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CHARLOCK, THOMAS PETER
AEROSOL EFFECTS ON CLIMATE: CALCULATIONS
WITH A TIME-DEPENDENT RADIATIVE-CONVECTIVE
MODEL.

THE UNIVERSITY OF ARIZONA, PH.D., 1979
AEROSOL EFFECTS ON CLIMATE: CALCULATIONS
WITH A TIME-DEPENDENT RADIATIVE-CONVECTIVE MODEL

by

Thomas Peter Charlock

A Dissertation Submitted to the Faculty of the
DEPARTMENT OF ATMOSPHERIC SCIENCES
In Partial Fulfillment of the Requirements
For the Degree of
DOCTOR OF PHILOSOPHY
In the Graduate College
THE UNIVERSITY OF ARIZONA

1979
I hereby recommend that this dissertation prepared under my direction
by Thomas Peter Charlock
entitled Aerosol Effects on Climate: Calculations with a
Time-Dependent Radiative-Convective Model
be accepted as fulfilling the dissertation requirement for the Degree
of Doctor of Philosophy.

William D. Ballhaus
Dissertation Director

As members of the Final Examination Committee, we certify that we have read this dissertation and agree that it may be presented for final defense.

Richard M. Schmalz
Benjamin M. Herman
Voy M. Enrooth
Herby Mahan

Final approval and acceptance of this dissertation is contingent on the candidate's adequate performance and defense thereof at the final oral examination.
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The assistance given by other members of the Institute of Atmospheric Physics, past and present, is appreciated. The research was conducted under National Science Foundation grant ATM75-19742 A02.
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ABSTRACT

A one-dimensional, vertical energy balance model of the earth atmosphere system has been constructed. The model solves for the temperature of the globe or a given latitude belt. Aerosol changes the model shortwave (solar) and infrared (terrestrial) radiation fields, and thus changes the model's temperature solution.

The model consists of twelve atmospheric layers and the surface; each has the averaged physical properties of either the globe or a selected latitude belt. Radiative-convective equilibrium is calculated. Radiatively active constituents include water vapor, carbon dioxide, ozone, clouds, and aerosol.

Radiative transfer at solar wavelengths is done with a unique approximation to the adding scheme. The infrared radiative transfer calculation includes a separate treatment of water vapor dimer and aerosol effects in the water vapor window. A novel approach to convective adjustment is used.

The model calculates annual (steady state) and monthly (time marching) solutions. In the monthly case, separate heat storage is computed in oceans, land, and air. When run for globally averaged conditions the model produces temperatures very close to the Standard Atmosphere, except
for the lower stratosphere, where model temperatures err by about 5 degrees K. The model simulation for the latitude belt 40 to 50 degrees North departs from climatology only about 1 or 2 degrees K each month at the surface.

Calculations have been made for the case of zero aerosol optical depth and aerosol optical depth of 0.125. The aerosol drops the temperature of the global model by 1.5 K, that of the monthly model at midlatitudes by 1.6 K, and 1.8 K for the annual midlatitude model; ice-albedo feedback in the midlatitudes increases this drop to 3.2 K.

Dynamical feedback is not accounted for in the above calculations. Runs for differing aerosol optical depths and physical properties have been made. The model simulation for the surface temperature perturbation in the tropics caused by the Agung volcanic eruption agrees with a calculation by other authors.
CHAPTER 1

INTRODUCTION

The temperature pattern in the earth-atmosphere system is largely determined by the disposition of solar and infrared (terrestrial) radiation. Most incoming solar radiation is at wavelengths less than 4 microns, and most infrared (terrestrial) is at wavelengths greater than 4 microns. Both the solar and infrared radiation fields are influenced by atmospheric gases, clouds, the surface, and the atmospheric aerosol.

Aerosol, which in this work refers to the suspension of noncloud solid and liquid particles in the atmosphere, is found near the surface in concentrations ranging from $10\,\text{cm}^{-3}$ to $10^5\,\text{cm}^{-3}$. The typical concentration is $10^3\,\text{cm}^{-3}$, and this number rapidly decreases with height. The aerosol consists of particles of sea salt, mineral dust, organic matter, and products of inorganic gas to particle reactions and combustion (Junge, 1976).

Aerosol can scatter and absorb solar radiation. Scattering generally increases planetary reflectivity. This tends to cool the earth. Aerosol scatterers overlaying a strongly reflecting surface (as ice) can increase the surface absorption by a light trapping effect (Herman and
Browning, 1975). Aerosol scattering also changes the angle of incidence of incoming photons, increasing or decreasing their probability of being absorbed or reflected by other atmospheric constituents. Thus increased aerosol scattering may not necessarily imply global cooling.

Absorption of solar radiation by the aerosol tends to increase atmospheric temperatures. But if the photon thus absorbed would have otherwise been absorbed by another constituent besides aerosol, atmospheric temperatures then may not increase. The aerosol absorption might produce heat at some level in the atmosphere which can dispose of that heat (infrared cooling) more efficiently than can other levels. Fairly detailed computations must be performed to determine whether aerosol effects on solar radiation heat or cool the earth.

Aerosol can affect the infrared cooling of the earth by increasing atmospheric opacity. Aerosol optical depths in the infrared are generally smaller than those in solar wavelengths (Toon and Pollack, 1976). But in the infrared most aerosol particles absorb more strongly than in solar wavelengths. Absorption/emission of infrared by aerosol is more significant than aerosol scattering of infrared.

Aerosol infrared effects are generally masked by those of carbon dioxide and water vapor. The combination of these gases is strongly opaque except in the atmospheric
window region of 8 to 13.3 micron. The aerosol can be significant at window wavelengths. Aerosol increases atmospheric window counter-radiation, warming the surface.

Clouds are the most optically active component of the atmosphere, both in solar and infrared wavelengths. Aerosol can act as an "optical amplifier" by modifying cloud absorption and reflection (Twomey, 1974). Thus aerosol effects on radiation through clouds may be more significant than the direct aerosol effect on radiation itself. Aerosol effects on clouds are too complex to be dealt with in this study.

**Climate Models**

A detailed evaluation of solar and infrared fluxes is made in this work. A climate model is used to calculate the temperature pattern expected from the radiative fluxes. The model temperature responds to changes in aerosol quantity or optical properties. Real climate and the climate model both include feedback mechanisms.

Temperature, as it relates to infrared emission, is a negative feedback mechanism for climate. If atmospheric optics is perturbed and solar absorption thereby increased, temperature will tend to increase. The increased temperature will result in increased infrared emission to space. This increase in infrared emission balances the original
increase in solar absorption, thus limiting the temperature increase.

Atmospheric water vapor is a positive feedback mechanism for climate. If the absorption of solar radiation is increased, resulting in increased temperatures, more water will be evaporated from the oceans into the atmosphere. The additional water vapor will radiate more infrared energy downward towards the surface. The surface will respond by increasing its temperature and evaporating even more water.

Besides temperature and water vapor, significant climate feedback mechanisms are heat storage, atmospheric and oceanic dynamical heat transport, cloudiness and cloud optics, and ice-albedo. A one-dimensional radiative-convective climate model can account for some of the above feedback mechanisms. This type of model was originally developed by Manabe and Strickler (1964). The radiative-convective model can also provide the requisite detail in the radiation computations to account for aerosol effects (see Fig. 1).

Each vertical level in the one-dimensional radiative-convective model used here has the mean properties of that level at the latitude belt being modeled. The levels contain ozone, carbon dioxide, water vapor, aerosol, and cloud. Solar radiation enters the top level at a specified angle of incidence and is reflected, transmitted, and absorbed
Upgoing and downgoing infrared and solar fluxes at each level.

Aerosol, cloud, Rayleigh scatterers, H$_2$O, O$_3$, and CO$_2$

Fig. 1. One-dimensional Radiative Convective Model Schematic.

Solar radiation enters at the top and is scattered and absorbed by the various layers, which also absorb and emit infrared radiation.
at different levels. The model uses a new scheme for calculating radiative transfer at solar wavelengths. Infrared radiative transfer is also calculated. Heat can be stored at each level. The vertical column is generally used to represent the properties of a single latitude belt. In the real earth-atmosphere system, atmospheric and oceanic dynamical processes (called dynamics) transport heat into and out of the side walls of any given latitude belt; the model simulates the effects of dynamics by adding (subtracting) heat to (from) each level in the vertical column.

When each level in the model emits an amount of infrared energy equal to the net input from all other sources, the system is in approximate radiative equilibrium. "Approximate" because energy sources other than radiation (as dynamics and heat storage, which are usually smaller) are present at each level. Temperatures at each level are obtained from the solution of energy balance (conservation) equations.

The model radiative equilibrium temperatures are convectively adjusted to obtain a more realistic vertical profile. The convective adjustment procedure simulates the turbulent vertical fluxes of sensible and latent heat found in the real atmosphere. A novel technique for calculating convective adjustment is employed.
The model has twelve atmospheric layers: 50 mb each between 1000-800 mb and 100 mb each on up. Higher resolution is employed in the lower troposphere because aerosol tends to be concentrated there.

The model is run in three versions, labeled Monthly, Annual, and Global. The Monthly version of the model solves for the temperature structure of a latitude belt in a given month. The model time steps to the next month, storing heat in both the surface and atmosphere. An Annual version of the model solves directly for the steady state mean annual temperature of a latitude belt. The Global version calculates a mean temperature pattern for the entire earth atmosphere system.

The model neglects feedback effects due to clouds and dynamics. Perturbations in atmospheric radiation may change the quantity of cloud cover or its optical properties; this, in turn, may dampen or amplify the original radiation perturbation. An attempt at modeling cloud feedback in a one-dimensional model has been made by Weare and Snell (1974).

Meridional transport of atmospheric and oceanic heat by dynamics is specified in the model. The dynamical transports calculated from the Manabe, Bryan, and Spelman (1978) general circulation model are used. The general circulation model is a highly complex, three-dimensional
simulation of the atmosphere and ocean. The transports supplied by the general circulation model are regarded here as fixed. Hence dynamical feedbacks are neglected.

Dynamical feedback could be included in the present one-dimensional (vertical) model by coupling it with a one-dimensional (horizontal) energy balance climate model (Fig. 2). Horizontal models can transport heat among various latitude belts; they were developed independently by Sellers (1969) and Budyko (1969). A later Monthly version (Sellers, 1973) would be more suitable for use, and at considerably greater economy than coupling the present model to a general circulation model.

Earlier studies of aerosol effects on climate with a one-dimensional radiative-convective model include those of Rasool and Schneider (1971) and Reck (1976). Both use simplified radiation schemes for calculating aerosol effects. They assumed no meridional dynamical heat transport or heat storage.

Aerosol Properties

Most calculations made with the model use aerosol optics specified by the global average model of Toon and Pollack (1976)(note Table 1). Toon and Pollack give aerosol optical properties between 0.25 and 100 micron, and they also specify the aerosol height distribution. The
Fig. 2. One-dimensional (Horizontal) Sellers-Budyko Energy Balance Climate Model.

Each box has the mean properties of a latitude belt. Heat (dynamical) is transported between the belts.
Table 1. Toon and Pollack Atmospheric Aerosol Model for the Troposphere and Lower Stratosphere.

N is the number of aerosol particles per cm$^3$. $r$ is the particle radius in cm. $\beta$, $\sigma$, and $r_m$ are parameters whose numerical values are cited in the table. The optical depths (at 0.55 micron wavelength) cited are those from the top of the atmosphere to the height in question.

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<th>Optical Depth</th>
<th>Composition</th>
<th>Size Distribution</th>
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<tr>
<td>0</td>
<td>0.125</td>
<td>15% NaCl</td>
<td>$dN(r)/dr \propto r^{-(\beta+1)}$</td>
</tr>
<tr>
<td>1</td>
<td>0.083</td>
<td>35% basalt</td>
<td>$r&lt;0.1\mu m, \beta=-1; -0.1&lt;r&lt;5\mu m, \beta=2;$</td>
</tr>
<tr>
<td>2</td>
<td>0.058</td>
<td>50% $(NH_4)_2SO_4$</td>
<td>$5\mu m&lt;r&lt;50\mu m, \beta=4; 50\mu m&lt;r, dN/dr=0$</td>
</tr>
<tr>
<td>3</td>
<td>0.042</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.034</td>
<td>40% basalt</td>
<td>$dN(r)/dr \propto r^{-(\beta+1)}$</td>
</tr>
<tr>
<td>5</td>
<td>0.028</td>
<td>60% $(NH_4)_2SO_4$</td>
<td>$r&lt;0.045\mu m, \beta=-1$</td>
</tr>
<tr>
<td>6</td>
<td>0.024</td>
<td></td>
<td>$0.045&lt;r&lt;5\mu m, \beta=2.6$</td>
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<tr>
<td>7</td>
<td>0.021</td>
<td></td>
<td>$5&lt;r&lt;30\mu m, \beta=4.6$</td>
</tr>
<tr>
<td>8</td>
<td>0.017</td>
<td></td>
<td>$30\mu m&lt;r, dN(r)/dr=0$</td>
</tr>
<tr>
<td>9</td>
<td>0.014</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>$5.0\times10^{-3}$</td>
<td>100% sulfate</td>
<td>$dN(r)/dr \propto \exp\left{[-\ln^2(r/r_m)]/(2\ln^2\sigma)\right}$</td>
</tr>
<tr>
<td>15</td>
<td>$3.3\times10^{-3}$</td>
<td>$(75% H_2SO_4, 25% H_2O$</td>
<td>$\sigma=2, r_m=0.035\mu m$</td>
</tr>
<tr>
<td>18</td>
<td>$1.8\times10^{-3}$</td>
<td>liquid solution</td>
<td></td>
</tr>
</tbody>
</table>
real aerosol is not fixed as assumed here, but rather varies with space and time.

Aerosol is usually strongly concentrated in the lower troposphere where its lifetime is 3-6 days (Junge, 1976). Upper tropospheric aerosol has a lifetime of about 20 days, and stratospheric aerosol even longer.

Most of the increase in aerosol in the upcoming decades can be expected in midlatitudes due to surface sources. Industrial activity is centered there, and the short aerosol lifetimes should prevent significant increases in optical depth at other latitudes. Hence much of the effort in this study is spent on the zone 40 to 50 degrees North (40-50N).

Calculations are also done to determine aerosol effects on the globe and on the tropics. This model, as that by Hansen, Wang, and Lacis (1978), is able to roughly simulate the climate perturbation caused by the 1963 Agung volcanic eruption. Results of calculations of aerosol effects on climatological data sets are also given.
The core of the climate model consists of thirteen energy balance equations, one for the surface and each of the twelve atmospheric layers. The energy balance equations account for the input of solar radiation, the horizontal flux of heat due to atmospheric and oceanic currents, terrestrial infrared radiation, and heat storage. The equations are temperature dependent, and their solution yields the tentative temperature pattern on the surface and in the atmosphere. Changes in solar radiation, aerosol concentration, dynamics, etc., modify the equations and result in different temperatures.

The surface energy balance equation is

\[ SW_s - DO_s - ST_s - IR_s = 0. \] (1)

The absorption of solar radiation by the surface (the difference of the downgoing and reflected upgoing solar fluxes at the surface) is \( SW_s \), the horizontal divergence of heat by ocean currents (positive for heat flow out to other latitude belts, this is usually a negative number for midlatitudes) is \( DO_s \), the storage of heat in land and sea (positive for heat stored by sending it down away from the surface)
is $ST_s$, and the net outgoing infrared radiation from the surface (the difference of the upgoing and downgoing infrared fluxes at the surface) is $IR_s$. $SW_s$ is positive as solar radiation adds heat to the surface. The other terms are positive when they balance $SW_s$ by removing heat from the surface by causing heat (a) to flow to other latitudes ($DO_s$), (b) to be stored beneath the surface ($ST_s$), or (c) to be radiated to the atmosphere or space ($IR_s$).

The energy balance equation for an arbitrary atmospheric layer is

$$SW_n - DH_n - ST_n - IR_n = 0 \quad (2)$$

The absorption of solar radiation by the atmospheric layer is $SW_n$, the horizontal divergence of heat to other latitudes by atmospheric dynamics (considered positive for heat flow out to other latitudes, this is usually a negative number for midlatitudes) is $DH_n$, the storage of sensible and latent heat in the layer (positive for heat stored) is $ST_n$, and the infrared divergence (the difference of the net upgoing infrared fluxes at the top and bottom of the layer, a positive number for infrared cooling) is $IR_n$.

The one equation (1) and the twelve equations (2) are solved each month to obtain the tentative temperature pattern in the latitude belt; the temperature pattern is then modified by the convective adjustment procedure. Convective adjustment accounts for the effects of the
vertical fluxes of sensible and latent heat; these fluxes are not explicitly accounted for in the energy balance equations.

In the following sections the terms in (1) and (2) are discussed.

**Solar Radiation**

The temperature structure of the atmosphere is ultimately determined by the disposition of energy arriving from the sun. The model accounts for scattering and absorption of shortwave (solar) radiation by clouds, aerosol, gases and the earth's surface. Water vapor, ozone, and carbon dioxide absorb solar radiation, and the other gaseous constituents, nitrogen and oxygen, function primarily as Rayleigh scatterers. Clouds, aerosol, and the earth's surface both absorb and scatter.

The amount of solar radiation absorbed by the surface and the twelve atmospheric layers is determined by a "direct-diffuse pseudo adding" scheme, referred to in the following as "pseudo". It is a highly modified version of the adding scheme used by Lacis and Hansen (1974). Pseudo splits incoming solar radiation into two streams, direct and diffuse, and then does an adding calculation for both. Direct radiation is at the original cosine of the angle of incidence $\mu_o$, and the scattered diffuse radiation is set at an effective cosine angle $\mu$. (In this work direction is
specified by the cosine of the angle in question. Here the angle is that between the beam and the vertical.) Reflectivity and transmissivity must be specified for each individual layer.

Exponential type transmissions are calculated for carbon dioxide and water vapor from formulae in Kerschgens, Raschke, and Reuter (1976) and Welch and Zdunkowski (1976). Ozone absorption is obtained from the Lacis and Hansen (1974) parameterization. Cloud optical properties are specified somewhat like those in general circulation model studies by Gates et al. (1971) and Manabe and Strickler (1964). Aerosol layer optics are calculated from the incident angle dependent two-stream model of Coakley and Chylek (1975), and aerosol physical properties are obtained from Toon and Pollack (1976).

Radiative transfer is performed in the following wavelength intervals (subsequent paragraphs explain what occurs in each interval):

L = 1  < 0.34 micron
L = 2  0.34 to 0.48
L = 3  0.48 to 0.72
L = 4  0.72 to 0.926
L = 5  > 0.926 water vapor absorbing bands
L = 6  > 0.926 water vapor nonabsorbing bands
\( SW_s \) and \( SW_n \) are the solar terms in energy balance equations (1) and (2). They are obtained by weighting the calculated absorptions with the solar flux in the wavelength interval. The solar fluxes are found in the *Handbook of Geophysics and Space Environments* (Air Force Cambridge Research Laboratories, 1965). Pseudo does not perform calculations for two different constituents, as aerosol and gas, in the same layer, so each of the twelve atmospheric layers is split, with aerosol on the bottom and gaseous constituents on the top (aerosol concentrations usually decrease with height).

Rayleigh scattering is calculated at wavelengths below 0.926 micron; at longer wavelengths the Rayleigh optical thickness of the atmosphere is negligible. Ozone absorption occurs at wavelengths less than 0.34 micron and between 0.48 and 0.72 micron. Water vapor absorption occurs in several wavelength intervals past 0.926 micron. Aerosol and cloud effects are calculated at all solar wavelength intervals.

Carbon dioxide absorption is calculated in four wavelength intervals independent of \( L=1 \) to \( L=6 \) above. Pseudo is run with split layers for carbon dioxide and aerosol, and then a separate run is made for aerosol only at the same carbon dioxide wavelength intervals. The difference determines the amount of absorption due to carbon dioxide corrected for aerosol overlap. The generally small
overlap of carbon dioxide and water vapor is neglected in solar wavelengths.

Direct-diffuse Pseudo Adding

The direct-diffuse pseudo adding radiative transfer scheme calculates net absorption in several layers due to various constituents. It accounts for the interaction of scattering and absorption, and it runs fairly quickly on the computer. The pseudo scheme is not the adding scheme itself but is rather an approximation inspired by it. The system is assumed to be azimuthally independent (this provides an adequate accounting of the flux terms).

We now refer to Fig. 3, similar to one used in Lacis and Hansen's (1974) explanation of the adding technique. This technique combines the optical properties of two layers to form one. Radiation at cosine angle \( \mu_0 \) hits the top of layer \( a \) with optical depth \( \tau_a \). We wish to determine the disposition of this radiation through the added layers \( a \) and \( b \) (more about layer \( c \) later).

Note that rays \( R \) are reflected by either layer \( a \) itself, or by some combination of layers \( a \) and \( b \) through multiple paths. Beams \( T \) have been intercepted but are still transmitted downwards. Beams \( D \) are bouncing between the two layers headed down, while \( U \) denotes those headed up.

An unmarked quantity of radiation free flights through \( a \) and \( b \) without being intercepted; it continues
Fig. 3. Adding Schematic Diagram.
directly at the original cosine angle of incidence $\mu_0$. Radiation which is scattered is assumed thereafter to be diffuse with effective cosine angle $\mu$. This last is a key assumption. The value of $\mu$ is discussed in Appendix A (see Fig. 4).

The pseudo scheme assumption of a single effective cosine angle $\mu$ for scattered radiation fixes the direction of the scattered beams in both the upward and downward hemispheres. The scattering angles are regarded as so random that they can be characterized by a single effective cosine angle $\mu$ in both hemispheres; this is analogous to the use of a single constant diffusion coefficient $D$ in Fickian diffusion. The value of $\mu$ selected is appropriate for nearly isotropic radiation, and the pseudo scheme is adjusted to account for the effects of highly nonisotropic scatterers (as sharply forward scattering aerosol).

