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MOMENT ROTATION CURVES FOR
TOP AND SEAT CONNECTIONS

by
Ismat A. A. Abul-Hamayel

A Thesis Submitted to the Faculty of the
DEPARTMENT OF CIVIL ENGINEERING AND ENGINEERING MECHANICS

In Partial Fulfillment of the Requirements
For the Degree of
MASTER OF SCIENCE
WITH A MAJOR IN CIVIL ENGINEERING

In the Graduate College
THE UNIVERSITY OF ARIZONA

1 9 8 5
STATEMENT BY AUTHOR

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This thesis has been approved on the date shown below:

R. M. RICHARD
Professor of Civil Engineering and Engineering Mechanics

Date
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ABSTRACT

Analytical moment rotation characteristics of bolted top and seat connections are presented. The Richard Equation is used to give the force deformation response of these connections for static loading.

Different gage lengths, angle thicknesses, and beam depth are considered. Comparisons with full-scale test results are made. Recommendations and observations are made for the effect of changing the connection parameters on the behavior of these connections.
CHAPTER 1

INTRODUCTION

Connections are very important in a steel frame, and although they occupy a very small space in the structure, they are the most expensive element to design and fabricate.

Predicting the behavior of a steel connection is an important consideration for the safety and economy of the structure. Full-scale tests of connections are very expensive and difficult to conduct. Developing a set of moment rotation curves from full-scale tests is not an easy task. Because there are many variables that can be changed in a connection, it is not practical to make a physical test for every case that may be used. Therefore, an analytical method that predicts the behavior of a connection is desirable.

Many efforts have been made to derive an expression to predict the moment rotation response of steel connections. An expression that is popular was developed by Frye and Morris [1].

In this study the moment rotation response of top and seat angle connections under static loading is presented using the Richard Equation. A connection similar to those
being investigated is shown in Figure 1. Comparisons between the analytical method, the Frye and Morris formulas, and full-scale tests results are made.

The connection parameters that affect the response of these connections were changed. These connection parameters are: the gage length, angle thickness, and beam depth. The length of the top angle is held constant.

A computer program was written to calculate all the data required to plot the moment rotation curves which are plotted using the program "XYPLOT," written by George Williams, Research Associate, Department of Civil Engineering and Engineering Mechanics, the University of Arizona.

Beam analyses using the developed moment rotation curves are presented; recommendations and observations concerning the use of the Richard Equation in predicting connection response are also made.
Figure 1. Top and seat connection.
CHAPTER 2

BACKGROUND

The American Institute of Steel Construction specifications divides connections into three types. Type 1 is a rigid connection in which the angle between the intersecting members is essentially unchanged. This connection is capable of resisting 90% or more of the fixed end beam moment. Type 2 is called a flexible connection which is designed to carry the end shear, and generally resist 20% or less of the fixed end beam moment. This type allows essentially free rotation between the joined members. Type 3 is a semi-rigid connection which is somewhere between the rigid and the flexible connections in resisting moment. The moment that can be transferred is between 20-90% of the fixed end beam moment. The moment rotation characteristics of each of the three types of connections, according to AISC classification [2], are shown in Figure 2.

Due to the difficulty of determining the degree of restraint, and therefore the moment that can be transferred, Type 3 connections generally have not been used. In order to determine the degree of restraint of this kind of connection, a dependable moment rotation curve must be
Figure 2. AISC connection types.
available. Developing an analytical procedure to find such a moment rotation curve is the primary objective of this study.

**Beam Line Theory**

To better understand connection types, the beam line theory is introduced here.

To take advantage of the moment developed by a semi-rigid connection, the magnitude of the moment must be determined. Consider the simply supported, uniformly loaded beam with superimposed end moments, as shown in Figure 3. The end rotation, $\theta$, for the beam can be shown to be

$$\theta = \frac{wL^3}{24EI} - \frac{M_s L}{2EI}$$

For a given beam and loading, the above equation is a straight line, that is, $\theta$ is a linear function of the superimposed moment. The above equation is called the beam line equation. There are two cases of interest in this equation. First, if the superimposed moments are zero, then the end rotation is that of a "simply supported" beam. Second, if the end rotation is zero, then the applied end moment is that of "fixed end" moment. These two cases are shown in Figure 4.

Consider a connection that has a given moment rotation relationship such as the one shown in Figure 5. If the moment rotation relationship for the connection and the
Figure 3. Simply supported beam with superimposed end moments.

Figure 4. Moment-rotation relationship for a beam shown in Figure 3.
Figure 5. A typical moment-rotation relationship for connection.
beam are superimposed, there is a point at which the moment and rotation coincide, which is the intersection point as shown in Figure 6.
Figure 6. Moment-rotation relationships for beam and connection superimposed.
CHAPTER 3

GENERATION OF MOMENT ROTATION CURVES

In this study a nonlinear moment rotation response for top and seat connections is generated for different connection parameters. This nonlinear response is mathematically presented using the Richard Equation.

First the load deformation response of a top and seat connection is found from laboratory tests and represented by the Richard Equation. Then the moment rotation response is found. This can be obtained by selecting the four parameters that uniquely describe each curve. These four parameters are: the elastic stiffness \(K\), the plastic stiffness \(K_p\), the reference load \(R_0\), and the Richard parameter \(n\) which describes the sharpness of the transition between the two linear portions of the curve.

Shown in Figure 7 are some of the possible load deformation responses; representations of the different possible responses are given.

The Richard Equation, which is used in this study, is shown below.

\[
R = \frac{(K - K_p)\Delta}{1 + \left| \frac{(K - K_p)\Delta}{R_0} n^{1/n} + K_p \right|} \quad (2)
\]
The Richard Equation and some possible load-deformation curves.

\[ R = \frac{(K-K_p) \Delta}{\left(1 + \left|\frac{(K-K_p) \Delta}{R_0}\right|^{n}\right)^{1/n}} + K_p \Delta \]

- \( K \) = Elastic Stiffness
- \( K_p \) = Plastic Stiffness
- \( R_0 \) = Reference Load
- \( n \) = Curve Shape Parameter

- \( K = 0 \), \( K_p > 0 \) (Gap Element)
- \( K > K_p > 0 \) (Strain Hardening)
- \( K_p > K > 0 \) (Strain Stiffening)
- \( K_p < 0 < K \) (Strain Softening)
where

\[ K = \text{elastic stiffness} \]
\[ K_p = \text{plastic stiffness} \]
\[ R_0 = \text{reference load} \]
\[ n = \text{curve shape parameter} \]
\[ \Delta = \text{displacement} \]
\[ R = \text{the force at each bolt} \]

The displacement, \( \Delta \), defined as the distance above the point of rotation times the rotation, is determined for a given rotation. The point of rotation is the point of contact between the beam and the seat angle, as shown in Figure 1.

