcal{S}$ does not contain any γ^μ matrices except γ_5 (see (3.26))which anticommutes with all other γ^μ,

$$[\,\gamma^\mu,\, \gamma_5\,]_+ = 0\ , \tag{3.35}$$

we further have

$$\gamma^\mu \mathcal{S} = \mathcal{S}^\dagger \gamma^\mu\ . \tag{3.36}$$

(ii).

$$\mathcal{V}_\mu = \mathcal{V}_\mu^\dagger\ , \tag{3.37}$$

$$\gamma^\mu \mathcal{V}_\mu = \overline{\mathcal{V}}_\mu \gamma^\mu\ . \tag{3.38}$$

Proof

$$\gamma^\mu \mathcal{V}_\mu = \overline{\gamma^\mu \mathcal{V}_\mu}$$

$$
\begin{aligned}
&= \overline{\mathcal{V}}_\mu \overline{\gamma^\mu} \\
&= \gamma_0 \mathcal{V}_\mu^\dagger \gamma_0 \gamma^\mu \qquad\qquad (= \overline{\mathcal{V}}_\mu \gamma^\mu\ ) \\
&= \gamma_0 \gamma_0 \gamma^\mu \mathcal{V}_\mu^\dagger \\
&= \gamma^\mu \mathcal{V}_\mu^\dagger \, .
\end{aligned} \tag{3.39}
$$

■

(iii).

$$\mathcal{T}_{\mu\nu} = \overline{\mathcal{T}}_{\mu\nu} \, . \tag{3.40}$$

Proof

$$
\begin{aligned}
\sigma^{\mu\nu} \mathcal{T}_{\mu\nu} &= \overline{\sigma^{\mu\nu} \mathcal{T}_{\mu\nu}} \\
&= \overline{\mathcal{T}}_{\mu\nu} \overline{\sigma}^{\mu\nu} \\
&= \overline{\mathcal{T}}_{\mu\nu} \sigma^{\mu\nu} \\
&= \sigma^{\mu\nu} \overline{\mathcal{T}}_{\mu\nu} \, .
\end{aligned} \tag{3.41}
$$

■

3.2.2 The locality of Bose fields

In the usual theory, the locality [78] [11] of Bose fields needs to be postulated. In the following, however, we shall show that this property is naturally implied by the consistency condition [12]

$$\mathcal{B}(x) = 8\pi^2 \lim_{z \to 0} \partial\!\!\!/^z \{T[\Psi(x+z)\overline{\Psi}(x-z)]\} z\!\!\!/^3 \qquad (\text{ for } z^2 \neq 0) \, , \tag{3.42}$$

[11] Locality of the fields $\phi(x)$ means that the field operators either commute or anticommute with each other at spacelike separated points, $[\,\phi_m(x),\ \phi_n(y)\,]_\pm = 0$ for $(x-y)^2 < 0$. Note that the strict locality of Dirac's field Ψ is evident from our postulates (3.3) and (3.4).

[12] In the following, the limit can be approached on any path which does not touch the light cone $z^2 = 0$, for instance keeping $z_0 = 0$.

which will follow from our basic postulates (3.2) through (3.4). It should be noted that the braces at (3.42) indicate that $\partial\!\!\!/^z := \gamma^\mu \frac{\partial}{\partial z^\mu}$ is to act on Ψ and $\overline{\Psi}$ but not on $z\!\!\!/^3 := (z_0^2 - \vec{z}^2)\gamma_\mu z^\mu$.

Now, let us derive the consistency condition (3.42). First of all, we introduce the time ordered bilocal quantum field $b(x,z)$ [13]

$$b(x,z) := (4\pi)^2 T[\Psi(x+z)\overline{\Psi}(x-z)] , \tag{3.43}$$

which satisfies the equation

$$T\{ \partial\!\!\!/^x + \partial\!\!\!/^z + 2i\,\mathcal{B}(x+z) \} b(x,z) = 2\pi^2 \delta^4(z) . \tag{3.44}$$

<u>Proof</u>

We start from (A.60)

$$T\{ \partial\!\!\!/^u + i\,\mathcal{B}(u) \} \Psi(u)\overline{\Psi}(v) = \delta^4(u-v) . \tag{3.45}$$

Let

$$\begin{aligned} u &:= x+z , \\ v &:= x-z , \end{aligned} \tag{3.46}$$

so

$$T\{ \partial\!\!\!/^{x+z} + i\,\mathcal{B}(x+z) \} \Psi(x+z)\overline{\Psi}(x-z) = \delta^4(2z) . \tag{3.47}$$

[13]Since Ψ is a local field due to (3.3) and (3.4), hence $b(x,z)$ is regarded as a bilocal quantum field.

As

$$\not{\partial}^{x+z}\{\,\Psi(x+z)\overline{\Psi}(x-z)\,\} \;=\; \not{\partial}^{x+z}\{\,\Psi(x+z)\,\}\overline{\Psi}(x-z)$$
$$=\; \frac{1}{2}(\,\not{\partial}^{x}+\not{\partial}^{z}\,)\{\,\Psi(x+z)\overline{\Psi}(x-z)\,\}\;, \qquad (3.48)$$

(3.47) becomes

$$\begin{aligned} T\{\,\not{\partial}^{x}+\not{\partial}^{z}+2i\,\mathcal{B}(x+z)\,\}(4\pi)^2\Psi(x+z)\overline{\Psi}(x-z) &= (4\pi)^2\,(2)\,\delta^4(2z) \\ &= (4\pi)^2\,(2)\,\frac{\delta^4(z)}{2^4} \\ &= 2\pi^2\delta^4(z)\;. \end{aligned} \qquad (3.49)$$

Using the fact that

$$T\,\{\,\mathcal{B}(x+z)\Psi(x+z)\overline{\Psi}(x-z)\,\} = T\,\{\,\mathcal{B}(x+z)\,T[\,\Psi(x+z)\overline{\Psi}(x-z)\,]\,\} \qquad (3.50)$$

and the definition of $b(x,z)$ from (3.43), we reach (3.44) finally.

■

Moreover, by checking the singularity of equation (3.44) for $z \to 0$, we conclude

$$\not{\partial}^{z} b(x,z) \;\xrightarrow{z\to 0}\; 2\pi^2\delta^4(z)\;. \qquad (3.51)$$

Proof

From the discussion in section (A.1), we know that the differential operator $\not{\partial}^{x}+\not{\partial}^{z}$ should act after the time ordering T. Hence

$$\begin{aligned} (\not{\partial}^{x}+\not{\partial}^{z}\,)b^{\zeta\eta}(x,z) &= (\,\not{\partial}^{x}+\not{\partial}^{z})\{\theta(2z_0)\Psi^{\zeta}(x+z)\overline{\Psi}^{\eta}(x-z) \\ &\quad -\theta(-2z_0)\overline{\Psi}^{\eta}(x-z)\Psi^{\zeta}(x+z)\}\;. \end{aligned} \qquad (3.52)$$

As

$$
\begin{aligned}
(\not{\partial}^x + \not{\partial}^z)\{\Psi(x+z)\overline{\Psi}(x-z)\} &= 2\not{\partial}^{x+z}\{\Psi(x+z)\overline{\Psi}(x-z)\}\ , \quad (3.53)\\
(\not{\partial}^x + \not{\partial}^z)\theta(\pm 2z_0) &= \gamma^0\partial_0^z\theta(\pm 2z_0)\ , \quad (3.54)
\end{aligned}
$$

it implies that $(\not{\partial}^x + \not{\partial}^z)b(x,z)$ is more singular than $b(x,z)$ itself.
Also, due to the Dirac equation (3.2)

$$
(i\not{\partial} - \not{B})\Psi = 0\ , \tag{3.55}
$$

we have

$$
i\int \not{B}(x)\Psi(x)\,dx\,f(x) = \gamma^\mu\int\Psi(x)\,dx\,\partial_\mu f(x) \quad \in \mathcal{A}\ . \tag{3.56}
$$

Thus the product $\not{B}(x+z)b(x+z)$ is no more singular than $b(x,z)$. Therefore, for $z\to 0$, the most singular term on left-hand side of (3.44) is $\not{\partial}^z b(x,z)$ while it is $2\pi^2\delta^4(z)$ on right-hand side.

■

However, we have the representation

$$
2\pi^2\delta^4(z) = i\not{\partial}^z \not{z}_-^{-3} \tag{3.57}
$$

with

$$
\begin{aligned}
\not{z}_-^{-3} &:= (z_-^{-2})^2\not{z}\ ,\\
z_-^{-2} &:= (z^2 - i\varepsilon)^{-1} \qquad (\varepsilon\to +0)\ .
\end{aligned}
\tag{3.58}
$$

Proof

As

$$
\begin{aligned}
\not{\partial}^z z_-^{-2} &= -2\gamma^\rho z_-^{-4} z_\rho = -2 \not{z}_-^{-3} \\
\Rightarrow \qquad \not{z}_-^{-3} &= \frac{-1}{2} \not{\partial}^z z_-^{-2} \,.
\end{aligned}
\tag{3.59}
$$

Hence

$$
\begin{aligned}
i\not{\partial}^z \not{z}_-^{-3} &\overset{3.59}{=} \frac{-i}{2} \not{\partial}^z \not{\partial}^z z_-^{-2} \\
&\overset{3.8}{=} \frac{-i}{2} \Box z_-^{-2} \\
&\overset{3.58}{=} \frac{-i}{2} \Box (z^2 - i\varepsilon)^{-1} \qquad (\varepsilon \to +0) \\
&= -i\partial_\mu \{ z^\mu (z^2 - i\varepsilon)^{-2} \} \\
&= 4\varepsilon (z^2 - i\varepsilon)^{-3} \,.
\end{aligned}
\tag{3.60}
$$

However, in Euclidean space, we know the integral

$$
\begin{aligned}
\int \frac{d^4 z_E}{(z_E^2 + \epsilon^2)^3} &= \int_0^{2\pi} d\phi \int_0^\pi sin\theta \, d\theta \int_0^\pi sin\theta' \, d\theta' \int_0^\infty \frac{z_E^3 dz_E}{(z_E^2 + \epsilon^2)^3} \\
&= \frac{\pi^2}{2} \int_0^\infty \frac{z_E^2 d(z_E^2)}{(z_E^2 + \epsilon^2)^3} \\
&= \frac{\pi^2}{2\epsilon^2}
\end{aligned}
\tag{3.61}
$$

or

$$
\frac{\epsilon^2}{(z_E^2 + \epsilon^2)^3} = \frac{\pi^2}{2} \delta^4(z_E) \qquad (\text{ for } \epsilon \to +0) \,. \tag{3.62}
$$

Thus, by using the facts that $(z_E)_0 = iz_0$, $z_E^2 = -z^2$ and the $i\varepsilon$ prescription in

Minkowski space, (3.62) becomes

$$\varepsilon(z^2 - i\varepsilon)^{-3} = \frac{\pi^2}{2}\delta^4(z) \qquad (\text{ for } \varepsilon \to +0)\ . \tag{3.63}$$

Using (3.63) in (3.60), we arrive at (3.57).

■

Note that we have defined $z_-^{-2} := (z^2 - i\varepsilon)^{-1}$ for $\varepsilon \to +0$ in (3.58) and have used it in the above calculation. In the following, the subscript '$-$' for the negative powers of $\not{z}$ will be dropped for brevity's sake, but it is implicit.

Now, with (3.51) and (3.57), we have

$$\not{\partial}^z b(x,z) \xrightarrow{z\to 0} i\not{\partial}^z \not{z}^{-3} \tag{3.64}$$

or

$$b(x,z) = i\not{z}^{-3} + r(x,z) \tag{3.65}$$

with

$$\lim_{z\to 0} \not{z}^3\, r(x,z) = 0\ . \tag{3.66}$$

Hence (3.44) becomes

$$\begin{aligned} T\{\, \not{\partial}^x + \not{\partial}^z + 2i\,\not{B}(x+z)\,\}\{\, i\not{z}^{-3} + r(x,z)\,\} &= 2\pi^2\delta^4(z) & (3.67)\\ \overset{3.57}{\Rightarrow} \qquad \{\, \not{\partial}^z r(x,z) - 2\,\not{B}(x+z)\not{z}^{-3}\,\}\not{z}^3 &= -2i\not{B}(x+z)r(x,z)\not{z}^3 & \\ & & (3.68)\end{aligned}$$

by using the fact that $\not{\partial}^x r(x,z)$ is less singular than $\not{\partial}^z r(x,z)$ and thus $\not{\partial}^x r(x,z)$ can be neglected when $z \to 0$.

Taking the limit on both sides of (3.68), we get

$$\begin{aligned}\lim_{z\to 0}\{\ \not{\partial}^z r(x,z) - 2\,\mathcal{B}(x+z)\,\not{z}^{-3}\,\}\,\not{z}^3 \quad &= \quad -2i\lim_{z\to 0}\mathcal{B}(x+z)r(x,z)\,\not{z}^3 \\ &\overset{3.66}{=} \quad 0\ . \end{aligned} \tag{3.69}$$

Also

$$\begin{aligned}\lim_{z\to 0}\not{\partial}^z(\,i\not{z}^{-3}\,)\,\not{z}^3 \quad &\overset{3.57}{=} \quad 2\pi^2\,\lim_{z\to 0}\delta^4(z)\,\not{z}^3 \\ &= \quad 0\ . \end{aligned} \tag{3.70}$$

Therefore,

$$\begin{aligned}\lim_{z\to 0}\{\ \not{\partial}^z b(x,z)\ \}\,\not{z}^3 \quad &\overset{3.65}{=} \quad \lim_{z\to 0}\{\ \not{\partial}^z r(x,z)\ \}\,\not{z}^3 \\ &\overset{3.69}{=} \quad 2\lim_{z\to 0}\mathcal{B}(x+z)\,\not{z}^{-3}\,\not{z}^3 \\ &= \quad 2\mathcal{B}(x) \end{aligned} \tag{3.71}$$

or

$$\begin{aligned}\mathcal{B}(x) \quad &= \quad \frac{1}{2}\lim_{z\to 0}\{\not{\partial}^z b(x,z)\ \}\,\not{z}^3 \\ &= \quad 8\pi^2\lim_{z\to 0}\not{\partial}^z\{T[\Psi(x+z)\overline{\Psi}(x-z)]\}\,\not{z}^3 \qquad (\text{ for } z^2 \neq 0\ )\ , \end{aligned} \tag{3.72}$$

which is our consistency condition (3.42).

Since in (3.42) the limit $z \to 0$ may be approached on any path which does not touch the cone $z^2 = 0$, we may choose the path $z_0 = 0$. By (3.3), (3.4) and (3.42), the components $\mathcal{B}^P(x)$ of $\mathcal{B}(x)$ then satisfy

$$\begin{array}{lrcll} & [\,\mathcal{B}^P(x),\,\Psi(0)\,]_- & = & 0 & \text{at } x^2<0\,, \qquad (3.73)\\ \text{and} & [\,\mathcal{B}^P(x),\,\mathcal{B}^Q(0)\,]_- & = & 0 & \text{at } x^2<0\,, \qquad (3.74)\end{array}$$

where $\mathcal{B}(x) := \mathbf{b}_P\mathcal{B}^P(x)$ with the constant matrices $\mathbf{b}_P$ which act on the spin, flavor, and color components of Dirac's field Ψ.

In other words, (3.72) proves that all components $\mathcal{B}^P(x)$ of $\mathcal{B}(x)$ show the local commutabilities; but we shall see in section 3.4 that they cannot be canonical. Moreover, due to the same result (3.72), the Bose field $\mathcal{B}(x)$ can be considered less fundamental than Dirac's in quantum induction programme. However, in (3.72) the field $\mathcal{B}$ should not be regarded as creating bound (or confined) fermions [79], since bound states require forces and form factors which do not enter in (3.42).

3.2.3 Absence of Pauli terms

In this section, we are going to show another consequence of quantum induction programme first, namely, $\mathcal{T}_{\mu\nu}=0$ as follows. [80, 81]

<u>Proof</u>

Note that in the following, the Bose field $\mathcal{B}(x)$ is such a non-canonical quantum field that the 'coincident' field products exist.
From (3.65), (3.66), $b(x,z)$ can be written as

$$b(x,z) \;\; := \;\; (4\pi)^2 T[\Psi(x+z)\overline{\Psi}(x-z)]$$

$$= \quad i\not{z}^{-3} \; + \; \{C^{(-2)} + C^{(-1)} + \overset{o}{C} + r^{(1)}\}(x,z) \; , \tag{3.75}$$

where $C^{(-2)}(x,z)$, $C^{(-1)}(x,z)$, $\overset{o}{C}(x,z)$ [14] are Bose fields localized at x and $C^{(-2)}(x,z)$, $C^{(-1)}(x,z)$, $C^{(0)}(x,z)$ can be made homogeneous in the sense of

$$C^{(h)}(x,\alpha z) = \alpha^h \, C^{(h)}(x,z) \qquad (\text{ for } h=-2,-1,0, \;\; \alpha \in \mathcal{C} \;) \; , \tag{3.76}$$

while the 'remainder' $r^{(1)}(x,z)$ is less singular than $C^{(-2)}(x,z)$, $C^{(-1)}(x,z)$, $\overset{o}{C}(x,z)$ and satisfies [15]

$$\lim_{z\to 0} r^{(1)}(x,z) \; = \; 0 \qquad (\text{ on a path with } z^2 \neq 0 \;) \; . \tag{3.77}$$

Moreover, using the Taylor expansion, we have

$$\mathcal{B}(x+z) = \mathcal{B}(x) \; + \; z^\mu \mathcal{B}_{,\mu}(x) \; + \; \frac{1}{2} z^\mu z^\nu \mathcal{B}_{,\mu\nu}(x) \; + \; R(x,z) \; , \tag{3.78}$$

where the remainder $R(x,z)$ of this Taylor polynomial vanishes at $z \; = \; 0$ faster than z^2.

Furthermore, by the definition (A.3) of the time ordered product, (3.44) becomes [16]

---

[14] We shall find shortly that $\overset{o}{C}(x,z)$ actually consists of a homogeneous term $C^{(0)}(x,z)$, a logarithmic term $L(x)ln(i\varepsilon \; - \; M^2 z^2\,)$ and a remainder $Q(x)$.

[15] In the following, we're not going to discuss in detail the $r^{(1)}(x,z)$ term which will be further analysed when we come to gravity [57]. The result (3.77) just comes from the fact that for small $|z^\mu|$, $r^{(1)}(x,z)$ behaves as $\not{z} ln(z^2)$.

[16] Note that in order to prevent the usual 'factor ordering' problems, we postulate that for any 'coincident' field product which exists, the succession of its factors is irrelevant. For instance $\mathcal{B}^{\zeta\eta}(x+z)\Psi^\eta(x+z) \; = \; \Psi^\eta(x+z)\mathcal{B}^{\zeta\eta}(x+z)$.

$$
\begin{aligned}
T\{\, \not\partial^x + \not\partial^z + 2i\mathcal{B}(x+z)\,\} b(x,z) &= 2\pi^2\delta^4(z) \\
&\overset{3.57}{=} i\not\partial^z \not z^{-3} \\
\overset{A.3}{\Rightarrow} \{\, \not\partial^x + \not\partial^z + 2i\mathcal{B}(x+z)\} b(x,z) \;+\; 2i\theta(-z_0)[\mathcal{B}(x+z),\, b(x,z)] &= i\not\partial^z \not z^{-3} \,.
\end{aligned}
\tag{3.79}
$$

Hence, inserting (3.75) and (3.78) into (3.79) and using dimensional analysis, we obtain the recursive set of differential equations for $C^{(-2)}$, $C^{(-1)}$ and $\overset{o}{C}(x,z)$

$$
\begin{aligned}
\not\partial^z C^{(-2)}(x,z) &= 2\mathcal{B}(x)\not z^{-3} \,, &(3.80)\\
\not\partial^z C^{(-1)}(x,z) &= 2z^\mu \mathcal{B}_{,\mu}(x)\not z^{-3} \;-\; \{\, 2i\mathcal{B}(x) \;+\; \not\partial^x \,\} C^{(-2)}(x,z) \,, &(3.81)\\
\not\partial^z \overset{o}{C}(x,z) &= z^\mu z^\nu \mathcal{B}_{,\mu\nu}(x)\not z^{-3} \;-\; 2iz^\mu \mathcal{B}_{,\mu}(x) C^{(-2)}(x,z) \\
&\quad - \{\, 2i\mathcal{B}(x) \;+\; \not\partial^x \,\} C^{(-1)}(x,z) \,, &(3.82)
\end{aligned}
$$

since they are all linearly independent.

Now, in order to solve (3.80), we need to use (3.34), (3.37), and (3.40) which are equivalent with

$$
\begin{aligned}
\mathcal{S}\gamma^\rho &= \gamma^\rho \mathcal{S}^\dagger \,, \\
\mathcal{V}_\mu \gamma^\rho &= \gamma^\rho \overline{\mathcal{V}}_\mu \,, \\
\mathcal{T}_{\mu\nu}\gamma^\rho &= \gamma^\rho \mathcal{T}^\dagger_{\mu\nu} \,.
\end{aligned}
\tag{3.83}
$$

Thus, with the help of (3.83), (3.80) can be written as

$$\begin{aligned}\not\partial^z C^{(-2)}(x,z) &= 2\mathcal{B}(x)\not z^{-3} \\ &\overset{3.25}{=} 2\gamma^\mu \not z^{-3}\overline{\mathcal{V}}_\mu(x) + 2\not z^{-3}\mathcal{S}^\dagger(x) + 2\sigma^{\mu\nu}\not z^{-3}\mathcal{T}^\dagger_{\mu\nu}(x)\ ;\end{aligned} \tag{3.84}$$

and its solution is

$$z^4 C^{(-2)}(x,z) = 2\not z z^\mu \overline{\mathcal{V}}_\mu(x) - z^2\mathcal{S}^\dagger(x) + \not z\sigma^{\mu\nu}\not z\mathcal{T}^\dagger_{\mu\nu}(x)\ . \tag{3.85}$$

When this is substituted into (3.81) with (3.25), we obtain

$$\begin{aligned}\not\partial^z C^{(-1)}(x,z) &= 2z^\mu\gamma^\rho\not z^{-3}(\ \overline{\mathcal{V}}_{\rho,\mu} - \overline{\mathcal{V}}_{\mu,\rho} - 2i\overline{\mathcal{V}}_\rho\overline{\mathcal{V}}_\mu\ ) \\ &+ z^{-2}\gamma^\rho(\ \mathcal{S}^\dagger_{,\rho} + 2i\mathcal{V}_\rho\mathcal{S}^\dagger\ ) + 2\not z^{-3}z^\rho(\ \mathcal{S}^\dagger_{,\rho} - 2i\mathcal{S}^\dagger\overline{\mathcal{V}}_\rho\ ) \\ &+ 2iz^{-2}\mathcal{S}\mathcal{S}^\dagger + 2i\gamma^{\mu\nu}\not z^{-3}\gamma^{\rho\sigma}\not z\mathcal{T}_{\mu\nu}\mathcal{T}_{\rho\sigma} \\ &+ \gamma^\rho\not z^{-3}\gamma^{\mu\nu}\not z(\ 2\mathcal{V}_\rho\mathcal{T}_{\mu\nu} - i\mathcal{T}_{\mu\nu,\rho}\ ) + 2\not z^{-3}\gamma^{\mu\nu}\not z\mathcal{S}\mathcal{T}_{\mu\nu} \\ &+ 2z^\rho\gamma^{\mu\nu}\not z^{-3}(\ 2\mathcal{T}_{\mu\nu}\overline{\mathcal{V}}_\rho + i\mathcal{T}_{\mu\nu,\rho}\ ) - 2z^{-2}\gamma^{\mu\nu}\mathcal{T}_{\mu\nu}\mathcal{S}^\dagger\end{aligned} \tag{3.86}$$

Here and below, we have used

$$\begin{aligned}\not z &:= z_\mu\gamma^\mu \\ \not z^\rho &:= z_\mu\gamma^{\mu\rho} \\ \not z^{\rho\sigma} &:= z_\mu\gamma^{\mu\rho\sigma}\end{aligned} \tag{3.87}$$

with [17]

$$
\begin{aligned}
\gamma^{\mu\rho} &:= \gamma^{[\mu}\gamma^{\rho]} \\
\gamma^{\mu\rho\sigma} &:= \gamma^{[\mu}\gamma^{\rho}\gamma^{\sigma]} \, .
\end{aligned}
\tag{3.88}
$$

Moreover, all the fields and their derivatives are localized at x, but for brevity's sake, the argument x has been suppressed.

With the help of the gauge covariant derivatives [18]

$$
\begin{aligned}
\mathcal{S}_\rho &:= \mathcal{S}_{,\rho} + i[\overline{\mathcal{V}}_\rho \, , \, \mathcal{S}]^- \;=\; \overline{\mathcal{S}}_\rho \\
\mathcal{S}_{\rho\sigma} &:= \mathcal{S}_{,\rho\sigma} + i[\overline{\mathcal{V}}_\sigma \, , \, \mathcal{S}_\rho]^- \;=\; \overline{\mathcal{S}}_{\rho\sigma} \\
\mathcal{V}_{\mu\rho} &:= \mathcal{V}_{\mu,\rho} \;-\; \mathcal{V}_{\rho,\mu} \;+\; i[\mathcal{V}_\rho \, , \, \mathcal{V}_\mu] \;=\; \mathcal{V}^\dagger_{\mu\rho} \\
\mathcal{V}_{\mu\rho\sigma} &:= \mathcal{V}_{\mu\rho,\sigma} \;+\; i[\mathcal{V}_\sigma \, , \, \mathcal{V}_{\mu\rho}] \;=\; \mathcal{V}^\dagger_{\mu\rho\sigma} \\
\mathcal{T}_{\rho\sigma\tau} &:= \mathcal{T}_{\rho\sigma,\tau} \;+\; i[\overline{\mathcal{V}}_\tau \, , \, \mathcal{T}_{\rho\sigma}]^- \, ,
\end{aligned}
\tag{3.89}
$$

it is tedious but straightforward to show that the solution of (3.86) is

$$
C^{(-1)}(x,z) \;=\; z^{-4}\{\, C^{(-1)}_+ \;+\; C^{(-1)}_0 \;+\; C^{(-1)}_- \,\}(x,z) \tag{3.90}
$$

with

$$
C^{(-1)}_+(x,z) \;:=\; z^2 \not{z}^{\rho\sigma}[\mathcal{S} \, , \, \mathcal{T}_{\rho\sigma}]^+ \;+\; 2i \not{z}^3 \mathcal{T}_{\rho\sigma}\mathcal{T}^{\rho\sigma}
$$

[17] The brackets here indicate that we are to average over all permutations of the indices within the brackets, with a plus or minus sign for even or odd permutations, respectively. For instance $\gamma^{[\mu}\gamma^{\rho}\gamma^{\sigma]} := \frac{1}{3!}[\gamma^\mu\gamma^\rho\gamma^\sigma - \gamma^\mu\gamma^\sigma\gamma^\rho - \gamma^\rho\gamma^\mu\gamma^\sigma + \gamma^\sigma\gamma^\mu\gamma^\rho + \gamma^\rho\gamma^\sigma\gamma^\mu - \gamma^\sigma\gamma^\rho\gamma^\mu]$.

[18] In the following, $[A \, , \, B]^\pm := AB \pm B\gamma_0 A\gamma_0$, and $A_{zz\ldots} := z^\rho z^\sigma \ldots A_{\rho\sigma\ldots}$.

$$
\begin{aligned}
& - 6i \not{z}^{\rho\sigma} \mathcal{T}_{z\rho} \mathcal{T}_{z\sigma} + 2iz^2 \gamma^{\rho\sigma\tau} [\mathcal{T}_{\rho\sigma}, \mathcal{T}_{z\tau}] \\
& - i(8 \not{z} \mathcal{T}_{z\mu} \mathcal{T}_z^{\mu} + z^2 \not{z}^{\rho\sigma} \mathcal{T}_{\rho\mu} \mathcal{T}_\sigma^{\mu}) , \qquad (3.91)
\end{aligned}
$$

$$
\begin{aligned}
C_0^{(-1)}(x,z) &:= z^2 (\not{z}^{\mu} \mathcal{S}_\mu + \frac{1}{2} \not{z}^{\rho\sigma} \overline{\mathcal{V}}_{\rho\sigma}) \\
&\quad + 2iz^2 [\mathcal{V}_z , \mathcal{S}^\dagger]^+ + i \not{z}^3 \mathcal{S}\mathcal{S}^\dagger , \qquad (3.92)
\end{aligned}
$$

$$
\begin{aligned}
C_-^{(-1)}(x,z) &:= z^2 \not{z}^{\rho\sigma\tau} [\mathcal{V}_\tau , \mathcal{T}_{\rho\sigma}]^+ + i(z^2 \gamma^{\rho\sigma} - 4 z^\rho \not{z}^\sigma) \mathcal{T}^\dagger_{\rho\sigma z} \\
&\quad + 2z^2 (\gamma^\mu [\mathcal{S} , \mathcal{T}_{\mu z}]^+ + [\mathcal{V}^\mu , \mathcal{T}_{z\mu}]^+) \\
&\quad + iz^2 \gamma^{\rho\sigma\tau} [\mathcal{T}_{\rho\sigma} , \mathcal{T}_{z\tau}]_+ . \qquad (3.93)
\end{aligned}
$$

Using (3.83) and the above solutions $C_\pm^{(-1)}$, $C_0^{(-1)}$, it is easy to obtain the Dirac adjoints

$$
\begin{aligned}
\overline{C_\pm^{(-1)}(x,z)} &= \pm C_\pm^{(-1)}(x,-z) , \\
\overline{C_0^{(-1)}(x,z)} &= C_0^{(-1)}(x,-z) .
\end{aligned} \qquad (3.94)
$$

Moreover, we find

$$
\begin{aligned}
T\bar{b}(x,-z) &= T[\gamma_0 b^\dagger(x,-z)\gamma_0] \\
&= T[\gamma_0 (4\pi)^2 T\{\Psi(x+z)\Psi^\dagger(x-z)\gamma_0\}^\dagger \gamma_0] \\
&= T[(4\pi)^2 T\{\Psi(x-z)\Psi^\dagger(x+z)\}\gamma_0] , \\
&= b(x,z) , \qquad (3.95)
\end{aligned}
$$

where T restores the time ordering which has been inverted by the hermitian conjugation in $\bar{b} = \gamma_0 b^\dagger \gamma_0$.

Then, with the representation of $b(x, z)$ (3.75)

$$b(x, z) = i\not{z}^{-3} + \{C^{(-2)} + C^{(-1)} + r^{(0)}\}(x, z) \tag{3.96}$$

and the reciprocity condition (3.95), we obtain

$$\begin{aligned} \overline{z^4 C^{(h)}(x, z)} &= z^4 C^{(h)}(x, -z) \qquad (\,for\ h = -2, -1\,)\ , \\ \overline{z^4 r^{(0)}(x, z)} &= z^4 r^{(0)}(x, -z)\ , \end{aligned} \tag{3.97}$$

since all the terms in (3.96) are linearly independent.

It follows from (3.90) and (3.94) that

$$\begin{aligned} -2C_-^{(-1)}(x, -z) &= \overline{z^4 C^{-1}(x, z)} - z^4 C^{-1}(x, -z) \\ &\overset{3.97}{=} z^4 C^{-1}(x, -z) - z^4 C^{-1}(x, -z) \\ &= 0\ . \end{aligned} \tag{3.98}$$

Hence,

$$C_-^{(-1)} = 0\ . \tag{3.99}$$

Since all the terms in (3.93) are linearly independent, thus we may conclude that the simplest solution to (3.99) is

$$\mathcal{T}_{\mu\nu} = 0\ . \tag{3.100}$$

■

Hence, we see that 'tensor potentials' $\mathcal{T}_{\mu\nu} \neq 0$ would violate a basic relation of $*$-algebras

$$\{\Psi(f)\Psi^\dagger(g)\}^\dagger = \Psi(g)\Psi^\dagger(f) \ , \tag{3.101}$$

which has been used in the derivation of reciprocity condition (3.95).

