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THE BETA CEPHEI STAR SPICA (α VIRGINIS):

A SPECTROGRAPHIC INVESTIGATION

by

Robert Jones Dukes, Jr.

A Dissertation Submitted to the Faculty of the

DEPARTMENT OF ASTRONOMY

In Partial Fulfillment of the Requirements
For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

THE UNIVERSITY OF ARIZONA

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THE UNIVERSITY OF ARIZONA

GRADUATE COLLEGE

I hereby recommend that this dissertation prepared under my direction by ROBERT JONES DUKES, JR. entitled THE BETA CEPHEI STAR SPICA (α VIRGINIS): A SPECTROGRAPHIC INVESTIGATION be accepted as fulfilling the dissertation requirement of the degree of DOCTOR OF PHILOSOPHY

Walter S Fitch
Dissertation Director

29 May 1973
Date

After inspection of the final copy of the dissertation, the following members of the Final Examination Committee concur in its approval and recommend its acceptance:*

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*This approval and acceptance is contingent on the candidate's adequate performance and defense of this dissertation at the final oral examination. The inclusion of this sheet bound into the library copy of the dissertation is evidence of satisfactory performance at the final examination.

STATEMENT BY AUTHOR

This Work is Dedicated

to

My Parents

in appreciation for their encouragement
during my years of schooling and for
being the people they are.

ACKNOWLEDGMENTS

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ABSTRACT

We have obtained 722 new velocities of the primary component of Spica (α Virginis) during the 1970 and 1971 observing seasons, as well as ten nights of spectral scans of the He 4471 line. The new measures were combined with published velocities to determine an improved orbit which has a significantly lower K_1 than that found in earlier work. The residuals of these velocities from the orbit were analyzed for the pulsational phenomenon. We found that, in addition to the known pulsational frequency (f_1 = 5.7542 c/d), there are three previously undetected frequencies (f_2 = 5.7854 c/d, f_3 = 4.1158 c/d, and f_4 = 3.6171 c/d). The existence of these new terms was verified by the presence of similar terms in the previously published photometric data. We also found a correlation between the pulsation amplitude and the tidal potential at the surface of the primary. Comparison of the observational results with pulsation models showed best agreement for the strongest frequency as the radial first overtone mode. The additional frequencies were identified as the weakly excited fundamental (f_3) and two non-radial modes.

CHAPTER 1

INTRODUCTION

The bright star Spica (α Virginis) has long been known to be a four-day spectroscopic binary (Vogel 1890) with an apsidal period of about 130 years (Struve and Ebbighausen 1934). Recently, Shobbrook et al. (1969) discovered photometrically that it was also a Beta Cephei star with a four-hour period. The Beta Cephei light variations were superimposed on those of an ellipsoidal variable with a period equal to the orbital period and an amplitude about twice that of the Beta Cephei variation. After this announcement, Smak (1970) analyzed the velocities which had previously been published by Baker (1910), Struve and Ebbighausen (1934), and Struve et al. (1958) for evidence of a short period variation. In doing this he found that it was necessary to redetermine the photometric short period found by Shobbrook et al. (1969). His conclusions were that no one short period would satisfy all of the observations but that it was necessary to introduce a short period decreasing with time. More recently, Spica has been studied with the intensity interferometer by Hanbury-Brown and his colleagues (Herbison-Evans et al. 1971). As a part of this study, they redetermined the spectroscopic binary orbit based on all available data. Finally additional photometric and spectroscopic observations, together with a reanalysis of all available data, were published by Shobbrook, Lomb and Herbison-Evans (1972). The short period was redetermined using three seasons' photometric data. Forty-three new velocities (31 of which were obtained on one night) were used to verify the velocity variation found in the earlier data. In addition the earlier velocity data was reanalyzed. Some of the periods which had been found by Smak were found to have been in error. Of especial importance was the absence of a four-hour pulsation period in the 1910 data. Instead one close to six hours was found. Also a period decrease of the magnitude found by Smak could not be confirmed. There was a definite period change but it was uncertain whether or not it was an increase or decrease.

The present work was begun in 1970 as a study of the short period velocity and line profile variation. Of special interest was the possibility of studying the variations of amplitude of the short period variation in connection with the tidal perturbation theory which has been used by Fitch (1967, 1969) to explain amplitude variations in the Beta Cephei stars σ Scorpii, β Cephei and 16 Lacertae. Since the binary period of Spica is much shorter than that of any of the previously studied stars and since the orbital eccentricity is moderate it was felt that the effects of tidal perturbations should be more pronounced than those found in the above mentioned stars.

CHAPTER 2

THE OBSERVATIONAL MATERIAL AND ITS REDUCTION

The primary observational material for this project consists of 622 spectra obtained in April, May and June of 1970 and 100 additional spectra obtained on three nights in April of 1971. All of the observations were made with the coudé spectrograph of the Steward Observatory 36" telescope. The majority of the spectra had a dispersion of 7.1 Å/mm, and were made with a 50 micron projected slit width which yielded a resolution of 0.35 Å, although a few early in the project had a dispersion of 2.6 Å/mm which, with the 50 micron slit, yielded a resolution of 0.13 Å. This latter combination was dropped after examination of the spectra revealed that the features were too broad to be easily measured and that there were too few measurable lines available. The exposure times for the 7.1 Å/mm spectra ranged from 2 to 20 minutes with an average exposure being about eight minutes. Also during March and April of 1971 we obtained ten nights of spectral scans of the He 4471 line, which were made using the 5-channel spectral scanner developed by R. L. Hilliard, and had a resolution of 0.16 Å and a length of 7.1 Å. The duration of each of the line profile observations using the scanner was about 15 minutes.

Because of the nearly four-day orbital period the observations obtained in any one season from one observing location only cover four rather limited phase ranges of the orbital period. This introduces many complications into the analysis as will be seen later.

The spectra were all measured on the Grant measuring engine of the Kitt Peak National Observatory. The lines measured together with their laboratory wavelengths are given in Table 1. An average of 13 lines were used in forming the velocity from one spectrum.

Table 1. Lines Measured in Spica.

Element	Wavelength (Å)
He I	4471.507
He I	4387.928
H γ	4340.468
He I	4143.759
He I	4120.857
H δ	4101.737
He I	4026.218
He I	4009.270
H ϵ	3970.074
H8	3889.051
H9	3835.386
He I	3819.637
H10	3797.900
H11	3770.632

It was very difficult to see the lines of the secondary on these spectra and consequently they were not measured. Tests showed that to measure the lines of both components required at least four times as much measuring time as was necessary to measure only the lines of the primary. In addition, the velocities obtained were of very poor quality. Since there were a large number of spectra to be measured and since the object of the study was the variable nature of the primary, the lines of the secondary were not measured.

The initial reductions were carried out on the CDC 6400 computer of the University of Arizona Computer Center using a program developed by N. B. Sanwal and modified by the present author. Later, time was made available on the Kitt Peak CDC 6400 and this was used for the balance of the reductions and much of the analysis.

Initially laboratory wavelengths were used for these reductions. However, because of blending problems, both between lines of the primary and secondary and between the line being measured and adjacent lines in the spectrum, these were not the best wavelengths to use. Corrections to the laboratory wavelengths, which minimized the residuals for all measures of each line, were determined separately for each of the four orbital phase groups covered by the observations and the data was re-reduced using these wavelengths. The corrections (in velocity units), which were derived in this manner and applied, are listed in Table 2 and the results of the measurements in Table 3.

Since this spectrograph had not been used previously for an extended velocity program, standard stars were observed to check on the

Table 2. Corrections to Laboratory Wavelengths for Lines Measured (in Velocity Units, Km S^{-1})

Orbital Phase Group	Line						
	4471	4387	4340	4143	4120	4101	4026
0.2	-10.6	13.0	0.8	17.5	-12.0	3.9	- 8.1
0.4	-24.7	- 3.0	5.3	0.4	-32.9	11.9	-15.1
0.6	-14.3	2.1	6.8	8.8	-18.2	8.2	- 8.6
0.8	-17.0	0.8	5.3	12.5	-23.9	9.2	-12.8
	Line						
	4009	3970	3889	3835	3819	3797	3770
0.2	12.3	0.5	- 1.5	- 6.5	10.2	-12.1	-11.5
0.4	2.9	10.2	16.3	11.3	- 6.2	4.4	10.0
0.6	5.4	9.3	4.2	3.4	- 8.0	- 2.1	3.1
0.8	1.9	10.7	7.8	5.4	- 4.4	- 0.4	5.2

velocity stability. These observations are discussed in detail in Appendix A. Briefly, we found that there was no indication of a velocity variation due to the telescope or spectrograph unless possibly a declination effect is present. As evidence for this we have that the correction to the observed velocity for Vega necessary to bring it into agreement with the Wilson (1953) catalog was -6.1 km/sec while that for Arcturus was -2.6 km/sec. Since we are primarily interested in differential and not absolute velocity measures this effect, if present, will not adversely affect most of this work. However, the gamma velocity we derive for the system will possibly be in error by a few kilometers per second.

Table 3. Observed Velocities.