A computer program "Steel.C" was written by the author to find the moment of top and seat connections for a given rotation. The program uses the Richard equation, as mentioned above, to solve for the forces, and then the moments. The input data for the program is the plastic stiffness (\( K_p \)) and the shape parameter (\( n \)) calculated from the report by Blewitt and Richard [3]. A summary of the results obtained by Blewitt and Richard is presented in Appendix A. Also the depth of the beam, the gage length, the thickness of the angle, and the length of the angle are entered as input data. These input data are required for the top angle and web angle (if present). The elastic stiffness (\( K \)) and the reference load (\( R_0 \)) are calculated
according to the gage length and angle thickness as given in the report by Blewitt and Richard [3], and summarized in Appendix A.

The force is then found for each bolt in the web angle and for the top angle. Then the moment is found as the summation of the bolt heights times the bolt forces. This procedure is entered in a loop and repeated for increments of the rotation, which gives a set of data points (rotations, moments) which represents the initial moment rotation curve.

An initial curve is then drawn through the initial set of data points. Next a Richard curve fit for these data points is plotted. The initial and final slopes of the curve are determined, and also the intercept of the final slope with the y-axis is found. Then the parameter (n) which defines the sharpness of the curve is determined as demonstrated in Reference [4]. The procedure to determine these four parameters that describe each curve is summarized in Appendix B. With these four parameters, a Richard curve fit may be drawn through these points to give the final moment rotation curve, which represents the nonlinear moment rotation response of the top and seat connection. To plot the initial and final moment rotation curves, the program "XYPLOT" was used. This program was written by George Williams, Research Associate, Department of Civil Engineering and Engineering Mechanics, the University of Arizona.
CHAPTER 4

COMPARISON WITH FULL-SCALE TESTING

In order to check the results of the analytical procedure, comparisons with data available from full-scale testing are made. Tests made by Rathbun in 1935 [5], Hechtman and Johnston in 1947 [6], and by Azizinamini, Bradburn and Radziminski in 1985 [7] were used. Results from these tests and analytical results that are generated by the use of the Richard Equation are shown in the same graph. Results from the formulas by Frye and Morris, introduced in 1975 [1], are also included in the same graphs for the top and seat connections. When examining these curves, the performance of the Richard Equation against both the full-scale tests and the expression by Frye and Morris can be seen.

Frye and Morris developed nondimensional formulas to predict the moment rotation response of seven types of connections [1]. One of the connections that they developed a moment rotation expression for was the top and seat angle connection, shown in Figure 1. The formula is

\[
\theta = 8.46(KM) \times 10^{-4} + 1.01(KM)^3 \times 10^{-4} + 1.24(KM)^5 \times 10^{-8}
\]

(3)
where

\[ \theta = \text{connection rotation} \]
\[ M = \text{moment} \]
\[ k = \text{standardization constant defined as} \]
\[ K = t^{-0.5} \times d^{-1.5} \times f^{-1.1} \times L^{-0.7} \]  \hspace{1cm} (4)

where

\[ t = \text{thickness} \]
\[ d = \text{depth of the beam} \]
\[ f = \text{diameter of the fastener} \]
\[ L = \text{length of the top angle} \]

Rathbun did his tests in the Material Testing Laboratory of the School of Technology of the College of the City of New York. The connections that he used were riveted (bolts were generally not used at that time). He tested three different types of connections. Two types are compared with the analytical prediction that is generated by the Richard Equation. The first type is top and seat connection, as shown earlier in Figure 1. The second type is top and seat connection with a web angle, as shown in Figure 8.

In all his tests, a 12-inch beam having a 3/8-inch thick top angle with a 2-1/2 inch gage was used. For the first type of connection, the only difference in the three tests was the length of the top angle; they were 6, 8, and
Figure 8. Top and seat connection with double web angle.
14 inches. For the second type of connection, a 3/8-inch thick web angle with three rivets with a 2-1/4 inch gage were used. The lengths of the top angles were 9 and 14 inches.

For all the curves shown, the symbols used are as follows:

(Δ) = experimental
(—) = Richard equation (analytical)
(--) = Frye and Morris (analytical)

Figures 9 through 13 contain the results of tests made by Rathbun, along with the analytical predictions. Figures 9 through 11 are for the first type of connection. Figures 9 and 10 do not show good agreement between the analytical prediction and experimental results, nor results from the Frye and Morris expression. Good agreement between the Richard analytical prediction and experimental results is shown in Figure 11. In all three figures, the divergence of the results by Frye and Morris's expression at high rotations is very noticeable. Figures 12 and 13 are for the second type of connection. It is shown in those figures that the prediction by the analytical method agrees very well with the experimental results.

Hechtman and Johnston did their tests at the Fritz Engineering Laboratory of Lehigh University. They tested riveted top and seat connections in all of their tests, as
Figure 9. Comparison with test 8 by Rathbun.

- $R_s = 190$
- $N = 2.10$
- $K = 61100$
- $K_p = 2600$
Figure 10. Comparison with test 9 by Rathbun.

- $R_b = 260$
- $N = 2.30$
- $K = 80000$
- $K_p = 3310$
Figure 11. Comparison with test 10 by Rathbun.

\[ \begin{align*}
R_o &= 445. \\
N &= 2.20 \\
K &= 133500. \\
K_p &= 6250. 
\end{align*} \]
Figure 12. Comparison with test 11 by Rathbun.

$R_o = 540.$
$N = 2.70$
$K = 146700.$
$K_p = 0.000.$
Figure 13. Comparison with test 12 by Rathbun.

\[ R_a = 700. \]
\[ N = 1.90 \]
\[ K = 230690. \]
\[ K_F = 12500. \]
shown in Figure 1. Eight of Hechtman and Johnston's tests are compared with the analytical prediction that is generated by the Richard Equation. A gage of 2-1/2 inches was used for all the specimens. For specimens 5 and 16, a 1/2-inch thick top angle was used; as for the other specimens, 5/8 inch thick angle was used. The beam depth varied from 12 to 18 inches, while the length of the top angle varied from 6-3/4 to 12-1/2 inches. Table 1 shows the test specimens numbers as they appear in the report by Hechtman and Johnston [6], along with the geometry of the specimens.