The real angular distribution of the scattered beams is determined by the angle of incidence of the incoming beams and the phase function of the scatterers; a precise radiative transfer calculation would account for the multiplicity of angles that the scattered beams actually assume. The diffuse $\mu$ assumption simplifies the radiative transfer calculation by accounting for only two angles (the direct and the scattered diffuse) for all the beams. In Appendix C this pseudo scheme is compared with more precise
Fig. 4. Direct-diffuse Schematic Diagram.

Radiation strikes an individual layer at cosine angle \( \mu_0 \). It either (1) continues unintercepted through a at \( \mu_0 \), (2) is scattered up or down at effective cosine angle \( \mu \), or (3) is absorbed.
radiation schemes; pseudo does well for the small aerosol optical depth cases that are of interest in this work.

In the following equations a prime (as \( R'_b \)) refers to a beam which is impinging directly at cosine angle \( \mu_0 \). The absence of a prime (as \( R_b \)) refers to a beam which is impinging diffusely at effective cosine angle \( \mu \). The asterisk (as \( R^*_a \)) refers to a beam which strikes (layer a) heading up. \( R_b \), for example, is the reflectivity of layer b to diffuse downcoming beams. \( R'_b \) is the reflectivity of layer b to direct incoming beams. \( R^*_a \) is the reflectivity of layer a to upgoing beams, which are considered to be diffuse. \( R', R, \) and \( R^* \) are specified for each layer by techniques given in subsequent sections.

\( S' \) is the fraction of the direct (unscattered) beam entering layer b which, by multiple scattering within layers a and b, is eventually reflected downward across the boundary between the two layers. \( S \) is the like fraction for diffuse (scattered) beams.

\[
S' = R_a^* R_b' \left[ 1 + \sum_{n=1}^{\infty} (R_a^* R_b')^n \right] = \frac{R_a^* R_b'/(1 - R_a^* R_b)}{}
\]

\( S = \frac{R_a^* R_b/(1 - R_a^* R_b)}{} \).

We also use a term \( T'_a \). It is not a transmissivity. \( T'_a \) specifies that fraction of the direct (\( \mu_0 \)) downcoming beam which is scattered downward by layer a. It does not
include that portion of the beam which proceeds through layer \( a \) unintercepted.

\[
T_a' = \left[ \text{transmissivity of layer } a \text{ to direct beam } \mu_0 \right] - e^{-(\tau_a/\mu_0)} .
\]  

(5)

In subsequent sections we explain how the transmissivities are calculated. Then \( T' \), \( T \), and \( T^* \) may be specified for each layer.

Assuming that we know \( R' \), \( T' \), etc., for individual layers we now give the pseudo adding prescription for calculating \( R' \), \( T' \) for a composite of layers.

The adding variable \( D' \) accounts for all diffuse radiation headed down between the two layers \( a \) and \( b \). The direct beam which strikes the upper layer is the original source of this diffuse radiation. The diffuse downgoing beams \( D' \) are (1) beams intercepted in \( a \) and scattered down, (2) beams free flighting through \( a \) and now headed down diffusely because of multiple scattering by \( b \) and \( a \), and (3) beams intercepted in \( a \) and scattered down, now heading down again because of multiple scatter by \( b \) and \( a \). These three terms are on the right of the following equation:

\[
D' = T_a' + S'e^{-(\tau_a/\mu_0)} + ST_a' .
\]  

(6)

Analogous to \( D' \), we have \( U' \) for beams between \( a \) and \( b \) headed up. The diffuse radiation \( U' \) headed up between \( a \) and \( b \) is (1) direct radiation which free flights down
through a and is reflected up by b and (2) diffuse downgoing radiation $D'$ reflected back up by b. These two terms are on the right in (7), as

$$U' = R_b'e^{-\left(\frac{\tau_a}{\mu_o}\right)} + R_b'D'. \quad (7)$$

Layers a and b are now combined to form one. This is done by specifying a new $R'$ and $T'$, as

$$R'(\tau_a + \tau_b) = R_a' + U'e^{-\left(\frac{\tau_a}{\mu}\right)} + T_a'U' \quad (8)$$

$$T'(\tau_a + \tau_b) = e^{-\left(\frac{\tau_b}{\mu}\right)}D' + T_b'e^{-\left(\frac{\tau_a}{\mu_o}\right)} + T_b'D'. \quad (9)$$

where the sources of $R'(\tau_a + \tau_b)$ on the right of (8) are (1) direct beams that are reflected by the upper layer a, (2) diffuse upgoing beams $U'$ between a and b that free flight through a, and (3) diffuse upgoing beams $U'$ between a and b that are scattered in the upward direction as they encounter a. The sources of $T'(\tau_a + \tau_b)$ are (1) diffuse downgoing beams $D'$ between a and b free flying through b, (2) direct beams free flying through a and being scattered down by b, and (3) diffuse downgoing beams $D'$ between a and b scattered down by b.

The process is continued. The added layers a and b are now added to layer c, and so on down to the ground. $R_a'$ and $T_a'$ are replaced by $R'(\tau_a + \tau_b)$ and $T'(\tau_a + \tau_b)$ in the system of equations given above. This is called adding, and is similar to the doubling procedure.
The system is made complete by sets of equations for diffuse upgoing radiation \((R^*, T^*)\) and diffuse downgoing radiation \((R, T)\), as well as those for direct downgoing radiation \((R', T')\) described above. These additional equations are given in Appendix A, as is the procedure for calculating absorption with the adding scheme.

The above pseudo system requires some modification to better account for aerosol scattering. If light at cosine angle \(\mu_o\) is intercepted by a strongly forward scattering aerosol, the system of equations will deflect the entire downward scattered beam to the effective diffuse cosine angle \(\mu\). In reality, most of the downward scattered beam retains a cosine angle close to \(\mu_o\).

We correct for this by adding to the term which accounts for free flight (no scattering or absorption) through the layer:

\[
e^{-\mu_a/\mu_o} + e^{-\mu_a/\mu_o} + T_a' C_a g = E_a. \tag{10}
\]

We subtract an equal amount from the term that accounts for interception and subsequent transfer into the diffuse beam:

\[
T_a' + T_a' - T_a' C_a g = T_a''. \tag{11}
\]

\(T_a' C_a g\) is an approximate correction term. \(T_a'\) specifies the quantity of radiation at the direct \(\mu_o\) scattered into the diffuse \(\mu\), \(C_a\) adjusts for multiple scattering effects,
and $g$ is the asymmetry factor for the scattering phase function $p(\cos \theta)$, as

$$g = \frac{1}{2} \int_{-1}^{1} p(\cos \theta) \cos \theta \, d(\cos \theta). \quad (12)$$

g appropriately multiplies the correction term by zero for isotropic scatterers and pure absorbers. $g$ retains the full correction factor for strongly forward scattering aerosol, for which $g$ approaches 1. The rationale for using the multiple scattering adjustment factor $C_a$ in the correction term $T_a \cdot C_a g$ is given below.

Some of the radiation specified as direct which strikes the layer will in reality be radiation which has already been scattered, but has been placed back in the direct beam by the correction term $T_a \cdot C_a g$. We wish to eliminate multiply scattered beams which have thus reentered the direct stream. Several scatters by even sharply forward scattering aerosol will result in a beam significantly displaced from the original cosine angle $\mu_0$. The most appropriate angle for these multiply scattered beams is the diffuse cosine angle $\mu$, not the direct $\mu_0$.

To adjust for the multiple scattering effects we first note that the probability of a photon traversing optical depth $x$ with no scattering is

$$P(0) = e^{-x}. \quad (13)$$
The probability of the photon scattering once is

\[ P(0) = xe^{-X}. \]  

(14)

(13) and (14) are due to Herman (1963). Then the fraction scattered at least once is

\[ 1 - P(0) = 1 - e^{-X} \]  

(15)

and the fraction scattered more than once is

\[ 1 - P(0) - P(1) = 1 - xe^{-X} - e^{-X}. \]  

(16)

\( C_a \) gives the fraction of total scattered photons which are scattered only once:

\[ C_a = \left[ (1 - e^{-X}) - (1 - xe^{-X} - e^{-X}) \right] / (1 - e^{-X}) \]

\[ = xe^{-X} / (1 - e^{-X}) \]  

(17)

\[ x = \sum_{i=1}^{a} \tau_i / \mu_0 \]  

(18)

where the summation for \( x \) includes those layers which have nonzero \( g \) only.

It is noted that \( C_a \) would be an inadequate correction factor if absorption dominates scattering for aerosol, as this would invalidate (15) and (16). Also, a theoretically perfectly forward scattering aerosol would be inadequately handled by \( C_a \); \( C_a \) would turn some of the scattered beam from the original \( \mu_0 \).

With (10), (11), (17), and (18) applied to the pseudo scheme we obtain
\[ D' = T_a'' + S'E_a + ST_a'' \]  
(19)

\[ U' = R_b'E_a + R_bD' \]  
(20)

\[ R'(\tau_a + \tau_b) = R_a' + U'e^{-(\tau_a/\mu)} + T_a'U' \]  
(21)

\[ T''(\tau_a + \tau_b) = e^{-(\tau_b/\mu)} D' + T_b''E_a + T_bD' \]  
(22)

\[ E_a = \prod_{i=1}^{a} E_i = \prod_{i=1}^{a} \left[ e^{-(\tau_i/\mu_o)} + T_i'C_i \right] . \]  
(23)

The above equations, and similar ones in Appendix A, are used to calculate radiative transfer in the model. This gives \( SW_s \) and \( SW_n \). In subsequent sections techniques for computing transmissivities and reflectivities for layers of aerosols, gases, and clouds are given. Pseudo variables \( R_a', T_a' \), etc., may then be obtained for each layer.

Radiative Properties of Aerosol

In solar wavelengths, the radiative properties of aerosol layers are calculated with the incident angle dependent two-stream formulation of Coakley and Chylek (1975). The Coakley and Chylek formulation is adequate for aerosol optical depths of a few tenths, even at fairly large angles of incidence. Aerosol optical depth \( \tau \), single scattering albedo \( \omega_o \), phase function \( p \), and the cosine \( \mu \) of the angle of incidence must be specified. Aerosol layer reflectivity \( r(\mu) \) and transmissivity \( t(\mu) \) may then be calculated from the following expressions:
\[
\begin{align*}
\mathbf{r}(\mu) &= \frac{(U+1)(U-1)}{(U+1)^2 e^{(e^{a\tau/\mu})} - (U-1)^2 e^{-(e^{a\tau/\mu})}} \\
\mathbf{t}(\mu) &= \frac{4U}{(U+1)^2 e^{(e^{a\tau/\mu})} - (U-1)^2 e^{-(e^{a\tau/\mu})}} \\

U &= (1-\omega_o)^{-1/2} \left[ 1 - \omega_o + 2\omega_o \beta(\mu) \right]^{1/2} \\
\alpha &= (1-\omega_o)^{1/2} \left[ 1 - \omega_o + 2\omega_o \beta(\mu) \right]^{1/2} \\
\beta(\mu) &= \frac{1}{2\omega_o} \int_0^1 p^o(\mu, -\mu') d\mu' 
\end{align*}
\]

where \(p^o\) is the azimuthally independent form of the aerosol phase function. All calculations in this model employ the Henyey-Greenstein phase function, the azimuthally independent form of which is given in Appendix B; the Henyey-Greenstein \(p^o\) is a function of the aerosol asymmetry factor \(g\).

Most calculations made with the model employ the global average aerosol properties given in Toon and Pollack (1976). Toon and Pollack specify the global average aerosol optical depth as 0.125 at 0.55 micron. They also specify the single scattering albedo \(\omega_o\), asymmetry factor \(g\), and \(Q_{\text{ext}}\) as functions of wavelength. From their \(Q_{\text{ext}}\) we may obtain optical depth \(\tau_\lambda\) as a function of wavelength \(\lambda\) as
where $\lambda_{\text{ref}}$ is the reference wavelength of 0.55 micron.

In the Toon and Pollack model aerosol, composition and size distribution are functions of height, and $Q_{\text{ext}}$, $\omega_0$, and $g$ are as well. Composition from 0 to 3 km is 15% sodium chloride, 35% basalt, and 50% ammonium sulfate. Tropospheric aerosol from 3 to 12 km is cited as 40% basalt and 60% ammonium sulfate. Stratospheric aerosol is taken as a 75% sulphuric acid solution. The size distribution is approximately that due to Elterman, Wexler, and Chang (1969).

The Toon and Pollack aerosol is highly scattering, with single scattering albedo exceeding 0.99 at 0.55 micron for the upper tropospheric aerosol. Physical properties are not specified as a function of relative humidity. Twomey, (personal communication,1978) has indicated that the asymmetry factor may be relative humidity dependent. Hanel (1976) shows how the optical depth and single scattering albedo of some aerosol samples are functions of relative humidity.

Rayleigh Scattering

The Rayleigh reflectivity of an atmospheric layer is given as

$$r(\mu) = (1-e^{-\tau/\mu})/2 .$$

(31)

This assumes that the Rayleigh scattering is isotropic; one-half of the radiation intercepted goes up, the rest down.
The Rayleigh optical depth at 0.55 micron is 0.145. This is apportioned among the twelve atmospheric layers by mass. The absorbing gas or aerosol is always assumed to underlie the Rayleigh particles in any given layer. Rayleigh optical depths in each wavelength interval are calculated with a method given in Appendix P. It is assumed that Rayleigh scattering is negligible at wavelengths above 0.926 micron.

Water Vapor Absorption

Water vapor is the most important clear sky atmospheric absorber, having several significant absorption bands beyond 0.926 micron. Rayleigh scattering is negligible at these wavelengths. Water vapor absorption is calculated with overlap due to aerosol and cloud effects included. A parameterization of water vapor transmissivity $t$ is given by Welch and Zdunkowski (1976) as

$$ t = \sum_{i=1}^{N} a_i e^{-b_i u} $$

where $u$ is the water vapor pathlength and $(a_i, b_i)$ are specified coefficients. The $i$ do not refer to wavelength intervals. Equation (33) calculates an energy weighted transmission over several water vapor absorbing bands at once, essentially grouping the $(a_i, b_i)$ by relative attenuation rather than by wavelength.

Equation (33) is inserted into the pseudo scheme (19) through (23) as follows. The term $\exp(-b_i u)$ is
inserted into the exponential expressions in (19) to (23). T, R, etc., are zero for a water vapor layer because water vapor scattering is small. The pseudo scheme is then executed N times. Net absorption is calculated by weighting the absorptions made by the N calculations according to the aᵢ and then summing.

A listing of the water vapor bands, the incident solar energies contained within them, and the (aᵢ, bᵢ) are given in Appendix D. Following the suggestion of Welch, Geleyn, Zdunkowski, and Korb (1976), who employ a similar parameterization for water vapor transmissivity, aerosol optical properties are weighted according to the energy in each water vapor band. Calculations are also made at water vapor nonabsorbing wavelengths beyond 0.926 micron, with appropriate weighting of aerosol optical properties.

Model calculations are made at constant surface relative humidity. The mass of water vapor in the model atmosphere fluctuates according to the surface temperature. This provides the water vapor temperature feedback discussed earlier; the effect is pronounced in the infrared. Sellers (1973) provides empirical formulas which permit the calculation of an approximate water vapor vertical distribution; these are given in Appendix E.
Carbon Dioxide Absorption

Carbon dioxide absorption of solar energy is calculated by making pseudo runs in four bands between 1.2 and 3.0 micron. As previously noted, carbon dioxide calculations are made independent of other solar radiation calculations, which cover the entire solar spectrum but neglect carbon dioxide effects. A set of runs is made in the carbon dioxide bands with carbon dioxide and aerosol in alternate layers, and another is made in the same bands with no carbon dioxide. The difference gives the absorption due to carbon dioxide.

Carbon dioxide optical properties are specified with expressions like (33). Coefficients are given in Appendix D. The carbon dioxide volume mixing ratio is \(330 \times 10^{-6}\).

The overlap of carbon dioxide and water vapor is neglected. It could be accounted for by splitting the water vapor parameterization into several bands compatible with the carbon dioxide calculation. This has been done by Kerschgens et al. (1976). The resultant doubling of computational time is judged not to be worthwhile.

Ozone Absorption

Ozone absorbs in the ultraviolet and visible regions. It is concentrated in the upper atmosphere. Ozone absorption of solar radiation is a significant determinant of stratospheric temperature.
An exponential type expression for transmissivity, as used for water vapor and carbon dioxide, is not available for ozone. Ozone absorption is calculated with a modified Lacis and Hansen (1974) parameterization. Lacis and Hansen develop expressions for ozone absorption in the visible (vis) and ultraviolet (uv) as a fraction of total solar incident flux. They assume that all reflection occurs below the ozone layer. The frequency integrated expressions for absorption as a function of pathlength $x$ are

\[
A_{\text{vis}}^{\text{oz}}(x) = \frac{0.02118x}{(1 + 0.042x + 0.000323x^2)}
\]

\[
A_{\text{uv}}^{\text{oz}}(x) = \frac{1.082x}{(1 + 138.6x)^{0.805}}
+ \frac{0.0658x}{[1 + (103.6x)^3]}.
\]

Lacis and Hansen sum these and then multiply by the solar flux. This model, however, retains the distinction between visible and ultraviolet; it weights each $A_{\text{oz}}$ by energy.

Expressions (34) do not fit in with the pseudo formulation, which permits only exponential type absorption. Hence the ozone calculation is auxiliary. For $L=1$ (less than 0.34 micron) and $L=3$ (0.48 to 0.72 micron) each of the twelve layers is split according to Fig. 5. Each layer is stacked to contain ozone, Rayleigh scatterers, and then aerosol.

Absorption in each layer is calculated with pseudo neglecting ozone. Each layer absorption is then reduced by
Fig. 5. Split Layers for Radiative Transfer at Ozone Absorbing Wavelengths.
(1 - A_{o_2}), where A_{o_2} is the ozone absorption (34) down to that point. This accounts for absorption in ozone wavelengths by aerosol and partially corrects for the aerosol-ozone overlap. Ozone absorption is then calculated for layer m as (visible or ultraviolet)

\[ A_{o_2}(x_m) - A_{o_2}(x_{m-1}) + R_{m,13} \left[ A_{o_2}(x_{m-1}^*) - A_{o_2}(x_m^*) \right] \]

(35)

where \( R_{m,13} \) is the reflectivity of layer m and all below it, and level 13 is the surface. \( x_m \) is the ozone pathlength from the top of the atmosphere down to the bottom of layer m. It is calculated from the ozone amount \( o_m \) and the magnification factor \( M \) (given by Lacis and Hansen) as

\[ x_m = o_m M \]

\[ M = 35(1244\mu_o^2 + 1)^{-1/2} \]

(36)

where \( M \) is the magnification factor. The magnification factor accounts for the fact that the downcoming direct solar beam is not vertical (it enters at a slant \( \mu_o \) direction) and also that it is refracted from its original angle of incidence by the atmosphere; for no refraction we would have \( M = 1/\mu_o \). \( x_m^* \) is the path down through the atmosphere and back up to m, as

\[ x_m^* = o_{total} M + 1.9(o_{total} - o_m) \]

(37)

where 1.9 is the effective magnification factor for diffuse upgoing radiation.
The total amount of ozone in a vertical column is obtained from Hoyt (1976), who compiles it on a seasonal, latitudinal basis. The ozone height distribution is due to Green (1964), quoted by Lacis and Hansen (1974). \( o(h) \) is the ozone amount (cm, NTP) in a vertical column above \( h \), given by

\[
o(h) = \left[ a + ae^{-b/c} \right] / \left\{ 1 + e^{(h-b)/c} \right\}
\]  

(38)

where \( h \) is altitude, \( a \) the total amount of ozone from the surface to the top of the atmosphere, and \( b \) the altitude at which ozone concentration is highest (20km is used here). The maximum ozone concentration is \( a(1-\exp(-b/c))/(4c) \), with \( c \) taken as 5 km.

Cloud Optics

Clouds are the most important optical component of the atmosphere in solar wavelengths. They cover roughly 40\% of the globe and reflect about half of the radiation incident on them. There is significant uncertainty both in our knowledge of the optics of clouds and in their amount; this is not the case with the absorbing gases described above. Hoyt (1978) has noted that it would require about 30 years of sampling to determine annual mean cloud cover to within 1\% with 95\% confidence. And a comparable cloud climatology for use in modeling would have to be enormously
complex; it would have to catalog the varied physical composition of clouds (from which optics may be calculated) as well as amount.

 Twomey (1976) has calculated the absorption of model clouds as up to 20% of incident solar radiation. He shows that changes in the droplet distribution of clouds can significantly alter their optics. Welch et al. (1976) calculate cloud optical properties but get less absorption than Twomey. Liou (1976) calculates the optics of several cloud types but notes that the values for absorption obtained are lower than those observed.

Several models of cloud optics have been proposed for use in general circulation models. Lacis and Hansen (1974) specify cloud reflectivity from the incident angle independent two-stream formulation of Sagan and Pollack (1967). Cloud absorption is then calculated with multiple scattering parameterization which is incompatible with the radiative transfer scheme used in this model.

 Gates et al. (1971) specify the reflectivities and equivalent water vapor absorption pathlengths of low and mid clouds for the Mintz-Arakawa general circulation model. Manabe and Strickler (1964) give the reflectivities and absorptivities of high, mid, and low clouds used in their radiative convective model. Reflectivities and equivalent water vapor pathlengths (for absorption) for three cloud
parameterizations used in this model are given below. (The
reflectivities are two-digit numbers followed by the equiva-
lent water vapor pathlengths in parentheses.)