Helio.JD 2440000+	Vel. (Km S^{-1})	Helio.JD 2440000+	Vel. (Km S^{-1})	Helio.JD 2440000+	Vel. (Km S^{-1})	Helio.JD 2440000+	Vel. (Km S^{-1})	Helio.JD 2440000+	Vel. (Km S^{-1})	Helio.JD 2440000+	Vel. (Km S^{-1})
697.690	14.1	699.746	14.4	700.871	96.0	706.900	-106.2	717.673	15.1	719.849	15.2
.697	5.7	.751	18.0	.879	97.9	.912	-108.2	.678	17.7	.863	9.2
.709	6.2	.757	20.6	.885	99.6	.919	-111.2	.692	14.8	.870	24.4
.715	8.0	.763	24.5	.890	101.4	.925	-113.2	.698	12.5	.879	10.4
.722	3.6	.769	23.6	.897	92.2	707.646	4.8	.703	14.3	.887	19.9
.735	.8	.775	25.5	.901	94.7	.657	.3	.708	7.3	.761	11.0
.742	1.0	.779	18.8	.913	94.5	.662	3.1	.714	12.2	.767	17.2
.748	4.9	.784	25.6	.918	91.3	.666	2.6	.719	15.6	.772	13.5
.755	- 9.4	.788	16.0	.938	100.2	.680	- 3.4	.734	- 2.0	.778	14.9
.760	- 11.0	.792	18.3	.947	94.7	.687	- 4.0	.740	6.5	.784	9.1
.773	- 8.3	.796	22.8	703.669	1.9	.697	2.1	.745	6.7	.796	15.8
.779	- 8.3	.805	14.3	.685	- 5.3	.703	1.1	.751	7.1	.804	20.3
.786	- 3.5	.810	8.9	.700	3.4	.709	1.3	.757	- 1.2	.810	5.9
.791	- 2.6	.814	4.2	.713	2.5	.715	- 2.3	.762	4.6	720.625	80.8
.798	- 10.8	.818	8.2	.722	7.6	.722	6.6	.768	1.5	.631	82.2
.806	- 13.6	.822	4.7	.730	5.4	.728	4.3	.777	4.1	.637	89.8
.813	- 11.9	.827	4.6	.737	8.9	.734	1.5	.782	- 1.2	.644	94.9
.820	- 10.2	.830	.3	.750	9.2	.797	17.1	.788	- 3.3	.652	82.9
.828	- 10.6	.835	7.3	.757	17.1	.808	17.4	.794	- 3.0	.661	83.6
.837	- 6.3	.840	4.3	.781	26.9	.815	8.3	.801	2.8	.669	99.5
.843	- 2.2	.845	9.3	.793	16.8	.821	1.2	.808	- 3.6	.676	82.7
.851	- 5.5	.849	2.0	.803	20.2	.828	2.9	.814	- 1.2	.687	101.2
.859	- 2.8	.858	- 7.1	.815	14.2	.834	0.0	.824	- 11.9	.693	94.7
.878	- 4.7	.864	- 11.5	.827	13.3	.841	1.2	.830	- 14.5	.701	112.2
.886	- 2.6	.868	- 6.9	.832	16.2	.851	1.6	.837	- 12.5	.707	90.6
.895	- 16.6	.872	- 4.9	.840	12.0	.859	6.6	.843	- 12.3	.715	97.7
.902	- 28.5	.876	- 11.2	.849	11.8	.866	8.0	.850	- 12.9	.722	95.9
.911	- 28.3	.881	- 7.7	.859	19.7	.872	10.1	.857	- 13.7	.728	97.6
.920	- 42.8	.887	5.4	.869	23.0	.879	5.5	.874	- 18.3	.739	91.2
.929	- 31.4	.898	4.5	.886	21.5	.888	15.4	718.636	-119.1	.746	86.3
.941	- 49.7	.905	2.5	.897	30.5	.898	14.3	.647	-126.8	.753	93.7
.955	- 49.8	.912	- .8	.906	26.2	.910	21.4	.658	-124.7	.761	106.0
698.712	-110.9	.919	16.9	.913	21.1	.917	27.1	.670	-125.5	.767	101.0
.733	-117.8	700.651	108.9	705.818	- 8.9	.925	31.7	.680	-124.9	.774	106.2
.743	-111.7	.658	98.0	.829	- 9.3	.934	27.3	.686	-124.5	.795	105.9
.754	-108.7	.666	106.9	.842	- 6.6	.944	32.8	.692	-121.0	.805	108.5
.773	-113.1	.674	111.3	.851	0.0	716.676	100.6	.707	-107.5	.816	106.2
.808	-120.8	.682	103.8	.862	- 11.1	.683	100.6	.714	-110.8	.826	104.9
.818	-118.5	.689	114.2	.871	- 16.1	.688	100.7	.721	-102.8	.839	101.8
.826	-125.3	.696	106.3	.881	- 18.5	.694	93.9	.729	-114.7	.848	106.4
.834	-112.9	.704	93.9	706.676	-126.1	.704	100.4	.737	-118.0	.857	100.2
.841	-118.6	.718	95.5	.686	-132.4	.711	95.7	.747	-107.9	.866	98.4
.848	-109.8	.733	114.4	.696	-118.4	.716	104.8	.759	-120.3	727.656	4.4
.856	-100.3	.739	95.6	.706	-119.0	.725	95.0	.774	- 84.5	.663	- .4
.862	-105.9	.744	106.6	.715	-127.2	.730	104.4	.781	- 92.8	.670	4.6
.870	-105.0	.749	105.8	.725	-131.9	.736	100.4	.787	-120.2	.678	14.8
.877	-101.4	.754	99.1	.733	-118.5	.742	99.9	.794	-110.5	.689	2.3
.885	- 99.8	.760	96.0	.741	-115.6	.750	109.3	.802	-101.6	.707	3.9
.896	-103.4	.765	108.8	.752	-122.1	.757	106.1	.809	- 99.1	.715	13.2
.904	- 89.7	.773	106.6	.763	-129.9	.763	105.4	.817	-113.7	.724	8.0
.912	- 88.9	.777	90.1	.769	-117.6	.769	105.9	.825	-104.1	.737	9.0
.919	- 87.1	.781	103.2	.776	-117.2	.775	111.5	.833	-103.9	.746	14.5
.928	- 89.3	.786	104.4	.783	-112.9	.781	106.4	.841	- 97.4	.757	8.3
.941	- 77.2	.789	104.4	.791	-117.2	.788	105.2	.854	-104.6	.766	13.2
.953	- 83.9	.795	114.6	.798	-122.7	.799	109.1	.871	-103.2	.775	8.3
699.671	4.8	.799	112.4	.810	-128.0	.806	98.8	.879	- 87.0	.784	9.6
.693	- 2.6	.804	106.4	.817	-129.6	.815	86.0	.888	-110.2	.795	13.8
.698	1.3	.812	102.6	.825	-129.4	.824	98.5	719.743	13.1	.812	19.9
.705	1.9	.817	106.4	.832	-129.3	.836	95.1	.748	9.1	.822	17.2
.710	.7	.822	103.6	.839	-122.6	.846	96.3	.755	14.5	.832	17.5
.715	4.0	.826	107.5	.846	-117.6	.859	81.0	.817	12.9	.842	13.7
.721	1.9	.830	105.0	.854	-119.5	717.644	22.0	.823	17.9	.852	8.0
.725	2.6	.835	106.6	.865	-109.3	.649	20.3	.829	19.5	.863	10.7
.731	8.6	.839	110.8	.881	-107.7	.661	19.1	.835	19.0	728.638	95.8
.740	14.1	.866	99.0	.890	-103.0	.667	21.3	.841	14.6	.655	106.1

Table 3--Continued.

Helio.JD 2440000+	Vel. (Km S^{-1})	Helio.JD 2440000+	Vel. (Km S^{-1})	Helio.JD 2440000+	Vel. (Km S^{-1})	Helio.JD 2440000+	Vel. (Km S^{-1})	Helio.JD 2440000+	Vel. (Km S^{-1})	Helio.JD 2440000+	Vel. (Km S^{-1})
728.668	107.8	734.654	-127.5	747.616	4.5	753.767	11.7	759.709	9.1	1073.846	86.0
.681	92.3	.669	-128.8	.622	- 7.8	.776	- .5	.722	8.0	.859	79.3
.693	97.1	.683	-129.6	.628	3.6	.784	- 9.0	.738	7.1	.874	94.5
.704	97.8	.695	-126.5	.635	5.7	.793	9.7	761.629	34.3	.890	94.6
.713	94.9	.708	-120.9	.642	3.4	754.631	-136.0	.639	44.9	.909	86.8
.728	94.3	.721	-122.5	.660	- 1.4	.640	-139.6	.649	46.6	1074.640	43.2
.736	98.3	.733	-127.2	.667	- 3.5	.647	-138.1	.660	50.3	.653	39.2
.743	98.0	.743	-125.1	.674	- .3	.653	-131.3	.671	46.1	.664	34.7
.749	94.3	.753	-133.5	.681	3.3	.660	-130.8	.683	38.9	.676	36.2
.758	95.5	.762	-131.1	.689	- 1.2	.668	-136.0	.694	38.1	.687	34.6
.767	100.0	.770	-128.3	.696	- 1.0	.677	-133.2	.707	32.6	.697	25.0
.774	96.4	.780	-128.9	.702	1.0	.686	-135.2	.723	14.6	.700	28.0
.786	96.6	735.640	6.4	.714	6.7	.693	-134.6	.742	15.9	.707	22.9
.798	105.1	.652	8.8	.742	5.1	.701	-139.1	1072.790	- 9.7	.711	26.9
.803	99.1	.662	2.6	.752	12.5	.711	-126.4	.795	- 4.2	.714	19.9
.813	104.9	.669	2.6	.764	7.1	.718	-132.1	.800	- 4.8	.718	23.3
.825	100.9	.683	1.9	.782	7.6	.726	-134.0	.806	- 4.6	.720	26.1
.840	97.4	.690	1.9	.805	7.4	.734	-131.4	.813	- 6.8	.733	28.1
.854	108.5	.696	7.8	752.630	100.1	.742	-139.5	.888	- 2.8	.737	19.4
729.626	32.1	.705	6.6	.636	95.5	.761	-129.4	.894	- .9	.740	28.4
.633	19.6	.712	8.3	.641	96.8	755.634	1.3	.901	- 3.7	.743	24.0
.639	28.1	.718	5.8	.647	97.1	.642	1.4	.907	.3	.746	26.2
.645	25.0	.725	11.6	.653	105.1	.650	3.4	.913	- 1.6	.749	25.9
.651	17.9	.732	9.7	.659	97.9	.658	5.1	.920	0.0	.751	19.5
.658	27.2	.739	5.1	.665	94.1	.665	- 2.5	.927	.5	.754	27.7
.660	18.9	.746	9.6	.673	97.3	.673	.7	.936	.4	.758	26.7
.678	15.1	.752	2.3	.681	100.6	.679	3.3	1073.624	68.6	.761	29.2
.685	20.0	.763	1.1	.688	93.1	.685	3.3	.628	63.8	.764	22.0
.693	21.0	.769	6.7	.696	98.3	.692	5.5	.633	63.2	.767	21.2
.702	12.3	.777	.9	.704	102.0	.697	- .7	.638	61.2	.776	14.3
.708	10.3	.784	2.1	.710	91.7	.705	3.8	.643	64.2	.780	32.6
.716	7.7	.791	1.9	.721	96.1	.711	1.0	.647	62.0	.783	20.5
.723	9.9	.799	6.9	.730	95.2	.717	.9	.652	72.6	.786	25.3
.730	8.6	.806	9.0	.737	99.7	.723	3.0	.657	78.0	.790	26.6
.743	8.1	.814	- .6	.744	97.7	.729	4.8	.661	73.8	.794	27.8
.749	10.3	745.638	29.6	.752	97.6	.736	.2	.666	75.3	.798	20.4
.757	5.4	.663	24.1	.761	93.8	.742	3.1	.672	67.0	.802	28.1
.763	5.0	.674	20.0	.768	109.0	.750	- 1.3	.677	73.6	.808	36.4
.770	3.0	.685	24.3	.777	99.8	.758	- 1.1	.707	75.8	.813	38.1
.778	5.9	.697	21.7	.787	103.4	756.661	99.7	.713	81.7	.819	36.9
.787	1.1	.709	8.4	.798	92.3	.669	97.7	.717	72.9	.824	36.4
.803	5.4	.723	12.7	753.630	46.7	.677	100.9	.722	73.5	.841	27.4
.811	4.2	.737	7.0	.637	40.6	.686	105.1	.727	68.3	.849	13.1
.820	9.1	.749	17.5	.647	29.8	.695	99.6	.733	78.0	.863	24.8
.831	- .3	.761	4.6	.655	35.4	.702	111.3	.740	76.3	.875	17.9
732.660	108.3	.776	8.5	.680	33.1	.710	108.1	.746	65.7	.883	27.0
.668	107.4	.790	13.2	.688	24.1	.719	105.1	.751	77.5	.888	16.9
.677	99.7	746.640	-128.8	.698	20.1	.727	102.3	.757	69.6	.894	20.2
.688	89.9	.650	-125.6	.706	15.9	.736	111.2	.763	72.6	.900	22.1
.701	97.5	.657	-119.0	.714	25.1	.744	102.1	.769	75.5	.907	17.2
.715	96.6	.668	-130.3	.721	21.8	.755	106.1	.783	83.6	.915	18.4
.732	96.8	.676	-122.2	.728	21.9	.766	106.2	.790	80.0	.924	16.7
.746	97.3	.685	-128.9	.736	5.6	.785	105.2	.801	81.6		
.758	103.9	.694	-114.7	.743	11.2	759.629	4.9	.812	68.0		
.787	104.1	.703	-117.9	.751	8.7	.645	- 1.5	.823	70.6		
.808	92.1	747.607	- 4.8	.761	2.2	.697	3.1	.834	70.2		