Shown in Figures 14 through 21 are the experimental results of the tests made by Hechtman and Johnston along with results generated by the analytical method and Frye and Morris formulas. Shown in Figures 15 and 17 are good agreement between the experimental data and the analytical prediction. These two tests were the only tests by Hechtman and Johnston in which a top angle thickness of 1/2 inch was used. In Figures 14, 16, 20 and 21, good agreement is also shown between the experimental results and the analytical results generated by the Richard Equation. Figures 18 and 19 show that the analytical results generated by the Richard Equation do not agree well with the experimental data. All the specimens used in these tests were riveted.

Azizinamini, Bradburn and Radziminiski made their tests in the Structures Laboratory of the Department of
Table 1. Geometry of the specimens used by Hechtman and Johnston (1947).

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Gage Length (inches)</th>
<th>Angle Thickness (inches)</th>
<th>Top Angle Length (inches)</th>
<th>Beam Depth (inches)</th>
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<td>2-1/2</td>
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<td>2-1/2</td>
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<td>2-1/2</td>
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<td>2-1/2</td>
<td>5/8</td>
<td>8-1/2</td>
<td>12</td>
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</table>
Figure 14. Comparison with test 2 by Hechtman and Johnston.

\[ R_o = 400. \]
\[ N = 2.00 \]
\[ K = 127380. \]
\[ K_p = 6875. \]
Figure 15. Comparison with test 5 by Hechtman and Johnston.

\[ R_0 = 880. \]
\[ N = 1.90 \]
\[ K = 280000. \]
\[ K_p = 17500. \]
Figure 16. Comparison with test 9 by Hechtman and Johnston.

\[ R_0 = 1130. \]
\[ N = 2.30 \]
\[ K = 387600. \]
\[ K_P = 24000. \]
Figure 17. Comparison with test 16 by Hechtman and Johnston.
Figure 18. Comparison with test 20 by Hechtman and Johnston.

\[ R_e = 910. \]
\[ N = 1.60 \]
\[ K = 338660. \]
\[ K_p = 14300. \]
Figure 19. Comparison with test 22 by Hechtman and Johnston.

\[ R_e = 1000. \]
\[ N = 1.95 \]
\[ K = 365710. \]
\[ K_p = 19000. \]
Figure 20. Comparison with test 24 by Hechtman and Johnston.

\[ R_e = 1259 \]
\[ N = 1.70 \]
\[ K = 480000 \]
\[ K_p = 19000 \]
Figure 21. Comparison with test 35 by Hechtman and Johnston.

R = 488
N = 1.65
K = 180000
K_p = 9500
Civil Engineering at the University of South Carolina. These connections were bolted top and seat connections with double web angles, as shown in Figure 8. Six tests are chosen from this study to be compared to the results predicted by the analytical method.

In all these tests, a 14-inch beam was used. For the top angles, the gage length was 2-1/2 inches; the angle thickness varied from 3/8-5/8 inches, the angle length was 8 inches. For the web angles, the gage length was 2-19/32 inches, the angle thickness varied from 1/4-3/8 inches, and the length of the angle was 8-1/2 inches. Tabulated in Table 2 are the labels of the tests and their geometry as they appear in the report by Azizinamini et al. [7]. Although 3/4- and 7/8-inch bolts were used in these connections, the size of the bolt is not a consideration in the Richard Equation.

Shown in Figures 22 through 27 are the results of tests made by Azizinamini et al. along with the analytical prediction.

Shown in Figures 23 and 24 are that the results from the analytical method agree very well with the results of the tests. The results in Figures 22, 25 and 26, however, are not good. Shown in Figure 27 is a poor prediction when the results of the analytical method and results of the test are compared. The connection shown in Figure 27 has a
Table 2. Geometry of the specimens used by Azizinamini et al. (1985).

<table>
<thead>
<tr>
<th>Test</th>
<th>Beam Depth (inches)</th>
<th>Length (inches)</th>
<th>Thickness (inches)</th>
<th>Gage Length (inches)</th>
<th>Gage Thickness (inches)</th>
<th>Gage (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14S1</td>
<td>14</td>
<td>8</td>
<td>3/8</td>
<td>2-1/2</td>
<td>8-1/2</td>
<td>1/4</td>
</tr>
<tr>
<td>14S2</td>
<td>14</td>
<td>8</td>
<td>1/2</td>
<td>2-1/2</td>
<td>8-1/2</td>
<td>1/4</td>
</tr>
<tr>
<td>14S4</td>
<td>14</td>
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<td>3/8</td>
<td>2-1/2</td>
<td>8-1/2</td>
<td>3/8</td>
</tr>
<tr>
<td>14S5</td>
<td>14</td>
<td>8</td>
<td>3/8</td>
<td>2-1/2</td>
<td>8-1/2</td>
<td>1/4</td>
</tr>
<tr>
<td>14S6</td>
<td>14</td>
<td>8</td>
<td>1/2</td>
<td>2-1/2</td>
<td>8-1/2</td>
<td>1/4</td>
</tr>
<tr>
<td>14S8</td>
<td>14</td>
<td>8</td>
<td>5/8</td>
<td>2-1/2</td>
<td>8-1/2</td>
<td>1/4</td>
</tr>
</tbody>
</table>
Figure 22. Comparison with test 14S1 by Azizinamini et al.
Figure 23. Comparison with test 14S2 by Azizinamini et al.

\[ R_0 = 750. \]
\[ N = 1.80 \]
\[ K = 231000. \]
\[ K_p = 10000. \]
Figure 24. Comparison with test 14S4 by Azizinamini et al.

\[
\begin{align*}
R_0 &= 600. \\
N &= 1.90 \\
K &= 187000. \\
K_p &= 8500.
\end{align*}
\]
Figure 25. Comparison with test 14S5 by Azizinamini et al.

\[ R_0 = 418. \]
\[ N = 1.50 \]
\[ K = 147000. \]
\[ K_P = 5625. \]
Figure 26. Comparison with test 14S6 by Azizinamini et al.

\[ R_0 = 540. \]
\[ N = 1.40 \]
\[ K = 216000. \]
\[ K_F = 9250. \]
Figure 27. Comparison with test 14S8 by Azizinamini et al.