Moreover, as a hypothetical $\mathcal{T}_{\mu\nu} \neq 0$ might consist of two independent terms

$$\mathcal{T}_{\mu\nu} = T'_{\mu\nu} + \kappa F_{\mu\nu} \ , \tag{3.102}$$

where $T'_{\mu\nu}$ is a basic tensor field and $\kappa F_{\mu\nu}$, with an arbitrary coefficient κ, forms the 'Pauli Term' $\sigma^{\mu\nu}\kappa F_{\mu\nu}\Psi$ [82] which would make the magnetic moments of quarks or leptons an adjustable parameter, (3.100) implies that not only the 'Pauli Term' must be zero in quantum induction programme, but also no basic tensor field should be observed. In fact, the 'Pauli Term' in quantum electrodynamics was strictly excluded because of its non-renormalizability which was validated by the excellent agreement of quantum electrodynamics with the observed magnetic moment of the electron [83] though it is consistent with all accepted invariance principles including Lorentz, CP and gauge invariances. [19]

[19] However, in recent years, the Standard Model is no longer regarded as a fundamental theory but an effective field theory [84, 85, 86, 87, 88, 89]. In other words, it is just a low energy approximation to some more general theory and the non-renormalizable interactions are thus all suppressed by powers of the underlying fundamental mass scale which, for instance, is the Planck mass scale. Hence, renormalizability is not a fundamental physical requirement anymore [90, 91] and the experimental success in QED merely indicates that nature still hides the expected discrepancy in 'higher decimals'. In this sense, if the "Pauli Term" is found to be nonvanishing, then our Quantum Induction Programme will be totally discarded.

3.2.4 Standard coupling matrices

From last section, we see that the Pauli terms must be absent. Thus (3.25) becomes

$$\mathcal{B}(x) = \mathcal{S}(x) + \gamma^\mu \mathcal{V}_\mu(x) \ . \tag{3.103}$$

Although $\mathcal{S}(x)$ and $\mathcal{V}_\mu(x)$ are 96×96 matrices, only a few of their elements differ from zero in practice. For instance, in the Standard Model with *minimal* Higgs contents, $\mathcal{S}(x)$ contains the $1+3$ Higgs fields

$$\mathcal{S}(x) = \mathbf{h}_0 \mathcal{S}^0(x) + \mathbf{h}_p \mathcal{S}^p(x) \qquad (\ p = 1,2,3 \ ) \ , \tag{3.104}$$

while the $1+3+8$ Yang-Mills fields are contained in

$$\mathcal{V}_\mu(x) = \mathbf{t}_0 \mathcal{V}^0_\mu(x) + \mathbf{t}_p \mathcal{V}^p_\mu(x) + \mathbf{t}_c \mathcal{V}^c_\mu(x) \qquad (\ p = 1,2,3 \ ; \ c = 1,\ldots 8 \ ) \ , \tag{3.105}$$

where $\mathcal{S}^0(x)$, $\mathcal{S}^p(x)$, $\mathcal{V}^0_\mu(x)$, $\mathcal{V}^p_\mu(x)$, $\mathcal{V}^c_\mu(x)$ are single field components and $\mathbf{h}_0$, $\mathbf{h}_p$, $\mathbf{t}_0$, $\mathbf{t}_p$, $\mathbf{t}_c$ are 96×96 matrices which contain coupling constants and generators of Lie groups. [20] In order to facilitate the further discussion, here we try to connect our field components and coupling matrices with the usual notations used in the literature by comparing with the lagrangian of the Minimal Standard model.

[20]Note that the coupling constants and the mass matrix in the 'Standard coupling' matrices are bare ones.

First of all, let us deal with the scalar field $\mathcal{S}(x)$. Due to (3.6) and (3.103), the fermion - scalar interaction can be described by

$$\begin{aligned}\mathcal{L}_y &= -\overline{\Psi}\mathcal{S}\Psi \\ &= -\overline{\Psi}\mathbf{h}_0\Psi\mathcal{S}^0 - \overline{\Psi}\mathbf{h}_p\Psi\mathcal{S}^p \end{aligned} \tag{3.106}$$

with the coupling matrices $\mathbf{h}_0$ and $\mathbf{h}_p$ defined as [21]

$$\mathbf{h}_0 := \frac{\mathbf{m}}{v}, \tag{3.107}$$

$$\mathbf{h}_p := i(\ \gamma_+\tau_p\mathbf{h}_0 - \gamma_-\mathbf{h}_0\tau_p\ ) \qquad (p = 1, 2, 3)\ , \tag{3.108}$$

where the 'mass matrix' $\mathbf{m}$ is a 24×24 diagonal matrix with all the bare quark and lepton masses m_f [22] as the diagonal elements. It acts on the color and flavor spaces only [23] and the Higgs field vacuum value v has the value $v = 246\ GeV$.[24] The Pauli isospin matrices τ_p [25] are defined as usual and the 'chirality projectors' are given by

$$\gamma_\pm := \frac{1}{2}(\ 1 \pm \gamma_5\ )\ . \tag{3.109}$$

Moreover, with the introduction of left-handed and right-handed Dirac fields

[21] The reason why $\mathbf{h}_0$ and $\mathbf{h}_p$ are chosen in this way will be seen shortly.

[22] Here, the generation mixing [92] is neglected for brevity's sake. Moreover, it should be noticed that under the finite 'renormalization' of quantum induction, the usual infinite m_f are well defined and calculable.

[23] Hence, a Dirac unit matrix is implied on the right-handed side of (3.107), i.e. $\mathbf{h}_0 := \frac{\mathbf{m}\otimes 1_{4\times 4}}{v}$. But for brevity's sake, the unit matrix will not be shown in the following.

[24] See eq. (2.68)

[25] See eq. (2.40)

$$\begin{aligned}\Psi_L &:= \gamma_-\Psi \\ \Psi_R &:= \gamma_+\Psi \ ,\end{aligned} \tag{3.110}$$

we have

$$\begin{aligned}\overline{\Psi}_L &= \overline{\Psi}\gamma_+ \\ \overline{\Psi}_R &= \overline{\Psi}\gamma_- \\ \Psi &= \Psi_L + \Psi_R \\ \overline{\Psi}\Psi &= \overline{\Psi}_L\Psi_R + \overline{\Psi}_R\Psi_L\end{aligned} \tag{3.111}$$

by using the relation (3.30) and the properties

$$\gamma_+\gamma_\mu = \gamma_\mu\gamma_- \ , \tag{3.112}$$

$$\gamma_\pm^2 = \gamma_\pm \ , \tag{3.113}$$

$$\gamma_+\gamma_- = 0 \ . \tag{3.114}$$

Hence $\mathcal{L}_y$ becomes [26]

$$\mathcal{L}_y = -\overline{\Psi}_L(\ \mathcal{S}^0 + i\tau_p\mathcal{S}^p\ )\frac{\mathbf{m}}{v}\Psi_R - \overline{\Psi}_R\frac{\mathbf{m}}{v}(\ \mathcal{S}^0 - i\mathcal{S}^p\tau_p\ )\Psi_L \ . \tag{3.115}$$

[26]Note that the mass matrix $\mathbf{m}$ commutes with the unit matrix.

Thus, for any ferimon family $\begin{pmatrix} U \\ D \end{pmatrix}$ [27], $\mathcal{L}_y$ becomes

$$\begin{aligned}
\mathcal{L}_y &= -\begin{pmatrix} \overline{U}_L & \overline{D}_L \end{pmatrix} \begin{pmatrix} \mathcal{S}^0 + i\mathcal{S}^3 & i\mathcal{S}^1 + \mathcal{S}^2 \\ i\mathcal{S}^1 - \mathcal{S}^2 & \mathcal{S}^0 - i\mathcal{S}^3 \end{pmatrix} \begin{pmatrix} \frac{m_U}{v} U_R \\ \frac{m_D}{v} D_R \end{pmatrix} \\
&\quad - \begin{pmatrix} \frac{m_U}{v}\overline{U}_R & \frac{m_D}{v}\overline{D}_R \end{pmatrix} \begin{pmatrix} \mathcal{S}^0 - i\mathcal{S}^3 & -i\mathcal{S}^1 - \mathcal{S}^2 \\ -i\mathcal{S}^1 + \mathcal{S}^2 & \mathcal{S}^0 + i\mathcal{S}^3 \end{pmatrix} \begin{pmatrix} U_L \\ D_L \end{pmatrix} \\
&= -\frac{m_U}{v} \begin{pmatrix} \overline{U}_L & \overline{D}_L \end{pmatrix} \begin{pmatrix} \mathcal{S}^0 + i\mathcal{S}^3 \\ i\mathcal{S}^1 - \mathcal{S}^2 \end{pmatrix} U_R \\
&\quad - \frac{m_U}{v}\overline{U}_R \begin{pmatrix} \mathcal{S}^0 - i\mathcal{S}^3 & -i\mathcal{S}^1 - i\mathcal{S}^2 \end{pmatrix} \begin{pmatrix} U_L \\ D_L \end{pmatrix} \\
&\quad - \frac{m_D}{v} \begin{pmatrix} \overline{U}_L & \overline{D}_L \end{pmatrix} \begin{pmatrix} i\mathcal{S}^1 + \mathcal{S}^2 \\ \mathcal{S}^0 - i\mathcal{S}^3 \end{pmatrix} D_R \\
&\quad - \frac{m_D}{v}\overline{D}_R \begin{pmatrix} -i\mathcal{S}^1 + \mathcal{S}^2 & \mathcal{S}^0 + i\mathcal{S}^3 \end{pmatrix} \begin{pmatrix} U_L \\ D_L \end{pmatrix} .
\end{aligned} \tag{3.116}$$

However, in the electroweak theory, the fermion - scalar interaction is described by the $SU(2) \otimes U(1)$ *Yukawa coupling* [94]

$$\mathcal{L}_y = -G_U\{ (\overline{\chi}_L\tilde{\phi})U_R + \overline{U}_R(\tilde{\phi}^\dagger\chi_L) \} - G_D\{ (\overline{\chi}_L\phi)D_R + \overline{D}_R(\phi^\dagger\chi_L) \} \tag{3.117}$$

with the empirical constants

$$G_U = \frac{m_U}{v} ,$$

[27] Here we use the notation $U := (\nu_e, \nu_\mu, \nu_\tau, u^\alpha, c^\alpha, t^\alpha)$ and $D := (e^-, \mu^-, \tau^-, d^\alpha, s^\alpha, b^\alpha)$ where the α index labels the color degree of freedom. In all cases each component belongs to a different generation. Moreover, in the following, neutrinos need not be massless.

$$G_D = \frac{m_D}{v}, \tag{3.118}$$

where $\chi_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}$ is an $SU(2)$ doublet, U_R and D_R are singlets. Here ϕ is the Higgs complex doublet

$$\phi = \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix}, \tag{3.119}$$

with weak hypercharge $Y(\phi) = 1$. This has been introduced in section (2.2), while the isodoublet

$$\tilde{\phi} := i\tau_2 \phi^* = \begin{pmatrix} \phi^{(0)*} \\ -\phi^{(+)*} \end{pmatrix} \tag{3.120}$$

has $Y(\tilde{\phi}) = -1$.

Comparison with (3.116) and (3.117) shows that the field components $\mathcal{S}^0$, $\mathcal{S}^p$ are related to the Higgs doublet components by [28]

$$\begin{aligned} \mathcal{S}^0 &\leftrightarrow \frac{1}{2}(\phi^{(0)} + \bar{\phi}^{(0)}) = \phi_3 \\ \mathcal{S}^1 &\leftrightarrow \frac{i}{2}(\phi^{(-)} - \phi^{(+)}) = \phi_2 \end{aligned}$$

[28] Note that $\mathcal{S}(x)$ are quantum fields while ϕ are *classical* fields. What we did is just to find out the classical counterpart of the quantum fields $\mathcal{S}(x)$. Moreover, according to (2.36), ϕ_3 is chosen as the physical Higgs component, while ϕ_1, ϕ_2, ϕ_4 are unphysical ones.

$$\mathcal{S}^2 \leftrightarrow \frac{1}{2}(\phi^{(+)} + \phi^{(-)}) = \phi_1$$
$$\mathcal{S}^3 \leftrightarrow \frac{i}{2}(\phi^{(0)} - \bar{\phi}^{(0)}) = -\phi_4 \; . \tag{3.121}$$

Thus, according to (2.48) with (2.36),

$$\begin{aligned} \langle|\mathcal{S}^0(x)|\rangle &= v \\ \langle|\mathcal{S}^p(x)|\rangle &= 0 \end{aligned} \tag{3.122}$$

and hence

$$\langle|\mathcal{S}(x)|\rangle = \mathbf{m} \; . \tag{3.123}$$

In short, if the coupling matrices $\mathbf{h}_0$, $\mathbf{h}_p$ are defined as in (3.107) and (3.108), the field components $\mathcal{S}^0(x)$ and $\mathcal{S}^p(x)$ are related to the physical and unphysical Higgs components respectively.

Before we further identify the coupling matrices $\mathbf{t}_0$, $\mathbf{t}_p$ and $\mathbf{t}_c$, let us first introduce the quark projector P_q and lepton projector P_l

$$P_q := \mathbf{0}_{24\times24} \oplus \mathbf{1}_{72\times72} \; , \tag{3.124}$$
$$P_l := \mathbf{1}_{24\times24} \oplus \mathbf{0}_{72\times72} \; . \tag{3.125}$$

From these there follows

$$P_q^2 = P_q \; , \tag{3.126}$$

$$P_l^2 = P_l \ , \tag{3.127}$$

$$P_l + P_q = \mathbf{1}_{96\times 96} \ , \tag{3.128}$$

$$P_l P_q = 0 \ . \tag{3.129}$$

Now we know that in quantum chromodynamics, the quark-gluon coupling is given by [96]

$$\mathcal{L}_{QCD}^{int} = -g_s \left(\bar{q}\gamma^\mu \frac{\lambda_c}{2} q \right) G_\mu^c \ , \tag{3.130}$$

with the Gell-Mann matrices λ_c and gluon fields G_μ^c.

However, in quantum induction, due to (3.6), (3.103) and (3.105), that interaction is described by

$$-\overline{\Psi} \left(\gamma^\mu \mathbf{t}_c \mathcal{V}_\mu^c \right) \Psi \ . \tag{3.131}$$

Hence, if the $\mathcal{V}_\mu^c$ are defined as

$$\mathcal{V}_\mu^c := G_\mu^c \qquad (c = 1, \ldots 8) \ , \tag{3.132}$$

then by comparing (3.130) with (3.131), we can make the identification

$$\mathbf{t}_c = \frac{1}{2} g_s P_q \lambda_c \ . \tag{3.133}$$

By the same token, if $\mathcal{V}^p_\mu$ are identified as the three $SU(2)_L$ gauge fields

$$\mathcal{V}^p_\mu := W^p_\mu \qquad (p = 1,2,3)\,, \tag{3.134}$$

then by comparing the isodoublet $\chi_L - W$ interaction[95]

$$\begin{aligned} \mathcal{L}_{\chi_L W} &= -g\overline{\chi}_L\,(\frac{\tau_p}{2})\gamma^\mu\chi_L W^p_\mu \\ &= -g\overline{\chi}\gamma_+\,(\frac{\tau_p}{2})\,\gamma^\mu\gamma_-\chi W^p_\mu \\ &\overset{3.112,3.113}{=} -g\overline{\chi}\,(\frac{\tau_p}{2})\,\gamma^\mu\gamma_-\chi W^p_\mu \end{aligned} \tag{3.135}$$

with its counterpart in QI

$$-\overline{\Psi}\,(\,\gamma^\mu \mathbf{t}_p \mathcal{V}^p_\mu\,)\,\Psi\,, \tag{3.136}$$

we have

$$\mathbf{t}_p = \frac{1}{2}g\gamma_-\tau_p\,. \tag{3.137}$$

Finally, with the examination of the fermion - B_μ interactions as contained in the $SU(2)_L \otimes U(1)_Y$ electroweak theory[95]

$$\mathcal{L}_{fB} = -\frac{1}{2}g'\,(\,\overline{\chi}_L\gamma^\mu Y_L B_\mu\chi_L + \overline{R}\gamma^\mu Y_R B_\mu R\,)\,, \tag{3.138}$$

$\mathcal{V}^0_\mu$ can be identified as the $U(1)_Y$ gauge field B_μ

$$\mathcal{V}^0_\mu \;:=\; B_\mu \;, \tag{3.139}$$

if $\mathbf{t}_0$ is written as

$$\mathbf{t}_0 \;=\; \frac{1}{2}g'\,(\,\frac{1}{3}P_q \;-\; P_l \;+\; \gamma_+\otimes\tau_3\,)\;. \tag{3.140}$$

In the following, let us just take the electron multiplet $\chi \;=\; \begin{pmatrix} \nu_e \\ e \end{pmatrix}$ to show that eq.(3.140) is proper.[29]

Proof

As the weak hypercharges Y for e_L, ν_e, e_R, are -1, -1, -2 repectively[95], (3.138) becomes

$$\begin{aligned}
\mathcal{L}_{fB} \;&=\; -\frac{1}{2}g'\,(\,-\overline{\chi}_L\gamma^\mu B_\mu\chi_L \;-\; 2\overline{R}\gamma^\mu B_\mu R\,) \\
&=\; -\frac{1}{2}g'\,\{\,-(\,\bar{\nu}_e\;\bar{e}_L\,)\,\gamma^\mu\begin{pmatrix}\nu_e \\ e_L\end{pmatrix} \;-\; 2\bar{e}_R\gamma^\mu e_R\,\}\,B_\mu \\
&=\; \frac{1}{2}g'\,\{\,\bar{\nu}_e\gamma^\mu\nu_e \;+\; \bar{e}_L\gamma^\mu e_L \;+\; 2\bar{e}_R\gamma^\mu e_R\,\}\,B_\mu\;.
\end{aligned} \tag{3.141}$$

Its counterpart in QI reads

$$-\overline{\Psi}\gamma^\mu\mathbf{t}_0\mathcal{V}^0_\mu\Psi \;=\; -\frac{1}{2}g'\overline{\Psi}\gamma^\mu\,(\,\frac{1}{3}P_q \;-\; P_l \;+\; \gamma_+\otimes\tau_3\,)\,\mathcal{V}^0_\mu\Psi\;. \tag{3.142}$$

[29] Reader may try other doublets to justify eq.(3.140).

Since we are dealing with leptons only, the first term in (3.142) has no contribution, and the second term gives

$$\frac{1}{2}g'\,(\,\bar{\nu}_e\gamma^\mu\nu_e \;+\; \bar{e}\gamma^\mu e\,)\,\mathcal{V}^0_\mu \overset{3.111}{=} \frac{1}{2}g'\,(\,\bar{\nu}_e\gamma^\mu\nu_e \;+\; \bar{e}_L\gamma^\mu e_L \;+\; \bar{e}_R\gamma^\mu e_R\,)\,\mathcal{V}^0_\mu\,, \quad (3.143)$$

while the last term of (3.142) contributes

$$\begin{aligned} -\frac{1}{2}g'\,(\,\bar{\nu}_e\gamma^\mu\gamma_+\nu_e \;-\; \bar{e}\gamma^\mu\gamma_+ e\,)\,\mathcal{V}^0_\mu &= \frac{1}{2}g'\bar{e}\gamma^\mu\gamma_+ e\mathcal{V}^0_\mu \\ &\overset{3.113}{=} \frac{1}{2}g'\bar{e}\gamma^\mu\gamma_+\gamma_+ e\mathcal{V}^0_\mu \\ &\overset{3.112}{=} \frac{1}{2}g'\bar{e}\gamma_-\gamma^\mu\gamma_+ e\mathcal{V}^0_\mu \\ &\overset{3.110}{=} \frac{1}{2}g'\bar{e}_R\gamma^\mu e_R\mathcal{V}^0_\mu\,, \end{aligned} \quad (3.144)$$

which with (3.143) together gives the result (3.141); thus (3.140) is verified.

■

Note that in Standard Model, $\mathcal{V}^0_\mu$ and $\mathcal{V}^3_\mu$ are mixed to form the neutral boson fields [30]

$$\begin{aligned} Z_\mu &:= \cos\theta\,\mathcal{V}^3_\mu \;-\; \sin\theta\,\mathcal{V}^0_\mu \\ A_\mu &:= \sin\theta\,\mathcal{V}^3_\mu \;+\; \cos\theta\,\mathcal{V}^0_\mu\,. \end{aligned} \quad (3.145)$$

[30] See equation (2.60). Note that the Weinberg angle θ is defined by $g\sin\theta \;=\; g'\cos\theta$.

Hence, by defining the charged boson fields [31]

$$
\begin{aligned}
W^+_\mu &:= \frac{1}{\sqrt{2}} (\mathcal{V}^1_\mu - i\mathcal{V}^2_\mu) , \\
W^-_\mu &:= \frac{1}{\sqrt{2}} (\mathcal{V}^1_\mu + i\mathcal{V}^2_\mu) ,
\end{aligned}
\tag{3.146}
$$

and with the introduction of the matrices $\mathbf{t}_A, \mathbf{t}_Z, \mathbf{t}_W, \mathbf{t}^\dagger_W$ as

$$
\begin{aligned}
\mathbf{t}_A &:= \mathbf{t}_0 \cos\theta + \mathbf{t}_3 \sin\theta , \\
\mathbf{t}_Z &:= \mathbf{t}_3 \cos\theta - \mathbf{t}_0 \sin\theta , \\
\mathbf{t}_W &:= \frac{1}{\sqrt{2}} (\mathbf{t}_1 + i\mathbf{t}_2) , \\
\mathbf{t}^\dagger_W &:= \frac{1}{\sqrt{2}} (\mathbf{t}_1 - i\mathbf{t}_2) ,
\end{aligned}
\tag{3.147}
$$

we can write

$$
\mathbf{t}_0 \mathcal{V}^0_\mu + \mathbf{t}_p \mathcal{V}^p_\mu = \mathbf{t}_A A_\mu + \mathbf{t}_Z Z_\mu + \mathbf{t}^\dagger_W W^-_\mu + \mathbf{t}_W W^+_\mu . \tag{3.148}
$$

Moreover, the constant matrices $\mathbf{h}_Y$ and $\mathbf{t}_Y$ are made "orthonormal" in the sense of

[31] See equation (2.59).

$$\begin{aligned} Tr(\mathbf{h}_H^\dagger \mathbf{h}_I) &= 2u\,\delta_{HI} & &(\text{ with } u \;:=\; \frac{1}{2}Tr(\mathbf{h}_0^\dagger \mathbf{h}_0))\ , & (3.149)\\ Tr(\mathbf{t}_Y \mathbf{t}_Z) &= t\,\delta_{YZ} & &(\text{ with } t \;:=\; 12g_i^2)\ , & (3.150)\end{aligned}$$

with [32]

$$\begin{aligned} g_1 &:= \sqrt{\frac{5}{3}}g'\ ,\\ g_2 &:= g\ ,\\ g_3 &:= g_s\ ,\end{aligned} \tag{3.151}$$

and $\mathbf{h}_0$, $\mathbf{h}_p$, $\mathbf{t}_0$, $\mathbf{t}_p$, $\mathbf{t}_c$ are defined as in (3.107), (3.108), (3.140), (3.137), and (3.133) respectively,

$$\begin{aligned} \mathbf{h}_0 &= \frac{\mathbf{m}}{v} = \mathbf{h}_0^\dagger\ , & (3.152)\\ \mathbf{h}_p &= i(\gamma_+\tau_p\mathbf{h}_0 - \gamma_-\mathbf{h}_0\tau_p)\ , & (3.153)\\ \mathbf{t}_0 &= \frac{1}{2}g'(\frac{1}{3}P_q - P_l + \gamma_+\tau_3)\ , & (3.154)\\ \mathbf{t}_p &= \frac{1}{2}g\gamma_-\tau_p\ , & (3.155)\\ \mathbf{t}_c &= \frac{1}{2}g_sP_q\lambda_c\ . & (3.156)\end{aligned}$$

Before the proofs are shown, I should mention some facts about traces in general : if A and B are any matrices,

[32] g', g, g_s are coupling constants for the gauge groups $U(1), SU(2)$ and $SU(3)$ respectively.

$$
\begin{aligned}
Tr\,(A + B) &= Tr\,(A) + Tr\,(B)\,, && (3.157)\\
Tr\,(AB) &= Tr\,(BA)\,, && (3.158)\\
Tr\,(A \otimes B) &= Tr\,(A) \times Tr\,(B)\,. && (3.159)
\end{aligned}
$$

Also, it is useful to recall that the Pauli isospin matrices $(\tau_p)_{2\times 2}$, Gell-Mann matrices $(\lambda_c)_{3\times 3}$ and gamma matrices $(\gamma_\mu)_{4\times 4}$, $(\gamma_\pm)_{4\times 4}$ act on different spaces, namely, isospinor, color and spinor spaces respectively. [33] In addition to this, it is noted that

$$
\begin{aligned}
Tr(\tau_p) &= Tr(\lambda_c) = Tr(\gamma_\mu) = 0\,, && (3.160)\\
Tr(\gamma_\pm) &= 2\,, && (3.161)\\
Tr(\lambda_\alpha \lambda_\beta) &= 2\delta_{\alpha\beta}\,. && (3.162)
\end{aligned}
$$

<u>**Proof**</u>

(i).

$$
\begin{aligned}
Tr(\mathbf{h}_0^\dagger \mathbf{h}_p) &= i\,Tr\{\mathbf{h}_0^\dagger \gamma_+ \tau_p \mathbf{h}_0 - \mathbf{h}_0^\dagger \gamma_- \mathbf{h}_0 \tau_p\}\\
&\overset{3.161}{=} 2i\,Tr\{\mathbf{h}_0^\dagger \tau_p \mathbf{h}_0 - \mathbf{h}_0^\dagger \mathbf{h}_0 \tau_p\}\\
&\overset{3.158}{=} 2i\,Tr\{\mathbf{h}_0^\dagger \tau_p \mathbf{h}_0 - \mathbf{h}_0 \tau_p \mathbf{h}_0^\dagger\}\\
&\overset{3.152}{=} 2i\,Tr\{\mathbf{h}_0^\dagger \tau_p \mathbf{h}_0 - \mathbf{h}_0^\dagger \tau_p \mathbf{h}_0\}\\
&= 0\,. && (3.163)
\end{aligned}
$$

(ii).

$$
Tr(\mathbf{h}_p^\dagger \mathbf{h}_q) = Tr\{(\gamma_+ \tau_p \mathbf{h}_0 - \gamma_- \mathbf{h}_0 \tau_p)^\dagger (\gamma_+ \tau_q \mathbf{h}_0 - \gamma_- \mathbf{h}_0 \tau_q)\}
$$

[33] Hence, direct product is implied for any multiplication among them.

$$\begin{aligned}
&= Tr\{(\mathbf{h}_0^\dagger\tau_p^\dagger\gamma_+^\dagger \,-\, \tau_p^\dagger\mathbf{h}_0^\dagger\gamma_-^\dagger)(\gamma_+\tau_q\mathbf{h}_0 \,-\, \gamma_-\mathbf{h}_0\tau_q)\} \\
&= Tr\{(\mathbf{h}_0^\dagger\tau_p\gamma_+ \,-\, \tau_p\mathbf{h}_0^\dagger\gamma_-)(\gamma_+\tau_q\mathbf{h}_0 \,-\, \gamma_-\mathbf{h}_0\tau_q)\} \\
&\overset{3.114}{=} Tr\{\gamma_+\mathbf{h}_0^\dagger\tau_p\tau_q\mathbf{h}_0 \,+\, \gamma_-\tau_p\mathbf{h}_0^\dagger\mathbf{h}_0\tau_q\} \\
&\overset{3.159,3.161}{=} (2)Tr(\mathbf{h}_0^\dagger\tau_p\tau_q\mathbf{h}_0 \,+\, \tau_p\mathbf{h}_0^\dagger\mathbf{h}_0\tau_q) \\
&= 2\; Tr\{\mathbf{h}_0^\dagger(2)(\delta_{pq}\mathbf{1}_{2\times2} \,+\, i\epsilon_{pqr}\tau_r)\mathbf{h}_0\} \\
&\overset{3.160}{=} 4\; Tr\{\mathbf{h}_0^\dagger\delta_{pq}\mathbf{1}_{2\times2}\mathbf{h}_0\} \\
&= 8\; Tr(\mathbf{h}_0^\dagger\mathbf{h}_0)\delta_{pq} \\
&= Tr(\mathbf{h}_0^\dagger\mathbf{h}_0\otimes\mathbf{1}_{8\times8})\delta_{pq} && (3.164) \\
&= 2u\delta_{pq}\;. && (3.165)
\end{aligned}$$

(iii).

$$\begin{aligned}
Tr(\mathbf{t}_0\mathbf{t}_0) &= \frac{1}{4}g'^2Tr(\,\frac{1}{9}P_q^2 \,-\, \frac{2}{3}P_qP_l \,+\, \frac{2}{3}P_q\gamma_+\tau_3 \,+\, P_l^2 \,-\, 2P_l\gamma_+\tau_3 \,+\, \gamma_+\otimes\mathbf{1}_{24\times24}\,) \\
&\overset{3.129,3.160}{=} \frac{1}{4}g'^2Tr(\,\frac{1}{9}P_q \,+\, P_l \,+\, \gamma_+\otimes\mathbf{1}_{24\times24}\,) \\
&= \frac{1}{4}g'^2\{\frac{1}{9}(72) \,+\, 24 \,+\, (2)(24)\} \\
&= 20g'^2 \\
&= 20(\frac{3}{5})g_1^2 \\
&= 12g_1^2\;. && (3.166)
\end{aligned}$$

(iv).

$$\begin{aligned}
Tr(\mathbf{t}_p\mathbf{t}_q) &= \frac{1}{4}g^2Tr(\,\gamma_-\tau_p\gamma_-\tau_q\,) \\
&= \frac{1}{4}g_2^2Tr(\,\gamma_-\gamma_-\tau_p\tau_q\,) \\
&\overset{3.113}{=} \frac{1}{4}g_2^2Tr\{\,\gamma_-(\delta_{pq} \,+\, i\epsilon_{pqr}\tau_r)\,\} \\
&\overset{3.160}{=} \frac{1}{4}g_2^2Tr(\,\gamma_-\delta_{pq}\,)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{4} g_2^2 Tr(\, \gamma_- \otimes \mathbf{1}_{24\times 24}\,) \delta_{pq} \\
&= \frac{1}{4} g_2^2 (2)(24) \delta_{pq} \\
&= 12 g_2^2 \delta_{pq} \ .
\end{aligned} \tag{3.167}
$$

(v).

$$
\begin{aligned}
Tr(\mathbf{t}_c \mathbf{t}_d) &= \frac{1}{4} g_s^2\, Tr(\, \mathbf{1}_{4\times 4} \otimes P_q^2 \lambda_c \lambda_d\,) \\
&\overset{3.126}{=} \frac{1}{4} g_s^2 (4)\, Tr(\, P_q\, \lambda_c \lambda_d\,) \\
&\overset{3.162}{=} 2 g_s^2 \delta_{cd}\, Tr(P_q) \\
&= 12 g_s^2 \delta_{cd} \\
&= 12 g_3^2 \delta_{cd} \ .
\end{aligned} \tag{3.168}
$$

(vi).

$$
\begin{aligned}
Tr(\mathbf{t}_0 \mathbf{t}_p) &= \frac{1}{4} g' g\, Tr\{\, (\frac{1}{3} P_q \, - \, P_l \, + \, \gamma_+ \tau_3)(\gamma_- \tau_p)\,) \\
&= \frac{1}{4} g' g\, Tr(\, \gamma_+ \tau_3 \gamma_- \tau_p\,\} \\
&= \frac{1}{4} g' g\, Tr(\, \gamma_+ \gamma_- \tau_3 \tau_p\,) \\
&\overset{3.114}{=} 0 \ .
\end{aligned} \tag{3.169}
$$

(vii).

$$
\begin{aligned}
Tr(\mathbf{t}_0 \mathbf{t}_c) &= \frac{1}{4} g' g_s Tr\{\, (\frac{1}{3} P_q \, - \, P_l \, + \, \gamma_+ \tau_3)(P_q \lambda_c)\,\} \\
&\overset{3.160}{=} 0 \ .
\end{aligned} \tag{3.170}
$$

(viii).

$$
Tr(\mathbf{t}_p \mathbf{t}_c) = \frac{1}{4} g g_s Tr(\, \gamma_- \tau_p\, P_q \lambda_c\,)
$$

$$\stackrel{3.160}{=} \quad 0 \; . \tag{3.171}$$

Thus, with the above results (i) to (viii), (3.149) and (3.150) are obtained.