CHAPTER 3

THE BINARY ORBIT

As a preliminary to the study of the pulsation it was necessary to remove the binary motion, which required the calculation of the spectroscopic binary orbit. This was expected to be a trivial matter but proved to be one of the most difficult and time consuming parts of the project.

This was due to several factors which greatly complicated the analysis. One of these was the nearly 4-day ($4\overset{d}{.}01452$) orbital period which means that during any one observing season only four rather limited ranges of orbital phase can be covered. These phase ranges for the current observations are shown schematically in Figure 1. The behavior of the star during the remainder of the orbit must be inferred by interpolating between the observed portions. Alternatively, data from several seasons can be combined to give complete coverage of the orbit. Unfortunately, the seasons so combined must cover several years since the difference between the orbital period and four days is enough to cause the observing window to shift by nearly one quarter of the orbital period in one year. But combining data spaced over a number of years brings in the complication of the apsidal motion and requires the introduction of another parameter, the apsidal period, into the solution.

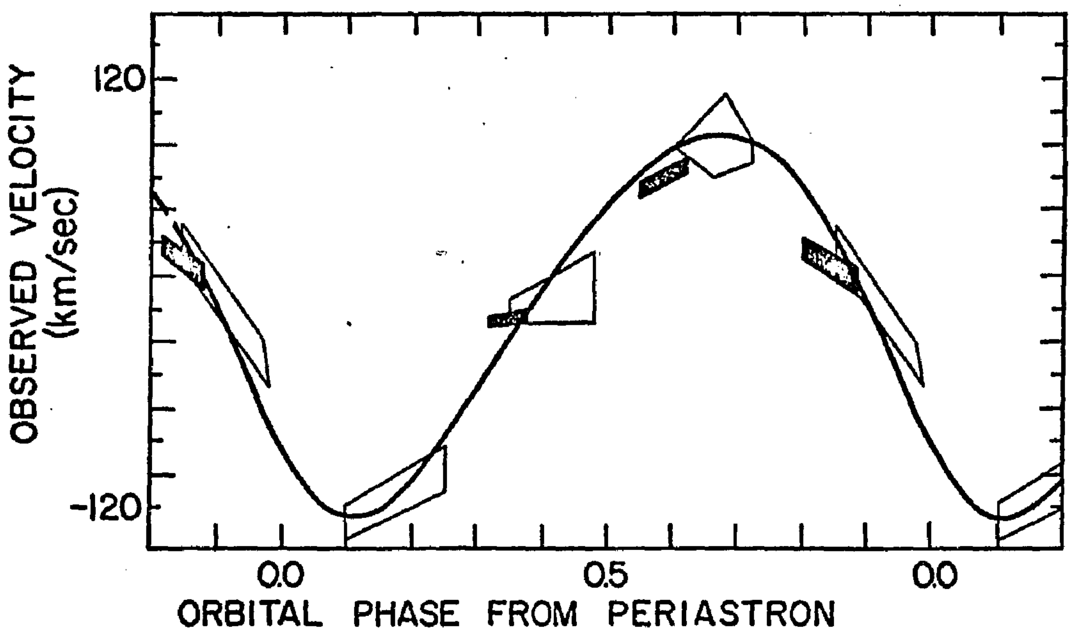

Figure 1. Observed and Computed Orbital Velocity Variation of Spica. -- Open regions show schematically the 1970 observation and solid regions the 1971 observation.

Another problem is the blending between the lines of the primary and secondary. This type of problem has been discussed in detail by Petrie, Andrews and Scarfe (1967) using artificial B-star spectra composed of two identical components. They gave corrections, as a function of the measured separation of the lines of the two components, which can be applied to the measured velocity to give the true orbital velocity. Unfortunately, these are not applicable to Spica since the two components are not of equal brightness but rather the primary is two magnitudes brighter than the secondary, and since, for reasons discussed previously, the lines of the secondary were not measured. The blending problem is seen by inspection to cause velocities near the γ-velocity axis to be shifted toward this axis. This has the effect of tilting the branches with respect to their true position and causing the value of K determined by a least squares orbital fitting procedure to be smaller than the true value. Shobbrook et al. (1972) solved this problem by eliminating those velocities which were not far enough away from the γ-axis to be relatively unaffected by the blending. However, this means that only those velocities near maxima and minima are suitable for use, which in turn means that the quantities (particularly e) that are strongly influenced by the γ-velocity crossing are poorly determined.

Rather than doing this we decided to adopt a procedure consisting of several steps to determine the orbit. First, we determined the orbital period as seen from the earth ($P_{\oplus}$) by locating the main peak on a periodogram calculated for frequencies in the neighborhood of 0.25 c/d from the data set consisting of all the published observations mentioned earlier plus a few published by Kao and Struve (1949) plus the current set. By then fitting frequencies around this main peak together with their first three harmonics we determined the frequency corresponding to this period to be $f_{\oplus} = 1/P_{\oplus} = 0.249115 \pm 0.000001$ c/d, where we arrived at the estimate of the error by computing a signal-to-noise ratio, defined as the square root of the sum of the squares of the amplitudes of the frequency and its harmonics divided by the mean error of the fit, for those frequencies used in the fittings. We set the error limits at those frequencies for which this quantity was ten percent lower than its maximum value. This period is related to the orbital period as seen from the coordinate system rotating with the line of nodes of the orbit (P_*) and the apsidal period (P_{aps}) by the following equation:

$$1/P_{\oplus} - 1/P_* = 1/P_{aps}. \tag{1}$$

We adopted the orbit determined by Shobbrook et al. (1972), including their value of the orbital period (P_*), as a starting point and found an orbital solution from our set of observations. We then found an improved value for the orbital period (P_*) by fitting by least squares the value of T_o from our orbit together with those found by Shobbrook et al. from the earlier data sets to a linear equation of the form:

$$T_{oN} = T_{zero} + E_N P, \tag{2}$$

where the E_N are integer cycle numbers and were calculated using the published orbital period. The values of T_o used together with the residuals from the fit are given in Table 4. We found the orbital period (P_*) to be 4.01452 ± 0.00002 days.

Table 4. Values of T_o and Residuals from Linear Fit.

T_o (Heliocentric JD)	(0 - C) (days)
2417955.84	-0.06
2426041.26	+0.11
2435563.57	-0.03
2440284.78	+0.10
2440690.04	-0.11

Next, we followed the procedure of Shobbrook et al. of eliminating all blended velocities (we chose to eliminate all velocities with $|v| < 50$ km/sec) from the data set used to compute the periodogram for finding $P_{\oplus}$. To conserve computer time we replaced our observations with mean values, each based on about ten observations. We used this data set, with proper allowance for apsidal motion, to determine an orbit while requiring that the two periods (P_* and P_{aps}) have the values found above. There were two orbits which were equally good, in the sense of having equal fitting errors, but which differed in K by 9 km/sec. We initially adopted the one with the larger K which agreed with that found by Shobbrook et al. (1972) but as a result of the fittings of the pulsation frequency to the data from individual nights, as described in the next section, we found that the nightly mean value determined from the orbital residuals varied in phase with the orbital frequency and in a sense which indicated that K was too large. We determined the correction to K from the amplitude of the variation and to γ from its centroid and found that the corrected quantities agreed very well with those of our second orbit. We therefore adopted this second orbit.