\( R_0 = 800 \).

\( N = 1.70 \).

\( K = 304000 \).

\( K_p = 10000 \).
5/8-inch thick top angle. Apparently this is the source of the error for the analytical prediction.

The "best fit" parameters for the Richard curve for all the curves which are produced for comparison with the experimental data are shown on each plot, and they are also tabulated in Tables 3 through 5 for easy reference when the reproduction of any curve is desired.

In general, for all the three groups of tests, the Richard Equation demonstrated that it can predict the moment rotation response of top and seat connections and also gives good results when compared to both the experimental data and Frye and Morris formulas. It seems that when the top angle thickness is 5/8 inch, the moment rotation response predicted by the Richard Equation does not agree well with the experimental data. This may be due to the extrapolation used to calculate the curve shape parameter (n) and the plastic stiffness (Kp) for the 5/8 inch thick angle. It is also noted that the results from the expression by Frye and Morris diverge at higher rotations.

The analytical prediction generated by the Richard Equation either agrees very well with the results obtained experimentally or gives lower values. This puts the prediction of the Richard Equation on the safe side, and allows us to utilize the moment provided by a connection in the design process. After all, the accuracy of these test
Table 3. Richard curve parameters for moment rotation curves for comparison with tests by Rathbun (1935).

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$K$ (k.in./rad)</th>
<th>$K_p$ (k.in./rad)</th>
<th>$M_o$ (k.in)</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>61,100</td>
<td>2,600</td>
<td>190</td>
<td>2.10</td>
</tr>
<tr>
<td>9</td>
<td>80,000</td>
<td>3,310</td>
<td>260</td>
<td>2.30</td>
</tr>
<tr>
<td>10</td>
<td>133,500</td>
<td>6,250</td>
<td>445</td>
<td>2.20</td>
</tr>
<tr>
<td>11</td>
<td>146,700</td>
<td>10,000</td>
<td>540</td>
<td>2.70</td>
</tr>
<tr>
<td>12</td>
<td>230,690</td>
<td>12,500</td>
<td>700</td>
<td>1.90</td>
</tr>
</tbody>
</table>
Table 4. Richard curve parameters for moment rotation curves for comparison with tests by Hechtman and Johnston (1947).

<table>
<thead>
<tr>
<th>Test No.</th>
<th>( K ) (k.in/rad)</th>
<th>( K_p ) (k.in/rad)</th>
<th>( Mo ) (k.in)</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
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<td>400</td>
<td>2.00</td>
</tr>
<tr>
<td>5</td>
<td>280,000</td>
<td>17,500</td>
<td>880</td>
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</tr>
<tr>
<td>9</td>
<td>387,600</td>
<td>24,000</td>
<td>1,130</td>
<td>2.30</td>
</tr>
<tr>
<td>16</td>
<td>100,000</td>
<td>5,000</td>
<td>315</td>
<td>1.60</td>
</tr>
<tr>
<td>20</td>
<td>338,660</td>
<td>14,300</td>
<td>910</td>
<td>1.60</td>
</tr>
<tr>
<td>22</td>
<td>365,710</td>
<td>19,000</td>
<td>1,000</td>
<td>1.95</td>
</tr>
<tr>
<td>24</td>
<td>480,000</td>
<td>19,000</td>
<td>1,259</td>
<td>1.70</td>
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<tr>
<td>35</td>
<td>180,000</td>
<td>9,500</td>
<td>488</td>
<td>1.65</td>
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</table>
Table 5. Richard curve parameters for moment rotation curves for comparison with tests by Azizinamini et al. (1985).

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$K$ (k.in./rad)</th>
<th>$K_p$ (k.in.$^2$/rad)</th>
<th>$M_o$ (k.in.)</th>
<th>$n$</th>
</tr>
</thead>
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<tr>
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<td>5,000</td>
<td>420</td>
<td>2.10</td>
</tr>
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<td>14S2</td>
<td>231,000</td>
<td>10,000</td>
<td>750</td>
<td>1.80</td>
</tr>
<tr>
<td>14S4</td>
<td>187,000</td>
<td>8,500</td>
<td>600</td>
<td>1.90</td>
</tr>
<tr>
<td>14S5</td>
<td>147,000</td>
<td>5,625</td>
<td>418</td>
<td>1.50</td>
</tr>
<tr>
<td>14S6</td>
<td>216,000</td>
<td>9,250</td>
<td>540</td>
<td>1.40</td>
</tr>
<tr>
<td>14S8</td>
<td>304,000</td>
<td>10,000</td>
<td>800</td>
<td>1.70</td>
</tr>
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</table>
results is not better than the results obtained from the Richard Equation, and that is due to the difficulties in collecting the data in the laboratory.

More investigations need to be done for the 5/8 angle thickness, when the Richard Equation is used to predict the moment rotation response of top and seat connection for static loading.
CHAPTER 5

ANALYSIS OF RESULTS

After developing the Richard Equation and how it predicts the moment rotation response, a routine to obtain moment rotation curves for top and seat connection was developed. Gage lengths of 2.5 and 3 inches were used. Top angle thickness of 1/4, 3/8 and 1/2 inches were used. Then the beam depth was increased from 10 to 24 inches by 2-inch increments. The length of the top angle was held constant at 6 inches. To get moment values for any length of the top angle other than the 6 inches specified, the moment must be multiplied by a factor, which is the actual length in inches divided by 6 inches.

Moment rotation curves for top and seat connections, generated by the Richard Equation, are shown in Appendix C of this report. The Richard curve parameters for "best fit" are printed on each curve, and they are also tabulated in Table 6. All these responses are for a top angle length of 6 inches.

By examining these curves, many observations can be made with regard to the behavior of top and seat connections. Some of the observations made about the behavior of
Table 6. Richard curve parameters for moment rotation curves.