■

3.2.5 The implied gauge invariance

In the usual theory, the gauge invariance of the theory is postulated. Then the gauge transformation rule for the Bose field is derived from that postulate [34]. In contrast to this familiar approach, we find that the gauge invariance of (3.2) through (3.4) and (3.6) is inevitable in Quantum Induction since by using the consistency condition (3.42), the local gauge transformation

$$\begin{aligned} \Psi(x) &\longrightarrow e^{-i\omega(x)}\Psi(x) \\ \overline{\Psi}(x) &\longrightarrow \overline{\Psi}(x)e^{i\overline{\omega}(x)} \end{aligned} \tag{3.172}$$

implies the transformation rule for Bose fields

$$\mathcal{S}(x) \longrightarrow e^{-i\overline{\omega}(x)}\mathcal{S}(x)e^{i\omega(x)} \; , \tag{3.173}$$

$$\mathcal{V}_\mu(x) \longrightarrow e^{-i\omega(x)}\left\{\mathcal{V}(x) - i\partial_\mu\right\}e^{i\omega(x)} \; . \tag{3.174}$$

Here we have used

$$\omega \; := \; \alpha\mathbf{1} + \beta\gamma_5 \; ,$$

[34] See section 2.1.1.

$$\overline{\omega} := \gamma_0 \omega^\dagger \gamma_0 \ , \tag{3.175}$$

where α, β are assumed hermitian and contain no Dirac matrix. [35]

Proof:
Our consistency condition (3.42) can be written

$$\begin{aligned}
\mathcal{B}(x) &= 8\pi^2 \lim_{z\to 0} \not{\partial}^z \{T[\Psi(x+z)\overline{\Psi}(x-z)]\} \not{z}^3 \\
&= 8\pi^2 \gamma^\mu \lim_{z\to 0} T[\Psi(x-z)\overleftrightarrow{\partial}^z_\mu \overline{\Psi}(x+z)] \not{z}^3 \qquad (\text{ for } z^2 \neq 0) \ .
\end{aligned} \tag{3.176}$$

Then under the gauge transformation

$$\begin{aligned}
\Psi(x-z) &\longrightarrow e^{-i\omega(x-z)}\Psi(x-z) \\
\overline{\Psi}(x+z) &\longrightarrow \overline{\Psi}(x+z)e^{i\overline{\omega}(x+z)} \ ,
\end{aligned} \tag{3.177}$$

and noting

$$\gamma^\mu \omega = \overline{\omega}\gamma^\mu \ , \tag{3.178}$$

the Bose field $\mathcal{B}(x)$ transforms as

$$\mathcal{B}(x) \longrightarrow 8\pi^2 \gamma^\mu \lim_{z\to 0}\{e^{-i\omega(x-z)} \ T[\Psi(x-z)\overleftrightarrow{\partial}^z_\mu \overline{\Psi}(x+z)] \ e^{i\overline{\omega}(x+z)}\} \not{z}^3$$

[35] Note that under this assumption, we have $\omega = \omega^\dagger$ and thus conformal transformations are not under our consideration.

$$\overset{3.178}{=} 8\pi^2 \lim_{z\to 0} e^{-i\overline{\omega}(y-z)}\gamma^\mu \{T[\Psi(x-z)\overset{\leftrightarrow}{\partial}{}^x_\mu \overline{\Psi}(x+z)]\} \not z^3 e^{i\omega(y+z)}|_{y=x}$$
$$+ 8\pi^2 \lim_{z\to 0} e^{-i\overline{\omega}(x-z)} (\not\partial^x - \overset{\leftarrow}{\not\partial^x})T[\Psi(y-z)\overline{\Psi}(y+z)] \not z^3 e^{i\omega(x+z)}|_{y=x} . \tag{3.179}$$

From (3.65) and (3.66) we have

$$(4\pi)^2 \lim_{z\to 0}\{T[\Psi(y-z)\overline{\Psi}(y+z)]\}\not z^3 = -i . \tag{3.180}$$

Hence

$$\begin{aligned} \mathcal{B}(x) \quad &\longrightarrow \quad e^{-i\overline{\omega}(y)}\,\mathcal{B}(x)\,e^{i\omega(y)}|_{y=x} - \frac{i}{2}e^{-i\overline{\omega}(x)}\,(\not\partial^x - \overset{\leftarrow}{\not\partial^x})e^{i\omega(x)} \\ &= \quad e^{-i\overline{\omega}(x)}\,\mathcal{B}(x)\,e^{i\omega(x)} + \frac{i}{2}\not\partial^x\,(e^{-i\omega(x)}e^{i\omega(x)}) - ie^{-i\overline{\omega}(x)}\,\not\partial^x e^{i\omega(x)} \\ &= \quad e^{-i\overline{\omega}(x)}\,\{\mathcal{B}(x) - i\not\partial\}e^{i\omega(x)} , \end{aligned} \tag{3.181}$$

from which (3.173) and (3.174) can be extracted.

■

However, one must realize that Quantum Induction provides only a framework, with the coupling matrices $\mathbf{h}_K$, $\mathbf{t}_Y$ to be filled in, for instance, by the Standard Model. In other words, the transformations (3.172) through (3.174) could be represented mathematically by the "maximal" gauge group $\mathcal{U}_F \times \mathcal{U}_F$; but the "physical" gauge group $\mathcal{G}$ will be much smaller, i.e. $U(1) \times SU(2) \times SU(3) \subset \mathcal{G} \subset \mathcal{U}_F \times \mathcal{U}_F$, since only the twelve Yang-Mills fields $\mathcal{V}^Y_\mu$ coupled by $\mathbf{t}_Y$ ((3.154) through (3.156)) have been observed. Hence, instead of having a complete basis of generators for the gauge group $\mathcal{U}_F \times \mathcal{U}_F$, only

$$\omega = \mathbf{t}_Y \omega^Y \tag{3.182}$$

is admitted in the transformations (3.172) through (3.174) where $\mathbf{t}_Y$ are the twelve coupling matrices which generate the gauge group $U(1) \times SU(2) \times SU(3)$. Moreover, when the symmetry breaking of $U(1) \times SU(2)$ caused by

$$\langle | \mathcal{S}(x) | \rangle = \mathbf{h}_0 v \tag{3.183}$$

is considered, further restriction will be imposed on ω.

As we know, the $\mathbf{h}_0$ from (3.107) is not a real diagonal matrix when the generation mixing of Kobayashi and Maskawa [92] is considered. However, this mixing does not mix electric charges or leptons and quarks and their colors. Thus (3.183) *only* commutes with the

$$\mathbf{t}_A \quad := \quad \mathbf{t}_0 \cos\theta \; + \; \mathbf{t}_3 \sin\theta \tag{3.184}$$

from (3.147) and the eight generators $\mathbf{t}_c$ of $SU(3)$ since $\mathbf{t}_A$ and $\mathbf{t}_c$ only distinguish charges and colors respectively. In other words, besides $\mathbf{t}_A$ and $\mathbf{t}_c$, $\mathbf{h}_0$ does not commute with the remaining combinations of $\mathbf{t}_0$, $\mathbf{t}_1$,$\mathbf{t}_2$,$\mathbf{t}_3$. Hence (3.182) does not leave $\mathbf{h}_0$ and thus (3.183) gauge invariant, unless it is restricted to

$$\omega = \mathbf{t}_A \omega^A \; + \; \mathbf{t}_c \omega^c \tag{3.185}$$

3.3 Induced (Quantum) Field Equations

In contrast with the usual theory where quantum field equations are never mentioned, they are induced by our basic postulates (3.2) through (3.4) in Quantum Induction. In the following, let us start to derive them by continuing our investigation on the recursive set of differential equation (3.82)

$$\begin{aligned} \partial\!\!\!/^{z} \overset{o}{C}(x,z) &= z^{\mu} z^{\nu} \mathcal{B}_{,\mu\nu}(x) z\!\!\!/^{-3} - 2iz^{\mu}\mathcal{B}_{,\mu}(x) C^{(-2)}(x,z) \\ &\quad - \{ 2i\mathcal{B}(x) + \partial\!\!\!/^{x} \} C^{(-1)}(x,z) \,. \end{aligned} \qquad (3.186)$$

As we know from section (3.2.4),

$$\mathcal{T}_{\mu\nu} = 0 \,. \qquad (3.187)$$

Hence, our $C^{(-2)}(x,z)$ in (3.85) and $C^{(-1)}(x,z)$ in (3.90) become

$$C^{(-2)}(x,z) = 2 z\!\!\!/^{-3} z^{\mu} \overline{\mathcal{V}}_{\mu}(x) - z^{-2} \mathcal{S}^{\dagger}(x) \,, \qquad (3.188)$$

$$C^{(-1)}(x,z) = z^{-2} (\mathcal{S}^{\dagger}_{z} + \frac{1}{2} z\!\!\!/^{\rho\sigma} \overline{\mathcal{V}}_{\rho\sigma}) + 2iz^{-2} [\mathcal{V}_{z}, \mathcal{S}^{\dagger}]^{+} + i z\!\!\!/^{-1} \mathcal{S}\mathcal{S}^{\dagger} \qquad (3.189)$$

By inserting (3.188), (3.189) in (3.186) and with the help of the gauge covariant derivatives (3.89), we have

$$\begin{aligned} \partial\!\!\!/^{z} \overset{o}{C}(x,z) &= 2iz^{-2} (z^{\rho} \mathcal{S}_{,\rho} \mathcal{S}^{\dagger} - z\!\!\!/^{\rho} \mathcal{S} \mathcal{S}^{\dagger}_{,\rho}) \\ &\quad + (z\!\!\!/^{-3} z^{\rho} z^{\sigma} - z^{-2} z^{\rho} \gamma^{\sigma} + z\!\!\!/^{-1} \eta^{\rho\sigma}) \mathcal{S}^{\dagger}_{,\rho\sigma} \end{aligned}$$

$$
\begin{aligned}
&+ 2\not{z}^{-1}\mathcal{S}^{\dagger}\mathcal{S}\mathcal{S}^{\dagger} + (\gamma^{\rho}\not{z}^{-3}z^{\mu}z^{\nu} + z^{-2}\gamma^{\mu}\not{z}^{\nu\rho})\overline{\mathcal{V}}_{\rho,\mu\nu} \\
&+ 2z^{-2}\gamma^{\mu}\not{z}^{\rho\sigma}\overline{\mathcal{V}}_{\mu}\overline{\mathcal{V}}_{\sigma}\overline{\mathcal{V}}_{\rho} \\
&+ 2i\,(z^{-2}\gamma^{\mu}\not{z}^{\rho\sigma} - 2\gamma^{\rho}\not{z}^{-3}z^{\mu}z^{\sigma})\,\overline{\mathcal{V}}_{\rho,\mu}\overline{\mathcal{V}}_{\sigma} \\
&+ 2iz^{-2}\,(\gamma^{\mu}\not{z}^{\rho\sigma} + \gamma^{\rho}z^{\mu\sigma})\,\overline{\mathcal{V}}_{\mu}\overline{\mathcal{V}}_{\sigma,\rho} \\
&+ 2z^{-2}\,(2z^{\rho} - \not{z}^{\rho})\,\mathcal{S}\mathcal{S}^{\dagger}\overline{\mathcal{V}}_{\rho} \\
&+ 2z^{-2}\,(2z^{\rho} + \not{z}^{\rho})\,\mathcal{S}\mathcal{V}_{\mu}\mathcal{S}^{\dagger} + 2\gamma^{\rho}\not{z}^{-1}\overline{\mathcal{V}}_{\rho}\mathcal{S}\mathcal{S}^{\dagger} \\
&+ 2z^{-2}\gamma^{\mu}\,(2z^{\rho} + \not{z}^{\rho})\,\mathcal{V}_{\mu}\mathcal{V}_{\rho}\mathcal{S}^{\dagger} \\
&+ 2z^{-2}\gamma^{\mu}\,(2z^{\rho} - \not{z}^{\rho})\,\mathcal{V}_{\mu}\mathcal{S}^{\dagger}\overline{\mathcal{V}}_{\rho} + 2z^{-2}\not{z}^{\rho\sigma}\mathcal{S}\overline{\mathcal{V}}_{\sigma}\overline{\mathcal{V}}_{\rho} \\
&+ iz^{-2}\,(2\gamma^{\rho}z^{\mu} - 2\gamma^{\mu}z^{\rho} - \gamma^{\mu}\not{z}^{\rho})\,\mathcal{V}_{\rho,\mu}\mathcal{S}^{\dagger} \\
&+ iz^{-2}\,(\gamma^{\mu}\not{z}^{\rho} - 2\gamma^{\mu}z^{\rho} + 2\not{z}^{\mu\rho})\,\mathcal{S}^{\dagger}\overline{\mathcal{V}}_{\rho,\mu} \\
&+ iz^{-2}\,(\gamma^{\mu}\not{z}^{\rho} - 2\gamma^{\mu}z^{\rho} - 4\not{z}^{-1}z^{\mu}z^{\rho})\,\mathcal{S}^{\dagger}_{,\mu}\overline{\mathcal{V}}_{\rho} \\
&- iz^{-2}\gamma^{\mu}\,(2z^{\rho} + \not{z}^{\rho})\,\mathcal{V}_{\rho}\mathcal{S}^{\dagger}_{,\mu}\,. \qquad (3.190)
\end{aligned}
$$

Its solution consists of a homogeneous term $C^{(0)}(x,z)$, a logarithmic term and a remainder $Q(x)$ [36]

$$\overset{o}{C}(x,z) = C^{(0)}(x,z) + L(x)\,ln(i\varepsilon - M^2z^2) + Q(x)\ , \qquad (3.191)$$

with $\varepsilon \to +0$ and M =const., where [37]

$$
\begin{aligned}
C^{(0)}(x,z) = {} & \not{z}^{-1}\,(\frac{1}{3}\mathcal{V}^{\mu}_{z\mu} + [\,\mathcal{V}_z, \mathcal{S}^{\dagger}\mathcal{S}\,]_{+}) \\
& + \not{z}^{-3}\,(\frac{1}{3})\,(\overline{\mathcal{V}}_{z,zz} - 4\overline{\mathcal{V}}_z\overline{\mathcal{V}}_z\overline{\mathcal{V}}_z + 2i\,[\,\overline{\mathcal{V}}_z\,,\,\overline{\mathcal{V}}_{z,z}\,])
\end{aligned}
$$

[36] In order to avoid the lengthy presentation of the derivation of the solution, only the results are quoted here. In fact, their verification merely requires to perform differentiations.

[37] In the following, $\mathcal{S}^{\mu}_{\mu}$ and $\mathcal{V}^{\mu}_{\rho\mu}$ stand for the *contractions* of the gauge covariant derivatives of second order. For instance, $\mathcal{S}^{\mu}_{\mu} := \eta^{\alpha\beta}\ \mathcal{S}_{\alpha\beta}$.

$$
\begin{aligned}
&+ z^{-2}\,(\frac{1}{3}\gamma^\rho \overline{\mathcal{V}}_{\rho zz} + \frac{i}{2}\not{z}^{\rho\sigma}\,[\overline{\mathcal{V}}_z\,,\,\overline{\mathcal{V}}_{\sigma\rho}\,]_+\,) \\
&+ z^{-2}\,(\frac{i}{2}\,[\,\mathcal{V}_{z,z}\,,\,\mathcal{S}^\dagger\,]^- + \frac{1}{2}\,[\mathcal{V}_z\mathcal{V}_z\,,\,\mathcal{S}^\dagger\,]^+ - i\not{z}^\rho\,[\,\mathcal{V}_z,\mathcal{S}^\dagger_\rho\,]^+ \\
&- \frac{1}{2}\mathcal{S}^\dagger_{zz} + \mathcal{V}_z\mathcal{S}^\dagger\overline{\mathcal{V}}_z\,)\;, \qquad (3.192)
\end{aligned}
$$

$$
L(x) = \frac{1}{2}\mathcal{F}^\dagger(x) - \frac{1}{3}\overline{\mathcal{F}}_\mu(x)\gamma^\mu + \frac{1}{4}[\,\overline{\mathcal{V}}_{\nu\mu},\,\mathcal{S}^\dagger\,]^\dagger(x)\sigma^{\mu\nu}\;, \qquad (3.193)
$$

with

$$
\begin{aligned}
\mathcal{F}(x) &:= (\,\mathcal{S}^\mu_\mu + 2\mathcal{S}\mathcal{S}^\dagger\mathcal{S}\,)(x)\;, && (3.194)\\
\mathcal{F}_\mu(x) &:= \mathcal{V}^\rho_{\mu\rho}(x) + \frac{3i}{2}(\,\mathcal{S}^\dagger_\mu\mathcal{S} - \mathcal{S}^\dagger\mathcal{S}_\mu\,)(x)\;. && (3.195)
\end{aligned}
$$

As for the remainder $Q(x)$, in the following we will see that it can be dug out of the short distance representation of the bilocal field $b(x,z)$ in another way, namely, using the familiar averaging procedure from perturbation theory [97] to simplify the directional-dependent terms.

Now, let us denote the Lorentz invariant average of any function $f(z)$ over the *directions* of z by $< f(z) >_{z^2}$. The following results which we need shortly are stated here first.

$$
\begin{aligned}
< z^\mu z^\nu >_{z^2} &= \frac{1}{4}\eta^{\mu\nu}z^2\;, && (3.196)\\
< z^\alpha z^\beta z^\rho z^\sigma >_{z^2} &= \frac{1}{24}\,(\,\eta^{\alpha\beta}\eta^{\rho\sigma} + \eta^{\alpha\beta}\eta^{\beta\sigma} + \eta^{\alpha\sigma}\eta^{\beta\rho}\,)\,z^4\;, && (3.197)\\
< g(z^2) >_{z^2} &= g(z^2)\;, && (3.198)\\
< h(z) >_{z^2} &= 0 \qquad (\,\text{for } h(z) = -h(-z)\,)\;. && (3.199)
\end{aligned}
$$

<u>**Proof**</u>

(i).

Since the only Lorentz invariant tensor of valence 2 is $\eta^{\mu\nu}$, hence

$$\begin{aligned}
< z^\mu z^\nu >_{z^2} &= \eta^{\mu\nu} f(z^2) \,, && (3.200)\\
\text{but} \qquad z^2 &= < z^2 >_{z^2} \\
&= \eta_{\mu\nu} < z^\mu z^\nu >_{z^2} \\
&\overset{3.200}{=} \eta_{\mu\nu}\eta^{\mu\nu} f(z^2) \\
&= 4 f(z^2) \,, \\
\text{or} \qquad f(z^2) &= \frac{1}{4} z^2 \,. && (3.201)
\end{aligned}$$

(ii).

Also

$$\begin{aligned}
< z^\alpha z^\beta z^\rho z^\sigma >_{z^2} &= (\eta^{\alpha\beta}\eta^{\rho\sigma} + \eta^{\alpha\rho}\eta^{\beta\sigma} + \eta^{\alpha\sigma}\eta^{\beta\rho}) f(z^4) \,, && (3.202)\\
\text{but} \qquad z^4 &= \eta_{\alpha\beta}\eta_{\rho\sigma} < z^\alpha z^\beta z^\rho z^\sigma >_{z^2} \\
&\overset{3.202}{=} \eta_{\alpha\beta}\eta_{\rho\sigma}(\eta^{\alpha\beta}\eta^{\rho\sigma} + \eta^{\alpha\rho}\eta^{\beta\sigma} + \eta^{\alpha\sigma}\eta^{\beta\rho}) f(z^4) \\
&= \{ (4)(4) + 4 + 4 \} f(z^4) \\
&= 24 f(z^4) \,, \\
\text{or} \qquad f(z^4) &= \frac{1}{24} z^4 \,. && (3.203)
\end{aligned}$$

■

From our previous discussion of the short distance representation of the bilocal field $b(x, z)$ (3.75) with (3.191), we know that

$$\begin{aligned}
Q(x) &= (4\pi)^2 T[\Psi(x+z)\overline{\Psi}(x-z)] - i\not{z}^{-3} - C^{(-2)}(x,z) - C^{(-1)}(x,z) \\
&\quad - C^{(0)}(x,z) - r^{(1)}(x,z) - L(x)\, ln\,(i\varepsilon - M^2 z^2) \,. \qquad (3.204)
\end{aligned}$$

As $r^{(1)}(x,z)$ vanishes for $z \to 0$ in the sense of (3.77), so after using the averaging procedure to remove the directional-dependent term, we have

$$\begin{aligned} Q(x) \;=\; & \lim_{z^2 \to 0} < (4\pi)^2 T[\Psi(x+z)\overline{\Psi}(x-z)] \;-\; C^{(-2)}(x,z) \;-\; C^{(0)}(x,z) \\ & \quad - \; L(x)\, ln\,(i\varepsilon - M^2 z^2) >_{z^2} . \end{aligned} \tag{3.205}$$

Using the fact that

$$\begin{aligned} 2Tr\{\mathbf{h}_K L(x)\} & \stackrel{3.193}{=} Tr\{\mathbf{h}_K \mathcal{F}^{\dagger}(x)\} \\ & = Tr\{\mathbf{h}_K \mathbf{h}_L^{\dagger} \mathcal{F}^L(x)\} \\ & \stackrel{3.149}{=} 2u\mathcal{F}^K(x) \ , \end{aligned} \tag{3.206}$$

$$\begin{aligned} -3Tr\{\gamma_\mu \mathbf{t}_Y L(x)\} & \stackrel{3.193}{=} Tr\{\mathbf{t}_Y \mathcal{F}_\mu(x)\} \\ & = Tr\{\mathbf{t}_Y \mathbf{t}_Z \mathcal{F}_\mu^Z(x)\} \\ & \stackrel{3.150}{=} t\mathcal{F}_\mu^Y(x) \\ & \stackrel{5.11}{=} 3u\mathcal{F}_\mu^Y(x) \ , \end{aligned} \tag{3.207}$$

and the notations

$$C^K(x,z^2) \;:=\; Tr\{\mathbf{h}_K < (C^{(-2)} + C^{(0)})(x,z) >_{z^2}\} \ , \tag{3.208}$$

$$C_\mu^Y(x,z^2) \;:=\; Tr\{\gamma_\mu \mathbf{t}_Y < (C^{(-2)} + C^{(0)})(x,z) >_{z^2}\} \ , \tag{3.209}$$

$$\kappa \;:=\; (4\pi)^{-2} \ , \tag{3.210}$$

$$\mu \;:=\; e^{\frac{-8\pi^2}{u}} M \ , \tag{3.211}$$

equation (3.205) implies that

$$\mathcal{F}^K + j^K = Q^K , \tag{3.212}$$

$$\mathcal{F}^Y_\mu - j^Y_\mu = Q^Y_\mu , \tag{3.213}$$

which are known as "Dirac induced" (quantum) field equations because they have followed from our basic postulates (3.2) through (3.4). [38] Here the scalar, vectorial "Dirac sources" are defined as

$$\begin{aligned} j^K(x) &:= \lim_{z^2 \to 0} < T[\overline{\Psi}(x-z)\mathbf{h}_K \Psi(x+z)] + \kappa C^K(x,z^2) \\ &\qquad + \kappa u \mathcal{F}^K(x)\, ln\,(i\varepsilon - \mu^2 z^2) >_{z^2} , \end{aligned} \tag{3.214}$$

$$\begin{aligned} j^Y_\mu(x) &:= \lim_{z^2 \to 0} < T[\overline{\Psi}(x-z)\gamma_\mu \mathbf{t}_Y \Psi(x+z)] + \kappa C^Y_\mu(x,z^2) \\ &\qquad - \kappa u \mathcal{F}^Y_\mu(x)\, ln\,(i\varepsilon - \mu^2 z^2) >_{z^2} , \end{aligned} \tag{3.215}$$

while the "purely bosonic" parts of the induced field equations $\mathcal{F}^K(x)$, $\mathcal{F}^Y_\mu(x)$ are defined as in equations (3.194) and (3.195)

$$\mathcal{F}^K(x) := \frac{1}{2u} Tr\{\, \mathbf{h}^\dagger_K \mathcal{F}(x)\,\}$$

[38]Note that $\mathcal{F}(x)$, $\mathcal{F}_\mu(x)$ contain $\mathcal{S}$ and $\mathcal{V}_\mu$ respectively. In turn, $\mathcal{S}$ contains all the (scalar and pseudoscalar) Higgs fields while $\mathcal{V}_\mu$ involves all (vectorial and axial) Yang-Mills fields. Hence, the induced quantum field equations (3.212) and (3.213) resemble those which would hold for such fields in other versions of quantum field theory if they were not basically renormalized. However, it should be noted that though $j^K(x)$ and $j^Y_\mu(x)$ resemble the 'currents' one obtains by usual 'point splitting' method [98], in contrast with theirs, ours (i.e. eqs. (3.214) and (3.215)) are determined uniquely. Moreover, our Dirac sources are convergent since the divergent second and third terms in Dirac sources are cancelled with their counterparts in the first term.

$$
\begin{aligned}
&= \frac{1}{2u} Tr\{\, \mathbf{h}_K [\, \mathcal{S}^{\mu}_{\mu} \;+\; 2\mathcal{S}\mathcal{S}^{\dagger}\mathcal{S}\,](x)\,\}\;, && (3.216)\\
\mathcal{F}^{Y}_{\mu}(x) \;&:=\; \frac{1}{3u} Tr\{\, \mathbf{t}_Y \mathcal{F}_{\mu}(x)\,\}\\
&= \frac{1}{3u} Tr\{\, \mathbf{t}_Y [\mathcal{V}^{\rho}_{\mu\rho}(x) \;+\; \frac{3i}{2}(\,\mathcal{S}^{\dagger}_{\mu}\mathcal{S} \;-\; \mathcal{S}^{\dagger}\mathcal{S}_{\mu}\,)(x)]\,\}\;, && (3.217)
\end{aligned}
$$

and the remainders are

$$
\begin{aligned}
Q^{K}(x) \;&:=\; -\kappa\, Tr\{\mathbf{h}_K Q(x)\}\\
&= \lim_{z^2\to 0} \kappa < T[\overline{\Psi}(x-z)\mathbf{h}_K\Psi(x+z)] \;+\; C^{K}(x,z^2)\\
&\qquad + u\mathcal{F}^{K}(x)\, ln\,(\, i\varepsilon \;-\; M^2 z^2\,) >_{z^2}\;,
\end{aligned}
\tag{3.218}
$$

$$
\begin{aligned}
Q^{Y}_{\mu}(x) \;&:=\; \kappa\, Tr\{\gamma_{\mu}\mathbf{t}_Y Q(x)\}\\
&= \lim_{z^2\to 0} -\kappa < T[\overline{\Psi}(x-z)\gamma_{\mu}\mathbf{t}_Y\Psi(x+z)] \;+\; C^{Y}_{\mu}(x,z^2)\\
&\qquad - u\mathcal{F}^{Y}_{\mu}(x)\, ln\,(\, i\varepsilon \;-\; M^2 z^2\,) >_{z^2}\;.
\end{aligned}
\tag{3.219}
$$

More discussions about the $Q^{K}(x)$, $Q^{Y}_{\mu}(x)$ and the "universal" mass M will be found in Appendix C.

3.4 The Bare Coupling Constants

In this section, we shall see that in order to keep our postulates possible, our Bose propagators should fade away faster than the usual canonical ones at asymptotics energies. In other words, our Bose propagators should behave non-canonical. Moreover, with the fact that at asymptotic energies, the complete vertex functions in Quantum Induction approach the bare ones, our couplings approach bare constants.

3.4.1 Non-canonical Bose propagators

In previous sections, we have seen that under the postulates (3.2) through (3.4), the implied recursive set of differential equations (3.80) through (3.82) is solved and $L(x)$ (3.193) is obtained. However, inspecting carefully the right-hand side of $L(x)$, we find that it contains the 'coincident' field products (as operator valued distributions) [39]

$$\begin{aligned} \mathcal{B}^P(x)\mathcal{B}^Q(x) &\in \mathcal{A}' \\ \mathcal{B}^P_{,\mu}(x)\mathcal{B}^Q(x) &\in \mathcal{A}' \,. \end{aligned} \tag{3.220}$$

Moreover, if the discussion is extended to gravity [57], it is found that the energy tensor involves the term $\mathcal{B}^P_{,\mu\nu}(x)\mathcal{B}^Q(x)$

$$\text{i.e.} \qquad \mathcal{B}^P_{,\mu\nu}(x)\mathcal{B}^Q(x) \in \mathcal{A}' \,. \tag{3.221}$$

[39] $\mathcal{A}'$ denotes the space of operator valued distributions, see section (3.1). Also note that $\mathcal{B}^P(x)$ is defined by $\mathbf{b}_P\mathcal{B}^P(x) = \mathcal{B}(x)$.

Since $L(x)$ must exist in order not to contradict our basic postulates (3.80) to (3.82), so the existence of these 'coincident' field products must be demanded as a secondary postulate which then in turn restricts the behavior of the *interacting* Bose propagators at extremely high energies, namely, they decrease *faster* than usually for $|k^2| \to \infty$. Now, let us first introduce the *interacting* scalar propagator [40]

$$\tilde{F}_s^{KL}(k^2) \;:=\; -i\int e^{ikx}\,dx\,\langle\,|T[\,H^K(x)H^L(0)\,]\,|\,\rangle \tag{3.222}$$

$$= \int_0^\infty \frac{d\zeta\rho_s^{KL}(\zeta)}{k^2 + i\varepsilon - \zeta}\,, \tag{3.223}$$

and the *interacting* vector propagator

$$\tilde{F}_{\mu\nu}^{YZ}(k) \;:=\; -i\int e^{ikx}\,dx\,\langle\,|T[\,\mathcal{V}_\mu^Y(x)\mathcal{V}_\nu^Z(0)\,]\,|\,\rangle\,. \tag{3.224}$$

in momentum space.

The "complete" *interacting* Dirac propagator in momentum space is defined as

$$\tilde{F}(\not{p}) \;:=\; -i\int e^{ipx}\,dx\langle|T[\,\Psi(x)\overline{\Psi}(0)\,]\,|\,\rangle \tag{3.225}$$

$$= \int_0^\infty \frac{\not{p}\rho_v(\eta) + \rho_s(\eta)}{p^2 + i\varepsilon - \eta}d\eta\,, \tag{3.226}$$

where ρ_s and ρ_v are the scalar and vectorial part of the spectral density distribution.

[40]In the following, the scalar field $H(x)$ is defined by $H(x) := \mathcal{S}(x) - \langle|\mathcal{S}(x)|\rangle = \mathcal{S}(x) - \mathbf{m}$, and thus $\langle|H(x)|\rangle = 0$. Moreover, using the constant standard coupling matrices $\mathbf{h}_K$ and the condition (3.149) to extract the *components* of $H(x)$, $H^K(x) := (2u)^{-1}Tr\{\,\mathbf{h}_K^\dagger H(x)\,\} = H^{K\dagger}(x)$, we obtain $H(x) = \mathbf{h}_K H^K(x)$.