Now we fixed γ, K, P_*, and P_{aps} at the values found above and, using the other orbital parameters of our adopted orbit as starting values, determined an orbit from the data set which included the blended velocities. This had the effect of changing the orbital parameters, T_o, e, and γ to give a better fit on the branches while preserving the fit at maximum and minimum. This final orbit is plotted in Figure 1 together with the schematic representation of our observations and its orbital parameters are given in Table 5.

As a check we computed new orbits from each individual data set holding K, γ, and P_* constant and verified that the T_o's had not changed value sufficiently to cause a change in P_*.

We then calculated residuals from our final orbit. Since over half of the current data set consists of blended velocities we had to eliminate the effect of this blending from the blended residuals before we could combine them with the unblended ones to form the final data set. We did this by observing that the portions of the branches covered by the blended velocities were small enough so that the velocities could be approximated by a straight line tilted with respect to the branch and by then fitting straight lines in orbital phase to each branch. Residuals from these phase groups were combined with the orbital residuals

Table 5. Adopted Orbital Solution and Revised Physical Parameters.

P_* (days)	4.01452 ± 0.00001
T_o (J.D.)	2440690.05 ± 0.07
γ (km/sec)	0.0 ± 1.0
K (km/sec)	116.0 ± 1.6
e	0.13 ± 0.01
ω (^{o})	129 ± 7
P_{aps} (years)	143 ± 20
$R_1/R_\odot$	7.9 ± 0.9
$M_1/M_\odot$	10.3 ± 0.8
$M_2/M_\odot$	6.1 ± 0.7
log g_1 (c.g.s.)	3.7 ± 0.1
D (pc)	81 ± 6
a (km)	$(1.88 \pm 0.08) \times 10^7$
log $L/L_\odot$	4.15 ± 0.17
M_{v_1}	-3.6 ± 0.1
M_{v_2}	-1.6 ± 0.2

from the phase groups near maximum and minimum and from the 31 velocities which Shobbrook et al. (1972) obtained on J.D. 2440283 to form our final data set.

In our comparisons in Section V of the observations with models we are going to need the rotational frequency of the star, which can be calculated given the rotational velocity, which we will take from the work of Watson (1972), and the radius. Now the radius has been given by Herbison-Evans et al. (1971) but was determined along with many other physical parameters of the system using the orbital elements of an orbit approximately the same as the one published by Shobbrook et al. (1972). We therefore have updated these quantities using our orbit and also list their values in Table 5. Using our radius, Watson's v sin i = 155 km/sec, and Herbison-Evans et al.'s i we find v = 170 km/sec and the rotational frequency, Ω = 0.425 c/d.

CHAPTER 4

THE PULSATION ANALYSIS

The first step in the pulsation analysis was to obtain a periodogram from the data for frequencies in the neighborhood of the published primary frequency $f_1 = 1/P_1$. Next, several frequencies near this maximum were fitted to the observations by least squares. Interpolating among the amplitudes and mean errors found in these fits gave a best fit frequency $f_1 = 5.7542$ c/d. Fitting f_1 to the measures on each night separately then yielded the amplitude A, mean value γ, and phase zero point α, for each night. These are given in Table 6. The amplitude is highly variable as can be seen in Figure 2 showing a large amplitude velocity curve obtained on J.D. 2440699 together with a small amplitude one from J.D. 2440755. Those phase zero points found on the small amplitude nights are meaningless since the amplitude is much less than the mean error of the fit.

In Figure 3 we compare the relative tidal potential at the center of the disk of the primary due to the secondary with the observed velocity and the light amplitudes, which were calculated from the data of Shobbrook et al. (1969, 1972) in the manner described above and are given in terms of percent mean light, as a function of orbital phase. The tidal potential, which was calculated from equation (9) of Fitch's paper (Fitch 1967), is expressed in units of the unperturbed potential

Table 6. Amplitudes and Phase Zero Points of the Velocity Variation.

Heliocentric JD (2440000 +)	γ (km s^{-1})	A (km s^{-1})	α (periods)
283.140	- 2.4	7.9	0.03
697.820	3.4	9.2	0.93
698.833	1.9	8.4	0.99
699.814	- 2.4	13.0	0.99
700.799	- 0.7	4.5	0.00
703.791	4.4	4.9	0.05
705.850	- 8.3	16.4	0.97
706.811	- 5.2	4.8	0.00
707.795	1.8	11.8	0.26
716.775	- 2.3	5.8	0.72
717.765	- 1.0	0.1	0.75
718.758	12.0	4.5	0.64
719.815	5.0	0.4	0.56
720.777	- 5.0	0.3	0.78
727.760	3.4	1.8	0.79
728.746	- 2.5	3.5	0.09
729.728	- 1.4	3.5	0.83
732.734	- 0.7	2.9	0.83
734.746	- 1.6	4.4	0.62
735.733	0.2	3.0	0.27
745.730	- 3.7	4.9	0.04
746.671	8.2	7.0	0.76
747.706	1.1	2.2	0.86
752.714	1.5	2.5	0.74
753.711	- 2.8	10.5	0.89
754.696	- 7.0	2.1	0.82
755.713	1.0	2.4	0.41
756.706	7.1	2.9	0.33
759.683	5.3	2.5	0.86
761.695	2.7	14.9	0.86
1072.863	- 8.9	0.6	0.65
1073.767	-15.6	3.4	0.27
1074.782	0.8	4.8	0.81

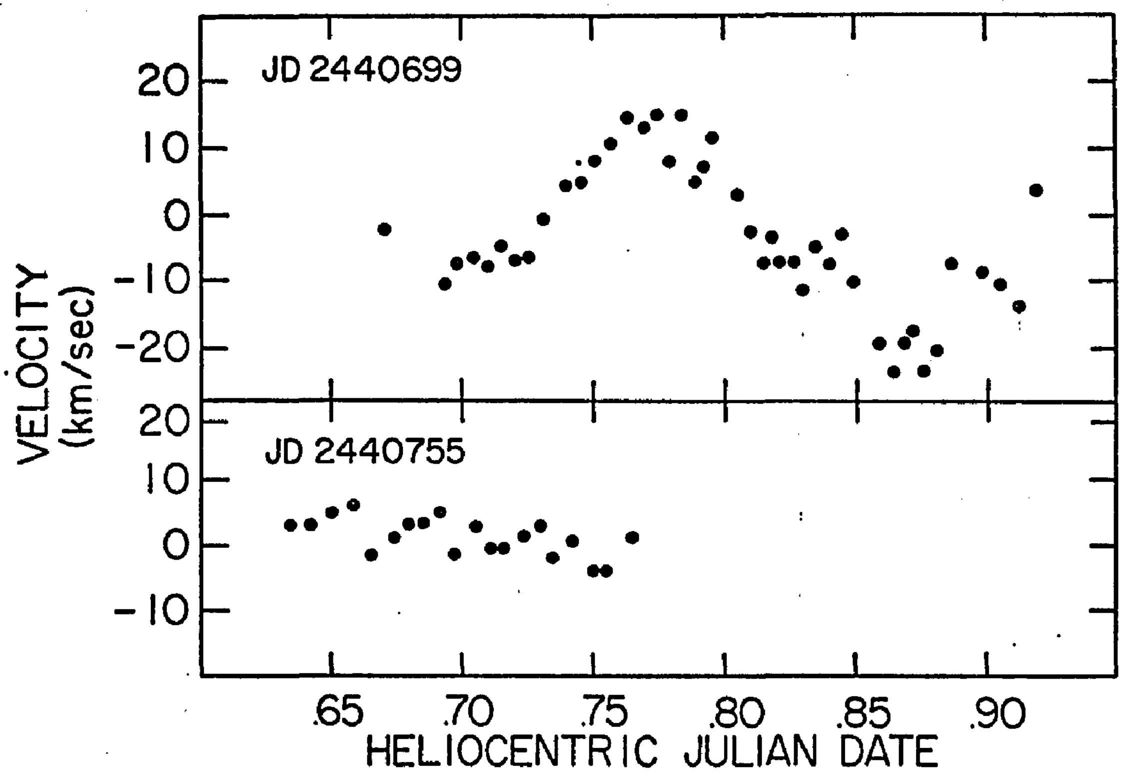

Figure 2. Pulsational Velocity Variation on Two Nights. -- Upper plot shows a typical large amplitude night while lower plot is a small amplitude night.