<table>
<thead>
<tr>
<th>Beam Depth (in.)</th>
<th>Gage Length (in.)</th>
<th>Top Angle Thickness (in.)</th>
<th>K (k.in./rad)</th>
<th>Kp (K.in./rad)</th>
<th>Mo (k.in)</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1/4</td>
<td>18,900</td>
<td>955</td>
<td>54</td>
<td>1.90</td>
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<tr>
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<td>56,245</td>
<td>2,045</td>
<td>157</td>
<td>1.75</td>
</tr>
<tr>
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<td>2.5</td>
<td>1/2</td>
<td>81,600</td>
<td>3,200</td>
<td>222</td>
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<td>407</td>
<td>38</td>
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<tr>
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<td>34,286</td>
<td>1,430</td>
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<td>3,570</td>
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<tr>
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<td>2.5</td>
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<td>1,071</td>
<td>69</td>
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</tr>
<tr>
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<td>1.65</td>
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<tr>
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<td>1.50</td>
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<td>19,600</td>
<td>500</td>
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Table 6—Continued

<table>
<thead>
<tr>
<th>Beam Depth (in.)</th>
<th>Gage Length (in.)</th>
<th>Top Angle Thickness (in.)</th>
<th>K (k.in./rad)</th>
<th>Kp (k.in./rad)</th>
<th>M₀ (k.in)</th>
<th>n</th>
</tr>
</thead>
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<td>3/8</td>
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<td>9,285</td>
<td>350</td>
<td>2.10</td>
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<td>6,665</td>
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<td>610</td>
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<td>254,810</td>
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</tbody>
</table>
top and seat connections are illustrated in Figures 28 through 30. Shown in Figure 28 is the effect of changing the angle thickness on moment rotation curves. For a 12 inch beam depth and a 2-1/2 inch gage, a thicker angle increases the moment. This increase in moment is expected because a thicker angle yields a stiffer connection. This is also true for any beam depth and gage. Curves for other depths and gages are presented in Appendix C. Shown in Figure 29 is the effect of changing the gage length on moment rotation curves. For a 12 inch beam depth and a 1/4 inch angle thickness, a larger gage gives a smaller moment. This is also expected because a larger gage makes the connection less stiff. This is also true for any beam depth and angle thickness. Curves for other depths and angle thicknesses are presented in Appendix C. Shown in Figure 30 is the effect of changing the beam depth on moment rotation curves. For a 2-1/2 inch gage and a 1/4 inch angle thickness, a deeper beam gives a larger moment. A deeper beam means a longer moment arm, and this produces a larger moment. This is also true for any gage and an angle thickness. Curves for other gages and angle thicknesses are presented in Appendix C.
Figure 28. Moment rotation curves (2.5 inch gage, 12 inch depth).
Figure 29. Moment rotation curves (1/4 inch angle, 12 inch depth).
Figure 30. Moment rotation curves (2.5 inch gage, 1/4 inch angle).
CHAPTER 6

EXAMPLES FOR BEAM LINE ANALYSIS

Beam line analyses are performed here using the moment rotation curves of top and seat connections that have been developed. This analysis determines the moment generated by a connection and the degree of restraint for that connection. The use of the beam line analysis was explained in Chapter 2. Four examples are solved to illustrate the use of the beam line analysis. The beam line and moment rotation curve for the connections used in these examples are shown in Figures 31 through 34. From these examples, it is clear that connections are capable of producing significant moment relative to fixed-end beam moments. The magnitude of this moment depends on the connection geometry and the loading. The percentages of connection moments to the fixed end moments range from 11-32%. Therefore, if a connection is capable of producing a significant percentage of the fixed end beam moment, it may be utilized in the design process.

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Example 6.1

Given: W12x53, A36 steel, $S = 70.6 \text{ in}^3$, $I = 425 \text{ in}^4$

Span: 240 in. laterally supported

Loading: Uniform with $W = 2\text{ k/ft.}$

Connection: Top and seat angle with top angle thickness of 1/2 inch and a gage of 2.5 inches

Solution:

1. $\theta_{\text{free end}} = \frac{wL^3}{24EI}$

   
   \[ = \frac{2(k/\text{ft}) \times (240 \text{ in})^3 \times (\text{ft/12 in})}{24 \times (30,000 \text{ kip/in}^2)(425 \text{ in}^4)} = .00753 \text{ rad.} \]

2. $M_{\text{fixed}} = \frac{wL^2}{12}$

   
   \[ = \frac{2(k/\text{ft}) \times (240 \text{ in})^2 \times (\text{ft/12 in})}{12} = 800 \text{ k-in.} \]

3. $M_{\text{conn.}} = 254 \text{ kip.in from Figure 31.}$

4. $\frac{M_{\text{conn.}}}{M_{\text{fixed}}} = \frac{254}{800} = 32\%$
Figure 31. Beamline analysis of W 12 x 53 section loaded to 2 k/ft.
Example 6.2

Given: W16X67, A36 steel, \( S = 117 \text{ in}^3 \), \( I = 954 \text{ in}^4 \)

Span: 320 in. laterally supported

Loading: Uniform with \( W = 2k/\text{ft} \).

Connection: Top and seat angle with top angle
thickness of 1/2 inch and a gage of 2.5 inches

Solution:

1. \( \theta_{\text{free end}} = \frac{wL^3}{24EI} \)
   
   \[ = \frac{2(k/\text{ft}) \times (320 \text{ in})^3 \times (\text{ft/12 in})}{24 \times (30,000 \text{ k/in}^2)(954 \text{ in}^4)} = 0.008 \text{ rad.} \]

2. \( M_{\text{fixed}} = \frac{wL^2}{12} \)
   
   \[ = \frac{2(k/\text{ft}) \times (320 \text{ in})^2 \times (\text{ft/12 in})}{12} = 1422 \text{ k-in.} \]

3. \( M_{\text{conn.}} = 362 \text{ k-in from Figure 32.} \)

4. \( \frac{M_{\text{conn.}}}{M_{\text{fixed}}} = \frac{362}{1422} = 25\% \)
Figure 32. Beamline analysis of W 16 x 67 section loaded to 2 k/ft.
Example 6.3

Given: W24X104, A36 steel, \( S = 258 \text{ in}^3 \), \( I = 3100 \text{ in}^4 \)

Span: 360 in. laterally supported

Loading: Uniform with \( W = 5 \text{k/ft} \).