[41] Furthermore, from the analysis shown in [103, 56], we know that the propagators of vector fields $\tilde{F}^{YZ}_{\mu\nu}(k)$ possess transversality. [42] Hence (3.224) can be rewritten as

$$\tilde{F}^{YZ}_{\mu\nu}(k) \;=\; (\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2 + i\varepsilon}) \int_0^\infty \frac{d\zeta \rho^{YZ}_v(\zeta)}{k^2 + i\varepsilon - \zeta} \qquad (\text{where } \varepsilon \to +0)\ , \quad (3.227)$$

which has the same dimension as the propagators of scalar fields (3.223). Thus, we will restrict our following discussion about Bose propagators to the scalar case only.

In order to simplify the notation in the following, let us just consider a typical component of $H(x)$, such as the "physical" Higgs field $H^0 = H^{0\dagger}$. Its *interacting* propagator is

$$\begin{aligned} \tilde{F}_s(k^2) \;&:=\; -i \int e^{ikx}\, dx \langle | T[\, H^0(x) H^0(0)\,] | \rangle && (3.228) \\ &=\; \int_0^\infty \frac{d\zeta \rho_s(\zeta)}{k^2 + i\varepsilon - \zeta}\ . && (3.229) \end{aligned}$$

Taking the Fourier transform of (3.228), we have

$$\begin{aligned} i \int e^{-ikx} \tilde{F}_s(k^2)\, dk \;&=\; (2\pi)^4\ \langle | T[\, H^0(x) H^0(0)\,] | \rangle \\ \Rightarrow \qquad i \int \tilde{F}_s(k^2)\, dk \;&=\; (2\pi)^4\ \langle | T[\, H^0(0) H^0(0)\,] | \rangle\ . && (3.230) \end{aligned}$$

Due to (3.220), (3.230) must be finite. So, it means

$$k^4 \tilde{F}_s(k^2) \;\longrightarrow\; 0 \qquad \text{for } k \to \infty\ . \qquad (3.231)$$

[41] For a simple introduction of the above Lehmann–Källén spectral representation [99, 100] in the interacting theory, refs. [101, 102] may be consulted.

[42] Here we mean that $\tilde{F}^{YZ}_{\mu\nu}(k)$ is transverse at μ, i.e. $k^\mu \tilde{F}^{YZ}_{\mu\nu}(k) = 0$, (and thus also in ν).

Moreover, if condition (3.221) is taken into consideration, proceeding in a similar manner, we further obtain

$$
\begin{aligned}
& (2\pi)^4 \,\langle |T[\,H^0(x)H^0(0)\,]|\rangle &=& \; i\int e^{-ikx}\tilde{F}_s(k^2)\,dk \\
\Rightarrow \quad & (2\pi)^4 \,\langle |T[\,\Box H^0(x)H^0(0)\,]|\rangle &=& \; -i\int e^{-ikx}k^2\tilde{F}_s(k^2)\,dk \\
\Rightarrow \quad & (2\pi)^4 \,\langle |T[\,\Box H^0(0)H^0(0)\,]|\rangle &=& \; -i\int k^2\tilde{F}_s(k^2)\,dk \qquad \neq \infty\,. \qquad (3.232)
\end{aligned}
$$

Hence we may conclude

$$k^6\tilde{F}_s(k^2) \longrightarrow 0 \qquad\qquad \text{for } k \to \infty\,. \qquad (3.233)$$

In other words, the *existence* of such 'coincident field products' requires the *non-canonical* Bose propagators to fade away faster than the usual canonical ones for $k \to \infty$. Hence their spectral density distributions must fulfill 'moment conditions' [43]

$$\int_0^\infty \rho_s(\zeta)\,d\zeta = 0\,, \qquad (3.234)$$

$$\int_0^\infty \zeta\rho_s(\zeta)\,d\zeta = 0\,, \qquad (3.235)$$

$$\int_0^\infty \zeta^2\rho_s(\zeta)\,d\zeta = 0\,. \qquad (3.236)$$

<u>**Proof**</u>

(i).

[43] Note that these 'moment conditions' hold for the spectral density distributions $\rho_v^{YZ}(\zeta)$ of vector propagators as well.

(3.231) and (3.229) give

$$\begin{aligned} & \lim_{k\to\infty}\int_0^\infty \frac{k^4}{k^2+i\varepsilon-\zeta}\rho_s(\zeta)d\zeta &=& \ 0 \\ \Rightarrow\quad & \lim_{k\to\infty}\int_0^\infty \frac{k^2(k^2-\zeta)+k^2\zeta}{k^2+i\varepsilon-\zeta}\rho_s(\zeta)d\zeta &=& \ 0 \\ \Rightarrow\quad & \lim_{k\to\infty} k^2\int_0^\infty \rho_s(\zeta)d\zeta + \int_0^\infty \zeta\rho_s(\zeta)d\zeta &=& \ 0\,. \end{aligned} \tag{3.237}$$

Therefore, (3.234) and (3.235) are obtained.

(ii).
Similarly, (3.233) and (3.229) give

$$\begin{aligned} & \lim_{k\to\infty}\int_0^\infty \{ k^2(k^2+i\varepsilon-\zeta)^{-1}\zeta^2 + k^2\zeta + k^2\}\,\rho_s(\zeta)\,d\zeta &=& \ 0 \\ \overset{3.234,3.235}{\Longrightarrow}\quad & \lim_{k\to\infty}\int_0^\infty k^2(k^2+i\varepsilon-\zeta)^{-1}\zeta^2\rho_s(\zeta)\,d\zeta &=& \ 0 \\ \Longrightarrow\quad & \int_0^\infty \zeta^2\rho_s(\zeta)d\zeta &=& \ 0 \end{aligned} \tag{3.238}$$

■

For completeness, let us also discuss the *interacting* Dirac propagator (3.226) whose spectral density distributions fulfill the following 'moment conditions'.

$$\int \rho_s(\eta)d\eta = \mathbf{m}^\dagger = v\mathbf{h}_0\,, \tag{3.239}$$

$$\int \rho_v(\eta)d\eta = 1\,, \tag{3.240}$$

$$\int \eta\rho_v(\eta)d\eta \approx v^2\mathbf{h}_0^2\,. \tag{3.241}$$

Proof

(3.204) suggests that

$$
\begin{aligned}
\lim_{z\to 0} - z\,\langle\,|\{C^{(-2)} + C^{(-1)}\}(x,z)|\,\rangle \quad &= \quad \lim_{z\to 0}\{\, i\not{z}^{-2} - (4\pi)^2 z\,\langle\,|\,T\,[\,\Psi(x+z)\overline{\Psi}(x-z)]|\,\rangle\,\} \\
&\overset{3.225}{=} \quad \lim_{z\to 0}\{\, i\not{z}^{-2} - \frac{iz}{\pi^2}\int e^{-2ipz}\tilde{F}(\not{p})dp\,\} \\
&\overset{3.226}{=} \quad \lim_{z\to 0}\{\, i\not{z}^{-2} - \frac{iz}{\pi^2}\int e^{-2ipz}\frac{\not{p}\rho_v(\eta) + \rho_s(\eta)}{p^2 + i\varepsilon - \eta}d\eta dp\,\}\ .
\end{aligned}
\tag{3.242}
$$

However, from (3.188), (3.189)

$$
\begin{aligned}
C^{(-2)}(x,z) \quad &\overset{3.188}{=} \quad 2\not{z}^{-3}z^{\mu}\overline{\mathcal{V}}_{\mu}(x) - z^{-2}\mathcal{S}^{\dagger}(x) \\
\Rightarrow \qquad -z\,\langle\,|\,C^{(-2)}(x,z)|\,\rangle \quad &= \quad z^{-1}\langle\,|\,\mathcal{S}^{\dagger}(x)\,|\,\rangle \\
&\overset{3.123}{=} \quad z^{-1}\mathbf{m}^{\dagger} \\
&\overset{3.107}{=} \quad z^{-1}v\mathbf{h}_0\ ,
\end{aligned}
\tag{3.243}
$$

$$
-z\,\langle\,|\,C^{(-1)}(x,z)|\,\rangle \quad \overset{3.189,3.183}{\approx} \quad -iz\not{z}^{-1}v^2\mathbf{h}_0^2\ , \tag{3.244}
$$

and by the fact that

$$
e^{-2ipz}\not{p} = \frac{i}{2}\not{\partial}^{z}e^{-2ipz}\ , \tag{3.245}
$$

(3.242) then becomes

$$
\lim_{z\to 0}\{z^{-1}v\mathbf{h}_0 - iz\not{z}^{-1}v^2\mathbf{h}_0^2\}
$$

$$= \lim_{z \to 0} \{ i \not{z}^{-2} - \frac{iz}{\pi^2} \int d\eta \, [\frac{i}{2} \rho_v(\eta) \not{\partial}^z + \rho_s(\eta)] \int \frac{e^{-2ipz} dp}{p^2 + i\varepsilon - \eta} \} . \quad (3.246)$$

By inserting the expansion (B.2)

$$\frac{i}{\pi^2} \int \frac{e^{-2ipz} dp}{p^2 + i\varepsilon - \eta} = -z^{-2} + \eta \{ 1 - ln (i\varepsilon - e^{2\gamma} \eta z^2) \} + \frac{5}{4} \eta^2 z^2 + \ldots \qquad (\varepsilon \to +0, \gamma \approx 0.577) , \quad (3.247)$$

we obtain

$$\begin{aligned} v\mathbf{h}_0 - \int \rho_s(\eta) d\eta &= 0 , \\ \lim_{z \to 0} i \not{z}^{-2} \{ 1 - \int \rho_v(\eta) d\eta \} &= 0 , \\ \lim_{z \to 0} iz \not{z}^{-1} \{ v^2 \mathbf{h}_0^2 - \int \eta \rho_v(\eta) d\eta \} &= 0 . \end{aligned} \quad (3.248)$$

Thus (3.239), (3.240), (3.241) are established.

■

These 'moment conditions' yield the result

$$\begin{aligned} \lim_{p \to \infty} \not{p} \tilde{F}(\not{p}) &= \int_0^\infty \rho_v(\eta) d\eta \\ &= 1 . \end{aligned} \quad (3.249)$$

All this can be summarized by the remark that the Feynman diagrams with loops free of boson lines cancel up to finite parts because of (3.239) and (3.240). On the other hand, the loop integrals with Bose propagators converge by their fading properties (3.233). Hence all those integrals will vanish for rising external momenta and only the tree diagrams survive at high energies. Thus, in contrast with other theories where the complete vertices either diverge for Abelian interactions or tend to zero for non-Abelian couplings at asymptotic energies, ours approach bare ones for $k \to \infty$.

3.4.2 Bare complete vertex functions

In section 3.4.1., we find that the interaction at high energies is tied to the *bare* couplings. Actually, they approach bare constants at asymptotic energies as in the following, we shall see that the *complete* vertex functions $\Gamma_J(k)$ and $\Gamma^\mu_Y(k)$ in quantum induction approach the bare ones in the sense of

$$\Gamma_J(p,q) \quad \to \quad \mathbf{h}_J \,, \tag{3.250}$$

$$\Gamma^\mu_Y(p,q) \quad \to \quad \gamma^\mu \mathbf{t}_Y \qquad (\text{ for } |(p-q)^2| \to \infty) \,. \tag{3.251}$$

<u>**Proof**</u>

As usual, with (3.222) through (3.226), the *complete* vertex functions $\Gamma_J(p,q)$ and $\Gamma^\mu_Y(p,q)$ can be written

$$(2\pi)^8\langle|T[\Psi(u)\overline{\Psi}(v)H^K(0)]|\rangle \;=\; -\int_0^\infty e^{i(qv-pu)}\tilde{F}(\not{p})\Gamma_J(p,q)\tilde{F}^{JK}_s((p-q)^2)\tilde{F}(\not{q})dpdq\,, \tag{3.252}$$

$$(2\pi)^8\langle|T[\Psi(u)\overline{\Psi}(v)\mathcal{V}^Z_\rho(0)]|\rangle \;=\; -\int_0^\infty e^{i(qv-pu)}\tilde{F}(\not{p})\Gamma^\mu_Y(p,q)\tilde{F}^{YZ}_{\mu\rho}(p-q)\tilde{F}(\not{q})dpdq\,. \tag{3.253}$$

Here, let us define the local limit

$$
\begin{aligned}
D(x) \quad &:= \quad \lim_{z\to 0} \{ z^2 C^{(-2)}(x,z) \,+\, i\not{z}^{-1} \,-\, (4\pi)^2\, T[z^2\Psi(x\,+\,z)\overline{\Psi}(x\,-\,z)]\,\} \\
&\overset{3.204}{=} \quad 0 \;, \qquad (3.254)
\end{aligned}
$$

where

$$
\begin{aligned}
C^{(-2)}(x,z) \quad &= \quad 2\not{z}^{-3} z^\mu \overline{\mathcal{V}}_\mu(x) \,-\, z^{-2}\mathcal{S}^\dagger(x) \\
&= \quad 2\not{z}^{-3} z^\mu \overline{\mathcal{V}}_\mu(x) \,-\, z^{-2}H^\dagger(x) \,-\, z^{-2}\mathbf{m}^\dagger \;. \qquad (3.255)
\end{aligned}
$$

Using the fact that

$$
\begin{aligned}
\langle\,|\,H^K(0)\,|\,\rangle \quad &= \quad 0 \;, \qquad (3.256) \\
\langle\,|\,\mathcal{V}^Z_\rho(0)\,|\,\rangle \quad &= \quad 0 \;, \qquad (3.257)
\end{aligned}
$$

and approximating the 'mixed' propagators by the bare

$$
\text{i.e.} \qquad \langle\,|T\,[\,H^K(x)\mathcal{V}^Z_\rho(0)\,]\,|\,\rangle \;=\; 0 \;, \qquad (3.258)
$$

we have

$$\begin{aligned} 0 &= \int_0^\infty e^{ikx}dx \, \langle \, | \, T \, [\, D(x) H^K(0) \,] \, | \, \rangle \\ &= \lim_{z\to 0} \int_0^\infty e^{ikx}dx \, \langle \, | \, T \, [(H^\dagger(x) \, + \, (4\pi)^2 z^2 \Psi(x \, + \, z) \overline{\Psi}(x \, - \, z) \,) H^K(0) \,] | \, \rangle \\ &= \lim_{z\to 0} \{ \, i\mathbf{h}_J^\dagger \, - \, z^2 G_J(k,z) \, \} \tilde{F}_s^{JK}(k^2) \; , \end{aligned} \tag{3.259}$$

or

$$\lim_{z\to 0} z^2 G_J(k,z) \quad = \quad i\mathbf{h}_J^\dagger \; , \tag{3.260}$$

where

$$z^2 G_J(k,z) \quad := \quad \frac{z^2}{\pi^2} \int_0^\infty e^{-2irz} \tilde{F}(\not{r} \, + \, \frac{1}{2} \not{k}) \Gamma_J(r + \frac{1}{2}k, r - \frac{1}{2}k) \tilde{F}(\not{r} \, - \, \frac{1}{2} \not{k}) dr \; . \tag{3.261}$$

Also,

$$\begin{aligned} 0 &= \int_0^\infty e^{ikx}dx \, \langle \, | \, T \, [\, D(x) \mathcal{V}_\rho^Z(0) \,] \, | \, \rangle \\ &= \lim_{z\to 0} \int_0^\infty e^{ikx}dx \, \langle \, | \, T \, [(2z^{-1} z^\mu \overline{\mathcal{V}}_\mu(x) \, - \, (4\pi)^2 z^2 \Psi(x \, + \, z) \overline{\Psi}(x \, - \, z) \,) \mathcal{V}_\rho^Z(0)] \, | \, \rangle \\ &= \lim_{z\to 0} \{ \, 2i \not{z}^{-1} z^\mu \overline{\mathfrak{t}}_Y \, + \, z^2 G_Y^\mu(k,z) \, \} \tilde{F}_{\mu\rho}^{YZ}(k) \; , \end{aligned} \tag{3.262}$$

where

$$z^2 G_Y^\mu(k,z) \quad := \quad \frac{z^2}{\pi^2} \int_0^\infty e^{-2irz} \tilde{F}(\not{r} \, + \, \frac{1}{2} \not{k}) \Gamma_Y^\mu(r + \frac{1}{2}k, r - \frac{1}{2}k) \tilde{F}(\not{r} \, - \, \frac{1}{2} \not{k}) dr \; . \tag{3.263}$$

However, as we noted above, all interactions *fade away* at asymptotic energies. Hence the *complete* vertex functions will tend to bare ones, so that for $|k^2| \to \infty$,

$$\Gamma_J(r + \frac{1}{2}k, r - \frac{1}{2}k) \quad \longrightarrow \quad \mathbf{c}_J \ , \tag{3.264}$$

$$\Gamma_Y^{\mu}(r + \frac{1}{2}k, r - \frac{1}{2}k) \quad \longrightarrow \quad \gamma^{\mu}\mathbf{c}_Y \ , \tag{3.265}$$

where $\mathbf{c}_J, \mathbf{c}_Y$ are constant matrices, not containing any Dirac's matrices except γ_5. Since (3.261) and (3.263) have the same form, so let $\mathbf{c}$ represent either $\mathbf{c}_J$ or $\gamma^{\mu}\mathbf{c}_Y$ and do the evaluation below. Thus with the definition of the *interacting* Dirac propagators (3.226), the right-hand sides of (3.261) and (3.263) become [44]

$$\frac{z^2}{\pi^2}\int_0^{\infty} e^{-2irz} \frac{(\not{r} + \frac{1}{2}\not{k})\rho_v^{\eta} + \rho_s^{\eta}}{(r + \frac{1}{2}k)^2 + i\varepsilon - \eta}\, \mathbf{c}\, \frac{(\not{r} - \frac{1}{2}\not{k})\rho_v^{\zeta} + \rho_s^{\zeta}}{(r - \frac{1}{2}k)^2 + i\varepsilon - \zeta} dr d\eta d\zeta \ . \tag{3.266}$$

By the Feynman formula [104] [45]

$$\frac{1}{ab} = \int_0^1 \frac{dt}{[at + b(1 - t)]^2} \ , \tag{3.267}$$

the denominator of (3.266) can be simplified to

$$[((r + \frac{1}{2}k)^2 - \eta)t + ((r - \frac{1}{2}k)^2 - \zeta)(1 - t)]^2 = (l^2 - \omega)^2 \ , \tag{3.268}$$

[44] In the following, ρ_s^{η} is used as the short-hand notation for $\rho_s(\eta)$.

[45] Formula (3.267) can be obtained by writing $\frac{1}{ab} = \frac{1}{b-a}(\frac{1}{a} - \frac{1}{b}) = \frac{1}{b-a}\int_a^b \frac{dz}{z^2}$ and making the change of variable $z = at + b(1 - t)$.

where

$$l \quad := \quad r + (t - \frac{1}{2})k \, , \tag{3.269}$$

$$\omega \quad := \quad \eta t + \zeta(1 - t) + t(t - 1)k^2 - i\varepsilon \, . \tag{3.270}$$

Hence (3.266) becomes

$$\begin{aligned} &\frac{z^2}{\pi^2}\int_0^1 dt \int_0^\infty d\zeta d\eta \, e^{-i(1-2t)kz} \{ (\not{l} - t\not{k} + \not{k})\rho_v^\eta \mathbf{c} (\not{l} - t\not{k})\rho_v^\zeta \\ &\quad + (\not{l} - t\not{k} + \not{k})\rho_v^\eta \mathbf{c}\rho_s^\zeta + \rho_s^\eta \mathbf{c} (\not{l} - t\not{k})\rho_v^\zeta \\ &\quad + \rho_s^\eta \mathbf{c}\rho_s^\zeta \} \, e^{-2ilz}(l^2 - \omega)^{-2} dl \, . \end{aligned} \tag{3.271}$$

However, by using the expansions (B.27) through (B.29), only the terms with $l^\mu l^\nu$ survive. Then, by omitting the terms vanishing at $z = 0$, (3.271) can be further reduced to

$$i\int_0^1 dt \int_0^\infty d\zeta d\eta \, \gamma_\mu \rho_v^\eta \mathbf{c} \gamma_\nu \rho_v^\zeta \{ \frac{1}{2}\eta^{\mu\nu} - z^\mu z^\nu z^{-2} \} + \mathcal{O}(z^{(0)}) \, . \tag{3.272}$$

Here, for simplicity, we let ρ_v^η and ρ_v^ζ commute with all other matrices in (3.272), so according to (3.240), we obtain

$$\int_0^1 dt \int_0^\infty d\zeta d\eta \, \rho_v^\eta \rho_v^\zeta = 1 \, . \tag{3.273}$$

This then makes (3.272) become

$$i\gamma_\mu \mathbf{c}\gamma_\nu \left(\frac{1}{2}\eta^{\mu\nu} - z^\mu z^\nu z^{-2} \right) + \mathcal{O}(z^{(0)}) \,. \tag{3.274}$$

(i).

Inserting (3.264) in (3.270), for (3.261) we obtain [46]

$$\begin{aligned} z^2 G_J(k,z) &= i\gamma_\mu \mathbf{c}_J \gamma_\nu \left(\frac{1}{2}\eta^{\mu\nu} - z^\mu z^\nu z^{-2} \right) + \mathcal{O}(z^{(0)}) \\ \overset{3.260}{\Rightarrow} \qquad i\mathbf{h}_J^\dagger &= 2i\bar{\mathbf{c}}_J^\dagger - i\bar{\mathbf{c}}_J^\dagger \not{z}\not{z} z^{-2} \\ &= 2i\bar{\mathbf{c}}_J^\dagger - i\bar{\mathbf{c}}_J^\dagger \\ &= i\bar{\mathbf{c}}_J^\dagger \,. \end{aligned} \tag{3.275}$$

Therefore, we conclude that

$$\begin{aligned} \bar{\mathbf{c}}_J^\dagger &= \mathbf{h}_J^\dagger \\ \Rightarrow \qquad \bar{\mathbf{c}}_J &= \mathbf{h}_J \\ &\overset{3.34}{=} \bar{\mathbf{s}}_J \,. \end{aligned} \tag{3.276}$$

(ii).

Proceeding in a similiar manner, inserting (3.265) in (3.274), for (3.263) we get

$$z^2 G_Y^\sigma(k,z) = i\gamma_\mu \gamma^\sigma \mathbf{c}_Y \gamma_\nu \left(\frac{1}{2}\eta^{\mu\nu} - z^\mu z^\nu z^{-2} \right) + \mathcal{O}(z^{(0)})$$

[46] It should be noted that since $\mathbf{c}_J$ (as well as $\mathbf{c}_Y$) does not contain Dirac matrices except in γ_5, then $\mathbf{c}_J \leftrightarrow \bar{\mathbf{c}}_J^\dagger$ for $\gamma_5 \leftrightarrow -\gamma_5$.

$$
\begin{aligned}
&= i\gamma_\mu\gamma^\sigma(\frac{\gamma^\mu}{2} - z^\mu \not{z}^{-1})\bar{\mathbf{c}}_Y^\dagger + \mathcal{O}(z^{(0)}) \\
&= i(\gamma^\sigma\gamma_\mu - 2\eta^\sigma_\mu)(\frac{\gamma^\mu}{2} - z^\mu \not{z}^{-1})\bar{\mathbf{c}}_Y^\dagger + \mathcal{O}(z^{(0)}) \\
&= 2iz^\sigma \not{z}^{-1}\bar{\mathbf{c}}_Y^\dagger + \mathcal{O}(z^{(0)}) \ . \qquad (3.277)
\end{aligned}
$$

Comparison with (3.262) suggests

$$\bar{\mathbf{c}}_Y^\dagger = \bar{\mathbf{t}}_Y \ . \qquad (3.278)$$

However, according to (3.37)

$$\mathbf{t}_Y^\dagger = \mathbf{t}_Y \ , \qquad (3.279)$$

thus

$$\mathbf{c}_Y = \mathbf{t}_Y \ . \qquad (3.280)$$

■

CHAPTER 4

THE DYNAMICAL HIGGS MECHANISM

4.1 The 'Mexican Hat' Shaped Potential

In chapter two, we have discussed the usual Higgs mechanism by the 'Mexican hat' shaped potential (2.37), for the Higgs mass obtaining formula (2.64)

$$m_H^2 \;=\; \frac{\lambda}{3}v^2 \; . \tag{4.1}$$

Now, let us briefly review how we got the Higgs mass (4.1) in chapter two with the 'Mexican hat' shaped potential. Here, the reformulation will be done by the method of effective potentials [40, 41]. We start from the Standard Model with minimal Higgs contents. Because only the physical Higgs field is related to the Higgs mass, let us for simplicity restrict ourselves to the single physical [1] Higgs field $\phi(x)$ in the following discussion.

In the tree approximation, the effective potential is given by

$$V_{eff}(\phi) \;=\; (\frac{m_\phi^2}{2})\phi^2 \;+\; (\frac{\lambda}{4!})\phi^4 \tag{4.2}$$

which is just the usual 'Mexican hat' shaped potential (2.37). We use this to break

[1] Note that in consistency with our previous usage (2.36) and (2.47), the physical Higgs component should be denoted by $\phi_3(x)$. However, in the following the subscript 3 will be dropped for brevity's sake.

the symmetry by giving the physical Higgs field the vacuum expectation value

$$\langle\, |\phi|\, \rangle \;=\; v \;=\; 246 \text{ GeV} \; . \tag{4.3}$$

In other words, the vacuum expectation value v gives the minimum value of the effective potential

$$V'_{eff}(v) \;=\; 0 \; . \tag{4.4}$$

Thus, with (4.2), it leads to

$$v^2 \;=\; -(\frac{6}{\lambda})m_\phi^2 \; , \tag{4.5}$$

which obviously shows that the contribution to v just comes from the *Higgs field only.*[2] Now, the mass m_H of the physical Higgs particle is then given by

$$m_H^2 \;=\; \frac{\partial^2 V_{eff}(v)}{\partial \phi^2} \;=\; \frac{\lambda}{3} v^2 \; , \tag{4.6}$$

which is what we are all familiar with.

Here, it should be noted that the 'Mexican hat' shaped potential (2.37) is decided ad hoc in spite of several attempts to derive it from more fundamental principles. For instance, under the Coleman-Weinberg method [105, 106], the Higgs

[2]Note that this conclusion directly follows from our "ad hoc" potential and no deeper explanation is offered within the usual framework. However, we shall later see that this can be explained by our "dynamical" symmetry breaking in Quantum Induction.

potential which breaks the symmetry spontaneously can be induced by quantum radiative corrections. Only because of its prediction of a low Higgs mass ($m_H \approx 11$ GeV), this method is experimentally ruled out. However, as we shall see in a subsequent section, under a newly proposed *dynamical* Higgs mechanism, the conventional connection (4.1) between $\lambda(m_Z)$ and m_H can be recovered with a more fundamental theoretical reason in contrast to the usual approach where (4.1) is obtained by using the 'Mexican hat' shaped potential which is totally inserted by hand.

4.2 The Dynamical Higgs Mechanism

4.2.1 The implied Higgs field equation

In Quantum Induction, the issue is addressed in a different approach, namely, by using the implied quantum field equation. In section 3.3, we have already derived the Dirac induced field equations. One of them is (3.212)

$$\begin{aligned} \mathcal{F}^K + j^K &= Q^K \,, && (4.7)\\ \text{with} \qquad \mathcal{F}^K &= \frac{1}{2u} Tr[\,\mathbf{h}_K^\dagger \{\, \mathcal{S}_\mu^\mu + 2\mathcal{S}\mathcal{S}^\dagger\mathcal{S}\,\}(x)\,]\,, && (4.8) \end{aligned}$$

where $\mathcal{S}(x)$ is the *quantum* counterpart of the Higgs field. From the analysis done in section 3.2.4, $\mathcal{S}^0(x)$ and $\mathcal{S}^P(x)$ are known as the *physical* and *unphysical* Higgs components respectively. In order to extract them from (4.7), we consider the following way.

(i). Simple use of the definition of $\mathcal{S}_\mu^\mu$ in (3.89) with (3.149) shows

$$\begin{aligned} Tr\,[\,\mathbf{h}_K^\dagger \mathcal{S}_\mu^\mu\,] &= 2u\Box\mathcal{S}^K + Tr[\mathbf{h}_K^\dagger(\,2\overline{\mathcal{V}}_\mu\mathcal{S}\mathcal{V}^\mu - \overline{\mathcal{V}}_\mu\overline{\mathcal{V}}^\mu\mathcal{S} - \mathcal{S}\mathcal{V}_\mu\mathcal{V}^\mu\,)]\\ &\quad + i\,Tr[\mathbf{h}_K^\dagger(\,\overline{\mathcal{V}}^\mu_{,\mu}\mathcal{S} - \mathcal{S}\mathcal{V}^\mu_{,\mu} + 2\overline{\mathcal{V}}^\mu\mathcal{S}_{,\mu} - 2\mathcal{S}_{,\mu}\mathcal{V}^\mu\,)]\,. \qquad (4.9) \end{aligned}$$

(ii). Using the assumption that the bare top quark mass m_t will greatly exceed the other bare quark and lepton masses m_f, we have

$$\begin{aligned} 2u &\overset{3.149}{=} Tr\,[\,\mathbf{h}_0^\dagger\mathbf{h}_0\,]\\ &\overset{3.152}{=} Tr\,[\,\frac{\mathbf{m}^2}{v^2}\,] \end{aligned}$$

$$
\begin{aligned}
&= \frac{4\sum_f m_f^2}{v^2} \\
&\approx \frac{12 m_t^2}{v^2}\,, \qquad (4.10) \\
\text{or} \qquad m_t^2 &= \frac{uv^2}{6}\,. \qquad (4.11)
\end{aligned}
$$

Thus,

$$
\begin{aligned}
Tr\,[\,\mathbf{h}_K^\dagger \mathcal{S}\mathcal{S}^\dagger \mathcal{S}\,] &\overset{3.164}{=} \mathcal{S}^K \mathcal{S}_L \mathcal{S}^L\, Tr[\,\mathbf{h}_0^\dagger \mathbf{h}_0 \mathbf{h}_0^\dagger \mathbf{h}_0\,] \\
&\overset{3.152}{=} \mathcal{S}^K \mathcal{S}_L \mathcal{S}^L\, Tr\,[\,\frac{\mathbf{m}^4}{v^4}\,] \\
&\approx 12 \mathcal{S}^K \mathcal{S}_L \mathcal{S}^L \,\frac{m_t^4}{v^4} \\
&\approx \frac{u^2}{3} \mathcal{S}^K \mathcal{S}_L \mathcal{S}^L\,. \qquad (4.12)
\end{aligned}
$$

(iii). Inserting (4.9) and (4.12) into (4.7), by straightforward computation we get

$$
\begin{aligned}
Q^K(x) \;=\; & \Box \mathcal{S}^K + \frac{u}{3}\mathcal{S}^K \mathcal{S}_L \mathcal{S}^L + \frac{1}{2u} Tr[\mathbf{h}_K^\dagger (\,2\overline{\mathcal{V}}_\mu \mathcal{S}\mathcal{V}^\mu - \overline{\mathcal{V}}_\mu \overline{\mathcal{V}}^\mu \mathcal{S} - \mathcal{S}\mathcal{V}_\mu \mathcal{V}^\mu\,)] \\
& + \frac{i}{2u} Tr[\mathbf{h}_K^\dagger (\,\overline{\mathcal{V}}^\mu_{,\mu} \mathcal{S} - \mathcal{S}\mathcal{V}^\mu_{,\mu} + 2\overline{\mathcal{V}}^\mu \mathcal{S}_{,\mu} - 2\mathcal{S}_{,\mu}\mathcal{V}^\mu\,)] + j^K(x)\,. \qquad (4.13)
\end{aligned}
$$

where $j^K(x)$ is the scalar Dirac source given by (3.214).