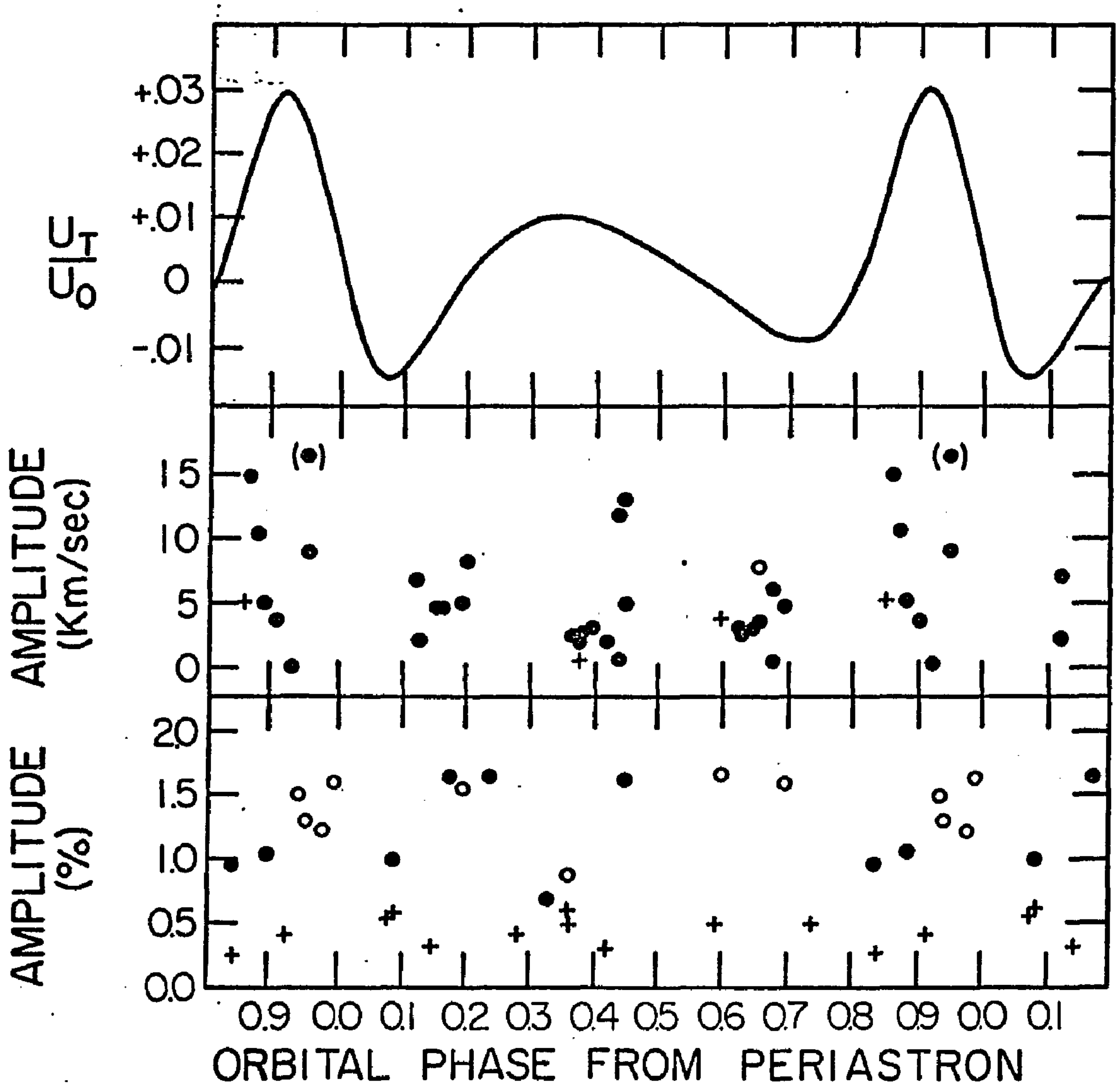

Figure 3. Tidal Potential with Observed Velocity and Light Amplitudes. -- The smooth curve is the variation in tidal potential at the center of the disk of the primary due to the secondary. The middle plot is the variation of the velocity amplitude with orbital phase while the lower curve is the variation of the light amplitude. The meanings of the symbols are discussed in the text.

of a single, non-rotating star with the same mass and radius as the primary. The open circle in the plot of the velocity amplitudes is for Shobbrook et al.'s data on J.D. 2440283, while the filled circles and crosses are our 1970 and 1971 data respectively, with the point in parentheses at orbital phase 0.95 representing the night of J.D. 2440705 which only had seven velocities. In the light amplitude the open circles refer to the 1968, the closed circles the 1969, and the crosses the 1970 observing seasons.

Comparing first the plots of tidal potential and velocity amplitudes we see that our largest amplitudes occurred near the maxima in tidal potential. Obviously the simple tidal effects model used by Fitch (1967, 1969) in his work on σ Scorpi and β Cephei is not applicable since we observe low amplitudes occurring in the same phase range as the high amplitudes but the upper envelope of the observed amplitudes does bear a strong resemblance to the tidal potential curve.

Turning to the light amplitudes we see that the case is not as clear. Most of the large amplitude pulsation observed occurred in 1968. This plot is slightly misleading since an inspection of the 1968 light curves, most of which cover more than one pulsation period, reveals rapid amplitude changes; up to 0.01 magnitudes in about one-tenth of a day or one-half of a pulsation period. This variation then, which is 50 percent of the observed amplitude, occurs in a range in orbital phase of 0.01. Thus if we could know the continuous behavior of the star during this period our plot of observed amplitude versus orbital phase might be much different. If these 1968 observations are ignored the remaining observations more closely resemble the velocity amplitude behavior. We conclude that the tidal potential does influence the pulsation amplitude as has been found by Fitch (1967, 1969) for other Beta Cephei stars.

Shobbrook et al. (1972) have concluded that the amplitude of pulsation secularly decreased from 1968 through 1971. This may be the case for the average amplitude or the frequency of high amplitude occurrence but the current observations show that large amplitude pulsation was still occurring in 1970.

Returning to the periodogram analysis of the velocity data, the amplitude-frequency spectrum was recalculated after subtracting the nightly variation in γ from the data, with the results that f_1 was unchanged but now the noise background in the spectrum was greatly reduced. In the following discussion all calculations were made on the data set already whitened for variations due to γ. Further, at each step in the computations variations due to each frequency, f_i, already found were removed before searching for the next term. The details of the steps involved in this search are given in Appendix B. Briefly, to conserve computer time we first calculated low resolution spectra using a short time step, as described by Fitch (1967), and then high resolution spectra were calculated for short frequency intervals in the neighborhood of any main peaks and their diurnal sidelobes. Finally, least squares fittings were used around the main peaks in these high resolution spectra to determine the best frequency for the term involved. In this manner we isolated three additional terms: f_2 = 5.7854 c/d, f_3 = 4.1158 c/d, and f_4 = 3.6171 c/d. Our final velocity solution is given in Table 7

Table 7. Adopted Velocity and Light Solutions.

Term	Frequency(c/d)		A		α(Periods)		Signal/Noise		A_V/A_ℓ	$\Delta\alpha(V-\ell)$
	R.V.	Light	(km S^{-1})	(% Light)	R.V.	Light	R.V.	Light		
f_1	5.7542	5.7550	3.8	0.90	0.01	0.74	0.65	1.11	4.2	+0.27
f_2	5.7854	5.7845	2.8	0.31	0.57	0.35	0.48	0.38	9.0	+0.22
f_3	4.1158	4.1158	1.8	0.16	0.23	0.20	0.31	0.20	11.3	+0.03
f_4	3.6171	3.6173	2.1	0.31	0.01	0.81	0.36	0.38	9.0	+0.20

together with the light solution discussed below. For each term in the solution we give the frequency of the best fit, the amplitude found in the fit, the phase zero point of the term, and the signal-to-noise ratio for the term, defined as the amplitude of the term divided by the mean error of the fit.

Since the signal-to-noise ratio was poor for all terms, ranging from 0.65 for the primary to 0.31 for f_3, we attempted to verify our results by examining the published photometry of Shobbrook et al. (1969, 1972) by least squares fittings in the neighborhood of each of these frequencies. As can be seen from the table there were slight frequency differences between the velocity and light terms. In particular the light f_1 is very close to what Shobbrook et al. found in their 1969 paper. However, they later (1972) modified this to f_1 = 5.7542 c/d in agreement with our velocity data. They attributed the change to their improper removal of the orbital light variation in the earlier paper. Since we removed this by fitting the orbital frequency and its first five harmonics to the light data while they finally used least squares optimization of the parameters involved in their equation (1) expressing the light variation due to the ellipsoidal effect (Shobbrook et al. 1972) the difference in our pulsation frequencies is probably explained. These slight frequency differences should not affect the comparison of the light and velocity terms.

To determine if a term was real we applied three of the four criteria suggested by Fitch (1969). These are that a maximum in the amplitude occur simultaneously with a minimum in the mean error as found by least squares fittings for real as opposed to spurious terms and that all real terms should show the same relation between their velocity and light phase zero points. We give the velocity to light amplitude ratios (Av/A_ℓ) and the relative phase zero points in the sense α(vel) $-$ α(light) in the last two columns of Table 7. Normally, it would be meaningless to compare the phases of terms of different frequencies. However, for our velocity data slight changes in the frequency, such as are involved here, cause only minor changes in the phases. Therefore in this case it is permissible to compare the relative phases for the light and velocity solutions and find that, except for f_3, they are all about +0.25 in agreement with the 90^o phase lag between velocity and light found in other Beta Cephei stars.

The agreement is not nearly as good for the f_3 term which we were especially interested in since it lies in the same frequency region as the strong term which Shobbrook et al. (1972) reported finding in the 1908 data set of Baker and which has since apparently disappeared. We therefore performed a careful examination of the amplitude-frequency spectra in this region from both the recent light and velocity data as well as from Baker's data. Using least squares fittings around the simultaneous maxima found in both the light and velocity data we found a possibility with better phase agreement and a slightly better fit to the velocity data at 3.9535 c/d but the amplitude of the corresponding light term was very small causing us to discard this possibility. The details of the preceding search are also given in Appendix B.

The results of our analysis of the Baker observations differed significantly from those of Shobbrook et al. (1972). Using the residuals from all of Baker's velocities calculated with our orbit and performing straight least squares fittings as they did in the regions 3.6 to 3.65, 3.9 to 4.2, and 5.7 to 5.8 c/d we found 21 peaks of significantly higher amplitude and lower mean error than the 3.96 c/d term they picked. The highest of these was at 5.7565 c/d and had an amplitude 2.5 times greater than the 3.96 c/d peak. The reason for this is most certainly due to the differences in the orbits and the small number (32) and the poor quality of the velocities. We redid the least squares fittings for the same frequency ranges using residuals from an orbit very similar to the one used by Shobbrook et al. (1972) and found that the peak structure had changed significantly with the 3.96 c/d peak now being among the highest present.

We cannot conceive of a reason for a real change in K of the magnitude required by the differences in the two orbits, except for perhaps large scale mass exchange which would be visible spectroscopically. Therefore, we do not feel that it is justified to use significantly different values of K or e for calculating residuals from the 1908 and current data sets. We did not carry this investigation any further since we feel that the problem is that 32 observations are not enough to permit disentangling the orbital and pulsational effects. It is entirely possible that a term close to 4 c/d was the strongest pulsational mode in 1908 and, as we shall see in the following section, this would have very interesting implications for our adopted model but we feel that the case for this has not yet been conclusively made and it will probably never be possible to do so.