<table>
<thead>
<tr>
<th>Low</th>
<th>Mid</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.69(0.048)</td>
<td>0.48(0.015)</td>
</tr>
<tr>
<td>2</td>
<td>0.65(108.4)</td>
<td>0.60(12.62)</td>
</tr>
<tr>
<td>3</td>
<td>0.69(108.4)</td>
<td>0.48(12.62)</td>
</tr>
</tbody>
</table>

1 is the Manabe and Strickler cloud optics with cloud absorp-
tivity converted to an equivalent water vapor pathlength.
2 is a modification of the Mintz-Arakawa cloud optics with high cloud the same as 1. 3 is judged an appropriate com-
bination of 1 and 2 and is used for most calculations with this model.

Cloud absorption in 1 was judged too small when compared to the theoretical calculations cited above. 2 was discarded because the reflectivities for mid and low cloud are too similar; mid clouds are less dense and theory indi-
cates they should have a lower reflectivity. High cloud absorption seems low. High cloud albedo, however, agrees with Hunt's (1973) calculation of 0.20 ± 0.05 for the 0.6 micron albedo of cirrus.

Calculations made with a globally averaged form of this model and cloud optics 1, 2, and 3 gave a spread of less than 0.5 K surface temperature. \( \frac{dT}{d\tau} \), the variation of surface temperature with respect to aerosol optical
depth, was almost identical for the three cloud parameteri-
zations. Thus an inadequacy in model cloud optics may cause
an error in the evaluation of absolute temperature $T$, but
it probably won't cause an error in the evaluation of climate
sensitivity $dT/d\tau$.

Here cloud albedo is assumed to be incident angle
independent and wavelength independent. Both assumptions are
significant model drawbacks. Ohring and Adler (1978) devel­
oped a simple formula to account for variation of cloud
albedo with incident angle. Experimentation indicated it
was not satisfactory for use with this particular model, however.

Some calculations of cloud optical properties were
made with pseudo and the Coakley and Chylek (1975) two-
stream; they did not agree well with published results and
were not incorporated into the model cloud optics. To
obtain a tractable, new scheme for cloud optics would be a
major study in itself and is outside the scope of this work.

**Infrared Radiation**

The earth atmosphere system is a strong emitter in
the infrared region between 4 and 50 microns. Infrared
emission and absorption by gases, clouds, and aerosol are
as significant a determinant of atmospheric temperature as
is the incoming solar radiation. In this section we develop
a detailed method for the computation of infrared fluxes as functions of temperature, concentrations of water vapor, carbon dioxide, ozone, aerosol, and clouds.

Rodgers (1967) and Sasamori (1968a) have developed infrared transfer schemes which have been used by a number of modelers. Rodgers' method compares well with the more detailed Goody statistical model; it is used here for broadband water vapor calculations. The Sasamori scheme is based on the Yamamoto radiation chart; we use it to calculate fluxes in the 9.6 micron ozone and 14 micron carbon dioxide bands. Modifications to both schemes are made to account for cloud effects, aerosol, and water vapor e-type absorption.

Water vapor has an infrared window to space: a weakly absorbing region between 8 and 13.3 micron. Experimental work cited by Bignell (1970) showed a new window effect; for water vapor partial pressure e exceeding 15 mb significant, unaccounted for absorption was occurring in the window. The Rodgers scheme does not include it. Bignell speculated that this e-type absorption was due to the water vapor dimer (H₂O)₂, whose amount would be a function of the partial pressure e of water vapor. Coffey (1977) has confirmed that the effect is due to a dimer.

Cox (1973) has indicated that the water vapor e-type effects can result in up to 30 percent more infrared
cooling by atmospheric layers. This pronounced effect is largely confined to the lower troposphere, especially in the tropics, where high e values are obtained. In the upper troposphere and in cold climates, e values are small, and the 8 to 13.3 micron region becomes a window again. E-type infrared parameters used in this model are taken from Lee (1973).

Infrared effects due to aerosol are generally negligible when compared to water vapor at nonwindow wavelengths. Hence aerosol effects are calculated across the infrared broadband (denoting the entire terrestrial spectrum) only in the stratosphere, where there is insufficient water vapor to mask aerosol properties. In the window, aerosol can be significant when compared to water vapor, even in the troposphere. Grassl (1973) compares the importance of aerosol and e-type effects in the window. This model includes aerosol infrared effects in the 8 to 13.3 micron window region.

The model adjusts infrared aerosol optical depth as

$$\tau + \tau(1-\omega_o)$$ \hspace{1cm} (39)

The optical depth is discounted by the single scattering albedo as the calculation assumes that the aerosol is an infrared absorber/emitter only. Aerosol scattering is neglected in the infrared because aerosol single scattering
albedo is small there; the scattering that occurs is mostly forward. Welch and Zdunkowski (1976), however, feel that aerosol scattering is important in the window; they neglect e-type and aerosol absorption there instead.

Cloud effects, as well as aerosol and e-type effects, are not directly provided for in the Rodgers or Sasamori scheme. The infrared effects of clouds are parameterized in terms of emissivity $\varepsilon$ and reflectivity $r$. $\varepsilon$ and $r$ are selected from a review of the literature.

Hunt (1973) calculates the infrared properties of clouds and finds them to be a function of the droplet size distribution. Hunt's cirrus (ice) clouds have lower emissivities than water clouds: for cirrus under 500 m thick emissivity is less than 0.20.

Yamamoto, Tanaka, and Asano (1970) included the effects of both droplets and water vapor in calculations of cloud infrared optics. The Yamamoto results are used in this model. For an infinitely thick (> 200m) cloud emissivity is 0.97 and reflectivity 0.03; these values are adopted for low and mid clouds, broadband. In the window and in the 14 micron carbon dioxide band we average Yamamoto's values for $\infty$ and 50m depths. 14 micron emissivity is 0.97, reflectivity 0.02. Cloud window optics are given in Appendix G; these are also due to Yamamoto et al.

Zdunkowski and Crandall (1971) use different water vapor optical data than Yamamoto and obtain higher
reflectivities, even exceeding 10% at 9 micron. Hunt obtains different reflectivities than Zdunkowski and Cran dall. Radiometric observations by Cox (1976) give cloud effective emissivities between 0.41 and 0.84, sharply contrasting with the above calculations. Gorodetskiy et al. (1977) measured low and mid clouds between 10.5 and 12.0 micron and obtained emissivities of 0.96 ± 0.04, which supports our selection.

The model assumes that a cloud "swamps" the infrared properties of a layer. Cloud infrared optics are taken to so dominate a layer that the gases and aerosol in it may be neglected. This is probably a good assumption for the high emissivity low and mid clouds. The infrared effects of cirrus are less pronounced.

Platt (1973, 1975) measured cirrus emissivities as about 0.3. Fleming and Cox (1974) cite emissivities between 0.2 and 0.8. The swamping assumption indicates that as cirrus does not approach blackness, its effective emissivity must be increased to account for the infrared active gases it supposedly masks. Cirrus emissivity and reflectivity are selected as one-half the values for low and mid cloud. Manabe and Strickler (1964) assumed that low cloud is black and cirrus half black.
Infrared Formulas

The modified Rodgers and Sasamori formulas for the calculation of infrared fluxes are given below. The discussion is detailed. In the subsequent sections dealing with radiative equilibrium we will explain how these fluxes are used to calculate $I_{R_s}$ and $I_{R_n}$, the infrared terms in energy balance equations (1) and (2).

The downward infrared flux at level $z$ is given by:

$$ F_{↓}(z) = F_{↓A}(z) - F_{↓B}(z) + F_{↓C}(z) + F_{↓D}(z) + F_{↓E}(z). $$  \hspace{1cm} (40)

$F_{↓A}(z)$ is the broadband infrared flux due to water vapor calculated with the Rodgers (1967) formulation. The expression for clear skies is

$$ F_{↓A}(z) = \int \int_{z}^{\infty} \sigma T^4(z') \, d\varepsilon_2(z,z')/dz' \, dz' $$ \hspace{1cm} (41)

where $\sigma$ is the Stefan-Boltzmann constant. $\varepsilon_2$ is an effective downgoing emissivity developed by Rodgers; it is a function of water vapor pathlength. Rodgers notes that calculations made with (41) agree well with Goody's statistical model. Rodgers new values for $\varepsilon_2$ are quoted by Welch and Zdunkowski (1976).

If aerosol is present in the top layer its effects over the infrared broadband are accounted for with $F_{↓A}(z)$. An effective water vapor pathlength is calculated for the
aerosol, and this is added to the existing water vapor path-length in the top layer. The procedure is outlined in Appendix H.

Temperature is of course specified in the above calculation, and it is slightly altered when cloud is present to account for cloud reflectivity. When calculating the downward stream $F_{\downarrow A}(z)$ under cloud, the temperature at cloud level is increased to roughly match the amount of upgoing radiation that would be reflected by the cloud, as

$$T_{\text{cloud}}^4 = T_{\text{cloud}}^4 + F_{\downarrow A}(z) r_{\text{cloud}}/\sigma$$  \hspace{1cm} (42)

where $r_{\text{cloud}}$ is cloud reflectivity. Equation (42) assumes that the cloud is nearly a blackbody. Its temperature fictitiously increases in the calculation to account for the extra radiation [the second term on the right of (42)] that is reflected.

The full expression for $F_{\downarrow A}(z)$ under a cloud of emissivity $\varepsilon$ and reflectivity $r$ is

$$F_{\downarrow A}(z) = (1-\varepsilon-r)F_{\downarrow A}(z) + \varepsilon \{ c T_{\text{cloud}}^4 [1-\varepsilon_2(\text{cloud}, z)] \}
+ \int \sigma T^4(z') \, d\varepsilon_2(z,z')/dz' \, dz'$$

where the term $F_{\downarrow A}(z)$ on the right hand side of (43) is just (41), the clear sky value of $F_{\downarrow A}(z)$ which penetrates the cloud.
We return to (40) and note the second term, $F_{+B}(z)$, which is subtracted from the downward flux. The broadband $F_{+A}(z)$ calculation includes the effects of the water vapor continuum in the window. $F_{+B}(z)$ subtracts out this effect on the window. With $F_{+C}(z)$ the window will be added back in, but with additional constituents (aerosol and e-type absorption). These constituents are strong enough to be significant in the relatively empty water vapor window, but they would be masked by water vapor outside the window.

This subtraction-addition technique for $F_{+A}(z)$, $F_{+B}(z)$, and $F_{+C}(z)$ was used by Welch and Zdunkowski (1976). With $B_n$ the Plankian function per unit wavenumber $n$ and $\varepsilon_w(z,z')$ the effective water vapor continuum emissivity in the window, we have for clear skies

$$F_{+B}(z) = \int \left[ \frac{1250}{750 \text{cm}^{-1}} \right] B_n(T) dn \left[ \frac{1}{z} \frac{\varepsilon_w(z,z')}{dz'} dz' \right] \quad (44)$$

where the bracketed term is the blackbody flux in the window. It is comparable to $\sigma T^4$ for the broadband calculation (41). A full discussion of (44) is given in Appendix I.

The next term in (40) is $F_{+C}(z)$. This adds the window back into the infrared calculation, this time including the effects of aerosol and water vapor e-type absorption as well as the water vapor continuum. The expression for flux is similar to that used by Elsasser and Culbertson (1960):
\[ F_{\pm C}(z) = - \int_{-750}^{1250} \int B_n(T) \frac{d\tau_{Fw,n}(z,z')}{dz'} dz'dn \]  

(45)

where \( \tau_{Fw,n} \) is the flux transmissivity in the window at wave-number \( n \) due to aerosol, water vapor, and water vapor dimers. Equation (45) is discussed in Appendix I.

We have covered the first three terms in (40), accounting for the infrared fluxes due to water vapor and aerosol. Now we use the Sasamori formulation (1968a) to calculate the fourth term \( F_{\pm D}(z) \). This is the flux due to carbon dioxide, corrected for overlap with water vapor, as

\[ F_{\pm D}(z) = 4\sigma \int_{T(z_t)}^{T(z)} \bar{A}_{CO_2} [y_p(T) - y_p(T')] \tau_{F,CO_2,H_2O} [u_p(T) - u_p(T')] T'\,dT' + aT^{4}(z_t)\bar{A}_{CO_2} [y_p(T) - y_p[T(z_t)]] \tau_{F,CO_2,H_2O} [u_p(T) - u_p[T(z_t)]] \]  

(46)

where \( T(z_t) \) is the temperature of the level \( z_t \), above which the quantity of absorber changes only slightly. The second term in (46) accounts for emission from the level \( z_t \). \( \bar{A} \) and \( \bar{A} \) are normalized absorptivities; they are functions of the pathlength of the absorbing gas. Sasamori gives expressions for \( \bar{A} \) and \( \bar{A} \) and explains how the infrared flux takes the form of (46) above. \( y_p(T) \) is the pressure corrected amount.
of carbon dioxide above the level at which the temperature is $T$; $u_p(T)$ is the same for water vapor. $\tau_{F, CO_2, H_2O}$ is the flux transmissivity of water vapor in the 14 micron carbon dioxide band; it is specified by Sasamori and is used to correct for the carbon dioxide–water vapor overlap. A full discussion of (46) is found in Appendix I.

The last term in (40), $F_{PE}(z)$, specifies the downward flux due to ozone. Using Sasamori’s formulation and accounting for overlap due to water vapor and aerosol, we have

$$ F_{PE}(z) = 4\sigma \int_{T(z_t)}^{T(z)} A_{03} \left[ y_p(T) - y_p(T') \right] T_{Fw, 1050} $$

$$ \cdot (z, z') T' \frac{dT'}{dT} $$

$$ + \sigma T^4(z_t) A_{03} \left\{ y_p(T) - y_p[T(z_t)] \right\} T_{Fw, 1050}(z, z_t) $$

(47)

where $T_{Fw, 1050}$ is the flux transmissivity due to water vapor and aerosol at window wavenumber 1050 cm$^{-1}$. It was used in (45). Here $y_p(T)$ is the pressure corrected amount of ozone above the level at which the temperature is $T$. Details on (47) are given in Appendix I. Appendix J gives expressions for upgoing fluxes.
Heat Storage and Dynamical Heat Fluxes

In this section we explain how the model allows for latent and sensible heat storage in the atmosphere ($ST_n$), heat storage in the land and sea ($ST_s$), and meridional heat fluxes due to atmospheric and oceanic dynamics ($DH_n$ and $DO_s$).

Latent heat storage in the atmosphere is given as the product $L \Delta U$. $L$ is the latent heat of vaporization and $\Delta U$ is the difference in total precipitable water for the current and previous months. $U$ and its variation with height are obtained from the expressions of Appendix E. Latent heat storage is apportioned by height according to the amount of precipitable water in a given layer.

Sensible heat storage in the atmosphere is given as $C_p \Delta T$. $C_p$ is the total heat capacity of the atmosphere (constant pressure heat capacity $C_p$ is used as the total atmospheric pressure changes little from month to month), and $\Delta T$ is the difference in surface temperatures for the current and previous month. Sensible heat storage is apportioned according to the mass in each layer.

(The sensible heat storage in the atmosphere should be more properly accounted for with a $\Delta T$ for each level in the atmosphere, rather than just the surface. The use of a surface $\Delta T$ does not cause a significant error since monthly temperature variations in the troposphere, which contains most of the atmospheric mass, approximate those on the surface.)
The amount of latent and sensible heat stored $ST_n$ in atmospheric layer $n$ is thus

$$ST_n = L\Delta U \frac{u(n)}{U} + C_p \frac{\Delta T \Delta p}{1000 \text{mb}}$$

(48)

where $u(n)$ is the water vapor in layer $n$ and $\Delta p$ is the pressure difference between top and bottom of the layer.

Heat storage in land and sea is calculated with the method of Sellers (1973). Assuming a sinusoidal surface temperature variation, Sellers expressed surface heat storage as

$$ST_s = \left[\frac{\pi}{P}C\lambda\right]^{1/2} \left[ T - T_{\text{annual}} + \frac{(P/2\pi)}{C\lambda} \frac{\partial T}{\partial t} \right]$$

(49)

where $P$ is the 12 month period of oscillation, $C$ is the heat capacity, and $\lambda$ the thermal conductivity. The product $(C\lambda)^{1/2}$ has dimensions cal cm$^{-2}$ K$^{-1}$ sec$^{-1/2}$ and is called the thermal property; Sellers' values of 0.04 for land and 2.0 for sea are used. $T_{\text{annual}}$ is the annual average surface temperature.

This model does not calculate atmospheric or oceanic dynamics. Meridional heat fluxes are specified on a monthly basis (fixed for each month) from two other models. Hence this model has no dynamical feedback; the dynamical heat transports are fixed and do not respond to changes in other climatic variables as temperature and water vapor do.

Stratospheric meridional heat fluxes have been computed using the Geophysical Fluid Dynamics Laboratory general
circulation model by Manabe and Mahlman (1976). Their values for fluxes at 110 mb at various seasons have been crudely extrapolated for use from 0-200 mb at each month in the model.

Total atmospheric meridional fluxes have been calculated monthly in another general circulation model run by Manabe, Bryan, and Spelman (1978). The difference of these fluxes and the stratospheric fluxes of Manabe and Mahlman are apportioned to the layers from 200-1000 mb by mass. The oceanic meridional fluxes calculated by Manabe, Bryan, and Spelman are used in this model. Atmospheric and oceanic fluxes are input to the model energy balance equations (1) and (2) as $D_{n}$ and $D_{o}$ respectively.

Radiative-convective Equilibrium

We have discussed all of the radiation terms in energy balance equations (1) for the surface and (2) for the twelve atmospheric layers. If we are given sun angle, ground albedo, distribution of aerosol, absorbing gases, clouds, and temperature, we may calculate the radiation terms in (1) and (2). However in the current problem, temperature is not given. The following radiative-convective equilibrium procedure gives a prescription for calculating the temperature at each level in the model.

In radiative equilibrium the surface and each layer of the atmosphere emit enough infrared to balance the
absorbed solar radiation. In the modified form of radiative equilibrium used in this model, each layer has additional sources or sinks of energy from heat storage and horizontal convergence due to climatological dynamics. Noting Fig. 6, we see that the surface in an earth-atmosphere system in strict radiative equilibrium is exceedingly hot: 330 K is typical. The radiative equilibrium vertical temperature lapse rate (-dT/dz) is superadiabatic near the surface, and the lower layers are unstable with respect to convective processes.

The convective adjustment procedure, which eliminates this unrealistic temperature, follows. A lapse rate is empirically specified (dT/dz = -6.5 K/km is used for the Global model). The temperature of the surface and the lower layers are adjusted to conform with the selected lapse rate, subject to the constraint that the combined surface-lower layer system remains in radiative equilibrium. The procedure is continued on up until the next higher layer is stable with respect to the selected lapse rate. In practice the entire troposphere is usually convectively adjusted. The result is a realistic surface temperature.

In the case of radiative equilibrium (or modified radiative equilibrium, where heat storage and dynamics are included) each layer individually balances energy inputs with outputs by adjusting its temperature (infrared
Fig. 6. Radiative Equilibrium and Radiative Equilibrium with a Convective Adjustment.
emission). Individual layers no longer balance their own energies once they are convectively adjusted, however. The convectively adjusted layers balance energy as a composite, the sum of the energy inputs to the convectively adjusted layers equaling the sum of the outputs from them.

The total infrared emission from an earth-atmosphere column in radiative equilibrium (or modified radiative equilibrium) remains the same when the column is convectively adjusted. The temperature structures in the radiative equilibrium case and in the convectively adjusted case are, of course, different. This does not indicate that an atmosphere with given energy inputs can have more than one temperature structure when in radiative equilibrium. The convectively adjusted case yields different temperatures because it is a form of constrained radiative equilibrium, with the relation between temperatures (dT/dz) in the lower layers fixed.

The convective adjustment procedure does not specify the energy used in the convection process. It merely sets the vertical lapse rate of temperature under the constraint of net energy balance. Hence the exclusion of vertical fluxes of turbulent sensible heat and latent heat in the model.

Manabe and Strickler (1964) performed the first radiative-convective calculations. Their model marched forward in time and required hundreds of days of iterations to
reach a steady state. Different procedures for calculating radiative-convective equilibrium are described by Ramanathan (1976) and Coakley (1977). The procedure used here is somewhat similar to Coakley's. It was developed independently by the author and solves directly for a steady state.

We describe the procedure for calculating radiative equilibrium first, and then convective adjustment. In the radiative equilibrium calculation all energy inputs into the surface and atmospheric layers are specified except for the infrared fluxes. The infrared fluxes are functions of the unknown temperature structure. The fluxes emerging from layers (1) and (2) may be approximated as

\[ F_\uparrow(1) = d_1^\varepsilon \sigma T_1^4 \]  
\[ F_\uparrow(2) = d_2^\varepsilon \sigma T_2^4 + (1-d_2^\varepsilon')F_\uparrow(1) \]  

where \( d_1^\varepsilon \) and \( d_2^\varepsilon \) are the effective downgoing emissivities of layers (1) and (2). The primes on the \( d_2^\varepsilon \)' term in (51) indicate that the effective emissivity for transmission \( d_2^\varepsilon \)' may differ from that for emission \( d_2^\varepsilon \). If \( T_1, T_2, F_\uparrow(1), F_\uparrow(2) \) are known and \( d_2^\varepsilon \)' can be calculated from known infrared transmission properties of layer (2) constituents, then \( d_1^\varepsilon \) and \( d_2^\varepsilon \) can be calculated. The set \( d_1^\varepsilon, d_2^\varepsilon, d_2^\varepsilon' \) can then be used to compute approximate values for \( F_\uparrow(1) \) and \( F_\uparrow(2) \) for temperatures in the neighborhood of the original \( T_1 \) and \( T_2 \).
By first guessing temperatures \( T_1, \ldots, T_{12} \) we can use (40), the full-blown infrared treatment, to compute \( F_\downarrow(1), \ldots, F_\downarrow(12) \). These are used to infer a set of effective emissivities for downgoing fluxes \( d\varepsilon_1', \ldots, d\varepsilon_{12}' \), \( d\varepsilon_1, \ldots, d\varepsilon_{12} \). A comparable set of upgoing emissivities \( u\varepsilon_1', \ldots, u\varepsilon_{12}', u\varepsilon_1, \ldots, u\varepsilon_{12} \) is also obtained. Expressions for infrared flux at all levels can be written as (50) and (51), as functions of temperature and emissivity.