Eq. (4.13) is known as the implied quantum field equation for Higgs fields $\mathcal{S}^K$ in Quantum Induction and is taken as the starting point for our following discussion. Here it should be noted that 'quantum field equations' have never been discussed in the usual quantum field theory [3]. However, their existence has followed from our

[3] In the usual theory, the classical field equations are not generalized to quantum field equations, but replaced by the infinitely renormalized S-matrix and the effective action.

basic postulates of Quantum Induction. In fact, everything we have done so far for the *usual* Higgs mechanism is purely classical, but using *quantum-mechanical language*. Moreover, applying the Euler-Lagrange equation [107]

$$\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) = \frac{\partial \mathcal{L}}{\partial \phi} \tag{4.14}$$

to ϕ with the usual "Higgs Lagrangian density" given by [4]

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{m_\phi^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 \,, \tag{4.15}$$

we have

$$\Box\phi + m_\phi^2 \phi + \frac{\lambda}{6}\phi^3 = 0 \,. \tag{4.16}$$

Hence, by comparing the above classical field equation derived from (4.15) for the *physical* Higgs field ϕ with its quantum counterpart extracted from (4.13) for $\mathcal{S}^0$, [5] we see that the customary λ corresponds to the parameter $2u$ in Quantum Induction

$$u = \frac{\lambda}{2} \,, \tag{4.17}$$

i.e. u is related with the 'self coupling' strength λ of the Higgs field by (4.17). Here,

[4] $\mathcal{L} := T - V$, where T is the kinetic term and V is just the "Mexican hat" shaped potential.

[5] In order to find the quantum counterpart of equation (4.16) within the conventional Higgs mechanism, we let $K = 0$ in (4.13) first, and then neglect the Yang-Mills fields $\mathcal{V}_\mu$ as well as terms which are nonlinear in $\mathcal{S}_L$. Finally, $j^0(\mathrm{x})$ is ignored completely under the "free" propagation consideration.

λ is its asymptotic value $\lambda(\infty)$. It is just like what we shall see in (5.24) where u determines the *bare* Yang-Mills coupling constants which are approached at high energies.

Now, since the VEV of $\mathcal{S}^0$ is non-zero,

$$v \;=\; \langle\, |\mathcal{S}^0(x)|\, \rangle \;\neq\; 0\;, \tag{4.18}$$

so as usual, we split $\mathcal{S}^0$ into v and its quantum fluctuations H^0

$$\text{i.e.} \qquad \mathcal{S}^0(x) \;=\; v + H^0(x)\;, \tag{4.19}$$

$$\text{where} \qquad \langle\, |H^0(x)|\, \rangle \;=\; 0\;. \tag{4.20}$$

In other words, [6]

$$\mathcal{S}^K(x) \;=\; \delta_0^K v \,+\, H^K(x)\;. \tag{4.21}$$

Then inserting (4.21) into (4.13) with (4.17), we have

$$Q^K(x) \;=\; \Box H^K(x) \,+\, \frac{\lambda}{6} v^2 \,\{\, \delta_0^K v \,+\, H^K(x) \,+\, 2\delta_0^K H^0(x)\,\} \,+\, J^K(x)\;, \tag{4.22}$$

where

$$J^K(x) \quad := \quad J^K_{bose}(x) \,+\, j^K(x)\;, \tag{4.23}$$

[6] See the footnote 38 in chapter 3.

$$
\begin{aligned}
J^K_{bose}(x) \quad &:= \quad \frac{u}{3}(\,2vH_KH^0 \,+\, v\delta^0_KH_LH^L \,+\, H_KH_LH^L\,) \\
&\qquad + \frac{1}{2u}Tr[\mathbf{h}^\dagger_K(\,2\overline{\mathcal{V}}_\mu\mathcal{S}\mathcal{V}^\mu \,-\, \overline{\mathcal{V}}_\mu\overline{\mathcal{V}}^\mu\mathcal{S} \,-\, \mathcal{S}\mathcal{V}_\mu\mathcal{V}^\mu\,)] \\
&\qquad + \frac{i}{2u}Tr[\mathbf{h}^\dagger_K(\,\overline{\mathcal{V}}^\mu_{,\mu}\mathcal{S} \,-\, \mathcal{S}\mathcal{V}^\mu_{,\mu} \,+\, 2\overline{\mathcal{V}}^\mu\mathcal{S}_{,\mu} \,-\, 2\mathcal{S}_{,\mu}\mathcal{V}^\mu\,)] \quad (4.24) \\
j^K(x) \quad &\overset{3.214}{:=} \quad \lim_{z^2\to 0} < T[\overline{\Psi}(x-z)\mathbf{h}_K\Psi(x+z)] \,+\, \kappa C^K(x,z^2) \\
&\qquad + \kappa u\mathcal{F}^K(x)\,ln\,(\,i\varepsilon \,-\, \mu^2z^2\,) >_{z^2} . \quad (4.25)
\end{aligned}
$$

Now $J^K(x)$ will be approximated by [7]

$$
\begin{aligned}
J^K(x) \quad &\approx \quad (\,a_1\Box \,-\, a_2\,)\mathcal{S}^K(x) \\
&\overset{4.21}{=} \quad (\,a_1\Box \,-\, a_2\,)H^K(x) \,-\, a_2v\delta^K_0 \,, \quad (4.26)
\end{aligned}
$$

where a_1 and a_2 are constants.

Since $\langle\,|H^K(x)|\,\rangle \,=\, 0$ and $\langle\,|Q^K(x)|\,\rangle \,=\, 0$, [8] taking the VEV of (4.22) with the approximation (4.26) yields

$$
a_2 \,=\, \frac{\lambda}{6}v^2 \,. \quad (4.27)
$$

Substituting (4.26) and (4.27) into (4.22), we obtain [9]

$$
Q^K(x) \,=\, (\,1 \,+\, a_2\,)\Box H^K(x) \,+\, \frac{\lambda}{3}v^2\delta^K_0H^0(x) \,. \quad (4.28)
$$

[7] Note that the approximation (4.26) leaves (4.22) covariant under O_4. One might think of ignoring $J^K(x)$ completely, as in the conventional Higgs mechanism. However, $J^K(x) \,=\, 0$ would make $v \,=\, 0$ by (4.27). As for approximating $J^K(x)$ by a constant c^K, this would make the unphysical Higgs components H^1, H^2, H^3 massive.

[8] In Appendix C, we shall see that Q^K(x) behaves as functional differentiator, so its vacuum expectation value is zero.

[9] More discussion will appear in Appendix C.

Moreover, the "unphysical" Higgs components H^1, H^2, H^3 are massless as we see that the second term of (4.28) *vanishes* when $K \neq 0$. This agrees with the literature since Quantum Induction resembles the conventional theory in the *Landau* gauge.

Now, let us consider the physical Higgs propagator $\tilde{F}^{00}(k^2)$, because its pole determines the physical Higgs mass. Since

$$\begin{array}{rrcl} & Q^K(x) & \overset{C.20}{=} & -i\int \mathcal{M}(x-y)\,dy\frac{\delta}{\delta H^K(y)} \\ \Longrightarrow & Q^K(x)H^J(0) & = & -i\int \mathcal{M}(x-y)\,dy\delta(y)\,\delta^{KJ} \\ & & = & -i\delta^{KJ}\mathcal{M}(x) \\ \Longrightarrow & i\int e^{ikx}Q^K(x)H^J(0)\,dx & = & \delta^{KJ}\tilde{\mathcal{M}}(k^2) \\ \Longrightarrow & i\int e^{ikx}\langle|T[Q^K(x)H^J(0)]|\rangle\,dx & = & \delta^{KJ}\tilde{\mathcal{M}}(k^2) \\ \overset{4.28}{\Longrightarrow} & (1+a_2)k^2\tilde{F}^{KJ}(k^2)\;-\;\frac{\lambda v^2}{3}\delta_0^K\tilde{F}^{0J}(k^2) & = & \delta^{KJ}\tilde{\mathcal{M}}(k^2)\;, \end{array} \tag{4.29}$$

the *complete* Higgs propagators $\tilde{F}^{KJ}(k^2)$ are given by

$$\tilde{F}^{KJ}(k^2)\;:=\;-i\int e^{ikx}\langle dx\,\langle|T[H^K(x)H^J(0)]|\rangle\;. \tag{4.30}$$

Now, in order to look for the Higgs mass, let us limit ourselves to the low energy case. Then (4.29) becomes

$$(1+a_2)k^2\tilde{F}^{KJ}(k^2)\;-\;\frac{\lambda v^2}{3}\delta_0^K\tilde{F}^{0J}(k^2)\;\overset{C.22}{=}\;\delta^{KJ}\qquad \text{for }\;|k^2|\;\ll\;M_{PL}^2 \tag{4.31}$$

where $\lambda = \lambda(m_z)$ can be reached by the sliding $\lambda(\mu)$ when the equations of the renormalization group are integrated. [10] Moreover, in deriving (4.29), one actually notes how the perturbation affects the vertex functions as well as the propagator. Thus the usual sliding of coupling constants such as $\lambda(\mu)$ most likely *includes* the renormalization of propagators. In other words,

$$\lambda(m_Z) \longrightarrow \frac{\lambda(m_Z)}{Z_H} \,. \tag{4.32}$$

Hence, instead of equation (4.31), we shall consider

$$(1+a_2)\{\, k^2 \tilde{F}^{KJ}(k^2) \;-\; \frac{\lambda v^2}{3} \delta_0^K \tilde{F}^{0J}(k^2)\,\} = \delta^{KJ} \qquad \text{for} \;\; |k^2| \;\ll\; M_{PL}^2 \,. \tag{4.33}$$

For the physical component $K = J = 0$, we have

$$\tilde{F}^{00}(k^2) \;=\; \frac{Z_H}{k^2 \;-\; \frac{\lambda(m_Z)}{3} v^2} \qquad \text{for} \;\; |k^2| \;\ll\; M_{PL}^2 \;; \tag{4.34}$$

thus the Higgs mass is given by

$$m_H^2 \;=\; \frac{\lambda(m_Z)}{3} v^2 \,. \tag{4.35}$$

which agrees with the conventional relation appeared in (4.6).

[10] See section 5.2 .

From (4.26) we get

$$\begin{aligned} \langle |J^K(x)| \rangle &= -a_2 v \delta_0^K \\ &\overset{4.27}{=} -\frac{\lambda}{6} v^3 \delta_0^K \,. \end{aligned} \tag{4.36}$$

Hence, in contrast to (4.5), (4.36) emphasizes the important role played by J^0 to trigger the value v which is regarded as most essential in the Higgs mechanism. In other words, we believe that Bose fields as well as Fermi fields are responsible for the Higgs mechanism. Thus in turn they [11] should give some contribution to the Higgs mass and this is the main difference between the usual Higgs mechanism and ours. In fact, since the Higgs field gives masses to quarks as well as gauge bosons and to itself, it actually affects all *interactions*. Hence, it seems very naive when one tries to derive its main properties (v and m_H) only from the self-coupling (4.2) without considering the contribution from other fields, for instance, Yang-Mills fields.

4.2.2 The dynamical Higgs mechanism

So far, our discussion just concentrates upon the Higgs mass, but the most fundamental question has not been touched. Why is our Higgs mechanism called *dynamical* ?

As we mentioned before, the spontaneous symmetry breaking is due to elementary Higgs scalars in the ususal Higgs mechanism. In contrast to this, we propose

[11] In section 4.2.2 ,we shall see that the contribution from Fermi fields to the Higgs mass is very small compared with the one from Bose fields.

that the symmetry breaking is mainly brought on *dynamically* through the interactions of the Bose fields with the Higgs fields. The contribution from Fermi fields is very small compared with the one from Bose fields. Here, let us first compare the magnitudes of $\langle | j^0(x) | \rangle$ and $\langle | J^0_{bose}(x) | \rangle$.

From (4.25) we have

$$\begin{aligned} j^0(x) &= \lim_{z^2 \to 0} < T[\overline{\Psi}(x-z)\mathbf{h}_0\Psi(x+z)] \\ &\qquad + \kappa C^0(x,z^2) + \kappa u \mathcal{F}^0(x)\, ln\,(\,i\varepsilon\, -\, \mu^2 z^2\,) >_{z^2} , \end{aligned} \tag{4.37}$$

where $C^0(x, z^2)$ is given by (3.208)

$$C^0(x,z^2) \quad := \quad Tr[\,\mathbf{h}_0 < (C^{(-2)} + C^{(0)})(x,z) \ >_{z^2}] \, . \tag{4.38}$$

By the results at (3.188) for $C^{(-2)}(x,z)$ and (3.192) for $C^{(0)}(x,z)$, $j^0(x)$ becomes [12]

$$\begin{aligned} j^0(x) &= \lim_{z^2 \to 0} < T[\overline{\Psi}(x-z)\mathbf{h}_0\Psi(x+z)] + \kappa u \mathcal{F}^0(x)\, ln\,(\,i\varepsilon\, -\, \mu^2 z^2\,) \\ &\qquad - 2\kappa u v z^{-2} >_{z^2} - G(x) \, , \end{aligned} \tag{4.39}$$

where

$$\langle | G(x) | \rangle = \frac{\kappa}{4} Tr[\mathbf{h}_0 \langle | \mathcal{S}\mathcal{V}_\mu \mathcal{V}^\mu + \overline{\mathcal{V}}_\mu \overline{\mathcal{V}}^\mu \mathcal{S} + i\mathcal{S}\mathcal{V}^\mu_{,\mu} - i\overline{\mathcal{V}}^\mu_{,\mu}\mathcal{S} | \rangle] \, . \tag{4.40}$$

Hence

$$\langle | j^0(x) + G(x) | \rangle = \kappa\, Tr[\mathbf{h}_0 \int \rho^\eta_s \, d\eta\, \eta \ ln(\frac{e^{1-2\gamma}\eta}{\mu^2})] \, . \tag{4.41}$$

[12]Note that here the Dirac source $j^0(x)$ resembles the "currents" one obtains by "points splitting" in the usual theory. However, as we see in equation (4.39), ours are determined *uniquely* in contrast with the usual approach.

Proof

By using the *interacting* Dirac propagator (3.226):

$$(2\pi)^4 \langle | T [\Psi(2z)\overline{\Psi}(0)] | \rangle = i \int e^{-2ipz} dp \tilde{F}(\not{p}) \, , \tag{4.42}$$

$$\text{where} \qquad \tilde{F}(\not{p}) = \int_0^\infty \frac{\not{p}\rho_v^\eta + \rho_s^\eta}{p^2 + i\varepsilon - \eta} d\eta \, , \tag{4.43}$$

and the short distance expansion (B.2):

$$i\pi^{-2} \int \frac{e^{-2ipz} dp}{p^2 + i\varepsilon - \eta} = -z^{-2} + \eta \, ln(e^{2\gamma}\eta z^2) + \dots \, , \tag{4.44}$$

$$i\pi^{-2} \int \frac{e^{-2ipz} p^\mu dp}{p^2 + i\varepsilon - \eta} = iz^{-4} z^\mu + i\eta z^{-2} z^\mu + \dots \, , \tag{4.45}$$

for $z \to 0$ (at $z^2 \neq 0$), we have

$$(4\pi)^2 \langle | T [\Psi(2z)\overline{\Psi}(0)] | \rangle = i \int d\eta \, \eta \rho_v^\eta \not{z}^{-1} + i \int d\eta \, \rho_v^\eta \not{z}^{-3} + \int d\eta \, \eta \rho_s^\eta \, ln(e^{2\gamma}\eta z^2) - \int d\eta \, \rho_s^\eta z^{-2} + \dots \, . \tag{4.46}$$

The dots in the above equations symbolize terms which vanish for $z \to 0$. Now, since

$$Tr \{ \mathbf{h}_0 \, T [\Psi(2z)\overline{\Psi}(0)] \} = -T [\overline{\Psi}(0) \mathbf{h}_0 \Psi(2z)] \tag{4.47}$$

and recalling the 'moment conditions' (3.239)

$$\begin{aligned}\int d\eta\, \rho_s^\eta &= \mathbf{m}^\dagger \\ &\overset{3.107}{=} v\mathbf{h}_0^\dagger\,, \end{aligned} \tag{4.48}$$

we can rewrite (4.46) as

$$\begin{aligned}(4\pi)^2 \langle\, |\, T\,[\,\overline{\Psi}(0)\mathbf{h}_0\Psi(2z)\,]\,|\,\rangle &= -Tr\,\{\,\mathbf{h}_0 i \int d\eta\,\eta\rho_v^\eta z^{-1} + \mathbf{h}_0 i \int d\eta\,\rho_v^\eta z^{-3} \\ &\quad + \mathbf{h}_0 \int d\eta\,\eta\,\rho_s^\eta\, ln(e^{2\gamma}\eta z^2) \\ &\quad - \mathbf{h}_0\mathbf{h}_0^\dagger z^{-2} v + \ldots\}\,. \end{aligned} \tag{4.49}$$

As the first two terms of the right-hand side of (4.49) are odd functions in z, they will vanish in the average over the direction of z. Hence we obtain [13]

$$\begin{aligned} &\lim_{z^2\to 0} \; < \langle\, |\, T\,[\,\overline{\Psi}(x-z)\mathbf{h}_0\Psi(x+z)\,]\,|\rangle >_{z^2} \\ &= -\kappa \lim_{z^2\to 0} \; < Tr[\mathbf{h}_0 \int d\eta\,\eta\rho_s^\eta\, ln(e^{2\gamma}\eta z^2) - \mathbf{h}_0\mathbf{h}_0^\dagger z^{-2} v] >_{z^2}\,, \end{aligned} \tag{4.50}$$

where

$$\kappa := (4\pi)^{-2}\,. \tag{4.51}$$

Here, it should be noted that $j^0(x)$ is finite as we see from equation (4.7). In other

[13]Note that $T\,[\,\overline{\Psi}(0)\mathbf{h}_0\Psi(2z)\,] = T\,[\,\overline{\Psi}(x-z)\mathbf{h}_0\Psi(x+z)\,]$ because of translational invariance.

words, $j^0(x)$ should be convergent when $z \longrightarrow 0$. Therefore, after inserting (4.50) into (4.39),

$$\begin{aligned} \langle |j^0(x) + G(x)|\rangle &= \kappa \lim_{z^2 \to 0} < Tr[\mathbf{h}_0 \int d\eta\, \eta \rho_s^\eta\, ln(e^{2\gamma}\eta z^2) - \mathbf{h}_0 \mathbf{h}_0^\dagger z^{-2} v] \\ &\quad + u\langle |\mathcal{F}^0(x)|\rangle\, ln\,(i\varepsilon - \mu^2 z^2) - 2uvz^{-2} >_{z^2} \end{aligned} \tag{4.52}$$

the terms with z^{-2} cancel. This gives the condition

$$2uv = Tr\,[\mathbf{h}_0 \mathbf{h}_0^\dagger v] \tag{4.53}$$

which agrees with (3.149). Similarly, the terms with $ln(z^2)$ must cancel and this gives another consistency condition

$$u\,\langle |\mathcal{F}^0(x)|\rangle = Tr[\,\mathbf{h}_0 \int d\eta\, \eta\, \rho_s^\eta\,]\ . \tag{4.54}$$

Thus substituting (4.53) and (4.54) into (4.52), we obtain

$$\langle |j^0(x) + G(x)|\rangle = \kappa\, Tr[\,\mathbf{h}_0 \int \rho_s^\eta\, d\eta\, \eta\, ln(\frac{e^{1-2\gamma}\eta}{\mu^2})]\ . \tag{4.55}$$

■

Here, by using dimensional analysis, we note

$$\langle |j^0(x) + G(x)|\rangle \sim \kappa v^3 \tag{4.56}$$

since

$$\rho_s^\eta \eta \sim v^3 \tag{4.57}$$

when ρ_s^η is given by sharp (bare) mass approximation [14]

$$\rho_s^\eta = \sqrt{\eta}\,\delta(\eta - m_f^2)\ . \tag{4.58}$$

Moreover, from (4.40), we also learn that

$$\langle |G(x)| \rangle \sim \kappa v^3\ . \tag{4.59}$$

Therefore,

$$\langle |j^0(x)| \rangle \leq \kappa\, v^3\ , \tag{4.60}$$

which follows from (4.56) and (4.59). However,

$$\begin{aligned} \langle |J^0(x)| \rangle &\overset{4.23}{:=} \langle |J^0_{bose}(x) + j^0(x)| \rangle \\ &\overset{4.36}{=} -\frac{\lambda}{6} v^3 \\ &\overset{5.29}{=} -0.4 v^3\ . \end{aligned} \tag{4.61}$$

[14]In contrast with the usual theory, the ultraviolet divergencies are avoided in Quantum Induction and hence the *bare* coupling constants and mass are all meaningful and tied to each other (see section 5.1). Moreover, the *bare* mass here means the one at asymptotic energies.

Thus,

$$| \langle | J^0_{bose}(x) | \rangle | \;\gg\; | \langle | j^0(x) | \rangle | \,. \tag{4.62}$$

where $\langle | J^0_{bose}(x) | \rangle$ is given by (4.24)

$$\begin{aligned} \langle | J^0_{bose}(x) | \rangle \;:=\; & \frac{1}{6} Tr[\mathbf{h}_0 \langle |\, 2vH_0H^0 + vH_LH^L + H_0H_LH^L \,| \rangle] \\ & + \frac{1}{2u} Tr[\mathbf{h}_0 \langle |\, 2\overline{\mathcal{V}}_\mu \mathcal{S} \mathcal{V}^\mu - \overline{\mathcal{V}}_\mu \overline{\mathcal{V}}^\mu \mathcal{S} - \mathcal{S}\mathcal{V}_\mu \mathcal{V}^\mu \\ & - i\overline{\mathcal{V}}^\mu_{,\mu}\mathcal{S} + i\mathcal{S}\mathcal{V}^\mu_{,\mu} \,| \rangle] \,. \end{aligned} \tag{4.63}$$

Though we have shown that $| \langle | j^0(x) | \rangle |$ is very small, comparing (4.60) with the familiar quark condensate [109]

$$\begin{aligned} c^Q \;&\approx\; (\,0.25 \text{ GeV}\,)^3 \\ &\ll\; | \langle | j^0(x) | \rangle | \end{aligned} \tag{4.64}$$

leads to the conclusion that the Higgs mechanism is not influenced significantly by this quark condensate c^Q.

CHAPTER 5

ASYMPTOTIC FREEDOM AND HIGGS MASS

One truly may be thankful to nature for this :
limiting itself to something that we can compute.

5.1 Couplings at Asymptotic Energy Regime

As we know, the standard theory which describes the strong and electroweak interactions is based on the gauge group $SU(3) \otimes SU(2) \otimes U(1)$ [111, 112, 37, 38, 39]. This gauge group has three different couplings: the non-Abelian couplings $g_s(\equiv g_3)$, $g(\equiv g_2)$ which decrease at short distances and the Abelian coupling $g'(\equiv \sqrt{\frac{3}{5}}g_1)$which increases at short distances. Here, one has to realize that the above properties can be verified only in the energy range $< 10^2$ GeV where measurements are made. Though different grand-unified theories [1], for instance $SU(4) \otimes SU(4)$ [114] or $SU(5)$ [115], attempted to combine the $SU(3) \otimes SU(2) \otimes U(1)$ gauge group into a single gauge group with one coupling at some unification energy ($\approx 10^{15}$ GeV), there is still no real description about these couplings at asymptotic energy regimes. However, in section 3.4, we have seen that the Bose fields *fade* asymptotically and the couplings tend to *bare* constants. In the following, we shall show that these bare constants can be related as

$$g_i^2(\infty) = \frac{u}{4} \,. \tag{5.1}$$

[1]For a pedagogical introduction to grand unification, see [113].

Proof

First of all, let us recall the "purely bosonic" parts of the equations (3.212) and (3.213),

$$\begin{aligned} \mathcal{F} &:= (\mathcal{S}^{\mu}_{\mu} + 2\mathcal{S}\mathcal{S}^{\dagger}\mathcal{S})(x)\ , && (5.2)\\ \mathcal{F}_{\mu} &:= \mathcal{V}^{\rho}_{\mu\rho}(x) + \frac{3i}{2}\{\mathcal{S}^{\dagger}_{\mu}\mathcal{S} - \mathcal{S}^{\dagger}\mathcal{S}_{\mu}\}(x)\ . && (5.3)\end{aligned}$$

$$\begin{aligned} \text{Also} \quad \delta\int Tr[\mathcal{S}^{\dagger}_{\mu}\mathcal{S}^{\mu}] &= 2\int Tr[\mathcal{S}^{\dagger}_{\mu}\delta\,(\mathcal{S}^{,\mu} + i\overline{\mathcal{V}}^{\mu}\mathcal{S} - i\mathcal{S}\mathcal{V}^{\mu})] \\ &= 2\int Tr[-\mathcal{S}^{\dagger,\mu}_{\mu}\delta\mathcal{S} + i\mathcal{S}^{\dagger}_{\mu}\overline{\mathcal{V}}^{\mu}\delta\mathcal{S} + i\mathcal{S}\mathcal{S}^{\dagger}_{\mu}\delta\overline{\mathcal{V}}^{\mu} \\ &\qquad - i\mathcal{V}^{\mu}\mathcal{S}^{\dagger}_{\mu}\delta\mathcal{S} - i\mathcal{S}^{\dagger}_{\mu}\mathcal{S}\delta\mathcal{V}^{\mu}] \\ &= 2\int Tr[-\mathcal{S}^{\mu\dagger}_{\mu}\delta\mathcal{S} + i(\mathcal{S}^{\dagger}\mathcal{S}_{\mu} - \mathcal{S}^{\dagger}_{\mu}\mathcal{S})\delta\mathcal{V}^{\mu}]\ , && (5.4)\\ \delta\int Tr[\mathcal{V}_{\mu\rho}\mathcal{V}^{\mu\rho}] &= 4\int Tr[\mathcal{V}_{\mu\rho}\delta(\mathcal{V}^{\mu,\rho} + i\mathcal{V}^{\rho}\mathcal{V}^{\mu})] \\ &= 4\int Tr[\mathcal{V}^{\mu}_{\mu\rho}\delta\mathcal{V}^{\rho}]\ , && (5.5)\\ \delta\int Tr[\mathcal{S}^{\dagger}\mathcal{S}\mathcal{S}^{\dagger}\mathcal{S}] &= 4\int Tr[\mathcal{S}^{\dagger}\mathcal{S}\mathcal{S}^{\dagger}\delta\mathcal{S}]\ , && (5.6)\end{aligned}$$

where $\delta\mathcal{S}, \delta\mathcal{V}_{\rho} \in \mathcal{D}$. Hence the equations (5.2) and (5.3) can be recollected from the variation

$$\delta\mathbf{S}^{part} = -\int Tr[\frac{2}{3}\mathcal{F}^{\rho}\delta\mathcal{V}_{\rho} + \mathcal{F}^{\dagger}\delta\mathcal{S}] \qquad (5.7)$$

of the "partial" action

$$\mathbf{S}^{part} := \int Tr\,[\,\frac{1}{2}\mathcal{S}^{\dagger}_{\mu}\mathcal{S}^{\mu} - \frac{1}{2}\mathcal{S}^{\dagger}\mathcal{S}\mathcal{S}^{\dagger}\mathcal{S} - \frac{1}{6}\mathcal{V}_{\mu\rho}\mathcal{V}^{\mu\rho}\,]\;. \tag{5.8}$$

Now, in order to compare with the usual action $\mathbf{S}^{usual}$, where the Higgs and Yang-Mills components $\mathcal{S}^K, \mathcal{V}^Y$ contribute

$$\mathbf{S}^{usual} = \frac{1}{2}\int(\mathcal{S}^K_{,\mu}\mathcal{S}^{,\mu}_K - \mathcal{V}^Y_{\mu,\rho}\mathcal{V}^{\mu,\rho}_Y + \mathcal{V}^{\mu}_{Y,\mu}\mathcal{V}^{Y\rho}_{,\rho} - \ldots)\;, \tag{5.9}$$

we rewrite (5.8) as

$$\begin{aligned}\mathbf{S}^{part} \quad &\approx \quad \int(\,\frac{1}{2}Tr\,[\,\mathbf{s}^{\dagger}_K\mathbf{s}_L\,]\,\mathcal{S}^K_{,\mu}\mathcal{S}^{,\mu}_L - \frac{1}{3}Tr\,[\,\mathbf{t}_X\mathbf{t}_Y\,]\,\mathcal{V}^X_{\mu,\rho}\mathcal{V}^{\mu,\rho}_Y \\ &\qquad + \frac{1}{3}Tr\,[\,\mathbf{t}_X\mathbf{t}_Y\,]\,\mathcal{V}^{\mu}_{X,\mu}\mathcal{V}^{Y\rho}_{,\rho} + \ldots) \\ &\overset{3.149,3.150}{\approx} \quad \int(\,u\,\delta_{KL}\,\mathcal{S}^K_{,\mu}\mathcal{S}^{,\mu}_L - \frac{1}{3}t\,\delta_{YZ}\,\mathcal{V}^X_{\mu,\rho}\mathcal{V}^{\mu,\rho}_Y \\ &\qquad + \frac{1}{3}t\,\delta_{YZ}\,\mathcal{V}^{\mu}_{X,\mu}\mathcal{V}^{Y\rho}_{,\rho} + \ldots)\;.\end{aligned} \tag{5.10}$$

Hence, by comparing (5.9) and (5.10), we conclude that

$$u = \frac{t}{3}\;. \tag{5.11}$$

However, with (3.150)

$$t := 12g_i^2(\infty)\;, \tag{5.12}$$

we therefore obtain

$$g_i^2(\infty) = \frac{u}{4} . \tag{5.13}$$

■

The usual "Yukawa action" of the top quark is here measured by h and related to the top bare mass by [116]

$$m_t^2(\infty) = \frac{1}{2} h^2(\infty) v^2 . \tag{5.14}$$

But (4.11) gives

$$m_t^2(\infty) = \frac{uv^2}{6} \tag{5.15}$$

so that

$$h^2(\infty) = \frac{u}{3} . \tag{5.16}$$

Moreover, in chapter four, we have already found that the "self coupling" λ of the Higgs field is given by

$$\lambda(\infty) = 2u ; \tag{5.17}$$

thus these five "bare" couplings are connected by

$$4g_i^2(\infty) \;=\; 3h^2(\infty) \;=\; \frac{\lambda(\infty)}{2} \;=\; u \tag{5.18}$$

with a *single* empirical parameter u which will be determined in the next section.

5.2 The Sliding Couplings and Modified RGE's

5.2.1 The sliding Yang-Mill couplings

Up to this point, we know that the Higgs mass is related to $\lambda(m_Z)$ by equation (4.35) and $\lambda(\infty)$ is related to other bare coupling constants through equation (5.18). All the cards are now on the table; it remains only to find out the connection between them. The simplest guess is the usual renormalization group equations (RGE's) with some modifications. Here we need to admit that there is no compelling reason within Quantum Induction framework for us to invoke such RGE's. The main justification lies on the validity of our prediction.