In addition to the velocity data we had the spectral scans mentioned in Section II. These were of marginal value because in order to achieve adequate time resolution we had not covered enough spectral range to insure uniform placement of the continuum from night to night and because of difficulties with the scanner which caused noise spikes to be inserted into the signal occasionally. The reduction and analysis of this data is discussed in Appendix C. There were obvious variations in the central depth of the line on at least two nights. Also, by comparing each profile with a nightly mean profile we detected possible smaller changes. Because of our inability to determine the continuum properly there was no way to insure that the nightly variations had the same zero point. Since our experience with the removal of the nightly mean velocity from the velocity data had indicated that the presence of a variable zero offset is to increase the noise in the amplitude-frequency spectrum without changing the peak frequency, we attempted a periodogram calculation from this data and found some indication of a periodicity of around 4 c/d. The poor quality and short time span of the data prevented us from carrying this investigation further in an attempt to verify that this was the same as the periodicity in the light investigation. We feel that this possibility does warrant further study at a later time.

CHAPTER 5

DISCUSSION

There have been several models proposed for the observed pulsation phenomena in the Beta Cephei stars, which have utilized both radial and non-radial pulsations and the effects on the pulsation of both rotation and the tidal influence of a secondary. Non-radial oscillations have been invoked since they seem to explain the large velocity to light amplitude ratio observed in Beta Cephei stars as compared to the Cepheid strip variables and the line profile variations which characterize these stars.

Watson (1971a) has recently shown that it is possible to explain the large velocity to light amplitude ratio by taking into account the much larger bolometric correction necessary for these stars than for the Cepheid strip variables.

He has also (Watson 1972) calculated a P- log g - θ_e relation from his and other observations of the classical Beta Cephei stars, fitting scanner data from model atmospheres to obtain log g. He (Watson 1971b) used this relation and his scanner observations of Spica to predict a period for it which he found to be in good agreement with that determined by Shobbrook et al. (1969). We were led to redo this calculation after we had calculated a pulsation constant, Q, from our results combined with those of Herbison-Evans et al. (1971) and found it to be much smaller than the mean value Watson (1972) found for the classical Beta Cephei stars. Since the line profiles of Spica are very difficult to work with we calculated the predicted period using his value of θ_e but calculated log g from the presumably higher quality orbital and interferometric mass and radius. We found that instead of the predicted period being close to 5.75 c/d it was instead in the region of 4 c/d. Thus the strongest pulsation in Spica does not agree well with that observed in other Beta Cephei stars.

If we consider the radial pulsation model and identify the strong f_1 term as the first overtone then we have two possibilities for a weakly excited fundamental. Choosing the 3.6171 c/d term we find a period ratio, $f_4/f_1 = 0.626$, while choosing $f_3 = 4.1158$ c/d gives a period ratio, $f_3/f_1 = 0.715$. This latter choice gives the pulsation constant, $Q_o = 0.035$, compared to Watson's value of 0.037 for the mean of the classical Beta Cephei stars while the former choice gives $Q_o = 0.040$. Comparing these with model calculations should help us make the proper choice. We find that there have been two sets of extensive radial pulsation models calculated in the proper mass ranges; these are the 15, 20, and 30 $M_\odot$ models by Stothers (1965) who used a linear, adiabatic pulsation calculation and included the effects of a semi-convection zone, and the more recent linear, non-adiabatic calculations by Davey (1973) who neglected the effects of the semi-convection zone. The period ratios found by Davey rapidly approach 0.75 as the model evolves from the main sequence for both the 10 and 20 $M_\odot$ sequences while the period ratios of Stothers model approach 0.713, 0.703, and 0.690 for the 15, 20, and 30 $M_{\odot}$ sequences, respectively. We note that the period ratios in Davey's models are essentially independent of mass while those of Stothers show a slight mass dependence.

Returning to our observed period ratios we note that there are none of the 15 $M_{\odot}$ models of Stothers or the 10 $M_{\odot}$ models of Davey which have a period ratio close to the 0.626 value at a radius close to 7.9 $R_{\odot}$ but our 0.715 period ratio is closely approximated by the terminal value of the Stothers sequence and lies relatively close to that found by Davey. Comparing the observed fundamental (f_3), first overtone (f_1), and period ratio (f_3/f_1) we find that a model between either models 3 and 4 or 4 and 5 of Stothers' 15 $M_{\odot}$ sequence or between models 8 and 9 of Davey's 10 $M_{\odot}$ sequence will approximately fit all three. There are two other regions in Davey's sequence where the more accurately determined first overtone is matched, between models 9 and 10 and between models 12 and 13, but the observed value of the fundamental (if our identification of this term is correct) does not lie between the corresponding model values. It is beyond the scope of this paper to attempt to reconcile the differences between the models; most especially the large difference in period ratios. We shall only note that all of the models mentioned lie in the S-bend region of their respective sequences and thus it is probable that Spica, as is expected for a Beta Cephei star, also lies in the S-bend.

The identification of our f_2 and f_4 terms are not as clear. We note that, as mentioned previously, f_4 is probably tidally coupled to the fundamental, f_3, and is possibly a non-radial mode, either one of the f or low-frequency g modes. f_2 is also probably a non-radial mode. Harper and Rose (1970) in their 10 $M_\odot$ sequence of models, which were not evolved far enough to be comparable to the observed radial periods, found that the p_o mode had a frequency of 5.8 c/d for their most evolved model. This is in the right region to correspond to f_2 but we cannot be certain since it is unknown how the frequency of the p_o mode will change as evolution progresses.

There have been two sets of investigations of models utilizing purely non-radial or a combination of radial and non-radial modes, both of which invoke rotation as a means of lifting degeneracy. In our discussion of these we are going to use the rotational frequency of Spica, $\Omega = 0.425$ c/d which we have calculated earlier.

The first of these was originally proposed by Ledoux (1951) and most recently discussed by Osaki (1971). It involves the excitation of the P_2 harmonic of the f or Kelvin mode which is five fold degenerate in the absence of rotation. Rotation lifts this degeneracy resulting in five possible oscillations. The ones which can be used to explain both amplitude and line profile variations are either the $P_2^{\,o}$ and $P_2^{\,-2}$ or $P_2^{\,+2}$. The calculated splitting, which is given by equation (15) in Osaki's paper, is independent of the combination chosen. Using our value of 0.425 c/d for Ω and 0.179 as the constant C we find a predicted splitting $\Delta f = 0.68$ which bears no resemblance to any of the frequency differences we have observed. We should point out that the constant C, which was calculated by Ledoux (1951) for the f mode of the standard model (n = 3 polytrope) is not strongly dependent on the model chosen and is always less than 0.25 meaning that the limits on the predicted splitting are between 0.58 and 0.78 for our rotational frequency. Therefore, we will not consider this model further.

The other of the models was first proposed by Chandrasekhar and Lebovitz (1962) and was expanded by Clement (1965, 1967). It involves an accidental degeneracy between the fundamental radial mode and the second harmonic (P_2) f mode which occurs for a certain value of the ratio of specific heats. Rotation lifts this degeneracy resulting in two highly non-radial modes whose frequencies depend on the unperturbed frequency and the rotational velocity. Clement has calculated the necessary quantities for polytropes of varying indices. Using the value we found earlier for the rotational frequency and interpolating in Tables 3 and 4 of his 1967 paper, we find that our f_1 and f_3 fit well to the results for an effective polytropic index of about 3.2 and with an unperturbed frequency of about 4.3 c/d. Again we can only remark that the f_2 and f_4 we observe are possibly non-radial modes similar to those discussed in the consideration of the radial pulsation model. Our identifications for the observed terms for these two models are summarized in Table 8.

There are two problems connected with this model which we have not considered. The first is that Stothers (1965) found that the effective polytropic index for his models is everywhere less than 3 which cast doubts on the validity of these identifications which require an effective polytropic index of about 3.2. The second is the recent work of Denis (1972) which shows that considering the case in which the

Table 8. Summary of Frequency Identifications.

Term	Frequency (c/d)	Identification	
		Radial Pulsation Model	Clement Model
f_1	5.7542	First Overtone	Non-radial S-mode
f_2	5.7584	Non-radial (Possibly p_o)	Non-radial (Possibly p_o)
f_3	4.1158	Fundamental	Non-radial R-mode
f_4	3.6171	Non-radial (Possibly f or g_o)	Non-radial (Possibly g_o)

variable is the synchronously rotating primary of a binary star system, the tidal effects increased the splitting due to rotation alone by a factor of 5 for the homogeneous and compressible model (n = 1 polytrope). Until this work is extended to include models more nearly resembling real stars any attempt at applying it to observations of real stars is meaningless but it is probable that the splitting due to tidal effects will act as the models become more centrally condensed. Thus the fit of our frequencies to the model neglecting tidal effects is probably fortuitous.

We will therefore adopt the radial mode model with possible non-radial complications as our final one.

CHAPTER 6

SUMMARY AND CONCLUSIONS

In this paper we have combined new velocities with those previously published to determine an improved orbit for Spica, changing at least one of the elements (K_1) significantly. This is important since the physical parameters of Spica are probably one of the best sets currently existing for any star and are being used to check on such things as the apsidal motion predictions of theory (Mathias and Odell 1973).

We also analyzed the new velocities for pulsation and identified four terms, three of which were previously undetected. We have found these same three terms in the published photometric data and thus verified their existence. We have, in addition, shown that some of the amplitude modulation phenomena observed may be tidal in nature.

We have compared our results with models and have concluded that the best fit results from assuming that the primary pulsation in Spica is radial in the first overtone with the fundamental and two non-radial modes weakly excited.

If Spica is an overtone pulsator then it differs from the classical Beta Cephei stars which obey Watson's (1972) period-temperature-gravity relationship and probably have the fundamental most strongly excited. The difference in pulsational modes could possibly be due to the high rotational velocity or the evolutionary state of the star. We note that the change in log g which we have found from Watson's value moves Spica just outside and above the Beta Cephei strip which Watson (1972) found in the θ_e-log g plane. As the evolutionary tracks leave the strip momentarily at the top of the S-bend, it could be that Spica is in this region. Thus it is possible that the pulsations are in the process of being damped out or that alternatively overtone pulsators occupy a slightly different region of this plane.