These expressions will be approximate. They are most valid when the temperatures are close to those for which the emissivities were calculated. The expression for downward flux at an arbitrary level \( m \) is

\[
F_\downarrow(m) = d\varepsilon_m \sigma T_m^4 + \sum_{n=1}^{m-1} d\varepsilon_n \sigma T_n^4 \left( 1 - d\varepsilon_p' \right)
\]

where \( F_\downarrow(m) \) is a linear function of \( T_1^4, \ldots, T_m^4 \). Comparable equations for upward infrared fluxes, as \( F_\uparrow(m) \), are linear functions of \( T_m^4, \ldots, T_{12}^4 \).

These flux expressions are inserted into energy balance equations (1) and (2) as

\[
SW_s - DO_s - ST_s - IR_s
= SW_s - DO_s - ST_s - F_\downarrow(13) + F_\downarrow(12) = 0 \quad (1)
\]

\[
SW_m - DH_m - ST_m - IR_m
= SW_m - DH_m - ST_m - F_\downarrow(m) + F_\downarrow(m+1) + F_\downarrow(m)
- F_\downarrow(m-1) = 0 \quad (2)
\]
where all terms in the one equation (1) and the twelve equations (2) are fixed, except for the $F$ terms, which are functions of the thirteen temperatures $T_1, \ldots, T_{13}$. The system is solved for these temperatures. These are the radiative equilibrium temperatures (or for nonzero $D_0, D_H, S_T$, and $S_{T_m}$, the modified radiative equilibrium temperatures).

The temperature lapse rate in the lowest layers is always found to be convectively unstable. This calls for convective adjustment. We rewrite equations (1) and (2), using the term $F_m$ to express the infrared divergence from each layer:

$$SW_1 - DH_1 - ST_1 - F_1 (T_1^4, \ldots, T_{11}^4, T_{12}, T_{13}^4) = 0 \quad (53-1)$$

$$SW_{12} - DH_{12} - ST_{12} - F_{12} (T_1^4, \ldots, T_{11}^4, T_{12}, T_{13}^4) = 0 \quad (53-12)$$

$$SW_s - DO_s - ST_s - F_{13} (T_1^4, \ldots, T_{11}^4, T_{12}, T_{13}^4) = 0 \quad (53-13)$$

To convectively adjust the temperature of the lowest layer 12 we express $T_{12}$ as a function of $T_{13}$, the surface temperature, as

$$T_{12} = T_{13} + (dT/dp)(p_{12} - p_{13}) \quad (54)$$

where $p$ designates pressure, and $(dT/dp)$ is specified.

Equations (53) call for $T_{12}^4$, which can be expanded in binomial form as
\[ T_{12}^4 = T_{13}^4 + D \]  

\[ D = 4T_{13}^3 \left[ (dT/dp)(p_{12}-p_{13}) \right] + 6 \left[ T_{13}(dT/dp)(p_{12}-p_{13}) \right]^2 \]

\[ + 4T_{13} \left[ (dT/dp)(p_{12}-p_{13}) \right]^3 + \left[ (dT/dp)(p_{12}-p_{13}) \right]^4 \]

where \( D \) is computed with the value for \( T_{13} \) obtained in the previous calculation. \( D \) is then fixed. We substitute the above expression for \( T_{12}^4 \) into the set of equations (53). Each \( F_m \) term has one less variable, as

\[ F_m(T_1^4, \ldots, T_{12}^4, T_{13}^4) \rightarrow F_m(T_1^4, \ldots, T_{11}^4, T_{13}^4) \]

The equations (53-13) and (53-12) for the surface and lowest atmospheric layer are then added. The result is a system of twelve equations and twelve unknowns, which is then solved to obtain a set of (constrained) radiative equilibrium temperatures with the bottom two \( (T_{12} \text{ and } T_{13}) \) convectively adjusted. The solution yields \( T_1, \ldots, T_{11}, T_{13} \) and (54) yields \( T_{12} \).

The result is not exact, however. It does not conserve energy. When (55) was used to substitute for \( T_{12}^4 \), we fixed \( D \). Realistically, the three \( T_{13}^4 \) terms in \( D \) should be permitted to vary, as does \( T_{13}^4 \). We therefore iterate. The new value of \( T_{13}^4 \), convectively adjusted, is closer to the "true" \( T_{13}^4 \) for use in (55). The model typically
iterates eight times from the series (53) on; energy is then conserved to five significant figures.

Layers 11, 10, 9, ..., 1 are examined and convectively adjusted if necessary. The result is a tentative set of radiative-convective equilibrium temperatures $T_1$, $\ldots$, $T_{13}$.

Even this last set of temperatures requires adjustment. Recall that the effective emissivities $d^\varepsilon_m$, $d^\varepsilon_m'$, $u^\varepsilon_m$, $u^\varepsilon_m'$ were obtained from flux calculations made with first guess temperatures. We now use our last sequence $T_1$, $\ldots$, $T_{13}$ described above to recalculate the infrared fluxes. This produces a new set of emissivities for the whole calculation. The final iteration is done at least four times.

A summary of the radiative-convective procedure is given below. A more complete mathematical explanation is given in Appendix K.

1. All terms in energy balance equations (1) and (2) are calculated and fixed, except for the infrared terms.

2. Using the full infrared treatment described in the previous section, and a first guess of the temperature field, upward and downward fluxes are calculated at all levels. Effective emissivities for transmission $u^\varepsilon_m$, $d^\varepsilon_m$ are inferred directly from transmissivity terms in the full infrared calculation.
3. Using (52) and the $F_\downarrow(m)$ calculated in (2), we infer the $d_\varepsilon_m$. We also calculate the $u_\varepsilon_m$.

4. Emissivity expressions for infrared fluxes, linear in $T_m^4$, are inserted into energy balance equations (1) and (2). The thirteen equations are solved for the thirteen unknowns $T_m^4$.

5. If layer 12 is convectively unstable, express $T_{12}^4$ as a function of $T_{13}^4$ using (55). Then add the energy balance equation for layer 12 to that for the surface and resolve the system of equations for $T_1, \ldots, T_{11}, T_{13}$. Calculate $T_{12}$ with (54). Iterate to conserve energy.

6. Examine each layer for convective instability and repeat 5 if necessary.

7. The thirteen new $T_m$ are the new temperature field. Iterate on steps 2 on down.

Climate Calculation

Now we explain how all the previous sections are used in the calculation of the Monthly climate model. The Monthly model calculates the temperature of a given latitude belt each month; meridional heat fluxes are fixed to monthly climatological values but heat storage varies. The simpler Annual and Global models are explained at the end of this section.

The Monthly model uses fixed, monthly surface albedoes. Sea surface albedo as a function of month and
latitude is taken from Budyko (1974). Ground albedo is obtained from Sellers' (1973) temperature dependent formulation using fixed climatological temperatures. Bare ground albedo is taken as 0.15, and snow as 0.80, and the fractional area of land covered by snow $A_s$ is

$$A_s = 10.89 - \frac{T_{annual}(T_L')}{7100}.$$  

(57)

$A_s$ is used by Sellers to specify the snow cover of a given latitude belt on a monthly basis. $T_{annual}$, the mean annual surface temperature, and $T_L'$, the mean of the current and previous month's temperature, are both specified in degrees C here.

The Monthly model begins with a guess of the mean land-sea surface temperature for the month. On a first run this is the climatological temperature, and on other runs the previous estimate of the temperature for that month. This temperature is then used with the empirical formulas of Appendix E to calculate the distribution of water vapor in the vertical.

The Monthly model then calculates radiative transfer at solar wavelengths, separately for land and sea. The net solar absorption in each vertical layer is obtained by weighting the separate land and sea absorptions at each level. This gives the solar absorption terms $SW_s$ and $SW_n$ in energy balance equations (1) for the surface and (2) for each atmospheric layer.
Meridional transports $D_{0s}$ and $D_{Hn}$ are specified as noted in the previous section. Equations (48) and (49), and the first guess temperatures are used to compute heat storage terms $ST_n$ and $ST_s$. (The mean land-sea temperature has been guessed. Separate land and sea temperatures are needed to compute heat storage. We utilize the fixed climatological temperature difference between mean $T$ and sea $T$ and also mean $T$ and land $T$ each month. In the model the mean land-sea temperature may vary, but the difference between land and sea temperatures is fixed.)

Each term in the energy balance equations is now specified, except for the infrared flux divergences $IR_s$ and $IR_n$. These are obtained with steps 1 through 7 in the section on Radiative Convective Equilibrium, which calculates the vertical temperature distribution.

The surface temperature thus calculated will generally not be the same as the one which was first guessed and used to compute heat storage, so iteration is required. The first guess temperature is modified, heat storage and convective adjustment are recalculated, and a new temperature obtained. After fourteen iterations the final temperature calculated and the most previously "guessed" temperature almost always differ by less than 0.01 K. This is accepted as the final temperature for that month.
The Monthly model then goes on to the next month. The average annual temperature $T_{\text{annual}}$, used in the heat storage calculation, is modified monthly. The model has been run for midlatitude conditions and in a somewhat cruder form for the tropics.

Calculations are also made with the simplified Annual and Global models. The Annual model solves directly for a steady state, obtaining the average annual vertical temperature distribution in a given latitude belt. Averaged annual cloud, gaseous absorber concentration, solar input, meridional heat transport, and albedo are used. There is no heat storage in the Annual model; this cuts out a time-consuming iteration on the computer. The implications of using annually (as opposed to monthly) averaged radiation in a climate model are discussed in the Annual model results section.

The Global model solves directly for average annual global conditions as a steady state. There is no meridional heat transport in the Global model.
The results of studies made with the Monthly, Annual, and Global models are now presented. The Monthly model is run for midlatitude conditions with optical depths of 0.125 and 0.0; it is also used to simulate the climate perturbation in the tropics due to the 1963 Agung volcanic eruption. The Annual (for midlatitudes) and Global models calculate the effects of varying aerosol optical depths and aerosol physical properties. Also, aerosol effects on the solar and infrared radiation fields are calculated using midlatitude climatological temperatures as a base.

The physical properties of the aerosol used in the Monthly, Annual, and Global models are those specified in Toon and Pollack's (1976) study unless otherwise noted. Toon and Pollack gave a value of 0.125 for the globally averaged aerosol optical depth \( \tau \) at a reference wavelength of 0.55 micron; the aerosol optical depths quoted in this work are for the wavelength 0.55 micron.

**Monthly Model**

Figure 7 shows a run of the Monthly model for 40-50N latitude conditions with aerosol optical depth 0.125. Model surface temperatures are plotted each month along
Fig. 7. Surface Temperature (40-50N) Each Month for Model (τ=0.125, year 1) and Observed.
with the climatological temperatures from Sellers (1978 data). This is a first-year run with climatological temperatures specified as an initial condition.

A phase error is apparent in the model output. Model maximum temperature occurs two to three weeks too early. The model spring is too warm and the fall too cold. The mean annual temperature is 282.98 K. The climatological value is 282.80 K.

The Monthly model for 40-50N and optical depth 0.125 approaches a steady state when run for 12 years. A plot of the 12th year is given in Fig. 8. Also shown is a 4th-year run for aerosol optical depth 0.0, which approaches a steady state more rapidly; climatological temperatures are used as initial conditions here too. Figure 9 shows the march of annual temperature in these runs.

The model with an optical depth of 0.125 (year 12) has an annual mean temperature of 281.61 K and the model with an optical depth of 0.0 (year 4) a mean of 283.21 K. The difference of 1.6 K is the same in each month. With monthly surface albedoes fixed, no enhanced seasonal effect of the added aerosol is noted.

Reck (1976) calculated a drop of about 0.4 K in surface temperature for the latitude 45N in April with the addition of aerosol with an optical depth of 0.065. She employed the Manabe and Wetherald (1967) one-dimensional model. She repeated her calculation with a more weakly
Fig. 8. Surface Temperature (40-50N) Each Month for Model ($\tau=0.125$, year 12) and Another Model ($\tau=0.0$, year 4).
Fig. 9. Model Generated Annual Temperatures (40-50N) for Each Model Year.
absorbing aerosol (the mean visible refractive index was 1.55-0.0155i, still more absorbant than the Toon and Pollack aerosol used in the Monthly model) and reported an additional 0.2 K cooling. This gives a cooling of 0.4+0.2 = 0.6 K for the aerosol optical depth 0.065; it would correspond to a drop of about 1.2 K, not too far from our 1.6 K, for an aerosol optical depth of 0.125. Reck's (1976) radiation model was based on an earlier one (Reck, 1974) which used an incident angle independent formulation for aerosol optics (Sagan and Pollack, 1967); the aerosol was considered a cloud at one level and its optical properties were calculated at one solar and one infrared wavelength; heat storage and meridional transport due to dynamics were neglected.

As Toon and Pollack specify global average aerosol depth as 0.125, the Monthly model calculation for that optical depth should be considered as the model's attempt to reproduce climatology at 40-50N. The 12-year model generated annual temperature is 1.19 K (282.80-281.61) lower than the climatological value. The radiation calculation is the most likely physical source of error.

The albedoes for the surface or clouds may be inappropriate. Errors in cloud climatology or cloud infrared properties are likely. Only a single sun angle for an entire month is considered in the solar radiation calculation. The outgoing infrared radiation is calculated from one vertical temperature structure each month. Real day-night
temperature differences could introduce nonlinearities. There may also be some error in specified aerosol optical depth or composition for 40-50N.

Another possible source of error in the calculation of annual temperature is atmospheric/oceanic dynamics. If the general circulation model meridional heat transports are incorrect the temperatures inferred from them may be wrong as well.

We note that the phase error, apparent in the first year of the model run, persists in the longer period runs. The phase error is probably not due to dynamics. It occurs when the model is run with dynamics generated by Sellers' (1973) parameterization, as well as with the general circulation model dynamics. The oceanic heat storage parameterization is the most likely source of the phase error.

Surface heat storage is calculated with (49). This is probably the best simple expression available. Its assumption of a sinusoidal surface temperature variation tends to generate one. Figures 10 and 11 for year one Monthly model runs at 50-60N and 30-40N exhibit sinusoidal phase errors similar to 40-50N. The phase error persists even if we increase the thermal property (like a heat capacity) in (49).
Fig. 10. Surface Temperature (50–60N) Each Month for Model (τ=0.125, year 1) and Observed.
Fig. 11. Surface Temperature (30-40N) Each Month for Model ($\tau=0.125$, year 1) and Observed.
Equation (49) essentially assumes a constant mixed layer depth for the ocean. The real ocean mixed layer depth decreases in summer. In June ocean surface temperatures exceed their average annual values, forming a temperature inversion. This inversion reduces the mixing of cooler, deeper water and permits Northern Hemisphere ocean surface temperatures to increase into August.

A variable mixed layer depth would improve the model and possibly eliminate the phase error. It must be kept in mind, however, that while land and sea heat storage are computed separately in the model, a mean land-sea temperature is the final result of the model computation; the phase error may be an artifact of this land-sea averaging assumption.

Figure 12 shows a vertical temperature sounding with the March (year 12, optical depth 0.125) model output plotted with March 40-50N observations (Oort and Rasmusson, 1971). The generally good agreement of computed and observed tropospheric temperatures attests to the validity of Rennick's (1977) lapse rate parameterization, which is employed here.

The simulation of stratospheric temperature is not good. Meridional heat transport is critical to the determination of stratospheric temperature. The Monthly model uses the results of Manabe and Mahlman's (1976) excellent study of the stratosphere. Their values for transport at
Fig. 12. March (40-50N) Temperature Soundings for the Model ($\tau=0.125$, year 12) and Observations.
one level for four seasons are extrapolated out for 0-200 mb at each month in the model. More precision than was available is needed.

Figure 13 shows the Monthly model September (year 12, optical depth 0.125) and September observations. Figure 14 plots the September run for aerosol optical depth 0.125 and also that for optical depth 0.0. The change in stratospheric temperature with the addition of the aerosol is small. The aerosol is concentrated in the lower tropospheric layers, 67% in the lowest 300 mb. The entire troposphere has been convectively adjusted, and this procedure has distributed the aerosol-induced cooling through the troposphere fairly uniformly.

Agung Eruption

The Mount Agung volcanic eruption in 1963 drastically increased stratospheric aerosol content and probably perturbed the climate. This climate perturbation can be examined with the Monthly model.

Hansen, Wang, and Lacis (1978) have noted how tropical temperatures were changed following Agung. Figure 15 illustrates the drop in tropical temperatures, given as the departure of each month's temperature from its climatological mean.

Hansen, Wang, and Lacis reviewed the literature on the Agung eruption and expressed the stratospheric aerosol
Fig. 13. September (40-50N) Temperature Soundings for the Model ($\tau=0.125$, year 12) and Observations.
Fig. 14. September (40-50N) Temperature Soundings for Model Runs with \((\tau=0.125, \text{year 12})\) and \((\tau=0.0, \text{year 4})\).
Fig. 15. Departure of Tropical Surface Temperatures from the Climatological Mean as a Function of Time.

Observations (solid) are cited by Hansen, Wang, and Lacis (1978). The results of the Monthly model (dotted) and Hansen model (dashed) are shown, and the time of the Agung volcanic eruption is marked.
optical depth perturbation as

\[
\Delta \tau = 0.2 f(t)
\]

\[
f(t) = \begin{cases} 
   t/120 & 0 < t < 120 \\
   1 & 120 < t < 240 \\
   e^{-(t-240)/240} & 240 < t 
\end{cases} 
\] (58)

where \( t \) is the number of days after the eruption. They assumed that the volcanic aerosol was composed of sulfuric acid droplets. The results of their radiative convective model simulation of the effects of Agung on the tropical troposphere is shown in Fig. 15. Figure 16 gives their results for the stratospheric temperature perturbation.

The Monthly model, in a more primitive form, was run with the above stratospheric optical depth perturbation. Calculations were performed for a single average latitude belt 30N-30S. The thermal properties (49) of land and sea were averaged. The emissivity correction to stratospheric ozone flux was not made.

Meridional transports for the Monthly model were taken from Sellers' (1978) new global climatic model. The Monthly model was run with no volcanic aerosol to establish a base line for each month's temperature. These base line temperatures were then compared with those obtained from a Monthly model run made with the volcanic aerosol optical depth as specified in (57). The temperature change
Fig. 16. Departure of Tropical Stratospheric Temperatures from the Climatological Mean as a Function of Time.

The observations (solid) are an Australian radiosonde station's reports from the 60 mb level. The results of the Monthly model (dotted) are for 0-100 mb and those of the Hansen model (dashed) are for 55 mb.
for the surface and the stratosphere are shown in Figs. 15 and 16.

Both the Monthly model and the Hansen model simulate the gross features of the Agung temperature perturbation. The models give the correct sign and magnitude of the perturbation for both the surface and stratosphere. In the models and in observations, the stratosphere responds and relaxes faster than the surface.

Calculations made with the Monthly model and those reported by Hansen, Wang, and Lacis (1978) are not in complete agreement. The Monthly model surface temperature drops more rapidly; this is probably due to the heat storage parameterization. The Monthly model stratospheric temperature warms more than the Hansen model. This could be due to an inadequate parameterization of absorption in the pseudo radiative transfer scheme. Solar absorption due to the stratospheric aerosol is small, though. The stratospheric heating is mostly due to the infrared effects of the aerosol, which are computed differently in the two models. Also, the Hansen model has finer resolution in the stratosphere than the present model, so different results should be expected.

The Agung eruption study points toward some validity for the Monthly model concepts. In the following Annual section similar calculations are performed more economically.
**Annual Model**

The Annual model solves directly for a steady state mean annual temperature at a given latitude belt. As previously noted, it neglects heat storage but includes dynamics. We compare runs with the Annual model and the Monthly model for 40-50N:

<table>
<thead>
<tr>
<th>Model</th>
<th>Mean Annual T</th>
<th>ΔT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>τ = 0.0 (year 4)</td>
<td>283.209</td>
<td>-1.603</td>
</tr>
<tr>
<td>τ = 0.125 (year 12)</td>
<td>281.606</td>
<td></td>
</tr>
<tr>
<td>Annual model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>τ = 0.0</td>
<td>283.806</td>
<td>-1.808</td>
</tr>
<tr>
<td>τ = 0.125</td>
<td>281.998</td>
<td></td>
</tr>
</tbody>
</table>

(The above temperatures are given to three decimal places. This does not, of course, imply that the model can calculate the state of the actual climate to such precision.)

The economical Annual model is seen to yield mean annual temperatures close to those obtained with the Monthly model. The ΔT for the addition of aerosol is only slightly different for the two.

The summed annual absorption of solar radiation for the Monthly model slightly exceeds that for the Annual model. This is because the Monthly model has high surface albedo in winter coincident with lower winter inputs of solar radiation. With the Annual model the monthly surface
albedoes are averaged, and more of the mean annual solar radiation is reflected.