At the electroweak unification scale $m_Z \approx 91.16$ GeV, the experimentally accepted values for $U(1)$, $SU(2)$ and $SU(3)$ couplings are respectively given as [21]

$$\begin{aligned} g_1^2(m_Z) &\approx \frac{4\pi}{59.0}, \\ g_2^2(m_Z) &\approx \frac{4\pi}{29.6}, \\ g_3^2(m_Z) &\approx \frac{4\pi}{8.4}. \end{aligned} \tag{5.19}$$

As we mentioned before, the coupling g_1 increases as the energy increases while the couplings g_2 and g_3 decrease with increasing energy. This trend can be extrapolated to higher energies μ from the energy of m_Z upward via the standard one-loop RGE's of the form [2]

[2]It should be noted that equation (5.20) is assumed valid at energy scale which is *far below* Planck scale. The Planck scale is the one at which the quantized gravity becomes important. It is given by the Planck mass $M_{Planck} = \sqrt{\frac{8\pi}{G_N}} \approx 6.12 \times 10^{19}$ GeV.

$$g_i^{-2}(\mu) \;=\; g_i^{-2}(m_Z) \;-\; \frac{c_i}{(4\pi)^2} ln(\frac{m_Z^2}{\mu^2}) \qquad (\text{for } m_Z < \mu \ll M_{Planck}) \;, \quad (5.20)$$

where [3]

$$c_1 \;=\; -\frac{41}{10},\; c_2 \;=\; \frac{19}{6},\; c_3 \;=\; 7\;. \quad (5.21)$$

However, it would be difficult to determine the transitions between (5.20) and (5.18) in detail. Therefore, here we just modify equation (5.20) by the simple interpolation

$$g_i^{-2}(\mu) \;\approx\; g_i^{-2}(m_Z) \;-\; \frac{c_i}{(4\pi)^2} ln\,(\frac{m_Z^2}{\mu^2} + \frac{m_Z^2}{\mu_i^2}) \quad (\text{for } m_Z < \mu < \infty) \quad (5.22)$$

with the masses μ_i = constant $\gg$ m_Z where the transitions occur. Hence, for $\mu \ll \mu_i$, (5.22) reduce to (5.20) and for $\mu \to \infty$, (5.22) approach the common asymptote given by (5.18).

Now in order to determine the u which appeared in (5.18), we take $\mu \to \infty$, so that (5.22) becomes

$$g_i^{-2}(\infty) \;\approx\; g_i^{-2}(m_Z) \;+\; \frac{c_i}{8\pi^2} ln\,(\frac{\mu_i}{m_Z})\;. \quad (5.23)$$

By using (5.18)

$$g_i^{-2}(\infty) \;=\; \frac{4}{u}\;, \quad (5.24)$$

[3]Note that the coefficients c_i depend on the matter contents of the theory. The values given at equation (5.21) are for the Standard Model.

we obtain

$$\begin{aligned} T_i(u) &:= ln(\frac{\mu_i}{m_Z}) \\ &= \frac{8\pi^2}{c_i}\{\frac{4}{u} - g_i^{-2}(m_Z)\} . \end{aligned} \tag{5.25}$$

The sketch of $T_i(u)$ in figure 5.1 shows that the three $T_i(u)$ come close together, though they do not actually meet at a sharp point. They form such small triangle that we can expect

$$u \approx 1.18 , \tag{5.26}$$

where

$$T_1(u) \approx T_2(u) \approx T_3(u) \approx T \approx 29 . \tag{5.27}$$

Hence, from (5.18), we get

$$h(\infty) = 0.625 , \tag{5.28}$$

$$\lambda(\infty) = 2.34 . \tag{5.29}$$

5.2.2 Couplings of the Higgs field

Conventionally, one finds (5.20) by integrating the differential equation

$$(4\pi)^2 \frac{dg_i}{dt} \;=\; -c_i g_i^3 \tag{5.30}$$

with

$$t \;:= ln(\frac{\mu}{m_Z}) \;. \tag{5.31}$$

Moreover, our "modified" equation (5.22) can also be recovered after intergrating the same differential equation (5.30) but with different variables τ_i. In other words, (5.22) gives

$$(4\pi)^2 \frac{dg_i}{d\tau_i} \;=\; -c_i g_i^3 \tag{5.32}$$

$$\text{with} \qquad \tau_i \;:=\; -\frac{1}{2} ln(\frac{m_Z^2}{\mu^2} + \frac{m_Z^2}{\mu_i^2}) \;=\; -\frac{1}{2} ln(e^{-2t} + e^{-2T_i}) \;. \tag{5.33}$$

Proceeding in a similar manner for the familiar one-loop RG functions of the sliding $h(\mu)$ and $\lambda(\mu)$ [116], but merely replacing the variable t by

$$\tau \;:=\; -\frac{1}{2} ln(e^{-2t} + e^{-2T}) \qquad \text{(with } T \;\approx\; 29) \;, \tag{5.34}$$

which agrees with (5.33) except that the average value T of T_1, T_2, T_3 is used. Then we obtain our "modified" equations for $h(\mu)$ and $\lambda(\mu)$,

$$
\begin{aligned}
(8\pi)^2 \frac{dh}{d\tau} &= (\, 18h^2 - 32g_3^2 - 9g_2^2 - \frac{17}{5}g_1^2 \,)h \;, \qquad (5.35)\\
(2\pi)^2 \frac{d\lambda}{d\tau} &= \lambda^2 + 3\lambda h^2 - 9h^4 - \frac{9}{4}(\, g_2^2 + \frac{1}{5}g_1^2 \,)\lambda \\
&\quad + \frac{27}{40}(\, \frac{5}{2}g_2^4 + g_2^2 g_1^2 + \frac{3}{10}g_1^4 \,) \quad (\text{ for } \tau > 0\,) \;. \qquad (5.36)
\end{aligned}
$$

The $g_i(\mu)$ from (5.22) enter here with

$$
\mu \overset{5.31,5.34}{=} \frac{m_Z}{\sqrt{e^{-2\tau} - e^{-2T}}} \qquad (\text{where } T \approx 29) \;. \qquad (5.37)
$$

Note that $\tau = T$ and $\tau = 0$ correspond to $\mu = \infty$ and $\mu = m_Z$ repectively. Now in order to find out $h(m_Z)$ and $\lambda(m_Z)$, equations (5.36), (5.37) are treated *numerically* and are integrated from $\tau = T$ downwards until $\tau = 0$ is reached. By using (5.28) and (5.29), we thus conclude

$$
\begin{aligned}
h(m_Z) &\approx 1.03 \;, \qquad (5.38)\\
\lambda(m_Z) &\approx 1.74 \;. \qquad (5.39)
\end{aligned}
$$

5.3 Top and Higgs Masses

Since the observed top mass is given by [116]

$$m_t = \frac{h(m_Z)v}{\sqrt{2}} , \tag{5.40}$$

with (5.38), we obtain

$$m_t \approx 179\,\mathrm{GeV} \tag{5.41}$$

which agrees with the observed value [21].

For the Higgs mass, using (4.35) and (5.39), we predict

$$m_H \approx 190\,\mathrm{GeV} . \tag{5.42}$$

However, it should be noted that our calculated $h(m_Z)$ and $\lambda(m_Z)$ depend on where we take the values u and T. In other words,

$$\begin{aligned} m_t &= m_t(u,T) , \\ m_H &= m_H(u,T) . \end{aligned} \tag{5.43}$$

Therefore, we can estimate the uncertainties of our predictions by contour plots of $m_t(u,T)$ and $m_H(u,T)$ in the (u,T)-plane. In figure 5.1, the top quark masses are indicated by dashed lines while the solid lines are for Higgs scalar masses. When we ask for those Higgs masses which due to figure 5.1 are compatible with most of the triangle formed by the curves $T_i(u)$, we find

$$180\,\mathrm{GeV} \;<\; m_H \;<\; 200\,\mathrm{GeV}\;. \tag{5.44}$$

In a similar manner, our estimate of the top quark mass becomes

$$165\,\mathrm{GeV} \;<\; m_t \;<\; 195\,\mathrm{GeV}\;. \tag{5.45}$$

Finally, let us make a summary by using fig. 5.2. Since in Quantum Induction, the bare couplings approach constants at asymptotics energies, so our modified RGE's for $g_i^{-2}(\mu)$ can be devised. With the help of the observed $g_i^{-2}(m_Z)$ given in (5.19), u can be estimated. Thus $\lambda(\infty)$ and $h(\infty)$ will then be known through our connection of couplings given in (5.18). Inspired by the modified RGE's for $g_i^{-2}(\mu)$, we can construct similar modified RGE's for $\lambda(\mu)$ and $h(\mu)$, hence $\lambda(m_Z)$, $h(m_Z)$ can be estimated and in turns top quark and Higgs masses are calculated.

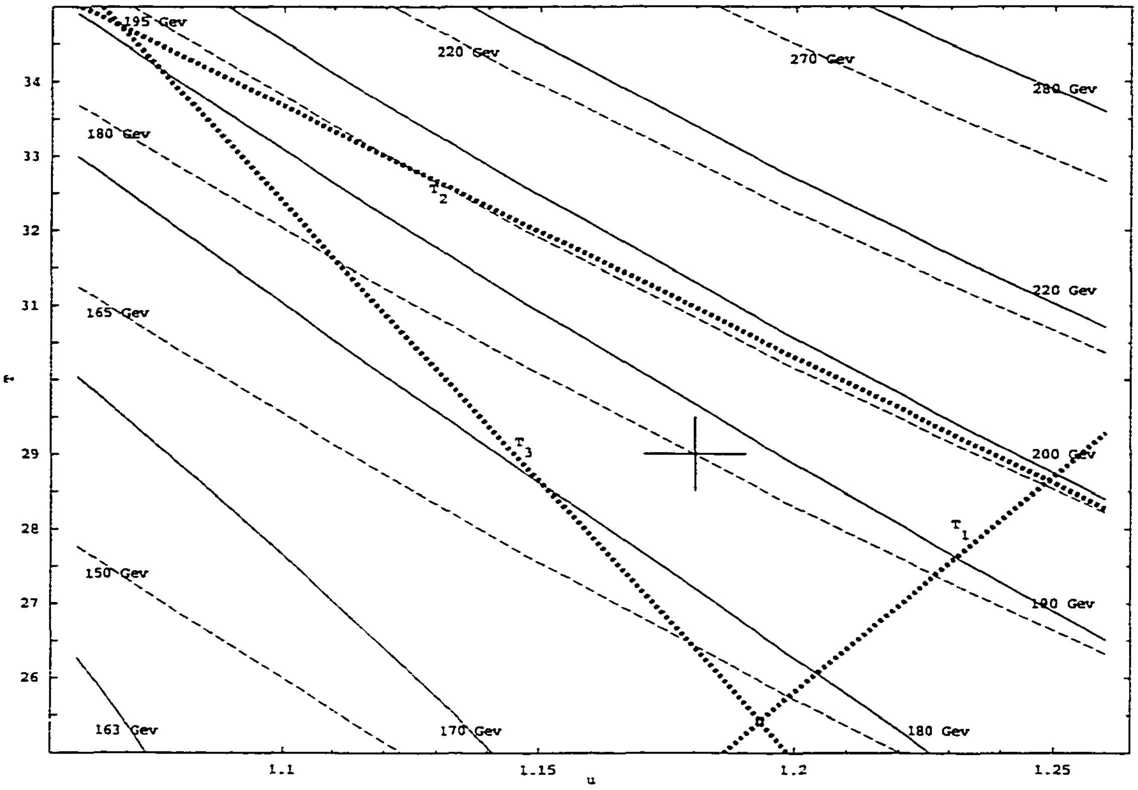

Figure 5.1: The figure shows the contour plots of $m_t(u,T)$ and $m_H(u,T)$. The curves $T_i(u)$ are plotted by using (5.25). They form a small triangle which suggests our chosen u (5.26) and the average T (5.27). The solid and dashed lines indicate the Higgs masses $m_H(u,T)$ and top quark masses $m_t(u,T)$ respectively of the contour plots in the (u,T)-plane.

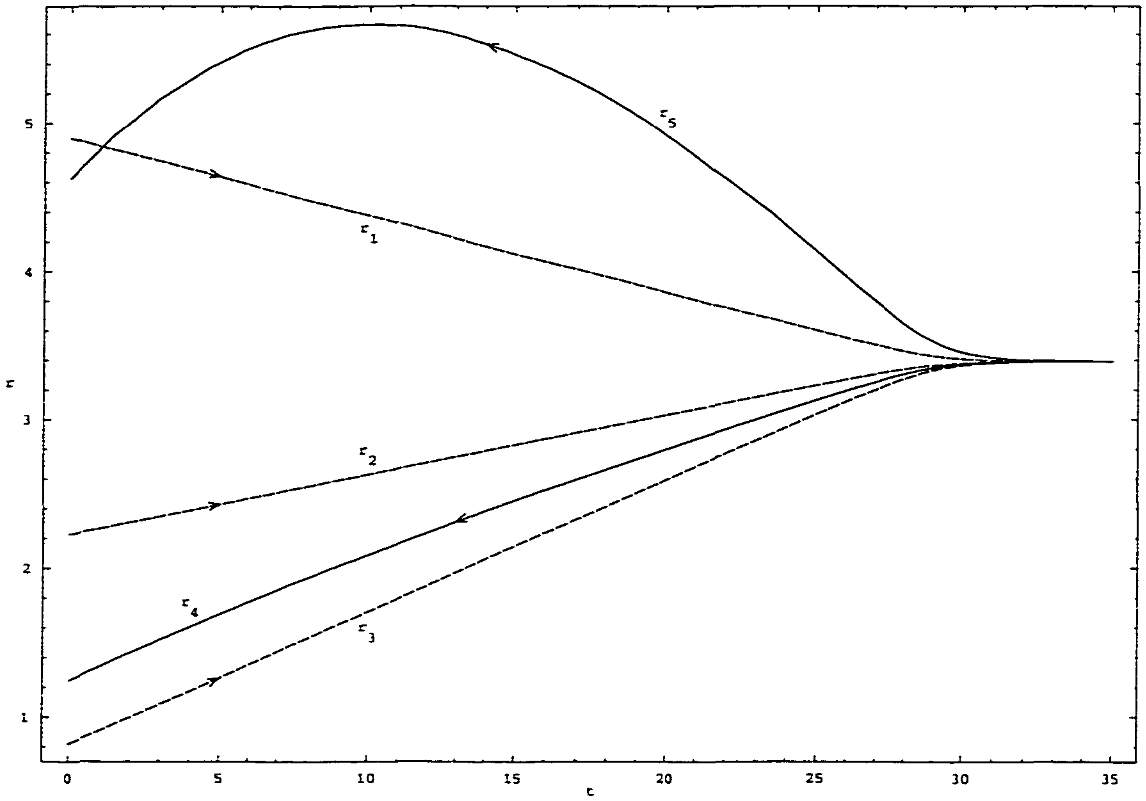

Figure 5.2: We plot $r_i \equiv g_i^{-2}$, $r_4 \equiv \frac{3}{8}h^{-2}$ and $r_5 \equiv 8\lambda^{-1}$ as functions of $t = ln(\frac{\mu}{m_Z})$. The arrows indicate the logical procedure of our calculation of Higgs and top quark masses.

APPENDIX A

ACTION PRINCIPLE FOR QUANTUM INDUCTION

In this appendix, we shall see how postulates (3.2) and (3.3) relate to the action principle

$$T(\mathcal{P}\delta\mathbf{S} - i\delta\mathcal{P}) = 0 \tag{A.1}$$

with

$$\mathbf{S} := \int dx\, \overline{\Psi}(i\partial\!\!\!/ - \mathcal{B})\Psi \qquad (\text{for } \delta\Psi, \delta\overline{\Psi} \in \mathcal{D}) \tag{A.2}$$

which may be regarded as our primary postulate. Before we show the derivation, let us discuss an important technical point related to the commutability of the time ordering operator T.

A.1 Time Ordering and Commutability

As usual, the time ordering operation T is defined to order any number of local fields chronologically by putting 'later' operators to the left and 'earlier' ones to the right. For instance, using the step functions, the time ordered product of two local fields can be written [1]

$$T(\phi_1(x)\phi_2(y)) := \theta(x^0 - y^0)\phi_1(x)\phi_2(y) \pm \theta(y^0 - x^0)\phi_2(y)\phi_1(x) \ . \tag{A.3}$$

However, we must be careful when differentiated operators are present in time ordering. Nambu [117] has shown that the differentiation should be carried out *before* (*after*) the time ordering T in *Hamiltonian* (*Lagrangian*) formalism. In other words, the operators ∂_μ and T in Lagrangian formalism must be related by [2]

$$T\partial_\mu \ := \ \partial_\mu T \tag{A.4}$$

such that for any fields $\phi(x)$ and $\chi(y)$

$$T\left[\phi_{,\mu}(x)\chi(y)\right] \ := \ \partial_\mu^x T\left[\phi(x)\chi(y)\right] \ . \tag{A.5}$$

Here we take "scalar electrodynamics" [122, 123], i.e. the theory of interacting Klein-Gordon and Maxwell fields, as an illustration of the above-mentioned ideas by working out the lowest order perturbation expansion of the theory explicitly.

The Lagrangian of "scalar electrodynamics" is given by

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{em} + \mathcal{L}_{int} \ , \tag{A.6}$$

where

[1] The minus sign is for Fermi fields.

[2] In some literature, the notation T^* [118, 119] or $\hat{T}$ [120] is used to denote the present T.

$$\mathcal{L}_0 = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi \,, \tag{A.7}$$

$$\mathcal{L}_{em} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\mu^2}{2} A_\sigma A^\sigma - \frac{\zeta}{2} (\partial_\sigma A^\sigma)^2 \tag{A.8}$$

[3] are the free Lagrangian density of a complex Klein-Gordon field and Maxwell field respectively. As usual, by performing the minimal coupling prescription

$$\partial_\mu \longrightarrow \partial_\mu + ieA_\mu \,, \tag{A.9}$$

the interaction Lagrangian density can be obtained by

$$\mathcal{L}_{int} = -ieA^\mu (\phi^* \overleftrightarrow{\partial}_\mu \phi) + e^2 A_\mu A^\mu \phi^* \phi \,, \tag{A.10}$$

[4] and hence the canonical momenta are

$$\pi^* := \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi^*)} = \partial^0 \phi + ieA^0 \phi \,, \tag{A.11}$$

$$\pi := \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)} = \partial^0 \phi^* - ieA^0 \phi^* \,. \tag{A.12}$$

Making use of (A.11) and (A.12), the Hamiltonian density then reads

$$\mathcal{H} := \pi \dot{\phi} + \pi^* \dot{\phi}^* - \mathcal{L} \tag{A.13}$$

$$= \mathcal{H}_0 + \mathcal{H}_{em} + \mathcal{H}_{int} \,, \tag{A.14}$$

[3] μ^2 is denoted as the photon mass squared here, and the last term of (A.8) is called a gauge-fixing term.

[4] Note that $\overleftrightarrow{\partial}_\mu := \partial_\mu - \overleftarrow{\partial}_\mu$.

where

$$\begin{aligned}
\mathcal{H}_0 &= (\pi^*\pi + \nabla\phi^*\cdot\nabla\phi + m^2\phi^*\phi) && \text{(A.15)}\\
\mathcal{H}_{int} &= ieA^0(\pi^*\phi^* - \pi\phi) + ieA^k\phi^*\overleftrightarrow{\partial}_k\phi - e^2A_\mu A^\mu\phi^*\phi + e^2(A^0)^2\phi^*\phi && \text{(A.16)}\\
&= -\mathcal{L}_{int} - e^2(A^0)^2\phi^*\phi\,. && \text{(A.17)}
\end{aligned}$$

A quantization of the theory is as usual achieved by going over to field operators $\hat{\phi}$, $\hat{\phi}^\dagger$, $\hat{\pi}$, $\hat{\pi}^\dagger$, $\hat{A}_\mu$. For the scalar fields, these are required to satisfy the commutation relations at equal time,

$$\begin{aligned}
[\hat{\phi}(\vec{x},t),\hat{\pi}(\vec{x}',t)] &= [\hat{\phi}^\dagger(\vec{x},t),\hat{\pi}^\dagger(\vec{x}',t)]\\
&= i\delta^3(\vec{x}-\vec{x}') && \text{(A.18)}
\end{aligned}$$

while the other equal-time commutators are required to vanish.

As a starting point for the development of a perturbation theory one performs a canonical transformation

$$\hat{\mathcal{O}}(t) \longrightarrow \hat{\mathcal{U}}(t)\hat{\mathcal{O}}(t)\hat{\mathcal{U}}^{-1}(t)\,, \qquad \text{(A.19)}$$

with [5]

$$\hat{\mathcal{U}}(t) = e^{i\hat{H}_0^s t}e^{-i\hat{H}t}\,, \qquad \text{(A.20)}$$

[5]The superscript s indicates that the operator $\hat{H}_0^s$ is in Schrodinger picture, and it is related to its counterpart in Heisenberg picture by $\hat{H}_0^h = e^{i\hat{H}t}\hat{H}_0^s e^{-i\hat{H}t}$.

$$\hat{H}_0^s := \int d^3x \, \hat{\mathcal{H}}_0^s \,, \tag{A.21}$$

$$\hat{H} := \int d^3x \, \hat{\mathcal{H}} \,, \tag{A.22}$$

from the Heisenberg representation to the interaction representation. Using (A.18) and the equation of motion for a Heisenberg field operator

$$\frac{\partial \hat{\phi}}{\partial t} = -i[\hat{\phi}, \hat{H}] \,, \tag{A.23}$$

one can easily derive the canonical momenta in the interaction representation [6]

$$\hat{\pi}^\dagger = \partial^0 \hat{\phi} \,, \tag{A.24}$$

$$\hat{\pi} = \partial^0 \hat{\phi}^\dagger \,. \tag{A.25}$$

Thus, equipped with (A.24) (A.25), the interaction Hamiltonian (A.16) in this representation reads

$$\begin{aligned} \hat{\mathcal{H}}_{int} &= ie\hat{A}^0(\hat{\pi}^\dagger\hat{\phi}^\dagger - \hat{\pi}\hat{\phi}) + ie\hat{A}^k(\hat{\phi}^\dagger \overleftrightarrow{\partial}_k \hat{\phi}) - e^2\hat{A}_\mu\hat{A}^\mu\hat{\phi}^\dagger\hat{\phi} + e^2(\hat{A}^0)^2\hat{\phi}^\dagger\hat{\phi} & \text{(A.26)} \\ &= ie\hat{A}^\mu(\hat{\phi}^\dagger \overleftrightarrow{\partial}_\mu \hat{\phi}) - e^2\hat{A}_\mu\hat{A}^\mu\hat{\phi}^\dagger\hat{\phi} + e^2(\hat{A}^0)^2\hat{\phi}^\dagger\hat{\phi} & \text{(A.27)} \\ &= -\hat{\mathcal{L}}_{int} + e^2(\hat{A}^0)^2\hat{\phi}^\dagger\hat{\phi} \,. & \text{(A.28)} \end{aligned}$$

Hence the normal ordered Interaction Hamiltonian density, the basis of perturbation expansion, is found to be

[6] It is important to keep in mind that the operators discussed below are in the interaction picture.

$$\hat{\mathcal{H}}_{int} = ie :\hat{A}^{\mu}(\hat{\phi}^{\dagger}\overleftrightarrow{\partial}_{\mu}\hat{\phi}): - e^2 :\hat{A}_{\mu}\hat{A}^{\mu}\hat{\phi}^{\dagger}\hat{\phi}: + e^2 :(\hat{A}^0)^2\hat{\phi}^{\dagger}\hat{\phi}: \ . \tag{A.29}$$

Note that the last term in $\hat{\mathcal{H}}_{int}$ is noncovariant. Now, consider the 1st order of $\hat{S}$ matrix of the theory

$$\begin{aligned} \hat{S}^{(1)} &= -i\int d^4x\, T[\hat{\mathcal{H}}_{int}(x)] & \text{(A.30)} \\ &= \int d^4x \{ e :\hat{A}^{\mu}(\hat{\phi}^{\dagger}\overleftrightarrow{\partial}_{\mu}\hat{\phi}): + ie^2 :\hat{A}_{\mu}\hat{A}^{\mu}\hat{\phi}^{\dagger}\hat{\phi}: - ie^2 :(\hat{A}^0)^2\hat{\phi}^{\dagger}\hat{\phi}: \} \ . & \text{(A.31)} \end{aligned}$$

The last term is noncovariant and represents an unobservable processes which should be eliminated. In the following, we shall see this noncovariant term will be cancelled by a term which comes from the 2nd order expansion of $\hat{S}$ matrix

$$\hat{S}^{(2)} = \frac{(-i)^2}{2!}\int d^4x d^4y\, T[\hat{\mathcal{H}}_{int}(x)\hat{\mathcal{H}}_{int}(y)] \ , \tag{A.32}$$

namely,

$$\begin{aligned} \hat{S}^{(2)}_p = e^2 \int d^4x d^4y \ :\hat{A}^{\mu}(x)\hat{A}^{\nu}(y): \ :\{ &\hat{\phi}^{\dagger}(x)\langle 0|T[\partial_{\mu}\hat{\phi}(x)\hat{\phi}^{\dagger}(y)]|0\rangle\partial_{\nu}\hat{\phi}(y) \\ &+ \partial_{\mu}\hat{\phi}^{\dagger}(x)\langle 0|T[(\hat{\phi}(x)\partial_{\nu}\hat{\phi}^{\dagger}(y)]|0\rangle\hat{\phi}(y) \\ &- \partial_{\mu}\hat{\phi}^{\dagger}(x)\langle 0|T[\hat{\phi}(x)\hat{\phi}^{\dagger}(y)]|0\rangle\partial_{\nu}\hat{\phi}(y) \\ &- \hat{\phi}^{\dagger}(x)\langle 0|T[\partial_{\mu}\hat{\phi}(x)\partial_{\nu}\hat{\phi}^{\dagger}(y)]|0\rangle\hat{\phi}(y) \}: \ . \end{aligned} \tag{A.33}$$

As

$$\langle 0|T[\hat{\phi}(x)\hat{\phi}^{\dagger}(y)]|0\rangle = i\triangle_F(x-y) \tag{A.34}$$

$$= i\int \frac{d^4p}{(2\pi)^4}\frac{e^{-ip(x-y)}}{p^2-m^2+i\varepsilon}\,, \tag{A.35}$$

where $\triangle_F(x-y)$ is the Feynman propagator. Hence, making use of (A.3),

$$\begin{aligned}\partial^y_\nu\langle 0|T[\hat\phi(x)\hat\phi^\dagger(y)]|0\rangle &= \partial^y_\nu(\langle 0|\theta(x^0-y^0)\hat\phi(x)\hat\phi^\dagger(y)|0\rangle \\ &\quad + \langle 0|\theta(y^0-x^0)\hat\phi^\dagger(y)\hat\phi(x)|0\rangle) \\ &= \langle 0|T[\hat\phi(x)\partial_\nu\hat\phi^\dagger(y)]|0\rangle \\ &\quad - g_{\nu 0}\delta(x_0-y_0)\langle 0|[\hat\phi(x),\hat\phi^\dagger(y)]_-|0\rangle \\ &= \langle 0|T[\hat\phi(x)\partial_\nu\hat\phi^\dagger(y)]|0\rangle \end{aligned} \tag{A.36}$$

since

$$[\hat\phi(x),\hat\phi^\dagger(y)]_- = \int \frac{d^3p}{(2\pi)^3 2\omega_p}(e^{-ip(x-y)}-e^{ip(x-y)})\,, \tag{A.37}$$

and for $x_0=y_0$, the integrand (A.37) is an odd function in $\vec{\mathbf{p}}$; thus it vanishes.

Moreover, making use of (A.36) and (A.37) gives

$$\begin{aligned}\partial^x_\mu\partial^y_\nu\langle 0|T[\hat\phi(x)\hat\phi^\dagger(y)]|0\rangle &= \partial^x_\mu(\theta(x_0-y_0)\langle 0|\hat\phi(x)\partial_\nu\hat\phi^\dagger(y)|0\rangle \\ &\quad + \theta(y_0-x_0)\langle 0|\partial_\nu\hat\phi^\dagger(y)\hat\phi(x)|0\rangle) \\ &= \langle 0|T[\partial_\mu\hat\phi(x)\partial_\nu\hat\phi^\dagger(y)]|0\rangle \\ &\quad + g_{\mu 0}\delta(x_0-y_0)\partial^y_\nu\langle 0|[\hat\phi(x),\hat\phi^\dagger(y)]|0\rangle \\ &= \langle 0|T[\partial_\mu\hat\phi(x)\partial_\nu\hat\phi^\dagger(y)]|0\rangle + ig_{\mu 0}g_{\nu 0}\delta^4(x-y) \end{aligned} \tag{A.38}$$

Or

$$\langle 0|T[\partial_\mu\hat\phi(x)\partial_\nu\hat\phi^\dagger(y)]|0\rangle = \partial^x_\mu\partial^y_\nu\langle 0|T[\hat\phi(x)\hat\phi^\dagger(y)]|0\rangle - ig_{\mu 0}g_{\nu 0}\delta^4(x-y)\,. \tag{A.39}$$

Substitituting (A.36), (A.39) into (A.33), we obtain

$$\hat{S}_p^{(2)} = e^2 \int d^4x d^4y \, :\hat{A}^\mu(x)\hat{A}^\nu(y): \, :\hat{\phi}^\dagger(x) \overleftrightarrow{\partial}_\mu^x \, i\triangle_F(x-y) \overleftrightarrow{\partial}_\nu^y \hat{\phi}(y): + ie^2 \int d^4x \, :(\hat{A}^0(x))^2 \hat{\phi}^\dagger(x)\hat{\phi}(x): \; . \tag{A.40}$$

The last term (noncovariant) just cancels the one apperaed in (A.31). Although our proof is in lowest-order, but those noncovariant terms will always be cancelled in the theory [124, 125, 126]. More importantly, we note that the noncovariant term appeared in $\hat{S}^{(1)}$ (A.31) is due to the noncovariant term in the Hamiltonian density (A.28). On the other hand, the noncovariant term appearing in $\hat{S}_p^{(2)}$ (A.40) is due to the noncommutability of T operator and ∂_μ operator in (A.38).

Since the $\hat{S}$ matrix is covariant, so if the Hamiltonian density (A.28) is replaced by a covariant expansion

$$\hat{\mathcal{H}}_{int} = -\hat{\mathcal{L}}_{int} \; , \tag{A.41}$$

i.e. in Lagrangian formalism, we still get the correct result if at the same time T satifies

$$T\partial_\mu := \partial_\mu T \; . \tag{A.42}$$

A.2 The Action Principle

We start from the action principle [7] with Dirac field $\Psi(x)$ which describes the 24 leptons and quarks as discussed in section (3.1)

$$T[\mathcal{P}\delta\mathbf{S} - i\delta\mathcal{P}] = 0 \tag{A.43}$$

with action [8]

$$\mathbf{S} := \int dx\overline{\Psi}(x)(i\partial\!\!\!/ - \mathcal{B}(x))\Psi(x) \qquad (\text{for } \delta\Psi, \delta\overline{\Psi} \in \mathcal{D}) \ . \tag{A.44}$$

Here $\mathcal{P} \in \mathcal{A}$ is any polynomial of the smeared field [9]

$$C + \sum_{i=1}^{n} \Psi[f_1^{(i)}] \ldots \Psi[f_i^{(i)}] \tag{A.45}$$

where $f \in \mathcal{D}$ are the test functions, C is any complex constant and as usual

$$\partial\!\!\!/ := \gamma^\mu \frac{\partial}{\partial x^\mu} \ . \tag{A.46}$$

For $\mathcal{P}$ = constant, (A.43) demands that

$$\delta\mathbf{S} = 0 \ . \tag{A.47}$$

[7] A general discussion about the relation between the Feynman's path integral and the action principle which is known as 'canonical postulate' can be found in [121, 127].

[8] Here $\mathbf{S}$ is a functional of the quantum fields Ψ to be determined, rather than of classical fields to be "quantized" later.