Additional observations of Spica are necessary to confirm the existence of the terms identified in the current paper and to shed some light on the nature of the non-radial modes, to confirm or deny the secular decrease in pulsation amplitude, and to verify the suspected line profile variations. Ideally observations made during the same day at widely spaced observatories could extend the coverage of the pulsation phenomena over a greater orbital phase range than is possible from a single location during one season.

The other Beta Cephei stars, especially any rapid rotators which are confirmed to be members of the Beta Cephei class, should be carefully examined for evidence of overtone rather than fundamental pulsation. One such possibility is Beta Centauri with a rotational velocity (v sin i) of 105 km/sec (Watson 1972). Watson (1971b) has predicted a period of $0\overset{d}{.}195$ for this star while Shobbrook and Robertson (1968) found that a period of $0\overset{d}{.}1348$ best fitted their velocity observations. If the predicted period is that of fundamental pulsation and the observed the first overtone then the period ratio would be 0.69 which is close, considering the rough nature of this calculation, to that found from models and in Spica. Certainly this star warrants further observation.

APPENDIX A

STANDARD STAR OBSERVATIONS

Since the current work is the first extensive velocity program attempted with the Steward Observatory 36" telescope it was necessary to investigate the velocity stability of the instrument. A number of stars were observed during the course of the project for possible use as velocity standards. Although it would have been preferable to obtain several exposures of two or three different standard stars at the beginning, midpoint, and end of each night, this was found to be too time-consuming, especially at the beginning of a night when the variable was favorably placed for observation. Therefore, most of the spectra of standard stars were obtained during the latter part of the night after the variable had set.

Although spectra of a number of stars were obtained for possible use as standards it was found feasible to measure only those of Vega and Arcturus. The results of each of these sets of measures are summarized in the following paragraphs.

There were a total of 15 measurable spectra of Arcturus. These were each measured twice using 12 lines whose wavelengths were initially taken from Petrie and Fletcher (1967). After reduction of all measures it was apparent that systematic corrections to the wavelengths of the stellar lines would significantly improve the internal probable errors.

After these corrections were applied the data was re-reduced using the revised wavelengths, resulting in the reduction of the mean internal probable error of a single observation from 1.2 to 0.3 km/sec. It is apparent from this factor of four reduction that the wavelengths quoted by Petrie and Fletcher (1967) are not valid for the resolution being used for the current work. Also, since the spectrograph used by them had a resolution which was about five times better than that of the spectrograph used in the current project it is not surprising that the internal errors they quote are about five times better.

The original and revised wavelengths used are listed in Table 9 while the velocities measured are given in Table 10.

The mean velocity for all spectra of Arcturus is -2.6 km/sec as compared to a value of -5.2 km/sec as given in the Wilson catalog. This implies a small negative correction between the 36" coudé spectrograph and the Wilson catalog.

The 34 useable spectra of Vega were each measured several times. The results of the initial measurements, which were made using those lines listed in Table 11, are given in Table 12. These were especially disappointing as there was a large scatter, even between repeated measurements of the same spectrum. Several methods of measuring were tried in an attempt to increase the measuring accuracy but the results obtained were no better than the original ones. Finally it was realized that the overlapping of the comparison spectrum and the stellar spectrum, which caused asymmetrical line profiles for some of the stellar lines was one of the chief sources of difficulty. Most of the spectra were

Table 9. Wavelengths of Lines Measured in Arcturus.

Element	Original Wavelength	Revised Wavelength
Fe	4445.479	4445.551
Fe	4442.838	4443.072
Fe	4442.349	4442.387
Ti^{+}	4417.723	4417.596
Fe^{+}	4416.827	4416.613
Sc^{+}	4415.563	4415.191
V	4389.988	4389.966
Fe	4389.254	4389.302
V	4379.238	4379.233
Fe	4375.944	4375.914
Cr	4344.511	4344.456
Cr	4339.456	4339.589

Table 10. Velocities Measured for Arcturus.

Plate	Date	Measure 1	Measure 2	Mean
109-11	4/20/72	-4.2 km/sec	-2.8 km/sec	-2.1 km/sec
110-1	4/21/70	-1.2	-1.9	-1.6
112-9	4/21/70	-2.3	-2.0	-2.1
120-9	4/22/70	-0.3	0.3	0.0
126-8	4/23/70	-4.5	-3.9	-4.2
133-5	4/26/70	-0.6	0.2	-0.4
149-6	4/30/70	-1.4	-1.8	-1.6
155-3	5/09/70	-6.1	-6.4	-6.3
162-4	5/10/70	-4.9	-2.5	-3.7
167-8	5/11/70	-3.0	-2.8	-2.9
170-6	5/12/70	-3.2	-2.2	-2.7
175-5	5/13/70	-5.1	-2.9	-4.0
184-2	5/21/70	-5.2	-5.1	-5.1
192-5	5/27/70	-2.0	-2.0	-2.0

Table 11. Lines Measured in Vega (First Set).

Wavelength	Element
3711.973	H I
3721.940	H I
3734.370	H I
3750.154	H I
3770.632	H I
3797.900	H I
3835.741	H I
3889.051	H I
3970.074	H I
4101.737	H I
4340.468	H I
4481.22	Mg II

Table 12. Velocities Measured for Vega (Using Lines Given in Table 11).

Plate	Date	Measure 1	2	3	4	Mean
SC112-10	4/21/70	- 3.8				- 3.8
SC120-08	4/22/70	- 5.8	- 1.4			- 3.6
SC120-09	4/22/70	-10.7	-10.4			-10.6
SC120-10	4/22/70	-12.1	-12.4			-12.3
SC128-07	4/23/70	- 9.3				- 9.3
SC149-07	4/30/70	-11.1	- 7.6			- 9.4
SC149-08	4/30/70	- 8.2	- 9.6			- 8.9
SC149-09	4/30/70	- 7.6	- 8.9			- 8.3
SC149-10	4/30/70	-11.1	-10.9			-11.0
SC155-04	5/09/70	- 5.8	- 7.3	- 7.3		- 6.8
SC155-05	5/09/70	- 7.2	- 8.1	- 7.7		- 7.7
SC162-05	5/10/70	- 4.2	- 9.1	- 9.8		- 7.7
SC167-10	5/11/70	- 6.2				- 6.2
SC170-07	5/12/70	- 8.7	- 8.7	- 8.8		- 8.7
SC175-06	5/13/70	1.6	- 6.7	- 4.1	- 6.1	- 3.8
SC175-07	5/13/70	-10.9	- 8.9	-10.4	- 8.4	- 9.7
SC179-08	5/20/70	- 1.3	- 1.8			- 1.6
SC184-03	5/21/70	- 6.1				- 6.1
SC188-05	5/22/70	-10.5	-17.0	- 8.5	- 6.2	-10.6
SC188-06	5/22/70	-10.5	-12.3	-12.1	- 6.3	-10.3
SC190-06	5/24/70	- 8.3				- 8.3
SC192-06	5/27/70	- 9.5	-12.5			-11.0
SC199-03	6/07/70	- 4.5	-12.2			- 8.4
SC203-07	6/09/70	- 8.8				- 8.8
SC206-06	6/14/70	- 6.0				- 6.0
SC208-11	6/15/70	- 9.0				- 9.0
SC210-08	6/16/70	- 3.2	- 7.1			- 5.2
SC213-09	6/16/70	- 9.2	- 6.3	- 8.6	- 6.5	- 7.7
SC221-03	6/21/70	- 8.6				- 8.6
SC222-03	6/23/70	- 9.1	- 6.2	- 7.9		- 7.9

then remeasured using the lines given in Table 13, which are primarily the weaker metal lines and which were chosen to be well removed from any strong comparison lines. An attempt was made to improve the wavelengths of the stellar lines to minimize the internal probable errors in the same manner as was done for Arcturus but the corrections obtained were not significant.

The results of this second set of measurements are given in Table 14. The mean velocity obtained from all measures was -7.8 km/sec compared with the Wilson catalog value of -13.9 km/sec again indicating a negative correction between the 36" coude spectrograph and the Wilson catalog.

The average internal probable error for the second set of measures was improved by a factor of two over those measures using the hydrogen lines and the consistency of the remeasures of a spectrum was greatly improved. In spite of this, there was still a large scatter between velocities obtained from different spectra as evidenced by the external probable error of 2.3 km/sec. The possibility of the scatter being caused by an hour angle effect was eliminated by the presence of two spectra taken ten minutes apart on April 20, 1970, with a velocity difference of 10 km/sec. There was also no apparent correlation of the observed velocity with date such as might be due to a gradual shift in telescope optics. Since the wide slit tends to cause large guiding errors and since the average exposure time for Vega was only three minutes it was assumed that the external scatter was mostly due to guiding errors.

Table 13. Lines Measured in Vega (Second Set).

Wavelength	Element
4443.89	Ti II
4468.47	Ti II
4481.22	Mg II
4501.24	Ti II
4508.26	Fe II
4515.31	Fe II
4520.21	Fe II
4522.59	Fe II
4533.98	Ti II
4549.51	Fe II, Ti II
4555.84	Fe II
4558.60	Cr II
4563.70	Ti II
4571.92	Ti II
4583.79	Fe II
4588.16	Cr II

Table 14. Velocities Measured for Vega (Using Lines Given in Table 13).