Despite its lower solar absorption, the temperature in the Annual model slightly exceeds that for the annual mean of the Monthly model. The Annual model atmosphere is then a less efficient infrared emitter than the summed Monthly model. This is probably due to nonlinearities in the relations of Annual and summed Monthly surface temperatures and cloud cover to lapse rate, water vapor distribution, and ultimately infrared emission. These are similar to the relations of incoming solar radiation and seasonal surface albedo mentioned above.

Annual averaging of solar radiation does, however, cause the Annual model to react more strongly to the addition of aerosol than does the Monthly model. With the Coakley and Chylek (1975) two stream formulation, aerosol reflectivity increases with increasing slant angle of incidence. Slant angle is greatest in winter, giving the Monthly model peak aerosol reflectivities during those months with low solar inputs. The Annual model averages the slant angle. This gives higher aerosol reflectivities for the higher annual input of solar radiation. The Annual model thus reacts more strongly to the addition of reflecting aerosol than does the Monthly model.
Figure 17 shows a series of Annual (40-50N) runs for different amounts of Toon and Pollack aerosol. The "floating albedo" case accounts for ice-albedo feedback; this about doubles the sensitivity of the belt to changes in aerosol. \( \frac{dT}{d\tau} \) is the derivative of surface temperature with respect to optical depth. \( \frac{dT}{d\tau} \) for the fixed albedo case is -14.5, while that for the floating albedo is -25.3.

Albedo was calculated using Sellers' (1973) formulation. If the temperature dropped 2 K, the temperature each month was dropped 2 K and its surface albedo recalculated for the annually averaged albedo [see (57)]. The floating albedo calculation is iterative; a new temperature is calculated until the temperature and albedo are in adjustment. For optical depth 0.5 the Annual model at 40-50N produces an ice-covered belt. Dynamical feedback mechanisms in the real atmosphere would probably prevent this from occurring, provided that the optical depth increase was confined to only the 10 degree wide latitude belt. Strong cooling in one latitude belt would increase meridional transport into it from other belts.

Aerosol amount in Fig. 17 varies uniformly in the vertical, except for the (dashed) stratospheric calculation. The top layer (stratospheric) has a base optical depth of 0.0025. Here additional aerosol beyond 0.125 optical depth was placed into the stratosphere as sulfuric acid droplets.
Fig. 17. Surface Temperature (Annual 40-50N) as a Function of Aerosol Optical Depth for Fixed and Floating Albedoes.

The dashed line refers to a calculation which places all aerosol in excess of $\tau=0.125$ into the stratosphere as $H_2SO_4$ droplets.
The change from considering the additional aerosol as tropospheric is small for tropospheric temperatures. Shifting aerosol optical depth in excess of 0.125 into the stratosphere significantly affects stratospheric temperatures, however.

Figure 18 displays the difference of surface and stratospheric temperature as a function of total aerosol optical depth. As in Fig. 17, the dashed line refers to the case of the aerosol optical depth beyond 0.125 being placed in the stratosphere, a change of composition (H₂SO₄) as well as position. Adding aerosol to the stratosphere, as opposed to adding it mostly to the troposphere, causes more relative heating of the stratosphere.

The effects of varying aerosol composition throughout the entire column is shown in Fig. 19. Single scattering albedo \( \omega_0 \) is varied with fixed surface albedo \( A=0.13872 \). Aerosol asymmetry factor and optical depth relative to that at a reference wavelength remain as before. The strongly absorbing aerosol with \( \omega_0=0.75 \) has \( dT/d\tau=+4.9 \). The conservatively scattering (nonabsorbing) aerosol with \( \omega_0=1.0 \) has \( dT/d\tau=-15.4 \), almost the same as for the Toon and Pollack aerosol. It is seen that adding aerosol with \( \omega_0=0.81 \) would effect no change in the computed annual temperature.

An Annual run made with the Hanel (1976) continental aerosol, optical depth 0.125, gives an annual temperature of 282.853 K, \( dT/d\tau=-7.6 \). This European continental aerosol
Fig. 18. Temperature Difference between the Surface (T_{13}) and Stratosphere (T_1) as a Function of Aerosol Optical Depth for the Annual Model (40-50N).

The dashed line refers to a calculation which places all aerosol in excess of \( \tau = 0.125 \) into the stratosphere as \( \text{H}_2\text{SO}_4 \) droplets.
Fig. 19. Surface Temperature (Annual 40-50N) as a Function of Aerosol Optical Depth at Various Single Scattering Albedoes $\omega_0$.

The dashed line refers to the Toon and Pollack aerosol.
would cause cooling, but only half that of the globally averaged Toon and Pollack aerosol. Aerosol optical depth and single scattering albedo are functions of relative humidity in the Hanel model. Swelling of the aerosol by absorption of water vapor changes its optical properties; this relative humidity effect is not accounted for in the Toon and Pollack model. The optical depth 0.125 refers to the dry, unswollen aerosol. An important swelling effect may be the change in the aerosol asymmetry factor $g$. As Hanel does not specify $g$, calculations with the Hanel model use the Toon and Pollack $g$, which is not a function of relative humidity.

Figure 20 shows temperature changes in the Annual model due to changes in the quantity of ozone and cloud. This sets the effects of changes in aerosol in perspective with those of other climatic parameters. Ozone has a small effect on surface temperature. Cloudiness is a more significant parameter, at least with the cloud optics used in this model. A 10% variation in cloudiness roughly corresponds to a 0.1 change in Toon and Pollack aerosol optical depth.

**Global Model**

By eliminating meridional heat fluxes and changing solar input, surface albedo, etc., to global values, the
Fig. 20. Surface Temperature (Annual 40-50N) as a Function of Percent Change in the Amounts of Ozone and Cloudiness from Their Climatological Values.
Annual model is changed to the Global annual model. The Global model is a true one-dimensional radiative-convective model with constant relative humidity. As such it should be compared with the original Manabe and Wetherald (1967) study. The Manabe and Wetherald model did not include aerosol.

Using different techniques for radiative transfer and convective adjustment, Manabe and Wetherald calculated a surface temperature of 288.4 K for a surface albedo of 0.102. The Global model gives 291.9 K with a no aerosol condition. The Manabe and Wetherald cloud cover was 0.63, while the Hoyt (1976) B cloud cover used here is 0.41. The solar wavelength cloud optics used in both models is similar. The Manabe infrared calculation in the window includes no water vapor dimer effects or special provision for cloud.

When calculations are done for a more recently determined global average surface albedo of 0.154 (Otterman, 1977) and the global average aerosol optical depth 0.125, Global temperature is 287.5. Temperatures in this run closely match those of the U. S. Standard Atmosphere, except for one point (150 mb) above tropopause level, where Global is too cold by 5 K. The Manabe model is about 3 K too cold at that level.
A doubling of carbon dioxide concentration in this model increases temperature by 1.5 K, compared to 2.4 K for the Manabe and Wetherald model. A recent calculation by Ramanathan, quoted by Schneider (1975), also gave 1.5 K. Increasing the solar constant by 1% increases temperatures by 0.9 and 1.3 K in the Global and Manabe models, respectively.

Calculations made with the Global model use the Manabe surface albedo of 0.102 unless otherwise noted. Both Global and Manabe employ a 6.5 K/km lapse rate. Cloud overlaps (Appendix 0) were neglected in the Global model, reducing surface temperatures by about 0.8 K (291.216-290.402).

Figure 21 shows aerosol effects on the Global model. Aerosol is seen to have a nearly linear effect on surface temperature out to 0.5 optical depth. The Global model surface temperature is less sensitive to the addition of aerosol than either of the midlatitude models. Global surface temperature drops 1.5 K for the addition of 0.125 optical depth of Toon and Pollack aerosol; Annual drops 1.8 K. The higher mean sun angle of the Global model gives smaller aerosol reflectivity.

The angle effect is also significant for a strongly absorbing aerosol, single scattering albedo 0.75; the Global run temperature increases 0.4 K for the addition of 0.125 optical depth while the midlatitude Annual temperature
Fig. 21. Surface Temperature (Global) as a Function of Aerosol Optical Depth for Various Single Scattering Albedoes $\omega_0$.

The dashed line refers to the Toon and Pollack aerosol.
increases 0.6 K. Aerosol with a single scattering albedo of 0.81 seems to have little effect on the surface temperature of either the Global or Annual models. The critical value 0.81 for heating/cooling is the same for global and midlatitude cases.

Figure 22 gives stratospheric aerosol effects on the Global surface temperature. As with the midlatitude Annual model, stratospheric and tropospheric aerosol have almost the same effect on the surface temperature. The addition of stratospheric aerosol lowers the temperature difference of surface and stratosphere. Figure 23 shows significant increases in stratospheric temperature for the addition of stratospheric aerosol. The stratospheric aerosol consists of sulfuric acid droplets placed between 0 and 100 mb.

Aerosol Infrared Effects

The model calculates aerosol absorption and emission, but not scattering, in the infrared. Case A below has no aerosol, case B 0.125 optical depth of aerosol, and Case C aerosol in solar but not in infrared wavelengths. The aerosol optical depth in the top 100 mb (stratospheric) layer is only 0.0025 in cases B and C.
Fig. 22. Surface Temperature (Global) as a Function of Stratospheric Aerosol Optical Depth.
Fig. 23. Stratospheric Temperature (Global) as a Function of Stratospheric Aerosol Optical Depth.
On the surface the solar radiation effects of the aerosol (generally cooling) are seen to dominate the infrared (heating). Adding the aerosol in solar wavelengths alone (C) depresses surface temperature 1.9 K, while including infrared effects (B) cuts this to 1.5 K. The same applies to the stratosphere. But stratospheric temperature is less sensitive to the addition of the largely tropospheric aerosol than is the tropospheric temperature.

Stratospheric aerosol can strongly influence stratospheric temperature. In cases D, E, and F below tropospheric aerosol optical depth is 0.1225 (0.125-0.0025). Stratospheric optical depth is 0.0025 in D (the unperturbed 0-100 mb amount) and 0.1275 (0.125+0.0025) in E and F. F has no stratospheric aerosol infrared effect.

As with tropospheric aerosol, stratospheric aerosol solar wavelength effects overcome infrared effects and cool the surface. But stratospheric aerosol infrared effects are
more significant for stratospheric temperature than are stratospheric aerosol solar wavelength effects.

Comparing D and F we note that the addition of stratospheric aerosol in only the solar wavelengths cools the stratosphere. While the aerosol causes more solar absorption in the stratosphere, it also increases planetary reflectivity overall, and the surface cools. The stratosphere also cools as the upgoing infrared that it absorbs from the lower layers is reduced more than the local solar absorption is increased. The sulfuric acid droplets which comprise the model stratospheric aerosol have negligible absorption below 2.0 micron, hence their small absorption of solar radiation. At wavelengths greater than 4 micron the single scattering albedo of the sulfuric acid droplets is almost zero.

Case E includes stratospheric aerosol infrared effects. These cause stratospheric heating. Stratospheric temperature is more sensitive to perturbations in stratospheric aerosol optical depth than is tropospheric temperature.

The stratosphere generally cools in the infrared. But an increase in stratospheric aerosol, which increases infrared opacity, heats the stratosphere. This is a window effect. Across the infrared spectrum broadband, the stratosphere is undergoing radiative exchange with layers not far beneath itself and only a few degrees warmer; the
stratosphere also emits directly to space. But the stratospheric window is usually transparent. Addition of a window active constituent (as aerosol) to the stratosphere places the stratosphere in radiative exchange with much warmer layers further below. Because $T^4$ is not linear (the infrared emission of a layer is roughly proportional to $T^4$), stratospheric window absorption from one direction (from where it is warm) can easily exceed stratospheric window emission in two directions (from where it is cold).

Climate Stability

Rough assessments of climate stability can be made with the Global model. Schneider and Mass (1975) defined a climate sensitivity parameter as

$$\beta = S_o \frac{dT_{13}}{dS}$$

(59)

where $S_o$ is the present solar constant and $T_{13}$ surface temperature. $S$ may vary with sunspot activity.

The Manabe and Wetherald (1967) one-dimensional model gives a $\beta$ of about 130 K. Cess (1976) notes that the Budyko (1969) one-dimensional lateral model, which includes ice albedo and dynamical feedbacks, yields a $\beta$ of 400 K. Cess (1976) estimates $\beta$ as 145 K from an analysis of climatological and satellite data.

The Global $\beta$ is 91.3 K for a surface albedo of 0.102, aerosol optical depth 0.125, and surface relative
humidity 0.79. $\beta$ is strongly dependent on the humidity and its vertical structure. The Manabe and Global models distribute humidity differently. $\beta$ is calculated as 143.3 K with Global at a surface relative humidity of 0.70.

A $\beta$ of 91.3 K is taken as our "unperturbed" Global climate sensitivity. An increase in cloud cover by 50% reduces this to 82.7 K. A surface albedo of 0.154 gives a $\beta$ of 89.7 K.

Aerosol has a small effect on the climate sensitivity parameter. Discarding the aerosol increases $\beta$ to 94.2 K. Adding 0.125 optical depth to the stratosphere decreases $\beta$ to 88.5 K. When aerosol infrared effects are neglected in the unperturbed case, $\beta$ is reduced only to 90.5 K.

The Global model $\beta$ increases from 91.3 K to 125 K when Sellers' (1974) temperature dependent global average albedo is used. This is consistent with Lian and Cess (1977), who conclude that (temperature dependent) ice-albedo feedback amplifies global climate sensitivity by roughly 25%. Sellers' (1974) global average model has a $\beta$ of about 120 K (his model d).

Climatological Calculations

Temperature changes due to the addition of aerosol have been computed by the Monthly, Annual, and Global models. The temperature changes have been inferred from calculated radiative diabatic heating or cooling by the aerosol. Here
model radiation subroutines are used to calculate the diabatic effects of aerosol on climatological, rather than model generated, temperature patterns.

Figures 24-29 show diabatic heating/cooling at solar and infrared wavelengths for the 40-50N latitude belt. Temperature and water vapor are from Oort and Rasmusson (1971). Calculations have been made for each month. Results are presented for aerosol optical depth 0.0 and also 0.250, twice the global average amount.

Figure 24 gives solar absorption for the entire surface-atmosphere column for each month. This tracks the sun closely. The reflecting properties of the aerosol dominate the absorbing. Aerosol also reduces the infrared cooling of the total column (Fig. 25). The infrared curve tracks the monthly variation in surface temperature. Figure 26 gives the net solar absorbed minus net outgoing infrared. It confirms earlier model calculations which indicated cooling for added aerosol.

Figure 27 shows the absorption of solar radiation in the atmospheric slab between 950-1000 mb. For a total column optical depth of 0.250 this layer has an optical depth of 0.0450. Aerosol increases the absorption for all months except November and December. Maximum absorption is obtained in July, the compromise of peak incoming solar radiation (June) and peak precipitable water (August).
Fig. 24. Solar Radiation Absorbed Each Month by the Surface-atmosphere Column (40-50N) with Optical Depths $\tau=0.0$ and $\tau=0.250$. 
Fig. 25. Infrared Energy Emitted Each Month by the Surface-atmosphere Column (40-50N) with Optical Depths $\tau=0.0$ and $\tau=0.250$. 
Fig. 26. Net Radiation Each Month for the Earth-atmosphere Column (40-50N) with Optical Depths $\tau=0.0$ and $\tau=0.250$. 
Fig. 27. Solar Energy Heating (40-50N) of the Atmospheric Slab between 950 and 1000 mb for Optical Depths 0.0 and 0.250 Each Month.

For the case $\tau=0.250$ the 950-1000 mb slab has a $\tau$ of 0.045.
Fig. 28. Infrared Cooling Rates (40-50N) of the Atmospheric Slab between 950 and 1000 mb for Optical Depths 0.0 and 0.25 Each Month.

For the case $\tau=0.250$ the 950-1000 mb slab has a $\tau$ of 0.045.
Fig. 29. Radiational Cooling (Infrared Out Minus Solar In) of the Atmospheric Slab between 950 and 1000 mb for Optical Depths 0.0 and 0.250 Each Month (40-50N).

For the case $\tau=0.250$ the 950-1000 mb slab has a $\tau$ of 0.045.
The net infrared cooling (divergence) in the 950-1000 mb layer, plotted in Fig. 28, is more erratic. The aerosol causes less infrared cooling of this layer in July and August, despite the increased opacity of the layer caused by the presence of the aerosol. We now explain this.

During July and August the water vapor dimer effect is pronounced at low levels, swamping the aerosol effect in the window. Adding aerosol only slightly increases layer emission. Both aerosol and water vapor concentration fall off with height. The dimer infrared effect is reduced with height more rapidly than the aerosol effect, however, as the dimer effect depends on the square of the water vapor amount (note Appendix I and Lee, 1973). Aerosol infrared counter radiation from intermediate levels sends additional flux to the 950-1000 mb layer, causing it to cool less.

Net radiative cooling in the 950-1000 mb slab is shown in Fig. 29. The aerosol has a proportionately larger effect on a low-level atmospheric slab than on the total surface-atmosphere system.

Aerosol diabatic effects diminish with height. In the layer 800-850 mb in April, aerosol of total column optical depth 0.250 increases solar heating by 1.5% and decreases infrared cooling by 1.0%. The comparable figures for the 950-1000 mb slab are an increase of 5.9% and an increase of 12.5%.
The monthly radiation calculation may be summed to give annual values. For optical depth 0.125, solar radiation absorbed by the 40-50N belt is $2.071 \times 10^5$ erg cm$^{-2}$ sec$^{-1}$ and infrared output is $2.388 \times 10^5$ erg cm$^{-2}$ sec$^{-1}$. The radiative energy deficit (solar absorbed minus infrared emitted) is $2.670 \times 10^4$ erg cm$^{-2}$ sec$^{-1}$.

The deficit should be made up by atmospheric and oceanic heat transports. The Manabe, Bryan, and Spelman (1978) general circulation model convergence into the belt is $3.0 \times 10^4$ erg cm$^{-2}$ sec$^{-1}$. Calculated radiative deficit and general circulation model dynamics differ by only 12%. This is considered fortuitous as the general circulation model ocean transport may be off as much as a few hundred percent (Leetmaa, personal communication, 1979).

For no aerosol, solar absorption in the belt is calculated as $2.116 \times 10^5$ erg cm$^{-2}$ sec$^{-1}$ and infrared output is $2.346 \times 10^5$ erg cm$^{-2}$ sec$^{-1}$. Hoyt's (1976) comparable figures are $2.064 \times 10^5$ and $2.358 \times 10^5$, respectively.
CHAPTER 4

CONCLUSIONS

Here the significant results of the model are restated. The temperature drops for the addition of aerosol with an optical depth of 0.125 are 1.6 K, 1.8 K, and 1.5 K for the Monthly midlatitude model, the Annual midlatitude model, and the Global average model, respectively. With ice-albedo feedback the Annual model temperature drops 3.2 K.

Reck's (1976) calculations can be interpreted to yield a temperature drop of about 1.2 K at 45N for the addition of aerosol with an optical depth of 0.125. Rasool and Schneider's (1971) calculations have an aerosol optical depth of 0.125 reducing global temperature about 2.0 K. These two models use simplified radiative treatments of the aerosol but are in rough agreement with the results cited here.

We should not, however, conclude that our results give the temperature changes for the addition of aerosol to the real atmosphere. Feedback mechanisms due to dynam-ice and clouds are not accounted for. The addition of aerosol with an optical depth of 0.125 reduces the absorbed
solar energy by about 2%. Sellers' (1969) one-dimensional (horizontal) model accounts for dynamics and ice-albedo feedback. Reducing the solar input by 2-5% triggers a Sellers' model ice age and a much larger temperature drop.

Hence the inclusion of coupled dynamical and ice-albedo feedback is probably essential to an attempt to infer temperature perturbations from changes in the atmospheric aerosol. We say this despite the model's approximate simulation of the tropical temperature perturbation due to the Agung eruption. Paleoclimatic data indicates that the tropics have smaller temperature changes than midlatitudes during ice ages (CLIMAP, 1976). Tropical temperatures are probably less sensitive to changes in dynamics than mid-latitude temperatures. Our neglect of dynamical feedback (as with the Agung perturbation) may be a fair assumption for the tropics, but it is likely to be a poor one for mid and high latitudes.

Part of the reason for the successful simulation of Agung effects was the short-term nature of the phenomenon—about a year. A longer-term aerosol perturbation would involve heating or cooling of very deep ocean layers, something likely not well-simulated with the current model. Changes in the temperature of the deep ocean could significantly alter ocean dynamics, and even the present ocean dynamical processes are only roughly understood.
Feedback due to cloud amount and cloud optics is neglected in this and most other climate models. Manabe, Bryan, and Spelman (1978), simulated global climate very well by assuming time independent cloud cover. This suggests that cloudiness may not be as critical a factor in climate simulation as previously thought. Cess' (1976) climate analysis concludes that cloud amount feedback is not important zonally or globally. But Twomey's (1977) computations show that changes in aerosol concentration would affect cloud optics to at least some degree.

The temperature output of the model is not the sole critical climatic variable. Perturbations in radiation by changes in the solar constant (Wetherald and Manabe, 1975) or surface albedo (Charney, 1975) affect precipitation patterns in general circulation models more significantly than temperature patterns. Aerosol effects on such models would likely be similar.