[9] See section 3.1

Hence , with (A.44) follows that [10]

$$\int dx\ \delta\overline{\Psi}(i\partial\!\!\!/ - \mathcal{B})\Psi - \int dx\ \overline{\Psi}(i\overleftarrow{\partial\!\!\!/} + \mathcal{B})\delta\Psi - \int dx\ \overline{\Psi}\delta\mathcal{B}\Psi = 0\ . \tag{A.48}$$

As $\delta\overline{\Psi}$, $\delta\Psi$ and δB are assumed to be independent, we obtain Dirac's equation

$$(i\partial\!\!\!/ - \mathcal{B})\Psi = 0 \tag{A.49}$$

and its adjoint

$$\overline{\Psi}(i\overleftarrow{\partial\!\!\!/} + \mathcal{B}) = 0\ . \tag{A.50}$$

Now, we take $\mathcal{P} = \Psi(0)$ with (A.44); then (A.43) becomes

$$\begin{aligned} T\int dx\ \Psi(0)\delta\overline{\Psi}(x)(i\partial\!\!\!/^x - \mathcal{B}(x))\Psi(x) \\ -T\int dx\ [\Psi(0)\overline{\Psi}(x)(i\overleftarrow{\partial\!\!\!/^x} + \mathcal{B}(x)) + i\delta(x)]\delta\Psi(x) = 0\ . \end{aligned} \tag{A.51}$$

Since $\delta\Psi, \delta\overline{\Psi} \in \mathcal{D}$ are independent and arbitrary, (A.51) demands [11]

[10] In contrast to the usual theory where $\overline{\Psi} := \Psi^\dagger\gamma_0$ is assumed from the start and one lets Ψ and $\overline{\Psi}$ vary independently without any explanation (see, for instance, [77]), here $\Psi(x)$ and $\overline{\Psi}(x)$ are treated as two different fields initially so that their variations are independent. However, we shall find that $\overline{\Psi} = \Psi^\dagger\gamma_0$ later by demanding the consistency of the Dirac's equation (A.49) and its adjoint (A.50) (See section 3.2). Moreover, it should be noted that the variations $\delta\Psi$, $\delta\overline{\Psi}$ are infinitely differentiable and of compact support (i.e. $\delta\Psi, \delta\overline{\Psi} \in \mathcal{D}$) and non-quantized. It is because (A.43) is to define the basic quantum field Ψ, therefore no quantum field has been defined earlier. More justifications about this can be found in ref.[128].

[11] $\mathbf{1}$ stands for Dirac unit matrix, but for brevity it will be dropped in the following.

$$T\,[\,(i\partial\!\!\!/^x - \mathcal{B}(x))\Psi(x)\Psi^T(0)\,] \;=\; 0\;, \tag{A.52}$$

$$-iT\,[\,\Psi(0)\overline{\Psi}(x)(i\overleftarrow{\partial\!\!\!/^x} + \mathcal{B}(x))\,] \;=\; \delta(x)\mathbf{1}\;. \tag{A.53}$$

With spinor indices, (A.52) can be written

$$T\,[\,(i\partial\!\!\!/^x - \mathcal{B}(x))^{\alpha\zeta}\Psi^\zeta(x)\Psi^\eta(0)\,] \;=\; 0^{\alpha\eta}\;. \tag{A.54}$$

Invoking the property of the time ordering (A.42), we can write (A.54)

$$(i\partial\!\!\!/^x - \mathcal{B}(x))^{\alpha\zeta}T\,[\,\Psi^\zeta(x)\Psi^\eta(0)\,] \;=\; 0^{\alpha\eta}\;. \tag{A.55}$$

As

$$T\,[\,\Psi^\zeta(x)\Psi^\eta(0)\,] \;=\; \theta(x^0)\Psi^\zeta(x)\Psi^\eta(0) \;-\; \theta(-x^0)\Psi^\eta(0)\Psi^\zeta(x)\;, \tag{A.56}$$

it follows that

$$\begin{aligned}\{\partial\!\!\!/^x\theta(x^0)\}\Psi^\zeta(x)\Psi^\eta(0) + \theta(x^0)\{\partial\!\!\!/^x\Psi^\zeta(x)\}\Psi^\eta(0) - \{\partial\!\!\!/^x\theta(-x^0)\}\Psi^\eta(0)\Psi^\zeta(x)\\ -\theta(-x^0)\Psi^\eta(0)\{\partial\!\!\!/^x\Psi^\zeta(x)\} + T[i\mathcal{B}(x)\Psi^\zeta(x)\Psi^\eta(0)] = 0^{\zeta\eta}\;.\end{aligned} \tag{A.57}$$

Using Dirac's equations (A.49) and (A.50), the second term and the fourth term cancel with the fifth term, thus

$$\begin{aligned}
\gamma^0\delta(x^0)\{\Psi^\zeta(x)\Psi^\eta(0)+\Psi^\eta(0)\Psi^\zeta(x)\} &= 0^{\zeta\eta} \\
\Rightarrow \qquad \gamma^0\delta(x^0)[\Psi^\zeta(x),\Psi^\eta(0)]_+ &= 0^{\zeta\eta} \\
\Rightarrow \qquad \gamma^0\delta(x^0)[\Psi(x),\Psi^T(0)]_+^{\zeta\eta} &= 0^{\zeta\eta} \ ,
\end{aligned} \tag{A.58}$$

or

$$[\Psi(x),\Psi^T(0)]_+\delta(x^0)=0 \ , \tag{A.59}$$

which is one of the canonical equal time relations.

Moreover, taking the adjoint of (A.53) with using (3.30), (3.31),we get

$$T\,[\,(\not{\partial}^x+i\mathcal{B}(x))\Psi(x)\overline{\Psi}(0)\,] \;=\; \delta(x) \ . \tag{A.60}$$

Proceeding similarly as above, we obtain the further canonical relation

$$[\Psi(x),\Psi^\dagger(0)]_+\delta(x^0)=\delta(x) \ . \tag{A.61}$$

APPENDIX B

SHORT DISTANCE EXPANSION

(I). In this appendix, a formal derivation of the expansion

$$\begin{aligned} F(z^2,\zeta) &:= \frac{i}{\pi^2}\int \frac{e^{-2ilz}\,dl}{l^2 + \zeta} \qquad\qquad (B.1)\\ &= -z^{-2} + \zeta\{1 - ln(e^{2\gamma}\zeta z^2)\} + \frac{5}{4}\zeta^2 z^2 \\ &\qquad - \frac{1}{2}\zeta^2 z^2 ln(e^{2\gamma}\zeta z^2) + \ldots\ldots \qquad (\text{with } \gamma \approx 0.577)(B.2) \end{aligned}$$

is given. First of all, it should be noted that the function $F(z^2,\zeta)$ can be expressed in terms of a single variable function $f(\zeta s)$,

i.e. $$F(z^2,\zeta) = \zeta f(\zeta s) \qquad (\text{where } s := z^2)\ . \qquad (B.3)$$

Proof

We can check (B.3) by the following arguments.

$$\begin{aligned} F(a^2 z^2,\zeta) &= \frac{i}{\pi^2}\int \frac{e^{-2ilaz}\,dl}{l^2 + \zeta} \\ &= \frac{i}{a^2\pi^2}\int \frac{e^{-2ipz}\,dp}{p^2 + a^2\zeta} \\ &= \frac{1}{a^2}\,F(z^2,a^2\zeta) \end{aligned}$$

$$\stackrel{B.3}{\Rightarrow} \qquad \zeta f(a^2\zeta s) = (\frac{1}{a^2})\,(a^2\zeta)\,f(a^2\zeta s)$$
$$= \zeta f(a^2\zeta s)\;. \tag{B.4}$$

■

Moreover, the function $f(\zeta s)$ satisfies the following equation

$$\zeta s f''(\zeta s) \;+\; 2f'(\zeta s) \;-\; f(\zeta s) \;=\; 0 \qquad (s \neq 0)\;. \tag{B.5}$$

Proof

As usual,

$$\begin{aligned} s &:= z^2 \\ &= z_\mu z^\mu \\ &= z_0^2 \;-\; \vec{z}^{\,2}\;. \end{aligned} \tag{B.6}$$

Hence,

$$\begin{aligned} \Box f(\zeta s) &= \partial_\mu \{\, \partial^\mu f(\zeta z^2)\,\} \\ &= \zeta\,\partial_\mu \{\, f'(\zeta z^2)\cdot \partial^\mu z^2\,\} \\ &= \{\, \partial_\mu(2z^\mu)\,\}\,\zeta f'(\zeta z^2) \;+\; 2\zeta z^\mu \partial_\mu f'(\zeta z^2) \\ &= 2\,\zeta\,\delta^\mu_\mu f'(\zeta z^2) \;+\; (2)(2)\zeta^2 z_\mu z^\mu f''(\zeta z^2) \\ &= 8\zeta f'(\zeta s) \;+\; 4\zeta^2 s f''(\zeta s)\;, \end{aligned} \tag{B.7}$$

$$\begin{aligned} \text{or} \quad \zeta s f''(\zeta s) \;+\; 2f'(\zeta s) \;-\; f(\zeta s) &= \frac{1}{4\zeta}(\Box \;-\; 4\zeta) f(\zeta s) \\ &\stackrel{B.1}{=} \frac{i}{4\zeta^2\pi^2}(\Box \;-\; 4\zeta)\int \frac{e^{-2ilz}\,dl}{l^2 \;+\; \zeta} \\ &= \frac{-i}{\zeta^2\pi^2}\int e^{-2ilz}\,dl \\ &= \frac{-i}{\zeta^2\pi^2}(2\pi)^4\delta^4(2z) \\ &= \frac{-i\pi^2}{\zeta^2}\delta^4(z)\;. \end{aligned} \tag{B.8}$$

■

By comparing with the differential equations satisfied by the special Bessel function $k_1(\eta)$ [129]

$$\eta k_1''(\eta) \; + \; 2k_1'(\eta) \; - \; k_1(\eta) \; = \; 0 \; , \tag{B.9}$$

we may conclude that $F(z^2, \zeta)$ is equal to $k_1(\zeta z^2)$ up to a normalization factor,

$$\text{i.e.} \qquad F(z^2, \zeta) \; \sim \; k_1(\zeta z^2) \; . \tag{B.10}$$

In order to find out the normalization factor, we need to invoke the fact that

$$F(z^2, 0) \; = \; -\frac{1}{z^2} \; . \tag{B.11}$$

Proof

In the following, our evaluation will be done in Euclidean space with the Euclidean 4-vector variable z_E and 4-momentum variable l_E defined as

$$\begin{aligned}
z &:= (\, iz_E^0 \, , \, \mathbf{z}_E \,) \; , && \text{(B.12)} \\
l &:= \frac{1}{2}(\, il_E^0 \, , \, \mathbf{l}_E \,) \; , && \text{(B.13)} \\
\text{and} \qquad lz &= -\frac{1}{2} l_E z_E \; , && \text{(B.14)} \\
l^2 &= -\frac{1}{4} l_E^2 \; , && \text{(B.15)} \\
dl &= \frac{1}{16} l dl_E \; . && \text{(B.16)}
\end{aligned}$$

Hence,

$$
\begin{aligned}
F(z^2,0) &= \frac{1}{4\pi^2}\int \frac{e^{il_E z_E} dl_E}{\frac{1}{4}l_E^2 - \zeta}\Big|_{\zeta=0} \\
&= \frac{1}{(2\pi)^2}\int \frac{e^{il_E z_E} dl_E}{l_E^2} \\
&= \frac{c}{z_E^2} \qquad\qquad > 0\,.
\end{aligned} \tag{B.17}
$$

In order to determine the constant c, let us first take the inverse Fourier transform of (B.17), obtaining

$$
\begin{aligned}
& \int e^{-iz_E l_E}(2\pi)^2 c\, z_E^{-2} dz_E &=& \;(2\pi)^4 l_E^{-2} \\
\Rightarrow\quad & \int e^{-iz_E l_E}(2\pi)^2 z_E^{-2} dz_E &=& \;(2\pi)^4 c^{-1}\, l_E^{-2}\,.
\end{aligned} \tag{B.18}
$$

Since (B.17) can be rewritten as

$$\int e^{iz_E l_E} z_E^{-2} dz_E = (2\pi)^4 c\, l_E^{-2}\,, \tag{B.19}$$

by taking complex conjugate of (B.19), we then find

$$\int e^{-iz_E l_E} z_E^{-2} dz_E = (2\pi)^4 c^\dagger\, l_E^{-2}\,. \tag{B.20}$$

Thus, from (B.18) and (B.20), we know

$$c^{-1} = c^\dagger \qquad > 0\,, \tag{B.21}$$

or

$$c = 1 \, . \tag{B.22}$$

So

$$\begin{aligned} F(z^2, 0) &= \frac{1}{z_E^2} \\ &= -\frac{1}{z^2} \, . \end{aligned} \tag{B.23}$$

■

Now, with (B.10), (B.11) and the following property of $k_1(\zeta z^2)$ [129]

$$\lim_{\zeta z^2 \to 0} 2\zeta z^2 k_1(\zeta z^2) = 1 \, , \tag{B.24}$$

we find

$$F(z^2, \zeta) = -2\zeta k_1(\zeta z^2) \, , \tag{B.25}$$

where $k_1(\zeta z^2)$ is given by [129]

$$k_1(\zeta z^2) = (\frac{1}{2} + \frac{1}{4}\zeta z^2 + \frac{1}{24}\zeta z^4) ln(e^{2\gamma}\zeta z^2) + \frac{1}{2\zeta} z^{-2} - \frac{1}{2} - \frac{5}{8}\zeta z^2 + \ldots \tag{B.26}$$

By inserting (B.26) into (B.25), we then have (B.2).

(II). The following expansions

$$z^2 \int \frac{e^{-2ilz}\, dl}{(l^2 - \omega)^2} = 0 + \ldots\ldots \tag{B.27}$$

$$z^2 \int \frac{e^{-2ilz}\, l^\mu\, dl}{(l^2 - \omega)^2} = 0 + \ldots\ldots \tag{B.28}$$

$$z^2 \int \frac{e^{-2ilz}\, l^\mu l^\nu\, dl}{(l^2 - \omega)^2} = i\pi^2 (\frac{1}{2}\eta^{\mu\nu} - z^\mu z^\nu z^{-2} + \ldots) \tag{B.29}$$

are used in section (3.4). The dots symbolize terms which vanish for $z \to 0$.

Proof

(i).

From (B.2), we have

$$\begin{aligned} \int \frac{e^{-2ilz}\, dl}{l^2 - \omega} &= i\pi^2 \{ z^{-2} + \omega\,(1 - ln(i\varepsilon - e^{2\gamma}\omega z^2)) \\ &\quad - \frac{1}{2}\omega^2 z^2 (\frac{5}{2} - ln\,(i\varepsilon - e^{2\gamma}\omega z^2)) \\ &\quad + \ldots\ldots \} \qquad (\text{with } \gamma \approx 0.577)\ . \end{aligned} \tag{B.30}$$

Differentiating (B.30) with respect to ω gives

$$\begin{aligned} \int \frac{e^{-2ilz}\, dl}{(l^2 - \omega)^2} &= i\pi^2 \{ -ln(i\varepsilon - e^{2\gamma}\omega z^2) \\ &\quad + \omega z^2 (\, ln\,(i\varepsilon - e^{2\gamma}\omega z^2\,) - 2\,) \\ &\quad + \ldots\ldots \}\ . \end{aligned} \tag{B.31}$$

Hence

$$z^2 \int \frac{e^{-2ilz}\, dl}{(l^2 - \omega)^2} = i\pi^2 z^2 \{ -ln\,(i\varepsilon - e^{2\gamma}\omega z^2)$$

$$+ \omega z^2 (ln (i\varepsilon - e^{2\gamma} \omega z^2) - 2) + \ldots\ldots \} \,. \qquad \text{(B.32)}$$

Omitting all the terms which vanish for $z \to 0$, we obtain (B.27).

(ii).
Differentiating (B.31) with respect to z_μ, we get

$$\int \frac{e^{-2ilz}\, l^\mu \, dl}{(l^2 - \omega)^2} = \pi^2 \{ z^\mu z^{-2} - \omega z^\mu (ln (i\varepsilon - e^{2\gamma} \omega z^2) - 1) + \ldots\ldots \} \,. \qquad \text{(B.33)}$$

Thus multiplying both sides by z^2 and omitting all the terms which vanish for $z \to 0$ gives (B.28).

(iii).
Differentiating (B.33) with respect to z_ν gives

$$\begin{aligned} \int \frac{e^{-2ilz}\, l^\mu l^\nu \, dl}{(l^2 - \omega)^2} = \; & i\pi^2 \{ \frac{1}{2} \eta^{\mu\nu} z^{-2} - z^\mu z^\nu z^{-4} \\ & - \frac{1}{2} \eta^{\mu\nu} \omega (ln (i\varepsilon - e^{2\gamma} \omega z^2) - 1) \\ & - \omega z^\mu z^\nu z^{-2} + \ldots \} \,. \end{aligned} \qquad \text{(B.34)}$$

Then multiplying both sides by z^2 and omitting all the terms which vanish for $z \to 0$ gives (B.29).

■

APPENDIX C

DETERMINATION OF THE QUANTIZATION TERM Q AND THE UNIVERSAL MASS M

In the following, it will be found that $Q^K(x)$ and $Q^Y_\mu(x)$ behave as functional differentiators

$$
\begin{aligned}
iQ^K(x) &:= \int \mathcal{M}(x-y)dy \, \frac{\delta}{\delta H^K(y)} \, , && \text{(C.1)} \\
iQ^Y_\rho(x) &:= \int \mathcal{M}^\mu_\rho(x-y)dy \, \frac{\delta}{\delta \mathcal{V}^Y_\mu(y)} \, , && \text{(C.2)}
\end{aligned}
$$

while the "universal" mass M has the value

$$
M \approx 8.7 \times 10^7 \, \text{kg} \, . \qquad \text{(C.3)}
$$

C.1 Quantization Term Q

In section (3.3), our quantum field equations are given by

$$
\begin{aligned}
\mathcal{F}^K + j^K &= Q^K \, , && \text{(C.4)} \\
\mathcal{F}^Y_\mu - j^Y_\mu &= Q^Y_\mu \, . && \text{(C.5)}
\end{aligned}
$$

In order to find Q^K and Q^Y_μ, let us first investigate how their free counterparts $Q^{fK}(x)$, Q^{fY}_μ look like.

After dropping all interaction terms in (C.4), (C.5) and replacing H^K, $\mathcal{V}^Y_\mu$ by their free counterparts H^{fK}, $\mathcal{V}^{fY}_\mu$, we have

$$(\Box + m_K^2)\, H^{fK} \;=\; Q^{fK}\,, \tag{C.6}$$

$$(\Box + m_Y^2)\, \mathcal{V}^{fY}_\mu \;-\; \mathcal{V}^{fY\rho}_{,\rho\mu} \;=\; Q^{fY}_\mu\,. \tag{C.7}$$

As H^{fK} and $\mathcal{V}^{fY}_\mu$ are strictly canonical, so in Landau gauge, their propagators are

$$\begin{aligned} \tilde{F}^{fKI}(k^2) &:= -i\int e^{ikx}\,dx\,\langle\,|\,T[\,H^{fK}(x)H^{fI}(0)\,]\,|\,\rangle \\ &= \frac{\delta^{KI}}{k^2 + i\varepsilon - m_K^2}\,, \end{aligned} \tag{C.8}$$

$$\begin{aligned} \tilde{F}^{fXY}_{\mu\rho}(k) &:= -i\int e^{ikx}\,dx\,\langle\,|\,T[\,\mathcal{V}^{fX}_\mu(x)\mathcal{V}^{fY}_\rho(0)\,]\,|\,\rangle \\ &= -\frac{\delta^{XY}\tilde{\Delta}_{\mu\rho}(k)}{k^2 + i\varepsilon - m_Y^2}\,, \end{aligned} \tag{C.9}$$

with $$\tilde{\Delta}_{\mu\rho}(k) := \delta_{\mu\rho} - \frac{k_\mu k_\rho}{k^2 + i\varepsilon}\,. \tag{C.10}$$

Hence,

$$\begin{aligned} \delta^{KI} &\overset{C.8}{=} (k^2 + i\varepsilon - m_K^2)\tilde{F}^{fKI}(k^2) \\ &= \int e^{ikx}dx\,\langle\,|\,T\,[\,i(\Box + m_K^2)\,H^{fK}(x)H^{fI}(0)\,]\,|\,\rangle \\ &\overset{C.6}{=} \int e^{ikx}dx\,\langle\,|\,T\,[\,iQ^{fK}(x)H^{fI}(0)\,]\,|\,\rangle\,, \end{aligned} \tag{C.11}$$

from which $Q^{fK}(x)$ can be easily identified as the functional differentiator,

$$iQ^{fK}(x) \;=\; \frac{\delta}{\delta H^{fK}(x)} \,. \tag{C.12}$$

Proceeding in a similiar manner, we have

$$iQ^{fY}_{\mu} = (\,\Box^{-1}\partial_{\mu}\partial_{\rho} \,-\, \delta_{\mu\rho}\,)\,\frac{\delta}{\delta \mathcal{V}^{fY}_{\rho}(x)} \,. \tag{C.13}$$

Now, when "interactions" are admitted, the equations (C.6), (C.7) are modified by keeping the "free" differential operator and introducing the generalized free fields [130, 131, 132] H^{gK} ,$\mathcal{V}^{gY}_{\mu}$ so that

$$(\,\Box \,+\, m_K^2\,)\, H^{gK} \;=\; Q^K \,, \tag{C.14}$$

$$(\,\Box \,+\, m_Y^2\,)\, \mathcal{V}^{gY}_{\mu} \,-\, \mathcal{V}^{gY\rho}_{,\rho\mu} \;=\; Q^Y_{\mu} \,. \tag{C.15}$$

It must be noted that at low energies, $Q^K(x)$ and $Q^Y_{\mu}(x)$ would become $Q^{fK}(x)$ and $Q^{fY}_{\mu}(x)$ respectively, while at asymptotic energies, we have found in section (3.4) that

$$k^6 \tilde{F}^{KI}(k^2) \;\longrightarrow\; 0 \qquad \text{for } \; |k^2| \;\rightarrow\; \infty \,, \tag{C.16}$$

$$k^6 \tilde{F}^{XY}_{\mu\rho}(k) \;\longrightarrow\; 0 \qquad \text{for } \; |k^2| \;\rightarrow\; \infty \,. \tag{C.17}$$

Here $\tilde{F}^{KI}(k^2)$ is the interacting scalar propagator defined in (3.222)

$$\tilde{F}^{KI}(k^2) \quad := \quad -i \int e^{ikx}\, dx \, \langle \, | \, T[\, H^K(x) H^I(0) \,] \, | \, \rangle \, , \tag{C.18}$$

and $\tilde{F}^{XY}_{\mu\rho}(k)$ is the interacting vector propagator defined in (3.224)

$$\tilde{F}^{XY}_{\mu\rho}(k) \; := \; -i \int e^{ikx}\, dx \, \langle \, | \, T[\, \mathcal{V}^X_\mu(x) \mathcal{V}^Y_\rho(0) \,] \, | \, \rangle \, . \tag{C.19}$$

Hence, in order to satisfy the above-mentioned conditions at low and asymptotic energies, we need

$$iQ^K(x) \;=\; \int \mathcal{M}(x-y)\, dy \frac{\delta}{\delta H^K(y)} \, , \tag{C.20}$$

$$iQ^Y_\mu(x) \;=\; \int \mathcal{M}^\rho_\mu(x-y)\, dy \frac{\delta}{\delta \mathcal{V}^Y_\rho(y)} \, , \tag{C.21}$$

where the "mollifiers" $\mathcal{M}(x)$, $\mathcal{M}^\rho_\mu(x)$ must satisfy [1]

$$\tilde{\mathcal{M}}(k^2) \;\approx\; 1 \qquad \text{for } |k^2| \ll M^2_{PL} \, , \tag{C.22}$$

$$\text{and} \qquad k^4 \tilde{\mathcal{M}}(k^2) \;\longrightarrow\; 0 \qquad \text{for } |k^2| \to \infty \, , \tag{C.23}$$

[1] The conditions (C.22) and (C.23) can also be expressed by a spectral density $\mu(\zeta)$. Both the "mollifiers" and propagators share the same analyticity which we shall see in (C.38). Hence the spectral representation of $\tilde{\mathcal{M}}(k^2)$ is given by $\tilde{\mathcal{M}}(k^2) = \int \frac{\mu(\zeta)\, d\zeta}{(\zeta - k^2 - i\epsilon)}$. Thus the asymptotic "fading" in (C.23) imposes the same "moments conditions" $\int \mu(\zeta) d\zeta = \int \zeta \mu(\zeta) d\zeta = 0$ and $\int \zeta^2 \mu(\zeta) \, d\zeta \neq \infty$ as the propagators did. Furthermore, (C.22) is assured by $\mu(\zeta) = 0$ for $\zeta < \mu^2_{GUT}$ and $\int^\infty_{\mu^2_{GUT}} \zeta^{-1} \mu(\zeta) d\zeta = 1$.

with $\tilde{\mathcal{M}}(k^2)$, $\tilde{\mathcal{M}}^{\rho}_{\mu}(k)$ defined as the inverse Fourier transforms

$$\tilde{\mathcal{M}}(k^2) := \int e^{ikx} dx\, \mathcal{M}(x) \ , \tag{C.24}$$

$$\begin{aligned} \tilde{\mathcal{M}}^{\rho}_{\mu}(k) &:= \int e^{ikx} dx\, \mathcal{M}^{\rho}_{\mu}(x) \\ &= \tilde{\Delta}^{\rho}_{\mu}(k)\tilde{\mathcal{M}}(k^2) \ . \end{aligned} \tag{C.25}$$

This follows by observing that if (C.22) is inserted into (C.24), $\mathcal{M}(x)$ behaves as the delta function $\delta(x-y)$. When it is substituted into (C.20), $iQ^K(x)$ will become $\frac{\delta}{\delta H^K(x)}$ at low energies. This is consistent with equation (C.12). Moreover, it is easy to see that (C.23) follows from (C.17) and (C.38) since the operator $\tilde{D}^{Y\mu}_{\rho X}$ in (C.40) has the dimension k^2.

C.2 Universal Mass M

In this section, we try to estimate the "universal" mass constant M contained in (3.191) by using the photon propagator $\tilde{F}^{AA}$. First of all, let us consider the vacuum values

$$\tilde{Q}^{YZ}_{\rho\sigma}(k) \quad := \quad i\int e^{ikx}\, dx \,\langle\, |\, T[\, Q^Y_\rho(x)\mathcal{V}^Z_\sigma(0)\,]\,|\,\rangle \tag{C.26}$$

$$\stackrel{C.21}{=} \quad \delta^{YZ}\tilde{\mathcal{M}}_{\rho\sigma}(k)\ . \tag{C.27}$$

Inserting (3.219), for (C.26) we get

$$\begin{aligned}\tilde{Q}^{YZ}_{\rho\sigma}(k) \;=\; -i\lim_{z^2\to 0}\int e^{ikx}\, dx\langle\,|\, T \;<\; \overline{\Psi}(x-z)\gamma_\rho \mathbf{t}_Y\Psi(x+z) \;+\; \kappa C^Y_\rho(x,z^2) \\ -\;\kappa u\mathcal{F}^Y_\rho(x)\, ln\,(\, i\varepsilon \;-\; M^2z^2\,) >_{z^2} \mathcal{V}^Z_\sigma(0)\,|\,\rangle\ .\end{aligned} \tag{C.28}$$

Employing the complete amputated vertex function $\Gamma^\sigma_X(p,q)$ defined in (3.253), we have

$$\begin{aligned}&(2\pi)^4\int e^{ikx}dx\,\langle\,|\, T\,[\,\overline{\Psi}(x-z)\gamma_\rho \mathbf{t}_Y\Psi(x+z)\,\mathcal{V}^Z_\sigma(0)\,]|\,\rangle \\ =\;& Tr\,\gamma_\rho \mathbf{t}_Y\int e^{-2irz}\tilde{F}(\not r + \frac{1}{2}\not k)\Gamma^\mu_X(r+\frac{1}{2}k, r-\frac{1}{2}k)\tilde{F}(\not r - \frac{1}{2}\not k)\tilde{F}^{XZ}_{\mu\sigma}(k)dr\end{aligned} \tag{C.29}$$

$$=\; Tr\,\gamma_\rho \mathbf{t}_Y\int e^{-2irz}\tilde{F}(\not r + \frac{1}{2}\not k)\gamma^\mu \mathbf{t}_X\tilde{F}(\not r - \frac{1}{2}\not k)\tilde{F}^{XZ}_{\mu\sigma}(k)dr \tag{C.30}$$

$$:=\; -i\pi^2\,\tilde{\mathcal{G}}^{Y\mu}_{\rho X}(k)\,\tilde{F}^{XZ}_{\mu\sigma}(k)\ , \tag{C.31}$$

where we have approximated the vertex function by constant matrices [2]

[2]We have shown in (3.251) that $\Gamma^\mu_X(k) \;\to\; \gamma^\mu \mathbf{t}_X$ for $|k^2| \;\to\; \infty$. Here, in order to make (C.29) manageable, Γ^μ_X must be approximated by constant matrices. Hence, under Lorentz and gauge

$$\Gamma^{\mu}_{X}(k) \approx \gamma^{\mu}\mathbf{t}_X \, . \tag{C.32}$$

Since (C.31) contains the "interacting" Bose propagator $\tilde{F}^{XZ}_{\mu\sigma}(k)$ as factor, we likewise define $\tilde{\mathcal{F}}^{Y\mu}_{\rho X}$ and $\tilde{\mathcal{C}}^{Y\mu}_{\rho X}$ by

$$\tilde{\mathcal{F}}^{Y\mu}_{\rho X}(k)\tilde{F}^{XZ}_{\mu\sigma} \;:=\; i\int e^{ikx}\,dx\,\langle\,|\,T\,[\,\mathcal{F}^{Y}_{\rho}(x)\mathcal{V}^{Z}_{\sigma}(0)\,]\,|\,\rangle \tag{C.33}$$

$$\tilde{\mathcal{C}}^{Y\mu}_{\rho X}(k)\tilde{F}^{XZ}_{\mu\sigma} \;:=\; i\int e^{ikx}dx\,\langle\,|\,T\,\{Tr\,[\,\gamma_\rho\mathbf{t}_Y < C^{(0)}(x,z) >_{z^2}]\,\mathcal{V}^{Z}_{\sigma}(0)\}\,|\,\rangle \tag{C.34}$$

Also

$$\begin{aligned}
< C^{(-2)}(x,z) >_{z^2} &\overset{3.188}{=} 2< z^{-4}\gamma^{\rho}z_\rho z^{\mu} >_{z^2}\overline{\mathcal{V}}_{\mu}(x) \;-\; < z^{-2} >_{z^2}\mathcal{S}^{\dagger}(x)\\
&\overset{3.198}{=} 2z^{-4}\gamma^{\rho}< z_\rho z^{\mu} >_{z^2}\overline{\mathcal{V}}_{\mu}(x) \;-\; z^{-2}\mathcal{S}^{\dagger}(x)\\
&\overset{3.196}{=} \frac{1}{2}z^{-2}\gamma^{\mu}\overline{\mathcal{V}}_{\mu}(x) \;-\; z^{-2}\mathcal{S}^{\dagger}(x)\,;
\end{aligned} \tag{C.35}$$

then

$$\begin{aligned}
Tr\,\{\gamma_\rho\mathbf{t}_Y < C^{(-2)}(x,z) >_{z^2}\} &\overset{3.20}{=} \frac{1}{2}z^{-2}\,Tr\,\{\gamma_\rho\mathbf{t}_Y\gamma^{\mu}\overline{\mathbf{t}}_X\mathcal{V}^{X}_{\mu}(x)\}\\
&\overset{3.150,5.11}{=} \frac{3u}{2}z^{-2}\,\delta^{\mu}_{\rho}\delta^{Y}_{X}\mathcal{V}^{X}_{\mu}(x)
\end{aligned} \tag{C.36}$$

and

$$i\int e^{ikx}\,dx\,\langle\,|\,T\,[\,C^{Y}_{\rho}(x,z^2)\mathcal{V}^{Z}_{\sigma}(0)\,]\,|\,\rangle$$

invariance, (C.32) results. More justification for this "bare" approximation will be seen later in this section.

$$\begin{aligned}&\overset{3.209}{=} && i\int e^{ikx}dx\,\langle\,|\,T\,[Tr\,\{\,\gamma_\rho \mathbf{t}_Y < (C^{(-2)} + C^{(0)})(x,z) >_{z^2}\}\,\mathcal{V}^Z_\sigma(0)\,]\,|\,\rangle \\ &\overset{C.34,C.36}{=} && \{\tilde{\mathcal{C}}^{Y\mu}_{\rho X}(k) - \frac{3u}{2}z^{-2}\delta^\mu_\rho\delta^Y_X\}\tilde{F}^{XZ}_{\mu\sigma}(k)\,. \qquad \text{(C.37)}\end{aligned}$$

Therefore, with the help of (C.31), (C.33) and (C.37), (C.27) can be rewritten as

$$\tilde{\mathcal{D}}^{Y\mu}_{\rho X}(k)\,\tilde{F}^{XZ}_{\mu\sigma}(k) = \delta^{YZ}\tilde{\mathcal{M}}^\sigma_\rho(k) \qquad \text{(C.38)}$$

where

$$\begin{aligned}(4\pi)^2\,\tilde{\mathcal{D}}^{Y\mu}_{\rho X}(k) \;&:=\; \lim_{z^2\to 0} < \tilde{\mathcal{G}}^{Y\mu}_{\rho X}(k,z) + \tilde{\mathcal{C}}^{Y\mu}_{\rho X}(k) \\ &\qquad - \frac{3u}{2}z^{-2}\delta^\mu_\rho\delta^Y_X - u\tilde{\mathcal{F}}^{Y\mu}_{\rho X}(k)\,ln\,(\,i\varepsilon - M^2z^2\,) >_{z^2} \qquad \text{(C.39)}\end{aligned}$$

Here, $\tilde{\mathcal{D}}^{Y\mu}_{\rho X}$ is identified as the inverse of the "interacting" vector propagator $\tilde{F}^{XZ}_{\mu\sigma}$. It is noted that as $\tilde{\mathcal{G}}^{Y\mu}_{\rho X}(k,z)$ contained in (C.39) is only needed in the limit, in the following, those terms which vanish for $z \longrightarrow 0$ can be omitted and the non-vanishing terms are averaged over the directions of z during our intermediate calculations of the term $\tilde{\mathcal{G}}^{Y\mu}_{\rho X}$.