Connection: Top and seat angle with top angle

thicknes of 3/8 inch and a gage of 2.5 inches

Solution:

1. \( \theta_{\text{free end}} = \frac{wL^3}{24EI} \)

   \[ \frac{5(\text{k/ft}) \times (360 \text{ in})^3 \times (\text{ft/12 in})}{24(30,000 \text{ k/in}^2) \times (3100 \text{ in}^4)} = 0.0087 \text{ rad.} \]

2. \( M_{\text{fixed}} = \frac{wL^2}{12} \)

   \[ \frac{5(\text{k/ft}) \times (360 \text{ in})^2 \times (\text{ft/12 in})}{12} = 4500 \text{ k-in.} \]

3. \( M_{\text{conn.}} = 500 \text{ k/in} \) from Figure 33.

4. \( \frac{M_{\text{conn.}}}{M_{\text{fixed}}} = \frac{500}{4500} = 11\% \)
Figure 33. Beamline analysis of W 24 x 104 section loaded to 5 k/ft.
Example 6.4

Given: W24X84, A36 steel, S = 196 in$^3$, I = 2370 in$^4$

Span: 480 in. laterally supported

Loading: Uniform with $W = 2k/ft$.

Connection: Top and seat angle with top angle thickness of 1/2 inch and a gage of 3.0 inches

Solution:

1. \[ \theta_{\text{free end}} = \frac{WL^3}{24EI} \]
   \[ = \frac{2(k/ft) \times (480 \text{ in})^3 \times (\text{ft/12 in})}{24 \times (30,000 \text{ k/in}^2) \times (2370 \text{ in}^4)} = 0.011 \text{ rad.} \]

2. \[ M_{\text{fixed}} = \frac{wL^2}{12} \]
   \[ = \frac{2(k/ft) \times (480 \text{ in})^2 \times (\text{ft/12 in})}{12} = 3200 \text{ k-in.} \]

3. \[ M_{\text{conn.}} = 550 \text{ k-in from Figure 34.} \]

4. \[ \frac{M_{\text{conn.}}}{M_{\text{fixed}}} = \frac{550}{3200} = 17\% \]
Figure 34. Beamline analysis of W 24 x 84 section loaded to 2 k/ft.
CHAPTER 7

SUMMARY AND CONCLUSIONS

Moment rotation curves for top and seat connections for static loading have been generated and presented using the Richard Equation and plotted using the program "XYPLOT," written by George William.

Comparisons between experimental results, results from formulas by Frye and Morris, and results from an analytical procedure are made. Also, the beam line theory is presented, and its use with the moment rotation curves is also illustrated.

In this study, when the top angle thickness of 5/8 inch was used results did not agree well. This apparently was because extrapolation was needed to find the equation parameters for this angle size. This extrapolation may be the source of error and the reason that inconsistent results were gotten when comparisons with experimental results were made. The analytical procedure predicted the moment rotation response for bolted connections, even though the size of the bolt was not a factor in calculations. Rivets were used in the specimens used in the experimental tests performed by Rathbun and Hechtman and Johnston.
It is apparent that more investigation is needed for the 5/8 inch top angle thickness, so that more dependable values for the Richard Equation parameters may be determined.

Significant moment is generated by top and seat connections. This moment has not been utilized because a dependable moment rotation curve for such connections has not been developed to date.

Since either Type 1 or Type 2 connection only is used in the design, the moment generated by the Type 3 connection is not utilized. The primary objective of this study was to develop an analytical procedure which utilizes the moment produced by Type 3 connections so that a smaller section may be used without affecting the safety of the structure.
APPENDIX A

SUMMARY OF THE RESULTS BY BLEWITT AND RICHARD
SUMMARY OF THE RESULTS BY BLEWITT AND RICHARD

Blewitt and Richard did their tests at the University of Arizona. Nineteen double angle tension tests, two compression tests, and one shear test were performed. The objective of their study was to define the force-deformation characteristics for a wide range of double-angle connection geometries. A summary of the results for the tests in tension is presented.

For a three-inch element of a double-angle framing connection loaded in tension, the elastic stiffness, $K$, and the reference load, $R_0$, are summarized as follows:

$$K_T = 18 \times 10^4 \left(\frac{t_1}{g}\right)^3 \text{ kip/in.}$$

$R_{04} = 16.0 - 4.0 \text{ g}$

$R_{05} = 32.0 - 8.4 \text{ g}$

$R_{06} = 52.0 - 14.2 \text{ g}$

$R_{07} = 63.0 - 16.0 \text{ g}$

$R_{08} = 72.1 - 17.5 \text{ g}$

$R_{09} = 81.4 - 19.2 \text{ g}$

$R_{010} = 90.6 - 20.8 \text{ g}$

$t_1$ = thickness of the angle leg, in.

$g$ = gage length, in.

The subscripts of $R_0$ indicate the number of sixteenths of an inch of angle thickness.
The values of the Richard parameter, \( n \), and the plastic stiffness, \( K_p \), are presented in Table A.1 and Figure A.1, respectively.
Table A.1. Recommended Richard Equation parameters for three inch double angle segments (units in kips and inches).

<table>
<thead>
<tr>
<th>Gage Length</th>
<th>Angle Thickness</th>
<th>K</th>
<th>K_p</th>
<th>R_0</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1/2</td>
<td>850</td>
<td>22</td>
<td>20</td>
<td>1.1</td>
</tr>
<tr>
<td>3</td>
<td>3/8</td>
<td>350</td>
<td>12</td>
<td>10</td>
<td>2.0</td>
</tr>
<tr>
<td>2-1/4</td>
<td>1/2</td>
<td>2000</td>
<td>23</td>
<td>32</td>
<td>0.7</td>
</tr>
<tr>
<td>2-1/4</td>
<td>3/8</td>
<td>850</td>
<td>20</td>
<td>19</td>
<td>1.2</td>
</tr>
<tr>
<td>2-1/4</td>
<td>1/4</td>
<td>250</td>
<td>11</td>
<td>7</td>
<td>2.5</td>
</tr>
<tr>
<td>1-3/4</td>
<td>1/2</td>
<td>4200</td>
<td>31</td>
<td>42</td>
<td>1.0</td>
</tr>
<tr>
<td>1-3/4</td>
<td>3/8</td>
<td>1800</td>
<td>27</td>
<td>28</td>
<td>0.9</td>
</tr>
<tr>
<td>1-3/4</td>
<td>1/4</td>
<td>550</td>
<td>24</td>
<td>9</td>
<td>3.8</td>
</tr>
</tbody>
</table>
Figure A.1. $K_p$ for tension as a function of the gage length for various angle thicknesses.
APPENDIX B

PROCEDURE TO DETERMINE THE FOUR PARAMETERS THAT DESCRIBE A CURVE
PROCEDURE TO DETERMINE THE FOUR PARAMETERS
THAT DESCRIBE A CURVE

The four parameters that describe each curve are:

1. The elastic stiffness, $K$.
2. The plastic stiffness, $K_p$.
3. The reference load, $R_0$.
4. The curve shape parameter, $n$.