Lastly, we note the limitations of the aerosol model. Toon and Pollack's (1976) global survey is probably the best in the field to date. But the global sampling of aerosols, upon which that study is based, has been scant. And our knowledge of their radiative properties is dependent upon assumptions of particle sphericity, homogeneity, and dryness which generally do not occur in the real atmosphere.
Inadequate parameterization of dynamical feedback may result in a climate model erring in the magnitude of perturbation by 50%. But inadequate specification of aerosol optical properties can lead to prediction of a climate perturbation of the wrong sign. To predict aerosol effects on climate better, we need better specification of the physical properties and geographical distribution of the tropospheric aerosol.
APPENDIX A

DIRECT-DIFFUSE PSEUDO ADDING

Equations (19) through (23) specify the direct-diffuse pseudo adding method for downgoing beams at incident cosine angle $\mu_o$ as

\[
D' = T_a'' + S'E_a + ST_a'' \tag{19}
\]

\[
U' = R_b'E_a + R_bT' \tag{20}
\]

\[
R'(\tau_a + \tau_b) = R_a' + U'e^{-(\tau_a/\mu)} + T_a'U' \tag{21}
\]

\[
T''(\tau_a + \tau_b) = e^{-(\tau_b/\mu)}D' + T_b'E_a + T_bT' \tag{22}
\]

\[
E_a = \prod_{i=1}^{n} [e^{-\tau_i/\mu_o} + T_i'C_iG] \tag{23}
\]

where we recall that the primes stand for incident radiation at direct cosine angle $\mu_o$. Computational schemes may also be written for diffuse downgoing radiation $(R,T)$ and diffuse upgoing radiation $(R*,T*)$. Again, these are similar to the Lacis and Hansen (1974) development, as

\[
D = T_a + Se^{-(\tau_a/\mu)} + ST_a \tag{A1}
\]

\[
U = R_b'e^{-(\tau_a/\mu)} + R_bT \tag{A2}
\]

\[
R(\tau_a + \tau_b) = R_a + e^{-(\tau_a/\mu)}U + T_a*U \tag{A3}
\]

\[
T(\tau_a + \tau_b) = e^{-(\tau_b/\mu)}D + T_b'e^{-(\tau_a/\mu)} + T_bT \tag{A4}
\]
\[ S^* = \frac{R_b R_a^*}{1 - R_b^* R_a} \quad (A5) \]
\[ U^* = T_b^* + S^* e^{-\tau_b/\mu} + S^* T_b^* \quad (A6) \]
\[ D^* = R_a^* e^{-\tau_b/\mu} + R_a^* U^* \quad (A7) \]
\[ R^*(\tau_a + \tau_b) = R_b^* + e^{-\tau_b/\mu} D^* + T_b D^* \quad (A8) \]
\[ T^*(\tau_a + \tau_b) = e^{-\tau_a/\mu} U^* + T_a^* e^{-\tau_b/\mu} + T_a U^* \quad (A9) \]

where equations (A8) and (A9) for composite R* and T* differ from the Lacis and Hansen formulation in that upgoing (reflected) radiation is assigned the diffuse path \( \mu \) rather than the direct \( \mu_0 \).

The expression for \( \text{ABS}(1,m) \), the absorption of radiation in the layers 1 through m, is obtained by considering the disposition of beams at the boundaries (Fig. 30) of \( (1,m) \), as

\[ \text{ABS}(1,m) = 1 - R'_{1,M+1} + U' - D' - E_{1,m} \quad (A10) \]

where unit quantity 1 of radiation enters the top of \( (1,m) \) and \( R'_{1,M+1} \) is headed out of the top. Diffuse beams \( U' \) are headed into the bottom of \( (1,m) \), while diffuse beams \( D' \) and free flying beams \( E_{1,m} \) are headed out the bottom. Note that \( R'_{1,M+1} \) is the reflectivity of the earth atmosphere system.
Fig. 30. Notation for Adding Method.
To calculate $\text{ABS}(l,m)$ we must specify $U'$ and $D'$ for the composite $(l,m)$ as

$$S' = R^*_{l,m} R'_{m+1,M+1}/(1-R^*_{l,m} R_{m+1,M+1})$$  \hspace{1cm} (A11)$$

$$S = R^*_{l,m} R_{m+1,M+1}/(1-R^*_{l,m} R_{m+1,M+1})$$  \hspace{1cm} (A12)$$

$$D' = T''_{l,m} + S'E_{l,m} + ST''_{l,m}$$  \hspace{1cm} (A13)$$

$$U' = R'_{m+1,M+1} E_{l,m} + R_{m+1,M+1} D'$$  \hspace{1cm} (A14)$$

where $R^*_{l,m}$ is the reflectivity to diffuse upgoing radiation of layers $l$ through $m$. $R^*_{l,m}$ is calculated using (A5) through (A9) to add layers 1 and 2, then (1 and 2) with 3, and so on down to $m$.

$R'_{m+1,M+1}$ is the reflectivity to direct downgoing beams of layers $m+1$ through $M+1$. It is calculated by using equations (19) through (23) to add layers $M$ and $M+1$, and then on up to $m+1$.

$R_{m+1,M+1}$ is like $R'_{m+1,M+1}$ except that it is for diffuse beams. It is calculated with (A1) through (A4).

$R_{m+1,M+1}$ is needed as $R_D$ when calculating the (19) through (23) series of equations adding on up.

$T'_{l,m}$ applies to direct beams striking layers $l$ through $m$. (19) through (23) calculates it for 1 and 2 on down to $m$. 


With the absorption ABS(l,m) of layers 1 through m known, the absorption of an individual layer can be calculated with differencing. Thus we obtain SW_n and SW_s, the solar inputs for the energy balance equations (1) and (2).

Diffuse radiation in this model is assumed to have an effective cosine angle of \( \mu = 1/1.9 \). This was calculated by Lacis and Hansen (1974) as an effective magnification factor for diffuse upgoing radiation, including multiple scattering effects. This differs from the Elsasser and Culbertson (1960) value of 1/1.6, which is used for infrared radiation.
APPENDIX B

EVALUATION OF \( \beta(\mu) \) IN THE TWO-STREAM FORMULATION

The Coakley and Chylek (1975) two-stream formulation requires the evaluation of a term \( \beta(\mu) \), as

\[
\beta(\mu) = \frac{1}{2\omega_c} \int_0^1 d\mu' \ p^0(\mu, -\mu')
\]

where \( p^0 \) is the azimuthally independent form of the phase function. The Henyey-Greenstein phase function is used in this model. Sobolev (1975) notes that the Henyey-Greenstein \( p^0 \) may be expanded in Legendre polynomials \( P(\mu) \) as

\[
p^0(\mu, \mu') = \sum_{i=0}^{\infty} (2i + 1)g^{i} P_i(\mu)P_i(\mu') .
\]
Working on the integral, we have

\[
\int_0^1 p_0^0(u,-u')du' = \int_0^\infty \sum_{i=0}^{\infty} (2i + 1)g^i P_i(u)P_i(-u')du' \\
= \sum_{i=0}^{\infty} (2i + 1)g^i P_i(u) \int_0^1 P_i(-u')du' \\
= \sum_{i=0}^{\infty} (2i + 1)g^i P_i(u)(-1)^i \int_0^1 P_i(u')du' \\
= g^0 P_0(u) \int_0^1 P_0(u')du' \\
+ \sum_{i=1}^{\infty} (2i+1)g^i P_i(u) (-1)^i \\
\left[ \frac{P_{i+1}(u')-P_{i-1}(u')}{2i+1} \right]_{u'=0}^{u'=1} \\
= 1 + \sum_{i=1}^{\infty} g^i P_i(u)(-1) \\
\left[ P_{i-1}(0) - P_{i+1}(0) \right]
\]

where a relation given in Spiegel (1968) has been used, as

\[
\int P_n(x)dx = \left[ P_{n+1}(x) - P_{n-1}(x) \right]/(2n+1) .
\]

The summation above is terminated after \( i=50 \). Even \( i \) contribute zero to the summation. \( P(u) \) values are calculated from a standard recursion formula.
APPENDIX C

COMPARISON OF DIRECT-DIFFUSE PSEUDO ADDING WITH PUBLISHED RESULTS

Calculations with the pseudo direct-diffuse adding scheme are here compared with the published results of Coakley and Chylek (1975) and Herman and Browning (1975).

In conjunction with the development of their two-stream formulation, Coakley and Chylek used the doubling technique to calculate the net reflectivity of an aerosol overlying a surface. Using an asymmetry factor of 0.844 and a single scattering albedo of 1.0, they calculated reflectivity as a function of incident angle for surface albedoes of 0.7 and 0.1.

The Coakley and Chylek results for optical depth 0.5 are shown in Figs. 31 and 32. The doubling calculation is solid, while their own incident angle dependent two-stream (their model 1) is dashed. The two-stream calculation places all of the aerosol in one layer. There is a significant difference between the doubling "standard" and the two-stream approximation for small $\mu_o$ (cosine of the angle of incidence).

These calculations have been repeated with the pseudo scheme (dotted). Here pseudo was run for twelve layers, the optical properties of each specified with the
Fig. 31. Reflectivity as a Function of $\mu_o$ with $\tau=0.5$ and $A=0.1$ as Calculated with Doubling, Two-stream, and Pseudo.

Doubling is solid, two-stream is dashed, and pseudo is dotted. $\mu_o$ is the cosine of the angle of incidence, $\tau$ is the aerosol optical depth, and $A$ is the surface albedo. The asymmetry factor is 0.844 and the single scattering albedo is 1.0.
Fig. 32. Reflectivity as a Function of $\mu_o$ with $\tau=0.5$ and $A=0.7$ as Calculated with Doubling, Two-stream, and Pseudo.

Doubling is solid, two-stream is dashed, and pseudo is dotted. $\mu_o$ is the cosine of the angle of incidence, $\tau$ is the aerosol optical depth, and $A$ is the surface albedo. The asymmetry factor is 0.844 and the single scattering albedo is 1.0.
Coakley and Chylek two-stream. Pseudo corresponds more closely with the doubling than does the two-stream by itself.

The accuracy of the two-stream calculation increases as optical depth decreases. Layering the optical depth and inserting into pseudo improves the accuracy of the computation.

In Figs. 33 and 34 we again compare the pseudo (dotted) with the doubling (solid), but this time with the scatter correction to pseudo omitted. The pseudo scatter correction is given in (10) through (23). We note that this uncorrected pseudo differs from the standard doubling computation as much as the straight (single layer) two-stream does.

Most calculations done with the model use an optical depth less than 0.5. In Figs. 35 and 36 the doubling and pseudo computations are compared for an optical depth of 0.125. It is noted that a straight two-stream calculation also closely agrees with doubling for optical depth 0.125.

Herman and Browning (1975) used sophisticated radiative transfer techniques to calculate the reflectivity of a system containing both aerosol and Rayleigh scatterers. Their results for nonabsorbing aerosol in an atmosphere with a Rayleigh optical depth of 0.145 are given in Fig. 37 and 38 (solid). Reflectivity is plotted as a function of aerosol optical depth for various surface albedoes.
Fig. 33. Reflectivity as a Function of $\mu_o$ with $\tau=0.5$ and $A=0.1$ as Calculated with Doubling, Two-stream, and Uncorrected Pseudo.

Doubling is solid, two-stream is dashed, and uncorrected pseudo is dotted.
Fig. 34. Reflectivity as a Function of $\mu_o$ with $\tau=0.5$ and $A=0.7$ as Calculated with Doubling, Two-stream, and Uncorrected Pseudo.

Doubling is solid, two-stream is dashed, and uncorrected pseudo is dotted.
Fig. 35. Reflectivity as a Function of $\mu_0$ with $\tau=0.125$ and $A=0.1$ as Calculated with Doubling and Pseudo.

Doubling is solid and pseudo is dotted.
Fig. 36. Reflectivity as a Function of $\mu_o$ with $\tau=0.125$ and $A=0.7$ as Calculated with Doubling and Pseudo.

Doubling is solid and pseudo is dotted.
Fig. 37. Reflectivity and Aerosol $\tau$ as Calculated by Herman (Solid) and Pseudo (Dotted) for Various Albedoes with $\mu_o = 0.9659$. 

Herman and Pseudo coincide for $A=0.8$.

$\tau_{\text{Rayleigh}} = 0.145$

$\mu_o = 0.9659$
Fig. 38. Reflectivity and Aerosol $\tau$ as Calculated by Herman (Solid) and Pseudo (Dotted) for Various Surface Albedoes with $\mu_o = 0.5736$. 

$\tau_{\text{Rayleigh}} = 0.145$

$\mu_o = 0.5736$

(Herman and Pseudo coincide for $A=0.8$)
The same calculations, made with the pseudo technique, are also plotted (dotted). The effective asymmetry factor $g$ of the aerosol used by Herman and Browning could not be easily obtained. The pseudo calculations were made with $g$ equal to 0.70, regarded as typical by scanning Toon and Pollack (1976). The results for a $g$ of 0.65 are in slightly closer agreement with Herman and Browning, but they are not plotted.

It thus appears that the direct diffuse pseudo adding formulation satisfactorily determines net reflectivity for small aerosol optical depths and moderate incident angles; it compares well with more accurate techniques used by Coakley and Chylek (doubling) and also Herman and Browning. Pseudo has not been tested against standard techniques with absorbing aerosols, however.
APPENDIX D

H₂O AND CO₂ SOLAR WAVELENGTH TRANSMISSION FUNCTIONS

The transmissions of H₂O and CO₂ are calculated as

\[ t = \sum_{i=1}^{N} a_i e^{-b_i u} \]

where \( u \) is the H₂O or CO₂ pathlength and \( a_i, b_i \) are given below.

H₂O absorption occurs in the following wavelength intervals:
- 0.926-0.978
- 1.06-1.22
- 1.22-1.612
- 1.63-2.05
- 2.23-2.995
- 2.995-3.722
- 4.58-9.70 micron

We now list \( a_i, b_i \) for H₂O.

<table>
<thead>
<tr>
<th>( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_i )</td>
<td>0.0609</td>
<td>0.0840</td>
<td>0.0243</td>
<td>0.0865</td>
<td>0.0429</td>
<td>0.5461</td>
<td>0.1553</td>
</tr>
<tr>
<td>( b_i )</td>
<td>0.8839</td>
<td>0.0300</td>
<td>0.921.034</td>
<td>16.3932</td>
<td>122.8607</td>
<td>0.1001</td>
<td>1.7655</td>
</tr>
</tbody>
</table>

These are due to Welch and Zdunkowski (1976).

CO₂ transmission is specified in four intervals by Kerschgens et al. (1976).

1.277-1.575 micron

<table>
<thead>
<tr>
<th>( i )</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_i )</td>
<td>1.0</td>
</tr>
<tr>
<td>( b_i )</td>
<td>0.000045</td>
</tr>
</tbody>
</table>
1.575-1.670 micron
\[ a_i = 0.9778 \pm 0.0222 \]
\[ b_i = 0.00003 \pm 0.02 \]

1.670-2.110 micron
\[ a_i = 0.8007 \pm 0.1993 \]
\[ b_i = 0.00012 \pm 0.0131 \]

2.199-2.990 micron
\[ a_i = 0.8149 \pm 0.1467 \pm 0.0384 \]
\[ b_i = 0.00023 \pm 0.12 \pm 0.9 \]

In the case of CO\textsubscript{2}, separate transmissions are calculated for each of the four intervals.
APPENDIX E

EMPIRICAL FORMULAS FOR WATER VAPOR
HEIGHT DISTRIBUTION

The relative humidity \( r \) in any given run in the model is fixed. Here we want to specify the temperature and relative humidity, and then obtain the water vapor distribution as a function of height. Sellers (1973) has provided several empirical formulas which are useful. Lowe's (1977) formula is used to obtain the saturation vapor pressure \( e_s(T) \) in millibars (mb). From \( e_s \) and \( r \) we obtain surface vapor pressure \( e_o \), and then (with one of Seller's expressions) the precipitable water \( U \), as

\[
e_o = e_s r
\]

\[
U = (0.123 + 0.152e_o)p_o/1000
\]

where the surface pressure \( p_o \) is taken as 1000 mb. Using the coefficient \( a_1 \), mixing ratio \( q \) may be given as a function of the pressure \( p \), the surface mixing ratio \( q_o \), and the gravitational constant \( g \), as

\[
q = q_o(p/p_o)^a_1
\]

\[
a_1 = 622e_o/(gU) - 1
\]

\[
q_o = 622e_o/p_o
\]

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The corrected optical pathlength of water vapor between two layers is $u_p$:

$$u_p(n) = \frac{1}{g} \int_{p_{n-1}}^{p_n} q(p) (p/p_0)^\kappa dp$$

$$u_p(n) = \left\{ \frac{[q_o p^{-(\kappa+a_1)}]}{[g(1+\kappa+a_1)]]} \right\} \left[ p_n^{(1+\kappa+a_1)} - p_{n-1}^{(1+\kappa+a_1)} \right]$$

We use a $\kappa$ of 0.9 (McClatchey et al., 1971). The above expression is used to calculate $u_p$ for levels 3 to 12 (up to 200 mb).

The uncorrected pathlength of water vapor in model levels 1 and 2 is fixed at $3 \times 10^{-4}$ gm cm$^{-2}$. This corresponds to Manabe and Wetherald's (1967) stratospheric mixing ratio of $3 \times 10^{-6}$ for water vapor. Stratospheric temperature is relatively sensitive to small changes in the local water vapor amount. The stratospheric mixing ratio is more constant than the tropospheric, and the previous expressions were derived for tropospheric conditions.
APPENDIX F

CARBON DIOXIDE HEIGHT DISTRIBUTION

The carbon dioxide volume mixing ratio is $330 \times 10^{-6}$, so the mass mixing ratio $q$ is given by

$$q = 330 \times 10^{-6} \times \frac{44}{28.966} = 501.28 \times 10^{-6}$$

where 44 is the molecular weight of carbon dioxide and 28.966 is the molecular weight of dry air. The corrected mass of CO$_2$ (gm cm$^{-2}$) between two pressure levels is then

$$\rho_i \left( \frac{1}{g} \right) \int_{p_i}^{p_{i+1}} q \left( \frac{p}{p_0} \right)^{\kappa} dp = q \left[ \frac{p_i^{\kappa+1} - p_{i+1}^{\kappa+1}}{K \rho_o (\kappa+1)} \right]$$

where $\kappa$ for carbon dioxide is 0.75 (McClatchey et al., 1971). This mass is then multiplied by $1.96 \times 10^{-7}$ gm cm$^{-3}$ (carbon dioxide density at STP) to obtain the pathlength in cm reduced to STP.
Cloud infrared optical properties are listed below as

emissivity (broadband) 0.97
reflectivity (broadband) 0.03
emissivity (14 micron) 0.97
reflectivity (14 micron) 0.02

The window properties are given as

\[
\begin{array}{cccccc}
\text{wavenumber (cm}^{-1}\text{)} & 800 & 900 & 1000 & 1100 & 1200 \\
\text{emissivity} & 0.98 & 0.95 & 0.92 & 0.88 & 0.85 \\
\text{reflectivity} & 0.015 & 0.015 & 0.03 & 0.04 & 0.05 \\
\end{array}
\]

The window extends from 750-1250 cm\(^{-1}\) (13.3-8.0 micron).

The above quantities are obtained from Yamamoto et al. (1970).
APPENDIX H

EQUIVALENT WATER VAPOR PATHLENGTH FOR AEROSOL

Equation (41) occasionally requires a water vapor equivalent pathlength for stratospheric aerosol. This is calculated by determining a broadband effective emissivity for the aerosol, and then inferring an effective water vapor pathlength from that. \( B_n \) is the Planckian blackbody function per unit wavenumber \( n \), and \( \tau_F \) is the flux transmissivity at aerosol optical depth \( \tau_n \), as

\[
\varepsilon = \frac{\int_{500 \text{ cm}^{-1}}^{2300 \text{ cm}^{-1}} B_n \left[ 1 - \tau_F(\tau_n) \right] \, dn}{\int_{500 \text{ cm}^{-1}}^{2300 \text{ cm}^{-1}} B_n \, dn}
\]

\[
B_n = \frac{2\pi \hbar c n^3}{(e^{\hbar c n/kT} - 1)}
\]

Staley and Jurica (1970) have used a similar expression for the emissivity of a gas. \( \tau_F \) is calculated with the exponential integral \( E_3 \), which accounts for the diffuse nature of the beam (Elsasser and Culbertson, 1960), as

\[
\tau_F(x) = 2E_3(kx) = 2\int_1^\infty e^{-kx\eta} \eta^{-3} \, d\eta, \quad \eta = 1/\nu
\]

A curve fit to the above is made from numerical values in Abramowitz and Stegun (1964):
\[ x < 0.02 \quad E_3(x) = [-0.952x + 0.5] \]
\[ 0.02 \leq x \leq 0.2 \quad E_3(x) = [0.497133e^{-1.748015x}] \]
\[ 0.2 < x \quad E_3(x) = [0.444032e^{-1.383818x}] \]

We then infer an equivalent water vapor pathlength from
\[
u_{EQ} = 10 \left[ (\ln e - \ln 66.5039) / 1.62716 \right]
\]

This is an exponential curve fit to a Staley-Jurica type emissivity for water vapor.
APPENDIX I

DOWNGOING INFRARED FLUXES: FURTHER EXPLANATIONS AND FINITE DIFFERENCE FORMS

The equation for downgoing fluxes is

\[ F_+(z) = F_{+A}(z) - F_{+B}(z) + F_{+C}(z) + F_{+D}(z) + F_{+E}(z). \]  

(40)

We now evaluate the first term as

\[ F_{+A}(z) = \int_{z}^{\infty} \sigma T^4(z') \, d\varepsilon_2(z,z')/dz' \, dz'. \]  

(41)

Rodgers' new \( \varepsilon_2 \) values are quoted in Welch and Zdunkowski (1976) as

\[
\begin{align*}
\varepsilon_2(x) &= 8.321x^{1/2} - 194.8x + 4674x^{3/2} - 55103x^2 & x < 0.001 \\
\varepsilon_2(x) &= 0.6554 + 0.1213 \ln(x) + 1.552x10^{-2} [\ln(x)]^2 \\
&\quad + 1.856 [\ln(x)]^3 + 9.4297x10^{-5} [\ln(x)]^4 & x > 0.001
\end{align*}
\]

Each layer \( m \) has temperature \( T_m \) and water vapor corrected optical pathlength \( u_p \). \( u_p(m,j) \) is the optical pathlength between layers \( m \) and \( j \) (including \( m \) and \( j \)). So we approximate (41) as

\[
F_{+A}(m) = \sigma T^4_m \varepsilon_2[u_p(m,m)] + \sum_{j=1}^{m-1} \sigma T^4_j \{\varepsilon_2[u_p(m,j)] \\
- \varepsilon_2[u_p(m,j+1)]\}
\]
where the first term is the contribution of the layer $m$ emission to the downgoing flux at the bottom of $m$.