From (C.31), $\tilde{\mathcal{G}}^{Y\mu}_{\rho X}$ is given by [3]

$$\begin{aligned}\tilde{\mathcal{G}}^{Y\mu}_{\rho X} \;&:=\; i\pi^{-2}\,Tr\,\gamma_\rho \mathbf{t}_Y\int e^{-2irz}\tilde{F}(\not r + \frac{1}{2}\not k)\gamma^\mu \mathbf{t}_X\tilde{F}(\not r - \frac{1}{2}\not k)dr \qquad \text{(C.40)}\\ &\overset{3.226}{=}\; i\pi^{-2}\,Tr\,\gamma_\rho \mathbf{t}_Y\int e^{-2irz}\frac{(\not r + \frac{1}{2}\not k)\rho^\eta_v + \rho^\eta_s}{(r+\frac{1}{2}k)^2 + i\varepsilon - \eta}\gamma^\mu \mathbf{t}_X\,\frac{(\not r - \frac{1}{2}\not k)\rho^\zeta_v + \rho^\zeta_s}{(r-\frac{1}{2}k)^2 + i\varepsilon - \zeta}dr\,d\eta\,d\zeta\,. \qquad \text{(C.41)}\end{aligned}$$

[3]In the following, the abbreviations $\rho^\eta_v := \rho_v(\eta)$ and $\rho^\eta_s := \rho_s(\eta)$ are used for brevity's sake.

By the Feynman formula [104]

$$\frac{1}{ab} = \int_0^1 \frac{dt}{[at + b(1 - t)]^2}, \tag{C.42}$$

the denominator of (3.266) can be simplified as

$$[((r + \frac{1}{2}k)^2 - \eta)t + ((r - \frac{1}{2}k)^2 - \zeta)(1 - t)]^2 = (l^2 - \omega)^2, \tag{C.43}$$

where

$$\begin{aligned} l &:= r + (t - \frac{1}{2})k, && \text{(C.44)} \\ \omega &:= \eta t + \zeta(1 - t) + t(t - 1)k^2 - i\varepsilon, && \text{(C.45)} \\ &:= \sigma + t(t - 1)k^2 - i\varepsilon. && \text{(C.46)} \end{aligned}$$

Hence (C.41) becomes

$$\begin{aligned} \tilde{\mathcal{G}}^{Y\mu}_{\rho X} = \; & i\pi^{-2} Tr\, \gamma_\rho \mathbf{t}_Y \int_0^1 dt \int_0^\infty d\zeta d\eta\, dl\, e^{-i(1-2t)kz}\, e^{-2ilz} (l^2 - \omega)^{-2} \\ & \{ (\not{l} - t\not{k} + \not{k})\rho^\eta_v \gamma^\mu \mathbf{t}_X (\not{l} - t\not{k})\rho^\zeta_v + (\not{l} - t\not{k} + \not{k})\rho^\eta_v \gamma^\mu \mathbf{t}_X \rho^\zeta_s \\ & + \rho^\eta_s \gamma^\mu \mathbf{t}_X (\not{l} - t\not{k})\rho^\zeta_v + \rho^\eta_s \gamma^\mu \mathbf{t}_X \rho^\zeta_s \}. \end{aligned} \tag{C.47}$$

With the help of the identities

$$Tr(A \otimes B) = Tr(A) \times Tr(B) , \quad (C.48)$$

$$Tr(\gamma_\rho \gamma^\mu) = \delta_\rho^\mu Tr(\mathbf{1}) , \quad (C.49)$$

$$Tr(\gamma_\rho \not{l} \gamma^\mu \not{k}) = \{ 2l_\rho k^\mu - lk\delta_\rho^\mu \} Tr(\mathbf{1}) \quad (C.50)$$

and the abbreviations

$$V_{YX}^{\eta\zeta} := Tr[\bar{\mathbf{t}}_Y \rho_v^\eta \bar{\mathbf{t}}_X \rho_v^\zeta] , \quad (C.51)$$

$$S_{YX}^{\eta\zeta} := Tr[\mathbf{t}_Y \rho_s^{\eta\dagger} \bar{\mathbf{t}}_X \rho_s^\zeta] , \quad (C.52)$$

$$N_\rho^\mu(l,k,t) := 2l_\rho l^\mu + (1-2t)(l_\rho k^\mu + k_\rho l^\mu) + 2t(t-1)k_\rho k^\mu - N(l,k,t)\delta_\rho^\mu , \quad (C.53)$$

$$N(l,k,t) := l^2 + (1-2t)kl + t(t-1)k^2 , \quad (C.54)$$

$$d(l,k,z,\omega,t) := i\pi^{-2} e^{-i(1-2t)kz} e^{-2ilz} (l^2 - \omega)^{-2} dl , \quad (C.55)$$

(C.47) can be further reduced to

$$\tilde{\mathcal{G}}_{\rho X}^{Y\mu} = \int_0^1 dt \int d(l,k,z,\omega,t) \{ N_\rho^\mu(l,k,t) V_{YX}^{\eta\zeta} + \delta_\rho^\mu S_{YX}^{\eta\zeta} \} d\eta d\zeta . \quad (C.56)$$

Now, let us denote

$$\{ I, I_\rho, I_\rho^\sigma \}(\omega, z) := i\pi^{-2} \int dl \, e^{-2ilz} (l^2 - \omega)^{-2} \{ 1, l_\rho, l_\rho l^\sigma \} . \quad (C.57)$$

Since from (B.28), (B.30) and (B.31), we know that under $z \longrightarrow 0$ the non-vanishing terms in I, I_ρ, I_ρ^σ are

$$\begin{aligned} I(\omega,z) &= ln(i\varepsilon - e^{2\gamma}\gamma z^{-2})\ , && \text{(C.58)}\\ I_\rho(\omega,z) &= iz^{-2}z_\rho\ , && \text{(C.59)}\\ I_\rho^\sigma(\omega,z) &= (z^{-4} + \omega z^{-2})z_\rho z_\sigma + \frac{1}{2}\{\omega\, ln(i\varepsilon - e^{2\gamma}\omega z^2) - z^{-2}\}\eta_\rho^\sigma\ , && \text{(C.60)}\end{aligned}$$

we have

$$\begin{aligned} \lim_{z^2\to 0} < e^{i(2t-1)kz}\, I(\omega,z) >_{z^2} &= ln(i\varepsilon - e^{2\gamma}\gamma z^{-2})\ , && \text{(C.61)}\\ \lim_{z^2\to 0} < e^{i(2t-1)kz}\, I_\rho(\omega,z) >_{z^2} &= \frac{1}{4}(1-2t)\,k_\rho\ , && \text{(C.62)}\\ \lim_{z^2\to 0} < e^{i(2t-1)kz}\, I_\rho^\sigma(\omega,z) >_{z^2} &= \frac{1}{4}\{2\omega\, ln(i\varepsilon - e^{2\gamma}\omega z^2) - z^{-2} - \omega\}\eta_\rho^\sigma \\ &\quad + \frac{1}{2}(t-\frac{1}{2})^2 k^2\delta_\rho^\sigma\ . && \text{(C.63)}\end{aligned}$$

With the above results,

$$\begin{aligned} &\lim_{z^2\to 0} < \int d(l,k,z,\omega,t) N_\rho^\mu(l,k,t) >_{z^2} \\ &= (t-\frac{1}{2})^2 (2k_\rho k^\mu - \frac{1}{3}K_\rho^\mu) \\ &\quad + \{t(t-1)(2k_\rho k^\mu - k^2\delta_\rho^\mu) - \omega\delta_\rho^\mu\}\, ln(i\varepsilon - e^{2\gamma}\omega z^2) \\ &\quad + \frac{1}{2}\{z^{-2} + \omega - 3(t-\frac{1}{2})^2 k^2\}\delta_\rho^\mu\ , \qquad \text{(C.64)}\end{aligned}$$

where

$$K_\rho^\mu := k_\rho k^\mu - k^2\delta_\rho^\mu\ . \qquad \text{(C.65)}$$

Therefore,

$$\begin{aligned}
&\lim_{z^2\to 0} < \int_0^1 dt \int d(l,k,z,\omega,t) N^\mu_\rho(l,k,t) >_{z^2} \\
= \;& \frac{5}{36} k_\rho k^\mu + \frac{1}{4}(2z^2 + \eta + \zeta - \frac{13}{18}k^2)\delta^\mu_\rho \\
&+ \int_0^1 dt \{2t(t-1)K^\mu_\rho - \sigma\delta^\mu_\rho\} ln(i\varepsilon - e^{2\gamma}\omega z^2)\,. \qquad \text{(C.66)}
\end{aligned}$$

Using the fact that

$$\begin{aligned}
\int d\eta d\zeta\, V^{\eta\zeta}_{YX} &\overset{C.51,3.240,3.150}{=} 3u\delta_{YX}\,, && \text{(C.67)} \\
V_{YX} &:= \int d\eta d\zeta\,(\eta + \zeta) V^{\eta\zeta}_{YX} && \text{(C.68)} \\
&\overset{C.51}{=} \int d\eta d\zeta\,(\eta+\zeta)\, Tr[\bar{\mathbf{t}}_Y \rho^\eta_v \bar{\mathbf{t}}_X \rho^\zeta_v] \\
&\overset{3.241}{\approx} v^2\, Tr[\mathbf{h}_0^2(\mathbf{t}_Y\mathbf{t}_X + \mathbf{t}_X\mathbf{t}_Y)]\,, && \text{(C.69)}
\end{aligned}$$

we finally have

$$\begin{aligned}
&\lim_{z^2\to 0} < \tilde{\mathcal{G}}^{Y\mu}_{\rho X}(k,z) >_{z^2} \\
= \;& u(\frac{5}{12}k_\rho k^\mu - \frac{13}{24}k^2\delta^\mu_\rho + \frac{3}{2}z^{-2}\delta^\mu_\rho)\delta^Y_X + \frac{1}{4}\delta^\mu_\rho V_{YX} \\
&+ \int_0^1 dt \int_0^\infty d\eta d\zeta\,[\delta^\mu_\rho S^{\eta\zeta}_{YX} + \{2t(t-1)K^\mu_\rho - \sigma\delta^\mu_\rho\} V^{\eta\zeta}_{YX}] ln(i\varepsilon - e^{2\gamma}\omega z^2)\,. \\
&\hfill \text{(C.70)}
\end{aligned}$$

Now, by the linear approximations, $\tilde{\mathcal{F}}^{Y\mu}_{\rho X}(k)$ and $\tilde{\mathcal{C}}^{Y\mu}_{\rho X}(k)$ are given by

$$
\begin{aligned}
u\tilde{\mathcal{F}}^{Y\mu}_{\rho X} &= \delta^{\mu}_{\rho} S_{YX} - uK^{\mu}_{\rho}\delta^{Y}_{X} - \frac{1}{2}\delta^{\mu}_{\rho}V_{YX} , & \text{(C.71)} \\
\tilde{\mathcal{C}}^{Y\mu}_{\rho X} &= \frac{u}{24}(13k^2\delta^{\mu}_{\rho} - 10k_{\rho}k^{\mu})\delta^{Y}_{X} - \frac{1}{4}\delta^{\mu}_{\rho}V_{YX} . & \text{(C.72)}
\end{aligned}
$$

Proof

First of all, let us perform the linearization in $\mathcal{V}_\mu$ so that all the terms which are not linear in $\mathcal{V}_\mu$ are neglected. Hence, from (3.89), we obtain

$$
\begin{aligned}
\mathcal{V}_{\rho\sigma\tau} &\approx \mathcal{V}_{\rho,\sigma\tau} - \mathcal{V}_{\sigma,\rho\tau} , & \text{(C.73)} \\
i\mathcal{S}_{\tau} &\approx \mathcal{S}\mathcal{V}_{\tau} - \overline{\mathcal{V}}_{\tau}\mathcal{S} , & \text{(C.74)}
\end{aligned}
$$

and thus (3.195) and (3.192) become

$$
\begin{aligned}
\mathcal{F}_{\rho} &\approx \Box\mathcal{V}_{\rho} - \mathcal{V}^{\mu}_{,\mu\rho} + \frac{3}{2}[\mathcal{S}^{\dagger}\mathcal{S},\, \mathcal{V}_{\rho}]_{+} - 3\mathcal{S}^{\dagger}\overline{\mathcal{V}}_{\rho}\mathcal{S} , & \text{(C.75)} \\
3C^{(0)}(x,z) &\approx \not{z}^{-1}z^{\rho}(\Box\mathcal{V}_{\rho} - \mathcal{V}^{\mu}_{,\mu\rho} + 3[\mathcal{V}_{\rho},\, \mathcal{S}^{\dagger}\mathcal{S}]_{+}) \\
&\quad + \not{z}^{-3}z^{\mu}z^{\rho}z^{\sigma}\mathcal{V}_{\mu,\rho\sigma} + z^{-2}\gamma^{\rho}z^{\mu}z^{\sigma}(\mathcal{V}_{\rho,\mu\sigma} - \mathcal{V}_{\mu,\rho\sigma}) & \text{(C.76)}
\end{aligned}
$$

respectively.

Moreover, for the Higgs field using the approximations

$$
\begin{aligned}
\mathcal{S}(x) &= H(x) + \mathbf{h}_0 v \\
&\approx \mathbf{h}_0 v , & \text{(C.77)} \\
\text{and} \qquad \mathbf{h}_0^{\dagger} &= \mathbf{h}_0 , & \text{(C.78)}
\end{aligned}
$$

we have

$$\mathcal{F}_\rho \approx \Box\mathcal{V}_\rho - \mathcal{V}^\mu_{,\mu\rho} + 3v^2\{\frac{1}{2}, [\mathbf{h}_0^2, \mathcal{V}_\rho]_+ - \mathbf{h}_0\overline{\mathcal{V}}_\rho\mathbf{h}_0\}, \quad (C.79)$$

$$12< C^{(0)}(x,z) >_{z^2} \approx 2\gamma^\rho(\Box\mathcal{V}_\rho - \mathcal{V}^\mu_{,\mu\rho}) + 3v^2\gamma^\rho[\mathcal{V}_\rho, \mathbf{h}_0^2]_+ + \frac{1}{6}\gamma^\rho(\Box\mathcal{V}_\rho - 2\mathcal{V}^\mu_{,\mu\rho}) . \quad (C.80)$$

Furthermore, by invoking the relations

$$Tr[\mathbf{h}_K^\dagger\mathbf{h}_I] = 2u\delta_{KI} , \quad (C.81)$$

$$Tr[\mathbf{t}_X\mathbf{t}_Y] = 3u\delta_{XY} , \quad (C.82)$$

$$V_{YX} = v^2 Tr[\mathbf{h}_0^2(\mathbf{t}_Y\mathbf{t}_X + \mathbf{t}_X\mathbf{t}_Y)] , \quad (C.83)$$

$$S_{YX} := \int d\eta d\zeta S^{\eta\zeta}_{YX} \quad (C.84)$$

$$\overset{C.52}{=} \int d\eta d\zeta Tr[\mathbf{t}_Y\rho_s^{\eta\dagger}\overline{\mathbf{t}}_X\rho_s^\zeta]$$

$$\overset{3.239}{=} v^2 Tr[\mathbf{t}_Y\mathbf{h}_0\overline{\mathbf{t}}_X\mathbf{h}_0] , \quad (C.85)$$

we obtain

$$\mathcal{F}^Y_\rho(x) = (3u)^{-1}Tr[\mathbf{t}_Y\mathcal{F}_\rho(x)]$$

$$= \Box\mathcal{V}^Y_\rho - \mathcal{V}^{Y\mu}_{,\mu\rho} + u^{-1}(\frac{1}{2}V_{YX} - S_{YX})\mathcal{V}^X_\rho, \quad (C.86)$$

$$24Tr[\gamma_\rho\mathbf{t}_Y\langle C^{(0)}(x,z)\rangle_{z^2}] = u(13\Box\mathcal{V}^Y_\rho - 10\mathcal{V}^{Y\mu}_{,\mu\rho})(x) + 6V_{YX}\mathcal{V}^X_\rho . \quad (C.87)$$

After inserting (C.86), (C.87) into (C.33), (C.34) respectively and using the Bose propagator $\tilde{F}^{XZ}_{\mu\sigma}(k)$ defined in (3.224), we reach (C.71), (C.72).

■

Substituting (C.70) and (C.72) into (C.39) yields [4]

$$(4\pi)^2\,\tilde{\mathcal{D}}^{Y\mu}_{\rho X}(k) \;=\; \lim_{z^2\to 0} < \int_0^1 dt \int_0^\infty d\eta d\zeta\,[\,\delta^\mu_\rho S^{\eta\zeta}_{YX} + \{2t(t-1)K^\mu_\rho - \sigma\delta^\mu_\rho\}V^{\eta\zeta}_{YX}\,] \\ ln(\,i\varepsilon - e^{2\gamma}\omega z^2\,) - u\tilde{\mathcal{F}}^{Y\mu}_{\rho X}(k)\,ln\,(\,i\varepsilon - M^2 z^2\,) >_{z^2} . \tag{C.88}$$

Since $\tilde{\mathcal{D}}^{Y\mu}_{\rho X}(k)$ should be convergent for $z \longrightarrow 0$, the terms with z^{-2} cancel. Thus we get the consistency condition

$$u\tilde{\mathcal{F}}^{Y\mu}_{\rho X} \;=\; \int_0^1 dt \int_0^\infty d\eta d\zeta\,[\,\delta^\mu_\rho S^{\eta\zeta}_{YX} + \{2t(t-1)K^\mu_\rho - \sigma\delta^\mu_\rho\}V^{\eta\zeta}_{YX}\,]\;, \tag{C.89}$$

while (C.88) becomes

$$(4\pi)^2\,\tilde{\mathcal{D}}^{Y\mu}_{\rho X}(k) \;=\; -u\tilde{\mathcal{F}}^{Y\mu}_{\rho X}\,ln(\,M^2 e^{-2\gamma}\omega^{-1}\,) \\ \stackrel{C.89}{=} -\int_0^1 dt \int_0^\infty d\eta d\zeta\,[\,\delta^\mu_\rho S^{\eta\zeta}_{YX} + \{2t(t-1)K^\mu_\rho - \sigma\delta^\mu_\rho\}V^{\eta\zeta}_{YX}\,] \\ ln(\,M^2 e^{-2\gamma}\omega^{-1}\,)\;. \tag{C.90}$$

[4]Note that the $i\varepsilon$ in the logarithmic term of (C.88) is just used to determine the sign of the imaginary part.

Here, we note that the linearized approximation (C.71) agrees with our consistency condition (C.89) since

$$\begin{aligned} u\tilde{\mathcal{F}}^{Y\mu}_{\rho X} &= \int_0^1 dt \int_0^\infty d\eta d\zeta \, [\, \delta^\mu_\rho S^{\eta\zeta}_{YX} + \{2t(t-1)K^\mu_\rho - \sigma\delta^\mu_\rho\} V^{\eta\zeta}_{YX} \,] \\ &\overset{C.68,C.84}{=} \delta^\mu_\rho S_{YX} - uK^\mu_\rho \delta^Y_X - \frac{1}{2}\delta^\mu_\rho V_{YX} \, . \end{aligned} \tag{C.91}$$

However, they come from different origins. Equation (C.71) is obtained by linearizing the Yang - Mills equation (3.195) in $\mathcal{V}_\mu$ while the consistency condition (C.89) follows from (C.28) where the complete vertex function $\Gamma^\mu_X(k)$ are approximated by the bare ones $\gamma^\mu \mathbf{t}_X$. Hence, this coincidence signifies that the "bare approximation" (C.32) we used in (C.28) *corresponds* to the linearization in (3.195).

Now, in order to find the "universal" mass M, let us first derive the photon propagator $\tilde{F}^{AA}(k)$. As we know, the photon and gluon fields are coupled with Dirac's Ψ by matrices $\mathbf{t}_Y = \mathbf{t}_A, \mathbf{t}_c$. They are diagonal in the flavor spaces and do not contain any γ_5 factor. Thus they commute with the matrix $\mathbf{h}_0$ and satisfy $\bar{\mathbf{t}}_Y = \mathbf{t}_Y$. Hence

$$\begin{aligned} \frac{1}{2}V_{YX} - S_{YX} &\overset{C.69,C.85}{=} \frac{v^2}{2} Tr[\, \mathbf{h}_0^2(\mathbf{t}_Y\mathbf{t}_X - \mathbf{t}_X\mathbf{t}_Y) - 2\mathbf{h}_0\mathbf{t}_Y\mathbf{h}_0\bar{\mathbf{t}}_X \,] \\ &= 0 \, . \end{aligned} \tag{C.92}$$

This does not mean that photons and gluons contribute nothing to the term $S^{\eta\zeta}_{YX} - \sigma V^{\eta\zeta}_{YX}$ since

$$\begin{aligned} &\int_0^1 dt \int_0^\infty d\eta d\zeta \, [\, \delta^\mu_\rho S^{\eta\zeta}_{YX} - \delta^\mu_\rho \sigma V^{\eta\zeta}_{YX} \,] ln(\, M^2 e^{-2\gamma} \omega^{-1} \,) \\ \neq \; & \{\, \delta^\mu_\rho S_{YX} - \frac{1}{2}\delta^\mu_\rho V_{YX} \,\} ln(\, M^2 e^{-2\gamma} \omega^{-1} \,) \end{aligned}$$

$$\overset{C.92}{=} \quad 0 \,, \tag{C.93}$$

because the ω contained in $ln(M^2 e^{-2\gamma} \omega^{-1})$ also depends on the variables t, η, ζ. Yet we may assume that if we make an approximation as in (C.93), the errors can be compensated by those we made in the linear approximation of $\tilde{\mathcal{C}}^{Y\mu}_{\rho X}$. [5] Thus we have

$$(4\pi)^2 \, \tilde{\mathcal{D}}^{Y\mu}_{\rho X}(k) \; = 2K^{\mu}_{\rho} \int_0^1 t(1-t)dt \int_0^{\infty} d\eta d\zeta \, V^{\eta\zeta}_{YX} \, ln(M^2 e^{-2\gamma} \omega^{-1}) \; . \tag{C.94}$$

However, for photons, our linearization in $\mathcal{V}^A_\mu \; = \; A_\mu$ should be best at low energy. As from (C.45)

$$\omega \;:=\; \sigma + t(t - 1)k^2 - i\varepsilon \tag{C.95}$$

$$\text{where} \qquad \sigma \;:=\; \eta t + \zeta(1 - t) \,, \tag{C.96}$$

so when $|k^2| \; << \; m_e^2$, [6]

$$\sigma >> |t(t - 1)k^2| \,. \tag{C.97}$$

because we shall use the sharp fermion masses approximation in which $\eta = \zeta = m_f^2$.

Hence, the function $ln(M^2 e^{-2\gamma} \omega^{-1})$ can be approximated by its Taylor polynomial at $\omega = \sigma$

[5] We made this assumption so that $\tilde{\mathcal{D}}^{Y\mu}_{\rho X}$ is proportional to K^{μ}_{ρ}, thus the photon is kept massless.

[6] In the following, we shall discuss the vacuum polarization which occurs at $|k^2| \; << \; m_e^2$.

$$ln(\frac{M^2 e^{-2\gamma}}{\omega}) \approx ln(\frac{M^2 e^{-2\gamma}}{\sigma}) + \frac{t(1-t)k^2}{\sigma} \,. \tag{C.98}$$

Moreover, using the *sharp* fermion masses (m_f) approximation

$$\begin{aligned} & \rho_v^\eta && \approx && \mathrm{diag}\{\rho^\nu, \rho^e, \ldots \rho^t, \rho^b\}(\eta) \\ \text{with} \quad & \rho^f(\eta) && := && \delta(\eta - m_f^2) \\ \text{where} \quad & f && = && \nu, e, \ldots t, b. \,, \end{aligned} \tag{C.99}$$

for any smooth function $\psi(\sigma)$ of σ we have

$$\int_0^\infty d\eta d\zeta \, \rho_v^\eta \rho_v^\zeta \, \psi(\sigma) = \sum_f \psi(m_f^2) \,. \tag{C.100}$$

Meanwhile, as [7]

$$\mathbf{t}_A = \mathrm{diag}[\, e(\infty)\,\{0, -1, 0, 1, \ldots \frac{-1}{3}\}] \otimes \mathbf{1} \,, \tag{C.101}$$

we obtain

$$Tr[\,(\bar{\mathbf{t}}_A)^2\,] = 4 \sum_f e_f^2(\infty) \,. \tag{C.102}$$

[7] It is noted that as in all basic calculations in this paper, bare coupling constants have been used. Hence we use $\mathbf{t}_A$ which contains the bare charges $e(\infty)$ instead of using $\mathbf{t}_A^{obs.}$ of observed charges e.

Using the results obtained in (C.100) and (C.102), we have

$$\int_0^\infty d\eta d\zeta\, V_{AA}^{\eta\zeta}\,\psi(\sigma) \overset{C.51}{=} \int_0^\infty d\eta d\zeta\, Tr\,[\bar{\mathbf{t}}_A\rho_v^\eta\bar{\mathbf{t}}_A\rho_v^\zeta\,]\psi(\sigma)$$
$$\overset{C.100,C.102}{=} 4\sum_f e_f^2(\infty)\psi(m_f^2)\ . \qquad \text{(C.103)}$$

Inserting (C.98), (C.103) into (C.94) and using the definitions and properties

$$\alpha := \frac{e^2}{4\pi} = \frac{1}{137}\ , \qquad \text{(C.104)}$$
$$\alpha(\infty) := \frac{e^2(\infty)}{4\pi}\ , \qquad \text{(C.105)}$$
$$\frac{e_f(\infty)}{e(\infty)} = \frac{e_f}{e}\ , \qquad \text{(C.106)}$$

we obtain

$$\tilde{\mathcal{D}}_{\rho A}^{A\mu} \approx K_\rho^\mu Z_3^{-1}(1+\frac{k^2}{m_{pol}^2}) \qquad \text{for} \quad |k^2| << m_e^2\ . \qquad \text{(C.107)}$$

Here Z_3 and m_{pol} are given by

$$Z_3^{-1} := \frac{2\alpha(\infty)}{3\pi}\sum\nolimits_f(\frac{e_f}{e})^2 ln(\frac{Me^{-\gamma}}{m_f})\ , \qquad \text{(C.108)}$$
$$Z_3^{-1}(\frac{m_e}{m_{pol}})^2 := \frac{\alpha(\infty)}{15\pi}\sum\nolimits_f(\frac{e_f}{e})^2(\frac{m_e}{m_f})^2$$
$$\approx \alpha(\infty)\ . \qquad \text{(C.109)}$$

As we know, a *transverse* photon propagator is given by

$$\begin{aligned}\tilde{F}^{AA}_{\mu\rho} &= -\tilde{\Delta}_{\mu\rho}(k)\tilde{F}^{AA}(k^2) \\ &= K_{\mu\rho}k^{-2}\tilde{F}^{AA}(k^2) \ ,\end{aligned} \tag{C.110}$$

and from (C.22), the "mollifiers" $\tilde{\mathcal{M}}(k^2) \approx 1$ at low energies, thus substituting (C.107) into (C.38), we conclude

$$\tilde{\mathcal{F}}^{AA}(k^2) \approx Z_3(1 - \frac{k^2}{m^2_{pol}}) \qquad \text{for} \quad |k^2| << m^2_e \ . \tag{C.111}$$

Now, comparing with the usual Uehling's constant of vacuum polarization [133], we have

$$m^2_{pol} = \frac{15\pi}{\alpha} m^2_e \ . \tag{C.112}$$

Moreover, from our previous calculation,

$$\begin{aligned} Tr[\mathbf{t}_A\mathbf{t}_A] &= 3u \\ &\overset{C.101}{=} 4\sum_f e^2_f(\infty) \\ &\overset{C.106}{=} 4e^2(\infty)\sum_f (\frac{e_f}{e})^2 \\ &= 32e^2(\infty) \ ; \end{aligned} \tag{C.113}$$

hence,

$$\begin{aligned}\alpha(\infty) &:= \frac{e^2(\infty)}{4\pi} \\ &\stackrel{C.113}{=} \frac{3u}{128\pi} \\ &= 8.8 \times 10^{-3} \,. \end{aligned} \tag{C.114}$$

Inserting (C.112) and (C.114) into (C.109), we obtain

$$\begin{aligned} Z_3 &= \frac{\alpha}{\alpha(\infty)} \\ &= 0.83 \,. \end{aligned} \tag{C.115}$$

With this Z_3 , (C.108) becomes

$$\begin{aligned} \frac{3\pi}{2\alpha} &= \sum_f \Big(\frac{e_f}{e}\Big)^2 \{\, ln\Big(\frac{Me^{-\gamma}}{m_t}\Big) + ln\Big(\frac{m_t}{m_f}\Big)\} \\ &\approx 8\, ln\Big(\frac{M}{m_t}\Big) - (8)(0.577) + 52.8 \\ &\approx 8\, ln\Big(\frac{M}{m_t}\Big) + 48.16 \,. \end{aligned} \tag{C.116}$$

Thus we obtain

$$ln\Big(\frac{M}{m_t}\Big) \approx 74.7$$

$$\begin{aligned}
\text{and} \qquad M &\approx 2.8 \times 10^{32}\, m_t \\
&\approx 2.01 \times 10^{32}\, v \\
&\approx 8.7 \times 10^{7}\, kg \,, && \text{(C.117)} \\
\text{hence} \qquad \mu &\approx 10^{-29}\, M \\
&\approx 2010\, v \,. && \text{(C.118)}
\end{aligned}$$

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