For a given moment rotation curve, the procedure for determining the four parameters that uniquely describe each curve is as follows:

1. Find the initial slope of the curve which represents $K$.
2. Find the final slope of the curve which represents $K_p$.
3. Find the intercept of the final slope with the $y$-axis which represents $R_0$.
4. Choose a point $(\theta_a, M_a)$ near the knee of the curve and a corresponding point $(\theta_b, M_b)$ where $\theta_b = 2\theta_a$. See Figure B.1.
5. Calculate $A$ and $B$ from the following equations.

$$A = \frac{(K - K_p)}{M_a} = \frac{\theta_a - K_p}{\theta_a}$$  \hspace{1cm} (1)

$$B = \frac{(K - K_p)}{M_b} = \frac{\theta_b - K_p}{\theta_b}$$  \hspace{1cm} (2)
Figure B.1. Richard curve parameters.
6. Use Figures B.2 and B.3 to obtain the curve shape parameter, n.

The Richard curve fit for the curve in Figure B.1 is shown in Figure C.37, with the four parameters printed on the figure.
Figure B.2. Shape parameter $n$ vs $A$. 
Figure B.3. Shape parameter $n$ vs $A$. 
APPENDIX C

MOMENT ROTATION CURVES FOR TOP AND SEAT CONNECTIONS
MOMENT ROTATION CURVES FOR TOP AND SEAT CONNECTIONS

These curves are generated using the Richard Equation:

\[
R = \frac{(K - K_p)\Delta}{\left(1 + \left|\frac{(K - K_p)\Delta}{R_0}\right|^{1/n}\right)} + K_p\Delta
\]

where

- \(R_0\) = reference load
- \(N\) = curve shape parameter
- \(K\) = elastic stiffness
- \(K_p\) = plastic stiffness
Figure C.1. Moment rotation Richard curve fit (2.5 inch gage, 1/4 inch angle, 10 inch depth).

\[ R_{o} = 54. \]
\[ N = 1.90 \]
\[ K = 18900. \]
\[ K_p = 955. \]
Figure C.2. Moment rotation Richard curve fit (2.5 inch gage, 3/8 inch angle, 10 inch depth).
Figure C.3. Moment rotation Richard curve fit (2.5 inch gage, 1/2 inch angle, 10 inch depth).

\[ R_0 = 222, \]
\[ N = 1.50, \]
\[ K = 81600, \]
\[ K_P = 3200. \]
Figure C.4. Moment rotation Richard curve fit (3.0 inch gage, 1/4 inch angle, 10 inch depth).

\[ R_e = 38. \]
\[ N = 3.80 \]
\[ K = 10700. \]
\[ K_p = 407. \]
Figure C.5. Moment rotation Richard curve fit (3.0 inch gage, 3/8 inch angle, 10 inch depth).

\[ R_n = 91. \]
\[ N = 2.40 \]
\[ K = 34286. \]
\[ k_p = 1430. \]
Figure C.6. Moment rotation Richard curve fit (3.0 inch gage, 1/2 inch angle, 10 inch depth).

$R_0 = 169.$
$N = 1.45$
$K = 69524.$
$K_p = 3570.$
Figure C.7. Moment rotation Richard curve fit (2.5 inch gage, 1/4 inch angle, 12 inch depth).
Figure C.8. Moment rotation Richard curve fit (2.5 inch gage, 3/8 inch angle, 12 inch depth).

\[ R_0 = 194. \]
\[ N = 1.65 \]
\[ K = 82657. \]
\[ K_F = 2976. \]
Figure C.9. Moment rotation Richard curve fit (2.5 inch gage, 1/2 inch angle, 12 inch depth).

\[ R_0 = 245. \]
\[ N = 1.80 \]
\[ K = 113840. \]
\[ K_p = 6730. \]
Figure C.10. Moment rotation Richard curve fit (3.0 inch gage, 1/4 inch angle, 12 inch depth).

\[ R_b = 45. \]
\[ N = 3.00 \]
\[ K = 16222. \]
\[ K_p = 556. \]
Figure C.11. Moment rotation Richard curve fit (3.0 inch gage, 3/8 inch angle, 12 inch depth).
Figure C.12. Moment rotation Richard curve fit (3.0 inch gage, 1/2 inch angle, 12 inch depth).
Figure C.13. Moment rotation Richard curve fit (2.5 inch gage, 1/4 inch angle, 14 inch depth).

\[ R_0 = 78. \]
\[ N = 2.20 \]
\[ K = 33330. \]
\[ K_F = 1530. \]
Figure C.14. Moment rotation Richard curve fit (2.5 inch gage, 3/8 inch angle, 14 inch depth).

\[ R_0 = 230. \]
\[ N = 1.75 \]
\[ K = 102780. \]
\[ K_p = 3470. \]
Figure C.15. Moment rotation Richard curve fit (2.5 inch gage, 1/2 inch angle, 14 inch depth).
Figure C.16. Moment rotation Richard curve fit (3.0 inch gage, 1/4 inch angle, 14 inch depth).

\[ R_a = 55, \quad N = 3.80, \quad K = 19600, \quad K_p = 500. \]
Figure C.17. Moment rotation Richard curve fit (3.0 inch gage, 3/8 inch angle, 14 inch depth).

\[ R_0 = 130. \]
\[ N = 2.10 \]
\[ K = 65524. \]
\[ K_p = 2620. \]
Figure C.18. Moment rotation Richard curve fit (3.0 inch gage, 1/2 inch angle, 14 inch depth).
Figure C.19. Moment rotation Richard curve fit (2.5 inch gage, 1/4 inch angle, 16 inch depth).