For $m=1$

$$F_{\downarrow A}(1) = \sigma T_1^4 \varepsilon_2 U_p(1,1)$$

The expression (43) for cloudy skies is evaluated analogously.

We now discuss and evaluate the second term in (40), $F_{\downarrow B}(z)$, as

$$F_{\downarrow B}(z) = \int \int \frac{1}{750 \text{cm}^{-1}} B_n(T) d\eta \frac{d\varepsilon_w(z,z')}{dz} dz' \ (44)$$

where $\varepsilon_w$ is calculated from $\tau_{Fw}$, the flux transmissivity for the water vapor continuum at pathlength $u_p$, as

$$\varepsilon_w(u_p) = 1 - \tau_{Fw}(u_p) = 1 - 2E_3(0.1u_p).$$

The relation between $\tau_{Fw}$ and the exponential integral $E_3$ is explained in Appendix H, which also shows how $E_3$ is evaluated. We have assumed an absorption $k$ of 0.1/gm for water vapor across the window.

When cloud is present the effective cloud $\varepsilon$ and $r$ are calculated as averages across the entire window. An effective water vapor pathlength for the cloud $u_p(\text{cloud})$ is then calculated from
\[ \tau_p(\text{cloud}) = 1 - \varepsilon - r = 2E_3[0.1u_p(\text{cloud})] = e^{-1.66[0.1u_p(\text{cloud})]} \]

\[ u_p(\text{cloud}) = -\ln(1 - \varepsilon - r)/0.166 \]

where the exponential integral is evaluated differently than in Appendix H. We infer a new window cloud temperature from

\[ T^4(\text{cloud}) \to T^4(\text{cloud}) + F_B(z)r/\sigma . \]

With the new \( u_p(\text{cloud}) \) and \( T(\text{cloud}) \) we recompute (44). The evaluation of (44) follows. For each \( T_m \) at layer \( m \) we have

\[ B(T) = \int_{750\text{cm}^{-1}}^{1250\text{cm}^{-1}} B_n(T)dn = \sum_{n=800\text{cm}^{-1}}^{1200\text{cm}^{-1}} \left[ 2\pi hc^2n^3(100\text{cm}^{-1})/ \right. \]

\[ \left. (e^{hc_0/nkT-1}) \right] \]

\[ \varepsilon_w[U_p(m,j)] = 1 - 2E_3[0.1U_p(m,j)] \]

where \( E_3 \) is evaluated by the method of Appendix H. The finite difference form of (44) is

\[ F_B(m) = B(T_m)\varepsilon_w[U_p(m,m)] + \sum_{j=1}^{m-1} B(T_j)\{\varepsilon_w[U_p(m,j)] - \varepsilon_w[U_p(m,j+1)]\} \]

\[ F_B(1) = B(T_1)\varepsilon_w[U_p(1,1)]. \]
The third term in (40) is $F_\uparrow C(z)$:

$$F_\uparrow C(z) = - \int_{750\text{cm}^{-1}}^{1250} \left[ B_n(T) \frac{d\tau_{Fw,n}(z,z')}{dz'} dz' \right] \ dn$$

(45)

where $\tau_{Fw,n}$ is the flux transmissivity in the window at wavenumber $n$ due to aerosol, water vapor, and water vapor dimers; it is calculated with the exponential integral as

$$\tau_{Fw,n}(z,z') = 2E_3[\tau_n(z,z')]$$

where $\tau_n(z,z')$ is the optical depth at wavenumber $n$ between $z$ and $z'$. $\tau_n$ in each layer is a function of pressure corrected water vapor pathlength $u_p$, uncorrected pathlength $u$, water vapor partial pressure $e$, temperature $T$, and aerosol optical depth $\tau_a$, as

$$\tau_n = 0.1(T/273.15)^{1/2}u_p + 0.001e^{\exp[1745/T - 5.89]}u + k_n \tau_a$$

where $k_n$ corrects the aerosol optical depth $\tau_a$ at 0.55 micron to that at the appropriate wavenumber $n$; $k_n$ is obtained from Appendix P. The above expression for $\tau_n$ includes the effect of water vapor e-type absorption as reported by Lee (1973).

Cloud effects are handled by calculating an effective optical depth at wavenumber $n$ for the cloud layer. We invert the following, solving for $\tau_n(\text{cloud})$, as

$$1 - \epsilon_n - r_n = 2E_3[\tau_n(\text{cloud})]$$
where \( \varepsilon_n \) and \( r_n \) are cloud emissivity and reflectivity at wavenumber \( n \). Cloud reflectivity is accounted for in a method similar to that used for \( F_B(z) \).

To evaluate (45) we shift notation a bit. The expression \( \tau_{Fw, n}(m, j) \) is used to denote flux transmissivity at wavenumber \( n \) between layers \( m \) and \( j \), including \( m \) and \( j \), as

\[
F_{+C}(1) = \sum_{n=800}^{1200} B_n(T_1) \left[ 1 - \tau_{Fw, n}(1, 1) \right] (100 \text{cm}^{-1})
\]

\[
F_{+C}(m) = \sum_{n=800}^{1200} B_n(T_m) \left[ 1 - \tau_{Fw, n}(m, m) \right] (100 \text{cm}^{-1})
\]

\[
+ \sum_{n=800}^{1200} \sum_{j=1}^{m-1} B_n(T_j) \left[ \tau_{Fw, n}(m, j+1) - \tau_{Fw, n}(m, j) \right] (100 \text{cm}^{-1})
\]

\( F_{+D}(z) \) is the next term in (40):

\[
F_{+D}(z) = 4\sigma \int_{T(z)} \bar{A}_{CO_2} \left[ y_p(T) - y_p(T') \right] \tau_{F, CO_2, H_2O} [u_p(T) - u_p(T')] T' dT'
\]

\[
+ \sigma T_4(z_t) \bar{A}_{CO_2} \left\{ y_p(T) - y_p[T(z_t)] \right\} \tau_{F, CO_2, H_2O} [u_p(T) - u_p[T(z_t)]]
\]

We recall that \( y_p \) is the carbon dioxide pathlength and that \( \tau_{F, CO_2, H_2O} \) is the water vapor transmissivity in the carbon dioxide band.
Cloud effects in (46) are calculated by specifying a cloud transmissivity from cloud emissivity and reflectivity. Then we invert the expression for $\tau_{\text{F,CO}_2,\text{H}_2\text{O}}$ given below, to obtain an effective water vapor pathlength for the cloud $u_p(\text{cloud})$. Calculations with this $u_p(\text{cloud})$ then include the effect of the cloud on the carbon dioxide-water vapor overlap. 14 micron cloud optics are obtained from Yamamoto, Tanaka, and Asano (1970). Cloud infrared reflectivity is accounted for as in cases $F_{\downarrow B}(z)$ and $F_{\downarrow C}(z)$.

Sasamori (1968a) provides

$$A_{\text{CO}_2}(x) = 0.0546 \log(x) + 0.0581$$

$$A_{\text{CO}_2}(x) = 0.0461 \log(x) + 0.074$$

$$\tau_{\text{F,CO}_2,\text{H}_2\text{O}}(x) = 1.33 - 0.832(x + 0.0286)^{0.260}$$

$z_t$ in (46) is the level above which the pathlength changes only slightly. For $m>1$, $z_t$ will be considered to extend up through layer 1. It is noted that

$$T_{m+1/2} = \frac{T_m + T_{m+1}}{2}$$

where $T_{m+1/2}$ is the temperature at the bottom of layer $m$; $T_{m-1/2}$ is the temperature at the top. The first term on the right hand side of (46) is split:

$$T(z) = T_{m+1/2} T_{m-1/2} T_{m+1/2}$$

$$f \cdot \frac{1}{f} = f + f$$

$$T(z_t) = T_{1+1/2} T_{1+1/2} T_{m-1/2}$$
The second integral is approximated as

\[\frac{T_{m+1/2}}{4\sigma} \int_{T_{m-1/2}} A \tau T' \, dT' = \sigma \bar{A}_{\text{CO}_2} \left[ Y_p(m,m) \right] \tau_{F,\text{CO}_2,\text{H}_2\text{O}} \left[ U_p(m,m) \right] \]

\[\frac{T_{m-1/2}^3}{T_{m+1/2}^3 + T_{m-1/2}^3} \left( T_{m+1/2}^4 - T_{m-1/2}^4 \right)\]

and the first as

\[\frac{T_{m-1/2}}{4\sigma} \int_{T_{1+1/2}} A \tau T' \, dT' = \sigma \sum_{n=2}^{m-1} \left\{ \bar{A}_{\text{CO}_2} \left[ Y_p(m,n-1) \right] \tau_{F,\text{CO}_2,\text{H}_2\text{O}} \left[ U_p(m,n-1) \right] T_{n-1/2}^3 \right. \]

\[+ \bar{A}_{\text{CO}_2} \left[ Y(m,n) \right] \tau_{F,\text{CO}_2,\text{H}_2\text{O}} \left[ U_p(m,n-1) \right] T_{n+1/2}^3 \left\} \frac{T_{n+1/2}^4 - T_{n-1/2}^4}{T_{n-1/2}^3 - T_{n+1/2}^3} \right.\]

In the evaluation of both integrals we have factored out \( \bar{A}\tau \) after weighting by \( T' \, \tau \); we then integrated \( T' \, \tau \). The second integral is zero if \( m = 2 \). For \( m = 1 \) the second integral is zero again, and the first is evaluated assuming \( T_{m-1/2} = T_1 \). \( Y_p(m,j) \) is used to denote the carbon dioxide pathlength between layers \( m \) and \( j \), including \( m \) and \( j \).
Returning to (46), we express the second ($\sigma T^4$) term

\[ \sigma T_1^4 \bar{A}_{CO_2} \left[ Y_p(m, l) \right] \tau_{F, CO_2, H_2O} \left[ U_p(m, l) \right] . \]

At last we come to $F_{\text{E}}(z)$, the end term of (40). It accounts for ozone flux corrected for overlap with water vapor and aerosol, as

\[
F_{\text{E}}(z) = 4\sigma \int_{T(z_t)}^{T(z)} \bar{A}_{O_3} \left[ y_p(T) - y_p(T') \right] \tau_{Fw, 1050}(z, z') T^3 dT' + \sigma T^4(z_t) \bar{A}_{O_3} \left[ y_p(T) - y_p(T(z_t)) \right] \tau_{Fw, 1050}(z, z_t)
\]

where $y_p$ is the pressure corrected ozone pathlength.

$\tau_{Fw, 1050}$ is the aerosol and water vapor transmissivity at ozone wavelengths. A $\kappa$ of 0.4 has been used in correcting ozone pathlength (McClatchey et al., 1971).

Sasamori (1968a) specifies

\[
x > 0.01 \quad \bar{A}_{O_3}(x) = 0.0212 \log(x) + 0.0748
\]

\[
x < 0.01 \quad \bar{A}_{O_3}(x) = 0.209(x + 7 \times 10^{-5})^{0.436} - 0.00321
\]

\[
\bar{A}_{O_3}(x) = 0.0122 \log(x + 6.5 \times 10^{-4}) + 0.0385
\]

For $m>1$ the expression in (47) is evaluated as is (46) but with the $A$ and $\tau$ terms replaced by their ozone counterparts.

Equation (47) was found to yield unacceptable values for ozone fluxes in the top layer 1. This was especially true for (47)'s upward flux counterpart. The two gave a
divergence of infrared in layer 1 due to ozone. Ozone, however, causes infrared convergence in the stratosphere. Ramanathan and Grose (1978) show that ozone generally causes heating (convergence) in the infrared between 60N and 60S at 120 mb.

Equation (47) was originally developed by Sasamori (1968a) for use in the NCAR general circulation model. The GCM actually employs a somewhat modified form, however (Sasamori, 1968b). The ozone flux error became apparent when globally averaged versions of the model produced a stratospheric temperature that was too low. The error may have been due to (1) improper encoding, (2) an inadequate finite difference approximation to the Sasamori formulation, or (3) a slip-up in the original (1968a) scheme.

Equation (47) and its upward counterpart are then replaced by a simple emissivity formulation for layer 1 alone. This formulation yields infrared convergence due to ozone in the 9.6 micron band in layer 1. Quite simply

\[ F_{\varepsilon}(1) = \sigma \varepsilon [y_p(1), T_1] T_1^4 \]

where the emissivity \( \varepsilon \) is a function of ozone pathlength and temperature. It is given as a linear interpolation from the Staley and Jurica (1970) ozone emissivity tables at -70 C and -40 C as

\[ \varepsilon(x,T) = (233.16 - T)(0.01033x + 0.03333) \]

\[ + (T-203.16)(0.017x + 0.0524)/30 \]
APPENDIX J

UPGOING INFRARED FLUXES AND THEIR FINITE DIFFERENCE FORMS

The upgoing infrared flux at level \( z \) is expressed as

\[
F_{+}(z) = F_{+A}(z) - F_{+B}(z) + F_{+C}(z) + F_{+D}(z) + F_{+E}(z). 
\]

We now discuss and evaluate the individual terms.

\( F_{+A}(z) \) is the Rodgers water vapor flux. Its downward counterpart is (41), as

\[
F_{+A}(z) = \sigma T_{13}^4 \left[ 1 - \epsilon_1(0, z) \right] + \int_0^z \sigma T^4(z') \frac{d\epsilon_1(z', z)}{dz'} dz'.
\]

\( x < 0.001 \)

\[
\epsilon_1(x) = 12.292x^{1/2} - 1069.7x + 40483x^{3/2} - 504860x^2
\]

\( x > 0.001 \)

\[
\epsilon_1(x) = 0.60390 + 0.15134\ln(x) + 2.8234\times 10^{-2} \left\{ \ln(x) \right\}^2
+ 4.10631\times 10^{-3} \left\{ \ln(x) \right\}^3 + 2.38083\times 10^{-4} \left\{ \ln(x) \right\}^4
\]

\( F_{+A}(m) \) is the finite difference form, giving the upward flux at the top of layer \( m \) as

\[
F_{+A}(m) = \sigma T_{13}^4 \left\{ 1 - \epsilon_1 \left[ U_p(12, m) \right] \right\} + \sigma \epsilon_1 \left[ U_p(m, m) \right] T_m^4
+ \sum_{n=m+1}^{12} \frac{\sigma}{n} \left\{ \epsilon_1 \left[ U_p(n, m) \right] - \epsilon_1 \left[ U_p(n-1, m) \right] \right\}
\]

where the summation term is not evaluated for \( m=12 \).
The expression for cloudy skies is

\[ F_{\uparrow A}(z) = (1 - \varepsilon - r) F_{\uparrow A}(z) + \sigma T^4(\text{cloud}) \left[ 1 - \varepsilon_1(z, \text{cloud top}) \right] \]

\[ \text{cloud} + \varepsilon \int_{z} \sigma T^4(z') \, d\varepsilon_1(z', z)/dz' \, dz' \]

where \( T(\text{cloud}) \), \( \varepsilon \), and \( r \) refer to cloud parameters. \( F_{\uparrow A}(z) \) on the right is the flux for clear skies.

The second term in the upward flux equation is \( F_{\uparrow B}(z) \). This is the counterpart of (44), as

\[ F_{\uparrow B}(z) = \left[ 1 - \varepsilon_w(0, z) \right] \int_{750}^{1250} B_n(T_{13}) \, dn \]

\[ + \int_{z} \left[ \int_{750}^{1250} B_n(T) \, dn \right] d\varepsilon_w(z', z)/dz' \, dz' \]

With \( B(T) \) defined in the discussion of (44) previously, we evaluate the above as

\[ F_{\uparrow B}(m) = \{ 1 - \varepsilon_w[U_p(12, m)] \} B(T_{13}) + \varepsilon_w[U_p(m, m)] B(T_m) \]

\[ + \sum_{n=m+1}^{12} B(T_n) \{ \varepsilon_w[U_p(n, m)] - \varepsilon_w[U_p(n-1, m)] \} \]

where the summation is not evaluated for \( m = 12 \).

The component \( F_{\uparrow C}(z) \) of the upward flux is

\[ F_{\uparrow C}(z) = \int_{750}^{1250} B_n(T_{13}) \tau_{Fw, n}(0, z) \, dn + \int_{0}^{z} \int_{750}^{1250} B_n(z') \, d\tau_{Fw, n}(z', z) / dz' \, dn \, dz' \]
The finite difference form of the above is

\[
F_{+C}(m) = \sum_{n=800}^{1200} B_n(T_{13}) \tau_{Fw,n}(13,m)(100\text{cm}^{-1})
\]

\[
+ \sum_{n=800}^{1200} \sum_{j=m+1}^{12} B_n(T_j)[\tau_{Fw,n}(j-1,m) - \tau_{Fw,n}(j,m)](100\text{cm}^{-1})
\]

We now give \( F_{+D}(z) \) and its finite difference form as

\[
F_{+D}(z) = 4\sigma \int A_{CO_2} [y_p(T') - y_p(T)] \tau_{F,CO_2,\text{H}_2\text{O}} [u_p(T') - u_p(T)] T_1^3 dT'
\]

\[
F_{+D}(m) = \sigma \overline{A}_{CO_2} [y_p(m,m)] \tau_{F,CO_2,\text{H}_2\text{O}} [u_p(m,m)]
\]

\[
\left[ \frac{T_{m+1/2}^3}{T_{m+1/2}^3 + T_{m-1/2}^3} \right] (T_{m-1/2}^4 - T_{m+1/2}^4)
\]

\[
+ \sigma \sum_{n=m}^{11} (T_{n+1/2}^4 - T_{n+3/2}^4) \left\{ \overline{A}_{CO_2} [y_p(n,m)] \tau_{F,CO_2,\text{H}_2\text{O}} [u_p(n,m)]
\]

\[
\cdot T_{n+1/2}^3/(T_{n+1/2}^3 + T_{n-1/2}^3) + \overline{A}_{CO_2} [y_p(n+1,m)]
\]

\[
\cdot \tau_{F,CO_2,\text{H}_2\text{O}} [u_p(n+1,m)] T_{n+3/2}^3/(T_{n+3/2}^3 + T_{n+1/2}^3)
\]

where the summation is not evaluated for \( m=12 \).
$F_{+E}(z)$ is the last term in the upward flux. For $m\neq 1$, we have

$$F_{+E}(z) = 4\sigma \int T(z) \left[ \bar{A}_0 \left[ y_p(T') - y_p(T) \right] \tau_{Fw,1050}(z',z) T^{'} \right] \frac{dT'}{T_{13}}$$

The finite difference form is the same as $F_{+D}(z)$ but with $\bar{A}_{CO_2}$ replaced by $\bar{A}_{O_3}$ and $\tau_{F,CO_2,H_2O}$ replaced by $\tau_{Fw,1050}$. For $m=1$, the remarks in Appendix I apply. The flux is then

$$F_{+E}(1) = F_{+E}(2) \left[ 1 - \epsilon \left[ y_p(1,1),T_1 \right] \right] + \epsilon \left[ y_p(1,1),T_1 \right] o T_1^4.$$
Noting steps 1 to 7 outlined in the main text, the discussion here begins with step 2, the evaluation of effective emissivities for transmission \( u_{\varepsilon'} \), \( d_{\varepsilon'} \). These are inferred directly from the main infrared calculation (40):

\[
u_{\varepsilon'} = d_{\varepsilon'} = \varepsilon_A - \varepsilon_B + \varepsilon_C + \varepsilon_D + \varepsilon_E.
\]

In the following, pathlengths refer to individual layers, as

\[
\varepsilon_A' = \left[ \varepsilon_1(u_p) + \varepsilon_2(u_p) \right] / 2
\]

\[
\varepsilon_B' = \left[ \varepsilon_w(u_p) / \sigma T^4 \right] \sum_{800}^{1200} B_n(T)(100\text{cm}^{-1})
\]

\[
\varepsilon_C' = \left( \sum_{800}^{1200} B_n(T) [1 - \tau_{FW,n}(u_p)](100\text{cm}^{-1}) \right) / \sigma T^4
\]

\[
\varepsilon_D' = \bar{A}_{CO_2} [y_p(CO_2)]
\]

\[
\varepsilon_E' = \bar{A}_{O_3} [y_p(O_3)].
\]

\( d_{\varepsilon} \) and \( u_{\varepsilon} \) are calculated in step 3 as

\[
d_{\varepsilon_m} = \left[ F_+^{(m)} - (1 - d_{\varepsilon_m}) F_+^{(m-1)} \right] / \sigma T_m^4
\]

\[
u_{\varepsilon_m} = \left[ F_+^{(m)} - (1 - u_{\varepsilon_m}) F_+^{(m+1)} \right] / \sigma T_m^4
\]
where all terms on the right-hand side have been evaluated in the main infrared calculation by the methods of Appendices I and J.