Plate	Date	Measure 1	Measure 2	Mean
SC120-08	4/22/70	- 3.1	- 1.3	- 2.2
SC120-09	4/22/70	- 6.5	- 4.4	- 5.5
SC120-10	4/22/70	- 7.2		- 7.2
SC149-07	4/30/70	- 8.8		- 8.8
SC149-08	4/30/70	-18.5	-17.4	-18.0
SC149-09	4/30/70	- 9.6	-10.1	- 9.9
SC155-04	5/09/70	- 7.6	- 8.3	- 8.0
SC155-05	5/09/70	- 6.7	- 6.9	- 6.8
SC162-05	5/10/70	- 7.8	- 8.2	- 8.0
SC167-09	5/11/70	- 5.4	- 5.2	- 5.3
SC167-10	5/11/70	-10.9	- 9.7	-10.3
SC167-11	5/11/70	- 4.0	2.4	- 0.8
SC175-06	5/13/70	- 3.7	- 4.6	- 4.2
SC175-07	5/13/70	- 7.6	- 7.9	- 7.8
SC184-03	5/13/70	- 5.9	- 5.8	- 5.9
SC188-05	5/22/70	- 7.4	-11.6	- 9.5
SC188-06	5/22/70	-10.6	- 9.9	-10.3
SC192-06	5/27/70	- 7.1	- 6.3	- 6.7
SC199-03	6/07/70	- 8.7	- 7.1	- 7.9
SC206-06	6/14/70	- 8.9	- 8.4	- 8.7
SC208-11	6/15/70	- 9.9	-10.0	-10.0

The difference between the corrections necessary to apply to the measured velocities in order to agree with the Wilson catalog as determined from Arcturus and Vega could be due to a declination effect which could lead to a significant difference in the correction that should be applied to the velocities for Spica but since in studying the pulsation phenomenon we are only interested in relative velocities this does not affect the current investigation. However, we should realize that the gamma velocity we determine is not necessarily valid.

APPENDIX B

THE PERIODOGRAM ANALYSIS OF THE PULSATION

After isolating f_1 as described in the main text we then whitened the data set for it and proceeded to search for additional periodicities by calculating low resolution spectra and then examining the main peaks of these in detail. Figure 4 shows a portion of the first broadened spectrum calculated after whitening for f_1, and Table 16 gives the improved values of frequencies and amplitudes obtained from maximum resolution spectra for the broad peaks (except the unlikely one at 2.60 c/d) listed in Table 15. Since f_2 = 5.7854 c/d is obviously the most important term here, the data set was re-whitened for both f_1 and f_2 simultaneously.

Next, a new spectrum showed the third strongest frequency to be near 4.13 c/d instead of 3.61 as suggested by Table 16. This value was improved at high resolution to f_3 = 4.1184 c/d (or its first sidelobe with 95 percent amplitude at 4.1158 c/d). After whitening for f_1, f_2, and f_3 simultaneously, a fourth term f_4 = 3.6171 c/d was found.

Finally, all four frequencies were fitted simultaneously to the data giving the results in Table 7 of the main text. Now it is apparent that 3.6171 c/d is stronger than 4.1158 c/d.

As we discussed in the main text we attempted to verify these frequencies using the published light data and found good agreement for

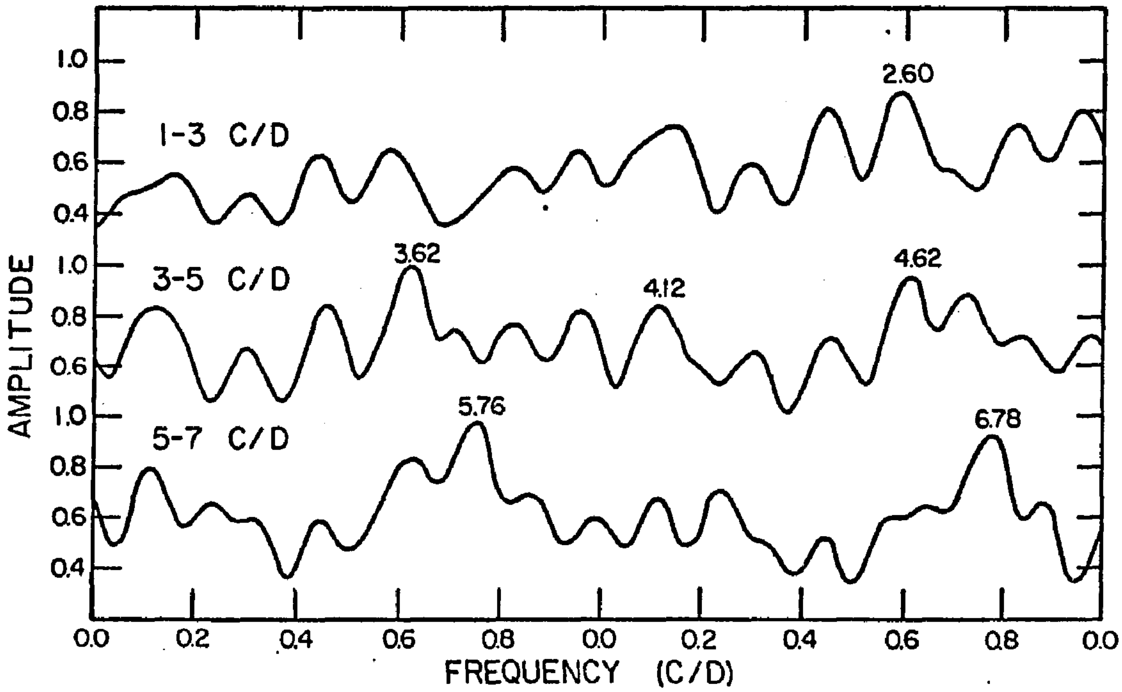

Figure 4. Periodogram After Removal of Strongest Term. -- The peaks labeled are those discussed in the text.

Table 15. Main Peaks of Low Resolution Periodogram.

Frequency (c/d)	Amplitude (% of maximum)
2.60	88
3.62	100
4.62	97
5.64	84
5.76	98
6.78	92

Table 16. Main Peaks of High Resolution Periodogram.

Frequency (c/d)	Amplitude (km sec^{-1})
3.6168	2.38
4.6194	2.36
5.6410	2.18
5.7853	2.61
6.7314	2.06

all terms except f_3. In an attempt to isolate some other nearby frequency that could have f_3 as one of its aliases we constructed maximum resolution amplitude-frequency spectra of the region from 3.9 to 4.2 c/d for Baker's 1908 observations as well as for the 1969-1971 velocity and light observations. Then we examined, by simultaneous least squares fittings of the three previously determined frequencies plus ones being considered, the neighborhoods of the peaks in the spectra of the recent observations which lay close to the peaks in the spectrum of Baker's observations. There were several peaks which satisfied the criteria relating to the simultaneous maxima and minima in the amplitude and mean errors respectively, but these did not satisfy the phase relationship any better than f_3 and did not give any better representation of the light and velocity data. There was one peak, in the neighborhood of 3.9534 c/d, which satisfied the phase criteria much better than f_3 but the amplitude of the light term was very small. Because of the small amplitude of the light term we also eliminated it leaving with us with our original solution involving $f_3 = 4.1158$ c/d.

APPENDIX C

THE ANALYSIS OF THE LINE PROFILE DATA

The line profiles we obtained were all of a 7 Å region centered near He 4471. The small spectral region covered meant that it was impossible to identify the continuum level with any confidence. Also, on two nights the center of the region was displaced from this line center which further complicated the analysis. The profiles were normalized to a "pseudo-continuum" identified as the five to ten highest points on a mean profile covering the entire night's observations. Then the normalized profiles were compared with the nightly mean profiles in a search for short period variations. The only obvious ones occurred on J.D. 2441024 when one profile was significantly deeper than the mean and J.D. 2441029 when a series of four profiles became shallow and then deepened again. To check for less obvious variations each profile was compared visually with the mean and the difference in central depths, which was usually around one to five percent was noted. These measurements are given in Table 17. The profiles on the last of the nights were excluded from further analysis because they were exceptionally noisy.

Since there was some unknown night-to-night variation in the mean central depths due to our failure to identify the true continuum it might be expected that it would be impossible to find a meaningful period. However, our experience with the analogous problem of finding the period

Table 17. Line Profile Central Depths.

Julian Date 2440000 +	Δ I (%)	Julian Date 2440000 +	Δ I (%)	Julian Date 2440000 +	Δ I (%)
1021.966	- 3	1024.907	- 4	1029.959	- 1
.982	- 6	.920	- 1	.976	+ 3
.996	- 2	.935	0	.992	+ 1
1022.010	0	.953	+ 1	1030.015	- 3
.037	0	.972	+ 2	1031.736	- 2
1022.055	+ 2	1024.986	+ 3	1031.753	0
.783	+ 1	1025.000	+ 3	.769	- 4
.797	+ 1	.015	+ 1	.785	0
.820	0	.029	+ 4	.803	- 3
.834	0	.042	+ 2	.820	0
1022.848	- 1	1028.790	0	1031.835	- 2
.860	- 1	.805	- 2	.852	- 1
.873	- 2	.821	- 2	.868	- 2
.888	- 2	.841	- 3	.882	- 3
.902	0	.861	- 3	.898	0
1022.916	- 2	1028.875	- 2	1031.913	0
.930	0	.897	- 3	.925	- 4
.944	0	.910	- 2	.940	0
1023.907	0	.924	- 1	.954	0
.923	- 2	.939	- 1	.968	- 1
1023.941	0	1028.954	- 2	1031.981	0
.956	0	.967	0	.995	0
.970	0	.988	0	1032.011	0
.991	+ 2	1029.002	0	.024	+ 3
1024.004	+ 2	.012	+ 2	.037	+ 2
1024.016	+ 2	1029.030	+ 5		
.846	0	.904	- 4		
.861	- 3	.917	- 3		
.878	- 3	.931	- 2		
.893	-12	.946	- 1		

NOTE: Δ I is the difference in intensity of the line center in the sense (observed - mean).

in the velocity data both before and after subtracting the nightly mean variation from the data and finding them the same caused us to feel that a search for periodicities in our differential depth measurements might give us some useful information. We constructed a maximum resolution amplitude-frequency spectrum for the region from 1.5 to 7 c/d and found the main peak to be at 3.09 c/d with a diurnal sidelobe at 4.09 c/d. Since the latter is indistinguishable from the frequency used at the resolution involved is possible that this is the real variation, not the 3.09 c/d term, and that it is the same as that identified in the velocities.

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