- $R_0 = 100$
- $N = 1.80$
- $K = 42670$
- $K_p = 1500$
Figure C.20. Moment rotation Richard curve fit (2.5 inch gage, 3/8 inch angle, 16 inch depth).

\[ R_s = 267. \]
\[ N = 1.80 \]
\[ K = 127330. \]
\[ K_P = 4167. \]
Figure C.21. Moment rotation Richard curve fit (2.5 inch gage, 1/2 inch angle, 16 inch depth).

\[ R_e = 378. \]
\[ N = 1.40 \]
\[ K = 186670. \]
\[ K_p = 7640. \]
Figure C.22. Moment rotation Richard curve fit (3.0 inch gage, 1/4 inch angle, 16 inch depth).

\[ R_0 = 62. \]
\[ N = 3.90 \]
\[ K = 25185. \]
\[ K_p = 648. \]
Figure C.23. Moment rotation Richard curve fit (3.0 inch gage, 3/8 inch angle, 16 inch depth).
Figure C.24. Moment rotation Richard curve fit (3.0 inch gage, 1/2 inch angle, 16 inch depth).
Figure C.25. Moment rotation Richard curve fit (2.5 inch gage, 1/4 inch angle, 18 inch depth).

\[ R_o = 109. \\
N = 2.20 \\
K = 52000. \\
K_p = 2040. \]
Figure C.26. Moment rotation Richard curve fit (2.5 inch gage, 3/8 inch angle, 18 inch depth).

\[ R_e = 295. \]
\[ N = 1.90 \]
\[ K = 161900. \]
\[ K_p = 6550. \]
Figure C.27. Moment rotation Richard curve fit (2.5 inch gage, 1/2 inch angle, 18 inch depth).

\[ R_0 = 400. \]
\[ N = 1.75 \]
\[ K = 237330. \]
\[ K_p = 11670. \]
Figure C.28. Moment rotation Richard curve fit (3.0 inch gage, 1/4 inch angle, 18 inch depth).

\[ R_0 = 71. \]
\[ N = 3.10 \]
\[ K = 34480. \]
\[ K_p = 714. \]
Figure C.29. Moment rotation Richard curve fit (3.0 inch gage, 3/8 inch angle, 18 inch depth).

\[ R_a = 173. \]
\[ N = 2.50 \]
\[ K = 92380. \]
\[ K_F = 4170. \]
Figure C.30. Moment rotation Richard curve fit (3.0 inch gage, 1/2 inch angle, 18 inch depth).

\[ R_0 = 300. \]
\[ N = 1.80 \]
\[ K = 165710. \]
\[ K_F = 8930. \]
Figure C.31. Moment rotation Richard curve fit (2.5 inch gage, 1/4 inch angle, 20 inch depth).

\[ R_g = 122. \]
\[ N = 1.90 \]
\[ K = 65000. \]
\[ K_p = 2290. \]
Figure C.32. Moment rotation Richard curve fit (2.5 inch gage, 3/8 inch angle, 20 inch depth).

\[ R_a = 325. \]
\[ N = 2.00 \]
\[ K = 180000. \]
\[ K_p = 7140. \]
Figure C.33. Moment rotation Richard curve fit (2.5 inch gage, 1/2 inch angle, 20 inch depth).
Figure C.34. Moment rotation Richard curve fit (3.0 inch gage, 1/4 inch angle, 20 inch depth).
Figure C.35. Moment rotation Richard curve fit (3.0 inch gage, 3/8 inch angle, 20 inch depth).

\[ R_0 = 190 \]
\[ N = 2.60 \]
\[ K = 114285 \]
\[ K_P = 5000 \]
Figure C.36. Moment rotation Richard curve fit (3.0 inch gage, 1/2 inch angle, 20 inch depth).

\[ R_0 = 350. \]
\[ N = 2.00 \]
\[ K = 204570. \]
\[ K_P = 11430. \]
Figure C.37. Moment rotation Richard curve fit (2.5 inch gage, 1/4 inch angle, 22 inch depth).

\[ R_0 = 130. \]
\[ N = 2.30 \]
\[ K = 71620. \]
\[ K_p = 3095. \]
**Figure C.38. Moment rotation Richard curve fit (2.5 inch gage, 3/8 inch angle, 22 inch depth).**

Given values:

- \( R_s = 350 \)
- \( N = 2.10 \)
- \( K = 205710 \)
- \( K_p = 9285 \)
Figure C.39. Moment rotation Richard curve fit (2.5 inch gage, 1/2 inch angle, 22 inch depth).

$R_e = 559.$

$N = 1.55$

$K = 285280.$

$K_p = 12260.$
Figure C.40. Moment rotation Richard curve fit (3.0 inch gage, 1/4 inch angle, 22 inch depth).

\[ R_0 = 90. \]
\[ N = 2.80 \]
\[ K = 46860. \]
\[ K_p = 857. \]
Figure C.41. Moment rotation Richard curve fit (3.0 inch gage, 3/8 inch angle, 22 inch depth).

$R_0 = 205.$

$N = 3.50$

$K = 133330.$

$K_p = 6665.$
Figure C.42. Moment rotation Richard curve fit (3.0 inch gage, 1/2 inch angle, 22 inch depth).

\[ R_e = 380. \]
\[ N = 1.90 \]
\[ K = 220000. \]
\[ K_p = 13330. \]
Figure C.43. Moment rotation Richard curve fit (2.5 inch gage, 1/4 inch angle, 24 inch depth).
Figure C.44. Moment rotation Richard curve fit (2.5 inch gage, 3/8 inch angle, 24 inch depth).

\[ R_0 = 380. \]
\[ N = 2.10 \]
\[ K = 237780. \]
\[ K_p = 11110. \]
Figure C.45. Moment rotation Richard curve fit (2.5 inch gage, 1/2 inch angle, 24 inch depth).
Figure C.46. Moment rotation Richard curve fit (3.0 inch gage, 1/4 inch angle, 24 inch depth).

\[ R_e = 95. \]
\[ N = 3.90 \]
\[ K = 56000. \]
\[ K_p = 1266. \]
Figure C.47. Moment rotation Richard curve fit (3.0 inch gage, 3/8 inch angle, 24 inch depth).

\[ R_e = 240, \quad N = 2.95, \quad K = 146670, \quad K_P = 6940. \]
Figure C.48. Moment rotation Richard curve fit (3.0 inch gage, 1/2 inch angle, 24 inch depth).
REFERENCES