In step 4, the energy balance equations (53-1) to (53-13) are written in terms of $T^4$ and then expressed in matrix form. This is done by first working out expressions for $F_\uparrow(m)$ and $F_\downarrow(m)$ in terms of $T^4$, as

$$F_\uparrow(m) = \frac{\sigma}{13} \sum_{n=12}^{m} (1-u_{n}'') + \sum_{n=12}^{m} \sigma T^4_n u_{n}'' \sum_{p=n-1}^{m} (1-u_{p}')$$

$$F_\downarrow(m) = \frac{\sigma}{d} \sum_{n=1}^{m-1} \sum_{p=n+1}^{m} (1-d_{p}')$$

$U_{m,j}$ is the coefficient of $T^4_j$ in $F_\uparrow(m)$, as

$$x_m = 1$$

$$U_{m,j} = u_j x_j^\sigma$$

$$x_{j+1} = x_j (1-u_j')$$

$$U_{m,j} = 0 \text{ if } j<m$$

and $D_{m,j}$ is the coefficient of $T^4_j$ in $F_\downarrow(m)$, as

$$y_m = 1$$

$$D_{m,j} = d_j y_j^\sigma$$

$$y_{j-1} = y_j (1-d_j')$$

$$D_{m,j} = 0 \text{ if } j>m$$
The infrared divergence of a layer may be written as

\[ F_m(T_{13}, \ldots, T_{13}) = F_1(m) - F_1(m-1) - F_1(m+1) + F_1(m) \]

\[ F_{13}(T_{13}, \ldots, T_{13}) = \sigma T_{13}^4 - F_1(12) \]

\[ F_1(T_{13}, \ldots, T_{13}) = F_1(1) - F_1(2) + F_1(1) \]

The coefficient of \( T_j^4 \) in the infrared divergence of layer \( m \) is \( A_m,j \):

\[ A_m,j = U_{m,j} - D_{m-1,j} - U_{m+1,j} + D_{m,j} \]

\[ A_{l,j} = U_{l,j} - U_{2,j} + D_{1,j} \]

\[ A_{13,j} = U_{13,j} - D_{12,j} \]

We can now express the energy balance Equations (53-1) to (53-13) in matrix form as

\[ A_{1,1}T_{1}^4 + \cdots + A_{1,11}T_{11}^4 + A_{1,12}T_{12}^4 + A_{1,13}T_{13}^4 = A_{1,14} \]

\[ \vdots \]

\[ A_{13,1}T_{1}^4 + \cdots + A_{13,11}T_{11}^4 + A_{13,12}T_{12}^4 + A_{13,13}T_{13}^4 = A_{13,14} \]

\[ A_{13,14} = SW_s - DO_s - ST_s \]

\[ A_{m,14} = SW_m - DH_m - ST_m \quad (m \neq 13) \]

The above system of equations is solved to give the radiative equilibrium temperatures \( T_{13}^4, \ldots, T_{13}^4 \).
In step 5, these temperatures are convectively adjusted. Using the lapse rate parameterization (note Appendix L), the temperature at level 12 may be expressed as

\[ T_{12} = T(p_{12}) = (p_{12}/p_0)^\kappa T_{13} - (p_{12}/p_{oo})^\kappa g_0(p_0 - p_{12}) \]

\[ p_0 = p_{oo} = 1000\text{mb} \]
\[ p_{12} = 975\text{mb} \].

We wish to express \( T_{12}^4 \) as a function of \( T_{13}^4 \), as above, and then resolve the * system of energy balance equations subject to this new constraint. We set

\[ T_{12}^4 = (p_{12}/p_0)^4\kappa \left[T_{13}^4 - 4T_{13}^3x + 6T_{13}^2x^2 - 4T_{13}x^3 + x^4 \right] \]

\[ x = g_0(p_0 - p_{12}) \].

Note that \( T_{13}^2, T_{13}^3, \) and \( T_{13}^4 \) in the above expression are fixed; they do not vary as does \( T_{13}^4 \), for which we solve in the next solution to the equations. We eliminate \( T_{12}^4 \) from the * system with the following sequence. For \( m=1,13 \), we replace

\[ A_{m,14} = A_{m,14} - A_{m,13}(p_{12}/p_{13})^{4\kappa} \left[-4T_{13}^3x + 6T_{13}^2x^2 - 4T_{13}x^3 + x^4 \right] \]

\[ A_{m,12} = A_{m,12}(p_{12}/p_{13})^{4\kappa} \]

\[ A_{m,12} = A_{m,12} + A_{m,13} \]
\[ A_{m,13} = A_{m,14} \]
and reduce the size of the matrix:

\[ A_{12,j} = A_{12,j} + A_{13,j} \quad j=1,13 \]

The result is a new system of equations:

\[ B_{m,j} = A_{m,j} \]

\[ B_{1,1}T_1^4 + \ldots + B_{1,11}T_{11}^4 + B_{1,12}T_{13}^4 = B_{1,13} \]

\[ \vdots \]

\[ B_{12,1}T_1^4 + \ldots + B_{12,11}T_{11}^4 + B_{12,12}T_{13}^4 = B_{12,13} \]

This system of twelve equations is solved for the twelve unknowns \( T_1^4, \ldots, T_{11}^4, T_{13}^4 \). \( T_{12} \) is then obtained from the lapse rate formulation. We iterate from the \( * \) system to this point eight times to bring \( T_{12} \) into adjustment with the other \( T_m \) and conserve energy.

Layers \( 11, 10, 9, \ldots \) are successively adjusted until a stable lapse rate results (step 6). With the final \( T_1, \ldots, T_{13} \) obtained, we iterate over the whole scheme (step 7).
APPENDIX L

LAPSE RATE PARAMETERIZATION

Model runs at specific latitude belts (as opposed to the entire globe) employ a lapse rate parameterization due to Rennick (1977). Rennick found that temperature aloft could be expressed as a function of pressure $p$, surface temperature $T_{13}$, and a term $g_o$. Her results compare well with observed lapse rates at different latitudes and seasons. The expressions are

\[
T(p) = \left(\frac{p}{p_0}\right)^\kappa T_{13} - \left(\frac{p}{p_\infty}\right)^\kappa g_o(p_\infty - p)
\]

\[
\kappa = \frac{R}{c_p}
\]

\[
p_0 = p_\infty = 1000\text{mb}
\]

Rennick has tabulated $g_o$ as a function of latitude for annually averaged conditions. This is used in the Annual model. In the Monthly model we employ Rennick's parameterization of $g_o$ as a function of $T$:

\[
g_o = (r_o + r_1 T_{13} + r_2 T_{13}^2 + r_3 T_{13}^3)g_s
\]

\[
r_o = 2.83600971 \times 10^3
\]

\[
r_1 = -2.92491718 \times 10^1
\]

\[
r_2 = 1.00660823 \times 10^{-1}
\]

\[
r_3 = -1.1554555 \times 10^{-4}
\]
where $g_s$ is the pseudoadiabatic lapse rate of potential
temperature $\vartheta$ and

\[
\frac{\vartheta}{\vartheta p} = (\vartheta/\vartheta T)(\vartheta T/\vartheta p) = \left[ \frac{\vartheta}{T} - \frac{(\vartheta/\vartheta T) \kappa (\vartheta p/\vartheta T)}{\vartheta T/\vartheta p} \right] \frac{\vartheta T/\vartheta p}{\vartheta p/\vartheta T} = \frac{(\vartheta T/\vartheta p)}{(\vartheta T/\vartheta z)(\vartheta z/\vartheta p)} - \frac{\kappa}{\vartheta T/\vartheta p}
\]

at 1000 mb

We may obtain the pseudoadiabatic lapse rate from a formula
in the Smithsonian Meteorological Tables, Smithsonian Institution (1958), as

\[
dT/d\phi = -Y/X \quad \text{deg K/gpm}
\]

\[
X = c_p + q(c_w + dL_v/dT - L_v/T + 1/k(dlne_w/dT)(RdT + qL_v + kL_v))
\]

\[
L_v = \left[ 2500 - 2.27(T-273.15) \right] \times 10^7 \text{ erg g}^{-1}
\]

\[
k = 0.62197
\]

\[
c_p = 1.005 \times 10^7 \text{ erg g}^{-1} \text{ K}^{-1}
\]

\[
q = \text{mixing ratio}
\]

\[
Y = 9.8 \times 10^6 (1+q)(RT + qL_v)/RT
\]

\[
dlne_w/dT = 1/T^2 \left[ 6790.5 - 5.02808T + (4916.8)10^{-0.0304T}T^2 
\right. 
\left. + (174209)10^{-1302.88/T} \right]
\]

\[
\vartheta T/\vartheta z = dT/d\phi \times 10^{-4}
\]

\[
\vartheta z/\vartheta p = -1/(\rho g) = -RT/pg
\]
APPENDIX M

CLIMATE CALCULATION

This appendix fills in the details of the Climate Calculation section explanation of the Monthly model.

1. A first guess of the composite land-sea surface temperature is made and used (Appendix E) to calculate the distribution of water vapor in the vertical. The water vapor is then fixed for the month's calculation. Albedo, cloud, quantity of gaseous absorbers, and meridional heat transport are specified from climatology or from a general circulation model output.

2. Solar wavelength radiative transfer is done separately for land and sea surface albedoes, and for conditions of clear, low, mid, and high cloud. Cloud-clear averaging is done with the methods of Appendix O, and the land and sea cases are averaged to obtain net solar absorption in each land-sea composite layer.

3. Latent heat storage in the atmosphere is calculated, based on the precipitable water obtained in 1 and on the precipitable water from the previous month.

4. Sensible heat storage in the atmosphere is calculated. In subsequent iterations, the sensible heat storage
is recomputed using a smoothly advanced temperature (for the atmospheric heat storage only), as

$$T_{\text{new}} = 0.85T_{\text{old}} + 0.15T_{\text{new}}$$

5. We compute heat storage in land and sea separately. Land temperature is obtained by differencing the calculated composite land-sea temperature and the (fixed) climatological difference between the land T and mean land-sea T. Sea temperature is obtained similarly.

With $T_{\text{annual}}$ as the annual temperature, $T$ as current temperature, and $T_p$ as previous month's temperature, surface heat storage is given by a finite difference approximation to (49), as

$$S_{T_S} = \left[\frac{\pi}{P}C\lambda\right]^{1/2} \left[ T - T_{\text{annual}} + \frac{6}{\pi}(T-T_p) \right].$$

6. Modified radiative convective equilibrium, including heat storage and meridional transport, is calculated in steps. We iterate on steps 2 through 7 in the Radiative Convective Equilibrium section.

With each iteration, we obtain a new vertical temperature distribution. The surface temperature, used to compute heat storage, is advanced with smoothing:

$$T_{\text{new}} = 0.8T_{\text{old}} + 0.2T_{\text{new}}.$$
It is further damped by a factor of one-half after the first iteration. In the earlier iterations, \( T_{\text{new}} \) is not permitted to depart from \( T_{\text{old}} \) by more than 7 degrees.

The new \( T \) is used to recompute heat storage steps 4 through 6 of the present section with each run of the Radiative Convective Equilibrium (2 to 7). Fourteen iterations are performed, after which the \( T \) used to calculate heat storage matches the output from the Radiative Convective Equilibrium by less than 0.01 degree. This is accepted as the new \( T \) for the month.
The incident angle of the sun is specified as a time-area average for each month. The average cosine of the solar zenith angle from noon to midnight $\mu_s$ multiplies the solar constant to give the incoming flux at the top of the atmosphere. The average cosine of the solar zenith angle from noon to sunset $\mu_o$ divides the optical depth in scattering and absorption calculations. $\mu_s$ and $\mu_o$ are taken from Newquist (1976) as

$$
\mu_o = \frac{1}{\pi} \left\{ \sin \phi_e \sin \phi_s \sin \left[ \cos^{-1} \left( \frac{-\cos \phi_e \cos \phi_S}{\sin \phi_e / \sin \phi_S} \right) \right] \\
+ \cos \phi_e \cos \phi_s \cos^{-1} \left( \frac{-\cos \phi_e \cos \phi_S}{\sin \phi_e / \sin \phi_S} \right) \right\}
$$

$$
\mu_s = \left\{ \sin \phi_e \sin \phi_s \sin \left[ \cos^{-1} \left( \frac{-\cos \phi_e \cos \phi_S}{\sin \phi_e / \sin \phi_S} \right) \right] \\
+ \cos \phi_e \cos \phi_s \cos^{-1} \left( \frac{-\cos \phi_e \cos \phi_S}{\sin \phi_e / \sin \phi_S} \right) \right\} \\
/ \left[ \cos^{-1} \left( \frac{-\cos \phi_e \cos \phi_S}{\sin \phi_e / \sin \phi_S} \right) \right]
$$

where $\phi_e$ is colatitude and $\phi_s$ is the codeclination angle.

The solar declination angle $\delta$ is assumed to vary as

$$
\delta = 23.5 \sin \left[ \frac{(\text{day}-80)360}{365} \right]
$$

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Because $\mu_s$ multiplies the solar constant, it is convenient to include in it a correction for the solar radius vector $R$. $R$ is evaluated from the American Ephemeris and Nautical Almanac, United States Naval Observatory (1956). On January 4, 1958, the vector was at extreme displacement .9832619; this is assumed to vary sinusoidally with date. An adjustment to $\mu_s$ is then made:

$$\mu_s \rightarrow \frac{\mu_s}{R^2}$$

$\mu_o$, $\mu_s$, $\delta$, and $R$ are evaluated 600 times during the year; each month has an average for these variables based on 50 values. (Each month has an equal number of days.) $\mu_o$ and $\mu_s$ are calculated for each degree of latitude in a ten degree belt. $\mu_o$ and $\mu_s$ are summed and cosine (area) averaged to yield the time-area average $\mu_o$ and $\mu_s$ for a belt in a given month.
APPENDIX O

CLOUD OVERLAPS

The model accounts for low, mid, and high clouds. Clouds often overlap (two or three cloud levels present at the same time). The eight possible permutations of cloud/clear conditions are shown in Fig. 39. Cloud amounts for radiation calculations are specified as fractions of low CL, mid CM, and high CH from Hoyt's (1976, model B) seasonal, latitudinal climatology.

With each run of the model, radiative transfer is calculated for cases 1 through 4. Case 5 is then assigned fluxes in each layer as one-half those of case 2 and one-half those of case 3. Case 8 is assigned as one-third each of cases 2, 3, and 4. Total flux at each layer in the system is then obtained by weighting the flux for each case by the number beneath it, which is just the probability of occurrence for the case. The probability of case 1 is just one minus the sum of the other probabilities.

This procedure is less physically realistic than fully computing radiative transfer for each of the eight cases, but it reduces computational time by a factor of two.
Fig. 39. Cloud Scheme
APPENDIX P

OPTICAL PROPERTIES OF AEROSOL
AND RAYLEIGH SCATTERERS

Here we catalog wavelength dependent properties of Rayleigh scatterers and aerosol. These are given at wavelength intervals $L=1$ to $L=6$, which cover the entire solar spectrum, and $L=7$ to $L=10$, which cover the auxiliary CO$_2$ wavelengths. We catalog the energy $E$ contained within the interval $L$ as a fraction of that in the whole solar spectrum, the Rayleigh optical depth $\tau$, the Toon and Pollack (1976) aerosol function $Q_{\text{ext}}$, the aerosol asymmetry factor $g$, and $(1-\omega_0)$ for aerosol single scattering albedo $\omega_0$.

$L=1$ 0.0 to 0.3400 micron (O$_3$ absorbing)
$L=2$ 0.340 to 0.480 micron
$L=3$ 0.480 to 0.720 micron (O$_3$ absorbing)
$L=4$ 0.720 to 0.926 micron
$L=5$ > 0.926 micron (H$_2$O nonabsorbing)
$L=6$ > 0.926 micron (H$_2$O absorbing)
$L=7$ 1.277 to 1.575 micron (CO$_2$ absorbing)
$L=8$ 1.575 to 1.670 micron (CO$_2$ absorbing)
$L=9$ 1.670 to 2.110 micron (CO$_2$ absorbing)
$L=10$ 2.199 to 2.990 micron (CO$_2$ absorbing)
Parameters are listed as X(L):

<table>
<thead>
<tr>
<th>X(1)</th>
<th>X(2)</th>
<th>X(3)</th>
<th>X(4)</th>
<th>X(5)</th>
<th>X(6)</th>
<th>X(7)</th>
<th>X(8)</th>
<th>X(9)</th>
<th>X(10)</th>
</tr>
</thead>
</table>

Energy E contained within wavelength interval

|           | 0.0386132 | 0.167457 | 0.305105 | 0.160655 | 0.0593183 | 0.268851 | 0.067675 | 0.01417 | 0.04132 | 0.025569 |

Rayleigh optical depth $\tau$

|          | 10.93    | 3.226    | 0.8503   | 0.2174   | 0.0     | 0.0    | 0.0    | 0.0    | 0.0    | 0.0      |

(The method of calculating the Rayleigh optical depth is given at the end of this section.)

$q_{ext}$, lower tropospheric aerosol

|          | 1.08     | 1.04     | 0.989    | 0.943    | 0.824   | 0.829  | 0.8409 | 0.8068 | 0.7273 | 0.6818   |

$q_{ext}$, upper tropospheric aerosol

|          | 1.38     | 1.17     | 0.931    | 0.776    | 0.568   | 0.527  | 0.5172 | 0.4414 | 0.4     | 0.3397   |

$q_{ext}$, stratospheric aerosol

|          | 2.2      | 1.6      | 1.0      | 0.7      | 0.36    | 0.18   | 0.26   | 0.18   | 0.10    | 0.005    |

Asymmetry factor $g$, lower tropospheric aerosol

|          | 0.714    | 0.71     | 0.709    | 0.704    | 0.701   | 0.702  | 0.7    | 0.7    | 0.7     | 0.7      |
Asymmetry factor $g$, upper tropospheric aerosol

\[
\begin{array}{cccccc}
0.654 & 0.654 & 0.654 & 0.658 & 0.666 \\
0.667 & 0.666 & 0.666 & 0.666 & 0.666 \\
\end{array}
\]

Asymmetry factor $g$, stratospheric aerosol

\[
\begin{array}{cccc}
0.7 & 0.7 & 0.7 & 0.67 & 0.54 \\
0.53 & 0.55 & 0.52 & 0.5 & 0.4 \\
\end{array}
\]

$(1 - \omega_0)$, lower tropospheric aerosol

\[
\begin{array}{cccccc}
0.0175 & 0.0130 & 0.0122 & 0.0136 & 0.0242 \\
0.0264 & 0.01435 & 0.01655 & 0.01605 & 0.0234 \\
\end{array}
\]

$(1 - \omega_0)$, upper tropospheric aerosol

\[
\begin{array}{cccccc}
0.0076 & 0.0068 & 0.0072 & 0.008 & 0.0294 \\
0.0190 & 0.0096 & 0.0116 & 0.012 & 0.0132 \\
\end{array}
\]

$(1 - \omega_0)$, stratospheric aerosol

\[
\begin{array}{cccccc}
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.075 \\
0.0620 & 0.0 & 0.002 & 0.01 & 0.2 \\
\end{array}
\]

The Rayleigh optical depth $\tau_L$ in the wavelength intervals $L=1$ to $L=4$ is calculated with the weighting scheme

\[
\tau_L = \left[ \sum \Delta E_\lambda \tau_{\text{ref}}(\lambda^{-4}/\lambda_{\text{ref}}^{-4}) \right] / \left( \sum \Delta E_\lambda \right)
\]

where $\Delta E_\lambda$ is the solar energy in the subinterval grouped about wavelength $\lambda$, and ref denotes the reference wavelength of 0.55 micron. It is noted that the optical depth in the
Rayleigh region is proportional to the inverse fourth power of the wavelength. The summations are over the wavelength limits of the interval, as 0.34 to 0.48 micron for \( L=2 \). Below 0.6 micron, the summation increment is .005 micron, and above 0.6 micron, the increment is 0.01 micron. The Rayleigh optical depth of the atmosphere, which is 0.145 at 0.55 micron, is negligible above 0.926 micron.

We now catalog aerosol infrared properties. These are cited as \( k_n = (1 - \omega_0, n) \tau_n \), where \( \omega_0, n \) and \( \tau_n \) are, respectively, the single scattering albedo and optical depth parameter at wavenumber \( n \). The following \( k_n \) are for lower tropospheric, upper tropospheric, and stratospheric aerosol:

<table>
<thead>
<tr>
<th>Wavenumber (cm(^{-1}))</th>
<th>800</th>
<th>900</th>
<th>1000</th>
<th>1100</th>
<th>1200</th>
</tr>
</thead>
<tbody>
<tr>
<td>lower troposphere</td>
<td>.1256</td>
<td>.2027</td>
<td>.2482</td>
<td>.4494</td>
<td>.1818</td>
</tr>
<tr>
<td>upper troposphere</td>
<td>.0683</td>
<td>.1075</td>
<td>.1828</td>
<td>.4655</td>
<td>.1267</td>
</tr>
<tr>
<td>stratosphere</td>
<td>.055</td>
<td>.065</td>
<td>.090</td>
<td>.200</td>
<td>.200</td>
</tr>
</tbody>
</table>

\( \tau_n \) above is a reference number, giving optical depth at wavenumber \( n \) relative to optical depth at 0.55 micron. The effective optical depth at wavenumber \( n \) is obtained by multiplying \( k_n \) by the optical depth at 0.55 micron. \( \tau_n \) is obtained by noting the equivalent wavelength \( \lambda = 1/n \) of wavenumber \( n \) and then manipulating equation (30) as

\[
\tau_n = \frac{\tau_\lambda}{\tau_{\lambda_{\text{ref}}}} = \frac{Q_{\text{ext}}(\lambda)}{Q_{\text{ext}}(\lambda_{\text{ref}})}
\]

All of the aerosol optical properties referenced in this appendix are from the Toon and Pollack (1976) model.
LIST OF REFERENCES


Hänel, G., 1976: The properties of atmospheric aerosol particles as functions of the relative humidity at thermodynamic equilibrium with the surrounding moist air. Advances in Geophysics, 19, 74-188.


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